Rules for integrands of the form \((a + bx)^m (c + dx)^n\)

0: \(\int (a + bx)^m (c + dx) \, dx \) when \(ad - bc(m + 2) = 0\)

Derivation: Algebraic expansion

Basis: If \(ad - bc(m + 2) = 0\), then \(c + dx = \frac{d(a + b(m+2)x)}{b(n+2)}\)

Rule 1.1.1.2.0: If \(ad - bc(m + 2) = 0\), then

\[
\int (a + bx)^m (c + dx) \, dx \rightarrow \frac{d}{b(m + 2)} \int (a + bx)^m (a + b(m + 2)x) \, dx \rightarrow \frac{d}{b(m + 2)} (a + bx)^{m+1}
\]

```
Int[(a_+b_+x_)^m_*(c_+d_+x__),x_Symbol] :=
  d*x*(a+b*x)^(m+1)/(b*(m+2));
FreeQ[{a,b,c,d,m,x} && EqQ[a*d-b*c*(m+2),0]]
```
1. \( \int (a + bx)^m (c + dx)^n \, dx \) when \( bc \neq ad \neq 0 \) \land m + n + 2 = 0

1. \( \int \frac{1}{(a + bx) (c + dx)} \, dx \) when \( bc \neq ad \neq 0 \)

1: \( \int \frac{1}{(a + bx) (c + dx)} \, dx \) when \( bc + ad = 0 \)

Derivation: Algebraic simplification

Basis: If \( bc + ad = 0 \), then \( (a + bx) (c + dx) = ac + bd x^2 \)

Rule 1.1.1.2.1.1: If \( bc + ad = 0 \), then

\[
\int \frac{1}{(a + bx) (c + dx)} \, dx \rightarrow \int \frac{1}{ac + bd x^2} \, dx
\]

Program code:

```math
Int[1/((a_+b_.*x_)*(c_+d_.*x_)),x_Symbol] :=
  Int[1/(a*c+b*d+x^2),x] /;
FreeQ[{a,b,c,d},x] && EqQ[b*c+a*d,0]
```
2: \[ \int \frac{1}{(a + bx)(c + dx)} \, dx \text{ when } bc - ad \neq 0 \]

**Derivation: Algebraic expansion**

**Basis:**

\[ \frac{1}{(a + bx)(c + dx)} = \frac{b}{(b - a)(a + bx)} - \frac{d}{(b - a)(c + dx)} \]

**Rule 1.1.1.2.1.1.2:** If \( bc - ad \neq 0 \), then

\[
\int \frac{1}{(a + bx)(c + dx)} \, dx \rightarrow \frac{b}{b - a} \int \frac{1}{a + bx} \, dx - \frac{d}{b - a} \int \frac{1}{c + dx} \, dx
\]

**Program code:**

```plaintext
Int[1/((a_.+b_.*x_)*(c_.+d_.*x_)),x_Symbol] :=
  b/(b*c-a*d)*Int[1/(a+b*x),x] - d/(b*c-a*d)*Int[1/(c+d*x),x] /;
FreeQ[{a,b,c,d},x] && NeQ[b*c-a*d,0]
```
2: \[ \int (a + bx)^m (c + dx)^n \, dx \] when \( bc - ad \neq 0 \land m + n + 2 = 0 \land m \neq -1 \)

Reference: G&R 2.155, CRC 59a with \( m + n + 2 = 0 \)

Reference: G&R 2.110.2 or 2.110.6 with \( k = 1 \) and \( m + n + 2 = 0 \)

Derivation: Linear recurrence 3 with \( m + n + 2 = 0 \)

- Rule 1.1.2.1.2: If \( bc - ad \neq 0 \land m + n + 2 = 0 \land m \neq -1 \), then

\[
\int (a + bx)^m (c + dx)^n \, dx \rightarrow \frac{(a + bx)^{m+1} (c + dx)^{n+1}}{(bc - ad) (m + 1)}
\]

- Program code:

```plaintext
Int[(a_.+b_.*x_.)^m_.*(c_.+d_.*x_.)^n_.,x_Symbol] :=
(a+b*x)^(m+1) *(c+d*x)^(n+1)/( (b*c-a*d)* (m+1)) /;
FreeQ[{a,b,c,d,m,n},x] && NeQ[b*c-a*d,0] && EqQ[m+n+2,0] && NeQ[m,-1]
```
2. \( \int (a + bx)^m (c + dx)^n \, dx \) when \( bc + ad = 0 \) and \( m = n \)

1: \( \int (a + bx)^n (c + dx)^n \, dx \) when \( bc + ad = 0 \) and \( m + \frac{1}{2} \in \mathbb{Z}^+ \)

**Derivation: Inverted integration by parts**

**Rule 1.1.1.2.2.1: If** \( bc + ad = 0 \) and \( m + \frac{1}{2} \in \mathbb{Z}^+ \), **then**

\[
\int (a + bx)^n (c + dx)^n \, dx \rightarrow \frac{x (a + bx)^n (c + dx)^n}{2m+1} + \frac{2acm}{2m+1} \int (a + bx)^{n-1} (c + dx)^{m-1} \, dx
\]

**Program code:**

```mathematica
Int[(a + b x)^m (c + d x)^m, x_Symbol] := x*(a+b x)^m*(c+d x)^m/(2*m+1) + 2*a*c*m/(2*m+1)*Int[(a+b x)^(m-1)*(c+d x)^(m-1), x] //;
Int[1/((a+b x)^(3/2)*(c+d x)^(3/2)), x_Symbol] := x/(a+c*Sqrt[a+b x]*Sqrt[c+d x]) //;
FreeQ[{a,b,c,d,x} & EqQ[b*c+a*d,0] & IGtQ[m+1/2,0]
```

2. \( \int (a + bx)^m (c + dx)^n \, dx \) when \( bc + ad = 0 \) and \( m + \frac{1}{2} \in \mathbb{Z}^- \)

1: \( \int \frac{1}{(a + bx)^{3/2} (c + dx)^{3/2}} \, dx \) when \( bc + ad = 0 \)

**Rule 1.1.1.2.2.2.1: If** \( bc + ad = 0 \), **then**

\[
\int \frac{1}{(a + bx)^{3/2} (c + dx)^{3/2}} \, dx \rightarrow \frac{x}{a \, c \sqrt[3]{a + bx} \, \sqrt{c + dx}}
\]

