Rules for integrands of the form $(d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p$ when $e\,f-d\,g\neq 0$

0: $\int (e\,x)^m\,(f+g\,x)\,\left(b\,x+c\,x^2\right)^p\,dx$ when $b\,g\,(m+p+1)-c\,f\,(m+2\,p+2)=0\;\land\;m+2\,p+2\neq 0$

- **Rule 1.2.1.3.0: If $b\,g\,(m+p+1)-c\,f\,(m+2\,p+2)=0\;\land\;m+2\,p+2\neq 0$, then**

$$\int (e\,x)^m\,(f+g\,x)\,\left(b\,x+c\,x^2\right)^p\,dx\;\longrightarrow\;\frac{g\,(e\,x)^m\,\left(b\,x+c\,x^2\right)^{p+1}}{c\,(m+2\,p+2)}$$

- **Program code:**

```
Int[(e_.*x_)^m_.*(f_+g_.*x_)*(b_.*x_+c_.*x_^2)^p_.,x_Symbol] :=
  g*(e*x)^m*(b*x+c*x^2)^(p+1)/(c*(m+2*p+2)) /;
FreeQ[{b,c,e,f,g,m,p},x] && EqQ[b*g*(m+p+1)-c*f*(m+2*p+2),0] && NeQ[m+2*p+2,0]
```

1: $\int x^m\,(f+g\,x)\,\left(a+c\,x^2\right)^p\,dx$ when $m\in\mathbb{Z}\;\land\;2\,p\notin\mathbb{Z}$

- **Derivation: Algebraic expansion**
- **Rule 1.2.1.3.1: If $m\in\mathbb{Z}\;\land\;2\,p\notin\mathbb{Z}$, then**

$$\int x^m\,(f+g\,x)\,\left(a+c\,x^2\right)^p\,dx\;\longrightarrow\;f\int x^m\,\left(a+c\,x^2\right)^p\,dx+g\int x^{m+1}\,\left(a+c\,x^2\right)^p\,dx$$

- **Program code:**

```
Int[x_^m_.*(f_+g_.*x_)*(a_+c_.*x_^2)^p_.,x_Symbol] :=
  f*Int[x^m*(a+c*x^2)^p,x] + g*Int[x^(m+1)*(a+c*x^2)^p,x] /;
FreeQ[{a,c,f,g,p},x] && IntegerQ[m] && Not[IntegerQ[2*p]]
```

2: $\int (e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx$ when $p\in\mathbb{Z}\,\land\,(p>0\,\lor\,a==0\land m\in\mathbb{Z})$

- **Derivation: Algebraic expansion**
- **Rule 1.2.1.3.2: If** $p\in\mathbb{Z}\,\land\,(p>0\,\lor\,a==0\land m\in\mathbb{Z})$**, then**

$$\int (e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx\,\longrightarrow\,\int \text{ExpandIntegrand}\left[(e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p,\,x\right]dx$$

- **Program code:**

```
Int[(e_.*x_)^m_.*(f_.+g_.*x_)*(a_.+b_.*x_+c_.*x_^2)^p_.,x_Symbol] :=
  Int[ExpandIntegrand[(e*x)^m*(f+g*x)*(a+b*x+c*x^2)^p,x],x] /;
FreeQ[{a,b,c,e,f,g,m},x] && IntegerQ[p] && (GtQ[p,0] || EqQ[a,0] && IntegerQ[m])
```

```
Int[(e_.*x_)^m_.*(f_.+g_.*x_)*(a_+c_.*x_^2)^p_.,x_Symbol] :=
  Int[ExpandIntegrand[(e*x)^m*(f+g*x)*(a+c*x^2)^p,x],x] /;
FreeQ[{a,c,e,f,g,m},x] && IGtQ[p,0]
```

3: $\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx$ when $b^2-4\,a\,c==0\,\land\,m+2\,p+3==0\,\land\,2\,c\,f-b\,g==0$

- **Derivation: Quadratic recurrence 2a with** $2\,c\,f-b\,g==0$ **: square quadratic recurrence 3b with** $m+2\,p+3==0$
- **Rule 1.2.1.3.3: If** $b^2-4\,a\,c==0\,\land\,m+2\,p+3==0\,\land\,2\,c\,f-b\,g==0$**, then**

$$\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx\,\longrightarrow\,-\frac{f\,g\,(d+e\,x)^{m+1}\,\left(a+b\,x+c\,x^2\right)^{p+1}}{b\,(p+1)\,(e\,f-d\,g)}$$

- **Program code:**

```
Int[(d_.+e_.*x_)^m_.*(f_+g_.*x_)*(a_+b_.*x_+c_.*x_^2)^p_.,x_Symbol] :=
  -f*g*(d+e*x)^(m+1)*(a+b*x+c*x^2)^(p+1)/(b*(p+1)*(e*f-d*g)) /;
FreeQ[{a,b,c,d,e,f,g,m,p},x] && EqQ[b^2-4*a*c,0] && EqQ[m+2*p+3,0] && EqQ[2*c*f-b*g,0]
```

4: $\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx$ when $2\,c\,f-b\,g==0\,\wedge\,p<-1\,\wedge\,m>0$

Derivation: Integration by parts

- **Basis: If** $2\,c\,f-b\,g==0$, **then** $\partial_x\frac{g\,(a+b\,x+c\,x^2)^{p+1}}{2\,c\,(p+1)}==(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p$

Rule 1.2.1.3.4: If $2\,c\,f-b\,g==0\,\wedge\,p<-1\,\wedge\,m>0$, **then**

$$\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx\,\longrightarrow\,\frac{g\,(d+e\,x)^m\,\left(a+b\,x+c\,x^2\right)^{p+1}}{2\,c\,(p+1)}-\frac{e\,g\,m}{2\,c\,(p+1)}\int (d+e\,x)^{m-1}\,\left(a+b\,x+c\,x^2\right)^{p+1}\,dx$$

Program code:

```
Int[(d_.+e_.*x_)^m_.*(f_.+g_.*x_)*(a_.+b_.*x_+c_.*x_^2)^p_,x_Symbol] :=
  g*(d+e*x)^m*(a+b*x+c*x^2)^(p+1)/(2*c*(p+1)) -
  e*g*m/(2*c*(p+1))*Int[(d+e*x)^(m-1)*(a+b*x+c*x^2)^(p+1),x] /;
FreeQ[{a,b,c,d,e,f,g},x] && EqQ[2*c*f-b*g,0] && LtQ[p,-1] && GtQ[m,0]
```

5. $\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx$ when $b^2-4\,a\,c\neq 0$

1: $\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx$ when $b^2-4\,a\,c==0\,\wedge\,m+2\,p+3==0\,\wedge\,2\,c\,f-b\,g\neq 0\,\wedge\,2\,c\,d-b\,e\neq 0$

Derivation: Algebraic expansion

- **Basis:** $f+g\,x==\frac{(2\,c\,f-b\,g)\,(d+e\,x)}{2\,c\,d-b\,e}-\frac{(e\,f-d\,g)\,(b+2\,c\,x)}{2\,c\,d-b\,e}$

Rule 1.2.1.3.5: If $b^2-4\,a\,c==0\,\wedge\,m+2\,p+3==0\,\wedge\,2\,c\,f-b\,g\neq 0\,\wedge\,2\,c\,d-b\,e\neq 0$, **then**

$$\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx\,\longrightarrow$$

$$-\frac{2\,c\,(e\,f-d\,g)\,(d+e\,x)^{m+1}\,\left(a+b\,x+c\,x^2\right)^{p+1}}{(p+1)\,(2\,c\,d-b\,e)^2}+\frac{2\,c\,f-b\,g}{2\,c\,d-b\,e}\int (d+e\,x)^{m+1}\,\left(a+b\,x+c\,x^2\right)^p\,dx$$

Program code:

```
Int[(d_.+e_.*x_)^m_.*(f_.+g_.*x_)*(a_+b_.*x_+c_.*x_^2)^p_,x_Symbol] :=
  -2*c*(e*f-d*g)*(d+e*x)^(m+1)*(a+b*x+c*x^2)^(p+1)/((p+1)*(2*c*d-b*e)^2) +
  (2*c*f-b*g)/(2*c*d-b*e)*Int[(d+e*x)^(m+1)*(a+b*x+c*x^2)^p,x] /;
FreeQ[{a,b,c,d,e,f,g,m,p},x] && EqQ[b^2-4*a*c,0] && EqQ[m+2*p+3,0] && NeQ[2*c*f-b*g,0] && NeQ[2*c*d-b*e,0]
```

2: $\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx$ when $b^2-4\,a\,c=0$

Derivation: Piecewise constant extraction

- **Basis: If** $b^2-4\,a\,c=0$**, then** $\partial_x \frac{\left(a+b\,x+c\,x^2\right)^p}{\left(\frac{b}{2}+c\,x\right)^{2\,p}}=0$

- **Rule 1.2.1.3.6: If** $b^2-4\,a\,c=0$**, then**

$$\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx\;\longrightarrow\;\frac{\left(a+b\,x+c\,x^2\right)^{\text{FracPart}[p]}}{c^{\text{IntPart}[p]}\,\left(\frac{b}{2}+c\,x\right)^{2\,\text{FracPart}[p]}}\int (d+e\,x)^m\,(f+g\,x)\,\left(\frac{b}{2}+c\,x\right)^{2\,p}\,dx$$

Program code:

```
Int[(d_.+e_.*x_)^m_.*(f_.+g_.*x_)*(a_.+b_.*x_+c_.*x_^2)^p_,x_Symbol] :=
  (a+b*x+c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2+c*x)^(2*FracPart[p]))*Int[(d+e*x)^m*(f+g*x)*(b/2+c*x)^(2*p),x] /;
FreeQ[{a,b,c,d,e,f,g,m},x] && EqQ[b^2-4*a*c,0]
```

6: $\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx$ when $b^2-4\,a\,c\neq 0\;\wedge\;p\in\mathbb{Z}\;\wedge\;(p>0\;\vee\;a=0\wedge m\in\mathbb{Z})$

Derivation: Algebraic expansion

Rule 1.2.1.3.6: If $b^2-4\,a\,c\neq 0\;\wedge\;p\in\mathbb{Z}^+$**, then**

$$\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx\;\longrightarrow\;\int \text{ExpandIntegrand}\left[(d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p,\,x\right]dx$$

Program code:

```
Int[(d_.+e_.*x_)^m_.*(f_.+g_.*x_)*(a_.+b_.*x_+c_.*x_^2)^p_.,x_Symbol] :=
  Int[ExpandIntegrand[(d+e*x)^m*(f+g*x)*(a+b*x+c*x^2)^p,x],x] /;
FreeQ[{a,b,c,d,e,f,g,m},x] && NeQ[b^2-4*a*c,0] && IntegerQ[p] && (GtQ[p,0] || EqQ[a,0] && IntegerQ[m])
```

```
Int[(d_.+e_.*x_)^m_.*(f_.+g_.*x_)*(a_+c_.*x_^2)^p_.,x_Symbol] :=
  Int[ExpandIntegrand[(d+e*x)^m*(f+g*x)*(a+c*x^2)^p,x],x] /;
FreeQ[{a,c,d,e,f,g,m},x] && IGtQ[p,0]
```

7. $\int (d+e\,x)\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx$ when $b^2-4\,a\,c\neq 0$

1: $\int \frac{(d+e\,x)\,(f+g\,x)}{a+b\,x+c\,x^2}\,dx$ when $b^2-4\,a\,c\neq 0$

Derivation: Algebraic expansion

Rule 1.2.1.3.7.1: If $b^2-4\,a\,c\neq 0$, then

$$\int \frac{(d+e\,x)\,(f+g\,x)}{a+b\,x+c\,x^2}\,dx \longrightarrow \frac{e\,g\,x}{c}+\frac{1}{c}\int \frac{c\,d\,f-a\,e\,g+(c\,e\,f+c\,d\,g-b\,e\,g)\,x}{a+b\,x+c\,x^2}\,dx$$

Program code:

```
Int[(d_.+e_.*x_)*(f_+g_.*x_)/(a_.+b_.*x_+c_.*x_^2),x_Symbol] :=
  e*g*x/c + 1/c*Int[(c*d*f-a*e*g+(c*e*f+c*d*g-b*e*g)*x)/(a+b*x+c*x^2),x] /;
FreeQ[{a,b,c,d,e,f,g},x] && NeQ[b^2-4*a*c,0]
```

```
Int[(d_.+e_.*x_)*(f_+g_.*x_)/(a_+c_.*x_^2),x_Symbol] :=
  e*g*x/c + 1/c*Int[(c*d*f-a*e*g+c*(e*f+d*g)*x)/(a+c*x^2),x] /;
FreeQ[{a,c,d,e,f,g},x]
```

2: $\int (d+e\,x)\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx$ when $b^2-4\,a\,c\neq 0\,\land\,b^2\,e\,g\,(p+2)-2\,a\,c\,e\,g+c\,(2\,c\,d\,f-b\,(e\,f+d\,g))\,(2\,p+3)=0\,\land\,p\neq -1$

Derivation: ???

Note: If $b^2-4\,a\,c\neq 0\,\land\,b^2\,e\,g\,(p+2)-2\,a\,c\,e\,g+c\,(2\,c\,d\,f-b\,(e\,f+d\,g))\,(2\,p+3)=0$, then $p\neq -\frac{3}{2}$.

