Rules for integrands involving inverse tangents and cotangents

1. $\int u \,\text{ArcTan}[a+b\,x^n]\,dx$

1: $\int \text{ArcTan}[a+b\,x^n]\,dx$

- **Derivation: Integration by parts**
- **Rule:**

$$\int \text{ArcTan}[a+b\,x^n]\,dx \longrightarrow x\,\text{ArcTan}[a+b\,x^n]-b\,n\int \frac{x^n}{1+a^2+2\,a\,b\,x^n+b^2\,x^{2\,n}}\,dx$$

- **Program code:**

```
Int[ArcTan[a_+b_.*x_^n_],x_Symbol] :=
  x*ArcTan[a+b*x^n] -
  b*n*Int[x^n/(1+a^2+2*a*b*x^n+b^2*x^(2*n)),x] /;
FreeQ[{a,b,n},x]
```

```
Int[ArcCot[a_+b_.*x_^n_],x_Symbol] :=
  x*ArcCot[a+b*x^n] +
  b*n*Int[x^n/(1+a^2+2*a*b*x^n+b^2*x^(2*n)),x] /;
FreeQ[{a,b,n},x]
```

2. $\int x^m \text{ArcTan}[a + b x^n] \, dx$

1: $\int \frac{\text{ArcTan}[a + b x^n]}{x} \, dx$

- **Derivation: Algebraic expansion**
- **Basis:** $\text{ArcTan}[z] = \frac{1}{2} i \text{Log}[1 - i z] - \frac{1}{2} i \text{Log}[1 + i z]$
- **Rule:**

$$\int \frac{\text{ArcTan}[a + b x^n]}{x} \, dx \longrightarrow \frac{i}{2} \int \frac{\text{Log}[1 - i a - i b x^n]}{x} \, dx - \frac{i}{2} \int \frac{\text{Log}[1 + i a + i b x^n]}{x} \, dx$$

- **Program code:**

```
Int[ArcTan[a_.+b_.*x_^n_]/x_,x_Symbol] :=
  I/2*Int[Log[1-I*a-I*b*x^n]/x,x] -
  I/2*Int[Log[1+I*a+I*b*x^n]/x,x] /;
FreeQ[{a,b,n},x]
```

```
Int[ArcCot[a_.+b_.*x_^n_]/x_,x_Symbol] :=
  I/2*Int[Log[1-I/(a+b*x^n)]/x,x] -
  I/2*Int[Log[1+I/(a+b*x^n)]/x,x] /;
FreeQ[{a,b,n},x]
```

2: $\int x^m \operatorname{ArcTan}[a + b x^n] \, dx$ when $(m \mid n) \in \mathbb{Q} \land m + 1 \neq 0 \land m + 1 \neq n$

Reference: G&R 2.851, CRC 456, A&S 4.4.69

Reference: G&R 2.852, CRC 458, A&S 4.4.71

Derivation: Integration by parts

Rule: If $(m \mid n) \in \mathbb{Q} \land m + 1 \neq 0 \land m + 1 \neq n$, then

$$\int x^m \operatorname{ArcTan}[a + b x^n] \, dx \longrightarrow \frac{x^{m+1} \operatorname{ArcTan}[a + b x^n]}{m + 1} - \frac{b n}{m + 1} \int \frac{x^{m+n}}{1 + a^2 + 2 a b x^n + b^2 x^{2n}} \, dx$$

Program code:

```
Int[x_^m_.*ArcTan[a_+b_.*x_^n_],x_Symbol] :=
  x^(m+1)*ArcTan[a+b*x^n]/(m+1) -
  b*n/(m+1)*Int[x^(m+n)/(1+a^2+2*a*b*x^n+b^2*x^(2*n)),x] /;
FreeQ[{a,b},x] && RationalQ[m,n] && m+1≠0 && m+1≠n
```

```
Int[x_^m_.*ArcCot[a_+b_.*x_^n_],x_Symbol] :=
  x^(m+1)*ArcCot[a+b*x^n]/(m+1) +
  b*n/(m+1)*Int[x^(m+n)/(1+a^2+2*a*b*x^n+b^2*x^(2*n)),x] /;
FreeQ[{a,b},x] && RationalQ[m,n] && m+1≠0 && m+1≠n
```

2. $\int u\,\text{ArcTan}\left[a+b\,f^{c+d\,x}\right]\,dx$

1: $\int \text{ArcTan}\left[a+b\,f^{c+d\,x}\right]\,dx$

Derivation: Algebraic expansion

Basis: $\text{ArcTan}[z] == \frac{1}{2}\,i\,\text{Log}[1-i\,z]-\frac{1}{2}\,i\,\text{Log}[1+i\,z]$

Rule:

$$\int \text{ArcTan}\left[a+b\,f^{c+d\,x}\right]\,dx \longrightarrow \frac{i}{2}\int \text{Log}\left[1-i\left(a+b\,f^{c+d\,x}\right)\right]\,dx-\frac{i}{2}\int \text{Log}\left[1+i\left(a+b\,f^{c+d\,x}\right)\right]\,dx$$

Program code:

```
Int[ArcTan[a_.+b_.*f_^(c_.+d_.*x_)],x_Symbol] :=
  I/2*Int[Log[1-I*a-I*b*f^(c+d*x)],x] -
  I/2*Int[Log[1+I*a+I*b*f^(c+d*x)],x] /;
FreeQ[{a,b,c,d,f},x]
```

```
Int[ArcCot[a_.+b_.*f_^(c_.+d_.*x_)],x_Symbol] :=
  I/2*Int[Log[1-I/(a+b*f^(c+d*x))],x] -
  I/2*Int[Log[1+I/(a+b*f^(c+d*x))],x] /;
FreeQ[{a,b,c,d,f},x]
```

2: $\int x^m \text{ArcTan}\left[a+b f^{c+d x}\right] dx$ when $m \in \mathbb{Z} \wedge m > 0$

Derivation: Algebraic expansion

Basis: $\text{ArcTan}[z] = \frac{1}{2} i \text{Log}[1 - i z] - \frac{1}{2} i \text{Log}[1 + i z]$

Rule: If $m \in \mathbb{Z} \wedge m > 0$**, then**

$$\int x^m \text{ArcTan}\left[a+b f^{c+d x}\right] dx \longrightarrow \frac{i}{2} \int x^m \text{Log}\left[1 - i \left(a + b f^{c+d x}\right)\right] dx - \frac{i}{2} \int x^m \text{Log}\left[1 + i \left(a + b f^{c+d x}\right)\right] dx$$

Program code:

```
Int[x_^m_.*ArcTan[a_.+b_.*f_^(c_.+d_.*x_)],x_Symbol] :=
  I/2*Int[x^m*Log[1-I*a-I*b*f^(c+d*x)],x] -
  I/2*Int[x^m*Log[1+I*a+I*b*f^(c+d*x)],x] /;
FreeQ[{a,b,c,d,f},x] && IntegerQ[m] && m>0
```

```
Int[x_^m_.*ArcCot[a_.+b_.*f_^(c_.+d_.*x_)],x_Symbol] :=
  I/2*Int[x^m*Log[1-I/(a+b*f^(c+d*x))],x] -
  I/2*Int[x^m*Log[1+I/(a+b*f^(c+d*x))],x] /;
FreeQ[{a,b,c,d,f},x] && IntegerQ[m] && m>0
```

3: $\int u \, \text{ArcTan}\left[\frac{c}{a + b x^n}\right]^m dx$

Derivation: Algebraic simplification

Basis: $\text{ArcTan}[z] = \text{ArcCot}\left[\frac{1}{z}\right]$

Rule:

$$\int u \, \text{ArcTan}\left[\frac{c}{a + b x^n}\right]^m dx \longrightarrow \int u \, \text{ArcCot}\left[\frac{a}{c} + \frac{b x^n}{c}\right]^m dx$$

Program code:

```
Int[u_.*ArcTan[c_./(a_.+b_.*x_^n_.)]^m_.,x_Symbol] :=
  Int[u*ArcCot[a/c+b*x^n/c]^m,x] /;
FreeQ[{a,b,c,n,m},x]
```

```
Int[u_.*ArcCot[c_./(a_.+b_.*x_^n_.)]^m_.,x_Symbol] :=
  Int[u*ArcTan[a/c+b*x^n/c]^m,x] /;
FreeQ[{a,b,c,n,m},x]
```

4. $\int u\,\text{ArcTan}\left[\frac{c\,x}{\sqrt{a+b\,x^2}}\right]dx$ when $b+c^2=0$

1: $\int \text{ArcTan}\left[\frac{c\,x}{\sqrt{a+b\,x^2}}\right]dx$ when $b+c^2=0$