**Program code:**

```mathematica
Int[1/((a+b x)^(3/2)*(c+d x)^(3/2)), x_Symbol] := x/(a+c*Sqrt[a+b x]*Sqrt[c+d x]) //;
FreeQ[{a,b,c,d,x} & EqQ[b*c+a*d,0]
```
2: \[ \int (a + b x)^m (c + d x)^n \, dx \] when \( b c + a d = 0 \) and \( m + \frac{3}{2} \in \mathbb{Z}^+ \\

Derivation: Integration by parts

Basis: \( (a + b x)^m (c + d x)^m = x^{2(m+1)} \frac{(a + b x)^m (c + d x)^n}{x^{2(m+1)}} + 1 \)

Basis: If \( b c + a d = 0 \), then \( \int \frac{(a + b x)^m (c + d x)^n}{x^{2(m+1)}} \, dx = - \frac{(a + b x)^{m+1} (c + d x)^{n+1}}{2 a c (m+1)} \)

- Rule 1.1.2.2.2.2: If \( b c + a d = 0 \) and \( m + \frac{3}{2} \in \mathbb{Z}^- \), then

\[
\int (a + b x)^m (c + d x)^n \, dx \longrightarrow - \frac{x (a + b x)^{m+1} (c + d x)^{n+1}}{2 a c (m+1)} + \frac{2 m + 3}{2 a c (m+1)} \int (a + b x)^{m+1} (c + d x)^{n+1} \, dx
\]

Program code:

```
Int[(a_.+b_.)*x_]*((c_.+d_.)*x_)^m_,x_Symbol] :=
- x*(a+b*x)^(m+1)*(c+d*x)^(m+1)/(2*a*c*(m+1)) +
(2*m+3)/(2*a*c*(m+1))*(Int[(a+b*x)^(m+1)*(c+d*x)^(m+1),x] /;
FreeQ[{a,b,c,d},x] && EqQ[b*c+a*d,0] && ILtQ[m+3/2,0]
```
3: \[ \int (a + b x)^m (c + d x)^n \, dx \text{ when } b c + a d = 0 \land (m \in \mathbb{Z} \lor a > 0 \land c > 0) \]

Derivation: Algebraic simplification

Basis: If \( b c + a d = 0 \land (m \in \mathbb{Z} \lor a > 0 \land c > 0) \), then \( (a + b x)^m (c + d x)^n = (a c + b d x^2)^n \)

Rule 1.1.2.2.3: If \( b c + a d = 0 \land (m \in \mathbb{Z} \lor a > 0 \land c > 0) \), then

\[ \int (a + b x)^m (c + d x)^n \, dx \rightarrow \int (a c + b d x^2)^n \, dx \]

Program code:

```
Int[(a_+b_+x_)^m_.*(c_+d_+x_)^n_.,x_Symbol] :=
   Int[(a*c+b*d+x^2)^m_.,x] /;
FreeQ[{a,b,c,d,m},x] && EqQ[b*c+a*d,0] && (IntegerQ[m] || GtQ[a,0] && GtQ[c,0])
```
4: \[ \int (a + b x)^m (c + d x)^n \, dx \text{ when } b c + a d = 0 \land 2 m \notin \mathbb{Z} \]

Derivation: Piecewise constant extraction

Basis: If \( b c + a d = 0 \), then \( \partial_x \left( \frac{(a + b x)^m (c + d x)^n}{(a + b d x^2)^m} \right) = 0 \)

Basis: If \( b c + a d = 0 \), then \( \frac{(a + b x)^m (c + d x)^n}{(a + b d x^2)^m} = \frac{(a + b x)^\text{FracPart}[m] (c + d x)^\text{FracPart}[m]}{(a + b d x^2)^\text{FracPart}[m]} \)

Rule 1.1.2.2.4: If \( b c + a d = 0 \land 2 m \notin \mathbb{Z} \), then

\[
\int (a + b x)^m (c + d x)^n \, dx \rightarrow \frac{(a + b x)^\text{FracPart}[m] (c + d x)^\text{FracPart}[m]}{(a + b d x^2)^\text{FracPart}[m]} \int (a + b d x^2)^n \, dx
\]

Program code:

```mathematica
Int[(a_+b_+c_+d_+x_)^m_*(c_+d_+x_)^n_,x_Symbol] :=
  (a+b x)^\text{FracPart}[m] *(c+d x)^\text{FracPart}[m] / (a+c+b d x^2)^\text{FracPart}[m] *Int[(a+c b d x^2)^m_, x] /;
FreeQ[{a,b,c,d,m},x] && EqQ[b*c+a*d,0] && Not[IntegerQ[2*m]]
```

Reference: G&R 2.110.3 or 2.110.4 with \( k = 1 \)

Reference: G&R 2.110.3 or 2.110.4 with \( k = 1 \)

Derivation: Integration by parts

Basis: \( (a + b x)^m = \partial_x \frac{(a + b x)^{m-1}}{b (m-1)} \)

Rule 1.1.2.5.1: If \( b c - a d \neq 0 \land m + 1 \in \mathbb{Z}^\land n \notin \mathbb{Z}^\land n > 0 \).
\[ \int (a + bx)^m (c + dx)^n \, dx \rightarrow \frac{(a + bx)^{m+1} (c + dx)^n}{b (m + 1)} - \frac{d}{b (m + 1)} \int (a + bx)^{m+1} (c + dx)^{n-1} \, dx \]

**Program code:**

```mathematica
Int[(a_ + b_.*x_)^m_.* (c_ + d_.*x_)^n_, x_Symbol] :=
(a + b x)^m (c + d x)^n / (b (m + 1)) -
d^m / (b (m + 1)) * Int[(a + b x)^m (c + d x)^n, x] /;
FreeQ[{a, b, c, d, n}, x] && NeQ[b + c - a d, 0] && IntegerQ[n, 0] && GtQ[n, 0]
```

2: \[ \int (a + bx)^m (c + dx)^n \, dx \text{ when } b c - a d \neq 0 \land m + 1 \in \mathbb{Z}^- \land n \notin \mathbb{Z} \land n < 0 \]

Reference: G&R 2.155, CRC 59a

Reference: G&R 2.110.2 or 2.110.6 with \( k = 1 \)

Derivation: Integration by parts

**Basis:** \( (a + bx)^m (c + dx)^n = (c + dx)^{m+2} \frac{(a + bx)^n}{(c + dx)^{n+2}} \)

**Basis:** \( \frac{(a + bx)^n}{(c + dx)^{n+2}} = \partial_x \frac{(a + bx)^{n+1}}{(b + c a d) (m + 1) (c + dx)^{n+1}} \)