Rule 1.2.1.3.7.2: If $b^2-4\,a\,c\neq 0\,\land\,b^2\,e\,g\,(p+2)-2\,a\,c\,e\,g+c\,(2\,c\,d\,f-b\,(e\,f+d\,g))\,(2\,p+3)=0\,\land\,p\neq -1$, then

$$\int (d+e\,x)\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx \longrightarrow -\frac{(b\,e\,g\,(p+2)-c\,(e\,f+d\,g)\,(2\,p+3)-2\,c\,e\,g\,(p+1)\,x)\,\left(a+b\,x+c\,x^2\right)^{p+1}}{2\,c^2\,(p+1)\,(2\,p+3)}$$

Program code:

```
Int[(d_.+e_.*x_)*(f_.+g_.*x_)*(a_.+b_.*x_+c_.*x_^2)^p_,x_Symbol] :=
  -(b*e*g*(p+2)-c*(e*f+d*g)*(2*p+3)-2*c*e*g*(p+1)*x)*(a+b*x+c*x^2)^(p+1)/(2*c^2*(p+1)*(2*p+3)) /;
FreeQ[{a,b,c,d,e,f,g,p},x] && NeQ[b^2-4*a*c,0] && EqQ[b^2*e*g*(p+2)-2*a*c*e*g+c*(2*c*d*f-b*(e*f+d*g))*(2*p+3),0] && NeQ[p,-1]
```

```
Int[(d_.+e_.*x_)*(f_.+g_.*x_)*(a_+c_.*x_^2)^p_,x_Symbol] :=
  ((e*f+d*g)*(2*p+3)+2*e*g*(p+1)*x)*(a+c*x^2)^(p+1)/(2*c*(p+1)*(2*p+3)) /;
FreeQ[{a,c,d,e,f,g,p},x] && EqQ[a*e*g-c*d*f*(2*p+3),0] && NeQ[p,-1]
```

3: $\int (d+e\,x)\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx$ when $b^2-4\,a\,c\neq 0\,\land\,p<-1$

Derivation: ???

Rule 1.2.1.3.7.3: If $b^2-4\,a\,c\neq 0\,\land\,p<-1$**, then**

$$\int (d+e\,x)\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx\,\longrightarrow$$

$$-\left(\left(2\,a\,c\,(e\,f+d\,g)-b\,(c\,d\,f+a\,e\,g)-\left(b^2\,e\,g-b\,c\,(e\,f+d\,g)+2\,c\,(c\,d\,f-a\,e\,g)\right)\,x\right)\,\left(a+b\,x+c\,x^2\right)^{p+1}\right)\Big/\left(c\,(p+1)\,\left(b^2-4\,a\,c\right)\right)-\frac{b^2\,e\,g\,(p+2)-2\,a\,c\,e\,g+c\,(2\,c\,d\,f-b\,(e\,f+d\,g))\,(2\,p+3)}{c\,(p+1)\,\left(b^2-4\,a\,c\right)}\int\left(a+b\,x+c\,x^2\right)^{p+1}\,dx$$

Program code:

```
Int[(d_.+e_.*x_)*(f_.+g_.*x_)*(a_.+b_.*x_+c_.*x_^2)^p_,x_Symbol] :=
  -(2*a*c*(e*f+d*g)-b*(c*d*f+a*e*g)-(b^2*e*g-b*c*(e*f+d*g)+2*c*(c*d*f-a*e*g))*x)*(a+b*x+c*x^2)^(p+1)/(c*(p+1)*(b^2-4*a*c)) -
  (b^2*e*g*(p+2)-2*a*c*e*g+c*(2*c*d*f-b*(e*f+d*g))*(2*p+3))/(c*(p+1)*(b^2-4*a*c))*Int[(a+b*x+c*x^2)^(p+1),x] /;
FreeQ[{a,b,c,d,e,f,g},x] && NeQ[b^2-4*a*c,0] && LtQ[p,-1]
```

```
Int[(d_.+e_.*x_)*(f_.+g_.*x_)*(a_+c_.*x_^2)^p_,x_Symbol] :=
  (a*(e*f+d*g)-(c*d*f-a*e*g)*x)*(a+c*x^2)^(p+1)/(2*a*c*(p+1)) -
  (a*e*g-c*d*f*(2*p+3))/(2*a*c*(p+1))*Int[(a+c*x^2)^(p+1),x] /;
FreeQ[{a,c,d,e,f,g},x] && LtQ[p,-1]
```

4: $\int (d+e\,x)\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx$ when $b^2-4\,a\,c\neq 0\,\wedge\,p\nleq -1$

Derivation: ???

Rule 1.2.1.3.7.4: If $b^2-4\,a\,c\neq 0\,\wedge\,p\nleq -1$, then

$$\int (d+e\,x)\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx\,\longrightarrow$$

$$-\left((b\,e\,g\,(p+2)-c\,(e\,f+d\,g)\,(2\,p+3)-2\,c\,e\,g\,(p+1)\,x)\,\left(a+b\,x+c\,x^2\right)^{p+1}\right)\Big/\left(2\,c^2\,(p+1)\,(2\,p+3)\right)+\frac{b^2\,e\,g\,(p+2)-2\,a\,c\,e\,g+c\,(2\,c\,d\,f-b\,(e\,f+d\,g))\,(2\,p+3)}{2\,c^2\,(2\,p+3)}\int\left(a+b\,x+c\,x^2\right)^p\,dx$$

Program code:

```
Int[(d_.+e_.*x_)*(f_.+g_.*x_)*(a_.+b_.*x_+c_.*x_^2)^p_,x_Symbol] :=
  -(b*e*g*(p+2)-c*(e*f+d*g)*(2*p+3)-2*c*e*g*(p+1)*x)*(a+b*x+c*x^2)^(p+1)/(2*c^2*(p+1)*(2*p+3)) +
  (b^2*e*g*(p+2)-2*a*c*e*g+c*(2*c*d*f-b*(e*f+d*g))*(2*p+3))/(2*c^2*(2*p+3))*Int[(a+b*x+c*x^2)^p,x] /;
FreeQ[{a,b,c,d,e,f,g,p},x] && NeQ[b^2-4*a*c,0] && Not[LeQ[p,-1]]
```

```
Int[(d_.+e_.*x_)*(f_.+g_.*x_)*(a_+c_.*x_^2)^p_,x_Symbol] :=
  ((e*f+d*g)*(2*p+3)+2*e*g*(p+1)*x)*(a+c*x^2)^(p+1)/(2*c*(p+1)*(2*p+3)) -
  (a*e*g-c*d*f*(2*p+3))/(c*(2*p+3))*Int[(a+c*x^2)^p,x] /;
FreeQ[{a,c,d,e,f,g,p},x] && Not[LeQ[p,-1]]
```

8. $\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx$ when $b^2-4\,a\,c\neq 0\,\wedge\,c\,d^2-b\,d\,e+a\,e^2==0$

1. $\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx$ when $b^2-4\,a\,c\neq 0\,\wedge\,c\,d^2-b\,d\,e+a\,e^2==0\,\wedge\,p\in\mathbb{Z}$

1: $\int (e\,x)^m\,(f+g\,x)\,\left(b\,x+c\,x^2\right)^p\,dx$ when $p\in\mathbb{Z}$

Derivation: Algebraic simplification

Rule 1.2.1.2.8.1.1: If $p\in\mathbb{Z}$, then

$$\int (e\,x)^m\,(f+g\,x)\,\left(b\,x+c\,x^2\right)^p\,dx\;\longrightarrow\;\frac{1}{e^p}\int (e\,x)^{m+p}\,(f+g\,x)\,(b+c\,x)^p\,dx$$

Program code:

```
Int[(e_.*x_)^m_.*(f_.+g_.*x_)*(b_.*x_+c_.*x_^2)^p_.,x_Symbol] :=
  1/e^p*Int[(e*x)^(m+p)*(f+g*x)*(b+c*x)^p,x] /;
FreeQ[{b,c,e,f,g,m},x] && IntegerQ[p]
```

2: $\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx$ when $b^2-4\,a\,c\neq 0\,\wedge\,c\,d^2-b\,d\,e+a\,e^2==0\,\wedge\,p\in\mathbb{Z}$

Derivation: Algebraic simplification

Basis: If $c\,d^2-b\,d\,e+a\,e^2==0$, then $a+b\,x+c\,x^2==(d+e\,x)\,\left(\frac{a}{d}+\frac{c\,x}{e}\right)$

Rule 1.2.1.3.8.1.2: If $b^2-4\,a\,c\neq 0\,\wedge\,c\,d^2-b\,d\,e+a\,e^2==0\,\wedge\,p\in\mathbb{Z}$, then

$$\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx\;\longrightarrow\;\int (d+e\,x)^{m+p}\,(f+g\,x)\,\left(\frac{a}{d}+\frac{c\,x}{e}\right)^p\,dx$$

Program code:

```
Int[(d_+e_.*x_)^m_*(f_.+g_.*x_)*(a_.+b_.*x_+c_.*x_^2)^p_.,x_Symbol] :=
  Int[(d+e*x)^(m+p)*(f+g*x)*(a/d+c/e*x)^p,x] /;
FreeQ[{a,b,c,d,e,f,g,m},x] && NeQ[b^2-4*a*c,0] && EqQ[c*d^2-b*d*e+a*e^2,0] && IntegerQ[p]
```

```
Int[(d_+e_.*x_)^m_*(f_.+g_.*x_)*(a_+c_.*x_^2)^p_.,x_Symbol] :=
  Int[(d+e*x)^(m+p)*(f+g*x)*(a/d+c/e*x)^p,x] /;
FreeQ[{a,c,d,e,f,g,m},x] && EqQ[c*d^2+a*e^2,0] && (IntegerQ[p] || GtQ[a,0] && GtQ[d,0] && EqQ[m+p,0])
```

2. $\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx$ when $b^2-4\,a\,c\neq 0\,\wedge\,c\,d^2-b\,d\,e+a\,e^2==0\,\wedge\,p\notin\mathbb{Z}$

0: $\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx$ when $b^2-4\,a\,c\neq 0\,\wedge\,c\,d^2-b\,d\,e+a\,e^2==0\,\wedge\,p\notin\mathbb{Z}\,\wedge\,m\in\mathbb{Z}^-$?? ???

Derivation: Algebraic simplification

- **Basis: If** $c\,d^2-b\,d\,e+a\,e^2==0$, **then** $d+e\,x==\frac{d\,e\,(a+b\,x+c\,x^2)}{a\,e+c\,d\,x}$
- **Basis: If** $c\,d^2+a\,e^2==0$, **then** $d+e\,x==\frac{d^2\,(a+c\,x^2)}{a\,(d-e\,x)}$

Rule 1.2.1.3.8.2.0: If $b^2-4\,a\,c\neq 0\,\wedge\,c\,d^2-b\,d\,e+a\,e^2==0\,\wedge\,m\in\mathbb{Z}^-$, **then**

$$\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx\,\longrightarrow\,d^m\,e^m\int\frac{(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^{m+p}}{(a\,e+c\,d\,x)^m}\,dx$$

Program code:

```
Int[(d_+e_.*x_)^m_*(f_.+g_.*x_)*(a_.+b_.*x_+c_.*x_^2)^p_,x_Symbol] :=
  d^m*e^m*Int[(f+g*x)*(a+b*x+c*x^2)^(m+p)/(a*e+c*d*x)^m,x] /;
FreeQ[{a,b,c,d,e,f,g,p},x] && NeQ[b^2-4*a*c,0] && EqQ[c*d^2-b*d*e+a*e^2,0] && Not[IntegerQ[2*p]] && ILtQ[m,0]
```

```
Int[x_*(d_+e_.*x_)^m_*(a_+c_.*x_^2)^p_,x_Symbol] :=
  d^m*e^m*Int[x*(a+c*x^2)^(m+p)/(a*e+c*d*x)^m,x] /;
FreeQ[{a,c,d,e,p},x] && EqQ[c*d^2+a*e^2,0] && Not[IntegerQ[p]] && ILtQ[m,0]          && EqQ[m,-1] && Not[ILtQ[p-1/2,0]]
```

1: $\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx$ when $b^2-4\,a\,c\neq 0\,\wedge\,c\,d^2-b\,d\,e+a\,e^2==0\,\wedge\,m\,(g\,(c\,d-b\,e)+c\,e\,f)+e\,(p+1)\,(2\,c\,f-b\,g)==0$

- **Derivation: Quadratic recurrence 3a with** $c\,d^2-b\,d\,e+a\,e^2==0$ **and** $m\,(g\,(c\,d-b\,e)+c\,e\,f)+e\,(p+1)\,(2\,c\,f-b\,g)==0$
- **Note: If** $b^2-4\,a\,c\neq 0\,\wedge\,c\,d^2-b\,d\,e+a\,e^2==0\,\wedge\,m\,(g\,(c\,d-b\,e)+c\,e\,f)+e\,(p+1)\,(2\,c\,f-b\,g)==0$**, then** $m+2\,p+2\neq 0$**.**
- **Rule 1.2.1.3.8.2.1: If** $b^2-4\,a\,c\neq 0\,\wedge\,c\,d^2-b\,d\,e+a\,e^2==0\,\wedge\,m\,(g\,(c\,d-b\,e)+c\,e\,f)+e\,(p+1)\,(2\,c\,f-b\,g)==0$**, then**

$$\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx\,\longrightarrow\,\frac{g\,(d+e\,x)^m\,\left(a+b\,x+c\,x^2\right)^{p+1}}{c\,(m+2\,p+2)}$$