Derivation: Integration by parts

- **Basis: If** $b+c^2=0$**, then** $\partial_x \text{ArcTan}\left[\frac{c\,x}{\sqrt{a+b\,x^2}}\right]=\frac{c}{\sqrt{a+b\,x^2}}$

- **Rule: If** $b+c^2=0$**, then**

$$\int \text{ArcTan}\left[\frac{c\,x}{\sqrt{a+b\,x^2}}\right]dx \to x\,\text{ArcTan}\left[\frac{c\,x}{\sqrt{a+b\,x^2}}\right]-c\int\frac{x}{\sqrt{a+b\,x^2}}\,dx$$

- **Program code:**

```
Int[ArcTan[c_.*x_/Sqrt[a_.+b_.*x_^2]],x_Symbol] :=
  x*ArcTan[(c*x)/Sqrt[a+b*x^2]] - c*Int[x/Sqrt[a+b*x^2],x] /;
FreeQ[{a,b,c},x] && EqQ[b+c^2,0]
```

```
Int[ArcCot[c_.*x_/Sqrt[a_.+b_.*x_^2]],x_Symbol] :=
  x*ArcCot[(c*x)/Sqrt[a+b*x^2]] + c*Int[x/Sqrt[a+b*x^2],x] /;
FreeQ[{a,b,c},x] && EqQ[b+c^2,0]
```

2. $\int (d\,x)^m\,\text{ArcTan}\left[\frac{c\,x}{\sqrt{a+b\,x^2}}\right]dx$ when $b+c^2=0$

1: $\int \frac{\text{ArcTan}\left[\frac{c\,x}{\sqrt{a+b\,x^2}}\right]}{x}\,dx$ when $b+c^2=0$

Derivation: Integration by parts

- **Basis: If** $b+c^2=0$**, then** $\partial_x \text{ArcTan}\left[\frac{c\,x}{\sqrt{a+b\,x^2}}\right]=\frac{c}{\sqrt{a+b\,x^2}}$

- **Rule: If** $b+c^2=0$**, then**

$$\int \frac{\text{ArcTan}\left[\frac{c\,x}{\sqrt{a+b\,x^2}}\right]}{x}\,dx \longrightarrow \text{ArcTan}\left[\frac{c\,x}{\sqrt{a+b\,x^2}}\right]\text{Log}[x]-c\int \frac{\text{Log}[x]}{\sqrt{a+b\,x^2}}\,dx$$

Program code:

```
Int[ArcTan[c_.*x_/Sqrt[a_.+b_.*x_^2]]/x_,x_Symbol] :=
  ArcTan[c*x/Sqrt[a+b*x^2]]*Log[x] - c*Int[Log[x]/Sqrt[a+b*x^2],x] /;
FreeQ[{a,b,c},x] && EqQ[b+c^2,0]
```

```
Int[ArcCot[c_.*x_/Sqrt[a_.+b_.*x_^2]]/x_,x_Symbol] :=
  ArcCot[c*x/Sqrt[a+b*x^2]]*Log[x] + c*Int[Log[x]/Sqrt[a+b*x^2],x] /;
FreeQ[{a,b,c},x] && EqQ[b+c^2,0]
```

2: $\int (d\,x)^m\,\text{ArcTan}\left[\frac{c\,x}{\sqrt{a+b\,x^2}}\right]dx$ when $b+c^2==0\,\land\,m\neq-1$

Derivation: Integration by parts

Basis: If $b+c^2==0$, **then** $\partial_x\,\text{ArcTan}\left[\frac{c\,x}{\sqrt{a+b\,x^2}}\right]==\frac{c}{\sqrt{a+b\,x^2}}$

Rule: If $b+c^2==0\,\land\,m\neq-1$, **then**

$$\int (d\,x)^m\,\text{ArcTan}\left[\frac{c\,x}{\sqrt{a+b\,x^2}}\right]dx \longrightarrow \frac{(d\,x)^{m+1}\,\text{ArcTan}\left[\frac{c\,x}{\sqrt{a+b\,x^2}}\right]}{d\,(m+1)}-\frac{c}{d\,(m+1)}\int \frac{(d\,x)^{m+1}}{\sqrt{a+b\,x^2}}\,dx$$

Program code:

```
Int[(d_.*x_)^m_.*ArcTan[c_.*x_/Sqrt[a_.+b_.*x_^2]],x_Symbol] :=
  (d*x)^(m+1)*ArcTan[(c*x)/Sqrt[a+b*x^2]]/(d*(m+1)) - c/(d*(m+1))*Int[(d*x)^(m+1)/Sqrt[a+b*x^2],x] /;
FreeQ[{a,b,c,d,m},x] && EqQ[b+c^2,0] && NeQ[m,-1]
```

```
Int[(d_.*x_)^m_.*ArcCot[c_.*x_/Sqrt[a_.+b_.*x_^2]],x_Symbol] :=
  (d*x)^(m+1)*ArcCot[(c*x)/Sqrt[a+b*x^2]]/(d*(m+1)) + c/(d*(m+1))*Int[(d*x)^(m+1)/Sqrt[a+b*x^2],x] /;
FreeQ[{a,b,c,d,m},x] && EqQ[b+c^2,0] && NeQ[m,-1]
```

3. $\int \frac{\text{ArcTan}\left[\frac{c\,x}{\sqrt{a+b\,x^2}}\right]^m}{\sqrt{d+e\,x^2}}\,dx$ when $b+c^2==0\,\land\,b\,d-a\,e==0$

1. $\int \frac{\text{ArcTan}\left[\frac{c\,x}{\sqrt{a+b\,x^2}}\right]^m}{\sqrt{a+b\,x^2}}\,dx$ when $b+c^2==0$

1: $\int \frac{1}{\sqrt{a+b\,x^2}\,\text{ArcTan}\left[\frac{c\,x}{\sqrt{a+b\,x^2}}\right]}\,dx$ when $b+c^2==0$

Derivation: Reciprocal rule for integration

Basis: If $b+c^2==0$**, then** $\partial_x\,\text{ArcTan}\left[\frac{c\,x}{\sqrt{a+b\,x^2}}\right]==\frac{c}{\sqrt{a+b\,x^2}}$

Rule: If $b+c^2==0$**, then**

$$\int \frac{1}{\sqrt{a+b\,x^2}\,\text{ArcTan}\left[\frac{c\,x}{\sqrt{a+b\,x^2}}\right]}\,dx \longrightarrow \frac{1}{c}\,\text{Log}\left[\text{ArcTan}\left[\frac{c\,x}{\sqrt{a+b\,x^2}}\right]\right]$$

Program code:

```
Int[1/(Sqrt[a_.+b_.*x_^2]*ArcTan[c_.*x_/Sqrt[a_.+b_.*x_^2]]),x_Symbol] :=
  1/c*Log[ArcTan[c*x/Sqrt[a+b*x^2]]] /;
FreeQ[{a,b,c},x] && EqQ[b+c^2,0]
```

```
Int[1/(Sqrt[a_.+b_.*x_^2]*ArcCot[c_.*x_/Sqrt[a_.+b_.*x_^2]]),x_Symbol] :=
  -1/c*Log[ArcCot[c*x/Sqrt[a+b*x^2]]] /;
FreeQ[{a,b,c},x] && EqQ[b+c^2,0]
```

2: $\int \frac{\text{ArcTan}\left[\frac{c\,x}{\sqrt{a+b\,x^2}}\right]^m}{\sqrt{a+b\,x^2}}\,dx$ when $b+c^2==0\wedge m\neq-1$

- **Derivation: Power rule for integration**

- **Basis: If** $b+c^2==0$**, then** $\partial_x \text{ArcTan}\left[\frac{c\,x}{\sqrt{a+b\,x^2}}\right]==\frac{c}{\sqrt{a+b\,x^2}}$

- **Rule: If** $b+c^2==0\wedge m\neq-1$**, then**

$$\int \frac{\text{ArcTan}\left[\frac{c\,x}{\sqrt{a+b\,x^2}}\right]^m}{\sqrt{a+b\,x^2}}\,dx\longrightarrow\frac{\text{ArcTan}\left[\frac{c\,x}{\sqrt{a+b\,x^2}}\right]^{m+1}}{c\,(m+1)}$$

- **Program code:**

```
Int[ArcTan[c_.*x_/Sqrt[a_.+b_.*x_^2]]^m_./Sqrt[a_.+b_.*x_^2],x_Symbol] :=
  ArcTan[c*x/Sqrt[a+b*x^2]]^(m+1)/(c*(m+1)) /;
FreeQ[{a,b,c,m},x] && EqQ[b+c^2,0] && NeQ[m,-1]
```

```
Int[ArcCot[c_.*x_/Sqrt[a_.+b_.*x_^2]]^m_./Sqrt[a_.+b_.*x_^2],x_Symbol] :=
  -ArcCot[c*x/Sqrt[a+b*x^2]]^(m+1)/(c*(m+1)) /;
FreeQ[{a,b,c,m},x] && EqQ[b+c^2,0] && NeQ[m,-1]
```

