Maple 2018.2 Integration Test Results
on the problems in "6 Hyperbolic functions/6.4 Hyperbolic cotangent"

Test results for the 19 problems in "6.4.1 (c+d x)^m (a+b coth)^n.txt"

Problem 1: Result more than twice size of optimal antiderivative.

$$\int x^3 \coth(b x + a)\, dx$$

Optimal(type 4, 79 leaves, 6 steps):

$$-\frac{x^4}{4} + \frac{x^3 \ln(1 - e^{2 b x + 2 a})}{b} + \frac{3 x^2 \operatorname{polylog}(2, e^{2 b x + 2 a})}{2 b^2} - \frac{3 x \operatorname{polylog}(3, e^{2 b x + 2 a})}{2 b^3} + \frac{3 \operatorname{polylog}(4, e^{2 b x + 2 a})}{4 b^4}$$

Result(type 4, 199 leaves):

$$-\frac{x^4}{4} - \frac{3 a^4}{2 b^4} + \frac{3 \operatorname{polylog}(2, -e^{b x + a}) x^2}{b^2} - \frac{2 a^3 x}{b^3} - \frac{6 \operatorname{polylog}(3, -e^{b x + a}) x}{b^3} + \frac{\ln(1 - e^{b x + a}) x^3}{b} + \frac{3 \operatorname{polylog}(2, e^{b x + a}) x^2}{b^2} - \frac{6 \operatorname{polylog}(3, e^{b x + a}) x}{b^3}$$
$$+ \frac{\ln(1 + e^{b x + a}) x^3}{b} + \frac{6 \operatorname{polylog}(4, e^{b x + a})}{b^4} + \frac{6 \operatorname{polylog}(4, -e^{b x + a})}{b^4} + \frac{\ln(1 - e^{b x + a}) a^3}{b^4} + \frac{2 a^3 \ln(e^{b x + a})}{b^4} - \frac{a^3 \ln(e^{b x + a} - 1)}{b^4}$$

Problem 2: Result more than twice size of optimal antiderivative.

$$\int x \coth(b x + a)\, dx$$

Optimal(type 4, 41 leaves, 4 steps):

$$-\frac{x^2}{2} + \frac{x \ln(1 - e^{2 b x + 2 a})}{b} + \frac{\operatorname{polylog}(2, e^{2 b x + 2 a})}{2 b^2}$$

Result(type 4, 121 leaves):

$$-\frac{x^2}{2} - \frac{2 a x}{b} - \frac{a^2}{b^2} + \frac{\ln(1 - e^{b x + a}) x}{b} + \frac{a \ln(1 - e^{b x + a})}{b^2} + \frac{\operatorname{polylog}(2, e^{b x + a})}{b^2} + \frac{\ln(1 + e^{b x + a}) x}{b} + \frac{\operatorname{polylog}(2, -e^{b x + a})}{b^2} + \frac{2 a \ln(e^{b x + a})}{b^2}$$
$$- \frac{a \ln(e^{b x + a} - 1)}{b^2}$$

Problem 3: Result more than twice size of optimal antiderivative.

$$\int x^2 \coth(b x + a)^2\, dx$$

Optimal(type 4, 63 leaves, 6 steps):

$$-\frac{x^2}{b} + \frac{x^3}{3} - \frac{x^2 \coth(b x + a)}{b} + \frac{2 x \ln(1 - e^{2 b x + 2 a})}{b^2} + \frac{\operatorname{polylog}(2, e^{2 b x + 2 a})}{b^3}$$

Result(type 4, 155 leaves):

$$\frac{x^3}{3}-\frac{2x^2}{b\left(e^{2bx+2a}-1\right)}-\frac{2x^2}{b}-\frac{4ax}{b^2}-\frac{2a^2}{b^3}+\frac{2\ln\left(1-e^{bx+a}\right)x}{b^2}+\frac{2\ln\left(1-e^{bx+a}\right)a}{b^3}+\frac{2\operatorname{polylog}\left(2,e^{bx+a}\right)}{b^3}+\frac{2\ln\left(1+e^{bx+a}\right)x}{b^2}+\frac{2\operatorname{polylog}\left(2,-e^{bx+a}\right)}{b^3}+\frac{4a\ln\left(e^{bx+a}\right)}{b^3}-\frac{2a\ln\left(e^{bx+a}-1\right)}{b^3}$$

Problem 6: Result more than twice size of optimal antiderivative.

$$\int\frac{(dx+c)^3}{a+a\coth(fx+e)}\,dx$$

Optimal(type 3, 153 leaves, 5 steps):

$$\frac{3d^3x}{8af^3}+\frac{3d(dx+c)^2}{8af^2}+\frac{(dx+c)^3}{4af}+\frac{(dx+c)^4}{8ad}-\frac{3d^3}{8f^4(a+a\coth(fx+e))}-\frac{3d^2(dx+c)}{4f^3(a+a\coth(fx+e))}-\frac{3d(dx+c)^2}{4f^2(a+a\coth(fx+e))}-\frac{(dx+c)^3}{2f(a+a\coth(fx+e))}$$

Result(type 3, 928 leaves):

$$\frac{1}{f^4a}\Bigg(-d^3\left(\frac{(fx+e)^3\cosh(fx+e)\sinh(fx+e)}{2}-\frac{(fx+e)^4}{8}-\frac{3(fx+e)^2\cosh(fx+e)^2}{4}+\frac{3(fx+e)\cosh(fx+e)\sinh(fx+e)}{4}+\frac{3(fx+e)^2}{8}-\frac{3\cosh(fx+e)^2}{8}\right)-3cd^2f\left(\frac{(fx+e)^2\cosh(fx+e)\sinh(fx+e)}{2}-\frac{(fx+e)^3}{6}-\frac{(fx+e)\cosh(fx+e)^2}{2}+\frac{\sinh(fx+e)\cosh(fx+e)}{4}+\frac{fx}{4}+\frac{e}{4}\right)+3d^3e\left(\frac{(fx+e)^2\cosh(fx+e)\sinh(fx+e)}{2}-\frac{(fx+e)^3}{6}-\frac{(fx+e)\cosh(fx+e)^2}{2}+\frac{\sinh(fx+e)\cosh(fx+e)}{4}+\frac{fx}{4}+\frac{e}{4}\right)-3c^2df^2\left(\frac{(fx+e)\cosh(fx+e)\sinh(fx+e)}{2}-\frac{(fx+e)^2}{4}-\frac{\cosh(fx+e)^2}{4}\right)+6cd^2ef\left(\frac{(fx+e)\cosh(fx+e)\sinh(fx+e)}{2}-\frac{(fx+e)^2}{4}-\frac{\cosh(fx+e)^2}{4}\right)-3d^3e^2\left(\frac{(fx+e)\cosh(fx+e)\sinh(fx+e)}{2}-\frac{(fx+e)^2}{4}-\frac{\cosh(fx+e)^2}{4}\right)-c^3f^3\left(\frac{\sinh(fx+e)\cosh(fx+e)}{2}-\frac{fx}{2}-\frac{e}{2}\right)+3c^2def^2\left(\frac{\sinh(fx+e)\cosh(fx+e)}{2}-\frac{fx}{2}-\frac{e}{2}\right)-3cd^2e^2f\left(\frac{\sinh(fx+e)\cosh(fx+e)}{2}-\frac{fx}{2}-\frac{e}{2}\right)+d^3e^3\left(\frac{\sinh(fx+e)\cosh(fx+e)}{2}-\frac{fx}{2}-\frac{e}{2}\right)+d^3\left(\frac{(fx+e)^3\cosh(fx+e)^2}{2}-\frac{3(fx+e)^2\cosh(fx+e)\sinh(fx+e)}{4}-\frac{(fx+e)^3}{4}+\frac{3(fx+e)\cosh(fx+e)^2}{4}-\frac{3\sinh(fx+e)\cosh(fx+e)}{8}-\frac{3fx}{8}-\frac{3e}{8}\right)+3cd^2f\left(\frac{(fx+e)^2\cosh(fx+e)^2}{2}-\frac{(fx+e)\cosh(fx+e)\sinh(fx+e)}{2}-\frac{(fx+e)^2}{4}+\frac{\cosh(fx+e)^2}{4}\right)-3d^3e\left(\frac{(fx+e)^2\cosh(fx+e)^2}{2}-\frac{(fx+e)\cosh(fx+e)\sinh(fx+e)}{2}-\frac{(fx+e)^2}{4}+\frac{\cosh(fx+e)^2}{4}\right)+3c^2df^2\left(\frac{(fx+e)\cosh(fx+e)^2}{2}-\frac{\sinh(fx+e)\cosh(fx+e)}{4}-\frac{fx}{4}-\frac{e}{4}\right)-6cd^2ef\left(\frac{(fx+e)\cosh(fx+e)^2}{2}-\frac{\sinh(fx+e)\cosh(fx+e)}{4}-\frac{fx}{4}-\frac{e}{4}\right)+3d^3e^2\left(\frac{(fx+e)\cosh(fx+e)^2}{2}-\frac{\sinh(fx+e)\cosh(fx+e)}{4}-\frac{fx}{4}-\frac{e}{4}\right)+\frac{c^3f^3\cosh(fx+e)^2}{2}-\frac{3c^2def^2\cosh(fx+e)^2}{2}$$

$$+\frac{3 c d^2 e^2 f \cosh(fx+e)^2}{2} - \frac{d^3 e^3 \cosh(fx+e)^2}{2}\Bigg)$$

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Problem 7: Result more than twice size of optimal antiderivative.
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$$\int \frac{(dx+c)^2}{a+a\coth(fx+e)}\,dx$$

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Optimal(type 3, 110 leaves, 4 steps):
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$$\frac{d^2 x}{4 a f^2} + \frac{(dx+c)^2}{4 a f} + \frac{(dx+c)^3}{6 a d} - \frac{d^2}{4 f^3 (a+a\coth(fx+e))} - \frac{d(dx+c)}{2 f^2 (a+a\coth(fx+e))} - \frac{(dx+c)^2}{2 f (a+a\coth(fx+e))}$$

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Result(type 3, 448 leaves):
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$$\frac{1}{f^3 a}\Bigg(-d^2\left(\frac{(fx+e)^2\cosh(fx+e)\sinh(fx+e)}{2} - \frac{(fx+e)^3}{6} - \frac{(fx+e)\cosh(fx+e)^2}{2} + \frac{\sinh(fx+e)\cosh(fx+e)}{4} + \frac{fx}{4} + \frac{e}{4}\right)$$
$$-2cdf\left(\frac{(fx+e)\cosh(fx+e)\sinh(fx+e)}{2} - \frac{(fx+e)^2}{4} - \frac{\cosh(fx+e)^2}{4}\right) + 2d^2 e\left(\frac{(fx+e)\cosh(fx+e)\sinh(fx+e)}{2} - \frac{(fx+e)^2}{4}\right.$$
$$\left.- \frac{\cosh(fx+e)^2}{4}\right) - c^2 f^2\left(\frac{\sinh(fx+e)\cosh(fx+e)}{2} - \frac{fx}{2} - \frac{e}{2}\right) + 2cdef\left(\frac{\sinh(fx+e)\cosh(fx+e)}{2} - \frac{fx}{2} - \frac{e}{2}\right)$$
$$-d^2 e^2\left(\frac{\sinh(fx+e)\cosh(fx+e)}{2} - \frac{fx}{2} - \frac{e}{2}\right) + d^2\left(\frac{(fx+e)^2\cosh(fx+e)^2}{2} - \frac{(fx+e)\cosh(fx+e)\sinh(fx+e)}{2} - \frac{(fx+e)^2}{4}\right.$$
$$\left.+ \frac{\cosh(fx+e)^2}{4}\right) + 2cdf\left(\frac{(fx+e)\cosh(fx+e)^2}{2} - \frac{\sinh(fx+e)\cosh(fx+e)}{4} - \frac{fx}{4} - \frac{e}{4}\right) - 2d^2 e\left(\frac{(fx+e)\cosh(fx+e)^2}{2}\right.$$
$$\left.- \frac{\sinh(fx+e)\cosh(fx+e)}{4} - \frac{fx}{4} - \frac{e}{4}\right) + \frac{c^2 f^2\cosh(fx+e)^2}{2} - cdef\cosh(fx+e)^2 + \frac{d^2 e^2\cosh(fx+e)^2}{2}\Bigg)$$