- **Program code:**

```
Int[(d_.+e_.*x_)^m_*(f_.+g_.*x_)*(a_.+b_.*x_+c_.*x_^2)^p_,x_Symbol] :=
  g*(d+e*x)^m*(a+b*x+c*x^2)^(p+1)/(c*(m+2*p+2)) /;
FreeQ[{a,b,c,d,e,f,g,m,p},x] && NeQ[b^2-4*a*c,0] && EqQ[c*d^2-b*d*e+a*e^2,0] && EqQ[m*(g*(c*d-b*e)+c*e*f)+e*(p+1)*(2*c*f-b*g),0]
```

```
Int[(d_+e_.*x_)^m_*(f_.+g_.*x_)*(a_+c_.*x_^2)^p_,x_Symbol] :=
  g*(d+e*x)^m*(a+c*x^2)^(p+1)/(c*(m+2*p+2)) /;
FreeQ[{a,c,d,e,f,g,m,p},x] && EqQ[c*d^2+a*e^2,0] && EqQ[m*(d*g+e*f)+2*e*f*(p+1),0]
```

2: $\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx$ when $b^2-4\,a\,c\neq 0\,\wedge\,c\,d^2-b\,d\,e+a\,e^2==0\,\wedge\,p<-1\,\wedge\,m>0$

- **Derivation: Quadratic recurrence 3a with $c\,d^2-b\,d\,e+a\,e^2=0$: special quadratic recurrence 2b**
- **Note: If $b^2-4\,a\,c\neq 0\,\wedge\,c\,d^2-b\,d\,e+a\,e^2==0$, then $2\,c\,d-b\,e\neq 0$.**
- **Rule 1.2.1.3.8.2.2: If $b^2-4\,a\,c\neq 0\,\wedge\,c\,d^2-b\,d\,e+a\,e^2==0\,\wedge\,p<-1\,\wedge\,m>0$, then**

$$\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx\,\longrightarrow$$

$$\frac{(g\,(c\,d-b\,e)+c\,e\,f)\,(d+e\,x)^m\,\left(a+b\,x+c\,x^2\right)^{p+1}}{c\,(p+1)\,(2\,c\,d-b\,e)}-\frac{e\,(m\,(g\,(c\,d-b\,e)+c\,e\,f)+e\,(p+1)\,(2\,c\,f-b\,g))}{c\,(p+1)\,(2\,c\,d-b\,e)}\int (d+e\,x)^{m-1}\,\left(a+b\,x+c\,x^2\right)^{p+1}\,dx$$

Program code:

```
Int[(d_.+e_.*x_)^m_*(f_.+g_.*x_)*(a_.+b_.*x_+c_.*x_^2)^p_,x_Symbol] :=
  (g*(c*d-b*e)+c*e*f)*(d+e*x)^m*(a+b*x+c*x^2)^(p+1)/(c*(p+1)*(2*c*d-b*e)) -
  e*(m*(g*(c*d-b*e)+c*e*f)+e*(p+1)*(2*c*f-b*g))/(c*(p+1)*(2*c*d-b*e))*
    Int[(d+e*x)^(m-1)*(a+b*x+c*x^2)^(p+1),x] /;
FreeQ[{a,b,c,d,e,f,g},x] && NeQ[b^2-4*a*c,0] && EqQ[c*d^2-b*d*e+a*e^2,0] && LtQ[p,-1] && GtQ[m,0]
```

```
Int[(d_.+e_.*x_)^m_*(f_.+g_.*x_)*(a_+c_.*x_^2)^p_,x_Symbol] :=
  (d*g+e*f)*(d+e*x)^m*(a+c*x^2)^(p+1)/(2*c*d*(p+1)) -
  e*(m*(d*g+e*f)+2*e*f*(p+1))/(2*c*d*(p+1))*Int[(d+e*x)^(m-1)*(a+c*x^2)^(p+1),x] /;
FreeQ[{a,c,d,e,f,g},x] && EqQ[c*d^2+a*e^2,0] && LtQ[p,-1] && GtQ[m,0]
```

```
Int[(d_.+e_.*x_)^m_*(f_.+g_.*x_)*(a_.+b_.*x_+c_.*x_^2)^p_,x_Symbol] :=
  (g*(c*d-b*e)+c*e*f)*(d+e*x)^m*(a+b*x+c*x^2)^(p+1)/(c*(p+1)*(2*c*d-b*e)) -
  e*(m*(g*(c*d-b*e)+c*e*f)+e*(p+1)*(2*c*f-b*g))/(c*(p+1)*(2*c*d-b*e))*
    Int[(d+e*x)^Simplify[m-1]*(a+b*x+c*x^2)^Simplify[p+1],x] /;
FreeQ[{a,b,c,d,e,f,g,m,p},x] && NeQ[b^2-4*a*c,0] && EqQ[c*d^2-b*d*e+a*e^2,0] && SumSimplerQ[p,1] && SumSimplerQ[m,-1] && NeQ[p,-
```

```
Int[(d_+e_.*x_)^m_*(f_.+g_.*x_)*(a_+c_.*x_^2)^p_,x_Symbol] :=
  (d*g+e*f)*(d+e*x)^m*(a+c*x^2)^(p+1)/(2*c*d*(p+1)) -
  e*(m*(d*g+e*f)+2*e*f*(p+1))/(2*c*d*(p+1))*Int[(d+e*x)^Simplify[m-1]*(a+c*x^2)^Simplify[p+1],x] /;
FreeQ[{a,c,d,e,f,g,m,p},x] && EqQ[c*d^2+a*e^2,0] && SumSimplerQ[p,1] && SumSimplerQ[m,-1] && NeQ[p,-1] && Not[IGtQ[m,0]]
```

3: $\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx$ when $b^2-4\,a\,c\neq 0\ \land\ c\,d^2-b\,d\,e+a\,e^2==0\ \land\ (m\leq -1\ \lor\ m+2\,p+2==0)\ \land\ m+p+1\neq 0$

- **Derivation: Quadratic recurrence 3a with $c\,d^2-b\,d\,e+a\,e^2=0$**
- **Note: If $b^2-4\,a\,c\neq 0\ \land\ c\,d^2-b\,d\,e+a\,e^2==0$, then $2\,c\,d-b\,e\neq 0$.**
- **Rule 1.2.1.3.8.2.3: If $b^2-4\,a\,c\neq 0\ \land\ c\,d^2-b\,d\,e+a\,e^2==0\ \land\ (m\leq -1\ \lor\ m+2\,p+2==0)\ \land\ m+p+1\neq 0$, then**

$$\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx\ \longrightarrow$$

$$\frac{(d\,g-e\,f)\,(d+e\,x)^m\,\left(a+b\,x+c\,x^2\right)^{p+1}}{(2\,c\,d-b\,e)\,(m+p+1)}+\frac{m\,(g\,(c\,d-b\,e)+c\,e\,f)+e\,(p+1)\,(2\,c\,f-b\,g)}{e\,(2\,c\,d-b\,e)\,(m+p+1)}\int (d+e\,x)^{m+1}\,\left(a+b\,x+c\,x^2\right)^p\,dx$$

Program code:

```
Int[(d_.+e_.*x_)^m_*(f_.+g_.*x_)*(a_.+b_.*x_+c_.*x_^2)^p_,x_Symbol] :=
  (d*g-e*f)*(d+e*x)^m*(a+b*x+c*x^2)^(p+1)/((2*c*d-b*e)*(m+p+1)) +
  (m*(g*(c*d-b*e)+c*e*f)+e*(p+1)*(2*c*f-b*g))/(e*(2*c*d-b*e)*(m+p+1))*Int[(d+e*x)^(m+1)*(a+b*x+c*x^2)^p,x] /;
FreeQ[{a,b,c,d,e,f,g,m,p},x] && NeQ[b^2-4*a*c,0] && EqQ[c*d^2-b*d*e+a*e^2,0] &&
  (LtQ[m,-1] && Not[IGtQ[m+p+1,0]] || LtQ[m,0] && LtQ[p,-1] || EqQ[m+2*p+2,0]) && NeQ[m+p+1,0]
```

```
Int[(d_+e_.*x_)^m_*(f_.+g_.*x_)*(a_+c_.*x_^2)^p_,x_Symbol] :=
  (d*g-e*f)*(d+e*x)^m*(a+c*x^2)^(p+1)/(2*c*d*(m+p+1)) +
  (m*(g*c*d+c*e*f)+2*e*c*f*(p+1))/(e*(2*c*d)*(m+p+1))*Int[(d+e*x)^(m+1)*(a+c*x^2)^p,x] /;
FreeQ[{a,c,d,e,f,g,m,p},x] && EqQ[c*d^2+a*e^2,0] &&
  (LtQ[m,-1] && Not[IGtQ[m+p+1,0]] || LtQ[m,0] && LtQ[p,-1] || EqQ[m+2*p+2,0]) && NeQ[m+p+1,0]
```

4: $\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx$ when $b^2-4\,a\,c\neq 0\,\wedge\,c\,d^2-b\,d\,e+a\,e^2==0\,\wedge\,m+2\,p+2\neq 0$

Derivation: Quadratic recurrence 3a with $c\,d^2-b\,d\,e+a\,e^2==0$

Rule 1.2.1.3.8.2.4: If $b^2-4\,a\,c\neq 0\,\wedge\,c\,d^2-b\,d\,e+a\,e^2==0\,\wedge\,m+2\,p+2\neq 0$**, then**

$$\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx\,\longrightarrow$$

$$\frac{g\,(d+e\,x)^m\,\left(a+b\,x+c\,x^2\right)^{p+1}}{c\,(m+2\,p+2)}+\frac{m\,(g\,(c\,d-b\,e)+c\,e\,f)+e\,(p+1)\,(2\,c\,f-b\,g)}{c\,e\,(m+2\,p+2)}\int (d+e\,x)^m\,\left(a+b\,x+c\,x^2\right)^p\,dx$$

Program code:

```
Int[(d_.+e_.*x_)^m_*(f_.+g_.*x_)*(a_.+b_.*x_+c_.*x_^2)^p_,x_Symbol] :=
  g*(d+e*x)^m*(a+b*x+c*x^2)^(p+1)/(c*(m+2*p+2)) +
  (m*(g*(c*d-b*e)+c*e*f)+e*(p+1)*(2*c*f-b*g))/(c*e*(m+2*p+2))*Int[(d+e*x)^m*(a+b*x+c*x^2)^p,x] /;
FreeQ[{a,b,c,d,e,f,g,m,p},x] && NeQ[b^2-4*a*c,0] && EqQ[c*d^2-b*d*e+a*e^2,0] && NeQ[m+2*p+2,0] && (NeQ[m,2] || EqQ[d,0])
```

```
Int[(d_+e_.*x_)^m_*(f_.+g_.*x_)*(a_+c_.*x_^2)^p_,x_Symbol] :=
  g*(d+e*x)^m*(a+c*x^2)^(p+1)/(c*(m+2*p+2)) +
  (m*(d*g+e*f)+2*e*f*(p+1))/(e*(m+2*p+2))*Int[(d+e*x)^m*(a+c*x^2)^p,x] /;
FreeQ[{a,c,d,e,f,g,m,p},x] && EqQ[c*d^2+a*e^2,0] && NeQ[m+2*p+2,0] && NeQ[m,2]
```

5. $\int x^2 (f + g x) \left(a + c x^2\right)^p dx$ when $a g^2 + f^2 c == 0$

1: $\int x^2 (f + g x) \left(a + c x^2\right)^p dx$ when $a g^2 + f^2 c == 0 \land p < -2$

Derivation: Quadratic recurrence 2a

Rule 1.2.1.3.8.2.5.1: If $a g^2 + f^2 c == 0 \land p < -2$, then

$$\int x^2 (f + g x) \left(a + c x^2\right)^p dx \longrightarrow \frac{x^2 (a g - c f x) \left(a + c x^2\right)^{p+1}}{2 a c (p + 1)} - \frac{1}{2 a c (p + 1)} \int x (2 a g - c f (2 p + 5) x) \left(a + c x^2\right)^{p+1} dx$$

Program code:

```
Int[x_^2*(f_+g_.*x_)*(a_+c_.*x_^2)^p_,x_Symbol] :=
  x^2*(a*g-c*f*x)*(a+c*x^2)^(p+1)/(2*a*c*(p+1)) -
  1/(2*a*c*(p+1))*Int[x*Simp[2*a*g-c*f*(2*p+5)*x,x]*(a+c*x^2)^(p+1),x] /;
FreeQ[{a,c,f,g},x] && EqQ[a*g^2+f^2*c,0] && LtQ[p,-2]
```

2: $\int x^2 (f + g x) \left(a + c x^2\right)^p dx$ when $a g^2 + f^2 c == 0$

Derivation: Algebraic expansion

Basis: $x^2 (f + g x) == \frac{(f+g x) (a+c x^2)}{c} - \frac{a (f+g x)}{c}$

Rule 1.2.1.3.8.2.5.2: If $a g^2 + f^2 c == 0$, then

$$\int x^2 (f + g x) \left(a + c x^2\right)^p dx \longrightarrow \frac{1}{c} \int (f + g x) \left(a + c x^2\right)^{p+1} dx - \frac{a}{c} \int (f + g x) \left(a + c x^2\right)^p dx$$