2: $\int \frac{\text{ArcTan}\left[\frac{c\,x}{\sqrt{a+b\,x^2}}\right]^m}{\sqrt{d+e\,x^2}}\,dx$ when $b+c^2==0\,\wedge\,b\,d-a\,e==0$

– **Derivation: Piecewise constant extraction**

- **Basis: If** $b\,d-a\,e==0$, **then** $\partial_x \frac{\sqrt{a+b\,x^2}}{\sqrt{d+e\,x^2}}==0$

– **Rule: If** $b+c^2==0\,\wedge\,b\,d-a\,e==0$, **then**

$$\int \frac{\text{ArcTan}\left[\frac{c\,x}{\sqrt{a+b\,x^2}}\right]^m}{\sqrt{d+e\,x^2}}\,dx\,\longrightarrow\,\frac{\sqrt{a+b\,x^2}}{\sqrt{d+e\,x^2}}\int \frac{\text{ArcTan}\left[\frac{c\,x}{\sqrt{a+b\,x^2}}\right]^m}{\sqrt{a+b\,x^2}}\,dx$$

Program code:

```
Int[ArcTan[c_.*x_/Sqrt[a_.+b_.*x_^2]]^m_./Sqrt[d_.+e_.*x_^2],x_Symbol] :=
  Sqrt[a+b*x^2]/Sqrt[d+e*x^2]*Int[ArcTan[c*x/Sqrt[a+b*x^2]]^m/Sqrt[a+b*x^2],x] /;
FreeQ[{a,b,c,d,e,m},x] && EqQ[b+c^2,0] && EqQ[b*d-a*e,0]
```

```
Int[ArcCot[c_.*x_/Sqrt[a_.+b_.*x_^2]]^m_./Sqrt[d_.+e_.*x_^2],x_Symbol] :=
  Sqrt[a+b*x^2]/Sqrt[d+e*x^2]*Int[ArcCot[c*x/Sqrt[a+b*x^2]]^m/Sqrt[a+b*x^2],x] /;
FreeQ[{a,b,c,d,e,m},x] && EqQ[b+c^2,0] && EqQ[b*d-a*e,0]
```

5: $\int u\,\text{ArcTan}\left[v+s\sqrt{v^2+1}\right]dx$ when $s^2 == 1$

Derivation: Algebraic simplification

- **Basis: If** $s^2 == 1$**, then** $\text{ArcTan}\left[z+s\sqrt{z^2+1}\right] == \frac{\pi s}{4}+\frac{\text{ArcTan}[z]}{2}$
- **Basis: If** $s^2 == 1$**, then** $\text{ArcCot}\left[z+s\sqrt{z^2+1}\right] == \frac{\pi s}{4}-\frac{\text{ArcTan}[z]}{2}$
- **Rule: If** $s^2 == 1$**, then**

$$\int u\,\text{ArcTan}\left[v+s\sqrt{v^2+1}\right]dx \longrightarrow \frac{\pi s}{4}\int u\,dx+\frac{1}{2}\int u\,\text{ArcTan}[v]\,dx$$

- **Program code:**

```
Int[u_.*ArcTan[v_+s_.*Sqrt[w_]],x_Symbol] :=
  Pi*s/4*Int[u,x] + 1/2*Int[u*ArcTan[v],x] /;
EqQ[s^2,1] && EqQ[w,v^2+1]
```

```
Int[u_.*ArcCot[v_+s_.*Sqrt[w_]],x_Symbol] :=
  Pi*s/4*Int[u,x] - 1/2*Int[u*ArcTan[v],x] /;
EqQ[s^2,1] && EqQ[w,v^2+1]
```

6: $\int \frac{f[x, \text{ArcTan}[a\,x]]}{1+(a+b\,x)^2}\,dx$

Derivation: Integration by substitution

- Basis: $\frac{f[z]}{1+z^2} = f[\text{Tan}[\text{ArcTan}[z]]]\ \text{ArcTan}'[z]$
- Basis: $r+s\,x+t\,x^2 = -\frac{s^2-4\,r\,t}{4\,t}\left(1-\frac{(s+2\,t\,x)^2}{s^2-4\,r\,t}\right)$
- Basis: $1+\text{Tan}[z]^2 = \text{Sec}[z]^2$

Rule:

$$\int \frac{f[x, \text{ArcTan}[a+b\,x]]}{1+(a+b\,x)^2}\,dx \longrightarrow \frac{1}{b}\,\text{Subst}\left[\int f\left[-\frac{a}{b}+\frac{\text{Tan}[x]}{b},\,x\right]dx,\,x,\,\text{ArcTan}[a+b\,x]\right]$$

Program code:

```
If[TrueQ[$LoadShowSteps],

Int[u_*v_^n_.,x_Symbol] :=
  With[{tmp=InverseFunctionOfLinear[u,x]},
  ShowStep["","Int[f[x,ArcTan[a+b*x]]/(1+(a+b*x)^2),x]",
           "Subst[Int[f[-a/b+Tan[x]/b,x],x],x,ArcTan[a+b*x]]/b",Hold[
  (-Discriminant[v,x]/(4*Coefficient[v,x,2]))^n/Coefficient[tmp[[1]],x,1]*
    Subst[Int[SimplifyIntegrand[SubstForInverseFunction[u,tmp,x]*Sec[x]^(2*(n+1)),x],x], x, tmp]]] /;
 Not[FalseQ[tmp]] && EqQ[Head[tmp],ArcTan] && EqQ[Discriminant[v,x]*tmp[[1]]^2+D[v,x]^2,0]] /;
SimplifyFlag && QuadraticQ[v,x] && ILtQ[n,0] && NegQ[Discriminant[v,x]] && MatchQ[u,r_.*f_^w_ /; FreeQ[f,x]],

Int[u_*v_^n_.,x_Symbol] :=
  With[{tmp=InverseFunctionOfLinear[u,x]},
  (-Discriminant[v,x]/(4*Coefficient[v,x,2]))^n/Coefficient[tmp[[1]],x,1]*
    Subst[Int[SimplifyIntegrand[SubstForInverseFunction[u,tmp,x]*Sec[x]^(2*(n+1)),x],x], x, tmp] /;
 Not[FalseQ[tmp]] && EqQ[Head[tmp],ArcTan] && EqQ[Discriminant[v,x]*tmp[[1]]^2+D[v,x]^2,0]] /;
QuadraticQ[v,x] && ILtQ[n,0] && NegQ[Discriminant[v,x]] && MatchQ[u,r_.*f_^w_ /; FreeQ[f,x]]]
```

```
If[TrueQ[$LoadShowSteps],

Int[u_*v_^n_.,x_Symbol] :=
  With[{tmp=InverseFunctionOfLinear[u,x]},
  ShowStep["","Int[f[x,ArcCot[a+b*x]]/(1+(a+b*x)^2),x]",
           "-Subst[Int[f[-a/b+Cot[x]/b,x],x],x,ArcCot[a+b*x]]/b",Hold[
  -(-Discriminant[v,x]/(4*Coefficient[v,x,2]))^n/Coefficient[tmp[[1]],x,1]*
    Subst[Int[SimplifyIntegrand[SubstForInverseFunction[u,tmp,x]*Csc[x]^(2*(n+1)),x],x], x, tmp]]] /;
 Not[FalseQ[tmp]] && EqQ[Head[tmp],ArcCot] && EqQ[Discriminant[v,x]*tmp[[1]]^2+D[v,x]^2,0]] /;
SimplifyFlag && QuadraticQ[v,x] && ILtQ[n,0] && NegQ[Discriminant[v,x]] && MatchQ[u,r_.*f_^w_ /; FreeQ[f,x]],

Int[u_*v_^n_.,x_Symbol] :=
  With[{tmp=InverseFunctionOfLinear[u,x]},
  -(-Discriminant[v,x]/(4*Coefficient[v,x,2]))^n/Coefficient[tmp[[1]],x,1]*
    Subst[Int[SimplifyIntegrand[SubstForInverseFunction[u,tmp,x]*Csc[x]^(2*(n+1)),x],x], x, tmp] /;
 Not[FalseQ[tmp]] && EqQ[Head[tmp],ArcCot] && EqQ[Discriminant[v,x]*tmp[[1]]^2+D[v,x]^2,0]] /;
QuadraticQ[v,x] && ILtQ[n,0] && NegQ[Discriminant[v,x]] && MatchQ[u,r_.*f_^w_ /; FreeQ[f,x]]]
```