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Problem 8: Result more than twice size of optimal antiderivative.
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$$\int \frac{dx+c}{a+a\coth(fx+e)}\,dx$$

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Optimal(type 3, 69 leaves, 3 steps):
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$$\frac{dx}{4af} + \frac{(dx+c)^2}{4ad} - \frac{d}{4f^2(a+a\coth(fx+e))} + \frac{-dx-c}{2f(a+a\coth(fx+e))}$$

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Result(type 3, 164 leaves):
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$$\frac{1}{f^2 a}\Bigg(-d\left(\frac{(fx+e)\cosh(fx+e)\sinh(fx+e)}{2} - \frac{(fx+e)^2}{4} - \frac{\cosh(fx+e)^2}{4}\right) - cf\left(\frac{\sinh(fx+e)\cosh(fx+e)}{2} - \frac{fx}{2} - \frac{e}{2}\right)$$
$$+de\left(\frac{\sinh(fx+e)\cosh(fx+e)}{2} - \frac{fx}{2} - \frac{e}{2}\right) + d\left(\frac{(fx+e)\cosh(fx+e)^2}{2} - \frac{\sinh(fx+e)\cosh(fx+e)}{4} - \frac{fx}{4} - \frac{e}{4}\right) + \frac{cf\cosh(fx+e)^2}{2}$$
$$-\frac{de\cosh(fx+e)^2}{2}\Bigg)$$

Problem 10: Result more than twice size of optimal antiderivative.

$$\int \frac{(dx+c)^2}{(a+a\coth(fx+e))^2}\,dx$$

Optimal(type 3, 150 leaves, 8 steps):

$$-\frac{d^2 e^{-4fx-4e}}{128a^2f^3}+\frac{d^2 e^{-2fx-2e}}{8a^2f^3}-\frac{d e^{-4fx-4e}(dx+c)}{32a^2f^2}+\frac{d e^{-2fx-2e}(dx+c)}{4a^2f^2}-\frac{e^{-4fx-4e}(dx+c)^2}{16a^2f}+\frac{e^{-2fx-2e}(dx+c)^2}{4a^2f}+\frac{(dx+c)^3}{12a^2d}$$

Result(type 3, 1056 leaves):

$$\frac{1}{f^3a^2}\Bigg(2d^2\left(\frac{(fx+e)^2\sinh(fx+e)\cosh(fx+e)^3}{4}-\frac{(fx+e)^2\cosh(fx+e)\sinh(fx+e)}{8}-\frac{(fx+e)^3}{24}-\frac{(fx+e)\sinh(fx+e)^2\cosh(fx+e)^2}{8}\right.$$
$$\left.+\frac{\cosh(fx+e)^3\sinh(fx+e)}{32}-\frac{\sinh(fx+e)\cosh(fx+e)}{64}-\frac{fx}{64}-\frac{e}{64}\right)+4cdf\left(\frac{(fx+e)\sinh(fx+e)\cosh(fx+e)^3}{4}\right.$$
$$\left.-\frac{(fx+e)\cosh(fx+e)\sinh(fx+e)}{8}-\frac{(fx+e)^2}{16}-\frac{\cosh(fx+e)^2\sinh(fx+e)^2}{16}\right)-4d^2e\left(\frac{(fx+e)\sinh(fx+e)\cosh(fx+e)^3}{4}\right.$$
$$\left.-\frac{(fx+e)\cosh(fx+e)\sinh(fx+e)}{8}-\frac{(fx+e)^2}{16}-\frac{\cosh(fx+e)^2\sinh(fx+e)^2}{16}\right)+2c^2f^2\left(\frac{\cosh(fx+e)^3\sinh(fx+e)}{4}\right.$$
$$\left.-\frac{\sinh(fx+e)\cosh(fx+e)}{8}-\frac{fx}{8}-\frac{e}{8}\right)-4cdef\left(\frac{\cosh(fx+e)^3\sinh(fx+e)}{4}-\frac{\sinh(fx+e)\cosh(fx+e)}{8}-\frac{fx}{8}-\frac{e}{8}\right)$$
$$+2d^2e^2\left(\frac{\cosh(fx+e)^3\sinh(fx+e)}{4}-\frac{\sinh(fx+e)\cosh(fx+e)}{8}-\frac{fx}{8}-\frac{e}{8}\right)-2d^2\left(\frac{(fx+e)^2\sinh(fx+e)^2\cosh(fx+e)^2}{4}\right.$$
$$-\frac{(fx+e)^2\cosh(fx+e)^2}{4}-\frac{(fx+e)\sinh(fx+e)\cosh(fx+e)^3}{8}+\frac{5(fx+e)\cosh(fx+e)\sinh(fx+e)}{16}+\frac{5(fx+e)^2}{32}$$
$$\left.+\frac{\cosh(fx+e)^2\sinh(fx+e)^2}{32}-\frac{\cosh(fx+e)^2}{8}\right)-4cdf\left(\frac{(fx+e)\sinh(fx+e)^2\cosh(fx+e)^2}{4}-\frac{(fx+e)\cosh(fx+e)^2}{4}\right.$$
$$\left.-\frac{\cosh(fx+e)^3\sinh(fx+e)}{16}+\frac{5\sinh(fx+e)\cosh(fx+e)}{32}+\frac{5fx}{32}+\frac{5e}{32}\right)+4d^2e\left(\frac{(fx+e)\sinh(fx+e)^2\cosh(fx+e)^2}{4}\right.$$
$$\left.-\frac{(fx+e)\cosh(fx+e)^2}{4}-\frac{\cosh(fx+e)^3\sinh(fx+e)}{16}+\frac{5\sinh(fx+e)\cosh(fx+e)}{32}+\frac{5fx}{32}+\frac{5e}{32}\right)-2c^2f^2\left(\frac{\cosh(fx+e)^2\sinh(fx+e)^2}{4}\right.$$
$$\left.-\frac{\cosh(fx+e)^2}{4}\right)+4cdef\left(\frac{\cosh(fx+e)^2\sinh(fx+e)^2}{4}-\frac{\cosh(fx+e)^2}{4}\right)-2d^2e^2\left(\frac{\cosh(fx+e)^2\sinh(fx+e)^2}{4}-\frac{\cosh(fx+e)^2}{4}\right)$$
$$-d^2\left(\frac{(fx+e)^2\cosh(fx+e)\sinh(fx+e)}{2}-\frac{(fx+e)^3}{6}-\frac{(fx+e)\cosh(fx+e)^2}{2}+\frac{\sinh(fx+e)\cosh(fx+e)}{4}+\frac{fx}{4}+\frac{e}{4}\right)$$
$$-2cdf\left(\frac{(fx+e)\cosh(fx+e)\sinh(fx+e)}{2}-\frac{(fx+e)^2}{4}-\frac{\cosh(fx+e)^2}{4}\right)+2d^2e\left(\frac{(fx+e)\cosh(fx+e)\sinh(fx+e)}{2}-\frac{(fx+e)^2}{4}\right.$$
$$\left.-\frac{\cosh(fx+e)^2}{4}\right)-c^2f^2\left(\frac{\sinh(fx+e)\cosh(fx+e)}{2}-\frac{fx}{2}-\frac{e}{2}\right)+2cdef\left(\frac{\sinh(fx+e)\cosh(fx+e)}{2}-\frac{fx}{2}-\frac{e}{2}\right)$$
$$-d^2e^2\left(\frac{\sinh(fx+e)\cosh(fx+e)}{2}-\frac{fx}{2}-\frac{e}{2}\right)\Bigg)$$

Problem 11: Result more than twice size of optimal antiderivative.

$$\int \frac{dx+c}{(a+a\coth(fx+e))^3}\,dx$$

Optimal(type 3, 174 leaves, 11 steps):

$$\frac{11dx}{96a^3f}-\frac{dx^2}{16a^3}+\frac{x(dx+c)}{8a^3}-\frac{d}{36f^2(a+a\coth(fx+e))^3}+\frac{-dx-c}{6f(a+a\coth(fx+e))^3}-\frac{5d}{96af^2(a+a\coth(fx+e))^2}+\frac{-dx-c}{8af(a+a\coth(fx+e))^2}$$
$$-\frac{11d}{96f^2(a^3+a^3\coth(fx+e))}+\frac{-dx-c}{8f(a^3+a^3\coth(fx+e))}$$

Result(type 3, 770 leaves):

$$\frac{1}{f^2a^3}\Bigg(4d\left(\frac{(fx+e)\sinh(fx+e)^2\cosh(fx+e)^4}{6}-\frac{(fx+e)\sinh(fx+e)^2\cosh(fx+e)^2}{12}-\frac{(fx+e)\cosh(fx+e)^2}{12}-\frac{\sinh(fx+e)\cosh(fx+e)^5}{36}\right.$$
$$\left.+\frac{\cosh(fx+e)^3\sinh(fx+e)}{36}+\frac{\sinh(fx+e)\cosh(fx+e)}{24}+\frac{fx}{24}+\frac{e}{24}\right)+4cf\left(\frac{\sinh(fx+e)^2\cosh(fx+e)^4}{6}-\frac{\cosh(fx+e)^2\sinh(fx+e)^2}{12}\right.$$
$$\left.-\frac{\cosh(fx+e)^2}{12}\right)-4de\left(\frac{\sinh(fx+e)^2\cosh(fx+e)^4}{6}-\frac{\cosh(fx+e)^2\sinh(fx+e)^2}{12}-\frac{\cosh(fx+e)^2}{12}\right)$$
$$-4d\left(\frac{(fx+e)\sinh(fx+e)^3\cosh(fx+e)^3}{6}-\frac{(fx+e)\sinh(fx+e)\cosh(fx+e)^3}{8}+\frac{(fx+e)\cosh(fx+e)\sinh(fx+e)}{16}+\frac{(fx+e)^2}{32}\right.$$
$$\left.-\frac{\sinh(fx+e)^2\cosh(fx+e)^4}{36}+\frac{13\cosh(fx+e)^2\sinh(fx+e)^2}{288}+\frac{\cosh(fx+e)^2}{72}\right)-4cf\left(\frac{\sinh(fx+e)^3\cosh(fx+e)^3}{6}\right.$$
$$\left.-\frac{\cosh(fx+e)^3\sinh(fx+e)}{8}+\frac{\sinh(fx+e)\cosh(fx+e)}{16}+\frac{fx}{16}+\frac{e}{16}\right)+4de\left(\frac{\sinh(fx+e)^3\cosh(fx+e)^3}{6}-\frac{\cosh(fx+e)^3\sinh(fx+e)}{8}\right.$$
$$\left.+\frac{\sinh(fx+e)\cosh(fx+e)}{16}+\frac{fx}{16}+\frac{e}{16}\right)-3d\left(\frac{(fx+e)\sinh(fx+e)^2\cosh(fx+e)^2}{4}-\frac{(fx+e)\cosh(fx+e)^2}{4}-\frac{\cosh(fx+e)^3\sinh(fx+e)}{16}\right.$$
$$\left.+\frac{5\sinh(fx+e)\cosh(fx+e)}{32}+\frac{5fx}{32}+\frac{5e}{32}\right)-3cf\left(\frac{\cosh(fx+e)^2\sinh(fx+e)^2}{4}-\frac{\cosh(fx+e)^2}{4}\right)+3de\left(\frac{\cosh(fx+e)^2\sinh(fx+e)^2}{4}\right.$$
$$\left.-\frac{\cosh(fx+e)^2}{4}\right)+d\left(\frac{(fx+e)\cosh(fx+e)\sinh(fx+e)^3}{4}-\frac{3(fx+e)\cosh(fx+e)\sinh(fx+e)}{8}+\frac{3(fx+e)^2}{16}-\frac{\cosh(fx+e)^2\sinh(fx+e)^2}{16}\right.$$
$$\left.+\frac{\cosh(fx+e)^2}{4}\right)+cf\left(\left(\frac{\sinh(fx+e)^3}{4}-\frac{3\sinh(fx+e)}{8}\right)\cosh(fx+e)+\frac{3fx}{8}+\frac{3e}{8}\right)-de\left(\left(\frac{\sinh(fx+e)^3}{4}-\frac{3\sinh(fx+e)}{8}\right)\cosh(fx+e)\right.$$
$$\left.+\frac{3fx}{8}+\frac{3e}{8}\right)\Bigg)$$

