

# Mathematica 11.3 Integration Test Results

on the problems in the test-suite directory "1 Algebraic functions\1.1 Binomial products\1.1.1 Linear"

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Test results for the 1917 problems in "1.1.1.2  $(a+b x)^m (c+d x)^{n.m}$ "

Problem 73: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^3}{x^5} dx$$

Optimal (type 1, 17 leaves, 1 step) :

$$-\frac{(a + b x)^4}{4 a x^4}$$

Result (type 1, 39 leaves) :

$$-\frac{a^3}{4 x^4} - \frac{a^2 b}{x^3} - \frac{3 a b^2}{2 x^2} - \frac{b^3}{x}$$

Problem 82: Result more than twice size of optimal antiderivative.

$$\int x (a + b x)^5 dx$$

Optimal (type 1, 30 leaves, 2 steps) :

$$-\frac{a (a + b x)^6}{6 b^2} + \frac{(a + b x)^7}{7 b^2}$$

Result (type 1, 67 leaves) :

$$\frac{a^5 x^2}{2} + \frac{5}{3} a^4 b x^3 + \frac{5}{2} a^3 b^2 x^4 + 2 a^2 b^3 x^5 + \frac{5}{6} a b^4 x^6 + \frac{b^5 x^7}{7}$$

### Problem 90: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^5}{x^7} dx$$

Optimal (type 1, 17 leaves, 1 step) :

$$-\frac{(a + b x)^6}{6 a x^6}$$

Result (type 1, 65 leaves) :

$$-\frac{a^5}{6 x^6} - \frac{a^4 b}{x^5} - \frac{5 a^3 b^2}{2 x^4} - \frac{10 a^2 b^3}{3 x^3} - \frac{5 a b^4}{2 x^2} - \frac{b^5}{x}$$

### Problem 105: Result more than twice size of optimal antiderivative.

$$\int x (a + b x)^7 dx$$

Optimal (type 1, 30 leaves, 2 steps) :

$$-\frac{a (a + b x)^8}{8 b^2} + \frac{(a + b x)^9}{9 b^2}$$

Result (type 1, 91 leaves) :

$$\frac{a^7 x^2}{2} + \frac{7}{3} a^6 b x^3 + \frac{21}{4} a^5 b^2 x^4 + 7 a^4 b^3 x^5 + \frac{35}{6} a^3 b^4 x^6 + 3 a^2 b^5 x^7 + \frac{7}{8} a b^6 x^8 + \frac{b^7 x^9}{9}$$

### Problem 115: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^7}{x^9} dx$$

Optimal (type 1, 17 leaves, 1 step) :

$$-\frac{(a + b x)^8}{8 a x^8}$$

Result (type 1, 87 leaves) :

$$-\frac{a^7}{8 x^8} - \frac{a^6 b}{x^7} - \frac{7 a^5 b^2}{2 x^6} - \frac{7 a^4 b^3}{x^5} - \frac{35 a^3 b^4}{4 x^4} - \frac{7 a^2 b^5}{x^3} - \frac{7 a b^6}{2 x^2} - \frac{b^7}{x}$$

### Problem 116: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^7}{x^{10}} dx$$

Optimal (type 1, 36 leaves, 2 steps) :

$$-\frac{(a + b x)^8}{9 a x^9} + \frac{b (a + b x)^8}{72 a^2 x^8}$$

Result (type 1, 91 leaves) :

$$-\frac{a^7}{9 x^9} - \frac{7 a^6 b}{8 x^8} - \frac{3 a^5 b^2}{x^7} - \frac{35 a^4 b^3}{6 x^6} - \frac{7 a^3 b^4}{x^5} - \frac{21 a^2 b^5}{4 x^4} - \frac{7 a b^6}{3 x^3} - \frac{b^7}{2 x^2}$$

### Problem 132: Result more than twice size of optimal antiderivative.

$$\int x^2 (a + b x)^{10} dx$$

Optimal (type 1, 47 leaves, 2 steps) :

$$\frac{a^2 (a + b x)^{11}}{11 b^3} - \frac{a (a + b x)^{12}}{6 b^3} + \frac{(a + b x)^{13}}{13 b^3}$$

Result (type 1, 126 leaves) :

$$\frac{a^{10} x^3}{3} + \frac{5}{2} a^9 b x^4 + 9 a^8 b^2 x^5 + 20 a^7 b^3 x^6 + 30 a^6 b^4 x^7 + \frac{63}{2} a^5 b^5 x^8 + \frac{70}{3} a^4 b^6 x^9 + 12 a^3 b^7 x^{10} + \frac{45}{11} a^2 b^8 x^{11} + \frac{5}{6} a b^9 x^{12} + \frac{b^{10} x^{13}}{13}$$

### Problem 133: Result more than twice size of optimal antiderivative.

$$\int x (a + b x)^{10} dx$$

Optimal (type 1, 30 leaves, 2 steps) :

$$-\frac{a (a + b x)^{11}}{11 b^2} + \frac{(a + b x)^{12}}{12 b^2}$$

Result (type 1, 128 leaves) :

$$\frac{a^{10} x^2}{2} + \frac{10}{3} a^9 b x^3 + \frac{45}{4} a^8 b^2 x^4 + 24 a^7 b^3 x^5 + 35 a^6 b^4 x^6 + 36 a^5 b^5 x^7 + \frac{105}{4} a^4 b^6 x^8 + \frac{40}{3} a^3 b^7 x^9 + \frac{9}{2} a^2 b^8 x^{10} + \frac{10}{11} a b^9 x^{11} + \frac{b^{10} x^{12}}{12}$$

### Problem 146: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^{10}}{x^{12}} dx$$

Optimal (type 1, 17 leaves, 1 step) :

$$-\frac{(a + b x)^{11}}{11 a x^{11}}$$

Result (type 1, 114 leaves) :

$$-\frac{a^{10}}{11 x^{11}} - \frac{a^9 b}{x^{10}} - \frac{5 a^8 b^2}{x^9} - \frac{15 a^7 b^3}{x^8} - \frac{30 a^6 b^4}{x^7} - \frac{42 a^5 b^5}{x^6} - \frac{42 a^4 b^6}{x^5} - \frac{30 a^3 b^7}{x^4} - \frac{15 a^2 b^8}{x^3} - \frac{5 a b^9}{x^2} - \frac{b^{10}}{x}$$

### Problem 147: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^{10}}{x^{13}} dx$$

Optimal (type 1, 36 leaves, 2 steps) :

$$-\frac{(a + b x)^{11}}{12 a x^{12}} + \frac{b (a + b x)^{11}}{132 a^2 x^{11}}$$

Result (type 1, 128 leaves) :

$$-\frac{a^{10}}{12 x^{12}} - \frac{10 a^9 b}{11 x^{11}} - \frac{9 a^8 b^2}{2 x^{10}} - \frac{40 a^7 b^3}{3 x^9} - \frac{105 a^6 b^4}{4 x^8} - \frac{36 a^5 b^5}{x^7} - \frac{35 a^4 b^6}{x^6} - \frac{24 a^3 b^7}{x^5} - \frac{45 a^2 b^8}{4 x^4} - \frac{10 a b^9}{3 x^3} - \frac{b^{10}}{2 x^2}$$

### Problem 148: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^{10}}{x^{14}} dx$$

Optimal (type 1, 56 leaves, 3 steps) :

$$-\frac{(a + b x)^{11}}{13 a x^{13}} + \frac{b (a + b x)^{11}}{78 a^2 x^{12}} - \frac{b^2 (a + b x)^{11}}{858 a^3 x^{11}}$$

Result (type 1, 126 leaves) :

$$-\frac{a^{10}}{13 x^{13}} - \frac{5 a^9 b}{6 x^{12}} - \frac{45 a^8 b^2}{11 x^{11}} - \frac{12 a^7 b^3}{x^{10}} - \frac{70 a^6 b^4}{3 x^9} - \frac{63 a^5 b^5}{2 x^8} - \frac{30 a^4 b^6}{x^7} - \frac{20 a^3 b^7}{x^6} - \frac{9 a^2 b^8}{x^5} - \frac{5 a b^9}{2 x^4} - \frac{b^{10}}{3 x^3}$$

Problem 212: Result more than twice size of optimal antiderivative.

$$\int \frac{x^5}{(a + b x)^7} dx$$

Optimal (type 1, 17 leaves, 1 step):

$$\frac{x^6}{6 a (a + b x)^6}$$

Result (type 1, 64 leaves):

$$\frac{a^5 + 6 a^4 b x + 15 a^3 b^2 x^2 + 20 a^2 b^3 x^3 + 15 a b^4 x^4 + 6 b^5 x^5}{6 b^6 (a + b x)^6}$$

Problem 226: Result more than twice size of optimal antiderivative.

$$\int \frac{x^8}{(a + b x)^{10}} dx$$

Optimal (type 1, 17 leaves, 1 step):

$$\frac{x^9}{9 a (a + b x)^9}$$

Result (type 1, 97 leaves):

$$\frac{a^8 + 9 a^7 b x + 36 a^6 b^2 x^2 + 84 a^5 b^3 x^3 + 126 a^4 b^4 x^4 + 126 a^3 b^5 x^5 + 84 a^2 b^6 x^6 + 36 a b^7 x^7 + 9 b^8 x^8}{9 b^9 (a + b x)^9}$$

Problem 227: Result more than twice size of optimal antiderivative.

$$\int \frac{x^7}{(a + b x)^{10}} dx$$

Optimal (type 1, 35 leaves, 2 steps):

$$\frac{x^8}{9 a (a + b x)^9} + \frac{x^8}{72 a^2 (a + b x)^8}$$

Result (type 1, 86 leaves):

$$-\frac{a^7 + 9 a^6 b x + 36 a^5 b^2 x^2 + 84 a^4 b^3 x^3 + 126 a^3 b^4 x^4 + 126 a^2 b^5 x^5 + 84 a b^6 x^6 + 36 b^7 x^7}{72 b^8 (a + b x)^9}$$

**Problem 243:** Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^8}{x^{10}} dx$$

Optimal (type 1, 17 leaves, 1 step) :

$$-\frac{(a + b x)^9}{9 a x^9}$$

Result (type 1, 96 leaves) :

$$-\frac{a^8}{9 x^9} - \frac{a^7 b}{x^8} - \frac{4 a^6 b^2}{x^7} - \frac{28 a^5 b^3}{3 x^6} - \frac{14 a^4 b^4}{x^5} - \frac{14 a^3 b^5}{x^4} - \frac{28 a^2 b^6}{3 x^3} - \frac{4 a b^7}{x^2} - \frac{b^8}{x}$$

**Problem 244:** Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^7}{x^{10}} dx$$

Optimal (type 1, 36 leaves, 2 steps) :

$$-\frac{(a + b x)^8}{9 a x^9} + \frac{b (a + b x)^8}{72 a^2 x^8}$$

Result (type 1, 91 leaves) :

$$-\frac{a^7}{9 x^9} - \frac{7 a^6 b}{8 x^8} - \frac{3 a^5 b^2}{x^7} - \frac{35 a^4 b^3}{6 x^6} - \frac{7 a^3 b^4}{x^5} - \frac{21 a^2 b^5}{4 x^4} - \frac{7 a b^6}{3 x^3} - \frac{b^7}{2 x^2}$$

**Problem 368:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{x^{-1+m} (2 a m + b (-1 + 2 m) x)}{2 (a + b x)^{3/2}} dx$$

Optimal (type 3, 13 leaves, 2 steps) :

$$\frac{x^m}{\sqrt{a + b x}}$$

Result (type 5, 100 leaves):

$$\frac{1}{2 a^2 (1+m) \sqrt{1+\frac{bx}{a}}} x^m \sqrt{a+bx} \left( 2a (1+m) \text{Hypergeometric2F1}\left[-\frac{1}{2}, m, 1+m, -\frac{bx}{a}\right] - b x \left( 2m \text{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, 2+m, -\frac{bx}{a}\right] + \text{Hypergeometric2F1}\left[\frac{3}{2}, 1+m, 2+m, -\frac{bx}{a}\right] \right) \right)$$

Problem 369: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \left( -\frac{bx^m}{2(a+bx)^{3/2}} + \frac{mx^{-1+m}}{\sqrt{a+bx}} \right) dx$$

Optimal (type 3, 13 leaves, ? steps):

$$\frac{x^m}{\sqrt{a+bx}}$$

Result (type 5, 100 leaves):

$$\frac{1}{2 a^2 (1+m) \sqrt{1+\frac{bx}{a}}} x^m \sqrt{a+bx} \left( 2a (1+m) \text{Hypergeometric2F1}\left[-\frac{1}{2}, m, 1+m, -\frac{bx}{a}\right] - b x \left( 2m \text{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, 2+m, -\frac{bx}{a}\right] + \text{Hypergeometric2F1}\left[\frac{3}{2}, 1+m, 2+m, -\frac{bx}{a}\right] \right) \right)$$

Problem 375: Result unnecessarily involves higher level functions.

$$\int \frac{(a+bx)^{1/3}}{x} dx$$

Optimal (type 3, 91 leaves, 5 steps):

$$3 (a+bx)^{1/3} - \sqrt{3} a^{1/3} \text{ArcTan}\left[\frac{a^{1/3} + 2 (a+bx)^{1/3}}{\sqrt{3} a^{1/3}}\right] - \frac{1}{2} a^{1/3} \text{Log}[x] + \frac{3}{2} a^{1/3} \text{Log}[a^{1/3} - (a+bx)^{1/3}]$$

Result (type 5, 57 leaves):

$$\frac{6 (a+bx) - 3 a \left(1 + \frac{a}{bx}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{a}{bx}\right]}{2 (a+bx)^{2/3}}$$

### Problem 376: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{1/3}}{x^2} dx$$

Optimal (type 3, 97 leaves, 5 steps):

$$-\frac{(a + b x)^{1/3}}{x} - \frac{b \operatorname{ArcTan}\left[\frac{a^{1/3} + 2(a + b x)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} a^{2/3}} - \frac{b \operatorname{Log}[x]}{6 a^{2/3}} + \frac{b \operatorname{Log}[a^{1/3} - (a + b x)^{1/3}]}{2 a^{2/3}}$$

Result (type 5, 61 leaves):

$$\frac{-2(a + b x) - b \left(1 + \frac{a}{b x}\right)^{2/3} x \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{a}{b x}\right]}{2 x (a + b x)^{2/3}}$$

### Problem 377: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{1/3}}{x^3} dx$$

Optimal (type 3, 127 leaves, 6 steps):

$$-\frac{(a + b x)^{1/3}}{2 x^2} - \frac{b (a + b x)^{1/3}}{6 a x} + \frac{b^2 \operatorname{ArcTan}\left[\frac{a^{1/3} + 2(a + b x)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{3 \sqrt{3} a^{5/3}} + \frac{b^2 \operatorname{Log}[x]}{18 a^{5/3}} - \frac{b^2 \operatorname{Log}[a^{1/3} - (a + b x)^{1/3}]}{6 a^{5/3}}$$

Result (type 5, 78 leaves):

$$\frac{-3 a^2 - 4 a b x - b^2 x^2 + b^2 \left(1 + \frac{a}{b x}\right)^{2/3} x^2 \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{a}{b x}\right]}{6 a x^2 (a + b x)^{2/3}}$$

### Problem 382: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{2/3}}{x} dx$$

Optimal (type 3, 92 leaves, 5 steps):

$$\frac{3}{2} (a + b x)^{2/3} + \sqrt{3} a^{2/3} \operatorname{ArcTan}\left[\frac{a^{1/3} + 2(a + b x)^{1/3}}{\sqrt{3} a^{1/3}}\right] - \frac{1}{2} a^{2/3} \operatorname{Log}[x] + \frac{3}{2} a^{2/3} \operatorname{Log}[a^{1/3} - (a + b x)^{1/3}]$$

Result (type 5, 57 leaves):

$$\frac{3 (a + b x) - 6 a \left(1 + \frac{a}{b x}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, -\frac{a}{b x}\right]}{2 (a + b x)^{1/3}}$$

**Problem 383:** Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{2/3}}{x^2} dx$$

Optimal (type 3, 94 leaves, 5 steps):

$$-\frac{(a + b x)^{2/3}}{x} + \frac{2 b \text{ArcTan}\left[\frac{a^{1/3} + 2 (a + b x)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} a^{1/3}} - \frac{b \text{Log}[x]}{3 a^{1/3}} + \frac{b \text{Log}\left[a^{1/3} - (a + b x)^{1/3}\right]}{a^{1/3}}$$

Result (type 5, 58 leaves):

$$\frac{-a - b x - 2 b \left(1 + \frac{a}{b x}\right)^{1/3} x \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, -\frac{a}{b x}\right]}{x (a + b x)^{1/3}}$$

**Problem 384:** Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{2/3}}{x^3} dx$$

Optimal (type 3, 127 leaves, 6 steps):

$$-\frac{(a + b x)^{2/3}}{2 x^2} - \frac{b (a + b x)^{2/3}}{3 a x} - \frac{b^2 \text{ArcTan}\left[\frac{a^{1/3} + 2 (a + b x)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{3 \sqrt{3} a^{4/3}} + \frac{b^2 \text{Log}[x]}{18 a^{4/3}} - \frac{b^2 \text{Log}\left[a^{1/3} - (a + b x)^{1/3}\right]}{6 a^{4/3}}$$

Result (type 5, 79 leaves):

$$\frac{-3 a^2 - 5 a b x - 2 b^2 x^2 + 2 b^2 \left(1 + \frac{a}{b x}\right)^{1/3} x^2 \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, -\frac{a}{b x}\right]}{6 a x^2 (a + b x)^{1/3}}$$

**Problem 389:** Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{4/3}}{x} dx$$

Optimal (type 3, 105 leaves, 6 steps):

$$\frac{3 a (a + b x)^{1/3} + \frac{3}{4} (a + b x)^{4/3} - \sqrt{3} a^{4/3} \operatorname{ArcTan}\left[\frac{a^{1/3} + 2 (a + b x)^{1/3}}{\sqrt{3} a^{1/3}}\right] - \frac{1}{2} a^{4/3} \operatorname{Log}[x] + \frac{3}{2} a^{4/3} \operatorname{Log}[a^{1/3} - (a + b x)^{1/3}]}{2}$$

Result (type 5, 74 leaves) :

$$\left(\frac{15 a}{4} + \frac{3 b x}{4}\right) (a + b x)^{1/3} - \frac{3 a^2 \left(\frac{a+b x}{b x}\right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{a}{b x}\right]}{2 (a + b x)^{2/3}}$$

Problem 390: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{4/3}}{x^2} dx$$

Optimal (type 3, 107 leaves, 6 steps) :

$$4 b (a + b x)^{1/3} - \frac{(a + b x)^{4/3}}{x} - \frac{4 a^{1/3} b \operatorname{ArcTan}\left[\frac{a^{1/3} + 2 (a + b x)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3}} - \frac{2}{3} a^{1/3} b \operatorname{Log}[x] + 2 a^{1/3} b \operatorname{Log}[a^{1/3} - (a + b x)^{1/3}]$$

Result (type 5, 64 leaves) :

$$\frac{\left(3 b - \frac{a}{x}\right) (a + b x) - 2 a b \left(1 + \frac{a}{b x}\right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{a}{b x}\right]}{(a + b x)^{2/3}}$$

Problem 391: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{4/3}}{x^3} dx$$

Optimal (type 3, 124 leaves, 6 steps) :

$$-\frac{2 b (a + b x)^{1/3}}{3 x} - \frac{(a + b x)^{4/3}}{2 x^2} - \frac{2 b^2 \operatorname{ArcTan}\left[\frac{a^{1/3} + 2 (a + b x)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{3 \sqrt{3} a^{2/3}} - \frac{b^2 \operatorname{Log}[x]}{9 a^{2/3}} + \frac{b^2 \operatorname{Log}[a^{1/3} - (a + b x)^{1/3}]}{3 a^{2/3}}$$

Result (type 5, 76 leaves) :

$$\frac{-3 a^2 - 10 a b x - 7 b^2 x^2 - 2 b^2 \left(1 + \frac{a}{b x}\right)^{2/3} x^2 \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{a}{b x}\right]}{6 x^2 (a + b x)^{2/3}}$$

### Problem 396: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x(a + bx)^{1/3}} dx$$

Optimal (type 3, 79 leaves, 4 steps):

$$\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{a^{1/3}+2(a+b x)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{a^{1/3}} - \frac{\operatorname{Log}[x]}{2 a^{1/3}} + \frac{3 \operatorname{Log}\left[a^{1/3}-(a+b x)^{1/3}\right]}{2 a^{1/3}}$$

Result (type 5, 46 leaves):

$$-\frac{3 \left(\frac{a+b x}{b x}\right)^{1/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, -\frac{a}{b x}\right]}{(a+b x)^{1/3}}$$

### Problem 397: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^2(a + bx)^{1/3}} dx$$

Optimal (type 3, 100 leaves, 5 steps):

$$-\frac{(a+b x)^{2/3}}{a x} - \frac{b \operatorname{ArcTan}\left[\frac{a^{1/3}+2(a+b x)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} a^{4/3}} + \frac{b \operatorname{Log}[x]}{6 a^{4/3}} - \frac{b \operatorname{Log}\left[a^{1/3}-(a+b x)^{1/3}\right]}{2 a^{4/3}}$$

Result (type 5, 60 leaves):

$$-\frac{a-b x+b\left(1+\frac{a}{b x}\right)^{1/3} x \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, -\frac{a}{b x}\right]}{a x(a+b x)^{1/3}}$$

### Problem 398: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^3(a + bx)^{1/3}} dx$$

Optimal (type 3, 130 leaves, 6 steps):

$$-\frac{(a+b x)^{2/3}}{2 a x^2} + \frac{2 b (a+b x)^{2/3}}{3 a^2 x} + \frac{2 b^2 \operatorname{ArcTan}\left[\frac{a^{1/3}+2(a+b x)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{3 \sqrt{3} a^{7/3}} - \frac{b^2 \operatorname{Log}[x]}{9 a^{7/3}} + \frac{b^2 \operatorname{Log}\left[a^{1/3}-(a+b x)^{1/3}\right]}{3 a^{7/3}}$$

Result (type 5, 78 leaves) :

$$\frac{-3 a^2 + a b x + 4 b^2 x^2 - 4 b^2 \left(1 + \frac{a}{b x}\right)^{1/3} x^2 \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, -\frac{a}{b x}\right]}{6 a^2 x^2 (a + b x)^{1/3}}$$

Problem 404: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^2 (-a + b x)^{1/3}} dx$$

Optimal (type 3, 103 leaves, 5 steps) :

$$\frac{(-a + b x)^{2/3}}{a x} - \frac{b \text{ArcTan}\left[\frac{a^{1/3} - 2(-a + b x)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} a^{4/3}} + \frac{b \text{Log}[x]}{6 a^{4/3}} - \frac{b \text{Log}\left[a^{1/3} + (-a + b x)^{1/3}\right]}{2 a^{4/3}}$$

Result (type 5, 62 leaves) :

$$\frac{-a + b x - b \left(1 - \frac{a}{b x}\right)^{1/3} x \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{a}{b x}\right]}{a x (-a + b x)^{1/3}}$$

Problem 405: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^3 (-a + b x)^{1/3}} dx$$

Optimal (type 3, 136 leaves, 6 steps) :

$$\frac{(-a + b x)^{2/3}}{2 a x^2} + \frac{2 b (-a + b x)^{2/3}}{3 a^2 x} - \frac{2 b^2 \text{ArcTan}\left[\frac{a^{1/3} - 2(-a + b x)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{3 \sqrt{3} a^{7/3}} + \frac{b^2 \text{Log}[x]}{9 a^{7/3}} - \frac{b^2 \text{Log}\left[a^{1/3} + (-a + b x)^{1/3}\right]}{3 a^{7/3}}$$

Result (type 5, 81 leaves) :

$$\frac{-3 a^2 - a b x + 4 b^2 x^2 - 4 b^2 \left(1 - \frac{a}{b x}\right)^{1/3} x^2 \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{a}{b x}\right]}{6 a^2 x^2 (-a + b x)^{1/3}}$$

Problem 410: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x (a + b x)^{2/3}} dx$$

Optimal (type 3, 80 leaves, 4 steps):

$$-\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{a^{1/3}+2(a+b x)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{a^{2/3}} - \frac{\operatorname{Log}[x]}{2 a^{2/3}} + \frac{3 \operatorname{Log}\left[a^{1/3}-\left(a+b x\right)^{1/3}\right]}{2 a^{2/3}}$$

Result (type 5, 48 leaves):

$$-\frac{3\left(\frac{a+b x}{b x}\right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{a}{b x}\right]}{2(a+b x)^{2/3}}$$

Problem 411: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^2(a+b x)^{2/3}} dx$$

Optimal (type 3, 98 leaves, 5 steps):

$$-\frac{(a+b x)^{1/3}}{a x} + \frac{2 b \operatorname{ArcTan}\left[\frac{a^{1/3}+2(a+b x)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} a^{5/3}} + \frac{b \operatorname{Log}[x]}{3 a^{5/3}} - \frac{b \operatorname{Log}\left[a^{1/3}-\left(a+b x\right)^{1/3}\right]}{a^{5/3}}$$

Result (type 5, 60 leaves):

$$-\frac{a-b x+b\left(1+\frac{a}{b x}\right)^{2/3} x \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{a}{b x}\right]}{a x(a+b x)^{2/3}}$$

Problem 412: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^3(a+b x)^{2/3}} dx$$

Optimal (type 3, 130 leaves, 6 steps):

$$-\frac{(a+b x)^{1/3}}{2 a x^2} + \frac{5 b(a+b x)^{1/3}}{6 a^2 x} - \frac{5 b^2 \operatorname{ArcTan}\left[\frac{a^{1/3}+2(a+b x)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{3 \sqrt{3} a^{8/3}} - \frac{5 b^2 \operatorname{Log}[x]}{18 a^{8/3}} + \frac{5 b^2 \operatorname{Log}\left[a^{1/3}-\left(a+b x\right)^{1/3}\right]}{6 a^{8/3}}$$

Result (type 5, 79 leaves):

$$-\frac{3 a^2+2 a b x+5 b^2 x^2-5 b^2\left(1+\frac{a}{b x}\right)^{2/3} x^2 \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{a}{b x}\right]}{6 a^2 x^2(a+b x)^{2/3}}$$

### Problem 417: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x(a+b x)^{4/3}} dx$$

Optimal (type 3, 93 leaves, 5 steps):

$$\frac{3}{a(a+b x)^{1/3}} + \frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{a^{1/3}+2(a+b x)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{a^{4/3}} - \frac{\operatorname{Log}[x]}{2 a^{4/3}} + \frac{3 \operatorname{Log}\left[a^{1/3}-(a+b x)^{1/3}\right]}{2 a^{4/3}}$$

Result (type 5, 50 leaves):

$$\frac{3-3\left(1+\frac{a}{b x}\right)^{1/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, -\frac{a}{b x}\right]}{a(a+b x)^{1/3}}$$

### Problem 418: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^2(a+b x)^{4/3}} dx$$

Optimal (type 3, 113 leaves, 6 steps):

$$-\frac{4 b}{a^2(a+b x)^{1/3}} - \frac{1}{a x(a+b x)^{1/3}} - \frac{4 b \operatorname{ArcTan}\left[\frac{a^{1/3}+2(a+b x)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} a^{7/3}} + \frac{2 b \operatorname{Log}[x]}{3 a^{7/3}} - \frac{2 b \operatorname{Log}\left[a^{1/3}-(a+b x)^{1/3}\right]}{a^{7/3}}$$

Result (type 5, 61 leaves):

$$\frac{-a-4 b x+4 b\left(1+\frac{a}{b x}\right)^{1/3} x \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, -\frac{a}{b x}\right]}{a^2 x(a+b x)^{1/3}}$$

### Problem 419: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^3(a+b x)^{4/3}} dx$$

Optimal (type 3, 149 leaves, 7 steps):

$$\frac{14 b^2}{3 a^3(a+b x)^{1/3}} - \frac{1}{2 a x^2(a+b x)^{1/3}} + \frac{7 b}{6 a^2 x(a+b x)^{1/3}} + \frac{14 b^2 \operatorname{ArcTan}\left[\frac{a^{1/3}+2(a+b x)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{3 \sqrt{3} a^{10/3}} - \frac{7 b^2 \operatorname{Log}[x]}{9 a^{10/3}} + \frac{7 b^2 \operatorname{Log}\left[a^{1/3}-(a+b x)^{1/3}\right]}{3 a^{10/3}}$$

Result (type 5, 79 leaves) :

$$\frac{-3 a^2 + 7 a b x + 28 b^2 x^2 - 28 b^2 \left(1 + \frac{a}{b x}\right)^{1/3} x^2 \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, -\frac{a}{b x}\right]}{6 a^3 x^2 (a + b x)^{1/3}}$$

Problem 648: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{1-x} \sqrt{x}} dx$$

Optimal (type 3, 8 leaves, 3 steps) :

$$-\text{ArcSin}[1 - 2 x]$$

Result (type 3, 38 leaves) :

$$\frac{2 \sqrt{-1+x} \sqrt{x} \log \left[\sqrt{-1+x}+\sqrt{x}\right]}{\sqrt{-(-1+x) x}}$$

Problem 707: Result more than twice size of optimal antiderivative.

$$\int x^m (a + b x)^{5/2} dx$$

Optimal (type 5, 48 leaves, 2 steps) :

$$\frac{2 x^m \left(-\frac{b x}{a}\right)^{-m} (a + b x)^{7/2} \text{Hypergeometric2F1}\left[\frac{7}{2}, -m, \frac{9}{2}, 1 + \frac{b x}{a}\right]}{7 b}$$

Result (type 5, 125 leaves) :

$$\begin{aligned} & \left(x^{1+m} \sqrt{a+b x} \left(a^2 (6+5 m+m^2) \text{Hypergeometric2F1}\left[-\frac{1}{2}, 1+m, 2+m, -\frac{b x}{a}\right] +\right.\right. \\ & b (1+m) \times \left(2 a (3+m) \text{Hypergeometric2F1}\left[-\frac{1}{2}, 2+m, 3+m, -\frac{b x}{a}\right] + b (2+m) \times \text{Hypergeometric2F1}\left[-\frac{1}{2}, 3+m, 4+m, -\frac{b x}{a}\right]\right)\left.\right)\Bigg) \Bigg/ \left((1+m) (2+m) (3+m) \sqrt{1+\frac{b x}{a}}\right) \end{aligned}$$

**Problem 713:** Result more than twice size of optimal antiderivative.

$$\int \frac{x^{2+m}}{\sqrt{a+bx}} dx$$

Optimal (type 5, 51 leaves, 2 steps):

$$\frac{2 a^2 x^m \left(-\frac{b x}{a}\right)^{-m} \sqrt{a+b x} \text{Hypergeometric2F1}\left[\frac{1}{2}, -2-m, \frac{3}{2}, 1+\frac{b x}{a}\right]}{b^3}$$

Result (type 5, 109 leaves):

$$\begin{aligned} & \left( x^{1+m} \sqrt{a+b x} \left( -a (2+m) \text{Hypergeometric2F1}\left[-\frac{1}{2}, 1+m, 2+m, -\frac{b x}{a}\right] + \right. \right. \\ & \left. \left. b (1+m) x \text{Hypergeometric2F1}\left[-\frac{1}{2}, 2+m, 3+m, -\frac{b x}{a}\right] + a (2+m) \text{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, 2+m, -\frac{b x}{a}\right] \right) \right) \Big/ \left( b^2 (1+m) (2+m) \sqrt{1+\frac{b x}{a}} \right) \end{aligned}$$

**Problem 717:** Result more than twice size of optimal antiderivative.

$$\int \frac{x^{-2+m}}{\sqrt{a+bx}} dx$$

Optimal (type 5, 49 leaves, 2 steps):

$$\frac{2 b x^m \left(-\frac{b x}{a}\right)^{-m} \sqrt{a+b x} \text{Hypergeometric2F1}\left[\frac{1}{2}, 2-m, \frac{3}{2}, 1+\frac{b x}{a}\right]}{a^2}$$

Result (type 5, 114 leaves):

$$\begin{aligned} & \frac{1}{a^3 m (-1+m^2)} x^{-1+m} \sqrt{a+b x} \left( a^2 m (1+m) \text{Hypergeometric2F1}\left[-\frac{1}{2}, -1+m, m, -\frac{b x}{a}\right] - \right. \\ & \left. b (-1+m) x \left( a (1+m) \text{Hypergeometric2F1}\left[-\frac{1}{2}, m, 1+m, -\frac{b x}{a}\right] - b m x \text{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, 2+m, -\frac{b x}{a}\right] \right) \right) \end{aligned}$$

**Problem 718:** Result more than twice size of optimal antiderivative.

$$\int \frac{x^{-3+m}}{\sqrt{a+bx}} dx$$

Optimal (type 5, 51 leaves, 2 steps):

$$\frac{2 b^2 x^m \left(-\frac{b x}{a}\right)^{-m} \sqrt{a+b x} \text{Hypergeometric2F1}\left[\frac{1}{2}, 3-m, \frac{3}{2}, 1+\frac{b x}{a}\right]}{a^3}$$

Result (type 5, 156 leaves):

$$\left( x^{-2+m} \sqrt{1 + \frac{bx}{a}} \left( a^3 m (-1 + m^2) \text{Hypergeometric2F1}\left[-\frac{1}{2}, -2+m, -1+m, -\frac{bx}{a}\right] - b (-2+m) x \left( a^2 m (1+m) \text{Hypergeometric2F1}\left[-\frac{1}{2}, -1+m, m, -\frac{bx}{a}\right] + b (-1+m) x \left( -a (1+m) \text{Hypergeometric2F1}\left[-\frac{1}{2}, m, 1+m, -\frac{bx}{a}\right] + b m x \text{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, 2+m, -\frac{bx}{a}\right] \right) \right) \right) / \left( a^3 (-2+m) (-1+m) m (1+m) \sqrt{a+b x} \right)$$

Problem 1162: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{3-x} \sqrt{-2+x}} dx$$

Optimal (type 3, 8 leaves, 3 steps):

$$-\text{ArcSin}[5-2x]$$

Result (type 3, 36 leaves):

$$\frac{2 \sqrt{-3+x} \sqrt{-2+x} \text{ArcSinh}\left[\sqrt{-3+x}\right]}{\sqrt{-(-3+x) (-2+x)}}$$

Problem 1170: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(6-3e x)^{1/4} (2+e x)^{3/4}} dx$$

Optimal (type 3, 241 leaves, 11 steps):

$$\frac{\sqrt{2} \text{ArcTan}\left[1-\frac{\sqrt{2} (2-e x)^{1/4}}{(2+e x)^{1/4}}\right]}{3^{1/4} e}-\frac{\sqrt{2} \text{ArcTan}\left[1+\frac{\sqrt{2} (2-e x)^{1/4}}{(2+e x)^{1/4}}\right]}{3^{1/4} e}-\frac{\text{Log}\left[\frac{\sqrt{6-3 e x}-\sqrt{6} (2-e x)^{1/4} (2+e x)^{1/4}+\sqrt{3} \sqrt{2+e x}}{\sqrt{2+e x}}\right]}{\sqrt{2} 3^{1/4} e}+\frac{\text{Log}\left[\frac{\sqrt{6-3 e x}+\sqrt{6} (2-e x)^{1/4} (2+e x)^{1/4}+\sqrt{3} \sqrt{2+e x}}{\sqrt{2+e x}}\right]}{\sqrt{2} 3^{1/4} e}$$

Result (type 5, 43 leaves):

$$\frac{2 \sqrt{2} (2 + e x)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \frac{1}{4} (2 + e x)\right]}{3^{1/4} e}$$

**Problem 1171:** Result unnecessarily involves higher level functions.

$$\int \frac{(a - i a x)^{7/4}}{(a + i a x)^{1/4}} dx$$

Optimal (type 4, 144 leaves, 6 steps):

$$\frac{14 a^2 x}{5 (a - i a x)^{1/4} (a + i a x)^{1/4}} - \frac{14}{15} i (a - i a x)^{3/4} (a + i a x)^{3/4} - \frac{2 i (a - i a x)^{7/4} (a + i a x)^{3/4}}{5 a} - \frac{14 a^2 (1 + x^2)^{1/4} \text{EllipticE}\left[\frac{\text{ArcTan}[x]}{2}, 2\right]}{5 (a - i a x)^{1/4} (a + i a x)^{1/4}}$$

Result (type 5, 84 leaves):

$$\frac{2 a (a - i a x)^{3/4} \left(-10 i + 7 x - 3 i x^2 + 7 i 2^{3/4} (1 + i x)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1}{2} - \frac{i x}{2}\right]\right)}{15 (a + i a x)^{1/4}}$$

**Problem 1172:** Result unnecessarily involves higher level functions.

$$\int \frac{(a - i a x)^{3/4}}{(a + i a x)^{1/4}} dx$$

Optimal (type 4, 106 leaves, 5 steps):

$$\frac{2 a x}{(a - i a x)^{1/4} (a + i a x)^{1/4}} - \frac{2 i (a - i a x)^{3/4} (a + i a x)^{3/4}}{3 a} - \frac{2 a (1 + x^2)^{1/4} \text{EllipticE}\left[\frac{\text{ArcTan}[x]}{2}, 2\right]}{(a - i a x)^{1/4} (a + i a x)^{1/4}}$$

Result (type 5, 74 leaves):

$$\frac{2 (a - i a x)^{3/4} \left(-i + x + i 2^{3/4} (1 + i x)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1}{2} - \frac{i x}{2}\right]\right)}{3 (a + i a x)^{1/4}}$$

**Problem 1173:** Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a - i a x)^{1/4} (a + i a x)^{1/4}} dx$$

Optimal (type 4, 71 leaves, 4 steps):

$$\frac{2x}{(a - \text{i}ax)^{1/4} (a + \text{i}ax)^{1/4}} - \frac{2(1+x^2)^{1/4} \text{EllipticE}\left[\frac{\text{ArcTan}[x]}{2}, 2\right]}{(a - \text{i}ax)^{1/4} (a + \text{i}ax)^{1/4}}$$

Result (type 5, 70 leaves) :

$$\frac{2\text{i} 2^{3/4} (1 + \text{i}x)^{1/4} (a - \text{i}ax)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1}{2} - \frac{\text{i}x}{2}\right]}{3a (a + \text{i}ax)^{1/4}}$$

Problem 1174: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a - \text{i}ax)^{5/4} (a + \text{i}ax)^{1/4}} dx$$

Optimal (type 4, 78 leaves, 4 steps) :

$$-\frac{2\text{i}}{a (a - \text{i}ax)^{1/4} (a + \text{i}ax)^{1/4}} + \frac{2(1+x^2)^{1/4} \text{EllipticE}\left[\frac{\text{ArcTan}[x]}{2}, 2\right]}{a (a - \text{i}ax)^{1/4} (a + \text{i}ax)^{1/4}}$$

Result (type 5, 82 leaves) :

$$\frac{-6\text{i} + 6x - 2 \times 2^{3/4} (1 + \text{i}x)^{1/4} (\text{i} + x) \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1}{2} - \frac{\text{i}x}{2}\right]}{3a (a - \text{i}ax)^{1/4} (a + \text{i}ax)^{1/4}}$$

Problem 1175: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a - \text{i}ax)^{9/4} (a + \text{i}ax)^{1/4}} dx$$

Optimal (type 4, 82 leaves, 4 steps) :

$$-\frac{4\text{i}}{5a (a - \text{i}ax)^{5/4} (a + \text{i}ax)^{1/4}} + \frac{2(1+x^2)^{1/4} \text{EllipticE}\left[\frac{\text{ArcTan}[x]}{2}, 2\right]}{5a^2 (a - \text{i}ax)^{1/4} (a + \text{i}ax)^{1/4}}$$

Result (type 5, 97 leaves) :

$$\frac{6 (2 + \text{i}x + x^2) - 2 \times 2^{3/4} (1 + \text{i}x)^{1/4} (\text{i} + x)^2 \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1}{2} - \frac{\text{i}x}{2}\right]}{15a^2 (\text{i} + x) (a - \text{i}ax)^{1/4} (a + \text{i}ax)^{1/4}}$$

### Problem 1176: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a - \text{i} a x)^{13/4} (a + \text{i} a x)^{1/4}} dx$$

Optimal (type 4, 115 leaves, 5 steps):

$$-\frac{4 \text{i}}{15 a^2 (a - \text{i} a x)^{5/4} (a + \text{i} a x)^{1/4}} - \frac{2 \text{i} (a + \text{i} a x)^{3/4}}{9 a^2 (a - \text{i} a x)^{9/4}} + \frac{2 (1 + x^2)^{1/4} \text{EllipticE}\left[\frac{\text{ArcTan}[x]}{2}, 2\right]}{15 a^3 (a - \text{i} a x)^{1/4} (a + \text{i} a x)^{1/4}}$$

Result (type 5, 103 leaves):

$$\frac{22 \text{i} - 4 x + 12 \text{i} x^2 + 6 x^3 - 2 \times 2^{3/4} (1 + \text{i} x)^{1/4} (\text{i} + x)^3 \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1}{2} - \frac{\text{i} x}{2}\right]}{45 a^3 (\text{i} + x)^2 (a - \text{i} a x)^{1/4} (a + \text{i} a x)^{1/4}}$$

### Problem 1177: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a - \text{i} a x)^{17/4} (a + \text{i} a x)^{1/4}} dx$$

Optimal (type 4, 148 leaves, 6 steps):

$$-\frac{4 \text{i}}{39 a^3 (a - \text{i} a x)^{5/4} (a + \text{i} a x)^{1/4}} - \frac{2 \text{i} (a + \text{i} a x)^{3/4}}{13 a^2 (a - \text{i} a x)^{13/4}} - \frac{10 \text{i} (a + \text{i} a x)^{3/4}}{117 a^3 (a - \text{i} a x)^{9/4}} + \frac{2 (1 + x^2)^{1/4} \text{EllipticE}\left[\frac{\text{ArcTan}[x]}{2}, 2\right]}{39 a^4 (a - \text{i} a x)^{1/4} (a + \text{i} a x)^{1/4}}$$

Result (type 5, 102 leaves):

$$\frac{2 \left(20 + 8 x^2 - 9 \text{i} x^3 - 3 x^4 + 2^{3/4} (1 + \text{i} x)^{1/4} (\text{i} + x)^4 \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1}{2} - \frac{\text{i} x}{2}\right]\right)}{117 a^4 (\text{i} + x)^3 (a - \text{i} a x)^{1/4} (a + \text{i} a x)^{1/4}}$$

### Problem 1178: Result unnecessarily involves higher level functions.

$$\int \frac{(a - \text{i} a x)^{1/4}}{(a + \text{i} a x)^{1/4}} dx$$

Optimal (type 3, 256 leaves, 12 steps):

$$\begin{aligned}
 & -\frac{\frac{i}{2} (a - i a x)^{1/4} (a + i a x)^{3/4}}{a} - \frac{i \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (a - i a x)^{1/4}}{(a + i a x)^{1/4}}\right]}{\sqrt{2}} + \\
 & \frac{i \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (a - i a x)^{1/4}}{(a + i a x)^{1/4}}\right]}{\sqrt{2}} - \frac{i \operatorname{Log}\left[1 + \frac{\sqrt{a - i a x}}{\sqrt{a + i a x}} - \frac{\sqrt{2} (a - i a x)^{1/4}}{(a + i a x)^{1/4}}\right]}{2 \sqrt{2}} + \frac{i \operatorname{Log}\left[1 + \frac{\sqrt{a - i a x}}{\sqrt{a + i a x}} + \frac{\sqrt{2} (a - i a x)^{1/4}}{(a + i a x)^{1/4}}\right]}{2 \sqrt{2}}
 \end{aligned}$$

Result (type 5, 71 leaves):

$$\frac{(a - i a x)^{1/4} \left( -\frac{i}{2} + x + \frac{i}{2} 2^{3/4} (1 + i x)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \frac{1}{2} - \frac{i x}{2}\right] \right)}{(a + i a x)^{1/4}}$$

Problem 1179: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a - i a x)^{3/4} (a + i a x)^{1/4}} dx$$

Optimal (type 3, 233 leaves, 11 steps):

$$\begin{aligned}
 & -\frac{\frac{i \sqrt{2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (a - i a x)^{1/4}}{(a + i a x)^{1/4}}\right]}{a}}{a} + \frac{\frac{i \sqrt{2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (a - i a x)^{1/4}}{(a + i a x)^{1/4}}\right]}{a}}{a} - \frac{i \operatorname{Log}\left[1 + \frac{\sqrt{a - i a x}}{\sqrt{a + i a x}} - \frac{\sqrt{2} (a - i a x)^{1/4}}{(a + i a x)^{1/4}}\right]}{\sqrt{2} a} + \frac{i \operatorname{Log}\left[1 + \frac{\sqrt{a - i a x}}{\sqrt{a + i a x}} + \frac{\sqrt{2} (a - i a x)^{1/4}}{(a + i a x)^{1/4}}\right]}{\sqrt{2} a}
 \end{aligned}$$

Result (type 5, 68 leaves):

$$\frac{2 \frac{i}{2} 2^{3/4} (1 + i x)^{1/4} (a - i a x)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \frac{1}{2} - \frac{i x}{2}\right]}{a (a + i a x)^{1/4}}$$

Problem 1184: Result unnecessarily involves higher level functions.

$$\int \frac{(a - i a x)^{3/4}}{(a + i a x)^{3/4}} dx$$

Optimal (type 3, 256 leaves, 12 steps):

$$\begin{aligned}
 & -\frac{\frac{i}{2} (a - i a x)^{3/4} (a + i a x)^{1/4}}{a} - \frac{3 i \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (a - i a x)^{1/4}}{(a + i a x)^{1/4}}\right]}{\sqrt{2}} + \\
 & \frac{3 i \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (a - i a x)^{1/4}}{(a + i a x)^{1/4}}\right]}{\sqrt{2}} + \frac{3 i \operatorname{Log}\left[1 + \frac{\sqrt{a - i a x}}{\sqrt{a + i a x}} - \frac{\sqrt{2} (a - i a x)^{1/4}}{(a + i a x)^{1/4}}\right]}{2 \sqrt{2}} - \frac{3 i \operatorname{Log}\left[1 + \frac{\sqrt{a - i a x}}{\sqrt{a + i a x}} + \frac{\sqrt{2} (a - i a x)^{1/4}}{(a + i a x)^{1/4}}\right]}{2 \sqrt{2}}
 \end{aligned}$$

Result (type 5, 71 leaves) :

$$\frac{(\mathbf{a} - \mathbf{i} \mathbf{a} \mathbf{x})^{3/4} \left( -\mathbf{i} + \mathbf{x} + \mathbf{i} \sqrt{2}^{1/4} (1 + \mathbf{i} \mathbf{x})^{3/4} \text{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1}{2} - \frac{\mathbf{i} \mathbf{x}}{2}\right] \right)}{(\mathbf{a} + \mathbf{i} \mathbf{a} \mathbf{x})^{3/4}}$$

Problem 1185: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(\mathbf{a} - \mathbf{i} \mathbf{a} \mathbf{x})^{1/4} (\mathbf{a} + \mathbf{i} \mathbf{a} \mathbf{x})^{3/4}} d\mathbf{x}$$

Optimal (type 3, 233 leaves, 11 steps) :

$$-\frac{\frac{\mathbf{i} \sqrt{2} \text{ArcTan}\left[1 - \frac{\sqrt{2} (\mathbf{a} - \mathbf{i} \mathbf{a} \mathbf{x})^{1/4}}{(\mathbf{a} + \mathbf{i} \mathbf{a} \mathbf{x})^{1/4}}\right]}{\mathbf{a}} + \frac{\mathbf{i} \sqrt{2} \text{ArcTan}\left[1 + \frac{\sqrt{2} (\mathbf{a} - \mathbf{i} \mathbf{a} \mathbf{x})^{1/4}}{(\mathbf{a} + \mathbf{i} \mathbf{a} \mathbf{x})^{1/4}}\right]}{\mathbf{a}} + \frac{\mathbf{i} \text{Log}\left[1 + \frac{\sqrt{\mathbf{a} - \mathbf{i} \mathbf{a} \mathbf{x}}}{\sqrt{\mathbf{a} + \mathbf{i} \mathbf{a} \mathbf{x}}} - \frac{\sqrt{2} (\mathbf{a} - \mathbf{i} \mathbf{a} \mathbf{x})^{1/4}}{(\mathbf{a} + \mathbf{i} \mathbf{a} \mathbf{x})^{1/4}}\right]}{\sqrt{2} \mathbf{a}} - \frac{\mathbf{i} \text{Log}\left[1 + \frac{\sqrt{\mathbf{a} - \mathbf{i} \mathbf{a} \mathbf{x}}}{\sqrt{\mathbf{a} + \mathbf{i} \mathbf{a} \mathbf{x}}} + \frac{\sqrt{2} (\mathbf{a} - \mathbf{i} \mathbf{a} \mathbf{x})^{1/4}}{(\mathbf{a} + \mathbf{i} \mathbf{a} \mathbf{x})^{1/4}}\right]}{\sqrt{2} \mathbf{a}}}{\mathbf{a}}$$

Result (type 5, 70 leaves) :

$$\frac{2 \mathbf{i} 2^{1/4} (1 + \mathbf{i} \mathbf{x})^{3/4} (\mathbf{a} - \mathbf{i} \mathbf{a} \mathbf{x})^{3/4} \text{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1}{2} - \frac{\mathbf{i} \mathbf{x}}{2}\right]}{3 \mathbf{a} (\mathbf{a} + \mathbf{i} \mathbf{a} \mathbf{x})^{3/4}}$$

Problem 1189: Result unnecessarily involves higher level functions.

$$\int \frac{(\mathbf{a} - \mathbf{i} \mathbf{a} \mathbf{x})^{5/4}}{(\mathbf{a} + \mathbf{i} \mathbf{a} \mathbf{x})^{3/4}} d\mathbf{x}$$

Optimal (type 4, 112 leaves, 5 steps) :

$$-\frac{10}{3} \mathbf{i} (\mathbf{a} - \mathbf{i} \mathbf{a} \mathbf{x})^{1/4} (\mathbf{a} + \mathbf{i} \mathbf{a} \mathbf{x})^{1/4} - \frac{2 \mathbf{i} (\mathbf{a} - \mathbf{i} \mathbf{a} \mathbf{x})^{5/4} (\mathbf{a} + \mathbf{i} \mathbf{a} \mathbf{x})^{1/4}}{3 \mathbf{a}} + \frac{10 \mathbf{a}^2 (1 + \mathbf{x}^2)^{3/4} \text{EllipticF}\left[\frac{\text{ArcTan}[\mathbf{x}]}{2}, 2\right]}{3 (\mathbf{a} - \mathbf{i} \mathbf{a} \mathbf{x})^{3/4} (\mathbf{a} + \mathbf{i} \mathbf{a} \mathbf{x})^{3/4}}$$

Result (type 5, 80 leaves) :

$$-\frac{2 \mathbf{i} \mathbf{a} (\mathbf{a} - \mathbf{i} \mathbf{a} \mathbf{x})^{1/4} \left( 6 + 5 \mathbf{i} \mathbf{x} + \mathbf{x}^2 - 5 \times 2^{1/4} (1 + \mathbf{i} \mathbf{x})^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{1}{2} - \frac{\mathbf{i} \mathbf{x}}{2}\right] \right)}{3 (\mathbf{a} + \mathbf{i} \mathbf{a} \mathbf{x})^{3/4}}$$

Problem 1190: Result unnecessarily involves higher level functions.

$$\int \frac{(\mathbf{a} - \mathbf{i} \mathbf{a} \mathbf{x})^{1/4}}{(\mathbf{a} + \mathbf{i} \mathbf{a} \mathbf{x})^{3/4}} d\mathbf{x}$$

Optimal (type 4, 76 leaves, 4 steps):

$$-\frac{2 \text{i} (a - \text{i} a x)^{1/4} (a + \text{i} a x)^{1/4}}{a} + \frac{2 a (1 + x^2)^{3/4} \text{EllipticF}\left[\frac{\text{ArcTan}[x]}{2}, 2\right]}{(a - \text{i} a x)^{3/4} (a + \text{i} a x)^{3/4}}$$

Result (type 5, 72 leaves):

$$\frac{2 (a - \text{i} a x)^{1/4} \left(-\text{i} + x + \text{i} 2^{1/4} (1 + \text{i} x)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{1}{2} - \frac{\text{i} x}{2}\right]\right)}{(a + \text{i} a x)^{3/4}}$$

Problem 1191: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a - \text{i} a x)^{3/4} (a + \text{i} a x)^{3/4}} dx$$

Optimal (type 4, 43 leaves, 3 steps):

$$\frac{2 (1 + x^2)^{3/4} \text{EllipticF}\left[\frac{\text{ArcTan}[x]}{2}, 2\right]}{(a - \text{i} a x)^{3/4} (a + \text{i} a x)^{3/4}}$$

Result (type 5, 68 leaves):

$$\frac{2 \text{i} 2^{1/4} (1 + \text{i} x)^{3/4} (a - \text{i} a x)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{1}{2} - \frac{\text{i} x}{2}\right]}{a (a + \text{i} a x)^{3/4}}$$

Problem 1192: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a - \text{i} a x)^{7/4} (a + \text{i} a x)^{3/4}} dx$$

Optimal (type 4, 82 leaves, 4 steps):

$$-\frac{2 \text{i} (a + \text{i} a x)^{1/4}}{3 a^2 (a - \text{i} a x)^{3/4}} + \frac{2 (1 + x^2)^{3/4} \text{EllipticF}\left[\frac{\text{ArcTan}[x]}{2}, 2\right]}{3 a (a - \text{i} a x)^{3/4} (a + \text{i} a x)^{3/4}}$$

Result (type 5, 79 leaves):

$$\frac{2 \left(-\text{i} + x + 2^{1/4} (1 + \text{i} x)^{3/4} (\text{i} + x) \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{1}{2} - \frac{\text{i} x}{2}\right]\right)}{3 a (a - \text{i} a x)^{3/4} (a + \text{i} a x)^{3/4}}$$

### Problem 1193: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a - i a x)^{11/4} (a + i a x)^{3/4}} dx$$

Optimal (type 4, 115 leaves, 5 steps):

$$-\frac{2 i (a + i a x)^{1/4}}{7 a^2 (a - i a x)^{7/4}} - \frac{2 i (a + i a x)^{1/4}}{7 a^3 (a - i a x)^{3/4}} + \frac{2 (1 + x^2)^{3/4} \text{EllipticF}\left[\frac{\text{ArcTan}[x]}{2}, 2\right]}{7 a^2 (a - i a x)^{3/4} (a + i a x)^{3/4}}$$

Result (type 5, 93 leaves):

$$\frac{2 \left(2 + i x + x^2 + 2^{1/4} (1 + i x)^{3/4} (i + x)^2 \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{1}{2} - \frac{i x}{2}\right]\right)}{7 a^2 (i + x) (a - i a x)^{3/4} (a + i a x)^{3/4}}$$

### Problem 1194: Result unnecessarily involves higher level functions.

$$\int \frac{(a - i a x)^{7/4}}{(a + i a x)^{7/4}} dx$$

Optimal (type 3, 291 leaves, 13 steps):

$$\begin{aligned} & \frac{4 i (a - i a x)^{7/4}}{3 a (a + i a x)^{3/4}} + \frac{7 i (a - i a x)^{3/4} (a + i a x)^{1/4}}{3 a} + \frac{7 i \text{ArcTan}\left[1 - \frac{\sqrt{2} (a - i a x)^{1/4}}{(a + i a x)^{1/4}}\right]}{\sqrt{2}} - \\ & \frac{7 i \text{ArcTan}\left[1 + \frac{\sqrt{2} (a - i a x)^{1/4}}{(a + i a x)^{1/4}}\right]}{\sqrt{2}} - \frac{7 i \text{Log}\left[1 + \frac{\sqrt{a - i a x}}{\sqrt{a + i a x}} - \frac{\sqrt{2} (a - i a x)^{1/4}}{(a + i a x)^{1/4}}\right]}{2 \sqrt{2}} + \frac{7 i \text{Log}\left[1 + \frac{\sqrt{a - i a x}}{\sqrt{a + i a x}} + \frac{\sqrt{2} (a - i a x)^{1/4}}{(a + i a x)^{1/4}}\right]}{2 \sqrt{2}} \end{aligned}$$

Result (type 5, 76 leaves):

$$\frac{(a - i a x)^{3/4} \left(11 i - 3 x - 7 i 2^{1/4} (1 + i x)^{3/4} \text{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1}{2} - \frac{i x}{2}\right]\right)}{3 (a + i a x)^{3/4}}$$

### Problem 1195: Result unnecessarily involves higher level functions.

$$\int \frac{(a - i a x)^{3/4}}{(a + i a x)^{7/4}} dx$$

Optimal (type 3, 266 leaves, 12 steps):

$$\frac{4 \text{i} (a - \text{i} a x)^{3/4}}{3 a (a + \text{i} a x)^{3/4}} + \frac{\text{i} \sqrt{2} \operatorname{ArcTan} \left[ 1 - \frac{\sqrt{2} (a - \text{i} a x)^{1/4}}{(a + \text{i} a x)^{1/4}} \right]}{a} -$$

$$\frac{\text{i} \sqrt{2} \operatorname{ArcTan} \left[ 1 + \frac{\sqrt{2} (a - \text{i} a x)^{1/4}}{(a + \text{i} a x)^{1/4}} \right]}{a} - \frac{\text{i} \operatorname{Log} \left[ 1 + \frac{\sqrt{a - \text{i} a x}}{\sqrt{a + \text{i} a x}} - \frac{\sqrt{2} (a - \text{i} a x)^{1/4}}{(a + \text{i} a x)^{1/4}} \right]}{\sqrt{2} a} + \frac{\text{i} \operatorname{Log} \left[ 1 + \frac{\sqrt{a - \text{i} a x}}{\sqrt{a + \text{i} a x}} + \frac{\sqrt{2} (a - \text{i} a x)^{1/4}}{(a + \text{i} a x)^{1/4}} \right]}{\sqrt{2} a}$$

Result (type 5, 73 leaves):

$$-\frac{2 \text{i} (a - \text{i} a x)^{3/4} \left( -2 + 2^{1/4} (1 + \text{i} x)^{3/4} \operatorname{Hypergeometric2F1} \left[ \frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1}{2} - \frac{\text{i} x}{2} \right] \right)}{3 a (a + \text{i} a x)^{3/4}}$$

Problem 1199: Result unnecessarily involves higher level functions.

$$\int \frac{(a - \text{i} a x)^{9/4}}{(a + \text{i} a x)^{7/4}} dx$$

Optimal (type 4, 139 leaves, 6 steps):

$$\frac{4 \text{i} (a - \text{i} a x)^{9/4}}{3 a (a + \text{i} a x)^{3/4}} + 10 \text{i} (a - \text{i} a x)^{1/4} (a + \text{i} a x)^{1/4} + \frac{2 \text{i} (a - \text{i} a x)^{5/4} (a + \text{i} a x)^{1/4}}{a} - \frac{10 a^2 (1 + x^2)^{3/4} \operatorname{EllipticF} \left[ \frac{\operatorname{ArcTan}[x]}{2}, 2 \right]}{(a - \text{i} a x)^{3/4} (a + \text{i} a x)^{3/4}}$$

Result (type 5, 80 leaves):

$$\frac{2 \text{i} a (a - \text{i} a x)^{1/4} \left( 20 + 11 \text{i} x + x^2 - 15 \times 2^{1/4} (1 + \text{i} x)^{3/4} \operatorname{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{1}{2} - \frac{\text{i} x}{2} \right] \right)}{3 (a + \text{i} a x)^{3/4}}$$

Problem 1200: Result unnecessarily involves higher level functions.

$$\int \frac{(a - \text{i} a x)^{5/4}}{(a + \text{i} a x)^{7/4}} dx$$

Optimal (type 4, 113 leaves, 5 steps):

$$\frac{4 \text{i} (a - \text{i} a x)^{5/4}}{3 a (a + \text{i} a x)^{3/4}} + \frac{10 \text{i} (a - \text{i} a x)^{1/4} (a + \text{i} a x)^{1/4}}{3 a} - \frac{10 a (1 + x^2)^{3/4} \operatorname{EllipticF} \left[ \frac{\operatorname{ArcTan}[x]}{2}, 2 \right]}{3 (a - \text{i} a x)^{3/4} (a + \text{i} a x)^{3/4}}$$

Result (type 5, 76 leaves):

$$-\frac{2 (a - \frac{i}{2} a x)^{1/4} \left(-7 \frac{i}{2} + 3 x + 5 \frac{i}{2} 2^{1/4} (1 + \frac{i}{2} x)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{1}{2} - \frac{i}{2} x\right]\right)}{3 (a + \frac{i}{2} a x)^{3/4}}$$

**Problem 1201:** Result unnecessarily involves higher level functions.

$$\int \frac{(a - \frac{i}{2} a x)^{1/4}}{(a + \frac{i}{2} a x)^{7/4}} dx$$

Optimal (type 4, 79 leaves, 4 steps) :

$$\frac{4 i (a - \frac{i}{2} a x)^{1/4}}{3 a (a + \frac{i}{2} a x)^{3/4}} - \frac{2 (1 + x^2)^{3/4} \text{EllipticF}\left[\frac{\text{ArcTan}[x]}{2}, 2\right]}{3 (a - \frac{i}{2} a x)^{3/4} (a + \frac{i}{2} a x)^{3/4}}$$

Result (type 5, 73 leaves) :

$$-\frac{2 i (a - \frac{i}{2} a x)^{1/4} \left(-2 + 2^{1/4} (1 + \frac{i}{2} x)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{1}{2} - \frac{i}{2} x\right]\right)}{3 a (a + \frac{i}{2} a x)^{3/4}}$$

**Problem 1202:** Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a - \frac{i}{2} a x)^{3/4} (a + \frac{i}{2} a x)^{7/4}} dx$$

Optimal (type 4, 82 leaves, 4 steps) :

$$\frac{2 i (a - \frac{i}{2} a x)^{1/4}}{3 a^2 (a + \frac{i}{2} a x)^{3/4}} + \frac{2 (1 + x^2)^{3/4} \text{EllipticF}\left[\frac{\text{ArcTan}[x]}{2}, 2\right]}{3 a (a - \frac{i}{2} a x)^{3/4} (a + \frac{i}{2} a x)^{3/4}}$$

Result (type 5, 73 leaves) :

$$\frac{2 i (a - \frac{i}{2} a x)^{1/4} \left(1 + 2^{1/4} (1 + \frac{i}{2} x)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{1}{2} - \frac{i}{2} x\right]\right)}{3 a^2 (a + \frac{i}{2} a x)^{3/4}}$$

**Problem 1203:** Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a - \frac{i}{2} a x)^{7/4} (a + \frac{i}{2} a x)^{7/4}} dx$$

Optimal (type 4, 81 leaves, 4 steps) :

$$\frac{2x}{3a^2(a - \text{i}ax)^{3/4}(a + \text{i}ax)^{3/4}} + \frac{2(1+x^2)^{3/4}\text{EllipticF}\left[\frac{\text{ArcTan}[x]}{2}, 2\right]}{3a^2(a - \text{i}ax)^{3/4}(a + \text{i}ax)^{3/4}}$$

Result (type 5, 76 leaves) :

$$\frac{2(x + 2^{1/4}(1 + \text{i}x)^{3/4}(\text{i} + x)\text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{1}{2} - \frac{\text{i}x}{2}\right])}{3a^2(a - \text{i}ax)^{3/4}(a + \text{i}ax)^{3/4}}$$

Problem 1204: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a - \text{i}ax)^{11/4}(a + \text{i}ax)^{7/4}} dx$$

Optimal (type 4, 114 leaves, 5 steps) :

$$-\frac{2\text{i}}{7a^2(a - \text{i}ax)^{7/4}(a + \text{i}ax)^{3/4}} + \frac{10x}{21a^3(a - \text{i}ax)^{3/4}(a + \text{i}ax)^{3/4}} + \frac{10(1+x^2)^{3/4}\text{EllipticF}\left[\frac{\text{ArcTan}[x]}{2}, 2\right]}{21a^3(a - \text{i}ax)^{3/4}(a + \text{i}ax)^{3/4}}$$

Result (type 5, 96 leaves) :

$$\frac{2(3 + 5\text{i}x + 5x^2 + 5 \times 2^{1/4}(1 + \text{i}x)^{3/4}(\text{i} + x)^2\text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{1}{2} - \frac{\text{i}x}{2}\right])}{21a^3(\text{i} + x)(a - \text{i}ax)^{3/4}(a + \text{i}ax)^{3/4}}$$

Problem 1205: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a - \text{i}ax)^{15/4}(a + \text{i}ax)^{7/4}} dx$$

Optimal (type 4, 147 leaves, 6 steps) :

$$-\frac{2\text{i}}{11a^2(a - \text{i}ax)^{11/4}(a + \text{i}ax)^{3/4}} - \frac{2\text{i}}{11a^3(a - \text{i}ax)^{7/4}(a + \text{i}ax)^{3/4}} + \frac{10x}{33a^4(a - \text{i}ax)^{3/4}(a + \text{i}ax)^{3/4}} + \frac{10(1+x^2)^{3/4}\text{EllipticF}\left[\frac{\text{ArcTan}[x]}{2}, 2\right]}{33a^4(a - \text{i}ax)^{3/4}(a + \text{i}ax)^{3/4}}$$

Result (type 5, 103 leaves) :

$$\frac{2(6\text{i} - 2x + 10\text{i}x^2 + 5x^3 + 5 \times 2^{1/4}(1 + \text{i}x)^{3/4}(\text{i} + x)^3\text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{1}{2} - \frac{\text{i}x}{2}\right])}{33a^4(\text{i} + x)^2(a - \text{i}ax)^{3/4}(a + \text{i}ax)^{3/4}}$$

### Problem 1206: Result unnecessarily involves higher level functions.

$$\int \frac{(a - i a x)^{7/4}}{(a + i a x)^{5/4}} dx$$

Optimal (type 4, 137 leaves, 6 steps):

$$-\frac{14 a x}{(a - i a x)^{1/4} (a + i a x)^{1/4}} + \frac{4 i (a - i a x)^{7/4}}{a (a + i a x)^{1/4}} + \frac{14 i (a - i a x)^{3/4} (a + i a x)^{3/4}}{3 a} + \frac{14 a (1 + x^2)^{1/4} \text{EllipticE}\left[\frac{\text{ArcTan}[x]}{2}, 2\right]}{(a - i a x)^{1/4} (a + i a x)^{1/4}}$$

Result (type 5, 74 leaves):

$$-\frac{2 (a - i a x)^{3/4} \left(-13 i + x + 7 i 2^{3/4} (1 + i x)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1}{2} - \frac{i x}{2}\right]\right)}{3 (a + i a x)^{1/4}}$$

### Problem 1207: Result unnecessarily involves higher level functions.

$$\int \frac{(a - i a x)^{3/4}}{(a + i a x)^{5/4}} dx$$

Optimal (type 4, 102 leaves, 5 steps):

$$-\frac{6 x}{(a - i a x)^{1/4} (a + i a x)^{1/4}} + \frac{4 i (a - i a x)^{3/4}}{a (a + i a x)^{1/4}} + \frac{6 (1 + x^2)^{1/4} \text{EllipticE}\left[\frac{\text{ArcTan}[x]}{2}, 2\right]}{(a - i a x)^{1/4} (a + i a x)^{1/4}}$$

Result (type 5, 71 leaves):

$$-\frac{2 i (a - i a x)^{3/4} \left(-2 + 2^{3/4} (1 + i x)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1}{2} - \frac{i x}{2}\right]\right)}{a (a + i a x)^{1/4}}$$

### Problem 1208: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a - i a x)^{1/4} (a + i a x)^{5/4}} dx$$

Optimal (type 4, 78 leaves, 4 steps):

$$\frac{2 i}{a (a - i a x)^{1/4} (a + i a x)^{1/4}} + \frac{2 (1 + x^2)^{1/4} \text{EllipticE}\left[\frac{\text{ArcTan}[x]}{2}, 2\right]}{a (a - i a x)^{1/4} (a + i a x)^{1/4}}$$

Result (type 5, 73 leaves) :

$$\frac{2 \text{i} (a - \text{i} a x)^{3/4} \left( -3 + 2^{3/4} (1 + \text{i} x)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1}{2} - \frac{\text{i} x}{2} \right] \right)}{3 a^2 (a + \text{i} a x)^{1/4}}$$

Problem 1209: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a - \text{i} a x)^{5/4} (a + \text{i} a x)^{5/4}} dx$$

Optimal (type 4, 46 leaves, 3 steps) :

$$\frac{2 (1 + x^2)^{1/4} \text{EllipticE}\left[\frac{\text{ArcTan}[x]}{2}, 2\right]}{a^2 (a - \text{i} a x)^{1/4} (a + \text{i} a x)^{1/4}}$$

Result (type 5, 79 leaves) :

$$\frac{6 x - 2 \times 2^{3/4} (1 + \text{i} x)^{1/4} (\text{i} + x) \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1}{2} - \frac{\text{i} x}{2} \right]}{3 a^2 (a - \text{i} a x)^{1/4} (a + \text{i} a x)^{1/4}}$$

Problem 1210: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a - \text{i} a x)^{9/4} (a + \text{i} a x)^{5/4}} dx$$

Optimal (type 4, 82 leaves, 4 steps) :

$$\frac{2 \text{i}}{5 a^2 (a - \text{i} a x)^{5/4} (a + \text{i} a x)^{1/4}} + \frac{6 (1 + x^2)^{1/4} \text{EllipticE}\left[\frac{\text{ArcTan}[x]}{2}, 2\right]}{5 a^3 (a - \text{i} a x)^{1/4} (a + \text{i} a x)^{1/4}}$$

Result (type 5, 96 leaves) :

$$\frac{2 + 6 \text{i} x + 6 x^2 - 2 \times 2^{3/4} (1 + \text{i} x)^{1/4} (\text{i} + x)^2 \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1}{2} - \frac{\text{i} x}{2} \right]}{5 a^3 (\text{i} + x) (a - \text{i} a x)^{1/4} (a + \text{i} a x)^{1/4}}$$

Problem 1211: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a - \text{i} a x)^{13/4} (a + \text{i} a x)^{5/4}} dx$$

Optimal (type 4, 115 leaves, 5 steps) :

$$-\frac{2 \text{i}}{9 a^2 (a - \text{i} a x)^{9/4} (a + \text{i} a x)^{1/4}} - \frac{2 \text{i}}{9 a^3 (a - \text{i} a x)^{5/4} (a + \text{i} a x)^{1/4}} + \frac{2 (1 + x^2)^{1/4} \text{EllipticE}\left[\frac{\text{ArcTan}[x]}{2}, 2\right]}{3 a^4 (a - \text{i} a x)^{1/4} (a + \text{i} a x)^{1/4}}$$

Result (type 5, 103 leaves) :

$$\frac{4 \text{i} - 4 x + 12 \text{i} x^2 + 6 x^3 - 2 \times 2^{3/4} (1 + \text{i} x)^{1/4} (\text{i} + x)^3 \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1}{2} - \frac{\text{i} x}{2}\right]}{9 a^4 (\text{i} + x)^2 (a - \text{i} a x)^{1/4} (a + \text{i} a x)^{1/4}}$$

Problem 1212: Result unnecessarily involves higher level functions.

$$\int \frac{(a - \text{i} a x)^{5/4}}{(a + \text{i} a x)^{5/4}} dx$$

Optimal (type 3, 287 leaves, 13 steps) :

$$\begin{aligned} & \frac{4 \text{i} (a - \text{i} a x)^{5/4}}{a (a + \text{i} a x)^{1/4}} + \frac{5 \text{i} (a - \text{i} a x)^{1/4} (a + \text{i} a x)^{3/4}}{a} + \frac{5 \text{i} \text{ArcTan}\left[1 - \frac{\sqrt{2} (a - \text{i} a x)^{1/4}}{(a + \text{i} a x)^{1/4}}\right]}{\sqrt{2}} - \\ & \frac{5 \text{i} \text{ArcTan}\left[1 + \frac{\sqrt{2} (a - \text{i} a x)^{1/4}}{(a + \text{i} a x)^{1/4}}\right]}{\sqrt{2}} + \frac{5 \text{i} \text{Log}\left[1 + \frac{\sqrt{a - \text{i} a x}}{\sqrt{a + \text{i} a x}} - \frac{\sqrt{2} (a - \text{i} a x)^{1/4}}{(a + \text{i} a x)^{1/4}}\right]}{2 \sqrt{2}} - \frac{5 \text{i} \text{Log}\left[1 + \frac{\sqrt{a - \text{i} a x}}{\sqrt{a + \text{i} a x}} + \frac{\sqrt{2} (a - \text{i} a x)^{1/4}}{(a + \text{i} a x)^{1/4}}\right]}{2 \sqrt{2}} \end{aligned}$$

Result (type 5, 72 leaves) :

$$-\frac{(a - \text{i} a x)^{1/4} \left(-9 \text{i} + x + 5 \text{i} 2^{3/4} (1 + \text{i} x)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \frac{1}{2} - \frac{\text{i} x}{2}\right]\right)}{(a + \text{i} a x)^{1/4}}$$

Problem 1213: Result unnecessarily involves higher level functions.

$$\int \frac{(a - \text{i} a x)^{1/4}}{(a + \text{i} a x)^{5/4}} dx$$

Optimal (type 3, 264 leaves, 12 steps) :

$$\begin{aligned} & \frac{4 \text{i} (a - \text{i} a x)^{1/4}}{a (a + \text{i} a x)^{1/4}} + \frac{\text{i} \sqrt{2} \text{ArcTan}\left[1 - \frac{\sqrt{2} (a - \text{i} a x)^{1/4}}{(a + \text{i} a x)^{1/4}}\right]}{a} - \\ & \frac{\text{i} \sqrt{2} \text{ArcTan}\left[1 + \frac{\sqrt{2} (a - \text{i} a x)^{1/4}}{(a + \text{i} a x)^{1/4}}\right]}{a} + \frac{\text{i} \text{Log}\left[1 + \frac{\sqrt{a - \text{i} a x}}{\sqrt{a + \text{i} a x}} - \frac{\sqrt{2} (a - \text{i} a x)^{1/4}}{(a + \text{i} a x)^{1/4}}\right]}{\sqrt{2} a} - \frac{\text{i} \text{Log}\left[1 + \frac{\sqrt{a - \text{i} a x}}{\sqrt{a + \text{i} a x}} + \frac{\sqrt{2} (a - \text{i} a x)^{1/4}}{(a + \text{i} a x)^{1/4}}\right]}{\sqrt{2} a} \end{aligned}$$

Result (type 5, 71 leaves) :

$$-\frac{2 \text{i} \left(a - \text{i} a x\right)^{1/4} \left(-2 + 2^{3/4} \left(1 + \text{i} x\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \frac{1}{2} - \frac{\text{i} x}{2}\right]\right)}{a \left(a + \text{i} a x\right)^{1/4}}$$

Problem 1217: Result unnecessarily involves higher level functions.

$$\int \frac{(a - \text{i} a x)^{7/4}}{(a + \text{i} a x)^{9/4}} dx$$

Optimal (type 4, 141 leaves, 6 steps) :

$$\frac{4 \text{i} \left(a - \text{i} a x\right)^{7/4}}{5 a \left(a + \text{i} a x\right)^{5/4}} + \frac{42 x}{5 \left(a - \text{i} a x\right)^{1/4} \left(a + \text{i} a x\right)^{1/4}} - \frac{28 \text{i} \left(a - \text{i} a x\right)^{3/4}}{5 a \left(a + \text{i} a x\right)^{1/4}} - \frac{42 \left(1 + x^2\right)^{1/4} \text{EllipticE}\left[\frac{\text{ArcTan}[x]}{2}, 2\right]}{5 \left(a - \text{i} a x\right)^{1/4} \left(a + \text{i} a x\right)^{1/4}}$$

Result (type 5, 84 leaves) :

$$\frac{2 \left(a - \text{i} a x\right)^{3/4} \left(-12 - 16 \text{i} x + 7 \times 2^{3/4} \left(1 + \text{i} x\right)^{5/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1}{2} - \frac{\text{i} x}{2}\right]\right)}{5 a \left(-\text{i} + x\right) \left(a + \text{i} a x\right)^{1/4}}$$

Problem 1218: Result unnecessarily involves higher level functions.

$$\int \frac{(a - \text{i} a x)^{3/4}}{(a + \text{i} a x)^{9/4}} dx$$

Optimal (type 4, 115 leaves, 5 steps) :

$$\frac{4 \text{i} \left(a - \text{i} a x\right)^{3/4}}{5 a \left(a + \text{i} a x\right)^{5/4}} - \frac{6 \text{i}}{5 a \left(a - \text{i} a x\right)^{1/4} \left(a + \text{i} a x\right)^{1/4}} - \frac{6 \left(1 + x^2\right)^{1/4} \text{EllipticE}\left[\frac{\text{ArcTan}[x]}{2}, 2\right]}{5 a \left(a - \text{i} a x\right)^{1/4} \left(a + \text{i} a x\right)^{1/4}}$$

Result (type 5, 83 leaves) :

$$\frac{2 \left(a - \text{i} a x\right)^{3/4} \left(-1 - 3 \text{i} x + 2^{3/4} \left(1 + \text{i} x\right)^{5/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1}{2} - \frac{\text{i} x}{2}\right]\right)}{5 a^2 \left(-\text{i} + x\right) \left(a + \text{i} a x\right)^{1/4}}$$

Problem 1219: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(a - \text{i} a x\right)^{1/4} \left(a + \text{i} a x\right)^{9/4}} dx$$

Optimal (type 4, 82 leaves, 4 steps):

$$\frac{4 \text{i}}{5 a (a - \text{i} a x)^{1/4} (a + \text{i} a x)^{5/4}} + \frac{2 (1 + x^2)^{1/4} \text{EllipticE}\left[\frac{\text{ArcTan}[x]}{2}, 2\right]}{5 a^2 (a - \text{i} a x)^{1/4} (a + \text{i} a x)^{1/4}}$$

Result (type 5, 84 leaves):

$$\frac{2 (a - \text{i} a x)^{3/4} \left(6 + 3 \text{i} x - 2^{3/4} (1 + \text{i} x)^{5/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1}{2} - \frac{\text{i} x}{2}\right]\right)}{15 a^3 (-\text{i} + x) (a + \text{i} a x)^{1/4}}$$

Problem 1220: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a - \text{i} a x)^{5/4} (a + \text{i} a x)^{9/4}} dx$$

Optimal (type 4, 82 leaves, 5 steps):

$$\frac{2 \text{i}}{5 a^2 (a - \text{i} a x)^{1/4} (a + \text{i} a x)^{5/4}} + \frac{6 (1 + x^2)^{1/4} \text{EllipticE}\left[\frac{\text{ArcTan}[x]}{2}, 2\right]}{5 a^3 (a - \text{i} a x)^{1/4} (a + \text{i} a x)^{1/4}}$$

Result (type 5, 94 leaves):

$$\frac{2 - 6 \text{i} x + 6 x^2 - 2 \times 2^{3/4} (1 + \text{i} x)^{1/4} (1 + x^2) \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1}{2} - \frac{\text{i} x}{2}\right]}{5 a^3 (-\text{i} + x) (a - \text{i} a x)^{1/4} (a + \text{i} a x)^{1/4}}$$

Problem 1221: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a - \text{i} a x)^{9/4} (a + \text{i} a x)^{9/4}} dx$$

Optimal (type 4, 88 leaves, 4 steps):

$$\frac{2 x}{5 a^4 (a - \text{i} a x)^{1/4} (a + \text{i} a x)^{1/4} (1 + x^2)} + \frac{6 (1 + x^2)^{1/4} \text{EllipticE}\left[\frac{\text{ArcTan}[x]}{2}, 2\right]}{5 a^4 (a - \text{i} a x)^{1/4} (a + \text{i} a x)^{1/4}}$$

Result (type 5, 98 leaves):

$$\frac{8 x + 6 x^3 - 2 \times 2^{3/4} (1 + \text{i} x)^{1/4} (-\text{i} + x) (\text{i} + x)^2 \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1}{2} - \frac{\text{i} x}{2}\right]}{5 a^4 (a - \text{i} a x)^{1/4} (a + \text{i} a x)^{1/4} (1 + x^2)}$$

### Problem 1222: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a - \frac{i}{a}ax)^{13/4} (a + \frac{i}{a}ax)^{9/4}} dx$$

Optimal (type 4, 121 leaves, 5 steps):

$$-\frac{2 \frac{i}{a}}{9 a^2 (a - \frac{i}{a}ax)^{9/4} (a + \frac{i}{a}ax)^{5/4}} + \frac{14 x}{45 a^5 (a - \frac{i}{a}ax)^{1/4} (a + \frac{i}{a}ax)^{1/4} (1+x^2)} + \frac{14 (1+x^2)^{1/4} \text{EllipticE}\left[\frac{\text{ArcTan}[x]}{2}, 2\right]}{15 a^5 (a - \frac{i}{a}ax)^{1/4} (a + \frac{i}{a}ax)^{1/4}}$$

Result (type 5, 120 leaves):

$$\left( 2 \left( 5 + 28 \frac{i}{a}x + 28 x^2 + 21 \frac{i}{a}x^3 + 21 x^4 - 7 \times 2^{3/4} (1 + \frac{i}{a}x)^{1/4} (-\frac{i}{a} + x) (\frac{i}{a} + x)^3 \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1}{2} - \frac{i}{2}x\right] \right) \right) / \\ (45 a^5 (-\frac{i}{a} + x) (\frac{i}{a} + x)^2 (a - \frac{i}{a}ax)^{1/4} (a + \frac{i}{a}ax)^{1/4})$$

### Problem 1223: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a - \frac{i}{a}ax)^{17/4} (a + \frac{i}{a}ax)^{9/4}} dx$$

Optimal (type 4, 154 leaves, 6 steps):

$$-\frac{2 \frac{i}{a}}{13 a^2 (a - \frac{i}{a}ax)^{13/4} (a + \frac{i}{a}ax)^{5/4}} - \frac{2 \frac{i}{a}}{13 a^3 (a - \frac{i}{a}ax)^{9/4} (a + \frac{i}{a}ax)^{5/4}} + \\ \frac{14 x}{65 a^6 (a - \frac{i}{a}ax)^{1/4} (a + \frac{i}{a}ax)^{1/4} (1+x^2)} + \frac{42 (1+x^2)^{1/4} \text{EllipticE}\left[\frac{\text{ArcTan}[x]}{2}, 2\right]}{65 a^6 (a - \frac{i}{a}ax)^{1/4} (a + \frac{i}{a}ax)^{1/4}}$$

Result (type 5, 127 leaves):

$$\left( 2 \left( 10 \frac{i}{a} - 23 x + 56 \frac{i}{a}x^2 + 7 x^3 + 42 \frac{i}{a}x^4 + 21 x^5 - 7 \times 2^{3/4} (1 + \frac{i}{a}x)^{1/4} (-\frac{i}{a} + x) (\frac{i}{a} + x)^4 \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1}{2} - \frac{i}{2}x\right] \right) \right) / \\ (65 a^6 (-\frac{i}{a} + x) (\frac{i}{a} + x)^3 (a - \frac{i}{a}ax)^{1/4} (a + \frac{i}{a}ax)^{1/4})$$

### Problem 1224: Result unnecessarily involves higher level functions.

$$\int \frac{(a - \frac{i}{a}ax)^{5/4}}{(a + \frac{i}{a}ax)^{9/4}} dx$$

Optimal (type 3, 297 leaves, 13 steps):

$$\frac{\frac{4 \text{i} (a - \text{i} a x)^{5/4}}{5 a (a + \text{i} a x)^{5/4}} - \frac{4 \text{i} (a - \text{i} a x)^{1/4}}{a (a + \text{i} a x)^{1/4}} - \frac{\text{i} \sqrt{2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (a - \text{i} a x)^{1/4}}{(a + \text{i} a x)^{1/4}}\right]}{a} + \frac{\text{i} \sqrt{2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (a - \text{i} a x)^{1/4}}{(a + \text{i} a x)^{1/4}}\right]}{a} - \frac{\text{i} \operatorname{Log}\left[1 + \frac{\sqrt{a - \text{i} a x}}{\sqrt{a + \text{i} a x}} - \frac{\sqrt{2} (a - \text{i} a x)^{1/4}}{(a + \text{i} a x)^{1/4}}\right]}{\sqrt{2} a} + \frac{\text{i} \operatorname{Log}\left[1 + \frac{\sqrt{a - \text{i} a x}}{\sqrt{a + \text{i} a x}} + \frac{\sqrt{2} (a - \text{i} a x)^{1/4}}{(a + \text{i} a x)^{1/4}}\right]}{\sqrt{2} a}}$$

Result (type 5, 84 leaves) :

$$\frac{2 (a - \text{i} a x)^{1/4} \left(-8 - 12 \text{i} x + 5 \times 2^{3/4} (1 + \text{i} x)^{5/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \frac{1}{2} - \frac{\text{i} x}{2}\right]\right)}{5 a (-\text{i} + x) (a + \text{i} a x)^{1/4}}$$

Problem 1235: Result more than twice size of optimal antiderivative.

$$\int (3 - 6x)^m (2 + 4x)^m dx$$

Optimal (type 5, 20 leaves, 2 steps) :

$$6^m x \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -m, \frac{3}{2}, 4x^2\right]$$

Result (type 5, 42 leaves) :

$$(3 - 6x)^m x (2 + 4x)^m (1 - 4x^2)^{-m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -m, \frac{3}{2}, 4x^2\right]$$

Problem 1236: Result more than twice size of optimal antiderivative.

$$\int (a + bx)^4 (c + dx) dx$$

Optimal (type 1, 38 leaves, 2 steps) :

$$\frac{(b c - a d) (a + b x)^5}{5 b^2} + \frac{d (a + b x)^6}{6 b^2}$$

Result (type 1, 84 leaves) :

$$\frac{1}{30} x (15 a^4 (2 c + d x) + 20 a^3 b x (3 c + 2 d x) + 15 a^2 b^2 x^2 (4 c + 3 d x) + 6 a b^3 x^3 (5 c + 4 d x) + b^4 x^4 (6 c + 5 d x))$$

### Problem 1246: Result more than twice size of optimal antiderivative.

$$\int (a + bx)^4 (c + dx)^2 dx$$

Optimal (type 1, 65 leaves, 2 steps) :

$$\frac{(bc - ad)^2 (a + bx)^5}{5b^3} + \frac{d(bc - ad)(a + bx)^6}{3b^3} + \frac{d^2(a + bx)^7}{7b^3}$$

Result (type 1, 148 leaves) :

$$\begin{aligned} & a^4 c^2 x + a^3 c (2bc + ad) x^2 + \frac{1}{3} a^2 (6b^2 c^2 + 8abc + a^2 d^2) x^3 + \\ & ab (b^2 c^2 + 3abc + a^2 d^2) x^4 + \frac{1}{5} b^2 (b^2 c^2 + 8abc + 6a^2 d^2) x^5 + \frac{1}{3} b^3 d (bc + 2ad) x^6 + \frac{1}{7} b^4 d^2 x^7 \end{aligned}$$

### Problem 1258: Result more than twice size of optimal antiderivative.

$$\int (a + bx)^5 (c + dx)^3 dx$$

Optimal (type 1, 92 leaves, 2 steps) :

$$\frac{(bc - ad)^3 (a + bx)^6}{6b^4} + \frac{3d(bc - ad)^2 (a + bx)^7}{7b^4} + \frac{3d^2(bc - ad)(a + bx)^8}{8b^4} + \frac{d^3(a + bx)^9}{9b^4}$$

Result (type 1, 235 leaves) :

$$\begin{aligned} & \frac{1}{504} x (126a^5 (4c^3 + 6c^2dx + 4cd^2x^2 + d^3x^3) + 126a^4bx (10c^3 + 20c^2dx + 15cd^2x^2 + 4d^3x^3) + \\ & 84a^3b^2x^2 (20c^3 + 45c^2dx + 36cd^2x^2 + 10d^3x^3) + 36a^2b^3x^3 (35c^3 + 84c^2dx + 70cd^2x^2 + 20d^3x^3) + \\ & 9ab^4x^4 (56c^3 + 140c^2dx + 120cd^2x^2 + 35d^3x^3) + b^5x^5 (84c^3 + 216c^2dx + 189cd^2x^2 + 56d^3x^3)) \end{aligned}$$

### Problem 1259: Result more than twice size of optimal antiderivative.

$$\int (a + bx)^4 (c + dx)^3 dx$$

Optimal (type 1, 92 leaves, 2 steps) :

$$\frac{(bc - ad)^3 (a + bx)^5}{5b^4} + \frac{d(bc - ad)^2 (a + bx)^6}{2b^4} + \frac{3d^2(bc - ad)(a + bx)^7}{7b^4} + \frac{d^3(a + bx)^8}{8b^4}$$

Result (type 1, 217 leaves) :

$$\begin{aligned} & a^4 c^3 x + \frac{1}{2} a^3 c^2 (4 b c + 3 a d) x^2 + a^2 c (2 b^2 c^2 + 4 a b c d + a^2 d^2) x^3 + \frac{1}{4} a (4 b^3 c^3 + 18 a b^2 c^2 d + 12 a^2 b c d^2 + a^3 d^3) x^4 + \\ & \frac{1}{5} b (b^3 c^3 + 12 a b^2 c^2 d + 18 a^2 b c d^2 + 4 a^3 d^3) x^5 + \frac{1}{2} b^2 d (b^2 c^2 + 4 a b c d + 2 a^2 d^2) x^6 + \frac{1}{7} b^3 d^2 (3 b c + 4 a d) x^7 + \frac{1}{8} b^4 d^3 x^8 \end{aligned}$$

Problem 1268: Result more than twice size of optimal antiderivative.

$$\int \frac{(c + d x)^3}{(a + b x)^5} dx$$

Optimal (type 1, 28 leaves, 1 step) :

$$-\frac{(c + d x)^4}{4 (b c - a d) (a + b x)^4}$$

Result (type 1, 91 leaves) :

$$-\frac{a^3 d^3 + a^2 b d^2 (c + 4 d x) + a b^2 d (c^2 + 4 c d x + 6 d^2 x^2) + b^3 (c^3 + 4 c^2 d x + 6 c d^2 x^2 + 4 d^3 x^3)}{4 b^4 (a + b x)^4}$$

Problem 1273: Result more than twice size of optimal antiderivative.

$$\int (a + b x)^9 (c + d x)^7 dx$$

Optimal (type 1, 200 leaves, 2 steps) :

$$\begin{aligned} & \frac{(b c - a d)^7 (a + b x)^{10}}{10 b^8} + \frac{7 d (b c - a d)^6 (a + b x)^{11}}{11 b^8} + \frac{7 d^2 (b c - a d)^5 (a + b x)^{12}}{4 b^8} + \frac{35 d^3 (b c - a d)^4 (a + b x)^{13}}{13 b^8} + \\ & \frac{5 d^4 (b c - a d)^3 (a + b x)^{14}}{2 b^8} + \frac{7 d^5 (b c - a d)^2 (a + b x)^{15}}{5 b^8} + \frac{7 d^6 (b c - a d) (a + b x)^{16}}{16 b^8} + \frac{d^7 (a + b x)^{17}}{17 b^8} \end{aligned}$$

Result (type 1, 993 leaves) :

$$\begin{aligned}
& a^9 c^7 x + \frac{1}{2} a^8 c^6 (9 b c + 7 a d) x^2 + a^7 c^5 (12 b^2 c^2 + 21 a b c d + 7 a^2 d^2) x^3 + \\
& \frac{7}{4} a^6 c^4 (12 b^3 c^3 + 36 a b^2 c^2 d + 27 a^2 b c d^2 + 5 a^3 d^3) x^4 + \frac{7}{5} a^5 c^3 (18 b^4 c^4 + 84 a b^3 c^3 d + 108 a^2 b^2 c^2 d^2 + 45 a^3 b c d^3 + 5 a^4 d^4) x^5 + \\
& \frac{7}{2} a^4 c^2 (6 b^5 c^5 + 42 a b^4 c^4 d + 84 a^2 b^3 c^3 d^2 + 60 a^3 b^2 c^2 d^3 + 15 a^4 b c d^4 + a^5 d^5) x^6 + \\
& a^3 c (12 b^6 c^6 + 126 a b^5 c^5 d + 378 a^2 b^4 c^4 d^2 + 420 a^3 b^3 c^3 d^3 + 180 a^4 b^2 c^2 d^4 + 27 a^5 b c d^5 + a^6 d^6) x^7 + \\
& \frac{1}{8} a^2 (36 b^7 c^7 + 588 a b^6 c^6 d + 2646 a^2 b^5 c^5 d^2 + 4410 a^3 b^4 c^4 d^3 + 2940 a^4 b^3 c^3 d^4 + 756 a^5 b^2 c^2 d^5 + 63 a^6 b c d^6 + a^7 d^7) x^8 + \\
& a b (b^7 c^7 + 28 a b^6 c^6 d + 196 a^2 b^5 c^5 d^2 + 490 a^3 b^4 c^4 d^3 + 490 a^4 b^3 c^3 d^4 + 196 a^5 b^2 c^2 d^5 + 28 a^6 b c d^6 + a^7 d^7) x^9 + \\
& \frac{1}{10} b^2 (b^7 c^7 + 63 a b^6 c^6 d + 756 a^2 b^5 c^5 d^2 + 2940 a^3 b^4 c^4 d^3 + 4410 a^4 b^3 c^3 d^4 + 2646 a^5 b^2 c^2 d^5 + 588 a^6 b c d^6 + 36 a^7 d^7) x^{10} + \\
& \frac{7}{11} b^3 d (b^6 c^6 + 27 a b^5 c^5 d + 180 a^2 b^4 c^4 d^2 + 420 a^3 b^3 c^3 d^3 + 378 a^4 b^2 c^2 d^4 + 126 a^5 b c d^5 + 12 a^6 d^6) x^{11} + \\
& \frac{7}{4} b^4 d^2 (b^5 c^5 + 15 a b^4 c^4 d + 60 a^2 b^3 c^3 d^2 + 84 a^3 b^2 c^2 d^3 + 42 a^4 b c d^4 + 6 a^5 d^5) x^{12} + \\
& \frac{7}{13} b^5 d^3 (5 b^4 c^4 + 45 a b^3 c^3 d + 108 a^2 b^2 c^2 d^2 + 84 a^3 b c d^3 + 18 a^4 d^4) x^{13} + \frac{1}{2} b^6 d^4 (5 b^3 c^3 + 27 a b^2 c^2 d + 36 a^2 b c d^2 + 12 a^3 d^3) x^{14} + \\
& \frac{1}{5} b^7 d^5 (7 b^2 c^2 + 21 a b c d + 12 a^2 d^2) x^{15} + \frac{1}{16} b^8 d^6 (7 b c + 9 a d) x^{16} + \frac{1}{17} b^9 d^7 x^{17}
\end{aligned}$$

Problem 1274: Result more than twice size of optimal antiderivative.

$$\int (a + b x)^8 (c + d x)^7 dx$$

Optimal (type 1, 200 leaves, 2 steps):

$$\begin{aligned}
& \frac{(b c - a d)^7 (a + b x)^9}{9 b^8} + \frac{7 d (b c - a d)^6 (a + b x)^{10}}{10 b^8} + \frac{21 d^2 (b c - a d)^5 (a + b x)^{11}}{11 b^8} + \frac{35 d^3 (b c - a d)^4 (a + b x)^{12}}{12 b^8} + \\
& \frac{35 d^4 (b c - a d)^3 (a + b x)^{13}}{13 b^8} + \frac{3 d^5 (b c - a d)^2 (a + b x)^{14}}{2 b^8} + \frac{7 d^6 (b c - a d) (a + b x)^{15}}{15 b^8} + \frac{d^7 (a + b x)^{16}}{16 b^8}
\end{aligned}$$

Result (type 1, 897 leaves):

$$\begin{aligned}
& a^8 c^7 x + \frac{1}{2} a^7 c^6 (8 b c + 7 a d) x^2 + \frac{7}{3} a^6 c^5 (4 b^2 c^2 + 8 a b c d + 3 a^2 d^2) x^3 + \\
& \frac{7}{4} a^5 c^4 (8 b^3 c^3 + 28 a b^2 c^2 d + 24 a^2 b c d^2 + 5 a^3 d^3) x^4 + \frac{7}{5} a^4 c^3 (10 b^4 c^4 + 56 a b^3 c^3 d + 84 a^2 b^2 c^2 d^2 + 40 a^3 b c d^3 + 5 a^4 d^4) x^5 + \\
& \frac{7}{6} a^3 c^2 (8 b^5 c^5 + 70 a b^4 c^4 d + 168 a^2 b^3 c^3 d^2 + 140 a^3 b^2 c^2 d^3 + 40 a^4 b c d^4 + 3 a^5 d^5) x^6 + \\
& a^2 c (4 b^6 c^6 + 56 a b^5 c^5 d + 210 a^2 b^4 c^4 d^2 + 280 a^3 b^3 c^3 d^3 + 140 a^4 b^2 c^2 d^4 + 24 a^5 b c d^5 + a^6 d^6) x^7 + \\
& \frac{1}{8} a (8 b^7 c^7 + 196 a b^6 c^6 d + 1176 a^2 b^5 c^5 d^2 + 2450 a^3 b^4 c^4 d^3 + 1960 a^4 b^3 c^3 d^4 + 588 a^5 b^2 c^2 d^5 + 56 a^6 b c d^6 + a^7 d^7) x^8 + \\
& \frac{1}{9} b (b^7 c^7 + 56 a b^6 c^6 d + 588 a^2 b^5 c^5 d^2 + 1960 a^3 b^4 c^4 d^3 + 2450 a^4 b^3 c^3 d^4 + 1176 a^5 b^2 c^2 d^5 + 196 a^6 b c d^6 + 8 a^7 d^7) x^9 + \\
& \frac{7}{10} b^2 d (b^6 c^6 + 24 a b^5 c^5 d + 140 a^2 b^4 c^4 d^2 + 280 a^3 b^3 c^3 d^3 + 210 a^4 b^2 c^2 d^4 + 56 a^5 b c d^5 + 4 a^6 d^6) x^{10} + \\
& \frac{7}{11} b^3 d^2 (3 b^5 c^5 + 40 a b^4 c^4 d + 140 a^2 b^3 c^3 d^2 + 168 a^3 b^2 c^2 d^3 + 70 a^4 b c d^4 + 8 a^5 d^5) x^{11} + \\
& \frac{7}{12} b^4 d^3 (5 b^4 c^4 + 40 a b^3 c^3 d + 84 a^2 b^2 c^2 d^2 + 56 a^3 b c d^3 + 10 a^4 d^4) x^{12} + \frac{7}{13} b^5 d^4 (5 b^3 c^3 + 24 a b^2 c^2 d + 28 a^2 b c d^2 + 8 a^3 d^3) x^{13} + \\
& \frac{1}{2} b^6 d^5 (3 b^2 c^2 + 8 a b c d + 4 a^2 d^2) x^{14} + \frac{1}{15} b^7 d^6 (7 b c + 8 a d) x^{15} + \frac{1}{16} b^8 d^7 x^{16}
\end{aligned}$$

**Problem 1275: Result more than twice size of optimal antiderivative.**

$$\int (a + b x)^7 (c + d x)^7 dx$$

Optimal (type 1, 200 leaves, 2 steps):

$$\begin{aligned}
& \frac{(b c - a d)^7 (a + b x)^8}{8 b^8} + \frac{7 d (b c - a d)^6 (a + b x)^9}{9 b^8} + \frac{21 d^2 (b c - a d)^5 (a + b x)^{10}}{10 b^8} + \frac{35 d^3 (b c - a d)^4 (a + b x)^{11}}{11 b^8} + \\
& \frac{35 d^4 (b c - a d)^3 (a + b x)^{12}}{12 b^8} + \frac{21 d^5 (b c - a d)^2 (a + b x)^{13}}{13 b^8} + \frac{d^6 (b c - a d) (a + b x)^{14}}{2 b^8} + \frac{d^7 (a + b x)^{15}}{15 b^8}
\end{aligned}$$

Result (type 1, 785 leaves):

$$\begin{aligned}
& a^7 c^7 x + \frac{7}{2} a^6 c^6 (b c + a d) x^2 + \frac{7}{3} a^5 c^5 (3 b^2 c^2 + 7 a b c d + 3 a^2 d^2) x^3 + \\
& \frac{7}{4} a^4 c^4 (5 b^3 c^3 + 21 a b^2 c^2 d + 21 a^2 b c d^2 + 5 a^3 d^3) x^4 + \frac{7}{5} a^3 c^3 (5 b^4 c^4 + 35 a b^3 c^3 d + 63 a^2 b^2 c^2 d^2 + 35 a^3 b c d^3 + 5 a^4 d^4) x^5 + \\
& \frac{7}{6} a^2 c^2 (3 b^5 c^5 + 35 a b^4 c^4 d + 105 a^2 b^3 c^3 d^2 + 105 a^3 b^2 c^2 d^3 + 35 a^4 b c d^4 + 3 a^5 d^5) x^6 + \\
& a c (b^6 c^6 + 21 a b^5 c^5 d + 105 a^2 b^4 c^4 d^2 + 175 a^3 b^3 c^3 d^3 + 105 a^4 b^2 c^2 d^4 + 21 a^5 b c d^5 + a^6 d^6) x^7 + \\
& \frac{1}{8} (b^7 c^7 + 49 a b^6 c^6 d + 441 a^2 b^5 c^5 d^2 + 1225 a^3 b^4 c^4 d^3 + 1225 a^4 b^3 c^3 d^4 + 441 a^5 b^2 c^2 d^5 + 49 a^6 b c d^6 + a^7 d^7) x^8 + \\
& \frac{7}{9} b d (b^6 c^6 + 21 a b^5 c^5 d + 105 a^2 b^4 c^4 d^2 + 175 a^3 b^3 c^3 d^3 + 105 a^4 b^2 c^2 d^4 + 21 a^5 b c d^5 + a^6 d^6) x^9 + \\
& \frac{7}{10} b^2 d^2 (3 b^5 c^5 + 35 a b^4 c^4 d + 105 a^2 b^3 c^3 d^2 + 105 a^3 b^2 c^2 d^3 + 35 a^4 b c d^4 + 3 a^5 d^5) x^{10} + \\
& \frac{7}{11} b^3 d^3 (5 b^4 c^4 + 35 a b^3 c^3 d + 63 a^2 b^2 c^2 d^2 + 35 a^3 b c d^3 + 5 a^4 d^4) x^{11} + \frac{7}{12} b^4 d^4 (5 b^3 c^3 + 21 a b^2 c^2 d + 21 a^2 b c d^2 + 5 a^3 d^3) x^{12} + \\
& \frac{7}{13} b^5 d^5 (3 b^2 c^2 + 7 a b c d + 3 a^2 d^2) x^{13} + \frac{1}{2} b^6 d^6 (b c + a d) x^{14} + \frac{1}{15} b^7 d^7 x^{15}
\end{aligned}$$

Problem 1276: Result more than twice size of optimal antiderivative.

$$\int (a + b x)^6 (c + d x)^7 dx$$

Optimal (type 1, 173 leaves, 2 steps):

$$\begin{aligned}
& \frac{(b c - a d)^6 (c + d x)^8}{8 d^7} - \frac{2 b (b c - a d)^5 (c + d x)^9}{3 d^7} + \frac{3 b^2 (b c - a d)^4 (c + d x)^{10}}{2 d^7} - \\
& \frac{20 b^3 (b c - a d)^3 (c + d x)^{11}}{11 d^7} + \frac{5 b^4 (b c - a d)^2 (c + d x)^{12}}{4 d^7} - \frac{6 b^5 (b c - a d) (c + d x)^{13}}{13 d^7} + \frac{b^6 (c + d x)^{14}}{14 d^7}
\end{aligned}$$

Result (type 1, 684 leaves):

$$\begin{aligned}
& a^6 c^7 x + \frac{1}{2} a^5 c^6 (6 b c + 7 a d) x^2 + a^4 c^5 (5 b^2 c^2 + 14 a b c d + 7 a^2 d^2) x^3 + \\
& \frac{1}{4} a^3 c^4 (20 b^3 c^3 + 105 a b^2 c^2 d + 126 a^2 b c d^2 + 35 a^3 d^3) x^4 + a^2 c^3 (3 b^4 c^4 + 28 a b^3 c^3 d + 63 a^2 b^2 c^2 d^2 + 42 a^3 b c d^3 + 7 a^4 d^4) x^5 + \\
& \frac{1}{2} a c^2 (2 b^5 c^5 + 35 a b^4 c^4 d + 140 a^2 b^3 c^3 d^2 + 175 a^3 b^2 c^2 d^3 + 70 a^4 b c d^4 + 7 a^5 d^5) x^6 + \\
& \frac{1}{7} c (b^6 c^6 + 42 a b^5 c^5 d + 315 a^2 b^4 c^4 d^2 + 700 a^3 b^3 c^3 d^3 + 525 a^4 b^2 c^2 d^4 + 126 a^5 b c d^5 + 7 a^6 d^6) x^7 + \\
& \frac{1}{8} d (7 b^6 c^6 + 126 a b^5 c^5 d + 525 a^2 b^4 c^4 d^2 + 700 a^3 b^3 c^3 d^3 + 315 a^4 b^2 c^2 d^4 + 42 a^5 b c d^5 + a^6 d^6) x^8 + \\
& \frac{1}{3} b d^2 (7 b^5 c^5 + 70 a b^4 c^4 d + 175 a^2 b^3 c^3 d^2 + 140 a^3 b^2 c^2 d^3 + 35 a^4 b c d^4 + 2 a^5 d^5) x^9 + \\
& \frac{1}{2} b^2 d^3 (7 b^4 c^4 + 42 a b^3 c^3 d + 63 a^2 b^2 c^2 d^2 + 28 a^3 b c d^3 + 3 a^4 d^4) x^{10} + \frac{1}{11} b^3 d^4 (35 b^3 c^3 + 126 a b^2 c^2 d + 105 a^2 b c d^2 + 20 a^3 d^3) x^{11} + \\
& \frac{1}{4} b^4 d^5 (7 b^2 c^2 + 14 a b c d + 5 a^2 d^2) x^{12} + \frac{1}{13} b^5 d^6 (7 b c + 6 a d) x^{13} + \frac{1}{14} b^6 d^7 x^{14}
\end{aligned}$$

**Problem 1277:** Result more than twice size of optimal antiderivative.

$$\int (a + b x)^5 (c + d x)^7 dx$$

Optimal (type 1, 144 leaves, 2 steps):

$$\begin{aligned}
& -\frac{(b c - a d)^5 (c + d x)^8}{8 d^6} + \frac{5 b (b c - a d)^4 (c + d x)^9}{9 d^6} - \\
& \frac{b^2 (b c - a d)^3 (c + d x)^{10}}{d^6} + \frac{10 b^3 (b c - a d)^2 (c + d x)^{11}}{11 d^6} - \frac{5 b^4 (b c - a d) (c + d x)^{12}}{12 d^6} + \frac{b^5 (c + d x)^{13}}{13 d^6}
\end{aligned}$$

Result (type 1, 574 leaves):

$$\begin{aligned}
& a^5 c^7 x + \frac{1}{2} a^4 c^6 (5 b c + 7 a d) x^2 + \frac{1}{3} a^3 c^5 (10 b^2 c^2 + 35 a b c d + 21 a^2 d^2) x^3 + \frac{5}{4} a^2 c^4 (2 b^3 c^3 + 14 a b^2 c^2 d + 21 a^2 b c d^2 + 7 a^3 d^3) x^4 + \\
& a c^3 (b^4 c^4 + 14 a b^3 c^3 d + 42 a^2 b^2 c^2 d^2 + 35 a^3 b c d^3 + 7 a^4 d^4) x^5 + \frac{1}{6} c^2 (b^5 c^5 + 35 a b^4 c^4 d + 210 a^2 b^3 c^3 d^2 + 350 a^3 b^2 c^2 d^3 + 175 a^4 b c d^4 + 21 a^5 d^5) x^6 + \\
& c d (b^5 c^5 + 15 a b^4 c^4 d + 50 a^2 b^3 c^3 d^2 + 50 a^3 b^2 c^2 d^3 + 15 a^4 b c d^4 + a^5 d^5) x^7 + \\
& \frac{1}{8} d^2 (21 b^5 c^5 + 175 a b^4 c^4 d + 350 a^2 b^3 c^3 d^2 + 210 a^3 b^2 c^2 d^3 + 35 a^4 b c d^4 + a^5 d^5) x^8 + \\
& \frac{5}{9} b d^3 (7 b^4 c^4 + 35 a b^3 c^3 d + 42 a^2 b^2 c^2 d^2 + 14 a^3 b c d^3 + a^4 d^4) x^9 + \frac{1}{2} b^2 d^4 (7 b^3 c^3 + 21 a b^2 c^2 d + 14 a^2 b c d^2 + 2 a^3 d^3) x^{10} + \\
& \frac{1}{11} b^3 d^5 (21 b^2 c^2 + 35 a b c d + 10 a^2 d^2) x^{11} + \frac{1}{12} b^4 d^6 (7 b c + 5 a d) x^{12} + \frac{1}{13} b^5 d^7 x^{13}
\end{aligned}$$

### Problem 1278: Result more than twice size of optimal antiderivative.

$$\int (a + bx)^4 (c + dx)^7 \, dx$$

Optimal (type 1, 119 leaves, 2 steps):

$$\frac{(bc - ad)^4 (c + dx)^8}{8d^5} - \frac{4b(bc - ad)^3 (c + dx)^9}{9d^5} + \frac{3b^2(bc - ad)^2 (c + dx)^{10}}{5d^5} - \frac{4b^3(bc - ad) (c + dx)^{11}}{11d^5} + \frac{b^4 (c + dx)^{12}}{12d^5}$$

Result (type 1, 473 leaves):

$$\begin{aligned} & a^4 c^7 x + \frac{1}{2} a^3 c^6 (4bc + 7ad) x^2 + \frac{1}{3} a^2 c^5 (6b^2 c^2 + 28abc + 21a^2 d^2) x^3 + \frac{1}{4} a c^4 (4b^3 c^3 + 42ab^2 c^2 d + 84a^2 bc d^2 + 35a^3 d^3) x^4 + \\ & \frac{1}{5} c^3 (b^4 c^4 + 28ab^3 c^3 d + 126a^2 b^2 c^2 d^2 + 140a^3 bc d^3 + 35a^4 d^4) x^5 + \frac{7}{6} c^2 d (b^4 c^4 + 12ab^3 c^3 d + 30a^2 b^2 c^2 d^2 + 20a^3 bc d^3 + 3a^4 d^4) x^6 + \\ & c d^2 (3b^4 c^4 + 20ab^3 c^3 d + 30a^2 b^2 c^2 d^2 + 12a^3 bc d^3 + a^4 d^4) x^7 + \frac{1}{8} d^3 (35b^4 c^4 + 140ab^3 c^3 d + 126a^2 b^2 c^2 d^2 + 28a^3 bc d^3 + a^4 d^4) x^8 + \\ & \frac{1}{9} bd^4 (35b^3 c^3 + 84ab^2 c^2 d + 42a^2 bc d^2 + 4a^3 d^3) x^9 + \frac{1}{10} b^2 d^5 (21b^2 c^2 + 28abc + 6a^2 d^2) x^{10} + \frac{1}{11} b^3 d^6 (7bc + 4ad) x^{11} + \frac{1}{12} b^4 d^7 x^{12} \end{aligned}$$

### Problem 1279: Result more than twice size of optimal antiderivative.

$$\int (a + bx)^3 (c + dx)^7 \, dx$$

Optimal (type 1, 92 leaves, 2 steps):

$$-\frac{(bc - ad)^3 (c + dx)^8}{8d^4} + \frac{b(bc - ad)^2 (c + dx)^9}{3d^4} - \frac{3b^2(bc - ad) (c + dx)^{10}}{10d^4} + \frac{b^3 (c + dx)^{11}}{11d^4}$$

Result (type 1, 360 leaves):

$$\begin{aligned} & a^3 c^7 x + \frac{1}{2} a^2 c^6 (3bc + 7ad) x^2 + ac^5 (b^2 c^2 + 7abc + 7a^2 d^2) x^3 + \frac{1}{4} c^4 (b^3 c^3 + 21ab^2 c^2 d + 63a^2 bc d^2 + 35a^3 d^3) x^4 + \\ & \frac{7}{5} c^3 d (b^3 c^3 + 9ab^2 c^2 d + 15a^2 bc d^2 + 5a^3 d^3) x^5 + \frac{7}{2} c^2 d^2 (b^3 c^3 + 5ab^2 c^2 d + 5a^2 bc d^2 + a^3 d^3) x^6 + c d^3 (5b^3 c^3 + 15ab^2 c^2 d + 9a^2 bc d^2 + a^3 d^3) x^7 + \\ & \frac{1}{8} d^4 (35b^3 c^3 + 63ab^2 c^2 d + 21a^2 bc d^2 + a^3 d^3) x^8 + \frac{1}{3} bd^5 (7b^2 c^2 + 7abc + a^2 d^2) x^9 + \frac{1}{10} b^2 d^6 (7bc + 3ad) x^{10} + \frac{1}{11} b^3 d^7 x^{11} \end{aligned}$$

### Problem 1280: Result more than twice size of optimal antiderivative.

$$\int (a + bx)^2 (c + dx)^7 \, dx$$

Optimal (type 1, 65 leaves, 2 steps):

$$\frac{(bc - ad)^2 (c + dx)^8}{8d^3} - \frac{2b(bc - ad)(c + dx)^9}{9d^3} + \frac{b^2(c + dx)^{10}}{10d^3}$$

Result (type 1, 261 leaves):

$$a^2 c^7 x + \frac{1}{2} a c^6 (2bc + 7ad) x^2 + \frac{1}{3} c^5 (b^2 c^2 + 14abc + 21a^2 d^2) x^3 + \frac{7}{4} c^4 d (b^2 c^2 + 6abc + 5a^2 d^2) x^4 + \frac{7}{5} c^3 d^2 (3b^2 c^2 + 10abc + 5a^2 d^2) x^5 + \frac{7}{6} c^2 d^3 (5b^2 c^2 + 10abc + 3a^2 d^2) x^6 + c d^4 (5b^2 c^2 + 6abc + a^2 d^2) x^7 + \frac{1}{8} d^5 (21b^2 c^2 + 14abc + a^2 d^2) x^8 + \frac{1}{9} b d^6 (7bc + 2ad) x^9 + \frac{1}{10} b^2 d^7 x^{10}$$

**Problem 1281:** Result more than twice size of optimal antiderivative.

$$\int (a + bx) (c + dx)^7 dx$$

Optimal (type 1, 38 leaves, 2 steps):

$$-\frac{(bc - ad)(c + dx)^8}{8d^2} + \frac{b(c + dx)^9}{9d^2}$$

Result (type 1, 151 leaves):

$$a c^7 x + \frac{1}{2} c^6 (bc + 7ad) x^2 + \frac{7}{3} c^5 d (bc + 3ad) x^3 + \frac{7}{4} c^4 d^2 (3bc + 5ad) x^4 + 7c^3 d^3 (bc + ad) x^5 + \frac{7}{6} c^2 d^4 (5bc + 3ad) x^6 + c d^5 (3bc + ad) x^7 + \frac{1}{8} d^6 (7bc + ad) x^8 + \frac{1}{9} b d^7 x^9$$

**Problem 1284:** Result more than twice size of optimal antiderivative.

$$\int \frac{(c + dx)^7}{(a + bx)^2} dx$$

Optimal (type 3, 187 leaves, 2 steps):

$$\begin{aligned} & \frac{21d^2(bc - ad)^5 x}{b^7} - \frac{(bc - ad)^7}{b^8(a + bx)} + \frac{35d^3(bc - ad)^4(a + bx)^2}{2b^8} + \frac{35d^4(bc - ad)^3(a + bx)^3}{3b^8} + \\ & \frac{21d^5(bc - ad)^2(a + bx)^4}{4b^8} + \frac{7d^6(bc - ad)(a + bx)^5}{5b^8} + \frac{d^7(a + bx)^6}{6b^8} + \frac{7d(bc - ad)^6 \log[a + bx]}{b^8} \end{aligned}$$

Result (type 3, 388 leaves):

$$\frac{1}{60 b^8 (a + b x)} \left( 60 a^7 d^7 - 60 a^6 b d^6 (7 c + 6 d x) + 210 a^5 b^2 d^5 (6 c^2 + 10 c d x - d^2 x^2) + 70 a^4 b^3 d^4 (-30 c^3 - 72 c^2 d x + 18 c d^2 x^2 + d^3 x^3) - 35 a^3 b^4 d^3 (-60 c^4 - 180 c^3 d x + 90 c^2 d^2 x^2 + 12 c d^3 x^3 + d^4 x^4) + 21 a^2 b^5 d^2 (-60 c^5 - 200 c^4 d x + 200 c^3 d^2 x^2 + 50 c^2 d^3 x^3 + 10 c d^4 x^4 + d^5 x^5) - 7 a b^6 d (-60 c^6 - 180 c^5 d x + 450 c^4 d^2 x^2 + 200 c^3 d^3 x^3 + 75 c^2 d^4 x^4 + 18 c d^5 x^5 + 2 d^6 x^6) + b^7 (-60 c^7 + 1260 c^5 d^2 x^2 + 1050 c^4 d^3 x^3 + 700 c^3 d^4 x^4 + 315 c^2 d^5 x^5 + 84 c d^6 x^6 + 10 d^7 x^7) + 420 d (b c - a d)^6 (a + b x) \operatorname{Log}[a + b x] \right)$$

**Problem 1285:** Result more than twice size of optimal antiderivative.

$$\int \frac{(c + d x)^7}{(a + b x)^3} dx$$

Optimal (type 3, 185 leaves, 2 steps):

$$\begin{aligned} & \frac{35 d^3 (b c - a d)^4 x}{b^7} - \frac{(b c - a d)^7}{2 b^8 (a + b x)^2} - \frac{7 d (b c - a d)^6}{b^8 (a + b x)} + \frac{35 d^4 (b c - a d)^3 (a + b x)^2}{2 b^8} + \\ & \frac{7 d^5 (b c - a d)^2 (a + b x)^3}{b^8} + \frac{7 d^6 (b c - a d) (a + b x)^4}{4 b^8} + \frac{d^7 (a + b x)^5}{5 b^8} + \frac{21 d^2 (b c - a d)^5 \operatorname{Log}[a + b x]}{b^8} \end{aligned}$$

Result (type 3, 389 leaves):

$$\begin{aligned} & \frac{1}{20 b^8 (a + b x)^2} \left( -130 a^7 d^7 + 10 a^6 b d^6 (77 c + 16 d x) + 10 a^5 b^2 d^5 (-189 c^2 - 56 c d x + 50 d^2 x^2) + 70 a^4 b^3 d^4 (35 c^3 + 6 c^2 d x - 34 c d^2 x^2 + 2 d^3 x^3) - 35 a^3 b^4 d^3 (50 c^4 - 20 c^3 d x - 126 c^2 d^2 x^2 + 20 c d^3 x^3 + d^4 x^4) + 7 a^2 b^5 d^2 (90 c^5 - 200 c^4 d x - 550 c^3 d^2 x^2 + 200 c^2 d^3 x^3 + 25 c d^4 x^4 + 2 d^5 x^5) - 7 a b^6 d (10 c^6 - 120 c^5 d x - 200 c^4 d^2 x^2 + 200 c^3 d^3 x^3 + 50 c^2 d^4 x^4 + 10 c d^5 x^5 + d^6 x^6) + b^7 (-10 c^7 - 140 c^6 d x + 700 c^4 d^3 x^3 + 350 c^3 d^4 x^4 + 140 c^2 d^5 x^5 + 35 c d^6 x^6 + 4 d^7 x^7) - 420 d^2 (-b c + a d)^5 (a + b x)^2 \operatorname{Log}[a + b x] \right) \end{aligned}$$

**Problem 1288:** Result more than twice size of optimal antiderivative.

$$\int \frac{(c + d x)^7}{(a + b x)^6} dx$$

Optimal (type 3, 181 leaves, 2 steps):

$$\begin{aligned} & \frac{d^6 (7 b c - 6 a d) x}{b^7} + \frac{d^7 x^2}{2 b^6} - \frac{(b c - a d)^7}{5 b^8 (a + b x)^5} - \frac{7 d (b c - a d)^6}{4 b^8 (a + b x)^4} - \frac{7 d^2 (b c - a d)^5}{b^8 (a + b x)^3} - \frac{35 d^3 (b c - a d)^4}{2 b^8 (a + b x)^2} - \frac{35 d^4 (b c - a d)^3}{b^8 (a + b x)} + \frac{21 d^5 (b c - a d)^2 \operatorname{Log}[a + b x]}{b^8} \end{aligned}$$

Result (type 3, 389 leaves):

$$\frac{1}{20 b^8 (a + b x)^5} \left( 459 a^7 d^7 + 3 a^6 b d^6 (-406 c + 625 d x) + a^5 b^2 d^5 (959 c^2 - 5250 c d x + 2700 d^2 x^2) + 5 a^4 b^3 d^4 (-28 c^3 + 875 c^2 d x - 1680 c d^2 x^2 + 260 d^3 x^3) - 5 a^3 b^4 d^3 (7 c^4 + 140 c^3 d x - 1540 c^2 d^2 x^2 + 1120 c d^3 x^3 + 80 d^4 x^4) - a^2 b^5 d^2 (14 c^5 + 175 c^4 d x + 1400 c^3 d^2 x^2 - 6300 c^2 d^3 x^3 + 700 c d^4 x^4 + 500 d^5 x^5) - 7 a b^6 d (c^6 + 10 c^5 d x + 50 c^4 d^2 x^2 + 200 c^3 d^3 x^3 - 300 c^2 d^4 x^4 - 100 c d^5 x^5 + 10 d^6 x^6) - b^7 (4 c^7 + 35 c^6 d x + 140 c^5 d^2 x^2 + 350 c^4 d^3 x^3 + 700 c^3 d^4 x^4 - 140 c d^6 x^6 - 10 d^7 x^7) + 420 d^5 (b c - a d)^2 (a + b x)^5 \operatorname{Log}[a + b x] \right)$$

Problem 1289: Result more than twice size of optimal antiderivative.

$$\int \frac{(c + d x)^7}{(a + b x)^7} dx$$

Optimal (type 3, 186 leaves, 2 steps):

$$\frac{d^7 x}{b^7} - \frac{(b c - a d)^7}{6 b^8 (a + b x)^6} - \frac{7 d (b c - a d)^6}{5 b^8 (a + b x)^5} - \frac{21 d^2 (b c - a d)^5}{4 b^8 (a + b x)^4} - \frac{35 d^3 (b c - a d)^4}{3 b^8 (a + b x)^3} - \frac{35 d^4 (b c - a d)^3}{2 b^8 (a + b x)^2} - \frac{21 d^5 (b c - a d)^2}{b^8 (a + b x)} + \frac{7 d^6 (b c - a d) \operatorname{Log}[a + b x]}{b^8}$$

Result (type 3, 390 leaves):

$$-\frac{1}{60 b^8 (a + b x)^6} \left( 669 a^7 d^7 + 3 a^6 b d^6 (-343 c + 1198 d x) + 3 a^5 b^2 d^5 (70 c^2 - 1918 c d x + 2575 d^2 x^2) + 5 a^4 b^3 d^4 (14 c^3 + 252 c^2 d x - 2625 c d^2 x^2 + 1640 d^3 x^3) + 5 a^3 b^4 d^3 (7 c^4 + 84 c^3 d x + 630 c^2 d^2 x^2 - 3080 c d^3 x^3 + 810 d^4 x^4) + 3 a^2 b^5 d^2 (7 c^5 + 70 c^4 d x + 350 c^3 d^2 x^2 + 1400 c^2 d^3 x^3 - 3150 c d^4 x^4 + 120 d^5 x^5) + a b^6 d (14 c^6 + 126 c^5 d x + 525 c^4 d^2 x^2 + 1400 c^3 d^3 x^3 + 3150 c^2 d^4 x^4 - 2520 c d^5 x^5 - 360 d^6 x^6) + b^7 (10 c^7 + 84 c^6 d x + 315 c^5 d^2 x^2 + 700 c^4 d^3 x^3 + 1050 c^3 d^4 x^4 + 1260 c^2 d^5 x^5 - 60 d^7 x^7) + 420 d^6 (-b c + a d) (a + b x)^6 \operatorname{Log}[a + b x] \right)$$

Problem 1291: Result more than twice size of optimal antiderivative.

$$\int \frac{(c + d x)^7}{(a + b x)^9} dx$$

Optimal (type 1, 28 leaves, 1 step):

$$-\frac{(c + d x)^8}{8 (b c - a d) (a + b x)^8}$$

Result (type 1, 353 leaves):

$$\begin{aligned}
& - \frac{1}{8 b^8 (a + b x)^8} (a^7 d^7 + a^6 b d^6 (c + 8 d x) + a^5 b^2 d^5 (c^2 + 8 c d x + 28 d^2 x^2) + \\
& \quad a^4 b^3 d^4 (c^3 + 8 c^2 d x + 28 c d^2 x^2 + 56 d^3 x^3) + a^3 b^4 d^3 (c^4 + 8 c^3 d x + 28 c^2 d^2 x^2 + 56 c d^3 x^3 + 70 d^4 x^4) + \\
& \quad a^2 b^5 d^2 (c^5 + 8 c^4 d x + 28 c^3 d^2 x^2 + 56 c^2 d^3 x^3 + 70 c d^4 x^4 + 56 d^5 x^5) + a b^6 d (c^6 + 8 c^5 d x + 28 c^4 d^2 x^2 + 56 c^3 d^3 x^3 + 70 c^2 d^4 x^4 + 56 c d^5 x^5 + 28 d^6 x^6) + \\
& \quad b^7 (c^7 + 8 c^6 d x + 28 c^5 d^2 x^2 + 56 c^4 d^3 x^3 + 70 c^3 d^4 x^4 + 56 c^2 d^5 x^5 + 28 c d^6 x^6 + 8 d^7 x^7))
\end{aligned}$$

**Problem 1292:** Result more than twice size of optimal antiderivative.

$$\int \frac{(c + d x)^7}{(a + b x)^{10}} dx$$

Optimal (type 1, 58 leaves, 2 steps):

$$\begin{aligned}
& - \frac{(c + d x)^8}{9 (b c - a d) (a + b x)^9} + \frac{d (c + d x)^8}{72 (b c - a d)^2 (a + b x)^8}
\end{aligned}$$

Result (type 1, 367 leaves):

$$\begin{aligned}
& - \frac{1}{72 b^8 (a + b x)^9} (a^7 d^7 + a^6 b d^6 (2 c + 9 d x) + 3 a^5 b^2 d^5 (c^2 + 6 c d x + 12 d^2 x^2) + a^4 b^3 d^4 (4 c^3 + 27 c^2 d x + 72 c d^2 x^2 + 84 d^3 x^3) + \\
& \quad a^3 b^4 d^3 (5 c^4 + 36 c^3 d x + 108 c^2 d^2 x^2 + 168 c d^3 x^3 + 126 d^4 x^4) + 3 a^2 b^5 d^2 (2 c^5 + 15 c^4 d x + 48 c^3 d^2 x^2 + 84 c^2 d^3 x^3 + 84 c d^4 x^4 + 42 d^5 x^5) + \\
& \quad a b^6 d (7 c^6 + 54 c^5 d x + 180 c^4 d^2 x^2 + 336 c^3 d^3 x^3 + 378 c^2 d^4 x^4 + 252 c d^5 x^5 + 84 d^6 x^6) + \\
& \quad b^7 (8 c^7 + 63 c^6 d x + 216 c^5 d^2 x^2 + 420 c^4 d^3 x^3 + 504 c^3 d^4 x^4 + 378 c^2 d^5 x^5 + 168 c d^6 x^6 + 36 d^7 x^7)
\end{aligned}$$

**Problem 1293:** Result more than twice size of optimal antiderivative.

$$\int \frac{(c + d x)^7}{(a + b x)^{11}} dx$$

Optimal (type 1, 89 leaves, 3 steps):

$$\begin{aligned}
& - \frac{(c + d x)^8}{10 (b c - a d) (a + b x)^{10}} + \frac{d (c + d x)^8}{45 (b c - a d)^2 (a + b x)^9} - \frac{d^2 (c + d x)^8}{360 (b c - a d)^3 (a + b x)^8}
\end{aligned}$$

Result (type 1, 371 leaves):

$$\begin{aligned}
& - \frac{1}{360 b^8 (a + b x)^{10}} (a^7 d^7 + a^6 b d^6 (3 c + 10 d x) + 3 a^5 b^2 d^5 (2 c^2 + 10 c d x + 15 d^2 x^2) + 5 a^4 b^3 d^4 (2 c^3 + 12 c^2 d x + 27 c d^2 x^2 + 24 d^3 x^3) + \\
& \quad 5 a^3 b^4 d^3 (3 c^4 + 20 c^3 d x + 54 c^2 d^2 x^2 + 72 c d^3 x^3 + 42 d^4 x^4) + 3 a^2 b^5 d^2 (7 c^5 + 50 c^4 d x + 150 c^3 d^2 x^2 + 240 c^2 d^3 x^3 + 210 c d^4 x^4 + 84 d^5 x^5) + \\
& \quad a b^6 d (28 c^6 + 210 c^5 d x + 675 c^4 d^2 x^2 + 1200 c^3 d^3 x^3 + 1260 c^2 d^4 x^4 + 756 c d^5 x^5 + 210 d^6 x^6) + \\
& \quad b^7 (36 c^7 + 280 c^6 d x + 945 c^5 d^2 x^2 + 1800 c^4 d^3 x^3 + 2100 c^3 d^4 x^4 + 1512 c^2 d^5 x^5 + 630 c d^6 x^6 + 120 d^7 x^7)
\end{aligned}$$

### Problem 1294: Result more than twice size of optimal antiderivative.

$$\int \frac{(c + d x)^7}{(a + b x)^{12}} dx$$

Optimal (type 1, 120 leaves, 4 steps):

$$-\frac{(c + d x)^8}{11 (b c - a d) (a + b x)^{11}} + \frac{3 d (c + d x)^8}{110 (b c - a d)^2 (a + b x)^{10}} - \frac{d^2 (c + d x)^8}{165 (b c - a d)^3 (a + b x)^9} + \frac{d^3 (c + d x)^8}{1320 (b c - a d)^4 (a + b x)^8}$$

Result (type 1, 369 leaves):

$$\begin{aligned} & -\frac{1}{1320 b^8 (a + b x)^{11}} (a^7 d^7 + a^6 b d^6 (4 c + 11 d x) + a^5 b^2 d^5 (10 c^2 + 44 c d x + 55 d^2 x^2) + 5 a^4 b^3 d^4 (4 c^3 + 22 c^2 d x + 44 c d^2 x^2 + 33 d^3 x^3) + \\ & 5 a^3 b^4 d^3 (7 c^4 + 44 c^3 d x + 110 c^2 d^2 x^2 + 132 c d^3 x^3 + 66 d^4 x^4) + a^2 b^5 d^2 (56 c^5 + 385 c^4 d x + 1100 c^3 d^2 x^2 + 1650 c^2 d^3 x^3 + 1320 c d^4 x^4 + 462 d^5 x^5) + \\ & a b^6 d (84 c^6 + 616 c^5 d x + 1925 c^4 d^2 x^2 + 3300 c^3 d^3 x^3 + 3300 c^2 d^4 x^4 + 1848 c d^5 x^5 + 462 d^6 x^6) + \\ & b^7 (120 c^7 + 924 c^6 d x + 3080 c^5 d^2 x^2 + 5775 c^4 d^3 x^3 + 6600 c^3 d^4 x^4 + 4620 c^2 d^5 x^5 + 1848 c d^6 x^6 + 330 d^7 x^7) ) \end{aligned}$$

### Problem 1295: Result more than twice size of optimal antiderivative.

$$\int \frac{(c + d x)^7}{(a + b x)^{13}} dx$$

Optimal (type 1, 151 leaves, 5 steps):

$$-\frac{(c + d x)^8}{12 (b c - a d) (a + b x)^{12}} + \frac{d (c + d x)^8}{33 (b c - a d)^2 (a + b x)^{11}} - \frac{d^2 (c + d x)^8}{110 (b c - a d)^3 (a + b x)^{10}} + \frac{d^3 (c + d x)^8}{495 (b c - a d)^4 (a + b x)^9} - \frac{d^4 (c + d x)^8}{3960 (b c - a d)^5 (a + b x)^8}$$

Result (type 1, 371 leaves):

$$\begin{aligned} & -\frac{1}{3960 b^8 (a + b x)^{12}} (a^7 d^7 + a^6 b d^6 (5 c + 12 d x) + 3 a^5 b^2 d^5 (5 c^2 + 20 c d x + 22 d^2 x^2) + 5 a^4 b^3 d^4 (7 c^3 + 36 c^2 d x + 66 c d^2 x^2 + 44 d^3 x^3) + \\ & 5 a^3 b^4 d^3 (14 c^4 + 84 c^3 d x + 198 c^2 d^2 x^2 + 220 c d^3 x^3 + 99 d^4 x^4) + 3 a^2 b^5 d^2 (42 c^5 + 280 c^4 d x + 770 c^3 d^2 x^2 + 1100 c^2 d^3 x^3 + 825 c d^4 x^4 + 264 d^5 x^5) + \\ & a b^6 d (210 c^6 + 1512 c^5 d x + 4620 c^4 d^2 x^2 + 7700 c^3 d^3 x^3 + 7425 c^2 d^4 x^4 + 3960 c d^5 x^5 + 924 d^6 x^6) + \\ & b^7 (330 c^7 + 2520 c^6 d x + 8316 c^5 d^2 x^2 + 15400 c^4 d^3 x^3 + 17325 c^3 d^4 x^4 + 11880 c^2 d^5 x^5 + 4620 c d^6 x^6 + 792 d^7 x^7) ) \end{aligned}$$

### Problem 1299: Result more than twice size of optimal antiderivative.

$$\int (a + b x)^{12} (c + d x)^{10} dx$$

Optimal (type 1, 275 leaves, 2 steps):

$$\begin{aligned}
 & \frac{(bc - ad)^{10} (a + bx)^{13}}{13b^{11}} + \frac{5d(bc - ad)^9 (a + bx)^{14}}{7b^{11}} + \frac{3d^2(bc - ad)^8 (a + bx)^{15}}{b^{11}} + \\
 & \frac{15d^3(bc - ad)^7 (a + bx)^{16}}{2b^{11}} + \frac{210d^4(bc - ad)^6 (a + bx)^{17}}{17b^{11}} + \frac{14d^5(bc - ad)^5 (a + bx)^{18}}{b^{11}} + \frac{210d^6(bc - ad)^4 (a + bx)^{19}}{19b^{11}} + \\
 & \frac{6d^7(bc - ad)^3 (a + bx)^{20}}{b^{11}} + \frac{15d^8(bc - ad)^2 (a + bx)^{21}}{7b^{11}} + \frac{5d^9(bc - ad) (a + bx)^{22}}{11b^{11}} + \frac{d^{10}(a + bx)^{23}}{23b^{11}}
 \end{aligned}$$

Result (type 1, 1817 leaves):

$$\begin{aligned}
 & a^{12}c^{10}x + a^{11}c^9(6bc + 5ad)x^2 + a^{10}c^8(22b^2c^2 + 40abc + 15a^2d^2)x^3 + \\
 & 5a^9c^7(11b^3c^3 + 33ab^2c^2d + 27a^2bc + d^2 + 6a^3d^3)x^4 + a^8c^6(99b^4c^4 + 440ab^3c^3d + 594a^2b^2c^2d^2 + 288a^3bc + d^3 + 42a^4d^4)x^5 + \\
 & 3a^7c^5(44b^5c^5 + 275ab^4c^4d + 550a^2b^3c^3d^2 + 440a^3b^2c^2d^3 + 140a^4bc + d^4 + 14a^5d^5)x^6 + \\
 & \frac{3}{7}a^6c^4(308b^6c^6 + 2640ab^5c^5d + 7425a^2b^4c^4d^2 + 8800a^3b^3c^3d^3 + 4620a^4b^2c^2d^4 + 1008a^5bc + d^5 + 70a^6d^6)x^7 + \\
 & 3a^5c^3(33b^7c^7 + 385ab^6c^6d + 1485a^2b^5c^5d^2 + 2475a^3b^4c^4d^3 + 1925a^4b^3c^3d^4 + 693a^5b^2c^2d^5 + 105a^6bc + d^6 + 5a^7d^7)x^8 + \\
 & 5a^4c^2(11b^8c^8 + 176ab^7c^7d + 924a^2b^6c^6d^2 + 2112a^3b^5c^5d^3 + 2310a^4b^4c^4d^4 + 1232a^5b^3c^3d^5 + 308a^6b^2c^2d^6 + 32a^7bc + d^7 + a^8d^8)x^9 + a^3c \\
 & (22b^9c^9 + 495ab^8c^8d + 3564a^2b^7c^7d^2 + 11088a^3b^6c^6d^3 + 16632a^4b^5c^5d^4 + 12474a^5b^4c^4d^5 + 4620a^6b^3c^3d^6 + 792a^7b^2c^2d^7 + 54a^8bc + d^8 + a^9d^9) \\
 & x^{10} + \frac{1}{11}a^2(66b^{10}c^{10} + 2200ab^9c^9d + 22275a^2b^8c^8d^2 + 95040a^3b^7c^7d^3 + 194040a^4b^6c^6d^4 + \\
 & 199584a^5b^5c^5d^5 + 103950a^6b^4c^4d^6 + 26400a^7b^3c^3d^7 + 2970a^8b^2c^2d^8 + 120a^9bc + d^9 + a^{10}d^{10})x^{11} + \\
 & ab(b^{10}c^{10} + 55ab^9c^9d + 825a^2b^8c^8d^2 + 4950a^3b^7c^7d^3 + 13860a^4b^6c^6d^4 + 19404a^5b^5c^5d^5 + 13860a^6b^4c^4d^6 + 4950a^7b^3c^3d^7 + \\
 & 825a^8b^2c^2d^8 + 55a^9bc + d^9 + a^{10}d^{10})x^{12} + \frac{1}{13}b^2(b^{10}c^{10} + 120ab^9c^9d + 2970a^2b^8c^8d^2 + 26400a^3b^7c^7d^3 + 103950a^4b^6c^6d^4 + \\
 & 199584a^5b^5c^5d^5 + 194040a^6b^4c^4d^6 + 95040a^7b^3c^3d^7 + 22275a^8b^2c^2d^8 + 2200a^9bc + d^9 + 66a^{10}d^{10})x^{13} + \frac{5}{7}b^3d \\
 & (b^9c^9 + 54ab^8c^8d + 792a^2b^7c^7d^2 + 4620a^3b^6c^6d^3 + 12474a^4b^5c^5d^4 + 16632a^5b^4c^4d^5 + 11088a^6b^3c^3d^6 + 3564a^7b^2c^2d^7 + 495a^8bc + d^8 + 22a^9d^9) \\
 & x^{14} + 3b^4d^2(b^8c^8 + 32ab^7c^7d + 308a^2b^6c^6d^2 + 1232a^3b^5c^5d^3 + 2310a^4b^4c^4d^4 + 2112a^5b^3c^3d^5 + 924a^6b^2c^2d^6 + 176a^7bc + d^7 + 11a^8d^8)x^{15} + \\
 & \frac{3}{2}b^5d^3(5b^7c^7 + 105ab^6c^6d + 693a^2b^5c^5d^2 + 1925a^3b^4c^4d^3 + 2475a^4b^3c^3d^4 + 1485a^5b^2c^2d^5 + 385a^6bc + d^6 + 33a^7d^7)x^{16} + \\
 & \frac{3}{17}b^6d^4(70b^6c^6 + 1008ab^5c^5d + 4620a^2b^4c^4d^2 + 8800a^3b^3c^3d^3 + 7425a^4b^2c^2d^4 + 2640a^5bc + d^5 + 308a^6d^6)x^{17} + \\
 & b^7d^5(14b^5c^5 + 140ab^4c^4d + 440a^2b^3c^3d^2 + 550a^3b^2c^2d^3 + 275a^4bc + d^4 + 44a^5d^5)x^{18} + \\
 & \frac{5}{19}b^8d^6(42b^4c^4 + 288ab^3c^3d + 594a^2b^2c^2d^2 + 440a^3bc + d^3 + 99a^4d^4)x^{19} + b^9d^7(6b^3c^3 + 27ab^2c^2d + 33a^2bc + d^2 + 11a^3d^3)x^{20} + \\
 & \frac{1}{7}b^{10}d^8(15b^2c^2 + 40abc + 22a^2d^2)x^{21} + \frac{1}{11}b^{11}d^9(5bc + 6ad)x^{22} + \frac{1}{23}b^{12}d^{10}x^{23}
 \end{aligned}$$

Problem 1300: Result more than twice size of optimal antiderivative.

$$\int (a + bx)^{11} (c + dx)^{10} dx$$

Optimal (type 1, 279 leaves, 2 steps):

$$\begin{aligned} & \frac{(bc - ad)^{10} (a + bx)^{12}}{12 b^{11}} + \frac{10d (bc - ad)^9 (a + bx)^{13}}{13 b^{11}} + \frac{45d^2 (bc - ad)^8 (a + bx)^{14}}{14 b^{11}} + \\ & \frac{8d^3 (bc - ad)^7 (a + bx)^{15}}{b^{11}} + \frac{105d^4 (bc - ad)^6 (a + bx)^{16}}{8 b^{11}} + \frac{252d^5 (bc - ad)^5 (a + bx)^{17}}{17 b^{11}} + \frac{35d^6 (bc - ad)^4 (a + bx)^{18}}{3 b^{11}} + \\ & \frac{120d^7 (bc - ad)^3 (a + bx)^{19}}{19 b^{11}} + \frac{9d^8 (bc - ad)^2 (a + bx)^{20}}{4 b^{11}} + \frac{10d^9 (bc - ad) (a + bx)^{21}}{21 b^{11}} + \frac{d^{10} (a + bx)^{22}}{22 b^{11}} \end{aligned}$$

Result (type 1, 1702 leaves):

$$\begin{aligned}
& a^{11} c^{10} x + \frac{1}{2} a^{10} c^9 (11 b c + 10 a d) x^2 + \frac{5}{3} a^9 c^8 (11 b^2 c^2 + 22 a b c d + 9 a^2 d^2) x^3 + \\
& \frac{5}{4} a^8 c^7 (33 b^3 c^3 + 110 a b^2 c^2 d + 99 a^2 b c d^2 + 24 a^3 d^3) x^4 + 3 a^7 c^6 (22 b^4 c^4 + 110 a b^3 c^3 d + 165 a^2 b^2 c^2 d^2 + 88 a^3 b c d^3 + 14 a^4 d^4) x^5 + \\
& \frac{1}{2} a^6 c^5 (154 b^5 c^5 + 1100 a b^4 c^4 d + 2475 a^2 b^3 c^3 d^2 + 2200 a^3 b^2 c^2 d^3 + 770 a^4 b c d^4 + 84 a^5 d^5) x^6 + \\
& \frac{6}{7} a^5 c^4 (77 b^6 c^6 + 770 a b^5 c^5 d + 2475 a^2 b^4 c^4 d^2 + 3300 a^3 b^3 c^3 d^3 + 1925 a^4 b^2 c^2 d^4 + 462 a^5 b c d^5 + 35 a^6 d^6) x^7 + \\
& \frac{15}{4} a^4 c^3 (11 b^7 c^7 + 154 a b^6 c^6 d + 693 a^2 b^5 c^5 d^2 + 1320 a^3 b^4 c^4 d^3 + 1155 a^4 b^3 c^3 d^4 + 462 a^5 b^2 c^2 d^5 + 77 a^6 b c d^6 + 4 a^7 d^7) x^8 + \\
& \frac{5}{3} a^3 c^2 (11 b^8 c^8 + 220 a b^7 c^7 d + 1386 a^2 b^6 c^6 d^2 + 3696 a^3 b^5 c^5 d^3 + 4620 a^4 b^4 c^4 d^4 + 2772 a^5 b^3 c^3 d^5 + 770 a^6 b^2 c^2 d^6 + 88 a^7 b c d^7 + 3 a^8 d^8) x^9 + \\
& \frac{1}{2} a^2 c (11 b^9 c^9 + 330 a b^8 c^8 d + 2970 a^2 b^7 c^7 d^2 + 11088 a^3 b^6 c^6 d^3 + 19404 a^4 b^5 c^5 d^4 + 16632 a^5 b^4 c^4 d^5 + 6930 a^6 b^3 c^3 d^6 + \\
& 1320 a^7 b^2 c^2 d^7 + 99 a^8 b c d^8 + 2 a^9 d^9) x^{10} + \frac{1}{11} a (11 b^{10} c^{10} + 550 a b^9 c^9 d + 7425 a^2 b^8 c^8 d^2 + 39600 a^3 b^7 c^7 d^3 + \\
& 97020 a^4 b^6 c^6 d^4 + 116424 a^5 b^5 c^5 d^5 + 69300 a^6 b^4 c^4 d^6 + 19800 a^7 b^3 c^3 d^7 + 2475 a^8 b^2 c^2 d^8 + 110 a^9 b c d^9 + a^{10} d^{10}) x^{11} + \\
& \frac{1}{12} b (b^{10} c^{10} + 110 a b^9 c^9 d + 2475 a^2 b^8 c^8 d^2 + 19800 a^3 b^7 c^7 d^3 + 69300 a^4 b^6 c^6 d^4 + 116424 a^5 b^5 c^5 d^5 + 97020 a^6 b^4 c^4 d^6 + \\
& 39600 a^7 b^3 c^3 d^7 + 7425 a^8 b^2 c^2 d^8 + 550 a^9 b c d^9 + 11 a^{10} d^{10}) x^{12} + \frac{5}{13} b^2 d (2 b^9 c^9 + 99 a b^8 c^8 d + 1320 a^2 b^7 c^7 d^2 + \\
& 6930 a^3 b^6 c^6 d^3 + 16632 a^4 b^5 c^5 d^4 + 19404 a^5 b^4 c^4 d^5 + 11088 a^6 b^3 c^3 d^6 + 2970 a^7 b^2 c^2 d^7 + 330 a^8 b c d^8 + 11 a^9 d^9) x^{13} + \\
& \frac{15}{14} b^3 d^2 (3 b^8 c^8 + 88 a b^7 c^7 d + 770 a^2 b^6 c^6 d^2 + 2772 a^3 b^5 c^5 d^3 + 4620 a^4 b^4 c^4 d^4 + 3696 a^5 b^3 c^3 d^5 + 1386 a^6 b^2 c^2 d^6 + 220 a^7 b c d^7 + 11 a^8 d^8) x^{14} + \\
& 2 b^4 d^3 (4 b^7 c^7 + 77 a b^6 c^6 d + 462 a^2 b^5 c^5 d^2 + 1155 a^3 b^4 c^4 d^3 + 1320 a^4 b^3 c^3 d^4 + 693 a^5 b^2 c^2 d^5 + 154 a^6 b c d^6 + 11 a^7 d^7) x^{15} + \\
& \frac{3}{8} b^5 d^4 (35 b^6 c^6 + 462 a b^5 c^5 d + 1925 a^2 b^4 c^4 d^2 + 3300 a^3 b^3 c^3 d^3 + 2475 a^4 b^2 c^2 d^4 + 770 a^5 b c d^5 + 77 a^6 d^6) x^{16} + \\
& \frac{3}{17} b^6 d^5 (84 b^5 c^5 + 770 a b^4 c^4 d + 2200 a^2 b^3 c^3 d^2 + 2475 a^3 b^2 c^2 d^3 + 1100 a^4 b c d^4 + 154 a^5 d^5) x^{17} + \\
& \frac{5}{6} b^7 d^6 (14 b^4 c^4 + 88 a b^3 c^3 d + 165 a^2 b^2 c^2 d^2 + 110 a^3 b c d^3 + 22 a^4 d^4) x^{18} + \frac{5}{19} b^8 d^7 (24 b^3 c^3 + 99 a b^2 c^2 d + 110 a^2 b c d^2 + 33 a^3 d^3) x^{19} + \\
& \frac{1}{4} b^9 d^8 (9 b^2 c^2 + 22 a b c d + 11 a^2 d^2) x^{20} + \frac{1}{21} b^{10} d^9 (10 b c + 11 a d) x^{21} + \frac{1}{22} b^{11} d^{10} x^{22}
\end{aligned}$$

Problem 1301: Result more than twice size of optimal antiderivative.

$$\int (a + b x)^{10} (c + d x)^{10} dx$$

Optimal (type 1, 279 leaves, 2 steps):

$$\begin{aligned}
& \frac{(b c - a d)^{10} (a + b x)^{11}}{11 b^{11}} + \frac{5 d (b c - a d)^9 (a + b x)^{12}}{6 b^{11}} + \frac{45 d^2 (b c - a d)^8 (a + b x)^{13}}{13 b^{11}} + \\
& \frac{60 d^3 (b c - a d)^7 (a + b x)^{14}}{7 b^{11}} + \frac{14 d^4 (b c - a d)^6 (a + b x)^{15}}{b^{11}} + \frac{63 d^5 (b c - a d)^5 (a + b x)^{16}}{4 b^{11}} + \frac{210 d^6 (b c - a d)^4 (a + b x)^{17}}{17 b^{11}} + \\
& \frac{20 d^7 (b c - a d)^3 (a + b x)^{18}}{3 b^{11}} + \frac{45 d^8 (b c - a d)^2 (a + b x)^{19}}{19 b^{11}} + \frac{d^9 (b c - a d) (a + b x)^{20}}{2 b^{11}} + \frac{d^{10} (a + b x)^{21}}{21 b^{11}}
\end{aligned}$$

Result (type 1, 1539 leaves):

$$\begin{aligned}
& a^{10} c^{10} x + 5 a^9 c^9 (b c + a d) x^2 + \frac{5}{3} a^8 c^8 (9 b^2 c^2 + 20 a b c d + 9 a^2 d^2) x^3 + \\
& \frac{15}{2} a^7 c^7 (4 b^3 c^3 + 15 a b^2 c^2 d + 15 a^2 b c d^2 + 4 a^3 d^3) x^4 + 3 a^6 c^6 (14 b^4 c^4 + 80 a b^3 c^3 d + 135 a^2 b^2 c^2 d^2 + 80 a^3 b c d^3 + 14 a^4 d^4) x^5 + \\
& 2 a^5 c^5 (21 b^5 c^5 + 175 a b^4 c^4 d + 450 a^2 b^3 c^3 d^2 + 450 a^3 b^2 c^2 d^3 + 175 a^4 b c d^4 + 21 a^5 d^5) x^6 + \\
& \frac{30}{7} a^4 c^4 (7 b^6 c^6 + 84 a b^5 c^5 d + 315 a^2 b^4 c^4 d^2 + 480 a^3 b^3 c^3 d^3 + 315 a^4 b^2 c^2 d^4 + 84 a^5 b c d^5 + 7 a^6 d^6) x^7 + \\
& \frac{15}{2} a^3 c^3 (2 b^7 c^7 + 35 a b^6 c^6 d + 189 a^2 b^5 c^5 d^2 + 420 a^3 b^4 c^4 d^3 + 420 a^4 b^3 c^3 d^4 + 189 a^5 b^2 c^2 d^5 + 35 a^6 b c d^6 + 2 a^7 d^7) x^8 + \\
& \frac{5}{3} a^2 c^2 (3 b^8 c^8 + 80 a b^7 c^7 d + 630 a^2 b^6 c^6 d^2 + 2016 a^3 b^5 c^5 d^3 + 2940 a^4 b^4 c^4 d^4 + 2016 a^5 b^3 c^3 d^5 + 630 a^6 b^2 c^2 d^6 + 80 a^7 b c d^7 + 3 a^8 d^8) x^9 + \\
& a c (b^9 c^9 + 45 a b^8 c^8 d + 540 a^2 b^7 c^7 d^2 + 2520 a^3 b^6 c^6 d^3 + 5292 a^4 b^5 c^5 d^4 + 5292 a^5 b^4 c^4 d^5 + 2520 a^6 b^3 c^3 d^6 + 540 a^7 b^2 c^2 d^7 + 45 a^8 b c d^8 + a^9 d^9) x^{10} + \\
& \frac{1}{11} (b^{10} c^{10} + 100 a b^9 c^9 d + 2025 a^2 b^8 c^8 d^2 + 14400 a^3 b^7 c^7 d^3 + 44100 a^4 b^6 c^6 d^4 + \\
& 63504 a^5 b^5 c^5 d^5 + 44100 a^6 b^4 c^4 d^6 + 14400 a^7 b^3 c^3 d^7 + 2025 a^8 b^2 c^2 d^8 + 100 a^9 b c d^9 + a^{10} d^{10}) x^{11} + \\
& \frac{5}{6} b d (b^9 c^9 + 45 a b^8 c^8 d + 540 a^2 b^7 c^7 d^2 + 2520 a^3 b^6 c^6 d^3 + 5292 a^4 b^5 c^5 d^4 + 5292 a^5 b^4 c^4 d^5 + 2520 a^6 b^3 c^3 d^6 + 540 a^7 b^2 c^2 d^7 + 45 a^8 b c d^8 + a^9 d^9) x^{12} + \\
& \frac{15}{13} b^2 d^2 (3 b^8 c^8 + 80 a b^7 c^7 d + 630 a^2 b^6 c^6 d^2 + 2016 a^3 b^5 c^5 d^3 + 2940 a^4 b^4 c^4 d^4 + 2016 a^5 b^3 c^3 d^5 + 630 a^6 b^2 c^2 d^6 + 80 a^7 b c d^7 + 3 a^8 d^8) x^{13} + \\
& \frac{30}{7} b^3 d^3 (2 b^7 c^7 + 35 a b^6 c^6 d + 189 a^2 b^5 c^5 d^2 + 420 a^3 b^4 c^4 d^3 + 420 a^4 b^3 c^3 d^4 + 189 a^5 b^2 c^2 d^5 + 35 a^6 b c d^6 + 2 a^7 d^7) x^{14} + \\
& 2 b^4 d^4 (7 b^6 c^6 + 84 a b^5 c^5 d + 315 a^2 b^4 c^4 d^2 + 480 a^3 b^3 c^3 d^3 + 315 a^4 b^2 c^2 d^4 + 84 a^5 b c d^5 + 7 a^6 d^6) x^{15} + \\
& \frac{3}{4} b^5 d^5 (21 b^5 c^5 + 175 a b^4 c^4 d + 450 a^2 b^3 c^3 d^2 + 450 a^3 b^2 c^2 d^3 + 175 a^4 b c d^4 + 21 a^5 d^5) x^{16} + \\
& \frac{15}{17} b^6 d^6 (14 b^4 c^4 + 80 a b^3 c^3 d + 135 a^2 b^2 c^2 d^2 + 80 a^3 b c d^3 + 14 a^4 d^4) x^{17} + \frac{5}{3} b^7 d^7 (4 b^3 c^3 + 15 a b^2 c^2 d + 15 a^2 b c d^2 + 4 a^3 d^3) x^{18} + \\
& \frac{5}{19} b^8 d^8 (9 b^2 c^2 + 20 a b c d + 9 a^2 d^2) x^{19} + \frac{1}{2} b^9 d^9 (b c + a d) x^{20} + \frac{1}{21} b^{10} d^{10} x^{21}
\end{aligned}$$

### Problem 1302: Result more than twice size of optimal antiderivative.

$$\int (a + b x)^9 (c + d x)^{10} dx$$

Optimal (type 1, 250 leaves, 2 steps):

$$\begin{aligned} & -\frac{(b c - a d)^9 (c + d x)^{11}}{11 d^{10}} + \frac{3 b (b c - a d)^8 (c + d x)^{12}}{4 d^{10}} - \frac{36 b^2 (b c - a d)^7 (c + d x)^{13}}{13 d^{10}} + \frac{6 b^3 (b c - a d)^6 (c + d x)^{14}}{d^{10}} - \frac{42 b^4 (b c - a d)^5 (c + d x)^{15}}{5 d^{10}} + \\ & \frac{63 b^5 (b c - a d)^4 (c + d x)^{16}}{8 d^{10}} - \frac{84 b^6 (b c - a d)^3 (c + d x)^{17}}{17 d^{10}} + \frac{2 b^7 (b c - a d)^2 (c + d x)^{18}}{d^{10}} - \frac{9 b^8 (b c - a d) (c + d x)^{19}}{19 d^{10}} + \frac{b^9 (c + d x)^{20}}{20 d^{10}} \end{aligned}$$

Result (type 1, 1397 leaves):

$$\begin{aligned} & a^9 c^{10} x + \frac{1}{2} a^8 c^9 (9 b c + 10 a d) x^2 + 3 a^7 c^8 (4 b^2 c^2 + 10 a b c d + 5 a^2 d^2) x^3 + \\ & \frac{3}{4} a^6 c^7 (28 b^3 c^3 + 120 a b^2 c^2 d + 135 a^2 b c d^2 + 40 a^3 d^3) x^4 + \frac{6}{5} a^5 c^6 (21 b^4 c^4 + 140 a b^3 c^3 d + 270 a^2 b^2 c^2 d^2 + 180 a^3 b c d^3 + 35 a^4 d^4) x^5 + \\ & 3 a^4 c^5 (7 b^5 c^5 + 70 a b^4 c^4 d + 210 a^2 b^3 c^3 d^2 + 240 a^3 b^2 c^2 d^3 + 105 a^4 b c d^4 + 14 a^5 d^5) x^6 + \\ & 6 a^3 c^4 (2 b^6 c^6 + 30 a b^5 c^5 d + 135 a^2 b^4 c^4 d^2 + 240 a^3 b^3 c^3 d^3 + 180 a^4 b^2 c^2 d^4 + 54 a^5 b c d^5 + 5 a^6 d^6) x^7 + \\ & \frac{3}{4} a^2 c^3 (6 b^7 c^7 + 140 a b^6 c^6 d + 945 a^2 b^5 c^5 d^2 + 2520 a^3 b^4 c^4 d^3 + 2940 a^4 b^3 c^3 d^4 + 1512 a^5 b^2 c^2 d^5 + 315 a^6 b c d^6 + 20 a^7 d^7) x^8 + \\ & a c^2 (b^8 c^8 + 40 a b^7 c^7 d + 420 a^2 b^6 c^6 d^2 + 1680 a^3 b^5 c^5 d^3 + 2940 a^4 b^4 c^4 d^4 + 2352 a^5 b^3 c^3 d^5 + 840 a^6 b^2 c^2 d^6 + 120 a^7 b c d^7 + 5 a^8 d^8) x^9 + \\ & \frac{1}{10} c (b^9 c^9 + 90 a b^8 c^8 d + 1620 a^2 b^7 c^7 d^2 + 10080 a^3 b^6 c^6 d^3 + 26460 a^4 b^5 c^5 d^4 + 31752 a^5 b^4 c^4 d^5 + \\ & 17640 a^6 b^3 c^3 d^6 + 4320 a^7 b^2 c^2 d^7 + 405 a^8 b c d^8 + 10 a^9 d^9) x^{10} + \frac{1}{11} d (10 b^9 c^9 + 405 a b^8 c^8 d + 4320 a^2 b^7 c^7 d^2 + \\ & 17640 a^3 b^6 c^6 d^3 + 31752 a^4 b^5 c^5 d^4 + 26460 a^5 b^4 c^4 d^5 + 10080 a^6 b^3 c^3 d^6 + 1620 a^7 b^2 c^2 d^7 + 90 a^8 b c d^8 + a^9 d^9) x^{11} + \\ & \frac{3}{4} b d^2 (5 b^8 c^8 + 120 a b^7 c^7 d + 840 a^2 b^6 c^6 d^2 + 2352 a^3 b^5 c^5 d^3 + 2940 a^4 b^4 c^4 d^4 + 1680 a^5 b^3 c^3 d^5 + 420 a^6 b^2 c^2 d^6 + 40 a^7 b c d^7 + a^8 d^8) x^{12} + \\ & \frac{6}{13} b^2 d^3 (20 b^7 c^7 + 315 a b^6 c^6 d + 1512 a^2 b^5 c^5 d^2 + 2940 a^3 b^4 c^4 d^3 + 2520 a^4 b^3 c^3 d^4 + 945 a^5 b^2 c^2 d^5 + 140 a^6 b c d^6 + 6 a^7 d^7) x^{13} + \\ & 3 b^3 d^4 (5 b^6 c^6 + 54 a b^5 c^5 d + 180 a^2 b^4 c^4 d^2 + 240 a^3 b^3 c^3 d^3 + 135 a^4 b^2 c^2 d^4 + 30 a^5 b c d^5 + 2 a^6 d^6) x^{14} + \\ & \frac{6}{5} b^4 d^5 (14 b^5 c^5 + 105 a b^4 c^4 d + 240 a^2 b^3 c^3 d^2 + 210 a^3 b^2 c^2 d^3 + 70 a^4 b c d^4 + 7 a^5 d^5) x^{15} + \\ & \frac{3}{8} b^5 d^6 (35 b^4 c^4 + 180 a b^3 c^3 d + 270 a^2 b^2 c^2 d^2 + 140 a^3 b c d^3 + 21 a^4 d^4) x^{16} + \frac{3}{17} b^6 d^7 (40 b^3 c^3 + 135 a b^2 c^2 d + 120 a^2 b c d^2 + 28 a^3 d^3) x^{17} + \\ & \frac{1}{2} b^7 d^8 (5 b^2 c^2 + 10 a b c d + 4 a^2 d^2) x^{18} + \frac{1}{19} b^8 d^9 (10 b c + 9 a d) x^{19} + \frac{1}{20} b^9 d^{10} x^{20} \end{aligned}$$

### Problem 1303: Result more than twice size of optimal antiderivative.

$$\int (a + b x)^8 (c + d x)^{10} dx$$

Optimal (type 1, 225 leaves, 2 steps):

$$\begin{aligned} & \frac{(b c - a d)^8 (c + d x)^{11}}{11 d^9} - \frac{2 b (b c - a d)^7 (c + d x)^{12}}{3 d^9} + \frac{28 b^2 (b c - a d)^6 (c + d x)^{13}}{13 d^9} - \frac{4 b^3 (b c - a d)^5 (c + d x)^{14}}{d^9} + \\ & \frac{14 b^4 (b c - a d)^4 (c + d x)^{15}}{3 d^9} - \frac{7 b^5 (b c - a d)^3 (c + d x)^{16}}{2 d^9} + \frac{28 b^6 (b c - a d)^2 (c + d x)^{17}}{17 d^9} - \frac{4 b^7 (b c - a d) (c + d x)^{18}}{9 d^9} + \frac{b^8 (c + d x)^{19}}{19 d^9} \end{aligned}$$

Result (type 1, 1241 leaves):

$$\begin{aligned} & a^8 c^{10} x + a^7 c^9 (4 b c + 5 a d) x^2 + \frac{1}{3} a^6 c^8 (28 b^2 c^2 + 80 a b c d + 45 a^2 d^2) x^3 + \\ & 2 a^5 c^7 (7 b^3 c^3 + 35 a b^2 c^2 d + 45 a^2 b c d^2 + 15 a^3 d^3) x^4 + 2 a^4 c^6 (7 b^4 c^4 + 56 a b^3 c^3 d + 126 a^2 b^2 c^2 d^2 + 96 a^3 b c d^3 + 21 a^4 d^4) x^5 + \\ & \frac{14}{3} a^3 c^5 (2 b^5 c^5 + 25 a b^4 c^4 d + 90 a^2 b^3 c^3 d^2 + 120 a^3 b^2 c^2 d^3 + 60 a^4 b c d^4 + 9 a^5 d^5) x^6 + \\ & 2 a^2 c^4 (2 b^6 c^6 + 40 a b^5 c^5 d + 225 a^2 b^4 c^4 d^2 + 480 a^3 b^3 c^3 d^3 + 420 a^4 b^2 c^2 d^4 + 144 a^5 b c d^5 + 15 a^6 d^6) x^7 + \\ & a c^3 (b^7 c^7 + 35 a b^6 c^6 d + 315 a^2 b^5 c^5 d^2 + 1050 a^3 b^4 c^4 d^3 + 1470 a^4 b^3 c^3 d^4 + 882 a^5 b^2 c^2 d^5 + 210 a^6 b c d^6 + 15 a^7 d^7) x^8 + \\ & \frac{1}{9} c^2 (b^8 c^8 + 80 a b^7 c^7 d + 1260 a^2 b^6 c^6 d^2 + 6720 a^3 b^5 c^5 d^3 + 14700 a^4 b^4 c^4 d^4 + 14112 a^5 b^3 c^3 d^5 + 5880 a^6 b^2 c^2 d^6 + 960 a^7 b c d^7 + 45 a^8 d^8) x^9 + \\ & c d (b^8 c^8 + 36 a b^7 c^7 d + 336 a^2 b^6 c^6 d^2 + 1176 a^3 b^5 c^5 d^3 + 1764 a^4 b^4 c^4 d^4 + 1176 a^5 b^3 c^3 d^5 + 336 a^6 b^2 c^2 d^6 + 36 a^7 b c d^7 + a^8 d^8) x^{10} + \\ & \frac{1}{11} d^2 (45 b^8 c^8 + 960 a b^7 c^7 d + 5880 a^2 b^6 c^6 d^2 + 14112 a^3 b^5 c^5 d^3 + 14700 a^4 b^4 c^4 d^4 + 6720 a^5 b^3 c^3 d^5 + 1260 a^6 b^2 c^2 d^6 + 80 a^7 b c d^7 + a^8 d^8) x^{11} + \\ & \frac{2}{3} b d^3 (15 b^7 c^7 + 210 a b^6 c^6 d + 882 a^2 b^5 c^5 d^2 + 1470 a^3 b^4 c^4 d^3 + 1050 a^4 b^3 c^3 d^4 + 315 a^5 b^2 c^2 d^5 + 35 a^6 b c d^6 + a^7 d^7) x^{12} + \\ & \frac{14}{13} b^2 d^4 (15 b^6 c^6 + 144 a b^5 c^5 d + 420 a^2 b^4 c^4 d^2 + 480 a^3 b^3 c^3 d^3 + 225 a^4 b^2 c^2 d^4 + 40 a^5 b c d^5 + 2 a^6 d^6) x^{13} + \\ & 2 b^3 d^5 (9 b^5 c^5 + 60 a b^4 c^4 d + 120 a^2 b^3 c^3 d^2 + 90 a^3 b^2 c^2 d^3 + 25 a^4 b c d^4 + 2 a^5 d^5) x^{14} + \\ & \frac{2}{3} b^4 d^6 (21 b^4 c^4 + 96 a b^3 c^3 d + 126 a^2 b^2 c^2 d^2 + 56 a^3 b c d^3 + 7 a^4 d^4) x^{15} + \frac{1}{2} b^5 d^7 (15 b^3 c^3 + 45 a b^2 c^2 d + 35 a^2 b c d^2 + 7 a^3 d^3) x^{16} + \\ & \frac{1}{17} b^6 d^8 (45 b^2 c^2 + 80 a b c d + 28 a^2 d^2) x^{17} + \frac{1}{9} b^7 d^9 (5 b c + 4 a d) x^{18} + \frac{1}{19} b^8 d^{10} x^{19} \end{aligned}$$

### Problem 1304: Result more than twice size of optimal antiderivative.

$$\int (a + b x)^7 (c + d x)^{10} dx$$

Optimal (type 1, 200 leaves, 2 steps):

$$\begin{aligned}
& - \frac{(b c - a d)^7 (c + d x)^{11}}{11 d^8} + \frac{7 b (b c - a d)^6 (c + d x)^{12}}{12 d^8} - \frac{21 b^2 (b c - a d)^5 (c + d x)^{13}}{13 d^8} + \frac{5 b^3 (b c - a d)^4 (c + d x)^{14}}{2 d^8} - \\
& \frac{7 b^4 (b c - a d)^3 (c + d x)^{15}}{3 d^8} + \frac{21 b^5 (b c - a d)^2 (c + d x)^{16}}{16 d^8} - \frac{7 b^6 (b c - a d) (c + d x)^{17}}{17 d^8} + \frac{b^7 (c + d x)^{18}}{18 d^8}
\end{aligned}$$

Result (type 1, 1105 leaves):

$$\begin{aligned}
& a^7 c^{10} x + \frac{1}{2} a^6 c^9 (7 b c + 10 a d) x^2 + \frac{1}{3} a^5 c^8 (21 b^2 c^2 + 70 a b c d + 45 a^2 d^2) x^3 + \\
& \frac{5}{4} a^4 c^7 (7 b^3 c^3 + 42 a b^2 c^2 d + 63 a^2 b c d^2 + 24 a^3 d^3) x^4 + 7 a^3 c^6 (b^4 c^4 + 10 a b^3 c^3 d + 27 a^2 b^2 c^2 d^2 + 24 a^3 b c d^3 + 6 a^4 d^4) x^5 + \\
& \frac{7}{6} a^2 c^5 (3 b^5 c^5 + 50 a b^4 c^4 d + 225 a^2 b^3 c^3 d^2 + 360 a^3 b^2 c^2 d^3 + 210 a^4 b c d^4 + 36 a^5 d^5) x^6 + \\
& a c^4 (b^6 c^6 + 30 a b^5 c^5 d + 225 a^2 b^4 c^4 d^2 + 600 a^3 b^3 c^3 d^3 + 630 a^4 b^2 c^2 d^4 + 252 a^5 b c d^5 + 30 a^6 d^6) x^7 + \\
& \frac{1}{8} c^3 (b^7 c^7 + 70 a b^6 c^6 d + 945 a^2 b^5 c^5 d^2 + 4200 a^3 b^4 c^4 d^3 + 7350 a^4 b^3 c^3 d^4 + 5292 a^5 b^2 c^2 d^5 + 1470 a^6 b c d^6 + 120 a^7 d^7) x^8 + \\
& \frac{5}{9} c^2 d (2 b^7 c^7 + 63 a b^6 c^6 d + 504 a^2 b^5 c^5 d^2 + 1470 a^3 b^4 c^4 d^3 + 1764 a^4 b^3 c^3 d^4 + 882 a^5 b^2 c^2 d^5 + 168 a^6 b c d^6 + 9 a^7 d^7) x^9 + \\
& \frac{1}{2} c d^2 (9 b^7 c^7 + 168 a b^6 c^6 d + 882 a^2 b^5 c^5 d^2 + 1764 a^3 b^4 c^4 d^3 + 1470 a^4 b^3 c^3 d^4 + 504 a^5 b^2 c^2 d^5 + 63 a^6 b c d^6 + 2 a^7 d^7) x^{10} + \\
& \frac{1}{11} d^3 (120 b^7 c^7 + 1470 a b^6 c^6 d + 5292 a^2 b^5 c^5 d^2 + 7350 a^3 b^4 c^4 d^3 + 4200 a^4 b^3 c^3 d^4 + 945 a^5 b^2 c^2 d^5 + 70 a^6 b c d^6 + a^7 d^7) x^{11} + \\
& \frac{7}{12} b d^4 (30 b^6 c^6 + 252 a b^5 c^5 d + 630 a^2 b^4 c^4 d^2 + 600 a^3 b^3 c^3 d^3 + 225 a^4 b^2 c^2 d^4 + 30 a^5 b c d^5 + a^6 d^6) x^{12} + \\
& \frac{7}{13} b^2 d^5 (36 b^5 c^5 + 210 a b^4 c^4 d + 360 a^2 b^3 c^3 d^2 + 225 a^3 b^2 c^2 d^3 + 50 a^4 b c d^4 + 3 a^5 d^5) x^{13} + \\
& \frac{5}{2} b^3 d^6 (6 b^4 c^4 + 24 a b^3 c^3 d + 27 a^2 b^2 c^2 d^2 + 10 a^3 b c d^3 + a^4 d^4) x^{14} + \frac{1}{3} b^4 d^7 (24 b^3 c^3 + 63 a b^2 c^2 d + 42 a^2 b c d^2 + 7 a^3 d^3) x^{15} + \\
& \frac{1}{16} b^5 d^8 (45 b^2 c^2 + 70 a b c d + 21 a^2 d^2) x^{16} + \frac{1}{17} b^6 d^9 (10 b c + 7 a d) x^{17} + \frac{1}{18} b^7 d^{10} x^{18}
\end{aligned}$$

Problem 1305: Result more than twice size of optimal antiderivative.

$$\int (a + b x)^6 (c + d x)^{10} dx$$

Optimal (type 1, 170 leaves, 2 steps):

$$\begin{aligned} & \frac{(b c - a d)^6 (c + d x)^{11}}{11 d^7} - \frac{b (b c - a d)^5 (c + d x)^{12}}{2 d^7} + \frac{15 b^2 (b c - a d)^4 (c + d x)^{13}}{13 d^7} - \\ & \frac{10 b^3 (b c - a d)^3 (c + d x)^{14}}{7 d^7} + \frac{b^4 (b c - a d)^2 (c + d x)^{15}}{d^7} - \frac{3 b^5 (b c - a d) (c + d x)^{16}}{8 d^7} + \frac{b^6 (c + d x)^{17}}{17 d^7} \end{aligned}$$

Result (type 1, 939 leaves):

$$\begin{aligned} & a^6 c^{10} x + a^5 c^9 (3 b c + 5 a d) x^2 + 5 a^4 c^8 (b^2 c^2 + 4 a b c d + 3 a^2 d^2) x^3 + \\ & \frac{5}{2} a^3 c^7 (2 b^3 c^3 + 15 a b^2 c^2 d + 27 a^2 b c d^2 + 12 a^3 d^3) x^4 + a^2 c^6 (3 b^4 c^4 + 40 a b^3 c^3 d + 135 a^2 b^2 c^2 d^2 + 144 a^3 b c d^3 + 42 a^4 d^4) x^5 + \\ & a c^5 (b^5 c^5 + 25 a b^4 c^4 d + 150 a^2 b^3 c^3 d^2 + 300 a^3 b^2 c^2 d^3 + 210 a^4 b c d^4 + 42 a^5 d^5) x^6 + \\ & \frac{1}{7} c^4 (b^6 c^6 + 60 a b^5 c^5 d + 675 a^2 b^4 c^4 d^2 + 2400 a^3 b^3 c^3 d^3 + 3150 a^4 b^2 c^2 d^4 + 1512 a^5 b c d^5 + 210 a^6 d^6) x^7 + \\ & \frac{5}{4} c^3 d (b^6 c^6 + 27 a b^5 c^5 d + 180 a^2 b^4 c^4 d^2 + 420 a^3 b^3 c^3 d^3 + 378 a^4 b^2 c^2 d^4 + 126 a^5 b c d^5 + 12 a^6 d^6) x^8 + \\ & 5 c^2 d^2 (b^6 c^6 + 16 a b^5 c^5 d + 70 a^2 b^4 c^4 d^2 + 112 a^3 b^3 c^3 d^3 + 70 a^4 b^2 c^2 d^4 + 16 a^5 b c d^5 + a^6 d^6) x^9 + \\ & c d^3 (12 b^6 c^6 + 126 a b^5 c^5 d + 378 a^2 b^4 c^4 d^2 + 420 a^3 b^3 c^3 d^3 + 180 a^4 b^2 c^2 d^4 + 27 a^5 b c d^5 + a^6 d^6) x^{10} + \\ & \frac{1}{11} d^4 (210 b^6 c^6 + 1512 a b^5 c^5 d + 3150 a^2 b^4 c^4 d^2 + 2400 a^3 b^3 c^3 d^3 + 675 a^4 b^2 c^2 d^4 + 60 a^5 b c d^5 + a^6 d^6) x^{11} + \\ & \frac{1}{2} b d^5 (42 b^5 c^5 + 210 a b^4 c^4 d + 300 a^2 b^3 c^3 d^2 + 150 a^3 b^2 c^2 d^3 + 25 a^4 b c d^4 + a^5 d^5) x^{12} + \\ & \frac{5}{13} b^2 d^6 (42 b^4 c^4 + 144 a b^3 c^3 d + 135 a^2 b^2 c^2 d^2 + 40 a^3 b c d^3 + 3 a^4 d^4) x^{13} + \frac{5}{7} b^3 d^7 (12 b^3 c^3 + 27 a b^2 c^2 d + 15 a^2 b c d^2 + 2 a^3 d^3) x^{14} + \\ & b^4 d^8 (3 b^2 c^2 + 4 a b c d + a^2 d^2) x^{15} + \frac{1}{8} b^5 d^9 (5 b c + 3 a d) x^{16} + \frac{1}{17} b^6 d^{10} x^{17} \end{aligned}$$

Problem 1306: Result more than twice size of optimal antiderivative.

$$\int (a + b x)^5 (c + d x)^{10} dx$$

Optimal (type 1, 146 leaves, 2 steps):

$$\begin{aligned} & -\frac{(b c - a d)^5 (c + d x)^{11}}{11 d^6} + \frac{5 b (b c - a d)^4 (c + d x)^{12}}{12 d^6} - \\ & \frac{10 b^2 (b c - a d)^3 (c + d x)^{13}}{13 d^6} + \frac{5 b^3 (b c - a d)^2 (c + d x)^{14}}{7 d^6} - \frac{b^4 (b c - a d) (c + d x)^{15}}{3 d^6} + \frac{b^5 (c + d x)^{16}}{16 d^6} \end{aligned}$$

Result (type 1, 811 leaves):

$$\begin{aligned}
& a^5 c^{10} x + \frac{5}{2} a^4 c^9 (b c + 2 a d) x^2 + \frac{5}{3} a^3 c^8 (2 b^2 c^2 + 10 a b c d + 9 a^2 d^2) x^3 + \\
& \frac{5}{4} a^2 c^7 (2 b^3 c^3 + 20 a b^2 c^2 d + 45 a^2 b c d^2 + 24 a^3 d^3) x^4 + a c^6 (b^4 c^4 + 20 a b^3 c^3 d + 90 a^2 b^2 c^2 d^2 + 120 a^3 b c d^3 + 42 a^4 d^4) x^5 + \\
& \frac{1}{6} c^5 (b^5 c^5 + 50 a b^4 c^4 d + 450 a^2 b^3 c^3 d^2 + 1200 a^3 b^2 c^2 d^3 + 1050 a^4 b c d^4 + 252 a^5 d^5) x^6 + \\
& \frac{5}{7} c^4 d (2 b^5 c^5 + 45 a b^4 c^4 d + 240 a^2 b^3 c^3 d^2 + 420 a^3 b^2 c^2 d^3 + 252 a^4 b c d^4 + 42 a^5 d^5) x^7 + \\
& \frac{15}{8} c^3 d^2 (3 b^5 c^5 + 40 a b^4 c^4 d + 140 a^2 b^3 c^3 d^2 + 168 a^3 b^2 c^2 d^3 + 70 a^4 b c d^4 + 8 a^5 d^5) x^8 + \\
& \frac{5}{3} c^2 d^3 (8 b^5 c^5 + 70 a b^4 c^4 d + 168 a^2 b^3 c^3 d^2 + 140 a^3 b^2 c^2 d^3 + 40 a^4 b c d^4 + 3 a^5 d^5) x^9 + \\
& \frac{1}{2} c d^4 (42 b^5 c^5 + 252 a b^4 c^4 d + 420 a^2 b^3 c^3 d^2 + 240 a^3 b^2 c^2 d^3 + 45 a^4 b c d^4 + 2 a^5 d^5) x^{10} + \\
& \frac{1}{11} d^5 (252 b^5 c^5 + 1050 a b^4 c^4 d + 1200 a^2 b^3 c^3 d^2 + 450 a^3 b^2 c^2 d^3 + 50 a^4 b c d^4 + a^5 d^5) x^{11} + \\
& \frac{5}{12} b d^6 (42 b^4 c^4 + 120 a b^3 c^3 d + 90 a^2 b^2 c^2 d^2 + 20 a^3 b c d^3 + a^4 d^4) x^{12} + \frac{5}{13} b^2 d^7 (24 b^3 c^3 + 45 a b^2 c^2 d + 20 a^2 b c d^2 + 2 a^3 d^3) x^{13} + \\
& \frac{5}{14} b^3 d^8 (9 b^2 c^2 + 10 a b c d + 2 a^2 d^2) x^{14} + \frac{1}{3} b^4 d^9 (2 b c + a d) x^{15} + \frac{1}{16} b^5 d^{10} x^{16}
\end{aligned}$$

**Problem 1307: Result more than twice size of optimal antiderivative.**

$$\int (a + b x)^4 (c + d x)^{10} dx$$

Optimal (type 1, 119 leaves, 2 steps):

$$\frac{(b c - a d)^4 (c + d x)^{11}}{11 d^5} - \frac{b (b c - a d)^3 (c + d x)^{12}}{3 d^5} + \frac{6 b^2 (b c - a d)^2 (c + d x)^{13}}{13 d^5} - \frac{2 b^3 (b c - a d) (c + d x)^{14}}{7 d^5} + \frac{b^4 (c + d x)^{15}}{15 d^5}$$

Result (type 1, 660 leaves):

$$\begin{aligned}
& a^4 c^{10} x + a^3 c^9 (2 b c + 5 a d) x^2 + \frac{1}{3} a^2 c^8 (6 b^2 c^2 + 40 a b c d + 45 a^2 d^2) x^3 + \\
& a c^7 (b^3 c^3 + 15 a b^2 c^2 d + 45 a^2 b c d^2 + 30 a^3 d^3) x^4 + \frac{1}{5} c^6 (b^4 c^4 + 40 a b^3 c^3 d + 270 a^2 b^2 c^2 d^2 + 480 a^3 b c d^3 + 210 a^4 d^4) x^5 + \\
& \frac{1}{3} c^5 d (5 b^4 c^4 + 90 a b^3 c^3 d + 360 a^2 b^2 c^2 d^2 + 420 a^3 b c d^3 + 126 a^4 d^4) x^6 + \frac{3}{7} c^4 d^2 (15 b^4 c^4 + 160 a b^3 c^3 d + 420 a^2 b^2 c^2 d^2 + 336 a^3 b c d^3 + 70 a^4 d^4) x^7 + \\
& 3 c^3 d^3 (5 b^4 c^4 + 35 a b^3 c^3 d + 63 a^2 b^2 c^2 d^2 + 35 a^3 b c d^3 + 5 a^4 d^4) x^8 + \frac{1}{3} c^2 d^4 (70 b^4 c^4 + 336 a b^3 c^3 d + 420 a^2 b^2 c^2 d^2 + 160 a^3 b c d^3 + 15 a^4 d^4) x^9 + \\
& \frac{1}{5} c d^5 (126 b^4 c^4 + 420 a b^3 c^3 d + 360 a^2 b^2 c^2 d^2 + 90 a^3 b c d^3 + 5 a^4 d^4) x^{10} + \frac{1}{11} d^6 (210 b^4 c^4 + 480 a b^3 c^3 d + 270 a^2 b^2 c^2 d^2 + 40 a^3 b c d^3 + a^4 d^4) x^{11} + \\
& \frac{1}{3} b d^7 (30 b^3 c^3 + 45 a b^2 c^2 d + 15 a^2 b c d^2 + a^3 d^3) x^{12} + \frac{1}{13} b^2 d^8 (45 b^2 c^2 + 40 a b c d + 6 a^2 d^2) x^{13} + \frac{1}{7} b^3 d^9 (5 b c + 2 a d) x^{14} + \frac{1}{15} b^4 d^{10} x^{15}
\end{aligned}$$

**Problem 1308:** Result more than twice size of optimal antiderivative.

$$\int (a + b x)^3 (c + d x)^{10} dx$$

Optimal (type 1, 92 leaves, 2 steps):

$$-\frac{(b c - a d)^3 (c + d x)^{11}}{11 d^4} + \frac{b (b c - a d)^2 (c + d x)^{12}}{4 d^4} - \frac{3 b^2 (b c - a d) (c + d x)^{13}}{13 d^4} + \frac{b^3 (c + d x)^{14}}{14 d^4}$$

Result (type 1, 511 leaves):

$$\begin{aligned}
& a^3 c^{10} x + \frac{1}{2} a^2 c^9 (3 b c + 10 a d) x^2 + a c^8 (b^2 c^2 + 10 a b c d + 15 a^2 d^2) x^3 + \frac{1}{4} c^7 (b^3 c^3 + 30 a b^2 c^2 d + 135 a^2 b c d^2 + 120 a^3 d^3) x^4 + \\
& c^6 d (2 b^3 c^3 + 27 a b^2 c^2 d + 72 a^2 b c d^2 + 42 a^3 d^3) x^5 + \frac{3}{2} c^5 d^2 (5 b^3 c^3 + 40 a b^2 c^2 d + 70 a^2 b c d^2 + 28 a^3 d^3) x^6 + \\
& \frac{6}{7} c^4 d^3 (20 b^3 c^3 + 105 a b^2 c^2 d + 126 a^2 b c d^2 + 35 a^3 d^3) x^7 + \frac{3}{4} c^3 d^4 (35 b^3 c^3 + 126 a b^2 c^2 d + 105 a^2 b c d^2 + 20 a^3 d^3) x^8 + \\
& c^2 d^5 (28 b^3 c^3 + 70 a b^2 c^2 d + 40 a^2 b c d^2 + 5 a^3 d^3) x^9 + \frac{1}{2} c d^6 (42 b^3 c^3 + 72 a b^2 c^2 d + 27 a^2 b c d^2 + 2 a^3 d^3) x^{10} + \\
& \frac{1}{11} d^7 (120 b^3 c^3 + 135 a b^2 c^2 d + 30 a^2 b c d^2 + a^3 d^3) x^{11} + \frac{1}{4} b d^8 (15 b^2 c^2 + 10 a b c d + a^2 d^2) x^{12} + \frac{1}{13} b^2 d^9 (10 b c + 3 a d) x^{13} + \frac{1}{14} b^3 d^{10} x^{14}
\end{aligned}$$

**Problem 1309:** Result more than twice size of optimal antiderivative.

$$\int (a + b x)^2 (c + d x)^{10} dx$$

Optimal (type 1, 65 leaves, 2 steps):

$$\frac{(b c - a d)^2 (c + d x)^{11}}{11 d^3} - \frac{b (b c - a d) (c + d x)^{12}}{6 d^3} + \frac{b^2 (c + d x)^{13}}{13 d^3}$$

Result (type 1, 358 leaves):

$$\begin{aligned} & a^2 c^{10} x + a c^9 (b c + 5 a d) x^2 + \frac{1}{3} c^8 (b^2 c^2 + 20 a b c d + 45 a^2 d^2) x^3 + \\ & \frac{5}{2} c^7 d (b^2 c^2 + 9 a b c d + 12 a^2 d^2) x^4 + 3 c^6 d^2 (3 b^2 c^2 + 16 a b c d + 14 a^2 d^2) x^5 + 2 c^5 d^3 (10 b^2 c^2 + 35 a b c d + 21 a^2 d^2) x^6 + \\ & 6 c^4 d^4 (5 b^2 c^2 + 12 a b c d + 5 a^2 d^2) x^7 + \frac{3}{2} c^3 d^5 (21 b^2 c^2 + 35 a b c d + 10 a^2 d^2) x^8 + \frac{5}{3} c^2 d^6 (14 b^2 c^2 + 16 a b c d + 3 a^2 d^2) x^9 + \\ & c d^7 (12 b^2 c^2 + 9 a b c d + a^2 d^2) x^{10} + \frac{1}{11} d^8 (45 b^2 c^2 + 20 a b c d + a^2 d^2) x^{11} + \frac{1}{6} b d^9 (5 b c + a d) x^{12} + \frac{1}{13} b^2 d^{10} x^{13} \end{aligned}$$

Problem 1310: Result more than twice size of optimal antiderivative.

$$\int (a + b x) (c + d x)^{10} dx$$

Optimal (type 1, 38 leaves, 2 steps):

$$\frac{(b c - a d) (c + d x)^{11}}{11 d^2} + \frac{b (c + d x)^{12}}{12 d^2}$$

Result (type 1, 220 leaves):

$$\begin{aligned} & a c^{10} x + \frac{1}{2} c^9 (b c + 10 a d) x^2 + \frac{5}{3} c^8 d (2 b c + 9 a d) x^3 + \frac{15}{4} c^7 d^2 (3 b c + 8 a d) x^4 + 6 c^6 d^3 (4 b c + 7 a d) x^5 + 7 c^5 d^4 (5 b c + 6 a d) x^6 + \\ & 6 c^4 d^5 (6 b c + 5 a d) x^7 + \frac{15}{4} c^3 d^6 (7 b c + 4 a d) x^8 + \frac{5}{3} c^2 d^7 (8 b c + 3 a d) x^9 + \frac{1}{2} c d^8 (9 b c + 2 a d) x^{10} + \frac{1}{11} d^9 (10 b c + a d) x^{11} + \frac{1}{12} b d^{10} x^{12} \end{aligned}$$

Problem 1312: Result more than twice size of optimal antiderivative.

$$\int \frac{(c + d x)^{10}}{a + b x} dx$$

Optimal (type 3, 241 leaves, 2 steps):

$$\begin{aligned} & \frac{d (b c - a d)^9 x}{b^{10}} + \frac{(b c - a d)^8 (c + d x)^2}{2 b^9} + \frac{(b c - a d)^7 (c + d x)^3}{3 b^8} + \frac{(b c - a d)^6 (c + d x)^4}{4 b^7} + \frac{(b c - a d)^5 (c + d x)^5}{5 b^6} + \\ & \frac{(b c - a d)^4 (c + d x)^6}{6 b^5} + \frac{(b c - a d)^3 (c + d x)^7}{7 b^4} + \frac{(b c - a d)^2 (c + d x)^8}{8 b^3} + \frac{(b c - a d) (c + d x)^9}{9 b^2} + \frac{(c + d x)^{10}}{10 b} + \frac{(b c - a d)^{10} \text{Log}[a + b x]}{b^{11}} \end{aligned}$$

Result (type 3, 591 leaves):

$$\frac{1}{2520 b^{10}} d x \left( -2520 a^9 d^9 + 1260 a^8 b d^8 (20 c + d x) - 840 a^7 b^2 d^7 (135 c^2 + 15 c d x + d^2 x^2) + 210 a^6 b^3 d^6 (1440 c^3 + 270 c^2 d x + 40 c d^2 x^2 + 3 d^3 x^3) - 252 a^5 b^4 d^5 (2100 c^4 + 600 c^3 d x + 150 c^2 d^2 x^2 + 25 c d^3 x^3 + 2 d^4 x^4) + 210 a^4 b^5 d^4 (3024 c^5 + 1260 c^4 d x + 480 c^3 d^2 x^2 + 135 c^2 d^3 x^3 + 24 c d^4 x^4 + 2 d^5 x^5) - 120 a^3 b^6 d^3 (4410 c^6 + 2646 c^5 d x + 1470 c^4 d^2 x^2 + 630 c^3 d^3 x^3 + 189 c^2 d^4 x^4 + 35 c d^5 x^5 + 3 d^6 x^6) + 45 a^2 b^7 d^2 (6720 c^7 + 5880 c^6 d x + 4704 c^5 d^2 x^2 + 2940 c^4 d^3 x^3 + 1344 c^3 d^4 x^4 + 420 c^2 d^5 x^5 + 80 c d^6 x^6 + 7 d^7 x^7) - 10 a b^8 d (11340 c^8 + 15120 c^7 d x + 17640 c^6 d^2 x^2 + 15876 c^5 d^3 x^3 + 10584 c^4 d^4 x^4 + 5040 c^3 d^5 x^5 + 1620 c^2 d^6 x^6 + 315 c d^7 x^7 + 28 d^8 x^8) + b^9 (25200 c^9 + 56700 c^8 d x + 100800 c^7 d^2 x^2 + 132300 c^6 d^3 x^3 + 127008 c^5 d^4 x^4 + 88200 c^4 d^5 x^5 + 43200 c^3 d^6 x^6 + 14175 c^2 d^7 x^7 + 2800 c d^8 x^8 + 252 d^9 x^9) \right) + \frac{(b c - a d)^{10} \operatorname{Log}[a + b x]}{b^{11}}$$

Problem 1313: Result more than twice size of optimal antiderivative.

$$\int \frac{(c + d x)^{10}}{(a + b x)^2} dx$$

Optimal (type 3, 258 leaves, 2 steps):

$$\begin{aligned} & \frac{45 d^2 (b c - a d)^8 x}{b^{10}} - \frac{(b c - a d)^{10}}{b^{11} (a + b x)} + \frac{60 d^3 (b c - a d)^7 (a + b x)^2}{b^{11}} + \frac{70 d^4 (b c - a d)^6 (a + b x)^3}{b^{11}} + \\ & \frac{63 d^5 (b c - a d)^5 (a + b x)^4}{b^{11}} + \frac{42 d^6 (b c - a d)^4 (a + b x)^5}{b^{11}} + \frac{20 d^7 (b c - a d)^3 (a + b x)^6}{b^{11}} + \\ & \frac{45 d^8 (b c - a d)^2 (a + b x)^7}{7 b^{11}} + \frac{5 d^9 (b c - a d) (a + b x)^8}{4 b^{11}} + \frac{d^{10} (a + b x)^9}{9 b^{11}} + \frac{10 d (b c - a d)^9 \operatorname{Log}[a + b x]}{b^{11}} \end{aligned}$$

Result (type 3, 708 leaves):

$$\begin{aligned} & \frac{1}{252 b^{11} (a + b x)} \left( -252 a^{10} d^{10} + 252 a^9 b d^9 (10 c + 9 d x) + 1260 a^8 b^2 d^8 (-9 c^2 - 16 c d x + d^2 x^2) - 420 a^7 b^3 d^7 (-72 c^3 - 189 c^2 d x + 27 c d^2 x^2 + d^3 x^3) + \right. \\ & 210 a^6 b^4 d^6 (-252 c^4 - 864 c^3 d x + 216 c^2 d^2 x^2 + 18 c d^3 x^3 + d^4 x^4) - 126 a^5 b^5 d^5 (-504 c^5 - 2100 c^4 d x + 840 c^3 d^2 x^2 + 120 c^2 d^3 x^3 + 15 c d^4 x^4 + d^5 x^5) + \\ & 42 a^4 b^6 d^4 (-1260 c^6 - 6048 c^5 d x + 3780 c^4 d^2 x^2 + 840 c^3 d^3 x^3 + 180 c^2 d^4 x^4 + 27 c d^5 x^5 + 2 d^6 x^6) - \\ & 12 a^3 b^7 d^3 (-2520 c^7 - 13230 c^6 d x + 13230 c^5 d^2 x^2 + 4410 c^4 d^3 x^3 + 1470 c^3 d^4 x^4 + 378 c^2 d^5 x^5 + 63 c d^6 x^6 + 5 d^7 x^7) + \\ & 9 a^2 b^8 d^2 (-1260 c^8 - 6720 c^7 d x + 11760 c^6 d^2 x^2 + 5880 c^5 d^3 x^3 + 2940 c^4 d^4 x^4 + 1176 c^3 d^5 x^5 + 336 c^2 d^6 x^6 + 60 c d^7 x^7 + 5 d^8 x^8) - \\ & a b^9 d (-2520 c^9 - 11340 c^8 d x + 45360 c^7 d^2 x^2 + 35280 c^6 d^3 x^3 + 26460 c^5 d^4 x^4 + 15876 c^4 d^5 x^5 + 7056 c^3 d^6 x^6 + 2160 c^2 d^7 x^7 + 405 c d^8 x^8 + 35 d^9 x^9) + \\ & b^{10} (-252 c^{10} + 11340 c^8 d^2 x^2 + 15120 c^7 d^3 x^3 + 17640 c^6 d^4 x^4 + 15876 c^5 d^5 x^5 + 10584 c^4 d^6 x^6 + 5040 c^3 d^7 x^7 + 1620 c^2 d^8 x^8 + 315 c d^9 x^9 + 28 d^{10} x^{10}) - \\ & \left. 2520 d (-b c + a d)^9 (a + b x) \operatorname{Log}[a + b x] \right) \end{aligned}$$

### Problem 1314: Result more than twice size of optimal antiderivative.

$$\int \frac{(c + d x)^{10}}{(a + b x)^3} dx$$

Optimal (type 3, 262 leaves, 2 steps):

$$\begin{aligned} & \frac{120 d^3 (b c - a d)^7 x}{b^{10}} - \frac{(b c - a d)^{10}}{2 b^{11} (a + b x)^2} - \frac{10 d (b c - a d)^9}{b^{11} (a + b x)} + \frac{105 d^4 (b c - a d)^6 (a + b x)^2}{b^{11}} + \frac{84 d^5 (b c - a d)^5 (a + b x)^3}{b^{11}} + \frac{105 d^6 (b c - a d)^4 (a + b x)^4}{2 b^{11}} + \\ & \frac{24 d^7 (b c - a d)^3 (a + b x)^5}{b^{11}} + \frac{15 d^8 (b c - a d)^2 (a + b x)^6}{2 b^{11}} + \frac{10 d^9 (b c - a d) (a + b x)^7}{7 b^{11}} + \frac{d^{10} (a + b x)^8}{8 b^{11}} + \frac{45 d^2 (b c - a d)^8 \log[a + b x]}{b^{11}} \end{aligned}$$

Result (type 3, 708 leaves):

$$\begin{aligned} & \frac{1}{56 b^{11} (a + b x)^2} \left( 532 a^{10} d^{10} - 56 a^9 b d^9 (85 c + 26 d x) + 28 a^8 b^2 d^8 (675 c^2 + 380 c d x - 116 d^2 x^2) - 280 a^7 b^3 d^7 (156 c^3 + 117 c^2 d x - 91 c d^2 x^2 + 3 d^3 x^3) + \right. \\ & 210 a^6 b^4 d^6 (308 c^4 + 256 c^3 d x - 414 c^2 d^2 x^2 + 32 c d^3 x^3 + d^4 x^4) - 84 a^5 b^5 d^5 (756 c^5 + 560 c^4 d x - 2000 c^3 d^2 x^2 + 280 c^2 d^3 x^3 + 20 c d^4 x^4 + d^5 x^5) + \\ & 42 a^4 b^6 d^4 (980 c^6 + 336 c^5 d x - 4760 c^4 d^2 x^2 + 1120 c^3 d^3 x^3 + 140 c^2 d^4 x^4 + 16 c d^5 x^5 + d^6 x^6) - \\ & 24 a^3 b^7 d^3 (700 c^7 - 490 c^6 d x - 6174 c^5 d^2 x^2 + 2450 c^4 d^3 x^3 + 490 c^3 d^4 x^4 + 98 c^2 d^5 x^5 + 14 c d^6 x^6 + d^7 x^7) + \\ & 3 a^2 b^8 d^2 (1260 c^8 - 4480 c^7 d x - 21560 c^6 d^2 x^2 + 15680 c^5 d^3 x^3 + 4900 c^4 d^4 x^4 + 1568 c^3 d^5 x^5 + 392 c^2 d^6 x^6 + 64 c d^7 x^7 + 5 d^8 x^8) - \\ & 2 a b^9 d (140 c^9 - 2520 c^8 d x - 6720 c^7 d^2 x^2 + 11760 c^6 d^3 x^3 + 5880 c^5 d^4 x^4 + 2940 c^4 d^5 x^5 + 1176 c^3 d^6 x^6 + 336 c^2 d^7 x^7 + 60 c d^8 x^8 + 5 d^9 x^9) + \\ & b^{10} (-28 c^{10} - 560 c^9 d x + 6720 c^7 d^3 x^3 + 5880 c^6 d^4 x^4 + 4704 c^5 d^5 x^5 + 2940 c^4 d^6 x^6 + 1344 c^3 d^7 x^7 + 420 c^2 d^8 x^8 + 80 c d^9 x^9 + 7 d^{10} x^{10}) + \\ & \left. 2520 d^2 (b c - a d)^8 (a + b x)^2 \log[a + b x] \right) \end{aligned}$$

### Problem 1320: Result more than twice size of optimal antiderivative.

$$\int \frac{(c + d x)^{10}}{(a + b x)^9} dx$$

Optimal (type 3, 258 leaves, 2 steps):

$$\begin{aligned} & \frac{d^9 (10 b c - 9 a d) x}{b^{10}} + \frac{d^{10} x^2}{2 b^9} - \frac{(b c - a d)^{10}}{8 b^{11} (a + b x)^8} - \frac{10 d (b c - a d)^9}{7 b^{11} (a + b x)^7} - \frac{15 d^2 (b c - a d)^8}{2 b^{11} (a + b x)^6} - \frac{24 d^3 (b c - a d)^7}{b^{11} (a + b x)^5} - \\ & \frac{105 d^4 (b c - a d)^6}{2 b^{11} (a + b x)^4} - \frac{84 d^5 (b c - a d)^5}{b^{11} (a + b x)^3} - \frac{105 d^6 (b c - a d)^4}{b^{11} (a + b x)^2} - \frac{120 d^7 (b c - a d)^3}{b^{11} (a + b x)} + \frac{45 d^8 (b c - a d)^2 \log[a + b x]}{b^{11}} \end{aligned}$$

Result (type 3, 712 leaves):

$$\frac{1}{56 b^{11} (a + b x)^8} \left( 3601 a^{10} d^{10} + 2 a^9 b d^9 (-4609 c + 13144 d x) + a^8 b^2 d^8 (6849 c^2 - 68704 c d x + 81928 d^2 x^2) + 8 a^7 b^3 d^7 (-105 c^3 + 6534 c^2 d x - 27538 c d^2 x^2 + 17542 d^3 x^3) + 14 a^6 b^4 d^6 (-15 c^4 - 480 c^3 d x + 12348 c^2 d^2 x^2 - 28112 c d^3 x^3 + 10010 d^4 x^4) - 28 a^5 b^5 d^5 (3 c^5 + 60 c^4 d x + 840 c^3 d^2 x^2 - 11508 c^2 d^3 x^3 + 15050 c d^4 x^4 - 2744 d^5 x^5) - 14 a^4 b^6 d^4 (3 c^6 + 48 c^5 d x + 420 c^4 d^2 x^2 + 3360 c^3 d^3 x^3 - 26250 c^2 d^4 x^4 + 19040 c d^5 x^5 - 1064 d^6 x^6) - 8 a^3 b^7 d^3 (3 c^7 + 42 c^6 d x + 294 c^5 d^2 x^2 + 1470 c^4 d^3 x^3 + 7350 c^3 d^4 x^4 - 32340 c^2 d^5 x^5 + 10780 c d^6 x^6 + 728 d^7 x^7) - a^2 b^8 d^2 (15 c^8 + 192 c^7 d x + 1176 c^6 d^2 x^2 + 4704 c^5 d^3 x^3 + 14700 c^4 d^4 x^4 + 47040 c^3 d^5 x^5 - 105840 c^2 d^6 x^6 + 4480 c d^7 x^7 + 3248 d^8 x^8) - 2 a b^9 d (5 c^9 + 60 c^8 d x + 336 c^7 d^2 x^2 + 1176 c^6 d^3 x^3 + 2940 c^5 d^4 x^4 + 5880 c^4 d^5 x^5 + 11760 c^3 d^6 x^6 - 10080 c^2 d^7 x^7 - 2240 c d^8 x^8 + 140 d^9 x^9) - b^{10} (7 c^{10} + 80 c^9 d x + 420 c^8 d^2 x^2 + 1344 c^7 d^3 x^3 + 2940 c^6 d^4 x^4 + 4704 c^5 d^5 x^5 + 5880 c^4 d^6 x^6 + 6720 c^3 d^7 x^7 - 560 c d^9 x^9 - 28 d^{10} x^{10}) + 2520 d^8 (b c - a d)^2 (a + b x)^8 \text{Log}[a + b x] \right)$$

**Problem 1321: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c + d x)^{10}}{(a + b x)^{10}} dx$$

Optimal (type 3, 257 leaves, 2 steps):

$$\begin{aligned} & \frac{d^{10} x}{b^{10}} - \frac{(b c - a d)^{10}}{9 b^{11} (a + b x)^9} - \frac{5 d (b c - a d)^9}{4 b^{11} (a + b x)^8} - \frac{45 d^2 (b c - a d)^8}{7 b^{11} (a + b x)^7} - \frac{20 d^3 (b c - a d)^7}{b^{11} (a + b x)^6} - \frac{42 d^4 (b c - a d)^6}{b^{11} (a + b x)^5} - \\ & \frac{63 d^5 (b c - a d)^5}{b^{11} (a + b x)^4} - \frac{70 d^6 (b c - a d)^4}{b^{11} (a + b x)^3} - \frac{60 d^7 (b c - a d)^3}{b^{11} (a + b x)^2} - \frac{45 d^8 (b c - a d)^2}{b^{11} (a + b x)} + \frac{10 d^9 (b c - a d) \text{Log}[a + b x]}{b^{11}} \end{aligned}$$

Result (type 3, 708 leaves):

$$\begin{aligned} & -\frac{1}{252 b^{11} (a + b x)^9} \left( 4861 a^{10} d^{10} + a^9 b d^9 (-7129 c + 41229 d x) + 9 a^8 b^2 d^8 (140 c^2 - 6849 c d x + 17064 d^2 x^2) + 12 a^7 b^3 d^7 (35 c^3 + 945 c^2 d x - 19602 c d^2 x^2 + 27342 d^3 x^3) + 42 a^6 b^4 d^6 (5 c^4 + 90 c^3 d x + 1080 c^2 d^2 x^2 - 12348 c d^3 x^3 + 10458 d^4 x^4) + 126 a^5 b^5 d^5 (c^5 + 15 c^4 d x + 120 c^3 d^2 x^2 + 840 c^2 d^3 x^3 - 5754 c d^4 x^4 + 2982 d^5 x^5) + 42 a^4 b^6 d^4 (2 c^6 + 27 c^5 d x + 180 c^4 d^2 x^2 + 840 c^3 d^3 x^3 + 3780 c^2 d^4 x^4 - 15750 c d^5 x^5 + 4704 d^6 x^6) + 12 a^3 b^7 d^3 (5 c^7 + 63 c^6 d x + 378 c^5 d^2 x^2 + 1470 c^4 d^3 x^3 + 4410 c^3 d^4 x^4 + 13230 c^2 d^5 x^5 - 32340 c d^6 x^6 + 4536 d^7 x^7) + 9 a^2 b^8 d^2 (5 c^8 + 60 c^7 d x + 336 c^6 d^2 x^2 + 1176 c^5 d^3 x^3 + 2940 c^4 d^4 x^4 + 5880 c^3 d^5 x^5 + 11760 c^2 d^6 x^6 - 15120 c d^7 x^7 + 252 d^8 x^8) + a b^9 d (35 c^9 + 405 c^8 d x + 2160 c^7 d^2 x^2 + 7056 c^6 d^3 x^3 + 15876 c^5 d^4 x^4 + 26460 c^4 d^5 x^5 + 35280 c^3 d^6 x^6 + 45360 c^2 d^7 x^7 - 22680 c d^8 x^8 - 2268 d^9 x^9) + b^{10} (28 c^{10} + 315 c^9 d x + 1620 c^8 d^2 x^2 + 5040 c^7 d^3 x^3 + 10584 c^6 d^4 x^4 + 15876 c^5 d^5 x^5 + 17640 c^4 d^6 x^6 + 15120 c^3 d^7 x^7 + 11340 c^2 d^8 x^8 - 252 d^{10} x^{10}) + 2520 d^9 (-b c + a d) (a + b x)^9 \text{Log}[a + b x] \right) \end{aligned}$$

### Problem 1322: Result more than twice size of optimal antiderivative.

$$\int \frac{(c + d x)^{10}}{(a + b x)^{11}} dx$$

Optimal (type 3, 271 leaves, 2 steps):

$$\begin{aligned} & -\frac{(b c - a d)^{10}}{10 b^{11} (a + b x)^{10}} - \frac{10 d (b c - a d)^9}{9 b^{11} (a + b x)^9} - \frac{45 d^2 (b c - a d)^8}{8 b^{11} (a + b x)^8} - \frac{120 d^3 (b c - a d)^7}{7 b^{11} (a + b x)^7} - \frac{35 d^4 (b c - a d)^6}{b^{11} (a + b x)^6} - \\ & \frac{252 d^5 (b c - a d)^5}{5 b^{11} (a + b x)^5} - \frac{105 d^6 (b c - a d)^4}{2 b^{11} (a + b x)^4} - \frac{40 d^7 (b c - a d)^3}{b^{11} (a + b x)^3} - \frac{45 d^8 (b c - a d)^2}{2 b^{11} (a + b x)^2} - \frac{10 d^9 (b c - a d)}{b^{11} (a + b x)} + \frac{d^{10} \log[a + b x]}{b^{11}} \end{aligned}$$

Result (type 3, 591 leaves):

$$\begin{aligned} & -\frac{1}{2520 b^{11} (a + b x)^{10}} (b c - a d) (7381 a^9 d^9 + a^8 b d^8 (4861 c + 71290 d x) + a^7 b^2 d^7 (3601 c^2 + 46090 c d x + 308205 d^2 x^2) + a^6 b^3 d^6 \\ & (2761 c^3 + 33490 c^2 d x + 194805 c d^2 x^2 + 784080 d^3 x^3) + a^5 b^4 d^5 (2131 c^4 + 25090 c^3 d x + 138105 c^2 d^2 x^2 + 481680 c d^3 x^3 + 1296540 d^4 x^4) + \\ & a^4 b^5 d^4 (1627 c^5 + 18790 c^4 d x + 100305 c^3 d^2 x^2 + 330480 c^2 d^3 x^3 + 767340 c d^4 x^4 + 1450008 d^5 x^5) + \\ & a^3 b^6 d^3 (1207 c^6 + 13750 c^5 d x + 71955 c^4 d^2 x^2 + 229680 c^3 d^3 x^3 + 502740 c^2 d^4 x^4 + 814968 c d^5 x^5 + 1102500 d^6 x^6) + \\ & a^2 b^7 d^2 (847 c^7 + 9550 c^6 d x + 49275 c^5 d^2 x^2 + 154080 c^4 d^3 x^3 + 326340 c^3 d^4 x^4 + 497448 c^2 d^5 x^5 + 573300 c d^6 x^6 + 554400 d^7 x^7) + \\ & a b^8 d (532 c^8 + 5950 c^7 d x + 30375 c^6 d^2 x^2 + 93600 c^5 d^3 x^3 + 194040 c^4 d^4 x^4 + 285768 c^3 d^5 x^5 + 308700 c^2 d^6 x^6 + 252000 c d^7 x^7 + 170100 d^8 x^8) + \\ & b^9 (252 c^9 + 2800 c^8 d x + 14175 c^7 d^2 x^2 + 43200 c^6 d^3 x^3 + 88200 c^5 d^4 x^4 + 127008 c^4 d^5 x^5 + \\ & 132300 c^3 d^6 x^6 + 100800 c^2 d^7 x^7 + 56700 c d^8 x^8 + 25200 d^9 x^9) ) + \frac{d^{10} \log[a + b x]}{b^{11}} \end{aligned}$$

### Problem 1323: Result more than twice size of optimal antiderivative.

$$\int \frac{(c + d x)^{10}}{(a + b x)^{12}} dx$$

Optimal (type 1, 28 leaves, 1 step):

$$-\frac{(c + d x)^{11}}{11 (b c - a d) (a + b x)^{11}}$$

Result (type 1, 665 leaves):

$$\begin{aligned}
& - \frac{1}{11 b^{11} (a + b x)^{11}} (a^{10} d^{10} + a^9 b d^9 (c + 11 d x) + a^8 b^2 d^8 (c^2 + 11 c d x + 55 d^2 x^2) + a^7 b^3 d^7 (c^3 + 11 c^2 d x + 55 c d^2 x^2 + 165 d^3 x^3) + \\
& \quad a^6 b^4 d^6 (c^4 + 11 c^3 d x + 55 c^2 d^2 x^2 + 165 c d^3 x^3 + 330 d^4 x^4) + a^5 b^5 d^5 (c^5 + 11 c^4 d x + 55 c^3 d^2 x^2 + 165 c^2 d^3 x^3 + 330 c d^4 x^4 + 462 d^5 x^5) + \\
& \quad a^4 b^6 d^4 (c^6 + 11 c^5 d x + 55 c^4 d^2 x^2 + 165 c^3 d^3 x^3 + 330 c^2 d^4 x^4 + 462 c d^5 x^5 + 462 d^6 x^6) + \\
& \quad a^3 b^7 d^3 (c^7 + 11 c^6 d x + 55 c^5 d^2 x^2 + 165 c^4 d^3 x^3 + 330 c^3 d^4 x^4 + 462 c^2 d^5 x^5 + 462 c d^6 x^6 + 330 d^7 x^7) + \\
& \quad a^2 b^8 d^2 (c^8 + 11 c^7 d x + 55 c^6 d^2 x^2 + 165 c^5 d^3 x^3 + 330 c^4 d^4 x^4 + 462 c^3 d^5 x^5 + 462 c^2 d^6 x^6 + 330 c d^7 x^7 + 165 d^8 x^8) + \\
& \quad a^1 b^9 d (c^9 + 11 c^8 d x + 55 c^7 d^2 x^2 + 165 c^6 d^3 x^3 + 330 c^5 d^4 x^4 + 462 c^4 d^5 x^5 + 462 c^3 d^6 x^6 + 330 c^2 d^7 x^7 + 165 c d^8 x^8 + 55 d^9 x^9) + \\
& \quad b^{10} (c^{10} + 11 c^9 d x + 55 c^8 d^2 x^2 + 165 c^7 d^3 x^3 + 330 c^6 d^4 x^4 + 462 c^5 d^5 x^5 + 462 c^4 d^6 x^6 + 330 c^3 d^7 x^7 + 165 c^2 d^8 x^8 + 55 c d^9 x^9 + 11 d^{10} x^{10})
\end{aligned}$$

**Problem 1324:** Result more than twice size of optimal antiderivative.

$$\int \frac{(c + d x)^{10}}{(a + b x)^{13}} dx$$

Optimal (type 1, 58 leaves, 2 steps):

$$\begin{aligned}
& - \frac{(c + d x)^{11}}{12 (b c - a d) (a + b x)^{12}} + \frac{d (c + d x)^{11}}{132 (b c - a d)^2 (a + b x)^{11}}
\end{aligned}$$

Result (type 1, 684 leaves):

$$\begin{aligned}
& - \frac{1}{132 b^{11} (a + b x)^{12}} (a^{10} d^{10} + 2 a^9 b d^9 (c + 6 d x) + 3 a^8 b^2 d^8 (c^2 + 8 c d x + 22 d^2 x^2) + 4 a^7 b^3 d^7 (c^3 + 9 c^2 d x + 33 c d^2 x^2 + 55 d^3 x^3) + \\
& \quad a^6 b^4 d^6 (5 c^4 + 48 c^3 d x + 198 c^2 d^2 x^2 + 440 c d^3 x^3 + 495 d^4 x^4) + 6 a^5 b^5 d^5 (c^5 + 10 c^4 d x + 44 c^3 d^2 x^2 + 110 c^2 d^3 x^3 + 165 c d^4 x^4 + 132 d^5 x^5) + \\
& \quad a^4 b^6 d^4 (7 c^6 + 72 c^5 d x + 330 c^4 d^2 x^2 + 880 c^3 d^3 x^3 + 1485 c^2 d^4 x^4 + 1584 c d^5 x^5 + 924 d^6 x^6) + \\
& \quad 4 a^3 b^7 d^3 (2 c^7 + 21 c^6 d x + 99 c^5 d^2 x^2 + 275 c^4 d^3 x^3 + 495 c^3 d^4 x^4 + 594 c^2 d^5 x^5 + 462 c d^6 x^6 + 198 d^7 x^7) + \\
& \quad 3 a^2 b^8 d^2 (3 c^8 + 32 c^7 d x + 154 c^6 d^2 x^2 + 440 c^5 d^3 x^3 + 825 c^4 d^4 x^4 + 1056 c^3 d^5 x^5 + 924 c^2 d^6 x^6 + 528 c d^7 x^7 + 165 d^8 x^8) + \\
& \quad 2 a b^9 d (5 c^9 + 54 c^8 d x + 264 c^7 d^2 x^2 + 770 c^6 d^3 x^3 + 1485 c^5 d^4 x^4 + 1980 c^4 d^5 x^5 + 1848 c^3 d^6 x^6 + 1188 c^2 d^7 x^7 + 495 c d^8 x^8 + 110 d^9 x^9) + b^{10} \\
& \quad (11 c^{10} + 120 c^9 d x + 594 c^8 d^2 x^2 + 1760 c^7 d^3 x^3 + 3465 c^6 d^4 x^4 + 4752 c^5 d^5 x^5 + 4620 c^4 d^6 x^6 + 3168 c^3 d^7 x^7 + 1485 c^2 d^8 x^8 + 440 c d^9 x^9 + 66 d^{10} x^{10})
\end{aligned}$$

**Problem 1325:** Result more than twice size of optimal antiderivative.

$$\int \frac{(c + d x)^{10}}{(a + b x)^{14}} dx$$

Optimal (type 1, 89 leaves, 3 steps):

$$\begin{aligned}
& - \frac{(c + d x)^{11}}{13 (b c - a d) (a + b x)^{13}} + \frac{d (c + d x)^{11}}{78 (b c - a d)^2 (a + b x)^{12}} - \frac{d^2 (c + d x)^{11}}{858 (b c - a d)^3 (a + b x)^{11}}
\end{aligned}$$

Result (type 1, 690 leaves):

$$\begin{aligned}
& - \frac{1}{858 b^{11} (a + b x)^{13}} (a^{10} d^{10} + a^9 b d^9 (3 c + 13 d x) + 3 a^8 b^2 d^8 (2 c^2 + 13 c d x + 26 d^2 x^2) + 2 a^7 b^3 d^7 (5 c^3 + 39 c^2 d x + 117 c d^2 x^2 + 143 d^3 x^3) + \\
& \quad a^6 b^4 d^6 (15 c^4 + 130 c^3 d x + 468 c^2 d^2 x^2 + 858 c d^3 x^3 + 715 d^4 x^4) + 3 a^5 b^5 d^5 (7 c^5 + 65 c^4 d x + 260 c^3 d^2 x^2 + 572 c^2 d^3 x^3 + 715 c d^4 x^4 + 429 d^5 x^5) + \\
& \quad a^4 b^6 d^4 (28 c^6 + 273 c^5 d x + 1170 c^4 d^2 x^2 + 2860 c^3 d^3 x^3 + 4290 c^2 d^4 x^4 + 3861 c d^5 x^5 + 1716 d^6 x^6) + \\
& \quad 2 a^3 b^7 d^3 (18 c^7 + 182 c^6 d x + 819 c^5 d^2 x^2 + 2145 c^4 d^3 x^3 + 3575 c^3 d^4 x^4 + 3861 c^2 d^5 x^5 + 2574 c d^6 x^6 + 858 d^7 x^7) + \\
& \quad 3 a^2 b^8 d^2 (15 c^8 + 156 c^7 d x + 728 c^6 d^2 x^2 + 2002 c^5 d^3 x^3 + 3575 c^4 d^4 x^4 + 4290 c^3 d^5 x^5 + 3432 c^2 d^6 x^6 + 1716 c d^7 x^7 + 429 d^8 x^8) + \\
& \quad a b^9 d (55 c^9 + 585 c^8 d x + 2808 c^7 d^2 x^2 + 8008 c^6 d^3 x^3 + 15015 c^5 d^4 x^4 + 19305 c^4 d^5 x^5 + 17160 c^3 d^6 x^6 + 10296 c^2 d^7 x^7 + 3861 c d^8 x^8 + 715 d^9 x^9) + \\
& \quad b^{10} (66 c^{10} + 715 c^9 d x + 3510 c^8 d^2 x^2 + 10296 c^7 d^3 x^3 + 20020 c^6 d^4 x^4 + \\
& \quad 27027 c^5 d^5 x^5 + 25740 c^4 d^6 x^6 + 17160 c^3 d^7 x^7 + 7722 c^2 d^8 x^8 + 2145 c d^9 x^9 + 286 d^{10} x^{10}) )
\end{aligned}$$

**Problem 1326:** Result more than twice size of optimal antiderivative.

$$\int \frac{(c + d x)^{10}}{(a + b x)^{15}} dx$$

Optimal (type 1, 120 leaves, 4 steps):

$$\begin{aligned}
& - \frac{(c + d x)^{11}}{14 (b c - a d) (a + b x)^{14}} + \frac{3 d (c + d x)^{11}}{182 (b c - a d)^2 (a + b x)^{13}} - \frac{d^2 (c + d x)^{11}}{364 (b c - a d)^3 (a + b x)^{12}} + \frac{d^3 (c + d x)^{11}}{4004 (b c - a d)^4 (a + b x)^{11}}
\end{aligned}$$

Result (type 1, 692 leaves):

$$\begin{aligned}
& - \frac{1}{4004 b^{11} (a + b x)^{14}} (a^{10} d^{10} + 2 a^9 b d^9 (2 c + 7 d x) + a^8 b^2 d^8 (10 c^2 + 56 c d x + 91 d^2 x^2) + 4 a^7 b^3 d^7 (5 c^3 + 35 c^2 d x + 91 c d^2 x^2 + 91 d^3 x^3) + \\
& \quad 7 a^6 b^4 d^6 (5 c^4 + 40 c^3 d x + 130 c^2 d^2 x^2 + 208 c d^3 x^3 + 143 d^4 x^4) + 14 a^5 b^5 d^5 (4 c^5 + 35 c^4 d x + 130 c^3 d^2 x^2 + 260 c^2 d^3 x^3 + 286 c d^4 x^4 + 143 d^5 x^5) + \\
& \quad 7 a^4 b^6 d^4 (12 c^6 + 112 c^5 d x + 455 c^4 d^2 x^2 + 1040 c^3 d^3 x^3 + 1430 c^2 d^4 x^4 + 1144 c d^5 x^5 + 429 d^6 x^6) + \\
& \quad 4 a^3 b^7 d^3 (30 c^7 + 294 c^6 d x + 1274 c^5 d^2 x^2 + 3185 c^4 d^3 x^3 + 5005 c^3 d^4 x^4 + 5005 c^2 d^5 x^5 + 3003 c d^6 x^6 + 858 d^7 x^7) + \\
& \quad a^2 b^8 d^2 (165 c^8 + 1680 c^7 d x + 7644 c^6 d^2 x^2 + 20384 c^5 d^3 x^3 + 35035 c^4 d^4 x^4 + 40040 c^3 d^5 x^5 + 30030 c^2 d^6 x^6 + 13728 c d^7 x^7 + 3003 d^8 x^8) + 2 a b^9 d \\
& \quad (110 c^9 + 1155 c^8 d x + 5460 c^7 d^2 x^2 + 15288 c^6 d^3 x^3 + 28028 c^5 d^4 x^4 + 35035 c^4 d^5 x^5 + 30030 c^3 d^6 x^6 + 17160 c^2 d^7 x^7 + 6006 c d^8 x^8 + 1001 d^9 x^9) + \\
& \quad b^{10} (286 c^{10} + 3080 c^9 d x + 15015 c^8 d^2 x^2 + 43680 c^7 d^3 x^3 + 84084 c^6 d^4 x^4 + 112112 c^5 d^5 x^5 + \\
& \quad 105105 c^4 d^6 x^6 + 68640 c^3 d^7 x^7 + 30030 c^2 d^8 x^8 + 8008 c d^9 x^9 + 1001 d^{10} x^{10}) )
\end{aligned}$$

**Problem 1327:** Result more than twice size of optimal antiderivative.

$$\int \frac{(c + d x)^{10}}{(a + b x)^{16}} dx$$

Optimal (type 1, 151 leaves, 5 steps):

$$-\frac{(c + d x)^{11}}{15 (b c - a d) (a + b x)^{15}} + \frac{2 d (c + d x)^{11}}{105 (b c - a d)^2 (a + b x)^{14}} - \frac{2 d^2 (c + d x)^{11}}{455 (b c - a d)^3 (a + b x)^{13}} + \frac{d^3 (c + d x)^{11}}{1365 (b c - a d)^4 (a + b x)^{12}} - \frac{d^4 (c + d x)^{11}}{15015 (b c - a d)^5 (a + b x)^{11}}$$

Result (type 1, 690 leaves):

$$\begin{aligned} & -\frac{1}{15015 b^{11} (a + b x)^{15}} (a^{10} d^{10} + 5 a^9 b d^9 (c + 3 d x) + 15 a^8 b^2 d^8 (c^2 + 5 c d x + 7 d^2 x^2) + 5 a^7 b^3 d^7 (7 c^3 + 45 c^2 d x + 105 c d^2 x^2 + 91 d^3 x^3) + \\ & 35 a^6 b^4 d^6 (2 c^4 + 15 c^3 d x + 45 c^2 d^2 x^2 + 65 c d^3 x^3 + 39 d^4 x^4) + 21 a^5 b^5 d^5 (6 c^5 + 50 c^4 d x + 175 c^3 d^2 x^2 + 325 c^2 d^3 x^3 + 325 c d^4 x^4 + 143 d^5 x^5) + \\ & 35 a^4 b^6 d^4 (6 c^6 + 54 c^5 d x + 210 c^4 d^2 x^2 + 455 c^3 d^3 x^3 + 585 c^2 d^4 x^4 + 429 c d^5 x^5 + 143 d^6 x^6) + \\ & 5 a^3 b^7 d^3 (66 c^7 + 630 c^6 d x + 2646 c^5 d^2 x^2 + 6370 c^4 d^3 x^3 + 9555 c^3 d^4 x^4 + 9009 c^2 d^5 x^5 + 5005 c d^6 x^6 + 1287 d^7 x^7) + \\ & 15 a^2 b^8 d^2 (33 c^8 + 330 c^7 d x + 1470 c^6 d^2 x^2 + 3822 c^5 d^3 x^3 + 6370 c^4 d^4 x^4 + 7007 c^3 d^5 x^5 + 5005 c^2 d^6 x^6 + 2145 c d^7 x^7 + 429 d^8 x^8) + 5 a b^9 d \\ & (143 c^9 + 1485 c^8 d x + 6930 c^7 d^2 x^2 + 19110 c^6 d^3 x^3 + 34398 c^5 d^4 x^4 + 42042 c^4 d^5 x^5 + 35035 c^3 d^6 x^6 + 19305 c^2 d^7 x^7 + 6435 c d^8 x^8 + 1001 d^9 x^9) + \\ & b^{10} (1001 c^{10} + 10725 c^9 d x + 51975 c^8 d^2 x^2 + 150150 c^7 d^3 x^3 + 286650 c^6 d^4 x^4 + 378378 c^5 d^5 x^5 + \\ & 350350 c^4 d^6 x^6 + 225225 c^3 d^7 x^7 + 96525 c^2 d^8 x^8 + 25025 c d^9 x^9 + 3003 d^{10} x^{10}) ) \end{aligned}$$

### Problem 1328: Result more than twice size of optimal antiderivative.

$$\int \frac{(c + d x)^{10}}{(a + b x)^{17}} dx$$

Optimal (type 1, 182 leaves, 6 steps):

$$\begin{aligned} & -\frac{(c + d x)^{11}}{16 (b c - a d) (a + b x)^{16}} + \frac{d (c + d x)^{11}}{48 (b c - a d)^2 (a + b x)^{15}} - \frac{d^2 (c + d x)^{11}}{168 (b c - a d)^3 (a + b x)^{14}} + \\ & \frac{d^3 (c + d x)^{11}}{728 (b c - a d)^4 (a + b x)^{13}} - \frac{d^4 (c + d x)^{11}}{4368 (b c - a d)^5 (a + b x)^{12}} + \frac{d^5 (c + d x)^{11}}{48048 (b c - a d)^6 (a + b x)^{11}} \end{aligned}$$

Result (type 1, 694 leaves):

$$\begin{aligned} & -\frac{1}{48048 b^{11} (a + b x)^{16}} (a^{10} d^{10} + 2 a^9 b d^9 (3 c + 8 d x) + 3 a^8 b^2 d^8 (7 c^2 + 32 c d x + 40 d^2 x^2) + 8 a^7 b^3 d^7 (7 c^3 + 42 c^2 d x + 90 c d^2 x^2 + 70 d^3 x^3) + \\ & 14 a^6 b^4 d^6 (9 c^4 + 64 c^3 d x + 180 c^2 d^2 x^2 + 240 c d^3 x^3 + 130 d^4 x^4) + 84 a^5 b^5 d^5 (3 c^5 + 24 c^4 d x + 80 c^3 d^2 x^2 + 140 c^2 d^3 x^3 + 130 c d^4 x^4 + 52 d^5 x^5) + \\ & 14 a^4 b^6 d^4 (33 c^6 + 288 c^5 d x + 1080 c^4 d^2 x^2 + 2240 c^3 d^3 x^3 + 2730 c^2 d^4 x^4 + 1872 c d^5 x^5 + 572 d^6 x^6) + \\ & 8 a^3 b^7 d^3 (99 c^7 + 924 c^6 d x + 3780 c^5 d^2 x^2 + 8820 c^4 d^3 x^3 + 12740 c^3 d^4 x^4 + 11466 c^2 d^5 x^5 + 6006 c d^6 x^6 + 1430 d^7 x^7) + \\ & 3 a^2 b^8 d^2 (429 c^8 + 4224 c^7 d x + 18480 c^6 d^2 x^2 + 47040 c^5 d^3 x^3 + 76440 c^4 d^4 x^4 + 81536 c^3 d^5 x^5 + 56056 c^2 d^6 x^6 + 22880 c d^7 x^7 + 4290 d^8 x^8) + \\ & 2 a b^9 d (1001 c^9 + 10296 c^8 d x + 47520 c^7 d^2 x^2 + 129360 c^6 d^3 x^3 + 229320 c^5 d^4 x^4 + 275184 c^4 d^5 x^5 + 224224 c^3 d^6 x^6 + \\ & 120120 c^2 d^7 x^7 + 38610 c d^8 x^8 + 5720 d^9 x^9) + b^{10} (3003 c^{10} + 32032 c^9 d x + 154440 c^8 d^2 x^2 + 443520 c^7 d^3 x^3 + \\ & 840840 c^6 d^4 x^4 + 1100736 c^5 d^5 x^5 + 1009008 c^4 d^6 x^6 + 640640 c^3 d^7 x^7 + 270270 c^2 d^8 x^8 + 68640 c d^9 x^9 + 8008 d^{10} x^{10}) ) \end{aligned}$$

### Problem 1329: Result more than twice size of optimal antiderivative.

$$\int \frac{(c + dx)^{10}}{(a + bx)^{18}} dx$$

Optimal (type 1, 213 leaves, 7 steps):

$$\begin{aligned} & -\frac{(c + dx)^{11}}{17(b c - a d)(a + b x)^{17}} + \frac{3d(c + dx)^{11}}{136(b c - a d)^2(a + b x)^{16}} - \frac{d^2(c + dx)^{11}}{136(b c - a d)^3(a + b x)^{15}} + \\ & \frac{d^3(c + dx)^{11}}{476(b c - a d)^4(a + b x)^{14}} - \frac{3d^4(c + dx)^{11}}{6188(b c - a d)^5(a + b x)^{13}} + \frac{d^5(c + dx)^{11}}{12376(b c - a d)^6(a + b x)^{12}} - \frac{d^6(c + dx)^{11}}{136136(b c - a d)^7(a + b x)^{11}} \end{aligned}$$

Result (type 1, 690 leaves):

$$\begin{aligned} & -\frac{1}{136136 b^{11} (a + b x)^{17}} \\ & (a^{10} d^{10} + a^9 b d^9 (7c + 17d x) + a^8 b^2 d^8 (28c^2 + 119c d x + 136d^2 x^2) + 4a^7 b^3 d^7 (21c^3 + 119c^2 d x + 238c d^2 x^2 + 170d^3 x^3) + 14a^6 b^4 d^6 \\ & (15c^4 + 102c^3 d x + 272c^2 d^2 x^2 + 340c d^3 x^3 + 170d^4 x^4) + 14a^5 b^5 d^5 (33c^5 + 255c^4 d x + 816c^3 d^2 x^2 + 1360c^2 d^3 x^3 + 1190c d^4 x^4 + 442d^5 x^5) + \\ & 14a^4 b^6 d^4 (66c^6 + 561c^5 d x + 2040c^4 d^2 x^2 + 4080c^3 d^3 x^3 + 4760c^2 d^4 x^4 + 3094c d^5 x^5 + 884d^6 x^6) + \\ & 4a^3 b^7 d^3 (429c^7 + 3927c^6 d x + 15708c^5 d^2 x^2 + 35700c^4 d^3 x^3 + 49980c^3 d^4 x^4 + 43316c^2 d^5 x^5 + 21658c d^6 x^6 + 4862d^7 x^7) + a^2 b^8 d^2 \\ & (3003c^8 + 29172c^7 d x + 125664c^6 d^2 x^2 + 314160c^5 d^3 x^3 + 499800c^4 d^4 x^4 + 519792c^3 d^5 x^5 + 346528c^2 d^6 x^6 + 136136c d^7 x^7 + 24310d^8 x^8) + \\ & a b^9 d (5005c^9 + 51051c^8 d x + 233376c^7 d^2 x^2 + 628320c^6 d^3 x^3 + 1099560c^5 d^4 x^4 + 1299480c^4 d^5 x^5 + 1039584c^3 d^6 x^6 + \\ & 544544c^2 d^7 x^7 + 170170c d^8 x^8 + 24310d^9 x^9) + b^{10} (8008c^{10} + 85085c^9 d x + 408408c^8 d^2 x^2 + 1166880c^7 d^3 x^3 + \\ & 2199120c^6 d^4 x^4 + 2858856c^5 d^5 x^5 + 2598960c^4 d^6 x^6 + 1633632c^3 d^7 x^7 + 680680c^2 d^8 x^8 + 170170c d^9 x^9 + 19448d^{10} x^{10}) \end{aligned}$$

### Problem 1330: Result more than twice size of optimal antiderivative.

$$\int \frac{(c + dx)^{10}}{(a + bx)^{19}} dx$$

Optimal (type 1, 244 leaves, 8 steps):

$$\begin{aligned} & -\frac{(c + dx)^{11}}{18(b c - a d)(a + b x)^{18}} + \frac{7d(c + dx)^{11}}{306(b c - a d)^2(a + b x)^{17}} - \frac{7d^2(c + dx)^{11}}{816(b c - a d)^3(a + b x)^{16}} + \frac{7d^3(c + dx)^{11}}{2448(b c - a d)^4(a + b x)^{15}} - \\ & \frac{d^4(c + dx)^{11}}{1224(b c - a d)^5(a + b x)^{14}} + \frac{d^5(c + dx)^{11}}{5304(b c - a d)^6(a + b x)^{13}} - \frac{d^6(c + dx)^{11}}{31824(b c - a d)^7(a + b x)^{12}} + \frac{d^7(c + dx)^{11}}{350064(b c - a d)^8(a + b x)^{11}} \end{aligned}$$

Result (type 1, 694 leaves):

$$\begin{aligned}
& - \frac{1}{350064 b^{11} (a + b x)^{18}} \\
& \left( a^{10} d^{10} + 2 a^9 b d^9 (4 c + 9 d x) + 9 a^8 b^2 d^8 (4 c^2 + 16 c d x + 17 d^2 x^2) + 24 a^7 b^3 d^7 (5 c^3 + 27 c^2 d x + 51 c d^2 x^2 + 34 d^3 x^3) + 6 a^6 b^4 d^6 \right. \\
& \left( 55 c^4 + 360 c^3 d x + 918 c^2 d^2 x^2 + 1088 c d^3 x^3 + 510 d^4 x^4 \right) + 36 a^5 b^5 d^5 (22 c^5 + 165 c^4 d x + 510 c^3 d^2 x^2 + 816 c^2 d^3 x^3 + 680 c d^4 x^4 + 238 d^5 x^5) + \\
& 6 a^4 b^6 d^4 (286 c^6 + 2376 c^5 d x + 8415 c^4 d^2 x^2 + 16320 c^3 d^3 x^3 + 18360 c^2 d^4 x^4 + 11424 c d^5 x^5 + 3094 d^6 x^6) + \\
& 24 a^3 b^7 d^3 (143 c^7 + 1287 c^6 d x + 5049 c^5 d^2 x^2 + 11220 c^4 d^3 x^3 + 15300 c^3 d^4 x^4 + 12852 c^2 d^5 x^5 + 6188 c d^6 x^6 + 1326 d^7 x^7) + \\
& 9 a^2 b^8 d^2 (715 c^8 + 6864 c^7 d x + 29172 c^6 d^2 x^2 + 71808 c^5 d^3 x^3 + 112200 c^4 d^4 x^4 + 114240 c^3 d^5 x^5 + 74256 c^2 d^6 x^6 + 28288 c d^7 x^7 + 4862 d^8 x^8) + \\
& 2 a b^9 d (5720 c^9 + 57915 c^8 d x + 262548 c^7 d^2 x^2 + 700128 c^6 d^3 x^3 + 1211760 c^5 d^4 x^4 + 1413720 c^4 d^5 x^5 + 1113840 c^3 d^6 x^6 + \\
& 572832 c^2 d^7 x^7 + 175032 c d^8 x^8 + 24310 d^9 x^9) + b^{10} (19448 c^{10} + 205920 c^9 d x + 984555 c^8 d^2 x^2 + 2800512 c^7 d^3 x^3 + \\
& 5250960 c^6 d^4 x^4 + 6785856 c^5 d^5 x^5 + 6126120 c^4 d^6 x^6 + 3818880 c^3 d^7 x^7 + 1575288 c^2 d^8 x^8 + 388960 c d^9 x^9 + 43758 d^{10} x^{10})
\end{aligned}$$

**Problem 1331:** Result more than twice size of optimal antiderivative.

$$\int \frac{(c + d x)^{10}}{(a + b x)^{20}} dx$$

Optimal (type 1, 273 leaves, 2 steps):

$$\begin{aligned}
& \frac{(b c - a d)^{10}}{19 b^{11} (a + b x)^{19}} - \frac{5 d (b c - a d)^9}{9 b^{11} (a + b x)^{18}} - \frac{45 d^2 (b c - a d)^8}{17 b^{11} (a + b x)^{17}} - \frac{15 d^3 (b c - a d)^7}{2 b^{11} (a + b x)^{16}} - \frac{14 d^4 (b c - a d)^6}{b^{11} (a + b x)^{15}} - \\
& \frac{18 d^5 (b c - a d)^5}{b^{11} (a + b x)^{14}} - \frac{210 d^6 (b c - a d)^4}{13 b^{11} (a + b x)^{13}} - \frac{10 d^7 (b c - a d)^3}{b^{11} (a + b x)^{12}} - \frac{45 d^8 (b c - a d)^2}{11 b^{11} (a + b x)^{11}} - \frac{d^9 (b c - a d)}{b^{11} (a + b x)^{10}} - \frac{d^{10}}{9 b^{11} (a + b x)^9}
\end{aligned}$$

Result (type 1, 692 leaves):

$$\begin{aligned}
& - \frac{1}{831402 b^{11} (a + b x)^{19}} \left( a^{10} d^{10} + a^9 b d^9 (9 c + 19 d x) + 9 a^8 b^2 d^8 (5 c^2 + 19 c d x + 19 d^2 x^2) + \right. \\
& 3 a^7 b^3 d^7 (55 c^3 + 285 c^2 d x + 513 c d^2 x^2 + 323 d^3 x^3) + 3 a^6 b^4 d^6 (165 c^4 + 1045 c^3 d x + 2565 c^2 d^2 x^2 + 2907 c d^3 x^3 + 1292 d^4 x^4) + \\
& 9 a^5 b^5 d^5 (143 c^5 + 1045 c^4 d x + 3135 c^3 d^2 x^2 + 4845 c^2 d^3 x^3 + 3876 c d^4 x^4 + 1292 d^5 x^5) + \\
& 3 a^4 b^6 d^4 (1001 c^6 + 8151 c^5 d x + 28215 c^4 d^2 x^2 + 53295 c^3 d^3 x^3 + 58140 c^2 d^4 x^4 + 34884 c d^5 x^5 + 9044 d^6 x^6) + \\
& 3 a^3 b^7 d^3 (2145 c^7 + 19019 c^6 d x + 73359 c^5 d^2 x^2 + 159885 c^4 d^3 x^3 + 213180 c^3 d^4 x^4 + 174420 c^2 d^5 x^5 + 81396 c d^6 x^6 + 16796 d^7 x^7) + \\
& 9 a^2 b^8 d^2 (1430 c^8 + 13585 c^7 d x + 57057 c^6 d^2 x^2 + 138567 c^5 d^3 x^3 + 213180 c^4 d^4 x^4 + 213180 c^3 d^5 x^5 + 135660 c^2 d^6 x^6 + 50388 c d^7 x^7 + 8398 d^8 x^8) + \\
& a b^9 d (24310 c^9 + 244530 c^8 d x + 1100385 c^7 d^2 x^2 + 2909907 c^6 d^3 x^3 + 4988412 c^5 d^4 x^4 + 5755860 c^4 d^5 x^5 + 4476780 c^3 d^6 x^6 + \\
& 2267460 c^2 d^7 x^7 + 680238 c d^8 x^8 + 92378 d^9 x^9) + b^{10} (43758 c^{10} + 461890 c^9 d x + 2200770 c^8 d^2 x^2 + 6235515 c^7 d^3 x^3 + \\
& \left. 11639628 c^6 d^4 x^4 + 14965236 c^5 d^5 x^5 + 13430340 c^4 d^6 x^6 + 8314020 c^3 d^7 x^7 + 3401190 c^2 d^8 x^8 + 831402 c d^9 x^9 + 92378 d^{10} x^{10} \right)
\end{aligned}$$

### Problem 1332: Result more than twice size of optimal antiderivative.

$$\int \frac{(c + d x)^{10}}{(a + b x)^{21}} dx$$

Optimal (type 1, 279 leaves, 2 steps):

$$\begin{aligned} & -\frac{(b c - a d)^{10}}{20 b^{11} (a + b x)^{20}} - \frac{10 d (b c - a d)^9}{19 b^{11} (a + b x)^{19}} - \frac{5 d^2 (b c - a d)^8}{2 b^{11} (a + b x)^{18}} - \frac{120 d^3 (b c - a d)^7}{17 b^{11} (a + b x)^{17}} - \frac{105 d^4 (b c - a d)^6}{8 b^{11} (a + b x)^{16}} - \\ & \frac{84 d^5 (b c - a d)^5}{5 b^{11} (a + b x)^{15}} - \frac{15 d^6 (b c - a d)^4}{b^{11} (a + b x)^{14}} - \frac{120 d^7 (b c - a d)^3}{13 b^{11} (a + b x)^{13}} - \frac{15 d^8 (b c - a d)^2}{4 b^{11} (a + b x)^{12}} - \frac{10 d^9 (b c - a d)}{11 b^{11} (a + b x)^{11}} - \frac{d^{10}}{10 b^{11} (a + b x)^{10}} \end{aligned}$$

Result (type 1, 692 leaves):

$$\begin{aligned} & -\frac{1}{1847560 b^{11} (a + b x)^{20}} (a^{10} d^{10} + 10 a^9 b d^9 (c + 2 d x) + 5 a^8 b^2 d^8 (11 c^2 + 40 c d x + 38 d^2 x^2) + \\ & 20 a^7 b^3 d^7 (11 c^3 + 55 c^2 d x + 95 c d^2 x^2 + 57 d^3 x^3) + 5 a^6 b^4 d^6 (143 c^4 + 880 c^3 d x + 2090 c^2 d^2 x^2 + 2280 c d^3 x^3 + 969 d^4 x^4) + \\ & 2 a^5 b^5 d^5 (1001 c^5 + 7150 c^4 d x + 20900 c^3 d^2 x^2 + 31350 c^2 d^3 x^3 + 24225 c d^4 x^4 + 7752 d^5 x^5) + \\ & 5 a^4 b^6 d^4 (1001 c^6 + 8008 c^5 d x + 27170 c^4 d^2 x^2 + 50160 c^3 d^3 x^3 + 53295 c^2 d^4 x^4 + 31008 c d^5 x^5 + 7752 d^6 x^6) + \\ & 20 a^3 b^7 d^3 (572 c^7 + 5005 c^6 d x + 19019 c^5 d^2 x^2 + 40755 c^4 d^3 x^3 + 53295 c^3 d^4 x^4 + 42636 c^2 d^5 x^5 + 19380 c d^6 x^6 + 3876 d^7 x^7) + 5 a^2 b^8 d^2 \\ & (4862 c^8 + 45760 c^7 d x + 190190 c^6 d^2 x^2 + 456456 c^5 d^3 x^3 + 692835 c^4 d^4 x^4 + 682176 c^3 d^5 x^5 + 426360 c^2 d^6 x^6 + 155040 c d^7 x^7 + 25194 d^8 x^8) + \\ & 10 a b^9 d (4862 c^9 + 48620 c^8 d x + 217360 c^7 d^2 x^2 + 570570 c^6 d^3 x^3 + 969969 c^5 d^4 x^4 + 1108536 c^4 d^5 x^5 + 852720 c^3 d^6 x^6 + \\ & 426360 c^2 d^7 x^7 + 125970 c d^8 x^8 + 16796 d^9 x^9) + b^{10} (92378 c^{10} + 972400 c^9 d x + 4618900 c^8 d^2 x^2 + 13041600 c^7 d^3 x^3 + \\ & 24249225 c^6 d^4 x^4 + 31039008 c^5 d^5 x^5 + 27713400 c^4 d^6 x^6 + 17054400 c^3 d^7 x^7 + 6928350 c^2 d^8 x^8 + 1679600 c d^9 x^9 + 184756 d^{10} x^{10}) \end{aligned}$$

### Problem 1333: Result more than twice size of optimal antiderivative.

$$\int \frac{(c + d x)^{10}}{(a + b x)^{22}} dx$$

Optimal (type 1, 279 leaves, 2 steps):

$$\begin{aligned} & -\frac{(b c - a d)^{10}}{21 b^{11} (a + b x)^{21}} - \frac{d (b c - a d)^9}{2 b^{11} (a + b x)^{20}} - \frac{45 d^2 (b c - a d)^8}{19 b^{11} (a + b x)^{19}} - \frac{20 d^3 (b c - a d)^7}{3 b^{11} (a + b x)^{18}} - \frac{210 d^4 (b c - a d)^6}{17 b^{11} (a + b x)^{17}} - \\ & \frac{63 d^5 (b c - a d)^5}{4 b^{11} (a + b x)^{16}} - \frac{14 d^6 (b c - a d)^4}{b^{11} (a + b x)^{15}} - \frac{60 d^7 (b c - a d)^3}{7 b^{11} (a + b x)^{14}} - \frac{45 d^8 (b c - a d)^2}{13 b^{11} (a + b x)^{13}} - \frac{5 d^9 (b c - a d)}{6 b^{11} (a + b x)^{12}} - \frac{d^{10}}{11 b^{11} (a + b x)^{11}} \end{aligned}$$

Result (type 1, 692 leaves):

$$\begin{aligned}
& - \frac{1}{3879876 b^{11} (a + b x)^{21}} \left( a^{10} d^{10} + a^9 b d^9 (11 c + 21 d x) + 3 a^8 b^2 d^8 (22 c^2 + 77 c d x + 70 d^2 x^2) + \right. \\
& \quad 2 a^7 b^3 d^7 (143 c^3 + 693 c^2 d x + 1155 c d^2 x^2 + 665 d^3 x^3) + 7 a^6 b^4 d^6 (143 c^4 + 858 c^3 d x + 1980 c^2 d^2 x^2 + 2090 c d^3 x^3 + 855 d^4 x^4) + \\
& \quad 21 a^5 b^5 d^5 (143 c^5 + 1001 c^4 d x + 2860 c^3 d^2 x^2 + 4180 c^2 d^3 x^3 + 3135 c d^4 x^4 + 969 d^5 x^5) + \\
& \quad 7 a^4 b^6 d^4 (1144 c^6 + 9009 c^5 d x + 30030 c^4 d^2 x^2 + 54340 c^3 d^3 x^3 + 56430 c^2 d^4 x^4 + 31977 c d^5 x^5 + 7752 d^6 x^6) + \\
& \quad 2 a^3 b^7 d^3 (9724 c^7 + 84084 c^6 d x + 315315 c^5 d^2 x^2 + 665665 c^4 d^3 x^3 + 855855 c^3 d^4 x^4 + 671517 c^2 d^5 x^5 + 298452 c d^6 x^6 + 58140 d^7 x^7) + \\
& \quad 3 a^2 b^8 d^2 (14586 c^8 + 136136 c^7 d x + 560560 c^6 d^2 x^2 + 1331330 c^5 d^3 x^3 + 1996995 c^4 d^4 x^4 + 1939938 c^3 d^5 x^5 + \\
& \quad 1193808 c^2 d^6 x^6 + 426360 c d^7 x^7 + 67830 d^8 x^8) + a b^9 d (92378 c^9 + 918918 c^8 d x + 4084080 c^7 d^2 x^2 + 10650640 c^6 d^3 x^3 + \\
& \quad 17972955 c^5 d^4 x^4 + 20369349 c^4 d^5 x^5 + 15519504 c^3 d^6 x^6 + 7674480 c^2 d^7 x^7 + 2238390 c d^8 x^8 + 293930 d^9 x^9) + \\
& \left. b^{10} (184756 c^{10} + 1939938 c^9 d x + 9189180 c^8 d^2 x^2 + 25865840 c^7 d^3 x^3 + 47927880 c^6 d^4 x^4 + 61108047 c^5 d^5 x^5 + \right. \\
& \quad \left. 54318264 c^4 d^6 x^6 + 33256080 c^3 d^7 x^7 + 13430340 c^2 d^8 x^8 + 3233230 c d^9 x^9 + 352716 d^{10} x^{10}) \right)
\end{aligned}$$

**Problem 1362: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b x)^9}{(c + d x)^8} dx$$

Optimal (type 3, 232 leaves, 2 steps):

$$\begin{aligned}
& - \frac{b^8 (8 b c - 9 a d) x}{d^9} + \frac{b^9 x^2}{2 d^8} + \frac{(b c - a d)^9}{7 d^{10} (c + d x)^7} - \frac{3 b (b c - a d)^8}{2 d^{10} (c + d x)^6} + \frac{36 b^2 (b c - a d)^7}{5 d^{10} (c + d x)^5} - \\
& \frac{21 b^3 (b c - a d)^6}{d^{10} (c + d x)^4} + \frac{42 b^4 (b c - a d)^5}{d^{10} (c + d x)^3} - \frac{63 b^5 (b c - a d)^4}{d^{10} (c + d x)^2} + \frac{84 b^6 (b c - a d)^3}{d^{10} (c + d x)} + \frac{36 b^7 (b c - a d)^2 \operatorname{Log}[c + d x]}{d^{10}}
\end{aligned}$$

Result (type 3, 584 leaves):

$$\begin{aligned}
& - \frac{1}{70 d^{10} (c + d x)^7} \left( 10 a^9 d^9 + 15 a^8 b d^8 (c + 7 d x) + 24 a^7 b^2 d^7 (c^2 + 7 c d x + 21 d^2 x^2) + 42 a^6 b^3 d^6 (c^3 + 7 c^2 d x + 21 c d^2 x^2 + 35 d^3 x^3) + \right. \\
& \quad 84 a^5 b^4 d^5 (c^4 + 7 c^3 d x + 21 c^2 d^2 x^2 + 35 c d^3 x^3 + 35 d^4 x^4) + 210 a^4 b^5 d^4 (c^5 + 7 c^4 d x + 21 c^3 d^2 x^2 + 35 c^2 d^3 x^3 + 35 c d^4 x^4 + 21 d^5 x^5) + \\
& \quad 840 a^3 b^6 d^3 (c^6 + 7 c^5 d x + 21 c^4 d^2 x^2 + 35 c^3 d^3 x^3 + 35 c^2 d^4 x^4 + 21 c d^5 x^5 + 7 d^6 x^6) - \\
& \quad 6 a^2 b^7 c d^2 (1089 c^6 + 7203 c^5 d x + 20139 c^4 d^2 x^2 + 30625 c^3 d^3 x^3 + 26950 c^2 d^4 x^4 + 13230 c d^5 x^5 + 2940 d^6 x^6) + \\
& \quad 6 a b^8 d (1443 c^8 + 9261 c^7 d x + 24843 c^6 d^2 x^2 + 35525 c^5 d^3 x^3 + 28175 c^4 d^4 x^4 + 11025 c^3 d^5 x^5 + 735 c^2 d^6 x^6 - 735 c d^7 x^7 - 105 d^8 x^8) - \\
& \quad b^9 (3349 c^9 + 20923 c^8 d x + 53949 c^7 d^2 x^2 + 72275 c^6 d^3 x^3 + 50225 c^5 d^4 x^4 + 12495 c^4 d^5 x^5 - 4655 c^3 d^6 x^6 - 3185 c^2 d^7 x^7 - 315 c d^8 x^8 + 35 d^9 x^9) - \\
& \quad \left. 2520 b^7 (b c - a d)^2 (c + d x)^7 \operatorname{Log}[c + d x] \right)
\end{aligned}$$

**Problem 1363:** Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^8}{(c + d x)^8} dx$$

Optimal (type 3, 209 leaves, 2 steps):

$$\begin{aligned} & \frac{b^8 x}{d^8} - \frac{(b c - a d)^8}{7 d^9 (c + d x)^7} + \frac{4 b (b c - a d)^7}{3 d^9 (c + d x)^6} - \frac{28 b^2 (b c - a d)^6}{5 d^9 (c + d x)^5} + \frac{14 b^3 (b c - a d)^5}{d^9 (c + d x)^4} - \\ & \frac{70 b^4 (b c - a d)^4}{3 d^9 (c + d x)^3} + \frac{28 b^5 (b c - a d)^3}{d^9 (c + d x)^2} - \frac{28 b^6 (b c - a d)^2}{d^9 (c + d x)} - \frac{8 b^7 (b c - a d) \operatorname{Log}[c + d x]}{d^9} \end{aligned}$$

Result (type 3, 474 leaves):

$$\begin{aligned} & -\frac{1}{105 d^9 (c + d x)^7} (15 a^8 d^8 + 20 a^7 b d^7 (c + 7 d x) + 28 a^6 b^2 d^6 (c^2 + 7 c d x + 21 d^2 x^2) + 42 a^5 b^3 d^5 (c^3 + 7 c^2 d x + 21 c d^2 x^2 + 35 d^3 x^3) + \\ & 70 a^4 b^4 d^4 (c^4 + 7 c^3 d x + 21 c^2 d^2 x^2 + 35 c d^3 x^3 + 35 d^4 x^4) + 140 a^3 b^5 d^3 (c^5 + 7 c^4 d x + 21 c^3 d^2 x^2 + 35 c^2 d^3 x^3 + 35 c d^4 x^4 + 21 d^5 x^5) + \\ & 420 a^2 b^6 d^2 (c^6 + 7 c^5 d x + 21 c^4 d^2 x^2 + 35 c^3 d^3 x^3 + 35 c^2 d^4 x^4 + 21 c d^5 x^5 + 7 d^6 x^6) - \\ & 2 a b^7 c d (1089 c^6 + 7203 c^5 d x + 20139 c^4 d^2 x^2 + 30625 c^3 d^3 x^3 + 26950 c^2 d^4 x^4 + 13230 c d^5 x^5 + 2940 d^6 x^6) + \\ & b^8 (1443 c^8 + 9261 c^7 d x + 24843 c^6 d^2 x^2 + 35525 c^5 d^3 x^3 + 28175 c^4 d^4 x^4 + 11025 c^3 d^5 x^5 + 735 c^2 d^6 x^6 - 735 c d^7 x^7 - 105 d^8 x^8) + \\ & 840 b^7 (b c - a d) (c + d x)^7 \operatorname{Log}[c + d x] \end{aligned}$$

**Problem 1365:** Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^6}{(c + d x)^8} dx$$

Optimal (type 1, 28 leaves, 1 step):

$$\frac{(a + b x)^7}{7 (b c - a d) (c + d x)^7}$$

Result (type 1, 271 leaves):

$$\begin{aligned} & -\frac{1}{7 d^7 (c + d x)^7} (a^6 d^6 + a^5 b d^5 (c + 7 d x) + a^4 b^2 d^4 (c^2 + 7 c d x + 21 d^2 x^2) + \\ & a^3 b^3 d^3 (c^3 + 7 c^2 d x + 21 c d^2 x^2 + 35 d^3 x^3) + a^2 b^4 d^2 (c^4 + 7 c^3 d x + 21 c^2 d^2 x^2 + 35 c d^3 x^3 + 35 d^4 x^4) + \\ & a b^5 d (c^5 + 7 c^4 d x + 21 c^3 d^2 x^2 + 35 c^2 d^3 x^3 + 35 c d^4 x^4 + 21 d^5 x^5) + b^6 (c^6 + 7 c^5 d x + 21 c^4 d^2 x^2 + 35 c^3 d^3 x^3 + 35 c^2 d^4 x^4 + 21 c d^5 x^5 + 7 d^6 x^6) ) \end{aligned}$$

**Problem 1366:** Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^5}{(c + d x)^8} dx$$

Optimal (type 1, 58 leaves, 2 steps) :

$$\frac{(a + b x)^6}{7 (b c - a d) (c + d x)^7} + \frac{b (a + b x)^6}{42 (b c - a d)^2 (c + d x)^6}$$

Result (type 1, 205 leaves) :

$$-\frac{1}{42 d^6 (c + d x)^7} (6 a^5 d^5 + 5 a^4 b d^4 (c + 7 d x) + 4 a^3 b^2 d^3 (c^2 + 7 c d x + 21 d^2 x^2) + 3 a^2 b^3 d^2 (c^3 + 7 c^2 d x + 21 c d^2 x^2 + 35 d^3 x^3) + 2 a b^4 d (c^4 + 7 c^3 d x + 21 c^2 d^2 x^2 + 35 c d^3 x^3 + 35 d^4 x^4) + b^5 (c^5 + 7 c^4 d x + 21 c^3 d^2 x^2 + 35 c^2 d^3 x^3 + 35 c d^4 x^4 + 21 d^5 x^5))$$

**Problem 1453:** Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(-2 + x) \sqrt{2 + x}} dx$$

Optimal (type 3, 14 leaves, 2 steps) :

$$-\text{ArcTanh}\left[\frac{\sqrt{2+x}}{2}\right]$$

Result (type 3, 31 leaves) :

$$\frac{1}{2} \text{Log}[2 - \sqrt{2 + x}] - \frac{1}{2} \text{Log}[2 + \sqrt{2 + x}]$$

**Problem 1458:** Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + b x) (c + d x)^{1/3}} dx$$

Optimal (type 3, 139 leaves, 4 steps) :

$$\frac{\sqrt{3} \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} (c+d x)^{1/3}}{(b c-a d)^{1/3}}}{\sqrt{3}}\right]}{b^{2/3} (b c-a d)^{1/3}} - \frac{\text{Log}[a+b x]}{2 b^{2/3} (b c-a d)^{1/3}} + \frac{3 \text{Log}\left[(b c-a d)^{1/3}-b^{1/3} (c+d x)^{1/3}\right]}{2 b^{2/3} (b c-a d)^{1/3}}$$

Result (type 5, 47 leaves) :

$$-\frac{3 (c + d x)^{2/3} \text{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{b (c+d x)}{b c-a d}\right]}{2 b c - 2 a d}$$

Problem 1459: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + b x) (c + d x)^{2/3}} dx$$

Optimal (type 3, 140 leaves, 4 steps) :

$$-\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1+\frac{2 b^{1/3} (c+d x)^{1/3}}{(b c-a d)^{1/3}}}{\sqrt{3}}\right]}{b^{1/3} (b c-a d)^{2/3}} - \frac{\operatorname{Log}[a+b x]}{2 b^{1/3} (b c-a d)^{2/3}} + \frac{3 \operatorname{Log}\left[(b c-a d)^{1/3}-b^{1/3} (c+d x)^{1/3}\right]}{2 b^{1/3} (b c-a d)^{2/3}}$$

Result (type 5, 46 leaves) :

$$-\frac{3 (c + d x)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{b (c+d x)}{b c-a d}\right]}{b c - a d}$$

Problem 1539: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-b x} \sqrt{2+b x}} dx$$

Optimal (type 3, 10 leaves, 3 steps) :

$$\frac{\operatorname{ArcSin}[1+b x]}{b}$$

Result (type 3, 51 leaves) :

$$\frac{2 \sqrt{x} \sqrt{2+b x} \operatorname{ArcSinh}\left[\frac{\sqrt{b} \sqrt{x}}{\sqrt{2}}\right]}{\sqrt{b} \sqrt{-b x (2+b x)}}$$

Problem 1540: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-1-b x} \sqrt{2+b x}} dx$$

Optimal (type 3, 11 leaves, 3 steps) :

$\text{ArcSin}[3 + 2 b x]$  $b$ 

Result (type 3, 49 leaves) :

$$\frac{2 \sqrt{1+b x} \sqrt{2+b x} \text{ArcSinh}[\sqrt{1+b x}]}{b \sqrt{- (1+b x) (2+b x)}}$$

Problem 1550: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-4+b x} \sqrt{4+b x}} dx$$

Optimal (type 3, 11 leaves, 1 step) :

 $\text{ArcCosh}[\frac{b x}{4}]$  $b$ 

Result (type 3, 24 leaves) :

$$\frac{2 \text{ArcSinh}[\frac{\sqrt{-4+b x}}{2 \sqrt{2}}]}{b}$$

Problem 1555: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{4-x} \sqrt{x}} dx$$

Optimal (type 3, 10 leaves, 3 steps) :

 $-\text{ArcSin}\left[1 - \frac{x}{2}\right]$ 

Result (type 3, 38 leaves) :

$$\frac{2 \sqrt{-4+x} \sqrt{x} \text{Log}[\sqrt{-4+x} + \sqrt{x}]}{\sqrt{-(-4+x) x}}$$

Problem 1558: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{a-b x} \sqrt{c+d x}} dx$$

Optimal (type 3, 43 leaves, 3 steps):

$$-\frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{d} \sqrt{a-b x}}{\sqrt{b} \sqrt{c+d x}}\right]}{\sqrt{b} \sqrt{d}}$$

Result (type 3, 64 leaves):

$$\frac{\frac{i}{2} \log \left[2 \sqrt{a-b x} \sqrt{c+d x}-\frac{i (b c-a d+2 b d x)}{\sqrt{b} \sqrt{d}}\right]}{\sqrt{b} \sqrt{d}}$$

**Problem 1559: Result unnecessarily involves higher level functions.**

$$\int (a+b x)^{3/2} (c+d x)^{1/3} dx$$

Optimal (type 4, 457 leaves, 5 steps):

$$\begin{aligned} & -\frac{108 (b c-a d)^2 \sqrt{a+b x} (c+d x)^{1/3}}{935 b d^2} + \frac{12 (b c-a d) (a+b x)^{3/2} (c+d x)^{1/3}}{187 b d} + \\ & \frac{6 (a+b x)^{5/2} (c+d x)^{1/3}}{17 b} - \left( \frac{108 \times 3^{3/4} \sqrt{2-\sqrt{3}} (b c-a d)^3 ((b c-a d)^{1/3} - b^{1/3} (c+d x)^{1/3})}{\sqrt{\frac{(b c-a d)^{2/3} + b^{1/3} (b c-a d)^{1/3} (c+d x)^{1/3} + b^{2/3} (c+d x)^{2/3}}{\left((1-\sqrt{3}) (b c-a d)^{1/3} - b^{1/3} (c+d x)^{1/3}\right)^2}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{\left(1+\sqrt{3}\right) (b c-a d)^{1/3} - b^{1/3} (c+d x)^{1/3}}{\left(1-\sqrt{3}\right) (b c-a d)^{1/3} - b^{1/3} (c+d x)^{1/3}}\right], -7+4 \sqrt{3}] \right) / \\ & \left( \frac{935 b^{4/3} d^3 \sqrt{a+b x}}{\sqrt{-\frac{(b c-a d)^{1/3} ((b c-a d)^{1/3} - b^{1/3} (c+d x)^{1/3})}{\left((1-\sqrt{3}) (b c-a d)^{1/3} - b^{1/3} (c+d x)^{1/3}\right)^2}}} \right) \end{aligned}$$

Result (type 5, 142 leaves):

$$\begin{aligned} & -\frac{1}{935 b d^3 \sqrt{a+b x}} 6 (c+d x)^{1/3} \left( -d (a+b x) (27 a^2 d^2 + 2 a b d (23 c + 50 d x) + b^2 (-18 c^2 + 10 c d x + 55 d^2 x^2)) - \right. \\ & \left. 27 (b c-a d)^3 \sqrt{\frac{d (a+b x)}{-b c+a d}} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, \frac{b (c+d x)}{b c-a d}\right] \right) \end{aligned}$$

### Problem 1560: Result unnecessarily involves higher level functions.

$$\int \sqrt{a + b x} (c + d x)^{1/3} dx$$

Optimal (type 4, 419 leaves, 4 steps):

$$\begin{aligned} & \frac{12 (b c - a d) \sqrt{a + b x} (c + d x)^{1/3}}{55 b d} + \frac{6 (a + b x)^{3/2} (c + d x)^{1/3}}{11 b} + \\ & \left( \frac{12 \times 3^{3/4} \sqrt{2 - \sqrt{3}} (b c - a d)^2 ((b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3})}{\sqrt{\left( (1 - \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right)^2}} \right. \\ & \left. \text{EllipticF}[\text{ArcSin}\left[\frac{(1 + \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}}{(1 - \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}}\right], -7 + 4\sqrt{3}] \right) / \\ & \left( \frac{55 b^{4/3} d^2 \sqrt{a + b x}}{\sqrt{-\frac{(b c - a d)^{1/3} ((b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3})}{\left( (1 - \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right)^2}}} \right) \end{aligned}$$

Result (type 5, 110 leaves):

$$\frac{1}{55 b d^2 \sqrt{a + b x}} 6 (c + d x)^{1/3} \left( d (a + b x) (2 b c + 3 a d + 5 b d x) - 3 (b c - a d)^2 \sqrt{\frac{d (a + b x)}{-b c + a d}} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, \frac{b (c + d x)}{b c - a d}\right] \right)$$

### Problem 1561: Result unnecessarily involves higher level functions.

$$\int \frac{(c + d x)^{1/3}}{\sqrt{a + b x}} dx$$

Optimal (type 4, 381 leaves, 3 steps):

$$\begin{aligned}
& \frac{6 \sqrt{a+b x} (c+d x)^{1/3}}{5 b} - \\
& \left( 4 \times 3^{3/4} \sqrt{2-\sqrt{3}} (b c-a d) \left( (b c-a d)^{1/3} - b^{1/3} (c+d x)^{1/3} \right) \sqrt{\frac{(b c-a d)^{2/3} + b^{1/3} (b c-a d)^{1/3} (c+d x)^{1/3} + b^{2/3} (c+d x)^{2/3}}{\left( (1-\sqrt{3}) (b c-a d)^{1/3} - b^{1/3} (c+d x)^{1/3} \right)^2}} \right. \\
& \left. \text{EllipticF}[\text{ArcSin}\left[\frac{(1+\sqrt{3}) (b c-a d)^{1/3} - b^{1/3} (c+d x)^{1/3}}{(1-\sqrt{3}) (b c-a d)^{1/3} - b^{1/3} (c+d x)^{1/3}}\right], -7+4\sqrt{3}] \right) / \\
& \left( 5 b^{4/3} d \sqrt{a+b x} \sqrt{-\frac{(b c-a d)^{1/3} \left( (b c-a d)^{1/3} - b^{1/3} (c+d x)^{1/3} \right)}{\left( (1-\sqrt{3}) (b c-a d)^{1/3} - b^{1/3} (c+d x)^{1/3} \right)^2}} \right)
\end{aligned}$$

Result (type 5, 93 leaves):

$$\frac{6 (c+d x)^{1/3} \left( d (a+b x) + (b c-a d) \sqrt{\frac{d (a+b x)}{-b c+a d}} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, \frac{b (c+d x)}{b c-a d}\right] \right)}{5 b d \sqrt{a+b x}}$$

**Problem 1562:** Result unnecessarily involves higher level functions.

$$\int \frac{(c+d x)^{1/3}}{(a+b x)^{3/2}} dx$$

Optimal (type 4, 366 leaves, 3 steps):

$$\begin{aligned}
& -\frac{2 (c+d x)^{1/3}}{b \sqrt{a+b x}} - \left( 4 \sqrt{2-\sqrt{3}} \left( (b c-a d)^{1/3} - b^{1/3} (c+d x)^{1/3} \right) \right. \\
& \left. \sqrt{\frac{(b c-a d)^{2/3} + b^{1/3} (b c-a d)^{1/3} (c+d x)^{1/3} + b^{2/3} (c+d x)^{2/3}}{\left( (1-\sqrt{3}) (b c-a d)^{1/3} - b^{1/3} (c+d x)^{1/3} \right)^2}} \text{EllipticF}[\text{ArcSin}\left[\frac{(1+\sqrt{3}) (b c-a d)^{1/3} - b^{1/3} (c+d x)^{1/3}}{(1-\sqrt{3}) (b c-a d)^{1/3} - b^{1/3} (c+d x)^{1/3}}\right], -7+4\sqrt{3}] \right) / \\
& \left( 3^{1/4} b^{4/3} \sqrt{a+b x} \sqrt{-\frac{(b c-a d)^{1/3} \left( (b c-a d)^{1/3} - b^{1/3} (c+d x)^{1/3} \right)}{\left( (1-\sqrt{3}) (b c-a d)^{1/3} - b^{1/3} (c+d x)^{1/3} \right)^2}} \right)
\end{aligned}$$

Result (type 5, 74 leaves):

$$\frac{2 (c + d x)^{1/3} \left(-1 + \sqrt{\frac{d (a+b x)}{-b c+a d}} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, \frac{b (c+d x)}{b c-a d}\right]\right)}{b \sqrt{a+b x}}$$

**Problem 1563:** Result unnecessarily involves higher level functions.

$$\int \frac{(c + d x)^{1/3}}{(a + b x)^{5/2}} dx$$

Optimal (type 4, 417 leaves, 4 steps):

$$\begin{aligned} & -\frac{2 (c + d x)^{1/3}}{3 b (a + b x)^{3/2}} - \frac{4 d (c + d x)^{1/3}}{9 b (b c - a d) \sqrt{a + b x}} + \\ & \left( 4 \sqrt{2 - \sqrt{3}} d \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right) \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + d x)^{1/3} + b^{2/3} (c + d x)^{2/3}}{\left(1 - \sqrt{3}\right) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}}} \right. \\ & \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1 + \sqrt{3}\right) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}}{\left(1 - \sqrt{3}\right) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}}\right], -7 + 4 \sqrt{3}\right]\right) / \\ & \left( 9 \times 3^{1/4} b^{4/3} (b c - a d) \sqrt{a + b x} \sqrt{-\frac{(b c - a d)^{1/3} \left((b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}\right)}{\left(\left(1 - \sqrt{3}\right) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}\right)^2}} \right) \end{aligned}$$

Result (type 5, 104 leaves):

$$\frac{2 (c + d x)^{1/3} \left(3 b c - a d + 2 b d x + d (a + b x) \sqrt{\frac{d (a+b x)}{-b c+a d}} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, \frac{b (c+d x)}{b c-a d}\right]\right)}{9 b (-b c + a d) (a + b x)^{3/2}}$$

**Problem 1564:** Result unnecessarily involves higher level functions.

$$\int \frac{(c + d x)^{1/3}}{(a + b x)^{7/2}} dx$$

Optimal (type 4, 457 leaves, 5 steps):

$$\begin{aligned}
& - \frac{2 (c + d x)^{1/3}}{5 b (a + b x)^{5/2}} - \frac{4 d (c + d x)^{1/3}}{45 b (b c - a d) (a + b x)^{3/2}} + \frac{28 d^2 (c + d x)^{1/3}}{135 b (b c - a d)^2 \sqrt{a + b x}} - \\
& \left( 28 \sqrt{2 - \sqrt{3}} d^2 ((b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}) \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + d x)^{1/3} + b^{2/3} (c + d x)^{2/3}}{\left( (1 - \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right)^2}} \right. \\
& \left. \text{EllipticF}[\text{ArcSin}\left[\frac{(1 + \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}}{(1 - \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}}\right], -7 + 4 \sqrt{3}] \right) / \\
& \left( 135 \times 3^{1/4} b^{4/3} (b c - a d)^2 \sqrt{a + b x} \sqrt{-\frac{(b c - a d)^{1/3} ((b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3})}{\left( (1 - \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right)^2}} \right)
\end{aligned}$$

Result (type 5, 140 leaves):

$$\begin{aligned}
& \left( 2 (c + d x)^{1/3} \left( -7 a^2 d^2 + 2 a b d (24 c + 17 d x) + b^2 (-27 c^2 - 6 c d x + 14 d^2 x^2) + \right. \right. \\
& \left. \left. 7 d^2 (a + b x)^2 \sqrt{\frac{d (a + b x)}{-b c + a d}} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, \frac{b (c + d x)}{b c - a d}\right] \right) \right) / \left( 135 b (b c - a d)^2 (a + b x)^{5/2} \right)
\end{aligned}$$

Problem 1565: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{3/2}}{(c + d x)^{1/3}} dx$$

Optimal (type 4, 839 leaves, 6 steps):

$$\begin{aligned}
& - \frac{54 (b c - a d) \sqrt{a + b x} (c + d x)^{2/3}}{91 d^2} + \frac{6 (a + b x)^{3/2} (c + d x)^{2/3}}{13 d} - \\
& \frac{162 (b c - a d)^2 \sqrt{a + b x}}{91 b^{2/3} d^2 ((1 - \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3})} + \left( 81 \times 3^{1/4} \sqrt{2 + \sqrt{3}} (b c - a d)^{7/3} ((b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}) \right. \\
& \left. \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + d x)^{1/3} + b^{2/3} (c + d x)^{2/3}}{((1 - \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3})^2}} \operatorname{EllipticE}[\operatorname{ArcSin}\left[\frac{(1 + \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}}{(1 - \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}}\right], -7 + 4\sqrt{3}] \right) / \\
& \left( 91 b^{2/3} d^3 \sqrt{a + b x} \sqrt{-\frac{(b c - a d)^{1/3} ((b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3})}{((1 - \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3})^2}} - \left( 54 \sqrt{2} 3^{3/4} (b c - a d)^{7/3} ((b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}) \right. \right. \\
& \left. \left. \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + d x)^{1/3} + b^{2/3} (c + d x)^{2/3}}{((1 - \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3})^2}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{(1 + \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}}{(1 - \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}}\right], -7 + 4\sqrt{3}] \right) / \\
& \left( 91 b^{2/3} d^3 \sqrt{a + b x} \sqrt{-\frac{(b c - a d)^{1/3} ((b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3})}{((1 - \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3})^2}} \right)
\end{aligned}$$

Result (type 5, 108 leaves):

$$\frac{1}{182 d^3 \sqrt{a + b x}} 3 (c + d x)^{2/3} \left( 4 d (a + b x) (-9 b c + 16 a d + 7 b d x) + 27 (b c - a d)^2 \sqrt{\frac{d (a + b x)}{-b c + a d}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, \frac{b (c + d x)}{b c - a d}\right] \right)$$

Problem 1566: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{a + b x}}{(c + d x)^{1/3}} dx$$

Optimal (type 4, 804 leaves, 5 steps):

$$\begin{aligned}
& \frac{6 \sqrt{a+b x} (c+d x)^{2/3}}{7 d} + \frac{18 (b c - a d) \sqrt{a+b x}}{7 b^{2/3} d \left( (1-\sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c+d x)^{1/3} \right)} - \\
& \left( 9 \times 3^{1/4} \sqrt{2+\sqrt{3}} (b c - a d)^{4/3} \left( (b c - a d)^{1/3} - b^{1/3} (c+d x)^{1/3} \right) \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c+d x)^{1/3} + b^{2/3} (c+d x)^{2/3}}{\left( (1-\sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c+d x)^{1/3} \right)^2}} \right. \\
& \left. \text{EllipticE} \left[ \text{ArcSin} \left[ \frac{(1+\sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c+d x)^{1/3}}{(1-\sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c+d x)^{1/3}} \right], -7+4\sqrt{3} \right] \right) / \\
& \left( 7 b^{2/3} d^2 \sqrt{a+b x} \sqrt{-\frac{(b c - a d)^{1/3} \left( (b c - a d)^{1/3} - b^{1/3} (c+d x)^{1/3} \right)}{\left( (1-\sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c+d x)^{1/3} \right)^2}} + \left( 6 \sqrt{2} 3^{3/4} (b c - a d)^{4/3} \left( (b c - a d)^{1/3} - b^{1/3} (c+d x)^{1/3} \right) \right. \right. \\
& \left. \left. \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c+d x)^{1/3} + b^{2/3} (c+d x)^{2/3}}{\left( (1-\sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c+d x)^{1/3} \right)^2}} \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{(1+\sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c+d x)^{1/3}}{(1-\sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c+d x)^{1/3}} \right], -7+4\sqrt{3} \right] \right) / \\
& \left( 7 b^{2/3} d^2 \sqrt{a+b x} \sqrt{-\frac{(b c - a d)^{1/3} \left( (b c - a d)^{1/3} - b^{1/3} (c+d x)^{1/3} \right)}{\left( (1-\sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c+d x)^{1/3} \right)^2}} \right)
\end{aligned}$$

Result (type 5, 77 leaves):

$$\frac{3 \sqrt{a+b x} (c+d x)^{2/3} \left( 4 + \frac{3 \text{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{2}{3}, \frac{5}{3}, \frac{b (c+d x)}{b c - a d} \right]}{\sqrt{\frac{d (a+b x)}{-b c + a d}}} \right)}{14 d}$$

Problem 1567: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\sqrt{a+b x} (c+d x)^{1/3}} dx$$

Optimal (type 4, 762 leaves, 4 steps):

$$\begin{aligned}
& - \frac{6 \sqrt{a+b x}}{b^{2/3} \left( (1-\sqrt{3}) (b c-a d)^{1/3} - b^{1/3} (c+d x)^{1/3} \right)} + \\
& \left( 3 \times 3^{1/4} \sqrt{2+\sqrt{3}} (b c-a d)^{1/3} \left( (b c-a d)^{1/3} - b^{1/3} (c+d x)^{1/3} \right) \sqrt{\frac{(b c-a d)^{2/3} + b^{1/3} (b c-a d)^{1/3} (c+d x)^{1/3} + b^{2/3} (c+d x)^{2/3}}{\left( (1-\sqrt{3}) (b c-a d)^{1/3} - b^{1/3} (c+d x)^{1/3} \right)^2}} \right. \\
& \left. \text{EllipticE} \left[ \text{ArcSin} \left[ \frac{(1+\sqrt{3}) (b c-a d)^{1/3} - b^{1/3} (c+d x)^{1/3}}{(1-\sqrt{3}) (b c-a d)^{1/3} - b^{1/3} (c+d x)^{1/3}} \right], -7+4\sqrt{3} \right] \right) / \\
& \left( b^{2/3} d \sqrt{a+b x} \sqrt{-\frac{(b c-a d)^{1/3} \left( (b c-a d)^{1/3} - b^{1/3} (c+d x)^{1/3} \right)}{\left( (1-\sqrt{3}) (b c-a d)^{1/3} - b^{1/3} (c+d x)^{1/3} \right)^2}} \right) - \left( 2 \sqrt{2} 3^{3/4} (b c-a d)^{1/3} \left( (b c-a d)^{1/3} - b^{1/3} (c+d x)^{1/3} \right) \right. \\
& \left. \sqrt{\frac{(b c-a d)^{2/3} + b^{1/3} (b c-a d)^{1/3} (c+d x)^{1/3} + b^{2/3} (c+d x)^{2/3}}{\left( (1-\sqrt{3}) (b c-a d)^{1/3} - b^{1/3} (c+d x)^{1/3} \right)^2}} \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{(1+\sqrt{3}) (b c-a d)^{1/3} - b^{1/3} (c+d x)^{1/3}}{(1-\sqrt{3}) (b c-a d)^{1/3} - b^{1/3} (c+d x)^{1/3}} \right], -7+4\sqrt{3} \right] \right) / \\
& \left( b^{2/3} d \sqrt{a+b x} \sqrt{-\frac{(b c-a d)^{1/3} \left( (b c-a d)^{1/3} - b^{1/3} (c+d x)^{1/3} \right)}{\left( (1-\sqrt{3}) (b c-a d)^{1/3} - b^{1/3} (c+d x)^{1/3} \right)^2}} \right)
\end{aligned}$$

Result (type 5, 73 leaves) :

$$\frac{3 \sqrt{\frac{d (a+b x)}{-b c+a d}} (c+d x)^{2/3} \text{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{2}{3}, \frac{5}{3}, \frac{b (c+d x)}{b c-a d} \right]}{2 d \sqrt{a+b x}}$$

Problem 1568: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a+b x)^{3/2} (c+d x)^{1/3}} dx$$

Optimal (type 4, 796 leaves, 5 steps) :

$$\begin{aligned}
& - \frac{2 (c + d x)^{2/3}}{(b c - a d) \sqrt{a + b x}} - \frac{2 d \sqrt{a + b x}}{b^{2/3} (b c - a d) \left( (1 - \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right)} + \\
& \left( 3^{1/4} \sqrt{2 + \sqrt{3}} \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right) \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + d x)^{1/3} + b^{2/3} (c + d x)^{2/3}}{\left( (1 - \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right)^2}} \right. \\
& \left. \text{EllipticE} \left[ \text{ArcSin} \left[ \frac{(1 + \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}}{(1 - \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}} \right], -7 + 4 \sqrt{3} \right] \right) / \\
& \left( b^{2/3} (b c - a d)^{2/3} \sqrt{a + b x} \sqrt{-\frac{(b c - a d)^{1/3} ((b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3})}{\left( (1 - \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right)^2}} \right. \\
& \left( 2 \sqrt{2} \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right) \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + d x)^{1/3} + b^{2/3} (c + d x)^{2/3}}{\left( (1 - \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right)^2}} \right. \\
& \left. \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{(1 + \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}}{(1 - \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}} \right], -7 + 4 \sqrt{3} \right] \right) / \\
& \left( 3^{1/4} b^{2/3} (b c - a d)^{2/3} \sqrt{a + b x} \sqrt{-\frac{(b c - a d)^{1/3} ((b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3})}{\left( (1 - \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right)^2}} \right)
\end{aligned}$$

Result (type 5, 83 leaves):

$$\frac{(c + d x)^{2/3} \left( -4 + \sqrt{\frac{d (a + b x)}{-b c + a d}} \text{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{2}{3}, \frac{5}{3}, \frac{b (c + d x)}{b c - a d} \right] \right)}{2 (b c - a d) \sqrt{a + b x}}$$