Program code:

```
Int[x_^2*(f_+g_.*x_)*(a_+c_.*x_^2)^p_,x_Symbol] :=
  1/c*Int[(f+g*x)*(a+c*x^2)^(p+1),x] - a/c*Int[(f+g*x)*(a+c*x^2)^p,x] /;
FreeQ[{a,c,f,g,p},x] && EqQ[a*g^2+f^2*c,0]
```

?: $\int (d+e x)^m (f+g x) \left(a+b x+c x^2\right)^p dx$ when $b^2-4 a c \neq 0 \land c f^2-b f g+a g^2 == 0 \land p \in \mathbb{Z}$

- **Derivation: Algebraic simplification**
- **Basis: If** $c f^2-b f g+a g^2 == 0$**, then** $a+b x+c x^2 == (f+g x) \left(\frac{a}{f}+\frac{c x}{g}\right)$
- **Rule 1.2.1.3.8.1.2: If** $b^2-4 a c \neq 0 \land c f^2-b f g+a g^2 == 0 \land p \in \mathbb{Z}$**, then**

$$\int (d+e x)^m (f+g x) \left(a+b x+c x^2\right)^p dx \longrightarrow \int (d+e x)^m (f+g x)^{p+1} \left(\frac{a}{f}+\frac{c x}{g}\right)^p dx$$

- **Program code:**

```
Int[(d_+e_.*x_)^m_*(f_.+g_.*x_)*(a_.+b_.*x_+c_.*x_^2)^p_.,x_Symbol] :=
  Int[(d+e*x)^m*(f+g*x)^(p+1)*(a/f+c/g*x)^p,x] /;
FreeQ[{a,b,c,d,e,f,g,m},x] && NeQ[b^2-4*a*c,0] && EqQ[c*f^2-b*f*g+a*g^2,0] && IntegerQ[p]
```

```
Int[(d_+e_.*x_)^m_*(f_.+g_.*x_)*(a_.+c_.*x_^2)^p_.,x_Symbol] :=
  Int[(d+e*x)^m*(f+g*x)^(p+1)*(a/f+c/g*x)^p,x] /;
FreeQ[{a,c,d,e,f,g,m},x] && EqQ[c*f^2+a*g^2,0] && (IntegerQ[p] || GtQ[a,0] && GtQ[f,0] && EqQ[p,-1])
```

9: $\int \frac{(d+e x)^m (f+g x)}{a+b x+c x^2} dx$ when $b^2-4 a c \neq 0 \land c d^2-b d e+a e^2 \neq 0 \land m \in \mathbb{Z}$

Derivation: Algebraic expansion

- **Rule 1.2.1.3.9: If** $b^2-4 a c \neq 0 \land c d^2-b d e+a e^2 \neq 0 \land m \in \mathbb{Z}$**, then**

$$\int \frac{(d+e x)^m (f+g x)}{a+b x+c x^2} dx \longrightarrow \int \text{ExpandIntegrand}\left[\frac{(d+e x)^m (f+g x)}{a+b x+c x^2}, x\right] dx$$

- **Program code:**

```
Int[(d_.+e_.*x_)^m_*(f_.+g_.*x_)/(a_.+b_.*x_+c_.*x_^2),x_Symbol] :=
  Int[ExpandIntegrand[(d+e*x)^m*(f+g*x)/(a+b*x+c*x^2),x],x] /;
FreeQ[{a,b,c,d,e,f,g},x] && NeQ[b^2-4*a*c,0] && NeQ[c*d^2-b*d*e+a*e^2,0] && IntegerQ[m]
```

```
Int[(d_.+e_.*x_)^m_*(f_.+g_.*x_)/(a_+c_.*x_^2),x_Symbol] :=
  Int[ExpandIntegrand[(d+e*x)^m*(f+g*x)/(a+c*x^2),x],x] /;
FreeQ[{a,c,d,e,f,g},x] && NeQ[c*d^2+a*e^2,0] && IntegerQ[m]
```

10. $\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx$ when $b^2-4\,a\,c\neq 0\,\land\,c\,d^2-b\,d\,e+a\,e^2\neq 0\,\land\,m+2\,p+3==0$

1: $\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx$ when $b^2-4\,a\,c\neq 0\,\land\,c\,d^2-b\,d\,e+a\,e^2\neq 0\,\land\,m+2\,p+3==0\,\land\,b\,(e\,f+d\,g)-2\,(c\,d\,f+a\,e\,g)==0$

Derivation: Quadratic recurrence 3b

Rule 1.2.1.3.10.1: If $b^2-4\,a\,c\neq 0\,\land\,c\,d^2-b\,d\,e+a\,e^2\neq 0\,\land\,m+2\,p+3==0\,\land\,p\neq -1\,\land\,b\,(e\,f+d\,g)-2\,(c\,d\,f+a\,e\,g)==0$**, then**

$$\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx\;\longrightarrow\;-\frac{(e\,f-d\,g)\,(d+e\,x)^{m+1}\,\left(a+b\,x+c\,x^2\right)^{p+1}}{2\,(p+1)\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}$$

Program code:

```
Int[(d_.+e_.*x_)^m_*(f_.+g_.*x_)*(a_.+b_.*x_+c_.*x_^2)^p_.,x_Symbol] :=
  -(e*f-d*g)*(d+e*x)^(m+1)*(a+b*x+c*x^2)^(p+1)/(2*(p+1)*(c*d^2-b*d*e+a*e^2)) /;
FreeQ[{a,b,c,d,e,f,g,m,p},x] && NeQ[b^2-4*a*c,0] && NeQ[c*d^2-b*d*e+a*e^2,0] && EqQ[Simplify[m+2*p+3],0] && EqQ[b*(e*f+d*g)-2*(c
```

```
Int[(d_.+e_.*x_)^m_*(f_.+g_.*x_)*(a_+c_.*x_^2)^p_.,x_Symbol] :=
  -(e*f-d*g)*(d+e*x)^(m+1)*(a+c*x^2)^(p+1)/(2*(p+1)*(c*d^2+a*e^2)) /;
FreeQ[{a,c,d,e,f,g,m,p},x] && NeQ[c*d^2+a*e^2,0] && EqQ[Simplify[m+2*p+3],0] && EqQ[c*d*f+a*e*g,0]
```

2: $\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx$ when $b^2-4\,a\,c\neq 0\,\land\,c\,d^2-b\,d\,e+a\,e^2\neq 0\,\land\,m+2\,p+3==0\,\land\,p<-1$

Derivation: Quadratic recurrence 2a

Rule 1.2.1.3.10.2: If $b^2-4\,a\,c\neq 0\,\land\,c\,d^2-b\,d\,e+a\,e^2\neq 0\,\land\,m+2\,p+3==0\,\land\,p<-1$**, then**

$$\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx\;\longrightarrow$$

$$\frac{(d+e\,x)^m\,\left(a+b\,x+c\,x^2\right)^{p+1}\,(b\,f-2\,a\,g+(2\,c\,f-b\,g)\,x)}{(p+1)\,\left(b^2-4\,a\,c\right)}+\frac{m\,(b\,(e\,f+d\,g)-2\,(c\,d\,f+a\,e\,g))}{(p+1)\,\left(b^2-4\,a\,c\right)}\int (d+e\,x)^{m-1}\,\left(a+b\,x+c\,x^2\right)^{p+1}\,dx$$

Program code:

```
Int[(d_.+e_.*x_)^m_*(f_.+g_.*x_)*(a_.+b_.*x_+c_.*x_^2)^p_,x_Symbol] :=
  (d+e*x)^m*(a+b*x+c*x^2)^(p+1)*(b*f-2*a*g+(2*c*f-b*g)*x)/((p+1)*(b^2-4*a*c)) -
  m*(b*(e*f+d*g)-2*(c*d*f+a*e*g))/((p+1)*(b^2-4*a*c))*Int[(d+e*x)^(m-1)*(a+b*x+c*x^2)^(p+1),x] /;
FreeQ[{a,b,c,d,e,f,g},x] && NeQ[b^2-4*a*c,0] && NeQ[c*d^2-b*d*e+a*e^2,0] && EqQ[Simplify[m+2*p+3],0] && LtQ[p,-1]
```

```
Int[(d_.+e_.*x_)^m_*(f_.+g_.*x_)*(a_+c_.*x_^2)^p_,x_Symbol] :=
  (d+e*x)^m*(a+c*x^2)^(p+1)*(a*g-c*f*x)/(2*a*c*(p+1)) -
  m*(c*d*f+a*e*g)/(2*a*c*(p+1))*Int[(d+e*x)^(m-1)*(a+c*x^2)^(p+1),x] /;
FreeQ[{a,c,d,e,f,g},x] && NeQ[c*d^2+a*e^2,0] && EqQ[Simplify[m+2*p+3],0] && LtQ[p,-1]
```

3: $\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx$ when $b^2-4\,a\,c\neq 0\,\wedge\,c\,d^2-b\,d\,e+a\,e^2\neq 0\,\wedge\,m+2\,p+3==0\,\wedge\,p\nleq -1$

Derivation: Quadratic recurrence 3b

Rule 1.2.1.3.10.3: If $b^2-4\,a\,c\neq 0\,\wedge\,c\,d^2-b\,d\,e+a\,e^2\neq 0\,\wedge\,m+2\,p+3==0\,\wedge\,p\nleq -1$**, then**

$$\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx\,\longrightarrow$$

$$-\frac{(e\,f-d\,g)\,(d+e\,x)^{m+1}\,\left(a+b\,x+c\,x^2\right)^{p+1}}{2\,(p+1)\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}-\frac{b\,(e\,f+d\,g)-2\,(c\,d\,f+a\,e\,g)}{2\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}\int (d+e\,x)^{m+1}\,\left(a+b\,x+c\,x^2\right)^p\,dx$$

Program code:

```
Int[(d_.+e_.*x_)^m_*(f_.+g_.*x_)*(a_.+b_.*x_+c_.*x_^2)^p_.,x_Symbol] :=
  -(e*f-d*g)*(d+e*x)^(m+1)*(a+b*x+c*x^2)^(p+1)/(2*(p+1)*(c*d^2-b*d*e+a*e^2)) -
  (b*(e*f+d*g)-2*(c*d*f+a*e*g))/(2*(c*d^2-b*d*e+a*e^2))*Int[(d+e*x)^(m+1)*(a+b*x+c*x^2)^p,x] /;
FreeQ[{a,b,c,d,e,f,g,m,p},x] && NeQ[b^2-4*a*c,0] && NeQ[c*d^2-b*d*e+a*e^2,0] && EqQ[Simplify[m+2*p+3],0]
```

```
Int[(d_.+e_.*x_)^m_*(f_.+g_.*x_)*(a_+c_.*x_^2)^p_.,x_Symbol] :=
  -(e*f-d*g)*(d+e*x)^(m+1)*(a+c*x^2)^(p+1)/(2*(p+1)*(c*d^2+a*e^2)) +
  (c*d*f+a*e*g)/(c*d^2+a*e^2)*Int[(d+e*x)^(m+1)*(a+c*x^2)^p,x] /;
FreeQ[{a,c,d,e,f,g,m,p},x] && NeQ[c*d^2+a*e^2,0] && EqQ[Simplify[m+2*p+3],0]
```

11: $\int (e\,x)^m\,(f+g\,x)\,\left(a+c\,x^2\right)^p\,dx$ when $m\notin\mathbb{Q}\,\wedge\,p\notin\mathbb{Z}^+$

Derivation: Algebraic expansion

Rule 1.2.1.3.11: If $m\notin\mathbb{Q}\,\wedge\,p\notin\mathbb{Z}^+$, then

$$\int (e\,x)^m\,(f+g\,x)\,\left(a+c\,x^2\right)^p\,dx\;\longrightarrow\;f\int (e\,x)^m\,\left(a+c\,x^2\right)^p\,dx+\frac{g}{e}\int (e\,x)^{m+1}\,\left(a+c\,x^2\right)^p\,dx$$

Program code:

```
Int[(e_.*x_)^m_*(f_+g_.*x_)*(a_+c_.*x_^2)^p_,x_Symbol] :=
  f*Int[(e*x)^m*(a+c*x^2)^p,x] + g/e*Int[(e*x)^(m+1)*(a+c*x^2)^p,x] /;
FreeQ[{a,c,e,f,g,p},x] && Not[RationalQ[m]] && Not[IGtQ[p,0]]
```

12: $\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx$ when $b^2-4\,a\,c\neq 0\,\wedge\,c\,d^2-b\,d\,e+a\,e^2\neq 0\,\wedge\,m==p\,\wedge\,b\,d+a\,e==0\,\wedge\,c\,d+b\,e==0$

Derivation: Piecewise constant extraction

- **Basis: If $b\,d+a\,e==0\,\wedge\,c\,d+b\,e==0$, then $\partial_x\frac{(d+e\,x)^p\,(a+b\,x+c\,x^2)^p}{(a\,d+c\,e\,x^3)^p}==0$**

Rule 1.2.1.3.12: If $m==p\,\wedge\,b\,d+a\,e==0\,\wedge\,c\,d+b\,e==0$, then

$$\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx\;\longrightarrow\;\frac{(d+e\,x)^{\text{FracPart}[p]}\,\left(a+b\,x+c\,x^2\right)^{\text{FracPart}[p]}}{\left(a\,d+c\,e\,x^3\right)^{\text{FracPart}[p]}}\int (f+g\,x)\,\left(a\,d+c\,e\,x^3\right)^p\,dx$$

Program code:

```
Int[(d_.+e_.*x_)^m_*(f_.+g_.*x_)*(a_+b_.*x_+c_.*x_^2)^p_,x_Symbol] :=
  (d+e*x)^FracPart[p]*(a+b*x+c*x^2)^FracPart[p]/(a*d+c*e*x^3)^FracPart[p]*Int[(f+g*x)*(a*d+c*e*x^3)^p,x] /;
FreeQ[{a,b,c,d,e,f,g,m,p},x] && EqQ[m,p] && EqQ[b*d+a*e,0] && EqQ[c*d+b*e,0]
```

13. $\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx$ when $b^2-4\,a\,c\neq 0\,\wedge\,c\,d^2-b\,d\,e+a\,e^2\neq 0\,\wedge\,p>0$

1: $\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx$ when $b^2-4\,a\,c\neq 0\,\wedge\,c\,d^2-b\,d\,e+a\,e^2\neq 0\,\wedge\,p>0\,\wedge\,m<-2$

Derivation: ???