Problem 12: Unable to integrate problem.

$$\int \frac{(dx+c)^m}{(a+a\coth(fx+e))^2}\,dx$$

Optimal(type 4, 148 leaves, 4 steps):

$$\frac{(dx+c)^{1+m}}{4a^2 d(1+m)}+\frac{2^{-2-m}\mathrm{e}^{-2e+\frac{2cf}{d}}(dx+c)^m\Gamma\left(1+m,\frac{2f(dx+c)}{d}\right)}{a^2 f\left(\frac{f(dx+c)}{d}\right)^m}-\frac{4^{-2-m}\mathrm{e}^{-4e+\frac{4cf}{d}}(dx+c)^m\Gamma\left(1+m,\frac{4f(dx+c)}{d}\right)}{a^2 f\left(\frac{f(dx+c)}{d}\right)^m}$$

Result(type 8, 22 leaves):

$$\int\frac{(dx+c)^m}{(a+a\coth(fx+e))^2}\,\mathrm{d}x$$

Problem 13: Result more than twice size of optimal antiderivative.

$$\int(dx+c)(a+b\coth(fx+e))\,\mathrm{d}x$$

Optimal(type 4, 69 leaves, 6 steps):

$$\frac{a(dx+c)^2}{2d}-\frac{b(dx+c)^2}{2d}+\frac{b(dx+c)\ln\left(1-\mathrm{e}^{2fx+2e}\right)}{f}+\frac{bd\,\mathrm{polylog}\left(2,\mathrm{e}^{2fx+2e}\right)}{2f^2}$$

Result(type 4, 200 leaves):

$$\frac{adx^2}{2}-\frac{bdx^2}{2}+cax+bcx-\frac{2bc\ln\left(\mathrm{e}^{fx+e}\right)}{f}+\frac{bc\ln\left(1+\mathrm{e}^{fx+e}\right)}{f}+\frac{bc\ln\left(\mathrm{e}^{fx+e}-1\right)}{f}-\frac{2bdex}{f}-\frac{bde^2}{f^2}+\frac{bd\ln\left(1-\mathrm{e}^{fx+e}\right)x}{f}$$
$$+\frac{bd\ln\left(1-\mathrm{e}^{fx+e}\right)e}{f^2}+\frac{bd\,\mathrm{polylog}\left(2,\mathrm{e}^{fx+e}\right)}{f^2}+\frac{bd\ln\left(1+\mathrm{e}^{fx+e}\right)x}{f}+\frac{bd\,\mathrm{polylog}\left(2,-\mathrm{e}^{fx+e}\right)}{f^2}+\frac{2bde\ln\left(\mathrm{e}^{fx+e}\right)}{f^2}-\frac{bde\ln\left(\mathrm{e}^{fx+e}-1\right)}{f^2}$$

Problem 16: Result more than twice size of optimal antiderivative.

$$\int(dx+c)^3(a+b\coth(fx+e))^3\,\mathrm{d}x$$

Optimal(type 4, 524 leaves, 28 steps):

$$-\frac{3b^3d(dx+c)^2}{2f^2}-\frac{3ab^2(dx+c)^3}{f}+\frac{b^3(dx+c)^3}{2f}+\frac{a^3(dx+c)^4}{4d}-\frac{3a^2b(dx+c)^4}{4d}+\frac{3ab^2(dx+c)^4}{4d}-\frac{b^3(dx+c)^4}{4d}$$
$$-\frac{3b^3d(dx+c)^2\coth(fx+e)}{2f^2}-\frac{3ab^2(dx+c)^3\coth(fx+e)}{f}-\frac{b^3(dx+c)^3\coth(fx+e)^2}{2f}+\frac{3b^3d^2(dx+c)\ln\left(1-\mathrm{e}^{2fx+2e}\right)}{f^3}$$
$$+\frac{9ab^2d(dx+c)^2\ln\left(1-\mathrm{e}^{2fx+2e}\right)}{f^2}+\frac{3a^2b(dx+c)^3\ln\left(1-\mathrm{e}^{2fx+2e}\right)}{f}+\frac{b^3(dx+c)^3\ln\left(1-\mathrm{e}^{2fx+2e}\right)}{f}+\frac{3b^3d^3\,\mathrm{polylog}\left(2,\mathrm{e}^{2fx+2e}\right)}{2f^4}$$
$$+\frac{9ab^2d^2(dx+c)\,\mathrm{polylog}\left(2,\mathrm{e}^{2fx+2e}\right)}{f^3}+\frac{9a^2bd(dx+c)^2\,\mathrm{polylog}\left(2,\mathrm{e}^{2fx+2e}\right)}{2f^2}+\frac{3b^3d(dx+c)^2\,\mathrm{polylog}\left(2,\mathrm{e}^{2fx+2e}\right)}{2f^2}$$
$$-\frac{9ab^2d^3\,\mathrm{polylog}\left(3,\mathrm{e}^{2fx+2e}\right)}{2f^4}-\frac{9a^2bd^2(dx+c)\,\mathrm{polylog}\left(3,\mathrm{e}^{2fx+2e}\right)}{2f^3}-\frac{3b^3d^2(dx+c)\,\mathrm{polylog}\left(3,\mathrm{e}^{2fx+2e}\right)}{2f^3}+\frac{9a^2bd^3\,\mathrm{polylog}\left(4,\mathrm{e}^{2fx+2e}\right)}{4f^4}$$
$$+\frac{3b^3d^3\,\mathrm{polylog}\left(4,\mathrm{e}^{2fx+2e}\right)}{4f^4}$$

Result(type ?, 2776 leaves): Display of huge result suppressed!

Problem 17: Result more than twice size of optimal antiderivative.

$$\int (d x + c)\ (a + b \coth(f x + e))^3\, dx$$

Optimal(type 4, 243 leaves, 16 steps):

$$3 a b^2 c x + \frac{b^3 d x}{2 f} + \frac{3 a b^2 d x^2}{2} + \frac{a^3 (d x + c)^2}{2 d} - \frac{3 a^2 b (d x + c)^2}{2 d} - \frac{b^3 (d x + c)^2}{2 d} - \frac{b^3 d \coth(f x + e)}{2 f^2} - \frac{3 a b^2 (d x + c) \coth(f x + e)}{f} - \frac{b^3 (d x + c) \coth(f x + e)^2}{2 f} + \frac{3 a^2 b (d x + c) \ln(1 - e^{2 f x + 2 e})}{f} + \frac{b^3 (d x + c) \ln(1 - e^{2 f x + 2 e})}{f} + \frac{3 a b^2 d \ln(\sinh(f x + e))}{f^2} + \frac{3 a^2 b d \operatorname{polylog}(2, e^{2 f x + 2 e})}{2 f^2} + \frac{b^3 d \operatorname{polylog}(2, e^{2 f x + 2 e})}{2 f^2}$$

Result(type 4, 650 leaves):

$$-\frac{6 b a^2 d e x}{f} + \frac{3 b \ln(1 - e^{f x + e}) a^2 d x}{f} + \frac{3 b \ln(1 + e^{f x + e}) a^2 d x}{f} + \frac{3 b \ln(1 - e^{f x + e}) a^2 d e}{f^2} + \frac{6 b a^2 d e \ln(e^{f x + e})}{f^2} - \frac{3 b a^2 d e \ln(e^{f x + e} - 1)}{f^2} + \frac{b^3 \ln(1 - e^{f x + e}) d e}{f^2} - \frac{6 b^2 a d \ln(e^{f x + e})}{f^2} + \frac{3 b^2 a d \ln(1 + e^{f x + e})}{f^2} + \frac{3 b^2 a d \ln(e^{f x + e} - 1)}{f^2} + \frac{2 b^3 d e \ln(e^{f x + e})}{f^2} - \frac{b^3 d e \ln(e^{f x + e} - 1)}{f^2} - \frac{6 b a^2 c \ln(e^{f x + e})}{f} + \frac{3 b a^2 c \ln(1 + e^{f x + e})}{f} + \frac{3 b a^2 c \ln(e^{f x + e} - 1)}{f} - \frac{2 b^3 d e x}{f} - \frac{3 b a^2 d e^2}{f^2} + \frac{b^3 \ln(1 - e^{f x + e}) d x}{f} + \frac{b^3 \ln(1 + e^{f x + e}) d x}{f} + \frac{3 b a^2 d \operatorname{polylog}(2, e^{f x + e})}{f^2} + \frac{3 b a^2 d \operatorname{polylog}(2, -e^{f x + e})}{f^2} - \frac{b^3 d e^2}{f^2} + \frac{b^3 d \operatorname{polylog}(2, e^{f x + e})}{f^2} + \frac{b^3 d \operatorname{polylog}(2, -e^{f x + e})}{f^2} - \frac{2 b^3 c \ln(e^{f x + e})}{f} + \frac{b^3 c \ln(1 + e^{f x + e})}{f} + \frac{b^3 c \ln(e^{f x + e} - 1)}{f} - \frac{b^2 (6 a d f x e^{2 f x + 2 e} + 2 b d f x e^{2 f x + 2 e} + 6 a c f e^{2 f x + 2 e} + 2 b c f e^{2 f x + 2 e} - 6 a d f x + b d e^{2 f x + 2 e} - 6 a c f - b d)}{f^2 (e^{2 f x + 2 e} - 1)^2} - \frac{3 b a^2 x^2 d}{2} + 3 b a^2 c x + 3 a b^2 c x + \frac{3 a b^2 d x^2}{2} + \frac{a^3 d x^2}{2} - \frac{b^3 x^2 d}{2} + c a^3 x + b^3 c x$$

Problem 19: Result more than twice size of optimal antiderivative.

$$\int \frac{d x + c}{(a + b \coth(f x + e))^2}\, dx$$

Optimal(type 4, 195 leaves, 5 steps):

$$-\frac{(d x + c)^2}{2 (a^2 - b^2) d} + \frac{(-2 a d f x - 2 a c f + b d)^2}{4 a (a - b) (a + b)^2 d f^2} + \frac{b (d x + c)}{(a^2 - b^2) f (a + b \coth(f x + e))} + \frac{b (-2 a d f x - 2 a c f + b d) \ln\left(1 + \frac{-a + b}{(a + b) e^{2 f x + 2 e}}\right)}{(a^2 - b^2)^2 f^2}$$

$$+\frac{a b d \operatorname{polylog}\left(2, \frac{a-b}{(a+b) e^{2 f x+2 e}}\right)}{\left(a^{2}-b^{2}\right)^{2} f^{2}}$$