7. $\int u \,\text{ArcTan}[c + d \,\text{Tan}[a + b x]] \,dx$

   1. $\int \text{ArcTan}[c + d \,\text{Tan}[a + b x]] \,dx$

      1: $\int \text{ArcTan}[c + d \,\text{Tan}[a + b x]] \,dx$ when $(c + i d)^2 == -1$

Derivation: Integration by parts

- **Basis: If** $(c + i d)^2 == -1$**, then** $\partial_x \text{ArcTan}[c + d \,\text{Tan}[a + b x]] == \frac{i b}{c + i d + c e^{2 i (a+b x)}}$

- **Rule: If** $(c + i d)^2 == -1$**, then**

$$\int \text{ArcTan}[c + d \,\text{Tan}[a + b x]] \,dx \longrightarrow x \,\text{ArcTan}[c + d \,\text{Tan}[a + b x]] - i b \int \frac{x}{c + i d + c e^{2 i a + 2 i b x}} \,dx$$

Program code:

```
Int[ArcTan[c_.+d_.*Tan[a_.+b_.*x_]],x_Symbol] :=
  x*ArcTan[c+d*Tan[a+b*x]] -
  I*b*Int[x/(c+I*d+c*E^(2*I*a+2*I*b*x)),x] /;
FreeQ[{a,b,c,d},x] && EqQ[(c+I*d)^2,-1]
```

```
Int[ArcCot[c_.+d_.*Tan[a_.+b_.*x_]],x_Symbol] :=
  x*ArcCot[c+d*Tan[a+b*x]] +
  I*b*Int[x/(c+I*d+c*E^(2*I*a+2*I*b*x)),x] /;
FreeQ[{a,b,c,d},x] && EqQ[(c+I*d)^2,-1]
```

```
Int[ArcTan[c_.+d_.*Cot[a_.+b_.*x_]],x_Symbol] :=
  x*ArcTan[c+d*Cot[a+b*x]] -
  I*b*Int[x/(c-I*d-c*E^(2*I*a+2*I*b*x)),x] /;
FreeQ[{a,b,c,d},x] && EqQ[(c-I*d)^2,-1]
```

```
Int[ArcCot[c_.+d_.*Cot[a_.+b_.*x_]],x_Symbol] :=
  x*ArcCot[c+d*Cot[a+b*x]] +
  I*b*Int[x/(c-I*d-c*E^(2*I*a+2*I*b*x)),x] /;
FreeQ[{a,b,c,d},x] && EqQ[(c-I*d)^2,-1]
```

2: $\int \text{ArcTan}[c + d \,\text{Tan}[a + b\, x]]\, dx$ when $(c + i\, d)^2 \neq -1$

Derivation: Integration by parts

- **Basis:** $\partial_x \text{ArcTan}[c + d\,\text{Tan}[a + b\,x]] = \frac{b\,(1+i\,c+d)\,e^{2\,i\,a+2\,i\,b\,x}}{1+i\,c-d+(1+i\,c+d)\,e^{2\,i\,a+2\,i\,b\,x}} - \frac{b\,(1-i\,c-d)\,e^{2\,i\,a+2\,i\,b\,x}}{1-i\,c+d+(1-i\,c-d)\,e^{2\,i\,a+2\,i\,b\,x}}$

- **Rule:** If $(c + i\,d)^2 \neq -1$, then

$$\int \text{ArcTan}[c + d\,\text{Tan}[a + b\,x]]\, dx \longrightarrow$$

$$x\,\text{ArcTan}[c + d\,\text{Tan}[a + b\,x]] - b\,(1 + i\,c + d) \int \frac{x\,e^{2\,i\,a+2\,i\,b\,x}}{1 + i\,c - d + (1 + i\,c + d)\,e^{2\,i\,a+2\,i\,b\,x}}\,dx + b\,(1 - i\,c - d) \int \frac{x\,e^{2\,i\,a+2\,i\,b\,x}}{1 - i\,c + d + (1 - i\,c - d)\,e^{2\,i\,a+2\,i\,b\,x}}\,dx$$

Program code:

```
Int[ArcTan[c_.+d_.*Tan[a_.+b_.*x_]],x_Symbol] :=
  x*ArcTan[c+d*Tan[a+b*x]] -
  b*(1+I*c+d)*Int[x*E^(2*I*a+2*I*b*x)/(1+I*c-d+(1+I*c+d)*E^(2*I*a+2*I*b*x)),x] +
  b*(1-I*c-d)*Int[x*E^(2*I*a+2*I*b*x)/(1-I*c+d+(1-I*c-d)*E^(2*I*a+2*I*b*x)),x] /;
FreeQ[{a,b,c,d},x] && NeQ[(c+I*d)^2,-1]
```

```
Int[ArcCot[c_.+d_.*Tan[a_.+b_.*x_]],x_Symbol] :=
  x*ArcCot[c+d*Tan[a+b*x]] +
   b*(1+I*c+d)*Int[x*E^(2*I*a+2*I*b*x)/(1+I*c-d+(1+I*c+d)*E^(2*I*a+2*I*b*x)),x] -
  b*(1-I*c-d)*Int[x*E^(2*I*a+2*I*b*x)/(1-I*c+d+(1-I*c-d)*E^(2*I*a+2*I*b*x)),x] /;
FreeQ[{a,b,c,d},x] && NeQ[(c+I*d)^2,-1]
```

```
Int[ArcTan[c_.+d_.*Cot[a_.+b_.*x_]],x_Symbol] :=
  x*ArcTan[c+d*Cot[a+b*x]] +
   b*(1+I*c-d)*Int[x*E^(2*I*a+2*I*b*x)/(1+I*c+d-(1+I*c-d)*E^(2*I*a+2*I*b*x)),x] -
  b*(1-I*c+d)*Int[x*E^(2*I*a+2*I*b*x)/(1-I*c-d-(1-I*c+d)*E^(2*I*a+2*I*b*x)),x] /;
FreeQ[{a,b,c,d},x] && NeQ[(c+I*d)^2,-1]
```

```
Int[ArcCot[c_.+d_.*Cot[a_.+b_.*x_]],x_Symbol] :=
  x*ArcCot[c+d*Cot[a+b*x]] -
   b*(1+I*c-d)*Int[x*E^(2*I*a+2*I*b*x)/(1+I*c+d-(1+I*c-d)*E^(2*I*a+2*I*b*x)),x] +
  b*(1-I*c+d)*Int[x*E^(2*I*a+2*I*b*x)/(1-I*c-d-(1-I*c+d)*E^(2*I*a+2*I*b*x)),x] /;
FreeQ[{a,b,c,d},x] && NeQ[(c-I*d)^2,-1]
```

2. $\int (e+f\,x)^m \operatorname{ArcTan}[c+d\operatorname{Tan}[a+b\,x]]\,dx$ when $m \in \mathbb{Z}^+$

1: $\int (e+f\,x)^m \operatorname{ArcTan}[c+d\operatorname{Tan}[a+b\,x]]\,dx$ when $m \in \mathbb{Z}^+ \bigwedge (c+i\,d)^2 == -1$

Derivation: Integration by parts

- **Basis: If** $(c+i\,d)^2 == -1$**, then** $\partial_x \operatorname{ArcTan}[c+d\operatorname{Tan}[a+b\,x]] == \frac{i\,b}{c+i\,d+c\,e^{2\,i\,(a+b\,x)}}$

- **Rule: If** $m \in \mathbb{Z}^+ \bigwedge (c+i\,d)^2 == -1$**, then**

$$\int (e+f\,x)^m \operatorname{ArcTan}[c+d\operatorname{Tan}[a+b\,x]]\,dx \longrightarrow \frac{(e+f\,x)^{m+1}\operatorname{ArcTan}[c+d\operatorname{Tan}[a+b\,x]]}{f\,(m+1)} - \frac{i\,b}{f\,(m+1)}\int \frac{(e+f\,x)^{m+1}}{c+i\,d+c\,e^{2\,i\,a+2\,i\,b\,x}}\,dx$$