Problem 1569: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + b x)^{5/2} (c + d x)^{1/3}} dx$$

Optimal (type 4, 842 leaves, 6 steps):

$$\begin{aligned}
& - \frac{2 (c + d x)^{2/3}}{3 (b c - a d) (a + b x)^{3/2}} + \frac{10 d (c + d x)^{2/3}}{9 (b c - a d)^2 \sqrt{a + b x}} + \frac{10 d^2 \sqrt{a + b x}}{9 b^{2/3} (b c - a d)^2 \left( (1 - \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right)} - \\
& \left( 5 \sqrt{2 + \sqrt{3}} d \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right) \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + d x)^{1/3} + b^{2/3} (c + d x)^{2/3}}{\left( (1 - \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right)^2}} \right. \\
& \left. \text{EllipticE} \left[ \text{ArcSin} \left[ \frac{(1 + \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}}{(1 - \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}} \right], -7 + 4\sqrt{3} \right] \right) / \\
& \left( 3 \times 3^{3/4} b^{2/3} (b c - a d)^{5/3} \sqrt{a + b x} \sqrt{-\frac{(b c - a d)^{1/3} \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right)}{\left( (1 - \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right)^2}} + \right. \\
& \left( 10 \sqrt{2} d \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right) \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + d x)^{1/3} + b^{2/3} (c + d x)^{2/3}}{\left( (1 - \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right)^2}} \right. \\
& \left. \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{(1 + \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}}{(1 - \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}} \right], -7 + 4\sqrt{3} \right] \right) / \\
& \left( 9 \times 3^{1/4} b^{2/3} (b c - a d)^{5/3} \sqrt{a + b x} \sqrt{-\frac{(b c - a d)^{1/3} \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right)}{\left( (1 - \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right)^2}} \right)
\end{aligned}$$

Result (type 5, 105 leaves):

$$\frac{(c + d x)^{2/3} \left( 4 (-3 b c + 8 a d + 5 b d x) - 5 d (a + b x) \sqrt{\frac{d (a + b x)}{-b c + a d}} \text{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{2}{3}, \frac{5}{3}, \frac{b (c + d x)}{b c - a d} \right] \right)}{18 (b c - a d)^2 (a + b x)^{3/2}}$$

Problem 1570: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{3/2}}{(c + d x)^{2/3}} dx$$

Optimal (type 4, 416 leaves, 4 steps):

$$\begin{aligned}
& - \frac{54 (b c - a d) \sqrt{a + b x} (c + d x)^{1/3}}{55 d^2} + \frac{6 (a + b x)^{3/2} (c + d x)^{1/3}}{11 d} - \\
& \left( 54 \times 3^{3/4} \sqrt{2 - \sqrt{3}} (b c - a d)^2 \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right) \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + d x)^{1/3} + b^{2/3} (c + d x)^{2/3}}{\left( (1 - \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right)^2}} \right. \\
& \left. \text{EllipticF}[\text{ArcSin}\left[\frac{(1 + \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}}{(1 - \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}}\right], -7 + 4 \sqrt{3}] \right) / \\
& \left( 55 b^{1/3} d^3 \sqrt{a + b x} \sqrt{-\frac{(b c - a d)^{1/3} \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right)}{\left( (1 - \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right)^2}} \right)
\end{aligned}$$

Result (type 5, 108 leaves):

$$\frac{1}{55 d^3 \sqrt{a + b x}} 3 (c + d x)^{1/3} \left( 2 d (a + b x) (-9 b c + 14 a d + 5 b d x) + 27 (b c - a d)^2 \sqrt{\frac{d (a + b x)}{-b c + a d}} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, \frac{b (c + d x)}{b c - a d}\right] \right)$$

Problem 1571: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{a + b x}}{(c + d x)^{2/3}} dx$$

Optimal (type 4, 381 leaves, 3 steps):

$$\begin{aligned}
& \frac{6 \sqrt{a + b x} (c + d x)^{1/3}}{5 d} + \\
& \left( 6 \times 3^{3/4} \sqrt{2 - \sqrt{3}} (b c - a d) \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right) \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + d x)^{1/3} + b^{2/3} (c + d x)^{2/3}}{\left( (1 - \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right)^2}} \right. \\
& \left. \text{EllipticF}[\text{ArcSin}\left[\frac{(1 + \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}}{(1 - \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}}\right], -7 + 4 \sqrt{3}] \right) / \\
& \left( 5 b^{1/3} d^2 \sqrt{a + b x} \sqrt{-\frac{(b c - a d)^{1/3} \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right)}{\left( (1 - \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right)^2}} \right)
\end{aligned}$$

Result (type 5, 77 leaves) :

$$\frac{3 \sqrt{a+b x} (c+d x)^{1/3}}{5 d} \left( 2 + \frac{3 \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, \frac{b(c+d x)}{b c-a d}\right]}{\sqrt{\frac{d(a+b x)}{-b c+a d}}} \right)$$

Problem 1572: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\sqrt{a+b x} (c+d x)^{2/3}} dx$$

Optimal (type 4, 345 leaves, 2 steps) :

$$-\left( 2 \times 3^{3/4} \sqrt{2 - \sqrt{3}} \left( (b c - a d)^{1/3} - b^{1/3} (c+d x)^{1/3} \right) \right. \\ \left. \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c+d x)^{1/3} + b^{2/3} (c+d x)^{2/3}}{\left( (1 - \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c+d x)^{1/3} \right)^2}} \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\left( 1 + \sqrt{3} \right) (b c - a d)^{1/3} - b^{1/3} (c+d x)^{1/3}}{\left( 1 - \sqrt{3} \right) (b c - a d)^{1/3} - b^{1/3} (c+d x)^{1/3}} \right], -7 + 4 \sqrt{3} \right] \right) / \\ \left( b^{1/3} d \sqrt{a+b x} \sqrt{-\frac{(b c - a d)^{1/3} \left( (b c - a d)^{1/3} - b^{1/3} (c+d x)^{1/3} \right)}{\left( (1 - \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c+d x)^{1/3} \right)^2}} \right)$$

Result (type 5, 71 leaves) :

$$\frac{3 \sqrt{\frac{d(a+b x)}{-b c+a d}} (c+d x)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, \frac{b(c+d x)}{b c-a d}\right]}{d \sqrt{a+b x}}$$

Problem 1573: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a+b x)^{3/2} (c+d x)^{2/3}} dx$$

Optimal (type 4, 383 leaves, 3 steps) :

$$\begin{aligned}
& - \frac{2 (c + d x)^{1/3}}{(b c - a d) \sqrt{a + b x}} + \left( 2 \sqrt{2 - \sqrt{3}} \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right) \right. \\
& \left. \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + d x)^{1/3} + b^{2/3} (c + d x)^{2/3}}{\left( (1 - \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right)^2}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[ \frac{\left( 1 + \sqrt{3} \right) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}}{\left( 1 - \sqrt{3} \right) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}} \right], -7 + 4\sqrt{3}] \right) / \\
& \left( 3^{1/4} b^{1/3} (b c - a d) \sqrt{a + b x} \sqrt{-\frac{(b c - a d)^{1/3} ((b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3})}{\left( (1 - \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right)^2}}} \right)
\end{aligned}$$

Result (type 5, 81 leaves):

$$\begin{aligned}
& - \frac{(c + d x)^{1/3} \left( 2 + \sqrt{\frac{d (a + b x)}{-b c + a d}} \operatorname{Hypergeometric2F1}\left[ \frac{1}{3}, \frac{1}{2}, \frac{4}{3}, \frac{b (c + d x)}{b c - a d} \right] \right)}{(b c - a d) \sqrt{a + b x}}
\end{aligned}$$

Problem 1574: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + b x)^{5/2} (c + d x)^{2/3}} dx$$

Optimal (type 4, 421 leaves, 4 steps):

$$\begin{aligned}
& - \frac{2 (c + d x)^{1/3}}{3 (b c - a d) (a + b x)^{3/2}} + \frac{14 d (c + d x)^{1/3}}{9 (b c - a d)^2 \sqrt{a + b x}} - \\
& \left( 14 \sqrt{2 - \sqrt{3}} d \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right) \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + d x)^{1/3} + b^{2/3} (c + d x)^{2/3}}{\left( (1 - \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right)^2}} \right. \\
& \left. \operatorname{EllipticF}[\operatorname{ArcSin}\left[ \frac{\left( 1 + \sqrt{3} \right) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}}{\left( 1 - \sqrt{3} \right) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}} \right], -7 + 4\sqrt{3}] \right) / \\
& \left( 9 \times 3^{1/4} b^{1/3} (b c - a d)^2 \sqrt{a + b x} \sqrt{-\frac{(b c - a d)^{1/3} ((b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3})}{\left( (1 - \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right)^2}}} \right)
\end{aligned}$$

Result (type 5, 102 leaves):

$$\frac{(c + d x)^{1/3} \left( -6 b c + 20 a d + 14 b d x + 7 d (a + b x) \sqrt{\frac{d (a+b x)}{-b c+a d}} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, \frac{b (c+d x)}{b c-a d}\right] \right)}{9 (b c - a d)^2 (a + b x)^{3/2}}$$

**Problem 1575:** Result unnecessarily involves higher level functions.

$$\int (a + b x)^{2/3} (c + d x)^{1/3} dx$$

Optimal (type 3, 219 leaves, 3 steps):

$$\begin{aligned} & \frac{(b c - a d) (a + b x)^{2/3} (c + d x)^{1/3}}{6 b d} + \frac{(a + b x)^{5/3} (c + d x)^{1/3}}{2 b} + \\ & \frac{(b c - a d)^2 \text{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 d^{1/3} (a+b x)^{1/3}}{\sqrt{3} b^{1/3} (c+d x)^{1/3}}\right]}{3 \sqrt{3} b^{4/3} d^{5/3}} + \frac{(b c - a d)^2 \text{Log}[c + d x]}{18 b^{4/3} d^{5/3}} + \frac{(b c - a d)^2 \text{Log}\left[-1 + \frac{d^{1/3} (a+b x)^{1/3}}{b^{1/3} (c+d x)^{1/3}}\right]}{6 b^{4/3} d^{5/3}} \end{aligned}$$

Result (type 5, 109 leaves):

$$\frac{1}{6 b d^2 (a + b x)^{1/3}} (c + d x)^{1/3} \left( d (a + b x) (2 a d + b (c + 3 d x)) - 2 (b c - a d)^2 \left(\frac{d (a + b x)}{-b c + a d}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{b (c + d x)}{b c - a d}\right] \right)$$

**Problem 1576:** Result unnecessarily involves higher level functions.

$$\int \frac{(c + d x)^{1/3}}{(a + b x)^{1/3}} dx$$

Optimal (type 3, 172 leaves, 2 steps):

$$\begin{aligned} & \frac{(a + b x)^{2/3} (c + d x)^{1/3}}{b} - \frac{(b c - a d) \text{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 d^{1/3} (a+b x)^{1/3}}{\sqrt{3} b^{1/3} (c+d x)^{1/3}}\right]}{\sqrt{3} b^{4/3} d^{2/3}} - \frac{(b c - a d) \text{Log}[c + d x]}{6 b^{4/3} d^{2/3}} - \frac{(b c - a d) \text{Log}\left[-1 + \frac{d^{1/3} (a+b x)^{1/3}}{b^{1/3} (c+d x)^{1/3}}\right]}{2 b^{4/3} d^{2/3}} \end{aligned}$$

Result (type 5, 90 leaves):

$$\frac{(c + d x)^{1/3} \left( d (a + b x) + (b c - a d) \left(\frac{d (a+b x)}{-b c+a d}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{b (c+d x)}{b c-a d}\right] \right)}{b d (a + b x)^{1/3}}$$

### Problem 1577: Result unnecessarily involves higher level functions.

$$\int \frac{(c + d x)^{1/3}}{(a + b x)^{4/3}} dx$$

Optimal (type 3, 149 leaves, 2 steps):

$$-\frac{3 (c + d x)^{1/3}}{b (a + b x)^{1/3}} - \frac{\sqrt{3} d^{1/3} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 d^{1/3} (a+b x)^{1/3}}{\sqrt{3} b^{1/3} (c+d x)^{1/3}}\right]}{b^{4/3}} - \frac{d^{1/3} \operatorname{Log}[c + d x]}{2 b^{4/3}} - \frac{3 d^{1/3} \operatorname{Log}\left[-1 + \frac{d^{1/3} (a+b x)^{1/3}}{b^{1/3} (c+d x)^{1/3}}\right]}{2 b^{4/3}}$$

Result (type 5, 74 leaves):

$$\frac{3 (c + d x)^{1/3} \left(-1 + \left(\frac{d (a+b x)}{-b c+a d}\right)^{1/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{b (c+d x)}{b c-a d}\right]\right)}{b (a + b x)^{1/3}}$$

### Problem 1582: Result unnecessarily involves higher level functions.

$$\int (a + b x)^{4/3} (c + d x)^{1/3} dx$$

Optimal (type 4, 655 leaves, 6 steps):

$$\begin{aligned} & -\frac{3 (b c - a d)^2 (a + b x)^{1/3} (c + d x)^{1/3}}{20 b d^2} + \frac{3 (b c - a d) (a + b x)^{4/3} (c + d x)^{1/3}}{40 b d} + \frac{3 (a + b x)^{7/3} (c + d x)^{1/3}}{8 b} + \\ & \left( 3^{3/4} \sqrt{2 + \sqrt{3}} (b c - a d)^3 ((a + b x) (c + d x))^{2/3} \sqrt{(b c + a d + 2 b d x)^2} \left( (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right) \right. \\ & \quad \left. \sqrt{\left( (b c - a d)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (b c - a d)^{2/3} ((a + b x) (c + d x))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a + b x) (c + d x))^{2/3} \right) / \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right)^2} \right) \\ & \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}}{\left(1 + \sqrt{3}\right) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}}\right], -7 - 4 \sqrt{3}\right] / \left( 10 \times 2^{2/3} b^{4/3} d^{7/3} (a + b x)^{2/3} \right. \\ & \quad \left. (c + d x)^{2/3} (b c + a d + 2 b d x) \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})}{((1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})^2}} \sqrt{(a d + b (c + 2 d x))^2} \right) \end{aligned}$$

Result (type 5, 140 leaves):

$$-\frac{1}{40 b d^3 (a + b x)^{2/3}} 3 (c + d x)^{1/3} \\ \left( -d (a + b x) (2 a^2 d^2 + a b d (5 c + 9 d x) + b^2 (-2 c^2 + c d x + 5 d^2 x^2)) - 2 (b c - a d)^3 \left( \frac{d (a + b x)}{-b c + a d} \right)^{2/3} \text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{b (c + d x)}{b c - a d} \right] \right)$$

**Problem 1583:** Result unnecessarily involves higher level functions.

$$\int (a + b x)^{1/3} (c + d x)^{1/3} dx$$

Optimal (type 4, 617 leaves, 5 steps):

$$\frac{3 (b c - a d) (a + b x)^{1/3} (c + d x)^{1/3}}{10 b d} + \frac{3 (a + b x)^{4/3} (c + d x)^{1/3}}{5 b} - \\ \left( 3^{3/4} \sqrt{2 + \sqrt{3}} (b c - a d)^2 ((a + b x) (c + d x))^{2/3} \sqrt{(b c + a d + 2 b d x)^2} \left( (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right) \right. \\ \left. \sqrt{\left( ((b c - a d)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (b c - a d)^{2/3} ((a + b x) (c + d x))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a + b x) (c + d x))^{2/3}) / \right.} \right. \\ \left. \left. \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right)^2 \right) \right. \\ \left. \left. \text{EllipticF} [\text{ArcSin} \left[ \frac{(1 - \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}}{(1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}} \right], -7 - 4 \sqrt{3}] \right) / \left( 5 \times 2^{2/3} b^{4/3} d^{4/3} (a + b x)^{2/3} \right. \\ \left. (c + d x)^{2/3} (b c + a d + 2 b d x) \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})}{((1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})^2}} \sqrt{(a d + b (c + 2 d x))^2} \right)$$

Result (type 5, 108 leaves):

$$\frac{1}{10 b d^2 (a + b x)^{2/3}} 3 (c + d x)^{1/3} \left( d (a + b x) (a d + b (c + 2 d x)) - (b c - a d)^2 \left( \frac{d (a + b x)}{-b c + a d} \right)^{2/3} \text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{b (c + d x)}{b c - a d} \right] \right)$$

**Problem 1584:** Result unnecessarily involves higher level functions.

$$\int \frac{(c + d x)^{1/3}}{(a + b x)^{2/3}} dx$$

Optimal (type 4, 576 leaves, 4 steps):

$$\begin{aligned}
& \frac{3 (a + b x)^{1/3} (c + d x)^{1/3}}{2 b} + \\
& \left( 3^{3/4} \sqrt{2 + \sqrt{3}} (b c - a d) ((a + b x) (c + d x))^{2/3} \sqrt{(b c + a d + 2 b d x)^2} \left( (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right) \right. \\
& \quad \left. \sqrt{\left( ((b c - a d)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (b c - a d)^{2/3} ((a + b x) (c + d x))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a + b x) (c + d x))^{2/3}) \right.} \right. \\
& \quad \left. \left. \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right)^2 \right) \right) \\
& \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}}{\left(1 + \sqrt{3}\right) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}}\right], -7 - 4\sqrt{3}\right] \Bigg) \Bigg/ \left( 2^{2/3} b^{4/3} d^{1/3} (a + b x)^{2/3} \right. \\
& \quad \left. (c + d x)^{2/3} (b c + a d + 2 b d x) \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})}{((1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})^2}} \sqrt{(a d + b (c + 2 d x))^2} \right)
\end{aligned}$$

Result (type 5, 93 leaves):

$$\frac{3 (c + d x)^{1/3} \left( d (a + b x) + (b c - a d) \left( \frac{d (a + b x)}{-b c + a d} \right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{b (c + d x)}{b c - a d}\right] \right)}{2 b d (a + b x)^{2/3}}$$

Problem 1585: Result unnecessarily involves higher level functions.

$$\int \frac{(c + d x)^{1/3}}{(a + b x)^{5/3}} dx$$

Optimal (type 4, 568 leaves, 4 steps):

$$\begin{aligned}
& - \frac{3 (c + d x)^{1/3}}{2 b (a + b x)^{2/3}} + \left( 3^{3/4} \sqrt{2 + \sqrt{3}} d^{2/3} ((a + b x) (c + d x))^{2/3} \sqrt{(b c + a d + 2 b d x)^2} \left( (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right) \right. \\
& \quad \left. \sqrt{\left( (b c - a d)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (b c - a d)^{2/3} ((a + b x) (c + d x))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a + b x) (c + d x))^{2/3} \right) /} \right. \\
& \quad \left. \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right)^2 \right) \\
& \quad \text{EllipticF}[\text{ArcSin}\left[ \frac{(1 - \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}}{(1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}} \right], -7 - 4\sqrt{3}] \Bigg) / \\
& \quad \left( 2^{2/3} b^{4/3} (a + b x)^{2/3} (c + d x)^{2/3} (b c + a d + 2 b d x) \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})}{((1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})^2}} \sqrt{(a d + b (c + 2 d x))^2} \right)
\end{aligned}$$

Result (type 5, 76 leaves):

$$\frac{3 (c + d x)^{1/3} \left( -1 + \left( \frac{d (a + b x)}{-b c + a d} \right)^{2/3} \text{Hypergeometric2F1}\left[ \frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{b (c + d x)}{b c - a d} \right] \right)}{2 b (a + b x)^{2/3}}$$

Problem 1586: Result unnecessarily involves higher level functions.

$$\int \frac{(c + d x)^{1/3}}{(a + b x)^{8/3}} dx$$

Optimal (type 4, 617 leaves, 5 steps):

$$\begin{aligned}
& - \frac{3 (c + d x)^{1/3}}{5 b (a + b x)^{5/3}} - \frac{3 d (c + d x)^{1/3}}{10 b (b c - a d) (a + b x)^{2/3}} - \\
& \left( 3^{3/4} \sqrt{2 + \sqrt{3}} d^{5/3} ((a + b x) (c + d x))^{2/3} \sqrt{(b c + a d + 2 b d x)^2} \left( (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right) \right. \\
& \left. \sqrt{\left( (b c - a d)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (b c - a d)^{2/3} ((a + b x) (c + d x))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a + b x) (c + d x))^{2/3} \right) /} \right. \\
& \left. \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right)^2 \right) \\
& \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{(1 - \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}}{(1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}} \right], -7 - 4\sqrt{3} \right] / \left( 5 \times 2^{2/3} b^{4/3} (b c - a d) \right. \\
& \left. (a + b x)^{2/3} (c + d x)^{2/3} (b c + a d + 2 b d x) \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})}{((1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})^2}} \sqrt{(a d + b (c + 2 d x))^2} \right)
\end{aligned}$$

Result (type 5, 103 leaves):

$$\frac{3 (c + d x)^{1/3} \left( 2 b c - a d + b d x + d (a + b x) \left( \frac{d (a + b x)}{-b c + a d} \right)^{2/3} \text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{b (c + d x)}{b c - a d} \right] \right)}{10 b (-b c + a d) (a + b x)^{5/3}}$$

### Problem 1587: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{4/3}}{(c + d x)^{1/3}} dx$$

Optimal (type 3, 216 leaves, 3 steps):

$$\begin{aligned}
& - \frac{2 (b c - a d) (a + b x)^{1/3} (c + d x)^{2/3}}{3 d^2} + \frac{(a + b x)^{4/3} (c + d x)^{2/3}}{2 d} - \\
& \frac{2 (b c - a d)^2 \text{ArcTan} \left[ \frac{1}{\sqrt{3}} + \frac{2 b^{1/3} (c + d x)^{1/3}}{\sqrt{3} d^{1/3} (a + b x)^{1/3}} \right]}{3 \sqrt{3} b^{2/3} d^{7/3}} - \frac{(b c - a d)^2 \text{Log} [a + b x]}{9 b^{2/3} d^{7/3}} - \frac{(b c - a d)^2 \text{Log} [-1 + \frac{b^{1/3} (c + d x)^{1/3}}{d^{1/3} (a + b x)^{1/3}}]}{3 b^{2/3} d^{7/3}}
\end{aligned}$$

Result (type 5, 107 leaves):

$$\frac{1}{6 d^3 (a + b x)^{2/3}} (c + d x)^{2/3} \left( d (a + b x) (-4 b c + 7 a d + 3 b d x) + 2 (b c - a d)^2 \left( \frac{d (a + b x)}{-b c + a d} \right)^{2/3} \text{Hypergeometric2F1} \left[ \frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b (c + d x)}{b c - a d} \right] \right)$$

### Problem 1588: Result unnecessarily involves higher level functions.

$$\int \frac{(a+b x)^{1/3}}{(c+d x)^{1/3}} dx$$

Optimal (type 3, 171 leaves, 2 steps):

$$\frac{(a+b x)^{1/3} (c+d x)^{2/3}}{d} + \frac{(b c - a d) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 b^{1/3} (c+d x)^{1/3}}{\sqrt{3} d^{1/3} (a+b x)^{1/3}}\right]}{\sqrt{3} b^{2/3} d^{4/3}} + \frac{(b c - a d) \operatorname{Log}[a+b x]}{6 b^{2/3} d^{4/3}} + \frac{(b c - a d) \operatorname{Log}\left[-1 + \frac{b^{1/3} (c+d x)^{1/3}}{d^{1/3} (a+b x)^{1/3}}\right]}{2 b^{2/3} d^{4/3}}$$

Result (type 5, 76 leaves):

$$\frac{(a+b x)^{1/3} (c+d x)^{2/3} \left(2 + \frac{\operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b (c+d x)}{b c - a d}\right]}{\left(\frac{d (a+b x)}{-b c + a d}\right)^{1/3}}\right)}{2 d}$$

### Problem 1589: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a+b x)^{2/3} (c+d x)^{1/3}} dx$$

Optimal (type 3, 126 leaves, 1 step):

$$-\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 b^{1/3} (c+d x)^{1/3}}{\sqrt{3} d^{1/3} (a+b x)^{1/3}}\right]}{b^{2/3} d^{1/3}} - \frac{\operatorname{Log}[a+b x]}{2 b^{2/3} d^{1/3}} - \frac{3 \operatorname{Log}\left[-1 + \frac{b^{1/3} (c+d x)^{1/3}}{d^{1/3} (a+b x)^{1/3}}\right]}{2 b^{2/3} d^{1/3}}$$

Result (type 5, 73 leaves):

$$\frac{3 \left(\frac{d (a+b x)}{-b c + a d}\right)^{2/3} (c+d x)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b (c+d x)}{b c - a d}\right]}{2 d (a+b x)^{2/3}}$$

### Problem 1594: Result unnecessarily involves higher level functions.

$$\int \frac{(a+b x)^{8/3}}{(c+d x)^{1/3}} dx$$

Optimal (type 4, 1365 leaves, 8 steps):

$$\begin{aligned}
& \frac{3 (b c - a d)^2 (a + b x)^{2/3} (c + d x)^{2/3}}{7 d^3} - \frac{12 (b c - a d) (a + b x)^{5/3} (c + d x)^{2/3}}{35 d^2} + \\
& \frac{3 (a + b x)^{8/3} (c + d x)^{2/3}}{10 d} - \left( 3 \times 2^{2/3} (b c - a d)^3 ((a + b x) (c + d x))^{1/3} \sqrt{(b c + a d + 2 b d x)^2} \sqrt{(a d + b (c + 2 d x))^2} \right) / \\
& \left( 7 b^{2/3} d^{11/3} (a + b x)^{1/3} (c + d x)^{1/3} (b c + a d + 2 b d x) \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right) \right) + \\
& \left( 3 \times 3^{1/4} \sqrt{2 - \sqrt{3}} (b c - a d)^{11/3} ((a + b x) (c + d x))^{1/3} \sqrt{(b c + a d + 2 b d x)^2} \left( (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right) \right. \\
& \left. \sqrt{\left( ((b c - a d)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (b c - a d)^{2/3} ((a + b x) (c + d x))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a + b x) (c + d x))^{2/3}) / \right.} \right. \\
& \left. \left. \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right)^2 \right) \right) \\
& \text{EllipticE}[\text{ArcSin}\left[\frac{(1 - \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}}{(1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}}\right], -7 - 4 \sqrt{3}] \Bigg) / \left( 7 \times 2^{1/3} b^{2/3} d^{11/3} (a + b x)^{1/3} \right. \\
& (c + d x)^{1/3} (b c + a d + 2 b d x) \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})}{((1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})^2}} \sqrt{(a d + b (c + 2 d x))^2} \Bigg) - \\
& \left( 2 \times 2^{1/6} \times 3^{3/4} (b c - a d)^{11/3} ((a + b x) (c + d x))^{1/3} \sqrt{(b c + a d + 2 b d x)^2} ((b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}) \right. \\
& \left. \sqrt{\left( ((b c - a d)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (b c - a d)^{2/3} ((a + b x) (c + d x))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a + b x) (c + d x))^{2/3}) / \right.} \right. \\
& \left. \left. \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right)^2 \right) \right) \\
& \text{EllipticF}[\text{ArcSin}\left[\frac{(1 - \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}}{(1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}}\right], -7 - 4 \sqrt{3}] \Bigg) / \left( 7 b^{2/3} d^{11/3} (a + b x)^{1/3} \right. \\
& (c + d x)^{1/3} (b c + a d + 2 b d x) \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})}{((1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})^2}} \sqrt{(a d + b (c + 2 d x))^2} \Bigg)
\end{aligned}$$

Result (type 5, 138 leaves):

$$\frac{1}{70 d^4 (a + b x)^{1/3}} 3 (c + d x)^{2/3} \left( d (a + b x) (25 a^2 d^2 + 2 a b d (-14 c + 11 d x) + b^2 (10 c^2 - 8 c d x + 7 d^2 x^2)) - 10 (b c - a d)^3 \left( \frac{d (a + b x)}{-b c + a d} \right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b (c + d x)}{b c - a d}\right] \right)$$

Problem 1595: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{5/3}}{(c + d x)^{1/3}} dx$$

Optimal (type 4, 1330 leaves, 7 steps):

$$\begin{aligned}
& - \frac{15 (b c - a d) (a + b x)^{2/3} (c + d x)^{2/3}}{28 d^2} + \frac{3 (a + b x)^{5/3} (c + d x)^{2/3}}{7 d} + \\
& \left( 15 (b c - a d)^2 ((a + b x) (c + d x))^{1/3} \sqrt{(b c + a d + 2 b d x)^2} \sqrt{(a d + b (c + 2 d x))^2} \right) / \\
& \left( 14 \times 2^{1/3} b^{2/3} d^{8/3} (a + b x)^{1/3} (c + d x)^{1/3} (b c + a d + 2 b d x) \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right) \right) - \\
& \left( 15 \times 3^{1/4} \sqrt{2 - \sqrt{3}} (b c - a d)^{8/3} ((a + b x) (c + d x))^{1/3} \sqrt{(b c + a d + 2 b d x)^2} \left( (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right) \right. \\
& \left. \sqrt{\left( (b c - a d)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (b c - a d)^{2/3} ((a + b x) (c + d x))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a + b x) (c + d x))^{2/3} \right)} / \right. \\
& \left. \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right)^2 \right) \\
& \text{EllipticE}[\text{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}}{\left(1 + \sqrt{3}\right) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}}, -7 - 4 \sqrt{3}\right]] / \left( 28 \times 2^{1/3} b^{2/3} d^{8/3} (a + b x)^{1/3} \right. \\
& (c + d x)^{1/3} (b c + a d + 2 b d x) \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})}{\left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right)^2}} \sqrt{(a d + b (c + 2 d x))^2} \right) + \\
& \left( 5 \times 3^{3/4} (b c - a d)^{8/3} ((a + b x) (c + d x))^{1/3} \sqrt{(b c + a d + 2 b d x)^2} \left( (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right) \right. \\
& \left. \sqrt{\left( (b c - a d)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (b c - a d)^{2/3} ((a + b x) (c + d x))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a + b x) (c + d x))^{2/3} \right)} / \right. \\
& \left. \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right)^2 \right) \\
& \text{EllipticF}[\text{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}}{\left(1 + \sqrt{3}\right) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}}, -7 - 4 \sqrt{3}\right]] / \left( 7 \times 2^{5/6} b^{2/3} d^{8/3} (a + b x)^{1/3} \right. \\
& (c + d x)^{1/3} (b c + a d + 2 b d x) \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})}{\left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right)^2}} \sqrt{(a d + b (c + 2 d x))^2} \right)
\end{aligned}$$

### Result (type 5, 107 leaves):

$$\frac{1}{28 d^3 (a + b x)^{1/3}} 3 (c + d x)^{2/3} \left( d (a + b x) (-5 b c + 9 a d + 4 b d x) + 5 (b c - a d)^2 \left( \frac{d (a + b x)}{-b c + a d} \right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b (c + d x)}{b c - a d}\right] \right)$$

### Problem 1596: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{2/3}}{(c + d x)^{1/3}} dx$$

Optimal (type 4, 1293 leaves, 6 steps):

$$\begin{aligned} & \frac{3 (a + b x)^{2/3} (c + d x)^{2/3}}{4 d} - \left( 3 (b c - a d) ((a + b x) (c + d x))^{1/3} \sqrt{(b c + a d + 2 b d x)^2} \sqrt{(a d + b (c + 2 d x))^2} \right) / \\ & \left( 2 \times 2^{1/3} b^{2/3} d^{5/3} (a + b x)^{1/3} (c + d x)^{1/3} (b c + a d + 2 b d x) \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right) + \right. \\ & \left( 3 \times 3^{1/4} \sqrt{2 - \sqrt{3}} (b c - a d)^{5/3} ((a + b x) (c + d x))^{1/3} \sqrt{(b c + a d + 2 b d x)^2} \left( (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right) \right. \\ & \left. \sqrt{\left( (b c - a d)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (b c - a d)^{2/3} ((a + b x) (c + d x))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a + b x) (c + d x))^{2/3} \right) /} \right. \\ & \left. \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right)^2 \right) \\ & \text{EllipticE} [\text{ArcSin} \left[ \frac{(1 - \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}}{(1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}} \right], -7 - 4 \sqrt{3}] / \left( 4 \times 2^{1/3} b^{2/3} d^{5/3} (a + b x)^{1/3} \right. \\ & (c + d x)^{1/3} (b c + a d + 2 b d x) \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})}{((1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})^2}} \sqrt{(a d + b (c + 2 d x))^2} \Bigg) - \\ & \left( 3^{3/4} (b c - a d)^{5/3} ((a + b x) (c + d x))^{1/3} \sqrt{(b c + a d + 2 b d x)^2} \left( (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right) \right. \\ & \left. \sqrt{\left( (b c - a d)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (b c - a d)^{2/3} ((a + b x) (c + d x))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a + b x) (c + d x))^{2/3} \right) /} \right. \\ & \left. \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right)^2 \right) \\ & \text{EllipticF} [\text{ArcSin} \left[ \frac{(1 - \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}}{(1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}} \right], -7 - 4 \sqrt{3}] / \left( 2^{5/6} b^{2/3} d^{5/3} (a + b x)^{1/3} \right. \\ & (c + d x)^{1/3} (b c + a d + 2 b d x) \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})}{((1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})^2}} \sqrt{(a d + b (c + 2 d x))^2} \Bigg) \end{aligned}$$

Result (type 5, 76 leaves) :

$$\frac{3 (a + b x)^{2/3} (c + d x)^{2/3} \left( 1 + \frac{\text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b (c+d x)}{b c - a d}\right]}{\left(\frac{d (a+b x)}{-b c + a d}\right)^{2/3}} \right)}{4 d}$$

Problem 1597: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + b x)^{1/3} (c + d x)^{1/3}} dx$$

Optimal (type 4, 1257 leaves, 5 steps) :

$$\begin{aligned}
& \left( 3 \left( (a+b x) (c+d x) \right)^{1/3} \sqrt{(b c + a d + 2 b d x)^2} \sqrt{(a d + b (c+2 d x))^2} \right) / \\
& \left( 2^{1/3} b^{2/3} d^{2/3} (a+b x)^{1/3} (c+d x)^{1/3} (b c + a d + 2 b d x) \left( (1+\sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+b x) (c+d x))^{1/3} \right) - \right. \\
& \left. \left( 3 \times 3^{1/4} \sqrt{2-\sqrt{3}} (b c - a d)^{2/3} ((a+b x) (c+d x))^{1/3} \sqrt{(b c + a d + 2 b d x)^2} \left( (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+b x) (c+d x))^{1/3} \right) \right. \right. \\
& \left. \left. \sqrt{\left( (b c - a d)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (b c - a d)^{2/3} ((a+b x) (c+d x))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a+b x) (c+d x))^{2/3} \right) / \right. \right. \\
& \left. \left. \left( (1+\sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+b x) (c+d x))^{1/3} \right)^2 \right) \right. \\
& \text{EllipticE} \left[ \text{ArcSin} \left[ \frac{(1-\sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+b x) (c+d x))^{1/3}}{(1+\sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+b x) (c+d x))^{1/3}} \right], -7 - 4 \sqrt{3} \right] / \left( 2 \times 2^{1/3} b^{2/3} d^{2/3} (a+b x)^{1/3} \right. \\
& (c+d x)^{1/3} (b c + a d + 2 b d x) \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+b x) (c+d x))^{1/3})}{((1+\sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+b x) (c+d x))^{1/3})^2}} \sqrt{(a d + b (c+2 d x))^2} \right) + \\
& \left( 2^{1/6} \times 3^{3/4} (b c - a d)^{2/3} ((a+b x) (c+d x))^{1/3} \sqrt{(b c + a d + 2 b d x)^2} \left( (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+b x) (c+d x))^{1/3} \right) \right. \\
& \left. \sqrt{\left( (b c - a d)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (b c - a d)^{2/3} ((a+b x) (c+d x))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a+b x) (c+d x))^{2/3} \right) / \right. \\
& \left. \left( (1+\sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+b x) (c+d x))^{1/3} \right)^2 \right) \\
& \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{(1-\sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+b x) (c+d x))^{1/3}}{(1+\sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+b x) (c+d x))^{1/3}} \right], -7 - 4 \sqrt{3} \right] / \\
& \left( b^{2/3} d^{2/3} (a+b x)^{1/3} (c+d x)^{1/3} (b c + a d + 2 b d x) \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+b x) (c+d x))^{1/3})}{((1+\sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+b x) (c+d x))^{1/3})^2}} \sqrt{(a d + b (c+2 d x))^2} \right)
\end{aligned}$$

Result (type 5, 73 leaves):

$$\frac{3 \left( \frac{d (a+b x)}{-b c+a d} \right)^{1/3} (c+d x)^{2/3} \text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b (c+d x)}{b c-a d} \right]}{2 d (a+b x)^{1/3}}$$

### Problem 1598: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + b x)^{4/3} (c + d x)^{1/3}} dx$$

Optimal (type 4, 1297 leaves, 6 steps):

$$-\frac{3 (c + d x)^{2/3}}{(b c - a d) (a + b x)^{1/3}} + \left( 3 d^{1/3} ((a + b x) (c + d x))^{1/3} \sqrt{(b c + a d + 2 b d x)^2} \sqrt{(a d + b (c + 2 d x))^2} \right) /$$

$$- \left( 2^{1/3} b^{2/3} (b c - a d) (a + b x)^{1/3} (c + d x)^{1/3} (b c + a d + 2 b d x) \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right) \right) -$$

$$\left( 3 \times 3^{1/4} \sqrt{2 - \sqrt{3}} d^{1/3} ((a + b x) (c + d x))^{1/3} \sqrt{(b c + a d + 2 b d x)^2} \left( (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right) \right)$$

$$\sqrt{\left( (b c - a d)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (b c - a d)^{2/3} ((a + b x) (c + d x))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a + b x) (c + d x))^{2/3} \right) /}$$

$$\left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right)^2$$

$$\text{EllipticE}[\text{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}}{\left(1 + \sqrt{3}\right) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}}\right], -7 - 4 \sqrt{3}] / \left( 2 \times 2^{1/3} b^{2/3} (b c - a d)^{1/3} \right)$$

$$(a + b x)^{1/3} (c + d x)^{1/3} (b c + a d + 2 b d x) \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})}{\left((1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}\right)^2}} \sqrt{(a d + b (c + 2 d x))^2} +$$

$$\left( 2^{1/6} \times 3^{3/4} d^{1/3} ((a + b x) (c + d x))^{1/3} \sqrt{(b c + a d + 2 b d x)^2} \left( (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right) \right)$$

$$\sqrt{\left( (b c - a d)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (b c - a d)^{2/3} ((a + b x) (c + d x))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a + b x) (c + d x))^{2/3} \right) /} \\ \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right)^2$$

$$\text{EllipticF}[\text{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}}{\left(1 + \sqrt{3}\right) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}}\right], -7 - 4 \sqrt{3}] / \left( b^{2/3} (b c - a d)^{1/3} (a + b x)^{1/3} \right)$$

$$(c + d x)^{1/3} (b c + a d + 2 b d x) \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})}{\left((1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}\right)^2}} \sqrt{(a d + b (c + 2 d x))^2}$$

Result (type 5, 83 leaves) :

$$\frac{3 (c + d x)^{2/3} \left( -2 + \left( \frac{d (a+b x)}{-b c+a d} \right)^{1/3} \text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b (c+d x)}{b c-a d} \right] \right)}{2 (b c - a d) (a + b x)^{1/3}}$$

Problem 1599: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + b x)^{7/3} (c + d x)^{1/3}} dx$$

Optimal (type 4, 1335 leaves, 7 steps) :

$$\begin{aligned}
& - \frac{3 (c + d x)^{2/3}}{4 (b c - a d) (a + b x)^{4/3}} + \frac{3 d (c + d x)^{2/3}}{2 (b c - a d)^2 (a + b x)^{1/3}} - \left( 3 d^{4/3} ((a + b x) (c + d x))^{1/3} \sqrt{(b c + a d + 2 b d x)^2} \sqrt{(a d + b (c + 2 d x))^2} \right) / \\
& \left( 2 \times 2^{1/3} b^{2/3} (b c - a d)^2 (a + b x)^{1/3} (c + d x)^{1/3} (b c + a d + 2 b d x) \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right) \right) + \\
& \left( 3 \times 3^{1/4} \sqrt{2 - \sqrt{3}} d^{4/3} ((a + b x) (c + d x))^{1/3} \sqrt{(b c + a d + 2 b d x)^2} \left( (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right) \right. \\
& \left. \sqrt{\left( ((b c - a d)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (b c - a d)^{2/3} ((a + b x) (c + d x))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a + b x) (c + d x))^{2/3}) / \right.} \right. \\
& \left. \left. \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right)^2 \right) \right)
\end{aligned}$$

$$\text{EllipticE}[\text{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}}{\left(1 + \sqrt{3}\right) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}}\right], -7 - 4 \sqrt{3}] / \left(4 \times 2^{1/3} b^{2/3} (b c - a d)^{4/3}\right)$$

$$(a + b x)^{1/3} (c + d x)^{1/3} (b c + a d + 2 b d x) \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})}{\left((1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}\right)^2}} \sqrt{(a d + b (c + 2 d x))^2} -$$

$$\begin{aligned}
& \left( 3^{3/4} d^{4/3} ((a + b x) (c + d x))^{1/3} \sqrt{(b c + a d + 2 b d x)^2} ((b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}) \right. \\
& \left. \sqrt{\left( ((b c - a d)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (b c - a d)^{2/3} ((a + b x) (c + d x))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a + b x) (c + d x))^{2/3}) / \right.} \right. \\
& \left. \left. \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right)^2 \right) \right)
\end{aligned}$$

$$\text{EllipticF}[\text{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}}{\left(1 + \sqrt{3}\right) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}}\right], -7 - 4 \sqrt{3}] / \left(2^{5/6} b^{2/3} (b c - a d)^{4/3}\right)$$

$$(a + b x)^{1/3} (c + d x)^{1/3} (b c + a d + 2 b d x) \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})}{\left((1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}\right)^2}} \sqrt{(a d + b (c + 2 d x))^2}$$

Result (type 5, 100 leaves):

$$-\frac{3 (c + d x)^{2/3} \left(-3 a d + b (c - 2 d x) + d (a + b x) \left(\frac{d (a + b x)}{-b c + a d}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b (c + d x)}{b c - a d}\right]\right)}{4 (b c - a d)^2 (a + b x)^{4/3}}$$

Problem 1600: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + b x)^{10/3} (c + d x)^{1/3}} dx$$

Optimal (type 4, 1372 leaves, 8 steps):

$$\begin{aligned}
& - \frac{3 (c + d x)^{2/3}}{7 (b c - a d) (a + b x)^{7/3}} + \frac{15 d (c + d x)^{2/3}}{28 (b c - a d)^2 (a + b x)^{4/3}} - \frac{15 d^2 (c + d x)^{2/3}}{14 (b c - a d)^3 (a + b x)^{1/3}} + \\
& \left( \frac{15 d^{7/3} ((a + b x) (c + d x))^{1/3} \sqrt{(b c + a d + 2 b d x)^2} \sqrt{(a d + b (c + 2 d x))^2}}{(14 \times 2^{1/3} b^{2/3} (b c - a d)^3 (a + b x)^{1/3} (c + d x)^{1/3} (b c + a d + 2 b d x) ((1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})} \right) - \\
& \left( \frac{15 \times 3^{1/4} \sqrt{2 - \sqrt{3}} d^{7/3} ((a + b x) (c + d x))^{1/3} \sqrt{(b c + a d + 2 b d x)^2} ((b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})}{\sqrt{\left( ((b c - a d)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (b c - a d)^{2/3} ((a + b x) (c + d x))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a + b x) (c + d x))^{2/3}) / ((1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})^2}} \right) \\
& \text{EllipticE} \left[ \text{ArcSin} \left[ \frac{(1 - \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}}{(1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}} \right], -7 - 4\sqrt{3} \right] / \left( 28 \times 2^{1/3} b^{2/3} (b c - a d)^{7/3} \right) \\
& (a + b x)^{1/3} (c + d x)^{1/3} (b c + a d + 2 b d x) \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})}{((1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})^2}} \sqrt{(a d + b (c + 2 d x))^2} + \\
& \left( 5 \times 3^{3/4} d^{7/3} ((a + b x) (c + d x))^{1/3} \sqrt{(b c + a d + 2 b d x)^2} ((b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}) \right. \\
& \left. \sqrt{\left( ((b c - a d)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (b c - a d)^{2/3} ((a + b x) (c + d x))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a + b x) (c + d x))^{2/3}) / ((1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})^2}} \right) \\
& \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{(1 - \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}}{(1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}} \right], -7 - 4\sqrt{3} \right] / \left( 7 \times 2^{5/6} b^{2/3} (b c - a d)^{7/3} \right) \\
& (a + b x)^{1/3} (c + d x)^{1/3} (b c + a d + 2 b d x) \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})}{((1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})^2}} \sqrt{(a d + b (c + 2 d x))^2}
\end{aligned}$$

Result (type 5, 136 leaves):

$$\frac{1}{28 (b c - a d)^3 (a + b x)^{7/3}} 3 (c + d x)^{2/3} \left( -19 a^2 d^2 + a b d (13 c - 25 d x) + b^2 (-4 c^2 + 5 c d x - 10 d^2 x^2) + 5 d^2 (a + b x)^2 \left( \frac{d (a + b x)}{-b c + a d} \right)^{1/3} \text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b (c + d x)}{b c - a d} \right] \right)$$

**Problem 1601:** Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{5/3}}{(c + d x)^{2/3}} dx$$

Optimal (type 3, 216 leaves, 3 steps):

$$-\frac{5 (b c - a d) (a + b x)^{2/3} (c + d x)^{1/3}}{6 d^2} + \frac{(a + b x)^{5/3} (c + d x)^{1/3}}{2 d} - \frac{5 (b c - a d)^2 \text{ArcTan} \left[ \frac{1}{\sqrt{3}} + \frac{2 d^{1/3} (a + b x)^{1/3}}{\sqrt{3} b^{1/3} (c + d x)^{1/3}} \right]}{3 \sqrt{3} b^{1/3} d^{8/3}} - \frac{5 (b c - a d)^2 \text{Log} [c + d x]}{18 b^{1/3} d^{8/3}} - \frac{5 (b c - a d)^2 \text{Log} \left[ -1 + \frac{d^{1/3} (a + b x)^{1/3}}{b^{1/3} (c + d x)^{1/3}} \right]}{6 b^{1/3} d^{8/3}}$$

Result (type 5, 107 leaves):

$$\frac{1}{6 d^3 (a + b x)^{1/3}} (c + d x)^{1/3} \left( d (a + b x) (-5 b c + 8 a d + 3 b d x) + 10 (b c - a d)^2 \left( \frac{d (a + b x)}{-b c + a d} \right)^{1/3} \text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{b (c + d x)}{b c - a d} \right] \right)$$

**Problem 1602:** Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{2/3}}{(c + d x)^{2/3}} dx$$

Optimal (type 3, 169 leaves, 2 steps):

$$\frac{(a + b x)^{2/3} (c + d x)^{1/3}}{d} + \frac{2 (b c - a d) \text{ArcTan} \left[ \frac{1}{\sqrt{3}} + \frac{2 d^{1/3} (a + b x)^{1/3}}{\sqrt{3} b^{1/3} (c + d x)^{1/3}} \right]}{\sqrt{3} b^{1/3} d^{5/3}} + \frac{(b c - a d) \text{Log} [c + d x]}{3 b^{1/3} d^{5/3}} + \frac{(b c - a d) \text{Log} \left[ -1 + \frac{d^{1/3} (a + b x)^{1/3}}{b^{1/3} (c + d x)^{1/3}} \right]}{b^{1/3} d^{5/3}}$$

Result (type 5, 74 leaves):

$$\frac{(a + b x)^{2/3} (c + d x)^{1/3}}{d} \left( 1 + \frac{2 \text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{b (c + d x)}{b c - a d} \right]}{\left( \frac{d (a + b x)}{-b c + a d} \right)^{2/3}} \right)$$

### Problem 1603: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + b x)^{1/3} (c + d x)^{2/3}} dx$$

Optimal (type 3, 126 leaves, 1 step):

$$-\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2 d^{1/3} (a+b x)^{1/3}}{\sqrt{3} b^{1/3} (c+d x)^{1/3}}\right]}{b^{1/3} d^{2/3}}-\frac{\operatorname{Log}[c+d x]}{2 b^{1/3} d^{2/3}}-\frac{3 \operatorname{Log}\left[-1+\frac{d^{1/3} (a+b x)^{1/3}}{b^{1/3} (c+d x)^{1/3}}\right]}{2 b^{1/3} d^{2/3}}$$

Result (type 5, 71 leaves):

$$\frac{3 \left(\frac{d (a+b x)}{-b c+a d}\right)^{1/3} (c+d x)^{1/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{b (c+d x)}{b c-a d}\right]}{d (a+b x)^{1/3}}$$

### Problem 1608: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{7/3}}{(c + d x)^{2/3}} dx$$

Optimal (type 4, 649 leaves, 6 steps):

$$\begin{aligned} & \frac{21 (b c - a d)^2 (a + b x)^{1/3} (c + d x)^{1/3}}{20 d^3} - \frac{21 (b c - a d) (a + b x)^{4/3} (c + d x)^{1/3}}{40 d^2} + \frac{3 (a + b x)^{7/3} (c + d x)^{1/3}}{8 d} - \\ & \left( 7 \times 3^{3/4} \sqrt{2 + \sqrt{3}} (b c - a d)^3 ((a + b x) (c + d x))^{2/3} \sqrt{(b c + a d + 2 b d x)^2} \left( (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right) \right. \\ & \quad \sqrt{\left( ((b c - a d)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (b c - a d)^{2/3} ((a + b x) (c + d x))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a + b x) (c + d x))^{2/3}) \right.} \\ & \quad \left. \left. \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right)^2 \right) \right) \\ & \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}}{\left(1 + \sqrt{3}\right) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}}\right], -7 - 4 \sqrt{3}\right] \Bigg/ \left( 10 \times 2^{2/3} b^{1/3} d^{10/3} (a + b x)^{2/3} \right. \\ & \quad \left. (c + d x)^{2/3} (b c + a d + 2 b d x) \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})}{((1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})^2}} \sqrt{(a d + b (c + 2 d x))^2} \right) \end{aligned}$$

Result (type 5, 137 leaves):

$$\frac{1}{40 d^4 (a + b x)^{2/3}} 3 (c + d x)^{1/3} \left( d (a + b x) (26 a^2 d^2 + a b d (-35 c + 17 d x) + b^2 (14 c^2 - 7 c d x + 5 d^2 x^2)) - 14 (b c - a d)^3 \left( \frac{d (a + b x)}{-b c + a d} \right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{b (c + d x)}{b c - a d}\right] \right)$$

Problem 1609: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{4/3}}{(c + d x)^{2/3}} dx$$

Optimal (type 4, 614 leaves, 5 steps):

$$\begin{aligned} & -\frac{6 (b c - a d) (a + b x)^{1/3} (c + d x)^{1/3}}{5 d^2} + \frac{3 (a + b x)^{4/3} (c + d x)^{1/3}}{5 d} + \\ & \left( 2 \times 2^{1/3} \times 3^{3/4} \sqrt{2 + \sqrt{3}} (b c - a d)^2 ((a + b x) (c + d x))^{2/3} \sqrt{(b c + a d + 2 b d x)^2} \left( (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right) \right. \\ & \quad \left. \sqrt{\left( ((b c - a d)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (b c - a d)^{2/3} ((a + b x) (c + d x))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a + b x) (c + d x))^{2/3}) / \right.} \right. \\ & \quad \left. \left. \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right)^2 \right) \right. \\ & \quad \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{(1 - \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}}{(1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}}\right], -7 - 4 \sqrt{3}\right]\right) / \\ & \quad \left( 5 b^{1/3} d^{7/3} (a + b x)^{2/3} (c + d x)^{2/3} (b c + a d + 2 b d x) \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})}{((1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})^2}} \sqrt{(a d + b (c + 2 d x))^2} \right) \end{aligned}$$

Result (type 5, 106 leaves):

$$\frac{1}{5 d^3 (a + b x)^{2/3}} 3 (c + d x)^{1/3} \left( d (a + b x) (-2 b c + 3 a d + b d x) + 2 (b c - a d)^2 \left( \frac{d (a + b x)}{-b c + a d} \right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{b (c + d x)}{b c - a d}\right] \right)$$

Problem 1610: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{1/3}}{(c + d x)^{2/3}} dx$$

Optimal (type 4, 577 leaves, 4 steps):

$$\begin{aligned}
& \frac{3 (a + b x)^{1/3} (c + d x)^{1/3}}{2 d} - \\
& \left( 3^{3/4} \sqrt{2 + \sqrt{3}} (b c - a d) ((a + b x) (c + d x))^{2/3} \sqrt{(b c + a d + 2 b d x)^2} ((b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}) \right. \\
& \quad \left. \sqrt{\left( ((b c - a d)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (b c - a d)^{2/3} ((a + b x) (c + d x))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a + b x) (c + d x))^{2/3}) / \right.} \right. \\
& \quad \left. \left. \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right)^2 \right) \right) \\
& \text{EllipticF}\left[\text{ArcSin}\left[\frac{(1 - \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}}{(1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}}\right], -7 - 4\sqrt{3}\right] / \left( 2^{2/3} b^{1/3} d^{4/3} (a + b x)^{2/3} \right. \\
& \quad \left. (c + d x)^{2/3} (b c + a d + 2 b d x) \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})}{((1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})^2}} \sqrt{(a d + b (c + 2 d x))^2} \right)
\end{aligned}$$

Result (type 5, 76 leaves):

$$\frac{3 (a + b x)^{1/3} (c + d x)^{1/3} \left( 1 + \frac{\text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{b (c + d x)}{b c - a d}\right]}{\left(\frac{d (a + b x)}{-b c + a d}\right)^{1/3}} \right)}{2 d}$$

Problem 1611: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + b x)^{2/3} (c + d x)^{2/3}} dx$$

Optimal (type 4, 542 leaves, 3 steps):

$$\begin{aligned}
& \left( 2^{1/3} \times 3^{3/4} \sqrt{2 + \sqrt{3}} \right) \left( (a + b x) (c + d x) \right)^{2/3} \sqrt{(b c + a d + 2 b d x)^2} \left( (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} \left( (a + b x) (c + d x) \right)^{1/3} \right) \\
& \sqrt{\left( (b c - a d)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (b c - a d)^{2/3} \left( (a + b x) (c + d x) \right)^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} \left( (a + b x) (c + d x) \right)^{2/3} \right) /} \\
& \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} \left( (a + b x) (c + d x) \right)^{1/3} \right)^2 \\
& \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{(1 - \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} \left( (a + b x) (c + d x) \right)^{1/3}}{(1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} \left( (a + b x) (c + d x) \right)^{1/3}} \right], -7 - 4\sqrt{3} \right] / \\
& \left( b^{1/3} d^{1/3} (a + b x)^{2/3} (c + d x)^{2/3} (b c + a d + 2 b d x) \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})}{((1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})^2}} \sqrt{(a d + b (c + 2 d x))^2} \right)
\end{aligned}$$

Result (type 5, 71 leaves):

$$\frac{3 \left( \frac{d (a + b x)}{-b c + a d} \right)^{2/3} (c + d x)^{1/3} \text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{b (c + d x)}{b c - a d} \right]}{d (a + b x)^{2/3}}$$

Problem 1612: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + b x)^{5/3} (c + d x)^{2/3}} dx$$

Optimal (type 4, 586 leaves, 4 steps):

$$\begin{aligned}
& -\frac{3 (c + d x)^{1/3}}{2 (b c - a d) (a + b x)^{2/3}} - \left( 3^{3/4} \sqrt{2 + \sqrt{3}} d^{2/3} ((a + b x) (c + d x))^{2/3} \sqrt{(b c + a d + 2 b d x)^2} \left( (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right) \right. \\
& \sqrt{\left( (b c - a d)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (b c - a d)^{2/3} ((a + b x) (c + d x))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a + b x) (c + d x))^{2/3} \right) /} \\
& \left. \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right)^2 \right) \\
& \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{(1 - \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} \left( (a + b x) (c + d x) \right)^{1/3}}{(1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} \left( (a + b x) (c + d x) \right)^{1/3}} \right], -7 - 4\sqrt{3} \right] / \left( 2^{2/3} b^{1/3} (b c - a d) \right) \\
& (a + b x)^{2/3} (c + d x)^{2/3} (b c + a d + 2 b d x) \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})}{((1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})^2}} \sqrt{(a d + b (c + 2 d x))^2}
\end{aligned}$$

Result (type 5, 83 leaves):

$$-\frac{3 (c + d x)^{1/3} \left(1 + \left(\frac{d (a+b x)}{-b c+a d}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{b (c+d x)}{b c-a d}\right]\right)}{2 (b c - a d) (a + b x)^{2/3}}$$

Problem 1613: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + b x)^{8/3} (c + d x)^{2/3}} dx$$

Optimal (type 4, 621 leaves, 5 steps):

$$\begin{aligned} & -\frac{3 (c + d x)^{1/3}}{5 (b c - a d) (a + b x)^{5/3}} + \frac{6 d (c + d x)^{1/3}}{5 (b c - a d)^2 (a + b x)^{2/3}} + \\ & \left( 2 \times 2^{1/3} \times 3^{3/4} \sqrt{2 + \sqrt{3}} d^{5/3} ((a + b x) (c + d x))^{2/3} \sqrt{(b c + a d + 2 b d x)^2} \left( (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right) \right. \\ & \left. \sqrt{\left( (b c - a d)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (b c - a d)^{2/3} ((a + b x) (c + d x))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a + b x) (c + d x))^{2/3} \right) / \right. \\ & \left. \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right)^2 \right) \\ & \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\left(1 - \sqrt{3}\right) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}}{\left(1 + \sqrt{3}\right) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}} \right], -7 - 4 \sqrt{3} \right] / \left( 5 b^{1/3} (b c - a d)^2 (a + b x)^{2/3} \right. \\ & \left. (c + d x)^{2/3} (b c + a d + 2 b d x) \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})}{((1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})^2}} \sqrt{(a d + b (c + 2 d x))^2} \right) \end{aligned}$$

Result (type 5, 102 leaves):

$$\begin{aligned} & \frac{3 (c + d x)^{1/3} \left(-b c + 3 a d + 2 b d x + 2 d (a + b x) \left(\frac{d (a+b x)}{-b c+a d}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{b (c+d x)}{b c-a d}\right]\right)}{5 (b c - a d)^2 (a + b x)^{5/3}} \end{aligned}$$

Problem 1614: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + b x)^{11/3} (c + d x)^{2/3}} dx$$

Optimal (type 4, 656 leaves, 6 steps):

$$\begin{aligned}
& - \frac{3 (c + d x)^{1/3}}{8 (b c - a d) (a + b x)^{8/3}} + \frac{21 d (c + d x)^{1/3}}{40 (b c - a d)^2 (a + b x)^{5/3}} - \frac{21 d^2 (c + d x)^{1/3}}{20 (b c - a d)^3 (a + b x)^{2/3}} - \\
& \left( 7 \times 3^{3/4} \sqrt{2 + \sqrt{3}} d^{8/3} ((a + b x) (c + d x))^{2/3} \sqrt{(b c + a d + 2 b d x)^2} \left( (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right) \right. \\
& \left. \sqrt{\left( (b c - a d)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (b c - a d)^{2/3} ((a + b x) (c + d x))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a + b x) (c + d x))^{2/3} \right) /} \right. \\
& \left. \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right)^2 \right) \\
& \text{EllipticF}[\text{ArcSin}\left[\frac{(1 - \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}}{(1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}}\right], -7 - 4\sqrt{3}] \Bigg) / \left( 10 \times 2^{2/3} b^{1/3} (b c - a d)^3 \right. \\
& \left. (a + b x)^{2/3} (c + d x)^{2/3} (b c + a d + 2 b d x) \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})}{((1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})^2}} \sqrt{(a d + b (c + 2 d x))^2} \right)
\end{aligned}$$

Result (type 5, 136 leaves):

$$\begin{aligned}
& \left( 3 (c + d x)^{1/3} \left( 26 a^2 d^2 + a b d (-17 c + 35 d x) + b^2 (5 c^2 - 7 c d x + 14 d^2 x^2) + \right. \right. \\
& \left. \left. 14 d^2 (a + b x)^2 \left( \frac{d (a + b x)}{-b c + a d} \right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{b (c + d x)}{b c - a d}\right] \right) \right) / (40 (-b c + a d)^3 (a + b x)^{8/3})
\end{aligned}$$

Problem 1615: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{7/3}}{(c + d x)^{4/3}} dx$$

Optimal (type 3, 241 leaves, 4 steps):

$$\begin{aligned}
& - \frac{3 (a + b x)^{7/3}}{d (c + d x)^{1/3}} - \frac{14 b (b c - a d) (a + b x)^{1/3} (c + d x)^{2/3}}{3 d^3} + \frac{7 b (a + b x)^{4/3} (c + d x)^{2/3}}{2 d^2} - \\
& \frac{14 b^{1/3} (b c - a d)^2 \text{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 b^{1/3} (c + d x)^{1/3}}{\sqrt{3} d^{1/3} (a + b x)^{1/3}}\right]}{3 \sqrt{3} d^{10/3}} - \frac{7 b^{1/3} (b c - a d)^2 \text{Log}[a + b x]}{9 d^{10/3}} - \frac{7 b^{1/3} (b c - a d)^2 \text{Log}\left[-1 + \frac{b^{1/3} (c + d x)^{1/3}}{d^{1/3} (a + b x)^{1/3}}\right]}{3 d^{10/3}}
\end{aligned}$$

Result (type 5, 132 leaves):

$$\frac{1}{6 d^4 (a + b x)^{2/3}} \\ (c + d x)^{2/3} \left( d (a + b x) \left( b (-10 b c + 13 a d) + 3 b^2 d x - \frac{18 (b c - a d)^2}{c + d x} \right) + 14 b (b c - a d)^2 \left( \frac{d (a + b x)}{-b c + a d} \right)^{2/3} \text{Hypergeometric2F1} \left[ \frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b (c + d x)}{b c - a d} \right] \right)$$

**Problem 1616:** Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{4/3}}{(c + d x)^{4/3}} dx$$

Optimal (type 3, 195 leaves, 3 steps):

$$-\frac{3 (a + b x)^{4/3}}{d (c + d x)^{1/3}} + \frac{4 b (a + b x)^{1/3} (c + d x)^{2/3}}{d^2} + \frac{4 b^{1/3} (b c - a d) \text{ArcTan} \left[ \frac{1}{\sqrt{3}} + \frac{2 b^{1/3} (c + d x)^{1/3}}{\sqrt{3} d^{1/3} (a + b x)^{1/3}} \right]}{\sqrt{3} d^{7/3}} + \\ \frac{2 b^{1/3} (b c - a d) \text{Log}[a + b x]}{3 d^{7/3}} + \frac{2 b^{1/3} (b c - a d) \text{Log}[-1 + \frac{b^{1/3} (c + d x)^{1/3}}{d^{1/3} (a + b x)^{1/3}}]}{d^{7/3}}$$

Result (type 5, 95 leaves):

$$\frac{(a + b x)^{1/3} (c + d x)^{2/3} \left( \frac{4 b c - 3 a d + b d x}{c + d x} + \frac{2 b \text{Hypergeometric2F1} \left[ \frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b (c + d x)}{b c - a d} \right]}{\left( \frac{d (a + b x)}{-b c + a d} \right)^{1/3}} \right)}{d^2}$$

**Problem 1617:** Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{1/3}}{(c + d x)^{4/3}} dx$$

Optimal (type 3, 149 leaves, 2 steps):

$$-\frac{3 (a + b x)^{1/3}}{d (c + d x)^{1/3}} - \frac{\sqrt{3} b^{1/3} \text{ArcTan} \left[ \frac{1}{\sqrt{3}} + \frac{2 b^{1/3} (c + d x)^{1/3}}{\sqrt{3} d^{1/3} (a + b x)^{1/3}} \right]}{d^{4/3}} - \frac{b^{1/3} \text{Log}[a + b x]}{2 d^{4/3}} - \frac{3 b^{1/3} \text{Log}[-1 + \frac{b^{1/3} (c + d x)^{1/3}}{d^{1/3} (a + b x)^{1/3}}]}{2 d^{4/3}}$$

Result (type 5, 90 leaves):

$$\frac{-6 d (a + b x) + 3 b \left( \frac{d (a + b x)}{-b c + a d} \right)^{2/3} (c + d x) \text{Hypergeometric2F1} \left[ \frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b (c + d x)}{b c - a d} \right]}{2 d^2 (a + b x)^{2/3} (c + d x)^{1/3}}$$

### Problem 1622: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{8/3}}{(c + d x)^{4/3}} dx$$

Optimal (type 4, 1355 leaves, 8 steps):

$$\begin{aligned}
& - \frac{3 (a + b x)^{8/3}}{d (c + d x)^{1/3}} - \frac{30 b (b c - a d) (a + b x)^{2/3} (c + d x)^{2/3}}{7 d^3} + \frac{24 b (a + b x)^{5/3} (c + d x)^{2/3}}{7 d^2} + \\
& \left( \frac{30 \times 2^{2/3} b^{1/3} (b c - a d)^2 ((a + b x) (c + d x))^{1/3} \sqrt{(b c + a d + 2 b d x)^2} \sqrt{(a d + b (c + 2 d x))^2}}{7 d^{11/3} (a + b x)^{1/3} (c + d x)^{1/3} (b c + a d + 2 b d x) \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right)} \right) - \\
& \left( 15 \times 2^{2/3} \times 3^{1/4} \sqrt{2 - \sqrt{3}} b^{1/3} (b c - a d)^{8/3} ((a + b x) (c + d x))^{1/3} \sqrt{(b c + a d + 2 b d x)^2} \left( (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right) \right. \\
& \quad \left. \sqrt{\left( (b c - a d)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (b c - a d)^{2/3} ((a + b x) (c + d x))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a + b x) (c + d x))^{2/3} \right) /} \right. \\
& \quad \left. \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right)^2 \right) \\
& \quad \text{EllipticE}[\text{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}}{\left(1 + \sqrt{3}\right) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}}\right], -7 - 4\sqrt{3}] \Bigg) / \\
& \left( 7 d^{11/3} (a + b x)^{1/3} (c + d x)^{1/3} (b c + a d + 2 b d x) \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})}{((1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})^2}} \sqrt{(a d + b (c + 2 d x))^2} \right) + \\
& \left( 20 \times 2^{1/6} \times 3^{3/4} b^{1/3} (b c - a d)^{8/3} ((a + b x) (c + d x))^{1/3} \sqrt{(b c + a d + 2 b d x)^2} \left( (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right) \right. \\
& \quad \left. \sqrt{\left( (b c - a d)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (b c - a d)^{2/3} ((a + b x) (c + d x))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a + b x) (c + d x))^{2/3} \right) /} \right. \\
& \quad \left. \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right)^2 \right) \\
& \quad \text{EllipticF}[\text{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}}{\left(1 + \sqrt{3}\right) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}}\right], -7 - 4\sqrt{3}] \Bigg) / \\
& \left( 7 d^{11/3} (a + b x)^{1/3} (c + d x)^{1/3} (b c + a d + 2 b d x) \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})}{((1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})^2}} \sqrt{(a d + b (c + 2 d x))^2} \right)
\end{aligned}$$

Result (type 5, 131 leaves):

$$\frac{1}{7 d^4 (a + b x)^{1/3}} - \frac{3 (c + d x)^{2/3} \left( d (a + b x) \left( b (-3 b c + 4 a d) + b^2 d x - \frac{7 (b c - a d)^2}{c + d x} \right) + 10 b (b c - a d)^2 \left( \frac{d (a + b x)}{-b c + a d} \right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b (c + d x)}{b c - a d}\right]\right)}{(c + d x)^{4/3}}$$

Problem 1623: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{5/3}}{(c + d x)^{4/3}} dx$$

Optimal (type 4, 1317 leaves, 7 steps):

$$\begin{aligned}
& - \frac{3 (a+b x)^{5/3}}{d (c+d x)^{1/3}} + \frac{15 b (a+b x)^{2/3} (c+d x)^{2/3}}{4 d^2} - \left( 15 b^{1/3} (b c - a d) ((a+b x) (c+d x))^{1/3} \sqrt{(b c + a d + 2 b d x)^2} \sqrt{(a d + b (c+2 d x))^2} \right) / \\
& \left( 2 \times 2^{1/3} d^{8/3} (a+b x)^{1/3} (c+d x)^{1/3} (b c + a d + 2 b d x) \left( (1+\sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+b x) (c+d x))^{1/3} \right) \right) + \\
& \left( 15 \times 3^{1/4} \sqrt{2-\sqrt{3}} b^{1/3} (b c - a d)^{5/3} ((a+b x) (c+d x))^{1/3} \sqrt{(b c + a d + 2 b d x)^2} ((b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+b x) (c+d x))^{1/3}) \right) / \\
& \sqrt{\left( ((b c - a d)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (b c - a d)^{2/3} ((a+b x) (c+d x))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a+b x) (c+d x))^{2/3}) / \right.} \\
& \left. \left( (1+\sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+b x) (c+d x))^{1/3} \right)^2 \right) \\
& \text{EllipticE}[\text{ArcSin}\left[\frac{(1-\sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+b x) (c+d x))^{1/3}}{(1+\sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+b x) (c+d x))^{1/3}}\right], -7 - 4 \sqrt{3}] \Bigg) / \left( 4 \times 2^{1/3} d^{8/3} (a+b x)^{1/3} \right. \\
& (c+d x)^{1/3} (b c + a d + 2 b d x) \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+b x) (c+d x))^{1/3})}{((1+\sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+b x) (c+d x))^{1/3})^2}} \sqrt{(a d + b (c+2 d x))^2} \Bigg) - \\
& \left( 5 \times 3^{3/4} b^{1/3} (b c - a d)^{5/3} ((a+b x) (c+d x))^{1/3} \sqrt{(b c + a d + 2 b d x)^2} ((b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+b x) (c+d x))^{1/3}) \right. \\
& \sqrt{\left( ((b c - a d)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (b c - a d)^{2/3} ((a+b x) (c+d x))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a+b x) (c+d x))^{2/3}) / \right.} \\
& \left. \left( (1+\sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+b x) (c+d x))^{1/3} \right)^2 \right) \\
& \text{EllipticF}[\text{ArcSin}\left[\frac{(1-\sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+b x) (c+d x))^{1/3}}{(1+\sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+b x) (c+d x))^{1/3}}\right], -7 - 4 \sqrt{3}] \Bigg) / \\
& \left( 2^{5/6} d^{8/3} (a+b x)^{1/3} (c+d x)^{1/3} (b c + a d + 2 b d x) \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+b x) (c+d x))^{1/3})}{((1+\sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+b x) (c+d x))^{1/3})^2}} \sqrt{(a d + b (c+2 d x))^2} \right)
\end{aligned}$$

Result (type 5, 98 leaves):

$$\frac{3 (a+b x)^{2/3} (c+d x)^{2/3} \left( \frac{5 b c - 4 a d + b d x}{c+d x} + \frac{5 b \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b (c+d x)}{b c - a d}\right]}{\left(\frac{d (a+b x)}{-b c + a d}\right)^{2/3}} \right)}{4 d^2}$$

### Problem 1624: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{2/3}}{(c + d x)^{4/3}} dx$$

Optimal (type 4, 1279 leaves, 6 steps):

$$\begin{aligned}
& -\frac{3 (a + b x)^{2/3}}{d (c + d x)^{1/3}} + \left( 3 \times 2^{2/3} b^{1/3} ((a + b x) (c + d x))^{1/3} \sqrt{(b c + a d + 2 b d x)^2} \sqrt{(a d + b (c + 2 d x))^2} \right) / \\
& \left( d^{5/3} (a + b x)^{1/3} (c + d x)^{1/3} (b c + a d + 2 b d x) \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right) - \right. \\
& \left( 3 \times 3^{1/4} \sqrt{2 - \sqrt{3}} b^{1/3} (b c - a d)^{2/3} ((a + b x) (c + d x))^{1/3} \sqrt{(b c + a d + 2 b d x)^2} \left( (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right) \right. \\
& \left. \sqrt{\left( ((b c - a d)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (b c - a d)^{2/3} ((a + b x) (c + d x))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a + b x) (c + d x))^{2/3}) / \right.} \right. \\
& \left. \left. \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right)^2 \right) \right. \\
& \left. \text{EllipticE} \left[ \text{ArcSin} \left[ \frac{(1 - \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}}{(1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}} \right], -7 - 4 \sqrt{3} \right] \right) / \\
& \left( 2^{1/3} d^{5/3} (a + b x)^{1/3} (c + d x)^{1/3} (b c + a d + 2 b d x) \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})}{((1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})^2}} \sqrt{(a d + b (c + 2 d x))^2} \right) + \\
& \left( 2 \times 2^{1/6} \times 3^{3/4} b^{1/3} (b c - a d)^{2/3} ((a + b x) (c + d x))^{1/3} \sqrt{(b c + a d + 2 b d x)^2} \left( (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right) \right. \\
& \left. \sqrt{\left( ((b c - a d)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (b c - a d)^{2/3} ((a + b x) (c + d x))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a + b x) (c + d x))^{2/3}) / \right.} \right. \\
& \left. \left. \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right)^2 \right) \right. \\
& \left. \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{(1 - \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}}{(1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}} \right], -7 - 4 \sqrt{3} \right] \right) / \\
& \left( d^{5/3} (a + b x)^{1/3} (c + d x)^{1/3} (b c + a d + 2 b d x) \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})}{((1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})^2}} \sqrt{(a d + b (c + 2 d x))^2} \right)
\end{aligned}$$

Result (type 5, 87 leaves):

$$\frac{-3d(a + bx) + 3b\left(\frac{d(a+bx)}{-bc+ad}\right)^{1/3}(c + dx)\text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b(c+dx)}{bc-ad}\right]}{d^2(a + bx)^{1/3}(c + dx)^{1/3}}$$

Problem 1625: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + bx)^{1/3} (c + dx)^{4/3}} dx$$

Optimal (type 4, 1298 leaves, 6 steps):

$$\begin{aligned}
& \frac{3 (a+b x)^{2/3}}{(b c - a d) (c + d x)^{1/3}} - \left( 3 b^{1/3} ((a+b x) (c+d x))^{1/3} \sqrt{(b c + a d + 2 b d x)^2} \sqrt{(a d + b (c+2 d x))^2} \right) / \\
& \left( 2^{1/3} d^{2/3} (b c - a d) (a+b x)^{1/3} (c+d x)^{1/3} (b c + a d + 2 b d x) \left( (1+\sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+b x) (c+d x))^{1/3} \right) \right) + \\
& \left( 3 \times 3^{1/4} \sqrt{2 - \sqrt{3}} b^{1/3} ((a+b x) (c+d x))^{1/3} \sqrt{(b c + a d + 2 b d x)^2} \left( (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+b x) (c+d x))^{1/3} \right) \right. \\
& \left. \sqrt{\left( ((b c - a d)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (b c - a d)^{2/3} ((a+b x) (c+d x))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a+b x) (c+d x))^{2/3}) / \right.} \right. \\
& \left. \left. \left( (1+\sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+b x) (c+d x))^{1/3} \right)^2 \right) \right) / \\
& \text{EllipticE} \left[ \text{ArcSin} \left[ \frac{(1-\sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+b x) (c+d x))^{1/3}}{(1+\sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+b x) (c+d x))^{1/3}} \right], -7 - 4 \sqrt{3} \right] / \left( 2 \times 2^{1/3} d^{2/3} (b c - a d)^{1/3} \right. \\
& (a+b x)^{1/3} (c+d x)^{1/3} (b c + a d + 2 b d x) \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+b x) (c+d x))^{1/3})}{((1+\sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+b x) (c+d x))^{1/3})^2}} \sqrt{(a d + b (c+2 d x))^2} \Bigg) - \\
& \left( 2^{1/6} \times 3^{3/4} b^{1/3} ((a+b x) (c+d x))^{1/3} \sqrt{(b c + a d + 2 b d x)^2} ((b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+b x) (c+d x))^{1/3}) \right. \\
& \left. \sqrt{\left( ((b c - a d)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (b c - a d)^{2/3} ((a+b x) (c+d x))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a+b x) (c+d x))^{2/3}) / \right.} \right. \\
& \left. \left. \left( (1+\sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+b x) (c+d x))^{1/3} \right)^2 \right) \right) / \\
& \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{(1-\sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+b x) (c+d x))^{1/3}}{(1+\sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+b x) (c+d x))^{1/3}} \right], -7 - 4 \sqrt{3} \right] / \left( d^{2/3} (b c - a d)^{1/3} (a+b x)^{1/3} \right. \\
& (c+d x)^{1/3} (b c + a d + 2 b d x) \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+b x) (c+d x))^{1/3})}{((1+\sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+b x) (c+d x))^{1/3})^2}} \sqrt{(a d + b (c+2 d x))^2} \Bigg)
\end{aligned}$$

Result (type 5, 100 leaves):

$$\frac{6 d (a+b x) - 3 b \left( \frac{d (a+b x)}{-b c + a d} \right)^{1/3} (c+d x) \text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b (c+d x)}{b c - a d} \right]}{2 d (b c - a d) (a+b x)^{1/3} (c+d x)^{1/3}}$$

Problem 1626: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + b x)^{4/3} (c + d x)^{4/3}} dx$$

Optimal (type 4, 1327 leaves, 7 steps):

$$\begin{aligned}
& - \frac{3}{(b c - a d) (a + b x)^{1/3} (c + d x)^{1/3}} - \frac{6 d (a + b x)^{2/3}}{(b c - a d)^2 (c + d x)^{1/3}} + \\
& \left( 3 \times 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \sqrt{(b c + a d + 2 b d x)^2} \sqrt{(a d + b (c + 2 d x))^2} \right) / \\
& \left( (b c - a d)^2 (a + b x)^{1/3} (c + d x)^{1/3} (b c + a d + 2 b d x) \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right) \right) - \\
& \left( 3 \times 3^{1/4} \sqrt{2 - \sqrt{3}} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \sqrt{(b c + a d + 2 b d x)^2} \left( (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right) \right) \\
& \sqrt{\left( ((b c - a d)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (b c - a d)^{2/3} ((a + b x) (c + d x))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a + b x) (c + d x))^{2/3}) \right) /} \\
& \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right)^2 \\
& \text{EllipticE} \left[ \text{ArcSin} \left[ \frac{(1 - \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}}{(1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}} \right], -7 - 4\sqrt{3} \right] / \left( 2^{1/3} (b c - a d)^{4/3} (a + b x)^{1/3} \right. \\
& \left. (c + d x)^{1/3} (b c + a d + 2 b d x) \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})}{((1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})^2}} \sqrt{(a d + b (c + 2 d x))^2} \right) + \\
& \left( 2 \times 2^{1/6} \times 3^{3/4} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \sqrt{(b c + a d + 2 b d x)^2} \left( (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right) \right. \\
& \left. \sqrt{\left( ((b c - a d)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (b c - a d)^{2/3} ((a + b x) (c + d x))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a + b x) (c + d x))^{2/3}) \right) /} \right. \\
& \left. \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right)^2 \right) \\
& \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{(1 - \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}}{(1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}} \right], -7 - 4\sqrt{3} \right] / \left( (b c - a d)^{4/3} (a + b x)^{1/3} \right. \\
& \left. (c + d x)^{1/3} (b c + a d + 2 b d x) \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})}{((1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})^2}} \sqrt{(a d + b (c + 2 d x))^2} \right)
\end{aligned}$$

Result (type 5, 98 leaves):

$$\begin{aligned}
& - \frac{3 \left( a d + b (c + 2 d x) - b \left( \frac{d (a + b x)}{-b c + a d} \right)^{1/3} (c + d x) \text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b (c + d x)}{b c - a d} \right] \right)}{(b c - a d)^2 (a + b x)^{1/3} (c + d x)^{1/3}}
\end{aligned}$$

Problem 1627: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + b x)^{7/3} (c + d x)^{4/3}} dx$$

Optimal (type 4, 1370 leaves, 8 steps):

$$\begin{aligned}
& - \frac{3}{4 (b c - a d) (a + b x)^{4/3} (c + d x)^{1/3}} + \frac{15 d}{4 (b c - a d)^2 (a + b x)^{1/3} (c + d x)^{1/3}} + \\
& \frac{15 d^2 (a + b x)^{2/3}}{2 (b c - a d)^3 (c + d x)^{1/3}} - \left( \frac{15 b^{1/3} d^{4/3} ((a + b x) (c + d x))^{1/3} \sqrt{(b c + a d + 2 b d x)^2} \sqrt{(a d + b (c + 2 d x))^2}}{\left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right)} \right) / \\
& \left( 2 \times 2^{1/3} (b c - a d)^3 (a + b x)^{1/3} (c + d x)^{1/3} (b c + a d + 2 b d x) \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right) \right) + \\
& \left( 15 \times 3^{1/4} \sqrt{2 - \sqrt{3}} b^{1/3} d^{4/3} ((a + b x) (c + d x))^{1/3} \sqrt{(b c + a d + 2 b d x)^2} \left( (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right) \right) \\
& \sqrt{\left( (b c - a d)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (b c - a d)^{2/3} ((a + b x) (c + d x))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a + b x) (c + d x))^{2/3} \right) /} \\
& \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right)^2 \\
& \text{EllipticE} \left[ \text{ArcSin} \left[ \frac{(1 - \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}}{(1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}} \right], -7 - 4\sqrt{3} \right] / \left( 4 \times 2^{1/3} (b c - a d)^{7/3} \right) \\
& (a + b x)^{1/3} (c + d x)^{1/3} (b c + a d + 2 b d x) \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})}{\left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right)^2}} \sqrt{(a d + b (c + 2 d x))^2} - \\
& \left( 5 \times 3^{3/4} b^{1/3} d^{4/3} ((a + b x) (c + d x))^{1/3} \sqrt{(b c + a d + 2 b d x)^2} \left( (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right) \right. \\
& \left. \sqrt{\left( (b c - a d)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (b c - a d)^{2/3} ((a + b x) (c + d x))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a + b x) (c + d x))^{2/3} \right) /} \right. \\
& \left. \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right)^2 \right) \\
& \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{(1 - \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}}{(1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}} \right], -7 - 4\sqrt{3} \right] / \left( 2^{5/6} (b c - a d)^{7/3} (a + b x)^{1/3} \right) \\
& (c + d x)^{1/3} (b c + a d + 2 b d x) \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})}{\left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right)^2}} \sqrt{(a d + b (c + 2 d x))^2}
\end{aligned}$$

Result (type 5, 138 leaves):

$$-\left( \left( 3 \left( 4 a^2 d^2 + a b d (7 c + 15 d x) + b^2 (-c^2 + 5 c d x + 10 d^2 x^2) - 5 b d (a + b x) \left( \frac{d (a + b x)}{-b c + a d} \right)^{1/3} (c + d x) \text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b (c + d x)}{b c - a d} \right] \right) \right) \Big/ \left( 4 (-b c + a d)^3 (a + b x)^{4/3} (c + d x)^{1/3} \right) \right)$$

**Problem 1628: Result unnecessarily involves higher level functions.**

$$\int \frac{(-1+x)^{1/3}}{(1+x)^{1/3}} dx$$

Optimal (type 3, 77 leaves, 2 steps):

$$(-1+x)^{1/3} (1+x)^{2/3} + \frac{2 \text{ArcTan} \left[ \frac{1}{\sqrt{3}} + \frac{2 (1+x)^{1/3}}{\sqrt{3} (-1+x)^{1/3}} \right]}{\sqrt{3}} + \frac{1}{3} \text{Log}[-1+x] + \text{Log} \left[ -1 + \frac{(1+x)^{1/3}}{(-1+x)^{1/3}} \right]$$

Result (type 5, 50 leaves):

$$\left( \frac{-1+x}{1+x} \right)^{1/3} \left( 1+x - 2^{2/3} (1+x)^{1/3} \text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{1-x}{2} \right] \right)$$

**Problem 1629: Result unnecessarily involves higher level functions.**

$$\int (a+b x)^{3/2} (c+d x)^{1/4} dx$$

Optimal (type 4, 185 leaves, 6 steps):

$$-\frac{8 (b c - a d)^2 \sqrt{a + b x} (c + d x)^{1/4}}{77 b d^2} + \frac{4 (b c - a d) (a + b x)^{3/2} (c + d x)^{1/4}}{77 b d} + \frac{4 (a + b x)^{5/2} (c + d x)^{1/4}}{11 b} + \frac{16 (b c - a d)^{13/4} \sqrt{-\frac{d (a + b x)}{b c - a d}} \text{EllipticF}[\text{ArcSin}[\frac{b^{1/4} (c + d x)^{1/4}}{(b c - a d)^{1/4}}], -1]}{77 b^{5/4} d^3 \sqrt{a + b x}}$$

Result (type 5, 140 leaves):

$$-\frac{1}{77 b d^3 \sqrt{a + b x}} 4 (c + d x)^{1/4} \left( -d (a + b x) (4 a^2 d^2 + a b d (5 c + 13 d x) + b^2 (-2 c^2 + c d x + 7 d^2 x^2)) - 4 (b c - a d)^3 \sqrt{\frac{d (a + b x)}{-b c + a d}} \text{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{b (c + d x)}{b c - a d} \right] \right)$$

### Problem 1630: Result unnecessarily involves higher level functions.

$$\int \sqrt{a + b x} (c + d x)^{1/4} dx$$

Optimal (type 4, 147 leaves, 5 steps):

$$\frac{4 (b c - a d) \sqrt{a + b x} (c + d x)^{1/4}}{21 b d} + \frac{4 (a + b x)^{3/2} (c + d x)^{1/4}}{7 b} - \frac{8 (b c - a d)^{9/4} \sqrt{-\frac{d (a + b x)}{b c - a d}} \text{EllipticF}[\text{ArcSin}\left[\frac{b^{1/4} (c + d x)^{1/4}}{(b c - a d)^{1/4}}\right], -1]}{21 b^{5/4} d^2 \sqrt{a + b x}}$$

Result (type 5, 109 leaves):

$$\frac{1}{21 b d^2 \sqrt{a + b x}} 4 (c + d x)^{1/4} \left( d (a + b x) (2 a d + b (c + 3 d x)) - 2 (b c - a d)^2 \sqrt{\frac{d (a + b x)}{-b c + a d}} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{b (c + d x)}{b c - a d}\right] \right)$$

### Problem 1631: Result unnecessarily involves higher level functions.

$$\int \frac{(c + d x)^{1/4}}{\sqrt{a + b x}} dx$$

Optimal (type 4, 111 leaves, 4 steps):

$$\frac{4 \sqrt{a + b x} (c + d x)^{1/4}}{3 b} + \frac{4 (b c - a d)^{5/4} \sqrt{-\frac{d (a + b x)}{b c - a d}} \text{EllipticF}[\text{ArcSin}\left[\frac{b^{1/4} (c + d x)^{1/4}}{(b c - a d)^{1/4}}\right], -1]}{3 b^{5/4} d \sqrt{a + b x}}$$

Result (type 5, 93 leaves):

$$\frac{4 (c + d x)^{1/4} \left( d (a + b x) + (b c - a d) \sqrt{\frac{d (a + b x)}{-b c + a d}} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{b (c + d x)}{b c - a d}\right] \right)}{3 b d \sqrt{a + b x}}$$

### Problem 1632: Result unnecessarily involves higher level functions.

$$\int \frac{(c + d x)^{1/4}}{(a + b x)^{3/2}} dx$$

Optimal (type 4, 104 leaves, 4 steps):

$$-\frac{2 (c+d x)^{1/4}}{b \sqrt{a+b x}} + \frac{2 (b c-a d)^{1/4} \sqrt{-\frac{d (a+b x)}{b c-a d}} \text{EllipticF}[\text{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c-a d)^{1/4}}\right], -1]}{b^{5/4} \sqrt{a+b x}}$$

Result (type 5, 74 leaves):

$$\frac{2 (c+d x)^{1/4} \left(-1 + \sqrt{\frac{d (a+b x)}{-b c+a d}} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{b (c+d x)}{b c-a d}\right]\right)}{b \sqrt{a+b x}}$$

**Problem 1633:** Result unnecessarily involves higher level functions.

$$\int \frac{(c+d x)^{1/4}}{(a+b x)^{5/2}} dx$$

Optimal (type 4, 145 leaves, 5 steps):

$$-\frac{2 (c+d x)^{1/4}}{3 b (a+b x)^{3/2}} - \frac{d (c+d x)^{1/4}}{3 b (b c-a d) \sqrt{a+b x}} - \frac{d \sqrt{-\frac{d (a+b x)}{b c-a d}} \text{EllipticF}[\text{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c-a d)^{1/4}}\right], -1]}{3 b^{5/4} (b c-a d)^{3/4} \sqrt{a+b x}}$$

Result (type 5, 103 leaves):

$$\frac{(c+d x)^{1/4} \left(2 b c-a d+b d x+d (a+b x) \sqrt{\frac{d (a+b x)}{-b c+a d}} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{b (c+d x)}{b c-a d}\right]\right)}{3 b (-b c+a d) (a+b x)^{3/2}}$$

**Problem 1634:** Result unnecessarily involves higher level functions.

$$\int \frac{(c+d x)^{1/4}}{(a+b x)^{7/2}} dx$$

Optimal (type 4, 185 leaves, 6 steps):

$$-\frac{2 (c+d x)^{1/4}}{5 b (a+b x)^{5/2}} - \frac{d (c+d x)^{1/4}}{15 b (b c-a d) (a+b x)^{3/2}} + \frac{d^2 (c+d x)^{1/4}}{6 b (b c-a d)^2 \sqrt{a+b x}} + \frac{d^2 \sqrt{-\frac{d (a+b x)}{b c-a d}} \text{EllipticF}[\text{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c-a d)^{1/4}}\right], -1]}{6 b^{5/4} (b c-a d)^{7/4} \sqrt{a+b x}}$$

Result (type 5, 140 leaves):

$$\left( (c + d x)^{1/4} \left( -5 a^2 d^2 + 2 a b d (11 c + 6 d x) + b^2 (-12 c^2 - 2 c d x + 5 d^2 x^2) + 5 d^2 (a + b x)^2 \sqrt{\frac{d (a + b x)}{-b c + a d}} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{b (c + d x)}{b c - a d}\right] \right) \right) / (30 b (b c - a d)^2 (a + b x)^{5/2})$$

**Problem 1635:** Result unnecessarily involves higher level functions.

$$\int (a + b x)^{3/2} (c + d x)^{3/4} dx$$

Optimal (type 4, 270 leaves, 10 steps):

$$-\frac{8 (b c - a d)^2 \sqrt{a + b x} (c + d x)^{3/4}}{65 b d^2} + \frac{4 (b c - a d) (a + b x)^{3/2} (c + d x)^{3/4}}{39 b d} + \frac{4 (a + b x)^{5/2} (c + d x)^{3/4}}{13 b} + \frac{16 (b c - a d)^{15/4} \sqrt{-\frac{d (a + b x)}{b c - a d}} \text{EllipticE}[\text{ArcSin}\left[\frac{b^{1/4} (c + d x)^{1/4}}{(b c - a d)^{1/4}}\right], -1]}{65 b^{7/4} d^3 \sqrt{a + b x}} - \frac{16 (b c - a d)^{15/4} \sqrt{-\frac{d (a + b x)}{b c - a d}} \text{EllipticF}[\text{ArcSin}\left[\frac{b^{1/4} (c + d x)^{1/4}}{(b c - a d)^{1/4}}\right], -1]}{65 b^{7/4} d^3 \sqrt{a + b x}}$$

Result (type 5, 141 leaves):

$$-\frac{1}{195 b d^3 \sqrt{a + b x}} 4 (c + d x)^{3/4} \left( -d (a + b x) (4 a^2 d^2 + a b d (17 c + 25 d x) + b^2 (-6 c^2 + 5 c d x + 15 d^2 x^2)) - 4 (b c - a d)^3 \sqrt{\frac{d (a + b x)}{-b c + a d}} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \frac{b (c + d x)}{b c - a d}\right] \right)$$

**Problem 1636:** Result unnecessarily involves higher level functions.

$$\int \sqrt{a + b x} (c + d x)^{3/4} dx$$

Optimal (type 4, 232 leaves, 9 steps):

$$\frac{4(b c - a d) \sqrt{a + b x} (c + d x)^{3/4}}{15 b d} + \frac{4(a + b x)^{3/2} (c + d x)^{3/4}}{9 b} -$$

$$\frac{8(b c - a d)^{11/4} \sqrt{-\frac{d(a+b x)}{b c-a d}} \text{EllipticE}[\text{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c-a d)^{1/4}}\right], -1]}{15 b^{7/4} d^2 \sqrt{a + b x}} -$$

$$\frac{8(b c - a d)^{11/4} \sqrt{-\frac{d(a+b x)}{b c-a d}} \text{EllipticF}[\text{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c-a d)^{1/4}}\right], -1]}{15 b^{7/4} d^2 \sqrt{a + b x}}$$

Result (type 5, 110 leaves):

$$\frac{1}{45 b d^2 \sqrt{a + b x}} 4(c + d x)^{3/4} \left( d(a + b x)(3 b c + 2 a d + 5 b d x) - 2(b c - a d)^2 \sqrt{\frac{d(a + b x)}{-b c + a d}} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \frac{b(c + d x)}{b c - a d}\right] \right)$$

**Problem 1637:** Result unnecessarily involves higher level functions.

$$\int \frac{(c + d x)^{3/4}}{\sqrt{a + b x}} dx$$

Optimal (type 4, 196 leaves, 8 steps):

$$\frac{4 \sqrt{a + b x} (c + d x)^{3/4}}{5 b} + \frac{12(b c - a d)^{7/4} \sqrt{-\frac{d(a+b x)}{b c-a d}} \text{EllipticE}[\text{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c-a d)^{1/4}}\right], -1]}{5 b^{7/4} d \sqrt{a + b x}} -$$

$$\frac{12(b c - a d)^{7/4} \sqrt{-\frac{d(a+b x)}{b c-a d}} \text{EllipticF}[\text{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c-a d)^{1/4}}\right], -1]}{5 b^{7/4} d \sqrt{a + b x}}$$

Result (type 5, 93 leaves):

$$\frac{4(c + d x)^{3/4} \left( d(a + b x) + (b c - a d) \sqrt{\frac{d(a + b x)}{-b c + a d}} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \frac{b(c + d x)}{b c - a d}\right] \right)}{5 b d \sqrt{a + b x}}$$

**Problem 1638:** Result unnecessarily involves higher level functions.

$$\int \frac{(c + d x)^{3/4}}{(a + b x)^{3/2}} dx$$

Optimal (type 4, 184 leaves, 8 steps):

$$\frac{2 (c + d x)^{3/4}}{b \sqrt{a + b x}} + \frac{6 (b c - a d)^{3/4} \sqrt{-\frac{d (a+b x)}{b c-a d}} \text{EllipticE}[\text{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c-a d)^{1/4}}\right], -1]}{b^{7/4} \sqrt{a + b x}} - \frac{6 (b c - a d)^{3/4} \sqrt{-\frac{d (a+b x)}{b c-a d}} \text{EllipticF}[\text{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c-a d)^{1/4}}\right], -1]}{b^{7/4} \sqrt{a + b x}}$$

Result (type 5, 74 leaves):

$$\frac{2 (c + d x)^{3/4} \left(-1 + \sqrt{\frac{d (a+b x)}{-b c+a d}} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \frac{b (c+d x)}{b c-a d}\right]\right)}{b \sqrt{a + b x}}$$

Problem 1639: Result unnecessarily involves higher level functions.

$$\int \frac{(c + d x)^{3/4}}{(a + b x)^{5/2}} dx$$

Optimal (type 4, 221 leaves, 9 steps):

$$\frac{2 (c + d x)^{3/4}}{3 b (a + b x)^{3/2}} - \frac{d (c + d x)^{3/4}}{b (b c - a d) \sqrt{a + b x}} + \frac{d \sqrt{-\frac{d (a+b x)}{b c-a d}} \text{EllipticE}[\text{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c-a d)^{1/4}}\right], -1]}{b^{7/4} (b c - a d)^{1/4} \sqrt{a + b x}} - \frac{d \sqrt{-\frac{d (a+b x)}{b c-a d}} \text{EllipticF}[\text{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c-a d)^{1/4}}\right], -1]}{b^{7/4} (b c - a d)^{1/4} \sqrt{a + b x}}$$

Result (type 5, 104 leaves):

$$\frac{(c + d x)^{3/4} \left(2 b c + a d + 3 b d x - d (a + b x) \sqrt{\frac{d (a+b x)}{-b c+a d}} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \frac{b (c+d x)}{b c-a d}\right]\right)}{3 b (-b c + a d) (a + b x)^{3/2}}$$

Problem 1640: Result unnecessarily involves higher level functions.

$$\int \frac{(c + d x)^{3/4}}{(a + b x)^{7/2}} dx$$

Optimal (type 4, 270 leaves, 10 steps):

$$\begin{aligned} & \frac{2 (c + d x)^{3/4}}{5 b (a + b x)^{5/2}} - \frac{d (c + d x)^{3/4}}{5 b (b c - a d) (a + b x)^{3/2}} + \frac{3 d^2 (c + d x)^{3/4}}{10 b (b c - a d)^2 \sqrt{a + b x}} - \\ & \frac{3 d^2 \sqrt{-\frac{d (a+b x)}{b c-a d}} \text{EllipticE}[\text{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c-a d)^{1/4}}\right], -1]}{10 b^{7/4} (b c - a d)^{5/4} \sqrt{a + b x}} + \frac{3 d^2 \sqrt{-\frac{d (a+b x)}{b c-a d}} \text{EllipticF}[\text{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c-a d)^{1/4}}\right], -1]}{10 b^{7/4} (b c - a d)^{5/4} \sqrt{a + b x}} \end{aligned}$$

Result (type 5, 140 leaves):

$$\left( \left( c + d x \right)^{3/4} \left( a^2 d^2 + 2 a b d (3 c + 4 d x) - b^2 (4 c^2 + 2 c d x - 3 d^2 x^2) - d^2 (a + b x)^2 \sqrt{\frac{d (a + b x)}{-b c + a d}} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \frac{b (c + d x)}{b c - a d}\right] \right) \right) / \\ \left( 10 b (b c - a d)^2 (a + b x)^{5/2} \right)$$

Problem 1641: Result unnecessarily involves higher level functions.

$$\int (a + b x)^{3/2} (c + d x)^{5/4} dx$$

Optimal (type 4, 220 leaves, 7 steps):

$$-\frac{8 (b c - a d)^3 \sqrt{a + b x} (c + d x)^{1/4}}{231 b^2 d^2} + \frac{4 (b c - a d)^2 (a + b x)^{3/2} (c + d x)^{1/4}}{231 b^2 d} + \frac{4 (b c - a d) (a + b x)^{5/2} (c + d x)^{1/4}}{33 b^2} + \\ \frac{4 (a + b x)^{5/2} (c + d x)^{5/4}}{15 b} + \frac{16 (b c - a d)^{17/4} \sqrt{-\frac{d (a + b x)}{b c - a d}} \text{EllipticF}[\text{ArcSin}\left[\frac{b^{1/4} (c + d x)^{1/4}}{(b c - a d)^{1/4}}\right], -1]}{231 b^{9/4} d^3 \sqrt{a + b x}}$$

Result (type 5, 182 leaves):

$$\frac{1}{1155 b^2 d^3 \sqrt{a + b x}} \\ 4 (c + d x)^{1/4} \left( -d (a + b x) (20 a^3 d^3 - 12 a^2 b d^2 (6 c + d x) - a b^2 d (35 c^2 + 214 c d x + 119 d^2 x^2) + b^3 (10 c^3 - 5 c^2 d x - 112 c d^2 x^2 - 77 d^3 x^3)) + \right. \\ \left. 20 (b c - a d)^4 \sqrt{\frac{d (a + b x)}{-b c + a d}} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{b (c + d x)}{b c - a d}\right] \right)$$

Problem 1642: Result unnecessarily involves higher level functions.

$$\int \sqrt{a + b x} (c + d x)^{5/4} dx$$

Optimal (type 4, 182 leaves, 6 steps):

$$\frac{20 (b c - a d)^2 \sqrt{a + b x} (c + d x)^{1/4}}{231 b^2 d} + \frac{20 (b c - a d) (a + b x)^{3/2} (c + d x)^{1/4}}{77 b^2} +$$

$$\frac{4 (a + b x)^{3/2} (c + d x)^{5/4}}{11 b} - \frac{40 (b c - a d)^{13/4} \sqrt{-\frac{d (a+b x)}{b c-a d}} \text{EllipticF}[\text{ArcSin}[\frac{b^{1/4} (c+d x)^{1/4}}{(b c-a d)^{1/4}}], -1]}{231 b^{9/4} d^2 \sqrt{a + b x}}$$

Result (type 5, 143 leaves):

$$\frac{1}{231 b^2 d^2 \sqrt{a + b x}} 4 (c + d x)^{1/4} \left( -d (a + b x) (10 a^2 d^2 - 2 a b d (13 c + 3 d x) - b^2 (5 c^2 + 36 c d x + 21 d^2 x^2)) - \right.$$

$$\left. 10 (b c - a d)^3 \sqrt{\frac{d (a + b x)}{-b c + a d}} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{b (c + d x)}{b c - a d}\right] \right)$$

Problem 1643: Result unnecessarily involves higher level functions.

$$\int \frac{(c + d x)^{5/4}}{\sqrt{a + b x}} dx$$

Optimal (type 4, 144 leaves, 5 steps):

$$\frac{20 (b c - a d) \sqrt{a + b x} (c + d x)^{1/4}}{21 b^2} + \frac{4 \sqrt{a + b x} (c + d x)^{5/4}}{7 b} + \frac{20 (b c - a d)^{9/4} \sqrt{-\frac{d (a+b x)}{b c-a d}} \text{EllipticF}[\text{ArcSin}[\frac{b^{1/4} (c+d x)^{1/4}}{(b c-a d)^{1/4}}], -1]}{21 b^{9/4} d \sqrt{a + b x}}$$

Result (type 5, 111 leaves):

$$\frac{1}{21 b^2 d \sqrt{a + b x}} 4 (c + d x)^{1/4} \left( -d (a + b x) (-8 b c + 5 a d - 3 b d x) + 5 (b c - a d)^2 \sqrt{\frac{d (a + b x)}{-b c + a d}} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{b (c + d x)}{b c - a d}\right] \right)$$

Problem 1644: Result unnecessarily involves higher level functions.

$$\int \frac{(c + d x)^{5/4}}{(a + b x)^{3/2}} dx$$

Optimal (type 4, 132 leaves, 5 steps):

$$\frac{10 d \sqrt{a+b x} (c+d x)^{1/4}}{3 b^2} - \frac{2 (c+d x)^{5/4}}{b \sqrt{a+b x}} + \frac{10 (b c-a d)^{5/4} \sqrt{-\frac{d (a+b x)}{b c-a d}} \text{EllipticF}[\text{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c-a d)^{1/4}}\right], -1]}{3 b^{9/4} \sqrt{a+b x}}$$

Result (type 5, 95 leaves):

$$\frac{2 (c+d x)^{1/4} \left(3 b c - 5 a d - 2 b d x + \frac{5 d (a+b x) \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{b (c+d x)}{b c-a d}\right]}{\sqrt{\frac{d (a+b x)}{-b c+a d}}}\right)}{3 b^2 \sqrt{a+b x}}$$

Problem 1645: Result unnecessarily involves higher level functions.

$$\int \frac{(c+d x)^{5/4}}{(a+b x)^{5/2}} dx$$

Optimal (type 4, 135 leaves, 5 steps):

$$\frac{5 d (c+d x)^{1/4}}{3 b^2 \sqrt{a+b x}} - \frac{2 (c+d x)^{5/4}}{3 b (a+b x)^{3/2}} + \frac{5 d (b c-a d)^{1/4} \sqrt{-\frac{d (a+b x)}{b c-a d}} \text{EllipticF}[\text{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c-a d)^{1/4}}\right], -1]}{3 b^{9/4} \sqrt{a+b x}}$$

Result (type 5, 95 leaves):

$$\frac{(c+d x)^{1/4} \left(-2 b c - 5 a d - 7 b d x + 5 d (a+b x) \sqrt{\frac{d (a+b x)}{-b c+a d}} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{b (c+d x)}{b c-a d}\right]\right)}{3 b^2 (a+b x)^{3/2}}$$

Problem 1646: Result unnecessarily involves higher level functions.

$$\int \frac{(c+d x)^{5/4}}{(a+b x)^{7/2}} dx$$

Optimal (type 4, 175 leaves, 6 steps):

$$\frac{d (c+d x)^{1/4}}{3 b^2 (a+b x)^{3/2}} - \frac{d^2 (c+d x)^{1/4}}{6 b^2 (b c-a d) \sqrt{a+b x}} - \frac{2 (c+d x)^{5/4}}{5 b (a+b x)^{5/2}} - \frac{d^2 \sqrt{-\frac{d (a+b x)}{b c-a d}} \text{EllipticF}[\text{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c-a d)^{1/4}}\right], -1]}{6 b^{9/4} (b c-a d)^{3/4} \sqrt{a+b x}}$$

Result (type 5, 138 leaves):

$$\left( \left( c + d x \right)^{1/4} \left( -5 a^2 d^2 - 2 a b d (c + 6 d x) + b^2 (12 c^2 + 22 c d x + 5 d^2 x^2) + 5 d^2 (a + b x)^2 \sqrt{\frac{d (a + b x)}{-b c + a d}} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{b (c + d x)}{b c - a d}\right] \right) \right) / \\ (30 b^2 (-b c + a d) (a + b x)^{5/2})$$

**Problem 1647:** Result unnecessarily involves higher level functions.

$$\int \frac{(c + d x)^{5/4}}{(a + b x)^{9/2}} dx$$

Optimal (type 4, 213 leaves, 7 steps):

$$\frac{d (c + d x)^{1/4}}{7 b^2 (a + b x)^{5/2}} - \frac{d^2 (c + d x)^{1/4}}{42 b^2 (b c - a d) (a + b x)^{3/2}} + \frac{5 d^3 (c + d x)^{1/4}}{84 b^2 (b c - a d)^2 \sqrt{a + b x}} - \frac{2 (c + d x)^{5/4}}{7 b (a + b x)^{7/2}} + \frac{5 d^3 \sqrt{-\frac{d (a + b x)}{b c - a d}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4} (c + d x)^{1/4}}{(b c - a d)^{1/4}}\right], -1\right]}{84 b^{9/4} (b c - a d)^{7/4} \sqrt{a + b x}}$$

Result (type 5, 181 leaves):

$$\left( (c + d x)^{1/4} \left( -5 a^3 d^3 - a^2 b d^2 (2 c + 17 d x) + a b^2 d (36 c^2 + 68 c d x + 17 d^2 x^2) - b^3 (24 c^3 + 36 c^2 d x + 2 c d^2 x^2 - 5 d^3 x^3) + 5 d^3 (a + b x)^3 \sqrt{\frac{d (a + b x)}{-b c + a d}} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{b (c + d x)}{b c - a d}\right] \right) \right) / (84 b^2 (b c - a d)^2 (a + b x)^{7/2})$$

**Problem 1648:** Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{5/2}}{(c + d x)^{1/4}} dx$$

Optimal (type 4, 264 leaves, 10 steps):

$$\frac{16 (b c - a d)^2 \sqrt{a + b x} (c + d x)^{3/4}}{39 d^3} - \frac{40 (b c - a d) (a + b x)^{3/2} (c + d x)^{3/4}}{117 d^2} + \frac{4 (a + b x)^{5/2} (c + d x)^{3/4}}{13 d} - \frac{32 (b c - a d)^{15/4} \sqrt{-\frac{d (a + b x)}{b c - a d}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{b^{1/4} (c + d x)^{1/4}}{(b c - a d)^{1/4}}\right], -1\right]}{39 b^{3/4} d^4 \sqrt{a + b x}} + \frac{32 (b c - a d)^{15/4} \sqrt{-\frac{d (a + b x)}{b c - a d}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4} (c + d x)^{1/4}}{(b c - a d)^{1/4}}\right], -1\right]}{39 b^{3/4} d^4 \sqrt{a + b x}}$$

Result (type 5, 138 leaves):

$$\frac{1}{117 d^4 \sqrt{a+b x}} 4 (c+d x)^{3/4} \left( d (a+b x) (31 a^2 d^2 + 2 a b d (-17 c + 14 d x) + b^2 (12 c^2 - 10 c d x + 9 d^2 x^2)) - 8 (b c - a d)^3 \sqrt{\frac{d (a+b x)}{-b c + a d}} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \frac{b (c+d x)}{b c - a d}\right] \right)$$

Problem 1649: Result unnecessarily involves higher level functions.

$$\int \frac{(a+b x)^{3/2}}{(c+d x)^{1/4}} dx$$

Optimal (type 4, 229 leaves, 9 steps):

$$\begin{aligned} & -\frac{8 (b c - a d) \sqrt{a+b x} (c+d x)^{3/4}}{15 d^2} + \frac{4 (a+b x)^{3/2} (c+d x)^{3/4}}{9 d} + \\ & \frac{16 (b c - a d)^{11/4} \sqrt{-\frac{d (a+b x)}{b c - a d}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c - a d)^{1/4}}\right], -1\right]}{15 b^{3/4} d^3 \sqrt{a+b x}} - \frac{16 (b c - a d)^{11/4} \sqrt{-\frac{d (a+b x)}{b c - a d}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c - a d)^{1/4}}\right], -1\right]}{15 b^{3/4} d^3 \sqrt{a+b x}} \end{aligned}$$

Result (type 5, 107 leaves):

$$\frac{1}{45 d^3 \sqrt{a+b x}} 4 (c+d x)^{3/4} \left( d (a+b x) (-6 b c + 11 a d + 5 b d x) + 4 (b c - a d)^2 \sqrt{\frac{d (a+b x)}{-b c + a d}} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \frac{b (c+d x)}{b c - a d}\right] \right)$$

Problem 1650: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{a+b x}}{(c+d x)^{1/4}} dx$$

Optimal (type 4, 196 leaves, 8 steps):

$$\frac{4 \sqrt{a+b x} (c+d x)^{3/4}}{5 d} - \frac{8 (b c-a d)^{7/4} \sqrt{-\frac{d (a+b x)}{b c-a d}} \text{EllipticE}[\text{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c-a d)^{1/4}}\right], -1]}{5 b^{3/4} d^2 \sqrt{a+b x}} +$$

$$\frac{8 (b c-a d)^{7/4} \sqrt{-\frac{d (a+b x)}{b c-a d}} \text{EllipticF}[\text{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c-a d)^{1/4}}\right], -1]}{5 b^{3/4} d^2 \sqrt{a+b x}}$$

Result (type 5, 77 leaves) :

$$\frac{4 \sqrt{a+b x} (c+d x)^{3/4} \left(3 + \frac{2 \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \frac{b (c+d x)}{b c-a d}\right]}{\sqrt{\frac{d (a+b x)}{-b c+a d}}}\right)}{15 d}$$

Problem 1651: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\sqrt{a+b x} (c+d x)^{1/4}} dx$$

Optimal (type 4, 167 leaves, 7 steps) :

$$\frac{4 (b c-a d)^{3/4} \sqrt{-\frac{d (a+b x)}{b c-a d}} \text{EllipticE}[\text{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c-a d)^{1/4}}\right], -1]}{b^{3/4} d \sqrt{a+b x}} - \frac{4 (b c-a d)^{3/4} \sqrt{-\frac{d (a+b x)}{b c-a d}} \text{EllipticF}[\text{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c-a d)^{1/4}}\right], -1]}{b^{3/4} d \sqrt{a+b x}}$$

Result (type 5, 73 leaves) :

$$\frac{4 \sqrt{\frac{d (a+b x)}{-b c+a d}} (c+d x)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \frac{b (c+d x)}{b c-a d}\right]}{3 d \sqrt{a+b x}}$$

Problem 1652: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a+b x)^{3/2} (c+d x)^{1/4}} dx$$

Optimal (type 4, 191 leaves, 8 steps) :

$$-\frac{2(c+dx)^{3/4}}{(bc-ad)\sqrt{a+bx}} + \frac{2\sqrt{-\frac{d(a+bx)}{bc-ad}} \text{EllipticE}[\text{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1]}{b^{3/4}(bc-ad)^{1/4}\sqrt{a+bx}} - \frac{2\sqrt{-\frac{d(a+bx)}{bc-ad}} \text{EllipticF}[\text{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1]}{b^{3/4}(bc-ad)^{1/4}\sqrt{a+bx}}$$

Result (type 5, 83 leaves):

$$\frac{2(c+dx)^{3/4} \left( -3 + \sqrt{\frac{d(a+bx)}{-bc+ad}} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \frac{b(c+dx)}{bc-ad}\right] \right)}{3(bc-ad)\sqrt{a+bx}}$$

Problem 1653: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a+bx)^{5/2}(c+dx)^{1/4}} dx$$

Optimal (type 4, 224 leaves, 9 steps):

$$-\frac{2(c+dx)^{3/4}}{3(bc-ad)(a+bx)^{3/2}} + \frac{d(c+dx)^{3/4}}{(bc-ad)^2\sqrt{a+bx}} - \frac{d\sqrt{-\frac{d(a+bx)}{bc-ad}} \text{EllipticE}[\text{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1]}{b^{3/4}(bc-ad)^{5/4}\sqrt{a+bx}} + \frac{d\sqrt{-\frac{d(a+bx)}{bc-ad}} \text{EllipticF}[\text{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1]}{b^{3/4}(bc-ad)^{5/4}\sqrt{a+bx}}$$

Result (type 5, 102 leaves):

$$\frac{(c+dx)^{3/4} \left( -2bc + 5ad + 3bdx - d(a+bx) \sqrt{\frac{d(a+bx)}{-bc+ad}} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \frac{b(c+dx)}{bc-ad}\right] \right)}{3(bc-ad)^2(a+bx)^{3/2}}$$

Problem 1654: Result unnecessarily involves higher level functions.

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{3/4}} dx$$

Optimal (type 4, 144 leaves, 5 steps):

$$-\frac{8(bc-ad)\sqrt{a+bx}(c+dx)^{1/4}}{7d^2} + \frac{4(a+bx)^{3/2}(c+dx)^{1/4}}{7d} + \frac{16(bc-ad)^{9/4}\sqrt{-\frac{d(a+bx)}{bc-ad}} \text{EllipticF}[\text{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1]}{7b^{1/4}d^3\sqrt{a+bx}}$$

Result (type 5, 106 leaves):

$$\frac{1}{7 d^3 \sqrt{a+b x}} 4 (c+d x)^{1/4} \left( d (a+b x) (-2 b c + 3 a d + b d x) + 4 (b c - a d)^2 \sqrt{\frac{d (a+b x)}{-b c + a d}} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{b (c+d x)}{b c - a d}\right] \right)$$

Problem 1655: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{a+b x}}{(c+d x)^{3/4}} dx$$

Optimal (type 4, 111 leaves, 4 steps):

$$\frac{4 \sqrt{a+b x} (c+d x)^{1/4}}{3 d} - \frac{8 (b c - a d)^{5/4} \sqrt{-\frac{d (a+b x)}{b c - a d}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c - a d)^{1/4}}\right], -1\right]}{3 b^{1/4} d^2 \sqrt{a+b x}}$$

Result (type 5, 77 leaves):

$$\frac{4 \sqrt{a+b x} (c+d x)^{1/4} \left( 1 + \frac{2 \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{b (c+d x)}{b c - a d}\right]}{\sqrt{\frac{d (a+b x)}{-b c + a d}}} \right)}{3 d}$$

Problem 1656: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\sqrt{a+b x} (c+d x)^{3/4}} dx$$

Optimal (type 4, 83 leaves, 3 steps):

$$\frac{4 (b c - a d)^{1/4} \sqrt{-\frac{d (a+b x)}{b c - a d}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c - a d)^{1/4}}\right], -1\right]}{b^{1/4} d \sqrt{a+b x}}$$

Result (type 5, 71 leaves):

$$\frac{4 \sqrt{\frac{d (a+b x)}{-b c + a d}} (c+d x)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{b (c+d x)}{b c - a d}\right]}{d \sqrt{a+b x}}$$

### Problem 1657: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + b x)^{3/2} (c + d x)^{3/4}} dx$$

Optimal (type 4, 111 leaves, 4 steps):

$$-\frac{2 (c + d x)^{1/4}}{(b c - a d) \sqrt{a + b x}} - \frac{2 \sqrt{-\frac{d (a + b x)}{b c - a d}} \text{EllipticF}[\text{ArcSin}\left[\frac{b^{1/4} (c + d x)^{1/4}}{(b c - a d)^{1/4}}\right], -1]}{b^{1/4} (b c - a d)^{3/4} \sqrt{a + b x}}$$

Result (type 5, 81 leaves):

$$-\frac{2 (c + d x)^{1/4} \left(1 + \sqrt{\frac{d (a + b x)}{-b c + a d}} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{b (c + d x)}{b c - a d}\right]\right)}{(b c - a d) \sqrt{a + b x}}$$

### Problem 1658: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + b x)^{5/2} (c + d x)^{3/4}} dx$$

Optimal (type 4, 149 leaves, 5 steps):

$$-\frac{2 (c + d x)^{1/4}}{3 (b c - a d) (a + b x)^{3/2}} + \frac{5 d (c + d x)^{1/4}}{3 (b c - a d)^2 \sqrt{a + b x}} + \frac{5 d \sqrt{-\frac{d (a + b x)}{b c - a d}} \text{EllipticF}[\text{ArcSin}\left[\frac{b^{1/4} (c + d x)^{1/4}}{(b c - a d)^{1/4}}\right], -1]}{3 b^{1/4} (b c - a d)^{7/4} \sqrt{a + b x}}$$

Result (type 5, 102 leaves):

$$\frac{(c + d x)^{1/4} \left(-2 b c + 7 a d + 5 b d x + 5 d (a + b x) \sqrt{\frac{d (a + b x)}{-b c + a d}} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{b (c + d x)}{b c - a d}\right]\right)}{3 (b c - a d)^2 (a + b x)^{3/2}}$$

### Problem 1659: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{5/2}}{(c + d x)^{5/4}} dx$$

Optimal (type 4, 254 leaves, 10 steps):

$$\begin{aligned}
 & -\frac{4 (a+b x)^{5/2}}{d (c+d x)^{1/4}} - \frac{16 b (b c - a d) \sqrt{a+b x} (c+d x)^{3/4}}{3 d^3} + \frac{40 b (a+b x)^{3/2} (c+d x)^{3/4}}{9 d^2} + \\
 & \frac{32 b^{1/4} (b c - a d)^{11/4} \sqrt{-\frac{d (a+b x)}{b c - a d}} \text{EllipticE}[\text{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c - a d)^{1/4}}\right], -1]}{3 d^4 \sqrt{a+b x}} - \frac{32 b^{1/4} (b c - a d)^{11/4} \sqrt{-\frac{d (a+b x)}{b c - a d}} \text{EllipticF}[\text{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c - a d)^{1/4}}\right], -1]}{3 d^4 \sqrt{a+b x}}
 \end{aligned}$$

Result (type 5, 131 leaves):

$$\begin{aligned}
 & \frac{1}{9 d^4 \sqrt{a+b x}} \\
 & 4 (c+d x)^{3/4} \left( d (a+b x) \left( b (-3 b c + 4 a d) + b^2 d x - \frac{9 (b c - a d)^2}{c+d x} \right) + 8 b (b c - a d)^2 \sqrt{\frac{d (a+b x)}{-b c + a d}} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \frac{b (c+d x)}{b c - a d}\right] \right)
 \end{aligned}$$

Problem 1660: Result unnecessarily involves higher level functions.

$$\int \frac{(a+b x)^{3/2}}{(c+d x)^{5/4}} dx$$

Optimal (type 4, 220 leaves, 9 steps):

$$\begin{aligned}
 & -\frac{4 (a+b x)^{3/2}}{d (c+d x)^{1/4}} + \frac{24 b \sqrt{a+b x} (c+d x)^{3/4}}{5 d^2} - \frac{48 b^{1/4} (b c - a d)^{7/4} \sqrt{-\frac{d (a+b x)}{b c - a d}} \text{EllipticE}[\text{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c - a d)^{1/4}}\right], -1]}{5 d^3 \sqrt{a+b x}} + \\
 & \frac{48 b^{1/4} (b c - a d)^{7/4} \sqrt{-\frac{d (a+b x)}{b c - a d}} \text{EllipticF}[\text{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c - a d)^{1/4}}\right], -1]}{5 d^3 \sqrt{a+b x}}
 \end{aligned}$$

Result (type 5, 98 leaves):

$$\begin{aligned}
 & 4 \sqrt{a+b x} (c+d x)^{3/4} \left( \frac{6 b c - 5 a d + b d x}{c+d x} + \frac{4 b \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \frac{b (c+d x)}{b c - a d}\right]}{\sqrt{\frac{d (a+b x)}{-b c + a d}}} \right) \\
 & 5 d^2
 \end{aligned}$$

### Problem 1661: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{a+b x}}{(c+d x)^{5/4}} dx$$

Optimal (type 4, 190 leaves, 8 steps):

$$\begin{aligned} & -\frac{4 \sqrt{a+b x}}{d (c+d x)^{1/4}} + \frac{8 b^{1/4} (b c-a d)^{3/4} \sqrt{-\frac{d (a+b x)}{b c-a d}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c-a d)^{1/4}}\right], -1\right]}{d^2 \sqrt{a+b x}} - \\ & \frac{8 b^{1/4} (b c-a d)^{3/4} \sqrt{-\frac{d (a+b x)}{b c-a d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c-a d)^{1/4}}\right], -1\right]}{d^2 \sqrt{a+b x}} \end{aligned}$$

Result (type 5, 90 leaves):

$$\begin{aligned} & -12 d (a+b x) + 8 b \sqrt{\frac{d (a+b x)}{-b c+a d}} (c+d x) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \frac{b (c+d x)}{b c-a d}\right] \\ & \frac{3 d^2 \sqrt{a+b x} (c+d x)^{1/4}}{} \end{aligned}$$

### Problem 1662: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\sqrt{a+b x} (c+d x)^{5/4}} dx$$

Optimal (type 4, 197 leaves, 8 steps):

$$\begin{aligned} & \frac{4 \sqrt{a+b x}}{(b c-a d) (c+d x)^{1/4}} - \frac{4 b^{1/4} \sqrt{\frac{d (a+b x)}{b c-a d}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c-a d)^{1/4}}\right], -1\right]}{d (b c-a d)^{1/4} \sqrt{a+b x}} + \frac{4 b^{1/4} \sqrt{\frac{d (a+b x)}{b c-a d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c-a d)^{1/4}}\right], -1\right]}{d (b c-a d)^{1/4} \sqrt{a+b x}} \end{aligned}$$

Result (type 5, 100 leaves):

$$\begin{aligned} & 12 d (a+b x) - 4 b \sqrt{\frac{d (a+b x)}{-b c+a d}} (c+d x) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \frac{b (c+d x)}{b c-a d}\right] \\ & \frac{3 d (b c-a d) \sqrt{a+b x} (c+d x)^{1/4}}{} \end{aligned}$$

### Problem 1663: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + b x)^{3/2} (c + d x)^{5/4}} dx$$

Optimal (type 4, 222 leaves, 9 steps):

$$\begin{aligned} & -\frac{2}{(b c - a d) \sqrt{a + b x} (c + d x)^{1/4}} - \frac{6 d \sqrt{a + b x}}{(b c - a d)^2 (c + d x)^{1/4}} + \\ & \frac{6 b^{1/4} \sqrt{-\frac{d(a+b x)}{b c - a d}} \text{EllipticE}[\text{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c - a d)^{1/4}}\right], -1]}{(b c - a d)^{5/4} \sqrt{a + b x}} - \frac{6 b^{1/4} \sqrt{-\frac{d(a+b x)}{b c - a d}} \text{EllipticF}[\text{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c - a d)^{1/4}}\right], -1]}{(b c - a d)^{5/4} \sqrt{a + b x}} \end{aligned}$$

Result (type 5, 99 leaves):

$$\frac{-4 a d - 2 b (c + 3 d x) + 2 b \sqrt{\frac{d(a+b x)}{-b c + a d}} (c + d x) \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \frac{b(c+d x)}{b c - a d}\right]}{(b c - a d)^2 \sqrt{a + b x} (c + d x)^{1/4}}$$

### Problem 1664: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + b x)^{5/2} (c + d x)^{5/4}} dx$$

Optimal (type 4, 261 leaves, 10 steps):

$$\begin{aligned} & -\frac{2}{3 (b c - a d) (a + b x)^{3/2} (c + d x)^{1/4}} + \frac{7 d}{3 (b c - a d)^2 \sqrt{a + b x} (c + d x)^{1/4}} + \frac{7 d^2 \sqrt{a + b x}}{(b c - a d)^3 (c + d x)^{1/4}} - \\ & \frac{7 b^{1/4} d \sqrt{-\frac{d(a+b x)}{b c - a d}} \text{EllipticE}[\text{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c - a d)^{1/4}}\right], -1]}{(b c - a d)^{9/4} \sqrt{a + b x}} + \frac{7 b^{1/4} d \sqrt{-\frac{d(a+b x)}{b c - a d}} \text{EllipticF}[\text{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c - a d)^{1/4}}\right], -1]}{(b c - a d)^{9/4} \sqrt{a + b x}} \end{aligned}$$

Result (type 5, 139 leaves):

$$\begin{aligned} & \left( -12 a^2 d^2 - a b d (11 c + 35 d x) + b^2 (2 c^2 - 7 c d x - 21 d^2 x^2) + 7 b d (a + b x) \sqrt{\frac{d(a+b x)}{-b c + a d}} (c + d x) \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \frac{b(c+d x)}{b c - a d}\right] \right) / \\ & \left( 3 (-b c + a d)^3 (a + b x)^{3/2} (c + d x)^{1/4} \right) \end{aligned}$$

### Problem 1665: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{7/2}}{(c + d x)^{7/4}} dx$$

Optimal (type 4, 207 leaves, 7 steps):

$$\begin{aligned} & -\frac{4 (a + b x)^{7/2}}{3 d (c + d x)^{3/4}} + \frac{160 b (b c - a d)^2 \sqrt{a + b x} (c + d x)^{1/4}}{33 d^4} - \frac{80 b (b c - a d) (a + b x)^{3/2} (c + d x)^{1/4}}{33 d^3} + \\ & \frac{56 b (a + b x)^{5/2} (c + d x)^{1/4}}{33 d^2} - \frac{320 b^{3/4} (b c - a d)^{13/4} \sqrt{-\frac{d (a + b x)}{b c - a d}} \text{EllipticF}[\text{ArcSin}\left[\frac{b^{1/4} (c + d x)^{1/4}}{(b c - a d)^{1/4}}\right], -1]}{33 d^5 \sqrt{a + b x}} \end{aligned}$$

Result (type 5, 181 leaves):

$$\begin{aligned} & \frac{1}{33 d^5 \sqrt{a + b x}} \\ & 4 (c + d x)^{1/4} \left( \frac{1}{c + d x} d (a + b x) \left( 11 (b c - a d)^3 + b (29 b^2 c^2 - 67 a b c d + 41 a^2 d^2) (c + d x) - 3 b^2 d (3 b c - 5 a d) \times (c + d x) + 3 b^3 d^2 x^2 (c + d x) \right) - \right. \\ & \left. 80 b (b c - a d)^3 \sqrt{\frac{d (a + b x)}{-b c + a d}} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{b (c + d x)}{b c - a d}\right] \right) \end{aligned}$$

### Problem 1666: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{3/2}}{(c + d x)^{7/4}} dx$$

Optimal (type 4, 137 leaves, 5 steps):

$$\begin{aligned} & -\frac{4 (a + b x)^{3/2}}{3 d (c + d x)^{3/4}} + \frac{8 b \sqrt{a + b x} (c + d x)^{1/4}}{3 d^2} - \frac{16 b^{3/4} (b c - a d)^{5/4} \sqrt{-\frac{d (a + b x)}{b c - a d}} \text{EllipticF}[\text{ArcSin}\left[\frac{b^{1/4} (c + d x)^{1/4}}{(b c - a d)^{1/4}}\right], -1]}{3 d^3 \sqrt{a + b x}} \end{aligned}$$

Result (type 5, 98 leaves):

$$\frac{4 \sqrt{a+b x} (c+d x)^{1/4} \left( \frac{2 b c - a d + b d x}{c+d x} + \frac{4 b \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{b (c+d x)}{b c - a d}\right]}{\sqrt{\frac{d (a+b x)}{-b c + a d}}} \right)}{3 d^2}$$

**Problem 1667:** Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{a+b x}}{(c+d x)^{7/4}} dx$$

Optimal (type 4, 111 leaves, 4 steps):

$$-\frac{4 \sqrt{a+b x}}{3 d (c+d x)^{3/4}} + \frac{8 b^{3/4} (b c - a d)^{1/4} \sqrt{-\frac{d (a+b x)}{b c - a d}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c - a d)^{1/4}}\right], -1\right]}{3 d^2 \sqrt{a+b x}}$$

Result (type 5, 90 leaves):

$$\frac{-4 d (a+b x) + 8 b \sqrt{\frac{d (a+b x)}{-b c + a d}} (c+d x) \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{b (c+d x)}{b c - a d}\right]}{3 d^2 \sqrt{a+b x} (c+d x)^{3/4}}$$

**Problem 1668:** Result unnecessarily involves higher level functions.

$$\int \frac{1}{\sqrt{a+b x} (c+d x)^{7/4}} dx$$

Optimal (type 4, 118 leaves, 4 steps):

$$\frac{4 \sqrt{a+b x}}{3 (b c - a d) (c+d x)^{3/4}} + \frac{4 b^{3/4} \sqrt{-\frac{d (a+b x)}{b c - a d}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4} (c-d x)^{1/4}}{(b c - a d)^{1/4}}\right], -1\right]}{3 d (b c - a d)^{3/4} \sqrt{a+b x}}$$

Result (type 5, 98 leaves):

$$\frac{4 \left( d (a+b x) + b \sqrt{\frac{d (a+b x)}{-b c + a d}} (c+d x) \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{b (c+d x)}{b c - a d}\right]\right)}{3 d (b c - a d) \sqrt{a+b x} (c+d x)^{3/4}}$$

### Problem 1669: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + b x)^{3/2} (c + d x)^{7/4}} dx$$

Optimal (type 4, 146 leaves, 5 steps):

$$-\frac{2}{(b c - a d) \sqrt{a + b x} (c + d x)^{3/4}} - \frac{10 d \sqrt{a + b x}}{3 (b c - a d)^2 (c + d x)^{3/4}} - \frac{\frac{10 b^{3/4}}{\sqrt{-\frac{d (a+b x)}{b c - a d}}} \text{EllipticF}[\text{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c - a d)^{1/4}}\right], -1]}{3 (b c - a d)^{7/4} \sqrt{a + b x}}$$

Result (type 5, 102 leaves):

$$-\frac{2 \left(3 b c + 2 a d + 5 b d x + 5 b \sqrt{\frac{d (a+b x)}{-b c + a d}} (c + d x) \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{b (c+d x)}{b c - a d}\right]\right)}{3 (b c - a d)^2 \sqrt{a + b x} (c + d x)^{3/4}}$$

### Problem 1670: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + b x)^{5/2} (c + d x)^{7/4}} dx$$

Optimal (type 4, 178 leaves, 6 steps):

$$-\frac{2}{3 (b c - a d) (a + b x)^{3/2} (c + d x)^{3/4}} + \frac{3 d}{(b c - a d)^2 \sqrt{a + b x} (c + d x)^{3/4}} + \\ \frac{\frac{5 d^2 \sqrt{a + b x}}{(b c - a d)^3 (c + d x)^{3/4}} + \frac{5 b^{3/4} d \sqrt{-\frac{d (a+b x)}{b c - a d}} \text{EllipticF}[\text{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c - a d)^{1/4}}\right], -1]}{(b c - a d)^{11/4} \sqrt{a + b x}}}$$

Result (type 5, 139 leaves):

$$\left( -4 a^2 d^2 - a b d (13 c + 21 d x) + b^2 (2 c^2 - 9 c d x - 15 d^2 x^2) - 15 b d (a + b x) \sqrt{\frac{d (a + b x)}{-b c + a d}} (c + d x) \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{b (c + d x)}{b c - a d}\right] \right) / \\ \left( 3 (-b c + a d)^3 (a + b x)^{3/2} (c + d x)^{3/4} \right)$$

### Problem 1671: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{7/2}}{(c + d x)^{9/4}} dx$$

Optimal (type 4, 286 leaves, 11 steps):

$$\begin{aligned} & -\frac{4 (a + b x)^{7/2}}{5 d (c + d x)^{5/4}} - \frac{56 b (a + b x)^{5/2}}{5 d^2 (c + d x)^{1/4}} - \frac{224 b^2 (b c - a d) \sqrt{a + b x} (c + d x)^{3/4}}{15 d^4} + \frac{112 b^2 (a + b x)^{3/2} (c + d x)^{3/4}}{9 d^3} + \\ & \frac{448 b^{5/4} (b c - a d)^{11/4} \sqrt{-\frac{d (a + b x)}{b c - a d}} \text{EllipticE}[\text{ArcSin}\left[\frac{b^{1/4} (c + d x)^{1/4}}{(b c - a d)^{1/4}}\right], -1]}{15 d^5 \sqrt{a + b x}} - \frac{448 b^{5/4} (b c - a d)^{11/4} \sqrt{-\frac{d (a + b x)}{b c - a d}} \text{EllipticF}[\text{ArcSin}\left[\frac{b^{1/4} (c + d x)^{1/4}}{(b c - a d)^{1/4}}\right], -1]}{15 d^5 \sqrt{a + b x}} \end{aligned}$$

Result (type 5, 169 leaves):

$$\begin{aligned} & \frac{1}{45 d^5 \sqrt{a + b x}} 4 (c + d x)^{3/4} \left( \frac{1}{(c + d x)^2} d (a + b x) \left( 9 (b c - a d)^3 - 153 b (b c - a d)^2 (c + d x) - b^2 (24 b c - 29 a d) (c + d x)^2 + 5 b^3 d x (c + d x)^2 \right) + \right. \\ & \left. 112 b^2 (b c - a d)^2 \sqrt{\frac{d (a + b x)}{-b c + a d}} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \frac{b (c + d x)}{b c - a d}\right] \right) \end{aligned}$$

### Problem 1672: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{5/2}}{(c + d x)^{9/4}} dx$$

Optimal (type 4, 248 leaves, 10 steps):

$$\begin{aligned} & -\frac{4 (a + b x)^{5/2}}{5 d (c + d x)^{5/4}} - \frac{8 b (a + b x)^{3/2}}{d^2 (c + d x)^{1/4}} + \frac{48 b^2 \sqrt{a + b x} (c + d x)^{3/4}}{5 d^3} - \\ & \frac{96 b^{5/4} (b c - a d)^{7/4} \sqrt{-\frac{d (a + b x)}{b c - a d}} \text{EllipticE}[\text{ArcSin}\left[\frac{b^{1/4} (c + d x)^{1/4}}{(b c - a d)^{1/4}}\right], -1]}{5 d^4 \sqrt{a + b x}} + \frac{96 b^{5/4} (b c - a d)^{7/4} \sqrt{-\frac{d (a + b x)}{b c - a d}} \text{EllipticF}[\text{ArcSin}\left[\frac{b^{1/4} (c + d x)^{1/4}}{(b c - a d)^{1/4}}\right], -1]}{5 d^4 \sqrt{a + b x}} \end{aligned}$$

Result (type 5, 141 leaves):

$$\frac{1}{5 d^4 \sqrt{a+b x}} 4 (c+d x)^{3/4} \\ \left( -\frac{d (a+b x) \left( (b c-a d)^2 - 12 b (b c-a d) (c+d x) - b^2 (c+d x)^2 \right)}{(c+d x)^2} - 8 b^2 (b c-a d) \sqrt{\frac{d (a+b x)}{-b c+a d}} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \frac{b (c+d x)}{b c-a d}\right] \right)$$

Problem 1673: Result unnecessarily involves higher level functions.

$$\int \frac{(a+b x)^{3/2}}{(c+d x)^{9/4}} dx$$

Optimal (type 4, 222 leaves, 9 steps):

$$\begin{aligned} & -\frac{4 (a+b x)^{3/2}}{5 d (c+d x)^{5/4}} - \frac{24 b \sqrt{a+b x}}{5 d^2 (c+d x)^{1/4}} + \frac{48 b^{5/4} (b c-a d)^{3/4} \sqrt{-\frac{d (a+b x)}{b c-a d}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c-a d)^{1/4}}\right], -1\right]}{5 d^3 \sqrt{a+b x}} - \\ & \frac{48 b^{5/4} (b c-a d)^{3/4} \sqrt{-\frac{d (a+b x)}{b c-a d}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c-a d)^{1/4}}\right], -1\right]}{5 d^3 \sqrt{a+b x}} \end{aligned}$$

Result (type 5, 107 leaves):

$$\frac{-4 d (a+b x) (6 b c+a d+7 b d x)+16 b^2 \sqrt{\frac{d (a+b x)}{-b c+a d}} (c+d x)^2 \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \frac{b (c+d x)}{b c-a d}\right]}{5 d^3 \sqrt{a+b x} (c+d x)^{5/4}}$$

Problem 1674: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{a+b x}}{(c+d x)^{9/4}} dx$$

Optimal (type 4, 232 leaves, 9 steps):

$$\begin{aligned}
 & -\frac{4 \sqrt{a+b x}}{5 d (c+d x)^{5/4}} + \frac{8 b \sqrt{a+b x}}{5 d (b c-a d) (c+d x)^{1/4}} - \\
 & \frac{8 b^{5/4} \sqrt{-\frac{d (a+b x)}{b c-a d}} \text{EllipticE}[\text{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c-a d)^{1/4}}\right], -1]}{5 d^2 (b c-a d)^{1/4} \sqrt{a+b x}} + \frac{8 b^{5/4} \sqrt{-\frac{d (a+b x)}{b c-a d}} \text{EllipticF}[\text{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c-a d)^{1/4}}\right], -1]}{5 d^2 (b c-a d)^{1/4} \sqrt{a+b x}}
 \end{aligned}$$

Result (type 5, 116 leaves):

$$\frac{-12 d (a+b x) (a d+b (c+2 d x)) + 8 b^2 \sqrt{\frac{d (a+b x)}{-b c+a d}} (c+d x)^2 \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \frac{b (c+d x)}{b c-a d}\right]}{15 d^2 (-b c+a d) \sqrt{a+b x} (c+d x)^{5/4}}$$

Problem 1675: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\sqrt{a+b x} (c+d x)^{9/4}} dx$$

Optimal (type 4, 236 leaves, 9 steps):

$$\begin{aligned}
 & \frac{4 \sqrt{a+b x}}{5 (b c-a d) (c+d x)^{5/4}} + \frac{12 b \sqrt{a+b x}}{5 (b c-a d)^2 (c+d x)^{1/4}} - \\
 & \frac{12 b^{5/4} \sqrt{-\frac{d (a+b x)}{b c-a d}} \text{EllipticE}[\text{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c-a d)^{1/4}}\right], -1]}{5 d (b c-a d)^{5/4} \sqrt{a+b x}} + \frac{12 b^{5/4} \sqrt{-\frac{d (a+b x)}{b c-a d}} \text{EllipticF}[\text{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c-a d)^{1/4}}\right], -1]}{5 d (b c-a d)^{5/4} \sqrt{a+b x}}
 \end{aligned}$$

Result (type 5, 115 leaves):

$$\begin{aligned}
 & -\frac{4 \left(d (a+b x) (-4 b c+a d-3 b d x)+b^2 \sqrt{\frac{d (a+b x)}{-b c+a d}} (c+d x)^2 \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \frac{b (c+d x)}{b c-a d}\right]\right)}{5 d (b c-a d)^2 \sqrt{a+b x} (c+d x)^{5/4}}
 \end{aligned}$$

Problem 1676: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a+b x)^{3/2} (c+d x)^{9/4}} dx$$

Optimal (type 4, 262 leaves, 10 steps):

$$\begin{aligned}
& - \frac{2}{(b c - a d) \sqrt{a + b x} (c + d x)^{5/4}} - \frac{14 d \sqrt{a + b x}}{5 (b c - a d)^2 (c + d x)^{5/4}} - \frac{42 b d \sqrt{a + b x}}{5 (b c - a d)^3 (c + d x)^{1/4}} + \\
& \frac{42 b^{5/4} \sqrt{-\frac{d (a + b x)}{b c - a d}} \text{EllipticE}[\text{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c-a d)^{1/4}}\right], -1]}{5 (b c - a d)^{9/4} \sqrt{a + b x}} - \frac{42 b^{5/4} \sqrt{-\frac{d (a + b x)}{b c - a d}} \text{EllipticF}[\text{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c-a d)^{1/4}}\right], -1]}{5 (b c - a d)^{9/4} \sqrt{a + b x}}
\end{aligned}$$

Result (type 5, 138 leaves):

$$\left( -4 a^2 d^2 + 4 a b d (9 c + 7 d x) + 2 b^2 (5 c^2 + 28 c d x + 21 d^2 x^2) - 14 b^2 \sqrt{\frac{d (a + b x)}{-b c + a d}} (c + d x)^2 \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \frac{b (c + d x)}{b c - a d}\right] \right) / \\
\left( 5 (-b c + a d)^3 \sqrt{a + b x} (c + d x)^{5/4} \right)$$

Problem 1677: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + b x)^{5/2} (c + d x)^{9/4}} dx$$

Optimal (type 4, 303 leaves, 11 steps):

$$\begin{aligned}
& - \frac{2}{3 (b c - a d) (a + b x)^{3/2} (c + d x)^{5/4}} + \frac{11 d}{3 (b c - a d)^2 \sqrt{a + b x} (c + d x)^{5/4}} + \frac{77 d^2 \sqrt{a + b x}}{15 (b c - a d)^3 (c + d x)^{5/4}} + \frac{77 b d^2 \sqrt{a + b x}}{5 (b c - a d)^4 (c + d x)^{1/4}} - \\
& \frac{77 b^{5/4} d \sqrt{-\frac{d (a + b x)}{b c - a d}} \text{EllipticE}[\text{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c-a d)^{1/4}}\right], -1]}{5 (b c - a d)^{13/4} \sqrt{a + b x}} + \frac{77 b^{5/4} d \sqrt{-\frac{d (a + b x)}{b c - a d}} \text{EllipticF}[\text{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c-a d)^{1/4}}\right], -1]}{5 (b c - a d)^{13/4} \sqrt{a + b x}}
\end{aligned}$$

Result (type 5, 156 leaves):

$$\begin{aligned}
& \frac{1}{15 (b c - a d)^4 \sqrt{a + b x}} (c + d x)^{3/4} \\
& \left( 75 b^2 d - \frac{10 b^2 (b c - a d)}{a + b x} + \frac{12 d^2 (b c - a d) (a + b x)}{(c + d x)^2} + \frac{156 b d^2 (a + b x)}{c + d x} - 77 b^2 d \sqrt{\frac{d (a + b x)}{-b c + a d}} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \frac{b (c + d x)}{b c - a d}\right] \right)
\end{aligned}$$

Problem 1678: Result unnecessarily involves higher level functions.

$$\int (a + b x)^{3/4} (c + d x)^{5/4} dx$$

Optimal (type 3, 205 leaves, 8 steps):

$$\begin{aligned} & \frac{5 (b c - a d)^2 (a + b x)^{3/4} (c + d x)^{1/4}}{96 b^2 d} + \frac{5 (b c - a d) (a + b x)^{7/4} (c + d x)^{1/4}}{24 b^2} + \\ & \frac{(a + b x)^{7/4} (c + d x)^{5/4}}{3 b} + \frac{5 (b c - a d)^3 \operatorname{ArcTan}\left[\frac{d^{1/4} (a+b x)^{1/4}}{b^{1/4} (c+d x)^{1/4}}\right]}{64 b^{9/4} d^{7/4}} - \frac{5 (b c - a d)^3 \operatorname{ArcTanh}\left[\frac{d^{1/4} (a+b x)^{1/4}}{b^{1/4} (c+d x)^{1/4}}\right]}{64 b^{9/4} d^{7/4}} \end{aligned}$$

Result (type 5, 143 leaves):

$$\begin{aligned} & \frac{1}{96 b^2 d^2 (a + b x)^{1/4}} (c + d x)^{1/4} \left( -d (a + b x) (15 a^2 d^2 - 6 a b d (7 c + 2 d x) - b^2 (5 c^2 + 52 c d x + 32 d^2 x^2)) - \right. \\ & \left. 15 (b c - a d)^3 \left( \frac{d (a + b x)}{-b c + a d} \right)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \frac{b (c + d x)}{b c - a d}\right] \right) \end{aligned}$$

Problem 1679: Result unnecessarily involves higher level functions.

$$\int \frac{(c + d x)^{5/4}}{(a + b x)^{1/4}} dx$$

Optimal (type 3, 167 leaves, 7 steps):

$$\begin{aligned} & \frac{5 (b c - a d) (a + b x)^{3/4} (c + d x)^{1/4}}{8 b^2} + \frac{(a + b x)^{3/4} (c + d x)^{5/4}}{2 b} - \frac{5 (b c - a d)^2 \operatorname{ArcTan}\left[\frac{d^{1/4} (a+b x)^{1/4}}{b^{1/4} (c+d x)^{1/4}}\right]}{16 b^{9/4} d^{3/4}} + \frac{5 (b c - a d)^2 \operatorname{ArcTanh}\left[\frac{d^{1/4} (a+b x)^{1/4}}{b^{1/4} (c+d x)^{1/4}}\right]}{16 b^{9/4} d^{3/4}} \end{aligned}$$

Result (type 5, 111 leaves):

$$\frac{1}{8 b^2 d (a + b x)^{1/4}} (c + d x)^{1/4} \left( -d (a + b x) (-9 b c + 5 a d - 4 b d x) + 5 (b c - a d)^2 \left( \frac{d (a + b x)}{-b c + a d} \right)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \frac{b (c + d x)}{b c - a d}\right] \right)$$

Problem 1680: Result unnecessarily involves higher level functions.

$$\int \frac{(c + d x)^{5/4}}{(a + b x)^{5/4}} dx$$

Optimal (type 3, 152 leaves, 7 steps):

$$\begin{aligned} & \frac{5 d (a + b x)^{3/4} (c + d x)^{1/4}}{b^2} - \frac{4 (c + d x)^{5/4}}{b (a + b x)^{1/4}} - \frac{5 d^{1/4} (b c - a d) \operatorname{ArcTan}\left[\frac{d^{1/4} (a+b x)^{1/4}}{b^{1/4} (c+d x)^{1/4}}\right]}{2 b^{9/4}} + \frac{5 d^{1/4} (b c - a d) \operatorname{ArcTanh}\left[\frac{d^{1/4} (a+b x)^{1/4}}{b^{1/4} (c+d x)^{1/4}}\right]}{2 b^{9/4}} \end{aligned}$$

Result (type 5, 93 leaves):

$$\frac{(c+d x)^{1/4} \left(-4 b c+5 a d+b d x+5 (b c-a d) \left(\frac{d (a+b x)}{-b c+a d}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \frac{b (c+d x)}{b c-a d}\right]\right)}{b^2 (a+b x)^{1/4}}$$

**Problem 1681:** Result unnecessarily involves higher level functions.

$$\int \frac{(c+d x)^{5/4}}{(a+b x)^{9/4}} dx$$

Optimal (type 3, 134 leaves, 7 steps):

$$-\frac{4 d (c+d x)^{1/4}}{b^2 (a+b x)^{1/4}} - \frac{4 (c+d x)^{5/4}}{5 b (a+b x)^{5/4}} - \frac{2 d^{5/4} \text{ArcTan}\left[\frac{d^{1/4} (a+b x)^{1/4}}{b^{1/4} (c+d x)^{1/4}}\right]}{b^{9/4}} + \frac{2 d^{5/4} \text{ArcTanh}\left[\frac{d^{1/4} (a+b x)^{1/4}}{b^{1/4} (c+d x)^{1/4}}\right]}{b^{9/4}}$$

Result (type 5, 94 leaves):

$$-\frac{4 (c+d x)^{1/4} \left(5 a d+b (c+6 d x)-5 d (a+b x) \left(\frac{d (a+b x)}{-b c+a d}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \frac{b (c+d x)}{b c-a d}\right]\right)}{5 b^2 (a+b x)^{5/4}}$$

**Problem 1686:** Result unnecessarily involves higher level functions.

$$\int (a+b x)^{5/4} (c+d x)^{5/4} dx$$

Optimal (type 4, 408 leaves, 7 steps):

$$\begin{aligned} & -\frac{5 (b c-a d)^3 (a+b x)^{1/4} (c+d x)^{1/4}}{84 b^2 d^2} + \frac{(b c-a d)^2 (a+b x)^{5/4} (c+d x)^{1/4}}{42 b^2 d} + \frac{(b c-a d) (a+b x)^{9/4} (c+d x)^{1/4}}{7 b^2} + \\ & \frac{2 (a+b x)^{9/4} (c+d x)^{5/4}}{7 b} + \left( 5 (b c-a d)^{9/2} ((a+b x) (c+d x))^{3/4} \sqrt{(b c+a d+2 b d x)^2} \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a+b x) (c+d x)}}{b c-a d} \right) \right. \\ & \left. \sqrt{\frac{(a d+b (c+2 d x))^2}{(b c-a d)^2 \left(1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a+b x) (c+d x)}}{b c-a d}\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{\sqrt{2} b^{1/4} d^{1/4} ((a+b x) (c+d x))^{1/4}}{\sqrt{b c-a d}}\right], \frac{1}{2}\right] \right) / \\ & \left( 168 \sqrt{2} b^{9/4} d^{9/4} (a+b x)^{3/4} (c+d x)^{3/4} (b c+a d+2 b d x) \sqrt{(a d+b (c+2 d x))^2} \right) \end{aligned}$$

Result (type 5, 183 leaves):

$$\frac{1}{84 b^2 d^3 (a + b x)^{3/4}} \\ (c + d x)^{1/4} \left( -d (a + b x) (5 a^3 d^3 - a^2 b d^2 (17 c + 2 d x) - a b^2 d (17 c^2 + 68 c d x + 36 d^2 x^2) + b^3 (5 c^3 - 2 c^2 d x - 36 c d^2 x^2 - 24 d^3 x^3) \right) + \\ 5 (b c - a d)^4 \left( \frac{d (a + b x)}{-b c + a d} \right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{b (c + d x)}{b c - a d}\right]$$

Problem 1687: Result unnecessarily involves higher level functions.

$$\int (a + b x)^{1/4} (c + d x)^{5/4} dx$$

Optimal (type 4, 370 leaves, 6 steps):

$$\frac{(b c - a d)^2 (a + b x)^{1/4} (c + d x)^{1/4}}{6 b^2 d} + \frac{(b c - a d) (a + b x)^{5/4} (c + d x)^{1/4}}{3 b^2} + \frac{2 (a + b x)^{5/4} (c + d x)^{5/4}}{5 b} - \\ \left( (b c - a d)^{7/2} ((a + b x) (c + d x))^{3/4} \sqrt{(b c + a d + 2 b d x)^2} \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + b x) (c + d x)}}{b c - a d} \right) \right. \\ \left. \sqrt{\frac{(a d + b (c + 2 d x))^2}{(b c - a d)^2 \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + b x) (c + d x)}}{b c - a d} \right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{\sqrt{2} b^{1/4} d^{1/4} ((a + b x) (c + d x))^{1/4}}{\sqrt{b c - a d}}\right], \frac{1}{2}\right] \right) / \\ \left( 12 \sqrt{2} b^{9/4} d^{5/4} (a + b x)^{3/4} (c + d x)^{3/4} (b c + a d + 2 b d x) \sqrt{(a d + b (c + 2 d x))^2} \right)$$

Result (type 5, 142 leaves):

$$\frac{1}{30 b^2 d^2 (a + b x)^{3/4}} (c + d x)^{1/4} \\ (-d (a + b x) (5 a^2 d^2 - 2 a b d (6 c + d x) - b^2 (5 c^2 + 22 c d x + 12 d^2 x^2)) - 5 (b c - a d)^3 \left( \frac{d (a + b x)}{-b c + a d} \right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{b (c + d x)}{b c - a d}\right])$$

### Problem 1688: Result unnecessarily involves higher level functions.

$$\int \frac{(c + dx)^{5/4}}{(a + bx)^{3/4}} dx$$

Optimal (type 4, 332 leaves, 5 steps):

$$\frac{5(b c - a d) (a + b x)^{1/4} (c + d x)^{1/4}}{3 b^2} + \frac{2 (a + b x)^{1/4} (c + d x)^{5/4}}{3 b} +$$

$$\left( \frac{5 (b c - a d)^{5/2} ((a + b x) (c + d x))^{3/4} \sqrt{(b c + a d + 2 b d x)^2}}{(b c - a d)^2 \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + b x) (c + d x)}}{b c - a d} \right)^2} \right)$$

$$\left. \left( \frac{(a d + b (c + 2 d x))^2}{(b c - a d)^2 \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + b x) (c + d x)}}{b c - a d} \right)^2} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{\sqrt{2} b^{1/4} d^{1/4} ((a + b x) (c + d x))^{1/4}}{\sqrt{b c - a d}}\right], \frac{1}{2}\right] \right) \middle/ \right.$$

$$\left. \left( 6 \sqrt{2} b^{9/4} d^{1/4} (a + b x)^{3/4} (c + d x)^{3/4} (b c + a d + 2 b d x) \sqrt{(a d + b (c + 2 d x))^2} \right) \right)$$

Result (type 5, 111 leaves):

$$\frac{1}{3 b^2 d (a + b x)^{3/4}} (c + d x)^{1/4} \left( -d (a + b x) (-7 b c + 5 a d - 2 b d x) + 5 (b c - a d)^2 \left( \frac{d (a + b x)}{-b c + a d} \right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{b (c + d x)}{b c - a d}\right] \right)$$

### Problem 1689: Result unnecessarily involves higher level functions.

$$\int \frac{(c + dx)^{5/4}}{(a + bx)^{7/4}} dx$$

Optimal (type 4, 325 leaves, 5 steps):

$$\frac{10 d \left(a+b x\right)^{1/4} \left(c+d x\right)^{1/4}}{3 b^2} - \frac{4 \left(c+d x\right)^{5/4}}{3 b \left(a+b x\right)^{3/4}} +$$

$$\left( 5 d^{3/4} (b c - a d)^{3/2} ((a+b x) (c+d x))^{3/4} \sqrt{(b c + a d + 2 b d x)^2} \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a+b x) (c+d x)}}{b c - a d} \right) \sqrt{\frac{(a d + b (c+2 d x))^2}{(b c - a d)^2 \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a+b x) (c+d x)}}{b c - a d} \right)^2}} \right.$$

$$\left. \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{\sqrt{2} b^{1/4} d^{1/4} ((a+b x) (c+d x))^{1/4}}{\sqrt{b c - a d}}\right], \frac{1}{2}\right]\right) / \left( 3 \sqrt{2} b^{9/4} (a+b x)^{3/4} (c+d x)^{3/4} (b c + a d + 2 b d x) \sqrt{(a d + b (c+2 d x))^2} \right)$$

Result (type 5, 95 leaves):

$$-\frac{2 (c+d x)^{1/4} \left( 2 b c - 5 a d - 3 b d x + \frac{5 d (a+b x) \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{b (c+d x)}{b c - a d}\right]}{\left(\frac{d (a+b x)}{-b c + a d}\right)^{1/4}} \right)}{3 b^2 (a+b x)^{3/4}}$$

Problem 1690: Result unnecessarily involves higher level functions.

$$\int \frac{(c+d x)^{5/4}}{(a+b x)^{11/4}} dx$$

Optimal (type 4, 325 leaves, 5 steps):

$$-\frac{20 d (c+d x)^{1/4}}{21 b^2 (a+b x)^{3/4}} - \frac{4 (c+d x)^{5/4}}{7 b (a+b x)^{7/4}} +$$

$$\left( 5 \sqrt{2} d^{7/4} \sqrt{b c - a d} ((a+b x) (c+d x))^{3/4} \sqrt{(b c + a d + 2 b d x)^2} \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a+b x) (c+d x)}}{b c - a d} \right) \right.$$

$$\left. \sqrt{\frac{(a d + b (c+2 d x))^2}{(b c - a d)^2 \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a+b x) (c+d x)}}{b c - a d} \right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{\sqrt{2} b^{1/4} d^{1/4} ((a+b x) (c+d x))^{1/4}}{\sqrt{b c - a d}}\right], \frac{1}{2}\right]\right) /$$

$$\left( 21 b^{9/4} (a+b x)^{3/4} (c+d x)^{3/4} (b c + a d + 2 b d x) \sqrt{(a d + b (c+2 d x))^2} \right)$$

Result (type 5, 95 leaves):

$$-\frac{4 (c + d x)^{1/4} \left(3 b c + 5 a d + 8 b d x - 5 d (a + b x) \left(\frac{d (a + b x)}{-b c + a d}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{b (c + d x)}{b c - a d}\right]\right)}{21 b^2 (a + b x)^{7/4}}$$

Problem 1691: Result unnecessarily involves higher level functions.

$$\int \frac{(c + d x)^{5/4}}{(a + b x)^{15/4}} dx$$

Optimal (type 4, 363 leaves, 6 steps):

$$\begin{aligned} & -\frac{20 d (c + d x)^{1/4}}{77 b^2 (a + b x)^{7/4}} - \frac{20 d^2 (c + d x)^{1/4}}{231 b^2 (b c - a d) (a + b x)^{3/4}} - \frac{4 (c + d x)^{5/4}}{11 b (a + b x)^{11/4}} - \\ & \left( 10 \sqrt{2} d^{11/4} ((a + b x) (c + d x))^{3/4} \sqrt{(b c + a d + 2 b d x)^2} \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + b x) (c + d x)}}{b c - a d} \right) \right. \\ & \left. \sqrt{\frac{(a d + b (c + 2 d x))^2}{(b c - a d)^2 \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + b x) (c + d x)}}{b c - a d} \right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{\sqrt{2} b^{1/4} d^{1/4} ((a + b x) (c + d x))^{1/4}}{\sqrt{b c - a d}}\right], \frac{1}{2}\right] \right) / \\ & \left( 231 b^{9/4} \sqrt{b c - a d} (a + b x)^{3/4} (c + d x)^{3/4} (b c + a d + 2 b d x) \sqrt{(a d + b (c + 2 d x))^2} \right) \end{aligned}$$

Result (type 5, 140 leaves):

$$\begin{aligned} & \left( 4 (c + d x)^{1/4} \left( -10 a^2 d^2 - 2 a b d (3 c + 13 d x) + b^2 (21 c^2 + 36 c d x + 5 d^2 x^2) + \right. \right. \\ & \left. \left. 10 d^2 (a + b x)^2 \left( \frac{d (a + b x)}{-b c + a d} \right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{b (c + d x)}{b c - a d}\right] \right) \right) / (231 b^2 (-b c + a d) (a + b x)^{11/4}) \end{aligned}$$

Problem 1692: Result unnecessarily involves higher level functions.

$$\int \frac{(c + d x)^{5/4}}{(a + b x)^{19/4}} dx$$

Optimal (type 4, 401 leaves, 7 steps):

$$\begin{aligned}
 & -\frac{4 d (c + d x)^{1/4}}{33 b^2 (a + b x)^{11/4}} - \frac{4 d^2 (c + d x)^{1/4}}{231 b^2 (b c - a d) (a + b x)^{7/4}} + \frac{8 d^3 (c + d x)^{1/4}}{231 b^2 (b c - a d)^2 (a + b x)^{3/4}} - \\
 & \frac{4 (c + d x)^{5/4}}{15 b (a + b x)^{15/4}} + \left( 4 \sqrt{2} d^{15/4} ((a + b x) (c + d x))^{3/4} \sqrt{(b c + a d + 2 b d x)^2} \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + b x) (c + d x)}}{b c - a d} \right) \right. \\
 & \left. \sqrt{\frac{(a d + b (c + 2 d x))^2}{(b c - a d)^2 \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + b x) (c + d x)}}{b c - a d} \right)^2}} \text{EllipticF}\left[ 2 \text{ArcTan}\left[ \frac{\sqrt{2} b^{1/4} d^{1/4} ((a + b x) (c + d x))^{1/4}}{\sqrt{b c - a d}} \right], \frac{1}{2} \right] \right) / \\
 & \left( 231 b^{9/4} (b c - a d)^{3/2} (a + b x)^{3/4} (c + d x)^{3/4} (b c + a d + 2 b d x) \sqrt{(a d + b (c + 2 d x))^2} \right)
 \end{aligned}$$

Result (type 5, 179 leaves):

$$\begin{aligned}
 & \left( 4 (c + d x)^{1/4} \left( -20 a^3 d^3 - 12 a^2 b d^2 (c + 6 d x) + a b^2 d (119 c^2 + 214 c d x + 35 d^2 x^2) - b^3 (77 c^3 + 112 c^2 d x + 5 c d^2 x^2 - 10 d^3 x^3) + \right. \right. \\
 & \left. \left. 20 d^3 (a + b x)^3 \left( \frac{d (a + b x)}{-b c + a d} \right)^{3/4} \text{Hypergeometric2F1}\left[ \frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{b (c + d x)}{b c - a d} \right] \right) \right) / (1155 b^2 (b c - a d)^2 (a + b x)^{15/4})
 \end{aligned}$$

Problem 1693: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{5/4}}{(c + d x)^{1/4}} dx$$

Optimal (type 3, 167 leaves, 7 steps):

$$\begin{aligned}
 & -\frac{5 (b c - a d) (a + b x)^{1/4} (c + d x)^{3/4}}{8 d^2} + \frac{(a + b x)^{5/4} (c + d x)^{3/4}}{2 d} + \frac{5 (b c - a d)^2 \text{ArcTan}\left[ \frac{d^{1/4} (a + b x)^{1/4}}{b^{1/4} (c + d x)^{1/4}} \right]}{16 b^{3/4} d^{9/4}} + \frac{5 (b c - a d)^2 \text{ArcTanh}\left[ \frac{d^{1/4} (a + b x)^{1/4}}{b^{1/4} (c + d x)^{1/4}} \right]}{16 b^{3/4} d^{9/4}}
 \end{aligned}$$

Result (type 5, 108 leaves):

$$\frac{1}{24 d^3 (a + b x)^{3/4}} (c + d x)^{3/4} \left( 3 d (a + b x) (-5 b c + 9 a d + 4 b d x) + 5 (b c - a d)^2 \left( \frac{d (a + b x)}{-b c + a d} \right)^{3/4} \text{Hypergeometric2F1}\left[ \frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b (c + d x)}{b c - a d} \right] \right)$$

### Problem 1694: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{1/4}}{(c + d x)^{1/4}} dx$$

Optimal (type 3, 127 leaves, 6 steps):

$$\frac{(a + b x)^{1/4} (c + d x)^{3/4}}{d} - \frac{(b c - a d) \operatorname{ArcTan}\left[\frac{d^{1/4} (a+b x)^{1/4}}{b^{1/4} (c+d x)^{1/4}}\right]}{2 b^{3/4} d^{5/4}} - \frac{(b c - a d) \operatorname{ArcTanh}\left[\frac{d^{1/4} (a+b x)^{1/4}}{b^{1/4} (c+d x)^{1/4}}\right]}{2 b^{3/4} d^{5/4}}$$

Result (type 5, 76 leaves):

$$\frac{(a + b x)^{1/4} (c + d x)^{3/4} \left( 3 + \frac{\operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b(c+d x)}{-b c+a d}\right]}{\left(\frac{d(a+b x)}{-b c+a d}\right)^{1/4}} \right)}{3 d}$$

### Problem 1695: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + b x)^{3/4} (c + d x)^{1/4}} dx$$

Optimal (type 3, 85 leaves, 5 steps):

$$\frac{2 \operatorname{ArcTan}\left[\frac{d^{1/4} (a+b x)^{1/4}}{b^{1/4} (c+d x)^{1/4}}\right]}{b^{3/4} d^{1/4}} + \frac{2 \operatorname{ArcTanh}\left[\frac{d^{1/4} (a+b x)^{1/4}}{b^{1/4} (c+d x)^{1/4}}\right]}{b^{3/4} d^{1/4}}$$

Result (type 5, 73 leaves):

$$\frac{4 \left(\frac{d(a+b x)}{-b c+a d}\right)^{3/4} (c + d x)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b(c+d x)}{b c-a d}\right]}{3 d (a + b x)^{3/4}}$$

### Problem 1700: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{7/4}}{(c + d x)^{1/4}} dx$$

Optimal (type 4, 751 leaves, 7 steps):

$$\frac{\frac{7(b c - a d) (a + b x)^{3/4} (c + d x)^{3/4}}{15 d^2} + \frac{2(a + b x)^{7/4} (c + d x)^{3/4}}{5 d} + \frac{7(b c - a d) \sqrt{(a + b x) (c + d x)} \sqrt{(b c + a d + 2 b d x)^2} \sqrt{(a d + b (c + 2 d x))^2}}{10 \sqrt{b} d^{5/2} (a + b x)^{1/4} (c + d x)^{1/4} (b c + a d + 2 b d x) \left(1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a+b x) (c+d x)}}{b c - a d}\right)} -$$

$$\left( 7(b c - a d)^{7/2} ((a + b x) (c + d x))^{1/4} \sqrt{(b c + a d + 2 b d x)^2} \left(1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + b x) (c + d x)}}{b c - a d}\right) \right.$$

$$\left. \sqrt{\frac{(a d + b (c + 2 d x))^2}{(b c - a d)^2 \left(1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a+b x) (c+d x)}}{b c - a d}\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{\sqrt{2} b^{1/4} d^{1/4} ((a + b x) (c + d x))^{1/4}}{\sqrt{b c - a d}}\right], \frac{1}{2}\right]\right)$$

$$\left( 10 \sqrt{2} b^{3/4} d^{11/4} (a + b x)^{1/4} (c + d x)^{1/4} (b c + a d + 2 b d x) \sqrt{(a d + b (c + 2 d x))^2} \right) +$$

$$\left( 7(b c - a d)^{7/2} ((a + b x) (c + d x))^{1/4} \sqrt{(b c + a d + 2 b d x)^2} \left(1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + b x) (c + d x)}}{b c - a d}\right) \right.$$

$$\left. \sqrt{\frac{(a d + b (c + 2 d x))^2}{(b c - a d)^2 \left(1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a+b x) (c+d x)}}{b c - a d}\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{\sqrt{2} b^{1/4} d^{1/4} ((a + b x) (c + d x))^{1/4}}{\sqrt{b c - a d}}\right], \frac{1}{2}\right]\right)$$

$$\left( 20 \sqrt{2} b^{3/4} d^{11/4} (a + b x)^{1/4} (c + d x)^{1/4} (b c + a d + 2 b d x) \sqrt{(a d + b (c + 2 d x))^2} \right)$$

Result (type 5, 107 leaves):

$$\frac{1}{15 d^3 (a + b x)^{1/4}} (c + d x)^{3/4} \left( d (a + b x) (-7 b c + 13 a d + 6 b d x) + 7 (b c - a d)^2 \left(\frac{d (a + b x)}{-b c + a d}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b (c + d x)}{b c - a d}\right] \right)$$

**Problem 1701: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + b x)^{3/4}}{(c + d x)^{1/4}} dx$$

Optimal (type 4, 705 leaves, 6 steps):

$$\begin{aligned}
 & \frac{2 (a+b x)^{3/4} (c+d x)^{3/4}}{3 d} - \frac{\sqrt{(a+b x) (c+d x)} \sqrt{(b c + a d + 2 b d x)^2} \sqrt{(a d + b (c + 2 d x))^2}}{\sqrt{b} d^{3/2} (a+b x)^{1/4} (c+d x)^{1/4} (b c + a d + 2 b d x) \left(1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a+b x) (c+d x)}}{b c - a d}\right)} + \\
 & \left( (b c - a d)^{5/2} ((a+b x) (c+d x))^{1/4} \sqrt{(b c + a d + 2 b d x)^2} \left(1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a+b x) (c+d x)}}{b c - a d}\right) \right. \\
 & \left. \sqrt{\frac{(a d + b (c + 2 d x))^2}{(b c - a d)^2 \left(1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a+b x) (c+d x)}}{b c - a d}\right)^2} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{\sqrt{2} b^{1/4} d^{1/4} ((a+b x) (c+d x))^{1/4}}{\sqrt{b c - a d}}\right], \frac{1}{2}\right]} \right) / \\
 & \left( \sqrt{2} b^{3/4} d^{7/4} (a+b x)^{1/4} (c+d x)^{1/4} (b c + a d + 2 b d x) \sqrt{(a d + b (c + 2 d x))^2} \right) - \\
 & \left( (b c - a d)^{5/2} ((a+b x) (c+d x))^{1/4} \sqrt{(b c + a d + 2 b d x)^2} \left(1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a+b x) (c+d x)}}{b c - a d}\right) \right. \\
 & \left. \sqrt{\frac{(a d + b (c + 2 d x))^2}{(b c - a d)^2 \left(1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a+b x) (c+d x)}}{b c - a d}\right)^2} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{\sqrt{2} b^{1/4} d^{1/4} ((a+b x) (c+d x))^{1/4}}{\sqrt{b c - a d}}\right], \frac{1}{2}\right]} \right) / \\
 & \left( 2 \sqrt{2} b^{3/4} d^{7/4} (a+b x)^{1/4} (c+d x)^{1/4} (b c + a d + 2 b d x) \sqrt{(a d + b (c + 2 d x))^2} \right)
 \end{aligned}$$

Result (type 5, 76 leaves):

$$\frac{2 (a+b x)^{3/4} (c+d x)^{3/4} \left(1 + \frac{\text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b (c+d x)}{b c - a d}\right]}{\left(\frac{d (a+b x)}{-b c + a d}\right)^{3/4}}\right)}{3 d}$$

### Problem 1702: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + b x)^{1/4} (c + d x)^{1/4}} dx$$

Optimal (type 4, 688 leaves, 5 steps):

$$\begin{aligned} & \frac{2 \sqrt{(a + b x) (c + d x)} \sqrt{(b c + a d + 2 b d x)^2} \sqrt{(a d + b (c + 2 d x))^2}}{\sqrt{b} \sqrt{d} (b c - a d) (a + b x)^{1/4} (c + d x)^{1/4} (b c + a d + 2 b d x) \left(1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + b x) (c + d x)}}{b c - a d}\right)} - \\ & \left( \sqrt{2} (b c - a d)^{3/2} ((a + b x) (c + d x))^{1/4} \sqrt{(b c + a d + 2 b d x)^2} \left(1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + b x) (c + d x)}}{b c - a d}\right) \right) \sqrt{\frac{(a d + b (c + 2 d x))^2}{(b c - a d)^2 \left(1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + b x) (c + d x)}}{b c - a d}\right)^2}} \\ & \text{EllipticE} \left[ 2 \text{ArcTan} \left[ \frac{\sqrt{2} b^{1/4} d^{1/4} ((a + b x) (c + d x))^{1/4}}{\sqrt{b c - a d}} \right], \frac{1}{2} \right] / \left( b^{3/4} d^{3/4} (a + b x)^{1/4} (c + d x)^{1/4} (b c + a d + 2 b d x) \sqrt{(a d + b (c + 2 d x))^2} \right) + \\ & \left( b c - a d \right)^{3/2} ((a + b x) (c + d x))^{1/4} \sqrt{(b c + a d + 2 b d x)^2} \left(1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + b x) (c + d x)}}{b c - a d}\right) \\ & \sqrt{\frac{(a d + b (c + 2 d x))^2}{(b c - a d)^2 \left(1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + b x) (c + d x)}}{b c - a d}\right)^2}} \text{EllipticF} \left[ 2 \text{ArcTan} \left[ \frac{\sqrt{2} b^{1/4} d^{1/4} ((a + b x) (c + d x))^{1/4}}{\sqrt{b c - a d}} \right], \frac{1}{2} \right] / \\ & \left( \sqrt{2} b^{3/4} d^{3/4} (a + b x)^{1/4} (c + d x)^{1/4} (b c + a d + 2 b d x) \sqrt{(a d + b (c + 2 d x))^2} \right) \end{aligned}$$

Result (type 5, 73 leaves):

$$\frac{4 \left(\frac{d (a + b x)}{-b c + a d}\right)^{1/4} (c + d x)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b (c + d x)}{b c - a d}\right]}{3 d (a + b x)^{1/4}}$$

### Problem 1703: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + bx)^{5/4} (c + dx)^{1/4}} dx$$

Optimal (type 4, 718 leaves, 6 steps):

$$\begin{aligned}
& -\frac{4 (c + dx)^{3/4}}{(bc - ad) (a + bx)^{1/4}} + \frac{4 \sqrt{d} \sqrt{(a + bx) (c + dx)} \sqrt{(bc + ad + 2bdx)^2} \sqrt{(ad + b(c + 2dx))^2}}{\sqrt{b} (bc - ad)^2 (a + bx)^{1/4} (c + dx)^{1/4} (bc + ad + 2bdx) \left(1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + bx) (c + dx)}}{bc - ad}\right)} - \\
& \left( 2 \sqrt{2} d^{1/4} \sqrt{bc - ad} ((a + bx) (c + dx))^{1/4} \sqrt{(bc + ad + 2bdx)^2} \left(1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + bx) (c + dx)}}{bc - ad}\right) \right. \\
& \left. \sqrt{\frac{(ad + b(c + 2dx))^2}{(bc - ad)^2 \left(1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + bx) (c + dx)}}{bc - ad}\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{\sqrt{2} b^{1/4} d^{1/4} ((a + bx) (c + dx))^{1/4}}{\sqrt{bc - ad}}\right], \frac{1}{2}\right] \right) / \\
& \left( b^{3/4} (a + bx)^{1/4} (c + dx)^{1/4} (bc + ad + 2bdx) \sqrt{(ad + b(c + 2dx))^2} \right) + \\
& \left( \sqrt{2} d^{1/4} \sqrt{bc - ad} ((a + bx) (c + dx))^{1/4} \sqrt{(bc + ad + 2bdx)^2} \left(1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + bx) (c + dx)}}{bc - ad}\right) \sqrt{\frac{(ad + b(c + 2dx))^2}{(bc - ad)^2 \left(1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + bx) (c + dx)}}{bc - ad}\right)^2}} \right. \\
& \left. \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{\sqrt{2} b^{1/4} d^{1/4} ((a + bx) (c + dx))^{1/4}}{\sqrt{bc - ad}}\right], \frac{1}{2}\right] \right) / \left( b^{3/4} (a + bx)^{1/4} (c + dx)^{1/4} (bc + ad + 2bdx) \sqrt{(ad + b(c + 2dx))^2} \right)
\end{aligned}$$

Result (type 5, 84 leaves):

$$\frac{4 (c + dx)^{3/4} \left(-3 + 2 \left(\frac{d(a + bx)}{-bc + ad}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b(c + dx)}{bc - ad}\right]\right)}{3 (bc - ad) (a + bx)^{1/4}}$$

Problem 1704: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + bx)^{9/4} (c + dx)^{1/4}} dx$$

Optimal (type 4, 760 leaves, 7 steps):

$$\begin{aligned}
& -\frac{4 (c + dx)^{3/4}}{5 (bc - ad) (a + bx)^{5/4}} + \frac{8d (c + dx)^{3/4}}{5 (bc - ad)^2 (a + bx)^{1/4}} - \frac{8d^{3/2} \sqrt{(a + bx) (c + dx)} \sqrt{(bc + ad + 2bdx)^2} \sqrt{(ad + b(c + 2dx))^2}}{5\sqrt{b} (bc - ad)^3 (a + bx)^{1/4} (c + dx)^{1/4} (bc + ad + 2bdx) \left(1 + \frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}\right)} + \\
& \left( 4\sqrt{2} d^{5/4} ((a + bx) (c + dx))^{1/4} \sqrt{(bc + ad + 2bdx)^2} \left(1 + \frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}\right) \right. \\
& \left. \sqrt{\frac{(ad + b(c + 2dx))^2}{(bc - ad)^2 \left(1 + \frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{\sqrt{2} b^{1/4} d^{1/4} ((a + bx) (c + dx))^{1/4}}{\sqrt{bc - ad}}\right], \frac{1}{2}\right] \right) / \\
& \left( 5b^{3/4} \sqrt{bc - ad} (a + bx)^{1/4} (c + dx)^{1/4} (bc + ad + 2bdx) \sqrt{(ad + b(c + 2dx))^2} \right) - \\
& \left( 2\sqrt{2} d^{5/4} ((a + bx) (c + dx))^{1/4} \sqrt{(bc + ad + 2bdx)^2} \left(1 + \frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}\right) \right. \\
& \left. \sqrt{\frac{(ad + b(c + 2dx))^2}{(bc - ad)^2 \left(1 + \frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{\sqrt{2} b^{1/4} d^{1/4} ((a + bx) (c + dx))^{1/4}}{\sqrt{bc - ad}}\right], \frac{1}{2}\right] \right) /
\end{aligned}$$

Result (type 5, 102 leaves):

$$-\frac{4 (c+d x)^{3/4} \left(-9 a d+3 b \left(c-2 d x\right)+4 d \left(a+b x\right) \left(\frac{d (a+b x)}{-b c+a d}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b (c+d x)}{b c-a d}\right]\right)}{15 \left(b c-a d\right)^2 \left(a+b x\right)^{5/4}}$$

**Problem 1705:** Result unnecessarily involves higher level functions.

$$\int \frac{(a+b x)^{7/4}}{(c+d x)^{3/4}} dx$$

Optimal (type 3, 167 leaves, 7 steps):

$$-\frac{7 (b c-a d) (a+b x)^{3/4} (c+d x)^{1/4}}{8 d^2} + \frac{(a+b x)^{7/4} (c+d x)^{1/4}}{2 d} - \frac{21 (b c-a d)^2 \text{ArcTan}\left[\frac{d^{1/4} (a+b x)^{1/4}}{b^{1/4} (c+d x)^{1/4}}\right]}{16 b^{1/4} d^{11/4}} + \frac{21 (b c-a d)^2 \text{ArcTanh}\left[\frac{d^{1/4} (a+b x)^{1/4}}{b^{1/4} (c+d x)^{1/4}}\right]}{16 b^{1/4} d^{11/4}}$$

Result (type 5, 107 leaves):

$$\frac{1}{8 d^3 (a+b x)^{1/4}} (c+d x)^{1/4} \left(d (a+b x) (-7 b c+11 a d+4 b d x)+21 (b c-a d)^2 \left(\frac{d (a+b x)}{-b c+a d}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \frac{b (c+d x)}{b c-a d}\right]\right)$$

**Problem 1706:** Result unnecessarily involves higher level functions.

$$\int \frac{(a+b x)^{3/4}}{(c+d x)^{3/4}} dx$$

Optimal (type 3, 127 leaves, 6 steps):

$$\frac{(a+b x)^{3/4} (c+d x)^{1/4}}{d} + \frac{3 (b c-a d) \text{ArcTan}\left[\frac{d^{1/4} (a+b x)^{1/4}}{b^{1/4} (c+d x)^{1/4}}\right]}{2 b^{1/4} d^{7/4}} - \frac{3 (b c-a d) \text{ArcTanh}\left[\frac{d^{1/4} (a+b x)^{1/4}}{b^{1/4} (c+d x)^{1/4}}\right]}{2 b^{1/4} d^{7/4}}$$

Result (type 5, 74 leaves):

$$\frac{(a+b x)^{3/4} (c+d x)^{1/4} \left(1+\frac{3 \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \frac{b (c+d x)}{b c-a d}\right]}{\left(\frac{d (a+b x)}{-b c+a d}\right)^{3/4}}\right)}{d}$$

**Problem 1707:** Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a+b x)^{1/4} (c+d x)^{3/4}} dx$$

Optimal (type 3, 85 leaves, 5 steps):

$$-\frac{2 \operatorname{ArcTan}\left[\frac{d^{1/4} (a+b x)^{1/4}}{b^{1/4} (c+d x)^{1/4}}\right]}{b^{1/4} d^{3/4}} + \frac{2 \operatorname{ArcTanh}\left[\frac{d^{1/4} (a+b x)^{1/4}}{b^{1/4} (c+d x)^{1/4}}\right]}{b^{1/4} d^{3/4}}$$

Result (type 5, 71 leaves) :

$$\frac{4 \left(\frac{d (a+b x)}{-b c+a d}\right)^{1/4} (c+d x)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \frac{b (c+d x)}{b c-a d}\right]}{d (a+b x)^{1/4}}$$

Problem 1712: Result unnecessarily involves higher level functions.

$$\int \frac{(a+b x)^{5/4}}{(c+d x)^{3/4}} dx$$

Optimal (type 4, 332 leaves, 5 steps) :

$$\begin{aligned} & -\frac{5 (b c - a d) (a+b x)^{1/4} (c+d x)^{1/4}}{3 d^2} + \frac{2 (a+b x)^{5/4} (c+d x)^{1/4}}{3 d} + \\ & \left( 5 (b c - a d)^{5/2} ((a+b x) (c+d x))^{3/4} \sqrt{(b c + a d + 2 b d x)^2} \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a+b x) (c+d x)}}{b c - a d} \right) \right. \\ & \left. \sqrt{\frac{(a d + b (c + 2 d x))^2}{(b c - a d)^2 \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a+b x) (c+d x)}}{b c - a d} \right)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{\sqrt{2} b^{1/4} d^{1/4} ((a+b x) (c+d x))^{1/4}}{\sqrt{b c - a d}}\right], \frac{1}{2}\right] \right) / \\ & \left( 6 \sqrt{2} b^{1/4} d^{9/4} (a+b x)^{3/4} (c+d x)^{3/4} (b c + a d + 2 b d x) \sqrt{(a d + b (c + 2 d x))^2} \right) \end{aligned}$$

Result (type 5, 107 leaves) :

$$\frac{1}{3 d^3 (a+b x)^{3/4}} (c+d x)^{1/4} \left( d (a+b x) (-5 b c + 7 a d + 2 b d x) + 5 (b c - a d)^2 \left( \frac{d (a+b x)}{-b c + a d} \right)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{b (c+d x)}{b c - a d}\right] \right)$$

Problem 1713: Result unnecessarily involves higher level functions.

$$\int \frac{(a+b x)^{1/4}}{(c+d x)^{3/4}} dx$$

Optimal (type 4, 295 leaves, 4 steps):

$$\frac{2 (a + b x)^{1/4} (c + d x)^{1/4}}{d} -$$

$$\left( \begin{aligned} & (b c - a d)^{3/2} ((a + b x) (c + d x))^{3/4} \sqrt{(b c + a d + 2 b d x)^2} \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + b x) (c + d x)}}{b c - a d} \right) \sqrt{\frac{(a d + b (c + 2 d x))^2}{(b c - a d)^2 \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + b x) (c + d x)}}{b c - a d} \right)^2}} \\ & \text{EllipticF} \left[ 2 \text{ArcTan} \left[ \frac{\sqrt{2} b^{1/4} d^{1/4} ((a + b x) (c + d x))^{1/4}}{\sqrt{b c - a d}} \right], \frac{1}{2} \right] \end{aligned} \right) / \\ & \left( \sqrt{2} b^{1/4} d^{5/4} (a + b x)^{3/4} (c + d x)^{3/4} (b c + a d + 2 b d x) \sqrt{(a d + b (c + 2 d x))^2} \right)$$

Result (type 5, 74 leaves):

$$\frac{2 (a + b x)^{1/4} (c + d x)^{1/4}}{d} \left( 1 + \frac{\text{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{b (c + d x)}{b c - a d} \right]}{\left( \frac{d (a + b x)}{-b c + a d} \right)^{1/4}} \right)$$

Problem 1714: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + b x)^{3/4} (c + d x)^{3/4}} dx$$

Optimal (type 4, 270 leaves, 3 steps):

$$\left( \begin{aligned} & \sqrt{2} \sqrt{b c - a d} ((a + b x) (c + d x))^{3/4} \sqrt{(b c + a d + 2 b d x)^2} \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + b x) (c + d x)}}{b c - a d} \right) \sqrt{\frac{(a d + b (c + 2 d x))^2}{(b c - a d)^2 \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + b x) (c + d x)}}{b c - a d} \right)^2}} \\ & \text{EllipticF} \left[ 2 \text{ArcTan} \left[ \frac{\sqrt{2} b^{1/4} d^{1/4} ((a + b x) (c + d x))^{1/4}}{\sqrt{b c - a d}} \right], \frac{1}{2} \right] \end{aligned} \right) / \\ & \left( b^{1/4} d^{1/4} (a + b x)^{3/4} (c + d x)^{3/4} (b c + a d + 2 b d x) \sqrt{(a d + b (c + 2 d x))^2} \right)$$

Result (type 5, 71 leaves) :

$$\frac{4 \left( \frac{d(a+b x)}{-b c+a d} \right)^{3/4} (c+d x)^{1/4} \text{Hypergeometric2F1}\left[ \frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{b(c+d x)}{b c-a d} \right]}{d (a+b x)^{3/4}}$$

Problem 1715: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a+b x)^{7/4} (c+d x)^{3/4}} dx$$

Optimal (type 4, 306 leaves, 4 steps) :

$$\begin{aligned} & -\frac{4 (c+d x)^{1/4}}{3 (b c-a d) (a+b x)^{3/4}} - \\ & \left( 2 \sqrt{2} d^{3/4} (a+b x) (c+d x)^{3/4} \sqrt{(b c+a d+2 b d x)^2} \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a+b x) (c+d x)}}{b c-a d} \right) \sqrt{\frac{(a d+b (c+2 d x))^2}{(b c-a d)^2 \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a+b x) (c+d x)}}{b c-a d} \right)^2}} \right. \\ & \left. \text{EllipticF}\left[ 2 \text{ArcTan}\left[ \frac{\sqrt{2} b^{1/4} d^{1/4} ((a+b x) (c+d x))^{1/4}}{\sqrt{b c-a d}} \right], \frac{1}{2} \right] \right) / \\ & \left( 3 b^{1/4} \sqrt{b c-a d} (a+b x)^{3/4} (c+d x)^{3/4} (b c+a d+2 b d x) \sqrt{(a d+b (c+2 d x))^2} \right) \end{aligned}$$

Result (type 5, 84 leaves) :

$$\frac{4 (c+d x)^{1/4} \left( 1 + 2 \left( \frac{d(a+b x)}{-b c+a d} \right)^{3/4} \text{Hypergeometric2F1}\left[ \frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{b(c+d x)}{b c-a d} \right] \right)}{3 (b c-a d) (a+b x)^{3/4}}$$

Problem 1716: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a+b x)^{11/4} (c+d x)^{3/4}} dx$$

Optimal (type 4, 339 leaves, 5 steps) :

$$\begin{aligned}
& - \frac{4 (c + d x)^{1/4}}{7 (b c - a d) (a + b x)^{7/4}} + \frac{8 d (c + d x)^{1/4}}{7 (b c - a d)^2 (a + b x)^{3/4}} + \\
& \left( 4 \sqrt{2} d^{7/4} ((a + b x) (c + d x))^{3/4} \sqrt{(b c + a d + 2 b d x)^2} \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + b x) (c + d x)}}{b c - a d} \right) \right) \sqrt{\frac{(a d + b (c + 2 d x))^2}{(b c - a d)^2 \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + b x) (c + d x)}}{b c - a d} \right)^2}} \\
& \text{EllipticF} \left[ 2 \text{ArcTan} \left[ \frac{\sqrt{2} b^{1/4} d^{1/4} ((a + b x) (c + d x))^{1/4}}{\sqrt{b c - a d}} \right], \frac{1}{2} \right] / \\
& \left( 7 b^{1/4} (b c - a d)^{3/2} (a + b x)^{3/4} (c + d x)^{3/4} (b c + a d + 2 b d x) \sqrt{(a d + b (c + 2 d x))^2} \right)
\end{aligned}$$

Result (type 5, 102 leaves):

$$\frac{4 (c + d x)^{1/4} \left( -b c + 3 a d + 2 b d x + 4 d (a + b x) \left( \frac{d (a+b x)}{-b c+a d} \right)^{3/4} \text{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{b (c+d x)}{b c-a d} \right] \right)}{7 (b c - a d)^2 (a + b x)^{7/4}}$$

Problem 1717: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{5/4}}{(c + d x)^{5/4}} dx$$

Optimal (type 3, 152 leaves, 7 steps):

$$-\frac{4 (a + b x)^{5/4}}{d (c + d x)^{1/4}} + \frac{5 b (a + b x)^{1/4} (c + d x)^{3/4}}{d^2} - \frac{5 b^{1/4} (b c - a d) \text{ArcTan} \left[ \frac{d^{1/4} (a+b x)^{1/4}}{b^{1/4} (c+d x)^{1/4}} \right]}{2 d^{9/4}} - \frac{5 b^{1/4} (b c - a d) \text{ArcTanh} \left[ \frac{d^{1/4} (a+b x)^{1/4}}{b^{1/4} (c+d x)^{1/4}} \right]}{2 d^{9/4}}$$

Result (type 5, 99 leaves):

$$\frac{(a + b x)^{1/4} (c + d x)^{3/4} \left( \frac{3 (5 b c - 4 a d + b d x)}{c + d x} + \frac{5 b \text{Hypergeometric2F1} \left[ \frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b (c+d x)}{b c-a d} \right]}{\left( \frac{d (a+b x)}{-b c+a d} \right)^{1/4}} \right)}{3 d^2}$$

**Problem 1718:** Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{1/4}}{(c + d x)^{5/4}} dx$$

Optimal (type 3, 108 leaves, 6 steps):

$$-\frac{4 (a + b x)^{1/4}}{d (c + d x)^{1/4}} + \frac{2 b^{1/4} \operatorname{ArcTan}\left[\frac{d^{1/4} (a+b x)^{1/4}}{b^{1/4} (c+d x)^{1/4}}\right]}{d^{5/4}} + \frac{2 b^{1/4} \operatorname{ArcTanh}\left[\frac{d^{1/4} (a+b x)^{1/4}}{b^{1/4} (c+d x)^{1/4}}\right]}{d^{5/4}}$$

Result (type 5, 89 leaves):

$$\frac{4 \left(-3 d (a + b x) + b \left(\frac{d (a+b x)}{-b c+a d}\right)^{3/4} (c + d x) \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b (c+d x)}{b c-a d}\right]\right)}{3 d^2 (a + b x)^{3/4} (c + d x)^{1/4}}$$

**Problem 1723:** Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{11/4}}{(c + d x)^{5/4}} dx$$

Optimal (type 4, 776 leaves, 8 steps):

$$\begin{aligned}
& - \frac{4 (a + b x)^{11/4}}{d (c + d x)^{1/4}} - \frac{77 b (b c - a d) (a + b x)^{3/4} (c + d x)^{3/4}}{15 d^3} + \frac{22 b (a + b x)^{7/4} (c + d x)^{3/4}}{5 d^2} + \\
& \frac{77 \sqrt{b} (b c - a d) \sqrt{(a + b x) (c + d x)} \sqrt{(b c + a d + 2 b d x)^2} \sqrt{(a d + b (c + 2 d x))^2}}{10 d^{7/2} (a + b x)^{1/4} (c + d x)^{1/4} (b c + a d + 2 b d x) \left(1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + b x) (c + d x)}}{b c - a d}\right)} - \\
& \left( 77 b^{1/4} (b c - a d)^{7/2} ((a + b x) (c + d x))^{1/4} \sqrt{(b c + a d + 2 b d x)^2} \left(1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + b x) (c + d x)}}{b c - a d}\right) \right. \\
& \left. \sqrt{\frac{(a d + b (c + 2 d x))^2}{(b c - a d)^2 \left(1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + b x) (c + d x)}}{b c - a d}\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{\sqrt{2} b^{1/4} d^{1/4} ((a + b x) (c + d x))^{1/4}}{\sqrt{b c - a d}}\right], \frac{1}{2}\right] \right) / \\
& \left( 10 \sqrt{2} d^{15/4} (a + b x)^{1/4} (c + d x)^{1/4} (b c + a d + 2 b d x) \sqrt{(a d + b (c + 2 d x))^2} \right) + \\
& \left( 77 b^{1/4} (b c - a d)^{7/2} ((a + b x) (c + d x))^{1/4} \sqrt{(b c + a d + 2 b d x)^2} \left(1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + b x) (c + d x)}}{b c - a d}\right) \right. \\
& \left. \sqrt{\frac{(a d + b (c + 2 d x))^2}{(b c - a d)^2 \left(1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + b x) (c + d x)}}{b c - a d}\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{\sqrt{2} b^{1/4} d^{1/4} ((a + b x) (c + d x))^{1/4}}{\sqrt{b c - a d}}\right], \frac{1}{2}\right] \right) / \\
& \left( 20 \sqrt{2} d^{15/4} (a + b x)^{1/4} (c + d x)^{1/4} (b c + a d + 2 b d x) \sqrt{(a d + b (c + 2 d x))^2} \right)
\end{aligned}$$

Result (type 5, 132 leaves):

$$\begin{aligned}
& \frac{1}{15 d^4 (a + b x)^{1/4}} \\
& (c + d x)^{3/4} \left( d (a + b x) \left( b (-17 b c + 23 a d) + 6 b^2 d x - \frac{60 (b c - a d)^2}{c + d x} \right) + 77 b (b c - a d)^2 \left( \frac{d (a + b x)}{-b c + a d} \right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b (c + d x)}{b c - a d}\right] \right)
\end{aligned}$$

### Problem 1724: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{7/4}}{(c + d x)^{5/4}} dx$$

Optimal (type 4, 730 leaves, 7 steps):

$$\begin{aligned}
& -\frac{4 (a + b x)^{7/4}}{d (c + d x)^{1/4}} + \frac{14 b (a + b x)^{3/4} (c + d x)^{3/4}}{3 d^2} - \frac{7 \sqrt{b} \sqrt{(a + b x) (c + d x)} \sqrt{(b c + a d + 2 b d x)^2} \sqrt{(a d + b (c + 2 d x))^2}}{d^{5/2} (a + b x)^{1/4} (c + d x)^{1/4} (b c + a d + 2 b d x) \left(1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + b x) (c + d x)}}{b c - a d}\right)} + \\
& \left( 7 b^{1/4} (b c - a d)^{5/2} ((a + b x) (c + d x))^{1/4} \sqrt{(b c + a d + 2 b d x)^2} \left(1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + b x) (c + d x)}}{b c - a d}\right) \right. \\
& \left. \sqrt{\frac{(a d + b (c + 2 d x))^2}{(b c - a d)^2 \left(1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + b x) (c + d x)}}{b c - a d}\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{\sqrt{2} b^{1/4} d^{1/4} ((a + b x) (c + d x))^{1/4}}{\sqrt{b c - a d}}\right], \frac{1}{2}\right] \right) / \\
& \left( \sqrt{2} d^{11/4} (a + b x)^{1/4} (c + d x)^{1/4} (b c + a d + 2 b d x) \sqrt{(a d + b (c + 2 d x))^2} \right) - \\
& \left( 7 b^{1/4} (b c - a d)^{5/2} ((a + b x) (c + d x))^{1/4} \sqrt{(b c + a d + 2 b d x)^2} \left(1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + b x) (c + d x)}}{b c - a d}\right) \right. \\
& \left. \sqrt{\frac{(a d + b (c + 2 d x))^2}{(b c - a d)^2 \left(1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + b x) (c + d x)}}{b c - a d}\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{\sqrt{2} b^{1/4} d^{1/4} ((a + b x) (c + d x))^{1/4}}{\sqrt{b c - a d}}\right], \frac{1}{2}\right] \right) / \\
& \left( 2 \sqrt{2} d^{11/4} (a + b x)^{1/4} (c + d x)^{1/4} (b c + a d + 2 b d x) \sqrt{(a d + b (c + 2 d x))^2} \right)
\end{aligned}$$

Result (type 5, 98 leaves):

$$\frac{2 (a + b x)^{3/4} (c + d x)^{3/4} \left( \frac{7 b c - 6 a d + b d x}{c + d x} + \frac{7 b \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b (c+d x)}{b c - a d}\right]}{\left(\frac{d (a+b x)}{-b c + a d}\right)^{3/4}} \right)}{3 d^2}$$

Problem 1725: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{3/4}}{(c + d x)^{5/4}} dx$$

Optimal (type 4, 712 leaves, 6 steps):

$$\begin{aligned} & -\frac{4 (a + b x)^{3/4}}{d (c + d x)^{1/4}} + \frac{6 \sqrt{b} \sqrt{(a + b x) (c + d x)} \sqrt{(b c + a d + 2 b d x)^2} \sqrt{(a d + b (c + 2 d x))^2}}{d^{3/2} (b c - a d) (a + b x)^{1/4} (c + d x)^{1/4} (b c + a d + 2 b d x) \left(1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a+b x) (c+d x)}}{b c - a d}\right)} - \\ & \left( 3 \sqrt{2} b^{1/4} (b c - a d)^{3/2} ((a + b x) (c + d x))^{1/4} \sqrt{(b c + a d + 2 b d x)^2} \left(1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a+b x) (c+d x)}}{b c - a d}\right) \right. \\ & \left. \sqrt{\frac{(a d + b (c + 2 d x))^2}{(b c - a d)^2 \left(1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a+b x) (c+d x)}}{b c - a d}\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{\sqrt{2} b^{1/4} d^{1/4} ((a + b x) (c + d x))^{1/4}}{\sqrt{b c - a d}}\right], \frac{1}{2}\right] \right) / \\ & \left( d^{7/4} (a + b x)^{1/4} (c + d x)^{1/4} (b c + a d + 2 b d x) \sqrt{(a d + b (c + 2 d x))^2} \right) + \\ & \left( 3 b^{1/4} (b c - a d)^{3/2} ((a + b x) (c + d x))^{1/4} \sqrt{(b c + a d + 2 b d x)^2} \left(1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a+b x) (c+d x)}}{b c - a d}\right) \sqrt{\frac{(a d + b (c + 2 d x))^2}{(b c - a d)^2 \left(1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a+b x) (c+d x)}}{b c - a d}\right)^2}} \right. \\ & \left. \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{\sqrt{2} b^{1/4} d^{1/4} ((a + b x) (c + d x))^{1/4}}{\sqrt{b c - a d}}\right], \frac{1}{2}\right] \right) / \left( \sqrt{2} d^{7/4} (a + b x)^{1/4} (c + d x)^{1/4} (b c + a d + 2 b d x) \sqrt{(a d + b (c + 2 d x))^2} \right) \end{aligned}$$

Result (type 5, 87 leaves):

$$\frac{-4 d (a + b x) + 4 b \left(\frac{d (a+b x)}{-b c+a d}\right)^{1/4} (c + d x) \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b (c+d x)}{b c-a d}\right]}{d^2 (a + b x)^{1/4} (c + d x)^{1/4}}$$

Problem 1726: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + b x)^{1/4} (c + d x)^{5/4}} dx$$

Optimal (type 4, 719 leaves, 6 steps):

$$\begin{aligned} & \frac{4 (a + b x)^{3/4}}{(b c - a d) (c + d x)^{1/4}} - \frac{4 \sqrt{b} \sqrt{(a + b x) (c + d x)} \sqrt{(b c + a d + 2 b d x)^2} \sqrt{(a d + b (c + 2 d x))^2}}{\sqrt{d} (b c - a d)^2 (a + b x)^{1/4} (c + d x)^{1/4} (b c + a d + 2 b d x) \left(1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + b x) (c + d x)}}{b c - a d}\right)} + \\ & \left(2 \sqrt{2} b^{1/4} \sqrt{b c - a d} ((a + b x) (c + d x))^{1/4} \sqrt{(b c + a d + 2 b d x)^2} \left(1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + b x) (c + d x)}}{b c - a d}\right)\right. \\ & \left. \sqrt{\frac{(a d + b (c + 2 d x))^2}{(b c - a d)^2 \left(1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + b x) (c + d x)}}{b c - a d}\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{\sqrt{2} b^{1/4} d^{1/4} ((a + b x) (c + d x))^{1/4}}{\sqrt{b c - a d}}\right], \frac{1}{2}\right]\right) / \\ & \left(\sqrt{2} b^{1/4} \sqrt{b c - a d} ((a + b x) (c + d x))^{1/4} \sqrt{(b c + a d + 2 b d x)^2} \sqrt{(a d + b (c + 2 d x))^2}\right) - \\ & \left(\sqrt{2} b^{1/4} \sqrt{b c - a d} ((a + b x) (c + d x))^{1/4} \sqrt{(b c + a d + 2 b d x)^2} \left(1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + b x) (c + d x)}}{b c - a d}\right) \sqrt{\frac{(a d + b (c + 2 d x))^2}{(b c - a d)^2 \left(1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + b x) (c + d x)}}{b c - a d}\right)^2}}\right. \\ & \left. \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{\sqrt{2} b^{1/4} d^{1/4} ((a + b x) (c + d x))^{1/4}}{\sqrt{b c - a d}}\right], \frac{1}{2}\right]\right) / \left(d^{3/4} (a + b x)^{1/4} (c + d x)^{1/4} (b c + a d + 2 b d x) \sqrt{(a d + b (c + 2 d x))^2}\right) \end{aligned}$$

Result (type 5, 100 leaves):

$$\frac{12 d \left(a+b x\right)-8 b \left(\frac{d \left(a+b x\right)}{-b c+a d}\right)^{1/4} \left(c+d x\right) \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b \left(c+d x\right)}{b c-a d}\right]}{3 d \left(b c-a d\right) \left(a+b x\right)^{1/4} \left(c+d x\right)^{1/4}}$$

**Problem 1727:** Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a+b x)^{5/4} (c+d x)^{5/4}} dx$$

Optimal (type 4, 750 leaves, 7 steps):

$$-\frac{4}{(b c-a d) \left(a+b x\right)^{1/4} \left(c+d x\right)^{1/4}}-\frac{8 d \left(a+b x\right)^{3/4}}{\left(b c-a d\right)^2 \left(c+d x\right)^{1/4}}+\frac{8 \sqrt{b} \sqrt{d} \sqrt{(a+b x) (c+d x)} \sqrt{(b c+a d+2 b d x)^2} \sqrt{(a d+b (c+2 d x))^2}}{\left(b c-a d\right)^3 \left(a+b x\right)^{1/4} \left(c+d x\right)^{1/4} (b c+a d+2 b d x) \left(1+\frac{2 \sqrt{b} \sqrt{d} \sqrt{(a+b x) (c+d x)}}{b c-a d}\right)}$$

$$\left(4 \sqrt{2} b^{1/4} d^{1/4} \left((a+b x) (c+d x)\right)^{1/4} \sqrt{(b c+a d+2 b d x)^2} \left(1+\frac{2 \sqrt{b} \sqrt{d} \sqrt{(a+b x) (c+d x)}}{b c-a d}\right)\right)$$

$$\left.\left(\frac{(a d+b (c+2 d x))^2}{(b c-a d)^2 \left(1+\frac{2 \sqrt{b} \sqrt{d} \sqrt{(a+b x) (c+d x)}}{b c-a d}\right)^2} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{\sqrt{2} b^{1/4} d^{1/4} \left((a+b x) (c+d x)\right)^{1/4}}{\sqrt{b c-a d}}\right], \frac{1}{2}\right]\right)/\right.$$

$$\left.\left(\sqrt{b c-a d} \left(a+b x\right)^{1/4} \left(c+d x\right)^{1/4} (b c+a d+2 b d x) \sqrt{(a d+b (c+2 d x))^2}\right)+\right.$$

$$\left.\left(2 \sqrt{2} b^{1/4} d^{1/4} \left((a+b x) (c+d x)\right)^{1/4} \sqrt{(b c+a d+2 b d x)^2} \left(1+\frac{2 \sqrt{b} \sqrt{d} \sqrt{(a+b x) (c+d x)}}{b c-a d}\right)\right)$$

$$\left.\left(\frac{(a d+b (c+2 d x))^2}{(b c-a d)^2 \left(1+\frac{2 \sqrt{b} \sqrt{d} \sqrt{(a+b x) (c+d x)}}{b c-a d}\right)^2} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{\sqrt{2} b^{1/4} d^{1/4} \left((a+b x) (c+d x)\right)^{1/4}}{\sqrt{b c-a d}}\right], \frac{1}{2}\right]\right)/\right.$$

$$\left.\left(\sqrt{b c-a d} \left(a+b x\right)^{1/4} \left(c+d x\right)^{1/4} (b c+a d+2 b d x) \sqrt{(a d+b (c+2 d x))^2}\right)\right)$$

Result (type 5, 102 leaves):

$$-\frac{4 \left(3 a d+3 b \left(c+2 d x\right)-4 b \left(\frac{d (a+b x)}{-b c+a d}\right)^{1/4} (c+d x) \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b (c+d x)}{b c-a d}\right]\right)}{3 \left(b c-a d\right)^2 (a+b x)^{1/4} (c+d x)^{1/4}}$$

Problem 1728: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a+b x)^{9/4} (c+d x)^{5/4}} dx$$

Optimal (type 4, 795 leaves, 8 steps):

$$\begin{aligned}
& - \frac{4}{5 (b c - a d) (a + b x)^{5/4} (c + d x)^{1/4}} + \frac{24 d}{5 (b c - a d)^2 (a + b x)^{1/4} (c + d x)^{1/4}} + \\
& - \frac{48 d^2 (a + b x)^{3/4}}{5 (b c - a d)^3 (c + d x)^{1/4}} - \frac{48 \sqrt{b} d^{3/2} \sqrt{(a + b x) (c + d x)} \sqrt{(b c + a d + 2 b d x)^2} \sqrt{(a d + b (c + 2 d x))^2}}{5 (b c - a d)^4 (a + b x)^{1/4} (c + d x)^{1/4} (b c + a d + 2 b d x) \left(1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + b x) (c + d x)}}{b c - a d}\right)} + \\
& \left( 24 \sqrt{2} b^{1/4} d^{5/4} ((a + b x) (c + d x))^{1/4} \sqrt{(b c + a d + 2 b d x)^2} \left(1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + b x) (c + d x)}}{b c - a d}\right) \right. \\
& \left. \sqrt{\frac{(a d + b (c + 2 d x))^2}{(b c - a d)^2 \left(1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + b x) (c + d x)}}{b c - a d}\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{\sqrt{2} b^{1/4} d^{1/4} ((a + b x) (c + d x))^{1/4}}{\sqrt{b c - a d}}\right], \frac{1}{2}\right] \right) / \\
& \left( 5 (b c - a d)^{3/2} (a + b x)^{1/4} (c + d x)^{1/4} (b c + a d + 2 b d x) \sqrt{(a d + b (c + 2 d x))^2} \right) - \\
& \left( 12 \sqrt{2} b^{1/4} d^{5/4} ((a + b x) (c + d x))^{1/4} \sqrt{(b c + a d + 2 b d x)^2} \left(1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + b x) (c + d x)}}{b c - a d}\right) \right. \\
& \left. \sqrt{\frac{(a d + b (c + 2 d x))^2}{(b c - a d)^2 \left(1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + b x) (c + d x)}}{b c - a d}\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{\sqrt{2} b^{1/4} d^{1/4} ((a + b x) (c + d x))^{1/4}}{\sqrt{b c - a d}}\right], \frac{1}{2}\right] \right) / \\
& \left( 5 (b c - a d)^{3/2} (a + b x)^{1/4} (c + d x)^{1/4} (b c + a d + 2 b d x) \sqrt{(a d + b (c + 2 d x))^2} \right)
\end{aligned}$$

Result (type 5, 139 leaves):

$$\begin{aligned}
& - \left( \left( 4 \left( 5 a^2 d^2 + 2 a b d (4 c + 9 d x) + b^2 (-c^2 + 6 c d x + 12 d^2 x^2) \right) - \right. \right. \\
& \left. \left. 8 b d (a + b x) \left( \frac{d (a + b x)}{-b c + a d} \right)^{1/4} (c + d x) \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b (c + d x)}{b c - a d}\right] \right) \right) / \left( 5 (-b c + a d)^3 (a + b x)^{5/4} (c + d x)^{1/4} \right)
\end{aligned}$$

### Problem 1729: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(1 - ax)^{1/4} (1 + bx)^{3/4}} dx$$

Optimal (type 3, 279 leaves, 11 steps):

$$\begin{aligned} & \frac{\sqrt{2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} b^{1/4} (1-ax)^{1/4}}{a^{1/4} (1+bx)^{1/4}}\right]}{a^{1/4} b^{3/4}} - \frac{\sqrt{2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} b^{1/4} (1-ax)^{1/4}}{a^{1/4} (1+bx)^{1/4}}\right]}{a^{1/4} b^{3/4}} - \\ & \frac{\operatorname{Log}\left[\sqrt{a} + \frac{\sqrt{b} \sqrt{1-ax}}{\sqrt{1+bx}} - \frac{\sqrt{2} a^{1/4} b^{1/4} (1-ax)^{1/4}}{(1+bx)^{1/4}}\right]}{\sqrt{2} a^{1/4} b^{3/4}} + \frac{\operatorname{Log}\left[\sqrt{a} + \frac{\sqrt{b} \sqrt{1-ax}}{\sqrt{1+bx}} + \frac{\sqrt{2} a^{1/4} b^{1/4} (1-ax)^{1/4}}{(1+bx)^{1/4}}\right]}{\sqrt{2} a^{1/4} b^{3/4}} \end{aligned}$$

Result (type 5, 63 leaves):

$$\frac{4 (1 + bx)^{1/4} \left(\frac{b-a}{a+b} x\right)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \frac{a+a b x}{a+b}\right]}{b (1 - ax)^{1/4}}$$

### Problem 1730: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(1 - ax)^{1/4} (1 + ax)^{3/4}} dx$$

Optimal (type 3, 193 leaves, 11 steps):

$$\begin{aligned} & \frac{\sqrt{2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{a} - \frac{\sqrt{2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{a} - \frac{\operatorname{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2} (1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{\sqrt{2} a} + \frac{\operatorname{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} + \frac{\sqrt{2} (1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{\sqrt{2} a} \end{aligned}$$

Result (type 5, 38 leaves):

$$\frac{2 \times 2^{3/4} (1 + ax)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \frac{1}{2} (1 + ax)\right]}{a}$$

### Problem 1736: Result unnecessarily involves higher level functions.

$$\int (a + bx)^{5/2} (c + dx)^{1/6} dx$$

Optimal (type 4, 487 leaves, 6 steps):

$$\begin{aligned}
& \frac{81 (b c - a d)^3 \sqrt{a + b x} (c + d x)^{1/6}}{1408 b d^3} - \frac{9 (b c - a d)^2 (a + b x)^{3/2} (c + d x)^{1/6}}{352 b d^2} + \frac{3 (b c - a d) (a + b x)^{5/2} (c + d x)^{1/6}}{176 b d} + \\
& \frac{3 (a + b x)^{7/2} (c + d x)^{1/6}}{11 b} - \left( 81 \times 3^{3/4} (b c - a d)^{11/3} (c + d x)^{1/6} \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right) \right. \\
& \left. \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + d x)^{1/3} + b^{2/3} (c + d x)^{2/3}}{\left( (b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3} \right)^2}} \operatorname{EllipticF}[\operatorname{ArcCos}\left[ \frac{(b c - a d)^{1/3} - (1 - \sqrt{3}) b^{1/3} (c + d x)^{1/3}}{(b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3}} \right], \frac{1}{4} (2 + \sqrt{3})] \right) / \\
& \left( 2816 b d^4 \sqrt{a + b x} \sqrt{-\frac{b^{1/3} (c + d x)^{1/3} \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right)}{\left( (b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3} \right)^2}} \right)
\end{aligned}$$

Result (type 5, 181 leaves):

$$\begin{aligned}
& -\frac{1}{1408 b d^4 \sqrt{a + b x}} \\
& 3 (c + d x)^{1/6} \left( -d (a + b x) (81 a^3 d^3 + a^2 b d^2 (113 c + 356 d x) + a b^2 d (-93 c^2 + 40 c d x + 376 d^2 x^2) + b^3 (27 c^3 - 12 c^2 d x + 8 c d^2 x^2 + 128 d^3 x^3)) + \right. \\
& \left. 81 (b c - a d)^4 \sqrt{\frac{d (a + b x)}{-b c + a d}} \operatorname{Hypergeometric2F1}\left[ \frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \frac{b (c + d x)}{b c - a d} \right] \right)
\end{aligned}$$

Problem 1737: Result unnecessarily involves higher level functions.

$$\int (a + b x)^{3/2} (c + d x)^{1/6} dx$$

Optimal (type 4, 449 leaves, 5 steps):

$$\begin{aligned}
& - \frac{27 (b c - a d)^2 \sqrt{a + b x} (c + d x)^{1/6}}{320 b d^2} + \frac{3 (b c - a d) (a + b x)^{3/2} (c + d x)^{1/6}}{80 b d} + \\
& \frac{3 (a + b x)^{5/2} (c + d x)^{1/6}}{8 b} + \left( 27 \times 3^{3/4} (b c - a d)^{8/3} (c + d x)^{1/6} \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right) \right. \\
& \left. \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + d x)^{1/3} + b^{2/3} (c + d x)^{2/3}}{\left( (b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3} \right)^2}} \operatorname{EllipticF}[\operatorname{ArcCos}\left[ \frac{(b c - a d)^{1/3} - (1 - \sqrt{3}) b^{1/3} (c + d x)^{1/3}}{(b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3}} \right], \frac{1}{4} (2 + \sqrt{3})] \right) / \\
& \left( 640 b d^3 \sqrt{a + b x} \sqrt{-\frac{b^{1/3} (c + d x)^{1/3} \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right)}{\left( (b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3} \right)^2}} \right)
\end{aligned}$$

Result (type 5, 142 leaves):

$$\begin{aligned}
& - \frac{1}{320 b d^3 \sqrt{a + b x}} 3 (c + d x)^{1/6} \left( -d (a + b x) (27 a^2 d^2 + 2 a b d (11 c + 38 d x) + b^2 (-9 c^2 + 4 c d x + 40 d^2 x^2)) - \right. \\
& \left. 27 (b c - a d)^3 \sqrt{\frac{d (a + b x)}{-b c + a d}} \operatorname{Hypergeometric2F1}\left[ \frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \frac{b (c + d x)}{b c - a d} \right] \right)
\end{aligned}$$

Problem 1738: Result unnecessarily involves higher level functions.

$$\int \sqrt{a + b x} (c + d x)^{1/6} dx$$

Optimal (type 4, 411 leaves, 4 steps):

$$\begin{aligned}
& \frac{3 (b c - a d) \sqrt{a + b x} (c + d x)^{1/6}}{20 b d} + \frac{3 (a + b x)^{3/2} (c + d x)^{1/6}}{5 b} - \\
& \left( 3 \times 3^{3/4} (b c - a d)^{5/3} (c + d x)^{1/6} \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right) \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + d x)^{1/3} + b^{2/3} (c + d x)^{2/3}}{\left( (b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3} \right)^2}} \right. \\
& \left. \text{EllipticF}[\text{ArcCos}\left[\frac{(b c - a d)^{1/3} - (1 - \sqrt{3}) b^{1/3} (c + d x)^{1/3}}{(b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3}}\right], \frac{1}{4} (2 + \sqrt{3})] \right) / \\
& \left( 40 b d^2 \sqrt{a + b x} \sqrt{-\frac{b^{1/3} (c + d x)^{1/3} \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right)}{\left( (b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3} \right)^2}} \right)
\end{aligned}$$

Result (type 5, 109 leaves):

$$\frac{1}{20 b d^2 \sqrt{a + b x}} 3 (c + d x)^{1/6} \left( d (a + b x) (3 a d + b (c + 4 d x)) - 3 (b c - a d)^2 \sqrt{\frac{d (a + b x)}{-b c + a d}} \text{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \frac{b (c + d x)}{b c - a d}\right] \right)$$

Problem 1739: Result unnecessarily involves higher level functions.

$$\int \frac{(c + d x)^{1/6}}{\sqrt{a + b x}} dx$$

Optimal (type 4, 375 leaves, 3 steps):

$$\begin{aligned}
& \frac{3 \sqrt{a + b x} (c + d x)^{1/6}}{2 b} + \\
& \left( 3^{3/4} (b c - a d)^{2/3} (c + d x)^{1/6} \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right) \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + d x)^{1/3} + b^{2/3} (c + d x)^{2/3}}{\left( (b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3} \right)^2}} \right. \\
& \left. \text{EllipticF}[\text{ArcCos}\left[\frac{(b c - a d)^{1/3} - (1 - \sqrt{3}) b^{1/3} (c + d x)^{1/3}}{(b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3}}\right], \frac{1}{4} (2 + \sqrt{3})] \right) / \\
& \left( 4 b d \sqrt{a + b x} \sqrt{-\frac{b^{1/3} (c + d x)^{1/3} \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right)}{\left( (b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3} \right)^2}} \right)
\end{aligned}$$

Result (type 5, 93 leaves) :

$$\frac{3 (c + d x)^{1/6} \left( d (a + b x) + (b c - a d) \sqrt{\frac{d (a+b x)}{-b c+a d}} \text{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \frac{b (c+d x)}{b c-a d}\right]\right)}{2 b d \sqrt{a + b x}}$$

Problem 1740: Result unnecessarily involves higher level functions.

$$\int \frac{(c + d x)^{1/6}}{(a + b x)^{3/2}} dx$$

Optimal (type 4, 367 leaves, 3 steps) :

$$-\frac{2 (c + d x)^{1/6}}{b \sqrt{a + b x}} + \left( (c + d x)^{1/6} \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right) \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + d x)^{1/3} + b^{2/3} (c + d x)^{2/3}}{\left( (b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3} \right)^2}} \text{EllipticF} \left[ \text{ArcCos} \left[ \frac{(b c - a d)^{1/3} - (1 - \sqrt{3}) b^{1/3} (c + d x)^{1/3}}{(b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3}} \right], \frac{1}{4} (2 + \sqrt{3}) \right] \right) / \\ \left( 3^{1/4} b (b c - a d)^{1/3} \sqrt{a + b x} \sqrt{-\frac{b^{1/3} (c + d x)^{1/3} \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right)}{\left( (b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3} \right)^2}} \right)$$

Result (type 5, 74 leaves) :

$$\frac{2 (c + d x)^{1/6} \left( -1 + \sqrt{\frac{d (a+b x)}{-b c+a d}} \text{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \frac{b (c+d x)}{b c-a d}\right]\right)}{b \sqrt{a + b x}}$$

Problem 1741: Result unnecessarily involves higher level functions.

$$\int \frac{(c + d x)^{1/6}}{(a + b x)^{5/2}} dx$$

Optimal (type 4, 409 leaves, 4 steps) :

$$\begin{aligned}
& - \frac{2 (c + d x)^{1/6}}{3 b (a + b x)^{3/2}} - \frac{2 d (c + d x)^{1/6}}{9 b (b c - a d) \sqrt{a + b x}} - \\
& \left( 2 d (c + d x)^{1/6} \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right) \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + d x)^{1/3} + b^{2/3} (c + d x)^{2/3}}{(b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3}}^2} \right. \\
& \left. \text{EllipticF} \left[ \text{ArcCos} \left[ \frac{(b c - a d)^{1/3} - (1 - \sqrt{3}) b^{1/3} (c + d x)^{1/3}}{(b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3}} \right], \frac{1}{4} (2 + \sqrt{3}) \right] \right) / \\
& \left( 9 \times 3^{1/4} b (b c - a d)^{4/3} \sqrt{a + b x} \sqrt{-\frac{b^{1/3} (c + d x)^{1/3} ((b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3})}{((b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3})^2}} \right)
\end{aligned}$$

Result (type 5, 104 leaves):

$$\frac{2 (c + d x)^{1/6} \left( 3 b c - 2 a d + b d x + 2 d (a + b x) \sqrt{\frac{d (a + b x)}{-b c + a d}} \text{Hypergeometric2F1} \left[ \frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \frac{b (c + d x)}{b c - a d} \right] \right)}{9 b (-b c + a d) (a + b x)^{3/2}}$$

Problem 1742: Result unnecessarily involves higher level functions.

$$\int (a + b x)^{3/2} (c + d x)^{5/6} dx$$

Optimal (type 4, 896 leaves, 7 steps):

$$\begin{aligned}
& - \frac{27 (b c - a d)^2 \sqrt{a + b x} (c + d x)^{5/6}}{224 b d^2} + \frac{3 (b c - a d) (a + b x)^{3/2} (c + d x)^{5/6}}{28 b d} + \\
& \frac{3 (a + b x)^{5/2} (c + d x)^{5/6}}{10 b} - \frac{81 (1 + \sqrt{3}) (b c - a d)^3 \sqrt{a + b x} (c + d x)^{1/6}}{448 b^{5/3} d^2 ((b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3})} - \\
& \left( 81 \times 3^{1/4} (b c - a d)^{10/3} (c + d x)^{1/6} ((b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}) \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + d x)^{1/3} + b^{2/3} (c + d x)^{2/3}}{((b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3})^2}} \right. \\
& \left. \text{EllipticE} [\text{ArcCos} \left[ \frac{(b c - a d)^{1/3} - (1 - \sqrt{3}) b^{1/3} (c + d x)^{1/3}}{(b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3}}, \frac{1}{4} (2 + \sqrt{3}) \right]] \right) / \\
& \left( 448 b^{5/3} d^3 \sqrt{a + b x} \sqrt{- \frac{b^{1/3} (c + d x)^{1/3} ((b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3})}{((b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3})^2}} \right) - \\
& \left( 27 \times 3^{3/4} (1 - \sqrt{3}) (b c - a d)^{10/3} (c + d x)^{1/6} ((b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}) \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + d x)^{1/3} + b^{2/3} (c + d x)^{2/3}}{((b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3})^2}} \right. \\
& \left. \text{EllipticF} [\text{ArcCos} \left[ \frac{(b c - a d)^{1/3} - (1 - \sqrt{3}) b^{1/3} (c + d x)^{1/3}}{(b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3}}, \frac{1}{4} (2 + \sqrt{3}) \right]] \right) / \\
& \left( 896 b^{5/3} d^3 \sqrt{a + b x} \sqrt{- \frac{b^{1/3} (c + d x)^{1/3} ((b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3})}{((b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3})^2}} \right)
\end{aligned}$$

Result (type 5, 142 leaves):

$$\begin{aligned}
& - \frac{1}{1120 b d^3 \sqrt{a + b x}} 3 (c + d x)^{5/6} \left( -d (a + b x) (27 a^2 d^2 + 2 a b d (65 c + 92 d x) + b^2 (-45 c^2 + 40 c d x + 112 d^2 x^2)) - \right. \\
& \left. 27 (b c - a d)^3 \sqrt{\frac{d (a + b x)}{-b c + a d}} \text{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \frac{b (c + d x)}{b c - a d} \right] \right)
\end{aligned}$$

### Problem 1743: Result unnecessarily involves higher level functions.

$$\int \sqrt{a + b x} (c + d x)^{5/6} dx$$

Optimal (type 4, 858 leaves, 6 steps):

$$\begin{aligned}
& \frac{15 (b c - a d) \sqrt{a + b x} (c + d x)^{5/6}}{56 b d} + \frac{3 (a + b x)^{3/2} (c + d x)^{5/6}}{7 b} + \\
& \frac{45 (1 + \sqrt{3}) (b c - a d)^2 \sqrt{a + b x} (c + d x)^{1/6}}{112 b^{5/3} d ((b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3})} + \left( \frac{45 \times 3^{1/4} (b c - a d)^{7/3} (c + d x)^{1/6} ((b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3})}{\sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + d x)^{1/3} + b^{2/3} (c + d x)^{2/3}}{((b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3})^2}} \operatorname{EllipticE}[\operatorname{ArcCos}\left[\frac{(b c - a d)^{1/3} - (1 - \sqrt{3}) b^{1/3} (c + d x)^{1/3}}{(b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3}}\right], \frac{1}{4} (2 + \sqrt{3})] \right) / \\
& \left( \frac{112 b^{5/3} d^2 \sqrt{a + b x}}{\sqrt{-\frac{b^{1/3} (c + d x)^{1/3} ((b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3})}{((b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3})^2}}} + \right. \\
& \left. \left( \frac{15 \times 3^{3/4} (1 - \sqrt{3}) (b c - a d)^{7/3} (c + d x)^{1/6} ((b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3})}{\sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + d x)^{1/3} + b^{2/3} (c + d x)^{2/3}}{((b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3})^2}}} \right. \right. \\
& \left. \left. \operatorname{EllipticF}[\operatorname{ArcCos}\left[\frac{(b c - a d)^{1/3} - (1 - \sqrt{3}) b^{1/3} (c + d x)^{1/3}}{(b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3}}\right], \frac{1}{4} (2 + \sqrt{3})] \right) / \right. \\
& \left. \left( \frac{224 b^{5/3} d^2 \sqrt{a + b x}}{\sqrt{-\frac{b^{1/3} (c + d x)^{1/3} ((b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3})}{((b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3})^2}}} \right) \right)
\end{aligned}$$

Result (type 5, 110 leaves):

$$\frac{1}{56 b d^2 \sqrt{a + b x}} 3 (c + d x)^{5/6} \left( d (a + b x) (5 b c + 3 a d + 8 b d x) - 3 (b c - a d)^2 \sqrt{\frac{d (a + b x)}{-b c + a d}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \frac{b (c + d x)}{b c - a d}\right] \right)$$

### Problem 1744: Result unnecessarily involves higher level functions.

$$\int \frac{(c + d x)^{5/6}}{\sqrt{a + b x}} dx$$

Optimal (type 4, 817 leaves, 5 steps):

$$\begin{aligned} & \frac{3 \sqrt{a + b x} (c + d x)^{5/6}}{4 b} - \frac{15 (1 + \sqrt{3}) (b c - a d) \sqrt{a + b x} (c + d x)^{1/6}}{8 b^{5/3} ((b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3})} - \\ & \left( 15 \times 3^{1/4} (b c - a d)^{4/3} (c + d x)^{1/6} ((b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}) \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + d x)^{1/3} + b^{2/3} (c + d x)^{2/3}}{((b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3})^2}} \right. \\ & \left. \text{EllipticE} [\text{ArcCos} \left[ \frac{(b c - a d)^{1/3} - (1 - \sqrt{3}) b^{1/3} (c + d x)^{1/3}}{(b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3}} \right], \frac{1}{4} (2 + \sqrt{3})] \right) / \\ & \left( 8 b^{5/3} d \sqrt{a + b x} \sqrt{-\frac{b^{1/3} (c + d x)^{1/3} ((b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3})}{((b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3})^2}} \right) - \\ & \left( 5 \times 3^{3/4} (1 - \sqrt{3}) (b c - a d)^{4/3} (c + d x)^{1/6} ((b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}) \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + d x)^{1/3} + b^{2/3} (c + d x)^{2/3}}{((b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3})^2}} \right. \\ & \left. \text{EllipticF} [\text{ArcCos} \left[ \frac{(b c - a d)^{1/3} - (1 - \sqrt{3}) b^{1/3} (c + d x)^{1/3}}{(b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3}} \right], \frac{1}{4} (2 + \sqrt{3})] \right) / \\ & \left( 16 b^{5/3} d \sqrt{a + b x} \sqrt{-\frac{b^{1/3} (c + d x)^{1/3} ((b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3})}{((b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3})^2}} \right) \end{aligned}$$

Result (type 5, 93 leaves):

$$\frac{3 (c + d x)^{5/6} \left( d (a + b x) + (b c - a d) \sqrt{\frac{d (a + b x)}{-b c + a d}} \text{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \frac{b (c + d x)}{b c - a d} \right] \right)}{4 b d \sqrt{a + b x}}$$

### Problem 1745: Result unnecessarily involves higher level functions.

$$\int \frac{(c + dx)^{5/6}}{(a + bx)^{3/2}} dx$$

Optimal (type 4, 798 leaves, 5 steps):

$$\begin{aligned}
& - \frac{2(c + dx)^{5/6}}{b\sqrt{a + bx}} - \frac{5(1 + \sqrt{3})d\sqrt{a + bx}(c + dx)^{1/6}}{b^{5/3}((bc - ad)^{1/3} - (1 + \sqrt{3})b^{1/3}(c + dx)^{1/3})} - \\
& \left( 5 \times 3^{1/4} (bc - ad)^{1/3} (c + dx)^{1/6} ((bc - ad)^{1/3} - b^{1/3}(c + dx)^{1/3}) \sqrt{\frac{(bc - ad)^{2/3} + b^{1/3}(bc - ad)^{1/3}(c + dx)^{1/3} + b^{2/3}(c + dx)^{2/3}}{((bc - ad)^{1/3} - (1 + \sqrt{3})b^{1/3}(c + dx)^{1/3})^2}} \right. \\
& \left. \text{EllipticE} \left[ \text{ArcCos} \left[ \frac{(bc - ad)^{1/3} - (1 - \sqrt{3})b^{1/3}(c + dx)^{1/3}}{(bc - ad)^{1/3} - (1 + \sqrt{3})b^{1/3}(c + dx)^{1/3}} \right], \frac{1}{4}(2 + \sqrt{3}) \right] \right) / \\
& \left( b^{5/3}\sqrt{a + bx} \sqrt{-\frac{b^{1/3}(c + dx)^{1/3}((bc - ad)^{1/3} - b^{1/3}(c + dx)^{1/3})}{((bc - ad)^{1/3} - (1 + \sqrt{3})b^{1/3}(c + dx)^{1/3})^2}} - \left( 5(1 - \sqrt{3})(bc - ad)^{1/3}(c + dx)^{1/6}((bc - ad)^{1/3} - b^{1/3}(c + dx)^{1/3}) \right. \right. \\
& \left. \left. \sqrt{\frac{(bc - ad)^{2/3} + b^{1/3}(bc - ad)^{1/3}(c + dx)^{1/3} + b^{2/3}(c + dx)^{2/3}}{((bc - ad)^{1/3} - (1 + \sqrt{3})b^{1/3}(c + dx)^{1/3})^2}} \text{EllipticF} \left[ \text{ArcCos} \left[ \frac{(bc - ad)^{1/3} - (1 - \sqrt{3})b^{1/3}(c + dx)^{1/3}}{(bc - ad)^{1/3} - (1 + \sqrt{3})b^{1/3}(c + dx)^{1/3}} \right], \frac{1}{4}(2 + \sqrt{3}) \right] \right) / \\
& \left( 2 \times 3^{1/4} b^{5/3} \sqrt{a + bx} \sqrt{-\frac{b^{1/3}(c + dx)^{1/3}((bc - ad)^{1/3} - b^{1/3}(c + dx)^{1/3})}{((bc - ad)^{1/3} - (1 + \sqrt{3})b^{1/3}(c + dx)^{1/3})^2}} \right)
\end{aligned}$$

Result (type 5, 74 leaves):

$$\frac{2(c + dx)^{5/6} \left( -1 + \sqrt{\frac{d(a + bx)}{-bc + ad}} \text{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \frac{b(c + dx)}{bc - ad} \right] \right)}{b\sqrt{a + bx}}$$

Problem 1746: Result unnecessarily involves higher level functions.

$$\int \frac{(c + d x)^{5/6}}{(a + b x)^{5/2}} dx$$

Optimal (type 4, 854 leaves, 6 steps):

$$\begin{aligned} & -\frac{2 (c + d x)^{5/6}}{3 b (a + b x)^{3/2}} - \frac{10 d (c + d x)^{5/6}}{9 b (b c - a d) \sqrt{a + b x}} - \frac{10 (1 + \sqrt{3}) d^2 \sqrt{a + b x} (c + d x)^{1/6}}{9 b^{5/3} (b c - a d) ((b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3})} - \\ & \left( 10 d (c + d x)^{1/6} ((b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}) \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + d x)^{1/3} + b^{2/3} (c + d x)^{2/3}}{((b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3})^2}} \right. \\ & \left. \text{EllipticE} \left[ \text{ArcCos} \left[ \frac{(b c - a d)^{1/3} - (1 - \sqrt{3}) b^{1/3} (c + d x)^{1/3}}{(b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3}} \right], \frac{1}{4} (2 + \sqrt{3}) \right] \right) / \\ & \left( 3 \times 3^{3/4} b^{5/3} (b c - a d)^{2/3} \sqrt{a + b x} \sqrt{-\frac{b^{1/3} (c + d x)^{1/3} ((b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3})}{((b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3})^2}} \right) - \\ & \left( 5 (1 - \sqrt{3}) d (c + d x)^{1/6} ((b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}) \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + d x)^{1/3} + b^{2/3} (c + d x)^{2/3}}{((b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3})^2}} \right. \\ & \left. \text{EllipticF} \left[ \text{ArcCos} \left[ \frac{(b c - a d)^{1/3} - (1 - \sqrt{3}) b^{1/3} (c + d x)^{1/3}}{(b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3}} \right], \frac{1}{4} (2 + \sqrt{3}) \right] \right) / \\ & \left( 9 \times 3^{1/4} b^{5/3} (b c - a d)^{2/3} \sqrt{a + b x} \sqrt{-\frac{b^{1/3} (c + d x)^{1/3} ((b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3})}{((b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3})^2}} \right) \end{aligned}$$

Result (type 5, 105 leaves):

$$\begin{aligned} & -\frac{2 (c + d x)^{5/6} \left( 3 b c + 2 a d + 5 b d x - 2 d (a + b x) \sqrt{\frac{d (a + b x)}{-b c + a d}} \text{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \frac{b (c + d x)}{b c - a d} \right] \right)}{9 b (b c - a d) (a + b x)^{3/2}} \end{aligned}$$

### Problem 1747: Result unnecessarily involves higher level functions.

$$\int \frac{(c + dx)^{5/6}}{(a + bx)^{7/2}} dx$$

Optimal (type 4, 896 leaves, 7 steps):

$$\begin{aligned}
& -\frac{2(c + dx)^{5/6}}{5b(a + bx)^{5/2}} - \frac{2d(c + dx)^{5/6}}{9b(b c - ad)(a + bx)^{3/2}} + \frac{8d^2(c + dx)^{5/6}}{27b(b c - ad)^2 \sqrt{a + bx}} + \\
& \frac{8(1 + \sqrt{3})d^3 \sqrt{a + bx} (c + dx)^{1/6}}{27b^{5/3}(b c - ad)^2 ((b c - ad)^{1/3} - (1 + \sqrt{3})b^{1/3}(c + dx)^{1/3})} + \left( 8d^2(c + dx)^{1/6} ((b c - ad)^{1/3} - b^{1/3}(c + dx)^{1/3}) \right. \\
& \left. \sqrt{\frac{(b c - ad)^{2/3} + b^{1/3}(b c - ad)^{1/3}(c + dx)^{1/3} + b^{2/3}(c + dx)^{2/3}}{((b c - ad)^{1/3} - (1 + \sqrt{3})b^{1/3}(c + dx)^{1/3})^2}} \operatorname{EllipticE}[\operatorname{ArcCos}\left[\frac{(b c - ad)^{1/3} - (1 - \sqrt{3})b^{1/3}(c + dx)^{1/3}}{(b c - ad)^{1/3} - (1 + \sqrt{3})b^{1/3}(c + dx)^{1/3}}\right], \frac{1}{4}(2 + \sqrt{3})] \right) / \\
& \left( 9 \times 3^{3/4} b^{5/3} (b c - ad)^{5/3} \sqrt{a + bx} \sqrt{-\frac{b^{1/3}(c + dx)^{1/3} ((b c - ad)^{1/3} - b^{1/3}(c + dx)^{1/3})}{((b c - ad)^{1/3} - (1 + \sqrt{3})b^{1/3}(c + dx)^{1/3})^2}} \right. \\
& \left. + 4(1 - \sqrt{3})d^2(c + dx)^{1/6} ((b c - ad)^{1/3} - b^{1/3}(c + dx)^{1/3}) \sqrt{\frac{(b c - ad)^{2/3} + b^{1/3}(b c - ad)^{1/3}(c + dx)^{1/3} + b^{2/3}(c + dx)^{2/3}}{((b c - ad)^{1/3} - (1 + \sqrt{3})b^{1/3}(c + dx)^{1/3})^2}} \right. \\
& \left. \operatorname{EllipticF}[\operatorname{ArcCos}\left[\frac{(b c - ad)^{1/3} - (1 - \sqrt{3})b^{1/3}(c + dx)^{1/3}}{(b c - ad)^{1/3} - (1 + \sqrt{3})b^{1/3}(c + dx)^{1/3}}\right], \frac{1}{4}(2 + \sqrt{3})] \right) / \\
& \left( 27 \times 3^{1/4} b^{5/3} (b c - ad)^{5/3} \sqrt{a + bx} \sqrt{-\frac{b^{1/3}(c + dx)^{1/3} ((b c - ad)^{1/3} - b^{1/3}(c + dx)^{1/3})}{((b c - ad)^{1/3} - (1 + \sqrt{3})b^{1/3}(c + dx)^{1/3})^2}} \right)
\end{aligned}$$

Result (type 5, 140 leaves):

$$- \left( \left( 2 (c + d x)^{5/6} \left( -8 a^2 d^2 - a b d (39 c + 55 d x) + b^2 (27 c^2 + 15 c d x - 20 d^2 x^2) + \right. \right. \right. \\
 \left. \left. \left. 8 d^2 (a + b x)^2 \sqrt{\frac{d (a + b x)}{-b c + a d}} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \frac{b (c + d x)}{b c - a d}\right] \right) \right) \Big/ \left( 135 b (b c - a d)^2 (a + b x)^{5/2} \right)$$

**Problem 1748: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + b x)^{5/2}}{(c + d x)^{1/6}} dx$$

Optimal (type 4, 890 leaves, 7 steps):

$$\begin{aligned}
& \frac{81 (b c - a d)^2 \sqrt{a + b x} (c + d x)^{5/6}}{224 d^3} - \frac{9 (b c - a d) (a + b x)^{3/2} (c + d x)^{5/6}}{28 d^2} + \\
& \frac{3 (a + b x)^{5/2} (c + d x)^{5/6}}{10 d} + \frac{243 (1 + \sqrt{3}) (b c - a d)^3 \sqrt{a + b x} (c + d x)^{1/6}}{448 b^{2/3} d^3 ((b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3})} + \\
& \left( 243 \times 3^{1/4} (b c - a d)^{10/3} (c + d x)^{1/6} ((b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}) \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + d x)^{1/3} + b^{2/3} (c + d x)^{2/3}}{((b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3})^2}} \right. \\
& \text{EllipticE} [\text{ArcCos} \left[ \frac{(b c - a d)^{1/3} - (1 - \sqrt{3}) b^{1/3} (c + d x)^{1/3}}{(b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3}} \right], \frac{1}{4} (2 + \sqrt{3})] \Bigg) / \\
& \left( 448 b^{2/3} d^4 \sqrt{a + b x} \sqrt{-\frac{b^{1/3} (c + d x)^{1/3} ((b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3})}{((b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3})^2}} \right) + \\
& \left( 81 \times 3^{3/4} (1 - \sqrt{3}) (b c - a d)^{10/3} (c + d x)^{1/6} ((b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}) \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + d x)^{1/3} + b^{2/3} (c + d x)^{2/3}}{((b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3})^2}} \right. \\
& \text{EllipticF} [\text{ArcCos} \left[ \frac{(b c - a d)^{1/3} - (1 - \sqrt{3}) b^{1/3} (c + d x)^{1/3}}{(b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3}} \right], \frac{1}{4} (2 + \sqrt{3})] \Bigg) / \\
& \left( 896 b^{2/3} d^4 \sqrt{a + b x} \sqrt{-\frac{b^{1/3} (c + d x)^{1/3} ((b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3})}{((b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3})^2}} \right)
\end{aligned}$$

Result (type 5, 138 leaves):

$$\begin{aligned}
& \frac{1}{1120 d^4 \sqrt{a + b x}} 3 (c + d x)^{5/6} \left( d (a + b x) (367 a^2 d^2 + 2 a b d (-195 c + 172 d x) + b^2 (135 c^2 - 120 c d x + 112 d^2 x^2)) - \right. \\
& \left. 81 (b c - a d)^3 \sqrt{\frac{d (a + b x)}{-b c + a d}} \text{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \frac{b (c + d x)}{b c - a d} \right] \right)
\end{aligned}$$

Problem 1749: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{3/2}}{(c + d x)^{1/6}} dx$$

Optimal (type 4, 855 leaves, 6 steps):

$$\begin{aligned} & \frac{27 (b c - a d) \sqrt{a + b x} (c + d x)^{5/6}}{56 d^2} + \frac{3 (a + b x)^{3/2} (c + d x)^{5/6}}{7 d} - \\ & \frac{81 (1 + \sqrt{3}) (b c - a d)^2 \sqrt{a + b x} (c + d x)^{1/6}}{112 b^{2/3} d^2 ((b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3})} - \left( 81 \times 3^{1/4} (b c - a d)^{7/3} (c + d x)^{1/6} ((b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}) \right. \\ & \left. \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + d x)^{1/3} + b^{2/3} (c + d x)^{2/3}}{((b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3})^2}} \operatorname{EllipticE}[\operatorname{ArcCos}\left[\frac{(b c - a d)^{1/3} - (1 - \sqrt{3}) b^{1/3} (c + d x)^{1/3}}{(b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3}}\right], \frac{1}{4} (2 + \sqrt{3})] \right) / \\ & \left( 112 b^{2/3} d^3 \sqrt{a + b x} \sqrt{-\frac{b^{1/3} (c + d x)^{1/3} ((b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3})}{((b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3})^2}} \right. - \\ & \left( 27 \times 3^{3/4} (1 - \sqrt{3}) (b c - a d)^{7/3} (c + d x)^{1/6} ((b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}) \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + d x)^{1/3} + b^{2/3} (c + d x)^{2/3}}{((b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3})^2}} \right. \\ & \left. \operatorname{EllipticF}[\operatorname{ArcCos}\left[\frac{(b c - a d)^{1/3} - (1 - \sqrt{3}) b^{1/3} (c + d x)^{1/3}}{(b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3}}\right], \frac{1}{4} (2 + \sqrt{3})] \right) / \\ & \left( 224 b^{2/3} d^3 \sqrt{a + b x} \sqrt{-\frac{b^{1/3} (c + d x)^{1/3} ((b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3})}{((b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3})^2}} \right) \end{aligned}$$

Result (type 5, 108 leaves):

$$\frac{1}{280 d^3 \sqrt{a + b x}} 3 (c + d x)^{5/6} \left( 5 d (a + b x) (-9 b c + 17 a d + 8 b d x) + 27 (b c - a d)^2 \sqrt{\frac{d (a + b x)}{-b c + a d}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \frac{b (c + d x)}{b c - a d}\right] \right)$$

## Problem 1750: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{a+b x}}{(c+d x)^{1/6}} d x$$

Optimal (type 4, 820 leaves, 5 steps):

$$\begin{aligned} & \frac{3 \sqrt{a+b x} (c+d x)^{5/6}}{4 d} + \frac{9 (1+\sqrt{3}) (b c-a d) \sqrt{a+b x} (c+d x)^{1/6}}{8 b^{2/3} d ((b c-a d)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+d x)^{1/3})} + \\ & \left( 9 \times 3^{1/4} (b c-a d)^{4/3} (c+d x)^{1/6} ((b c-a d)^{1/3} - b^{1/3} (c+d x)^{1/3}) \sqrt{\frac{(b c-a d)^{2/3} + b^{1/3} (b c-a d)^{1/3} (c+d x)^{1/3} + b^{2/3} (c+d x)^{2/3}}{((b c-a d)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+d x)^{1/3})^2}} \right. \\ & \left. \text{EllipticE} \left[ \text{ArcCos} \left[ \frac{(b c-a d)^{1/3} - (1-\sqrt{3}) b^{1/3} (c+d x)^{1/3}}{(b c-a d)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+d x)^{1/3}} \right], \frac{1}{4} (2+\sqrt{3}) \right] \right) / \\ & \left( 8 b^{2/3} d^2 \sqrt{a+b x} \sqrt{-\frac{b^{1/3} (c+d x)^{1/3} ((b c-a d)^{1/3} - b^{1/3} (c+d x)^{1/3})}{((b c-a d)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+d x)^{1/3})^2}} \right) + \\ & \left( 3 \times 3^{3/4} (1-\sqrt{3}) (b c-a d)^{4/3} (c+d x)^{1/6} ((b c-a d)^{1/3} - b^{1/3} (c+d x)^{1/3}) \sqrt{\frac{(b c-a d)^{2/3} + b^{1/3} (b c-a d)^{1/3} (c+d x)^{1/3} + b^{2/3} (c+d x)^{2/3}}{((b c-a d)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+d x)^{1/3})^2}} \right. \\ & \left. \text{EllipticF} \left[ \text{ArcCos} \left[ \frac{(b c-a d)^{1/3} - (1-\sqrt{3}) b^{1/3} (c+d x)^{1/3}}{(b c-a d)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+d x)^{1/3}} \right], \frac{1}{4} (2+\sqrt{3}) \right] \right) / \\ & \left( 16 b^{2/3} d^2 \sqrt{a+b x} \sqrt{-\frac{b^{1/3} (c+d x)^{1/3} ((b c-a d)^{1/3} - b^{1/3} (c+d x)^{1/3})}{((b c-a d)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+d x)^{1/3})^2}} \right) \end{aligned}$$

Result (type 5, 77 leaves):

$$3 \sqrt{a+b x} (c+d x)^{5/6} \left( 5 + \frac{3 \text{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \frac{b (c+d x)}{-b c+a d} \right]}{\sqrt{\frac{d (a+b x)}{-b c+a d}}} \right)$$

### Problem 1751: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\sqrt{a + bx} (c + dx)^{1/6}} dx$$

Optimal (type 4, 780 leaves, 4 steps):

$$\begin{aligned}
 & - \frac{3 (1 + \sqrt{3}) \sqrt{a + bx} (c + dx)^{1/6}}{b^{2/3} ((bc - ad)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + dx)^{1/3})} - \\
 & \left( 3 \times 3^{1/4} (bc - ad)^{1/3} (c + dx)^{1/6} ((bc - ad)^{1/3} - b^{1/3} (c + dx)^{1/3}) \sqrt{\frac{(bc - ad)^{2/3} + b^{1/3} (bc - ad)^{1/3} (c + dx)^{1/3} + b^{2/3} (c + dx)^{2/3}}{((bc - ad)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + dx)^{1/3})^2}} \right. \\
 & \left. \text{EllipticE} [\text{ArcCos} \left[ \frac{(bc - ad)^{1/3} - (1 - \sqrt{3}) b^{1/3} (c + dx)^{1/3}}{(bc - ad)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + dx)^{1/3}} \right], \frac{1}{4} (2 + \sqrt{3})] \right) / \\
 & \left( b^{2/3} d \sqrt{a + bx} \sqrt{-\frac{b^{1/3} (c + dx)^{1/3} ((bc - ad)^{1/3} - b^{1/3} (c + dx)^{1/3})}{((bc - ad)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + dx)^{1/3})^2}} \right) - \\
 & \left( 3^{3/4} (1 - \sqrt{3}) (bc - ad)^{1/3} (c + dx)^{1/6} ((bc - ad)^{1/3} - b^{1/3} (c + dx)^{1/3}) \sqrt{\frac{(bc - ad)^{2/3} + b^{1/3} (bc - ad)^{1/3} (c + dx)^{1/3} + b^{2/3} (c + dx)^{2/3}}{((bc - ad)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + dx)^{1/3})^2}} \right. \\
 & \left. \text{EllipticF} [\text{ArcCos} \left[ \frac{(bc - ad)^{1/3} - (1 - \sqrt{3}) b^{1/3} (c + dx)^{1/3}}{(bc - ad)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + dx)^{1/3}} \right], \frac{1}{4} (2 + \sqrt{3})] \right) / \\
 & \left( 2 b^{2/3} d \sqrt{a + bx} \sqrt{-\frac{b^{1/3} (c + dx)^{1/3} ((bc - ad)^{1/3} - b^{1/3} (c + dx)^{1/3})}{((bc - ad)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + dx)^{1/3})^2}} \right)
 \end{aligned}$$

Result (type 5, 73 leaves):

$$\frac{6 \sqrt{\frac{d(a+bx)}{-bc+ad}} (c + dx)^{5/6} \text{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \frac{b(c+dx)}{bc-ad} \right]}{5 d \sqrt{a + bx}}$$

### Problem 1752: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + bx)^{3/2} (c + dx)^{1/6}} dx$$

Optimal (type 4, 813 leaves, 5 steps):

$$\begin{aligned} & -\frac{2 (c + dx)^{5/6}}{(bc - ad) \sqrt{a + bx}} - \frac{2 (1 + \sqrt{3}) d \sqrt{a + bx} (c + dx)^{1/6}}{b^{2/3} (bc - ad) ((bc - ad)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + dx)^{1/3})} - \\ & \left( 2 \times 3^{1/4} (c + dx)^{1/6} ((bc - ad)^{1/3} - b^{1/3} (c + dx)^{1/3}) \sqrt{\frac{(bc - ad)^{2/3} + b^{1/3} (bc - ad)^{1/3} (c + dx)^{1/3} + b^{2/3} (c + dx)^{2/3}}{((bc - ad)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + dx)^{1/3})^2}} \right. \\ & \left. \text{EllipticE} \left[ \text{ArcCos} \left[ \frac{(bc - ad)^{1/3} - (1 - \sqrt{3}) b^{1/3} (c + dx)^{1/3}}{(bc - ad)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + dx)^{1/3}} \right], \frac{1}{4} (2 + \sqrt{3}) \right] \right) / \\ & \left( b^{2/3} (bc - ad)^{2/3} \sqrt{a + bx} \sqrt{-\frac{b^{1/3} (c + dx)^{1/3} ((bc - ad)^{1/3} - b^{1/3} (c + dx)^{1/3})}{((bc - ad)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + dx)^{1/3})^2}} \right) - \\ & \left( (1 - \sqrt{3}) (c + dx)^{1/6} ((bc - ad)^{1/3} - b^{1/3} (c + dx)^{1/3}) \sqrt{\frac{(bc - ad)^{2/3} + b^{1/3} (bc - ad)^{1/3} (c + dx)^{1/3} + b^{2/3} (c + dx)^{2/3}}{((bc - ad)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + dx)^{1/3})^2}} \right. \\ & \left. \text{EllipticF} \left[ \text{ArcCos} \left[ \frac{(bc - ad)^{1/3} - (1 - \sqrt{3}) b^{1/3} (c + dx)^{1/3}}{(bc - ad)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + dx)^{1/3}} \right], \frac{1}{4} (2 + \sqrt{3}) \right] \right) / \\ & \left( 3^{1/4} b^{2/3} (bc - ad)^{2/3} \sqrt{a + bx} \sqrt{-\frac{b^{1/3} (c + dx)^{1/3} ((bc - ad)^{1/3} - b^{1/3} (c + dx)^{1/3})}{((bc - ad)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + dx)^{1/3})^2}} \right) \end{aligned}$$

Result (type 5, 84 leaves):

$$\frac{2 (c + dx)^{5/6} \left( -5 + 2 \sqrt{\frac{d (a + bx)}{-bc + ad}} \text{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \frac{b (c + dx)}{bc - ad} \right] \right)}{5 (bc - ad) \sqrt{a + bx}}$$

### Problem 1753: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + bx)^{5/2} (c + dx)^{1/6}} dx$$

Optimal (type 4, 858 leaves, 6 steps):

$$\begin{aligned}
& -\frac{2(c + dx)^{5/6}}{3(b c - a d) (a + b x)^{3/2}} + \frac{8d(c + dx)^{5/6}}{9(b c - a d)^2 \sqrt{a + b x}} + \frac{8(1 + \sqrt{3}) d^2 \sqrt{a + b x} (c + dx)^{1/6}}{9b^{2/3} (b c - a d)^2 ((b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + dx)^{1/3})} + \\
& \left( 8d(c + dx)^{1/6} ((b c - a d)^{1/3} - b^{1/3} (c + dx)^{1/3}) \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + dx)^{1/3} + b^{2/3} (c + dx)^{2/3}}{((b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + dx)^{1/3})^2}} \right. \\
& \left. \text{EllipticE} \left[ \text{ArcCos} \left[ \frac{(b c - a d)^{1/3} - (1 - \sqrt{3}) b^{1/3} (c + dx)^{1/3}}{(b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + dx)^{1/3}} \right], \frac{1}{4} (2 + \sqrt{3}) \right] \right) / \\
& \left( 3 \times 3^{3/4} b^{2/3} (b c - a d)^{5/3} \sqrt{a + b x} \sqrt{-\frac{b^{1/3} (c + dx)^{1/3} ((b c - a d)^{1/3} - b^{1/3} (c + dx)^{1/3})}{((b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + dx)^{1/3})^2}} \right) + \\
& \left( 4(1 - \sqrt{3}) d (c + dx)^{1/6} ((b c - a d)^{1/3} - b^{1/3} (c + dx)^{1/3}) \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + dx)^{1/3} + b^{2/3} (c + dx)^{2/3}}{((b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + dx)^{1/3})^2}} \right. \\
& \left. \text{EllipticF} \left[ \text{ArcCos} \left[ \frac{(b c - a d)^{1/3} - (1 - \sqrt{3}) b^{1/3} (c + dx)^{1/3}}{(b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + dx)^{1/3}} \right], \frac{1}{4} (2 + \sqrt{3}) \right] \right) / \\
& \left( 9 \times 3^{1/4} b^{2/3} (b c - a d)^{5/3} \sqrt{a + b x} \sqrt{-\frac{b^{1/3} (c + dx)^{1/3} ((b c - a d)^{1/3} - b^{1/3} (c + dx)^{1/3})}{((b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + dx)^{1/3})^2}} \right)
\end{aligned}$$

Result (type 5, 105 leaves):

$$-\frac{1}{45 (b c - a d)^2 (a + b x)^{3/2}} 2 (c + dx)^{5/6} \left( -5 (-3 b c + 7 a d + 4 b d x) + 8 d (a + b x) \sqrt{\frac{d (a + b x)}{-b c + a d}} \text{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \frac{b (c + dx)}{b c - a d} \right] \right)$$

### Problem 1754: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{5/2}}{(c + d x)^{5/6}} dx$$

Optimal (type 4, 440 leaves, 5 steps):

$$\begin{aligned} & \frac{81 (b c - a d)^2 \sqrt{a + b x} (c + d x)^{1/6}}{64 d^3} - \frac{9 (b c - a d) (a + b x)^{3/2} (c + d x)^{1/6}}{16 d^2} + \\ & \frac{3 (a + b x)^{5/2} (c + d x)^{1/6}}{8 d} - \left( 81 \times 3^{3/4} (b c - a d)^{8/3} (c + d x)^{1/6} \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right) \right. \\ & \left. \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + d x)^{1/3} + b^{2/3} (c + d x)^{2/3}}{\left( (b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3} \right)^2}} \text{EllipticF} \left[ \text{ArcCos} \left[ \frac{(b c - a d)^{1/3} - (1 - \sqrt{3}) b^{1/3} (c + d x)^{1/3}}{(b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3}} \right], \frac{1}{4} (2 + \sqrt{3}) \right] \right) / \\ & \left( 128 d^4 \sqrt{a + b x} \sqrt{-\frac{b^{1/3} (c + d x)^{1/3} \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right)}{\left( (b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3} \right)^2}} \right) \end{aligned}$$

Result (type 5, 138 leaves):

$$\begin{aligned} & \frac{1}{64 d^4 \sqrt{a + b x}} 3 (c + d x)^{1/6} \left( d (a + b x) (47 a^2 d^2 + 2 a b d (-33 c + 14 d x) + b^2 (27 c^2 - 12 c d x + 8 d^2 x^2)) - \right. \\ & \left. 81 (b c - a d)^3 \sqrt{\frac{d (a + b x)}{-b c + a d}} \text{Hypergeometric2F1} \left[ \frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \frac{b (c + d x)}{b c - a d} \right] \right) \end{aligned}$$

### Problem 1755: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{3/2}}{(c + d x)^{5/6}} dx$$

Optimal (type 4, 405 leaves, 4 steps):

$$\begin{aligned}
& - \frac{27 (b c - a d) \sqrt{a + b x} (c + d x)^{1/6}}{20 d^2} + \frac{3 (a + b x)^{3/2} (c + d x)^{1/6}}{5 d} + \\
& \left( 27 \times 3^{3/4} (b c - a d)^{5/3} (c + d x)^{1/6} \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right) \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + d x)^{1/3} + b^{2/3} (c + d x)^{2/3}}{\left( (b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3} \right)^2}} \right. \\
& \left. \text{EllipticF} \left[ \text{ArcCos} \left[ \frac{(b c - a d)^{1/3} - (1 - \sqrt{3}) b^{1/3} (c + d x)^{1/3}}{(b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3}} \right], \frac{1}{4} (2 + \sqrt{3}) \right] \right) / \\
& \left( 40 d^3 \sqrt{a + b x} \sqrt{-\frac{b^{1/3} (c + d x)^{1/3} \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right)}{\left( (b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3} \right)^2}} \right)
\end{aligned}$$

Result (type 5, 107 leaves):

$$\frac{1}{20 d^3 \sqrt{a + b x}} 3 (c + d x)^{1/6} \left( d (a + b x) (-9 b c + 13 a d + 4 b d x) + 27 (b c - a d)^2 \sqrt{\frac{d (a + b x)}{-b c + a d}} \text{Hypergeometric2F1} \left[ \frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \frac{b (c + d x)}{b c - a d} \right] \right)$$

Problem 1756: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{a + b x}}{(c + d x)^{5/6}} dx$$

Optimal (type 4, 372 leaves, 3 steps):

$$\begin{aligned}
& \frac{3 \sqrt{a + b x} (c + d x)^{1/6}}{2 d} - \\
& \left( 3 \times 3^{3/4} (b c - a d)^{2/3} (c + d x)^{1/6} \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right) \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + d x)^{1/3} + b^{2/3} (c + d x)^{2/3}}{\left( (b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3} \right)^2}} \right. \\
& \left. \text{EllipticF} \left[ \text{ArcCos} \left[ \frac{(b c - a d)^{1/3} - (1 - \sqrt{3}) b^{1/3} (c + d x)^{1/3}}{(b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3}} \right], \frac{1}{4} (2 + \sqrt{3}) \right] \right) / \\
& \left( 4 d^2 \sqrt{a + b x} \sqrt{-\frac{b^{1/3} (c + d x)^{1/3} \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right)}{\left( (b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3} \right)^2}} \right)
\end{aligned}$$

Result (type 5, 77 leaves) :

$$\frac{3 \sqrt{a+b x} (c+d x)^{1/6}}{2 d} \left( 1 + \frac{3 \text{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \frac{b(c+d x)}{b c-a d}\right]}{\sqrt{\frac{d(a+b x)}{-b c+a d}}} \right)$$

Problem 1757: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\sqrt{a+b x} (c+d x)^{5/6}} dx$$

Optimal (type 4, 343 leaves, 2 steps) :

$$\begin{aligned} & \left( 3^{3/4} (c+d x)^{1/6} \left( (b c - a d)^{1/3} - b^{1/3} (c+d x)^{1/3} \right) \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c+d x)^{1/3} + b^{2/3} (c+d x)^{2/3}}{(b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c+d x)^{1/3}^2}} \right. \\ & \left. \text{EllipticF} \left[ \text{ArcCos} \left[ \frac{(b c - a d)^{1/3} - (1 - \sqrt{3}) b^{1/3} (c+d x)^{1/3}}{(b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c+d x)^{1/3}} \right], \frac{1}{4} (2 + \sqrt{3}) \right] \right) / \\ & \left( d (b c - a d)^{1/3} \sqrt{a+b x} \sqrt{-\frac{b^{1/3} (c+d x)^{1/3} ((b c - a d)^{1/3} - b^{1/3} (c+d x)^{1/3})}{((b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c+d x)^{1/3})^2}} \right) \end{aligned}$$

Result (type 5, 71 leaves) :

$$\frac{6 \sqrt{\frac{d(a+b x)}{-b c+a d}} (c+d x)^{1/6} \text{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \frac{b(c+d x)}{b c-a d}\right]}{d \sqrt{a+b x}}$$

Problem 1758: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a+b x)^{3/2} (c+d x)^{5/6}} dx$$

Optimal (type 4, 372 leaves, 3 steps) :

$$\begin{aligned}
& - \frac{2 (c + d x)^{1/6}}{(b c - a d) \sqrt{a + b x}} - \left( 2 (c + d x)^{1/6} \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right) \right. \\
& \left. \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + d x)^{1/3} + b^{2/3} (c + d x)^{2/3}}{\left( (b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3} \right)^2}} \operatorname{EllipticF}[\operatorname{ArcCos}\left[ \frac{(b c - a d)^{1/3} - (1 - \sqrt{3}) b^{1/3} (c + d x)^{1/3}}{(b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3}} \right], \frac{1}{4} (2 + \sqrt{3})] \right) / \\
& \left( 3^{1/4} (b c - a d)^{4/3} \sqrt{a + b x} \sqrt{-\frac{b^{1/3} (c + d x)^{1/3} \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right)}{\left( (b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3} \right)^2}} \right)
\end{aligned}$$

Result (type 5, 82 leaves):

$$\begin{aligned}
& \frac{2 (c + d x)^{1/6} \left( 1 + 2 \sqrt{\frac{d (a + b x)}{-b c + a d}} \operatorname{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \frac{b (c + d x)}{b c - a d}\right] \right)}{(b c - a d) \sqrt{a + b x}}
\end{aligned}$$

Problem 1759: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + b x)^{5/2} (c + d x)^{5/6}} dx$$

Optimal (type 4, 410 leaves, 4 steps):

$$\begin{aligned}
& - \frac{2 (c + d x)^{1/6}}{3 (b c - a d) (a + b x)^{3/2}} + \frac{16 d (c + d x)^{1/6}}{9 (b c - a d)^2 \sqrt{a + b x}} + \\
& \left( 16 d (c + d x)^{1/6} \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right) \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + d x)^{1/3} + b^{2/3} (c + d x)^{2/3}}{\left( (b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3} \right)^2}} \right. \\
& \left. \operatorname{EllipticF}[\operatorname{ArcCos}\left[ \frac{(b c - a d)^{1/3} - (1 - \sqrt{3}) b^{1/3} (c + d x)^{1/3}}{(b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3}} \right], \frac{1}{4} (2 + \sqrt{3})] \right) / \\
& \left( 9 \times 3^{1/4} (b c - a d)^{7/3} \sqrt{a + b x} \sqrt{-\frac{b^{1/3} (c + d x)^{1/3} \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right)}{\left( (b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3} \right)^2}} \right)
\end{aligned}$$

Result (type 5, 102 leaves):

$$\frac{2 (c + d x)^{1/6} \left( -3 b c + 11 a d + 8 b d x + 16 d (a + b x) \sqrt{\frac{d (a + b x)}{-b c + a d}} \text{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \frac{b (c + d x)}{b c - a d}\right] \right)}{9 (b c - a d)^2 (a + b x)^{3/2}}$$

Problem 1760: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{5/2}}{(c + d x)^{7/6}} dx$$

Optimal (type 4, 880 leaves, 7 steps):

$$\begin{aligned} & -\frac{6 (a + b x)^{5/2}}{d (c + d x)^{1/6}} - \frac{405 b (b c - a d) \sqrt{a + b x} (c + d x)^{5/6}}{56 d^3} + \frac{45 b (a + b x)^{3/2} (c + d x)^{5/6}}{7 d^2} - \\ & \frac{1215 (1 + \sqrt{3}) b^{1/3} (b c - a d)^2 \sqrt{a + b x} (c + d x)^{1/6}}{112 d^3 ((b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3})} - \left( \frac{1215 \times 3^{1/4} b^{1/3} (b c - a d)^{7/3} (c + d x)^{1/6} ((b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3})}{((b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3})^2} \text{EllipticE} \left[ \text{ArcCos} \left[ \frac{(b c - a d)^{1/3} - (1 - \sqrt{3}) b^{1/3} (c + d x)^{1/3}}{(b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3}} \right], \frac{1}{4} (2 + \sqrt{3}) \right] \right) / \\ & \left( \frac{112 d^4 \sqrt{a + b x}}{\sqrt{-\frac{b^{1/3} (c + d x)^{1/3} ((b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3})}{((b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3})^2}}} - \right. \\ & \left. \left( \frac{405 \times 3^{3/4} (1 - \sqrt{3}) b^{1/3} (b c - a d)^{7/3} (c + d x)^{1/6} ((b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3})}{((b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3})^2} \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + d x)^{1/3} + b^{2/3} (c + d x)^{2/3}}{((b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3})^2}} \right. \right. \\ & \left. \left. \text{EllipticF} \left[ \text{ArcCos} \left[ \frac{(b c - a d)^{1/3} - (1 - \sqrt{3}) b^{1/3} (c + d x)^{1/3}}{(b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3}} \right], \frac{1}{4} (2 + \sqrt{3}) \right] \right) / \\ & \left( \frac{224 d^4 \sqrt{a + b x}}{\sqrt{-\frac{b^{1/3} (c + d x)^{1/3} ((b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3})}{((b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3})^2}}} \right) \end{aligned}$$

Result (type 5, 132 leaves):

$$\frac{1}{56 d^4 \sqrt{a+b x}} 3 (c+d x)^{5/6} \\ \left( d (a+b x) \left( b (-23 b c + 31 a d) + 8 b^2 d x - \frac{112 (b c - a d)^2}{c+d x} \right) + 81 b (b c - a d)^2 \sqrt{\frac{d (a+b x)}{-b c + a d}} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \frac{b (c+d x)}{b c - a d}\right] \right)$$

**Problem 1761:** Result unnecessarily involves higher level functions.

$$\int \frac{(a+b x)^{3/2}}{(c+d x)^{7/6}} dx$$

Optimal (type 4, 844 leaves, 6 steps):

$$-\frac{6 (a+b x)^{3/2}}{d (c+d x)^{1/6}} + \frac{27 b \sqrt{a+b x} (c+d x)^{5/6}}{4 d^2} + \frac{81 (1+\sqrt{3}) b^{1/3} (b c - a d) \sqrt{a+b x} (c+d x)^{1/6}}{8 d^2 ((b c - a d)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+d x)^{1/3})} + \\ \left( 81 \times 3^{1/4} b^{1/3} (b c - a d)^{4/3} (c+d x)^{1/6} ((b c - a d)^{1/3} - b^{1/3} (c+d x)^{1/3}) \right. \\ \left. \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c+d x)^{1/3} + b^{2/3} (c+d x)^{2/3}}{((b c - a d)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+d x)^{1/3})^2}} \text{EllipticE}\left[\text{ArcCos}\left[\frac{(b c - a d)^{1/3} - (1-\sqrt{3}) b^{1/3} (c+d x)^{1/3}}{(b c - a d)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+d x)^{1/3}}\right], \frac{1}{4} (2+\sqrt{3})\right]\right) / \\ \left( 8 d^3 \sqrt{a+b x} \sqrt{-\frac{b^{1/3} (c+d x)^{1/3} ((b c - a d)^{1/3} - b^{1/3} (c+d x)^{1/3})}{((b c - a d)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+d x)^{1/3})^2}} \right) + \\ \left( 27 \times 3^{3/4} (1-\sqrt{3}) b^{1/3} (b c - a d)^{4/3} (c+d x)^{1/6} ((b c - a d)^{1/3} - b^{1/3} (c+d x)^{1/3}) \right) \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c+d x)^{1/3} + b^{2/3} (c+d x)^{2/3}}{((b c - a d)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+d x)^{1/3})^2}} \\ \text{EllipticF}\left[\text{ArcCos}\left[\frac{(b c - a d)^{1/3} - (1-\sqrt{3}) b^{1/3} (c+d x)^{1/3}}{(b c - a d)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+d x)^{1/3}}\right], \frac{1}{4} (2+\sqrt{3})\right]\right) / \\ \left( 16 d^3 \sqrt{a+b x} \sqrt{-\frac{b^{1/3} (c+d x)^{1/3} ((b c - a d)^{1/3} - b^{1/3} (c+d x)^{1/3})}{((b c - a d)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+d x)^{1/3})^2}} \right)$$

Result (type 5, 99 leaves):

$$\frac{3 \sqrt{a+b x} (c+d x)^{5/6}}{20 d^2} \left( \frac{\frac{5 (9 b c - 8 a d + b d x)}{c+d x} + \frac{27 b \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \frac{b (c+d x)}{b c-a d}\right]}{\sqrt{\frac{d (a+b x)}{-b c+a d}}}}{\sqrt{\frac{d (a+b x)}{-b c+a d}}} \right)$$

Problem 1762: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{a+b x}}{(c+d x)^{7/6}} dx$$

Optimal (type 4, 806 leaves, 5 steps):

$$-\frac{6 \sqrt{a+b x}}{d (c+d x)^{1/6}} - \frac{9 (1+\sqrt{3}) b^{1/3} \sqrt{a+b x} (c+d x)^{1/6}}{d ((b c - a d)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+d x)^{1/3})} -$$

$$\left( 9 \times 3^{1/4} b^{1/3} (b c - a d)^{1/3} (c+d x)^{1/6} \left( (b c - a d)^{1/3} - b^{1/3} (c+d x)^{1/3} \right) \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c+d x)^{1/3} + b^{2/3} (c+d x)^{2/3}}{((b c - a d)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+d x)^{1/3})^2}} \right)$$

$$\text{EllipticE} \left[ \text{ArcCos} \left[ \frac{(b c - a d)^{1/3} - (1-\sqrt{3}) b^{1/3} (c+d x)^{1/3}}{(b c - a d)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+d x)^{1/3}} \right], \frac{1}{4} (2+\sqrt{3}) \right] /$$

$$\left( d^2 \sqrt{a+b x} \sqrt{-\frac{b^{1/3} (c+d x)^{1/3} \left( (b c - a d)^{1/3} - b^{1/3} (c+d x)^{1/3} \right)}{((b c - a d)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+d x)^{1/3})^2}} \right) -$$

$$\left( 3 \times 3^{3/4} (1-\sqrt{3}) b^{1/3} (b c - a d)^{1/3} (c+d x)^{1/6} \left( (b c - a d)^{1/3} - b^{1/3} (c+d x)^{1/3} \right) \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c+d x)^{1/3} + b^{2/3} (c+d x)^{2/3}}{((b c - a d)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+d x)^{1/3})^2}} \right)$$

$$\text{EllipticF} \left[ \text{ArcCos} \left[ \frac{(b c - a d)^{1/3} - (1-\sqrt{3}) b^{1/3} (c+d x)^{1/3}}{(b c - a d)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+d x)^{1/3}} \right], \frac{1}{4} (2+\sqrt{3}) \right] /$$

$$\left( 2 d^2 \sqrt{a+b x} \sqrt{-\frac{b^{1/3} (c+d x)^{1/3} \left( (b c - a d)^{1/3} - b^{1/3} (c+d x)^{1/3} \right)}{((b c - a d)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+d x)^{1/3})^2}} \right)$$

Result (type 5, 90 leaves):

$$\frac{-30 d (a + b x) + 18 b \sqrt{\frac{d (a+b x)}{-b c+a d}} (c + d x) \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \frac{b (c+d x)}{b c-a d}\right]}{5 d^2 \sqrt{a + b x} (c + d x)^{1/6}}$$

Problem 1763: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\sqrt{a + b x} (c + d x)^{7/6}} dx$$

Optimal (type 4, 817 leaves, 5 steps):

$$\begin{aligned} & \frac{6 \sqrt{a + b x}}{(b c - a d) (c + d x)^{1/6}} + \frac{6 \left(1 + \sqrt{3}\right) b^{1/3} \sqrt{a + b x} (c + d x)^{1/6}}{(b c - a d) \left((b c - a d)^{1/3} - \left(1 + \sqrt{3}\right) b^{1/3} (c + d x)^{1/3}\right)} + \\ & \left(6 \times 3^{1/4} b^{1/3} (c + d x)^{1/6} \left((b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}\right) \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + d x)^{1/3} + b^{2/3} (c + d x)^{2/3}}{\left((b c - a d)^{1/3} - \left(1 + \sqrt{3}\right) b^{1/3} (c + d x)^{1/3}\right)^2}} \right. \\ & \left. \text{EllipticE} \left[ \text{ArcCos} \left[ \frac{\left(b c - a d\right)^{1/3} - \left(1 - \sqrt{3}\right) b^{1/3} (c + d x)^{1/3}}{\left(b c - a d\right)^{1/3} - \left(1 + \sqrt{3}\right) b^{1/3} (c + d x)^{1/3}} \right], \frac{1}{4} \left(2 + \sqrt{3}\right) \right] \right) / \\ & \left( d (b c - a d)^{2/3} \sqrt{a + b x} \sqrt{-\frac{b^{1/3} (c + d x)^{1/3} \left((b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}\right)}{\left((b c - a d)^{1/3} - \left(1 + \sqrt{3}\right) b^{1/3} (c + d x)^{1/3}\right)^2}} \right) + \\ & \left( 3^{3/4} \left(1 - \sqrt{3}\right) b^{1/3} (c + d x)^{1/6} \left((b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}\right) \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + d x)^{1/3} + b^{2/3} (c + d x)^{2/3}}{\left((b c - a d)^{1/3} - \left(1 + \sqrt{3}\right) b^{1/3} (c + d x)^{1/3}\right)^2}} \right. \\ & \left. \text{EllipticF} \left[ \text{ArcCos} \left[ \frac{\left(b c - a d\right)^{1/3} - \left(1 - \sqrt{3}\right) b^{1/3} (c + d x)^{1/3}}{\left(b c - a d\right)^{1/3} - \left(1 + \sqrt{3}\right) b^{1/3} (c + d x)^{1/3}} \right], \frac{1}{4} \left(2 + \sqrt{3}\right) \right] \right) / \\ & \left( d (b c - a d)^{2/3} \sqrt{a + b x} \sqrt{-\frac{b^{1/3} (c + d x)^{1/3} \left((b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}\right)}{\left((b c - a d)^{1/3} - \left(1 + \sqrt{3}\right) b^{1/3} (c + d x)^{1/3}\right)^2}} \right) \end{aligned}$$

Result (type 5, 100 leaves):

$$\frac{6 \left( 5 d (a + b x) - 2 b \sqrt{\frac{d (a + b x)}{-b c + a d}} (c + d x) \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \frac{b (c + d x)}{b c - a d}\right] \right)}{5 d (b c - a d) \sqrt{a + b x} (c + d x)^{1/6}}$$

Problem 1764: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + b x)^{3/2} (c + d x)^{7/6}} dx$$

Optimal (type 4, 844 leaves, 6 steps):

$$\begin{aligned}
& -\frac{2}{(b c - a d) \sqrt{a + b x} (c + d x)^{1/6}} - \frac{8 d \sqrt{a + b x}}{(b c - a d)^2 (c + d x)^{1/6}} - \\
& \frac{8 (1 + \sqrt{3}) b^{1/3} d \sqrt{a + b x} (c + d x)^{1/6}}{(b c - a d)^2 ((b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3})} - \left( 8 \times 3^{1/4} b^{1/3} (c + d x)^{1/6} ((b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}) \right. \\
& \left. \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + d x)^{1/3} + b^{2/3} (c + d x)^{2/3}}{((b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3})^2}} \text{EllipticE} \left[ \text{ArcCos} \left[ \frac{(b c - a d)^{1/3} - (1 - \sqrt{3}) b^{1/3} (c + d x)^{1/3}}{(b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3}} \right], \frac{1}{4} (2 + \sqrt{3}) \right] \right) / \\
& \left( (b c - a d)^{5/3} \sqrt{a + b x} \sqrt{-\frac{b^{1/3} (c + d x)^{1/3} ((b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3})}{((b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3})^2}} \right. - \\
& \left( 4 (1 - \sqrt{3}) b^{1/3} (c + d x)^{1/6} ((b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}) \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + d x)^{1/3} + b^{2/3} (c + d x)^{2/3}}{((b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3})^2}} \right. \\
& \left. \text{EllipticF} \left[ \text{ArcCos} \left[ \frac{(b c - a d)^{1/3} - (1 - \sqrt{3}) b^{1/3} (c + d x)^{1/3}}{(b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3}} \right], \frac{1}{4} (2 + \sqrt{3}) \right] \right) / \\
& \left( 3^{1/4} (b c - a d)^{5/3} \sqrt{a + b x} \sqrt{-\frac{b^{1/3} (c + d x)^{1/3} ((b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3})}{((b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3})^2}} \right)
\end{aligned}$$

Result (type 5, 102 leaves):

$$-\frac{2 \left(15 \, a \, d + 5 \, b \, (c + 4 \, d \, x) - 8 \, b \, \sqrt{\frac{d \, (a + b \, x)}{-b \, c + a \, d}} \, (c + d \, x) \, \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \frac{b \, (c + d \, x)}{b \, c - a \, d}\right]\right)}{5 \, (b \, c - a \, d)^2 \, \sqrt{a + b \, x} \, (c + d \, x)^{1/6}}$$

Problem 1765: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + b \, x)^{5/2} \, (c + d \, x)^{7/6}} \, dx$$

Optimal (type 4, 893 leaves, 7 steps):

$$\begin{aligned}
& -\frac{2}{3 \, (b \, c - a \, d) \, (a + b \, x)^{3/2} \, (c + d \, x)^{1/6}} + \frac{20 \, d}{9 \, (b \, c - a \, d)^2 \, \sqrt{a + b \, x} \, (c + d \, x)^{1/6}} + \\
& \frac{80 \, d^2 \, \sqrt{a + b \, x}}{9 \, (b \, c - a \, d)^3 \, (c + d \, x)^{1/6}} + \frac{80 \, (1 + \sqrt{3}) \, b^{1/3} \, d^2 \, \sqrt{a + b \, x} \, (c + d \, x)^{1/6}}{9 \, (b \, c - a \, d)^3 \, ((b \, c - a \, d)^{1/3} - (1 + \sqrt{3}) \, b^{1/3} \, (c + d \, x)^{1/3})} + \\
& \left(80 \, b^{1/3} \, d \, (c + d \, x)^{1/6} \, ((b \, c - a \, d)^{1/3} - b^{1/3} \, (c + d \, x)^{1/3}) \sqrt{\frac{(b \, c - a \, d)^{2/3} + b^{1/3} \, (b \, c - a \, d)^{1/3} \, (c + d \, x)^{1/3} + b^{2/3} \, (c + d \, x)^{2/3}}{((b \, c - a \, d)^{1/3} - (1 + \sqrt{3}) \, b^{1/3} \, (c + d \, x)^{1/3})^2}}\right. \\
& \left. \text{EllipticE}\left[\text{ArcCos}\left[\frac{(b \, c - a \, d)^{1/3} - (1 - \sqrt{3}) \, b^{1/3} \, (c + d \, x)^{1/3}}{(b \, c - a \, d)^{1/3} - (1 + \sqrt{3}) \, b^{1/3} \, (c + d \, x)^{1/3}}\right], \frac{1}{4} \left(2 + \sqrt{3}\right)\right]\right) / \\
& \left(3 \times 3^{3/4} \, (b \, c - a \, d)^{8/3} \, \sqrt{a + b \, x} \sqrt{-\frac{b^{1/3} \, (c + d \, x)^{1/3} \, ((b \, c - a \, d)^{1/3} - b^{1/3} \, (c + d \, x)^{1/3})}{((b \, c - a \, d)^{1/3} - (1 + \sqrt{3}) \, b^{1/3} \, (c + d \, x)^{1/3})^2}}\right) + \\
& \left(40 \, (1 - \sqrt{3}) \, b^{1/3} \, d \, (c + d \, x)^{1/6} \, ((b \, c - a \, d)^{1/3} - b^{1/3} \, (c + d \, x)^{1/3}) \sqrt{\frac{(b \, c - a \, d)^{2/3} + b^{1/3} \, (b \, c - a \, d)^{1/3} \, (c + d \, x)^{1/3} + b^{2/3} \, (c + d \, x)^{2/3}}{((b \, c - a \, d)^{1/3} - (1 + \sqrt{3}) \, b^{1/3} \, (c + d \, x)^{1/3})^2}}\right. \\
& \left. \text{EllipticF}\left[\text{ArcCos}\left[\frac{(b \, c - a \, d)^{1/3} - (1 - \sqrt{3}) \, b^{1/3} \, (c + d \, x)^{1/3}}{(b \, c - a \, d)^{1/3} - (1 + \sqrt{3}) \, b^{1/3} \, (c + d \, x)^{1/3}}\right], \frac{1}{4} \left(2 + \sqrt{3}\right)\right]\right) / \\
& \left(9 \times 3^{1/4} \, (b \, c - a \, d)^{8/3} \, \sqrt{a + b \, x} \sqrt{-\frac{b^{1/3} \, (c + d \, x)^{1/3} \, ((b \, c - a \, d)^{1/3} - b^{1/3} \, (c + d \, x)^{1/3})}{((b \, c - a \, d)^{1/3} - (1 + \sqrt{3}) \, b^{1/3} \, (c + d \, x)^{1/3})^2}}\right)
\end{aligned}$$

Result (type 5, 139 leaves):

$$-\left( \left( 2 \left( 27 a^2 d^2 + 2 a b d (8 c + 35 d x) + b^2 (-3 c^2 + 10 c d x + 40 d^2 x^2) - \right. \right. \right. \\ \left. \left. \left. 16 b d (a + b x) \sqrt{\frac{d (a + b x)}{-b c + a d}} (c + d x) \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \frac{b (c + d x)}{b c - a d}\right] \right) \right) \Bigg/ \left( 9 (-b c + a d)^3 (a + b x)^{3/2} (c + d x)^{1/6} \right)$$

Problem 1766: Result more than twice size of optimal antiderivative.

$$\int (a + b x)^{1/6} (c + d x)^{13/6} dx$$

Optimal (type 5, 84 leaves, 2 steps):

$$\frac{6 (b c - a d)^2 (a + b x)^{7/6} (c + d x)^{1/6} \text{Hypergeometric2F1}\left[-\frac{13}{6}, \frac{7}{6}, \frac{13}{6}, -\frac{d (a + b x)}{b c - a d}\right]}{7 b^3 \left(\frac{b (c + d x)}{b c - a d}\right)^{1/6}}$$