Result(type 4, 523 leaves):

$$\frac{d x^{2}}{2\left(a^{2}+2 b a+b^{2}\right)}+\frac{c x}{a^{2}+2 b a+b^{2}}-\frac{2 b^{2}(d x+c)}{(a-b) f\left(a^{2}+2 b a+b^{2}\right)\left(a e^{2 f x+2 e}+b e^{2 f x+2 e}-a+b\right)}-\frac{2 b^{2} d \ln \left(e^{f x+e}\right)}{(a+b)^{2}(a-b)^{2} f^{2}}$$
$$+\frac{b^{2} d \ln \left(a e^{2 f x+2 e}+b e^{2 f x+2 e}-a+b\right)}{(a+b)^{2}(a-b)^{2} f^{2}}+\frac{4 b a c \ln \left(e^{f x+e}\right)}{(a+b)^{2}(a-b)^{2} f}-\frac{2 b a c \ln \left(a e^{2 f x+2 e}+b e^{2 f x+2 e}-a+b\right)}{(a+b)^{2}(a-b)^{2} f}-\frac{4 b a d e \ln \left(e^{f x+e}\right)}{(a+b)^{2}(a-b)^{2} f^{2}}$$
$$+\frac{2 b a d e \ln \left(a e^{2 f x+2 e}+b e^{2 f x+2 e}-a+b\right)}{(a+b)^{2}(a-b)^{2} f^{2}}-\frac{2 b a d \ln \left(1-\frac{(a+b) e^{2 f x+2 e}}{a-b}\right) x}{(a+b)^{2}(a-b)^{2} f}-\frac{2 b a d \ln \left(1-\frac{(a+b) e^{2 f x+2 e}}{a-b}\right) e}{(a+b)^{2}(a-b)^{2} f^{2}}+\frac{2 b a d x^{2}}{(a+b)^{2}(a-b)^{2}}$$
$$+\frac{4 b a d e x}{(a+b)^{2}(a-b)^{2} f}+\frac{2 b a d e^{2}}{(a+b)^{2}(a-b)^{2} f^{2}}-\frac{b a d \operatorname{polylog}\left(2, \frac{(a+b) e^{2 f x+2 e}}{a-b}\right)}{(a+b)^{2}(a-b)^{2} f^{2}}$$

Test results for the 58 problems in "6.4.2 Hyperbolic cotangent functions.txt"

Problem 5: Unable to integrate problem.

$$\int \coth (b x+a)^{n} d x$$

Optimal(type 5, 41 leaves, 2 steps):

$$\frac{\coth (b x+a)^{1+n} \operatorname{hypergeom}\left(\left[1, \frac{1}{2}+\frac{n}{2}\right],\left[\frac{3}{2}+\frac{n}{2}\right], \coth (b x+a)^{2}\right)}{b(1+n)}$$

Result(type 8, 10 leaves):

$$\int \coth (b x+a)^{n} d x$$

Problem 6: Unable to integrate problem.

$$\int(b \coth (d x+c))^{n} d x$$

Optimal(type 5, 46 leaves, 2 steps):

$$\frac{(b \coth (d x+c))^{1+n} \operatorname{hypergeom}\left(\left[1, \frac{1}{2}+\frac{n}{2}\right],\left[\frac{3}{2}+\frac{n}{2}\right], \coth (d x+c)^{2}\right)}{b d(1+n)}$$

Result(type 8, 12 leaves):

$$\int(b \coth (d x+c))^{n} d x$$

Problem 7: Unable to integrate problem.

$$\int \left(b \coth(d x + c)^{2}\right)^{n} \, dx$$

Optimal(type 5, 47 leaves, 3 steps):

$$\frac{\coth(d x + c) \left(b \coth(d x + c)^{2}\right)^{n} \operatorname{hypergeom}\left(\left[1, \frac{1}{2} + n\right], \left[\frac{3}{2} + n\right], \coth(d x + c)^{2}\right)}{d (1 + 2 n)}$$

Result(type 8, 14 leaves):

$$\int \left(b \coth(d x + c)^{2}\right)^{n} \, dx$$

Problem 10: Unable to integrate problem.

$$\int \left(b \coth(d x + c)^{2}\right)^{2/3} \, dx$$

Optimal(type 3, 239 leaves, 14 steps):

$$\frac{\operatorname{arctanh}\left(\coth(d x + c)^{1/3}\right) \left(b \coth(d x + c)^{2}\right)^{2/3}}{d \coth(d x + c)^{4/3}} - \frac{\left(b \coth(d x + c)^{2}\right)^{2/3} \ln\left(1 - \coth(d x + c)^{1/3} + \coth(d x + c)^{2/3}\right)}{4 d \coth(d x + c)^{4/3}}$$
$$+ \frac{\left(b \coth(d x + c)^{2}\right)^{2/3} \ln\left(1 + \coth(d x + c)^{1/3} + \coth(d x + c)^{2/3}\right)}{4 d \coth(d x + c)^{4/3}} - \frac{\arctan\left(\frac{\left(1 - 2 \coth(d x + c)^{1/3}\right) \sqrt{3}}{3}\right) \left(b \coth(d x + c)^{2}\right)^{2/3} \sqrt{3}}{2 d \coth(d x + c)^{4/3}}$$
$$+ \frac{\arctan\left(\frac{\left(1 + 2 \coth(d x + c)^{1/3}\right) \sqrt{3}}{3}\right) \left(b \coth(d x + c)^{2}\right)^{2/3} \sqrt{3}}{2 d \coth(d x + c)^{4/3}} - \frac{3 \left(b \coth(d x + c)^{2}\right)^{2/3} \tanh(d x + c)}{d}$$

Result(type 8, 14 leaves):

$$\int \left(b \coth(d x + c)^{2}\right)^{2/3} \, dx$$

Problem 12: Result more than twice size of optimal antiderivative.

$$\int \left(b \coth(d x + c)^{3}\right)^{1/3} \, dx$$

Optimal(type 3, 29 leaves, 2 steps):

$$\frac{\left(b \coth(d x + c)^{3}\right)^{1/3} \ln(\sinh(d x + c)) \tanh(d x + c)}{d}$$

Result(type 3, 191 leaves):

$$\frac{\left(\frac{b \left(e^{2 d x + 2 c} + 1\right)^{3}}{\left(e^{2 d x + 2 c} - 1\right)^{3}}\right)^{1/3} \left(e^{2 d x + 2 c} - 1\right) x}{e^{2 d x + 2 c} + 1} - \frac{2 \left(\frac{b \left(e^{2 d x + 2 c} + 1\right)^{3}}{\left(e^{2 d x + 2 c} - 1\right)^{3}}\right)^{1/3} \left(e^{2 d x + 2 c} - 1\right) (d x + c)}{\left(e^{2 d x + 2 c} + 1\right) d}$$

$$+\frac{\left(\frac{b\left(e^{2dx+2c}+1\right)^3}{\left(e^{2dx+2c}-1\right)^3}\right)^{1/3}\left(e^{2dx+2c}-1\right)\ln\left(e^{2dx+2c}-1\right)}{\left(e^{2dx+2c}+1\right)d}$$

Problem 13: Unable to integrate problem.

$$\int\left(b\coth(dx+c)^4\right)^n\,dx$$

Optimal(type 5, 51 leaves, 3 steps):

$$\frac{\coth(dx+c)\left(b\coth(dx+c)^4\right)^n\operatorname{hypergeom}\left(\left[1,\frac{1}{2}+2n\right],\left[\frac{3}{2}+2n\right],\coth(dx+c)^2\right)}{d(1+4n)}$$

Result(type 8, 14 leaves):

$$\int\left(b\coth(dx+c)^4\right)^n\,dx$$

Problem 15: Unable to integrate problem.

$$\int\left(b\coth(dx+c)^4\right)^{2/3}\,dx$$

Optimal(type 3, 239 leaves, 14 steps):

$$\frac{\operatorname{arctanh}\left(\coth(dx+c)^{1/3}\right)\left(b\coth(dx+c)^4\right)^{2/3}}{d\coth(dx+c)^{8/3}}-\frac{\left(b\coth(dx+c)^4\right)^{2/3}\ln\left(1-\coth(dx+c)^{1/3}+\coth(dx+c)^{2/3}\right)}{4d\coth(dx+c)^{8/3}}$$
$$+\frac{\left(b\coth(dx+c)^4\right)^{2/3}\ln\left(1+\coth(dx+c)^{1/3}+\coth(dx+c)^{2/3}\right)}{4d\coth(dx+c)^{8/3}}+\frac{\arctan\left(\frac{\left(1-2\coth(dx+c)^{1/3}\right)\sqrt{3}}{3}\right)\left(b\coth(dx+c)^4\right)^{2/3}\sqrt{3}}{2d\coth(dx+c)^{8/3}}$$
$$-\frac{\arctan\left(\frac{\left(1+2\coth(dx+c)^{1/3}\right)\sqrt{3}}{3}\right)\left(b\coth(dx+c)^4\right)^{2/3}\sqrt{3}}{2d\coth(dx+c)^{8/3}}-\frac{3\left(b\coth(dx+c)^4\right)^{2/3}\tanh(dx+c)}{5d}$$

Result(type 8, 14 leaves):

$$\int\left(b\coth(dx+c)^4\right)^{2/3}\,dx$$

Problem 16: Unable to integrate problem.

$$\int\frac{1}{\left(b\coth(dx+c)^4\right)^{1/3}}\,dx$$

Optimal(type 3, 239 leaves, 14 steps):

$$-\frac{3\coth(dx+c)}{d\left(b\coth(dx+c)^4\right)^{1/3}}+\frac{\operatorname{arctanh}\left(\coth(dx+c)^{1/3}\right)\coth(dx+c)^{4/3}}{d\left(b\coth(dx+c)^4\right)^{1/3}}-\frac{\coth(dx+c)^{4/3}\ln\left(1-\coth(dx+c)^{1/3}+\coth(dx+c)^{2/3}\right)}{4d\left(b\coth(dx+c)^4\right)^{1/3}}$$
$$+\frac{\coth(dx+c)^{4/3}\ln\left(1+\coth(dx+c)^{1/3}+\coth(dx+c)^{2/3}\right)}{4d\left(b\coth(dx+c)^4\right)^{1/3}}+\frac{\arctan\left(\frac{\left(1-2\coth(dx+c)^{1/3}\right)\sqrt{3}}{3}\right)\coth(dx+c)^{4/3}\sqrt{3}}{2d\left(b\coth(dx+c)^4\right)^{1/3}}$$
$$-\frac{\arctan\left(\frac{\left(1+2\coth(dx+c)^{1/3}\right)\sqrt{3}}{3}\right)\coth(dx+c)^{4/3}\sqrt{3}}{2d\left(b\coth(dx+c)^4\right)^{1/3}}$$

Result(type 8, 14 leaves):

$$\int\frac{1}{\left(b\coth(dx+c)^4\right)^{1/3}}\,dx$$

Problem 17: Unable to integrate problem.

$$\int\frac{1}{\left(b\coth(dx+c)^4\right)^{4/3}}\,dx$$

Optimal(type 3, 311 leaves, 16 steps):

$$-\frac{3\coth(dx+c)}{bd\left(b\coth(dx+c)^4\right)^{1/3}}+\frac{\operatorname{arctanh}\left(\coth(dx+c)^{1/3}\right)\coth(dx+c)^{4/3}}{bd\left(b\coth(dx+c)^4\right)^{1/3}}-\frac{\coth(dx+c)^{4/3}\ln\left(1-\coth(dx+c)^{1/3}+\coth(dx+c)^{2/3}\right)}{4bd\left(b\coth(dx+c)^4\right)^{1/3}}$$
$$+\frac{\coth(dx+c)^{4/3}\ln\left(1+\coth(dx+c)^{1/3}+\coth(dx+c)^{2/3}\right)}{4bd\left(b\coth(dx+c)^4\right)^{1/3}}+\frac{\arctan\left(\frac{\left(1-2\coth(dx+c)^{1/3}\right)\sqrt{3}}{3}\right)\coth(dx+c)^{4/3}\sqrt{3}}{2bd\left(b\coth(dx+c)^4\right)^{1/3}}$$
$$-\frac{\arctan\left(\frac{\left(1+2\coth(dx+c)^{1/3}\right)\sqrt{3}}{3}\right)\coth(dx+c)^{4/3}\sqrt{3}}{2bd\left(b\coth(dx+c)^4\right)^{1/3}}-\frac{3\tanh(dx+c)}{7bd\left(b\coth(dx+c)^4\right)^{1/3}}-\frac{3\tanh(dx+c)^3}{13bd\left(b\coth(dx+c)^4\right)^{1/3}}$$