Program code:

```
Int[(e_.+f_.*x_)^m_.*ArcTan[c_.+d_.*Tan[a_.+b_.*x_]],x_Symbol] :=
  (e+f*x)^(m+1)*ArcTan[c+d*Tan[a+b*x]]/(f*(m+1)) -
  I*b/(f*(m+1))*Int[(e+f*x)^(m+1)/(c+I*d+c*E^(2*I*a+2*I*b*x)),x] /;
FreeQ[{a,b,c,d,e,f},x] && IGtQ[m,0] && EqQ[(c+I*d)^2,-1]
```

```
Int[(e_.+f_.*x_)^m_.*ArcCot[c_.+d_.*Tan[a_.+b_.*x_]],x_Symbol] :=
  (e+f*x)^(m+1)*ArcCot[c+d*Tan[a+b*x]]/(f*(m+1)) +
  I*b/(f*(m+1))*Int[(e+f*x)^(m+1)/(c+I*d+c*E^(2*I*a+2*I*b*x)),x] /;
FreeQ[{a,b,c,d,e,f},x] && IGtQ[m,0] && EqQ[(c+I*d)^2,-1]
```

```
Int[(e_.+f_.*x_)^m_.*ArcTan[c_.+d_.*Cot[a_.+b_.*x_]],x_Symbol] :=
  (e+f*x)^(m+1)*ArcTan[c+d*Cot[a+b*x]]/(f*(m+1)) -
  I*b/(f*(m+1))*Int[(e+f*x)^(m+1)/(c-I*d-c*E^(2*I*a+2*I*b*x)),x] /;
FreeQ[{a,b,c,d,e,f},x] && IGtQ[m,0] && EqQ[(c-I*d)^2,-1]
```

```
Int[(e_.+f_.*x_)^m_.*ArcCot[c_.+d_.*Cot[a_.+b_.*x_]],x_Symbol] :=
  (e+f*x)^(m+1)*ArcCot[c+d*Cot[a+b*x]]/(f*(m+1)) +
  I*b/(f*(m+1))*Int[(e+f*x)^(m+1)/(c-I*d-c*E^(2*I*a+2*I*b*x)),x] /;
FreeQ[{a,b,c,d,e,f},x] && IGtQ[m,0] && EqQ[(c-I*d)^2,-1]
```

2: $\int (e+f\,x)^m \operatorname{ArcTan}[c+d\operatorname{Tan}[a+b\,x]]\,dx$ when $m\in\mathbb{Z}^+ \wedge (c+i\,d)^2\neq -1$

Derivation: Integration by parts

- **Basis:** $\partial_x \operatorname{ArcTan}[c+d\operatorname{Tan}[a+b\,x]] == \frac{b\,(1+i\,c+d)\,e^{2\,i\,a+2\,i\,b\,x}}{1+i\,c-d+(1+i\,c+d)\,e^{2\,i\,a+2\,i\,b\,x}} - \frac{b\,(1-i\,c-d)\,e^{2\,i\,a+2\,i\,b\,x}}{1-i\,c+d+(1-i\,c-d)\,e^{2\,i\,a+2\,i\,b\,x}}$
- **Rule: If** $m\in\mathbb{Z}^+ \wedge (c+i\,d)^2\neq -1$**, then**

$$\int (e+f\,x)^m \operatorname{ArcTan}[c+d\operatorname{Tan}[a+b\,x]]\,dx \longrightarrow$$

$$\frac{(e+f\,x)^{m+1}\operatorname{ArcTan}[c+d\operatorname{Tan}[a+b\,x]]}{f\,(m+1)} - \frac{b\,(1+i\,c+d)}{f\,(m+1)}\int \frac{(e+f\,x)^{m+1}\,e^{2\,i\,a+2\,i\,b\,x}}{1+i\,c-d+(1+i\,c+d)\,e^{2\,i\,a+2\,i\,b\,x}}\,dx + \frac{b\,(1-i\,c-d)}{f\,(m+1)}\int \frac{(e+f\,x)^{m+1}\,e^{2\,i\,a+2\,i\,b\,x}}{1-i\,c+d+(1-i\,c-d)\,e^{2\,i\,a+2\,i\,b\,x}}\,dx$$

Program code:

```
Int[(e_.+f_.*x_)^m_.*ArcTan[c_.+d_.*Tan[a_.+b_.*x_]],x_Symbol] :=
  (e+f*x)^(m+1)*ArcTan[c+d*Tan[a+b*x]]/(f*(m+1)) -
  b*(1+I*c+d)/(f*(m+1))*Int[(e+f*x)^(m+1)*E^(2*I*a+2*I*b*x)/(1+I*c-d+(1+I*c+d)*E^(2*I*a+2*I*b*x)),x] +
  b*(1-I*c-d)/(f*(m+1))*Int[(e+f*x)^(m+1)*E^(2*I*a+2*I*b*x)/(1-I*c+d+(1-I*c-d)*E^(2*I*a+2*I*b*x)),x] /;
FreeQ[{a,b,c,d,e,f},x] && IGtQ[m,0] && NeQ[(c+I*d)^2,-1]
```

```
Int[(e_.+f_.*x_)^m_.*ArcCot[c_.+d_.*Tan[a_.+b_.*x_]],x_Symbol] :=
  (e+f*x)^(m+1)*ArcCot[c+d*Tan[a+b*x]]/(f*(m+1)) +
  b*(1+I*c+d)/(f*(m+1))*Int[(e+f*x)^(m+1)*E^(2*I*a+2*I*b*x)/(1+I*c-d+(1+I*c+d)*E^(2*I*a+2*I*b*x)),x] -
  b*(1-I*c-d)/(f*(m+1))*Int[(e+f*x)^(m+1)*E^(2*I*a+2*I*b*x)/(1-I*c+d+(1-I*c-d)*E^(2*I*a+2*I*b*x)),x] /;
FreeQ[{a,b,c,d,e,f},x] && IGtQ[m,0] && NeQ[(c+I*d)^2,-1]
```

```
Int[(e_.+f_.*x_)^m_.*ArcTan[c_.+d_.*Cot[a_.+b_.*x_]],x_Symbol] :=
  (e+f*x)^(m+1)*ArcTan[c+d*Cot[a+b*x]]/(f*(m+1)) +
  b*(1+I*c-d)/(f*(m+1))*Int[(e+f*x)^(m+1)*E^(2*I*a+2*I*b*x)/(1+I*c+d-(1+I*c-d)*E^(2*I*a+2*I*b*x)),x] -
  b*(1-I*c+d)/(f*(m+1))*Int[(e+f*x)^(m+1)*E^(2*I*a+2*I*b*x)/(1-I*c-d-(1-I*c+d)*E^(2*I*a+2*I*b*x)),x] /;
FreeQ[{a,b,c,d,e,f},x] && IGtQ[m,0] && NeQ[(c-I*d)^2,-1]
```

```
Int[(e_.+f_.*x_)^m_.*ArcCot[c_.+d_.*Cot[a_.+b_.*x_]],x_Symbol] :=
  (e+f*x)^(m+1)*ArcCot[c+d*Cot[a+b*x]]/(f*(m+1)) -
  b*(1+I*c-d)/(f*(m+1))*Int[(e+f*x)^(m+1)*E^(2*I*a+2*I*b*x)/(1+I*c+d-(1+I*c-d)*E^(2*I*a+2*I*b*x)),x] +
  b*(1-I*c+d)/(f*(m+1))*Int[(e+f*x)^(m+1)*E^(2*I*a+2*I*b*x)/(1-I*c-d-(1-I*c+d)*E^(2*I*a+2*I*b*x)),x] /;
FreeQ[{a,b,c,d,e,f},x] && IGtQ[m,0] && NeQ[(c-I*d)^2,-1]
```

8. $\int u\,\text{ArcTan}[c+d\,\text{Tanh}[a+b\,x]]\,dx$

1. $\int u\,\text{ArcTan}[\text{Tanh}[a+b\,x]]\,dx$

1: $\int \text{ArcTan}[\text{Tanh}[a+b\,x]]\,dx$

–

Derivation: Integration by parts

Basis: $\partial_x \text{ArcTan}[\text{Tanh}[a+b\,x]] == b\,\text{Sech}[2\,a+2\,b\,x]$

Rule:

$$\int \text{ArcTan}[\text{Tanh}[a+b\,x]]\,dx \longrightarrow x\,\text{ArcTan}[\text{Tanh}[a+b\,x]] - b\int x\,\text{Sech}[2\,a+2\,b\,x]\,dx$$

Program code:

```
Int[ArcTan[Tanh[a_.+b_.*x_]],x_Symbol] :=
  x*ArcTan[Tanh[a+b*x]] - b*Int[x*Sech[2*a+2*b*x],x] /;
FreeQ[{a,b},x]
```

```
Int[ArcCot[Tanh[a_.+b_.*x_]],x_Symbol] :=
  x*ArcCot[Tanh[a+b*x]] + b*Int[x*Sech[2*a+2*b*x],x] /;
FreeQ[{a,b},x]
```

```
Int[ArcTan[Coth[a_.+b_.*x_]],x_Symbol] :=
  x*ArcTan[Coth[a+b*x]] + b*Int[x*Sech[2*a+2*b*x],x] /;
FreeQ[{a,b},x]
```

```
Int[ArcCot[Coth[a_.+b_.*x_]],x_Symbol] :=
  x*ArcCot[Coth[a+b*x]] - b*Int[x*Sech[2*a+2*b*x],x] /;
FreeQ[{a,b},x]
```