Rule 1.1.1.2.4: If \( b c - a d \neq 0 \land m + 1 \in \mathbb{Z}^- \land n \notin \mathbb{Z} \land n < 0 \), then

\[ \int (a + bx)^m (c + dx)^n \, dx \rightarrow \frac{(a + bx)^{m+1} (c + dx)^{n+1}}{(b + c a d) (m + 1)} - \frac{d\ (m + n + 2)}{(b + c a d) (m + 1)} \int (a + bx)^{m+1} (c + dx)^{n-1} \, dx \]

**Program code:**

```mathematica
Int[(a_ + b_.*x_)^m_.* (c_ + d_.*x_)^n_, x_Symbol] :=
(a + b x)^m (c + d x)^n / (b (m + 1)) -
d^m / (b (m + 1)) * Int[(a + b x)^m (c + d x)^n, x] /;
FreeQ[{a, b, c, d, n}, x] && NeQ[b + c - a d, 0] && IntegerQ[n, 0] && LtQ[n, 0]
```
3. \( \int (a + bx)^n (c + dx)^n \, dx \) when \( b \, c - a \, d \neq 0 \wedge m \in \mathbb{Z} \)

1: \( \int (a + bx)^n (c + dx)^n \, dx \) when \( b \, c - a \, d \neq 0 \wedge m \in \mathbb{Z}^+ \)

**Derivation: Algebraic expansion**

**Rule 1.1.1.2.3.1:** If \( b \, c - a \, d \neq 0 \wedge m \in \mathbb{Z}^+ \), then

\[
\int (a + bx)^n (c + dx)^n \, dx \quad \rightarrow \quad \int \text{ExpandIntegrand} [(a + bx)^n (c + dx)^n, x] \, dx
\]

**Program code:**

```plaintext
Int[(a_ + b_. + x_.)^m_. + (c_ + d_. + x_.)^n_. , x_Symbol] :=
Int[ExpandIntegrand[(a + b x)^m (c + d x)^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && NeQ[b + c - a + d, 0] && IGtQ[m, 0] &&
(Not[IntegerQ[n]] || EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0] || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

2: \( \int (a + bx)^n (c + dx)^n \, dx \) when \( b \, c - a \, d \neq 0 \wedge m \in \mathbb{Z}^- \wedge n \in \mathbb{Z} \)

**Derivation: Algebraic expansion**

**Rule 1.1.1.2.3.2:** If \( b \, c - a \, d \neq 0 \wedge m \in \mathbb{Z}^- \wedge n \in \mathbb{Z} \), then

\[
\int (a + bx)^n (c + dx)^n \, dx \quad \rightarrow \quad \int \text{ExpandIntegrand} [(a + bx)^n (c + dx)^n, x] \, dx
\]

**Program code:**

```plaintext
Int[(a_ + b_. + x_.)^m_. + (c_ + d_. + x_.)^n_. , x_Symbol] :=
Int[ExpandIntegrand[(a + b x)^m (c + d x)^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && NeQ[b + c - a + d, 0] && ILtQ[m, 0] && IGtQ[n, 0] &&
IntegerQ[n] && Not[IGtQ[n, 0] && LtQ[m + n + 2, 0]]
```
4: \[ \int (a + b x)^m (c + d x)^n \, dx \text{ when } b c - a d \neq 0 \land m + n + 2 \in \mathbb{Z}^- \land m \neq -1 \]

Reference: G&R 2.155, CRC 59a

Reference: G&R 2.110.2 or 2.110.6 with \( k = 1 \)

Derivation: Linear recurrence 3

Derivation: Integration by parts

Basis: \( (a + b x)^m (c + d x)^n = (c + d x)^{m+n+2} \frac{(a+b x)^n}{(c+d x)^{m+2}} \)

Rule 1.1.1.2.4: If \( b c - a d \neq 0 \land m + n + 2 \in \mathbb{Z}^- \land m \neq -1 \), then

\[ \int (a + b x)^m (c + d x)^n \, dx \rightarrow \frac{(a + b x)^{m+1} (c + d x)^{n+1}}{(b c - a d) (m+1)} - \frac{d (m + n + 2)}{(b c - a d) (m+1)} \int (a + b x)^{m+1} (c + d x)^n \, dx \]

Program code:

```plaintext
Int[(-b_)*x_]*((a_)*(-d_)*x_)^n_*x_Symbol] :=
(a+b*x)^(m+1)*((c+d*x)^(n+1)) / ((b*c-a*d)*(m+1)) -
d*Simplify[m+n+2]/((b*c-a*d)*(m+1)) * Int[(a+b*x)^(Simplify[m+1]*(c+d*x)^n,x) ];
FreeQ[{a,b,c,d,m,n},x] && NeQ[b*c-a*d,0] && LtQ[Simplify[m+n+2],0] && NeQ[m,-1] &&
Not[LtQ[m,-1] && LtQ[n,-1] && (EqQ[a,0] || NeQ[c,0] && LtQ[m-n,0] && IntegerQ[n])) &&
(SumSimplerQ[m,1] || Not[SumSimplerQ[n,1]])
```
5. \( \int (a + bx)^m (c + dx)^n \, dx \) when \( bc - ad \neq 0 \land n > 0 \)

1: \( \int (a + bx)^m (c + dx)^n \, dx \) when \( bc - ad \neq 0 \land n > 0 \land m < -1 \)

Reference: G&R 2.110.3 or 2.110.4 with \( k = 1 \)

Derivation: Integration by parts

Basis: \( (a + bx)^m = \partial_x \frac{(a+bx)^{m-1}}{b(m+1)} \)

Note: If \( n \in \mathbb{Z} \) and \( m \not\in \mathbb{Z} \), there is no need to drive \( m \) toward \( 0 \) along with \( n \).

Rule 1.1.1.2.5.1: If \( bc - ad \neq 0 \land n > 0 \land m < -1 \), then

\[
\int (a + bx)^m (c + dx)^n \, dx \rightarrow \frac{(a+bx)^{n-1}(c+dx)^n}{b(m+1)} - \frac{d}{b(m+1)} \int (a + bx)^m (c + dx)^{n-1} \, dx
\]

Program code:
2: \[\int (a + b x)^n (c + d x)^n \, dx \quad \text{when} b c - a d \neq 0 \land n > 0 \land m + n + 1 \neq 0\]

Reference: G&R 2.151, CRC 59b

Reference: G&R 2.110.1 or 2.110.5 with \(k = 1\)

Derivation: Linear recurrence 2

Derivation: Inverted integration by parts

Rule 1.1.1.2.5.2: If \(b c - a d \neq 0 \land n > 0 \land m + n + 1 \neq 0\), then

\[
\int (a + b x)^n (c + d x)^n \, dx \rightarrow \frac{(a + b x)^{n+1} (c + d x)^n}{b (m + n + 1)} + \frac{n (b c - a d)}{b (m + n + 1)} \int (a + b x)^n (c + d x)^{n-1} \, dx
\]

Program code:

```
Int[1/((a + b* x_^)^m*(c + d* x_^)^n), x_Symbol] :=
-2/(b*(a+b*x)^m*(c+d*x)^n) + c*Int[1/((a+b*x)^m*(c+d*x)^n), x] /;
FreeQ[{a,b,c,d},x] && EqQ[b*c+a*d,0] && NegQ[a^2*b^2]

Int[(a + b * x_)*^m*(c + d * x_)*^n, x_Symbol] :=
(a+b*x)^m*(c+d*x)^n/(b*(m+n+1)) +
2*c*n/(m+n+1)*Int[(a+b*x)^m*(c+d*x)^n, x] /;
FreeQ[{a,b,c,d},x] && EqQ[b*c+a*d,0] && IgQ[m+n+1/2,0] && IgQ[n+1/2,0] && LtQ[m,n]

Int[(a + b * x_)*^m*(c + d * x_)*^n, x_Symbol] :=
(a+b*x)^m*(c+d*x)^n/(b*(m+n+1)) +
n*(b+c-a*d)/(b*(m+n+1))*Int[(a+b*x)^m*(c+d*x)^n, x] /;
FreeQ[{a,b,c,d},x] && NeQ[b*c+a*d,0] && GtQ[n,0] && NeQ[m+n+1,0] &&
Not[IgQ[m,0]] && (Not[IntegerQ[n]] || GtQ[m,0] && LtQ[m-n,0]) && Not[IltQ[m+n+2,0]] && IntLinearQ[a,b,c,d,m,n,x]
```
6: \[ \int (a + b x)^n (c + d x)^n \, dx \quad \text{when} \quad b c - a d \neq 0 \land m < -1 \]

Reference: G&R 2.155, CRC 59a

Reference: G&R 2.110.2 or 2.110.6 with \( k = 1 \)

Derivation: Linear recurrence 3

Derivation: Integration by parts

Basis: \((a + b x)^m (c + d x)^n = (c + d x)^{m+n+2} \frac{(a + b x)^n}{(c + d x)^{m+2}}\)

Rule 1.1.1.2.6: If \( b c - a d \neq 0 \land m < -1 \), then

\[ \int (a + b x)^n (c + d x)^n \, dx \rightarrow \frac{(a + b x)^{n+1}}{(b c - a d) (m+1)} (c + d x)^{n+1} - \frac{d (m+n+2)}{(b c - a d) (m+1)} \int (a + b x)^{m+1} (c + d x)^n \, dx \]

Program code:

```mathematica
Int[(a_+b_+c_+d_+x_)^m_*(c_+d_+x_)^n_,x_Symbol] :=
    (a+b*x)^(m+1)*(c+d*x)^(n+1)/((b*c-a*d)*(m+1)) -
    d*(m+n+2)/((b*c-a*d)*(m+1))*Int[(a+b*x)^(m+1)*(c+d*x)^n,x] /;
FreeQ[{a,b,c,d,n},x] && NeQ[b*c-a*d,0] && LtQ[m,-1] &&
Not[LtQ[n,-1] && (EqQ[a,0] || NeQ[c,0] && LtQ[m-n,0] && IntegerQ[n])] && IntLinearQ[a,b,c,d,m,n,x]
```

7. \[ \int (a + b x)^n (c + d x)^n \, dx \quad \text{when} \quad b c - a d \neq 0 \land -1 \leq m < 0 \land -1 < n < 0 \]

1. \[ \int \frac{1}{\sqrt{a + b x} \sqrt{c + d x}} \, dx \quad \text{when} \quad b c - a d \neq 0 \]

1: \[ \int \frac{1}{\sqrt{a + b x} \sqrt{c + d x}} \, dx \quad \text{when} \quad a + c = 0 \land b - d = 0 \land a > 0 \]

Rule 1.1.1.2.7.1.1: If \( a + c = 0 \land b - d = 0 \land a > 0 \), then
Rules for integrands of the form \((a + bx)^m (c + dx)^n\)

\[
\int \frac{1}{\sqrt{a + bx} \sqrt{c + dx}} \, dx \rightarrow \frac{1}{b} \text{ArcCosh} \left[ \frac{bx}{a} \right]
\]

Program code:

```
Int[1/(Sqrt[a + b.*x_] + Sqrt[c + d.*x_]), x_Symbol] :=
   ArcCosh[b*x/a]/b /;
FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b - d, 0] && GtQ[a, 0]
```

2: \[
\int \frac{1}{\sqrt{a + bx} \sqrt{c + dx}} \, dx \text{ when } b + d = 0 \land a + c > 0
\]

Derivation: Algebraic simplification

- Basis: If \(a + c > 0\), then \((a + bx)^n (c - bx)^n = ((a + bx) (c - bx))^n = (a c - b (a - c) x - b^2 x^2)^n\)

- Rule 1.1.2.7.1.2: If \(b + d = 0 \land a + c > 0\), then

\[
\int \frac{1}{\sqrt{a + bx} \sqrt{c + dx}} \, dx \rightarrow \int \frac{1}{\sqrt{a c - b (a - c) x - b^2 x^2}} \, dx
\]

Program code:

```
Int[1/(Sqrt[a + b.*x_] + Sqrt[c + d.*x_]), x_Symbol] :=
   Int[1/Sqrt[a*c - b*(a-c)*x-b^2*x^2], x] /;
FreeQ[{a, b, c, d}, x] && EqQ[b + d, 0] && GtQ[a + c, 0]
```
Derivation: Integration by substitution

Basis: If \( b > 0 \), then
\[
\int \frac{1}{\sqrt{a + bx} \cdot \sqrt{c + dx}} \, dx = \frac{2}{\sqrt{b}} \text{Subst}\left[ \int \frac{1}{\sqrt{b c - a d + d x^2}} \, dx, x, \sqrt{a + bx} \right] \partial x \sqrt{a + bx}
\]

Rule 1.1.1.2.7.1.3: If \( b c - a d > 0 \) \( \land \) \( b > 0 \), then
\[
\int \frac{1}{\sqrt{a + bx} \cdot \sqrt{c + dx}} \, dx \rightarrow \frac{2}{\sqrt{b}} \text{Subst}\left[ \int \frac{1}{\sqrt{b c - a d + d x^2}} \, dx, x, \sqrt{a + bx} \right]
\]

Program code:

```plaintext
Int[1/ (Sqrt[a_. + b_. * x_] * Sqrt[c_. + d_. * x_]), x, Symbol] :=
2/Sqrt[b] * Subst[Int[1/ Sqrt[b * c - a * d + d * x^2]], x, Sqrt[a + b * x]] ;
FreeQ[{a, b, c, d}, x] \&\& GtQ[b * c - a * d, 0] \&\& GtQ[b, 0]
```
2. \[
\int \frac{1}{(a + bx)(c + dx)^{1/3}} \, dx \quad \text{when } b - a d \neq 0
\]

1. \[
\int \frac{1}{(a + bx)(c + dx)^{1/3}} \, dx \quad \text{when } \frac{b - a d}{b} > 0
\]

Derivation: Integration by substitution

Basis: Let \( q = \left( \frac{b - a d}{b} \right)^{1/3} \), then

\[\frac{1}{(a + bx)(c + dx)^{1/3}} \, dx \rightarrow -\frac{\log[a + bx]}{2 b q} - \frac{3}{2 b q} \text{Subst} \left[ \int \frac{1}{q - x} \, dx, x, (c + dx)^{1/3} \right] + \frac{3}{2 b} \text{Subst} \left[ \int \frac{1}{q^2 + q x + x^2} \, dx, x, (c + dx)^{1/3} \right]\]

Program code:

```mathematica
Int[1/((a_+b_*x_)*(c_+d_*x_))^(1/3)),x_Symbol] :=
    With[{q=Rt[(b*c-a*d)/b,3]},
        -Log[RemoveContent[a+b*x,x]]/(2*b*q) -
        3/(2*b*q)*Subst[Int[1/(q-x),x,(c+d*x)^13],q-x] +
        3/(2*b)*Subst[Int[1/(q^2+q*x+x^2),x,x^3],x,(c+d*x)^13]] /
    FreeQ[{a,b,c,d},x] && PosQ[(b*c-a*d)/b]
```

2. \[
\int \frac{1}{(a + bx)(c + dx)^{1/3}} \, dx \quad \text{when } \frac{b - a d}{b} \neq 0
\]

Derivation: Integration by substitution

Basis: Let \( q = \left( -\frac{b - a d}{b} \right)^{1/3} \), then

\[\frac{1}{(a + bx)(c + dx)^{1/3}} \, dx \rightarrow -\frac{\log[a + bx]}{2 b q} - \frac{3}{2 b q} \text{Subst} \left[ \int \frac{1}{q + x} \, dx, x, (c + dx)^{1/3} \right] + \frac{3}{2 b} \text{Subst} \left[ \int \frac{1}{q^2 - q x + x^2} \, dx, x, (c + dx)^{1/3} \right]\]

Rule 1.1.1.2.7.2.2: If \( \frac{b - a d}{b} \neq 0 \), let \( q = \left( -\frac{b - a d}{b} \right)^{1/3} \), then
Rules for integrands of the form \((a+b\,x)^m\,(c+d\,x)^n\)

\[
\int \frac{1}{(a + b\,x)\,(c + d\,x)^{1/3}} \, dx \to
\]

\[
\frac{\log [a + b\,x]}{2\,b\,q} - \frac{3}{2\,b\,q} \text{Subst} \left[ \int \frac{1}{q + x} \, dx, x, (c + d\,x)^{1/3} \right] + \frac{3}{2\,b} \text{Subst} \left[ \int \frac{1}{q^2 - q\,x + x^2} \, dx, x, (c + d\,x)^{1/3} \right]
\]

Program code:

```plaintext
Int[1/((a_+.b_.*x_)*(c_+.d_.*x_)^(1/3)),x_Symbol] :=
  With[{q=Rt[-(b*c-a*d)/b,3]},
    Log[RemoveContent[a+b*x,x]]/(2*b*q) -
    3/(2*b*q)*Subst[Int[1/(q+x),x,(c+d*x)^(1/3)] +
    3/(2*b)*Subst[Int[1/(q^2-q*x+x^2),x,(c+d*x)^(1/3)]]/;
    FreeQ[{a,b,c,d},x] && NegQ[(b*c-a*d)/b]]
```
3. \( \int \frac{1}{(a + bx) \ (c + dx)^{2/3}} \, dx \) when \( b \ c - a \ d \neq 0 \)

1: \( \int \frac{1}{(a + bx) \ (c + dx)^{2/3}} \, dx \) when \( \frac{b \ c - a \ d}{b} > 0 \)

Derivation: Integration by substitution

Basis: Let \( q = \left( \frac{b \ c - a \ d}{b} \right)^{1/3} \), then

\[ \int \frac{1}{(a + bx) \ (c + dx)^{2/3}} \, dx \rightarrow \]

\[ -\frac{\log[a + bx]}{2 \ b q^2} - \frac{3}{2 \ b q} \ \text{Subst} \left[ \int \frac{1}{q - x} \, dx, x, (c + dx)^{1/3} \right] - \frac{3}{2 \ b q} \ \text{Subst} \left[ \int \frac{1}{q^2 + q x + x^2} \, dx, x, (c + dx)^{1/3} \right] \]

Program code:

```mathematica
Int[1/((a_+b_)*x_)*((c_+d_)*x_)^(2/3)),x_Symbol] :=
  With[{q=Rt[(b*c-a*d)/b,3]},
    -Log[RemoveContent[a+b*x,x]]/(2*b*q^2) -
    3/(2+b*q^2)*Subst[Int[1/(q-x),x,(c+d*x)^(1/3)]] -
    3/(2+b*q)*Subst[Int[1/(q^2+q*x+x^2),x,(c+d*x)^(1/3)]]];
FreeQ[{a,b,c,d},x] && PosQ[(b*c-a*d)/b]
```

2: \( \int \frac{1}{(a + bx) \ (c + dx)^{2/3}} \, dx \) when \( \frac{b \ c - a \ d}{b} \neq 0 \)

Derivation: Integration by substitution

Basis: Let \( q = \left( -\frac{b \ c - a \ d}{b} \right)^{1/3} \), then

\[ \int \frac{1}{(a + bx) \ (c + dx)^{2/3}} \, dx \rightarrow \]

\[ -\frac{\log[a + bx]}{2 \ b q^2} + \frac{3}{2 \ b q} \ \text{Subst} \left[ \int \frac{1}{q - x} \, dx, x, (c + dx)^{1/3} \right] + \frac{3}{2 \ b q} \ \text{Subst} \left[ \int \frac{1}{q^2 + q x + x^2} \, dx, x, (c + dx)^{1/3} \right] \]

Rule 1.1.2.7.3.2: If \( \frac{b \ c - a \ d}{b} \neq 0 \), let \( q = \left( -\frac{b \ c - a \ d}{b} \right)^{1/3} \), then
4. \[ \int \frac{1}{(a + b x)^{1/3} (c + d x)^{2/3}} \, dx \] when \( b c - a d \neq 0 \)

1: \[ \int \frac{1}{(a + b x)^{1/3} (c + d x)^{2/3}} \, dx \] when \( b c - a d \neq 0 \) \( \land \frac{c}{b} > 0 \)

**Rule 1.1.1.2.7.4.1:** If \( b c - a d \neq 0 \) \( \land \frac{c}{b} > 0 \), let \( q = \left( \frac{d}{b} \right)^{1/3} \), then

\[ \int \frac{1}{(a + b x)^{1/3} (c + d x)^{2/3}} \, dx \rightarrow -\frac{\sqrt{3}}{d} \text{ArcTan} \left[ \frac{2q (a + b x)^{1/3} + \sqrt{3}}{(c + d x)^{1/3}} \right] - \frac{q}{2d} \log [c + d x] - \frac{3q}{2d} \log \left[ \frac{q (a + b x)^{1/3}}{(c + d x)^{1/3}} - 1 \right] \]

**Program code:**

```mathematica
Int[1/((a_.+b_.)*x_)*(c_.+d_.)*x_]^(2/3), x_Symbol] :=
With[{q=Root[-(b*c-a*d)/b, 3]},

-Log[RemoveContent[a*b*x,x]]/(2*b*q^2) +
3/(2*b*q^2)*Subst[Int[1/(q+x), x, (c+d*x)^(1/3)] +
3/(2*b*q)*Subst[Int[1/(q^2-q*x+x^2), x, (c+d*x)^(1/3)]]] /;
FreeQ[{a,b,c,d},x] \& NegQ[(b*c-a*d)/b]
```

Int[1/((a_.+b_.)*x_)*(c_.+d_.)*x_]^(1/3), x_Symbol] :=
With[{q=Root[d/b, 3]},

-Sqrt[3]*q/d*ArcTan[2*q*(a+b*x)^(1/3)/(Sqrt[3]*(c+d*x)^(1/3)) + 1/Sqrt[3]] -
q/(2*d)*Log[c+d*x] -
3*q/(2*d)*Log[q*(a+b*x)^(1/3)/(c+d*x)^(1/3)-1] /;
FreeQ[{a,b,c,d},x] \& NeQ[b*c-a*d, 0] \& PosQ[d/b]
```
Rule 1.1.2.7.4.2: If \( b \frac{c - a}{d} \neq 0 \) \( \wedge \frac{d}{b} \neq 0 \), let \( q = \left( -\frac{d}{b} \right)^{1/3} \), then

\[
\int \frac{1}{(a + bx)^{1/3} (c + dx)^{2/3}} \, dx \rightarrow \frac{\sqrt{3} q}{d} \text{ArcTan} \left[ \frac{1}{\sqrt{3}} - \frac{2 q (a + bx)^{1/3}}{\sqrt{3} (c + dx)^{1/3}} \right] + \frac{q}{2 d} \log[c + d x] + \frac{3 q}{2 d} \log[\frac{q (a + bx)^{1/3}}{(c + dx)^{1/3}} + 1] + \frac{1}{3 q (2 d)} \log[\frac{q (a + bx)^{1/3} (1/3)}{(c + dx)^{1/3} + 1}]
\]

Program code:

```mathematica
Int[1/((a_ + b_ x_)^(1/3) * (c_ + d_ x_)^(2/3)), x_Symbol] :=
With[{q = Sqrt[-d/b]},
   + q/(2 d) * Log[c + d x]
   + 3 q/(2 d) * Log[q * (a + b x)^(1/3) / (c + d x)^(1/3) + 1] /;
FreeQ[{a,b,c,d},x] && NeQ[b c - a d,0] && NegQ[d/b]
```
5: \int (a + b x)^m (c + d x)^n \, dx \text{ when } b c - a d \neq 0 \land -1 < m < 0 \land n = m \land 3 \leq \text{Denominator}[m] \leq 4

Derivation: Piecewise constant extraction

Basis: \frac{(a+b x)^m (c+d x)^n}{(a+b x) (c+d x)^m} = 0

Rule 1.1.2.7.5: If \( b c - a d \neq 0 \land -1 < m < 0 \land \frac{3}{m} \leq \text{Denominator}[m] \leq 4 \), then

\int (a + b x)^m (c + d x)^n \, dx \rightarrow \frac{(a + b x)^m (c + d x)^n}{(a c + (b c + a d) x + b d x^2)^m} \int (a c + (b c + a d) x + b d x^2)^n \, dx

\int (a + b x)^m (c + d x)^n \, dx \rightarrow \frac{(a + b x)^m (c + d x)^n}{(a + b x) (c + d x)^m} \int \frac{(a c + (b c + a d) x + b d x^2)^n}{(a + b x) (c + d x)^m} \, dx

Program code:

```mathematica
Int[(a_ + b_ x)^m_*(c_ + d_ x)^n_]*x_Symbol] := 
(a+b x)^m*(c+d x)^n/(a+c+(b c+a d) x+b d x^2)^m*Int[(a+c+(b c+a d) x+b d x^2)^n,m,x] /;
FreeQ[{a,b,c,d,x},x] && NeQ[b c-a d,0] && LtQ[-1,m,0] && LeQ[3,Denominator[m],4] && AtomQ[b c+a d]

Int[(a_ + b_ x)^m_*(c_ + d_ x)^n_]*x_Symbol] := 
(a+b x)^m*(c+d x)^n/((a+b x)*(c+d x))^m*Int[(a+c+(b c+a d)*x+b d x^2)^n,m,x] /;
FreeQ[{a,b,c,d,x},x] && NeQ[b c-a d,0] && LtQ[-1,m,0] && LeQ[3,Denominator[m],4]
```
6: \[ \int (a + b x)^n (c + d x)^n \, dx \] when \( b c - a d \neq 0 \land -1 < m < 0 \land -1 \leq n < 0 \)