Result(type 8, 14 leaves):

$$\int\frac{1}{\left(b\coth(dx+c)^4\right)^{4/3}}\,dx$$

Problem 18: Unable to integrate problem.

$$\int\left(b\coth(dx+c)^m\right)^{2/3}\,dx$$

Optimal(type 5, 52 leaves, 3 steps):

$$\frac{3\coth(dx+c)\left(b\coth(dx+c)^m\right)^{2/3}\operatorname{hypergeom}\left(\left[1,\frac{1}{2}+\frac{m}{3}\right],\left[\frac{3}{2}+\frac{m}{3}\right],\coth(dx+c)^2\right)}{d(3+2m)}$$

Result(type 8, 14 leaves):

$$\int \left(b \coth(d x+c)^{m}\right)^{2/3} dx$$

Problem 19: Unable to integrate problem.

$$\int \frac{1}{\left(b \coth(d x+c)^{m}\right)^{2/3}} dx$$

Optimal(type 5, 52 leaves, 3 steps):

$$\frac{3 \coth(d x+c) \operatorname{hypergeom}\left(\left[1, \frac{1}{2}-\frac{m}{3}\right], \left[\frac{3}{2}-\frac{m}{3}\right], \coth(d x+c)^{2}\right)}{d (3-2 m) \left(b \coth(d x+c)^{m}\right)^{2/3}}$$

Result(type 8, 14 leaves):

$$\int \frac{1}{\left(b \coth(d x+c)^{m}\right)^{2/3}} dx$$

Problem 24: Result more than twice size of optimal antiderivative.

$$\int (a+b \coth(d x+c))^{2} dx$$

Optimal(type 3, 38 leaves, 2 steps):

$$\left(a^{2}+b^{2}\right) x-\frac{b^{2} \coth(d x+c)}{d}+\frac{2 a b \ln(\sinh(d x+c))}{d}$$

Result(type 3, 115 leaves):

$$-\frac{b^{2} \coth(d x+c)}{d}-\frac{\ln(\coth(d x+c)-1) a^{2}}{2 d}-\frac{\ln(\coth(d x+c)-1) b a}{d}-\frac{\ln(\coth(d x+c)-1) b^{2}}{2 d}+\frac{\ln(\coth(d x+c)+1) a^{2}}{2 d}$$
$$-\frac{\ln(\coth(d x+c)+1) b a}{d}+\frac{\ln(\coth(d x+c)+1) b^{2}}{2 d}$$

Problem 26: Result more than twice size of optimal antiderivative.

$$\int \frac{\sinh(x)^{2}}{1+\coth(x)} dx$$

Optimal(type 3, 30 leaves, 4 steps):

$$-\frac{3 x}{8}-\frac{1}{8 (1-\coth(x))}+\frac{1}{8 (1+\coth(x))^{2}}+\frac{1}{4 (1+\coth(x))}$$

Result(type 3, 69 leaves):

$$\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)^4}-\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^3}+\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{8}+\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}+\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{8}$$

```
Problem 27: Result more than twice size of optimal antiderivative.
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$$\int \frac{\sinh(x)}{1+\coth(x)}\,dx$$

```
Optimal(type 3, 15 leaves, 2 steps):
```

$$\frac{2\cosh(x)}{3}-\frac{\sinh(x)}{3\,(1+\coth(x))}$$

```
Result(type 3, 39 leaves):
```

$$-\frac{2}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^3}+\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}+\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)}$$

```
Problem 31: Unable to integrate problem.
```

$$\int \operatorname{sech}(x)^2\sqrt{1+\coth(x)}\,dx$$

```
Optimal(type 3, 17 leaves, 4 steps):
```

$$\operatorname{arctanh}\left(\sqrt{1+\coth(x)}\right)+\sqrt{1+\coth(x)}\,\tanh(x)$$

```
Result(type 8, 13 leaves):
```

$$\int \operatorname{sech}(x)^2\sqrt{1+\coth(x)}\,dx$$

```
Problem 35: Result more than twice size of optimal antiderivative.
```

$$\int \frac{\operatorname{sech}(x)^3}{a+b\coth(x)}\,dx$$

```
Optimal(type 3, 75 leaves, 9 steps):
```

$$\frac{\arctan(\sinh(x))}{2\,a}-\frac{b^2\arctan(\sinh(x))}{a^3}-\frac{b\operatorname{sech}(x)}{a^2}+\frac{b\operatorname{arctanh}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)\sqrt{a^2-b^2}}{a^3}+\frac{\operatorname{sech}(x)\tanh(x)}{2\,a}$$