Result (type 5, 182 leaves):

$$-\frac{1}{2240 b^3 d^2 (a + b x)^{5/6}} \\ 3 (c + d x)^{1/6} \left( -d (a + b x) (91 a^3 d^3 - 13 a^2 b d^2 (23 c + 2 d x) + a b^2 d (341 c^2 + 84 c d x + 16 d^2 x^2) + b^3 (91 c^3 + 614 c^2 d x + 656 c d^2 x^2 + 224 d^3 x^3) \right) + \\ 91 (b c - a d)^4 \left( \frac{d (a + b x)}{-b c + a d} \right)^{5/6} \text{Hypergeometric2F1}\left[\frac{1}{6}, \frac{5}{6}, \frac{7}{6}, \frac{b (c + d x)}{b c - a d}\right]$$

Problem 1772: Result unnecessarily involves higher level functions.

$$\int (a + b x)^{1/6} (c + d x)^{5/6} dx$$

Optimal (type 3, 427 leaves, 14 steps):

$$\begin{aligned}
& \frac{5 (b c - a d) (a + b x)^{1/6} (c + d x)^{5/6}}{12 b d} + \frac{(a + b x)^{7/6} (c + d x)^{5/6}}{2 b} + \frac{5 (b c - a d)^2 \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2 d^{1/6} (a+b x)^{1/6}}{\sqrt{3} b^{1/6} (c+d x)^{1/6}}\right]}{24 \sqrt{3} b^{11/6} d^{7/6}} - \\
& \frac{5 (b c - a d)^2 \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 d^{1/6} (a+b x)^{1/6}}{\sqrt{3} b^{1/6} (c+d x)^{1/6}}\right]}{24 \sqrt{3} b^{11/6} d^{7/6}} - \frac{5 (b c - a d)^2 \operatorname{ArcTanh}\left[\frac{d^{1/6} (a+b x)^{1/6}}{b^{1/6} (c+d x)^{1/6}}\right]}{36 b^{11/6} d^{7/6}} + \\
& \frac{5 (b c - a d)^2 \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3} (a+b x)^{1/3}}{(c+d x)^{1/3}} - \frac{b^{1/6} d^{1/6} (a+b x)^{1/6}}{(c+d x)^{1/6}}\right]}{144 b^{11/6} d^{7/6}} - \frac{5 (b c - a d)^2 \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3} (a+b x)^{1/3}}{(c+d x)^{1/3}} + \frac{b^{1/6} d^{1/6} (a+b x)^{1/6}}{(c+d x)^{1/6}}\right]}{144 b^{11/6} d^{7/6}}
\end{aligned}$$

Result (type 5, 109 leaves):

$$\frac{1}{12 b d^2 (a+b x)^{5/6}} (c+d x)^{5/6} \left( d (a+b x) (5 b c + a d + 6 b d x) - (b c - a d)^2 \left( \frac{d (a+b x)}{-b c + a d} \right)^{5/6} \operatorname{Hypergeometric2F1}\left[\frac{5}{6}, \frac{5}{6}, \frac{11}{6}, \frac{b (c+d x)}{b c - a d}\right] \right)$$

Problem 1773: Result unnecessarily involves higher level functions.

$$\int \frac{(a+b x)^{1/6}}{(c+d x)^{1/6}} dx$$

Optimal (type 3, 378 leaves, 13 steps):

$$\begin{aligned}
& \frac{(a+b x)^{1/6} (c+d x)^{5/6}}{d} + \frac{(b c - a d) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2 d^{1/6} (a+b x)^{1/6}}{\sqrt{3} b^{1/6} (c+d x)^{1/6}}\right]}{2 \sqrt{3} b^{5/6} d^{7/6}} - \frac{(b c - a d) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 d^{1/6} (a+b x)^{1/6}}{\sqrt{3} b^{1/6} (c+d x)^{1/6}}\right]}{2 \sqrt{3} b^{5/6} d^{7/6}} - \\
& \frac{(b c - a d) \operatorname{ArcTanh}\left[\frac{d^{1/6} (a+b x)^{1/6}}{b^{1/6} (c+d x)^{1/6}}\right]}{3 b^{5/6} d^{7/6}} + \frac{(b c - a d) \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3} (a+b x)^{1/3}}{(c+d x)^{1/3}} - \frac{b^{1/6} d^{1/6} (a+b x)^{1/6}}{(c+d x)^{1/6}}\right]}{12 b^{5/6} d^{7/6}} - \frac{(b c - a d) \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3} (a+b x)^{1/3}}{(c+d x)^{1/3}} + \frac{b^{1/6} d^{1/6} (a+b x)^{1/6}}{(c+d x)^{1/6}}\right]}{12 b^{5/6} d^{7/6}}
\end{aligned}$$

Result (type 5, 76 leaves):

$$\frac{(a+b x)^{1/6} (c+d x)^{5/6}}{5 d} \left( 5 + \frac{\operatorname{Hypergeometric2F1}\left[\frac{5}{6}, \frac{5}{6}, \frac{11}{6}, \frac{b (c+d x)}{b c - a d}\right]}{\left(\frac{d (a+b x)}{-b c + a d}\right)^{1/6}} \right)$$

Problem 1774: Result unnecessarily involves higher level functions.

$$\int \frac{(a+b x)^{1/6}}{(c+d x)^{7/6}} dx$$

Optimal (type 3, 332 leaves, 13 steps):

$$\begin{aligned}
& - \frac{6 (a+b x)^{1/6}}{d (c+d x)^{1/6}} - \frac{\sqrt{3} b^{1/6} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2 d^{1/6} (a+b x)^{1/6}}{\sqrt{3} b^{1/6} (c+d x)^{1/6}}\right]}{d^{7/6}} + \frac{\sqrt{3} b^{1/6} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 d^{1/6} (a+b x)^{1/6}}{\sqrt{3} b^{1/6} (c+d x)^{1/6}}\right]}{d^{7/6}} + \\
& \frac{2 b^{1/6} \operatorname{ArcTanh}\left[\frac{d^{1/6} (a+b x)^{1/6}}{b^{1/6} (c+d x)^{1/6}}\right]}{d^{7/6}} - \frac{b^{1/6} \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3} (a+b x)^{1/3}}{(c+d x)^{1/3}} - \frac{b^{1/6} d^{1/6} (a+b x)^{1/6}}{(c+d x)^{1/6}}\right]}{2 d^{7/6}} + \frac{b^{1/6} \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3} (a+b x)^{1/3}}{(c+d x)^{1/3}} + \frac{b^{1/6} d^{1/6} (a+b x)^{1/6}}{(c+d x)^{1/6}}\right]}{2 d^{7/6}}
\end{aligned}$$

Result (type 5, 89 leaves) :

$$\begin{aligned}
& \frac{6 \left(-5 d (a+b x) + b \left(\frac{d (a+b x)}{-b c+a d}\right)^{5/6} (c+d x) \operatorname{Hypergeometric2F1}\left[\frac{5}{6}, \frac{5}{6}, \frac{11}{6}, \frac{b (c+d x)}{b c-a d}\right]\right)}{5 d^2 (a+b x)^{5/6} (c+d x)^{1/6}}
\end{aligned}$$

Problem 1779: Result unnecessarily involves higher level functions.

$$\int (a+b x)^{5/6} (c+d x)^{1/6} dx$$

Optimal (type 3, 427 leaves, 14 steps) :

$$\begin{aligned}
& \frac{(b c - a d) (a+b x)^{5/6} (c+d x)^{1/6}}{12 b d} + \frac{(a+b x)^{11/6} (c+d x)^{1/6}}{2 b} - \frac{5 (b c - a d)^2 \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2 d^{1/6} (a+b x)^{1/6}}{\sqrt{3} b^{1/6} (c+d x)^{1/6}}\right]}{24 \sqrt{3} b^{7/6} d^{11/6}} + \\
& \frac{5 (b c - a d)^2 \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 d^{1/6} (a+b x)^{1/6}}{\sqrt{3} b^{1/6} (c+d x)^{1/6}}\right]}{24 \sqrt{3} b^{7/6} d^{11/6}} - \frac{5 (b c - a d)^2 \operatorname{ArcTanh}\left[\frac{d^{1/6} (a+b x)^{1/6}}{b^{1/6} (c+d x)^{1/6}}\right]}{36 b^{7/6} d^{11/6}} + \\
& \frac{5 (b c - a d)^2 \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3} (a+b x)^{1/3}}{(c+d x)^{1/3}} - \frac{b^{1/6} d^{1/6} (a+b x)^{1/6}}{(c+d x)^{1/6}}\right]}{144 b^{7/6} d^{11/6}} - \frac{5 (b c - a d)^2 \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3} (a+b x)^{1/3}}{(c+d x)^{1/3}} + \frac{b^{1/6} d^{1/6} (a+b x)^{1/6}}{(c+d x)^{1/6}}\right]}{144 b^{7/6} d^{11/6}}
\end{aligned}$$

Result (type 5, 109 leaves) :

$$\frac{1}{12 b d^2 (a+b x)^{1/6}} (c+d x)^{1/6} \left( d (a+b x) (5 a d + b (c+6 d x)) - 5 (b c - a d)^2 \left(\frac{d (a+b x)}{-b c+a d}\right)^{1/6} \operatorname{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{6}, \frac{7}{6}, \frac{b (c+d x)}{b c-a d}\right] \right)$$

Problem 1780: Result unnecessarily involves higher level functions.

$$\int \frac{(a+b x)^{5/6}}{(c+d x)^{5/6}} dx$$

Optimal (type 3, 378 leaves, 13 steps) :

$$\begin{aligned} & \frac{(a+b x)^{5/6} (c+d x)^{1/6}}{d} - \frac{5 (b c - a d) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2 d^{1/6} (a+b x)^{1/6}}{\sqrt{3} b^{1/6} (c+d x)^{1/6}}\right]}{2 \sqrt{3} b^{1/6} d^{11/6}} + \frac{5 (b c - a d) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 d^{1/6} (a+b x)^{1/6}}{\sqrt{3} b^{1/6} (c+d x)^{1/6}}\right]}{2 \sqrt{3} b^{1/6} d^{11/6}} - \\ & \frac{5 (b c - a d) \operatorname{Arctanh}\left[\frac{d^{1/6} (a+b x)^{1/6}}{b^{1/6} (c+d x)^{1/6}}\right]}{3 b^{1/6} d^{11/6}} + \frac{5 (b c - a d) \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3} (a+b x)^{1/3}}{(c+d x)^{1/3}} - \frac{b^{1/6} d^{1/6} (a+b x)^{1/6}}{(c+d x)^{1/6}}\right]}{12 b^{1/6} d^{11/6}} - \frac{5 (b c - a d) \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3} (a+b x)^{1/3}}{(c+d x)^{1/3}} + \frac{b^{1/6} d^{1/6} (a+b x)^{1/6}}{(c+d x)^{1/6}}\right]}{12 b^{1/6} d^{11/6}} \end{aligned}$$

Result (type 5, 74 leaves):

$$\frac{(a+b x)^{5/6} (c+d x)^{1/6} \left(1 + \frac{5 \operatorname{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{6}, \frac{7}{6}, \frac{b (c+d x)}{b c-a d}\right]}{\left(\frac{d (a+b x)}{-b c+a d}\right)^{5/6}}\right)}{d}$$

Problem 1781: Result unnecessarily involves higher level functions.

$$\int \frac{(a+b x)^{5/6}}{(c+d x)^{11/6}} dx$$

Optimal (type 3, 334 leaves, 13 steps):

$$\begin{aligned} & -\frac{6 (a+b x)^{5/6}}{5 d (c+d x)^{5/6}} + \frac{\sqrt{3} b^{5/6} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2 d^{1/6} (a+b x)^{1/6}}{\sqrt{3} b^{1/6} (c+d x)^{1/6}}\right]}{d^{11/6}} - \frac{\sqrt{3} b^{5/6} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 d^{1/6} (a+b x)^{1/6}}{\sqrt{3} b^{1/6} (c+d x)^{1/6}}\right]}{d^{11/6}} + \\ & \frac{2 b^{5/6} \operatorname{Arctanh}\left[\frac{d^{1/6} (a+b x)^{1/6}}{b^{1/6} (c+d x)^{1/6}}\right]}{d^{11/6}} - \frac{b^{5/6} \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3} (a+b x)^{1/3}}{(c+d x)^{1/3}} - \frac{b^{1/6} d^{1/6} (a+b x)^{1/6}}{(c+d x)^{1/6}}\right]}{2 d^{11/6}} + \frac{b^{5/6} \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3} (a+b x)^{1/3}}{(c+d x)^{1/3}} + \frac{b^{1/6} d^{1/6} (a+b x)^{1/6}}{(c+d x)^{1/6}}\right]}{2 d^{11/6}} \end{aligned}$$

Result (type 5, 90 leaves):

$$\frac{-6 d (a+b x) + 30 b \left(\frac{d (a+b x)}{-b c+a d}\right)^{1/6} (c+d x) \operatorname{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{6}, \frac{7}{6}, \frac{b (c+d x)}{b c-a d}\right]}{5 d^2 (a+b x)^{1/6} (c+d x)^{5/6}}$$

Problem 1792: Result more than twice size of optimal antiderivative.

$$\int (a+b x)^{7/6} (c+d x)^{13/6} dx$$

Optimal (type 5, 84 leaves, 2 steps):

$$\frac{6 (b c - a d)^2 (a+b x)^{13/6} (c+d x)^{1/6} \operatorname{Hypergeometric2F1}\left[-\frac{13}{6}, \frac{13}{6}, \frac{19}{6}, -\frac{d (a+b x)}{b c-a d}\right]}{13 b^3 \left(\frac{b (c+d x)}{b c-a d}\right)^{1/6}}$$

Result (type 5, 234 leaves):

$$-\frac{1}{8320 b^3 d^3 (a + b x)^{5/6}} 3 (c + d x)^{1/6} \\ \left( -d (a + b x) (91 a^4 d^4 - 26 a^3 b d^3 (15 c + d x) + 2 a^2 b^2 d^2 (320 c^2 + 55 c d x + 8 d^2 x^2) + 2 a b^3 d (195 c^3 + 1225 c^2 d x + 1280 c d^2 x^2 + 432 d^3 x^3) + b^4 (-91 c^4 + 26 c^3 d x + 1264 c^2 d^2 x^2 + 1696 c d^3 x^3 + 640 d^4 x^4)) - 91 (b c - a d)^5 \left( \frac{d (a + b x)}{-b c + a d} \right)^{5/6} \text{Hypergeometric2F1} \left[ \frac{1}{6}, \frac{5}{6}, \frac{7}{6}, \frac{b (c + d x)}{b c - a d} \right] \right)$$

Problem 1793: Result more than twice size of optimal antiderivative.

$$\int (a + b x)^{7/6} (c + d x)^{7/6} dx$$

Optimal (type 5, 82 leaves, 2 steps):

$$\frac{6 (b c - a d) (a + b x)^{13/6} (c + d x)^{1/6} \text{Hypergeometric2F1} \left[ -\frac{7}{6}, \frac{13}{6}, \frac{19}{6}, -\frac{d (a + b x)}{b c - a d} \right]}{13 b^2 \left( \frac{b (c + d x)}{b c - a d} \right)^{1/6}}$$

Result (type 5, 183 leaves):

$$\frac{1}{320 b^2 d^3 (a + b x)^{5/6}} \\ 3 (c + d x)^{1/6} \left( -d (a + b x) (7 a^3 d^3 - a^2 b d^2 (23 c + 2 d x) - a b^2 d (23 c^2 + 92 c d x + 48 d^2 x^2) + b^3 (7 c^3 - 2 c^2 d x - 48 c d^2 x^2 - 32 d^3 x^3)) + 7 (b c - a d)^4 \left( \frac{d (a + b x)}{-b c + a d} \right)^{5/6} \text{Hypergeometric2F1} \left[ \frac{1}{6}, \frac{5}{6}, \frac{7}{6}, \frac{b (c + d x)}{b c - a d} \right] \right)$$

Problem 1798: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{7/6}}{(c + d x)^{1/6}} dx$$

Optimal (type 3, 424 leaves, 14 steps):

$$\begin{aligned}
& - \frac{7(b c - a d) (a + b x)^{1/6} (c + d x)^{5/6}}{12 d^2} + \frac{(a + b x)^{7/6} (c + d x)^{5/6}}{2 d} - \frac{7(b c - a d)^2 \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2 d^{1/6} (a+b x)^{1/6}}{\sqrt{3} b^{1/6} (c+d x)^{1/6}}\right]}{24 \sqrt{3} b^{5/6} d^{13/6}} + \\
& \frac{7(b c - a d)^2 \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 d^{1/6} (a+b x)^{1/6}}{\sqrt{3} b^{1/6} (c+d x)^{1/6}}\right]}{24 \sqrt{3} b^{5/6} d^{13/6}} + \frac{7(b c - a d)^2 \operatorname{ArcTanh}\left[\frac{d^{1/6} (a+b x)^{1/6}}{b^{1/6} (c+d x)^{1/6}}\right]}{36 b^{5/6} d^{13/6}} - \\
& \frac{7(b c - a d)^2 \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3} (a+b x)^{1/3}}{(c+d x)^{1/3}} - \frac{b^{1/6} d^{1/6} (a+b x)^{1/6}}{(c+d x)^{1/6}}\right]}{144 b^{5/6} d^{13/6}} + \frac{7(b c - a d)^2 \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3} (a+b x)^{1/3}}{(c+d x)^{1/3}} + \frac{b^{1/6} d^{1/6} (a+b x)^{1/6}}{(c+d x)^{1/6}}\right]}{144 b^{5/6} d^{13/6}}
\end{aligned}$$

Result (type 5, 108 leaves):

$$\frac{1}{60 d^3 (a + b x)^{5/6}} (c + d x)^{5/6} \left( 5 d (a + b x) (-7 b c + 13 a d + 6 b d x) + 7 (b c - a d)^2 \left( \frac{d (a + b x)}{-b c + a d} \right)^{5/6} \operatorname{Hypergeometric2F1}\left[\frac{5}{6}, \frac{5}{6}, \frac{11}{6}, \frac{b (c + d x)}{b c - a d}\right] \right)$$

Problem 1799: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{7/6}}{(c + d x)^{7/6}} dx$$

Optimal (type 3, 403 leaves, 14 steps):

$$\begin{aligned}
& - \frac{6 (a + b x)^{7/6}}{d (c + d x)^{1/6}} + \frac{7 b (a + b x)^{1/6} (c + d x)^{5/6}}{d^2} + \frac{7 b^{1/6} (b c - a d) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2 d^{1/6} (a+b x)^{1/6}}{\sqrt{3} b^{1/6} (c+d x)^{1/6}}\right]}{2 \sqrt{3} d^{13/6}} - \\
& \frac{7 b^{1/6} (b c - a d) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 d^{1/6} (a+b x)^{1/6}}{\sqrt{3} b^{1/6} (c+d x)^{1/6}}\right]}{2 \sqrt{3} d^{13/6}} - \frac{7 b^{1/6} (b c - a d) \operatorname{ArcTanh}\left[\frac{d^{1/6} (a+b x)^{1/6}}{b^{1/6} (c+d x)^{1/6}}\right]}{3 d^{13/6}} + \\
& \frac{7 b^{1/6} (b c - a d) \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3} (a+b x)^{1/3}}{(c+d x)^{1/3}} - \frac{b^{1/6} d^{1/6} (a+b x)^{1/6}}{(c+d x)^{1/6}}\right]}{12 d^{13/6}} - \frac{7 b^{1/6} (b c - a d) \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3} (a+b x)^{1/3}}{(c+d x)^{1/3}} + \frac{b^{1/6} d^{1/6} (a+b x)^{1/6}}{(c+d x)^{1/6}}\right]}{12 d^{13/6}}
\end{aligned}$$

Result (type 5, 99 leaves):

$$\frac{(a + b x)^{1/6} (c + d x)^{5/6}}{5 d^2} \left( \frac{5 (7 b c - 6 a d + b d x)}{c + d x} + \frac{7 b \operatorname{Hypergeometric2F1}\left[\frac{5}{6}, \frac{5}{6}, \frac{11}{6}, \frac{b (c + d x)}{b c - a d}\right]}{\left(\frac{d (a + b x)}{-b c + a d}\right)^{1/6}} \right)$$

### Problem 1800: Result unnecessarily involves higher level functions.

$$\int \frac{(a+b x)^{7/6}}{(c+d x)^{13/6}} dx$$

Optimal (type 3, 358 leaves, 14 steps):

$$\begin{aligned} & -\frac{6 (a+b x)^{7/6}}{7 d (c+d x)^{7/6}} - \frac{6 b (a+b x)^{1/6}}{d^2 (c+d x)^{1/6}} - \frac{\sqrt{3} b^{7/6} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2 d^{1/6} (a+b x)^{1/6}}{\sqrt{3} b^{1/6} (c+d x)^{1/6}}\right]}{d^{13/6}} + \frac{\sqrt{3} b^{7/6} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 d^{1/6} (a+b x)^{1/6}}{\sqrt{3} b^{1/6} (c+d x)^{1/6}}\right]}{d^{13/6}} + \\ & \frac{2 b^{7/6} \operatorname{ArcTanh}\left[\frac{d^{1/6} (a+b x)^{1/6}}{b^{1/6} (c+d x)^{1/6}}\right]}{d^{13/6}} - \frac{b^{7/6} \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3} (a+b x)^{1/3}}{(c+d x)^{1/3}} - \frac{b^{1/6} d^{1/6} (a+b x)^{1/6}}{(c+d x)^{1/6}}\right]}{2 d^{13/6}} + \frac{b^{7/6} \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3} (a+b x)^{1/3}}{(c+d x)^{1/3}} + \frac{b^{1/6} d^{1/6} (a+b x)^{1/6}}{(c+d x)^{1/6}}\right]}{2 d^{13/6}} \end{aligned}$$

Result (type 5, 107 leaves):

$$\begin{aligned} & \frac{-30 d (a+b x) (7 b c + a d + 8 b d x) + 42 b^2 \left(\frac{d (a+b x)}{-b c + a d}\right)^{5/6} (c+d x)^2 \operatorname{Hypergeometric2F1}\left[\frac{5}{6}, \frac{5}{6}, \frac{11}{6}, \frac{b (c+d x)}{b c - a d}\right]}{35 d^3 (a+b x)^{5/6} (c+d x)^{7/6}} \end{aligned}$$

### Problem 1805: Result unnecessarily involves higher level functions.

$$\int \frac{(c+d x)^{7/6}}{(a+b x)^{1/6}} dx$$

Optimal (type 3, 424 leaves, 14 steps):

$$\begin{aligned} & \frac{7 (b c - a d) (a+b x)^{5/6} (c+d x)^{1/6}}{12 b^2} + \frac{(a+b x)^{5/6} (c+d x)^{7/6}}{2 b} + \frac{7 (b c - a d)^2 \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2 d^{1/6} (a+b x)^{1/6}}{\sqrt{3} b^{1/6} (c+d x)^{1/6}}\right]}{24 \sqrt{3} b^{13/6} d^{5/6}} - \\ & \frac{7 (b c - a d)^2 \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 d^{1/6} (a+b x)^{1/6}}{\sqrt{3} b^{1/6} (c+d x)^{1/6}}\right]}{24 \sqrt{3} b^{13/6} d^{5/6}} + \frac{7 (b c - a d)^2 \operatorname{ArcTanh}\left[\frac{d^{1/6} (a+b x)^{1/6}}{b^{1/6} (c+d x)^{1/6}}\right]}{36 b^{13/6} d^{5/6}} - \\ & \frac{7 (b c - a d)^2 \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3} (a+b x)^{1/3}}{(c+d x)^{1/3}} - \frac{b^{1/6} d^{1/6} (a+b x)^{1/6}}{(c+d x)^{1/6}}\right]}{144 b^{13/6} d^{5/6}} + \frac{7 (b c - a d)^2 \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3} (a+b x)^{1/3}}{(c+d x)^{1/3}} + \frac{b^{1/6} d^{1/6} (a+b x)^{1/6}}{(c+d x)^{1/6}}\right]}{144 b^{13/6} d^{5/6}} \end{aligned}$$

Result (type 5, 111 leaves):

$$\frac{1}{12 b^2 d (a+b x)^{1/6}} (c+d x)^{1/6} \left( -d (a+b x) (-13 b c + 7 a d - 6 b d x) + 7 (b c - a d)^2 \left(\frac{d (a+b x)}{-b c + a d}\right)^{1/6} \operatorname{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{6}, \frac{7}{6}, \frac{b (c+d x)}{b c - a d}\right] \right)$$

### Problem 1806: Result unnecessarily involves higher level functions.

$$\int \frac{(c + d x)^{1/6}}{(a + b x)^{1/6}} dx$$

Optimal (type 3, 378 leaves, 13 steps):

$$\begin{aligned} & \frac{(a + b x)^{5/6} (c + d x)^{1/6}}{b} + \frac{(b c - a d) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2 d^{1/6} (a+b x)^{1/6}}{\sqrt{3} b^{1/6} (c+d x)^{1/6}}\right]}{2 \sqrt{3} b^{7/6} d^{5/6}} - \frac{(b c - a d) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 d^{1/6} (a+b x)^{1/6}}{\sqrt{3} b^{1/6} (c+d x)^{1/6}}\right]}{2 \sqrt{3} b^{7/6} d^{5/6}} + \\ & \frac{(b c - a d) \operatorname{ArcTanh}\left[\frac{d^{1/6} (a+b x)^{1/6}}{b^{1/6} (c+d x)^{1/6}}\right]}{3 b^{7/6} d^{5/6}} - \frac{(b c - a d) \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3} (a+b x)^{1/3}}{(c+d x)^{1/3}} - \frac{b^{1/6} d^{1/6} (a+b x)^{1/6}}{(c+d x)^{1/6}}\right]}{12 b^{7/6} d^{5/6}} + \frac{(b c - a d) \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3} (a+b x)^{1/3}}{(c+d x)^{1/3}} + \frac{b^{1/6} d^{1/6} (a+b x)^{1/6}}{(c+d x)^{1/6}}\right]}{12 b^{7/6} d^{5/6}} \end{aligned}$$

Result (type 5, 90 leaves):

$$\frac{(c + d x)^{1/6} \left(d (a + b x) + (b c - a d) \left(\frac{d (a+b x)}{-b c+a d}\right)^{1/6} \operatorname{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{6}, \frac{7}{6}, \frac{b (c+d x)}{b c-a d}\right]\right)}{b d (a + b x)^{1/6}}$$

### Problem 1807: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + b x)^{1/6} (c + d x)^{5/6}} dx$$

Optimal (type 3, 309 leaves, 12 steps):

$$\begin{aligned} & \frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2 d^{1/6} (a+b x)^{1/6}}{\sqrt{3} b^{1/6} (c+d x)^{1/6}}\right]}{b^{1/6} d^{5/6}} - \frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 d^{1/6} (a+b x)^{1/6}}{\sqrt{3} b^{1/6} (c+d x)^{1/6}}\right]}{b^{1/6} d^{5/6}} + \\ & \frac{2 \operatorname{ArcTanh}\left[\frac{d^{1/6} (a+b x)^{1/6}}{b^{1/6} (c+d x)^{1/6}}\right]}{b^{1/6} d^{5/6}} - \frac{\operatorname{Log}\left[b^{1/3} + \frac{d^{1/3} (a+b x)^{1/3}}{(c+d x)^{1/3}} - \frac{b^{1/6} d^{1/6} (a+b x)^{1/6}}{(c+d x)^{1/6}}\right]}{2 b^{1/6} d^{5/6}} + \frac{\operatorname{Log}\left[b^{1/3} + \frac{d^{1/3} (a+b x)^{1/3}}{(c+d x)^{1/3}} + \frac{b^{1/6} d^{1/6} (a+b x)^{1/6}}{(c+d x)^{1/6}}\right]}{2 b^{1/6} d^{5/6}} \end{aligned}$$

Result (type 5, 71 leaves):

$$\frac{6 \left(\frac{d (a+b x)}{-b c+a d}\right)^{1/6} (c + d x)^{1/6} \operatorname{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{6}, \frac{7}{6}, \frac{b (c+d x)}{b c-a d}\right]}{d (a + b x)^{1/6}}$$

### Problem 1824: Result unnecessarily involves higher level functions.

$$\int \frac{(c + d x)^{11/6}}{(a + b x)^{5/6}} dx$$

Optimal (type 3, 424 leaves, 14 steps):

$$\begin{aligned} & \frac{11 (b c - a d) (a + b x)^{1/6} (c + d x)^{5/6}}{12 b^2} + \frac{(a + b x)^{1/6} (c + d x)^{11/6}}{2 b} - \frac{55 (b c - a d)^2 \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2 d^{1/6} (a + b x)^{1/6}}{\sqrt{3} b^{1/6} (c + d x)^{1/6}}\right]}{24 \sqrt{3} b^{17/6} d^{1/6}} + \\ & \frac{55 (b c - a d)^2 \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 d^{1/6} (a + b x)^{1/6}}{\sqrt{3} b^{1/6} (c + d x)^{1/6}}\right]}{24 \sqrt{3} b^{17/6} d^{1/6}} + \frac{55 (b c - a d)^2 \operatorname{ArcTanh}\left[\frac{d^{1/6} (a + b x)^{1/6}}{b^{1/6} (c + d x)^{1/6}}\right]}{36 b^{17/6} d^{1/6}} - \\ & \frac{55 (b c - a d)^2 \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3} (a + b x)^{1/3}}{(c + d x)^{1/3}} - \frac{b^{1/6} d^{1/6} (a + b x)^{1/6}}{(c + d x)^{1/6}}\right]}{144 b^{17/6} d^{1/6}} + \frac{55 (b c - a d)^2 \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3} (a + b x)^{1/3}}{(c + d x)^{1/3}} + \frac{b^{1/6} d^{1/6} (a + b x)^{1/6}}{(c + d x)^{1/6}}\right]}{144 b^{17/6} d^{1/6}} \end{aligned}$$

Result (type 5, 111 leaves):

$$\frac{1}{12 b^2 d (a + b x)^{5/6}} (c + d x)^{5/6} \left( -d (a + b x) (-17 b c + 11 a d - 6 b d x) + 11 (b c - a d)^2 \left( \frac{d (a + b x)}{-b c + a d} \right)^{5/6} \operatorname{Hypergeometric2F1}\left[\frac{5}{6}, \frac{5}{6}, \frac{11}{6}, \frac{b (c + d x)}{b c - a d}\right] \right)$$

### Problem 1825: Result unnecessarily involves higher level functions.

$$\int \frac{(c + d x)^{5/6}}{(a + b x)^{5/6}} dx$$

Optimal (type 3, 378 leaves, 13 steps):

$$\begin{aligned} & \frac{(a + b x)^{1/6} (c + d x)^{5/6}}{b} - \frac{5 (b c - a d) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2 d^{1/6} (a + b x)^{1/6}}{\sqrt{3} b^{1/6} (c + d x)^{1/6}}\right]}{2 \sqrt{3} b^{11/6} d^{1/6}} + \frac{5 (b c - a d) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 d^{1/6} (a + b x)^{1/6}}{\sqrt{3} b^{1/6} (c + d x)^{1/6}}\right]}{2 \sqrt{3} b^{11/6} d^{1/6}} + \\ & \frac{5 (b c - a d) \operatorname{ArcTanh}\left[\frac{d^{1/6} (a + b x)^{1/6}}{b^{1/6} (c + d x)^{1/6}}\right]}{3 b^{11/6} d^{1/6}} - \frac{5 (b c - a d) \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3} (a + b x)^{1/3}}{(c + d x)^{1/3}} - \frac{b^{1/6} d^{1/6} (a + b x)^{1/6}}{(c + d x)^{1/6}}\right]}{12 b^{11/6} d^{1/6}} + \frac{5 (b c - a d) \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3} (a + b x)^{1/3}}{(c + d x)^{1/3}} + \frac{b^{1/6} d^{1/6} (a + b x)^{1/6}}{(c + d x)^{1/6}}\right]}{12 b^{11/6} d^{1/6}} \end{aligned}$$

Result (type 5, 90 leaves):

$$\begin{aligned} & (c + d x)^{5/6} \left( d (a + b x) + (b c - a d) \left( \frac{d (a + b x)}{-b c + a d} \right)^{5/6} \operatorname{Hypergeometric2F1}\left[\frac{5}{6}, \frac{5}{6}, \frac{11}{6}, \frac{b (c + d x)}{b c - a d}\right] \right) \\ & b d (a + b x)^{5/6} \end{aligned}$$

### Problem 1826: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + b x)^{5/6} (c + d x)^{1/6}} dx$$

Optimal (type 3, 309 leaves, 12 steps):

$$\begin{aligned} & -\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2 d^{1/6} (a+b x)^{1/6}}{\sqrt{3} b^{1/6} (c+d x)^{1/6}}\right]}{b^{5/6} d^{1/6}} + \frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 d^{1/6} (a+b x)^{1/6}}{\sqrt{3} b^{1/6} (c+d x)^{1/6}}\right]}{b^{5/6} d^{1/6}} + \\ & \frac{2 \operatorname{ArcTanh}\left[\frac{d^{1/6} (a+b x)^{1/6}}{b^{1/6} (c+d x)^{1/6}}\right]}{b^{5/6} d^{1/6}} - \frac{\operatorname{Log}\left[b^{1/3} + \frac{d^{1/3} (a+b x)^{1/3}}{(c+d x)^{1/3}} - \frac{b^{1/6} d^{1/6} (a+b x)^{1/6}}{(c+d x)^{1/6}}\right]}{2 b^{5/6} d^{1/6}} + \frac{\operatorname{Log}\left[b^{1/3} + \frac{d^{1/3} (a+b x)^{1/3}}{(c+d x)^{1/3}} + \frac{b^{1/6} d^{1/6} (a+b x)^{1/6}}{(c+d x)^{1/6}}\right]}{2 b^{5/6} d^{1/6}} \end{aligned}$$

Result (type 5, 73 leaves):

$$\frac{6 \left(\frac{d (a+b x)}{-b c+a d}\right)^{5/6} (c+d x)^{5/6} \operatorname{Hypergeometric2F1}\left[\frac{5}{6}, \frac{5}{6}, \frac{11}{6}, \frac{b (c+d x)}{b c-a d}\right]}{5 d (a+b x)^{5/6}}$$

### Problem 1831: Result unnecessarily involves higher level functions.

$$\int \frac{(c + d x)^{13/6}}{(a + b x)^{7/6}} dx$$

Optimal (type 3, 449 leaves, 15 steps):

$$\begin{aligned} & \frac{91 d (b c - a d) (a+b x)^{5/6} (c+d x)^{1/6}}{12 b^3} + \frac{13 d (a+b x)^{5/6} (c+d x)^{7/6}}{2 b^2} - \frac{6 (c+d x)^{13/6}}{b (a+b x)^{1/6}} + \\ & \frac{91 d^{1/6} (b c - a d)^2 \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2 d^{1/6} (a+b x)^{1/6}}{\sqrt{3} b^{1/6} (c+d x)^{1/6}}\right]}{24 \sqrt{3} b^{19/6}} - \frac{91 d^{1/6} (b c - a d)^2 \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 d^{1/6} (a+b x)^{1/6}}{\sqrt{3} b^{1/6} (c+d x)^{1/6}}\right]}{24 \sqrt{3} b^{19/6}} + \frac{91 d^{1/6} (b c - a d)^2 \operatorname{ArcTanh}\left[\frac{d^{1/6} (a+b x)^{1/6}}{b^{1/6} (c+d x)^{1/6}}\right]}{36 b^{19/6}} - \\ & \frac{91 d^{1/6} (b c - a d)^2 \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3} (a+b x)^{1/3}}{(c+d x)^{1/3}} - \frac{b^{1/6} d^{1/6} (a+b x)^{1/6}}{(c+d x)^{1/6}}\right]}{144 b^{19/6}} + \frac{91 d^{1/6} (b c - a d)^2 \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3} (a+b x)^{1/3}}{(c+d x)^{1/3}} + \frac{b^{1/6} d^{1/6} (a+b x)^{1/6}}{(c+d x)^{1/6}}\right]}{144 b^{19/6}} \end{aligned}$$

Result (type 5, 129 leaves):

$$\begin{aligned} & \frac{1}{12 b^3 (a+b x)^{1/6}} (c+d x)^{1/6} \\ & \left( -91 a^2 d^2 - 13 a b d (-13 c + d x) + b^2 (-72 c^2 + 25 c d x + 6 d^2 x^2) + 91 (b c - a d)^2 \left(\frac{d (a+b x)}{-b c+a d}\right)^{1/6} \operatorname{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{6}, \frac{7}{6}, \frac{b (c+d x)}{b c-a d}\right] \right) \end{aligned}$$

### Problem 1832: Result unnecessarily involves higher level functions.

$$\int \frac{(c+dx)^{7/6}}{(a+bx)^{7/6}} dx$$

Optimal (type 3, 403 leaves, 14 steps):

$$\begin{aligned} & \frac{7d(a+bx)^{5/6}(c+dx)^{1/6}}{b^2} - \frac{6(c+dx)^{7/6}}{b(a+bx)^{1/6}} + \frac{7d^{1/6}(bc-ad)\operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2d^{1/6}(a+bx)^{1/6}}{\sqrt{3}b^{1/6}(c+dx)^{1/6}}\right]}{2\sqrt{3}b^{13/6}} - \\ & \frac{7d^{1/6}(bc-ad)\operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2d^{1/6}(a+bx)^{1/6}}{\sqrt{3}b^{1/6}(c+dx)^{1/6}}\right]}{2\sqrt{3}b^{13/6}} + \frac{7d^{1/6}(bc-ad)\operatorname{ArcTanh}\left[\frac{d^{1/6}(a+bx)^{1/6}}{b^{1/6}(c+dx)^{1/6}}\right]}{3b^{13/6}} - \\ & \frac{7d^{1/6}(bc-ad)\operatorname{Log}\left[b^{1/3} + \frac{d^{1/3}(a+bx)^{1/3}}{(c+dx)^{1/3}} - \frac{b^{1/6}d^{1/6}(a+bx)^{1/6}}{(c+dx)^{1/6}}\right]}{12b^{13/6}} + \frac{7d^{1/6}(bc-ad)\operatorname{Log}\left[b^{1/3} + \frac{d^{1/3}(a+bx)^{1/3}}{(c+dx)^{1/3}} + \frac{b^{1/6}d^{1/6}(a+bx)^{1/6}}{(c+dx)^{1/6}}\right]}{12b^{13/6}} \end{aligned}$$

Result (type 5, 93 leaves):

$$\frac{(c+dx)^{1/6} \left( -6bc + 7ad + bdx + 7(bc-ad) \left( \frac{d(a+bx)}{-bc+ad} \right)^{1/6} \operatorname{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{6}, \frac{7}{6}, \frac{b(c+dx)}{bc-ad} \right] \right)}{b^2(a+bx)^{1/6}}$$

### Problem 1833: Result unnecessarily involves higher level functions.

$$\int \frac{(c+dx)^{1/6}}{(a+bx)^{7/6}} dx$$

Optimal (type 3, 332 leaves, 13 steps):

$$\begin{aligned} & -\frac{6(c+dx)^{1/6}}{b(a+bx)^{1/6}} + \frac{\sqrt{3}d^{1/6}\operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2d^{1/6}(a+bx)^{1/6}}{\sqrt{3}b^{1/6}(c+dx)^{1/6}}\right]}{b^{7/6}} - \frac{\sqrt{3}d^{1/6}\operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2d^{1/6}(a+bx)^{1/6}}{\sqrt{3}b^{1/6}(c+dx)^{1/6}}\right]}{b^{7/6}} + \\ & \frac{2d^{1/6}\operatorname{ArcTanh}\left[\frac{d^{1/6}(a+bx)^{1/6}}{b^{1/6}(c+dx)^{1/6}}\right]}{b^{7/6}} - \frac{d^{1/6}\operatorname{Log}\left[b^{1/3} + \frac{d^{1/3}(a+bx)^{1/3}}{(c+dx)^{1/3}} - \frac{b^{1/6}d^{1/6}(a+bx)^{1/6}}{(c+dx)^{1/6}}\right]}{2b^{7/6}} + \frac{d^{1/6}\operatorname{Log}\left[b^{1/3} + \frac{d^{1/3}(a+bx)^{1/3}}{(c+dx)^{1/3}} + \frac{b^{1/6}d^{1/6}(a+bx)^{1/6}}{(c+dx)^{1/6}}\right]}{2b^{7/6}} \end{aligned}$$

Result (type 5, 74 leaves):

$$\frac{6(c+dx)^{1/6} \left( -1 + \left( \frac{d(a+bx)}{-bc+ad} \right)^{1/6} \operatorname{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{6}, \frac{7}{6}, \frac{b(c+dx)}{bc-ad} \right] \right)}{b(a+bx)^{1/6}}$$

**Problem 1843:** Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + b x)^{7/6} (c + d x)^{19/6}} dx$$

Optimal (type 5, 82 leaves, 2 steps):

$$\frac{\frac{6 b^2 \left(\frac{b (c+d x)}{b c-a d}\right)^{1/6} \text{Hypergeometric2F1}\left[-\frac{1}{6}, \frac{19}{6}, \frac{5}{6}, -\frac{d (a+b x)}{b c-a d}\right]}{(b c-a d)^3 (a+b x)^{1/6} (c+d x)^{1/6}}}{}$$

Result (type 5, 179 leaves):

$$\left( -30 (a^3 d^3 - a^2 b d^2 (5 c + 2 d x) + a b^2 d (23 c^2 + 36 c d x + 16 d^2 x^2) + b^3 (13 c^3 + 62 c^2 d x + 80 c d^2 x^2 + 32 d^3 x^3)) + \frac{768 b^3 \left(\frac{d (a+b x)}{-b c+a d}\right)^{1/6} (c+d x)^3 \text{Hypergeometric2F1}\left[\frac{1}{6}, \frac{5}{6}, \frac{11}{6}, \frac{b (c+d x)}{b c-a d}\right]}{(b c-a d)^4 (a+b x)^{1/6} (c+d x)^{13/6}} \right)$$

**Problem 1850:** Unable to integrate problem.

$$\int \frac{(a+b x)^m}{(c+d x)^2} dx$$

Optimal (type 5, 52 leaves, 1 step):

$$\frac{b (a+b x)^{1+m} \text{Hypergeometric2F1}\left[2, 1+m, 2+m, -\frac{d (a+b x)}{b c-a d}\right]}{(b c-a d)^2 (1+m)}$$

Result (type 8, 17 leaves):

$$\int \frac{(a+b x)^m}{(c+d x)^2} dx$$

**Problem 1851:** Unable to integrate problem.

$$\int \frac{(a+b x)^m}{(c+d x)^3} dx$$

Optimal (type 5, 54 leaves, 1 step):

$$\frac{b^2 (a + b x)^{1+m} \text{Hypergeometric2F1}[3, 1+m, 2+m, -\frac{d (a+b x)}{b c-a d}]}{(b c - a d)^3 (1+m)}$$

Result (type 8, 17 leaves) :

$$\int \frac{(a + b x)^m}{(c + d x)^3} dx$$

Problem 1857: Unable to integrate problem.

$$\int \frac{(c + d x)^n}{(a + b x)^2} dx$$

Optimal (type 5, 51 leaves, 1 step) :

$$\frac{d (c + d x)^{1+n} \text{Hypergeometric2F1}[2, 1+n, 2+n, \frac{b (c+d x)}{b c-a d}]}{(b c - a d)^2 (1+n)}$$

Result (type 8, 17 leaves) :

$$\int \frac{(c + d x)^n}{(a + b x)^2} dx$$

Problem 1858: Unable to integrate problem.

$$\int \frac{(c + d x)^n}{(a + b x)^3} dx$$

Optimal (type 5, 54 leaves, 1 step) :

$$\frac{d^2 (c + d x)^{1+n} \text{Hypergeometric2F1}[3, 1+n, 2+n, \frac{b (c+d x)}{b c-a d}]}{(b c - a d)^3 (1+n)}$$

Result (type 8, 17 leaves) :

$$\int \frac{(c + d x)^n}{(a + b x)^3} dx$$

Problem 1864: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (a + b x)^{1+n} (c + d x)^{-n} dx$$

Optimal (type 5, 72 leaves, 2 steps):

$$\frac{(a + b x)^{2+n} (c + d x)^{-n} \left( \frac{b(c+d x)}{b c - a d} \right)^n \text{Hypergeometric2F1}[n, 2+n, 3+n, -\frac{d(a+b x)}{b c - a d}]}{b (2+n)}$$

Result (type 6, 200 leaves):

$$a (a + b x)^n (c + d x)^{-n} \left( \left( 3 b c x^2 \text{AppellF1}[2, -n, n, 3, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \left( 6 a c \text{AppellF1}[2, -n, n, 3, -\frac{b x}{a}, -\frac{d x}{c}] + 2 n x \left( b c \text{AppellF1}[3, 1-n, n, 4, -\frac{b x}{a}, -\frac{d x}{c}] - a d \text{AppellF1}[3, -n, 1+n, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) \right) - \frac{\left( \frac{d(a+b x)}{-b c + a d} \right)^{-n} (c + d x) \text{Hypergeometric2F1}[1-n, -n, 2-n, \frac{b(c+d x)}{b c - a d}]}{d (-1+n)} \right)$$

Problem 1865: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (a + b x)^{2+n} (c + d x)^{-n} dx$$

Optimal (type 5, 72 leaves, 2 steps):

$$\frac{(a + b x)^{3+n} (c + d x)^{-n} \left( \frac{b(c+d x)}{b c - a d} \right)^n \text{Hypergeometric2F1}[n, 3+n, 4+n, -\frac{d(a+b x)}{b c - a d}]}{b (3+n)}$$

Result (type 6, 317 leaves):

$$\begin{aligned}
& a (a + b x)^n (c + d x)^{-n} \left( \left( 3 a b c x^2 \text{AppellF1}[2, -n, n, 3, -\frac{b x}{a}, -\frac{d x}{c}] \right) \right. \\
& \left. \left( 3 a c \text{AppellF1}[2, -n, n, 3, -\frac{b x}{a}, -\frac{d x}{c}] + n x \left( b c \text{AppellF1}[3, 1-n, n, 4, -\frac{b x}{a}, -\frac{d x}{c}] - a d \text{AppellF1}[3, -n, 1+n, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) \right) + \right. \\
& \left. \left( 4 b^2 c x^3 \text{AppellF1}[3, -n, n, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) \right) / \\
& \left( 12 a c \text{AppellF1}[3, -n, n, 4, -\frac{b x}{a}, -\frac{d x}{c}] + 3 b c n x \text{AppellF1}[4, 1-n, n, 5, -\frac{b x}{a}, -\frac{d x}{c}] - 3 a d n x \text{AppellF1}[4, -n, 1+n, 5, -\frac{b x}{a}, -\frac{d x}{c}] \right) - \\
& \frac{a \left( \frac{d(a+b x)}{-b c+a d} \right)^{-n} (c + d x) \text{Hypergeometric2F1}[1-n, -n, 2-n, \frac{b(c+d x)}{b c-a d}]}{d (-1+n)}
\end{aligned}$$

**Problem 1882: Result unnecessarily involves higher level functions.**

$$\int (a + b x)^m (a c (1 + m) + b c (2 + m) x)^{-3-m} dx$$

Optimal (type 3, 95 leaves, 2 steps):

$$-\frac{(a + b x)^{1+m} (a c (1 + m) + b c (2 + m) x)^{-2-m}}{a b c (2 + m)} + \frac{(a + b x)^{1+m} (a c (1 + m) + b c (2 + m) x)^{-1-m}}{a^2 b c^2 (1 + m) (2 + m)}$$

Result (type 5, 82 leaves):

$$-\frac{1}{a^3 b c^3 (1 + m)} (a + b x)^{1+m} (c (a (1 + m) + b (2 + m) x))^{-m} \left( -1 - m - \frac{b (2 + m) x}{a} \right)^m \text{Hypergeometric2F1}[1+m, 3+m, 2+m, \frac{(2+m)(a+b x)}{a}]$$

**Problem 1884: Result unnecessarily involves higher level functions.**

$$\int (a + b x)^{\frac{-2 b c + a d}{b c - a d}} (c + d x)^{\frac{b c - 2 a d}{b c - a d}} dx$$

Optimal (type 3, 97 leaves, 2 steps):

$$-\frac{(a + b x)^{\frac{-b c}{b c - a d}} (c + d x)^{\frac{a d}{b c - a d}}}{b c} + \frac{(a + b x)^{\frac{a d}{b c - a d}} (c + d x)^{\frac{a d}{b c - a d}}}{a b c}$$

Result (type 5, 159 leaves):

$$\frac{1}{a d^2} (b c - a d) (a + b x)^{\frac{-2 b c + a d}{b c - a d}} \left( \frac{d (a + b x)}{-b c + a d} \right)^{\frac{-2 b c + a d}{b c - a d}} (c + d x)^{\frac{a d}{b c - a d}} \text{Hypergeometric2F1}\left[ \frac{a d}{b c - a d}, \frac{-2 b c + a d}{-b c + a d}, \frac{b c}{b c - a d}, \frac{b (c + d x)}{b c - a d} \right]$$

## Test results for the 3201 problems in "1.1.1.3 (a+b x)^m (c+d x)^n (e+f x)^p.m"

**Problem 7:** Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x) (a c - b c x)^3}{x^3} dx$$

Optimal (type 1, 18 leaves, 1 step):

$$-\frac{c^3 (a - b x)^4}{2 x^2}$$

Result (type 1, 41 leaves):

$$c^3 \left( -\frac{a^4}{2 x^2} + \frac{2 a^3 b}{x} + 2 a b^3 x - \frac{b^4 x^2}{2} \right)$$

**Problem 24:** Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x) (a c - b c x)^4}{x^7} dx$$

Optimal (type 1, 41 leaves, 2 steps):

$$-\frac{c^4 (a - b x)^5}{6 x^6} - \frac{7 b c^4 (a - b x)^5}{30 a x^5}$$

Result (type 1, 85 leaves):

$$-\frac{a^5 c^4}{6 x^6} + \frac{3 a^4 b c^4}{5 x^5} - \frac{a^3 b^2 c^4}{2 x^4} - \frac{2 a^2 b^3 c^4}{3 x^3} + \frac{3 a b^4 c^4}{2 x^2} - \frac{b^5 c^4}{x}$$

**Problem 35:** Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x) (a c - b c x)^5}{x^4} dx$$

Optimal (type 1, 18 leaves, 1 step):

$$-\frac{c^5 (a - b x)^6}{3 x^3}$$

Result (type 1, 63 leaves):

$$c^5 \left( -\frac{a^6}{3x^3} + \frac{2a^5b}{x^2} - \frac{5a^4b^2}{x} - 5a^2b^4x + 2ab^5x^2 - \frac{b^6x^3}{3} \right)$$

Problem 45: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+bx)(ac-bcx)^6}{x^9} dx$$

Optimal (type 1, 41 leaves, 2 steps):

$$-\frac{c^6(a-bx)^7}{8x^8} - \frac{9bc^6(a-bx)^7}{56ax^7}$$

Result (type 1, 112 leaves):

$$-\frac{a^7c^6}{8x^8} + \frac{5a^6bc^6}{7x^7} - \frac{3a^5b^2c^6}{2x^6} + \frac{a^4b^3c^6}{x^5} + \frac{5a^3b^4c^6}{4x^4} - \frac{3a^2b^5c^6}{x^3} + \frac{5ab^6c^6}{2x^2} - \frac{b^7c^6}{x}$$

Problem 61: Result unnecessarily involves higher level functions.

$$\int \frac{(ex)^m}{(2-2ax)^4(1+ax)^3} dx$$

Optimal (type 5, 86 leaves, 5 steps):

$$\frac{(ex)^{1+m} \text{Hypergeometric2F1}\left[4, \frac{1+m}{2}, \frac{3+m}{2}, a^2 x^2\right]}{16e(1+m)} + \frac{a(ex)^{2+m} \text{Hypergeometric2F1}\left[4, \frac{2+m}{2}, \frac{4+m}{2}, a^2 x^2\right]}{16e^2(2+m)}$$

Result (type 6, 120 leaves):

$$\left( (2+m)x(ex)^m \text{AppellF1}[1+m, 4, 3, 2+m, ax, -ax] \right) / \left( 16(1+m)(-1+ax)^4(1+ax)^3 \left( (2+m) \text{AppellF1}[1+m, 4, 3, 2+m, ax, -ax] + a x \left( 4 \text{AppellF1}[2+m, 5, 3, 3+m, ax, -ax] - 3 \text{HypergeometricPFQ}\left[\left\{4, 1+\frac{m}{2}\right\}, \left\{2+\frac{m}{2}\right\}, a^2 x^2\right] \right) \right) \right)$$

Problem 66: Result unnecessarily involves higher level functions.

$$\int \frac{(ex)^m}{(a+bx)^2(a d - b d x)^3} dx$$

Optimal (type 5, 98 leaves, 5 steps):

$$\frac{(e x)^{1+m} \text{Hypergeometric2F1}\left[3, \frac{1+m}{2}, \frac{3+m}{2}, \frac{b^2 x^2}{a^2}\right]}{a^5 d^3 e (1+m)} + \frac{b (e x)^{2+m} \text{Hypergeometric2F1}\left[3, \frac{2+m}{2}, \frac{4+m}{2}, \frac{b^2 x^2}{a^2}\right]}{a^6 d^3 e^2 (2+m)}$$

Result (type 6, 144 leaves):

$$\left( a (2+m) x (e x)^m \text{AppellF1}\left[1+m, 3, 2, 2+m, \frac{b x}{a}, -\frac{b x}{a}\right] \right) / \left( d^3 (1+m) (a-bx)^3 (a+bx)^2 \left( a (2+m) \text{AppellF1}\left[1+m, 3, 2, 2+m, \frac{b x}{a}, -\frac{b x}{a}\right] + b x \left( 3 \text{AppellF1}\left[2+m, 4, 2, 3+m, \frac{b x}{a}, -\frac{b x}{a}\right] - 2 \text{HypergeometricPFQ}\left[\{3, 1+\frac{m}{2}\}, \{2+\frac{m}{2}\}, \frac{b^2 x^2}{a^2}\right] \right) \right)$$

Problem 67: Result unnecessarily involves higher level functions.

$$\int \frac{(e x)^m}{(a+b x)^3 (a d - b d x)^4} dx$$

Optimal (type 5, 98 leaves, 5 steps):

$$\frac{(e x)^{1+m} \text{Hypergeometric2F1}\left[4, \frac{1+m}{2}, \frac{3+m}{2}, \frac{b^2 x^2}{a^2}\right]}{a^7 d^4 e (1+m)} + \frac{b (e x)^{2+m} \text{Hypergeometric2F1}\left[4, \frac{2+m}{2}, \frac{4+m}{2}, \frac{b^2 x^2}{a^2}\right]}{a^8 d^4 e^2 (2+m)}$$

Result (type 6, 144 leaves):

$$\left( a (2+m) x (e x)^m \text{AppellF1}\left[1+m, 4, 3, 2+m, \frac{b x}{a}, -\frac{b x}{a}\right] \right) / \left( d^4 (1+m) (a-bx)^4 (a+bx)^3 \left( a (2+m) \text{AppellF1}\left[1+m, 4, 3, 2+m, \frac{b x}{a}, -\frac{b x}{a}\right] + b x \left( 4 \text{AppellF1}\left[2+m, 5, 3, 3+m, \frac{b x}{a}, -\frac{b x}{a}\right] - 3 \text{HypergeometricPFQ}\left[\{4, 1+\frac{m}{2}\}, \{2+\frac{m}{2}\}, \frac{b^2 x^2}{a^2}\right] \right) \right)$$

Problem 124: Result more than twice size of optimal antiderivative.

$$\int (a+b x)^5 (A+B x) dx$$

Optimal (type 1, 38 leaves, 2 steps):

$$\frac{(A b - a B) (a+b x)^6}{6 b^2} + \frac{B (a+b x)^7}{7 b^2}$$

Result (type 1, 109 leaves):

$$a^5 A x + \frac{1}{2} a^4 (5 A b + a B) x^2 + \frac{5}{3} a^3 b (2 A b + a B) x^3 + \frac{5}{2} a^2 b^2 (A b + a B) x^4 + a b^3 (A b + 2 a B) x^5 + \frac{1}{6} b^4 (A b + 5 a B) x^6 + \frac{1}{7} b^5 B x^7$$

### Problem 132: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + bx)^5 (A + Bx)}{x^8} dx$$

Optimal (type 1, 44 leaves, 2 steps):

$$-\frac{A(a + bx)^6}{7ax^7} + \frac{(Ab - 7aB)(a + bx)^6}{42a^2x^6}$$

Result (type 1, 104 leaves):

$$-\frac{1}{42x^7} (21b^5x^5(A + 2Bx) + 35ab^4x^4(2A + 3Bx) + 35a^2b^3x^3(3A + 4Bx) + 21a^3b^2x^2(4A + 5Bx) + 7a^4bx(5A + 6Bx) + a^5(6A + 7Bx))$$

### Problem 144: Result more than twice size of optimal antiderivative.

$$\int x^3(a + bx)^{10}(A + Bx) dx$$

Optimal (type 1, 112 leaves, 2 steps):

$$-\frac{a^3(ab - aB)(a + bx)^{11}}{11b^5} + \frac{a^2(3Ab - 4aB)(a + bx)^{12}}{12b^5} - \frac{3a(ab - 2aB)(a + bx)^{13}}{13b^5} + \frac{(Ab - 4aB)(a + bx)^{14}}{14b^5} + \frac{B(a + bx)^{15}}{15b^5}$$

Result (type 1, 231 leaves):

$$\begin{aligned} & \frac{1}{4}a^{10}Ax^4 + \frac{1}{5}a^9(10Ab + aB)x^5 + \frac{5}{6}a^8b(9Ab + 2aB)x^6 + \frac{15}{7}a^7b^2(8Ab + 3aB)x^7 + \frac{15}{4}a^6b^3(7Ab + 4aB)x^8 + \frac{14}{3}a^5b^4(6Ab + 5aB)x^9 + \\ & \frac{21}{5}a^4b^5(5Ab + 6aB)x^{10} + \frac{30}{11}a^3b^6(4Ab + 7aB)x^{11} + \frac{5}{4}a^2b^7(3Ab + 8aB)x^{12} + \frac{5}{13}a^1b^8(2Ab + 9aB)x^{13} + \frac{1}{14}b^9(Ab + 10aB)x^{14} + \frac{1}{15}b^{10}Bx^{15} \end{aligned}$$

### Problem 145: Result more than twice size of optimal antiderivative.

$$\int x^2(a + bx)^{10}(A + Bx) dx$$

Optimal (type 1, 87 leaves, 2 steps):

$$\frac{a^2(ab - aB)(a + bx)^{11}}{11b^4} - \frac{a(2Ab - 3aB)(a + bx)^{12}}{12b^4} + \frac{(Ab - 3aB)(a + bx)^{13}}{13b^4} + \frac{B(a + bx)^{14}}{14b^4}$$

Result (type 1, 226 leaves):

$$\begin{aligned} & \frac{1}{3} a^{10} A x^3 + \frac{1}{4} a^9 (10 A b + a B) x^4 + a^8 b (9 A b + 2 a B) x^5 + \frac{5}{2} a^7 b^2 (8 A b + 3 a B) x^6 + \frac{30}{7} a^6 b^3 (7 A b + 4 a B) x^7 + \frac{21}{4} a^5 b^4 (6 A b + 5 a B) x^8 + \\ & \frac{14}{3} a^4 b^5 (5 A b + 6 a B) x^9 + 3 a^3 b^6 (4 A b + 7 a B) x^{10} + \frac{15}{11} a^2 b^7 (3 A b + 8 a B) x^{11} + \frac{5}{12} a b^8 (2 A b + 9 a B) x^{12} + \frac{1}{13} b^9 (A b + 10 a B) x^{13} + \frac{1}{14} b^{10} B x^{14} \end{aligned}$$

**Problem 146:** Result more than twice size of optimal antiderivative.

$$\int x (a + b x)^{10} (A + B x) dx$$

Optimal (type 1, 61 leaves, 2 steps):

$$-\frac{a (A b - a B) (a + b x)^{11}}{11 b^3} + \frac{(A b - 2 a B) (a + b x)^{12}}{12 b^3} + \frac{B (a + b x)^{13}}{13 b^3}$$

Result (type 1, 218 leaves):

$$\begin{aligned} & \frac{1}{6} a^{10} x^2 (3 A + 2 B x) + \frac{5}{6} a^9 b x^3 (4 A + 3 B x) + \frac{9}{4} a^8 b^2 x^4 (5 A + 4 B x) + 4 a^7 b^3 x^5 (6 A + 5 B x) + 5 a^6 b^4 x^6 (7 A + 6 B x) + \frac{9}{2} a^5 b^5 x^7 (8 A + 7 B x) + \\ & \frac{35}{12} a^4 b^6 x^8 (9 A + 8 B x) + \frac{4}{3} a^3 b^7 x^9 (10 A + 9 B x) + \frac{9}{22} a^2 b^8 x^{10} (11 A + 10 B x) + \frac{5}{66} a b^9 x^{11} (12 A + 11 B x) + \frac{1}{156} b^{10} x^{12} (13 A + 12 B x) \end{aligned}$$

**Problem 147:** Result more than twice size of optimal antiderivative.

$$\int (a + b x)^{10} (A + B x) dx$$

Optimal (type 1, 38 leaves, 2 steps):

$$\frac{(A b - a B) (a + b x)^{11}}{11 b^2} + \frac{B (a + b x)^{12}}{12 b^2}$$

Result (type 1, 198 leaves):

$$\begin{aligned} & \frac{1}{132} x \\ & (66 a^{10} (2 A + B x) + 220 a^9 b x (3 A + 2 B x) + 495 a^8 b^2 x^2 (4 A + 3 B x) + 792 a^7 b^3 x^3 (5 A + 4 B x) + 924 a^6 b^4 x^4 (6 A + 5 B x) + 792 a^5 b^5 x^5 (7 A + 6 B x) + \\ & 495 a^4 b^6 x^6 (8 A + 7 B x) + 220 a^3 b^7 x^7 (9 A + 8 B x) + 66 a^2 b^8 x^8 (10 A + 9 B x) + 12 a b^9 x^9 (11 A + 10 B x) + b^{10} x^{10} (12 A + 11 B x)) \end{aligned}$$

**Problem 160:** Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^{10} (A + B x)}{x^{13}} dx$$

Optimal (type 1, 44 leaves, 2 steps):

$$-\frac{A(a + bx)^{11}}{12ax^{12}} + \frac{(Ab - 12aB)(a + bx)^{11}}{132a^2x^{11}}$$

Result (type 1, 199 leaves):

$$-\frac{1}{132x^{12}}(66b^{10}x^{10}(A + 2Bx) + 220ab^9x^9(2A + 3Bx) + 495a^2b^8x^8(3A + 4Bx) + 792a^3b^7x^7(4A + 5Bx) + 924a^4b^6x^6(5A + 6Bx) + 792a^5b^5x^5(6A + 7Bx) + 495a^6b^4x^4(7A + 8Bx) + 220a^7b^3x^3(8A + 9Bx) + 66a^8b^2x^2(9A + 10Bx) + 12a^9bx(10A + 11Bx) + a^{10}(11A + 12Bx))$$

**Problem 161:** Result more than twice size of optimal antiderivative.

$$\int \frac{(a + bx)^{10}(A + Bx)}{x^{14}} dx$$

Optimal (type 1, 72 leaves, 3 steps):

$$-\frac{A(a + bx)^{11}}{13ax^{13}} + \frac{(2Ab - 13aB)(a + bx)^{11}}{156a^2x^{12}} - \frac{b(2Ab - 13aB)(a + bx)^{11}}{1716a^3x^{11}}$$

Result (type 1, 202 leaves):

$$-\frac{1}{1716x^{13}}(286b^{10}x^{10}(2A + 3Bx) + 1430ab^9x^9(3A + 4Bx) + 3861a^2b^8x^8(4A + 5Bx) + 6864a^3b^7x^7(5A + 6Bx) + 8580a^4b^6x^6(6A + 7Bx) + 7722a^5b^5x^5(7A + 8Bx) + 5005a^6b^4x^4(8A + 9Bx) + 2288a^7b^3x^3(9A + 10Bx) + 702a^8b^2x^2(10A + 11Bx) + 130a^9bx(11A + 12Bx) + 11a^{10}(12A + 13Bx))$$

**Problem 169:** Result more than twice size of optimal antiderivative.

$$\int x^3(a + bx)(c + dx)^{16} dx$$

Optimal (type 1, 114 leaves, 2 steps):

$$\frac{c^3(bc - ad)(c + dx)^{17}}{17d^5} - \frac{c^2(4bc - 3ad)(c + dx)^{18}}{18d^5} + \frac{3c(2bc - ad)(c + dx)^{19}}{19d^5} - \frac{(4bc - ad)(c + dx)^{20}}{20d^5} + \frac{b(c + dx)^{21}}{21d^5}$$

Result (type 1, 359 leaves):

$$\begin{aligned}
& \frac{1}{4} a c^{16} x^4 + \frac{1}{5} c^{15} (b c + 16 a d) x^5 + \frac{4}{3} c^{14} d (2 b c + 15 a d) x^6 + \frac{40}{7} c^{13} d^2 (3 b c + 14 a d) x^7 + \frac{35}{2} c^{12} d^3 (4 b c + 13 a d) x^8 + \\
& \frac{364}{9} c^{11} d^4 (5 b c + 12 a d) x^9 + \frac{364}{5} c^{10} d^5 (6 b c + 11 a d) x^{10} + 104 c^9 d^6 (7 b c + 10 a d) x^{11} + \frac{715}{6} c^8 d^7 (8 b c + 9 a d) x^{12} + \\
& 110 c^7 d^8 (9 b c + 8 a d) x^{13} + \frac{572}{7} c^6 d^9 (10 b c + 7 a d) x^{14} + \frac{728}{15} c^5 d^{10} (11 b c + 6 a d) x^{15} + \frac{91}{4} c^4 d^{11} (12 b c + 5 a d) x^{16} + \\
& \frac{140}{17} c^3 d^{12} (13 b c + 4 a d) x^{17} + \frac{20}{9} c^2 d^{13} (14 b c + 3 a d) x^{18} + \frac{8}{19} c d^{14} (15 b c + 2 a d) x^{19} + \frac{1}{20} d^{15} (16 b c + a d) x^{20} + \frac{1}{21} b d^{16} x^{21}
\end{aligned}$$

**Problem 170:** Result more than twice size of optimal antiderivative.

$$\int x^2 (a + b x) (c + d x)^{16} dx$$

Optimal (type 1, 88 leaves, 2 steps):

$$-\frac{c^2 (b c - a d) (c + d x)^{17}}{17 d^4} + \frac{c (3 b c - 2 a d) (c + d x)^{18}}{18 d^4} - \frac{(3 b c - a d) (c + d x)^{19}}{19 d^4} + \frac{b (c + d x)^{20}}{20 d^4}$$

Result (type 1, 355 leaves):

$$\begin{aligned}
& \frac{1}{3} a c^{16} x^3 + \frac{1}{4} c^{15} (b c + 16 a d) x^4 + \frac{8}{5} c^{14} d (2 b c + 15 a d) x^5 + \frac{20}{3} c^{13} d^2 (3 b c + 14 a d) x^6 + 20 c^{12} d^3 (4 b c + 13 a d) x^7 + \\
& \frac{91}{2} c^{11} d^4 (5 b c + 12 a d) x^8 + \frac{728}{9} c^{10} d^5 (6 b c + 11 a d) x^9 + \frac{572}{5} c^9 d^6 (7 b c + 10 a d) x^{10} + 130 c^8 d^7 (8 b c + 9 a d) x^{11} + \\
& \frac{715}{6} c^7 d^8 (9 b c + 8 a d) x^{12} + 88 c^6 d^9 (10 b c + 7 a d) x^{13} + 52 c^5 d^{10} (11 b c + 6 a d) x^{14} + \frac{364}{15} c^4 d^{11} (12 b c + 5 a d) x^{15} + \\
& \frac{35}{4} c^3 d^{12} (13 b c + 4 a d) x^{16} + \frac{40}{17} c^2 d^{13} (14 b c + 3 a d) x^{17} + \frac{4}{9} c d^{14} (15 b c + 2 a d) x^{18} + \frac{1}{19} d^{15} (16 b c + a d) x^{19} + \frac{1}{20} b d^{16} x^{20}
\end{aligned}$$

**Problem 171:** Result more than twice size of optimal antiderivative.

$$\int x (a + b x) (c + d x)^{16} dx$$

Optimal (type 1, 62 leaves, 2 steps):

$$\frac{c (b c - a d) (c + d x)^{17}}{17 d^3} - \frac{(2 b c - a d) (c + d x)^{18}}{18 d^3} + \frac{b (c + d x)^{19}}{19 d^3}$$

Result (type 1, 347 leaves):

$$\begin{aligned}
& \frac{1}{2} a c^{16} x^2 + \frac{1}{3} c^{15} (b c + 16 a d) x^3 + 2 c^{14} d (2 b c + 15 a d) x^4 + 8 c^{13} d^2 (3 b c + 14 a d) x^5 + \frac{70}{3} c^{12} d^3 (4 b c + 13 a d) x^6 + \\
& 52 c^{11} d^4 (5 b c + 12 a d) x^7 + 91 c^{10} d^5 (6 b c + 11 a d) x^8 + \frac{1144}{9} c^9 d^6 (7 b c + 10 a d) x^9 + 143 c^8 d^7 (8 b c + 9 a d) x^{10} + \\
& 130 c^7 d^8 (9 b c + 8 a d) x^{11} + \frac{286}{3} c^6 d^9 (10 b c + 7 a d) x^{12} + 56 c^5 d^{10} (11 b c + 6 a d) x^{13} + 26 c^4 d^{11} (12 b c + 5 a d) x^{14} + \\
& \frac{28}{3} c^3 d^{12} (13 b c + 4 a d) x^{15} + \frac{5}{2} c^2 d^{13} (14 b c + 3 a d) x^{16} + \frac{8}{17} c d^{14} (15 b c + 2 a d) x^{17} + \frac{1}{18} d^{15} (16 b c + a d) x^{18} + \frac{1}{19} b d^{16} x^{19}
\end{aligned}$$

**Problem 172:** Result more than twice size of optimal antiderivative.

$$\int (a + b x) (c + d x)^{16} dx$$

Optimal (type 1, 38 leaves, 2 steps):

$$-\frac{(b c - a d) (c + d x)^{17}}{17 d^2} + \frac{b (c + d x)^{18}}{18 d^2}$$

Result (type 1, 342 leaves):

$$\begin{aligned}
& a c^{16} x + \frac{1}{2} c^{15} (b c + 16 a d) x^2 + \frac{8}{3} c^{14} d (2 b c + 15 a d) x^3 + 10 c^{13} d^2 (3 b c + 14 a d) x^4 + 28 c^{12} d^3 (4 b c + 13 a d) x^5 + \\
& \frac{182}{3} c^{11} d^4 (5 b c + 12 a d) x^6 + 104 c^{10} d^5 (6 b c + 11 a d) x^7 + 143 c^9 d^6 (7 b c + 10 a d) x^8 + \frac{1430}{9} c^8 d^7 (8 b c + 9 a d) x^9 + \\
& 143 c^7 d^8 (9 b c + 8 a d) x^{10} + 104 c^6 d^9 (10 b c + 7 a d) x^{11} + \frac{182}{3} c^5 d^{10} (11 b c + 6 a d) x^{12} + 28 c^4 d^{11} (12 b c + 5 a d) x^{13} + \\
& 10 c^3 d^{12} (13 b c + 4 a d) x^{14} + \frac{8}{3} c^2 d^{13} (14 b c + 3 a d) x^{15} + \frac{1}{2} c d^{14} (15 b c + 2 a d) x^{16} + \frac{1}{17} d^{15} (16 b c + a d) x^{17} + \frac{1}{18} b d^{16} x^{18}
\end{aligned}$$

**Problem 173:** Result more than twice size of optimal antiderivative.

$$\int x^2 (2 + x)^5 (2 + 3 x) dx$$

Optimal (type 1, 12 leaves, 1 step):

$$\frac{1}{3} x^3 (2 + x)^6$$

Result (type 1, 42 leaves):

$$\frac{64 x^3}{3} + 64 x^4 + 80 x^5 + \frac{160 x^6}{3} + 20 x^7 + 4 x^8 + \frac{x^9}{3}$$

### Problem 202: Result more than twice size of optimal antiderivative.

$$\int x^3 (a + bx)^2 (c + dx)^{16} dx$$

Optimal (type 1, 177 leaves, 2 steps):

$$\begin{aligned} & -\frac{c^3 (b c - a d)^2 (c + d x)^{17}}{17 d^6} + \frac{c^2 (5 b c - 3 a d) (b c - a d) (c + d x)^{18}}{18 d^6} - \\ & \frac{c (10 b^2 c^2 - 12 a b c d + 3 a^2 d^2) (c + d x)^{19}}{19 d^6} + \frac{(10 b^2 c^2 - 8 a b c d + a^2 d^2) (c + d x)^{20}}{20 d^6} - \frac{b (5 b c - 2 a d) (c + d x)^{21}}{21 d^6} + \frac{b^2 (c + d x)^{22}}{22 d^6} \end{aligned}$$

Result (type 1, 589 leaves):

$$\begin{aligned} & \frac{1}{4} a^2 c^{16} x^4 + \frac{2}{5} a c^{15} (b c + 8 a d) x^5 + \frac{1}{6} c^{14} (b^2 c^2 + 32 a b c d + 120 a^2 d^2) x^6 + \frac{16}{7} c^{13} d (b^2 c^2 + 15 a b c d + 35 a^2 d^2) x^7 + \\ & \frac{5}{2} c^{12} d^2 (6 b^2 c^2 + 56 a b c d + 91 a^2 d^2) x^8 + \frac{56}{9} c^{11} d^3 (10 b^2 c^2 + 65 a b c d + 78 a^2 d^2) x^9 + \frac{182}{5} c^{10} d^4 (5 b^2 c^2 + 24 a b c d + 22 a^2 d^2) x^{10} + \\ & \frac{208}{11} c^9 d^5 (21 b^2 c^2 + 77 a b c d + 55 a^2 d^2) x^{11} + \frac{143}{6} c^8 d^6 (28 b^2 c^2 + 80 a b c d + 45 a^2 d^2) x^{12} + \\ & 220 c^7 d^7 (4 b^2 c^2 + 9 a b c d + 4 a^2 d^2) x^{13} + \frac{143}{7} c^6 d^8 (45 b^2 c^2 + 80 a b c d + 28 a^2 d^2) x^{14} + \frac{208}{15} c^5 d^9 (55 b^2 c^2 + 77 a b c d + 21 a^2 d^2) x^{15} + \\ & \frac{91}{4} c^4 d^{10} (22 b^2 c^2 + 24 a b c d + 5 a^2 d^2) x^{16} + \frac{56}{17} c^3 d^{11} (78 b^2 c^2 + 65 a b c d + 10 a^2 d^2) x^{17} + \frac{10}{9} c^2 d^{12} (91 b^2 c^2 + 56 a b c d + 6 a^2 d^2) x^{18} + \\ & \frac{16}{19} c d^{13} (35 b^2 c^2 + 15 a b c d + a^2 d^2) x^{19} + \frac{1}{20} d^{14} (120 b^2 c^2 + 32 a b c d + a^2 d^2) x^{20} + \frac{2}{21} b d^{15} (8 b c + a d) x^{21} + \frac{1}{22} b^2 d^{16} x^{22} \end{aligned}$$

### Problem 203: Result more than twice size of optimal antiderivative.

$$\int x^2 (a + bx)^2 (c + dx)^{16} dx$$

Optimal (type 1, 137 leaves, 2 steps):

$$\begin{aligned} & \frac{c^2 (b c - a d)^2 (c + d x)^{17}}{17 d^5} - \frac{c (b c - a d) (2 b c - a d) (c + d x)^{18}}{9 d^5} + \frac{(6 b^2 c^2 - 6 a b c d + a^2 d^2) (c + d x)^{19}}{19 d^5} - \frac{b (2 b c - a d) (c + d x)^{20}}{10 d^5} + \frac{b^2 (c + d x)^{21}}{21 d^5} \end{aligned}$$

Result (type 1, 585 leaves):

$$\begin{aligned}
& \frac{1}{3} a^2 c^{16} x^3 + \frac{1}{2} a c^{15} (b c + 8 a d) x^4 + \frac{1}{5} c^{14} (b^2 c^2 + 32 a b c d + 120 a^2 d^2) x^5 + \\
& \frac{8}{3} c^{13} d (b^2 c^2 + 15 a b c d + 35 a^2 d^2) x^6 + \frac{20}{7} c^{12} d^2 (6 b^2 c^2 + 56 a b c d + 91 a^2 d^2) x^7 + 7 c^{11} d^3 (10 b^2 c^2 + 65 a b c d + 78 a^2 d^2) x^8 + \\
& \frac{364}{9} c^{10} d^4 (5 b^2 c^2 + 24 a b c d + 22 a^2 d^2) x^9 + \frac{104}{5} c^9 d^5 (21 b^2 c^2 + 77 a b c d + 55 a^2 d^2) x^{10} + 26 c^8 d^6 (28 b^2 c^2 + 80 a b c d + 45 a^2 d^2) x^{11} + \\
& \frac{715}{3} c^7 d^7 (4 b^2 c^2 + 9 a b c d + 4 a^2 d^2) x^{12} + 22 c^6 d^8 (45 b^2 c^2 + 80 a b c d + 28 a^2 d^2) x^{13} + \frac{104}{7} c^5 d^9 (55 b^2 c^2 + 77 a b c d + 21 a^2 d^2) x^{14} + \\
& \frac{364}{15} c^4 d^{10} (22 b^2 c^2 + 24 a b c d + 5 a^2 d^2) x^{15} + \frac{7}{2} c^3 d^{11} (78 b^2 c^2 + 65 a b c d + 10 a^2 d^2) x^{16} + \frac{20}{17} c^2 d^{12} (91 b^2 c^2 + 56 a b c d + 6 a^2 d^2) x^{17} + \\
& \frac{8}{9} c d^{13} (35 b^2 c^2 + 15 a b c d + a^2 d^2) x^{18} + \frac{1}{19} d^{14} (120 b^2 c^2 + 32 a b c d + a^2 d^2) x^{19} + \frac{1}{10} b d^{15} (8 b c + a d) x^{20} + \frac{1}{21} b^2 d^{16} x^{21}
\end{aligned}$$

**Problem 204: Result more than twice size of optimal antiderivative.**

$$\int x (a + b x)^2 (c + d x)^{16} dx$$

Optimal (type 1, 98 leaves, 2 steps):

$$\begin{aligned}
& \frac{c (b c - a d)^2 (c + d x)^{17}}{17 d^4} + \frac{(b c - a d) (3 b c - a d) (c + d x)^{18}}{18 d^4} - \frac{b (3 b c - 2 a d) (c + d x)^{19}}{19 d^4} + \frac{b^2 (c + d x)^{20}}{20 d^4}
\end{aligned}$$

Result (type 1, 583 leaves):

$$\begin{aligned}
& \frac{1}{2} a^2 c^{16} x^2 + \frac{2}{3} a c^{15} (b c + 8 a d) x^3 + \frac{1}{4} c^{14} (b^2 c^2 + 32 a b c d + 120 a^2 d^2) x^4 + \frac{16}{5} c^{13} d (b^2 c^2 + 15 a b c d + 35 a^2 d^2) x^5 + \\
& \frac{10}{3} c^{12} d^2 (6 b^2 c^2 + 56 a b c d + 91 a^2 d^2) x^6 + 8 c^{11} d^3 (10 b^2 c^2 + 65 a b c d + 78 a^2 d^2) x^7 + \frac{91}{2} c^{10} d^4 (5 b^2 c^2 + 24 a b c d + 22 a^2 d^2) x^8 + \\
& \frac{208}{9} c^9 d^5 (21 b^2 c^2 + 77 a b c d + 55 a^2 d^2) x^9 + \frac{143}{5} c^8 d^6 (28 b^2 c^2 + 80 a b c d + 45 a^2 d^2) x^{10} + \\
& 260 c^7 d^7 (4 b^2 c^2 + 9 a b c d + 4 a^2 d^2) x^{11} + \frac{143}{6} c^6 d^8 (45 b^2 c^2 + 80 a b c d + 28 a^2 d^2) x^{12} + 16 c^5 d^9 (55 b^2 c^2 + 77 a b c d + 21 a^2 d^2) x^{13} + \\
& 26 c^4 d^{10} (22 b^2 c^2 + 24 a b c d + 5 a^2 d^2) x^{14} + \frac{56}{15} c^3 d^{11} (78 b^2 c^2 + 65 a b c d + 10 a^2 d^2) x^{15} + \frac{5}{4} c^2 d^{12} (91 b^2 c^2 + 56 a b c d + 6 a^2 d^2) x^{16} + \\
& \frac{16}{17} c d^{13} (35 b^2 c^2 + 15 a b c d + a^2 d^2) x^{17} + \frac{1}{18} d^{14} (120 b^2 c^2 + 32 a b c d + a^2 d^2) x^{18} + \frac{2}{19} b d^{15} (8 b c + a d) x^{19} + \frac{1}{20} b^2 d^{16} x^{20}
\end{aligned}$$

**Problem 383: Result unnecessarily involves higher level functions.**

$$\int \frac{x^m}{(a + b x)^2 (c + d x)^2} dx$$

Optimal (type 5, 125 leaves, 4 steps):

$$-\frac{d x^{1+m}}{c (b c - a d) (c + d x)} + \frac{b^2 x^{1+m} \text{Hypergeometric2F1}[1, 1+m, 2+m, -\frac{b x}{a}]}{a (b c - a d)^2 (1+m)} - \frac{d (b c (1-m) + a d m) x^{1+m} \text{Hypergeometric2F1}[1, 1+m, 2+m, -\frac{d x}{c}]}{c^2 (b c - a d)^2 (1+m)}$$

Result (type 6, 142 leaves):

$$\left( a c (2+m) x^{1+m} \text{AppellF1}[1+m, 2, 1, 2+m, -\frac{d x}{c}, -\frac{b x}{a}] \right) / \left( (1+m) (a+b x) (c+d x)^2 \left( a c (2+m) \text{AppellF1}[1+m, 2, 1, 2+m, -\frac{d x}{c}, -\frac{b x}{a}] - x (b c \text{AppellF1}[2+m, 2, 2, 3+m, -\frac{d x}{c}, -\frac{b x}{a}] + 2 a d \text{AppellF1}[2+m, 3, 1, 3+m, -\frac{d x}{c}, -\frac{b x}{a}]) \right) \right)$$

Problem 384: Result unnecessarily involves higher level functions.

$$\int \frac{x^m}{(a+b x) (c+d x)^3} dx$$

Optimal (type 5, 206 leaves, 5 steps):

$$-\frac{d x^{1+m}}{2 c (b c - a d) (c + d x)^2} + \frac{d (a d (1-m) - b c (3-m)) x^{1+m}}{2 c^2 (b c - a d)^2 (c + d x)} + \frac{b^3 x^{1+m} \text{Hypergeometric2F1}[1, 1+m, 2+m, -\frac{b x}{a}]}{a (b c - a d)^3 (1+m)} + \frac{d (a^2 d^2 (1-m) m - 2 a b c d (2-m) m - b^2 c^2 (2-3 m + m^2)) x^{1+m} \text{Hypergeometric2F1}[1, 1+m, 2+m, -\frac{d x}{c}]}{2 c^3 (b c - a d)^3 (1+m)}$$

Result (type 6, 142 leaves):

$$\left( a c (2+m) x^{1+m} \text{AppellF1}[1+m, 3, 1, 2+m, -\frac{d x}{c}, -\frac{b x}{a}] \right) / \left( (1+m) (a+b x) (c+d x)^3 \left( a c (2+m) \text{AppellF1}[1+m, 3, 1, 2+m, -\frac{d x}{c}, -\frac{b x}{a}] - x (b c \text{AppellF1}[2+m, 3, 2, 3+m, -\frac{d x}{c}, -\frac{b x}{a}] + 3 a d \text{AppellF1}[2+m, 4, 1, 3+m, -\frac{d x}{c}, -\frac{b x}{a}]) \right) \right)$$

Problem 463: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^{1/3} \sqrt{c+d x} (4 c + d x)} dx$$

Optimal (type 3, 199 leaves, 2 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{3} c^{1/6} (c^{1/3}+2^{1/3} d^{1/3} x^{1/3})}{\sqrt{c+d x}}\right]}{2^{2/3} \sqrt{3} c^{5/6} d^{2/3}} + \frac{\text{ArcTan}\left[\frac{\sqrt{c+d x}}{\sqrt{3} \sqrt{c}}\right]}{2^{2/3} \sqrt{3} c^{5/6} d^{2/3}} - \frac{\text{ArcTanh}\left[\frac{c^{1/6} (c^{1/3}-2^{1/3} d^{1/3} x^{1/3})}{\sqrt{c+d x}}\right]}{2^{2/3} c^{5/6} d^{2/3}} + \frac{\text{ArcTanh}\left[\frac{\sqrt{c+d x}}{\sqrt{c}}\right]}{3 \times 2^{2/3} c^{5/6} d^{2/3}}$$

Result (type 6, 147 leaves):

$$\left( \frac{30 c x^{2/3} \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{d x}{c}, -\frac{d x}{4 c}\right]}{3} \right) / \left( \sqrt{c + d x} (4 c + d x) \right)$$

$$\left( 20 c \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{d x}{c}, -\frac{d x}{4 c}\right] - 3 d x \left( \operatorname{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 2, \frac{8}{3}, -\frac{d x}{c}, -\frac{d x}{4 c}\right] + 2 \operatorname{AppellF1}\left[\frac{5}{3}, \frac{3}{2}, 1, \frac{8}{3}, -\frac{d x}{c}, -\frac{d x}{4 c}\right] \right) \right)$$

Problem 464: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^{1/3} (8 c - d x) \sqrt{c + d x}} dx$$

Optimal (type 3, 143 leaves, 9 steps):

$$-\frac{\operatorname{ArcTan}\left[\frac{\sqrt{3} c^{1/6} (c^{1/3} + d^{1/3} x^{1/3})}{\sqrt{c + d x}}\right]}{2 \sqrt{3} c^{5/6} d^{2/3}} + \frac{\operatorname{ArcTanh}\left[\frac{(c^{1/3} + d^{1/3} x^{1/3})^2}{3 c^{1/6} \sqrt{c + d x}}\right]}{6 c^{5/6} d^{2/3}} - \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{c + d x}}{3 \sqrt{c}}\right]}{6 c^{5/6} d^{2/3}}$$

Result (type 6, 148 leaves):

$$\left( 60 c x^{2/3} \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{d x}{c}, \frac{d x}{8 c}\right] \right) / \left( (8 c - d x) \sqrt{c + d x} \right)$$

$$\left( 40 c \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{d x}{c}, \frac{d x}{8 c}\right] + 3 d x \left( \operatorname{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 2, \frac{8}{3}, -\frac{d x}{c}, \frac{d x}{8 c}\right] - 4 \operatorname{AppellF1}\left[\frac{5}{3}, \frac{3}{2}, 1, \frac{8}{3}, -\frac{d x}{c}, \frac{d x}{8 c}\right] \right) \right)$$

Problem 728: Result more than twice size of optimal antiderivative.

$$\int \frac{(1+x)^{3/2}}{\sqrt{1-x} x} dx$$

Optimal (type 3, 43 leaves, 6 steps):

$$-\sqrt{1-x} \sqrt{1+x} + 2 \operatorname{ArcSin}[x] - \operatorname{ArcTanh}[\sqrt{1-x} \sqrt{1+x}]$$

Result (type 3, 96 leaves):

$$-\sqrt{1-x^2} + 4 \operatorname{ArcSin}\left[\frac{\sqrt{1+x}}{\sqrt{2}}\right] + \operatorname{Log}\left[1 - \sqrt{1+x}\right] - \operatorname{Log}\left[2 + \sqrt{1-x} - \sqrt{1+x}\right] - \operatorname{Log}\left[1 + \sqrt{1+x}\right] + \operatorname{Log}\left[2 + \sqrt{1-x} + \sqrt{1+x}\right]$$

Problem 758: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x \sqrt{1-a-bx} \sqrt{1+ax+bx}} dx$$

Optimal (type 3, 54 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTanh} \left[ \frac{\sqrt{1-a} \sqrt{1+a+b x}}{\sqrt{1+a} \sqrt{1-a-b x}} \right]}{\sqrt{1-a^2}}$$

Result (type 3, 107 leaves):

$$\frac{i \sqrt{-1+a+b x} \sqrt{1+a+b x} \operatorname{Log} \left[ \frac{2 \sqrt{-1+a+b x} \sqrt{1+a+b x}}{x} + \frac{2 i (-1+a^2+a b x)}{\sqrt{1-a^2} x} \right]}{\sqrt{1-a^2} \sqrt{-(-1+a+b x) (1+a+b x)}}$$

Problem 838: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-1+x} \sqrt{1+x}} dx$$

Optimal (type 3, 2 leaves, 1 step):

$$\operatorname{ArcCosh}[x]$$

Result (type 3, 16 leaves):

$$\frac{2 \operatorname{ArcSinh} \left[ \frac{\sqrt{-1+x}}{\sqrt{2}} \right]}{\sqrt{2}}$$

Problem 855: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{x} \sqrt{2-b x} \sqrt{2+b x}} dx$$

Optimal (type 4, 30 leaves, 1 step):

$$\frac{\sqrt{2} \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \frac{\sqrt{b} \sqrt{x}}{\sqrt{2}} \right], -1 \right]}{\sqrt{b}}$$

Result (type 4, 70 leaves):

$$\frac{2 i \sqrt{-\frac{1}{b}} b \sqrt{1-\frac{4}{b^2 x^2}} \times \operatorname{EllipticF} \left[ i \operatorname{ArcSinh} \left[ \frac{\sqrt{2} \sqrt{-\frac{1}{b}}}{\sqrt{x}} \right], -1 \right]}{\sqrt{8-2 b^2 x^2}}$$

**Problem 856:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-x} \sqrt{2-bx} \sqrt{2+bx}} dx$$

Optimal (type 4, 33 leaves, 1 step) :

$$\frac{\sqrt{2} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{b} \sqrt{-x}}{\sqrt{2}}\right], -1\right]}{\sqrt{b}}$$

Result (type 4, 78 leaves) :

$$\frac{2 \pm \sqrt{-\frac{1}{b}} b \sqrt{1 - \frac{4}{b^2 x^2}} \sqrt{-x^2} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{1}{b}}}{\sqrt{x}}\right], -1\right]}{\sqrt{8 - 2 b^2 x^2}}$$

**Problem 857:** Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{ex} \sqrt{2-bx} \sqrt{2+bx}} dx$$

Optimal (type 4, 42 leaves, 1 step) :

$$\frac{\sqrt{2} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{b} \sqrt{ex}}{\sqrt{2} \sqrt{e}}\right], -1\right]}{\sqrt{b} \sqrt{e}}$$

Result (type 4, 81 leaves) :

$$\frac{2 \pm \sqrt{-\frac{1}{b}} b \sqrt{1 - \frac{4}{b^2 x^2}} x^{3/2} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{1}{b}}}{\sqrt{x}}\right], -1\right]}{\sqrt{ex} \sqrt{8 - 2 b^2 x^2}}$$

**Problem 861:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{1-x} \sqrt{x} \sqrt{1+x}} dx$$

Optimal (type 4, 10 leaves, 1 step) :

$$2 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{x}\right], -1\right]$$

Result (type 4, 66 leaves) :

$$\frac{2 \pm \sqrt{1 + \frac{1}{-1+x}} \sqrt{1 + \frac{2}{-1+x}} (-1+x)^{3/2} \text{EllipticF}\left[\pm \text{ArcSinh}\left[\frac{1}{\sqrt{-1+x}}\right], 2\right]}{\sqrt{-(-1+x)x} \sqrt{1+x}}$$

Problem 862: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{1+x} \sqrt{x-x^2}} dx$$

Optimal (type 4, 10 leaves, 2 steps) :

$$2 \text{EllipticF}[\text{ArcSin}[\sqrt{x}], -1]$$

Result (type 4, 66 leaves) :

$$\frac{2 \pm \sqrt{1 + \frac{1}{-1+x}} \sqrt{1 + \frac{2}{-1+x}} (-1+x)^{3/2} \text{EllipticF}\left[\pm \text{ArcSinh}\left[\frac{1}{\sqrt{-1+x}}\right], 2\right]}{\sqrt{-(-1+x)x} \sqrt{1+x}}$$

Problem 863: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{bx} \sqrt{1-cx} \sqrt{1+cx}} dx$$

Optimal (type 4, 33 leaves, 1 step) :

$$\frac{2 \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{bx}}{\sqrt{b}}\right], -1]}{\sqrt{b} \sqrt{c}}$$

Result (type 4, 76 leaves) :

$$\frac{2 \pm \sqrt{-\frac{1}{c}} c \sqrt{1 - \frac{1}{c^2 x^2}} x^{3/2} \text{EllipticF}\left[\pm \text{ArcSinh}\left[\frac{\sqrt{-\frac{1}{c}}}{\sqrt{x}}\right], -1\right]}{\sqrt{bx} \sqrt{1 - c^2 x^2}}$$

**Problem 864:** Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{bx} \sqrt{1-cx} \sqrt{1+dx}} dx$$

Optimal (type 4, 38 leaves, 1 step) :

$$\frac{2 \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{\sqrt{c} \sqrt{bx}}{\sqrt{b}}\right], -\frac{d}{c}]}{\sqrt{b} \sqrt{c}}$$

Result (type 4, 89 leaves) :

$$\frac{2 \sqrt{\frac{c-\frac{1}{x}}{c}} \sqrt{\frac{d+\frac{1}{x}}{d}} x^{3/2} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{1}{c}}}{\sqrt{x}}\right], -\frac{c}{d}]}{\sqrt{\frac{1}{c}} \sqrt{bx} \sqrt{1-cx} \sqrt{1+dx}}$$

**Problem 865:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1+x}}{\sqrt{1-x} \sqrt{x}} dx$$

Optimal (type 4, 10 leaves, 1 step) :

$$2 \operatorname{EllipticE}[\operatorname{ArcSin}[\sqrt{x}], -1]$$

Result (type 4, 104 leaves) :

$$\frac{2 \sqrt{\frac{-1+x}{1+x}} \sqrt{\frac{1+x}{-1+x}} \left(\sqrt{-1+x} x \sqrt{\frac{1+x}{-1+x}} + \frac{i \sqrt{2} x \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{2}}{\sqrt{-1+x}}\right], \frac{1}{2}\right]}{\sqrt{\frac{x}{-1+x}}}\right)}{\sqrt{-(-1+x)x}}$$

**Problem 866:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1+x}}{\sqrt{x-x^2}} dx$$

Optimal (type 4, 10 leaves, 2 steps) :

$$\frac{2 \operatorname{EllipticE}[\operatorname{ArcSin}[\sqrt{x}], -1]}{\sqrt{-(-1+x)x}}$$

Result (type 4, 104 leaves) :

$$\frac{2 \sqrt{\frac{-1+x}{1+x}} \sqrt{\frac{1+x}{-1+x}} \left( \sqrt{-1+x} x \sqrt{\frac{1+x}{-1+x}} + \frac{i \sqrt{2} \times \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{2}}{\sqrt{-1+x}}\right], \frac{1}{2}\right]}{\sqrt{\frac{x}{-1+x}}} \right)}{\sqrt{-(-1+x)x}}$$

Problem 867: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1+c x}}{\sqrt{b x} \sqrt{1-c x}} dx$$

Optimal (type 4, 33 leaves, 1 step) :

$$\frac{2 \operatorname{EllipticE}[\operatorname{ArcSin}\left[\frac{\sqrt{c} \sqrt{b x}}{\sqrt{b}}\right], -1]}{\sqrt{b} \sqrt{c}}$$

Result (type 4, 119 leaves) :

$$\frac{-2 \sqrt{-\frac{1}{c}} (-1+c x) \left( \sqrt{-\frac{1}{c}} \sqrt{1-\frac{1}{c x}} (1+c x) - \sqrt{1+\frac{1}{c x}} \sqrt{x} \operatorname{EllipticE}[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{1}{c}}}{\sqrt{x}}\right], -1] \right)}{\sqrt{1-\frac{1}{c x}} \sqrt{b x} \sqrt{1-c^2 x^2}}$$

Problem 868: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1+c x}}{\sqrt{b x} \sqrt{1-d x}} dx$$

Optimal (type 4, 38 leaves, 1 step) :

$$\frac{2 \operatorname{EllipticE}[\operatorname{ArcSin}\left[\frac{\sqrt{d} \sqrt{b x}}{\sqrt{b}}\right], -\frac{c}{d}]}{\sqrt{b} \sqrt{d}}$$

Result (type 4, 102 leaves) :

$$\frac{2 \sqrt{1-d x} \left(-1-c x+\frac{\sqrt{1+\frac{1}{c x}} \sqrt{x} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{1}{c}}}{\sqrt{x}}\right],-\frac{c}{d}\right]}{\sqrt{-\frac{1}{c}} \sqrt{1-\frac{1}{d x}}}\right)}{d \sqrt{b x} \sqrt{1+c x}}$$

**Problem 871:** Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1-c x}}{\sqrt{b x} \sqrt{1+c x}} dx$$

Optimal (type 4, 37 leaves, 1 step) :

$$\frac{2 \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c} \sqrt{b x}}{\sqrt{-b}}\right],-1\right]}{\sqrt{-b} \sqrt{c}}$$

Result (type 4, 77 leaves) :

$$\frac{2 c \left(\frac{1}{c^2}-x^2-\sqrt{\frac{1}{c}} \sqrt{1-\frac{1}{c^2 x^2}} x^{3/2} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{1}{c}}}{\sqrt{x}}\right],-1\right]\right)}{\sqrt{b x} \sqrt{1-c^2 x^2}}$$

**Problem 872:** Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1-c x}}{\sqrt{b x} \sqrt{1+d x}} dx$$

Optimal (type 4, 42 leaves, 1 step) :

$$\frac{2 \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{d} \sqrt{b x}}{\sqrt{-b}}\right],-\frac{c}{d}\right]}{\sqrt{-b} \sqrt{d}}$$

Result (type 4, 112 leaves) :

$$\frac{-\frac{2 \sqrt{\frac{1}{c}} (-1+c x) (1+d x)}{d}-2 \sqrt{1-\frac{1}{c x}} \sqrt{1+\frac{1}{d x}} x^{3/2} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{1}{c}}}{\sqrt{x}}\right],-\frac{c}{d}\right]}{\sqrt{\frac{1}{c}} \sqrt{b x} \sqrt{1-c x} \sqrt{1+d x}}$$

Problem 874: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{d+ex}}{\sqrt{2-3x} \sqrt{x}} dx$$

Optimal (type 4, 51 leaves, 2 steps):

$$\frac{2\sqrt{d+ex} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\frac{3}{2}}\sqrt{x}\right], -\frac{2e}{3d}\right]}{\sqrt{3} \sqrt{1+\frac{ex}{d}}}$$

Result (type 4, 125 leaves):

$$\frac{2\sqrt{x} \left( \frac{3(d+ex)}{\sqrt{2-3x}} - \frac{(3d+2e)\sqrt{\frac{d+ex}{e(-2+3x)}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\frac{2+\frac{3d}{e}}{2-3x}}\right], \frac{2e}{3d+2e}\right]}{\sqrt{2+\frac{3d}{e}} \sqrt{\frac{x}{-2+3x}}} \right)}{3\sqrt{d+ex}}$$

Problem 875: Result unnecessarily involves higher level functions.

$$\int \frac{x^4}{(1-x)^{1/3} (2-x)^{1/3}} dx$$

Optimal (type 4, 752 leaves, 8 steps):

$$\begin{aligned}
& \frac{99}{130} (1-x)^{2/3} (2-x)^{2/3} x^2 + \frac{3}{13} (1-x)^{2/3} (2-x)^{2/3} x^3 + \frac{27}{455} (1-x)^{2/3} (2-x)^{2/3} (89+34x) - \\
& \frac{891 \times 2^{2/3} \sqrt{(3-2x)^2} \sqrt{(-3+2x)^2} (2-3x+x^2)^{1/3}}{91 (3-2x) (1-x)^{1/3} (2-x)^{1/3} (1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})} + \left( \frac{891 \times 3^{1/4} \sqrt{2-\sqrt{3}} \sqrt{(-3+2x)^2} (2-3x+x^2)^{1/3} (1+2^{2/3} (2-3x+x^2)^{1/3})}{\sqrt{\frac{1-2^{2/3} (2-3x+x^2)^{1/3} + 2 \times 2^{1/3} (2-3x+x^2)^{2/3}}{(1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})^2}} \operatorname{EllipticE}[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3}}{1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3}}\right], -7-4\sqrt{3}]} \right) / \\
& \left( \frac{91 \times 2^{1/3} (3-2x) \sqrt{(3-2x)^2} (1-x)^{1/3} (2-x)^{1/3} \sqrt{\frac{1+2^{2/3} (2-3x+x^2)^{1/3}}{(1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})^2}}}{594 \times 2^{1/6} \times 3^{3/4} \sqrt{(-3+2x)^2} (2-3x+x^2)^{1/3} (1+2^{2/3} (2-3x+x^2)^{1/3})} \right. \\
& \left. \sqrt{\frac{1-2^{2/3} (2-3x+x^2)^{1/3} + 2 \times 2^{1/3} (2-3x+x^2)^{2/3}}{(1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})^2}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3}}{1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3}}\right], -7-4\sqrt{3}] \right) / \\
& \left( \frac{91 (3-2x) \sqrt{(3-2x)^2} (1-x)^{1/3} (2-x)^{1/3} \sqrt{\frac{1+2^{2/3} (2-3x+x^2)^{1/3}}{(1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})^2}}}{\sqrt{\frac{1-2^{2/3} (2-3x+x^2)^{1/3} + 2 \times 2^{1/3} (2-3x+x^2)^{2/3}}{(1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})^2}}} \right)
\end{aligned}$$

Result (type 5, 54 leaves) :

$$\frac{3}{910} (1-x)^{2/3} \left( (2-x)^{2/3} (1602 + 612x + 231x^2 + 70x^3) - 2970 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, -1+x\right] \right)$$

**Problem 876:** Result unnecessarily involves higher level functions.

$$\int \frac{x^3}{(1-x)^{1/3} (2-x)^{1/3}} dx$$

Optimal (type 4, 727 leaves, 7 steps) :

$$\begin{aligned}
& \frac{3}{10} (1-x)^{2/3} (2-x)^{2/3} x^2 + \frac{9}{70} (1-x)^{2/3} (2-x)^{2/3} (23+8x) - \frac{81 \sqrt{(3-2x)^2} \sqrt{(-3+2x)^2} (2-3x+x^2)^{1/3}}{7 \times 2^{1/3} (3-2x) (1-x)^{1/3} (2-x)^{1/3} (1+\sqrt{3}+2^{2/3}(2-3x+x^2)^{1/3})} + \\
& \left( 81 \times 3^{1/4} \sqrt{2-\sqrt{3}} \sqrt{(-3+2x)^2} (2-3x+x^2)^{1/3} (1+2^{2/3}(2-3x+x^2)^{1/3}) \right. \\
& \left. \sqrt{\frac{1-2^{2/3}(2-3x+x^2)^{1/3}+2\times 2^{1/3}(2-3x+x^2)^{2/3}}{(1+\sqrt{3}+2^{2/3}(2-3x+x^2)^{1/3})^2} \text{EllipticE}[\text{ArcSin}\left[\frac{1-\sqrt{3}+2^{2/3}(2-3x+x^2)^{1/3}}{1+\sqrt{3}+2^{2/3}(2-3x+x^2)^{1/3}}\right], -7-4\sqrt{3}]} \right) / \\
& \left( 14 \times 2^{1/3} (3-2x) \sqrt{(3-2x)^2} (1-x)^{1/3} (2-x)^{1/3} \sqrt{\frac{1+2^{2/3}(2-3x+x^2)^{1/3}}{(1+\sqrt{3}+2^{2/3}(2-3x+x^2)^{1/3})^2}} - \right. \\
& \left. \left( 27 \times 2^{1/6} \times 3^{3/4} \sqrt{(-3+2x)^2} (2-3x+x^2)^{1/3} (1+2^{2/3}(2-3x+x^2)^{1/3}) \right. \right. \\
& \left. \left. \sqrt{\frac{1-2^{2/3}(2-3x+x^2)^{1/3}+2\times 2^{1/3}(2-3x+x^2)^{2/3}}{(1+\sqrt{3}+2^{2/3}(2-3x+x^2)^{1/3})^2} \text{EllipticF}[\text{ArcSin}\left[\frac{1-\sqrt{3}+2^{2/3}(2-3x+x^2)^{1/3}}{1+\sqrt{3}+2^{2/3}(2-3x+x^2)^{1/3}}\right], -7-4\sqrt{3}]} \right) / \\
& \left( 7 (3-2x) \sqrt{(3-2x)^2} (1-x)^{1/3} (2-x)^{1/3} \sqrt{\frac{1+2^{2/3}(2-3x+x^2)^{1/3}}{(1+\sqrt{3}+2^{2/3}(2-3x+x^2)^{1/3})^2}} \right)
\end{aligned}$$

Result (type 5, 49 leaves) :

$$\frac{3}{70} (1-x)^{2/3} \left( (2-x)^{2/3} (69+24x+7x^2) - 135 \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, -1+x\right] \right)$$

**Problem 877:** Result unnecessarily involves higher level functions.

$$\int \frac{x^2}{(1-x)^{1/3} (2-x)^{1/3}} dx$$

Optimal (type 4, 720 leaves, 7 steps) :

$$\begin{aligned}
& \frac{45}{28} (1-x)^{2/3} (2-x)^{2/3} + \frac{3}{7} (1-x)^{2/3} (2-x)^{2/3} x - \frac{99 \sqrt{(3-2x)^2} \sqrt{(-3+2x)^2} (2-3x+x^2)^{1/3}}{14 \times 2^{1/3} (3-2x) (1-x)^{1/3} (2-x)^{1/3} (1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})} + \\
& \left( 99 \times 3^{1/4} \sqrt{2-\sqrt{3}} \sqrt{(-3+2x)^2} (2-3x+x^2)^{1/3} (1+2^{2/3} (2-3x+x^2)^{1/3}) \right. \\
& \left. \sqrt{\frac{1-2^{2/3} (2-3x+x^2)^{1/3} + 2 \times 2^{1/3} (2-3x+x^2)^{2/3}}{(1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})^2}} \text{EllipticE}[\text{ArcSin}\left[\frac{1-\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3}}{1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3}}\right], -7-4\sqrt{3}] \right) / \\
& \left( 28 \times 2^{1/3} (3-2x) \sqrt{(3-2x)^2} (1-x)^{1/3} (2-x)^{1/3} \sqrt{\frac{1+2^{2/3} (2-3x+x^2)^{1/3}}{(1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})^2}} - \right. \\
& \left. \left( 33 \times 3^{3/4} \sqrt{(-3+2x)^2} (2-3x+x^2)^{1/3} (1+2^{2/3} (2-3x+x^2)^{1/3}) \sqrt{\frac{1-2^{2/3} (2-3x+x^2)^{1/3} + 2 \times 2^{1/3} (2-3x+x^2)^{2/3}}{(1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})^2}} \right. \right. \\
& \left. \left. \text{EllipticF}[\text{ArcSin}\left[\frac{1-\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3}}{1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3}}\right], -7-4\sqrt{3}] \right) / \right. \\
& \left. \left( 7 \times 2^{5/6} (3-2x) \sqrt{(3-2x)^2} (1-x)^{1/3} (2-x)^{1/3} \sqrt{\frac{1+2^{2/3} (2-3x+x^2)^{1/3}}{(1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})^2}} \right) \right)
\end{aligned}$$

Result (type 5, 44 leaves) :

$$\frac{3}{28} (1-x)^{2/3} \left( (2-x)^{2/3} (15+4x) - 33 \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, -1+x\right] \right)$$

Problem 878: Result unnecessarily involves higher level functions.

$$\int \frac{x}{(1-x)^{1/3} (2-x)^{1/3}} dx$$

Optimal (type 4, 695 leaves, 6 steps) :

$$\begin{aligned}
& \frac{3}{4} (1-x)^{2/3} (2-x)^{2/3} - \frac{9 \sqrt{(3-2x)^2} \sqrt{(-3+2x)^2} (2-3x+x^2)^{1/3}}{2 \times 2^{1/3} (3-2x) (1-x)^{1/3} (2-x)^{1/3} (1+\sqrt{3} + 2^{2/3} (2-3x+x^2)^{1/3})} + \\
& \left( 9 \times 3^{1/4} \sqrt{2-\sqrt{3}} \sqrt{(-3+2x)^2} (2-3x+x^2)^{1/3} (1+2^{2/3} (2-3x+x^2)^{1/3}) \right. \\
& \left. \sqrt{\frac{1-2^{2/3} (2-3x+x^2)^{1/3} + 2 \times 2^{1/3} (2-3x+x^2)^{2/3}}{(1+\sqrt{3} + 2^{2/3} (2-3x+x^2)^{1/3})^2}} \text{EllipticE}[\text{ArcSin}\left[\frac{1-\sqrt{3} + 2^{2/3} (2-3x+x^2)^{1/3}}{1+\sqrt{3} + 2^{2/3} (2-3x+x^2)^{1/3}}\right], -7-4\sqrt{3}] \right) / \\
& \left( 4 \times 2^{1/3} (3-2x) \sqrt{(3-2x)^2} (1-x)^{1/3} (2-x)^{1/3} \sqrt{\frac{1+2^{2/3} (2-3x+x^2)^{1/3}}{(1+\sqrt{3} + 2^{2/3} (2-3x+x^2)^{1/3})^2}} \right. \\
& \left. \left( 3 \times 3^{3/4} \sqrt{(-3+2x)^2} (2-3x+x^2)^{1/3} (1+2^{2/3} (2-3x+x^2)^{1/3}) \sqrt{\frac{1-2^{2/3} (2-3x+x^2)^{1/3} + 2 \times 2^{1/3} (2-3x+x^2)^{2/3}}{(1+\sqrt{3} + 2^{2/3} (2-3x+x^2)^{1/3})^2}} \right. \right. \\
& \left. \left. \text{EllipticF}[\text{ArcSin}\left[\frac{1-\sqrt{3} + 2^{2/3} (2-3x+x^2)^{1/3}}{1+\sqrt{3} + 2^{2/3} (2-3x+x^2)^{1/3}}\right], -7-4\sqrt{3}] \right) / \right. \\
& \left. \left( 2^{5/6} (3-2x) \sqrt{(3-2x)^2} (1-x)^{1/3} (2-x)^{1/3} \sqrt{\frac{1+2^{2/3} (2-3x+x^2)^{1/3}}{(1+\sqrt{3} + 2^{2/3} (2-3x+x^2)^{1/3})^2}} \right) \right)
\end{aligned}$$

Result (type 5, 38 leaves):

$$\frac{3}{4} (1-x)^{2/3} \left( (2-x)^{2/3} - 3 \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, -1+x\right] \right)$$

Problem 879: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(1-x)^{1/3} (2-x)^{1/3}} dx$$

Optimal (type 4, 671 leaves, 5 steps):

$$\begin{aligned}
& - \frac{3 \sqrt{(3-2x)^2} \sqrt{(-3+2x)^2} (2-3x+x^2)^{1/3}}{2^{1/3} (3-2x) (1-x)^{1/3} (2-x)^{1/3} (1+\sqrt{3} + 2^{2/3} (2-3x+x^2)^{1/3})} + \\
& \left( 3 \times 3^{1/4} \sqrt{2-\sqrt{3}} \sqrt{(-3+2x)^2} (2-3x+x^2)^{1/3} (1+2^{2/3} (2-3x+x^2)^{1/3}) \sqrt{\frac{1-2^{2/3} (2-3x+x^2)^{1/3} + 2 \times 2^{1/3} (2-3x+x^2)^{2/3}}{(1+\sqrt{3} + 2^{2/3} (2-3x+x^2)^{1/3})^2}} \right. \\
& \left. \text{EllipticE}[\text{ArcSin}\left[\frac{1-\sqrt{3} + 2^{2/3} (2-3x+x^2)^{1/3}}{1+\sqrt{3} + 2^{2/3} (2-3x+x^2)^{1/3}}\right], -7-4\sqrt{3}] \right) / \\
& \left( 2 \times 2^{1/3} (3-2x) \sqrt{(3-2x)^2} (1-x)^{1/3} (2-x)^{1/3} \sqrt{\frac{1+2^{2/3} (2-3x+x^2)^{1/3}}{(1+\sqrt{3} + 2^{2/3} (2-3x+x^2)^{1/3})^2}} \right) - \\
& \left( 2^{1/6} \times 3^{3/4} \sqrt{(-3+2x)^2} (2-3x+x^2)^{1/3} (1+2^{2/3} (2-3x+x^2)^{1/3}) \sqrt{\frac{1-2^{2/3} (2-3x+x^2)^{1/3} + 2 \times 2^{1/3} (2-3x+x^2)^{2/3}}{(1+\sqrt{3} + 2^{2/3} (2-3x+x^2)^{1/3})^2}} \right. \\
& \left. \text{EllipticF}[\text{ArcSin}\left[\frac{1-\sqrt{3} + 2^{2/3} (2-3x+x^2)^{1/3}}{1+\sqrt{3} + 2^{2/3} (2-3x+x^2)^{1/3}}\right], -7-4\sqrt{3}] \right) / \\
& \left( (3-2x) \sqrt{(3-2x)^2} (1-x)^{1/3} (2-x)^{1/3} \sqrt{\frac{1+2^{2/3} (2-3x+x^2)^{1/3}}{(1+\sqrt{3} + 2^{2/3} (2-3x+x^2)^{1/3})^2}} \right)
\end{aligned}$$

Result (type 5, 26 leaves):

$$-\frac{3}{2} (1-x)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, -1+x\right]$$

Problem 880: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(1-x)^{1/3} (2-x)^{1/3} x} dx$$

Optimal (type 3, 99 leaves, 1 step):

$$\begin{aligned}
& - \frac{\sqrt{3} \text{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2^{1/3} (2-x)^{2/3}}{\sqrt{3} (1-x)^{1/3}}\right]}{2 \times 2^{1/3}} + \frac{3 \text{Log}\left[-(1-x)^{1/3} + \frac{(2-x)^{2/3}}{2^{2/3}}\right]}{4 \times 2^{1/3}} - \frac{\text{Log}[x]}{2 \times 2^{1/3}}
\end{aligned}$$

Result (type 6, 115 leaves):

$$\left( \frac{15 (1-x)^{2/3} \text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, 1, \frac{5}{3}, -1+x, 1-x\right]}{3} \right) / \left( 2 (2-x)^{1/3} x \right. \\ \left. \left( -5 \text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, 1, \frac{5}{3}, -1+x, 1-x\right] + (-1+x) \left( 3 \text{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, 2, \frac{8}{3}, -1+x, 1-x\right] - \text{AppellF1}\left[\frac{5}{3}, \frac{4}{3}, 1, \frac{8}{3}, -1+x, 1-x\right] \right) \right) \right)$$

Problem 881: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(1-x)^{1/3} (2-x)^{1/3} x^2} dx$$

Optimal (type 4, 796 leaves, 8 steps):

$$\begin{aligned} & -\frac{(1-x)^{2/3} (2-x)^{2/3}}{2x} - \frac{\sqrt{(3-2x)^2} \sqrt{(-3+2x)^2} (2-3x+x^2)^{1/3}}{2 \times 2^{1/3} (3-2x) (1-x)^{1/3} (2-x)^{1/3} (1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})} - \\ & \frac{\sqrt{3} \text{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2^{1/3} (2-x)^{2/3}}{\sqrt{3} (1-x)^{1/3}}\right]}{4 \times 2^{1/3}} + \left( 3^{1/4} \sqrt{2-\sqrt{3}} \sqrt{(-3+2x)^2} (2-3x+x^2)^{1/3} (1+2^{2/3} (2-3x+x^2)^{1/3}) \right. \\ & \left. \sqrt{\frac{1-2^{2/3} (2-3x+x^2)^{1/3} + 2 \times 2^{1/3} (2-3x+x^2)^{2/3}}{(1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3}}{1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3}}\right], -7-4\sqrt{3}\right] \right) / \\ & \left( 4 \times 2^{1/3} (3-2x) \sqrt{(3-2x)^2} (1-x)^{1/3} (2-x)^{1/3} \sqrt{\frac{1+2^{2/3} (2-3x+x^2)^{1/3}}{(1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})^2}} - \right. \\ & \left( \sqrt{(-3+2x)^2} (2-3x+x^2)^{1/3} (1+2^{2/3} (2-3x+x^2)^{1/3}) \sqrt{\frac{1-2^{2/3} (2-3x+x^2)^{1/3} + 2 \times 2^{1/3} (2-3x+x^2)^{2/3}}{(1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})^2}} \right. \\ & \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3}}{1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3}}\right], -7-4\sqrt{3}\right] \right) / \\ & \left( 2^{5/6} \times 3^{1/4} (3-2x) \sqrt{(3-2x)^2} (1-x)^{1/3} (2-x)^{1/3} \sqrt{\frac{1+2^{2/3} (2-3x+x^2)^{1/3}}{(1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})^2}} + \frac{3 \log\left[-(1-x)^{1/3} + \frac{(2-x)^{2/3}}{2^{2/3}}\right]}{8 \times 2^{1/3}} - \frac{\log[x]}{4 \times 2^{1/3}} \right) \end{aligned}$$

Result (type 6, 219 leaves):

$$\begin{aligned} & \frac{1}{10 (2-x)^{1/3} x} (1-x)^{2/3} \left( 5 (-2+x) - \left( 50 \text{AppellF1} \left[ \frac{2}{3}, \frac{1}{3}, 1, \frac{5}{3}, -1+x, 1-x \right] \right) \right) / \\ & \left( 5 \text{AppellF1} \left[ \frac{2}{3}, \frac{1}{3}, 1, \frac{5}{3}, -1+x, 1-x \right] - (-1+x) \left( 3 \text{AppellF1} \left[ \frac{5}{3}, \frac{1}{3}, 2, \frac{8}{3}, -1+x, 1-x \right] - \text{AppellF1} \left[ \frac{5}{3}, \frac{4}{3}, 1, \frac{8}{3}, -1+x, 1-x \right] \right) \right) + \\ & \left( 8 (-1+x) \text{AppellF1} \left[ \frac{5}{3}, \frac{1}{3}, 1, \frac{8}{3}, -1+x, 1-x \right] \right) / \\ & \left( -8 \text{AppellF1} \left[ \frac{5}{3}, \frac{1}{3}, 1, \frac{8}{3}, -1+x, 1-x \right] + (-1+x) \left( 3 \text{AppellF1} \left[ \frac{8}{3}, \frac{1}{3}, 2, \frac{11}{3}, -1+x, 1-x \right] - \text{AppellF1} \left[ \frac{8}{3}, \frac{4}{3}, 1, \frac{11}{3}, -1+x, 1-x \right] \right) \right) \end{aligned}$$

**Problem 882: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(1-x)^{1/3} (2-x)^{1/3} x^3} dx$$

Optimal (type 4, 821 leaves, 9 steps):

$$\begin{aligned}
& - \frac{(1-x)^{2/3} (2-x)^{2/3}}{4x^2} - \frac{(1-x)^{2/3} (2-x)^{2/3}}{2x} - \frac{\sqrt{(3-2x)^2} \sqrt{(-3+2x)^2} (2-3x+x^2)^{1/3}}{2 \times 2^{1/3} (3-2x) (1-x)^{1/3} (2-x)^{1/3} (1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})} - \\
& \frac{\text{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2^{1/3} (2-x)^{2/3}}{\sqrt{3} (1-x)^{1/3}}\right]}{2 \times 2^{1/3} \sqrt{3}} + \left( 3^{1/4} \sqrt{2-\sqrt{3}} \sqrt{(-3+2x)^2} (2-3x+x^2)^{1/3} (1+2^{2/3} (2-3x+x^2)^{1/3}) \right. \\
& \left. \sqrt{\frac{1-2^{2/3} (2-3x+x^2)^{1/3} + 2 \times 2^{1/3} (2-3x+x^2)^{2/3}}{(1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3}}{1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3}}\right], -7-4\sqrt{3}\right] \right) / \\
& \left( 4 \times 2^{1/3} (3-2x) \sqrt{(3-2x)^2} (1-x)^{1/3} (2-x)^{1/3} \sqrt{\frac{1+2^{2/3} (2-3x+x^2)^{1/3}}{(1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})^2}} \right. \\
& \left( \sqrt{(-3+2x)^2} (2-3x+x^2)^{1/3} (1+2^{2/3} (2-3x+x^2)^{1/3}) \sqrt{\frac{1-2^{2/3} (2-3x+x^2)^{1/3} + 2 \times 2^{1/3} (2-3x+x^2)^{2/3}}{(1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})^2}} \right. \\
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3}}{1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3}}\right], -7-4\sqrt{3}\right] \right) / \\
& \left( 2^{5/6} \times 3^{1/4} (3-2x) \sqrt{(3-2x)^2} (1-x)^{1/3} (2-x)^{1/3} \sqrt{\frac{1+2^{2/3} (2-3x+x^2)^{1/3}}{(1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})^2}} \right) + \frac{\text{Log}\left[-(1-x)^{1/3} + \frac{(2-x)^{2/3}}{2^{2/3}}\right]}{4 \times 2^{1/3}} - \frac{\text{Log}[x]}{6 \times 2^{1/3}}
\end{aligned}$$

Result (type 6, 225 leaves):

$$\begin{aligned}
& \frac{1}{20 (2-x)^{1/3} x^2} (1-x)^{2/3} \left( 5 (-2+x) (1+2x) + \left( 75 \times \text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, 1, \frac{5}{3}, -1+x, 1-x\right] \right) / \right. \\
& \left. \left( -5 \text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, 1, \frac{5}{3}, -1+x, 1-x\right] + (-1+x) \left( 3 \text{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, 2, \frac{8}{3}, -1+x, 1-x\right] - \text{AppellF1}\left[\frac{5}{3}, \frac{4}{3}, 1, \frac{8}{3}, -1+x, 1-x\right] \right) \right) + \\
& \left( 16 (-1+x) \times \text{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, 1, \frac{8}{3}, -1+x, 1-x\right] \right) / \\
& \left. \left( -8 \text{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, 1, \frac{8}{3}, -1+x, 1-x\right] + (-1+x) \left( 3 \text{AppellF1}\left[\frac{8}{3}, \frac{1}{3}, 2, \frac{11}{3}, -1+x, 1-x\right] - \text{AppellF1}\left[\frac{8}{3}, \frac{4}{3}, 1, \frac{11}{3}, -1+x, 1-x\right] \right) \right)
\end{aligned}$$

### Problem 883: Result unnecessarily involves higher level functions.

$$\int \frac{x^3 (a + b x)^{1/4}}{(c + d x)^{1/4}} dx$$

Optimal (type 3, 340 leaves, 8 steps):

$$\begin{aligned} & -\frac{(195 b^3 c^3 + 135 a b^2 c^2 d + 105 a^2 b c d^2 + 77 a^3 d^3) (a + b x)^{1/4} (c + d x)^{3/4}}{512 b^3 d^4} + \\ & \frac{x^2 (a + b x)^{5/4} (c + d x)^{3/4}}{4 b d} + \frac{(a + b x)^{5/4} (c + d x)^{3/4} (117 b^2 c^2 + 94 a b c d + 77 a^2 d^2 - 8 b d (13 b c + 11 a d)) x}{384 b^3 d^3} + \\ & \frac{(b c - a d) (195 b^3 c^3 + 135 a b^2 c^2 d + 105 a^2 b c d^2 + 77 a^3 d^3) \operatorname{ArcTan}\left[\frac{d^{1/4} (a+b x)^{1/4}}{b^{1/4} (c+d x)^{1/4}}\right]}{1024 b^{15/4} d^{17/4}} + \\ & \frac{(b c - a d) (195 b^3 c^3 + 135 a b^2 c^2 d + 105 a^2 b c d^2 + 77 a^3 d^3) \operatorname{ArcTanh}\left[\frac{d^{1/4} (a+b x)^{1/4}}{b^{1/4} (c+d x)^{1/4}}\right]}{1024 b^{15/4} d^{17/4}} \end{aligned}$$

Result (type 5, 221 leaves):

$$\begin{aligned} & \frac{1}{1536 b^3 d^5 (a + b x)^{3/4}} \\ & (c + d x)^{3/4} \left( d (a + b x) (77 a^3 d^3 + a^2 b d^2 (61 c - 44 d x) + a b^2 d (63 c^2 - 40 c d x + 32 d^2 x^2) + b^3 (-585 c^3 + 468 c^2 d x - 416 c d^2 x^2 + 384 d^3 x^3)) - \right. \\ & \left. (-195 b^4 c^4 + 60 a b^3 c^3 d + 30 a^2 b^2 c^2 d^2 + 28 a^3 b c d^3 + 77 a^4 d^4) \left( \frac{d (a + b x)}{-b c + a d} \right)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b (c + d x)}{b c - a d}\right] \right) \end{aligned}$$

### Problem 884: Result unnecessarily involves higher level functions.

$$\int \frac{x^2 (a + b x)^{1/4}}{(c + d x)^{1/4}} dx$$

Optimal (type 3, 268 leaves, 8 steps):

$$\begin{aligned} & \frac{(15 b^2 c^2 + 10 a b c d + 7 a^2 d^2) (a + b x)^{1/4} (c + d x)^{3/4}}{32 b^2 d^3} - \frac{(9 b c + 7 a d) (a + b x)^{5/4} (c + d x)^{3/4}}{24 b^2 d^2} + \frac{x (a + b x)^{5/4} (c + d x)^{3/4}}{3 b d} - \\ & \frac{(b c - a d) (15 b^2 c^2 + 10 a b c d + 7 a^2 d^2) \operatorname{ArcTan}\left[\frac{d^{1/4} (a+b x)^{1/4}}{b^{1/4} (c+d x)^{1/4}}\right]}{64 b^{11/4} d^{13/4}} - \frac{(b c - a d) (15 b^2 c^2 + 10 a b c d + 7 a^2 d^2) \operatorname{ArcTanh}\left[\frac{d^{1/4} (a+b x)^{1/4}}{b^{1/4} (c+d x)^{1/4}}\right]}{64 b^{11/4} d^{13/4}} \end{aligned}$$

Result (type 5, 168 leaves) :

$$\frac{1}{96 b^2 d^4 (a + b x)^{3/4}} (c + d x)^{3/4} \left( -d (a + b x) (7 a^2 d^2 + 2 a b d (3 c - 2 d x) + b^2 (-45 c^2 + 36 c d x - 32 d^2 x^2)) + (-15 b^3 c^3 + 5 a b^2 c^2 d + 3 a^2 b c d^2 + 7 a^3 d^3) \left( \frac{d (a + b x)}{-b c + a d} \right)^{3/4} \text{Hypergeometric2F1} \left[ \frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b (c + d x)}{b c - a d} \right] \right)$$

Problem 885: Result unnecessarily involves higher level functions.

$$\int \frac{x (a + b x)^{1/4}}{(c + d x)^{1/4}} dx$$

Optimal (type 3, 188 leaves, 7 steps) :

$$\begin{aligned} & -\frac{(5 b c + 3 a d) (a + b x)^{1/4} (c + d x)^{3/4}}{8 b d^2} + \frac{(a + b x)^{5/4} (c + d x)^{3/4}}{2 b d} + \\ & \frac{(b c - a d) (5 b c + 3 a d) \text{ArcTan} \left[ \frac{d^{1/4} (a + b x)^{1/4}}{b^{1/4} (c + d x)^{1/4}} \right]}{16 b^{7/4} d^{9/4}} + \frac{(b c - a d) (5 b c + 3 a d) \text{ArcTanh} \left[ \frac{d^{1/4} (a + b x)^{1/4}}{b^{1/4} (c + d x)^{1/4}} \right]}{16 b^{7/4} d^{9/4}} \end{aligned}$$

Result (type 5, 122 leaves) :

$$\frac{1}{24 b d^3 (a + b x)^{3/4}} (c + d x)^{3/4} \left( 3 d (a + b x) (-5 b c + a d + 4 b d x) + (5 b^2 c^2 - 2 a b c d - 3 a^2 d^2) \left( \frac{d (a + b x)}{-b c + a d} \right)^{3/4} \text{Hypergeometric2F1} \left[ \frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b (c + d x)}{b c - a d} \right] \right)$$

Problem 886: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{1/4}}{(c + d x)^{1/4}} dx$$

Optimal (type 3, 127 leaves, 6 steps) :

$$\frac{(a + b x)^{1/4} (c + d x)^{3/4}}{d} - \frac{(b c - a d) \text{ArcTan} \left[ \frac{d^{1/4} (a + b x)^{1/4}}{b^{1/4} (c + d x)^{1/4}} \right]}{2 b^{3/4} d^{5/4}} - \frac{(b c - a d) \text{ArcTanh} \left[ \frac{d^{1/4} (a + b x)^{1/4}}{b^{1/4} (c + d x)^{1/4}} \right]}{2 b^{3/4} d^{5/4}}$$

Result (type 5, 76 leaves) :

$$\frac{(a+b x)^{1/4} (c+d x)^{3/4}}{3 d} \left( 3 + \frac{\text{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b (c+d x)}{b c-a d}\right]}{\left(\frac{d (a+b x)}{-b c+a d}\right)^{1/4}} \right)$$

**Problem 887:** Result unnecessarily involves higher level functions.

$$\int \frac{(a+b x)^{1/4}}{x (c+d x)^{1/4}} dx$$

Optimal (type 3, 169 leaves, 11 steps):

$$-\frac{2 a^{1/4} \text{ArcTan}\left[\frac{c^{1/4} (a+b x)^{1/4}}{a^{1/4} (c+d x)^{1/4}}\right]}{c^{1/4}} + \frac{2 b^{1/4} \text{ArcTan}\left[\frac{d^{1/4} (a+b x)^{1/4}}{b^{1/4} (c+d x)^{1/4}}\right]}{d^{1/4}} - \frac{2 a^{1/4} \text{ArcTanh}\left[\frac{c^{1/4} (a+b x)^{1/4}}{a^{1/4} (c+d x)^{1/4}}\right]}{c^{1/4}} + \frac{2 b^{1/4} \text{ArcTanh}\left[\frac{d^{1/4} (a+b x)^{1/4}}{b^{1/4} (c+d x)^{1/4}}\right]}{d^{1/4}}$$

Result (type 6, 216 leaves):

$$\begin{aligned} & \left( 36 a (b c - a d) (a+b x)^{5/4} \text{AppellF1}\left[\frac{5}{4}, \frac{1}{4}, 1, \frac{9}{4}, \frac{d (a+b x)}{-b c + a d}, 1 + \frac{b x}{a}\right] \right) / \\ & \left( 5 b x (c+d x)^{1/4} \left( 9 a (b c - a d) \text{AppellF1}\left[\frac{5}{4}, \frac{1}{4}, 1, \frac{9}{4}, \frac{d (a+b x)}{-b c + a d}, 1 + \frac{b x}{a}\right] - \right. \right. \\ & \left. \left. (a+b x) \left( (-4 b c + 4 a d) \text{AppellF1}\left[\frac{9}{4}, \frac{1}{4}, 2, \frac{13}{4}, \frac{d (a+b x)}{-b c + a d}, 1 + \frac{b x}{a}\right] + a d \text{AppellF1}\left[\frac{9}{4}, \frac{5}{4}, 1, \frac{13}{4}, \frac{d (a+b x)}{-b c + a d}, 1 + \frac{b x}{a}\right] \right) \right) \right) \end{aligned}$$

**Problem 888:** Result unnecessarily involves higher level functions.

$$\int \frac{(a+b x)^{1/4}}{x^2 (c+d x)^{1/4}} dx$$

Optimal (type 3, 131 leaves, 5 steps):

$$-\frac{(a+b x)^{1/4} (c+d x)^{3/4}}{c x} - \frac{(b c - a d) \text{ArcTan}\left[\frac{c^{1/4} (a+b x)^{1/4}}{a^{1/4} (c+d x)^{1/4}}\right]}{2 a^{3/4} c^{5/4}} - \frac{(b c - a d) \text{ArcTanh}\left[\frac{c^{1/4} (a+b x)^{1/4}}{a^{1/4} (c+d x)^{1/4}}\right]}{2 a^{3/4} c^{5/4}}$$

Result (type 6, 176 leaves):

$$\begin{aligned} & - (a+b x) (c+d x) + \frac{2 b d (b c - a d) x^2 \text{AppellF1}\left[1, \frac{3}{4}, \frac{1}{4}, 2, -\frac{a}{b x}, -\frac{c}{d x}\right]}{-8 b d x \text{AppellF1}\left[1, \frac{3}{4}, \frac{1}{4}, 2, -\frac{a}{b x}, -\frac{c}{d x}\right] + b c \text{AppellF1}\left[2, \frac{3}{4}, \frac{5}{4}, 3, -\frac{a}{b x}, -\frac{c}{d x}\right] + 3 a d \text{AppellF1}\left[2, \frac{7}{4}, \frac{1}{4}, 3, -\frac{a}{b x}, -\frac{c}{d x}\right]} \\ & c x (a+b x)^{3/4} (c+d x)^{1/4} \end{aligned}$$

### Problem 889: Result unnecessarily involves higher level functions.

$$\int \frac{(a+b x)^{1/4}}{x^3 (c+d x)^{1/4}} dx$$

Optimal (type 3, 194 leaves, 6 steps):

$$\begin{aligned} & \frac{(3 b c + 5 a d) (a+b x)^{1/4} (c+d x)^{3/4}}{8 a c^2 x} - \frac{(a+b x)^{5/4} (c+d x)^{3/4}}{2 a c x^2} + \\ & \frac{(b c - a d) (3 b c + 5 a d) \operatorname{ArcTan}\left[\frac{c^{1/4} (a+b x)^{1/4}}{a^{1/4} (c+d x)^{1/4}}\right]}{16 a^{7/4} c^{9/4}} + \frac{(b c - a d) (3 b c + 5 a d) \operatorname{ArcTanh}\left[\frac{c^{1/4} (a+b x)^{1/4}}{a^{1/4} (c+d x)^{1/4}}\right]}{16 a^{7/4} c^{9/4}} \end{aligned}$$

Result (type 6, 211 leaves):

$$\begin{aligned} & \left( (a+b x) (c+d x) (-4 a c - b c x + 5 a d x) + \right. \\ & \left. \left( 2 b d (-3 b^2 c^2 - 2 a b c d + 5 a^2 d^2) x^3 \operatorname{AppellF1}\left[1, \frac{3}{4}, \frac{1}{4}, 2, -\frac{a}{b x}, -\frac{c}{d x}\right] \right) / \left( -8 b d x \operatorname{AppellF1}\left[1, \frac{3}{4}, \frac{1}{4}, 2, -\frac{a}{b x}, -\frac{c}{d x}\right] + \right. \right. \\ & \left. \left. b c \operatorname{AppellF1}\left[2, \frac{3}{4}, \frac{5}{4}, 3, -\frac{a}{b x}, -\frac{c}{d x}\right] + 3 a d \operatorname{AppellF1}\left[2, \frac{7}{4}, \frac{1}{4}, 3, -\frac{a}{b x}, -\frac{c}{d x}\right] \right) / \left( 8 a c^2 x^2 (a+b x)^{3/4} (c+d x)^{1/4} \right) \right) \end{aligned}$$

### Problem 890: Result unnecessarily involves higher level functions.

$$\int \frac{(a+b x)^{1/4}}{x^4 (c+d x)^{1/4}} dx$$

Optimal (type 3, 266 leaves, 8 steps):

$$\begin{aligned} & - \frac{(a+b x)^{1/4} (c+d x)^{3/4}}{3 c x^3} - \frac{(b c - 9 a d) (a+b x)^{1/4} (c+d x)^{3/4}}{24 a c^2 x^2} + \frac{(7 b c - 15 a d) (b c + 3 a d) (a+b x)^{1/4} (c+d x)^{3/4}}{96 a^2 c^3 x} - \\ & \frac{(b c - a d) (7 b^2 c^2 + 10 a b c d + 15 a^2 d^2) \operatorname{ArcTan}\left[\frac{c^{1/4} (a+b x)^{1/4}}{a^{1/4} (c+d x)^{1/4}}\right]}{64 a^{11/4} c^{13/4}} - \frac{(b c - a d) (7 b^2 c^2 + 10 a b c d + 15 a^2 d^2) \operatorname{ArcTanh}\left[\frac{c^{1/4} (a+b x)^{1/4}}{a^{1/4} (c+d x)^{1/4}}\right]}{64 a^{11/4} c^{13/4}} \end{aligned}$$

Result (type 6, 260 leaves):

$$\begin{aligned} & \left( - (a+b x) (c+d x) (-7 b^2 c^2 x^2 + 2 a b c x (2 c - 3 d x) + a^2 (32 c^2 - 36 c d x + 45 d^2 x^2)) + \right. \\ & \left. \left( 6 b d (7 b^3 c^3 + 3 a b^2 c^2 d + 5 a^2 b c d^2 - 15 a^3 d^3) x^4 \operatorname{AppellF1}\left[1, \frac{3}{4}, \frac{1}{4}, 2, -\frac{a}{b x}, -\frac{c}{d x}\right] \right) / \left( -8 b d x \operatorname{AppellF1}\left[1, \frac{3}{4}, \frac{1}{4}, 2, -\frac{a}{b x}, -\frac{c}{d x}\right] + \right. \right. \\ & \left. \left. b c \operatorname{AppellF1}\left[2, \frac{3}{4}, \frac{5}{4}, 3, -\frac{a}{b x}, -\frac{c}{d x}\right] + 3 a d \operatorname{AppellF1}\left[2, \frac{7}{4}, \frac{1}{4}, 3, -\frac{a}{b x}, -\frac{c}{d x}\right] \right) \right) / \left( 96 a^2 c^3 x^3 (a+b x)^{3/4} (c+d x)^{1/4} \right) \end{aligned}$$

### Problem 891: Result unnecessarily involves higher level functions.

$$\int \frac{(a+b x)^{1/4}}{x^5 (c+d x)^{1/4}} dx$$

Optimal (type 3, 368 leaves, 9 steps):

$$\begin{aligned} & -\frac{(a+b x)^{1/4} (c+d x)^{3/4}}{4 c x^4} - \frac{(b c - 13 a d) (a+b x)^{1/4} (c+d x)^{3/4}}{48 a c^2 x^3} + \\ & \frac{(11 b^2 c^2 + 10 a b c d - 117 a^2 d^2) (a+b x)^{1/4} (c+d x)^{3/4}}{384 a^2 c^3 x^2} - \frac{(77 b^3 c^3 + 61 a b^2 c^2 d + 63 a^2 b c d^2 - 585 a^3 d^3) (a+b x)^{1/4} (c+d x)^{3/4}}{1536 a^3 c^4 x} + \\ & \frac{(b c - a d) (77 b^3 c^3 + 105 a b^2 c^2 d + 135 a^2 b c d^2 + 195 a^3 d^3) \operatorname{ArcTan}\left[\frac{c^{1/4} (a+b x)^{1/4}}{a^{1/4} (c+d x)^{1/4}}\right]}{1024 a^{15/4} c^{17/4}} + \\ & \frac{(b c - a d) (77 b^3 c^3 + 105 a b^2 c^2 d + 135 a^2 b c d^2 + 195 a^3 d^3) \operatorname{ArcTanh}\left[\frac{c^{1/4} (a+b x)^{1/4}}{a^{1/4} (c+d x)^{1/4}}\right]}{1024 a^{15/4} c^{17/4}} \end{aligned}$$

Result (type 6, 315 leaves):

$$\begin{aligned} & \left( (a+b x) (c+d x) (-77 b^3 c^3 x^3 + a b^2 c^2 x^2 (44 c - 61 d x) + a^2 b c x (-32 c^2 + 40 c d x - 63 d^2 x^2) + a^3 (-384 c^3 + 416 c^2 d x - 468 c d^2 x^2 + 585 d^3 x^3)) - \right. \\ & \left. \left( 6 b d (77 b^4 c^4 + 28 a b^3 c^3 d + 30 a^2 b^2 c^2 d^2 + 60 a^3 b c d^3 - 195 a^4 d^4) x^5 \operatorname{AppellF1}\left[1, \frac{3}{4}, \frac{1}{4}, 2, -\frac{a}{b x}, -\frac{c}{d x}\right] \right) / \right. \\ & \left. \left( -8 b d x \operatorname{AppellF1}\left[1, \frac{3}{4}, \frac{1}{4}, 2, -\frac{a}{b x}, -\frac{c}{d x}\right] + b c \operatorname{AppellF1}\left[2, \frac{3}{4}, \frac{5}{4}, 3, -\frac{a}{b x}, -\frac{c}{d x}\right] + 3 a d \operatorname{AppellF1}\left[2, \frac{7}{4}, \frac{1}{4}, 3, -\frac{a}{b x}, -\frac{c}{d x}\right] \right) \right) / \\ & (1536 a^3 c^4 x^4 (a+b x)^{3/4} (c+d x)^{1/4}) \end{aligned}$$

### Problem 892: Result unnecessarily involves higher level functions.

$$\int \frac{x^2 (1+x)^{1/4}}{(1-x)^{1/4}} dx$$

Optimal (type 3, 234 leaves, 14 steps):

$$\begin{aligned} & -\frac{3}{8} (1-x)^{3/4} (1+x)^{1/4} - \frac{1}{12} (1-x)^{3/4} (1+x)^{5/4} - \frac{1}{3} (1-x)^{3/4} x (1+x)^{5/4} + \\ & \frac{3 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (1-x)^{1/4}}{(1+x)^{1/4}}\right]}{8 \sqrt{2}} - \frac{3 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (1-x)^{1/4}}{(1+x)^{1/4}}\right]}{8 \sqrt{2}} - \frac{3 \operatorname{Log}\left[1 + \frac{\sqrt{1-x}}{\sqrt{1+x}} - \frac{\sqrt{2} (1-x)^{1/4}}{(1+x)^{1/4}}\right]}{16 \sqrt{2}} + \frac{3 \operatorname{Log}\left[1 + \frac{\sqrt{1-x}}{\sqrt{1+x}} + \frac{\sqrt{2} (1-x)^{1/4}}{(1+x)^{1/4}}\right]}{16 \sqrt{2}} \end{aligned}$$

Result (type 5, 57 leaves) :

$$\frac{1}{24} (1+x)^{1/4} \left( - (1-x)^{3/4} (11 + 10x + 8x^2) + 9 \times 2^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \frac{1+x}{2}\right] \right)$$

Problem 893: Result unnecessarily involves higher level functions.

$$\int \frac{x (1+x)^{1/4}}{(1-x)^{1/4}} dx$$

Optimal (type 3, 213 leaves, 13 steps) :

$$\begin{aligned} & -\frac{1}{4} (1-x)^{3/4} (1+x)^{1/4} - \frac{1}{2} (1-x)^{3/4} (1+x)^{5/4} + \frac{\text{ArcTan}\left[1 - \frac{\sqrt{2} (1-x)^{1/4}}{(1+x)^{1/4}}\right]}{4 \sqrt{2}} - \\ & \frac{\text{ArcTan}\left[1 + \frac{\sqrt{2} (1-x)^{1/4}}{(1+x)^{1/4}}\right]}{4 \sqrt{2}} - \frac{\text{Log}\left[1 + \frac{\sqrt{1-x}}{\sqrt{1+x}} - \frac{\sqrt{2} (1-x)^{1/4}}{(1+x)^{1/4}}\right]}{8 \sqrt{2}} + \frac{\text{Log}\left[1 + \frac{\sqrt{1-x}}{\sqrt{1+x}} + \frac{\sqrt{2} (1-x)^{1/4}}{(1+x)^{1/4}}\right]}{8 \sqrt{2}} \end{aligned}$$

Result (type 5, 51 leaves) :

$$\frac{1}{4} (1+x)^{1/4} \left( - (1-x)^{3/4} (3 + 2x) + 2^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \frac{1+x}{2}\right] \right)$$

Problem 894: Result unnecessarily involves higher level functions.

$$\int \frac{(1+x)^{1/4}}{(1-x)^{1/4}} dx$$

Optimal (type 3, 186 leaves, 12 steps) :

$$\begin{aligned} & - (1-x)^{3/4} (1+x)^{1/4} + \frac{\text{ArcTan}\left[1 - \frac{\sqrt{2} (1-x)^{1/4}}{(1+x)^{1/4}}\right]}{\sqrt{2}} - \frac{\text{ArcTan}\left[1 + \frac{\sqrt{2} (1-x)^{1/4}}{(1+x)^{1/4}}\right]}{\sqrt{2}} - \frac{\text{Log}\left[1 + \frac{\sqrt{1-x}}{\sqrt{1+x}} - \frac{\sqrt{2} (1-x)^{1/4}}{(1+x)^{1/4}}\right]}{2 \sqrt{2}} + \frac{\text{Log}\left[1 + \frac{\sqrt{1-x}}{\sqrt{1+x}} + \frac{\sqrt{2} (1-x)^{1/4}}{(1+x)^{1/4}}\right]}{2 \sqrt{2}} \end{aligned}$$

Result (type 5, 43 leaves) :

$$(1+x)^{1/4} \left( - (1-x)^{3/4} + 2^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \frac{1+x}{2}\right] \right)$$

### Problem 895: Result unnecessarily involves higher level functions.

$$\int \frac{(1+x)^{1/4}}{(1-x)^{1/4} x} dx$$

Optimal (type 3, 203 leaves, 16 steps):

$$\begin{aligned} & -2 \operatorname{ArcTan}\left[\frac{(1+x)^{1/4}}{(1-x)^{1/4}}\right] + \sqrt{2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (1-x)^{1/4}}{(1+x)^{1/4}}\right] - \sqrt{2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (1-x)^{1/4}}{(1+x)^{1/4}}\right] - \\ & 2 \operatorname{Arctanh}\left[\frac{(1+x)^{1/4}}{(1-x)^{1/4}}\right] - \frac{\operatorname{Log}\left[1 + \frac{\sqrt{1-x}}{\sqrt{1+x}} - \frac{\sqrt{2} (1-x)^{1/4}}{(1+x)^{1/4}}\right]}{\sqrt{2}} + \frac{\operatorname{Log}\left[1 + \frac{\sqrt{1-x}}{\sqrt{1+x}} + \frac{\sqrt{2} (1-x)^{1/4}}{(1+x)^{1/4}}\right]}{\sqrt{2}} \end{aligned}$$

Result (type 6, 119 leaves):

$$\begin{aligned} & \left(72 (1+x)^{5/4} \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{4}, 1, \frac{9}{4}, \frac{1+x}{2}, 1+x\right]\right) / \\ & \left(5 (1-x)^{1/4} \times \left(18 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{4}, 1, \frac{9}{4}, \frac{1+x}{2}, 1+x\right] + (1+x) \left(8 \operatorname{AppellF1}\left[\frac{9}{4}, \frac{1}{4}, 2, \frac{13}{4}, \frac{1+x}{2}, 1+x\right] + \operatorname{AppellF1}\left[\frac{9}{4}, \frac{5}{4}, 1, \frac{13}{4}, \frac{1+x}{2}, 1+x\right]\right)\right)\right) \end{aligned}$$

### Problem 896: Result unnecessarily involves higher level functions.

$$\int \frac{(1+x)^{1/4}}{(1-x)^{1/4} x^2} dx$$

Optimal (type 3, 62 leaves, 5 steps):

$$-\frac{(1-x)^{3/4} (1+x)^{1/4}}{x} - \operatorname{ArcTan}\left[\frac{(1+x)^{1/4}}{(1-x)^{1/4}}\right] - \operatorname{Arctanh}\left[\frac{(1+x)^{1/4}}{(1-x)^{1/4}}\right]$$

Result (type 6, 106 leaves):

$$\begin{aligned} & -1 + x^2 - \frac{4 x^2 \operatorname{AppellF1}\left[1, \frac{1}{4}, \frac{3}{4}, 2, \frac{1}{x}, -\frac{1}{x}\right]}{8 \operatorname{AppellF1}\left[1, \frac{1}{4}, \frac{3}{4}, 2, \frac{1}{x}, -\frac{1}{x}\right] - 3 \operatorname{AppellF1}\left[2, \frac{1}{4}, \frac{7}{4}, 3, \frac{1}{x}, -\frac{1}{x}\right] + \operatorname{AppellF1}\left[2, \frac{5}{4}, \frac{3}{4}, 3, \frac{1}{x}, -\frac{1}{x}\right]} \\ & \quad (1-x)^{1/4} x (1+x)^{3/4} \end{aligned}$$

### Problem 897: Result unnecessarily involves higher level functions.

$$\int \frac{(1+x)^{1/4}}{(1-x)^{1/4} x^3} dx$$

Optimal (type 3, 91 leaves, 6 steps):

$$-\frac{(1-x)^{3/4}(1+x)^{1/4}}{4x} - \frac{(1-x)^{3/4}(1+x)^{5/4}}{2x^2} - \frac{1}{4} \text{ArcTan}\left[\frac{(1+x)^{1/4}}{(1-x)^{1/4}}\right] - \frac{1}{4} \text{ArcTanh}\left[\frac{(1+x)^{1/4}}{(1-x)^{1/4}}\right]$$

Result (type 6, 114 leaves):

$$\frac{2 - \frac{2}{x^2} - \frac{3}{x} + 3x - \frac{4 \times \text{AppellF1}\left[1, \frac{1}{4}, \frac{3}{4}, 2, \frac{1}{x}, -\frac{1}{x}\right]}{8 \times \text{AppellF1}\left[1, \frac{1}{4}, \frac{3}{4}, 2, \frac{1}{x}, -\frac{1}{x}\right] - 3 \times \text{AppellF1}\left[2, \frac{1}{4}, \frac{7}{4}, 3, \frac{1}{x}, -\frac{1}{x}\right] + \text{AppellF1}\left[2, \frac{5}{4}, \frac{3}{4}, 3, \frac{1}{x}, -\frac{1}{x}\right]}}{4(1-x)^{1/4}(1+x)^{3/4}}$$

Problem 898: Result unnecessarily involves higher level functions.

$$\int \frac{(1+x)^{1/4}}{(1-x)^{1/4}x^4} dx$$

Optimal (type 3, 114 leaves, 8 steps):

$$-\frac{(1-x)^{3/4}(1+x)^{1/4}}{3x^3} - \frac{5(1-x)^{3/4}(1+x)^{1/4}}{12x^2} - \frac{11(1-x)^{3/4}(1+x)^{1/4}}{24x} - \frac{3}{8} \text{ArcTan}\left[\frac{(1+x)^{1/4}}{(1-x)^{1/4}}\right] - \frac{3}{8} \text{ArcTanh}\left[\frac{(1+x)^{1/4}}{(1-x)^{1/4}}\right]$$

Result (type 6, 119 leaves):

$$\frac{10 - \frac{8}{x^3} - \frac{10}{x^2} - \frac{3}{x} + 11x - \frac{36 \times \text{AppellF1}\left[1, \frac{1}{4}, \frac{3}{4}, 2, \frac{1}{x}, -\frac{1}{x}\right]}{8 \times \text{AppellF1}\left[1, \frac{1}{4}, \frac{3}{4}, 2, \frac{1}{x}, -\frac{1}{x}\right] - 3 \times \text{AppellF1}\left[2, \frac{1}{4}, \frac{7}{4}, 3, \frac{1}{x}, -\frac{1}{x}\right] + \text{AppellF1}\left[2, \frac{5}{4}, \frac{3}{4}, 3, \frac{1}{x}, -\frac{1}{x}\right]}}{24(1-x)^{1/4}(1+x)^{3/4}}$$

Problem 899: Result unnecessarily involves higher level functions.

$$\int \frac{(1+x)^{1/4}}{(1-x)^{1/4}x^5} dx$$

Optimal (type 3, 137 leaves, 9 steps):

$$-\frac{(1-x)^{3/4}(1+x)^{1/4}}{4x^4} - \frac{7(1-x)^{3/4}(1+x)^{1/4}}{24x^3} - \frac{29(1-x)^{3/4}(1+x)^{1/4}}{96x^2} - \frac{83(1-x)^{3/4}(1+x)^{1/4}}{192x} - \frac{11}{64} \text{ArcTan}\left[\frac{(1+x)^{1/4}}{(1-x)^{1/4}}\right] - \frac{11}{64} \text{ArcTanh}\left[\frac{(1+x)^{1/4}}{(1-x)^{1/4}}\right]$$

Result (type 6, 124 leaves):

$$\frac{58 - \frac{48}{x^4} - \frac{56}{x^3} - \frac{10}{x^2} - \frac{27}{x} + 83x - \frac{132 \times \text{AppellF1}\left[1, \frac{1}{4}, \frac{3}{4}, 2, \frac{1}{x}, -\frac{1}{x}\right]}{8 \times \text{AppellF1}\left[1, \frac{1}{4}, \frac{3}{4}, 2, \frac{1}{x}, -\frac{1}{x}\right] - 3 \times \text{AppellF1}\left[2, \frac{1}{4}, \frac{7}{4}, 3, \frac{1}{x}, -\frac{1}{x}\right] + \text{AppellF1}\left[2, \frac{5}{4}, \frac{3}{4}, 3, \frac{1}{x}, -\frac{1}{x}\right]}}{192(1-x)^{1/4}(1+x)^{3/4}}$$

### Problem 900: Result unnecessarily involves higher level functions.

$$\int \frac{x^3}{(a + b x)^{3/4} (c + d x)^{1/4}} dx$$

Optimal (type 3, 259 leaves, 7 steps):

$$\begin{aligned} & \frac{x^2 (a + b x)^{1/4} (c + d x)^{3/4}}{3 b d} + \frac{(a + b x)^{1/4} (c + d x)^{3/4} (45 b^2 c^2 + 54 a b c d + 77 a^2 d^2 - 4 b d (9 b c + 11 a d) x)}{96 b^3 d^3} - \\ & \frac{(15 b^3 c^3 + 15 a b^2 c^2 d + 21 a^2 b c d^2 + 77 a^3 d^3) \operatorname{ArcTan}\left[\frac{d^{1/4} (a+b x)^{1/4}}{b^{1/4} (c+d x)^{1/4}}\right]}{64 b^{15/4} d^{13/4}} - \frac{(15 b^3 c^3 + 15 a b^2 c^2 d + 21 a^2 b c d^2 + 77 a^3 d^3) \operatorname{ArcTanh}\left[\frac{d^{1/4} (a+b x)^{1/4}}{b^{1/4} (c+d x)^{1/4}}\right]}{64 b^{15/4} d^{13/4}} \end{aligned}$$

Result (type 5, 168 leaves):

$$\begin{aligned} & \frac{1}{96 b^3 d^4 (a + b x)^{3/4}} (c + d x)^{3/4} \left( d (a + b x) (77 a^2 d^2 + 2 a b d (27 c - 22 d x) + b^2 (45 c^2 - 36 c d x + 32 d^2 x^2)) - \right. \\ & \left. (15 b^3 c^3 + 15 a b^2 c^2 d + 21 a^2 b c d^2 + 77 a^3 d^3) \left(\frac{d (a + b x)}{-b c + a d}\right)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b (c + d x)}{b c - a d}\right] \right) \end{aligned}$$

### Problem 901: Result unnecessarily involves higher level functions.

$$\int \frac{x^2}{(a + b x)^{3/4} (c + d x)^{1/4}} dx$$

Optimal (type 3, 201 leaves, 7 steps):

$$\begin{aligned} & - \frac{(5 b c + 7 a d) (a + b x)^{1/4} (c + d x)^{3/4}}{8 b^2 d^2} + \frac{x (a + b x)^{1/4} (c + d x)^{3/4}}{2 b d} + \\ & \frac{(5 b^2 c^2 + 6 a b c d + 21 a^2 d^2) \operatorname{ArcTan}\left[\frac{d^{1/4} (a+b x)^{1/4}}{b^{1/4} (c+d x)^{1/4}}\right]}{16 b^{11/4} d^{9/4}} + \frac{(5 b^2 c^2 + 6 a b c d + 21 a^2 d^2) \operatorname{ArcTanh}\left[\frac{d^{1/4} (a+b x)^{1/4}}{b^{1/4} (c+d x)^{1/4}}\right]}{16 b^{11/4} d^{9/4}} \end{aligned}$$

Result (type 5, 123 leaves):

$$\begin{aligned} & \frac{1}{24 b^2 d^3 (a + b x)^{3/4}} \\ & (c + d x)^{3/4} \left( -3 d (a + b x) (5 b c + 7 a d - 4 b d x) + (5 b^2 c^2 + 6 a b c d + 21 a^2 d^2) \left(\frac{d (a + b x)}{-b c + a d}\right)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b (c + d x)}{b c - a d}\right] \right) \end{aligned}$$

### Problem 902: Result unnecessarily involves higher level functions.

$$\int \frac{x}{(a + b x)^{3/4} (c + d x)^{1/4}} dx$$

Optimal (type 3, 130 leaves, 6 steps) :

$$\frac{(a + b x)^{1/4} (c + d x)^{3/4}}{b d} - \frac{(b c + 3 a d) \operatorname{ArcTan}\left[\frac{d^{1/4} (a+b x)^{1/4}}{b^{1/4} (c+d x)^{1/4}}\right]}{2 b^{7/4} d^{5/4}} - \frac{(b c + 3 a d) \operatorname{ArcTanh}\left[\frac{d^{1/4} (a+b x)^{1/4}}{b^{1/4} (c+d x)^{1/4}}\right]}{2 b^{7/4} d^{5/4}}$$

Result (type 5, 95 leaves) :

$$\frac{(c + d x)^{3/4} \left(3 d (a + b x) - (b c + 3 a d) \left(\frac{d (a+b x)}{-b c+a d}\right)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b (c+d x)}{b c-a d}\right]\right)}{3 b d^2 (a + b x)^{3/4}}$$

### Problem 903: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + b x)^{3/4} (c + d x)^{1/4}} dx$$

Optimal (type 3, 85 leaves, 5 steps) :

$$\frac{2 \operatorname{ArcTan}\left[\frac{d^{1/4} (a+b x)^{1/4}}{b^{1/4} (c+d x)^{1/4}}\right]}{b^{3/4} d^{1/4}} + \frac{2 \operatorname{ArcTanh}\left[\frac{d^{1/4} (a+b x)^{1/4}}{b^{1/4} (c+d x)^{1/4}}\right]}{b^{3/4} d^{1/4}}$$

Result (type 5, 73 leaves) :

$$\frac{4 \left(\frac{d (a+b x)}{-b c+a d}\right)^{3/4} (c + d x)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b (c+d x)}{b c-a d}\right]}{3 d (a + b x)^{3/4}}$$

### Problem 904: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x (a + b x)^{3/4} (c + d x)^{1/4}} dx$$

Optimal (type 3, 85 leaves, 4 steps) :

$$-\frac{2 \operatorname{ArcTan}\left[\frac{c^{1/4} (a+b x)^{1/4}}{a^{1/4} (c+d x)^{1/4}}\right]}{a^{3/4} c^{1/4}} - \frac{2 \operatorname{ArcTanh}\left[\frac{c^{1/4} (a+b x)^{1/4}}{a^{1/4} (c+d x)^{1/4}}\right]}{a^{3/4} c^{1/4}}$$

Result (type 6, 146 leaves) :

$$\left( \frac{8 b d x \operatorname{AppellF1}\left[1, \frac{3}{4}, \frac{1}{4}, 2, -\frac{a}{b x}, -\frac{c}{d x}\right]}{\left(-8 b d x \operatorname{AppellF1}\left[1, \frac{3}{4}, \frac{1}{4}, 2, -\frac{a}{b x}, -\frac{c}{d x}\right] + b c \operatorname{AppellF1}\left[2, \frac{3}{4}, \frac{5}{4}, 3, -\frac{a}{b x}, -\frac{c}{d x}\right] + 3 a d \operatorname{AppellF1}\left[2, \frac{7}{4}, \frac{1}{4}, 3, -\frac{a}{b x}, -\frac{c}{d x}\right]\right)} \right) \Bigg/ \left( \left(a + b x\right)^{3/4} \left(c + d x\right)^{1/4} \right)$$

**Problem 905:** Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^2 (a + b x)^{3/4} (c + d x)^{1/4}} dx$$

Optimal (type 3, 134 leaves, 5 steps):

$$-\frac{(a + b x)^{1/4} (c + d x)^{3/4}}{a c x} + \frac{(3 b c + a d) \operatorname{ArcTan}\left[\frac{c^{1/4} (a+b x)^{1/4}}{a^{1/4} (c+d x)^{1/4}}\right]}{2 a^{7/4} c^{5/4}} + \frac{(3 b c + a d) \operatorname{ArcTanh}\left[\frac{c^{1/4} (a+b x)^{1/4}}{a^{1/4} (c+d x)^{1/4}}\right]}{2 a^{7/4} c^{5/4}}$$

Result (type 6, 180 leaves):

$$-\frac{(a + b x) (c + d x)}{a c x (a + b x)^{3/4} (c + d x)^{1/4}} + \frac{2 b d (3 b c + a d) x^2 \operatorname{AppellF1}\left[1, \frac{3}{4}, \frac{1}{4}, 2, -\frac{a}{b x}, -\frac{c}{d x}\right]}{8 b d x \operatorname{AppellF1}\left[1, \frac{3}{4}, \frac{1}{4}, 2, -\frac{a}{b x}, -\frac{c}{d x}\right] - b c \operatorname{AppellF1}\left[2, \frac{3}{4}, \frac{5}{4}, 3, -\frac{a}{b x}, -\frac{c}{d x}\right] - 3 a d \operatorname{AppellF1}\left[2, \frac{7}{4}, \frac{1}{4}, 3, -\frac{a}{b x}, -\frac{c}{d x}\right]}$$

**Problem 906:** Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^3 (a + b x)^{3/4} (c + d x)^{1/4}} dx$$

Optimal (type 3, 206 leaves, 7 steps):

$$-\frac{(a + b x)^{1/4} (c + d x)^{3/4}}{2 a c x^2} + \frac{(7 b c + 5 a d) (a + b x)^{1/4} (c + d x)^{3/4}}{8 a^2 c^2 x} - \frac{(21 b^2 c^2 + 6 a b c d + 5 a^2 d^2) \operatorname{ArcTan}\left[\frac{c^{1/4} (a+b x)^{1/4}}{a^{1/4} (c+d x)^{1/4}}\right]}{16 a^{11/4} c^{9/4}} - \frac{(21 b^2 c^2 + 6 a b c d + 5 a^2 d^2) \operatorname{ArcTanh}\left[\frac{c^{1/4} (a+b x)^{1/4}}{a^{1/4} (c+d x)^{1/4}}\right]}{16 a^{11/4} c^{9/4}}$$

Result (type 6, 211 leaves):

$$\begin{aligned} & \left( (a + b x) (c + d x) (-4 a c + 7 b c x + 5 a d x) + \right. \\ & \left. \left( 2 b d (21 b^2 c^2 + 6 a b c d + 5 a^2 d^2) x^3 \operatorname{AppellF1}\left[1, \frac{3}{4}, \frac{1}{4}, 2, -\frac{a}{b x}, -\frac{c}{d x}\right] \right) \Bigg/ \left( -8 b d x \operatorname{AppellF1}\left[1, \frac{3}{4}, \frac{1}{4}, 2, -\frac{a}{b x}, -\frac{c}{d x}\right] + \right. \right. \\ & \left. \left. b c \operatorname{AppellF1}\left[2, \frac{3}{4}, \frac{5}{4}, 3, -\frac{a}{b x}, -\frac{c}{d x}\right] + 3 a d \operatorname{AppellF1}\left[2, \frac{7}{4}, \frac{1}{4}, 3, -\frac{a}{b x}, -\frac{c}{d x}\right] \right) \Bigg/ \left( 8 a^2 c^2 x^2 (a + b x)^{3/4} (c + d x)^{1/4} \right) \right) \end{aligned}$$

### Problem 907: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^4 (a + b x)^{3/4} (c + d x)^{1/4}} dx$$

Optimal (type 3, 288 leaves, 8 steps):

$$\begin{aligned} & -\frac{(a + b x)^{1/4} (c + d x)^{3/4}}{3 a c x^3} + \frac{(11 b c + 9 a d) (a + b x)^{1/4} (c + d x)^{3/4}}{24 a^2 c^2 x^2} - \frac{(77 b^2 c^2 + 54 a b c d + 45 a^2 d^2) (a + b x)^{1/4} (c + d x)^{3/4}}{96 a^3 c^3 x} + \\ & \frac{(77 b^3 c^3 + 21 a b^2 c^2 d + 15 a^2 b c d^2 + 15 a^3 d^3) \operatorname{ArcTan}\left[\frac{c^{1/4} (a+b x)^{1/4}}{a^{1/4} (c+d x)^{1/4}}\right]}{64 a^{15/4} c^{13/4}} + \frac{(77 b^3 c^3 + 21 a b^2 c^2 d + 15 a^2 b c d^2 + 15 a^3 d^3) \operatorname{ArcTanh}\left[\frac{c^{1/4} (a+b x)^{1/4}}{a^{1/4} (c+d x)^{1/4}}\right]}{64 a^{15/4} c^{13/4}} \end{aligned}$$

Result (type 6, 259 leaves):

$$\begin{aligned} & -\left( \left( (a + b x) (c + d x) (77 b^2 c^2 x^2 + 2 a b c x (-22 c + 27 d x) + a^2 (32 c^2 - 36 c d x + 45 d^2 x^2)) \right) + \right. \\ & \left( 6 b d (77 b^3 c^3 + 21 a b^2 c^2 d + 15 a^2 b c d^2 + 15 a^3 d^3) x^4 \operatorname{AppellF1}\left[1, \frac{3}{4}, \frac{1}{4}, 2, -\frac{a}{b x}, -\frac{c}{d x}\right] \right) / \left( -8 b d x \operatorname{AppellF1}\left[1, \frac{3}{4}, \frac{1}{4}, 2, -\frac{a}{b x}, -\frac{c}{d x}\right] + \right. \\ & \left. \left. b c \operatorname{AppellF1}\left[2, \frac{3}{4}, \frac{5}{4}, 3, -\frac{a}{b x}, -\frac{c}{d x}\right] + 3 a d \operatorname{AppellF1}\left[2, \frac{7}{4}, \frac{1}{4}, 3, -\frac{a}{b x}, -\frac{c}{d x}\right] \right) \right) / \left( 96 a^3 c^3 x^3 (a + b x)^{3/4} (c + d x)^{1/4} \right) \end{aligned}$$

### Problem 908: Result unnecessarily involves higher level functions.

$$\int \frac{(e x)^{3/2}}{(1-x)^{1/4} (1+x)^{1/4}} dx$$

Optimal (type 3, 244 leaves, 13 steps):

$$\begin{aligned} & -\frac{1}{2} e \sqrt{e x} (1-x^2)^{3/4} - \frac{e^{3/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e x}}{\sqrt{e} (1-x^2)^{1/4}}\right]}{4 \sqrt{2}} + \\ & \frac{e^{3/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e x}}{\sqrt{e} (1-x^2)^{1/4}}\right]}{4 \sqrt{2}} - \frac{e^{3/2} \operatorname{Log}\left[\sqrt{e} + \frac{\sqrt{e} x}{\sqrt{1-x^2}} - \frac{\sqrt{2} \sqrt{e x}}{(1-x^2)^{1/4}}\right]}{8 \sqrt{2}} + \frac{e^{3/2} \operatorname{Log}\left[\sqrt{e} + \frac{\sqrt{e} x}{\sqrt{1-x^2}} + \frac{\sqrt{2} \sqrt{e x}}{(1-x^2)^{1/4}}\right]}{8 \sqrt{2}} \end{aligned}$$

Result (type 5, 39 leaves):

$$\frac{1}{2} e \sqrt{e x} \left( - (1-x^2)^{3/4} + \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, x^2\right] \right)$$

### Problem 909: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(1-x)^{1/4} \sqrt{e x} (1+x)^{1/4}} dx$$

Optimal (type 3, 216 leaves, 12 steps):

$$-\frac{\text{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e} x}{\sqrt{e} (1-x^2)^{1/4}}\right]}{\sqrt{2} \sqrt{e}} + \frac{\text{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e} x}{\sqrt{e} (1-x^2)^{1/4}}\right]}{\sqrt{2} \sqrt{e}} - \frac{\text{Log}\left[\sqrt{e} + \frac{\sqrt{e} x}{\sqrt{1-x^2}} - \frac{\sqrt{2} \sqrt{e} x}{(1-x^2)^{1/4}}\right]}{2 \sqrt{2} \sqrt{e}} + \frac{\text{Log}\left[\sqrt{e} + \frac{\sqrt{e} x}{\sqrt{1-x^2}} + \frac{\sqrt{2} \sqrt{e} x}{(1-x^2)^{1/4}}\right]}{2 \sqrt{2} \sqrt{e}}$$

Result (type 5, 23 leaves):

$$\frac{2 \times \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, x^2\right]}{\sqrt{e x}}$$

### Problem 913: Result unnecessarily involves higher level functions.

$$\int \frac{(e x)^{5/2}}{(1-x)^{1/4} (1+x)^{1/4}} dx$$

Optimal (type 4, 93 leaves, 6 steps):

$$-\frac{e^3 (1-x^2)^{3/4}}{2 \sqrt{e x}} - \frac{1}{3} e (e x)^{3/2} (1-x^2)^{3/4} + \frac{e^2 \left(1 - \frac{1}{x^2}\right)^{1/4} \sqrt{e x} \text{EllipticE}\left[\frac{\text{ArcCsc}[x]}{2}, 2\right]}{2 (1-x^2)^{1/4}}$$

Result (type 5, 39 leaves):

$$-\frac{1}{3} e (e x)^{3/2} \left( (1-x^2)^{3/4} - \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, x^2\right] \right)$$

### Problem 914: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{e x}}{(1-x)^{1/4} (1+x)^{1/4}} dx$$

Optimal (type 4, 60 leaves, 5 steps):

$$-\frac{e (1-x^2)^{3/4}}{\sqrt{e x}} + \frac{\left(1 - \frac{1}{x^2}\right)^{1/4} \sqrt{e x} \text{EllipticE}\left[\frac{\text{ArcCsc}[x]}{2}, 2\right]}{(1-x^2)^{1/4}}$$

Result (type 5, 25 leaves):

$$\frac{2}{3} x \sqrt{e x} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, x^2\right]$$

**Problem 915:** Result unnecessarily involves higher level functions.

$$\int \frac{1}{(1-x)^{1/4} (e x)^{3/2} (1+x)^{1/4}} dx$$

Optimal (type 4, 42 leaves, 4 steps):

$$-\frac{2 \left(1 - \frac{1}{x^2}\right)^{1/4} \sqrt{e x} \text{EllipticE}\left[\frac{\text{ArcCsc}[x]}{2}, 2\right]}{e^2 (1-x^2)^{1/4}}$$

Result (type 5, 44 leaves):

$$-\frac{2 x \left(3 (1-x^2)^{3/4} + 2 x^2 \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, x^2\right]\right)}{3 (e x)^{3/2}}$$

**Problem 916:** Result unnecessarily involves higher level functions.

$$\int \frac{1}{(1-x)^{1/4} (e x)^{7/2} (1+x)^{1/4}} dx$$

Optimal (type 4, 70 leaves, 5 steps):

$$-\frac{2 (1-x^2)^{3/4}}{5 e (e x)^{5/2}} - \frac{4 \left(1 - \frac{1}{x^2}\right)^{1/4} \sqrt{e x} \text{EllipticE}\left[\frac{\text{ArcCsc}[x]}{2}, 2\right]}{5 e^4 (1-x^2)^{1/4}}$$

Result (type 5, 51 leaves):

$$\frac{x \left(-6 (1-x^2)^{3/4} (1+2 x^2) - 8 x^4 \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, x^2\right]\right)}{15 (e x)^{7/2}}$$

**Problem 917:** Result unnecessarily involves higher level functions.

$$\int \frac{1}{(1-x)^{1/4} (e x)^{11/2} (1+x)^{1/4}} dx$$

Optimal (type 4, 95 leaves, 6 steps):

$$-\frac{2 \left(1-x^2\right)^{3/4}}{9 e \left(e x\right)^{9/2}} - \frac{4 \left(1-x^2\right)^{3/4}}{15 e^3 \left(e x\right)^{5/2}} - \frac{8 \left(1-\frac{1}{x^2}\right)^{1/4} \sqrt{e x} \text{EllipticE}\left[\frac{\text{ArcCsc}[x]}{2}, 2\right]}{15 e^6 \left(1-x^2\right)^{1/4}}$$

Result (type 5, 60 leaves) :

$$-\frac{2 \sqrt{e x} \left(\left(1-x^2\right)^{3/4} \left(5+6 x^2+12 x^4\right)+8 x^6 \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, x^2\right]\right)}{45 e^6 x^5}$$

Problem 942: Result unnecessarily involves higher level functions.

$$\int \frac{x^3 (a+b x)^n}{(c+d x)^2} dx$$

Optimal (type 5, 203 leaves, 3 steps) :

$$\begin{aligned} & \frac{x^2 (a+b x)^{1+n}}{b d (2+n) (c+d x)} - \left( (a+b x)^{1+n} (c (b c (2+n) (a d + b c (3+n)) - a d (a d + b c (5+3 n))) + d (b c - a d) (a d + b c (3+n)) x) \right) / \\ & (b^2 d^3 (b c - a d) (1+n) (2+n) (c+d x)) - \frac{c^2 (3 a d - b c (3+n)) (a+b x)^{1+n} \text{Hypergeometric2F1}\left[1, 1+n, 2+n, -\frac{d (a+b x)}{b c-a d}\right]}{d^3 (b c - a d)^2 (1+n)} \end{aligned}$$

Result (type 6, 126 leaves) :

$$\begin{aligned} & \left(5 a c x^4 (a+b x)^n \text{AppellF1}\left[4, -n, 2, 5, -\frac{b x}{a}, -\frac{d x}{c}\right]\right) / \left(4 (c+d x)^2\right. \\ & \left.\left(5 a c \text{AppellF1}\left[4, -n, 2, 5, -\frac{b x}{a}, -\frac{d x}{c}\right] + b c n x \text{AppellF1}\left[5, 1-n, 2, 6, -\frac{b x}{a}, -\frac{d x}{c}\right] - 2 a d x \text{AppellF1}\left[5, -n, 3, 6, -\frac{b x}{a}, -\frac{d x}{c}\right]\right)\right) \end{aligned}$$

Problem 943: Result unnecessarily involves higher level functions.

$$\int \frac{x^2 (a+b x)^n}{(c+d x)^2} dx$$

Optimal (type 5, 122 leaves, 3 steps) :

$$\begin{aligned} & \frac{(a+b x)^{1+n}}{b d^2 (1+n)} + \frac{c^2 (a+b x)^{1+n}}{d^2 (b c - a d) (c+d x)} + \frac{c (2 a d - b c (2+n)) (a+b x)^{1+n} \text{Hypergeometric2F1}\left[1, 1+n, 2+n, -\frac{d (a+b x)}{b c-a d}\right]}{d^2 (b c - a d)^2 (1+n)} \end{aligned}$$

Result (type 6, 126 leaves) :

$$\left( \frac{4 a c x^3 (a + b x)^n \text{AppellF1}[3, -n, 2, 4, -\frac{b x}{a}, -\frac{d x}{c}]}{\left( 4 a c \text{AppellF1}[3, -n, 2, 4, -\frac{b x}{a}, -\frac{d x}{c}] + b c n x \text{AppellF1}[4, 1-n, 2, 5, -\frac{b x}{a}, -\frac{d x}{c}] - 2 a d x \text{AppellF1}[4, -n, 3, 5, -\frac{b x}{a}, -\frac{d x}{c}] \right)} \right) / \left( 3 (c + d x)^2 \right)$$

**Problem 944:** Result unnecessarily involves higher level functions.

$$\int \frac{x (a + b x)^n}{(c + d x)^2} dx$$

Optimal (type 5, 99 leaves, 2 steps):

$$-\frac{c (a + b x)^{1+n}}{d (b c - a d) (c + d x)} - \frac{(a d - b c (1+n)) (a + b x)^{1+n} \text{Hypergeometric2F1}[1, 1+n, 2+n, -\frac{d (a+b x)}{b c-a d}]}{d (b c - a d)^2 (1+n)}$$

Result (type 6, 126 leaves):

$$\left( 3 a c x^2 (a + b x)^n \text{AppellF1}[2, -n, 2, 3, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \left( 2 (c + d x)^2 \right)$$

$$\left( 3 a c \text{AppellF1}[2, -n, 2, 3, -\frac{b x}{a}, -\frac{d x}{c}] + b c n x \text{AppellF1}[3, 1-n, 2, 4, -\frac{b x}{a}, -\frac{d x}{c}] - 2 a d x \text{AppellF1}[3, -n, 3, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right)$$

**Problem 945:** Unable to integrate problem.

$$\int \frac{(a + b x)^n}{(c + d x)^2} dx$$

Optimal (type 5, 52 leaves, 1 step):

$$\frac{b (a + b x)^{1+n} \text{Hypergeometric2F1}[2, 1+n, 2+n, -\frac{d (a+b x)}{b c-a d}]}{(b c - a d)^2 (1+n)}$$

Result (type 8, 17 leaves):

$$\int \frac{(a + b x)^n}{(c + d x)^2} dx$$

**Problem 946:** Unable to integrate problem.

$$\int \frac{(a + b x)^n}{x (c + d x)^2} dx$$

Optimal (type 5, 139 leaves, 4 steps):

$$\begin{aligned} & -\frac{d(a+b x)^{1+n}}{c(b c-a d)(c+d x)} + \frac{d(a d-b c(1-n))(a+b x)^{1+n} \text{Hypergeometric2F1}[1, 1+n, 2+n, -\frac{d(a+b x)}{b c-a d}]}{c^2(b c-a d)^2(1+n)} - \\ & \frac{(a+b x)^{1+n} \text{Hypergeometric2F1}[1, 1+n, 2+n, 1+\frac{b x}{a}]}{a c^2(1+n)} \end{aligned}$$

Result (type 8, 20 leaves):

$$\int \frac{(a+b x)^n}{x(c+d x)^2} dx$$

**Problem 947:** Unable to integrate problem.

$$\int \frac{(a+b x)^n}{x^2(c+d x)^2} dx$$

Optimal (type 5, 190 leaves, 5 steps):

$$\begin{aligned} & -\frac{d(b c-2 a d)(a+b x)^{1+n}}{a c^2(b c-a d)(c+d x)} - \frac{(a+b x)^{1+n}}{a c x(c+d x)} - \frac{d^2(2 a d-b c(2-n))(a+b x)^{1+n} \text{Hypergeometric2F1}[1, 1+n, 2+n, -\frac{d(a+b x)}{b c-a d}]}{c^3(b c-a d)^2(1+n)} + \\ & \frac{(2 a d-b c n)(a+b x)^{1+n} \text{Hypergeometric2F1}[1, 1+n, 2+n, 1+\frac{b x}{a}]}{a^2 c^3(1+n)} \end{aligned}$$

Result (type 8, 20 leaves):

$$\int \frac{(a+b x)^n}{x^2(c+d x)^2} dx$$

**Problem 950:** Result more than twice size of optimal antiderivative.

$$\int \frac{(b x)^{5/2}(c+d x)^n}{e+f x} dx$$

Optimal (type 6, 61 leaves, 3 steps):

$$\frac{2(b x)^{7/2}(c+d x)^n \left(1+\frac{d x}{c}\right)^{-n} \text{AppellF1}[\frac{7}{2}, -n, 1, \frac{9}{2}, -\frac{d x}{c}, -\frac{f x}{e}]}{7 b e}$$

Result (type 6, 239 leaves):

$$\frac{1}{15 f^3 x^2}$$

$$2(bx)^{5/2} (c+dx)^n \left( - \left( \left( 45 c e^4 \text{AppellF1} \left[ \frac{1}{2}, -n, 1, \frac{3}{2}, -\frac{dx}{c}, -\frac{fx}{e} \right] \right) \middle/ \left( (e+fx) \left( 3 c e \text{AppellF1} \left[ \frac{1}{2}, -n, 1, \frac{3}{2}, -\frac{dx}{c}, -\frac{fx}{e} \right] + 2 d e n x \text{AppellF1} \left[ \frac{3}{2}, 1-n, 1, \frac{5}{2}, -\frac{dx}{c}, -\frac{fx}{e} \right] - 2 c f x \text{AppellF1} \left[ \frac{3}{2}, -n, 2, \frac{5}{2}, -\frac{dx}{c}, -\frac{fx}{e} \right] \right) \right) + \left( 1 + \frac{dx}{c} \right)^{-n} \right)$$

$$\left( 15 e^2 \text{Hypergeometric2F1} \left[ \frac{1}{2}, -n, \frac{3}{2}, -\frac{dx}{c} \right] + f x \left( -5 e \text{Hypergeometric2F1} \left[ \frac{3}{2}, -n, \frac{5}{2}, -\frac{dx}{c} \right] + 3 f x \text{Hypergeometric2F1} \left[ \frac{5}{2}, -n, \frac{7}{2}, -\frac{dx}{c} \right] \right) \right)$$

**Problem 951:** Result more than twice size of optimal antiderivative.

$$\int \frac{(bx)^{5/2} (c+dx)^n}{(e+fx)^2} dx$$

Optimal (type 6, 61 leaves, 3 steps):

$$\frac{2(bx)^{7/2} (c+dx)^n \left( 1 + \frac{dx}{c} \right)^{-n} \text{AppellF1} \left[ \frac{7}{2}, -n, 2, \frac{9}{2}, -\frac{dx}{c}, -\frac{fx}{e} \right]}{7 b e^2}$$

Result (type 6, 345 leaves):

$$\frac{1}{3 f^3}$$

$$2 b^2 \sqrt{bx} (c+dx)^n \left( \left( 27 c e^3 \text{AppellF1} \left[ \frac{1}{2}, -n, 1, \frac{3}{2}, -\frac{dx}{c}, -\frac{fx}{e} \right] \right) \middle/ \left( (e+fx) \left( 3 c e \text{AppellF1} \left[ \frac{1}{2}, -n, 1, \frac{3}{2}, -\frac{dx}{c}, -\frac{fx}{e} \right] + 2 d e n x \text{AppellF1} \left[ \frac{3}{2}, 1-n, 1, \frac{5}{2}, -\frac{dx}{c}, -\frac{fx}{e} \right] - 2 c f x \text{AppellF1} \left[ \frac{3}{2}, -n, 2, \frac{5}{2}, -\frac{dx}{c}, -\frac{fx}{e} \right] \right) \right) -$$

$$\left( 9 c e^4 \text{AppellF1} \left[ \frac{1}{2}, -n, 2, \frac{3}{2}, -\frac{dx}{c}, -\frac{fx}{e} \right] \right) \middle/ \left( (e+fx)^2 \left( 3 c e \text{AppellF1} \left[ \frac{1}{2}, -n, 2, \frac{3}{2}, -\frac{dx}{c}, -\frac{fx}{e} \right] + 2 d e n x \text{AppellF1} \left[ \frac{3}{2}, 1-n, 2, \frac{5}{2}, -\frac{dx}{c}, -\frac{fx}{e} \right] - 4 c f x \text{AppellF1} \left[ \frac{3}{2}, -n, 3, \frac{5}{2}, -\frac{dx}{c}, -\frac{fx}{e} \right] \right) \right) +$$

$$\left( 1 + \frac{dx}{c} \right)^{-n} \left( -6 e \text{Hypergeometric2F1} \left[ \frac{1}{2}, -n, \frac{3}{2}, -\frac{dx}{c} \right] + f x \text{Hypergeometric2F1} \left[ \frac{3}{2}, -n, \frac{5}{2}, -\frac{dx}{c} \right] \right)$$

**Problem 954:** Result more than twice size of optimal antiderivative.

$$\int \frac{(bx)^m (c+dx)^n}{e+fx} dx$$

Optimal (type 6, 63 leaves, 2 steps):

$$\frac{(bx)^{1+m} (c+dx)^n \left(1 + \frac{dx}{c}\right)^{-n} \text{AppellF1}[1+m, -n, 1, 2+m, -\frac{dx}{c}, -\frac{fx}{e}]}{b e (1+m)}$$

Result (type 6, 153 leaves):

$$\begin{aligned} & \left( c e (2+m) \times (bx)^m (c+dx)^n \text{AppellF1}[1+m, -n, 1, 2+m, -\frac{dx}{c}, -\frac{fx}{e}] \right) / \\ & \left( (1+m) (e+fx) \left( c e (2+m) \text{AppellF1}[1+m, -n, 1, 2+m, -\frac{dx}{c}, -\frac{fx}{e}] + \right. \right. \\ & \left. \left. \times \left( d e n \text{AppellF1}[2+m, 1-n, 1, 3+m, -\frac{dx}{c}, -\frac{fx}{e}] - c f \text{AppellF1}[2+m, -n, 2, 3+m, -\frac{dx}{c}, -\frac{fx}{e}] \right) \right) \right) \end{aligned}$$

Problem 955: Result more than twice size of optimal antiderivative.

$$\int \frac{(bx)^m (c+dx)^n}{(e+fx)^2} dx$$

Optimal (type 6, 63 leaves, 2 steps):

$$\frac{(bx)^{1+m} (c+dx)^n \left(1 + \frac{dx}{c}\right)^{-n} \text{AppellF1}[1+m, -n, 2, 2+m, -\frac{dx}{c}, -\frac{fx}{e}]}{b e^2 (1+m)}$$

Result (type 6, 153 leaves):

$$\begin{aligned} & \left( c e (2+m) \times (bx)^m (c+dx)^n \text{AppellF1}[1+m, -n, 2, 2+m, -\frac{dx}{c}, -\frac{fx}{e}] \right) / \\ & \left( (1+m) (e+fx)^2 \left( c e (2+m) \text{AppellF1}[1+m, -n, 2, 2+m, -\frac{dx}{c}, -\frac{fx}{e}] + \right. \right. \\ & \left. \left. \times \left( d e n \text{AppellF1}[2+m, 1-n, 2, 3+m, -\frac{dx}{c}, -\frac{fx}{e}] - 2 c f \text{AppellF1}[2+m, -n, 3, 3+m, -\frac{dx}{c}, -\frac{fx}{e}] \right) \right) \right) \end{aligned}$$

Problem 956: Result more than twice size of optimal antiderivative.

$$\int (bx)^m (c+dx)^n (e+fx)^p dx$$

Optimal (type 6, 81 leaves, 3 steps):

$$\frac{(bx)^{1+m} (c+dx)^n \left(1 + \frac{dx}{c}\right)^{-n} (e+fx)^p \left(1 + \frac{fx}{e}\right)^{-p} \text{AppellF1}[1+m, -n, -p, 2+m, -\frac{dx}{c}, -\frac{fx}{e}]}{b (1+m)}$$

Result (type 6, 163 leaves):

$$\begin{aligned} & \left( c e (2+m) x (b x)^m (c+d x)^n (e+f x)^p \text{AppellF1}[1+m, -n, -p, 2+m, -\frac{d x}{c}, -\frac{f x}{e}] \right) / \\ & \left( (1+m) \left( c e (2+m) \text{AppellF1}[1+m, -n, -p, 2+m, -\frac{d x}{c}, -\frac{f x}{e}] \right) + \right. \\ & \left. x \left( d e n \text{AppellF1}[2+m, 1-n, -p, 3+m, -\frac{d x}{c}, -\frac{f x}{e}] + c f p \text{AppellF1}[2+m, -n, 1-p, 3+m, -\frac{d x}{c}, -\frac{f x}{e}] \right) \right) \end{aligned}$$

**Problem 958: Result unnecessarily involves higher level functions.**

$$\int x^2 (a+b x)^n (c+d x)^p dx$$

Optimal (type 5, 206 leaves, 4 steps):

$$\begin{aligned} & - \frac{(b c (2+n) + a d (2+p)) (a+b x)^{1+n} (c+d x)^{1+p}}{b^2 d^2 (2+n+p) (3+n+p)} + \frac{x (a+b x)^{1+n} (c+d x)^{1+p}}{b d (3+n+p)} - \\ & \left( (b^2 c^2 (2+3 n+n^2) + 2 a b c d (1+n) (1+p) + a^2 d^2 (2+3 p+p^2)) (a+b x)^{1+n} (c+d x)^{1+p} \text{Hypergeometric2F1}[1, 2+n+p, 2+p, \frac{b (c+d x)}{b c-a d}] \right) / \\ & (b^2 d^2 (b c - a d) (1+p) (2+n+p) (3+n+p)) \end{aligned}$$

Result (type 6, 136 leaves):

$$\begin{aligned} & \left( 4 a c x^3 (a+b x)^n (c+d x)^p \text{AppellF1}[3, -n, -p, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \\ & \left( 3 \left( 4 a c \text{AppellF1}[3, -n, -p, 4, -\frac{b x}{a}, -\frac{d x}{c}] + b c n x \text{AppellF1}[4, 1-n, -p, 5, -\frac{b x}{a}, -\frac{d x}{c}] + a d p x \text{AppellF1}[4, -n, 1-p, 5, -\frac{b x}{a}, -\frac{d x}{c}] \right) \right) \end{aligned}$$

**Problem 959: Result unnecessarily involves higher level functions.**

$$\int x (a+b x)^n (c+d x)^p dx$$

Optimal (type 5, 117 leaves, 3 steps):

$$\frac{(a+b x)^{1+n} (c+d x)^{1+p}}{b d (2+n+p)} + \frac{(b c (1+n) + a d (1+p)) (a+b x)^{1+n} (c+d x)^{1+p} \text{Hypergeometric2F1}[1, 2+n+p, 2+p, \frac{b (c+d x)}{b c-a d}]}{b d (b c - a d) (1+p) (2+n+p)}$$

Result (type 6, 136 leaves):

$$\begin{aligned} & \left( 3 a c x^2 (a+b x)^n (c+d x)^p \text{AppellF1}[2, -n, -p, 3, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \\ & \left( 6 a c \text{AppellF1}[2, -n, -p, 3, -\frac{b x}{a}, -\frac{d x}{c}] + 2 b c n x \text{AppellF1}[3, 1-n, -p, 4, -\frac{b x}{a}, -\frac{d x}{c}] + 2 a d p x \text{AppellF1}[3, -n, 1-p, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) \end{aligned}$$

### Problem 961: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^n (c + d x)^p}{x} dx$$

Optimal (type 6, 85 leaves, 2 steps):

$$-\frac{b (a + b x)^{1+n} (c + d x)^p \left(\frac{b (c+d x)}{b c-a d}\right)^{-p} \text{AppellF1}[1+n, -p, 1, 2+n, -\frac{d (a+b x)}{b c-a d}, \frac{a+b x}{a}]}{a (1+n)}$$

Result (type 6, 214 leaves):

$$\begin{aligned} & \left( b d (-1+n+p) \times (a + b x)^n (c + d x)^p \text{AppellF1}[-n-p, -n, -p, 1-n-p, -\frac{a}{b x}, -\frac{c}{d x}] \right) / \\ & \left( (n+p) \left( b d (-1+n+p) \times \text{AppellF1}[-n-p, -n, -p, 1-n-p, -\frac{a}{b x}, -\frac{c}{d x}] - \right. \right. \\ & \quad \left. \left. a d n \text{AppellF1}[1-n-p, 1-n, -p, 2-n-p, -\frac{a}{b x}, -\frac{c}{d x}] - b c p \text{AppellF1}[1-n-p, -n, 1-p, 2-n-p, -\frac{a}{b x}, -\frac{c}{d x}] \right) \right) \end{aligned}$$

### Problem 962: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^n (c + d x)^p}{x^2} dx$$

Optimal (type 6, 85 leaves, 2 steps):

$$-\frac{b (a + b x)^{1+n} (c + d x)^p \left(\frac{b (c+d x)}{b c-a d}\right)^{-p} \text{AppellF1}[1+n, -p, 2, 2+n, -\frac{d (a+b x)}{b c-a d}, \frac{a+b x}{a}]}{a^2 (1+n)}$$

Result (type 6, 216 leaves):

$$\begin{aligned} & \left( b d (-2+n+p) (a + b x)^n (c + d x)^p \text{AppellF1}[1-n-p, -n, -p, 2-n-p, -\frac{a}{b x}, -\frac{c}{d x}] \right) / \\ & \left( (-1+n+p) \left( b d (-2+n+p) \times \text{AppellF1}[1-n-p, -n, -p, 2-n-p, -\frac{a}{b x}, -\frac{c}{d x}] - \right. \right. \\ & \quad \left. \left. a d n \text{AppellF1}[2-n-p, 1-n, -p, 3-n-p, -\frac{a}{b x}, -\frac{c}{d x}] - b c p \text{AppellF1}[2-n-p, -n, 1-p, 3-n-p, -\frac{a}{b x}, -\frac{c}{d x}] \right) \right) \end{aligned}$$

### Problem 963: Result more than twice size of optimal antiderivative.

$$\int (b x)^{3/2} (c + d x)^n (e + f x)^p dx$$

Optimal (type 6, 79 leaves, 3 steps):

$$\frac{2 \left(b x\right)^{5/2} \left(c+d x\right)^n \left(1+\frac{d x}{c}\right)^{-n} \left(e+f x\right)^p \left(1+\frac{f x}{e}\right)^{-p} \text{AppellF1}\left[\frac{5}{2}, -n, -p, \frac{7}{2}, -\frac{d x}{c}, -\frac{f x}{e}\right]}{5 b}$$

Result (type 6, 159 leaves):

$$\begin{aligned} & \left(14 c e x \left(b x\right)^{3/2} \left(c+d x\right)^n \left(e+f x\right)^p \text{AppellF1}\left[\frac{5}{2}, -n, -p, \frac{7}{2}, -\frac{d x}{c}, -\frac{f x}{e}\right]\right) / \left(5 \left(7 c e \text{AppellF1}\left[\frac{5}{2}, -n, -p, \frac{7}{2}, -\frac{d x}{c}, -\frac{f x}{e}\right] + \right.\right. \\ & \left.\left.2 x \left(d e n \text{AppellF1}\left[\frac{7}{2}, 1-n, -p, \frac{9}{2}, -\frac{d x}{c}, -\frac{f x}{e}\right] + c f p \text{AppellF1}\left[\frac{7}{2}, -n, 1-p, \frac{9}{2}, -\frac{d x}{c}, -\frac{f x}{e}\right]\right)\right)\right) \end{aligned}$$

**Problem 965:** Result more than twice size of optimal antiderivative.

$$\int \frac{\left(c+d x\right)^n \left(e+f x\right)^p}{\sqrt{b x}} dx$$

Optimal (type 6, 77 leaves, 3 steps):

$$\frac{2 \sqrt{b x} \left(c+d x\right)^n \left(1+\frac{d x}{c}\right)^{-n} \left(e+f x\right)^p \left(1+\frac{f x}{e}\right)^{-p} \text{AppellF1}\left[\frac{1}{2}, -n, -p, \frac{3}{2}, -\frac{d x}{c}, -\frac{f x}{e}\right]}{b}$$

Result (type 6, 157 leaves):

$$\begin{aligned} & \left(6 c e x \left(c+d x\right)^n \left(e+f x\right)^p \text{AppellF1}\left[\frac{1}{2}, -n, -p, \frac{3}{2}, -\frac{d x}{c}, -\frac{f x}{e}\right]\right) / \\ & \left(\sqrt{b x} \left(3 c e \text{AppellF1}\left[\frac{1}{2}, -n, -p, \frac{3}{2}, -\frac{d x}{c}, -\frac{f x}{e}\right] + 2 d e n x \text{AppellF1}\left[\frac{3}{2}, 1-n, -p, \frac{5}{2}, -\frac{d x}{c}, -\frac{f x}{e}\right] + \right.\right. \\ & \left.\left.2 c f p x \text{AppellF1}\left[\frac{3}{2}, -n, 1-p, \frac{5}{2}, -\frac{d x}{c}, -\frac{f x}{e}\right]\right)\right) \end{aligned}$$

**Problem 966:** Result more than twice size of optimal antiderivative.

$$\int (b x)^m (\pi + d x)^n (e + f x)^p dx$$

Optimal (type 6, 49 leaves, 1 step):

$$\frac{e^p \pi^n \left(b x\right)^{1+m} \text{AppellF1}\left[1+m, -n, -p, 2+m, -\frac{d x}{\pi}, -\frac{f x}{e}\right]}{b (1+m)}$$

Result (type 6, 163 leaves):

$$\begin{aligned} & \left( e (2+m) \pi x (bx)^m (\pi+dx)^n (e+fx)^p \text{AppellF1}[1+m, -n, -p, 2+m, -\frac{dx}{\pi}, -\frac{fx}{e}] \right) / \\ & \left( (1+m) \left( e (2+m) \pi \text{AppellF1}[1+m, -n, -p, 2+m, -\frac{dx}{\pi}, -\frac{fx}{e}] + \right. \right. \\ & \left. \left. x \left( d \in n \text{AppellF1}[2+m, 1-n, -p, 3+m, -\frac{dx}{\pi}, -\frac{fx}{e}] + f p \pi \text{AppellF1}[2+m, -n, 1-p, 3+m, -\frac{dx}{\pi}, -\frac{fx}{e}] \right) \right) \right) \end{aligned}$$

**Problem 967: Result more than twice size of optimal antiderivative.**

$$\int (bx)^m (\pi+dx)^n (e+fx)^p dx$$

Optimal (type 6, 65 leaves, 2 steps):

$$\frac{\pi^n (bx)^{1+m} (e+fx)^p \left( 1 + \frac{fx}{e} \right)^{-p} \text{AppellF1}[1+m, -n, -p, 2+m, -\frac{dx}{\pi}, -\frac{fx}{e}]}{b (1+m)}$$

Result (type 6, 163 leaves):

$$\begin{aligned} & \left( e (2+m) \pi x (bx)^m (\pi+dx)^n (e+fx)^p \text{AppellF1}[1+m, -n, -p, 2+m, -\frac{dx}{\pi}, -\frac{fx}{e}] \right) / \\ & \left( (1+m) \left( e (2+m) \pi \text{AppellF1}[1+m, -n, -p, 2+m, -\frac{dx}{\pi}, -\frac{fx}{e}] + \right. \right. \\ & \left. \left. x \left( d \in n \text{AppellF1}[2+m, 1-n, -p, 3+m, -\frac{dx}{\pi}, -\frac{fx}{e}] + f p \pi \text{AppellF1}[2+m, -n, 1-p, 3+m, -\frac{dx}{\pi}, -\frac{fx}{e}] \right) \right) \right) \end{aligned}$$

**Problem 968: Result more than twice size of optimal antiderivative.**

$$\int (bx)^{5/2} (\pi+dx)^n (e+fx)^p dx$$

Optimal (type 6, 47 leaves, 1 step):

$$\frac{2 e^p \pi^n (bx)^{7/2} \text{AppellF1}[\frac{7}{2}, -n, -p, \frac{9}{2}, -\frac{dx}{\pi}, -\frac{fx}{e}]}{7 b}$$

Result (type 6, 159 leaves):

$$\begin{aligned} & \left( 18 e \pi x (bx)^{5/2} (\pi+dx)^n (e+fx)^p \text{AppellF1}[\frac{7}{2}, -n, -p, \frac{9}{2}, -\frac{dx}{\pi}, -\frac{fx}{e}] \right) / \left( 7 \left( 9 e \pi \text{AppellF1}[\frac{7}{2}, -n, -p, \frac{9}{2}, -\frac{dx}{\pi}, -\frac{fx}{e}] + \right. \right. \\ & \left. \left. 2 x \left( d \in n \text{AppellF1}[\frac{9}{2}, 1-n, -p, \frac{11}{2}, -\frac{dx}{\pi}, -\frac{fx}{e}] + f p \pi \text{AppellF1}[\frac{9}{2}, -n, 1-p, \frac{11}{2}, -\frac{dx}{\pi}, -\frac{fx}{e}] \right) \right) \right) \end{aligned}$$

### Problem 969: Result more than twice size of optimal antiderivative.

$$\int (bx)^{5/2} (\pi + dx)^n (e + fx)^p dx$$

Optimal (type 6, 63 leaves, 2 steps):

$$\frac{2\pi^n (bx)^{7/2} (e + fx)^p \left(1 + \frac{fx}{e}\right)^{-p} \text{AppellF1}\left[\frac{7}{2}, -n, -p, \frac{9}{2}, -\frac{dx}{\pi}, -\frac{fx}{e}\right]}{7b}$$

Result (type 6, 159 leaves):

$$\left(18e\pi x(bx)^{5/2}(\pi + dx)^n(e + fx)^p \text{AppellF1}\left[\frac{7}{2}, -n, -p, \frac{9}{2}, -\frac{dx}{\pi}, -\frac{fx}{e}\right]\right) / \left(7\left(9e\pi \text{AppellF1}\left[\frac{7}{2}, -n, -p, \frac{9}{2}, -\frac{dx}{\pi}, -\frac{fx}{e}\right] + 2x\left(den \text{AppellF1}\left[\frac{9}{2}, 1-n, -p, \frac{11}{2}, -\frac{dx}{\pi}, -\frac{fx}{e}\right] + fp\pi \text{AppellF1}\left[\frac{9}{2}, -n, 1-p, \frac{11}{2}, -\frac{dx}{\pi}, -\frac{fx}{e}\right]\right)\right)\right)$$

### Problem 970: Result unnecessarily involves higher level functions.

$$\int x^3 (a + bx)^n (c + dx)^{-n} dx$$

Optimal (type 5, 295 leaves, 4 steps):

$$\begin{aligned} & \frac{x^2 (a + bx)^{1+n} (c + dx)^{1-n}}{4bd} + \frac{1}{24b^3d^3} \\ & (a + bx)^{1+n} (c + dx)^{1-n} (2abc d (3 - n^2) + a^2 d^2 (6 - 5n + n^2) + b^2 c^2 (6 + 5n + n^2) - 2bd (ad (3 - n) + bc (3 + n))x) - \\ & \frac{1}{24b^4d^3(1+n)} (3ab^2c^2d (2 + n - 2n^2 - n^3) + a^3d^3 (6 - 11n + 6n^2 - n^3) + 3a^2bc d^2 (2 - n - 2n^2 + n^3) + b^3c^3 (6 + 11n + 6n^2 + n^3)) \\ & (a + bx)^{1+n} (c + dx)^{-n} \left( \frac{b(c + dx)}{bc - ad} \right)^n \text{Hypergeometric2F1}[n, 1+n, 2+n, -\frac{d(a + bx)}{bc - ad}] \end{aligned}$$

Result (type 6, 130 leaves):

$$\begin{aligned} & \left(5acx^4 (a + bx)^n (c + dx)^{-n} \text{AppellF1}[4, -n, n, 5, -\frac{bx}{a}, -\frac{dx}{c}]\right) / \\ & \left(20ac \text{AppellF1}[4, -n, n, 5, -\frac{bx}{a}, -\frac{dx}{c}] + 4bcnx \text{AppellF1}[5, 1-n, n, 6, -\frac{bx}{a}, -\frac{dx}{c}] - 4adnx \text{AppellF1}[5, -n, 1+n, 6, -\frac{bx}{a}, -\frac{dx}{c}]\right) \end{aligned}$$

### Problem 971: Result unnecessarily involves higher level functions.

$$\int x^2 (a + bx)^n (c + dx)^{-n} dx$$

Optimal (type 5, 199 leaves, 4 steps):

$$\begin{aligned} & -\frac{(a d (2-n) + b c (2+n)) (a+b x)^{1+n} (c+d x)^{1-n}}{6 b^2 d^2} + \frac{x (a+b x)^{1+n} (c+d x)^{1-n}}{3 b d} + \frac{1}{6 b^3 d^2 (1+n)} \\ & (2 a b c d (1-n^2) + a^2 d^2 (2-3 n+n^2) + b^2 c^2 (2+3 n+n^2)) (a+b x)^{1+n} (c+d x)^{-n} \left( \frac{b (c+d x)}{b c-a d} \right)^n \text{Hypergeometric2F1}[n, 1+n, 2+n, -\frac{d (a+b x)}{b c-a d}] \end{aligned}$$

Result (type 6, 130 leaves):

$$\begin{aligned} & \left( 4 a c x^3 (a+b x)^n (c+d x)^{-n} \text{AppellF1}[3, -n, n, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \\ & \left( 12 a c \text{AppellF1}[3, -n, n, 4, -\frac{b x}{a}, -\frac{d x}{c}] + 3 b c n x \text{AppellF1}[4, 1-n, n, 5, -\frac{b x}{a}, -\frac{d x}{c}] - 3 a d n x \text{AppellF1}[4, -n, 1+n, 5, -\frac{b x}{a}, -\frac{d x}{c}] \right) \end{aligned}$$

**Problem 972:** Result unnecessarily involves higher level functions.

$$\int x (a+b x)^n (c+d x)^{-n} dx$$

Optimal (type 5, 124 leaves, 3 steps):

$$\frac{(a+b x)^{1+n} (c+d x)^{1-n}}{2 b d} - \frac{(a d (1-n) + b c (1+n)) (a+b x)^{1+n} (c+d x)^{-n} \left( \frac{b (c+d x)}{b c-a d} \right)^n \text{Hypergeometric2F1}[n, 1+n, 2+n, -\frac{d (a+b x)}{b c-a d}]}{2 b^2 d (1+n)}$$

Result (type 6, 130 leaves):

$$\begin{aligned} & \left( 3 a c x^2 (a+b x)^n (c+d x)^{-n} \text{AppellF1}[2, -n, n, 3, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \\ & \left( 6 a c \text{AppellF1}[2, -n, n, 3, -\frac{b x}{a}, -\frac{d x}{c}] + 2 n x \left( b c \text{AppellF1}[3, 1-n, n, 4, -\frac{b x}{a}, -\frac{d x}{c}] - a d \text{AppellF1}[3, -n, 1+n, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) \right) \end{aligned}$$

**Problem 974:** Result unnecessarily involves higher level functions.

$$\int \frac{(a+b x)^n (c+d x)^{-n}}{x} dx$$

Optimal (type 5, 108 leaves, 5 steps):

$$\begin{aligned} & \frac{(a+b x)^n (c+d x)^{-n} \text{Hypergeometric2F1}[1, n, 1+n, \frac{c (a+b x)}{a (c+d x)}]}{n} + \frac{(a+b x)^n (c+d x)^{-n} \left( \frac{b (c+d x)}{b c-a d} \right)^n \text{Hypergeometric2F1}[n, n, 1+n, -\frac{d (a+b x)}{b c-a d}]}{n} \end{aligned}$$

Result (type 6, 216 leaves):

$$\begin{aligned} & \left( a (-b c + a d) (2+n) (a+b x)^{1+n} (c+d x)^{-n} \text{AppellF1}[1+n, n, 1, 2+n, \frac{d(a+b x)}{-b c + a d}, 1 + \frac{b x}{a}] \right) / \\ & \left( b (1+n) \times \left( a (-b c + a d) (2+n) \text{AppellF1}[1+n, n, 1, 2+n, \frac{d(a+b x)}{-b c + a d}, 1 + \frac{b x}{a}] + \right. \right. \\ & \left. \left. (a+b x) \left( (-b c + a d) \text{AppellF1}[2+n, n, 2, 3+n, \frac{d(a+b x)}{-b c + a d}, 1 + \frac{b x}{a}] + a d n \text{AppellF1}[2+n, 1+n, 1, 3+n, \frac{d(a+b x)}{-b c + a d}, 1 + \frac{b x}{a}] \right) \right) \right) \end{aligned}$$

**Problem 975:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(a+b x)^n (c+d x)^{-n}}{x^2} dx$$

Optimal (type 5, 62 leaves, 1 step):

$$\frac{(b c - a d) (a+b x)^{1+n} (c+d x)^{-1-n} \text{Hypergeometric2F1}[2, 1+n, 2+n, \frac{c(a+b x)}{a(c+d x)}]}{a^2 (1+n)}$$

Result (type 6, 141 leaves):

$$\begin{aligned} & - \left( \left( 2 b d (a+b x)^n (c+d x)^{-n} \text{AppellF1}[1, -n, n, 2, -\frac{a}{b x}, -\frac{c}{d x}] \right) / \right. \\ & \left. \left( 2 b d x \text{AppellF1}[1, -n, n, 2, -\frac{a}{b x}, -\frac{c}{d x}] + a d n \text{AppellF1}[2, 1-n, n, 3, -\frac{a}{b x}, -\frac{c}{d x}] - b c n \text{AppellF1}[2, -n, 1+n, 3, -\frac{a}{b x}, -\frac{c}{d x}] \right) \right) \end{aligned}$$

**Problem 976:** Result unnecessarily involves higher level functions.

$$\int \frac{(a+b x)^n (c+d x)^{-n}}{x^3} dx$$

Optimal (type 5, 117 leaves, 2 steps):

$$\frac{(a+b x)^{1+n} (c+d x)^{1-n}}{2 a c x^2} - \frac{(b c - a d) (a d (1+n) + b (c - c n)) (a+b x)^{1+n} (c+d x)^{-1-n} \text{Hypergeometric2F1}[2, 1+n, 2+n, \frac{c(a+b x)}{a(c+d x)}]}{2 a^3 c (1+n)}$$

Result (type 6, 146 leaves):

$$\begin{aligned} & - \left( \left( 3 b d (a+b x)^n (c+d x)^{-n} \text{AppellF1}[2, -n, n, 3, -\frac{a}{b x}, -\frac{c}{d x}] \right) / \right. \\ & \left. \left( 6 b d x^2 \text{AppellF1}[2, -n, n, 3, -\frac{a}{b x}, -\frac{c}{d x}] + 2 a d n x \text{AppellF1}[3, 1-n, n, 4, -\frac{a}{b x}, -\frac{c}{d x}] - 2 b c n x \text{AppellF1}[3, -n, 1+n, 4, -\frac{a}{b x}, -\frac{c}{d x}] \right) \right) \end{aligned}$$

### Problem 977: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^n (c + d x)^{-n}}{x^4} dx$$

Optimal (type 5, 194 leaves, 4 steps):

$$-\frac{(a + b x)^{1+n} (c + d x)^{1-n}}{3 a c x^3} + \frac{(b c (2 - n) + a d (2 + n)) (a + b x)^{1+n} (c + d x)^{1-n}}{6 a^2 c^2 x^2} + \frac{1}{6 a^4 c^2 (1 + n)} \\ (b c - a d) (2 a b c d (1 - n^2) + b^2 c^2 (2 - 3 n + n^2) + a^2 d^2 (2 + 3 n + n^2)) (a + b x)^{1+n} (c + d x)^{-1-n} \text{Hypergeometric2F1}[2, 1 + n, 2 + n, \frac{c (a + b x)}{a (c + d x)}]$$

Result (type 6, 146 leaves):

$$-\left( \left( 4 b d (a + b x)^n (c + d x)^{-n} \text{AppellF1}[3, -n, n, 4, -\frac{a}{b x}, -\frac{c}{d x}] \right) / \right. \\ \left. \left( 3 x^2 \left( 4 b d x \text{AppellF1}[3, -n, n, 4, -\frac{a}{b x}, -\frac{c}{d x}] + a d n \text{AppellF1}[4, 1 - n, n, 5, -\frac{a}{b x}, -\frac{c}{d x}] - b c n \text{AppellF1}[4, -n, 1 + n, 5, -\frac{a}{b x}, -\frac{c}{d x}] \right) \right) \right)$$

### Problem 978: Result unnecessarily involves higher level functions.

$$\int (1 - x)^n x^3 (1 + x)^{-n} dx$$

Optimal (type 5, 105 leaves, 3 steps):

$$-\frac{1}{4} (1 - x)^{1+n} x^2 (1 + x)^{1-n} - \frac{1}{12} (1 - x)^{1+n} (1 + x)^{1-n} (3 + 2 n^2 - 2 n x) + \frac{2^{-n} n (2 + n^2) (1 - x)^{1+n} \text{Hypergeometric2F1}[n, 1 + n, 2 + n, \frac{1-x}{2}]}{3 (1 + n)}$$

Result (type 6, 79 leaves):

$$(5 (1 - x)^n x^4 (1 + x)^{-n} \text{AppellF1}[4, -n, n, 5, x, -x]) / \\ (4 (5 \text{AppellF1}[4, -n, n, 5, x, -x] - n x (\text{AppellF1}[5, 1 - n, n, 6, x, -x] + \text{AppellF1}[5, -n, 1 + n, 6, x, -x])))$$

### Problem 979: Result unnecessarily involves higher level functions.

$$\int (1 - x)^n x^2 (1 + x)^{-n} dx$$

Optimal (type 5, 94 leaves, 3 steps):

$$\frac{1}{3} n (1 - x)^{1+n} (1 + x)^{1-n} - \frac{1}{3} (1 - x)^{1+n} x (1 + x)^{1-n} - \frac{2^{-n} (1 + 2 n^2) (1 - x)^{1+n} \text{Hypergeometric2F1}[n, 1 + n, 2 + n, \frac{1-x}{2}]}{3 (1 + n)}$$

Result (type 6, 79 leaves) :

$$\begin{aligned} & \left( 4 (1-x)^n x^3 (1+x)^{-n} \text{AppellF1}[3, -n, n, 4, x, -x] \right) / \\ & \left( 3 (4 \text{AppellF1}[3, -n, n, 4, x, -x] - n x (\text{AppellF1}[4, 1-n, n, 5, x, -x] + \text{AppellF1}[4, -n, 1+n, 5, x, -x])) \right) \end{aligned}$$

Problem 980: Result unnecessarily involves higher level functions.

$$\int (1-x)^n x (1+x)^{-n} dx$$

Optimal (type 5, 61 leaves, 2 steps) :

$$-\frac{1}{2} (1-x)^{1+n} (1+x)^{1-n} + \frac{2^{-n} n (1-x)^{1+n} \text{Hypergeometric2F1}[n, 1+n, 2+n, \frac{1-x}{2}]}{1+n}$$

Result (type 6, 79 leaves) :

$$\begin{aligned} & \left( 3 (1-x)^n x^2 (1+x)^{-n} \text{AppellF1}[2, -n, n, 3, x, -x] \right) / \\ & \left( 2 (3 \text{AppellF1}[2, -n, n, 3, x, -x] - n x (\text{AppellF1}[3, 1-n, n, 4, x, -x] + \text{AppellF1}[3, -n, 1+n, 4, x, -x])) \right) \end{aligned}$$

Problem 982: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(1-x)^n (1+x)^{-n}}{x} dx$$

Optimal (type 5, 68 leaves, 3 steps) :

$$-\frac{(1-x)^n (1+x)^{-n} \text{Hypergeometric2F1}[1, n, 1+n, \frac{1-x}{1+x}]}{n} + \frac{2^{-n} (1-x)^n \text{Hypergeometric2F1}[n, n, 1+n, \frac{1-x}{2}]}{n}$$

Result (type 6, 140 leaves) :

$$\begin{aligned} & \left( 2 (2+n) (1-x)^{1+n} (1+x)^{-n} \text{AppellF1}[1+n, n, 1, 2+n, \frac{1-x}{2}, 1-x] \right) / \left( (1+n) x \left( -2 (2+n) \text{AppellF1}[1+n, n, 1, 2+n, \frac{1-x}{2}, 1-x] + \right. \right. \\ & \left. \left. (-1+x) \left( 2 \text{AppellF1}[2+n, n, 2, 3+n, \frac{1-x}{2}, 1-x] + n \text{AppellF1}[2+n, 1+n, 1, 3+n, \frac{1-x}{2}, 1-x] \right) \right) \right) \end{aligned}$$

Problem 983: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(1-x)^n (1+x)^{-n}}{x^2} dx$$

Optimal (type 5, 44 leaves, 1 step):

$$\frac{2 (1-x)^{1+n} (1+x)^{-1-n} \text{Hypergeometric2F1}[2, 1+n, 2+n, \frac{1-x}{1+x}]}{1+n}$$

Result (type 6, 90 leaves):

$$-\left( \left( 2 (1-x)^n (1+x)^{-n} \text{AppellF1}[1, -n, n, 2, \frac{1}{x}, -\frac{1}{x}] \right) \middle/ \right. \\ \left. \left( 2 x \text{AppellF1}[1, -n, n, 2, \frac{1}{x}, -\frac{1}{x}] - n \left( \text{AppellF1}[2, 1-n, n, 3, \frac{1}{x}, -\frac{1}{x}] + \text{AppellF1}[2, -n, 1+n, 3, \frac{1}{x}, -\frac{1}{x}] \right) \right) \right)$$

**Problem 984:** Result unnecessarily involves higher level functions.

$$\int \frac{(1-x)^n (1+x)^{-n}}{x^3} dx$$

Optimal (type 5, 71 leaves, 2 steps):

$$-\frac{(1-x)^{1+n} (1+x)^{1-n}}{2 x^2} + \frac{2 n (1-x)^{1+n} (1+x)^{-1-n} \text{Hypergeometric2F1}[2, 1+n, 2+n, \frac{1-x}{1+x}]}{1+n}$$

Result (type 6, 95 leaves):

$$-\left( \left( 3 (1-x)^n (1+x)^{-n} \text{AppellF1}[2, -n, n, 3, \frac{1}{x}, -\frac{1}{x}] \right) \middle/ \right. \\ \left. \left( 2 x \left( 3 x \text{AppellF1}[2, -n, n, 3, \frac{1}{x}, -\frac{1}{x}] - n \left( \text{AppellF1}[3, 1-n, n, 4, \frac{1}{x}, -\frac{1}{x}] + \text{AppellF1}[3, -n, 1+n, 4, \frac{1}{x}, -\frac{1}{x}] \right) \right) \right) \right)$$

**Problem 985:** Result unnecessarily involves higher level functions.

$$\int \frac{(1-x)^n (1+x)^{-n}}{x^4} dx$$

Optimal (type 5, 105 leaves, 4 steps):

$$-\frac{(1-x)^{1+n} (1+x)^{1-n}}{3 x^3} + \frac{n (1-x)^{1+n} (1+x)^{1-n}}{3 x^2} - \frac{2 (1+2 n^2) (1-x)^{1+n} (1+x)^{-1-n} \text{Hypergeometric2F1}[2, 1+n, 2+n, \frac{1-x}{1+x}]}{3 (1+n)}$$

Result (type 6, 95 leaves):

$$\begin{aligned}
& - \left( \left( 4 (1-x)^n (1+x)^{-n} \text{AppellF1}[3, -n, n, 4, \frac{1}{x}, -\frac{1}{x}] \right) \right. \\
& \left. \left( 3x^2 \left( 4x \text{AppellF1}[3, -n, n, 4, \frac{1}{x}, -\frac{1}{x}] - n \left( \text{AppellF1}[4, 1-n, n, 5, \frac{1}{x}, -\frac{1}{x}] + \text{AppellF1}[4, -n, 1+n, 5, \frac{1}{x}, -\frac{1}{x}] \right) \right) \right)
\end{aligned}$$

**Problem 993:** Result unnecessarily involves higher level functions.

$$\int x^m (3 - 2ax)^{-1+n} (6 + 4ax)^n dx$$

Optimal (type 5, 104 leaves, 5 steps):

$$\frac{2^n \times 3^{-1+2n} x^{1+m} \text{Hypergeometric2F1}\left[\frac{1+m}{2}, 1-n, \frac{3+m}{2}, \frac{4a^2 x^2}{9}\right]}{1+m} + \frac{2^{1+n} \times 9^{-1+n} a x^{2+m} \text{Hypergeometric2F1}\left[\frac{2+m}{2}, 1-n, \frac{4+m}{2}, \frac{4a^2 x^2}{9}\right]}{2+m}$$

Result (type 6, 168 leaves):

$$\begin{aligned}
& - \left( \left( 3 (2+m) x^{1+m} (18 - 8a^2 x^2)^n \text{AppellF1}[1+m, 1-n, -n, 2+m, \frac{2ax}{3}, -\frac{2ax}{3}] \right) \right. \\
& \left. \left( (1+m) (-3 + 2ax) \left( 3 (2+m) \text{AppellF1}[1+m, 1-n, -n, 2+m, \frac{2ax}{3}, -\frac{2ax}{3}] + \right. \right. \right. \\
& \left. \left. \left. 2ax \left( -(-1+n) \text{AppellF1}[2+m, 2-n, -n, 3+m, \frac{2ax}{3}, -\frac{2ax}{3}] + n \text{HypergeometricPFQ}\left[\left\{1+\frac{m}{2}, 1-n\right\}, \left\{2+\frac{m}{2}\right\}, \frac{4a^2 x^2}{9}\right] \right) \right) \right)
\end{aligned}$$

**Problem 994:** Result unnecessarily involves higher level functions.

$$\int x^m (3 - 2ax)^{-2+n} (6 + 4ax)^n dx$$

Optimal (type 5, 158 leaves, 8 steps):

$$\begin{aligned}
& \frac{2^n \times 9^{-1+n} x^{1+m} \text{Hypergeometric2F1}\left[\frac{1+m}{2}, 2-n, \frac{3+m}{2}, \frac{4a^2 x^2}{9}\right]}{1+m} + \\
& \frac{2^{2+n} \times 3^{-3+2n} a x^{2+m} \text{Hypergeometric2F1}\left[\frac{2+m}{2}, 2-n, \frac{4+m}{2}, \frac{4a^2 x^2}{9}\right]}{2+m} + \frac{2^{2+n} \times 9^{-2+n} a^2 x^{3+m} \text{Hypergeometric2F1}\left[\frac{3+m}{2}, 2-n, \frac{5+m}{2}, \frac{4a^2 x^2}{9}\right]}{3+m}
\end{aligned}$$

Result (type 6, 163 leaves):

$$\begin{aligned} & \left( 3 (2 + m) x^{1+m} (3 - 2 a x)^{-2+n} (6 + 4 a x)^n \text{AppellF1}[1 + m, 2 - n, -n, 2 + m, \frac{2 a x}{3}, -\frac{2 a x}{3}] \right) / \\ & \left( (1 + m) \left( 3 (2 + m) \text{AppellF1}[1 + m, 2 - n, -n, 2 + m, \frac{2 a x}{3}, -\frac{2 a x}{3}] + \right. \right. \\ & \left. \left. 2 a x \left( n \text{AppellF1}[2 + m, 2 - n, 1 - n, 3 + m, \frac{2 a x}{3}, -\frac{2 a x}{3}] - (-2 + n) \text{AppellF1}[2 + m, 3 - n, -n, 3 + m, \frac{2 a x}{3}, -\frac{2 a x}{3}] \right) \right) \right) \end{aligned}$$

**Problem 995: Result more than twice size of optimal antiderivative.**

$$\int x^m (a + b x)^{1+n} (c + d x)^n dx$$

Optimal (type 6, 79 leaves, 3 steps):

$$\frac{a x^{1+m} (a + b x)^n \left(1 + \frac{b x}{a}\right)^{-n} (c + d x)^n \left(1 + \frac{d x}{c}\right)^{-n} \text{AppellF1}[1 + m, -1 - n, -n, 2 + m, -\frac{b x}{a}, -\frac{d x}{c}]}{1 + m}$$

Result (type 6, 308 leaves):

$$\begin{aligned} & \frac{1}{2 + m} a c x^{1+m} (a + b x)^n (c + d x)^n \\ & \left( \left( a (2 + m)^2 \text{AppellF1}[1 + m, -n, -n, 2 + m, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \left( (1 + m) \left( a c (2 + m) \text{AppellF1}[1 + m, -n, -n, 2 + m, -\frac{b x}{a}, -\frac{d x}{c}] + \right. \right. \right. \right. \\ & \left. \left. \left. \left. n x \left( b c \text{AppellF1}[2 + m, 1 - n, -n, 3 + m, -\frac{b x}{a}, -\frac{d x}{c}] + a d \text{AppellF1}[2 + m, -n, 1 - n, 3 + m, -\frac{b x}{a}, -\frac{d x}{c}] \right) \right) \right) + \right. \\ & \left( b (3 + m) x \text{AppellF1}[2 + m, -n, -n, 3 + m, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \left( a c (3 + m) \text{AppellF1}[2 + m, -n, -n, 3 + m, -\frac{b x}{a}, -\frac{d x}{c}] + \right. \\ & \left. \left. \left. n x \left( b c \text{AppellF1}[3 + m, 1 - n, -n, 4 + m, -\frac{b x}{a}, -\frac{d x}{c}] + a d \text{AppellF1}[3 + m, -n, 1 - n, 4 + m, -\frac{b x}{a}, -\frac{d x}{c}] \right) \right) \right) \right) \end{aligned}$$

**Problem 998: Result unnecessarily involves higher level functions.**

$$\int \frac{\left(1 - \frac{x}{a}\right)^{-n/2} \left(1 + \frac{x}{a}\right)^{n/2}}{x^2} dx$$

Optimal (type 5, 70 leaves, 1 step):

$$\frac{4 \left(1 - \frac{x}{a}\right)^{1-\frac{n}{2}} \left(1 + \frac{x}{a}\right)^{\frac{1}{2}(-2+n)} \text{Hypergeometric2F1}[2, 1 - \frac{n}{2}, 2 - \frac{n}{2}, \frac{a-x}{a+x}]}{a (2 - n)}$$

Result (type 6, 139 leaves):

$$\left( \left( 4 \left( \frac{a+x}{a} \right)^{n/2} \left( 1 - \frac{x}{a} \right)^{-n/2} \text{AppellF1}[1, -\frac{n}{2}, \frac{n}{2}, 2, -\frac{a}{x}, \frac{a}{x}] \right) / \right. \\ \left. \left( 4 \times \text{AppellF1}[1, -\frac{n}{2}, \frac{n}{2}, 2, -\frac{a}{x}, \frac{a}{x}] + a n \left( \text{AppellF1}[2, 1 - \frac{n}{2}, \frac{n}{2}, 3, -\frac{a}{x}, \frac{a}{x}] + \text{AppellF1}[2, -\frac{n}{2}, \frac{2+n}{2}, 3, -\frac{a}{x}, \frac{a}{x}] \right) \right) \right)$$

**Problem 1000:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(1 - ax)^{-n} (1 + ax)^n}{x} dx$$

Optimal (type 5, 86 leaves, 3 steps):

$$\frac{(1 - ax)^{-n} (1 + ax)^n \text{Hypergeometric2F1}[1, -n, 1 - n, \frac{1 - ax}{1 + ax}]}{n} - \frac{2^n (1 - ax)^{-n} \text{Hypergeometric2F1}[-n, -n, 1 - n, \frac{1}{2} (1 - ax)]}{n}$$

Result (type 6, 182 leaves):

$$\left( 2 (-2 + n) (1 - ax)^{1-n} (1 + ax)^n \text{AppellF1}[1 - n, -n, 1, 2 - n, \frac{1}{2} (1 - ax), 1 - ax] \right) / \\ \left( a (1 - n) \times \left( -2 (-2 + n) \text{AppellF1}[1 - n, -n, 1, 2 - n, \frac{1}{2} (1 - ax), 1 - ax] + \right. \right. \\ \left. \left. (-1 + ax) \left( n \text{AppellF1}[2 - n, 1 - n, 1, 3 - n, \frac{1}{2} (1 - ax), 1 - ax] - 2 \text{AppellF1}[2 - n, -n, 2, 3 - n, \frac{1}{2} (1 - ax), 1 - ax] \right) \right) \right)$$

**Problem 1001:** Result unnecessarily involves higher level functions.

$$\int \frac{(1 - ax)^{1-n} (1 + ax)^{1+n}}{x^2} dx$$

Optimal (type 5, 106 leaves, 3 steps):

$$\frac{2 a (1 - ax)^{1-n} (1 + ax)^{-1+n} \text{Hypergeometric2F1}[2, 1 - n, 2 - n, \frac{1 - ax}{1 + ax}]}{1 - n} + \frac{2^n a (1 - ax)^{1-n} \text{Hypergeometric2F1}[1 - n, -n, 2 - n, \frac{1}{2} (1 - ax)]}{1 - n}$$

Result (type 6, 158 leaves):

$$\frac{a (1 + ax)^n \left( - \left( \left( 2 (1 - ax)^{-n} \text{AppellF1}[1, n, -n, 2, \frac{1}{ax}, -\frac{1}{ax}] \right) / \right. \right.}{1+n} \\ \left. \left. \left( 2 ax \text{AppellF1}[1, n, -n, 2, \frac{1}{ax}, -\frac{1}{ax}] + n \left( \text{AppellF1}[2, n, 1-n, 3, \frac{1}{ax}, -\frac{1}{ax}] + \text{AppellF1}[2, 1+n, -n, 3, \frac{1}{ax}, -\frac{1}{ax}] \right) \right) \right) - \\ \frac{2^{-n} (1 + ax) \text{Hypergeometric2F1}[n, 1+n, 2+n, \frac{1}{2} (1 + ax)]}{1+n} \right)$$

Problem 1006: Result unnecessarily involves higher level functions.

$$\int \frac{(a - bx)^{-n} (a + bx)^{1+n}}{x} dx$$

Optimal (type 5, 142 leaves, 6 steps):

$$\frac{(a - bx)^{1-n} (a + bx)^n}{2n} - \frac{a (a - bx)^{-n} (a + bx)^n \text{Hypergeometric2F1}[1, n, 1+n, \frac{a+bx}{a-bx}]}{n} + \\ \frac{2^{-1-n} (1+2n) (a - bx)^{-n} \left(\frac{a-bx}{a}\right)^n (a + bx)^{1+n} \text{Hypergeometric2F1}[n, 1+n, 2+n, \frac{a+bx}{2a}]}{n (1+n)}$$

Result (type 6, 262 leaves):

$$(a - bx)^{-n} (a + bx)^n \left( \left( 2 a^2 (-2+n) (a - bx) \text{AppellF1}[1-n, -n, 1, 2-n, \frac{a-bx}{2a}, 1-\frac{b}{a}] \right) / \right. \\ \left( b (-1+n) x \left( 2 a (-2+n) \text{AppellF1}[1-n, -n, 1, 2-n, \frac{a-bx}{2a}, 1-\frac{b}{a}] + (a - bx) \left( n \text{AppellF1}[2-n, 1-n, 1, 3-n, \frac{a-bx}{2a}, 1-\frac{b}{a}] - \right. \right. \right. \\ \left. \left. \left. 2 \text{AppellF1}[2-n, -n, 2, 3-n, \frac{a-bx}{2a}, 1-\frac{b}{a}] \right) \right) \right) + \frac{(a + bx) \left( 1 - \frac{a+b}{2a} \right)^n \text{Hypergeometric2F1}[n, 1+n, 2+n, \frac{a+bx}{2a}]}{1+n} \right)$$

Problem 1007: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(a - bx)^{-n} (a + bx)^{1+n}}{x^2} dx$$

Optimal (type 5, 140 leaves, 5 steps):

$$-\frac{\left(a - bx\right)^{-n} \left(a + bx\right)^{1+n}}{x} + \frac{b \left(1 + 2 n\right) \left(a - bx\right)^{-n} \left(a + bx\right)^n \text{Hypergeometric2F1}\left[1, -n, 1 - n, \frac{a - bx}{a + bx}\right]}{n} -$$

$$\frac{2^n b \left(a - bx\right)^{-n} \left(a + bx\right)^n \left(\frac{a + bx}{a}\right)^{-n} \text{Hypergeometric2F1}\left[-n, -n, 1 - n, \frac{a - bx}{2a}\right]}{n}$$

### Result (type 6, 324 leaves):

$$2 a \left(a - b x\right)^{-n} \left(a + b x\right)^n \left(- \left(\left(b \text{AppellF1}\left[1, n, -n, 2, \frac{a}{b x}, -\frac{a}{b x}\right]\right) \middle/ \right.\right.$$

$$\left.\left(2 b x \text{AppellF1}\left[1, n, -n, 2, \frac{a}{b x}, -\frac{a}{b x}\right] + a n \left(\text{AppellF1}\left[2, n, 1-n, 3, \frac{a}{b x}, -\frac{a}{b x}\right] + \text{AppellF1}\left[2, 1+n, -n, 3, \frac{a}{b x}, -\frac{a}{b x}\right]\right)\right) \middle) + \right.$$

$$\left((-2+n) \left(a - b x\right) \text{AppellF1}\left[1-n, -n, 1, 2-n, \frac{a - b x}{2 a}, 1 - \frac{b x}{a}\right]\right) \middle/ \left((-1+n) x \left(2 a (-2+n) \text{AppellF1}\left[1-n, -n, 1, 2-n, \frac{a - b x}{2 a}, 1 - \frac{b x}{a}\right] + \right.\right.$$

$$\left.\left.(a - b x) \left(n \text{AppellF1}\left[2-n, 1-n, 1, 3-n, \frac{a - b x}{2 a}, 1 - \frac{b x}{a}\right] - 2 \text{AppellF1}\left[2-n, -n, 2, 3-n, \frac{a - b x}{2 a}, 1 - \frac{b x}{a}\right]\right)\right)\right)$$

Problem 1008: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(a - bx)^{-n} (a + bx)^{1+n}}{x^3} dx$$

Optimal (type 5, 62 leaves, 1 step) :

$$-\frac{4 b^2 (a - b x)^{1-n} (a + b x)^{-1+n} \text{Hypergeometric2F1}\left[3, 1-n, 2-n, \frac{a-b x}{a+b x}\right]}{a (1-n)}$$

Result (type 6, 254 leaves):

$$\frac{1}{2} b \left(a - b x\right)^{-n} \left(a + b x\right)^n \left(- \left(\left(4 b \text{AppellF1}\left[1, n, -n, 2, \frac{a}{b x}, -\frac{a}{b x}\right]\right) / \left(2 b x \text{AppellF1}\left[1, n, -n, 2, \frac{a}{b x}, -\frac{a}{b x}\right]\right) + a n \left(\text{AppellF1}\left[2, n, 1-n, 3, \frac{a}{b x}, -\frac{a}{b x}\right] + \text{AppellF1}\left[2, 1+n, -n, 3, \frac{a}{b x}, -\frac{a}{b x}\right]\right)\right) - \left(3 a \text{AppellF1}\left[2, n, -n, 3, \frac{a}{b x}, -\frac{a}{b x}\right]\right) / \left(x \left(3 b x \text{AppellF1}\left[2, n, -n, 3, \frac{a}{b x}, -\frac{a}{b x}\right] + a n \left(\text{AppellF1}\left[3, n, 1-n, 4, \frac{a}{b x}, -\frac{a}{b x}\right] + \text{AppellF1}\left[3, 1+n, -n, 4, \frac{a}{b x}, -\frac{a}{b x}\right]\right)\right)\right)$$

Problem 1009: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(a - b x)^{-n} (a + b x)^{1+n}}{x^4} dx$$

Optimal (type 5, 101 leaves, 2 steps):

$$\frac{(a - bx)^{1-n} (a + bx)^{2+n}}{3a^2 x^3} - \frac{4b^3 (1+2n) (a - bx)^{1-n} (a + bx)^{-1+n} \text{Hypergeometric2F1}[3, 1-n, 2-n, \frac{a-bx}{a+bx}]}{3a^2 (1-n)}$$

Result (type 6, 255 leaves):

$$\begin{aligned} & \frac{1}{6x^2} b (a - bx)^{-n} (a + bx)^n \left( - \left( 9b x \text{AppellF1}[2, n, -n, 3, \frac{a}{bx}, -\frac{a}{bx}] \right) \right. \\ & \quad \left. \left( 3b x \text{AppellF1}[2, n, -n, 3, \frac{a}{bx}, -\frac{a}{bx}] + a n \left( \text{AppellF1}[3, n, 1-n, 4, \frac{a}{bx}, -\frac{a}{bx}] + \text{AppellF1}[3, 1+n, -n, 4, \frac{a}{bx}, -\frac{a}{bx}] \right) \right) \right) - \\ & \left( 8a \text{AppellF1}[3, n, -n, 4, \frac{a}{bx}, -\frac{a}{bx}] \right) \left/ \left( 4b x \text{AppellF1}[3, n, -n, 4, \frac{a}{bx}, -\frac{a}{bx}] + \right. \right. \\ & \quad \left. \left. a n \left( \text{AppellF1}[4, n, 1-n, 5, \frac{a}{bx}, -\frac{a}{bx}] + \text{AppellF1}[4, 1+n, -n, 5, \frac{a}{bx}, -\frac{a}{bx}] \right) \right) \right) \end{aligned}$$

Problem 1010: Result unnecessarily involves higher level functions.

$$\int \frac{(a - bx)^{-n} (a + bx)^{1+n}}{x^5} dx$$

Optimal (type 5, 139 leaves, 4 steps):

$$\frac{(a - bx)^{1-n} (a + bx)^{2+n}}{4a^2 x^4} - \frac{b (1+2n) (a - bx)^{1-n} (a + bx)^{2+n}}{12a^3 x^3} - \frac{4b^4 (1+n+n^2) (a - bx)^{1-n} (a + bx)^{-1+n} \text{Hypergeometric2F1}[3, 1-n, 2-n, \frac{a-bx}{a+bx}]}{3a^3 (1-n)}$$

Result (type 6, 255 leaves):

$$\begin{aligned} & \frac{1}{12x^3} b (a - bx)^{-n} (a + bx)^n \left( - \left( 16b x \text{AppellF1}[3, n, -n, 4, \frac{a}{bx}, -\frac{a}{bx}] \right) \right. \\ & \quad \left. \left( 4b x \text{AppellF1}[3, n, -n, 4, \frac{a}{bx}, -\frac{a}{bx}] + a n \left( \text{AppellF1}[4, n, 1-n, 5, \frac{a}{bx}, -\frac{a}{bx}] + \text{AppellF1}[4, 1+n, -n, 5, \frac{a}{bx}, -\frac{a}{bx}] \right) \right) \right) - \\ & \left( 15a \text{AppellF1}[4, n, -n, 5, \frac{a}{bx}, -\frac{a}{bx}] \right) \left/ \left( 5b x \text{AppellF1}[4, n, -n, 5, \frac{a}{bx}, -\frac{a}{bx}] + \right. \right. \\ & \quad \left. \left. a n \left( \text{AppellF1}[5, n, 1-n, 6, \frac{a}{bx}, -\frac{a}{bx}] + \text{AppellF1}[5, 1+n, -n, 6, \frac{a}{bx}, -\frac{a}{bx}] \right) \right) \right) \end{aligned}$$

Problem 1011: Result more than twice size of optimal antiderivative.

$$\int (a + bx) (A + Bx) (d + ex)^4 dx$$

Optimal (type 1, 77 leaves, 2 steps):

$$\frac{(b d - a e) (B d - A e) (d + e x)^5}{5 e^3} - \frac{(2 b B d - A b e - a B e) (d + e x)^6}{6 e^3} + \frac{b B (d + e x)^7}{7 e^3}$$

Result (type 1, 172 leaves):

$$a A d^4 x + \frac{1}{2} d^3 (A b d + a B d + 4 a A e) x^2 + \frac{1}{3} d^2 (2 a e (2 B d + 3 A e) + b d (B d + 4 A e)) x^3 + \\ \frac{1}{2} d e (a e (3 B d + 2 A e) + b d (2 B d + 3 A e)) x^4 + \frac{1}{5} e^2 (a e (4 B d + A e) + 2 b d (3 B d + 2 A e)) x^5 + \frac{1}{6} e^3 (4 b B d + A b e + a B e) x^6 + \frac{1}{7} b B e^4 x^7$$

Problem 1022: Result more than twice size of optimal antiderivative.

$$\int (a + b x)^2 (A + B x) (d + e x)^4 dx$$

Optimal (type 1, 120 leaves, 2 steps):

$$-\frac{(b d - a e)^2 (B d - A e) (d + e x)^5}{5 e^4} + \frac{(b d - a e) (3 b B d - 2 A b e - a B e) (d + e x)^6}{6 e^4} - \frac{b (3 b B d - A b e - 2 a B e) (d + e x)^7}{7 e^4} + \frac{b^2 B (d + e x)^8}{8 e^4}$$

Result (type 1, 283 leaves):

$$a^2 A d^4 x + \frac{1}{2} a d^3 (2 A b d + a B d + 4 a A e) x^2 + \frac{1}{3} d^2 (2 a B d (b d + 2 a e) + A (b^2 d^2 + 8 a b d e + 6 a^2 e^2)) x^3 + \\ \frac{1}{4} d (2 a^2 e^2 (3 B d + 2 A e) + 4 a b d e (2 B d + 3 A e) + b^2 d^2 (B d + 4 A e)) x^4 + \frac{1}{5} e (a^2 e^2 (4 B d + A e) + 4 a b d e (3 B d + 2 A e) + 2 b^2 d^2 (2 B d + 3 A e)) x^5 + \\ \frac{1}{6} e^2 (a^2 B e^2 + 2 a b e (4 B d + A e) + 2 b^2 d (3 B d + 2 A e)) x^6 + \frac{1}{7} b e^3 (4 b B d + A b e + 2 a B e) x^7 + \frac{1}{8} b^2 B e^4 x^8$$

Problem 1035: Result more than twice size of optimal antiderivative.

$$\int (a + b x)^3 (A + B x) (d + e x)^5 dx$$

Optimal (type 1, 163 leaves, 2 steps):

$$\frac{(b d - a e)^3 (B d - A e) (d + e x)^6}{6 e^5} - \frac{(b d - a e)^2 (4 b B d - 3 A b e - a B e) (d + e x)^7}{7 e^5} + \\ \frac{3 b (b d - a e) (2 b B d - A b e - a B e) (d + e x)^8}{8 e^5} - \frac{b^2 (4 b B d - A b e - 3 a B e) (d + e x)^9}{9 e^5} + \frac{b^3 B (d + e x)^{10}}{10 e^5}$$

Result (type 1, 471 leaves):

$$\begin{aligned}
& a^3 A d^5 x + \frac{1}{2} a^2 d^4 (3 A b d + a B d + 5 a A e) x^2 + \frac{1}{3} a d^3 (a B d (3 b d + 5 a e) + A (3 b^2 d^2 + 15 a b d e + 10 a^2 e^2)) x^3 + \\
& \frac{1}{4} d^2 (a B d (3 b^2 d^2 + 15 a b d e + 10 a^2 e^2) + A (b^3 d^3 + 15 a b^2 d^2 e + 30 a^2 b d e^2 + 10 a^3 e^3)) x^4 + \\
& \frac{1}{5} d (30 a^2 b d e^2 (B d + A e) + 5 a^3 e^3 (2 B d + A e) + 15 a b^2 d^2 e (B d + 2 A e) + b^3 d^3 (B d + 5 A e)) x^5 + \\
& \frac{1}{6} e (30 a b^2 d^2 e (B d + A e) + 15 a^2 b d e^2 (2 B d + A e) + a^3 e^3 (5 B d + A e) + 5 b^3 d^3 (B d + 2 A e)) x^6 + \\
& \frac{1}{7} e^2 (a^3 B e^3 + 10 b^3 d^2 (B d + A e) + 15 a b^2 d e (2 B d + A e) + 3 a^2 b e^2 (5 B d + A e)) x^7 + \\
& \frac{1}{8} b e^3 (3 a^2 B e^2 + 5 b^2 d (2 B d + A e) + 3 a b e (5 B d + A e)) x^8 + \frac{1}{9} b^2 e^4 (5 b B d + A b e + 3 a B e) x^9 + \frac{1}{10} b^3 B e^5 x^{10}
\end{aligned}$$

**Problem 1036: Result more than twice size of optimal antiderivative.**

$$\int (a + b x)^3 (A + B x) (d + e x)^4 dx$$

Optimal (type 1, 163 leaves, 2 steps):

$$\begin{aligned}
& \frac{(b d - a e)^3 (B d - A e) (d + e x)^5}{5 e^5} - \frac{(b d - a e)^2 (4 b B d - 3 A b e - a B e) (d + e x)^6}{6 e^5} + \\
& \frac{3 b (b d - a e) (2 b B d - A b e - a B e) (d + e x)^7}{7 e^5} - \frac{b^2 (4 b B d - A b e - 3 a B e) (d + e x)^8}{8 e^5} + \frac{b^3 B (d + e x)^9}{9 e^5}
\end{aligned}$$

Result (type 1, 397 leaves):

$$\begin{aligned}
& a^3 A d^4 x + \frac{1}{2} a^2 d^3 (3 A b d + a B d + 4 a A e) x^2 + \frac{1}{3} a d^2 (a B d (3 b d + 4 a e) + 3 A (b^2 d^2 + 4 a b d e + 2 a^2 e^2)) x^3 + \\
& \frac{1}{4} d (3 a B d (b^2 d^2 + 4 a b d e + 2 a^2 e^2) + A (b^3 d^3 + 12 a b^2 d^2 e + 18 a^2 b d e^2 + 4 a^3 e^3)) x^4 + \\
& \frac{1}{5} (a^3 e^3 (4 B d + A e) + 6 a^2 b d e^2 (3 B d + 2 A e) + 6 a b^2 d^2 e (2 B d + 3 A e) + b^3 d^3 (B d + 4 A e)) x^5 + \\
& \frac{1}{6} e (a^3 B e^3 + 3 a^2 b e^2 (4 B d + A e) + 6 a b^2 d e (3 B d + 2 A e) + 2 b^3 d^2 (2 B d + 3 A e)) x^6 + \\
& \frac{1}{7} b e^2 (3 a^2 B e^2 + 3 a b e (4 B d + A e) + 2 b^2 d (3 B d + 2 A e)) x^7 + \frac{1}{8} b^2 e^3 (4 b B d + A b e + 3 a B e) x^8 + \frac{1}{9} b^3 B e^4 x^9
\end{aligned}$$

**Problem 1046: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + bx)^3 (A + Bx)}{(d + ex)^6} dx$$

Optimal (type 1, 86 leaves, 2 steps) :

$$-\frac{(Bd - Ae)(a + bx)^4}{5e(bd - ae)(d + ex)^5} + \frac{(4bd^2 + Ab - 5ab^2)(a + bx)^4}{20e(bd - ae)^2(d + ex)^4}$$

Result (type 1, 211 leaves) :

$$-\frac{1}{20e^5(d + ex)^5}(a^3e^3(4Ae + B(d + 5ex)) + a^2be^2(3Ae(d + 5ex) + 2B(d^2 + 5de + 10e^2x^2)) + ab^2e(2Ae(d^2 + 5de + 10e^2x^2) + 3B(d^3 + 5d^2ex + 10de^2x^2 + 10e^3x^3)) + b^3(Ae(d^3 + 5d^2ex + 10de^2x^2 + 10e^3x^3) + 4B(d^4 + 5d^3ex + 10d^2e^2x^2 + 10de^3x^3 + 5e^4x^4))$$

**Problem 1051: Result more than twice size of optimal antiderivative.**

$$\int (a + bx)^6 (A + Bx) (d + ex)^8 dx$$

Optimal (type 1, 292 leaves, 2 steps) :

$$-\frac{(bd - ae)^6(Bd - Ae)(d + ex)^9}{9e^8} + \frac{(bd - ae)^5(7bd^2 - 6abe - abe)(d + ex)^{10}}{10e^8} - \frac{3b(bd - ae)^4(7bd^2 - 5abe - 2abe)(d + ex)^{11}}{11e^8} + \frac{5b^2(bd - ae)^3(7bd^2 - 4abe - 3abe)(d + ex)^{12}}{12e^8} - \frac{5b^3(bd - ae)^2(7bd^2 - 3abe - 4abe)(d + ex)^{13}}{13e^8} + \frac{3b^4(bd - ae)(7bd^2 - 2abe - 5abe)(d + ex)^{14}}{14e^8} - \frac{b^5(7bd^2 - Abe - 6abe)(d + ex)^{15}}{15e^8} + \frac{b^6B(d + ex)^{16}}{16e^8}$$

Result (type 1, 1385 leaves) :

$$\begin{aligned}
& a^6 A d^8 x + \frac{1}{2} a^5 d^7 (6 A b d + a B d + 8 a A e) x^2 + \frac{1}{3} a^4 d^6 (2 a B d (3 b d + 4 a e) + A (15 b^2 d^2 + 48 a b d e + 28 a^2 e^2)) x^3 + \\
& \frac{1}{4} a^3 d^5 (a B d (15 b^2 d^2 + 48 a b d e + 28 a^2 e^2) + 4 A (5 b^3 d^3 + 30 a b^2 d^2 e + 42 a^2 b d e^2 + 14 a^3 e^3)) x^4 + \\
& \frac{1}{5} a^2 d^4 (4 a B d (5 b^3 d^3 + 30 a b^2 d^2 e + 42 a^2 b d e^2 + 14 a^3 e^3) + A (15 b^4 d^4 + 160 a b^3 d^3 e + 420 a^2 b^2 d^2 e^2 + 336 a^3 b d e^3 + 70 a^4 e^4)) x^5 + \\
& \frac{1}{6} a d^3 (a B d (15 b^4 d^4 + 160 a b^3 d^3 e + 420 a^2 b^2 d^2 e^2 + 336 a^3 b d e^3 + 70 a^4 e^4) + \\
& 2 A (3 b^5 d^5 + 60 a b^4 d^4 e + 280 a^2 b^3 d^3 e^2 + 420 a^3 b^2 d^2 e^3 + 210 a^4 b d e^4 + 28 a^5 e^5)) x^6 + \\
& \frac{1}{7} d^2 (2 a B d (3 b^5 d^5 + 60 a b^4 d^4 e + 280 a^2 b^3 d^3 e^2 + 420 a^3 b^2 d^2 e^3 + 210 a^4 b d e^4 + 28 a^5 e^5) + \\
& A (b^6 d^6 + 48 a b^5 d^5 e + 420 a^2 b^4 d^4 e^2 + 1120 a^3 b^3 d^3 e^3 + 1050 a^4 b^2 d^2 e^4 + 336 a^5 b d e^5 + 28 a^6 e^6)) x^7 + \\
& \frac{1}{8} d (168 a^5 b d e^5 (2 B d + A e) + 420 a^2 b^4 d^4 e^2 (B d + 2 A e) + 4 a^6 e^6 (7 B d + 2 A e) + 210 a^4 b^2 d^2 e^4 (5 B d + 4 A e) + \\
& 280 a^3 b^3 d^3 e^3 (4 B d + 5 A e) + 24 a b^5 d^5 e (2 B d + 7 A e) + b^6 d^6 (B d + 8 A e)) x^8 + \\
& \frac{1}{9} e (420 a^4 b^2 d^2 e^4 (2 B d + A e) + a^6 e^6 (8 B d + A e) + 168 a b^5 d^5 e (B d + 2 A e) + 24 a^5 b d e^5 (7 B d + 2 A e) + \\
& 280 a^3 b^3 d^3 e^3 (5 B d + 4 A e) + 210 a^2 b^4 d^4 e^2 (4 B d + 5 A e) + 4 b^6 d^6 (2 B d + 7 A e)) x^9 + \\
& \frac{1}{10} e^2 (a^6 B e^6 + 560 a^3 b^3 d^2 e^3 (2 B d + A e) + 6 a^5 b e^5 (8 B d + A e) + 28 b^6 d^5 (B d + 2 A e) + 60 a^4 b^2 d e^4 (7 B d + 2 A e) + \\
& 210 a^2 b^4 d^3 e^2 (5 B d + 4 A e) + 84 a b^5 d^4 e (4 B d + 5 A e)) x^{10} + \frac{1}{11} b e^3 \\
& (6 a^5 B e^5 + 420 a^2 b^3 d^2 e^2 (2 B d + A e) + 15 a^4 b e^4 (8 B d + A e) + 80 a^3 b^2 d e^3 (7 B d + 2 A e) + 84 a b^4 d^3 e (5 B d + 4 A e) + 14 b^5 d^4 (4 B d + 5 A e)) x^{11} + \\
& \frac{1}{12} b^2 e^4 (15 a^4 B e^4 + 168 a b^3 d^2 e (2 B d + A e) + 20 a^3 b e^3 (8 B d + A e) + 60 a^2 b^2 d e^2 (7 B d + 2 A e) + 14 b^4 d^3 (5 B d + 4 A e)) x^{12} + \\
& \frac{1}{13} b^3 e^5 (20 a^3 B e^3 + 28 b^3 d^2 (2 B d + A e) + 15 a^2 b e^2 (8 B d + A e) + 24 a b^2 d e (7 B d + 2 A e)) x^{13} + \\
& \frac{1}{14} b^4 e^6 (15 a^2 B e^2 + 6 a b e (8 B d + A e) + 4 b^2 d (7 B d + 2 A e)) x^{14} + \\
& \frac{1}{15} b^5 e^7 (8 b B d + A b e + 6 a B e) x^{15} + \frac{1}{16} b^6 B e^8 x^{16}
\end{aligned}$$

**Problem 1052: Result more than twice size of optimal antiderivative.**

$$\int (a + b x)^6 (A + B x) (d + e x)^7 dx$$

Optimal (type 1, 292 leaves, 2 steps):

$$\begin{aligned}
& - \frac{(b d - a e)^6 (B d - A e) (d + e x)^8}{8 e^8} + \frac{(b d - a e)^5 (7 b B d - 6 A b e - a B e) (d + e x)^9}{9 e^8} - \frac{3 b (b d - a e)^4 (7 b B d - 5 A b e - 2 a B e) (d + e x)^{10}}{10 e^8} + \\
& \frac{5 b^2 (b d - a e)^3 (7 b B d - 4 A b e - 3 a B e) (d + e x)^{11}}{11 e^8} - \frac{5 b^3 (b d - a e)^2 (7 b B d - 3 A b e - 4 a B e) (d + e x)^{12}}{12 e^8} + \\
& \frac{3 b^4 (b d - a e) (7 b B d - 2 A b e - 5 a B e) (d + e x)^{13}}{13 e^8} - \frac{b^5 (7 b B d - A b e - 6 a B e) (d + e x)^{14}}{14 e^8} + \frac{b^6 B (d + e x)^{15}}{15 e^8}
\end{aligned}$$

Result (type 1, 1224 leaves):

$$\begin{aligned}
& a^6 A d^7 x + \frac{1}{2} a^5 d^6 (6 A b d + a B d + 7 a A e) x^2 + \frac{1}{3} a^4 d^5 (a B d (6 b d + 7 a e) + 3 A (5 b^2 d^2 + 14 a b d e + 7 a^2 e^2)) x^3 + \\
& \frac{1}{4} a^3 d^4 (3 a B d (5 b^2 d^2 + 14 a b d e + 7 a^2 e^2) + A (20 b^3 d^3 + 105 a b^2 d^2 e + 126 a^2 b d e^2 + 35 a^3 e^3)) x^4 + \\
& \frac{1}{5} a^2 d^3 (a B d (20 b^3 d^3 + 105 a b^2 d^2 e + 126 a^2 b d e^2 + 35 a^3 e^3) + 5 A (3 b^4 d^4 + 28 a b^3 d^3 e + 63 a^2 b^2 d^2 e^2 + 42 a^3 b d e^3 + 7 a^4 e^4)) x^5 + \frac{1}{6} a d^2 \\
& (5 a B d (3 b^4 d^4 + 28 a b^3 d^3 e + 63 a^2 b^2 d^2 e^2 + 42 a^3 b d e^3 + 7 a^4 e^4) + 3 A (2 b^5 d^5 + 35 a b^4 d^4 e + 140 a^2 b^3 d^3 e^2 + 175 a^3 b^2 d^2 e^3 + 70 a^4 b d e^4 + 7 a^5 e^5)) \\
& x^6 + \frac{1}{7} d (3 a B d (2 b^5 d^5 + 35 a b^4 d^4 e + 140 a^2 b^3 d^3 e^2 + 175 a^3 b^2 d^2 e^3 + 70 a^4 b d e^4 + 7 a^5 e^5)) + \\
& A (b^6 d^6 + 42 a b^5 d^5 e + 315 a^2 b^4 d^4 e^2 + 700 a^3 b^3 d^3 e^3 + 525 a^4 b^2 d^2 e^4 + 126 a^5 b d e^5 + 7 a^6 e^6) x^7 + \\
& \frac{1}{8} (700 a^3 b^3 d^3 e^3 (B d + A e) + 42 a^5 b d e^5 (3 B d + A e) + a^6 e^6 (7 B d + A e) + 42 a b^5 d^5 e (B d + 3 A e) + 105 a^4 b^2 d^2 e^4 (5 B d + 3 A e) + \\
& 105 a^2 b^4 d^4 e^2 (3 B d + 5 A e) + b^6 d^6 (B d + 7 A e)) x^8 + \frac{1}{9} e (a^6 B e^6 + 525 a^2 b^4 d^3 e^2 (B d + A e) + 105 a^4 b^2 d e^4 (3 B d + A e) + \\
& 6 a^5 b e^5 (7 B d + A e) + 7 b^6 d^5 (B d + 3 A e) + 140 a^3 b^3 d^2 e^3 (5 B d + 3 A e) + 42 a b^5 d^4 e (3 B d + 5 A e)) x^9 + \frac{1}{10} b e^2 \\
& (6 a^5 B e^5 + 210 a b^4 d^3 e (B d + A e) + 140 a^3 b^2 d e^3 (3 B d + A e) + 15 a^4 b e^4 (7 B d + A e) + 105 a^2 b^3 d^2 e^2 (5 B d + 3 A e) + 7 b^5 d^4 (3 B d + 5 A e)) x^{10} + \\
& \frac{1}{11} b^2 e^3 (15 a^4 B e^4 + 35 b^4 d^3 (B d + A e) + 105 a^2 b^2 d e^2 (3 B d + A e) + 20 a^3 b e^3 (7 B d + A e) + 42 a b^3 d^2 e (5 B d + 3 A e)) x^{11} + \\
& \frac{1}{12} b^3 e^4 (20 a^3 B e^3 + 42 a b^2 d e (3 B d + A e) + 15 a^2 b e^2 (7 B d + A e) + 7 b^3 d^2 (5 B d + 3 A e)) x^{12} + \\
& \frac{1}{13} b^4 e^5 (15 a^2 B e^2 + 7 b^2 d (3 B d + A e) + 6 a b e (7 B d + A e)) x^{13} + \frac{1}{14} b^5 e^6 (7 b B d + A b e + 6 a B e) x^{14} + \frac{1}{15} b^6 B e^7 x^{15}
\end{aligned}$$

Problem 1053: Result more than twice size of optimal antiderivative.

$$\int (a + b x)^6 (A + B x) (d + e x)^6 dx$$

Optimal (type 1, 290 leaves, 2 steps):

$$\begin{aligned}
& \frac{(A b - a B) (b d - a e)^6 (a + b x)^7}{7 b^8} + \frac{(b d - a e)^5 (b B d + 6 A b e - 7 a B e) (a + b x)^8}{8 b^8} + \frac{e (b d - a e)^4 (2 b B d + 5 A b e - 7 a B e) (a + b x)^9}{3 b^8} + \\
& \frac{e^2 (b d - a e)^3 (3 b B d + 4 A b e - 7 a B e) (a + b x)^{10}}{2 b^8} + \frac{5 e^3 (b d - a e)^2 (4 b B d + 3 A b e - 7 a B e) (a + b x)^{11}}{11 b^8} + \\
& \frac{e^4 (b d - a e) (5 b B d + 2 A b e - 7 a B e) (a + b x)^{12}}{4 b^8} + \frac{e^5 (6 b B d + A b e - 7 a B e) (a + b x)^{13}}{13 b^8} + \frac{B e^6 (a + b x)^{14}}{14 b^8}
\end{aligned}$$

Result (type 1, 1069 leaves):

$$\begin{aligned}
& a^6 A d^6 x + \frac{1}{2} a^5 d^5 (a B d + 6 A (b d + a e)) x^2 + a^4 d^4 (2 a B d (b d + a e) + A (5 b^2 d^2 + 12 a b d e + 5 a^2 e^2)) x^3 + \\
& \frac{1}{4} a^3 d^3 (3 a B d (5 b^2 d^2 + 12 a b d e + 5 a^2 e^2) + 10 A (2 b^3 d^3 + 9 a b^2 d^2 e + 9 a^2 b d e^2 + 2 a^3 e^3)) x^4 + \\
& a^2 d^2 (2 a B d (2 b^3 d^3 + 9 a b^2 d^2 e + 9 a^2 b d e^2 + 2 a^3 e^3) + 3 A (b^4 d^4 + 8 a b^3 d^3 e + 15 a^2 b^2 d^2 e^2 + 8 a^3 b d e^3 + a^4 e^4)) x^5 + \\
& \frac{1}{2} a d (5 a B d (b^4 d^4 + 8 a b^3 d^3 e + 15 a^2 b^2 d^2 e^2 + 8 a^3 b d e^3 + a^4 e^4) + 2 A (b^5 d^5 + 15 a b^4 d^4 e + 50 a^2 b^3 d^3 e^2 + 50 a^3 b^2 d^2 e^3 + 15 a^4 b d e^4 + a^5 e^5)) x^6 + \\
& \frac{1}{7} (6 a B d (b^5 d^5 + 15 a b^4 d^4 e + 50 a^2 b^3 d^3 e^2 + 50 a^3 b^2 d^2 e^3 + 15 a^4 b d e^4 + a^5 e^5)) + \\
& A (b^6 d^6 + 36 a b^5 d^5 e + 225 a^2 b^4 d^4 e^2 + 400 a^3 b^3 d^3 e^3 + 225 a^4 b^2 d^2 e^4 + 36 a^5 b d e^5 + a^6 e^6) x^7 + \frac{1}{8} (a^6 B e^6 + 6 a^5 b e^5 (6 B d + A e)) + \\
& 45 a^4 b^2 d e^4 (5 B d + 2 A e) + 100 a^3 b^3 d^2 e^3 (4 B d + 3 A e) + 75 a^2 b^4 d^3 e^2 (3 B d + 4 A e) + 18 a b^5 d^4 e (2 B d + 5 A e) + b^6 d^5 (B d + 6 A e) x^8 + \\
& \frac{1}{3} b e (2 a^5 B e^5 + 5 a^4 b e^4 (6 B d + A e) + 20 a^3 b^2 d e^3 (5 B d + 2 A e) + 25 a^2 b^3 d^2 e^2 (4 B d + 3 A e) + 10 a b^4 d^3 e (3 B d + 4 A e) + b^5 d^4 (2 B d + 5 A e)) x^9 + \\
& \frac{1}{2} b^2 e^2 (3 a^4 B e^4 + 4 a^3 b e^3 (6 B d + A e) + 9 a^2 b^2 d e^2 (5 B d + 2 A e) + 6 a b^3 d^2 e (4 B d + 3 A e) + b^4 d^3 (3 B d + 4 A e)) x^{10} + \\
& \frac{1}{11} b^3 e^3 (20 a^3 B e^3 + 15 a^2 b e^2 (6 B d + A e) + 18 a b^2 d e (5 B d + 2 A e) + 5 b^3 d^2 (4 B d + 3 A e)) x^{11} + \\
& \frac{1}{4} b^4 e^4 (5 a^2 B e^2 + 2 a b e (6 B d + A e) + b^2 d (5 B d + 2 A e)) x^{12} + \frac{1}{13} b^5 e^5 (6 b B d + A b e + 6 a B e) x^{13} + \frac{1}{14} b^6 B e^6 x^{14}
\end{aligned}$$

Problem 1054: Result more than twice size of optimal antiderivative.

$$\int (a + b x)^6 (A + B x) (d + e x)^5 dx$$

Optimal (type 1, 240 leaves, 2 steps):

$$\begin{aligned}
& \frac{(A b - a B) (b d - a e)^5 (a + b x)^7}{7 b^7} + \frac{(b d - a e)^4 (b B d + 5 A b e - 6 a B e) (a + b x)^8}{8 b^7} + \\
& \frac{5 e (b d - a e)^3 (b B d + 2 A b e - 3 a B e) (a + b x)^9}{9 b^7} + \frac{e^2 (b d - a e)^2 (b B d + A b e - 2 a B e) (a + b x)^{10}}{b^7} + \\
& \frac{5 e^3 (b d - a e) (2 b B d + A b e - 3 a B e) (a + b x)^{11}}{11 b^7} + \frac{e^4 (5 b B d + A b e - 6 a B e) (a + b x)^{12}}{12 b^7} + \frac{B e^5 (a + b x)^{13}}{13 b^7}
\end{aligned}$$

Result (type 1, 907 leaves):

$$\begin{aligned}
& a^6 A d^5 x + \frac{1}{2} a^5 d^4 (6 A b d + a B d + 5 a A e) x^2 + \frac{1}{3} a^4 d^3 (a B d (6 b d + 5 a e) + 5 A (3 b^2 d^2 + 6 a b d e + 2 a^2 e^2)) x^3 + \\
& \frac{5}{4} a^3 d^2 (a B d (3 b^2 d^2 + 6 a b d e + 2 a^2 e^2) + A (4 b^3 d^3 + 15 a b^2 d^2 e + 12 a^2 b d e^2 + 2 a^3 e^3)) x^4 + \\
& a^2 d (a B d (4 b^3 d^3 + 15 a b^2 d^2 e + 12 a^2 b d e^2 + 2 a^3 e^3) + A (3 b^4 d^4 + 20 a b^3 d^3 e + 30 a^2 b^2 d^2 e^2 + 12 a^3 b d e^3 + a^4 e^4)) x^5 + \\
& \frac{1}{6} a (5 a B d (3 b^4 d^4 + 20 a b^3 d^3 e + 30 a^2 b^2 d^2 e^2 + 12 a^3 b d e^3 + a^4 e^4) + A (6 b^5 d^5 + 75 a b^4 d^4 e + 200 a^2 b^3 d^3 e^2 + 150 a^3 b^2 d^2 e^3 + 30 a^4 b d e^4 + a^5 e^5)) x^6 + \\
& \frac{1}{7} (a B (6 b^5 d^5 + 75 a b^4 d^4 e + 200 a^2 b^3 d^3 e^2 + 150 a^3 b^2 d^2 e^3 + 30 a^4 b d e^4 + a^5 e^5) + \\
& A b (b^5 d^5 + 30 a b^4 d^4 e + 150 a^2 b^3 d^3 e^2 + 200 a^3 b^2 d^2 e^3 + 75 a^4 b d e^4 + 6 a^5 e^5)) x^7 + \\
& \frac{1}{8} b (6 a^5 B e^5 + 150 a^2 b^3 d^2 e^2 (B d + A e) + 100 a^3 b^2 d e^3 (2 B d + A e) + 15 a^4 b e^4 (5 B d + A e) + 30 a b^4 d^3 e (B d + 2 A e) + b^5 d^4 (B d + 5 A e)) x^8 + \\
& \frac{5}{9} b^2 e (3 a^4 B e^4 + 12 a b^3 d^2 e (B d + A e) + 15 a^2 b^2 d e^2 (2 B d + A e) + 4 a^3 b e^3 (5 B d + A e) + b^4 d^3 (B d + 2 A e)) x^9 + \\
& \frac{1}{2} b^3 e^2 (4 a^3 B e^3 + 2 b^3 d^2 (B d + A e) + 6 a b^2 d e (2 B d + A e) + 3 a^2 b e^2 (5 B d + A e)) x^{10} + \\
& \frac{1}{11} b^4 e^3 (15 a^2 B e^2 + 5 b^2 d (2 B d + A e) + 6 a b e (5 B d + A e)) x^{11} + \frac{1}{12} b^5 e^4 (5 b B d + A b e + 6 a B e) x^{12} + \frac{1}{13} b^6 B e^5 x^{13}
\end{aligned}$$

Problem 1055: Result more than twice size of optimal antiderivative.

$$\int (a + b x)^6 (A + B x) (d + e x)^4 dx$$

Optimal (type 1, 204 leaves, 2 steps):

$$\begin{aligned}
& \frac{(A b - a B) (b d - a e)^4 (a + b x)^7}{7 b^6} + \frac{(b d - a e)^3 (b B d + 4 A b e - 5 a B e) (a + b x)^8}{8 b^6} + \frac{2 e (b d - a e)^2 (2 b B d + 3 A b e - 5 a B e) (a + b x)^9}{9 b^6} + \\
& \frac{e^2 (b d - a e) (3 b B d + 2 A b e - 5 a B e) (a + b x)^{10}}{5 b^6} + \frac{e^3 (4 b B d + A b e - 5 a B e) (a + b x)^{11}}{11 b^6} + \frac{B e^4 (a + b x)^{12}}{12 b^6}
\end{aligned}$$

Result (type 1, 762 leaves):

$$\begin{aligned}
& a^6 A d^4 x + \frac{1}{2} a^5 d^3 (6 A b d + a B d + 4 a A e) x^2 + \frac{1}{3} a^4 d^2 (2 a B d (3 b d + 2 a e) + 3 A (5 b^2 d^2 + 8 a b d e + 2 a^2 e^2)) x^3 + \\
& \frac{1}{4} a^3 d (3 a B d (5 b^2 d^2 + 8 a b d e + 2 a^2 e^2) + 4 A (5 b^3 d^3 + 15 a b^2 d^2 e + 9 a^2 b d e^2 + a^3 e^3)) x^4 + \\
& \frac{1}{5} a^2 (4 a B d (5 b^3 d^3 + 15 a b^2 d^2 e + 9 a^2 b d e^2 + a^3 e^3) + A (15 b^4 d^4 + 80 a b^3 d^3 e + 90 a^2 b^2 d^2 e^2 + 24 a^3 b d e^3 + a^4 e^4)) x^5 + \\
& \frac{1}{6} a (6 A b (b^4 d^4 + 10 a b^3 d^3 e + 20 a^2 b^2 d^2 e^2 + 10 a^3 b d e^3 + a^4 e^4) + a B (15 b^4 d^4 + 80 a b^3 d^3 e + 90 a^2 b^2 d^2 e^2 + 24 a^3 b d e^3 + a^4 e^4)) x^6 + \\
& \frac{1}{7} b (6 a B (b^4 d^4 + 10 a b^3 d^3 e + 20 a^2 b^2 d^2 e^2 + 10 a^3 b d e^3 + a^4 e^4) + A b (b^4 d^4 + 24 a b^3 d^3 e + 90 a^2 b^2 d^2 e^2 + 80 a^3 b d e^3 + 15 a^4 e^4)) x^7 + \\
& \frac{1}{8} b^2 (15 a^4 B e^4 + 20 a^3 b e^3 (4 B d + A e) + 30 a^2 b^2 d e^2 (3 B d + 2 A e) + 12 a b^3 d^2 e (2 B d + 3 A e) + b^4 d^3 (B d + 4 A e)) x^8 + \\
& \frac{1}{9} b^3 e (20 a^3 B e^3 + 15 a^2 b e^2 (4 B d + A e) + 12 a b^2 d e (3 B d + 2 A e) + 2 b^3 d^2 (2 B d + 3 A e)) x^9 + \\
& \frac{1}{10} b^4 e^2 (15 a^2 B e^2 + 6 a b e (4 B d + A e) + 2 b^2 d (3 B d + 2 A e)) x^{10} + \frac{1}{11} b^5 e^3 (4 b B d + A b e + 6 a B e) x^{11} + \frac{1}{12} b^6 B e^4 x^{12}
\end{aligned}$$

**Problem 1056:** Result more than twice size of optimal antiderivative.

$$\int (a + b x)^6 (A + B x) (d + e x)^3 dx$$

Optimal (type 1, 159 leaves, 2 steps):

$$\begin{aligned}
& \frac{(A b - a B) (b d - a e)^3 (a + b x)^7}{7 b^5} + \frac{(b d - a e)^2 (b B d + 3 A b e - 4 a B e) (a + b x)^8}{8 b^5} + \\
& \frac{e (b d - a e) (b B d + A b e - 2 a B e) (a + b x)^9}{3 b^5} + \frac{e^2 (3 b B d + A b e - 4 a B e) (a + b x)^{10}}{10 b^5} + \frac{B e^3 (a + b x)^{11}}{11 b^5}
\end{aligned}$$

Result (type 1, 586 leaves):

$$\begin{aligned}
& a^6 A d^3 x + \frac{1}{2} a^5 d^2 (6 A b d + a B d + 3 a A e) x^2 + a^4 d (a B d (2 b d + a e) + A (5 b^2 d^2 + 6 a b d e + a^2 e^2)) x^3 + \\
& \frac{1}{4} a^3 (3 a B d (5 b^2 d^2 + 6 a b d e + a^2 e^2) + A (20 b^3 d^3 + 45 a b^2 d^2 e + 18 a^2 b d e^2 + a^3 e^3)) x^4 + \\
& \frac{1}{5} a^2 (a B (20 b^3 d^3 + 45 a b^2 d^2 e + 18 a^2 b d e^2 + a^3 e^3) + 3 A b (5 b^3 d^3 + 20 a b^2 d^2 e + 15 a^2 b d e^2 + 2 a^3 e^3)) x^5 + \\
& \frac{1}{2} a b (a B (5 b^3 d^3 + 20 a b^2 d^2 e + 15 a^2 b d e^2 + 2 a^3 e^3) + A b (2 b^3 d^3 + 15 a b^2 d^2 e + 20 a^2 b d e^2 + 5 a^3 e^3)) x^6 + \\
& \frac{1}{7} b^2 (3 a B (2 b^3 d^3 + 15 a b^2 d^2 e + 20 a^2 b d e^2 + 5 a^3 e^3) + A b (b^3 d^3 + 18 a b^2 d^2 e + 45 a^2 b d e^2 + 20 a^3 e^3)) x^7 + \\
& \frac{1}{8} b^3 (20 a^3 B e^3 + 18 a b^2 d e (B d + A e) + 15 a^2 b e^2 (3 B d + A e) + b^3 d^2 (B d + 3 A e)) x^8 + \\
& \frac{1}{3} b^4 e (5 a^2 B e^2 + b^2 d (B d + A e) + 2 a b e (3 B d + A e)) x^9 + \frac{1}{10} b^5 e^2 (3 b B d + A b e + 6 a B e) x^{10} + \frac{1}{11} b^6 B e^3 x^{11}
\end{aligned}$$

**Problem 1057:** Result more than twice size of optimal antiderivative.

$$\int (a + b x)^6 (A + B x) (d + e x)^2 dx$$

Optimal (type 1, 118 leaves, 2 steps):

$$\frac{(A b - a B) (b d - a e)^2 (a + b x)^7}{7 b^4} + \frac{(b d - a e) (b B d + 2 A b e - 3 a B e) (a + b x)^8}{8 b^4} + \frac{e (2 b B d + A b e - 3 a B e) (a + b x)^9}{9 b^4} + \frac{B e^2 (a + b x)^{10}}{10 b^4}$$

Result (type 1, 386 leaves):

$$\begin{aligned}
& \frac{1}{2520} x (210 a^6 (4 A (3 d^2 + 3 d e x + e^2 x^2) + B x (6 d^2 + 8 d e x + 3 e^2 x^2)) + 252 a^5 b x (5 A (6 d^2 + 8 d e x + 3 e^2 x^2) + 2 B x (10 d^2 + 15 d e x + 6 e^2 x^2)) + \\
& 630 a^4 b^2 x^2 (2 A (10 d^2 + 15 d e x + 6 e^2 x^2) + B x (15 d^2 + 24 d e x + 10 e^2 x^2)) + \\
& 120 a^3 b^3 x^3 (7 A (15 d^2 + 24 d e x + 10 e^2 x^2) + 4 B x (21 d^2 + 35 d e x + 15 e^2 x^2)) + \\
& 45 a^2 b^4 x^4 (8 A (21 d^2 + 35 d e x + 15 e^2 x^2) + 5 B x (28 d^2 + 48 d e x + 21 e^2 x^2)) + 30 a b^5 x^5 \\
& (3 A (28 d^2 + 48 d e x + 21 e^2 x^2) + 2 B x (36 d^2 + 63 d e x + 28 e^2 x^2)) + b^6 x^6 (10 A (36 d^2 + 63 d e x + 28 e^2 x^2) + 7 B x (45 d^2 + 80 d e x + 36 e^2 x^2))
\end{aligned}$$

**Problem 1058:** Result more than twice size of optimal antiderivative.

$$\int (a + b x)^6 (A + B x) (d + e x) dx$$

Optimal (type 1, 75 leaves, 2 steps):

$$\frac{(A b - a B) (b d - a e) (a + b x)^7}{7 b^3} + \frac{(b B d + A b e - 2 a B e) (a + b x)^8}{8 b^3} + \frac{B e (a + b x)^9}{9 b^3}$$

Result (type 1, 231 leaves):

$$\begin{aligned} \frac{1}{504} x \left( 84 a^6 (3 A (2 d + e x) + B x (3 d + 2 e x)) + 126 a^4 b^2 x^2 (5 A (4 d + 3 e x) + 3 B x (5 d + 4 e x)) + \right. \\ \left. 252 a^5 b x (B x (4 d + 3 e x) + A (6 d + 4 e x)) + 168 a^3 b^3 x^3 (3 A (5 d + 4 e x) + 2 B x (6 d + 5 e x)) + \right. \\ \left. 36 a^2 b^4 x^4 (7 A (6 d + 5 e x) + 5 B x (7 d + 6 e x)) + 18 a b^5 x^5 (4 A (7 d + 6 e x) + 3 B x (8 d + 7 e x)) + b^6 x^6 (9 A (8 d + 7 e x) + 7 B x (9 d + 8 e x)) \right) \end{aligned}$$

Problem 1059: Result more than twice size of optimal antiderivative.

$$\int (a + b x)^6 (A + B x) dx$$

Optimal (type 1, 38 leaves, 2 steps):

$$\frac{(A b - a B) (a + b x)^7}{7 b^2} + \frac{B (a + b x)^8}{8 b^2}$$

Result (type 1, 122 leaves):

$$\begin{aligned} \frac{1}{56} x \left( 28 a^6 (2 A + B x) + 56 a^5 b x (3 A + 2 B x) + 70 a^4 b^2 x^2 (4 A + 3 B x) + \right. \\ \left. 56 a^3 b^3 x^3 (5 A + 4 B x) + 28 a^2 b^4 x^4 (6 A + 5 B x) + 8 a b^5 x^5 (7 A + 6 B x) + b^6 x^6 (8 A + 7 B x) \right) \end{aligned}$$

Problem 1060: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^6 (A + B x)}{d + e x} dx$$

Optimal (type 3, 220 leaves, 2 steps):

$$\begin{aligned} \frac{b (b d - a e)^5 (B d - A e) x}{e^7} - \frac{(b d - a e)^4 (B d - A e) (a + b x)^2}{2 e^6} + \frac{(b d - a e)^3 (B d - A e) (a + b x)^3}{3 e^5} - \frac{(b d - a e)^2 (B d - A e) (a + b x)^4}{4 e^4} + \\ \frac{(b d - a e) (B d - A e) (a + b x)^5}{5 e^3} - \frac{(B d - A e) (a + b x)^6}{6 e^2} + \frac{B (a + b x)^7}{7 b e} - \frac{(b d - a e)^6 (B d - A e) \text{Log}[d + e x]}{e^8} \end{aligned}$$

Result (type 3, 501 leaves):

$$\frac{1}{420 e^8} \left( e x (420 a^6 B e^6 + 1260 a^5 b e^5 (-2 B d + 2 A e + B e x) + 1050 a^4 b^2 e^4 (3 A e (-2 d + e x) + B (6 d^2 - 3 d e x + 2 e^2 x^2)) + 700 a^3 b^3 e^3 (2 A e (6 d^2 - 3 d e x + 2 e^2 x^2) + B (-12 d^3 + 6 d^2 e x - 4 d e^2 x^2 + 3 e^3 x^3)) + 105 a^2 b^4 e^2 (5 A e (-12 d^3 + 6 d^2 e x - 4 d e^2 x^2 + 3 e^3 x^3) + B (60 d^4 - 30 d^3 e x + 20 d^2 e^2 x^2 - 15 d e^3 x^3 + 12 e^4 x^4)) + 42 a b^5 e (A e (60 d^4 - 30 d^3 e x + 20 d^2 e^2 x^2 - 15 d e^3 x^3 + 12 e^4 x^4) + B (-60 d^5 + 30 d^4 e x - 20 d^3 e^2 x^2 + 15 d^2 e^3 x^3 - 12 d e^4 x^4 + 10 e^5 x^5)) + b^6 (7 A e (-60 d^5 + 30 d^4 e x - 20 d^3 e^2 x^2 + 15 d^2 e^3 x^3 - 12 d e^4 x^4 + 10 e^5 x^5) + B (420 d^6 - 210 d^5 e x + 140 d^4 e^2 x^2 - 105 d^3 e^3 x^3 + 84 d^2 e^4 x^4 - 70 d e^5 x^5 + 60 e^6 x^6)) ) - 420 (b d - a e)^6 (B d - A e) \operatorname{Log}[d + e x] \right)$$

**Problem 1061:** Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^6 (A + B x)}{(d + e x)^2} dx$$

Optimal (type 3, 277 leaves, 2 steps):

$$\begin{aligned} & \frac{3 b (b d - a e)^4 (7 b B d - 5 A b e - 2 a B e) x}{e^7} + \frac{(b d - a e)^6 (B d - A e)}{e^8 (d + e x)} + \frac{5 b^2 (b d - a e)^3 (7 b B d - 4 A b e - 3 a B e) (d + e x)^2}{2 e^8} - \\ & \frac{5 b^3 (b d - a e)^2 (7 b B d - 3 A b e - 4 a B e) (d + e x)^3}{3 e^8} + \frac{3 b^4 (b d - a e) (7 b B d - 2 A b e - 5 a B e) (d + e x)^4}{4 e^8} - \\ & \frac{b^5 (7 b B d - A b e - 6 a B e) (d + e x)^5}{5 e^8} + \frac{b^6 B (d + e x)^6}{6 e^8} + \frac{(b d - a e)^5 (7 b B d - 6 A b e - a B e) \operatorname{Log}[d + e x]}{e^8} \end{aligned}$$