2: $\int (e + f x)^m \text{ArcTan}[\text{Tanh}[a + b x]] \, dx$ when $m \in \mathbb{Z}^+$

Derivation: Integration by parts

Basis: $\partial_x \text{ArcTan}[\text{Tanh}[a + b x]] == b \, \text{Sech}[2 a + 2 b x]$

Rule: If $m \in \mathbb{Z}^+$**, then**

$$\int (e + f x)^m \text{ArcTan}[\text{Tanh}[a + b x]] \, dx \longrightarrow \frac{(e + f x)^{m+1} \text{ArcTan}[\text{Tanh}[a + b x]]}{f (m + 1)} - \frac{b}{f (m + 1)} \int (e + f x)^{m+1} \text{Sech}[2 a + 2 b x] \, dx$$

Program code:

```
Int[(e_.+f_.*x_)^m_.*ArcTan[Tanh[a_.+b_.*x_]],x_Symbol] :=
  (e+f*x)^(m+1)*ArcTan[Tanh[a+b*x]]/(f*(m+1)) - b/(f*(m+1))*Int[(e+f*x)^(m+1)*Sech[2*a+2*b*x],x] /;
FreeQ[{a,b,e,f},x] && IGtQ[m,0]
```

```
Int[(e_.+f_.*x_)^m_.*ArcCot[Tanh[a_.+b_.*x_]],x_Symbol] :=
  (e+f*x)^(m+1)*ArcCot[Tanh[a+b*x]]/(f*(m+1)) + b/(f*(m+1))*Int[(e+f*x)^(m+1)*Sech[2*a+2*b*x],x] /;
FreeQ[{a,b,e,f},x] && IGtQ[m,0]
```

```
Int[(e_.+f_.*x_)^m_.*ArcTan[Coth[a_.+b_.*x_]],x_Symbol] :=
  (e+f*x)^(m+1)*ArcTan[Coth[a+b*x]]/(f*(m+1)) + b/(f*(m+1))*Int[(e+f*x)^(m+1)*Sech[2*a+2*b*x],x] /;
FreeQ[{a,b,e,f},x] && IGtQ[m,0]
```

```
Int[(e_.+f_.*x_)^m_.*ArcCot[Coth[a_.+b_.*x_]],x_Symbol] :=
  (e+f*x)^(m+1)*ArcCot[Coth[a+b*x]]/(f*(m+1)) - b/(f*(m+1))*Int[(e+f*x)^(m+1)*Sech[2*a+2*b*x],x] /;
FreeQ[{a,b,e,f},x] && IGtQ[m,0]
```

2. $\int u \, \text{ArcTan}[c + d \, \text{Tanh}[a + b \, x]] \, dx$

   1. $\int \text{ArcTan}[c + d \, \text{Tanh}[a + b \, x]] \, dx$

      1: $\int \text{ArcTan}[c + d \, \text{Tanh}[a + b \, x]] \, dx$ when $(c - d)^2 == -1$

Derivation: Integration by parts

Basis: If $(c - d)^2 == -1$**, then** $\partial_x \text{ArcTan}[c + d \, \text{Tanh}[a + b \, x]] == \frac{b}{c-d+c \, e^{2 a+2 b x}}$

Rule: If $(c - d)^2 == -1$**, then**

$$\int \text{ArcTan}[c + d \, \text{Tanh}[a + b \, x]] \, dx \longrightarrow x \, \text{ArcTan}[c + d \, \text{Tanh}[a + b \, x]] - b \int \frac{x}{c - d + c \, e^{2 a+2 b x}} \, dx$$

Program code:

```
Int[ArcTan[c_.+d_.*Tanh[a_.+b_.*x_]],x_Symbol] :=
  x*ArcTan[c+d*Tanh[a+b*x]] -
  b*Int[x/(c-d+c*E^(2*a+2*b*x)),x] /;
FreeQ[{a,b,c,d},x] && EqQ[(c-d)^2,-1]
```

```
Int[ArcCot[c_.+d_.*Tanh[a_.+b_.*x_]],x_Symbol] :=
  x*ArcCot[c+d*Tanh[a+b*x]] +
  b*Int[x/(c-d+c*E^(2*a+2*b*x)),x] /;
FreeQ[{a,b,c,d},x] && EqQ[(c-d)^2,-1]
```

```
Int[ArcTan[c_.+d_.*Coth[a_.+b_.*x_]],x_Symbol] :=
  x*ArcTan[c+d*Coth[a+b*x]] -
  b*Int[x/(c-d-c*E^(2*a+2*b*x)),x] /;
FreeQ[{a,b,c,d},x] && EqQ[(c-d)^2,-1]
```

```
Int[ArcCot[c_.+d_.*Coth[a_.+b_.*x_]],x_Symbol] :=
  x*ArcCot[c+d*Coth[a+b*x]] +
  b*Int[x/(c-d-c*E^(2*a+2*b*x)),x] /;
FreeQ[{a,b,c,d},x] && EqQ[(c-d)^2,-1]
```

2: $\int \text{ArcTan}[c + d\,\text{Tanh}[a + b\,x]]\,dx$ when $(c - d)^2 \neq -1$

Derivation: Integration by parts

- **Basis:** $\partial_x \text{ArcTan}[c + d\,\text{Tanh}[a + b\,x]] == -\frac{i\,b\,(i-c-d)\,e^{2a+2bx}}{i-c+d+(i-c-d)\,e^{2a+2bx}} + \frac{i\,b\,(i+c+d)\,e^{2a+2bx}}{i+c-d+(i+c+d)\,e^{2a+2bx}}$

- **Rule: If** $(c - d)^2 \neq -1$**, then**

$$\int \text{ArcTan}[c + d\,\text{Tanh}[a + b\,x]]\,dx \longrightarrow$$

$$x\,\text{ArcTan}[c + d\,\text{Tanh}[a + b\,x]] + i\,b\,(i - c - d)\int \frac{x\,e^{2a+2bx}}{i - c + d + (i - c - d)\,e^{2a+2bx}}\,dx - i\,b\,(i + c + d)\int \frac{x\,e^{2a+2bx}}{i + c - d + (i + c + d)\,e^{2a+2bx}}\,dx$$

- **Program code:**

```
Int[ArcTan[c_.+d_.*Tanh[a_.+b_.*x_]],x_Symbol] :=
  x*ArcTan[c+d*Tanh[a+b*x]] +
  I*b*(I-c-d)*Int[x*E^(2*a+2*b*x)/(I-c+d+(I-c-d)*E^(2*a+2*b*x)),x] -
  I*b*(I+c+d)*Int[x*E^(2*a+2*b*x)/(I+c-d+(I+c+d)*E^(2*a+2*b*x)),x] /;
FreeQ[{a,b,c,d},x] && NeQ[(c-d)^2,-1]
```

```
Int[ArcCot[c_.+d_.*Tanh[a_.+b_.*x_]],x_Symbol] :=
  x*ArcCot[c+d*Tanh[a+b*x]] -
  I*b*(I-c-d)*Int[x*E^(2*a+2*b*x)/(I-c+d+(I-c-d)*E^(2*a+2*b*x)),x] +
  I*b*(I+c+d)*Int[x*E^(2*a+2*b*x)/(I+c-d+(I+c+d)*E^(2*a+2*b*x)),x] /;
FreeQ[{a,b,c,d},x] && NeQ[(c-d)^2,-1]
```

```
Int[ArcTan[c_.+d_.*Coth[a_.+b_.*x_]],x_Symbol] :=
  x*ArcTan[c+d*Coth[a+b*x]] -
  I*b*(I-c-d)*Int[x*E^(2*a+2*b*x)/(I-c+d-(I-c-d)*E^(2*a+2*b*x)),x] +
  I*b*(I+c+d)*Int[x*E^(2*a+2*b*x)/(I+c-d-(I+c+d)*E^(2*a+2*b*x)),x] /;
FreeQ[{a,b,c,d},x] && NeQ[(c-d)^2,-1]
```

```
Int[ArcCot[c_.+d_.*Coth[a_.+b_.*x_]],x_Symbol] :=
  x*ArcCot[c+d*Coth[a+b*x]] +
  I*b*(I-c-d)*Int[x*E^(2*a+2*b*x)/(I-c+d-(I-c-d)*E^(2*a+2*b*x)),x] -
  I*b*(I+c+d)*Int[x*E^(2*a+2*b*x)/(I+c-d-(I+c+d)*E^(2*a+2*b*x)),x] /;
FreeQ[{a,b,c,d},x] && NeQ[(c-d)^2,-1]
```

2. $\int (e + f x)^m \operatorname{ArcTan}[c + d \operatorname{Tanh}[a + b x]] \, dx$