```
Result(type 3, 186 leaves):
```

$$-\frac{2 b \arctan\left(\frac{2 \tanh\left(\frac{x}{2}\right) b+2 a}{2 \sqrt{-a^2+b^2}}\right)}{a \sqrt{-a^2+b^2}}+\frac{2 b^3 \arctan\left(\frac{2 \tanh\left(\frac{x}{2}\right) b+2 a}{2 \sqrt{-a^2+b^2}}\right)}{a^3 \sqrt{-a^2+b^2}}-\frac{\tanh\left(\frac{x}{2}\right)^3}{a\left(\tanh\left(\frac{x}{2}\right)^2+1\right)^2}-\frac{2 \tanh\left(\frac{x}{2}\right)^2 b}{a^2\left(\tanh\left(\frac{x}{2}\right)^2+1\right)^2}+\frac{\tanh\left(\frac{x}{2}\right)}{a\left(\tanh\left(\frac{x}{2}\right)^2+1\right)^2}$$

$$-\frac{2 b}{a^2\left(\tanh\left(\frac{x}{2}\right)^2+1\right)^2}+\frac{\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}-\frac{2 \arctan\left(\tanh\left(\frac{x}{2}\right)\right) b^2}{a^3}$$

Problem 37: Result more than twice size of optimal antiderivative.

$$\int \frac{\tanh(x)^3}{1+\coth(x)}\, dx$$

Optimal(type 3, 31 leaves, 5 steps):

$$-\frac{3 x}{2}+2 \ln(\cosh(x))+\frac{3 \tanh(x)}{2}-\tanh(x)^2+\frac{\tanh(x)^2}{2(1+\coth(x))}$$

Result(type 3, 79 leaves):

$$-\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}+\frac{1}{\tanh\left(\frac{x}{2}\right)+1}-\frac{7 \ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2}+\frac{2\left(\tanh\left(\frac{x}{2}\right)^3-\tanh\left(\frac{x}{2}\right)^2+\tanh\left(\frac{x}{2}\right)\right)}{\left(\tanh\left(\frac{x}{2}\right)^2+1\right)^2}+2 \ln\left(\tanh\left(\frac{x}{2}\right)^2+1\right)$$

$$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2}$$

Problem 38: Result more than twice size of optimal antiderivative.

$$\int \frac{\tanh(x)}{1+\coth(x)}\, dx$$

Optimal(type 3, 15 leaves, 4 steps):

$$-\frac{x}{2}+\frac{1}{2(1+\coth(x))}+\ln(\cosh(x))$$

Result(type 3, 46 leaves):

$$-\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}+\frac{1}{\tanh\left(\frac{x}{2}\right)+1}-\frac{3 \ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2}+\ln\left(\tanh\left(\frac{x}{2}\right)^2+1\right)-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2}$$

Problem 44: Result more than twice size of optimal antiderivative.

$$\int x^2 \coth(a+2\ln(x))\, dx$$

Optimal(type 3, 35 leaves, 5 steps):

$$\frac{x^3}{3}+\frac{\arctan\left(e^{\frac{a}{2}}x\right)}{e^{\frac{3a}{2}}}-\frac{\operatorname{arctanh}\left(e^{\frac{a}{2}}x\right)}{e^{\frac{3a}{2}}}$$

Result(type 3, 82 leaves):

$$\frac{x^3}{3}+\frac{\ln\left((-e^a)^{3/2}-x\,e^{2a}\right)}{2\,(-e^a)^{3/2}}-\frac{\ln\left((-e^a)^{3/2}+x\,e^{2a}\right)}{2\,(-e^a)^{3/2}}+\frac{\ln\left(-x\sqrt{e^a}+1\right)}{2\,(e^a)^{3/2}}-\frac{\ln\left(x\sqrt{e^a}+1\right)}{2\,(e^a)^{3/2}}$$

Problem 45: Result more than twice size of optimal antiderivative.

$$\int \frac{\coth(a+2\ln(x))}{x}\, dx$$

Optimal(type 3, 10 leaves, 2 steps):

$$\frac{\ln(\sinh(a+2\ln(x)))}{2}$$

Result(type 3, 25 leaves):

$$-\frac{\ln(\coth(a+2\ln(x))-1)}{4}-\frac{\ln(\coth(a+2\ln(x))+1)}{4}$$

Problem 46: Result more than twice size of optimal antiderivative.

$$\int \frac{\coth(a+2\ln(x))}{x^2}\, dx$$

Optimal(type 3, 29 leaves, 5 steps):

$$\frac{1}{x}+e^{\frac{a}{2}}\arctan\left(e^{\frac{a}{2}}x\right)-e^{\frac{a}{2}}\operatorname{arctanh}\left(e^{\frac{a}{2}}x\right)$$

Result(type 3, 92 leaves):

$$\frac{1}{x}+\frac{\sqrt{e^a}\,\ln\left((e^a)^{3/2}-x\,e^{2a}\right)}{2}-\frac{\sqrt{e^a}\,\ln\left(-(e^a)^{3/2}-x\,e^{2a}\right)}{2}+\frac{\sqrt{-e^a}\,\ln\left((-e^a)^{3/2}-x\,e^{2a}\right)}{2}-\frac{\sqrt{-e^a}\,\ln\left(-(-e^a)^{3/2}-x\,e^{2a}\right)}{2}$$

Problem 47: Unable to integrate problem.

$$\int (e\,x)^m \coth(a+2\ln(x))\, dx$$

Optimal(type 5, 56 leaves, 3 steps):

$$\frac{(ex)^{1+m}}{e(1+m)} - \frac{2(ex)^{1+m}\operatorname{hypergeom}\left(\left[1, \frac{1}{4}+\frac{m}{4}\right], \left[\frac{5}{4}+\frac{m}{4}\right], e^{2a}x^4\right)}{e(1+m)}$$

Result(type 8, 15 leaves):

$$\int (ex)^m \coth(a+2\ln(x))\, dx$$

Problem 48: Unable to integrate problem.

$$\int (ex)^m \coth(a+2\ln(x))^3\, dx$$

Optimal(type 5, 162 leaves, 5 steps):

$$\frac{(3+m)(5+m)(ex)^{1+m}}{8e(1+m)} - \frac{(ex)^{1+m}(1+e^{2a}x^4)^2}{4e(1-e^{2a}x^4)^2} - \frac{(ex)^{1+m}(e^{2a}(3-m)-e^{4a}(5+m)x^4)}{8ee^{2a}(1-e^{2a}x^4)}$$
$$- \frac{(m^2+2m+9)(ex)^{1+m}\operatorname{hypergeom}\left(\left[1, \frac{1}{4}+\frac{m}{4}\right], \left[\frac{5}{4}+\frac{m}{4}\right], e^{2a}x^4\right)}{4e(1+m)}$$

Result(type 8, 17 leaves):

$$\int (ex)^m \coth(a+2\ln(x))^3\, dx$$

Problem 49: Result more than twice size of optimal antiderivative.

$$\int \frac{\coth(d(a+b\ln(cx^n)))^2}{x}\, dx$$

Optimal(type 3, 28 leaves, 3 steps):

$$-\frac{\coth(ad+bd\ln(cx^n))}{bdn} + \ln(x)$$

Result(type 3, 79 leaves):

$$-\frac{\coth(d(a+b\ln(cx^n)))}{bdn} - \frac{\ln(\coth(d(a+b\ln(cx^n)))-1)}{2bdn} + \frac{\ln(\coth(d(a+b\ln(cx^n)))+1)}{2bdn}$$

Problem 50: Unable to integrate problem.

$$\int \frac{\coth(d(a+b\ln(cx^n)))^2}{x^3}\, dx$$

Optimal(type 5, 132 leaves, 5 steps):

$$\frac{-bdn+2}{2bdnx^2} + \frac{1+e^{2ad}(cx^n)^{2bd}}{bdnx^2(1-e^{2ad}(cx^n)^{2bd})} - \frac{2\operatorname{hypergeom}\left(\left[1, -\frac{1}{bdn}\right], \left[1-\frac{1}{bdn}\right], e^{2ad}(cx^n)^{2bd}\right)}{bdnx^2}$$

Result(type 8, 188 leaves):

$$-\frac{1}{2x^2}-\frac{2}{d\,b\,n\,x^2\left(\left(e^{d\left(a+b\left(\ln(c)+\ln(e^{n\ln(x)})-\frac{I\pi\,\mathrm{csgn}(I\,c\,e^{n\ln(x)})\,(-\mathrm{csgn}(I\,c\,e^{n\ln(x)})+\mathrm{csgn}(I\,c))\,(-\mathrm{csgn}(I\,c\,e^{n\ln(x)})+\mathrm{csgn}(I\,e^{n\ln(x)}))}{2}\right)\right)}\right)^2-1\right)}+\int -\frac{4}{d\,b\,n\,x^3\left(\left(e^{d\left(a+b\left(\ln(c)+\ln(e^{n\ln(x)})-\frac{I\pi\,\mathrm{csgn}(I\,c\,e^{n\ln(x)})\,(-\mathrm{csgn}(I\,c\,e^{n\ln(x)})+\mathrm{csgn}(I\,c))\,(-\mathrm{csgn}(I\,c\,e^{n\ln(x)})+\mathrm{csgn}(I\,e^{n\ln(x)}))}{2}\right)\right)}\right)^2-1\right)}\,dx$$

Problem 51: Unable to integrate problem.

$$\int \coth\left(d\left(a+b\ln(c\,x^n)\right)\right)^p\,dx$$

Optimal(type 6, 107 leaves, 4 steps):

$$\frac{x\left(-1-e^{2ad}\left(c\,x^n\right)^{2bd}\right)^p \mathit{AppellF1}\left(\frac{1}{2\,b\,d\,n},p,-p,1+\frac{1}{2\,b\,d\,n},e^{2ad}\left(c\,x^n\right)^{2bd},-e^{2ad}\left(c\,x^n\right)^{2bd}\right)}{\left(1+e^{2ad}\left(c\,x^n\right)^{2bd}\right)^p}$$

Result(type 8, 17 leaves):

$$\int \coth\left(d\left(a+b\ln(c\,x^n)\right)\right)^p\,dx$$

Problem 55: Unable to integrate problem.

$$\int \frac{\tanh(x)}{\sqrt{a+b\coth(x)^2+c\coth(x)^4}}\,dx$$

Optimal(type 3, 86 leaves, 8 steps):

$$-\frac{\operatorname{arctanh}\left(\frac{2a+b\coth(x)^2}{2\sqrt{a}\sqrt{a+b\coth(x)^2+c\coth(x)^4}}\right)}{2\sqrt{a}}+\frac{\operatorname{arctanh}\left(\frac{2a+b+(b+2c)\coth(x)^2}{2\sqrt{a+b+c}\sqrt{a+b\coth(x)^2+c\coth(x)^4}}\right)}{2\sqrt{a+b+c}}$$

Result(type 8, 21 leaves):

$$\int \frac{\tanh(x)}{\sqrt{a+b\coth(x)^2+c\coth(x)^4}}\,dx$$

Test results for the 17 problems in "6.4.7 (d hyper)^m (a+b (c coth)^n)^p.txt"

Problem 1: Result more than twice size of optimal antiderivative.

$$\int \left(a+b\coth(d\,x+c)^2\right)^5\,dx$$

Optimal(type 3, 152 leaves, 4 steps):

$$(a+b)^5 x-\frac{b\left(5a^4+10a^3b+10a^2b^2+5ab^3+b^4\right)\coth(dx+c)}{d}-\frac{b^2\left(10a^3+10a^2b+5ab^2+b^3\right)\coth(dx+c)^3}{3d}-\frac{b^3\left(10a^2+5ba+b^2\right)\coth(dx+c)^5}{5d}-\frac{b^4(5a+b)\coth(dx+c)^7}{7d}-\frac{b^5\coth(dx+c)^9}{9d}$$

Result(type 3, 471 leaves):