Result (type 3, 643 leaves):

$$\begin{aligned} & \frac{1}{60 e^8 (d + e x)} \\ & (60 a^6 e^6 (B d - A e) + 360 a^5 b e^5 (A d e + B (-d^2 + d e x + e^2 x^2)) + 450 a^4 b^2 e^4 (2 A e (-d^2 + d e x + e^2 x^2) + B (2 d^3 - 4 d^2 e x - 3 d e^2 x^2 + e^3 x^3)) + \\ & 200 a^3 b^3 e^3 (3 A e (2 d^3 - 4 d^2 e x - 3 d e^2 x^2 + e^3 x^3) + 2 B (-3 d^4 + 9 d^3 e x + 6 d^2 e^2 x^2 - 2 d e^3 x^3 + e^4 x^4)) + \\ & 75 a^2 b^4 e^2 (4 A e (-3 d^4 + 9 d^3 e x + 6 d^2 e^2 x^2 - 2 d e^3 x^3 + e^4 x^4) + B (12 d^5 - 48 d^4 e x - 30 d^3 e^2 x^2 + 10 d^2 e^3 x^3 - 5 d e^4 x^4 + 3 e^5 x^5)) + 6 a b^5 e \\ & (5 A e (12 d^5 - 48 d^4 e x - 30 d^3 e^2 x^2 + 10 d^2 e^3 x^3 - 5 d e^4 x^4 + 3 e^5 x^5) - 6 B (10 d^6 - 50 d^5 e x - 30 d^4 e^2 x^2 + 10 d^3 e^3 x^3 - 5 d^2 e^4 x^4 + 3 d e^5 x^5 - 2 e^6 x^6)) + \\ & b^6 (6 A e (-10 d^6 + 50 d^5 e x + 30 d^4 e^2 x^2 - 10 d^3 e^3 x^3 + 5 d^2 e^4 x^4 - 3 d e^5 x^5 + 2 e^6 x^6) + \\ & B (60 d^7 - 360 d^6 e x - 210 d^5 e^2 x^2 + 70 d^4 e^3 x^3 - 35 d^3 e^4 x^4 + 21 d^2 e^5 x^5 - 14 d e^6 x^6 + 10 e^7 x^7)) + \\ & 60 (b d - a e)^5 (7 b B d - 6 A b e - a B e) (d + e x) \operatorname{Log}[d + e x] \end{aligned}$$

**Problem 1065:** Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^6 (A + B x)}{(d + e x)^6} dx$$

Optimal (type 3, 272 leaves, 2 steps):

$$\begin{aligned}
 & -\frac{b^5 (6 b B d - A b e - 6 a B e) x}{e^7} + \frac{b^6 B x^2}{2 e^6} + \frac{(b d - a e)^6 (B d - A e)}{5 e^8 (d + e x)^5} - \frac{(b d - a e)^5 (7 b B d - 6 A b e - a B e)}{4 e^8 (d + e x)^4} + \frac{b (b d - a e)^4 (7 b B d - 5 A b e - 2 a B e)}{e^8 (d + e x)^3} - \\
 & \frac{5 b^2 (b d - a e)^3 (7 b B d - 4 A b e - 3 a B e)}{2 e^8 (d + e x)^2} + \frac{5 b^3 (b d - a e)^2 (7 b B d - 3 A b e - 4 a B e)}{e^8 (d + e x)} + \frac{3 b^4 (b d - a e) (7 b B d - 2 A b e - 5 a B e) \log[d + e x]}{e^8}
 \end{aligned}$$

Result (type 3, 633 leaves):

$$\begin{aligned}
 & \frac{1}{20 e^8 (d + e x)^5} \left( -a^6 e^6 (4 A e + B (d + 5 e x)) - 2 a^5 b e^5 (3 A e (d + 5 e x) + 2 B (d^2 + 5 d e x + 10 e^2 x^2)) - \right. \\
 & 5 a^4 b^2 e^4 (2 A e (d^2 + 5 d e x + 10 e^2 x^2) + 3 B (d^3 + 5 d^2 e x + 10 d e^2 x^2 + 10 e^3 x^3)) - \\
 & 20 a^3 b^3 e^3 (A e (d^3 + 5 d^2 e x + 10 d e^2 x^2 + 10 e^3 x^3) + 4 B (d^4 + 5 d^3 e x + 10 d^2 e^2 x^2 + 10 d e^3 x^3 + 5 e^4 x^4)) + \\
 & 5 a^2 b^4 e^2 (-12 A e (d^4 + 5 d^3 e x + 10 d^2 e^2 x^2 + 10 d e^3 x^3 + 5 e^4 x^4) + B d (137 d^4 + 625 d^3 e x + 1100 d^2 e^2 x^2 + 900 d e^3 x^3 + 300 e^4 x^4)) + \\
 & 2 a b^5 e (A d e (137 d^4 + 625 d^3 e x + 1100 d^2 e^2 x^2 + 900 d e^3 x^3 + 300 e^4 x^4) - \\
 & 6 B (87 d^6 + 375 d^5 e x + 600 d^4 e^2 x^2 + 400 d^3 e^3 x^3 + 50 d^2 e^4 x^4 - 50 d e^5 x^5 - 10 e^6 x^6)) + \\
 & b^6 (-2 A e (87 d^6 + 375 d^5 e x + 600 d^4 e^2 x^2 + 400 d^3 e^3 x^3 + 50 d^2 e^4 x^4 - 50 d e^5 x^5 - 10 e^6 x^6) + \\
 & B (459 d^7 + 1875 d^6 e x + 2700 d^5 e^2 x^2 + 1300 d^4 e^3 x^3 - 400 d^3 e^4 x^4 - 500 d^2 e^5 x^5 - 70 d e^6 x^6 + 10 e^7 x^7)) + \\
 & \left. 60 b^4 (b d - a e) (7 b B d - 2 A b e - 5 a B e) (d + e x)^5 \log[d + e x] \right)
 \end{aligned}$$

Problem 1066: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^6 (A + B x)}{(d + e x)^7} dx$$

Optimal (type 3, 278 leaves, 2 steps):

$$\begin{aligned}
 & \frac{b^6 B x}{e^7} + \frac{(b d - a e)^6 (B d - A e)}{6 e^8 (d + e x)^6} - \frac{(b d - a e)^5 (7 b B d - 6 A b e - a B e)}{5 e^8 (d + e x)^5} + \\
 & \frac{3 b (b d - a e)^4 (7 b B d - 5 A b e - 2 a B e)}{4 e^8 (d + e x)^4} - \frac{5 b^2 (b d - a e)^3 (7 b B d - 4 A b e - 3 a B e)}{3 e^8 (d + e x)^3} + \\
 & \frac{5 b^3 (b d - a e)^2 (7 b B d - 3 A b e - 4 a B e)}{2 e^8 (d + e x)^2} - \frac{3 b^4 (b d - a e) (7 b B d - 2 A b e - 5 a B e)}{e^8 (d + e x)} - \frac{b^5 (7 b B d - A b e - 6 a B e) \log[d + e x]}{e^8}
 \end{aligned}$$

Result (type 3, 619 leaves):

$$\begin{aligned}
& - \frac{1}{60 e^8 (d + e x)^6} \left( 2 a^6 e^6 (5 A e + B (d + 6 e x)) + \right. \\
& \quad 6 a^5 b e^5 (2 A e (d + 6 e x) + B (d^2 + 6 d e x + 15 e^2 x^2)) + 15 a^4 b^2 e^4 (A e (d^2 + 6 d e x + 15 e^2 x^2) + B (d^3 + 6 d^2 e x + 15 d e^2 x^2 + 20 e^3 x^3)) + \\
& \quad 20 a^3 b^3 e^3 (A e (d^3 + 6 d^2 e x + 15 d e^2 x^2 + 20 e^3 x^3) + 2 B (d^4 + 6 d^3 e x + 15 d^2 e^2 x^2 + 20 d e^3 x^3 + 15 e^4 x^4)) + \\
& \quad 30 a^2 b^4 e^2 (A e (d^4 + 6 d^3 e x + 15 d^2 e^2 x^2 + 20 d e^3 x^3 + 15 e^4 x^4) + 5 B (d^5 + 6 d^4 e x + 15 d^3 e^2 x^2 + 20 d^2 e^3 x^3 + 15 d e^4 x^4 + 6 e^5 x^5)) - \\
& \quad 6 a b^5 e (-10 A e (d^5 + 6 d^4 e x + 15 d^3 e^2 x^2 + 20 d^2 e^3 x^3 + 15 d e^4 x^4 + 6 e^5 x^5)) + \\
& \quad B d (147 d^5 + 822 d^4 e x + 1875 d^3 e^2 x^2 + 2200 d^2 e^3 x^3 + 1350 d e^4 x^4 + 360 e^5 x^5) - \\
& \quad b^6 (A d e (147 d^5 + 822 d^4 e x + 1875 d^3 e^2 x^2 + 2200 d^2 e^3 x^3 + 1350 d e^4 x^4 + 360 e^5 x^5) - B (669 d^7 + 3594 d^6 e x + 7725 d^5 e^2 x^2 + \\
& \quad 8200 d^4 e^3 x^3 + 4050 d^3 e^4 x^4 + 360 d^2 e^5 x^5 - 360 d e^6 x^6 - 60 e^7 x^7)) + 60 b^5 (7 b B d - A b e - 6 a B e) (d + e x)^6 \log [d + e x] \Big)
\end{aligned}$$

**Problem 1067:** Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^6 (A + B x)}{(d + e x)^8} dx$$

Optimal (type 3, 213 leaves, 3 steps):

$$\begin{aligned}
& - \frac{(B d - A e) (a + b x)^7}{7 e (b d - a e) (d + e x)^7} - \frac{B (b d - a e)^6}{6 e^8 (d + e x)^6} + \frac{6 b B (b d - a e)^5}{5 e^8 (d + e x)^5} - \\
& \frac{15 b^2 B (b d - a e)^4}{4 e^8 (d + e x)^4} + \frac{20 b^3 B (b d - a e)^3}{3 e^8 (d + e x)^3} - \frac{15 b^4 B (b d - a e)^2}{2 e^8 (d + e x)^2} + \frac{6 b^5 B (b d - a e)}{e^8 (d + e x)} + \frac{b^6 B \log [d + e x]}{e^8}
\end{aligned}$$

Result (type 3, 615 leaves):

$$\begin{aligned}
& - \frac{1}{420 e^8 (d + e x)^7} \left( 10 a^6 e^6 (6 A e + B (d + 7 e x)) + 12 a^5 b e^5 (5 A e (d + 7 e x) + 2 B (d^2 + 7 d e x + 21 e^2 x^2)) + \right. \\
& \quad 15 a^4 b^2 e^4 (4 A e (d^2 + 7 d e x + 21 e^2 x^2) + 3 B (d^3 + 7 d^2 e x + 21 d e^2 x^2 + 35 e^3 x^3)) + \\
& \quad 20 a^3 b^3 e^3 (3 A e (d^3 + 7 d^2 e x + 21 d e^2 x^2 + 35 e^3 x^3) + 4 B (d^4 + 7 d^3 e x + 21 d^2 e^2 x^2 + 35 d e^3 x^3 + 35 e^4 x^4)) + \\
& \quad 30 a^2 b^4 e^2 (2 A e (d^4 + 7 d^3 e x + 21 d^2 e^2 x^2 + 35 d e^3 x^3 + 35 e^4 x^4) + 5 B (d^5 + 7 d^4 e x + 21 d^3 e^2 x^2 + 35 d^2 e^3 x^3 + 35 d e^4 x^4 + 21 e^5 x^5)) + 60 a b^5 e \\
& \quad (A e (d^5 + 7 d^4 e x + 21 d^3 e^2 x^2 + 35 d^2 e^3 x^3 + 35 d e^4 x^4 + 21 e^5 x^5) + 6 B (d^6 + 7 d^5 e x + 21 d^4 e^2 x^2 + 35 d^3 e^3 x^3 + 35 d^2 e^4 x^4 + 21 d e^5 x^5 + 7 e^6 x^6)) - \\
& \quad b^6 (60 A e (d^6 + 7 d^5 e x + 21 d^4 e^2 x^2 + 35 d^3 e^3 x^3 + 35 d^2 e^4 x^4 + 21 d e^5 x^5 + 7 e^6 x^6)) - \\
& \quad B d (1089 d^6 + 7203 d^5 e x + 20139 d^4 e^2 x^2 + 30625 d^3 e^3 x^3 + 26950 d^2 e^4 x^4 + 13230 d e^5 x^5 + 2940 e^6 x^6) - 420 b^6 B (d + e x)^7 \log [d + e x] \Big)
\end{aligned}$$

**Problem 1068:** Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^6 (A + B x)}{(d + e x)^9} dx$$

Optimal (type 1, 86 leaves, 2 steps):

$$-\frac{(B d - A e) (a + b x)^7}{8 e (b d - a e) (d + e x)^8} + \frac{(7 b B d + A b e - 8 a B e) (a + b x)^7}{56 e (b d - a e)^2 (d + e x)^7}$$

Result (type 1, 597 leaves):

$$\begin{aligned} & -\frac{1}{56 e^8 (d + e x)^8} (a^6 e^6 (7 A e + B (d + 8 e x)) + \\ & 2 a^5 b e^5 (3 A e (d + 8 e x) + B (d^2 + 8 d e x + 28 e^2 x^2)) + a^4 b^2 e^4 (5 A e (d^2 + 8 d e x + 28 e^2 x^2) + 3 B (d^3 + 8 d^2 e x + 28 d e^2 x^2 + 56 e^3 x^3)) + \\ & 4 a^3 b^3 e^3 (A e (d^3 + 8 d^2 e x + 28 d e^2 x^2 + 56 e^3 x^3) + B (d^4 + 8 d^3 e x + 28 d^2 e^2 x^2 + 56 d e^3 x^3 + 70 e^4 x^4)) + \\ & a^2 b^4 e^2 (3 A e (d^4 + 8 d^3 e x + 28 d^2 e^2 x^2 + 56 d e^3 x^3 + 70 e^4 x^4) + 5 B (d^5 + 8 d^4 e x + 28 d^3 e^2 x^2 + 56 d^2 e^3 x^3 + 70 d e^4 x^4 + 56 e^5 x^5)) + 2 a b^5 e \\ & (A e (d^5 + 8 d^4 e x + 28 d^3 e^2 x^2 + 56 d^2 e^3 x^3 + 70 d e^4 x^4 + 56 e^5 x^5) + 3 B (d^6 + 8 d^5 e x + 28 d^4 e^2 x^2 + 56 d^3 e^3 x^3 + 70 d^2 e^4 x^4 + 56 d e^5 x^5 + 28 e^6 x^6)) + \\ & b^6 (A e (d^6 + 8 d^5 e x + 28 d^4 e^2 x^2 + 56 d^3 e^3 x^3 + 70 d^2 e^4 x^4 + 56 d e^5 x^5 + 28 e^6 x^6) + \\ & 7 B (d^7 + 8 d^6 e x + 28 d^5 e^2 x^2 + 56 d^4 e^3 x^3 + 70 d^3 e^4 x^4 + 56 d^2 e^5 x^5 + 28 d e^6 x^6 + 8 e^7 x^7)) ) \end{aligned}$$

Problem 1069: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^6 (A + B x)}{(d + e x)^{10}} dx$$

Optimal (type 1, 135 leaves, 3 steps):

$$-\frac{(B d - A e) (a + b x)^7}{9 e (b d - a e) (d + e x)^9} + \frac{(7 b B d + 2 A b e - 9 a B e) (a + b x)^7}{72 e (b d - a e)^2 (d + e x)^8} + \frac{b (7 b B d + 2 A b e - 9 a B e) (a + b x)^7}{504 e (b d - a e)^3 (d + e x)^7}$$

Result (type 1, 603 leaves):

$$\begin{aligned} & -\frac{1}{504 e^8 (d + e x)^9} (7 a^6 e^6 (8 A e + B (d + 9 e x)) + \\ & 6 a^5 b e^5 (7 A e (d + 9 e x) + 2 B (d^2 + 9 d e x + 36 e^2 x^2)) + 15 a^4 b^2 e^4 (2 A e (d^2 + 9 d e x + 36 e^2 x^2) + B (d^3 + 9 d^2 e x + 36 d e^2 x^2 + 84 e^3 x^3)) + \\ & 4 a^3 b^3 e^3 (5 A e (d^3 + 9 d^2 e x + 36 d e^2 x^2 + 84 e^3 x^3) + 4 B (d^4 + 9 d^3 e x + 36 d^2 e^2 x^2 + 84 d e^3 x^3 + 126 e^4 x^4)) + \\ & 3 a^2 b^4 e^2 (4 A e (d^4 + 9 d^3 e x + 36 d^2 e^2 x^2 + 84 d e^3 x^3 + 126 e^4 x^4) + 5 B (d^5 + 9 d^4 e x + 36 d^3 e^2 x^2 + 84 d^2 e^3 x^3 + 126 d e^4 x^4 + 126 e^5 x^5)) + \\ & 6 a b^5 e (A e (d^5 + 9 d^4 e x + 36 d^3 e^2 x^2 + 84 d^2 e^3 x^3 + 126 d e^4 x^4 + 126 e^5 x^5) + \\ & 2 B (d^6 + 9 d^5 e x + 36 d^4 e^2 x^2 + 84 d^3 e^3 x^3 + 126 d^2 e^4 x^4 + 126 d e^5 x^5 + 84 e^6 x^6)) + \\ & b^6 (2 A e (d^6 + 9 d^5 e x + 36 d^4 e^2 x^2 + 84 d^3 e^3 x^3 + 126 d^2 e^4 x^4 + 126 d e^5 x^5 + 84 e^6 x^6) + \\ & 7 B (d^7 + 9 d^6 e x + 36 d^5 e^2 x^2 + 84 d^4 e^3 x^3 + 126 d^3 e^4 x^4 + 126 d^2 e^5 x^5 + 84 d e^6 x^6 + 36 e^7 x^7)) ) \end{aligned}$$

Problem 1070: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^6 (A + B x)}{(d + e x)^{11}} dx$$

Optimal (type 1, 185 leaves, 4 steps):

$$-\frac{(Bd-Ae)(a+bx)^7}{10e(bd-ae)(d+ex)^{10}} + \frac{(7bBd+3AbE-10aBe)(a+bx)^7}{90e(bd-ae)^2(d+ex)^9} + \frac{b(7bBd+3AbE-10aBe)(a+bx)^7}{360e(bd-ae)^3(d+ex)^8} + \frac{b^2(7bBd+3AbE-10aBe)(a+bx)^7}{2520e(bd-ae)^4(d+ex)^7}$$

Result (type 1, 602 leaves):

$$\begin{aligned} & -\frac{1}{2520e^8(d+ex)^{10}} (28a^6e^6(9Ae+B(d+10ex)) + 42a^5be^5(4Ae(d+10ex) + B(d^2+10dex+45e^2x^2))) + \\ & 15a^4b^2e^4(7Ae(d^2+10dex+45e^2x^2) + 3B(d^3+10d^2ex+45d^2e^2x^2+120e^3x^3)) + \\ & 20a^3b^3e^3(3Ae(d^3+10d^2ex+45d^2e^2x^2+120e^3x^3) + 2B(d^4+10d^3ex+45d^2e^2x^2+120dex^3+210e^4x^4)) + \\ & 30a^2b^4e^2(Ae(d^4+10d^3ex+45d^2e^2x^2+120dex^3+210e^4x^4) + B(d^5+10d^4ex+45d^3e^2x^2+120d^2e^3x^3+210dex^4+252e^5x^5)) + \\ & 6ab^5e(2Ae(d^5+10d^4ex+45d^3e^2x^2+120dex^3+210dex^4+252e^5x^5)) + \\ & 3B(d^6+10d^5ex+45d^4e^2x^2+120d^3e^3x^3+210dex^4+252dex^5+210e^6x^6)) + \\ & b^6(3Ae(d^6+10d^5ex+45d^4e^2x^2+120d^3e^3x^3+210dex^4+252dex^5+210e^6x^6)) + \\ & 7B(d^7+10d^6ex+45d^5e^2x^2+120d^4e^3x^3+210dex^4+252dex^5+210dex^6+120e^7x^7)) \end{aligned}$$

**Problem 1071: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+bx)^6(A+Bx)}{(d+ex)^{12}} dx$$

Optimal (type 1, 235 leaves, 5 steps):

$$\begin{aligned} & -\frac{(Bd-Ae)(a+bx)^7}{11e(bd-ae)(d+ex)^{11}} + \frac{(7bBd+4AbE-11aBe)(a+bx)^7}{110e(bd-ae)^2(d+ex)^{10}} + \\ & \frac{b(7bBd+4AbE-11aBe)(a+bx)^7}{330e(bd-ae)^3(d+ex)^9} + \frac{b^2(7bBd+4AbE-11aBe)(a+bx)^7}{1320e(bd-ae)^4(d+ex)^8} + \frac{b^3(7bBd+4AbE-11aBe)(a+bx)^7}{9240e(bd-ae)^5(d+ex)^7} \end{aligned}$$

Result (type 1, 605 leaves):

$$\begin{aligned} & -\frac{1}{9240e^8(d+ex)^{11}} (84a^6e^6(10Ae+B(d+11ex)) + 56a^5be^5(9Ae(d+11ex) + 2B(d^2+11dex+55e^2x^2))) + \\ & 35a^4b^2e^4(8Ae(d^2+11dex+55e^2x^2) + 3B(d^3+11d^2ex+55dex^2+165e^3x^3)) + \\ & 20a^3b^3e^3(7Ae(d^3+11d^2ex+55dex^2+165e^3x^3) + 4B(d^4+11d^3ex+55d^2e^2x^2+165dex^3+330e^4x^4)) + \\ & 10a^2b^4e^2(6Ae(d^4+11d^3ex+55d^2e^2x^2+165dex^3+330e^4x^4) + 5B(d^5+11d^4ex+55d^3e^2x^2+165d^2e^3x^3+330dex^4+462e^5x^5)) + \\ & 4ab^5e(5Ae(d^5+11d^4ex+55d^3e^2x^2+165dex^3+330dex^4+462e^5x^5)) + \\ & 6B(d^6+11d^5ex+55d^4e^2x^2+165d^3e^3x^3+330dex^4+462dex^5+462e^6x^6)) + \\ & b^6(4Ae(d^6+11d^5ex+55d^4e^2x^2+165d^3e^3x^3+330dex^4+462dex^5+462e^6x^6)) + \\ & 7B(d^7+11d^6ex+55d^5e^2x^2+165d^4e^3x^3+330dex^4+462dex^5+462e^6x^6+330e^7x^7)) \end{aligned}$$

### Problem 1072: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + bx)^6 (A + Bx)}{(d + ex)^{13}} dx$$

Optimal (type 1, 292 leaves, 2 steps):

$$\begin{aligned} & \frac{(bd - ae)^6 (Bd - Ae)}{12e^8 (d + ex)^{12}} - \frac{(bd - ae)^5 (7bd - 6abe - abe)}{11e^8 (d + ex)^{11}} + \frac{3b(bd - ae)^4 (7bd - 5abe - 2abe)}{10e^8 (d + ex)^{10}} - \frac{5b^2(bd - ae)^3 (7bd - 4abe - 3abe)}{9e^8 (d + ex)^9} + \\ & \frac{5b^3(bd - ae)^2 (7bd - 3abe - 4abe)}{8e^8 (d + ex)^8} - \frac{3b^4(bd - ae) (7bd - 2abe - 5abe)}{7e^8 (d + ex)^7} + \frac{b^5 (7bd - Abe - 6abe)}{6e^8 (d + ex)^6} - \frac{b^6 B}{5e^8 (d + ex)^5} \end{aligned}$$

Result (type 1, 600 leaves):

$$\begin{aligned} & -\frac{1}{27720e^8(d + ex)^{12}} (210a^6e^6(11Ae + B(d + 12ex)) + 252a^5be^5(5Ae(d + 12ex) + B(d^2 + 12dex + 66e^2x^2)) + \\ & 210a^4b^2e^4(3Ae(d^2 + 12dex + 66e^2x^2) + B(d^3 + 12d^2ex + 66dex^2 + 220e^3x^3)) + \\ & 140a^3b^3e^3(2Ae(d^3 + 12d^2ex + 66dex^2 + 220e^3x^3) + B(d^4 + 12d^3ex + 66d^2e^2x^2 + 220de^3x^3 + 495e^4x^4)) + \\ & 15a^2b^4e^2(7Ae(d^4 + 12d^3ex + 66d^2e^2x^2 + 220de^3x^3 + 495e^4x^4) + 5B(d^5 + 12d^4ex + 66d^3e^2x^2 + 220d^2e^3x^3 + 495de^4x^4 + 792e^5x^5)) + \\ & 30ab^5e(Ae(d^5 + 12d^4ex + 66d^3e^2x^2 + 220d^2e^3x^3 + 495de^4x^4 + 792e^5x^5) + \\ & B(d^6 + 12d^5ex + 66d^4e^2x^2 + 220d^3e^3x^3 + 495d^2e^4x^4 + 792de^5x^5 + 924e^6x^6)) + \\ & b^6(5Ae(d^6 + 12d^5ex + 66d^4e^2x^2 + 220d^3e^3x^3 + 495d^2e^4x^4 + 792de^5x^5 + 924e^6x^6) + \\ & 7B(d^7 + 12d^6ex + 66d^5e^2x^2 + 220d^4e^3x^3 + 495d^3e^4x^4 + 792d^2e^5x^5 + 924de^6x^6 + 792e^7x^7)) \end{aligned}$$

### Problem 1073: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + bx)^6 (A + Bx)}{(d + ex)^{14}} dx$$

Optimal (type 1, 292 leaves, 2 steps):

$$\begin{aligned} & \frac{(bd - ae)^6 (Bd - Ae)}{13e^8 (d + ex)^{13}} - \frac{(bd - ae)^5 (7bd - 6abe - abe)}{12e^8 (d + ex)^{12}} + \frac{3b(bd - ae)^4 (7bd - 5abe - 2abe)}{11e^8 (d + ex)^{11}} - \frac{b^2(bd - ae)^3 (7bd - 4abe - 3abe)}{2e^8 (d + ex)^{10}} + \\ & \frac{5b^3(bd - ae)^2 (7bd - 3abe - 4abe)}{9e^8 (d + ex)^9} - \frac{3b^4(bd - ae) (7bd - 2abe - 5abe)}{8e^8 (d + ex)^8} + \frac{b^5 (7bd - Abe - 6abe)}{7e^8 (d + ex)^7} - \frac{b^6 B}{6e^8 (d + ex)^6} \end{aligned}$$

Result (type 1, 605 leaves):

$$\begin{aligned}
& - \frac{1}{72072 e^8 (d + e x)^{13}} (462 a^6 e^6 (12 A e + B (d + 13 e x)) + 252 a^5 b e^5 (11 A e (d + 13 e x) + 2 B (d^2 + 13 d e x + 78 e^2 x^2)) + \\
& \quad 126 a^4 b^2 e^4 (10 A e (d^2 + 13 d e x + 78 e^2 x^2) + 3 B (d^3 + 13 d^2 e x + 78 d e^2 x^2 + 286 e^3 x^3)) + \\
& \quad 56 a^3 b^3 e^3 (9 A e (d^3 + 13 d^2 e x + 78 d e^2 x^2 + 286 e^3 x^3) + 4 B (d^4 + 13 d^3 e x + 78 d^2 e^2 x^2 + 286 d e^3 x^3 + 715 e^4 x^4)) + \\
& \quad 21 a^2 b^4 e^2 (8 A e (d^4 + 13 d^3 e x + 78 d^2 e^2 x^2 + 286 d e^3 x^3 + 715 e^4 x^4) + 5 B (d^5 + 13 d^4 e x + 78 d^3 e^2 x^2 + 286 d^2 e^3 x^3 + 715 d e^4 x^4 + 1287 e^5 x^5)) + \\
& \quad 6 a b^5 e (7 A e (d^5 + 13 d^4 e x + 78 d^3 e^2 x^2 + 286 d^2 e^3 x^3 + 715 d e^4 x^4 + 1287 e^5 x^5)) + \\
& \quad 6 B (d^6 + 13 d^5 e x + 78 d^4 e^2 x^2 + 286 d^3 e^3 x^3 + 715 d^2 e^4 x^4 + 1287 d e^5 x^5 + 1716 e^6 x^6)) + \\
& b^6 (6 A e (d^6 + 13 d^5 e x + 78 d^4 e^2 x^2 + 286 d^3 e^3 x^3 + 715 d^2 e^4 x^4 + 1287 d e^5 x^5 + 1716 e^6 x^6)) + \\
& 7 B (d^7 + 13 d^6 e x + 78 d^5 e^2 x^2 + 286 d^4 e^3 x^3 + 715 d^3 e^4 x^4 + 1287 d^2 e^5 x^5 + 1716 d e^6 x^6 + 1716 e^7 x^7))
\end{aligned}$$

**Problem 1074:** Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^6 (A + B x)}{(d + e x)^{15}} dx$$

Optimal (type 1, 292 leaves, 2 steps):

$$\begin{aligned}
& \frac{(b d - a e)^6 (B d - A e)}{14 e^8 (d + e x)^{14}} - \frac{(b d - a e)^5 (7 b B d - 6 A b e - a B e)}{13 e^8 (d + e x)^{13}} + \frac{b (b d - a e)^4 (7 b B d - 5 A b e - 2 a B e)}{4 e^8 (d + e x)^{12}} - \frac{5 b^2 (b d - a e)^3 (7 b B d - 4 A b e - 3 a B e)}{11 e^8 (d + e x)^{11}} + \\
& \frac{b^3 (b d - a e)^2 (7 b B d - 3 A b e - 4 a B e)}{2 e^8 (d + e x)^{10}} - \frac{b^4 (b d - a e) (7 b B d - 2 A b e - 5 a B e)}{3 e^8 (d + e x)^9} + \frac{b^5 (7 b B d - A b e - 6 a B e)}{8 e^8 (d + e x)^8} - \frac{b^6 B}{7 e^8 (d + e x)^7}
\end{aligned}$$

Result (type 1, 602 leaves):

$$\begin{aligned}
& - \frac{1}{24024 e^8 (d + e x)^{14}} (132 a^6 e^6 (13 A e + B (d + 14 e x)) + 132 a^5 b e^5 (6 A e (d + 14 e x) + B (d^2 + 14 d e x + 91 e^2 x^2)) + \\
& \quad 30 a^4 b^2 e^4 (11 A e (d^2 + 14 d e x + 91 e^2 x^2) + 3 B (d^3 + 14 d^2 e x + 91 d e^2 x^2 + 364 e^3 x^3)) + \\
& \quad 24 a^3 b^3 e^3 (5 A e (d^3 + 14 d^2 e x + 91 d e^2 x^2 + 364 e^3 x^3) + 2 B (d^4 + 14 d^3 e x + 91 d^2 e^2 x^2 + 364 d e^3 x^3 + 1001 e^4 x^4)) + \\
& \quad 4 a^2 b^4 e^2 (9 A e (d^4 + 14 d^3 e x + 91 d^2 e^2 x^2 + 364 d e^3 x^3 + 1001 e^4 x^4) + 5 B (d^5 + 14 d^4 e x + 91 d^3 e^2 x^2 + 364 d^2 e^3 x^3 + 1001 d e^4 x^4 + 2002 e^5 x^5)) + \\
& \quad 2 a b^5 e (4 A e (d^5 + 14 d^4 e x + 91 d^3 e^2 x^2 + 364 d^2 e^3 x^3 + 1001 d e^4 x^4 + 2002 e^5 x^5)) + \\
& \quad 3 B (d^6 + 14 d^5 e x + 91 d^4 e^2 x^2 + 364 d^3 e^3 x^3 + 1001 d^2 e^4 x^4 + 2002 d e^5 x^5 + 3003 e^6 x^6)) + \\
& b^6 (A e (d^6 + 14 d^5 e x + 91 d^4 e^2 x^2 + 364 d^3 e^3 x^3 + 1001 d^2 e^4 x^4 + 2002 d e^5 x^5 + 3003 e^6 x^6) + \\
& B (d^7 + 14 d^6 e x + 91 d^5 e^2 x^2 + 364 d^4 e^3 x^3 + 1001 d^3 e^4 x^4 + 2002 d^2 e^5 x^5 + 3003 d e^6 x^6 + 3432 e^7 x^7))
\end{aligned}$$

**Problem 1075:** Result more than twice size of optimal antiderivative.

$$\int (a + b x)^{10} (A + B x) (d + e x)^{13} dx$$

Optimal (type 1, 464 leaves, 2 steps):

$$\begin{aligned}
& - \frac{(b d - a e)^{10} (B d - A e) (d + e x)^{14}}{14 e^{12}} + \frac{(b d - a e)^9 (11 b B d - 10 A b e - a B e) (d + e x)^{15}}{15 e^{12}} - \frac{5 b (b d - a e)^8 (11 b B d - 9 A b e - 2 a B e) (d + e x)^{16}}{16 e^{12}} + \\
& \frac{15 b^2 (b d - a e)^7 (11 b B d - 8 A b e - 3 a B e) (d + e x)^{17}}{17 e^{12}} - \frac{5 b^3 (b d - a e)^6 (11 b B d - 7 A b e - 4 a B e) (d + e x)^{18}}{3 e^{12}} + \\
& \frac{42 b^4 (b d - a e)^5 (11 b B d - 6 A b e - 5 a B e) (d + e x)^{19}}{19 e^{12}} - \frac{21 b^5 (b d - a e)^4 (11 b B d - 5 A b e - 6 a B e) (d + e x)^{20}}{10 e^{12}} + \\
& \frac{10 b^6 (b d - a e)^3 (11 b B d - 4 A b e - 7 a B e) (d + e x)^{21}}{7 e^{12}} - \frac{15 b^7 (b d - a e)^2 (11 b B d - 3 A b e - 8 a B e) (d + e x)^{22}}{22 e^{12}} + \\
& \frac{5 b^8 (b d - a e) (11 b B d - 2 A b e - 9 a B e) (d + e x)^{23}}{23 e^{12}} - \frac{b^9 (11 b B d - A b e - 10 a B e) (d + e x)^{24}}{24 e^{12}} + \frac{b^{10} B (d + e x)^{25}}{25 e^{12}}
\end{aligned}$$

Result (type 1, 3532 leaves):

$$\begin{aligned}
& a^{10} A d^{13} x + \frac{1}{2} a^9 d^{12} (10 A b d + a B d + 13 a A e) x^2 + \frac{1}{3} a^8 d^{11} (a B d (10 b d + 13 a e) + A (45 b^2 d^2 + 130 a b d e + 78 a^2 e^2)) x^3 + \\
& \frac{1}{4} a^7 d^{10} (a B d (45 b^2 d^2 + 130 a b d e + 78 a^2 e^2) + A (120 b^3 d^3 + 585 a b^2 d^2 e + 780 a^2 b d e^2 + 286 a^3 e^3)) x^4 + \\
& \frac{1}{5} a^6 d^9 (a B d (120 b^3 d^3 + 585 a b^2 d^2 e + 780 a^2 b d e^2 + 286 a^3 e^3) + 5 A (42 b^4 d^4 + 312 a b^3 d^3 e + 702 a^2 b^2 d^2 e^2 + 572 a^3 b d e^3 + 143 a^4 e^4)) x^5 + \\
& \frac{1}{6} a^5 d^8 (5 a B d (42 b^4 d^4 + 312 a b^3 d^3 e + 702 a^2 b^2 d^2 e^2 + 572 a^3 b d e^3 + 143 a^4 e^4) + \\
& A (252 b^5 d^5 + 2730 a b^4 d^4 e + 9360 a^2 b^3 d^3 e^2 + 12870 a^3 b^2 d^2 e^3 + 7150 a^4 b d e^4 + 1287 a^5 e^5)) x^6 + \\
& \frac{1}{7} a^4 d^7 (a B d (252 b^5 d^5 + 2730 a b^4 d^4 e + 9360 a^2 b^3 d^3 e^2 + 12870 a^3 b^2 d^2 e^3 + 7150 a^4 b d e^4 + 1287 a^5 e^5) + \\
& 3 A (70 b^6 d^6 + 1092 a b^5 d^5 e + 5460 a^2 b^4 d^4 e^2 + 11440 a^3 b^3 d^3 e^3 + 10725 a^4 b^2 d^2 e^4 + 4290 a^5 b d e^5 + 572 a^6 e^6)) x^7 + \\
& \frac{3}{8} a^3 d^6 (a B d (70 b^6 d^6 + 1092 a b^5 d^5 e + 5460 a^2 b^4 d^4 e^2 + 11440 a^3 b^3 d^3 e^3 + 10725 a^4 b^2 d^2 e^4 + 4290 a^5 b d e^5 + 572 a^6 e^6) + \\
& A (40 b^7 d^7 + 910 a b^6 d^6 e + 6552 a^2 b^5 d^5 e^2 + 20020 a^3 b^4 d^4 e^3 + 28600 a^4 b^3 d^3 e^4 + 19305 a^5 b^2 d^2 e^5 + 5720 a^6 b d e^6 + 572 a^7 e^7)) x^8 + \\
& \frac{1}{3} a^2 d^5 (a B d (40 b^7 d^7 + 910 a b^6 d^6 e + 6552 a^2 b^5 d^5 e^2 + 20020 a^3 b^4 d^4 e^3 + 28600 a^4 b^3 d^3 e^4 + 19305 a^5 b^2 d^2 e^5 + 5720 a^6 b d e^6 + 572 a^7 e^7) + \\
& A (15 b^8 d^8 + 520 a b^7 d^7 e + 5460 a^2 b^6 d^6 e^2 + 24024 a^3 b^5 d^5 e^3 + 50050 a^4 b^4 d^4 e^4 + 51480 a^5 b^3 d^3 e^5 + 25740 a^6 b^2 d^2 e^6 + 5720 a^7 b d e^7 + 429 a^8 e^8)) \\
& x^9 + \frac{1}{10} a d^4 (3 a B d (15 b^8 d^8 + 520 a b^7 d^7 e + 5460 a^2 b^6 d^6 e^2 + 24024 a^3 b^5 d^5 e^3 + 50050 a^4 b^4 d^4 e^4 + 51480 a^5 b^3 d^3 e^5 + \\
& 25740 a^6 b^2 d^2 e^6 + 5720 a^7 b d e^7 + 429 a^8 e^8) + 5 A (2 b^9 d^9 + 117 a b^8 d^8 e + 1872 a^2 b^7 d^7 e^2 + 12012 a^3 b^6 d^6 e^3 + \\
& 36036 a^4 b^5 d^5 e^4 + 54054 a^5 b^4 d^4 e^5 + 41184 a^6 b^3 d^3 e^6 + 15444 a^7 b^2 d^2 e^7 + 2574 a^8 b d e^8 + 143 a^9 e^9)) x^{10} + \\
& \frac{1}{11} d^3 (5 a B d (2 b^9 d^9 + 117 a b^8 d^8 e + 1872 a^2 b^7 d^7 e^2 + 12012 a^3 b^6 d^6 e^3 + 36036 a^4 b^5 d^5 e^4 + 54054 a^5 b^4 d^4 e^5 + 41184 a^6 b^3 d^3 e^6 + \\
& 15444 a^7 b^2 d^2 e^7 + 2574 a^8 b d e^8 + 143 a^9 e^9) + A (b^{10} d^{10} + 130 a b^9 d^9 e + 3510 a^2 b^8 d^8 e^2 + 34320 a^3 b^7 d^7 e^3 + 150150 a^4 b^6 d^6 e^4 + \\
& 324324 a^5 b^5 d^5 e^5 + 360360 a^6 b^4 d^4 e^6 + 205920 a^7 b^3 d^3 e^7 + 57915 a^8 b^2 d^2 e^8 + 7150 a^9 b d e^9 + 286 a^{10} e^{10})) x^{11} +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{12} d^2 (360360 a^6 b^4 d^4 e^6 (B d + A e) + 1430 a^9 b d e^9 (5 B d + 2 A e) + 51480 a^7 b^3 d^3 e^7 (4 B d + 3 A e) + 26 a^{10} e^{10} (11 B d + 3 A e) + \\
& 108108 a^5 b^5 d^5 e^5 (3 B d + 4 A e) + 17160 a^3 b^7 d^7 e^3 (2 B d + 5 A e) + 6435 a^8 b^2 d^2 e^8 (9 B d + 5 A e) + \\
& 130 a b^9 d^9 e (B d + 6 A e) + 30030 a^4 b^6 d^6 e^4 (5 B d + 9 A e) + 1170 a^2 b^8 d^8 e^2 (3 B d + 11 A e) + b^{10} d^{10} (B d + 13 A e)) x^{12} + \\
& d e (33264 a^5 b^5 d^5 e^5 (B d + A e) + a^{10} e^{10} (6 B d + A e) + 495 a^8 b^2 d^2 e^8 (5 B d + 2 A e) + 6930 a^6 b^4 d^4 e^6 (4 B d + 3 A e) + \\
& 20 a^9 b d e^9 (11 B d + 3 A e) + 6930 a^4 b^6 d^6 e^4 (3 B d + 4 A e) + 495 a^2 b^8 d^8 e^2 (2 B d + 5 A e) + \\
& 1320 a^7 b^3 d^3 e^7 (9 B d + 5 A e) + b^{10} d^{10} (B d + 6 A e) + 1320 a^3 b^7 d^7 e^3 (5 B d + 9 A e) + 20 a b^9 d^9 e (3 B d + 11 A e)) x^{13} + \\
& \frac{1}{14} e^2 (360360 a^4 b^6 d^6 e^4 (B d + A e) + 130 a^9 b d e^9 (6 B d + A e) + a^{10} e^{10} (13 B d + A e) + 17160 a^7 b^3 d^3 e^7 (5 B d + 2 A e) + \\
& 108108 a^5 b^5 d^5 e^5 (4 B d + 3 A e) + 1170 a^8 b^2 d^2 e^8 (11 B d + 3 A e) + 51480 a^3 b^7 d^7 e^3 (3 B d + 4 A e) + \\
& 1430 a b^9 d^9 e (2 B d + 5 A e) + 30030 a^6 b^4 d^4 e^6 (9 B d + 5 A e) + 6435 a^2 b^8 d^8 e^2 (5 B d + 9 A e) + 26 b^{10} d^{10} (3 B d + 11 A e)) x^{14} + \\
& \frac{1}{15} e^3 (a^{10} B e^{10} + 205920 a^3 b^7 d^6 e^3 (B d + A e) + 585 a^8 b^2 d e^8 (6 B d + A e) + 10 a^9 b e^9 (13 B d + A e) + 30030 a^6 b^4 d^3 e^6 (5 B d + 2 A e) + \\
& 90090 a^4 b^6 d^5 e^4 (4 B d + 3 A e) + 3120 a^7 b^3 d^2 e^7 (11 B d + 3 A e) + 19305 a^2 b^8 d^7 e^2 (3 B d + 4 A e) + \\
& 143 b^{10} d^9 (2 B d + 5 A e) + 36036 a^5 b^5 d^4 e^5 (9 B d + 5 A e) + 1430 a b^9 d^8 e (5 B d + 9 A e)) x^{15} + \\
& \frac{1}{16} b e^4 (10 a^9 B e^9 + 77220 a^2 b^7 d^6 e^2 (B d + A e) + 1560 a^7 b^2 d e^7 (6 B d + A e) + 45 a^8 b e^8 (13 B d + A e) + 36036 a^5 b^4 d^3 e^5 (5 B d + 2 A e) + 51480 a^3 b^6 \\
& d^5 e^3 (4 B d + 3 A e) + 5460 a^6 b^3 d^2 e^6 (11 B d + 3 A e) + 4290 a b^8 d^7 e (3 B d + 4 A e) + 30030 a^4 b^5 d^4 e^4 (9 B d + 5 A e) + 143 b^9 d^8 (5 B d + 9 A e)) x^{16} + \\
& \frac{3}{17} b^2 e^5 (15 a^8 B e^8 + 5720 a b^7 d^6 e (B d + A e) + 910 a^6 b^2 d e^6 (6 B d + A e) + 40 a^7 b e^7 (13 B d + A e) + 10010 a^4 b^4 d^3 e^4 (5 B d + 2 A e) + \\
& 6435 a^2 b^6 d^5 e^2 (4 B d + 3 A e) + 2184 a^5 b^3 d^2 e^5 (11 B d + 3 A e) + 143 b^8 d^7 (3 B d + 4 A e) + 5720 a^3 b^5 d^4 e^3 (9 B d + 5 A e)) x^{17} + \\
& \frac{1}{6} b^3 e^6 (40 a^7 B e^7 + 572 b^7 d^6 (B d + A e) + 1092 a^5 b^2 d e^5 (6 B d + A e) + 70 a^6 b e^6 (13 B d + A e) + 5720 a^3 b^4 d^3 e^3 (5 B d + 2 A e) + \\
& 1430 a b^6 d^5 e (4 B d + 3 A e) + 1820 a^4 b^3 d^2 e^4 (11 B d + 3 A e) + 2145 a^2 b^5 d^4 e^2 (9 B d + 5 A e)) x^{18} + \\
& \frac{1}{19} b^4 e^7 (210 a^6 B e^6 + 2730 a^4 b^2 d e^4 (6 B d + A e) + 252 a^5 b e^5 (13 B d + A e) + 6435 a^2 b^4 d^3 e^2 (5 B d + 2 A e) + \\
& 429 b^6 d^5 (4 B d + 3 A e) + 3120 a^3 b^3 d^2 e^3 (11 B d + 3 A e) + 1430 a b^5 d^4 e (9 B d + 5 A e)) x^{19} + \\
& \frac{1}{20} b^5 e^8 (252 a^5 B e^5 + 1560 a^3 b^2 d e^3 (6 B d + A e) + 210 a^4 b e^4 (13 B d + A e) + 1430 a b^4 d^3 e (5 B d + 2 A e) + \\
& 1170 a^2 b^3 d^2 e^2 (11 B d + 3 A e) + 143 b^5 d^4 (9 B d + 5 A e)) x^{20} + \\
& \frac{1}{21} b^6 e^9 (210 a^4 B e^4 + 585 a^2 b^2 d e^2 (6 B d + A e) + 120 a^3 b e^3 (13 B d + A e) + 143 b^4 d^3 (5 B d + 2 A e) + 260 a b^3 d^2 e (11 B d + 3 A e)) x^{21} + \\
& \frac{1}{22} b^7 e^{10} (120 a^3 B e^3 + 130 a b^2 d e (6 B d + A e) + 45 a^2 b e^2 (13 B d + A e) + 26 b^3 d^2 (11 B d + 3 A e)) x^{22} + \\
& \frac{1}{23} b^8 e^{11} (45 a^2 B e^2 + 13 b^2 d (6 B d + A e) + 10 a b e (13 B d + A e)) x^{23} + \\
& \frac{1}{24} b^9 e^{12} (13 b B d + A b e + 10 a B e) x^{24} + \frac{1}{25} b^{10} B e^{13} x^{25}
\end{aligned}$$

### Problem 1076: Result more than twice size of optimal antiderivative.

$$\int (a + bx)^{10} (A + Bx) (d + ex)^{12} dx$$

Optimal (type 1, 464 leaves, 2 steps):

$$\begin{aligned} & - \frac{(bd - ae)^{10} (Bd - Ae) (d + ex)^{13}}{13e^{12}} + \frac{(bd - ae)^9 (11bBd - 10AbE - aBe) (d + ex)^{14}}{14e^{12}} - \frac{b(bd - ae)^8 (11bBd - 9AbE - 2aBe) (d + ex)^{15}}{3e^{12}} + \\ & \frac{15b^2 (bd - ae)^7 (11bBd - 8AbE - 3aBe) (d + ex)^{16}}{16e^{12}} - \frac{30b^3 (bd - ae)^6 (11bBd - 7AbE - 4aBe) (d + ex)^{17}}{17e^{12}} + \\ & \frac{7b^4 (bd - ae)^5 (11bBd - 6AbE - 5aBe) (d + ex)^{18}}{3e^{12}} - \frac{42b^5 (bd - ae)^4 (11bBd - 5AbE - 6aBe) (d + ex)^{19}}{19e^{12}} + \\ & \frac{3b^6 (bd - ae)^3 (11bBd - 4AbE - 7aBe) (d + ex)^{20}}{2e^{12}} - \frac{5b^7 (bd - ae)^2 (11bBd - 3AbE - 8aBe) (d + ex)^{21}}{7e^{12}} + \\ & \frac{5b^8 (bd - ae) (11bBd - 2AbE - 9aBe) (d + ex)^{22}}{22e^{12}} - \frac{b^9 (11bBd - AbE - 10aBe) (d + ex)^{23}}{23e^{12}} + \frac{b^{10}B (d + ex)^{24}}{24e^{12}} \end{aligned}$$

Result (type 1, 3320 leaves):

$$\begin{aligned} & a^{10} Ad^{12} x + \frac{1}{2} a^9 d^{11} (a Bd + 2A (5bd + 6ae)) x^2 + \frac{1}{3} a^8 d^{10} (2aBd (5bd + 6ae) + 3A (15b^2 d^2 + 40abd + 22a^2 e^2)) x^3 + \\ & \frac{1}{4} a^7 d^9 (3aBd (15b^2 d^2 + 40abd + 22a^2 e^2) + 20A (6b^3 d^3 + 27ab^2 d^2 e + 33a^2 bd e^2 + 11a^3 e^3)) x^4 + \\ & a^6 d^8 (4aBd (6b^3 d^3 + 27ab^2 d^2 e + 33a^2 bd e^2 + 11a^3 e^3) + A (42b^4 d^4 + 288ab^3 d^3 e + 594a^2 b^2 d^2 e^2 + 440a^3 bd e^3 + 99a^4 e^4)) x^5 + \\ & \frac{1}{6} a^5 d^7 (5aBd (42b^4 d^4 + 288ab^3 d^3 e + 594a^2 b^2 d^2 e^2 + 440a^3 bd e^3 + 99a^4 e^4)) + \\ & 18A (14b^5 d^5 + 140ab^4 d^4 e + 440a^2 b^3 d^3 e^2 + 550a^3 b^2 d^2 e^3 + 275a^4 bd e^4 + 44a^5 e^5) x^6 + \\ & \frac{3}{7} a^4 d^6 (6aBd (14b^5 d^5 + 140ab^4 d^4 e + 440a^2 b^3 d^3 e^2 + 550a^3 b^2 d^2 e^3 + 275a^4 bd e^4 + 44a^5 e^5)) + \\ & A (70b^6 d^6 + 1008ab^5 d^5 e + 4620a^2 b^4 d^4 e^2 + 8800a^3 b^3 d^3 e^3 + 7425a^4 b^2 d^2 e^4 + 2640a^5 bd e^5 + 308a^6 e^6) x^7 + \\ & \frac{3}{8} a^3 d^5 (aBd (70b^6 d^6 + 1008ab^5 d^5 e + 4620a^2 b^4 d^4 e^2 + 8800a^3 b^3 d^3 e^3 + 7425a^4 b^2 d^2 e^4 + 2640a^5 bd e^5 + 308a^6 e^6)) + \\ & 8A (5b^7 d^7 + 105ab^6 d^6 e + 693a^2 b^5 d^5 e^2 + 1925a^3 b^4 d^4 e^3 + 2475a^4 b^3 d^3 e^4 + 1485a^5 b^2 d^2 e^5 + 385a^6 bd e^6 + 33a^7 e^7) x^8 + \\ & \frac{1}{3} a^2 d^4 (8aBd (5b^7 d^7 + 105ab^6 d^6 e + 693a^2 b^5 d^5 e^2 + 1925a^3 b^4 d^4 e^3 + 2475a^4 b^3 d^3 e^4 + 1485a^5 b^2 d^2 e^5 + 385a^6 bd e^6 + 33a^7 e^7)) + \\ & 15A (b^8 d^8 + 32ab^7 d^7 e + 308a^2 b^6 d^6 e^2 + 1232a^3 b^5 d^5 e^3 + 2310a^4 b^4 d^4 e^4 + 2112a^5 b^3 d^3 e^5 + 924a^6 b^2 d^2 e^6 + 176a^7 bd e^7 + 11a^8 e^8) x^9 + \frac{1}{2} a d^3 \\ & (9aBd (b^8 d^8 + 32ab^7 d^7 e + 308a^2 b^6 d^6 e^2 + 1232a^3 b^5 d^5 e^3 + 2310a^4 b^4 d^4 e^4 + 2112a^5 b^3 d^3 e^5 + 924a^6 b^2 d^2 e^6 + 176a^7 bd e^7 + 11a^8 e^8) + 2A (b^9 d^9 + \\ & 54ab^8 d^8 e + 792a^2 b^7 d^7 e^2 + 4620a^3 b^6 d^6 e^3 + 12474a^4 b^5 d^5 e^4 + 16632a^5 b^4 d^4 e^5 + 11088a^6 b^3 d^3 e^6 + 3564a^7 b^2 d^2 e^7 + 495a^8 bd e^8 + 22a^9 e^9)) \end{aligned}$$

$$\begin{aligned}
& x^{10} + \frac{1}{11} d^2 \left( 10 a B d \left( b^9 d^9 + 54 a b^8 d^8 e + 792 a^2 b^7 d^7 e^2 + 4620 a^3 b^6 d^6 e^3 + 12474 a^4 b^5 d^5 e^4 + 16632 a^5 b^4 d^4 e^5 + 11088 a^6 b^3 d^3 e^6 + 3564 a^7 b^2 d^2 e^7 + 495 a^8 b d e^8 + 22 a^9 e^9 \right) + A \left( b^{10} d^{10} + 120 a b^9 d^9 e + 2970 a^2 b^8 d^8 e^2 + 26400 a^3 b^7 d^7 e^3 + 103950 a^4 b^6 d^6 e^4 + 199584 a^5 b^5 d^5 e^5 + 194040 a^6 b^4 d^4 e^6 + 95040 a^7 b^3 d^3 e^7 + 22275 a^8 b^2 d^2 e^8 + 2200 a^9 b d e^9 + 66 a^{10} e^{10} \right) \right) x^{11} + \\
& \frac{1}{12} d \left( 6 a^{10} e^{10} (11 B d + 2 A e) + 220 a^9 b d e^9 (10 B d + 3 A e) + 2475 a^8 b^2 d^2 e^8 (9 B d + 4 A e) + 11880 a^7 b^3 d^3 e^7 (8 B d + 5 A e) + 27720 a^6 b^4 d^4 e^6 (7 B d + 6 A e) + 33264 a^5 b^5 d^5 e^5 (6 B d + 7 A e) + 20790 a^4 b^6 d^6 e^4 (5 B d + 8 A e) + 6600 a^3 b^7 d^7 e^3 (4 B d + 9 A e) + 990 a^2 b^8 d^8 e^2 (3 B d + 10 A e) + 60 a b^9 d^9 e (2 B d + 11 A e) + b^{10} d^{10} (B d + 12 A e) \right) x^{12} + \\
& \frac{1}{13} e \left( a^{10} e^{10} (12 B d + A e) + 60 a^9 b d e^9 (11 B d + 2 A e) + 990 a^8 b^2 d^2 e^8 (10 B d + 3 A e) + 6600 a^7 b^3 d^3 e^7 (9 B d + 4 A e) + 20790 a^6 b^4 d^4 e^6 (8 B d + 5 A e) + 33264 a^5 b^5 d^5 e^5 (7 B d + 6 A e) + 27720 a^4 b^6 d^6 e^4 (6 B d + 7 A e) + 11880 a^3 b^7 d^7 e^3 (5 B d + 8 A e) + 2475 a^2 b^8 d^8 e^2 (4 B d + 9 A e) + 220 a b^9 d^9 e (3 B d + 10 A e) + 6 b^{10} d^{10} (2 B d + 11 A e) \right) x^{13} + \\
& \frac{1}{14} e^2 \left( a^{10} B e^{10} + 10 a^9 b e^9 (12 B d + A e) + 270 a^8 b^2 d e^8 (11 B d + 2 A e) + 2640 a^7 b^3 d^2 e^7 (10 B d + 3 A e) + 11550 a^6 b^4 d^3 e^6 (9 B d + 4 A e) + 24948 a^5 b^5 d^4 e^5 (8 B d + 5 A e) + 27720 a^4 b^6 d^5 e^4 (7 B d + 6 A e) + 15840 a^3 b^7 d^6 e^3 (6 B d + 7 A e) + 4455 a^2 b^8 d^7 e^2 (5 B d + 8 A e) + 550 a b^9 d^8 e (4 B d + 9 A e) + 22 b^{10} d^9 (3 B d + 10 A e) \right) x^{14} + \\
& \frac{1}{3} b e^3 \left( 2 a^9 B e^9 + 9 a^8 b e^8 (12 B d + A e) + 144 a^7 b^2 d e^7 (11 B d + 2 A e) + 924 a^6 b^3 d^2 e^6 (10 B d + 3 A e) + 2772 a^5 b^4 d^3 e^5 (9 B d + 4 A e) + 4158 a^4 b^5 d^4 e^4 (8 B d + 5 A e) + 3168 a^3 b^6 d^5 e^3 (7 B d + 6 A e) + 1188 a^2 b^7 d^6 e^2 (6 B d + 7 A e) + 198 a b^8 d^7 e (5 B d + 8 A e) + 11 b^9 d^8 (4 B d + 9 A e) \right) x^{15} + \frac{3}{16} b^2 e^4 \left( 15 a^8 B e^8 + 40 a^7 b e^7 (12 B d + A e) + 420 a^6 b^2 d e^6 (11 B d + 2 A e) + 1848 a^5 b^3 d^2 e^5 (10 B d + 3 A e) + 3850 a^4 b^4 d^3 e^4 (9 B d + 4 A e) + 3960 a^3 b^5 d^4 e^3 (8 B d + 5 A e) + 1980 a^2 b^6 d^5 e^2 (7 B d + 6 A e) + 440 a b^7 d^6 e (6 B d + 7 A e) + 33 b^8 d^7 (5 B d + 8 A e) \right) x^{16} + \\
& \frac{3}{17} b^3 e^5 \left( 40 a^7 B e^7 + 70 a^6 b e^6 (12 B d + A e) + 504 a^5 b^2 d e^5 (11 B d + 2 A e) + 1540 a^4 b^3 d^2 e^4 (10 B d + 3 A e) + 2200 a^3 b^4 d^3 e^3 (9 B d + 4 A e) + 1485 a^2 b^5 d^4 e^2 (8 B d + 5 A e) + 440 a b^6 d^5 e (7 B d + 6 A e) + 44 b^7 d^6 (6 B d + 7 A e) \right) x^{17} + \\
& \frac{1}{6} b^4 e^6 \left( 70 a^6 B e^6 + 84 a^5 b e^5 (12 B d + A e) + 420 a^4 b^2 d e^4 (11 B d + 2 A e) + 880 a^3 b^3 d^2 e^3 (10 B d + 3 A e) + 825 a^2 b^4 d^3 e^2 (9 B d + 4 A e) + 330 a b^5 d^4 e (8 B d + 5 A e) + 44 b^6 d^5 (7 B d + 6 A e) \right) x^{18} + \\
& \frac{1}{19} b^5 e^7 \left( 252 a^5 B e^5 + 210 a^4 b e^4 (12 B d + A e) + 720 a^3 b^2 d e^3 (11 B d + 2 A e) + 990 a^2 b^3 d^2 e^2 (10 B d + 3 A e) + 550 a b^4 d^3 e (9 B d + 4 A e) + 99 b^5 d^4 (8 B d + 5 A e) \right) x^{19} + \\
& \frac{1}{4} b^6 e^8 \left( 42 a^4 B e^4 + 24 a^3 b e^3 (12 B d + A e) + 54 a^2 b^2 d e^2 (11 B d + 2 A e) + 44 a b^3 d^2 e (10 B d + 3 A e) + 11 b^4 d^3 (9 B d + 4 A e) \right) x^{20} + \\
& \frac{1}{21} b^7 e^9 \left( 120 a^3 B e^3 + 45 a^2 b e^2 (12 B d + A e) + 60 a b^2 d e (11 B d + 2 A e) + 22 b^3 d^2 (10 B d + 3 A e) \right) x^{21} + \\
& \frac{1}{22} b^8 e^{10} \left( 45 a^2 B e^2 + 10 a b e (12 B d + A e) + 6 b^2 d (11 B d + 2 A e) \right) x^{22} + \\
& \frac{1}{23} b^9 e^{11} \left( 12 b B d + A b e + 10 a B e \right) x^{23} + \frac{1}{24} b^{10} B e^{12} x^{24}
\end{aligned}$$

### Problem 1077: Result more than twice size of optimal antiderivative.

$$\int (a + bx)^{10} (A + Bx) (d + ex)^{11} dx$$

Optimal (type 1, 461 leaves, 2 steps):

$$\begin{aligned} & \frac{(bd - ae)^{10} (Bd - Ae) (d + ex)^{12}}{12e^{12}} + \frac{(bd - ae)^9 (11bBd - 10AbE - aBe) (d + ex)^{13}}{13e^{12}} - \frac{5b(bd - ae)^8 (11bBd - 9AbE - 2aBe) (d + ex)^{14}}{14e^{12}} + \\ & \frac{b^2 (bd - ae)^7 (11bBd - 8AbE - 3aBe) (d + ex)^{15}}{e^{12}} - \frac{15b^3 (bd - ae)^6 (11bBd - 7AbE - 4aBe) (d + ex)^{16}}{8e^{12}} + \\ & \frac{42b^4 (bd - ae)^5 (11bBd - 6AbE - 5aBe) (d + ex)^{17}}{17e^{12}} - \frac{7b^5 (bd - ae)^4 (11bBd - 5AbE - 6aBe) (d + ex)^{18}}{3e^{12}} + \\ & \frac{30b^6 (bd - ae)^3 (11bBd - 4AbE - 7aBe) (d + ex)^{19}}{19e^{12}} - \frac{3b^7 (bd - ae)^2 (11bBd - 3AbE - 8aBe) (d + ex)^{20}}{4e^{12}} + \\ & \frac{5b^8 (bd - ae) (11bBd - 2AbE - 9aBe) (d + ex)^{21}}{21e^{12}} - \frac{b^9 (11bBd - AbE - 10aBe) (d + ex)^{22}}{22e^{12}} + \frac{b^{10}B(d + ex)^{23}}{23e^{12}} \end{aligned}$$

Result (type 1, 3018 leaves):

$$\begin{aligned} & a^{10}Ad^{11}x + \frac{1}{2}a^9d^{10}(10Abd + aBd + 11aAe)x^2 + \frac{1}{3}a^8d^9(aBd(10bd + 11ae) + 5A(9b^2d^2 + 22abd + 11a^2e^2))x^3 + \\ & \frac{5}{4}a^7d^8(aBd(9b^2d^2 + 22abd + 11a^2e^2) + A(24b^3d^3 + 99ab^2d^2e + 110a^2bd^2e^2 + 33a^3e^3))x^4 + \\ & a^6d^7(aBd(24b^3d^3 + 99ab^2d^2e + 110a^2bd^2e^2 + 33a^3e^3) + 3A(14b^4d^4 + 88ab^3d^3e + 165a^2b^2d^2e^2 + 110a^3bd^2e^3 + 22a^4e^4))x^5 + \\ & \frac{1}{2}a^5d^6(5aBd(14b^4d^4 + 88ab^3d^3e + 165a^2b^2d^2e^2 + 110a^3bd^2e^3 + 22a^4e^4)) + \\ & A(84b^5d^5 + 770ab^4d^4e + 2200a^2b^3d^3e^2 + 2475a^3b^2d^2e^3 + 1100a^4bd^2e^4 + 154a^5e^5)x^6 + \\ & \frac{3}{7}a^4d^5(aBd(84b^5d^5 + 770ab^4d^4e + 2200a^2b^3d^3e^2 + 2475a^3b^2d^2e^3 + 1100a^4bd^2e^4 + 154a^5e^5)) + \\ & 2A(35b^6d^6 + 462ab^5d^5e + 1925a^2b^4d^4e^2 + 3300a^3b^3d^3e^3 + 2475a^4b^2d^2e^4 + 770a^5bd^2e^5 + 77a^6e^6)x^7 + \\ & \frac{3}{4}a^3d^4(aBd(35b^6d^6 + 462ab^5d^5e + 1925a^2b^4d^4e^2 + 3300a^3b^3d^3e^3 + 2475a^4b^2d^2e^4 + 770a^5bd^2e^5 + 77a^6e^6)) + \\ & 5A(4b^7d^7 + 77ab^6d^6e + 462a^2b^5d^5e^2 + 1155a^3b^4d^4e^3 + 1320a^4b^3d^3e^4 + 693a^5b^2d^2e^5 + 154a^6bd^2e^6 + 11a^7e^7)x^8 + \\ & \frac{5}{3}a^2d^3(2aBd(4b^7d^7 + 77ab^6d^6e + 462a^2b^5d^5e^2 + 1155a^3b^4d^4e^3 + 1320a^4b^3d^3e^4 + 693a^5b^2d^2e^5 + 154a^6bd^2e^6 + 11a^7e^7)) + \\ & A(3b^8d^8 + 88ab^7d^7e + 770a^2b^6d^6e^2 + 2772a^3b^5d^5e^3 + 4620a^4b^4d^4e^4 + 3696a^5b^3d^3e^5 + 1386a^6b^2d^2e^6 + 220a^7bd^2e^7 + 11a^8e^8)x^9 + \\ & \frac{1}{2}ad^2(3aBd(3b^8d^8 + 88ab^7d^7e + 770a^2b^6d^6e^2 + 2772a^3b^5d^5e^3 + 4620a^4b^4d^4e^4 + 3696a^5b^3d^3e^5 + 1386a^6b^2d^2e^6 + 220a^7bd^2e^7 + 11a^8e^8)) + \\ & A(2b^9d^9 + 99ab^8d^8e + 1320a^2b^7d^7e^2 + 6930a^3b^6d^6e^3 + 16632a^4b^5d^5e^4 + \\ & 19404a^5b^4d^4e^5 + 11088a^6b^3d^3e^6 + 2970a^7b^2d^2e^7 + 330a^8bd^2e^8 + 11a^9e^9)x^{10} \end{aligned}$$

$$\begin{aligned}
& \frac{1}{11} d \left( 5 a B d \left( 2 b^9 d^9 + 99 a b^8 d^8 e + 1320 a^2 b^7 d^7 e^2 + 6930 a^3 b^6 d^6 e^3 + 16632 a^4 b^5 d^5 e^4 + 19404 a^5 b^4 d^4 e^5 + 11088 a^6 b^3 d^3 e^6 + 2970 a^7 b^2 d^2 e^7 + 330 a^8 b d e^8 + 11 a^9 e^9 \right) + A \left( b^{10} d^{10} + 110 a b^9 d^9 e + 2475 a^2 b^8 d^8 e^2 + 19800 a^3 b^7 d^7 e^3 + 69300 a^4 b^6 d^6 e^4 + 116424 a^5 b^5 d^5 e^5 + 97020 a^6 b^4 d^4 e^6 + 39600 a^7 b^3 d^3 e^7 + 7425 a^8 b^2 d^2 e^8 + 550 a^9 b d e^9 + 11 a^{10} e^{10} \right) \right) x^{11} + \\
& \frac{1}{12} \left( 116424 a^5 b^5 d^5 e^5 (B d + A e) + 19800 a^7 b^3 d^3 e^7 (2 B d + A e) + 2475 a^8 b^2 d^2 e^8 (3 B d + A e) + 110 a^9 b d e^9 (5 B d + A e) + a^{10} e^{10} (11 B d + A e) + 19800 a^3 b^7 d^7 e^3 (B d + 2 A e) + 2475 a^2 b^8 d^8 e^2 (B d + 3 A e) + 110 a b^9 d^9 e (B d + 5 A e) + 13860 a^6 b^4 d^4 e^6 (7 B d + 5 A e) + 13860 a^4 b^6 d^6 e^4 (5 B d + 7 A e) + b^{10} d^{10} (B d + 11 A e) \right) x^{12} + \\
& \frac{1}{13} e \left( a^{10} B e^{10} + 97020 a^4 b^6 d^5 e^4 (B d + A e) + 34650 a^6 b^4 d^3 e^6 (2 B d + A e) + 6600 a^7 b^3 d^2 e^7 (3 B d + A e) + 495 a^8 b^2 d e^8 (5 B d + A e) + 10 a^9 b e^9 (11 B d + A e) + 7425 a^2 b^8 d^7 e^2 (B d + 2 A e) + 550 a b^9 d^8 e (B d + 3 A e) + 11 b^{10} d^9 (B d + 5 A e) + 16632 a^5 b^5 d^4 e^5 (7 B d + 5 A e) + 7920 a^3 b^7 d^6 e^3 (5 B d + 7 A e) \right) x^{13} + \\
& \frac{5}{14} b e^2 \left( 2 a^9 B e^9 + 11088 a^3 b^6 d^5 e^3 (B d + A e) + 8316 a^5 b^4 d^3 e^5 (2 B d + A e) + 2310 a^6 b^3 d^2 e^6 (3 B d + A e) + 264 a^7 b^2 d e^7 (5 B d + A e) + 9 a^8 b e^8 (11 B d + A e) + 330 a b^8 d^7 e (B d + 2 A e) + 11 b^9 d^8 (B d + 3 A e) + 2772 a^4 b^5 d^4 e^4 (7 B d + 5 A e) + 594 a^2 b^7 d^6 e^2 (5 B d + 7 A e) \right) x^{14} + b^2 e^3 \left( 3 a^8 B e^8 + 1386 a^2 b^6 d^5 e^2 (B d + A e) + 2310 a^4 b^4 d^3 e^4 (2 B d + A e) + 924 a^5 b^3 d^2 e^5 (3 B d + A e) + 154 a^6 b^2 d e^6 (5 B d + A e) + 8 a^7 b e^7 (11 B d + A e) + 11 b^8 d^7 (B d + 2 A e) + 528 a^3 b^5 d^4 e^3 (7 B d + 5 A e) + 44 a b^7 d^6 e (5 B d + 7 A e) \right) x^{15} + \\
& \frac{3}{8} b^3 e^4 \left( 20 a^7 B e^7 + 770 a b^6 d^5 e (B d + A e) + 3300 a^3 b^4 d^3 e^3 (2 B d + A e) + 1925 a^4 b^3 d^2 e^4 (3 B d + A e) + 462 a^5 b^2 d e^5 (5 B d + A e) + 35 a^6 b e^6 (11 B d + A e) + 495 a^2 b^5 d^4 e^2 (7 B d + 5 A e) + 11 b^7 d^6 (5 B d + 7 A e) \right) x^{16} + \\
& \frac{3}{17} b^4 e^5 \left( 70 a^6 B e^6 + 154 b^6 d^5 (B d + A e) + 2475 a^2 b^4 d^3 e^2 (2 B d + A e) + 2200 a^3 b^3 d^2 e^3 (3 B d + A e) + 770 a^4 b^2 d e^4 (5 B d + A e) + 84 a^5 b e^5 (11 B d + A e) + 220 a b^5 d^4 e (7 B d + 5 A e) \right) x^{17} + \frac{1}{6} b^5 e^6 \left( 84 a^5 B e^5 + 550 a b^4 d^3 e (2 B d + A e) + 825 a^2 b^3 d^2 e^2 (3 B d + A e) + 440 a^3 b^2 d e^3 (5 B d + A e) + 70 a^4 b e^4 (11 B d + A e) + 22 b^5 d^4 (7 B d + 5 A e) \right) x^{18} + \frac{5}{19} b^6 e^7 \left( 42 a^4 B e^4 + 33 b^4 d^3 (2 B d + A e) + 110 a b^3 d^2 e (3 B d + A e) + 99 a^2 b^2 d e^2 (5 B d + A e) + 24 a^3 b e^3 (11 B d + A e) \right) x^{19} + \frac{1}{4} b^7 e^8 \left( 24 a^3 B e^3 + 11 b^3 d^2 (3 B d + A e) + 22 a b^2 d e (5 B d + A e) + 9 a^2 b e^2 (11 B d + A e) \right) x^{20} + \frac{1}{21} b^8 e^9 \left( 45 a^2 B e^2 + 11 b^2 d (5 B d + A e) + 10 a b e (11 B d + A e) \right) x^{21} + \frac{1}{22} b^9 e^{10} \left( 11 b B d + A b e + 10 a B e \right) x^{22} + \frac{1}{23} b^{10} B e^{11} x^{23}
\end{aligned}$$

**Problem 1078: Result more than twice size of optimal antiderivative.**

$$\int (a + b x)^{10} (A + B x) (d + e x)^{10} dx$$

Optimal (type 1, 460 leaves, 2 steps):

$$\begin{aligned}
& \frac{(A b - a B) (b d - a e)^{10} (a + b x)^{11}}{11 b^{12}} + \frac{(b d - a e)^9 (b B d + 10 A b e - 11 a B e) (a + b x)^{12}}{12 b^{12}} + \frac{5 e (b d - a e)^8 (2 b B d + 9 A b e - 11 a B e) (a + b x)^{13}}{13 b^{12}} + \\
& \frac{15 e^2 (b d - a e)^7 (3 b B d + 8 A b e - 11 a B e) (a + b x)^{14}}{14 b^{12}} + \frac{2 e^3 (b d - a e)^6 (4 b B d + 7 A b e - 11 a B e) (a + b x)^{15}}{b^{12}} + \\
& \frac{21 e^4 (b d - a e)^5 (5 b B d + 6 A b e - 11 a B e) (a + b x)^{16}}{8 b^{12}} + \frac{42 e^5 (b d - a e)^4 (6 b B d + 5 A b e - 11 a B e) (a + b x)^{17}}{17 b^{12}} + \\
& \frac{5 e^6 (b d - a e)^3 (7 b B d + 4 A b e - 11 a B e) (a + b x)^{18}}{3 b^{12}} + \frac{15 e^7 (b d - a e)^2 (8 b B d + 3 A b e - 11 a B e) (a + b x)^{19}}{19 b^{12}} + \\
& \frac{e^8 (b d - a e) (9 b B d + 2 A b e - 11 a B e) (a + b x)^{20}}{4 b^{12}} + \frac{e^9 (10 b B d + A b e - 11 a B e) (a + b x)^{21}}{21 b^{12}} + \frac{B e^{10} (a + b x)^{22}}{22 b^{12}}
\end{aligned}$$

Result (type 1, 2815 leaves):

$$\begin{aligned}
& a^{10} A d^{10} x + \frac{1}{2} a^9 d^9 (a B d + 10 A (b d + a e)) x^2 + \frac{5}{3} a^8 d^8 (2 a B d (b d + a e) + A (9 b^2 d^2 + 20 a b d e + 9 a^2 e^2)) x^3 + \\
& \frac{5}{4} a^7 d^7 (a B d (9 b^2 d^2 + 20 a b d e + 9 a^2 e^2) + 6 A (4 b^3 d^3 + 15 a b^2 d^2 e + 15 a^2 b d e^2 + 4 a^3 e^3)) x^4 + \\
& 3 a^6 d^6 (2 a B d (4 b^3 d^3 + 15 a b^2 d^2 e + 15 a^2 b d e^2 + 4 a^3 e^3) + A (14 b^4 d^4 + 80 a b^3 d^3 e + 135 a^2 b^2 d^2 e^2 + 80 a^3 b d e^3 + 14 a^4 e^4)) x^5 + \\
& \frac{1}{2} a^5 d^5 (5 a B d (14 b^4 d^4 + 80 a b^3 d^3 e + 135 a^2 b^2 d^2 e^2 + 80 a^3 b d e^3 + 14 a^4 e^4)) + \\
& 4 A (21 b^5 d^5 + 175 a b^4 d^4 e + 450 a^2 b^3 d^3 e^2 + 450 a^3 b^2 d^2 e^3 + 175 a^4 b d e^4 + 21 a^5 e^5) x^6 + \\
& \frac{6}{7} a^4 d^4 (2 a B d (21 b^5 d^5 + 175 a b^4 d^4 e + 450 a^2 b^3 d^3 e^2 + 450 a^3 b^2 d^2 e^3 + 175 a^4 b d e^4 + 21 a^5 e^5)) + \\
& 5 A (7 b^6 d^6 + 84 a b^5 d^5 e + 315 a^2 b^4 d^4 e^2 + 480 a^3 b^3 d^3 e^3 + 315 a^4 b^2 d^2 e^4 + 84 a^5 b d e^5 + 7 a^6 e^6) x^7 + \\
& \frac{15}{4} a^3 d^3 (a B d (7 b^6 d^6 + 84 a b^5 d^5 e + 315 a^2 b^4 d^4 e^2 + 480 a^3 b^3 d^3 e^3 + 315 a^4 b^2 d^2 e^4 + 84 a^5 b d e^5 + 7 a^6 e^6)) + \\
& A (4 b^7 d^7 + 70 a b^6 d^6 e + 378 a^2 b^5 d^5 e^2 + 840 a^3 b^4 d^4 e^3 + 840 a^4 b^3 d^3 e^4 + 378 a^5 b^2 d^2 e^5 + 70 a^6 b d e^6 + 4 a^7 e^7) x^8 + \\
& \frac{5}{3} a^2 d^2 (4 a B d (2 b^7 d^7 + 35 a b^6 d^6 e + 189 a^2 b^5 d^5 e^2 + 420 a^3 b^4 d^4 e^3 + 420 a^4 b^3 d^3 e^4 + 189 a^5 b^2 d^2 e^5 + 35 a^6 b d e^6 + 2 a^7 e^7)) + \\
& A (3 b^8 d^8 + 80 a b^7 d^7 e + 630 a^2 b^6 d^6 e^2 + 2016 a^3 b^5 d^5 e^3 + 2940 a^4 b^4 d^4 e^4 + 2016 a^5 b^3 d^3 e^5 + 630 a^6 b^2 d^2 e^6 + 80 a^7 b d e^7 + 3 a^8 e^8) x^9 + \\
& \frac{1}{2} a d (3 a B d (3 b^8 d^8 + 80 a b^7 d^7 e + 630 a^2 b^6 d^6 e^2 + 2016 a^3 b^5 d^5 e^3 + 2940 a^4 b^4 d^4 e^4 + 2016 a^5 b^3 d^3 e^5 + 630 a^6 b^2 d^2 e^6 + 80 a^7 b d e^7 + 3 a^8 e^8)) + \\
& 2 A (b^9 d^9 + 45 a b^8 d^8 e + 540 a^2 b^7 d^7 e^2 + 2520 a^3 b^6 d^6 e^3 + 5292 a^4 b^5 d^5 e^4 + 5292 a^5 b^4 d^4 e^5 + 2520 a^6 b^3 d^3 e^6 + 540 a^7 b^2 d^2 e^7 + 45 a^8 b d e^8 + a^9 e^9) x^{10} + \\
& \frac{1}{11} (10 a B d (b^9 d^9 + 45 a b^8 d^8 e + 540 a^2 b^7 d^7 e^2 + 2520 a^3 b^6 d^6 e^3 + 5292 a^4 b^5 d^5 e^4 + 5292 a^5 b^4 d^4 e^5 + 2520 a^6 b^3 d^3 e^6 + \\
& 540 a^7 b^2 d^2 e^7 + 45 a^8 b d e^8 + a^9 e^9) + A (b^{10} d^{10} + 100 a b^9 d^9 e + 2025 a^2 b^8 d^8 e^2 + 14400 a^3 b^7 d^7 e^3 + 44100 a^4 b^6 d^6 e^4 + \\
& 63504 a^5 b^5 d^5 e^5 + 44100 a^6 b^4 d^4 e^6 + 14400 a^7 b^3 d^3 e^7 + 2025 a^8 b^2 d^2 e^8 + 100 a^9 b d e^9 + a^{10} e^{10}) x^{11} + \\
& \frac{1}{12} (a^{10} B e^{10} + 10 a^9 b e^9 (10 B d + A e) + 225 a^8 b^2 d e^8 (9 B d + 2 A e) + 1800 a^7 b^3 d^2 e^7 (8 B d + 3 A e) + 6300 a^6 b^4 d^3 e^6 (7 B d + 4 A e) + \\
& 10584 a^5 b^5 d^4 e^5 (6 B d + 5 A e) + 8820 a^4 b^6 d^5 e^4 (5 B d + 6 A e) + 3600 a^3 b^7 d^6 e^3 (4 B d + 7 A e) +
\end{aligned}$$

$$\begin{aligned}
& 675 a^2 b^8 d^7 e^2 (3 B d + 8 A e) + 50 a b^9 d^8 e (2 B d + 9 A e) + b^{10} d^9 (B d + 10 A e) ) x^{12} + \\
& \frac{5}{13} b e (2 a^9 B e^9 + 9 a^8 b e^8 (10 B d + A e) + 120 a^7 b^2 d e^7 (9 B d + 2 A e) + 630 a^6 b^3 d^2 e^6 (8 B d + 3 A e) + 1512 a^5 b^4 d^3 e^5 (7 B d + 4 A e) + \\
& 1764 a^4 b^5 d^4 e^4 (6 B d + 5 A e) + 1008 a^3 b^6 d^5 e^3 (5 B d + 6 A e) + 270 a^2 b^7 d^6 e^2 (4 B d + 7 A e) + 30 a b^8 d^7 e (3 B d + 8 A e) + b^9 d^8 (2 B d + 9 A e) ) x^{13} + \\
& \frac{15}{14} b^2 e^2 (3 a^8 B e^8 + 8 a^7 b e^7 (10 B d + A e) + 70 a^6 b^2 d e^6 (9 B d + 2 A e) + 252 a^5 b^3 d^2 e^5 (8 B d + 3 A e) + 420 a^4 b^4 d^3 e^4 (7 B d + 4 A e) + \\
& 336 a^3 b^5 d^4 e^3 (6 B d + 5 A e) + 126 a^2 b^6 d^5 e^2 (5 B d + 6 A e) + 20 a b^7 d^6 e (4 B d + 7 A e) + b^8 d^7 (3 B d + 8 A e) ) x^{14} + \\
& 2 b^3 e^3 (4 a^7 B e^7 + 7 a^6 b e^6 (10 B d + A e) + 42 a^5 b^2 d e^5 (9 B d + 2 A e) + 105 a^4 b^3 d^2 e^4 (8 B d + 3 A e) + \\
& 120 a^3 b^4 d^3 e^3 (7 B d + 4 A e) + 63 a^2 b^5 d^4 e^2 (6 B d + 5 A e) + 14 a b^6 d^5 e (5 B d + 6 A e) + b^7 d^6 (4 B d + 7 A e) ) x^{15} + \\
& \frac{3}{8} b^4 e^4 (35 a^6 B e^6 + 42 a^5 b e^5 (10 B d + A e) + 175 a^4 b^2 d e^4 (9 B d + 2 A e) + 300 a^3 b^3 d^2 e^3 (8 B d + 3 A e) + \\
& 225 a^2 b^4 d^3 e^2 (7 B d + 4 A e) + 70 a b^5 d^4 e (6 B d + 5 A e) + 7 b^6 d^5 (5 B d + 6 A e) ) x^{16} + \frac{3}{17} b^5 e^5 \\
& (84 a^5 B e^5 + 70 a^4 b e^4 (10 B d + A e) + 200 a^3 b^2 d e^3 (9 B d + 2 A e) + 225 a^2 b^3 d^2 e^2 (8 B d + 3 A e) + 100 a b^4 d^3 e (7 B d + 4 A e) + 14 b^5 d^4 (6 B d + 5 A e) ) \\
& x^{17} + \frac{5}{6} b^6 e^6 (14 a^4 B e^4 + 8 a^3 b e^3 (10 B d + A e) + 15 a^2 b^2 d e^2 (9 B d + 2 A e) + 10 a b^3 d^2 e (8 B d + 3 A e) + 2 b^4 d^3 (7 B d + 4 A e) ) x^{18} + \\
& \frac{5}{19} b^7 e^7 (24 a^3 B e^3 + 9 a^2 b e^2 (10 B d + A e) + 10 a b^2 d e (9 B d + 2 A e) + 3 b^3 d^2 (8 B d + 3 A e) ) x^{19} + \\
& \frac{1}{4} b^8 e^8 (9 a^2 B e^2 + 2 a b e (10 B d + A e) + b^2 d (9 B d + 2 A e) ) x^{20} + \\
& \frac{1}{21} b^9 e^9 (10 b B d + A b e + 10 a B e) x^{21} + \frac{1}{22} b^{10} B e^{10} x^{22}
\end{aligned}$$

**Problem 1079: Result more than twice size of optimal antiderivative.**

$$\int (a + b x)^{10} (A + B x) (d + e x)^9 dx$$

Optimal (type 1, 415 leaves, 2 steps):

$$\begin{aligned}
& \frac{(A b - a B) (b d - a e)^9 (a + b x)^{11}}{11 b^{11}} + \frac{(b d - a e)^8 (b B d + 9 A b e - 10 a B e) (a + b x)^{12}}{12 b^{11}} + \\
& \frac{9 e (b d - a e)^7 (b B d + 4 A b e - 5 a B e) (a + b x)^{13}}{13 b^{11}} + \frac{6 e^2 (b d - a e)^6 (3 b B d + 7 A b e - 10 a B e) (a + b x)^{14}}{7 b^{11}} + \\
& \frac{14 e^3 (b d - a e)^5 (2 b B d + 3 A b e - 5 a B e) (a + b x)^{15}}{5 b^{11}} + \frac{63 e^4 (b d - a e)^4 (b B d + A b e - 2 a B e) (a + b x)^{16}}{8 b^{11}} + \\
& \frac{42 e^5 (b d - a e)^3 (3 b B d + 2 A b e - 5 a B e) (a + b x)^{17}}{17 b^{11}} + \frac{2 e^6 (b d - a e)^2 (7 b B d + 3 A b e - 10 a B e) (a + b x)^{18}}{3 b^{11}} + \\
& \frac{9 e^7 (b d - a e) (4 b B d + A b e - 5 a B e) (a + b x)^{19}}{19 b^{11}} + \frac{e^8 (9 b B d + A b e - 10 a B e) (a + b x)^{20}}{20 b^{11}} + \frac{B e^9 (a + b x)^{21}}{21 b^{11}}
\end{aligned}$$

Result (type 1, 2553 leaves):

$$\begin{aligned}
 & a^{10} A d^9 x + \frac{1}{2} a^9 d^8 (10 A b d + a B d + 9 a A e) x^2 + \frac{1}{3} a^8 d^7 (a B d (10 b d + 9 a e) + 9 A (5 b^2 d^2 + 10 a b d e + 4 a^2 e^2)) x^3 + \\
 & \frac{3}{4} a^7 d^6 (3 a B d (5 b^2 d^2 + 10 a b d e + 4 a^2 e^2) + A (40 b^3 d^3 + 135 a b^2 d^2 e + 120 a^2 b d e^2 + 28 a^3 e^3)) x^4 + \\
 & \frac{3}{5} a^6 d^5 (a B d (40 b^3 d^3 + 135 a b^2 d^2 e + 120 a^2 b d e^2 + 28 a^3 e^3) + A (70 b^4 d^4 + 360 a b^3 d^3 e + 540 a^2 b^2 d^2 e^2 + 280 a^3 b d e^3 + 42 a^4 e^4)) x^5 + \\
 & a^5 d^4 (a B d (35 b^4 d^4 + 180 a b^3 d^3 e + 270 a^2 b^2 d^2 e^2 + 140 a^3 b d e^3 + 21 a^4 e^4) + \\
 & 3 A (14 b^5 d^5 + 105 a b^4 d^4 e + 240 a^2 b^3 d^3 e^2 + 210 a^3 b^2 d^2 e^3 + 70 a^4 b d e^4 + 7 a^5 e^5)) x^6 + \\
 & \frac{6}{7} a^4 d^3 (3 a B d (14 b^5 d^5 + 105 a b^4 d^4 e + 240 a^2 b^3 d^3 e^2 + 210 a^3 b^2 d^2 e^3 + 70 a^4 b d e^4 + 7 a^5 e^5)) + \\
 & 7 A (5 b^6 d^6 + 54 a b^5 d^5 e + 180 a^2 b^4 d^4 e^2 + 240 a^3 b^3 d^3 e^3 + 135 a^4 b^2 d^2 e^4 + 30 a^5 b d e^5 + 2 a^6 e^6) x^7 + \\
 & \frac{3}{4} a^3 d^2 (7 a B d (5 b^6 d^6 + 54 a b^5 d^5 e + 180 a^2 b^4 d^4 e^2 + 240 a^3 b^3 d^3 e^3 + 135 a^4 b^2 d^2 e^4 + 30 a^5 b d e^5 + 2 a^6 e^6) + \\
 & A (20 b^7 d^7 + 315 a b^6 d^6 e + 1512 a^2 b^5 d^5 e^2 + 2940 a^3 b^4 d^4 e^3 + 2520 a^4 b^3 d^3 e^4 + 945 a^5 b^2 d^2 e^5 + 140 a^6 b d e^6 + 6 a^7 e^7)) x^8 + \\
 & \frac{1}{3} a^2 d (2 a B d (20 b^7 d^7 + 315 a b^6 d^6 e + 1512 a^2 b^5 d^5 e^2 + 2940 a^3 b^4 d^4 e^3 + 2520 a^4 b^3 d^3 e^4 + 945 a^5 b^2 d^2 e^5 + 140 a^6 b d e^6 + 6 a^7 e^7)) + \\
 & 3 A (5 b^8 d^8 + 120 a b^7 d^7 e + 840 a^2 b^6 d^6 e^2 + 2352 a^3 b^5 d^5 e^3 + 2940 a^4 b^4 d^4 e^4 + 1680 a^5 b^3 d^3 e^5 + 420 a^6 b^2 d^2 e^6 + 40 a^7 b d e^7 + a^8 e^8) x^9 + \frac{1}{10} a \\
 & (9 a B d (5 b^8 d^8 + 120 a b^7 d^7 e + 840 a^2 b^6 d^6 e^2 + 2352 a^3 b^5 d^5 e^3 + 2940 a^4 b^4 d^4 e^4 + 1680 a^5 b^3 d^3 e^5 + 420 a^6 b^2 d^2 e^6 + 40 a^7 b d e^7 + a^8 e^8) + A (10 b^9 d^9 + \\
 & 405 a b^8 d^8 e + 4320 a^2 b^7 d^7 e^2 + 17640 a^3 b^6 d^6 e^3 + 31752 a^4 b^5 d^5 e^4 + 26460 a^5 b^4 d^4 e^5 + 10080 a^6 b^3 d^3 e^6 + 1620 a^7 b^2 d^2 e^7 + 90 a^8 b d e^8 + a^9 e^9)) \\
 & x^{10} + \frac{1}{11} (a B (10 b^9 d^9 + 405 a b^8 d^8 e + 4320 a^2 b^7 d^7 e^2 + 17640 a^3 b^6 d^6 e^3 + 31752 a^4 b^5 d^5 e^4 + 26460 a^5 b^4 d^4 e^5 + 10080 a^6 b^3 d^3 e^6 + \\
 & 1620 a^7 b^2 d^2 e^7 + 90 a^8 b d e^8 + a^9 e^9) + A b (b^9 d^9 + 90 a b^8 d^8 e + 1620 a^2 b^7 d^7 e^2 + 10080 a^3 b^6 d^6 e^3 + \\
 & 26460 a^4 b^5 d^5 e^4 + 31752 a^5 b^4 d^4 e^5 + 17640 a^6 b^3 d^3 e^6 + 4320 a^7 b^2 d^2 e^7 + 405 a^8 b d e^8 + 10 a^9 e^9)) x^{11} + \\
 & \frac{1}{12} b (10 a^9 B e^9 + 26460 a^4 b^5 d^4 e^4 (B d + A e) + 1080 a^7 b^2 d e^7 (4 B d + A e) + 45 a^8 b e^8 (9 B d + A e) + 10584 a^5 b^4 d^3 e^5 (3 B d + 2 A e) + \\
 & 5040 a^3 b^6 d^5 e^3 (2 B d + 3 A e) + 2520 a^6 b^3 d^2 e^6 (7 B d + 3 A e) + 90 a b^8 d^7 e (B d + 4 A e) + 540 a^2 b^7 d^6 e^2 (3 B d + 7 A e) + b^9 d^8 (B d + 9 A e)) x^{12} + \\
 & \frac{3}{13} b^2 e (15 a^8 B e^8 + 5040 a^3 b^5 d^4 e^3 (B d + A e) + 630 a^6 b^2 d e^6 (4 B d + A e) + 40 a^7 b e^7 (9 B d + A e) + 2940 a^4 b^4 d^3 e^4 (3 B d + 2 A e) + \\
 & 630 a^2 b^6 d^5 e^2 (2 B d + 3 A e) + 1008 a^5 b^3 d^2 e^5 (7 B d + 3 A e) + 3 b^8 d^7 (B d + 4 A e) + 40 a b^7 d^6 e (3 B d + 7 A e)) x^{13} + \\
 & \frac{3}{7} b^3 e^2 (20 a^7 B e^7 + 945 a^2 b^5 d^4 e^2 (B d + A e) + 378 a^5 b^2 d e^5 (4 B d + A e) + 35 a^6 b e^6 (9 B d + A e) + 840 a^3 b^4 d^3 e^3 (3 B d + 2 A e) + \\
 & 70 a b^6 d^5 e (2 B d + 3 A e) + 420 a^4 b^3 d^2 e^4 (7 B d + 3 A e) + 2 b^7 d^6 (3 B d + 7 A e)) x^{14} + \\
 & \frac{2}{5} b^4 e^3 (35 a^6 B e^6 + 210 a b^5 d^4 e (B d + A e) + 315 a^4 b^2 d e^4 (4 B d + A e) + 42 a^5 b e^5 (9 B d + A e) + \\
 & 315 a^2 b^4 d^3 e^2 (3 B d + 2 A e) + 7 b^6 d^5 (2 B d + 3 A e) + 240 a^3 b^3 d^2 e^3 (7 B d + 3 A e)) x^{15} + \frac{3}{8} b^5 e^4 \\
 & (42 a^5 B e^5 + 21 b^5 d^4 (B d + A e) + 180 a^3 b^2 d e^3 (4 B d + A e) + 35 a^4 b e^4 (9 B d + A e) + 70 a b^4 d^3 e (3 B d + 2 A e) + 90 a^2 b^3 d^2 e^2 (7 B d + 3 A e)) x^{16} +
 \end{aligned}$$

$$\begin{aligned}
& \frac{3}{17} b^6 e^5 (70 a^4 B e^4 + 135 a^2 b^2 d e^2 (4 B d + A e) + 40 a^3 b e^3 (9 B d + A e) + 14 b^4 d^3 (3 B d + 2 A e) + 40 a b^3 d^2 e (7 B d + 3 A e)) x^{17} + \\
& \frac{1}{6} b^7 e^6 (40 a^3 B e^3 + 30 a b^2 d e (4 B d + A e) + 15 a^2 b e^2 (9 B d + A e) + 4 b^3 d^2 (7 B d + 3 A e)) x^{18} + \\
& \frac{1}{19} b^8 e^7 (45 a^2 B e^2 + 9 b^2 d (4 B d + A e) + 10 a b e (9 B d + A e)) x^{19} + \\
& \frac{1}{20} b^9 e^8 (9 b B d + A b e + 10 a B e) x^{20} + \frac{1}{21} b^{10} B e^9 x^{21}
\end{aligned}$$

**Problem 1080: Result more than twice size of optimal antiderivative.**

$$\int (a + b x)^{10} (A + B x) (d + e x)^8 dx$$

Optimal (type 1, 372 leaves, 2 steps):

$$\begin{aligned}
& \frac{(A b - a B) (b d - a e)^8 (a + b x)^{11}}{11 b^{10}} + \frac{(b d - a e)^7 (b B d + 8 A b e - 9 a B e) (a + b x)^{12}}{12 b^{10}} + \frac{4 e (b d - a e)^6 (2 b B d + 7 A b e - 9 a B e) (a + b x)^{13}}{13 b^{10}} + \\
& \frac{2 e^2 (b d - a e)^5 (b B d + 2 A b e - 3 a B e) (a + b x)^{14}}{b^{10}} + \frac{14 e^3 (b d - a e)^4 (4 b B d + 5 A b e - 9 a B e) (a + b x)^{15}}{15 b^{10}} + \\
& \frac{7 e^4 (b d - a e)^3 (5 b B d + 4 A b e - 9 a B e) (a + b x)^{16}}{8 b^{10}} + \frac{28 e^5 (b d - a e)^2 (2 b B d + A b e - 3 a B e) (a + b x)^{17}}{17 b^{10}} + \\
& \frac{2 e^6 (b d - a e) (7 b B d + 2 A b e - 9 a B e) (a + b x)^{18}}{9 b^{10}} + \frac{e^7 (8 b B d + A b e - 9 a B e) (a + b x)^{19}}{19 b^{10}} + \frac{B e^8 (a + b x)^{20}}{20 b^{10}}
\end{aligned}$$

Result (type 1, 2307 leaves):

$$\begin{aligned}
& a^{10} A d^8 x + \frac{1}{2} a^9 d^7 (10 A b d + a B d + 8 a A e) x^2 + \frac{1}{3} a^8 d^6 (2 a B d (5 b d + 4 a e) + A (45 b^2 d^2 + 80 a b d e + 28 a^2 e^2)) x^3 + \\
& \frac{1}{4} a^7 d^5 (a B d (45 b^2 d^2 + 80 a b d e + 28 a^2 e^2) + 8 A (15 b^3 d^3 + 45 a b^2 d^2 e + 35 a^2 b d e^2 + 7 a^3 e^3)) x^4 + \\
& \frac{2}{5} a^6 d^4 (4 a B d (15 b^3 d^3 + 45 a b^2 d^2 e + 35 a^2 b d e^2 + 7 a^3 e^3) + 5 A (21 b^4 d^4 + 96 a b^3 d^3 e + 126 a^2 b^2 d^2 e^2 + 56 a^3 b d e^3 + 7 a^4 e^4)) x^5 + \\
& \frac{1}{3} a^5 d^3 (5 a B d (21 b^4 d^4 + 96 a b^3 d^3 e + 126 a^2 b^2 d^2 e^2 + 56 a^3 b d e^3 + 7 a^4 e^4) + \\
& 14 A (9 b^5 d^5 + 60 a b^4 d^4 e + 120 a^2 b^3 d^3 e^2 + 90 a^3 b^2 d^2 e^3 + 25 a^4 b d e^4 + 2 a^5 e^5)) x^6 + \\
& 2 a^4 d^2 (2 a B d (9 b^5 d^5 + 60 a b^4 d^4 e + 120 a^2 b^3 d^3 e^2 + 90 a^3 b^2 d^2 e^3 + 25 a^4 b d e^4 + 2 a^5 e^5) + \\
& A (15 b^6 d^6 + 144 a b^5 d^5 e + 420 a^2 b^4 d^4 e^2 + 480 a^3 b^3 d^3 e^3 + 225 a^4 b^2 d^2 e^4 + 40 a^5 b d e^5 + 2 a^6 e^6)) x^7 + \\
& \frac{1}{4} a^3 d (7 a B d (15 b^6 d^6 + 144 a b^5 d^5 e + 420 a^2 b^4 d^4 e^2 + 480 a^3 b^3 d^3 e^3 + 225 a^4 b^2 d^2 e^4 + 40 a^5 b d e^5 + 2 a^6 e^6) + \\
& 4 A (15 b^7 d^7 + 210 a b^6 d^6 e + 882 a^2 b^5 d^5 e^2 + 1470 a^3 b^4 d^4 e^3 + 1050 a^4 b^3 d^3 e^4 + 315 a^5 b^2 d^2 e^5 + 35 a^6 b d e^6 + a^7 e^7)) x^8 +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{9} a^2 (8 a B d (15 b^7 d^7 + 210 a b^6 d^6 e + 882 a^2 b^5 d^5 e^2 + 1470 a^3 b^4 d^4 e^3 + 1050 a^4 b^3 d^3 e^4 + 315 a^5 b^2 d^2 e^5 + 35 a^6 b d e^6 + a^7 e^7) + \\
& A (45 b^8 d^8 + 960 a b^7 d^7 e + 5880 a^2 b^6 d^6 e^2 + 14112 a^3 b^5 d^5 e^3 + 14700 a^4 b^4 d^4 e^4 + 6720 a^5 b^3 d^3 e^5 + 1260 a^6 b^2 d^2 e^6 + 80 a^7 b d e^7 + a^8 e^8)) x^9 + \\
& \frac{1}{10} a (10 A b (b^8 d^8 + 36 a b^7 d^7 e + 336 a^2 b^6 d^6 e^2 + 1176 a^3 b^5 d^5 e^3 + 1764 a^4 b^4 d^4 e^4 + 1176 a^5 b^3 d^3 e^5 + 336 a^6 b^2 d^2 e^6 + 36 a^7 b d e^7 + a^8 e^8) + \\
& a B (45 b^8 d^8 + 960 a b^7 d^7 e + 5880 a^2 b^6 d^6 e^2 + 14112 a^3 b^5 d^5 e^3 + 14700 a^4 b^4 d^4 e^4 + 6720 a^5 b^3 d^3 e^5 + 1260 a^6 b^2 d^2 e^6 + 80 a^7 b d e^7 + a^8 e^8)) x^{10} + \\
& \frac{1}{11} b (10 a B (b^8 d^8 + 36 a b^7 d^7 e + 336 a^2 b^6 d^6 e^2 + 1176 a^3 b^5 d^5 e^3 + 1764 a^4 b^4 d^4 e^4 + 1176 a^5 b^3 d^3 e^5 + 336 a^6 b^2 d^2 e^6 + 36 a^7 b d e^7 + a^8 e^8) + \\
& A b (b^8 d^8 + 80 a b^7 d^7 e + 1260 a^2 b^6 d^6 e^2 + 6720 a^3 b^5 d^5 e^3 + 14700 a^4 b^4 d^4 e^4 + 14112 a^5 b^3 d^3 e^5 + 5880 a^6 b^2 d^2 e^6 + 960 a^7 b d e^7 + 45 a^8 e^8)) x^{11} + \\
& \frac{1}{12} b^2 (45 a^8 B e^8 + 7056 a^5 b^3 d^2 e^5 (2 B d + A e) + 120 a^7 b e^7 (8 B d + A e) + 1260 a^2 b^6 d^5 e^2 (B d + 2 A e) + 840 a^6 b^2 d e^6 (7 B d + 2 A e) + \\
& 2940 a^4 b^4 d^3 e^4 (5 B d + 4 A e) + 1680 a^3 b^5 d^4 e^3 (4 B d + 5 A e) + 40 a b^7 d^6 e (2 B d + 7 A e) + b^8 d^7 (B d + 8 A e)) x^{12} + \\
& \frac{2}{13} b^3 e (60 a^7 B e^7 + 2940 a^4 b^3 d^2 e^4 (2 B d + A e) + 105 a^6 b e^6 (8 B d + A e) + 140 a b^6 d^5 e (B d + 2 A e) + 504 a^5 b^2 d e^5 (7 B d + 2 A e) + \\
& 840 a^3 b^4 d^3 e^3 (5 B d + 4 A e) + 315 a^2 b^5 d^4 e^2 (4 B d + 5 A e) + 2 b^7 d^6 (2 B d + 7 A e)) x^{13} + \\
& b^4 e^2 (15 a^6 B e^6 + 240 a^3 b^3 d^2 e^3 (2 B d + A e) + 18 a^5 b e^5 (8 B d + A e) + 2 b^6 d^5 (B d + 2 A e) + 60 a^4 b^2 d e^4 (7 B d + 2 A e) + \\
& 45 a^2 b^4 d^3 e^2 (5 B d + 4 A e) + 10 a b^5 d^4 e (4 B d + 5 A e)) x^{14} + \frac{2}{15} b^5 e^3 \\
& (126 a^5 B e^5 + 630 a^2 b^3 d^2 e^2 (2 B d + A e) + 105 a^4 b e^4 (8 B d + A e) + 240 a^3 b^2 d e^3 (7 B d + 2 A e) + 70 a b^4 d^3 e (5 B d + 4 A e) + 7 b^5 d^4 (4 B d + 5 A e)) x^{15} + \\
& \frac{1}{8} b^6 e^4 (105 a^4 B e^4 + 140 a b^3 d^2 e (2 B d + A e) + 60 a^3 b e^3 (8 B d + A e) + 90 a^2 b^2 d e^2 (7 B d + 2 A e) + 7 b^4 d^3 (5 B d + 4 A e)) x^{16} + \\
& \frac{1}{17} b^7 e^5 (120 a^3 B e^3 + 28 b^3 d^2 (2 B d + A e) + 45 a^2 b e^2 (8 B d + A e) + 40 a b^2 d e (7 B d + 2 A e)) x^{17} + \\
& \frac{1}{18} b^8 e^6 (45 a^2 B e^2 + 10 a b e (8 B d + A e) + 4 b^2 d (7 B d + 2 A e)) x^{18} + \\
& \frac{1}{19} b^9 e^7 (8 b B d + A b e + 10 a B e) x^{19} + \frac{1}{20} b^{10} B e^8 x^{20}
\end{aligned}$$

**Problem 1081: Result more than twice size of optimal antiderivative.**

$$\int (a + b x)^{10} (A + B x) (d + e x)^7 dx$$

Optimal (type 1, 329 leaves, 2 steps):

$$\begin{aligned}
& \frac{(A b - a B) (b d - a e)^7 (a + b x)^{11}}{11 b^9} + \frac{(b d - a e)^6 (b B d + 7 A b e - 8 a B e) (a + b x)^{12}}{12 b^9} + \\
& \frac{7 e (b d - a e)^5 (b B d + 3 A b e - 4 a B e) (a + b x)^{13}}{13 b^9} + \frac{e^2 (b d - a e)^4 (3 b B d + 5 A b e - 8 a B e) (a + b x)^{14}}{2 b^9} + \\
& \frac{7 e^3 (b d - a e)^3 (b B d + A b e - 2 a B e) (a + b x)^{15}}{3 b^9} + \frac{7 e^4 (b d - a e)^2 (5 b B d + 3 A b e - 8 a B e) (a + b x)^{16}}{16 b^9} + \\
& \frac{7 e^5 (b d - a e) (3 b B d + A b e - 4 a B e) (a + b x)^{17}}{17 b^9} + \frac{e^6 (7 b B d + A b e - 8 a B e) (a + b x)^{18}}{18 b^9} + \frac{B e^7 (a + b x)^{19}}{19 b^9}
\end{aligned}$$

Result (type 1, 2034 leaves):

$$\begin{aligned}
& a^{10} A d^7 x + \frac{1}{2} a^9 d^6 (10 A b d + a B d + 7 a A e) x^2 + \frac{1}{3} a^8 d^5 (a B d (10 b d + 7 a e) + A (45 b^2 d^2 + 70 a b d e + 21 a^2 e^2)) x^3 + \\
& \frac{1}{4} a^7 d^4 (a B d (45 b^2 d^2 + 70 a b d e + 21 a^2 e^2) + 5 A (24 b^3 d^3 + 63 a b^2 d^2 e + 42 a^2 b d e^2 + 7 a^3 e^3)) x^4 + \\
& a^6 d^3 (a B d (24 b^3 d^3 + 63 a b^2 d^2 e + 42 a^2 b d e^2 + 7 a^3 e^3) + 7 A (6 b^4 d^4 + 24 a b^3 d^3 e + 27 a^2 b^2 d^2 e^2 + 10 a^3 b d e^3 + a^4 e^4)) x^5 + \frac{7}{6} a^5 d^2 \\
& (5 a B d (6 b^4 d^4 + 24 a b^3 d^3 e + 27 a^2 b^2 d^2 e^2 + 10 a^3 b d e^3 + a^4 e^4) + A (36 b^5 d^5 + 210 a b^4 d^4 e + 360 a^2 b^3 d^3 e^2 + 225 a^3 b^2 d^2 e^3 + 50 a^4 b d e^4 + 3 a^5 e^5)) + \\
& A (30 b^6 d^6 + 252 a b^5 d^5 e + 630 a^2 b^4 d^4 e^2 + 600 a^3 b^3 d^3 e^3 + 225 a^4 b^2 d^2 e^4 + 30 a^5 b d e^5 + a^6 e^6)) x^7 + \\
& \frac{1}{8} a^3 (7 a B d (30 b^6 d^6 + 252 a b^5 d^5 e + 630 a^2 b^4 d^4 e^2 + 600 a^3 b^3 d^3 e^3 + 225 a^4 b^2 d^2 e^4 + 30 a^5 b d e^5 + a^6 e^6)) + \\
& A (120 b^7 d^7 + 1470 a b^6 d^6 e + 5292 a^2 b^5 d^5 e^2 + 7350 a^3 b^4 d^4 e^3 + 4200 a^4 b^3 d^3 e^4 + 945 a^5 b^2 d^2 e^5 + 70 a^6 b d e^6 + a^7 e^7) x^8 + \\
& \frac{1}{9} a^2 (a B (120 b^7 d^7 + 1470 a b^6 d^6 e + 5292 a^2 b^5 d^5 e^2 + 7350 a^3 b^4 d^4 e^3 + 4200 a^4 b^3 d^3 e^4 + 945 a^5 b^2 d^2 e^5 + 70 a^6 b d e^6 + a^7 e^7)) + \\
& 5 A b (9 b^7 d^7 + 168 a b^6 d^6 e + 882 a^2 b^5 d^5 e^2 + 1764 a^3 b^4 d^4 e^3 + 1470 a^4 b^3 d^3 e^4 + 504 a^5 b^2 d^2 e^5 + 63 a^6 b d e^6 + 2 a^7 e^7) x^9 + \\
& \frac{1}{2} a b (a B (9 b^7 d^7 + 168 a b^6 d^6 e + 882 a^2 b^5 d^5 e^2 + 1764 a^3 b^4 d^4 e^3 + 1470 a^4 b^3 d^3 e^4 + 504 a^5 b^2 d^2 e^5 + 63 a^6 b d e^6 + 2 a^7 e^7)) + \\
& A b (2 b^7 d^7 + 63 a b^6 d^6 e + 504 a^2 b^5 d^5 e^2 + 1470 a^3 b^4 d^4 e^3 + 1764 a^4 b^3 d^3 e^4 + 882 a^5 b^2 d^2 e^5 + 168 a^6 b d e^6 + 9 a^7 e^7) x^{10} + \\
& \frac{1}{11} b^2 (5 a B (2 b^7 d^7 + 63 a b^6 d^6 e + 504 a^2 b^5 d^5 e^2 + 1470 a^3 b^4 d^4 e^3 + 1764 a^4 b^3 d^3 e^4 + 882 a^5 b^2 d^2 e^5 + 168 a^6 b d e^6 + 9 a^7 e^7)) + \\
& A b (b^7 d^7 + 70 a b^6 d^6 e + 945 a^2 b^5 d^5 e^2 + 4200 a^3 b^4 d^4 e^3 + 7350 a^4 b^3 d^3 e^4 + 5292 a^5 b^2 d^2 e^5 + 1470 a^6 b d e^6 + 120 a^7 e^7) x^{11} + \\
& \frac{1}{12} b^3 (120 a^7 B e^7 + 4200 a^3 b^4 d^3 e^3 (B d + A e) + 1764 a^5 b^2 d e^5 (3 B d + A e) + 210 a^6 b e^6 (7 B d + A e) + 70 a b^6 d^5 e (B d + 3 A e) + \\
& 1470 a^4 b^3 d^2 e^4 (5 B d + 3 A e) + 315 a^2 b^5 d^4 e^2 (3 B d + 5 A e) + b^7 d^6 (B d + 7 A e)) x^{12} + \\
& \frac{7}{13} b^4 e (30 a^6 B e^6 + 225 a^2 b^4 d^3 e^2 (B d + A e) + 210 a^4 b^2 d e^4 (3 B d + A e) + 36 a^5 b e^5 (7 B d + A e)) + \\
& b^6 d^5 (B d + 3 A e) + 120 a^3 b^3 d^2 e^3 (5 B d + 3 A e) + 10 a b^5 d^4 e (3 B d + 5 A e)) x^{13} + \\
& \frac{1}{2} b^5 e^2 (36 a^5 B e^5 + 50 a b^4 d^3 e (B d + A e) + 120 a^3 b^2 d e^3 (3 B d + A e) + 30 a^4 b e^4 (7 B d + A e) + 45 a^2 b^3 d^2 e^2 (5 B d + 3 A e) + b^5 d^4 (3 B d + 5 A e)) x^{14} + \\
& \frac{1}{3} b^6 e^3 (42 a^4 B e^4 + 7 b^4 d^3 (B d + A e) + 63 a^2 b^2 d e^2 (3 B d + A e) + 24 a^3 b e^3 (7 B d + A e) + 14 a b^3 d^2 e (5 B d + 3 A e)) x^{15} + \\
& \frac{1}{16} b^7 e^4 (120 a^3 B e^3 + 70 a b^2 d e (3 B d + A e) + 45 a^2 b e^2 (7 B d + A e) + 7 b^3 d^2 (5 B d + 3 A e)) x^{16} + \\
& \frac{1}{17} b^8 e^5 (45 a^2 B e^2 + 7 b^2 d (3 B d + A e) + 10 a b e (7 B d + A e)) x^{17} + \\
& \frac{1}{18} b^9 e^6 (7 b B d + A b e + 10 a B e) x^{18} + \frac{1}{19} b^{10} B e^7 x^{19}
\end{aligned}$$

Problem 1082: Result more than twice size of optimal antiderivative.

$$\int (a + bx)^{10} (A + Bx) (d + ex)^6 dx$$

Optimal (type 1, 290 leaves, 2 steps):

$$\begin{aligned} & \frac{(Ab - aB)(bd - ae)^6 (a + bx)^{11}}{11b^8} + \frac{(bd - ae)^5 (bBd + 6Ab e - 7aBe)(a + bx)^{12}}{12b^8} + \frac{3e(bd - ae)^4 (2bBd + 5Ab e - 7aBe)(a + bx)^{13}}{13b^8} + \\ & \frac{5e^2(bd - ae)^3 (3bBd + 4Ab e - 7aBe)(a + bx)^{14}}{14b^8} + \frac{e^3(bd - ae)^2 (4bBd + 3Ab e - 7aBe)(a + bx)^{15}}{3b^8} + \\ & \frac{3e^4(bd - ae)(5bBd + 2Ab e - 7aBe)(a + bx)^{16}}{16b^8} + \frac{e^5(6bBd + Ab e - 7aBe)(a + bx)^{17}}{17b^8} + \frac{Be^6(a + bx)^{18}}{18b^8} \end{aligned}$$

Result (type 1, 1788 leaves):

$$\begin{aligned}
& a^{10} A d^6 x + \frac{1}{2} a^9 d^5 (10 A b d + a B d + 6 a A e) x^2 + \frac{1}{3} a^8 d^4 (2 a B d (5 b d + 3 a e) + 15 A (3 b^2 d^2 + 4 a b d e + a^2 e^2)) x^3 + \\
& \frac{5}{4} a^7 d^3 (3 a B d (3 b^2 d^2 + 4 a b d e + a^2 e^2) + A (24 b^3 d^3 + 54 a b^2 d^2 e + 30 a^2 b d e^2 + 4 a^3 e^3)) x^4 + \\
& a^6 d^2 (2 a B d (12 b^3 d^3 + 27 a b^2 d^2 e + 15 a^2 b d e^2 + 2 a^3 e^3) + A (42 b^4 d^4 + 144 a b^3 d^3 e + 135 a^2 b^2 d^2 e^2 + 40 a^3 b d e^3 + 3 a^4 e^4)) x^5 + \\
& \frac{1}{6} a^5 d (5 a B d (42 b^4 d^4 + 144 a b^3 d^3 e + 135 a^2 b^2 d^2 e^2 + 40 a^3 b d e^3 + 3 a^4 e^4)) + \\
& 6 A (42 b^5 d^5 + 210 a b^4 d^4 e + 300 a^2 b^3 d^3 e^2 + 150 a^3 b^2 d^2 e^3 + 25 a^4 b d e^4 + a^5 e^5) x^6 + \\
& \frac{1}{7} a^4 (6 a B d (42 b^5 d^5 + 210 a b^4 d^4 e + 300 a^2 b^3 d^3 e^2 + 150 a^3 b^2 d^2 e^3 + 25 a^4 b d e^4 + a^5 e^5)) + \\
& A (210 b^6 d^6 + 1512 a b^5 d^5 e + 3150 a^2 b^4 d^4 e^2 + 2400 a^3 b^3 d^3 e^3 + 675 a^4 b^2 d^2 e^4 + 60 a^5 b d e^5 + a^6 e^6) x^7 + \\
& \frac{1}{8} a^3 (10 A b (12 b^6 d^6 + 126 a b^5 d^5 e + 378 a^2 b^4 d^4 e^2 + 420 a^3 b^3 d^3 e^3 + 180 a^4 b^2 d^2 e^4 + 27 a^5 b d e^5 + a^6 e^6)) + \\
& a B (210 b^6 d^6 + 1512 a b^5 d^5 e + 3150 a^2 b^4 d^4 e^2 + 2400 a^3 b^3 d^3 e^3 + 675 a^4 b^2 d^2 e^4 + 60 a^5 b d e^5 + a^6 e^6) x^8 + \\
& \frac{5}{9} a^2 b (9 A b (b^6 d^6 + 16 a b^5 d^5 e + 70 a^2 b^4 d^4 e^2 + 112 a^3 b^3 d^3 e^3 + 70 a^4 b^2 d^2 e^4 + 16 a^5 b d e^5 + a^6 e^6)) + \\
& 2 a B (12 b^6 d^6 + 126 a b^5 d^5 e + 378 a^2 b^4 d^4 e^2 + 420 a^3 b^3 d^3 e^3 + 180 a^4 b^2 d^2 e^4 + 27 a^5 b d e^5 + a^6 e^6) x^9 + \\
& \frac{1}{2} a b^2 (9 a B (b^6 d^6 + 16 a b^5 d^5 e + 70 a^2 b^4 d^4 e^2 + 112 a^3 b^3 d^3 e^3 + 70 a^4 b^2 d^2 e^4 + 16 a^5 b d e^5 + a^6 e^6)) + \\
& 2 A b (b^6 d^6 + 27 a b^5 d^5 e + 180 a^2 b^4 d^4 e^2 + 420 a^3 b^3 d^3 e^3 + 378 a^4 b^2 d^2 e^4 + 126 a^5 b d e^5 + 12 a^6 e^6) x^{10} + \\
& \frac{1}{11} b^3 (10 a B (b^6 d^6 + 27 a b^5 d^5 e + 180 a^2 b^4 d^4 e^2 + 420 a^3 b^3 d^3 e^3 + 378 a^4 b^2 d^2 e^4 + 126 a^5 b d e^5 + 12 a^6 e^6)) + \\
& A b (b^6 d^6 + 60 a b^5 d^5 e + 675 a^2 b^4 d^4 e^2 + 2400 a^3 b^3 d^3 e^3 + 3150 a^4 b^2 d^2 e^4 + 1512 a^5 b d e^5 + 210 a^6 e^6) x^{11} + \\
& \frac{1}{12} b^4 (210 a^6 B e^6 + 252 a^5 b e^5 (6 B d + A e) + 630 a^4 b^2 d e^4 (5 B d + 2 A e) + 600 a^3 b^3 d^2 e^3 (4 B d + 3 A e) + \\
& 225 a^2 b^4 d^3 e^2 (3 B d + 4 A e) + 30 a b^5 d^4 e (2 B d + 5 A e) + b^6 d^5 (B d + 6 A e)) x^{12} + \frac{1}{13} b^5 e \\
& (252 a^5 B e^5 + 210 a^4 b e^4 (6 B d + A e) + 360 a^3 b^2 d e^3 (5 B d + 2 A e) + 225 a^2 b^3 d^2 e^2 (4 B d + 3 A e) + 50 a b^4 d^3 e (3 B d + 4 A e) + 3 b^5 d^4 (2 B d + 5 A e)) \\
& x^{13} + \frac{5}{14} b^6 e^2 (42 a^4 B e^4 + 24 a^3 b e^3 (6 B d + A e) + 27 a^2 b^2 d e^2 (5 B d + 2 A e) + 10 a b^3 d^2 e (4 B d + 3 A e) + b^4 d^3 (3 B d + 4 A e)) x^{14} + \\
& \frac{1}{3} b^7 e^3 (24 a^3 B e^3 + 9 a^2 b e^2 (6 B d + A e) + 6 a b^2 d e (5 B d + 2 A e) + b^3 d^2 (4 B d + 3 A e)) x^{15} + \\
& \frac{1}{16} b^8 e^4 (45 a^2 B e^2 + 10 a b e (6 B d + A e) + 3 b^2 d (5 B d + 2 A e)) x^{16} + \\
& \frac{1}{17} b^9 e^5 (6 b B d + A b e + 10 a B e) x^{17} + \frac{1}{18} b^{10} B e^6 x^{18}
\end{aligned}$$

Problem 1083: Result more than twice size of optimal antiderivative.

$$\int (a + bx)^{10} (A + Bx) (d + ex)^5 dx$$

Optimal (type 1, 243 leaves, 2 steps):

$$\begin{aligned} & \frac{(Ab - aB)(bd - ae)^5 (a + bx)^{11}}{11b^7} + \frac{(bd - ae)^4 (bBd + 5AbE - 6aBe)(a + bx)^{12}}{12b^7} + \\ & \frac{5e(bd - ae)^3 (bBd + 2AbE - 3aBe)(a + bx)^{13}}{13b^7} + \frac{5e^2(bd - ae)^2 (bBd + AbE - 2aBe)(a + bx)^{14}}{7b^7} + \\ & \frac{e^3(bd - ae)(2bBd + AbE - 3aBe)(a + bx)^{15}}{3b^7} + \frac{e^4(5bBd + AbE - 6aBe)(a + bx)^{16}}{16b^7} + \frac{Be^5(a + bx)^{17}}{17b^7} \end{aligned}$$

Result (type 1, 1509 leaves):

$$\begin{aligned}
& a^{10} A d^5 x + \frac{1}{2} a^9 d^4 (a B d + 5 A (2 b d + a e)) x^2 + \frac{5}{3} a^8 d^3 (a B d (2 b d + a e) + A (9 b^2 d^2 + 10 a b d e + 2 a^2 e^2)) x^3 + \\
& \frac{5}{4} a^7 d^2 (a B d (9 b^2 d^2 + 10 a b d e + 2 a^2 e^2) + A (24 b^3 d^3 + 45 a b^2 d^2 e + 20 a^2 b d e^2 + 2 a^3 e^3)) x^4 + \\
& a^6 d (a B d (24 b^3 d^3 + 45 a b^2 d^2 e + 20 a^2 b d e^2 + 2 a^3 e^3) + A (42 b^4 d^4 + 120 a b^3 d^3 e + 90 a^2 b^2 d^2 e^2 + 20 a^3 b d e^3 + a^4 e^4)) x^5 + \\
& \frac{1}{6} a^5 (5 a B d (42 b^4 d^4 + 120 a b^3 d^3 e + 90 a^2 b^2 d^2 e^2 + 20 a^3 b d e^3 + a^4 e^4) + \\
& A (252 b^5 d^5 + 1050 a b^4 d^4 e + 1200 a^2 b^3 d^3 e^2 + 450 a^3 b^2 d^2 e^3 + 50 a^4 b d e^4 + a^5 e^5)) x^6 + \\
& \frac{1}{7} a^4 (a B (252 b^5 d^5 + 1050 a b^4 d^4 e + 1200 a^2 b^3 d^3 e^2 + 450 a^3 b^2 d^2 e^3 + 50 a^4 b d e^4 + a^5 e^5) + \\
& 5 A b (42 b^5 d^5 + 252 a b^4 d^4 e + 420 a^2 b^3 d^3 e^2 + 240 a^3 b^2 d^2 e^3 + 45 a^4 b d e^4 + 2 a^5 e^5)) x^7 + \\
& \frac{5}{8} a^3 b (a B (42 b^5 d^5 + 252 a b^4 d^4 e + 420 a^2 b^3 d^3 e^2 + 240 a^3 b^2 d^2 e^3 + 45 a^4 b d e^4 + 2 a^5 e^5) + \\
& 3 A b (8 b^5 d^5 + 70 a b^4 d^4 e + 168 a^2 b^3 d^3 e^2 + 140 a^3 b^2 d^2 e^3 + 40 a^4 b d e^4 + 3 a^5 e^5)) x^8 + \\
& \frac{5}{3} a^2 b^2 (a B (8 b^5 d^5 + 70 a b^4 d^4 e + 168 a^2 b^3 d^3 e^2 + 140 a^3 b^2 d^2 e^3 + 40 a^4 b d e^4 + 3 a^5 e^5) + \\
& A b (3 b^5 d^5 + 40 a b^4 d^4 e + 140 a^2 b^3 d^3 e^2 + 168 a^3 b^2 d^2 e^3 + 70 a^4 b d e^4 + 8 a^5 e^5)) x^9 + \\
& \frac{1}{2} a b^3 (3 a B (3 b^5 d^5 + 40 a b^4 d^4 e + 140 a^2 b^3 d^3 e^2 + 168 a^3 b^2 d^2 e^3 + 70 a^4 b d e^4 + 8 a^5 e^5) + \\
& A b (2 b^5 d^5 + 45 a b^4 d^4 e + 240 a^2 b^3 d^3 e^2 + 420 a^3 b^2 d^2 e^3 + 252 a^4 b d e^4 + 42 a^5 e^5)) x^{10} + \\
& \frac{1}{11} b^4 (5 a B (2 b^5 d^5 + 45 a b^4 d^4 e + 240 a^2 b^3 d^3 e^2 + 420 a^3 b^2 d^2 e^3 + 252 a^4 b d e^4 + 42 a^5 e^5) + \\
& A b (b^5 d^5 + 50 a b^4 d^4 e + 450 a^2 b^3 d^3 e^2 + 1200 a^3 b^2 d^2 e^3 + 1050 a^4 b d e^4 + 252 a^5 e^5)) x^{11} + \\
& \frac{1}{12} b^5 (252 a^5 B e^5 + 450 a^2 b^3 d^2 e^2 (B d + A e) + 600 a^3 b^2 d e^3 (2 B d + A e) + 210 a^4 b e^4 (5 B d + A e) + 50 a b^4 d^3 e (B d + 2 A e) + b^5 d^4 (B d + 5 A e)) x^{12} + \\
& \frac{5}{13} b^6 e (42 a^4 B e^4 + 20 a b^3 d^2 e (B d + A e) + 45 a^2 b^2 d e^2 (2 B d + A e) + 24 a^3 b e^3 (5 B d + A e) + b^4 d^3 (B d + 2 A e)) x^{13} + \\
& \frac{5}{14} b^7 e^2 (24 a^3 B e^3 + 2 b^3 d^2 (B d + A e) + 10 a b^2 d e (2 B d + A e) + 9 a^2 b e^2 (5 B d + A e)) x^{14} + \\
& \frac{1}{3} b^8 e^3 (9 a^2 B e^2 + b^2 d (2 B d + A e) + 2 a b e (5 B d + A e)) x^{15} + \\
& \frac{1}{16} b^9 e^4 (5 b B d + A b e + 10 a B e) x^{16} + \frac{1}{17} b^{10} B e^5 x^{17}
\end{aligned}$$

Problem 1084: Result more than twice size of optimal antiderivative.

$$\int (a + b x)^{10} (A + B x) (d + e x)^4 dx$$

Optimal (type 1, 204 leaves, 2 steps):

$$\begin{aligned} & \frac{(A b - a B) (b d - a e)^4 (a + b x)^{11}}{11 b^6} + \frac{(b d - a e)^3 (b B d + 4 A b e - 5 a B e) (a + b x)^{12}}{12 b^6} + \frac{2 e (b d - a e)^2 (2 b B d + 3 A b e - 5 a B e) (a + b x)^{13}}{13 b^6} + \\ & \frac{e^2 (b d - a e) (3 b B d + 2 A b e - 5 a B e) (a + b x)^{14}}{7 b^6} + \frac{e^3 (4 b B d + A b e - 5 a B e) (a + b x)^{15}}{15 b^6} + \frac{B e^4 (a + b x)^{16}}{16 b^6} \end{aligned}$$

Result (type 1, 1098 leaves):

$$\begin{aligned} & \frac{1}{240240} x \left( 8008 a^{10} (6 A (5 d^4 + 10 d^3 e x + 10 d^2 e^2 x^2 + 5 d e^3 x^3 + e^4 x^4) + B x (15 d^4 + 40 d^3 e x + 45 d^2 e^2 x^2 + 24 d e^3 x^3 + 5 e^4 x^4)) + \right. \\ & 11440 a^9 b x (7 A (15 d^4 + 40 d^3 e x + 45 d^2 e^2 x^2 + 24 d e^3 x^3 + 5 e^4 x^4) + 2 B x (35 d^4 + 105 d^3 e x + 126 d^2 e^2 x^2 + 70 d e^3 x^3 + 15 e^4 x^4)) + \\ & 12870 a^8 b^2 x^2 (8 A (35 d^4 + 105 d^3 e x + 126 d^2 e^2 x^2 + 70 d e^3 x^3 + 15 e^4 x^4) + 3 B x (70 d^4 + 224 d^3 e x + 280 d^2 e^2 x^2 + 160 d e^3 x^3 + 35 e^4 x^4)) + \\ & 11440 a^7 b^3 x^3 (9 A (70 d^4 + 224 d^3 e x + 280 d^2 e^2 x^2 + 160 d e^3 x^3 + 35 e^4 x^4) + 4 B x (126 d^4 + 420 d^3 e x + 540 d^2 e^2 x^2 + 315 d e^3 x^3 + 70 e^4 x^4)) + \\ & 40040 a^6 b^4 x^4 (2 A (126 d^4 + 420 d^3 e x + 540 d^2 e^2 x^2 + 315 d e^3 x^3 + 70 e^4 x^4) + B x (210 d^4 + 720 d^3 e x + 945 d^2 e^2 x^2 + 560 d e^3 x^3 + 126 e^4 x^4)) + \\ & 4368 a^5 b^5 x^5 (11 A (210 d^4 + 720 d^3 e x + 945 d^2 e^2 x^2 + 560 d e^3 x^3 + 126 e^4 x^4) + 6 B x (330 d^4 + 1155 d^3 e x + 1540 d^2 e^2 x^2 + 924 d e^3 x^3 + 210 e^4 x^4)) + \\ & 1820 a^4 b^6 x^6 (12 A (330 d^4 + 1155 d^3 e x + 1540 d^2 e^2 x^2 + 924 d e^3 x^3 + 210 e^4 x^4) + 7 B x (495 d^4 + 1760 d^3 e x + 2376 d^2 e^2 x^2 + 1440 d e^3 x^3 + 330 e^4 x^4)) + \\ & 560 a^3 b^7 x^7 (13 A (495 d^4 + 1760 d^3 e x + 2376 d^2 e^2 x^2 + 1440 d e^3 x^3 + 330 e^4 x^4) + 8 B x (715 d^4 + 2574 d^3 e x + 3510 d^2 e^2 x^2 + 2145 d e^3 x^3 + 495 e^4 x^4)) + \\ & 120 a^2 b^8 x^8 (14 A (715 d^4 + 2574 d^3 e x + 3510 d^2 e^2 x^2 + 2145 d e^3 x^3 + 495 e^4 x^4) + 9 B x (1001 d^4 + 3640 d^3 e x + 5005 d^2 e^2 x^2 + 3080 d e^3 x^3 + 715 e^4 x^4)) + 80 a b^9 \\ & x^9 (3 A (1001 d^4 + 3640 d^3 e x + 5005 d^2 e^2 x^2 + 3080 d e^3 x^3 + 715 e^4 x^4) + 2 B x (1365 d^4 + 5005 d^3 e x + 6930 d^2 e^2 x^2 + 4290 d e^3 x^3 + 1001 e^4 x^4)) + b^{10} \\ & x^{10} (16 A (1365 d^4 + 5005 d^3 e x + 6930 d^2 e^2 x^2 + 4290 d e^3 x^3 + 1001 e^4 x^4) + 11 B x (1820 d^4 + 6720 d^3 e x + 9360 d^2 e^2 x^2 + 5824 d e^3 x^3 + 1365 e^4 x^4)) \end{aligned}$$

Problem 1085: Result more than twice size of optimal antiderivative.

$$\int (a + b x)^{10} (A + B x) (d + e x)^3 dx$$

Optimal (type 1, 159 leaves, 2 steps):

$$\begin{aligned} & \frac{(A b - a B) (b d - a e)^3 (a + b x)^{11}}{11 b^5} + \frac{(b d - a e)^2 (b B d + 3 A b e - 4 a B e) (a + b x)^{12}}{12 b^5} + \\ & \frac{3 e (b d - a e) (b B d + A b e - 2 a B e) (a + b x)^{13}}{13 b^5} + \frac{e^2 (3 b B d + A b e - 4 a B e) (a + b x)^{14}}{14 b^5} + \frac{B e^3 (a + b x)^{15}}{15 b^5} \end{aligned}$$

Result (type 1, 855 leaves):

$$\begin{aligned}
& \frac{1}{60060} x \left( 3003 a^{10} (5 A (4 d^3 + 6 d^2 e x + 4 d e^2 x^2 + e^3 x^3) + B x (10 d^3 + 20 d^2 e x + 15 d e^2 x^2 + 4 e^3 x^3)) + \right. \\
& 10010 a^9 b x (3 A (10 d^3 + 20 d^2 e x + 15 d e^2 x^2 + 4 e^3 x^3) + B x (20 d^3 + 45 d^2 e x + 36 d e^2 x^2 + 10 e^3 x^3)) + \\
& 6435 a^8 b^2 x^2 (7 A (20 d^3 + 45 d^2 e x + 36 d e^2 x^2 + 10 e^3 x^3) + 3 B x (35 d^3 + 84 d^2 e x + 70 d e^2 x^2 + 20 e^3 x^3)) + \\
& 25740 a^7 b^3 x^3 (2 A (35 d^3 + 84 d^2 e x + 70 d e^2 x^2 + 20 e^3 x^3) + B x (56 d^3 + 140 d^2 e x + 120 d e^2 x^2 + 35 e^3 x^3)) + \\
& 5005 a^6 b^4 x^4 (9 A (56 d^3 + 140 d^2 e x + 120 d e^2 x^2 + 35 e^3 x^3) + 5 B x (84 d^3 + 216 d^2 e x + 189 d e^2 x^2 + 56 e^3 x^3)) + \\
& 6006 a^5 b^5 x^5 (5 A (84 d^3 + 216 d^2 e x + 189 d e^2 x^2 + 56 e^3 x^3) + 3 B x (120 d^3 + 315 d^2 e x + 280 d e^2 x^2 + 84 e^3 x^3)) + \\
& 1365 a^4 b^6 x^6 (11 A (120 d^3 + 315 d^2 e x + 280 d e^2 x^2 + 84 e^3 x^3) + 7 B x (165 d^3 + 440 d^2 e x + 396 d e^2 x^2 + 120 e^3 x^3)) + \\
& 1820 a^3 b^7 x^7 (3 A (165 d^3 + 440 d^2 e x + 396 d e^2 x^2 + 120 e^3 x^3) + 2 B x (220 d^3 + 594 d^2 e x + 540 d e^2 x^2 + 165 e^3 x^3)) + \\
& 105 a^2 b^8 x^8 (13 A (220 d^3 + 594 d^2 e x + 540 d e^2 x^2 + 165 e^3 x^3) + 9 B x (286 d^3 + 780 d^2 e x + 715 d e^2 x^2 + 220 e^3 x^3)) + \\
& 30 a b^9 x^9 (7 A (286 d^3 + 780 d^2 e x + 715 d e^2 x^2 + 220 e^3 x^3) + 5 B x (364 d^3 + 1001 d^2 e x + 924 d e^2 x^2 + 286 e^3 x^3)) + \\
& b^{10} x^{10} (15 A (364 d^3 + 1001 d^2 e x + 924 d e^2 x^2 + 286 e^3 x^3) + 11 B x (455 d^3 + 1260 d^2 e x + 1170 d e^2 x^2 + 364 e^3 x^3))
\end{aligned}$$

**Problem 1086: Result more than twice size of optimal antiderivative.**

$$\int (a + b x)^{10} (A + B x) (d + e x)^2 dx$$

Optimal (type 1, 118 leaves, 2 steps):

$$\frac{(A B - a B) (b d - a e)^2 (a + b x)^{11}}{11 b^4} + \frac{(b d - a e) (b B d + 2 A b e - 3 a B e) (a + b x)^{12}}{12 b^4} + \frac{e (2 b B d + A b e - 3 a B e) (a + b x)^{13}}{13 b^4} + \frac{B e^2 (a + b x)^{14}}{14 b^4}$$

Result (type 1, 614 leaves):

$$\begin{aligned}
& \frac{1}{12012} x \left( 1001 a^{10} (4 A (3 d^2 + 3 d e x + e^2 x^2) + B x (6 d^2 + 8 d e x + 3 e^2 x^2)) + 2002 a^9 b x (5 A (6 d^2 + 8 d e x + 3 e^2 x^2) + 2 B x (10 d^2 + 15 d e x + 6 e^2 x^2)) + \right. \\
& 9009 a^8 b^2 x^2 (2 A (10 d^2 + 15 d e x + 6 e^2 x^2) + B x (15 d^2 + 24 d e x + 10 e^2 x^2)) + \\
& 3432 a^7 b^3 x^3 (7 A (15 d^2 + 24 d e x + 10 e^2 x^2) + 4 B x (21 d^2 + 35 d e x + 15 e^2 x^2)) + \\
& 3003 a^6 b^4 x^4 (8 A (21 d^2 + 35 d e x + 15 e^2 x^2) + 5 B x (28 d^2 + 48 d e x + 21 e^2 x^2)) + \\
& 6006 a^5 b^5 x^5 (3 A (28 d^2 + 48 d e x + 21 e^2 x^2) + 2 B x (36 d^2 + 63 d e x + 28 e^2 x^2)) + \\
& 1001 a^4 b^6 x^6 (10 A (36 d^2 + 63 d e x + 28 e^2 x^2) + 7 B x (45 d^2 + 80 d e x + 36 e^2 x^2)) + \\
& 364 a^3 b^7 x^7 (11 A (45 d^2 + 80 d e x + 36 e^2 x^2) + 8 B x (55 d^2 + 99 d e x + 45 e^2 x^2)) + \\
& 273 a^2 b^8 x^8 (4 A (55 d^2 + 99 d e x + 45 e^2 x^2) + 3 B x (66 d^2 + 120 d e x + 55 e^2 x^2)) + \\
& 14 a b^9 x^9 (13 A (66 d^2 + 120 d e x + 55 e^2 x^2) + 10 B x (78 d^2 + 143 d e x + 66 e^2 x^2)) + \\
& b^{10} x^{10} (14 A (78 d^2 + 143 d e x + 66 e^2 x^2) + 11 B x (91 d^2 + 168 d e x + 78 e^2 x^2))
\end{aligned}$$

**Problem 1087: Result more than twice size of optimal antiderivative.**

$$\int (a + b x)^{10} (A + B x) (d + e x) dx$$

Optimal (type 1, 75 leaves, 2 steps):

$$\frac{(A b - a B) (b d - a e) (a + b x)^{11}}{11 b^3} + \frac{(b B d + A b e - 2 a B e) (a + b x)^{12}}{12 b^3} + \frac{B e (a + b x)^{13}}{13 b^3}$$

Result (type 1, 383 leaves):

$$\begin{aligned} & \frac{1}{66} a b^9 x^{10} (66 A d + 60 B d x + 60 A e x + 55 B e x^2) + \frac{1}{22} a^2 b^8 x^9 (110 A d + 99 B d x + 99 A e x + 90 B e x^2) + \frac{1}{6} a^{10} x (3 A (2 d + e x) + B x (3 d + 2 e x)) + \\ & \frac{3}{4} a^8 b^2 x^3 (5 A (4 d + 3 e x) + 3 B x (5 d + 4 e x)) + \frac{5}{6} a^9 b x^2 (B x (4 d + 3 e x) + A (6 d + 4 e x)) + 2 a^7 b^3 x^4 (3 A (5 d + 4 e x) + 2 B x (6 d + 5 e x)) + \\ & a^6 b^4 x^5 (7 A (6 d + 5 e x) + 5 B x (7 d + 6 e x)) + \frac{3}{2} a^5 b^5 x^6 (4 A (7 d + 6 e x) + 3 B x (8 d + 7 e x)) + \frac{5}{12} a^4 b^6 x^7 (9 A (8 d + 7 e x) + 7 B x (9 d + 8 e x)) + \\ & \frac{1}{3} a^3 b^7 x^8 (5 A (9 d + 8 e x) + 4 B x (10 d + 9 e x)) + \frac{b^{10} x^{11} (13 A (12 d + 11 e x) + 11 B x (13 d + 12 e x))}{1716} \end{aligned}$$

Problem 1088: Result more than twice size of optimal antiderivative.

$$\int (a + b x)^{10} (A + B x) dx$$

Optimal (type 1, 38 leaves, 2 steps):

$$\frac{(A b - a B) (a + b x)^{11}}{11 b^2} + \frac{B (a + b x)^{12}}{12 b^2}$$

Result (type 1, 198 leaves):

$$\begin{aligned} & \frac{1}{132} x \\ & (66 a^{10} (2 A + B x) + 220 a^9 b x (3 A + 2 B x) + 495 a^8 b^2 x^2 (4 A + 3 B x) + 792 a^7 b^3 x^3 (5 A + 4 B x) + 924 a^6 b^4 x^4 (6 A + 5 B x) + 792 a^5 b^5 x^5 (7 A + 6 B x) + \\ & 495 a^4 b^6 x^6 (8 A + 7 B x) + 220 a^3 b^7 x^7 (9 A + 8 B x) + 66 a^2 b^8 x^8 (10 A + 9 B x) + 12 a b^9 x^9 (11 A + 10 B x) + b^{10} x^{10} (12 A + 11 B x)) \end{aligned}$$

Problem 1089: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^{10} (A + B x)}{d + e x} dx$$

Optimal (type 3, 348 leaves, 2 steps):

$$\begin{aligned}
& \frac{b(b d - a e)^9 (B d - A e) x}{e^{11}} - \frac{(b d - a e)^8 (B d - A e) (a + b x)^2}{2 e^{10}} + \frac{(b d - a e)^7 (B d - A e) (a + b x)^3}{3 e^9} - \frac{(b d - a e)^6 (B d - A e) (a + b x)^4}{4 e^8} + \\
& \frac{(b d - a e)^5 (B d - A e) (a + b x)^5}{5 e^7} - \frac{(b d - a e)^4 (B d - A e) (a + b x)^6}{6 e^6} + \frac{(b d - a e)^3 (B d - A e) (a + b x)^7}{7 e^5} - \frac{(b d - a e)^2 (B d - A e) (a + b x)^8}{8 e^4} + \\
& \frac{(b d - a e) (B d - A e) (a + b x)^9}{9 e^3} - \frac{(B d - A e) (a + b x)^{10}}{10 e^2} + \frac{B (a + b x)^{11}}{11 b e} - \frac{(b d - a e)^{10} (B d - A e) \log[d + e x]}{e^{12}}
\end{aligned}$$

Result (type 3, 1252 leaves):

$$\begin{aligned}
& \frac{1}{27720 e^{11}} x (27720 a^{10} B e^{10} + 138600 a^9 b e^9 (-2 B d + 2 A e + B e x) + 207900 a^8 b^2 e^8 (3 A e (-2 d + e x) + B (6 d^2 - 3 d e x + 2 e^2 x^2)) + \\
& 277200 a^7 b^3 e^7 (2 A e (6 d^2 - 3 d e x + 2 e^2 x^2) + B (-12 d^3 + 6 d^2 e x - 4 d e^2 x^2 + 3 e^3 x^3)) + \\
& 97020 a^6 b^4 e^6 (5 A e (-12 d^3 + 6 d^2 e x - 4 d e^2 x^2 + 3 e^3 x^3) + B (60 d^4 - 30 d^3 e x + 20 d^2 e^2 x^2 - 15 d e^3 x^3 + 12 e^4 x^4)) + \\
& 116424 a^5 b^5 e^5 (A e (60 d^4 - 30 d^3 e x + 20 d^2 e^2 x^2 - 15 d e^3 x^3 + 12 e^4 x^4) + B (-60 d^5 + 30 d^4 e x - 20 d^3 e^2 x^2 + 15 d^2 e^3 x^3 - 12 d e^4 x^4 + 10 e^5 x^5)) + \\
& 13860 a^4 b^6 e^4 (7 A e (-60 d^5 + 30 d^4 e x - 20 d^3 e^2 x^2 + 15 d^2 e^3 x^3 - 12 d e^4 x^4 + 10 e^5 x^5)) + \\
& B (420 d^6 - 210 d^5 e x + 140 d^4 e^2 x^2 - 105 d^3 e^3 x^3 + 84 d^2 e^4 x^4 - 70 d e^5 x^5 + 60 e^6 x^6)) + \\
& 3960 a^3 b^7 e^3 (2 A e (420 d^6 - 210 d^5 e x + 140 d^4 e^2 x^2 - 105 d^3 e^3 x^3 + 84 d^2 e^4 x^4 - 70 d e^5 x^5 + 60 e^6 x^6)) + \\
& B (-840 d^7 + 420 d^6 e x - 280 d^5 e^2 x^2 + 210 d^4 e^3 x^3 - 168 d^3 e^4 x^4 + 140 d^2 e^5 x^5 - 120 d e^6 x^6 + 105 e^7 x^7)) + \\
& 495 a^2 b^8 e^2 (3 A e (-840 d^7 + 420 d^6 e x - 280 d^5 e^2 x^2 + 210 d^4 e^3 x^3 - 168 d^3 e^4 x^4 + 140 d^2 e^5 x^5 - 120 d e^6 x^6 + 105 e^7 x^7)) + \\
& B (2520 d^8 - 1260 d^7 e x + 840 d^6 e^2 x^2 - 630 d^5 e^3 x^3 + 504 d^4 e^4 x^4 - 420 d^3 e^5 x^5 + 360 d^2 e^6 x^6 - 315 d e^7 x^7 + 280 e^8 x^8)) + \\
& 110 a b^9 e (A e (2520 d^8 - 1260 d^7 e x + 840 d^6 e^2 x^2 - 630 d^5 e^3 x^3 + 504 d^4 e^4 x^4 - 420 d^3 e^5 x^5 + 360 d^2 e^6 x^6 - 315 d e^7 x^7 + 280 e^8 x^8)) + \\
& B (-2520 d^9 + 1260 d^8 e x - 840 d^7 e^2 x^2 + 630 d^6 e^3 x^3 - 504 d^5 e^4 x^4 + 420 d^4 e^5 x^5 - 360 d^3 e^6 x^6 + 315 d^2 e^7 x^7 - 280 d e^8 x^8 + 252 e^9 x^9)) + \\
& b^{10} (11 A e (-2520 d^9 + 1260 d^8 e x - 840 d^7 e^2 x^2 + 630 d^6 e^3 x^3 - 504 d^5 e^4 x^4 + 420 d^4 e^5 x^5 - 360 d^3 e^6 x^6 + 315 d^2 e^7 x^7 - 280 d e^8 x^8 + 252 e^9 x^9)) + \\
& B (27720 d^{10} - 13860 d^9 e x + 9240 d^8 e^2 x^2 - 6930 d^7 e^3 x^3 + 5544 d^6 e^4 x^4 - 4620 d^5 e^5 x^5 + 3960 d^4 e^6 x^6 - \\
& 3465 d^3 e^7 x^7 + 3080 d^2 e^8 x^8 - 2772 d e^9 x^9 + 2520 e^{10} x^{10})) + \frac{(b d - a e)^{10} (-B d + A e) \log[d + e x]}{e^{12}}
\end{aligned}$$

Problem 1090: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^{10} (A + B x)}{(d + e x)^2} dx$$

Optimal (type 3, 445 leaves, 2 steps):

$$\begin{aligned}
& - \frac{5 b (b d - a e)^8 (11 b B d - 9 A b e - 2 a B e) x}{e^{11}} + \frac{(b d - a e)^{10} (B d - A e)}{e^{12} (d + e x)} + \frac{15 b^2 (b d - a e)^7 (11 b B d - 8 A b e - 3 a B e) (d + e x)^2}{2 e^{12}} \\
& - \frac{10 b^3 (b d - a e)^6 (11 b B d - 7 A b e - 4 a B e) (d + e x)^3}{e^{12}} + \frac{21 b^4 (b d - a e)^5 (11 b B d - 6 A b e - 5 a B e) (d + e x)^4}{2 e^{12}} \\
& - \frac{42 b^5 (b d - a e)^4 (11 b B d - 5 A b e - 6 a B e) (d + e x)^5}{5 e^{12}} + \frac{5 b^6 (b d - a e)^3 (11 b B d - 4 A b e - 7 a B e) (d + e x)^6}{e^{12}} \\
& - \frac{15 b^7 (b d - a e)^2 (11 b B d - 3 A b e - 8 a B e) (d + e x)^7}{7 e^{12}} + \frac{5 b^8 (b d - a e) (11 b B d - 2 A b e - 9 a B e) (d + e x)^8}{8 e^{12}} \\
& + \frac{b^9 (11 b B d - A b e - 10 a B e) (d + e x)^9}{9 e^{12}} + \frac{b^{10} B (d + e x)^{10}}{10 e^{12}} + \frac{(b d - a e)^9 (11 b B d - 10 A b e - a B e) \text{Log}[d + e x]}{e^{12}}
\end{aligned}$$

Result (type 3, 1486 leaves):

$$\begin{aligned}
& \frac{1}{2520 e^{12} (d + e x)} \left( -2520 a^{10} e^{10} (-B d + A e) + \right. \\
& 25200 a^9 b e^9 (A d e + B (-d^2 + d e x + e^2 x^2)) + 56700 a^8 b^2 e^8 (2 A e (-d^2 + d e x + e^2 x^2) + B (2 d^3 - 4 d^2 e x - 3 d e^2 x^2 + e^3 x^3)) + \\
& 50400 a^7 b^3 e^7 (3 A e (2 d^3 - 4 d^2 e x - 3 d e^2 x^2 + e^3 x^3) + 2 B (-3 d^4 + 9 d^3 e x + 6 d^2 e^2 x^2 - 2 d e^3 x^3 + e^4 x^4)) + \\
& 44100 a^6 b^4 e^6 (4 A e (-3 d^4 + 9 d^3 e x + 6 d^2 e^2 x^2 - 2 d e^3 x^3 + e^4 x^4) + B (12 d^5 - 48 d^4 e x - 30 d^3 e^2 x^2 + 10 d^2 e^3 x^3 - 5 d e^4 x^4 + 3 e^5 x^5)) + 10584 a^5 b^5 e^5 \\
& (5 A e (12 d^5 - 48 d^4 e x - 30 d^3 e^2 x^2 + 10 d^2 e^3 x^3 - 5 d e^4 x^4 + 3 e^5 x^5) - 6 B (10 d^6 - 50 d^5 e x - 30 d^4 e^2 x^2 + 10 d^3 e^3 x^3 - 5 d^2 e^4 x^4 + 3 d e^5 x^5 - 2 e^6 x^6)) + \\
& 8820 a^4 b^6 e^4 (6 A e (-10 d^6 + 50 d^5 e x + 30 d^4 e^2 x^2 - 10 d^3 e^3 x^3 + 5 d^2 e^4 x^4 - 3 d e^5 x^5 + 2 e^6 x^6)) + \\
& B (60 d^7 - 360 d^6 e x - 210 d^5 e^2 x^2 + 70 d^4 e^3 x^3 - 35 d^3 e^4 x^4 + 21 d^2 e^5 x^5 - 14 d e^6 x^6 + 10 e^7 x^7) + \\
& 720 a^3 b^7 e^3 (7 A e (60 d^7 - 360 d^6 e x - 210 d^5 e^2 x^2 + 70 d^4 e^3 x^3 - 35 d^3 e^4 x^4 + 21 d^2 e^5 x^5 - 14 d e^6 x^6 + 10 e^7 x^7) - \\
& 4 B (105 d^8 - 735 d^7 e x - 420 d^6 e^2 x^2 + 140 d^5 e^3 x^3 - 70 d^4 e^4 x^4 + 42 d^3 e^5 x^5 - 28 d^2 e^6 x^6 + 20 d e^7 x^7 - 15 e^8 x^8)) + \\
& 135 a^2 b^8 e^2 (8 A e (-105 d^8 + 735 d^7 e x + 420 d^6 e^2 x^2 - 140 d^5 e^3 x^3 + 70 d^4 e^4 x^4 - 42 d^3 e^5 x^5 + 28 d^2 e^6 x^6 - 20 d e^7 x^7 + 15 e^8 x^8)) + \\
& 3 B (280 d^9 - 2240 d^8 e x - 1260 d^7 e^2 x^2 + 420 d^6 e^3 x^3 - 210 d^5 e^4 x^4 + 126 d^4 e^5 x^5 - 84 d^3 e^6 x^6 + 60 d^2 e^7 x^7 - 45 d e^8 x^8 + 35 e^9 x^9) + \\
& 10 a b^9 e (9 A e (280 d^9 - 2240 d^8 e x - 1260 d^7 e^2 x^2 + 420 d^6 e^3 x^3 - 210 d^5 e^4 x^4 + 126 d^4 e^5 x^5 - 84 d^3 e^6 x^6 + 60 d^2 e^7 x^7 - 45 d e^8 x^8 + 35 e^9 x^9) - 10 B \\
& (252 d^{10} - 2268 d^9 e x - 1260 d^8 e^2 x^2 + 420 d^7 e^3 x^3 - 210 d^6 e^4 x^4 + 126 d^5 e^5 x^5 - 84 d^4 e^6 x^6 + 60 d^3 e^7 x^7 - 45 d^2 e^8 x^8 + 35 d e^9 x^9 - 28 e^{10} x^{10})) + b^{10} \\
& (10 A e (-252 d^{10} + 2268 d^9 e x + 1260 d^8 e^2 x^2 - 420 d^7 e^3 x^3 + 210 d^6 e^4 x^4 - 126 d^5 e^5 x^5 + 84 d^4 e^6 x^6 - 60 d^3 e^7 x^7 + 45 d^2 e^8 x^8 - 35 d e^9 x^9 + 28 e^{10} x^{10})) + \\
& B (2520 d^{11} - 25200 d^{10} e x - 13860 d^9 e^2 x^2 + 4620 d^8 e^3 x^3 - 2310 d^7 e^4 x^4 + 1386 d^6 e^5 x^5 - 924 d^5 e^6 x^6 + 660 d^4 e^7 x^7 - \\
& 495 d^3 e^8 x^8 + 385 d^2 e^9 x^9 - 308 d e^{10} x^{10} + 252 e^{11} x^{11}) + 2520 (b d - a e)^9 (11 b B d - 10 A b e - a B e) (d + e x) \text{Log}[d + e x]
\end{aligned}$$

Problem 1091: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^{10} (A + B x)}{(d + e x)^3} dx$$

Optimal (type 3, 445 leaves, 2 steps):

$$\begin{aligned}
& \frac{15 b^2 (b d - a e)^7 (11 b B d - 8 A b e - 3 a B e) x}{e^{11}} + \frac{(b d - a e)^{10} (B d - A e)}{2 e^{12} (d + e x)^2} - \frac{(b d - a e)^9 (11 b B d - 10 A b e - a B e)}{e^{12} (d + e x)} - \\
& \frac{15 b^3 (b d - a e)^6 (11 b B d - 7 A b e - 4 a B e) (d + e x)^2}{e^{12}} + \frac{14 b^4 (b d - a e)^5 (11 b B d - 6 A b e - 5 a B e) (d + e x)^3}{e^{12}} - \\
& \frac{21 b^5 (b d - a e)^4 (11 b B d - 5 A b e - 6 a B e) (d + e x)^4}{2 e^{12}} + \frac{6 b^6 (b d - a e)^3 (11 b B d - 4 A b e - 7 a B e) (d + e x)^5}{e^{12}} - \\
& \frac{5 b^7 (b d - a e)^2 (11 b B d - 3 A b e - 8 a B e) (d + e x)^6}{2 e^{12}} + \frac{5 b^8 (b d - a e) (11 b B d - 2 A b e - 9 a B e) (d + e x)^7}{7 e^{12}} - \\
& \frac{b^9 (11 b B d - A b e - 10 a B e) (d + e x)^8}{8 e^{12}} + \frac{b^{10} B (d + e x)^9}{9 e^{12}} - \frac{5 b (b d - a e)^8 (11 b B d - 9 A b e - 2 a B e) \text{Log}[d + e x]}{e^{12}}
\end{aligned}$$

Result (type 3, 1480 leaves):

$$\begin{aligned}
& \frac{1}{504 e^{12} (d + e x)^2} (-252 a^{10} e^{10} (A e + B (d + 2 e x)) - \\
& 2520 a^9 b e^9 (A e (d + 2 e x) - B d (3 d + 4 e x)) + 11340 a^8 b^2 e^8 (A d e (3 d + 4 e x) + B (-5 d^3 - 4 d^2 e x + 4 d e^2 x^2 + 2 e^3 x^3)) + \\
& 30240 a^7 b^3 e^7 (A e (-5 d^3 - 4 d^2 e x + 4 d e^2 x^2 + 2 e^3 x^3) + B (7 d^4 + 2 d^3 e x - 11 d^2 e^2 x^2 - 4 d e^3 x^3 + e^4 x^4)) + \\
& 17640 a^6 b^4 e^6 (3 A e (7 d^4 + 2 d^3 e x - 11 d^2 e^2 x^2 - 4 d e^3 x^3 + e^4 x^4) + B (-27 d^5 + 6 d^4 e x + 63 d^3 e^2 x^2 + 20 d^2 e^3 x^3 - 5 d e^4 x^4 + 2 e^5 x^5)) + 10584 a^5 b^5 e^5 \\
& (2 A e (-27 d^5 + 6 d^4 e x + 63 d^3 e^2 x^2 + 20 d^2 e^3 x^3 - 5 d e^4 x^4 + 2 e^5 x^5) + 3 B (22 d^6 - 16 d^5 e x - 68 d^4 e^2 x^2 - 20 d^3 e^3 x^3 + 5 d^2 e^4 x^4 - 2 d e^5 x^5 + e^6 x^6)) + \\
& 5292 a^4 b^6 e^4 (5 A e (22 d^6 - 16 d^5 e x - 68 d^4 e^2 x^2 - 20 d^3 e^3 x^3 + 5 d^2 e^4 x^4 - 2 d e^5 x^5 + e^6 x^6)) + \\
& B (-130 d^7 + 160 d^6 e x + 500 d^5 e^2 x^2 + 140 d^4 e^3 x^3 - 35 d^3 e^4 x^4 + 14 d^2 e^5 x^5 - 7 d e^6 x^6 + 4 e^7 x^7)) + \\
& 1008 a^3 b^7 e^3 (3 A e (-130 d^7 + 160 d^6 e x + 500 d^5 e^2 x^2 + 140 d^4 e^3 x^3 - 35 d^3 e^4 x^4 + 14 d^2 e^5 x^5 - 7 d e^6 x^6 + 4 e^7 x^7)) + \\
& 2 B (225 d^8 - 390 d^7 e x - 1035 d^6 e^2 x^2 - 280 d^5 e^3 x^3 + 70 d^4 e^4 x^4 - 28 d^3 e^5 x^5 + 14 d^2 e^6 x^6 - 8 d e^7 x^7 + 5 e^8 x^8)) + \\
& 108 a^2 b^8 e^2 (7 A e (225 d^8 - 390 d^7 e x - 1035 d^6 e^2 x^2 - 280 d^5 e^3 x^3 + 70 d^4 e^4 x^4 - 28 d^3 e^5 x^5 + 14 d^2 e^6 x^6 - 8 d e^7 x^7 + 5 e^8 x^8)) - \\
& 3 B (595 d^9 - 1330 d^8 e x - 3185 d^7 e^2 x^2 - 840 d^6 e^3 x^3 + 210 d^5 e^4 x^4 - 84 d^4 e^5 x^5 + 42 d^3 e^6 x^6 - 24 d^2 e^7 x^7 + 15 d e^8 x^8 - 10 e^9 x^9)) + \\
& 18 a b^9 e (4 A e (-595 d^9 + 1330 d^8 e x + 3185 d^7 e^2 x^2 + 840 d^6 e^3 x^3 - 210 d^5 e^4 x^4 + 84 d^4 e^5 x^5 - 42 d^3 e^6 x^6 + 24 d^2 e^7 x^7 - 15 d e^8 x^8 + 10 e^9 x^9)) + \\
& 5 B (532 d^{10} - 1456 d^9 e x - 3248 d^8 e^2 x^2 - 840 d^7 e^3 x^3 + 210 d^6 e^4 x^4 - 84 d^5 e^5 x^5 + 42 d^4 e^6 x^6 - 24 d^3 e^7 x^7 + 15 d^2 e^8 x^8 - 10 d e^9 x^9 + 7 e^{10} x^{10})) + \\
& b^{10} (9 A e (532 d^{10} - 1456 d^9 e x - 3248 d^8 e^2 x^2 - 840 d^7 e^3 x^3 + 210 d^6 e^4 x^4 - 84 d^5 e^5 x^5 + 42 d^4 e^6 x^6 - 24 d^3 e^7 x^7 + 15 d^2 e^8 x^8 - 10 d e^9 x^9 + 7 e^{10} x^{10}) + \\
& B (-5292 d^{11} + 17136 d^{10} e x + 36288 d^9 e^2 x^2 + 9240 d^8 e^3 x^3 - 2310 d^7 e^4 x^4 + 924 d^6 e^5 x^5 - 462 d^5 e^6 x^6 + 264 d^4 e^7 x^7 - \\
& 165 d^3 e^8 x^8 + 110 d^2 e^9 x^9 - 77 d e^{10} x^{10} + 56 e^{11} x^{11})) - 2520 b (b d - a e)^8 (11 b B d - 9 A b e - 2 a B e) (d + e x)^2 \text{Log}[d + e x]
\end{aligned}$$

Problem 1098: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^{10} (A + B x)}{(d + e x)^{10}} dx$$

Optimal (type 3, 441 leaves, 2 steps):

$$\begin{aligned}
& - \frac{b^9 (10 b B d - A b e - 10 a B e) x}{e^{11}} + \frac{b^{10} B x^2}{2 e^{10}} + \frac{(b d - a e)^{10} (B d - A e)}{9 e^{12} (d + e x)^9} - \frac{(b d - a e)^9 (11 b B d - 10 A b e - a B e)}{8 e^{12} (d + e x)^8} + \\
& \frac{5 b (b d - a e)^8 (11 b B d - 9 A b e - 2 a B e)}{7 e^{12} (d + e x)^7} - \frac{5 b^2 (b d - a e)^7 (11 b B d - 8 A b e - 3 a B e)}{2 e^{12} (d + e x)^6} + \frac{6 b^3 (b d - a e)^6 (11 b B d - 7 A b e - 4 a B e)}{e^{12} (d + e x)^5} - \\
& \frac{21 b^4 (b d - a e)^5 (11 b B d - 6 A b e - 5 a B e)}{2 e^{12} (d + e x)^4} + \frac{14 b^5 (b d - a e)^4 (11 b B d - 5 A b e - 6 a B e)}{e^{12} (d + e x)^3} - \frac{15 b^6 (b d - a e)^3 (11 b B d - 4 A b e - 7 a B e)}{e^{12} (d + e x)^2} + \\
& \frac{15 b^7 (b d - a e)^2 (11 b B d - 3 A b e - 8 a B e)}{e^{12} (d + e x)} + \frac{5 b^8 (b d - a e) (11 b B d - 2 A b e - 9 a B e) \operatorname{Log}[d + e x]}{e^{12}}
\end{aligned}$$

Result (type 3, 1460 leaves):

$$\begin{aligned}
& - \frac{1}{504 e^{12} (d + e x)^9} (7 a^{10} e^{10} (8 A e + B (d + 9 e x)) + \\
& 10 a^9 b e^9 (7 A e (d + 9 e x) + 2 B (d^2 + 9 d e x + 36 e^2 x^2)) + 45 a^8 b^2 e^8 (2 A e (d^2 + 9 d e x + 36 e^2 x^2) + B (d^3 + 9 d^2 e x + 36 d e^2 x^2 + 84 e^3 x^3)) + \\
& 24 a^7 b^3 e^7 (5 A e (d^3 + 9 d^2 e x + 36 d e^2 x^2 + 84 e^3 x^3) + 4 B (d^4 + 9 d^3 e x + 36 d^2 e^2 x^2 + 84 d e^3 x^3 + 126 e^4 x^4)) + \\
& 42 a^6 b^4 e^6 (4 A e (d^4 + 9 d^3 e x + 36 d^2 e^2 x^2 + 84 d e^3 x^3 + 126 e^4 x^4) + 5 B (d^5 + 9 d^4 e x + 36 d^3 e^2 x^2 + 84 d^2 e^3 x^3 + 126 d e^4 x^4 + 126 e^5 x^5)) + \\
& 252 a^5 b^5 e^5 (A e (d^5 + 9 d^4 e x + 36 d^3 e^2 x^2 + 84 d^2 e^3 x^3 + 126 d e^4 x^4 + 126 e^5 x^5) + \\
& 2 B (d^6 + 9 d^5 e x + 36 d^4 e^2 x^2 + 84 d^3 e^3 x^3 + 126 d^2 e^4 x^4 + 126 d e^5 x^5 + 84 e^6 x^6)) + \\
& 210 a^4 b^6 e^4 (2 A e (d^6 + 9 d^5 e x + 36 d^4 e^2 x^2 + 84 d^3 e^3 x^3 + 126 d^2 e^4 x^4 + 126 d e^5 x^5 + 84 e^6 x^6) + \\
& 7 B (d^7 + 9 d^6 e x + 36 d^5 e^2 x^2 + 84 d^4 e^3 x^3 + 126 d^3 e^4 x^4 + 126 d^2 e^5 x^5 + 84 d e^6 x^6 + 36 e^7 x^7)) + \\
& 840 a^3 b^7 e^3 (A e (d^7 + 9 d^6 e x + 36 d^5 e^2 x^2 + 84 d^4 e^3 x^3 + 126 d^3 e^4 x^4 + 126 d^2 e^5 x^5 + 84 d e^6 x^6 + 36 e^7 x^7) + \\
& 8 B (d^8 + 9 d^7 e x + 36 d^6 e^2 x^2 + 84 d^5 e^3 x^3 + 126 d^4 e^4 x^4 + 126 d^3 e^5 x^5 + 84 d^2 e^6 x^6 + 36 d e^7 x^7 + 9 e^8 x^8)) - \\
& 9 a^2 b^8 e^2 (-280 A e (d^8 + 9 d^7 e x + 36 d^6 e^2 x^2 + 84 d^5 e^3 x^3 + 126 d^4 e^4 x^4 + 126 d^3 e^5 x^5 + 84 d^2 e^6 x^6 + 36 d e^7 x^7 + 9 e^8 x^8) + B d \\
& (7129 d^8 + 61 641 d^7 e x + 235 224 d^6 e^2 x^2 + 518 616 d^5 e^3 x^3 + 725 004 d^4 e^4 x^4 + 661 500 d^3 e^5 x^5 + 388 080 d^2 e^6 x^6 + 136 080 d e^7 x^7 + 22 680 e^8 x^8)) - \\
& 2 a b^9 e (A d e (7129 d^8 + 61 641 d^7 e x + 235 224 d^6 e^2 x^2 + 518 616 d^5 e^3 x^3 + 725 004 d^4 e^4 x^4 + 661 500 d^3 e^5 x^5 + 388 080 d^2 e^6 x^6 + \\
& 136 080 d e^7 x^7 + 22 680 e^8 x^8) - 10 B (4861 d^{10} + 41 229 d^9 e x + 153 576 d^8 e^2 x^2 + 328 104 d^7 e^3 x^3 + \\
& 439 236 d^6 e^4 x^4 + 375 732 d^5 e^5 x^5 + 197 568 d^4 e^6 x^6 + 54 432 d^3 e^7 x^7 + 2268 d^2 e^8 x^8 - 2268 d e^9 x^9 - 252 e^{10} x^{10})) - \\
& b^{10} (-2 A e (4861 d^{10} + 41 229 d^9 e x + 153 576 d^8 e^2 x^2 + 328 104 d^7 e^3 x^3 + 439 236 d^6 e^4 x^4 + 375 732 d^5 e^5 x^5 + 197 568 d^4 e^6 x^6 + \\
& 54 432 d^3 e^7 x^7 + 2268 d^2 e^8 x^8 - 2268 d e^9 x^9 - 252 e^{10} x^{10}) + B (42 131 d^{11} + 351 459 d^{10} e x + 1 281 096 d^9 e^2 x^2 + 2 656 584 d^8 e^3 x^3 + \\
& 3 402 756 d^7 e^4 x^4 + 2 704 212 d^6 e^5 x^5 + 1 220 688 d^5 e^6 x^6 + 190 512 d^4 e^7 x^7 - 77 112 d^3 e^8 x^8 - 36 288 d^2 e^9 x^9 - 2772 d e^{10} x^{10} + 252 e^{11} x^{11})) - \\
& 2520 b^8 (b d - a e) (11 b B d - 2 A b e - 9 a B e) (d + e x)^9 \operatorname{Log}[d + e x])
\end{aligned}$$

Problem 1099: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^{10} (A + B x)}{(d + e x)^{11}} dx$$

Optimal (type 3, 446 leaves, 2 steps):

$$\begin{aligned}
 & \frac{b^{10} B x}{e^{11}} + \frac{(b d - a e)^{10} (B d - A e)}{10 e^{12} (d + e x)^{10}} - \frac{(b d - a e)^9 (11 b B d - 10 A b e - a B e)}{9 e^{12} (d + e x)^9} + \frac{5 b (b d - a e)^8 (11 b B d - 9 A b e - 2 a B e)}{8 e^{12} (d + e x)^8} - \\
 & \frac{15 b^2 (b d - a e)^7 (11 b B d - 8 A b e - 3 a B e)}{7 e^{12} (d + e x)^7} + \frac{5 b^3 (b d - a e)^6 (11 b B d - 7 A b e - 4 a B e)}{e^{12} (d + e x)^6} - \frac{42 b^4 (b d - a e)^5 (11 b B d - 6 A b e - 5 a B e)}{5 e^{12} (d + e x)^5} + \\
 & \frac{21 b^5 (b d - a e)^4 (11 b B d - 5 A b e - 6 a B e)}{2 e^{12} (d + e x)^4} - \frac{10 b^6 (b d - a e)^3 (11 b B d - 4 A b e - 7 a B e)}{e^{12} (d + e x)^3} + \\
 & \frac{15 b^7 (b d - a e)^2 (11 b B d - 3 A b e - 8 a B e)}{2 e^{12} (d + e x)^2} - \frac{5 b^8 (b d - a e) (11 b B d - 2 A b e - 9 a B e)}{e^{12} (d + e x)} - \frac{b^9 (11 b B d - A b e - 10 a B e) \text{Log}[d + e x]}{e^{12}}
 \end{aligned}$$

Result (type 3, 1447 leaves):

$$\begin{aligned}
 & -\frac{1}{2520 e^{12} (d + e x)^{10}} \left( 28 a^{10} e^{10} (9 A e + B (d + 10 e x)) + 70 a^9 b e^9 (4 A e (d + 10 e x) + B (d^2 + 10 d e x + 45 e^2 x^2)) + \right. \\
 & 45 a^8 b^2 e^8 (7 A e (d^2 + 10 d e x + 45 e^2 x^2) + 3 B (d^3 + 10 d^2 e x + 45 d e^2 x^2 + 120 e^3 x^3)) + \\
 & 120 a^7 b^3 e^7 (3 A e (d^3 + 10 d^2 e x + 45 d e^2 x^2 + 120 e^3 x^3) + 2 B (d^4 + 10 d^3 e x + 45 d^2 e^2 x^2 + 120 d e^3 x^3 + 210 e^4 x^4)) + \\
 & 420 a^6 b^4 e^6 (A e (d^4 + 10 d^3 e x + 45 d^2 e^2 x^2 + 120 d e^3 x^3 + 210 e^4 x^4) + B (d^5 + 10 d^4 e x + 45 d^3 e^2 x^2 + 120 d^2 e^3 x^3 + 210 d e^4 x^4 + 252 e^5 x^5)) + \\
 & 252 a^5 b^5 e^5 (2 A e (d^5 + 10 d^4 e x + 45 d^3 e^2 x^2 + 120 d^2 e^3 x^3 + 210 d e^4 x^4 + 252 e^5 x^5)) + \\
 & 3 B (d^6 + 10 d^5 e x + 45 d^4 e^2 x^2 + 120 d^3 e^3 x^3 + 210 d^2 e^4 x^4 + 252 d e^5 x^5 + 210 e^6 x^6)) + \\
 & 210 a^4 b^6 e^4 (3 A e (d^6 + 10 d^5 e x + 45 d^4 e^2 x^2 + 120 d^3 e^3 x^3 + 210 d^2 e^4 x^4 + 252 d e^5 x^5 + 210 e^6 x^6)) + \\
 & 7 B (d^7 + 10 d^6 e x + 45 d^5 e^2 x^2 + 120 d^4 e^3 x^3 + 210 d^3 e^4 x^4 + 252 d^2 e^5 x^5 + 210 d e^6 x^6 + 120 e^7 x^7)) + \\
 & 840 a^3 b^7 e^3 (A e (d^7 + 10 d^6 e x + 45 d^5 e^2 x^2 + 120 d^4 e^3 x^3 + 210 d^3 e^4 x^4 + 252 d^2 e^5 x^5 + 210 d e^6 x^6 + 120 e^7 x^7)) + \\
 & 4 B (d^8 + 10 d^7 e x + 45 d^6 e^2 x^2 + 120 d^5 e^3 x^3 + 210 d^4 e^4 x^4 + 252 d^3 e^5 x^5 + 210 d^2 e^6 x^6 + 120 d e^7 x^7 + 45 e^8 x^8)) + \\
 & 1260 a^2 b^8 e^2 (A e (d^8 + 10 d^7 e x + 45 d^6 e^2 x^2 + 120 d^5 e^3 x^3 + 210 d^4 e^4 x^4 + 252 d^3 e^5 x^5 + 210 d^2 e^6 x^6 + 120 d e^7 x^7 + 45 e^8 x^8)) + \\
 & 9 B (d^9 + 10 d^8 e x + 45 d^7 e^2 x^2 + 120 d^6 e^3 x^3 + 210 d^5 e^4 x^4 + 252 d^4 e^5 x^5 + 210 d^3 e^6 x^6 + 120 d^2 e^7 x^7 + 45 d e^8 x^8 + 10 e^9 x^9)) - \\
 & 10 a b^9 e (-252 A e (d^9 + 10 d^8 e x + 45 d^7 e^2 x^2 + 120 d^6 e^3 x^3 + 210 d^5 e^4 x^4 + 252 d^4 e^5 x^5 + 210 d^3 e^6 x^6 + 120 d^2 e^7 x^7 + 45 d e^8 x^8 + 10 e^9 x^9)) + \\
 & B d (7381 d^9 + 71290 d^8 e x + 308205 d^7 e^2 x^2 + 784080 d^6 e^3 x^3 + 1296540 d^5 e^4 x^4 + 1450008 d^4 e^5 x^5 + 1102500 d^3 e^6 x^6 + \\
 & 554400 d^2 e^7 x^7 + 170100 d e^8 x^8 + 25200 e^9 x^9)) - b^{10} (A d e (7381 d^9 + 71290 d^8 e x + 308205 d^7 e^2 x^2 + 784080 d^6 e^3 x^3 + \\
 & 1296540 d^5 e^4 x^4 + 1450008 d^4 e^5 x^5 + 1102500 d^3 e^6 x^6 + 554400 d^2 e^7 x^7 + 170100 d e^8 x^8 + 25200 e^9 x^9)) - \\
 & B (55991 d^{11} + 532190 d^{10} e x + 2256255 d^9 e^2 x^2 + 5600880 d^8 e^3 x^3 + 8969940 d^7 e^4 x^4 + 9599688 d^6 e^5 x^5 + 6835500 d^5 e^6 x^6 + 3074400 d^4 e^7 x^7 + \\
 & 737100 d^3 e^8 x^8 + 25200 d^2 e^9 x^9 - 25200 d e^{10} x^{10} - 2520 e^{11} x^{11})) + 2520 b^9 (11 b B d - A b e - 10 a B e) (d + e x)^{10} \text{Log}[d + e x]
 \end{aligned}$$

**Problem 1100: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b x)^{10} (A + B x)}{(d + e x)^{12}} dx$$

Optimal (type 3, 321 leaves, 3 steps):

$$\begin{aligned}
 & -\frac{(B d - A e) (a + b x)^{11}}{11 e (b d - a e) (d + e x)^{11}} - \frac{B (b d - a e)^{10}}{10 e^{12} (d + e x)^{10}} + \frac{10 b B (b d - a e)^9}{9 e^{12} (d + e x)^9} - \frac{45 b^2 B (b d - a e)^8}{8 e^{12} (d + e x)^8} + \frac{120 b^3 B (b d - a e)^7}{7 e^{12} (d + e x)^7} - \frac{35 b^4 B (b d - a e)^6}{e^{12} (d + e x)^6} + \\
 & \frac{252 b^5 B (b d - a e)^5}{5 e^{12} (d + e x)^5} - \frac{105 b^6 B (b d - a e)^4}{2 e^{12} (d + e x)^4} + \frac{40 b^7 B (b d - a e)^3}{e^{12} (d + e x)^3} - \frac{45 b^8 B (b d - a e)^2}{2 e^{12} (d + e x)^2} + \frac{10 b^9 B (b d - a e)}{e^{12} (d + e x)} + \frac{b^{10} B \log[d + e x]}{e^{12}}
 \end{aligned}$$

Result (type 3, 1443 leaves):

$$\begin{aligned}
 & -\frac{1}{27720 e^{12} (d + e x)^{11}} \left( 252 a^{10} e^{10} (10 A e + B (d + 11 e x)) + 280 a^9 b e^9 (9 A e (d + 11 e x) + 2 B (d^2 + 11 d e x + 55 e^2 x^2)) + \right. \\
 & 315 a^8 b^2 e^8 (8 A e (d^2 + 11 d e x + 55 e^2 x^2) + 3 B (d^3 + 11 d^2 e x + 55 d e^2 x^2 + 165 e^3 x^3)) + \\
 & 360 a^7 b^3 e^7 (7 A e (d^3 + 11 d^2 e x + 55 d e^2 x^2 + 165 e^3 x^3) + 4 B (d^4 + 11 d^3 e x + 55 d^2 e^2 x^2 + 165 d e^3 x^3 + 330 e^4 x^4)) + \\
 & 420 a^6 b^4 e^6 (6 A e (d^4 + 11 d^3 e x + 55 d^2 e^2 x^2 + 165 d e^3 x^3 + 330 e^4 x^4) + 5 B (d^5 + 11 d^4 e x + 55 d^3 e^2 x^2 + 165 d^2 e^3 x^3 + 330 d e^4 x^4 + 462 e^5 x^5)) + \\
 & 504 a^5 b^5 e^5 (5 A e (d^5 + 11 d^4 e x + 55 d^3 e^2 x^2 + 165 d^2 e^3 x^3 + 330 d e^4 x^4 + 462 e^5 x^5)) + \\
 & 6 B (d^6 + 11 d^5 e x + 55 d^4 e^2 x^2 + 165 d^3 e^3 x^3 + 330 d^2 e^4 x^4 + 462 d e^5 x^5 + 462 e^6 x^6)) + \\
 & 630 a^4 b^6 e^4 (4 A e (d^6 + 11 d^5 e x + 55 d^4 e^2 x^2 + 165 d^3 e^3 x^3 + 330 d^2 e^4 x^4 + 462 d e^5 x^5 + 462 e^6 x^6)) + \\
 & 7 B (d^7 + 11 d^6 e x + 55 d^5 e^2 x^2 + 165 d^4 e^3 x^3 + 330 d^3 e^4 x^4 + 462 d^2 e^5 x^5 + 462 d e^6 x^6 + 330 e^7 x^7)) + \\
 & 840 a^3 b^7 e^3 (3 A e (d^7 + 11 d^6 e x + 55 d^5 e^2 x^2 + 165 d^4 e^3 x^3 + 330 d^3 e^4 x^4 + 462 d^2 e^5 x^5 + 462 d e^6 x^6 + 330 e^7 x^7)) + \\
 & 8 B (d^8 + 11 d^7 e x + 55 d^6 e^2 x^2 + 165 d^5 e^3 x^3 + 330 d^4 e^4 x^4 + 462 d^3 e^5 x^5 + 462 d^2 e^6 x^6 + 330 d e^7 x^7 + 165 e^8 x^8)) + \\
 & 1260 a^2 b^8 e^2 (2 A e (d^8 + 11 d^7 e x + 55 d^6 e^2 x^2 + 165 d^5 e^3 x^3 + 330 d^4 e^4 x^4 + 462 d^3 e^5 x^5 + 462 d^2 e^6 x^6 + 330 d e^7 x^7 + 165 e^8 x^8)) + \\
 & 9 B (d^9 + 11 d^8 e x + 55 d^7 e^2 x^2 + 165 d^6 e^3 x^3 + 330 d^5 e^4 x^4 + 462 d^4 e^5 x^5 + 462 d^3 e^6 x^6 + 330 d^2 e^7 x^7 + 165 d e^8 x^8 + 55 e^9 x^9)) + \\
 & 2520 a b^9 e (A e (d^9 + 11 d^8 e x + 55 d^7 e^2 x^2 + 165 d^6 e^3 x^3 + 330 d^5 e^4 x^4 + 462 d^4 e^5 x^5 + 462 d^3 e^6 x^6 + 330 d^2 e^7 x^7 + 165 d e^8 x^8 + 55 e^9 x^9)) + \\
 & 10 B (d^{10} + 11 d^9 e x + 55 d^8 e^2 x^2 + 165 d^7 e^3 x^3 + 330 d^6 e^4 x^4 + 462 d^5 e^5 x^5 + 462 d^4 e^6 x^6 + 330 d^3 e^7 x^7 + 165 d^2 e^8 x^8 + 55 d e^9 x^9 + 11 e^{10} x^{10})) + \\
 & b^{10} (2520 A e (d^{10} + 11 d^9 e x + 55 d^8 e^2 x^2 + 165 d^7 e^3 x^3 + 330 d^6 e^4 x^4 + 462 d^5 e^5 x^5 + 462 d^4 e^6 x^6 + 330 d^3 e^7 x^7 + 165 d^2 e^8 x^8 + 55 d e^9 x^9 + 11 e^{10} x^{10})) - \\
 & B d (83711 d^{10} + 893101 d^9 e x + 4313045 d^8 e^2 x^2 + 12430935 d^7 e^3 x^3 + 23718420 d^6 e^4 x^4 + 31376268 d^5 e^5 x^5 + 29241828 d^4 e^6 x^6 + \\
 & 19057500 d^3 e^7 x^7 + 8385300 d^2 e^8 x^8 + 2286900 d e^9 x^9 + 304920 e^{10} x^{10}) - 27720 b^{10} B (d + e x)^{11} \log[d + e x] \Big)
 \end{aligned}$$

Problem 1101: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^{10} (A + B x)}{(d + e x)^{13}} dx$$

Optimal (type 1, 86 leaves, 2 steps):

$$\begin{aligned}
 & -\frac{(B d - A e) (a + b x)^{11}}{12 e (b d - a e) (d + e x)^{12}} + \frac{(11 b B d + A b e - 12 a B e) (a + b x)^{11}}{132 e (b d - a e)^2 (d + e x)^{11}}
 \end{aligned}$$

Result (type 1, 1421 leaves):

$$\begin{aligned}
& - \frac{1}{132 e^{12} (d + e x)^{12}} (a^{10} e^{10} (11 A e + B (d + 12 e x)) + \\
& 2 a^9 b e^9 (5 A e (d + 12 e x) + B (d^2 + 12 d e x + 66 e^2 x^2)) + 3 a^8 b^2 e^8 (3 A e (d^2 + 12 d e x + 66 e^2 x^2) + B (d^3 + 12 d^2 e x + 66 d e^2 x^2 + 220 e^3 x^3)) + \\
& 4 a^7 b^3 e^7 (2 A e (d^3 + 12 d^2 e x + 66 d e^2 x^2 + 220 e^3 x^3) + B (d^4 + 12 d^3 e x + 66 d^2 e^2 x^2 + 220 d e^3 x^3 + 495 e^4 x^4)) + \\
& a^6 b^4 e^6 (7 A e (d^4 + 12 d^3 e x + 66 d^2 e^2 x^2 + 220 d e^3 x^3 + 495 e^4 x^4) + 5 B (d^5 + 12 d^4 e x + 66 d^3 e^2 x^2 + 220 d^2 e^3 x^3 + 495 d e^4 x^4 + 792 e^5 x^5)) + \\
& 6 a^5 b^5 e^5 (A e (d^5 + 12 d^4 e x + 66 d^3 e^2 x^2 + 220 d^2 e^3 x^3 + 495 d e^4 x^4 + 792 e^5 x^5) + \\
& B (d^6 + 12 d^5 e x + 66 d^4 e^2 x^2 + 220 d^3 e^3 x^3 + 495 d^2 e^4 x^4 + 792 d e^5 x^5 + 924 e^6 x^6)) + \\
& a^4 b^6 e^4 (5 A e (d^6 + 12 d^5 e x + 66 d^4 e^2 x^2 + 220 d^3 e^3 x^3 + 495 d^2 e^4 x^4 + 792 d e^5 x^5 + 924 e^6 x^6)) + \\
& 7 B (d^7 + 12 d^6 e x + 66 d^5 e^2 x^2 + 220 d^4 e^3 x^3 + 495 d^3 e^4 x^4 + 792 d^2 e^5 x^5 + 924 d e^6 x^6 + 792 e^7 x^7)) + \\
& 4 a^3 b^7 e^3 (A e (d^7 + 12 d^6 e x + 66 d^5 e^2 x^2 + 220 d^4 e^3 x^3 + 495 d^3 e^4 x^4 + 792 d^2 e^5 x^5 + 924 d e^6 x^6 + 792 e^7 x^7)) + \\
& 2 B (d^8 + 12 d^7 e x + 66 d^6 e^2 x^2 + 220 d^5 e^3 x^3 + 495 d^4 e^4 x^4 + 792 d^3 e^5 x^5 + 924 d^2 e^6 x^6 + 792 d e^7 x^7 + 495 e^8 x^8)) + \\
& 3 a^2 b^8 e^2 (A e (d^8 + 12 d^7 e x + 66 d^6 e^2 x^2 + 220 d^5 e^3 x^3 + 495 d^4 e^4 x^4 + 792 d^3 e^5 x^5 + 924 d^2 e^6 x^6 + 792 d e^7 x^7 + 495 e^8 x^8)) + \\
& 3 B (d^9 + 12 d^8 e x + 66 d^7 e^2 x^2 + 220 d^6 e^3 x^3 + 495 d^5 e^4 x^4 + 792 d^4 e^5 x^5 + 924 d^3 e^6 x^6 + 792 d^2 e^7 x^7 + 495 d e^8 x^8 + 220 e^9 x^9)) + \\
& 2 a b^9 e (A e (d^9 + 12 d^8 e x + 66 d^7 e^2 x^2 + 220 d^6 e^3 x^3 + 495 d^5 e^4 x^4 + 792 d^4 e^5 x^5 + 924 d^3 e^6 x^6 + 792 d^2 e^7 x^7 + 495 d e^8 x^8 + 220 e^9 x^9)) + \\
& 5 B (d^{10} + 12 d^9 e x + 66 d^8 e^2 x^2 + 220 d^7 e^3 x^3 + 495 d^6 e^4 x^4 + 792 d^5 e^5 x^5 + 924 d^4 e^6 x^6 + 792 d^3 e^7 x^7 + 495 d^2 e^8 x^8 + 220 d e^9 x^9 + 66 e^{10} x^{10})) + \\
& b^{10} (A e (d^{10} + 12 d^9 e x + 66 d^8 e^2 x^2 + 220 d^7 e^3 x^3 + 495 d^6 e^4 x^4 + 792 d^5 e^5 x^5 + 924 d^4 e^6 x^6 + 792 d^3 e^7 x^7 + 495 d^2 e^8 x^8 + 220 d e^9 x^9 + 66 e^{10} x^{10})) + \\
& 11 B (d^{11} + 12 d^{10} e x + 66 d^9 e^2 x^2 + 220 d^8 e^3 x^3 + 495 d^7 e^4 x^4 + 792 d^6 e^5 x^5 + \\
& 924 d^5 e^6 x^6 + 792 d^4 e^7 x^7 + 495 d^3 e^8 x^8 + 220 d^2 e^9 x^9 + 66 d e^{10} x^{10} + 12 e^{11} x^{11})) )
\end{aligned}$$

**Problem 1102: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b x)^{10} (A + B x)}{(d + e x)^{14}} dx$$

Optimal (type 1, 135 leaves, 3 steps):

$$-\frac{(B d - A e) (a + b x)^{11}}{13 e (b d - a e) (d + e x)^{13}} + \frac{(11 b B d + 2 A b e - 13 a B e) (a + b x)^{11}}{156 e (b d - a e)^2 (d + e x)^{12}} + \frac{b (11 b B d + 2 A b e - 13 a B e) (a + b x)^{11}}{1716 e (b d - a e)^3 (d + e x)^{11}}$$

Result (type 1, 1433 leaves):

$$\begin{aligned}
& - \frac{1}{1716 e^{12} (d + e x)^{13}} (11 a^{10} e^{10} (12 A e + B (d + 13 e x)) + 10 a^9 b e^9 (11 A e (d + 13 e x) + 2 B (d^2 + 13 d e x + 78 e^2 x^2)) + \\
& \quad 9 a^8 b^2 e^8 (10 A e (d^2 + 13 d e x + 78 e^2 x^2) + 3 B (d^3 + 13 d^2 e x + 78 d e^2 x^2 + 286 e^3 x^3)) + \\
& \quad 8 a^7 b^3 e^7 (9 A e (d^3 + 13 d^2 e x + 78 d e^2 x^2 + 286 e^3 x^3) + 4 B (d^4 + 13 d^3 e x + 78 d^2 e^2 x^2 + 286 d e^3 x^3 + 715 e^4 x^4)) + \\
& \quad 7 a^6 b^4 e^6 (8 A e (d^4 + 13 d^3 e x + 78 d^2 e^2 x^2 + 286 d e^3 x^3 + 715 e^4 x^4) + 5 B (d^5 + 13 d^4 e x + 78 d^3 e^2 x^2 + 286 d^2 e^3 x^3 + 715 d e^4 x^4 + 1287 e^5 x^5)) + \\
& \quad 6 a^5 b^5 e^5 (7 A e (d^5 + 13 d^4 e x + 78 d^3 e^2 x^2 + 286 d^2 e^3 x^3 + 715 d e^4 x^4 + 1287 e^5 x^5)) + \\
& \quad 6 B (d^6 + 13 d^5 e x + 78 d^4 e^2 x^2 + 286 d^3 e^3 x^3 + 715 d^2 e^4 x^4 + 1287 d e^5 x^5 + 1716 e^6 x^6)) + \\
& \quad 5 a^4 b^6 e^4 (6 A e (d^6 + 13 d^5 e x + 78 d^4 e^2 x^2 + 286 d^3 e^3 x^3 + 715 d^2 e^4 x^4 + 1287 d e^5 x^5 + 1716 e^6 x^6)) + \\
& \quad 7 B (d^7 + 13 d^6 e x + 78 d^5 e^2 x^2 + 286 d^4 e^3 x^3 + 715 d^3 e^4 x^4 + 1287 d^2 e^5 x^5 + 1716 d e^6 x^6 + 1716 e^7 x^7)) + \\
& \quad 4 a^3 b^7 e^3 (5 A e (d^7 + 13 d^6 e x + 78 d^5 e^2 x^2 + 286 d^4 e^3 x^3 + 715 d^3 e^4 x^4 + 1287 d^2 e^5 x^5 + 1716 d e^6 x^6 + 1716 e^7 x^7)) + \\
& \quad 8 B (d^8 + 13 d^7 e x + 78 d^6 e^2 x^2 + 286 d^5 e^3 x^3 + 715 d^4 e^4 x^4 + 1287 d^3 e^5 x^5 + 1716 d^2 e^6 x^6 + 1716 d e^7 x^7 + 1287 e^8 x^8)) + \\
& \quad 3 a^2 b^8 e^2 (4 A e (d^8 + 13 d^7 e x + 78 d^6 e^2 x^2 + 286 d^5 e^3 x^3 + 715 d^4 e^4 x^4 + 1287 d^3 e^5 x^5 + 1716 d^2 e^6 x^6 + 1716 d e^7 x^7 + 1287 e^8 x^8)) + \\
& \quad 9 B (d^9 + 13 d^8 e x + 78 d^7 e^2 x^2 + 286 d^6 e^3 x^3 + 715 d^5 e^4 x^4 + 1287 d^4 e^5 x^5 + 1716 d^3 e^6 x^6 + 1716 d^2 e^7 x^7 + 1287 d e^8 x^8 + 715 e^9 x^9)) + \\
& \quad 2 a b^9 e (3 A e (d^9 + 13 d^8 e x + 78 d^7 e^2 x^2 + 286 d^6 e^3 x^3 + 715 d^5 e^4 x^4 + 1287 d^4 e^5 x^5 + 1716 d^3 e^6 x^6 + 1716 d^2 e^7 x^7 + 1287 d e^8 x^8 + 715 e^9 x^9) + 10 \\
& \quad B (d^{10} + 13 d^9 e x + 78 d^8 e^2 x^2 + 286 d^7 e^3 x^3 + 715 d^6 e^4 x^4 + 1287 d^5 e^5 x^5 + 1716 d^4 e^6 x^6 + 1716 d^3 e^7 x^7 + 1287 d^2 e^8 x^8 + 715 d e^9 x^9 + 286 e^{10} x^{10})) + \\
& \quad b^{10} (2 A e (d^{10} + 13 d^9 e x + 78 d^8 e^2 x^2 + 286 d^7 e^3 x^3 + 715 d^6 e^4 x^4 + 1287 d^5 e^5 x^5 + 1716 d^4 e^6 x^6 + 1716 d^3 e^7 x^7 + 1287 d^2 e^8 x^8 + \\
& \quad 715 d e^9 x^9 + 286 e^{10} x^{10}) + 11 B (d^{11} + 13 d^{10} e x + 78 d^9 e^2 x^2 + 286 d^8 e^3 x^3 + 715 d^7 e^4 x^4 + \\
& \quad 1287 d^6 e^5 x^5 + 1716 d^5 e^6 x^6 + 1716 d^4 e^7 x^7 + 1287 d^3 e^8 x^8 + 715 d^2 e^9 x^9 + 286 d e^{10} x^{10} + 78 e^{11} x^{11})) )
\end{aligned}$$

**Problem 1103: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b x)^{10} (A + B x)}{(d + e x)^{15}} dx$$

Optimal (type 1, 185 leaves, 4 steps):

$$\begin{aligned}
& - \frac{(B d - A e) (a + b x)^{11}}{14 e (b d - a e) (d + e x)^{14}} + \frac{(11 b B d + 3 A b e - 14 a B e) (a + b x)^{11}}{182 e (b d - a e)^2 (d + e x)^{13}} + \\
& \frac{b (11 b B d + 3 A b e - 14 a B e) (a + b x)^{11}}{1092 e (b d - a e)^3 (d + e x)^{12}} + \frac{b^2 (11 b B d + 3 A b e - 14 a B e) (a + b x)^{11}}{12012 e (b d - a e)^4 (d + e x)^{11}}
\end{aligned}$$

Result (type 1, 1430 leaves):

$$\begin{aligned}
& - \frac{1}{12012 e^{12} (d + e x)^{14}} (66 a^{10} e^{10} (13 A e + B (d + 14 e x)) + 110 a^9 b e^9 (6 A e (d + 14 e x) + B (d^2 + 14 d e x + 91 e^2 x^2)) + \\
& 45 a^8 b^2 e^8 (11 A e (d^2 + 14 d e x + 91 e^2 x^2) + 3 B (d^3 + 14 d^2 e x + 91 d e^2 x^2 + 364 e^3 x^3)) + \\
& 72 a^7 b^3 e^7 (5 A e (d^3 + 14 d^2 e x + 91 d e^2 x^2 + 364 e^3 x^3) + 2 B (d^4 + 14 d^3 e x + 91 d^2 e^2 x^2 + 364 d e^3 x^3 + 1001 e^4 x^4)) + \\
& 28 a^6 b^4 e^6 (9 A e (d^4 + 14 d^3 e x + 91 d^2 e^2 x^2 + 364 d e^3 x^3 + 1001 e^4 x^4) + 5 B (d^5 + 14 d^4 e x + 91 d^3 e^2 x^2 + 364 d^2 e^3 x^3 + 1001 d e^4 x^4 + 2002 e^5 x^5)) + \\
& 42 a^5 b^5 e^5 (4 A e (d^5 + 14 d^4 e x + 91 d^3 e^2 x^2 + 364 d^2 e^3 x^3 + 1001 d e^4 x^4 + 2002 e^5 x^5)) + \\
& 3 B (d^6 + 14 d^5 e x + 91 d^4 e^2 x^2 + 364 d^3 e^3 x^3 + 1001 d^2 e^4 x^4 + 2002 d e^5 x^5 + 3003 e^6 x^6) + \\
& 105 a^4 b^6 e^4 (A e (d^6 + 14 d^5 e x + 91 d^4 e^2 x^2 + 364 d^3 e^3 x^3 + 1001 d^2 e^4 x^4 + 2002 d e^5 x^5 + 3003 e^6 x^6) + \\
& B (d^7 + 14 d^6 e x + 91 d^5 e^2 x^2 + 364 d^4 e^3 x^3 + 1001 d^3 e^4 x^4 + 2002 d^2 e^5 x^5 + 3003 d e^6 x^6 + 3432 e^7 x^7)) + \\
& 20 a^3 b^7 e^3 (3 A e (d^7 + 14 d^6 e x + 91 d^5 e^2 x^2 + 364 d^4 e^3 x^3 + 1001 d^3 e^4 x^4 + 2002 d^2 e^5 x^5 + 3003 d e^6 x^6 + 3432 e^7 x^7) + \\
& 4 B (d^8 + 14 d^7 e x + 91 d^6 e^2 x^2 + 364 d^5 e^3 x^3 + 1001 d^4 e^4 x^4 + 2002 d^3 e^5 x^5 + 3003 d^2 e^6 x^6 + 3432 d e^7 x^7 + 3003 e^8 x^8)) + \\
& 6 a^2 b^8 e^2 (5 A e (d^8 + 14 d^7 e x + 91 d^6 e^2 x^2 + 364 d^5 e^3 x^3 + 1001 d^4 e^4 x^4 + 2002 d^3 e^5 x^5 + 3003 d^2 e^6 x^6 + 3432 d e^7 x^7 + 3003 e^8 x^8) + \\
& 9 B (d^9 + 14 d^8 e x + 91 d^7 e^2 x^2 + 364 d^6 e^3 x^3 + 1001 d^5 e^4 x^4 + 2002 d^4 e^5 x^5 + 3003 d^3 e^6 x^6 + 3432 d^2 e^7 x^7 + 3003 d e^8 x^8 + 2002 e^9 x^9) + 6 a b^9 e \\
& (2 A e (d^9 + 14 d^8 e x + 91 d^7 e^2 x^2 + 364 d^6 e^3 x^3 + 1001 d^5 e^4 x^4 + 2002 d^4 e^5 x^5 + 3003 d^3 e^6 x^6 + 3432 d^2 e^7 x^7 + 3003 d e^8 x^8 + 2002 e^9 x^9) + 5 B (d^{10} + \\
& 14 d^9 e x + 91 d^8 e^2 x^2 + 364 d^7 e^3 x^3 + 1001 d^6 e^4 x^4 + 2002 d^5 e^5 x^5 + 3003 d^4 e^6 x^6 + 3432 d^3 e^7 x^7 + 3003 d^2 e^8 x^8 + 2002 d e^9 x^9 + 1001 e^{10} x^{10})) + \\
& b^{10} (3 A e (d^{10} + 14 d^9 e x + 91 d^8 e^2 x^2 + 364 d^7 e^3 x^3 + 1001 d^6 e^4 x^4 + 2002 d^5 e^5 x^5 + 3003 d^4 e^6 x^6 + 3432 d^3 e^7 x^7 + 3003 d^2 e^8 x^8 + \\
& 2002 d e^9 x^9 + 1001 e^{10} x^{10}) + 11 B (d^{11} + 14 d^{10} e x + 91 d^9 e^2 x^2 + 364 d^8 e^3 x^3 + 1001 d^7 e^4 x^4 + \\
& 2002 d^6 e^5 x^5 + 3003 d^5 e^6 x^6 + 3432 d^4 e^7 x^7 + 3003 d^3 e^8 x^8 + 2002 d^2 e^9 x^9 + 1001 d e^{10} x^{10} + 364 e^{11} x^{11})) )
\end{aligned}$$

**Problem 1104:** Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^{10} (A + B x)}{(d + e x)^{16}} dx$$

Optimal (type 1, 235 leaves, 5 steps):

$$\begin{aligned}
& - \frac{(B d - A e) (a + b x)^{11}}{15 e (b d - a e) (d + e x)^{15}} + \frac{(11 b B d + 4 A b e - 15 a B e) (a + b x)^{11}}{210 e (b d - a e)^2 (d + e x)^{14}} + \\
& \frac{b (11 b B d + 4 A b e - 15 a B e) (a + b x)^{11}}{910 e (b d - a e)^3 (d + e x)^{13}} + \frac{b^2 (11 b B d + 4 A b e - 15 a B e) (a + b x)^{11}}{5460 e (b d - a e)^4 (d + e x)^{12}} + \frac{b^3 (11 b B d + 4 A b e - 15 a B e) (a + b x)^{11}}{60060 e (b d - a e)^5 (d + e x)^{11}}
\end{aligned}$$

Result (type 1, 1430 leaves):

$$\begin{aligned}
& - \frac{1}{60060 e^{12} (d + e x)^{15}} (286 a^{10} e^{10} (14 A e + B (d + 15 e x)) + 220 a^9 b e^9 (13 A e (d + 15 e x) + 2 B (d^2 + 15 d e x + 105 e^2 x^2)) + \\
& 495 a^8 b^2 e^8 (4 A e (d^2 + 15 d e x + 105 e^2 x^2) + B (d^3 + 15 d^2 e x + 105 d e^2 x^2 + 455 e^3 x^3)) + \\
& 120 a^7 b^3 e^7 (11 A e (d^3 + 15 d^2 e x + 105 d e^2 x^2 + 455 e^3 x^3) + 4 B (d^4 + 15 d^3 e x + 105 d^2 e^2 x^2 + 455 d e^3 x^3 + 1365 e^4 x^4)) + \\
& 420 a^6 b^4 e^6 (2 A e (d^4 + 15 d^3 e x + 105 d^2 e^2 x^2 + 455 d e^3 x^3 + 1365 e^4 x^4) + B (d^5 + 15 d^4 e x + 105 d^3 e^2 x^2 + 455 d^2 e^3 x^3 + 1365 d e^4 x^4 + 3003 e^5 x^5)) + \\
& 168 a^5 b^5 e^5 (3 A e (d^5 + 15 d^4 e x + 105 d^3 e^2 x^2 + 455 d^2 e^3 x^3 + 1365 d e^4 x^4 + 3003 e^5 x^5)) + \\
& 2 B (d^6 + 15 d^5 e x + 105 d^4 e^2 x^2 + 455 d^3 e^3 x^3 + 1365 d^2 e^4 x^4 + 3003 d e^5 x^5 + 5005 e^6 x^6)) + \\
& 35 a^4 b^6 e^4 (8 A e (d^6 + 15 d^5 e x + 105 d^4 e^2 x^2 + 455 d^3 e^3 x^3 + 1365 d^2 e^4 x^4 + 3003 d e^5 x^5 + 5005 e^6 x^6)) + \\
& 7 B (d^7 + 15 d^6 e x + 105 d^5 e^2 x^2 + 455 d^4 e^3 x^3 + 1365 d^3 e^4 x^4 + 3003 d^2 e^5 x^5 + 5005 d e^6 x^6 + 6435 e^7 x^7)) + \\
& 20 a^3 b^7 e^3 (7 A e (d^7 + 15 d^6 e x + 105 d^5 e^2 x^2 + 455 d^4 e^3 x^3 + 1365 d^3 e^4 x^4 + 3003 d^2 e^5 x^5 + 5005 d e^6 x^6 + 6435 e^7 x^7)) + \\
& 8 B (d^8 + 15 d^7 e x + 105 d^6 e^2 x^2 + 455 d^5 e^3 x^3 + 1365 d^4 e^4 x^4 + 3003 d^3 e^5 x^5 + 5005 d^2 e^6 x^6 + 6435 d e^7 x^7 + 6435 e^8 x^8)) + \\
& 30 a^2 b^8 e^2 (2 A e (d^8 + 15 d^7 e x + 105 d^6 e^2 x^2 + 455 d^5 e^3 x^3 + 1365 d^4 e^4 x^4 + 3003 d^3 e^5 x^5 + 5005 d^2 e^6 x^6 + 6435 d e^7 x^7 + 6435 e^8 x^8)) + \\
& 3 B (d^9 + 15 d^8 e x + 105 d^7 e^2 x^2 + 455 d^6 e^3 x^3 + 1365 d^5 e^4 x^4 + 3003 d^4 e^5 x^5 + 5005 d^3 e^6 x^6 + 6435 d^2 e^7 x^7 + 6435 d e^8 x^8 + 5005 e^9 x^9) + 20 a b^9 e \\
& (A e (d^9 + 15 d^8 e x + 105 d^7 e^2 x^2 + 455 d^6 e^3 x^3 + 1365 d^5 e^4 x^4 + 3003 d^4 e^5 x^5 + 5005 d^3 e^6 x^6 + 6435 d^2 e^7 x^7 + 6435 d e^8 x^8 + 5005 e^9 x^9) + 2 B (d^{10} + \\
& 15 d^9 e x + 105 d^8 e^2 x^2 + 455 d^7 e^3 x^3 + 1365 d^6 e^4 x^4 + 3003 d^5 e^5 x^5 + 5005 d^4 e^6 x^6 + 6435 d^3 e^7 x^7 + 6435 d^2 e^8 x^8 + 5005 d e^9 x^9 + 3003 e^{10} x^{10})) + \\
& b^{10} (4 A e (d^{10} + 15 d^9 e x + 105 d^8 e^2 x^2 + 455 d^7 e^3 x^3 + 1365 d^6 e^4 x^4 + 3003 d^5 e^5 x^5 + 5005 d^4 e^6 x^6 + 6435 d^3 e^7 x^7 + 6435 d^2 e^8 x^8 + \\
& 5005 d e^9 x^9 + 3003 e^{10} x^{10}) + 11 B (d^{11} + 15 d^{10} e x + 105 d^9 e^2 x^2 + 455 d^8 e^3 x^3 + 1365 d^7 e^4 x^4 + \\
& 3003 d^6 e^5 x^5 + 5005 d^5 e^6 x^6 + 6435 d^4 e^7 x^7 + 6435 d^3 e^8 x^8 + 5005 d^2 e^9 x^9 + 3003 d e^{10} x^{10} + 1365 e^{11} x^{11})) )
\end{aligned}$$

**Problem 1105: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b x)^{10} (A + B x)}{(d + e x)^{17}} dx$$

Optimal (type 1, 285 leaves, 6 steps):

$$\begin{aligned}
& - \frac{(B d - A e) (a + b x)^{11}}{16 e (b d - a e) (d + e x)^{16}} + \frac{(11 b B d + 5 A b e - 16 a B e) (a + b x)^{11}}{240 e (b d - a e)^2 (d + e x)^{15}} + \frac{b (11 b B d + 5 A b e - 16 a B e) (a + b x)^{11}}{840 e (b d - a e)^3 (d + e x)^{14}} + \\
& \frac{b^2 (11 b B d + 5 A b e - 16 a B e) (a + b x)^{11}}{3640 e (b d - a e)^4 (d + e x)^{13}} + \frac{b^3 (11 b B d + 5 A b e - 16 a B e) (a + b x)^{11}}{21840 e (b d - a e)^5 (d + e x)^{12}} + \frac{b^4 (11 b B d + 5 A b e - 16 a B e) (a + b x)^{11}}{240240 e (b d - a e)^6 (d + e x)^{11}}
\end{aligned}$$

Result (type 1, 1429 leaves):

$$\begin{aligned}
& - \frac{1}{240240 e^{12} (d + e x)^{16}} \left( 1001 a^{10} e^{10} (15 A e + B (d + 16 e x)) + 1430 a^9 b e^9 (7 A e (d + 16 e x) + B (d^2 + 16 d e x + 120 e^2 x^2)) + \right. \\
& \quad 495 a^8 b^2 e^8 (13 A e (d^2 + 16 d e x + 120 e^2 x^2) + 3 B (d^3 + 16 d^2 e x + 120 d e^2 x^2 + 560 e^3 x^3)) + \\
& \quad 1320 a^7 b^3 e^7 (3 A e (d^3 + 16 d^2 e x + 120 d e^2 x^2 + 560 e^3 x^3) + B (d^4 + 16 d^3 e x + 120 d^2 e^2 x^2 + 560 d e^3 x^3 + 1820 e^4 x^4)) + \\
& \quad 210 a^6 b^4 e^6 (11 A e (d^4 + 16 d^3 e x + 120 d^2 e^2 x^2 + 560 d e^3 x^3 + 1820 e^4 x^4) + 5 B (d^5 + 16 d^4 e x + 120 d^3 e^2 x^2 + 560 d^2 e^3 x^3 + 1820 d e^4 x^4 + 4368 e^5 x^5)) + \\
& \quad 252 a^5 b^5 e^5 (5 A e (d^5 + 16 d^4 e x + 120 d^3 e^2 x^2 + 560 d^2 e^3 x^3 + 1820 d e^4 x^4 + 4368 e^5 x^5)) + \\
& \quad 3 B (d^6 + 16 d^5 e x + 120 d^4 e^2 x^2 + 560 d^3 e^3 x^3 + 1820 d^2 e^4 x^4 + 4368 d e^5 x^5 + 8008 e^6 x^6)) + \\
& \quad 70 a^4 b^6 e^4 (9 A e (d^6 + 16 d^5 e x + 120 d^4 e^2 x^2 + 560 d^3 e^3 x^3 + 1820 d^2 e^4 x^4 + 4368 d e^5 x^5 + 8008 e^6 x^6)) + \\
& \quad 7 B (d^7 + 16 d^6 e x + 120 d^5 e^2 x^2 + 560 d^4 e^3 x^3 + 1820 d^3 e^4 x^4 + 4368 d^2 e^5 x^5 + 8008 d e^6 x^6 + 11440 e^7 x^7) + \\
& \quad 280 a^3 b^7 e^3 (A e (d^7 + 16 d^6 e x + 120 d^5 e^2 x^2 + 560 d^4 e^3 x^3 + 1820 d^3 e^4 x^4 + 4368 d^2 e^5 x^5 + 8008 d e^6 x^6 + 11440 e^7 x^7) + \\
& \quad B (d^8 + 16 d^7 e x + 120 d^6 e^2 x^2 + 560 d^5 e^3 x^3 + 1820 d^4 e^4 x^4 + 4368 d^3 e^5 x^5 + 8008 d^2 e^6 x^6 + 11440 d e^7 x^7 + 12870 e^8 x^8)) + \\
& \quad 15 a^2 b^8 e^2 (7 A e (d^8 + 16 d^7 e x + 120 d^6 e^2 x^2 + 560 d^5 e^3 x^3 + 1820 d^4 e^4 x^4 + 4368 d^3 e^5 x^5 + 8008 d^2 e^6 x^6 + 11440 d e^7 x^7 + 12870 e^8 x^8) + \\
& \quad 9 B (d^9 + 16 d^8 e x + 120 d^7 e^2 x^2 + 560 d^6 e^3 x^3 + 1820 d^5 e^4 x^4 + 4368 d^4 e^5 x^5 + 8008 d^3 e^6 x^6 + 11440 d^2 e^7 x^7 + 12870 d e^8 x^8 + 11440 e^9 x^9)) + \\
& \quad 10 a b^9 e (3 A e (d^9 + 16 d^8 e x + 120 d^7 e^2 x^2 + 560 d^6 e^3 x^3 + 1820 d^5 e^4 x^4 + 4368 d^4 e^5 x^5 + 8008 d^3 e^6 x^6 + 11440 d^2 e^7 x^7 + 12870 d e^8 x^8 + 11440 e^9 x^9)) + \\
& \quad 5 B (d^{10} + 16 d^9 e x + 120 d^8 e^2 x^2 + 560 d^7 e^3 x^3 + 1820 d^6 e^4 x^4 + 4368 d^5 e^5 x^5 + \\
& \quad 8008 d^4 e^6 x^6 + 11440 d^3 e^7 x^7 + 12870 d^2 e^8 x^8 + 11440 d e^9 x^9 + 8008 e^{10} x^{10})) + \\
& \quad b^{10} (5 A e (d^{10} + 16 d^9 e x + 120 d^8 e^2 x^2 + 560 d^7 e^3 x^3 + 1820 d^6 e^4 x^4 + 4368 d^5 e^5 x^5 + 8008 d^4 e^6 x^6 + 11440 d^3 e^7 x^7 + 12870 d^2 e^8 x^8 + \\
& \quad 11440 d e^9 x^9 + 8008 e^{10} x^{10}) + 11 B (d^{11} + 16 d^{10} e x + 120 d^9 e^2 x^2 + 560 d^8 e^3 x^3 + 1820 d^7 e^4 x^4 + 4368 d^6 e^5 x^5 + \\
& \quad 8008 d^5 e^6 x^6 + 11440 d^4 e^7 x^7 + 12870 d^3 e^8 x^8 + 11440 d^2 e^9 x^9 + 8008 d e^{10} x^{10} + 4368 e^{11} x^{11})) )
\end{aligned}$$

**Problem 1106: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b x)^{10} (A + B x)}{(d + e x)^{18}} dx$$

Optimal (type 1, 335 leaves, 7 steps):

$$\begin{aligned}
& - \frac{(B d - A e) (a + b x)^{11}}{17 e (b d - a e) (d + e x)^{17}} + \frac{(11 b B d + 6 A b e - 17 a B e) (a + b x)^{11}}{272 e (b d - a e)^2 (d + e x)^{16}} + \\
& \frac{b (11 b B d + 6 A b e - 17 a B e) (a + b x)^{11}}{816 e (b d - a e)^3 (d + e x)^{15}} + \frac{b^2 (11 b B d + 6 A b e - 17 a B e) (a + b x)^{11}}{2856 e (b d - a e)^4 (d + e x)^{14}} + \\
& \frac{b^3 (11 b B d + 6 A b e - 17 a B e) (a + b x)^{11}}{12376 e (b d - a e)^5 (d + e x)^{13}} + \frac{b^4 (11 b B d + 6 A b e - 17 a B e) (a + b x)^{11}}{74256 e (b d - a e)^6 (d + e x)^{12}} + \frac{b^5 (11 b B d + 6 A b e - 17 a B e) (a + b x)^{11}}{816816 e (b d - a e)^7 (d + e x)^{11}}
\end{aligned}$$

Result (type 1, 1433 leaves):

$$\begin{aligned}
& - \frac{1}{816816 e^{12} (d+e x)^{17}} (3003 a^{10} e^{10} (16 A e + B (d + 17 e x)) + 2002 a^9 b e^9 (15 A e (d + 17 e x) + 2 B (d^2 + 17 d e x + 136 e^2 x^2)) + \\
& 1287 a^8 b^2 e^8 (14 A e (d^2 + 17 d e x + 136 e^2 x^2) + 3 B (d^3 + 17 d^2 e x + 136 d e^2 x^2 + 680 e^3 x^3)) + \\
& 792 a^7 b^3 e^7 (13 A e (d^3 + 17 d^2 e x + 136 d e^2 x^2 + 680 e^3 x^3) + 4 B (d^4 + 17 d^3 e x + 136 d^2 e^2 x^2 + 680 d e^3 x^3 + 2380 e^4 x^4)) + \\
& 462 a^6 b^4 e^6 (12 A e (d^4 + 17 d^3 e x + 136 d^2 e^2 x^2 + 680 d e^3 x^3 + 2380 e^4 x^4) + 5 B (d^5 + 17 d^4 e x + 136 d^3 e^2 x^2 + 680 d^2 e^3 x^3 + 2380 d e^4 x^4 + 6188 e^5 x^5)) + \\
& 252 a^5 b^5 e^5 (11 A e (d^5 + 17 d^4 e x + 136 d^3 e^2 x^2 + 680 d^2 e^3 x^3 + 2380 d e^4 x^4 + 6188 e^5 x^5)) + \\
& 6 B (d^6 + 17 d^5 e x + 136 d^4 e^2 x^2 + 680 d^3 e^3 x^3 + 2380 d^2 e^4 x^4 + 6188 d e^5 x^5 + 12376 e^6 x^6)) + \\
& 126 a^4 b^6 e^4 (10 A e (d^6 + 17 d^5 e x + 136 d^4 e^2 x^2 + 680 d^3 e^3 x^3 + 2380 d^2 e^4 x^4 + 6188 d e^5 x^5 + 12376 e^6 x^6)) + \\
& 7 B (d^7 + 17 d^6 e x + 136 d^5 e^2 x^2 + 680 d^4 e^3 x^3 + 2380 d^3 e^4 x^4 + 6188 d^2 e^5 x^5 + 12376 d e^6 x^6 + 19448 e^7 x^7)) + \\
& 56 a^3 b^7 e^3 (9 A e (d^7 + 17 d^6 e x + 136 d^5 e^2 x^2 + 680 d^4 e^3 x^3 + 2380 d^3 e^4 x^4 + 6188 d^2 e^5 x^5 + 12376 d e^6 x^6 + 19448 e^7 x^7)) + \\
& 8 B (d^8 + 17 d^7 e x + 136 d^6 e^2 x^2 + 680 d^5 e^3 x^3 + 2380 d^4 e^4 x^4 + 6188 d^3 e^5 x^5 + 12376 d^2 e^6 x^6 + 19448 d e^7 x^7 + 24310 e^8 x^8)) + \\
& 21 a^2 b^8 e^2 (8 A e (d^8 + 17 d^7 e x + 136 d^6 e^2 x^2 + 680 d^5 e^3 x^3 + 2380 d^4 e^4 x^4 + 6188 d^3 e^5 x^5 + 12376 d^2 e^6 x^6 + 19448 d e^7 x^7 + 24310 e^8 x^8)) + \\
& 9 B (d^9 + 17 d^8 e x + 136 d^7 e^2 x^2 + 680 d^6 e^3 x^3 + 2380 d^5 e^4 x^4 + 6188 d^4 e^5 x^5 + 12376 d^3 e^6 x^6 + 19448 d^2 e^7 x^7 + 24310 d e^8 x^8 + 24310 e^9 x^9)) + \\
& 6 a b^9 e (7 A e (d^9 + 17 d^8 e x + 136 d^7 e^2 x^2 + 680 d^6 e^3 x^3 + 2380 d^5 e^4 x^4 + 6188 d^4 e^5 x^5 + 12376 d^3 e^6 x^6 + 19448 d^2 e^7 x^7 + 24310 d e^8 x^8 + 24310 e^9 x^9)) + \\
& 10 B (d^{10} + 17 d^9 e x + 136 d^8 e^2 x^2 + 680 d^7 e^3 x^3 + 2380 d^6 e^4 x^4 + 6188 d^5 e^5 x^5 + \\
& 12376 d^4 e^6 x^6 + 19448 d^3 e^7 x^7 + 24310 d^2 e^8 x^8 + 24310 d e^9 x^9 + 19448 e^{10} x^{10})) + \\
& b^{10} (6 A e (d^{10} + 17 d^9 e x + 136 d^8 e^2 x^2 + 680 d^7 e^3 x^3 + 2380 d^6 e^4 x^4 + 6188 d^5 e^5 x^5 + 12376 d^4 e^6 x^6 + 19448 d^3 e^7 x^7 + 24310 d^2 e^8 x^8 + \\
& 24310 d e^9 x^9 + 19448 e^{10} x^{10}) + 11 B (d^{11} + 17 d^{10} e x + 136 d^9 e^2 x^2 + 680 d^8 e^3 x^3 + 2380 d^7 e^4 x^4 + 6188 d^6 e^5 x^5 + \\
& 12376 d^5 e^6 x^6 + 19448 d^4 e^7 x^7 + 24310 d^3 e^8 x^8 + 24310 d^2 e^9 x^9 + 19448 d e^{10} x^{10} + 12376 e^{11} x^{11})) )
\end{aligned}$$

**Problem 1107: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+b x)^{10} (A+B x)}{(d+e x)^{19}} dx$$

Optimal (type 1, 385 leaves, 8 steps):

$$\begin{aligned}
& - \frac{(B d - A e) (a+b x)^{11}}{18 e (b d - a e) (d+e x)^{18}} + \frac{(11 b B d + 7 A b e - 18 a B e) (a+b x)^{11}}{306 e (b d - a e)^2 (d+e x)^{17}} + \frac{b (11 b B d + 7 A b e - 18 a B e) (a+b x)^{11}}{816 e (b d - a e)^3 (d+e x)^{16}} + \\
& \frac{b^2 (11 b B d + 7 A b e - 18 a B e) (a+b x)^{11}}{2448 e (b d - a e)^4 (d+e x)^{15}} + \frac{b^3 (11 b B d + 7 A b e - 18 a B e) (a+b x)^{11}}{8568 e (b d - a e)^5 (d+e x)^{14}} + \\
& \frac{b^4 (11 b B d + 7 A b e - 18 a B e) (a+b x)^{11}}{37128 e (b d - a e)^6 (d+e x)^{13}} + \frac{b^5 (11 b B d + 7 A b e - 18 a B e) (a+b x)^{11}}{222768 e (b d - a e)^7 (d+e x)^{12}} + \frac{b^6 (11 b B d + 7 A b e - 18 a B e) (a+b x)^{11}}{2450448 e (b d - a e)^8 (d+e x)^{11}}
\end{aligned}$$

Result (type 1, 1428 leaves):

$$\begin{aligned}
& - \frac{1}{2450448 e^{12} (d+e x)^{18}} (8008 a^{10} e^{10} (17 A e + B (d + 18 e x)) + 10010 a^9 b e^9 (8 A e (d + 18 e x) + B (d^2 + 18 d e x + 153 e^2 x^2)) + \\
& 9009 a^8 b^2 e^8 (5 A e (d^2 + 18 d e x + 153 e^2 x^2) + B (d^3 + 18 d^2 e x + 153 d e^2 x^2 + 816 e^3 x^3)) + \\
& 3432 a^7 b^3 e^7 (7 A e (d^3 + 18 d^2 e x + 153 d e^2 x^2 + 816 e^3 x^3) + 2 B (d^4 + 18 d^3 e x + 153 d^2 e^2 x^2 + 816 d e^3 x^3 + 3060 e^4 x^4)) + \\
& 924 a^6 b^4 e^6 (13 A e (d^4 + 18 d^3 e x + 153 d^2 e^2 x^2 + 816 d e^3 x^3 + 3060 e^4 x^4) + 5 B (d^5 + 18 d^4 e x + 153 d^3 e^2 x^2 + 816 d^2 e^3 x^3 + 3060 d e^4 x^4 + 8568 e^5 x^5)) + \\
& 2772 a^5 b^5 e^5 (2 A e (d^5 + 18 d^4 e x + 153 d^3 e^2 x^2 + 816 d^2 e^3 x^3 + 3060 d e^4 x^4 + 8568 e^5 x^5) + \\
& B (d^6 + 18 d^5 e x + 153 d^4 e^2 x^2 + 816 d^3 e^3 x^3 + 3060 d^2 e^4 x^4 + 8568 d e^5 x^5 + 18564 e^6 x^6)) + \\
& 210 a^4 b^6 e^4 (11 A e (d^6 + 18 d^5 e x + 153 d^4 e^2 x^2 + 816 d^3 e^3 x^3 + 3060 d^2 e^4 x^4 + 8568 d e^5 x^5 + 18564 e^6 x^6) + \\
& 7 B (d^7 + 18 d^6 e x + 153 d^5 e^2 x^2 + 816 d^4 e^3 x^3 + 3060 d^3 e^4 x^4 + 8568 d^2 e^5 x^5 + 18564 d e^6 x^6 + 31824 e^7 x^7)) + \\
& 168 a^3 b^7 e^3 (5 A e (d^7 + 18 d^6 e x + 153 d^5 e^2 x^2 + 816 d^4 e^3 x^3 + 3060 d^3 e^4 x^4 + 8568 d^2 e^5 x^5 + 18564 d e^6 x^6 + 31824 e^7 x^7) + \\
& 4 B (d^8 + 18 d^7 e x + 153 d^6 e^2 x^2 + 816 d^5 e^3 x^3 + 3060 d^4 e^4 x^4 + 8568 d^3 e^5 x^5 + 18564 d^2 e^6 x^6 + 31824 d e^7 x^7 + 43758 e^8 x^8)) + \\
& 252 a^2 b^8 e^2 (A e (d^8 + 18 d^7 e x + 153 d^6 e^2 x^2 + 816 d^5 e^3 x^3 + 3060 d^4 e^4 x^4 + 8568 d^3 e^5 x^5 + 18564 d^2 e^6 x^6 + 31824 d e^7 x^7 + 43758 e^8 x^8) + \\
& B (d^9 + 18 d^8 e x + 153 d^7 e^2 x^2 + 816 d^6 e^3 x^3 + 3060 d^5 e^4 x^4 + 8568 d^4 e^5 x^5 + 18564 d^3 e^6 x^6 + 31824 d^2 e^7 x^7 + 43758 d e^8 x^8 + 48620 e^9 x^9)) + 14 a b^9 \\
& e (4 A e (d^9 + 18 d^8 e x + 153 d^7 e^2 x^2 + 816 d^6 e^3 x^3 + 3060 d^5 e^4 x^4 + 8568 d^4 e^5 x^5 + 18564 d^3 e^6 x^6 + 31824 d^2 e^7 x^7 + 43758 d e^8 x^8 + 48620 e^9 x^9) + \\
& 5 B (d^{10} + 18 d^9 e x + 153 d^8 e^2 x^2 + 816 d^7 e^3 x^3 + 3060 d^6 e^4 x^4 + 8568 d^5 e^5 x^5 + \\
& 18564 d^4 e^6 x^6 + 31824 d^3 e^7 x^7 + 43758 d^2 e^8 x^8 + 48620 d e^9 x^9 + 43758 e^{10} x^{10})) + \\
& b^{10} (7 A e (d^{10} + 18 d^9 e x + 153 d^8 e^2 x^2 + 816 d^7 e^3 x^3 + 3060 d^6 e^4 x^4 + 8568 d^5 e^5 x^5 + 18564 d^4 e^6 x^6 + 31824 d^3 e^7 x^7 + 43758 d^2 e^8 x^8 + \\
& 48620 d e^9 x^9 + 43758 e^{10} x^{10}) + 11 B (d^{11} + 18 d^{10} e x + 153 d^9 e^2 x^2 + 816 d^8 e^3 x^3 + 3060 d^7 e^4 x^4 + 8568 d^6 e^5 x^5 + \\
& 18564 d^5 e^6 x^6 + 31824 d^4 e^7 x^7 + 43758 d^3 e^8 x^8 + 48620 d^2 e^9 x^9 + 43758 d e^{10} x^{10} + 31824 e^{11} x^{11})) )
\end{aligned}$$

**Problem 1108: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+b x)^{10} (A+B x)}{(d+e x)^{20}} dx$$

Optimal (type 1, 460 leaves, 2 steps):

$$\begin{aligned}
& \frac{(b d - a e)^{10} (B d - A e)}{19 e^{12} (d+e x)^{19}} - \frac{(b d - a e)^9 (11 b B d - 10 A b e - a B e)}{18 e^{12} (d+e x)^{18}} + \frac{5 b (b d - a e)^8 (11 b B d - 9 A b e - 2 a B e)}{17 e^{12} (d+e x)^{17}} - \\
& \frac{15 b^2 (b d - a e)^7 (11 b B d - 8 A b e - 3 a B e)}{16 e^{12} (d+e x)^{16}} + \frac{2 b^3 (b d - a e)^6 (11 b B d - 7 A b e - 4 a B e)}{e^{12} (d+e x)^{15}} - \\
& \frac{3 b^4 (b d - a e)^5 (11 b B d - 6 A b e - 5 a B e)}{e^{12} (d+e x)^{14}} + \frac{42 b^5 (b d - a e)^4 (11 b B d - 5 A b e - 6 a B e)}{13 e^{12} (d+e x)^{13}} - \frac{5 b^6 (b d - a e)^3 (11 b B d - 4 A b e - 7 a B e)}{2 e^{12} (d+e x)^{12}} + \\
& \frac{15 b^7 (b d - a e)^2 (11 b B d - 3 A b e - 8 a B e)}{11 e^{12} (d+e x)^{11}} - \frac{b^8 (b d - a e) (11 b B d - 2 A b e - 9 a B e)}{2 e^{12} (d+e x)^{10}} + \frac{b^9 (11 b B d - A b e - 10 a B e)}{9 e^{12} (d+e x)^9} - \frac{b^{10} B}{8 e^{12} (d+e x)^8}
\end{aligned}$$

Result (type 1, 1433 leaves):

$$\begin{aligned}
& - \frac{1}{6651216 e^{12} (d+e x)^{19}} (19448 a^{10} e^{10} (18 A e + B (d + 19 e x)) + 11440 a^9 b e^9 (17 A e (d + 19 e x) + 2 B (d^2 + 19 d e x + 171 e^2 x^2)) + \\
& 6435 a^8 b^2 e^8 (16 A e (d^2 + 19 d e x + 171 e^2 x^2) + 3 B (d^3 + 19 d^2 e x + 171 d e^2 x^2 + 969 e^3 x^3)) + \\
& 3432 a^7 b^3 e^7 (15 A e (d^3 + 19 d^2 e x + 171 d e^2 x^2 + 969 e^3 x^3) + 4 B (d^4 + 19 d^3 e x + 171 d^2 e^2 x^2 + 969 d e^3 x^3 + 3876 e^4 x^4)) + 1716 a^6 b^4 e^6 \\
& (14 A e (d^4 + 19 d^3 e x + 171 d^2 e^2 x^2 + 969 d e^3 x^3 + 3876 e^4 x^4) + 5 B (d^5 + 19 d^4 e x + 171 d^3 e^2 x^2 + 969 d^2 e^3 x^3 + 3876 d e^4 x^4 + 11628 e^5 x^5)) + \\
& 792 a^5 b^5 e^5 (13 A e (d^5 + 19 d^4 e x + 171 d^3 e^2 x^2 + 969 d^2 e^3 x^3 + 3876 d e^4 x^4 + 11628 e^5 x^5)) + \\
& 6 B (d^6 + 19 d^5 e x + 171 d^4 e^2 x^2 + 969 d^3 e^3 x^3 + 3876 d^2 e^4 x^4 + 11628 d e^5 x^5 + 27132 e^6 x^6)) + \\
& 330 a^4 b^6 e^4 (12 A e (d^6 + 19 d^5 e x + 171 d^4 e^2 x^2 + 969 d^3 e^3 x^3 + 3876 d^2 e^4 x^4 + 11628 d e^5 x^5 + 27132 e^6 x^6)) + \\
& 7 B (d^7 + 19 d^6 e x + 171 d^5 e^2 x^2 + 969 d^4 e^3 x^3 + 3876 d^3 e^4 x^4 + 11628 d^2 e^5 x^5 + 27132 d e^6 x^6 + 50388 e^7 x^7)) + \\
& 120 a^3 b^7 e^3 (11 A e (d^7 + 19 d^6 e x + 171 d^5 e^2 x^2 + 969 d^4 e^3 x^3 + 3876 d^3 e^4 x^4 + 11628 d^2 e^5 x^5 + 27132 d e^6 x^6 + 50388 e^7 x^7)) + \\
& 8 B (d^8 + 19 d^7 e x + 171 d^6 e^2 x^2 + 969 d^5 e^3 x^3 + 3876 d^4 e^4 x^4 + 11628 d^3 e^5 x^5 + 27132 d^2 e^6 x^6 + 50388 d e^7 x^7 + 75582 e^8 x^8)) + \\
& 36 a^2 b^8 e^2 (10 A e (d^8 + 19 d^7 e x + 171 d^6 e^2 x^2 + 969 d^5 e^3 x^3 + 3876 d^4 e^4 x^4 + 11628 d^3 e^5 x^5 + 27132 d^2 e^6 x^6 + 50388 d e^7 x^7 + 75582 e^8 x^8)) + \\
& 9 B (d^9 + 19 d^8 e x + 171 d^7 e^2 x^2 + 969 d^6 e^3 x^3 + 3876 d^5 e^4 x^4 + 11628 d^4 e^5 x^5 + 27132 d^3 e^6 x^6 + 50388 d^2 e^7 x^7 + 75582 d e^8 x^8 + 92378 e^9 x^9) + 8 a \\
& b^9 e (9 A e (d^9 + 19 d^8 e x + 171 d^7 e^2 x^2 + 969 d^6 e^3 x^3 + 3876 d^5 e^4 x^4 + 11628 d^4 e^5 x^5 + 27132 d^3 e^6 x^6 + 50388 d^2 e^7 x^7 + 75582 d e^8 x^8 + 92378 e^9 x^9)) + \\
& 10 B (d^{10} + 19 d^9 e x + 171 d^8 e^2 x^2 + 969 d^7 e^3 x^3 + 3876 d^6 e^4 x^4 + 11628 d^5 e^5 x^5 + \\
& 27132 d^4 e^6 x^6 + 50388 d^3 e^7 x^7 + 75582 d^2 e^8 x^8 + 92378 d e^9 x^9 + 92378 e^{10} x^{10})) + \\
& b^{10} (8 A e (d^{10} + 19 d^9 e x + 171 d^8 e^2 x^2 + 969 d^7 e^3 x^3 + 3876 d^6 e^4 x^4 + 11628 d^5 e^5 x^5 + 27132 d^4 e^6 x^6 + 50388 d^3 e^7 x^7 + \\
& 75582 d^2 e^8 x^8 + 92378 d e^9 x^9 + 92378 e^{10} x^{10}) + 11 B (d^{11} + 19 d^{10} e x + 171 d^9 e^2 x^2 + 969 d^8 e^3 x^3 + 3876 d^7 e^4 x^4 + \\
& 11628 d^6 e^5 x^5 + 27132 d^5 e^6 x^6 + 50388 d^4 e^7 x^7 + 75582 d^3 e^8 x^8 + 92378 d^2 e^9 x^9 + 92378 d e^{10} x^{10} + 75582 e^{11} x^{11})) )
\end{aligned}$$

**Problem 1109: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+b x)^{10} (A+B x)}{(d+e x)^{21}} dx$$

Optimal (type 1, 462 leaves, 2 steps):

$$\begin{aligned}
& \frac{(b d - a e)^{10} (B d - A e)}{20 e^{12} (d+e x)^{20}} - \frac{(b d - a e)^9 (11 b B d - 10 A b e - a B e)}{19 e^{12} (d+e x)^{19}} + \frac{5 b (b d - a e)^8 (11 b B d - 9 A b e - 2 a B e)}{18 e^{12} (d+e x)^{18}} - \\
& \frac{15 b^2 (b d - a e)^7 (11 b B d - 8 A b e - 3 a B e)}{17 e^{12} (d+e x)^{17}} + \frac{15 b^3 (b d - a e)^6 (11 b B d - 7 A b e - 4 a B e)}{8 e^{12} (d+e x)^{16}} - \\
& \frac{14 b^4 (b d - a e)^5 (11 b B d - 6 A b e - 5 a B e)}{5 e^{12} (d+e x)^{15}} + \frac{3 b^5 (b d - a e)^4 (11 b B d - 5 A b e - 6 a B e)}{e^{12} (d+e x)^{14}} - \frac{30 b^6 (b d - a e)^3 (11 b B d - 4 A b e - 7 a B e)}{13 e^{12} (d+e x)^{13}} + \\
& \frac{5 b^7 (b d - a e)^2 (11 b B d - 3 A b e - 8 a B e)}{4 e^{12} (d+e x)^{12}} - \frac{5 b^8 (b d - a e) (11 b B d - 2 A b e - 9 a B e)}{11 e^{12} (d+e x)^{11}} + \frac{b^9 (11 b B d - A b e - 10 a B e)}{10 e^{12} (d+e x)^{10}} - \frac{b^{10} B}{9 e^{12} (d+e x)^9}
\end{aligned}$$

Result (type 1, 1428 leaves):

$$\begin{aligned}
& - \frac{1}{16628040 e^{12} (d + e x)^{20}} (43758 a^{10} e^{10} (19 A e + B (d + 20 e x)) + 48620 a^9 b e^9 (9 A e (d + 20 e x) + B (d^2 + 20 d e x + 190 e^2 x^2)) + \\
& 12870 a^8 b^2 e^8 (17 A e (d^2 + 20 d e x + 190 e^2 x^2) + 3 B (d^3 + 20 d^2 e x + 190 d e^2 x^2 + 1140 e^3 x^3)) + \\
& 25740 a^7 b^3 e^7 (4 A e (d^3 + 20 d^2 e x + 190 d e^2 x^2 + 1140 e^3 x^3) + B (d^4 + 20 d^3 e x + 190 d^2 e^2 x^2 + 1140 d e^3 x^3 + 4845 e^4 x^4)) + 15015 a^6 b^4 e^6 \\
& (3 A e (d^4 + 20 d^3 e x + 190 d^2 e^2 x^2 + 1140 d e^3 x^3 + 4845 e^4 x^4) + B (d^5 + 20 d^4 e x + 190 d^3 e^2 x^2 + 1140 d^2 e^3 x^3 + 4845 d e^4 x^4 + 15504 e^5 x^5)) + \\
& 2574 a^5 b^5 e^5 (7 A e (d^5 + 20 d^4 e x + 190 d^3 e^2 x^2 + 1140 d^2 e^3 x^3 + 4845 d e^4 x^4 + 15504 e^5 x^5)) + \\
& 3 B (d^6 + 20 d^5 e x + 190 d^4 e^2 x^2 + 1140 d^3 e^3 x^3 + 4845 d^2 e^4 x^4 + 15504 d e^5 x^5 + 38760 e^6 x^6)) + \\
& 495 a^4 b^6 e^4 (13 A e (d^6 + 20 d^5 e x + 190 d^4 e^2 x^2 + 1140 d^3 e^3 x^3 + 4845 d^2 e^4 x^4 + 15504 d e^5 x^5 + 38760 e^6 x^6)) + \\
& 7 B (d^7 + 20 d^6 e x + 190 d^5 e^2 x^2 + 1140 d^4 e^3 x^3 + 4845 d^3 e^4 x^4 + 15504 d^2 e^5 x^5 + 38760 d e^6 x^6 + 77520 e^7 x^7)) + \\
& 660 a^3 b^7 e^3 (3 A e (d^7 + 20 d^6 e x + 190 d^5 e^2 x^2 + 1140 d^4 e^3 x^3 + 4845 d^3 e^4 x^4 + 15504 d^2 e^5 x^5 + 38760 d e^6 x^6 + 77520 e^7 x^7)) + \\
& 2 B (d^8 + 20 d^7 e x + 190 d^6 e^2 x^2 + 1140 d^5 e^3 x^3 + 4845 d^4 e^4 x^4 + 15504 d^3 e^5 x^5 + 38760 d^2 e^6 x^6 + 77520 d e^7 x^7 + 125970 e^8 x^8)) + \\
& 45 a^2 b^8 e^2 (11 A e (d^8 + 20 d^7 e x + 190 d^6 e^2 x^2 + 1140 d^5 e^3 x^3 + 4845 d^4 e^4 x^4 + 15504 d^3 e^5 x^5 + 38760 d^2 e^6 x^6 + 77520 d e^7 x^7 + 125970 e^8 x^8)) + \\
& 9 B (d^9 + 20 d^8 e x + 190 d^7 e^2 x^2 + 1140 d^6 e^3 x^3 + 4845 d^5 e^4 x^4 + 15504 d^4 e^5 x^5 + 38760 d^3 e^6 x^6 + 77520 d^2 e^7 x^7 + 125970 d e^8 x^8 + 167960 e^9 x^9)) + \\
& 90 a b^9 e (A e (d^9 + 20 d^8 e x + 190 d^7 e^2 x^2 + 1140 d^6 e^3 x^3 + 4845 d^5 e^4 x^4 + 15504 d^4 e^5 x^5 + 38760 d^3 e^6 x^6 + 77520 d^2 e^7 x^7 + \\
& 125970 d e^8 x^8 + 167960 e^9 x^9) + B (d^{10} + 20 d^9 e x + 190 d^8 e^2 x^2 + 1140 d^7 e^3 x^3 + 4845 d^6 e^4 x^4 + \\
& 15504 d^5 e^5 x^5 + 38760 d^4 e^6 x^6 + 77520 d^3 e^7 x^7 + 125970 d^2 e^8 x^8 + 167960 d e^9 x^9 + 184756 e^{10} x^{10})) + \\
& b^{10} (9 A e (d^{10} + 20 d^9 e x + 190 d^8 e^2 x^2 + 1140 d^7 e^3 x^3 + 4845 d^6 e^4 x^4 + 15504 d^5 e^5 x^5 + 38760 d^4 e^6 x^6 + 77520 d^3 e^7 x^7 + \\
& 125970 d^2 e^8 x^8 + 167960 d e^9 x^9 + 184756 e^{10} x^{10}) + 11 B (d^{11} + 20 d^{10} e x + 190 d^9 e^2 x^2 + 1140 d^8 e^3 x^3 + 4845 d^7 e^4 x^4 + \\
& 15504 d^6 e^5 x^5 + 38760 d^5 e^6 x^6 + 77520 d^4 e^7 x^7 + 125970 d^3 e^8 x^8 + 167960 d^2 e^9 x^9 + 184756 d e^{10} x^{10} + 167960 e^{11} x^{11})) )
\end{aligned}$$

**Problem 1110: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b x)^{10} (A + B x)}{(d + e x)^{22}} dx$$

Optimal (type 1, 464 leaves, 2 steps):

$$\begin{aligned}
& \frac{(b d - a e)^{10} (B d - A e)}{21 e^{12} (d + e x)^{21}} - \frac{(b d - a e)^9 (11 b B d - 10 A b e - a B e)}{20 e^{12} (d + e x)^{20}} + \frac{5 b (b d - a e)^8 (11 b B d - 9 A b e - 2 a B e)}{19 e^{12} (d + e x)^{19}} - \\
& \frac{5 b^2 (b d - a e)^7 (11 b B d - 8 A b e - 3 a B e)}{6 e^{12} (d + e x)^{18}} + \frac{30 b^3 (b d - a e)^6 (11 b B d - 7 A b e - 4 a B e)}{17 e^{12} (d + e x)^{17}} - \frac{21 b^4 (b d - a e)^5 (11 b B d - 6 A b e - 5 a B e)}{8 e^{12} (d + e x)^{16}} + \\
& \frac{14 b^5 (b d - a e)^4 (11 b B d - 5 A b e - 6 a B e)}{5 e^{12} (d + e x)^{15}} - \frac{15 b^6 (b d - a e)^3 (11 b B d - 4 A b e - 7 a B e)}{7 e^{12} (d + e x)^{14}} + \frac{15 b^7 (b d - a e)^2 (11 b B d - 3 A b e - 8 a B e)}{13 e^{12} (d + e x)^{13}} - \\
& \frac{5 b^8 (b d - a e) (11 b B d - 2 A b e - 9 a B e)}{12 e^{12} (d + e x)^{12}} + \frac{b^9 (11 b B d - A b e - 10 a B e)}{11 e^{12} (d + e x)^{11}} - \frac{b^{10} B}{10 e^{12} (d + e x)^{10}}
\end{aligned}$$

Result (type 1, 1431 leaves):

$$\begin{aligned}
& - \frac{1}{38798760 e^{12} (d + e x)^{21}} \left( 92378 a^{10} e^{10} (20 A e + B (d + 21 e x)) + 48620 a^9 b e^9 (19 A e (d + 21 e x) + 2 B (d^2 + 21 d e x + 210 e^2 x^2)) + \right. \\
& \quad 72930 a^8 b^2 e^8 (6 A e (d^2 + 21 d e x + 210 e^2 x^2) + B (d^3 + 21 d^2 e x + 210 d e^2 x^2 + 1330 e^3 x^3)) + \\
& \quad 11440 a^7 b^3 e^7 (17 A e (d^3 + 21 d^2 e x + 210 d e^2 x^2 + 1330 e^3 x^3) + 4 B (d^4 + 21 d^3 e x + 210 d^2 e^2 x^2 + 1330 d e^3 x^3 + 5985 e^4 x^4)) + 5005 a^6 b^4 e^6 \\
& \quad (16 A e (d^4 + 21 d^3 e x + 210 d^2 e^2 x^2 + 1330 d e^3 x^3 + 5985 e^4 x^4) + 5 B (d^5 + 21 d^4 e x + 210 d^3 e^2 x^2 + 1330 d^2 e^3 x^3 + 5985 d e^4 x^4 + 20349 e^5 x^5)) + \\
& \quad 6006 a^5 b^5 e^5 (5 A e (d^5 + 21 d^4 e x + 210 d^3 e^2 x^2 + 1330 d^2 e^3 x^3 + 5985 d e^4 x^4 + 20349 e^5 x^5) + \\
& \quad 2 B (d^6 + 21 d^5 e x + 210 d^4 e^2 x^2 + 1330 d^3 e^3 x^3 + 5985 d^2 e^4 x^4 + 20349 d e^5 x^5 + 54264 e^6 x^6)) + \\
& \quad 5005 a^4 b^6 e^4 (2 A e (d^6 + 21 d^5 e x + 210 d^4 e^2 x^2 + 1330 d^3 e^3 x^3 + 5985 d^2 e^4 x^4 + 20349 d e^5 x^5 + 54264 e^6 x^6) + \\
& \quad B (d^7 + 21 d^6 e x + 210 d^5 e^2 x^2 + 1330 d^4 e^3 x^3 + 5985 d^3 e^4 x^4 + 20349 d^2 e^5 x^5 + 54264 d e^6 x^6 + 116280 e^7 x^7)) + \\
& \quad 220 a^3 b^7 e^3 (13 A e (d^7 + 21 d^6 e x + 210 d^5 e^2 x^2 + 1330 d^4 e^3 x^3 + 5985 d^3 e^4 x^4 + 20349 d^2 e^5 x^5 + 54264 d e^6 x^6 + 116280 e^7 x^7) + \\
& \quad 8 B (d^8 + 21 d^7 e x + 210 d^6 e^2 x^2 + 1330 d^5 e^3 x^3 + 5985 d^4 e^4 x^4 + 20349 d^3 e^5 x^5 + 54264 d^2 e^6 x^6 + 116280 d e^7 x^7 + 203490 e^8 x^8)) + \\
& \quad 165 a^2 b^8 e^2 (4 A e (d^8 + 21 d^7 e x + 210 d^6 e^2 x^2 + 1330 d^5 e^3 x^3 + 5985 d^4 e^4 x^4 + 20349 d^3 e^5 x^5 + 54264 d^2 e^6 x^6 + 116280 d e^7 x^7 + 203490 e^8 x^8) + \\
& \quad 3 B (d^9 + 21 d^8 e x + 210 d^7 e^2 x^2 + 1330 d^6 e^3 x^3 + 5985 d^5 e^4 x^4 + 20349 d^4 e^5 x^5 + 54264 d^3 e^6 x^6 + 116280 d^2 e^7 x^7 + 203490 d e^8 x^8 + 293930 e^9 x^9)) + \\
& \quad 10 a b^9 e (11 A e (d^9 + 21 d^8 e x + 210 d^7 e^2 x^2 + 1330 d^6 e^3 x^3 + 5985 d^5 e^4 x^4 + 20349 d^4 e^5 x^5 + 54264 d^3 e^6 x^6 + 116280 d^2 e^7 x^7 + \\
& \quad 203490 d e^8 x^8 + 293930 e^9 x^9) + 10 B (d^{10} + 21 d^9 e x + 210 d^8 e^2 x^2 + 1330 d^7 e^3 x^3 + 5985 d^6 e^4 x^4 + \\
& \quad 20349 d^5 e^5 x^5 + 54264 d^4 e^6 x^6 + 116280 d^3 e^7 x^7 + 203490 d^2 e^8 x^8 + 293930 d e^9 x^9 + 352716 e^{10} x^{10})) + \\
& \quad b^{10} (10 A e (d^{10} + 21 d^9 e x + 210 d^8 e^2 x^2 + 1330 d^7 e^3 x^3 + 5985 d^6 e^4 x^4 + 20349 d^5 e^5 x^5 + 54264 d^4 e^6 x^6 + 116280 d^3 e^7 x^7 + \\
& \quad 203490 d^2 e^8 x^8 + 293930 d e^9 x^9 + 352716 e^{10} x^{10}) + 11 B (d^{11} + 21 d^{10} e x + 210 d^9 e^2 x^2 + 1330 d^8 e^3 x^3 + 5985 d^7 e^4 x^4 + \\
& \quad 20349 d^6 e^5 x^5 + 54264 d^5 e^6 x^6 + 116280 d^4 e^7 x^7 + 203490 d^3 e^8 x^8 + 293930 d^2 e^9 x^9 + 352716 d e^{10} x^{10} + 352716 e^{11} x^{11})) )
\end{aligned}$$

**Problem 1122: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + B x) (d + e x)^5}{(a + b x)^2} dx$$

Optimal (type 3, 227 leaves, 2 steps):

$$\begin{aligned}
& \frac{5 e (b d - a e)^3 (b B d + 2 A b e - 3 a B e) x}{b^6} - \frac{(A b - a B) (b d - a e)^5}{b^7 (a + b x)} + \\
& \frac{5 e^2 (b d - a e)^2 (b B d + A b e - 2 a B e) (a + b x)^2}{b^7} + \frac{5 e^3 (b d - a e) (2 b B d + A b e - 3 a B e) (a + b x)^3}{3 b^7} + \\
& \frac{e^4 (5 b B d + A b e - 6 a B e) (a + b x)^4}{4 b^7} + \frac{B e^5 (a + b x)^5}{5 b^7} + \frac{(b d - a e)^4 (b B d + 5 A b e - 6 a B e) \text{Log}[a + b x]}{b^7}
\end{aligned}$$

Result (type 3, 500 leaves):

$$\frac{1}{60 b^7 (a + b x)} \left( B \left( -60 a^6 e^5 + 300 a^5 b e^4 (d + e x) + 60 a^4 b^2 e^3 (-10 d^2 - 20 d e x + 3 e^2 x^2) + 30 a^3 b^3 e^2 (20 d^3 + 60 d^2 e x - 25 d e^2 x^2 - 2 e^3 x^3) + 10 a^2 b^4 e (-30 d^4 - 120 d^3 e x + 120 d^2 e^2 x^2 + 25 d e^3 x^3 + 3 e^4 x^4) + b^6 e x^2 (300 d^4 + 300 d^3 e x + 200 d^2 e^2 x^2 + 75 d e^3 x^3 + 12 e^4 x^4) + a b^5 (60 d^5 + 300 d^4 e x - 900 d^3 e^2 x^2 - 400 d^2 e^3 x^3 - 125 d e^4 x^4 - 18 e^5 x^5) \right) - 5 A b \left( -12 a^5 e^5 + 12 a^4 b e^4 (5 d + 4 e x) + 30 a^3 b^2 e^3 (-4 d^2 - 6 d e x + e^2 x^2) - 10 a^2 b^3 e^2 (-12 d^3 - 24 d^2 e x + 12 d e^2 x^2 + e^3 x^3) + 5 a b^4 e (-12 d^4 - 24 d^3 e x + 36 d^2 e^2 x^2 + 8 d e^3 x^3 + e^4 x^4) + b^5 (12 d^5 - 120 d^3 e^2 x^2 - 60 d^2 e^3 x^3 - 20 d e^4 x^4 - 3 e^5 x^5) \right) + 60 (b d - a e)^4 (b B d + 5 A b e - 6 a B e) (a + b x) \operatorname{Log}[a + b x] \right)$$

**Problem 1192:** Result more than twice size of optimal antiderivative.

$$\int (5 - 2x)^6 (2 + 3x)^3 (-16 + 33x) dx$$

Optimal (type 1, 18 leaves, 1 step) :

$$-\frac{1}{2} (5 - 2x)^7 (2 + 3x)^4$$

Result (type 1, 56 leaves) :

$$-2000000 x - 37500 x^2 + 3987500 x^3 - \frac{98125 x^4}{2} - 3816225 x^5 + 1497230 x^6 + 1235404 x^7 - 1256376 x^8 + 452304 x^9 - 76896 x^{10} + 5184 x^{11}$$

**Problem 2517:** Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{a + b x} (e + f x) \sqrt{2 b e - a f + b f x}} dx$$

Optimal (type 3, 59 leaves, 2 steps) :

$$\frac{\operatorname{ArcTan}\left[\frac{\sqrt{f} \sqrt{a+b x} \sqrt{2 b e-a f+b f x}}{b e-a f}\right]}{\sqrt{f} (b e-a f)}$$

Result (type 3, 81 leaves) :

$$\frac{\frac{2 i \sqrt{f} (-b e+a f)}{e+f x} + \frac{2 f \sqrt{a+b x} \sqrt{2 b e-a f+b f x}}{e+f x}}{\sqrt{f} (b e-a f)}$$

Problem 2637: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{a + bx} \sqrt{c + \frac{b(-1+c)x}{a}} \sqrt{e + \frac{b(-1+e)x}{a}}} dx$$

Optimal (type 4, 58 leaves, 1 step):

$$\frac{2\sqrt{a} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{1-c} \sqrt{a+b x}}{\sqrt{a}}\right], \frac{1-e}{1-c}\right]}{b \sqrt{1-c}}$$

Result (type 4, 129 leaves):

$$-\frac{2(a+bx) \sqrt{\frac{-1+c+\frac{a}{a+bx}}{-1+c}} \sqrt{\frac{-1+e+\frac{a}{a+bx}}{-1+e}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{-a}{-1+c}}}{\sqrt{a+b x}}\right], \frac{-1+c}{-1+e}\right]}{b \sqrt{-\frac{a}{-1+c}} \sqrt{c+\frac{b(-1+c)x}{a}} \sqrt{e+\frac{b(-1+e)x}{a}}}$$

Problem 2640: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{e + \frac{b(-1+e)x}{a}}}{\sqrt{a + bx} \sqrt{c + \frac{b(-1+c)x}{a}}} dx$$

Optimal (type 4, 58 leaves, 1 step):

$$\frac{2\sqrt{a} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{1-c} \sqrt{a+b x}}{\sqrt{a}}\right], \frac{1-e}{1-c}\right]}{b \sqrt{1-c}}$$

Result (type 4, 191 leaves):

$$-\frac{2(a+bx)^{3/2} \left( -\frac{\sqrt{\frac{-a}{-1+e}} (-1+c+\frac{a}{a+bx}) (-1+e+\frac{a}{a+bx})}{-1+c} + \frac{a \sqrt{\frac{-1+c+\frac{a}{a+bx}}{-1+e}} \sqrt{\frac{-1+e+\frac{a}{a+bx}}{-1+e}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{-a}{-1+e}}}{\sqrt{a+b x}}\right], \frac{-1+e}{-1+c}\right]}{\sqrt{a+b x}} \right)}{a b \sqrt{-\frac{a}{-1+e}} \sqrt{c+\frac{b(-1+c)x}{a}} \sqrt{e+\frac{b(-1+e)x}{a}}}$$

Problem 2641: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{c + dx}}{\sqrt{a + bx} \sqrt{e + \frac{b(-1+e)x}{a}}} dx$$

Optimal (type 4, 96 leaves, 2 steps):

$$\frac{2\sqrt{a}\sqrt{c+dx}\text{EllipticE}[\text{ArcSin}\left[\frac{\sqrt{1-e}\sqrt{a+bx}}{\sqrt{a}}\right], -\frac{ad}{(bc-ad)(1-e)}]}{b\sqrt{1-e}\sqrt{\frac{b(c+dx)}{bc-ad}}}$$

Result (type 4, 200 leaves):

$$\left(2\sqrt{\frac{-1+e+\frac{a}{a+bx}}{-1+e}} \left(b\sqrt{a-\frac{bc}{d}}\sqrt{a+bx}(c+dx)\sqrt{\frac{ae+b(-1+e)x}{(-1+e)(a+bx)}} - (bc-ad)(a+bx)\sqrt{\frac{b(c+dx)}{d(a+bx)}}\text{EllipticE}[\text{ArcSin}\left[\frac{\sqrt{a-\frac{bc}{d}}}{\sqrt{a+bx}}\right], \frac{ad}{(bc-ad)(-1+e)}]\right)\right) / \left(b^2\sqrt{a-\frac{bc}{d}}\sqrt{c+dx}\sqrt{e+\frac{b(-1+e)x}{a}}\right)$$

Problem 2642: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a+bx}}{\sqrt{c+\frac{b(-1+c)x}{a}} \sqrt{e+\frac{b(-1+e)x}{a}}} dx$$

Optimal (type 4, 162 leaves, 2 steps):

$$\frac{2a\sqrt{c-e}\sqrt{a+bx}\sqrt{-\frac{(1-c)(ae-b(1-e)x)}{a(c-e)}}\text{EllipticE}[\text{ArcSin}\left[\frac{\sqrt{1-e}\sqrt{\frac{c-b(1-c)x}{a}}}{\sqrt{c-e}}\right], \frac{c-e}{1-e}]}{b(1-c)\sqrt{1-e}\sqrt{\frac{(1-c)(a+bx)}{a}}\sqrt{e-\frac{b(1-e)x}{a}}}$$

Result (type 4, 103 leaves):

$$\begin{aligned}
 & -\frac{1}{b(-1+e)\sqrt{\frac{(-1+c)(a+b x)}{a}}} \\
 & 2 \pm a \sqrt{a+b x} \left( \text{EllipticE}\left[\pm \text{ArcSinh}\left[\sqrt{\frac{(-1+c)(a+b x)}{a}}\right], \frac{-1+e}{-1+c}\right] - \text{EllipticF}\left[\pm \text{ArcSinh}\left[\sqrt{\frac{(-1+c)(a+b x)}{a}}\right], \frac{-1+e}{-1+c}\right] \right)
 \end{aligned}$$

**Problem 2674:** Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1-2x}}{\sqrt{-3-5x} \sqrt{2+3x}} dx$$

Optimal (type 4, 31 leaves, 1 step) :

$$\frac{2}{3} \sqrt{\frac{7}{5}} \text{EllipticE}\left[\text{ArcSin}\left[\sqrt{5} \sqrt{2+3x}\right], \frac{2}{35}\right]$$

Result (type 4, 109 leaves) :

$$\begin{aligned}
 & -\frac{2 \left( \frac{3(-3+x+10x^2)}{\sqrt{2+3x}} + \sqrt{35} \sqrt{\frac{-1+2x}{2+3x}} (2+3x) \sqrt{\frac{3+5x}{2+3x}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{7}{2}}}{\sqrt{2+3x}}\right], \frac{2}{35}\right] \right)}{15 \sqrt{-3-5x} \sqrt{1-2x}}
 \end{aligned}$$

**Problem 2678:** Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1-2x}}{\sqrt{2+3x} \sqrt{3+5x}} dx$$

Optimal (type 4, 49 leaves, 2 steps) :

$$\begin{aligned}
 & \frac{2 \sqrt{\frac{7}{5}} \sqrt{-3-5x} \text{EllipticE}\left[\text{ArcSin}\left[\sqrt{5} \sqrt{2+3x}\right], \frac{2}{35}\right]}{3 \sqrt{3+5x}}
 \end{aligned}$$

Result (type 4, 121 leaves) :

$$\left( \frac{2 \sqrt{1-2x} \left( 5 \sqrt{3+5x} (-2+x+6x^2) + \sqrt{33} \sqrt{\frac{-1+2x}{3+5x}} \sqrt{\frac{2+3x}{3+5x}} (3+5x)^2 \text{EllipticE}[\text{ArcSin}\left[\frac{\sqrt{\frac{11}{2}}}{\sqrt{3+5x}}\right], -\frac{2}{33}] \right)}{(15 \sqrt{2+3x} (-3+x+10x^2))} \right)$$

**Problem 2819:** Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{3+5x}}{\sqrt{1-2x} (2+3x)^{3/2}} dx$$

Optimal (type 4, 81 leaves, 4 steps):

$$-\frac{2 \sqrt{1-2x} \sqrt{3+5x}}{7 \sqrt{2+3x}} + \frac{2 \sqrt{\frac{5}{7}} \sqrt{-3-5x} \text{EllipticE}[\text{ArcSin}[\sqrt{5} \sqrt{2+3x}], \frac{2}{35}]}{3 \sqrt{3+5x}}$$

Result (type 4, 70 leaves):

$$\frac{-6 \sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x} - 2 \pm \sqrt{33} (2+3x) \text{EllipticE}[\pm \text{ArcSinh}[\sqrt{9+15x}], -\frac{2}{33}]}{42+63x}$$

**Problem 2841:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{1+x} \sqrt{2+x} \sqrt{3+x}} dx$$

Optimal (type 4, 12 leaves, 1 step):

$$-2 \text{EllipticF}[\text{ArcSin}\left[\frac{1}{\sqrt{3+x}}\right], 2]$$

Result (type 4, 55 leaves):

$$\frac{2 \pm \sqrt{1+\frac{1}{1+x}} \text{EllipticF}[\pm \text{ArcSinh}[\frac{1}{\sqrt{1+x}}], 2]}{\sqrt{\frac{2+x}{3+x}} \sqrt{\frac{3+x}{1+x}}}$$

Problem 2842: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{3-x} \sqrt{1+x} \sqrt{2+x}} dx$$

Optimal (type 4, 16 leaves, 1 step) :

$$2 \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{\sqrt{1+x}}{2}\right], -4]$$

Result (type 4, 74 leaves) :

$$\frac{\frac{i}{4} \sqrt{1+\frac{4}{-3+x}} \sqrt{1+\frac{5}{-3+x}} (-3+x)^{3/2} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{2}{\sqrt{-3+x}}\right], \frac{5}{4}\right]}{\sqrt{-(-3+x)(1+x)} \sqrt{2+x}}$$

Problem 2843: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{2-x} \sqrt{1+x} \sqrt{3+x}} dx$$

Optimal (type 4, 24 leaves, 1 step) :

$$\sqrt{2} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{\sqrt{1+x}}{\sqrt{3}}\right], -\frac{3}{2}]$$

Result (type 4, 67 leaves) :

$$-\frac{2 (3+x) \sqrt{1-\frac{5}{3+x}} \sqrt{1-\frac{2}{3+x}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{5}}{\sqrt{3+x}}\right], \frac{2}{5}\right]}{\sqrt{-50+35 (3+x)-5 (3+x)^2}}$$

Problem 2844: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{2-x} \sqrt{3-x} \sqrt{1+x}} dx$$

Optimal (type 4, 18 leaves, 1 step) :

$$\operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{\sqrt{1+x}}{\sqrt{3}}\right], \frac{3}{4}]$$

Result (type 4, 65 leaves) :

$$-\frac{2 \pm \sqrt{1 - \frac{3}{2-x}} \sqrt{1 + \frac{1}{2-x}} (2-x) \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{1}{\sqrt{2-x}}\right], -3\right]}{\sqrt{-(-3+x)(1+x)}}$$

Problem 2845: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{1-x} \sqrt{2+x} \sqrt{3+x}} dx$$

Optimal (type 4, 18 leaves, 1 step) :

$$2 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{2+x}}{\sqrt{3}}\right], -3\right]$$

Result (type 4, 78 leaves) :

$$-\frac{2 \pm \sqrt{-(-1+x)(2+x)} \sqrt{3+x} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{3}}{\sqrt{-1+x}}\right], \frac{4}{3}\right]}{\sqrt{3+\frac{9}{-1+x}} (-1+x)^{3/2} \sqrt{\frac{3+x}{-1+x}}}$$

Problem 2846: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{1-x} \sqrt{3-x} \sqrt{2+x}} dx$$

Optimal (type 4, 25 leaves, 1 step) :

$$\frac{2 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{2+x}}{\sqrt{3}}\right], \frac{3}{5}\right]}{\sqrt{5}}$$

Result (type 4, 68 leaves) :

$$-\frac{2 \sqrt{\frac{-3+x}{-1+x}} (-1+x) \sqrt{\frac{2+x}{-1+x}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{3}}{\sqrt{1-x}}\right], -\frac{2}{3}\right]}{\sqrt{3} \sqrt{6+x-x^2}}$$

Problem 2847: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{1-x} \sqrt{2-x} \sqrt{3+x}} dx$$

Optimal (type 4, 23 leaves, 1 step) :

$$\frac{2 \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{\sqrt{3+x}}{2}\right], \frac{4}{5}]}{\sqrt{5}}$$

Result (type 4, 65 leaves) :

$$\frac{2 i \sqrt{1-\frac{4}{1-x}} \sqrt{1+\frac{1}{1-x}} (1-x) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{1}{\sqrt{1-x}}\right], -4\right]}{\sqrt{-(-2+x) (3+x)}}$$

Problem 2848: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{1-x} \sqrt{2-x} \sqrt{3-x}} dx$$

Optimal (type 4, 14 leaves, 1 step) :

$$\frac{2 \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{1}{\sqrt{3-x}}\right], 2]}{}$$

Result (type 4, 67 leaves) :

$$\frac{2 i \sqrt{\frac{-3+x}{-1+x}} \sqrt{\frac{-2+x}{-1+x}} (-1+x) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{1}{\sqrt{1-x}}\right], 2\right]}{\sqrt{2-x} \sqrt{3-x}}$$

Problem 2849: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-3+x} \sqrt{-2+x} \sqrt{-1+x}} dx$$

Optimal (type 4, 12 leaves, 1 step) :

$$\frac{-2 \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{1}{\sqrt{-1+x}}\right], 2]}{}$$

Result (type 4, 59 leaves) :

$$\frac{2 \pm \sqrt{1 + \frac{1}{-3+x}} \sqrt{1 + \frac{2}{-3+x}} (-3+x) \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{1}{\sqrt{-3+x}}\right], 2\right]}{\sqrt{-2+x} \sqrt{-1+x}}$$

Problem 2851: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-2-x} \sqrt{-3+x} \sqrt{-1+x}} dx$$

Optimal (type 4, 41 leaves, 2 steps) :

$$\frac{2\sqrt{2+x} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1}{\sqrt{\frac{2+x}{3}}}\right], \frac{5}{3}\right]}{\sqrt{3} \sqrt{-2-x}}$$

Result (type 4, 72 leaves) :

$$\frac{2 \pm \sqrt{\frac{-3+x}{-1+x}} \sqrt{\frac{-1+x}{2+x}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{3}}{\sqrt{-2-x}}\right], \frac{5}{3}\right]}{\sqrt{3} \sqrt{\frac{-3+x}{2+x}}}$$

Problem 2853: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-1-x} \sqrt{-3+x} \sqrt{-2+x}} dx$$

Optimal (type 4, 41 leaves, 2 steps) :

$$\frac{2\sqrt{1+x} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1}{\sqrt{\frac{1+x}{3}}}\right], \frac{4}{3}\right]}{\sqrt{3} \sqrt{-1-x}}$$

Result (type 4, 72 leaves) :

$$\frac{2 \pm \sqrt{\frac{-3+x}{-2+x}} \sqrt{\frac{-2+x}{1+x}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{3}}{\sqrt{-1-x}}\right], \frac{4}{3}\right]}{\sqrt{3} \sqrt{\frac{-3+x}{1+x}}}$$

Problem 2854: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-3-x} \sqrt{-1-x} \sqrt{-2+x}} dx$$

Optimal (type 4, 57 leaves, 3 steps):

$$-\frac{2 \sqrt{1+x} \sqrt{3+x} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1}{\sqrt{\frac{3+x}{5}}}\right], \frac{2}{5}\right]}{\sqrt{5} \sqrt{-3-x} \sqrt{-1-x}}$$

Result (type 4, 75 leaves):

$$\frac{2 i \sqrt{1+\frac{3}{-2+x}} \sqrt{1+\frac{5}{-2+x}} (-2+x) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{3}}{\sqrt{-2+x}}\right], \frac{5}{3}\right]}{\sqrt{-15-3 (-2+x)} \sqrt{-1-x}}$$

Problem 2855: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-2-x} \sqrt{-1-x} \sqrt{-3+x}} dx$$

Optimal (type 4, 57 leaves, 3 steps):

$$-\frac{2 \sqrt{1+x} \sqrt{2+x} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1}{\sqrt{\frac{2+x}{5}}}\right], \frac{1}{5}\right]}{\sqrt{5} \sqrt{-2-x} \sqrt{-1-x}}$$

Result (type 4, 69 leaves):

$$\frac{i \sqrt{1+\frac{4}{-3+x}} \sqrt{1+\frac{5}{-3+x}} (-3+x) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{2}{\sqrt{-3+x}}\right], \frac{5}{4}\right]}{\sqrt{-2-x} \sqrt{-1-x}}$$

Problem 2856: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-3-x} \sqrt{-2-x} \sqrt{-1-x}} dx$$

Optimal (type 4, 14 leaves, 1 step):

$$2 \text{EllipticF}[\text{ArcSin}\left[\frac{1}{\sqrt{-1-x}}\right], 2]$$

Result (type 4, 67 leaves) :

$$\frac{2 i \sqrt{\frac{1+x}{3+x}} \sqrt{\frac{2+x}{3+x}} (3+x) \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{1}{\sqrt{-3-x}}\right], 2\right]}{\sqrt{-2-x} \sqrt{-1-x}}$$

Problem 2857: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(a+b x)^{3/2} \sqrt{c+d x} \sqrt{e+f x}} dx$$

Optimal (type 4, 204 leaves, 4 steps) :

$$-\frac{2 b \sqrt{c+d x} \sqrt{e+f x}}{(b c-a d) (b e-a f) \sqrt{a+b x}} + \frac{2 \sqrt{f} \sqrt{-d e+c f} \sqrt{a+b x} \sqrt{\frac{d (e+f x)}{d e-c f}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{f} \sqrt{c+d x}}{\sqrt{-d e+c f}}\right], -\frac{b (d e-c f)}{(b c-a d) f}\right]}{(b c-a d) (b e-a f) \sqrt{-\frac{d (a+b x)}{b c-a d}} \sqrt{e+f x}}$$

Result (type 4, 201 leaves) :

$$\begin{aligned} & \left(2 b \sqrt{c+d x} \sqrt{e+f x} \left(-1 - \frac{1}{\sqrt{\frac{b (e+f x)}{b e-a f}}} \right.\right. \\ & \left.\left. \left. \left. \frac{i}{\sqrt{\frac{d (a+b x)}{b (c+d x)}} \left(\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{d (a+b x)}{b c-a d}}\right], \frac{b c f-a d f}{b d e-a d f}\right] - \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{d (a+b x)}{b c-a d}}\right], \frac{b c f-a d f}{b d e-a d f}\right]\right)\right) \right) \right) \right) \Bigg/ ((b c - \\ & a d) (b e - a f) \sqrt{a+b x}) \end{aligned}$$

Problem 2858: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(a+b x)^{5/2} \sqrt{c+d x} \sqrt{e+f x}} dx$$

Optimal (type 4, 437 leaves, 8 steps):

$$\begin{aligned}
 & -\frac{2 b \sqrt{c+d x} \sqrt{e+f x}}{3 (b c-a d) (b e-a f) (a+b x)^{3/2}} + \frac{4 b (b d e+b c f-2 a d f) \sqrt{c+d x} \sqrt{e+f x}}{3 (b c-a d)^2 (b e-a f)^2 \sqrt{a+b x}} - \\
 & \frac{4 \sqrt{d} (b d e+b c f-2 a d f) \sqrt{\frac{b (c+d x)}{b c-a d}} \sqrt{e+f x} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{d} \sqrt{a+b x}}{\sqrt{-b c+a d}}\right], \frac{(b c-a d) f}{d (b e-a f)}\right]}{3 (-b c+a d)^{3/2} (b e-a f)^2 \sqrt{c+d x} \sqrt{\frac{b (e+f x)}{b e-a f}}} + \\
 & \frac{2 \sqrt{d} (2 b d e+b c f-3 a d f) \sqrt{\frac{b (c+d x)}{b c-a d}} \sqrt{\frac{b (e+f x)}{b e-a f}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{d} \sqrt{a+b x}}{\sqrt{-b c+a d}}\right], \frac{(b c-a d) f}{d (b e-a f)}\right]}{3 b (-b c+a d)^{3/2} (b e-a f) \sqrt{c+d x} \sqrt{e+f x}}
 \end{aligned}$$

Result (type 4, 449 leaves):

$$\begin{aligned}
 & -\frac{1}{3 b \sqrt{-a+\frac{b c}{d}} (b c-a d)^2 (b e-a f)^2 (a+b x)^{3/2} \sqrt{c+d x} \sqrt{e+f x}} \\
 & 2 \left( b^2 \sqrt{-a+\frac{b c}{d}} (c+d x) (e+f x) ((b c-a d) (b e-a f) - 2 (b d e+b c f-2 a d f) (a+b x)) + \right. \\
 & (a+b x) \left( 2 b^2 \sqrt{-a+\frac{b c}{d}} (b d e+b c f-2 a d f) (c+d x) (e+f x) + \right. \\
 & 2 \pm (b c-a d) f (b d e+b c f-2 a d f) (a+b x)^{3/2} \sqrt{\frac{b (c+d x)}{d (a+b x)}} \sqrt{\frac{b (e+f x)}{f (a+b x)}} \operatorname{EllipticE}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{-a+\frac{b c}{d}}}{\sqrt{a+b x}}\right], \frac{b d e-a d f}{b c f-a d f}\right] - \\
 & \left. \pm (b c-a d) f (b d e+2 b c f-3 a d f) (a+b x)^{3/2} \sqrt{\frac{b (c+d x)}{d (a+b x)}} \sqrt{\frac{b (e+f x)}{f (a+b x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{-a+\frac{b c}{d}}}{\sqrt{a+b x}}\right], \frac{b d e-a d f}{b c f-a d f}\right] \right)
 \end{aligned}$$

Problem 2862: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{2+3x}}{\sqrt{1-2x} \sqrt{3+5x}} dx$$

Optimal (type 4, 31 leaves, 1 step):

$$-\sqrt{\frac{7}{5}} \text{EllipticE}\left[\text{ArcSin}\left[\sqrt{\frac{5}{11}} \sqrt{1-2x}\right], \frac{33}{35}\right]$$

Result (type 4, 129 leaves):

$$\frac{\sqrt{2+3x} \sqrt{\frac{-1+2x}{3+5x}} \left(5 \sqrt{\frac{-1+2x}{3+5x}} \sqrt{\frac{2+3x}{3+5x}} \sqrt{3+5x} + i \sqrt{2} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{1}{\sqrt{9+15x}}\right], -\frac{33}{2}\right]\right)}{5 \sqrt{1-2x} \sqrt{\frac{2+3x}{3+5x}}}$$

Problem 2863: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}} dx$$

Optimal (type 4, 29 leaves, 1 step):

$$-\frac{2 \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{3}{7}} \sqrt{1-2x}\right], \frac{35}{33}\right]}{\sqrt{33}}$$

Result (type 4, 74 leaves):

$$\frac{i \sqrt{2+3x} \sqrt{\frac{-2+4x}{3+5x}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{1}{\sqrt{9+15x}}\right], -\frac{33}{2}\right]}{\sqrt{1-2x} \sqrt{\frac{2+3x}{3+5x}}}$$

Problem 2870: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{2+3x}}{\sqrt{1-2x} (3+5x)^{3/2}} dx$$

Optimal (type 4, 81 leaves, 4 steps):

$$-\frac{2 \sqrt{1-2x} \sqrt{2+3x}}{11 \sqrt{3+5x}} + \frac{2 \sqrt{\frac{7}{5}} \sqrt{-3-5x} \operatorname{EllipticE}[\operatorname{ArcSin}[\sqrt{5} \sqrt{2+3x}], \frac{2}{35}]}{11 \sqrt{3+5x}}$$

Result (type 4, 61 leaves):

$$\frac{2}{55} \left( -\frac{5 \sqrt{1-2x} \sqrt{2+3x}}{\sqrt{3+5x}} - i \sqrt{33} \operatorname{EllipticE}[i \operatorname{ArcSinh}[\sqrt{9+15x}], -\frac{2}{33}] \right)$$

Problem 2884: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{x}}{\sqrt{a+2x} \sqrt{c+2x}} dx$$

Optimal (type 4, 86 leaves, 3 steps):

$$\frac{\sqrt{a-c} \sqrt{x} \sqrt{-\frac{c+2x}{a-c}} \operatorname{EllipticE}[\operatorname{ArcSin}[\frac{\sqrt{a+2x}}{\sqrt{a-c}}], 1-\frac{c}{a}]}{\sqrt{2} \sqrt{-\frac{x}{a}} \sqrt{c+2x}}$$

Result (type 4, 120 leaves):

$$-\left( \left( i c \sqrt{1+\frac{2x}{a}} \sqrt{1+\frac{2x}{c}} \left( \operatorname{EllipticE}[i \operatorname{ArcSinh}[\sqrt{2} \sqrt{\frac{1}{a}} \sqrt{x}], \frac{a}{c}] - \operatorname{EllipticF}[i \operatorname{ArcSinh}[\sqrt{2} \sqrt{\frac{1}{a}} \sqrt{x}], \frac{a}{c}] \right) \right) / \right. \\ \left. \left( \sqrt{2} \sqrt{\frac{1}{a}} \sqrt{a+2x} \sqrt{c+2x} \right) \right)$$

Problem 2885: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{4-x} \sqrt{5-x} \sqrt{-3+x}} dx$$

Optimal (type 4, 18 leaves, 1 step):

$$\sqrt{2} \operatorname{EllipticF}[\operatorname{ArcSin}[\sqrt{-3+x}], \frac{1}{2}]$$

Result (type 4, 46 leaves) :

$$\frac{2 \sqrt{-15 + 8x - x^2} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1}{\sqrt{4-x}}\right], -1\right]}{\sqrt{1 - \frac{1}{(-4+x)^2} (-4+x)}}$$

Problem 2886: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{4-x} \sqrt{(5-x)(-3+x)}} dx$$

Optimal (type 4, 14 leaves, 3 steps) :

$$-2 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{4-x}\right], -1\right]$$

Result (type 4, 46 leaves) :

$$\frac{2 \sqrt{-15 + 8x - x^2} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1}{\sqrt{4-x}}\right], -1\right]}{\sqrt{1 - \frac{1}{(-4+x)^2} (-4+x)}}$$

Problem 2887: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{4-x} \sqrt{-15 + 8x - x^2}} dx$$

Optimal (type 4, 14 leaves, 2 steps) :

$$-2 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{4-x}\right], -1\right]$$

Result (type 4, 44 leaves) :

$$\frac{2 \sqrt{1 - \frac{1}{(-4+x)^2} (-4+x)} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1}{\sqrt{4-x}}\right], -1\right]}{\sqrt{-15 + 8x - x^2}}$$

Problem 2888: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{6-x} \sqrt{-2+x} \sqrt{-1+x}} dx$$

Optimal (type 4, 16 leaves, 1 step) :

$$\frac{2 \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{\sqrt{-2+x}}{2}\right], -4]}{x}$$

Result (type 4, 74 leaves) :

$$\frac{\frac{i}{\sqrt{1 + \frac{4}{-6+x}} \sqrt{1 + \frac{5}{-6+x}} (-6+x)^{3/2} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{2}{\sqrt{-6+x}}\right], \frac{5}{4}\right]}}{\sqrt{-(-6+x)(-2+x)} \sqrt{-1+x}}$$

Problem 2889: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{(6-x)(-2+x)} \sqrt{-1+x}} dx$$

Optimal (type 4, 25 leaves, 3 steps) :

$$\frac{\frac{2 \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{\sqrt{6-x}}{2}\right], \frac{4}{5}]}{\sqrt{5}}}{x}$$

Result (type 4, 74 leaves) :

$$\frac{\frac{i}{\sqrt{1 + \frac{4}{-6+x}} \sqrt{1 + \frac{5}{-6+x}} (-6+x)^{3/2} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{2}{\sqrt{-6+x}}\right], \frac{5}{4}\right]}}{\sqrt{-(-6+x)(-2+x)} \sqrt{-1+x}}$$

Problem 2890: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-1+x} \sqrt{-12+8x-x^2}} dx$$

Optimal (type 4, 25 leaves, 2 steps) :

$$\frac{\frac{2 \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{\sqrt{6-x}}{2}\right], \frac{4}{5}]}{\sqrt{5}}}{x}$$

Result (type 4, 68 leaves) :

$$-\frac{2 \sqrt{\frac{-6+x}{-1+x}} \sqrt{\frac{-2+x}{-1+x}} (-1+x) \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{5}}{\sqrt{-1+x}}\right], \frac{1}{5}\right]}{\sqrt{5} \sqrt{-12+8x-x^2}}$$

Problem 2922: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(1-2x)^{3/2} \sqrt{2+3x} \sqrt{3+5x}} dx$$

Optimal (type 4, 81 leaves, 4 steps):

$$\frac{4 \sqrt{2+3x} \sqrt{3+5x}}{77 \sqrt{1-2x}} + \frac{2 \sqrt{\frac{5}{7}} \sqrt{-3-5x} \text{EllipticE}\left[\text{ArcSin}\left[\sqrt{5} \sqrt{2+3x}\right], \frac{2}{35}\right]}{11 \sqrt{3+5x}}$$

Result (type 4, 61 leaves):

$$\frac{2}{77} \left( \frac{2 \sqrt{2+3x} \sqrt{3+5x}}{\sqrt{1-2x}} - i \sqrt{33} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{9+15x}\right], -\frac{2}{33}\right] \right)$$

Problem 3001: Result unnecessarily involves higher level functions.

$$\int (a+bx)^{1/3} (c+dx)^{2/3} (e+fx)^2 dx$$

Optimal (type 3, 571 leaves, 5 steps):

$$\begin{aligned}
& \frac{(b c - a d) (10 a^2 d^2 f^2 - 10 a b d f (3 d e - c f) + b^2 (27 d^2 e^2 - 24 c d e f + 7 c^2 f^2)) (a + b x)^{1/3} (c + d x)^{2/3}}{81 b^3 d^3} + \\
& \frac{(10 a^2 d^2 f^2 - 10 a b d f (3 d e - c f) + b^2 (27 d^2 e^2 - 24 c d e f + 7 c^2 f^2)) (a + b x)^{4/3} (c + d x)^{2/3}}{54 b^3 d^2} + \\
& \frac{f (15 b d e - 7 b c f - 8 a d f) (a + b x)^{4/3} (c + d x)^{5/3}}{36 b^2 d^2} + \frac{f (a + b x)^{4/3} (c + d x)^{5/3} (e + f x)}{4 b d} + \frac{1}{81 \sqrt{3} b^{11/3} d^{10/3}} \\
& (b c - a d)^2 (10 a^2 d^2 f^2 - 10 a b d f (3 d e - c f) + b^2 (27 d^2 e^2 - 24 c d e f + 7 c^2 f^2)) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 b^{1/3} (c + d x)^{1/3}}{\sqrt{3} d^{1/3} (a + b x)^{1/3}}\right] + \\
& \frac{(b c - a d)^2 (10 a^2 d^2 f^2 - 10 a b d f (3 d e - c f) + b^2 (27 d^2 e^2 - 24 c d e f + 7 c^2 f^2)) \operatorname{Log}[a + b x]}{486 b^{11/3} d^{10/3}} + \frac{1}{162 b^{11/3} d^{10/3}} \\
& (b c - a d)^2 (10 a^2 d^2 f^2 - 10 a b d f (3 d e - c f) + b^2 (27 d^2 e^2 - 24 c d e f + 7 c^2 f^2)) \operatorname{Log}\left[-1 + \frac{b^{1/3} (c + d x)^{1/3}}{d^{1/3} (a + b x)^{1/3}}\right]
\end{aligned}$$