Rule 1.2.1.3.13.1: If $b^2-4\,a\,c\neq 0\,\wedge\,c\,d^2-b\,d\,e+a\,e^2\neq 0\,\wedge\,p>0\,\wedge\,m<-2$, then

$$\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx\;\longrightarrow$$

$$-\frac{(d+e\,x)^{m+1}\,\left(a+b\,x+c\,x^2\right)^p}{e^2\,(m+1)\,(m+2)\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}\cdot$$

$$\left((d\,g-e\,f\,(m+2))\,\left(c\,d^2-b\,d\,e+a\,e^2\right)-d\,p\,(2\,c\,d-b\,e)\,(e\,f-d\,g)-e\,\left(g\,(m+1)\,\left(c\,d^2-b\,d\,e+a\,e^2\right)+p\,(2\,c\,d-b\,e)\,(e\,f-d\,g)\right)\,x\right)-$$

$$\frac{p}{e^2\,(m+1)\,(m+2)\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}\int (d+e\,x)^{m+2}\,\left(a+b\,x+c\,x^2\right)^{p-1}\cdot$$

$$\left(2\,a\,c\,e\,(e\,f-d\,g)\,(m+2)+b^2\,e\,(d\,g\,(p+1)-e\,f\,(m+p+2))+b\,\left(a\,e^2\,g\,(m+1)-c\,d\,(d\,g\,(2\,p+1)-e\,f\,(m+2\,p+2))\right)-\right.$$
$$\left.c\,(2\,c\,d\,(d\,g\,(2\,p+1)-e\,f\,(m+2\,p+2))-e\,(2\,a\,e\,g\,(m+1)-b\,(d\,g\,(m-2\,p)+e\,f\,(m+2\,p+2))))\,x\right)\,dx$$

Program code:

```
Int[(d_.+e_.*x_)^m_*(f_.+g_.*x_)*(a_.+b_.*x_+c_.*x_^2)^p_.,x_Symbol] :=
  -(d+e*x)^(m+1)*(a+b*x+c*x^2)^p/(e^2*(m+1)*(m+2)*(c*d^2-b*d*e+a*e^2))*
    ((d*g-e*f*(m+2))*(c*d^2-b*d*e+a*e^2)-d*p*(2*c*d-b*e)*(e*f-d*g)-e*(g*(m+1)*(c*d^2-b*d*e+a*e^2)+p*(2*c*d-b*e)*(e*f-d*g))*x) -
  p/(e^2*(m+1)*(m+2)*(c*d^2-b*d*e+a*e^2))*Int[(d+e*x)^(m+2)*(a+b*x+c*x^2)^(p-1)*
    Simp[2*a*c*e*(e*f-d*g)*(m+2)+b^2*e*(d*g*(p+1)-e*f*(m+p+2))+b*(a*e^2*g*(m+1)-c*d*(d*g*(2*p+1)-e*f*(m+2*p+2)))-
      c*(2*c*d*(d*g*(2*p+1)-e*f*(m+2*p+2))-e*(2*a*e*g*(m+1)-b*(d*g*(m-2*p)+e*f*(m+2*p+2))))*x,x],x] /;
FreeQ[{a,b,c,d,e,f,g},x] && NeQ[b^2-4*a*c,0] && NeQ[c*d^2-b*d*e+a*e^2,0] &&
  GtQ[p,0] && LtQ[m,-2] && LtQ[m+2*p,0] && Not[ILtQ[m+2*p+3,0]]
```

```
Int[(d_.+e_.*x_)^m_*(f_.+g_.*x_)*(a_+c_.*x_^2)^p_.,x_Symbol] :=
  -(d+e*x)^(m+1)*(a+c*x^2)^p/(e^2*(m+1)*(m+2)*(c*d^2+a*e^2))*
    ((d*g-e*f*(m+2))*(c*d^2+a*e^2)-2*c*d^2*p*(e*f-d*g)-e*(g*(m+1)*(c*d^2+a*e^2)+2*c*d*p*(e*f-d*g))*x) -
  p/(e^2*(m+1)*(m+2)*(c*d^2+a*e^2))*Int[(d+e*x)^(m+2)*(a+c*x^2)^(p-1)*
    Simp[2*a*c*e*(e*f-d*g)*(m+2)-c*(2*c*d*(d*g*(2*p+1)-e*f*(m+2*p+2))-2*a*e^2*g*(m+1))*x,x],x] /;
FreeQ[{a,c,d,e,f,g},x] && NeQ[c*d^2+a*e^2,0] &&
  GtQ[p,0] && LtQ[m,-2] && LtQ[m+2*p,0] && Not[ILtQ[m+2*p+3,0]]
```

2: $\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx$ when $b^2-4\,a\,c\neq 0\;\land\;c\,d^2-b\,d\,e+a\,e^2\neq 0\;\land\;p>0\;\land\;m<-1\;\land\;m+2\,p+1\notin\mathbb{Z}^-$

Derivation: Quadratic recurrence 1a

Rule 1.2.1.3.13.2: If $b^2-4\,a\,c\neq 0\;\land\;c\,d^2-b\,d\,e+a\,e^2\neq 0\;\land\;p>0\;\land\;m<-1\;\land\;m+2\,p+1\notin\mathbb{Z}^-$**, then**

$$\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx\;\longrightarrow$$

$$\left((d+e\,x)^{m+1}\,(f\,e\,(m+2\,p+2)-g\,d\,(2\,p+1)+e\,g\,(m+1)\,x)\,\left(a+b\,x+c\,x^2\right)^p\right)\Big/\left(e^2\,(m+1)\,(m+2\,p+2)\right)+$$
$$\frac{p}{e^2\,(m+1)\,(m+2\,p+2)}\int (d+e\,x)^{m+1}\,\left(a+b\,x+c\,x^2\right)^{p-1}\cdot$$

$$(g\,(b\,d+2\,a\,e+2\,a\,e\,m+2\,b\,d\,p)-f\,b\,e\,(m+2\,p+2)+(g\,(2\,c\,d+b\,e+b\,e\,m+4\,c\,d\,p)-2\,c\,e\,f\,(m+2\,p+2))\,x)\,dx$$

- Program code:

```
Int[(d_.+e_.*x_)^m_*(f_.+g_.*x_)*(a_.+b_.*x_+c_.*x_^2)^p_.,x_Symbol] :=
  (d+e*x)^(m+1)*(e*f*(m+2*p+2)-d*g*(2*p+1)+e*g*(m+1)*x)*(a+b*x+c*x^2)^p/(e^2*(m+1)*(m+2*p+2)) +
  p/(e^2*(m+1)*(m+2*p+2))*Int[(d+e*x)^(m+1)*(a+b*x+c*x^2)^(p-1)*
    Simp[g*(b*d+2*a*e+2*a*e*m+2*b*d*p)-f*b*e*(m+2*p+2)+(g*(2*c*d+b*e+b*e*m+4*c*d*p)-2*c*e*f*(m+2*p+2))*x,x],x] /;
FreeQ[{a,b,c,d,e,f,g,m},x] && NeQ[b^2-4*a*c,0] && NeQ[c*d^2-b*d*e+a*e^2,0] && RationalQ[p] && p>0 &&
  (LtQ[m,-1] || EqQ[p,1] || IntegerQ[p] && Not[RationalQ[m]]) && NeQ[m,-1] && Not[ILtQ[m+2*p+1,0]] &&
  (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m,2*p])
```

```
Int[(d_.+e_.*x_)^m_*(f_.+g_.*x_)*(a_+c_.*x_^2)^p_.,x_Symbol] :=
  (d+e*x)^(m+1)*(e*f*(m+2*p+2)-d*g*(2*p+1)+e*g*(m+1)*x)*(a+c*x^2)^p/(e^2*(m+1)*(m+2*p+2)) +
  p/(e^2*(m+1)*(m+2*p+2))*Int[(d+e*x)^(m+1)*(a+c*x^2)^(p-1)*
    Simp[g*(2*a*e+2*a*e*m)+(g*(2*c*d+4*c*d*p)-2*c*e*f*(m+2*p+2))*x,x],x] /;
FreeQ[{a,c,d,e,f,g,m},x] && NeQ[c*d^2+a*e^2,0] && RationalQ[p] && p>0 &&
  (LtQ[m,-1] || EqQ[p,1] || IntegerQ[p] && Not[RationalQ[m]]) && NeQ[m,-1] && Not[ILtQ[m+2*p+1,0]] &&
  (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m,2*p])
```

3: $\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx$ when $b^2-4\,a\,c\neq 0\,\wedge\,c\,d^2-b\,d\,e+a\,e^2\neq 0\,\wedge\,p>0\,\wedge\,-1\leq m<0\,\wedge\,m+2\,p\notin\mathbb{Z}^-$

Derivation: Quadratic recurrence 1b

Rule 1.2.1.3.13.3: If $b^2-4\,a\,c\neq 0\,\wedge\,c\,d^2-b\,d\,e+a\,e^2\neq 0\,\wedge\,p>0\,\wedge\,-1\leq m<0\,\wedge\,m+2\,p\notin\mathbb{Z}^-$**, then**

$$\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx\,\longrightarrow$$

$$\left((d+e\,x)^{m+1}\,(c\,e\,f\,(m+2\,p+2)-g\,(c\,d+2\,c\,d\,p-b\,e\,p)+g\,c\,e\,(m+2\,p+1)\,x)\,\left(a+b\,x+c\,x^2\right)^p\right)\Big/\left(c\,e^2\,(m+2\,p+1)\,(m+2\,p+2)\right)-$$
$$\frac{p}{c\,e^2\,(m+2\,p+1)\,(m+2\,p+2)}\int (d+e\,x)^m\,\left(a+b\,x+c\,x^2\right)^{p-1}\,\cdot$$
$$\left(c\,e\,f\,(b\,d-2\,a\,e)\,(m+2\,p+2)+g\,(a\,e\,(b\,e-2\,c\,d\,m+b\,e\,m)+b\,d\,(b\,e\,p-c\,d-2\,c\,d\,p))\,+\right.$$
$$\left.\left(c\,e\,f\,(2\,c\,d-b\,e)\,(m+2\,p+2)+g\,\left(b^2\,e^2\,(p+m+1)-2\,c^2\,d^2\,(1+2\,p)-c\,e\,(b\,d\,(m-2\,p)+2\,a\,e\,(m+2\,p+1))\right)\right)x\right)dx$$

Program code:

```
Int[(d_.+e_.*x_)^m_*(f_.+g_.*x_)*(a_.+b_.*x_+c_.*x_^2)^p_.,x_Symbol] :=
  (d+e*x)^(m+1)*(c*e*f*(m+2*p+2)-g*(c*d+2*c*d*p-b*e*p)+g*c*e*(m+2*p+1)*x)*(a+b*x+c*x^2)^p/
    (c*e^2*(m+2*p+1)*(m+2*p+2)) -
  p/(c*e^2*(m+2*p+1)*(m+2*p+2))*Int[(d+e*x)^m*(a+b*x+c*x^2)^(p-1)*
    Simp[c*e*f*(b*d-2*a*e)*(m+2*p+2)+g*(a*e*(b*e-2*c*d*m+b*e*m)+b*d*(b*e*p-c*d-2*c*d*p))+
      (c*e*f*(2*c*d-b*e)*(m+2*p+2)+g*(b^2*e^2*(p+m+1)-2*c^2*d^2*(1+2*p)-c*e*(b*d*(m-2*p)+2*a*e*(m+2*p+1))))*x,x],x] /;
FreeQ[{a,b,c,d,e,f,g,m},x] && NeQ[b^2-4*a*c,0] && NeQ[c*d^2-b*d*e+a*e^2,0] &&
  GtQ[p,0] && (IntegerQ[p] || Not[RationalQ[m]] || GeQ[m,-1] && LtQ[m,0]) && Not[ILtQ[m+2*p,0]] &&
  (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m,2*p])
```

```
Int[(d_.+e_.*x_)^m_*(f_.+g_.*x_)*(a_+c_.*x_^2)^p_.,x_Symbol] :=
  (d+e*x)^(m+1)*(c*e*f*(m+2*p+2)-g*c*d*(2*p+1)+g*c*e*(m+2*p+1)*x)*(a+c*x^2)^p/
    (c*e^2*(m+2*p+1)*(m+2*p+2)) +
  2*p/(c*e^2*(m+2*p+1)*(m+2*p+2))*Int[(d+e*x)^m*(a+c*x^2)^(p-1)*
    Simp[f*a*c*e^2*(m+2*p+2)+a*c*d*e*g*m-(c^2*f*d*e*(m+2*p+2)-g*(c^2*d^2*(2*p+1)+a*c*e^2*(m+2*p+1)))*x,x],x] /;
FreeQ[{a,c,d,e,f,g,m},x] && NeQ[c*d^2+a*e^2,0] &&
  GtQ[p,0] && (IntegerQ[p] || Not[RationalQ[m]] || GeQ[m,-1] && LtQ[m,0]) && Not[ILtQ[m+2*p,0]] &&
  (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m,2*p])
```