Derivation: Integration by substitution

Basis: If \( p \in \mathbb{Z}^+ \), then \( (a + b x)^m (c + d x)^n = \frac{p}{b} \text{Subst} \left[ x^{p(m+1)-1} \left( c - \frac{a d}{b} + \frac{d}{b} x^p \right)^n, x, (a + b x)^{1/p} \right] \partial x (a + b x)^{1/p} \)

- Rule 1.1.2.7.7: If \( b c - a d \neq 0 \land -1 < m < 0 \land -1 \leq n < 0 \), let \( p = \text{Denominator}[m] \), then

\[ \int (a + b x)^n (c + d x)^n \, dx \rightarrow \frac{p}{b} \text{Subst} \left[ \int x^{p(m+1)-1} \left( c - \frac{a d}{b} + \frac{d}{b} x^p \right)^n, x, (a + b x)^{1/p} \right] \]

Program code:

```mathematica
Int[(a_.+b_.*x_)*(c_.+d_.*x_)]^m_*(c_.+d_.*x_)]^n_,x_Symbol] :=
   With[{p=Denominator[m]},
   p/b*Subst[Int[x^(p*(m+1)-1)*(c-a*d/b+d*x^p/b)^n,x],x,(a+b*x)^(1/p)] /;
   FreeQ[{a,b,c,d},x] \&\& NeQ[b*c-a*d,0] \&\& LtQ[-1,m,0] \&\& LeQ[-1,n,0] \&\& LeQ[Denominator[n],Denominator[m]] \&\&
   IntLinearQ[a,b,c,d,m,n,x]
```
Rules for integrands of the form \((a + bx)^m (c + dx)^n\)

H. \[\int (a + bx)^m (c + dx)^n \, dx \quad \text{when} \quad b \, c - a \, d \neq 0\]

1. \[\int (b x)^m (c + d x)^n \, dx \]

1: \[\int (b x)^m (c + d x)^n \, dx \quad \text{when} \quad m \notin \mathbb{Z} \land \ (n \in \mathbb{Z} \lor c > 0)\]

Rule 1.1.1.2.H.1.1: If \(m \notin \mathbb{Z} \land \ (n \in \mathbb{Z} \lor c > 0)\), then

\[
\int (b x)^m (c + d x)^n \, dx \rightarrow \frac{c^n (b x)^{m+1}}{b (m+1)} \text{Hypergeometric2F1}[-n, m+1, m+2, -\frac{d x}{c}] \]

Program code:
```plaintext
Int[(b_. x_)^m_*(c_+d_. x_)^n_,x_Symbol] :=
c^n* (b*x)^(m+1) / (b*(m+1)) * Hypergeometric2F1[-n, m+1, m+2, -d*x/c] /;
FreeQ[{b,c,d,m,n},x] && Not[IntegerQ[m]] && (IntegerQ[n] || GtQ[c,0] && Not[EqQ[n,-1/2] && EqQ[c^2-d^2,0] && GtQ[-d/(b*c),0]])
```

2: \[\int (b x)^m (c + d x)^n \, dx \quad \text{when} \quad n \notin \mathbb{Z} \land \ (m \in \mathbb{Z} \lor -\frac{d}{b c} > 0)\]

Rule 1.1.1.2.H.1.2: If \(n \notin \mathbb{Z} \land \ (m \in \mathbb{Z} \lor -\frac{d}{b c} > 0)\), then

\[
\int (b x)^m (c + d x)^n \, dx \rightarrow \frac{(c + d x)^{n+1}}{d (n+1) \left(-\frac{d}{b c}\right)^m} \text{Hypergeometric2F1}[-m, n+1, n+2, 1+\frac{d x}{c}] \]

Program code:
```plaintext
Int[(b_. x_)^m_*(c_+d_. x_)^n_,x_Symbol] :=
(c+d*x)^(n+1) / (d*(n+1)*(-d/(b*c))^m) * Hypergeometric2F1[-m, n+1, n+2, 1+d*x/c] /;
FreeQ[{b,c,d,m,n},x] && Not[IntegerQ[n]] && (IntegerQ[m] || GtQ[-d/(b*c),0])
```
3. \( \int (b x)^m (c + d x)^n \, dx \) when \( m \notin \mathbb{Z} \land n \notin \mathbb{Z} \land c \neq 0 \land -\frac{d}{b c} \neq 0 \)

1: \( \int (b x)^m (c + d x)^n \, dx \) when \( m \notin \mathbb{Z} \land n \notin \mathbb{Z} \land c \neq 0 \land -\frac{d}{b c} \neq 0 \land (m \in \mathbb{R} \lor n \notin \mathbb{R}) \)

Derivation: Piecewise constant extraction

Basis: \( \frac{\partial}{\partial x} \frac{(c + d x)^n}{(1 + \frac{d x}{c})^n} = 0 \)

Rule 1.1.1.2.H.1.3.1: If \( m \notin \mathbb{Z} \land n \notin \mathbb{Z} \land c \neq 0 \land -\frac{d}{b c} \neq 0 \land (m \in \mathbb{R} \lor n \notin \mathbb{R}) \), then

\[
\int (b x)^m (c + d x)^n \, dx \rightarrow \frac{c \text{IntPart}[n] (c + d x)^\text{FracPart}[n]}{(1 + \frac{d x}{c})^\text{FracPart}[n]} \int (b x)^m \left(1 + \frac{d x}{c}\right)^n \, dx
\]

Program code:

```mathematica
Int[(b_. x_)^m_*(c_+d_. x_)^n_, x_Symbol] :=
c^\text{IntPart}[n] (c+d x)^\text{FracPart}[n]/(1+d x/c)^\text{FracPart}[n]*Int[(b x)^m*(1+d x/c)^n,x] /;
FreeQ[{b,c,d,m,n},x] && Not[IntegerQ[m]] && Not[IntegerQ[n]] && Not[GtQ[c,0]] && Not[GtQ[-d/(b*c),0]] &&
(RationalQ[m] && Not[EqQ[n,-1/2] && EqQ[c^2-d^2,0]] || Not[RationalQ[n]])
```
2: \[ \int (b \, x)^m \, (c + d \, x)^n \, d x \quad \text{when} \quad m \notin \mathbb{Z} \land n \notin \mathbb{Z} \land c \neq 0 \land -\frac{d}{b \, c} \neq 0 \land \neg (m \in \mathbb{R} \lor n \in \mathbb{R}) \]

- Derivation: Piecewise constant extraction

- Basis: \[ \partial_x \left( \frac{(b \, x)^m}{(-\frac{d}{b \, c})^n} \right) = 0 \]