Result (type 5, 311 leaves):

$$\begin{aligned}
& \frac{1}{324 b^3 d^4 (a + b x)^{2/3}} \\
& (c + d x)^{2/3} \left( d (a + b x) (20 a^3 d^3 f^2 - 12 a^2 b d^2 f (5 d e + c f + d f x) + 3 a b^2 d (-3 c^2 f^2 + 2 c d f (8 e + f x) + 3 d^2 (6 e^2 + 4 e f x + f^2 x^2))) + \right. \\
& \left. b^3 (28 c^3 f^2 - 3 c^2 d f (32 e + 7 f x) + 18 c d^2 (6 e^2 + 4 e f x + f^2 x^2) + 27 d^3 x (6 e^2 + 8 e f x + 3 f^2 x^2)) \right) - 2 (b c - a d)^2 \\
& (10 a^2 d^2 f^2 + 10 a b d f (-3 d e + c f) + b^2 (27 d^2 e^2 - 24 c d e f + 7 c^2 f^2)) \left( \frac{d (a + b x)}{-b c + a d} \right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b (c + d x)}{b c - a d}\right]
\end{aligned}$$

Problem 3002: Result unnecessarily involves higher level functions.

$$\int (a + b x)^{1/3} (c + d x)^{2/3} (e + f x) dx$$

Optimal (type 3, 331 leaves, 4 steps):

$$\begin{aligned}
& \frac{(b c - a d) (9 b d e - 4 b c f - 5 a d f) (a + b x)^{1/3} (c + d x)^{2/3}}{27 b^2 d^2} + \frac{(9 b d e - 4 b c f - 5 a d f) (a + b x)^{4/3} (c + d x)^{2/3}}{18 b^2 d} + \\
& \frac{f (a + b x)^{4/3} (c + d x)^{5/3}}{3 b d} + \frac{(b c - a d)^2 (9 b d e - 4 b c f - 5 a d f) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 b^{1/3} (c+d x)^{1/3}}{\sqrt{3} d^{1/3} (a+b x)^{1/3}}\right]}{27 \sqrt{3} b^{8/3} d^{7/3}} + \\
& \frac{(b c - a d)^2 (9 b d e - 4 b c f - 5 a d f) \operatorname{Log}[a + b x]}{162 b^{8/3} d^{7/3}} + \frac{(b c - a d)^2 (9 b d e - 4 b c f - 5 a d f) \operatorname{Log}\left[-1 + \frac{b^{1/3} (c+d x)^{1/3}}{d^{1/3} (a+b x)^{1/3}}\right]}{54 b^{8/3} d^{7/3}}
\end{aligned}$$

Result (type 5, 175 leaves):

$$\begin{aligned}
& \frac{1}{54 b^2 d^3 (a + b x)^{2/3}} (c + d x)^{2/3} \left( d (a + b x) (-5 a^2 d^2 f + a b d (9 d e + 4 c f + 3 d f x) + b^2 (-8 c^2 f + 6 c d (3 e + f x) + 9 d^2 x (3 e + 2 f x))) + \right. \\
& \left. (b c - a d)^2 (-9 b d e + 4 b c f + 5 a d f) \left( \frac{d (a + b x)}{-b c + a d} \right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b (c + d x)}{b c - a d}\right] \right)
\end{aligned}$$

Problem 3003: Result unnecessarily involves higher level functions.

$$\int (a + b x)^{1/3} (c + d x)^{2/3} dx$$

Optimal (type 3, 219 leaves, 3 steps):

$$\begin{aligned}
& \frac{(b c - a d) (a + b x)^{1/3} (c + d x)^{2/3}}{3 b d} + \frac{(a + b x)^{4/3} (c + d x)^{2/3}}{2 b} + \\
& \frac{(b c - a d)^2 \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 b^{1/3} (c+d x)^{1/3}}{\sqrt{3} d^{1/3} (a+b x)^{1/3}}\right]}{3 \sqrt{3} b^{5/3} d^{4/3}} + \frac{(b c - a d)^2 \operatorname{Log}[a + b x]}{18 b^{5/3} d^{4/3}} + \frac{(b c - a d)^2 \operatorname{Log}\left[-1 + \frac{b^{1/3} (c+d x)^{1/3}}{d^{1/3} (a+b x)^{1/3}}\right]}{6 b^{5/3} d^{4/3}}
\end{aligned}$$

Result (type 5, 109 leaves):

$$\frac{1}{6 b d^2 (a + b x)^{2/3}} (c + d x)^{2/3} \left( d (a + b x) (2 b c + a d + 3 b d x) - (b c - a d)^2 \left( \frac{d (a + b x)}{-b c + a d} \right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b (c + d x)}{b c - a d}\right] \right)$$

Problem 3004: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{1/3} (c + d x)^{2/3}}{e + f x} dx$$

Optimal (type 3, 409 leaves, 4 steps):

$$\begin{aligned}
& \frac{(a+b x)^{1/3} (c+d x)^{2/3}}{f} + \frac{(3 b d e - 2 b c f - a d f) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 b^{1/3} (c+d x)^{1/3}}{\sqrt{3} d^{1/3} (a+b x)^{1/3}}\right]}{\sqrt{3} b^{2/3} d^{1/3} f^2} - \\
& \frac{\sqrt{3} (b e - a f)^{1/3} (d e - c f)^{2/3} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 (b e - a f)^{1/3} (c+d x)^{1/3}}{\sqrt{3} (d e - c f)^{1/3} (a+b x)^{1/3}}\right]}{f^2} + \frac{(3 b d e - 2 b c f - a d f) \operatorname{Log}[a+b x]}{6 b^{2/3} d^{1/3} f^2} + \frac{(b e - a f)^{1/3} (d e - c f)^{2/3} \operatorname{Log}[e+f x]}{2 f^2} - \\
& \frac{3 (b e - a f)^{1/3} (d e - c f)^{2/3} \operatorname{Log}\left[-(a+b x)^{1/3} + \frac{(b e - a f)^{1/3} (c+d x)^{1/3}}{(d e - c f)^{1/3}}\right]}{2 f^2} + \frac{(3 b d e - 2 b c f - a d f) \operatorname{Log}\left[-1 + \frac{b^{1/3} (c+d x)^{1/3}}{d^{1/3} (a+b x)^{1/3}}\right]}{2 b^{2/3} d^{1/3} f^2}
\end{aligned}$$

Result (type 6, 541 leaves):

$$\begin{aligned}
& \frac{1}{5 f (a+b x)^{2/3}} \\
& (c+d x)^{2/3} \left( 5 (a+b x) - \frac{1}{d^2 (e+f x)} 4 (b c - a d) \left( - \left( \left( 5 b f (-d e + c f) (c+d x) \operatorname{AppellF1}\left[1, \frac{2}{3}, 1, 2, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c+d x)}\right] \right) \right) \right) \right) / \left( 6 b \right. \\
& f (c+d x) \operatorname{AppellF1}\left[1, \frac{2}{3}, 1, 2, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c+d x)}\right] + \\
& b (-3 d e + 3 c f) \operatorname{AppellF1}\left[2, \frac{2}{3}, 2, 3, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c+d x)}\right] + 2 (b c - a d) f \operatorname{AppellF1}\left[2, \frac{5}{3}, 1, 3, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c+d x)}\right] \left. \right) \\
& \left( 2 (d e - c f) (3 b d e - 2 b c f - a d f) \operatorname{AppellF1}\left[\frac{5}{3}, \frac{2}{3}, 1, \frac{8}{3}, \frac{b (c+d x)}{b c - a d}, \frac{f (c+d x)}{-d e + c f}\right] \right) / \\
& \left( \frac{8 (b c - a d) (-d e + c f) \operatorname{AppellF1}\left[\frac{5}{3}, \frac{2}{3}, 1, \frac{8}{3}, \frac{b (c+d x)}{b c - a d}, \frac{f (c+d x)}{-d e + c f}\right]}{c+d x} + 3 (b c - a d) f \operatorname{AppellF1}\left[\frac{8}{3}, \frac{2}{3}, 2, \frac{11}{3}, \frac{b (c+d x)}{b c - a d}, \frac{f (c+d x)}{-d e + c f}\right] + \right. \\
& \left. 2 b (-d e + c f) \operatorname{AppellF1}\left[\frac{8}{3}, \frac{5}{3}, 1, \frac{11}{3}, \frac{b (c+d x)}{b c - a d}, \frac{f (c+d x)}{-d e + c f}\right] \right)
\end{aligned}$$

Problem 3005: Result unnecessarily involves higher level functions.

$$\int \frac{(a+b x)^{1/3} (c+d x)^{2/3}}{(e+f x)^2} dx$$

Optimal (type 3, 417 leaves, 4 steps):

$$\begin{aligned}
& - \frac{(a+b x)^{1/3} (c+d x)^{2/3}}{f (e+f x)} - \frac{\sqrt{3} b^{1/3} d^{2/3} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 b^{1/3} (c+d x)^{1/3}}{\sqrt{3} d^{1/3} (a+b x)^{1/3}}\right]}{f^2} + \\
& \frac{(3 b d e - b c f - 2 a d f) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 (b e - a f)^{1/3} (c+d x)^{1/3}}{\sqrt{3} (d e - c f)^{1/3} (a+b x)^{1/3}}\right]}{\sqrt{3} f^2 (b e - a f)^{2/3} (d e - c f)^{1/3}} - \frac{b^{1/3} d^{2/3} \operatorname{Log}[a+b x]}{2 f^2} - \frac{(3 b d e - b c f - 2 a d f) \operatorname{Log}[e+f x]}{6 f^2 (b e - a f)^{2/3} (d e - c f)^{1/3}} + \\
& \frac{(3 b d e - b c f - 2 a d f) \operatorname{Log}\left[-(a+b x)^{1/3} + \frac{(b e - a f)^{1/3} (c+d x)^{1/3}}{(d e - c f)^{1/3}}\right]}{2 f^2 (b e - a f)^{2/3} (d e - c f)^{1/3}} - \frac{3 b^{1/3} d^{2/3} \operatorname{Log}\left[-1 + \frac{b^{1/3} (c+d x)^{1/3}}{d^{1/3} (a+b x)^{1/3}}\right]}{2 f^2}
\end{aligned}$$

Result (type 6, 743 leaves):

$$\begin{aligned}
& \frac{1}{5 f (a+b x)^{2/3} (e+f x)} (c+d x)^{2/3} \left( -5 (a+b x) - \right. \\
& \frac{1}{d} \frac{4 b}{d} \left( - \left( 5 b c f (c+d x) \operatorname{AppellF1}\left[1, \frac{2}{3}, 1, 2, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c+d x)}\right] \right) / \left( 6 b f (c+d x) \operatorname{AppellF1}\left[1, \frac{2}{3}, 1, 2, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c+d x)}\right] + \right. \right. \\
& b (-3 d e + 3 c f) \operatorname{AppellF1}\left[2, \frac{2}{3}, 2, 3, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c+d x)}\right] + 2 (b c - a d) f \operatorname{AppellF1}\left[2, \frac{5}{3}, 1, 3, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c+d x)}\right] \left. \right) - \\
& \left( 5 a d f (c+d x) \operatorname{AppellF1}\left[1, \frac{2}{3}, 1, 2, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c+d x)}\right] \right) / \left( -6 b f (c+d x) \operatorname{AppellF1}\left[1, \frac{2}{3}, 1, 2, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c+d x)}\right] + \right. \\
& 3 b (d e - c f) \operatorname{AppellF1}\left[2, \frac{2}{3}, 2, 3, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c+d x)}\right] + 2 (-b c + a d) f \operatorname{AppellF1}\left[2, \frac{5}{3}, 1, 3, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c+d x)}\right] \left. \right) - \\
& \left( 6 (b c - a d) (-d e + c f) \operatorname{AppellF1}\left[\frac{5}{3}, \frac{2}{3}, 1, \frac{8}{3}, \frac{b (c+d x)}{b c - a d}, \frac{f (c+d x)}{-d e + c f}\right] \right) / \\
& \left( \frac{8 (b c - a d) (-d e + c f) \operatorname{AppellF1}\left[\frac{5}{3}, \frac{2}{3}, 1, \frac{8}{3}, \frac{b (c+d x)}{b c - a d}, \frac{f (c+d x)}{-d e + c f}\right]}{c+d x} + 3 (b c - a d) f \operatorname{AppellF1}\left[\frac{8}{3}, \frac{2}{3}, 2, \frac{11}{3}, \frac{b (c+d x)}{b c - a d}, \frac{f (c+d x)}{-d e + c f}\right] + \right. \\
& \left. \left. 2 b (-d e + c f) \operatorname{AppellF1}\left[\frac{8}{3}, \frac{5}{3}, 1, \frac{11}{3}, \frac{b (c+d x)}{b c - a d}, \frac{f (c+d x)}{-d e + c f}\right] \right) \right)
\end{aligned}$$

Problem 3006: Result unnecessarily involves higher level functions.

$$\int \frac{(a+b x)^{1/3} (c+d x)^{2/3}}{(e+f x)^3} dx$$

Optimal (type 3, 325 leaves, 3 steps):

$$\begin{aligned} & \frac{(a+b x)^{1/3} (c+d x)^{5/3}}{2 (d e - c f) (e + f x)^2} - \frac{(b c - a d) (a+b x)^{1/3} (c+d x)^{2/3}}{6 (b e - a f) (d e - c f) (e + f x)} + \frac{(b c - a d)^2 \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 (b e - a f)^{1/3} (c+d x)^{1/3}}{\sqrt{3} (d e - c f)^{1/3} (a+b x)^{1/3}}\right]}{3 \sqrt{3} (b e - a f)^{5/3} (d e - c f)^{4/3}} - \\ & \frac{(b c - a d)^2 \operatorname{Log}[e + f x]}{18 (b e - a f)^{5/3} (d e - c f)^{4/3}} + \frac{(b c - a d)^2 \operatorname{Log}\left[-(a+b x)^{1/3} + \frac{(b e - a f)^{1/3} (c+d x)^{1/3}}{(d e - c f)^{1/3}}\right]}{6 (b e - a f)^{5/3} (d e - c f)^{4/3}} \end{aligned}$$

Result (type 5, 196 leaves):

$$\left( (a+b x)^{1/3} \left( f (b e - a f) (c+d x) (-3 a c f + a d (e - 2 f x) + b (2 c e + 3 d e x - c f x)) - 2 (b c - a d)^2 f \left( \frac{(b e - a f) (c+d x)}{(b c - a d) (e + f x)} \right)^{1/3} (e + f x)^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{(-d e + c f) (a+b x)}{(b c - a d) (e + f x)}\right] \right) \right) / \left( 6 f (b e - a f)^2 (d e - c f) (c+d x)^{1/3} (e + f x)^2 \right)$$

Problem 3007: Result unnecessarily involves higher level functions.

$$\int \frac{(a+b x)^{1/3} (c+d x)^{2/3}}{(e + f x)^4} dx$$

Optimal (type 3, 465 leaves, 4 steps):

$$\begin{aligned} & - \frac{f (a+b x)^{4/3} (c+d x)^{5/3}}{3 (b e - a f) (d e - c f) (e + f x)^3} + \frac{(9 b d e - 5 b c f - 4 a d f) (a+b x)^{1/3} (c+d x)^{5/3}}{18 (b e - a f) (d e - c f)^2 (e + f x)^2} - \\ & \frac{(b c - a d) (9 b d e - 5 b c f - 4 a d f) (a+b x)^{1/3} (c+d x)^{2/3}}{54 (b e - a f)^2 (d e - c f)^2 (e + f x)} + \frac{(b c - a d)^2 (9 b d e - 5 b c f - 4 a d f) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 (b e - a f)^{1/3} (c+d x)^{1/3}}{\sqrt{3} (d e - c f)^{1/3} (a+b x)^{1/3}}\right]}{27 \sqrt{3} (b e - a f)^{8/3} (d e - c f)^{7/3}} - \\ & \frac{(b c - a d)^2 (9 b d e - 5 b c f - 4 a d f) \operatorname{Log}[e + f x]}{162 (b e - a f)^{8/3} (d e - c f)^{7/3}} + \frac{(b c - a d)^2 (9 b d e - 5 b c f - 4 a d f) \operatorname{Log}\left[-(a+b x)^{1/3} + \frac{(b e - a f)^{1/3} (c+d x)^{1/3}}{(d e - c f)^{1/3}}\right]}{54 (b e - a f)^{8/3} (d e - c f)^{7/3}} \end{aligned}$$

Result (type 5, 304 leaves):

$$\frac{1}{54 f (b e - a f)^3 (d e - c f)^2 (c + d x)^{1/3} (e + f x)^3} \\ (a + b x)^{1/3} \left( - (b e - a f) (c + d x) \left( 18 (b e - a f)^2 (d e - c f)^2 - 3 (b e - a f) (d e - c f) (3 b d e - b c f - 2 a d f) (e + f x) - \right. \right. \\ \left. \left. (8 a^2 d^2 f^2 - 4 a b d f (3 d e + c f) + b^2 (9 d^2 e^2 - 6 c d e f + 5 c^2 f^2)) (e + f x)^2 \right) + \right. \\ \left. 2 (b c - a d)^2 f (-9 b d e + 5 b c f + 4 a d f) \left( \frac{(b e - a f) (c + d x)}{(b c - a d) (e + f x)} \right)^{1/3} (e + f x)^3 \text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{(-d e + c f) (a + b x)}{(b c - a d) (e + f x)} \right] \right)$$

**Problem 3008: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + b x)^{1/3} (e + f x)^2}{(c + d x)^{1/3}} dx$$

Optimal (type 3, 475 leaves, 4 steps):

$$\frac{(5 a^2 d^2 f^2 - 2 a b d f (9 d e - 4 c f) + b^2 (27 d^2 e^2 - 36 c d e f + 14 c^2 f^2)) (a + b x)^{1/3} (c + d x)^{2/3}}{27 b^2 d^3} + \\ \frac{f (12 b d e - 7 b c f - 5 a d f) (a + b x)^{4/3} (c + d x)^{2/3}}{18 b^2 d^2} + \frac{f (a + b x)^{4/3} (c + d x)^{2/3} (e + f x)}{3 b d} + \frac{1}{27 \sqrt{3} b^{8/3} d^{10/3}} \\ (b c - a d) (5 a^2 d^2 f^2 - 2 a b d f (9 d e - 4 c f) + b^2 (27 d^2 e^2 - 36 c d e f + 14 c^2 f^2)) \text{ArcTan} \left[ \frac{1}{\sqrt{3}} + \frac{2 b^{1/3} (c + d x)^{1/3}}{\sqrt{3} d^{1/3} (a + b x)^{1/3}} \right] + \\ \frac{(b c - a d) (5 a^2 d^2 f^2 - 2 a b d f (9 d e - 4 c f) + b^2 (27 d^2 e^2 - 36 c d e f + 14 c^2 f^2)) \text{Log}[a + b x]}{162 b^{8/3} d^{10/3}} + \\ \frac{(b c - a d) (5 a^2 d^2 f^2 - 2 a b d f (9 d e - 4 c f) + b^2 (27 d^2 e^2 - 36 c d e f + 14 c^2 f^2)) \text{Log}[-1 + \frac{b^{1/3} (c + d x)^{1/3}}{d^{1/3} (a + b x)^{1/3}}]}{54 b^{8/3} d^{10/3}}$$

Result (type 5, 229 leaves):

$$\frac{1}{54 b^2 d^4 (a + b x)^{2/3}} \\ (c + d x)^{2/3} \left( d (a + b x) \left( -5 a^2 d^2 f^2 + a b d f (-5 c f + 3 d (6 e + f x)) + b^2 (28 c^2 f^2 - 3 c d f (24 e + 7 f x) + 18 d^2 (3 e^2 + 3 e f x + f^2 x^2)) \right) - \right. \\ \left. (b c - a d) (5 a^2 d^2 f^2 + 2 a b d f (-9 d e + 4 c f) + b^2 (27 d^2 e^2 - 36 c d e f + 14 c^2 f^2)) \left( \frac{d (a + b x)}{-b c + a d} \right)^{2/3} \text{Hypergeometric2F1} \left[ \frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b (c + d x)}{b c - a d} \right] \right)$$

### Problem 3009: Result unnecessarily involves higher level functions.

$$\int \frac{(a+b x)^{1/3} (e+f x)}{(c+d x)^{1/3}} dx$$

Optimal (type 3, 273 leaves, 3 steps):

$$\begin{aligned} & \frac{(3 b d e - 2 b c f - a d f) (a+b x)^{1/3} (c+d x)^{2/3}}{3 b d^2} + \frac{f (a+b x)^{4/3} (c+d x)^{2/3}}{2 b d} + \frac{(b c - a d) (3 b d e - 2 b c f - a d f) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 b^{1/3} (c+d x)^{1/3}}{\sqrt{3} d^{1/3} (a+b x)^{1/3}}\right]}{3 \sqrt{3} b^{5/3} d^{7/3}} + \\ & \frac{(b c - a d) (3 b d e - 2 b c f - a d f) \operatorname{Log}[a+b x]}{18 b^{5/3} d^{7/3}} + \frac{(b c - a d) (3 b d e - 2 b c f - a d f) \operatorname{Log}\left[-1 + \frac{b^{1/3} (c+d x)^{1/3}}{d^{1/3} (a+b x)^{1/3}}\right]}{6 b^{5/3} d^{7/3}} \end{aligned}$$

Result (type 5, 129 leaves):

$$\begin{aligned} & \frac{1}{6 b d^3 (a+b x)^{2/3}} (c+d x)^{2/3} \\ & \left( d (a+b x) (a d f + b (6 d e - 4 c f + 3 d f x)) + (b c - a d) (-3 b d e + 2 b c f + a d f) \left(\frac{d (a+b x)}{-b c + a d}\right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b (c+d x)}{b c - a d}\right] \right) \end{aligned}$$

### Problem 3010: Result unnecessarily involves higher level functions.

$$\int \frac{(a+b x)^{1/3}}{(c+d x)^{1/3}} dx$$

Optimal (type 3, 171 leaves, 2 steps):

$$\begin{aligned} & \frac{(a+b x)^{1/3} (c+d x)^{2/3}}{d} + \frac{(b c - a d) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 b^{1/3} (c+d x)^{1/3}}{\sqrt{3} d^{1/3} (a+b x)^{1/3}}\right]}{\sqrt{3} b^{2/3} d^{4/3}} + \frac{(b c - a d) \operatorname{Log}[a+b x]}{6 b^{2/3} d^{4/3}} + \frac{(b c - a d) \operatorname{Log}\left[-1 + \frac{b^{1/3} (c+d x)^{1/3}}{d^{1/3} (a+b x)^{1/3}}\right]}{2 b^{2/3} d^{4/3}} \end{aligned}$$

Result (type 5, 76 leaves):

$$\frac{(a+b x)^{1/3} (c+d x)^{2/3} \left(2 + \frac{\operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b (c+d x)}{b c - a d}\right]}{\left(\frac{d (a+b x)}{-b c + a d}\right)^{1/3}}\right)}{2 d}$$

### Problem 3011: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{1/3}}{(c + d x)^{1/3} (e + f x)} dx$$

Optimal (type 3, 339 leaves, 4 steps):

$$\begin{aligned} & -\frac{\sqrt{3} b^{1/3} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 b^{1/3} (c+d x)^{1/3}}{\sqrt{3} d^{1/3} (a+b x)^{1/3}}\right]}{d^{1/3} f} + \frac{\sqrt{3} (b e - a f)^{1/3} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 (b e - a f)^{1/3} (c+d x)^{1/3}}{\sqrt{3} (d e - c f)^{1/3} (a+b x)^{1/3}}\right]}{f (d e - c f)^{1/3}} - \frac{b^{1/3} \operatorname{Log}[a + b x]}{2 d^{1/3} f} - \\ & \frac{(b e - a f)^{1/3} \operatorname{Log}[e + f x]}{2 f (d e - c f)^{1/3}} + \frac{3 (b e - a f)^{1/3} \operatorname{Log}\left[-(a + b x)^{1/3} + \frac{(b e - a f)^{1/3} (c+d x)^{1/3}}{(d e - c f)^{1/3}}\right]}{2 f (d e - c f)^{1/3}} - \frac{3 b^{1/3} \operatorname{Log}\left[-1 + \frac{b^{1/3} (c+d x)^{1/3}}{d^{1/3} (a+b x)^{1/3}}\right]}{2 d^{1/3} f} \end{aligned}$$

Result (type 6, 290 leaves):

$$\begin{aligned} & -\left(\left(21 (b c - a d) (b e - a f)^2 (a + b x)^{4/3} \operatorname{AppellF1}\left[\frac{4}{3}, \frac{1}{3}, 1, \frac{7}{3}, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}\right]\right) / \right. \\ & \left.\left(4 b (-b e + a f) (c + d x)^{1/3} (e + f x) \left(7 (b c - a d) (b e - a f) \operatorname{AppellF1}\left[\frac{4}{3}, \frac{1}{3}, 1, \frac{7}{3}, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}\right] + (a + b x) \left((-3 b c f + 3 a d f)\right.\right.\right.\right. \\ & \left.\left.\left.\left.\operatorname{AppellF1}\left[\frac{7}{3}, \frac{1}{3}, 2, \frac{10}{3}, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}\right] + d (-b e + a f) \operatorname{AppellF1}\left[\frac{7}{3}, \frac{4}{3}, 1, \frac{10}{3}, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}\right]\right)\right)\right) \end{aligned}$$

### Problem 3012: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{1/3}}{(c + d x)^{1/3} (e + f x)^2} dx$$

Optimal (type 3, 256 leaves, 2 steps):

$$\begin{aligned} & \frac{(a + b x)^{1/3} (c + d x)^{2/3}}{(d e - c f) (e + f x)} + \frac{(b c - a d) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 (b e - a f)^{1/3} (c+d x)^{1/3}}{\sqrt{3} (d e - c f)^{1/3} (a+b x)^{1/3}}\right]}{\sqrt{3} (b e - a f)^{2/3} (d e - c f)^{4/3}} - \\ & \frac{(b c - a d) \operatorname{Log}[e + f x]}{6 (b e - a f)^{2/3} (d e - c f)^{4/3}} + \frac{(b c - a d) \operatorname{Log}\left[-(a + b x)^{1/3} + \frac{(b e - a f)^{1/3} (c+d x)^{1/3}}{(d e - c f)^{1/3}}\right]}{2 (b e - a f)^{2/3} (d e - c f)^{4/3}} \end{aligned}$$

Result (type 5, 124 leaves):

$$\frac{\left( a + b x \right)^{1/3} \left( c + d x \right)^{2/3} \left( \frac{1}{d e - c f} + \frac{\text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{(-d e + c f)(a + b x)}{(b c - a d)(e + f x)}\right]}{(-d e + c f) \left(\frac{(b e - a f)(c + d x)}{(b c - a d)(e + f x)}\right)^{2/3}} \right)}{e + f x}$$

**Problem 3013:** Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{1/3}}{(c + d x)^{1/3} (e + f x)^3} dx$$

Optimal (type 3, 386 leaves, 3 steps):

$$\begin{aligned} & -\frac{f (a + b x)^{4/3} (c + d x)^{2/3}}{2 (b e - a f) (d e - c f) (e + f x)^2} + \frac{(3 b d e - b c f - 2 a d f) (a + b x)^{1/3} (c + d x)^{2/3}}{3 (b e - a f) (d e - c f)^2 (e + f x)} + \\ & \frac{(b c - a d) (3 b d e - b c f - 2 a d f) \text{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 (b e - a f)^{1/3} (c + d x)^{1/3}}{\sqrt{3} (d e - c f)^{1/3} (a + b x)^{1/3}}\right]}{3 \sqrt{3} (b e - a f)^{5/3} (d e - c f)^{7/3}} - \frac{(b c - a d) (3 b d e - b c f - 2 a d f) \text{Log}[e + f x]}{18 (b e - a f)^{5/3} (d e - c f)^{7/3}} + \\ & \frac{(b c - a d) (3 b d e - b c f - 2 a d f) \text{Log}\left[-(a + b x)^{1/3} + \frac{(b e - a f)^{1/3} (c + d x)^{1/3}}{(d e - c f)^{1/3}}\right]}{6 (b e - a f)^{5/3} (d e - c f)^{7/3}} \end{aligned}$$

Result (type 5, 212 leaves):

$$\left( (a + b x)^{1/3} \left( (b e - a f) (c + d x) (3 (b e - a f) (d e - c f) + (3 b d e + b c f - 4 a d f) (e + f x)) + 2 (b c - a d) (-3 b d e + b c f + 2 a d f) \left( \frac{(b e - a f) (c + d x)}{(b c - a d) (e + f x)} \right)^{1/3} (e + f x)^2 \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{(-d e + c f) (a + b x)}{(b c - a d) (e + f x)}\right] \right) \right) / \left( 6 (b e - a f)^2 (d e - c f)^2 (c + d x)^{1/3} (e + f x)^2 \right)$$

**Problem 3014:** Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{1/3}}{(c + d x)^{1/3} (e + f x)^4} dx$$

Optimal (type 3, 591 leaves, 5 steps):

$$\begin{aligned}
& \frac{(a+b x)^{1/3} (c+d x)^{2/3}}{3 (d e - c f) (e + f x)^3} + \frac{(6 b d e + b c f - 7 a d f) (a+b x)^{1/3} (c+d x)^{2/3}}{18 (b e - a f) (d e - c f)^2 (e + f x)^2} + \\
& \frac{(28 a^2 d^2 f^2 - a b d f (51 d e + 5 c f) + b^2 (18 d^2 e^2 + 15 c d e f - 5 c^2 f^2)) (a+b x)^{1/3} (c+d x)^{2/3}}{54 (b e - a f)^2 (d e - c f)^3 (e + f x)} + \\
& \left( (b c - a d) (14 a^2 d^2 f^2 - 4 a b d f (9 d e - 2 c f) + b^2 (27 d^2 e^2 - 18 c d e f + 5 c^2 f^2)) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 (b e - a f)^{1/3} (c+d x)^{1/3}}{\sqrt{3} (d e - c f)^{1/3} (a+b x)^{1/3}}\right] \right) / \\
& \left( 27 \sqrt{3} (b e - a f)^{8/3} (d e - c f)^{10/3} \right) - \frac{(b c - a d) (14 a^2 d^2 f^2 - 4 a b d f (9 d e - 2 c f) + b^2 (27 d^2 e^2 - 18 c d e f + 5 c^2 f^2)) \operatorname{Log}[e + f x]}{162 (b e - a f)^{8/3} (d e - c f)^{10/3}} + \\
& \left( (b c - a d) (14 a^2 d^2 f^2 - 4 a b d f (9 d e - 2 c f) + b^2 (27 d^2 e^2 - 18 c d e f + 5 c^2 f^2)) \operatorname{Log}\left[-(a+b x)^{1/3} + \frac{(b e - a f)^{1/3} (c+d x)^{1/3}}{(d e - c f)^{1/3}}\right] \right) / \\
& (54 (b e - a f)^{8/3} (d e - c f)^{10/3})
\end{aligned}$$

Result (type 5, 334 leaves):

$$\begin{aligned}
& \frac{1}{54 (b e - a f)^3 (d e - c f)^3 (c + d x)^{1/3} (e + f x)^3} \\
& (a+b x)^{1/3} \left( (b e - a f) (c + d x) \left( 18 (b e - a f)^2 (d e - c f)^2 + 3 (b e - a f) (d e - c f) (6 b d e + b c f - 7 a d f) (e + f x) + \right. \right. \\
& \left. \left. (28 a^2 d^2 f^2 - a b d f (51 d e + 5 c f) + b^2 (18 d^2 e^2 + 15 c d e f - 5 c^2 f^2)) (e + f x)^2 \right) - \right. \\
& 2 (b c - a d) (14 a^2 d^2 f^2 + 4 a b d f (-9 d e + 2 c f) + b^2 (27 d^2 e^2 - 18 c d e f + 5 c^2 f^2)) \left( \frac{(b e - a f) (c + d x)}{(b c - a d) (e + f x)} \right)^{1/3} \\
& (e + f x)^3 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{(-d e + c f) (a + b x)}{(b c - a d) (e + f x)}\right]
\end{aligned}$$

Problem 3015: Result unnecessarily involves higher level functions.

$$\int \frac{(e + f x)^3}{(a + b x)^{1/3} (c + d x)^{2/3}} dx$$

Optimal (type 3, 587 leaves, 3 steps):

$$\begin{aligned}
& \frac{f (a + b x)^{2/3} (c + d x)^{1/3} (e + f x)^2}{3 b d} + \frac{1}{54 b^3 d^3} \\
& f (a + b x)^{2/3} (c + d x)^{1/3} (28 a^2 d^2 f^2 - a b d f (108 d e - 31 c f) + b^2 (144 d^2 e^2 - 135 c d e f + 40 c^2 f^2) + 3 b d f (15 b d e - 8 b c f - 7 a d f) x) + \\
& \frac{1}{27 \sqrt{3} b^{10/3} d^{11/3}} (14 a^3 d^3 f^3 - 6 a^2 b d^2 f^2 (9 d e - 2 c f) + 3 a b^2 d f (27 d^2 e^2 - 18 c d e f + 5 c^2 f^2) - b^3 (81 d^3 e^3 - 162 c d^2 e^2 f + 135 c^2 d e f^2 - 40 c^3 f^3)) \\
& \text{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 d^{1/3} (a + b x)^{1/3}}{\sqrt{3} b^{1/3} (c + d x)^{1/3}}\right] + \frac{1}{162 b^{10/3} d^{11/3}} \\
& (14 a^3 d^3 f^3 - 6 a^2 b d^2 f^2 (9 d e - 2 c f) + 3 a b^2 d f (27 d^2 e^2 - 18 c d e f + 5 c^2 f^2) - b^3 (81 d^3 e^3 - 162 c d^2 e^2 f + 135 c^2 d e f^2 - 40 c^3 f^3)) \text{Log}[c + d x] + \\
& \frac{1}{54 b^{10/3} d^{11/3}} (14 a^3 d^3 f^3 - 6 a^2 b d^2 f^2 (9 d e - 2 c f) + 3 a b^2 d f (27 d^2 e^2 - 18 c d e f + 5 c^2 f^2) - b^3 (81 d^3 e^3 - 162 c d^2 e^2 f + 135 c^2 d e f^2 - 40 c^3 f^3)) \\
& \text{Log}\left[-1 + \frac{d^{1/3} (a + b x)^{1/3}}{b^{1/3} (c + d x)^{1/3}}\right]
\end{aligned}$$

Result (type 5, 275 leaves):

$$\begin{aligned}
& \frac{1}{54 b^3 d^4 (a + b x)^{1/3}} \\
& (c + d x)^{1/3} \left( d f (a + b x) (28 a^2 d^2 f^2 + a b d f (31 c f - 3 d (36 e + 7 f x))) + b^2 (40 c^2 f^2 - 3 c d f (45 e + 8 f x) + 9 d^2 (18 e^2 + 9 e f x + 2 f^2 x^2)) \right) + \\
& 2 (-14 a^3 d^3 f^3 + 6 a^2 b d^2 f^2 (9 d e - 2 c f) - 3 a b^2 d f (27 d^2 e^2 - 18 c d e f + 5 c^2 f^2) + b^3 (81 d^3 e^3 - 162 c d^2 e^2 f + 135 c^2 d e f^2 - 40 c^3 f^3)) \\
& \left( \frac{d (a + b x)}{-b c + a d} \right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{b (c + d x)}{b c - a d}\right]
\end{aligned}$$

Problem 3016: Result unnecessarily involves higher level functions.

$$\int \frac{(e + f x)^2}{(a + b x)^{1/3} (c + d x)^{2/3}} dx$$

Optimal (type 3, 369 leaves, 3 steps):

$$\frac{f (9 b d e - 5 b c f - 4 a d f) (a + b x)^{2/3} (c + d x)^{1/3}}{6 b^2 d^2} + \frac{f (a + b x)^{2/3} (c + d x)^{1/3} (e + f x)}{2 b d} -$$

$$\frac{(2 a^2 d^2 f^2 - 2 a b d f (3 d e - c f) + b^2 (9 d^2 e^2 - 12 c d e f + 5 c^2 f^2)) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 d^{1/3} (a+b x)^{1/3}}{\sqrt{3} b^{1/3} (c+d x)^{1/3}}\right]}{3 \sqrt{3} b^{7/3} d^{8/3}} -$$

$$\frac{(2 a^2 d^2 f^2 - 2 a b d f (3 d e - c f) + b^2 (9 d^2 e^2 - 12 c d e f + 5 c^2 f^2)) \operatorname{Log}[c + d x]}{18 b^{7/3} d^{8/3}} -$$

$$\frac{(2 a^2 d^2 f^2 - 2 a b d f (3 d e - c f) + b^2 (9 d^2 e^2 - 12 c d e f + 5 c^2 f^2)) \operatorname{Log}\left[-1 + \frac{d^{1/3} (a+b x)^{1/3}}{b^{1/3} (c+d x)^{1/3}}\right]}{6 b^{7/3} d^{8/3}}$$

Result (type 5, 162 leaves):

$$\frac{1}{6 b^2 d^3 (a + b x)^{1/3}} (c + d x)^{1/3} \left( d f (a + b x) (-5 b c f - 4 a d f + 3 b d (4 e + f x)) + \right.$$

$$\left. 2 (2 a^2 d^2 f^2 + 2 a b d f (-3 d e + c f) + b^2 (9 d^2 e^2 - 12 c d e f + 5 c^2 f^2)) \left( \frac{d (a + b x)}{-b c + a d} \right)^{1/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{b (c + d x)}{b c - a d}\right] \right)$$

Problem 3017: Result unnecessarily involves higher level functions.

$$\int \frac{e + f x}{(a + b x)^{1/3} (c + d x)^{2/3}} dx$$

Optimal (type 3, 200 leaves, 2 steps):

$$\frac{f (a + b x)^{2/3} (c + d x)^{1/3}}{b d} - \frac{(3 b d e - 2 b c f - a d f) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 d^{1/3} (a+b x)^{1/3}}{\sqrt{3} b^{1/3} (c+d x)^{1/3}}\right]}{\sqrt{3} b^{4/3} d^{5/3}} -$$

$$\frac{(3 b d e - 2 b c f - a d f) \operatorname{Log}[c + d x]}{6 b^{4/3} d^{5/3}} - \frac{(3 b d e - 2 b c f - a d f) \operatorname{Log}\left[-1 + \frac{d^{1/3} (a+b x)^{1/3}}{b^{1/3} (c+d x)^{1/3}}\right]}{2 b^{4/3} d^{5/3}}$$

Result (type 5, 99 leaves):

$$\frac{(c + d x)^{1/3} \left( d f (a + b x) + (3 b d e - 2 b c f - a d f) \left( \frac{d (a + b x)}{-b c + a d} \right)^{1/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{b (c + d x)}{b c - a d}\right] \right)}{b d^2 (a + b x)^{1/3}}$$

### Problem 3018: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + b x)^{1/3} (c + d x)^{2/3}} dx$$

Optimal (type 3, 126 leaves, 1 step):

$$-\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2 d^{1/3} (a+b x)^{1/3}}{\sqrt{3} b^{1/3} (c+d x)^{1/3}}\right]}{b^{1/3} d^{2/3}}-\frac{\operatorname{Log}[c+d x]}{2 b^{1/3} d^{2/3}}-\frac{3 \operatorname{Log}\left[-1+\frac{d^{1/3} (a+b x)^{1/3}}{b^{1/3} (c+d x)^{1/3}}\right]}{2 b^{1/3} d^{2/3}}$$

Result (type 5, 71 leaves):

$$\frac{3 \left(\frac{d (a+b x)}{-b c+a d}\right)^{1/3} (c+d x)^{1/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{b (c+d x)}{b c-a d}\right]}{d (a+b x)^{1/3}}$$

### Problem 3019: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + b x)^{1/3} (c + d x)^{2/3} (e + f x)} dx$$

Optimal (type 3, 197 leaves, 1 step):

$$-\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2 (d e-c f)^{1/3} (a+b x)^{1/3}}{\sqrt{3} (b e-a f)^{1/3} (c+d x)^{1/3}}\right]}{(b e-a f)^{1/3} (d e-c f)^{2/3}}+\frac{\operatorname{Log}[e+f x]}{2 (b e-a f)^{1/3} (d e-c f)^{2/3}}-\frac{3 \operatorname{Log}\left[\frac{(d e-c f)^{1/3} (a+b x)^{1/3}}{(b e-a f)^{1/3}}-(c+d x)^{1/3}\right]}{2 (b e-a f)^{1/3} (d e-c f)^{2/3}}$$

Result (type 5, 108 leaves):

$$\frac{3 (a+b x)^{2/3} \left(\frac{(b e-a f) (c+d x)}{(b c-a d) (e+f x)}\right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(-d e+c f) (a+b x)}{(b c-a d) (e+f x)}\right]}{2 (b e-a f) (c+d x)^{2/3}}$$

### Problem 3020: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + b x)^{1/3} (c + d x)^{2/3} (e + f x)^2} dx$$

Optimal (type 3, 293 leaves, 2 steps):

$$\begin{aligned}
& - \frac{f (a+b x)^{2/3} (c+d x)^{1/3}}{(b e-a f) (d e-c f) (e+f x)} - \frac{\left(3 b d e-b c f-2 a d f\right) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2 (d e-c f)^{1/3} (a+b x)^{1/3}}{\sqrt{3} (b e-a f)^{1/3} (c+d x)^{1/3}}\right]}{\sqrt{3} (b e-a f)^{4/3} (d e-c f)^{5/3}} + \\
& \frac{\left(3 b d e-b c f-2 a d f\right) \operatorname{Log}[e+f x]}{6 (b e-a f)^{4/3} (d e-c f)^{5/3}} - \frac{\left(3 b d e-b c f-2 a d f\right) \operatorname{Log}\left[\frac{(d e-c f)^{1/3} (a+b x)^{1/3}}{(b e-a f)^{1/3}}-\left(c+d x\right)^{1/3}\right]}{2 (b e-a f)^{4/3} (d e-c f)^{5/3}}
\end{aligned}$$

Result (type 5, 171 leaves):

$$\frac{\left(a+b x\right)^{2/3} \left(\frac{2 f (c+d x)}{(-d e+c f) (e+f x)}+\frac{\left(3 b d e-b c f-2 a d f\right)\left(\frac{(b e-a f) (c+d x)}{(b c-a d) (e+f x)}\right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(-d e+c f) (a+b x)}{(b c-a d) (e+f x)}\right]}{(b e-a f) (d e-c f)}\right)}{2 (b e-a f) (c+d x)^{2/3}}$$

Problem 3021: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a+b x)^{1/3} (c+d x)^{2/3} (e+f x)^3} dx$$

Optimal (type 3, 477 leaves, 4 steps):

$$\begin{aligned}
& - \frac{f (a+b x)^{2/3} (c+d x)^{1/3}}{2 (b e-a f) (d e-c f) (e+f x)^2} - \frac{f (9 b d e-4 b c f-5 a d f) (a+b x)^{2/3} (c+d x)^{1/3}}{6 (b e-a f)^2 (d e-c f)^2 (e+f x)} - \\
& \frac{\left(5 a^2 d^2 f^2-2 a b d f (6 d e-c f)+b^2 (9 d^2 e^2-6 c d e f+2 c^2 f^2)\right) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2 (d e-c f)^{1/3} (a+b x)^{1/3}}{\sqrt{3} (b e-a f)^{1/3} (c+d x)^{1/3}}\right]}{3 \sqrt{3} (b e-a f)^{7/3} (d e-c f)^{8/3}} + \\
& \frac{\left(5 a^2 d^2 f^2-2 a b d f (6 d e-c f)+b^2 (9 d^2 e^2-6 c d e f+2 c^2 f^2)\right) \operatorname{Log}[e+f x]}{18 (b e-a f)^{7/3} (d e-c f)^{8/3}} - \\
& \frac{\left(5 a^2 d^2 f^2-2 a b d f (6 d e-c f)+b^2 (9 d^2 e^2-6 c d e f+2 c^2 f^2)\right) \operatorname{Log}\left[\frac{(d e-c f)^{1/3} (a+b x)^{1/3}}{(b e-a f)^{1/3}}-\left(c+d x\right)^{1/3}\right]}{6 (b e-a f)^{7/3} (d e-c f)^{8/3}}
\end{aligned}$$

Result (type 5, 244 leaves):

$$\begin{aligned} & \left( (a + b x)^{2/3} \left( -f (b e - a f) (c + d x) (3 (b e - a f) (d e - c f) + (9 b d e - 4 b c f - 5 a d f) (e + f x)) + \right. \right. \\ & \left. \left. (5 a^2 d^2 f^2 + 2 a b d f (-6 d e + c f) + b^2 (9 d^2 e^2 - 6 c d e f + 2 c^2 f^2)) \left( \frac{(b e - a f) (c + d x)}{(b c - a d) (e + f x)} \right)^{2/3} (e + f x)^2 \right. \right. \\ & \left. \left. \text{Hypergeometric2F1} \left[ \frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(-d e + c f) (a + b x)}{(b c - a d) (e + f x)} \right] \right] \right) \Big/ \left( 6 (b e - a f)^3 (d e - c f)^2 (c + d x)^{2/3} (e + f x)^2 \right) \end{aligned}$$

Problem 3022: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^3}{(c + d x)^{1/3} (b c + a d + 2 b d x)^{1/3}} dx$$

Optimal (type 4, 1389 leaves, 7 steps):

$$\begin{aligned} & \frac{3 (a + b x)^2 (c + d x)^{2/3} (b c + a d + 2 b d x)^{2/3}}{20 d^2} + \frac{9 (b c - a d) (c + d x)^{2/3} (23 b c - 39 a d - 16 b d x) (b c + a d + 2 b d x)^{2/3}}{560 d^4} - \\ & \left( 81 (b c - a d)^3 ((c + d x) (b c + a d + 2 b d x))^{1/3} \sqrt{d^2 (3 b c + a d + 4 b d x)^2} \sqrt{(d (3 b c + a d) + 4 b d^2 x)^2} \right) \Big/ \\ & \left( 112 b^{2/3} d^6 (c + d x)^{1/3} (b c + a d + 2 b d x)^{1/3} (3 b c + a d + 4 b d x) \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} \right) \right. + \\ & \left. \left( 81 \times 3^{1/4} \sqrt{2 - \sqrt{3}} (b c - a d)^{11/3} ((c + d x) (b c + a d + 2 b d x))^{1/3} \right. \right. \\ & \left. \left. \sqrt{(d (3 b c + a d) + 4 b d^2 x)^2} ((b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3}) \right. \right. \\ & \left. \left. \sqrt{\left( ((b c - a d)^{4/3} - 2 b^{1/3} (b c - a d)^{2/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} + 4 b^{2/3} ((c + d x) (a d + b (c + 2 d x)))^{2/3}\right) / \right. \right. \\ & \left. \left. \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3}\right)^2 \right) \right. \right. \\ & \left. \left. \text{EllipticE} [\text{ArcSin} \left[ \frac{(1 - \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3}}{(1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3}}, -7 - 4 \sqrt{3} \right], \right. \right. \end{aligned}$$

$$\left. \left. 224 b^{2/3} d^4 (c + d x)^{1/3} (b c + a d + 2 b d x)^{1/3} (3 b c + a d + 4 b d x) \sqrt{d^2 (3 b c + a d + 4 b d x)^2} \right. \right)$$

$$\begin{aligned}
& \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3})}{((1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3})^2}} - \\
& \left( 27 \times 3^{3/4} (b c - a d)^{11/3} ((c + d x) (b c + a d + 2 b d x))^{1/3} \sqrt{(d (3 b c + a d) + 4 b d^2 x)^2} ((b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3}) \right. \\
& \left. \sqrt{\left( ((b c - a d)^{4/3} - 2 b^{1/3} (b c - a d)^{2/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} + 4 b^{2/3} ((c + d x) (a d + b (c + 2 d x)))^{2/3}) / \right.} \right. \\
& \left. \left. \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} \right)^2 \right) \right. \\
& \left. \text{EllipticF}[\text{ArcSin}\left[\frac{(1 - \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3}}{(1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3}}\right], -7 - 4\sqrt{3}] \right) / \\
& \left( 56 \sqrt{2} b^{2/3} d^4 (c + d x)^{1/3} (b c + a d + 2 b d x)^{1/3} (3 b c + a d + 4 b d x) \sqrt{d^2 (3 b c + a d + 4 b d x)^2} \right. \\
& \left. \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3})}{((1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3})^2}} \right)
\end{aligned}$$

Result (type 5, 160 leaves):

$$\begin{aligned}
& -\frac{1}{1120 b d^4 (c + d x)^{1/3}} 3 (a d + b (c + 2 d x))^{2/3} \left( -2 b (c + d x) (145 a^2 d^2 + 2 a b d (-93 c + 52 d x) + b^2 (69 c^2 - 48 c d x + 28 d^2 x^2)) + \right. \\
& \left. 135 \times 2^{1/3} (b c - a d)^3 \left( \frac{b (c + d x)}{b c - a d} \right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{a d + b (c + 2 d x)}{-b c + a d}\right] \right)
\end{aligned}$$

Problem 3023: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^2}{(c + d x)^{1/3} (b c + a d + 2 b d x)^{1/3}} dx$$

Optimal (type 4, 1373 leaves, 7 steps):

$$-\frac{45 (b c - a d) (c + d x)^{2/3} (b c + a d + 2 b d x)^{2/3}}{112 d^3} + \frac{3 (a + b x) (c + d x)^{2/3} (b c + a d + 2 b d x)^{2/3}}{14 d^2} +$$

$$\begin{aligned}
& \left( 99 (b c - a d)^2 ((c + d x) (b c + a d + 2 b d x))^{1/3} \sqrt{d^2 (3 b c + a d + 4 b d x)^2} \sqrt{(d (3 b c + a d) + 4 b d^2 x)^2} \right) / \\
& \left( 112 b^{2/3} d^5 (c + d x)^{1/3} (b c + a d + 2 b d x)^{1/3} (3 b c + a d + 4 b d x) \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} \right) \right) - \\
& \left( 99 \times 3^{1/4} \sqrt{2 - \sqrt{3}} (b c - a d)^{8/3} ((c + d x) (b c + a d + 2 b d x))^{1/3} \sqrt{(d (3 b c + a d) + 4 b d^2 x)^2} \right. \\
& \left. \left( (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} \right) \sqrt{\left( (b c - a d)^{4/3} - 2 b^{1/3} (b c - a d)^{2/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} + \right.} \right. \\
& \left. \left. 4 b^{2/3} ((c + d x) (a d + b (c + 2 d x)))^{2/3} \right) / \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} \right)^2 \right) \\
& \text{EllipticE} \left[ \text{ArcSin} \left[ \frac{(1 - \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3}}{(1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3}} \right], -7 - 4 \sqrt{3} \right] / \\
& \left( 224 b^{2/3} d^3 (c + d x)^{1/3} (b c + a d + 2 b d x)^{1/3} (3 b c + a d + 4 b d x) \sqrt{d^2 (3 b c + a d + 4 b d x)^2} \right. \\
& \left. \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3})}{((1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3})^2}} \right) + \\
& \left( 33 \times 3^{3/4} (b c - a d)^{8/3} ((c + d x) (b c + a d + 2 b d x))^{1/3} \sqrt{(d (3 b c + a d) + 4 b d^2 x)^2} \left( (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} \right) \right. \\
& \left. \sqrt{\left( (b c - a d)^{4/3} - 2 b^{1/3} (b c - a d)^{2/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} + 4 b^{2/3} ((c + d x) (a d + b (c + 2 d x)))^{2/3} \right) /} \right. \\
& \left. \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} \right)^2 \right) \\
& \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{(1 - \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3}}{(1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3}} \right], -7 - 4 \sqrt{3} \right] / \\
& \left( 56 \sqrt{2} b^{2/3} d^3 (c + d x)^{1/3} (b c + a d + 2 b d x)^{1/3} (3 b c + a d + 4 b d x) \sqrt{d^2 (3 b c + a d + 4 b d x)^2} \right. \\
& \left. \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3})}{((1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3})^2}} \right)
\end{aligned}$$

Result (type 5, 129 leaves):

$$\frac{1}{224 b d^3 (c + d x)^{1/3}} 3 (a d + b (c + 2 d x))^{2/3} \left( -2 b (c + d x) (15 b c - 23 a d - 8 b d x) + 33 \times 2^{1/3} (b c - a d)^2 \left( \frac{b (c + d x)}{b c - a d} \right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{a d + b (c + 2 d x)}{-b c + a d}\right] \right)$$

Problem 3024: Result unnecessarily involves higher level functions.

$$\int \frac{a + b x}{(c + d x)^{1/3} (b c + a d + 2 b d x)^{1/3}} dx$$

Optimal (type 4, 1326 leaves, 6 steps):

$$\begin{aligned} & \frac{3 (c + d x)^{2/3} (b c + a d + 2 b d x)^{2/3}}{8 d^2} - \left( 9 (b c - a d) ((c + d x) (b c + a d + 2 b d x))^{1/3} \sqrt{d^2 (3 b c + a d + 4 b d x)^2} \sqrt{(d (3 b c + a d) + 4 b d^2 x)^2} \right) / \\ & \left( 8 b^{2/3} d^4 (c + d x)^{1/3} (b c + a d + 2 b d x)^{1/3} (3 b c + a d + 4 b d x) \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} \right) + \right. \\ & \left( 9 \times 3^{1/4} \sqrt{2 - \sqrt{3}} (b c - a d)^{5/3} ((c + d x) (b c + a d + 2 b d x))^{1/3} \right. \\ & \left. \sqrt{(d (3 b c + a d) + 4 b d^2 x)^2} \left( (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} \right) \right. \\ & \left. \sqrt{\left( (b c - a d)^{4/3} - 2 b^{1/3} (b c - a d)^{2/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} + 4 b^{2/3} ((c + d x) (a d + b (c + 2 d x)))^{2/3} \right) /} \right. \\ & \left. \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} \right)^2 \right) \\ & \left. \text{EllipticE}[\text{ArcSin}\left[\frac{(1 - \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3}}{(1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3}}, -7 - 4 \sqrt{3}\right]] \right) / \\ & \left( 16 b^{2/3} d^2 (c + d x)^{1/3} (b c + a d + 2 b d x)^{1/3} (3 b c + a d + 4 b d x) \sqrt{d^2 (3 b c + a d + 4 b d x)^2} \right. \\ & \left. \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3})}{((1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3})^2}} \right) - \\ & \left( 3 \times 3^{3/4} (b c - a d)^{5/3} ((c + d x) (b c + a d + 2 b d x))^{1/3} \sqrt{(d (3 b c + a d) + 4 b d^2 x)^2} \left( (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} \right) \right) \end{aligned}$$

$$\begin{aligned}
& \sqrt{\left( \left( (b c - a d)^{4/3} - 2 b^{1/3} (b c - a d)^{2/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} + 4 b^{2/3} ((c + d x) (a d + b (c + 2 d x)))^{2/3} \right) / \right.} \\
& \left. \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} \right)^2 \right) \\
& \text{EllipticF}[\text{ArcSin}\left[ \frac{(1 - \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3}}{(1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3}} \right], -7 - 4 \sqrt{3}] \Bigg) / \\
& \left( 4 \sqrt{2} b^{2/3} d^2 (c + d x)^{1/3} (b c + a d + 2 b d x)^{1/3} (3 b c + a d + 4 b d x) \sqrt{d^2 (3 b c + a d + 4 b d x)^2} \right. \\
& \left. \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3})}{((1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3})^2}} \right)
\end{aligned}$$

Result (type 5, 95 leaves):

$$-\frac{3 (c + d x)^{2/3} (a d + b (c + 2 d x))^{2/3} \left( -2 + \frac{3 \times 2^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{a d + b (c + 2 d x)}{-b c + a d}\right]}{\left(\frac{b (c + d x)}{b c - a d}\right)^{2/3}} \right)}{16 d^2}$$

Problem 3025: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(c + d x)^{1/3} (b c + a d + 2 b d x)^{1/3}} dx$$

Optimal (type 4, 1283 leaves, 5 steps):

$$\begin{aligned}
& \left( 3 ((c + d x) (b c + a d + 2 b d x))^{1/3} \sqrt{d^2 (3 b c + a d + 4 b d x)^2} \sqrt{(d (3 b c + a d) + 4 b d^2 x)^2} \right) / \\
& \left( 2 b^{2/3} d^3 (c + d x)^{1/3} (b c + a d + 2 b d x)^{1/3} (3 b c + a d + 4 b d x) \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} \right) - \right. \\
& \left. \left( 3 \times 3^{1/4} \sqrt{2 - \sqrt{3}} (b c - a d)^{2/3} ((c + d x) (b c + a d + 2 b d x))^{1/3} \right. \right. \\
& \left. \left. \sqrt{(d (3 b c + a d) + 4 b d^2 x)^2 ((b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3})} \right. \right. \\
& \left. \left. \sqrt{\left( ((b c - a d)^{4/3} - 2 b^{1/3} (b c - a d)^{2/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} + 4 b^{2/3} ((c + d x) (a d + b (c + 2 d x)))^{2/3}\right) / \right.} \right. \\
& \left. \left. \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} \right)^2 \right) \right)
\end{aligned}$$

$$\begin{aligned} & \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) \left(b c-a d\right)^{2/3}+2 b^{1/3} \left(\left(c+d x\right) \left(a d+b \left(c+2 d x\right)\right)\right)^{1/3}}{\left(1+\sqrt{3}\right) \left(b c-a d\right)^{2/3}+2 b^{1/3} \left(\left(c+d x\right) \left(a d+b \left(c+2 d x\right)\right)\right)^{1/3}}\right], -7-4 \sqrt{3}\right]\Bigg] \\ & \left(4 b^{2/3} d \left(c+d x\right)^{1/3} \left(b c+a d+2 b d x\right)^{1/3} \left(3 b c+a d+4 b d x\right) \sqrt{d^2 \left(3 b c+a d+4 b d x\right)^2}\right. \\ & \left.\sqrt{\frac{\left(b c-a d\right)^{2/3} \left(\left(b c-a d\right)^{2/3}+2 b^{1/3} \left(\left(c+d x\right) \left(a d+b \left(c+2 d x\right)\right)\right)^{1/3}\right)}{\left(\left(1+\sqrt{3}\right) \left(b c-a d\right)^{2/3}+2 b^{1/3} \left(\left(c+d x\right) \left(a d+b \left(c+2 d x\right)\right)\right)^{1/3}\right)^2}}\right)+ \end{aligned}$$

$$\left(3^{3/4} \left(b c-a d\right)^{2/3} \left(\left(c+d x\right) \left(b c+a d+2 b d x\right)\right)^{1/3} \sqrt{\left(d \left(3 b c+a d\right)+4 b d^2 x\right)^2} \left(\left(b c-a d\right)^{2/3}+2 b^{1/3} \left(\left(c+d x\right) \left(a d+b \left(c+2 d x\right)\right)\right)^{1/3}\right)\right.$$

$$\left.\sqrt{\left(\left(b c-a d\right)^{4/3}-2 b^{1/3} \left(b c-a d\right)^{2/3} \left(\left(c+d x\right) \left(a d+b \left(c+2 d x\right)\right)\right)^{1/3}+4 b^{2/3} \left(\left(c+d x\right) \left(a d+b \left(c+2 d x\right)\right)\right)^{2/3}\right)/\right. \\ \left.\left(\left(1+\sqrt{3}\right) \left(b c-a d\right)^{2/3}+2 b^{1/3} \left(\left(c+d x\right) \left(a d+b \left(c+2 d x\right)\right)\right)^{1/3}\right)^2\right)$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) \left(b c-a d\right)^{2/3}+2 b^{1/3} \left(\left(c+d x\right) \left(a d+b \left(c+2 d x\right)\right)\right)^{1/3}}{\left(1+\sqrt{3}\right) \left(b c-a d\right)^{2/3}+2 b^{1/3} \left(\left(c+d x\right) \left(a d+b \left(c+2 d x\right)\right)\right)^{1/3}}\right], -7-4 \sqrt{3}\right]\Bigg]$$

$$\left(\sqrt{2} b^{2/3} d \left(c+d x\right)^{1/3} \left(b c+a d+2 b d x\right)^{1/3} \left(3 b c+a d+4 b d x\right) \sqrt{d^2 \left(3 b c+a d+4 b d x\right)^2}\right. \\ \left.\sqrt{\frac{\left(b c-a d\right)^{2/3} \left(\left(b c-a d\right)^{2/3}+2 b^{1/3} \left(\left(c+d x\right) \left(a d+b \left(c+2 d x\right)\right)\right)^{1/3}\right)}{\left(\left(1+\sqrt{3}\right) \left(b c-a d\right)^{2/3}+2 b^{1/3} \left(\left(c+d x\right) \left(a d+b \left(c+2 d x\right)\right)\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 94 leaves):

$$\frac{3 \left(\frac{b (c+d x)}{b c-a d}\right)^{1/3} \left(a d+b \left(c+2 d x\right)\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{a d+b \left(c+2 d x\right)}{-b c+a d}\right]}{2 \times 2^{2/3} b d \left(c+d x\right)^{1/3}}$$

Problem 3026: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a+b x) \left(c+d x\right)^{1/3} \left(b c+a d+2 b d x\right)^{1/3}} dx$$

Optimal (type 3, 178 leaves, 1 step):

$$-\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2 b^{2/3} (c+d x)^{2/3}}{\sqrt{3} (b c-a d)^{1/3} (b c+a d+2 b d x)^{1/3}}\right]}{2 b^{2/3} (b c-a d)^{2/3}}-\frac{\operatorname{Log}[a+b x]}{2 b^{2/3} (b c-a d)^{2/3}}+\frac{3 \operatorname{Log}\left[\frac{b^{2/3} (c+d x)^{2/3}}{(b c-a d)^{1/3}}-\left(b c+a d+2 b d x\right)^{1/3}\right]}{4 b^{2/3} (b c-a d)^{2/3}}$$

### Result (type 6, 276 leaves):

$$-\left( \left( 15 d (a + b x) \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \frac{-b c + a d}{d (a + b x)}, -\frac{b c - a d}{2 a d + 2 b d x} \right] \right) / \right. \\ \left. \left( b (c + d x)^{1/3} (a d + b (c + 2 d x))^{1/3} \left( 10 d (a + b x) \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \frac{-b c + a d}{d (a + b x)}, -\frac{b c - a d}{2 a d + 2 b d x} \right] - \right. \right. \right. \\ \left. \left. \left. (b c - a d) \left( \operatorname{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \frac{-b c + a d}{d (a + b x)}, -\frac{b c - a d}{2 a d + 2 b d x} \right] + 2 \operatorname{AppellF1}\left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \frac{-b c + a d}{d (a + b x)}, -\frac{b c - a d}{2 a d + 2 b d x} \right] \right) \right) \right) \right)$$

## Problem 3027: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + b x)^2 (c + d x)^{1/3} (b c + a d + 2 b d x)^{1/3}} dx$$

Optimal (type 4, 1510 leaves, 8 steps):

$$\begin{aligned}
& - \frac{(c + d x)^{2/3} (b c + a d + 2 b d x)^{2/3}}{(b c - a d)^2 (a + b x)} + \left( \left( (c + d x) (b c + a d + 2 b d x) \right)^{1/3} \sqrt{d^2 (3 b c + a d + 4 b d x)^2} \sqrt{(d (3 b c + a d) + 4 b d^2 x)^2} \right) / \\
& \left( b^{2/3} d (b c - a d)^2 (c + d x)^{1/3} (b c + a d + 2 b d x)^{1/3} (3 b c + a d + 4 b d x) \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} \right) \right) + \\
& \frac{\sqrt{3} d \operatorname{ArcTan} \left[ \frac{1}{\sqrt{3}} + \frac{2 b^{2/3} (c + d x)^{2/3}}{\sqrt{3} (b c - a d)^{1/3} (b c + a d + 2 b d x)^{1/3}} \right]}{2 b^{2/3} (b c - a d)^{5/3}} - \\
& \left( 3^{1/4} \sqrt{2 - \sqrt{3}} d ((c + d x) (b c + a d + 2 b d x))^{1/3} \sqrt{(d (3 b c + a d) + 4 b d^2 x)^2} \left( (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} \right) \right. \\
& \left. \sqrt{\left( (b c - a d)^{4/3} - 2 b^{1/3} (b c - a d)^{2/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} + 4 b^{2/3} ((c + d x) (a d + b (c + 2 d x)))^{2/3} \right) /} \right. \\
& \left. \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} \right)^2 \right) \\
& \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \frac{(1 - \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3}}{(1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3}} \right], -7 - 4 \sqrt{3} \right] / \\
& \left( 2 b^{2/3} (b c - a d)^{4/3} (c + d x)^{1/3} (b c + a d + 2 b d x)^{1/3} (3 b c + a d + 4 b d x) \sqrt{d^2 (3 b c + a d + 4 b d x)^2} \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3})}{((1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3})^2}} + \\
& \left( \sqrt{2} d ((c + d x) (b c + a d + 2 b d x))^{1/3} \sqrt{(d (3 b c + a d) + 4 b d^2 x)^2} ((b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3}) \right. \\
& \left. \sqrt{\left( ((b c - a d)^{4/3} - 2 b^{1/3} (b c - a d)^{2/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} + 4 b^{2/3} ((c + d x) (a d + b (c + 2 d x)))^{2/3}) / \right.} \right. \\
& \left. \left. \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} \right)^2 \right) \right) \\
& \text{EllipticF}[\text{ArcSin}\left[\frac{(1 - \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3}}{(1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3}}\right], -7 - 4 \sqrt{3}] \Bigg) / \\
& \left( 3^{1/4} b^{2/3} (b c - a d)^{4/3} (c + d x)^{1/3} (b c + a d + 2 b d x)^{1/3} (3 b c + a d + 4 b d x) \sqrt{d^2 (3 b c + a d + 4 b d x)^2} \right. \\
& \left. \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3})}{((1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3})^2}} \right. \\
& \left. \frac{d \log[a + b x]}{2 b^{2/3} (b c - a d)^{5/3}} - \frac{3 d \log\left[\frac{b^{2/3} (c + d x)^{2/3}}{(b c - a d)^{1/3}} - (b c + a d + 2 b d x)^{1/3}\right]}{4 b^{2/3} (b c - a d)^{5/3}} \right)
\end{aligned}$$

Result (type 6, 593 leaves):

$$\begin{aligned}
& \frac{1}{5 (b c - a d)^2} (c + d x)^{2/3} (a d + b (c + 2 d x))^{2/3} \\
& \left( -\frac{5}{a + b x} + \frac{1}{b c + a d + 2 b d x} d \left( 10 - \frac{5 c}{c + d x} + \frac{5 a d}{b c + b d x} + \left( 100 b (b c - a d) (c + d x) \text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, 1, \frac{5}{3}, \frac{b c - a d}{2 b c + 2 b d x}, \frac{b c - a d}{b c + b d x}\right] \right) / \right. \\
& \left. \left( d (a + b x) \left( 10 b (c + d x) \text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, 1, \frac{5}{3}, \frac{b c - a d}{2 b c + 2 b d x}, \frac{b c - a d}{b c + b d x}\right] + (b c - a d) \right. \right. \right. \\
& \left. \left. \left. \left( 6 \text{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, 2, \frac{8}{3}, \frac{b c - a d}{2 b c + 2 b d x}, \frac{b c - a d}{b c + b d x}\right] + \text{AppellF1}\left[\frac{5}{3}, \frac{4}{3}, 1, \frac{8}{3}, \frac{b c - a d}{2 b c + 2 b d x}, \frac{b c - a d}{b c + b d x}\right] \right) \right) - \left( 16 (b c - a d)^2 \right. \right. \\
& \left. \left. \left. \text{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, 1, \frac{8}{3}, \frac{b c - a d}{2 b c + 2 b d x}, \frac{b c - a d}{b c + b d x}\right] \right) / \left( d (a + b x) \left( 16 b (c + d x) \text{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, 1, \frac{8}{3}, \frac{b c - a d}{2 b c + 2 b d x}, \frac{b c - a d}{b c + b d x}\right] + \right. \right. \right. \\
& \left. \left. \left. (b c - a d) \left( 6 \text{AppellF1}\left[\frac{8}{3}, \frac{1}{3}, 2, \frac{11}{3}, \frac{b c - a d}{2 b c + 2 b d x}, \frac{b c - a d}{b c + b d x}\right] + \text{AppellF1}\left[\frac{8}{3}, \frac{4}{3}, 1, \frac{11}{3}, \frac{b c - a d}{2 b c + 2 b d x}, \frac{b c - a d}{b c + b d x}\right] \right) \right) \right) \right)
\end{aligned}$$

### Problem 3028: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + b x)^3 (c + d x)^{1/3} (b c + a d + 2 b d x)^{1/3}} dx$$

Optimal (type 4, 1558 leaves, 9 steps):

$$\begin{aligned}
& -\frac{(c + d x)^{2/3} (b c + a d + 2 b d x)^{2/3}}{2 (b c - a d)^2 (a + b x)^2} + \frac{2 d (c + d x)^{2/3} (b c + a d + 2 b d x)^{2/3}}{(b c - a d)^3 (a + b x)} - \\
& \left( 2 ((c + d x) (b c + a d + 2 b d x))^{1/3} \sqrt{d^2 (3 b c + a d + 4 b d x)^2} \sqrt{(d (3 b c + a d) + 4 b d^2 x)^2} \right) / \\
& \left( b^{2/3} (b c - a d)^3 (c + d x)^{1/3} (b c + a d + 2 b d x)^{1/3} (3 b c + a d + 4 b d x) \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} \right) - \right. \\
& \left. \frac{2 d^2 \text{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 b^{2/3} (c+d x)^{2/3}}{\sqrt{3} (b c - a d)^{1/3} (b c + a d + 2 b d x)^{1/3}}\right]}{\sqrt{3} b^{2/3} (b c - a d)^{8/3}} + \right. \\
& \left. \left( 3^{1/4} \sqrt{2 - \sqrt{3}} d^2 ((c + d x) (b c + a d + 2 b d x))^{1/3} \sqrt{(d (3 b c + a d) + 4 b d^2 x)^2} \left( (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} \right) \right. \right. \\
& \left. \left. \sqrt{\left( ((b c - a d)^{4/3} - 2 b^{1/3} (b c - a d)^{2/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} + 4 b^{2/3} ((c + d x) (a d + b (c + 2 d x)))^{2/3}\right) / \right.} \right. \\
& \left. \left. \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} \right)^2 \right) \right. \\
& \left. \left. \text{EllipticE}[\text{ArcSin}\left[\frac{(1 - \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3}}{(1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3}}\right], -7 - 4 \sqrt{3} \right] \right) / \\
& \left( b^{2/3} (b c - a d)^{7/3} (c + d x)^{1/3} (b c + a d + 2 b d x)^{1/3} (3 b c + a d + 4 b d x) \sqrt{d^2 (3 b c + a d + 4 b d x)^2} \right. \\
& \left. \left. \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3})}{((1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3})^2}} \right) - \\
& \left( 2 \sqrt{2} d^2 ((c + d x) (b c + a d + 2 b d x))^{1/3} \sqrt{(d (3 b c + a d) + 4 b d^2 x)^2} \left( (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} \right) \right. \\
& \left. \left. \sqrt{\left( ((b c - a d)^{4/3} - 2 b^{1/3} (b c - a d)^{2/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} + 4 b^{2/3} ((c + d x) (a d + b (c + 2 d x)))^{2/3}\right) / \right.} \right)
\end{aligned}$$

$$\begin{aligned}
& \left( \left( 1 + \sqrt{3} \right) (b c - a d)^{2/3} + 2 b^{1/3} \left( (c + d x) (a d + b (c + 2 d x))^{1/3} \right)^2 \right) \\
& \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\left( 1 - \sqrt{3} \right) (b c - a d)^{2/3} + 2 b^{1/3} \left( (c + d x) (a d + b (c + 2 d x))^{1/3} \right)}{\left( 1 + \sqrt{3} \right) (b c - a d)^{2/3} + 2 b^{1/3} \left( (c + d x) (a d + b (c + 2 d x))^{1/3} \right)} \right], -7 - 4 \sqrt{3} \right] \Bigg) \\
& \left( \frac{3^{1/4} b^{2/3} (b c - a d)^{7/3} (c + d x)^{1/3} (b c + a d + 2 b d x)^{1/3} (3 b c + a d + 4 b d x) \sqrt{d^2 (3 b c + a d + 4 b d x)^2}}{\sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x))^{1/3}))^2}{\left( 1 + \sqrt{3} \right) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x))^{1/3})^2}}} \right. \\
& \left. - \frac{2 d^2 \text{Log}[a + b x]}{3 b^{2/3} (b c - a d)^{8/3}} + \frac{d^2 \text{Log} \left[ \frac{b^{2/3} (c + d x)^{2/3}}{(b c - a d)^{1/3}} - (b c + a d + 2 b d x)^{1/3} \right]}{b^{2/3} (b c - a d)^{8/3}} \right)
\end{aligned}$$

Result (type 6, 620 leaves):

$$\begin{aligned}
& \frac{1}{10} (c + d x)^{2/3} (a d + b (c + 2 d x))^{2/3} \\
& \left( \frac{5 (-b c + 5 a d + 4 b d x)}{(b c - a d)^3 (a + b x)^2} + \left( 4 d^2 \left( 10 - \frac{5 c}{c + d x} + \frac{5 a d}{b c + b d x} + \left( 75 b (b c - a d) (c + d x) \text{AppellF1} \left[ \frac{2}{3}, \frac{1}{3}, 1, \frac{5}{3}, \frac{b c - a d}{2 b c + 2 b d x}, \frac{b c - a d}{b c + b d x} \right] \right) \right) \right. \\
& \left. \left( d (a + b x) \left( 10 b (c + d x) \text{AppellF1} \left[ \frac{2}{3}, \frac{1}{3}, 1, \frac{5}{3}, \frac{b c - a d}{2 b c + 2 b d x}, \frac{b c - a d}{b c + b d x} \right] + (b c - a d) \left( 6 \text{AppellF1} \left[ \frac{5}{3}, \frac{1}{3}, 2, \frac{8}{3}, \frac{b c - a d}{2 b c + 2 b d x}, \frac{b c - a d}{b c + b d x} \right] + \text{AppellF1} \left[ \frac{5}{3}, \frac{4}{3}, 1, \frac{8}{3}, \frac{b c - a d}{2 b c + 2 b d x}, \frac{b c - a d}{b c + b d x} \right] \right) \right) \right. \\
& \left. \left( 16 (b c - a d)^2 \text{AppellF1} \left[ \frac{5}{3}, \frac{1}{3}, 1, \frac{8}{3}, \frac{b c - a d}{2 b c + 2 b d x}, \frac{b c - a d}{b c + b d x} \right] \right) \right) \Bigg/ \left( d (a + b x) \right. \\
& \left. \left( 16 b (c + d x) \text{AppellF1} \left[ \frac{5}{3}, \frac{1}{3}, 1, \frac{8}{3}, \frac{b c - a d}{2 b c + 2 b d x}, \frac{b c - a d}{b c + b d x} \right] + (b c - a d) \left( 6 \text{AppellF1} \left[ \frac{8}{3}, \frac{1}{3}, 2, \frac{11}{3}, \frac{b c - a d}{2 b c + 2 b d x}, \frac{b c - a d}{b c + b d x} \right] + \text{AppellF1} \left[ \frac{8}{3}, \frac{4}{3}, 1, \frac{11}{3}, \frac{b c - a d}{2 b c + 2 b d x}, \frac{b c - a d}{b c + b d x} \right] \right) \right) \right) \Bigg) \Bigg/ \left( (-b c + a d)^3 (b c + a d + 2 b d x) \right)
\end{aligned}$$

Problem 3029: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^3}{(c + d x)^{1/3} (b c + a d + 2 b d x)^{4/3}} dx$$

Optimal (type 4, 1388 leaves, 7 steps):

$$\begin{aligned}
& \frac{3 (a + b x)^2 (c + d x)^{2/3}}{14 d^2 (b c + a d + 2 b d x)^{1/3}} + \frac{9 (b c - a d) (c + d x)^{2/3} (b c - 7 a d - 6 b d x)}{112 d^4 (b c + a d + 2 b d x)^{1/3}} + \\
& \left( \frac{81 (b c - a d)^2 ((c + d x) (b c + a d + 2 b d x))^{1/3} \sqrt{d^2 (3 b c + a d + 4 b d x)^2}}{\sqrt{(d (3 b c + a d) + 4 b d^2 x)^2}} \right) / \\
& \left( 112 b^{2/3} d^6 (c + d x)^{1/3} (b c + a d + 2 b d x)^{1/3} (3 b c + a d + 4 b d x) \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} \right) - \right. \\
& \left. \left( 81 \times 3^{1/4} \sqrt{2 - \sqrt{3}} (b c - a d)^{8/3} ((c + d x) (b c + a d + 2 b d x))^{1/3} \right. \right. \\
& \left. \left. \sqrt{(d (3 b c + a d) + 4 b d^2 x)^2} ((b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3}) \right. \right. \\
& \left. \left. \sqrt{\left( (b c - a d)^{4/3} - 2 b^{1/3} (b c - a d)^{2/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} + 4 b^{2/3} ((c + d x) (a d + b (c + 2 d x)))^{2/3} \right) / \right. \right. \\
& \left. \left. \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} \right)^2 \right) \right. \right. \\
& \left. \left. \text{EllipticE}[\text{ArcSin}\left[\frac{(1 - \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3}}{(1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3}}\right], -7 - 4 \sqrt{3}] \right) / \\
& \left( 224 b^{2/3} d^4 (c + d x)^{1/3} (b c + a d + 2 b d x)^{1/3} (3 b c + a d + 4 b d x) \sqrt{d^2 (3 b c + a d + 4 b d x)^2} \right. \\
& \left. \left. \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3})}{((1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3})^2}} \right) + \\
& \left( 27 \times 3^{3/4} (b c - a d)^{8/3} ((c + d x) (b c + a d + 2 b d x))^{1/3} \sqrt{(d (3 b c + a d) + 4 b d^2 x)^2} ((b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3}) \right. \\
& \left. \sqrt{\left( (b c - a d)^{4/3} - 2 b^{1/3} (b c - a d)^{2/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} + 4 b^{2/3} ((c + d x) (a d + b (c + 2 d x)))^{2/3} \right) / \right. \right. \\
& \left. \left. \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} \right)^2 \right) \right. \right. \\
& \left. \left. \text{EllipticF}[\text{ArcSin}\left[\frac{(1 - \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3}}{(1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3}}\right], -7 - 4 \sqrt{3}] \right) / \\
& \left( 56 \sqrt{2} b^{2/3} d^4 (c + d x)^{1/3} (b c + a d + 2 b d x)^{1/3} (3 b c + a d + 4 b d x) \sqrt{d^2 (3 b c + a d + 4 b d x)^2} \right)
\end{aligned}$$

$$\sqrt{\frac{(\mathbf{b} \mathbf{c} - \mathbf{a} \mathbf{d})^{2/3} ((\mathbf{b} \mathbf{c} - \mathbf{a} \mathbf{d})^{2/3} + 2 \mathbf{b}^{1/3} ((\mathbf{c} + \mathbf{d} \mathbf{x}) (\mathbf{a} \mathbf{d} + \mathbf{b} (\mathbf{c} + 2 \mathbf{d} \mathbf{x})))^{1/3})}{((1 + \sqrt{3}) (\mathbf{b} \mathbf{c} - \mathbf{a} \mathbf{d})^{2/3} + 2 \mathbf{b}^{1/3} ((\mathbf{c} + \mathbf{d} \mathbf{x}) (\mathbf{a} \mathbf{d} + \mathbf{b} (\mathbf{c} + 2 \mathbf{d} \mathbf{x})))^{1/3})^2}}$$

Result (type 5, 157 leaves):

$$\frac{1}{224 (\mathbf{c} + \mathbf{d} \mathbf{x})^{1/3}} (\mathbf{a} \mathbf{d} + \mathbf{b} (\mathbf{c} + 2 \mathbf{d} \mathbf{x}))^{2/3} \\ \left( \frac{6 (\mathbf{c} + \mathbf{d} \mathbf{x}) (-11 \mathbf{b} \mathbf{c} + 15 \mathbf{a} \mathbf{d} + 4 \mathbf{b} \mathbf{d} \mathbf{x} + \frac{14 (\mathbf{b} \mathbf{c} - \mathbf{a} \mathbf{d})^2}{\mathbf{a} \mathbf{d} + \mathbf{b} (\mathbf{c} + 2 \mathbf{d} \mathbf{x})})}{\mathbf{d}^4} + \frac{81 \times 2^{1/3} (\mathbf{b} \mathbf{c} - \mathbf{a} \mathbf{d})^2 \left(\frac{\mathbf{b} (\mathbf{c} + \mathbf{d} \mathbf{x})}{\mathbf{b} \mathbf{c} - \mathbf{a} \mathbf{d}}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{\mathbf{a} \mathbf{d} + \mathbf{b} (\mathbf{c} + 2 \mathbf{d} \mathbf{x})}{-\mathbf{b} \mathbf{c} + \mathbf{a} \mathbf{d}}\right]}{\mathbf{b} \mathbf{d}^4} \right)$$

Problem 3030: Result unnecessarily involves higher level functions.

$$\int \frac{(\mathbf{a} + \mathbf{b} \mathbf{x})^2}{(\mathbf{c} + \mathbf{d} \mathbf{x})^{1/3} (\mathbf{b} \mathbf{c} + \mathbf{a} \mathbf{d} + 2 \mathbf{b} \mathbf{d} \mathbf{x})^{4/3}} d\mathbf{x}$$

Optimal (type 4, 1366 leaves, 7 steps):

$$-\frac{3 (\mathbf{b} \mathbf{c} - \mathbf{a} \mathbf{d}) (\mathbf{c} + \mathbf{d} \mathbf{x})^{2/3}}{4 \mathbf{d}^3 (\mathbf{b} \mathbf{c} + \mathbf{a} \mathbf{d} + 2 \mathbf{b} \mathbf{d} \mathbf{x})^{1/3}} + \frac{3 (\mathbf{c} + \mathbf{d} \mathbf{x})^{2/3} (\mathbf{b} \mathbf{c} + \mathbf{a} \mathbf{d} + 2 \mathbf{b} \mathbf{d} \mathbf{x})^{2/3}}{16 \mathbf{d}^3} - \\ \left( \frac{9 (\mathbf{b} \mathbf{c} - \mathbf{a} \mathbf{d}) ((\mathbf{c} + \mathbf{d} \mathbf{x}) (\mathbf{b} \mathbf{c} + \mathbf{a} \mathbf{d} + 2 \mathbf{b} \mathbf{d} \mathbf{x}))^{1/3} \sqrt{\mathbf{d}^2 (3 \mathbf{b} \mathbf{c} + \mathbf{a} \mathbf{d} + 4 \mathbf{b} \mathbf{d} \mathbf{x})^2}}{\sqrt{(\mathbf{d} (3 \mathbf{b} \mathbf{c} + \mathbf{a} \mathbf{d}) + 4 \mathbf{b} \mathbf{d}^2 \mathbf{x})^2}} \right) / \\ (16 \mathbf{b}^{2/3} \mathbf{d}^5 (\mathbf{c} + \mathbf{d} \mathbf{x})^{1/3} (\mathbf{b} \mathbf{c} + \mathbf{a} \mathbf{d} + 2 \mathbf{b} \mathbf{d} \mathbf{x})^{1/3} (3 \mathbf{b} \mathbf{c} + \mathbf{a} \mathbf{d} + 4 \mathbf{b} \mathbf{d} \mathbf{x}) ((1 + \sqrt{3}) (\mathbf{b} \mathbf{c} - \mathbf{a} \mathbf{d})^{2/3} + 2 \mathbf{b}^{1/3} ((\mathbf{c} + \mathbf{d} \mathbf{x}) (\mathbf{a} \mathbf{d} + \mathbf{b} (\mathbf{c} + 2 \mathbf{d} \mathbf{x})))^{1/3})) + \\ \left( 9 \times 3^{1/4} \sqrt{2 - \sqrt{3}} (\mathbf{b} \mathbf{c} - \mathbf{a} \mathbf{d})^{5/3} ((\mathbf{c} + \mathbf{d} \mathbf{x}) (\mathbf{b} \mathbf{c} + \mathbf{a} \mathbf{d} + 2 \mathbf{b} \mathbf{d} \mathbf{x}))^{1/3} \sqrt{(\mathbf{d} (3 \mathbf{b} \mathbf{c} + \mathbf{a} \mathbf{d}) + 4 \mathbf{b} \mathbf{d}^2 \mathbf{x})^2} \right. \\ \left. ((\mathbf{b} \mathbf{c} - \mathbf{a} \mathbf{d})^{2/3} + 2 \mathbf{b}^{1/3} ((\mathbf{c} + \mathbf{d} \mathbf{x}) (\mathbf{a} \mathbf{d} + \mathbf{b} (\mathbf{c} + 2 \mathbf{d} \mathbf{x})))^{1/3}) \sqrt{\left( ((\mathbf{b} \mathbf{c} - \mathbf{a} \mathbf{d})^{4/3} - 2 \mathbf{b}^{1/3} (\mathbf{b} \mathbf{c} - \mathbf{a} \mathbf{d})^{2/3} ((\mathbf{c} + \mathbf{d} \mathbf{x}) (\mathbf{a} \mathbf{d} + \mathbf{b} (\mathbf{c} + 2 \mathbf{d} \mathbf{x})))^{1/3} + 4 \mathbf{b}^{2/3} ((\mathbf{c} + \mathbf{d} \mathbf{x}) (\mathbf{a} \mathbf{d} + \mathbf{b} (\mathbf{c} + 2 \mathbf{d} \mathbf{x})))^{2/3}) / ((1 + \sqrt{3}) (\mathbf{b} \mathbf{c} - \mathbf{a} \mathbf{d})^{2/3} + 2 \mathbf{b}^{1/3} ((\mathbf{c} + \mathbf{d} \mathbf{x}) (\mathbf{a} \mathbf{d} + \mathbf{b} (\mathbf{c} + 2 \mathbf{d} \mathbf{x})))^{1/3})^2} \right) / \\ \text{EllipticE}[\text{ArcSin}\left[\frac{(1 - \sqrt{3}) (\mathbf{b} \mathbf{c} - \mathbf{a} \mathbf{d})^{2/3} + 2 \mathbf{b}^{1/3} ((\mathbf{c} + \mathbf{d} \mathbf{x}) (\mathbf{a} \mathbf{d} + \mathbf{b} (\mathbf{c} + 2 \mathbf{d} \mathbf{x})))^{1/3}}{(1 + \sqrt{3}) (\mathbf{b} \mathbf{c} - \mathbf{a} \mathbf{d})^{2/3} + 2 \mathbf{b}^{1/3} ((\mathbf{c} + \mathbf{d} \mathbf{x}) (\mathbf{a} \mathbf{d} + \mathbf{b} (\mathbf{c} + 2 \mathbf{d} \mathbf{x})))^{1/3}}\right], -7 - 4 \sqrt{3}] \right) / \\ \left( 32 \mathbf{b}^{2/3} \mathbf{d}^3 (\mathbf{c} + \mathbf{d} \mathbf{x})^{1/3} (\mathbf{b} \mathbf{c} + \mathbf{a} \mathbf{d} + 2 \mathbf{b} \mathbf{d} \mathbf{x})^{1/3} (3 \mathbf{b} \mathbf{c} + \mathbf{a} \mathbf{d} + 4 \mathbf{b} \mathbf{d} \mathbf{x}) \sqrt{\mathbf{d}^2 (3 \mathbf{b} \mathbf{c} + \mathbf{a} \mathbf{d} + 4 \mathbf{b} \mathbf{d} \mathbf{x})^2} \right)$$

$$\begin{aligned}
& \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3})}{((1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3})^2}} - \\
& \left( 3 \times 3^{3/4} (b c - a d)^{5/3} ((c + d x) (b c + a d + 2 b d x))^{1/3} \sqrt{(d (3 b c + a d) + 4 b d^2 x)^2} ((b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3}) \right. \\
& \left. \sqrt{\left( ((b c - a d)^{4/3} - 2 b^{1/3} (b c - a d)^{2/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} + 4 b^{2/3} ((c + d x) (a d + b (c + 2 d x)))^{2/3}\right) / \right. \\
& \left. \left. \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3}\right)^2 \right) \\
& \text{EllipticF}[\text{ArcSin}\left[\frac{(1 - \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3}}{(1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3}}\right], -7 - 4 \sqrt{3}] \right) / \\
& \left( 8 \sqrt{2} b^{2/3} d^3 (c + d x)^{1/3} (b c + a d + 2 b d x)^{1/3} (3 b c + a d + 4 b d x) \sqrt{d^2 (3 b c + a d + 4 b d x)^2} \right. \\
& \left. \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3})}{((1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3})^2}} \right)
\end{aligned}$$

Result (type 5, 119 leaves):

$$\frac{3 (c + d x)^{2/3} \left( 6 b c - 10 a d - 4 b d x + \frac{3 \times 2^{1/3} (a d + b (c + 2 d x)) \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{a d + b (c + 2 d x)}{-b c + a d}\right]}{\left(\frac{b (c + d x)}{b c - a d}\right)^{2/3}} \right)}{32 d^3 (a d + b (c + 2 d x))^{1/3}}$$

Problem 3032: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(c + d x)^{1/3} (b c + a d + 2 b d x)^{4/3}} dx$$

Optimal (type 4, 1333 leaves, 6 steps):

$$\begin{aligned}
& - \frac{3 (c + d x)^{2/3}}{d (b c - a d) (b c + a d + 2 b d x)^{1/3}} + \left( 3 ((c + d x) (b c + a d + 2 b d x))^{1/3} \sqrt{d^2 (3 b c + a d + 4 b d x)^2} \sqrt{(d (3 b c + a d) + 4 b d^2 x)^2} \right) / \\
& \left( 2 b^{2/3} d^3 (b c - a d) (c + d x)^{1/3} (b c + a d + 2 b d x)^{1/3} (3 b c + a d + 4 b d x) \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} \right) \right) - \\
& \left( 3 \times 3^{1/4} \sqrt{2 - \sqrt{3}} ((c + d x) (b c + a d + 2 b d x))^{1/3} \sqrt{(d (3 b c + a d) + 4 b d^2 x)^2} \left( (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} \right) \right. \\
& \left. \sqrt{\left( (b c - a d)^{4/3} - 2 b^{1/3} (b c - a d)^{2/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} + 4 b^{2/3} ((c + d x) (a d + b (c + 2 d x)))^{2/3} \right) /} \right. \\
& \left. \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} \right)^2 \right) \\
& \text{EllipticE} \left[ \text{ArcSin} \left[ \frac{(1 - \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3}}{(1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3}} \right], -7 - 4\sqrt{3} \right] / \\
& \left( 4 b^{2/3} d (b c - a d)^{1/3} (c + d x)^{1/3} (b c + a d + 2 b d x)^{1/3} (3 b c + a d + 4 b d x) \sqrt{d^2 (3 b c + a d + 4 b d x)^2} \right. \\
& \left. \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3})}{((1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3})^2}} \right) + \\
& \left( 3^{3/4} ((c + d x) (b c + a d + 2 b d x))^{1/3} \sqrt{(d (3 b c + a d) + 4 b d^2 x)^2} \left( (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} \right) \right. \\
& \left. \sqrt{\left( (b c - a d)^{4/3} - 2 b^{1/3} (b c - a d)^{2/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} + 4 b^{2/3} ((c + d x) (a d + b (c + 2 d x)))^{2/3} \right) /} \right. \\
& \left. \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} \right)^2 \right) \\
& \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{(1 - \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3}}{(1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3}} \right], -7 - 4\sqrt{3} \right] / \\
& \left( \sqrt{2} b^{2/3} d (b c - a d)^{1/3} (c + d x)^{1/3} (b c + a d + 2 b d x)^{1/3} (3 b c + a d + 4 b d x) \sqrt{d^2 (3 b c + a d + 4 b d x)^2} \right. \\
& \left. \sqrt{\frac{(b c - a d)^{2/3} ((b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3})}{((1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3})^2}} \right)
\end{aligned}$$

Result (type 5, 127 leaves):

$$\frac{12 b (c + d x) - 3 \times 2^{1/3} \left( \frac{b (c+d x)}{b c-a d} \right)^{1/3} (a d + b (c + 2 d x)) \text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{a d+b (c+2 d x)}{-b c+a d} \right]}{4 b d (-b c + a d) (c + d x)^{1/3} (a d + b (c + 2 d x))^{1/3}}$$

Problem 3033: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + b x) (c + d x)^{1/3} (b c + a d + 2 b d x)^{4/3}} dx$$

Optimal (type 6, 113 leaves, 2 steps):

$$\frac{3 (c + d x)^{2/3} \left( -\frac{b c+a d+2 b d x}{b c-a d} \right)^{1/3} \text{AppellF1} \left[ \frac{2}{3}, \frac{4}{3}, 1, \frac{5}{3}, \frac{2 b (c+d x)}{b c-a d}, \frac{b (c+d x)}{b c-a d} \right]}{2 (b c - a d)^2 (b c + a d + 2 b d x)^{1/3}}$$

Result (type 6, 395 leaves):

$$\begin{aligned} & \left( -15 b (c + d x) \text{AppellF1} \left[ \frac{2}{3}, -\frac{2}{3}, 1, \frac{5}{3}, \frac{b c - a d}{2 b c + 2 b d x}, \frac{b c - a d}{b c + b d x} \right] + \right. \\ & 6 d (a + b x) \left( 3 \text{AppellF1} \left[ \frac{5}{3}, -\frac{2}{3}, 2, \frac{8}{3}, \frac{b c - a d}{2 b c + 2 b d x}, \frac{b c - a d}{b c + b d x} \right] - \text{AppellF1} \left[ \frac{5}{3}, \frac{1}{3}, 1, \frac{8}{3}, \frac{b c - a d}{2 b c + 2 b d x}, \frac{b c - a d}{b c + b d x} \right] \right) \Bigg) / \\ & \left( 2 b d (a + b x) (c + d x)^{1/3} (a d + b (c + 2 d x))^{1/3} \left( 5 b (c + d x) \text{AppellF1} \left[ \frac{2}{3}, -\frac{2}{3}, 1, \frac{5}{3}, \frac{b c - a d}{2 b c + 2 b d x}, \frac{b c - a d}{b c + b d x} \right] + \right. \right. \\ & \left. \left. (b c - a d) \left( 3 \text{AppellF1} \left[ \frac{5}{3}, -\frac{2}{3}, 2, \frac{8}{3}, \frac{b c - a d}{2 b c + 2 b d x}, \frac{b c - a d}{b c + b d x} \right] - \text{AppellF1} \left[ \frac{5}{3}, \frac{1}{3}, 1, \frac{8}{3}, \frac{b c - a d}{2 b c + 2 b d x}, \frac{b c - a d}{b c + b d x} \right] \right) \right) \right) \end{aligned}$$

Problem 3034: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + b x)^2 (c + d x)^{1/3} (b c + a d + 2 b d x)^{4/3}} dx$$

Optimal (type 6, 114 leaves, 2 steps):

$$\frac{3 d (c + d x)^{2/3} \left( -\frac{b c+a d+2 b d x}{b c-a d} \right)^{1/3} \text{AppellF1} \left[ \frac{2}{3}, \frac{4}{3}, 2, \frac{5}{3}, \frac{2 b (c+d x)}{b c-a d}, \frac{b (c+d x)}{b c-a d} \right]}{2 (b c - a d)^3 (b c + a d + 2 b d x)^{1/3}}$$

Result (type 6, 605 leaves):

$$\begin{aligned}
& \frac{1}{5 (a d + b (c + 2 d x))^{1/3}} \\
& \left( c + d x \right)^{2/3} \left( -\frac{5 (13 a d + b (c + 14 d x))}{(b c - a d)^3 (a + b x)} + \frac{1}{(-b c + a d)^3} d \left( - \left( \left( 400 b (b c - a d) (c + d x) \text{AppellF1} \left[ \frac{2}{3}, \frac{1}{3}, 1, \frac{5}{3}, \frac{b c - a d}{2 b c + 2 b d x}, \frac{b c - a d}{b c + b d x} \right] \right) / \right. \right. \right. \\
& \left. \left. \left. \left( d (a + b x) \left( 10 b (c + d x) \text{AppellF1} \left[ \frac{2}{3}, \frac{1}{3}, 1, \frac{5}{3}, \frac{b c - a d}{2 b c + 2 b d x}, \frac{b c - a d}{b c + b d x} \right] + \right. \right. \right. \right. \\
& \left. \left. \left. \left. (b c - a d) \left( 6 \text{AppellF1} \left[ \frac{5}{3}, \frac{1}{3}, 2, \frac{8}{3}, \frac{b c - a d}{2 b c + 2 b d x}, \frac{b c - a d}{b c + b d x} \right] + \text{AppellF1} \left[ \frac{5}{3}, \frac{4}{3}, 1, \frac{8}{3}, \frac{b c - a d}{2 b c + 2 b d x}, \frac{b c - a d}{b c + b d x} \right] \right) \right) \right) + \right. \\
& 7 \left( -10 + \frac{5 c}{c + d x} - \frac{5 a d}{b c + b d x} + \left( 16 (b c - a d)^2 \text{AppellF1} \left[ \frac{5}{3}, \frac{1}{3}, 1, \frac{8}{3}, \frac{b c - a d}{2 b c + 2 b d x}, \frac{b c - a d}{b c + b d x} \right] \right) / \right. \\
& \left. \left( d (a + b x) \left( 16 b (c + d x) \text{AppellF1} \left[ \frac{5}{3}, \frac{1}{3}, 1, \frac{8}{3}, \frac{b c - a d}{2 b c + 2 b d x}, \frac{b c - a d}{b c + b d x} \right] + (b c - a d) \right. \right. \right. \\
& \left. \left. \left. \left( 6 \text{AppellF1} \left[ \frac{8}{3}, \frac{1}{3}, 2, \frac{11}{3}, \frac{b c - a d}{2 b c + 2 b d x}, \frac{b c - a d}{b c + b d x} \right] + \text{AppellF1} \left[ \frac{8}{3}, \frac{4}{3}, 1, \frac{11}{3}, \frac{b c - a d}{2 b c + 2 b d x}, \frac{b c - a d}{b c + b d x} \right] \right) \right) \right) \right) \right)
\end{aligned}$$

Problem 3035: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + b x)^3 (c + d x)^{1/3} (b c + a d + 2 b d x)^{4/3}} dx$$

Optimal (type 6, 116 leaves, 2 steps):

$$\begin{aligned}
& \frac{3 d^2 (c + d x)^{2/3} \left( -\frac{b c + a d + 2 b d x}{b c - a d} \right)^{1/3} \text{AppellF1} \left[ \frac{2}{3}, \frac{4}{3}, 3, \frac{5}{3}, \frac{2 b (c + d x)}{b c - a d}, \frac{b (c + d x)}{b c - a d} \right]}{2 (b c - a d)^4 (b c + a d + 2 b d x)^{1/3}}
\end{aligned}$$

Result (type 6, 638 leaves):

$$\begin{aligned}
& \frac{1}{10 (b c - a d)^4} (c + d x)^{2/3} (a d + b (c + 2 d x))^{2/3} \left( 5 \left( \frac{-b c + a d}{(a + b x)^2} + \frac{8 d}{a + b x} + \frac{48 d^2}{b c + a d + 2 b d x} \right) - \right. \\
& \frac{1}{a d + b (c + 2 d x)} 4 d^2 \left( \left( 475 b (b c - a d) (c + d x) \text{AppellF1} \left[ \frac{2}{3}, \frac{1}{3}, 1, \frac{5}{3}, \frac{b c - a d}{2 b c + 2 b d x}, \frac{b c - a d}{b c + b d x} \right] \right) / \right. \\
& \left( d (a + b x) \left( 10 b (c + d x) \text{AppellF1} \left[ \frac{2}{3}, \frac{1}{3}, 1, \frac{5}{3}, \frac{b c - a d}{2 b c + 2 b d x}, \frac{b c - a d}{b c + b d x} \right] + \right. \right. \\
& (b c - a d) \left( 6 \text{AppellF1} \left[ \frac{5}{3}, \frac{1}{3}, 2, \frac{8}{3}, \frac{b c - a d}{2 b c + 2 b d x}, \frac{b c - a d}{b c + b d x} \right] + \text{AppellF1} \left[ \frac{5}{3}, \frac{4}{3}, 1, \frac{8}{3}, \frac{b c - a d}{2 b c + 2 b d x}, \frac{b c - a d}{b c + b d x} \right] \right) \left. \right) + \\
& 8 \left( 10 - \frac{5 c}{c + d x} + \frac{5 a d}{b c + b d x} - \left( 16 (b c - a d)^2 \text{AppellF1} \left[ \frac{5}{3}, \frac{1}{3}, 1, \frac{8}{3}, \frac{b c - a d}{2 b c + 2 b d x}, \frac{b c - a d}{b c + b d x} \right] \right) / \right. \\
& \left. \left( d (a + b x) \left( 16 b (c + d x) \text{AppellF1} \left[ \frac{5}{3}, \frac{1}{3}, 1, \frac{8}{3}, \frac{b c - a d}{2 b c + 2 b d x}, \frac{b c - a d}{b c + b d x} \right] + (b c - a d) \right. \right. \right. \\
& \left. \left. \left. \left( 6 \text{AppellF1} \left[ \frac{8}{3}, \frac{1}{3}, 2, \frac{11}{3}, \frac{b c - a d}{2 b c + 2 b d x}, \frac{b c - a d}{b c + b d x} \right] + \text{AppellF1} \left[ \frac{8}{3}, \frac{4}{3}, 1, \frac{11}{3}, \frac{b c - a d}{2 b c + 2 b d x}, \frac{b c - a d}{b c + b d x} \right] \right) \right) \right) \right) \right)
\end{aligned}$$

**Problem 3036: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(d - 3 e x)^{1/3} (d + e x) (d + 3 e x)^{1/3}} dx$$

Optimal (type 3, 120 leaves, 1 step):

$$\frac{\sqrt{3} \text{ArcTan} \left[ \frac{1}{\sqrt{3}} - \frac{(d - 3 e x)^{2/3}}{\sqrt{3} d^{1/3} (d + 3 e x)^{1/3}} \right]}{4 d^{2/3} e} + \frac{\text{Log}[d + e x]}{4 d^{2/3} e} - \frac{3 \text{Log} \left[ - \frac{(d - 3 e x)^{2/3}}{2 d^{1/3}} - (d + 3 e x)^{1/3} \right]}{8 d^{2/3} e}$$

Result (type 6, 196 leaves):

$$\begin{aligned}
& - \left( \left( 45 (d + e x) \text{AppellF1} \left[ \frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \frac{4 d}{3 (d + e x)}, \frac{2 d}{3 (d + e x)} \right] \right) / \right. \\
& \left( 2 e (d - 3 e x)^{1/3} (d + 3 e x)^{1/3} \left( 15 (d + e x) \text{AppellF1} \left[ \frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \frac{4 d}{3 (d + e x)}, \frac{2 d}{3 (d + e x)} \right] + \right. \right. \\
& \left. \left. 2 d \left( \text{AppellF1} \left[ \frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \frac{4 d}{3 (d + e x)}, \frac{2 d}{3 (d + e x)} \right] + 2 \text{AppellF1} \left[ \frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \frac{4 d}{3 (d + e x)}, \frac{2 d}{3 (d + e x)} \right] \right) \right) \right)
\end{aligned}$$

**Problem 3037: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + b x)^{4/3} (e + f x)^2}{(c + d x)^{4/3}} dx$$

Optimal (type 3, 562 leaves, 5 steps):

$$\begin{aligned}
 & \frac{3 (d e - c f)^2 (a + b x)^{7/3}}{d^2 (b c - a d) (c + d x)^{1/3}} - \frac{4 (a^2 d^2 f^2 - a b d f (9 d e - 7 c f) - b^2 (27 d^2 e^2 - 63 c d e f + 35 c^2 f^2)) (a + b x)^{1/3} (c + d x)^{2/3}}{27 b d^4} + \\
 & \frac{(a^2 d^2 f^2 - a b d f (9 d e - 7 c f) - b^2 (27 d^2 e^2 - 63 c d e f + 35 c^2 f^2)) (a + b x)^{4/3} (c + d x)^{2/3}}{9 b d^3 (b c - a d)} + \frac{f^2 (a + b x)^{7/3} (c + d x)^{2/3}}{3 b d^2} - \\
 & \frac{1}{27 \sqrt{3} b^{5/3} d^{13/3}} 4 (b c - a d) (a^2 d^2 f^2 - a b d f (9 d e - 7 c f) - b^2 (27 d^2 e^2 - 63 c d e f + 35 c^2 f^2)) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 b^{1/3} (c + d x)^{1/3}}{\sqrt{3} d^{1/3} (a + b x)^{1/3}}\right] - \\
 & \frac{2 (b c - a d) (a^2 d^2 f^2 - a b d f (9 d e - 7 c f) - b^2 (27 d^2 e^2 - 63 c d e f + 35 c^2 f^2)) \operatorname{Log}[a + b x]}{81 b^{5/3} d^{13/3}} - \\
 & \frac{2 (b c - a d) (a^2 d^2 f^2 - a b d f (9 d e - 7 c f) - b^2 (27 d^2 e^2 - 63 c d e f + 35 c^2 f^2)) \operatorname{Log}\left[-1 + \frac{b^{1/3} (c + d x)^{1/3}}{d^{1/3} (a + b x)^{1/3}}\right]}{27 b^{5/3} d^{13/3}}
 \end{aligned}$$

Result (type 5, 282 leaves):

$$\begin{aligned}
 & \frac{1}{27 b d^4} (a + b x)^{1/3} (c + d x)^{2/3} \\
 & \left( \frac{1}{c + d x} (2 a^2 d^2 f^2 (c + d x) + b^2 (140 c^3 f^2 + 7 c^2 d f (-36 e + 5 f x) + 3 c d^2 (36 e^2 - 21 e f x - 5 f^2 x^2) + 9 d^3 x (3 e^2 + 3 e f x + f^2 x^2))) + \right. \\
 & a b d (-133 c^2 f^2 + c d f (225 e - 37 f x) + d^2 (-81 e^2 + 63 e f x + 15 f^2 x^2)) + \frac{1}{\left(\frac{d (a+b x)}{-b c+a d}\right)^{1/3}} \\
 & \left. 2 (-a^2 d^2 f^2 + a b d f (9 d e - 7 c f) + b^2 (27 d^2 e^2 - 63 c d e f + 35 c^2 f^2)) \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b (c + d x)}{b c - a d}\right] \right)
 \end{aligned}$$

Problem 3038: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{4/3} (e + f x)}{(c + d x)^{4/3}} dx$$

Optimal (type 3, 328 leaves, 4 steps):

$$\begin{aligned}
& \frac{3 (d e - c f) (a + b x)^{7/3}}{d (b c - a d) (c + d x)^{1/3}} + \frac{2 (6 b d e - 7 b c f + a d f) (a + b x)^{1/3} (c + d x)^{2/3}}{3 d^3} - \\
& \frac{(6 b d e - 7 b c f + a d f) (a + b x)^{4/3} (c + d x)^{2/3}}{2 d^2 (b c - a d)} + \frac{2 (b c - a d) (6 b d e - 7 b c f + a d f) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 b^{1/3} (c+d x)^{1/3}}{\sqrt{3} d^{1/3} (a+b x)^{1/3}}\right]}{3 \sqrt{3} b^{2/3} d^{10/3}} + \\
& \frac{(b c - a d) (6 b d e - 7 b c f + a d f) \operatorname{Log}[a + b x]}{9 b^{2/3} d^{10/3}} + \frac{(b c - a d) (6 b d e - 7 b c f + a d f) \operatorname{Log}\left[-1 + \frac{b^{1/3} (c+d x)^{1/3}}{d^{1/3} (a+b x)^{1/3}}\right]}{3 b^{2/3} d^{10/3}}
\end{aligned}$$

Result (type 5, 137 leaves):

$$\begin{aligned}
& \frac{1}{6 d^3} (a + b x)^{1/3} (c + d x)^{2/3} \\
& \left( 6 b d e - 10 b c f + 7 a d f + 3 b d f x - \frac{18 (b c - a d) (-d e + c f)}{c + d x} + \frac{2 (6 b d e - 7 b c f + a d f) \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b (c+d x)}{b c - a d}\right]}{\left(\frac{d (a+b x)}{-b c + a d}\right)^{1/3}} \right)
\end{aligned}$$

Problem 3039: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{4/3}}{(c + d x)^{4/3}} dx$$

Optimal (type 3, 195 leaves, 3 steps):

$$\begin{aligned}
& -\frac{3 (a + b x)^{4/3}}{d (c + d x)^{1/3}} + \frac{4 b (a + b x)^{1/3} (c + d x)^{2/3}}{d^2} + \frac{4 b^{1/3} (b c - a d) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 b^{1/3} (c+d x)^{1/3}}{\sqrt{3} d^{1/3} (a+b x)^{1/3}}\right]}{\sqrt{3} d^{7/3}} + \\
& \frac{2 b^{1/3} (b c - a d) \operatorname{Log}[a + b x]}{3 d^{7/3}} + \frac{2 b^{1/3} (b c - a d) \operatorname{Log}\left[-1 + \frac{b^{1/3} (c+d x)^{1/3}}{d^{1/3} (a+b x)^{1/3}}\right]}{d^{7/3}}
\end{aligned}$$

Result (type 5, 95 leaves):

$$\frac{(a + b x)^{1/3} (c + d x)^{2/3} \left( \frac{4 b c - 3 a d + b d x}{c + d x} + \frac{2 b \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b (c+d x)}{b c - a d}\right]}{\left(\frac{d (a+b x)}{-b c + a d}\right)^{1/3}} \right)}{d^2}$$

### Problem 3040: Result unnecessarily involves higher level functions.

$$\int \frac{(a+b x)^{4/3}}{(c+d x)^{4/3} (e+f x)} dx$$

Optimal (type 3, 380 leaves, 4 steps):

$$\begin{aligned} & \frac{3 (b c - a d) (a + b x)^{1/3}}{d (d e - c f) (c + d x)^{1/3}} - \frac{\sqrt{3} b^{4/3} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 b^{1/3} (c+d x)^{1/3}}{\sqrt{3} d^{1/3} (a+b x)^{1/3}}\right]}{d^{4/3} f} + \frac{\sqrt{3} (b e - a f)^{4/3} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 (b e - a f)^{1/3} (c+d x)^{1/3}}{\sqrt{3} (d e - c f)^{1/3} (a+b x)^{1/3}}\right]}{f (d e - c f)^{4/3}} - \\ & \frac{b^{4/3} \operatorname{Log}[a + b x]}{2 d^{4/3} f} - \frac{(b e - a f)^{4/3} \operatorname{Log}[e + f x]}{2 f (d e - c f)^{4/3}} + \frac{3 (b e - a f)^{4/3} \operatorname{Log}\left[-(a + b x)^{1/3} + \frac{(b e - a f)^{1/3} (c+d x)^{1/3}}{(d e - c f)^{1/3}}\right]}{2 f (d e - c f)^{4/3}} - \frac{3 b^{4/3} \operatorname{Log}\left[-1 + \frac{b^{1/3} (c+d x)^{1/3}}{d^{1/3} (a+b x)^{1/3}}\right]}{2 d^{4/3} f} \end{aligned}$$

Result (type 6, 559 leaves):

$$\begin{aligned} & \frac{1}{5 d^2 (d e - c f) (a + b x)^{2/3} (c + d x)^{1/3}} 3 \left( -5 d (-b c + a d) (a + b x) - \frac{1}{d (e + f x)} \right. \\ & 2 b (b c - a d) (c + d x) \left( 5 f (-2 b d e + b c f + a d f) (c + d x) \operatorname{AppellF1}\left[1, \frac{2}{3}, 1, 2, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] \right) / \\ & \left( 6 b f (c + d x) \operatorname{AppellF1}\left[1, \frac{2}{3}, 1, 2, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] + b (-3 d e + 3 c f) \operatorname{AppellF1}\left[2, \frac{2}{3}, 2, 3, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] + \right. \\ & 2 (b c - a d) f \operatorname{AppellF1}\left[2, \frac{5}{3}, 1, 3, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] \left. - \left( 4 b (d e - c f)^2 \operatorname{AppellF1}\left[\frac{5}{3}, \frac{2}{3}, 1, \frac{8}{3}, \frac{b (c + d x)}{b c - a d}, \frac{f (c + d x)}{-d e + c f}\right] \right) / \right. \\ & \left. - \frac{8 (b c - a d) (-d e + c f) \operatorname{AppellF1}\left[\frac{5}{3}, \frac{2}{3}, 1, \frac{8}{3}, \frac{b (c + d x)}{b c - a d}, \frac{f (c + d x)}{-d e + c f}\right]}{c + d x} + (-3 b c f + 3 a d f) \right. \\ & \left. \operatorname{AppellF1}\left[\frac{8}{3}, \frac{2}{3}, 2, \frac{11}{3}, \frac{b (c + d x)}{b c - a d}, \frac{f (c + d x)}{-d e + c f}\right] + 2 b (d e - c f) \operatorname{AppellF1}\left[\frac{8}{3}, \frac{5}{3}, 1, \frac{11}{3}, \frac{b (c + d x)}{b c - a d}, \frac{f (c + d x)}{-d e + c f}\right] \right) \end{aligned}$$

### Problem 3041: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{4/3}}{(c + d x)^{4/3} (e + f x)^2} dx$$

Optimal (type 3, 301 leaves, 3 steps):

$$\begin{aligned}
& - \frac{3 (a+b x)^{4/3}}{(d e - c f) (c + d x)^{1/3} (e + f x)} + \frac{4 (b e - a f) (a + b x)^{1/3} (c + d x)^{2/3}}{(d e - c f)^2 (e + f x)} + \frac{4 (b c - a d) (b e - a f)^{1/3} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 (b e - a f)^{1/3} (c + d x)^{1/3}}{\sqrt{3} (d e - c f)^{1/3} (a + b x)^{1/3}}\right]}{\sqrt{3} (d e - c f)^{7/3}} - \\
& \frac{2 (b c - a d) (b e - a f)^{1/3} \operatorname{Log}[e + f x]}{3 (d e - c f)^{7/3}} + \frac{2 (b c - a d) (b e - a f)^{1/3} \operatorname{Log}\left[-(a + b x)^{1/3} + \frac{(b e - a f)^{1/3} (c + d x)^{1/3}}{(d e - c f)^{1/3}}\right]}{(d e - c f)^{7/3}}
\end{aligned}$$

Result (type 5, 160 leaves):

$$\left( (a + b x)^{1/3} \left( b (4 c e + d e x + 3 c f x) - a (3 d e + c f + 4 d f x) - \right. \right. \\
\left. \left. 4 (b c - a d) \left( \frac{(b e - a f) (c + d x)}{(b c - a d) (e + f x)} \right)^{1/3} (e + f x) \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{(-d e + c f) (a + b x)}{(b c - a d) (e + f x)}\right] \right) \right) / \left( (d e - c f)^2 (c + d x)^{1/3} (e + f x) \right)$$

Problem 3042: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{4/3}}{(c + d x)^{4/3} (e + f x)^3} dx$$

Optimal (type 3, 434 leaves, 4 steps):

$$\begin{aligned}
& \frac{3 d (a + b x)^{7/3}}{(b c - a d) (d e - c f) (c + d x)^{1/3} (e + f x)^2} - \frac{(6 b d e + b c f - 7 a d f) (a + b x)^{4/3} (c + d x)^{2/3}}{2 (b c - a d) (d e - c f)^2 (e + f x)^2} + \\
& \frac{2 (6 b d e + b c f - 7 a d f) (a + b x)^{1/3} (c + d x)^{2/3}}{3 (d e - c f)^3 (e + f x)} + \frac{2 (b c - a d) (6 b d e + b c f - 7 a d f) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 (b e - a f)^{1/3} (c + d x)^{1/3}}{\sqrt{3} (d e - c f)^{1/3} (a + b x)^{1/3}}\right]}{3 \sqrt{3} (b e - a f)^{2/3} (d e - c f)^{10/3}} - \\
& \frac{(b c - a d) (6 b d e + b c f - 7 a d f) \operatorname{Log}[e + f x]}{9 (b e - a f)^{2/3} (d e - c f)^{10/3}} + \frac{(b c - a d) (6 b d e + b c f - 7 a d f) \operatorname{Log}\left[-(a + b x)^{1/3} + \frac{(b e - a f)^{1/3} (c + d x)^{1/3}}{(d e - c f)^{1/3}}\right]}{3 (b e - a f)^{2/3} (d e - c f)^{10/3}}
\end{aligned}$$

Result (type 5, 208 leaves):

$$\begin{aligned}
& \frac{1}{6 (d e - c f)^3 (c + d x)^{1/3}} (a + b x)^{1/3} \left( 18 d (b c - a d) + \frac{3 (b e - a f) (d e - c f) (c + d x)}{(e + f x)^2} + \right. \\
& \left. \frac{(3 b d e + 7 b c f - 10 a d f) (c + d x)}{e + f x} - \frac{4 (6 b d e + b c f - 7 a d f) (c + d x) \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{(-d e + c f) (a + b x)}{(b c - a d) (e + f x)}\right]}{\left(\frac{(b e - a f) (c + d x)}{(b c - a d) (e + f x)}\right)^{2/3} (e + f x)} \right)
\end{aligned}$$

### Problem 3043: Result unnecessarily involves higher level functions.

$$\int \frac{(a + bx)^{4/3}}{(c + dx)^{4/3} (e + fx)^4} dx$$

Optimal (type 3, 645 leaves, 6 steps):

$$\begin{aligned} & \frac{3(bc - ad)(a + bx)^{1/3}}{d(de - cf)(c + dx)^{1/3}(e + fx)^3} + \frac{(bde + 9bcf - 10adf)(a + bx)^{1/3}(c + dx)^{2/3}}{3d(de - cf)^2(e + fx)^3} + \frac{(3bde + 32bcf - 35adf)(a + bx)^{1/3}(c + dx)^{2/3}}{9(de - cf)^3(e + fx)^2} + \\ & \frac{(140a^2d^2f^2 - 7abdf(21de + 19cf) + b^2(9d^2e^2 + 129cdef + 2c^2f^2))(a + bx)^{1/3}(c + dx)^{2/3}}{27(be - af)(de - cf)^4(e + fx)} + \\ & \left( \frac{4(bc - ad)(35a^2d^2f^2 - 7abdf(9de + cf) + b^2(27d^2e^2 + 9cdef - c^2f^2)) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2(be - af)^{1/3}(c + dx)^{1/3}}{\sqrt{3}(de - cf)^{1/3}(a + bx)^{1/3}}\right]}{27\sqrt{3}(be - af)^{5/3}(de - cf)^{13/3}} - \frac{2(bc - ad)(35a^2d^2f^2 - 7abdf(9de + cf) + b^2(27d^2e^2 + 9cdef - c^2f^2)) \operatorname{Log}[e + fx]}{81(be - af)^{5/3}(de - cf)^{13/3}} + \right. \\ & \left. \left( 2(bc - ad)(35a^2d^2f^2 - 7abdf(9de + cf) + b^2(27d^2e^2 + 9cdef - c^2f^2)) \operatorname{Log}\left[-(a + bx)^{1/3} + \frac{(be - af)^{1/3}(c + dx)^{1/3}}{(de - cf)^{1/3}}\right] \right) \right/ \\ & (27(be - af)^{5/3}(de - cf)^{13/3}) \end{aligned}$$

Result (type 5, 371 leaves):

$$\begin{aligned} & \frac{1}{27(be - af)^2(de - cf)^4(c + dx)^{1/3}(e + fx)^3} \\ & (a + bx)^{1/3} \left( (be - af) \left( 9(be - af)^2(de - cf)^2(c + dx) + 3(be - af)(de - cf)(3bde + 5bcf - 8adf)(c + dx)(e + fx) + \right. \right. \\ & (59a^2d^2f^2 - 2abdf(33de + 26cf) + b^2(9d^2e^2 + 48cdef + 2c^2f^2))(c + dx)(e + fx)^2 + 81d^2(bc - ad)(be - af)(e + fx)^3 \right) + \\ & 4(bc - ad)(-35a^2d^2f^2 + 7abdf(9de + cf) + b^2(-27d^2e^2 - 9cdef + c^2f^2)) \left( \frac{(be - af)(c + dx)}{(bc - ad)(e + fx)} \right)^{1/3} \\ & (e + fx)^3 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{(-de + cf)(a + bx)}{(bc - ad)(e + fx)}\right] \end{aligned}$$

### Problem 3044: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + bx)\sqrt{c + dx}(e + fx)^{1/4}} dx$$

Optimal (type 4, 266 leaves, 5 steps):

$$\frac{2 (d e - c f)^{1/4} \sqrt{-\frac{f (c+d x)}{d e - c f}} \text{EllipticPi}\left[-\frac{\sqrt{b} \sqrt{d e - c f}}{\sqrt{d} \sqrt{b e - a f}}, \text{ArcSin}\left[\frac{d^{1/4} (e+f x)^{1/4}}{(d e - c f)^{1/4}}\right], -1\right]}{\sqrt{b} d^{1/4} \sqrt{b e - a f} \sqrt{c + d x}} -$$

$$\frac{2 (d e - c f)^{1/4} \sqrt{-\frac{f (c+d x)}{d e - c f}} \text{EllipticPi}\left[\frac{\sqrt{b} \sqrt{d e - c f}}{\sqrt{d} \sqrt{b e - a f}}, \text{ArcSin}\left[\frac{d^{1/4} (e+f x)^{1/4}}{(d e - c f)^{1/4}}\right], -1\right]}{\sqrt{b} d^{1/4} \sqrt{b e - a f} \sqrt{c + d x}}$$

Result (type 6, 270 leaves):

$$-\left(\left(28 d f (a+b x) \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, \frac{1}{4}, \frac{7}{4}, \frac{-b c+a d}{d (a+b x)}, \frac{-b e+a f}{f (a+b x)}\right]\right) / \right. \\ \left.\left(3 b \sqrt{c+d x} (e+f x)^{1/4} \left(7 d f (a+b x) \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, \frac{1}{4}, \frac{7}{4}, \frac{-b c+a d}{d (a+b x)}, \frac{-b e+a f}{f (a+b x)}\right] + \right.\right.\right. \\ \left.\left.\left.(-b d e+a d f) \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, \frac{5}{4}, \frac{11}{4}, \frac{-b c+a d}{d (a+b x)}, \frac{-b e+a f}{f (a+b x)}\right] + 2 (-b c+a d) f \text{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, \frac{1}{4}, \frac{11}{4}, \frac{-b c+a d}{d (a+b x)}, \frac{-b e+a f}{f (a+b x)}\right]\right)\right)\right)$$

Problem 3045: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a+b x) \sqrt{c+d x} (e+f x)^{3/4}} dx$$

Optimal (type 4, 252 leaves, 5 steps):

$$\frac{2 (d e - c f)^{1/4} \sqrt{-\frac{f (c+d x)}{d e - c f}} \text{EllipticPi}\left[-\frac{\sqrt{b} \sqrt{d e - c f}}{\sqrt{d} \sqrt{b e - a f}}, \text{ArcSin}\left[\frac{d^{1/4} (e+f x)^{1/4}}{(d e - c f)^{1/4}}\right], -1\right]}{d^{1/4} (b e - a f) \sqrt{c + d x}} -$$

$$\frac{2 (d e - c f)^{1/4} \sqrt{-\frac{f (c+d x)}{d e - c f}} \text{EllipticPi}\left[\frac{\sqrt{b} \sqrt{d e - c f}}{\sqrt{d} \sqrt{b e - a f}}, \text{ArcSin}\left[\frac{d^{1/4} (e+f x)^{1/4}}{(d e - c f)^{1/4}}\right], -1\right]}{d^{1/4} (b e - a f) \sqrt{c + d x}}$$

Result (type 6, 271 leaves):

$$\begin{aligned}
& - \left( \left( 36 d f (a + b x) \text{AppellF1} \left[ \frac{5}{4}, \frac{1}{2}, \frac{3}{4}, \frac{9}{4}, \frac{-b c + a d}{d (a + b x)}, \frac{-b e + a f}{f (a + b x)} \right] \right) \right. \\
& \left. \left( 5 b \sqrt{c + d x} (e + f x)^{3/4} \left( 9 d f (a + b x) \text{AppellF1} \left[ \frac{5}{4}, \frac{1}{2}, \frac{3}{4}, \frac{9}{4}, \frac{-b c + a d}{d (a + b x)}, \frac{-b e + a f}{f (a + b x)} \right] + \right. \right. \right. \\
& \left. \left. \left. (-3 b d e + 3 a d f) \text{AppellF1} \left[ \frac{9}{4}, \frac{1}{2}, \frac{7}{4}, \frac{13}{4}, \frac{-b c + a d}{d (a + b x)}, \frac{-b e + a f}{f (a + b x)} \right] + 2 (-b c + a d) f \text{AppellF1} \left[ \frac{9}{4}, \frac{3}{2}, \frac{3}{4}, \frac{13}{4}, \frac{-b c + a d}{d (a + b x)}, \frac{-b e + a f}{f (a + b x)} \right] \right) \right) \right)
\end{aligned}$$

**Problem 3047: Result unnecessarily involves higher level functions.**

$$\int (a + b x) (c + d x)^n (e + f x)^{-n} dx$$

Optimal (type 5, 134 leaves, 3 steps):

$$\begin{aligned}
& \frac{b (c + d x)^{1+n} (e + f x)^{1-n}}{2 d f} + \frac{1}{2 d^2 f (1+n)} \\
& (2 a d f - b (c f (1-n) + d e (1+n))) (c + d x)^{1+n} (e + f x)^{-n} \left( \frac{d (e + f x)}{d e - c f} \right)^n \text{Hypergeometric2F1}[n, 1+n, 2+n, -\frac{f (c + d x)}{d e - c f}]
\end{aligned}$$

Result (type 6, 201 leaves):

$$\begin{aligned}
& (c + d x)^n (e + f x)^{-n} \left( \left( 3 b c e x^2 \text{AppellF1}[2, -n, n, 3, -\frac{d x}{c}, -\frac{f x}{e}] \right) \right. \\
& \left. \left( 6 c e \text{AppellF1}[2, -n, n, 3, -\frac{d x}{c}, -\frac{f x}{e}] + 2 n x \left( d e \text{AppellF1}[3, 1-n, n, 4, -\frac{d x}{c}, -\frac{f x}{e}] - c f \text{AppellF1}[3, -n, 1+n, 4, -\frac{d x}{c}, -\frac{f x}{e}] \right) \right) - \right. \\
& \left. \frac{a \left( \frac{f (c+d x)}{-d e+c f} \right)^{-n} (e + f x) \text{Hypergeometric2F1}[1-n, -n, 2-n, \frac{d (e+f x)}{d e-c f}]}{f (-1+n)} \right)
\end{aligned}$$

**Problem 3053: Result unnecessarily involves higher level functions.**

$$\int (a + b x)^{-n} (c + d x) (e + f x)^n dx$$

Optimal (type 5, 135 leaves, 3 steps):

$$\begin{aligned}
& \frac{d (a + b x)^{1-n} (e + f x)^{1+n}}{2 b f} + \frac{1}{2 b f^2 (1+n)} \\
& (b (2 c f - d e (1-n)) - a d f (1+n)) (a + b x)^{-n} \left( -\frac{f (a + b x)}{b e - a f} \right)^n (e + f x)^{1+n} \text{Hypergeometric2F1}[n, 1+n, 2+n, \frac{b (e + f x)}{b e - a f}]
\end{aligned}$$

Result (type 6, 192 leaves):

$$\begin{aligned} & \left( (a + b x)^{-n} (e + f x)^n \left( \left( 3 a d e x^2 \text{AppellF1}[2, n, -n, 3, -\frac{b x}{a}, -\frac{f x}{e}] \right) \right) \right) / \\ & \left( 6 a e \text{AppellF1}[2, n, -n, 3, -\frac{b x}{a}, -\frac{f x}{e}] + 2 n x \left( a f \text{AppellF1}[3, n, 1-n, 4, -\frac{b x}{a}, -\frac{f x}{e}] - b e \text{AppellF1}[3, 1+n, -n, 4, -\frac{b x}{a}, -\frac{f x}{e}] \right) \right) + \\ & \frac{c \left( \frac{f(a+b x)}{-b e+a f} \right)^n (e + f x) \text{Hypergeometric2F1}[n, 1+n, 2+n, \frac{b(e+f x)}{b e-a f}]}{f (1+n)} \end{aligned}$$

Problem 3059: Result more than twice size of optimal antiderivative.

$$\int (a + b x)^m (c + d x)^{-m} (e + f x)^p dx$$

Optimal (type 6, 121 leaves, 3 steps):

$$\frac{(a + b x)^{1+m} (c + d x)^{-m} \left( \frac{b(c+d x)}{b c - a d} \right)^m (e + f x)^p \left( \frac{b(e+f x)}{b e - a f} \right)^{-p} \text{AppellF1}[1+m, m, -p, 2+m, -\frac{d(a+b x)}{b c - a d}, -\frac{f(a+b x)}{b e - a f}]}{b (1+m)}$$

Result (type 6, 290 leaves):

$$\begin{aligned} & \left( (b c - a d) (b e - a f) (2 + m) (a + b x)^{1+m} (c + d x)^{-m} (e + f x)^p \text{AppellF1}[1 + m, m, -p, 2 + m, \frac{d(a+b x)}{-b c + a d}, \frac{f(a+b x)}{-b e + a f}] \right) / \\ & \left( b (1 + m) \left( (b c - a d) (b e - a f) (2 + m) \text{AppellF1}[1 + m, m, -p, 2 + m, \frac{d(a+b x)}{-b c + a d}, \frac{f(a+b x)}{-b e + a f}] - (a + b x) \left( (-b c + a d) f p \right. \right. \right. \right. \\ & \left. \left. \left. \left. \text{AppellF1}[2 + m, m, 1 - p, 3 + m, \frac{d(a+b x)}{-b c + a d}, \frac{f(a+b x)}{-b e + a f}] + d (b e - a f) m \text{AppellF1}[2 + m, 1 + m, -p, 3 + m, \frac{d(a+b x)}{-b c + a d}, \frac{f(a+b x)}{-b e + a f}] \right) \right) \right) \right) \end{aligned}$$

Problem 3060: Result unnecessarily involves higher level functions.

$$\int (5 - 4 x)^4 (1 + 2 x)^{-m} (2 + 3 x)^m dx$$

Optimal (type 5, 188 leaves, 4 steps):

$$\begin{aligned}
& -\frac{1}{45} (88-m) (5-4x)^2 (1+2x)^{1-m} (2+3x)^{1+m} - \frac{2}{15} (5-4x)^3 (1+2x)^{1-m} (2+3x)^{1+m} - \\
& \frac{(1+2x)^{1-m} (2+3x)^{1+m} (386850 - 25441m + 426m^2 - 2m^3 - 24(4359 - 154m + m^2)x)}{1215} + \frac{1}{1215(1-m)} \\
& 2^{-1-m} (3528363 - 639760m + 29050m^2 - 440m^3 + 2m^4) (1+2x)^{1-m} \text{Hypergeometric2F1}[1-m, -m, 2-m, -3(1+2x)]
\end{aligned}$$

Result (type 6, 155 leaves):

$$\begin{aligned}
& \left( 483 \times 2^{-1-m} (-5+4x)^5 (2+4x)^{-m} (8+12x)^m \text{AppellF1}[5, -m, m, 6, \frac{3}{23}(5-4x), \frac{1}{7}(5-4x)] \right) / \\
& \left( 5 \left( 966 \text{AppellF1}[5, -m, m, 6, \frac{3}{23}(5-4x), \frac{1}{7}(5-4x)] + \right. \right. \\
& \left. \left. m(-5+4x) \left( 21 \text{AppellF1}[6, 1-m, m, 7, \frac{3}{23}(5-4x), \frac{1}{7}(5-4x)] - 23 \text{AppellF1}[6, -m, 1+m, 7, \frac{3}{23}(5-4x), \frac{1}{7}(5-4x)] \right) \right) \right)
\end{aligned}$$

Problem 3061: Result unnecessarily involves higher level functions.

$$\int (a+bx)^m (c+dx)^{-m} (e+fx)^3 dx$$

Optimal (type 5, 432 leaves, 4 steps):

$$\begin{aligned}
& \frac{f(a+bx)^{1+m} (c+dx)^{1-m} (e+fx)^2}{4bd} + \frac{1}{24b^3d^3} \\
& f(a+bx)^{1+m} (c+dx)^{1-m} (a^2 d^2 f^2 (6-5m+m^2) - 2abd f (6de(2-m) - cf(3-m^2))) + b^2 (30d^2 e^2 - 12cd e f (2+m) + c^2 f^2 (6+5m+m^2)) - \\
& 2bdf (adf(3-m) - b(6de - cf(3+m)))x - \frac{1}{24b^4d^3(1+m)} \\
& (a^3 d^3 f^3 (6-11m+6m^2-m^3) - 3a^2 b d^2 f^2 (2-3m+m^2) (4de - cf(1+m)) + 3ab^2 df (1-m) (12d^2 e^2 - 8cd e f (1+m) + c^2 f^2 (2+3m+m^2))) - \\
& b^3 (24d^3 e^3 - 36cd^2 e^2 f (1+m) + 12c^2 d e f^2 (2+3m+m^2) - c^3 f^3 (6+11m+6m^2+m^3))) \\
& (a+bx)^{1+m} (c+dx)^{-m} \left( \frac{b(c+dx)}{bc-ad} \right)^m \text{Hypergeometric2F1}[m, 1+m, 2+m, -\frac{d(a+bx)}{bc-ad}]
\end{aligned}$$

Result (type 6, 440 leaves):

$$\begin{aligned}
& \left( (a + b x)^m (c + d x)^{-m} \left( \left( 9 a c e^2 f x^2 \text{AppellF1}[2, -m, m, 3, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \right. \right. \\
& \left. \left( 6 a c \text{AppellF1}[2, -m, m, 3, -\frac{b x}{a}, -\frac{d x}{c}] + 2 m x \left( b c \text{AppellF1}[3, 1-m, m, 4, -\frac{b x}{a}, -\frac{d x}{c}] - a d \text{AppellF1}[3, -m, 1+m, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) \right) + \right. \\
& \left( 4 a c e f^2 x^3 \text{AppellF1}[3, -m, m, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \\
& \left( 4 a c \text{AppellF1}[3, -m, m, 4, -\frac{b x}{a}, -\frac{d x}{c}] + m x \left( b c \text{AppellF1}[4, 1-m, m, 5, -\frac{b x}{a}, -\frac{d x}{c}] - a d \text{AppellF1}[4, -m, 1+m, 5, -\frac{b x}{a}, -\frac{d x}{c}] \right) \right) + \\
& \left( 5 a c f^3 x^4 \text{AppellF1}[4, -m, m, 5, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \\
& \left. \left( 20 a c \text{AppellF1}[4, -m, m, 5, -\frac{b x}{a}, -\frac{d x}{c}] + 4 b c m x \text{AppellF1}[5, 1-m, m, 6, -\frac{b x}{a}, -\frac{d x}{c}] - 4 a d m x \text{AppellF1}[5, -m, 1+m, 6, -\frac{b x}{a}, -\frac{d x}{c}] \right) - \right. \\
& \left. \left. e^3 \left( \frac{d(a+b x)}{-b c+a d} \right)^{-m} (c + d x) \text{Hypergeometric2F1}[1-m, -m, 2-m, \frac{b(c+d x)}{b c-a d}] \right) / d (-1+m) \right)
\end{aligned}$$

Problem 3062: Result unnecessarily involves higher level functions.

$$\int (a + b x)^m (c + d x)^{-m} (e + f x)^2 dx$$

Optimal (type 5, 250 leaves, 4 steps):

$$\begin{aligned}
& - \frac{f (a d f (2-m) - b (4 d e - c f (2+m))) (a + b x)^{1+m} (c + d x)^{1-m}}{6 b^2 d^2} + \frac{f (a + b x)^{1+m} (c + d x)^{1-m} (e + f x)}{3 b d} + \frac{1}{6 b^3 d^2 (1+m)} \\
& (a^2 d^2 f^2 (2-3m+m^2) - 2 a b d f (1-m) (3 d e - c f (1+m)) + b^2 (6 d^2 e^2 - 6 c d e f (1+m) + c^2 f^2 (2+3m+m^2))) \\
& (a + b x)^{1+m} (c + d x)^{-m} \left( \frac{b (c + d x)}{b c - a d} \right)^m \text{Hypergeometric2F1}[m, 1+m, 2+m, -\frac{d (a + b x)}{b c - a d}]
\end{aligned}$$

Result (type 6, 320 leaves):

$$\begin{aligned}
& \left( (a + b x)^m (c + d x)^{-m} \left( \left( 3 a c e f x^2 \text{AppellF1}[2, -m, m, 3, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \right. \right. \\
& \left. \left( 3 a c \text{AppellF1}[2, -m, m, 3, -\frac{b x}{a}, -\frac{d x}{c}] + m x \left( b c \text{AppellF1}[3, 1-m, m, 4, -\frac{b x}{a}, -\frac{d x}{c}] - a d \text{AppellF1}[3, -m, 1+m, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) \right) + \right. \\
& \left( 4 a c f^2 x^3 \text{AppellF1}[3, -m, m, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \\
& \left. \left( 12 a c \text{AppellF1}[3, -m, m, 4, -\frac{b x}{a}, -\frac{d x}{c}] + 3 b c m x \text{AppellF1}[4, 1-m, m, 5, -\frac{b x}{a}, -\frac{d x}{c}] - 3 a d m x \text{AppellF1}[4, -m, 1+m, 5, -\frac{b x}{a}, -\frac{d x}{c}] \right) - \right. \\
& \left. \left. e^{2 \left( \frac{d (a+b x)}{-b c+a d} \right) -m} (c + d x) \text{Hypergeometric2F1}[1-m, -m, 2-m, \frac{b (c+d x)}{b c-a d}] \right) / d (-1+m) \right)
\end{aligned}$$

Problem 3063: Result unnecessarily involves higher level functions.

$$\int (a + b x)^m (c + d x)^{-m} (e + f x) dx$$

Optimal (type 5, 135 leaves, 3 steps):

$$\begin{aligned}
& \frac{f (a + b x)^{1+m} (c + d x)^{1-m}}{2 b d} - \frac{1}{2 b^2 d (1+m)} \\
& (a d f (1-m) - b (2 d e - c f (1+m))) (a + b x)^{1+m} (c + d x)^{-m} \left( \frac{b (c + d x)}{b c - a d} \right)^m \text{Hypergeometric2F1}[m, 1+m, 2+m, -\frac{d (a + b x)}{b c - a d}]
\end{aligned}$$

Result (type 6, 201 leaves):

$$\begin{aligned}
& (a + b x)^m (c + d x)^{-m} \left( \left( 3 a c f x^2 \text{AppellF1}[2, -m, m, 3, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \right. \\
& \left( 6 a c \text{AppellF1}[2, -m, m, 3, -\frac{b x}{a}, -\frac{d x}{c}] + 2 m x \left( b c \text{AppellF1}[3, 1-m, m, 4, -\frac{b x}{a}, -\frac{d x}{c}] - a d \text{AppellF1}[3, -m, 1+m, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) \right) - \\
& \left. \left. e^{\left( \frac{d (a+b x)}{-b c+a d} \right) -m} (c + d x) \text{Hypergeometric2F1}[1-m, -m, 2-m, \frac{b (c+d x)}{b c-a d}] \right) / d (-1+m) \right)
\end{aligned}$$

Problem 3065: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^m (c + d x)^{-m}}{e + f x} dx$$

Optimal (type 5, 128 leaves, 5 steps):

$$\frac{\left( (a+b x)^m (c+d x)^{-m} \text{Hypergeometric2F1}[1, m, 1+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}] \right)}{f m} + \frac{\left( (a+b x)^m (c+d x)^{-m} \left( \frac{b (c+d x)}{b c - a d} \right)^m \text{Hypergeometric2F1}[m, m, 1+m, -\frac{d (a+b x)}{b c - a d}] \right)}{f m}$$

Result (type 6, 292 leaves):

$$\begin{aligned} & - \left( \left( (b c - a d) (b e - a f)^2 (2 + m) (a + b x)^{1+m} (c + d x)^{-m} \text{AppellF1}[1 + m, m, 1, 2 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] \right) \right. \\ & \quad \left. \left( b (-b e + a f) (1 + m) (e + f x) \left( (b c - a d) (b e - a f) (2 + m) \text{AppellF1}[1 + m, m, 1, 2 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] + (a + b x) \left( (-b c f + a d f) \right. \right. \right. \right. \\ & \quad \left. \left. \left. \left. \text{AppellF1}[2 + m, m, 2, 3 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] + d (-b e + a f) m \text{AppellF1}[2 + m, 1 + m, 1, 3 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] \right) \right) \right) \right) \end{aligned}$$

Problem 3067: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+b x)^m (c+d x)^{-m}}{(e+f x)^3} dx$$

Optimal (type 5, 174 leaves, 2 steps):

$$\begin{aligned} & - \frac{f (a + b x)^{1+m} (c + d x)^{1-m}}{2 (b e - a f) (d e - c f) (e + f x)^2} + \\ & \left( (b c - a d) (b (2 d e - c f (1 - m)) - a d f (1 + m)) (a + b x)^{1+m} (c + d x)^{-1-m} \text{Hypergeometric2F1}[2, 1 + m, 2 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}] \right) \Big/ \\ & \left( 2 (b e - a f)^3 (d e - c f) (1 + m) \right) \end{aligned}$$

Result (type 5, 432 leaves):

$$\begin{aligned}
& \left( (b e - a f)^4 (a + b x)^{1+m} (c + d x)^{-m} \right. \\
& \left( (-2 b e + a f (1+m) + b f (-1+m) x) \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] - 2 (a f (1+m) + b (-e + f m x)) \right. \\
& \left. \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1+m \right] + f (1+m) (a + b x) \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2+m \right] \right) \Bigg) / \\
& \left( (2 b e - 2 a f) (-b e + a f)^3 (e + f x)^2 \left( (b e - a f) (-a f (1+m) + b (e - f m x)) \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] + \right. \right. \\
& \frac{1}{c + d x} (a + b x) \left( (a f (1+m) (-2 c f + d (e - f x)) + b (c f (e (2+m) - f m x) + d e (-e + f (1+2m) x))) \right. \\
& \left. \left. \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1+m \right] + f (-d e + c f) (1+m) (a + b x) \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2+m \right] \right) \right)
\end{aligned}$$

Problem 3068: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^m (c + d x)^{-m}}{(e + f x)^4} dx$$

Optimal (type 5, 309 leaves, 4 steps):

$$\begin{aligned}
& -\frac{f (a + b x)^{1+m} (c + d x)^{1-m}}{3 (b e - a f) (d e - c f) (e + f x)^3} - \frac{f (b (4 d e - c f (2-m)) - a d f (2+m)) (a + b x)^{1+m} (c + d x)^{1-m}}{6 (b e - a f)^2 (d e - c f)^2 (e + f x)^2} - \\
& \left( (b c - a d) (2 a b d f (3 d e - c f (1-m)) (1+m) - a^2 d^2 f^2 (2+3m+m^2) - b^2 (6 d^2 e^2 - 6 c d e f (1-m) + c^2 f^2 (2-3m+m^2))) \right. \\
& \left. (a + b x)^{1+m} (c + d x)^{-1-m} \text{Hypergeometric2F1} [2, 1+m, 2+m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}] \right) \Bigg/ \left( 6 (b e - a f)^4 (d e - c f)^2 (1+m) \right)
\end{aligned}$$

Result (type 5, 1697 leaves):

$$\begin{aligned}
& \left( (a + b x)^{1+m} (c + d x)^{1-m} \left( 6 (b e - a f)^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] + \right. \right. \\
& 6 (b e - a f)^2 m \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] + 6 f (b e - a f) (a + b x) \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] + \\
& 6 f (-b e + a f) m^2 (a + b x) \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] + 2 f^2 (a + b x)^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] - \\
& f^2 m (a + b x)^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] - 2 f^2 m^2 (a + b x)^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] +
\end{aligned}$$

$$\begin{aligned}
& f^2 m^3 (a+b x)^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}, 1, m\right] - 6 (b e - a f)^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}, 1, 1+m\right] - \\
& 6 (b e - a f)^2 m \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}, 1, 1+m\right] + 12 f (b e - a f) m (a+b x) \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}, 1, 1+m\right] + \\
& 12 f (b e - a f) m^2 (a+b x) \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}, 1, 1+m\right] + 3 f^2 m (a+b x)^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}, 1, 1+m\right] - \\
& 3 f^2 m^3 (a+b x)^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}, 1, 1+m\right] + 6 f (-b e + a f) (a+b x) \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}, 1, 2+m\right] + \\
& 12 f (-b e + a f) m (a+b x) \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}, 1, 2+m\right] + \\
& 6 f (-b e + a f) m^2 (a+b x) \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}, 1, 2+m\right] + 3 f^2 m (a+b x)^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}, 1, 2+m\right] + \\
& 6 f^2 m^2 (a+b x)^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}, 1, 2+m\right] + 3 f^2 m^3 (a+b x)^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}, 1, 2+m\right] - \\
& 2 f^2 (a+b x)^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}, 1, 3+m\right] - 5 f^2 m (a+b x)^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}, 1, 3+m\right] - \\
& 4 f^2 m^2 (a+b x)^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}, 1, 3+m\right] - f^2 m^3 (a+b x)^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}, 1, 3+m\right] \Big) \Big) \Big) / \\
& \left( 3 (1+m) (e+f x)^3 \left( (b e - a f) (c+d x) (a^2 f^2 (2+3m+m^2) + 2 a b f (1+m) (-2 e + f m x) + b^2 (2 e^2 - 4 e f m x + f^2 (-1+m) m x^2)) \right. \right. \\
& \left. \left. \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}, 1, m\right] - \right. \right. \\
& (a+b x) \left( (a^2 f^2 (2+3m+m^2) (-3 c f + d (e - 2 f x)) - 2 a b f (1+m) (c f (-e (6+m) + 2 f m x) + d (2 e^2 - 2 e f (2+m) x + f^2 m x^2))) + \right. \\
& b^2 (c f (-2 e^2 (3+2m) + 2 e f m (3+m) x - f^2 (-1+m) m x^2) + d e (2 e^2 - 4 e f (1+2m) x + f^2 m (1+3m) x^2)) \Big) \\
& \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}, 1, 1+m\right] + f (1+m) (a+b x) \\
& \left( (a f (2+m) (-2 d e + 3 c f + d f x) + b c f (-e (6+m) + 2 f m x) + b d e (4 e - f (2+3m) x)) \right. \\
& \left. \left. \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}, 1, 2+m\right] + f (d e - c f) (2+m) (a+b x) \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}, 1, 3+m\right] \right) \right) \Big)
\end{aligned}$$

Problem 3069: Result unnecessarily involves higher level functions.

$$\int \frac{(1+2x)^{-m} (2+3x)^m}{(5-4x)^5} dx$$

Optimal (type 5, 179 leaves, 5 steps):

$$\begin{aligned} & \frac{(1+2x)^{1-m} (2+3x)^{1+m}}{322 (5-4x)^4} + \frac{(66+m) (1+2x)^{1-m} (2+3x)^{1+m}}{77763 (5-4x)^3} + \frac{(4359+220m+2m^2) (1+2x)^{1-m} (2+3x)^{1+m}}{25039686 (5-4x)^2} + \\ & \frac{(32010+4358m+132m^2+m^3) (1+2x)^{1-m} (2+3x)^{-1+m} \text{Hypergeometric2F1}[2, 1-m, 2-m, \frac{23(1+2x)}{14(2+3x)}]}{2453889228 (1-m)} \end{aligned}$$

Result (type 6, 153 leaves):

$$\begin{aligned} & \left( 15 \times 2^{-4-m} (2+4x)^{-m} (8+12x)^m \text{AppellF1}[4, -m, m, 5, \frac{23}{15-12x}, \frac{7}{5-4x}] \right) / \\ & \left( (-5+4x)^3 \left( 15 (-5+4x) \text{AppellF1}[4, -m, m, 5, \frac{23}{15-12x}, \frac{7}{5-4x}] + m \right. \right. \\ & \left. \left. \left( 23 \text{AppellF1}[5, 1-m, m, 6, \frac{23}{15-12x}, \frac{7}{5-4x}] - 21 \text{AppellF1}[5, -m, 1+m, 6, \frac{23}{15-12x}, \frac{7}{5-4x}] \right) \right) \right) \end{aligned}$$

Problem 3070: Result more than twice size of optimal antiderivative.

$$\int (a+bx)^m (c+dx)^{-1-m} (e+fx)^p dx$$

Optimal (type 6, 130 leaves, 3 steps):

$$\frac{1}{(bc-ad)(1+m)} (a+bx)^{1+m} (c+dx)^{-m} \left( \frac{b(c+dx)}{bc-ad} \right)^m (e+fx)^p \left( \frac{b(e+fx)}{be-af} \right)^{-p} \text{AppellF1}[1+m, 1+m, -p, 2+m, -\frac{d(a+bx)}{bc-ad}, -\frac{f(a+bx)}{be-af}]$$

Result (type 6, 300 leaves):

$$\begin{aligned} & \left( (bc-ad)(be-af)(2+m)(a+bx)^{1+m}(c+dx)^{-1-m}(e+fx)^p \text{AppellF1}[1+m, 1+m, -p, 2+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}] \right) / \\ & \left( b(1+m) \left( (bc-ad)(be-af)(2+m) \text{AppellF1}[1+m, 1+m, -p, 2+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}] \right. \right. \\ & \left. \left. - (a+bx) \left( (-bc+ad)f p \text{AppellF1}[2+m, 1+m, 1-p, 3+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}] \right) + \right. \right. \\ & \left. \left. d(b e - a f) (1+m) \text{AppellF1}[2+m, 2+m, -p, 3+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}] \right) \right) \end{aligned}$$

Problem 3071: Result unnecessarily involves higher level functions and more than twice size of optimal

antiderivative.

$$\int (5 - 4x)^3 (1 + 2x)^{-1-m} (2 + 3x)^m dx$$

Optimal (type 5, 142 leaves, 3 steps):

$$\begin{aligned} & -\frac{2}{9} (5 - 4x)^2 (1 + 2x)^{-m} (2 + 3x)^{1+m} - \frac{(1 + 2x)^{-m} (2 + 3x)^{1+m} (9261 - 512m + 4m^2 - 4(109 - 2m)m)}{27m} + \\ & \frac{2^{-1-m} (27783 - 8324m + 390m^2 - 4m^3) (1 + 2x)^{1-m} \text{Hypergeometric2F1}[1-m, -m, 2-m, -3(1+2x)]}{27(1-m)m} \end{aligned}$$

Result (type 6, 395 leaves):

$$\begin{aligned} & \frac{7}{4} \left( \left( 483 (5 - 4x)^2 (4 + 8x)^{-m} (8 + 12x)^m \text{AppellF1}[2, -m, m, 3, \frac{3}{23}(5 - 4x), \frac{1}{7}(5 - 4x)] \right) / \left( 483 \text{AppellF1}[2, -m, m, 3, \frac{3}{23}(5 - 4x), \frac{1}{7}(5 - 4x)] + \right. \right. \\ & m(-5 + 4x) \left( 21 \text{AppellF1}[3, 1-m, m, 4, \frac{3}{23}(5 - 4x), \frac{1}{7}(5 - 4x)] - 23 \text{AppellF1}[3, -m, 1+m, 4, \frac{3}{23}(5 - 4x), \frac{1}{7}(5 - 4x)] \right) - \\ & \left( 23 \times 2^{3+m} (2 + 3x)^m (-5 + 4x)^3 (2 + 4x)^{-m} \text{AppellF1}[3, -m, m, 4, -\frac{3}{23}(-5 + 4x), \frac{1}{7}(5 - 4x)] \right) / \\ & \left( 3 \left( 644 \text{AppellF1}[3, -m, m, 4, \frac{3}{23}(5 - 4x), \frac{1}{7}(5 - 4x)] + \right. \right. \\ & m(-5 + 4x) \left( 21 \text{AppellF1}[4, 1-m, m, 5, \frac{3}{23}(5 - 4x), \frac{1}{7}(5 - 4x)] - 23 \text{AppellF1}[4, -m, 1+m, 5, \frac{3}{23}(5 - 4x), \frac{1}{7}(5 - 4x)] \right) \left. \right) + \\ & \left. \left. 7 \times 2^{2-m} (1 + 2x)^{1-m} \text{Hypergeometric2F1}[1-m, -m, 2-m, -3-6x] \right. \right. \\ & \left. \left. - 1 + m \right. \right. \\ & \left. \left. \frac{196 (-3 - 6x)^m (1 + 2x)^{-m} (2 + 3x)^{1+m} \text{Hypergeometric2F1}[1+m, 1+m, 2+m, 4+6x]}{1+m} \right) \right) \end{aligned}$$

Problem 3072: Result unnecessarily involves higher level functions.

$$\int (5 - 4x)^2 (1 + 2x)^{-1-m} (2 + 3x)^m dx$$

Optimal (type 5, 121 leaves, 3 steps):

$$\begin{aligned} & -\frac{7(21-m)(1+2x)^{-m}(2+3x)^{1+m}}{3m} - \frac{1}{3}(5-4x)(1+2x)^{-m}(2+3x)^{1+m} + \\ & \frac{2^{-1-m}(441-86m+2m^2)(1+2x)^{1-m}\text{Hypergeometric2F1}[1-m, -m, 2-m, -3(1+2x)]}{3(1-m)m} \end{aligned}$$

Result (type 6, 241 leaves):

$$\frac{7}{4} \left( \left( 69 (5 - 4x)^2 (4 + 8x)^{-m} (8 + 12x)^m \text{AppellF1}[2, -m, m, 3, \frac{3}{23} (5 - 4x), \frac{1}{7} (5 - 4x)] \right) / \left( 483 \text{AppellF1}[2, -m, m, 3, \frac{3}{23} (5 - 4x), \frac{1}{7} (5 - 4x)] + m (-5 + 4x) \left( 21 \text{AppellF1}[3, 1 - m, m, 4, \frac{3}{23} (5 - 4x), \frac{1}{7} (5 - 4x)] - 23 \text{AppellF1}[3, -m, 1 + m, 4, \frac{3}{23} (5 - 4x), \frac{1}{7} (5 - 4x)] \right) + \frac{2^{2-m} (1 + 2x)^{1-m} \text{Hypergeometric2F1}[1 - m, -m, 2 - m, -3 - 6x]}{-1 + m} - \frac{28 (-3 - 6x)^m (1 + 2x)^{-m} (2 + 3x)^{1+m} \text{Hypergeometric2F1}[1 + m, 1 + m, 2 + m, 4 + 6x]}{1 + m} \right)$$

Problem 3075: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(ax + bx)^m (cx + dx)^{-1-m}}{ex + fx} dx$$

Optimal (type 5, 72 leaves, 1 step):

$$\frac{(ax + bx)^m (cx + dx)^{-m} \text{Hypergeometric2F1}[1, -m, 1 - m, \frac{(be - af)(c+dx)}{(de - cf)(a+bx)}]}{(de - cf)m}$$

Result (type 6, 362 leaves):

$$\begin{aligned} & \frac{1}{de - cf} (ax + bx)^m (cx + dx)^{-m} \left( \left( (bc - ad)f (be - af)^2 (2 + m) (a + bx) \text{AppellF1}[1 + m, m, 1, 2 + m, \frac{d(a + bx)}{-bc + ad}, \frac{f(a + bx)}{-be + af}] \right) / \right. \\ & \left. \left( b(-be + af)(1 + m)(e + fx) \left( (bc - ad)(be - af)(2 + m) \text{AppellF1}[1 + m, m, 1, 2 + m, \frac{d(a + bx)}{-bc + ad}, \frac{f(a + bx)}{-be + af}] \right) + \right. \right. \\ & \left. \left. (a + bx) \left( (-bcf + adf) \text{AppellF1}[2 + m, m, 2, 3 + m, \frac{d(a + bx)}{-bc + ad}, \frac{f(a + bx)}{-be + af}] \right) + \right. \right. \\ & \left. \left. d(-be + af)m \text{AppellF1}[2 + m, 1 + m, 1, 3 + m, \frac{d(a + bx)}{-bc + ad}, \frac{f(a + bx)}{-be + af}] \right) \right) - \frac{\left( \frac{d(a + bx)}{-bc + ad} \right)^{-m} \text{Hypergeometric2F1}[-m, -m, 1 - m, \frac{b(c+dx)}{bc - ad}]}{m} \end{aligned}$$

### Problem 3077: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^m (c + d x)^{-1-m}}{(e + f x)^3} dx$$

Optimal (type 5, 283 leaves, 4 steps):

$$\begin{aligned} & -\frac{f (a + b x)^{1+m} (c + d x)^{-m}}{2 (b e - a f) (d e - c f) (e + f x)^2} - \frac{f (b (3 d e - c f (1 - m)) - a d f (2 + m)) (a + b x)^{1+m} (c + d x)^{-m}}{2 (b e - a f)^2 (d e - c f)^2 (e + f x)} + \\ & \left( (2 a b d f (1 + m) (2 d e + c f m) - b^2 (2 d^2 e^2 + 4 c d e f m - c^2 f^2 (1 - m) m) - a^2 d^2 f^2 (2 + 3 m + m^2)) \right. \\ & \left. (a + b x)^m (c + d x)^{-m} \text{Hypergeometric2F1}[1, -m, 1 - m, \frac{(b e - a f) (c + d x)}{(d e - c f) (a + b x)}] \right) / (2 (b e - a f)^2 (d e - c f)^3 m) \end{aligned}$$

Result (type 5, 2361 leaves):

$$\begin{aligned} & - \left( \left( (b e - a f)^3 (a + b x)^{1+m} (c + d x)^{-m} \right. \right. \\ & \left. \left. \left( 2 (b e - a f)^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m \right] + 2 (b e - a f)^2 m \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m \right] + \right. \right. \\ & 4 f (-b e + a f) m (a + b x) \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m \right] + 4 f (-b e + a f) m^2 (a + b x) \\ & \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m \right] - f^2 m (a + b x)^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m \right] + \\ & f^2 m^3 (a + b x)^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m \right] + 4 f (b e - a f) (a + b x) \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m \right] + \\ & 8 f (b e - a f) m (a + b x) \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m \right] + 4 f (b e - a f) m^2 (a + b x) \\ & \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m \right] - 2 f^2 m (a + b x)^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m \right] - \\ & 4 f^2 m^2 (a + b x)^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m \right] - 2 f^2 m^3 (a + b x)^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m \right] + \\ & 2 f^2 (a + b x)^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + 5 f^2 m (a + b x)^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + \\ & \left. \left. 4 f^2 m^2 (a + b x)^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + f^2 m^3 (a + b x)^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] \right) \right) \end{aligned}$$

$$\begin{aligned}
& \left( 2 (-b e + a f)^3 (1+m) (e + f x)^2 \left( b^3 c e^3 - a b^2 c e^2 f + b^3 d e^3 x + 2 b^3 c e^2 f x - a b^2 d e^2 f x - 2 a b^2 c e f^2 x + \right. \right. \\
& \quad \left. \left. 2 b^3 d e^2 f x^2 + b^3 c e^2 f^2 x^2 - 2 a b^2 d e f^2 x^2 - a b^2 c f^3 x^2 + b^3 d e f^2 x^3 - a b^2 d f^3 x^3 - \right. \right. \\
& \quad \left. \left. f (-b e + a f) (1+m) (a + b x) (c + d x) (a f (2+m) + b (-2 e + f m x)) \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1+m \right] + f (1+m) (a + b x)^2 \right. \right. \\
& \quad \left. \left. (a f (2+m) (-d e + 2 c f + d f x) + b c f (-e (4+m) + f m x) + 2 b d e (e - f (1+m) x)) \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2+m \right] + \right. \right. \\
& \quad \left. \left. 2 a^3 d e f^2 \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3+m \right] - 2 a^3 c f^3 \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3+m \right] + \right. \right. \\
& \quad \left. \left. 3 a^3 d e f^2 m \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3+m \right] - 3 a^3 c f^3 m \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3+m \right] + \right. \right. \\
& \quad \left. \left. a^3 d e f^2 m^2 \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3+m \right] - a^3 c f^3 m^2 \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3+m \right] + \right. \right. \\
& \quad \left. \left. 6 a^2 b d e f^2 x \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3+m \right] - 6 a^2 b c f^3 x \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3+m \right] + \right. \right. \\
& \quad \left. \left. 9 a^2 b d e f^2 m x \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3+m \right] - 9 a^2 b c f^3 m x \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3+m \right] + \right. \right. \\
& \quad \left. \left. 3 a^2 b d e f^2 m^2 x \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3+m \right] - 3 a^2 b c f^3 m^2 x \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3+m \right] + \right. \right. \\
& \quad \left. \left. 6 a b^2 d e f^2 x^2 \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3+m \right] - 6 a b^2 c f^3 x^2 \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3+m \right] + \right. \right. \\
& \quad \left. \left. 9 a b^2 d e f^2 m x^2 \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3+m \right] - 9 a b^2 c f^3 m x^2 \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3+m \right] + \right. \right. \\
& \quad \left. \left. 3 a b^2 d e f^2 m^2 x^2 \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3+m \right] - 3 a b^2 c f^3 m^2 x^2 \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3+m \right] + \right. \right. \\
& \quad \left. \left. 2 b^3 d e f^2 x^3 \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3+m \right] - 2 b^3 c f^3 x^3 \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3+m \right] + \right. \right. \\
& \quad \left. \left. 3 b^3 d e f^2 m x^3 \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3+m \right] - 3 b^3 c f^3 m x^3 \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3+m \right] + \right. \right. \\
& \quad \left. \left. b^3 d e f^2 m^2 x^3 \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3+m \right] - b^3 c f^3 m^2 x^3 \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3+m \right] \right) \right)
\end{aligned}$$

Problem 3078: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^m (c + d x)^{-1-m}}{(e + f x)^4} dx$$

Optimal (type 5, 498 leaves, 5 steps):

$$\begin{aligned} & -\frac{f (a + b x)^{1+m} (c + d x)^{-m}}{3 (b e - a f) (d e - c f) (e + f x)^3} - \frac{f (b (5 d e - c f (2 - m)) - a d f (3 + m)) (a + b x)^{1+m} (c + d x)^{-m}}{6 (b e - a f)^2 (d e - c f)^2 (e + f x)^2} - \\ & \left( f (a^2 d^2 f^2 (6 + 5 m + m^2) - a b d f (d e (15 + 8 m) - c f (3 - 2 m - 2 m^2)) + b^2 (11 d^2 e^2 - c d e f (7 - 8 m) + c^2 f^2 (2 - 3 m + m^2))) (a + b x)^{1+m} (c + d x)^{-m} \right) / \\ & \left( 6 (b e - a f)^3 (d e - c f)^3 (e + f x) \right) + \\ & \left( (3 a b^2 d f (1 + m) (6 d^2 e^2 + 6 c d e f m - c^2 f^2 (1 - m) m) - 3 a^2 b d^2 f^2 (3 d e + c f m) (2 + 3 m + m^2) + a^3 d^3 f^3 (6 + 11 m + 6 m^2 + m^3)) - \right. \\ & \left. b^3 (6 d^3 e^3 + 18 c d^2 e^2 f m - 9 c^2 d e f^2 (1 - m) m + c^3 f^3 m (2 - 3 m + m^2)) (a + b x)^m \right. \\ & \left. (c + d x)^{-m} \text{Hypergeometric2F1}[1, -m, 1 - m, \frac{(b e - a f) (c + d x)}{(d e - c f) (a + b x)}] \right) / \left( 6 (b e - a f)^3 (d e - c f)^4 m \right) \end{aligned}$$

Result (type 5, 7153 leaves):

$$\begin{aligned} & \left( (a + b x)^{1+m} (c + d x)^{-m} \left( - (a^3 f^3 (6 + 11 m + 6 m^2 + m^3) + 3 a^2 b f^2 (2 + 3 m + m^2) (-3 e + f m x) + 3 a b^2 f (1 + m) (6 e^2 - 6 e f m x + f^2 (-1 + m) m x^2) + \right. \right. \\ & \left. \left. b^3 (-6 e^3 + 18 e^2 f m x - 9 e f^2 (-1 + m) m x^2 + f^3 m (2 - 3 m + m^2) x^3) \right) \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m \right] + f (1 + m) (a + b x) \right. \\ & \left( 3 (a^2 f^2 (6 + 5 m + m^2) + 2 a b f (2 + m) (-3 e + f m x) + b^2 (6 e^2 - 6 e f m x + f^2 (-1 + m) m x^2)) \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m \right] - \right. \\ & \left. f (2 + m) (a + b x) \left( 3 (a f (3 + m) + b (-3 e + f m x)) \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] - \right. \right. \\ & \left. \left. f (3 + m) (a + b x) \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] \right) \right) \Bigg) / \\ & \left( 3 (e + f x)^3 \left( 2 b^4 c e^4 - 2 a b^3 c e^3 f + 2 b^4 d e^4 x + 6 b^4 c e^3 f x - 2 a b^3 d e^3 f x - 6 a b^3 c e^2 f^2 x + 6 b^4 d e^3 f x^2 + 6 b^4 c e^2 f^2 x^2 - \right. \right. \\ & \left. \left. 6 a b^3 d e^2 f^2 x^2 - 6 a b^3 c e f^3 x^2 + 6 b^4 d e^2 f^2 x^3 + 2 b^4 c e f^3 x^3 - 6 a b^3 d e f^3 x^3 - 2 a b^3 c f^4 x^3 + 2 b^4 d e f^3 x^4 - 2 a b^3 d f^4 x^4 - f (b e - a f) (1 + m) (a + b x) (c + d x) \right. \right. \\ & \left. \left. (a^2 f^2 (6 + 5 m + m^2) + 2 a b f (2 + m) (-3 e + f m x) + b^2 (6 e^2 - 6 e f m x + f^2 (-1 + m) m x^2)) \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m \right] - \right. \right. \\ & \left. \left. f (1 + m) (a + b x)^2 (a^2 f^2 (6 + 5 m + m^2) (-d e + 3 c f + 2 d f x) + 2 a b f (2 + m) (c f (-e (9 + m) + 2 f m x) + d (3 e^2 - 2 e f (3 + m) x + f^2 m x^2)) + b^2 (c f (6 e^2 (3 + m) - 2 e f m (5 + m) x + f^2 (-1 + m) m x^2) - 3 d e (2 e^2 - 4 e f (1 + m) x + f^2 m (1 + m) x^2)) \right) \right) \right) \end{aligned}$$

$$\begin{aligned}
& \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m\right] + 12 a^3 b d e^2 f^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] - \\
& 18 a^3 b c e f^3 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] - 12 a^4 d e f^3 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] + \\
& 18 a^4 c f^4 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] + 18 a^3 b d e^2 f^2 m \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] - \\
& 29 a^3 b c e f^3 m \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] - 22 a^4 d e f^3 m \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] + \\
& 33 a^4 c f^4 m \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] + 6 a^3 b d e^2 f^2 m^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] - \\
& 12 a^3 b c e f^3 m^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] - 12 a^4 d e f^3 m^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] + \\
& 18 a^4 c f^4 m^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] - a^3 b c e f^3 m^3 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] - \\
& 2 a^4 d e f^3 m^3 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] + 3 a^4 c f^4 m^3 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] + \\
& 36 a^2 b^2 d e^2 f^2 x \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] - 54 a^2 b^2 c e f^3 x \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] - \\
& 42 a^3 b d e f^3 x \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] + 54 a^3 b c f^4 x \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] + \\
& 6 a^4 d f^4 x \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] + 54 a^2 b^2 d e^2 f^2 m x \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] - \\
& 87 a^2 b^2 c e f^3 m x \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] - 81 a^3 b d e f^3 m x \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] + \\
& 103 a^3 b c f^4 m x \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] + 11 a^4 d f^4 m x \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] + \\
& 18 a^2 b^2 d e^2 f^2 m^2 x \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] - 36 a^2 b^2 c e f^3 m^2 x \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] - \\
& 48 a^3 b d e f^3 m^2 x \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] + 60 a^3 b c f^4 m^2 x \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] + \\
& 6 a^4 d f^4 m^2 x \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] - 3 a^2 b^2 c e f^3 m^3 x \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] -
\end{aligned}$$

$$\begin{aligned}
& 9 a^3 b d e f^3 m^3 x \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + 11 a^3 b c f^4 m^3 x \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + \\
& a^4 d f^4 m^3 x \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + 36 a b^3 d e^2 f^2 x^2 \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] - \\
& 54 a b^3 c e f^3 x^2 \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] - 54 a^2 b^2 d e f^3 x^2 \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + \\
& 54 a^2 b^2 c f^4 x^2 \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + 18 a^3 b d f^4 x^2 \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + \\
& 54 a b^3 d e^2 f^2 m x^2 \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] - 87 a b^3 c e f^3 m x^2 \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] - \\
& 111 a^2 b^2 d e f^3 m x^2 \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + 111 a^2 b^2 c f^4 m x^2 \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + \\
& 33 a^3 b d f^4 m x^2 \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + 18 a b^3 d e^2 f^2 m^2 x^2 \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] - \\
& 36 a b^3 c e f^3 m^2 x^2 \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] - 72 a^2 b^2 d e f^3 m^2 x^2 \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + \\
& 72 a^2 b^2 c f^4 m^2 x^2 \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + 18 a^3 b d f^4 m^2 x^2 \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] - \\
& 3 a b^3 c e f^3 m^3 x^2 \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] - 15 a^2 b^2 d e f^3 m^3 x^2 \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + \\
& 15 a^2 b^2 c f^4 m^3 x^2 \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + 3 a^3 b d f^4 m^3 x^2 \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + \\
& 12 b^4 d e^2 f^2 x^3 \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] - 18 b^4 c e f^3 x^3 \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] - \\
& 30 a b^3 d e f^3 x^3 \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + 18 a b^3 c f^4 x^3 \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + \\
& 18 a^2 b^2 d f^4 x^3 \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + 18 b^4 d e^2 f^2 m x^3 \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] - \\
& 29 b^4 c e f^3 m x^3 \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] - 67 a b^3 d e f^3 m x^3 \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + \\
& 45 a b^3 c f^4 m x^3 \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + 33 a^2 b^2 d f^4 m x^3 \operatorname{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] +
\end{aligned}$$

$$\begin{aligned}
& 6 b^4 d e^2 f^2 m^2 x^3 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] - 12 b^4 c e f^3 m^2 x^3 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] - \\
& 48 a b^3 d e f^3 m^2 x^3 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + 36 a b^3 c f^4 m^2 x^3 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + \\
& 18 a^2 b^2 d f^4 m^2 x^3 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] - b^4 c e f^3 m^3 x^3 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] - \\
& 11 a b^3 d e f^3 m^3 x^3 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + 9 a b^3 c f^4 m^3 x^3 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + \\
& 3 a^2 b^2 d f^4 m^3 x^3 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] - 6 b^4 d e f^3 x^4 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + \\
& 6 a b^3 d f^4 x^4 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] - 15 b^4 d e f^3 m x^4 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + \\
& 4 b^4 c f^4 m x^4 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + 11 a b^3 d f^4 m x^4 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] - \\
& 12 b^4 d e f^3 m^2 x^4 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + 6 b^4 c f^4 m^2 x^4 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + \\
& 6 a b^3 d f^4 m^2 x^4 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] - 3 b^4 d e f^3 m^3 x^4 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + \\
& 2 b^4 c f^4 m^3 x^4 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + a b^3 d f^4 m^3 x^4 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + \\
& 6 a^4 d e f^3 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] - 6 a^4 c f^4 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] + \\
& 11 a^4 d e f^3 m \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] - 11 a^4 c f^4 m \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] + \\
& 6 a^4 d e f^3 m^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] - 6 a^4 c f^4 m^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] + \\
& a^4 d e f^3 m^3 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] - a^4 c f^4 m^3 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] + \\
& 24 a^3 b d e f^3 x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] - 24 a^3 b c f^4 x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] + \\
& 44 a^3 b d e f^3 m x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] - 44 a^3 b c f^4 m x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] +
\end{aligned}$$

$$\begin{aligned}
& 24 a^3 b d e f^3 m^2 x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] - 24 a^3 b c f^4 m^2 x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] + \\
& 4 a^3 b d e f^3 m^3 x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] - 4 a^3 b c f^4 m^3 x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] + \\
& 36 a^2 b^2 d e f^3 x^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] - 36 a^2 b^2 c f^4 x^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] + \\
& 66 a^2 b^2 d e f^3 m x^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] - 66 a^2 b^2 c f^4 m x^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] + \\
& 36 a^2 b^2 d e f^3 m^2 x^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] - 36 a^2 b^2 c f^4 m^2 x^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] + \\
& 6 a^2 b^2 d e f^3 m^3 x^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] - 6 a^2 b^2 c f^4 m^3 x^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] + \\
& 24 a b^3 d e f^3 x^3 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] - 24 a b^3 c f^4 x^3 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] + \\
& 44 a b^3 d e f^3 m x^3 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] - 44 a b^3 c f^4 m x^3 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] + \\
& 24 a b^3 d e f^3 m^2 x^3 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] - 24 a b^3 c f^4 m^2 x^3 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] + \\
& 4 a b^3 d e f^3 m^3 x^3 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] - 4 a b^3 c f^4 m^3 x^3 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] + \\
& 6 b^4 d e f^3 x^4 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] - 6 b^4 c f^4 x^4 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] + \\
& 11 b^4 d e f^3 m x^4 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] - 11 b^4 c f^4 m x^4 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] + \\
& 6 b^4 d e f^3 m^2 x^4 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] - 6 b^4 c f^4 m^2 x^4 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] + \\
& b^4 d e f^3 m^3 x^4 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] - b^4 c f^4 m^3 x^4 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right]
\end{aligned}$$

Problem 3079: Result more than twice size of optimal antiderivative.

$$\int (a + b x)^m (c + d x)^{-2-m} (e + f x)^p dx$$

Optimal (type 6, 131 leaves, 3 steps):

$$\frac{1}{(bc - ad)^2 (1+m)} b (a + bx)^{1+m} (c + dx)^{-m} \left( \frac{b(c+dx)}{bc-ad} \right)^m (e + fx)^p \left( \frac{b(e+fx)}{be-af} \right)^{-p} \text{AppellF1}[1+m, 2+m, -p, 2+m, -\frac{d(a+bx)}{bc-ad}, -\frac{f(a+bx)}{be-af}]$$

Result (type 6, 300 leaves):

$$\begin{aligned} & \left( (bc - ad) (be - af) (2+m) (a + bx)^{1+m} (c + dx)^{-2-m} (e + fx)^p \text{AppellF1}[1+m, 2+m, -p, 2+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}] \right) / \\ & \left( b (1+m) \left( (bc - ad) (be - af) (2+m) \text{AppellF1}[1+m, 2+m, -p, 2+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}] \right) - \right. \\ & \left. (a + bx) \left( (-bc + ad) fp \text{AppellF1}[2+m, 2+m, 1-p, 3+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}] + \right. \right. \\ & \left. \left. d (be - af) (2+m) \text{AppellF1}[2+m, 3+m, -p, 3+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}] \right) \right) \end{aligned}$$

Problem 3080: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (5 - 4x)^3 (1 + 2x)^{-2-m} (2 + 3x)^m dx$$

Optimal (type 5, 132 leaves, 3 steps):

$$\begin{aligned} & -\frac{1}{3} (5 - 4x)^2 (1 + 2x)^{-1-m} (2 + 3x)^{1+m} - \frac{(1 + 2x)^{-1-m} (2 + 3x)^{1+m} (2768 - 315m + 4m^2 - 8(43 - m)(1 + m)x)}{9(1 + m)} + \\ & \frac{2^{-m} (1323 - 128m + 2m^2) (1 + 2x)^{-m}}{9m} \text{Hypergeometric2F1}[-m, -m, 1 - m, -3(1 + 2x)] \end{aligned}$$

Result (type 6, 273 leaves):

$$\begin{aligned}
& \frac{7}{2} \left( -\frac{98 (1+2x)^{-1-m} (2+3x)^{1+m}}{1+m} - \left( 69 (5-4x)^2 (2+4x)^{-m} (4+6x)^m \text{AppellF1}[2, -m, m, 3, -\frac{3}{23} (-5+4x), \frac{1}{7} (5-4x)] \right) \right) / \\
& \left( 483 \text{AppellF1}[2, -m, m, 3, \frac{3}{23} (5-4x), \frac{1}{7} (5-4x)] + \right. \\
& m (-5+4x) \left( 21 \text{AppellF1}[3, 1-m, m, 4, \frac{3}{23} (5-4x), \frac{1}{7} (5-4x)] - 23 \text{AppellF1}[3, -m, 1+m, 4, \frac{3}{23} (5-4x), \frac{1}{7} (5-4x)] \right) + \\
& \frac{2^{3-m} (1+2x)^{1-m} \text{Hypergeometric2F1}[1-m, -m, 2-m, -3-6x]}{1-m} + \\
& \left. \frac{84 (-1-2x)^m (1+2x)^{-m} (2+3x) (6+9x)^m \text{Hypergeometric2F1}[1+m, 1+m, 2+m, 4+6x]}{1+m} \right)
\end{aligned}$$

**Problem 3081: Result unnecessarily involves higher level functions.**

$$\int (a+b x)^m (c+d x)^{-2-m} (e+f x)^2 dx$$

Optimal (type 5, 204 leaves, 4 steps):

$$\begin{aligned}
& \frac{(d e - c f) (a d f (1+m) + b (d e - c f (2+m))) (a+b x)^{1+m} (c+d x)^{-1-m}}{b d^2 (b c - a d) (1+m)} + \frac{f (a+b x)^{1+m} (c+d x)^{-1-m} (e+f x)}{b d} - \frac{1}{b d^3 m} \\
& f (a d f m + b (2 d e - c f (2+m))) (a+b x)^m \left( -\frac{d (a+b x)}{b c - a d} \right)^{-m} (c+d x)^{-m} \text{Hypergeometric2F1}[-m, -m, 1-m, \frac{b (c+d x)}{b c - a d}]
\end{aligned}$$

Result (type 6, 300 leaves):

$$\begin{aligned}
& \frac{1}{3} (a+b x)^m (c+d x)^{-2-m} \\
& \left( \frac{3 e^2 (a+b x) (c+d x)}{(b c - a d) (1+m)} - \left( 9 a c e f x^2 \text{AppellF1}[2, -m, 2+m, 3, -\frac{b x}{a}, -\frac{d x}{c}] \right) \right) / \left( -3 a c \text{AppellF1}[2, -m, 2+m, 3, -\frac{b x}{a}, -\frac{d x}{c}] - \right. \\
& b c m x \text{AppellF1}[3, 1-m, 2+m, 4, -\frac{b x}{a}, -\frac{d x}{c}] + a d (2+m) x \text{AppellF1}[3, -m, 3+m, 4, -\frac{b x}{a}, -\frac{d x}{c}] \Big) - \\
& \left. \left( 4 a c f^2 x^3 \text{AppellF1}[3, -m, 2+m, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \left( -4 a c \text{AppellF1}[3, -m, 2+m, 4, -\frac{b x}{a}, -\frac{d x}{c}] - \right. \right. \\
& b c m x \text{AppellF1}[4, 1-m, 2+m, 5, -\frac{b x}{a}, -\frac{d x}{c}] + a d (2+m) x \text{AppellF1}[4, -m, 3+m, 5, -\frac{b x}{a}, -\frac{d x}{c}] \Big) \Big)
\end{aligned}$$

Problem 3084: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^m (c + d x)^{-2-m}}{e + f x} dx$$

Optimal (type 5, 120 leaves, 2 steps):

$$\frac{\frac{d (a + b x)^{1+m} (c + d x)^{-1-m}}{(b c - a d) (d e - c f) (1 + m)} + \frac{f (a + b x)^m (c + d x)^{-m} \text{Hypergeometric2F1}[1, -m, 1 - m, \frac{(b e - a f) (c + d x)}{(d e - c f) (a + b x)}]}{(d e - c f)^2 m}}$$

Result (type 5, 578 leaves):

$$\begin{aligned} & - \left( \left( (a + b x)^{1+m} (c + d x)^{-2-m} \left( 6 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m \right] + 5 m \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m \right] + \right. \right. \right. \\ & \quad \left. \left. \left. m^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m \right] - \frac{3 f (a + b x) \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m \right]}{-b e + a f} - \right. \right. \\ & \quad \left. \left. \left. \frac{f m (a + b x) \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m \right]}{-b e + a f} + \frac{(d e - c f) (a + b x) \text{Hypergeometric2F1}[2, 3 + m, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{(b e - a f) (c + d x)} - \right. \right. \\ & \quad \left. \left. \left. \frac{f (-d e + c f) (a + b x)^2 \text{Hypergeometric2F1}[2, 3 + m, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{(b e - a f)^2 (c + d x)} \right) \right) / \right. \\ & \quad \left( (-b e + a f) (3 + m) \left( \frac{-a d (1 + m) + b c (2 + m) + b d x}{b c - a d} - \frac{b (2 + m) (e + f x) \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m \right]}{b e - a f} + \right. \right. \\ & \quad \left. \left. \left. \frac{b (-d e + c f) (a + b x) (e + f x) \text{Hypergeometric2F1}[2, 3 + m, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{(b e - a f)^2 (3 + m) (c + d x)} \right) \right) \right) \end{aligned}$$

Problem 3085: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^m (c + d x)^{-2-m}}{(e + f x)^2} dx$$

Optimal (type 5, 233 leaves, 4 steps):

$$\begin{aligned}
 & -\frac{d(a d f (2+m) - b(d e + c f (1+m))) (a+b x)^{1+m} (c+d x)^{-1-m}}{(b c - a d) (b e - a f) (d e - c f)^2 (1+m)} - \frac{f (a+b x)^{1+m} (c+d x)^{-1-m}}{(b e - a f) (d e - c f) (e+f x)} - \\
 & \frac{f (a d f (2+m) - b(2 d e + c f m)) (a+b x)^m (c+d x)^{-m} \text{Hypergeometric2F1}[1, -m, 1-m, \frac{(b e - a f) (c+d x)}{(d e - c f) (a+b x)}]}{(b e - a f) (d e - c f)^3 m}
 \end{aligned}$$

Result (type 5, 21480 leaves) : Display of huge result suppressed!

**Problem 3086:** Result more than twice size of optimal antiderivative.

$$\int \frac{(a+b x)^m (c+d x)^{-2-m}}{(e+f x)^3} dx$$

Optimal (type 5, 432 leaves, 5 steps) :

$$\begin{aligned}
 & \left( d(a^2 d^2 f^2 (6+5m+m^2) + b^2 (2 d^2 e^2 + 5 c d e f (1+m) - c^2 f^2 (1-m^2)) - a b d f (d e (9+5m) + c f (3+5m+2m^2))) (a+b x)^{1+m} (c+d x)^{-1-m} \right) / \\
 & \left( 2(b c - a d) (b e - a f)^2 (d e - c f)^3 (1+m) \right) - \frac{f (a+b x)^{1+m} (c+d x)^{-1-m}}{2(b e - a f) (d e - c f) (e+f x)^2} - \frac{f (b (4 d e - c f (1-m)) - a d f (3+m)) (a+b x)^{1+m} (c+d x)^{-1-m}}{2(b e - a f)^2 (d e - c f)^2 (e+f x)} - \\
 & \left( f (2 a b d f (2+m) (3 d e + c f m) - b^2 (6 d^2 e^2 + 6 c d e f m - c^2 f^2 (1-m) m) - a^2 d^2 f^2 (6+5m+m^2)) (a+b x)^m \right. \\
 & \left. (c+d x)^{-m} \text{Hypergeometric2F1}[1, -m, 1-m, \frac{(b e - a f) (c+d x)}{(d e - c f) (a+b x)}] \right) / (2(b e - a f)^2 (d e - c f)^4 m)
 \end{aligned}$$

Result (type 5, 57971 leaves) : Display of huge result suppressed!

**Problem 3087:** Result more than twice size of optimal antiderivative.

$$\int (a+b x)^m (c+d x)^{-3-m} (e+f x)^p dx$$

Optimal (type 6, 133 leaves, 3 steps) :

$$\frac{1}{(b c - a d)^3 (1+m)} b^2 (a+b x)^{1+m} (c+d x)^{-m} \left( \frac{b (c+d x)}{b c - a d} \right)^m (e+f x)^p \left( \frac{b (e+f x)}{b e - a f} \right)^{-p} \text{AppellF1}[1+m, 3+m, -p, 2+m, -\frac{d (a+b x)}{b c - a d}, -\frac{f (a+b x)}{b e - a f}]$$

Result (type 6, 300 leaves) :

$$\begin{aligned} & \left( (b c - a d) (b e - a f) (2 + m) (a + b x)^{1+m} (c + d x)^{-3-m} (e + f x)^p \text{AppellF1}[1 + m, 3 + m, -p, 2 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] \right) / \\ & \left( b (1 + m) \left( (b c - a d) (b e - a f) (2 + m) \text{AppellF1}[1 + m, 3 + m, -p, 2 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] \right. \right. - \\ & (a + b x) \left( (-b c + a d) f p \text{AppellF1}[2 + m, 3 + m, 1 - p, 3 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] \right) + \\ & \left. \left. d (b e - a f) (3 + m) \text{AppellF1}[2 + m, 4 + m, -p, 3 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] \right) \right) \end{aligned}$$

**Problem 3088: Result unnecessarily involves higher level functions.**

$$\int (5 - 4 x)^4 (1 + 2 x)^{-3-m} (2 + 3 x)^m dx$$

Optimal (type 5, 188 leaves, 4 steps):

$$\begin{aligned} & -\frac{1}{9} (107 - 2 m) (5 - 4 x)^2 (1 + 2 x)^{-2-m} (2 + 3 x)^{1+m} - \frac{1}{3} (5 - 4 x)^3 (1 + 2 x)^{-2-m} (2 + 3 x)^{1+m} + \\ & \frac{7 (1 + 2 x)^{-2-m} (2 + 3 x)^{1+m} (3 (4638 + 485 m + 108 m^2 - 2 m^3) + 2 (15209 + 1882 m - 530 m^2 + 8 m^3) x)}{9 (2 + 3 m + m^2)} - \\ & \frac{2^{2-m} (1323 - 85 m + m^2) (1 + 2 x)^{-m} \text{Hypergeometric2F1}[-m, -m, 1 - m, -3 (1 + 2 x)]}{9 m} \end{aligned}$$

Result (type 6, 318 leaves):

$$\begin{aligned} & 21 \left( \frac{392 (1 + 2 x)^{-1-m} (2 + 3 x)^{1+m}}{3 + 3 m} + \left( 23 (5 - 4 x)^2 (2 + 4 x)^{-m} (4 + 6 x)^m \text{AppellF1}[2, -m, m, 3, -\frac{3}{23} (-5 + 4 x), \frac{1}{7} (5 - 4 x)] \right) / \right. \\ & \left( 483 \text{AppellF1}[2, -m, m, 3, \frac{3}{23} (5 - 4 x), \frac{1}{7} (5 - 4 x)] + \right. \\ & m (-5 + 4 x) \left( 21 \text{AppellF1}[3, 1 - m, m, 4, \frac{3}{23} (5 - 4 x), \frac{1}{7} (5 - 4 x)] - 23 \text{AppellF1}[3, -m, 1 + m, 4, \frac{3}{23} (5 - 4 x), \frac{1}{7} (5 - 4 x)] \right) + \\ & \frac{2^{2-m} (1 + 2 x)^{1-m} \text{Hypergeometric2F1}[1 - m, -m, 2 - m, -3 - 6 x]}{-1 + m} - \\ & \frac{56 (-3 - 6 x)^m (1 + 2 x)^{-m} (2 + 3 x)^{1+m} \text{Hypergeometric2F1}[1 + m, 1 + m, 2 + m, 4 + 6 x]}{1 + m} - \\ & \left. \frac{1029 (-1 - 2 x)^m (1 + 2 x)^{-m} (2 + 3 x) (6 + 9 x)^m \text{Hypergeometric2F1}[1 + m, 3 + m, 2 + m, 4 + 6 x]}{1 + m} \right) \end{aligned}$$

Problem 3090: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (a + b x)^m (c + d x)^{-3-m} (e + f x)^2 dx$$

Optimal (type 5, 205 leaves, 4 steps):

$$\frac{(d e - c f)^2 (a + b x)^{1+m} (c + d x)^{-2-m}}{d^2 (b c - a d) (2 + m)} - \frac{(d e - c f) (2 a d f (2 + m) - b (d e + c f (3 + 2 m))) (a + b x)^{1+m} (c + d x)^{-1-m}}{d^2 (b c - a d)^2 (1 + m) (2 + m)} -$$

$$\frac{f^2 (a + b x)^m \left(-\frac{d (a+b x)}{b c-a d}\right)^{-m} (c + d x)^{-m} \text{Hypergeometric2F1}[-m, -m, 1 - m, \frac{b (c+d x)}{b c-a d}]}{d^3 m}$$

Result (type 6, 426 leaves):

$$\frac{1}{3} (a + b x)^m (c + d x)^{-3-m} \left( \left( 6 e f \left( \frac{c (a + b x)}{a (c + d x)} \right)^{-m} (c + d x) \left( b^2 c^2 (1 + m) x^2 \left( \frac{c (a + b x)}{a (c + d x)} \right)^m - a b c x \left( \frac{c (a + b x)}{a (c + d x)} \right)^m (-c m + d (2 + m) x) \right) + \right. \right. \\ a^2 \left( d^2 x^2 - c^2 \left( -1 + \left( \frac{c (a + b x)}{a (c + d x)} \right)^m \right) - c d x \left( -2 + 2 \left( \frac{c (a + b x)}{a (c + d x)} \right)^m + m \left( \frac{c (a + b x)}{a (c + d x)} \right)^m \right) \right) \left) \right) / \left( c (b c - a d)^2 (1 + m) (2 + m) \right) - \\ \left( 4 a c f^2 x^3 \text{AppellF1}[3, -m, 3 + m, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \left( -4 a c \text{AppellF1}[3, -m, 3 + m, 4, -\frac{b x}{a}, -\frac{d x}{c}] - \right. \\ \left. b c m x \text{AppellF1}[4, 1 - m, 3 + m, 5, -\frac{b x}{a}, -\frac{d x}{c}] + a d (3 + m) x \text{AppellF1}[4, -m, 4 + m, 5, -\frac{b x}{a}, -\frac{d x}{c}] \right) - \\ \left. 3 e^2 \left( \frac{d (a+b x)}{-b c+a d} \right)^{-m} (c + d x) \text{Hypergeometric2F1}[-2 - m, -m, -1 - m, \frac{b (c+d x)}{b c-a d}] \right) / d (2 + m)$$

Problem 3093: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^m (c + d x)^{-3-m}}{e + f x} dx$$

Optimal (type 5, 196 leaves, 4 steps):

$$\frac{d (a + b x)^{1+m} (c + d x)^{-2-m}}{(b c - a d) (d e - c f) (2 + m)} + \frac{d (a d f (2 + m) + b (d e - c f (3 + m))) (a + b x)^{1+m} (c + d x)^{-1-m}}{(b c - a d)^2 (d e - c f)^2 (1 + m) (2 + m)} -$$

$$\frac{f^2 (a + b x)^m (c + d x)^{-m} \text{Hypergeometric2F1}[1, -m, 1 - m, \frac{(b e - a f) (c + d x)}{(d e - c f) (a + b x)}]}{(d e - c f)^3 m}$$

Result (type 5, 12578 leaves):

$$\begin{aligned}
 & \left( (a+b x)^{1+2m} (c+d x)^{-6-2m} \left( \frac{-b c - b d x}{-b c + a d} \right)^{3+m} (-b e - b f x) \left( 1 - \frac{d (a+b x)}{-b c + a d} \right)^{-3-m} \right. \\
 & \left( 24 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}, 1, 3+m \right] + 26 m \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}, 1, 3+m \right] + \right. \\
 & \quad 9 m^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}, 1, 3+m \right] + m^3 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}, 1, 3+m \right] + \\
 & \quad \frac{24 f (a+b x) \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}, 1, 3+m \right]}{b e - a f} + \frac{14 f m (a+b x) \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}, 1, 3+m \right]}{b e - a f} + \\
 & \quad \frac{2 f m^2 (a+b x) \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}, 1, 3+m \right]}{b e - a f} + \frac{8 f^2 (a+b x)^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}, 1, 3+m \right]}{(b e - a f)^2} + \\
 & \quad \frac{2 f^2 m (a+b x)^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}, 1, 3+m \right]}{(b e - a f)^2} + \frac{5 (d e - c f) (a+b x) \text{Hypergeometric2F1} \left[ 2, 4+m, 5+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)} \right]}{(b e - a f) (c+d x)} + \\
 & \quad \frac{2 (d e - c f) m (a+b x) \text{Hypergeometric2F1} \left[ 2, 4+m, 5+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)} \right]}{(b e - a f) (c+d x)} + \\
 & \quad \frac{8 f (d e - c f) (a+b x)^2 \text{Hypergeometric2F1} \left[ 2, 4+m, 5+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)} \right]}{(b e - a f)^2 (c+d x)} + \\
 & \quad \frac{2 f (d e - c f) m (a+b x)^2 \text{Hypergeometric2F1} \left[ 2, 4+m, 5+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)} \right]}{(b e - a f)^2 (c+d x)} + \\
 & \quad \frac{3 f^2 (d e - c f) (a+b x)^3 \text{Hypergeometric2F1} \left[ 2, 4+m, 5+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)} \right]}{(b e - a f)^3 (c+d x)} + \\
 & \quad \frac{(d e - c f) (a+b x) \text{HypergeometricPFQ} \left[ \{2, 2, 4+m\}, \{1, 5+m\}, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)} \right]}{(b e - a f) (c+d x)} + \\
 & \quad \frac{2 f (d e - c f) (a+b x)^2 \text{HypergeometricPFQ} \left[ \{2, 2, 4+m\}, \{1, 5+m\}, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)} \right]}{(b e - a f)^2 (c+d x)} + \\
 & \quad \left. \frac{f^2 (d e - c f) (a+b x)^3 \text{HypergeometricPFQ} \left[ \{2, 2, 4+m\}, \{1, 5+m\}, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)} \right]}{(b e - a f)^3 (c+d x)} \right) \Bigg)
 \end{aligned}$$

$$\begin{aligned}
& \left( b (-b e + a f) (1+m) (2+m) (4+m) (e+f x)^2 \left( \frac{1}{b (-b e + a f) (1+m) (2+m) (4+m) (e+f x)} (a+b x)^{1+m} (c+d x)^{-3-m} \left( \frac{-b c - b d x}{-b c + a d} \right)^{3+m} \right. \right. \\
& \left. \left. (-b e - b f x) \left( 1 - \frac{d (a+b x)}{-b c + a d} \right)^{-3-m} \left( \frac{24 b f \text{HurwitzLerchPhi}\left[ \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}, 1, 3+m \right]}{b e - a f} + \frac{14 b f m \text{HurwitzLerchPhi}\left[ \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}, 1, 3+m \right]}{b e - a f} + \right. \right. \\
& \left. \left. \frac{2 b f m^2 \text{HurwitzLerchPhi}\left[ \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}, 1, 3+m \right]}{b e - a f} + \frac{16 b f^2 (a+b x) \text{HurwitzLerchPhi}\left[ \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}, 1, 3+m \right]}{(b e - a f)^2} + \right. \right. \\
& \left. \left. \frac{4 b f^2 m (a+b x) \text{HurwitzLerchPhi}\left[ \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}, 1, 3+m \right]}{(b e - a f)^2} + \frac{1}{d e - c f} 24 f (c+d x) \left( -\frac{d (d e - c f) (a+b x)}{(b e - a f) (c+d x)^2} + \frac{b (d e - c f)}{(b e - a f) (c+d x)} \right) \right. \right. \\
& \left. \left. \left( \frac{1}{1 - \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}} + (-3-m) \text{HurwitzLerchPhi}\left[ \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}, 1, 3+m \right] \right) + \frac{1}{d e - c f} 14 f m (c+d x) \left( -\frac{d (d e - c f) (a+b x)}{(b e - a f) (c+d x)^2} + \right. \right. \\
& \left. \left. \frac{b (d e - c f)}{(b e - a f) (c+d x)} \right) \left( \frac{1}{1 - \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}} + (-3-m) \text{HurwitzLerchPhi}\left[ \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}, 1, 3+m \right] \right) + \frac{1}{d e - c f} 2 f m^2 (c+d x) \right. \right. \\
& \left. \left. \left( -\frac{d (d e - c f) (a+b x)}{(b e - a f) (c+d x)^2} + \frac{b (d e - c f)}{(b e - a f) (c+d x)} \right) \left( \frac{1}{1 - \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}} + (-3-m) \text{HurwitzLerchPhi}\left[ \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}, 1, 3+m \right] \right) + \right. \right. \\
& \left. \left. \frac{1}{(d e - c f) (a+b x)} 24 (b e - a f) (c+d x) \left( -\frac{d (d e - c f) (a+b x)}{(b e - a f) (c+d x)^2} + \frac{b (d e - c f)}{(b e - a f) (c+d x)} \right) \right. \right. \\
& \left. \left. \left( \frac{1}{1 - \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}} + (-3-m) \text{HurwitzLerchPhi}\left[ \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}, 1, 3+m \right] \right) + \frac{1}{(d e - c f) (a+b x)} 26 (b e - a f) m (c+d x) \right. \right. \\
& \left. \left. \left( -\frac{d (d e - c f) (a+b x)}{(b e - a f) (c+d x)^2} + \frac{b (d e - c f)}{(b e - a f) (c+d x)} \right) \left( \frac{1}{1 - \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}} + (-3-m) \text{HurwitzLerchPhi}\left[ \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}, 1, 3+m \right] \right) + \right. \right. \\
& \left. \left. \frac{1}{(d e - c f) (a+b x)} 9 (b e - a f) m^2 (c+d x) \left( -\frac{d (d e - c f) (a+b x)}{(b e - a f) (c+d x)^2} + \frac{b (d e - c f)}{(b e - a f) (c+d x)} \right) \right. \right. \\
& \left. \left. \left( \frac{1}{1 - \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}} + (-3-m) \text{HurwitzLerchPhi}\left[ \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}, 1, 3+m \right] \right) + \frac{1}{(d e - c f) (a+b x)} (b e - a f) m^3 (c+d x) \right. \right. 
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{d(d e - c f)(a + b x)}{(b e - a f)(c + d x)^2} + \frac{b(d e - c f)}{(b e - a f)(c + d x)} \right) \left( \frac{1}{1 - \frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}} + (-3 - m) \text{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, 3 + m\right] \right) + \\
& \frac{1}{(b e - a f)(d e - c f)} 8 f^2 (a + b x)(c + d x) \left( -\frac{d(d e - c f)(a + b x)}{(b e - a f)(c + d x)^2} + \frac{b(d e - c f)}{(b e - a f)(c + d x)} \right) \\
& \left( \frac{1}{1 - \frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}} + (-3 - m) \text{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, 3 + m\right] \right) + \frac{1}{(b e - a f)(d e - c f)} 2 f^2 m (a + b x)(c + d x) \\
& \left( -\frac{d(d e - c f)(a + b x)}{(b e - a f)(c + d x)^2} + \frac{b(d e - c f)}{(b e - a f)(c + d x)} \right) \left( \frac{1}{1 - \frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}} + (-3 - m) \text{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, 3 + m\right] \right) + \\
& 5(4 + m) \left( -\frac{d(d e - c f)(a + b x)}{(b e - a f)(c + d x)^2} + \frac{b(d e - c f)}{(b e - a f)(c + d x)} \right) \left( \frac{1}{\left(1 - \frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}\right)^2} - \right. \\
& \left. \text{Hypergeometric2F1}[2, 4 + m, 5 + m, \frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}] \right) + 2m(4 + m) \left( -\frac{d(d e - c f)(a + b x)}{(b e - a f)(c + d x)^2} + \frac{b(d e - c f)}{(b e - a f)(c + d x)} \right) \\
& \left( \frac{1}{\left(1 - \frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}\right)^2} - \text{Hypergeometric2F1}[2, 4 + m, 5 + m, \frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}] \right) + \frac{1}{b e - a f} 8 f (4 + m) (a + b x) \\
& \left( -\frac{d(d e - c f)(a + b x)}{(b e - a f)(c + d x)^2} + \frac{b(d e - c f)}{(b e - a f)(c + d x)} \right) \left( \frac{1}{\left(1 - \frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}\right)^2} - \text{Hypergeometric2F1}[2, 4 + m, 5 + m, \frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}] \right) + \\
& \frac{1}{b e - a f} 2 f m (4 + m) (a + b x) \left( -\frac{d(d e - c f)(a + b x)}{(b e - a f)(c + d x)^2} + \frac{b(d e - c f)}{(b e - a f)(c + d x)} \right) \\
& \left( \frac{1}{\left(1 - \frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}\right)^2} - \text{Hypergeometric2F1}[2, 4 + m, 5 + m, \frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}] \right) + \frac{1}{(b e - a f)^2} 3 f^2 (4 + m) (a + b x)^2 \\
& \left( -\frac{d(d e - c f)(a + b x)}{(b e - a f)(c + d x)^2} + \frac{b(d e - c f)}{(b e - a f)(c + d x)} \right) \left( \frac{1}{\left(1 - \frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}\right)^2} - \text{Hypergeometric2F1}[2, 4 + m, 5 + m, \frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}] \right) - \\
& 5 d(d e - c f)(a + b x) \text{Hypergeometric2F1}[2, 4 + m, 5 + m, \frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}]
\end{aligned}$$

$$\begin{aligned}
& \frac{2 d (d e - c f) m (a + b x) \text{Hypergeometric2F1}[2, 4 + m, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{(b e - a f) (c + d x)^2} - \\
& \frac{8 d f (d e - c f) (a + b x)^2 \text{Hypergeometric2F1}[2, 4 + m, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{(b e - a f)^2 (c + d x)^2} - \\
& \frac{2 d f (d e - c f) m (a + b x)^2 \text{Hypergeometric2F1}[2, 4 + m, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{(b e - a f)^2 (c + d x)^2} - \\
& \frac{3 d f^2 (d e - c f) (a + b x)^3 \text{Hypergeometric2F1}[2, 4 + m, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{(b e - a f)^3 (c + d x)^2} + \\
& \frac{5 b (d e - c f) \text{Hypergeometric2F1}[2, 4 + m, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{(b e - a f) (c + d x)} + \frac{2 b (d e - c f) m \text{Hypergeometric2F1}[2, 4 + m, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{(b e - a f) (c + d x)} + \\
& \frac{16 b f (d e - c f) (a + b x) \text{Hypergeometric2F1}[2, 4 + m, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{(b e - a f)^2 (c + d x)} + \\
& \frac{4 b f (d e - c f) m (a + b x) \text{Hypergeometric2F1}[2, 4 + m, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{(b e - a f)^2 (c + d x)} + \\
& \frac{9 b f^2 (d e - c f) (a + b x)^2 \text{Hypergeometric2F1}[2, 4 + m, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{(b e - a f)^3 (c + d x)} + \\
& (4 + m) \left( -\frac{d (d e - c f) (a + b x)}{(b e - a f) (c + d x)^2} + \frac{b (d e - c f)}{(b e - a f) (c + d x)} \right) \left( -\frac{(b e - a f)^2 (c + d x)^2 (b c e + a d e - 2 a c f + 2 b d e x - b c f x - a d f x)}{(-b c + a d)^3 (e + f x)^3} - \right. \\
& \quad \left. \text{HypergeometricPFQ}[\{2, 2, 4 + m\}, \{1, 5 + m\}, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}] \right) + \frac{1}{b e - a f} 2 f (4 + m) (a + b x) \\
& \left( -\frac{d (d e - c f) (a + b x)}{(b e - a f) (c + d x)^2} + \frac{b (d e - c f)}{(b e - a f) (c + d x)} \right) \left( -\frac{(b e - a f)^2 (c + d x)^2 (b c e + a d e - 2 a c f + 2 b d e x - b c f x - a d f x)}{(-b c + a d)^3 (e + f x)^3} - \right. \\
& \quad \left. \text{HypergeometricPFQ}[\{2, 2, 4 + m\}, \{1, 5 + m\}, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}] \right) + \frac{1}{(b e - a f)^2} f^2 (4 + m) (a + b x)^2 \\
& \left( -\frac{d (d e - c f) (a + b x)}{(b e - a f) (c + d x)^2} + \frac{b (d e - c f)}{(b e - a f) (c + d x)} \right) \left( -\frac{(b e - a f)^2 (c + d x)^2 (b c e + a d e - 2 a c f + 2 b d e x - b c f x - a d f x)}{(-b c + a d)^3 (e + f x)^3} - \right. \\
& \quad \left. \text{HypergeometricPFQ}[\{2, 2, 4 + m\}, \{1, 5 + m\}, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}] \right) - 
\end{aligned}$$

$$\begin{aligned}
& \frac{d (d e - c f) (a + b x) \text{HypergeometricPFQ}\left[\{2, 2, 4+m\}, \{1, 5+m\}, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}\right]}{(b e - a f) (c + d x)^2} - \\
& \frac{2 d f (d e - c f) (a + b x)^2 \text{HypergeometricPFQ}\left[\{2, 2, 4+m\}, \{1, 5+m\}, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}\right]}{(b e - a f)^2 (c + d x)^2} - \\
& \frac{d f^2 (d e - c f) (a + b x)^3 \text{HypergeometricPFQ}\left[\{2, 2, 4+m\}, \{1, 5+m\}, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}\right]}{(b e - a f)^3 (c + d x)^2} + \\
& \frac{b (d e - c f) \text{HypergeometricPFQ}\left[\{2, 2, 4+m\}, \{1, 5+m\}, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}\right]}{(b e - a f) (c + d x)} + \\
& \frac{4 b f (d e - c f) (a + b x) \text{HypergeometricPFQ}\left[\{2, 2, 4+m\}, \{1, 5+m\}, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}\right]}{(b e - a f)^2 (c + d x)} + \\
& \left. \frac{3 b f^2 (d e - c f) (a + b x)^2 \text{HypergeometricPFQ}\left[\{2, 2, 4+m\}, \{1, 5+m\}, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}\right]}{(b e - a f)^3 (c + d x)} \right\} - \\
& \frac{1}{(-b c + a d) (-b e + a f) (1+m) (2+m) (4+m) (e + f x)} d (-3-m) (a + b x)^{1+m} (c + d x)^{-3-m} \left( \frac{-b c - b d x}{-b c + a d} \right)^{3+m} (-b e - b f x) \\
& \left( 1 - \frac{d (a + b x)}{-b c + a d} \right)^{-4-m} \left( 24 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3+m\right] + 26 m \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3+m\right] + \right. \\
& 9 m^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3+m\right] + m^3 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3+m\right] + \\
& \frac{24 f (a + b x) \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3+m\right]}{b e - a f} + \frac{14 f m (a + b x) \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3+m\right]}{b e - a f} + \\
& \frac{2 f m^2 (a + b x) \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3+m\right]}{b e - a f} + \frac{8 f^2 (a + b x)^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3+m\right]}{(b e - a f)^2} + \\
& \frac{2 f^2 m (a + b x)^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3+m\right]}{(b e - a f)^2} + \frac{5 (d e - c f) (a + b x) \text{Hypergeometric2F1}\left[2, 4+m, 5+m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{(b e - a f) (c + d x)} + \\
& \frac{2 (d e - c f) m (a + b x) \text{Hypergeometric2F1}\left[2, 4+m, 5+m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{(b e - a f) (c + d x)}
\end{aligned}$$

$$\begin{aligned}
& \frac{8 f (d e - c f) (a + b x)^2 \text{Hypergeometric2F1}\left[2, 4 + m, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{(b e - a f)^2 (c + d x)} + \\
& \frac{2 f (d e - c f) m (a + b x)^2 \text{Hypergeometric2F1}\left[2, 4 + m, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{(b e - a f)^2 (c + d x)} + \\
& \frac{3 f^2 (d e - c f) (a + b x)^3 \text{Hypergeometric2F1}\left[2, 4 + m, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{(b e - a f)^3 (c + d x)} + \\
& \frac{(d e - c f) (a + b x) \text{HypergeometricPFQ}\left[\{2, 2, 4 + m\}, \{1, 5 + m\}, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{(b e - a f) (c + d x)} + \\
& \frac{2 f (d e - c f) (a + b x)^2 \text{HypergeometricPFQ}\left[\{2, 2, 4 + m\}, \{1, 5 + m\}, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{(b e - a f)^2 (c + d x)} + \\
& \left. \frac{f^2 (d e - c f) (a + b x)^3 \text{HypergeometricPFQ}\left[\{2, 2, 4 + m\}, \{1, 5 + m\}, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{(b e - a f)^3 (c + d x)} \right\} - \\
& \frac{1}{(-b e + a f) (1 + m) (2 + m) (4 + m) (e + f x)} f (a + b x)^{1+m} (c + d x)^{-3-m} \left(\frac{-b c - b d x}{-b c + a d}\right)^{3+m} \left(1 - \frac{d (a + b x)}{-b c + a d}\right)^{-3-m} \\
& \left( 24 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] + 26 m \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] + \right. \\
& 9 m^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] + m^3 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] + \\
& \frac{24 f (a + b x) \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right]}{b e - a f} + \frac{14 f m (a + b x) \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right]}{b e - a f} + \\
& \frac{2 f m^2 (a + b x) \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right]}{b e - a f} + \frac{8 f^2 (a + b x)^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right]}{(b e - a f)^2} + \\
& \frac{2 f^2 m (a + b x)^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right]}{(b e - a f)^2} + \frac{5 (d e - c f) (a + b x) \text{Hypergeometric2F1}\left[2, 4 + m, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{(b e - a f) (c + d x)} + \\
& \frac{2 (d e - c f) m (a + b x) \text{Hypergeometric2F1}\left[2, 4 + m, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{(b e - a f) (c + d x)}
\end{aligned}$$

$$\begin{aligned}
& \frac{8 f (d e - c f) (a + b x)^2 \text{Hypergeometric2F1}\left[2, 4 + m, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{(b e - a f)^2 (c + d x)} + \\
& \frac{2 f (d e - c f) m (a + b x)^2 \text{Hypergeometric2F1}\left[2, 4 + m, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{(b e - a f)^2 (c + d x)} + \\
& \frac{3 f^2 (d e - c f) (a + b x)^3 \text{Hypergeometric2F1}\left[2, 4 + m, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{(b e - a f)^3 (c + d x)} + \\
& \frac{(d e - c f) (a + b x) \text{HypergeometricPFQ}\left[\{2, 2, 4 + m\}, \{1, 5 + m\}, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{(b e - a f) (c + d x)} + \\
& \frac{2 f (d e - c f) (a + b x)^2 \text{HypergeometricPFQ}\left[\{2, 2, 4 + m\}, \{1, 5 + m\}, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{(b e - a f)^2 (c + d x)} + \\
& \left. \frac{f^2 (d e - c f) (a + b x)^3 \text{HypergeometricPFQ}\left[\{2, 2, 4 + m\}, \{1, 5 + m\}, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{(b e - a f)^3 (c + d x)} \right\} - \\
& \frac{1}{b (-b e + a f) (1 + m) (2 + m) (4 + m) (e + f x)^2} f (a + b x)^{1+m} (c + d x)^{-3-m} \left( \frac{-b c - b d x}{-b c + a d} \right)^{3+m} (-b e - b f x) \left( 1 - \frac{d (a + b x)}{-b c + a d} \right)^{-3-m} \\
& \left( 24 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] + 26 m \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] + \right. \\
& 9 m^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] + m^3 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] + \\
& \frac{24 f (a + b x) \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right]}{b e - a f} + \frac{14 f m (a + b x) \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right]}{b e - a f} + \\
& \frac{2 f m^2 (a + b x) \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right]}{b e - a f} + \frac{8 f^2 (a + b x)^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right]}{(b e - a f)^2} + \\
& \frac{2 f^2 m (a + b x)^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right]}{(b e - a f)^2} + \frac{5 (d e - c f) (a + b x) \text{Hypergeometric2F1}\left[2, 4 + m, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{(b e - a f) (c + d x)} + \\
& \frac{2 (d e - c f) m (a + b x) \text{Hypergeometric2F1}\left[2, 4 + m, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{(b e - a f) (c + d x)}
\end{aligned}$$

$$\begin{aligned}
& \frac{8 f (d e - c f) (a + b x)^2 \text{Hypergeometric2F1}[2, 4 + m, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{(b e - a f)^2 (c + d x)} + \\
& \frac{2 f (d e - c f) m (a + b x)^2 \text{Hypergeometric2F1}[2, 4 + m, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{(b e - a f)^2 (c + d x)} + \\
& \frac{3 f^2 (d e - c f) (a + b x)^3 \text{Hypergeometric2F1}[2, 4 + m, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{(b e - a f)^3 (c + d x)} + \\
& \frac{(d e - c f) (a + b x) \text{HypergeometricPFQ}[\{2, 2, 4 + m\}, \{1, 5 + m\}, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{(b e - a f) (c + d x)} + \\
& \frac{2 f (d e - c f) (a + b x)^2 \text{HypergeometricPFQ}[\{2, 2, 4 + m\}, \{1, 5 + m\}, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{(b e - a f)^2 (c + d x)} + \\
& \left. \frac{f^2 (d e - c f) (a + b x)^3 \text{HypergeometricPFQ}[\{2, 2, 4 + m\}, \{1, 5 + m\}, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{(b e - a f)^3 (c + d x)} \right) - \\
& \frac{1}{(-b c + a d) (-b e + a f) (1 + m) (2 + m) (4 + m) (e + f x)} d (3 + m) (a + b x)^{1+m} (c + d x)^{-3-m} \left( \frac{-b c - b d x}{-b c + a d} \right)^{2+m} (-b e - b f x) \\
& \left( 1 - \frac{d (a + b x)}{-b c + a d} \right)^{-3-m} \left( 24 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + 26 m \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + \right. \\
& 9 m^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + m^3 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + \\
& \frac{24 f (a + b x) \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right]}{b e - a f} + \frac{14 f m (a + b x) \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right]}{b e - a f} + \\
& \frac{2 f m^2 (a + b x) \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right]}{b e - a f} + \frac{8 f^2 (a + b x)^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right]}{(b e - a f)^2} + \\
& \frac{2 f^2 m (a + b x)^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right]}{(b e - a f)^2} + \frac{5 (d e - c f) (a + b x) \text{Hypergeometric2F1}[2, 4 + m, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{(b e - a f) (c + d x)} + \\
& \frac{2 (d e - c f) m (a + b x) \text{Hypergeometric2F1}[2, 4 + m, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{(b e - a f) (c + d x)} +
\end{aligned}$$

$$\begin{aligned}
& \frac{8 f (d e - c f) (a + b x)^2 \text{Hypergeometric2F1}\left[2, 4 + m, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{(b e - a f)^2 (c + d x)} + \\
& \frac{2 f (d e - c f) m (a + b x)^2 \text{Hypergeometric2F1}\left[2, 4 + m, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{(b e - a f)^2 (c + d x)} + \\
& \frac{3 f^2 (d e - c f) (a + b x)^3 \text{Hypergeometric2F1}\left[2, 4 + m, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{(b e - a f)^3 (c + d x)} + \\
& \frac{(d e - c f) (a + b x) \text{HypergeometricPFQ}\left[\{2, 2, 4 + m\}, \{1, 5 + m\}, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{(b e - a f) (c + d x)} + \\
& \frac{2 f (d e - c f) (a + b x)^2 \text{HypergeometricPFQ}\left[\{2, 2, 4 + m\}, \{1, 5 + m\}, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{(b e - a f)^2 (c + d x)} + \\
& \left. \frac{f^2 (d e - c f) (a + b x)^3 \text{HypergeometricPFQ}\left[\{2, 2, 4 + m\}, \{1, 5 + m\}, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{(b e - a f)^3 (c + d x)} \right\} + \\
& \frac{1}{b (-b e + a f) (1 + m) (2 + m) (4 + m) (e + f x)} d (-3 - m) (a + b x)^{1+m} (c + d x)^{-4-m} \left(\frac{-b c - b d x}{-b c + a d}\right)^{3+m} (-b e - b f x) \left(1 - \frac{d (a + b x)}{-b c + a d}\right)^{-3-m} \\
& \left( 24 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] + 26 m \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] + \right. \\
& 9 m^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] + m^3 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] + \\
& \frac{24 f (a + b x) \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right]}{b e - a f} + \frac{14 f m (a + b x) \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right]}{b e - a f} + \\
& \frac{2 f m^2 (a + b x) \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right]}{b e - a f} + \frac{8 f^2 (a + b x)^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right]}{(b e - a f)^2} + \\
& \frac{2 f^2 m (a + b x)^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right]}{(b e - a f)^2} + \frac{5 (d e - c f) (a + b x) \text{Hypergeometric2F1}\left[2, 4 + m, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{(b e - a f) (c + d x)} + \\
& \frac{2 (d e - c f) m (a + b x) \text{Hypergeometric2F1}\left[2, 4 + m, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{(b e - a f) (c + d x)}
\end{aligned}$$

$$\begin{aligned}
& \frac{8 f (d e - c f) (a + b x)^2 \text{Hypergeometric2F1}\left[2, 4 + m, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{(b e - a f)^2 (c + d x)} + \\
& \frac{2 f (d e - c f) m (a + b x)^2 \text{Hypergeometric2F1}\left[2, 4 + m, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{(b e - a f)^2 (c + d x)} + \\
& \frac{3 f^2 (d e - c f) (a + b x)^3 \text{Hypergeometric2F1}\left[2, 4 + m, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{(b e - a f)^3 (c + d x)} + \\
& \frac{(d e - c f) (a + b x) \text{HypergeometricPFQ}\left[\{2, 2, 4 + m\}, \{1, 5 + m\}, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{(b e - a f) (c + d x)} + \\
& \frac{2 f (d e - c f) (a + b x)^2 \text{HypergeometricPFQ}\left[\{2, 2, 4 + m\}, \{1, 5 + m\}, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{(b e - a f)^2 (c + d x)} + \\
& \left. \frac{f^2 (d e - c f) (a + b x)^3 \text{HypergeometricPFQ}\left[\{2, 2, 4 + m\}, \{1, 5 + m\}, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{(b e - a f)^3 (c + d x)} \right\} + \\
& \frac{1}{(-b e + a f) (2 + m) (4 + m) (e + f x)} (a + b x)^m (c + d x)^{-3-m} \left(\frac{-b c - b d x}{-b c + a d}\right)^{3+m} (-b e - b f x) \left(1 - \frac{d (a + b x)}{-b c + a d}\right)^{-3-m} \\
& \left( 24 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] + 26 m \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] + \right. \\
& 9 m^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] + m^3 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] + \\
& \frac{24 f (a + b x) \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right]}{b e - a f} + \frac{14 f m (a + b x) \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right]}{b e - a f} + \\
& \frac{2 f m^2 (a + b x) \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right]}{b e - a f} + \frac{8 f^2 (a + b x)^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right]}{(b e - a f)^2} + \\
& \frac{2 f^2 m (a + b x)^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right]}{(b e - a f)^2} + \frac{5 (d e - c f) (a + b x) \text{Hypergeometric2F1}\left[2, 4 + m, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{(b e - a f) (c + d x)} + \\
& \frac{2 (d e - c f) m (a + b x) \text{Hypergeometric2F1}\left[2, 4 + m, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{(b e - a f) (c + d x)}
\end{aligned}$$

$$\begin{aligned}
& \frac{8 f (d e - c f) (a + b x)^2 \text{Hypergeometric2F1}\left[2, 4 + m, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{(b e - a f)^2 (c + d x)} + \\
& \frac{2 f (d e - c f) m (a + b x)^2 \text{Hypergeometric2F1}\left[2, 4 + m, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{(b e - a f)^2 (c + d x)} + \\
& \frac{3 f^2 (d e - c f) (a + b x)^3 \text{Hypergeometric2F1}\left[2, 4 + m, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{(b e - a f)^3 (c + d x)} + \\
& \frac{(d e - c f) (a + b x) \text{HypergeometricPFQ}\left[\{2, 2, 4 + m\}, \{1, 5 + m\}, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{(b e - a f) (c + d x)} + \\
& \frac{2 f (d e - c f) (a + b x)^2 \text{HypergeometricPFQ}\left[\{2, 2, 4 + m\}, \{1, 5 + m\}, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{(b e - a f)^2 (c + d x)} + \\
& \frac{f^2 (d e - c f) (a + b x)^3 \text{HypergeometricPFQ}\left[\{2, 2, 4 + m\}, \{1, 5 + m\}, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{(b e - a f)^3 (c + d x)} \Bigg) \Bigg)
\end{aligned}$$

**Problem 3094: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b x)^m (c + d x)^{-3-m}}{(e + f x)^2} dx$$

Optimal (type 5, 384 leaves, 5 steps):

$$\begin{aligned}
& \frac{d (a d f (3 + m) - b (d e + c f (2 + m))) (a + b x)^{1+m} (c + d x)^{-2-m}}{(b c - a d) (b e - a f) (d e - c f)^2 (2 + m)} - \\
& \left( \frac{d (a^2 d^2 f^2 (6 + 5 m + m^2) - b^2 (d^2 e^2 - c d e f (5 + 2 m) - c^2 f^2 (2 + 3 m + m^2)) - a b d f (d e (3 + 2 m) + c f (9 + 8 m + 2 m^2))) (a + b x)^{1+m} (c + d x)^{-1-m}}{(b c - a d)^2 (b e - a f) (d e - c f)^3 (1 + m) (2 + m)} - \right. \\
& \left. \frac{f (a + b x)^{1+m} (c + d x)^{-2-m}}{(b e - a f) (d e - c f) (e + f x)} + \right. \\
& \left. \frac{f^2 (a d f (3 + m) - b (3 d e + c f m)) (a + b x)^m (c + d x)^{-m} \text{Hypergeometric2F1}\left[1, -m, 1 - m, \frac{(b e - a f) (c + d x)}{(d e - c f) (a + b x)}\right]}{(b e - a f) (d e - c f)^4 m} \right)
\end{aligned}$$

Result (type 5, 38673 leaves): Display of huge result suppressed!

**Problem 3095: Result more than twice size of optimal antiderivative.**

$$\int (a + bx)^m (c + dx)^{-4-m} (e + fx)^p dx$$

Optimal (type 6, 133 leaves, 3 steps):

$$\frac{1}{(bc - ad)^4 (1+m)} b^3 (a + bx)^{1+m} (c + dx)^{-m} \left( \frac{b(c + dx)}{bc - ad} \right)^m (e + fx)^p \left( \frac{b(e + fx)}{be - af} \right)^{-p} \text{AppellF1}[1 + m, 4 + m, -p, 2 + m, -\frac{d(a + bx)}{bc - ad}, -\frac{f(a + bx)}{be - af}]$$

Result (type 6, 300 leaves):

$$\begin{aligned} & \left( (bc - ad) (be - af) (2 + m) (a + bx)^{1+m} (c + dx)^{-4-m} (e + fx)^p \text{AppellF1}[1 + m, 4 + m, -p, 2 + m, \frac{d(a + bx)}{-bc + ad}, \frac{f(a + bx)}{-be + af}] \right) / \\ & \left( b(1 + m) \left( (bc - ad) (be - af) (2 + m) \text{AppellF1}[1 + m, 4 + m, -p, 2 + m, \frac{d(a + bx)}{-bc + ad}, \frac{f(a + bx)}{-be + af}] \right) - \right. \\ & (a + bx) \left( (-bc + ad) f p \text{AppellF1}[2 + m, 4 + m, 1 - p, 3 + m, \frac{d(a + bx)}{-bc + ad}, \frac{f(a + bx)}{-be + af}] \right) + \\ & \left. d(be - af) (4 + m) \text{AppellF1}[2 + m, 5 + m, -p, 3 + m, \frac{d(a + bx)}{-bc + ad}, \frac{f(a + bx)}{-be + af}] \right) \end{aligned}$$

**Problem 3097: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + bx)^m (c + dx)^{-4-m} (e + fx)^3 dx$$

Optimal (type 5, 406 leaves, 10 steps):

$$\begin{aligned} & \frac{(de - cf)^3 (a + bx)^{1+m} (c + dx)^{-3-m}}{d^3 (bc - ad) (3 + m)} + \frac{3f(de - cf)^2 (a + bx)^{1+m} (c + dx)^{-2-m}}{d^3 (bc - ad) (2 + m)} + \\ & \frac{2b(de - cf)^3 (a + bx)^{1+m} (c + dx)^{-2-m}}{d^3 (bc - ad)^2 (2 + m) (3 + m)} + \frac{3f^2(de - cf)(a + bx)^{1+m} (c + dx)^{-1-m}}{d^3 (bc - ad) (1 + m)} + \frac{3bf(de - cf)^2 (a + bx)^{1+m} (c + dx)^{-1-m}}{d^3 (bc - ad)^2 (1 + m) (2 + m)} + \\ & \frac{2b^2(de - cf)^3 (a + bx)^{1+m} (c + dx)^{-1-m}}{d^3 (bc - ad)^3 (1 + m) (2 + m) (3 + m)} - \frac{f^3 (a + bx)^m \left( -\frac{d(a + bx)}{bc - ad} \right)^{-m} (c + dx)^{-m}}{d^4 m} \text{Hypergeometric2F1}[-m, -m, 1 - m, \frac{b(c + dx)}{bc - ad}] \end{aligned}$$

Result (type 6, 1833 leaves):

$$\frac{1}{c (bc - ad)^3 (1 + m) (2 + m) (3 + m)}$$

$$\begin{aligned}
& 3 e f^2 (a + b x)^m \left( \frac{c (a + b x)}{a (c + d x)} \right)^{-m} (c + d x)^{-3-m} \left( b^3 c^3 (2 + 3 m + m^2) x^3 \left( \frac{c (a + b x)}{a (c + d x)} \right)^m - a b^2 c^2 (1 + m) x^2 \left( \frac{c (a + b x)}{a (c + d x)} \right)^m (-c m + 2 d (3 + m) x) + \right. \\
& \left. a^2 b c x \left( \frac{c (a + b x)}{a (c + d x)} \right)^m (-2 c^2 m - 2 c d m (3 + m) x + d^2 (6 + 5 m + m^2) x^2) + a^3 \left( -2 d^3 x^3 + 2 c^3 \left( -1 + \left( \frac{c (a + b x)}{a (c + d x)} \right)^m \right) \right. \right. \\
& \left. \left. 2 c^2 d x \left( -3 + 3 \left( \frac{c (a + b x)}{a (c + d x)} \right)^m + m \left( \frac{c (a + b x)}{a (c + d x)} \right)^m \right) + c d^2 x^2 \left( -6 + 6 \left( \frac{c (a + b x)}{a (c + d x)} \right)^m + 5 m \left( \frac{c (a + b x)}{a (c + d x)} \right)^m + m^2 \left( \frac{c (a + b x)}{a (c + d x)} \right)^m \right) \right) \right) - \\
& \left( 5 a c f^3 x^4 (a + b x)^m (c + d x)^{-4-m} \text{AppellF1}[4, -m, 4 + m, 5, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \left( 4 \left( -5 a c \text{AppellF1}[4, -m, 4 + m, 5, -\frac{b x}{a}, -\frac{d x}{c}] - \right. \right. \\
& \left. \left. b c m x \text{AppellF1}[5, 1 - m, 4 + m, 6, -\frac{b x}{a}, -\frac{d x}{c}] + a d (4 + m) x \text{AppellF1}[5, -m, 5 + m, 6, -\frac{b x}{a}, -\frac{d x}{c}] \right) \right) + \\
& \left( 3 e^2 f x^2 (a + b x)^m (c + d x)^{-4-m} \left( 1 + \frac{d x}{c} \right) \left( (c + d x) \left( b^3 c^3 m (1 + m) x^3 + a b^2 c^2 m x^2 (c (-3 + m) - 2 d (3 + m) x) - a^2 b c x \right. \right. \right. \\
& \left. \left. \left. \left( d^2 (3 + m) x^2 \left( -2 - m + 2 \left( \frac{a (c + d x)}{c (a + b x)} \right)^m \right) + 2 c d (3 + m) x \left( -2 + m + 2 \left( \frac{a (c + d x)}{c (a + b x)} \right)^m \right) + 2 c^2 \left( -3 + 2 m + 3 \left( \frac{a (c + d x)}{c (a + b x)} \right)^m + m \left( \frac{a (c + d x)}{c (a + b x)} \right)^m \right) \right) + \right. \right. \\
& \left. \left. \left. a^3 \left( 2 d^3 m x^3 \left( \frac{a (c + d x)}{c (a + b x)} \right)^m - 6 c^3 \left( -1 + \left( \frac{a (c + d x)}{c (a + b x)} \right)^m \right) + 2 c^2 d x \left( 6 - 6 \left( \frac{a (c + d x)}{c (a + b x)} \right)^m + m \left( 2 + \left( \frac{a (c + d x)}{c (a + b x)} \right)^m \right) \right) + \right. \right. \right. \\
& \left. \left. \left. c d^2 x^2 \left( 6 + m^2 - 6 \left( \frac{a (c + d x)}{c (a + b x)} \right)^m + m \left( 5 + 4 \left( \frac{a (c + d x)}{c (a + b x)} \right)^m \right) \right) \right) \right) \text{Gamma}[1 - m] + \right. \\
& \left. m (3 c + d x) \left( b^3 c^3 (2 + 3 m + m^2) x^3 + a b^2 c^2 (1 + m) x^2 (c m - 2 d (3 + m) x) + a^2 b c x (-2 c^2 m - 2 c d m (3 + m) x + d^2 (6 + 5 m + m^2) x^2) + \right. \right. \\
& \left. \left. a^3 \left( -2 d^3 x^3 \left( \frac{a (c + d x)}{c (a + b x)} \right)^m - 2 c^3 \left( -1 + \left( \frac{a (c + d x)}{c (a + b x)} \right)^m \right) - 2 c^2 d x \left( -3 - m + 3 \left( \frac{a (c + d x)}{c (a + b x)} \right)^m \right) - c d^2 x^2 \left( -6 - 5 m - m^2 + 6 \left( \frac{a (c + d x)}{c (a + b x)} \right)^m \right) \right) \right) \right. \\
& \left. \left. \text{Gamma}[-m] \right) \right) / \left( (c + d x) \left( b^3 c^3 m (2 + 3 m + m^2) x^3 - 3 a b^2 c^2 m (1 + m) x^2 (c + d (3 + m) x) + \right. \right. \\
& \left. \left. 3 a^2 b c m x (2 c^2 + 2 c d (3 + m) x + d^2 (6 + 5 m + m^2) x^2) + a^3 \left( 6 c^3 \left( -1 + \left( \frac{a (c + d x)}{c (a + b x)} \right)^m \right) + 6 c^2 d x \left( -3 - m + 3 \left( \frac{a (c + d x)}{c (a + b x)} \right)^m \right) + \right. \right. \right. \\
& \left. \left. \left. 3 c d^2 x^2 \left( -6 - 5 m - m^2 + 6 \left( \frac{a (c + d x)}{c (a + b x)} \right)^m \right) + d^3 x^3 \left( -6 - 11 m - 6 m^2 - m^3 + 6 \left( \frac{a (c + d x)}{c (a + b x)} \right)^m \right) \right) \right) \right) \text{Gamma}[1 - m] + \right. \\
& \left. m \left( b^3 c^3 (2 + 3 m + m^2) x^3 (3 c (2 + m) + d m x) - 3 a b^2 c^2 (1 + m) x^2 (c^2 m + c d (12 + 14 m + 3 m^2) x + d^2 m (3 + m) x^2) + \right. \right. \\
& \left. \left. 3 a^2 b c x (2 c^3 m + 2 c^2 d m (4 + m) x + c d^2 (12 + 34 m + 19 m^2 + 3 m^3) x^2 + d^3 m (6 + 5 m + m^2) x^3) + \right. \right. \\
& \left. \left. a^3 \left( 6 c^4 \left( -1 + \left( \frac{a (c + d x)}{c (a + b x)} \right)^m \right) + 6 c^3 d x \left( -4 - m + 4 \left( \frac{a (c + d x)}{c (a + b x)} \right)^m \right) + d^4 x^4 \left( -6 - 11 m - 6 m^2 - m^3 + 6 \left( \frac{a (c + d x)}{c (a + b x)} \right)^m \right) \right) + \right. \right)
\end{aligned}$$

$$\frac{e^3 (c+d x)^{-3-m} \left(a - \frac{b c}{d} + \frac{b (c+d x)}{d}\right)^m \left(1 + \frac{b (c+d x)}{\left(a - \frac{b c}{d}\right) d}\right)^{-m} \text{Hypergeometric2F1}\left[-3-m, -m, -2-m, -\frac{b (c+d x)}{\left(a - \frac{b c}{d}\right) d}\right]}{d (3+m)}$$

$$3 c d^3 x^3 \left(-12 - 16 m - 7 m^2 - m^3 + 8 \left(\frac{a (c + d x)}{c (a + b x)}\right)^m\right) + 3 c^2 d^2 x^2 \left(-7 m - m^2 + 12 \left(-1 + \left(\frac{a (c + d x)}{c (a + b x)}\right)^m\right)\right) \text{Gamma}[-m] -$$

**Problem 3101: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+b x)^m (c+d x)^{-4-m}}{e+f x} dx$$

Optimal (type 5, 330 leaves, 5 steps):

$$\begin{aligned} & \frac{d (a+b x)^{1+m} (c+d x)^{-3-m}}{(b c-a d) (d e-c f) (3+m)} + \frac{d (a d f (3+m) + b (2 d e - c f (5+m))) (a+b x)^{1+m} (c+d x)^{-2-m}}{(b c-a d)^2 (d e-c f)^2 (2+m) (3+m)} + \\ & \left( d (a^2 d^2 f^2 (6+5 m+m^2) + a b d f (3+m) (d e-c f (5+2 m)) + b^2 (2 d^2 e^2 - c d e f (7+m) + c^2 f^2 (11+6 m+m^2))) (a+b x)^{1+m} (c+d x)^{-1-m} \right) / \\ & \left( (b c-a d)^3 (d e-c f)^3 (1+m) (2+m) (3+m) \right) + \frac{f^3 (a+b x)^m (c+d x)^{-m} \text{Hypergeometric2F1}[1, -m, 1-m, \frac{(b e-a f) (c+d x)}{(d e-c f) (a+b x)}]}{(d e-c f)^4 m} \end{aligned}$$

Result (type 5, 26263 leaves): Display of huge result suppressed!

**Problem 3102: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+b x)^m (c+d x)^{-4-m}}{(e+f x)^2} dx$$

Optimal (type 5, 634 leaves, 6 steps):

$$\begin{aligned}
& - \frac{d (a d f (4 + m) - b (d e + c f (3 + m))) (a + b x)^{1+m} (c + d x)^{-3-m}}{(b c - a d) (b e - a f) (d e - c f)^2 (3 + m)} - \\
& \left( d (a^2 d^2 f^2 (12 + 7 m + m^2) - b^2 (2 d^2 e^2 - 2 c d e f (4 + m) - c^2 f^2 (6 + 5 m + m^2)) - 2 a b d f (d e (2 + m) + c f (10 + 6 m + m^2))) (a + b x)^{1+m} (c + d x)^{-2-m} \right) / \\
& \left( (b c - a d)^2 (b e - a f) (d e - c f)^3 (2 + m) (3 + m) \right) - \\
& \frac{1}{(b c - a d)^3 (b e - a f) (d e - c f)^4 (1 + m) (2 + m) (3 + m)} d (a^3 d^3 f^3 (24 + 26 m + 9 m^2 + m^3) - a^2 b d^2 f^2 (3 + m) (d e (4 + 3 m) + c f (20 + 15 m + 3 m^2)) - \\
& b^3 (2 d^3 e^3 - 2 c d^2 e^2 f (5 + m) + c^2 d e f^2 (26 + 17 m + 3 m^2) + c^3 f^3 (6 + 11 m + 6 m^2 + m^3)) - \\
& a b^2 d f (2 d^2 e^2 (2 + m) - 2 c d e f (16 + 15 m + 3 m^2) - c^2 f^2 (44 + 50 m + 21 m^2 + 3 m^3))) (a + b x)^{1+m} (c + d x)^{-1-m} - \\
& \frac{f^3 (a d f (4 + m) - b (4 d e + c f m)) (a + b x)^m (c + d x)^{-m} \text{Hypergeometric2F1}[1, -m, 1 - m, \frac{(b e - a f) (c + d x)}{(d e - c f) (a + b x)}]}{(b e - a f) (d e - c f) (e + f x)^5 m}
\end{aligned}$$

Result (type 5, 64 249 leaves) : Display of huge result suppressed!

### Problem 3103: Result more than twice size of optimal antiderivative.

$$\int (a + b x)^m (c + d x)^{-5-m} (e + f x)^p dx$$

Optimal (type 6, 133 leaves, 3 steps) :

$$\frac{1}{(b c - a d)^5 (1 + m)} b^4 (a + b x)^{1+m} (c + d x)^{-m} \left( \frac{b (c + d x)}{b c - a d} \right)^m (e + f x)^p \left( \frac{b (e + f x)}{b e - a f} \right)^{-p} \text{AppellF1}[1 + m, 5 + m, -p, 2 + m, -\frac{d (a + b x)}{b c - a d}, -\frac{f (a + b x)}{b e - a f}]$$

Result (type 6, 300 leaves) :

$$\begin{aligned}
& \left( (b c - a d) (b e - a f) (2 + m) (a + b x)^{1+m} (c + d x)^{-5-m} (e + f x)^p \text{AppellF1}[1 + m, 5 + m, -p, 2 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] \right) / \\
& \left( b (1 + m) \left( (b c - a d) (b e - a f) (2 + m) \text{AppellF1}[1 + m, 5 + m, -p, 2 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] - \right. \right. \\
& (a + b x) \left( (-b c + a d) f p \text{AppellF1}[2 + m, 5 + m, 1 - p, 3 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] + \right. \\
& \left. \left. d (b e - a f) (5 + m) \text{AppellF1}[2 + m, 6 + m, -p, 3 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] \right) \right)
\end{aligned}$$

### Problem 3105: Attempted integration timed out after 120 seconds.

$$\int (a + b x)^m (c + d x)^{-5-m} (e + f x)^4 dx$$

Optimal (type 5, 650 leaves, 14 steps):

$$\begin{aligned} & \frac{(d e - c f)^4 (a + b x)^{1+m} (c + d x)^{-4-m}}{d^4 (b c - a d) (4 + m)} + \frac{4 f (d e - c f)^3 (a + b x)^{1+m} (c + d x)^{-3-m}}{d^4 (b c - a d) (3 + m)} + \frac{3 b (d e - c f)^4 (a + b x)^{1+m} (c + d x)^{-3-m}}{d^4 (b c - a d)^2 (3 + m) (4 + m)} + \\ & \frac{6 f^2 (d e - c f)^2 (a + b x)^{1+m} (c + d x)^{-2-m}}{d^4 (b c - a d) (2 + m)} + \frac{8 b f (d e - c f)^3 (a + b x)^{1+m} (c + d x)^{-2-m}}{d^4 (b c - a d)^2 (2 + m) (3 + m)} + \frac{6 b^2 (d e - c f)^4 (a + b x)^{1+m} (c + d x)^{-2-m}}{d^4 (b c - a d)^3 (2 + m) (3 + m) (4 + m)} + \\ & \frac{4 f^3 (d e - c f) (a + b x)^{1+m} (c + d x)^{-1-m}}{d^4 (b c - a d) (1 + m)} + \frac{6 b f^2 (d e - c f)^2 (a + b x)^{1+m} (c + d x)^{-1-m}}{d^4 (b c - a d)^2 (1 + m) (2 + m)} + \frac{8 b^2 f (d e - c f)^3 (a + b x)^{1+m} (c + d x)^{-1-m}}{d^4 (b c - a d)^3 (1 + m) (2 + m) (3 + m)} + \\ & \frac{6 b^3 (d e - c f)^4 (a + b x)^{1+m} (c + d x)^{-1-m}}{d^4 (b c - a d)^4 (1 + m) (2 + m) (3 + m) (4 + m)} - \frac{f^4 (a + b x)^m \left(-\frac{d (a + b x)}{b c - a d}\right)^{-m} (c + d x)^{-m} \text{Hypergeometric2F1}[-m, -m, 1 - m, \frac{b (c + d x)}{b c - a d}]}{d^5 m} \end{aligned}$$

Result (type 1, 1 leaves):

???

Problem 3110: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^m (c + d x)^{-5-m}}{e + f x} dx$$

Optimal (type 5, 557 leaves, 6 steps):

$$\begin{aligned} & \frac{d (a + b x)^{1+m} (c + d x)^{-4-m}}{(b c - a d) (d e - c f) (4 + m)} + \frac{d (a d f (4 + m) + b (3 d e - c f (7 + m))) (a + b x)^{1+m} (c + d x)^{-3-m}}{(b c - a d)^2 (d e - c f)^2 (3 + m) (4 + m)} + \\ & \left( \frac{d (a^2 d^2 f^2 (12 + 7 m + m^2) + 2 a b d f (4 + m) (d e - c f (4 + m)) + b^2 (6 d^2 e^2 - 2 c d e f (10 + m) + c^2 f^2 (26 + 9 m + m^2))) (a + b x)^{1+m} (c + d x)^{-2-m}}{(b c - a d)^3 (d e - c f)^3 (2 + m) (3 + m) (4 + m)} \right) + \\ & \left( \frac{d (a^3 d^3 f^3 (24 + 26 m + 9 m^2 + m^3) + a^2 b d^2 f^2 (12 + 7 m + m^2) (d e - c f (7 + 3 m)) + a b^2 d f (4 + m) (2 d^2 e^2 - 2 c d e f (5 + m) + c^2 f^2 (26 + 17 m + 3 m^2))) + b^3 (6 d^3 e^3 - 2 c d^2 e^2 f (13 + m) + c^2 d e f^2 (46 + 11 m + m^2) - c^3 f^3 (50 + 35 m + 10 m^2 + m^3)) (a + b x)^{1+m} (c + d x)^{-1-m}}{(b c - a d)^4 (d e - c f)^4 (1 + m) (2 + m) (3 + m) (4 + m)} \right) - \frac{f^4 (a + b x)^m (c + d x)^{-m} \text{Hypergeometric2F1}[1, -m, 1 - m, \frac{(b e - a f) (c + d x)}{(d e - c f) (a + b x)}]}{(d e - c f)^5 m} \end{aligned}$$

Result (type 5, 50481 leaves): Display of huge result suppressed!

Problem 3111: Result more than twice size of optimal antiderivative.

$$\int (a + b x)^m (c + d x)^{1-m} (e + f x)^p dx$$

Optimal (type 6, 131 leaves, 3 steps):

$$\frac{1}{b^2 (1+m)} (b c - a d) (a + b x)^{1+m} (c + d x)^{-m} \left( \frac{b (c + d x)}{b c - a d} \right)^m (e + f x)^p \left( \frac{b (e + f x)}{b e - a f} \right)^{-p} \text{AppellF1}[1+m, -1+m, -p, 2+m, -\frac{d (a+b x)}{b c - a d}, -\frac{f (a+b x)}{b e - a f}]$$

Result (type 6, 298 leaves):

$$\begin{aligned} & \left( (b c - a d) (b e - a f) (2+m) (a + b x)^{1+m} (c + d x)^{1-m} (e + f x)^p \text{AppellF1}[1+m, -1+m, -p, 2+m, \frac{d (a+b x)}{-b c + a d}, \frac{f (a+b x)}{-b e + a f}] \right) / \\ & \left( b (1+m) \left( (b c - a d) (b e - a f) (2+m) \text{AppellF1}[1+m, -1+m, -p, 2+m, \frac{d (a+b x)}{-b c + a d}, \frac{f (a+b x)}{-b e + a f}] \right) - \right. \\ & (a + b x) \left( (-b c + a d) f p \text{AppellF1}[2+m, -1+m, 1-p, 3+m, \frac{d (a+b x)}{-b c + a d}, \frac{f (a+b x)}{-b e + a f}] \right) + \\ & \left. d (b e - a f) (-1+m) \text{AppellF1}[2+m, m, -p, 3+m, \frac{d (a+b x)}{-b c + a d}, \frac{f (a+b x)}{-b e + a f}] \right) \end{aligned}$$

Problem 3112: Result unnecessarily involves higher level functions.

$$\int (a + b x)^m (c + d x)^{1-m} (e + f x)^3 dx$$

Optimal (type 5, 445 leaves, 4 steps):

$$\begin{aligned} & \frac{f (a + b x)^{1+m} (c + d x)^{2-m} (e + f x)^2}{5 b d} + \frac{1}{60 b^3 d^3} \\ & f (a + b x)^{1+m} (c + d x)^{2-m} (a^2 d^2 f^2 (12 - 7 m + m^2) - a b d f (15 d e (3 - m) - c f (9 + 2 m - 2 m^2))) + b^2 (48 d^2 e^2 - 15 c d e f (2 + m) + c^2 f^2 (6 + 5 m + m^2)) - \\ & 3 b d f (a d f (4 - m) - b (7 d e - c f (3 + m))) x - \frac{1}{60 b^5 d^3 (1+m)} (b c - a d) \\ & (a^3 d^3 f^3 (24 - 26 m + 9 m^2 - m^3) - 3 a^2 b d^2 f^2 (6 - 5 m + m^2) (5 d e - c f (1 + m)) + 3 a b^2 d f (2 - m) (20 d^2 e^2 - 10 c d e f (1 + m) + c^2 f^2 (2 + 3 m + m^2)) - \\ & b^3 (60 d^3 e^3 - 60 c d^2 e^2 f (1 + m) + 15 c^2 d e f^2 (2 + 3 m + m^2) - c^3 f^3 (6 + 11 m + 6 m^2 + m^3))) \\ & (a + b x)^{1+m} (c + d x)^{-m} \left( \frac{b (c + d x)}{b c - a d} \right)^m \text{Hypergeometric2F1}[-1+m, 1+m, 2+m, -\frac{d (a+b x)}{b c - a d}] \end{aligned}$$

Result (type 6, 461 leaves):

$$\begin{aligned}
& \frac{1}{4} (a + b x)^m (c + d x)^{1-m} \left( \left( 18 a c e^2 f x^2 \text{AppellF1}[2, -m, -1+m, 3, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \left( 3 a c \text{AppellF1}[2, -m, -1+m, 3, -\frac{b x}{a}, -\frac{d x}{c}] \right. \right. + \\
& \quad b c m x \text{AppellF1}[3, 1-m, -1+m, 4, -\frac{b x}{a}, -\frac{d x}{c}] - a d (-1+m) x \text{AppellF1}[3, -m, m, 4, -\frac{b x}{a}, -\frac{d x}{c}] \Big) + \\
& \quad \left( 16 a c e f^2 x^3 \text{AppellF1}[3, -m, -1+m, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \left( 4 a c \text{AppellF1}[3, -m, -1+m, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right. + \\
& \quad b c m x \text{AppellF1}[4, 1-m, -1+m, 5, -\frac{b x}{a}, -\frac{d x}{c}] - a d (-1+m) x \text{AppellF1}[4, -m, m, 5, -\frac{b x}{a}, -\frac{d x}{c}] \Big) + \\
& \quad \left( 5 a c f^3 x^4 \text{AppellF1}[4, -m, -1+m, 5, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \left( 5 a c \text{AppellF1}[4, -m, -1+m, 5, -\frac{b x}{a}, -\frac{d x}{c}] \right. + \\
& \quad b c m x \text{AppellF1}[5, 1-m, -1+m, 6, -\frac{b x}{a}, -\frac{d x}{c}] - a d (-1+m) x \text{AppellF1}[5, -m, m, 6, -\frac{b x}{a}, -\frac{d x}{c}] \Big) - \\
& \quad \left. \frac{4 e^3 \left( \frac{d(a+b x)}{-b c+a d} \right)^{-m} (c+d x) \text{Hypergeometric2F1}[2-m, -m, 3-m, \frac{b(c+d x)}{b c-a d}]}{d(-2+m)} \right)
\end{aligned}$$

**Problem 3113: Result unnecessarily involves higher level functions.**

$$\int (a + b x)^m (c + d x)^{1-m} (e + f x)^2 dx$$

Optimal (type 5, 260 leaves, 4 steps):

$$\begin{aligned}
& - \frac{f (a d f (3-m) - b (5 d e - c f (2+m))) (a + b x)^{1+m} (c + d x)^{2-m}}{12 b^2 d^2} + \frac{f (a + b x)^{1+m} (c + d x)^{2-m} (e + f x)}{4 b d} + \frac{1}{12 b^4 d^2 (1+m)} \\
& (b c - a d) (a^2 d^2 f^2 (6 - 5 m + m^2) - 2 a b d f (2-m) (4 d e - c f (1+m)) + b^2 (12 d^2 e^2 - 8 c d e f (1+m) + c^2 f^2 (2 + 3 m + m^2))) \\
& (a + b x)^{1+m} (c + d x)^{-m} \left( \frac{b (c + d x)}{b c - a d} \right)^m \text{Hypergeometric2F1}[-1+m, 1+m, 2+m, -\frac{d (a + b x)}{b c - a d}]
\end{aligned}$$

Result (type 6, 510 leaves):

$$\begin{aligned}
& c (a + b x)^m (c + d x)^{-m} \left( \left( 3 a e (d e + 2 c f) x^2 \text{AppellF1}[2, -m, m, 3, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \right. \\
& \left( 6 a c \text{AppellF1}[2, -m, m, 3, -\frac{b x}{a}, -\frac{d x}{c}] + 2 m x \left( b c \text{AppellF1}[3, 1-m, m, 4, -\frac{b x}{a}, -\frac{d x}{c}] - a d \text{AppellF1}[3, -m, 1+m, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) \right) + \\
& \left( 4 a f (2 d e + c f) x^3 \text{AppellF1}[3, -m, m, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \\
& \left( 3 \left( 4 a c \text{AppellF1}[3, -m, m, 4, -\frac{b x}{a}, -\frac{d x}{c}] + b c m x \text{AppellF1}[4, 1-m, m, 5, -\frac{b x}{a}, -\frac{d x}{c}] - a d m x \text{AppellF1}[4, -m, 1+m, 5, -\frac{b x}{a}, -\frac{d x}{c}] \right) \right) + \\
& \left( 5 a d f^2 x^4 \text{AppellF1}[4, -m, m, 5, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \\
& \left( 20 a c \text{AppellF1}[4, -m, m, 5, -\frac{b x}{a}, -\frac{d x}{c}] + 4 b c m x \text{AppellF1}[5, 1-m, m, 6, -\frac{b x}{a}, -\frac{d x}{c}] - 4 a d m x \text{AppellF1}[5, -m, 1+m, 6, -\frac{b x}{a}, -\frac{d x}{c}] \right) - \\
& \left. \frac{c e^{2 \left( \frac{d (a+b x)}{-b c+a d} \right)^{-m}} \text{Hypergeometric2F1}[1-m, -m, 2-m, \frac{b (c+d x)}{b c-a d}]}{d (-1+m)} - \frac{e^{2 x \left( \frac{d (a+b x)}{-b c+a d} \right)^{-m}} \text{Hypergeometric2F1}[1-m, -m, 2-m, \frac{b (c+d x)}{b c-a d}]}{-1+m} \right)
\end{aligned}$$

**Problem 3114:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (a + b x)^m (c + d x)^{1-m} (e + f x) dx$$

Optimal (type 5, 145 leaves, 3 steps):

$$\begin{aligned}
& \frac{f (a + b x)^{1+m} (c + d x)^{2-m}}{3 b d} - \frac{1}{3 b^3 d (1+m)} \\
& (b c - a d) (a d f (2-m) - b (3 d e - c f (1+m))) (a + b x)^{1+m} (c + d x)^{-m} \left( \frac{b (c + d x)}{b c - a d} \right)^m \text{Hypergeometric2F1}[-1+m, 1+m, 2+m, -\frac{d (a + b x)}{b c - a d}]
\end{aligned}$$

Result (type 6, 322 leaves):

$$\begin{aligned}
& c (a + b x)^m (c + d x)^{-m} \left( \left( 3 a (d e + c f) x^2 \text{AppellF1}[2, -m, m, 3, -\frac{b x}{a}, -\frac{d x}{c}] \right) \right. \\
& \left. \left( 6 a c \text{AppellF1}[2, -m, m, 3, -\frac{b x}{a}, -\frac{d x}{c}] + 2 m x \left( b c \text{AppellF1}[3, 1-m, m, 4, -\frac{b x}{a}, -\frac{d x}{c}] - a d \text{AppellF1}[3, -m, 1+m, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) \right) + \right. \\
& \left. \left( 4 a d f x^3 \text{AppellF1}[3, -m, m, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) \right) / \\
& \left( 12 a c \text{AppellF1}[3, -m, m, 4, -\frac{b x}{a}, -\frac{d x}{c}] + 3 b c m x \text{AppellF1}[4, 1-m, m, 5, -\frac{b x}{a}, -\frac{d x}{c}] - 3 a d m x \text{AppellF1}[4, -m, 1+m, 5, -\frac{b x}{a}, -\frac{d x}{c}] \right) - \\
& \left. \frac{e \left( \frac{d(a+b x)}{-b c+a d} \right)^{-m} (c + d x) \text{Hypergeometric2F1}[1-m, -m, 2-m, \frac{b(c+d x)}{b c-a d}]}{d (-1+m)} \right)
\end{aligned}$$

**Problem 3115:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (a + b x)^m (c + d x)^{1-m} dx$$

Optimal (type 5, 82 leaves, 2 steps):

$$\frac{(b c - a d) (a + b x)^{1+m} (c + d x)^{-m} \left( \frac{b(c+d x)}{b c-a d} \right)^m \text{Hypergeometric2F1}[-1+m, 1+m, 2+m, -\frac{d(a+b x)}{b c-a d}]}{b^2 (1+m)}$$

Result (type 6, 202 leaves):

$$\begin{aligned}
& \frac{1}{d} c (a + b x)^m (c + d x)^{-m} \left( \left( 3 a d^2 x^2 \text{AppellF1}[2, -m, m, 3, -\frac{b x}{a}, -\frac{d x}{c}] \right) \right. \\
& \left. \left( 6 a c \text{AppellF1}[2, -m, m, 3, -\frac{b x}{a}, -\frac{d x}{c}] + 2 m x \left( b c \text{AppellF1}[3, 1-m, m, 4, -\frac{b x}{a}, -\frac{d x}{c}] - a d \text{AppellF1}[3, -m, 1+m, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) \right) - \right. \\
& \left. \left( \frac{d(a+b x)}{-b c+a d} \right)^{-m} (c + d x) \text{Hypergeometric2F1}[1-m, -m, 2-m, \frac{b(c+d x)}{b c-a d}] \right) \\
& - 1 + m
\end{aligned}$$

**Problem 3116:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^m (c + d x)^{1-m}}{e + f x} dx$$

Optimal (type 5, 230 leaves, 6 steps):

$$\begin{aligned} & -\frac{d (d e - c f) (a + b x)^{1+m} (c + d x)^{-m}}{(b c - a d) f^2 m} - \frac{(d e - c f) (a + b x)^m (c + d x)^{-m} \text{Hypergeometric2F1}[1, -m, 1 - m, \frac{(b e - a f) (c + d x)}{(d e - c f) (a + b x)}]}{f^2 m} + \frac{1}{b (b c - a d) f^2 m (1 + m)} \\ & d (b (d e - c f (1 - m)) - a d f m) (a + b x)^{1+m} (c + d x)^{-m} \left( \frac{b (c + d x)}{b c - a d} \right)^m \text{Hypergeometric2F1}[m, 1 + m, 2 + m, -\frac{d (a + b x)}{b c - a d}] \end{aligned}$$

Result (type 6, 622 leaves):

$$\begin{aligned} & \left( (a + b x)^m (c + d x)^{-m} \left( -d (-b c + a d) e (b e - a f) (-1 + m) (2 + m) (a + b x) \text{AppellF1}[1 + m, m, 1, 2 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] \right. \right. - \right. \\ & c (b c - a d) f (b e - a f) (-1 + m) (2 + m) (a + b x) \text{AppellF1}[1 + m, m, 1, 2 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] + \\ & b (1 + m) \left( \frac{d (a + b x)}{-b c + a d} \right)^{-m} (c + d x) (e + f x) \left( (b c - a d) (b e - a f) (2 + m) \text{AppellF1}[1 + m, m, 1, 2 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] + \right. \\ & (a + b x) \left( (-b c f + a d f) \text{AppellF1}[2 + m, m, 2, 3 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] + \right. \\ & \left. \left. \left. d (-b e + a f) m \text{AppellF1}[2 + m, 1 + m, 1, 3 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] \right) \right) \text{Hypergeometric2F1}[1 - m, -m, 2 - m, \frac{b (c + d x)}{b c - a d}] \right) / \\ & \left( b f (1 - m) (1 + m) (e + f x) \left( (b c - a d) (b e - a f) (2 + m) \text{AppellF1}[1 + m, m, 1, 2 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] + \right. \right. \\ & (a + b x) \left( (-b c f + a d f) \text{AppellF1}[2 + m, m, 2, 3 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] + \right. \\ & \left. \left. \left. d (-b e + a f) m \text{AppellF1}[2 + m, 1 + m, 1, 3 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] \right) \right) \right) \end{aligned}$$

Problem 3117: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^m (c + d x)^{1-m}}{(e + f x)^2} dx$$

Optimal (type 5, 190 leaves, 6 steps):

$$\begin{aligned}
& - \frac{(a+b x)^m (c+d x)^{1-m}}{f (e+f x)} + \frac{(a d f (1-m) - b (d e - c f m)) (a+b x)^m (c+d x)^{-m} \text{Hypergeometric2F1}[1, m, 1+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{f^2 (b e - a f) m} + \\
& \frac{d (a+b x)^m (c+d x)^{-m} \left( \frac{b (c+d x)}{b c - a d} \right)^m \text{Hypergeometric2F1}[m, m, 1+m, -\frac{d (a+b x)}{b c - a d}]}{f^2 m}
\end{aligned}$$

Result (type 6, 461 leaves):

$$\begin{aligned}
& \frac{1}{(b e - a f) (1+m) (e + f x)} \\
& (a+b x)^{1+m} (c+d x)^{-m} \left( - \left( \left( d (b c - a d) (b e - a f)^3 (2+m) \text{AppellF1}[1+m, m, 1, 2+m, \frac{d (a+b x)}{-b c + a d}, \frac{f (a+b x)}{-b e + a f}] \right) \right) \right) / \left( b f (-b e + a f) \right. \\
& \left( (b c - a d) (b e - a f) (2+m) \text{AppellF1}[1+m, m, 1, 2+m, \frac{d (a+b x)}{-b c + a d}, \frac{f (a+b x)}{-b e + a f}] + (a+b x) \left( (-b c f + a d f) \text{AppellF1}[2+m, \right. \right. \\
& \left. \left. m, 2, 3+m, \frac{d (a+b x)}{-b c + a d}, \frac{f (a+b x)}{-b e + a f}] + d (-b e + a f) m \text{AppellF1}[2+m, 1+m, 1, 3+m, \frac{d (a+b x)}{-b c + a d}, \frac{f (a+b x)}{-b e + a f}] \right) \right) \right) + \\
& c \left( \frac{(b e - a f) (c+d x)}{(b c - a d) (e + f x)} \right)^m \text{Hypergeometric2F1}[m, 1+m, 2+m, \frac{(-d e + c f) (a+b x)}{(b c - a d) (e + f x)}] - \\
& \left. \frac{d e \left( \frac{(b e - a f) (c+d x)}{(b c - a d) (e + f x)} \right)^m \text{Hypergeometric2F1}[m, 1+m, 2+m, \frac{(-d e + c f) (a+b x)}{(b c - a d) (e + f x)}]}{f} \right)
\end{aligned}$$

Problem 3118: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+b x)^m (c+d x)^{1-m}}{(e+f x)^3} dx$$

Optimal (type 5, 85 leaves, 1 step):

$$\frac{(b c - a d)^2 (a+b x)^{1+m} (c+d x)^{-1-m} \text{Hypergeometric2F1}[3, 1+m, 2+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{(b e - a f)^3 (1+m)}$$

Result (type 5, 933 leaves):

$$\begin{aligned}
& \left( (a + b x)^{1+m} (c + d x)^{-m} \right) \\
& \left( -d e (b e - a f)^2 (1+m) (c + d x) \left( (-2 b e + a f (1+m) + b f (-1+m) x) \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] - 2 (a f (1+m) + \right. \right. \\
& \quad \left. \left. b (-e + f m x) \right) \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1+m \right] + f (1+m) (a + b x) \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2+m \right] \right) + \\
& \quad c f (b e - a f)^2 (1+m) (c + d x) \left( (-2 b e + a f (1+m) + b f (-1+m) x) \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] - \right. \\
& \quad \left. 2 (a f (1+m) + b (-e + f m x)) \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1+m \right] + \right. \\
& \quad \left. f (1+m) (a + b x) \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2+m \right] \right) + d (2 b e - 2 a f) \left( \frac{(b e - a f) (c + d x)}{(b c - a d) (e + f x)} \right)^m \\
& (e + f x) \left( (b e - a f) (c + d x) (a f (1+m) + b (-e + f m x)) \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] - \right. \\
& \quad \left. (a + b x) \left( (a f (1+m) (-2 c f + d (e - f x)) + b (c f (e (2+m) - f m x) + d e (-e + f (1+2 m) x))) \right) \right. \\
& \quad \left. \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1+m \right] + f (-d e + c f) (1+m) (a + b x) \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2+m \right] \right) \right) \\
& \text{Hypergeometric2F1} [m, 1+m, 2+m, \frac{(-d e + c f) (a + b x)}{(b c - a d) (e + f x)}] \Bigg) \Bigg) / \left( f (2 b e - 2 a f) (b e - a f) (1+m) (e + f x)^2 \right. \\
& \quad \left( (b e - a f) (c + d x) (a f (1+m) + b (-e + f m x)) \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] - (a + b x) \right. \\
& \quad \left. \left( (a f (1+m) (-2 c f + d (e - f x)) + b (c f (e (2+m) - f m x) + d e (-e + f (1+2 m) x))) \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1+m \right] + \right. \right. \\
& \quad \left. \left. f (-d e + c f) (1+m) (a + b x) \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2+m \right] \right) \right)
\end{aligned}$$

## Problem 3119: Result more than twice size of optimal antiderivative

$$\int \frac{(ax + bx^m)^m (cx + dx^{1-m})}{(ex + fx^4)^4} dx$$

Optimal (type 5, 176 leaves, 2 steps):

$$\begin{aligned}
& - \frac{f (a + b x)^{1+m} (c + d x)^{2-m}}{3 (b e - a f) (d e - c f) (e + f x)^3} + \\
& \left( (b c - a d)^2 (b (3 d e - c f (2 - m)) - a d f (1 + m)) (a + b x)^{1+m} (c + d x)^{-1-m} \text{Hypergeometric2F1}[3, 1 + m, 2 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}] \right) / \\
& (3 (b e - a f)^4 (d e - c f) (1 + m))
\end{aligned}$$

Result (type 5, 3837 leaves):

$$\begin{aligned}
& \left( d (b e - a f)^4 (a + b x)^{1+m} (c + d x)^{-m} \right. \\
& \left( (-2 b e + a f (1 + m) + b f (-1 + m) x) \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m\right] - 2 (a f (1 + m) + b (-e + f m x)) \right. \\
& \left. \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m\right] + f (1 + m) (a + b x) \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m\right] \right) / \\
& \left( f (2 b e - 2 a f) (-b e + a f)^3 (e + f x)^2 \left( (b e - a f) (-a f (1 + m) + b (e - f m x)) \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m\right] + \right. \right. \\
& \left. \frac{1}{c + d x} (a + b x) \left( (a f (1 + m) (-2 c f + d (e - f x)) + b (c f (e (2 + m) - f m x) + d e (-e + f (1 + 2 m) x))) \right. \right. \\
& \left. \left. \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m\right] + f (-d e + c f) (1 + m) (a + b x) \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m\right] \right) \right) + \\
& \left( c (a + b x)^{1+m} (c + d x)^{1-m} \left( 6 (b e - a f)^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m\right] + 6 (b e - a f)^2 m \right. \right. \\
& \left. \left. \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m\right] + 6 f (b e - a f) (a + b x) \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m\right] + \right. \right. \\
& \left. \left. 6 f (-b e + a f) m^2 (a + b x) \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m\right] + 2 f^2 (a + b x)^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m\right] - \right. \right. \\
& \left. \left. f^2 m (a + b x)^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m\right] - 2 f^2 m^2 (a + b x)^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m\right] + \right. \right. \\
& \left. \left. f^2 m^3 (a + b x)^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m\right] - 6 (b e - a f)^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m\right] - \right. \right. \\
& \left. \left. 6 (b e - a f)^2 m \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m\right] + 12 f (b e - a f) m (a + b x) \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m\right] + \right. \right. \\
& \left. \left. 12 f (b e - a f) m^2 (a + b x) \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m\right] + \right. \right)
\end{aligned}$$

$$\begin{aligned}
& 3 f^2 m (a + b x)^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m\right] - 3 f^2 m^3 (a + b x)^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m\right] + \\
& 6 f (-b e + a f) (a + b x) \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m\right] + \\
& 12 f (-b e + a f) m (a + b x) \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m\right] + 6 f (-b e + a f) m^2 (a + b x) \\
& \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m\right] + 3 f^2 m (a + b x)^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m\right] + \\
& 6 f^2 m^2 (a + b x)^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m\right] + 3 f^2 m^3 (a + b x)^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m\right] - \\
& 2 f^2 (a + b x)^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] - 5 f^2 m (a + b x)^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] - \\
& 4 f^2 m^2 (a + b x)^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] - f^2 m^3 (a + b x)^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] \Bigg) / \\
& \left( 3 (1 + m) (e + f x)^3 \left( (b e - a f) (c + d x) (a^2 f^2 (2 + 3 m + m^2) + 2 a b f (1 + m) (-2 e + f m x) + b^2 (2 e^2 - 4 e f m x + f^2 (-1 + m) m x^2)) \right. \right. \\
& \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m\right] - \\
& (a + b x) \left( (a^2 f^2 (2 + 3 m + m^2) (-3 c f + d (e - 2 f x)) - 2 a b f (1 + m) (c f (-e (6 + m) + 2 f m x) + d (2 e^2 - 2 e f (2 + m) x + f^2 m x^2)) \right. + \\
& b^2 (c f (-2 e^2 (3 + 2 m) + 2 e f m (3 + m) x - f^2 (-1 + m) m x^2) + d e (2 e^2 - 4 e f (1 + 2 m) x + f^2 m (1 + 3 m) x^2)) \Big) \\
& \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m\right] + f (1 + m) (a + b x) \\
& \left( (a f (2 + m) (-2 d e + 3 c f + d f x) + b c f (-e (6 + m) + 2 f m x) + b d e (4 e - f (2 + 3 m) x)) \text{HurwitzLerchPhi}\left[ \right. \right. \\
& \left. \left. \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m\right] + f (d e - c f) (2 + m) (a + b x) \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] \right) \Big) - \\
& \left( d e (a + b x)^{1+m} (c + d x)^{1-m} \left( 6 (b e - a f)^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m\right] + 6 (b e - a f)^2 m \right. \right. \\
& \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m\right] + 6 f (b e - a f) (a + b x) \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m\right] + \\
& 6 f (-b e + a f) m^2 (a + b x) \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m\right] + 2 f^2 (a + b x)^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m\right] - 
\end{aligned}$$

$$\begin{aligned}
& f^2 m (a + b x)^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] - 2 f^2 m^2 (a + b x)^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] + \\
& f^2 m^3 (a + b x)^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] - 6 (b e - a f)^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1+m \right] - \\
& 6 (b e - a f)^2 m \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1+m \right] + 12 f (b e - a f) m (a + b x) \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1+m \right] + \\
& 12 f (b e - a f) m^2 (a + b x) \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1+m \right] + \\
& 3 f^2 m (a + b x)^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1+m \right] - 3 f^2 m^3 (a + b x)^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1+m \right] + \\
& 6 f (-b e + a f) (a + b x) \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2+m \right] + \\
& 12 f (-b e + a f) m (a + b x) \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2+m \right] + 6 f (-b e + a f) m^2 (a + b x) \\
& \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2+m \right] + 3 f^2 m (a + b x)^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2+m \right] + \\
& 6 f^2 m^2 (a + b x)^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2+m \right] + 3 f^2 m^3 (a + b x)^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2+m \right] - \\
& 2 f^2 (a + b x)^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3+m \right] - 5 f^2 m (a + b x)^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3+m \right] - \\
& 4 f^2 m^2 (a + b x)^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3+m \right] - f^2 m^3 (a + b x)^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3+m \right] \Bigg) \Bigg) / \\
& \left( 3 f (1+m) (e + f x)^3 \left( (b e - a f) (c + d x) (a^2 f^2 (2 + 3 m + m^2) + 2 a b f (1+m) (-2 e + f m x) + b^2 (2 e^2 - 4 e f m x + f^2 (-1+m) m x^2)) \right. \right. \\
& \left. \left. \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] - \right. \right. \\
& (a + b x) \left( (a^2 f^2 (2 + 3 m + m^2) (-3 c f + d (e - 2 f x)) - 2 a b f (1+m) (c f (-e (6 + m) + 2 f m x) + d (2 e^2 - 2 e f (2 + m) x + f^2 m (1 + 3 m) x^2))) + \right. \\
& b^2 (c f (-2 e^2 (3 + 2 m) + 2 e f m (3 + m) x - f^2 (-1 + m) m x^2) + d e (2 e^2 - 4 e f (1 + 2 m) x + f^2 m (1 + 3 m) x^2)) \Bigg) \\
& \left. \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1+m \right] + f (1+m) (a + b x) \right. \\
& \left. \left( (a f (2 + m) (-2 d e + 3 c f + d f x) + b c f (-e (6 + m) + 2 f m x) + b d e (4 e - f (2 + 3 m) x)) \right. \right)
\end{aligned}$$

$$\text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m\right] + f (d e - c f) (2 + m) (a + b x) \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right]\right)\right)$$

**Problem 3120:** Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^m (c + d x)^{1-m}}{(e + f x)^5} dx$$

Optimal (type 5, 311 leaves, 4 steps):

$$\begin{aligned} & -\frac{f (a + b x)^{1+m} (c + d x)^{2-m}}{4 (b e - a f) (d e - c f) (e + f x)^4} - \frac{f (b (5 d e - c f (3 - m)) - a d f (2 + m)) (a + b x)^{1+m} (c + d x)^{2-m}}{12 (b e - a f)^2 (d e - c f)^2 (e + f x)^3} - \\ & \left( (b c - a d)^2 (2 a b d f (4 d e - c f (2 - m)) (1 + m) - a^2 d^2 f^2 (2 + 3 m + m^2) - b^2 (12 d^2 e^2 - 8 c d e f (2 - m) + c^2 f^2 (6 - 5 m + m^2))) \right. \\ & \left. (a + b x)^{1+m} (c + d x)^{-1-m} \text{Hypergeometric2F1}[3, 1 + m, 2 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}] \right) / (12 (b e - a f)^5 (d e - c f)^2 (1 + m)) \end{aligned}$$

Result (type 5, 63464 leaves): Display of huge result suppressed!

**Problem 3121:** Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^m (c + d x)^{1-m}}{(e + f x)^6} dx$$

Optimal (type 5, 542 leaves, 5 steps):

$$\begin{aligned} & -\frac{f (a + b x)^{1+m} (c + d x)^{2-m}}{5 (b e - a f) (d e - c f) (e + f x)^5} - \frac{f (b (7 d e - c f (4 - m)) - a d f (3 + m)) (a + b x)^{1+m} (c + d x)^{2-m}}{20 (b e - a f)^2 (d e - c f)^2 (e + f x)^4} - \\ & (f (a^2 d^2 f^2 (6 + 5 m + m^2) - a b d f (3 d e (7 + 4 m) - c f (9 + 2 m - 2 m^2)) + b^2 (27 d^2 e^2 - 3 c d e f (11 - 4 m) + c^2 f^2 (12 - 7 m + m^2))) \\ & (a + b x)^{1+m} (c + d x)^{2-m}) / (60 (b e - a f)^3 (d e - c f)^3 (e + f x)^3) + \\ & \frac{1}{60 (b e - a f)^6 (d e - c f)^3 (1 + m)} (b c - a d)^2 (3 a^2 b d^2 f^2 (5 d e - c f (2 - m)) (2 + 3 m + m^2) - a^3 d^3 f^3 (6 + 11 m + 6 m^2 + m^3) - 3 a b^2 d f (1 + m) \\ & (20 d^2 e^2 - 10 c d e f (2 - m) + c^2 f^2 (6 - 5 m + m^2)) + b^3 (60 d^3 e^3 - 60 c d^2 e^2 f (2 - m) + 15 c^2 d e f^2 (6 - 5 m + m^2) - c^3 f^3 (24 - 26 m + 9 m^2 - m^3))) \\ & (a + b x)^{1+m} (c + d x)^{-1-m} \text{Hypergeometric2F1}[3, 1 + m, 2 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}] \end{aligned}$$

Result (type 5, 136671 leaves): Display of huge result suppressed!