1: $\int (e + f x)^m \operatorname{ArcTan}[c + d \operatorname{Tanh}[a + b x]] \, dx$ when $m \in \mathbb{Z}^+ \wedge (c - d)^2 == -1$

Derivation: Integration by parts

Basis: If $(c - d)^2 == -1$**, then** $\partial_x \operatorname{ArcTan}[c + d \operatorname{Tanh}[a + b x]] == \frac{b}{c-d+c\, e^{2a+2bx}}$

Rule: If $m \in \mathbb{Z}^+ \wedge (c - d)^2 == -1$**, then**

$$\int (e + f x)^m \operatorname{ArcTan}[c + d \operatorname{Tanh}[a + b x]] \, dx \longrightarrow \frac{(e + f x)^{m+1} \operatorname{ArcTan}[c + d \operatorname{Tanh}[a + b x]]}{f\,(m + 1)} - \frac{b}{f\,(m + 1)} \int \frac{(e + f x)^{m+1}}{c - d + c\, e^{2a+2bx}} \, dx$$

Program code:

```
Int[(e_.+f_.*x_)^m_.*ArcTan[c_.+d_.*Tanh[a_.+b_.*x_]],x_Symbol] :=
  (e+f*x)^(m+1)*ArcTan[c+d*Tanh[a+b*x]]/(f*(m+1)) -
  b/(f*(m+1))*Int[(e+f*x)^(m+1)/(c-d+c*E^(2*a+2*b*x)),x] /;
FreeQ[{a,b,c,d,e,f},x] && IGtQ[m,0] && EqQ[(c-d)^2,-1]
```

```
Int[(e_.+f_.*x_)^m_.*ArcCot[c_.+d_.*Tanh[a_.+b_.*x_]],x_Symbol] :=
  (e+f*x)^(m+1)*ArcCot[c+d*Tanh[a+b*x]]/(f*(m+1)) +
  b/(f*(m+1))*Int[(e+f*x)^(m+1)/(c-d+c*E^(2*a+2*b*x)),x] /;
FreeQ[{a,b,c,d,e,f},x] && IGtQ[m,0] && EqQ[(c-d)^2,-1]
```

```
Int[(e_.+f_.*x_)^m_.*ArcTan[c_.+d_.*Coth[a_.+b_.*x_]],x_Symbol] :=
  (e+f*x)^(m+1)*ArcTan[c+d*Coth[a+b*x]]/(f*(m+1)) -
  b/(f*(m+1))*Int[(e+f*x)^(m+1)/(c-d-c*E^(2*a+2*b*x)),x] /;
FreeQ[{a,b,c,d,e,f},x] && IGtQ[m,0] && EqQ[(c-d)^2,-1]
```

```
Int[(e_.+f_.*x_)^m_.*ArcCot[c_.+d_.*Coth[a_.+b_.*x_]],x_Symbol] :=
  (e+f*x)^(m+1)*ArcCot[c+d*Coth[a+b*x]]/(f*(m+1)) +
  b/(f*(m+1))*Int[(e+f*x)^(m+1)/(c-d-c*E^(2*a+2*b*x)),x] /;
FreeQ[{a,b,c,d,e,f},x] && IGtQ[m,0] && EqQ[(c-d)^2,-1]
```

2: $\int (e+f\,x)^m \operatorname{ArcTan}[c+d\operatorname{Tanh}[a+b\,x]]\,dx$ when $m \in \mathbb{Z}^+ \wedge (c-d)^2 \neq -1$

Derivation: Integration by parts

- **Basis:** $\partial_x \operatorname{ArcTan}[c+d\operatorname{Tanh}[a+b\,x]] == -\frac{i\,b\,(i-c-d)\,e^{2a+2bx}}{i-c+d+(i-c-d)\,e^{2a+2bx}} + \frac{i\,b\,(i+c+d)\,e^{2a+2bx}}{i+c-d+(i+c+d)\,e^{2a+2bx}}$

Rule: If $m \in \mathbb{Z}^+ \wedge (c-d)^2 \neq -1$, then

$$\int (e+f\,x)^m \operatorname{ArcTan}[c+d\operatorname{Tanh}[a+b\,x]]\,dx \longrightarrow$$

$$\frac{(e+f\,x)^{m+1}\operatorname{ArcTan}[c+d\operatorname{Tanh}[a+b\,x]]}{f\,(m+1)} + \frac{i\,b\,(i-c-d)}{f\,(m+1)}\int \frac{(e+f\,x)^{m+1}\,e^{2a+2bx}}{i-c+d+(i-c-d)\,e^{2a+2bx}}\,dx - \frac{i\,b\,(i+c+d)}{f\,(m+1)}\int \frac{(e+f\,x)^{m+1}\,e^{2a+2bx}}{i+c-d+(i+c+d)\,e^{2a+2bx}}\,dx$$

Program code:

```
Int[(e_.+f_.*x_)^m_.*ArcTan[c_.+d_.*Tanh[a_.+b_.*x_]],x_Symbol] :=
  (e+f*x)^(m+1)*ArcTan[c+d*Tanh[a+b*x]]/(f*(m+1)) +
  I*b*(I-c-d)/(f*(m+1))*Int[(e+f*x)^(m+1)*E^(2*a+2*b*x)/(I-c+d+(I-c-d)*E^(2*a+2*b*x)),x] -
  I*b*(I+c+d)/(f*(m+1))*Int[(e+f*x)^(m+1)*E^(2*a+2*b*x)/(I+c-d+(I+c+d)*E^(2*a+2*b*x)),x] /;
FreeQ[{a,b,c,d,e,f},x] && IGtQ[m,0] && NeQ[(c-d)^2,-1]
```

```
Int[(e_.+f_.*x_)^m_.*ArcCot[c_.+d_.*Tanh[a_.+b_.*x_]],x_Symbol] :=
  (e+f*x)^(m+1)*ArcCot[c+d*Tanh[a+b*x]]/(f*(m+1)) -
  I*b*(I-c-d)/(f*(m+1))*Int[(e+f*x)^(m+1)*E^(2*a+2*b*x)/(I-c+d+(I-c-d)*E^(2*a+2*b*x)),x] +
  I*b*(I+c+d)/(f*(m+1))*Int[(e+f*x)^(m+1)*E^(2*a+2*b*x)/(I+c-d+(I+c+d)*E^(2*a+2*b*x)),x] /;
FreeQ[{a,b,c,d,e,f},x] && IGtQ[m,0] && NeQ[(c-d)^2,-1]
```

```
Int[(e_.+f_.*x_)^m_.*ArcTan[c_.+d_.*Coth[a_.+b_.*x_]],x_Symbol] :=
  (e+f*x)^(m+1)*ArcTan[c+d*Coth[a+b*x]]/(f*(m+1)) -
  I*b*(I-c-d)/(f*(m+1))*Int[(e+f*x)^(m+1)*E^(2*a+2*b*x)/(I-c+d-(I-c-d)*E^(2*a+2*b*x)),x] +
  I*b*(I+c+d)/(f*(m+1))*Int[(e+f*x)^(m+1)*E^(2*a+2*b*x)/(I+c-d-(I+c+d)*E^(2*a+2*b*x)),x] /;
FreeQ[{a,b,c,d,e,f},x] && IGtQ[m,0] && NeQ[(c-d)^2,-1]
```

```
Int[(e_.+f_.*x_)^m_.*ArcCot[c_.+d_.*Coth[a_.+b_.*x_]],x_Symbol] :=
  (e+f*x)^(m+1)*ArcCot[c+d*Coth[a+b*x]]/(f*(m+1)) +
  I*b*(I-c-d)/(f*(m+1))*Int[(e+f*x)^(m+1)*E^(2*a+2*b*x)/(I-c+d-(I-c-d)*E^(2*a+2*b*x)),x] -
  I*b*(I+c+d)/(f*(m+1))*Int[(e+f*x)^(m+1)*E^(2*a+2*b*x)/(I+c-d-(I+c+d)*E^(2*a+2*b*x)),x] /;
FreeQ[{a,b,c,d,e,f},x] && IGtQ[m,0] && NeQ[(c-d)^2,-1]
```

9. $\int v\,(a+b\,\text{ArcTan}[u])\,dx$ when u is free of inverse functions

1: $\int \text{ArcTan}[u]\,dx$ when u is free of inverse functions

Derivation: Integration by parts

Rule: If u is free of inverse functions, then

$$\int \text{ArcTan}[u]\,dx \longrightarrow x\,\text{ArcTan}[u]-\int \frac{x\,\partial_x u}{1+u^2}\,dx$$

Program code:

```
Int[ArcTan[u_],x_Symbol] :=
  x*ArcTan[u] -
  Int[SimplifyIntegrand[x*D[u,x]/(1+u^2),x],x] /;
InverseFunctionFreeQ[u,x]
```

```
Int[ArcCot[u_],x_Symbol] :=
  x*ArcCot[u] +
  Int[SimplifyIntegrand[x*D[u,x]/(1+u^2),x],x] /;
InverseFunctionFreeQ[u,x]
```