14. $\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx$ when $b^2-4\,a\,c\neq0\,\wedge\,c\,d^2-b\,d\,e+a\,e^2\neq0\,\wedge\,p<-1$

1. $\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx$ when $b^2-4\,a\,c\neq0\,\wedge\,c\,d^2-b\,d\,e+a\,e^2\neq0\,\wedge\,p<-1\,\wedge\,m>1$

1: $\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx$ when $b^2-4\,a\,c\neq0\,\wedge\,c\,d^2-b\,d\,e+a\,e^2\neq0\,\wedge\,p<-1\,\wedge\,m\in\mathbb{Z}^+$

Derivation: Algebraic expansion

Rule 1.2.1.3.14.1.1: If $b^2-4\,a\,c\neq0\,\wedge\,c\,d^2-b\,d\,e+a\,e^2\neq0\,\wedge\,p<-1\,\wedge\,m\in\mathbb{Z}^+$**, then**

$$\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx\,\longrightarrow$$

$$\int \left(a+b\,x+c\,x^2\right)^p\,\text{ExpandIntegrand}\left[(d+e\,x)^m\,(f+g\,x),\,x\right]\,dx$$

Program code:

```
Int[(d_+e_.*x_)^m_*(f_+g_.*x_)*(a_.+b_.*x_+c_.*x_^2)^p_,x_Symbol] :=
  Int[(a+b*x+c*x^2)^p*ExpandIntegrand[(d+e*x)^m*(f+g*x),x],x] /;
FreeQ[{a,b,c,d,e,f,g},x] && NeQ[b^2-4*a*c,0] && NeQ[c*d^2-b*d*e+a*e^2,0] && ILtQ[p,-1] && IGtQ[m,0] && RationalQ[a,b,c,d,e,f,g]
```

```
Int[(d_+e_.*x_)^m_*(f_+g_.*x_)*(a_+c_.*x_^2)^p_,x_Symbol] :=
  Int[(a+c*x^2)^p*ExpandIntegrand[(d+e*x)^m*(f+g*x),x],x] /;
FreeQ[{a,c,d,e,f,g},x] && NeQ[c*d^2+a*e^2,0] && ILtQ[p,-1] && IGtQ[m,0] && RationalQ[a,c,d,e,f,g]
```

2: $\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx$ when $b^2-4\,a\,c\neq0\,\wedge\,c\,d^2-b\,d\,e+a\,e^2\neq0\,\wedge\,p<-1\,\wedge\,m>1$

Derivation: ???

Note: Although powerful, this rule results in more complicated coefficients unless $b=0\,\wedge\,d=0$ **or the parameters are all numeric.**

Rule 1.2.1.3.14.1.2: If $b^2-4\,a\,c\neq0\,\wedge\,c\,d^2-b\,d\,e+a\,e^2\neq0\,\wedge\,p<-1\,\wedge\,m>1$**, then**

$$\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx\,\longrightarrow$$

$$-\left((d+e\,x)^{m-1}\,\left(a+b\,x+c\,x^2\right)^{p+1}\,\left(2\,a\,c\,(e\,f+d\,g)-b\,(c\,d\,f+a\,e\,g)-\left(2\,c^2\,d\,f+b^2\,e\,g-c\,(b\,e\,f+b\,d\,g+2\,a\,e\,g)\right)\,x\right)\right)\Big/\left(c\,(p+1)\,\left(b^2-4\,a\,c\right)\right)-$$
$$\frac{1}{c\,(p+1)\,\left(b^2-4\,a\,c\right)}\int (d+e\,x)^{m-2}\,\left(a+b\,x+c\,x^2\right)^{p+1}\cdot$$
$$\left(2\,c^2\,d^2\,f\,(2\,p+3)+b\,e\,g\,(a\,e\,(m-1)+b\,d\,(p+2))-c\,(2\,a\,e\,(e\,f\,(m-1)+d\,g\,m)+b\,d\,(d\,g\,(2\,p+3)-e\,f\,(m-2\,p-4)))+\right.$$

$$e\left(b^2 e g (m+p+1)+2 c^2 d f (m+2 p+2)-c (2 a e g m+b (e f+d g) (m+2 p+2))\right) x) \, dx$$

Program code:

```
Int[(d_.+e_.*x_)^m_*(f_.+g_.*x_)*(a_.+b_.*x_+c_.*x_^2)^p_.,x_Symbol] :=
  -(d+e*x)^(m-1)*(a+b*x+c*x^2)^(p+1)*(2*a*c*(e*f+d*g)-b*(c*d*f+a*e*g)-(2*c^2*d*f+b^2*e*g-c*(b*e*f+b*d*g+2*a*e*g))*x)/
    (c*(p+1)*(b^2-4*a*c)) -
  1/(c*(p+1)*(b^2-4*a*c))*Int[(d+e*x)^(m-2)*(a+b*x+c*x^2)^(p+1)*
    Simp[2*c^2*d^2*f*(2*p+3)+b*e*g*(a*e*(m-1)+b*d*(p+2))-c*(2*a*e*(e*f*(m-1)+d*g*m)+b*d*(d*g*(2*p+3)-e*f*(m-2*p-4))) +
      e*(b^2*e*g*(m+p+1)+2*c^2*d*f*(m+2*p+2)-c*(2*a*e*g*m+b*(e*f+d*g)*(m+2*p+2)))*x,x],x] /;
FreeQ[{a,b,c,d,e,f,g},x] && NeQ[b^2-4*a*c,0] && NeQ[c*d^2-b*d*e+a*e^2,0] && LtQ[p,-1] && GtQ[m,1] &&
  (EqQ[m,2] && EqQ[p,-3] && RationalQ[a,b,c,d,e,f,g] || Not[ILtQ[m+2*p+3,0]])
```

```
Int[(d_.+e_.*x_)^m_*(f_.+g_.*x_)*(a_+c_.*x_^2)^p_.,x_Symbol] :=
  (d+e*x)^(m-1)*(a+c*x^2)^(p+1)*(a*(e*f+d*g)-(c*d*f-a*e*g)*x)/(2*a*c*(p+1)) -
  1/(2*a*c*(p+1))*Int[(d+e*x)^(m-2)*(a+c*x^2)^(p+1)*
    Simp[a*e*(e*f*(m-1)+d*g*m)-c*d^2*f*(2*p+3)+e*(a*e*g*m-c*d*f*(m+2*p+2))*x,x],x] /;
FreeQ[{a,c,d,e,f,g},x] && NeQ[c*d^2+a*e^2,0] && LtQ[p,-1] && GtQ[m,1] &&
  (EqQ[d,0] || EqQ[m,2] && EqQ[p,-3] && RationalQ[a,c,d,e,f,g] || Not[ILtQ[m+2*p+3,0]])
```

2: $\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx$ when $b^2-4\,a\,c\neq 0\,\wedge\,c\,d^2-b\,d\,e+a\,e^2\neq 0\,\wedge\,p<-1\,\wedge\,m>0$

Derivation: Quadratic recurrence 2a

Rule 1.2.1.3.14.2: If $b^2-4\,a\,c\neq 0\,\wedge\,c\,d^2-b\,d\,e+a\,e^2\neq 0\,\wedge\,p<-1\,\wedge\,m>0$**, then**

$$\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx\,\longrightarrow$$

$$\frac{(d+e\,x)^m\,\left(a+b\,x+c\,x^2\right)^{p+1}\,(f\,b-2\,a\,g+(2\,c\,f-b\,g)\,x)}{(p+1)\,\left(b^2-4\,a\,c\right)}+$$

$$\frac{1}{(p+1)\,\left(b^2-4\,a\,c\right)}\int (d+e\,x)^{m-1}\,\left(a+b\,x+c\,x^2\right)^{p+1}\,\cdot$$

$$(g\,(2\,a\,e\,m+b\,d\,(2\,p+3))-f\,(b\,e\,m+2\,c\,d\,(2\,p+3))-e\,(2\,c\,f-b\,g)\,(m+2\,p+3)\,x)\,dx$$

Program code:

```
Int[(d_.+e_.*x_)^m_*(f_.+g_.*x_)*(a_.+b_.*x_+c_.*x_^2)^p_,x_Symbol] :=
  (d+e*x)^m*(a+b*x+c*x^2)^(p+1)*(f*b-2*a*g+(2*c*f-b*g)*x)/((p+1)*(b^2-4*a*c)) +
  1/((p+1)*(b^2-4*a*c))*Int[(d+e*x)^(m-1)*(a+b*x+c*x^2)^(p+1)*
    Simp[g*(2*a*e*m+b*d*(2*p+3))-f*(b*e*m+2*c*d*(2*p+3))-e*(2*c*f-b*g)*(m+2*p+3)*x,x],x] /;
FreeQ[{a,b,c,d,e,f,g},x] && NeQ[b^2-4*a*c,0] && NeQ[c*d^2-b*d*e+a*e^2,0] && LtQ[p,-1] && GtQ[m,0] &&
  (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m,2*p])
```

```
Int[(d_.+e_.*x_)^m_*(f_.+g_.*x_)*(a_+c_.*x_^2)^p_,x_Symbol] :=
  (d+e*x)^m*(a+c*x^2)^(p+1)*(a*g-c*f*x)/(2*a*c*(p+1)) -
  1/(2*a*c*(p+1))*Int[(d+e*x)^(m-1)*(a+c*x^2)^(p+1)*Simp[a*e*g*m-c*d*f*(2*p+3)-c*e*f*(m+2*p+3)*x,x],x] /;
FreeQ[{a,c,d,e,f,g},x] && NeQ[c*d^2+a*e^2,0] && LtQ[p,-1] && GtQ[m,0] &&
  (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m,2*p])
```

3: $\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx$ when $b^2-4\,a\,c\neq 0\,\wedge\,c\,d^2-b\,d\,e+a\,e^2\neq 0\,\wedge\,p<-1$

Derivation: Quadratic recurrence 2b

Rule 1.2.1.3.14.3: If $b^2-4\,a\,c\neq 0\,\wedge\,c\,d^2-b\,d\,e+a\,e^2\neq 0\,\wedge\,p<-1$**, then**

$$\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx\,\longrightarrow$$

$$\left((d+e\,x)^{m+1}\,\left(f\,\left(b\,c\,d-b^2\,e+2\,a\,c\,e\right)-a\,g\,(2\,c\,d-b\,e)+c\,(f\,(2\,c\,d-b\,e)-g\,(b\,d-2\,a\,e))\,x\right)\,\left(a+b\,x+c\,x^2\right)^{p+1}\right)\Big/$$
$$\left((p+1)\,\left(b^2-4\,a\,c\right)\,\left(c\,d^2-b\,d\,e+a\,e^2\right)\right)+$$

$$\frac{1}{(p+1)\left(b^2-4ac\right)\left(c d^2-b d e+a e^2\right)}\int(d+e x)^m\left(a+b x+c x^2\right)^{p+1}\cdot$$
$$\left(f\left(b c d e\,(2p-m+2)+b^2 e^2\,(p+m+2)-2c^2 d^2\,(2p+3)-2 a c e^2\,(m+2p+3)\right)-g\,(a e\,(b e-2 c d m+b e m)-b d\,(3 c d-b e+2 c d p-b e p))+\right.$$
$$\left.c e\,(g\,(b d-2 a e)-f\,(2 c d-b e))\,(m+2p+4)\,x\right)\,dx$$