### Problem 3122: Result more than twice size of optimal antiderivative.

$$\int (a + bx)^m (c + dx)^{2-m} (e + fx)^p dx$$

Optimal (type 6, 133 leaves, 3 steps):

$$\frac{1}{b^3 (1+m)} (b c - a d)^2 (a + b x)^{1+m} (c + d x)^{-m} \left( \frac{b (c + d x)}{b c - a d} \right)^m (e + f x)^p \left( \frac{b (e + f x)}{b e - a f} \right)^{-p} \text{AppellF1}[1+m, -2+m, -p, 2+m, -\frac{d (a + b x)}{b c - a d}, -\frac{f (a + b x)}{b e - a f}]$$

Result (type 6, 300 leaves):

$$\begin{aligned} & \left( (b c - a d) (b e - a f) (2 + m) (a + b x)^{1+m} (c + d x)^{2-m} (e + f x)^p \text{AppellF1}[1+m, -2+m, -p, 2+m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] \right) / \\ & \left( b (1+m) \left( (b c - a d) (b e - a f) (2 + m) \text{AppellF1}[1+m, -2+m, -p, 2+m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] \right) - \right. \\ & (a + b x) \left( (-b c + a d) f p \text{AppellF1}[2+m, -2+m, 1-p, 3+m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] \right) + \\ & \left. d (b e - a f) (-2 + m) \text{AppellF1}[2+m, -1+m, -p, 3+m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] \right) \end{aligned}$$

### Problem 3123: Result unnecessarily involves higher level functions.

$$\int (a + bx)^m (c + dx)^{2-m} (e + fx)^3 dx$$

Optimal (type 5, 447 leaves, 4 steps):

$$\begin{aligned} & \frac{f (a + b x)^{1+m} (c + d x)^{3-m} (e + f x)^2}{6 b d} + \frac{1}{120 b^3 d^3} \\ & f (a + b x)^{1+m} (c + d x)^{3-m} (a^2 d^2 f^2 (20 - 9 m + m^2) - 2 a b d f (9 d e (4 - m) - c f (6 + 2 m - m^2)) + b^2 (70 d^2 e^2 - 18 c d e f (2 + m) + c^2 f^2 (6 + 5 m + m^2))) - \\ & 4 b d f (a d f (5 - m) - b (8 d e - c f (3 + m))) x - \frac{1}{120 b^6 d^3 (1 + m)} (b c - a d)^2 \\ & (a^3 d^3 f^3 (60 - 47 m + 12 m^2 - m^3) - 3 a^2 b d^2 f^2 (12 - 7 m + m^2) (6 d e - c f (1 + m)) + 3 a b^2 d f (3 - m) (30 d^2 e^2 - 12 c d e f (1 + m) + c^2 f^2 (2 + 3 m + m^2))) - \\ & b^3 (120 d^3 e^3 - 90 c d^2 e^2 f (1 + m) + 18 c^2 d e f^2 (2 + 3 m + m^2) - c^3 f^3 (6 + 11 m + 6 m^2 + m^3)) \\ & (a + b x)^{1+m} (c + d x)^{-m} \left( \frac{b (c + d x)}{b c - a d} \right)^m \text{Hypergeometric2F1}[-2+m, 1+m, 2+m, -\frac{d (a + b x)}{b c - a d}] \end{aligned}$$

Result (type 6, 467 leaves):

$$\begin{aligned}
& \frac{1}{4} (a + b x)^m (c + d x)^{2-m} \left( \left( 18 a c e^2 f x^2 \text{AppellF1}[2, -m, -2+m, 3, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \left( 3 a c \text{AppellF1}[2, -m, -2+m, 3, -\frac{b x}{a}, -\frac{d x}{c}] + \right. \right. \\
& \quad b c m x \text{AppellF1}[3, 1-m, -2+m, 4, -\frac{b x}{a}, -\frac{d x}{c}] - a d (-2+m) x \text{AppellF1}[3, -m, -1+m, 4, -\frac{b x}{a}, -\frac{d x}{c}] \Big) + \\
& \quad \left( 16 a c e f^2 x^3 \text{AppellF1}[3, -m, -2+m, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \left( 4 a c \text{AppellF1}[3, -m, -2+m, 4, -\frac{b x}{a}, -\frac{d x}{c}] + \right. \\
& \quad b c m x \text{AppellF1}[4, 1-m, -2+m, 5, -\frac{b x}{a}, -\frac{d x}{c}] - a d (-2+m) x \text{AppellF1}[4, -m, -1+m, 5, -\frac{b x}{a}, -\frac{d x}{c}] \Big) + \\
& \quad \left( 5 a c f^3 x^4 \text{AppellF1}[4, -m, -2+m, 5, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \left( 5 a c \text{AppellF1}[4, -m, -2+m, 5, -\frac{b x}{a}, -\frac{d x}{c}] + \right. \\
& \quad b c m x \text{AppellF1}[5, 1-m, -2+m, 6, -\frac{b x}{a}, -\frac{d x}{c}] - a d (-2+m) x \text{AppellF1}[5, -m, -1+m, 6, -\frac{b x}{a}, -\frac{d x}{c}] \Big) - \\
& \quad \left. \frac{4 e^3 \left( \frac{d(a+b x)}{-b c+a d} \right)^{-m} (c+d x) \text{Hypergeometric2F1}[3-m, -m, 4-m, \frac{b(c+d x)}{b c-a d}]}{d(-3+m)} \right)
\end{aligned}$$

**Problem 3124: Result unnecessarily involves higher level functions.**

$$\int (a + b x)^m (c + d x)^{2-m} (e + f x)^2 dx$$

Optimal (type 5, 262 leaves, 4 steps):

$$\begin{aligned}
& -\frac{f (a d f (4-m) - b (6 d e - c f (2+m))) (a + b x)^{1+m} (c + d x)^{3-m}}{20 b^2 d^2} + \frac{f (a + b x)^{1+m} (c + d x)^{3-m} (e + f x)}{5 b d} + \frac{1}{20 b^5 d^2 (1+m)} \\
& (b c - a d)^2 (a^2 d^2 f^2 (12 - 7 m + m^2) - 2 a b d f (3 - m) (5 d e - c f (1 + m)) + b^2 (20 d^2 e^2 - 10 c d e f (1 + m) + c^2 f^2 (2 + 3 m + m^2))) \\
& (a + b x)^{1+m} (c + d x)^{-m} \left( \frac{b (c + d x)}{b c - a d} \right)^m \text{Hypergeometric2F1}[-2+m, 1+m, 2+m, -\frac{d (a + b x)}{b c - a d}]
\end{aligned}$$

Result (type 6, 340 leaves):

$$\frac{1}{3} \left( a + b x \right)^m \left( c + d x \right)^{2-m} \left( \begin{array}{l} \left( 9 a c e f x^2 \text{AppellF1}[2, -m, -2+m, 3, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \left( 3 a c \text{AppellF1}[2, -m, -2+m, 3, -\frac{b x}{a}, -\frac{d x}{c}] + \right. \\ b c m x \text{AppellF1}[3, 1-m, -2+m, 4, -\frac{b x}{a}, -\frac{d x}{c}] - a d (-2+m) x \text{AppellF1}[3, -m, -1+m, 4, -\frac{b x}{a}, -\frac{d x}{c}] \Big) + \\ \left( 4 a c f^2 x^3 \text{AppellF1}[3, -m, -2+m, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \left( 4 a c \text{AppellF1}[3, -m, -2+m, 4, -\frac{b x}{a}, -\frac{d x}{c}] + \right. \\ b c m x \text{AppellF1}[4, 1-m, -2+m, 5, -\frac{b x}{a}, -\frac{d x}{c}] - a d (-2+m) x \text{AppellF1}[4, -m, -1+m, 5, -\frac{b x}{a}, -\frac{d x}{c}] \Big) - \\ \left. 3 e^{2 \left( \frac{d (a+b x)}{-b c+a d} \right) -m} (c+d x) \text{Hypergeometric2F1}[3-m, -m, 4-m, \frac{b (c+d x)}{b c-a d}] \right) \\ \left. d (-3+m) \right)$$

Problem 3125: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (a + b x)^m (c + d x)^{2-m} (e + f x) dx$$

Optimal (type 5, 147 leaves, 3 steps):

$$\frac{f (a + b x)^{1+m} (c + d x)^{3-m}}{4 b d} - \frac{1}{4 b^4 d (1+m)} \\ (b c - a d)^2 (a d f (3-m) - b (4 d e - c f (1+m))) (a + b x)^{1+m} (c + d x)^{-m} \left( \frac{b (c + d x)}{b c - a d} \right)^m \text{Hypergeometric2F1}[-2+m, 1+m, 2+m, -\frac{d (a + b x)}{b c - a d}]$$

Result (type 6, 509 leaves):

$$\begin{aligned}
& c (a + b x)^m (c + d x)^{-m} \left( \left( 3 a c (2 d e + c f) x^2 \text{AppellF1}[2, -m, m, 3, -\frac{b x}{a}, -\frac{d x}{c}] \right) \right. \\
& \left. + \left( 6 a c \text{AppellF1}[2, -m, m, 3, -\frac{b x}{a}, -\frac{d x}{c}] + 2 m x \left( b c \text{AppellF1}[3, 1-m, m, 4, -\frac{b x}{a}, -\frac{d x}{c}] - a d \text{AppellF1}[3, -m, 1+m, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) \right) + \\
& \left( 4 a d (d e + 2 c f) x^3 \text{AppellF1}[3, -m, m, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) \right. \\
& \left. + \left( 3 \left( 4 a c \text{AppellF1}[3, -m, m, 4, -\frac{b x}{a}, -\frac{d x}{c}] + b c m x \text{AppellF1}[4, 1-m, m, 5, -\frac{b x}{a}, -\frac{d x}{c}] - a d m x \text{AppellF1}[4, -m, 1+m, 5, -\frac{b x}{a}, -\frac{d x}{c}] \right) \right) + \\
& \left( 5 a d^2 f x^4 \text{AppellF1}[4, -m, m, 5, -\frac{b x}{a}, -\frac{d x}{c}] \right) \right. \\
& \left. - \left( 20 a c \text{AppellF1}[4, -m, m, 5, -\frac{b x}{a}, -\frac{d x}{c}] + 4 b c m x \text{AppellF1}[5, 1-m, m, 6, -\frac{b x}{a}, -\frac{d x}{c}] - 4 a d m x \text{AppellF1}[5, -m, 1+m, 6, -\frac{b x}{a}, -\frac{d x}{c}] \right) - \right. \\
& \left. \frac{c^2 e^{\left(\frac{d(a+b x)}{-b c+a d}\right)^{-m}} \text{Hypergeometric2F1}[1-m, -m, 2-m, \frac{b(c+d x)}{b c-a d}]}{d(-1+m)} - \frac{c e x^{\left(\frac{d(a+b x)}{-b c+a d}\right)^{-m}} \text{Hypergeometric2F1}[1-m, -m, 2-m, \frac{b(c+d x)}{b c-a d}]}{-1+m} \right)
\end{aligned}$$

Problem 3126: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (a + b x)^m (c + d x)^{2-m} dx$$

Optimal (type 5, 84 leaves, 2 steps):

$$\begin{aligned}
& \frac{(b c - a d)^2 (a + b x)^{1+m} (c + d x)^{-m} \left( \frac{b(c+d x)}{b c-a d} \right)^m \text{Hypergeometric2F1}[-2+m, 1+m, 2+m, -\frac{d(a+b x)}{b c-a d}]}{b^3 (1+m)}
\end{aligned}$$

Result (type 6, 319 leaves):

$$\frac{1}{d} c (a + b x)^m (c + d x)^{-m} \left( \left( 3 a c d^2 x^2 \text{AppellF1}[2, -m, m, 3, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \right.$$

$$\left( 3 a c \text{AppellF1}[2, -m, m, 3, -\frac{b x}{a}, -\frac{d x}{c}] + m x \left( b c \text{AppellF1}[3, 1-m, m, 4, -\frac{b x}{a}, -\frac{d x}{c}] - a d \text{AppellF1}[3, -m, 1+m, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) \right) +$$

$$\left( 4 a d^3 x^3 \text{AppellF1}[3, -m, m, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) /$$

$$\left( 12 a c \text{AppellF1}[3, -m, m, 4, -\frac{b x}{a}, -\frac{d x}{c}] + 3 b c m x \text{AppellF1}[4, 1-m, m, 5, -\frac{b x}{a}, -\frac{d x}{c}] - 3 a d m x \text{AppellF1}[4, -m, 1+m, 5, -\frac{b x}{a}, -\frac{d x}{c}] \right) -$$

$$\left. c \left( \frac{d(a+b x)}{-b c + a d} \right)^{-m} (c + d x) \text{Hypergeometric2F1}[1-m, -m, 2-m, \frac{b(c+d x)}{b c - a d}] \right) / (-1 + m)$$

**Problem 3127: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + b x)^m (c + d x)^{2-m}}{e + f x} dx$$

Optimal (type 5, 370 leaves, 6 steps):

$$-\frac{d (2 a b c d f^2 m - a^2 d^2 f^2 m - b^2 (2 d^2 e^2 - 4 c d e f + c^2 f^2 (2 + m))) (a + b x)^{1+m} (c + d x)^{-m}}{2 b^2 (b c - a d) f^3 m} +$$

$$\frac{d^2 (a + b x)^{2+m} (c + d x)^{-m}}{2 b^2 f} + \frac{(d e - c f)^2 (a + b x)^m (c + d x)^{-m} \text{Hypergeometric2F1}[1, -m, 1-m, \frac{(b e - a f) (c + d x)}{(d e - c f) (a + b x)}]}{f^3 m} +$$

$$\left( d (2 a b d f (d e - c f (2 - m)) m + a^2 d^2 f^2 (1 - m) m - b^2 (2 d^2 e^2 - 2 c d e f (2 - m) + c^2 f^2 (2 - 3 m + m^2))) \right.$$

$$\left. (a + b x)^{1+m} (c + d x)^{-m} \left( \frac{b (c + d x)}{b c - a d} \right)^m \text{Hypergeometric2F1}[m, 1+m, 2+m, -\frac{d (a + b x)}{b c - a d}] \right) / (2 b^2 (b c - a d) f^3 m (1 + m))$$

Result (type 6, 303 leaves):

$$\begin{aligned}
& - \left( \left( (b c - a d) (b e - a f)^2 (2 + m) (a + b x)^{1+m} (c + d x)^{2-m} \text{AppellF1}[1 + m, -2 + m, 1, 2 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] \right) \right. \\
& \left. \left( b (-b e + a f) (1 + m) (e + f x) \left( (b c - a d) (b e - a f) (2 + m) \text{AppellF1}[1 + m, -2 + m, 1, 2 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] \right) + \right. \right. \\
& (a + b x) \left( (-b c f + a d f) \text{AppellF1}[2 + m, -2 + m, 2, 3 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] \right) - \\
& \left. \left. \left. d (b e - a f) (-2 + m) \text{AppellF1}[2 + m, -1 + m, 1, 3 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] \right) \right) \right)
\end{aligned}$$

**Problem 3128: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + b x)^m (c + d x)^{2-m}}{(e + f x)^2} dx$$

Optimal (type 5, 316 leaves, 7 steps):

$$\begin{aligned}
& - \frac{2 d^2 (d e - c f) (a + b x)^{1+m} (c + d x)^{-m}}{(b c - a d) f^3 m} + \frac{(d e - c f)^2 (a + b x)^{1+m} (c + d x)^{-m}}{f^2 (b e - a f) (e + f x)} + \frac{1}{f^3 (b e - a f) m} \\
& (d e - c f) (a d f (2 - m) - b (2 d e - c f m)) (a + b x)^m (c + d x)^{-m} \text{Hypergeometric2F1}[1, -m, 1 - m, \frac{(b e - a f) (c + d x)}{(d e - c f) (a + b x)}] + \frac{1}{b (b c - a d) f^3 m (1 + m)} \\
& d^2 (b (2 d e - c f (2 - m)) - a d f m) (a + b x)^{1+m} (c + d x)^{-m} \left( \frac{b (c + d x)}{b c - a d} \right)^m \text{Hypergeometric2F1}[m, 1 + m, 2 + m, -\frac{d (a + b x)}{b c - a d}]
\end{aligned}$$

Result (type 6, 291 leaves):

$$\begin{aligned}
& \left( (b c - a d) (b e - a f) (2 + m) (a + b x)^{1+m} (c + d x)^{2-m} \text{AppellF1}[1 + m, -2 + m, 2, 2 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] \right) \right. \\
& \left. \left( b (1 + m) (e + f x)^2 \left( (b c - a d) (b e - a f) (2 + m) \text{AppellF1}[1 + m, -2 + m, 2, 2 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] \right) + \right. \right. \\
& (a + b x) \left( (-2 b c f + 2 a d f) \text{AppellF1}[2 + m, -2 + m, 3, 3 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] \right) - \\
& \left. \left. d (b e - a f) (-2 + m) \text{AppellF1}[2 + m, -1 + m, 2, 3 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] \right) \right)
\end{aligned}$$

### Problem 3129: Result unnecessarily involves higher level functions.

$$\int \frac{(a + bx)^m (c + dx)^{2-m}}{(e + fx)^3} dx$$

Optimal (type 5, 362 leaves, 7 steps):

$$\begin{aligned} & \frac{(be - af) (a + bx)^{-1+m} (c + dx)^{2-m}}{2f^2 (e + fx)^2} + \frac{(adf (2-m) - b(3de - cf(1+m))) (a + bx)^{-1+m} (c + dx)^{2-m}}{2f^2 (de - cf) (e + fx)} - \\ & \left( (2abdf (2-m) (de - cf)m) - b^2 (2d^2 e^2 - 2cd e f m - c^2 f^2 (1-m)m) - a^2 d^2 f^2 (2 - 3m + m^2) \right) (a + bx)^{-1+m} \\ & (c + dx)^{1-m} \text{Hypergeometric2F1}\left[1, -1+m, m, \frac{(de - cf) (a + bx)}{(be - af) (c + dx)}\right] / (2f^3 (be - af) (de - cf) (1-m)) - \\ & \frac{d (bc - ad) (a + bx)^{-1+m} (c + dx)^{-m} \left(\frac{b(c+dx)}{bc-ad}\right)^m \text{Hypergeometric2F1}\left[-1+m, -1+m, m, -\frac{d(a+bx)}{bc-ad}\right]}{f^3 (1-m)} \end{aligned}$$

Result (type 6, 304 leaves):

$$\begin{aligned} & - \left( \left( (bc - ad) (be - af)^4 (2+m) (a + bx)^{1+m} (c + dx)^{2-m} \text{AppellF1}\left[1+m, -2+m, 3, 2+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right] \right) / \right. \\ & \left. \left( b (-be + af)^3 (1+m) (e + fx)^3 \left( (bc - ad) (be - af) (2+m) \text{AppellF1}\left[1+m, -2+m, 3, 2+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right] + \right. \right. \right. \\ & \left. \left. \left. (a + bx) \left( (-3bcf + 3adf) \text{AppellF1}\left[2+m, -2+m, 4, 3+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right] - \right. \right. \right. \\ & \left. \left. \left. d (be - af) (-2+m) \text{AppellF1}\left[2+m, -1+m, 3, 3+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right] \right) \right) \right) \right) \end{aligned}$$

### Problem 3131: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + bx)^m (c + dx)^{2-m}}{(e + fx)^5} dx$$

Optimal (type 5, 176 leaves, 2 steps):

$$\begin{aligned}
& - \frac{f (a + b x)^{1+m} (c + d x)^{3-m}}{4 (b e - a f) (d e - c f) (e + f x)^4} + \\
& \left( (b c - a d)^3 (b (4 d e - c f (3 - m)) - a d f (1 + m)) (a + b x)^{1+m} (c + d x)^{-1-m} \text{Hypergeometric2F1}[4, 1 + m, 2 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}] \right) / \\
& (4 (b e - a f)^5 (d e - c f) (1 + m))
\end{aligned}$$

Result (type 5, 3314 leaves):

$$\begin{aligned}
& - \left( \left( (a + b x)^{1+m} (c + d x)^{3-m} \left( (-4 b e + a f (1 + m) + b f (-3 + m) x) \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, -2 + m \right] + 4 \right. \right. \right. \\
& (3 b e - a f (1 + m) - b f (-2 + m) x) \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, -1 + m \right] - 12 b e \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] + \\
& 6 a f \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] + 6 a f m \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] - \\
& 6 b f x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] + 6 b f m x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] + \\
& 4 b e \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m \right] - 4 a f \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m \right] - \\
& 4 a f m \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m \right] - 4 b f m x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m \right] + \\
& a f \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m \right] + a f m \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m \right] + \\
& b f x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m \right] + b f m x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m \right] \right) / \\
& \left( 4 (e + f x)^4 \left( (b e - a f) (c + d x) (3 b e - a f (1 + m) - b f (-2 + m) x) \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, -2 + m \right] + \right. \right. \\
& (a^2 f (1 + m) (-4 c f + d (e - 3 f x)) - b^2 (d e x (9 e + f (5 - 4 m) x) + c (6 e^2 - 3 e f m x + f^2 (-2 + m) x^2))) + \\
& a b (c f (3 e (4 + m) + f (4 - 5 m) x) + d (-3 e^2 + e f (8 + 5 m) x - 3 f^2 (-1 + m) x^2))) \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, -1 + m \right] + \\
& 3 b^2 c e^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] + 6 a b d e^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] - \\
& 12 a b c e f \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] - 3 a^2 d e f \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] +
\end{aligned}$$

$$\begin{aligned}
& 6 a^2 c f^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] - 3 a b c e f m \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] - \\
& 3 a^2 d e f m \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] + 6 a^2 c f^2 m \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] + \\
& 9 b^2 d e^2 x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] - 6 b^2 c e f x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] - \\
& 6 a b d e f x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] + 3 a^2 d f^2 x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] - \\
& 3 b^2 c e f m x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] - 9 a b d e f m x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] + \\
& 9 a b c f^2 m x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] + 3 a^2 d f^2 m x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] + \\
& 3 b^2 d e f x^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] - 3 b^2 c f^2 x^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] - \\
& 6 b^2 d e f m x^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] + 3 b^2 c f^2 m x^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] + \\
& 3 a b d f^2 m x^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] - 3 a b d e^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1+m \right] + \\
& 4 a b c e f \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1+m \right] + 3 a^2 d e f \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1+m \right] - \\
& 4 a^2 c f^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1+m \right] + a b c e f m \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1+m \right] + \\
& 3 a^2 d e f m \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1+m \right] - 4 a^2 c f^2 m \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1+m \right] - \\
& 3 b^2 d e^2 x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1+m \right] + 4 b^2 c e f x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1+m \right] + \\
& 4 a b d e f x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1+m \right] - 4 a b c f^2 x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1+m \right] - \\
& a^2 d f^2 x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1+m \right] + b^2 c e f m x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1+m \right] + \\
& 7 a b d e f m x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1+m \right] - 7 a b c f^2 m x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1+m \right] -
\end{aligned}$$

$$\begin{aligned}
& a^2 d f^2 m x \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m\right] + b^2 d e f x^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m\right] - \\
& a b d f^2 x^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m\right] + 4 b^2 d e f m x^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m\right] - \\
& 3 b^2 c f^2 m x^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m\right] - a b d f^2 m x^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m\right] - \\
& a^2 d e f \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m\right] + a^2 c f^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m\right] - \\
& a^2 d e f m \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m\right] + a^2 c f^2 m \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m\right] - \\
& 2 a b d e f x \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m\right] + 2 a b c f^2 x \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m\right] - \\
& 2 a b d e f m x \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m\right] + 2 a b c f^2 m x \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m\right] - \\
& b^2 d e f x^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m\right] + b^2 c f^2 x^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m\right] - \\
& b^2 d e f m x^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m\right] + b^2 c f^2 m x^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m\right]
\end{aligned}$$

**Problem 3132: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b x)^m (c + d x)^{2-m}}{(e + f x)^6} dx$$

Optimal (type 5, 311 leaves, 4 steps):

$$\begin{aligned}
& \frac{f (a + b x)^{1+m} (c + d x)^{3-m}}{5 (b e - a f) (d e - c f) (e + f x)^5} - \frac{f (b (6 d e - c f (4 - m)) - a d f (2 + m)) (a + b x)^{1+m} (c + d x)^{3-m}}{20 (b e - a f)^2 (d e - c f)^2 (e + f x)^4} - \\
& \left( (b c - a d)^3 (2 a b d f (5 d e - c f (3 - m)) (1 + m) - a^2 d^2 f^2 (2 + 3 m + m^2) - b^2 (20 d^2 e^2 - 10 c d e f (3 - m) + c^2 f^2 (12 - 7 m + m^2)) \right. \\
& \left. (a + b x)^{1+m} (c + d x)^{-1-m} \operatorname{Hypergeometric2F1}[4, 1 + m, 2 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}] \right) / \left( 20 (b e - a f)^6 (d e - c f)^2 (1 + m) \right)
\end{aligned}$$

Result (type 5, 29 088 leaves): Display of huge result suppressed!

### Problem 3133: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^m (c + d x)^{2-m}}{(e + f x)^7} dx$$

Optimal (type 5, 541 leaves, 5 steps):

$$\begin{aligned} & -\frac{f (a + b x)^{1+m} (c + d x)^{3-m}}{6 (b e - a f) (d e - c f) (e + f x)^6} - \frac{f (b (8 d e - c f (5 - m)) - a d f (3 + m)) (a + b x)^{1+m} (c + d x)^{3-m}}{30 (b e - a f)^2 (d e - c f)^2 (e + f x)^5} - \\ & \left( f (a^2 d^2 f^2 (6 + 5 m + m^2) - 2 a b d f (d e (12 + 7 m) - c f (6 + 2 m - m^2)) + b^2 (38 d^2 e^2 - 2 c d e f (26 - 7 m) + c^2 f^2 (20 - 9 m + m^2))) \right. \\ & \left. (a + b x)^{1+m} (c + d x)^{3-m} \right) / \left( 120 (b e - a f)^3 (d e - c f)^3 (e + f x)^4 \right) + \\ & \frac{1}{120 (b e - a f)^7 (d e - c f)^3 (1 + m)} (b c - a d)^3 (3 a^2 b d^2 f^2 (6 d e - c f (3 - m)) (2 + 3 m + m^2) - a^3 d^3 f^3 (6 + 11 m + 6 m^2 + m^3) - 3 a b^2 d f (1 + m) \\ & (30 d^2 e^2 - 12 c d e f (3 - m) + c^2 f^2 (12 - 7 m + m^2)) + b^3 (120 d^3 e^3 - 90 c d^2 e^2 f (3 - m) + 18 c^2 d e f^2 (12 - 7 m + m^2) - c^3 f^3 (60 - 47 m + 12 m^2 - m^3)) \\ & (a + b x)^{1+m} (c + d x)^{-1-m} \text{Hypergeometric2F1}[4, 1 + m, 2 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}] \end{aligned}$$

Result (type 5, 79140 leaves): Display of huge result suppressed!

### Problem 3134: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^m (c + d x)^{3-m}}{e + f x} dx$$

Optimal (type 5, 488 leaves, 7 steps):

$$\begin{aligned} & \frac{b (b e - a f)^3 (a + b x)^{-3+m} (c + d x)^{4-m}}{(b c - a d) f^4 (3 - m)} - \frac{b (b (3 d e - c f (1 - m)) - a d f (2 + m)) (a + b x)^{-2+m} (c + d x)^{4-m}}{6 d^2 f^2} + \\ & \frac{b (a + b x)^{-1+m} (c + d x)^{4-m}}{3 d f} - \frac{(b e - a f)^3 (a + b x)^{-3+m} (c + d x)^{3-m} \text{Hypergeometric2F1}[1, -3 + m, -2 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{f^4 (3 - m)} - \\ & \frac{1}{6 b^3 d^2 f^4 (2 - m) (3 - m)} (b c - a d)^2 (3 a^2 b d^2 f^2 (d e - c f (3 - m)) (1 - m) m + a^3 d^3 f^3 m (2 - 3 m + m^2) + \\ & 3 a b^2 d f m (2 d^2 e^2 - 2 c d e f (3 - m) + c^2 f^2 (6 - 5 m + m^2)) - b^3 (6 d^3 e^3 - 6 c d^2 e^2 f (3 - m) + 3 c^2 d e f^2 (6 - 5 m + m^2) - c^3 f^3 (6 - 11 m + 6 m^2 - m^3)) \\ & (a + b x)^{-2+m} (c + d x)^{-m} \left( \frac{b (c + d x)}{b c - a d} \right)^m \text{Hypergeometric2F1}[-3 + m, -2 + m, -1 + m, -\frac{d (a + b x)}{b c - a d}] \end{aligned}$$

Result (type 6, 303 leaves):

$$\begin{aligned}
& - \left( \left( (b c - a d) (b e - a f)^2 (2 + m) (a + b x)^{1+m} (c + d x)^{3-m} \text{AppellF1}[1 + m, -3 + m, 1, 2 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] \right) \right. \\
& \left. \left( b (-b e + a f) (1 + m) (e + f x) \left( (b c - a d) (b e - a f) (2 + m) \text{AppellF1}[1 + m, -3 + m, 1, 2 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] \right) + \right. \right. \\
& \left. (a + b x) \left( (-b c f + a d f) \text{AppellF1}[2 + m, -3 + m, 2, 3 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] \right) - \right. \\
& \left. \left. \left. d (b e - a f) (-3 + m) \text{AppellF1}[2 + m, -2 + m, 1, 3 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] \right) \right) \right)
\end{aligned}$$

**Problem 3135: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + b x)^m (c + d x)^{3-m}}{(e + f x)^2} dx$$

Optimal (type 5, 397 leaves, 2 steps):

$$\begin{aligned}
& \frac{3 b d (d e - c f)^2 (a + b x)^m (c + d x)^{1-m}}{(b c - a d) f^4 m} + \frac{d^2 (a + b x)^{1+m} (c + d x)^{1-m}}{2 b f^2} - \frac{(d e - c f)^2 (a + b x)^m (c + d x)^{1-m}}{f^3 (e + f x)} + \frac{1}{f^4 (b e - a f) m} \\
& (d e - c f)^2 (a d f (3 - m) - b (3 d e - c f m)) (a + b x)^m (c + d x)^{-m} \text{Hypergeometric2F1}[1, m, 1 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}] + \\
& \left( d^2 (2 a b d f (2 d e - c f (3 - m)) m + a^2 d^2 f^2 (1 - m) m - b^2 (6 d^2 e^2 - 4 c d e f (3 - m) + c^2 f^2 (6 - 5 m + m^2))) \right. \\
& \left. (a + b x)^{1+m} (c + d x)^{-m} \left( \frac{b (c + d x)}{b c - a d} \right)^m \text{Hypergeometric2F1}[m, 1 + m, 2 + m, -\frac{d (a + b x)}{b c - a d}] \right) / (2 b^2 (b c - a d) f^4 m (1 + m))
\end{aligned}$$

Result (type 6, 291 leaves):

$$\begin{aligned}
& \left( (b c - a d) (b e - a f) (2 + m) (a + b x)^{1+m} (c + d x)^{3-m} \text{AppellF1}[1 + m, -3 + m, 2, 2 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] \right) \right. \\
& \left. \left( b (1 + m) (e + f x)^2 \left( (b c - a d) (b e - a f) (2 + m) \text{AppellF1}[1 + m, -3 + m, 2, 2 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] \right) + \right. \right. \\
& \left. (a + b x) \left( (-2 b c f + 2 a d f) \text{AppellF1}[2 + m, -3 + m, 3, 3 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] \right) - \right. \\
& \left. \left. d (b e - a f) (-3 + m) \text{AppellF1}[2 + m, -2 + m, 2, 3 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] \right) \right)
\end{aligned}$$

### Problem 3136: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^m (c + d x)^{3-m}}{(e + f x)^3} dx$$

Optimal (type 5, 453 leaves, 8 steps):

$$\begin{aligned} & -\frac{3 d^3 (d e - c f) (a + b x)^{1+m} (c + d x)^{-m}}{(b c - a d) f^4 m} - \frac{(d e - c f)^3 (a + b x)^{1+m} (c + d x)^{-m}}{2 f^3 (b e - a f) (e + f x)^2} + \\ & \frac{(d e - c f)^2 (b (5 d e + c f (1 - m)) - a d f (6 - m)) (a + b x)^{1+m} (c + d x)^{-m}}{2 f^3 (b e - a f)^2 (e + f x)} + \frac{1}{2 f^4 (b e - a f)^2 m} \\ & (d e - c f) (2 a b d f (3 - m) (2 d e - c f m) - b^2 (6 d^2 e^2 - 4 c d e f m - c^2 f^2 (1 - m) m) - a^2 d^2 f^2 (6 - 5 m + m^2)) \\ & (a + b x)^m (c + d x)^{-m} \text{Hypergeometric2F1}[1, -m, 1 - m, \frac{(b e - a f) (c + d x)}{(d e - c f) (a + b x)}] + \frac{1}{b (b c - a d) f^4 m (1 + m)} \\ & d^3 (b (3 d e - c f (3 - m)) - a d f m) (a + b x)^{1+m} (c + d x)^{-m} \left( \frac{b (c + d x)}{b c - a d} \right)^m \text{Hypergeometric2F1}[m, 1 + m, 2 + m, -\frac{d (a + b x)}{b c - a d}] \end{aligned}$$

Result (type 6, 304 leaves):

$$\begin{aligned} & - \left( \left( (b c - a d) (b e - a f)^4 (2 + m) (a + b x)^{1+m} (c + d x)^{3-m} \text{AppellF1}[1 + m, -3 + m, 3, 2 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] \right) \right) / \\ & \left( b (-b e + a f)^3 (1 + m) (e + f x)^3 \left( (b c - a d) (b e - a f) (2 + m) \text{AppellF1}[1 + m, -3 + m, 3, 2 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] \right) + \right. \\ & (a + b x) \left( (-3 b c f + 3 a d f) \text{AppellF1}[2 + m, -3 + m, 4, 3 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] - \right. \\ & \left. \left. d (b e - a f) (-3 + m) \text{AppellF1}[2 + m, -2 + m, 3, 3 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] \right) \right) \right) \end{aligned}$$

### Problem 3137: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^{1-n} (c + d x)^{1+n}}{b c + a d + 2 b d x} dx$$

Optimal (type 5, 245 leaves, 6 steps):

$$\begin{aligned}
& \frac{(b c - a d) (3 - 2 n) (a + b x)^{2-n} (c + d x)^{-1+n}}{8 b^3 (1 - n)} + \frac{d (a + b x)^{3-n} (c + d x)^{-1+n}}{4 b^3} + \\
& \frac{(b c - a d)^2 (a + b x)^{1-n} (c + d x)^{-1+n} \text{Hypergeometric2F1}[1, -1 + n, n, -\frac{b (c+d x)}{d (a+b x)}]}{8 b^3 d (1 - n)} - \\
& \frac{(b c - a d)^2 (1 - 2 n^2) (a + b x)^{-n} \left(-\frac{d (a+b x)}{b c-a d}\right)^n (c + d x)^n \text{Hypergeometric2F1}[-1 + n, n, 1 + n, \frac{b (c+d x)}{b c-a d}]}{8 b^2 d^2 (1 - n) n}
\end{aligned}$$

Result (type 6, 1073 leaves):

$$\begin{aligned}
& \frac{1}{4} (a + b x)^{-n} (c + d x)^n \left( \left( 3 a c x^2 \text{AppellF1}[2, n, -n, 3, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \right. \\
& \left( 3 a c \text{AppellF1}[2, n, -n, 3, -\frac{b x}{a}, -\frac{d x}{c}] + n x \left( a d \text{AppellF1}[3, n, 1 - n, 4, -\frac{b x}{a}, -\frac{d x}{c}] - b c \text{AppellF1}[3, 1 + n, -n, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) \right) + \\
& \left( 2 a c (b c - a d) (-2 + n) (a + b x) \text{AppellF1}[1 - n, -n, 1, 2 - n, \frac{d (a + b x)}{-b c + a d}, \frac{2 d (a + b x)}{-b c + a d}] \right) / \\
& \left( b (-1 + n) (a d + b (c + 2 d x)) \left( - (b c - a d) (-2 + n) \text{AppellF1}[1 - n, -n, 1, 2 - n, \frac{d (a + b x)}{-b c + a d}, \frac{2 d (a + b x)}{-b c + a d}] + \right. \right. \\
& \left. \left. d (a + b x) \left( n \text{AppellF1}[2 - n, 1 - n, 1, 3 - n, \frac{d (a + b x)}{-b c + a d}, \frac{2 d (a + b x)}{-b c + a d}] - 2 \text{AppellF1}[2 - n, -n, 2, 3 - n, \frac{d (a + b x)}{-b c + a d}, \frac{2 d (a + b x)}{-b c + a d}] \right) \right) \right) - \\
& \left( c^2 (b c - a d) (-2 + n) (a + b x) \text{AppellF1}[1 - n, -n, 1, 2 - n, \frac{d (a + b x)}{-b c + a d}, \frac{2 d (a + b x)}{-b c + a d}] \right) / \\
& \left( d (-1 + n) (a d + b (c + 2 d x)) \left( - (b c - a d) (-2 + n) \text{AppellF1}[1 - n, -n, 1, 2 - n, \frac{d (a + b x)}{-b c + a d}, \frac{2 d (a + b x)}{-b c + a d}] + \right. \right. \\
& \left. \left. d (a + b x) \left( n \text{AppellF1}[2 - n, 1 - n, 1, 3 - n, \frac{d (a + b x)}{-b c + a d}, \frac{2 d (a + b x)}{-b c + a d}] - 2 \text{AppellF1}[2 - n, -n, 2, 3 - n, \frac{d (a + b x)}{-b c + a d}, \frac{2 d (a + b x)}{-b c + a d}] \right) \right) \right) + \\
& \left( a^2 d (-b c + a d) (-2 + n) (a + b x) \text{AppellF1}[1 - n, -n, 1, 2 - n, \frac{d (a + b x)}{-b c + a d}, \frac{2 d (a + b x)}{-b c + a d}] \right) / \\
& \left( b^2 (-1 + n) (a d + b (c + 2 d x)) \left( - (b c - a d) (-2 + n) \text{AppellF1}[1 - n, -n, 1, 2 - n, \frac{d (a + b x)}{-b c + a d}, \frac{2 d (a + b x)}{-b c + a d}] + \right. \right. \\
& \left. \left. d (a + b x) \left( n \text{AppellF1}[2 - n, 1 - n, 1, 3 - n, \frac{d (a + b x)}{-b c + a d}, \frac{2 d (a + b x)}{-b c + a d}] - 2 \text{AppellF1}[2 - n, -n, 2, 3 - n, \frac{d (a + b x)}{-b c + a d}, \frac{2 d (a + b x)}{-b c + a d}] \right) \right) \right) + \\
& c \left( \frac{d (a + b x)}{-b c + a d} \right)^n (c + d x) \text{Hypergeometric2F1}[n, 1 + n, 2 + n, \frac{b (c+d x)}{b c-a d}] + \frac{a \left( \frac{d (a+b x)}{-b c+a d} \right)^n (c + d x) \text{Hypergeometric2F1}[n, 1 + n, 2 + n, \frac{b (c+d x)}{b c-a d}]}{b d (1 + n)}
\end{aligned}$$

Problem 3138: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^{1-n} (c + d x)^{1+n}}{(b c + a d + 2 b d x)^2} dx$$

Optimal (type 5, 154 leaves, 4 steps):

$$-\frac{\left(b c - a d\right) \left(a + b x\right)^{1-n} \left(c + d x\right)^{-1+n} \text{Hypergeometric2F1}\left[2, 1-n, 2-n, -\frac{d (a+b x)}{b (c+d x)}\right]}{4 b^3 d \ (1-n)} +$$

$$\frac{\left(a + b x\right)^{-n} \left(-\frac{d (a+b x)}{b c-a d}\right)^n \left(c + d x\right)^{1+n} \text{Hypergeometric2F1}\left[n, 1+n, 2+n, \frac{b (c+d x)}{b c-a d}\right]}{4 b d^2 \ (1+n)}$$

## Result (type 6, 904 leaves):

$$\begin{aligned} & \left( (a+b x)^{-n} (c+d x)^n \left( -b^2 c^2 (-1+n) \operatorname{AppellF1}[1, -n, n, 2, \frac{-b c + a d}{a d + b (c+2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] + 2 a b c d (-1+n) \right. \right. \\ & \quad \left. \left. \operatorname{AppellF1}[1, -n, n, 2, \frac{-b c + a d}{a d + b (c+2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] - a^2 d^2 (-1+n) \operatorname{AppellF1}[1, -n, n, 2, \frac{-b c + a d}{a d + b (c+2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] \right) + \right. \\ & \quad a d \left( \frac{b (c+d x)}{b c - a d} \right)^{-n} \left( 2 (a d + b (c+2 d x)) \operatorname{AppellF1}[1, -n, n, 2, \frac{-b c + a d}{a d + b (c+2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] + (b c - a d) n \right. \\ & \quad \left. \left( \operatorname{AppellF1}[2, 1-n, n, 3, \frac{-b c + a d}{a d + b (c+2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] + \operatorname{AppellF1}[2, -n, 1+n, 3, \frac{-b c + a d}{a d + b (c+2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] \right) \right) \\ & \quad \operatorname{Hypergeometric2F1}[1-n, -n, 2-n, \frac{d (a+b x)}{-b c + a d}] + b d x \left( \frac{b (c+d x)}{b c - a d} \right)^{-n} \\ & \quad \left( 2 (a d + b (c+2 d x)) \operatorname{AppellF1}[1, -n, n, 2, \frac{-b c + a d}{a d + b (c+2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] + (b c - a d) n \right. \\ & \quad \left. \left( \operatorname{AppellF1}[2, 1-n, n, 3, \frac{-b c + a d}{a d + b (c+2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] + \operatorname{AppellF1}[2, -n, 1+n, 3, \frac{-b c + a d}{a d + b (c+2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] \right) \right) \\ & \quad \operatorname{Hypergeometric2F1}[1-n, -n, 2-n, \frac{d (a+b x)}{-b c + a d}] \Big) \Big) / \left( 4 b^2 d^2 (1-n) \right. \\ & \quad \left( 2 (a d + b (c+2 d x)) \operatorname{AppellF1}[1, -n, n, 2, \frac{-b c + a d}{a d + b (c+2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] + (b c - a d) n \right. \\ & \quad \left. \left( \operatorname{AppellF1}[2, 1-n, n, 3, \frac{-b c + a d}{a d + b (c+2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] + \operatorname{AppellF1}[2, -n, 1+n, 3, \frac{-b c + a d}{a d + b (c+2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] \right) \right) \Big) \end{aligned}$$

Problem 3139: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^{1-n} (c + d x)^{1+n}}{(b c + a d + 2 b d x)^3} dx$$

Optimal (type 5, 230 leaves, 7 steps):

$$\begin{aligned} & \frac{(b c - a d) (a + b x)^{1-n} (c + d x)^n}{8 b^2 d (b c + a d + 2 b d x)^2} - \frac{(1 + 2 n) (a + b x)^{1-n} (c + d x)^n}{8 b^2 d (b c + a d + 2 b d x)} - \frac{(1 - 2 n^2) (a + b x)^{-n} (c + d x)^n \text{Hypergeometric2F1}[1, n, 1 + n, -\frac{b (c + d x)}{d (a + b x)}]}{8 b^2 d^2 n} + \\ & \frac{(a + b x)^{-n} \left(-\frac{d (a + b x)}{b c - a d}\right)^n (c + d x)^n \text{Hypergeometric2F1}[n, n, 1 + n, \frac{b (c + d x)}{b c - a d}]}{8 b^2 d^2 n} \end{aligned}$$

Result (type 6, 1027 leaves):

$$\begin{aligned}
& \frac{1}{16(a d + b(c + 2 d x))} (a + b x)^{-n} (c + d x)^n \left( \left( 3 a^2 \text{AppellF1}[2, -n, n, 3, \frac{-b c + a d}{a d + b(c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] \right) / \right. \\
& \left. \left( b^2 \left( 3(a d + b(c + 2 d x)) \text{AppellF1}[2, -n, n, 3, \frac{-b c + a d}{a d + b(c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] + (b c - a d) n \right. \right. \right. \\
& \left. \left. \left. \left( \text{AppellF1}[3, 1-n, n, 4, \frac{-b c + a d}{a d + b(c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] + \text{AppellF1}[3, -n, 1+n, 4, \frac{-b c + a d}{a d + b(c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] \right) \right) + \right. \\
& \left. \left( 3 c^2 \text{AppellF1}[2, -n, n, 3, \frac{-b c + a d}{a d + b(c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] \right) / \right. \\
& \left. \left( d^2 \left( 3(a d + b(c + 2 d x)) \text{AppellF1}[2, -n, n, 3, \frac{-b c + a d}{a d + b(c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] + (b c - a d) n \right. \right. \right. \\
& \left. \left. \left. \left( \text{AppellF1}[3, 1-n, n, 4, \frac{-b c + a d}{a d + b(c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] + \text{AppellF1}[3, -n, 1+n, 4, \frac{-b c + a d}{a d + b(c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] \right) \right) \right) - \right. \\
& \left. \left( 6 a c \text{AppellF1}[2, -n, n, 3, \frac{-b c + a d}{a d + b(c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] \right) / \right. \\
& \left. \left( b d \left( 3(a d + b(c + 2 d x)) \text{AppellF1}[2, -n, n, 3, \frac{-b c + a d}{a d + b(c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] + (b c - a d) n \right. \right. \right. \\
& \left. \left. \left. \left( \text{AppellF1}[3, 1-n, n, 4, \frac{-b c + a d}{a d + b(c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] + \text{AppellF1}[3, -n, 1+n, 4, \frac{-b c + a d}{a d + b(c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] \right) \right) \right) + \right. \\
& \left. \left( 4(b c - a d)(2+n)(c + d x) \text{AppellF1}[1+n, n, 1, 2+n, \frac{b(c + d x)}{b c - a d}, \frac{2 b(c + d x)}{b c - a d}] \right) / \right. \\
& \left. \left( b d^2(1+n) \left( (b c - a d)(2+n) \text{AppellF1}[1+n, n, 1, 2+n, \frac{b(c + d x)}{b c - a d}, \frac{2 b(c + d x)}{b c - a d}] + \right. \right. \right. \\
& \left. \left. \left. b(c + d x) \left( 2 \text{AppellF1}[2+n, n, 2, 3+n, \frac{b(c + d x)}{b c - a d}, \frac{2 b(c + d x)}{b c - a d}] + n \text{AppellF1}[2+n, 1+n, 1, 3+n, \frac{b(c + d x)}{b c - a d}, \frac{2 b(c + d x)}{b c - a d}] \right) \right) \right) \right)
\end{aligned}$$

Problem 3140: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^{1-n} (c + d x)^{1+n}}{(b c + a d + 2 b d x)^4} dx$$

Optimal (type 5, 71 leaves, 1 step):

$$\frac{(a + b x)^{2-n} (c + d x)^{-2+n} \text{Hypergeometric2F1}[4, 2-n, 3-n, -\frac{d(a+b x)}{b(c+d x)}]}{b^4 (b c - a d) (2-n)}$$

Result (type 6, 543 leaves):

$$\begin{aligned} & \frac{1}{12 b^2 d^2} (a + b x)^{-n} (c + d x)^n \left( - \left( \left( 3 \text{AppellF1}[1, -n, n, 2, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] \right) \right) / \right. \\ & \left( 2 (a d + b (c + 2 d x)) \text{AppellF1}[1, -n, n, 2, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] + (b c - a d) n \right. \\ & \left. \left( \text{AppellF1}[2, 1-n, n, 3, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] + \text{AppellF1}[2, -n, 1+n, 3, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] \right) \right) + \\ & \left( 2 (b c - a d)^2 \text{AppellF1}[3, -n, n, 4, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] \right) / \\ & \left( (a d + b (c + 2 d x))^2 \left( 4 (a d + b (c + 2 d x)) \text{AppellF1}[3, -n, n, 4, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] + (b c - a d) n \right. \right. \\ & \left. \left. \left( \text{AppellF1}[4, 1-n, n, 5, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] + \text{AppellF1}[4, -n, 1+n, 5, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] \right) \right) \right) \end{aligned}$$

Problem 3141: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^m (c + d x)^{2-m}}{b c + a d + 2 b d x} dx$$

Optimal (type 5, 231 leaves, 6 steps):

$$\begin{aligned} & \frac{(b c - a d) (1 + 2 m) (a + b x)^{1+m} (c + d x)^{-m}}{8 b^3 m} + \frac{d (a + b x)^{2+m} (c + d x)^{-m}}{4 b^3} + \frac{(b c - a d)^2 (a + b x)^m (c + d x)^{-m} \text{Hypergeometric2F1}[1, -m, 1-m, -\frac{b(c+d x)}{d(a+b x)}]}{8 b^3 d m} - \\ & \frac{(b c - a d) (1 - 4 m + 2 m^2) (a + b x)^{1+m} (c + d x)^{-m} \left(\frac{b(c+d x)}{b c - a d}\right)^m \text{Hypergeometric2F1}[m, 1+m, 2+m, -\frac{d(a+b x)}{b c - a d}]}{8 b^3 m (1 + m)} \end{aligned}$$

Result (type 6, 269 leaves):

$$\begin{aligned} & - \left( \left( (b c - a d) (2 + m) (a + b x)^{1+m} (c + d x)^{2-m} \text{AppellF1}[1+m, -2+m, 1, 2+m, \frac{d(a+b x)}{-b c + a d}, \frac{2 d(a+b x)}{-b c + a d}] \right) \right) / \\ & \left( b (1 + m) (a d + b (c + 2 d x)) \left( - (b c - a d) (2 + m) \text{AppellF1}[1+m, -2+m, 1, 2+m, \frac{d(a+b x)}{-b c + a d}, \frac{2 d(a+b x)}{-b c + a d}] + d (a + b x) \right. \right. \\ & \left. \left. \left( 2 \text{AppellF1}[2+m, -2+m, 2, 3+m, \frac{d(a+b x)}{-b c + a d}, \frac{2 d(a+b x)}{-b c + a d}] + (-2 + m) \text{AppellF1}[2+m, -1+m, 1, 3+m, \frac{d(a+b x)}{-b c + a d}, \frac{2 d(a+b x)}{-b c + a d}] \right) \right) \right) \end{aligned}$$

Problem 3142: Result unnecessarily involves higher level functions and more than twice size of optimal

antiderivative.

$$\int \frac{(a + b x)^m (c + d x)^{2-m}}{(b c + a d + 2 b d x)^2} dx$$

Optimal (type 5, 144 leaves, 4 steps):

$$-\frac{\begin{aligned} & (b c - a d) (a + b x)^m (c + d x)^{-m} \text{Hypergeometric2F1}[2, m, 1 + m, -\frac{d (a+b x)}{b (c+d x)}] \\ & + \end{aligned}}{4 b^3 d m} +$$

$$\frac{\begin{aligned} & (b c - a d) (a + b x)^m (c + d x)^{-m} \left(\frac{b (c+d x)}{b c-a d}\right)^m \text{Hypergeometric2F1}[-1 + m, m, 1 + m, -\frac{d (a+b x)}{b c-a d}] \\ & \end{aligned}}{4 b^3 d m}$$

Result (type 6, 1377 leaves):

$$\begin{aligned}
& \frac{1}{4 b^3} (a + b x)^m (c + d x)^{-m} \left( \left( 2 a b c \text{AppellF1}[1, m, -m, 2, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] \right) / \right. \\
& \left( 2 (a d + b (c + 2 d x)) \text{AppellF1}[1, m, -m, 2, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] - (b c - a d) m \right. \\
& \left. \left( \text{AppellF1}[2, m, 1 - m, 3, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] + \text{AppellF1}[2, 1 + m, -m, 3, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] \right) \right) - \\
& \left( b^2 c^2 \text{AppellF1}[1, m, -m, 2, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] \right) / \\
& \left( d \left( 2 (a d + b (c + 2 d x)) \text{AppellF1}[1, m, -m, 2, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] - (b c - a d) m \right. \right. \\
& \left. \left. \left( \text{AppellF1}[2, m, 1 - m, 3, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] + \text{AppellF1}[2, 1 + m, -m, 3, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] \right) \right) \right) - \\
& \left( a^2 d \text{AppellF1}[1, m, -m, 2, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] \right) / \\
& \left( 2 (a d + b (c + 2 d x)) \text{AppellF1}[1, m, -m, 2, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] - (b c - a d) m \right. \\
& \left. \left( \text{AppellF1}[2, m, 1 - m, 3, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] + \text{AppellF1}[2, 1 + m, -m, 3, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] \right) \right) - \\
& \left( 2 b^2 c (b c - a d) (-2 + m) (c + d x) \text{AppellF1}[1 - m, -m, 1, 2 - m, \frac{b (c + d x)}{b c - a d}, \frac{2 b (c + d x)}{b c - a d}] \right) / \\
& \left( d (-1 + m) (a d + b (c + 2 d x)) \left( (b c - a d) (-2 + m) \text{AppellF1}[1 - m, -m, 1, 2 - m, \frac{b (c + d x)}{b c - a d}, \frac{2 b (c + d x)}{b c - a d}] + \right. \right. \\
& \left. \left. b (c + d x) \left( m \text{AppellF1}[2 - m, 1 - m, 1, 3 - m, \frac{b (c + d x)}{b c - a d}, \frac{2 b (c + d x)}{b c - a d}] - 2 \text{AppellF1}[2 - m, -m, 2, 3 - m, \frac{b (c + d x)}{b c - a d}, \frac{2 b (c + d x)}{b c - a d}] \right) \right) \right) - \\
& \left( 2 a b (-b c + a d) (-2 + m) (c + d x) \text{AppellF1}[1 - m, -m, 1, 2 - m, \frac{b (c + d x)}{b c - a d}, \frac{2 b (c + d x)}{b c - a d}] \right) / \\
& \left( (-1 + m) (a d + b (c + 2 d x)) \left( (b c - a d) (-2 + m) \text{AppellF1}[1 - m, -m, 1, 2 - m, \frac{b (c + d x)}{b c - a d}, \frac{2 b (c + d x)}{b c - a d}] + \right. \right. \\
& \left. \left. b (c + d x) \left( m \text{AppellF1}[2 - m, 1 - m, 1, 3 - m, \frac{b (c + d x)}{b c - a d}, \frac{2 b (c + d x)}{b c - a d}] - 2 \text{AppellF1}[2 - m, -m, 2, 3 - m, \frac{b (c + d x)}{b c - a d}, \frac{2 b (c + d x)}{b c - a d}] \right) \right) \right) + \\
& \left. \left( a + b x \right) \left( \frac{b (c + d x)}{b c - a d} \right)^m \text{Hypergeometric2F1}[m, 1 + m, 2 + m, \frac{d (a + b x)}{-b c + a d}] \right) / \\
& \quad 1 + m
\end{aligned}$$

Problem 3143: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(a + bx)^m (c + dx)^{2-m}}{(bc + ad + 2bdx)^3} dx$$

Optimal (type 5, 261 leaves, 7 steps):

$$\begin{aligned} & \frac{(bc - ad) (a + bx)^{-1+m} (c + dx)^{2-m}}{8b^2 (bc + ad + 2bdx)^2} + \frac{(1 - 2m) (a + bx)^{-1+m} (c + dx)^{2-m}}{8b^2 d^2 (bc + ad + 2bdx)} - \\ & \frac{(1 - 4m + 2m^2) (a + bx)^{-1+m} (c + dx)^{1-m} \text{Hypergeometric2F1}[1, -1 + m, m, -\frac{d(a+bx)}{b(c+dx)}]}{8b^2 d^2 (1 - m)} - \\ & \frac{(bc - ad) (a + bx)^{-1+m} (c + dx)^{-m} \left(\frac{b(c+dx)}{bc-ad}\right)^m \text{Hypergeometric2F1}[-1 + m, -1 + m, m, -\frac{d(a+bx)}{bc-ad}]}{8b^3 d^2 (1 - m)} \end{aligned}$$

Result (type 6, 1593 leaves):

$$\begin{aligned} & \frac{1}{16b^3} (a + bx)^m (c + dx)^{-m} \left( \left( 8a \text{AppellF1}[1, m, -m, 2, \frac{-bc + ad}{ad + b(c + 2dx)}, \frac{bc - ad}{bc + ad + 2bdx}] \right) / \right. \\ & \left( 2(ad + b(c + 2dx)) \text{AppellF1}[1, m, -m, 2, \frac{-bc + ad}{ad + b(c + 2dx)}, \frac{bc - ad}{bc + ad + 2bdx}] - (bc - ad)m \right. \\ & \left. \left( \text{AppellF1}[2, m, 1 - m, 3, \frac{-bc + ad}{ad + b(c + 2dx)}, \frac{bc - ad}{bc + ad + 2bdx}] + \text{AppellF1}[2, 1 + m, -m, 3, \frac{-bc + ad}{ad + b(c + 2dx)}, \frac{bc - ad}{bc + ad + 2bdx}] \right) - \right. \\ & \left. \left( 8bc \text{AppellF1}[1, m, -m, 2, \frac{-bc + ad}{ad + b(c + 2dx)}, \frac{bc - ad}{bc + ad + 2bdx}] \right) / \right. \\ & \left. \left( d \left( 2(ad + b(c + 2dx)) \text{AppellF1}[1, m, -m, 2, \frac{-bc + ad}{ad + b(c + 2dx)}, \frac{bc - ad}{bc + ad + 2bdx}] - (bc - ad)m \right. \right. \right. \\ & \left. \left. \left. \left( \text{AppellF1}[2, m, 1 - m, 3, \frac{-bc + ad}{ad + b(c + 2dx)}, \frac{bc - ad}{bc + ad + 2bdx}] + \text{AppellF1}[2, 1 + m, -m, 3, \frac{-bc + ad}{ad + b(c + 2dx)}, \frac{bc - ad}{bc + ad + 2bdx}] \right) \right) \right) + \right. \\ & \left. \left( 6abc \text{AppellF1}[2, m, -m, 3, \frac{-bc + ad}{ad + b(c + 2dx)}, \frac{bc - ad}{bc + ad + 2bdx}] \right) / \left( (ad + b(c + 2dx)) \right. \right. \\ & \left. \left. \left( 3(ad + b(c + 2dx)) \text{AppellF1}[2, m, -m, 3, \frac{-bc + ad}{ad + b(c + 2dx)}, \frac{bc - ad}{bc + ad + 2bdx}] - (bc - ad)m \right. \right. \right. \\ & \left. \left. \left. \left( \text{AppellF1}[3, m, 1 - m, 4, \frac{-bc + ad}{ad + b(c + 2dx)}, \frac{bc - ad}{bc + ad + 2bdx}] + \text{AppellF1}[3, 1 + m, -m, 4, \frac{-bc + ad}{ad + b(c + 2dx)}, \frac{bc - ad}{bc + ad + 2bdx}] \right) \right) \right) - \right. \end{aligned}$$

$$\begin{aligned}
& \left( 3 b^2 c^2 \text{AppellF1} \left[ 2, m, -m, 3, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x} \right] \right) / \left( d (a d + b (c + 2 d x)) \right) \\
& \left( 3 (a d + b (c + 2 d x)) \text{AppellF1} \left[ 2, m, -m, 3, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x} \right] - (b c - a d) m \right. \\
& \left. \left( \text{AppellF1} \left[ 3, m, 1-m, 4, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x} \right] + \text{AppellF1} \left[ 3, 1+m, -m, 4, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x} \right] \right) \right) - \\
& \left( 3 a^2 d \text{AppellF1} \left[ 2, m, -m, 3, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x} \right] \right) / \left( (a d + b (c + 2 d x)) \right. \\
& \left. \left( 3 (a d + b (c + 2 d x)) \text{AppellF1} \left[ 2, m, -m, 3, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x} \right] - (b c - a d) m \right. \right. \\
& \left. \left. \left( \text{AppellF1} \left[ 3, m, 1-m, 4, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x} \right] + \text{AppellF1} \left[ 3, 1+m, -m, 4, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x} \right] \right) \right) \right) + \\
& \left( 4 b (-b c + a d) (-2 + m) (c + d x) \text{AppellF1} \left[ 1-m, -m, 1, 2-m, \frac{b (c + d x)}{b c - a d}, \frac{2 b (c + d x)}{b c - a d} \right] \right) / \\
& \left( d (-1 + m) (a d + b (c + 2 d x)) \left( (b c - a d) (-2 + m) \text{AppellF1} \left[ 1-m, -m, 1, 2-m, \frac{b (c + d x)}{b c - a d}, \frac{2 b (c + d x)}{b c - a d} \right] + \right. \right. \\
& \left. \left. b (c + d x) \left( m \text{AppellF1} \left[ 2-m, 1-m, 1, 3-m, \frac{b (c + d x)}{b c - a d}, \frac{2 b (c + d x)}{b c - a d} \right] - 2 \text{AppellF1} \left[ 2-m, -m, 2, 3-m, \frac{b (c + d x)}{b c - a d}, \frac{2 b (c + d x)}{b c - a d} \right] \right) \right) \right)
\end{aligned}$$

**Problem 3144:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^m (c + d x)^{2-m}}{(b c + a d + 2 b d x)^4} dx$$

Optimal (type 5, 65 leaves, 1 step):

$$\frac{(a + b x)^{1+m} (c + d x)^{-1-m} \text{Hypergeometric2F1} \left[ 4, 1+m, 2+m, -\frac{d (a+b x)}{b (c+d x)} \right]}{b^4 (b c - a d) (1 + m)}$$

Result (type 6, 812 leaves):

$$\begin{aligned}
& \frac{1}{24 b^3 d} (a + b x)^m (c + d x)^{-m} \left( - \left( \left( 6 \text{AppellF1}[1, m, -m, 2, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] \right) \right. \right. \\
& \left. \left. + 2 (a d + b (c + 2 d x)) \text{AppellF1}[1, m, -m, 2, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] - (b c - a d) m \right. \right. \\
& \left. \left( \text{AppellF1}[2, m, 1-m, 3, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] + \text{AppellF1}[2, 1+m, -m, 3, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] \right) \right) + \\
& \frac{1}{(a d + b (c + 2 d x))^2} (b c - a d) \left( - \left( \left( 9 (a d + b (c + 2 d x)) \text{AppellF1}[2, m, -m, 3, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] \right) \right. \right. \\
& \left. \left. + 3 (a d + b (c + 2 d x)) \text{AppellF1}[2, m, -m, 3, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] - (b c - a d) m \left( \text{AppellF1}[3, m, 1-m, \right. \right. \\
& \left. \left. 4, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] + \text{AppellF1}[3, 1+m, -m, 4, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] \right) \right) \right) - \\
& \left( 4 (b c - a d) \text{AppellF1}[3, m, -m, 4, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] \right) \Big/ \left( 4 (a d + b (c + 2 d x)) \right. \\
& \left. \text{AppellF1}[3, m, -m, 4, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] - (b c - a d) m \right. \\
& \left. \left( \text{AppellF1}[4, m, 1-m, 5, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] + \text{AppellF1}[4, 1+m, -m, 5, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x}] \right) \right) \right)
\end{aligned}$$

**Problem 3145: Result more than twice size of optimal antiderivative.**

$$\int (a + b x)^m (c + d x)^{-m-n} (e + f x)^{n+p} dx$$

Optimal (type 6, 139 leaves, 3 steps):

$$\frac{1}{b (1+m)} (a + b x)^{1+m} (c + d x)^{-m-n} \left( \frac{b (c + d x)}{b c - a d} \right)^{m+n} (e + f x)^{n+p} \left( \frac{b (e + f x)}{b e - a f} \right)^{-n-p} \text{AppellF1}[1+m, m+n, -n-p, 2+m, -\frac{d (a + b x)}{b c - a d}, -\frac{f (a + b x)}{b e - a f}]$$

Result (type 6, 323 leaves):

$$\begin{aligned} & \left( (b c - a d) (b e - a f) (2 + m) (a + b x)^{1+m} (c + d x)^{-m-n} (e + f x)^{n+p} \text{AppellF1}[1 + m, m + n, -n - p, 2 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] \right) / \\ & \left( b (1 + m) \left( (b c - a d) (b e - a f) (2 + m) \text{AppellF1}[1 + m, m + n, -n - p, 2 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] \right) - \right. \\ & (a + b x) \left( - (b c - a d) f (n + p) \text{AppellF1}[2 + m, m + n, 1 - n - p, 3 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] \right) + \\ & \left. d (b e - a f) (m + n) \text{AppellF1}[2 + m, 1 + m + n, -n - p, 3 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] \right) \end{aligned}$$

**Problem 3146:** Result more than twice size of optimal antiderivative.

$$\int (a + b x)^m (c + d x)^{-m-n} (e + f x)^{1+n} dx$$

Optimal (type 6, 139 leaves, 3 steps):

$$\begin{aligned} & \frac{1}{b^2 (1 + m)} (b e - a f) (a + b x)^{1+m} (c + d x)^{-m-n} \left( \frac{b (c + d x)}{b c - a d} \right)^{m+n} \\ & (e + f x)^n \left( \frac{b (e + f x)}{b e - a f} \right)^{-n} \text{AppellF1}[1 + m, m + n, -1 - n, 2 + m, -\frac{d (a + b x)}{b c - a d}, -\frac{f (a + b x)}{b e - a f}] \end{aligned}$$

Result (type 6, 312 leaves):

$$\begin{aligned} & \left( (b c - a d) (b e - a f) (2 + m) (a + b x)^{1+m} (c + d x)^{-m-n} (e + f x)^{1+n} \text{AppellF1}[1 + m, m + n, -1 - n, 2 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] \right) / \\ & \left( b (1 + m) \left( (b c - a d) (b e - a f) (2 + m) \text{AppellF1}[1 + m, m + n, -1 - n, 2 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] \right) - \right. \\ & (a + b x) \left( - (b c - a d) f (1 + n) \text{AppellF1}[2 + m, m + n, -n, 3 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] \right) + \\ & \left. d (b e - a f) (m + n) \text{AppellF1}[2 + m, 1 + m + n, -1 - n, 3 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] \right) \end{aligned}$$

**Problem 3147:** Result more than twice size of optimal antiderivative.

$$\int (a + b x)^m (c + d x)^{-m-n} (e + f x)^n dx$$

Optimal (type 6, 129 leaves, 3 steps):

$$\frac{1}{b(1+m)} (a+bx)^{1+m} (c+dx)^{-m-n} \left( \frac{b(c+dx)}{bc-ad} \right)^{m+n} (e+fx)^n \left( \frac{b(e+fx)}{be-af} \right)^{-n} \text{AppellF1}[1+m, m+n, -n, 2+m, -\frac{d(a+bx)}{bc-ad}, -\frac{f(a+bx)}{be-af}]$$

Result (type 6, 303 leaves):

$$\begin{aligned} & \left( (bc-ad)(be-af)(2+m)(a+bx)^{1+m}(c+dx)^{-m-n}(e+fx)^n \text{AppellF1}[1+m, m+n, -n, 2+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}] \right) / \\ & \left( b(1+m) \left( (bc-ad)(be-af)(2+m) \text{AppellF1}[1+m, m+n, -n, 2+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}] \right) - \right. \\ & (a+bx) \left( (-bc+ad)f n \text{AppellF1}[2+m, m+n, 1-n, 3+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}] \right) + \\ & \left. d(be-af)(m+n) \text{AppellF1}[2+m, 1+m+n, -n, 3+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}] \right) \end{aligned}$$

Problem 3148: Result more than twice size of optimal antiderivative.

$$\int (a+bx)^m (c+dx)^{-m-n} (e+fx)^{-1+n} dx$$

Optimal (type 6, 138 leaves, 3 steps):

$$\frac{1}{(be-af)(1+m)} (a+bx)^{1+m} (c+dx)^{-m-n} \left( \frac{b(c+dx)}{bc-ad} \right)^{m+n} (e+fx)^n \left( \frac{b(e+fx)}{be-af} \right)^{-n} \text{AppellF1}[1+m, m+n, 1-n, 2+m, -\frac{d(a+bx)}{bc-ad}, -\frac{f(a+bx)}{be-af}]$$

Result (type 6, 315 leaves):

$$\begin{aligned} & - \left( \left( (bc-ad)(be-af)(2+m)(a+bx)^{1+m}(c+dx)^{-m-n}(e+fx)^{-1+n} \text{AppellF1}[1+m, m+n, 1-n, 2+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}] \right) / \right. \\ & \left( b(1+m) \left( -(bc-ad)(be-af)(2+m) \text{AppellF1}[1+m, m+n, 1-n, 2+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}] \right) + \right. \\ & (a+bx) \left( -(bc-ad)f(-1+n) \text{AppellF1}[2+m, m+n, 2-n, 3+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}] \right) + \\ & \left. \left. d(be-af)(m+n) \text{AppellF1}[2+m, 1+m+n, 1-n, 3+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}] \right) \right) \end{aligned}$$

Problem 3150: Result more than twice size of optimal antiderivative.

$$\int (a+bx)^m (c+dx)^{-m-n} (e+fx)^{-3+n} dx$$

Optimal (type 5, 237 leaves, 2 steps):

$$\begin{aligned}
 & -\frac{f(a+b x)^{1+m} (c+d x)^{1-m-n} (e+f x)^{-2+n}}{(b e-a f) (d e-c f) (2-n)} - \\
 & \left( (a d f (1+m) - b (d e (2-n) - c f (1-m-n))) (a+b x)^{1+m} (c+d x)^{-m-n} \left( \frac{(b e-a f) (c+d x)}{(b c-a d) (e+f x)} \right)^{m+n} (e+f x)^{-1+n} \right. \\
 & \left. \text{Hypergeometric2F1}[1+m, m+n, 2+m, -\frac{(d e-c f) (a+b x)}{(b c-a d) (e+f x)}] \right) / ((b e-a f)^2 (d e-c f) (1+m) (2-n))
 \end{aligned}$$

Result (type 5, 5197 leaves):

$$\begin{aligned}
 & \left( (a+b x)^{1+2 m} (c+d x)^{-2 m-2 n} \left( \frac{-b c-b d x}{-b c+a d} \right)^{m+n} (e+f x)^{-6+2 n} \left( \frac{-b e-b f x}{-b e+a f} \right)^{3-n} \right. \\
 & \left( 1 - \frac{d (a+b x)}{-b c+a d} \right)^{-m-n} \left( 1 - \frac{f (a+b x)}{-b e+a f} \right)^{-2+n} \text{Gamma}[2+m] \left( \frac{2 \text{Hypergeometric2F1}[1, m+n, 3+m, \frac{(d e-c f) (a+b x)}{(b e-a f) (c+d x)}]}{\text{Gamma}[3+m]} + \right. \\
 & \frac{m \text{Hypergeometric2F1}[1, m+n, 3+m, \frac{(d e-c f) (a+b x)}{(b e-a f) (c+d x)}]}{\text{Gamma}[3+m]} + \frac{f (a+b x) \text{Hypergeometric2F1}[1, m+n, 3+m, \frac{(d e-c f) (a+b x)}{(b e-a f) (c+d x)}]}{(b e-a f) \text{Gamma}[3+m]} + \\
 & \frac{(d e-c f) (a+b x) \text{Gamma}[1+m+n] \text{Hypergeometric2F1}[2, 1+m+n, 4+m, \frac{(d e-c f) (a+b x)}{(b e-a f) (c+d x)}]}{(b e-a f) (c+d x) \text{Gamma}[4+m] \text{Gamma}[m+n]} - \\
 & \left. \frac{f (-d e+c f) (a+b x)^2 \text{Gamma}[1+m+n] \text{Hypergeometric2F1}[2, 1+m+n, 4+m, \frac{(d e-c f) (a+b x)}{(b e-a f) (c+d x)}]}{(b e-a f)^2 (c+d x) \text{Gamma}[4+m] \text{Gamma}[m+n]} \right) / \\
 & \left( b (1+m) \left( -\frac{1}{(-b e+a f) (1+m)} f (-2+n) (a+b x)^{1+m} (c+d x)^{-m-n} \left( \frac{-b c-b d x}{-b c+a d} \right)^{m+n} (e+f x)^{-3+n} \left( \frac{-b e-b f x}{-b e+a f} \right)^{3-n} \right. \right. \\
 & \left( 1 - \frac{d (a+b x)}{-b c+a d} \right)^{-m-n} \left( 1 - \frac{f (a+b x)}{-b e+a f} \right)^{-3+n} \text{Gamma}[2+m] \left( \frac{2 \text{Hypergeometric2F1}[1, m+n, 3+m, \frac{(d e-c f) (a+b x)}{(b e-a f) (c+d x)}]}{\text{Gamma}[3+m]} + \right. \\
 & \frac{m \text{Hypergeometric2F1}[1, m+n, 3+m, \frac{(d e-c f) (a+b x)}{(b e-a f) (c+d x)}]}{\text{Gamma}[3+m]} + \frac{f (a+b x) \text{Hypergeometric2F1}[1, m+n, 3+m, \frac{(d e-c f) (a+b x)}{(b e-a f) (c+d x)}]}{(b e-a f) \text{Gamma}[3+m]} + \\
 & \frac{(d e-c f) (a+b x) \text{Gamma}[1+m+n] \text{Hypergeometric2F1}[2, 1+m+n, 4+m, \frac{(d e-c f) (a+b x)}{(b e-a f) (c+d x)}]}{(b e-a f) (c+d x) \text{Gamma}[4+m] \text{Gamma}[m+n]} - 
 \end{aligned}$$



$$\begin{aligned}
& \frac{m \text{Hypergeometric2F1}[1, m+n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{\Gamma[3+m]} + \frac{f (a+b x) \text{Hypergeometric2F1}[1, m+n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{(b e - a f) \Gamma[3+m]} + \\
& \frac{(d e - c f) (a+b x) \Gamma[1+m+n] \text{Hypergeometric2F1}[2, 1+m+n, 4+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{(b e - a f) (c+d x) \Gamma[4+m] \Gamma[m+n]} - \\
& \left. \frac{f (-d e + c f) (a+b x)^2 \Gamma[1+m+n] \text{Hypergeometric2F1}[2, 1+m+n, 4+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{(b e - a f)^2 (c+d x) \Gamma[4+m] \Gamma[m+n]} \right) - \\
& \frac{1}{(-b c + a d) (1+m)} d (m+n) (a+b x)^{1+m} (c+d x)^{-m-n} \left( \frac{-b c - b d x}{-b c + a d} \right)^{-1+m+n} (e+f x)^{-3+n} \left( \frac{-b e - b f x}{-b e + a f} \right)^{3-n} \left( 1 - \frac{d (a+b x)}{-b c + a d} \right)^{-m-n} \\
& \left( 1 - \frac{f (a+b x)}{-b e + a f} \right)^{-2+n} \Gamma[2+m] \left( \frac{2 \text{Hypergeometric2F1}[1, m+n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{\Gamma[3+m]} + \right. \\
& m \text{Hypergeometric2F1}[1, m+n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}] + \frac{f (a+b x) \text{Hypergeometric2F1}[1, m+n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{\Gamma[3+m]} + \\
& (b e - a f) \Gamma[3+m] \\
& \frac{(d e - c f) (a+b x) \Gamma[1+m+n] \text{Hypergeometric2F1}[2, 1+m+n, 4+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{(b e - a f) (c+d x) \Gamma[4+m] \Gamma[m+n]} - \\
& \left. \frac{f (-d e + c f) (a+b x)^2 \Gamma[1+m+n] \text{Hypergeometric2F1}[2, 1+m+n, 4+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{(b e - a f)^2 (c+d x) \Gamma[4+m] \Gamma[m+n]} \right) + \\
& \frac{1}{b (1+m)} d (-m-n) (a+b x)^{1+m} (c+d x)^{-1-m-n} \left( \frac{-b c - b d x}{-b c + a d} \right)^{m+n} (e+f x)^{-3+n} \left( \frac{-b e - b f x}{-b e + a f} \right)^{3-n} \left( 1 - \frac{d (a+b x)}{-b c + a d} \right)^{-m-n} \\
& \left( 1 - \frac{f (a+b x)}{-b e + a f} \right)^{-2+n} \Gamma[2+m] \left( \frac{2 \text{Hypergeometric2F1}[1, m+n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{\Gamma[3+m]} + \right. \\
& m \text{Hypergeometric2F1}[1, m+n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}] + \frac{f (a+b x) \text{Hypergeometric2F1}[1, m+n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{\Gamma[3+m]} + \\
& (b e - a f) \Gamma[3+m] \\
& \frac{(d e - c f) (a+b x) \Gamma[1+m+n] \text{Hypergeometric2F1}[2, 1+m+n, 4+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{(b e - a f) (c+d x) \Gamma[4+m] \Gamma[m+n]} - \\
& \left. \frac{f (-d e + c f) (a+b x)^2 \Gamma[1+m+n] \text{Hypergeometric2F1}[2, 1+m+n, 4+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{(b e - a f)^2 (c+d x) \Gamma[4+m] \Gamma[m+n]} \right)
\end{aligned}$$

$$\begin{aligned}
& \left( a + b x \right)^m \left( c + d x \right)^{-m-n} \left( \frac{-b c - b d x}{-b c + a d} \right)^{m+n} \left( e + f x \right)^{-3+n} \left( \frac{-b e - b f x}{-b e + a f} \right)^{3-n} \left( 1 - \frac{d (a + b x)}{-b c + a d} \right)^{-m-n} \left( 1 - \frac{f (a + b x)}{-b e + a f} \right)^{-2+n} \\
& \text{Gamma}[2+m] \left[ \frac{2 \text{Hypergeometric2F1}[1, m+n, 3+m, \frac{(d e-c f) (a+b x)}{(b e-a f) (c+d x)}]}{\text{Gamma}[3+m]} + \right. \\
& \frac{m \text{Hypergeometric2F1}[1, m+n, 3+m, \frac{(d e-c f) (a+b x)}{(b e-a f) (c+d x)}]}{\text{Gamma}[3+m]} + \frac{f (a + b x) \text{Hypergeometric2F1}[1, m+n, 3+m, \frac{(d e-c f) (a+b x)}{(b e-a f) (c+d x)}]}{(b e - a f) \text{Gamma}[3+m]} + \\
& \frac{(d e - c f) (a + b x) \text{Gamma}[1 + m + n] \text{Hypergeometric2F1}[2, 1 + m + n, 4 + m, \frac{(d e-c f) (a+b x)}{(b e-a f) (c+d x)}]}{(b e - a f) (c + d x) \text{Gamma}[4 + m] \text{Gamma}[m + n]} - \\
& \left. \frac{f (-d e + c f) (a + b x)^2 \text{Gamma}[1 + m + n] \text{Hypergeometric2F1}[2, 1 + m + n, 4 + m, \frac{(d e-c f) (a+b x)}{(b e-a f) (c+d x)}]}{(b e - a f)^2 (c + d x) \text{Gamma}[4 + m] \text{Gamma}[m + n]} \right] + \\
& \frac{1}{b (1+m)} (a + b x)^{1+m} (c + d x)^{-m-n} \left( \frac{-b c - b d x}{-b c + a d} \right)^{m+n} (e + f x)^{-3+n} \left( \frac{-b e - b f x}{-b e + a f} \right)^{3-n} \left( 1 - \frac{d (a + b x)}{-b c + a d} \right)^{-m-n} \\
& \left( 1 - \frac{f (a + b x)}{-b e + a f} \right)^{-2+n} \text{Gamma}[2+m] \left[ \frac{b f \text{Hypergeometric2F1}[1, m+n, 3+m, \frac{(d e-c f) (a+b x)}{(b e-a f) (c+d x)}]}{(b e - a f) \text{Gamma}[3+m]} + \right. \\
& \frac{2 (m+n) \left( -\frac{d (d e-c f) (a+b x)}{(b e-a f) (c+d x)^2} + \frac{b (d e-c f)}{(b e-a f) (c+d x)} \right) \text{Hypergeometric2F1}[2, 1 + m + n, 4 + m, \frac{(d e-c f) (a+b x)}{(b e-a f) (c+d x)}]}{(3+m) \text{Gamma}[3+m]} + \\
& \frac{m (m+n) \left( -\frac{d (d e-c f) (a+b x)}{(b e-a f) (c+d x)^2} + \frac{b (d e-c f)}{(b e-a f) (c+d x)} \right) \text{Hypergeometric2F1}[2, 1 + m + n, 4 + m, \frac{(d e-c f) (a+b x)}{(b e-a f) (c+d x)}]}{(3+m) \text{Gamma}[3+m]} + \\
& \left. \left( f (m+n) (a + b x) \left( -\frac{d (d e-c f) (a+b x)}{(b e-a f) (c+d x)^2} + \frac{b (d e-c f)}{(b e-a f) (c+d x)} \right) \text{Hypergeometric2F1}[2, 1 + m + n, 4 + m, \frac{(d e-c f) (a+b x)}{(b e-a f) (c+d x)}] \right) / \right. \\
& \left. \left( (b e - a f) (3 + m) \text{Gamma}[3 + m] \right) - \frac{d (d e - c f) (a + b x) \text{Gamma}[1 + m + n] \text{Hypergeometric2F1}[2, 1 + m + n, 4 + m, \frac{(d e-c f) (a+b x)}{(b e-a f) (c+d x)}]}{(b e - a f) (c + d x)^2 \text{Gamma}[4 + m] \text{Gamma}[m + n]} + \right. \\
& \frac{d f (-d e + c f) (a + b x)^2 \text{Gamma}[1 + m + n] \text{Hypergeometric2F1}[2, 1 + m + n, 4 + m, \frac{(d e-c f) (a+b x)}{(b e-a f) (c+d x)}]}{(b e - a f)^2 (c + d x)^2 \text{Gamma}[4 + m] \text{Gamma}[m + n]} + \\
& \frac{b (d e - c f) \text{Gamma}[1 + m + n] \text{Hypergeometric2F1}[2, 1 + m + n, 4 + m, \frac{(d e-c f) (a+b x)}{(b e-a f) (c+d x)}]}{(b e - a f) (c + d x) \text{Gamma}[4 + m] \text{Gamma}[m + n]} - \\
& \left. \left( 2 b f (-d e + c f) (a + b x) \text{Gamma}[1 + m + n] \text{Hypergeometric2F1}[2, 1 + m + n, 4 + m, \frac{(d e-c f) (a+b x)}{(b e-a f) (c+d x)}] \right) / \left( (b e - a f)^2 (c + d x) \right) \right]
\end{aligned}$$

$$\begin{aligned}
& \text{Gamma}[4+m] \text{Gamma}[m+n] \Big) + \left( 2 (d e - c f) (1+m+n) (a+b x) \left( -\frac{d (d e - c f) (a+b x)}{(b e - a f) (c+d x)^2} + \frac{b (d e - c f)}{(b e - a f) (c+d x)} \right) \text{Gamma}[1+m+n] \right. \\
& \left. \text{Hypergeometric2F1}[3, 2+m+n, 5+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}] \right) \Big/ \left( (b e - a f) (4+m) (c+d x) \text{Gamma}[4+m] \text{Gamma}[m+n] \right) - \\
& \left( 2 f (-d e + c f) (1+m+n) (a+b x)^2 \left( -\frac{d (d e - c f) (a+b x)}{(b e - a f) (c+d x)^2} + \frac{b (d e - c f)}{(b e - a f) (c+d x)} \right) \text{Gamma}[1+m+n] \right. \\
& \left. \text{Hypergeometric2F1}[3, 2+m+n, 5+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}] \right) \Big/ \left( (b e - a f)^2 (4+m) (c+d x) \text{Gamma}[4+m] \text{Gamma}[m+n] \right) \Bigg)
\end{aligned}$$

**Problem 3151:** Attempted integration timed out after 120 seconds.

$$\int (a+b x)^m (c+d x)^{-m-n} (e+f x)^{-4+n} dx$$

Optimal (type 5, 428 leaves, 4 steps):

$$\begin{aligned}
& -\frac{f (a+b x)^{1+m} (c+d x)^{1-m-n} (e+f x)^{-3+n}}{(b e - a f) (d e - c f) (3-n)} + \frac{f (a d f (2+m) - b (d e (4-n) - c f (2-m-n))) (a+b x)^{1+m} (c+d x)^{1-m-n} (e+f x)^{-2+n}}{(b e - a f)^2 (d e - c f)^2 (2-n) (3-n)} + \\
& \left( (a^2 d^2 f^2 (2+3 m+m^2) - 2 a b d f (1+m) (d e (3-n) - c f (1-m-n))) - \right. \\
& \left. b^2 (2 c d e f (3-n) (1-m-n) - d^2 e^2 (6-5 n+n^2) - c^2 f^2 (2+m^2-m (3-2 n) - 3 n+n^2)) \right) (a+b x)^{1+m} (c+d x)^{-m-n} \left( \frac{(b e - a f) (c+d x)}{(b c - a d) (e+f x)} \right)^{m+n} \\
& (e+f x)^{-1+n} \text{Hypergeometric2F1}[1+m, m+n, 2+m, -\frac{(d e - c f) (a+b x)}{(b c - a d) (e+f x)}] \Big) \Big/ \left( (b e - a f)^3 (d e - c f)^2 (1+m) (2-n) (3-n) \right)
\end{aligned}$$

Result (type 1, 1 leaves):

???

**Problem 3152:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (a+b x)^m (c+d x)^n \left( \frac{b c f + a d f + a d f m + b c f n}{b d (2+m+n)} + f x \right)^{-3-m-n} dx$$

Optimal (type 3, 88 leaves, 1 step):

$$\frac{b d (2 + m + n) (a + b x)^{1+m} (c + d x)^{1+n} \left( \frac{f (a d (1+m) + b c (1+n))}{b d (2+m+n)} + f x \right)^{-2-m-n}}{(b c - a d)^2 f (1+m) (1+n)}$$

Result (type 5, 5681 leaves):

$$\begin{aligned} & \left( (a + b x)^{1+2m} (c + d x)^{2n} \left( \frac{-b c - b d x}{-b c + a d} \right)^{-n} \left( -\frac{-b c - a d - a d m - b c n - 2 b d x - b d m x - b d n x}{(b c - a d) (1+n)} \right)^{3+m+n} \right. \\ & (b c + a d + a d m + b c n + 2 b d x + b d m x + b d n x)^{-3-m-n} \left( \frac{f (a d (1+m) + b c (1+n) + b d (2+m+n) x)}{b d (2+m+n)} \right)^{-3-m-n} \\ & \left( 1 - \frac{d (a + b x)}{-b c + a d} \right)^n \left( 1 + \frac{d (2 + m + n) (a + b x)}{(b c - a d) (1+n)} \right)^{-2-m-n} \text{Gamma}[2+m] \left( \frac{2 \text{Hypergeometric2F1}[1, -n, 3+m, -\frac{d (1+m) (a+b x)}{b (1+n) (c+d x)}]}{\text{Gamma}[3+m]} + \right. \\ & \left. \frac{m \text{Hypergeometric2F1}[1, -n, 3+m, -\frac{d (1+m) (a+b x)}{b (1+n) (c+d x)}]}{\text{Gamma}[3+m]} + \frac{d (2 + m + n) (a + b x) \text{Hypergeometric2F1}[1, -n, 3+m, -\frac{d (1+m) (a+b x)}{b (1+n) (c+d x)}]}{(b c - a d) (1+n) \text{Gamma}[3+m]} - \right. \\ & \left. \frac{d (1+m) (a + b x) \text{Gamma}[1-n] \text{Hypergeometric2F1}[2, 1-n, 4+m, -\frac{d (1+m) (a+b x)}{b (1+n) (c+d x)}]}{b (1+n) (c+d x) \text{Gamma}[4+m] \text{Gamma}[-n]} - \right. \\ & \left. \frac{d^2 (1+m) (2 + m + n) (a + b x)^2 \text{Gamma}[1-n] \text{Hypergeometric2F1}[2, 1-n, 4+m, -\frac{d (1+m) (a+b x)}{b (1+n) (c+d x)}]}{b (b c - a d) (1+n)^2 (c+d x) \text{Gamma}[4+m] \text{Gamma}[-n]} \right) \Bigg) / \\ & \left( b (1+m) \left( \frac{1}{(b c - a d) (1+m) (1+n)} d (-2 - m - n) (2 + m + n) (a + b x)^{1+m} (c + d x)^n \left( \frac{-b c - b d x}{-b c + a d} \right)^{-n} \right. \right. \\ & \left( -\frac{-b c - a d - a d m - b c n - 2 b d x - b d m x - b d n x}{(b c - a d) (1+n)} \right)^{3+m+n} (b c + a d + a d m + b c n + 2 b d x + b d m x + b d n x)^{-3-m-n} \\ & \left( 1 - \frac{d (a + b x)}{-b c + a d} \right)^n \left( 1 + \frac{d (2 + m + n) (a + b x)}{(b c - a d) (1+n)} \right)^{-3-m-n} \text{Gamma}[2+m] \left( \frac{2 \text{Hypergeometric2F1}[1, -n, 3+m, -\frac{d (1+m) (a+b x)}{b (1+n) (c+d x)}]}{\text{Gamma}[3+m]} + \right. \\ & \left. \frac{m \text{Hypergeometric2F1}[1, -n, 3+m, -\frac{d (1+m) (a+b x)}{b (1+n) (c+d x)}]}{\text{Gamma}[3+m]} + \frac{d (2 + m + n) (a + b x) \text{Hypergeometric2F1}[1, -n, 3+m, -\frac{d (1+m) (a+b x)}{b (1+n) (c+d x)}]}{(b c - a d) (1+n) \text{Gamma}[3+m]} - \right. \\ & \left. \frac{d (1+m) (a + b x) \text{Gamma}[1-n] \text{Hypergeometric2F1}[2, 1-n, 4+m, -\frac{d (1+m) (a+b x)}{b (1+n) (c+d x)}]}{b (1+n) (c+d x) \text{Gamma}[4+m] \text{Gamma}[-n]} - \right. \\ & \left. \frac{d^2 (1+m) (2 + m + n) (a + b x)^2 \text{Gamma}[1-n] \text{Hypergeometric2F1}[2, 1-n, 4+m, -\frac{d (1+m) (a+b x)}{b (1+n) (c+d x)}]}{b (b c - a d) (1+n)^2 (c+d x) \text{Gamma}[4+m] \text{Gamma}[-n]} \right) \end{aligned}$$

$$\begin{aligned}
& \frac{1}{(-b c + a d) (1+m)} d n (a + b x)^{1+m} (c + d x)^n \left( \frac{-b c - b d x}{-b c + a d} \right)^{-n} \left( -\frac{-b c - a d - a d m - b c n - 2 b d x - b d m x - b d n x}{(b c - a d) (1+n)} \right)^{3+m+n} \\
& (b c + a d + a d m + b c n + 2 b d x + b d m x + b d n x)^{-3-m-n} \left( 1 - \frac{d (a + b x)}{-b c + a d} \right)^{-1+n} \left( 1 + \frac{d (2 + m + n) (a + b x)}{(b c - a d) (1+n)} \right)^{-2-m-n} \text{Gamma}[2+m] \\
& \left( \frac{2 \text{Hypergeometric2F1}[1, -n, 3+m, -\frac{d (1+m) (a+b x)}{b (1+n) (c+d x)}]}{\text{Gamma}[3+m]} + \frac{m \text{Hypergeometric2F1}[1, -n, 3+m, -\frac{d (1+m) (a+b x)}{b (1+n) (c+d x)}]}{\text{Gamma}[3+m]} + \right. \\
& \frac{d (2 + m + n) (a + b x) \text{Hypergeometric2F1}[1, -n, 3+m, -\frac{d (1+m) (a+b x)}{b (1+n) (c+d x)}]}{(b c - a d) (1+n) \text{Gamma}[3+m]} - \\
& \frac{d (1+m) (a + b x) \text{Gamma}[1-n] \text{Hypergeometric2F1}[2, 1-n, 4+m, -\frac{d (1+m) (a+b x)}{b (1+n) (c+d x)}]}{b (1+n) (c + d x) \text{Gamma}[4+m] \text{Gamma}[-n]} - \\
& \left. \frac{d^2 (1+m) (2 + m + n) (a + b x)^2 \text{Gamma}[1-n] \text{Hypergeometric2F1}[2, 1-n, 4+m, -\frac{d (1+m) (a+b x)}{b (1+n) (c+d x)}]}{b (b c - a d) (1+n)^2 (c + d x) \text{Gamma}[4+m] \text{Gamma}[-n]} \right) + \\
& \frac{1}{b (1+m)} (-3 - m - n) (2 b d + b d m + b d n) (a + b x)^{1+m} (c + d x)^n \left( \frac{-b c - b d x}{-b c + a d} \right)^{-n} \left( -\frac{-b c - a d - a d m - b c n - 2 b d x - b d m x - b d n x}{(b c - a d) (1+n)} \right)^{3+m+n} \\
& (b c + a d + a d m + b c n + 2 b d x + b d m x + b d n x)^{-4-m-n} \left( 1 - \frac{d (a + b x)}{-b c + a d} \right)^n \left( 1 + \frac{d (2 + m + n) (a + b x)}{(b c - a d) (1+n)} \right)^{-2-m-n} \text{Gamma}[2+m] \\
& \left( \frac{2 \text{Hypergeometric2F1}[1, -n, 3+m, -\frac{d (1+m) (a+b x)}{b (1+n) (c+d x)}]}{\text{Gamma}[3+m]} + \frac{m \text{Hypergeometric2F1}[1, -n, 3+m, -\frac{d (1+m) (a+b x)}{b (1+n) (c+d x)}]}{\text{Gamma}[3+m]} + \right. \\
& \frac{d (2 + m + n) (a + b x) \text{Hypergeometric2F1}[1, -n, 3+m, -\frac{d (1+m) (a+b x)}{b (1+n) (c+d x)}]}{(b c - a d) (1+n) \text{Gamma}[3+m]} - \\
& \frac{d (1+m) (a + b x) \text{Gamma}[1-n] \text{Hypergeometric2F1}[2, 1-n, 4+m, -\frac{d (1+m) (a+b x)}{b (1+n) (c+d x)}]}{b (1+n) (c + d x) \text{Gamma}[4+m] \text{Gamma}[-n]} - \\
& \left. \frac{d^2 (1+m) (2 + m + n) (a + b x)^2 \text{Gamma}[1-n] \text{Hypergeometric2F1}[2, 1-n, 4+m, -\frac{d (1+m) (a+b x)}{b (1+n) (c+d x)}]}{b (b c - a d) (1+n)^2 (c + d x) \text{Gamma}[4+m] \text{Gamma}[-n]} \right) - \\
& \frac{1}{b (b c - a d) (1+m) (1+n)} (3 + m + n) (-2 b d - b d m - b d n) (a + b x)^{1+m} (c + d x)^n \left( \frac{-b c - b d x}{-b c + a d} \right)^{-n} \\
& \left( -\frac{-b c - a d - a d m - b c n - 2 b d x - b d m x - b d n x}{(b c - a d) (1+n)} \right)^{2+m+n} (b c + a d + a d m + b c n + 2 b d x + b d m x + b d n x)^{-3-m-n}
\end{aligned}$$

$$\begin{aligned}
& \left(1 - \frac{d(a+b x)}{-b c + a d}\right)^n \left(1 + \frac{d(2+m+n)(a+b x)}{(b c - a d)(1+n)}\right)^{-2-m-n} \frac{\text{Gamma}[2+m]}{\text{Gamma}[3+m]} \left( \frac{2 \text{Hypergeometric2F1}[1, -n, 3+m, -\frac{d(1+m)(a+b x)}{b(1+n)(c+d x)}]}{\text{Gamma}[3+m]} + \right. \\
& \left. \frac{m \text{Hypergeometric2F1}[1, -n, 3+m, -\frac{d(1+m)(a+b x)}{b(1+n)(c+d x)}]}{\text{Gamma}[3+m]} + \frac{d(2+m+n)(a+b x) \text{Hypergeometric2F1}[1, -n, 3+m, -\frac{d(1+m)(a+b x)}{b(1+n)(c+d x)}]}{(b c - a d)(1+n) \text{Gamma}[3+m]} - \right. \\
& \left. \frac{d(1+m)(a+b x) \text{Gamma}[1-n] \text{Hypergeometric2F1}[2, 1-n, 4+m, -\frac{d(1+m)(a+b x)}{b(1+n)(c+d x)}]}{b(1+n)(c+d x) \text{Gamma}[4+m] \text{Gamma}[-n]} - \right. \\
& \left. \frac{d^2(1+m)(2+m+n)(a+b x)^2 \text{Gamma}[1-n] \text{Hypergeometric2F1}[2, 1-n, 4+m, -\frac{d(1+m)(a+b x)}{b(1+n)(c+d x)}]}{b(b c - a d)(1+n)^2(c+d x) \text{Gamma}[4+m] \text{Gamma}[-n]} \right) + \\
& \frac{1}{(-b c + a d)(1+m)} d n (a+b x)^{1+m} (c+d x)^n \left( \frac{-b c - b d x}{-b c + a d} \right)^{-1-n} \left( -\frac{-b c - a d - a d m - b c n - 2 b d x - b d m x - b d n x}{(b c - a d)(1+n)} \right)^{3+m+n} \\
& (b c + a d + a d m + b c n + 2 b d x + b d m x + b d n x)^{-3-m-n} \left( 1 - \frac{d(a+b x)}{-b c + a d} \right)^n \left( 1 + \frac{d(2+m+n)(a+b x)}{(b c - a d)(1+n)} \right)^{-2-m-n} \frac{\text{Gamma}[2+m]}{\text{Gamma}[3+m]} \\
& \left( \frac{2 \text{Hypergeometric2F1}[1, -n, 3+m, -\frac{d(1+m)(a+b x)}{b(1+n)(c+d x)}]}{\text{Gamma}[3+m]} + \frac{m \text{Hypergeometric2F1}[1, -n, 3+m, -\frac{d(1+m)(a+b x)}{b(1+n)(c+d x)}]}{\text{Gamma}[3+m]} + \right. \\
& \left. \frac{d(2+m+n)(a+b x) \text{Hypergeometric2F1}[1, -n, 3+m, -\frac{d(1+m)(a+b x)}{b(1+n)(c+d x)}]}{(b c - a d)(1+n) \text{Gamma}[3+m]} - \right. \\
& \left. \frac{d(1+m)(a+b x) \text{Gamma}[1-n] \text{Hypergeometric2F1}[2, 1-n, 4+m, -\frac{d(1+m)(a+b x)}{b(1+n)(c+d x)}]}{b(1+n)(c+d x) \text{Gamma}[4+m] \text{Gamma}[-n]} - \right. \\
& \left. \frac{d^2(1+m)(2+m+n)(a+b x)^2 \text{Gamma}[1-n] \text{Hypergeometric2F1}[2, 1-n, 4+m, -\frac{d(1+m)(a+b x)}{b(1+n)(c+d x)}]}{b(b c - a d)(1+n)^2(c+d x) \text{Gamma}[4+m] \text{Gamma}[-n]} \right) + \\
& \frac{1}{b(1+m)} d n (a+b x)^{1+m} (c+d x)^{-1+n} \left( \frac{-b c - b d x}{-b c + a d} \right)^{-n} \left( -\frac{-b c - a d - a d m - b c n - 2 b d x - b d m x - b d n x}{(b c - a d)(1+n)} \right)^{3+m+n} \\
& (b c + a d + a d m + b c n + 2 b d x + b d m x + b d n x)^{-3-m-n} \left( 1 - \frac{d(a+b x)}{-b c + a d} \right)^n \left( 1 + \frac{d(2+m+n)(a+b x)}{(b c - a d)(1+n)} \right)^{-2-m-n} \frac{\text{Gamma}[2+m]}{\text{Gamma}[3+m]} \\
& \left( \frac{2 \text{Hypergeometric2F1}[1, -n, 3+m, -\frac{d(1+m)(a+b x)}{b(1+n)(c+d x)}]}{\text{Gamma}[3+m]} + \frac{m \text{Hypergeometric2F1}[1, -n, 3+m, -\frac{d(1+m)(a+b x)}{b(1+n)(c+d x)}]}{\text{Gamma}[3+m]} + \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{d(2+m+n)(a+b x) \text{Hypergeometric2F1}[1, -n, 3+m, -\frac{d(1+m)(a+b x)}{b(1+n)(c+d x)}]}{(b c - a d)(1+n) \text{Gamma}[3+m]} - \\
& \frac{d(1+m)(a+b x) \text{Gamma}[1-n] \text{Hypergeometric2F1}[2, 1-n, 4+m, -\frac{d(1+m)(a+b x)}{b(1+n)(c+d x)}]}{b(1+n)(c+d x) \text{Gamma}[4+m] \text{Gamma}[-n]} - \\
& \left. \frac{d^2(1+m)(2+m+n)(a+b x)^2 \text{Gamma}[1-n] \text{Hypergeometric2F1}[2, 1-n, 4+m, -\frac{d(1+m)(a+b x)}{b(1+n)(c+d x)}]}{b(b c - a d)(1+n)^2(c+d x) \text{Gamma}[4+m] \text{Gamma}[-n]} \right) + \\
& (a+b x)^m (c+d x)^n \left( \frac{-b c - b d x}{-b c + a d} \right)^{-n} \left( -\frac{-b c - a d - a d m - b c n - 2 b d x - b d m x - b d n x}{(b c - a d)(1+n)} \right)^{3+m+n} \\
& (b c + a d + a d m + b c n + 2 b d x + b d m x + b d n x)^{-3-m-n} \left( 1 - \frac{d(a+b x)}{-b c + a d} \right)^n \left( 1 + \frac{d(2+m+n)(a+b x)}{(b c - a d)(1+n)} \right)^{-2-m-n} \text{Gamma}[2+m] \\
& \left( \frac{2 \text{Hypergeometric2F1}[1, -n, 3+m, -\frac{d(1+m)(a+b x)}{b(1+n)(c+d x)}]}{\text{Gamma}[3+m]} + \frac{m \text{Hypergeometric2F1}[1, -n, 3+m, -\frac{d(1+m)(a+b x)}{b(1+n)(c+d x)}]}{\text{Gamma}[3+m]} + \right. \\
& \frac{d(2+m+n)(a+b x) \text{Hypergeometric2F1}[1, -n, 3+m, -\frac{d(1+m)(a+b x)}{b(1+n)(c+d x)}]}{(b c - a d)(1+n) \text{Gamma}[3+m]} - \\
& \frac{d(1+m)(a+b x) \text{Gamma}[1-n] \text{Hypergeometric2F1}[2, 1-n, 4+m, -\frac{d(1+m)(a+b x)}{b(1+n)(c+d x)}]}{b(1+n)(c+d x) \text{Gamma}[4+m] \text{Gamma}[-n]} - \\
& \left. \frac{d^2(1+m)(2+m+n)(a+b x)^2 \text{Gamma}[1-n] \text{Hypergeometric2F1}[2, 1-n, 4+m, -\frac{d(1+m)(a+b x)}{b(1+n)(c+d x)}]}{b(b c - a d)(1+n)^2(c+d x) \text{Gamma}[4+m] \text{Gamma}[-n]} \right) + \\
& \frac{1}{b(1+m)} (a+b x)^{1+m} (c+d x)^n \left( \frac{-b c - b d x}{-b c + a d} \right)^{-n} \left( -\frac{-b c - a d - a d m - b c n - 2 b d x - b d m x - b d n x}{(b c - a d)(1+n)} \right)^{3+m+n} \\
& (b c + a d + a d m + b c n + 2 b d x + b d m x + b d n x)^{-3-m-n} \left( 1 - \frac{d(a+b x)}{-b c + a d} \right)^n \left( 1 + \frac{d(2+m+n)(a+b x)}{(b c - a d)(1+n)} \right)^{-2-m-n} \text{Gamma}[2+m] \\
& \left( \frac{b d(2+m+n) \text{Hypergeometric2F1}[1, -n, 3+m, -\frac{d(1+m)(a+b x)}{b(1+n)(c+d x)}]}{(b c - a d)(1+n) \text{Gamma}[3+m]} - \right. \\
& \left. \frac{2 n \left( \frac{d^2(1+m)(a+b x)}{b(1+n)(c+d x)^2} - \frac{d(1+m)}{(1+n)(c+d x)} \right) \text{Hypergeometric2F1}[2, 1-n, 4+m, -\frac{d(1+m)(a+b x)}{b(1+n)(c+d x)}]}{(3+m) \text{Gamma}[3+m]} \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{m n \left( \frac{d^2 (1+m) (a+b x)}{b (1+n) (c+d x)^2} - \frac{d (1+m)}{(1+n) (c+d x)} \right) \text{Hypergeometric2F1}[2, 1-n, 4+m, -\frac{d (1+m) (a+b x)}{b (1+n) (c+d x)}] - }{(3+m) \text{Gamma}[3+m]} - \\
& \left( d n (2+m+n) (a+b x) \left( \frac{d^2 (1+m) (a+b x)}{b (1+n) (c+d x)^2} - \frac{d (1+m)}{(1+n) (c+d x)} \right) \text{Hypergeometric2F1}[2, 1-n, 4+m, -\frac{d (1+m) (a+b x)}{b (1+n) (c+d x)}] \right) / \\
& \left( (b c - a d) (3+m) (1+n) \text{Gamma}[3+m] \right) + \frac{d^2 (1+m) (a+b x) \text{Gamma}[1-n] \text{Hypergeometric2F1}[2, 1-n, 4+m, -\frac{d (1+m) (a+b x)}{b (1+n) (c+d x)}]}{b (1+n) (c+d x)^2 \text{Gamma}[4+m] \text{Gamma}[-n]} + \\
& \frac{d^3 (1+m) (2+m+n) (a+b x)^2 \text{Gamma}[1-n] \text{Hypergeometric2F1}[2, 1-n, 4+m, -\frac{d (1+m) (a+b x)}{b (1+n) (c+d x)}]}{b (b c - a d) (1+n)^2 (c+d x)^2 \text{Gamma}[4+m] \text{Gamma}[-n]} - \\
& \frac{d (1+m) \text{Gamma}[1-n] \text{Hypergeometric2F1}[2, 1-n, 4+m, -\frac{d (1+m) (a+b x)}{b (1+n) (c-d x)}]}{(1+n) (c+d x) \text{Gamma}[4+m] \text{Gamma}[-n]} - \\
& \frac{2 d^2 (1+m) (2+m+n) (a+b x) \text{Gamma}[1-n] \text{Hypergeometric2F1}[2, 1-n, 4+m, -\frac{d (1+m) (a+b x)}{b (1+n) (c+d x)}]}{(b c - a d) (1+n)^2 (c+d x) \text{Gamma}[4+m] \text{Gamma}[-n]} - \\
& \left( 2 d (1+m) (1-n) (a+b x) \left( \frac{d^2 (1+m) (a+b x)}{b (1+n) (c+d x)^2} - \frac{d (1+m)}{(1+n) (c+d x)} \right) \text{Gamma}[1-n] \right. \\
& \left. \text{Hypergeometric2F1}[3, 2-n, 5+m, -\frac{d (1+m) (a+b x)}{b (1+n) (c+d x)}] \right) / (b (4+m) (1+n) (c+d x) \text{Gamma}[4+m] \text{Gamma}[-n]) - \\
& \left( 2 d^2 (1+m) (1-n) (2+m+n) (a+b x)^2 \left( \frac{d^2 (1+m) (a+b x)}{b (1+n) (c+d x)^2} - \frac{d (1+m)}{(1+n) (c+d x)} \right) \text{Gamma}[1-n] \right. \\
& \left. \text{Hypergeometric2F1}[3, 2-n, 5+m, -\frac{d (1+m) (a+b x)}{b (1+n) (c+d x)}] \right) / (b (b c - a d) (4+m) (1+n)^2 (c+d x) \text{Gamma}[4+m] \text{Gamma}[-n]) \right) \Bigg)
\end{aligned}$$

**Problem 3153: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a+b x)^m (c+d x)^{-1-\frac{d(b e-a f)(1+m)}{b(d e-c f)}} (e+f x)^{-1+\frac{(b c-a d)f(1+m)}{b(d e-c f)}} d x$$

Optimal (type 3, 101 leaves, 1 step):

$$\frac{b (a+b x)^{1+m} (c+d x)^{-\frac{d(b e-a f)(1+m)}{b(d e-c f)}} (e+f x)^{\frac{(b c-a d)f(1+m)}{b(d e-c f)}}}{(b c - a d) (b e - a f) (1+m)}$$

Result (type 6, 1616 leaves):

$$\begin{aligned}
 & \frac{1}{1+m} (a + bx)^{1+m} (c + dx)^{-\frac{d(be - af)(1+m)}{b(de - cf)}} (e + fx)^{\frac{(bc - ad)f(1+m)}{b(de - cf)}} \\
 & \left( \left( f \text{AppellF1}[1+m, \frac{d(be - af)(1+m)}{b(de - cf)}, 1 + \frac{(bc - ad)f(1+m)}{b(-de + cf)}, 2+m, \frac{d(a + bx)}{-bc + ad}, \frac{f(a + bx)}{-be + af}] \right) \middle/ \left( (-de + cf) \right. \right. \\
 & \left. \left( \frac{1}{1+m} f \left( 1 + \frac{(bc - ad)f(1+m)}{b(-de + cf)} \right) (a + bx) \text{AppellF1}[1+m, \frac{d(be - af)(1+m)}{b(de - cf)}, 1 + \frac{(bc - ad)f(1+m)}{b(-de + cf)}, 2+m, \frac{d(a + bx)}{-bc + ad}, \frac{f(a + bx)}{-be + af}] \right. \right. \\
 & \left. \left. + \frac{1}{b(de - cf)(1+m)} f(-bde - adf(1+m) + bcf(2+m)) (a + bx) \text{AppellF1}[1+m, \frac{d(be - af)(1+m)}{b(de - cf)}, 1 + \frac{(bc - ad)f(1+m)}{b(-de + cf)}, 2+m, \right. \right. \\
 & \left. \left. \frac{d(a + bx)}{-bc + ad}, \frac{f(a + bx)}{-be + af}] + b(e + fx) \text{AppellF1}[1+m, \frac{d(be - af)(1+m)}{b(de - cf)}, 1 + \frac{(bc - ad)f(1+m)}{b(-de + cf)}, 2+m, \right. \right. \\
 & \left. \left. \frac{d(a + bx)}{-bc + ad}, \frac{f(a + bx)}{-be + af}] \right) \right. \left. \left( (a + bx)(e + fx) \left( f(bde + adf(1+m) - bcf(2+m)) \text{AppellF1}[2+m, \frac{d(be - af)(1+m)}{b(de - cf)}, 2 + \frac{(bc - ad)f(1+m)}{b(-de + cf)}, \right. \right. \right. \\
 & \left. \left. \left. 3+m, \frac{d(a + bx)}{-bc + ad}, \frac{f(a + bx)}{-be + af}] + \frac{1}{b(c - ad)} d^2 (be - af)^2 (1+m) \text{AppellF1}[2+m, 1 + \frac{d(be - af)(1+m)}{b(de - cf)}, \right. \right. \right. \\
 & \left. \left. \left. 1 + \frac{(bc - ad)f(1+m)}{b(-de + cf)}, 3+m, \frac{d(a + bx)}{-bc + ad}, \frac{f(a + bx)}{-be + af}] \right) \right) \right. \left. \left( (be - af)(-de + cf)(2+m) \right) \right) + \\
 & \left( d \text{AppellF1}[1+m, 1 + \frac{d(be - af)(1+m)}{b(de - cf)}, \frac{(bc - ad)f(1+m)}{b(-de + cf)}, 2+m, \frac{d(a + bx)}{-bc + ad}, \frac{f(a + bx)}{-be + af}] \right) \middle/ \left( (de - cf) \right. \\
 & \left( \frac{1}{1+m} d \left( 1 + \frac{d(be - af)(1+m)}{b(de - cf)} \right) (a + bx) \text{AppellF1}[1+m, 1 + \frac{d(be - af)(1+m)}{b(de - cf)}, \frac{(bc - ad)f(1+m)}{b(-de + cf)}, 2+m, \frac{d(a + bx)}{-bc + ad}, \frac{f(a + bx)}{-be + af}] \right) - \\
 & \left. \frac{1}{b(de - cf)(1+m)} d(-bcf - adf(1+m) + bde(2+m)) (a + bx) \text{AppellF1}[1+m, 1 + \frac{d(be - af)(1+m)}{b(de - cf)}, \frac{(bc - ad)f(1+m)}{b(-de + cf)}, 2+m, \right. \\
 & \left. \frac{d(a + bx)}{-bc + ad}, \frac{f(a + bx)}{-be + af}] + b(cx + dx) \text{AppellF1}[1+m, 1 + \frac{d(be - af)(1+m)}{b(de - cf)}, \frac{(bc - ad)f(1+m)}{b(-de + cf)}, 2+m, \frac{d(a + bx)}{-bc + ad}, \frac{f(a + bx)}{-be + af}] \right) + \\
 & \left( (a + bx)(c + dx) \left( \frac{1}{b(e - af)} (bc - ad)^2 f^2 (1+m) \text{AppellF1}[2+m, 1 + \frac{d(be - af)(1+m)}{b(de - cf)}, 1 + \frac{(bc - ad)f(1+m)}{b(-de + cf)}, \right. \right. \right. \\
 & \left. \left. \left. 3+m, \frac{d(a + bx)}{-bc + ad}, \frac{f(a + bx)}{-be + af}] - d(-bcf - adf(1+m) + bde(2+m)) \text{AppellF1}[2+m, 2 + \frac{d(be - af)(1+m)}{b(de - cf)}, \right. \right. \right. \\
 & \left. \left. \left. \frac{(bc - ad)f(1+m)}{b(-de + cf)}, 3+m, \frac{d(a + bx)}{-bc + ad}, \frac{f(a + bx)}{-be + af}] \right) \right) \right) \middle/ \left( (bc - ad)(de - cf)(2+m) \right) \right)
 \end{aligned}$$

**Problem 3154:** Result more than twice size of optimal antiderivative.

$$\int (a + bx)^m (c + dx)^n (e + fx)^{-m-n} dx$$

Optimal (type 6, 129 leaves, 3 steps):

$$\frac{1}{b(1+m)} (a + bx)^{1+m} (c + dx)^n \left( \frac{b(c + dx)}{bc - ad} \right)^{-n} (e + fx)^{-m-n} \left( \frac{b(e + fx)}{be - af} \right)^{m+n} \text{AppellF1}[1+m, -n, m+n, 2+m, -\frac{d(a+bx)}{bc-ad}, -\frac{f(a+bx)}{be-af}]$$

Result (type 6, 303 leaves):

$$\begin{aligned} & \left( (bc - ad) (be - af) (2+m) (a + bx)^{1+m} (c + dx)^n (e + fx)^{-m-n} \text{AppellF1}[1+m, -n, m+n, 2+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}] \right) / \\ & \left( b(1+m) \left( (bc - ad) (be - af) (2+m) \text{AppellF1}[1+m, -n, m+n, 2+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}] \right) - \right. \\ & (a + bx) \left( d(-be + af) n \text{AppellF1}[2+m, 1-n, m+n, 3+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}] \right) + \\ & \left. (bc - ad) f(m+n) \text{AppellF1}[2+m, -n, 1+m+n, 3+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}] \right) \end{aligned}$$

**Problem 3155:** Result more than twice size of optimal antiderivative.

$$\int (a + bx)^m (c + dx)^n (e + fx)^{-1-m-n} dx$$

Optimal (type 6, 137 leaves, 3 steps):

$$\frac{1}{(be - af)(1+m)} (a + bx)^{1+m} (c + dx)^n \left( \frac{b(c + dx)}{bc - ad} \right)^{-n} (e + fx)^{-m-n} \left( \frac{b(e + fx)}{be - af} \right)^{m+n} \text{AppellF1}[1+m, -n, 1+m+n, 2+m, -\frac{d(a+bx)}{bc-ad}, -\frac{f(a+bx)}{be-af}]$$

Result (type 6, 308 leaves):

$$\begin{aligned} & \left( (b c - a d) (b e - a f) (2 + m) (a + b x)^{1+m} (c + d x)^n (e + f x)^{-1-m-n} \text{AppellF1}[1 + m, -n, 1 + m + n, 2 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] \right) / \\ & \left( b (1 + m) \left( (b c - a d) (b e - a f) (2 + m) \text{AppellF1}[1 + m, -n, 1 + m + n, 2 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] - \right. \right. \\ & (a + b x) \left( d (-b e + a f) n \text{AppellF1}[2 + m, 1 - n, 1 + m + n, 3 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] + \right. \\ & \left. \left. (b c - a d) f (1 + m + n) \text{AppellF1}[2 + m, -n, 2 + m + n, 3 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] \right) \right) \end{aligned}$$

**Problem 3157: Result more than twice size of optimal antiderivative.**

$$\int (a + b x)^m (c + d x)^n (e + f x)^{-3-m-n} dx$$

Optimal (type 5, 227 leaves, 2 steps):

$$\begin{aligned} & -\frac{f (a + b x)^{1+m} (c + d x)^{1+n} (e + f x)^{-2-m-n}}{(b e - a f) (d e - c f) (2 + m + n)} - \\ & \left( (a d f (1 + m) + b (c f (1 + n) - d e (2 + m + n))) (a + b x)^{1+m} (c + d x)^n \left( \frac{(b e - a f) (c + d x)}{(b c - a d) (e + f x)} \right)^{-n} (e + f x)^{-1-m-n} \right. \\ & \left. \text{Hypergeometric2F1}[1 + m, -n, 2 + m, -\frac{(d e - c f) (a + b x)}{(b c - a d) (e + f x)}] \right) / ((b e - a f)^2 (d e - c f) (1 + m) (2 + m + n)) \end{aligned}$$

Result (type 5, 5212 leaves):

$$\begin{aligned} & \left( (a + b x)^{1+2 m} (c + d x)^{2 n} \left( \frac{-b c - b d x}{-b c + a d} \right)^{-n} (e + f x)^{-6-2 m-2 n} \left( \frac{-b e - b f x}{-b e + a f} \right)^{3+m+n} \right. \\ & \left( 1 - \frac{d (a + b x)}{-b c + a d} \right)^n \left( 1 - \frac{f (a + b x)}{-b e + a f} \right)^{-2-m-n} \text{Gamma}[2 + m] \left( \frac{2 \text{Hypergeometric2F1}[1, -n, 3 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{\text{Gamma}[3 + m]} + \right. \\ & \left. \frac{m \text{Hypergeometric2F1}[1, -n, 3 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{\text{Gamma}[3 + m]} + \frac{f (a + b x) \text{Hypergeometric2F1}[1, -n, 3 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{(b e - a f) \text{Gamma}[3 + m]} + \right. \\ & \left. \frac{(d e - c f) (a + b x) \text{Gamma}[1 - n] \text{Hypergeometric2F1}[2, 1 - n, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{(b e - a f) (c + d x) \text{Gamma}[4 + m] \text{Gamma}[-n]} - \right. \\ & \left. \frac{f (-d e + c f) (a + b x)^2 \text{Gamma}[1 - n] \text{Hypergeometric2F1}[2, 1 - n, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{(b e - a f)^2 (c + d x) \text{Gamma}[4 + m] \text{Gamma}[-n]} \right) \end{aligned}$$

$$\begin{aligned}
& \left( b (1+m) \left( -\frac{1}{(-b e + a f) (1+m)} f (-2-m-n) (a+b x)^{1+m} (c+d x)^n \left( \frac{-b c - b d x}{-b c + a d} \right)^{-n} (e+f x)^{-3-m-n} \left( \frac{-b e - b f x}{-b e + a f} \right)^{3+m+n} \right. \right. \\
& \left. \left. \left( 1 - \frac{d (a+b x)}{-b c + a d} \right)^n \left( 1 - \frac{f (a+b x)}{-b e + a f} \right)^{-3-m-n} \Gamma[2+m] \right) \frac{2 \text{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e-c f) (a+b x)}{(b e-a f) (c+d x)}]}{\Gamma[3+m]} + \right. \\
& \left. m \text{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e-c f) (a+b x)}{(b e-a f) (c+d x)}] + \frac{f (a+b x) \text{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e-c f) (a+b x)}{(b e-a f) (c+d x)}]}{(b e - a f) \Gamma[3+m]} + \right. \\
& \left. (d e - c f) (a+b x) \Gamma[1-n] \text{Hypergeometric2F1}[2, 1-n, 4+m, \frac{(d e-c f) (a+b x)}{(b e-a f) (c+d x)}] - \right. \\
& \left. (b e - a f) (c+d x) \Gamma[4+m] \Gamma[-n] \right. \\
& \left. f (-d e + c f) (a+b x)^2 \Gamma[1-n] \text{Hypergeometric2F1}[2, 1-n, 4+m, \frac{(d e-c f) (a+b x)}{(b e-a f) (c+d x)}] \right) - \\
& \left. \frac{1}{(-b c + a d) (1+m)} d n (a+b x)^{1+m} (c+d x)^n \left( \frac{-b c - b d x}{-b c + a d} \right)^{-n} (e+f x)^{-3-m-n} \left( \frac{-b e - b f x}{-b e + a f} \right)^{3+m+n} \left( 1 - \frac{d (a+b x)}{-b c + a d} \right)^{-1+n} \right. \\
& \left. \left( 1 - \frac{f (a+b x)}{-b e + a f} \right)^{-2-m-n} \Gamma[2+m] \right) \frac{2 \text{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e-c f) (a+b x)}{(b e-a f) (c+d x)}]}{\Gamma[3+m]} + \\
& \left. m \text{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e-c f) (a+b x)}{(b e-a f) (c+d x)}] + \frac{f (a+b x) \text{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e-c f) (a+b x)}{(b e-a f) (c+d x)}]}{(b e - a f) \Gamma[3+m]} + \right. \\
& \left. (d e - c f) (a+b x) \Gamma[1-n] \text{Hypergeometric2F1}[2, 1-n, 4+m, \frac{(d e-c f) (a+b x)}{(b e-a f) (c+d x)}] - \right. \\
& \left. (b e - a f) (c+d x) \Gamma[4+m] \Gamma[-n] \right. \\
& \left. f (-d e + c f) (a+b x)^2 \Gamma[1-n] \text{Hypergeometric2F1}[2, 1-n, 4+m, \frac{(d e-c f) (a+b x)}{(b e-a f) (c+d x)}] \right) - \\
& \left. \frac{1}{(-b e + a f) (1+m)} f (3+m+n) (a+b x)^{1+m} (c+d x)^n \left( \frac{-b c - b d x}{-b c + a d} \right)^{-n} (e+f x)^{-3-m-n} \left( \frac{-b e - b f x}{-b e + a f} \right)^{2+m+n} \left( 1 - \frac{d (a+b x)}{-b c + a d} \right)^n \right. \\
& \left. \left( 1 - \frac{f (a+b x)}{-b e + a f} \right)^{-2-m-n} \Gamma[2+m] \right) \frac{2 \text{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e-c f) (a+b x)}{(b e-a f) (c+d x)}]}{\Gamma[3+m]} + \\
& \left. m \text{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e-c f) (a+b x)}{(b e-a f) (c+d x)}] + \frac{f (a+b x) \text{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e-c f) (a+b x)}{(b e-a f) (c+d x)}]}{(b e - a f) \Gamma[3+m]} + \right. 
\end{aligned}$$

$$\begin{aligned}
& \frac{(d e - c f) (a + b x) \Gamma[1 - n] {}_2F1[2, 1 - n, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{(b e - a f) (c + d x) \Gamma[4 + m] \Gamma[-n]} - \\
& \left. \frac{f (-d e + c f) (a + b x)^2 \Gamma[1 - n] {}_2F1[2, 1 - n, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{(b e - a f)^2 (c + d x) \Gamma[4 + m] \Gamma[-n]} \right\} + \\
& \frac{1}{b (1 + m)} f (-3 - m - n) (a + b x)^{1+m} (c + d x)^n \left( \frac{-b c - b d x}{-b c + a d} \right)^{-n} (e + f x)^{-4-m-n} \left( \frac{-b e - b f x}{-b e + a f} \right)^{3+m+n} \left( 1 - \frac{d (a + b x)}{-b c + a d} \right)^n \\
& \left( 1 - \frac{f (a + b x)}{-b e + a f} \right)^{-2-m-n} \Gamma[2 + m] \left( \frac{2 {}_2F1[1, -n, 3 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{\Gamma[3 + m]} + \right. \\
& \left. \frac{m {}_2F1[1, -n, 3 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{\Gamma[3 + m]} + \frac{f (a + b x) {}_2F1[1, -n, 3 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{(b e - a f) \Gamma[3 + m]} + \right. \\
& \left. \frac{(d e - c f) (a + b x) \Gamma[1 - n] {}_2F1[2, 1 - n, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{(b e - a f) (c + d x) \Gamma[4 + m] \Gamma[-n]} - \right. \\
& \left. \frac{f (-d e + c f) (a + b x)^2 \Gamma[1 - n] {}_2F1[2, 1 - n, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{(b e - a f)^2 (c + d x) \Gamma[4 + m] \Gamma[-n]} \right\} + \\
& \frac{1}{(-b c + a d) (1 + m)} d n (a + b x)^{1+m} (c + d x)^n \left( \frac{-b c - b d x}{-b c + a d} \right)^{-1-n} (e + f x)^{-3-m-n} \left( \frac{-b e - b f x}{-b e + a f} \right)^{3+m+n} \left( 1 - \frac{d (a + b x)}{-b c + a d} \right)^n \\
& \left( 1 - \frac{f (a + b x)}{-b e + a f} \right)^{-2-m-n} \Gamma[2 + m] \left( \frac{2 {}_2F1[1, -n, 3 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{\Gamma[3 + m]} + \right. \\
& \left. \frac{m {}_2F1[1, -n, 3 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{\Gamma[3 + m]} + \frac{f (a + b x) {}_2F1[1, -n, 3 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{(b e - a f) \Gamma[3 + m]} + \right. \\
& \left. \frac{(d e - c f) (a + b x) \Gamma[1 - n] {}_2F1[2, 1 - n, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{(b e - a f) (c + d x) \Gamma[4 + m] \Gamma[-n]} - \right. \\
& \left. \frac{f (-d e + c f) (a + b x)^2 \Gamma[1 - n] {}_2F1[2, 1 - n, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{(b e - a f)^2 (c + d x) \Gamma[4 + m] \Gamma[-n]} \right\} + \\
& \frac{1}{b (1 + m)} d n (a + b x)^{1+m} (c + d x)^{-1+n} \left( \frac{-b c - b d x}{-b c + a d} \right)^{-n} (e + f x)^{-3-m-n} \left( \frac{-b e - b f x}{-b e + a f} \right)^{3+m+n} \left( 1 - \frac{d (a + b x)}{-b c + a d} \right)^n
\end{aligned}$$

$$\begin{aligned}
& \left(1 - \frac{f(a + b x)}{-b e + a f}\right)^{-2-m-n} \Gamma[2+m] \left( \frac{2 \text{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e - c f)(a+b x)}{(b e - a f)(c+d x)}]}{\Gamma[3+m]} + \right. \\
& \frac{m \text{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e - c f)(a+b x)}{(b e - a f)(c+d x)}]}{\Gamma[3+m]} + \frac{f(a + b x) \text{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e - c f)(a+b x)}{(b e - a f)(c+d x)}]}{(b e - a f) \Gamma[3+m]} + \\
& \frac{(d e - c f)(a + b x) \Gamma[1 - n] \text{Hypergeometric2F1}[2, 1 - n, 4 + m, \frac{(d e - c f)(a+b x)}{(b e - a f)(c+d x)}]}{(b e - a f)(c + d x) \Gamma[4 + m] \Gamma[-n]} - \\
& \left. \frac{f(-d e + c f)(a + b x)^2 \Gamma[1 - n] \text{Hypergeometric2F1}[2, 1 - n, 4 + m, \frac{(d e - c f)(a+b x)}{(b e - a f)(c+d x)}]}{(b e - a f)^2 (c + d x) \Gamma[4 + m] \Gamma[-n]} \right) + \\
& (a + b x)^m (c + d x)^n \left( \frac{-b c - b d x}{-b c + a d} \right)^{-n} (e + f x)^{-3-m-n} \left( \frac{-b e - b f x}{-b e + a f} \right)^{3+m+n} \left( 1 - \frac{d(a + b x)}{-b c + a d} \right)^n \left( 1 - \frac{f(a + b x)}{-b e + a f} \right)^{-2-m-n} \\
& \Gamma[2+m] \left( \frac{2 \text{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e - c f)(a+b x)}{(b e - a f)(c+d x)}]}{\Gamma[3+m]} + \right. \\
& \frac{m \text{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e - c f)(a+b x)}{(b e - a f)(c+d x)}]}{\Gamma[3+m]} + \frac{f(a + b x) \text{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e - c f)(a+b x)}{(b e - a f)(c+d x)}]}{(b e - a f) \Gamma[3+m]} + \\
& \frac{(d e - c f)(a + b x) \Gamma[1 - n] \text{Hypergeometric2F1}[2, 1 - n, 4 + m, \frac{(d e - c f)(a+b x)}{(b e - a f)(c+d x)}]}{(b e - a f)(c + d x) \Gamma[4 + m] \Gamma[-n]} - \\
& \left. \frac{f(-d e + c f)(a + b x)^2 \Gamma[1 - n] \text{Hypergeometric2F1}[2, 1 - n, 4 + m, \frac{(d e - c f)(a+b x)}{(b e - a f)(c+d x)}]}{(b e - a f)^2 (c + d x) \Gamma[4 + m] \Gamma[-n]} \right) + \\
& \frac{1}{b(1+m)} (a + b x)^{1+m} (c + d x)^n \left( \frac{-b c - b d x}{-b c + a d} \right)^{-n} (e + f x)^{-3-m-n} \left( \frac{-b e - b f x}{-b e + a f} \right)^{3+m+n} \left( 1 - \frac{d(a + b x)}{-b c + a d} \right)^n \left( 1 - \frac{f(a + b x)}{-b e + a f} \right)^{-2-m-n} \\
& \Gamma[2+m] \left( \frac{b f \text{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e - c f)(a+b x)}{(b e - a f)(c+d x)}]}{(b e - a f) \Gamma[3+m]} - \right. \\
& \frac{2 n \left( -\frac{d(d e - c f)(a+b x)}{(b e - a f)(c+d x)^2} + \frac{b(d e - c f)}{(b e - a f)(c+d x)} \right) \text{Hypergeometric2F1}[2, 1 - n, 4 + m, \frac{(d e - c f)(a+b x)}{(b e - a f)(c+d x)}]}{(3 + m) \Gamma[3 + m]} - \\
& \left. \frac{m n \left( -\frac{d(d e - c f)(a+b x)}{(b e - a f)(c+d x)^2} + \frac{b(d e - c f)}{(b e - a f)(c+d x)} \right) \text{Hypergeometric2F1}[2, 1 - n, 4 + m, \frac{(d e - c f)(a+b x)}{(b e - a f)(c+d x)}]}{(3 + m) \Gamma[3 + m]} \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{f n (a + b x) \left( -\frac{d (d e - c f) (a + b x)}{(b e - a f) (c + d x)^2} + \frac{b (d e - c f)}{(b e - a f) (c + d x)} \right) \text{Hypergeometric2F1}[2, 1 - n, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}] - \\
& \quad (b e - a f) (3 + m) \text{Gamma}[3 + m] }{(b e - a f) (c + d x)} - \\
& \frac{d (d e - c f) (a + b x) \text{Gamma}[1 - n] \text{Hypergeometric2F1}[2, 1 - n, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{(b e - a f) (c + d x)^2 \text{Gamma}[4 + m] \text{Gamma}[-n]} + \\
& \frac{d f (-d e + c f) (a + b x)^2 \text{Gamma}[1 - n] \text{Hypergeometric2F1}[2, 1 - n, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{(b e - a f)^2 (c + d x)^2 \text{Gamma}[4 + m] \text{Gamma}[-n]} + \\
& \frac{b (d e - c f) \text{Gamma}[1 - n] \text{Hypergeometric2F1}[2, 1 - n, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{(b e - a f) (c + d x) \text{Gamma}[4 + m] \text{Gamma}[-n]} - \\
& \frac{2 b f (-d e + c f) (a + b x) \text{Gamma}[1 - n] \text{Hypergeometric2F1}[2, 1 - n, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{(b e - a f)^2 (c + d x) \text{Gamma}[4 + m] \text{Gamma}[-n]} + \left( 2 (d e - c f) (1 - n) (a + b x) \right. \\
& \quad \left. \left( -\frac{d (d e - c f) (a + b x)}{(b e - a f) (c + d x)^2} + \frac{b (d e - c f)}{(b e - a f) (c + d x)} \right) \text{Gamma}[1 - n] \text{Hypergeometric2F1}[3, 2 - n, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}] \right) / \\
& \quad \left( (b e - a f) (4 + m) (c + d x) \text{Gamma}[4 + m] \text{Gamma}[-n] \right) - \left( 2 f (-d e + c f) (1 - n) (a + b x)^2 \left( -\frac{d (d e - c f) (a + b x)}{(b e - a f) (c + d x)^2} + \frac{b (d e - c f)}{(b e - a f) (c + d x)} \right) \right. \\
& \quad \left. \left. \text{Gamma}[1 - n] \text{Hypergeometric2F1}[3, 2 - n, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}] \right) / \left( (b e - a f)^2 (4 + m) (c + d x) \text{Gamma}[4 + m] \text{Gamma}[-n] \right) \right) \right)
\end{aligned}$$

**Problem 3158: Attempted integration timed out after 120 seconds.**

$$\int (a + b x)^m (c + d x)^n (e + f x)^{-4-m-n} dx$$

Optimal (type 5, 402 leaves, 4 steps):

$$\begin{aligned}
& -\frac{f (a + b x)^{1+m} (c + d x)^{1+n} (e + f x)^{-3-m-n}}{(b e - a f) (d e - c f) (3 + m + n)} + \frac{f (a d f (2 + m) + b (c f (2 + n) - d e (4 + m + n))) (a + b x)^{1+m} (c + d x)^{1+n} (e + f x)^{-2-m-n}}{(b e - a f)^2 (d e - c f)^2 (2 + m + n) (3 + m + n)} + \\
& \quad \left( (a^2 d^2 f^2 (2 + 3 m + m^2) + 2 a b d f (1 + m) (c f (1 + n) - d e (3 + m + n))) - \right. \\
& \quad \left. b^2 (2 c d e f (1 + n) (3 + m + n) - c^2 f^2 (2 + 3 n + n^2) - d^2 e^2 (6 + m^2 + 5 n + n^2 + m (5 + 2 n))) \right) (a + b x)^{1+m} (c + d x)^n \left( \frac{(b e - a f) (c + d x)}{(b c - a d) (e + f x)} \right)^{-n} \\
& \quad (e + f x)^{-1-m-n} \text{Hypergeometric2F1}[1 + m, -n, 2 + m, -\frac{(d e - c f) (a + b x)}{(b c - a d) (e + f x)}] \Big) / \left( (b e - a f)^3 (d e - c f)^2 (1 + m) (2 + m + n) (3 + m + n) \right)
\end{aligned}$$

Result (type 1, 1 leaves):

???

Problem 3159: Result more than twice size of optimal antiderivative.

$$\int (a + bx)^m (c + dx)^n (e + fx)^p dx$$

Optimal (type 6, 123 leaves, 3 steps):

$$\frac{1}{b(1+m)} (a + bx)^{1+m} (c + dx)^n \left( \frac{b(c + dx)}{bc - ad} \right)^{-n} (e + fx)^p \left( \frac{b(e + fx)}{be - af} \right)^{-p} \text{AppellF1}[1 + m, -n, -p, 2 + m, -\frac{d(a + bx)}{bc - ad}, -\frac{f(a + bx)}{be - af}]$$

Result (type 6, 296 leaves):

$$\begin{aligned} & \left( (bc - ad) (be - af) (2 + m) (a + bx)^{1+m} (c + dx)^n (e + fx)^p \text{AppellF1}[1 + m, -n, -p, 2 + m, \frac{d(a + bx)}{-bc + ad}, \frac{f(a + bx)}{-be + af}] \right) / \\ & \left( b(1+m) \left( (bc - ad) (be - af) (2 + m) \text{AppellF1}[1 + m, -n, -p, 2 + m, \frac{d(a + bx)}{-bc + ad}, \frac{f(a + bx)}{-be + af}] \right) - (a + bx) \left( d(-be + af) n \right. \right. \\ & \left. \left. \text{AppellF1}[2 + m, 1 - n, -p, 3 + m, \frac{d(a + bx)}{-bc + ad}, \frac{f(a + bx)}{-be + af}] \right) + (-bc + ad) fp \text{AppellF1}[2 + m, -n, 1 - p, 3 + m, \frac{d(a + bx)}{-bc + ad}, \frac{f(a + bx)}{-be + af}] \right) \end{aligned}$$

Problem 3160: Result unnecessarily involves higher level functions.

$$\int (a + bx)^m (c + dx)^n (e + fx)^2 dx$$

Optimal (type 5, 259 leaves, 4 steps):

$$\begin{aligned} & \frac{f(bde(4 + m + n) - f(bc(2 + m) + ad(2 + n))) (a + bx)^{1+m} (c + dx)^{1+n}}{b^2 d^2 (2 + m + n) (3 + m + n)} + \\ & \frac{f(a + bx)^{1+m} (c + dx)^{1+n} (e + fx)}{bd(3 + m + n)} + \left( (f(bc(1 + m) + ad(1 + n)) (bde(4 + m + n) - f(bc(2 + m) + ad(2 + n))) + \right. \\ & \left. bd(2 + m + n) (af(cf + de(1 + n)) + be(cf(1 + m) - de(3 + m + n))) (a + bx)^{1+m} \right. \\ & \left. (c + dx)^{1+n} \text{Hypergeometric2F1}[1, 2 + m + n, 2 + n, \frac{b(c + dx)}{bc - ad}] \right) / (b^2 d^2 (bc - ad) (1 + n) (2 + m + n) (3 + m + n)) \end{aligned}$$

Result (type 6, 330 leaves):

$$\frac{1}{3} (a + b x)^m (c + d x)^n \left( \left( 9 a c e f x^2 \text{AppellF1}[2, -m, -n, 3, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \right.$$

$$\left( 3 a c \text{AppellF1}[2, -m, -n, 3, -\frac{b x}{a}, -\frac{d x}{c}] + b c m x \text{AppellF1}[3, 1-m, -n, 4, -\frac{b x}{a}, -\frac{d x}{c}] + a d n x \text{AppellF1}[3, -m, 1-n, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) +$$

$$\left( 4 a c f^2 x^3 \text{AppellF1}[3, -m, -n, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) /$$

$$\left( 4 a c \text{AppellF1}[3, -m, -n, 4, -\frac{b x}{a}, -\frac{d x}{c}] + b c m x \text{AppellF1}[4, 1-m, -n, 5, -\frac{b x}{a}, -\frac{d x}{c}] + a d n x \text{AppellF1}[4, -m, 1-n, 5, -\frac{b x}{a}, -\frac{d x}{c}] \right) +$$

$$\left. \frac{3 e^2 \left( \frac{d(a+b x)}{-b c + a d} \right)^{-m} (c + d x) \text{Hypergeometric2F1}[-m, 1+n, 2+n, \frac{b(c+d x)}{b c - a d}]}{d (1+n)} \right)$$

**Problem 3161:** Result unnecessarily involves higher level functions.

$$\int (a + b x)^m (c + d x)^n (e + f x) dx$$

Optimal (type 5, 131 leaves, 3 steps):

$$\frac{f (a + b x)^{1+m} (c + d x)^{1+n}}{b d (2 + m + n)} -$$

$$\left( (b d e (2 + m + n) - f (b c (1 + m) + a d (1 + n))) (a + b x)^{1+m} (c + d x)^{1+n} \text{Hypergeometric2F1}[1, 2 + m + n, 2 + n, \frac{b (c + d x)}{b c - a d}]\right) /$$

$$(b d (b c - a d) (1 + n) (2 + m + n))$$

Result (type 6, 202 leaves):

$$(a + b x)^m (c + d x)^n \left( \left( 3 a c f x^2 \text{AppellF1}[2, -m, -n, 3, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \right.$$

$$\left( 6 a c \text{AppellF1}[2, -m, -n, 3, -\frac{b x}{a}, -\frac{d x}{c}] + 2 b c m x \text{AppellF1}[3, 1-m, -n, 4, -\frac{b x}{a}, -\frac{d x}{c}] + \right.$$

$$2 a d n x \text{AppellF1}[3, -m, 1-n, 4, -\frac{b x}{a}, -\frac{d x}{c}] \left. \right) +$$

$$\left. \frac{e \left( \frac{d(a+b x)}{-b c + a d} \right)^{-m} (c + d x) \text{Hypergeometric2F1}[-m, 1+n, 2+n, \frac{b(c+d x)}{b c - a d}]}{d (1+n)} \right)$$

**Problem 3163:** Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^m (c + d x)^n}{e + f x} dx$$

Optimal (type 6, 100 leaves, 2 steps):

$$\frac{(a+b x)^{1+m} (c+d x)^n \left(\frac{b (c+d x)}{b c-a d}\right)^{-n} \text{AppellF1}\left[1+m, -n, 1, 2+m, -\frac{d (a+b x)}{b c-a d}, -\frac{f (a+b x)}{b e-a f}\right]}{(b e-a f) (1+m)}$$

Result (type 6, 298 leaves):

$$-\left(\left(\left(b c-a d\right) (b e-a f)^2 (2+m) (a+b x)^{1+m} (c+d x)^n \text{AppellF1}\left[1+m, -n, 1, 2+m, \frac{d (a+b x)}{-b c+a d}, \frac{f (a+b x)}{-b e+a f}\right]\right)\right)/ \\ \left(b (-b e+a f) (1+m) (e+f x) \left(\left(b c-a d\right) (b e-a f) (2+m) \text{AppellF1}\left[1+m, -n, 1, 2+m, \frac{d (a+b x)}{-b c+a d}, \frac{f (a+b x)}{-b e+a f}\right] - (a+b x) \left(d (-b e+a f) n \text{AppellF1}\left[2+m, 1-n, 1, 3+m, \frac{d (a+b x)}{-b c+a d}, \frac{f (a+b x)}{-b e+a f}\right] + (b c-a d) f \text{AppellF1}\left[2+m, -n, 2, 3+m, \frac{d (a+b x)}{-b c+a d}, \frac{f (a+b x)}{-b e+a f}\right]\right)\right)\right)$$

Problem 3164: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+b x)^m (c+d x)^n}{(e+f x)^2} dx$$

Optimal (type 6, 101 leaves, 2 steps):

$$\frac{b (a+b x)^{1+m} (c+d x)^n \left(\frac{b (c+d x)}{b c-a d}\right)^{-n} \text{AppellF1}\left[1+m, -n, 2, 2+m, -\frac{d (a+b x)}{b c-a d}, -\frac{f (a+b x)}{b e-a f}\right]}{(b e-a f)^2 (1+m)}$$

Result (type 6, 286 leaves):

$$\left(\left(b c-a d\right) (b e-a f) (2+m) (a+b x)^{1+m} (c+d x)^n \text{AppellF1}\left[1+m, -n, 2, 2+m, \frac{d (a+b x)}{-b c+a d}, \frac{f (a+b x)}{-b e+a f}\right]\right)\right)/ \\ \left(b (1+m) (e+f x)^2 \left(\left(b c-a d\right) (b e-a f) (2+m) \text{AppellF1}\left[1+m, -n, 2, 2+m, \frac{d (a+b x)}{-b c+a d}, \frac{f (a+b x)}{-b e+a f}\right] - (a+b x) \left(d (-b e+a f) n \text{AppellF1}\left[2+m, 1-n, 2, 3+m, \frac{d (a+b x)}{-b c+a d}, \frac{f (a+b x)}{-b e+a f}\right] + 2 (b c-a d) f \text{AppellF1}\left[2+m, -n, 3, 3+m, \frac{d (a+b x)}{-b c+a d}, \frac{f (a+b x)}{-b e+a f}\right]\right)\right)\right)$$

Problem 3165: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+b x)^m (c+d x)^n}{(e+f x)^3} dx$$

Optimal (type 6, 103 leaves, 2 steps):

$$\frac{b^2 (a + b x)^{1+m} (c + d x)^n \left(\frac{b (c+d x)}{b c-a d}\right)^{-n} \text{AppellF1}\left[1+m, -n, 3, 2+m, -\frac{d (a+b x)}{b c-a d}, -\frac{f (a+b x)}{b e-a f}\right]}{(b e-a f)^3 (1+m)}$$

Result (type 6, 299 leaves):

$$-\left( \left( (b c - a d) (b e - a f)^4 (2 + m) (a + b x)^{1+m} (c + d x)^n \text{AppellF1}\left[1+m, -n, 3, 2+m, \frac{d (a+b x)}{-b c+a d}, \frac{f (a+b x)}{-b e+a f}\right] \right) / \right. \\ \left. \left( b (-b e + a f)^3 (1+m) (e + f x)^3 \left( (b c - a d) (b e - a f) (2 + m) \text{AppellF1}\left[1+m, -n, 3, 2+m, \frac{d (a+b x)}{-b c+a d}, \frac{f (a+b x)}{-b e+a f}\right] - (a + b x) \left( d (-b e + a f) n \right. \right. \right. \right. \\ \left. \left. \left. \left. \text{AppellF1}\left[2+m, 1-n, 3, 3+m, \frac{d (a+b x)}{-b c+a d}, \frac{f (a+b x)}{-b e+a f}\right] + 3 (b c - a d) f \text{AppellF1}\left[2+m, -n, 4, 3+m, \frac{d (a+b x)}{-b c+a d}, \frac{f (a+b x)}{-b e+a f}\right] \right) \right) \right)$$

Problem 3170: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^{4/3}}{\sqrt{c + d x} (e + f x)} dx$$

Optimal (type 6, 100 leaves, 2 steps):

$$\frac{3 (a + b x)^{7/3} \sqrt{\frac{b (c+d x)}{b c-a d}} \text{AppellF1}\left[\frac{7}{3}, \frac{1}{2}, 1, \frac{10}{3}, -\frac{d (a+b x)}{b c-a d}, -\frac{f (a+b x)}{b e-a f}\right]}{7 (b e - a f) \sqrt{c + d x}}$$

Result (type 6, 921 leaves):

$$\begin{aligned}
& \frac{1}{35 d^2 (a + b x)^{2/3}} \\
& 6 b \sqrt{c + d x} \left( \frac{7 d (a + b x)}{f} + \left( (c + d x) \left( -26 (b c - a d) (3 b d e + 2 b c f - 5 a d f) \text{AppellF1}\left[\frac{7}{6}, \frac{2}{3}, 1, \frac{13}{6}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] \right. \right. \right. \\
& \left. \left. \left. \left( b (-3 d e + 3 c f) \text{AppellF1}\left[\frac{7}{6}, \frac{2}{3}, 2, \frac{13}{6}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] + 2 (b c - a d) f \text{AppellF1}\left[\frac{7}{6}, \frac{5}{3}, 1, \frac{13}{6}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] \right) - \right. \right. \\
& 7 b (c + d x) \text{AppellF1}\left[\frac{1}{6}, \frac{2}{3}, 1, \frac{7}{6}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] \left( 13 f (5 a^2 d^2 f + a b d (-3 d e + 42 c f + 49 d f x) - b^2 (12 c^2 f + \right. \right. \\
& \left. \left. 35 d^2 e x + 2 c d (16 e + 7 f x)) \right) \text{AppellF1}\left[\frac{7}{6}, \frac{2}{3}, 1, \frac{13}{6}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] + 14 (5 b d e + 2 b c f - 7 a d f) \left( 3 b (d e - c f) \right. \right. \\
& \left. \left. \text{AppellF1}\left[\frac{13}{6}, \frac{2}{3}, 2, \frac{19}{6}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] + 2 (-b c + a d) f \text{AppellF1}\left[\frac{13}{6}, \frac{5}{3}, 1, \frac{19}{6}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] \right) \right) \right) / \\
& \left( d (e + f x) \left( 7 b f (c + d x) \text{AppellF1}\left[\frac{1}{6}, \frac{2}{3}, 1, \frac{7}{6}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] + b (-6 d e + 6 c f) \text{AppellF1}\left[\frac{7}{6}, \frac{2}{3}, 2, \frac{13}{6}, \right. \right. \right. \\
& \left. \left. \left. \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] + 4 (b c - a d) f \text{AppellF1}\left[\frac{7}{6}, \frac{5}{3}, 1, \frac{13}{6}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] \right) \right. \right. \\
& \left. \left. \left( 13 b f (c + d x) \text{AppellF1}\left[\frac{7}{6}, \frac{2}{3}, 1, \frac{13}{6}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] + b (-6 d e + 6 c f) \text{AppellF1}\left[\frac{13}{6}, \frac{2}{3}, 2, \frac{19}{6}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] + \right. \right. \\
& \left. \left. 4 (b c - a d) f \text{AppellF1}\left[\frac{13}{6}, \frac{5}{3}, 1, \frac{19}{6}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] \right) \right) \right)
\end{aligned}$$

**Problem 3171: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c + d x)^{2/5} (e + f x)^{3/5}}{\sqrt{a + b x}} dx$$

Optimal (type 6, 123 leaves, 3 steps):

$$\begin{aligned}
& \frac{2 \sqrt{a + b x} (c + d x)^{2/5} (e + f x)^{3/5} \text{AppellF1}\left[\frac{1}{2}, -\frac{2}{5}, -\frac{3}{5}, \frac{3}{2}, -\frac{d (a+b x)}{b c-a d}, -\frac{f (a+b x)}{b e-a f}\right]}{b \left(\frac{b (c+d x)}{b c-a d}\right)^{2/5} \left(\frac{b (e+f x)}{b e-a f}\right)^{3/5}}
\end{aligned}$$

Result (type 6, 661 leaves):

$$\begin{aligned}
& \frac{1}{45 b^3 (c + d x)^{3/5} (e + f x)^{2/5}} 2 \sqrt{a + b x} \left( 15 b^2 (c + d x) (e + f x) - \right. \\
& 2 (a + b x) \left( \left( 9 (25 a^2 d^2 f^2 - 10 a b d f (3 d e + 2 c f) + b^2 (3 d^2 e^2 + 24 c d e f - 2 c^2 f^2)) \text{AppellF1} \left[ \frac{1}{2}, \frac{3}{5}, \frac{2}{5}, \frac{3}{2}, \frac{-b c + a d}{d (a + b x)}, \frac{-b e + a f}{f (a + b x)} \right] \right) / \right. \\
& \left( 15 d f (a + b x) \text{AppellF1} \left[ \frac{1}{2}, \frac{3}{5}, \frac{2}{5}, \frac{3}{2}, \frac{-b c + a d}{d (a + b x)}, \frac{-b e + a f}{f (a + b x)} \right] + (-4 b d e + 4 a d f) \text{AppellF1} \left[ \frac{3}{2}, \frac{3}{5}, \frac{7}{5}, \frac{5}{2}, \frac{-b c + a d}{d (a + b x)}, \frac{-b e + a f}{f (a + b x)} \right] + \right. \\
& \left. 6 (-b c + a d) f \text{AppellF1} \left[ \frac{3}{2}, \frac{8}{5}, \frac{2}{5}, \frac{5}{2}, \frac{-b c + a d}{d (a + b x)}, \frac{-b e + a f}{f (a + b x)} \right] \right) + \frac{1}{(a + b x)^2} \\
& (3 b d e + 2 b c f - 5 a d f) \left( -\frac{3 b^2 (c + d x) (e + f x)}{d f} + \left( 25 (b c - a d) (b e - a f) (a + b x) \text{AppellF1} \left[ \frac{3}{2}, \frac{3}{5}, \frac{2}{5}, \frac{5}{2}, \frac{-b c + a d}{d (a + b x)}, \frac{-b e + a f}{f (a + b x)} \right] \right) / \right. \\
& \left. \left( 25 d f (a + b x) \text{AppellF1} \left[ \frac{3}{2}, \frac{3}{5}, \frac{2}{5}, \frac{5}{2}, \frac{-b c + a d}{d (a + b x)}, \frac{-b e + a f}{f (a + b x)} \right] + (-4 b d e + 4 a d f) \right. \right. \\
& \left. \left. \text{AppellF1} \left[ \frac{5}{2}, \frac{3}{5}, \frac{7}{5}, \frac{7}{2}, \frac{-b c + a d}{d (a + b x)}, \frac{-b e + a f}{f (a + b x)} \right] + 6 (-b c + a d) f \text{AppellF1} \left[ \frac{5}{2}, \frac{8}{5}, \frac{2}{5}, \frac{7}{2}, \frac{-b c + a d}{d (a + b x)}, \frac{-b e + a f}{f (a + b x)} \right] \right) \right) \right)
\end{aligned}$$

Problem 3172: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{a + b x} (e + f x)^n}{\sqrt{c + d x}} dx$$

Optimal (type 6, 123 leaves, 3 steps):

$$\frac{2 (a + b x)^{3/2} \sqrt{\frac{b (c + d x)}{b c - a d}} (e + f x)^n \left( \frac{b (e + f x)}{b e - a f} \right)^{-n} \text{AppellF1} \left[ \frac{3}{2}, \frac{1}{2}, -n, \frac{5}{2}, -\frac{d (a + b x)}{b c - a d}, -\frac{f (a + b x)}{b e - a f} \right]}{3 b \sqrt{c + d x}}$$

Result (type 6, 289 leaves):

$$\begin{aligned}
& \left( 10 (b c - a d) (b e - a f) (a + b x)^{3/2} (e + f x)^n \text{AppellF1} \left[ \frac{3}{2}, \frac{1}{2}, -n, \frac{5}{2}, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f} \right] \right) / \\
& \left( 3 b \sqrt{c + d x} \left( 5 (b c - a d) (b e - a f) \text{AppellF1} \left[ \frac{3}{2}, \frac{1}{2}, -n, \frac{5}{2}, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f} \right] - (a + b x) \right. \right. \\
& \left. \left. \left( 2 (-b c + a d) f n \text{AppellF1} \left[ \frac{5}{2}, \frac{1}{2}, 1 - n, \frac{7}{2}, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f} \right] + d (b e - a f) \text{AppellF1} \left[ \frac{5}{2}, \frac{3}{2}, -n, \frac{7}{2}, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f} \right] \right) \right) \right)
\end{aligned}$$

### Problem 3173: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{c+dx} (e+fx)^n}{\sqrt{a+bx}} dx$$

Optimal (type 6, 121 leaves, 3 steps):

$$\frac{2\sqrt{a+bx}\sqrt{c+dx}(e+fx)^n \left(\frac{b(e+fx)}{be-af}\right)^{-n} \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, -n, \frac{3}{2}, -\frac{d(a+bx)}{bc-ad}, -\frac{f(a+bx)}{be-af}\right]}{b\sqrt{\frac{b(c+dx)}{bc-ad}}}$$

Result (type 6, 287 leaves):

$$\begin{aligned} & \left( 6(bc-ad)(be-af)\sqrt{a+bx}\sqrt{c+dx}(e+fx)^n \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, -n, \frac{3}{2}, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right] \right) / \\ & \left( b \left( 3(bc-ad)(be-af) \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, -n, \frac{3}{2}, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right] - (a+bx) \right. \right. \\ & \left. \left. \left( 2(-bc+ad)f^n \text{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, 1-n, \frac{5}{2}, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right] + d(-be+af) \text{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, -n, \frac{5}{2}, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right] \right) \right) \right) \end{aligned}$$

### Problem 3174: Result more than twice size of optimal antiderivative.

$$\int \frac{(e+fx)^n}{\sqrt{a+bx}(c+dx)^{3/2}} dx$$

Optimal (type 6, 128 leaves, 3 steps):

$$\frac{2\sqrt{a+bx}\sqrt{\frac{b(c+dx)}{bc-ad}}(e+fx)^n\left(\frac{b(e+fx)}{be-af}\right)^{-n}\text{AppellF1}\left[\frac{1}{2}, \frac{3}{2}, -n, \frac{3}{2}, -\frac{d(a+bx)}{bc-ad}, -\frac{f(a+bx)}{be-af}\right]}{(bc-ad)\sqrt{c+dx}}$$

Result (type 6, 816 leaves):

$$\begin{aligned}
& \frac{1}{3(c+dx)^{3/2}} 2(b e - a f) \sqrt{a+b x} (e+f x)^n \left( \left( 9 b (c+d x)^2 \text{AppellF1} \left[ \frac{1}{2}, -\frac{1}{2}, -n, \frac{3}{2}, \frac{d(a+b x)}{-b c + a d}, \frac{f(a+b x)}{-b e + a f} \right] \right) \right. \\
& \left( (b c - a d) \left( 3(b c - a d) (b e - a f) \text{AppellF1} \left[ \frac{1}{2}, -\frac{1}{2}, -n, \frac{3}{2}, \frac{d(a+b x)}{-b c + a d}, \frac{f(a+b x)}{-b e + a f} \right] - (a+b x) \left( 2(-b c + a d) f n \right. \right. \right. \\
& \left. \left. \left. \text{AppellF1} \left[ \frac{3}{2}, -\frac{1}{2}, 1-n, \frac{5}{2}, \frac{d(a+b x)}{-b c + a d}, \frac{f(a+b x)}{-b e + a f} \right] + d(-b e + a f) \text{AppellF1} \left[ \frac{3}{2}, \frac{1}{2}, -n, \frac{5}{2}, \frac{d(a+b x)}{-b c + a d}, \frac{f(a+b x)}{-b e + a f} \right] \right) \right) - \\
& \left( 5 d (a+b x) (c+d x) \text{AppellF1} \left[ \frac{3}{2}, \frac{1}{2}, -n, \frac{5}{2}, \frac{d(a+b x)}{-b c + a d}, \frac{f(a+b x)}{-b e + a f} \right] \right) \Big/ \left( (b c - a d) \right. \\
& \left. \left( 5 (b c - a d) (b e - a f) \text{AppellF1} \left[ \frac{3}{2}, \frac{1}{2}, -n, \frac{5}{2}, \frac{d(a+b x)}{-b c + a d}, \frac{f(a+b x)}{-b e + a f} \right] - (a+b x) \left( 2(-b c + a d) f n \right. \right. \right. \\
& \left. \left. \left. \text{AppellF1} \left[ \frac{5}{2}, \frac{1}{2}, 1-n, \frac{7}{2}, \frac{d(a+b x)}{-b c + a d}, \frac{f(a+b x)}{-b e + a f} \right] + d(b e - a f) \text{AppellF1} \left[ \frac{5}{2}, \frac{3}{2}, -n, \frac{7}{2}, \frac{d(a+b x)}{-b c + a d}, \frac{f(a+b x)}{-b e + a f} \right] \right) \right) - \\
& \left( 5 d (a+b x) \text{AppellF1} \left[ \frac{3}{2}, \frac{3}{2}, -n, \frac{5}{2}, \frac{d(a+b x)}{-b c + a d}, \frac{f(a+b x)}{-b e + a f} \right] \right) \Big/ \left( b \left( 5 (b c - a d) (b e - a f) \text{AppellF1} \left[ \frac{3}{2}, \frac{3}{2}, -n, \frac{5}{2}, \right. \right. \right. \\
& \left. \left. \left. \frac{d(a+b x)}{-b c + a d}, \frac{f(a+b x)}{-b e + a f} \right] - (a+b x) \left( 2(-b c + a d) f n \text{AppellF1} \left[ \frac{5}{2}, \frac{3}{2}, 1-n, \frac{7}{2}, \frac{d(a+b x)}{-b c + a d}, \frac{f(a+b x)}{-b e + a f} \right] + \right. \right. \right. \\
& \left. \left. \left. 3 d (b e - a f) \text{AppellF1} \left[ \frac{5}{2}, \frac{5}{2}, -n, \frac{7}{2}, \frac{d(a+b x)}{-b c + a d}, \frac{f(a+b x)}{-b e + a f} \right] \right) \right) \right)
\end{aligned}$$

**Problem 3175: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e+f x)^n}{(a+b x)^{3/2} \sqrt{c+d x}} dx$$

Optimal (type 6, 121 leaves, 3 steps):

$$-\frac{2 \sqrt{\frac{b(c+d x)}{b c - a d}} (e+f x)^n \left(\frac{b(e+f x)}{b e - a f}\right)^{-n} \text{AppellF1} \left[-\frac{1}{2}, \frac{1}{2}, -n, \frac{1}{2}, -\frac{d(a+b x)}{b c - a d}, -\frac{f(a+b x)}{b e - a f}\right]}{b \sqrt{a+b x} \sqrt{c+d x}}$$

Result (type 6, 825 leaves):

$$\begin{aligned}
& \frac{1}{3(bc - ad) \sqrt{a+b}x \sqrt{c+d}x} 2(b e - af) (e + fx)^n \left( \left( 3(b c - a d)^2 (c + d x) \text{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -n, \frac{1}{2}, \frac{d(a+b)x}{-bc+ad}, \frac{f(a+b)x}{-be+af}\right] \right) \right. \\
& \left( (-bc+ad) \left( (bc-ad)(be-af) \text{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -n, \frac{1}{2}, \frac{d(a+b)x}{-bc+ad}, \frac{f(a+b)x}{-be+af}\right] - (a+b)x \left( 2(-bc+ad)f n \right. \right. \right. \\
& \left. \left. \left. \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, 1-n, \frac{3}{2}, \frac{d(a+b)x}{-bc+ad}, \frac{f(a+b)x}{-be+af}\right] + d(-be+af) \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -n, \frac{3}{2}, \frac{d(a+b)x}{-bc+ad}, \frac{f(a+b)x}{-be+af}\right] \right) \right) - \\
& \left( 9d(a+b)x(c+d x) \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, -n, \frac{3}{2}, \frac{d(a+b)x}{-bc+ad}, \frac{f(a+b)x}{-be+af}\right] \right) \Big/ \left( 3(bc-ad)(be-af) \right. \\
& \left. \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, -n, \frac{3}{2}, \frac{d(a+b)x}{-bc+ad}, \frac{f(a+b)x}{-be+af}\right] - (a+b)x \left( 2(-bc+ad)f n \text{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, 1-n, \frac{5}{2}, \frac{d(a+b)x}{-bc+ad}, \frac{f(a+b)x}{-be+af}\right] + \right. \right. \\
& \left. \left. d(-be+af) \text{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, -n, \frac{5}{2}, \frac{d(a+b)x}{-bc+ad}, \frac{f(a+b)x}{-be+af}\right] \right) \right) + \left( 5d^2(a+b)^2 \text{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, -n, \frac{5}{2}, \frac{d(a+b)x}{-bc+ad}, \frac{f(a+b)x}{-be+af}\right] \right) \Big/ \\
& \left( b \left( 5(bc-ad)(be-af) \text{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, -n, \frac{5}{2}, \frac{d(a+b)x}{-bc+ad}, \frac{f(a+b)x}{-be+af}\right] - (a+b)x \left( 2(-bc+ad)f n \right. \right. \right. \\
& \left. \left. \left. \text{AppellF1}\left[\frac{5}{2}, \frac{1}{2}, 1-n, \frac{7}{2}, \frac{d(a+b)x}{-bc+ad}, \frac{f(a+b)x}{-be+af}\right] + d(be-af) \text{AppellF1}\left[\frac{5}{2}, \frac{3}{2}, -n, \frac{7}{2}, \frac{d(a+b)x}{-bc+ad}, \frac{f(a+b)x}{-be+af}\right] \right) \right) \right)
\end{aligned}$$

**Problem 3176: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+b}x (c+d x)^{1/3}}{e + fx} dx$$

Optimal (type 6, 100 leaves, 2 steps):

$$\frac{2(a+b)^{3/2} (c+d x)^{1/3} \text{AppellF1}\left[\frac{3}{2}, -\frac{1}{3}, 1, \frac{5}{2}, -\frac{d(a+b)x}{bc-ad}, -\frac{f(a+b)x}{be-af}\right]}{3(b e - a f) \left(\frac{b(c+d x)}{b c - a d}\right)^{1/3}}$$

Result (type 6, 901 leaves):

$$\begin{aligned}
& \frac{1}{35 (c + dx)^{2/3}} \\
& 6 \sqrt{a + bx} \left( \frac{7 (c + dx)}{f} - \left( d (a + bx) \left( 78 (bc - ad) (be - af) \text{AppellF1}\left[\frac{7}{6}, \frac{2}{3}, 1, \frac{13}{6}, \frac{-bc+ad}{d(a+bx)}, \frac{-be+af}{f(a+bx)}\right] \right. \right. \right. \\
& \left. \left. \left. 3d (be - af) \text{AppellF1}\left[\frac{7}{6}, \frac{2}{3}, \right. \right. \right. \\
& \left. \left. \left. 2, \frac{13}{6}, \frac{-bc+ad}{d(a+bx)}, \frac{-be+af}{f(a+bx)} \right] + 2 (bc - ad) f \text{AppellF1}\left[\frac{7}{6}, \frac{5}{3}, 1, \frac{13}{6}, \frac{-bc+ad}{d(a+bx)}, \frac{-be+af}{f(a+bx)} \right] \right) - 7 (a + bx) \right. \\
& \left. \text{AppellF1}\left[\frac{1}{6}, \frac{2}{3}, 1, \frac{7}{6}, \frac{-bc+ad}{d(a+bx)}, \frac{-be+af}{f(a+bx)} \right] \left( 13df (3b^2c e - 3adf (6a + 7bx) + b (a (32de - 17cf) + 7b (5de - 2cf) x)) \right. \right. \\
& \left. \left. \text{AppellF1}\left[\frac{7}{6}, \frac{2}{3}, 1, \frac{13}{6}, \frac{-bc+ad}{d(a+bx)}, \frac{-be+af}{f(a+bx)} \right] - 14 (5bde - 2bcf - 3adf) \left( 3d (be - af) \right. \right. \\
& \left. \left. \text{AppellF1}\left[\frac{13}{6}, \frac{2}{3}, 2, \frac{19}{6}, \frac{-bc+ad}{d(a+bx)}, \frac{-be+af}{f(a+bx)} \right] + 2 (bc - ad) f \text{AppellF1}\left[\frac{13}{6}, \frac{5}{3}, 1, \frac{19}{6}, \frac{-bc+ad}{d(a+bx)}, \frac{-be+af}{f(a+bx)} \right] \right) \right) \right) / \\
& \left( b^2 (e + fx) \left( 7df (a + bx) \text{AppellF1}\left[\frac{1}{6}, \frac{2}{3}, 1, \frac{7}{6}, \frac{-bc+ad}{d(a+bx)}, \frac{-be+af}{f(a+bx)} \right] + (-6bde + 6adf) \text{AppellF1}\left[\frac{7}{6}, \frac{2}{3}, 2, \right. \right. \right. \\
& \left. \left. \left. \frac{13}{6}, \frac{-bc+ad}{d(a+bx)}, \frac{-be+af}{f(a+bx)} \right] + 4 (-bc + ad) f \text{AppellF1}\left[\frac{7}{6}, \frac{5}{3}, 1, \frac{13}{6}, \frac{-bc+ad}{d(a+bx)}, \frac{-be+af}{f(a+bx)} \right] \right) \right. \\
& \left. \left( 13df (a + bx) \text{AppellF1}\left[\frac{7}{6}, \frac{2}{3}, 1, \frac{13}{6}, \frac{-bc+ad}{d(a+bx)}, \frac{-be+af}{f(a+bx)} \right] + (-6bde + 6adf) \text{AppellF1}\left[\frac{13}{6}, \frac{2}{3}, 2, \frac{19}{6}, \frac{-bc+ad}{d(a+bx)}, \frac{-be+af}{f(a+bx)} \right] + \right. \right. \\
& \left. \left. 4 (-bc + ad) f \text{AppellF1}\left[\frac{13}{6}, \frac{5}{3}, 1, \frac{19}{6}, \frac{-bc+ad}{d(a+bx)}, \frac{-be+af}{f(a+bx)} \right] \right) \right)
\end{aligned}$$

**Problem 3177: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + bx)^{1/3} \sqrt{c + dx}}{e + fx} dx$$

Optimal (type 6, 100 leaves, 2 steps):

$$\begin{aligned}
& \frac{3 (a + bx)^{4/3} \sqrt{c + dx} \text{AppellF1}\left[\frac{4}{3}, -\frac{1}{2}, 1, \frac{7}{3}, -\frac{d(a+bx)}{bc-ad}, -\frac{f(a+bx)}{be-af}\right]}{4 (be - af) \sqrt{\frac{b(c+dx)}{bc-ad}}}
\end{aligned}$$

Result (type 6, 895 leaves):

$$\begin{aligned}
& \frac{1}{35 (a + b x)^{2/3}} \\
& 6 \sqrt{c + d x} \left( \frac{7 (a + b x)}{f} + \left( b (c + d x) \left( -78 (b c - a d) (d e - c f) \text{AppellF1}\left[\frac{7}{6}, \frac{2}{3}, 1, \frac{13}{6}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] \right. \right. \right. \\
& \left. \left. \left. \left( b (-3 d e + 3 c f) \text{AppellF1}\left[\frac{7}{6}, \frac{2}{3}, 2, \frac{13}{6}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] + 2 (b c - a d) f \text{AppellF1}\left[\frac{7}{6}, \frac{5}{3}, 1, \frac{13}{6}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] \right) - 7 (c + d x) \right. \right. \\
& \left. \left. \left. \text{AppellF1}\left[\frac{1}{6}, \frac{2}{3}, 1, \frac{7}{6}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] \left( 13 b f (a d (-3 d e + 17 c f + 14 d f x) + b (-32 c d e + 18 c^2 f - 35 d^2 e x + 21 c d f x)) \right) \right. \right. \\
& \left. \left. \left. \text{AppellF1}\left[\frac{7}{6}, \frac{2}{3}, 1, \frac{13}{6}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] + 14 (5 b d e - 3 b c f - 2 a d f) \left( 3 b (d e - c f) \right. \right. \right. \\
& \left. \left. \left. \text{AppellF1}\left[\frac{13}{6}, \frac{2}{3}, 2, \frac{19}{6}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] + 2 (-b c + a d) f \text{AppellF1}\left[\frac{13}{6}, \frac{5}{3}, 1, \frac{19}{6}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] \right) \right) \right) \right) / \\
& \left( d^2 (e + f x) \left( 7 b f (c + d x) \text{AppellF1}\left[\frac{1}{6}, \frac{2}{3}, 1, \frac{7}{6}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] + b (-6 d e + 6 c f) \text{AppellF1}\left[\frac{7}{6}, \frac{2}{3}, 2, \frac{13}{6}, \right. \right. \right. \\
& \left. \left. \left. \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] + 4 (b c - a d) f \text{AppellF1}\left[\frac{7}{6}, \frac{5}{3}, 1, \frac{13}{6}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] \right) \right. \\
& \left( 13 b f (c + d x) \text{AppellF1}\left[\frac{7}{6}, \frac{2}{3}, 1, \frac{13}{6}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] + b (-6 d e + 6 c f) \text{AppellF1}\left[\frac{13}{6}, \frac{2}{3}, 2, \frac{19}{6}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] + \right. \\
& \left. \left. \left. 4 (b c - a d) f \text{AppellF1}\left[\frac{13}{6}, \frac{5}{3}, 1, \frac{19}{6}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] \right) \right)
\end{aligned}$$

**Problem 3178: Result more than twice size of optimal antiderivative.**

$$\int \sqrt{a + b x} (c + d x)^{1/3} (e + f x)^{1/4} dx$$

Optimal (type 6, 125 leaves, 3 steps):

$$\begin{aligned}
& \frac{2 (a + b x)^{3/2} (c + d x)^{1/3} (e + f x)^{1/4} \text{AppellF1}\left[\frac{3}{2}, -\frac{1}{3}, -\frac{1}{4}, \frac{5}{2}, -\frac{d (a+b x)}{b c-a d}, -\frac{f (a+b x)}{b e-a f}\right]}{3 b \left(\frac{b (c+d x)}{b c-a d}\right)^{1/3} \left(\frac{b (e+f x)}{b e-a f}\right)^{1/4}}
\end{aligned}$$

Result (type 6, 1077 leaves):

$$\begin{aligned}
& \left( \frac{12 (3 b d e + 4 b c f + 6 a d f)}{325 b d f} + \frac{12 x}{25} \right) \sqrt{a+b x} (c+d x)^{1/3} (e+f x)^{1/4} - \frac{1}{82225 b^3 d f \left( c + \frac{(a+b x) (d - \frac{a d}{a+b x})}{b} \right)^{2/3} \left( e + \frac{(a+b x) (f - \frac{a f}{a+b x})}{b} \right)^{3/4}} \\
& 72 (a+b x)^{3/2} \left( 1058 (-21 a^3 d^3 f^3 + 9 a^2 b d^2 f^2 (3 d e + 4 c f) - a b^2 d f (20 d^2 e^2 + 14 c d e f + 29 c^2 f^2) + b^3 (5 d^3 e^3 + 5 c d^2 e^2 f + 2 c^2 d e f^2 + 9 c^3 f^3)) \right. \\
& \left. \text{AppellF1} \left[ \frac{11}{12}, \frac{2}{3}, \frac{3}{4}, \frac{23}{12}, \frac{-b c + a d}{d (a+b x)}, \frac{-b e + a f}{f (a+b x)} \right] \right) / \\
& \left( (a+b x) \left( -23 d f \text{AppellF1} \left[ \frac{11}{12}, \frac{2}{3}, \frac{3}{4}, \frac{23}{12}, \frac{-b c + a d}{d (a+b x)}, \frac{-b e + a f}{f (a+b x)} \right] + \frac{1}{a+b x} (9 d (b e - a f) \right. \right. \\
& \left. \left. \text{AppellF1} \left[ \frac{23}{12}, \frac{2}{3}, \frac{7}{4}, \frac{35}{12}, \frac{-b c + a d}{d (a+b x)}, \frac{-b e + a f}{f (a+b x)} \right] + 8 (b c - a d) f \text{AppellF1} \left[ \frac{23}{12}, \frac{5}{3}, \frac{3}{4}, \frac{35}{12}, \frac{-b c + a d}{d (a+b x)}, \frac{-b e + a f}{f (a+b x)} \right] \right) \right) + \\
& \left( 11 (7 a^2 d^2 f^2 - 2 a b d f (3 d e + 4 c f) + b^2 (5 d^2 e^2 - 4 c d e f + 6 c^2 f^2)) \left( 35 d f \left( \frac{b c \left( \frac{17 b e}{a+b x} + f \left( 23 - \frac{17 a}{a+b x} \right) \right)}{a+b x} + \right. \right. \right. \\
& \left. \left. \left. d \left( f \left( 23 + \frac{17 a^2}{(a+b x)^2} - \frac{46 a}{a+b x} \right) + \frac{b e \left( 23 - \frac{17 a}{a+b x} \right)}{a+b x} \right) \right) \text{AppellF1} \left[ \frac{23}{12}, \frac{2}{3}, \frac{3}{4}, \frac{35}{12}, \frac{-b c + a d}{d (a+b x)}, \frac{-b e + a f}{f (a+b x)} \right] - \frac{1}{a+b x} \right. \\
& \left. 23 \left( d + \frac{b c}{a+b x} - \frac{a d}{a+b x} \right) \left( f + \frac{b e}{a+b x} - \frac{a f}{a+b x} \right) \left( 9 d (b e - a f) \text{AppellF1} \left[ \frac{35}{12}, \frac{2}{3}, \frac{7}{4}, \frac{47}{12}, \frac{-b c + a d}{d (a+b x)}, \frac{-b e + a f}{f (a+b x)} \right] + \right. \right. \\
& \left. \left. 8 (b c - a d) f \text{AppellF1} \left[ \frac{35}{12}, \frac{5}{3}, \frac{3}{4}, \frac{47}{12}, \frac{-b c + a d}{d (a+b x)}, \frac{-b e + a f}{f (a+b x)} \right] \right) \right) \right) / \\
& \left( d f \left( 35 d f \text{AppellF1} \left[ \frac{23}{12}, \frac{2}{3}, \frac{3}{4}, \frac{35}{12}, \frac{-b c + a d}{d (a+b x)}, \frac{-b e + a f}{f (a+b x)} \right] + \frac{1}{a+b x} \left( (-9 b d e + 9 a d f) \text{AppellF1} \left[ \frac{35}{12}, \frac{2}{3}, \frac{7}{4}, \frac{47}{12}, \right. \right. \right. \right. \\
& \left. \left. \left. \left. \frac{-b c + a d}{d (a+b x)}, \frac{-b e + a f}{f (a+b x)} \right] + 8 (-b c + a d) f \text{AppellF1} \left[ \frac{35}{12}, \frac{5}{3}, \frac{3}{4}, \frac{47}{12}, \frac{-b c + a d}{d (a+b x)}, \frac{-b e + a f}{f (a+b x)} \right] \right) \right) \right)
\end{aligned}$$

Problem 3179: Result more than twice size of optimal antiderivative.

$$\int (a+b x)^{1/3} \sqrt{c+d x} (e+f x)^{1/4} dx$$

Optimal (type 6, 125 leaves, 3 steps):

$$\frac{3 \left(a + b x\right)^{4/3} \sqrt{c + d x} \left(e + f x\right)^{1/4} \text{AppellF1}\left[\frac{4}{3}, -\frac{1}{2}, -\frac{1}{4}, \frac{7}{3}, -\frac{d (a+b x)}{b c-a d}, -\frac{f (a+b x)}{b e-a f}\right]}{4 b \sqrt{\frac{b (c+d x)}{b c-a d}} \left(\frac{b (e+f x)}{b e-a f}\right)^{1/4}}$$

### Result (type 6, 1078 leaves):

$$\begin{aligned}
& \frac{1}{3575 (a+b x)^{2/3} (e+f x)^{3/4}} \\
& \sqrt{c+d x} \left( \frac{132 (a+b x) (e+f x) (4 a d f + b (3 d e + 6 c f + 13 d f x))}{b d f} - \left( 72 (c+d x) \left( -23 (b c - a d) (d e - c f) (3 b d e - 7 b c f + 4 a d f) \right. \right. \right. \right. \\
& \text{AppellF1}\left[\frac{11}{12}, \frac{2}{3}, \frac{3}{4}, \frac{23}{12}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] \left( 9 b (d e - c f) \text{AppellF1}\left[\frac{11}{12}, \frac{2}{3}, \frac{7}{4}, \frac{23}{12}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] + \right. \\
& \left. 8 (-b c + a d) f \text{AppellF1}\left[\frac{11}{12}, \frac{5}{3}, \frac{3}{4}, \frac{23}{12}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] \right) - 11 (c+d x) \text{AppellF1}\left[-\frac{1}{12}, \frac{2}{3}, \frac{3}{4}, \frac{11}{12}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] \\
& \left( 23 b f (-2 a^2 d^2 f (-2 d e + 35 c f + 33 d f x) - b^2 (7 c^2 f^2 (12 c + 11 d x) - 2 c d e f (38 c + 33 d x) + d^2 e^2 (58 c + 55 d x))) + a \right. \\
& \left. b d (99 c^2 f^2 + d^2 e (3 e + 44 f x) + 2 c d f (15 e + 44 f x)) \text{AppellF1}\left[\frac{11}{12}, \frac{2}{3}, \frac{3}{4}, \frac{23}{12}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] + \right. \\
& \left. 11 (6 a^2 d^2 f^2 - 4 a b d f (d e + 2 c f) + b^2 (5 d^2 e^2 - 6 c d e f + 7 c^2 f^2)) \left( 9 b (d e - c f) \text{AppellF1}\left[\frac{23}{12}, \frac{2}{3}, \frac{7}{4}, \frac{35}{12}, \right. \right. \right. \\
& \left. \left. \left. \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] + 8 (-b c + a d) f \text{AppellF1}\left[\frac{23}{12}, \frac{5}{3}, \frac{3}{4}, \frac{35}{12}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] \right) \right) \Bigg) / \\
& \left( d^3 \left( 11 b f (c+d x) \text{AppellF1}\left[-\frac{1}{12}, \frac{2}{3}, \frac{3}{4}, \frac{11}{12}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] + b (-9 d e + 9 c f) \text{AppellF1}\left[\frac{11}{12}, \frac{2}{3}, \frac{7}{4}, \frac{23}{12}, \right. \right. \right. \\
& \left. \left. \left. \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] + 8 (b c - a d) f \text{AppellF1}\left[\frac{11}{12}, \frac{5}{3}, \frac{3}{4}, \frac{23}{12}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] \right) \right. \\
& \left( 23 b f (c+d x) \text{AppellF1}\left[\frac{11}{12}, \frac{2}{3}, \frac{3}{4}, \frac{23}{12}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] + b (-9 d e + 9 c f) \text{AppellF1}\left[\frac{23}{12}, \frac{2}{3}, \frac{7}{4}, \frac{35}{12}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] + \right. \\
& \left. 8 (b c - a d) f \text{AppellF1}\left[\frac{23}{12}, \frac{5}{3}, \frac{3}{4}, \frac{35}{12}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] \right) \right)
\end{aligned}$$

**Problem 3180:** Result more than twice size of optimal antiderivative.

$$\int (\mathbf{a} + \mathbf{b} x)^4 (\mathbf{A} + \mathbf{B} x) (\mathbf{d} + \mathbf{e} x)^m dx$$

Optimal (type 3, 234 leaves, 2 steps):

$$\begin{aligned} & -\frac{(b d - a e)^4 (B d - A e) (d + e x)^{1+m}}{e^6 (1+m)} + \frac{(b d - a e)^3 (5 b B d - 4 A b e - a B e) (d + e x)^{2+m}}{e^6 (2+m)} - \frac{2 b (b d - a e)^2 (5 b B d - 3 A b e - 2 a B e) (d + e x)^{3+m}}{e^6 (3+m)} + \\ & \frac{2 b^2 (b d - a e) (5 b B d - 2 A b e - 3 a B e) (d + e x)^{4+m}}{e^6 (4+m)} - \frac{b^3 (5 b B d - A b e - 4 a B e) (d + e x)^{5+m}}{e^6 (5+m)} + \frac{b^4 B (d + e x)^{6+m}}{e^6 (6+m)} \end{aligned}$$

Result (type 3, 635 leaves):

$$\begin{aligned} & \frac{1}{e^6 (1+m) (2+m) (3+m) (4+m) (5+m) (6+m)} (d + e x)^{1+m} (a^4 e^4 (360 + 342 m + 119 m^2 + 18 m^3 + m^4) (-B d + A e (2+m) + B e (1+m) x) + \\ & 4 a^3 b e^3 (120 + 74 m + 15 m^2 + m^3) (A e (3+m) (-d + e (1+m) x) + B (2 d^2 - 2 d e (1+m) x + e^2 (2 + 3 m + m^2) x^2)) + 6 a^2 b^2 e^2 (30 + 11 m + m^2) \\ & (A e (4+m) (2 d^2 - 2 d e (1+m) x + e^2 (2 + 3 m + m^2) x^2) + B (-6 d^3 + 6 d^2 e (1+m) x - 3 d e^2 (2 + 3 m + m^2) x^2 + e^3 (6 + 11 m + 6 m^2 + m^3) x^3)) + \\ & 4 a b^3 e (6+m) (A e (5+m) (-6 d^3 + 6 d^2 e (1+m) x - 3 d e^2 (2 + 3 m + m^2) x^2 + e^3 (6 + 11 m + 6 m^2 + m^3) x^3) + \\ & B (24 d^4 - 24 d^3 e (1+m) x + 12 d^2 e^2 (2 + 3 m + m^2) x^2 - 4 d e^3 (6 + 11 m + 6 m^2 + m^3) x^3 + e^4 (24 + 50 m + 35 m^2 + 10 m^3 + m^4) x^4)) - \\ & b^4 (-A e (6+m) (24 d^4 - 24 d^3 e (1+m) x + 12 d^2 e^2 (2 + 3 m + m^2) x^2 - 4 d e^3 (6 + 11 m + 6 m^2 + m^3) x^3 + e^4 (24 + 50 m + 35 m^2 + 10 m^3 + m^4) x^4) + \\ & B (120 d^5 - 120 d^4 e (1+m) x + 60 d^3 e^2 (2 + 3 m + m^2) x^2 - 20 d^2 e^3 (6 + 11 m + 6 m^2 + m^3) x^3 + \\ & 5 d e^4 (24 + 50 m + 35 m^2 + 10 m^3 + m^4) x^4 - e^5 (120 + 274 m + 225 m^2 + 85 m^3 + 15 m^4 + m^5) x^5)) \end{aligned}$$

**Problem 3181: Result more than twice size of optimal antiderivative.**

$$\int (a + b x)^3 (A + B x) (d + e x)^m dx$$

Optimal (type 3, 186 leaves, 2 steps):

$$\begin{aligned} & \frac{(b d - a e)^3 (B d - A e) (d + e x)^{1+m}}{e^5 (1+m)} - \frac{(b d - a e)^2 (4 b B d - 3 A b e - a B e) (d + e x)^{2+m}}{e^5 (2+m)} + \\ & \frac{3 b (b d - a e) (2 b B d - A b e - a B e) (d + e x)^{3+m}}{e^5 (3+m)} - \frac{b^2 (4 b B d - A b e - 3 a B e) (d + e x)^{4+m}}{e^5 (4+m)} + \frac{b^3 B (d + e x)^{5+m}}{e^5 (5+m)} \end{aligned}$$

Result (type 3, 391 leaves):

$$\begin{aligned} & \frac{1}{e^5 (1+m) (2+m) (3+m) (4+m) (5+m)} (d + e x)^{1+m} (a^3 e^3 (60 + 47 m + 12 m^2 + m^3) (-B d + A e (2+m) + B e (1+m) x) + \\ & 3 a^2 b e^2 (20 + 9 m + m^2) (A e (3+m) (-d + e (1+m) x) + B (2 d^2 - 2 d e (1+m) x + e^2 (2 + 3 m + m^2) x^2)) + 3 a b^2 e (5+m) \\ & (A e (4+m) (2 d^2 - 2 d e (1+m) x + e^2 (2 + 3 m + m^2) x^2) + B (-6 d^3 + 6 d^2 e (1+m) x - 3 d e^2 (2 + 3 m + m^2) x^2 + e^3 (6 + 11 m + 6 m^2 + m^3) x^3)) + \\ & b^3 (A e (5+m) (-6 d^3 + 6 d^2 e (1+m) x - 3 d e^2 (2 + 3 m + m^2) x^2 + e^3 (6 + 11 m + 6 m^2 + m^3) x^3) + \\ & B (24 d^4 - 24 d^3 e (1+m) x + 12 d^2 e^2 (2 + 3 m + m^2) x^2 - 4 d e^3 (6 + 11 m + 6 m^2 + m^3) x^3 + e^4 (24 + 50 m + 35 m^2 + 10 m^3 + m^4) x^4)) \end{aligned}$$

### Problem 3186: Unable to integrate problem.

$$\int \frac{(A + Bx)(d + ex)^m}{(ax + bx)^2} dx$$

Optimal (type 5, 112 leaves, 2 steps):

$$-\frac{(Ab - aB)(d + ex)^{1+m}}{b(bd - ae)(ax + bx)} + \frac{(aB e (1+m) - b(Bd + Ae m)) (d + ex)^{1+m} \text{Hypergeometric2F1}[1, 1+m, 2+m, \frac{b(d+ex)}{bd-ae}]}{b(bd - ae)^2 (1+m)}$$

Result (type 8, 22 leaves):

$$\int \frac{(A + Bx)(d + ex)^m}{(ax + bx)^2} dx$$

### Problem 3193: Result more than twice size of optimal antiderivative.

$$\int \frac{(2 + 3x)^m (3 + 5x)^3}{1 - 2x} dx$$

Optimal (type 5, 90 leaves, 3 steps):

$$-\frac{5135 (2 + 3x)^{1+m}}{216 (1+m)} - \frac{725 (2 + 3x)^{2+m}}{108 (2+m)} - \frac{125 (2 + 3x)^{3+m}}{54 (3+m)} + \frac{1331 (2 + 3x)^{1+m} \text{Hypergeometric2F1}[1, 1+m, 2+m, \frac{2}{7} (2 + 3x)]}{56 (1+m)}$$

Result (type 5, 240 leaves):

$$\begin{aligned} & \frac{1}{432} (2 + 3x)^m \left( -\frac{32670 (2 + 3x)}{1 + m} + \frac{2475 (40 + 36x - 36x^2 - 7^{2+m} (4 + 6x)^{-m} - 6m(-2 + x + 6x^2))}{2 + 3m + m^2} + \frac{1}{(1 + m) (2 + m) (3 + m)} \right. \\ & 250 (4 + 6x)^{-m} (7^{3+m} - 316 (4 + 6x)^m - 162x (4 + 6x)^m + 324x^2 (4 + 6x)^m - 216x^3 (4 + 6x)^m - 9m^2 (1 - 2x)^2 (2 + 3x) (4 + 6x)^m - \\ & 3m (4 + 6x)^m (46 - 59x - 120x^2 + 108x^3) \Big) - \frac{35937 \left( \frac{4+6x}{-3+6x} \right)^{-m} \text{Hypergeometric2F1}[-m, -m, 1 - m, \frac{7}{3-6x}]}{m} \end{aligned}$$

### Problem 3199: Unable to integrate problem.

$$\int \frac{(a + bx)^m}{(e + fx)^2} dx$$

Optimal (type 5, 52 leaves, 1 step):

$$\frac{b (a + b x)^{1+m} \text{Hypergeometric2F1}[2, 1+m, 2+m, -\frac{f (a+b x)}{b e - a f}]}{(b e - a f)^2 (1+m)}$$

Result (type 8, 17 leaves) :

$$\int \frac{(a + b x)^m}{(e + f x)^2} dx$$

Problem 3200: Unable to integrate problem.

$$\int \frac{(a + b x)^m}{(c + d x) (e + f x)^2} dx$$

Optimal (type 5, 187 leaves, 4 steps) :

$$\begin{aligned} & -\frac{f (a + b x)^{1+m}}{(b e - a f) (d e - c f) (e + f x)} + \frac{d^2 (a + b x)^{1+m} \text{Hypergeometric2F1}[1, 1+m, 2+m, -\frac{d (a+b x)}{b c - a d}]}{(b c - a d) (d e - c f)^2 (1+m)} + \\ & \frac{f (a d f - b (d e (1-m) + c f m)) (a + b x)^{1+m} \text{Hypergeometric2F1}[1, 1+m, 2+m, -\frac{f (a+b x)}{b e - a f}]}{(b e - a f)^2 (d e - c f)^2 (1+m)} \end{aligned}$$

Result (type 8, 24 leaves) :

$$\int \frac{(a + b x)^m}{(c + d x) (e + f x)^2} dx$$

Problem 3201: Unable to integrate problem.

$$\int \frac{(a + b x)^m}{(c + d x)^2 (e + f x)^2} dx$$

Optimal (type 5, 281 leaves, 5 steps) :

$$\begin{aligned} & \frac{f (b d e + b c f - 2 a d f) (a + b x)^{1+m}}{(b c - a d) (b e - a f) (d e - c f)^2 (e + f x)} + \frac{d (a + b x)^{1+m}}{(b c - a d) (d e - c f) (c + d x) (e + f x)} + \\ & \frac{d^2 (2 a d f - b (c f (2-m) + d e m)) (a + b x)^{1+m} \text{Hypergeometric2F1}[1, 1+m, 2+m, -\frac{d (a+b x)}{b c - a d}]}{(b c - a d)^2 (d e - c f)^3 (1+m)} - \\ & \frac{f^2 (2 a d f - b (d e (2-m) + c f m)) (a + b x)^{1+m} \text{Hypergeometric2F1}[1, 1+m, 2+m, -\frac{f (a+b x)}{b e - a f}]}{(b e - a f)^2 (d e - c f)^3 (1+m)} \end{aligned}$$

Result (type 8, 24 leaves) :

$$\int \frac{(a + b x)^m}{(c + d x)^2 (e + f x)^2} dx$$

Test results for the 159 problems in "1.1.1.4 (a+b x)^m (c+d x)^n (e+f x)^p (g+h x)^q.m"

Problem 33: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{A + B x}{\sqrt{a + b x} \sqrt{c + \frac{b(-1+c)x}{a}}} dx$$

Optimal (type 4, 145 leaves, 3 steps) :

$$-\frac{2 a^{3/2} B \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-c} \sqrt{a+b x}}{\sqrt{a}}\right], \frac{1-e}{1-c}\right]}{b^2 \sqrt{1-c} (1-e)} + \frac{2 \sqrt{a} (a B e + A (b - b e)) \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-c} \sqrt{a+b x}}{\sqrt{a}}\right], \frac{1-e}{1-c}\right]}{b^2 \sqrt{1-c} (1-e)}$$

Result (type 4, 309 leaves) :

$$\begin{aligned}
& - \left( \left( 2 \sqrt{\frac{a}{-1+c}} (a+b x)^{3/2} \right. \right. \\
& \left. \left. - B \sqrt{\frac{a}{-1+c}} \left( -1+c + \frac{a}{a+b x} \right) \left( -1+e + \frac{a}{a+b x} \right) - \frac{i a B (-1+e) \sqrt{\frac{-1+c+\frac{a}{a+b x}}{-1+c}} \sqrt{\frac{-1+e+\frac{a}{a+b x}}{-1+e}} \text{EllipticE}[i \text{ArcSinh}\left[\frac{\sqrt{\frac{a}{-1+c}}}{\sqrt{a+b x}}\right], \frac{-1+c}{-1+e}] }{\sqrt{a+b x}} + \right. \right. \\
& \left. \left. \frac{i (a B c + A (b - b c)) (-1+e) \sqrt{\frac{-1+c+\frac{a}{a+b x}}{-1+c}} \sqrt{\frac{-1+e+\frac{a}{a+b x}}{-1+e}} \text{EllipticF}[i \text{ArcSinh}\left[\frac{\sqrt{\frac{a}{-1+c}}}{\sqrt{a+b x}}\right], \frac{-1+c}{-1+e}] }{\sqrt{a+b x}} \right) \right) / \\
& \left. \left( a b^2 (-1+e) \sqrt{c + \frac{b (-1+c) x}{a}} \sqrt{e + \frac{b (-1+e) x}{a}} \right) \right)
\end{aligned}$$

Problem 34: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A+B x}{\sqrt{a+b x} \sqrt{c+d x}} dx$$

Optimal (type 4, 221 leaves, 5 steps):

$$\begin{aligned}
& - \frac{2 a B \sqrt{-b c + a d} \sqrt{\frac{b (c+d x)}{b c-a d}} \text{EllipticE}[\text{ArcSin}\left[\frac{\sqrt{d} \sqrt{a+b x}}{\sqrt{-b c+a d}}\right], -\frac{(b c-a d) (1-e)}{a d}] }{b^2 \sqrt{d} (1-e) \sqrt{c+d x}} + \\
& \frac{2 \sqrt{a} (a B e + A (b - b e)) \sqrt{\frac{b (c+d x)}{b c-a d}} \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{1-e} \sqrt{a+b x}}{\sqrt{a}}\right], -\frac{a d}{(b c-a d) (1-e)}]}{b^2 (1-e)^{3/2} \sqrt{c+d x}}
\end{aligned}$$

Result (type 4, 312 leaves):

$$\begin{aligned}
 & - \left( \left( 2 \sqrt{\frac{a}{-1+e}} (a+b x)^{3/2} \left( -\frac{b B \sqrt{\frac{a}{-1+e}} (c+d x) (a e + b (-1+e) x)}{(a+b x)^2} - \frac{i a B d \sqrt{\frac{b (c+d x)}{d (a+b x)}} \sqrt{\frac{-1+e+\frac{a}{a+b x}}{-1+e}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{-1+e}}}{\sqrt{a+b x}}\right], \frac{(b c-a d) (-1+e)}{a d}\right]}{\sqrt{a+b x}} + \right. \right. \right. \\
 & \left. \left. \left. \frac{i d (a B e + A (b - b e)) \sqrt{\frac{b (c+d x)}{d (a+b x)}} \sqrt{\frac{-1+e+\frac{a}{a+b x}}{-1+e}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{-1+e}}}{\sqrt{a+b x}}\right], \frac{(b c-a d) (-1+e)}{a d}\right]}{\sqrt{a+b x}} \right) \right) \right) / \left( a b^2 d \sqrt{c+d x} \sqrt{e + \frac{b (-1+e) x}{a}} \right)
 \end{aligned}$$

Problem 43: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x}}{a+b x} dx$$

Optimal (type 4, 570 leaves, 12 steps):

$$\begin{aligned}
 & \frac{2 \sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x}}{3 b} - \\
 & \left( 2 \sqrt{-d e+c f} (3 a d f h - b (d f g + d e h + c f h)) \sqrt{\frac{d (e+f x)}{d e-c f}} \sqrt{g+h x} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{f} \sqrt{c+d x}}{\sqrt{-d e+c f}}\right], \frac{(d e-c f) h}{f (d g-c h)}\right] \right) / \\
 & \left( 3 b^2 d \sqrt{f} h \sqrt{e+f x} \sqrt{\frac{d (g+h x)}{d g-c h}} \right) + \left( 2 \sqrt{-d e+c f} (3 a^2 d f h^2 - 3 a b (d e+c f) h^2 - b^2 (d g (f g-e h) - c h (f g+2 e h))) \sqrt{\frac{d (e+f x)}{d e-c f}} \right. \\
 & \left. \sqrt{\frac{d (g+h x)}{d g-c h}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{f} \sqrt{c+d x}}{\sqrt{-d e+c f}}\right], \frac{(d e-c f) h}{f (d g-c h)}\right] \right) / \left( 3 b^3 d \sqrt{f} h \sqrt{e+f x} \sqrt{g+h x} \right) - \frac{1}{b^3 \sqrt{f} \sqrt{e+f x} \sqrt{g+h x}} \\
 & 2 (b e - a f) \sqrt{-d e+c f} (b g - a h) \sqrt{\frac{d (e+f x)}{d e-c f}} \sqrt{\frac{d (g+h x)}{d g-c h}} \operatorname{EllipticPi}\left[-\frac{b (d e-c f)}{(b c-a d) f}, \operatorname{ArcSin}\left[\frac{\sqrt{f} \sqrt{c+d x}}{\sqrt{-d e+c f}}\right], \frac{(d e-c f) h}{f (d g-c h)}\right]
 \end{aligned}$$

Result (type 4, 29892 leaves): Display of huge result suppressed!

### Problem 54: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x} \sqrt{1+4x}} dx$$

Optimal (type 4, 47 leaves, 2 steps):

$$\frac{\sqrt{\frac{11}{2}} \sqrt{5-2x} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{1+4x}}{\sqrt{11}}\right], 3\right]}{2 \sqrt{-5+2x}}$$

Result (type 4, 111 leaves):

$$\frac{\frac{2(-5+2x)(-2+3x)}{\sqrt{\frac{1}{2}+2x}} + \sqrt{11} \sqrt{\frac{-5+2x}{1+4x}} \sqrt{\frac{-2+3x}{1+4x}} (1+4x) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{11}{3}}}{\sqrt{1+4x}}\right], 3\right]}{2 \sqrt{2-3x} \sqrt{-10+4x}}$$

### Problem 58: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{c+dx}}{(a+bx) \sqrt{e+fx} \sqrt{g+hx}} dx$$

Optimal (type 4, 293 leaves, 8 steps):

$$\begin{aligned} & \frac{2 \sqrt{-de+cf} \sqrt{\frac{d(e+fx)}{de-cf}} \sqrt{\frac{d(g+hx)}{dg-ch}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{f} \sqrt{c+dx}}{\sqrt{-de+cf}}\right], \frac{(de-cf)h}{f(dg-ch)}\right]}{b \sqrt{f} \sqrt{e+fx} \sqrt{g+hx}} - \\ & \frac{2 \sqrt{-de+cf} \sqrt{\frac{d(e+fx)}{de-cf}} \sqrt{\frac{d(g+hx)}{dg-ch}} \operatorname{EllipticPi}\left[-\frac{b(de-cf)}{(bc-ad)f}, \operatorname{ArcSin}\left[\frac{\sqrt{f} \sqrt{c+dx}}{\sqrt{-de+cf}}\right], \frac{(de-cf)h}{f(dg-ch)}\right]}{b \sqrt{f} \sqrt{e+fx} \sqrt{g+hx}} \end{aligned}$$

Result (type 4, 202 leaves):

$$\begin{aligned}
& - \left( \left( 2 \pm \sqrt{c+dx} \right) \sqrt{\frac{d(g+hx)}{dg-ch}} \left( \text{EllipticF}\left[\pm \text{ArcSinh}\left[\sqrt{\frac{f(c+dx)}{de-cf}}\right], \frac{deh-cfh}{dfg-cfh}\right] - \right. \right. \\
& \left. \left. \text{EllipticPi}\left[\frac{b(-de+cf)}{(bc-ad)f}, \pm \text{ArcSinh}\left[\sqrt{\frac{f(c+dx)}{de-cf}}\right], \frac{deh-cfh}{dfg-cfh}\right]\right) \right) \Big/ \left( b \sqrt{\frac{f(c+dx)}{d(e+fx)}} \sqrt{e+fx} \sqrt{g+hx} \right)
\end{aligned}$$

**Problem 59: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(c+dx)^{3/2}}{(a+bx) \sqrt{e+fx} \sqrt{g+hx}} dx$$

Optimal (type 4, 449 leaves, 11 steps):

$$\begin{aligned}
& \frac{2d\sqrt{-fg+eh}\sqrt{c+dx}}{\sqrt{\frac{f(g+hx)}{fg-eh}}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{h}\sqrt{e+fx}}{\sqrt{-fg+eh}}\right], -\frac{d(fg-eh)}{(de-cf)h}\right] + \\
& b f \sqrt{h} \sqrt{-\frac{f(c+dx)}{de-cf}} \sqrt{g+hx} \\
& \frac{2(b c - a d) \sqrt{-d e + c f}}{\sqrt{\frac{d(e+fx)}{de-cf}}} \sqrt{\frac{d(g+hx)}{dg-ch}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right], \frac{(de-cf)h}{f(dg-ch)}\right] - \\
& b^2 \sqrt{f} \sqrt{e+fx} \sqrt{g+hx} \\
& \frac{2(b c - a d) \sqrt{-d e + c f}}{\sqrt{\frac{d(e+fx)}{de-cf}}} \sqrt{\frac{d(g+hx)}{dg-ch}} \text{EllipticPi}\left[-\frac{b(de-cf)}{(bc-ad)f}, \text{ArcSin}\left[\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right], \frac{(de-cf)h}{f(dg-ch)}\right]
\end{aligned}$$

Result (type 4, 381 leaves):

$$\begin{aligned}
& -\frac{1}{b^2 f h \sqrt{g+h x}} 2 \sqrt{c+d x} \left( -\frac{b d f (g+h x)}{\sqrt{e+f x}} + \left( \pm \sqrt{\frac{f (g+h x)}{h (e+f x)}} \right) \left( -b d^2 (b e - a f) (-f g + e h) \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-e + \frac{f g}{h}}}{\sqrt{e+f x}} \right], \frac{(d e - c f) h}{d (-f g + e h)} \right] + \right. \right. \\
& \left. \left. f \left( -b (a d^2 (-f g + e h) + b (d^2 e g - 2 c d e h + c^2 f h)) \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-e + \frac{f g}{h}}}{\sqrt{e+f x}} \right], \frac{(d e - c f) h}{d (-f g + e h)} \right] + \right. \right. \\
& \left. \left. (b c - a d)^2 f h \text{EllipticPi} \left[ \frac{(b e - a f) h}{b (-f g + e h)}, \pm \text{ArcSinh} \left[ \frac{\sqrt{-e + \frac{f g}{h}}}{\sqrt{e+f x}} \right], \frac{(d e - c f) h}{d (-f g + e h)} \right] \right) \right) \right) \Bigg) \Bigg) / \left( d (-b e + a f) \sqrt{-e + \frac{f g}{h}} \sqrt{\frac{f (c+d x)}{d (e+f x)}} \right)
\end{aligned}$$

Problem 68: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{c i + d i x}{\sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x}} dx$$

Optimal (type 4, 137 leaves, 3 steps):

$$\begin{aligned}
& \frac{2 \sqrt{-f g + e h} i \sqrt{c+d x} \sqrt{\frac{f (g+h x)}{f g - e h}} \text{EllipticE} \left[ \text{ArcSin} \left[ \frac{\sqrt{h} \sqrt{e+f x}}{\sqrt{-f g + e h}} \right], -\frac{d (f g - e h)}{(d e - c f) h} \right]}{f \sqrt{h} \sqrt{-\frac{f (c+d x)}{d e - c f}} \sqrt{g+h x}}
\end{aligned}$$

Result (type 4, 180 leaves):

$$\begin{aligned}
& - \left( \left( 2 \pm i \sqrt{c+d x} \sqrt{g+h x} \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \sqrt{\frac{f (c+d x)}{d e - c f}} \right], \frac{d e h - c f h}{d f g - c f h} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \sqrt{\frac{f (c+d x)}{d e - c f}} \right], \frac{d e h - c f h}{d f g - c f h} \right] \right) \right) \right) / \\
& \left( h \sqrt{\frac{f (c+d x)}{d (e+f x)}} \sqrt{e+f x} \sqrt{\frac{d (g+h x)}{d g - c h}} \right)
\end{aligned}$$

Problem 69: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a + b x}{\sqrt{c + d x} \sqrt{e + f x} \sqrt{g + h x}} dx$$

Optimal (type 4, 284 leaves, 6 steps):

$$\begin{aligned} & \frac{2 b \sqrt{-d e + c f} \sqrt{\frac{d(e+f x)}{d e - c f}} \sqrt{g + h x} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{f} \sqrt{c+d x}}{\sqrt{-d e+c f}}\right], \frac{(d e-c f) h}{f(d g-c h)}\right]}{d \sqrt{f} h \sqrt{e+f x} \sqrt{\frac{d(g+h x)}{d g-c h}}} \\ & \frac{2 \sqrt{-d e + c f} (b g - a h) \sqrt{\frac{d(e+f x)}{d e - c f}} \sqrt{\frac{d(g+h x)}{d g-c h}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{f} \sqrt{c+d x}}{\sqrt{-d e+c f}}\right], \frac{(d e-c f) h}{f(d g-c h)}\right]}{d \sqrt{f} h \sqrt{e+f x} \sqrt{g+h x}} \end{aligned}$$

Result (type 4, 319 leaves):

$$\begin{aligned} & - \left( \left( 2 \left( -b d^2 \sqrt{-c + \frac{d e}{f}} (e + f x) (g + h x) - \right. \right. \right. \\ & \left. \left. \left. \pm b (d e - c f) h (c + d x)^{3/2} \sqrt{\frac{d (e + f x)}{f (c + d x)}} \sqrt{\frac{d (g + h x)}{h (c + d x)}} \operatorname{EllipticE}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{-c + \frac{d e}{f}}}{\sqrt{c + d x}}\right], \frac{d f g - c f h}{d e h - c f h}\right] + \pm d (b e - a f) h (c + d x)^{3/2} \right. \right. \right. \\ & \left. \left. \left. \sqrt{\frac{d (e + f x)}{f (c + d x)}} \sqrt{\frac{d (g + h x)}{h (c + d x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{-c + \frac{d e}{f}}}{\sqrt{c + d x}}\right], \frac{d f g - c f h}{d e h - c f h}\right] \right) \right) \right) \Big/ \left( d^2 \sqrt{-c + \frac{d e}{f}} f h \sqrt{c + d x} \sqrt{e + f x} \sqrt{g + h x} \right) \right) \end{aligned}$$

Problem 70: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(a + b x) \sqrt{c + d x} \sqrt{e + f x} \sqrt{g + h x}} dx$$

Optimal (type 4, 165 leaves, 4 steps):

$$\frac{2 \sqrt{-d e + c f} \sqrt{\frac{d (e+f x)}{d e - c f}} \sqrt{\frac{d (g+h x)}{d g - c h}} \operatorname{EllipticPi}\left[-\frac{b (d e - c f)}{(b c - a d) f}, \operatorname{ArcSin}\left[\frac{\sqrt{f} \sqrt{c+d x}}{\sqrt{-d e + c f}}\right], \frac{(d e - c f) h}{f (d g - c h)}\right]}{(b c - a d) \sqrt{f} \sqrt{e + f x} \sqrt{g + h x}}$$

Result (type 4, 225 leaves):

$$\begin{aligned} & \left( 2 \operatorname{Int} (c + d x) \sqrt{\frac{d (e + f x)}{f (c + d x)}} \sqrt{\frac{d (g + h x)}{h (c + d x)}} \right. \\ & \left. \left( \operatorname{EllipticF}\left[\operatorname{Int} \operatorname{ArcSinh}\left[\frac{\sqrt{-c + \frac{d e}{f}}}{\sqrt{c + d x}}\right], \frac{d f g - c f h}{d e h - c f h}\right] - \operatorname{EllipticPi}\left[\frac{(b c - a d) f}{b (-d e + c f)}, \operatorname{Int} \operatorname{ArcSinh}\left[\frac{\sqrt{-c + \frac{d e}{f}}}{\sqrt{c + d x}}\right], \frac{d f g - c f h}{d e h - c f h}\right] \right) \right) / \\ & \left( (-b c + a d) \sqrt{-c + \frac{d e}{f}} \sqrt{e + f x} \sqrt{g + h x} \right) \end{aligned}$$

Problem 71: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(a + b x) (c + d x)^{3/2} \sqrt{e + f x} \sqrt{g + h x}} dx$$

Optimal (type 4, 393 leaves, 10 steps):

$$\begin{aligned} & \frac{2 d^2 \sqrt{e + f x} \sqrt{g + h x}}{(b c - a d) (d e - c f) (d g - c h) \sqrt{c + d x}} - \frac{2 d \sqrt{h} \sqrt{-f g + e h} \sqrt{c + d x} \sqrt{\frac{f (g+h x)}{f g - e h}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{h} \sqrt{e + f x}}{\sqrt{-f g + e h}}\right], -\frac{d (f g - e h)}{(d e - c f) h}\right]}{(b c - a d) (d e - c f) (d g - c h) \sqrt{-\frac{f (c+d x)}{d e - c f}} \sqrt{g + h x}} \\ & \frac{2 b \sqrt{-d e + c f} \sqrt{\frac{d (e+f x)}{d e - c f}} \sqrt{\frac{d (g+h x)}{d g - c h}} \operatorname{EllipticPi}\left[-\frac{b (d e - c f)}{(b c - a d) f}, \operatorname{ArcSin}\left[\frac{\sqrt{f} \sqrt{c+d x}}{\sqrt{-d e + c f}}\right], \frac{(d e - c f) h}{f (d g - c h)}\right]}{(b c - a d)^2 \sqrt{f} \sqrt{e + f x} \sqrt{g + h x}} \end{aligned}$$

Result (type 4, 321 leaves):

$$\begin{aligned}
& \left( 2 \pm (c + dx) \sqrt{\frac{d(e + fx)}{f(c + dx)}} \sqrt{\frac{d(g + hx)}{h(c + dx)}} \right. \\
& \left. \left( (bc - ad)f \operatorname{EllipticE} \left[ \pm \operatorname{ArcSinh} \left[ \frac{\sqrt{-c + \frac{dg}{h}}}{\sqrt{c + dx}} \right], \frac{deh - cfh}{dfg - cfh} \right] + (bd e - 2bc f + ad f) \operatorname{EllipticF} \left[ \pm \operatorname{ArcSinh} \left[ \frac{\sqrt{-c + \frac{dg}{h}}}{\sqrt{c + dx}} \right], \frac{deh - cfh}{dfg - cfh} \right] + \right. \right. \\
& \left. \left. b(-de + cf) \operatorname{EllipticPi} \left[ \frac{(bc - ad)h}{b(-dg + ch)}, \pm \operatorname{ArcSinh} \left[ \frac{\sqrt{-c + \frac{dg}{h}}}{\sqrt{c + dx}} \right], \frac{deh - cfh}{dfg - cfh} \right] \right) \right) / \left( (bc - ad)^2 (-de + cf) \sqrt{-c + \frac{dg}{h}} \sqrt{e + fx} \sqrt{g + hx} \right)
\end{aligned}$$

**Problem 72:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + bx)(c + dx)^{5/2} \sqrt{e + fx} \sqrt{g + hx}} dx$$

Optimal (type 4, 875 leaves, 18 steps):

$$\begin{aligned}
& \frac{2 d^2 \sqrt{e+f x} \sqrt{g+h x}}{3 (b c-a d) (d e-c f) (d g-c h) (c+d x)^{3/2}} + \frac{2 b d^2 \sqrt{e+f x} \sqrt{g+h x}}{(b c-a d)^2 (d e-c f) (d g-c h) \sqrt{c+d x}} - \\
& \frac{4 d^2 (d f g+d e h-2 c f h) \sqrt{e+f x} \sqrt{g+h x}}{3 (b c-a d) (d e-c f)^2 (d g-c h)^2 \sqrt{c+d x}} + \frac{4 d \sqrt{f} (d f g+d e h-2 c f h) \sqrt{\frac{d(e+f x)}{d e-c f}} \sqrt{g+h x} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{f} \sqrt{c+d x}}{\sqrt{-d e+c f}}\right], \frac{(d e-c f) h}{f (d g-c h)}\right]}{3 (b c-a d) (-d e+c f)^{3/2} (d g-c h)^2 \sqrt{e+f x} \sqrt{\frac{d(g+h x)}{d g-c h}}} - \\
& \frac{2 b d \sqrt{h} \sqrt{-f g+e h} \sqrt{c+d x} \sqrt{\frac{f(g+h x)}{f g-e h}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{h} \sqrt{e+f x}}{\sqrt{-f g+e h}}\right], -\frac{d(f g-e h)}{(d e-c f) h}\right]}{(b c-a d)^2 (d e-c f) (d g-c h) \sqrt{-\frac{f(c+d x)}{d e-c f}} \sqrt{g+h x}} - \\
& \frac{2 \sqrt{f} (2 d f g+d e h-3 c f h) \sqrt{\frac{d(e+f x)}{d e-c f}} \sqrt{\frac{d(g+h x)}{d g-c h}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{f} \sqrt{c+d x}}{\sqrt{-d e+c f}}\right], \frac{(d e-c f) h}{f (d g-c h)}\right]}{3 (b c-a d) (-d e+c f)^{3/2} (d g-c h) \sqrt{e+f x} \sqrt{g+h x}} - \\
& \frac{2 b^2 \sqrt{-d e+c f} \sqrt{\frac{d(e+f x)}{d e-c f}} \sqrt{\frac{d(g+h x)}{d g-c h}} \operatorname{EllipticPi}\left[-\frac{b(d e-c f)}{(b c-a d) f}, \operatorname{ArcSin}\left[\frac{\sqrt{f} \sqrt{c+d x}}{\sqrt{-d e+c f}}\right], \frac{(d e-c f) h}{f (d g-c h)}\right]}{(b c-a d)^3 \sqrt{f} \sqrt{e+f x} \sqrt{g+h x}}
\end{aligned}$$

Result (type 4, 12191 leaves):

$$\begin{aligned}
& \frac{\sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x}}{3 (-b c+a d)^2 (-d e+c f)^2 (-d g+c h)^2} + \frac{2 d^2 \left(3 b d^2 e g-5 b c d f g+2 a d^2 f g-5 b c d e h+2 a d^2 e h+7 b c^2 f h-4 a c d f h\right)}{3 (b c-a d)^2 (-d e+c f)^2 (-d g+c h)^2 (c+d x)} + \\
& \frac{1}{2 \left\{ \left( (-3 b d^2 e g+5 b c d f g-2 a d^2 f g+5 b c d e h-2 a d^2 e h-7 b c^2 f h+4 a c d f h) (c+d x)^{3/2} \left(f+\frac{d e}{c+d x}-\frac{c f}{c+d x}\right) \left(h+\frac{d g}{c+d x}-\frac{c h}{c+d x}\right) \right) \right\}}
\end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{e + \frac{(c+d x) \left(f - \frac{c f}{c+d x}\right)}{d}} \sqrt{g + \frac{(c+d x) \left(h - \frac{c h}{c+d x}\right)}{d}} \right) + \left( (c+d x) \left(-b + \frac{b c}{c+d x} - \frac{a d}{c+d x}\right) \sqrt{f + \frac{d e}{c+d x} - \frac{c f}{c+d x}} \right. \\
& \left. \sqrt{h + \frac{d g}{c+d x} - \frac{c h}{c+d x}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}} \right. \\
& \left. \left( (b c - a d) (-d e + c f) h (-b d^2 f g^2 + b d^2 e g h + a d^2 f g h - 3 b c d e h^2 + 2 a d^2 e h^2 + 3 b c^2 f h^2 - 3 a c d f h^2) \right) \right) / \\
& \left( d (b g - a h) \sqrt{f + \frac{d e}{c+d x} - \frac{c f}{c+d x}} \sqrt{h + \frac{d g}{c+d x} - \frac{c h}{c+d x}} + \frac{3 b d^3 e^2 g \sqrt{h + \frac{d g}{c+d x} - \frac{c h}{c+d x}}}{\sqrt{f + \frac{d e}{c+d x} - \frac{c f}{c+d x}}} - \frac{8 b c d^2 e f g \sqrt{h + \frac{d g}{c+d x} - \frac{c h}{c+d x}}}{\sqrt{f + \frac{d e}{c+d x} - \frac{c f}{c+d x}}} + \right. \\
& \left. \frac{2 a d^3 e f g \sqrt{h + \frac{d g}{c+d x} - \frac{c h}{c+d x}}}{\sqrt{f + \frac{d e}{c+d x} - \frac{c f}{c+d x}}} + \frac{5 b c^2 d f^2 g \sqrt{h + \frac{d g}{c+d x} - \frac{c h}{c+d x}}}{\sqrt{f + \frac{d e}{c+d x} - \frac{c f}{c+d x}}} - \frac{2 a c d^2 f^2 g \sqrt{h + \frac{d g}{c+d x} - \frac{c h}{c+d x}}}{\sqrt{f + \frac{d e}{c+d x} - \frac{c f}{c+d x}}} - \frac{5 b c d^2 e^2 h \sqrt{h + \frac{d g}{c+d x} - \frac{c h}{c+d x}}}{\sqrt{f + \frac{d e}{c+d x} - \frac{c f}{c+d x}}} + \right. \\
& \left. \frac{2 a d^3 e^2 h \sqrt{h + \frac{d g}{c+d x} - \frac{c h}{c+d x}}}{\sqrt{f + \frac{d e}{c+d x} - \frac{c f}{c+d x}}} + \frac{12 b c^2 d e f h \sqrt{h + \frac{d g}{c+d x} - \frac{c h}{c+d x}}}{\sqrt{f + \frac{d e}{c+d x} - \frac{c f}{c+d x}}} - \frac{6 a c d^2 e f h \sqrt{h + \frac{d g}{c+d x} - \frac{c h}{c+d x}}}{\sqrt{f + \frac{d e}{c+d x} - \frac{c f}{c+d x}}} - \frac{7 b c^3 f^2 h \sqrt{h + \frac{d g}{c+d x} - \frac{c h}{c+d x}}}{\sqrt{f + \frac{d e}{c+d x} - \frac{c f}{c+d x}}} + \right. \\
& \left. \frac{4 a c^2 d f^2 h \sqrt{h + \frac{d g}{c+d x} - \frac{c h}{c+d x}}}{\sqrt{f + \frac{d e}{c+d x} - \frac{c f}{c+d x}}} - \frac{3 b^3 (d e - c f)^2 (d g - c h)^2 \sqrt{h + \frac{d g}{c+d x} - \frac{c h}{c+d x}}}{d (b g - a h) \left(b - \frac{b c}{c+d x} + \frac{a d}{c+d x}\right) \sqrt{f + \frac{d e}{c+d x} - \frac{c f}{c+d x}}} \right) \left( \left( 3 i b d^2 e f g \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left( \text{EllipticE} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] - \text{EllipticF} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] \right) / \\
& \left( (b c - a d) (-d e + c f) \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2}} + \frac{d f g + d e h - 2 c f h}{c + d x} \right) - \left( 5 i b c d f^2 g \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \right. \\
& \left. \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \left( \text{EllipticE} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] - \text{EllipticF} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] \right) \right) / \\
& \left( (b c - a d) (-d e + c f) \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2}} + \frac{d f g + d e h - 2 c f h}{c + d x} \right) + \left( 2 i a d^2 f^2 g \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \right. \\
& \left. \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \left( \text{EllipticE} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] - \text{EllipticF} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] \right) \right) / \\
& \left( (b c - a d) (-d e + c f) \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2}} + \frac{d f g + d e h - 2 c f h}{c + d x} \right) - \left( 5 i b c d e f h \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \right. \\
& \left. \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \left( \text{EllipticE} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] - \text{EllipticF} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left( (b c - a d) (-d e + c f) \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2} + \frac{d f g + d e h - 2 c f h}{c + d x}} \right) + \left( 2 \pm a d^2 e f h \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \right. \\
& \left. \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] \right) \right) / \\
& \left( (b c - a d) (-d e + c f) \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2} + \frac{d f g + d e h - 2 c f h}{c + d x}} \right) + \left( 7 \pm b c^2 f^2 h \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \right. \\
& \left. \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] \right) \right) / \\
& \left( (b c - a d) (-d e + c f) \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2} + \frac{d f g + d e h - 2 c f h}{c + d x}} \right) - \left( 4 \pm a c d f^2 h \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \right. \\
& \left. \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] \right) \right) / \\
& \left( (b c - a d) (-d e + c f) \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2} + \frac{d f g + d e h - 2 c f h}{c + d x}} \right) - 
\end{aligned}$$

$$\frac{3 \pm b^2 d^2 e g \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{(b c - a d)^2 \sqrt{-\frac{d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}} +$$

$$\frac{5 \pm b^2 c d f g \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{(b c - a d)^2 \sqrt{-\frac{d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}} -$$

$$\frac{2 \pm a b d^2 f g \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{(b c - a d)^2 \sqrt{-\frac{d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}} -$$

$$\frac{2 \pm b d f g \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{(b c - a d) \sqrt{-\frac{d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}} +$$

$$\frac{5 \pm b^2 c d e h \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{(b c - a d)^2 \sqrt{-\frac{d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}} -$$

$$\frac{2 \pm a b d^2 e h \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{(b c - a d)^2 \sqrt{-\frac{d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}} -$$

$$\frac{2 \pm b d e h \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}]}{(b c - a d) \sqrt{-\frac{d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}} -$$

$$\frac{7 \pm b^2 c^2 f h \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}]}{(b c - a d)^2 \sqrt{-\frac{d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}} +$$

$$\frac{4 \pm a b c d f h \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}]}{(b c - a d)^2 \sqrt{-\frac{d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}} +$$

$$\frac{5 \pm b c f h \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}]}{(b c - a d) \sqrt{-\frac{d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}} -$$

$$\frac{\pm a d f h \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}]}{(b c - a d) \sqrt{-\frac{d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}} + \frac{1}{(b c - a d)^3}$$

$$3 b^3 d^2 e g \left( \frac{\pm c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{\sqrt{-\frac{d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} - \right)$$

$$\frac{\frac{1}{2} \operatorname{ad} \sqrt{1 - \frac{-de+cf}{f(c+dx)}} \sqrt{1 - \frac{-dg+ch}{h(c+dx)}} \operatorname{EllipticPi}\left[\frac{(bc-ad)h}{b(-dg+ch)}, \frac{1}{2} \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-dg+ch}{h}}}{\sqrt{c+dx}}\right], \frac{(-de+cf)h}{f(-dg+ch)}\right]}{b \sqrt{-\frac{dg+ch}{h}} \sqrt{fh + \frac{d^2 e g}{(c+dx)^2} - \frac{c d f g}{(c+dx)^2} - \frac{c d e h}{(c+dx)^2} + \frac{c^2 f h}{(c+dx)^2} + \frac{d f g}{c+dx} + \frac{d e h}{c+dx} - \frac{2 c f h}{c+dx}}} \quad \boxed{-}$$

$$\frac{1}{(b c - a d)^3} 5 b^3 c d f g \left( \frac{\frac{1}{2} c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \frac{1}{2} \text{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{\sqrt{\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \right)$$

$$\frac{\frac{1}{2} \operatorname{ad} \sqrt{1 - \frac{-de+cf}{f(c+dx)}} \sqrt{1 - \frac{-dg+ch}{h(c+dx)}} \operatorname{EllipticPi}\left[\frac{(bc-ad)h}{b(-dg+ch)}, \frac{1}{2} \operatorname{ArcSinh}\left(\frac{\sqrt{\frac{-dg+ch}{h}}}{\sqrt{c+dx}}\right), \frac{(-de+cf)h}{f(-dg+ch)}\right]}{b \sqrt{-\frac{-dg+ch}{h}} \sqrt{fh + \frac{d^2 eg}{(c+dx)^2} - \frac{cd fg}{(c+dx)^2} - \frac{cd eh}{(c+dx)^2} + \frac{c^2 fh}{(c+dx)^2} + \frac{df g}{c+dx} + \frac{de h}{c+dx} - \frac{2cfh}{c+dx}}}$$

$$\frac{1}{(b c - a d)^3} 2 a b^2 d^2 f g \left( \frac{\frac{i}{2} c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \frac{i}{2} \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{\sqrt{\frac{-d g + c h}{h}}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}} \right)$$

$$\frac{\text{d} \operatorname{ad} \sqrt{1 - \frac{-\text{d e} + \text{c f}}{\text{f} (\text{c} + \text{d x})}} \sqrt{1 - \frac{-\text{d g} + \text{c h}}{\text{h} (\text{c} + \text{d x})}} \operatorname{EllipticPi}\left[\frac{(\text{b c} - \text{a d}) \text{h}}{\text{b} (-\text{d g} + \text{c h})}, \text{d} \operatorname{ArcSinh}\left[\frac{\sqrt{-\frac{-\text{d g} + \text{c h}}{\text{h}}}}{\sqrt{\text{c} + \text{d x}}}\right], \frac{(-\text{d e} + \text{c f}) \text{h}}{\text{f} (-\text{d g} + \text{c h})}\right]}{\text{b} \sqrt{-\frac{-\text{d g} + \text{c h}}{\text{h}}} \sqrt{\text{f} \text{h} + \frac{\text{d}^2 \text{e g}}{(\text{c} + \text{d x})^2} - \frac{\text{c d f g}}{(\text{c} + \text{d x})^2} - \frac{\text{c d e h}}{(\text{c} + \text{d x})^2} + \frac{\text{c}^2 \text{f h}}{(\text{c} + \text{d x})^2} + \frac{\text{d f g}}{\text{c} + \text{d x}} + \frac{\text{d e h}}{\text{c} + \text{d x}} - \frac{2 \text{c f h}}{\text{c} + \text{d x}}}}$$

$$\frac{1}{(b c - a d)^2 b^2 d f g} \left( \begin{array}{l} \pm c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \pm \text{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right] \\ \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}} \end{array} \right)$$

$$\left. \frac{\frac{1}{2} a d \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}, \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{b \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \right] - \right.$$

$$\frac{1}{(b c - a d)^3} 5 b^3 c d e h \left( \frac{\frac{1}{2} c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}, \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{\sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \right] - \right.$$

$$\left. \frac{\frac{1}{2} a d \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}, \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{b \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \right] + \right)$$

$$\frac{1}{(b c - a d)^3} 2 a b^2 d^2 e h \left( \frac{\frac{1}{2} c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}, \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{\sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \right] - \right)$$

$$\left. \frac{\frac{1}{2} a d \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}, \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{b \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \right] + \right)$$

$$\frac{1}{(b c - a d)^2} 2 b^2 d e h \left( \frac{\frac{1}{2} c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}, \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{\sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \right] - \right)$$

$$\begin{aligned}
& \left. \frac{\frac{\text{i} \text{a} \text{d}}{\sqrt{1 - \frac{-\text{d e} + \text{c f}}{\text{f} (\text{c} + \text{d x})}}} \sqrt{1 - \frac{-\text{d g} + \text{c h}}{\text{h} (\text{c} + \text{d x})}} \text{EllipticPi}\left[\frac{(\text{b c} - \text{a d}) \text{h}}{\text{b} (-\text{d g} + \text{c h})}, \frac{\sqrt{\frac{-\text{d g} + \text{c h}}{\text{h}}}}{\sqrt{\text{c} + \text{d x}}}, \frac{(-\text{d e} + \text{c f}) \text{h}}{\text{f} (-\text{d g} + \text{c h})}\right]} + \right. \\
& \left. \frac{\text{b} \sqrt{\frac{-\text{d g} + \text{c h}}{\text{h}}} \sqrt{\text{f} \text{h} + \frac{\text{d}^2 \text{e g}}{(\text{c} + \text{d x})^2} - \frac{\text{c d f g}}{(\text{c} + \text{d x})^2} - \frac{\text{c d e h}}{(\text{c} + \text{d x})^2} + \frac{\text{c}^2 \text{f h}}{(\text{c} + \text{d x})^2} + \frac{\text{d f g}}{\text{c} + \text{d x}} + \frac{\text{d e h}}{\text{c} + \text{d x}} - \frac{2 \text{c f h}}{\text{c} + \text{d x}}}}}{\text{b}^7 \text{b}^3 \text{c}^2 \text{f} \text{h}} \right. \\
& \left. \frac{\frac{\text{i} \text{c} \sqrt{1 - \frac{-\text{d e} + \text{c f}}{\text{f} (\text{c} + \text{d x})}}} \sqrt{1 - \frac{-\text{d g} + \text{c h}}{\text{h} (\text{c} + \text{d x})}} \text{EllipticPi}\left[\frac{(\text{b c} - \text{a d}) \text{h}}{\text{b} (-\text{d g} + \text{c h})}, \frac{\sqrt{\frac{-\text{d g} + \text{c h}}{\text{h}}}}{\sqrt{\text{c} + \text{d x}}}, \frac{(-\text{d e} + \text{c f}) \text{h}}{\text{f} (-\text{d g} + \text{c h})}\right]} - \right. \\
& \left. \frac{\text{b} \sqrt{\frac{-\text{d g} + \text{c h}}{\text{h}}} \sqrt{\text{f} \text{h} + \frac{\text{d}^2 \text{e g}}{(\text{c} + \text{d x})^2} - \frac{\text{c d f g}}{(\text{c} + \text{d x})^2} - \frac{\text{c d e h}}{(\text{c} + \text{d x})^2} + \frac{\text{c}^2 \text{f h}}{(\text{c} + \text{d x})^2} + \frac{\text{d f g}}{\text{c} + \text{d x}} + \frac{\text{d e h}}{\text{c} + \text{d x}} - \frac{2 \text{c f h}}{\text{c} + \text{d x}}}}}{4 \text{a} \text{b}^2 \text{c} \text{d} \text{f} \text{h}} \right. \\
& \left. \frac{\frac{\text{i} \text{c} \sqrt{1 - \frac{-\text{d e} + \text{c f}}{\text{f} (\text{c} + \text{d x})}}} \sqrt{1 - \frac{-\text{d g} + \text{c h}}{\text{h} (\text{c} + \text{d x})}} \text{EllipticPi}\left[\frac{(\text{b c} - \text{a d}) \text{h}}{\text{b} (-\text{d g} + \text{c h})}, \frac{\sqrt{\frac{-\text{d g} + \text{c h}}{\text{h}}}}{\sqrt{\text{c} + \text{d x}}}, \frac{(-\text{d e} + \text{c f}) \text{h}}{\text{f} (-\text{d g} + \text{c h})}\right]} - \right. \\
& \left. \frac{\text{b} \sqrt{\frac{-\text{d g} + \text{c h}}{\text{h}}} \sqrt{\text{f} \text{h} + \frac{\text{d}^2 \text{e g}}{(\text{c} + \text{d x})^2} - \frac{\text{c d f g}}{(\text{c} + \text{d x})^2} - \frac{\text{c d e h}}{(\text{c} + \text{d x})^2} + \frac{\text{c}^2 \text{f h}}{(\text{c} + \text{d x})^2} + \frac{\text{d f g}}{\text{c} + \text{d x}} + \frac{\text{d e h}}{\text{c} + \text{d x}} - \frac{2 \text{c f h}}{\text{c} + \text{d x}}}}}{5 \text{b}^2 \text{c} \text{f} \text{h}} \right. \\
& \left. \frac{\frac{\text{i} \text{c} \sqrt{1 - \frac{-\text{d e} + \text{c f}}{\text{f} (\text{c} + \text{d x})}}} \sqrt{1 - \frac{-\text{d g} + \text{c h}}{\text{h} (\text{c} + \text{d x})}} \text{EllipticPi}\left[\frac{(\text{b c} - \text{a d}) \text{h}}{\text{b} (-\text{d g} + \text{c h})}, \frac{\sqrt{\frac{-\text{d g} + \text{c h}}{\text{h}}}}{\sqrt{\text{c} + \text{d x}}}, \frac{(-\text{d e} + \text{c f}) \text{h}}{\text{f} (-\text{d g} + \text{c h})}\right]} - \right. \\
& \left. \frac{\text{b} \sqrt{\frac{-\text{d g} + \text{c h}}{\text{h}}} \sqrt{\text{f} \text{h} + \frac{\text{d}^2 \text{e g}}{(\text{c} + \text{d x})^2} - \frac{\text{c d f g}}{(\text{c} + \text{d x})^2} - \frac{\text{c d e h}}{(\text{c} + \text{d x})^2} + \frac{\text{c}^2 \text{f h}}{(\text{c} + \text{d x})^2} + \frac{\text{d f g}}{\text{c} + \text{d x}} + \frac{\text{d e h}}{\text{c} + \text{d x}} - \frac{2 \text{c f h}}{\text{c} + \text{d x}}}} \right]
\end{aligned}$$

$$\begin{aligned}
& \frac{\frac{1}{c-a} d \sqrt{1 - \frac{-de+cf}{f(c+d x)}} \sqrt{1 - \frac{-dg+ch}{h(c+d x)}} \operatorname{EllipticPi}\left[\frac{(bc-ad) h}{b(-dg+ch)}, \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-dg+ch}{h}}}{\sqrt{c+d x}}\right], \frac{(-de+cf) h}{f(-dg+ch)}\right]}{+} \\
& \left. b \sqrt{-\frac{-dg+ch}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}\right\} \\
& \frac{1}{(c-a d)^2} a b d f h \left( \begin{array}{l} \frac{1}{c} \sqrt{1 - \frac{-de+cf}{f(c+d x)}} \sqrt{1 - \frac{-dg+ch}{h(c+d x)}} \operatorname{EllipticPi}\left[\frac{(bc-ad) h}{b(-dg+ch)}, \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-dg+ch}{h}}}{\sqrt{c+d x}}\right], \frac{(-de+cf) h}{f(-dg+ch)}\right] \\ \sqrt{-\frac{-dg+ch}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}} \end{array} \right. \\
& \frac{1}{c-a d} \sqrt{1 - \frac{-de+cf}{f(c+d x)}} \sqrt{1 - \frac{-dg+ch}{h(c+d x)}} \operatorname{EllipticPi}\left[\frac{(bc-ad) h}{b(-dg+ch)}, \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-dg+ch}{h}}}{\sqrt{c+d x}}\right], \frac{(-de+cf) h}{f(-dg+ch)}\right] \\
& \left. b \sqrt{-\frac{-dg+ch}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}} \right\} \\
& \frac{1}{1-a d} b f h \left( \begin{array}{l} \frac{1}{c} \sqrt{1 - \frac{-de+cf}{f(c+d x)}} \sqrt{1 - \frac{-dg+ch}{h(c+d x)}} \operatorname{EllipticPi}\left[\frac{(bc-ad) h}{b(-dg+ch)}, \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-dg+ch}{h}}}{\sqrt{c+d x}}\right], \frac{(-de+cf) h}{f(-dg+ch)}\right] \\ \sqrt{-\frac{-dg+ch}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}} \end{array} \right. \\
& \left. \frac{1}{c-a d} d \sqrt{1 - \frac{-de+cf}{f(c+d x)}} \sqrt{1 - \frac{-dg+ch}{h(c+d x)}} \operatorname{EllipticPi}\left[\frac{(bc-ad) h}{b(-dg+ch)}, \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-dg+ch}{h}}}{\sqrt{c+d x}}\right], \frac{(-de+cf) h}{f(-dg+ch)}\right] \right) \\
& \left. b \sqrt{-\frac{-dg+ch}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}} \right\} \\
& + \frac{3 b d^2 e g}{(c+d x)^2} - \frac{5 b c d f g}{(c+d x)^2} + \frac{2 a d^2 f g}{(c+d x)^2} - \frac{5 b c d e h}{(c+d x)^2} + \frac{2 a d^2 e h}{(c+d x)^2} + \frac{7 b c^2 f h}{(c+d x)^2} - \frac{4 a c d f h}{(c+d x)^2} + \frac{2 b d f g}{c+d x} + \frac{2 b d e h}{c+d x} - \frac{5 b c f h}{c+d x} + \frac{a d f h}{c+d x}
\end{aligned}$$

$$\left. \sqrt{e + \frac{(c + d x) \left(f - \frac{c f}{c+d x}\right)}{d}} \sqrt{g + \frac{(c + d x) \left(h - \frac{c h}{c+d x}\right)}{d}} \right\}$$

**Problem 73:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + b x) \sqrt{c + d x} \sqrt{1 - f x} \sqrt{1 + f x}} dx$$

Optimal (type 4, 74 leaves, 3 steps):

$$\frac{2 \sqrt{\frac{f(c+d x)}{d+c f}} \text{EllipticPi}\left[\frac{2 b}{b+a f}, \text{ArcSin}\left[\frac{\sqrt{1-f x}}{\sqrt{2}}\right], \frac{2 d}{d+c f}\right]}{(b+a f) \sqrt{c+d x}}$$

Result (type 4, 203 leaves):

$$\begin{aligned} & \left. \left( 2 \pm (c + d x) \sqrt{\frac{d (-1 + f x)}{f (c + d x)}} \sqrt{\frac{d + d f x}{c f + d f x}} \right. \right. \\ & \left. \left. \left. \text{EllipticF}\left[\pm \text{ArcSinh}\left[\frac{\sqrt{-\frac{d+c f}{f}}}{\sqrt{c+d x}}\right], \frac{-d+c f}{d+c f}\right] - \text{EllipticPi}\left[\frac{b c f - a d f}{b d + b c f}, \pm \text{ArcSinh}\left[\frac{\sqrt{-\frac{d+c f}{f}}}{\sqrt{c+d x}}\right], \frac{-d+c f}{d+c f}\right]\right)\right) / \\ & \left. \left( (-b c + a d) \sqrt{-\frac{d+c f}{f}} \sqrt{1 - f^2 x^2} \right) \right) \end{aligned}$$

**Problem 74:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + b x) \sqrt{c + d x} \sqrt{1 - f^2 x^2}} dx$$

Optimal (type 4, 74 leaves, 4 steps):

$$-\frac{2 \sqrt{\frac{f(c+dx)}{d+c f}} \operatorname{EllipticPi}\left[\frac{2 b}{b+a f}, \operatorname{ArcSin}\left[\frac{\sqrt{1-f x}}{\sqrt{2}}\right], \frac{2 d}{d+c f}\right]}{(b+a f) \sqrt{c+d x}}$$

Result (type 4, 203 leaves):

$$\begin{aligned} & \left(2 \pm (c+d x) \sqrt{\frac{d(-1+f x)}{f(c+d x)}} \sqrt{\frac{d+d f x}{c f+d f x}}\right. \\ & \left.\left(\operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{-\frac{d+c f}{f}}}{\sqrt{c+d x}}\right], \frac{-d+c f}{d+c f}\right]-\operatorname{EllipticPi}\left[\frac{b c f-a d f}{b d+b c f}, \pm \operatorname{ArcSinh}\left[\frac{\sqrt{-\frac{d+c f}{f}}}{\sqrt{c+d x}}\right], \frac{-d+c f}{d+c f}\right]\right)\right) / \\ & \left((-b c+a d) \sqrt{-\frac{d+c f}{f}} \sqrt{1-f^2 x^2}\right) \end{aligned}$$

Problem 75: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{(a+b x) \sqrt{c+d x} \sqrt{1-f^2 x} \sqrt{1+f^2 x}} dx$$

Optimal (type 4, 86 leaves, 3 steps):

$$-\frac{2 \sqrt{\frac{f^2(c+dx)}{d+c f^2}} \operatorname{EllipticPi}\left[\frac{2 b}{b+a f^2}, \operatorname{ArcSin}\left[\frac{\sqrt{1-f^2 x}}{\sqrt{2}}\right], \frac{2 d}{d+c f^2}\right]}{(b+a f^2) \sqrt{c+d x}}$$

Result (type 4, 218 leaves):

$$\begin{aligned}
 & \left( 2 \operatorname{i} (c + dx) \sqrt{\frac{d(-1 + f^2 x)}{f^2(c + dx)}} \sqrt{\frac{d(1 + f^2 x)}{f^2(c + dx)}} \right. \\
 & \left. \left. \left( \operatorname{EllipticF}\left[\operatorname{i} \operatorname{ArcSinh}\left[\frac{\sqrt{-c - \frac{d}{f^2}}}{\sqrt{c + dx}}\right], \frac{-d + c f^2}{d + c f^2}\right] - \operatorname{EllipticPi}\left[\frac{(b c - a d) f^2}{b (d + c f^2)}, \operatorname{i} \operatorname{ArcSinh}\left[\frac{\sqrt{-c - \frac{d}{f^2}}}{\sqrt{c + dx}}\right], \frac{-d + c f^2}{d + c f^2}\right]\right) \right) / \\
 & \left( (-b c + a d) \sqrt{-c - \frac{d}{f^2}} \sqrt{1 - f^4 x^2} \right)
 \end{aligned}$$

Problem 76: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + b x) \sqrt{c + d x} \sqrt{1 - f^4 x^2}} dx$$

Optimal (type 4, 86 leaves, 4 steps):

$$\frac{2 \sqrt{\frac{f^2(c+d x)}{d+c f^2}} \operatorname{EllipticPi}\left[\frac{2 b}{b+a f^2}, \operatorname{ArcSin}\left[\frac{\sqrt{1-f^2 x}}{\sqrt{2}}\right], \frac{2 d}{d+c f^2}\right]}{(b+a f^2) \sqrt{c+d x}}$$

Result (type 4, 218 leaves):

$$\left( 2 \pm (c + d x) \sqrt{\frac{d (-1 + f^2 x)}{f^2 (c + d x)}} \sqrt{\frac{d (1 + f^2 x)}{f^2 (c + d x)}} \right. \\ \left. \left( \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-c - \frac{d}{f^2}}}{\sqrt{c + d x}} \right], \frac{-d + c f^2}{d + c f^2} \right] - \text{EllipticPi} \left[ \frac{(b c - a d) f^2}{b (d + c f^2)}, \pm \text{ArcSinh} \left[ \frac{\sqrt{-c - \frac{d}{f^2}}}{\sqrt{c + d x}} \right], \frac{-d + c f^2}{d + c f^2} \right] \right) \right) / \\ \left( (-b c + a d) \sqrt{-c - \frac{d}{f^2}} \sqrt{1 - f^4 x^2} \right)$$

**Problem 97: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{2 - 3 x}}{\sqrt{-5 + 2 x} \sqrt{1 + 4 x} (7 + 5 x)^{3/2}} dx$$

Optimal (type 4, 60 leaves, 5 steps):

$$\frac{2 \sqrt{\frac{11}{39}} \sqrt{5 - 2 x} \text{EllipticE} \left[ \text{ArcSin} \left[ \frac{\sqrt{\frac{39}{22}} \sqrt{1+4 x}}{\sqrt{7+5 x}} \right], \frac{62}{39} \right]}{23 \sqrt{-5 + 2 x}}$$

Result (type 4, 237 leaves):

$$\left( \sqrt{-5 + 2 x} \sqrt{1 + 4 x} \right. \\ \left. - 1922 \sqrt{\frac{7 + 5 x}{-2 + 3 x}} (-5 - 18 x + 8 x^2) + 62 \sqrt{682} \sqrt{\frac{-5 - 18 x + 8 x^2}{(2 - 3 x)^2}} (-14 + 11 x + 15 x^2) \text{EllipticE} \left[ \text{ArcSin} \left[ \sqrt{\frac{31}{39}} \sqrt{\frac{-5 + 2 x}{-2 + 3 x}} \right], \frac{39}{62} \right] - \right. \\ \left. 23 \sqrt{682} \sqrt{\frac{-5 - 18 x + 8 x^2}{(2 - 3 x)^2}} (-14 + 11 x + 15 x^2) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{31}{39}} \sqrt{\frac{-5 + 2 x}{-2 + 3 x}} \right], \frac{39}{62} \right] \right) / \\ \left( 27807 \sqrt{2 - 3 x} \sqrt{7 + 5 x} \sqrt{\frac{7 + 5 x}{-2 + 3 x}} (-5 - 18 x + 8 x^2) \right)$$

### Problem 99: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{a+b x} \sqrt{c+d x}}{\sqrt{e+f x} \sqrt{g+h x}} dx$$

Optimal (type 4, 721 leaves, 7 steps):

$$\begin{aligned} & \frac{\sqrt{a+b x} \sqrt{c+d x} \sqrt{g+h x}}{h \sqrt{e+f x}} - \frac{\sqrt{d g-c h} \sqrt{f g-e h} \sqrt{a+b x} \sqrt{\frac{(d e-c f) (g+h x)}{(d g-c h) (e+f x)}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{f g-e h} \sqrt{c+d x}}{\sqrt{d g-c h} \sqrt{e+f x}}\right], \frac{(b e-a f) (d g-c h)}{(b c-a d) (f g-e h)}\right]}{f h \sqrt{-\frac{(d e-c f) (a+b x)}{(b c-a d) (e+f x)}} \sqrt{g+h x}} + \\ & \left( (d e-c f) (b f g+b e h-2 a f h) \sqrt{\frac{(b e-a f) (c+d x)}{(d e-c f) (a+b x)}} \sqrt{g+h x} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{b g-a h} \sqrt{e+f x}}{\sqrt{f g-e h} \sqrt{a+b x}}\right], -\frac{(b c-a d) (f g-e h)}{(d e-c f) (b g-a h)}\right] \right) / \\ & \left( f^2 h \sqrt{b g-a h} \sqrt{f g-e h} \sqrt{c+d x} \sqrt{-\frac{(b e-a f) (g+h x)}{(f g-e h) (a+b x)}} + \right. \\ & \left. \left( \sqrt{b g-a h} (a d f h-b (d f g+d e h-c f h)) \sqrt{\frac{(f g-e h) (a+b x)}{(b g-a h) (e+f x)}} \sqrt{\frac{(f g-e h) (c+d x)}{(d g-c h) (e+f x)}} (e+f x) \right. \right. \\ & \left. \left. \operatorname{EllipticPi}\left[\frac{f (b g-a h)}{(b e-a f) h}, \operatorname{ArcSin}\left[\frac{\sqrt{b e-a f} \sqrt{g+h x}}{\sqrt{b g-a h} \sqrt{e+f x}}\right], \frac{(d e-c f) (b g-a h)}{(b e-a f) (d g-c h)}\right]\right) / \left( f^2 \sqrt{b e-a f} h^2 \sqrt{a+b x} \sqrt{c+d x} \right) \right) \end{aligned}$$

Result (type 4, 6667 leaves):

$$\begin{aligned} & -\frac{1}{f} 2 \left( -\frac{\sqrt{e+f x} \left(h + \frac{f g}{e+f x} - \frac{e h}{e+f x}\right) \sqrt{a + \frac{(e+f x) \left(b - \frac{b e}{e+f x}\right)}{f}} \sqrt{c + \frac{(e+f x) \left(d - \frac{d e}{e+f x}\right)}{f}}}{2 h \sqrt{g + \frac{(e+f x) \left(h - \frac{e h}{e+f x}\right)}{f}}} + \right. \\ & \left. \frac{1}{2 h^2 \sqrt{e+f x} \left(b - \frac{b e}{e+f x} + \frac{a f}{e+f x}\right) \left(d - \frac{d e}{e+f x} + \frac{c f}{e+f x}\right) \sqrt{g + \frac{(e+f x) \left(h - \frac{e h}{e+f x}\right)}{f}}} f (b g-a h) (f g-e h) \right. \\ & \left. \sqrt{\left(b - \frac{b e}{e+f x} + \frac{a f}{e+f x}\right) \left(d - \frac{d e}{e+f x} + \frac{c f}{e+f x}\right) \left(h + \frac{f g}{e+f x} - \frac{e h}{e+f x}\right)} \sqrt{a + \frac{(e+f x) \left(b - \frac{b e}{e+f x}\right)}{f}} \sqrt{c + \frac{(e+f x) \left(d - \frac{d e}{e+f x}\right)}{f}} \right) \end{aligned}$$

$$\begin{aligned}
& \left( d \sqrt{\frac{-\frac{b}{b e - a f} + \frac{1}{e + f x}}{-\frac{b}{b e - a f} + \frac{h}{-f g + e h}}} \sqrt{\frac{-\frac{d}{d e - c f} + \frac{1}{e + f x}}{-\frac{d}{d e - c f} + \frac{h}{-f g + e h}}} \left( -\frac{h}{-f g + e h} + \frac{1}{e + f x} \right) \text{EllipticF}[\text{ArcSin}\left[ \sqrt{\frac{(d e - c f) \left(-h - \frac{f g}{e + f x} + \frac{e h}{e + f x}\right)}{f (-d g + c h)}} \right], \right. \\
& \left. \left. \frac{(b e - a f) \left(-d g + c h\right)}{(d e - c f) \left(-b g + a h\right)} \right] \right) / \left( \sqrt{\frac{-\frac{h}{-f g + e h} + \frac{1}{e + f x}}{\frac{d}{d e - c f} - \frac{h}{-f g + e h}}} \sqrt{\left(b + \frac{-b e + a f}{e + f x}\right) \left(d + \frac{-d e + c f}{e + f x}\right) \left(h + \frac{f g - e h}{e + f x}\right)} - \right. \\
& \left. \left( d e \sqrt{-\frac{(b e - a f) \left(-f g + e h\right) \left(-\frac{b}{b e - a f} + \frac{1}{e + f x}\right)}{-b f g + a f h}} \left(-\frac{d}{d e - c f} + \frac{1}{e + f x}\right) \sqrt{\frac{-\frac{h}{-f g + e h} + \frac{1}{e + f x}}{\frac{d}{d e - c f} - \frac{h}{-f g + e h}}} \right. \right. \\
& \left. \left. \left( -b f g + a f h \right) \text{EllipticE}[\text{ArcSin}\left[ \sqrt{\frac{(d e - c f) \left(-h - \frac{f g}{e + f x} + \frac{e h}{e + f x}\right)}{f (-d g + c h)}} \right], \frac{(b e - a f) \left(-d g + c h\right)}{(d e - c f) \left(-b g + a h\right)} \right] \right. \\
& \left. \left. \left. \left. \frac{(b e - a f) \left(-f g + e h\right)}{(b e - a f) \left(-f g + e h\right)} - b \text{EllipticF}[\text{ArcSin}\left[ \sqrt{\frac{(d e - c f) \left(-h - \frac{f g}{e + f x} + \frac{e h}{e + f x}\right)}{f (-d g + c h)}} \right], \frac{(b e - a f) \left(-d g + c h\right)}{(d e - c f) \left(-b g + a h\right)} \right] \right) \right) / \right. \\
& \left. \left( \sqrt{\frac{-\frac{d}{d e - c f} + \frac{1}{e + f x}}{-\frac{d}{d e - c f} + \frac{h}{-f g + e h}}} \sqrt{\left(b + \frac{-b e + a f}{e + f x}\right) \left(d + \frac{-d e + c f}{e + f x}\right) \left(h + \frac{f g - e h}{e + f x}\right)} + c f \sqrt{-\frac{(b e - a f) \left(-f g + e h\right) \left(-\frac{b}{b e - a f} + \frac{1}{e + f x}\right)}{-b f g + a f h}} \right. \\
& \left. \left. \left( -\frac{d}{d e - c f} + \frac{1}{e + f x} \right) \sqrt{\frac{-\frac{h}{-f g + e h} + \frac{1}{e + f x}}{\frac{d}{d e - c f} - \frac{h}{-f g + e h}}} \left( -b f g + a f h \right) \text{EllipticE}[\text{ArcSin}\left[ \sqrt{\frac{(d e - c f) \left(-h - \frac{f g}{e + f x} + \frac{e h}{e + f x}\right)}{f (-d g + c h)}} \right], \frac{(b e - a f) \left(-d g + c h\right)}{(d e - c f) \left(-b g + a h\right)} \right] \right. \\
& \left. \left. \left. \left. \frac{(b e - a f) \left(-f g + e h\right)}{(b e - a f) \left(-f g + e h\right)} - \right. \right) \right)
\end{aligned}$$

$$\left. \frac{b \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(-h - \frac{f g}{e+f x} + \frac{e h}{e+f x}\right)}{f (-d g + c h)}}\right], \frac{(b e - a f) \left(-d g + c h\right)}{(d e - c f) \left(-b g + a h\right)}]}{b e - a f} \right\}$$

$$\left. \left( \sqrt{\frac{-\frac{d}{d e - c f} + \frac{1}{e+f x}}{-\frac{d}{d e - c f} + \frac{h}{-f g + e h}}} \sqrt{\left(b + \frac{-b e + a f}{e+f x}\right) \left(d + \frac{-d e + c f}{e+f x}\right) \left(h + \frac{f g - e h}{e+f x}\right)} \right) - \right.$$

$$\frac{1}{2 d h \sqrt{e+f x} \left(b - \frac{b e}{e+f x} + \frac{a f}{e+f x}\right) \left(d - \frac{d e}{e+f x} + \frac{c f}{e+f x}\right) \sqrt{g + \frac{(e+f x) \left(h - \frac{e h}{e+f x}\right)}{f}}} (b c - a d) f (-d e + c f)$$

$$\sqrt{\left(b - \frac{b e}{e+f x} + \frac{a f}{e+f x}\right) \left(d - \frac{d e}{e+f x} + \frac{c f}{e+f x}\right) \left(h + \frac{f g}{e+f x} - \frac{e h}{e+f x}\right)}$$

$$\sqrt{a + \frac{(e+f x) \left(b - \frac{b e}{e+f x}\right)}{f}} \sqrt{c + \frac{(e+f x) \left(d - \frac{d e}{e+f x}\right)}{f}}$$

$$\left. \left( h \sqrt{\frac{-\frac{b}{b e - a f} + \frac{1}{e+f x}}{-\frac{b}{b e - a f} + \frac{h}{-f g + e h}}} \sqrt{\frac{-\frac{d}{d e - c f} + \frac{1}{e+f x}}{-\frac{d}{d e - c f} + \frac{h}{-f g + e h}}} \left(-\frac{h}{-f g + e h} + \frac{1}{e+f x}\right) \right) \right.$$

$$\operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(-h - \frac{f g}{e+f x} + \frac{e h}{e+f x}\right)}{f (-d g + c h)}}\right], \frac{(b e - a f) \left(-d g + c h\right)}{(d e - c f) \left(-b g + a h\right)}] \right\}$$

$$\left. \left( \sqrt{\frac{-\frac{h}{-f g + e h} + \frac{1}{e+f x}}{\frac{d}{d e - c f} - \frac{h}{-f g + e h}}} \sqrt{\left(b + \frac{-b e + a f}{e+f x}\right) \left(d + \frac{-d e + c f}{e+f x}\right) \left(h + \frac{f g - e h}{e+f x}\right)} \right) + f g \sqrt{-\frac{(b e - a f) \left(-f g + e h\right) \left(-\frac{b}{b e - a f} + \frac{1}{e+f x}\right)}{-b f g + a f h}} \right\}$$

$$\begin{aligned}
& \left( -\frac{d}{de - cf} + \frac{1}{e + fx} \right) \sqrt{\frac{-\frac{h}{-fg + eh} + \frac{1}{e+fx}}{\frac{d}{de - cf} - \frac{h}{-fg + eh}}} \left( \frac{(-bf g + af h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(de - cf)(-h - \frac{fg}{e+fx} + \frac{eh}{e+fx})}{f(-dg + ch)}}, \frac{(be - af)(-dg + ch)}{(de - cf)(-bg + ah)}\right]}]}{(be - af)(-fg + eh)} - \right. \\
& \left. \frac{b \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(de - cf)(-h - \frac{fg}{e+fx} + \frac{eh}{e+fx})}{f(-dg + ch)}}, \frac{(be - af)(-dg + ch)}{(de - cf)(-bg + ah)}\right]]}{be - af} \right) / \\
& \left( \sqrt{\frac{-\frac{d}{de - cf} + \frac{1}{e+fx}}{-\frac{d}{de - cf} + \frac{h}{-fg + eh}}} \sqrt{\left(b + \frac{-be + af}{e+fx}\right) \left(d + \frac{-de + cf}{e+fx}\right) \left(h + \frac{fg - eh}{e+fx}\right)} - e h \sqrt{-\frac{(be - af)(-fg + eh)\left(-\frac{b}{be - af} + \frac{1}{e+fx}\right)}{-bf g + af h}} \right. \\
& \left. \left( -\frac{d}{de - cf} + \frac{1}{e+fx} \right) \sqrt{\frac{-\frac{h}{-fg + eh} + \frac{1}{e+fx}}{\frac{d}{de - cf} - \frac{h}{-fg + eh}}} \left( \frac{(-bf g + af h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(de - cf)(-h - \frac{fg}{e+fx} + \frac{eh}{e+fx})}{f(-dg + ch)}}, \frac{(be - af)(-dg + ch)}{(de - cf)(-bg + ah)}\right]}]}{(be - af)(-fg + eh)} - \right. \right. \\
& \left. \left. \frac{b \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(de - cf)(-h - \frac{fg}{e+fx} + \frac{eh}{e+fx})}{f(-dg + ch)}}, \frac{(be - af)(-dg + ch)}{(de - cf)(-bg + ah)}\right]]}{be - af} \right) / \right. \\
& \left. \left( \sqrt{\frac{-\frac{d}{de - cf} + \frac{1}{e+fx}}{-\frac{d}{de - cf} + \frac{h}{-fg + eh}}} \sqrt{\left(b + \frac{-be + af}{e+fx}\right) \left(d + \frac{-de + cf}{e+fx}\right) \left(h + \frac{fg - eh}{e+fx}\right)} - \right. \right. \\
& \left. \left. \frac{1}{2 dh^2 \sqrt{e+fx} \left(b - \frac{be}{e+fx} + \frac{af}{e+fx}\right) \left(d - \frac{de}{e+fx} + \frac{cf}{e+fx}\right) \sqrt{g + \frac{(e+fx)(h - \frac{eh}{e+fx})}{f}}} (b d f g + b d e h - b c f h - a d f h) \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{a + \frac{(e+f x) \left(b - \frac{b e}{e+f x}\right)}{f}} \sqrt{c + \frac{(e+f x) \left(d - \frac{d e}{e+f x}\right)}{f}} \\
& \left( d f g \sqrt{\frac{-\frac{b}{b e-a f} + \frac{1}{e+f x}}{-\frac{b}{b e-a f} + \frac{h}{-f g+e h}}} \sqrt{\frac{-\frac{d}{d e-c f} + \frac{1}{e+f x}}{-\frac{d}{d e-c f} + \frac{h}{-f g+e h}}} \left( -\frac{h}{-f g+e h} + \frac{1}{e+f x} \right) \text{EllipticF}[\text{ArcSin}\left[ \sqrt{\frac{(d e-c f) \left(-h - \frac{f g}{e+f x} + \frac{e h}{e+f x}\right)}{f (-d g+c h)}} \right], \right. \\
& \left. \frac{(b e-a f) \left(-d g+c h\right)}{(d e-c f) \left(-b g+a h\right)} \right] \Bigg) / \left( \sqrt{\frac{-\frac{h}{-f g+e h} + \frac{1}{e+f x}}{\frac{d}{d e-c f} - \frac{h}{-f g+e h}}} \sqrt{\left( b + \frac{-b e+a f}{e+f x} \right) \left( d + \frac{-d e+c f}{e+f x} \right) \left( h + \frac{f g-e h}{e+f x} \right)} - \right. \\
& \left. 2 d e h \sqrt{\frac{-\frac{b}{b e-a f} + \frac{1}{e+f x}}{-\frac{b}{b e-a f} + \frac{h}{-f g+e h}}} \sqrt{\frac{-\frac{d}{d e-c f} + \frac{1}{e+f x}}{-\frac{d}{d e-c f} + \frac{h}{-f g+e h}}} \left( -\frac{h}{-f g+e h} + \frac{1}{e+f x} \right) \text{EllipticF}[\text{ArcSin}\left[ \sqrt{\frac{(d e-c f) \left(-h - \frac{f g}{e+f x} + \frac{e h}{e+f x}\right)}{f (-d g+c h)}} \right], \right. \\
& \left. \frac{(b e-a f) \left(-d g+c h\right)}{(d e-c f) \left(-b g+a h\right)} \right] \Bigg) / \left( \sqrt{\frac{-\frac{h}{-f g+e h} + \frac{1}{e+f x}}{\frac{d}{d e-c f} - \frac{h}{-f g+e h}}} \sqrt{\left( b + \frac{-b e+a f}{e+f x} \right) \left( d + \frac{-d e+c f}{e+f x} \right) \left( h + \frac{f g-e h}{e+f x} \right)} + \left( c f h \sqrt{\frac{-\frac{b}{b e-a f} + \frac{1}{e+f x}}{-\frac{b}{b e-a f} + \frac{h}{-f g+e h}}} \right. \right. \\
& \left. \left. \sqrt{\frac{-\frac{d}{d e-c f} + \frac{1}{e+f x}}{-\frac{d}{d e-c f} + \frac{h}{-f g+e h}}} \left( -\frac{h}{-f g+e h} + \frac{1}{e+f x} \right) \text{EllipticF}[\text{ArcSin}\left[ \sqrt{\frac{(d e-c f) \left(-h - \frac{f g}{e+f x} + \frac{e h}{e+f x}\right)}{f (-d g+c h)}} \right], \frac{(b e-a f) \left(-d g+c h\right)}{(d e-c f) \left(-b g+a h\right)} \right] \right) / \right. \\
& \left. \left( \sqrt{\frac{-\frac{h}{-f g+e h} + \frac{1}{e+f x}}{\frac{d}{d e-c f} - \frac{h}{-f g+e h}}} \sqrt{\left( b + \frac{-b e+a f}{e+f x} \right) \left( d + \frac{-d e+c f}{e+f x} \right) \left( h + \frac{f g-e h}{e+f x} \right)} - \left( d e f g \sqrt{-\frac{(b e-a f) \left(-f g+e h\right) \left(-\frac{b}{b e-a f} + \frac{1}{e+f x}\right)}{-b f g+a f h}} \right. \right. \right. \\
& \left. \left. \left. \left( -\frac{d}{d e-c f} + \frac{1}{e+f x} \right) \sqrt{\frac{-\frac{h}{-f g+e h} + \frac{1}{e+f x}}{\frac{d}{d e-c f} - \frac{h}{-f g+e h}}} \frac{(-b f g+a f h) \text{EllipticE}[\text{ArcSin}\left[ \sqrt{\frac{(d e-c f) \left(-h - \frac{f g}{e+f x} + \frac{e h}{e+f x}\right)}{f (-d g+c h)}} \right], \frac{(b e-a f) \left(-d g+c h\right)}{(d e-c f) \left(-b g+a h\right)}]}{(b e-a f) \left(-f g+e h\right)} \right) - \right. \right. \right. 
\end{aligned}$$

$$\begin{aligned}
& \left. \left. \left. \frac{b \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(-h - \frac{f g}{e+f x} + \frac{e h}{e+f x}\right)}{f (-d g+c h)}}\right], \frac{(b e - a f) \left(-d g+c h\right)}{(d e - c f) \left(-b g+a h\right)}]}{b e - a f} \right] \right\} / \right. \\
& \left( \sqrt{\frac{-\frac{d}{d e - c f} + \frac{1}{e+f x}}{-\frac{d}{d e - c f} + \frac{h}{-f g+e h}}} \sqrt{\left(b + \frac{-b e + a f}{e+f x}\right) \left(d + \frac{-d e + c f}{e+f x}\right) \left(h + \frac{f g - e h}{e+f x}\right)} + c f^2 g \sqrt{-\frac{(b e - a f) \left(-f g + e h\right) \left(-\frac{b}{b e - a f} + \frac{1}{e+f x}\right)}{-b f g + a f h}} \right. \\
& \left. \left( -\frac{d}{d e - c f} + \frac{1}{e+f x} \right) \sqrt{\frac{-\frac{h}{-f g+e h} + \frac{1}{e+f x}}{\frac{d}{d e - c f} - \frac{h}{-f g+e h}}} \left( \frac{(-b f g + a f h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(-h - \frac{f g}{e+f x} + \frac{e h}{e+f x}\right)}{f (-d g+c h)}}\right], \frac{(b e - a f) \left(-d g+c h\right)}{(d e - c f) \left(-b g+a h\right)}]}{(b e - a f) \left(-f g + e h\right)} - \right. \right. \\
& \left. \left. b \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(-h - \frac{f g}{e+f x} + \frac{e h}{e+f x}\right)}{f (-d g+c h)}}\right], \frac{(b e - a f) \left(-d g+c h\right)}{(d e - c f) \left(-b g+a h\right)}]}{b e - a f} \right] \right\} / \right. \\
& \left( \sqrt{\frac{-\frac{d}{d e - c f} + \frac{1}{e+f x}}{-\frac{d}{d e - c f} + \frac{h}{-f g+e h}}} \sqrt{\left(b + \frac{-b e + a f}{e+f x}\right) \left(d + \frac{-d e + c f}{e+f x}\right) \left(h + \frac{f g - e h}{e+f x}\right)} + d e^2 h \sqrt{-\frac{(b e - a f) \left(-f g + e h\right) \left(-\frac{b}{b e - a f} + \frac{1}{e+f x}\right)}{-b f g + a f h}} \right. \\
& \left. \left( -\frac{d}{d e - c f} + \frac{1}{e+f x} \right) \sqrt{\frac{-\frac{h}{-f g+e h} + \frac{1}{e+f x}}{\frac{d}{d e - c f} - \frac{h}{-f g+e h}}} \left( \frac{(-b f g + a f h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(-h - \frac{f g}{e+f x} + \frac{e h}{e+f x}\right)}{f (-d g+c h)}}\right], \frac{(b e - a f) \left(-d g+c h\right)}{(d e - c f) \left(-b g+a h\right)}]}{(b e - a f) \left(-f g + e h\right)} - \right. \right. \\
& \left. \left. b \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(-h - \frac{f g}{e+f x} + \frac{e h}{e+f x}\right)}{f (-d g+c h)}}\right], \frac{(b e - a f) \left(-d g+c h\right)}{(d e - c f) \left(-b g+a h\right)}]}{b e - a f} \right] \right\} /
\end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{\frac{-\frac{d}{de-cf} + \frac{1}{e+fx}}{-\frac{d}{de-cf} + \frac{h}{-fg+eh}}} \sqrt{\left( b + \frac{-be+af}{e+fx} \right) \left( d + \frac{-de+cf}{e+fx} \right) \left( h + \frac{fg-eh}{e+fx} \right)} \right) - \left( cefh \sqrt{-\frac{(be-af)(-fg+eh)\left(-\frac{b}{be-af} + \frac{1}{e+fx}\right)}{-bfg+afh}} \right. \\
& \left( -\frac{d}{de-cf} + \frac{1}{e+fx} \right) \sqrt{\frac{-\frac{h}{-fg+eh} + \frac{1}{e+fx}}{\frac{d}{de-cf} - \frac{h}{-fg+eh}}} \left( \frac{(-bfg+afh) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(de-cf)(-h-\frac{fg}{e+fx}+\frac{eh}{e+fx})}{f(-dg+ch)}}\right], \frac{(be-af)(-dg+ch)}{(de-cf)(-bg+ah)}]}{(be-af)(-fg+eh)} \right. \\
& \left. \left. \frac{b \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(de-cf)(-h-\frac{fg}{e+fx}+\frac{eh}{e+fx})}{f(-dg+ch)}}\right], \frac{(be-af)(-dg+ch)}{(de-cf)(-bg+ah)}]}{be-af} \right) \right) / \\
& \left( \sqrt{\frac{-\frac{d}{de-cf} + \frac{1}{e+fx}}{-\frac{d}{de-cf} + \frac{h}{-fg+eh}}} \sqrt{\left( b + \frac{-be+af}{e+fx} \right) \left( d + \frac{-de+cf}{e+fx} \right) \left( h + \frac{fg-eh}{e+fx} \right)} \right) - \\
& \left( d(-fg+eh) \left( -\frac{d}{de-cf} + \frac{h}{-fg+eh} \right) \sqrt{\frac{-\frac{b}{be-af} + \frac{1}{e+fx}}{-\frac{b}{be-af} + \frac{h}{-fg+eh}}} \sqrt{-\frac{\left(-\frac{d}{de-cf} + \frac{1}{e+fx}\right) \left(-\frac{h}{-fg+eh} + \frac{1}{e+fx}\right)}{\left(-\frac{d}{de-cf} + \frac{h}{-fg+eh}\right)^2}} \operatorname{EllipticPi}\left[-\frac{-dfg+cfh}{(de-cf)h}, \right. \right. \\
& \left. \left. \operatorname{ArcSin}\left[\sqrt{\frac{(de-cf)(-h-\frac{fg}{e+fx}+\frac{eh}{e+fx})}{f(-dg+ch)}}, \frac{(be-af)(-dg+ch)}{(de-cf)(-bg+ah)}\right]\right] / \sqrt{\left( b + \frac{-be+af}{e+fx} \right) \left( d + \frac{-de+cf}{e+fx} \right) \left( h + \frac{fg-eh}{e+fx} \right)} \right)
\end{aligned}$$

**Problem 107: Result more than twice size of optimal antiderivative.**

$$\int \frac{(ax+b)^{3/2}}{\sqrt{c+dx} \sqrt{e+fx} \sqrt{g+hx}} dx$$

Optimal (type 4, 968 leaves, 10 steps):

$$\begin{aligned}
& \frac{b \sqrt{a+b x} \sqrt{c+d x} \sqrt{g+h x}}{d h \sqrt{e+f x}} - \frac{b \sqrt{d g-c h} \sqrt{f g-e h} \sqrt{a+b x}}{\sqrt{\frac{(d e-c f) (g+h x)}{(d g-c h) (e+f x)}}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{f g-e h} \sqrt{c+d x}}{\sqrt{d g-c h} \sqrt{e+f x}}\right], \frac{(b e-a f) (d g-c h)}{(b c-a d) (f g-e h)}\right] \\
& + \frac{d f h \sqrt{-\frac{(d e-c f) (a+b x)}{(b c-a d) (e+f x)}} \sqrt{g+h x}}{} \\
& \left( b (d e-c f) (b f g+b e h-2 a f h) \sqrt{\frac{(b e-a f) (c+d x)}{(d e-c f) (a+b x)}} \sqrt{g+h x} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b g-a h} \sqrt{e+f x}}{\sqrt{f g-e h} \sqrt{a+b x}}\right], -\frac{(b c-a d) (f g-e h)}{(d e-c f) (b g-a h)}\right]\right) / \\
& \left( d f^2 h \sqrt{b g-a h} \sqrt{f g-e h} \sqrt{c+d x} \sqrt{-\frac{(b e-a f) (g+h x)}{(f g-e h) (a+b x)}}\right) + \\
& \left( b \sqrt{b g-a h} (a d f h - b (d f g+d e h-c f h)) \sqrt{\frac{(f g-e h) (a+b x)}{(b g-a h) (e+f x)}} \sqrt{\frac{(f g-e h) (c+d x)}{(d g-c h) (e+f x)}} (e+f x)\right. \\
& \left. \text{EllipticPi}\left[\frac{f (b g-a h)}{(b e-a f) h}, \text{ArcSin}\left[\frac{\sqrt{b e-a f} \sqrt{g+h x}}{\sqrt{b g-a h} \sqrt{e+f x}}\right], \frac{(d e-c f) (b g-a h)}{(b e-a f) (d g-c h)}\right]\right) / \left( d f^2 \sqrt{b e-a f} h^2 \sqrt{a+b x} \sqrt{c+d x}\right) - \\
& \frac{1}{d h \sqrt{c+d x} \sqrt{e+f x}} 2 \sqrt{b c-a d} \sqrt{-d g+c h} (a+b x) \sqrt{\frac{(b g-a h) (c+d x)}{(d g-c h) (a+b x)}} \sqrt{\frac{(b g-a h) (e+f x)}{(f g-e h) (a+b x)}} \\
& \text{EllipticPi}\left[-\frac{b (d g-c h)}{(b c-a d) h}, \text{ArcSin}\left[\frac{\sqrt{b c-a d} \sqrt{g+h x}}{\sqrt{-d g+c h} \sqrt{a+b x}}\right], \frac{(b e-a f) (d g-c h)}{(b c-a d) (f g-e h)}\right]
\end{aligned}$$

Result (type 4, 6638 leaves):

$$\begin{aligned}
& -\frac{1}{d^2} 2 \left( -\frac{b (c+d x)^{3/2} \left(f + \frac{d e}{c+d x} - \frac{c f}{c+d x}\right) \left(h + \frac{d g}{c+d x} - \frac{c h}{c+d x}\right) \sqrt{a + \frac{(c+d x) \left(b - \frac{b c}{c+d x}\right)}{d}}}{2 f h \sqrt{e + \frac{(c+d x) \left(f - \frac{c f}{c+d x}\right)}{d}}} \sqrt{g + \frac{(c+d x) \left(h - \frac{c h}{c+d x}\right)}{d}} + \right. \\
& \left. \left( d (b g-a h) (d g-c h) (b f g+b e h-2 a f h) \sqrt{c+d x} \sqrt{\left(b - \frac{b c}{c+d x} + \frac{a d}{c+d x}\right) \left(f + \frac{d e}{c+d x} - \frac{c f}{c+d x}\right) \left(h + \frac{d g}{c+d x} - \frac{c h}{c+d x}\right)} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{a + \frac{(c+d x) \left(b - \frac{b c}{c+d x}\right)}{d}} \left( d e \sqrt{-\frac{(b c - a d) (-d g + c h) \left(-\frac{b}{b c-a d} + \frac{1}{c+d x}\right)}{-b d g + a d h}} \left( -\frac{f}{-d e + c f} + \frac{1}{c + d x} \right) \right. \\
& \left. \sqrt{\frac{-\frac{h}{-d g+c h} + \frac{1}{c+d x}}{\frac{f}{-d e+c f} - \frac{h}{-d g+c h}}} \left( \frac{(-b d g + a d h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(d e-c f) \left(h+\frac{d g}{c+d x}-\frac{c h}{c+d x}\right)}{d (-f g+e h)}}\right], \frac{(b c-a d) (-f g+e h)}{(-d e+c f) (-b g+a h)}] - \right. \right. \\
& \left. \left. b \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(d e-c f) \left(h+\frac{d g}{c+d x}-\frac{c h}{c+d x}\right)}{d (-f g+e h)}}\right], \frac{(b c-a d) (-f g+e h)}{(-d e+c f) (-b g+a h)}]\right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-d e+c f} + \frac{1}{c+d x}}{-\frac{f}{-d e+c f} + \frac{h}{-d g+c h}}} \sqrt{\left(b + \frac{-b c + a d}{c + d x}\right) \left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} - \right. \left. c f \sqrt{-\frac{(b c - a d) (-d g + c h) \left(-\frac{b}{b c-a d} + \frac{1}{c+d x}\right)}{-b d g + a d h}} \right. \\
& \left( -\frac{f}{-d e + c f} + \frac{1}{c + d x} \right) \sqrt{\frac{-\frac{h}{-d g+c h} + \frac{1}{c+d x}}{\frac{f}{-d e+c f} - \frac{h}{-d g+c h}}} \left( \frac{(-b d g + a d h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(d e-c f) \left(h+\frac{d g}{c+d x}-\frac{c h}{c+d x}\right)}{d (-f g+e h)}}\right], \frac{(b c-a d) (-f g+e h)}{(-d e+c f) (-b g+a h)}] - \right. \right. \\
& \left. \left. b \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(d e-c f) \left(h+\frac{d g}{c+d x}-\frac{c h}{c+d x}\right)}{d (-f g+e h)}}\right], \frac{(b c-a d) (-f g+e h)}{(-d e+c f) (-b g+a h)}]\right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-d e+c f} + \frac{1}{c+d x}}{-\frac{f}{-d e+c f} + \frac{h}{-d g+c h}}} \sqrt{\left(b + \frac{-b c + a d}{c + d x}\right) \left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} + \right. \left. f \sqrt{\frac{-\frac{b}{b c-a d} + \frac{1}{c+d x}}{-\frac{b}{b c-a d} + \frac{h}{-d g+c h}}} \sqrt{\frac{-\frac{f}{-d e+c f} + \frac{1}{c+d x}}{-\frac{f}{-d e+c f} + \frac{h}{-d g+c h}}} \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{h}{-dg + ch} + \frac{1}{c + dx} \right) \text{EllipticF}[\text{ArcSin}\left[ \sqrt{\frac{(-de + cf) \left( -h - \frac{dg}{c+dx} + \frac{ch}{c+dx} \right)}{d (-fg + eh)}} \right], \frac{(bc - ad) \left( -fg + eh \right)}{(-de + cf) \left( -bg + ah \right)}] \Bigg) / \\
& \left( \sqrt{\frac{-\frac{h}{-dg + ch} + \frac{1}{c+dx}}{\frac{f}{-de + cf} - \frac{h}{-dg + ch}}} \sqrt{\left( b + \frac{-bc + ad}{c + dx} \right) \left( f + \frac{de - cf}{c + dx} \right) \left( h + \frac{dg - ch}{c + dx} \right)} \right) \Bigg) / \\
& \left( 2fh^2 (fg - eh) \left( b - \frac{bc}{c + dx} + \frac{ad}{c + dx} \right) \sqrt{e + \frac{(c + dx) \left( f - \frac{cf}{c+dx} \right)}{d}} \sqrt{g + \frac{(c + dx) \left( h - \frac{ch}{c+dx} \right)}{d}} \right) - \\
& \left( d (be - af) (de - cf) (bf g + be h - 2af h) \sqrt{c + dx} \sqrt{\left( b - \frac{bc}{c + dx} + \frac{ad}{c + dx} \right) \left( f + \frac{de}{c + dx} - \frac{cf}{c + dx} \right) \left( h + \frac{dg}{c + dx} - \frac{ch}{c + dx} \right)} \right. \\
& \left. \sqrt{a + \frac{(c + dx) \left( b - \frac{bc}{c+dx} \right)}{d}} \left( dg \sqrt{-\frac{(bc - ad) \left( -dg + ch \right) \left( -\frac{b}{bc-ad} + \frac{1}{c+dx} \right)}{-bdg + adh}} \left( -\frac{f}{-de + cf} + \frac{1}{c + dx} \right) \right. \right. \\
& \left. \left. \sqrt{-\frac{h}{-dg + ch} + \frac{1}{c+dx}} \frac{(-bdg + adh) \text{EllipticE}[\text{ArcSin}\left[ \sqrt{\frac{(de - cf) \left( h + \frac{dg}{c+dx} - \frac{ch}{c+dx} \right)}{d (-fg + eh)}} \right], \frac{(bc - ad) \left( -fg + eh \right)}{(-de + cf) \left( -bg + ah \right)}]}{(bc - ad) \left( -dg + ch \right)} - \right. \right. \\
& \left. \left. b \text{EllipticF}[\text{ArcSin}\left[ \sqrt{\frac{(de - cf) \left( h + \frac{dg}{c+dx} - \frac{ch}{c+dx} \right)}{d (-fg + eh)}} \right], \frac{(bc - ad) \left( -fg + eh \right)}{(-de + cf) \left( -bg + ah \right)}] \right) \right) \Bigg)
\end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{\frac{-\frac{f}{-de+cf} + \frac{1}{c+dx}}{-\frac{f}{-de+cf} + \frac{h}{-dg+ch}}} \sqrt{\left(b + \frac{-bc+ad}{c+dx}\right) \left(f + \frac{de-cf}{c+dx}\right) \left(h + \frac{dg-ch}{c+dx}\right)} - c h \sqrt{-\frac{(bc-ad)(-dg+ch)\left(-\frac{b}{bc-ad} + \frac{1}{c+dx}\right)}{-bdg+adh}} \right. \\
& \left( -\frac{f}{-de+cf} + \frac{1}{c+dx} \right) \sqrt{\frac{-\frac{h}{-dg+ch} + \frac{1}{c+dx}}{\frac{f}{-de+cf} - \frac{h}{-dg+ch}}} \left( \frac{(-bdg+adh) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(de-cf)(h+\frac{dg}{c+dx}-\frac{ch}{c+dx})}{d(-fg+eh)}}\right], \frac{(bc-ad)(-fg+eh)}{(-de+cf)(-bg+ah)}]}{(bc-ad)(-dg+ch)} - \right. \\
& \left. b \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(de-cf)(h+\frac{dg}{c+dx}-\frac{ch}{c+dx})}{d(-fg+eh)}}\right], \frac{(bc-ad)(-fg+eh)}{(-de+cf)(-bg+ah)}] \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-de+cf} + \frac{1}{c+dx}}{-\frac{f}{-de+cf} + \frac{h}{-dg+ch}}} \sqrt{\left(b + \frac{-bc+ad}{c+dx}\right) \left(f + \frac{de-cf}{c+dx}\right) \left(h + \frac{dg-ch}{c+dx}\right)} + h \sqrt{\frac{-\frac{b}{bc-ad} + \frac{1}{c+dx}}{-\frac{b}{bc-ad} + \frac{h}{-dg+ch}}} \sqrt{\frac{-\frac{f}{-de+cf} + \frac{1}{c+dx}}{-\frac{f}{-de+cf} + \frac{h}{-dg+ch}}} \right. \\
& \left( -\frac{h}{-dg+ch} + \frac{1}{c+dx} \right) \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(-de+cf)(-h-\frac{dg}{c+dx}+\frac{ch}{c+dx})}{d(-fg+eh)}}\right], \frac{(bc-ad)(-fg+eh)}{(-de+cf)(-bg+ah)}] \Big) / \\
& \left( \sqrt{\frac{-\frac{h}{-dg+ch} + \frac{1}{c+dx}}{\frac{f}{-de+cf} - \frac{h}{-dg+ch}}} \sqrt{\left(b + \frac{-bc+ad}{c+dx}\right) \left(f + \frac{de-cf}{c+dx}\right) \left(h + \frac{dg-ch}{c+dx}\right)} \right) / \\
& \left( 2 f^2 h (fg - eh) \left(b - \frac{bc}{c+dx} + \frac{ad}{c+dx}\right) \sqrt{e + \frac{(c+dx)(f - \frac{cf}{c+dx})}{d}} \sqrt{g + \frac{(c+dx)(h - \frac{ch}{c+dx})}{d}} - \right. \\
& \left. \frac{1}{2 f^2 h^2 \left(b - \frac{bc}{c+dx} + \frac{ad}{c+dx}\right) \sqrt{e + \frac{(c+dx)(f - \frac{cf}{c+dx})}{d}} \sqrt{g + \frac{(c+dx)(h - \frac{ch}{c+dx})}{d}}} \right. \\
& \left. b (bd fg + bdeh + bc fh - 3adfh) \sqrt{c+dx} \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\left(b - \frac{bc}{c+dx} + \frac{ad}{c+dx}\right) \left(f + \frac{de}{c+dx} - \frac{cf}{c+dx}\right) \left(h + \frac{dg}{c+dx} - \frac{ch}{c+dx}\right)} \\
& \sqrt{a + \frac{(c+dx) \left(b - \frac{bc}{c+dx}\right)}{d}} \\
& \left( d^2 e g \sqrt{-\frac{(bc-ad)(-dg+ch)\left(-\frac{b}{bc-ad} + \frac{1}{c+dx}\right)}{-bdg+adh}} \left( -\frac{f}{-de+cf} + \frac{1}{c+dx} \right) \sqrt{\frac{-\frac{h}{-dg+ch} + \frac{1}{c+dx}}{\frac{f}{-de+cf} - \frac{h}{-dg+ch}}} \right. \\
& \left. \left( -bdg+adh \right) \text{EllipticE} \left[ \text{ArcSin} \left[ \sqrt{\frac{(de-cf)\left(h+\frac{dg}{c+dx}-\frac{ch}{c+dx}\right)}{d(-fg+eh)}} \right], \frac{(bc-ad)(-fg+eh)}{(-de+cf)(-bg+ah)} \right] \right. \\
& \left. \left. \frac{b \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(de-cf)\left(h+\frac{dg}{c+dx}-\frac{ch}{c+dx}\right)}{d(-fg+eh)}} \right], \frac{(bc-ad)(-fg+eh)}{(-de+cf)(-bg+ah)} \right]}{bc-ad} \right) \right\} / \\
& \left( \sqrt{\frac{-\frac{f}{-de+cf} + \frac{1}{c+dx}}{-\frac{f}{-de+cf} + \frac{h}{-dg+ch}}} \sqrt{\left(b + \frac{-bc+ad}{c+dx}\right) \left(f + \frac{de-cf}{c+dx}\right) \left(h + \frac{dg-ch}{c+dx}\right)} - c df g \sqrt{-\frac{(bc-ad)(-dg+ch)\left(-\frac{b}{bc-ad} + \frac{1}{c+dx}\right)}{-bdg+adh}} \right. \\
& \left. \left( -\frac{f}{-de+cf} + \frac{1}{c+dx} \right) \sqrt{\frac{-\frac{h}{-dg+ch} + \frac{1}{c+dx}}{\frac{f}{-de+cf} - \frac{h}{-dg+ch}}} \left( -bdg+adh \right) \text{EllipticE} \left[ \text{ArcSin} \left[ \sqrt{\frac{(de-cf)\left(h+\frac{dg}{c+dx}-\frac{ch}{c+dx}\right)}{d(-fg+eh)}} \right], \frac{(bc-ad)(-fg+eh)}{(-de+cf)(-bg+ah)} \right] \right. \\
& \left. \left. \frac{b \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(de-cf)\left(h+\frac{dg}{c+dx}-\frac{ch}{c+dx}\right)}{d(-fg+eh)}} \right], \frac{(bc-ad)(-fg+eh)}{(-de+cf)(-bg+ah)} \right]}{bc-ad} \right) \right\} /
\end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{\frac{-\frac{f}{-de+cf} + \frac{1}{c+dx}}{-\frac{f}{-de+cf} + \frac{h}{-dg+ch}}} \sqrt{\left( b + \frac{-bc+ad}{c+dx} \right) \left( f + \frac{de-cf}{c+dx} \right) \left( h + \frac{dg-ch}{c+dx} \right)} \right) - \left( c deh \sqrt{-\frac{(bc-ad)(-dg+ch)\left(-\frac{b}{bc-ad} + \frac{1}{c+dx}\right)}{-bdg+adh}} \right. \\
& \left( -\frac{f}{-de+cf} + \frac{1}{c+dx} \right) \sqrt{\frac{-\frac{h}{-dg+ch} + \frac{1}{c+dx}}{\frac{f}{-de+cf} - \frac{h}{-dg+ch}}} \left( \frac{(-bdg+adh) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(de-cf)(h+\frac{dg}{c+dx}-\frac{ch}{c+dx})}{d(-fg+eh)}}\right], \frac{(bc-ad)(-fg+eh)}{(-de+cf)(-bg+ah)}]}{(bc-ad)(-dg+ch)} \right. \\
& \left. \left. \frac{b \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(de-cf)(h+\frac{dg}{c+dx}-\frac{ch}{c+dx})}{d(-fg+eh)}}\right], \frac{(bc-ad)(-fg+eh)}{(-de+cf)(-bg+ah)}]}{bc-ad} \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-de+cf} + \frac{1}{c+dx}}{-\frac{f}{-de+cf} + \frac{h}{-dg+ch}}} \sqrt{\left( b + \frac{-bc+ad}{c+dx} \right) \left( f + \frac{de-cf}{c+dx} \right) \left( h + \frac{dg-ch}{c+dx} \right)} \right) + \left( c^2 fh \sqrt{-\frac{(bc-ad)(-dg+ch)\left(-\frac{b}{bc-ad} + \frac{1}{c+dx}\right)}{-bdg+adh}} \right. \\
& \left( -\frac{f}{-de+cf} + \frac{1}{c+dx} \right) \sqrt{\frac{-\frac{h}{-dg+ch} + \frac{1}{c+dx}}{\frac{f}{-de+cf} - \frac{h}{-dg+ch}}} \left( \frac{(-bdg+adh) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(de-cf)(h+\frac{dg}{c+dx}-\frac{ch}{c+dx})}{d(-fg+eh)}}\right], \frac{(bc-ad)(-fg+eh)}{(-de+cf)(-bg+ah)}]}{(bc-ad)(-dg+ch)} \right. \\
& \left. \left. \frac{b \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(de-cf)(h+\frac{dg}{c+dx}-\frac{ch}{c+dx})}{d(-fg+eh)}}\right], \frac{(bc-ad)(-fg+eh)}{(-de+cf)(-bg+ah)}]}{bc-ad} \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-de+cf} + \frac{1}{c+dx}}{-\frac{f}{-de+cf} + \frac{h}{-dg+ch}}} \sqrt{\left( b + \frac{-bc+ad}{c+dx} \right) \left( f + \frac{de-cf}{c+dx} \right) \left( h + \frac{dg-ch}{c+dx} \right)} \right) + \left( dfg \sqrt{-\frac{-\frac{b}{bc-ad} + \frac{1}{c+dx}}{-\frac{b}{bc-ad} + \frac{h}{-dg+ch}}} \sqrt{-\frac{-\frac{f}{-de+cf} + \frac{1}{c+dx}}{-\frac{f}{-de+cf} + \frac{h}{-dg+ch}}} \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{h}{-dg + ch} + \frac{1}{c + dx} \right) \operatorname{EllipticF}[\operatorname{ArcSin}\left[ \sqrt{\frac{(-de + cf) \left( -h - \frac{dg}{c+dx} + \frac{ch}{c+dx} \right)}{d (-fg + eh)}} \right], \frac{(bc - ad) (-fg + eh)}{(-de + cf) (-bg + ah)}] \Bigg] \\
& \left( \sqrt{\frac{-\frac{h}{-dg + ch} + \frac{1}{c+dx}}{\frac{f}{-de + cf} - \frac{h}{-dg + ch}}} \sqrt{\left( b + \frac{-bc + ad}{c + dx} \right) \left( f + \frac{de - cf}{c + dx} \right) \left( h + \frac{dg - ch}{c + dx} \right)} + \right. \\
& \left. d e h \sqrt{\frac{-\frac{b}{bc - ad} + \frac{1}{c+dx}}{-\frac{b}{bc - ad} + \frac{h}{-dg + ch}}} \sqrt{\frac{-\frac{f}{-de + cf} + \frac{1}{c+dx}}{-\frac{f}{-de + cf} + \frac{h}{-dg + ch}}} \left( -\frac{h}{-dg + ch} + \frac{1}{c + dx} \right) \operatorname{EllipticF}[\operatorname{ArcSin}\left[ \sqrt{\frac{(-de + cf) \left( -h - \frac{dg}{c+dx} + \frac{ch}{c+dx} \right)}{d (-fg + eh)}} \right], \right. \\
& \left. \left. \frac{(bc - ad) (-fg + eh)}{(-de + cf) (-bg + ah)} \right] \Bigg] \Bigg) \Bigg/ \left( \sqrt{\frac{-\frac{h}{-dg + ch} + \frac{1}{c+dx}}{\frac{f}{-de + cf} - \frac{h}{-dg + ch}}} \sqrt{\left( b + \frac{-bc + ad}{c + dx} \right) \left( f + \frac{de - cf}{c + dx} \right) \left( h + \frac{dg - ch}{c + dx} \right)} - \right. \\
& \left. 2 c f h \sqrt{\frac{-\frac{b}{bc - ad} + \frac{1}{c+dx}}{-\frac{b}{bc - ad} + \frac{h}{-dg + ch}}} \sqrt{\frac{-\frac{f}{-de + cf} + \frac{1}{c+dx}}{-\frac{f}{-de + cf} + \frac{h}{-dg + ch}}} \left( -\frac{h}{-dg + ch} + \frac{1}{c + dx} \right) \operatorname{EllipticF}[\operatorname{ArcSin}\left[ \sqrt{\frac{(-de + cf) \left( -h - \frac{dg}{c+dx} + \frac{ch}{c+dx} \right)}{d (-fg + eh)}} \right], \right. \\
& \left. \left. \frac{(bc - ad) (-fg + eh)}{(-de + cf) (-bg + ah)} \right] \Bigg) \Bigg/ \left( \sqrt{\frac{-\frac{h}{-dg + ch} + \frac{1}{c+dx}}{\frac{f}{-de + cf} - \frac{h}{-dg + ch}}} \sqrt{\left( b + \frac{-bc + ad}{c + dx} \right) \left( f + \frac{de - cf}{c + dx} \right) \left( h + \frac{dg - ch}{c + dx} \right)} - \right. \\
& \left. f (-dg + ch) \left( -\frac{f}{-de + cf} + \frac{h}{-dg + ch} \right) \sqrt{\frac{-\frac{b}{bc - ad} + \frac{1}{c+dx}}{-\frac{b}{bc - ad} + \frac{h}{-dg + ch}}} \sqrt{-\frac{\left( -\frac{f}{-de + cf} + \frac{1}{c+dx} \right) \left( -\frac{h}{-dg + ch} + \frac{1}{c+dx} \right)}{\left( -\frac{f}{-de + cf} + \frac{h}{-dg + ch} \right)^2}} \operatorname{EllipticPi}\left[ -\frac{-dfg + deh}{(-de + cf) h}, \right. \right. \\
& \left. \left. \operatorname{ArcSin}\left[ \sqrt{\frac{(-de + cf) \left( -h - \frac{dg}{c+dx} + \frac{ch}{c+dx} \right)}{d (-fg + eh)}} \right], \frac{(bc - ad) (-fg + eh)}{(-de + cf) (-bg + ah)} \right] \Bigg) \Bigg/ \left( \sqrt{\left( b + \frac{-bc + ad}{c + dx} \right) \left( f + \frac{de - cf}{c + dx} \right) \left( h + \frac{dg - ch}{c + dx} \right)} \right) \Bigg)
\end{aligned}$$