- Rule 1.1.1.2.H.1.3.2: If \( m \notin \mathbb{Z} \land n \notin \mathbb{Z} \land c \neq 0 \land -\frac{d}{b \, c} \neq 0 \land \neg (m \in \mathbb{R} \lor n \in \mathbb{R}) \), then

\[ \int (b \, x)^m \, (c + d \, x)^n \, d x \rightarrow \left( \frac{-b \, x}{d} \right)^{\text{IntPart}[m]} (b \, x)^{\text{FracPart}[m]} \left( \frac{-d \, x}{c} \right)^{\text{FracPart}[m]} \int \left( \frac{-d \, x}{c} \right)^n (c + d \, x)^n \, d x \]

- Program code:

```mathematica
Int[(b_.*x_)^m_*(c_+d_.*x_)^n_,x_Symbol] := 
(-b*c/d)^IntPart[m]*(b*x)^FracPart[m]/(-d*x/c)^FracPart[m]*Int[(-d*x/c)^n*(c+d*x)^n,x] /;
FreeQ[{b,c,d,m,n},x] && Not[IntegerQ[m]] && Not[IntegerQ[n]] && Not[GreaterQ[c,0]] && Not[GreaterQ[-d/(b*c),0]]
```
2. \( \int (a + b x)^m (c + d x)^n \, dx \) when \( b c - a d \neq 0 \land m \notin \mathbb{Z} \)

1: \( \int (a + b x)^m (c + d x)^n \, dx \) when \( b c - a d \neq 0 \land m \notin \mathbb{Z} \land (n \in \mathbb{Z} \lor \frac{b}{b c - a d} > 0) \)

Rule 1.1.1.2.H.2.2.1: If \( b c - a d \neq 0 \land m \notin \mathbb{Z} \land n \in \mathbb{Z} \lor \frac{b}{b c - a d} > 0 \), then

\[
\int (a + b x)^m (c + d x)^n \, dx \rightarrow \frac{(a + b x)^{m+1}}{b (m + 1) \left( \frac{b}{b c - a d} \right)^n} \text{Hypergeometric2F1}[-n, m+1, m+2, -\frac{d (a + b x)}{b c - a d}]
\]

Program code:

```mathematica
Int[(a_+b_+x_)^m*(c_+d_+x_)^n_,x_Symbol] :=
    (b*c-a*d)^n*(a+b*x)^(m+1)/(b^(n+1)*(m+1))*Hypergeometric2F1[-n,m+1,m+2,-d*(a+b*x)/(b*c-a*d)] /;
FreeQ[{a,b,c,d,m},x] \&\& NeQ[b*c-a*d,0] \&\& Not[IntegerQ[m]] \&\& IntegerQ[n]

Int[(a_+b_+x_)^m*(c_+d_+x_)^n_,x_Symbol] :=
    (a+b*x)^(m+1)/(b*(m+1)*(b/(b*c-a*d))^n)*Hypergeometric2F1[-n,m+1,m+2,-d*(a+b*x)/(b*c-a*d)] /;
FreeQ[{a,b,c,d,m},x] \&\& NeQ[b*c-a*d,0] \&\& Not[IntegerQ[m]] \&\& Not[IntegerQ[n]] \&\& GtQ[b/(b*c-a*d),0] \&\&
    (RationalQ[m] \| Not[RationalQ[n]] \&\& GtQ[-d/(b*c-a*d),0])
```
2: \( \int (a + bx)^m (c + dx)^n \, dx \) when \( bc - ad \neq 0 \land m \notin \mathbb{Z} \land n \notin \mathbb{Z} \land \frac{b}{bc-ad} \neq 0 \)

**Derivation: Piecewise constant extraction**

**Basis:**
\[
\partial_x \left( \frac{(c + dx)^n}{bc-ad} \right)^n = 0
\]

**Rule 1.1.1.2.H.2.2.2:** If \( bc - ad \neq 0 \land m \notin \mathbb{Z} \land n \notin \mathbb{Z} \land \frac{b}{bc-ad} \neq 0 \), then

\[
\int (a + bx)^m (c + dx)^n \, dx \rightarrow \frac{(c + dx)^{\text{FractionPart}[n]}}{\text{IntegerPart}[n]} \left( \frac{b}{bc-ad} \right)^{\text{FractionPart}[n]} \int (a + bx)^m \left( \frac{bc}{bc-ad} + \frac{bdx}{bc-ad} \right)^n \, dx
\]

**Program code:**

```mathematica
Int[(a_ + b_ . x_.)^m_ (*c_ + d_ . x_.)^n_ , x_Symbol] :=
(c + d x)^{\text{FractionPart}[n]} / ((b/(b + c - a d))^\text{IntegerPart}[n] * (b*(c + d x)/(b + c - a d))^\text{FractionPart}[n]) *
Int[(a + b x)^m \text{Simp}[b c/(b + c - a d) + b d x/(b + c - a d), x]^n, x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[b + c - a d, 0] && Not[IntegerQ[m]] && Not[IntegerQ[n]] && (RationalQ[m] || Not[SimplerQ[n + 1, m + 1]])
```
S: \[ \int (a + b \cdot u)^m (c + d \cdot u)^n \, du \] when \( u = e + f \cdot x \)

**Derivation: Integration by substitution**

**Rule 1.1.1.2.S:** If \( u = e + f \cdot x \), then

\[ \int (a + b \cdot u)^m (c + d \cdot u)^n \, du \longrightarrow \frac{1}{f} \text{Subst} \left[ \int (a + b \cdot x)^m (c + d \cdot x)^n \, dx, x, u \right] \]

**Program code:**

```mathematica
Int[(a_.+b_.*u_.)^m_.*(c_.+d_.*u_.)^n_.,x_Symbol] :=
   1/Coefficient[u,x,1]*Subst[Int[(a+b*x)^m*(c+d*x)^n],x,u] /;
FreeQ[{a,b,c,d,m,n},x] && LinearQ[u,x] && NeQ[Coefficient[u,x,0],0]
```

(* IntLinearQ[a,b,c,d,m,n,x] returns True iff \((a+b*x)^m*(c+d*x)^n\) is integrable wrt \( x \) in terms of non-hypergeometric functions. *)

```mathematica
IntLinearQ[a_,b_,c_,d_,m_,n_,x_] :=
   IGtQ[m,0] || IGtQ[n,0] || IntegersQ[3*m,3*n] || IntegersQ[4*m,4*n] || IntegersQ[2*m,6*n] || IntegersQ[6*m,2*n] || ILtQ[m+n,-1] || IntegerQ[m+n,-1] || NeQ[m,-1] && NeQ[n,-1]
```