2: $\int (c+d\,x)^m\,(a+b\,\text{ArcTan}[u])\,dx$ when $m \neq -1 \land$ u is free of inverse functions

Derivation: Integration by parts

Rule: If $m \neq -1 \land$ u is free of inverse functions, then

$$\int (c+d\,x)^m\,(a+b\,\text{ArcTan}[u])\,dx \longrightarrow \frac{(c+d\,x)^{m+1}\,(a+b\,\text{ArcTan}[u])}{d\,(m+1)}-\frac{b}{d\,(m+1)}\int \frac{(c+d\,x)^{m+1}\,\partial_x u}{1+u^2}\,dx$$

Program code:

```
Int[(c_.+d_.*x_)^m_.*(a_.+b_.*ArcTan[u_]),x_Symbol] :=
  (c+d*x)^(m+1)*(a+b*ArcTan[u])/(d*(m+1)) -
  b/(d*(m+1))*Int[SimplifyIntegrand[(c+d*x)^(m+1)*D[u,x]/(1+u^2),x],x] /;
FreeQ[{a,b,c,d,m},x] && NeQ[m,-1] && InverseFunctionFreeQ[u,x] && Not[FunctionOfQ[(c+d*x)^(m+1),u,x]] && FalseQ[PowerVariableExpn[u
```

```
Int[(c_.+d_.*x_)^m_.*(a_.+b_.*ArcCot[u_]),x_Symbol] :=
  (c+d*x)^(m+1)*(a+b*ArcCot[u])/(d*(m+1)) +
  b/(d*(m+1))*Int[SimplifyIntegrand[(c+d*x)^(m+1)*D[u,x]/(1+u^2),x],x] /;
FreeQ[{a,b,c,d,m},x] && NeQ[m,-1] && InverseFunctionFreeQ[u,x] && Not[FunctionOfQ[(c+d*x)^(m+1),u,x]] && FalseQ[PowerVariableExpn[u
```

3: $\int v\,(a+b\,\text{ArcTan}[u])\,dx$ when u and $\int v\,dx$ are free of inverse functions

Derivation: Integration by parts

Rule: If u is free of inverse functions, let $w = \int v\,dx$, if w is free of inverse functions, then

$$\int v\,(a+b\,\text{ArcTan}[u])\,dx \longrightarrow w\,(a+b\,\text{ArcTan}[u])-b\int\frac{w\,\partial_x u}{1+u^2}\,dx$$

Program code:

```
Int[v_*(a_.+b_.*ArcTan[u_]),x_Symbol] :=
  With[{w=IntHide[v,x]},
  Dist[(a+b*ArcTan[u]),w,x] - b*Int[SimplifyIntegrand[w*D[u,x]/(1+u^2),x],x] /;
 InverseFunctionFreeQ[w,x]] /;
FreeQ[{a,b},x] && InverseFunctionFreeQ[u,x] && Not[MatchQ[v, (c_.+d_.*x)^m_. /; FreeQ[{c,d,m},x]]] && FalseQ[FunctionOfLinear[v*(a+
```

```
Int[v_*(a_.+b_.*ArcCot[u_]),x_Symbol] :=
  With[{w=IntHide[v,x]},
  Dist[(a+b*ArcCot[u]),w,x] + b*Int[SimplifyIntegrand[w*D[u,x]/(1+u^2),x],x] /;
 InverseFunctionFreeQ[w,x]] /;
FreeQ[{a,b},x] && InverseFunctionFreeQ[u,x] && Not[MatchQ[v, (c_.+d_.*x)^m_. /; FreeQ[{c,d,m},x]]] && FalseQ[FunctionOfLinear[v*(a+
```

10: $\int \frac{\text{ArcTan}[v]\,\text{Log}[w]}{a+b\,x}\,dx$ when $\partial_x \frac{v}{a+b\,x} == 0 \bigwedge \partial_x \frac{w}{a+b\,x} == 0$

Derivation: Algebraic expansion

Basis: $\text{ArcTan}[z] == \frac{i}{2}\,\text{Log}[1-i\,z] - \frac{i}{2}\,\text{Log}[1+i\,z]$

Rule: If $\partial_x \frac{v}{a+b\,x} == 0 \bigwedge \partial_x \frac{w}{a+b\,x} == 0$**, then**

$$\int \frac{\text{ArcTan}[v]\,\text{Log}[w]}{a+b\,x}\,dx \longrightarrow \frac{i}{2}\int \frac{\text{Log}[1-i\,v]\,\text{Log}[w]}{a+b\,x}\,dx - \frac{i}{2}\int \frac{\text{Log}[1+i\,v]\,\text{Log}[w]}{a+b\,x}\,dx$$

Program code:

```
Int[ArcTan[v_]*Log[w_]/(a_.+b_.*x_),x_Symbol] :=
  I/2*Int[Log[1-I*v]*Log[w]/(a+b*x),x] - I/2*Int[Log[1+I*v]*Log[w]/(a+b*x),x] /;
FreeQ[{a,b},x] && LinearQ[v,x] && LinearQ[w,x] && EqQ[Simplify[D[v/(a+b*x),x]],0] && EqQ[Simplify[D[w/(a+b*x),x]],0]
```

11. $\int u\,\text{ArcTan}[v]\,\text{Log}[w]\,dx$ when v, w and $\int u\,dx$ are free of inverse functions

1: $\int \text{ArcTan}[v]\,\text{Log}[w]\,dx$ when v and w are free of inverse functions

Derivation: Integration by parts

Rule: If v **and** w **are free of inverse functions, then**

$$\int \text{ArcTan}[v]\,\text{Log}[w]\,dx \longrightarrow x\,\text{ArcTan}[v]\,\text{Log}[w] - \int \frac{x\,\text{Log}[w]\,\partial_x v}{1+v^2}\,dx - \int \frac{x\,\text{ArcTan}[v]\,\partial_x w}{w}\,dx$$

Program code:

```
Int[ArcTan[v_]*Log[w_],x_Symbol] :=
  x*ArcTan[v]*Log[w] -
  Int[SimplifyIntegrand[x*Log[w]*D[v,x]/(1+v^2),x],x] -
  Int[SimplifyIntegrand[x*ArcTan[v]*D[w,x]/w,x],x] /;
InverseFunctionFreeQ[v,x] && InverseFunctionFreeQ[w,x]
```

```
Int[ArcCot[v_]*Log[w_],x_Symbol] :=
  x*ArcCot[v]*Log[w] +
  Int[SimplifyIntegrand[x*Log[w]*D[v,x]/(1+v^2),x],x] -
  Int[SimplifyIntegrand[x*ArcCot[v]*D[w,x]/w,x],x] /;
InverseFunctionFreeQ[v,x] && InverseFunctionFreeQ[w,x]
```

2: $\int u\,\text{ArcTan}[v]\,\text{Log}[w]\,dx$ when v, w and $\int u\,dx$ are free of inverse functions

- **Derivation: Integration by parts**
- **Rule: If v and w are free of inverse functions, let $z = \int u\,dx$, if z is free of inverse functions, then**

$$\int u\,\text{ArcTan}[v]\,\text{Log}[w]\,dx \longrightarrow z\,\text{ArcTan}[v]\,\text{Log}[w] - \int \frac{z\,\text{Log}[w]\,\partial_x v}{1+v^2}\,dx - \int \frac{z\,\text{ArcTan}[v]\,\partial_x w}{w}\,dx$$

- **Program code:**

```
Int[u_*ArcTan[v_]*Log[w_],x_Symbol] :=
  With[{z=IntHide[u,x]},
  Dist[ArcTan[v]*Log[w],z,x] -
  Int[SimplifyIntegrand[z*Log[w]*D[v,x]/(1+v^2),x],x] -
  Int[SimplifyIntegrand[z*ArcTan[v]*D[w,x]/w,x],x] /;
 InverseFunctionFreeQ[z,x]] /;
InverseFunctionFreeQ[v,x] && InverseFunctionFreeQ[w,x]
```

```
Int[u_*ArcCot[v_]*Log[w_],x_Symbol] :=
  With[{z=IntHide[u,x]},
  Dist[ArcCot[v]*Log[w],z,x] +
  Int[SimplifyIntegrand[z*Log[w]*D[v,x]/(1+v^2),x],x] -
  Int[SimplifyIntegrand[z*ArcCot[v]*D[w,x]/w,x],x] /;
 InverseFunctionFreeQ[z,x]] /;
InverseFunctionFreeQ[v,x] && InverseFunctionFreeQ[w,x]
```