–

Program code:

```
Int[(d_.+e_.*x_)^m_*(f_.+g_.*x_)*(a_.+b_.*x_+c_.*x_^2)^p_,x_Symbol] :=
  (d+e*x)^(m+1)*(f*(b*c*d-b^2*e+2*a*c*e)-a*g*(2*c*d-b*e)+c*(f*(2*c*d-b*e)-g*(b*d-2*a*e))*x)*(a+b*x+c*x^2)^(p+1)/
    ((p+1)*(b^2-4*a*c)*(c*d^2-b*d*e+a*e^2)) +
  1/((p+1)*(b^2-4*a*c)*(c*d^2-b*d*e+a*e^2))*Int[(d+e*x)^m*(a+b*x+c*x^2)^(p+1)*
    Simp[f*(b*c*d*e*(2*p-m+2)+b^2*e^2*(p+m+2)-2*c^2*d^2*(2*p+3)-2*a*c*e^2*(m+2*p+3))-
      g*(a*e*(b*e-2*c*d*m+b*e*m)-b*d*(3*c*d-b*e+2*c*d*p-b*e*p))+
      c*e*(g*(b*d-2*a*e)-f*(2*c*d-b*e))*(m+2*p+4)*x,x],x] /;
FreeQ[{a,b,c,d,e,f,g,m},x] && NeQ[b^2-4*a*c,0] && NeQ[c*d^2-b*d*e+a*e^2,0] && LtQ[p,-1] && (IntegerQ[m] || IntegerQ[p] || Intege
```

```
Int[(d_.+e_.*x_)^m_*(f_.+g_.*x_)*(a_+c_.*x_^2)^p_,x_Symbol] :=
  -(d+e*x)^(m+1)*(f*a*c*e-a*g*c*d+c*(c*d*f+a*e*g)*x)*(a+c*x^2)^(p+1)/(2*a*c*(p+1)*(c*d^2+a*e^2)) +
  1/(2*a*c*(p+1)*(c*d^2+a*e^2))*Int[(d+e*x)^m*(a+c*x^2)^(p+1)*
    Simp[f*(c^2*d^2*(2*p+3)+a*c*e^2*(m+2*p+3))-a*c*d*e*g*m+c*e*(c*d*f+a*e*g)*(m+2*p+4)*x,x],x] /;
FreeQ[{a,c,d,e,f,g},x] && NeQ[c*d^2+a*e^2,0] && LtQ[p,-1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m,2*p])
```

15. $\int \frac{(d+e\,x)^m\,(f+g\,x)}{a+b\,x+c\,x^2}\,dx$ when $b^2-4\,a\,c\neq 0\,\wedge\,c\,d^2-b\,d\,e+a\,e^2\neq 0\,\wedge\,m\notin\mathbb{Z}$

1. $\int \frac{(d+e\,x)^m\,(f+g\,x)}{a+b\,x+c\,x^2}\,dx$ when $b^2-4\,a\,c\neq 0\,\wedge\,c\,d^2-b\,d\,e+a\,e^2\neq 0\,\wedge\,m\in\mathbb{Q}$

1: $\int \frac{(d+e\,x)^m\,(f+g\,x)}{a+b\,x+c\,x^2}\,dx$ when $b^2-4\,a\,c\neq 0\,\wedge\,c\,d^2-b\,d\,e+a\,e^2\neq 0\,\wedge\,m\notin\mathbb{Z}\,\wedge\,m>0$

Derivation: Quadratic recurrence 3a with $p=-1$

Rule 1.2.1.3.15.1.1: If $b^2-4\,a\,c\neq 0\,\wedge\,c\,d^2-b\,d\,e+a\,e^2\neq 0\,\wedge\,m\notin\mathbb{Z}\,\wedge\,m>0$**, then**

$$\int \frac{(d+e\,x)^m\,(f+g\,x)}{a+b\,x+c\,x^2}\,dx\;\longrightarrow\;\frac{g\,(d+e\,x)^m}{c\,m}+\frac{1}{c}\int \frac{(d+e\,x)^{m-1}\,(c\,d\,f-a\,e\,g+(g\,c\,d-b\,e\,g+c\,e\,f)\,x)}{a+b\,x+c\,x^2}\,dx$$

Program code:

```
Int[(d_.+e_.*x_)^m_*(f_.+g_.*x_)/(a_.+b_.*x_+c_.*x_^2),x_Symbol] :=
  g*(d+e*x)^m/(c*m) +
  1/c*Int[(d+e*x)^(m-1)*Simp[c*d*f-a*e*g+(g*c*d-b*e*g+c*e*f)*x,x]/(a+b*x+c*x^2),x] /;
FreeQ[{a,b,c,d,e,f,g},x] && NeQ[b^2-4*a*c,0] && NeQ[c*d^2-b*d*e+a*e^2,0] && FractionQ[m] && GtQ[m,0]
```

```
Int[(d_.+e_.*x_)^m_*(f_.+g_.*x_)/(a_+c_.*x_^2),x_Symbol] :=
  g*(d+e*x)^m/(c*m) +
  1/c*Int[(d+e*x)^(m-1)*Simp[c*d*f-a*e*g+(g*c*d+c*e*f)*x,x]/(a+c*x^2),x] /;
FreeQ[{a,c,d,e,f,g},x] && NeQ[c*d^2+a*e^2,0] && FractionQ[m] && GtQ[m,0]
```

2. $\int \frac{(d+e\,x)^m\,(f+g\,x)}{a+b\,x+c\,x^2}\,dx$ when $b^2-4\,a\,c\neq 0\ \land\ c\,d^2-b\,d\,e+a\,e^2\neq 0\ \land\ m\notin\mathbb{Z}\ \land\ m<0$

1: $\int \frac{f+g\,x}{\sqrt{d+e\,x}\,\left(a+b\,x+c\,x^2\right)}\,dx$ when $b^2-4\,a\,c\neq 0\ \land\ c\,d^2-b\,d\,e+a\,e^2\neq 0$

Derivation: Integration by substitution

- **Basis:** $\frac{f+g\,x}{\sqrt{d+e\,x}\,\left(a+b\,x+c\,x^2\right)} == 2\,\text{Subst}\left[\frac{e\,f-d\,g+g\,x^2}{c\,d^2-b\,d\,e+a\,e^2-(2\,c\,d-b\,e)\,x^2+c\,x^4},\ x,\ \sqrt{d+e\,x}\right]\partial_x\sqrt{d+e\,x}$

- **Rule 1.2.1.3.15.1.2.1: If** $b^2-4\,a\,c\neq 0\ \land\ c\,d^2-b\,d\,e+a\,e^2\neq 0$**, then**

$$\int \frac{f+g\,x}{\sqrt{d+e\,x}\,\left(a+b\,x+c\,x^2\right)}\,dx\ \longrightarrow\ 2\,\text{Subst}\left[\int \frac{e\,f-d\,g+g\,x^2}{c\,d^2-b\,d\,e+a\,e^2-(2\,c\,d-b\,e)\,x^2+c\,x^4}\,dx,\ x,\ \sqrt{d+e\,x}\right]$$

- **Program code:**

```
Int[(f_.+g_.*x_)/(Sqrt[d_.+e_.*x_]*(a_.+b_.*x_+c_.*x_^2)),x_Symbol] :=
  2*Subst[Int[(e*f-d*g+g*x^2)/(c*d^2-b*d*e+a*e^2-(2*c*d-b*e)*x^2+c*x^4),x],x,Sqrt[d+e*x]] /;
FreeQ[{a,b,c,d,e,f,g},x] && NeQ[b^2-4*a*c,0] && NeQ[c*d^2-b*d*e+a*e^2,0]
```

```
Int[(f_.+g_.*x_)/(Sqrt[d_.+e_.*x_]*(a_+c_.*x_^2)),x_Symbol] :=
  2*Subst[Int[(e*f-d*g+g*x^2)/(c*d^2+a*e^2-2*c*d*x^2+c*x^4),x],x,Sqrt[d+e*x]] /;
FreeQ[{a,c,d,e,f,g},x] && NeQ[c*d^2+a*e^2,0]
```

2: $\int \frac{(d+e\,x)^m\,(f+g\,x)}{a+b\,x+c\,x^2}\,dx$ when $b^2-4\,a\,c\neq 0\,\wedge\,c\,d^2-b\,d\,e+a\,e^2\neq 0\,\wedge\,m\notin\mathbb{Z}\,\wedge\,m<-1$

Derivation: Quadratic recurrence 3b

Rule 1.2.1.3.15.1.2.2: If $b^2-4\,a\,c\neq 0\,\wedge\,c\,d^2-b\,d\,e+a\,e^2\neq 0\,\wedge\,m\notin\mathbb{Z}\,\wedge\,m<-1$**, then**

$$\int \frac{(d+e\,x)^m\,(f+g\,x)}{a+b\,x+c\,x^2}\,dx\,\longrightarrow\,\frac{(e\,f-d\,g)\,(d+e\,x)^{m+1}}{(m+1)\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}+\frac{1}{c\,d^2-b\,d\,e+a\,e^2}\int \frac{(d+e\,x)^{m+1}\,(c\,d\,f-f\,b\,e+a\,e\,g-c\,(e\,f-d\,g)\,x)}{a+b\,x+c\,x^2}\,dx$$

Program code:

```
Int[(d_.+e_.*x_)^m_*(f_.+g_.*x_)/(a_.+b_.*x_+c_.*x_^2),x_Symbol] :=
  (e*f-d*g)*(d+e*x)^(m+1)/((m+1)*(c*d^2-b*d*e+a*e^2)) +
  1/(c*d^2-b*d*e+a*e^2)*Int[(d+e*x)^(m+1)*Simp[c*d*f-f*b*e+a*e*g-c*(e*f-d*g)*x,x]/(a+b*x+c*x^2),x] /;
FreeQ[{a,b,c,d,e,f,g,m},x] && NeQ[b^2-4*a*c,0] && NeQ[c*d^2-b*d*e+a*e^2,0] && FractionQ[m] && LtQ[m,-1]
```

```
Int[(d_.+e_.*x_)^m_*(f_.+g_.*x_)/(a_+c_.*x_^2),x_Symbol] :=
  (e*f-d*g)*(d+e*x)^(m+1)/((m+1)*(c*d^2+a*e^2)) +
  1/(c*d^2+a*e^2)*Int[(d+e*x)^(m+1)*Simp[c*d*f+a*e*g-c*(e*f-d*g)*x,x]/(a+c*x^2),x] /;
FreeQ[{a,c,d,e,f,g,m},x] && NeQ[c*d^2+a*e^2,0] && FractionQ[m] && LtQ[m,-1]
```

2: $\int \frac{(d+e\,x)^m\,(f+g\,x)}{a+b\,x+c\,x^2}\,dx$ when $b^2-4\,a\,c\neq 0\,\wedge\,c\,d^2-b\,d\,e+a\,e^2\neq 0\,\wedge\,m\notin\mathbb{Q}$

Derivation: Algebraic expansion

Rule 1.2.1.3.15.2: If $b^2-4\,a\,c\neq 0\,\wedge\,c\,d^2-b\,d\,e+a\,e^2\neq 0\,\wedge\,m\notin\mathbb{Z}$**, then**

$$\int \frac{(d+e\,x)^m\,(f+g\,x)}{a+b\,x+c\,x^2}\,dx\,\longrightarrow\,\int (d+e\,x)^m\,\text{ExpandIntegrand}\left[\frac{f+g\,x}{a+b\,x+c\,x^2},\,x\right]dx$$

Program code:

```
Int[(d_.+e_.*x_)^m_*(f_.+g_.*x_)/(a_.+b_.*x_+c_.*x_^2),x_Symbol] :=
  Int[ExpandIntegrand[(d+e*x)^m,(f+g*x)/(a+b*x+c*x^2),x],x] /;
FreeQ[{a,b,c,d,e,f,g},x] && NeQ[b^2-4*a*c,0] && NeQ[c*d^2-b*d*e+a*e^2,0] && Not[RationalQ[m]]
```

```
Int[(d_.+e_.*x_)^m_*(f_.+g_.*x_)/(a_+c_.*x_^2),x_Symbol] :=
  Int[ExpandIntegrand[(d+e*x)^m,(f+g*x)/(a+c*x^2),x],x] /;
FreeQ[{a,c,d,e,f,g},x] && NeQ[c*d^2+a*e^2,0] && Not[RationalQ[m]]
```

16: $\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx$ when $b^2-4\,a\,c\neq0\,\wedge\,c\,d^2-b\,d\,e+a\,e^2\neq0\,\wedge\,m>0\,\wedge\,m+2\,p+2\neq0$

Derivation: Quadratic recurrence 3a

- **Note: The special case rule for $m=1$ and $p=-1$ eliminates the constant term $\frac{g\,d}{c}$ from the result.**

- **Rule 1.2.1.3.16: If $b^2-4\,a\,c\neq0\,\wedge\,c\,d^2-b\,d\,e+a\,e^2\neq0\,\wedge\,m>0\,\wedge\,m+2\,p+2\neq0$, then**

$$\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx\,\longrightarrow$$

$$\frac{g\,(d+e\,x)^m\,\left(a+b\,x+c\,x^2\right)^{p+1}}{c\,(m+2\,p+2)}+\frac{1}{c\,(m+2\,p+2)}\int (d+e\,x)^{m-1}\,\left(a+b\,x+c\,x^2\right)^p\cdot$$
$$(m\,(c\,d\,f-a\,e\,g)+d\,(2\,c\,f-b\,g)\,(p+1)\,+(m\,(c\,e\,f+c\,d\,g-b\,e\,g)+e\,(p+1)\,(2\,c\,f-b\,g))\,x)\,dx$$