$$-\frac{2\coth(dx+c)^5a^2b^3}{d}-\frac{\coth(dx+c)^5ab^4}{d}-\frac{10\coth(dx+c)^3a^3b^2}{3d}-\frac{10\coth(dx+c)^3a^2b^3}{3d}-\frac{5\coth(dx+c)^3ab^4}{3d}+\frac{5\ln(\coth(dx+c)+1)a^4b}{2d}+\frac{5\ln(\coth(dx+c)+1)a^3b^2}{d}+\frac{5\ln(\coth(dx+c)+1)a^2b^3}{d}+\frac{5\ln(\coth(dx+c)+1)ab^4}{2d}-\frac{5a^4b\coth(dx+c)}{d}-\frac{10a^3b^2\coth(dx+c)}{d}-\frac{10a^2b^3\coth(dx+c)}{d}-\frac{5ab^4\coth(dx+c)}{d}-\frac{5\ln(\coth(dx+c)-1)a^4b}{2d}-\frac{5\ln(\coth(dx+c)-1)a^3b^2}{d}-\frac{5\ln(\coth(dx+c)-1)a^2b^3}{d}-\frac{5\ln(\coth(dx+c)-1)ab^4}{2d}-\frac{5\coth(dx+c)^7ab^4}{7d}-\frac{\coth(dx+c)^7b^5}{7d}-\frac{\coth(dx+c)^5b^5}{5d}-\frac{\coth(dx+c)^3b^5}{3d}-\frac{\ln(\coth(dx+c)-1)a^5}{2d}-\frac{\ln(\coth(dx+c)-1)b^5}{2d}-\frac{b^5\coth(dx+c)}{d}+\frac{\ln(\coth(dx+c)+1)a^5}{2d}+\frac{\ln(\coth(dx+c)+1)b^5}{2d}-\frac{b^5\coth(dx+c)^9}{9d}$$

Problem 2: Result more than twice size of optimal antiderivative.

$$\int\left(a+b\coth(dx+c)^2\right)^3\,dx$$

Optimal(type 3, 70 leaves, 4 steps):

$$(a+b)^3x-\frac{b\left(3a^2+3ba+b^2\right)\coth(dx+c)}{d}-\frac{b^2(3a+b)\coth(dx+c)^3}{3d}-\frac{b^3\coth(dx+c)^5}{5d}$$

Result(type 3, 234 leaves):

$$\frac{\ln(\coth(dx+c)+1)a^3}{2d}+\frac{3\ln(\coth(dx+c)+1)a^2b}{2d}+\frac{3\ln(\coth(dx+c)+1)ab^2}{2d}+\frac{\ln(\coth(dx+c)+1)b^3}{2d}-\frac{\coth(dx+c)^3ab^2}{d}-\frac{3\coth(dx+c)a^2b}{d}-\frac{3\coth(dx+c)ab^2}{d}-\frac{\coth(dx+c)^3b^3}{3d}-\frac{\coth(dx+c)b^3}{d}-\frac{\ln(\coth(dx+c)-1)a^3}{2d}-\frac{3\ln(\coth(dx+c)-1)a^2b}{2d}-\frac{3\ln(\coth(dx+c)-1)ab^2}{2d}-\frac{\ln(\coth(dx+c)-1)b^3}{2d}-\frac{b^3\coth(dx+c)^5}{5d}$$

Problem 3: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{\left(a+b\coth(dx+c)^2\right)^3}\,dx$$

Optimal(type 3, 128 leaves, 6 steps):

$$\frac{x}{(a+b)^3}+\frac{b\coth(dx+c)}{4a(a+b)d\left(a+b\coth(dx+c)^2\right)^2}+\frac{b(7a+3b)\coth(dx+c)}{8a^2(a+b)^2d\left(a+b\coth(dx+c)^2\right)}-\frac{(15a^2+10ba+3b^2)\arctan\left(\frac{\sqrt{a}\tanh(dx+c)}{\sqrt{b}}\right)\sqrt{b}}{8a^{5/2}(a+b)^3d}$$

Result(type 3, 351 leaves):

$$\frac{\ln(\coth(dx+c)+1)}{2d(a+b)^3}+\frac{7b^2\coth(dx+c)^3}{8d(a+b)^3\left(a+b\coth(dx+c)^2\right)^2}+\frac{5b^3\coth(dx+c)^3}{4d(a+b)^3\left(a+b\coth(dx+c)^2\right)^2a}+\frac{3b^4\coth(dx+c)^3}{8d(a+b)^3\left(a+b\coth(dx+c)^2\right)^2a^2}$$
$$+\frac{9ba\coth(dx+c)}{8d(a+b)^3\left(a+b\coth(dx+c)^2\right)^2}+\frac{7b^2\coth(dx+c)}{4d(a+b)^3\left(a+b\coth(dx+c)^2\right)^2}+\frac{5b^3\coth(dx+c)}{8d(a+b)^3\left(a+b\coth(dx+c)^2\right)^2a}$$
$$+\frac{15b\arctan\left(\frac{\coth(dx+c)b}{\sqrt{ba}}\right)}{8d(a+b)^3\sqrt{ba}}+\frac{5b^2\arctan\left(\frac{\coth(dx+c)b}{\sqrt{ba}}\right)}{4d(a+b)^3a\sqrt{ba}}+\frac{3b^3\arctan\left(\frac{\coth(dx+c)b}{\sqrt{ba}}\right)}{8d(a+b)^3a^2\sqrt{ba}}-\frac{\ln(\coth(dx+c)-1)}{2d(a+b)^3}$$

Problem 6: Result more than twice size of optimal antiderivative.

$$\int\coth(x)^3\sqrt{a+b\coth(x)^2}\,dx$$

Optimal(type 3, 51 leaves, 6 steps):

$$-\frac{\left(a+b\coth(x)^2\right)^{3/2}}{3b}+\operatorname{arctanh}\left(\frac{\sqrt{a+b\coth(x)^2}}{\sqrt{a+b}}\right)\sqrt{a+b}-\sqrt{a+b\coth(x)^2}$$

Result(type 3, 252 leaves):

$$-\frac{\left(a+b\coth(x)^2\right)^{3/2}}{3b}-\frac{\sqrt{(\coth(x)-1)^2b+2(\coth(x)-1)b+a+b}}{2}$$
$$-\frac{\sqrt{b}\ln\left(\frac{(\coth(x)-1)b+b}{\sqrt{b}}+\sqrt{(\coth(x)-1)^2b+2(\coth(x)-1)b+a+b}\right)}{2}$$
$$+\frac{\sqrt{a+b}\ln\left(\frac{2a+2b+2(\coth(x)-1)b+2\sqrt{a+b}\sqrt{(\coth(x)-1)^2b+2(\coth(x)-1)b+a+b}}{\coth(x)-1}\right)}{2}$$
$$-\frac{\sqrt{(1+\coth(x))^2b-2(1+\coth(x))b+a+b}}{2}+\frac{\sqrt{b}\ln\left(\frac{(1+\coth(x))b-b}{\sqrt{b}}+\sqrt{(1+\coth(x))^2b-2(1+\coth(x))b+a+b}\right)}{2}$$
$$+\frac{\sqrt{a+b}\ln\left(\frac{2a+2b-2(1+\coth(x))b+2\sqrt{a+b}\sqrt{(1+\coth(x))^2b-2(1+\coth(x))b+a+b}}{1+\coth(x)}\right)}{2}$$

Problem 7: Result more than twice size of optimal antiderivative.

$$\int \coth(x)^2 \sqrt{a+b\coth(x)^2}\, dx$$

Optimal(type 3, 67 leaves, 7 steps):

$$-\frac{(a+2b)\operatorname{arctanh}\left(\frac{\coth(x)\sqrt{b}}{\sqrt{a+b\coth(x)^2}}\right)}{2\sqrt{b}}+\operatorname{arctanh}\left(\frac{\coth(x)\sqrt{a+b}}{\sqrt{a+b\coth(x)^2}}\right)\sqrt{a+b}-\frac{\coth(x)\sqrt{a+b\coth(x)^2}}{2}$$

Result(type 3, 275 leaves):

$$-\frac{\coth(x)\sqrt{a+b\coth(x)^2}}{2}-\frac{a\ln\left(\coth(x)\sqrt{b}+\sqrt{a+b\coth(x)^2}\right)}{2\sqrt{b}}-\frac{\sqrt{(\coth(x)-1)^2 b+2(\coth(x)-1)b+a+b}}{2}$$
$$-\frac{\sqrt{b}\ln\left(\frac{(\coth(x)-1)b+b}{\sqrt{b}}+\sqrt{(\coth(x)-1)^2 b+2(\coth(x)-1)b+a+b}\right)}{2}$$
$$+\frac{\sqrt{a+b}\ln\left(\frac{2a+2b+2(\coth(x)-1)b+2\sqrt{a+b}\sqrt{(\coth(x)-1)^2 b+2(\coth(x)-1)b+a+b}}{\coth(x)-1}\right)}{2}$$
$$+\frac{\sqrt{(1+\coth(x))^2 b-2(1+\coth(x))b+a+b}}{2}-\frac{\sqrt{b}\ln\left(\frac{(1+\coth(x))b-b}{\sqrt{b}}+\sqrt{(1+\coth(x))^2 b-2(1+\coth(x))b+a+b}\right)}{2}$$
$$-\frac{\sqrt{a+b}\ln\left(\frac{2a+2b-2(1+\coth(x))b+2\sqrt{a+b}\sqrt{(1+\coth(x))^2 b-2(1+\coth(x))b+a+b}}{1+\coth(x)}\right)}{2}$$

Problem 8: Result more than twice size of optimal antiderivative.

$$\int \coth(x)\sqrt{a+b\coth(x)^2}\, dx$$

Optimal(type 3, 36 leaves, 5 steps):

$$\operatorname{arctanh}\left(\frac{\sqrt{a+b\coth(x)^2}}{\sqrt{a+b}}\right)\sqrt{a+b}-\sqrt{a+b\coth(x)^2}$$

Result(type 3, 237 leaves):

$$-\frac{\sqrt{(\coth(x)-1)^2 b+2(\coth(x)-1)b+a+b}}{2}-\frac{\sqrt{b}\ln\left(\frac{(\coth(x)-1)b+b}{\sqrt{b}}+\sqrt{(\coth(x)-1)^2 b+2(\coth(x)-1)b+a+b}\right)}{2}$$

$$+\frac{\sqrt{a+b}\,\ln\left(\frac{2a+2b+2(\coth(x)-1)b+2\sqrt{a+b}\sqrt{(\coth(x)-1)^2 b+2(\coth(x)-1)b+a+b}}{\coth(x)-1}\right)}{2}$$

$$-\frac{\sqrt{(1+\coth(x))^2 b-2(1+\coth(x))b+a+b}}{2}+\frac{\sqrt{b}\,\ln\left(\frac{(1+\coth(x))b-b}{\sqrt{b}}+\sqrt{(1+\coth(x))^2 b-2(1+\coth(x))b+a+b}\right)}{2}$$

$$+\frac{\sqrt{a+b}\,\ln\left(\frac{2a+2b-2(1+\coth(x))b+2\sqrt{a+b}\sqrt{(1+\coth(x))^2 b-2(1+\coth(x))b+a+b}}{1+\coth(x)}\right)}{2}$$

Problem 9: Result more than twice size of optimal antiderivative.

$$\int\sqrt{a+b\coth(x)^2}\,dx$$

Optimal(type 3, 48 leaves, 6 steps):

$$-\operatorname{arctanh}\left(\frac{\coth(x)\sqrt{b}}{\sqrt{a+b\coth(x)^2}}\right)\sqrt{b}+\operatorname{arctanh}\left(\frac{\coth(x)\sqrt{a+b}}{\sqrt{a+b\coth(x)^2}}\right)\sqrt{a+b}$$

Result(type 3, 237 leaves):

$$-\frac{\sqrt{(\coth(x)-1)^2 b+2(\coth(x)-1)b+a+b}}{2}-\frac{\sqrt{b}\,\ln\left(\frac{(\coth(x)-1)b+b}{\sqrt{b}}+\sqrt{(\coth(x)-1)^2 b+2(\coth(x)-1)b+a+b}\right)}{2}$$

$$+\frac{\sqrt{a+b}\,\ln\left(\frac{2a+2b+2(\coth(x)-1)b+2\sqrt{a+b}\sqrt{(\coth(x)-1)^2 b+2(\coth(x)-1)b+a+b}}{\coth(x)-1}\right)}{2}$$

$$+\frac{\sqrt{(1+\coth(x))^2 b-2(1+\coth(x))b+a+b}}{2}-\frac{\sqrt{b}\,\ln\left(\frac{(1+\coth(x))b-b}{\sqrt{b}}+\sqrt{(1+\coth(x))^2 b-2(1+\coth(x))b+a+b}\right)}{2}$$

$$-\frac{\sqrt{a+b}\,\ln\left(\frac{2a+2b-2(1+\coth(x))b+2\sqrt{a+b}\sqrt{(1+\coth(x))^2 b-2(1+\coth(x))b+a+b}}{1+\coth(x)}\right)}{2}$$

Problem 10: Result more than twice size of optimal antiderivative.

$$\int\coth(x)^2\left(a+b\coth(x)^2\right)^{3/2}dx$$

Optimal(type 3, 101 leaves, 8 steps):

$$(a+b)^{3/2}\operatorname{arctanh}\left(\frac{\coth(x)\sqrt{a+b}}{\sqrt{a+b\coth(x)^2}}\right)-\frac{(3a^2+12ba+8b^2)\operatorname{arctanh}\left(\frac{\coth(x)\sqrt{b}}{\sqrt{a+b\coth(x)^2}}\right)}{8\sqrt{b}}-\frac{(5a+4b)\coth(x)\sqrt{a+b\coth(x)^2}}{8}$$
$$-\frac{b\coth(x)^3\sqrt{a+b\coth(x)^2}}{4}$$

Result(type 3, 632 leaves):