**Problem 108: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a + bx}}{\sqrt{c + dx} \sqrt{e + fx} \sqrt{g + hx}} dx$$

Optimal (type 4, 228 leaves, 2 steps):

$$\left( 2 \sqrt{-d g + c h} (a + b x) \sqrt{\frac{(b g - a h) (c + d x)}{(d g - c h) (a + b x)}} \sqrt{\frac{(b g - a h) (e + f x)}{(f g - e h) (a + b x)}} \right. \\ \left. \text{EllipticPi}\left[-\frac{b (d g - c h)}{(b c - a d) h}, \text{ArcSin}\left[\frac{\sqrt{b c - a d} \sqrt{g + h x}}{\sqrt{-d g + c h} \sqrt{a + b x}}\right], \frac{(b e - a f) (d g - c h)}{(b c - a d) (f g - e h)}\right] \right) / \left( \sqrt{b c - a d} h \sqrt{c + d x} \sqrt{e + f x} \right)$$

Result (type 4, 584 leaves):

$$- \left( 2 \sqrt{\frac{(d g - c h) (a + b x)}{(b g - a h) (c + d x)}} (c + d x)^{3/2} \left( \frac{a d \sqrt{\frac{(d g - c h) (e + f x)}{(f g - e h) (c + d x)}} (g + h x) \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{(-d e + c f) (g + h x)}{(f g - e h) (c + d x)}}\right], \frac{(b c - a d) (-f g + e h)}{(d e - c f) (b g - a h)}] + \right. \right. \\ \left. \left. (d g - c h) (c + d x) \sqrt{\frac{(-d e + c f) (g + h x)}{(f g - e h) (c + d x)}} \right) \right. \\ \left. \frac{b c \sqrt{\frac{(d g - c h) (e + f x)}{(f g - e h) (c + d x)}} (g + h x) \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{(-d e + c f) (g + h x)}{(f g - e h) (c + d x)}}\right], \frac{(b c - a d) (-f g + e h)}{(d e - c f) (b g - a h)}] + \right. \\ \left. (-d g + c h) (c + d x) \sqrt{\frac{(-d e + c f) (g + h x)}{(f g - e h) (c + d x)}} \right) \\ \frac{1}{(d e - c f) h} b (f g - e h) \sqrt{-\frac{(d e - c f) (d g - c h) (e + f x) (g + h x)}{(f g - e h)^2 (c + d x)^2}} \\ \left. \text{EllipticPi}\left[\frac{d (-f g + e h)}{(d e - c f) h}, \text{ArcSin}\left[\sqrt{\frac{(-d e + c f) (g + h x)}{(f g - e h) (c + d x)}}\right], \frac{(b c - a d) (-f g + e h)}{(d e - c f) (b g - a h)}\right] \right) \right) / \left( d \sqrt{a + b x} \sqrt{e + f x} \sqrt{g + h x} \right)$$

Problem 110: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + b x)^{3/2} \sqrt{c + d x} \sqrt{e + f x} \sqrt{g + h x}} dx$$

Optimal (type 4, 429 leaves, 5 steps):

$$\begin{aligned}
& - \frac{2 b \sqrt{f g - e h} \sqrt{c + d x}}{\sqrt{-\frac{(b e - a f) (g + h x)}{(f g - e h) (a + b x)}}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{b g - a h} \sqrt{e + f x}}{\sqrt{f g - e h} \sqrt{a + b x}}\right], -\frac{(b c - a d) (f g - e h)}{(d e - c f) (b g - a h)}\right] \\
& - \frac{(b c - a d) (b e - a f) \sqrt{b g - a h}}{\sqrt{\frac{(b e - a f) (c + d x)}{(d e - c f) (a + b x)}}} \sqrt{g + h x} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b g - a h} \sqrt{e + f x}}{\sqrt{f g - e h} \sqrt{a + b x}}\right], -\frac{(b c - a d) (f g - e h)}{(d e - c f) (b g - a h)}\right] \\
& - \frac{(b c - a d) \sqrt{b g - a h} \sqrt{f g - e h} \sqrt{c + d x}}{\sqrt{-\frac{(b e - a f) (g + h x)}{(f g - e h) (a + b x)}}}
\end{aligned}$$

Result (type 4, 3247 leaves):

$$\begin{aligned}
& - \frac{2 b^2 \sqrt{c + d x} \sqrt{e + f x} \sqrt{g + h x}}{(b c - a d) (b e - a f) (b g - a h) \sqrt{a + b x}} - \frac{1}{d (b c - a d) (b e - a f) (b g - a h)} \\
& 2 \left( - \frac{b (c + d x)^{3/2} \left(f + \frac{d e}{c + d x} - \frac{c f}{c + d x}\right) \left(h + \frac{d g}{c + d x} - \frac{c h}{c + d x}\right) \sqrt{a + \frac{(c + d x) \left(b - \frac{b c}{c + d x}\right)}{d}}}{\sqrt{e + \frac{(c + d x) \left(f - \frac{c f}{c + d x}\right)}{d}} \sqrt{g + \frac{(c + d x) \left(h - \frac{c h}{c + d x}\right)}{d}}} + \frac{1}{(f g - e h) \left(b - \frac{b c}{c + d x} + \frac{a d}{c + d x}\right) \sqrt{e + \frac{(c + d x) \left(f - \frac{c f}{c + d x}\right)}{d}} \sqrt{g + \frac{(c + d x) \left(h - \frac{c h}{c + d x}\right)}{d}}} \right. \\
& (b c - a d) f (b g - a h) (-d g + c h) \sqrt{c + d x} \sqrt{\left(b - \frac{b c}{c + d x} + \frac{a d}{c + d x}\right) \left(f + \frac{d e}{c + d x} - \frac{c f}{c + d x}\right) \left(h + \frac{d g}{c + d x} - \frac{c h}{c + d x}\right)} \\
& \sqrt{a + \frac{(c + d x) \left(b - \frac{b c}{c + d x}\right)}{d}} \left( d e \sqrt{-\frac{(b c - a d) (-d g + c h) \left(-\frac{b}{b c - a d} + \frac{1}{c + d x}\right)}{-b d g + a d h}} \left(-\frac{f}{-d e + c f} + \frac{1}{c + d x}\right) \right. \\
& \left. \sqrt{-\frac{h}{-d g + c h} + \frac{1}{c + d x}} \left( -b d g + a d h \right) \text{EllipticE}\left[\text{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(h + \frac{d g}{c + d x} - \frac{c h}{c + d x}\right)}{d (-f g + e h)}}\right], \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)}\right] \right. \\
& \left. \left. - \frac{b \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(h + \frac{d g}{c + d x} - \frac{c h}{c + d x}\right)}{d (-f g + e h)}}\right], \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)}\right]}{b c - a d} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{\frac{-\frac{f}{-de+cf} + \frac{1}{c+dx}}{-\frac{f}{-de+cf} + \frac{h}{-dg+ch}}} \sqrt{\left( b + \frac{-bc+ad}{c+dx} \right) \left( f + \frac{de-cf}{c+dx} \right) \left( h + \frac{dg-ch}{c+dx} \right)} \right) - \left( cf \sqrt{-\frac{(bc-ad)(-dg+ch)\left(-\frac{b}{bc-ad} + \frac{1}{c+dx}\right)}{-bdg+adh}} \right. \\
& \left( -\frac{f}{-de+cf} + \frac{1}{c+dx} \right) \sqrt{\frac{-\frac{h}{-dg+ch} + \frac{1}{c+dx}}{\frac{f}{-de+cf} - \frac{h}{-dg+ch}}} \left( -bdg+adh \right) \text{EllipticE} \left[ \text{ArcSin} \left[ \sqrt{\frac{(de-cf)\left(h+\frac{dg}{c+dx}-\frac{ch}{c+dx}\right)}{d(-fg+eh)}} \right], \frac{(bc-ad)(-fg+eh)}{(-de+cf)(-bg+ah)} \right] \\
& \left. \frac{b \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(de-cf)\left(h+\frac{dg}{c+dx}-\frac{ch}{c+dx}\right)}{d(-fg+eh)}} \right], \frac{(bc-ad)(-fg+eh)}{(-de+cf)(-bg+ah)} \right]}{bc-ad} \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-de+cf} + \frac{1}{c+dx}}{-\frac{f}{-de+cf} + \frac{h}{-dg+ch}}} \sqrt{\left( b + \frac{-bc+ad}{c+dx} \right) \left( f + \frac{de-cf}{c+dx} \right) \left( h + \frac{dg-ch}{c+dx} \right)} + \left( f \sqrt{\frac{-\frac{b}{bc-ad} + \frac{1}{c+dx}}{-\frac{b}{bc-ad} + \frac{h}{-dg+ch}}} \sqrt{\frac{-\frac{f}{-de+cf} + \frac{1}{c+dx}}{-\frac{f}{-de+cf} + \frac{h}{-dg+ch}}} \right. \right. \\
& \left. \left( -\frac{h}{-dg+ch} + \frac{1}{c+dx} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-de+cf)\left(-h-\frac{dg}{c+dx}+\frac{ch}{c+dx}\right)}{d(-fg+eh)}} \right], \frac{(bc-ad)(-fg+eh)}{(-de+cf)(-bg+ah)} \right] \right) / \\
& \left( \sqrt{\frac{-\frac{h}{-dg+ch} + \frac{1}{c+dx}}{-\frac{f}{-de+cf} - \frac{h}{-dg+ch}}} \sqrt{\left( b + \frac{-bc+ad}{c+dx} \right) \left( f + \frac{de-cf}{c+dx} \right) \left( h + \frac{dg-ch}{c+dx} \right)} \right) - \\
& \frac{1}{(fg-eh) \left( b - \frac{bc}{c+dx} + \frac{ad}{c+dx} \right) \sqrt{e + \frac{(c+dx)\left(f-\frac{cf}{c+dx}\right)}{d}} \sqrt{g + \frac{(c+dx)\left(h-\frac{ch}{c+dx}\right)}{d}}} (bc-ad)(be-af)(-de+cf)h\sqrt{c+dx} \\
& \sqrt{\left( b - \frac{bc}{c+dx} + \frac{ad}{c+dx} \right) \left( f + \frac{de}{c+dx} - \frac{cf}{c+dx} \right) \left( h + \frac{dg}{c+dx} - \frac{ch}{c+dx} \right)} \sqrt{a + \frac{(c+dx)\left(b-\frac{bc}{c+dx}\right)}{d}}
\end{aligned}$$

$$\begin{aligned}
& \left( d g \sqrt{-\frac{(b c - a d) (-d g + c h) \left(-\frac{b}{b c - a d} + \frac{1}{c+d x}\right)}{-b d g + a d h}} \left(-\frac{f}{-d e + c f} + \frac{1}{c+d x}\right) \sqrt{\frac{-\frac{h}{-d g + c h} + \frac{1}{c+d x}}{\frac{f}{-d e + c f} - \frac{h}{-d g + c h}}} \right. \\
& \left. \left( -b d g + a d h \right) \text{EllipticE} \left[ \text{ArcSin} \left[ \sqrt{\frac{(d e - c f) \left(h + \frac{d g}{c+d x} - \frac{c h}{c+d x}\right)}{d (-f g + e h)}} \right], \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)} \right] \right. \\
& \left. \left. - \frac{b \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(d e - c f) \left(h + \frac{d g}{c+d x} - \frac{c h}{c+d x}\right)}{d (-f g + e h)}} \right], \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)} \right]}{b c - a d} \right) \right\} / \\
& \left( \sqrt{\frac{-\frac{f}{-d e + c f} + \frac{1}{c+d x}}{-\frac{f}{-d e + c f} + \frac{h}{-d g + c h}}} \sqrt{\left(b + \frac{-b c + a d}{c+d x}\right) \left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)} - \left(c h \sqrt{-\frac{(b c - a d) (-d g + c h) \left(-\frac{b}{b c - a d} + \frac{1}{c+d x}\right)}{-b d g + a d h}} \right. \right. \\
& \left. \left( -\frac{f}{-d e + c f} + \frac{1}{c+d x} \right) \sqrt{\frac{-\frac{h}{-d g + c h} + \frac{1}{c+d x}}{\frac{f}{-d e + c f} - \frac{h}{-d g + c h}}} \left( -b d g + a d h \right) \text{EllipticE} \left[ \text{ArcSin} \left[ \sqrt{\frac{(d e - c f) \left(h + \frac{d g}{c+d x} - \frac{c h}{c+d x}\right)}{d (-f g + e h)}} \right], \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)} \right] \right. \\
& \left. \left. - b \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(d e - c f) \left(h + \frac{d g}{c+d x} - \frac{c h}{c+d x}\right)}{d (-f g + e h)}} \right], \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)} \right] \right) \right\} / \\
& \left( \sqrt{\frac{-\frac{f}{-d e + c f} + \frac{1}{c+d x}}{-\frac{f}{-d e + c f} + \frac{h}{-d g + c h}}} \sqrt{\left(b + \frac{-b c + a d}{c+d x}\right) \left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)} + \left(h \sqrt{-\frac{-\frac{b}{b c - a d} + \frac{1}{c+d x}}{-\frac{b}{b c - a d} + \frac{h}{-d g + c h}}} \sqrt{\frac{-\frac{f}{-d e + c f} + \frac{1}{c+d x}}{-\frac{f}{-d e + c f} + \frac{h}{-d g + c h}}} \right.
\end{aligned}$$

$$\left( -\frac{h}{-d g + c h} + \frac{1}{c + d x} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-d e + c f) \left( -h - \frac{d g}{c+d x} + \frac{c h}{c+d x} \right)}{d (-f g + e h)}} \right], \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)} \right] \Bigg/ \\ \left( \sqrt{\frac{-\frac{h}{-d g + c h} + \frac{1}{c+d x}}{\frac{f}{-d e + c f} - \frac{h}{-d g + c h}}} \sqrt{\left( b + \frac{-b c + a d}{c + d x} \right) \left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) \right]$$

Problem 111: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + b x)^{3/2} (c + d x)^{3/2} \sqrt{e + f x} \sqrt{g + h x}} dx$$

Optimal (type 4, 786 leaves, ? steps):

$$-\frac{2 d^3 \sqrt{a + b x} \sqrt{e + f x} \sqrt{g + h x}}{(b c - a d)^2 (d e - c f) (d g - c h) \sqrt{c + d x}} - \frac{2 b^3 \sqrt{c + d x} \sqrt{e + f x} \sqrt{g + h x}}{(b c - a d)^2 (b e - a f) (b g - a h) \sqrt{a + b x}} + \\ \frac{2 b (a^2 d^2 f h - a b d^2 (f g + e h) + b^2 (2 d^2 e g + c^2 f h - c d (f g + e h))) \sqrt{c + d x} \sqrt{e + f x} \sqrt{g + h x}}{(b c - a d)^2 (b e - a f) (d e - c f) (b g - a h) (d g - c h) \sqrt{a + b x}} - \\ \left( 2 \sqrt{f g - e h} (a^2 d^2 f h - a b d^2 (f g + e h) + b^2 (2 d^2 e g + c^2 f h - c d (f g + e h))) \right. \\ \left. \sqrt{c + d x} \sqrt{-\frac{(b e - a f) (g + h x)}{(f g - e h) (a + b x)}} \text{EllipticE} \left[ \text{ArcSin} \left[ \frac{\sqrt{b g - a h} \sqrt{e + f x}}{\sqrt{f g - e h} \sqrt{a + b x}} \right], -\frac{(b c - a d) (f g - e h)}{(d e - c f) (b g - a h)} \right] \right) \Bigg/ \\ \left( (b c - a d)^2 (b e - a f) (d e - c f) \sqrt{b g - a h} (d g - c h) \sqrt{\frac{(b e - a f) (c + d x)}{(d e - c f) (a + b x)}} \sqrt{g + h x} \right) - \\ \frac{4 b d \sqrt{\frac{(b e - a f) (c + d x)}{(d e - c f) (a + b x)}} \sqrt{g + h x} \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\sqrt{b g - a h} \sqrt{e + f x}}{\sqrt{f g - e h} \sqrt{a + b x}} \right], -\frac{(b c - a d) (f g - e h)}{(d e - c f) (b g - a h)} \right]}{(b c - a d)^2 \sqrt{b g - a h} \sqrt{f g - e h} \sqrt{c + d x} \sqrt{-\frac{(b e - a f) (g + h x)}{(f g - e h) (a + b x)}}}$$

Result (type 4, 7061 leaves):

$$\begin{aligned}
& \sqrt{a+b x} \sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x} \\
& \left( \frac{1}{\left(c - \frac{ad}{b}\right) (a+b x)} \left( - \frac{2 b^3 c d^2 e g}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} - \frac{2 a b^2 d^3 e g}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} + \right. \right. \\
& \quad \frac{2 b^3 c^2 d f g}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} + \frac{2 a^2 b d^3 f g}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} + \\
& \quad \frac{2 b^3 c^2 d e h}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} + \frac{2 a^2 b d^3 e h}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} - \\
& \quad \frac{2 b^3 c^3 f h}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} - \frac{2 a^3 d^3 f h}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} - \\
& \quad \frac{1}{b} a \left( - \frac{4 b^3 d^3 e g}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} + \right. \\
& \quad \frac{2 b^3 c d^2 f g}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} + \frac{2 a b^2 d^3 f g}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} + \\
& \quad \frac{2 b^3 c d^2 e h}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} + \frac{2 a b^2 d^3 e h}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} - \\
& \quad \left. \left. \frac{2 b^3 c^2 d f h}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} - \frac{2 a^2 b d^3 f h}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} \right) \right) + \\
& \frac{1}{\left(a - \frac{bc}{d}\right) (c + d x)} \left( - \frac{2 b^3 c d^2 e g}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} - \frac{2 a b^2 d^3 e g}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} + \right. \\
& \quad \frac{2 b^3 c^2 d f g}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} + \frac{2 a^2 b d^3 f g}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} + \\
& \quad \frac{2 b^3 c^2 d e h}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} + \frac{2 a^2 b d^3 e h}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} - \\
& \quad \left. \left. \frac{2 b^3 c^3 f h}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} - \frac{2 a^3 d^3 f h}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} - \frac{1}{d} \right) \right. \\
& c \left( - \frac{4 b^3 d^3 e g}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} + \frac{2 b^3 c d^2 f g}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} + \right. \\
& \quad \frac{2 a b^2 d^3 f g}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} + \frac{2 b^3 c d^2 e h}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} + \\
& \quad \frac{2 a b^2 d^3 e h}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} - \frac{2 b^3 c^2 d f h}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} - \\
& \quad \left. \left. \frac{2 b^3 c^2 d f h}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{2 a^2 b d^3 f h}{(b c - a d)^2 (b e - a f) (-d e + c f) (b g - a h) (-d g + c h)} \Bigg) \Bigg) - \frac{1}{b^2 (-b c + a d)^2 (-b e + a f) (-d e + c f) (-b g + a h) (-d g + c h)} \\
& 2 \left( \left( (-2 b^2 d^2 e g + b^2 c d f g + a b d^2 f g + b^2 c d e h + a b d^2 e h - b^2 c^2 f h - a^2 d^2 f h) (a + b x)^{5/2} \left( d + \frac{b c}{a + b x} - \frac{a d}{a + b x} \right) \left( f + \frac{b e}{a + b x} - \frac{a f}{a + b x} \right) \right. \right. \\
& \left. \left. \left( h + \frac{b g}{a + b x} - \frac{a h}{a + b x} \right) \right) \Big/ \left( \sqrt{c + \frac{(a + b x) \left( d - \frac{a d}{a + b x} \right)}{b}} \sqrt{e + \frac{(a + b x) \left( f - \frac{a f}{a + b x} \right)}{b}} \sqrt{g + \frac{(a + b x) \left( h - \frac{a h}{a + b x} \right)}{b}} \right. \right. \\
& \left. \left. \frac{1}{\sqrt{c + \frac{(a + b x) \left( d - \frac{a d}{a + b x} \right)}{b}} \sqrt{e + \frac{(a + b x) \left( f - \frac{a f}{a + b x} \right)}{b}} \sqrt{g + \frac{(a + b x) \left( h - \frac{a h}{a + b x} \right)}{b}}} (b c - a d) (b e - a f) (b g - a h) (a + b x)^{3/2} \right. \right. \\
& \left. \left. \sqrt{\left( d + \frac{b c}{a + b x} - \frac{a d}{a + b x} \right) \left( f + \frac{b e}{a + b x} - \frac{a f}{a + b x} \right) \left( h + \frac{b g}{a + b x} - \frac{a h}{a + b x} \right)} \left( 2 b^2 d^2 e g \sqrt{\frac{(b c - a d) (b g - a h) \left( -\frac{d}{-b c + a d} + \frac{1}{a + b x} \right)}{b d g - b c h}} \right. \right. \right. \\
& \left. \left. \left. \left( -\frac{f}{-b e + a f} + \frac{1}{a + b x} \right) \sqrt{\frac{-\frac{h}{-b g + a h} + \frac{1}{a + b x}}{\frac{f}{-b e + a f} - \frac{h}{-b g + a h}}} \left( -\frac{(b d g - b c h) \text{EllipticE}[\text{ArcSin}\left[\sqrt{\frac{(b e - a f) \left( h + \frac{b g}{a + b x} - \frac{a h}{a + b x} \right)}{b (-f g + e h)}\right], \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}]}{(b c - a d) (b g - a h)} \right. \right. \right. \\
& \left. \left. \left. \left. \frac{d \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{(b e - a f) \left( h + \frac{b g}{a + b x} - \frac{a h}{a + b x} \right)}{b (-f g + e h)}\right], \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}]}{-b c + a d} \right) \Big/ \right. \right. \right. \\
& \left. \left. \left. \left. \sqrt{\frac{-\frac{f}{-b e + a f} + \frac{1}{a + b x}}{-\frac{f}{-b e + a f} + \frac{h}{-b g + a h}}} \sqrt{\left( d + \frac{b c - a d}{a + b x} \right) \left( f + \frac{b e - a f}{a + b x} \right) \left( h + \frac{b g - a h}{a + b x} \right)} - \left( b^2 c d f g \sqrt{\frac{(b c - a d) (b g - a h) \left( -\frac{d}{-b c + a d} + \frac{1}{a + b x} \right)}{b d g - b c h}} \right. \right. \right. \right. 
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{f}{-be+af} + \frac{1}{a+bx} \right) \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+bx}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \left( -\frac{(bdg-bch) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}, \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}\right]}]}{(bc-ad)(bg-ah)} \right. \\
& \left. \left. - \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}, \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}\right]}]}{-bc+ad} \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+bx}}{\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+bx}\right) \left(f + \frac{be-af}{a+bx}\right) \left(h + \frac{bg-ah}{a+bx}\right)} - a b d^2 f g \sqrt{\frac{(bc-ad)(bg-ah) \left(-\frac{d}{-bc+ad} + \frac{1}{a+bx}\right)}{bdg-bch}} \right. \\
& \left. - \left( -\frac{f}{-be+af} + \frac{1}{a+bx} \right) \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+bx}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \left( -\frac{(bdg-bch) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}, \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}\right]}]}{(bc-ad)(bg-ah)} \right. \right. \\
& \left. \left. - \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}, \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}\right]}]}{-bc+ad} \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+bx}}{\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+bx}\right) \left(f + \frac{be-af}{a+bx}\right) \left(h + \frac{bg-ah}{a+bx}\right)} - b^2 c d e h \sqrt{\frac{(bc-ad)(bg-ah) \left(-\frac{d}{-bc+ad} + \frac{1}{a+bx}\right)}{bdg-bch}} \right. \\
& \left. - \left( -\frac{f}{-be+af} + \frac{1}{a+bx} \right) \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+bx}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \left( -\frac{(bdg-bch) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}, \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}\right]}]}{(bc-ad)(bg-ah)} \right. \right. \\
& \left. \left. - \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}, \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}\right]}]}{-bc+ad} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) (-f g+e h)}{(-b e+a f) (-d g+c h)}}}{-b c+a d}\right\} / \\
& \left( \sqrt{\frac{-\frac{f}{-b e+a f}+\frac{1}{a+b x}}{-\frac{f}{-b e+a f}+\frac{h}{-b g+a h}}} \sqrt{\left(d+\frac{b c-a d}{a+b x}\right)\left(f+\frac{b e-a f}{a+b x}\right)\left(h+\frac{b g-a h}{a+b x}\right)} - \left( a b d^2 e h \sqrt{\frac{(b c-a d) (b g-a h) \left(-\frac{d}{-b c+a d}+\frac{1}{a+b x}\right)}{b d g-b c h}} \right. \right. \\
& \left. \left. \left( -\frac{f}{-b e+a f}+\frac{1}{a+b x} \right) \sqrt{\frac{-\frac{h}{-b g+a h}+\frac{1}{a+b x}}{\frac{f}{-b e+a f}-\frac{h}{-b g+a h}}} \left( -\frac{(b d g-b c h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) (-f g+e h)}{(-b e+a f) (-d g+c h)}}}{(b c-a d) (b g-a h)} - \right. \right. \right. \\
& \left. \left. \left. d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) (-f g+e h)}{(-b e+a f) (-d g+c h)}}}{-b c+a d}\right) \right\} / \\
& \left( \sqrt{\frac{-\frac{f}{-b e+a f}+\frac{1}{a+b x}}{-\frac{f}{-b e+a f}+\frac{h}{-b g+a h}}} \sqrt{\left(d+\frac{b c-a d}{a+b x}\right)\left(f+\frac{b e-a f}{a+b x}\right)\left(h+\frac{b g-a h}{a+b x}\right)} + \left( b^2 c^2 f h \sqrt{\frac{(b c-a d) (b g-a h) \left(-\frac{d}{-b c+a d}+\frac{1}{a+b x}\right)}{b d g-b c h}} \right. \right. \\
& \left. \left. \left( -\frac{f}{-b e+a f}+\frac{1}{a+b x} \right) \sqrt{\frac{-\frac{h}{-b g+a h}+\frac{1}{a+b x}}{\frac{f}{-b e+a f}-\frac{h}{-b g+a h}}} \left( -\frac{(b d g-b c h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) (-f g+e h)}{(-b e+a f) (-d g+c h)}}}{(b c-a d) (b g-a h)} - \right. \right. \right. \\
& \left. \left. \left. d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) (-f g+e h)}{(-b e+a f) (-d g+c h)}}}{-b c+a d}\right) \right\}
\end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) + \left( a^2 d^2 f h \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+b x}\right)}{bdg-bch}} \right. \\
& \left( -\frac{f}{-be+af} + \frac{1}{a+b x} \right) \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+b x}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \left( -\frac{(bdg-bch) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg-ah}{a+b x})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{(bc-ad)(bg-ah)} \right. \\
& \left. \left. \left. \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg-ah}{a+b x})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{-bc+ad} \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) + \left( b d^2 f g \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+b x}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \right. \\
& \left( -\frac{h}{-bg+ah} + \frac{1}{a+b x} \right) \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(-be+af)(-h-\frac{bg}{a+b x} + \frac{ah}{a+b x})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] \right) / \\
& \left( \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+b x}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) + \left( b d^2 e h \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+b x}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \right. \\
& \left( -\frac{h}{-bg+ah} + \frac{1}{a+b x} \right) \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(-be+af)(-h-\frac{bg}{a+b x} + \frac{ah}{a+b x})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] \right) / \\
& \left( \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+b x}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) - \left( b c d f h \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+b x}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{h}{-bg + ah} + \frac{1}{a + bx} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-be + af) \left( -h - \frac{bg}{a+bx} + \frac{ah}{a+bx} \right)}{b (-fg + eh)}} \right], \frac{(-bc + ad) (-fg + eh)}{(-be + af) (-dg + ch)} \right] \Bigg] \\
& \left( \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+bx}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \sqrt{\left( d + \frac{bc - ad}{a + bx} \right) \left( f + \frac{be - af}{a + bx} \right) \left( h + \frac{bg - ah}{a + bx} \right)} \right) - \left( ad^2 fh \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+bx}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+bx}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \right. \\
& \left. \left( -\frac{h}{-bg + ah} + \frac{1}{a + bx} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-be + af) \left( -h - \frac{bg}{a+bx} + \frac{ah}{a+bx} \right)}{b (-fg + eh)}} \right], \frac{(-bc + ad) (-fg + eh)}{(-be + af) (-dg + ch)} \right] \right) \\
& \left( \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+bx}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \sqrt{\left( d + \frac{bc - ad}{a + bx} \right) \left( f + \frac{be - af}{a + bx} \right) \left( h + \frac{bg - ah}{a + bx} \right)} \right)
\end{aligned}$$

**Problem 112: Result unnecessarily involves higher level functions.**

$$\int \frac{x^4 (e + fx)^n}{(a + bx)(c + dx)} dx$$

Optimal (type 5, 319 leaves, 8 steps):

$$\begin{aligned}
& \frac{e^2 (e + fx)^{1+n}}{b d f^3 (1+n)} + \frac{(bc + ad) e (e + fx)^{1+n}}{b^2 d^2 f^2 (1+n)} + \frac{(b^2 c^2 + abcd + a^2 d^2) (e + fx)^{1+n}}{b^3 d^3 f (1+n)} - \frac{2e (e + fx)^{2+n}}{b d f^3 (2+n)} - \frac{(bc + ad) (e + fx)^{2+n}}{b^2 d^2 f^2 (2+n)} + \frac{(e + fx)^{3+n}}{b d f^3 (3+n)} - \\
& \frac{a^4 (e + fx)^{1+n} \text{Hypergeometric2F1}[1, 1+n, 2+n, \frac{b(e+fx)}{b e - a f}]}{b^3 (b c - a d) (b e - a f) (1+n)} + \frac{c^4 (e + fx)^{1+n} \text{Hypergeometric2F1}[1, 1+n, 2+n, \frac{d(e+fx)}{d e - c f}]}{d^3 (b c - a d) (d e - c f) (1+n)}
\end{aligned}$$

Result (type 6, 262 leaves):

$$\begin{aligned}
& \frac{6}{5} e x^5 (e + fx)^n \left( \left( a b \text{AppellF1}[5, -n, 1, 6, -\frac{fx}{e}, -\frac{bx}{a}] \right) \Bigg/ \left( (bc - ad) (a + bx) \right. \right. \\
& \left. \left. \left( 6 a e \text{AppellF1}[5, -n, 1, 6, -\frac{fx}{e}, -\frac{bx}{a}] + a f n x \text{AppellF1}[6, 1-n, 1, 7, -\frac{fx}{e}, -\frac{bx}{a}] - b e x \text{AppellF1}[6, -n, 2, 7, -\frac{fx}{e}, -\frac{bx}{a}] \right) \right) + \\
& \left( c d \text{AppellF1}[5, -n, 1, 6, -\frac{fx}{e}, -\frac{dx}{c}] \right) \Bigg/ \left( (-bc + ad) (c + dx) \right. \\
& \left. \left. \left( 6 c e \text{AppellF1}[5, -n, 1, 6, -\frac{fx}{e}, -\frac{dx}{c}] + c f n x \text{AppellF1}[6, 1-n, 1, 7, -\frac{fx}{e}, -\frac{dx}{c}] - d e x \text{AppellF1}[6, -n, 2, 7, -\frac{fx}{e}, -\frac{dx}{c}] \right) \right) \right)
\end{aligned}$$

### Problem 113: Result unnecessarily involves higher level functions.

$$\int \frac{x^3 (e + f x)^n}{(a + b x) (c + d x)} dx$$

Optimal (type 5, 216 leaves, 6 steps):

$$\begin{aligned} & -\frac{e (e + f x)^{1+n}}{b d f^2 (1+n)} - \frac{(b c + a d) (e + f x)^{1+n}}{b^2 d^2 f (1+n)} + \frac{(e + f x)^{2+n}}{b d f^2 (2+n)} + \\ & \frac{a^3 (e + f x)^{1+n} \text{Hypergeometric2F1}[1, 1+n, 2+n, \frac{b (e+f x)}{b e-a f}]}{b^2 (b c - a d) (b e - a f) (1+n)} - \frac{c^3 (e + f x)^{1+n} \text{Hypergeometric2F1}[1, 1+n, 2+n, \frac{d (e+f x)}{d e-c f}]}{d^2 (b c - a d) (d e - c f) (1+n)} \end{aligned}$$

Result (type 6, 262 leaves):

$$\begin{aligned} & \frac{5}{4} e x^4 (e + f x)^n \left( \left( a b \text{AppellF1}[4, -n, 1, 5, -\frac{f x}{e}, -\frac{b x}{a}] \right) / \left( (b c - a d) (a + b x) \right. \right. \\ & \left. \left. \left( 5 a e \text{AppellF1}[4, -n, 1, 5, -\frac{f x}{e}, -\frac{b x}{a}] + a f n x \text{AppellF1}[5, 1-n, 1, 6, -\frac{f x}{e}, -\frac{b x}{a}] - b e x \text{AppellF1}[5, -n, 2, 6, -\frac{f x}{e}, -\frac{b x}{a}] \right) \right) + \\ & \left( c d \text{AppellF1}[4, -n, 1, 5, -\frac{f x}{e}, -\frac{d x}{c}] \right) / \left( (-b c + a d) (c + d x) \right. \\ & \left. \left. \left( 5 c e \text{AppellF1}[4, -n, 1, 5, -\frac{f x}{e}, -\frac{d x}{c}] + c f n x \text{AppellF1}[5, 1-n, 1, 6, -\frac{f x}{e}, -\frac{d x}{c}] - d e x \text{AppellF1}[5, -n, 2, 6, -\frac{f x}{e}, -\frac{d x}{c}] \right) \right) \right) \end{aligned}$$

### Problem 120: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^m (c + d x) (e + f x)}{g + h x} dx$$

Optimal (type 5, 134 leaves, 2 steps):

$$\begin{aligned} & \frac{(a + b x)^{1+m} (a d f h + b (d f g - d e h - c f h) (2+m) - b d f h (1+m) x)}{b^2 h^2 (1+m) (2+m)} + \\ & \frac{(d g - c h) (f g - e h) (a + b x)^{1+m} \text{Hypergeometric2F1}[1, 1+m, 2+m, -\frac{h (a+b x)}{b g - a h}]}{h^2 (b g - a h) (1+m)} \end{aligned}$$

Result (type 6, 317 leaves):

$$\begin{aligned} & \frac{1}{6} (a + b x)^m \left( \left( 9 a (d e + c f) g x^2 \text{AppellF1}[2, -m, 1, 3, -\frac{b x}{a}, -\frac{h x}{g}] \right) \middle/ ((g + h x) \right. \\ & \quad \left. \left( 3 a g \text{AppellF1}[2, -m, 1, 3, -\frac{b x}{a}, -\frac{h x}{g}] + b g m x \text{AppellF1}[3, 1-m, 1, 4, -\frac{b x}{a}, -\frac{h x}{g}] - a h x \text{AppellF1}[3, -m, 2, 4, -\frac{b x}{a}, -\frac{h x}{g}] \right) \right) + \\ & \quad \left( 8 a d f g x^3 \text{AppellF1}[3, -m, 1, 4, -\frac{b x}{a}, -\frac{h x}{g}] \right) \middle/ \left( (g + h x) \left( 4 a g \text{AppellF1}[3, -m, 1, 4, -\frac{b x}{a}, -\frac{h x}{g}] + b g m x \text{AppellF1}[4, 1-m, \right. \right. \\ & \quad \left. \left. 1, 5, -\frac{b x}{a}, -\frac{h x}{g}] - a h x \text{AppellF1}[4, -m, 2, 5, -\frac{b x}{a}, -\frac{h x}{g}] \right) \right) + \frac{6 c e \left( \frac{h(a+b x)}{b(g+h x)} \right)^{-m} \text{Hypergeometric2F1}[-m, -m, 1-m, \frac{b g-a h}{b g+b h x}]}{h m} \end{aligned}$$

**Problem 121:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^m (c + d x)}{(e + f x) (g + h x)} dx$$

Optimal (type 5, 140 leaves, 3 steps):

$$-\frac{(d e - c f) (a + b x)^{1+m} \text{Hypergeometric2F1}[1, 1+m, 2+m, -\frac{f(a+b x)}{b e - a f}]}{(b e - a f) (f g - e h) (1+m)} + \frac{(d g - c h) (a + b x)^{1+m} \text{Hypergeometric2F1}[1, 1+m, 2+m, -\frac{h(a+b x)}{b g - a h}]}{(b g - a h) (f g - e h) (1+m)}$$

Result (type 6, 390 leaves):

$$\begin{aligned} & \frac{1}{2} (a + b x)^m \\ & \left( 3 a d x^2 \left( (e f \text{AppellF1}[2, -m, 1, 3, -\frac{b x}{a}, -\frac{f x}{e}]) \middle/ ((f g - e h) (e + f x) \left( 3 a e \text{AppellF1}[2, -m, 1, 3, -\frac{b x}{a}, -\frac{f x}{e}] + b e m x \text{AppellF1}[3, 1-m, 1, \right. \right. \right. \right. \\ & \quad \left. \left. \left. \left. 4, -\frac{b x}{a}, -\frac{f x}{e}] - a f x \text{AppellF1}[3, -m, 2, 4, -\frac{b x}{a}, -\frac{f x}{e}] \right) \right) + \left( g h \text{AppellF1}[2, -m, 1, 3, -\frac{b x}{a}, -\frac{h x}{g}] \right) \middle/ ((-f g + e h) (g + h x) \right. \\ & \quad \left. \left( 3 a g \text{AppellF1}[2, -m, 1, 3, -\frac{b x}{a}, -\frac{h x}{g}] + b g m x \text{AppellF1}[3, 1-m, 1, 4, -\frac{b x}{a}, -\frac{h x}{g}] - a h x \text{AppellF1}[3, -m, 2, 4, -\frac{b x}{a}, -\frac{h x}{g}] \right) \right) + \\ & \quad \frac{1}{f g m - e h m} \left( 2 c \left( \frac{f(a+b x)}{b(e+f x)} \right)^{-m} \text{Hypergeometric2F1}[-m, -m, 1-m, \frac{b e - a f}{b e + b f x}] - \right. \\ & \quad \left. 2 c \left( \frac{h(a+b x)}{b(g+h x)} \right)^{-m} \text{Hypergeometric2F1}[-m, -m, 1-m, \frac{b g - a h}{b g + b h x}] \right) \end{aligned}$$

### Problem 123: Result more than twice size of optimal antiderivative.

$$\int \frac{x^m (e + f x)^n}{(a + b x) (c + d x)} dx$$

Optimal (type 6, 140 leaves, 6 steps):

$$\frac{b x^{1+m} (e + f x)^n \left(1 + \frac{f x}{e}\right)^{-n} \text{AppellF1}[1 + m, -n, 1, 2 + m, -\frac{f x}{e}, -\frac{b x}{a}]}{a (b c - a d) (1 + m)} - \frac{d x^{1+m} (e + f x)^n \left(1 + \frac{f x}{e}\right)^{-n} \text{AppellF1}[1 + m, -n, 1, 2 + m, -\frac{f x}{e}, -\frac{d x}{c}]}{c (b c - a d) (1 + m)}$$

Result (type 6, 309 leaves):

$$\begin{aligned} & \frac{1}{1 + m} e (2 + m) x^{1+m} (e + f x)^n \\ & \left( - \left( \left( a b \text{AppellF1}[1 + m, -n, 1, 2 + m, -\frac{f x}{e}, -\frac{b x}{a}] \right) / \left( (-b c + a d) (a + b x) \left( a e (2 + m) \text{AppellF1}[1 + m, -n, 1, 2 + m, -\frac{f x}{e}, -\frac{b x}{a}] \right. \right. \right. \right. \right. + \\ & \quad \left. \left. \left. \left. \left. \left. + x \left( a f n \text{AppellF1}[2 + m, 1 - n, 1, 3 + m, -\frac{f x}{e}, -\frac{b x}{a}] - b e \text{AppellF1}[2 + m, -n, 2, 3 + m, -\frac{f x}{e}, -\frac{b x}{a}] \right) \right) \right) \right) - \\ & \left( c d \text{AppellF1}[1 + m, -n, 1, 2 + m, -\frac{f x}{e}, -\frac{d x}{c}] \right) / \left( (b c - a d) (c + d x) \left( c e (2 + m) \text{AppellF1}[1 + m, -n, 1, 2 + m, -\frac{f x}{e}, -\frac{d x}{c}] \right. \right. \right. \right. \right. + \\ & \quad \left. \left. \left. \left. \left. \left. + x \left( c f n \text{AppellF1}[2 + m, 1 - n, 1, 3 + m, -\frac{f x}{e}, -\frac{d x}{c}] - d e \text{AppellF1}[2 + m, -n, 2, 3 + m, -\frac{f x}{e}, -\frac{d x}{c}] \right) \right) \right) \right) \right) \end{aligned}$$

### Problem 124: Result unnecessarily involves higher level functions.

$$\int (a + b x)^m (c + d x)^n (e + f x) (g + h x) dx$$

Optimal (type 5, 266 leaves, 3 steps):

$$\begin{aligned} & - \frac{(a + b x)^{1+m} (c + d x)^{1+n} (b c f h (2 + m) + a d f h (2 + n) - b d (f g + e h) (3 + m + n) - b d f h (2 + m + n) x)}{b^2 d^2 (2 + m + n) (3 + m + n)} + \\ & \left( (a^2 d^2 f h (1 + n) (2 + n) + a b d (1 + n) (2 c f h (1 + m) - d (f g + e h) (3 + m + n)) + \right. \\ & \quad \left. b^2 (c^2 f h (1 + m) (2 + m) - c d (f g + e h) (1 + m) (3 + m + n) + d^2 e g (2 + m + n) (3 + m + n)) \right) (a + b x)^{1+m} \\ & (c + d x)^n \left( \frac{b (c + d x)}{b c - a d} \right)^{-n} \text{Hypergeometric2F1}[1 + m, -n, 2 + m, -\frac{d (a + b x)}{b c - a d}] \Big/ (b^3 d^2 (1 + m) (2 + m + n) (3 + m + n)) \end{aligned}$$

Result (type 6, 335 leaves):

$$\frac{1}{3} (a + b x)^m (c + d x)^n$$

$$\left( \left( 9 a c (f g + e h) x^2 \text{AppellF1}[2, -m, -n, 3, -\frac{b x}{a}, -\frac{d x}{c}] \right) \middle/ \left( 2 \left( 3 a c \text{AppellF1}[2, -m, -n, 3, -\frac{b x}{a}, -\frac{d x}{c}] + b c m x \text{AppellF1}[3, 1-m, -n, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) + \left( 4 a c f h x^3 \text{AppellF1}[3, -m, -n, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) \right) \right. \\ \left. + \left( 4 a c \text{AppellF1}[3, -m, -n, 4, -\frac{b x}{a}, -\frac{d x}{c}] + b c m x \text{AppellF1}[4, 1-m, -n, 5, -\frac{b x}{a}, -\frac{d x}{c}] + a d n x \text{AppellF1}[4, -m, 1-n, 5, -\frac{b x}{a}, -\frac{d x}{c}] \right) + \right. \\ \left. \frac{3 e g \left( \frac{d(a+b x)}{-b c + a d} \right)^{-m} (c + d x) \text{Hypergeometric2F1}[-m, 1+n, 2+n, \frac{b(c+d x)}{b c - a d}]}{d (1+n)} \right)$$

**Problem 125: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + b x)^m (c + d x)^{1-m} (e + f x) (g + h x) dx$$

Optimal (type 5, 245 leaves, 3 steps):

$$\frac{(a + b x)^{1+m} (c + d x)^{2-m} (4 b d (f g + e h) - a d f h (3 - m) - b c f h (2 + m) + 3 b d f h x)}{12 b^2 d^2} + \frac{1}{12 b^4 d^2 (1 + m)}$$

$$(b c - a d) (a^2 d^2 f h (6 - 5 m + m^2) - 2 a b d (2 - m) (2 d (f g + e h) - c f h (1 + m)) + b^2 (12 d^2 e g - 4 c d (f g + e h) (1 + m) + c^2 f h (2 + 3 m + m^2)))$$

$$(a + b x)^{1+m} (c + d x)^{-m} \left( \frac{b (c + d x)}{b c - a d} \right)^m \text{Hypergeometric2F1}[-1 + m, 1 + m, 2 + m, -\frac{d (a + b x)}{b c - a d}]$$

Result (type 6, 1043 leaves):

$$\begin{aligned}
& \left( 3 a c d e g x^2 (a + b x)^m (c + d x)^{-m} \text{AppellF1}[2, -m, m, 3, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \\
& \left( 6 a c \text{AppellF1}[2, -m, m, 3, -\frac{b x}{a}, -\frac{d x}{c}] + 2 m x \left( b c \text{AppellF1}[3, 1-m, m, 4, -\frac{b x}{a}, -\frac{d x}{c}] - a d \text{AppellF1}[3, -m, 1+m, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) \right) + \\
& \left( 3 a c^2 f g x^2 (a + b x)^m (c + d x)^{-m} \text{AppellF1}[2, -m, m, 3, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \\
& \left( 6 a c \text{AppellF1}[2, -m, m, 3, -\frac{b x}{a}, -\frac{d x}{c}] + 2 m x \left( b c \text{AppellF1}[3, 1-m, m, 4, -\frac{b x}{a}, -\frac{d x}{c}] - a d \text{AppellF1}[3, -m, 1+m, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) \right) + \\
& \left( 3 a c^2 e h x^2 (a + b x)^m (c + d x)^{-m} \text{AppellF1}[2, -m, m, 3, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \\
& \left( 6 a c \text{AppellF1}[2, -m, m, 3, -\frac{b x}{a}, -\frac{d x}{c}] + 2 m x \left( b c \text{AppellF1}[3, 1-m, m, 4, -\frac{b x}{a}, -\frac{d x}{c}] - a d \text{AppellF1}[3, -m, 1+m, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) \right) + \\
& \left( 4 a c d f g x^3 (a + b x)^m (c + d x)^{-m} \text{AppellF1}[3, -m, m, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \\
& \left( 12 a c \text{AppellF1}[3, -m, m, 4, -\frac{b x}{a}, -\frac{d x}{c}] + 3 b c m x \text{AppellF1}[4, 1-m, m, 5, -\frac{b x}{a}, -\frac{d x}{c}] - 3 a d m x \text{AppellF1}[4, -m, 1+m, 5, -\frac{b x}{a}, -\frac{d x}{c}] \right) + \\
& \left( 4 a c d e h x^3 (a + b x)^m (c + d x)^{-m} \text{AppellF1}[3, -m, m, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \\
& \left( 12 a c \text{AppellF1}[3, -m, m, 4, -\frac{b x}{a}, -\frac{d x}{c}] + 3 b c m x \text{AppellF1}[4, 1-m, m, 5, -\frac{b x}{a}, -\frac{d x}{c}] - 3 a d m x \text{AppellF1}[4, -m, 1+m, 5, -\frac{b x}{a}, -\frac{d x}{c}] \right) + \\
& \left( 4 a c^2 f h x^3 (a + b x)^m (c + d x)^{-m} \text{AppellF1}[3, -m, m, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \\
& \left( 12 a c \text{AppellF1}[3, -m, m, 4, -\frac{b x}{a}, -\frac{d x}{c}] + 3 b c m x \text{AppellF1}[4, 1-m, m, 5, -\frac{b x}{a}, -\frac{d x}{c}] - 3 a d m x \text{AppellF1}[4, -m, 1+m, 5, -\frac{b x}{a}, -\frac{d x}{c}] \right) + \\
& \left( 5 a c d f h x^4 (a + b x)^m (c + d x)^{-m} \text{AppellF1}[4, -m, m, 5, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \\
& \left( 20 a c \text{AppellF1}[4, -m, m, 5, -\frac{b x}{a}, -\frac{d x}{c}] + 4 b c m x \text{AppellF1}[5, 1-m, m, 6, -\frac{b x}{a}, -\frac{d x}{c}] - 4 a d m x \text{AppellF1}[5, -m, 1+m, 6, -\frac{b x}{a}, -\frac{d x}{c}] \right) - \\
& c e g (c + d x)^{1-m} \left( a - \frac{b c}{d} + \frac{b (c+d x)}{d} \right)^m \left( 1 + \frac{b (c+d x)}{\left( a - \frac{b c}{d} \right) d} \right)^{-m} \text{Hypergeometric2F1}[1-m, -m, 2-m, -\frac{b (c+d x)}{\left( a - \frac{b c}{d} \right) d}] \\
& \hline d (-1 + m)
\end{aligned}$$

**Problem 126: Result unnecessarily involves higher level functions.**

$$\int (a + b x)^m (c + d x)^{-m} (e + f x) (g + h x) dx$$

Optimal (type 5, 235 leaves, 3 steps):

$$\frac{(a+b x)^{1+m} (c+d x)^{1-m} (3 b d (f g + e h) - a d f h (2-m) - b c f h (2+m) + 2 b d f h x)}{6 b^2 d^2} + \frac{1}{6 b^3 d^2 (1+m)} \\ (a^2 d^2 f h (2-3 m+m^2) - a b d (1-m) (3 d (f g + e h) - 2 c f h (1+m)) + b^2 (6 d^2 e g - 3 c d (f g + e h) (1+m) + c^2 f h (2+3 m+m^2))) \\ (a+b x)^{1+m} (c+d x)^{-m} \left( \frac{b (c+d x)}{b c - a d} \right)^m \text{Hypergeometric2F1}[m, 1+m, 2+m, -\frac{d (a+b x)}{b c - a d}]$$

Result (type 6, 324 leaves):

$$(a+b x)^m (c+d x)^{-m} \left( \left( 3 a c (f g + e h) x^2 \text{AppellF1}[2, -m, m, 3, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \right. \\ \left( 6 a c \text{AppellF1}[2, -m, m, 3, -\frac{b x}{a}, -\frac{d x}{c}] + 2 m x \left( b c \text{AppellF1}[3, 1-m, m, 4, -\frac{b x}{a}, -\frac{d x}{c}] - a d \text{AppellF1}[3, -m, 1+m, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) \right) + \\ \left( 4 a c f h x^3 \text{AppellF1}[3, -m, m, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \\ \left( 12 a c \text{AppellF1}[3, -m, m, 4, -\frac{b x}{a}, -\frac{d x}{c}] + 3 b c m x \text{AppellF1}[4, 1-m, m, 5, -\frac{b x}{a}, -\frac{d x}{c}] - 3 a d m x \text{AppellF1}[4, -m, 1+m, 5, -\frac{b x}{a}, -\frac{d x}{c}] \right) - \\ \left. \frac{e g \left( \frac{d (a+b x)}{-b c + a d} \right)^{-m} (c+d x) \text{Hypergeometric2F1}[1-m, -m, 2-m, \frac{b (c+d x)}{b c - a d}]}{d (-1+m)} \right)$$

Problem 127: Result unnecessarily involves higher level functions.

$$\int (a+b x)^m (c+d x)^{-1-m} (e+f x) (g+h x) dx$$

Optimal (type 5, 261 leaves, 3 steps):

$$\frac{(a+b x)^{1+m} (c+d x)^{-m} (2 b d^2 e g + b c^2 f h (2+m) - c d (2 b (f g + e h) + a f h m) + d (b c - a d) f h m x)}{2 b d^2 (b c - a d) m} - \\ \left( (b^2 c^2 f h (1+m) (2+m) - 2 b c d (1+m) (b f g + b e h + a f h m) + d^2 (2 b^2 e g + 2 a b (f g + e h) m - a^2 f h (1-m) m) \right) \\ (a+b x)^{1+m} (c+d x)^{-m} \left( \frac{b (c+d x)}{b c - a d} \right)^m \text{Hypergeometric2F1}[m, 1+m, 2+m, -\frac{d (a+b x)}{b c - a d}] \Big/ (2 b^2 d^2 (b c - a d) m (1+m))$$

Result (type 6, 346 leaves):

$$\begin{aligned}
& \frac{1}{6} (a + b x)^m (c + d x)^{-m} \\
& \left( \left( 9 a c (f g + e h) x^2 \text{AppellF1}[2, -m, 1+m, 3, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \left( (c + d x) \left( 3 a c \text{AppellF1}[2, -m, 1+m, 3, -\frac{b x}{a}, -\frac{d x}{c}] + b c m \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. x \text{AppellF1}[3, 1-m, 1+m, 4, -\frac{b x}{a}, -\frac{d x}{c}] - a d (1+m) x \text{AppellF1}[3, -m, 2+m, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) \right) + \right. \\
& \left( 8 a c f h x^3 \text{AppellF1}[3, -m, 1+m, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \left( (c + d x) \left( 4 a c \text{AppellF1}[3, -m, 1+m, 4, -\frac{b x}{a}, -\frac{d x}{c}] + \right. \right. \\
& \quad \left. \left. b c m x \text{AppellF1}[4, 1-m, 1+m, 5, -\frac{b x}{a}, -\frac{d x}{c}] - a d (1+m) x \text{AppellF1}[4, -m, 2+m, 5, -\frac{b x}{a}, -\frac{d x}{c}] \right) \right) - \\
& \left. \frac{6 e g \left( \frac{d(a+b x)}{-b c + a d} \right)^{-m} \text{Hypergeometric2F1}[-m, -m, 1-m, \frac{b(c+d x)}{b c - a d}]}{d m} \right)
\end{aligned}$$

**Problem 128: Result unnecessarily involves higher level functions.**

$$\int (a + b x)^m (c + d x)^{-2-m} (e + f x) (g + h x) dx$$

Optimal (type 5, 203 leaves, 3 steps):

$$\begin{aligned}
& \frac{1}{b d^2 (b c - a d) (1+m)} (a + b x)^{1+m} (c + d x)^{-1-m} (b d^2 e g + b c^2 f h (2+m) - c d (b (f g + e h) + a f h (1+m)) + d (b c - a d) f h (1+m) x) - \\
& \frac{1}{b d^3 m} (a d f h m + b (d (f g + e h) - c f h (2+m))) (a + b x)^m \left( -\frac{d(a+b x)}{b c - a d} \right)^{-m} (c + d x)^{-m} \text{Hypergeometric2F1}[-m, -m, 1-m, \frac{b(c+d x)}{b c - a d}]
\end{aligned}$$

Result (type 6, 303 leaves):

$$\begin{aligned}
& \frac{1}{6} (a + b x)^m (c + d x)^{-2-m} \\
& \left( \frac{6 e g (a + b x) (c + d x)}{(b c - a d) (1+m)} - \left( 9 a c (f g + e h) x^2 \text{AppellF1}[2, -m, 2+m, 3, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \left( -3 a c \text{AppellF1}[2, -m, 2+m, 3, -\frac{b x}{a}, -\frac{d x}{c}] - \right. \right. \\
& \quad \left. \left. b c m x \text{AppellF1}[3, 1-m, 2+m, 4, -\frac{b x}{a}, -\frac{d x}{c}] + a d (2+m) x \text{AppellF1}[3, -m, 3+m, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) - \right. \\
& \left( 8 a c f h x^3 \text{AppellF1}[3, -m, 2+m, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \left( -4 a c \text{AppellF1}[3, -m, 2+m, 4, -\frac{b x}{a}, -\frac{d x}{c}] - \right. \\
& \quad \left. \left. b c m x \text{AppellF1}[4, 1-m, 2+m, 5, -\frac{b x}{a}, -\frac{d x}{c}] + a d (2+m) x \text{AppellF1}[4, -m, 3+m, 5, -\frac{b x}{a}, -\frac{d x}{c}] \right) \right)
\end{aligned}$$

**Problem 129:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (a + b x)^m (c + d x)^{-3-m} (e + f x) (g + h x) dx$$

Optimal (type 5, 246 leaves, 3 steps):

$$\begin{aligned} & - \left( \left( (a + b x)^{1+m} (c + d x)^{-2-m} (a^2 b c f h m - a^3 d f h (1 + m) - b^3 c e g (2 + m) + a b^2 (c (f g + e h) + d e g (1 + m)) - \right. \right. \\ & \quad \left. \left. b (a^2 d f h (3 + 2 m) + b^2 (d e g + c (f g + e h) (1 + m)) - a b (2 c f h (1 + m) + d (f g + e h) (2 + m))) x \right) \right) / \left( b^2 (b c - a d)^2 (1 + m) (2 + m) \right) + \\ & \frac{f h (a + b x)^{3+m} (c + d x)^{-m} \left( \frac{b (c + d x)}{b c - a d} \right)^m \text{Hypergeometric2F1}[3 + m, 3 + m, 4 + m, -\frac{d (a + b x)}{b c - a d}]}{(b c - a d)^3 (3 + m)} \end{aligned}$$

Result (type 6, 633 leaves):

$$\begin{aligned} & \frac{1}{3} (a + b x)^m (c + d x)^{-3-m} \left( \left( 3 f g \left( \frac{c (a + b x)}{a (c + d x)} \right)^{-m} (c + d x) \left( b^2 c^2 (1 + m) x^2 \left( \frac{c (a + b x)}{a (c + d x)} \right)^m - a b c x \left( \frac{c (a + b x)}{a (c + d x)} \right)^m (-c m + d (2 + m) x) + \right. \right. \right. \\ & \quad \left. \left. \left. a^2 \left( d^2 x^2 - c^2 \left( -1 + \left( \frac{c (a + b x)}{a (c + d x)} \right)^m \right) - c d x \left( -2 + 2 \left( \frac{c (a + b x)}{a (c + d x)} \right)^m + m \left( \frac{c (a + b x)}{a (c + d x)} \right)^m \right) \right) \right) \right) \right) / \left( c (b c - a d)^2 (1 + m) (2 + m) \right) + \\ & \left( 3 e h \left( \frac{c (a + b x)}{a (c + d x)} \right)^{-m} (c + d x) \left( b^2 c^2 (1 + m) x^2 \left( \frac{c (a + b x)}{a (c + d x)} \right)^m - a b c x \left( \frac{c (a + b x)}{a (c + d x)} \right)^m (-c m + d (2 + m) x) + \right. \right. \\ & \quad \left. \left. a^2 \left( d^2 x^2 - c^2 \left( -1 + \left( \frac{c (a + b x)}{a (c + d x)} \right)^m \right) - c d x \left( -2 + 2 \left( \frac{c (a + b x)}{a (c + d x)} \right)^m + m \left( \frac{c (a + b x)}{a (c + d x)} \right)^m \right) \right) \right) \right) \right) / \left( c (b c - a d)^2 (1 + m) (2 + m) \right) - \\ & \left( 4 a c f h x^3 \text{AppellF1}[3, -m, 3 + m, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \left( -4 a c \text{AppellF1}[3, -m, 3 + m, 4, -\frac{b x}{a}, -\frac{d x}{c}] - \right. \\ & \quad \left. b c m x \text{AppellF1}[4, 1 - m, 3 + m, 5, -\frac{b x}{a}, -\frac{d x}{c}] + a d (3 + m) x \text{AppellF1}[4, -m, 4 + m, 5, -\frac{b x}{a}, -\frac{d x}{c}] \right) - \\ & \frac{3 e g \left( \frac{d (a + b x)}{-b c + a d} \right)^{-m} (c + d x) \text{Hypergeometric2F1}[-2 - m, -m, -1 - m, \frac{b (c + d x)}{b c - a d}]}{d (2 + m)} \end{aligned}$$

**Problem 132:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (a + b x)^3 (c + d x)^{-4-m} (e + f x)^m (g + h x) dx$$

Optimal (type 5, 815 leaves, 10 steps):

$$\begin{aligned}
 & \frac{1}{d^4 f^2 (d e - c f) (3 + m)} (b c - a d)^2 (a d f + b (c f (2 + m) - d e (3 + m))) (c f h (4 + m) - d (f g + e h (3 + m))) (c + d x)^{-3-m} (e + f x)^{1+m} - \\
 & \frac{b (b c - a d) (c f h (4 + m) - d (f g + e h (3 + m))) (a + b x) (c + d x)^{-3-m} (e + f x)^{1+m}}{d^3 f^2} + \\
 & \frac{h (a + b x)^3 (c + d x)^{-3-m} (e + f x)^{1+m}}{d f} - \frac{(b c - a d)^2 (3 a d f h - b (c f h (4 + m) - d (f g + e h m))) (c + d x)^{-2-m} (e + f x)^{1+m}}{d^4 f (d e - c f) (2 + m)} + \\
 & ((b c - a d) (c f h (4 + m) - d (f g + e h (3 + m))) (2 a^2 d^2 f^2 + 2 a b d f (c f (1 + m) - d e (3 + m))) + \\
 & b^2 (c^2 f^2 (2 + 3 m + m^2) - 2 c d e f (3 + 4 m + m^2) + d^2 e^2 (6 + 5 m + m^2)) (c + d x)^{-2-m} (e + f x)^{1+m}) / (d^4 f^2 (d e - c f)^2 (2 + m) (3 + m)) - \\
 & ((b c - a d) (a d f - b (2 d e (2 + m) - c f (3 + 2 m))) (3 a d f h - b (c f h (4 + m) - d (f g + e h m))) (c + d x)^{-1-m} (e + f x)^{1+m}) / \\
 & (d^4 f (d e - c f)^2 (1 + m) (2 + m)) - ((b c - a d) (c f h (4 + m) - d (f g + e h (3 + m))) \\
 & (2 a^2 d^2 f^2 + 2 a b d f (c f (1 + m) - d e (3 + m)) + b^2 (c^2 f^2 (2 + 3 m + m^2) - 2 c d e f (3 + 4 m + m^2) + d^2 e^2 (6 + 5 m + m^2))) (c + d x)^{-1-m} (e + f x)^{1+m}) / \\
 & (d^4 f (d e - c f)^3 (1 + m) (2 + m)) - \frac{1}{d^5 f m} b^2 (3 a d f h - b (c f h (4 + m) - d (f g + e h m))) (c + d x)^{-m} \\
 & (e + f x)^m \left( \frac{d (e + f x)}{d e - c f} \right)^{-m} \text{Hypergeometric2F1}[-m, -m, 1 - m, -\frac{f (c + d x)}{d e - c f}]
 \end{aligned}$$

Result (type 6, 10997 leaves):

$$\begin{aligned}
 & (3 a b^2 g (c + d x)^{-3-m} (e + f x)^m \left( -2 d^3 e^3 x^3 \left( \frac{e (c + d x)}{c (e + f x)} \right)^m + c d^2 e^2 x^2 \left( f (6 + 5 m + m^2) x + e \left( 6 + 5 m + m^2 - 6 \left( \frac{e (c + d x)}{c (e + f x)} \right)^m \right) \right) \right) + \\
 & c^3 \left( -2 e^2 f m x + e f^2 m (1 + m) x^2 + f^3 (2 + 3 m + m^2) x^3 - 2 e^3 \left( -1 + \left( \frac{e (c + d x)}{c (e + f x)} \right)^m \right) \right) - \\
 & 2 c^2 d e x \left( e f m (3 + m) x + f^2 (3 + 4 m + m^2) x^2 + e^2 \left( -3 - m + 3 \left( \frac{e (c + d x)}{c (e + f x)} \right)^m \right) \right) / (c (-d e + c f)^3 (1 + m) (2 + m) (3 + m)) + \\
 & (3 a^2 b h (c + d x)^{-3-m} (e + f x)^m \left( -2 d^3 e^3 x^3 \left( \frac{e (c + d x)}{c (e + f x)} \right)^m + c d^2 e^2 x^2 \left( f (6 + 5 m + m^2) x + e \left( 6 + 5 m + m^2 - 6 \left( \frac{e (c + d x)}{c (e + f x)} \right)^m \right) \right) \right) + \\
 & c^3 \left( -2 e^2 f m x + e f^2 m (1 + m) x^2 + f^3 (2 + 3 m + m^2) x^3 - 2 e^3 \left( -1 + \left( \frac{e (c + d x)}{c (e + f x)} \right)^m \right) \right) - \\
 & 2 c^2 d e x \left( e f m (3 + m) x + f^2 (3 + 4 m + m^2) x^2 + e^2 \left( -3 - m + 3 \left( \frac{e (c + d x)}{c (e + f x)} \right)^m \right) \right) / \\
 & (c (-d e + c f)^3 (1 + m) (2 + m) (3 + m)) + \left( 5 b^3 c e g x^4 (c + d x)^{-4-m} (e + f x)^m \text{AppellF1}[4, 4 + m, -m, 5, -\frac{d x}{c}, -\frac{f x}{e}] \right) / \\
 & \left( 4 \left( 5 c e \text{AppellF1}[4, 4 + m, -m, 5, -\frac{d x}{c}, -\frac{f x}{e}] \right) +
 \right)
 \end{aligned}$$

$$\begin{aligned}
& c f m x \operatorname{AppellF1}\left[5, 4+m, 1-m, 6, -\frac{d x}{c}, -\frac{f x}{e}\right] - d e (4+m) \times \operatorname{AppellF1}\left[5, 5+m, -m, 6, -\frac{d x}{c}, -\frac{f x}{e}\right]\Big)\Big) + \\
& \left(15 a b^2 c e h x^4 (c+d x)^{-4-m} (e+f x)^m \operatorname{AppellF1}\left[4, 4+m, -m, 5, -\frac{d x}{c}, -\frac{f x}{e}\right]\right) / \left(4 \left(5 c e \operatorname{AppellF1}\left[4, 4+m, -m, 5, -\frac{d x}{c}, -\frac{f x}{e}\right]\right) + \right. \\
& \left.c f m x \operatorname{AppellF1}\left[5, 4+m, 1-m, 6, -\frac{d x}{c}, -\frac{f x}{e}\right] - d e (4+m) \times \operatorname{AppellF1}\left[5, 5+m, -m, 6, -\frac{d x}{c}, -\frac{f x}{e}\right]\right)\Big) + \\
& \left(6 b^3 c e h x^5 (c+d x)^{-4-m} (e+f x)^m \operatorname{AppellF1}\left[5, 4+m, -m, 6, -\frac{d x}{c}, -\frac{f x}{e}\right]\right) / \left(5 \left(6 c e \operatorname{AppellF1}\left[5, 4+m, -m, 6, -\frac{d x}{c}, -\frac{f x}{e}\right]\right) + \right. \\
& \left.c f m x \operatorname{AppellF1}\left[6, 4+m, 1-m, 7, -\frac{d x}{c}, -\frac{f x}{e}\right] - d e (4+m) \times \operatorname{AppellF1}\left[6, 5+m, -m, 7, -\frac{d x}{c}, -\frac{f x}{e}\right]\right)\Big) + \\
& \left(3 a^2 b e g x^2 (c+d x)^{-3-m} \left(\frac{c+d x}{c}\right)^{4+m} \left(1+\frac{d x}{c}\right)^{-4-m} (e+f x)^{-1+m} \left(\frac{e+f x}{e}\right)^{-m} \left(1+\frac{f x}{e}\right)^{1+m} \left(c (4+m) (3 e+f x)\right.\right. \\
& \left.\left.-2 d^3 e^3 x^3 + c^3 \left(-2 e^2 f m x \left(\frac{c (e+f x)}{e (c+d x)}\right)^m + e f^2 m (1+m) x^2 \left(\frac{c (e+f x)}{e (c+d x)}\right)^m + f^3 (2+3 m+m^2) x^3 \left(\frac{c (e+f x)}{e (c+d x)}\right)^m + 2 e^3 (-1+\left(\frac{c (e+f x)}{e (c+d x)}\right)^m)\right)\right) - \right. \\
& \left.2 c^2 d e x \left(e f m (3+m) \times \left(\frac{c (e+f x)}{e (c+d x)}\right)^m + f^2 (3+4 m+m^2) x^2 \left(\frac{c (e+f x)}{e (c+d x)}\right)^m - e^2 (-3+3 \left(\frac{c (e+f x)}{e (c+d x)}\right)^m + m \left(\frac{c (e+f x)}{e (c+d x)}\right)^m)\right)\right) + \right. \\
& \left.c d^2 e^2 x^2 \left(f (6+5 m+m^2) \times \left(\frac{c (e+f x)}{e (c+d x)}\right)^m + e (-6+6 \left(\frac{c (e+f x)}{e (c+d x)}\right)^m + 5 m \left(\frac{c (e+f x)}{e (c+d x)}\right)^m + m^2 \left(\frac{c (e+f x)}{e (c+d x)}\right)^m)\right)\right) \operatorname{Gamma}[4+m] - \right. \\
& \left(2 d^4 e^4 (1+m) x^4 - 2 c d^3 e^3 x^3 (-3 e m + f (4+m) x) + c^4 \left(e^2 f^2 (-5+m) m x^2 \left(\frac{c (e+f x)}{e (c+d x)}\right)^m + 2 e f^3 m (1+m) x^3 \left(\frac{c (e+f x)}{e (c+d x)}\right)^m + \right.\right. \\
& \left.f^4 (2+3 m+m^2) x^4 \left(\frac{c (e+f x)}{e (c+d x)}\right)^m + 6 e^4 (-1+\left(\frac{c (e+f x)}{e (c+d x)}\right)^m) - 2 e^3 f x (4+m-4 \left(\frac{c (e+f x)}{e (c+d x)}\right)^m + 2 m \left(\frac{c (e+f x)}{e (c+d x)}\right)^m)\right) - \right. \\
& \left.2 c^3 d e x \left(2 e f^2 m (4+m) x^2 \left(\frac{c (e+f x)}{e (c+d x)}\right)^m + f^3 (4+5 m+m^2) x^3 \left(\frac{c (e+f x)}{e (c+d x)}\right)^m + e^2 f (4+m) \times \left(3-3 \left(\frac{c (e+f x)}{e (c+d x)}\right)^m + m \left(\frac{c (e+f x)}{e (c+d x)}\right)^m\right)\right) - \right. \\
& \left.e^3 \left(-8+m+8 \left(\frac{c (e+f x)}{e (c+d x)}\right)^m + 2 m \left(\frac{c (e+f x)}{e (c+d x)}\right)^m\right) + c^2 d^2 e^2 x^2 \left(f^2 (12+7 m+m^2) x^2 \left(\frac{c (e+f x)}{e (c+d x)}\right)^m + 2 e f (4+m) \times \left(-3+3 \left(\frac{c (e+f x)}{e (c+d x)}\right)^m + m \left(\frac{c (e+f x)}{e (c+d x)}\right)^m\right)\right) + e^2 \left(m^2 \left(\frac{c (e+f x)}{e (c+d x)}\right)^m + 12 (-1+\left(\frac{c (e+f x)}{e (c+d x)}\right)^m) + m (6+7 \left(\frac{c (e+f x)}{e (c+d x)}\right)^m)\right)\right) \operatorname{Gamma}[5+m]\right) / \\
& \left(c \left(24 c^4 e^4 \operatorname{Gamma}[4+m] + 6 c^4 e^4 m \operatorname{Gamma}[4+m] + 96 c^3 d e^4 x \operatorname{Gamma}[4+m] + 24 c^3 d e^4 m x \operatorname{Gamma}[4+m] + 144 c^2 d^2 e^4 x^2 \operatorname{Gamma}[4+m] + \right.\right. \\
& 36 c^2 d^2 e^4 m x^2 \operatorname{Gamma}[4+m] + 96 c d^3 e^4 x^3 \operatorname{Gamma}[4+m] + 24 c d^3 e^4 m x^3 \operatorname{Gamma}[4+m] + 24 d^4 e^4 x^4 \operatorname{Gamma}[4+m] + \\
& 6 d^4 e^4 m x^4 \operatorname{Gamma}[4+m] - 24 c^4 e^4 \left(\frac{c (e+f x)}{e (c+d x)}\right)^m \operatorname{Gamma}[4+m] - 6 c^4 e^4 m \left(\frac{c (e+f x)}{e (c+d x)}\right)^m \operatorname{Gamma}[4+m] - \\
& 96 c^3 d e^4 x \left(\frac{c (e+f x)}{e (c+d x)}\right)^m \operatorname{Gamma}[4+m] - 48 c^3 d e^4 m x \left(\frac{c (e+f x)}{e (c+d x)}\right)^m \operatorname{Gamma}[4+m] + 24 c^4 e^3 f m x \left(\frac{c (e+f x)}{e (c+d x)}\right)^m \operatorname{Gamma}[4+m] - 
\end{aligned}$$

$$\begin{aligned}
& 6 c^3 d e^4 m^2 x \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 6 c^4 e^3 f m^2 x \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - 144 c^2 d^2 e^4 x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 120 c^2 d^2 e^4 m x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 96 c^3 d e^3 f m x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - 12 c^4 e^2 f^2 m x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 33 c^2 d^2 e^4 m^2 x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 48 c^3 d e^3 f m^2 x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - 15 c^4 e^2 f^2 m^2 x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 3 c^2 d^2 e^4 m^3 x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 6 c^3 d e^3 f m^3 x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - 3 c^4 e^2 f^2 m^3 x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 144 c d^3 e^4 x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 144 c^2 d^2 e^3 f x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - 144 c^3 d e^2 f^2 x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + \\
& 48 c^4 e f^3 x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - 228 c d^3 e^4 m x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 444 c^2 d^2 e^3 f m x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 348 c^3 d e^2 f^2 m x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 108 c^4 e f^3 m x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - 132 c d^3 e^4 m^2 x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + \\
& 330 c^2 d^2 e^3 f m^2 x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - 282 c^3 d e^2 f^2 m^2 x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + \\
& 84 c^4 e f^3 m^2 x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - 33 c d^3 e^4 m^3 x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 93 c^2 d^2 e^3 f m^3 x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 87 c^3 d e^2 f^2 m^3 x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 27 c^4 e f^3 m^3 x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - 3 c d^3 e^4 m^4 x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + \\
& 9 c^2 d^2 e^3 f m^4 x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - 9 c^3 d e^2 f^2 m^4 x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 3 c^4 e f^3 m^4 x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 96 c d^3 e^3 f x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 144 c^2 d^2 e^2 f^2 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - 96 c^3 d e f^3 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + \\
& 24 c^4 f^4 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - 128 c d^3 e^3 f m x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 264 c^2 d^2 e^2 f^2 m x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 192 c^3 d e f^3 m x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 50 c^4 f^4 m x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - 62 c d^3 e^3 f m^2 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + \\
& 153 c^2 d^2 e^2 f^2 m^2 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - 126 c^3 d e f^3 m^2 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 35 c^4 f^4 m^2 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 13 c d^3 e^3 f m^3 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 36 c^2 d^2 e^2 f^2 m^3 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - 33 c^3 d e f^3 m^3 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] +
\end{aligned}$$

$$\begin{aligned}
& 10 c^4 f^4 m^3 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - c d^3 e^3 f m^4 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 3 c^2 d^2 e^2 f^2 m^4 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 3 c^3 d e f^3 m^4 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + c^4 f^4 m^4 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - 6 c^4 e^4 \text{Gamma}[5 + m] - 24 c^3 d e^4 x \text{Gamma}[5 + m] - \\
& 36 c^2 d^2 e^4 x^2 \text{Gamma}[5 + m] - 24 c d^3 e^4 x^3 \text{Gamma}[5 + m] - 6 d^4 e^4 x^4 \text{Gamma}[5 + m] + 6 c^4 e^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + \\
& 24 c^3 d e^4 x \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + 6 c^3 d e^4 m x \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - 6 c^4 e^3 f m x \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + \\
& 36 c^2 d^2 e^4 x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + 21 c^2 d^2 e^4 m x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - 24 c^3 d e^3 f m x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + \\
& 3 c^4 e^2 f^2 m x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + 3 c^2 d^2 e^4 m^2 x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - 6 c^3 d e^3 f m^2 x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + \\
& 3 c^4 e^2 f^2 m^2 x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + 24 c d^3 e^4 x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + 26 c d^3 e^4 m x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - \\
& 36 c^2 d^2 e^3 f m x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + 12 c^3 d e^2 f^2 m x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - 2 c^4 e f^3 m x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + \\
& 9 c d^3 e^4 m^2 x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - 21 c^2 d^2 e^3 f m^2 x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + 15 c^3 d e^2 f^2 m^2 x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - \\
& 3 c^4 e f^3 m^2 x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + c d^3 e^4 m^3 x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - 3 c^2 d^2 e^3 f m^3 x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + \\
& 3 c^3 d e^2 f^2 m^3 x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - c^4 e f^3 m^3 x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + 24 c d^3 e^3 f x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - \\
& 36 c^2 d^2 e^2 f^2 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + 24 c^3 d e f^3 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - 6 c^4 f^4 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + \\
& 26 c d^3 e^3 f m x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - 57 c^2 d^2 e^2 f^2 m x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + 42 c^3 d e f^3 m x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - \\
& 11 c^4 f^4 m x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + 9 c d^3 e^3 f m^2 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - 24 c^2 d^2 e^2 f^2 m^2 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + \\
& 21 c^3 d e f^3 m^2 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - 6 c^4 f^4 m^2 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + c d^3 e^3 f m^3 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - \\
& 3 c^2 d^2 e^2 f^2 m^3 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + 3 c^3 d e f^3 m^3 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - c^4 f^4 m^3 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] \Big) + 
\end{aligned}$$

$$\begin{aligned}
& \left( a^3 e h x^2 (c + d x)^{-3-m} \left( \frac{c + d x}{c} \right)^{4+m} \left( 1 + \frac{d x}{c} \right)^{-4-m} (e + f x)^{-1+m} \left( \frac{e + f x}{e} \right)^{-m} \left( 1 + \frac{f x}{e} \right)^{1+m} \left( c (4 + m) (3 e + f x) \right. \right. \\
& \quad \left. \left. - 2 d^3 e^3 x^3 + c^3 \left( -2 e^2 f m x \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + e f^2 m (1 + m) x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + f^3 (2 + 3 m + m^2) x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \right) + 2 e^3 \left( -1 + \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \right) \right) - \right. \\
& \quad \left. 2 c^2 d e x \left( e f m (3 + m) \times \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + f^2 (3 + 4 m + m^2) x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m - e^2 (-3 + 3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + m \left( \frac{c (e + f x)}{e (c + d x)} \right)^m) \right) + \right. \\
& \quad \left. c d^2 e^2 x^2 \left( f (6 + 5 m + m^2) \times \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + e (-6 + 6 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + 5 m \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + m^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m) \right) \right) \text{Gamma}[4 + m] - \right. \\
& \quad \left. \left( 2 d^4 e^4 (1 + m) x^4 - 2 c d^3 e^3 x^3 (-3 e m + f (4 + m) x) + c^4 \left( e^2 f^2 (-5 + m) m x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + 2 e f^3 m (1 + m) x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \right) + \right. \right. \\
& \quad \left. \left. f^4 (2 + 3 m + m^2) x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + 6 e^4 \left( -1 + \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \right) - 2 e^3 f x (4 + m - 4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + 2 m \left( \frac{c (e + f x)}{e (c + d x)} \right)^m) \right) - \right. \\
& \quad \left. 2 c^3 d e x \left( 2 e f^2 m (4 + m) x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + f^3 (4 + 5 m + m^2) x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + e^2 f (4 + m) x (3 - 3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + m \left( \frac{c (e + f x)}{e (c + d x)} \right)^m) \right. \right. \\
& \quad \left. \left. - e^3 (-8 + m + 8 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + 2 m \left( \frac{c (e + f x)}{e (c + d x)} \right)^m) + c^2 d^2 e^2 x^2 \left( f^2 (12 + 7 m + m^2) x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + 2 e f (4 + m) x (-3 + 3 \right. \right. \right. \\
& \quad \left. \left. \left. \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + m \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \right) + e^2 (m^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + 12 \left( -1 + \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \right) + m (6 + 7 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m) \right) \right) \right) \text{Gamma}[5 + m] \right) \right) / \\
& \left( c \left( 24 c^4 e^4 \text{Gamma}[4 + m] + 6 c^4 e^4 m \text{Gamma}[4 + m] + 96 c^3 d e^4 x \text{Gamma}[4 + m] + 24 c^3 d e^4 m x \text{Gamma}[4 + m] + 144 c^2 d^2 e^4 x^2 \text{Gamma}[4 + m] + \right. \right. \\
& \quad \left. \left. 36 c^2 d^2 e^4 m x^2 \text{Gamma}[4 + m] + 96 c d^3 e^4 x^3 \text{Gamma}[4 + m] + 24 c d^3 e^4 m x^3 \text{Gamma}[4 + m] + 24 d^4 e^4 x^4 \text{Gamma}[4 + m] + \right. \right. \\
& \quad \left. \left. 6 d^4 e^4 m x^4 \text{Gamma}[4 + m] - 24 c^4 e^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - 6 c^4 e^4 m \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \right. \right. \\
& \quad \left. \left. 96 c^3 d e^4 x \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - 48 c^3 d e^4 m x \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 24 c^4 e^3 f m x \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \right. \right. \\
& \quad \left. \left. 6 c^3 d e^4 m^2 x \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 6 c^4 e^3 f m^2 x \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - 144 c^2 d^2 e^4 x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \right. \right. \\
& \quad \left. \left. 120 c^2 d^2 e^4 m x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 96 c^3 d e^3 f m x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - 12 c^4 e^2 f^2 m x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \right. \right. \\
& \quad \left. \left. 33 c^2 d^2 e^4 m^2 x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 48 c^3 d e^3 f m^2 x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - 15 c^4 e^2 f^2 m^2 x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \right. \right. \\
& \quad \left. \left. 3 c^2 d^2 e^4 m^3 x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 6 c^3 d e^3 f m^3 x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - 3 c^4 e^2 f^2 m^3 x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \right. \right)
\end{aligned}$$

$$\begin{aligned}
& 144 c d^3 e^4 x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 144 c^2 d^2 e^3 f x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - 144 c^3 d e^2 f^2 x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + \\
& 48 c^4 e f^3 x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - 228 c d^3 e^4 m x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 444 c^2 d^2 e^3 f m x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 348 c^3 d e^2 f^2 m x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 108 c^4 e f^3 m x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - 132 c d^3 e^4 m^2 x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + \\
& 330 c^2 d^2 e^3 f m^2 x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - 282 c^3 d e^2 f^2 m^2 x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + \\
& 84 c^4 e f^3 m^2 x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - 33 c d^3 e^4 m^3 x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 93 c^2 d^2 e^3 f m^3 x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 87 c^3 d e^2 f^2 m^3 x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 27 c^4 e f^3 m^3 x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - 3 c d^3 e^4 m^4 x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + \\
& 9 c^2 d^2 e^3 f m^4 x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - 9 c^3 d e^2 f^2 m^4 x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 3 c^4 e f^3 m^4 x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 96 c d^3 e^3 f x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 144 c^2 d^2 e^2 f^2 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - 96 c^3 d e f^3 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + \\
& 24 c^4 f^4 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - 128 c d^3 e^3 f m x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 264 c^2 d^2 e^2 f^2 m x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 192 c^3 d e f^3 m x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 50 c^4 f^4 m x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - 62 c d^3 e^3 f m^2 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + \\
& 153 c^2 d^2 e^2 f^2 m^2 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - 126 c^3 d e f^3 m^2 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 35 c^4 f^4 m^2 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 13 c d^3 e^3 f m^3 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 36 c^2 d^2 e^2 f^2 m^3 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - 33 c^3 d e f^3 m^3 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + \\
& 10 c^4 f^4 m^3 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - c d^3 e^3 f m^4 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + 3 c^2 d^2 e^2 f^2 m^4 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - \\
& 3 c^3 d e f^3 m^4 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] + c^4 f^4 m^4 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[4 + m] - 6 c^4 e^4 \text{Gamma}[5 + m] - 24 c^3 d e^4 x \text{Gamma}[5 + m] - \\
& 36 c^2 d^2 e^4 x^2 \text{Gamma}[5 + m] - 24 c d^3 e^4 x^3 \text{Gamma}[5 + m] - 6 d^4 e^4 x^4 \text{Gamma}[5 + m] + 6 c^4 e^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + \\
& 24 c^3 d e^4 x \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + 6 c^3 d e^4 m x \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - 6 c^4 e^3 f m x \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] +
\end{aligned}$$

$$\begin{aligned}
& 36 c^2 d^2 e^4 x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + 21 c^2 d^2 e^4 m x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - 24 c^3 d e^3 f m x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + \\
& 3 c^4 e^2 f^2 m x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + 3 c^2 d^2 e^4 m^2 x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - 6 c^3 d e^3 f m^2 x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + \\
& 3 c^4 e^2 f^2 m^2 x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + 24 c d^3 e^4 x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + 26 c d^3 e^4 m x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - \\
& 36 c^2 d^2 e^3 f m x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + 12 c^3 d e^2 f^2 m x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - 2 c^4 e f^3 m x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + \\
& 9 c d^3 e^4 m^2 x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - 21 c^2 d^2 e^3 f m^2 x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + 15 c^3 d e^2 f^2 m^2 x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - \\
& 3 c^4 e f^3 m^2 x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + c d^3 e^4 m^3 x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - 3 c^2 d^2 e^3 f m^3 x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + \\
& 3 c^3 d e^2 f^2 m^3 x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - c^4 e f^3 m^3 x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + 24 c d^3 e^3 f x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - \\
& 36 c^2 d^2 e^2 f^2 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + 24 c^3 d e f^3 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - 6 c^4 f^4 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + \\
& 26 c d^3 e^3 f m x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - 57 c^2 d^2 e^2 f^2 m x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + 42 c^3 d e f^3 m x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - \\
& 11 c^4 f^4 m x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + 9 c d^3 e^3 f m^2 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - 24 c^2 d^2 e^2 f^2 m^2 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + \\
& 21 c^3 d e f^3 m^2 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - 6 c^4 f^4 m^2 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + c d^3 e^3 f m^3 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - \\
& 3 c^2 d^2 e^2 f^2 m^3 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] + 3 c^3 d e f^3 m^3 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] - c^4 f^4 m^3 x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \text{Gamma}[5 + m] \Big) + \\
& \frac{a^3 f^3 g (e + f x)^{1+m} \left( c - \frac{d e}{f} + \frac{d (e+f x)}{f} \right)^{-m} \left( 1 + \frac{d (e+f x)}{\left( c - \frac{d e}{f} \right) f} \right)^m \text{Hypergeometric2F1}\left[ 1+m, 4+m, 2+m, -\frac{d (e+f x)}{\left( c - \frac{d e}{f} \right) f} \right]}{(-d e + c f)^4 (1+m)}
\end{aligned}$$

**Problem 133:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (a + b x)^2 (c + d x)^{-4-m} (e + f x)^m (g + h x) dx$$

Optimal (type 5, 572 leaves, 9 steps):

$$\begin{aligned}
 & \frac{(b c - a d) (d g - c h) (a d f + b (c f (2 + m) - d e (3 + m))) (c + d x)^{-3-m} (e + f x)^{1+m}}{d^3 f (d e - c f) (3 + m)} - \\
 & \frac{b (d g - c h) (a + b x) (c + d x)^{-3-m} (e + f x)^{1+m}}{d^2 f} - \frac{(b c - a d)^2 h (c + d x)^{-2-m} (e + f x)^{1+m}}{d^3 (d e - c f) (2 + m)} - \\
 & \left( (d g - c h) (b^2 (d e - c f) (2 + m) (c f (1 + m) - d e (3 + m)) - 2 d f (b^2 c e + a^2 d f + a b (c f (1 + m) - d e (3 + m))) (c + d x)^{-2-m} (e + f x)^{1+m}) \right) / \\
 & \left( d^3 f (d e - c f)^2 (2 + m) (3 + m) \right) - \frac{(b c - a d) h (a d f - b (2 d e (2 + m) - c f (3 + 2 m))) (c + d x)^{-1-m} (e + f x)^{1+m}}{d^3 (d e - c f)^2 (1 + m) (2 + m)} + \\
 & \left( (d g - c h) (b^2 (d e - c f) (2 + m) (c f (1 + m) - d e (3 + m)) - 2 d f (b^2 c e + a^2 d f + a b (c f (1 + m) - d e (3 + m))) (c + d x)^{-1-m} (e + f x)^{1+m}) \right) / \\
 & \left( d^3 (d e - c f)^3 (1 + m) (2 + m) (3 + m) \right) - \frac{b^2 h (c + d x)^{-m} (e + f x)^m \left( \frac{d (e + f x)}{d e - c f} \right)^{-m} \text{Hypergeometric2F1}[-m, -m, 1 - m, -\frac{f (c + d x)}{d e - c f}]}{d^4 m}
 \end{aligned}$$

Result (type 6, 5412 leaves):

$$\begin{aligned}
 & \left( b^2 g (c + d x)^{-3-m} (e + f x)^m \left( -2 d^3 e^3 x^3 \left( \frac{e (c + d x)}{c (e + f x)} \right)^m + c d^2 e^2 x^2 \left( f (6 + 5 m + m^2) x + e \left( 6 + 5 m + m^2 - 6 \left( \frac{e (c + d x)}{c (e + f x)} \right)^m \right) \right) \right) + \right. \\
 & \quad c^3 \left( -2 e^2 f m x + e f^2 m (1 + m) x^2 + f^3 (2 + 3 m + m^2) x^3 - 2 e^3 \left( -1 + \left( \frac{e (c + d x)}{c (e + f x)} \right)^m \right) \right) - \\
 & \quad \left. 2 c^2 d e x \left( e f m (3 + m) x + f^2 (3 + 4 m + m^2) x^2 + e^2 \left( -3 - m + 3 \left( \frac{e (c + d x)}{c (e + f x)} \right)^m \right) \right) \right) / \left( c (-d e + c f)^3 (1 + m) (2 + m) (3 + m) \right) + \\
 & \left( 2 a b h (c + d x)^{-3-m} (e + f x)^m \left( -2 d^3 e^3 x^3 \left( \frac{e (c + d x)}{c (e + f x)} \right)^m + c d^2 e^2 x^2 \left( f (6 + 5 m + m^2) x + e \left( 6 + 5 m + m^2 - 6 \left( \frac{e (c + d x)}{c (e + f x)} \right)^m \right) \right) \right) + \right. \\
 & \quad c^3 \left( -2 e^2 f m x + e f^2 m (1 + m) x^2 + f^3 (2 + 3 m + m^2) x^3 - 2 e^3 \left( -1 + \left( \frac{e (c + d x)}{c (e + f x)} \right)^m \right) \right) - \\
 & \quad \left. 2 c^2 d e x \left( e f m (3 + m) x + f^2 (3 + 4 m + m^2) x^2 + e^2 \left( -3 - m + 3 \left( \frac{e (c + d x)}{c (e + f x)} \right)^m \right) \right) \right) / \\
 & \left( c (-d e + c f)^3 (1 + m) (2 + m) (3 + m) \right) + \left( 5 b^2 c e h x^4 (c + d x)^{-4-m} (e + f x)^m \text{AppellF1}[4, 4 + m, -m, 5, -\frac{d x}{c}, -\frac{f x}{e}] \right) / \\
 & \left( 4 \left( 5 c e \text{AppellF1}[4, 4 + m, -m, 5, -\frac{d x}{c}, -\frac{f x}{e}] + c f m x \text{AppellF1}[5, 4 + m, 1 - m, 6, -\frac{d x}{c}, -\frac{f x}{e}] \right) - \right. \\
 & \quad d e (4 + m) x \text{AppellF1}[5, 5 + m, -m, 6, -\frac{d x}{c}, -\frac{f x}{e}] \left. \right) + \left( 2 a b g x^2 (c + d x)^{-3-m} (e + f x)^m \left( c (4 + m) (3 e + f x) \right. \right. \\
 & \quad \left. \left. - 2 d^3 e^3 x^3 + c^3 \left( -2 e^2 f m x \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + e f^2 m (1 + m) x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + f^3 (2 + 3 m + m^2) x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + 2 e^3 \left( -1 + \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \right) \right) \right) - 
 \end{aligned}$$

$$\begin{aligned}
& 2 c^2 d e x \left( e f m (3 + m) \times \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + f^2 (3 + 4 m + m^2) x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m - e^2 (-3 + 3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + m \left( \frac{c (e + f x)}{e (c + d x)} \right)^m) \right) + \\
& c d^2 e^2 x^2 \left( f (6 + 5 m + m^2) \times \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + e \left( -6 + 6 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + 5 m \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + m^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \right) \right) \text{Gamma}[4 + m] - \\
& \left( 2 d^4 e^4 (1 + m) x^4 - 2 c d^3 e^3 x^3 (-3 e m + f (4 + m) x) + c^4 \left( e^2 f^2 (-5 + m) m x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + 2 e f^3 m (1 + m) x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + \right. \right. \\
& f^4 (2 + 3 m + m^2) x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + 6 e^4 \left( -1 + \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \right) - 2 e^3 f x \left( 4 + m - 4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + 2 m \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \right) \left. \right) - \\
& 2 c^3 d e x \left( 2 e f^2 m (4 + m) x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + f^3 (4 + 5 m + m^2) x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + e^2 f (4 + m) x \left( 3 - 3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + m \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \right) \right. - \\
& e^3 \left( -8 + m + 8 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + 2 m \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \right) + c^2 d^2 e^2 x^2 \left( f^2 (12 + 7 m + m^2) x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + 2 e f (4 + m) x \left( -3 + 3 \right. \right. \\
& \left. \left. \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + m \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \right) + e^2 \left( m^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + 12 \left( -1 + \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \right) + m \left( 6 + 7 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \right) \right) \right) \text{Gamma}[5 + m] \right) / \\
& \left( c \left( (4 + m) \left( 6 d^4 e^4 x^4 + c^4 \left( 6 e^3 f m x \left( \frac{c (e + f x)}{e (c + d x)} \right)^m - 3 e^2 f^2 m (1 + m) x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + 3 e f^3 (1 + m) (2 + m)^2 x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \right) + \right. \right. \\
& f^4 (6 + 11 m + 6 m^2 + m^3) x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m - 6 e^4 \left( -1 + \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \right) \left. \right) - 3 c^3 d e x \left( -2 e^2 f m (4 + m) x \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + e f^2 (12 + 26 m + 17 \right. \\
& m^2 + 3 m^3) x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + f^3 (8 + 14 m + 7 m^2 + m^3) x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + 2 e^3 \left( -4 + 4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + m \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \right) \left. \right) - c d^3 e^3 x^3 \\
& \left. \left( f (24 + 26 m + 9 m^2 + m^3) x \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + 3 e \left( -8 + 12 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + 16 m \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + 7 m^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + m^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \right) \right) \right) + \\
& 3 c^2 d^2 e^2 x^2 \left( e f (12 + 34 m + 19 m^2 + 3 m^3) x \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + f^2 (12 + 19 m + 8 m^2 + m^3) x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m - \right. \\
& e^2 \left( 7 m \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + m^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + 12 \left( -1 + \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \right) \right) \right) \text{Gamma}[4 + m] + \\
& \left( -6 d^4 e^4 x^4 + c^4 \left( -6 e^3 f m x \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + 3 e^2 f^2 m (1 + m) x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m - e f^3 m (2 + 3 m + m^2) x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \right) - \right. \\
& f^4 (6 + 11 m + 6 m^2 + m^3) x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + 6 e^4 \left( -1 + \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \right) \left. \right) + 3 c^3 d e x \left( -2 e^2 f m (4 + m) x \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + \right. \\
& e f^2 m (4 + 5 m + m^2) x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + f^3 (8 + 14 m + 7 m^2 + m^3) x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + 2 e^3 \left( -4 + 4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + m \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \right) \left. \right) -
\end{aligned}$$

$$\begin{aligned}
& 3 c^2 d^2 e^2 x^2 \left( e f m (12 + 7 m + m^2) \times \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + f^2 (12 + 19 m + 8 m^2 + m^3) x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m - \right. \\
& \quad e^2 \left( 7 m \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + m^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + 12 \left( -1 + \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \right) \right) + c d^3 e^3 x^3 \left( f (24 + 26 m + 9 m^2 + m^3) \times \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + \right. \\
& \quad \left. e \left( 26 m \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + 9 m^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + m^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + 24 \left( -1 + \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \right) \right) \right) \text{Gamma}[5 + m] \Big) + \\
& \left( a^2 h x^2 (c + d x)^{-3-m} (e + f x)^m \left( c (4 + m) (3 e + f x) \left( -2 d^3 e^3 x^3 + c^3 \left( -2 e^2 f m x \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + e f^2 m (1 + m) x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. f^3 (2 + 3 m + m^2) x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + 2 e^3 \left( -1 + \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \right) \right) - \right. \right. \right. \right. \right. \\
& \quad 2 c^2 d e x \left( e f m (3 + m) \times \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + f^2 (3 + 4 m + m^2) x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m - e^2 (-3 + 3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + m \left( \frac{c (e + f x)}{e (c + d x)} \right)^m) \right) + \\
& \quad c d^2 e^2 x^2 \left( f (6 + 5 m + m^2) \times \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + e \left( -6 + 6 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + 5 m \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + m^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \right) \right) \text{Gamma}[4 + m] - \\
& \left( 2 d^4 e^4 (1 + m) x^4 - 2 c d^3 e^3 x^3 (-3 e m + f (4 + m) x) + c^4 \left( e^2 f^2 (-5 + m) m x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + 2 e f^3 m (1 + m) x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + \right. \right. \\
& \quad \left. \left. f^4 (2 + 3 m + m^2) x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + 6 e^4 \left( -1 + \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \right) - 2 e^3 f x (4 + m - 4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + 2 m \left( \frac{c (e + f x)}{e (c + d x)} \right)^m) \right) - \right. \\
& \quad 2 c^3 d e x \left( 2 e f^2 m (4 + m) x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + f^3 (4 + 5 m + m^2) x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + e^2 f (4 + m) x \left( 3 - 3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + m \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \right) - \right. \\
& \quad \left. e^3 \left( -8 + m + 8 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + 2 m \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \right) \right) + \\
& \quad c^2 d^2 e^2 x^2 \left( f^2 (12 + 7 m + m^2) x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + 2 e f (4 + m) x \left( -3 + 3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + m \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \right) + \right. \\
& \quad \left. e^2 \left( m^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + 12 \left( -1 + \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \right) + m \left( 6 + 7 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \right) \right) \text{Gamma}[5 + m] \right) / \\
& \left( c \left( (4 + m) \left( 6 d^4 e^4 x^4 + c^4 \left( 6 e^3 f m x \left( \frac{c (e + f x)}{e (c + d x)} \right)^m - 3 e^2 f^2 m (1 + m) x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + 3 e f^3 (1 + m) (2 + m)^2 x^3 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. f^4 (6 + 11 m + 6 m^2 + m^3) x^4 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m - 6 e^4 \left( -1 + \left( \frac{c (e + f x)}{e (c + d x)} \right)^m \right) \right) - \right. \right. \right. \right. \right. \\
& \quad 3 c^3 d e x \left( -2 e^2 f m (4 + m) x \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + e f^2 (12 + 26 m + 17 m^2 + 3 m^3) x^2 \left( \frac{c (e + f x)}{e (c + d x)} \right)^m + \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. \right) \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& f^3 (8 + 14m + 7m^2 + m^3) x^3 \left( \frac{c(e + fx)}{e(c + dx)} \right)^m + 2e^3 \left( -4 + 4 \left( \frac{c(e + fx)}{e(c + dx)} \right)^m + m \left( \frac{c(e + fx)}{e(c + dx)} \right)^m \right) - c d^3 e^3 x^3 \\
& \left( f (24 + 26m + 9m^2 + m^3) x \left( \frac{c(e + fx)}{e(c + dx)} \right)^m + 3e \left( -8 + 12 \left( \frac{c(e + fx)}{e(c + dx)} \right)^m + 16m \left( \frac{c(e + fx)}{e(c + dx)} \right)^m + 7m^2 \left( \frac{c(e + fx)}{e(c + dx)} \right)^m + m^3 \left( \frac{c(e + fx)}{e(c + dx)} \right)^m \right) \right) + \\
& 3c^2 d^2 e^2 x^2 \left( e f (12 + 34m + 19m^2 + 3m^3) x \left( \frac{c(e + fx)}{e(c + dx)} \right)^m + f^2 (12 + 19m + 8m^2 + m^3) x^2 \left( \frac{c(e + fx)}{e(c + dx)} \right)^m - \right. \\
& \left. e^2 \left( 7m \left( \frac{c(e + fx)}{e(c + dx)} \right)^m + m^2 \left( \frac{c(e + fx)}{e(c + dx)} \right)^m + 12 \left( -1 + \left( \frac{c(e + fx)}{e(c + dx)} \right)^m \right) \right) \right) \text{Gamma}[4 + m] + \\
& \left( -6d^4 e^4 x^4 + c^4 \left( -6e^3 f m x \left( \frac{c(e + fx)}{e(c + dx)} \right)^m + 3e^2 f^2 m (1 + m) x^2 \left( \frac{c(e + fx)}{e(c + dx)} \right)^m - e f^3 m (2 + 3m + m^2) x^3 \left( \frac{c(e + fx)}{e(c + dx)} \right)^m - \right. \right. \\
& \left. \left. f^4 (6 + 11m + 6m^2 + m^3) x^4 \left( \frac{c(e + fx)}{e(c + dx)} \right)^m + 6e^4 \left( -1 + \left( \frac{c(e + fx)}{e(c + dx)} \right)^m \right) \right) + 3c^3 d e x \left( -2e^2 f m (4 + m) x \left( \frac{c(e + fx)}{e(c + dx)} \right)^m + \right. \\
& \left. e f^2 m (4 + 5m + m^2) x^2 \left( \frac{c(e + fx)}{e(c + dx)} \right)^m + f^3 (8 + 14m + 7m^2 + m^3) x^3 \left( \frac{c(e + fx)}{e(c + dx)} \right)^m + 2e^3 \left( -4 + 4 \left( \frac{c(e + fx)}{e(c + dx)} \right)^m + m \left( \frac{c(e + fx)}{e(c + dx)} \right)^m \right) \right) - \\
& 3c^2 d^2 e^2 x^2 \left( e f m (12 + 7m + m^2) x \left( \frac{c(e + fx)}{e(c + dx)} \right)^m + f^2 (12 + 19m + 8m^2 + m^3) x^2 \left( \frac{c(e + fx)}{e(c + dx)} \right)^m - \right. \\
& \left. e^2 \left( 7m \left( \frac{c(e + fx)}{e(c + dx)} \right)^m + m^2 \left( \frac{c(e + fx)}{e(c + dx)} \right)^m + 12 \left( -1 + \left( \frac{c(e + fx)}{e(c + dx)} \right)^m \right) \right) + c d^3 e^3 x^3 \left( f (24 + 26m + 9m^2 + m^3) x \left( \frac{c(e + fx)}{e(c + dx)} \right)^m + \right. \right. \\
& \left. \left. e (26m \left( \frac{c(e + fx)}{e(c + dx)} \right)^m + 9m^2 \left( \frac{c(e + fx)}{e(c + dx)} \right)^m + m^3 \left( \frac{c(e + fx)}{e(c + dx)} \right)^m + 24 \left( -1 + \left( \frac{c(e + fx)}{e(c + dx)} \right)^m \right) \right) \right) \text{Gamma}[5 + m] \right) + \\
& \frac{a^2 f^3 g (e + fx)^{1+m} \left( c - \frac{d e}{f} + \frac{d(e+fx)}{f} \right)^{-m} \left( 1 + \frac{d(e+fx)}{(c-d e/f) f} \right)^m \text{Hypergeometric2F1}[1+m, 4+m, 2+m, -\frac{d(e+fx)}{(c-d e/f) f}]}{(-d e + c f)^4 (1+m)}
\end{aligned}$$

**Problem 136: Result more than twice size of optimal antiderivative.**

$$\int \frac{(A + Bx) (c + dx)^n (e + fx)^p}{a + bx} dx$$

Optimal (type 6, 177 leaves, 5 steps):

$$\begin{aligned}
& - \frac{(A b - a B) (c + d x)^{1+n} (e + f x)^p \left( \frac{d(e+f x)}{d e - c f} \right)^{-p} \text{AppellF1}[1+n, 1, -p, 2+n, \frac{b(c+d x)}{b c - a d}, -\frac{f(c+d x)}{d e - c f}]}{b (b c - a d) (1+n)} - \\
& \frac{B (c + d x)^{1+n} (e + f x)^{1+p} \text{Hypergeometric2F1}[1, 2+n+p, 2+p, \frac{d(e+f x)}{d e - c f}]}{b (d e - c f) (1+p)}
\end{aligned}$$

Result (type 6, 692 leaves):

$$\begin{aligned}
& \frac{1}{b^2 f} (c + d x)^n (e + f x)^p \left( \left( A b d f^2 (-1+n+p) (a + b x) \text{AppellF1}[-n-p, -n, -p, 1-n-p, \frac{-b c + a d}{d(a+b x)}, \frac{-b e + a f}{f(a+b x)}] \right) / \right. \\
& \left( (n+p) \left( d f (-1+n+p) (a + b x) \text{AppellF1}[-n-p, -n, -p, 1-n-p, \frac{-b c + a d}{d(a+b x)}, \frac{-b e + a f}{f(a+b x)}] + (-b c + a d) f n \text{AppellF1}[1-n-p, 1-n, \right. \right. \\
& \left. \left. -p, 2-n-p, \frac{-b c + a d}{d(a+b x)}, \frac{-b e + a f}{f(a+b x)}] + d (-b e + a f) p \text{AppellF1}[1-n-p, -n, 1-p, 2-n-p, \frac{-b c + a d}{d(a+b x)}, \frac{-b e + a f}{f(a+b x)}] \right) - \right. \\
& \left( a B d f^2 (-1+n+p) (a + b x) \text{AppellF1}[-n-p, -n, -p, 1-n-p, \frac{-b c + a d}{d(a+b x)}, \frac{-b e + a f}{f(a+b x)}] \right) / \\
& \left( (n+p) \left( d f (-1+n+p) (a + b x) \text{AppellF1}[-n-p, -n, -p, 1-n-p, \frac{-b c + a d}{d(a+b x)}, \frac{-b e + a f}{f(a+b x)}] + (-b c + a d) f n \text{AppellF1}[1-n-p, 1-n, \right. \right. \\
& \left. \left. -p, 2-n-p, \frac{-b c + a d}{d(a+b x)}, \frac{-b e + a f}{f(a+b x)}] + d (-b e + a f) p \text{AppellF1}[1-n-p, -n, 1-p, 2-n-p, \frac{-b c + a d}{d(a+b x)}, \frac{-b e + a f}{f(a+b x)}] \right) + \right. \\
& \left. \frac{b B \left( \frac{f(c+d x)}{-d e + c f} \right)^{-n} (e + f x) \text{Hypergeometric2F1}[-n, 1+p, 2+p, \frac{d(e+f x)}{d e - c f}]}{1+p} \right)
\end{aligned}$$

Problem 137: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^m (A + B x) (c + d x)^{-m}}{e + f x} dx$$

Optimal (type 5, 233 leaves, 5 steps):

$$\begin{aligned}
& - \frac{d (B e - A f) (a + b x)^{1+m} (c + d x)^{-m}}{(b c - a d) f^2 m} - \frac{(B e - A f) (a + b x)^m (c + d x)^{-m} \text{Hypergeometric2F1}[1, -m, 1-m, \frac{(b e - a f) (c + d x)}{(d e - c f) (a + b x)}]}{f^2 m} - \frac{1}{b (b c - a d) f^2 m (1+m)} \\
& (a B d f m - b (B d e - A d f + B c f m)) (a + b x)^{1+m} (c + d x)^{-m} \left( \frac{b (c + d x)}{b c - a d} \right)^m \text{Hypergeometric2F1}[m, 1+m, 2+m, -\frac{d (a + b x)}{b c - a d}]
\end{aligned}$$

Result (type 6, 627 leaves):

$$\begin{aligned}
 & \left( (a+b x)^m (c+d x)^{-m} \left( -B d (-b c + a d) e (b e - a f) (-1+m) (2+m) (a+b x) \text{AppellF1}[1+m, m, 1, 2+m, \frac{d(a+b x)}{-b c + a d}, \frac{f(a+b x)}{-b e + a f}] \right. \right. + \\
 & A d (-b c + a d) f (b e - a f) (-1+m) (2+m) (a+b x) \text{AppellF1}[1+m, m, 1, 2+m, \frac{d(a+b x)}{-b c + a d}, \frac{f(a+b x)}{-b e + a f}] + \\
 & b B (1+m) \left( \frac{d(a+b x)}{-b c + a d} \right)^{-m} (c+d x) (e+f x) \left( (b c - a d) (b e - a f) (2+m) \text{AppellF1}[1+m, m, 1, 2+m, \frac{d(a+b x)}{-b c + a d}, \frac{f(a+b x)}{-b e + a f}] \right. + \\
 & (a+b x) \left( (-b c f + a d f) \text{AppellF1}[2+m, m, 2, 3+m, \frac{d(a+b x)}{-b c + a d}, \frac{f(a+b x)}{-b e + a f}] \right. + \\
 & \left. \left. d (-b e + a f) m \text{AppellF1}[2+m, 1+m, 1, 3+m, \frac{d(a+b x)}{-b c + a d}, \frac{f(a+b x)}{-b e + a f}] \right) \right) \text{Hypergeometric2F1}[1-m, -m, 2-m, \frac{b(c+d x)}{b c - a d}] \Big) \Big) / \\
 & \left( b d f (1-m) (1+m) (e+f x) \left( (b c - a d) (b e - a f) (2+m) \text{AppellF1}[1+m, m, 1, 2+m, \frac{d(a+b x)}{-b c + a d}, \frac{f(a+b x)}{-b e + a f}] \right. \right. + \\
 & (a+b x) \left( (-b c f + a d f) \text{AppellF1}[2+m, m, 2, 3+m, \frac{d(a+b x)}{-b c + a d}, \frac{f(a+b x)}{-b e + a f}] \right. + \\
 & \left. \left. d (-b e + a f) m \text{AppellF1}[2+m, 1+m, 1, 3+m, \frac{d(a+b x)}{-b c + a d}, \frac{f(a+b x)}{-b e + a f}] \right) \right) \Big)
 \end{aligned}$$

Problem 138: Result more than twice size of optimal antiderivative.

$$\int \frac{(A+B x)^n (c+d x)^p (e+f x)^q}{\sqrt{a+b x}} dx$$

Optimal (type 6, 250 leaves, 7 steps):

$$\begin{aligned}
 & \frac{1}{b^2} 2 (A b - a B) \sqrt{a+b x} (c+d x)^n \left( \frac{b(c+d x)}{b c - a d} \right)^{-n} (e+f x)^p \left( \frac{b(e+f x)}{b e - a f} \right)^{-p} \text{AppellF1}\left[\frac{1}{2}, -n, -p, \frac{3}{2}, -\frac{d(a+b x)}{b c - a d}, -\frac{f(a+b x)}{b e - a f}\right] + \\
 & \frac{2 B (a+b x)^{3/2} (c+d x)^n \left( \frac{b(c+d x)}{b c - a d} \right)^{-n} (e+f x)^p \left( \frac{b(e+f x)}{b e - a f} \right)^{-p} \text{AppellF1}\left[\frac{3}{2}, -n, -p, \frac{5}{2}, -\frac{d(a+b x)}{b c - a d}, -\frac{f(a+b x)}{b e - a f}\right]}{3 b^2}
 \end{aligned}$$

Result (type 6, 551 leaves):

$$\begin{aligned}
& \frac{1}{3 b^2} 2 (b c - a d) (b e - a f) \sqrt{a + b x} (c + d x)^n (e + f x)^p \\
& \left( \left( 9 (A b - a B) \text{AppellF1} \left[ \frac{1}{2}, -n, -p, \frac{3}{2}, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f} \right] \right) / \left( 3 (b c - a d) (b e - a f) \text{AppellF1} \left[ \frac{1}{2}, -n, -p, \frac{3}{2}, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f} \right] - \right. \right. \\
& 2 (a + b x) \left( d (-b e + a f) n \text{AppellF1} \left[ \frac{3}{2}, 1 - n, -p, \frac{5}{2}, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f} \right] + (-b c + a d) f p \right. \\
& \left. \text{AppellF1} \left[ \frac{3}{2}, -n, 1 - p, \frac{5}{2}, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f} \right] \right) + \left( 5 B (a + b x) \text{AppellF1} \left[ \frac{3}{2}, -n, -p, \frac{5}{2}, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f} \right] \right) / \\
& \left( 5 (b c - a d) (b e - a f) \text{AppellF1} \left[ \frac{3}{2}, -n, -p, \frac{5}{2}, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f} \right] - 2 (a + b x) \left( d (-b e + a f) n \right. \right. \\
& \left. \left. \text{AppellF1} \left[ \frac{5}{2}, 1 - n, -p, \frac{7}{2}, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f} \right] + (-b c + a d) f p \text{AppellF1} \left[ \frac{5}{2}, -n, 1 - p, \frac{7}{2}, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f} \right] \right) \right)
\end{aligned}$$

**Problem 139: Unable to integrate problem.**

$$\int (a + b x)^m (c + d x)^n (e + f x)^p (g + h x)^3 dx$$

Optimal (type 6, 530 leaves, 31 steps):

$$\begin{aligned}
& \frac{1}{b^4 (1+m)} (b g - a h)^3 (a + b x)^{1+m} (c + d x)^n \left( \frac{b (c + d x)}{b c - a d} \right)^{-n} (e + f x)^p \left( \frac{b (e + f x)}{b e - a f} \right)^{-p} \text{AppellF1} [1 + m, -n, -p, 2 + m, -\frac{d (a + b x)}{b c - a d}, -\frac{f (a + b x)}{b e - a f}] + \\
& \frac{1}{b^4 (2+m)} 3 h (b g - a h)^2 (a + b x)^{2+m} (c + d x)^n \left( \frac{b (c + d x)}{b c - a d} \right)^{-n} (e + f x)^p \left( \frac{b (e + f x)}{b e - a f} \right)^{-p} \text{AppellF1} [2 + m, -n, -p, 3 + m, -\frac{d (a + b x)}{b c - a d}, -\frac{f (a + b x)}{b e - a f}] + \\
& \frac{1}{b^4 (3+m)} 3 h^2 (b g - a h) (a + b x)^{3+m} (c + d x)^n \left( \frac{b (c + d x)}{b c - a d} \right)^{-n} (e + f x)^p \left( \frac{b (e + f x)}{b e - a f} \right)^{-p} \text{AppellF1} [3 + m, -n, -p, 4 + m, -\frac{d (a + b x)}{b c - a d}, -\frac{f (a + b x)}{b e - a f}] + \\
& \frac{1}{b^4 (4+m)} h^3 (a + b x)^{4+m} (c + d x)^n \left( \frac{b (c + d x)}{b c - a d} \right)^{-n} (e + f x)^p \left( \frac{b (e + f x)}{b e - a f} \right)^{-p} \text{AppellF1} [4 + m, -n, -p, 5 + m, -\frac{d (a + b x)}{b c - a d}, -\frac{f (a + b x)}{b e - a f}]
\end{aligned}$$

Result (type 8, 31 leaves):

$$\int (a + b x)^m (c + d x)^n (e + f x)^p (g + h x)^3 dx$$

**Problem 140: Unable to integrate problem.**

$$\int (a + b x)^m (c + d x)^n (e + f x)^p (g + h x)^2 dx$$

Optimal (type 6, 393 leaves, 15 steps) :

$$\begin{aligned} & \frac{1}{b^3 (1+m)} (b g - a h)^2 (a + b x)^{1+m} (c + d x)^n \left( \frac{b (c + d x)}{b c - a d} \right)^{-n} (e + f x)^p \left( \frac{b (e + f x)}{b e - a f} \right)^{-p} \text{AppellF1}[1+m, -n, -p, 2+m, -\frac{d (a+b x)}{b c - a d}, -\frac{f (a+b x)}{b e - a f}] + \\ & \frac{1}{b^3 (2+m)} 2 h (b g - a h) (a + b x)^{2+m} (c + d x)^n \left( \frac{b (c + d x)}{b c - a d} \right)^{-n} (e + f x)^p \left( \frac{b (e + f x)}{b e - a f} \right)^{-p} \text{AppellF1}[2+m, -n, -p, 3+m, -\frac{d (a+b x)}{b c - a d}, -\frac{f (a+b x)}{b e - a f}] + \\ & \frac{1}{b^3 (3+m)} h^2 (a + b x)^{3+m} (c + d x)^n \left( \frac{b (c + d x)}{b c - a d} \right)^{-n} (e + f x)^p \left( \frac{b (e + f x)}{b e - a f} \right)^{-p} \text{AppellF1}[3+m, -n, -p, 4+m, -\frac{d (a+b x)}{b c - a d}, -\frac{f (a+b x)}{b e - a f}] \end{aligned}$$

Result (type 8, 31 leaves) :

$$\int (a + b x)^m (c + d x)^n (e + f x)^p (g + h x)^2 dx$$

Problem 141: Unable to integrate problem.

$$\int (a + b x)^m (c + d x)^n (e + f x)^p (g + h x) dx$$

Optimal (type 6, 256 leaves, 7 steps) :

$$\begin{aligned} & \frac{1}{b^2 (1+m)} (b g - a h) (a + b x)^{1+m} (c + d x)^n \left( \frac{b (c + d x)}{b c - a d} \right)^{-n} (e + f x)^p \left( \frac{b (e + f x)}{b e - a f} \right)^{-p} \text{AppellF1}[1+m, -n, -p, 2+m, -\frac{d (a+b x)}{b c - a d}, -\frac{f (a+b x)}{b e - a f}] + \\ & \frac{1}{b^2 (2+m)} h (a + b x)^{2+m} (c + d x)^n \left( \frac{b (c + d x)}{b c - a d} \right)^{-n} (e + f x)^p \left( \frac{b (e + f x)}{b e - a f} \right)^{-p} \text{AppellF1}[2+m, -n, -p, 3+m, -\frac{d (a+b x)}{b c - a d}, -\frac{f (a+b x)}{b e - a f}] \end{aligned}$$

Result (type 8, 29 leaves) :

$$\int (a + b x)^m (c + d x)^n (e + f x)^p (g + h x) dx$$

Problem 142: Result more than twice size of optimal antiderivative.

$$\int (a + b x)^m (c + d x)^n (e + f x)^p dx$$

Optimal (type 6, 123 leaves, 3 steps) :

$$\frac{1}{b (1+m)} (a + b x)^{1+m} (c + d x)^n \left( \frac{b (c + d x)}{b c - a d} \right)^{-n} (e + f x)^p \left( \frac{b (e + f x)}{b e - a f} \right)^{-p} \text{AppellF1}[1+m, -n, -p, 2+m, -\frac{d (a+b x)}{b c - a d}, -\frac{f (a+b x)}{b e - a f}]$$

Result (type 6, 296 leaves) :

$$\begin{aligned} & \left( (b c - a d) (b e - a f) (2 + m) (a + b x)^{1+m} (c + d x)^n (e + f x)^p \text{AppellF1}[1 + m, -n, -p, 2 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] \right) / \\ & \left( b (1 + m) \left( (b c - a d) (b e - a f) (2 + m) \text{AppellF1}[1 + m, -n, -p, 2 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] \right) - (a + b x) \left( d (-b e + a f) n \right. \right. \\ & \left. \left. \text{AppellF1}[2 + m, 1 - n, -p, 3 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] + (-b c + a d) f p \text{AppellF1}[2 + m, -n, 1 - p, 3 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}] \right) \right) \right) \end{aligned}$$

**Problem 144:** Unable to integrate problem.

$$\int (a + b x)^m (A + B x) (c + d x)^n (e + f x)^{-m-n} dx$$

Optimal (type 6, 268 leaves, 7 steps):

$$\begin{aligned} & \frac{1}{b^2 (1 + m)} (A b - a B) (a + b x)^{1+m} (c + d x)^n \left( \frac{b (c + d x)}{b c - a d} \right)^{-n} (e + f x)^{-m-n} \left( \frac{b (e + f x)}{b e - a f} \right)^{m+n} \text{AppellF1}[1 + m, -n, m + n, 2 + m, -\frac{d (a + b x)}{b c - a d}, -\frac{f (a + b x)}{b e - a f}] + \\ & \frac{1}{b^2 (2 + m)} B (a + b x)^{2+m} (c + d x)^n \left( \frac{b (c + d x)}{b c - a d} \right)^{-n} (e + f x)^{-m-n} \left( \frac{b (e + f x)}{b e - a f} \right)^{m+n} \text{AppellF1}[2 + m, -n, m + n, 3 + m, -\frac{d (a + b x)}{b c - a d}, -\frac{f (a + b x)}{b e - a f}] \end{aligned}$$

Result (type 8, 35 leaves):

$$\int (a + b x)^m (A + B x) (c + d x)^n (e + f x)^{-m-n} dx$$

**Problem 145:** Result more than twice size of optimal antiderivative.

$$\int (a + b x)^m (A + B x) (c + d x)^n (e + f x)^{-1-m-n} dx$$

Optimal (type 6, 283 leaves, 7 steps):

$$\begin{aligned} & \frac{1}{b f (1 + m)} B (a + b x)^{1+m} (c + d x)^n \left( \frac{b (c + d x)}{b c - a d} \right)^{-n} (e + f x)^{-m-n} \\ & \left( \frac{b (e + f x)}{b e - a f} \right)^{m+n} \text{AppellF1}[1 + m, -n, m + n, 2 + m, -\frac{d (a + b x)}{b c - a d}, -\frac{f (a + b x)}{b e - a f}] - \frac{1}{f (b e - a f) (1 + m)} \\ & (B e - A f) (a + b x)^{1+m} (c + d x)^n \left( \frac{b (c + d x)}{b c - a d} \right)^{-n} (e + f x)^{-m-n} \left( \frac{b (e + f x)}{b e - a f} \right)^{m+n} \text{AppellF1}[1 + m, -n, 1 + m + n, 2 + m, -\frac{d (a + b x)}{b c - a d}, -\frac{f (a + b x)}{b e - a f}] \end{aligned}$$

Result (type 6, 576 leaves):

$$\begin{aligned}
& \frac{1}{b(1+m)} (b c - a d) (b e - a f) (2 + m) (a + b x)^{1+m} (c + d x)^n (e + f x)^{-m-n} \left( \left[ B \text{AppellF1}[1 + m, -n, m + n, 2 + m, \frac{d(a + b x)}{-b c + a d}, \frac{f(a + b x)}{-b e + a f}] \right] \right) / \\
& \left( f \left( (b c - a d) (b e - a f) (2 + m) \text{AppellF1}[1 + m, -n, m + n, 2 + m, \frac{d(a + b x)}{-b c + a d}, \frac{f(a + b x)}{-b e + a f}] \right) - (a + b x) \left( d (-b e + a f) n \text{AppellF1}[2 + m, 1 - n, \right. \right. \\
& \left. \left. m + n, 3 + m, \frac{d(a + b x)}{-b c + a d}, \frac{f(a + b x)}{-b e + a f}] + (b c - a d) f (m + n) \text{AppellF1}[2 + m, -n, 1 + m + n, 3 + m, \frac{d(a + b x)}{-b c + a d}, \frac{f(a + b x)}{-b e + a f}] \right) \right) + \\
& \left( \left( A - \frac{B e}{f} \right) \text{AppellF1}[1 + m, -n, 1 + m + n, 2 + m, \frac{d(a + b x)}{-b c + a d}, \frac{f(a + b x)}{-b e + a f}] \right) / \left( (e + f x) \right. \\
& \left. \left( (b c - a d) (b e - a f) (2 + m) \text{AppellF1}[1 + m, -n, 1 + m + n, 2 + m, \frac{d(a + b x)}{-b c + a d}, \frac{f(a + b x)}{-b e + a f}] - (a + b x) \left( d (-b e + a f) n \text{AppellF1}[2 + m, 1 - n, \right. \right. \right. \\
& \left. \left. \left. 1 + m + n, 3 + m, \frac{d(a + b x)}{-b c + a d}, \frac{f(a + b x)}{-b e + a f}] + (b c - a d) f (1 + m + n) \text{AppellF1}[2 + m, -n, 2 + m + n, 3 + m, \frac{d(a + b x)}{-b c + a d}, \frac{f(a + b x)}{-b e + a f}] \right) \right) \right)
\end{aligned}$$

**Problem 147: Result more than twice size of optimal antiderivative.**

$$\int (a + b x)^m (A + B x) (c + d x)^n (e + f x)^{-3-m-n} dx$$

Optimal (type 5, 263 leaves, 3 steps):

$$\begin{aligned}
& \frac{(B e - A f) (a + b x)^{1+m} (c + d x)^{1+n} (e + f x)^{-2-m-n}}{(b e - a f) (d e - c f) (2 + m + n)} - \\
& \left( (b (B c e (1 + m) + A (c f (1 + n) - d e (2 + m + n))) + a (A d f (1 + m) + B (d e (1 + n) - c f (2 + m + n)))) (a + b x)^{1+m} (c + d x)^n \left( \frac{(b e - a f) (c + d x)}{(b c - a d) (e + f x)} \right)^{-n} \right. \\
& \left. (e + f x)^{-1-m-n} \text{Hypergeometric2F1}[1 + m, -n, 2 + m, -\frac{(d e - c f) (a + b x)}{(b c - a d) (e + f x)}] \right) / \left( (b e - a f)^2 (d e - c f) (1 + m) (2 + m + n) \right)
\end{aligned}$$

Result (type 5, 10558 leaves):

$$\begin{aligned}
& \left( A (a + b x)^{1+2m} (c + d x)^{2n} \left( \frac{-b c - b d x}{-b c + a d} \right)^{-n} (e + f x)^{-6-2m-2n} \left( \frac{-b e - b f x}{-b e + a f} \right)^{3+m+n} \right. \\
& \left( 1 - \frac{d (a + b x)}{-b c + a d} \right)^n \left( 1 - \frac{f (a + b x)}{-b e + a f} \right)^{-2-m-n} \text{Gamma}[2 + m] \left( \frac{2 \text{Hypergeometric2F1}[1, -n, 3 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{\text{Gamma}[3 + m]} + \right. \\
& \left. \left. m \text{Hypergeometric2F1}[1, -n, 3 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}] + \frac{f (a + b x) \text{Hypergeometric2F1}[1, -n, 3 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{(b e - a f) \text{Gamma}[3 + m]} + \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{\left( d e - c f \right) \left( a + b x \right) \Gamma[1 - n] {}_2F1[2, 1 - n, 4 + m, \frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}] - }{(b e - a f) (c + d x) \Gamma[-n]} \\
& \frac{f \left( -d e + c f \right) \left( a + b x \right)^2 \Gamma[1 - n] {}_2F1[2, 1 - n, 4 + m, \frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}] }{(b e - a f)^2 (c + d x) \Gamma[4 + m] \Gamma[-n]} \Bigg) \Bigg) / \\
& \left( b (1 + m) \left( -\frac{1}{(-b e + a f) (1 + m)} f (-2 - m - n) (a + b x)^{1+m} (c + d x)^n \left( \frac{-b c - b d x}{-b c + a d} \right)^{-n} (e + f x)^{-3-m-n} \left( \frac{-b e - b f x}{-b e + a f} \right)^{3+m+n} \right. \right. \right. \\
& \left. \left. \left. \left( 1 - \frac{d (a + b x)}{-b c + a d} \right)^n \left( 1 - \frac{f (a + b x)}{-b e + a f} \right)^{-3-m-n} \Gamma[2 + m] \right) \frac{2 {}_2F1[1, -n, 3 + m, \frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}]}{\Gamma[3 + m]} + \right. \right. \\
& \left. \left. \left. m {}_2F1[1, -n, 3 + m, \frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}] + \frac{f (a + b x) {}_2F1[1, -n, 3 + m, \frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}]}{(b e - a f) \Gamma[3 + m]} + \right. \right. \right. \\
& \left. \left. \left. (d e - c f) (a + b x) \Gamma[1 - n] {}_2F1[2, 1 - n, 4 + m, \frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}] - \right. \right. \right. \\
& \left. \left. \left. (b e - a f) (c + d x) \Gamma[4 + m] \Gamma[-n] \right) \frac{f (-d e + c f) (a + b x)^2 \Gamma[1 - n] {}_2F1[2, 1 - n, 4 + m, \frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}]}{(b e - a f)^2 (c + d x) \Gamma[4 + m] \Gamma[-n]} \right) - \\
& \frac{1}{(-b c + a d) (1 + m)} d n (a + b x)^{1+m} (c + d x)^n \left( \frac{-b c - b d x}{-b c + a d} \right)^{-n} (e + f x)^{-3-m-n} \left( \frac{-b e - b f x}{-b e + a f} \right)^{3+m+n} \left( 1 - \frac{d (a + b x)}{-b c + a d} \right)^{-1+n} \\
& \left( 1 - \frac{f (a + b x)}{-b e + a f} \right)^{-2-m-n} \Gamma[2 + m] \left( \frac{2 {}_2F1[1, -n, 3 + m, \frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}]}{\Gamma[3 + m]} + \right. \right. \\
& \left. \left. \left. m {}_2F1[1, -n, 3 + m, \frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}] + \frac{f (a + b x) {}_2F1[1, -n, 3 + m, \frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}]}{(b e - a f) \Gamma[3 + m]} + \right. \right. \right. \\
& \left. \left. \left. (d e - c f) (a + b x) \Gamma[1 - n] {}_2F1[2, 1 - n, 4 + m, \frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}] - \right. \right. \right. \\
& \left. \left. \left. (b e - a f) (c + d x) \Gamma[4 + m] \Gamma[-n] \right) \frac{f (-d e + c f) (a + b x)^2 \Gamma[1 - n] {}_2F1[2, 1 - n, 4 + m, \frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}]}{(b e - a f)^2 (c + d x) \Gamma[4 + m] \Gamma[-n]} \right) - \\
& \frac{1}{(-b e + a f) (1 + m)} f (3 + m + n) (a + b x)^{1+m} (c + d x)^n \left( \frac{-b c - b d x}{-b c + a d} \right)^{-n} (e + f x)^{-3-m-n} \left( \frac{-b e - b f x}{-b e + a f} \right)^{2+m+n} \left( 1 - \frac{d (a + b x)}{-b c + a d} \right)^n
\end{aligned}$$

$$\begin{aligned}
& \left(1 - \frac{f(a+b x)}{-b e + a f}\right)^{-2-m-n} \Gamma[2+m] \left( \frac{2 \text{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{\Gamma[3+m]} + \right. \\
& \frac{m \text{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{\Gamma[3+m]} + \frac{f(a+b x) \text{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{(b e - a f) \Gamma[3+m]} + \\
& \frac{(d e - c f) (a+b x) \Gamma[1-n] \text{Hypergeometric2F1}[2, 1-n, 4+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{(b e - a f) (c+d x) \Gamma[4+m] \Gamma[-n]} - \\
& \left. \frac{f(-d e + c f) (a+b x)^2 \Gamma[1-n] \text{Hypergeometric2F1}[2, 1-n, 4+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{(b e - a f)^2 (c+d x) \Gamma[4+m] \Gamma[-n]} \right) + \\
& \frac{1}{b(1+m)} f(-3-m-n) (a+b x)^{1+m} (c+d x)^n \left(\frac{-b c - b d x}{-b c + a d}\right)^{-n} (e+f x)^{-4-m-n} \left(\frac{-b e - b f x}{-b e + a f}\right)^{3+m+n} \left(1 - \frac{d(a+b x)}{-b c + a d}\right)^n \\
& \left(1 - \frac{f(a+b x)}{-b e + a f}\right)^{-2-m-n} \Gamma[2+m] \left( \frac{2 \text{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{\Gamma[3+m]} + \right. \\
& \frac{m \text{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{\Gamma[3+m]} + \frac{f(a+b x) \text{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{(b e - a f) \Gamma[3+m]} + \\
& \frac{(d e - c f) (a+b x) \Gamma[1-n] \text{Hypergeometric2F1}[2, 1-n, 4+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{(b e - a f) (c+d x) \Gamma[4+m] \Gamma[-n]} - \\
& \left. \frac{f(-d e + c f) (a+b x)^2 \Gamma[1-n] \text{Hypergeometric2F1}[2, 1-n, 4+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{(b e - a f)^2 (c+d x) \Gamma[4+m] \Gamma[-n]} \right) + \\
& \frac{1}{(-b c + a d)(1+m)} d n (a+b x)^{1+m} (c+d x)^n \left(\frac{-b c - b d x}{-b c + a d}\right)^{-1-n} (e+f x)^{-3-m-n} \left(\frac{-b e - b f x}{-b e + a f}\right)^{3+m+n} \left(1 - \frac{d(a+b x)}{-b c + a d}\right)^n \\
& \left(1 - \frac{f(a+b x)}{-b e + a f}\right)^{-2-m-n} \Gamma[2+m] \left( \frac{2 \text{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{\Gamma[3+m]} + \right. \\
& \frac{m \text{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{\Gamma[3+m]} + \frac{f(a+b x) \text{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{(b e - a f) \Gamma[3+m]} + \\
& \frac{(d e - c f) (a+b x) \Gamma[1-n] \text{Hypergeometric2F1}[2, 1-n, 4+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{(b e - a f) (c+d x) \Gamma[4+m] \Gamma[-n]} -
\end{aligned}$$

$$\begin{aligned}
& \frac{f(-d e + c f) (a + b x)^2 \text{Gamma}[1 - n] \text{Hypergeometric2F1}[2, 1 - n, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{(b e - a f)^2 (c + d x) \text{Gamma}[4 + m] \text{Gamma}[-n]} + \\
& \frac{1}{b (1 + m)} d n (a + b x)^{1+m} (c + d x)^{-1+n} \left( \frac{-b c - b d x}{-b c + a d} \right)^{-n} (e + f x)^{-3-m-n} \left( \frac{-b e - b f x}{-b e + a f} \right)^{3+m+n} \left( 1 - \frac{d (a + b x)}{-b c + a d} \right)^n \\
& \left( 1 - \frac{f (a + b x)}{-b e + a f} \right)^{-2-m-n} \text{Gamma}[2 + m] \left( \frac{2 \text{Hypergeometric2F1}[1, -n, 3 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{\text{Gamma}[3 + m]} + \right. \\
& \left. \frac{m \text{Hypergeometric2F1}[1, -n, 3 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{\text{Gamma}[3 + m]} + \frac{f (a + b x) \text{Hypergeometric2F1}[1, -n, 3 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{(b e - a f) \text{Gamma}[3 + m]} + \right. \\
& \left. \frac{(d e - c f) (a + b x) \text{Gamma}[1 - n] \text{Hypergeometric2F1}[2, 1 - n, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{(b e - a f) (c + d x) \text{Gamma}[4 + m] \text{Gamma}[-n]} - \right. \\
& \left. \frac{f (-d e + c f) (a + b x)^2 \text{Gamma}[1 - n] \text{Hypergeometric2F1}[2, 1 - n, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{(b e - a f)^2 (c + d x) \text{Gamma}[4 + m] \text{Gamma}[-n]} \right) + \\
& (a + b x)^m (c + d x)^n \left( \frac{-b c - b d x}{-b c + a d} \right)^{-n} (e + f x)^{-3-m-n} \left( \frac{-b e - b f x}{-b e + a f} \right)^{3+m+n} \left( 1 - \frac{d (a + b x)}{-b c + a d} \right)^n \left( 1 - \frac{f (a + b x)}{-b e + a f} \right)^{-2-m-n} \\
& \text{Gamma}[2 + m] \left( \frac{2 \text{Hypergeometric2F1}[1, -n, 3 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{\text{Gamma}[3 + m]} + \right. \\
& \left. \frac{m \text{Hypergeometric2F1}[1, -n, 3 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{\text{Gamma}[3 + m]} + \frac{f (a + b x) \text{Hypergeometric2F1}[1, -n, 3 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{(b e - a f) \text{Gamma}[3 + m]} + \right. \\
& \left. \frac{(d e - c f) (a + b x) \text{Gamma}[1 - n] \text{Hypergeometric2F1}[2, 1 - n, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{(b e - a f) (c + d x) \text{Gamma}[4 + m] \text{Gamma}[-n]} - \right. \\
& \left. \frac{f (-d e + c f) (a + b x)^2 \text{Gamma}[1 - n] \text{Hypergeometric2F1}[2, 1 - n, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{(b e - a f)^2 (c + d x) \text{Gamma}[4 + m] \text{Gamma}[-n]} \right) + \\
& \frac{1}{b (1 + m)} (a + b x)^{1+m} (c + d x)^n \left( \frac{-b c - b d x}{-b c + a d} \right)^{-n} (e + f x)^{-3-m-n} \left( \frac{-b e - b f x}{-b e + a f} \right)^{3+m+n} \left( 1 - \frac{d (a + b x)}{-b c + a d} \right)^n \\
& \left( 1 - \frac{f (a + b x)}{-b e + a f} \right)^{-2-m-n} \text{Gamma}[2 + m] \left( \frac{b f \text{Hypergeometric2F1}[1, -n, 3 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{(b e - a f) \text{Gamma}[3 + m]} - \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{2 n \left( -\frac{d (d e - c f) (a + b x)}{(b e - a f) (c + d x)^2} + \frac{b (d e - c f)}{(b e - a f) (c + d x)} \right) \text{Hypergeometric2F1}[2, 1 - n, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}] - \\
& \quad (3 + m) \text{Gamma}[3 + m]}{m n \left( -\frac{d (d e - c f) (a + b x)}{(b e - a f) (c + d x)^2} + \frac{b (d e - c f)}{(b e - a f) (c + d x)} \right) \text{Hypergeometric2F1}[2, 1 - n, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}] - \\
& \quad (3 + m) \text{Gamma}[3 + m]} - \\
& \left( f n (a + b x) \left( -\frac{d (d e - c f) (a + b x)}{(b e - a f) (c + d x)^2} + \frac{b (d e - c f)}{(b e - a f) (c + d x)} \right) \text{Hypergeometric2F1}[2, 1 - n, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}] \right) / \\
& \left( (b e - a f) (3 + m) \text{Gamma}[3 + m] \right) - \frac{d (d e - c f) (a + b x) \text{Gamma}[1 - n] \text{Hypergeometric2F1}[2, 1 - n, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{(b e - a f) (c + d x)^2 \text{Gamma}[4 + m] \text{Gamma}[-n]} + \\
& \frac{d f (-d e + c f) (a + b x)^2 \text{Gamma}[1 - n] \text{Hypergeometric2F1}[2, 1 - n, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{(b e - a f)^2 (c + d x)^2 \text{Gamma}[4 + m] \text{Gamma}[-n]} + \\
& \frac{b (d e - c f) \text{Gamma}[1 - n] \text{Hypergeometric2F1}[2, 1 - n, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{(b e - a f) (c + d x) \text{Gamma}[4 + m] \text{Gamma}[-n]} - \\
& \frac{2 b f (-d e + c f) (a + b x) \text{Gamma}[1 - n] \text{Hypergeometric2F1}[2, 1 - n, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}]}{(b e - a f)^2 (c + d x) \text{Gamma}[4 + m] \text{Gamma}[-n]} + \\
& \left( 2 (d e - c f) (1 - n) (a + b x) \left( -\frac{d (d e - c f) (a + b x)}{(b e - a f) (c + d x)^2} + \frac{b (d e - c f)}{(b e - a f) (c + d x)} \right) \text{Gamma}[1 - n] \right. \\
& \quad \left. \text{Hypergeometric2F1}[3, 2 - n, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}] \right) / \left( (b e - a f) (4 + m) (c + d x) \text{Gamma}[4 + m] \text{Gamma}[-n] \right) - \\
& \left( 2 f (-d e + c f) (1 - n) (a + b x)^2 \left( -\frac{d (d e - c f) (a + b x)}{(b e - a f) (c + d x)^2} + \frac{b (d e - c f)}{(b e - a f) (c + d x)} \right) \text{Gamma}[1 - n] \right. \\
& \quad \left. \text{Hypergeometric2F1}[3, 2 - n, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}] \right) / \left( (b e - a f)^2 (4 + m) (c + d x) \text{Gamma}[4 + m] \text{Gamma}[-n] \right) \Bigg) \Bigg) - \\
& \left( B e (a + b x)^{1+2m} (c + d x)^{2n} \left( \frac{-b c - b d x}{-b c + a d} \right)^{-n} (e + f x)^{-6-2m-2n} \left( \frac{-b e - b f x}{-b e + a f} \right)^{3+m+n} \left( 1 - \frac{d (a + b x)}{-b c + a d} \right)^n \right. \\
& \quad \left. \left( 1 - \frac{f (a + b x)}{-b e + a f} \right)^{-2-m-n} \right. \\
& \quad \left. \text{Gamma}[2 + m] \right)
\end{aligned}$$

$$\begin{aligned}
& \left( \frac{2 \operatorname{Hypergeometric2F1}[1, -n, 3+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}]}{\operatorname{Gamma}[3+m]} + \frac{m \operatorname{Hypergeometric2F1}[1, -n, 3+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}]}{\operatorname{Gamma}[3+m]} + \right. \\
& \frac{f(a+b x) \operatorname{Hypergeometric2F1}[1, -n, 3+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}]}{(be-af) \operatorname{Gamma}[3+m]} + \\
& \frac{(de-cf)(a+b x) \operatorname{Gamma}[1-n] \operatorname{Hypergeometric2F1}[2, 1-n, 4+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}]}{(be-af)(c+d x) \operatorname{Gamma}[4+m] \operatorname{Gamma}[-n]} - \\
& \left. \frac{f(-de+cf)(a+b x)^2 \operatorname{Gamma}[1-n] \operatorname{Hypergeometric2F1}[2, 1-n, 4+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}]}{(be-af)^2 (c+d x) \operatorname{Gamma}[4+m] \operatorname{Gamma}[-n]} \right) / \\
& \left( b f (1+m) \left( -\frac{1}{(-b e+a f) (1+m)} f(-2-m-n) (a+b x)^{1+m} (c+d x)^n \left( \frac{-b c-b d x}{-b c+a d} \right)^{-n} (e+f x)^{-3-m-n} \left( \frac{-b e-b f x}{-b e+a f} \right)^{3+m+n} \right. \right. \\
& \left. \left. \left( 1 - \frac{d(a+b x)}{-b c+a d} \right)^n \left( 1 - \frac{f(a+b x)}{-b e+a f} \right)^{-3-m-n} \operatorname{Gamma}[2+m] \left( \frac{2 \operatorname{Hypergeometric2F1}[1, -n, 3+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}]}{\operatorname{Gamma}[3+m]} + \right. \right. \right. \\
& \left. \left. \left. m \operatorname{Hypergeometric2F1}[1, -n, 3+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}] \right. \right. + \frac{f(a+b x) \operatorname{Hypergeometric2F1}[1, -n, 3+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}]}{(be-af) \operatorname{Gamma}[3+m]} + \right. \\
& \left. \left. \left. (de-cf)(a+b x) \operatorname{Gamma}[1-n] \operatorname{Hypergeometric2F1}[2, 1-n, 4+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}] \right. \right. - \right. \\
& \left. \left. \left. (be-af)(c+d x) \operatorname{Gamma}[4+m] \operatorname{Gamma}[-n] \right. \right. \right. \\
& \left. \left. \left. f(-de+cf)(a+b x)^2 \operatorname{Gamma}[1-n] \operatorname{Hypergeometric2F1}[2, 1-n, 4+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}] \right. \right. \right) - \\
& \left. \left. \left. \frac{1}{(-b c+a d) (1+m)} d n (a+b x)^{1+m} (c+d x)^n \left( \frac{-b c-b d x}{-b c+a d} \right)^{-n} (e+f x)^{-3-m-n} \left( \frac{-b e-b f x}{-b e+a f} \right)^{3+m+n} \left( 1 - \frac{d(a+b x)}{-b c+a d} \right)^{-1+n} \right. \right. \right. \\
& \left. \left. \left. \left( 1 - \frac{f(a+b x)}{-b e+a f} \right)^{-2-m-n} \operatorname{Gamma}[2+m] \left( \frac{2 \operatorname{Hypergeometric2F1}[1, -n, 3+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}]}{\operatorname{Gamma}[3+m]} + \right. \right. \right. \\
& \left. \left. \left. m \operatorname{Hypergeometric2F1}[1, -n, 3+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}] \right. \right. + \frac{f(a+b x) \operatorname{Hypergeometric2F1}[1, -n, 3+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}]}{(be-af) \operatorname{Gamma}[3+m]} + \right. \\
& \left. \left. \left. (de-cf)(a+b x) \operatorname{Gamma}[1-n] \operatorname{Hypergeometric2F1}[2, 1-n, 4+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}] \right. \right. - \right. \\
& \left. \left. \left. (be-af)(c+d x) \operatorname{Gamma}[4+m] \operatorname{Gamma}[-n] \right. \right. \right. \right)
\end{aligned}$$



$$\begin{aligned}
& \frac{m \operatorname{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{\Gamma[3+m]} + \frac{f (a+b x) \operatorname{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{(b e - a f) \Gamma[3+m]} + \\
& \frac{(d e - c f) (a+b x) \Gamma[1-n] \operatorname{Hypergeometric2F1}[2, 1-n, 4+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{(b e - a f) (c+d x) \Gamma[4+m] \Gamma[-n]} - \\
& \left. \frac{f (-d e + c f) (a+b x)^2 \Gamma[1-n] \operatorname{Hypergeometric2F1}[2, 1-n, 4+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{(b e - a f)^2 (c+d x) \Gamma[4+m] \Gamma[-n]} \right\} + \\
& \frac{1}{b (1+m)} d n (a+b x)^{1+m} (c+d x)^{-1+n} \left( \frac{-b c - b d x}{-b c + a d} \right)^{-n} (e+f x)^{-3-m-n} \left( \frac{-b e - b f x}{-b e + a f} \right)^{3+m+n} \left( 1 - \frac{d (a+b x)}{-b c + a d} \right)^n \\
& \left( 1 - \frac{f (a+b x)}{-b e + a f} \right)^{-2-m-n} \Gamma[2+m] \left( \frac{2 \operatorname{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{\Gamma[3+m]} + \right. \\
& \frac{m \operatorname{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{\Gamma[3+m]} + \frac{f (a+b x) \operatorname{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{(b e - a f) \Gamma[3+m]} + \\
& \frac{(d e - c f) (a+b x) \Gamma[1-n] \operatorname{Hypergeometric2F1}[2, 1-n, 4+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{(b e - a f) (c+d x) \Gamma[4+m] \Gamma[-n]} - \\
& \left. \frac{f (-d e + c f) (a+b x)^2 \Gamma[1-n] \operatorname{Hypergeometric2F1}[2, 1-n, 4+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{(b e - a f)^2 (c+d x) \Gamma[4+m] \Gamma[-n]} \right\} + \\
& (a+b x)^m (c+d x)^n \left( \frac{-b c - b d x}{-b c + a d} \right)^{-n} (e+f x)^{-3-m-n} \left( \frac{-b e - b f x}{-b e + a f} \right)^{3+m+n} \left( 1 - \frac{d (a+b x)}{-b c + a d} \right)^n \left( 1 - \frac{f (a+b x)}{-b e + a f} \right)^{-2-m-n} \\
& \Gamma[2+m] \left( \frac{2 \operatorname{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{\Gamma[3+m]} + \right. \\
& \frac{m \operatorname{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{\Gamma[3+m]} + \frac{f (a+b x) \operatorname{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{(b e - a f) \Gamma[3+m]} + \\
& \frac{(d e - c f) (a+b x) \Gamma[1-n] \operatorname{Hypergeometric2F1}[2, 1-n, 4+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{(b e - a f) (c+d x) \Gamma[4+m] \Gamma[-n]} - \\
& \left. \frac{f (-d e + c f) (a+b x)^2 \Gamma[1-n] \operatorname{Hypergeometric2F1}[2, 1-n, 4+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}]}{(b e - a f)^2 (c+d x) \Gamma[4+m] \Gamma[-n]} \right\}
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{b(1+m)} (a+b x)^{1+m} (c+d x)^n \left( \frac{-b c - b d x}{-b c + a d} \right)^{-n} (e+f x)^{-3-m-n} \left( \frac{-b e - b f x}{-b e + a f} \right)^{3+m+n} \left( 1 - \frac{d(a+b x)}{-b c + a d} \right)^n \\
& \left( 1 - \frac{f(a+b x)}{-b e + a f} \right)^{-2-m-n} \text{Gamma}[2+m] \left( \frac{b f \text{Hypergeometric2F1}[1, -n, 3+m, \frac{(d e - c f)(a+b x)}{(b e - a f)(c+d x)}]}{(b e - a f) \text{Gamma}[3+m]} - \right. \\
& \left. \frac{2 n \left( -\frac{d(d e - c f)(a+b x)}{(b e - a f)(c+d x)^2} + \frac{b(d e - c f)}{(b e - a f)(c+d x)} \right) \text{Hypergeometric2F1}[2, 1-n, 4+m, \frac{(d e - c f)(a+b x)}{(b e - a f)(c+d x)}]}{(3+m) \text{Gamma}[3+m]} - \right. \\
& \left. \frac{m n \left( -\frac{d(d e - c f)(a+b x)}{(b e - a f)(c+d x)^2} + \frac{b(d e - c f)}{(b e - a f)(c+d x)} \right) \text{Hypergeometric2F1}[2, 1-n, 4+m, \frac{(d e - c f)(a+b x)}{(b e - a f)(c+d x)}]}{(3+m) \text{Gamma}[3+m]} - \right. \\
& \left. \left( f n (a+b x) \left( -\frac{d(d e - c f)(a+b x)}{(b e - a f)(c+d x)^2} + \frac{b(d e - c f)}{(b e - a f)(c+d x)} \right) \text{Hypergeometric2F1}[2, 1-n, 4+m, \frac{(d e - c f)(a+b x)}{(b e - a f)(c+d x)}] \right) / \right. \\
& \left. \left( (b e - a f)(3+m) \text{Gamma}[3+m] \right) - \frac{d(d e - c f)(a+b x) \text{Gamma}[1-n] \text{Hypergeometric2F1}[2, 1-n, 4+m, \frac{(d e - c f)(a+b x)}{(b e - a f)(c+d x)}]}{(b e - a f)(c+d x)^2 \text{Gamma}[4+m] \text{Gamma}[-n]} + \right. \\
& \left. \frac{d f (-d e + c f)(a+b x)^2 \text{Gamma}[1-n] \text{Hypergeometric2F1}[2, 1-n, 4+m, \frac{(d e - c f)(a+b x)}{(b e - a f)(c+d x)}]}{(b e - a f)^2 (c+d x)^2 \text{Gamma}[4+m] \text{Gamma}[-n]} + \right. \\
& \left. \frac{b(d e - c f) \text{Gamma}[1-n] \text{Hypergeometric2F1}[2, 1-n, 4+m, \frac{(d e - c f)(a+b x)}{(b e - a f)(c+d x)}]}{(b e - a f)(c+d x) \text{Gamma}[4+m] \text{Gamma}[-n]} - \right. \\
& \left. \frac{2 b f (-d e + c f)(a+b x) \text{Gamma}[1-n] \text{Hypergeometric2F1}[2, 1-n, 4+m, \frac{(d e - c f)(a+b x)}{(b e - a f)(c+d x)}]}{(b e - a f)^2 (c+d x) \text{Gamma}[4+m] \text{Gamma}[-n]} + \right. \\
& \left. \left( 2(d e - c f)(1-n)(a+b x) \left( -\frac{d(d e - c f)(a+b x)}{(b e - a f)(c+d x)^2} + \frac{b(d e - c f)}{(b e - a f)(c+d x)} \right) \text{Gamma}[1-n] \right. \right. \\
& \left. \left. \text{Hypergeometric2F1}[3, 2-n, 5+m, \frac{(d e - c f)(a+b x)}{(b e - a f)(c+d x)}] \right) / \left( (b e - a f)(4+m)(c+d x) \text{Gamma}[4+m] \text{Gamma}[-n] \right) - \right. \\
& \left. \left( 2 f (-d e + c f)(1-n)(a+b x)^2 \left( -\frac{d(d e - c f)(a+b x)}{(b e - a f)(c+d x)^2} + \frac{b(d e - c f)}{(b e - a f)(c+d x)} \right) \text{Gamma}[1-n] \right. \right. \\
& \left. \left. \text{Hypergeometric2F1}[3, 2-n, 5+m, \frac{(d e - c f)(a+b x)}{(b e - a f)(c+d x)}] \right) / \left( (b e - a f)^2 (4+m)(c+d x) \text{Gamma}[4+m] \text{Gamma}[-n] \right) \right) \right) + \\
& \frac{1}{f(b e - a f)(1+m)} B(a+b x)^{1+m} (c+d x)^n \left( \frac{(b e - a f)(c+d x)}{(b c - a d)(e+f x)} \right)^{-n}
\end{aligned}$$

$$\frac{(\mathbf{e} + \mathbf{f} x)^{-1-\mathbf{m}-\mathbf{n}}}{(\mathbf{b} \mathbf{c} - \mathbf{a} \mathbf{d}) (\mathbf{e} + \mathbf{f} x)}$$

Hypergeometric2F1[  
 1 +  
 m, -n, 2 +  
 m,  
 $\frac{(-\mathbf{d} \mathbf{e} + \mathbf{c} \mathbf{f}) (\mathbf{a} + \mathbf{b} x)}{(\mathbf{b} \mathbf{c} - \mathbf{a} \mathbf{d}) (\mathbf{e} + \mathbf{f} x)}$ ]

Problem 148: Attempted integration timed out after 120 seconds.

$$\int (\mathbf{a} + \mathbf{b} x)^{\mathbf{m}} (\mathbf{A} + \mathbf{B} x) (\mathbf{c} + \mathbf{d} x)^{\mathbf{n}} (\mathbf{e} + \mathbf{f} x)^{-4-\mathbf{m}-\mathbf{n}} dx$$

Optimal (type 5, 558 leaves, 4 steps):

$$\begin{aligned} & \frac{(\mathbf{B} \mathbf{e} - \mathbf{A} \mathbf{f}) (\mathbf{a} + \mathbf{b} x)^{1+\mathbf{m}} (\mathbf{c} + \mathbf{d} x)^{1+\mathbf{n}} (\mathbf{e} + \mathbf{f} x)^{-3-\mathbf{m}-\mathbf{n}}}{(\mathbf{b} \mathbf{e} - \mathbf{a} \mathbf{f}) (\mathbf{d} \mathbf{e} - \mathbf{c} \mathbf{f}) (3 + \mathbf{m} + \mathbf{n})} + \\ & \left( (\mathbf{a} \mathbf{f} (\mathbf{A} \mathbf{d} \mathbf{f} (2 + \mathbf{m}) + \mathbf{B} (\mathbf{d} \mathbf{e} (1 + \mathbf{n}) - \mathbf{c} \mathbf{f} (3 + \mathbf{m} + \mathbf{n}))) + \mathbf{b} (\mathbf{B} \mathbf{e} (\mathbf{d} \mathbf{e} + \mathbf{c} \mathbf{f} (1 + \mathbf{m})) + \mathbf{A} \mathbf{f} (\mathbf{c} \mathbf{f} (2 + \mathbf{n}) - \mathbf{d} \mathbf{e} (4 + \mathbf{m} + \mathbf{n})))) (\mathbf{a} + \mathbf{b} x)^{1+\mathbf{m}} \right. \\ & \left. (\mathbf{c} + \mathbf{d} x)^{1+\mathbf{n}} (\mathbf{e} + \mathbf{f} x)^{-2-\mathbf{m}-\mathbf{n}} \right) / \left( (\mathbf{b} \mathbf{e} - \mathbf{a} \mathbf{f})^2 (\mathbf{d} \mathbf{e} - \mathbf{c} \mathbf{f})^2 (2 + \mathbf{m} + \mathbf{n}) (3 + \mathbf{m} + \mathbf{n}) \right) + \frac{1}{(\mathbf{b} \mathbf{e} - \mathbf{a} \mathbf{f})^3 (\mathbf{d} \mathbf{e} - \mathbf{c} \mathbf{f})^2 (1 + \mathbf{m}) (2 + \mathbf{m} + \mathbf{n}) (3 + \mathbf{m} + \mathbf{n})} \\ & ((2 + \mathbf{m} + \mathbf{n}) (\mathbf{a} \mathbf{b} \mathbf{c} \mathbf{d} \mathbf{f} (\mathbf{B} \mathbf{e} - \mathbf{A} \mathbf{f}) + \mathbf{b} \mathbf{d} \mathbf{e} ((\mathbf{a} \mathbf{B} \mathbf{c} \mathbf{f} + \mathbf{A} (\mathbf{b} \mathbf{d} \mathbf{e} - \mathbf{b} \mathbf{c} \mathbf{f} - \mathbf{a} \mathbf{d} \mathbf{f})) (3 + \mathbf{m} + \mathbf{n}) - (\mathbf{B} \mathbf{e} - \mathbf{A} \mathbf{f}) (\mathbf{b} \mathbf{c} (1 + \mathbf{m}) + \mathbf{a} \mathbf{d} (1 + \mathbf{n}))) - \\ & (\mathbf{b} \mathbf{c} + \mathbf{a} \mathbf{d}) \mathbf{f} ((\mathbf{a} \mathbf{B} \mathbf{c} \mathbf{f} + \mathbf{A} (\mathbf{b} \mathbf{d} \mathbf{e} - \mathbf{b} \mathbf{c} \mathbf{f} - \mathbf{a} \mathbf{d} \mathbf{f})) (3 + \mathbf{m} + \mathbf{n}) - (\mathbf{B} \mathbf{e} - \mathbf{A} \mathbf{f}) (\mathbf{b} \mathbf{c} (1 + \mathbf{m}) + \mathbf{a} \mathbf{d} (1 + \mathbf{n}))) - \\ & (\mathbf{b} \mathbf{c} (1 + \mathbf{m}) + \mathbf{a} \mathbf{d} (1 + \mathbf{n})) (\mathbf{a} \mathbf{f} (\mathbf{A} \mathbf{d} \mathbf{f} (2 + \mathbf{m}) + \mathbf{B} (\mathbf{d} \mathbf{e} (1 + \mathbf{n}) - \mathbf{c} \mathbf{f} (3 + \mathbf{m} + \mathbf{n}))) + \mathbf{b} (\mathbf{B} \mathbf{e} (\mathbf{d} \mathbf{e} + \mathbf{c} \mathbf{f} (1 + \mathbf{m})) + \mathbf{A} \mathbf{f} (\mathbf{c} \mathbf{f} (2 + \mathbf{n}) - \mathbf{d} \mathbf{e} (4 + \mathbf{m} + \mathbf{n})))) \right) \\ & (\mathbf{a} + \mathbf{b} x)^{1+\mathbf{m}} (\mathbf{c} + \mathbf{d} x)^{\mathbf{n}} \left( \frac{(\mathbf{b} \mathbf{e} - \mathbf{a} \mathbf{f}) (\mathbf{c} + \mathbf{d} x)}{(\mathbf{b} \mathbf{c} - \mathbf{a} \mathbf{d}) (\mathbf{e} + \mathbf{f} x)} \right)^{-\mathbf{n}} (\mathbf{e} + \mathbf{f} x)^{-1-\mathbf{m}-\mathbf{n}} \text{Hypergeometric2F1}[1 + \mathbf{m}, -n, 2 + \mathbf{m}, -\frac{(\mathbf{d} \mathbf{e} - \mathbf{c} \mathbf{f}) (\mathbf{a} + \mathbf{b} \mathbf{x})}{(\mathbf{b} \mathbf{c} - \mathbf{a} \mathbf{d}) (\mathbf{e} + \mathbf{f} \mathbf{x})}]] \end{aligned}$$

Result (type 1, 1 leaves):

???

### Test results for the 34 problems in "1.1.1.5 P(x) (a+b x)^m (c+d x)^n.m"

Problem 25: Result more than twice size of optimal antiderivative.

$$\int (\mathbf{a} + \mathbf{b} x)^3 (\mathbf{c} + \mathbf{d} x)^{\mathbf{n}} (\mathbf{A} + \mathbf{B} x + \mathbf{C} x^2 + \mathbf{D} x^3) dx$$

Optimal (type 3, 455 leaves, 2 steps):

$$\begin{aligned}
& - \frac{(b c - a d)^3 (c^2 C d - B c d^2 + A d^3 - c^3 D) (c + d x)^{1+n}}{d^7 (1+n)} - \frac{(b c - a d)^2 (a d (2 c C d - B d^2 - 3 c^2 D) - b (5 c^2 C d - 4 B c d^2 + 3 A d^3 - 6 c^3 D)) (c + d x)^{2+n}}{d^7 (2+n)} - \\
& \frac{1}{d^7 (3+n)} (b c - a d) (a^2 d^2 (C d - 3 c D) - a b d (8 c C d - 3 B d^2 - 15 c^2 D) + b^2 (10 c^2 C d - 6 B c d^2 + 3 A d^3 - 15 c^3 D)) (c + d x)^{3+n} + \\
& \frac{1}{d^7 (4+n)} (a^3 d^3 D + 3 a^2 b d^2 (C d - 4 c D) - 3 a b^2 d (4 c C d - B d^2 - 10 c^2 D) + b^3 (10 c^2 C d - 4 B c d^2 + A d^3 - 20 c^3 D)) (c + d x)^{4+n} + \\
& \frac{b (3 a^2 d^2 D + 3 a b d (C d - 5 c D) - b^2 (5 c C d - B d^2 - 15 c^2 D)) (c + d x)^{5+n}}{d^7 (5+n)} + \frac{b^2 (b C d - 6 b c D + 3 a d D) (c + d x)^{6+n}}{d^7 (6+n)} + \frac{b^3 D (c + d x)^{7+n}}{d^7 (7+n)}
\end{aligned}$$

Result (type 3, 977 leaves):

$$\begin{aligned}
& \frac{1}{d^7 (1+n) (2+n) (3+n) (4+n) (5+n) (6+n) (7+n)} \\
& (c + d x)^{1+n} (a^3 d^3 (210 + 107 n + 18 n^2 + n^3) (-6 c^3 D + 2 c^2 d (C (4+n) + 3 D (1+n) x) - c d^2 (B (12 + 7 n + n^2) + (1+n) x (2 C (4+n) + 3 D (2+n) x))) + \\
& d^3 (A (24 + 26 n + 9 n^2 + n^3) + (1+n) x (B (12 + 7 n + n^2) + (2+n) x (C (4+n) + D (3+n) x))) + \\
& 3 a^2 b d^2 (42 + 13 n + n^2) (24 c^4 D - 6 c^3 d (C (5+n) + 4 D (1+n) x) + 2 c^2 d^2 (B (20 + 9 n + n^2) + 3 (1+n) x (C (5+n) + 2 D (2+n) x))) - \\
& c d^3 (A (60 + 47 n + 12 n^2 + n^3) + (1+n) x (2 B (20 + 9 n + n^2) + (2+n) x (3 C (5+n) + 4 D (3+n) x))) + \\
& d^4 (1+n) x (A (60 + 47 n + 12 n^2 + n^3) + (2+n) x (B (20 + 9 n + n^2) + (3+n) x (C (5+n) + D (4+n) x))) + \\
& 3 a b^2 d (7+n) (-120 c^5 D + 24 c^4 d (C (6+n) + 5 D (1+n) x) - 6 c^3 d^2 (B (30 + 11 n + n^2) + 2 (1+n) x (2 C (6+n) + 5 D (2+n) x))) + \\
& 2 c^2 d^3 (A (120 + 74 n + 15 n^2 + n^3) + (1+n) x (3 B (30 + 11 n + n^2) + 2 (2+n) x (3 C (6+n) + 5 D (3+n) x))) - \\
& c d^4 (1+n) x (2 A (120 + 74 n + 15 n^2 + n^3) + (2+n) x (3 B (30 + 11 n + n^2) + (3+n) x (4 C (6+n) + 5 D (4+n) x))) + \\
& d^5 (2 + 3 n + n^2) x^2 (A (120 + 74 n + 15 n^2 + n^3) + (3+n) x (B (30 + 11 n + n^2) + (4+n) x (C (6+n) + D (5+n) x))) + \\
& b^3 (720 c^6 D - 120 c^5 d (C (7+n) + 6 D (1+n) x) + 24 c^4 d^2 (B (42 + 13 n + n^2) + 5 (1+n) x (C (7+n) + 3 D (2+n) x))) - \\
& 6 c^3 d^3 (A (210 + 107 n + 18 n^2 + n^3) + 2 (1+n) x (2 B (42 + 13 n + n^2) + 5 (2+n) x (C (7+n) + 2 D (3+n) x))) + \\
& 2 c^2 d^4 (1+n) x (3 A (210 + 107 n + 18 n^2 + n^3) + (2+n) x (6 B (42 + 13 n + n^2) + 5 (3+n) x (2 C (7+n) + 3 D (4+n) x))) - \\
& c d^5 (2 + 3 n + n^2) x^2 (3 A (210 + 107 n + 18 n^2 + n^3) + (3+n) x (4 B (42 + 13 n + n^2) + (4+n) x (5 C (7+n) + 6 D (5+n) x))) + \\
& d^6 (6 + 11 n + 6 n^2 + n^3) x^3 (A (210 + 107 n + 18 n^2 + n^3) + (4+n) x (B (42 + 13 n + n^2) + (5+n) x (C (7+n) + D (6+n) x)))
\end{aligned}$$

Problem 29: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(c + d x)^n (A + B x + C x^2 + D x^3)}{a + b x} dx$$

Optimal (type 5, 203 leaves, 3 steps):

$$\begin{aligned}
& \frac{(a^2 d^2 D - a b d (C d - c D) - b^2 (c C d - B d^2 - c^2 D)) (c + d x)^{1+n}}{b^3 d^3 (1+n)} + \frac{(b C d - 2 b c D - a d D) (c + d x)^{2+n}}{b^2 d^3 (2+n)} + \\
& \frac{D (c + d x)^{3+n}}{b d^3 (3+n)} - \frac{(A b^3 - a (b^2 B - a b C + a^2 D)) (c + d x)^{1+n} \text{Hypergeometric2F1}[1, 1+n, 2+n, \frac{b (c+d x)}{b c-a d}]}{b^3 (b c - a d) (1+n)}
\end{aligned}$$

Result (type 6, 414 leaves):

$$\begin{aligned} & \frac{1}{12} (c + d x)^n \left( \left( 18 a B c x^2 \text{AppellF1}[2, -n, 1, 3, -\frac{d x}{c}, -\frac{b x}{a}] \right) \middle/ \left( (a + b x) \right) \right. \\ & \quad \left. \left( 3 a c \text{AppellF1}[2, -n, 1, 3, -\frac{d x}{c}, -\frac{b x}{a}] + a d n x \text{AppellF1}[3, 1-n, 1, 4, -\frac{d x}{c}, -\frac{b x}{a}] - b c x \text{AppellF1}[3, -n, 2, 4, -\frac{d x}{c}, -\frac{b x}{a}] \right) \right) + \\ & \left( 16 a c C x^3 \text{AppellF1}[3, -n, 1, 4, -\frac{d x}{c}, -\frac{b x}{a}] \right) \middle/ \left( (a + b x) \left( 4 a c \text{AppellF1}[3, -n, 1, 4, -\frac{d x}{c}, -\frac{b x}{a}] + \right. \right. \\ & \quad \left. \left. a d n x \text{AppellF1}[4, 1-n, 1, 5, -\frac{d x}{c}, -\frac{b x}{a}] - b c x \text{AppellF1}[4, -n, 2, 5, -\frac{d x}{c}, -\frac{b x}{a}] \right) \right) + \\ & \left( 15 a c D x^4 \text{AppellF1}[4, -n, 1, 5, -\frac{d x}{c}, -\frac{b x}{a}] \right) \middle/ \left( (a + b x) \left( 5 a c \text{AppellF1}[4, -n, 1, 5, -\frac{d x}{c}, -\frac{b x}{a}] + a d n x \text{AppellF1}[5, \right. \right. \\ & \quad \left. \left. 1-n, 1, 6, -\frac{d x}{c}, -\frac{b x}{a}] - b c x \text{AppellF1}[5, -n, 2, 6, -\frac{d x}{c}, -\frac{b x}{a}] \right) \right) - \frac{12 A (c + d x) \text{Hypergeometric2F1}[1, 1+n, 2+n, \frac{b (c+d x)}{b c-a d}]}{(b c - a d) (1+n)} \end{aligned}$$

Problem 30: Unable to integrate problem.

$$\int \frac{(c + d x)^n (A + B x + C x^2 + D x^3)}{(a + b x)^2} dx$$

Optimal (type 5, 220 leaves, 4 steps):

$$\begin{aligned} & \frac{(b C d - b c D - 2 a d D) (c + d x)^{1+n}}{b^3 d^2 (1+n)} - \frac{\left( A - \frac{a (b^2 B - a b C + a^2 D)}{b^3} \right) (c + d x)^{1+n}}{(b c - a d) (a + b x)} + \frac{D (c + d x)^{2+n}}{b^2 d^2 (2+n)} + \frac{1}{b^3 (b c - a d)^2 (1+n)} \\ & (a^3 d D (3+n) - b^3 (B c + A d n) + a b^2 (2 c C + B d (1+n)) - a^2 b (3 c D + C d (2+n))) (c + d x)^{1+n} \text{Hypergeometric2F1}[1, 1+n, 2+n, \frac{b (c + d x)}{b c - a d}] \end{aligned}$$

Result (type 8, 32 leaves):

$$\int \frac{(c + d x)^n (A + B x + C x^2 + D x^3)}{(a + b x)^2} dx$$

Problem 31: Unable to integrate problem.

$$\int \frac{(c + d x)^n (A + B x + C x^2 + D x^3)}{(a + b x)^3} dx$$

Optimal (type 5, 329 leaves, 4 steps):

$$\frac{D \left(c + d x\right)^{1+n}}{b^3 d \left(1 + n\right)} - \frac{\left(A b^3 - a \left(b^2 B - a b C + a^2 D\right)\right) \left(c + d x\right)^{1+n}}{2 b^3 \left(b c - a d\right) \left(a + b x\right)^2} -$$

$$\frac{\left(b^3 \left(2 B c - A d \left(1 - n\right)\right) - a^3 d D \left(5 + n\right) - a b^2 \left(4 c C + B d \left(1 + n\right)\right) + a^2 b \left(6 c D + C d \left(3 + n\right)\right)\right) \left(c + d x\right)^{1+n}}{2 b^3 \left(b c - a d\right)^2 \left(a + b x\right)} - \frac{1}{2 b^3 \left(b c - a d\right)^3 \left(1 + n\right)}$$

$$\left(b^3 \left(2 c^2 C + 2 B c d n - A d^2 \left(1 - n\right) n\right) - a^3 d^2 D \left(6 + 5 n + n^2\right) + a^2 b d \left(2 + n\right) \left(6 c D + C d \left(1 + n\right)\right) - a b^2 \left(6 c^2 D + 4 c C d \left(1 + n\right) + B d^2 n \left(1 + n\right)\right)\right)$$

$$\left(c + d x\right)^{1+n} \text{Hypergeometric2F1}\left[1, 1 + n, 2 + n, \frac{b \left(c + d x\right)}{b c - a d}\right]$$

Result (type 8, 32 leaves):

$$\int \frac{\left(c + d x\right)^n \left(A + B x + C x^2 + D x^3\right)}{\left(a + b x\right)^3} dx$$

Problem 32: Result unnecessarily involves higher level functions.

$$\int \left(a + b x\right)^m \left(A + B x\right) \left(c + d x\right)^n dx$$

Optimal (type 5, 141 leaves, 3 steps):

$$\frac{B \left(a + b x\right)^{1+m} \left(c + d x\right)^{1+n}}{b d \left(2 + m + n\right)} + \frac{1}{b^2 d \left(1 + m\right) \left(2 + m + n\right)}$$

$$\left(A b d \left(2 + m + n\right) - B \left(b c \left(1 + m\right) + a d \left(1 + n\right)\right)\right) \left(a + b x\right)^{1+m} \left(c + d x\right)^n \left(\frac{b \left(c + d x\right)}{b c - a d}\right)^{-n} \text{Hypergeometric2F1}\left[1 + m, -n, 2 + m, -\frac{d \left(a + b x\right)}{b c - a d}\right]$$

Result (type 6, 202 leaves):

$$\left(a + b x\right)^m \left(c + d x\right)^n \left(\left(3 a B c x^2 \text{AppellF1}\left[2, -m, -n, 3, -\frac{b x}{a}, -\frac{d x}{c}\right]\right) / \right.$$

$$\left(6 a c \text{AppellF1}\left[2, -m, -n, 3, -\frac{b x}{a}, -\frac{d x}{c}\right] + 2 b c m x \text{AppellF1}\left[3, 1 - m, -n, 4, -\frac{b x}{a}, -\frac{d x}{c}\right] + \right.$$

$$\left.2 a d n x \text{AppellF1}\left[3, -m, 1 - n, 4, -\frac{b x}{a}, -\frac{d x}{c}\right]\right) + \frac{A \left(\frac{d \left(a + b x\right)}{-b c + a d}\right)^{-m} \left(c + d x\right) \text{Hypergeometric2F1}\left[-m, 1 + n, 2 + n, \frac{b \left(c + d x\right)}{b c - a d}\right]}{d \left(1 + n\right)}\right)$$

Problem 33: Result unnecessarily involves higher level functions.

$$\int \left(a + b x\right)^m \left(c + d x\right)^n \left(A + B x + C x^2\right) dx$$

Optimal (type 5, 268 leaves, 4 steps):

$$\begin{aligned} & -\frac{(a C d (4 + m + 2 n) + b (c C (2 + m) - B d (3 + m + n))) (a + b x)^{1+m} (c + d x)^{1+n}}{b^2 d^2 (2 + m + n) (3 + m + n)} + \frac{C (a + b x)^{2+m} (c + d x)^{1+n}}{b^2 d (3 + m + n)} - \\ & \left( (d (2 + m + n) (a b c C (2 + m) + a^2 C d (1 + n) - A b^2 d (3 + m + n)) - (b c (1 + m) + a d (1 + n)) (a C d (4 + m + 2 n) + b (c C (2 + m) - B d (3 + m + n)))) \right. \\ & \left. (a + b x)^{1+m} (c + d x)^n \left( \frac{b (c + d x)}{b c - a d} \right)^{-n} \text{Hypergeometric2F1}\left[1 + m, -n, 2 + m, -\frac{d (a + b x)}{b c - a d}\right] \right) / (b^3 d^2 (1 + m) (2 + m + n) (3 + m + n)) \end{aligned}$$

Result (type 6, 327 leaves):

$$\begin{aligned} & \frac{1}{3} (a + b x)^m (c + d x)^n \\ & \left( \left( 9 a B c x^2 \text{AppellF1}\left[2, -m, -n, 3, -\frac{b x}{a}, -\frac{d x}{c}\right] \right) / \left( 6 a c \text{AppellF1}\left[2, -m, -n, 3, -\frac{b x}{a}, -\frac{d x}{c}\right] + 2 b c m x \text{AppellF1}\left[3, 1 - m, -n, 4, -\frac{b x}{a}, -\frac{d x}{c}\right] \right. \right. \\ & \left. \left. + 2 a d n x \text{AppellF1}\left[3, -m, 1 - n, 4, -\frac{b x}{a}, -\frac{d x}{c}\right] + \left( 4 a c C x^3 \text{AppellF1}\left[3, -m, -n, 4, -\frac{b x}{a}, -\frac{d x}{c}\right] \right) / \right. \\ & \left. \left( 4 a c \text{AppellF1}\left[3, -m, -n, 4, -\frac{b x}{a}, -\frac{d x}{c}\right] + b c m x \text{AppellF1}\left[4, 1 - m, -n, 5, -\frac{b x}{a}, -\frac{d x}{c}\right] + a d n x \text{AppellF1}\left[4, -m, 1 - n, 5, -\frac{b x}{a}, -\frac{d x}{c}\right] \right) + \right. \\ & \left. \left. \frac{3 A \left( \frac{d (a + b x)}{-b c + a d} \right)^{-m} (c + d x) \text{Hypergeometric2F1}\left[-m, 1 + n, 2 + n, \frac{b (c + d x)}{b c - a d}\right]}{d (1 + n)} \right) \right) \end{aligned}$$

Problem 34: Result unnecessarily involves higher level functions.

$$\int (a + b x)^m (c + d x)^n (A + B x + C x^2 + D x^3) dx$$

Optimal (type 5, 610 leaves, 5 steps):

$$\begin{aligned}
& \frac{\left( (a^2 d^2 D (m^2 + m (8 + 3 n) + 3 (6 + 5 m + n^2)) + b^2 (c^2 D (6 + 5 m + m^2) - c C d (2 + m) (4 + m + n) + B d^2 (12 + m^2 + 7 n + n^2 + m (7 + 2 n))) \right. \\
& \quad \left. + a b d (c D (2 + m) (6 + m + 3 n) - C d (m^2 + m (8 + 3 n) + 2 (8 + 6 n + n^2))) \right) (a + b x)^{1+m} (c + d x)^{1+n}) / (b^3 d^3 (2 + m + n) (3 + m + n) (4 + m + n)) - \\
& \frac{(a d D (9 + 2 m + 3 n) + b (c D (3 + m) - C d (4 + m + n))) (a + b x)^{2+m} (c + d x)^{1+n}}{b^3 d^2 (3 + m + n) (4 + m + n)} + \frac{D (a + b x)^{3+m} (c + d x)^{1+n}}{b^3 d (4 + m + n)} + \\
& \frac{1}{b^4 d^3 (1 + m) (2 + m + n) (3 + m + n) (4 + m + n)} \\
& (d (2 + m + n) (a^3 d^2 D (1 + n) (6 + m + 2 n) + a b^2 c (2 + m) (c D (3 + m) - C d (4 + m + n)) + A b^3 d^2 (12 + m^2 + 7 n + n^2 + m (7 + 2 n)) - \\
& \quad a^2 b d (C d (1 + n) (4 + m + n) - c D (2 + m) (6 + m + 3 n))) - (b c (1 + m) + a d (1 + n)) \\
& (a^2 d^2 D (m^2 + m (8 + 3 n) + 3 (6 + 5 n + n^2)) + b^2 (c^2 D (6 + 5 m + m^2) - c C d (2 + m) (4 + m + n) + B d^2 (12 + m^2 + 7 n + n^2 + m (7 + 2 n))) + \\
& \quad a b d (c D (2 + m) (6 + m + 3 n) - C d (m^2 + m (8 + 3 n) + 2 (8 + 6 n + n^2)))) \\
& (a + b x)^{1+m} (c + d x)^n \left( \frac{b (c + d x)}{b c - a d} \right)^{-n} \text{Hypergeometric2F1}[1 + m, -n, 2 + m, -\frac{d (a + b x)}{b c - a d}]
\end{aligned}$$

Result (type 6, 446 leaves):

$$\begin{aligned}
& \frac{1}{12} (a + b x)^m (c + d x)^n \\
& \left( \left( 18 a B c x^2 \text{AppellF1}[2, -m, -n, 3, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \left( 3 a c \text{AppellF1}[2, -m, -n, 3, -\frac{b x}{a}, -\frac{d x}{c}] + b c m x \text{AppellF1}[3, 1 - m, -n, 4, -\frac{b x}{a}, -\frac{d x}{c}] + \right. \right. \\
& \quad a d n x \text{AppellF1}[3, -m, 1 - n, 4, -\frac{b x}{a}, -\frac{d x}{c}] \left. \right) + \left( 16 a c C x^3 \text{AppellF1}[3, -m, -n, 4, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \\
& \quad \left( 4 a c \text{AppellF1}[3, -m, -n, 4, -\frac{b x}{a}, -\frac{d x}{c}] + b c m x \text{AppellF1}[4, 1 - m, -n, 5, -\frac{b x}{a}, -\frac{d x}{c}] + a d n x \text{AppellF1}[4, -m, 1 - n, 5, -\frac{b x}{a}, -\frac{d x}{c}] \right) + \\
& \quad \left( 15 a c D x^4 \text{AppellF1}[4, -m, -n, 5, -\frac{b x}{a}, -\frac{d x}{c}] \right) / \\
& \quad \left( 5 a c \text{AppellF1}[4, -m, -n, 5, -\frac{b x}{a}, -\frac{d x}{c}] + b c m x \text{AppellF1}[5, 1 - m, -n, 6, -\frac{b x}{a}, -\frac{d x}{c}] + a d n x \text{AppellF1}[5, -m, 1 - n, 6, -\frac{b x}{a}, -\frac{d x}{c}] \right) + \\
& \quad \left. \left. 12 A \left( \frac{d (a + b x)}{-b c + a d} \right)^{-m} (c + d x) \text{Hypergeometric2F1}[-m, 1 + n, 2 + n, \frac{b (c + d x)}{b c - a d}] \right) / d (1 + n) \right)
\end{aligned}$$

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Test results for the 78 problems in "1.1.1.6 P(x) (a+b x)^m (c+d x)^n (e+f x)^p.m"

Problem 61: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \sqrt{a + b x} \sqrt{c + d x} \sqrt{e + f x} (A + B x + C x^2) dx$$

Optimal (type 4, 1182 leaves, 10 steps):

$$\begin{aligned}
 & \frac{1}{315 b^3 d^3 f^3} 2 (8 a^3 C d^3 f^3 + 3 a^2 b d^2 f^2 (C d e - c C f - 4 B d f) - 3 a b^2 d f^2 ((c^2 C - 7 A d^2) f + B d (d e - 2 c f))) - \\
 & b^3 (C (16 d^3 e^3 - 3 c^2 d e f^2 - 8 c^3 f^3) + 3 d f (7 A d f (2 d e - c f) - B (8 d^2 e^2 - c d e f - 4 c^2 f^2))) \sqrt{a+b x} \sqrt{c+d x} \sqrt{e+f x} - \frac{1}{105 b^2 d^2 f^3} \\
 & 2 (7 b d f (b c C e + a C d e + a c C f - 3 A b d f) + (a d f - 4 b (d e + c f)) (2 a C d f - b (3 B d f - 2 C (d e + c f)))) \sqrt{a+b x} \sqrt{c+d x} (e+f x)^{3/2} - \\
 & 2 (2 a C d f - b (3 B d f - 2 C (d e + c f))) \sqrt{a+b x} (c+d x)^{3/2} (e+f x)^{3/2} + \\
 & \frac{2 C (a+b x)^{3/2} (c+d x)^{3/2} (e+f x)^{3/2}}{9 b d f} - \frac{1}{315 b^4 d^{7/2} f^4 \sqrt{c+d x} \sqrt{\frac{b(e+f x)}{b e-a f}}} \\
 & 2 \sqrt{-b c + a d} (16 a^4 C d^4 f^4 - 8 a^3 b d^3 f^3 (C d e + c C f + 3 B d f) + 3 a^2 b^2 d^2 f^2 (d f (5 B d e + 5 B c f + 14 A d f) - 2 C (d^2 e^2 - c d e f + c^2 f^2)) - \\
 & a b^3 d f (C (8 d^3 e^3 - 6 c d^2 e^2 f - 6 c^2 d e f^2 + 8 c^3 f^3) + 3 d f (14 A d f (d e + c f) - B (5 d^2 e^2 - 6 c d e f + 5 c^2 f^2))) + \\
 & b^4 (2 C (8 d^4 e^4 - 4 c d^3 e^3 f - 3 c^2 d^2 e^2 f^2 - 4 c^3 d e f^3 + 8 c^4 f^4) + \\
 & 3 d f (14 A d f (d^2 e^2 - c d e f + c^2 f^2) - B (8 d^3 e^3 - 5 c d^2 e^2 f - 5 c^2 d e f^2 + 8 c^3 f^3)))) \\
 & \sqrt{\frac{b (c+d x)}{b c-a d}} \sqrt{e+f x} \text{EllipticE}[\text{ArcSin}\left[\frac{\sqrt{d} \sqrt{a+b x}}{\sqrt{-b c+a d}}\right], \frac{(b c-a d) f}{d (b e-a f)}] - \frac{1}{315 b^4 d^{7/2} f^4 \sqrt{c+d x} \sqrt{e+f x}} \\
 & 2 \sqrt{-b c + a d} (b e - a f) (d e - c f) (8 a^3 C d^3 f^3 + 3 a^2 b d^2 f^2 (C d e - c C f - 4 B d f) - 3 a b^2 d f^2 ((c^2 C - 7 A d^2) f + B d (d e - 2 c f))) - \\
 & b^3 (C (16 d^3 e^3 - 3 c^2 d e f^2 - 8 c^3 f^3) + 3 d f (7 A d f (2 d e - c f) - B (8 d^2 e^2 - c d e f - 4 c^2 f^2))) \\
 & \sqrt{\frac{b (c+d x)}{b c-a d}} \sqrt{\frac{b (e+f x)}{b e-a f}} \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{d} \sqrt{a+b x}}{\sqrt{-b c+a d}}\right], \frac{(b c-a d) f}{d (b e-a f)}]
 \end{aligned}$$

Result (type 4, 11933 leaves):

$$\begin{aligned}
 & \sqrt{a+b x} \sqrt{c+d x} \sqrt{e+f x} \\
 & \left( \frac{1}{315 b^3 d^3 f^3} 2 (8 b^3 C d^3 e^3 - 3 b^3 c C d^2 e^2 f - 12 b^3 B d^3 e^2 f - 3 a b^2 C d^3 e^2 f - 3 b^3 c^2 C d e f^2 + 6 b^3 B c d^2 e f^2 + 2 a b^2 c C d^2 e f^2 + \right. \\
 & 21 A b^3 d^3 e f^2 + 6 a b^2 B d^3 e f^2 - 3 a^2 b C d^3 e f^2 + 8 b^3 c^3 C f^3 - 12 b^3 B c^2 d f^3 - 3 a b^2 c^2 C d f^3 + \\
 & 21 A b^3 c d^2 f^3 + 6 a b^2 B c d^2 f^3 - 3 a^2 b c C d^2 f^3 + 21 a A b^2 d^3 f^3 - 12 a^2 b B d^3 f^3 + 8 a^3 C d^3 f^3) + \frac{1}{315 b^2 d^2 f^2} \\
 & 2 (-6 b^2 C d^2 e^2 + 2 b^2 c C d e f + 9 b^2 B d^2 e f + 2 a b C d^2 e f - 6 b^2 c^2 C f^2 + 9 b^2 B c d f^2 + 2 a b c C d f^2 + 63 A b^2 d^2 f^2 + 9 a b B d^2 f^2 - 6 a^2 C d^2 f^2) x + \\
 & \left. \frac{2 (b C d e + b c C f + 9 b B d f + a C d f) x^2}{63 b d f} + \frac{2 C x^3}{9} \right)
 \end{aligned}$$

$$\begin{aligned}
& \frac{1}{315 b^5 d^3 f^3} 2 \left( \frac{1}{d f \sqrt{c + \frac{(a+b x) (d - \frac{a d}{a+b x})}{b}}} \sqrt{e + \frac{(a+b x) (f - \frac{a f}{a+b x})}{b}} \right. \\
& \left. (16 b^4 C d^4 e^4 - 8 b^4 c C d^3 e^3 f - 24 b^4 B d^4 e^3 f - 8 a b^3 C d^4 e^3 f - 6 b^4 c^2 C d^2 e^2 f^2 + 15 b^4 B c d^3 e^2 f^2 + 6 a b^3 c C d^3 e^2 f^2 + 42 A b^4 d^4 e^2 f^2 + 15 a b^3 B d^4 e^2 f^2 - 6 a^2 b^2 C d^4 e^2 f^2 - 8 b^4 c^3 C d e f^3 + 15 b^4 B c^2 d^2 e f^3 + 6 a b^3 c^2 C d^2 e f^3 - 42 A b^4 c d^3 e f^3 - 18 a b^3 B c d^3 e f^3 + 6 a^2 b^2 c C d^3 e f^3 - 42 a A b^3 d^4 e f^3 + 15 a^2 b^2 B d^4 e f^3 - 8 a^3 b C d^4 e f^3 + 16 b^4 c^4 C f^4 - 24 b^4 B c^3 d f^4 - 8 a b^3 c^3 C d f^4 + 42 A b^4 c^2 d^2 f^4 + 15 a b^3 B c^2 d^2 f^4 - 6 a^2 b^2 c^2 C d^2 f^4 - 42 a A b^3 c d^3 f^4 + 15 a^2 b^2 B c d^3 f^4 - 8 a^3 b c C d^3 f^4 + 42 a^2 A b^2 d^4 f^4 - 24 a^3 b B d^4 f^4 + 16 a^4 C d^4 f^4) \right. \\
& \left. (a+b x)^{3/2} \left( d + \frac{b c}{a+b x} - \frac{a d}{a+b x} \right) \left( f + \frac{b e}{a+b x} - \frac{a f}{a+b x} \right) - \frac{1}{d f \sqrt{c + \frac{(a+b x) (d - \frac{a d}{a+b x})}{b}}} \sqrt{e + \frac{(a+b x) (f - \frac{a f}{a+b x})}{b}} \right) \\
& (-b c + a d) (-b e + a f) (a+b x) \sqrt{\left( d + \frac{b c}{a+b x} - \frac{a d}{a+b x} \right) \left( f + \frac{b e}{a+b x} - \frac{a f}{a+b x} \right)} \left( \begin{array}{l} 16 \pm b^4 C d^4 e^4 f \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \\ \text{EllipticE}[\pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a+b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f}] - \text{EllipticF}[\pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a+b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f}] \end{array} \right) / \\
& \left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left( d + \frac{b c - a d}{a+b x} \right) \left( f + \frac{b e - a f}{a+b x} \right)} - \left( 8 \pm b^4 c C d^3 e^3 f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \right. \right. \\
& \left. \left. \text{EllipticE}[\pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a+b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f}] - \text{EllipticF}[\pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a+b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f}] \right) \right) /
\end{aligned}$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 24 i b^4 B d^4 e^3 f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 8 i a b^3 C d^4 e^3 f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 6 i b^4 c^2 C d^2 e^2 f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 15 i b^4 B c d^3 e^2 f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}}, \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}}, \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 6 \frac{i a b^3 c C d^3 e^2 f^3}{d (a + b x)} \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}}, \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}}, \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 42 \frac{i A b^4 d^4 e^2 f^3}{d (a + b x)} \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}}, \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}}, \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 15 \frac{i a b^3 B d^4 e^2 f^3}{d (a + b x)} \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}}, \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}}, \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 6 \pm a^2 b^2 C d^4 e^2 f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 8 \pm b^4 c^3 C d e f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 15 \pm b^4 B c^2 d^2 e f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 6 \pm a b^3 c^2 C d^2 e f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 42 \pm A b^4 c d^3 e f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 18 \pm a b^3 B c d^3 e f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 6 \pm a^2 b^2 c C d^3 e f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 42 i a A b^3 d^4 e f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 15 i a^2 b^2 B d^4 e f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 8 i a^3 b C d^4 e f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 16 i b^4 c^4 C f^5 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 24 \pm b^4 B c^3 d f^5 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 8 \pm a b^3 c^3 C d f^5 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 42 \pm A b^4 c^2 d^2 f^5 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 15 \pm a b^3 B c^2 d^2 f^5 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 6 \pm a^2 b^2 c^2 C d^2 f^5 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 42 \pm a A b^3 c d^3 f^5 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 15 \pm a^2 b^2 B c d^3 f^5 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 8 \pm a^3 b c C d^3 f^5 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 42 \pm a^2 A b^2 d^4 f^5 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 24 \pm a^3 b B d^4 f^5 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\begin{aligned}
& \left( \sqrt{-\frac{-bc+ad}{d}} (-be+af) \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right)} \right) + \left( 16 \pm a^4 C d^4 f^5 \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \right. \\
& \left. \left. \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right] \right) \right) / \\
& \left( \sqrt{-\frac{-bc+ad}{d}} (-be+af) \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right)} \right) - \\
& \frac{8 \pm b^3 C d^4 e^3 f \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right]}{\sqrt{-\frac{-bc+ad}{d}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right)}} + \\
& \frac{3 \pm b^3 c C d^3 e^2 f^2 \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right]}{\sqrt{-\frac{-bc+ad}{d}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right)}} + \\
& \frac{12 \pm b^3 B d^4 e^2 f^2 \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right]}{\sqrt{-\frac{-bc+ad}{d}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right)}} - \\
& \frac{3 \pm a b^2 C d^4 e^2 f^2 \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right]}{\sqrt{-\frac{-bc+ad}{d}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right)}}
\end{aligned}$$

$$\frac{3 i b^3 c^2 C d^2 e f^3 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}}$$

$$\frac{6 i b^3 B c d^3 e f^3 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}}$$

$$\frac{21 i A b^3 d^4 e f^3 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} +$$

$$\frac{3 i a b^2 B d^4 e f^3 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}}$$

$$\frac{8 i b^3 c^3 C d f^4 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} +$$

$$\frac{12 i b^3 B c^2 d^2 f^4 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}}$$

$$\begin{aligned}
& \frac{3 i a b^2 c^2 C d^2 f^4 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} \\
\\
& 21 \pm A b^3 c d^3 f^4 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right] + \\
& \quad \sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)} \\
\\
& 3 i a b^2 B c d^3 f^4 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right] + \\
& \quad \sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)} \\
\\
& 42 \pm a A b^2 d^4 f^4 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right] - \\
& \quad \sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)} \\
\\
& 24 \pm a^2 b B d^4 f^4 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right] + \\
& \quad \sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)} \\
\\
& 16 \pm a^3 C d^4 f^4 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right] \Bigg)
\end{aligned}$$

Problem 62: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{c+dx} \sqrt{e+fx} (A+Bx+Cx^2)}{\sqrt{a+bx}} dx$$

Optimal (type 4, 774 leaves, 9 steps):

$$\begin{aligned} & -\frac{1}{105 b^3 d^2 f^2} \\ & 2 (5 b d f (3 a C (d e + c f) + b (c C e - 7 A d f)) - (2 b d e - b c f + 4 a d f) (6 a C d f - b (7 B d f - 4 C (d e + c f)))) \sqrt{a+b x} \sqrt{c+d x} \sqrt{e+f x} - \\ & 2 (6 a C d f - b (7 B d f - 4 C (d e + c f))) \sqrt{a+b x} \sqrt{c+d x} (e+f x)^{3/2} + \frac{2 C \sqrt{a+b x} (c+d x)^{3/2} (e+f x)^{3/2}}{7 b d f} - \frac{1}{105 b^4 d^{5/2} f^3 \sqrt{c+d x} \sqrt{\frac{b(e+f x)}{b e - a f}}} \\ & 2 \sqrt{-b c + a d} \left( 3 b d f (5 b c f (3 a C (d e + c f) + b (c C e - 7 A d f)) - (b c e + a d e + 3 a c f) (6 a C d f - b (7 B d f - 4 C (d e + c f)))) + \right. \\ & 2 \left( \frac{b d e}{2} - (b c + a d) f \right) (5 b d f (3 a C (d e + c f) + b (c C e - 7 A d f)) - (2 b d e - b c f + 4 a d f) (6 a C d f - b (7 B d f - 4 C (d e + c f)))) \Big) \\ & \sqrt{\frac{b (c+d x)}{b c - a d}} \sqrt{e+f x} \text{EllipticE}[\text{ArcSin}\left[\frac{\sqrt{d} \sqrt{a+b x}}{\sqrt{-b c + a d}}\right], \frac{(b c - a d) f}{d (b e - a f)}] - \left( 2 \sqrt{-b c + a d} (b e - a f) (d e - c f) \right. \\ & (24 a^2 C d^2 f^2 + a b d f (13 C d e - 5 c C f - 28 B d f) - b^2 (7 d f (2 B d e - B c f - 5 A d f) - C (8 d^2 e^2 - c d e f - 4 c^2 f^2))) \\ & \left. \sqrt{\frac{b (c+d x)}{b c - a d}} \sqrt{\frac{b (e+f x)}{b e - a f}} \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{d} \sqrt{a+b x}}{\sqrt{-b c + a d}}\right], \frac{(b c - a d) f}{d (b e - a f)}] \right) / (105 b^4 d^{5/2} f^3 \sqrt{c+d x} \sqrt{e+f x}) \end{aligned}$$

Result (type 4, 7297 leaves):

$$\begin{aligned} & \sqrt{a+b x} \sqrt{c+d x} \sqrt{e+f x} \left( \frac{1}{105 b^3 d^2 f^2} \right. \\ & 2 (-4 b^2 C d^2 e^2 + 2 b^2 c C d e f + 7 b^2 B d^2 e f - 5 a b C d^2 e f - 4 b^2 c^2 C f^2 + 7 b^2 B c d f^2 - 5 a b c C d f^2 + 35 A b^2 d^2 f^2 - 28 a b B d^2 f^2 + 24 a^2 C d^2 f^2) + \\ & 2 (b C d e + b c C f + 7 b B d f - 6 a C d f) x + \frac{2 C x^2}{7 b} \Big) - \\ & \frac{1}{105 b^5 d^2 f^2} 2 \left( \frac{1}{d f \sqrt{c + \frac{(a+b x) (d - \frac{a d}{a+b x})}{b}} \sqrt{e + \frac{(a+b x) (f - \frac{a f}{a+b x})}{b}}} \right. \\ & (-8 b^3 C d^3 e^3 + 5 b^3 c C d^2 e^2 f + 14 b^3 B d^3 e^2 f - 9 a b^2 C d^3 e^2 f + \\ & 5 b^3 c^2 C d e f^2 - 14 b^3 B c d^2 e f^2 + 8 a b^2 c C d^2 e f^2 - 35 A b^3 d^3 e f^2 + 21 a b^2 B d^3 e f^2 - 16 a^2 b C d^3 e f^2 - 8 b^3 c^3 C f^3 + \\ & 14 b^3 B c^2 d f^3 - 9 a b^2 c^2 C d f^3 - 35 A b^3 c d^2 f^3 + 21 a b^2 B c d^2 f^3 - 16 a^2 b c C d^2 f^3 + 70 a A b^2 d^3 f^3 - 56 a^2 b B d^3 f^3 + 48 a^3 C d^3 f^3) \end{aligned}$$

$$\begin{aligned}
& \left( a + b x \right)^{3/2} \left( d + \frac{b c}{a + b x} - \frac{a d}{a + b x} \right) \left( f + \frac{b e}{a + b x} - \frac{a f}{a + b x} \right) + \frac{1}{d f \sqrt{c + \frac{(a+b x)(d-a d)}{b}}} \sqrt{e + \frac{(a+b x)(f-a f)}{b}} \\
& (-b c + a d) (-b e + a f) (a + b x) \sqrt{\left( d + \frac{b c}{a + b x} - \frac{a d}{a + b x} \right) \left( f + \frac{b e}{a + b x} - \frac{a f}{a + b x} \right)} \left( \begin{array}{l} 8 i b^3 C d^3 e^3 f \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \\ \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \end{array} \right) / \\
& \left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left( d + \frac{b c - a d}{a + b x} \right) \left( f + \frac{b e - a f}{a + b x} \right)} - \left( 5 i b^3 c C d^2 e^2 f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \right. \\
& \left. \left. \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) / \\
& \left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left( d + \frac{b c - a d}{a + b x} \right) \left( f + \frac{b e - a f}{a + b x} \right)} - \left( 14 i b^3 B d^3 e^2 f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \right. \\
& \left. \left. \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /
\end{aligned}$$

$$\left( \sqrt{-\frac{b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 9 \pm a b^2 C d^3 e^2 f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 5 \pm b^3 c^2 C d e f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 14 \pm b^3 B c d^2 e f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 8 \pm a b^2 c C d^2 e f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} + \left( 35 i a b^3 d^3 e f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \right.$$

$$\left. \text{EllipticE}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} - \left( 21 i a b^2 B d^3 e f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \right.$$

$$\left. \text{EllipticE}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} + \left( 16 i a^2 b C d^3 e f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \right.$$

$$\left. \text{EllipticE}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\begin{aligned}
& \left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 8 \pm b^3 c^3 C f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \\
& \left. \left. \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) \right) / \\
& \left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 14 \pm b^3 B c^2 d f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \\
& \left. \left. \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) \right) / \\
& \left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 9 \pm a b^2 c^2 C d f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \\
& \left. \left. \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) \right) / \\
& \left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 35 \pm A b^3 c d^2 f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)
\end{aligned}$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 21 \pm a b^2 B c d^2 f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 16 \pm a^2 b c C d^2 f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 70 \pm a A b^2 d^3 f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\begin{aligned}
& \left( \sqrt{-\frac{b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 56 i a^2 b B d^3 f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \\
& \left. \left. \left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) \right) / \\
& \left( \sqrt{-\frac{b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 48 i a^3 C d^3 f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \\
& \left. \left. \left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) \right) / \\
& \left( \sqrt{-\frac{b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \\
& \frac{4 i b^2 C d^3 e^2 f \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right]}{+} \\
& \sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \\
& \frac{2 i b^2 c C d^2 e f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right]}{+} \\
& \sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)}
\end{aligned}$$

$$\frac{7 i b^2 B d^3 e f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}}$$

$$\frac{8 i a b C d^3 e f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}}$$

$$\frac{4 i b^2 c^2 C d f^3 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} +$$

$$\frac{7 i b^2 B c d^2 f^3 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}}$$

$$\frac{8 i a b c C d^2 f^3 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}}$$

$$\frac{70 i A b^2 d^3 f^3 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} +$$

$$\frac{56 \pm a b B d^3 f^3 \sqrt{1 - \frac{-b c + a d}{d(a+b x)}} \sqrt{1 - \frac{-b e + a f}{f(a+b x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d(-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} \\ \frac{48 \pm a^2 C d^3 f^3 \sqrt{1 - \frac{-b c + a d}{d(a+b x)}} \sqrt{1 - \frac{-b e + a f}{f(a+b x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d(-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} \right]$$

Problem 63: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{c+d x} \sqrt{e+f x} (A+B x+C x^2)}{(a+b x)^{3/2}} dx$$

Optimal (type 4, 706 leaves, 9 steps):

$$\begin{aligned} & \frac{1}{15 b^3 d f (b e - a f)} 2 (24 a^2 C d f^2 - a b f (7 C d e + c C f + 20 B d f) + b^2 (5 d f (B e + 3 A f) - C e (2 d e - c f))) \sqrt{a+b x} \sqrt{c+d x} \sqrt{e+f x} + \\ & \frac{2 (6 a^2 C d f + b^2 (c C e + 5 A d f) - a b (C d e + c C f + 5 B d f)) \sqrt{a+b x} \sqrt{c+d x} (e+f x)^{3/2}}{5 b^2 (b c - a d) f (b e - a f)} - \frac{2 (A b^2 - a (b B - a C)) (c+d x)^{3/2} (e+f x)^{3/2}}{b (b c - a d) (b e - a f) \sqrt{a+b x}} + \\ & \left( 2 \sqrt{-b c + a d} (48 a^2 C d^2 f^2 - 8 a b d f (C d e + c C f + 5 B d f) + b^2 (5 d f (B d e + B c f + 6 A d f) - 2 C (d^2 e^2 - c d e f + c^2 f^2))) \right. \\ & \left. \sqrt{\frac{b (c+d x)}{b c - a d}} \sqrt{e+f x} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{d} \sqrt{a+b x}}{\sqrt{-b c + a d}}\right], \frac{(b c - a d) f}{d (b e - a f)}\right] \right) / \left( 15 b^4 d^{3/2} f^2 \sqrt{c+d x} \sqrt{\frac{b (e+f x)}{b e - a f}} \right) - \\ & \left( 2 \sqrt{-b c + a d} (d e - c f) (24 a^2 C d f^2 - a b f (7 C d e + c C f + 20 B d f) + b^2 (5 d f (B e + 3 A f) - C e (2 d e - c f))) \right. \\ & \left. \sqrt{\frac{b (c+d x)}{b c - a d}} \sqrt{\frac{b (e+f x)}{b e - a f}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{d} \sqrt{a+b x}}{\sqrt{-b c + a d}}\right], \frac{(b c - a d) f}{d (b e - a f)}\right] \right) / \left( 15 b^4 d^{3/2} f^2 \sqrt{c+d x} \sqrt{e+f x} \right) \end{aligned}$$

Result (type 4, 9487 leaves):

$$\begin{aligned}
 & \sqrt{a+b x} \sqrt{c+d x} \sqrt{e+f x} \left( \frac{2(b C d e + b c C f + 5 b B d f - 9 a C d f)}{15 b^3 d f} + \frac{2 C x}{5 b^2} - \frac{2(A b^2 - a b B + a^2 C)}{b^3 (a+b x)} \right) + \frac{1}{15 b^5 d f} \\
 & 2 \left( \left( -2 b^2 C d^2 e^2 + 2 b^2 c C d e f + 5 b^2 B d^2 e f - 8 a b C d^2 e f - 2 b^2 c^2 C f^2 + 5 b^2 B c d f^2 - 8 a b c C d f^2 + 30 A b^2 d^2 f^2 - 40 a b B d^2 f^2 + 48 a^2 C d^2 f^2 \right) \right. \\
 & \quad \left. \left( a+b x \right)^{3/2} \left( d + \frac{b c}{a+b x} - \frac{a d}{a+b x} \right) \left( f + \frac{b e}{a+b x} - \frac{a f}{a+b x} \right) \right) / \left( d f \sqrt{c + \frac{(a+b x)(d - \frac{a d}{a+b x})}{b}} \sqrt{e + \frac{(a+b x)(f - \frac{a f}{a+b x})}{b}} \right) + \\
 & \frac{1}{d f \sqrt{c + \frac{(a+b x)(d - \frac{a d}{a+b x})}{b}} \sqrt{e + \frac{(a+b x)(f - \frac{a f}{a+b x})}{b}}} (a+b x) \sqrt{\left( d + \frac{b c}{a+b x} - \frac{a d}{a+b x} \right) \left( f + \frac{b e}{a+b x} - \frac{a f}{a+b x} \right)} \left( \begin{array}{l} 2 \frac{1}{2} b^4 c C d^2 e^3 f \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \\ \text{EllipticE}[\frac{1}{2} \text{ArcSinh}\left[\frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}] - \text{EllipticF}[\frac{1}{2} \text{ArcSinh}\left[\frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}] \end{array} \right) / \\
 & \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \left( \begin{array}{l} \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left( d + \frac{b c - a d}{a+b x} \right) \left( f + \frac{b e - a f}{a+b x} \right)} \\ \text{EllipticE}[\frac{1}{2} \text{ArcSinh}\left[\frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}] - \text{EllipticF}[\frac{1}{2} \text{ArcSinh}\left[\frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}] \end{array} \right) / \\
 & \left( \begin{array}{l} 2 \frac{1}{2} a b^3 C d^3 e^3 f \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \\ \text{EllipticE}[\frac{1}{2} \text{ArcSinh}\left[\frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}] - \text{EllipticF}[\frac{1}{2} \text{ArcSinh}\left[\frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}] \end{array} \right) /
 \end{aligned}$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 2 \pm b^4 c^2 C d e^2 f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 5 \pm b^4 B c d^2 e^2 f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 8 \pm a b^3 c C d^2 e^2 f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 5 \pm a b^3 B d^3 e^2 f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 6 i a^2 b^2 C d^3 e^2 f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 2 i b^4 c^3 C e f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 5 i b^4 B c^2 d e f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 8 i a b^3 c^2 C d e f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 30 i A b^4 c d^2 e f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 50 i a b^3 B c d^2 e f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 66 i a^2 b^2 c C d^2 e f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} + \left( 30 i a A b^3 d^3 e f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \right.$$

$$\left. \text{EllipticE}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} - \left( 45 i a^2 b^2 B d^3 e f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \right.$$

$$\left. \text{EllipticE}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} + \left( 56 i a^3 b C d^3 e f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \right.$$

$$\left. \text{EllipticE}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 2 \pm a b^3 c^3 C f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 5 \pm a b^3 B c^2 d f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 6 \pm a^2 b^2 c^2 C d f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 30 \pm a A b^3 c d^2 f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 45 \pm a^2 b^2 B c d^2 f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 56 \pm a^3 b c C d^2 f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 30 \pm a^2 A b^2 d^3 f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\begin{aligned}
& \left( \sqrt{-\frac{-bc+ad}{d}} (-be+af) \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right)} \right) + \left( 40 \pm a^3 b B d^3 f^4 \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \right. \\
& \left. \left. \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right] \right) \right) / \\
& \left( \sqrt{-\frac{-bc+ad}{d}} (-be+af) \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right)} \right) - \left( 48 \pm a^4 C d^3 f^4 \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \right. \\
& \left. \left. \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right] \right) \right) / \\
& \left( \sqrt{-\frac{-bc+ad}{d}} (-be+af) \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right)} \right) - \\
& \frac{\pm b^3 c C d^2 e^2 f \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right]}{+} \\
& \frac{\sqrt{-\frac{-bc+ad}{d}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right)}}{\pm a b^2 C d^3 e^2 f \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right]} -
\end{aligned}$$

$$\frac{\frac{1}{2} b^3 c^2 C d e f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[\frac{i \operatorname{ArcSinh}\left[\frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right]}{\sqrt{a+b x}}, \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{-b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} +$$

$$\frac{10 \frac{1}{2} b^3 B c d^2 e f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[\frac{i \operatorname{ArcSinh}\left[\frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right]}{\sqrt{a+b x}}, \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{-b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} -$$

$$\frac{16 \frac{1}{2} a b^2 c C d^2 e f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[\frac{i \operatorname{ArcSinh}\left[\frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right]}{\sqrt{a+b x}}, \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{-b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} +$$

$$\frac{15 \frac{1}{2} A b^3 d^3 e f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[\frac{i \operatorname{ArcSinh}\left[\frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right]}{\sqrt{a+b x}}, \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{-b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} -$$

$$\frac{25 \frac{1}{2} a b^2 B d^3 e f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[\frac{i \operatorname{ArcSinh}\left[\frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right]}{\sqrt{a+b x}}, \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{-b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} +$$

$$\frac{32 \frac{1}{2} a^2 b C d^3 e f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[\frac{i \operatorname{ArcSinh}\left[\frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right]}{\sqrt{a+b x}}, \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{-b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} +$$

$$\frac{i a b^2 c^2 C d f^3 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} +$$

$$\frac{15 i A b^3 c d^2 f^3 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} -$$

$$\frac{25 i a b^2 B c d^2 f^3 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} +$$

$$\frac{32 i a^2 b c C d^2 f^3 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} -$$

$$\frac{30 i a A b^2 d^3 f^3 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} +$$

$$\frac{40 i a^2 b B d^3 f^3 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} -$$

$$\frac{48 \pm a^3 C d^3 f^3 \sqrt{1 - \frac{-b c + a d}{d(a+b x)}} \sqrt{1 - \frac{-b e + a f}{f(a+b x)}} \text{EllipticF}\left[\pm \text{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d(-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}}$$

Problem 64: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{c+d x} \sqrt{e+f x} (A+B x+C x^2)}{(a+b x)^{5/2}} dx$$

Optimal (type 4, 687 leaves, 9 steps):

$$\begin{aligned} & \frac{2 (8 a^2 C d f + b^2 (c C e + 3 B c f + A d f) - a b (C d e + 7 c C f + 4 B d f)) \sqrt{a+b x} \sqrt{c+d x} \sqrt{e+f x}}{3 b^3 (b c - a d) (b e - a f)} - \\ & \frac{2 (b B - 2 a C) \sqrt{c+d x} (e+f x)^{3/2}}{b^2 (b e - a f) \sqrt{a+b x}} - \frac{2 (A b^2 - a (b B - a C)) (c+d x)^{3/2} (e+f x)^{3/2}}{3 b (b c - a d) (b e - a f) (a+b x)^{3/2}} + \\ & \left( 2 (16 a^3 C d^2 f^2 - 8 a^2 b d f (B d f + 2 C (d e + c f)) - b^3 (c^2 C e f + A d^2 e f + c d (C e^2 + 6 B e f + A f^2))) + a b^2 \right. \\ & \left. (d f (7 B d e + 7 B c f + 2 A d f) + C (d^2 e^2 + 16 c d e f + c^2 f^2))) \sqrt{\frac{b (c+d x)}{b c - a d}} \sqrt{e+f x} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{d} \sqrt{a+b x}}{\sqrt{-b c + a d}}\right], \frac{(b c - a d) f}{d (b e - a f)}\right] \right) / \\ & \left( 3 b^4 \sqrt{d} \sqrt{-b c + a d} f (b e - a f) \sqrt{c+d x} \sqrt{\frac{b (e+f x)}{b e - a f}} + \left( 2 (d e - c f) (8 a^2 C d f + b^2 (c C e + 3 B c f + A d f) - a b (C d e + 7 c C f + 4 B d f)) \right. \right. \\ & \left. \left. \sqrt{\frac{b (c+d x)}{b c - a d}} \sqrt{\frac{b (e+f x)}{b e - a f}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{d} \sqrt{a+b x}}{\sqrt{-b c + a d}}\right], \frac{(b c - a d) f}{d (b e - a f)}\right] \right) / (3 b^4 \sqrt{d} \sqrt{-b c + a d} f \sqrt{c+d x} \sqrt{e+f x}) \right) \end{aligned}$$

Result (type 4, 5831 leaves):

$$\begin{aligned} & \sqrt{a+b x} \sqrt{c+d x} \sqrt{e+f x} \left( \frac{2 C}{3 b^3} - \frac{2 (A b^2 - a b B + a^2 C)}{3 b^3 (a+b x)^2} - \right. \\ & \left. (2 (3 b^3 B c e - 6 a b^2 c C e + A b^3 d e - 4 a b^2 B d e + 7 a^2 b C d e + A b^3 c f - 4 a b^2 B c f + 7 a^2 b c C f - 2 a A b^2 d f + 5 a^2 b B d f - 8 a^3 C d f)) / \right. \end{aligned}$$

$$\begin{aligned}
& \left( 3 b^3 (b c - a d) (b e - a f) (a + b x) \right) - \frac{1}{3 b^5 (b c - a d) (b e - a f)} \\
2 & \left( \left( -b^3 c C d e^2 + a b^2 C d^2 e^2 - b^3 c^2 C e f - 6 b^3 B c d e f + 16 a b^2 c C d e f - A b^3 d^2 e f + 7 a b^2 B d^2 e f - 16 a^2 b C d^2 e f + a b^2 c^2 C f^2 - A b^3 c d f^2 + \right. \right. \\
& \quad \left. \left. 7 a b^2 B c d f^2 - 16 a^2 b c C d f^2 + 2 a A b^2 d^2 f^2 - 8 a^2 b B d^2 f^2 + 16 a^3 C d^2 f^2 \right) (a + b x)^{3/2} \left( d + \frac{b c}{a + b x} - \frac{a d}{a + b x} \right) \left( f + \frac{b e}{a + b x} - \frac{a f}{a + b x} \right) \right) / \\
& \left( d f \sqrt{c + \frac{(a + b x) \left( d - \frac{a d}{a + b x} \right)}{b}} \sqrt{e + \frac{(a + b x) \left( f - \frac{a f}{a + b x} \right)}{b}} \right) + \frac{1}{d f \sqrt{c + \frac{(a + b x) \left( d - \frac{a d}{a + b x} \right)}{b}} \sqrt{e + \frac{(a + b x) \left( f - \frac{a f}{a + b x} \right)}{b}}} \\
& (-b c + a d) (-b e + a f) (a + b x) \sqrt{\left( d + \frac{b c}{a + b x} - \frac{a d}{a + b x} \right) \left( f + \frac{b e}{a + b x} - \frac{a f}{a + b x} \right)} \left( \begin{array}{l} \pm b^3 c C d e^2 f \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \\ \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \end{array} \right) / \\
& \left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left( d + \frac{b c - a d}{a + b x} \right) \left( f + \frac{b e - a f}{a + b x} \right)} - \sqrt{\pm a b^2 C d^2 e^2 f \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}}} \right. \\
& \quad \left. \left( \begin{array}{l} \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \end{array} \right) \right) /
\end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{-\frac{-bc+ad}{d}} (-be+af) \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right)} \right) + \left( \pm b^3 c^2 C e f^2 \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \right. \\
& \left. \left. \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right] \right) \right) / \\
& \left( \sqrt{-\frac{-bc+ad}{d}} (-be+af) \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right)} \right) + \left( 6 \pm b^3 B c d e f^2 \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \right. \\
& \left. \left. \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right] \right) \right) / \\
& \left( \sqrt{-\frac{-bc+ad}{d}} (-be+af) \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right)} \right) - \left( 16 \pm a b^2 c C d e f^2 \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \right. \\
& \left. \left. \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right] \right) \right) / \\
& \left( \sqrt{-\frac{-bc+ad}{d}} (-be+af) \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right)} \right) + \left( \pm A b^3 d^2 e f^2 \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \right)
\end{aligned}$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 7 \pm a b^2 B d^2 e f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 16 \pm a^2 b C d^2 e f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( \pm a b^2 c^2 C f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\begin{aligned}
& \left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( \frac{i A b^3 c d f^3}{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \\
& \left. \left. \left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) \right) \right) / \\
& \left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 7 i a b^2 B c d f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \\
& \left. \left. \left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) \right) \right) / \\
& \left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 16 i a^2 b c C d f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \\
& \left. \left. \left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) \right) \right) / \\
& \left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 2 i a A b^2 d^2 f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)
\end{aligned}$$

$$\begin{aligned}
& \left( \frac{\operatorname{EllipticE}[\operatorname{ArcSinh}\left(\frac{\sqrt{-\frac{-b c+a d}{d}}}{\sqrt{a+b x}}\right), \frac{d (-b e+a f)}{(-b c+a d) f}] - \operatorname{EllipticF}[\operatorname{ArcSinh}\left(\frac{\sqrt{-\frac{-b c+a d}{d}}}{\sqrt{a+b x}}\right), \frac{d (-b e+a f)}{(-b c+a d) f}]}{\sqrt{-\frac{-b c+a d}{d}} (-b e+a f) \sqrt{\left(d+\frac{b c-a d}{a+b x}\right) \left(f+\frac{b e-a f}{a+b x}\right)}} \right) \\
& + \left( \sqrt{-\frac{-b c+a d}{d}} (-b e+a f) \sqrt{\left(d+\frac{b c-a d}{a+b x}\right) \left(f+\frac{b e-a f}{a+b x}\right)} + \left(8 i a^2 b B d^2 f^3 \sqrt{1-\frac{-b c+a d}{d (a+b x)}} \sqrt{1-\frac{-b e+a f}{f (a+b x)}} \right. \right. \\
& \left. \left. \operatorname{EllipticE}[\operatorname{ArcSinh}\left(\frac{\sqrt{-\frac{-b c+a d}{d}}}{\sqrt{a+b x}}\right), \frac{d (-b e+a f)}{(-b c+a d) f}] - \operatorname{EllipticF}[\operatorname{ArcSinh}\left(\frac{\sqrt{-\frac{-b c+a d}{d}}}{\sqrt{a+b x}}\right), \frac{d (-b e+a f)}{(-b c+a d) f}]} \right) \right) / \\
& \left( \sqrt{-\frac{-b c+a d}{d}} (-b e+a f) \sqrt{\left(d+\frac{b c-a d}{a+b x}\right) \left(f+\frac{b e-a f}{a+b x}\right)} - \left(16 i a^3 C d^2 f^3 \sqrt{1-\frac{-b c+a d}{d (a+b x)}} \sqrt{1-\frac{-b e+a f}{f (a+b x)}} \right. \right. \\
& \left. \left. \operatorname{EllipticE}[\operatorname{ArcSinh}\left(\frac{\sqrt{-\frac{-b c+a d}{d}}}{\sqrt{a+b x}}\right), \frac{d (-b e+a f)}{(-b c+a d) f}] - \operatorname{EllipticF}[\operatorname{ArcSinh}\left(\frac{\sqrt{-\frac{-b c+a d}{d}}}{\sqrt{a+b x}}\right), \frac{d (-b e+a f)}{(-b c+a d) f}]} \right) \right) / \\
& \left( \sqrt{-\frac{-b c+a d}{d}} (-b e+a f) \sqrt{\left(d+\frac{b c-a d}{a+b x}\right) \left(f+\frac{b e-a f}{a+b x}\right)} - \right. \\
& \left. \frac{2 i b^2 c C d e f \sqrt{1-\frac{-b c+a d}{d (a+b x)}} \sqrt{1-\frac{-b e+a f}{f (a+b x)}} \operatorname{EllipticF}[\operatorname{ArcSinh}\left(\frac{\sqrt{-\frac{-b c+a d}{d}}}{\sqrt{a+b x}}\right), \frac{d (-b e+a f)}{(-b c+a d) f}]}{\sqrt{-\frac{-b c+a d}{d}} \sqrt{\left(d+\frac{b c-a d}{a+b x}\right) \left(f+\frac{b e-a f}{a+b x}\right)}} \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{3 i b^2 B d^2 e f \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} + \\
& \frac{8 i a b C d^2 e f \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} - \\
& \frac{3 i b^2 B c d f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} + \\
& \frac{8 i a b c C d f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} - \\
& \frac{2 i A b^2 d^2 f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} + \\
& \frac{8 i a b B d^2 f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}}
\end{aligned}$$

$$\frac{16 \pm a^2 C d^2 f^2 \sqrt{1 - \frac{-b c + a d}{d(a+b x)}} \sqrt{1 - \frac{-b e + a f}{f(a+b x)}} \text{EllipticF}\left[\pm \text{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d(-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}}$$

**Problem 65: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{c+d x} \sqrt{e+f x} (A+B x+C x^2)}{(a+b x)^{7/2}} dx$$

Optimal (type 4, 964 leaves, 9 steps):

$$\begin{aligned} & \left(2 (24 a^3 C d^2 f - a^2 b d (23 C d e + 41 c C f + 4 B d f) - b^3 (15 c^2 C e - 2 A d^2 e + c d (5 B e + A f)) + a b^2 (15 c^2 C f + d^2 (3 B e - A f) + c (40 C d e + 6 B d f))) \right. \\ & \quad \left. \sqrt{c+d x} \sqrt{e+f x}\right) / \left(15 b^3 (b c - a d)^2 (b e - a f) \sqrt{a+b x}\right) + \\ & \left(2 (6 a^3 C d f + a b^2 (10 c C e + 3 B d e + 3 B c f - 4 A d f) - b^3 (5 B c e - 2 A (d e + c f)) - a^2 b (B d f + 8 C (d e + c f))) \right. \sqrt{c+d x} (e+f x)^{3/2}) / \\ & \quad \left(15 b^2 (b c - a d) (b e - a f)^2 (a+b x)^{3/2}\right) - \frac{2 (A b^2 - a (b B - a C)) (c+d x)^{3/2} (e+f x)^{3/2}}{5 b (b c - a d) (b e - a f) (a+b x)^{5/2}} + \\ & \quad \frac{1}{2 \sqrt{d} (48 a^4 C d^2 f^2 - 8 a^3 b d f (B d f + 11 C (d e + c f)))} - \\ & 15 b^4 (-b c + a d)^{3/2} (b e - a f)^2 \sqrt{c+d x} \sqrt{\frac{b (e+f x)}{b e - a f}} \\ & \quad b^4 (2 A d^2 e^2 - c d e (5 B e + 2 A f) - c^2 (30 C e^2 + 5 B e f - 2 A f^2)) - a b^3 (d^2 e (3 B e - 2 A f) + c^2 f (70 C e + 3 B f) + 2 c d (35 C e^2 + 11 B e f - A f^2)) + \\ & \quad a^2 b^2 (2 C (19 d^2 e^2 + 81 c d e f + 19 c^2 f^2) - d f (2 A d f - 13 B (d e + c f))) \\ & \quad \sqrt{\frac{b (c+d x)}{b c - a d}} \sqrt{e+f x} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{d} \sqrt{a+b x}}{\sqrt{-b c + a d}}\right], \frac{(b c - a d) f}{d (b e - a f)}\right] + \left[2 (d e - c f)\right. \\ & \quad \left.(24 a^3 C d^2 f - a^2 b d (23 C d e + 41 c C f + 4 B d f) - b^3 (15 c^2 C e - 2 A d^2 e + c d (5 B e + A f)) + a b^2 (15 c^2 C f + d^2 (3 B e - A f) + c (40 C d e + 6 B d f)))\right. \\ & \quad \left.\sqrt{\frac{b (c+d x)}{b c - a d}} \sqrt{\frac{b (e+f x)}{b e - a f}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{d} \sqrt{a+b x}}{\sqrt{-b c + a d}}\right], \frac{(b c - a d) f}{d (b e - a f)}\right]\right] / \left(15 b^4 \sqrt{d} (-b c + a d)^{3/2} (b e - a f) \sqrt{c+d x} \sqrt{e+f x}\right) \end{aligned}$$

Result (type 4, 9529 leaves):

$$\begin{aligned} & \sqrt{a+b x} \sqrt{c+d x} \sqrt{e+f x} \left( -\frac{2 (A b^2 - a b B + a^2 C)}{5 b^3 (a+b x)^3} - \right. \\ & \quad \left. (2 (5 b^3 B c e - 10 a b^2 c C e + A b^3 d e - 6 a b^2 B d e + 11 a^2 b C d e + A b^3 c f - 6 a b^2 B c f + 11 a^2 b c C f - 2 a A b^2 d f + 7 a^2 b B d f - 12 a^3 C d f)) / \right. \end{aligned}$$

$$\left( 15 b^3 (b c - a d) (b e - a f) (a + b x)^2 \right) - \frac{1}{15 b^3 (b c - a d)^2 (b e - a f)^2 (a + b x)}$$

$$2 \left( 15 b^4 c^2 C e^2 + 5 b^4 B c d e^2 - 40 a b^3 c C d e^2 - 2 A b^4 d^2 e^2 - 3 a b^3 B d^2 e^2 + 23 a^2 b^2 C d^2 e^2 + 5 b^4 B c^2 e f - 40 a b^3 c^2 C e f + 2 A b^4 c d e f - 22 a b^3 B c d e f + 102 a^2 b^2 c C d e f + 2 a A b^3 d^2 e f + 13 a^2 b^2 B d^2 e f - 58 a^3 b C d^2 e f - 2 A b^4 c^2 f^2 - 3 a b^3 B c^2 f^2 + 23 a^2 b^2 c^2 C f^2 + \right.$$

$$\left. 2 a A b^3 c d f^2 + 13 a^2 b^2 B c d f^2 - 58 a^3 b c C d f^2 - 2 a^2 A b^2 d^2 f^2 - 8 a^3 b B d^2 f^2 + 33 a^4 C d^2 f^2 \right) + \frac{1}{15 b^5 (b c - a d)^2 (b e - a f)^2}$$

$$2 \left( \frac{1}{\sqrt{c + \frac{(a+b x) (d - \frac{a d}{a+b x})}{b}}} \sqrt{e + \frac{(a+b x) (f - \frac{a f}{a+b x})}{b}} \left( 30 b^4 c^2 C e^2 + 5 b^4 B c d e^2 - 70 a b^3 c C d e^2 - 2 A b^4 d^2 e^2 - 3 a b^3 B d^2 e^2 + 38 a^2 b^2 C d^2 e^2 + \right. \right.$$

$$\left. \left. 5 b^4 B c^2 e f - 70 a b^3 c^2 C e f + 2 A b^4 c d e f - 22 a b^3 B c d e f + 162 a^2 b^2 c C d e f + 2 a A b^3 d^2 e f + 13 a^2 b^2 B d^2 e f - 88 a^3 b C d^2 e f - 2 A b^4 c^2 f^2 - 3 a b^3 B c^2 f^2 + 38 a^2 b^2 c^2 C f^2 + 2 a A b^3 c d f^2 + 13 a^2 b^2 B c d f^2 - 88 a^3 b c C d f^2 - 2 a^2 A b^2 d^2 f^2 - 8 a^3 b B d^2 f^2 + 48 a^4 C d^2 f^2 \right) \right)$$

$$(a + b x)^{3/2} \left( d + \frac{b c}{a + b x} - \frac{a d}{a + b x} \right) \left( f + \frac{b e}{a + b x} - \frac{a f}{a + b x} \right) - \frac{1}{\sqrt{c + \frac{(a+b x) (d - \frac{a d}{a+b x})}{b}}} \sqrt{e + \frac{(a+b x) (f - \frac{a f}{a+b x})}{b}}$$

$$(b c - a d) (b e - a f) (a + b x) \sqrt{\left( d + \frac{b c}{a + b x} - \frac{a d}{a + b x} \right) \left( f + \frac{b e}{a + b x} - \frac{a f}{a + b x} \right)} \left( \begin{array}{l} 30 \pm b^4 c^2 C e^2 f \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \\ \text{EllipticE}[\pm \text{ArcSinh}[\frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}}], \frac{d (-b e + a f)}{(-b c + a d) f}] - \text{EllipticF}[\pm \text{ArcSinh}[\frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}}], \frac{d (-b e + a f)}{(-b c + a d) f}] \end{array} \right) \right)$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left( d + \frac{b c - a d}{a + b x} \right) \left( f + \frac{b e - a f}{a + b x} \right)} + \left( 5 \pm b^4 B c d e^2 f \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \right)$$

$$\left. \left. \left. \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left( d + \frac{b c - a d}{a + b x} \right) \left( f + \frac{b e - a f}{a + b x} \right)} + \left( 5 \pm b^4 B c d e^2 f \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right) \right) \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 70 \pm a b^3 c C d e^2 f \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 2 \pm A b^4 d^2 e^2 f \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 3 \pm a b^3 B d^2 e^2 f \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 38 \pm a^2 b^2 C d^2 e^2 f \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 5 \pm b^4 B c^2 e f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 70 \pm a b^3 c^2 C e f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 2 \pm A b^4 c d e f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 22 i a b^3 B c d e f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right.$$

$$\left( \text{EllipticE}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 162 i a^2 b^2 c C d e f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 2 i a A b^3 d^2 e f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 13 \pm a^2 b^2 B d^2 e f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 88 \pm a^3 b C d^2 e f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 2 \pm A b^4 c^2 f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 3 \pm a b^3 B c^2 f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} + \left( 38 i a^2 b^2 c^2 C f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \right.$$

$$\left. \left. \text{EllipticE}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} + \left( 2 i a A b^3 c d f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \right.$$

$$\left. \left. \text{EllipticE}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} + \left( 13 i a^2 b^2 B c d f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \right.$$

$$\left. \left. \text{EllipticE}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 88 \pm a^3 b c C d f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 2 \pm a^2 A b^2 d^2 f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 8 \pm a^3 b B d^2 f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 48 \pm a^4 C d^2 f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\begin{aligned}
& \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a+b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a+b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) / \\
& \left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)} - \right. \\
& \frac{15 \pm b^3 c C d e^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a+b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right]}{\sqrt{-\frac{-b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} + \\
& \frac{15 \pm a b^2 C d^2 e^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a+b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right]}{\sqrt{-\frac{-b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} - \\
& \frac{15 \pm b^3 c^2 C e f \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a+b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right]}{\sqrt{-\frac{-b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} - \\
& \frac{10 \pm b^3 B c d e f \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a+b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right]}{\sqrt{-\frac{-b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} + \\
& \frac{80 \pm a b^2 c C d e f \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a+b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right]}{\sqrt{-\frac{-b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} +
\end{aligned}$$

$$\frac{\text{i} A b^3 d^2 e f \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[\text{i} \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} +$$

$$\frac{9 i a b^2 B d^2 e f \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[\text{i} \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} -$$

$$\frac{64 i a^2 b C d^2 e f \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[\text{i} \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} +$$

$$\frac{15 i a b^2 c^2 C f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[\text{i} \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} +$$

$$\frac{\text{i} A b^3 c d f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[\text{i} \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} +$$

$$\frac{9 i a b^2 B c d f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[\text{i} \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} -$$

$$\begin{aligned}
& \frac{64 i a^2 b c C d f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} \\
& \frac{2 i a A b^2 d^2 f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} \\
& \frac{8 i a^2 b B d^2 f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} + \\
& \frac{48 i a^3 C d^2 f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} \Bigg)
\end{aligned}$$

**Problem 66: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{c + d x} \sqrt{e + f x} (A + B x + C x^2)}{(a + b x)^{9/2}} dx$$

Optimal (type 4, 1716 leaves, 10 steps):

$$\begin{aligned}
& - \frac{1}{105 b^3 (b c - a d)^2 (b e - a f)^2 (a + b x)^{3/2}} \\
& \quad 2 (24 a^4 C d^2 f^2 - a^3 b d f (61 C d e + 43 c C f - 4 B d f) - 3 a b^3 (d^2 e (B e - 3 A f) + 2 c^2 f (7 C e - B f) + c d (28 C e^2 - 5 B e f + 5 A f^2)) - \\
& \quad b^4 (4 A d^2 e^2 - c d e (7 B e - A f) - c^2 (35 C e^2 - 14 B e f + 8 A f^2)) - 3 a^2 b^2 (d f (3 B d e + 2 B c f - A d f) - C (15 d^2 e^2 + 37 c d e f + 5 c^2 f^2))) \\
& \quad \sqrt{c + d x} \sqrt{e + f x} + \frac{1}{105 b^3 (b c - a d)^3 (b e - a f)^3 \sqrt{a + b x}} 2 (48 a^5 C d^3 f^3 + 8 a^4 b d^2 f^2 (B d f - 16 C (d e + c f)) - \\
& \quad b^5 (8 A d^3 e^3 - c d^2 e^2 (14 B e + 5 A f) + c^2 d e (35 C e^2 + 14 B e f - 5 A f^2) + c^3 f (35 C e^2 - 14 B e f + 8 A f^2)) - \\
& \quad a b^4 (d^3 e^2 (6 B e - 19 A f) - 6 c^3 f^2 (7 C e - B f) - c^2 d f (238 C e^2 - 19 f (B e - A f)) - c d^2 e (42 C e^2 - f (19 B e + 20 A f))) + \\
& \quad a^3 b^2 d f (C (103 d^2 e^2 + 344 c d e f + 103 c^2 f^2) + d f (6 A d f - 19 B (d e + c f))) - \\
& \quad 3 a^2 b^3 (C (5 d^3 e^3 + 94 c d^2 e^2 f + 94 c^2 d e f^2 + 5 c^3 f^3) + d f (3 A d f (d e + c f) - B (3 d^2 e^2 + 16 c d e f + 3 c^2 f^2))) \sqrt{c + d x} \sqrt{e + f x} + \\
& \left( 2 (6 a^3 C d f + a b^2 (14 c C e + 3 B d e + 3 B c f - 8 A d f) - b^3 (7 B c e - 4 A (d e + c f)) + a^2 b (B d f - 10 C (d e + c f))) \sqrt{c + d x} (e + f x)^{3/2} \right) / \\
& \left( 35 b^2 (b c - a d) (b e - a f)^2 (a + b x)^{5/2} \right) - \\
& \frac{2 (A b^2 - a (b B - a C)) (c + d x)^{3/2} (e + f x)^{3/2}}{7 b (b c - a d) (b e - a f) (a + b x)^{7/2}} + \\
& \frac{1}{105 b^4 (-b c + a d)^{5/2} (b e - a f)^3 \sqrt{c + d x} \sqrt{\frac{b (e + f x)}{b e - a f}}} \\
& 2 \sqrt{d} (48 a^5 C d^3 f^3 + 8 a^4 b d^2 f^2 (B d f - 16 C (d e + c f)) - \\
& b^5 (8 A d^3 e^3 - c d^2 e^2 (14 B e + 5 A f) + c^2 d e (35 C e^2 + 14 B e f - 5 A f^2) + c^3 f (35 C e^2 - 14 B e f + 8 A f^2)) - \\
& a b^4 (d^3 e^2 (6 B e - 19 A f) - 6 c^3 f^2 (7 C e - B f) - c^2 d f (238 C e^2 - 19 f (B e - A f)) - c d^2 e (42 C e^2 - f (19 B e + 20 A f))) + \\
& a^3 b^2 d f (C (103 d^2 e^2 + 344 c d e f + 103 c^2 f^2) + d f (6 A d f - 19 B (d e + c f))) - \\
& 3 a^2 b^3 (C (5 d^3 e^3 + 94 c d^2 e^2 f + 94 c^2 d e f^2 + 5 c^3 f^3) + d f (3 A d f (d e + c f) - B (3 d^2 e^2 + 16 c d e f + 3 c^2 f^2))) \\
& \sqrt{\frac{b (c + d x)}{b c - a d}} \sqrt{e + f x} \text{EllipticE}[\text{ArcSin}\left[\frac{\sqrt{d} \sqrt{a + b x}}{\sqrt{-b c + a d}}\right], \frac{(b c - a d) f}{d (b e - a f)}] + \frac{1}{105 b^4 (-b c + a d)^{5/2} (b e - a f)^2 \sqrt{c + d x} \sqrt{e + f x}} \\
& 2 \sqrt{d} (d e - c f) (24 a^4 C d^2 f^2 - a^3 b d f (43 C d e + 61 c C f - 4 B d f) + b^4 (8 A d^2 e^2 - c d e (14 B e + A f) + c^2 (35 C e^2 + 7 B e f - 4 A f^2)) + 3 a b^3 \\
& (d^2 e (2 B e - 5 A f) - c^2 f (28 C e + B f) - c d (14 C e^2 - 5 B e f - 3 A f^2)) - 3 a^2 b^2 (d f (2 B d e + 3 B c f - A d f) - C (5 d^2 e^2 + 37 c d e f + 15 c^2 f^2))) \\
& \sqrt{\frac{b (c + d x)}{b c - a d}} \sqrt{\frac{b (e + f x)}{b e - a f}} \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{d} \sqrt{a + b x}}{\sqrt{-b c + a d}}\right], \frac{(b c - a d) f}{d (b e - a f)}]
\end{aligned}$$

Result (type 4, 15719 leaves):

$$\begin{aligned}
& \sqrt{a + b x} \sqrt{c + d x} \sqrt{e + f x} \left( -\frac{2 (A b^2 - a b B + a^2 C)}{7 b^3 (a + b x)^4} - \right. \\
& \left. (2 (7 b^3 B c e - 14 a b^2 c C e + A b^3 d e - 8 a b^2 B d e + 15 a^2 b C d e + A b^3 c f - 8 a b^2 B c f + 15 a^2 b c C f - 2 a A b^2 d f + 9 a^2 b B d f - 16 a^3 C d f)) / \right)
\end{aligned}$$

$$\begin{aligned}
& \left( 35 b^3 (b c - a d) (b e - a f) (a + b x)^3 \right) - \frac{1}{105 b^3 (b c - a d)^2 (b e - a f)^2 (a + b x)^2} \\
& 2 \left( 35 b^4 c^2 C e^2 + 7 b^4 B c d e^2 - 84 a b^3 c C d e^2 - 4 A b^4 d^2 e^2 - 3 a b^3 B d^2 e^2 + 45 a^2 b^2 C d^2 e^2 + 7 b^4 B c^2 e f - 84 a b^3 c^2 C e f + 2 A b^4 c d e f - \right. \\
& \quad 30 a b^3 B c d e f + 198 a^2 b^2 c C d e f + 6 a A b^3 d^2 e f + 15 a^2 b^2 B d^2 e f - 106 a^3 b C d^2 e f - 4 A b^4 c^2 f^2 - 3 a b^3 B c^2 f^2 + 45 a^2 b^2 c^2 C f^2 + \\
& \quad 6 a A b^3 c d f^2 + 15 a^2 b^2 B c d f^2 - 106 a^3 b c C d f^2 - 6 a^2 A b^2 d^2 f^2 - 8 a^3 b B d^2 f^2 + 57 a^4 C d^2 f^2 \left. \right) - \frac{1}{105 b^3 (b c - a d)^3 (b e - a f)^3 (a + b x)} \\
& 2 \left( 35 b^5 c^2 C d e^3 - 14 b^5 B c d^2 e^3 - 42 a b^4 c C d^2 e^3 + 8 A b^5 d^3 e^3 + 6 a b^4 B d^3 e^3 + 15 a^2 b^3 C d^3 e^3 + 35 b^5 c^3 C e^2 f + 14 b^5 B c^2 d e^2 f - \right. \\
& \quad 238 a b^4 c^2 C d e^2 f - 5 A b^5 c d^2 e^2 f + 19 a b^4 B c d^2 e^2 f + 282 a^2 b^3 c C d^2 e^2 f - 19 a A b^4 d^3 e^2 f - 9 a^2 b^3 B d^3 e^2 f - 103 a^3 b^2 C d^3 e^2 f - \\
& \quad 14 b^5 B c^3 e f^2 - 42 a b^4 c^3 C e f^2 - 5 A b^5 c^2 d e f^2 + 19 a b^4 B c^2 d e f^2 + 282 a^2 b^3 c^2 C d e f^2 + 20 a A b^4 c d^2 e f^2 - 48 a^2 b^3 B c d^2 e f^2 - \\
& \quad 344 a^3 b^2 c C d^2 e f^2 + 9 a^2 A b^3 d^3 e f^2 + 19 a^3 b^2 B d^3 e f^2 + 128 a^4 b C d^3 e f^2 + 8 A b^5 c^3 f^3 + 6 a b^4 B c^3 f^3 + 15 a^2 b^3 c^3 C f^3 - 19 a A b^4 c^2 d f^3 - \\
& \quad 9 a^2 b^3 B c^2 d f^3 - 103 a^3 b^2 c^2 C d f^3 + 9 a^2 A b^3 c d^2 f^3 + 19 a^3 b^2 B c d^2 f^3 + 128 a^4 b c C d^2 f^3 - 6 a^3 A b^2 d^3 f^3 - 8 a^4 b B d^3 f^3 - 48 a^5 C d^3 f^3 \left. \right) - 
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{105 b^5 (b c - a d)^3 (b e - a f)^3} 2 d f \sqrt{\frac{1}{d f \sqrt{c + \frac{(a+b x)(d - \frac{a d}{a+b x})}{b}} \sqrt{e + \frac{(a+b x)(f - \frac{a f}{a+b x})}{b}}}} \\
& (-35 b^5 c^2 C d e^3 + 14 b^5 B c d^2 e^3 + 42 a b^4 c C d^2 e^3 - 8 A b^5 d^3 e^3 - 6 a b^4 B d^3 e^3 - 15 a^2 b^3 C d^3 e^3 - 35 b^5 c^3 C e^2 f - 14 b^5 B c^2 d e^2 f + \\
& \quad 238 a b^4 c^2 C d e^2 f + 5 A b^5 c d^2 e^2 f - 19 a b^4 B c d^2 e^2 f - 282 a^2 b^3 c C d^2 e^2 f + 19 a A b^4 d^3 e^2 f + 9 a^2 b^3 B d^3 e^2 f + 103 a^3 b^2 C d^3 e^2 f + \\
& \quad 14 b^5 B c^3 e f^2 + 42 a b^4 c^3 C e f^2 + 5 A b^5 c^2 d e f^2 - 19 a b^4 B c^2 d e f^2 - 282 a^2 b^3 c^2 C d e f^2 - 20 a A b^4 c d^2 e f^2 + 48 a^2 b^3 B c d^2 e f^2 + \\
& \quad 344 a^3 b^2 c C d^2 e f^2 - 9 a^2 A b^3 d^3 e f^2 - 19 a^3 b^2 B d^3 e f^2 - 128 a^4 b C d^3 e f^2 - 8 A b^5 c^3 f^3 - 6 a b^4 B c^3 f^3 - 15 a^2 b^3 c^3 C f^3 + 19 a A b^4 c^2 d f^3 + \\
& \quad 9 a^2 b^3 B c^2 d f^3 + 103 a^3 b^2 c^2 C d f^3 - 9 a^2 A b^3 c d^2 f^3 - 19 a^3 b^2 B c d^2 f^3 - 128 a^4 b c C d^2 f^3 + 6 a^3 A b^2 d^3 f^3 + 8 a^4 b B d^3 f^3 + 48 a^5 C d^3 f^3) \\
& (a + b x)^{3/2} \left( d + \frac{b c}{a + b x} - \frac{a d}{a + b x} \right) \left( f + \frac{b e}{a + b x} - \frac{a f}{a + b x} \right) + \frac{1}{d f \sqrt{c + \frac{(a+b x)(d - \frac{a d}{a+b x})}{b}} \sqrt{e + \frac{(a+b x)(f - \frac{a f}{a+b x})}{b}}} \\
& (-b c + a d) (-b e + a f) (a + b x) \sqrt{\left( d + \frac{b c}{a + b x} - \frac{a d}{a + b x} \right) \left( f + \frac{b e}{a + b x} - \frac{a f}{a + b x} \right)} \left( \begin{array}{l} 35 \pm b^5 c^2 C d e^3 f \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \\ \text{EllipticE}[\pm \text{ArcSinh}\left[ \frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f}] - \text{EllipticF}[\pm \text{ArcSinh}\left[ \frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f}] \end{array} \right) \Bigg)
\end{aligned}$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 14 \pm b^5 B c d^2 e^3 f \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 42 \pm a b^4 c C d^2 e^3 f \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 8 \pm A b^5 d^3 e^3 f \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 6 \pm a b^4 B d^3 e^3 f \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 15 i a^2 b^3 C d^3 e^3 f \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 35 i b^5 c^3 C e^2 f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 14 i b^5 B c^2 d e^2 f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 238 i a b^4 c^2 C d e^2 f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 5 i A b^5 c d^2 e^2 f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 19 i a b^4 B c d^2 e^2 f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 282 i a^2 b^3 c C d^2 e^2 f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 19 \text{i} a A b^4 d^3 e^2 f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 9 \text{i} a^2 b^3 B d^3 e^2 f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 103 \text{i} a^3 b^2 C d^3 e^2 f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 14 \pm b^5 B c^3 e f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 42 \pm a b^4 c^3 C e f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 5 \pm A b^5 c^2 d e f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 19 \pm a b^4 B c^2 d e f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 282 i a^2 b^3 c^2 C d e f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 20 i a A b^4 c d^2 e f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 48 i a^2 b^3 B c d^2 e f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 344 i a^3 b^2 c C d^2 e f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 9 i a^2 A b^3 d^3 e f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 19 i a^3 b^2 B d^3 e f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 128 i a^4 b C d^3 e f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} + \left( 8 \pm A b^5 c^3 f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \right.$$

$$\left. \left. \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} + \left( 6 \pm a b^4 B c^3 f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \right.$$

$$\left. \left. \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} + \left( 15 \pm a^2 b^3 c^3 C f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \right.$$

$$\left. \left. \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 19 \text{I} a A b^4 c^2 d f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{I} \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \text{I} \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 9 \text{I} a^2 b^3 B c^2 d f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{I} \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \text{I} \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 103 \text{I} a^3 b^2 c^2 C d f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{I} \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \text{I} \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 9 \text{I} a^2 A b^3 c d^2 f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 19 \pm a^3 b^2 B c d^2 f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 128 \pm a^4 b c C d^2 f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 6 \pm a^3 A b^2 d^3 f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\begin{aligned}
& \left( \sqrt{-\frac{-bc+ad}{d}} (-be+af) \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right)} \right) - \left( 8 \pm a^4 b B d^3 f^4 \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \right) \\
& \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right] \right) \\
& \left( \sqrt{-\frac{-bc+ad}{d}} (-be+af) \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right)} \right) - \left( 48 \pm a^5 C d^3 f^4 \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \right. \\
& \left. \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right] \right) \right) \\
& \left( \sqrt{-\frac{-bc+ad}{d}} (-be+af) \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right)} \right) - \\
& \frac{70 \pm b^4 c^2 C d e^2 f \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right]}{+} \\
& \quad \sqrt{-\frac{-bc+ad}{d}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right)} \\
& \frac{7 \pm b^4 B c d^2 e^2 f \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right]}{+} \\
& \quad \sqrt{-\frac{-bc+ad}{d}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right)}
\end{aligned}$$

$$\frac{126 \pm a b^3 c C d^2 e^2 f \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{-b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}}$$

$$\frac{4 \pm A b^4 d^3 e^2 f \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{-b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}}$$

$$\frac{3 \pm a b^3 B d^3 e^2 f \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{-b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}}$$

$$\frac{60 \pm a^2 b^2 C d^3 e^2 f \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{-b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} +$$

$$\frac{7 \pm b^4 B c^2 d e f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{-b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} +$$

$$\frac{126 \pm a b^3 c^2 C d e f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{-b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} +$$

$$\frac{2 i A b^4 c d^2 e f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}}$$

$$\frac{30 i a b^3 B c d^2 e f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}}$$

$$\frac{222 i a^2 b^2 c C d^2 e f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} +$$

$$\frac{6 i a A b^3 d^3 e f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} +$$

$$\frac{15 i a^2 b^2 B d^3 e f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} +$$

$$\frac{104 i a^3 b C d^3 e f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}}$$

$$\frac{4 i A b^4 c^2 d f^3 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}}$$

$$\frac{3 i a b^3 B c^2 d f^3 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}}$$

$$\frac{60 i a^2 b^2 c^2 C d f^3 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} +$$

$$\frac{6 i a A b^3 c d^2 f^3 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} +$$

$$\frac{15 i a^2 b^2 B c d^2 f^3 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} +$$

$$\frac{104 i a^3 b c C d^2 f^3 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}}$$

$$\begin{aligned}
& \frac{6 i a^2 A b^2 d^3 f^3 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{-b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} \\
& \frac{8 i a^3 b B d^3 f^3 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{-b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} \\
& \left. \frac{48 i a^4 C d^3 f^3 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{-b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} \right)
\end{aligned}$$

**Problem 67: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + b x)^{3/2} \sqrt{c + d x} (A + B x + C x^2)}{\sqrt{e + f x}} dx$$

Optimal (type 4, 1235 leaves, 10 steps):

$$\begin{aligned}
& - \frac{1}{945 b^2 d^3 f^4} 2 \left( 5 b d f (7 a d f (5 b c C e + 3 a C d e + a c C f - 9 A b d f) - (3 b c e + 3 a d e + a c f) (4 a C d f + b (8 C d e + 6 c C f - 9 B d f))) + \right. \\
& \quad \left. 2 \left( \frac{a d f}{2} - b (2 d e + c f) \right) (7 b d f (5 b c C e + 3 a C d e + a c C f - 9 A b d f) - (6 b d e + 4 b c f - 3 a d f) (4 a C d f + b (8 C d e + 6 c C f - 9 B d f))) \right) \\
& \quad \sqrt{a + b x} \sqrt{c + d x} \sqrt{e + f x} - \frac{1}{315 b d^3 f^3} 2 (7 b d f (5 b c C e + 3 a C d e + a c C f - 9 A b d f) - \\
& \quad (6 b d e + 4 b c f - 3 a d f) (4 a C d f + b (8 C d e + 6 c C f - 9 B d f))) \sqrt{a + b x} (c + d x)^{3/2} \sqrt{e + f x} - \\
& \quad \frac{2 (4 a C d f + b (8 C d e + 6 c C f - 9 B d f)) (a + b x)^{3/2} (c + d x)^{3/2} \sqrt{e + f x}}{63 b d^2 f^2} + \\
& \quad \frac{2 C (a + b x)^{5/2} (c + d x)^{3/2} \sqrt{e + f x}}{9 b d f} + \\
& \quad \frac{1}{315 b^3 d^{7/2} f^5 \sqrt{c + d x} \sqrt{\frac{b (e + f x)}{b e - a f}}} \\
& \quad 2 \sqrt{-b c + a d} (8 a^4 C d^4 f^4 + a^3 b d^3 f^3 (11 C d e - 7 c C f - 18 B d f) - 3 a^2 b^2 d^2 f^2 (3 d f (4 B d e - 3 B c f - 7 A d f) - C (9 d^2 e^2 - 5 c d e f - 3 c^2 f^2)) - \\
& \quad a b^3 d f (2 C (92 d^3 e^3 - 33 c d^2 e^2 f - 18 c^2 d e f^2 - 16 c^3 f^3) + 3 d f (7 A d f (13 d e - 7 c f) - B (72 d^2 e^2 - 29 c d e f - 19 c^2 f^2))) + \\
& \quad b^4 (C (128 d^4 e^4 - 40 c d^3 e^3 f - 21 c^2 d^2 e^2 f^2 - 16 c^3 d e f^3 - 16 c^4 f^4) + \\
& \quad 3 d f (7 A d f (8 d^2 e^2 - 3 c d e f - 2 c^2 f^2) - B (48 d^3 e^3 - 16 c d^2 e^2 f - 9 c^2 d e f^2 - 8 c^3 f^3))) \sqrt{\frac{b (c + d x)}{b c - a d}} \sqrt{e + f x} \\
& \quad \text{EllipticE} [\text{ArcSin} \left[ \frac{\sqrt{d} \sqrt{a + b x}}{\sqrt{-b c + a d}} \right], \frac{(b c - a d) f}{d (b e - a f)}] + \frac{1}{315 b^3 d^{7/2} f^5 \sqrt{c + d x} \sqrt{e + f x}} 2 \sqrt{-b c + a d} (b e - a f) (d e - c f) \\
& \quad (4 a^3 C d^3 f^3 + 3 a^2 b d^2 f^2 (3 C d e - c C f - 3 B d f) - 3 a b^2 d f (3 d f (16 B d e + 3 B c f - 21 A d f) - 5 C (8 d^2 e^2 + 2 c d e f + c^2 f^2)) - \\
& \quad b^3 (C (128 d^3 e^3 + 24 c d^2 e^2 f + 15 c^2 d e f^2 + 8 c^3 f^3) + 3 d f (7 A d f (8 d e + c f) - 4 B (12 d^2 e^2 + 2 c d e f + c^2 f^2))) \\
& \quad \sqrt{\frac{b (c + d x)}{b c - a d}} \sqrt{\frac{b (e + f x)}{b e - a f}} \text{EllipticF} [\text{ArcSin} \left[ \frac{\sqrt{d} \sqrt{a + b x}}{\sqrt{-b c + a d}} \right], \frac{(b c - a d) f}{d (b e - a f)}]
\end{aligned}$$

Result (type 4, 12483 leaves):

$$\begin{aligned}
& \sqrt{a + b x} \sqrt{c + d x} \sqrt{e + f x} \\
& \left( - \frac{1}{315 b^2 d^3 f^4} 2 (64 b^3 C d^3 e^3 - 12 b^3 c C d^2 e^2 f - 72 b^3 B d^3 e^2 f - 84 a b^2 C d^3 e^2 f - 9 b^3 c^2 C d e f^2 + 15 b^3 B c d^2 e f^2 + 19 a b^2 c C d^2 e f^2 + \right. \\
& \quad 84 A b^3 d^3 e f^2 + 99 a b^2 B d^3 e f^2 + 6 a^2 b C d^3 e f^2 - 8 b^3 c^3 C f^3 + 12 b^3 B c^2 d f^3 + 15 a b^2 c^2 C d f^3 - \\
& \quad 21 A b^3 c d^2 f^3 - 27 a b^2 B c d^2 f^3 - 3 a^2 b c C d^2 f^3 - 126 a A b^2 d^3 f^3 - 9 a^2 b B d^3 f^3 + 4 a^3 C d^3 f^3) + \frac{1}{315 b d^2 f^3} \\
& \quad \left. 2 (48 b^2 C d^2 e^2 - 7 b^2 c C d e f - 54 b^2 B d^2 e f - 61 a b C d^2 e f - 6 b^2 c^2 C f^2 + 9 b^2 B c d f^2 + 11 a b c C d f^2 + 63 A b^2 d^2 f^2 + 72 a b B d^2 f^2 + 3 a^2 C d^2 f^2) x + \right)
\end{aligned}$$

$$\frac{2 \left( -8 b C d e + b c C f + 9 b B d f + 10 a C d f \right) x^2}{63 d f^2} + \frac{2 b C x^3}{9 f} +$$

$$\frac{1}{315 b^4 d^3 f^4} 2 \left( \frac{1}{d f \sqrt{c + \frac{(a+b x) \left(d - \frac{a d}{a+b x}\right)}{b}}} \sqrt{e + \frac{(a+b x) \left(f - \frac{a f}{a+b x}\right)}{b}} \right)$$

$$(128 b^4 C d^4 e^4 - 40 b^4 c C d^3 e^3 f - 144 b^4 B d^4 e^3 f - 184 a b^3 C d^4 e^3 f - 21 b^4 c^2 C d^2 e^2 f^2 + 48 b^4 B c d^3 e^2 f^2 + 66 a b^3 c C d^3 e^2 f^2 + 168 A b^4 d^4 e^2 f^2 + 216 a b^3 B d^4 e^2 f^2 + 27 a^2 b^2 C d^4 e^2 f^2 - 16 b^4 c^3 C d e f^3 + 27 b^4 B c^2 d^2 e f^3 + 36 a b^3 c^2 C d^2 e f^3 - 63 A b^4 c d^3 e f^3 - 87 a b^3 B c d^3 e f^3 - 15 a^2 b^2 c C d^3 e f^3 - 273 a A b^3 d^4 e f^3 - 36 a^2 b^2 B d^4 e f^3 + 11 a^3 b C d^4 e f^3 - 16 b^4 c^4 C f^4 + 24 b^4 B c^3 d f^4 + 32 a b^3 c^3 C d f^4 - 42 A b^4 c^2 d^2 f^4 - 57 a b^3 B c^2 d^2 f^4 - 9 a^2 b^2 c^2 C d^2 f^4 + 147 a A b^3 c d^3 f^4 + 27 a^2 b^2 B c d^3 f^4 - 7 a^3 b c C d^3 f^4 + 63 a^2 A b^2 d^4 f^4 - 18 a^3 b B d^4 f^4 + 8 a^4 C d^4 f^4)$$

$$(a+b x)^{3/2} \left( d + \frac{b c}{a+b x} - \frac{a d}{a+b x} \right) \left( f + \frac{b e}{a+b x} - \frac{a f}{a+b x} \right) - \frac{1}{d f \sqrt{c + \frac{(a+b x) \left(d - \frac{a d}{a+b x}\right)}{b}}} \sqrt{e + \frac{(a+b x) \left(f - \frac{a f}{a+b x}\right)}{b}}$$

$$(-b c + a d) (-b e + a f) (a + b x) \sqrt{\left(d + \frac{b c}{a+b x} - \frac{a d}{a+b x}\right) \left(f + \frac{b e}{a+b x} - \frac{a f}{a+b x}\right)} \left( \begin{array}{l} 128 \pm b^4 C d^4 e^4 f \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \\ \text{EllipticE}[\pm \text{ArcSinh}[\frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a+b x}}], \frac{d (-b e + a f)}{(-b c + a d) f}] - \text{EllipticF}[\pm \text{ArcSinh}[\frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a+b x}}], \frac{d (-b e + a f)}{(-b c + a d) f}] \end{array} \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)} - \left( 40 \pm b^4 c C d^3 e^3 f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \right. \right.$$

$$\left. \left. \text{EllipticE}[\pm \text{ArcSinh}[\frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a+b x}}], \frac{d (-b e + a f)}{(-b c + a d) f}] - \text{EllipticF}[\pm \text{ArcSinh}[\frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a+b x}}], \frac{d (-b e + a f)}{(-b c + a d) f}] \right) \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 144 i b^4 B d^4 e^3 f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 184 i a b^3 C d^4 e^3 f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 21 i b^4 c^2 C d^2 e^2 f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 48 i b^4 B c d^3 e^2 f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 66 i a b^3 c C d^3 e^2 f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 168 i A b^4 d^4 e^2 f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 216 i a b^3 B d^4 e^2 f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\begin{aligned}
& \left( \sqrt{-\frac{-bc+ad}{d}} (-be+af) \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-ad}{a+b x}\right)} \right) + \left( 27 \pm a^2 b^2 C d^4 e^2 f^3 \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \right. \\
& \left. \left. \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right] \right) \right) / \\
& \left( \sqrt{-\frac{-bc+ad}{d}} (-be+af) \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-ad}{a+b x}\right)} \right) - \left( 16 \pm b^4 c^3 C d e f^4 \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \right. \\
& \left. \left. \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right] \right) \right) / \\
& \left( \sqrt{-\frac{-bc+ad}{d}} (-be+af) \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-ad}{a+b x}\right)} \right) + \left( 27 \pm b^4 B c^2 d^2 e f^4 \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \right. \\
& \left. \left. \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right] \right) \right) / \\
& \left( \sqrt{-\frac{-bc+ad}{d}} (-be+af) \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-ad}{a+b x}\right)} \right) + \left( 36 \pm a b^3 c^2 C d^2 e f^4 \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \right.
\end{aligned}$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 63 \text{i} a b^4 c d^3 e f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 87 \text{i} a b^3 B c d^3 e f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 15 \text{i} a^2 b^2 c C d^3 e f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 273 i a A b^3 d^4 e f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 36 i a^2 b^2 B d^4 e f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 11 i a^3 b C d^4 e f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 16 i b^4 c^4 C f^5 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} + \left( 24 i b^4 B c^3 d f^5 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \right.$$

$$\left. \text{EllipticE}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} + \left( 32 i a b^3 c^3 C d f^5 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \right.$$

$$\left. \text{EllipticE}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} - \left( 42 i A b^4 c^2 d^2 f^5 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \right.$$

$$\left. \text{EllipticE}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 57 \pm a b^3 B c^2 d^2 f^5 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 9 \pm a^2 b^2 c^2 C d^2 f^5 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 147 \pm a A b^3 c d^3 f^5 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 27 \pm a^2 b^2 B c d^3 f^5 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 7 \pm a^3 b c C d^3 f^5 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 63 \pm a^2 A b^2 d^4 f^5 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 18 \pm a^3 b B d^4 f^5 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\begin{aligned}
& \left( \sqrt{-\frac{-bc+ad}{d}} (-be+af) \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right)} \right) + \left( 8 \pm a^4 C d^4 f^5 \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \right. \\
& \left. \left. \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right] \right) \right) / \\
& \left( \sqrt{-\frac{-bc+ad}{d}} (-be+af) \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right)} \right) - \\
& \frac{64 \pm b^3 C d^4 e^3 f \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right]}{\sqrt{-\frac{-bc+ad}{d}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right)}} + \\
& \frac{12 \pm b^3 c C d^3 e^2 f^2 \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right]}{\sqrt{-\frac{-bc+ad}{d}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right)}} + \\
& \frac{72 \pm b^3 B d^4 e^2 f^2 \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right]}{\sqrt{-\frac{-bc+ad}{d}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right)}} + \\
& \frac{36 \pm a b^2 C d^4 e^2 f^2 \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right]}{\sqrt{-\frac{-bc+ad}{d}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right)}}
\end{aligned}$$

$$\frac{9 i b^3 c^2 C d^2 e f^3 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}}$$

$$\frac{15 i b^3 B c d^3 e f^3 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}}$$

$$\frac{12 i a b^2 c C d^3 e f^3 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}}$$

$$\frac{84 i A b^3 d^4 e f^3 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}}$$

$$\begin{aligned} & 45 i a b^2 B d^4 e f^3 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right] \\ & + \sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)} \end{aligned}$$

$$\begin{aligned} & 15 i a^2 b C d^4 e f^3 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right] \\ & + \sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)} \end{aligned}$$

$$\frac{8 i b^3 c^3 C d f^4 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}}$$

$$\frac{12 i b^3 B c^2 d^2 f^4 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}}$$

$$\frac{9 i a b^2 c^2 C d^2 f^4 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} +$$

$$\frac{21 i A b^3 c d^3 f^4 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} +$$

$$\frac{18 i a b^2 B c d^3 f^4 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}}$$

$$\frac{3 i a^2 b c C d^3 f^4 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} +$$

$$\begin{aligned}
& \frac{63 i a A b^2 d^4 f^4 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} \\
& + \frac{18 i a^2 b B d^4 f^4 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} \\
& + \left. \frac{8 i a^3 C d^4 f^4 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} \right)
\end{aligned}$$

Problem 68: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{a+b x} \sqrt{c+d x} (A+B x+C x^2)}{\sqrt{e+f x}} dx$$

Optimal (type 4, 766 leaves, 9 steps):

$$\begin{aligned}
& - \frac{1}{105 b^2 d^2 f^3} \\
& 2 (5 b d f (3 b c C e + 3 a C d e + a c C f - 7 A b d f) + (a d f - 2 b (2 d e + c f)) (4 a C d f + b (6 C d e + 4 c C f - 7 B d f))) \sqrt{a+b x} \sqrt{c+d x} \sqrt{e+f x} - \\
& \frac{2 (4 a C d f + b (6 C d e + 4 c C f - 7 B d f)) \sqrt{a+b x} (c+d x)^{3/2} \sqrt{e+f x}}{35 b d^2 f^2} + \frac{2 C (a+b x)^{3/2} (c+d x)^{3/2} \sqrt{e+f x}}{7 b d f} - \frac{1}{105 b^3 d^{5/2} f^4 \sqrt{c+d x} \sqrt{\frac{b(e+f x)}{b e-a f}}} \\
& 2 \sqrt{-b c + a d} \left( 3 b d f (5 a d f (3 b c C e + 3 a C d e + a c C f - 7 A b d f) - (b c e + 3 a d e + a c f) (4 a C d f + b (6 C d e + 4 c C f - 7 B d f))) + \right. \\
& \left. 2 \left( \frac{b c f}{2} - d (b e + a f) \right) (5 b d f (3 b c C e + 3 a C d e + a c C f - 7 A b d f) + (a d f - 2 b (2 d e + c f)) (4 a C d f + b (6 C d e + 4 c C f - 7 B d f))) \right) \\
& \sqrt{\frac{b (c+d x)}{b c - a d}} \sqrt{e+f x} \text{EllipticE} \left[ \text{ArcSin} \left[ \frac{\sqrt{d} \sqrt{a+b x}}{\sqrt{-b c + a d}} \right], \frac{(b c - a d) f}{d (b e - a f)} \right] + \left( 2 \sqrt{-b c + a d} (b e - a f) (d e - c f) \right. \\
& (4 a^2 C d^2 f^2 + a b d f (8 C d e - 2 c C f - 7 B d f) - b^2 (7 d f (8 B d e + B c f - 10 A d f) - 4 C (12 d^2 e^2 + 2 c d e f + c^2 f^2))) \\
& \left. \sqrt{\frac{b (c+d x)}{b c - a d}} \sqrt{\frac{b (e+f x)}{b e - a f}} \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\sqrt{d} \sqrt{a+b x}}{\sqrt{-b c + a d}} \right], \frac{(b c - a d) f}{d (b e - a f)} \right] \right) / (105 b^3 d^{5/2} f^4 \sqrt{c+d x} \sqrt{e+f x})
\end{aligned}$$

Result (type 4, 7297 leaves):

$$\begin{aligned}
& \sqrt{a+b x} \sqrt{c+d x} \sqrt{e+f x} \left( \frac{1}{105 b^2 d^2 f^3} \right. \\
& 2 (24 b^2 C d^2 e^2 - 5 b^2 c C d e f - 28 b^2 B d^2 e f - 5 a b C d^2 e f - 4 b^2 c^2 C f^2 + 7 b^2 B c d f^2 + 2 a b c C d f^2 + 35 A b^2 d^2 f^2 + 7 a b B d^2 f^2 - 4 a^2 C d^2 f^2) + \\
& \left. \frac{2 (-6 b C d e + b c C f + 7 b B d f + a C d f) x}{35 b d f^2} + \frac{2 C x^2}{7 f} \right) + \\
& \frac{1}{105 b^4 d^2 f^3} 2 \left( \frac{1}{d f \sqrt{c + \frac{(a+b x) (d - \frac{a d}{a+b x})}{b}} \sqrt{e + \frac{(a+b x) (f - \frac{a f}{a+b x})}{b}}} \right. \\
& (-48 b^3 C d^3 e^3 + 16 b^3 c C d^2 e^2 f + 56 b^3 B d^3 e^2 f + 16 a b^2 C d^3 e^2 f + \\
& 9 b^3 c^2 C d e f^2 - 21 b^3 B c d^2 e f^2 - 8 a b^2 c C d^2 e f^2 - 70 A b^3 d^3 e f^2 - 21 a b^2 B d^3 e f^2 + 9 a^2 b C d^3 e f^2 + 8 b^3 c^3 C f^3 - \\
& 14 b^3 B c^2 d f^3 - 5 a b^2 c^2 C d f^3 + 35 A b^3 c d^2 f^3 + 14 a b^2 B c d^2 f^3 - 5 a^2 b c C d^2 f^3 + 35 a A b^2 d^3 f^3 - 14 a^2 b B d^3 f^3 + 8 a^3 C d^3 f^3) \\
& (a+b x)^{3/2} \left( d + \frac{b c}{a+b x} - \frac{a d}{a+b x} \right) \left( f + \frac{b e}{a+b x} - \frac{a f}{a+b x} \right) + \frac{1}{d f \sqrt{c + \frac{(a+b x) (d - \frac{a d}{a+b x})}{b}} \sqrt{e + \frac{(a+b x) (f - \frac{a f}{a+b x})}{b}}}
\end{aligned}$$

$$\left( -b c + a d \right) \left( -b e + a f \right) \left( a + b x \right) \sqrt{\left( d + \frac{b c}{a + b x} - \frac{a d}{a + b x} \right) \left( f + \frac{b e}{a + b x} - \frac{a f}{a + b x} \right)} \left( \begin{array}{l} 48 \pm b^3 C d^3 e^3 f \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \\ \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \end{array} \right) \right)$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left( d + \frac{b c - a d}{a + b x} \right) \left( f + \frac{b e - a f}{a + b x} \right)} \right) - \left( 16 \pm b^3 c C d^2 e^2 f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right.$$

$$\left. \left( \begin{array}{l} \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \end{array} \right) \right)$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left( d + \frac{b c - a d}{a + b x} \right) \left( f + \frac{b e - a f}{a + b x} \right)} \right) - \left( 56 \pm b^3 B d^3 e^2 f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \begin{array}{l} \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \end{array} \right) \right)$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left( d + \frac{b c - a d}{a + b x} \right) \left( f + \frac{b e - a f}{a + b x} \right)} \right) - \left( 16 \pm a b^2 C d^3 e^2 f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left( d + \frac{b c - a d}{a + b x} \right) \left( f + \frac{b e - a f}{a + b x} \right)} - \left( 9 \pm b^3 c^2 C d e f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \right.$$

$$\left. \left. \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left( d + \frac{b c - a d}{a + b x} \right) \left( f + \frac{b e - a f}{a + b x} \right)} + \left( 21 \pm b^3 B c d^2 e f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \right.$$

$$\left. \left. \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left( d + \frac{b c - a d}{a + b x} \right) \left( f + \frac{b e - a f}{a + b x} \right)} + \left( 8 \pm a b^2 c C d^2 e f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \right.$$

$$\left. \left. \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 70 \text{i} A b^3 d^3 e f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i} \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \text{i} \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 21 \text{i} a b^2 B d^3 e f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i} \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \text{i} \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 9 \text{i} a^2 b C d^3 e f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i} \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \text{i} \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 8 \text{i} b^3 c^3 C f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 14 \pm b^3 B c^2 d f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 5 \pm a b^2 c^2 C d f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 35 \pm A b^3 c d^2 f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 14 \pm a b^2 B c d^2 f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 5 \pm a^2 b c C d^2 f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 35 \pm a A b^2 d^3 f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 14 \pm a^2 b B d^3 f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\begin{aligned}
& \left( \frac{\operatorname{EllipticE}[\operatorname{ArcSinh}\left(\frac{\sqrt{-\frac{-b c+a d}{d}}}{\sqrt{a+b x}}\right), \frac{d (-b e+a f)}{(-b c+a d) f}] - \operatorname{EllipticF}[\operatorname{ArcSinh}\left(\frac{\sqrt{-\frac{-b c+a d}{d}}}{\sqrt{a+b x}}\right), \frac{d (-b e+a f)}{(-b c+a d) f}]}{\sqrt{-\frac{-b c+a d}{d}} (-b e+a f) \sqrt{\left(d + \frac{b c-a d}{a+b x}\right) \left(f + \frac{b e-a f}{a+b x}\right)}} \right) / \\
& \left( \sqrt{-\frac{-b c+a d}{d}} (-b e+a f) \sqrt{\left(d + \frac{b c-a d}{a+b x}\right) \left(f + \frac{b e-a f}{a+b x}\right)} - \left(8 i a^3 C d^3 f^4 \sqrt{1 - \frac{-b c+a d}{d (a+b x)}} \sqrt{1 - \frac{-b e+a f}{f (a+b x)}} \right. \right. \\
& \left. \left. \operatorname{EllipticE}[\operatorname{ArcSinh}\left(\frac{\sqrt{-\frac{-b c+a d}{d}}}{\sqrt{a+b x}}\right), \frac{d (-b e+a f)}{(-b c+a d) f}] - \operatorname{EllipticF}[\operatorname{ArcSinh}\left(\frac{\sqrt{-\frac{-b c+a d}{d}}}{\sqrt{a+b x}}\right), \frac{d (-b e+a f)}{(-b c+a d) f}]} \right) / \\
& \left( \sqrt{-\frac{-b c+a d}{d}} (-b e+a f) \sqrt{\left(d + \frac{b c-a d}{a+b x}\right) \left(f + \frac{b e-a f}{a+b x}\right)} - \right. \\
& \left. \frac{24 i b^2 C d^3 e^2 f \sqrt{1 - \frac{-b c+a d}{d (a+b x)}} \sqrt{1 - \frac{-b e+a f}{f (a+b x)}} \operatorname{EllipticF}[\operatorname{ArcSinh}\left(\frac{\sqrt{-\frac{-b c+a d}{d}}}{\sqrt{a+b x}}\right), \frac{d (-b e+a f)}{(-b c+a d) f}]}{\sqrt{-\frac{-b c+a d}{d}} \sqrt{\left(d + \frac{b c-a d}{a+b x}\right) \left(f + \frac{b e-a f}{a+b x}\right)}} + \right. \\
& \left. \frac{5 i b^2 c C d^2 e f^2 \sqrt{1 - \frac{-b c+a d}{d (a+b x)}} \sqrt{1 - \frac{-b e+a f}{f (a+b x)}} \operatorname{EllipticF}[\operatorname{ArcSinh}\left(\frac{\sqrt{-\frac{-b c+a d}{d}}}{\sqrt{a+b x}}\right), \frac{d (-b e+a f)}{(-b c+a d) f}]}{\sqrt{-\frac{-b c+a d}{d}} \sqrt{\left(d + \frac{b c-a d}{a+b x}\right) \left(f + \frac{b e-a f}{a+b x}\right)}} + \right. \\
& \left. \frac{28 i b^2 B d^3 e f^2 \sqrt{1 - \frac{-b c+a d}{d (a+b x)}} \sqrt{1 - \frac{-b e+a f}{f (a+b x)}} \operatorname{EllipticF}[\operatorname{ArcSinh}\left(\frac{\sqrt{-\frac{-b c+a d}{d}}}{\sqrt{a+b x}}\right), \frac{d (-b e+a f)}{(-b c+a d) f}]}{\sqrt{-\frac{-b c+a d}{d}} \sqrt{\left(d + \frac{b c-a d}{a+b x}\right) \left(f + \frac{b e-a f}{a+b x}\right)}} - \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{13 i a b C d^3 e f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} + \\
& \frac{4 i b^2 c^2 C d f^3 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} - \\
& \frac{7 i b^2 B c d^2 f^3 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} + \\
& \frac{i a b c C d^2 f^3 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} - \\
& \frac{35 i A b^2 d^3 f^3 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} + \\
& \frac{14 i a b B d^3 f^3 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}}
\end{aligned}$$

$$\frac{8 i a^2 C d^3 f^3 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{-b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} \Bigg)$$

Problem 69: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{c + d x} (A + B x + C x^2)}{\sqrt{a + b x} \sqrt{e + f x}} dx$$

Optimal (type 4, 527 leaves, 8 steps):

$$\begin{aligned} & \frac{2 (4 a C d f + b (4 C d e + 2 c C f - 5 B d f)) \sqrt{a+b x} \sqrt{c+d x} \sqrt{e+f x}}{15 b^2 d f^2} + \frac{2 C \sqrt{a+b x} (c+d x)^{3/2} \sqrt{e+f x}}{5 b d f} - \\ & \left( 2 \sqrt{-b c + a d} (3 b d f (b c C e + 3 a C d e + a c C f - 5 A b d f) - (2 b d e - b c f + 2 a d f) (4 a C d f + b (4 C d e + 2 c C f - 5 B d f))) \right. \\ & \left. \sqrt{\frac{b (c + d x)}{b c - a d}} \sqrt{e + f x} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{d} \sqrt{a+b x}}{\sqrt{-b c + a d}}\right], \frac{(b c - a d) f}{d (b e - a f)}\right] \right) / \left( 15 b^3 d^{3/2} f^3 \sqrt{c+d x} \sqrt{\frac{b (e + f x)}{b e - a f}} \right) - \\ & \left( 2 \sqrt{-b c + a d} (d e - c f) (4 a^2 C d f^2 + a b f (3 C d e - c C f - 5 B d f) - b^2 (5 d f (2 B e - 3 A f) - C e (8 d e + c f))) \right. \\ & \left. \sqrt{\frac{b (c + d x)}{b c - a d}} \sqrt{\frac{b (e + f x)}{b e - a f}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{d} \sqrt{a+b x}}{\sqrt{-b c + a d}}\right], \frac{(b c - a d) f}{d (b e - a f)}\right] \right) / \left( 15 b^3 d^{3/2} f^3 \sqrt{c+d x} \sqrt{e+f x} \right) \end{aligned}$$

Result (type 4, 5393 leaves):

$$\sqrt{a+b x} \sqrt{c+d x} \left( \frac{2 (-4 b C d e + b c C f + 5 b B d f - 4 a C d f)}{15 b^2 d f^2} + \frac{2 C x}{5 b f} \right) \sqrt{e+f x} + \frac{1}{15 b^4 d f^2}$$

$$\begin{aligned}
& 2 \left( \left( 8 b^2 C d^2 e^2 - 3 b^2 c C d e f - 10 b^2 B d^2 e f + 7 a b C d^2 e f - 2 b^2 c^2 C f^2 + 5 b^2 B c d f^2 - 3 a b c C d f^2 + 15 A b^2 d^2 f^2 - 10 a b B d^2 f^2 + 8 a^2 C d^2 f^2 \right) \right. \\
& \quad \left. \left( a + b x \right)^{3/2} \left( d + \frac{b c}{a + b x} - \frac{a d}{a + b x} \right) \left( f + \frac{b e}{a + b x} - \frac{a f}{a + b x} \right) \right) / \\
& \quad \left( d f \sqrt{c + \frac{(a + b x) (d - \frac{a d}{a + b x})}{b}} \sqrt{e + \frac{(a + b x) (f - \frac{a f}{a + b x})}{b}} \right) + \frac{1}{d f \sqrt{c + \frac{(a + b x) (d - \frac{a d}{a + b x})}{b}} \sqrt{e + \frac{(a + b x) (f - \frac{a f}{a + b x})}{b}}} \\
& (-b c + a d) (a + b x) \sqrt{\left( d + \frac{b c}{a + b x} - \frac{a d}{a + b x} \right) \left( f + \frac{b e}{a + b x} - \frac{a f}{a + b x} \right)} \left( \begin{array}{l} 8 \pm b^3 C d^2 e^3 f \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \\ \text{EllipticE}[\pm \text{ArcSinh}[\frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}}, \frac{d (-b e + a f)}{(-b c + a d) f}] - \text{EllipticF}[\pm \text{ArcSinh}[\frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}}, \frac{d (-b e + a f)}{(-b c + a d) f}]] \end{array} \right) / \\
& \left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left( d + \frac{b c - a d}{a + b x} \right) \left( f + \frac{b e - a f}{a + b x} \right)} - \left( 3 \pm b^3 c C d e^2 f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \right. \\
& \quad \left. \left. \text{EllipticE}[\pm \text{ArcSinh}[\frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}}, \frac{d (-b e + a f)}{(-b c + a d) f}] - \text{EllipticF}[\pm \text{ArcSinh}[\frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}}, \frac{d (-b e + a f)}{(-b c + a d) f}]] \right) \right) /
\end{aligned}$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 10 \pm b^3 B d^2 e^2 f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( \pm a b^2 C d^2 e^2 f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 2 \pm b^3 c^2 C e f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 5 \pm b^3 B c d e f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}}, \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}}, \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} + \left( 15 i a b^3 d^2 e f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \right.$$

$$\left. \left. \text{EllipticE}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}}, \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}}, \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} + \left( i a^2 b C d^2 e f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \right.$$

$$\left. \left. \text{EllipticE}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}}, \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}}, \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} + \left( 2 i a b^2 c^2 C f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \right.$$

$$\left. \left. \text{EllipticE}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}}, \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}}, \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) \right) /$$

$$\left( \sqrt{-\frac{b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 5 \pm a b^2 B c d f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 3 \pm a^2 b c C d f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 15 \pm a A b^2 d^2 f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 10 \pm a^2 b B d^2 f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 8 i a^3 C d^2 f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right.$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) -$$

$$\frac{4 i b^2 C d^2 e^2 f \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right]}{\sqrt{-\frac{-b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)}} +$$

$$\frac{i b^2 c C d e f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right]}{\sqrt{-\frac{-b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)}} +$$

$$\frac{5 i b^2 B d^2 e f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right]}{\sqrt{-\frac{-b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)}} -$$

$$\frac{3 i a b C d^2 e f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}}$$

$$\frac{i a b c C d f^3 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}}$$

$$\frac{15 i A b^2 d^2 f^3 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} +$$

$$\frac{10 i a b B d^2 f^3 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}}$$

$$\left. \frac{8 i a^2 C d^2 f^3 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} \right)$$