Program code:

```
Int[(d_.+e_.*x_)^m_*(f_.+g_.*x_)*(a_.+b_.*x_+c_.*x_^2)^p_.,x_Symbol] :=
  g*(d+e*x)^m*(a+b*x+c*x^2)^(p+1)/(c*(m+2*p+2)) +
  1/(c*(m+2*p+2))*Int[(d+e*x)^(m-1)*(a+b*x+c*x^2)^p*
    Simp[m*(c*d*f-a*e*g)+d*(2*c*f-b*g)*(p+1)+(m*(c*e*f+c*d*g-b*e*g)+e*(p+1)*(2*c*f-b*g))*x,x],x] /;
FreeQ[{a,b,c,d,e,f,g,p},x] && NeQ[b^2-4*a*c,0] && NeQ[c*d^2-b*d*e+a*e^2,0] && GtQ[m,0] && NeQ[m+2*p+2,0] &&
  (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m,2*p]) && Not[IGtQ[m,0] && EqQ[f,0]]
```

```
Int[(d_.+e_.*x_)^m_*(f_.+g_.*x_)*(a_+c_.*x_^2)^p_.,x_Symbol] :=
  g*(d+e*x)^m*(a+c*x^2)^(p+1)/(c*(m+2*p+2)) +
  1/(c*(m+2*p+2))*Int[(d+e*x)^(m-1)*(a+c*x^2)^p*
    Simp[c*d*f*(m+2*p+2)-a*e*g*m+c*(e*f*(m+2*p+2)+d*g*m)*x,x],x] /;
FreeQ[{a,c,d,e,f,g,p},x] && NeQ[c*d^2+a*e^2,0] && GtQ[m,0] && NeQ[m+2*p+2,0] &&
  (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m,2*p]) && Not[IGtQ[m,0] && EqQ[f,0]]
```

17: $\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx$ when $b^2-4\,a\,c\neq0\,\wedge\,c\,d^2-b\,d\,e+a\,e^2\neq0\,\wedge\,m<-1$

Derivation: Quadratic recurrence 3b

- **Rule 1.2.1.3.17: If $b^2-4\,a\,c\neq0\,\wedge\,c\,d^2-b\,d\,e+a\,e^2\neq0\,\wedge\,m<-1$, then**

$$\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx\,\longrightarrow$$

$$\frac{(e f-d g)\,(d+e x)^{m+1}\left(a+b x+c x^2\right)^{p+1}}{(m+1)\left(c d^2-b d e+a e^2\right)}+$$

$$\frac{1}{(m+1)\left(c d^2-b d e+a e^2\right)}\int (d+e x)^{m+1}\left(a+b x+c x^2\right)^p\,((c d f-f b e+a e g)\,(m+1)+b\,(d g-e f)\,(p+1)-c\,(e f-d g)\,(m+2 p+3)\,x)\,dx$$

Program code:

```
Int[(d_.+e_.*x_)^m_*(f_.+g_.*x_)*(a_.+b_.*x_+c_.*x_^2)^p_.,x_Symbol] :=
  (e*f-d*g)*(d+e*x)^(m+1)*(a+b*x+c*x^2)^(p+1)/((m+1)*(c*d^2-b*d*e+a*e^2)) +
  1/((m+1)*(c*d^2-b*d*e+a*e^2))*Int[(d+e*x)^(m+1)*(a+b*x+c*x^2)^p*
    Simp[(c*d*f-f*b*e+a*e*g)*(m+1)+b*(d*g-e*f)*(p+1)-c*(e*f-d*g)*(m+2*p+3)*x,x],x] /;
FreeQ[{a,b,c,d,e,f,g,p},x] && NeQ[b^2-4*a*c,0] && NeQ[c*d^2-b*d*e+a*e^2,0] && LtQ[m,-1] && (IntegerQ[m] || IntegerQ[p] || Intege
```

```
Int[(d_.+e_.*x_)^m_*(f_.+g_.*x_)*(a_+c_.*x_^2)^p_.,x_Symbol] :=
  (e*f-d*g)*(d+e*x)^(m+1)*(a+c*x^2)^(p+1)/((m+1)*(c*d^2+a*e^2)) +
  1/((m+1)*(c*d^2+a*e^2))*Int[(d+e*x)^(m+1)*(a+c*x^2)^p*Simp[(c*d*f+a*e*g)*(m+1)-c*(e*f-d*g)*(m+2*p+3)*x,x],x] /;
FreeQ[{a,c,d,e,f,g,p},x] && NeQ[c*d^2+a*e^2,0] && LtQ[m,-1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m,2*p])
```

```
Int[(d_.+e_.*x_)^m_*(f_.+g_.*x_)*(a_.+b_.*x_+c_.*x_^2)^p_.,x_Symbol] :=
  (e*f-d*g)*(d+e*x)^(m+1)*(a+b*x+c*x^2)^(p+1)/((m+1)*(c*d^2-b*d*e+a*e^2)) +
  1/((m+1)*(c*d^2-b*d*e+a*e^2))*Int[(d+e*x)^(m+1)*(a+b*x+c*x^2)^p*
    Simp[(c*d*f-f*b*e+a*e*g)*(m+1)+b*(d*g-e*f)*(p+1)-c*(e*f-d*g)*(m+2*p+3)*x,x],x] /;
FreeQ[{a,b,c,d,e,f,g,m,p},x] && NeQ[b^2-4*a*c,0] && NeQ[c*d^2-b*d*e+a*e^2,0] && ILtQ[Simplify[m+2*p+3],0] && NeQ[m,-1]
```

```
Int[(d_.+e_.*x_)^m_*(f_.+g_.*x_)*(a_+c_.*x_^2)^p_.,x_Symbol] :=
  (e*f-d*g)*(d+e*x)^(m+1)*(a+c*x^2)^(p+1)/((m+1)*(c*d^2+a*e^2)) +
  1/((m+1)*(c*d^2+a*e^2))*Int[(d+e*x)^(m+1)*(a+c*x^2)^p*Simp[(c*d*f+a*e*g)*(m+1)-c*(e*f-d*g)*(m+2*p+3)*x,x],x] /;
FreeQ[{a,c,d,e,f,g,m,p},x] && NeQ[c*d^2+a*e^2,0] && ILtQ[Simplify[m+2*p+3],0] && NeQ[m,-1]
```

18: $\int \frac{f+g x}{(d+e x)\sqrt{a+b x+c x^2}}\,dx$ when $4 c\,(a-d)-(b-e)^2=0\land f e\,(b-e)-2 g\,(b d-a e)=0\land b d-a e\neq 0$

Derivation: Integration by substitution

- **Basis: If** $4 c\,(a-d)-(b-e)^2=0\land f e\,(b-e)-2 g\,(b d-a e)=0$**, then**
$\frac{f+g x}{(d+e x)\sqrt{a+b x+c x^2}}=\frac{4 f\,(a-d)}{b d-a e}\,\text{Subst}\left[\frac{1}{4\,(a-d)-x^2},\,x,\,\frac{2\,(a-d)+(b-e)\,x}{\sqrt{a+b x+c x^2}}\right]\partial_x\frac{2\,(a-d)+(b-e)\,x}{\sqrt{a+b x+c x^2}}$

- **Rule 1.2.1.3.18: If** $4 c\,(a-d)-(b-e)^2=0\land f e\,(b-e)-2 g\,(b d-a e)=0\land b d-a e\neq 0$**, then**

$$\int \frac{f+g\,x}{(d+e\,x)\,\sqrt{a+b\,x+c\,x^2}}\,dx \;\to\; \frac{4\,f\,(a-d)}{b\,d-a\,e}\,\text{Subst}\!\left[\int \frac{1}{4\,(a-d)-x^2}\,dx,\,x,\,\frac{2\,(a-d)+(b-e)\,x}{\sqrt{a+b\,x+c\,x^2}}\right]$$

Program code:

```
Int[(f_+g_.*x_)/((d_.+e_.*x_)*Sqrt[a_.+b_.*x_+c_.*x_^2]),x_Symbol] :=
  4*f*(a-d)/(b*d-a*e)*Subst[Int[1/(4*(a-d)-x^2),x],x,(2*(a-d)+(b-e)*x)/Sqrt[a+b*x+c*x^2]] /;
FreeQ[{a,b,c,d,e,f,g},x] && EqQ[4*c*(a-d)-(b-e)^2,0] && EqQ[e*f*(b-e)-2*g*(b*d-a*e),0] && NeQ[b*d-a*e,0]
```

19. $\int \frac{f+g\,x}{\sqrt{e\,x}\,\sqrt{a+b\,x+c\,x^2}}\,dx$ when $b^2-4\,a\,c\neq 0$

1: $\int \frac{f+g\,x}{\sqrt{x}\,\sqrt{a+b\,x+c\,x^2}}\,dx$ when $b^2-4\,a\,c\neq 0$

Derivation: Integration by substitution

- **Basis:** $x^m\,F[x] = 2\,\text{Subst}\!\left[x^{2\,m+1}\,F\!\left[x^2\right],\,x,\,\sqrt{x}\right]\partial_x\sqrt{x}$

Rule 1.2.1.3.19.1: If $b^2-4\,a\,c\neq 0$**, then**

$$\int \frac{f+g\,x}{\sqrt{x}\,\sqrt{a+b\,x+c\,x^2}}\,dx \;\to\; 2\,\text{Subst}\!\left[\int \frac{f+g\,x^2}{\sqrt{a+b\,x^2+c\,x^4}}\,dx,\,x,\,\sqrt{x}\right]$$

Program code:

```
Int[(f_+g_.*x_)/(Sqrt[x_]*Sqrt[a_+b_.*x_+c_.*x_^2]),x_Symbol] :=
  2*Subst[Int[(f+g*x^2)/Sqrt[a+b*x^2+c*x^4],x],x,Sqrt[x]] /;
FreeQ[{a,b,c,f,g},x] && NeQ[b^2-4*a*c,0]
```

```
Int[(f_+g_.*x_)/(Sqrt[x_]*Sqrt[a_+c_.*x_^2]),x_Symbol] :=
  2*Subst[Int[(f+g*x^2)/Sqrt[a+c*x^4],x],x,Sqrt[x]] /;
FreeQ[{a,c,f,g},x]
```

2: $\int \frac{f+g\,x}{\sqrt{e\,x}\,\sqrt{a+b\,x+c\,x^2}}\,dx$ when $b^2-4\,a\,c\neq 0$

- **Derivation: Piecewise constant extraction**
- **Basis:** $\partial_x \frac{\sqrt{x}}{\sqrt{e\,x}} == 0$
- **Rule 1.2.1.3.19.2: If** $b^2-4\,a\,c\neq 0$**, then**

$$\int \frac{f+g\,x}{\sqrt{e\,x}\,\sqrt{a+b\,x+c\,x^2}}\,dx \longrightarrow \frac{\sqrt{x}}{\sqrt{e\,x}}\int \frac{f+g\,x}{\sqrt{x}\,\sqrt{a+b\,x+c\,x^2}}\,dx$$

```
Int[(f_+g_.*x_)/(Sqrt[e_*x_]*Sqrt[a_+b_.*x_+c_.*x_^2]),x_Symbol] :=
  Sqrt[x]/Sqrt[e*x]*Int[(f+g*x)/(Sqrt[x]*Sqrt[a+b*x+c*x^2]),x] /;
FreeQ[{a,b,c,e,f,g},x] && NeQ[b^2-4*a*c,0]
```

```
Int[(f_+g_.*x_)/(Sqrt[e_*x_]*Sqrt[a_+c_.*x_^2]),x_Symbol] :=
  Sqrt[x]/Sqrt[e*x]*Int[(f+g*x)/(Sqrt[x]*Sqrt[a+c*x^2]),x] /;
FreeQ[{a,c,e,f,g},x]
```

20: $\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx$ when $b^2-4\,a\,c\neq 0\,\land\,c\,d^2-b\,d\,e+a\,e^2\neq 0$

- **Derivation: Algebraic expansion**
- **Basis:** $f+g\,x == \frac{g\,(d+e\,x)}{e}+\frac{e\,f-d\,g}{e}$
- **Rule 1.2.1.3.20: If** $b^2-4\,a\,c\neq 0\,\land\,c\,d^2-b\,d\,e+a\,e^2\neq 0$**, then**

$$\int (d+e\,x)^m\,(f+g\,x)\,\left(a+b\,x+c\,x^2\right)^p\,dx \longrightarrow$$
$$\frac{g}{e}\int (d+e\,x)^{m+1}\,\left(a+b\,x+c\,x^2\right)^p\,dx+\frac{e\,f-d\,g}{e}\int (d+e\,x)^m\,\left(a+b\,x+c\,x^2\right)^p\,dx$$

- **Program code:**

```
Int[(d_.+e_.*x_)^m_*(f_.+g_.*x_)*(a_.+b_.*x_+c_.*x_^2)^p_.,x_Symbol] :=
  g/e*Int[(d+e*x)^(m+1)*(a+b*x+c*x^2)^p,x] + (e*f-d*g)/e*Int[(d+e*x)^m*(a+b*x+c*x^2)^p,x] /;
FreeQ[{a,b,c,d,e,f,g,m,p},x] && NeQ[b^2-4*a*c,0] && NeQ[c*d^2-b*d*e+a*e^2,0] && Not[IGtQ[m,0]]
```

```
Int[(d_.+e_.*x_)^m_*(f_.+g_.*x_)*(a_+c_.*x_^2)^p_.,x_Symbol] :=
  g/e*Int[(d+e*x)^(m+1)*(a+c*x^2)^p,x] + (e*f-d*g)/e*Int[(d+e*x)^m*(a+c*x^2)^p,x] /;
FreeQ[{a,c,d,e,f,g,m,p},x] && NeQ[c*d^2+a*e^2,0] && Not[IGtQ[m,0]]
```