$$-\frac{\coth(x)\left(a+b\coth(x)^2\right)^{3/2}}{4}-\frac{3a\coth(x)\sqrt{a+b\coth(x)^2}}{8}-\frac{3a^2\ln\left(\coth(x)\sqrt{b}+\sqrt{a+b\coth(x)^2}\right)}{8\sqrt{b}}$$
$$-\frac{\left((\coth(x)-1)^2b+2(\coth(x)-1)b+a+b\right)^{3/2}}{6}-\frac{b\sqrt{(\coth(x)-1)^2b+2(\coth(x)-1)b+a+b}\coth(x)}{4}$$
$$-\frac{3\sqrt{b}\ln\left(\frac{(\coth(x)-1)b+b}{\sqrt{b}}+\sqrt{(\coth(x)-1)^2b+2(\coth(x)-1)b+a+b}\right)a}{4}$$
$$+\frac{\sqrt{a+b}\ln\left(\frac{2a+2b+2(\coth(x)-1)b+2\sqrt{a+b}\sqrt{(\coth(x)-1)^2b+2(\coth(x)-1)b+a+b}}{\coth(x)-1}\right)a}{2}$$
$$-\frac{\sqrt{(\coth(x)-1)^2b+2(\coth(x)-1)b+a+b}\,a}{2}-\frac{\ln\left(\frac{(\coth(x)-1)b+b}{\sqrt{b}}+\sqrt{(\coth(x)-1)^2b+2(\coth(x)-1)b+a+b}\right)b^{3/2}}{2}$$
$$+\frac{\sqrt{a+b}\ln\left(\frac{2a+2b+2(\coth(x)-1)b+2\sqrt{a+b}\sqrt{(\coth(x)-1)^2b+2(\coth(x)-1)b+a+b}}{\coth(x)-1}\right)b}{2}$$
$$-\frac{\sqrt{(\coth(x)-1)^2b+2(\coth(x)-1)b+a+b}\,b}{2}+\frac{\left((1+\coth(x))^2b-2(1+\coth(x))b+a+b\right)^{3/2}}{6}$$
$$-\frac{b\sqrt{(1+\coth(x))^2b-2(1+\coth(x))b+a+b}\coth(x)}{4}$$
$$-\frac{3\sqrt{b}\ln\left(\frac{(1+\coth(x))b-b}{\sqrt{b}}+\sqrt{(1+\coth(x))^2b-2(1+\coth(x))b+a+b}\right)a}{4}$$
$$-\frac{\ln\left(\frac{2a+2b-2(1+\coth(x))b+2\sqrt{a+b}\sqrt{(1+\coth(x))^2b-2(1+\coth(x))b+a+b}}{1+\coth(x)}\right)\sqrt{a+b}\,a}{2}$$
$$+\frac{\sqrt{(1+\coth(x))^2b-2(1+\coth(x))b+a+b}\,a}{2}-\frac{\ln\left(\frac{(1+\coth(x))b-b}{\sqrt{b}}+\sqrt{(1+\coth(x))^2b-2(1+\coth(x))b+a+b}\right)b^{3/2}}{2}$$

$$-\frac{\ln\left(\frac{2a+2b-2(1+\coth(x))b+2\sqrt{a+b}\sqrt{(1+\coth(x))^2 b-2(1+\coth(x))b+a+b}}{1+\coth(x)}\right)\sqrt{a+b}\,b}{2}$$

$$+\frac{\sqrt{(1+\coth(x))^2 b-2(1+\coth(x))b+a+b}\;b}{2}$$

Problem 11: Result more than twice size of optimal antiderivative.

$$\int\sqrt{-1-\coth(x)^2}\,dx$$

Optimal(type 3, 37 leaves, 6 steps):

$$\arctan\left(\frac{\coth(x)}{\sqrt{-1-\coth(x)^2}}\right)-\arctan\left(\frac{\coth(x)\sqrt{2}}{\sqrt{-1-\coth(x)^2}}\right)\sqrt{2}$$

Result(type 3, 141 leaves):

$$-\frac{\sqrt{-(\coth(x)-1)^2-2\coth(x)}}{2}+\frac{\arctan\left(\frac{\coth(x)}{\sqrt{-(\coth(x)-1)^2-2\coth(x)}}\right)}{2}+\frac{\sqrt{2}\arctan\left(\frac{(-2-2\coth(x))\sqrt{2}}{4\sqrt{-(\coth(x)-1)^2-2\coth(x)}}\right)}{2}$$

$$+\frac{\sqrt{-(1+\coth(x))^2+2\coth(x)}}{2}+\frac{\arctan\left(\frac{\coth(x)}{\sqrt{-(1+\coth(x))^2+2\coth(x)}}\right)}{2}-\frac{\sqrt{2}\arctan\left(\frac{(-2+2\coth(x))\sqrt{2}}{4\sqrt{-(1+\coth(x))^2+2\coth(x)}}\right)}{2}$$

Problem 12: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{\sqrt{a+b\coth(x)^2}}\,dx$$

Optimal(type 3, 25 leaves, 3 steps):

$$\frac{\operatorname{arctanh}\left(\frac{\coth(x)\sqrt{a+b}}{\sqrt{a+b\coth(x)^2}}\right)}{\sqrt{a+b}}$$

Result(type 3, 113 leaves):

$$\frac{\ln\left(\frac{2a+2b+2(\coth(x)-1)b+2\sqrt{a+b}\sqrt{(\coth(x)-1)^2 b+2(\coth(x)-1)b+a+b}}{\coth(x)-1}\right)}{2\sqrt{a+b}}$$

$$-\frac{\ln\left(\frac{2a+2b-2(1+\coth(x))b+2\sqrt{a+b}\sqrt{(1+\coth(x))^2 b-2(1+\coth(x))b+a+b}}{1+\coth(x)}\right)}{2\sqrt{a+b}}$$

Problem 13: Result more than twice size of optimal antiderivative.

$$\int \frac{\coth(x)^2}{\left(a+b\coth(x)^2\right)^{3/2}}\,dx$$

Optimal(type 3, 45 leaves, 4 steps):

$$\frac{\operatorname{arctanh}\left(\frac{\coth(x)\sqrt{a+b}}{\sqrt{a+b\coth(x)^2}}\right)}{(a+b)^{3/2}}-\frac{\coth(x)}{(a+b)\sqrt{a+b\coth(x)^2}}$$

Result(type 3, 288 leaves):

$$-\frac{\coth(x)}{a\sqrt{a+b\coth(x)^2}}-\frac{1}{2(a+b)\sqrt{(\coth(x)-1)^2 b+2(\coth(x)-1)b+a+b}}$$
$$+\frac{b(2(\coth(x)-1)b+2b)}{(a+b)\left(4b(a+b)-4b^2\right)\sqrt{(\coth(x)-1)^2 b+2(\coth(x)-1)b+a+b}}$$
$$+\frac{\ln\left(\frac{2a+2b+2(\coth(x)-1)b+2\sqrt{a+b}\sqrt{(\coth(x)-1)^2 b+2(\coth(x)-1)b+a+b}}{\coth(x)-1}\right)}{2(a+b)^{3/2}}$$
$$+\frac{1}{2(a+b)\sqrt{(1+\coth(x))^2 b-2(1+\coth(x))b+a+b}}+\frac{b(2(1+\coth(x))b-2b)}{(a+b)\left(4b(a+b)-4b^2\right)\sqrt{(1+\coth(x))^2 b-2(1+\coth(x))b+a+b}}$$
$$-\frac{\ln\left(\frac{2a+2b-2(1+\coth(x))b+2\sqrt{a+b}\sqrt{(1+\coth(x))^2 b-2(1+\coth(x))b+a+b}}{1+\coth(x)}\right)}{2(a+b)^{3/2}}$$

Problem 14: Unable to integrate problem.

$$\int \frac{\tanh(x)}{\left(a+b\coth(x)^2\right)^{3/2}}\,dx$$

Optimal(type 3, 64 leaves, 8 steps):

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\coth(x)^2}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\coth(x)^2}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}+\frac{b}{a(a+b)\sqrt{a+b\coth(x)^2}}$$

Result(type 8, 15 leaves):

$$\int \frac{\tanh(x)}{\left(a+b\coth(x)^2\right)^{3/2}}\,dx$$

Problem 15: Result more than twice size of optimal antiderivative.

$$\int \frac{\coth(x)}{\left(a+b\coth(x)^2\right)^{5/2}}\,dx$$

Optimal(type 3, 58 leaves, 6 steps):

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\coth(x)^2}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}-\frac{1}{3\,(a+b)\,\left(a+b\coth(x)^2\right)^{3/2}}-\frac{1}{(a+b)^2\sqrt{a+b\coth(x)^2}}$$

Result(type 3, 419 leaves):

$$-\frac{1}{6\,(a+b)\,\left((\coth(x)-1)^2 b+2\,(\coth(x)-1)\,b+a+b\right)^{3/2}}+\frac{b\coth(x)}{6\,(a+b)\,a\left((\coth(x)-1)^2 b+2\,(\coth(x)-1)\,b+a+b\right)^{3/2}}$$
$$+\frac{b\coth(x)}{3\,(a+b)\,a^2\sqrt{(\coth(x)-1)^2 b+2\,(\coth(x)-1)\,b+a+b}}-\frac{1}{2\,(a+b)^2\sqrt{(\coth(x)-1)^2 b+2\,(\coth(x)-1)\,b+a+b}}$$
$$+\frac{\coth(x)\,b}{2\,(a+b)^2 a\sqrt{(\coth(x)-1)^2 b+2\,(\coth(x)-1)\,b+a+b}}$$
$$+\frac{\ln\left(\frac{2a+2b+2\,(\coth(x)-1)\,b+2\sqrt{a+b}\,\sqrt{(\coth(x)-1)^2 b+2\,(\coth(x)-1)\,b+a+b}}{\coth(x)-1}\right)}{2\,(a+b)^{5/2}}$$
$$-\frac{1}{6\,(a+b)\,\left((1+\coth(x))^2 b-2\,(1+\coth(x))\,b+a+b\right)^{3/2}}-\frac{b\coth(x)}{6\,(a+b)\,a\left((1+\coth(x))^2 b-2\,(1+\coth(x))\,b+a+b\right)^{3/2}}$$
$$-\frac{b\coth(x)}{3\,(a+b)\,a^2\sqrt{(1+\coth(x))^2 b-2\,(1+\coth(x))\,b+a+b}}-\frac{1}{2\,(a+b)^2\sqrt{(1+\coth(x))^2 b-2\,(1+\coth(x))\,b+a+b}}$$
$$-\frac{\coth(x)\,b}{2\,(a+b)^2 a\sqrt{(1+\coth(x))^2 b-2\,(1+\coth(x))\,b+a+b}}$$
$$+\frac{\ln\left(\frac{2a+2b-2\,(1+\coth(x))\,b+2\sqrt{a+b}\,\sqrt{(1+\coth(x))^2 b-2\,(1+\coth(x))\,b+a+b}}{1+\coth(x)}\right)}{2\,(a+b)^{5/2}}$$

Problem 16: Unable to integrate problem.

$$\int \frac{\tanh(x)^2}{\left(a+b\coth(x)^2\right)^{5/2}}\, dx$$

Optimal(type 3, 113 leaves, 7 steps):

$$\frac{\operatorname{arctanh}\left(\frac{\coth(x)\sqrt{a+b}}{\sqrt{a+b\coth(x)^2}}\right)}{(a+b)^{5/2}}+\frac{b\tanh(x)}{3a(a+b)\left(a+b\coth(x)^2\right)^{3/2}}+\frac{b(7a+4b)\tanh(x)}{3a^2(a+b)^2\sqrt{a+b\coth(x)^2}}-\frac{(3a+2b)(a+4b)\sqrt{a+b\coth(x)^2}\tanh(x)}{3a^3(a+b)^2}$$

Result(type 8, 17 leaves):

$$\int \frac{\tanh(x)^2}{\left(a+b\coth(x)^2\right)^{5/2}}\, dx$$

Problem 17: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-1-\coth(x)^2}}\, dx$$

Optimal(type 3, 22 leaves, 3 steps):

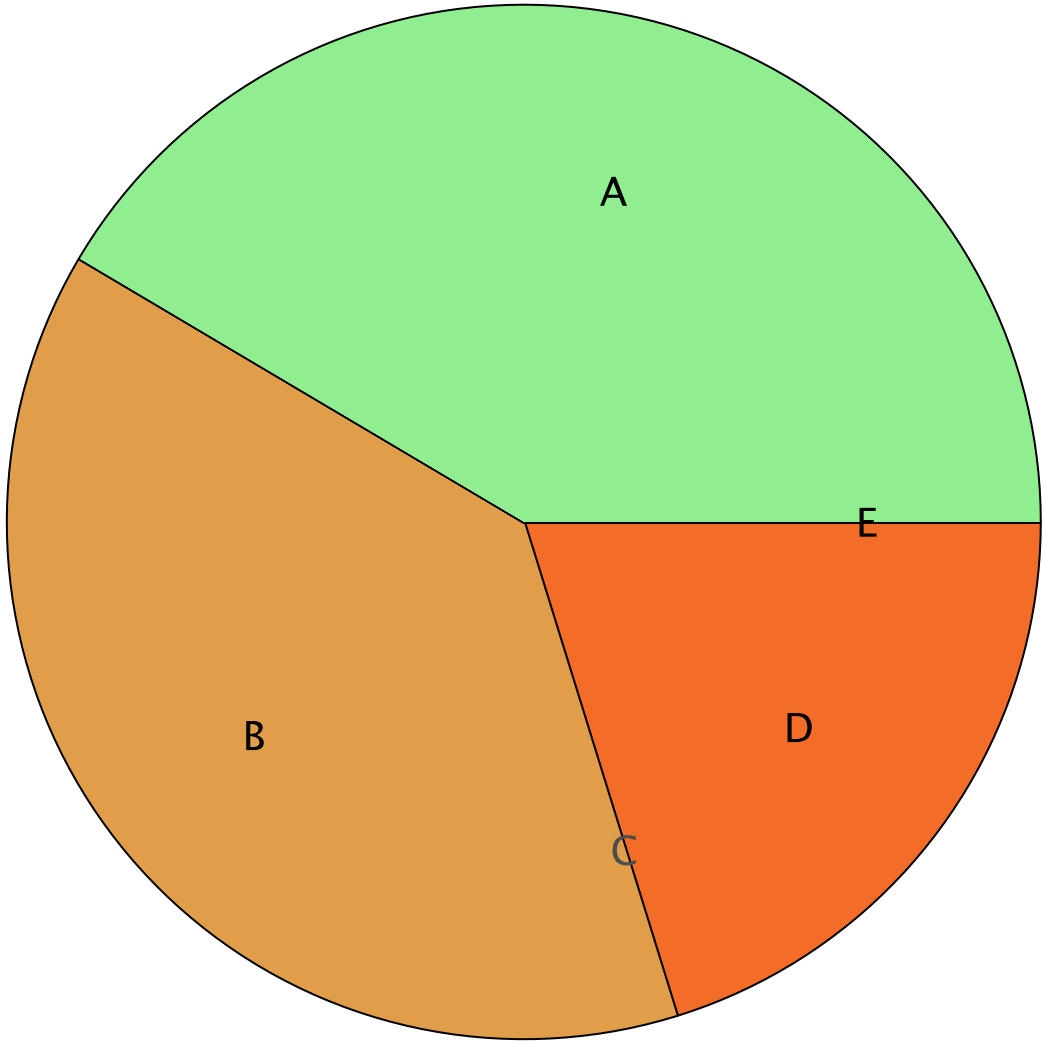
$$\frac{\arctan\left(\frac{\coth(x)\sqrt{2}}{\sqrt{-1-\coth(x)^2}}\right)\sqrt{2}}{2}$$

Result(type 3, 65 leaves):

$$-\frac{\sqrt{2}\arctan\left(\frac{(-2-2\coth(x))\sqrt{2}}{4\sqrt{-(\coth(x)-1)^2-2\coth(x)}}\right)}{4}+\frac{\sqrt{2}\arctan\left(\frac{(-2+2\coth(x))\sqrt{2}}{4\sqrt{-(1+\coth(x))^2+2\coth(x)}}\right)}{4}$$

Summary of Integration Test Results

94 integration problems



A - 39 optimal antiderivatives
B - 36 more than twice size of optimal antiderivatives
C - 0 unnecessarily complex antiderivatives
D - 19 unable to integrate problems
E - 0 integration timeouts