**Problem 70: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{c + d x} (A + B x + C x^2)}{(a + b x)^{3/2} \sqrt{e + f x}} dx$$

Optimal (type 4, 540 leaves, 8 steps):

$$\begin{aligned}
& \frac{2 (4 a^2 C d f + b^2 (c C e + 3 A d f) - a b (C d e + c C f + 3 B d f)) \sqrt{a+b x} \sqrt{c+d x} \sqrt{e+f x}}{3 b^2 (b c - a d) f (b e - a f)} - \frac{2 (A b^2 - a (b B - a C)) (c + d x)^{3/2} \sqrt{e+f x}}{b (b c - a d) (b e - a f) \sqrt{a+b x}} + \\
& \left( 2 \sqrt{-b c + a d} (8 a^2 C d f^2 - a b f (3 C d e + c C f + 6 B d f) + b^2 (3 d f (B e + A f) - C e (2 d e - c f))) \sqrt{\frac{b (c + d x)}{b c - a d}}} \right. \\
& \left. \sqrt{e+f x} \operatorname{EllipticE}[\operatorname{ArcSin}\left[\frac{\sqrt{d} \sqrt{a+b x}}{\sqrt{-b c + a d}}\right], \frac{(b c - a d) f}{d (b e - a f)}]\right) / \left( 3 b^3 \sqrt{d} f^2 (b e - a f) \sqrt{c+d x} \sqrt{\frac{b (e+f x)}{b e - a f}} \right) + \\
& \left( 2 \sqrt{-b c + a d} (d e - c f) (2 b C e - 3 b B f + 4 a C f) \sqrt{\frac{b (c + d x)}{b c - a d}} \sqrt{\frac{b (e+f x)}{b e - a f}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{\sqrt{d} \sqrt{a+b x}}{\sqrt{-b c + a d}}\right], \frac{(b c - a d) f}{d (b e - a f)}]\right) / \\
& (3 b^3 \sqrt{d} f^2 \sqrt{c+d x} \sqrt{e+f x})
\end{aligned}$$

Result (type 4, 5168 leaves):

$$\begin{aligned}
& \sqrt{a+b x} \sqrt{c+d x} \sqrt{e+f x} \left( \frac{2 C}{3 b^2 f} - \frac{2 (A b^2 - a b B + a^2 C)}{b^2 (b e - a f) (a + b x)} \right) + \\
& \frac{1}{3 b^4 f (b e - a f)} 2 \left( \left( -2 b^2 C d e^2 + b^2 c C e f + 3 b^2 B d e f - 3 a b C d e f - a b c C f^2 + 3 A b^2 d f^2 - 6 a b B d f^2 + 8 a^2 C d f^2 \right) \right. \\
& \left. (a + b x)^{3/2} \left( d + \frac{b c}{a + b x} - \frac{a d}{a + b x} \right) \left( f + \frac{b e}{a + b x} - \frac{a f}{a + b x} \right) \right) / \left( d f \sqrt{c + \frac{(a + b x) (d - \frac{a d}{a + b x})}{b}} \sqrt{e + \frac{(a + b x) (f - \frac{a f}{a + b x})}{b}} \right) + \\
& \frac{1}{d f \sqrt{c + \frac{(a+b x) (d - \frac{a d}{a+b x})}{b}} \sqrt{e + \frac{(a+b x) (f - \frac{a f}{a+b x})}{b}}} (-b e + a f) (a + b x) \sqrt{\left( d + \frac{b c}{a + b x} - \frac{a d}{a + b x} \right) \left( f + \frac{b e}{a + b x} - \frac{a f}{a + b x} \right)}
\end{aligned}$$

$$\begin{aligned}
& \left( - \left( \left( 2 \pm b^3 c C d e^2 f \sqrt{1 - \frac{-b c + a d}{d(a + b x)}} \sqrt{1 - \frac{-b e + a f}{f(a + b x)}} \right) \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d(-b e + a f)}{(-b c + a d)f} \right] - \right. \right. \right. \\
& \quad \left. \left. \left. \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d(-b e + a f)}{(-b c + a d)f} \right] \right) \right) / \left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \\
& \quad \left( 2 \pm a b^2 C d^2 e^2 f \sqrt{1 - \frac{-b c + a d}{d(a + b x)}} \sqrt{1 - \frac{-b e + a f}{f(a + b x)}} \right) \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d(-b e + a f)}{(-b c + a d)f} \right] - \right. \\
& \quad \left. \left. \left. \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d(-b e + a f)}{(-b c + a d)f} \right] \right) \right) / \\
& \quad \left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( \pm b^3 c^2 C e f^2 \sqrt{1 - \frac{-b c + a d}{d(a + b x)}} \sqrt{1 - \frac{-b e + a f}{f(a + b x)}} \right. \\
& \quad \left. \left. \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d(-b e + a f)}{(-b c + a d)f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d(-b e + a f)}{(-b c + a d)f} \right] \right) \right) / \\
& \quad \left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 3 \pm b^3 B c d e f^2 \sqrt{1 - \frac{-b c + a d}{d(a + b x)}} \sqrt{1 - \frac{-b e + a f}{f(a + b x)}} \right)
\end{aligned}$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 4 \pm a b^2 c C d e f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 3 \pm a b^2 B d^2 e f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 3 \pm a^2 b C d^2 e f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\begin{aligned}
& \left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( \frac{i a b^2 c^2 C f^3}{d (a + b x)} \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \\
& \left. \left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) \right) / \\
& \left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 3 i A b^3 c d f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \\
& \left. \left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) \right) / \\
& \left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 6 i a b^2 B c d f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \\
& \left. \left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) \right) / \\
& \left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 9 i a^2 b c C d f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)
\end{aligned}$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 3 \pm a A b^2 d^2 f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 6 \pm a^2 b B d^2 f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 8 \pm a^3 C d^2 f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) / \left( \sqrt{-\frac{-b c + a d}{d}} \right)$$

$$\begin{aligned}
& (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} + \frac{i b^2 c C d e f \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{-b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)}} - \\
& \frac{i a b C d^2 e f \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{-b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)}} - \\
& \frac{3 i b^2 B c d f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{-b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)}} + \\
& \frac{5 i a b c C d f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{-b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)}} - \\
& \frac{3 i A b^2 d^2 f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{-b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)}} + \\
& \frac{6 i a b B d^2 f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{-b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)}}
\end{aligned}$$

$$\frac{8 i a^2 C d^2 f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{-b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} \Bigg)$$

**Problem 71:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{c + d x} (A + B x + C x^2)}{(a + b x)^{5/2} \sqrt{e + f x}} dx$$

Optimal (type 4, 597 leaves, 8 steps):

$$\begin{aligned} & -\frac{2 (4 a^2 C f + b^2 (3 B e - 2 A f) - a b (6 C e + B f)) \sqrt{c + d x} \sqrt{e + f x}}{3 b^2 (b e - a f)^2 \sqrt{a + b x}} - \frac{2 (A b^2 - a (b B - a C)) (c + d x)^{3/2} \sqrt{e + f x}}{3 b (b c - a d) (b e - a f) (a + b x)^{3/2}} + \\ & \left( 2 \sqrt{d} (8 a^3 C d f^2 - a^2 b f (13 C d e + 7 c C f + 2 B d f) + a b^2 (3 C e (d e + 4 c f) + f (4 B d e + B c f - A d f))) - b^3 (A d e f + c (3 C e^2 + 3 B e f - 2 A f^2))) \right. \\ & \left. \sqrt{\frac{b (c + d x)}{b c - a d}} \sqrt{e + f x} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{d} \sqrt{a + b x}}{\sqrt{-b c + a d}}\right], \frac{(b c - a d) f}{d (b e - a f)}\right] \right) / \left( 3 b^3 \sqrt{-b c + a d} f (b e - a f)^2 \sqrt{c + d x} \sqrt{\frac{b (e + f x)}{b e - a f}} \right) + \\ & \left( 2 (d e - c f) (4 a^2 C d f + b^2 (3 c C e + A d f) - a b (B d f + 3 C (d e + c f))) \sqrt{\frac{b (c + d x)}{b c - a d}} \sqrt{\frac{b (e + f x)}{b e - a f}} \right. \\ & \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{d} \sqrt{a + b x}}{\sqrt{-b c + a d}}\right], \frac{(b c - a d) f}{d (b e - a f)}\right] \right) / \left( 3 b^3 \sqrt{d} \sqrt{-b c + a d} f (b e - a f) \sqrt{c + d x} \sqrt{e + f x} \right) \end{aligned}$$

Result (type 4, 5074 leaves):

$$\begin{aligned} & \sqrt{a + b x} \sqrt{c + d x} \sqrt{e + f x} \left( -\frac{2 (A b^2 - a b B + a^2 C)}{3 b^2 (b e - a f) (a + b x)^2} - \right. \\ & \left. \left( 2 (3 b^3 B c e - 6 a b^2 c C e + A b^3 d e - 4 a b^2 B d e + 7 a^2 b C d e - 2 A b^3 c f - a b^2 B c f + 4 a^2 b c C f + a A b^2 d f + 2 a^2 b B d f - 5 a^3 C d f) \right) / \right. \\ & \left. \left( 3 b^2 (b c - a d) (b e - a f)^2 (a + b x) \right) \right) - \frac{1}{3 b^4 (b c - a d) (b e - a f)^2} \end{aligned}$$

$$\begin{aligned}
& 2 \left( \left( -3 b^3 c C e^2 + 3 a b^2 C d e^2 - 3 b^3 B c e f + 12 a b^2 c C e f - A b^3 d e f + 4 a b^2 B d e f - 13 a^2 b C d e f + 2 A b^3 c f^2 + a b^2 B c f^2 - \right. \right. \\
& \quad \left. \left. 7 a^2 b c C f^2 - a A b^2 d f^2 - 2 a^2 b B d f^2 + 8 a^3 C d f^2 \right) (a + b x)^{3/2} \left( d + \frac{b c}{a + b x} - \frac{a d}{a + b x} \right) \left( f + \frac{b e}{a + b x} - \frac{a f}{a + b x} \right) \right) / \\
& \quad \left( f \sqrt{c + \frac{(a + b x) (d - \frac{a d}{a + b x})}{b}} \sqrt{e + \frac{(a + b x) (f - \frac{a f}{a + b x})}{b}} \right) - \frac{1}{f \sqrt{c + \frac{(a + b x) (d - \frac{a d}{a + b x})}{b}} \sqrt{e + \frac{(a + b x) (f - \frac{a f}{a + b x})}{b}}} \\
& (b c - a d) (-b e + a f) (a + b x) \sqrt{\left( d + \frac{b c}{a + b x} - \frac{a d}{a + b x} \right) \left( f + \frac{b e}{a + b x} - \frac{a f}{a + b x} \right)} \left( \left( 3 \frac{i}{2} b^3 c C e^2 f \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \right. \\
& \quad \left. \left. \text{EllipticE}[\pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f}] - \text{EllipticF}[\pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f}] \right) \right) / \\
& \left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left( d + \frac{b c - a d}{a + b x} \right) \left( f + \frac{b e - a f}{a + b x} \right)} - \left( 3 \frac{i}{2} a b^2 C d e^2 f \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \right. \\
& \quad \left. \left. \text{EllipticE}[\pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f}] - \text{EllipticF}[\pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f}] \right) \right) /
\end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{-\frac{-bc+ad}{d}} (-be+af) \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-ad}{a+b x}\right)} \right) + \left( 3 \pm b^3 B c e f^2 \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \right. \\
& \left. \left. \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right] \right) \right) / \\
& \left( \sqrt{-\frac{-bc+ad}{d}} (-be+af) \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-ad}{a+b x}\right)} \right) - \left( 12 \pm a b^2 c C e f^2 \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \right. \\
& \left. \left. \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right] \right) \right) / \\
& \left( \sqrt{-\frac{-bc+ad}{d}} (-be+af) \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-ad}{a+b x}\right)} \right) + \left( \pm A b^3 d e f^2 \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \right. \\
& \left. \left. \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right] \right) \right) / \\
& \left( \sqrt{-\frac{-bc+ad}{d}} (-be+af) \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-ad}{a+b x}\right)} \right) - \left( 4 \pm a b^2 B d e f^2 \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \right)
\end{aligned}$$

$$\left( \text{EllipticE}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} + \left( 13 \text{i} a^2 b C d e f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \right.$$

$$\left. \text{EllipticE}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} - \left( 2 \text{i} A b^3 c f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \right.$$

$$\left. \text{EllipticE}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} - \left( \text{i} a b^2 B c f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \right.$$

$$\left. \text{EllipticE}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\begin{aligned}
& \left( \sqrt{-\frac{-bc+ad}{d}} (-be+af) \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-ad}{a+b x}\right)} \right) + \left( 7 \pm a^2 b c C f^3 \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \right. \\
& \left. \left. \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right] \right) \right) \right) / \\
& \left( \sqrt{-\frac{-bc+ad}{d}} (-be+af) \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-ad}{a+b x}\right)} \right) + \left( \pm a A b^2 d f^3 \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \right. \\
& \left. \left. \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right] \right) \right) \right) / \\
& \left( \sqrt{-\frac{-bc+ad}{d}} (-be+af) \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-ad}{a+b x}\right)} \right) + \left( 2 \pm a^2 b B d f^3 \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \right. \\
& \left. \left. \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right] \right) \right) \right) / \\
& \left( \sqrt{-\frac{-bc+ad}{d}} (-be+af) \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-ad}{a+b x}\right)} \right) - \left( 8 \pm a^3 C d f^3 \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \right.
\end{aligned}$$

$$\begin{aligned}
& \left( \text{EllipticE} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a+b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a+b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) \Bigg/ \left( \sqrt{-\frac{-b c + a d}{d}} \right) \\
& (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} - \frac{3 i b^2 c C e f \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \text{EllipticF} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a+b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right]}{\sqrt{-\frac{-b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)}} - \\
& \frac{3 i b^2 B d e f \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \text{EllipticF} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a+b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right]}{\sqrt{-\frac{-b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)}} + \\
& \frac{9 i a b C d e f \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \text{EllipticF} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a+b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right]}{\sqrt{-\frac{-b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)}} + \\
& \frac{3 i a b c C f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \text{EllipticF} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a+b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right]}{\sqrt{-\frac{-b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)}} + \\
& \frac{i A b^2 d f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \text{EllipticF} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a+b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right]}{\sqrt{-\frac{-b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)}}
\end{aligned}$$

$$\frac{2 i a b B d f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} \\ \left. \frac{8 i a^2 C d f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}}\right]$$

**Problem 72:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{c + d x} (A + B x + C x^2)}{(a + b x)^{7/2} \sqrt{e + f x}} dx$$

Optimal (type 4, 1034 leaves, 9 steps):

$$\begin{aligned}
& \left( 2 (4 a^3 C d f - b^3 (5 B c e - 2 A d e - 4 A c f) + a b^2 (10 c C e + 3 B d e + B c f - 6 A d f) - a^2 b (8 C d e + 6 c C f - B d f)) \sqrt{c + d x} \sqrt{e + f x} \right) / \\
& \left( 15 b^2 (b c - a d) (b e - a f)^2 (a + b x)^{3/2} \right) - \frac{1}{15 b^2 (b c - a d)^2 (b e - a f)^3 \sqrt{a + b x}} \\
& 2 (8 a^4 C d^2 f^2 - a^3 b d f (23 C d e + 13 c C f - 2 B d f) - b^4 (2 A d^2 e^2 - c d e (5 B e - 3 A f) - c^2 (15 C e^2 - 10 B e f + 8 A f^2)) - a^2 b^2 \\
& (d f (7 B d e + 2 B c f - 3 A d f) - C (23 d^2 e^2 + 37 c d e f + 3 c^2 f^2)) - a b^3 (d^2 e (3 B e - 7 A f) + 2 c^2 f (5 C e - B f) + c d (40 C e^2 - 13 f (B e - A f))) ) \\
& \sqrt{c + d x} \sqrt{e + f x} - \frac{2 (A b^2 - a (b B - a C)) (c + d x)^{3/2} \sqrt{e + f x}}{5 b (b c - a d) (b e - a f) (a + b x)^{5/2}} + \frac{1}{15 b^3 (-b c + a d)^{3/2} (b e - a f)^3 \sqrt{c + d x} \sqrt{\frac{b (e + f x)}{b e - a f}}} \\
& 2 \sqrt{d} (8 a^4 C d^2 f^2 - a^3 b d f (23 C d e + 13 c C f - 2 B d f) - b^4 (2 A d^2 e^2 - c d e (5 B e - 3 A f) - c^2 (15 C e^2 - 10 B e f + 8 A f^2)) - a^2 b^2 \\
& (d f (7 B d e + 2 B c f - 3 A d f) - C (23 d^2 e^2 + 37 c d e f + 3 c^2 f^2)) - a b^3 (d^2 e (3 B e - 7 A f) + 2 c^2 f (5 C e - B f) + c d (40 C e^2 - 13 f (B e - A f))) ) \\
& \sqrt{\frac{b (c + d x)}{b c - a d}} \sqrt{e + f x} \text{EllipticE}[\text{ArcSin}\left[\frac{\sqrt{d} \sqrt{a + b x}}{\sqrt{-b c + a d}}\right], \frac{(b c - a d) f}{d (b e - a f)}] + \\
& \left( 2 \sqrt{d} (d e - c f) (4 a^3 C d f - b^3 (5 B c e - 2 A d e - 4 A c f) + a b^2 (10 c C e + 3 B d e + B c f - 6 A d f) - a^2 b (8 C d e + 6 c C f - B d f)) \right. \\
& \left. \sqrt{\frac{b (c + d x)}{b c - a d}} \sqrt{\frac{b (e + f x)}{b e - a f}} \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{d} \sqrt{a + b x}}{\sqrt{-b c + a d}}\right], \frac{(b c - a d) f}{d (b e - a f)}] \right) / (15 b^3 (-b c + a d)^{3/2} (b e - a f)^2 \sqrt{c + d x} \sqrt{e + f x})
\end{aligned}$$

Result (type 4, 9186 leaves):

$$\begin{aligned}
& \sqrt{a + b x} \sqrt{c + d x} \sqrt{e + f x} \left( -\frac{2 (A b^2 - a b B + a^2 C)}{5 b^2 (b e - a f) (a + b x)^3} - \right. \\
& (2 (5 b^3 B c e - 10 a b^2 c C e + A b^3 d e - 6 a b^2 B d e + 11 a^2 b C d e - 4 A b^3 c f - a b^2 B c f + 6 a^2 b c C f + 3 a A b^2 d f + 2 a^2 b B d f - 7 a^3 C d f)) / \\
& \left( 15 b^2 (b c - a d) (b e - a f)^2 (a + b x)^2 \right) - \frac{1}{15 b^2 (b c - a d)^2 (b e - a f)^3 (a + b x)} \\
& 2 (15 b^4 c^2 C e^2 + 5 b^4 B c d e^2 - 40 a b^3 c C d e^2 - 2 A b^4 d^2 e^2 - 3 a b^3 B d^2 e^2 + 23 a^2 b^2 C d^2 e^2 - 10 b^4 B c^2 e f - 10 a b^3 c^2 C e f - 3 A b^4 c d e f + \\
& 13 a b^3 B c d e f + 37 a^2 b^2 c C d e f + 7 a A b^3 d^2 e f - 7 a^2 b^2 B d^2 e f - 23 a^3 b C d^2 e f + 8 A b^4 c^2 f^2 + 2 a b^3 B c^2 f^2 + 3 a^2 b^2 c^2 C f^2 - \\
& \left. 13 a A b^3 c d f^2 - 2 a^2 b^2 B c d f^2 - 13 a^3 b c C d f^2 + 3 a^2 A b^2 d^2 f^2 + 2 a^3 b B d^2 f^2 + 8 a^4 C d^2 f^2 \right) + \frac{1}{15 b^4 (b c - a d)^2 (b e - a f)^3}
\end{aligned}$$

$$\begin{aligned}
& 2d \left( \frac{1}{d \sqrt{c + \frac{(a+b)x}{b} \left( d - \frac{ad}{a+b} \right)}} \sqrt{e + \frac{(a+b)x}{b} \left( f - \frac{af}{a+b} \right)} \right) \left( 15b^4 c^2 C e^2 + 5b^4 B c d e^2 - 40 a b^3 c C d e^2 - 2 A b^4 d^2 e^2 - 3 a b^3 B d^2 e^2 + 23 a^2 b^2 C d^2 e^2 - \right. \\
& \quad \left. 10 b^4 B c^2 e f - 10 a b^3 c^2 C e f - 3 A b^4 c d e f + 13 a b^3 B c d e f + 37 a^2 b^2 c C d e f + 7 a A b^3 d^2 e f - 7 a^2 b^2 B d^2 e f - 23 a^3 b C d^2 e f + \right. \\
& \quad \left. 8 A b^4 c^2 f^2 + 2 a b^3 B c^2 f^2 + 3 a^2 b^2 c^2 C f^2 - 13 a A b^3 c d f^2 - 2 a^2 b^2 B c d f^2 - 13 a^3 b c C d f^2 + 3 a^2 A b^2 d^2 f^2 + 2 a^3 b B d^2 f^2 + 8 a^4 C d^2 f^2 \right) \\
& (a+b)x)^{3/2} \left( d + \frac{b c}{a+b x} - \frac{a d}{a+b x} \right) \left( f + \frac{b e}{a+b x} - \frac{a f}{a+b x} \right) + \frac{1}{d \sqrt{c + \frac{(a+b)x}{b} \left( d - \frac{ad}{a+b} \right)}} \sqrt{e + \frac{(a+b)x}{b} \left( f - \frac{af}{a+b} \right)} \\
& (-b c + a d) (b e - a f) (a + b x) \sqrt{\left( d + \frac{b c}{a+b x} - \frac{a d}{a+b x} \right) \left( f + \frac{b e}{a+b x} - \frac{a f}{a+b x} \right)} \left( \begin{array}{l} 15 \pm b^4 c^2 C e^2 f \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \\ \text{EllipticE}[\pm \text{ArcSinh}\left(\frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right), \frac{d (-b e + a f)}{(-b c + a d) f}] - \text{EllipticF}[\pm \text{ArcSinh}\left(\frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right), \frac{d (-b e + a f)}{(-b c + a d) f}] \end{array} \right) / \\
& \left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left( d + \frac{b c - a d}{a+b x} \right) \left( f + \frac{b e - a f}{a+b x} \right)} + \left( 5 \pm b^4 B c d e^2 f \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \right. \right. \\
& \quad \left. \left. \text{EllipticE}[\pm \text{ArcSinh}\left(\frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right), \frac{d (-b e + a f)}{(-b c + a d) f}] - \text{EllipticF}[\pm \text{ArcSinh}\left(\frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right), \frac{d (-b e + a f)}{(-b c + a d) f}] \right) \right) / \\
& \left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left( d + \frac{b c - a d}{a+b x} \right) \left( f + \frac{b e - a f}{a+b x} \right)} - \left( 40 \pm a b^3 c C d e^2 f \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \right. \right. \\
& \quad \left. \left. \text{EllipticE}[\pm \text{ArcSinh}\left(\frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right), \frac{d (-b e + a f)}{(-b c + a d) f}] - \text{EllipticF}[\pm \text{ArcSinh}\left(\frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right), \frac{d (-b e + a f)}{(-b c + a d) f}] \right) \right)
\end{aligned}$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 2 \pm A b^4 d^2 e^2 f \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 3 \pm a b^3 B d^2 e^2 f \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 23 \pm a^2 b^2 C d^2 e^2 f \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 10 \pm b^4 B c^2 e f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 10 \pm a b^3 c^2 C e f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 3 \pm A b^4 c d e f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 13 \pm a b^3 B c d e f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 37 i a^2 b^2 c C d e f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 7 i a A b^3 d^2 e f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 7 i a^2 b^2 B d^2 e f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 23 \pm a^3 b C d^2 e f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 8 \pm A b^4 c^2 f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 2 \pm a b^3 B c^2 f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 3 \pm a^2 b^2 c^2 C f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 13 \pm a A b^3 c d f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 2 \pm a^2 b^2 B c d f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 13 \pm a^3 b c C d f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\begin{aligned}
& \left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 3 \pm a^2 A b^2 d^2 f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \\
& \left. \left. \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) \right) / \\
& \left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 2 \pm a^3 b B d^2 f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \\
& \left. \left. \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) \right) / \\
& \left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 8 \pm a^4 C d^2 f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \\
& \left. \left. \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) \right) / \\
& \left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) -
\end{aligned}$$

$$\frac{15 \pm b^3 c C d e^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} +$$

$$\frac{15 \pm a b^2 C d^2 e^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} +$$

$$\frac{5 i b^3 B c d e f \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} +$$

$$\frac{20 \pm a b^2 c C d e f \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} +$$

$$\frac{i A b^3 d^2 e f \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} -$$

$$\frac{6 i a b^2 B d^2 e f \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} -$$

$$\frac{19 \pm a^2 b C d^2 e f \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}}$$

$$\frac{4 \pm A b^3 c d f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}}$$

$$\frac{\pm a b^2 B c d f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}}$$

$$\frac{9 \pm a^2 b c C d f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} +$$

$$\frac{3 \pm a A b^2 d^2 f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} +$$

$$\frac{2 \pm a^2 b B d^2 f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} +$$

$$\frac{8 i a^3 C d^2 f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{-b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} \Bigg)$$

**Problem 73: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b x)^{3/2} (A+B x+C x^2)}{\sqrt{c+d x} \sqrt{e+f x}} dx$$

Optimal (type 4, 838 leaves, 9 steps):

$$\begin{aligned} & -\frac{1}{105 b d^3 f^3} \\ & 2 (5 b d f (5 b c C e + a C d e + a c C f - 7 A b d f) + (3 a d f - 4 b (d e + c f)) (2 a C d f - b (7 B d f - 6 C (d e + c f)))) \sqrt{a+b x} \sqrt{c+d x} \sqrt{e+f x} - \\ & \frac{2 (2 a C d f - b (7 B d f - 6 C (d e + c f))) (a+b x)^{3/2} \sqrt{c+d x} \sqrt{e+f x}}{35 b d^2 f^2} + \frac{2 C (a+b x)^{5/2} \sqrt{c+d x} \sqrt{e+f x}}{7 b d f} - \frac{1}{105 b^2 d^{7/2} f^4 \sqrt{c+d x} \sqrt{\frac{b (e+f x)}{b e-a f}}} \\ & 2 \sqrt{-b c + a d} \left(3 b d f (5 a d f (5 b c C e + a C d e + a c C f - 7 A b d f) - (3 b c e + a d e + a c f) (2 a C d f - b (7 B d f - 6 C (d e + c f)))) + \right. \\ & \left. 2 \left(\frac{a d f}{2} - b (d e + c f)\right) (5 b d f (5 b c C e + a C d e + a c C f - 7 A b d f) + (3 a d f - 4 b (d e + c f)) (2 a C d f - b (7 B d f - 6 C (d e + c f))))\right) \\ & \sqrt{\frac{b (c+d x)}{b c-a d}} \sqrt{e+f x} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{d} \sqrt{a+b x}}{\sqrt{-b c+a d}}\right], \frac{(b c-a d) f}{d (b e-a f)}\right] - \frac{1}{105 b^2 d^{7/2} f^4 \sqrt{c+d x} \sqrt{e+f x}} \\ & 2 \sqrt{-b c + a d} (b e - a f) (3 a^2 C d^2 f^2 (d e - c f) - 3 a b d f (7 d f (3 B d e + 2 B c f - 5 A d f) - C (16 d^2 e^2 + 8 c d e f + 11 c^2 f^2)) - \\ & b^2 (C (48 d^3 e^3 + 16 c d^2 e^2 f + 17 c^2 d e f^2 + 24 c^3 f^3) + 7 d f (5 A d f (2 d e + c f) - B (8 d^2 e^2 + 3 c d e f + 4 c^2 f^2))) \\ & \sqrt{\frac{b (c+d x)}{b c-a d}} \sqrt{\frac{b (e+f x)}{b e-a f}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{d} \sqrt{a+b x}}{\sqrt{-b c+a d}}\right], \frac{(b c-a d) f}{d (b e-a f)}\right] \end{aligned}$$

Result (type 4, 7300 leaves):

$$\begin{aligned} & \sqrt{a+b x} \sqrt{c+d x} \sqrt{e+f x} \\ & \left(\frac{1}{105 b d^3 f^3} - 2 (24 b^2 C d^2 e^2 + 23 b^2 c C d e f - 28 b^2 B d^2 e f - 33 a b C d^2 e f + 24 b^2 c^2 C f^2 - 28 b^2 B c d f^2 - 33 a b c C d f^2 + 35 A b^2 d^2 f^2 + \right. \end{aligned}$$

$$\begin{aligned}
& 42 a b B d^2 f^2 + 3 a^2 C d^2 f^2 \Big) + \frac{2 \left( -6 b C d e - 6 b c C f + 7 b B d f + 8 a C d f \right) x}{35 d^2 f^2} + \frac{2 b C x^2}{7 d f} \Big) + \\
& \frac{1}{105 b^3 d^3 f^3} 2 \sqrt{\frac{1}{d f \sqrt{c + \frac{(a+b x) \left(d - \frac{a d}{a+b x}\right)}{b}} \sqrt{e + \frac{(a+b x) \left(f - \frac{a f}{a+b x}\right)}{b}}}} \left( -48 b^3 C d^3 e^3 - 40 b^3 c C d^2 e^2 f + 56 b^3 B d^3 e^2 f + 72 a b^2 C d^3 e^2 f - \right. \\
& \left. 40 b^3 c^2 C d e f^2 + 49 b^3 B c d^2 e f^2 + 62 a b^2 c C d^2 e f^2 - 70 A b^3 d^3 e f^2 - 91 a b^2 B d^3 e f^2 - 12 a^2 b C d^3 e f^2 - 48 b^3 c^3 C f^3 + \right. \\
& \left. 56 b^3 B c^2 d f^3 + 72 a b^2 c^2 C d f^3 - 70 A b^3 c d^2 f^3 - 91 a b^2 B c d^2 f^3 - 12 a^2 b c C d^2 f^3 + 140 a A b^2 d^3 f^3 + 21 a^2 b B d^3 f^3 - 6 a^3 C d^3 f^3 \right) \\
& (a+b x)^{3/2} \left( d + \frac{b c}{a+b x} - \frac{a d}{a+b x} \right) \left( f + \frac{b e}{a+b x} - \frac{a f}{a+b x} \right) + \frac{1}{d f \sqrt{c + \frac{(a+b x) \left(d - \frac{a d}{a+b x}\right)}{b}} \sqrt{e + \frac{(a+b x) \left(f - \frac{a f}{a+b x}\right)}{b}}} \\
& (-b c + a d) (-b e + a f) (a+b x) \sqrt{\left( d + \frac{b c}{a+b x} - \frac{a d}{a+b x} \right) \left( f + \frac{b e}{a+b x} - \frac{a f}{a+b x} \right)} \left( \begin{array}{l} 48 \pm b^3 C d^3 e^3 f \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \\ \text{EllipticE}[\pm \text{ArcSinh}[\frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a+b x}}], \frac{d (-b e + a f)}{(-b c + a d) f}] - \text{EllipticF}[\pm \text{ArcSinh}[\frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a+b x}}], \frac{d (-b e + a f)}{(-b c + a d) f}] \end{array} \right) / \\
& \left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left( d + \frac{b c - a d}{a+b x} \right) \left( f + \frac{b e - a f}{a+b x} \right)} + \left( 40 \pm b^3 c C d^2 e^2 f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \right. \right. \\
& \left. \left. \text{EllipticE}[\pm \text{ArcSinh}[\frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a+b x}}], \frac{d (-b e + a f)}{(-b c + a d) f}] - \text{EllipticF}[\pm \text{ArcSinh}[\frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a+b x}}], \frac{d (-b e + a f)}{(-b c + a d) f}] \right) \right) /
\end{aligned}$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 56 i b^3 B d^3 e^2 f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 72 i a b^2 C d^3 e^2 f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 40 i b^3 c^2 C d e f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 49 i b^3 B c d^2 e f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 62 i a b^2 c C d^2 e f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right.$$

$$\left. \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 70 i A b^3 d^3 e f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right.$$

$$\left. \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 91 i a b^2 B d^3 e f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 12 \pm a^2 b C d^3 e f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 48 \pm b^3 c^3 C f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 56 \pm b^3 B c^2 d f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 72 \pm a b^2 c^2 C d f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} + \left( 70 \text{i} a b^3 c d^2 f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \right.$$

$$\left. \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} + \left( 91 \text{i} a b^2 B c d^2 f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \right.$$

$$\left. \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} + \left( 12 \text{i} a^2 b c C d^2 f^4 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \right.$$

$$\left. \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\begin{aligned}
& \left( \sqrt{-\frac{-bc+ad}{d}} (-be+af) \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-ad}{a+b x}\right)} \right) - \left( 140 \pm a A b^2 d^3 f^4 \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \right. \\
& \left. \left. \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right] \right) \right) / \\
& \left( \sqrt{-\frac{-bc+ad}{d}} (-be+af) \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-ad}{a+b x}\right)} \right) - \left( 21 \pm a^2 b B d^3 f^4 \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \right. \\
& \left. \left. \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right] \right) \right) / \\
& \left( \sqrt{-\frac{-bc+ad}{d}} (-be+af) \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-ad}{a+b x}\right)} \right) + \left( 6 \pm a^3 C d^3 f^4 \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \right. \\
& \left. \left. \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right] \right) \right) / \\
& \left( \sqrt{-\frac{-bc+ad}{d}} (-be+af) \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-ad}{a+b x}\right)} \right) -
\end{aligned}$$

$$\frac{24 \pm b^2 C d^3 e^2 f \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}}$$

$$\frac{23 \pm b^2 c C d^2 e f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} +$$

$$\frac{28 \pm b^2 B d^3 e f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} +$$

$$\frac{15 \pm a b C d^3 e f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} -$$

$$\frac{24 \pm b^2 c^2 C d f^3 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} +$$

$$\frac{28 \pm b^2 B c d^2 f^3 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} +$$

$$\begin{aligned}
& \frac{15 \text{i} \text{a} \text{b} \text{c} \text{C} \text{d}^2 \text{f}^3 \sqrt{1 - \frac{-\text{b} \text{c}+\text{a} \text{d}}{\text{d} (\text{a}+\text{b} \text{x})}} \sqrt{1 - \frac{-\text{b} \text{e}+\text{a} \text{f}}{\text{f} (\text{a}+\text{b} \text{x})}} \text{EllipticF}\left[\text{i} \text{ArcSinh}\left[\frac{\sqrt{\frac{-\text{b} \text{c}+\text{a} \text{d}}{\text{d}}}}{\sqrt{\text{a}+\text{b} \text{x}}}\right], \frac{\text{d} (-\text{b} \text{e}+\text{a} \text{f})}{(-\text{b} \text{c}+\text{a} \text{d}) \text{f}}\right]}{\sqrt{-\frac{\text{b} \text{c}+\text{a} \text{d}}{\text{d}}} \sqrt{\left(\text{d} + \frac{\text{b} \text{c}-\text{a} \text{d}}{\text{a}+\text{b} \text{x}}\right) \left(\text{f} + \frac{\text{b} \text{e}-\text{a} \text{f}}{\text{a}+\text{b} \text{x}}\right)}} \\
& \frac{35 \text{i} \text{A} \text{b}^2 \text{d}^3 \text{f}^3 \sqrt{1 - \frac{-\text{b} \text{c}+\text{a} \text{d}}{\text{d} (\text{a}+\text{b} \text{x})}} \sqrt{1 - \frac{-\text{b} \text{e}+\text{a} \text{f}}{\text{f} (\text{a}+\text{b} \text{x})}} \text{EllipticF}\left[\text{i} \text{ArcSinh}\left[\frac{\sqrt{\frac{-\text{b} \text{c}+\text{a} \text{d}}{\text{d}}}}{\sqrt{\text{a}+\text{b} \text{x}}}\right], \frac{\text{d} (-\text{b} \text{e}+\text{a} \text{f})}{(-\text{b} \text{c}+\text{a} \text{d}) \text{f}}\right]}{\sqrt{-\frac{\text{b} \text{c}+\text{a} \text{d}}{\text{d}}} \sqrt{\left(\text{d} + \frac{\text{b} \text{c}-\text{a} \text{d}}{\text{a}+\text{b} \text{x}}\right) \left(\text{f} + \frac{\text{b} \text{e}-\text{a} \text{f}}{\text{a}+\text{b} \text{x}}\right)}} \\
& \frac{21 \text{i} \text{a} \text{b} \text{B} \text{d}^3 \text{f}^3 \sqrt{1 - \frac{-\text{b} \text{c}+\text{a} \text{d}}{\text{d} (\text{a}+\text{b} \text{x})}} \sqrt{1 - \frac{-\text{b} \text{e}+\text{a} \text{f}}{\text{f} (\text{a}+\text{b} \text{x})}} \text{EllipticF}\left[\text{i} \text{ArcSinh}\left[\frac{\sqrt{\frac{-\text{b} \text{c}+\text{a} \text{d}}{\text{d}}}}{\sqrt{\text{a}+\text{b} \text{x}}}\right], \frac{\text{d} (-\text{b} \text{e}+\text{a} \text{f})}{(-\text{b} \text{c}+\text{a} \text{d}) \text{f}}\right]}{\sqrt{-\frac{\text{b} \text{c}+\text{a} \text{d}}{\text{d}}} \sqrt{\left(\text{d} + \frac{\text{b} \text{c}-\text{a} \text{d}}{\text{a}+\text{b} \text{x}}\right) \left(\text{f} + \frac{\text{b} \text{e}-\text{a} \text{f}}{\text{a}+\text{b} \text{x}}\right)}} + \\
& \left. \frac{6 \text{i} \text{a}^2 \text{C} \text{d}^3 \text{f}^3 \sqrt{1 - \frac{-\text{b} \text{c}+\text{a} \text{d}}{\text{d} (\text{a}+\text{b} \text{x})}} \sqrt{1 - \frac{-\text{b} \text{e}+\text{a} \text{f}}{\text{f} (\text{a}+\text{b} \text{x})}} \text{EllipticF}\left[\text{i} \text{ArcSinh}\left[\frac{\sqrt{\frac{-\text{b} \text{c}+\text{a} \text{d}}{\text{d}}}}{\sqrt{\text{a}+\text{b} \text{x}}}\right], \frac{\text{d} (-\text{b} \text{e}+\text{a} \text{f})}{(-\text{b} \text{c}+\text{a} \text{d}) \text{f}}\right]}{\sqrt{-\frac{\text{b} \text{c}+\text{a} \text{d}}{\text{d}}} \sqrt{\left(\text{d} + \frac{\text{b} \text{c}-\text{a} \text{d}}{\text{a}+\text{b} \text{x}}\right) \left(\text{f} + \frac{\text{b} \text{e}-\text{a} \text{f}}{\text{a}+\text{b} \text{x}}\right)}}\right]
\end{aligned}$$

**Problem 74:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{\text{a}+\text{b} \text{x}} (\text{A}+\text{B} \text{x}+\text{C} \text{x}^2)}{\sqrt{\text{c}+\text{d} \text{x}} \sqrt{\text{e}+\text{f} \text{x}}} \text{d}\text{x}$$

Optimal (type 4, 528 leaves, 8 steps):

$$\begin{aligned}
& - \frac{2 (2 a C d f - b (5 B d f - 4 C (d e + c f))) \sqrt{a+b x} \sqrt{c+d x} \sqrt{e+f x}}{15 b d^2 f^2} + \frac{2 C (a+b x)^{3/2} \sqrt{c+d x} \sqrt{e+f x}}{5 b d f} - \\
& \left( 2 \sqrt{-b c + a d} (3 b d f (3 b c C e + a C d e + a c C f - 5 A b d f) + (a d f - 2 b (d e + c f)) (2 a C d f - b (5 B d f - 4 C (d e + c f)))) \right. \\
& \left. \sqrt{\frac{b (c + d x)}{b c - a d}} \sqrt{e + f x} \text{EllipticE}[\text{ArcSin}\left[\frac{\sqrt{d} \sqrt{a + b x}}{\sqrt{-b c + a d}}\right], \frac{(b c - a d) f}{d (b e - a f)}] \right) / \left( 15 b^2 d^{5/2} f^3 \sqrt{c + d x} \sqrt{\frac{b (e + f x)}{b e - a f}} \right) - \\
& \left( 2 \sqrt{-b c + a d} (b e - a f) (a C d f (d e - c f) - b (5 d f (2 B d e + B c f - 3 A d f) - C (8 d^2 e^2 + 3 c d e f + 4 c^2 f^2))) \sqrt{\frac{b (c + d x)}{b c - a d}} \right. \\
& \left. \sqrt{\frac{b (e + f x)}{b e - a f}} \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{d} \sqrt{a + b x}}{\sqrt{-b c + a d}}\right], \frac{(b c - a d) f}{d (b e - a f)}] \right) / (15 b^2 d^{5/2} f^3 \sqrt{c + d x} \sqrt{e + f x})
\end{aligned}$$

Result (type 4, 3657 leaves):

$$\begin{aligned}
& \sqrt{a+b x} \sqrt{c+d x} \left( \frac{2 (-4 b C d e - 4 b c C f + 5 b B d f + a C d f)}{15 b d^2 f^2} + \frac{2 C x}{5 d f} \right) \sqrt{e+f x} - \frac{1}{15 b^3 d^2 f^2} \\
& 2 \left( \left( -8 b^2 C d^2 e^2 - 7 b^2 c C d e f + 10 b^2 B d^2 e f + 3 a b C d^2 e f - 8 b^2 c^2 C f^2 + 10 b^2 B c d f^2 + 3 a b c C d f^2 - 15 A b^2 d^2 f^2 - 5 a b B d^2 f^2 + 2 a^2 C d^2 f^2 \right) \right. \\
& \left. \left( a+b x \right)^{3/2} \left( d + \frac{b c}{a+b x} - \frac{a d}{a+b x} \right) \left( f + \frac{b e}{a+b x} - \frac{a f}{a+b x} \right) \right) / \left( d f \sqrt{c + \frac{(a+b x) (d - \frac{a d}{a+b x})}{b}} \sqrt{e + \frac{(a+b x) (f - \frac{a f}{a+b x})}{b}} \right) + \\
& \frac{1}{d f \sqrt{c + \frac{(a+b x) (d - \frac{a d}{a+b x})}{b}}} \sqrt{e + \frac{(a+b x) (f - \frac{a f}{a+b x})}{b}} \left( -b c + a d \right) \left( -b e + a f \right) \left( a+b x \right) \sqrt{\left( d + \frac{b c}{a+b x} - \frac{a d}{a+b x} \right) \left( f + \frac{b e}{a+b x} - \frac{a f}{a+b x} \right)}
\end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{\frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \\
& \left. \left( \text{EllipticE} \left[ \text{ArcSinh} \left[ \frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \text{ArcSinh} \left[ \frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) \right) / \\
& \left( \sqrt{\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left( d + \frac{b c - a d}{a + b x} \right) \left( f + \frac{b e - a f}{a + b x} \right)} + \sqrt{7 i b^2 c C d e f^2} \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \\
& \left. \left( \text{EllipticE} \left[ \text{ArcSinh} \left[ \frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \text{ArcSinh} \left[ \frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) \right) / \\
& \left( \sqrt{\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left( d + \frac{b c - a d}{a + b x} \right) \left( f + \frac{b e - a f}{a + b x} \right)} - \sqrt{10 i b^2 B d^2 e f^2} \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \\
& \left. \left( \text{EllipticE} \left[ \text{ArcSinh} \left[ \frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \text{ArcSinh} \left[ \frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) \right) / \\
& \left( \sqrt{\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left( d + \frac{b c - a d}{a + b x} \right) \left( f + \frac{b e - a f}{a + b x} \right)} - \sqrt{3 i a b C d^2 e f^2} \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)
\end{aligned}$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} + \left(8 i b^2 c^2 C f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}}\right. \right.$$

$$\left. \left. \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} - \left(10 i b^2 B c d f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}}\right. \right.$$

$$\left. \left. \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} - \left(3 i a b c C d f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}}\right. \right.$$

$$\left. \left. \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) \right) /$$

$$\begin{aligned}
& \left( \sqrt{-\frac{b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 15 \pm A b^2 d^2 f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \\
& \left. \left. \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) \right) / \\
& \left( \sqrt{-\frac{b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 5 \pm a b B d^2 f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \\
& \left. \left. \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) \right) / \\
& \left( \sqrt{-\frac{b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 2 \pm a^2 C d^2 f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \\
& \left. \left. \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) \right) / \left( \sqrt{-\frac{b c + a d}{d}} \right. \\
& \left. \left( -b e + a f \right) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \frac{4 \pm b C d^2 e f \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)}} -
\end{aligned}$$

$$\begin{aligned}
& \frac{4 i b c C d f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} + \\
& \frac{5 i b B d^2 f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} - \\
& \left. \frac{2 i a C d^2 f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} \right)
\end{aligned}$$

Problem 75: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A + B x + C x^2}{\sqrt{a + b x} \sqrt{c + d x} \sqrt{e + f x}} dx$$

Optimal (type 4, 387 leaves, 7 steps):

$$\begin{aligned}
& \frac{2 C \sqrt{a + b x} \sqrt{c + d x} \sqrt{e + f x}}{3 b d f} - \\
& \left( 2 \sqrt{-b c + a d} (2 a C d f - b (3 B d f - 2 C (d e + c f))) \sqrt{\frac{b (c + d x)}{b c - a d}} \sqrt{e + f x} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{d} \sqrt{a + b x}}{\sqrt{-b c + a d}}\right], \frac{(b c - a d) f}{d (b e - a f)}\right] \right. \\
& \left. + 3 b^2 d^{3/2} f^2 \sqrt{c + d x} \sqrt{\frac{b (e + f x)}{b e - a f}} + 2 \sqrt{-b c + a d} (a C f (d e - c f) - b (3 d f (B e - A f) - C e (2 d e + c f))) \right. \\
& \left. \sqrt{\frac{b (c + d x)}{b c - a d}} \sqrt{\frac{b (e + f x)}{b e - a f}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{d} \sqrt{a + b x}}{\sqrt{-b c + a d}}\right], \frac{(b c - a d) f}{d (b e - a f)}\right] \right) / (3 b^2 d^{3/2} f^2 \sqrt{c + d x} \sqrt{e + f x})
\end{aligned}$$

Result (type 4, 418 leaves):

$$\frac{1}{3 b^3 d^2 f^2 \sqrt{c + d x} \sqrt{e + f x}} \sqrt{a + b x} \left( 2 b^2 C d f (c + d x) (e + f x) - \frac{2 b^2 (-3 b B d f + 2 a C d f + 2 b C (d e + c f)) (c + d x) (e + f x)}{a + b x} + \right.$$

$$2 i \sqrt{-a + \frac{b c}{d}} d f (3 b B d f - 2 a C d f - 2 b C (d e + c f)) \sqrt{a + b x} \sqrt{\frac{b (c + d x)}{d (a + b x)}} \sqrt{\frac{b (e + f x)}{f (a + b x)}}$$

$$\text{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{-a + \frac{b c}{d}}}{\sqrt{a + b x}}\right], \frac{b d e - a d f}{b c f - a d f}\right] + \frac{1}{\sqrt{-a + \frac{b c}{d}}} 2 i b f (a C d (-d e + c f) + b (2 c^2 C f + 3 A d^2 f + c d (C e - 3 B f)))$$

$$\left. \sqrt{a + b x} \sqrt{\frac{b (c + d x)}{d (a + b x)}} \sqrt{\frac{b (e + f x)}{f (a + b x)}} \text{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{-a + \frac{b c}{d}}}{\sqrt{a + b x}}\right], \frac{b d e - a d f}{b c f - a d f}\right]\right)$$

Problem 76: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A + B x + C x^2}{(a + b x)^{3/2} \sqrt{c + d x} \sqrt{e + f x}} dx$$

Optimal (type 4, 422 leaves, 7 steps):

$$\begin{aligned}
& - \frac{2 (A b^2 - a (b B - a C)) \sqrt{c + d x} \sqrt{e + f x}}{b (b c - a d) (b e - a f) \sqrt{a + b x}} - \\
& \left( 2 (2 a^2 C d f + b^2 (c C e + A d f) - a b (C d e + c C f + B d f)) \sqrt{\frac{b (c + d x)}{b c - a d}} \sqrt{e + f x} \operatorname{EllipticE}[\operatorname{ArcSin}\left[\frac{\sqrt{d} \sqrt{a + b x}}{\sqrt{-b c + a d}}\right], \frac{(b c - a d) f}{d (b e - a f)}] \right) / \\
& \left( b^2 \sqrt{d} \sqrt{-b c + a d} f (b e - a f) \sqrt{c + d x} \sqrt{\frac{b (e + f x)}{b e - a f}} \right) - \\
& \frac{2 (a C (d e - c f) - b (c C e - B c f + A d f)) \sqrt{\frac{b (c + d x)}{b c - a d}} \sqrt{\frac{b (e + f x)}{b e - a f}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{\sqrt{d} \sqrt{a + b x}}{\sqrt{-b c + a d}}\right], \frac{(b c - a d) f}{d (b e - a f)}]}{b^2 \sqrt{d} \sqrt{-b c + a d} f \sqrt{c + d x} \sqrt{e + f x}}
\end{aligned}$$

Result (type 4, 477 leaves):

$$\begin{aligned}
& \frac{1}{b^3 (b c - a d) (b e - a f) \sqrt{a + b x} \sqrt{c + d x} \sqrt{e + f x}} \\
& 2 \left( -b^2 (A b^2 + a (-b B + a C)) (c + d x) (e + f x) + \frac{b^2 (2 a^2 C d f + b^2 (c C e + A d f) - a b (C d e + c C f + B d f)) (c + d x) (e + f x)}{d f} + \right. \\
& \left. \frac{1}{\sqrt{-a + \frac{b c}{d}} d} \right) \operatorname{EllipticE}\left[\operatorname{ArcSinh}\left[\frac{\sqrt{-a + \frac{b c}{d}}}{\sqrt{a + b x}}\right], \frac{b d e - a d f}{b c f - a d f}\right] + \frac{1}{\sqrt{-a + \frac{b c}{d}} d} b (-b c + a d) (a C (d e - c f) + b (c C e - B d e + A d f)) \\
& (a + b x)^{3/2} \sqrt{\frac{b (c + d x)}{d (a + b x)}} \sqrt{\frac{b (e + f x)}{f (a + b x)}} \operatorname{EllipticF}\left[\operatorname{ArcSinh}\left[\frac{\sqrt{-a + \frac{b c}{d}}}{\sqrt{a + b x}}\right], \frac{b d e - a d f}{b c f - a d f}\right]
\end{aligned}$$

Problem 77: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{A + Bx + Cx^2}{(ax + bx)^{5/2} \sqrt{c + dx} \sqrt{e + fx}} dx$$

Optimal (type 4, 642 leaves, 8 steps):

$$\begin{aligned} & -\frac{2(Ab^2 - a(bB - aC)) \sqrt{c+dx} \sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^{3/2}} + \\ & \left( \frac{2(2a^3CdF + ab^2(6cCe + BdE + Bcf - 4AdF) - b^3(3Bce - 2A(de + cf)) + a^2b(Bdf - 4C(de + cf))) \sqrt{c+dx} \sqrt{e+fx}}{(3b(bc-ad)^2(be-af)^2\sqrt{a+bx})} - \right. \\ & \left. \left( 2\sqrt{d}(2a^3CdF + ab^2(6cCe + BdE + Bcf - 4AdF) - b^3(3Bce - 2A(de + cf)) + a^2b(Bdf - 4C(de + cf))) \sqrt{\frac{b(c+dx)}{bc-ad}} \right. \right. \\ & \left. \left. \sqrt{e+fx} \operatorname{EllipticE}[\operatorname{ArcSin}\left[\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{-bc+ad}}\right], \frac{(bc-ad)f}{d(be-af)}] \right) / \left( 3b^2(-bc+ad)^{3/2}(be-af)^2\sqrt{c+dx} \sqrt{\frac{b(e+fx)}{be-af}} \right) - \right. \\ & \left. \left( 2(a^2Cd(de - cf) - b^2(3c^2Ce - 3BcdE + 2Ad^2e + Acdf) + ab(3(c^2C + Ad^2)f - Bd(de + 2cf))) \sqrt{\frac{b(c+dx)}{bc-ad}} \right. \right. \\ & \left. \left. \sqrt{\frac{b(e+fx)}{be-af}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{-bc+ad}}\right], \frac{(bc-ad)f}{d(be-af)}] \right) / (3b^2\sqrt{d}(-bc+ad)^{3/2}(be-af)\sqrt{c+dx}\sqrt{e+fx}) \right) \end{aligned}$$

Result (type 4, 4349 leaves):

$$\begin{aligned} & \sqrt{a+bx} \sqrt{c+dx} \sqrt{e+fx} \left( -\frac{2(Ab^2 - a(bB + a^2C))}{3b(bc-ad)(be-af)(a+bx)^2} - \right. \\ & \left. (2(3b^3Bce - 6ab^2CcE - 2Ab^3de - ab^2Bde + 4a^2bCdE - 2Ab^3cf - ab^2Bcf + 4a^2bcCf + 4aAb^2df - a^2bBdf - 2a^3CdF)) / \right. \\ & \left. (3b(bc-ad)^2(be-af)^2(a+bx)) \right) + \frac{1}{3b^3(bc-ad)^2(be-af)^2} \end{aligned}$$

$$\begin{aligned}
& 2 \left( \left( 3 b^3 B c e - 6 a b^2 c C e - 2 A b^3 d e - a b^2 B d e + 4 a^2 b C d e - 2 A b^3 c f - a b^2 B c f + 4 a^2 b c C f + 4 a A b^2 d f - a^2 b B d f - 2 a^3 C d f \right) \right. \\
& \quad \left. \left( a + b x \right)^{3/2} \left( d + \frac{b c}{a + b x} - \frac{a d}{a + b x} \right) \left( f + \frac{b e}{a + b x} - \frac{a f}{a + b x} \right) \right) / \left( \sqrt{c + \frac{(a + b x) \left( d - \frac{a d}{a + b x} \right)}{b}} \sqrt{e + \frac{(a + b x) \left( f - \frac{a f}{a + b x} \right)}{b}} \right) - \\
& \quad \frac{1}{\sqrt{c + \frac{(a + b x) \left( d - \frac{a d}{a + b x} \right)}{b}} \sqrt{e + \frac{(a + b x) \left( f - \frac{a f}{a + b x} \right)}{b}}} (b c - a d) (b e - a f) (a + b x) \sqrt{\left( d + \frac{b c}{a + b x} - \frac{a d}{a + b x} \right) \left( f + \frac{b e}{a + b x} - \frac{a f}{a + b x} \right)} \\
& \quad \left( 3 \pm b^3 B c e f \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \\
& \quad \left. \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) \right) / \\
& \quad \left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left( d + \frac{b c - a d}{a + b x} \right) \left( f + \frac{b e - a f}{a + b x} \right)} \right) - \left( 6 \pm a b^2 c C e f \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \\
& \quad \left. \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{-\frac{-bc+ad}{d}} (-be+af) \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-ad}{a+b x}\right)} \right) - \left( 2 \text{i} A b^3 d e f \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \right. \\
& \left. \left. \left( \text{EllipticE}[\text{i} \text{ArcSinh}\left[\frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}}\right], \frac{d(-be+af)}{(-bc+ad)f}] - \text{EllipticF}[\text{i} \text{ArcSinh}\left[\frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}}\right], \frac{d(-be+af)}{(-bc+ad)f}] \right) \right) \right) / \\
& \left( \sqrt{-\frac{-bc+ad}{d}} (-be+af) \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-ad}{a+b x}\right)} \right) - \left( \text{i} a b^2 B d e f \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \right. \\
& \left. \left( \text{EllipticE}[\text{i} \text{ArcSinh}\left[\frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}}\right], \frac{d(-be+af)}{(-bc+ad)f}] - \text{EllipticF}[\text{i} \text{ArcSinh}\left[\frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}}\right], \frac{d(-be+af)}{(-bc+ad)f}] \right) \right) \right) / \\
& \left( \sqrt{-\frac{-bc+ad}{d}} (-be+af) \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-ad}{a+b x}\right)} \right) + \left( 4 \text{i} a^2 b C d e f \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \right. \\
& \left. \left( \text{EllipticE}[\text{i} \text{ArcSinh}\left[\frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}}\right], \frac{d(-be+af)}{(-bc+ad)f}] - \text{EllipticF}[\text{i} \text{ArcSinh}\left[\frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}}\right], \frac{d(-be+af)}{(-bc+ad)f}] \right) \right) \right) / \\
& \left( \sqrt{-\frac{-bc+ad}{d}} (-be+af) \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-ad}{a+b x}\right)} \right) - \left( 2 \text{i} A b^3 c f^2 \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \right)
\end{aligned}$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( \text{i} a b^2 B c f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 4 \text{i} a^2 b c C f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 4 \text{i} a A b^2 d f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\begin{aligned}
& \left( \sqrt{-\frac{-bc+ad}{d}} (-be+af) \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right)} \right) - \left( \pm a^2 b B d f^2 \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \right. \\
& \left. \left. \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right] \right) \right) / \\
& \left( \sqrt{-\frac{-bc+ad}{d}} (-be+af) \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right)} \right) - \left( 2 \pm a^3 C d f^2 \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \right. \\
& \left. \left. \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right] \right) \right) / \left( \sqrt{-\frac{-bc+ad}{d}} \right. \\
& \left. \left. (-be+af) \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right)} \right) - \frac{3 \pm b^2 c C e \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right]}{\sqrt{-\frac{-bc+ad}{d}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right)}} + \\
& \frac{3 \pm a b C d e \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right]}{\sqrt{-\frac{-bc+ad}{d}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right)}} + \\
& \frac{3 \pm a b c C f \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}} \right], \frac{d(-be+af)}{(-bc+ad)f} \right]}{\sqrt{-\frac{-bc+ad}{d}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right)}} +
\end{aligned}$$

$$\begin{aligned}
& \frac{\text{i } A b^2 d f \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}[\text{i } \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} - \\
& \frac{\text{i } a b B d f \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}[\text{i } \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} - \\
& \left. \frac{2 \text{i } a^2 C d f \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}[\text{i } \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} \right)
\end{aligned}$$

**Problem 78: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B x + C x^2}{(a + b x)^{7/2} \sqrt{c + d x} \sqrt{e + f x}} dx$$

Optimal (type 4, 1116 leaves, 9 steps):

$$\begin{aligned}
& - \frac{2 (A b^2 - a (b B - a C)) \sqrt{c + d x} \sqrt{e + f x}}{5 b (b c - a d) (b e - a f) (a + b x)^{5/2}} + \\
& \left( 2 (2 a^3 C d f + a b^2 (10 c C e + B d e + B c f - 8 A d f) - b^3 (5 B c e - 4 A (d e + c f)) + 3 a^2 b (B d f - 2 C (d e + c f))) \sqrt{c + d x} \sqrt{e + f x} \right) / \\
& \left( 15 b (b c - a d)^2 (b e - a f)^2 (a + b x)^{3/2} \right) + \frac{1}{15 b (b c - a d)^3 (b e - a f)^3 \sqrt{a + b x}} \\
& 2 (2 a^4 C d^2 f^2 + a^3 b d f (3 B d f - 7 C (d e + c f)) - b^4 (8 A d^2 e^2 - c d e (10 B e - 7 A f) + c^2 (15 C e^2 - 10 B e f + 8 A f^2)) - \\
& a b^3 (d^2 e (2 B e - 23 A f) - 2 c^2 f (5 C e - B f) - c d (10 C e^2 - 33 B e f + 23 A f^2)) - \\
& a^2 b^2 (C (3 d^2 e^2 - 13 c d e f + 3 c^2 f^2) + d f (23 A d f - 7 B (d e + c f))) \sqrt{c + d x} \sqrt{e + f x} + \frac{1}{15 b^2 (-b c + a d)^{5/2} (b e - a f)^3 \sqrt{c + d x} \sqrt{\frac{b (e + f x)}{b e - a f}}} \\
& 2 \sqrt{d} (2 a^4 C d^2 f^2 + a^3 b d f (3 B d f - 7 C (d e + c f)) - b^4 (8 A d^2 e^2 - c d e (10 B e - 7 A f) + c^2 (15 C e^2 - 10 B e f + 8 A f^2)) - \\
& a b^3 (d^2 e (2 B e - 23 A f) - 2 c^2 f (5 C e - B f) - c d (10 C e^2 - 33 B e f + 23 A f^2)) - \\
& a^2 b^2 (C (3 d^2 e^2 - 13 c d e f + 3 c^2 f^2) + d f (23 A d f - 7 B (d e + c f))) \sqrt{\frac{b (c + d x)}{b c - a d}} \\
& \sqrt{e + f x} \text{EllipticE}[\text{ArcSin}\left[\frac{\sqrt{d} \sqrt{a + b x}}{\sqrt{-b c + a d}}\right], \frac{(b c - a d) f}{d (b e - a f)}] + \frac{1}{15 b^2 (-b c + a d)^{5/2} (b e - a f)^2 \sqrt{c + d x} \sqrt{e + f x}} \\
& 2 \sqrt{d} (a^3 C d f (d e - c f) + b^3 (8 A d^2 e^2 - c d e (10 B e - 3 A f) + c^2 (15 C e^2 - 5 B e f + 4 A f^2)) + a b^2 \\
& (d^2 e (2 B e - 19 A f) - c^2 f (20 C e - B f) - c d (10 C e^2 - 27 B e f + 11 A f^2)) - 3 a^2 b (d f (2 B d e + 3 B c f - 5 A d f) - C (d^2 e^2 + c d e f + 3 c^2 f^2))) \\
& \sqrt{\frac{b (c + d x)}{b c - a d}} \sqrt{\frac{b (e + f x)}{b e - a f}} \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{d} \sqrt{a + b x}}{\sqrt{-b c + a d}}\right], \frac{(b c - a d) f}{d (b e - a f)}]
\end{aligned}$$

Result (type 4, 8844 leaves):

$$\begin{aligned}
& \sqrt{a + b x} \sqrt{c + d x} \sqrt{e + f x} \left( - \frac{2 (A b^2 - a b B + a^2 C)}{5 b (b c - a d) (b e - a f) (a + b x)^3} - \right. \\
& \left( 2 (5 b^3 B c e - 10 a b^2 c C e - 4 A b^3 d e - a b^2 B d e + 6 a^2 b C d e - 4 A b^3 c f - a b^2 B c f + 6 a^2 b c C f + 8 a A b^2 d f - 3 a^2 b B d f - 2 a^3 C d f) \right) / \\
& \left( 15 b (b c - a d)^2 (b e - a f)^2 (a + b x)^2 \right) - \\
& \frac{1}{15 b (b c - a d)^3 (b e - a f)^3 (a + b x)} 2 (15 b^4 c^2 C e^2 - 10 b^4 B c d e^2 - 10 a b^3 c C d e^2 + 8 A b^4 d^2 e^2 + 2 a b^3 B d^2 e^2 + 3 a^2 b^2 C d^2 e^2 - \\
& 10 b^4 B c^2 e f - 10 a b^3 c^2 C e f + 7 A b^4 c d e f + 33 a b^3 B c d e f - 13 a^2 b^2 c C d e f - 23 a A b^3 d^2 e f - 7 a^2 b^2 B d^2 e f + 7 a^3 b C d^2 e f + \\
& 8 A b^4 c^2 f^2 + 2 a b^3 B c^2 f^2 + 3 a^2 b^2 c^2 C f^2 - 23 a A b^3 c d f^2 - 7 a^2 b^2 B c d f^2 + 7 a^3 b c C d f^2 + 23 a^2 A b^2 d^2 f^2 - 3 a^3 b B d^2 f^2 - 2 a^4 C d^2 f^2) \left. \right) +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{15 b^3 (b c - a d)^3 (b e - a f)^3} 2 d f \left( \frac{1}{d f \sqrt{c + \frac{(a+b x) (d - \frac{a d}{a+b x})}{b}}} \sqrt{e + \frac{(a+b x) (f - \frac{a f}{a+b x})}{b}} \right. \\
& \left. (15 b^4 c^2 C e^2 - 10 b^4 B c d e^2 - 10 a b^3 c C d e^2 + 8 A b^4 d^2 e^2 + 2 a b^3 B d^2 e^2 + 3 a^2 b^2 C d^2 e^2 - 10 b^4 B c^2 e f - 10 a b^3 c^2 C e f + 7 A b^4 c d e f + \right. \\
& \left. 33 a b^3 B c d e f - 13 a^2 b^2 c C d e f - 23 a A b^3 d^2 e f - 7 a^2 b^2 B d^2 e f + 7 a^3 b C d^2 e f + 8 A b^4 c^2 f^2 + 2 a b^3 B c^2 f^2 + 3 a^2 b^2 c^2 C f^2 - 23 a A b^3 c d f^2 - \right. \\
& \left. 7 a^2 b^2 B c d f^2 + 7 a^3 b c C d f^2 + 23 a^2 A b^2 d^2 f^2 - 3 a^3 b B d^2 f^2 - 2 a^4 C d^2 f^2) (a + b x)^{3/2} \left( d + \frac{b c}{a + b x} - \frac{a d}{a + b x} \right) \left( f + \frac{b e}{a + b x} - \frac{a f}{a + b x} \right) + \right. \\
& \left. \frac{1}{d f \sqrt{c + \frac{(a+b x) (d - \frac{a d}{a+b x})}{b}}} \sqrt{e + \frac{(a+b x) (f - \frac{a f}{a+b x})}{b}} (-b c + a d) (-b e + a f) (a + b x) \sqrt{\left( d + \frac{b c}{a + b x} - \frac{a d}{a + b x} \right) \left( f + \frac{b e}{a + b x} - \frac{a f}{a + b x} \right)} \right. \\
& \left. - \left( \left( 15 \pm b^4 c^2 C e^2 f \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right) \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \right. \right. \right. \\
& \left. \left. \left. \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) / \left( \sqrt{-\frac{b c + a d}{d}} (-b e + a f) \sqrt{\left( d + \frac{b c - a d}{a + b x} \right) \left( f + \frac{b e - a f}{a + b x} \right)} \right) + \right. \right. \\
& \left. \left. \left( 10 \pm b^4 B c d e^2 f \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right) \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \right. \right. \right. \\
& \left. \left. \left. \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) \right) / 
\end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{-\frac{-bc+ad}{d}} (-be+af) \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-ad}{a+b x}\right)} \right) + \left( 10 \frac{i}{\pi} a b^3 c C d e^2 f \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \right. \\
& \left. \left. \left( \text{EllipticE}\left[\frac{i}{\pi} \text{ArcSinh}\left[\frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}}\right], \frac{d(-be+af)}{(-bc+ad)f}\right] - \text{EllipticF}\left[\frac{i}{\pi} \text{ArcSinh}\left[\frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}}\right], \frac{d(-be+af)}{(-bc+ad)f}\right] \right) \right) / \\
& \left( \sqrt{-\frac{-bc+ad}{d}} (-be+af) \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-ad}{a+b x}\right)} \right) - \left( 8 \frac{i}{\pi} A b^4 d^2 e^2 f \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \right. \\
& \left. \left( \text{EllipticE}\left[\frac{i}{\pi} \text{ArcSinh}\left[\frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}}\right], \frac{d(-be+af)}{(-bc+ad)f}\right] - \text{EllipticF}\left[\frac{i}{\pi} \text{ArcSinh}\left[\frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}}\right], \frac{d(-be+af)}{(-bc+ad)f}\right] \right) \right) / \\
& \left( \sqrt{-\frac{-bc+ad}{d}} (-be+af) \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-ad}{a+b x}\right)} \right) - \left( 2 \frac{i}{\pi} a b^3 B d^2 e^2 f \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \right. \\
& \left. \left( \text{EllipticE}\left[\frac{i}{\pi} \text{ArcSinh}\left[\frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}}\right], \frac{d(-be+af)}{(-bc+ad)f}\right] - \text{EllipticF}\left[\frac{i}{\pi} \text{ArcSinh}\left[\frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}}\right], \frac{d(-be+af)}{(-bc+ad)f}\right] \right) \right) / \\
& \left( \sqrt{-\frac{-bc+ad}{d}} (-be+af) \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-ad}{a+b x}\right)} \right) - \left( 3 \frac{i}{\pi} a^2 b^2 C d^2 e^2 f \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \right)
\end{aligned}$$

$$\left( \text{EllipticE}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} + \left( 10 i b^4 B c^2 e f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \right.$$

$$\left. \text{EllipticE}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} + \left( 10 i a b^3 c^2 C e f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \right.$$

$$\left. \text{EllipticE}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} - \left( 7 i A b^4 c d e f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right. \right.$$

$$\left. \text{EllipticE}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 33 i a b^3 B c d e f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 13 i a^2 b^2 c C d e f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 23 i a A b^3 d^2 e f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 7 i a^2 b^2 B d^2 e f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 7 i a^3 b c d^2 e f^2 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 8 i A b^4 c^2 f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 2 i a b^3 B c^2 f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 3 \pm a^2 b^2 c^2 C f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 23 \pm a A b^3 c d f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 7 \pm a^2 b^2 B c d f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 7 \pm a^3 b c C d f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) - \left( 23 i a^2 A b^2 d^2 f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right.$$

$$\left. \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 3 i a^3 b B d^2 f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right.$$

$$\left. \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\left( \sqrt{-\frac{-b c + a d}{d}} (-b e + a f) \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right)} \right) + \left( 2 i a^4 C d^2 f^3 \sqrt{1 - \frac{-b c + a d}{d (a + b x)}} \sqrt{1 - \frac{-b e + a f}{f (a + b x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-b c + a d}{d}}}{\sqrt{a + b x}} \right], \frac{d (-b e + a f)}{(-b c + a d) f} \right] \right) /$$

$$\begin{aligned}
& \left( \sqrt{-\frac{-bc+ad}{d}} (-be+af) \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right)} \right) - \\
& \frac{5 i b^3 B c d e f \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}}\right], \frac{d(-be+af)}{(-bc+ad)f}\right]}{\sqrt{-\frac{-bc+ad}{d}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right)}} + \\
& \frac{10 i a b^2 c C d e f \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}}\right], \frac{d(-be+af)}{(-bc+ad)f}\right]}{\sqrt{-\frac{-bc+ad}{d}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right)}} + \\
& \frac{4 i A b^3 d^2 e f \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}}\right], \frac{d(-be+af)}{(-bc+ad)f}\right]}{\sqrt{-\frac{-bc+ad}{d}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right)}} + \\
& \frac{i a b^2 B d^2 e f \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}}\right], \frac{d(-be+af)}{(-bc+ad)f}\right]}{\sqrt{-\frac{-bc+ad}{d}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right)}} - \\
& \frac{6 i a^2 b C d^2 e f \sqrt{1 - \frac{-bc+ad}{d(a+b x)}} \sqrt{1 - \frac{-be+af}{f(a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{-\frac{-bc+ad}{d}}}{\sqrt{a+b x}}\right], \frac{d(-be+af)}{(-bc+ad)f}\right]}{\sqrt{-\frac{-bc+ad}{d}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right)}}
\end{aligned}$$

$$\begin{aligned}
& \frac{4 i A b^3 c d f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} + \\
& \frac{i a b^2 B c d f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} - \\
& \frac{6 i a^2 b c C d f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} - \\
& \frac{8 i a A b^2 d^2 f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} + \\
& \frac{3 i a^2 b B d^2 f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} + \\
& \left. \frac{2 i a^3 C d^2 f^2 \sqrt{1 - \frac{-b c + a d}{d (a+b x)}} \sqrt{1 - \frac{-b e + a f}{f (a+b x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-b c + a d}{d}}}{\sqrt{a+b x}}\right], \frac{d (-b e + a f)}{(-b c + a d) f}\right]}{\sqrt{-\frac{b c + a d}{d}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right)}} \right)
\end{aligned}$$

Test results for the 35 problems in "1.1.1.7 P(x) (a+b x)^m (c+d x)^n (e+f x)^p (g+h x)^q.m"

**Problem 1: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + b x)^2 (A + B x)}{\sqrt{c + d x} \sqrt{e + f x} \sqrt{g + h x}} dx$$

Optimal (type 4, 700 leaves, 9 steps):

$$\begin{aligned} & \frac{2 b (7 a B d f h + b (5 A d f h - 4 B (d f g + d e h + c f h))) \sqrt{c + d x} \sqrt{e + f x} \sqrt{g + h x}}{15 d^2 f^2 h^2} + \\ & \frac{2 b B (a + b x) \sqrt{c + d x} \sqrt{e + f x} \sqrt{g + h x}}{5 d f h} + \left( 2 \sqrt{-d e + c f} (15 a^2 B d^2 f^2 h^2 + 10 a b d f h (3 A d f h - 2 B (d f g + d e h + c f h))) - \right. \\ & b^2 (10 A d f h (d f g + d e h + c f h) - B (8 c^2 f^2 h^2 + 7 c d f h (f g + e h) + d^2 (8 f^2 g^2 + 7 e f g h + 8 e^2 h^2))) \\ & \left. \sqrt{\frac{d (e + f x)}{d e - c f}} \sqrt{g + h x} \text{EllipticE}[\text{ArcSin}\left[\frac{\sqrt{f} \sqrt{c + d x}}{\sqrt{-d e + c f}}\right], \frac{(d e - c f) h}{f (d g - c h)}]\right) / \left( 15 d^3 f^{5/2} h^3 \sqrt{e + f x} \sqrt{\frac{d (g + h x)}{d g - c h}} \right) - \\ & \frac{1}{15 d^3 f^{5/2} h^3 \sqrt{e + f x} \sqrt{g + h x}} 2 \sqrt{-d e + c f} (15 a^2 d^2 f^2 h^2 (B g - A h) + 10 a b d f h (3 A d f g h - B (c h (f g - e h) + d g (2 f g + e h))) - \\ & b^2 (5 A d f h (c h (f g - e h) + d g (2 f g + e h)) - B (4 c^2 f h^2 (f g - e h) + c d h (3 f^2 g^2 + e f g h - 4 e^2 h^2) + d^2 g (8 f^2 g^2 + 3 e f g h + 4 e^2 h^2))) \\ & \sqrt{\frac{d (e + f x)}{d e - c f}} \sqrt{\frac{d (g + h x)}{d g - c h}} \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{f} \sqrt{c + d x}}{\sqrt{-d e + c f}}\right], \frac{(d e - c f) h}{f (d g - c h)}] \end{aligned}$$

Result (type 4, 12443 leaves):

$$\begin{aligned} & \sqrt{c + d x} \sqrt{e + f x} \left( \frac{2 b (-4 b B d f g - 4 b B d e h - 4 b B c f h + 5 A b d f h + 10 a B d f h)}{15 d^2 f^2 h^2} + \frac{2 b^2 B x}{5 d f h} \right) \sqrt{g + h x} + \\ & \frac{1}{15 d^4 f^2 h^2} \left( 2 (8 b^2 B d^2 f^2 g^2 + 7 b^2 B d^2 e f g h + 7 b^2 B c d f^2 g h - 10 A b^2 d^2 f^2 g h - 20 a b B d^2 f^2 g h + 8 b^2 B d^2 e^2 h^2 + 7 b^2 B c d e f h^2 - \right. \\ & 10 A b^2 d^2 e f h^2 - 20 a b B d^2 e f h^2 + 8 b^2 B c^2 f^2 h^2 - 10 A b^2 c d f^2 h^2 - 20 a b B c d f^2 h^2 + 30 a A b d^2 f^2 h^2 + 15 a^2 B d^2 f^2 h^2) \\ & \left. (c + d x)^{3/2} \left( f + \frac{d e}{c + d x} - \frac{c f}{c + d x} \right) \left( h + \frac{d g}{c + d x} - \frac{c h}{c + d x} \right) \right) / \left( f h \sqrt{e + \frac{(c + d x) (f - \frac{c f}{c + d x})}{d}} \sqrt{g + \frac{(c + d x) (h - \frac{c h}{c + d x})}{d}} \right) - \end{aligned}$$

$$\begin{aligned}
& \frac{1}{f h \sqrt{e + \frac{(c+d x)(f - \frac{c f}{c+d x})}{d}} \sqrt{g + \frac{(c+d x)(h - \frac{c h}{c+d x})}{d}}} 2(c+d x) \sqrt{\left(f + \frac{d e}{c+d x} - \frac{c f}{c+d x}\right) \left(h + \frac{d g}{c+d x} - \frac{c h}{c+d x}\right)} \left(8 i b^2 B d^4 e f^2 g^3 h \sqrt{1 - \frac{-d e + c f}{f (c+d x)}}\right. \\
& \left. \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \left(\text{EllipticE}\left[\pm \text{ArcSinh}\left[\frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}\right] - \text{EllipticF}\left[\pm \text{ArcSinh}\left[\frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}\right]\right)\right) / \\
& \left(\sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)} - \left(8 i b^2 B c d^3 f^3 g^3 h \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}}\right.\right. \\
& \left.\left. \left(\text{EllipticE}\left[\pm \text{ArcSinh}\left[\frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}\right] - \text{EllipticF}\left[\pm \text{ArcSinh}\left[\frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}\right]\right)\right) / \\
& \left(\sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)} + \left(7 i b^2 B d^4 e^2 f g^2 h^2 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}}\right.\right. \\
& \left.\left. \left(\text{EllipticE}\left[\pm \text{ArcSinh}\left[\frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}\right] - \text{EllipticF}\left[\pm \text{ArcSinh}\left[\frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}\right]\right)\right) / \\
& \left(\sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)} - \left(8 i b^2 B c d^3 e f^2 g^2 h^2 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}}\right.\right.
\end{aligned}$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) - \left( 10 \text{i} A b^2 d^4 e f^2 g^2 h^2 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right.$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) - \left( 20 \text{i} a b B d^4 e f^2 g^2 h^2 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right.$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) + \left( \text{i} b^2 B c^2 d^2 f^3 g^2 h^2 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} + \left(10 \frac{i}{k} A b^2 c d^3 f^3 g^2 h^2 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}}\right. \right.$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) + \left( 20 \pm a b B c d^3 f^3 g^2 h^2 \sqrt{1 - \frac{d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} + \left(8 \frac{1}{2} b^2 B d^4 e^3 g h^3 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}}\right) \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) - \left( 8 \pm b^2 B c d^3 e^2 f g h^3 \sqrt{1 - \frac{d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) - \left( 10 \text{i} A b^2 d^4 e^2 f g h^3 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right.$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) - \left( 20 \text{i} a b B d^4 e^2 f g h^3 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) + \left( \text{i} b^2 B c^2 d^2 e f^2 g h^3 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) + \left( 10 \text{Integrate } b^2 c d^3 e f^2 g h^3 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) + \left( 20 \text{Integrate } a b B c d^3 e f^2 g h^3 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) + \left( 30 \text{Integrate } a A b d^4 e f^2 g h^3 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) + \left( 15 \text{Integrate } a^2 B d^4 e f^2 g h^3 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) - \left( i b^2 B c^3 d f^3 g h^3 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right.$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) - \left( 30 i a A b c d^3 f^3 g h^3 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) - \left( 15 i a^2 B c d^3 f^3 g h^3 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) - \left( 8 i b^2 B c d^3 e^3 h^4 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right.$$

$$\left. \left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) + \left( i b^2 B c^2 d^2 e^2 f h^4 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right.$$

$$\left. \left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) + \left( 10 i A b^2 c d^3 e^2 f h^4 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left. \left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) + \left( 20 i a b B c d^3 e^2 f h^4 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) - \left( i b^2 B c^3 d e f^2 h^4 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right.$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) - \left( 30 i a A b c d^3 e f^2 h^4 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) - \left( 15 i a^2 B c d^3 e f^2 h^4 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\begin{aligned}
& \left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) + \left( 8 i b^2 B c^4 f^3 h^4 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right. \\
& \left. \left. \left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) \right) / \\
& \left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) - \left( 10 i A b^2 c^3 d f^3 h^4 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right. \\
& \left. \left. \left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) \right) / \\
& \left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) - \left( 20 i a b B c^3 d f^3 h^4 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right. \\
& \left. \left. \left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) \right) / \\
& \left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) + \left( 30 i a A b c^2 d^2 f^3 h^4 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right.
\end{aligned}$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) + \left( 15 i a^2 B c^2 d^2 f^3 h^4 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right.$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) -$$

$$\frac{4 i b^2 B d^3 e f^2 g^2 h \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right]}{\sqrt{-\frac{-d e + c f}{f}} \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)}} +$$

$$\frac{4 i b^2 B c d^2 f^3 g^2 h \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right]}{\sqrt{-\frac{-d e + c f}{f}} \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)}} -$$

$$\frac{4 i b^2 B d^3 e^2 f g h^2 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right]}{\sqrt{-\frac{-d e + c f}{f}} \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)}} +$$

$$\begin{aligned}
& \frac{\text{i} b^2 B c d^2 e f^2 g h^2 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[\text{i} \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}\right]}{\sqrt{-\frac{d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}} + \\
& \frac{5 \text{i} A b^2 d^3 e f^2 g h^2 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[\text{i} \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}\right]}{\sqrt{-\frac{d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}} + \\
& \frac{10 \text{i} a b B d^3 e f^2 g h^2 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[\text{i} \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}\right]}{\sqrt{-\frac{d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}} + \\
& \frac{3 \text{i} b^2 B c^2 d f^3 g h^2 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[\text{i} \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}\right]}{\sqrt{-\frac{d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}} - \\
& \frac{5 \text{i} A b^2 c d^2 f^3 g h^2 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[\text{i} \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}\right]}{\sqrt{-\frac{d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}} - \\
& \frac{10 \text{i} a b B c d^2 f^3 g h^2 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[\text{i} \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}\right]}{\sqrt{-\frac{d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}} +
\end{aligned}$$

$$\frac{4 \pm b^2 B c d^2 e^2 f h^3 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}] + \sqrt{-\frac{d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}}}{\sqrt{-\frac{d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}}$$

$$\frac{3 \pm b^2 B c^2 d e f^2 h^3 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}] - \sqrt{-\frac{d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}}}{\sqrt{-\frac{d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}}$$

$$\frac{5 \pm A b^2 c d^2 e f^2 h^3 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}] - \sqrt{-\frac{d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}}}{\sqrt{-\frac{d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}}$$

$$\frac{10 \pm a b B c d^2 e f^2 h^3 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}] + \sqrt{-\frac{d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}}}{\sqrt{-\frac{d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}}$$

$$\frac{8 \pm b^2 B c^3 f^3 h^3 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}] - \sqrt{-\frac{d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}}}{\sqrt{-\frac{d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}}$$

$$\frac{10 \pm A b^2 c^2 d f^3 h^3 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}] - \sqrt{-\frac{d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}}}{\sqrt{-\frac{d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}}$$

$$\begin{aligned}
& \frac{20 \pm a b B c^2 d f^3 h^3 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}] + }{\sqrt{-\frac{d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}}} \\
& \frac{30 \pm a A b c d^2 f^3 h^3 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}] + }{\sqrt{-\frac{d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}}} \\
& \frac{15 \pm a^2 B c d^2 f^3 h^3 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}] - }{\sqrt{-\frac{d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}}} \\
& \left. \frac{15 \pm a^2 A d^3 f^3 h^3 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}]}{\sqrt{-\frac{d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}}} \right)
\end{aligned}$$

Problem 2: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b x)(A + B x)}{\sqrt{c + d x} \sqrt{e + f x} \sqrt{g + h x}} dx$$

Optimal (type 4, 405 leaves, 8 steps):

$$\begin{aligned}
& \frac{2 b B \sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x}}{3 d f h} + \\
& \left( 2 \sqrt{-d e+c f} (3 a B d f h + b (3 A d f h - 2 B (d f g + d e h + c f h))) \sqrt{\frac{d (e+f x)}{d e-c f}} \sqrt{g+h x} \operatorname{EllipticE}[\operatorname{ArcSin}\left[\frac{\sqrt{f} \sqrt{c+d x}}{\sqrt{-d e+c f}}\right], \frac{(d e-c f) h}{f (d g-c h)}] \right) / \\
& \left( 3 d^2 f^{3/2} h^2 \sqrt{e+f x} \sqrt{\frac{d (g+h x)}{d g-c h}} \right) - \left( 2 \sqrt{-d e+c f} (3 a d f h (B g - A h) + b (3 A d f g h - B (c h (f g - e h) + d g (2 f g + e h)))) \right. \\
& \left. \sqrt{\frac{d (e+f x)}{d e-c f}} \sqrt{\frac{d (g+h x)}{d g-c h}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{\sqrt{f} \sqrt{c+d x}}{\sqrt{-d e+c f}}\right], \frac{(d e-c f) h}{f (d g-c h)}] \right) / (3 d^2 f^{3/2} h^2 \sqrt{e+f x} \sqrt{g+h x})
\end{aligned}$$

Result (type 4, 450 leaves):

$$\begin{aligned}
& \frac{1}{3 d^3 f^2 h^2 \sqrt{e+f x} \sqrt{g+h x}} \\
& \sqrt{c+d x} \left( 2 b B d^2 f h (e+f x) (g+h x) - \frac{2 d^2 (-3 A b d f h - 3 a B d f h + 2 b B (d f g + d e h + c f h)) (e+f x) (g+h x)}{c+d x} + \frac{1}{\sqrt{-c+\frac{d e}{f}}} 2 \operatorname{i} (d e-c f) h \right. \\
& (3 A b d f h + 3 a B d f h - 2 b B (d f g + d e h + c f h)) \sqrt{c+d x} \sqrt{\frac{d (e+f x)}{f (c+d x)}} \sqrt{\frac{d (g+h x)}{h (c+d x)}} \operatorname{EllipticE}[\operatorname{i} \operatorname{ArcSinh}\left[\frac{\sqrt{-c+\frac{d e}{f}}}{\sqrt{c+d x}}\right], \frac{d f g - c f h}{d e h - c f h}] + \\
& \left. \frac{1}{\sqrt{-c+\frac{d e}{f}}} 2 \operatorname{i} d h (3 a d f (-B e + A f) h + b (-3 A d e f h + B c f (-f g + e h) + B d e (f g + 2 e h))) \sqrt{c+d x} \right. \\
& \left. \sqrt{\frac{d (e+f x)}{f (c+d x)}} \sqrt{\frac{d (g+h x)}{h (c+d x)}} \operatorname{EllipticF}[\operatorname{i} \operatorname{ArcSinh}\left[\frac{\sqrt{-c+\frac{d e}{f}}}{\sqrt{c+d x}}\right], \frac{d f g - c f h}{d e h - c f h}] \right)
\end{aligned}$$

### Problem 3: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A + Bx}{\sqrt{c + dx} \sqrt{e + fx} \sqrt{g + hx}} dx$$

Optimal (type 4, 284 leaves, 6 steps):

$$\begin{aligned} & \frac{2B\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}\operatorname{EllipticE}[\operatorname{ArcSin}[\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}],\frac{(de-cf)h}{f(dg-ch)}]}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}} \\ & \frac{2\sqrt{-de+cf}(Bg-Ah)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\operatorname{EllipticF}[\operatorname{ArcSin}[\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}],\frac{(de-cf)h}{f(dg-ch)}]}{d\sqrt{f}h\sqrt{e+fx}\sqrt{g+hx}} \end{aligned}$$

Result (type 4, 319 leaves):

$$\begin{aligned} & - \left( 2 \left( -B d^2 \sqrt{-c + \frac{d e}{f}} (e + f x) (g + h x) - \right. \right. \\ & \left. \left. \pm B (d e - c f) h (c + d x)^{3/2} \sqrt{\frac{d (e + f x)}{f (c + d x)}} \sqrt{\frac{d (g + h x)}{h (c + d x)}} \operatorname{EllipticE}[\pm \operatorname{ArcSinh}[\frac{\sqrt{-c + \frac{d e}{f}}}{\sqrt{c + d x}}, \frac{d f g - c f h}{d e h - c f h}] + \pm d (B e - A f) h (c + d x)^{3/2} \right. \right. \\ & \left. \left. \sqrt{\frac{d (e + f x)}{f (c + d x)}} \sqrt{\frac{d (g + h x)}{h (c + d x)}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}[\frac{\sqrt{-c + \frac{d e}{f}}}{\sqrt{c + d x}}, \frac{d f g - c f h}{d e h - c f h}] \right] \right) / \left( d^2 \sqrt{-c + \frac{d e}{f}} f h \sqrt{c + d x} \sqrt{e + f x} \sqrt{g + h x} \right) \right) \end{aligned}$$

### Problem 4: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A + Bx}{(a + bx) \sqrt{c + dx} \sqrt{e + fx} \sqrt{g + hx}} dx$$

Optimal (type 4, 313 leaves, 9 steps):

$$\frac{2 B \sqrt{-d e + c f} \sqrt{\frac{d (e+f x)}{d e - c f}} \sqrt{\frac{d (g+h x)}{d g - c h}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{f} \sqrt{c+d x}}{\sqrt{-d e+c f}}\right], \frac{(d e-c f) h}{f (d g-c h)}\right]}{b d \sqrt{f} \sqrt{e+f x} \sqrt{g+h x}} -$$

$$\frac{2 \left(A - \frac{a B}{b}\right) \sqrt{-d e + c f} \sqrt{\frac{d (e+f x)}{d e - c f}} \sqrt{\frac{d (g+h x)}{d g - c h}} \operatorname{EllipticPi}\left[-\frac{b (d e-c f)}{(b c-a d) f}, \operatorname{ArcSin}\left[\frac{\sqrt{f} \sqrt{c+d x}}{\sqrt{-d e+c f}}\right], \frac{(d e-c f) h}{f (d g-c h)}\right]}{(b c-a d) \sqrt{f} \sqrt{e+f x} \sqrt{g+h x}}$$

Result (type 4, 244 leaves):

$$\left(2 \pm \sqrt{e+f x} \sqrt{\frac{d (g+h x)}{h (c+d x)}} \left(b (-B c+A d) \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{-c+\frac{d e}{f}}}{\sqrt{c+d x}}\right], \frac{d f g-c f h}{d e h-c f h}\right] + \right.\right.$$

$$\left.\left.(-A b+a B) d \operatorname{EllipticPi}\left[\frac{(b c-a d) f}{b (-d e+c f)}, \pm \operatorname{ArcSinh}\left[\frac{\sqrt{-c+\frac{d e}{f}}}{\sqrt{c+d x}}\right], \frac{d f g-c f h}{d e h-c f h}\right]\right)\right) / \left(b (-b c+a d) \sqrt{-c+\frac{d e}{f}} f \sqrt{\frac{d (e+f x)}{f (c+d x)}} \sqrt{g+h x}\right)$$

Problem 5: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{A+B x}{(a+b x)^2 \sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x}} dx$$

Optimal (type 4, 678 leaves, 12 steps):

$$\begin{aligned}
& - \frac{b (A b - a B) \sqrt{c + d x} \sqrt{e + f x} \sqrt{g + h x}}{(b c - a d) (b e - a f) (b g - a h) (a + b x)} + \frac{(A b - a B) \sqrt{f} \sqrt{-d e + c f} \sqrt{\frac{d(e+f x)}{d e - c f}} \sqrt{g + h x} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{f} \sqrt{c+d x}}{\sqrt{-d e+c f}}\right], \frac{(d e-c f) h}{f (d g-c h)}\right]}{(b c - a d) (b e - a f) (b g - a h) \sqrt{e + f x} \sqrt{\frac{d(g+h x)}{d g-c h}}} - \\
& \frac{(A b - a B) \sqrt{f} \sqrt{-d e + c f} \sqrt{\frac{d(e+f x)}{d e - c f}} \sqrt{\frac{d(g+h x)}{d g-c h}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{f} \sqrt{c+d x}}{\sqrt{-d e+c f}}\right], \frac{(d e-c f) h}{f (d g-c h)}\right]}{b (b c - a d) (b e - a f) \sqrt{e + f x} \sqrt{g + h x}} + \\
& \left( \sqrt{-d e + c f} (3 a^2 A b d f h - a^3 B d f h - b^3 (2 B c e g - A (d e g + c f g + c e h)) + a b^2 (B (d e g + c f g + c e h) - 2 A (d f g + d e h + c f h))) \sqrt{\frac{d(e+f x)}{d e - c f}} \right. \\
& \left. \sqrt{\frac{d(g+h x)}{d g-c h}} \operatorname{EllipticPi}\left[-\frac{b (d e-c f)}{(b c - a d) f}, \operatorname{ArcSin}\left[\frac{\sqrt{f} \sqrt{c+d x}}{\sqrt{-d e+c f}}\right], \frac{(d e-c f) h}{f (d g-c h)}\right] \right) / (b (b c - a d)^2 \sqrt{f} (b e - a f) (b g - a h) \sqrt{e + f x} \sqrt{g + h x})
\end{aligned}$$

### Result (type 4, 14 516 leaves) :

$$\begin{aligned}
& - \frac{b(Ab - aB) \sqrt{c+dx} \sqrt{e+fx} \sqrt{g+hx}}{(bc - ad)(be - af)(bg - ah)(a+bx)} - \\
& \frac{1}{d(-bc + ad)(-be + af)(-bg + ah)} \left( \frac{(Ab - aB)(c+dx)^{3/2} \left( f + \frac{de}{c+dx} - \frac{cf}{c+dx} \right) \left( h + \frac{dg}{c+dx} - \frac{ch}{c+dx} \right)}{\sqrt{e + \frac{(c+dx)(f - \frac{cf}{c+dx})}{d}}} \sqrt{g + \frac{(c+dx)(h - \frac{ch}{c+dx})}{d}} \right) + \\
& \left( c+dx \right) \left( -b + \frac{bc}{c+dx} - \frac{ad}{c+dx} \right) \\
& \sqrt{f + \frac{de}{c+dx} - \frac{cf}{c+dx}} \sqrt{h + \frac{dg}{c+dx} - \frac{ch}{c+dx}} \sqrt{fh + \frac{d^2 eg}{(c+dx)^2} - \frac{cd fg}{(c+dx)^2} - \frac{cd eh}{(c+dx)^2} + \frac{c^2 fh}{(c+dx)^2} + \frac{df g}{c+dx} + \frac{de h}{c+dx} - \frac{2cfh}{c+dx}} \\
& \left( \frac{(bc - ad)h(-2bBe g + Abfg + Abfg + Abeh + Abeh - 2aa fh)}{(-bg + ah)\sqrt{f + \frac{de}{c+dx} - \frac{cf}{c+dx}}\sqrt{h + \frac{dg}{c+dx} - \frac{ch}{c+dx}}} - \frac{(Ab - aB)(de - cf)\sqrt{h + \frac{dg}{c+dx} - \frac{ch}{c+dx}}}{\sqrt{f + \frac{de}{c+dx} - \frac{cf}{c+dx}}} \right) + \\
& \left( 2b^3 B c e g - A b^3 d e g - a b^2 B d e g - A b^3 c f g - a b^2 B c f g + 2 a A b^2 d f g - A b^3 c e h - a b^2 B c e h + 2 a A b^2 d e h + 2 a A b^2 c f h - \right)
\end{aligned}$$

$$\frac{3 a^2 A b d f h + a^3 B d f h}{\sqrt{h + \frac{d g}{c + d x} - \frac{c h}{c + d x}}} \Bigg/ \left( \left( -b g + a h \right) \left( b - \frac{b c}{c + d x} + \frac{a d}{c + d x} \right) \sqrt{f + \frac{d e}{c + d x} - \frac{c f}{c + d x}} \right)$$

$$\left( i A b d^2 e f g \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \left[ \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] - \right. \right.$$

$$\left. \left. \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] \right] \right) \Big/$$

$$\left( (b c - a d) (-d e + c f) \sqrt{-\frac{d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2} + \frac{d f g + d e h - 2 c f h}{c + d x}} \right) - \left( i a B d^2 e f g \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \right.$$

$$\left. \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \left[ \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] \right] \right) \Big/$$

$$\left( (b c - a d) (-d e + c f) \sqrt{-\frac{d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2} + \frac{d f g + d e h - 2 c f h}{c + d x}} \right) - \left( i A b c d f^2 g \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \right.$$

$$\left. \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \left[ \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] \right] \right) \Big/$$

$$\left( (b c - a d) (-d e + c f) \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2} + \frac{d f g + d e h - 2 c f h}{c + d x}} \right) + \left( i a b c d f^2 g \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \right)$$

$$\sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \left( \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right] - \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]\right)$$

$$\left( (b c - a d) (-d e + c f) \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2} + \frac{d f g + d e h - 2 c f h}{c + d x}} \right) - \left( i a b c d e f h \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \right)$$

$$\sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \left( \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right] - \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]\right)$$

$$\left( (b c - a d) (-d e + c f) \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2} + \frac{d f g + d e h - 2 c f h}{c + d x}} \right) + \left( i a b c d e f h \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \right)$$

$$\sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \left( \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right] - \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]\right)$$

$$\left( (b c - a d) (-d e + c f) \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2} + \frac{d f g + d e h - 2 c f h}{c + d x}} \right) + \left( i a b c^2 f^2 h \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \right)$$

$$\sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] \right) /$$

$$\left( (b c - a d) (-d e + c f) \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2} + \frac{d f g + d e h - 2 c f h}{c + d x}} \right) - \left( \pm a B c^2 f^2 h \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \right)$$

$$\sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] \right) /$$

$$\left( (b c - a d) (-d e + c f) \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2} + \frac{d f g + d e h - 2 c f h}{c + d x}} \right) -$$

$$\frac{\pm A b^2 d^2 e g \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right]}{(b c - a d)^2 \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2} + \frac{d f g + d e h - 2 c f h}{c + d x}}} +$$

$$\frac{\pm a b B d^2 e g \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right]}{(b c - a d)^2 \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2} + \frac{d f g + d e h - 2 c f h}{c + d x}}} +$$

$$\frac{2 \pm b B d e g \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right]}{(b c - a d) \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2} + \frac{d f g + d e h - 2 c f h}{c + d x}}} +$$

$$\frac{i A b^2 c d f g \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{(b c - a d)^2 \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}}$$

$$\frac{i a b B c d f g \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{(b c - a d)^2 \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}}$$

$$\frac{2 i A b d f g \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{(b c - a d) \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}} +$$

$$\frac{i A b^2 c d e h \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{(b c - a d)^2 \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}}$$

$$\frac{i a b B c d e h \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{(b c - a d)^2 \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}}$$

$$\frac{2 i A b d e h \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{(b c - a d) \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}}$$

$$\frac{\text{i } A b^2 c^2 f h \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticF}[\text{i ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}] + (b c - a d)^2 \sqrt{-\frac{d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}}{}$$

$$\frac{\text{i } a b B c^2 f h \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticF}[\text{i ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}] + (b c - a d)^2 \sqrt{-\frac{d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}}{}$$

$$\frac{2 \text{i } A b c f h \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticF}[\text{i ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}] - (b c - a d) \sqrt{-\frac{d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}}{}$$

$$\frac{2 \text{i } a B c f h \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticF}[\text{i ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}] + (b c - a d) \sqrt{-\frac{d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}}{}$$

$$\frac{2 \text{i } a A d f h \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticF}[\text{i ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}] + (b c - a d) \sqrt{-\frac{d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}} + \frac{1}{(b c - a d)^3}}$$

$$A b^3 d^2 e g \left( \frac{\text{i } c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \text{i ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{\sqrt{-\frac{d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \right)$$

$$\left. \begin{aligned} & \frac{\pm a d \sqrt{1 - \frac{-d e + c f}{f(c+d x)}} \sqrt{1 - \frac{-d g + c h}{h(c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b(-d g + c h)}, \pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f(-d g + c h)}\right]}{\sqrt{b} \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \end{aligned} \right] -$$

$$\frac{1}{(b c - a d)^3} a b^2 B d^2 e g \left( \frac{\frac{i}{2} c \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \frac{i}{2} \text{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c + d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{\sqrt{\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c + d x)^2} - \frac{c d f g}{(c + d x)^2} - \frac{c d e h}{(c + d x)^2} + \frac{c^2 f h}{(c + d x)^2} + \frac{d f g}{c + d x} + \frac{d e h}{c + d x} - \frac{2 c f h}{c + d x}}} \right)$$

$$\left. \begin{aligned} & \frac{\frac{d}{dx} \operatorname{ad} \sqrt{1 - \frac{-de+cf}{f(c+dx)}} \sqrt{1 - \frac{-dg+ch}{h(c+dx)}} \operatorname{EllipticPi}\left[\frac{(bc-ad)h}{b(-dg+ch)}, \frac{\sqrt{\frac{-dg+ch}{h}}}{\sqrt{c+dx}}\right], \frac{(-de+cf)h}{f(-dg+ch)}}{b \sqrt{-\frac{-dg+ch}{h}} \sqrt{fh + \frac{d^2 eg}{(c+dx)^2} - \frac{cd fg}{(c+dx)^2} - \frac{cd eh}{(c+dx)^2} + \frac{c^2 fh}{(c+dx)^2} + \frac{df fg}{c+dx} + \frac{de h}{c+dx} - \frac{2cfh}{c+dx}}} \end{aligned} \right\} -$$

$$\frac{1}{(b c - a d)^2} 2 b^2 B d e g \left( \frac{\frac{i}{2} c \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \frac{i}{2} \text{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c + d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right] - \sqrt{\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c + d x)^2} - \frac{c d f g}{(c + d x)^2} - \frac{c d e h}{(c + d x)^2} + \frac{c^2 f h}{(c + d x)^2} + \frac{d f g}{c + d x} + \frac{d e h}{c + d x} - \frac{2 c f h}{c + d x}}}{\sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}}}$$

$$\left. \frac{\frac{d}{dx} \operatorname{ad} \sqrt{1 - \frac{-de+cf}{f(c+dx)}} \sqrt{1 - \frac{-dg+ch}{h(c+dx)}} \operatorname{EllipticPi}\left[\frac{(bc-ad)h}{b(-dg+ch)}, \frac{\sqrt{\frac{-dg+ch}{h}}}{\sqrt{c+dx}}\right], \frac{(-de+cf)h}{f(-dg+ch)}}{b \sqrt{-\frac{-dg+ch}{h}} \sqrt{fh + \frac{d^2 eg}{(c+dx)^2} - \frac{cd fg}{(c+dx)^2} - \frac{cdeh}{(c+dx)^2} + \frac{c^2 fh}{(c+dx)^2} + \frac{dfg}{c+dx} + \frac{deh}{c+dx} - \frac{2cfh}{c+dx}}} \right] -$$

$$\frac{1}{(b c - a d)^3} A b^3 c d f g \left( \frac{\frac{1}{2} c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \frac{1}{2} \text{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{\sqrt{-\frac{-d g + c h}{h}}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}} \right)$$

$$\left. \begin{aligned} & \frac{\pm a d \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{b \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \\ & + \end{aligned} \right]$$

$$\frac{1}{(b c - a d)^3} a b^2 B c d f g \left( \begin{aligned} & \frac{\pm c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{\sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \\ & - \end{aligned} \right)$$

$$\left. \begin{aligned} & \frac{\pm a d \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{b \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \\ & + \end{aligned} \right)$$

$$\frac{1}{(b c - a d)^2} 2 A b^2 d f g \left( \begin{aligned} & \frac{\pm c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{\sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \\ & - \end{aligned} \right)$$

$$\left. \begin{aligned} & \frac{\pm a d \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{b \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \\ & - \end{aligned} \right)$$

$$\frac{1}{(b c - a d)^3} A b^3 c d e h \left( \begin{aligned} & \frac{\pm c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{\sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \\ & - \end{aligned} \right)$$

$$\left. \begin{aligned} & \frac{\pm a d \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{b \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \\ & + \end{aligned} \right]$$

$$\frac{1}{(b c - a d)^3} a b^2 B c d e h \left( \begin{aligned} & \frac{\pm c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{\sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \\ & - \end{aligned} \right)$$

$$\left. \begin{aligned} & \frac{\pm a d \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{b \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \\ & + \end{aligned} \right)$$

$$\frac{1}{(b c - a d)^2} 2 A b^2 d e h \left( \begin{aligned} & \frac{\pm c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{\sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \\ & - \end{aligned} \right)$$

$$\left. \begin{aligned} & \frac{\pm a d \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{b \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \\ & + \end{aligned} \right)$$

$$\frac{1}{(b c - a d)^3} A b^3 c^2 f h \left( \begin{aligned} & \frac{\pm c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{\sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \\ & - \end{aligned} \right)$$

$$\begin{aligned}
& \left. \frac{\frac{1}{(b c - a d)^3} a b^2 B c^2 f h \left( \begin{array}{l} \frac{i a d \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticPi} \left[ \frac{(b c - a d) h}{b (-d g + c h)}, i \text{ArcSinh} \left[ \frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}, \frac{(-d e + c f) h}{f (-d g + c h)} \right] \right]} \\ - \\ \frac{b \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}}{\sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \end{array} \right) - \right. \\
& \left. \frac{i a d \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticPi} \left[ \frac{(b c - a d) h}{b (-d g + c h)}, i \text{ArcSinh} \left[ \frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}, \frac{(-d e + c f) h}{f (-d g + c h)} \right] \right]} \\ - \\ \frac{b \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}}{\sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \right) - \\
& \left. \frac{1}{(b c - a d)^2} 2 a b^2 c f h \left( \begin{array}{l} \frac{i c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticPi} \left[ \frac{(b c - a d) h}{b (-d g + c h)}, i \text{ArcSinh} \left[ \frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}, \frac{(-d e + c f) h}{f (-d g + c h)} \right] \right]} \\ - \\ \frac{b \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}}{\sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \end{array} \right) - \right. \\
& \left. \frac{i a d \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticPi} \left[ \frac{(b c - a d) h}{b (-d g + c h)}, i \text{ArcSinh} \left[ \frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}, \frac{(-d e + c f) h}{f (-d g + c h)} \right] \right]} \\ + \\ \frac{b \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}}{\sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \right) + \\
& \left. \frac{1}{(b c - a d)^2} 2 a b B c f h \left( \begin{array}{l} \frac{i c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticPi} \left[ \frac{(b c - a d) h}{b (-d g + c h)}, i \text{ArcSinh} \left[ \frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}, \frac{(-d e + c f) h}{f (-d g + c h)} \right] \right]} \\ - \\ \frac{b \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}}{\sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \end{array} \right) - \right.
\end{aligned}$$

$$\left. \frac{\frac{1}{2} a d \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \frac{\sqrt{\frac{-d g - c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}}{b \sqrt{-\frac{d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \right]$$

$$\frac{1}{(b c - a d)^2} 2 a A b d f h \left( \frac{i c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{\sqrt{-\frac{-d g + c h}{h}}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}} \right)$$

$$\frac{\frac{1}{2} \operatorname{ad} \sqrt{1 - \frac{-de+cf}{f(c+dx)}} \sqrt{1 - \frac{-dg+ch}{h(c+dx)}} \operatorname{EllipticPi}\left[\frac{(bc-ad)h}{b(-dg+ch)}, \frac{1}{2} \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-dg+ch}{h}}}{\sqrt{c+dx}}\right], \frac{(-de+cf)h}{f(-dg+ch)}\right]}{b \sqrt{-\frac{-dg+ch}{h}} \sqrt{fh + \frac{d^2 eg}{(c+dx)^2} - \frac{cd fg}{(c+dx)^2} - \frac{cd eh}{(c+dx)^2} + \frac{c^2 fh}{(c+dx)^2} + \frac{df g}{c+dx} + \frac{de h}{c+dx} - \frac{2cfh}{c+dx}}} +$$

$$\frac{1}{b c - a d} A b f h \left( \begin{array}{l} i c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticPi} \left[ \frac{(b c - a d) h}{b (-d g + c h)}, i \text{ArcSinh} \left[ \frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] \\ \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}} \end{array} \right)$$

$$\frac{\text{i a d} \sqrt{1 - \frac{-\text{d e} + \text{c f}}{\text{f} (\text{c} + \text{d x})}} \sqrt{1 - \frac{-\text{d g} + \text{c h}}{\text{h} (\text{c} + \text{d x})}} \text{EllipticPi}\left[\frac{(\text{b c} - \text{a d}) \text{h}}{\text{b} (-\text{d g} + \text{c h})}, \text{i ArcSinh}\left[\frac{\sqrt{\frac{-\text{d g} + \text{c h}}{\text{h}}}}{\sqrt{\text{c} + \text{d x}}}\right], \frac{(-\text{d e} + \text{c f}) \text{h}}{\text{f} (-\text{d g} + \text{c h})}\right]}{b \sqrt{-\frac{-\text{d g} + \text{c h}}{\text{h}}} \sqrt{\text{f} \text{h} + \frac{\text{d}^2 \text{e g}}{(\text{c} + \text{d x})^2} - \frac{\text{c d f g}}{(\text{c} + \text{d x})^2} - \frac{\text{c d e h}}{(\text{c} + \text{d x})^2} + \frac{\text{c}^2 \text{f h}}{(\text{c} + \text{d x})^2} + \frac{\text{d f g}}{\text{c} + \text{d x}} + \frac{\text{d e h}}{\text{c} + \text{d x}} - \frac{2 \text{c f h}}{\text{c} + \text{d x}}}}$$

$$\frac{1}{b c - a d} a B f h \left( \begin{array}{l} i c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticPi} \left[ \frac{(b c - a d) h}{b (-d g + c h)}, i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c+d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] \\ \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}} \end{array} \right)$$

**Problem 6:** Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^{3/2} (A + B x)}{\sqrt{c + d x} \sqrt{e + f x} \sqrt{g + h x}} dx$$

Optimal (type 4, 981 leaves, 10 steps):

$$\begin{aligned}
& \frac{(5 a B d f h + b (4 A d f h - 3 B (d f g + d e h + c f h))) \sqrt{a+b x} \sqrt{e+f x} \sqrt{g+h x}}{4 d f^2 h^2 \sqrt{c+d x}} + \frac{b B \sqrt{a+b x} \sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x}}{2 d f h} - \\
& \left( \sqrt{d g - c h} \sqrt{f g - e h} (5 a B d f h + b (4 A d f h - 3 B (d f g + d e h + c f h))) \sqrt{a+b x} \sqrt{-\frac{(d e - c f) (g + h x)}{(f g - e h) (c + d x)}} \right. \\
& \left. \text{EllipticE}[\text{ArcSin}\left[\frac{\sqrt{d g - c h} \sqrt{e + f x}}{\sqrt{f g - e h} \sqrt{c + d x}}\right], \frac{(b c - a d) (f g - e h)}{(b e - a f) (d g - c h)}]\right) / \left(4 d^2 f^2 h^2 \sqrt{\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}} \sqrt{g + h x}\right) - \\
& \left( (b e - a f) \sqrt{b g - a h} (3 a B d f h + b (4 A d f h - B (c f h + 3 d (f g + e h)))) \sqrt{\frac{(b e - a f) (c + d x)}{(d e - c f) (a + b x)}} \sqrt{g + h x}\right. \\
& \left. \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{b g - a h} \sqrt{e + f x}}{\sqrt{f g - e h} \sqrt{a + b x}}\right], -\frac{(b c - a d) (f g - e h)}{(d e - c f) (b g - a h)}]\right) / \left(4 b d f^2 h^2 \sqrt{f g - e h} \sqrt{c + d x} \sqrt{-\frac{(b e - a f) (g + h x)}{(f g - e h) (a + b x)}}\right) + \\
& \frac{1}{4 b d^2 \sqrt{b c - a d} f^2 h^3 \sqrt{c + d x} \sqrt{e + f x}} \sqrt{-d g + c h} (4 d f h (2 a (2 A b + a B) d f h - b B (b (d e g + c f g + c e h) + a (d f g + d e h + c f h))) - \\
& (a d f h + b (d f g + d e h + c f h)) (5 a B d f h + b (4 A d f h - 3 B (d f g + d e h + c f h))) (a + b x) \sqrt{\frac{(b g - a h) (c + d x)}{(d g - c h) (a + b x)}} \\
& \sqrt{\frac{(b g - a h) (e + f x)}{(f g - e h) (a + b x)}} \text{EllipticPi}\left[-\frac{b (d g - c h)}{(b c - a d) h}, \text{ArcSin}\left[\frac{\sqrt{b c - a d} \sqrt{g + h x}}{\sqrt{-d g + c h} \sqrt{a + b x}}\right], \frac{(b e - a f) (d g - c h)}{(b c - a d) (f g - e h)}\right]
\end{aligned}$$

Result (type 4, 21555 leaves): Display of huge result suppressed!

**Problem 7: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+b x} (A+B x)}{\sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x}} dx$$

Optimal (type 4, 736 leaves, 7 steps):

$$\begin{aligned}
& \frac{B \sqrt{d g - c h} \sqrt{f g - e h} \sqrt{a + b x}}{f h \sqrt{c + d x}} \sqrt{-\frac{(d e - c f) (g + h x)}{(f g - e h) (c + d x)}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{d g - c h} \sqrt{e + f x}}{\sqrt{f g - e h} \sqrt{c + d x}}\right], \frac{(b c - a d) (f g - e h)}{(b e - a f) (d g - c h)}\right] \\
& + \frac{d f h \sqrt{\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}} \sqrt{g + h x}}{B (b e - a f) \sqrt{b g - a h}} \sqrt{\frac{(b e - a f) (c + d x)}{(d e - c f) (a + b x)}} \sqrt{g + h x} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{b g - a h} \sqrt{e + f x}}{\sqrt{f g - e h} \sqrt{a + b x}}\right], -\frac{(b c - a d) (f g - e h)}{(d e - c f) (b g - a h)}\right] \\
& + \left( \sqrt{-d g + c h} (2 A b d f h + B (a d f h - b (d f g + d e h + c f h))) (a + b x) \sqrt{\frac{(b g - a h) (c + d x)}{(d g - c h) (a + b x)}} \sqrt{\frac{(b g - a h) (e + f x)}{(f g - e h) (a + b x)}} \right. \\
& \left. \operatorname{EllipticPi}\left[-\frac{b (d g - c h)}{(b c - a d) h}, \operatorname{ArcSin}\left[\frac{\sqrt{b c - a d} \sqrt{g + h x}}{\sqrt{-d g + c h} \sqrt{a + b x}}\right], \frac{(b e - a f) (d g - c h)}{(b c - a d) (f g - e h)}\right] \right) / \left( b d \sqrt{b c - a d} f h^2 \sqrt{c + d x} \sqrt{e + f x} \right)
\end{aligned}$$

Result (type 4, 6648 leaves):

$$\begin{aligned}
& -\frac{1}{d^2} 2 \left( -\frac{B (c + d x)^{3/2} \left(f + \frac{d e}{c + d x} - \frac{c f}{c + d x}\right) \left(h + \frac{d g}{c + d x} - \frac{c h}{c + d x}\right) \sqrt{a + \frac{(c + d x) \left(b - \frac{b c}{c + d x}\right)}{d}}}{2 f h \sqrt{e + \frac{(c + d x) \left(f - \frac{c f}{c + d x}\right)}{d}} \sqrt{g + \frac{(c + d x) \left(h - \frac{c h}{c + d x}\right)}{d}}} + \right. \\
& \left. d (b g - a h) (d g - c h) (B f g + B e h - 2 A f h) \sqrt{c + d x} \sqrt{\left(b - \frac{b c}{c + d x} + \frac{a d}{c + d x}\right) \left(f + \frac{d e}{c + d x} - \frac{c f}{c + d x}\right) \left(h + \frac{d g}{c + d x} - \frac{c h}{c + d x}\right)} \right. \\
& \left. \sqrt{a + \frac{(c + d x) \left(b - \frac{b c}{c + d x}\right)}{d}} \left(d e \sqrt{-\frac{(b c - a d) (-d g + c h) \left(-\frac{b}{b c - a d} + \frac{1}{c + d x}\right)}{-b d g + a d h}} \left(-\frac{f}{-d e + c f} + \frac{1}{c + d x}\right)\right) \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{-\frac{h}{-dg+ch} + \frac{1}{c+dx}}{\frac{f}{-de+cf} - \frac{h}{-dg+ch}}} \left( \frac{(-bdg+ad) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(de-cf)\left(h+\frac{dg}{c+dx}-\frac{ch}{c+dx}\right)}{d(-fg+eh)}}\right], \frac{(bc-ad)(-fg+eh)}{(-de+cf)(-bg+ah)}]}{(bc-ad)(-dg+ch)} - \right. \\
& \left. \frac{b \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(de-cf)\left(h+\frac{dg}{c+dx}-\frac{ch}{c+dx}\right)}{d(-fg+eh)}}\right], \frac{(bc-ad)(-fg+eh)}{(-de+cf)(-bg+ah)}]}{bc-ad} \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-de+cf} + \frac{1}{c+dx}}{-\frac{f}{-de+cf} + \frac{h}{-dg+ch}}} \sqrt{\left(b + \frac{-bc+ad}{c+dx}\right) \left(f + \frac{de-cf}{c+dx}\right) \left(h + \frac{dg-ch}{c+dx}\right)} - \right. \\
& \left. c f \sqrt{-\frac{(bc-ad)(-dg+ch)\left(-\frac{b}{bc-ad} + \frac{1}{c+dx}\right)}{-bdg+ad} h} \right. \\
& \left( -\frac{f}{-de+cf} + \frac{1}{c+dx} \right) \sqrt{\frac{-\frac{h}{-dg+ch} + \frac{1}{c+dx}}{\frac{f}{-de+cf} - \frac{h}{-dg+ch}}} \left( \frac{(-bdg+ad) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(de-cf)\left(h+\frac{dg}{c+dx}-\frac{ch}{c+dx}\right)}{d(-fg+eh)}}\right], \frac{(bc-ad)(-fg+eh)}{(-de+cf)(-bg+ah)}]}{(bc-ad)(-dg+ch)} - \right. \\
& \left. b \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(de-cf)\left(h+\frac{dg}{c+dx}-\frac{ch}{c+dx}\right)}{d(-fg+eh)}}\right], \frac{(bc-ad)(-fg+eh)}{(-de+cf)(-bg+ah)}] \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-de+cf} + \frac{1}{c+dx}}{-\frac{f}{-de+cf} + \frac{h}{-dg+ch}}} \sqrt{\left(b + \frac{-bc+ad}{c+dx}\right) \left(f + \frac{de-cf}{c+dx}\right) \left(h + \frac{dg-ch}{c+dx}\right)} + \right. \\
& \left. f \sqrt{\frac{-\frac{b}{bc-ad} + \frac{1}{c+dx}}{-\frac{b}{bc-ad} + \frac{h}{-dg+ch}}} \sqrt{\frac{-\frac{f}{-de+cf} + \frac{1}{c+dx}}{-\frac{f}{-de+cf} + \frac{h}{-dg+ch}}} \right. \\
& \left( -\frac{h}{-dg+ch} + \frac{1}{c+dx} \right) \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(-de+cf)\left(-h-\frac{dg}{c+dx}+\frac{ch}{c+dx}\right)}{d(-fg+eh)}}\right], \frac{(bc-ad)(-fg+eh)}{(-de+cf)(-bg+ah)}] \right)
\end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{\frac{-\frac{h}{-dg+ch} + \frac{1}{c+dx}}{\frac{f}{-de+cf} - \frac{h}{-dg+ch}}} \sqrt{\left( b + \frac{-bc+ad}{c+dx} \right) \left( f + \frac{de-ch}{c+dx} \right) \left( h + \frac{dg-ch}{c+dx} \right)} \right) / \\
& \left( 2fh^2(fg-eh) \left( b - \frac{bc}{c+dx} + \frac{ad}{c+dx} \right) \sqrt{e + \frac{(c+dx)(f-\frac{cf}{c+dx})}{d}} \sqrt{g + \frac{(c+dx)(h-\frac{ch}{c+dx})}{d}} \right) - \\
& \left( d(ba-fc)(de-cf)(Bfg+Beh-2Afh)\sqrt{c+dx} \sqrt{\left( b - \frac{bc}{c+dx} + \frac{ad}{c+dx} \right) \left( f + \frac{de}{c+dx} - \frac{cf}{c+dx} \right) \left( h + \frac{dg}{c+dx} - \frac{ch}{c+dx} \right)} \right. \\
& \left. \sqrt{a + \frac{(c+dx)(b-\frac{bc}{c+dx})}{d}} \left( dg \sqrt{-\frac{(bc-ad)(-dg+ch)(-\frac{b}{bc-ad} + \frac{1}{c+dx})}{-bdg+adh}} \left( -\frac{f}{-de+cf} + \frac{1}{c+dx} \right) \right. \right. \\
& \left. \left. \sqrt{\frac{-\frac{h}{-dg+ch} + \frac{1}{c+dx}}{\frac{f}{-de+cf} - \frac{h}{-dg+ch}}} \left( -b \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(de-cf)(h+\frac{dg}{c+dx}-\frac{ch}{c+dx})}{d(-fg+eh)}}\right], \frac{(bc-ad)(-fg+eh)}{(-de+cf)(-bg+ah)}] \right. \right. \\
& \left. \left. - \frac{(b \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(de-cf)(h+\frac{dg}{c+dx}-\frac{ch}{c+dx})}{d(-fg+eh)}\right], \frac{(bc-ad)(-fg+eh)}{(-de+cf)(-bg+ah)}]]}{bc-ad} \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-de+cf} + \frac{1}{c+dx}}{\frac{f}{-de+cf} + \frac{h}{-dg+ch}}} \sqrt{\left( b + \frac{-bc+ad}{c+dx} \right) \left( f + \frac{de-ch}{c+dx} \right) \left( h + \frac{dg-ch}{c+dx} \right)} \right) - \left( ch \sqrt{-\frac{(bc-ad)(-dg+ch)(-\frac{b}{bc-ad} + \frac{1}{c+dx})}{-bdg+adh}} \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{f}{-de+cf} + \frac{1}{c+dx} \right) \sqrt{\frac{-\frac{h}{-dg+ch} + \frac{1}{c+dx}}{\frac{f}{-de+cf} - \frac{h}{-dg+ch}}} \left( \frac{(-bdg+ad) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(de-cf)(h+\frac{dg}{c+dx}-\frac{ch}{c+dx})}{d(-fg+eh)}}, \frac{(bc-ad)(-fg+eh)}{(-de+cf)(-bg+ah)}\right]} - \right. \\
& \left. \frac{b \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(de-cf)(h+\frac{dg}{c+dx}-\frac{ch}{c+dx})}{d(-fg+eh)}}, \frac{(bc-ad)(-fg+eh)}{(-de+cf)(-bg+ah)}\right]}{b c - a d} \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-de+cf} + \frac{1}{c+dx}}{\frac{f}{-de+cf} + \frac{h}{-dg+ch}}} \sqrt{\left(b + \frac{-bc+ad}{c+dx}\right) \left(f + \frac{de-cf}{c+dx}\right) \left(h + \frac{dg-ch}{c+dx}\right)} + h \sqrt{\frac{-\frac{b}{bc-ad} + \frac{1}{c+dx}}{-\frac{b}{bc-ad} + \frac{h}{-dg+ch}}} \sqrt{\frac{-\frac{f}{-de+cf} + \frac{1}{c+dx}}{-\frac{f}{-de+cf} + \frac{h}{-dg+ch}}} \right. \\
& \left. \left( -\frac{h}{-dg+ch} + \frac{1}{c+dx} \right) \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(-de+cf)(-h-\frac{dg}{c+dx}+\frac{ch}{c+dx})}{d(-fg+eh)}}, \frac{(bc-ad)(-fg+eh)}{(-de+cf)(-bg+ah)}\right]] \right) / \\
& \left( \sqrt{\frac{-\frac{h}{-dg+ch} + \frac{1}{c+dx}}{\frac{f}{-de+cf} - \frac{h}{-dg+ch}}} \sqrt{\left(b + \frac{-bc+ad}{c+dx}\right) \left(f + \frac{de-cf}{c+dx}\right) \left(h + \frac{dg-ch}{c+dx}\right)} \right) / \\
& \left( 2f^2h(fg-eh) \left(b - \frac{bc}{c+dx} + \frac{ad}{c+dx}\right) \sqrt{e + \frac{(c+dx)\left(f - \frac{cf}{c+dx}\right)}{d}} \sqrt{g + \frac{(c+dx)\left(h - \frac{ch}{c+dx}\right)}{d}} \right) - \\
& \frac{1}{2f^2h^2 \left(b - \frac{bc}{c+dx} + \frac{ad}{c+dx}\right) \sqrt{e + \frac{(c+dx)\left(f - \frac{cf}{c+dx}\right)}{d}} \sqrt{g + \frac{(c+dx)\left(h - \frac{ch}{c+dx}\right)}{d}}} \\
& \left( bBdfg + bBdeh + bBc fh - 2Abdfh - aBdfh \right) \sqrt{c+dx} \\
& \sqrt{\left(b - \frac{bc}{c+dx} + \frac{ad}{c+dx}\right) \left(f + \frac{de}{c+dx} - \frac{cf}{c+dx}\right) \left(h + \frac{dg}{c+dx} - \frac{ch}{c+dx}\right)} \\
& \sqrt{a + \frac{(c+dx)\left(b - \frac{bc}{c+dx}\right)}{d}}
\end{aligned}$$

$$\begin{aligned}
& \left( d^2 e g \sqrt{-\frac{(b c - a d) (-d g + c h) \left(-\frac{b}{b c - a d} + \frac{1}{c + d x}\right)}{-b d g + a d h}} \left(-\frac{f}{-d e + c f} + \frac{1}{c + d x}\right) \sqrt{\frac{-\frac{h}{-d g + c h} + \frac{1}{c + d x}}{\frac{f}{-d e + c f} - \frac{h}{-d g + c h}}} \right. \\
& \left. \left( -b d g + a d h \right) \text{EllipticE} \left[ \text{ArcSin} \left[ \sqrt{\frac{(d e - c f) \left(h + \frac{d g}{c + d x} - \frac{c h}{c + d x}\right)}{d (-f g + e h)}} \right], \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)} \right] \right. \\
& \left. \left. - \frac{b \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(d e - c f) \left(h + \frac{d g}{c + d x} - \frac{c h}{c + d x}\right)}{d (-f g + e h)}} \right], \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)} \right]}{b c - a d} \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-d e + c f} + \frac{1}{c + d x}}{-\frac{f}{-d e + c f} + \frac{h}{-d g + c h}}} \sqrt{\left(b + \frac{-b c + a d}{c + d x}\right) \left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} - c d f g \sqrt{-\frac{(b c - a d) (-d g + c h) \left(-\frac{b}{b c - a d} + \frac{1}{c + d x}\right)}{-b d g + a d h}} \right. \\
& \left. \left( -\frac{f}{-d e + c f} + \frac{1}{c + d x} \right) \sqrt{\frac{-\frac{h}{-d g + c h} + \frac{1}{c + d x}}{\frac{f}{-d e + c f} - \frac{h}{-d g + c h}}} \left( -b d g + a d h \right) \text{EllipticE} \left[ \text{ArcSin} \left[ \sqrt{\frac{(d e - c f) \left(h + \frac{d g}{c + d x} - \frac{c h}{c + d x}\right)}{d (-f g + e h)}} \right], \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)} \right] \right. \\
& \left. \left. - b \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(d e - c f) \left(h + \frac{d g}{c + d x} - \frac{c h}{c + d x}\right)}{d (-f g + e h)}} \right], \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)} \right] \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-d e + c f} + \frac{1}{c + d x}}{-\frac{f}{-d e + c f} + \frac{h}{-d g + c h}}} \sqrt{\left(b + \frac{-b c + a d}{c + d x}\right) \left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} - c d e h \sqrt{-\frac{(b c - a d) (-d g + c h) \left(-\frac{b}{b c - a d} + \frac{1}{c + d x}\right)}{-b d g + a d h}} \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{f}{-de+cf} + \frac{1}{c+dx} \right) \sqrt{\frac{-\frac{h}{-dg+ch} + \frac{1}{c+dx}}{\frac{f}{-de+cf} - \frac{h}{-dg+ch}}} \left( \frac{(-bdg+ad) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(de-cf)(h+\frac{dg}{c+dx}-\frac{ch}{c+dx})}{d(-fg+eh)}}, \frac{(bc-ad)(-fg+eh)}{(-de+cf)(-bg+ah)}\right]}]}{(bc-ad)(-dg+ch)} \right. \\
& \left. - \frac{b \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(de-cf)(h+\frac{dg}{c+dx}-\frac{ch}{c+dx})}{d(-fg+eh)}}, \frac{(bc-ad)(-fg+eh)}{(-de+cf)(-bg+ah)}\right]]}{bc-ad} \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-de+cf} + \frac{1}{c+dx}}{\frac{f}{-de+cf} + \frac{h}{-dg+ch}}} \sqrt{\left( b + \frac{-bc+ad}{c+dx} \right) \left( f + \frac{de-cf}{c+dx} \right) \left( h + \frac{dg-ch}{c+dx} \right)} + c^2 fh \sqrt{-\frac{(bc-ad)(-dg+ch)\left(-\frac{b}{bc-ad} + \frac{1}{c+dx}\right)}{-bdg+ad}} \right. \\
& \left. - \left( -\frac{f}{-de+cf} + \frac{1}{c+dx} \right) \sqrt{\frac{-\frac{h}{-dg+ch} + \frac{1}{c+dx}}{\frac{f}{-de+cf} - \frac{h}{-dg+ch}}} \left( \frac{(-bdg+ad) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(de-cf)(h+\frac{dg}{c+dx}-\frac{ch}{c+dx})}{d(-fg+eh)}}, \frac{(bc-ad)(-fg+eh)}{(-de+cf)(-bg+ah)}\right]}]}{(bc-ad)(-dg+ch)} \right. \right. \\
& \left. \left. - \frac{b \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(de-cf)(h+\frac{dg}{c+dx}-\frac{ch}{c+dx})}{d(-fg+eh)}}, \frac{(bc-ad)(-fg+eh)}{(-de+cf)(-bg+ah)}\right]]}{bc-ad} \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-de+cf} + \frac{1}{c+dx}}{\frac{f}{-de+cf} + \frac{h}{-dg+ch}}} \sqrt{\left( b + \frac{-bc+ad}{c+dx} \right) \left( f + \frac{de-cf}{c+dx} \right) \left( h + \frac{dg-ch}{c+dx} \right)} + dfg \sqrt{\frac{-\frac{b}{bc-ad} + \frac{1}{c+dx}}{-\frac{b}{bc-ad} + \frac{h}{-dg+ch}}} \sqrt{\frac{-\frac{f}{-de+cf} + \frac{1}{c+dx}}{-\frac{f}{-de+cf} + \frac{h}{-dg+ch}}} \right. \\
& \left. - \left( -\frac{h}{-dg+ch} + \frac{1}{c+dx} \right) \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(-de+cf)(-h-\frac{dg}{c+dx}+\frac{ch}{c+dx})}{d(-fg+eh)}}, \frac{(bc-ad)(-fg+eh)}{(-de+cf)(-bg+ah)}\right]] \right) /
\end{aligned}$$

$$\begin{aligned}
& \left( d e h \sqrt{\frac{-\frac{b}{b c-a d} + \frac{1}{c+d x}}{-\frac{b}{b c-a d} + \frac{h}{-d g+c h}}} \sqrt{\frac{-\frac{f}{-d e+c f} + \frac{1}{c+d x}}{-\frac{f}{-d e+c f} + \frac{h}{-d g+c h}}} \left( -\frac{h}{-d g+c h} + \frac{1}{c+d x} \right) \text{EllipticF}[\text{ArcSin}\left[ \sqrt{\frac{(-d e+c f) \left( -h - \frac{d g}{c+d x} + \frac{c h}{c+d x} \right)}{d (-f g+e h)}} \right], \right. \\
& \left. \frac{(b c-a d) (-f g+e h)}{(-d e+c f) (-b g+a h)} \right] \Bigg) \Bigg/ \left( \sqrt{\frac{-\frac{h}{-d g+c h} + \frac{1}{c+d x}}{\frac{f}{-d e+c f} - \frac{h}{-d g+c h}}} \sqrt{\left( b + \frac{-b c+a d}{c+d x} \right) \left( f + \frac{d e-c f}{c+d x} \right) \left( h + \frac{d g-c h}{c+d x} \right)} - \right. \\
& \left. 2 c f h \sqrt{\frac{-\frac{b}{b c-a d} + \frac{1}{c+d x}}{-\frac{b}{b c-a d} + \frac{h}{-d g+c h}}} \sqrt{\frac{-\frac{f}{-d e+c f} + \frac{1}{c+d x}}{-\frac{f}{-d e+c f} + \frac{h}{-d g+c h}}} \left( -\frac{h}{-d g+c h} + \frac{1}{c+d x} \right) \text{EllipticF}[\text{ArcSin}\left[ \sqrt{\frac{(-d e+c f) \left( -h - \frac{d g}{c+d x} + \frac{c h}{c+d x} \right)}{d (-f g+e h)}} \right], \right. \\
& \left. \frac{(b c-a d) (-f g+e h)}{(-d e+c f) (-b g+a h)} \right] \Bigg) \Bigg/ \left( \sqrt{\frac{-\frac{h}{-d g+c h} + \frac{1}{c+d x}}{\frac{f}{-d e+c f} - \frac{h}{-d g+c h}}} \sqrt{\left( b + \frac{-b c+a d}{c+d x} \right) \left( f + \frac{d e-c f}{c+d x} \right) \left( h + \frac{d g-c h}{c+d x} \right)} - \right. \\
& \left. f (-d g+c h) \left( -\frac{f}{-d e+c f} + \frac{h}{-d g+c h} \right) \sqrt{\frac{-\frac{b}{b c-a d} + \frac{1}{c+d x}}{-\frac{b}{b c-a d} + \frac{h}{-d g+c h}}} \sqrt{-\frac{\left( -\frac{f}{-d e+c f} + \frac{1}{c+d x} \right) \left( -\frac{h}{-d g+c h} + \frac{1}{c+d x} \right)}{\left( -\frac{f}{-d e+c f} + \frac{h}{-d g+c h} \right)^2} \text{EllipticPi}\left[ -\frac{-d f g+d e h}{(-d e+c f) h}, \right.} \right. \\
& \left. \left. \text{ArcSin}\left[ \sqrt{\frac{(-d e+c f) \left( -h - \frac{d g}{c+d x} + \frac{c h}{c+d x} \right)}{d (-f g+e h)}} \right], \frac{(b c-a d) (-f g+e h)}{(-d e+c f) (-b g+a h)} \right] \Bigg) \Bigg/ \left( \sqrt{\left( b + \frac{-b c+a d}{c+d x} \right) \left( f + \frac{d e-c f}{c+d x} \right) \left( h + \frac{d g-c h}{c+d x} \right)} \right) \right)
\end{aligned}$$

Problem 9: Result more than twice size of optimal antiderivative.

$$\int \frac{A+Bx}{(a+bx)^{3/2} \sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x}} dx$$

Optimal (type 4, 606 leaves, 7 steps):

$$\begin{aligned}
& \frac{2 (A b - a B) d \sqrt{a+b x} \sqrt{e+f x} \sqrt{g+h x}}{(b c - a d) (b e - a f) (b g - a h) \sqrt{c+d x}} - \frac{2 b (A b - a B) \sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x}}{(b c - a d) (b e - a f) (b g - a h) \sqrt{a+b x}} - \\
& \left( 2 (A b - a B) \sqrt{d g - c h} \sqrt{f g - e h} \sqrt{a+b x} \sqrt{-\frac{(d e - c f) (g+h x)}{(f g - e h) (c+d x)}} \text{EllipticE}[\text{ArcSin}\left[\frac{\sqrt{d g - c h} \sqrt{e+f x}}{\sqrt{f g - e h} \sqrt{c+d x}}\right], \frac{(b c - a d) (f g - e h)}{(b e - a f) (d g - c h)}] \right) / \\
& \left( (b c - a d) (b e - a f) (b g - a h) \sqrt{\frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}} \sqrt{g+h x} \right) + \\
& \frac{2 (B c - A d) \sqrt{\frac{(b e - a f) (c+d x)}{(d e - c f) (a+b x)}} \sqrt{g+h x} \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{b g - a h} \sqrt{e+f x}}{\sqrt{f g - e h} \sqrt{a+b x}}\right], -\frac{(b c - a d) (f g - e h)}{(d e - c f) (b g - a h)}]}{(b c - a d) \sqrt{b g - a h} \sqrt{f g - e h} \sqrt{c+d x} \sqrt{-\frac{(b e - a f) (g+h x)}{(f g - e h) (a+b x)}}}
\end{aligned}$$

Result (type 4, 1749 leaves):

$$\begin{aligned}
& -\frac{2 b (A b - a B) \sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x}}{(b c - a d) (b e - a f) (b g - a h) \sqrt{a+b x}} + \\
& \frac{1}{b^2 (-b c + a d) (-b e + a f) (-b g + a h)} \left( -\frac{2 (A b - a B) (a+b x)^{5/2} \left(d + \frac{b c}{a+b x} - \frac{a d}{a+b x}\right) \left(f + \frac{b e}{a+b x} - \frac{a f}{a+b x}\right) \left(h + \frac{b g}{a+b x} - \frac{a h}{a+b x}\right)}{\sqrt{c + \frac{(a+b x) (d - \frac{a d}{a+b x})}{b}} \sqrt{e + \frac{(a+b x) (f - \frac{a f}{a+b x})}{b}} \sqrt{g + \frac{(a+b x) (h - \frac{a h}{a+b x})}{b}}} - \right. \\
& \frac{1}{\sqrt{c + \frac{(a+b x) (d - \frac{a d}{a+b x})}{b}} \sqrt{e + \frac{(a+b x) (f - \frac{a f}{a+b x})}{b}} \sqrt{g + \frac{(a+b x) (h - \frac{a h}{a+b x})}{b}}} 2 (b c - a d) (b e - a f) (b g - a h) (a+b x)^{3/2} \\
& \left. \sqrt{\left(d + \frac{b c}{a+b x} - \frac{a d}{a+b x}\right) \left(f + \frac{b e}{a+b x} - \frac{a f}{a+b x}\right) \left(h + \frac{b g}{a+b x} - \frac{a h}{a+b x}\right)} \left( -\frac{A b \sqrt{\frac{(b c - a d) (b g - a h) \left(-\frac{d}{-b c + a d} + \frac{1}{a+b x}\right)}{b d g - b c h}}}{b d g - b c h} \right. \right. \\
& \left. \left. \left( -\frac{f}{-b e + a f} + \frac{1}{a+b x} \right) \sqrt{\frac{-\frac{h}{-b g + a h} + \frac{1}{a+b x}}{\frac{f}{-b e + a f} - \frac{h}{-b g + a h}}} \right. \right. \\
& \left. \left. - \frac{(b d g - b c h) \text{EllipticE}[\text{ArcSin}\left[\sqrt{\frac{(b e - a f) (h + \frac{b g}{a+b x} - \frac{a h}{a+b x})}{b (-f g + e h)}}\right], \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}]}{(b c - a d) (b g - a h)} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) (-f g+e h)}{(-b e+a f) (-d g+c h)}}\right] }{-b c+a d} \Bigg) \Bigg) / \\
& \left( \sqrt{\frac{-\frac{f}{-b e+a f}+\frac{1}{a+b x}}{-\frac{f}{-b e+a f}+\frac{h}{-b g+a h}}} \sqrt{\left(d+\frac{b c-a d}{a+b x}\right)\left(f+\frac{b e-a f}{a+b x}\right)\left(h+\frac{b g-a h}{a+b x}\right)} \right) + \left( a B \sqrt{\frac{(b c-a d) (b g-a h) \left(-\frac{d}{-b c+a d}+\frac{1}{a+b x}\right)}{b d g-b c h}} \right. \\
& \left( -\frac{f}{-b e+a f}+\frac{1}{a+b x} \right) \sqrt{\frac{-\frac{h}{-b g+a h}+\frac{1}{a+b x}}{\frac{f}{-b e+a f}-\frac{h}{-b g+a h}}} \left( -\frac{(b d g-b c h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) (-f g+e h)}{(-b e+a f) (-d g+c h)}}}{(b c-a d) (b g-a h)} \right. - \\
& \left. \left. \left. d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) (-f g+e h)}{(-b e+a f) (-d g+c h)}}\right] \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-b e+a f}+\frac{1}{a+b x}}{-\frac{f}{-b e+a f}+\frac{h}{-b g+a h}}} \sqrt{\left(d+\frac{b c-a d}{a+b x}\right)\left(f+\frac{b e-a f}{a+b x}\right)\left(h+\frac{b g-a h}{a+b x}\right)} \right) - \left( B \sqrt{\frac{-\frac{d}{-b c+a d}+\frac{1}{a+b x}}{-\frac{d}{-b c+a d}+\frac{h}{-b g+a h}}} \sqrt{\frac{-\frac{f}{-b e+a f}+\frac{1}{a+b x}}{-\frac{f}{-b e+a f}+\frac{h}{-b g+a h}}} \right. \\
& \left( -\frac{h}{-b g+a h}+\frac{1}{a+b x} \right) \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(-b e+a f) \left(-h-\frac{b g}{a+b x}+\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) (-f g+e h)}{(-b e+a f) (-d g+c h)} \Bigg) \Bigg) / \\
& \left( \sqrt{\frac{-\frac{h}{-b g+a h}+\frac{1}{a+b x}}{\frac{f}{-b e+a f}-\frac{h}{-b g+a h}}} \sqrt{\left(d+\frac{b c-a d}{a+b x}\right)\left(f+\frac{b e-a f}{a+b x}\right)\left(h+\frac{b g-a h}{a+b x}\right)} \right) \Bigg)
\end{aligned}$$

**Problem 10:** Result more than twice size of optimal antiderivative.

$$\int \frac{A + Bx}{(ax + bx)^{5/2} \sqrt{c+dx} \sqrt{e+fx} \sqrt{g+hx}} dx$$

Optimal (type 4, 1081 leaves, 8 steps):

$$\begin{aligned}
& \left( 2d(3a^3Bdfh + b^3(3Bceg - 2A(deg + cfg + ceh))) - \right. \\
& \quad a^2b(B(deg + cfg + ceh) - 4A(dfh + deh + cfh)) - a^2b(6Adfh + B(dfh + deh + cfh)) \sqrt{a+bx} \sqrt{e+fx} \sqrt{g+hx} \Big) / \\
& \quad \left( 3(bc-ad)^2(be-af)^2(bg-ah)^2 \sqrt{c+dx} \right) - \frac{2b(Ab-abB) \sqrt{c+dx} \sqrt{e+fx} \sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}} - \\
& \left( 2b(3a^3Bdfh + b^3(3Bceg - 2A(deg + cfg + ceh))) - a^2b(B(deg + cfg + ceh) - 4A(dfh + deh + cfh)) - \right. \\
& \quad a^2b(6Adfh + B(dfh + deh + cfh)) \sqrt{c+dx} \sqrt{e+fx} \sqrt{g+hx} \Big) / \left( 3(bc-ad)^2(be-af)^2(bg-ah)^2 \sqrt{a+bx} \right) - \\
& \left( 2\sqrt{dg-ch} \sqrt{fg-eh} (3a^3Bdfh + b^3(3Bceg - 2A(deg + cfg + ceh))) - ab^2(B(deg + cfg + ceh) - 4A(dfh + deh + cfh)) - \right. \\
& \quad a^2b(6Adfh + B(dfh + deh + cfh)) \sqrt{a+bx} \sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}} \\
& \quad \text{EllipticE}[\text{ArcSin}\left[\frac{\sqrt{dg-ch} \sqrt{e+fx}}{\sqrt{fg-eh} \sqrt{c+dx}}\right], \frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}] \Big) / \left( 3(bc-ad)^2(be-af)^2(bg-ah)^2 \sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}} \sqrt{g+hx} \right) - \\
& \left( 2(3a^2d(bc-Ad)f h + b^2(3Bcddeg - A(2d^2eg - c^2fh + cd(fg+eh))) + ab(3Ad^2(fg+eh) - B(d^2eg + c^2fh + 2cd(fg+eh))) \right. \\
& \quad \left. \sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}} \sqrt{g+hx} \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{bg-ah} \sqrt{e+fx}}{\sqrt{fg-eh} \sqrt{a+bx}}\right], -\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}] \right) / \\
& \left( 3(bc-ad)^2(be-af)(bg-ah)^{3/2} \sqrt{fg-eh} \sqrt{c+dx} \sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}} \right)
\end{aligned}$$

Result (type 4, 10637 leaves):

$$\begin{aligned}
& \sqrt{a+bx} \sqrt{c+dx} \sqrt{e+fx} \sqrt{g+hx} \\
& \left( -\frac{2b(Ab-abB)}{3(bc-ad)(be-af)(bg-ah)(a+bx)^2} - (2b(3b^3Bceg - 2Ab^3deg - ab^2Bdeg - 2Ab^3cfg - ab^2Bcfg + 4aAb^2dfg - \right. \\
& \quad \left. a^2bBdfg - 2Ab^3ceh - ab^2Bceh + 4aAb^2deh - a^2bBdeh + 4aAb^2cfg - a^2bBcfg - 6a^2Abdfh + 3a^3Bdfh)) \right) /
\end{aligned}$$

$$\begin{aligned}
& \left( 3 (b c - a d)^2 (b e - a f)^2 (b g - a h)^2 (a + b x) \right) + \frac{1}{3 b^2 (-b c + a d)^2 (-b e + a f)^2 (-b g + a h)^2} \\
& 2 \left( \left( 3 b^3 B c e g - 2 A b^3 d e g - a b^2 B d e g - 2 A b^3 c f g - a b^2 B c f g + 4 a A b^2 d f g - a^2 b B d f g - 2 A b^3 c e h - a b^2 B c e h + 4 a A b^2 d e h - a^2 b B d e h + \right. \right. \\
& \quad \left. \left. 4 a A b^2 c f h - a^2 b B c f h - 6 a^2 A b d f h + 3 a^3 B d f h \right) (a + b x)^{5/2} \left( d + \frac{b c}{a + b x} - \frac{a d}{a + b x} \right) \left( f + \frac{b e}{a + b x} - \frac{a f}{a + b x} \right) \left( h + \frac{b g}{a + b x} - \frac{a h}{a + b x} \right) \right) / \\
& \left( \sqrt{c + \frac{(a + b x) (d - \frac{a d}{a + b x})}{b}} \sqrt{e + \frac{(a + b x) (f - \frac{a f}{a + b x})}{b}} \sqrt{g + \frac{(a + b x) (h - \frac{a h}{a + b x})}{b}} \right) + \\
& \frac{1}{\sqrt{c + \frac{(a + b x) (d - \frac{a d}{a + b x})}{b}} \sqrt{e + \frac{(a + b x) (f - \frac{a f}{a + b x})}{b}} \sqrt{g + \frac{(a + b x) (h - \frac{a h}{a + b x})}{b}}} (b c - a d) (b e - a f) (b g - a h) (a + b x)^{3/2} \\
& \sqrt{\left( d + \frac{b c}{a + b x} - \frac{a d}{a + b x} \right) \left( f + \frac{b e}{a + b x} - \frac{a f}{a + b x} \right) \left( h + \frac{b g}{a + b x} - \frac{a h}{a + b x} \right)} \left( - \left( 3 b^3 B c e g \sqrt{\frac{(b c - a d) (b g - a h) \left( -\frac{d}{-b c + a d} + \frac{1}{a + b x} \right)}{b d g - b c h}} \right. \right. \\
& \left. \left. \left( - \frac{f}{-b e + a f} + \frac{1}{a + b x} \right) \sqrt{\frac{-\frac{h}{-b g + a h} + \frac{1}{a + b x}}{\frac{f}{-b e + a f} - \frac{h}{-b g + a h}}} \right. \right. \\
& \left. \left. - \frac{(b d g - b c h) \text{EllipticE}[\text{ArcSin}\left[\sqrt{\frac{(b e - a f) \left( h + \frac{b g}{a + b x} - \frac{a h}{a + b x} \right)}{b (-f g + e h)}}\right], \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}] }{(b c - a d) (b g - a h)} \right. \right. \\
& \left. \left. \left. \frac{d \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{(b e - a f) \left( h + \frac{b g}{a + b x} - \frac{a h}{a + b x} \right)}{b (-f g + e h)}}\right], \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}]}{-b c + a d} \right) \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-b e + a f} + \frac{1}{a + b x}}{\frac{f}{-b e + a f} + \frac{h}{-b g + a h}}} \sqrt{\left( d + \frac{b c - a d}{a + b x} \right) \left( f + \frac{b e - a f}{a + b x} \right) \left( h + \frac{b g - a h}{a + b x} \right)} \right) + \left( 2 A b^3 d e g \sqrt{\frac{(b c - a d) (b g - a h) \left( -\frac{d}{-b c + a d} + \frac{1}{a + b x} \right)}{b d g - b c h}} \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{f}{-b e + a f} + \frac{1}{a + b x} \right) \sqrt{\frac{-\frac{h}{-b g + a h} + \frac{1}{a + b x}}{\frac{f}{-b e + a f} - \frac{h}{-b g + a h}}} \left( -\frac{(b d g - b c h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e - a f) \left(h + \frac{b g}{a + b x} - \frac{a h}{a + b x}\right)}{b (-f g + e h)}}, \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}\right]}]{(b c - a d) (b g - a h)} \right. \\
& \left. \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e - a f) \left(h + \frac{b g}{a + b x} - \frac{a h}{a + b x}\right)}{b (-f g + e h)}}, \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}\right]}]{-b c + a d} \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-b e + a f} + \frac{1}{a + b x}}{\frac{f}{-b e + a f} + \frac{h}{-b g + a h}}} \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right) \left(h + \frac{b g - a h}{a + b x}\right)} + \left(a b^2 B d e g \sqrt{\frac{(b c - a d) (b g - a h) \left(-\frac{d}{-b c + a d} + \frac{1}{a + b x}\right)}{b d g - b c h}} \right. \right. \\
& \left. \left. \left( -\frac{f}{-b e + a f} + \frac{1}{a + b x} \right) \sqrt{\frac{-\frac{h}{-b g + a h} + \frac{1}{a + b x}}{\frac{f}{-b e + a f} - \frac{h}{-b g + a h}}} \left( -\frac{(b d g - b c h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e - a f) \left(h + \frac{b g}{a + b x} - \frac{a h}{a + b x}\right)}{b (-f g + e h)}}, \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}\right]}]{(b c - a d) (b g - a h)} \right. \right. \\
& \left. \left. \left. \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e - a f) \left(h + \frac{b g}{a + b x} - \frac{a h}{a + b x}\right)}{b (-f g + e h)}}, \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}\right]}]{-b c + a d} \right) \right) / \right. \\
& \left( \sqrt{\frac{-\frac{f}{-b e + a f} + \frac{1}{a + b x}}{\frac{f}{-b e + a f} + \frac{h}{-b g + a h}}} \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right) \left(h + \frac{b g - a h}{a + b x}\right)} + \left(2 A b^3 c f g \sqrt{\frac{(b c - a d) (b g - a h) \left(-\frac{d}{-b c + a d} + \frac{1}{a + b x}\right)}{b d g - b c h}} \right. \right. \\
& \left. \left. \left( -\frac{f}{-b e + a f} + \frac{1}{a + b x} \right) \sqrt{\frac{-\frac{h}{-b g + a h} + \frac{1}{a + b x}}{\frac{f}{-b e + a f} - \frac{h}{-b g + a h}}} \left( -\frac{(b d g - b c h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e - a f) \left(h + \frac{b g}{a + b x} - \frac{a h}{a + b x}\right)}{b (-f g + e h)}}, \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}\right]}]{(b c - a d) (b g - a h)} \right. \right. \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) (-f g+e h)}{(-b e+a f) (-d g+c h)}}}{-b c+a d}\right\} / \\
& \left( \sqrt{\frac{-\frac{f}{-b e+a f}+\frac{1}{a+b x}}{-\frac{f}{-b e+a f}+\frac{h}{-b g+a h}}} \sqrt{\left(d+\frac{b c-a d}{a+b x}\right)\left(f+\frac{b e-a f}{a+b x}\right)\left(h+\frac{b g-a h}{a+b x}\right)} + \left(a b^2 b c f g \sqrt{\frac{(b c-a d) (b g-a h) \left(-\frac{d}{-b c+a d}+\frac{1}{a+b x}\right)}{b d g-b c h}} \right. \right. \\
& \left. \left. - \frac{(b d g-b c h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) (-f g+e h)}{(-b e+a f) (-d g+c h)}}}{(b c-a d) (b g-a h)} \right. \right. - \\
& \left. \left. \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) (-f g+e h)}{(-b e+a f) (-d g+c h)}}}{-b c+a d}\right\} / \right. \\
& \left( \sqrt{\frac{-\frac{f}{-b e+a f}+\frac{1}{a+b x}}{-\frac{f}{-b e+a f}+\frac{h}{-b g+a h}}} \sqrt{\left(d+\frac{b c-a d}{a+b x}\right)\left(f+\frac{b e-a f}{a+b x}\right)\left(h+\frac{b g-a h}{a+b x}\right)} - \left(4 a A b^2 d f g \sqrt{\frac{(b c-a d) (b g-a h) \left(-\frac{d}{-b c+a d}+\frac{1}{a+b x}\right)}{b d g-b c h}} \right. \right. \\
& \left. \left. - \frac{(b d g-b c h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) (-f g+e h)}{(-b e+a f) (-d g+c h)}}}{(b c-a d) (b g-a h)} \right. \right. - \\
& \left. \left. \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) (-f g+e h)}{(-b e+a f) (-d g+c h)}}}{-b c+a d}\right\} / \right)
\end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) + \left( a^2 b B d f g \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+b x}\right)}{bdg-bch}} \right. \\
& \left( -\frac{f}{-be+af} + \frac{1}{a+b x} \right) \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+b x}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \left( -\frac{(bdg-bch) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+b x}-\frac{ah}{a+b x})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{(bc-ad)(bg-ah)} \right. \\
& \left. \left. \left. d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+b x}-\frac{ah}{a+b x})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) + \left( 2 A b^3 c e h \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+b x}\right)}{bdg-bch}} \right. \\
& \left( -\frac{f}{-be+af} + \frac{1}{a+b x} \right) \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+b x}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \left( -\frac{(bdg-bch) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+b x}-\frac{ah}{a+b x})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{(bc-ad)(bg-ah)} \right. \\
& \left. \left. \left. d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+b x}-\frac{ah}{a+b x})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) + \left( a b^2 B c e h \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+b x}\right)}{bdg-bch}} \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{f}{-be+af} + \frac{1}{a+bx} \right) \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+bx}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \left( -\frac{(bdg-bch) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{(bc-ad)(bg-ah)} \right. \\
& \left. d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+bx}}{\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+bx}\right) \left(f + \frac{be-af}{a+bx}\right) \left(h + \frac{bg-ah}{a+bx}\right)} \right) - \left( 4ab^2deh \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+bx}\right)}{bdg-bch}} \right. \\
& \left. \left( -\frac{f}{-be+af} + \frac{1}{a+bx} \right) \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+bx}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \left( -\frac{(bdg-bch) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{(bc-ad)(bg-ah)} \right. \right. \\
& \left. \left. d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+bx}}{\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+bx}\right) \left(f + \frac{be-af}{a+bx}\right) \left(h + \frac{bg-ah}{a+bx}\right)} \right) + \left( a^2 b B deh \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+bx}\right)}{bdg-bch}} \right. \\
& \left. \left( -\frac{f}{-be+af} + \frac{1}{a+bx} \right) \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+bx}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \left( -\frac{(bdg-bch) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{(bc-ad)(bg-ah)} \right. \right. \\
& \left. \left. d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) (-f g+e h)}{(-b e+a f) (-d g+c h)}}]{-b c+a d}\right\} / \\
& \left( \sqrt{\frac{-\frac{f}{-b e+a f}+\frac{1}{a+b x}}{-\frac{f}{-b e+a f}+\frac{h}{-b g+a h}}} \sqrt{\left(d+\frac{b c-a d}{a+b x}\right)\left(f+\frac{b e-a f}{a+b x}\right)\left(h+\frac{b g-a h}{a+b x}\right)} - \left(4 a A b^2 c f h \sqrt{\frac{(b c-a d) (b g-a h) \left(-\frac{d}{-b c+a d}+\frac{1}{a+b x}\right)}{b d g-b c h}} \right.\right. \\
& \left. \left. \left(-\frac{f}{-b e+a f}+\frac{1}{a+b x}\right) \sqrt{\frac{-\frac{h}{-b g+a h}+\frac{1}{a+b x}}{\frac{f}{-b e+a f}-\frac{h}{-b g+a h}}} \left(-\frac{(b d g-b c h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) (-f g+e h)}{(-b e+a f) (-d g+c h)}}}{(b c-a d) (b g-a h)} - \right.\right. \right. \\
& \left. \left. \left.d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) (-f g+e h)}{(-b e+a f) (-d g+c h)}}\right]\right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-b e+a f}+\frac{1}{a+b x}}{-\frac{f}{-b e+a f}+\frac{h}{-b g+a h}}} \sqrt{\left(d+\frac{b c-a d}{a+b x}\right)\left(f+\frac{b e-a f}{a+b x}\right)\left(h+\frac{b g-a h}{a+b x}\right)} + \left(a^2 b B c f h \sqrt{\frac{(b c-a d) (b g-a h) \left(-\frac{d}{-b c+a d}+\frac{1}{a+b x}\right)}{b d g-b c h}} \right.\right. \\
& \left. \left. \left(-\frac{f}{-b e+a f}+\frac{1}{a+b x}\right) \sqrt{\frac{-\frac{h}{-b g+a h}+\frac{1}{a+b x}}{\frac{f}{-b e+a f}-\frac{h}{-b g+a h}}} \left(-\frac{(b d g-b c h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) (-f g+e h)}{(-b e+a f) (-d g+c h)}}}{(b c-a d) (b g-a h)} - \right.\right. \right. \\
& \left. \left. \left.d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) (-f g+e h)}{(-b e+a f) (-d g+c h)}}\right]\right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) + \left( 6 a^2 A b d f h \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+b x}\right)}{bdg-bch}} \right. \\
& \left( -\frac{f}{-be+af} + \frac{1}{a+b x} \right) \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+b x}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \left( -\frac{(bdg-bch) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)\left(h+\frac{bg}{a+b x}-\frac{ah}{a+b x}\right)}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{(bc-ad)(bg-ah)} \right. \\
& \left. \left. d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)\left(h+\frac{bg}{a+b x}-\frac{ah}{a+b x}\right)}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) - \left( 3 a^3 B d f h \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+b x}\right)}{bdg-bch}} \right. \\
& \left( -\frac{f}{-be+af} + \frac{1}{a+b x} \right) \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+b x}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \left( -\frac{(bdg-bch) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)\left(h+\frac{bg}{a+b x}-\frac{ah}{a+b x}\right)}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{(bc-ad)(bg-ah)} \right. \\
& \left. \left. d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)\left(h+\frac{bg}{a+b x}-\frac{ah}{a+b x}\right)}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) + \left( A b^2 d f g \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+b x}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{h}{-bg + ah} + \frac{1}{a + bx} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-be + af) \left( -h - \frac{bg}{a+bx} + \frac{ah}{a+bx} \right)}{b (-fg + eh)}} \right], \frac{(-bc + ad) (-fg + eh)}{(-be + af) (-dg + ch)} \right] \Bigg] \\
& \left( \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+bx}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \sqrt{\left( d + \frac{bc - ad}{a + bx} \right) \left( f + \frac{be - af}{a + bx} \right) \left( h + \frac{bg - ah}{a + bx} \right)} \right) - \left( abBdfg \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+bx}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+bx}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \right. \\
& \left. \left( -\frac{h}{-bg + ah} + \frac{1}{a + bx} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-be + af) \left( -h - \frac{bg}{a+bx} + \frac{ah}{a+bx} \right)}{b (-fg + eh)}} \right], \frac{(-bc + ad) (-fg + eh)}{(-be + af) (-dg + ch)} \right] \right] \\
& \left( \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+bx}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \sqrt{\left( d + \frac{bc - ad}{a + bx} \right) \left( f + \frac{be - af}{a + bx} \right) \left( h + \frac{bg - ah}{a + bx} \right)} \right) + \left( Ab^2deh \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+bx}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+bx}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \right. \\
& \left. \left( -\frac{h}{-bg + ah} + \frac{1}{a + bx} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-be + af) \left( -h - \frac{bg}{a+bx} + \frac{ah}{a+bx} \right)}{b (-fg + eh)}} \right], \frac{(-bc + ad) (-fg + eh)}{(-be + af) (-dg + ch)} \right] \right] \\
& \left( \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+bx}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \sqrt{\left( d + \frac{bc - ad}{a + bx} \right) \left( f + \frac{be - af}{a + bx} \right) \left( h + \frac{bg - ah}{a + bx} \right)} \right) - \left( abBdeh \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+bx}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+bx}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \right. \\
& \left. \left( -\frac{h}{-bg + ah} + \frac{1}{a + bx} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-be + af) \left( -h - \frac{bg}{a+bx} + \frac{ah}{a+bx} \right)}{b (-fg + eh)}} \right], \frac{(-bc + ad) (-fg + eh)}{(-be + af) (-dg + ch)} \right] \right] \\
& \left( \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+bx}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \sqrt{\left( d + \frac{bc - ad}{a + bx} \right) \left( f + \frac{be - af}{a + bx} \right) \left( h + \frac{bg - ah}{a + bx} \right)} \right) + \left( Ab^2cfh \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+bx}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+bx}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \right. \\
& \left. \left( -\frac{h}{-bg + ah} + \frac{1}{a + bx} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-be + af) \left( -h - \frac{bg}{a+bx} + \frac{ah}{a+bx} \right)}{b (-fg + eh)}} \right], \frac{(-bc + ad) (-fg + eh)}{(-be + af) (-dg + ch)} \right] \right]
\end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{\frac{-\frac{h}{bg+ah} + \frac{1}{a+bx}}{\frac{f}{-be+af} - \frac{h}{bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+bx}\right) \left(f + \frac{be-af}{a+bx}\right) \left(h + \frac{bg-ah}{a+bx}\right)} \right) - \left( abBc fh \sqrt{\frac{-\frac{d}{bc+ad} + \frac{1}{a+bx}}{-\frac{d}{bc+ad} + \frac{h}{bg+ah}}} \sqrt{\frac{-\frac{f}{be+af} + \frac{1}{a+bx}}{-\frac{f}{be+af} + \frac{h}{bg+ah}}} \right. \\
& \left. \left( -\frac{h}{bg+ah} + \frac{1}{a+bx} \right) \text{EllipticF}[\text{ArcSin}\left[ \sqrt{\frac{(-be+af)(-h-\frac{bg}{a+bx}+\frac{ah}{a+bx})}{b(-fg+eh)}} \right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] \right) / \\
& \left( \sqrt{\frac{-\frac{h}{bg+ah} + \frac{1}{a+bx}}{\frac{f}{-be+af} - \frac{h}{bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+bx}\right) \left(f + \frac{be-af}{a+bx}\right) \left(h + \frac{bg-ah}{a+bx}\right)} \right) - \left( 3abdfh \sqrt{\frac{-\frac{d}{bc+ad} + \frac{1}{a+bx}}{-\frac{d}{bc+ad} + \frac{h}{bg+ah}}} \sqrt{\frac{-\frac{f}{be+af} + \frac{1}{a+bx}}{-\frac{f}{be+af} + \frac{h}{bg+ah}}} \right. \\
& \left. \left( -\frac{h}{bg+ah} + \frac{1}{a+bx} \right) \text{EllipticF}[\text{ArcSin}\left[ \sqrt{\frac{(-be+af)(-h-\frac{bg}{a+bx}+\frac{ah}{a+bx})}{b(-fg+eh)}} \right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] \right) / \\
& \left( \sqrt{\frac{-\frac{h}{bg+ah} + \frac{1}{a+bx}}{\frac{f}{-be+af} - \frac{h}{bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+bx}\right) \left(f + \frac{be-af}{a+bx}\right) \left(h + \frac{bg-ah}{a+bx}\right)} \right) + \left( 3a^2 B d f h \sqrt{\frac{-\frac{d}{bc+ad} + \frac{1}{a+bx}}{-\frac{d}{bc+ad} + \frac{h}{bg+ah}}} \sqrt{\frac{-\frac{f}{be+af} + \frac{1}{a+bx}}{-\frac{f}{be+af} + \frac{h}{bg+ah}}} \right. \\
& \left. \left( -\frac{h}{bg+ah} + \frac{1}{a+bx} \right) \text{EllipticF}[\text{ArcSin}\left[ \sqrt{\frac{(-be+af)(-h-\frac{bg}{a+bx}+\frac{ah}{a+bx})}{b(-fg+eh)}} \right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] \right) / \\
& \left( \sqrt{\frac{-\frac{h}{bg+ah} + \frac{1}{a+bx}}{\frac{f}{-be+af} - \frac{h}{bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+bx}\right) \left(f + \frac{be-af}{a+bx}\right) \left(h + \frac{bg-ah}{a+bx}\right)} \right)
\end{aligned}$$

**Problem 11: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+bx)^{3/2} (de+cf+2dfx)}{\sqrt{c+dx} \sqrt{e+fx} \sqrt{g+hx}} dx$$

Optimal (type 4, 898 leaves, 10 steps):

$$\begin{aligned}
& \frac{(5 a d f h - b (3 d f g + d e h + c f h)) \sqrt{a+b x} \sqrt{e+f x} \sqrt{g+h x}}{2 f h^2 \sqrt{c+d x}} + \frac{b \sqrt{a+b x} \sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x}}{h} - \\
& \left( \sqrt{d g - c h} \sqrt{f g - e h} (5 a d f h - b (3 d f g + d e h + c f h)) \sqrt{a+b x} \sqrt{-\frac{(d e - c f) (g + h x)}{(f g - e h) (c + d x)}} \right. \\
& \left. \text{EllipticE}[\text{ArcSin}\left[\frac{\sqrt{d g - c h} \sqrt{e + f x}}{\sqrt{f g - e h} \sqrt{c + d x}}\right], \frac{(b c - a d) (f g - e h)}{(b e - a f) (d g - c h)}] \right) / \left( 2 d f h^2 \sqrt{\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}} \sqrt{g + h x} \right) - \\
& \left( (b e - a f) \sqrt{b g - a h} (3 a d f h + b (c f h - d (3 f g + e h))) \sqrt{\frac{(b e - a f) (c + d x)}{(d e - c f) (a + b x)}} \sqrt{g + h x} \right. \\
& \left. \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{b g - a h} \sqrt{e + f x}}{\sqrt{f g - e h} \sqrt{a + b x}}\right], -\frac{(b c - a d) (f g - e h)}{(d e - c f) (b g - a h)}] \right) / \left( 2 b f h^2 \sqrt{f g - e h} \sqrt{c + d x} \sqrt{-\frac{(b e - a f) (g + h x)}{(f g - e h) (a + b x)}} \right) - \\
& \left( \sqrt{-d g + c h} (6 a b d^2 f^2 g h - 3 a^2 d^2 f^2 h^2 + b^2 (2 c d e f h^2 - c^2 f^2 h^2 - d^2 (3 f^2 g^2 + e^2 h^2))) (a + b x) \sqrt{\frac{(b g - a h) (c + d x)}{(d g - c h) (a + b x)}} \sqrt{\frac{(b g - a h) (e + f x)}{(f g - e h) (a + b x)}} \right. \\
& \left. \text{EllipticPi}\left[-\frac{b (d g - c h)}{(b c - a d) h}, \text{ArcSin}\left[\frac{\sqrt{b c - a d} \sqrt{g + h x}}{\sqrt{-d g + c h} \sqrt{a + b x}}\right], \frac{(b e - a f) (d g - c h)}{(b c - a d) (f g - e h)}\right] \right) / \left( 2 b d \sqrt{b c - a d} f h^3 \sqrt{c + d x} \sqrt{e + f x} \right)
\end{aligned}$$

Result (type 4, 14 853 leaves):

$$\begin{aligned}
& \frac{b \sqrt{a+b x} \sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x}}{h} + \frac{1}{b^2 h} \left( \frac{(-3 b d f g - b d e h - b c f h + 5 a d f h) (a+b x)^{5/2} \left(d + \frac{b c}{a+b x} - \frac{a d}{a+b x}\right) \left(f + \frac{b e}{a+b x} - \frac{a f}{a+b x}\right) \left(h + \frac{b g}{a+b x} - \frac{a h}{a+b x}\right)}{2 d f h \sqrt{c + \frac{(a+b x) (d - \frac{a d}{a+b x})}{b}} \sqrt{e + \frac{(a+b x) (f - \frac{a f}{a+b x})}{b}} \sqrt{g + \frac{(a+b x) (h - \frac{a h}{a+b x})}{b}}} + \right. \\
& \left. \frac{1}{2 d f h \sqrt{c + \frac{(a+b x) (d - \frac{a d}{a+b x})}{b}} \sqrt{e + \frac{(a+b x) (f - \frac{a f}{a+b x})}{b}} \sqrt{g + \frac{(a+b x) (h - \frac{a h}{a+b x})}{b}}} \right. \\
& \left. (a+b x)^{3/2} \sqrt{\left(d + \frac{b c}{a+b x} - \frac{a d}{a+b x}\right) \left(f + \frac{b e}{a+b x} - \frac{a f}{a+b x}\right) \left(h + \frac{b g}{a+b x} - \frac{a h}{a+b x}\right)} \left(3 b^4 c d e f g^2 \sqrt{\frac{(b c - a d) (b g - a h) \left(-\frac{d}{-b c + a d} + \frac{1}{a+b x}\right)}{b d g - b c h}}\right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{f}{-be+af} + \frac{1}{a+bx} \right) \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+bx}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \left( -\frac{(bdg-bch) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{(bc-ad)(bg-ah)} \right. \\
& \left. - \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{-bc+ad} \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+bx}}{\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+bx}\right) \left(f + \frac{be-af}{a+bx}\right) \left(h + \frac{bg-ah}{a+bx}\right)} - 3ab^3d^2efg^2 \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+bx}\right)}{bdg-bch}} \right. \\
& \left. - \left( -\frac{f}{-be+af} + \frac{1}{a+bx} \right) \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+bx}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \left( -\frac{(bdg-bch) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{(bc-ad)(bg-ah)} \right. \right. \\
& \left. \left. - \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{-bc+ad} \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+bx}}{\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+bx}\right) \left(f + \frac{be-af}{a+bx}\right) \left(h + \frac{bg-ah}{a+bx}\right)} - 3ab^3cd^2g^2 \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+bx}\right)}{bdg-bch}} \right. \\
& \left. - \left( -\frac{f}{-be+af} + \frac{1}{a+bx} \right) \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+bx}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \left( -\frac{(bdg-bch) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{(bc-ad)(bg-ah)} \right. \right. \\
& \left. \left. - \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{-bc+ad} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{d \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(b e - a f) \left( h + \frac{b g}{a+b x} - \frac{a h}{a+b x} \right)}{b (-f g + e h)}} \right], \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)} \right]}{-b c + a d} \right\} / \\
& \left( \sqrt{\frac{-\frac{f}{-b e + a f} + \frac{1}{a+b x}}{-\frac{f}{-b e + a f} + \frac{h}{-b g + a h}}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right) \left(h + \frac{b g - a h}{a+b x}\right)} \right) + \left( 3 a^2 b^2 d^2 f^2 g^2 \sqrt{\frac{(b c - a d) (b g - a h) \left(-\frac{d}{-b c + a d} + \frac{1}{a+b x}\right)}{b d g - b c h}} \right. \\
& \left( -\frac{f}{-b e + a f} + \frac{1}{a+b x} \right) \sqrt{\frac{-\frac{h}{-b g + a h} + \frac{1}{a+b x}}{\frac{f}{-b e + a f} - \frac{h}{-b g + a h}}} \left( -\frac{(b d g - b c h) \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(b e - a f) \left( h + \frac{b g}{a+b x} - \frac{a h}{a+b x} \right)}{b (-f g + e h)}} \right], \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)} \right]}{(b c - a d) (b g - a h)} - \right. \\
& \left. \left. \frac{d \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(b e - a f) \left( h + \frac{b g}{a+b x} - \frac{a h}{a+b x} \right)}{b (-f g + e h)}} \right], \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)} \right]}{-b c + a d} \right) \right\} / \\
& \left( \sqrt{\frac{-\frac{f}{-b e + a f} + \frac{1}{a+b x}}{-\frac{f}{-b e + a f} + \frac{h}{-b g + a h}}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right) \left(h + \frac{b g - a h}{a+b x}\right)} \right) + \left( b^4 c d e^2 g h \sqrt{\frac{(b c - a d) (b g - a h) \left(-\frac{d}{-b c + a d} + \frac{1}{a+b x}\right)}{b d g - b c h}} \right. \\
& \left( -\frac{f}{-b e + a f} + \frac{1}{a+b x} \right) \sqrt{\frac{-\frac{h}{-b g + a h} + \frac{1}{a+b x}}{\frac{f}{-b e + a f} - \frac{h}{-b g + a h}}} \left( -\frac{(b d g - b c h) \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(b e - a f) \left( h + \frac{b g}{a+b x} - \frac{a h}{a+b x} \right)}{b (-f g + e h)}} \right], \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)} \right]}{(b c - a d) (b g - a h)} - \right. \\
& \left. \left. \frac{d \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(b e - a f) \left( h + \frac{b g}{a+b x} - \frac{a h}{a+b x} \right)}{b (-f g + e h)}} \right], \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)} \right]}{-b c + a d} \right) \right\}
\end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) - \left( ab^3 d^2 e^2 g h \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+b x}\right)}{bdg-bch}} \right. \\
& \quad \left( -\frac{f}{-be+af} + \frac{1}{a+b x} \right) \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+b x}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \left( -\frac{(bdg-bch)}{(bc-ad)(bg-ah)} \text{EllipticE}[\text{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg-ah}{a+b x})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] \right. \\
& \quad \left. \left. - \frac{d \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg-ah}{a+b x})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{-bc+ad} \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) + \left( b^4 c^2 e f g h \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+b x}\right)}{bdg-bch}} \right. \\
& \quad \left( -\frac{f}{-be+af} + \frac{1}{a+b x} \right) \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+b x}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \left( -\frac{(bdg-bch)}{(bc-ad)(bg-ah)} \text{EllipticE}[\text{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg-ah}{a+b x})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] \right. \\
& \quad \left. \left. - \frac{d \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg-ah}{a+b x})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{-bc+ad} \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) - \left( 10 a b^3 c d e f g h \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+b x}\right)}{bdg-bch}} \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{f}{-be+af} + \frac{1}{a+bx} \right) \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+bx}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \left( -\frac{(bdg-bch) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{(bc-ad)(bg-ah)} \right. \\
& \left. \left. - \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{-bc+ad} \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+bx}}{\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+bx}\right) \left(f + \frac{be-af}{a+bx}\right) \left(h + \frac{bg-ah}{a+bx}\right)} + \left(9a^2b^2d^2efgh\sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+bx}\right)}{bdg-bc}} \right. \right. \\
& \left. \left. - \frac{(bdg-bch) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{(bc-ad)(bg-ah)} \right. \right. \\
& \left. \left. - \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{-bc+ad} \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+bx}}{\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+bx}\right) \left(f + \frac{be-af}{a+bx}\right) \left(h + \frac{bg-ah}{a+bx}\right)} - \left(a b^3 c^2 f^2 g h \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+bx}\right)}{bdg-bc}} \right. \right. \\
& \left. \left. - \frac{(bdg-bch) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{(bc-ad)(bg-ah)} \right. \right. \\
& \left. \left. - \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{-bc+ad} \right) \right)
\end{aligned}$$

$$\left. \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}, \frac{(-b c+a d) \left(-f g+e h\right)}{(-b e+a f) \left(-d g+c h\right)}\right]}]{-b c+a d} \right\}$$

$$\left( \sqrt{\frac{-\frac{f}{-b e+a f}+\frac{1}{a+b x}}{-\frac{f}{-b e+a f}+\frac{h}{-b g+a h}}} \sqrt{\left(d+\frac{b c-a d}{a+b x}\right) \left(f+\frac{b e-a f}{a+b x}\right) \left(h+\frac{b g-a h}{a+b x}\right)} \right) + \left( 9 a^2 b^2 c d f^2 g h \sqrt{\frac{(b c-a d) (b g-a h) \left(-\frac{d}{-b c+a d}+\frac{1}{a+b x}\right)}{b d g-b c h}} \right)$$

$$\left( -\frac{f}{-b e+a f}+\frac{1}{a+b x} \right) \sqrt{\frac{-\frac{h}{-b g+a h}+\frac{1}{a+b x}}{\frac{f}{-b e+a f}-\frac{h}{-b g+a h}}} \left( -\frac{(b d g-b c h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}, \frac{(-b c+a d) \left(-f g+e h\right)}{(-b e+a f) \left(-d g+c h\right)}\right]}]{(b c-a d) (b g-a h)} \right) -$$

$$\left. \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}, \frac{(-b c+a d) \left(-f g+e h\right)}{(-b e+a f) \left(-d g+c h\right)}\right]}]{-b c+a d} \right\}$$

$$\left( \sqrt{\frac{-\frac{f}{-b e+a f}+\frac{1}{a+b x}}{-\frac{f}{-b e+a f}+\frac{h}{-b g+a h}}} \sqrt{\left(d+\frac{b c-a d}{a+b x}\right) \left(f+\frac{b e-a f}{a+b x}\right) \left(h+\frac{b g-a h}{a+b x}\right)} \right) - \left( 8 a^3 b d^2 f^2 g h \sqrt{\frac{(b c-a d) (b g-a h) \left(-\frac{d}{-b c+a d}+\frac{1}{a+b x}\right)}{b d g-b c h}} \right)$$

$$\left( -\frac{f}{-b e+a f}+\frac{1}{a+b x} \right) \sqrt{\frac{-\frac{h}{-b g+a h}+\frac{1}{a+b x}}{\frac{f}{-b e+a f}-\frac{h}{-b g+a h}}} \left( -\frac{(b d g-b c h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}, \frac{(-b c+a d) \left(-f g+e h\right)}{(-b e+a f) \left(-d g+c h\right)}\right]}]{(b c-a d) (b g-a h)} \right) -$$

$$\left. \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}, \frac{(-b c+a d) \left(-f g+e h\right)}{(-b e+a f) \left(-d g+c h\right)}\right]}]{-b c+a d} \right\}$$

$$\begin{aligned}
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) - \left( a b^3 c d e^2 h^2 \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+b x}\right)}{b d g - b c h}} \right. \\
& \left. \left( -\frac{f}{-be+af} + \frac{1}{a+b x} \right) \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+b x}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \left( -\frac{(bdg-bch)}{(bc-ad)(bg-ah)} \text{EllipticE}[\text{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg-ah}{a+b x})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] \right. \right. \\
& \left. \left. \left. \frac{d \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg-ah}{a+b x})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{-bc+ad} \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) + \left( a^2 b^2 d^2 e^2 h^2 \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+b x}\right)}{b d g - b c h}} \right. \\
& \left. \left( -\frac{f}{-be+af} + \frac{1}{a+b x} \right) \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+b x}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \left( -\frac{(bdg-bch)}{(bc-ad)(bg-ah)} \text{EllipticE}[\text{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg-ah}{a+b x})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] \right. \right. \\
& \left. \left. \left. \frac{d \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg-ah}{a+b x})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{-bc+ad} \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) - \left( a b^3 c^2 e f h^2 \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+b x}\right)}{b d g - b c h}} \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{f}{-be+af} + \frac{1}{a+bx} \right) \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+bx}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \left( -\frac{(bdg-bch) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{(bc-ad)(bg-ah)} \right. \\
& \left. - \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{-bc+ad} \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+bx}}{\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+bx}\right) \left(f + \frac{be-af}{a+bx}\right) \left(h + \frac{bg-ah}{a+bx}\right)} + \left(7a^2b^2cdefh^2 \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+bx}\right)}{bdg-bch}} \right. \right. \\
& \left. \left. - \frac{(bdg-bch) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{(bc-ad)(bg-ah)} \right. \right. \\
& \left. \left. - \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{-bc+ad} \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+bx}}{\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+bx}\right) \left(f + \frac{be-af}{a+bx}\right) \left(h + \frac{bg-ah}{a+bx}\right)} - \left(6a^3bd^2efh^2 \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+bx}\right)}{bdg-bch}} \right. \right. \\
& \left. \left. - \frac{(bdg-bch) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{(bc-ad)(bg-ah)} \right. \right. \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{d \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(b e - a f) \left( h + \frac{b g}{a+b x} - \frac{a h}{a+b x} \right)}{b (-f g + e h)}} \right], \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)} \right]}{-b c + a d} \right\} / \\
& \left( \sqrt{\frac{-\frac{f}{-b e + a f} + \frac{1}{a+b x}}{-\frac{f}{-b e + a f} + \frac{h}{-b g + a h}}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right) \left(h + \frac{b g - a h}{a+b x}\right)} \right) + \left( a^2 b^2 c^2 f^2 h^2 \sqrt{\frac{(b c - a d) (b g - a h) \left(-\frac{d}{-b c + a d} + \frac{1}{a+b x}\right)}{b d g - b c h}} \right. \\
& \left( -\frac{f}{-b e + a f} + \frac{1}{a+b x} \right) \sqrt{\frac{-\frac{h}{-b g + a h} + \frac{1}{a+b x}}{\frac{f}{-b e + a f} - \frac{h}{-b g + a h}}} \left( -\frac{(b d g - b c h) \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(b e - a f) \left( h + \frac{b g}{a+b x} - \frac{a h}{a+b x} \right)}{b (-f g + e h)}} \right], \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)} \right]}{(b c - a d) (b g - a h)} - \right. \\
& \left. \left. \frac{d \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(b e - a f) \left( h + \frac{b g}{a+b x} - \frac{a h}{a+b x} \right)}{b (-f g + e h)}} \right], \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)} \right]}{-b c + a d} \right) \right\} / \\
& \left( \sqrt{\frac{-\frac{f}{-b e + a f} + \frac{1}{a+b x}}{-\frac{f}{-b e + a f} + \frac{h}{-b g + a h}}} \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right) \left(h + \frac{b g - a h}{a+b x}\right)} \right) - \left( 6 a^3 b c d f^2 h^2 \sqrt{\frac{(b c - a d) (b g - a h) \left(-\frac{d}{-b c + a d} + \frac{1}{a+b x}\right)}{b d g - b c h}} \right. \\
& \left( -\frac{f}{-b e + a f} + \frac{1}{a+b x} \right) \sqrt{\frac{-\frac{h}{-b g + a h} + \frac{1}{a+b x}}{\frac{f}{-b e + a f} - \frac{h}{-b g + a h}}} \left( -\frac{(b d g - b c h) \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(b e - a f) \left( h + \frac{b g}{a+b x} - \frac{a h}{a+b x} \right)}{b (-f g + e h)}} \right], \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)} \right]}{(b c - a d) (b g - a h)} - \right. \\
& \left. \left. \frac{d \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(b e - a f) \left( h + \frac{b g}{a+b x} - \frac{a h}{a+b x} \right)}{b (-f g + e h)}} \right], \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)} \right]}{-b c + a d} \right) \right\}
\end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) + \left( 5 a^4 d^2 f^2 h^2 \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+b x}\right)}{b d g - b c h}} \right. \\
& \left. \left( -\frac{f}{-be+af} + \frac{1}{a+b x} \right) \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+b x}}{-\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \left( -\frac{(bdg-bch)}{(bc-ad)(bg-ah)} \text{EllipticE}[\text{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg-ah}{a+b x})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] \right. \right. \\
& \left. \left. - \frac{d \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg-ah}{a+b x})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{-bc+ad} \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) + \left( 2 b^3 c d e f g h \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+b x}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \right. \\
& \left. \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \left( -\frac{h}{-bg+ah} + \frac{1}{a+b x} \right) \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{(-be+af)(-h-\frac{bg}{a+b x}+\frac{ah}{a+b x})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] \right) / \\
& \left( \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+b x}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) - \left( 2 a b^2 d^2 e f g h \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+b x}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \right. \\
& \left. \left( -\frac{h}{-bg+ah} + \frac{1}{a+b x} \right) \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{(-be+af)(-h-\frac{bg}{a+b x}+\frac{ah}{a+b x})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] \right) / \\
& \left( \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+b x}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) - \left( 2 a b^2 c d f^2 g h \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+b x}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \right)
\end{aligned}$$

$$\left( -\frac{h}{-bg + ah} + \frac{1}{a + bx} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-be + af) \left( -h - \frac{bg}{a+bx} + \frac{ah}{a+bx} \right)}{b (-fg + eh)}}, \frac{(-bc + ad) \left( -fg + eh \right)}{(-be + af) \left( -dg + ch \right)} \right] \right]$$

$$\left( \sqrt{\frac{-\frac{h}{-bg + ah} + \frac{1}{a + bx}}{\frac{f}{-be + af} - \frac{h}{-bg + ah}}} \sqrt{\left( d + \frac{bc - ad}{a + bx} \right) \left( f + \frac{be - af}{a + bx} \right) \left( h + \frac{bg - ah}{a + bx} \right)} + \left( 2 a^2 b d^2 f^2 g h \sqrt{\frac{-\frac{d}{-bc + ad} + \frac{1}{a + bx}}{-\frac{d}{-bc + ad} + \frac{h}{-bg + ah}}} \sqrt{\frac{-\frac{f}{-be + af} + \frac{1}{a + bx}}{-\frac{f}{-be + af} + \frac{h}{-bg + ah}}} \right. \right.$$

$$\left. \left( -\frac{h}{-bg + ah} + \frac{1}{a + bx} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-be + af) \left( -h - \frac{bg}{a+bx} + \frac{ah}{a+bx} \right)}{b (-fg + eh)}}, \frac{(-bc + ad) \left( -fg + eh \right)}{(-be + af) \left( -dg + ch \right)} \right] \right] \right)$$

$$\left( \sqrt{\frac{-\frac{h}{-bg + ah} + \frac{1}{a + bx}}{\frac{f}{-be + af} - \frac{h}{-bg + ah}}} \sqrt{\left( d + \frac{bc - ad}{a + bx} \right) \left( f + \frac{be - af}{a + bx} \right) \left( h + \frac{bg - ah}{a + bx} \right)} - \left( 2 a b^2 c d e f h^2 \sqrt{\frac{-\frac{d}{-bc + ad} + \frac{1}{a + bx}}{-\frac{d}{-bc + ad} + \frac{h}{-bg + ah}}} \sqrt{\frac{-\frac{f}{-be + af} + \frac{1}{a + bx}}{-\frac{f}{-be + af} + \frac{h}{-bg + ah}}} \right. \right.$$

$$\left. \left( -\frac{h}{-bg + ah} + \frac{1}{a + bx} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-be + af) \left( -h - \frac{bg}{a+bx} + \frac{ah}{a+bx} \right)}{b (-fg + eh)}}, \frac{(-bc + ad) \left( -fg + eh \right)}{(-be + af) \left( -dg + ch \right)} \right] \right] \right)$$

$$\left( \sqrt{\frac{-\frac{h}{-bg + ah} + \frac{1}{a + bx}}{\frac{f}{-be + af} - \frac{h}{-bg + ah}}} \sqrt{\left( d + \frac{bc - ad}{a + bx} \right) \left( f + \frac{be - af}{a + bx} \right) \left( h + \frac{bg - ah}{a + bx} \right)} + \left( 2 a^2 b d^2 e f h^2 \sqrt{\frac{-\frac{d}{-bc + ad} + \frac{1}{a + bx}}{-\frac{d}{-bc + ad} + \frac{h}{-bg + ah}}} \sqrt{\frac{-\frac{f}{-be + af} + \frac{1}{a + bx}}{-\frac{f}{-be + af} + \frac{h}{-bg + ah}}} \right. \right.$$

$$\left. \left( -\frac{h}{-bg + ah} + \frac{1}{a + bx} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-be + af) \left( -h - \frac{bg}{a+bx} + \frac{ah}{a+bx} \right)}{b (-fg + eh)}}, \frac{(-bc + ad) \left( -fg + eh \right)}{(-be + af) \left( -dg + ch \right)} \right] \right] \right)$$

$$\left( \sqrt{\frac{-\frac{h}{-bg + ah} + \frac{1}{a + bx}}{\frac{f}{-be + af} - \frac{h}{-bg + ah}}} \sqrt{\left( d + \frac{bc - ad}{a + bx} \right) \left( f + \frac{be - af}{a + bx} \right) \left( h + \frac{bg - ah}{a + bx} \right)} + \left( 2 a^2 b c d f^2 h^2 \sqrt{\frac{-\frac{d}{-bc + ad} + \frac{1}{a + bx}}{-\frac{d}{-bc + ad} + \frac{h}{-bg + ah}}} \sqrt{\frac{-\frac{f}{-be + af} + \frac{1}{a + bx}}{-\frac{f}{-be + af} + \frac{h}{-bg + ah}}} \right. \right.$$

$$\left. \left( -\frac{h}{-bg + ah} + \frac{1}{a + bx} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-be + af) \left( -h - \frac{bg}{a+bx} + \frac{ah}{a+bx} \right)}{b (-fg + eh)}}, \frac{(-bc + ad) \left( -fg + eh \right)}{(-be + af) \left( -dg + ch \right)} \right] \right] \right)$$

$$\begin{aligned}
& \left( \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+b x}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) - \\
& \left( 2 a^3 d^2 f^2 h^2 \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+b x}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \left(-\frac{h}{-bg+ah} + \frac{1}{a+b x}\right) \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{(-be+af)(-h-\frac{bg}{a+b x}+\frac{ah}{a+b x})}{b(-fg+eh)}}\right], \right. \\
& \left. \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}\right] \right) / \left( \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+b x}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) - \\
& \left( 6 a b d^2 f^2 g (-bg+ah) \left(-\frac{f}{-be+af} + \frac{h}{-bg+ah}\right) \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+b x}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{-\frac{\left(-\frac{f}{-be+af} + \frac{1}{a+b x}\right) \left(-\frac{h}{-bg+ah} + \frac{1}{a+b x}\right)}{\left(-\frac{f}{-be+af} + \frac{h}{-bg+ah}\right)^2}} \right. \\
& \left. \text{EllipticPi}\left[-\frac{-bf g + be h}{(-be+af)h}, \text{ArcSin}\left[\sqrt{\frac{(-be+af)(-h-\frac{bg}{a+b x}+\frac{ah}{a+b x})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}\right]\right) / \\
& \left( \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} + \left( 3 b^2 d^2 f^2 g^2 (-bg+ah) \left(-\frac{f}{-be+af} + \frac{h}{-bg+ah}\right) \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+b x}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \right. \right. \\
& \left. \left. \sqrt{-\frac{\left(-\frac{f}{-be+af} + \frac{1}{a+b x}\right) \left(-\frac{h}{-bg+ah} + \frac{1}{a+b x}\right)}{\left(-\frac{f}{-be+af} + \frac{h}{-bg+ah}\right)^2}} \text{EllipticPi}\left[-\frac{-bf g + be h}{(-be+af)h}, \text{ArcSin}\left[\sqrt{\frac{(-be+af)(-h-\frac{bg}{a+b x}+\frac{ah}{a+b x})}{b(-fg+eh)}}\right], \right. \right. \\
& \left. \left. \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}\right]\right) / \left( h \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} + \right. \\
& \left. \left( b^2 d^2 e^2 h (-bg+ah) \left(-\frac{f}{-be+af} + \frac{h}{-bg+ah}\right) \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+b x}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{-\frac{\left(-\frac{f}{-be+af} + \frac{1}{a+b x}\right) \left(-\frac{h}{-bg+ah} + \frac{1}{a+b x}\right)}{\left(-\frac{f}{-be+af} + \frac{h}{-bg+ah}\right)^2}} \text{EllipticPi}\left[-\frac{-bf g + be h}{(-be+af)h}, \right. \right. \right. 
\end{aligned}$$

$$\begin{aligned}
& \text{ArcSin}\left[\sqrt{\frac{(-b e + a f) \left(-h - \frac{b g}{a+b x} + \frac{a h}{a+b x}\right)}{b (-f g + e h)}}\right], \frac{(-b c + a d) \left(-f g + e h\right)}{(-b e + a f) \left(-d g + c h\right)}\Bigg] \Bigg/ \left(\sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right) \left(h + \frac{b g - a h}{a+b x}\right)}\right) - \\
& \left(2 b^2 c d e f h (-b g + a h) \left(-\frac{f}{-b e + a f} + \frac{h}{-b g + a h}\right) \sqrt{\frac{-\frac{d}{-b c+a d} + \frac{1}{a+b x}}{-\frac{d}{-b c+a d} + \frac{h}{-b g+a h}}} \sqrt{-\frac{\left(-\frac{f}{-b e+a f} + \frac{1}{a+b x}\right) \left(-\frac{h}{-b g+a h} + \frac{1}{a+b x}\right)}{\left(-\frac{f}{-b e+a f} + \frac{h}{-b g+a h}\right)^2}}\right. \\
& \text{EllipticPi}\left[-\frac{-b f g + b e h}{(-b e + a f) h}, \text{ArcSin}\left[\sqrt{\frac{(-b e + a f) \left(-h - \frac{b g}{a+b x} + \frac{a h}{a+b x}\right)}{b (-f g + e h)}}\right], \frac{(-b c + a d) \left(-f g + e h\right)}{(-b e + a f) \left(-d g + c h\right)}\right]\Bigg) \Bigg/ \\
& \left(\sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right) \left(h + \frac{b g - a h}{a+b x}\right)} + \left(b^2 c^2 f^2 h (-b g + a h) \left(-\frac{f}{-b e + a f} + \frac{h}{-b g + a h}\right) \sqrt{\frac{-\frac{d}{-b c+a d} + \frac{1}{a+b x}}{-\frac{d}{-b c+a d} + \frac{h}{-b g+a h}}\right.\right.\right. \\
& \left.\left.\left.-\frac{\left(-\frac{f}{-b e+a f} + \frac{1}{a+b x}\right) \left(-\frac{h}{-b g+a h} + \frac{1}{a+b x}\right)}{\left(-\frac{f}{-b e+a f} + \frac{h}{-b g+a h}\right)^2}\right. \text{EllipticPi}\left[-\frac{-b f g + b e h}{(-b e + a f) h}, \text{ArcSin}\left[\sqrt{\frac{(-b e + a f) \left(-h - \frac{b g}{a+b x} + \frac{a h}{a+b x}\right)}{b (-f g + e h)}}\right],\right.\right. \\
& \left.\left.\left.\frac{(-b c + a d) \left(-f g + e h\right)}{(-b e + a f) \left(-d g + c h\right)}\right]\right)\Bigg/ \left(\sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right) \left(h + \frac{b g - a h}{a+b x}\right)} + \right. \\
& \left(3 a^2 d^2 f^2 h (-b g + a h) \left(-\frac{f}{-b e + a f} + \frac{h}{-b g + a h}\right) \sqrt{\frac{-\frac{d}{-b c+a d} + \frac{1}{a+b x}}{-\frac{d}{-b c+a d} + \frac{h}{-b g+a h}}} \sqrt{-\frac{\left(-\frac{f}{-b e+a f} + \frac{1}{a+b x}\right) \left(-\frac{h}{-b g+a h} + \frac{1}{a+b x}\right)}{\left(-\frac{f}{-b e+a f} + \frac{h}{-b g+a h}\right)^2}} \text{EllipticPi}\left[-\frac{-b f g + b e h}{(-b e + a f) h},\right.\right. \\
& \left.\left.\left.\text{ArcSin}\left[\sqrt{\frac{(-b e + a f) \left(-h - \frac{b g}{a+b x} + \frac{a h}{a+b x}\right)}{b (-f g + e h)}}\right], \frac{(-b c + a d) \left(-f g + e h\right)}{(-b e + a f) \left(-d g + c h\right)}\right]\right)\Bigg/ \left(\sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right) \left(h + \frac{b g - a h}{a+b x}\right)}\right)\right)
\end{aligned}$$

Problem 12: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{a+b x}}{\sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x}} dx$$

Optimal (type 4, 472 leaves, 5 steps):

$$\begin{aligned} & \frac{2 b \sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x}}{h \sqrt{a+b x}} - \frac{2 \sqrt{b g-a h} \sqrt{f g-e h} \sqrt{c+d x}}{\sqrt{-\frac{(b e-a f) (g+h x)}{(f g-e h) (a+b x)}}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{b g-a h} \sqrt{e+f x}}{\sqrt{f g-e h} \sqrt{a+b x}}\right], -\frac{(b c-a d) (f g-e h)}{(d e-c f) (b g-a h)}\right] \\ & \left(2 d (b g-a h)^{3/2} \sqrt{\frac{(f g-e h) (a+b x)}{(b g-a h) (e+f x)}} \sqrt{\frac{(f g-e h) (c+d x)}{(d g-c h) (e+f x)}} (e+f x)\right. \\ & \left. \text{EllipticPi}\left[\frac{f (b g-a h)}{(b e-a f) h}, \text{ArcSin}\left[\frac{\sqrt{b e-a f} \sqrt{g+h x}}{\sqrt{b g-a h} \sqrt{e+f x}}\right], \frac{(d e-c f) (b g-a h)}{(b e-a f) (d g-c h)}\right]\right) / \left(\sqrt{b e-a f} h^2 \sqrt{a+b x} \sqrt{c+d x}\right) \end{aligned}$$

Result (type 4, 6583 leaves):

$$\begin{aligned} & -\frac{1}{d} 2 \left( -\frac{(c+d x)^{3/2} \left(f + \frac{d e}{c+d x} - \frac{c f}{c+d x}\right) \left(h + \frac{d g}{c+d x} - \frac{c h}{c+d x}\right) \sqrt{a + \frac{(c+d x) \left(b - \frac{b c}{c+d x}\right)}{d}}}{h \sqrt{e + \frac{(c+d x) \left(f - \frac{c f}{c+d x}\right)}{d}} \sqrt{g + \frac{(c+d x) \left(h - \frac{c h}{c+d x}\right)}{d}}} - \right. \\ & \left. \left( f (b g-a h) (d g-c h)^2 \sqrt{c+d x} \sqrt{\left(b - \frac{b c}{c+d x} + \frac{a d}{c+d x}\right) \left(f + \frac{d e}{c+d x} - \frac{c f}{c+d x}\right) \left(h + \frac{d g}{c+d x} - \frac{c h}{c+d x}\right)} \right. \right. \\ & \left. \sqrt{a + \frac{(c+d x) \left(b - \frac{b c}{c+d x}\right)}{d}} \left(d e \sqrt{-\frac{(b c-a d) (-d g+c h) \left(-\frac{b}{b c-a d} + \frac{1}{c+d x}\right)}{-b d g+a d h}} \left(-\frac{f}{-d e+c f} + \frac{1}{c+d x}\right) \right. \right. \\ & \left. \left. \sqrt{-\frac{h}{-d g+c h} + \frac{1}{c+d x}} \frac{(-b d g+a d h) \text{EllipticE}\left[\text{ArcSin}\left[\sqrt{\frac{(d e-c f) \left(h + \frac{d g}{c+d x} - \frac{c h}{c+d x}\right)}{d (-f g+e h)}}\right], -\frac{(b c-a d) (-f g+e h)}{(-d e+c f) (-b g+a h)}\right]}{(b c-a d) (-d g+c h)} \right) \right) \end{aligned}$$

$$\begin{aligned}
& \left. \frac{b \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(h + \frac{d g}{c+d x} - \frac{c h}{c+d x}\right)}{d (-f g + e h)}}, \frac{(-b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)}]\right]}{b c - a d} \right\} / \\
& \left( \sqrt{\frac{-\frac{f}{-d e + c f} + \frac{1}{c+d x}}{-\frac{f}{-d e + c f} + \frac{h}{-d g + c h}}} \sqrt{\left(b + \frac{-b c + a d}{c+d x}\right) \left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)} - c f \sqrt{-\frac{(b c - a d) (-d g + c h) \left(-\frac{b}{b c - a d} + \frac{1}{c+d x}\right)}{-b d g + a d h}} \right. \\
& \left( -\frac{f}{-d e + c f} + \frac{1}{c+d x} \right) \sqrt{\frac{-\frac{h}{-d g + c h} + \frac{1}{c+d x}}{\frac{f}{-d e + c f} - \frac{h}{-d g + c h}}} \left( \frac{(-b d g + a d h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(h + \frac{d g}{c+d x} - \frac{c h}{c+d x}\right)}{d (-f g + e h)}}, \frac{(-b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)}]\right]}{(b c - a d) (-d g + c h)} - \right. \\
& \left. b \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(h + \frac{d g}{c+d x} - \frac{c h}{c+d x}\right)}{d (-f g + e h)}}, \frac{(-b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)}]\right]}{b c - a d} \right\} / \\
& \left( \sqrt{\frac{-\frac{f}{-d e + c f} + \frac{1}{c+d x}}{-\frac{f}{-d e + c f} + \frac{h}{-d g + c h}}} \sqrt{\left(b + \frac{-b c + a d}{c+d x}\right) \left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)} + f \sqrt{\frac{-\frac{b}{b c - a d} + \frac{1}{c+d x}}{-\frac{b}{b c - a d} + \frac{h}{-d g + c h}}} \sqrt{\frac{-\frac{f}{-d e + c f} + \frac{1}{c+d x}}{-\frac{f}{-d e + c f} + \frac{h}{-d g + c h}}} \right. \\
& \left( -\frac{h}{-d g + c h} + \frac{1}{c+d x} \right) \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(-d e + c f) \left(-h - \frac{d g}{c+d x} + \frac{c h}{c+d x}\right)}{d (-f g + e h)}}, \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)}]\right] \right\} / \\
& \left( \sqrt{\frac{-\frac{h}{-d g + c h} + \frac{1}{c+d x}}{\frac{f}{-d e + c f} - \frac{h}{-d g + c h}}} \sqrt{\left(b + \frac{-b c + a d}{c+d x}\right) \left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)} \right\} / \\
& \left( h^2 (-f g + e h) \left(b - \frac{b c}{c+d x} + \frac{a d}{c+d x}\right) \sqrt{e + \frac{(c+d x) \left(f - \frac{c f}{c+d x}\right)}{d}} \sqrt{g + \frac{(c+d x) \left(h - \frac{c h}{c+d x}\right)}{d}} \right) +
\end{aligned}$$

$$\begin{aligned}
& \left( (b e - a f) (d e - c f) (d g - c h) \sqrt{c + d x} \right) \sqrt{\left( b - \frac{b c}{c + d x} + \frac{a d}{c + d x} \right) \left( f + \frac{d e}{c + d x} - \frac{c f}{c + d x} \right) \left( h + \frac{d g}{c + d x} - \frac{c h}{c + d x} \right)} \\
& \sqrt{a + \frac{(c + d x) \left( b - \frac{b c}{c + d x} \right)}{d}} \left( \begin{array}{l} d g \sqrt{-\frac{(b c - a d) (-d g + c h) \left( -\frac{b}{b c - a d} + \frac{1}{c + d x} \right)}{-b d g + a d h}} \left( -\frac{f}{-d e + c f} + \frac{1}{c + d x} \right) \\
\sqrt{\frac{-\frac{h}{-d g + c h} + \frac{1}{c + d x}}{\frac{f}{-d e + c f} - \frac{h}{-d g + c h}}} \left( \frac{(-b d g + a d h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(d e - c f) \left( h + \frac{d g}{c + d x} - \frac{c h}{c + d x} \right)}{d (-f g + e h)}}, \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)}\right]} - \right. \right. \\
\left. \left. \frac{b \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(d e - c f) \left( h + \frac{d g}{c + d x} - \frac{c h}{c + d x} \right)}{d (-f g + e h)}}, \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)}\right]} - \right. \right. \\
\left. \left. \left( -\frac{f}{-d e + c f} + \frac{1}{c + d x} \right) \sqrt{\left( b + \frac{-b c + a d}{c + d x} \right) \left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} - c h \sqrt{-\frac{(b c - a d) (-d g + c h) \left( -\frac{b}{b c - a d} + \frac{1}{c + d x} \right)}{-b d g + a d h}} \right. \right. \\
\left. \left. \left( -\frac{f}{-d e + c f} + \frac{1}{c + d x} \right) \sqrt{\frac{-\frac{h}{-d g + c h} + \frac{1}{c + d x}}{\frac{f}{-d e + c f} - \frac{h}{-d g + c h}}} \left( \frac{(-b d g + a d h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(d e - c f) \left( h + \frac{d g}{c + d x} - \frac{c h}{c + d x} \right)}{d (-f g + e h)}}, \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)}\right]} - \right. \right. \\
\left. \left. \frac{b \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(d e - c f) \left( h + \frac{d g}{c + d x} - \frac{c h}{c + d x} \right)}{d (-f g + e h)}}, \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)}\right]} - \right. \right. \end{array} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{\frac{-\frac{f}{-de+cf} + \frac{1}{c+dx}}{-\frac{f}{-de+cf} + \frac{h}{-dg+ch}}} \sqrt{\left( b + \frac{-bc+ad}{c+dx} \right) \left( f + \frac{de-cf}{c+dx} \right) \left( h + \frac{dg-ch}{c+dx} \right)} + \left( h \sqrt{\frac{-\frac{b}{bc-ad} + \frac{1}{c+dx}}{-\frac{b}{bc-ad} + \frac{h}{-dg+ch}}} \sqrt{\frac{-\frac{f}{-de+cf} + \frac{1}{c+dx}}{-\frac{f}{-de+cf} + \frac{h}{-dg+ch}}} \right. \right. \\
& \left. \left. \left( -\frac{h}{-dg+ch} + \frac{1}{c+dx} \right) \text{EllipticF}[\text{ArcSin}\left[ \sqrt{\frac{(-de+cf)(-h-\frac{dg}{c+dx}+\frac{ch}{c+dx})}{d(-fg+eh)}}, \frac{(bc-ad)(-fg+eh)}{(-de+cf)(-bg+ah)} \right]] \right) / \right. \\
& \left. \left( \sqrt{\frac{-\frac{h}{-dg+ch} + \frac{1}{c+dx}}{\frac{f}{-de+cf} - \frac{h}{-dg+ch}}} \sqrt{\left( b + \frac{-bc+ad}{c+dx} \right) \left( f + \frac{de-cf}{c+dx} \right) \left( h + \frac{dg-ch}{c+dx} \right)} \right) \right) / \\
& \left( h (-fg+eh) \left( b - \frac{bc}{c+dx} + \frac{ad}{c+dx} \right) \sqrt{e + \frac{(c+dx)(f-\frac{cf}{c+dx})}{d}} \sqrt{g + \frac{(c+dx)(h-\frac{ch}{c+dx})}{d}} - \right. \\
& \left. \frac{1}{h^2 \left( b - \frac{bc}{c+dx} + \frac{ad}{c+dx} \right) \sqrt{e + \frac{(c+dx)(f-\frac{cf}{c+dx})}{d}} \sqrt{g + \frac{(c+dx)(h-\frac{ch}{c+dx})}{d}}} \right. \\
& \left. d (bg-ah) \sqrt{c+dx} \right. \\
& \left. \sqrt{\left( b - \frac{bc}{c+dx} + \frac{ad}{c+dx} \right) \left( f + \frac{de}{c+dx} - \frac{cf}{c+dx} \right) \left( h + \frac{dg}{c+dx} - \frac{ch}{c+dx} \right)} \right. \\
& \left. \sqrt{a + \frac{(c+dx)(b-\frac{bc}{c+dx})}{d}} \right. \\
& \left( \left( d^2 eg \sqrt{-\frac{(bc-ad)(-dg+ch)(-\frac{b}{bc-ad}+\frac{1}{c+dx})}{-bdg+adh}} \left( -\frac{f}{-de+cf} + \frac{1}{c+dx} \right) \sqrt{\frac{-\frac{h}{-dg+ch} + \frac{1}{c+dx}}{\frac{f}{-de+cf} - \frac{h}{-dg+ch}}} \right. \right. \\
& \left. \left. (-bdg+adh) \text{EllipticE}[\text{ArcSin}\left[ \sqrt{\frac{(de-cf)(h+\frac{dg}{c+dx}-\frac{ch}{c+dx})}{d(-fg+eh)}}, \frac{(bc-ad)(-fg+eh)}{(-de+cf)(-bg+ah)} \right]] \right) / \right. \\
& \left. (bc-ad)(-dg+ch) \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{b \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(h + \frac{d g}{c+d x} - \frac{c h}{c+d x}\right)}{d (-f g + e h)}}, \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)}\right]}]}{b c - a d} \right\} / \\
& \left( \sqrt{\frac{-\frac{f}{-d e + c f} + \frac{1}{c+d x}}{-\frac{f}{-d e + c f} + \frac{h}{-d g + c h}}} \sqrt{\left(b + \frac{-b c + a d}{c+d x}\right) \left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)} \right) - \left( c d f g \sqrt{-\frac{(b c - a d) (-d g + c h) \left(-\frac{b}{b c - a d} + \frac{1}{c+d x}\right)}{-b d g + a d h}} \right. \\
& \left. \left( -\frac{f}{-d e + c f} + \frac{1}{c+d x} \right) \sqrt{\frac{-\frac{h}{-d g + c h} + \frac{1}{c+d x}}{\frac{f}{-d e + c f} - \frac{h}{-d g + c h}}} \left( -b d g + a d h \right) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(h + \frac{d g}{c+d x} - \frac{c h}{c+d x}\right)}{d (-f g + e h)}}, \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)}\right]} \right. \\
& \left. \left. \left. (b c - a d) (-d g + c h)\right] \right) - b c - a d \right) \\
& \left. \frac{b \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(h + \frac{d g}{c+d x} - \frac{c h}{c+d x}\right)}{d (-f g + e h)}}, \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)}\right]}]}{b c - a d} \right\} / \\
& \left( \sqrt{\frac{-\frac{f}{-d e + c f} + \frac{1}{c+d x}}{-\frac{f}{-d e + c f} + \frac{h}{-d g + c h}}} \sqrt{\left(b + \frac{-b c + a d}{c+d x}\right) \left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)} \right) - \left( c d e h \sqrt{-\frac{(b c - a d) (-d g + c h) \left(-\frac{b}{b c - a d} + \frac{1}{c+d x}\right)}{-b d g + a d h}} \right. \\
& \left. \left( -\frac{f}{-d e + c f} + \frac{1}{c+d x} \right) \sqrt{\frac{-\frac{h}{-d g + c h} + \frac{1}{c+d x}}{\frac{f}{-d e + c f} - \frac{h}{-d g + c h}}} \left( -b d g + a d h \right) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(h + \frac{d g}{c+d x} - \frac{c h}{c+d x}\right)}{d (-f g + e h)}}, \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)}\right]} \right. \\
& \left. \left. \left. (b c - a d) (-d g + c h)\right] \right) - b c - a d \right) \\
& \left. \frac{b \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(h + \frac{d g}{c+d x} - \frac{c h}{c+d x}\right)}{d (-f g + e h)}}, \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)}\right]}]}{b c - a d} \right\} /
\end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{\frac{-\frac{f}{-de+cf} + \frac{1}{c+dx}}{-\frac{f}{-de+cf} + \frac{h}{-dg+ch}}} \sqrt{\left( b + \frac{-bc+ad}{c+dx} \right) \left( f + \frac{de-cf}{c+dx} \right) \left( h + \frac{dg-ch}{c+dx} \right)} \right) + \left( c^2 fh \sqrt{-\frac{(bc-ad)(-dg+ch)\left(-\frac{b}{bc-ad} + \frac{1}{c+dx}\right)}{-bdg+adh}} \right. \\
& \left( -\frac{f}{-de+cf} + \frac{1}{c+dx} \right) \sqrt{\frac{-\frac{h}{-dg+ch} + \frac{1}{c+dx}}{\frac{f}{-de+cf} - \frac{h}{-dg+ch}}} \left( \frac{(-bdg+adh) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(de-cf)(h+\frac{dg}{c+dx}-\frac{ch}{c+dx})}{d(-fg+eh)}}\right], \frac{(bc-ad)(-fg+eh)}{(-de+cf)(-bg+ah)}]}{(bc-ad)(-dg+ch)} \right. \\
& \left. \left. \left. \frac{b \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(de-cf)(h+\frac{dg}{c+dx}-\frac{ch}{c+dx})}{d(-fg+eh)}}\right], \frac{(bc-ad)(-fg+eh)}{(-de+cf)(-bg+ah)}]}{bc-ad} \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-de+cf} + \frac{1}{c+dx}}{-\frac{f}{-de+cf} + \frac{h}{-dg+ch}}} \sqrt{\left( b + \frac{-bc+ad}{c+dx} \right) \left( f + \frac{de-cf}{c+dx} \right) \left( h + \frac{dg-ch}{c+dx} \right)} \right) + \left( dfgh \sqrt{\frac{-\frac{b}{bc-ad} + \frac{1}{c+dx}}{-\frac{b}{bc-ad} + \frac{h}{-dg+ch}}} \sqrt{\frac{-\frac{f}{-de+cf} + \frac{1}{c+dx}}{-\frac{f}{-de+cf} + \frac{h}{-dg+ch}}} \right. \\
& \left( -\frac{h}{-dg+ch} + \frac{1}{c+dx} \right) \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(-de+cf)(-h-\frac{dg}{c+dx}+\frac{ch}{c+dx})}{d(-fg+eh)}}\right], \frac{(bc-ad)(-fg+eh)}{(-de+cf)(-bg+ah)}] \right) / \\
& \left( \sqrt{\frac{-\frac{h}{-dg+ch} + \frac{1}{c+dx}}{\frac{f}{-de+cf} - \frac{h}{-dg+ch}}} \sqrt{\left( b + \frac{-bc+ad}{c+dx} \right) \left( f + \frac{de-cf}{c+dx} \right) \left( h + \frac{dg-ch}{c+dx} \right)} \right) + \\
& \left( deh \sqrt{\frac{-\frac{b}{bc-ad} + \frac{1}{c+dx}}{-\frac{b}{bc-ad} + \frac{h}{-dg+ch}}} \sqrt{\frac{-\frac{f}{-de+cf} + \frac{1}{c+dx}}{-\frac{f}{-de+cf} + \frac{h}{-dg+ch}}} \left( -\frac{h}{-dg+ch} + \frac{1}{c+dx} \right) \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(-de+cf)(-h-\frac{dg}{c+dx}+\frac{ch}{c+dx})}{d(-fg+eh)}}\right], \right. \\
& \left. \left. \left. \frac{(bc-ad)(-fg+eh)}{(-de+cf)(-bg+ah)} \right] \right) / \left( \sqrt{\frac{-\frac{h}{-dg+ch} + \frac{1}{c+dx}}{\frac{f}{-de+cf} - \frac{h}{-dg+ch}}} \sqrt{\left( b + \frac{-bc+ad}{c+dx} \right) \left( f + \frac{de-cf}{c+dx} \right) \left( h + \frac{dg-ch}{c+dx} \right)} \right) - 
\end{aligned}$$

$$\begin{aligned}
& \left( 2 c f h \sqrt{\frac{-\frac{b}{b c - a d} + \frac{1}{c+d x}}{-\frac{b}{b c - a d} + \frac{h}{-d g + c h}}} \sqrt{\frac{-\frac{f}{-d e + c f} + \frac{1}{c+d x}}{-\frac{f}{-d e + c f} + \frac{h}{-d g + c h}}} \left( -\frac{h}{-d g + c h} + \frac{1}{c+d x} \right) \text{EllipticF}[\text{ArcSin}\left[ \sqrt{\frac{(-d e + c f) \left( -h - \frac{d g}{c+d x} + \frac{c h}{c+d x} \right)}{d (-f g + e h)}} \right], \right. \\
& \left. \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)} \right] \Bigg) \Bigg/ \left( \sqrt{\frac{-\frac{h}{-d g + c h} + \frac{1}{c+d x}}{\frac{f}{-d e + c f} - \frac{h}{-d g + c h}}} \sqrt{\left( b + \frac{-b c + a d}{c+d x} \right) \left( f + \frac{d e - c f}{c+d x} \right) \left( h + \frac{d g - c h}{c+d x} \right)} \right) - \\
& \left( f (-d g + c h) \left( -\frac{f}{-d e + c f} + \frac{h}{-d g + c h} \right) \sqrt{\frac{-\frac{b}{b c - a d} + \frac{1}{c+d x}}{-\frac{b}{b c - a d} + \frac{h}{-d g + c h}}} \sqrt{-\frac{\left( -\frac{f}{-d e + c f} + \frac{1}{c+d x} \right) \left( -\frac{h}{-d g + c h} + \frac{1}{c+d x} \right)}{\left( -\frac{f}{-d e + c f} + \frac{h}{-d g + c h} \right)^2}} \text{EllipticPi}\left[ -\frac{-d f g + d e h}{(-d e + c f) h}, \right. \right. \\
& \left. \left. \text{ArcSin}\left[ \sqrt{\frac{(-d e + c f) \left( -h - \frac{d g}{c+d x} + \frac{c h}{c+d x} \right)}{d (-f g + e h)}} \right], \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)} \right] \Bigg) \Bigg/ \left( \sqrt{\left( b + \frac{-b c + a d}{c+d x} \right) \left( f + \frac{d e - c f}{c+d x} \right) \left( h + \frac{d g - c h}{c+d x} \right)} \right) \right)
\end{aligned}$$

**Problem 13: Result more than twice size of optimal antiderivative.**

$$\int \frac{d e + c f + 2 d f x}{\sqrt{a + b x} \sqrt{c + d x} \sqrt{e + f x} \sqrt{g + h x}} dx$$

Optimal (type 4, 449 leaves, 5 steps):

$$\begin{aligned}
& 2 (b d e + b c f - 2 a d f) \sqrt{\frac{(b e - a f) (c+d x)}{(d e - c f) (a+b x)}} \sqrt{g + h x} \text{EllipticF}[\text{ArcSin}\left[ \frac{\sqrt{b g - a h} \sqrt{e + f x}}{\sqrt{f g - e h} \sqrt{a + b x}} \right], -\frac{(b c - a d) (f g - e h)}{(d e - c f) (b g - a h)}] + \\
& b \sqrt{b g - a h} \sqrt{f g - e h} \sqrt{c + d x} \sqrt{-\frac{(b e - a f) (g + h x)}{(f g - e h) (a + b x)}} \\
& \left( 4 d f \sqrt{-d g + c h} (a + b x) \sqrt{\frac{(b g - a h) (c + d x)}{(d g - c h) (a + b x)}} \sqrt{\frac{(b g - a h) (e + f x)}{(f g - e h) (a + b x)}} \right. \\
& \left. \text{EllipticPi}\left[ -\frac{b (d g - c h)}{(b c - a d) h}, \text{ArcSin}\left[ \frac{\sqrt{b c - a d} \sqrt{g + h x}}{\sqrt{-d g + c h} \sqrt{a + b x}} \right], \frac{(b e - a f) (d g - c h)}{(b c - a d) (f g - e h)} \right] \right) \Bigg/ \left( b \sqrt{b c - a d} h \sqrt{c + d x} \sqrt{e + f x} \right)
\end{aligned}$$

Result (type 4, 1529 leaves):

$$\begin{aligned}
& \frac{1}{b^2 \sqrt{c + \frac{(a+b)x}{b} \left( d - \frac{ad}{a+b} \right)}} \sqrt{e + \frac{(a+b)x}{b} \left( f - \frac{af}{a+b} \right)} \sqrt{g + \frac{(a+b)x}{b} \left( h - \frac{ah}{a+b} \right)} 2 (a+b)^{3/2} \sqrt{\left( d + \frac{bc}{a+b} - \frac{ad}{a+b} \right) \left( f + \frac{be}{a+b} - \frac{af}{a+b} \right) \left( h + \frac{bg}{a+b} - \frac{ah}{a+b} \right)} \\
& \left( - \left( bde \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+b}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \left( -\frac{h}{-bg+ah} + \frac{1}{a+b} \right) \text{EllipticF}[\text{ArcSin}\left[ \sqrt{\frac{(-be+af)(-h - \frac{bg}{a+b} + \frac{ah}{a+b})}{b(-fg+eh)}} \right], \right. \right. \\
& \left. \left. \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)} \right] \right) / \left( \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+b}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \sqrt{\left( d + \frac{bc-ad}{a+b} \right) \left( f + \frac{be-af}{a+b} \right) \left( h + \frac{bg-ah}{a+b} \right)} \right) - \\
& \left( bcf \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+b}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \left( -\frac{h}{-bg+ah} + \frac{1}{a+b} \right) \text{EllipticF}[\text{ArcSin}\left[ \sqrt{\frac{(-be+af)(-h - \frac{bg}{a+b} + \frac{ah}{a+b})}{b(-fg+eh)}} \right], \right. \right. \\
& \left. \left. \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)} \right] \right) / \left( \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+b}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \sqrt{\left( d + \frac{bc-ad}{a+b} \right) \left( f + \frac{be-af}{a+b} \right) \left( h + \frac{bg-ah}{a+b} \right)} \right) + \\
& \left( 2adf \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+b}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \left( -\frac{h}{-bg+ah} + \frac{1}{a+b} \right) \text{EllipticF}[\text{ArcSin}\left[ \sqrt{\frac{(-be+af)(-h - \frac{bg}{a+b} + \frac{ah}{a+b})}{b(-fg+eh)}} \right], \right. \right. \\
& \left. \left. \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)} \right] \right) / \left( \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+b}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \sqrt{\left( d + \frac{bc-ad}{a+b} \right) \left( f + \frac{be-af}{a+b} \right) \left( h + \frac{bg-ah}{a+b} \right)} \right) + \\
& \left( 2df(-bg+ah) \left( -\frac{f}{-be+af} + \frac{h}{-bg+ah} \right) \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+b}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{-\frac{\left( -\frac{f}{-be+af} + \frac{1}{a+b} \right) \left( -\frac{h}{-bg+ah} + \frac{1}{a+b} \right)}{\left( -\frac{f}{-be+af} + \frac{h}{-bg+ah} \right)^2}} \text{EllipticPi}\left[ -\frac{-bfg+bef}{(-be+af)h}, \right. \right. \\
& \left. \left. \text{ArcSin}\left[ \sqrt{\frac{(-be+af)(-h - \frac{bg}{a+b} + \frac{ah}{a+b})}{b(-fg+eh)}} \right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)} \right] \right) / \left( h \sqrt{\left( d + \frac{bc-ad}{a+b} \right) \left( f + \frac{be-af}{a+b} \right) \left( h + \frac{bg-ah}{a+b} \right)} \right)
\end{aligned}$$

### Problem 14: Result more than twice size of optimal antiderivative.

$$\int \frac{d e + c f + 2 d f x}{(a + b x)^{3/2} \sqrt{c + d x} \sqrt{e + f x} \sqrt{g + h x}} dx$$

Optimal (type 4, 625 leaves, 7 steps):

$$\begin{aligned} & \frac{2 d (b d e + b c f - 2 a d f) \sqrt{a + b x} \sqrt{e + f x} \sqrt{g + h x}}{(b c - a d) (b e - a f) (b g - a h) \sqrt{c + d x}} - \frac{2 b (b d e + b c f - 2 a d f) \sqrt{c + d x} \sqrt{e + f x} \sqrt{g + h x}}{(b c - a d) (b e - a f) (b g - a h) \sqrt{a + b x}} - \\ & \left( 2 (b d e + b c f - 2 a d f) \sqrt{d g - c h} \sqrt{f g - e h} \sqrt{a + b x} \sqrt{-\frac{(d e - c f) (g + h x)}{(f g - e h) (c + d x)}} \right. \\ & \left. \text{EllipticE} \left[ \text{ArcSin} \left[ \frac{\sqrt{d g - c h} \sqrt{e + f x}}{\sqrt{f g - e h} \sqrt{c + d x}} \right], \frac{(b c - a d) (f g - e h)}{(b e - a f) (d g - c h)} \right] \right) / \left( (b c - a d) (b e - a f) (b g - a h) \sqrt{\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}} \sqrt{g + h x} \right) - \\ & \frac{2 d (d e - c f) \sqrt{\frac{(b e - a f) (c + d x)}{(d e - c f) (a + b x)}} \sqrt{g + h x} \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\sqrt{b g - a h} \sqrt{e + f x}}{\sqrt{f g - e h} \sqrt{a + b x}} \right], -\frac{(b c - a d) (f g - e h)}{(d e - c f) (b g - a h)} \right]}{(b c - a d) \sqrt{b g - a h} \sqrt{f g - e h} \sqrt{c + d x} \sqrt{-\frac{(b e - a f) (g + h x)}{(f g - e h) (a + b x)}}} \end{aligned}$$

Result (type 4, 2236 leaves):

$$\begin{aligned} & \frac{2 b (b d e + b c f - 2 a d f) \sqrt{c + d x} \sqrt{e + f x} \sqrt{g + h x}}{(b c - a d) (b e - a f) (b g - a h) \sqrt{a + b x}} + \\ & \frac{1}{b^2 (-b c + a d) (-b e + a f) (-b g + a h)} 2 \left( \frac{(-b d e - b c f + 2 a d f) (a + b x)^{5/2} \left( d + \frac{b c}{a + b x} - \frac{a d}{a + b x} \right) \left( f + \frac{b e}{a + b x} - \frac{a f}{a + b x} \right) \left( h + \frac{b g}{a + b x} - \frac{a h}{a + b x} \right)}{\sqrt{c + \frac{(a + b x) (d - \frac{a d}{a + b x})}{b}}} \sqrt{e + \frac{(a + b x) (f - \frac{a f}{a + b x})}{b}} \sqrt{g + \frac{(a + b x) (h - \frac{a h}{a + b x})}{b}} - \right. \\ & \left. \frac{1}{\sqrt{c + \frac{(a + b x) (d - \frac{a d}{a + b x})}{b}}} \sqrt{e + \frac{(a + b x) (f - \frac{a f}{a + b x})}{b}} \sqrt{g + \frac{(a + b x) (h - \frac{a h}{a + b x})}{b}} (b c - a d) (b e - a f) (b g - a h) (a + b x)^{3/2} \right) \end{aligned}$$

$$\begin{aligned}
& \sqrt{\left(d + \frac{bc}{a+bx} - \frac{ad}{a+bx}\right) \left(f + \frac{be}{a+bx} - \frac{af}{a+bx}\right) \left(h + \frac{bg}{a+bx} - \frac{ah}{a+bx}\right)} \left( - \left( b d e \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+bx}\right)}{bdg-bch}} \right. \right. \\
& \left. \left. - \frac{(bdg-bch) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] - \right. \right. \\
& \left. \left. \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{-bc+ad} \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+bx}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+bx}\right) \left(f + \frac{be-af}{a+bx}\right) \left(h + \frac{bg-ah}{a+bx}\right)} \right) - \left( b c f \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+bx}\right)}{bdg-bch}} \right. \\
& \left. \left( - \frac{(bdg-bch) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] - \right. \right. \\
& \left. \left. \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{-bc+ad} \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+bx}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+bx}\right) \left(f + \frac{be-af}{a+bx}\right) \left(h + \frac{bg-ah}{a+bx}\right)} \right) + \left( 2 a d f \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+bx}\right)}{bdg-bch}} \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{f}{-be+af} + \frac{1}{a+bx} \right) \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+bx}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \left( -\frac{(bdg-bch) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{(bc-ad)(bg-ah)} \right. \\
& \left. \left. - \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{-bc+ad} \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+bx}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+bx}\right) \left(f + \frac{be-af}{a+bx}\right) \left(h + \frac{bg-ah}{a+bx}\right)} - \left(2df \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+bx}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+bx}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \right. \right. \\
& \left. \left. \left( -\frac{h}{-bg+ah} + \frac{1}{a+bx} \right) \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(-be+af)(-h-\frac{bg}{a+bx}+\frac{ah}{a+bx})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] \right) \right) / \\
& \left( \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+bx}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+bx}\right) \left(f + \frac{be-af}{a+bx}\right) \left(h + \frac{bg-ah}{a+bx}\right)} \right) \right)
\end{aligned}$$

**Problem 15: Result more than twice size of optimal antiderivative.**

$$\int \frac{de + cf + 2dfx}{(a+bx)^{5/2} \sqrt{c+dx} \sqrt{e+fx} \sqrt{g+hx}} dx$$

Optimal (type 4, 1090 leaves, 8 steps):

$$\begin{aligned}
& \left( 4 d (3 a^3 d^2 f^2 h - a^2 b d f (d f g + 4 d e h + 4 c f h)) - \right. \\
& \quad \left. b^3 (d^2 e^2 g - c d e (f g - e h) + c^2 f (f g + e h)) + a b^2 (2 c^2 f^2 h + d^2 e (f g + 2 e h) + c d f (f g + 3 e h)) \right) \sqrt{a+b x} \sqrt{e+f x} \sqrt{g+h x} \Big/ \\
& \quad \left( 3 (b c - a d)^2 (b e - a f)^2 (b g - a h)^2 \sqrt{c+d x} \right) - \frac{2 b (b d e + b c f - 2 a d f) \sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x}}{3 (b c - a d) (b e - a f) (b g - a h) (a+b x)^{3/2}} - \\
& \left( 4 b (3 a^3 d^2 f^2 h - a^2 b d f (d f g + 4 d e h + 4 c f h)) - b^3 (d^2 e^2 g - c d e (f g - e h) + c^2 f (f g + e h)) + \right. \\
& \quad \left. a b^2 (2 c^2 f^2 h + d^2 e (f g + 2 e h) + c d f (f g + 3 e h)) \right) \sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x} \Big/ \left( 3 (b c - a d)^2 (b e - a f)^2 (b g - a h)^2 \sqrt{a+b x} \right) - \\
& \left( 4 \sqrt{d g - c h} \sqrt{f g - e h} (3 a^3 d^2 f^2 h - a^2 b d f (d f g + 4 d e h + 4 c f h)) - b^3 (d^2 e^2 g - c d e (f g - e h) + c^2 f (f g + e h)) + \right. \\
& \quad \left. a b^2 (2 c^2 f^2 h + d^2 e (f g + 2 e h) + c d f (f g + 3 e h)) \right) \sqrt{a+b x} \sqrt{-\frac{(d e - c f) (g + h x)}{(f g - e h) (c + d x)}} \\
& \text{EllipticE}[\text{ArcSin}\left[\frac{\sqrt{d g - c h} \sqrt{e + f x}}{\sqrt{f g - e h} \sqrt{c + d x}}\right], \frac{(b c - a d) (f g - e h)}{(b e - a f) (d g - c h)}] \Big/ \left( 3 (b c - a d)^2 (b e - a f)^2 (b g - a h)^2 \sqrt{\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}} \sqrt{g + h x} \right) + \\
& \left( 2 (d e - c f) (3 a^2 d^2 f h - a b d (d f g + 3 d e h + 2 c f h)) + b^2 (2 d^2 e g - c d f g + c d e h + c^2 f h) \right) \sqrt{\frac{(b e - a f) (c + d x)}{(d e - c f) (a + b x)}} \\
& \quad \sqrt{g + h x} \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{b g - a h} \sqrt{e + f x}}{\sqrt{f g - e h} \sqrt{a + b x}}\right], -\frac{(b c - a d) (f g - e h)}{(d e - c f) (b g - a h)}] \Big/ \\
& \left( 3 (b c - a d)^2 (b e - a f) (b g - a h)^{3/2} \sqrt{f g - e h} \sqrt{c + d x} \sqrt{-\frac{(b e - a f) (g + h x)}{(f g - e h) (a + b x)}} \right)
\end{aligned}$$

Result (type 4, 10601 leaves):

$$\begin{aligned}
& \sqrt{a+b x} \sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x} \\
& \left( -\frac{2 b (b d e + b c f - 2 a d f)}{3 (b c - a d) (b e - a f) (b g - a h) (a+b x)^2} + \left( 4 b (b^3 d^2 e^2 g - b^3 c d e f g - a b^2 d^2 e f g + b^3 c^2 f^2 g - a b^2 c d f^2 g + a^2 b d^2 f^2 g + \right. \right. \\
& \quad \left. \left. b^3 c d e^2 h - 2 a b^2 d^2 e^2 h + b^3 c^2 e f h - 3 a b^2 c d e f h + 4 a^2 b d^2 e f h - 2 a b^2 c^2 f^2 h + 4 a^2 b c d f^2 h - 3 a^3 d^2 f^2 h \right) \right) / \\
& \left( 3 (b c - a d)^2 (b e - a f)^2 (b g - a h)^2 (a+b x) \right) + \frac{1}{3 b^2 (-b c + a d)^2 (-b e + a f)^2 (-b g + a h)^2}
\end{aligned}$$

$$\begin{aligned}
& 2 \left( - \left( 2 \left( b^3 d^2 e^2 g - b^3 c d e f g - a b^2 d^2 e f g + b^3 c^2 f^2 g - a b^2 c d f^2 g + a^2 b d^2 f^2 g + b^3 c d e^2 h - 2 a b^2 d^2 e^2 h + b^3 c^2 e f h - 3 a b^2 c d e f h + \right. \right. \right. \\
& \quad \left. \left. \left. 4 a^2 b d^2 e f h - 2 a b^2 c^2 f^2 h + 4 a^2 b c d f^2 h - 3 a^3 d^2 f^2 h \right) (a + b x)^{5/2} \left( d + \frac{b c}{a + b x} - \frac{a d}{a + b x} \right) \left( f + \frac{b e}{a + b x} - \frac{a f}{a + b x} \right) \right. \\
& \quad \left. \left. \left( h + \frac{b g}{a + b x} - \frac{a h}{a + b x} \right) \right) / \left( \sqrt{c + \frac{(a + b x) \left( d - \frac{a d}{a + b x} \right)}{b}} \sqrt{e + \frac{(a + b x) \left( f - \frac{a f}{a + b x} \right)}{b}} \sqrt{g + \frac{(a + b x) \left( h - \frac{a h}{a + b x} \right)}{b}} \right) + \right. \\
& \quad \frac{1}{\sqrt{c + \frac{(a + b x) \left( d - \frac{a d}{a + b x} \right)}{b}} \sqrt{e + \frac{(a + b x) \left( f - \frac{a f}{a + b x} \right)}{b}} \sqrt{g + \frac{(a + b x) \left( h - \frac{a h}{a + b x} \right)}{b}}} (b c - a d) (b e - a f) (b g - a h) (a + b x)^{3/2} \\
& \quad \left. \sqrt{\left( d + \frac{b c}{a + b x} - \frac{a d}{a + b x} \right) \left( f + \frac{b e}{a + b x} - \frac{a f}{a + b x} \right) \left( h + \frac{b g}{a + b x} - \frac{a h}{a + b x} \right)} \left( 2 b^3 d^2 e^2 g \sqrt{\frac{(b c - a d) (b g - a h) \left( -\frac{d}{-b c + a d} + \frac{1}{a + b x} \right)}{b d g - b c h}} \right. \right. \\
& \quad \left. \left. \left( -\frac{f}{-b e + a f} + \frac{1}{a + b x} \right) \sqrt{\frac{-\frac{h}{-b g + a h} + \frac{1}{a + b x}}{\frac{f}{-b e + a f} - \frac{h}{-b g + a h}}} \right. \right. \left( -\frac{(b d g - b c h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e - a f) \left( h + \frac{b g}{a + b x} - \frac{a h}{a + b x} \right)}{b (-f g + e h)}}, \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}\right]}{(b c - a d) (b g - a h)} \right. \right. \\
& \quad \left. \left. \left. \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e - a f) \left( h + \frac{b g}{a + b x} - \frac{a h}{a + b x} \right)}{b (-f g + e h)}}, \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}\right]}{-b c + a d} \right) \right) \right) / \right. \\
& \quad \left. \left( \sqrt{\frac{-\frac{f}{-b e + a f} + \frac{1}{a + b x}}{-\frac{f}{-b e + a f} + \frac{h}{-b g + a h}}} \sqrt{\left( d + \frac{b c - a d}{a + b x} \right) \left( f + \frac{b e - a f}{a + b x} \right) \left( h + \frac{b g - a h}{a + b x} \right)} - \left( 2 b^3 c d e f g \sqrt{\frac{(b c - a d) (b g - a h) \left( -\frac{d}{-b c + a d} + \frac{1}{a + b x} \right)}{b d g - b c h}} \right. \right. \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{f}{-b e + a f} + \frac{1}{a + b x} \right) \sqrt{\frac{-\frac{h}{-b g + a h} + \frac{1}{a + b x}}{\frac{f}{-b e + a f} - \frac{h}{-b g + a h}}} \left( -\frac{(b d g - b c h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e - a f) \left(h + \frac{b g}{a + b x} - \frac{a h}{a + b x}\right)}{b (-f g + e h)}}, \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}\right]}]{(b c - a d) (b g - a h)} \right. \\
& \left. \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e - a f) \left(h + \frac{b g}{a + b x} - \frac{a h}{a + b x}\right)}{b (-f g + e h)}}, \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}\right]}]{-b c + a d} \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-b e + a f} + \frac{1}{a + b x}}{\frac{f}{-b e + a f} + \frac{h}{-b g + a h}}} \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right) \left(h + \frac{b g - a h}{a + b x}\right)} \right) - \left( 2 a b^2 d^2 e f g \sqrt{\frac{(b c - a d) (b g - a h) \left(-\frac{d}{-b c + a d} + \frac{1}{a + b x}\right)}{b d g - b c h}} \right. \\
& \left. \left( -\frac{f}{-b e + a f} + \frac{1}{a + b x} \right) \sqrt{\frac{-\frac{h}{-b g + a h} + \frac{1}{a + b x}}{\frac{f}{-b e + a f} - \frac{h}{-b g + a h}}} \left( -\frac{(b d g - b c h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e - a f) \left(h + \frac{b g}{a + b x} - \frac{a h}{a + b x}\right)}{b (-f g + e h)}}, \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}\right]}]{(b c - a d) (b g - a h)} \right. \right. \\
& \left. \left. \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e - a f) \left(h + \frac{b g}{a + b x} - \frac{a h}{a + b x}\right)}{b (-f g + e h)}}, \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}\right]}]{-b c + a d} \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-b e + a f} + \frac{1}{a + b x}}{\frac{f}{-b e + a f} + \frac{h}{-b g + a h}}} \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right) \left(h + \frac{b g - a h}{a + b x}\right)} \right) + \left( 2 b^3 c^2 f^2 g \sqrt{\frac{(b c - a d) (b g - a h) \left(-\frac{d}{-b c + a d} + \frac{1}{a + b x}\right)}{b d g - b c h}} \right. \\
& \left. \left( -\frac{f}{-b e + a f} + \frac{1}{a + b x} \right) \sqrt{\frac{-\frac{h}{-b g + a h} + \frac{1}{a + b x}}{\frac{f}{-b e + a f} - \frac{h}{-b g + a h}}} \left( -\frac{(b d g - b c h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e - a f) \left(h + \frac{b g}{a + b x} - \frac{a h}{a + b x}\right)}{b (-f g + e h)}}, \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}\right]}]{(b c - a d) (b g - a h)} \right. \right. \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) (-f g+e h)}{(-b e+a f) (-d g+c h)}}}{-b c+a d}\right\} / \\
& \left( \sqrt{\frac{-\frac{f}{-b e+a f}+\frac{1}{a+b x}}{-\frac{f}{-b e+a f}+\frac{h}{-b g+a h}}} \sqrt{\left(d+\frac{b c-a d}{a+b x}\right)\left(f+\frac{b e-a f}{a+b x}\right)\left(h+\frac{b g-a h}{a+b x}\right)} - \left(2 a b^2 c d f^2 g \sqrt{\frac{(b c-a d) (b g-a h) \left(-\frac{d}{-b c+a d}+\frac{1}{a+b x}\right)}{b d g-b c h}} \right. \right. \\
& \left. \left. \left(-\frac{f}{-b e+a f}+\frac{1}{a+b x}\right) \sqrt{\frac{-\frac{h}{-b g+a h}+\frac{1}{a+b x}}{\frac{f}{-b e+a f}-\frac{h}{-b g+a h}}} \left(-\frac{(b d g-b c h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) (-f g+e h)}{(-b e+a f) (-d g+c h)}}}{(b c-a d) (b g-a h)} - \right. \right. \right. \\
& \left. \left. \left.d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) (-f g+e h)}{(-b e+a f) (-d g+c h)}}}{-b c+a d}\right)\right\} / \\
& \left( \sqrt{\frac{-\frac{f}{-b e+a f}+\frac{1}{a+b x}}{-\frac{f}{-b e+a f}+\frac{h}{-b g+a h}}} \sqrt{\left(d+\frac{b c-a d}{a+b x}\right)\left(f+\frac{b e-a f}{a+b x}\right)\left(h+\frac{b g-a h}{a+b x}\right)} + \left(2 a^2 b d^2 f^2 g \sqrt{\frac{(b c-a d) (b g-a h) \left(-\frac{d}{-b c+a d}+\frac{1}{a+b x}\right)}{b d g-b c h}} \right. \right. \\
& \left. \left. \left(-\frac{f}{-b e+a f}+\frac{1}{a+b x}\right) \sqrt{\frac{-\frac{h}{-b g+a h}+\frac{1}{a+b x}}{\frac{f}{-b e+a f}-\frac{h}{-b g+a h}}} \left(-\frac{(b d g-b c h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) (-f g+e h)}{(-b e+a f) (-d g+c h)}}}{(b c-a d) (b g-a h)} - \right. \right. \right. \\
& \left. \left. \left.d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) (-f g+e h)}{(-b e+a f) (-d g+c h)}}}{-b c+a d}\right)\right\}
\end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+bx}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+bx}\right) \left(f + \frac{be-af}{a+bx}\right) \left(h + \frac{bg-ah}{a+bx}\right)} \right) + \left( 2b^3 c d e^2 h \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+bx}\right)}{b d g - b c h}} \right. \\
& \left( -\frac{f}{-be+af} + \frac{1}{a+bx} \right) \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+bx}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \left( -\frac{(bdg-bch) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)\left(h+\frac{bg}{a+bx}-\frac{ah}{a+bx}\right)}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{(bc-ad)(bg-ah)} \right. \\
& \left. \left. \left. d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)\left(h+\frac{bg}{a+bx}-\frac{ah}{a+bx}\right)}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] \right) \right) / \right. \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+bx}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+bx}\right) \left(f + \frac{be-af}{a+bx}\right) \left(h + \frac{bg-ah}{a+bx}\right)} \right) - \left( 4ab^2 d^2 e^2 h \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+bx}\right)}{b d g - b c h}} \right. \\
& \left( -\frac{f}{-be+af} + \frac{1}{a+bx} \right) \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+bx}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \left( -\frac{(bdg-bch) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)\left(h+\frac{bg}{a+bx}-\frac{ah}{a+bx}\right)}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{(bc-ad)(bg-ah)} \right. \\
& \left. \left. \left. d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)\left(h+\frac{bg}{a+bx}-\frac{ah}{a+bx}\right)}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] \right) \right) / \right. \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+bx}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+bx}\right) \left(f + \frac{be-af}{a+bx}\right) \left(h + \frac{bg-ah}{a+bx}\right)} \right) + \left( 2b^3 c^2 e f h \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+bx}\right)}{b d g - b c h}} \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{f}{-be+af} + \frac{1}{a+bx} \right) \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+bx}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \left( -\frac{(bdg-bch) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}, \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}\right]}]}{(bc-ad)(bg-ah)} \right. \\
& \left. \left. - \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}, \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}\right]}]}{-bc+ad} \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+bx}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+bx}\right) \left(f + \frac{be-af}{a+bx}\right) \left(h + \frac{bg-ah}{a+bx}\right)} - \left( 6ab^2cdlefh \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+bx}\right)}{bdg-bch}} \right. \right. \\
& \left. \left. - \frac{(bdg-bch) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}, \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}\right]}]}{(bc-ad)(bg-ah)} \right. \right. \\
& \left. \left. - \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}, \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}\right]}]}{-bc+ad} \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+bx}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+bx}\right) \left(f + \frac{be-af}{a+bx}\right) \left(h + \frac{bg-ah}{a+bx}\right)} + \left( 8a^2bd^2efh \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+bx}\right)}{bdg-bch}} \right. \right. \\
& \left. \left. - \frac{(bdg-bch) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}, \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}\right]}]}{(bc-ad)(bg-ah)} \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) (-f g+e h)}{(-b e+a f) (-d g+c h)}}}{-b c+a d}\right\} / \\
& \left( \sqrt{\frac{-\frac{f}{-b e+a f}+\frac{1}{a+b x}}{-\frac{f}{-b e+a f}+\frac{h}{-b g+a h}}} \sqrt{\left(d+\frac{b c-a d}{a+b x}\right)\left(f+\frac{b e-a f}{a+b x}\right)\left(h+\frac{b g-a h}{a+b x}\right)} - \left(4 a b^2 c^2 f^2 h \sqrt{\frac{(b c-a d) (b g-a h) \left(-\frac{d}{-b c+a d}+\frac{1}{a+b x}\right)}{b d g-b c h}} \right.\right. \\
& \left. \left. \left(-\frac{f}{-b e+a f}+\frac{1}{a+b x}\right) \sqrt{\frac{-\frac{h}{-b g+a h}+\frac{1}{a+b x}}{\frac{f}{-b e+a f}-\frac{h}{-b g+a h}}} \left(-\frac{(b d g-b c h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) (-f g+e h)}{(-b e+a f) (-d g+c h)}}}{(b c-a d) (b g-a h)} - \right.\right. \right. \\
& \left. \left. \left.d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) (-f g+e h)}{(-b e+a f) (-d g+c h)}}\right]\right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-b e+a f}+\frac{1}{a+b x}}{-\frac{f}{-b e+a f}+\frac{h}{-b g+a h}}} \sqrt{\left(d+\frac{b c-a d}{a+b x}\right)\left(f+\frac{b e-a f}{a+b x}\right)\left(h+\frac{b g-a h}{a+b x}\right)} + \left(8 a^2 b c d f^2 h \sqrt{\frac{(b c-a d) (b g-a h) \left(-\frac{d}{-b c+a d}+\frac{1}{a+b x}\right)}{b d g-b c h}} \right.\right. \\
& \left. \left. \left(-\frac{f}{-b e+a f}+\frac{1}{a+b x}\right) \sqrt{\frac{-\frac{h}{-b g+a h}+\frac{1}{a+b x}}{\frac{f}{-b e+a f}-\frac{h}{-b g+a h}}} \left(-\frac{(b d g-b c h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) (-f g+e h)}{(-b e+a f) (-d g+c h)}}}{(b c-a d) (b g-a h)} - \right.\right. \right. \\
& \left. \left. \left.d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) (-f g+e h)}{(-b e+a f) (-d g+c h)}}\right]\right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) - \left( 6 a^3 d^2 f^2 h \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+b x}\right)}{bdg-bch}} \right. \\
& \left( -\frac{f}{-be+af} + \frac{1}{a+b x} \right) \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+b x}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \left( -\frac{(bdg-bch) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg-ah}{a+b x})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{(bc-ad)(bg-ah)} \right. \\
& \left. \left. \left. \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg-ah}{a+b x})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{-bc+ad} \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} + \left(b^2 d^2 e f g \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+b x}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \right. \right. \\
& \left. \left. \left( -\frac{h}{-bg+ah} + \frac{1}{a+b x} \right) \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(-be+af)(-h-\frac{bg}{a+b x}+\frac{ah}{a+b x})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] \right) \right) / \\
& \left( \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+b x}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} + \left(b^2 c d f^2 g \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+b x}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \right. \right. \\
& \left. \left. \left( -\frac{h}{-bg+ah} + \frac{1}{a+b x} \right) \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(-be+af)(-h-\frac{bg}{a+b x}+\frac{ah}{a+b x})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] \right) \right) / \\
& \left( \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+b x}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} - \left( 2 a b d^2 f^2 g \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+b x}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{h}{-bg + ah} + \frac{1}{a + bx} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-be + af) \left( -h - \frac{bg}{a+bx} + \frac{ah}{a+bx} \right)}{b (-fg + eh)}} \right], \frac{(-bc + ad) (-fg + eh)}{(-be + af) (-dg + ch)} \right] \Bigg] \\
& \left( \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+bx}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \sqrt{\left( d + \frac{bc - ad}{a + bx} \right) \left( f + \frac{be - af}{a + bx} \right) \left( h + \frac{bg - ah}{a + bx} \right)} \right) + \left( b^2 d^2 e^2 h \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+bx}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+bx}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \right. \\
& \left. \left( -\frac{h}{-bg + ah} + \frac{1}{a + bx} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-be + af) \left( -h - \frac{bg}{a+bx} + \frac{ah}{a+bx} \right)}{b (-fg + eh)}} \right], \frac{(-bc + ad) (-fg + eh)}{(-be + af) (-dg + ch)} \right] \right] \\
& \left( \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+bx}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \sqrt{\left( d + \frac{bc - ad}{a + bx} \right) \left( f + \frac{be - af}{a + bx} \right) \left( h + \frac{bg - ah}{a + bx} \right)} \right) + \left( 2 b^2 c d e f h \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+bx}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+bx}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \right. \\
& \left. \left( -\frac{h}{-bg + ah} + \frac{1}{a + bx} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-be + af) \left( -h - \frac{bg}{a+bx} + \frac{ah}{a+bx} \right)}{b (-fg + eh)}} \right], \frac{(-bc + ad) (-fg + eh)}{(-be + af) (-dg + ch)} \right] \right] \\
& \left( \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+bx}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \sqrt{\left( d + \frac{bc - ad}{a + bx} \right) \left( f + \frac{be - af}{a + bx} \right) \left( h + \frac{bg - ah}{a + bx} \right)} \right) - \left( 5 a b d^2 e f h \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+bx}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+bx}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \right. \\
& \left. \left( -\frac{h}{-bg + ah} + \frac{1}{a + bx} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-be + af) \left( -h - \frac{bg}{a+bx} + \frac{ah}{a+bx} \right)}{b (-fg + eh)}} \right], \frac{(-bc + ad) (-fg + eh)}{(-be + af) (-dg + ch)} \right] \right] \\
& \left( \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+bx}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \sqrt{\left( d + \frac{bc - ad}{a + bx} \right) \left( f + \frac{be - af}{a + bx} \right) \left( h + \frac{bg - ah}{a + bx} \right)} \right) + \left( b^2 c^2 f^2 h \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+bx}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+bx}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \right. \\
& \left. \left( -\frac{h}{-bg + ah} + \frac{1}{a + bx} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-be + af) \left( -h - \frac{bg}{a+bx} + \frac{ah}{a+bx} \right)}{b (-fg + eh)}} \right], \frac{(-bc + ad) (-fg + eh)}{(-be + af) (-dg + ch)} \right] \right]
\end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+bx}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+bx}\right) \left(f + \frac{be-af}{a+bx}\right) \left(h + \frac{bg-ah}{a+bx}\right)} \right) - \left( 5abcdf^2h \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+bx}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+bx}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \right. \\
& \left. \left( -\frac{h}{-bg+ah} + \frac{1}{a+bx} \right) \text{EllipticF}[\text{ArcSin}\left[ \sqrt{\frac{(-be+af)(-h-\frac{bg}{a+bx}+\frac{ah}{a+bx})}{b(-fg+eh)}} \right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] \right) / \\
& \left( \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+bx}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+bx}\right) \left(f + \frac{be-af}{a+bx}\right) \left(h + \frac{bg-ah}{a+bx}\right)} \right) + \left( 6a^2d^2f^2h \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+bx}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+bx}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \right. \\
& \left. \left( -\frac{h}{-bg+ah} + \frac{1}{a+bx} \right) \text{EllipticF}[\text{ArcSin}\left[ \sqrt{\frac{(-be+af)(-h-\frac{bg}{a+bx}+\frac{ah}{a+bx})}{b(-fg+eh)}} \right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] \right) / \\
& \left( \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+bx}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+bx}\right) \left(f + \frac{be-af}{a+bx}\right) \left(h + \frac{bg-ah}{a+bx}\right)} \right)
\end{aligned}$$

**Problem 16:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(a+bx)(abB - a^2C + b^2Bx + b^2Cx^2)}{\sqrt{c+dx} \sqrt{e+fx} \sqrt{g+hx}} dx$$

Optimal (type 4, 721 leaves, 8 steps):

$$\begin{aligned}
& \frac{2 b^2 (5 b B d f h + 2 C (a d f h - 2 b (d f g + d e h + c f h))) \sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x}}{15 d^2 f^2 h^2} + \\
& \frac{2 b^2 C (a+b x) \sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x}}{5 d f h} - \left( 2 b \sqrt{-d e + c f} (15 a^2 C d^2 f^2 h^2 - 10 a b d f h (3 B d f h - C (d f g + d e h + c f h))) + \right. \\
& \left. b^2 (10 B d f h (d f g + d e h + c f h) - C (8 c^2 f^2 h^2 + 7 c d f h (f g + e h) + d^2 (8 f^2 g^2 + 7 e f g h + 8 e^2 h^2))) \right) \sqrt{\frac{d (e+f x)}{d e - c f} \sqrt{g+h x}} \\
& \text{EllipticE} \left[ \text{ArcSin} \left[ \frac{\sqrt{f} \sqrt{c+d x}}{\sqrt{-d e + c f}}, \frac{(d e - c f) h}{f (d g - c h)} \right], \frac{1}{15 d^3 f^{5/2} h^3 \sqrt{e+f x} \sqrt{\frac{d (g+h x)}{d g - c h}}} \right] - \frac{1}{15 d^3 f^{5/2} h^3 \sqrt{e+f x} \sqrt{g+h x}} \\
& 2 \sqrt{-d e + c f} (15 a^3 C d^2 f^2 h^3 - 15 a^2 b d^2 f^2 h^2 (C g + B h) + 5 a b^2 d f h (6 B d f g h - C (c h (f g - e h) + d g (2 f g + e h))) - \\
& b^3 (5 B d f h (c h (f g - e h) + d g (2 f g + e h)) - C (4 c^2 f h^2 (f g - e h) + c d h (3 f^2 g^2 + e f g h - 4 e^2 h^2) + d^2 g (8 f^2 g^2 + 3 e f g h + 4 e^2 h^2))) \\
& \sqrt{\frac{d (e+f x)}{d e - c f}} \sqrt{\frac{d (g+h x)}{d g - c h}} \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\sqrt{f} \sqrt{c+d x}}{\sqrt{-d e + c f}}, \frac{(d e - c f) h}{f (d g - c h)} \right], \frac{1}{15 d^4 f^2 h^2} \right]
\end{aligned}$$

Result (type 4, 12 665 leaves) :

$$\begin{aligned}
& \sqrt{c+d x} \sqrt{e+f x} \left( \frac{2 b^2 (-4 b C d f g - 4 b C d e h - 4 b c C f h + 5 b B d f h + 5 a C d f h)}{15 d^2 f^2 h^2} + \frac{2 b^3 C x}{5 d f h} \right) \sqrt{g+h x} + \\
& \frac{1}{15 d^4 f^2 h^2} 2 \left( - \frac{1}{f h \sqrt{e + \frac{(c+d x) (f - \frac{c f}{c+d x})}{d}} \sqrt{g + \frac{(c+d x) (h - \frac{c h}{c+d x})}{d}}} \right. \\
& b (-8 b^2 C d^2 f^2 g^2 - 7 b^2 C d^2 e f g h - 7 b^2 c C d f^2 g h + 10 b^2 B d^2 f^2 g h + 10 a b C d^2 f^2 g h - 8 b^2 C d^2 e^2 h^2 - 7 b^2 c C d e f h^2 + \\
& 10 b^2 B d^2 e f h^2 + 10 a b C d^2 e f h^2 - 8 b^2 c^2 C f^2 h^2 + 10 b^2 B c d f^2 h^2 + 10 a b c C d f^2 h^2 - 30 a b B d^2 f^2 h^2 + 15 a^2 C d^2 f^2 h^2) \\
& (c+d x)^{3/2} \left( f + \frac{d e}{c+d x} - \frac{c f}{c+d x} \right) \left( h + \frac{d g}{c+d x} - \frac{c h}{c+d x} \right) - \frac{1}{f h \sqrt{e + \frac{(c+d x) (f - \frac{c f}{c+d x})}{d}} \sqrt{g + \frac{(c+d x) (h - \frac{c h}{c+d x})}{d}}}
\end{aligned}$$

$$\begin{aligned}
& \left( c + d x \right) \sqrt{\left( f + \frac{d e}{c + d x} - \frac{c f}{c + d x} \right) \left( h + \frac{d g}{c + d x} - \frac{c h}{c + d x} \right)} \left( 8 \pm b^3 C d^4 e f^2 g^3 h \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right. \\
& \left. \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) \right) / \\
& \left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} - 8 \pm b^3 c C d^3 f^3 g^3 h \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right. \\
& \left. \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) \right) / \\
& \left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} + 7 \pm b^3 C d^4 e^2 f g^2 h^2 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right. \\
& \left. \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) \right) / \\
& \left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} - 8 \pm b^3 c C d^3 e f^2 g^2 h^2 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)
\end{aligned}$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) - \left( 10 \text{i} b^3 B d^4 e f^2 g^2 h^2 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right.$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) - \left( 10 \text{i} a b^2 C d^4 e f^2 g^2 h^2 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) + \left( \text{i} b^3 c^2 C d^2 f^3 g^2 h^2 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) + \left( 10 \pm b^3 B c d^3 f^3 g^2 h^2 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) + \left( 10 \pm a b^2 c C d^3 f^3 g^2 h^2 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) + \left( 8 \pm b^3 C d^4 e^3 g h^3 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) - \left( 8 \pm b^3 c C d^3 e^2 f g h^3 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) - \left( 10 \text{i} b^3 B d^4 e^2 f g h^3 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right.$$

$$\left. \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) - \left( 10 \text{i} a b^2 C d^4 e^2 f g h^3 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right.$$

$$\left. \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) + \left( \text{i} b^3 c^2 C d^2 e f^2 g h^3 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left. \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) + \left( 10 \pm b^3 B c d^3 e f^2 g h^3 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) + \left( 10 \pm a b^2 c C d^3 e f^2 g h^3 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) + \left( 30 \pm a b^2 B d^4 e f^2 g h^3 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) - \left( 15 \pm a^2 b C d^4 e f^2 g h^3 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) - \left( i b^3 c^3 C d f^3 g h^3 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right.$$

$$\left. \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) - \left( 30 i a b^2 B c d^3 f^3 g h^3 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right.$$

$$\left. \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) + \left( 15 i a^2 b c C d^3 f^3 g h^3 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right.$$

$$\left. \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) - \left( 8 i b^3 c C d^3 e^3 h^4 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) + \left( i b^3 c^2 C d^2 e^2 f h^4 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) + \left( 10 i b^3 B c d^3 e^2 f h^4 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) + \left( 10 i a b^2 c C d^3 e^2 f h^4 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) - \left( i b^3 c^3 C d e f^2 h^4 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right.$$

$$\left. \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) - \left( 30 i a b^2 B c d^3 e f^2 h^4 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right.$$

$$\left. \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) + \left( 15 i a^2 b c C d^3 e f^2 h^4 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right.$$

$$\left. \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\begin{aligned}
& \left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) + \left( 8 i b^3 c^4 C f^3 h^4 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right. \\
& \left. \left. \left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) \right) / \\
& \left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) - \left( 10 i b^3 B c^3 d f^3 h^4 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right. \\
& \left. \left. \left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) \right) / \\
& \left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) - \left( 10 i a b^2 c^3 C d f^3 h^4 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right. \\
& \left. \left. \left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) \right) / \\
& \left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) + \left( 30 i a b^2 B c^2 d^2 f^3 h^4 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right.
\end{aligned}$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) - \left( 15 \pm a^2 b c^2 C d^2 f^3 h^4 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right.$$

$$\left. \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) -$$

$$\frac{4 i b^3 C d^3 e f^2 g^2 h \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right]}{\sqrt{-\frac{-d e + c f}{f}} \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)}} +$$

$$\frac{4 i b^3 c C d^2 f^3 g^2 h \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right]}{\sqrt{-\frac{-d e + c f}{f}} \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)}} -$$

$$\frac{4 i b^3 C d^3 e^2 f g h^2 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right]}{\sqrt{-\frac{-d e + c f}{f}} \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)}} +$$

$$\frac{i b^3 c C d^2 e f^2 g h^2 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}\right]}{\sqrt{-\frac{d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}} +$$

$$\frac{5 i b^3 B d^3 e f^2 g h^2 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}\right]}{\sqrt{-\frac{d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}} +$$

$$\frac{5 i a b^2 C d^3 e f^2 g h^2 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}\right]}{\sqrt{-\frac{d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}} +$$

$$\frac{3 i b^3 c^2 C d f^3 g h^2 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}\right]}{\sqrt{-\frac{d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}} -$$

$$\frac{5 i b^3 B c d^2 f^3 g h^2 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}\right]}{\sqrt{-\frac{d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}} -$$

$$\frac{5 i a b^2 c C d^2 f^3 g h^2 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}\right]}{\sqrt{-\frac{d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}} +$$

$$\frac{4 i b^3 c C d^2 e^2 f h^3 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}\right]}{\sqrt{-\frac{d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}} +$$

$$\frac{3 i b^3 c^2 C d e f^2 h^3 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}\right]}{\sqrt{-\frac{d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}} -$$

$$\frac{5 i b^3 B c d^2 e f^2 h^3 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}\right]}{\sqrt{-\frac{d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}} -$$

$$\frac{5 i a b^2 c C d^2 e f^2 h^3 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}\right]}{\sqrt{-\frac{d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}} +$$

$$\frac{8 i b^3 c^3 C f^3 h^3 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}\right]}{\sqrt{-\frac{d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}} -$$

$$\frac{10 i b^3 B c^2 d f^3 h^3 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}\right]}{\sqrt{-\frac{d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}} -$$

$$\begin{aligned}
& \frac{10 \pm a b^2 c^2 C d f^3 h^3 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}]}{\sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}} + \\
& \frac{30 \pm a b^2 B c d^2 f^3 h^3 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}]}{\sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}} - \\
& \frac{15 \pm a^2 b c C d^2 f^3 h^3 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}]}{\sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}} - \\
& \frac{15 \pm a^2 b B d^3 f^3 h^3 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}]}{\sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}} + \\
& \frac{15 \pm a^3 C d^3 f^3 h^3 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}]}{\sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}} \Bigg)
\end{aligned}$$

**Problem 17: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{a b B - a^2 C + b^2 B x + b^2 C x^2}{\sqrt{c + d x} \sqrt{e + f x} \sqrt{g + h x}} dx$$

Optimal (type 4, 410 leaves, 7 steps):

$$\begin{aligned}
& \frac{2 b^2 C \sqrt{c + d x} \sqrt{e + f x} \sqrt{g + h x}}{3 d f h} + \\
& \left( 2 b^2 \sqrt{-d e + c f} (3 B d f h - 2 C (d f g + d e h + c f h)) \sqrt{\frac{d (e + f x)}{d e - c f}} \sqrt{g + h x} \operatorname{EllipticE}[\operatorname{ArcSin}\left[\frac{\sqrt{f} \sqrt{c + d x}}{\sqrt{-d e + c f}}\right], \frac{(d e - c f) h}{f (d g - c h)}] \right) / \\
& \left( 3 d^2 f^{3/2} h^2 \sqrt{e + f x} \sqrt{\frac{d (g + h x)}{d g - c h}} \right) + \left( 2 \sqrt{-d e + c f} (3 a b B d f h^2 - 3 a^2 C d f h^2 - b^2 (3 B d f g h - C (c h (f g - e h) + d g (2 f g + e h)))) \right. \\
& \left. \sqrt{\frac{d (e + f x)}{d e - c f}} \sqrt{\frac{d (g + h x)}{d g - c h}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{\sqrt{f} \sqrt{c + d x}}{\sqrt{-d e + c f}}\right], \frac{(d e - c f) h}{f (d g - c h)}] \right) / (3 d^2 f^{3/2} h^2 \sqrt{e + f x} \sqrt{g + h x})
\end{aligned}$$

Result (type 4, 569 leaves):

$$\begin{aligned}
& \frac{2 b^2 C \sqrt{c + d x} \sqrt{e + f x} \sqrt{g + h x}}{3 d f h} + \frac{1}{3 d^3 \sqrt{-c + \frac{d e}{f}} f^2 h^2 \sqrt{e + \frac{(c+d x) (f - \frac{c f}{c+d x})}{d}} \sqrt{g + \frac{(c+d x) (h - \frac{c h}{c+d x})}{d}}} \\
& (c + d x)^{3/2} \left( -2 b^2 \sqrt{-c + \frac{d e}{f}} (-3 B d f h + 2 C (d f g + d e h + c f h)) \left( f + \frac{d e}{c + d x} - \frac{c f}{c + d x} \right) \left( h + \frac{d g}{c + d x} - \frac{c h}{c + d x} \right) - \frac{1}{\sqrt{c + d x}} 2 \pm b^2 (-d e + c f) h \right. \\
& (3 B d f h - 2 C (d f g + d e h + c f h)) \sqrt{1 - \frac{c}{c + d x} + \frac{d e}{f (c + d x)}} \sqrt{1 - \frac{c}{c + d x} + \frac{d g}{h (c + d x)}} \operatorname{EllipticE}[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{-c + \frac{d e}{f}}}{\sqrt{c + d x}}\right], \frac{d f g - c f h}{d e h - c f h}] + \\
& \left. \frac{1}{\sqrt{c + d x}} 2 \pm d h (3 a b B d f^2 h - 3 a^2 C d f^2 h + b^2 (-3 B d e f h + c C f (-f g + e h) + C d e (f g + 2 e h))) \right) \\
& \sqrt{1 - \frac{c}{c + d x} + \frac{d e}{f (c + d x)}} \sqrt{1 - \frac{c}{c + d x} + \frac{d g}{h (c + d x)}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{-c + \frac{d e}{f}}}{\sqrt{c + d x}}\right], \frac{d f g - c f h}{d e h - c f h}]
\end{aligned}$$

### Problem 18: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a b B - a^2 C + b^2 B x + b^2 C x^2}{(a + b x) \sqrt{c + d x} \sqrt{e + f x} \sqrt{g + h x}} dx$$

Optimal (type 4, 291 leaves, 7 steps):

$$\begin{aligned} & \frac{2 b C \sqrt{-d e + c f} \sqrt{\frac{d(e+f x)}{d e - c f}} \sqrt{g + h x} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{f} \sqrt{c+d x}}{\sqrt{-d e+c f}}\right], \frac{(d e-c f) h}{f(d g-c h)}\right]}{d \sqrt{f} h \sqrt{e+f x} \sqrt{\frac{d(g+h x)}{d g-c h}}} - \\ & \frac{2 \sqrt{-d e + c f} (b C g - b B h + a C h) \sqrt{\frac{d(e+f x)}{d e - c f}} \sqrt{\frac{d(g+h x)}{d g-c h}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{f} \sqrt{c+d x}}{\sqrt{-d e+c f}}\right], \frac{(d e-c f) h}{f(d g-c h)}\right]}{d \sqrt{f} h \sqrt{e+f x} \sqrt{g+h x}} \end{aligned}$$

Result (type 4, 326 leaves):

$$\begin{aligned} & \left( 2 \left( b C d^2 \sqrt{-c + \frac{d e}{f}} (e + f x) (g + h x) + \right. \right. \\ & \left. \left. \pm b C (d e - c f) h (c + d x)^{3/2} \sqrt{\frac{d (e + f x)}{f (c + d x)}} \sqrt{\frac{d (g + h x)}{h (c + d x)}} \operatorname{EllipticE}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{-c + \frac{d e}{f}}}{\sqrt{c + d x}}\right], \frac{d f g - c f h}{d e h - c f h}\right] - \pm d (b C e - b B f + a C f) h \right. \right. \\ & \left. \left. (c + d x)^{3/2} \sqrt{\frac{d (e + f x)}{f (c + d x)}} \sqrt{\frac{d (g + h x)}{h (c + d x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{-c + \frac{d e}{f}}}{\sqrt{c + d x}}\right], \frac{d f g - c f h}{d e h - c f h}\right] \right) \right) / \left( d^2 \sqrt{-c + \frac{d e}{f}} f h \sqrt{c + d x} \sqrt{e + f x} \sqrt{g + h x} \right) \end{aligned}$$

### Problem 19: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a b B - a^2 C + b^2 B x + b^2 C x^2}{(a + b x)^2 \sqrt{c + d x} \sqrt{e + f x} \sqrt{g + h x}} dx$$

Optimal (type 4, 309 leaves, 10 steps):

$$\frac{2 C \sqrt{-d e + c f} \sqrt{\frac{d (e+f x)}{d e - c f}} \sqrt{\frac{d (g+h x)}{d g - c h}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{f} \sqrt{c+d x}}{\sqrt{-d e+c f}}\right], \frac{(d e-c f) h}{f (d g-c h)}\right]}{d \sqrt{f} \sqrt{e+f x} \sqrt{g+h x}} -$$

$$\frac{2 (b B - 2 a C) \sqrt{-d e + c f} \sqrt{\frac{d (e+f x)}{d e - c f}} \sqrt{\frac{d (g+h x)}{d g - c h}} \operatorname{EllipticPi}\left[-\frac{b (d e-c f)}{(b c-a d) f}, \operatorname{ArcSin}\left[\frac{\sqrt{f} \sqrt{c+d x}}{\sqrt{-d e+c f}}\right], \frac{(d e-c f) h}{f (d g-c h)}\right]}{(b c-a d) \sqrt{f} \sqrt{e+f x} \sqrt{g+h x}}$$

Result (type 4, 248 leaves):

$$\left( 2 \pm \sqrt{e+f x} \sqrt{\frac{d (g+h x)}{h (c+d x)}} \left( - (b c C - b B d + a C d) \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{-c + \frac{d e}{f}}}{\sqrt{c+d x}}\right], \frac{d f g - c f h}{d e h - c f h}\right] + \right. \right. \right. \\ \left. \left. \left. (-b B + 2 a C) d \operatorname{EllipticPi}\left[\frac{(b c - a d) f}{b (-d e + c f)}, \pm \operatorname{ArcSinh}\left[\frac{\sqrt{-c + \frac{d e}{f}}}{\sqrt{c+d x}}\right], \frac{d f g - c f h}{d e h - c f h}\right]\right) \right) \right) / \left( (-b c + a d) \sqrt{-c + \frac{d e}{f}} f \sqrt{\frac{d (e+f x)}{f (c+d x)}} \sqrt{g+h x} \right)$$

Problem 20: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{a b B - a^2 C + b^2 B x + b^2 C x^2}{(a + b x)^3 \sqrt{c + d x} \sqrt{e + f x} \sqrt{g + h x}} dx$$

Optimal (type 4, 680 leaves, 13 steps):

$$\begin{aligned}
& - \frac{b^2 (b B - 2 a C) \sqrt{c + d x} \sqrt{e + f x} \sqrt{g + h x}}{(b c - a d) (b e - a f) (b g - a h) (a + b x)} + \frac{b (b B - 2 a C) \sqrt{f} \sqrt{-d e + c f} \sqrt{\frac{d (e + f x)}{d e - c f}} \sqrt{g + h x} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{f} \sqrt{c + d x}}{\sqrt{-d e + c f}}\right], \frac{(d e - c f) h}{f (d g - c h)}\right]}{(b c - a d) (b e - a f) (b g - a h) \sqrt{e + f x} \sqrt{\frac{d (g + h x)}{d g - c h}}} - \\
& \frac{(b B - 2 a C) \sqrt{f} \sqrt{-d e + c f} \sqrt{\frac{d (e + f x)}{d e - c f}} \sqrt{\frac{d (g + h x)}{d g - c h}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{f} \sqrt{c + d x}}{\sqrt{-d e + c f}}\right], \frac{(d e - c f) h}{f (d g - c h)}\right]}{(b c - a d) (b e - a f) \sqrt{e + f x} \sqrt{g + h x}} - \\
& \left( \sqrt{-d e + c f} (4 a^3 C d f h + 2 a b^2 B (d f g + d e h + c f h) - b^3 (B d e g - c (2 C e g - B f g - B e h)) - a^2 b (3 B d f h + 2 C (d f g + d e h + c f h))) \right. \\
& \left. \sqrt{\frac{d (e + f x)}{d e - c f}} \sqrt{\frac{d (g + h x)}{d g - c h}} \operatorname{EllipticPi}\left[-\frac{b (d e - c f)}{(b c - a d) f}, \operatorname{ArcSin}\left[\frac{\sqrt{f} \sqrt{c + d x}}{\sqrt{-d e + c f}}\right], \frac{(d e - c f) h}{f (d g - c h)}\right] \right) / \\
& ((b c - a d)^2 \sqrt{f} (b e - a f) (b g - a h) \sqrt{e + f x} \sqrt{g + h x})
\end{aligned}$$

Result (type 4, 16821 leaves):

$$\begin{aligned}
& - \frac{b^2 (b B - 2 a C) \sqrt{c + d x} \sqrt{e + f x} \sqrt{g + h x}}{(b c - a d) (b e - a f) (b g - a h) (a + b x)} - \\
& \frac{1}{d (-b c + a d) (-b e + a f) (-b g + a h)} \left( \frac{b (b B - 2 a C) (c + d x)^{3/2} \left(f + \frac{d e}{c + d x} - \frac{c f}{c + d x}\right) \left(h + \frac{d g}{c + d x} - \frac{c h}{c + d x}\right)}{\sqrt{e + \frac{(c + d x) (f - \frac{c f}{c + d x})}{d}} \sqrt{g + \frac{(c + d x) (h - \frac{c h}{c + d x})}{d}}} + \left( (c + d x) \left(-b + \frac{b c}{c + d x} - \frac{a d}{c + d x}\right) \right. \right. \\
& \left. \sqrt{f + \frac{d e}{c + d x} - \frac{c f}{c + d x}} \sqrt{h + \frac{d g}{c + d x} - \frac{c h}{c + d x}} \sqrt{f h + \frac{d^2 e g}{(c + d x)^2} - \frac{c d f g}{(c + d x)^2} - \frac{c d e h}{(c + d x)^2} + \frac{c^2 f h}{(c + d x)^2} + \frac{d f g}{c + d x} + \frac{d e h}{c + d x} - \frac{2 c f h}{c + d x}} \right. \\
& \left. \left( (b c - a d) h (-2 b^2 C e g + b^2 B f g + b^2 B e h - 2 a b B f h + 2 a^2 C f h) \right. \right. \\
& \left. \left. - \frac{b (b B - 2 a C) (d e - c f) \sqrt{h + \frac{d g}{c + d x} - \frac{c h}{c + d x}}}{\sqrt{f + \frac{d e}{c + d x} - \frac{c f}{c + d x}}} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left( b \left( 2 b^3 c C e g - b^3 B d e g - b^3 B c f g + 2 a b^2 B d f g - 2 a^2 b C d f g - b^3 B c e h + 2 a b^2 B d e h - 2 a^2 b C d e h + 2 a b^2 B c f h - 2 a^2 b c C f h - \right. \right. \\
& \quad \left. \left. 3 a^2 b B d f h + 4 a^3 C d f h \right) \sqrt{h + \frac{d g}{c + d x} - \frac{c h}{c + d x}} \right) / \left( (-b g + a h) \left( b - \frac{b c}{c + d x} + \frac{a d}{c + d x} \right) \sqrt{f + \frac{d e}{c + d x} - \frac{c f}{c + d x}} \right) \\
& \left( \left( \pm b^2 B d^2 e f g \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] - \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] \right) \right) / \right. \\
& \quad \left( (b c - a d) (-d e + c f) \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2} + \frac{d f g + d e h - 2 c f h}{c + d x}} \right) - \left( 2 \pm a b C d^2 e f g \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \right. \right. \\
& \quad \left. \left. \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] \right) \right) \right)
\end{aligned}$$

$$\sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \left( \text{EllipticE} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] - \text{EllipticF} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] \right)$$

$$\left( (b c - a d) (-d e + c f) \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2}} + \frac{d f g + d e h - 2 c f h}{c + d x} \right) + \left( 2 i a b c C d f^2 g \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \right)$$

$$\sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \left( \text{EllipticE} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] - \text{EllipticF} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] \right)$$

$$\left( (b c - a d) (-d e + c f) \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2}} + \frac{d f g + d e h - 2 c f h}{c + d x} \right) - \left( i b^2 B c d e f h \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \right)$$

$$\sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \left( \text{EllipticE} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] - \text{EllipticF} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] \right)$$

$$\left( (b c - a d) (-d e + c f) \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2}} + \frac{d f g + d e h - 2 c f h}{c + d x} \right) + \left( 2 i a b c C d e f h \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \right)$$

$$\sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \left( \text{EllipticE} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] - \text{EllipticF} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] \right)$$

$$\begin{aligned}
& \left( (b c - a d) (-d e + c f) \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2} + \frac{d f g + d e h - 2 c f h}{c + d x}} \right) + \left( \pm b^2 B c^2 f^2 h \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \right. \\
& \left. \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] \right) \right) / \\
& \left( (b c - a d) (-d e + c f) \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2} + \frac{d f g + d e h - 2 c f h}{c + d x}} \right) - \left( 2 \pm a b c^2 C f^2 h \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \right. \\
& \left. \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] \right) \right) / \\
& \left( (b c - a d) (-d e + c f) \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2} + \frac{d f g + d e h - 2 c f h}{c + d x}} \right) - \\
& \frac{\pm b^3 B d^2 e g \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right]}{(b c - a d)^2 \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2} + \frac{d f g + d e h - 2 c f h}{c + d x}}} + \\
& \frac{2 \pm a b^2 C d^2 e g \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right]}{(b c - a d)^2 \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2} + \frac{d f g + d e h - 2 c f h}{c + d x}}} +
\end{aligned}$$

$$\frac{2 i b^2 C d e g \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{(b c - a d) \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}} +$$

$$\frac{i b^3 B c d f g \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{(b c - a d)^2 \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}} -$$

$$\frac{2 i a b^2 c C d f g \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{(b c - a d)^2 \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}} -$$

$$\frac{2 i b^2 B d f g \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{(b c - a d) \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}} +$$

$$\frac{2 i a b C d f g \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{(b c - a d) \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}} +$$

$$\frac{i b^3 B c d e h \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{(b c - a d)^2 \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}} -$$

$$\frac{2 i a b^2 c C d e h \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[\frac{i \operatorname{ArcSinh}\left[\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right]}{\sqrt{c+d x}}, \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{(b c - a d)^2 \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}}$$

$$\frac{2 i b^2 B d e h \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[\frac{i \operatorname{ArcSinh}\left[\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right]}{\sqrt{c+d x}}, \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{(b c - a d) \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}} +$$

$$\frac{2 i a b C d e h \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[\frac{i \operatorname{ArcSinh}\left[\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right]}{\sqrt{c+d x}}, \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{(b c - a d) \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}} -$$

$$\frac{i b^3 B c^2 f h \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[\frac{i \operatorname{ArcSinh}\left[\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right]}{\sqrt{c+d x}}, \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{(b c - a d)^2 \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}} +$$

$$\frac{2 i a b^2 c^2 C f h \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[\frac{i \operatorname{ArcSinh}\left[\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right]}{\sqrt{c+d x}}, \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{(b c - a d)^2 \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}} +$$

$$\frac{2 i b^2 B c f h \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[\frac{i \operatorname{ArcSinh}\left[\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right]}{\sqrt{c+d x}}, \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{(b c - a d) \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}} -$$

$$\frac{4 i a b c C f h \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{(b c - a d) \sqrt{-\frac{d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}} +$$

$$\frac{2 i a b B d f h \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{(b c - a d) \sqrt{-\frac{d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}} -$$

$$\frac{2 i a^2 C d f h \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{(b c - a d) \sqrt{-\frac{d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}} + \frac{1}{(b c - a d)^3}$$

$$b^4 B d^2 e g \left( \frac{i c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, i \text{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{\sqrt{-\frac{d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} - \right. \\ \left. \frac{i a d \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, i \text{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{b \sqrt{-\frac{d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \right)$$

$$\frac{1}{(b c - a d)^3} 2 a b^3 C d^2 e g \left( \frac{i c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, i \text{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{\sqrt{-\frac{d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} - \right.$$

$$\left. \frac{\frac{1}{2} a d \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \frac{i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}}\right]}{b \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}}\right] - \right.$$

$$\frac{1}{(b c - a d)^2} 2 b^3 C d e g \left( \frac{\frac{1}{2} c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \frac{i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}}\right]}{\sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}}\right] - \right.$$

$$\left. \frac{\frac{1}{2} a d \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \frac{i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}}\right]}{b \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}}\right] - \right.$$

$$\frac{1}{(b c - a d)^3} b^4 B c d f g \left( \frac{\frac{1}{2} c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \frac{i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}}\right]}{\sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}}\right] - \right.$$

$$\left. \frac{\frac{1}{2} a d \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \frac{i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}}\right]}{b \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}}\right] + \right.$$

$$\frac{1}{(b c - a d)^3} 2 a b^3 c C d f g \left( \frac{\frac{1}{2} c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \frac{i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}}\right]}{\sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}}\right] - \right.$$

$$\begin{aligned}
& \frac{\pm a d}{c - a d} \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \pm \text{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c + d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right] \\
& + b \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c + d x)^2} - \frac{c d f g}{(c + d x)^2} - \frac{c d e h}{(c + d x)^2} + \frac{c^2 f h}{(c + d x)^2} + \frac{d f g}{c + d x} + \frac{d e h}{c + d x} - \frac{2 c f h}{c + d x}} \\
& \frac{1}{(c - a d)^2} 2 b^3 B d f g \left( \frac{\pm c \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \pm \text{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c + d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{\sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c + d x)^2} - \frac{c d f g}{(c + d x)^2} - \frac{c d e h}{(c + d x)^2} + \frac{c^2 f h}{(c + d x)^2} + \frac{d f g}{c + d x} + \frac{d e h}{c + d x} - \frac{2 c f h}{c + d x}}} \right. \\
& \left. \pm a d \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \pm \text{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c + d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right] \right) \\
& - b \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c + d x)^2} - \frac{c d f g}{(c + d x)^2} - \frac{c d e h}{(c + d x)^2} + \frac{c^2 f h}{(c + d x)^2} + \frac{d f g}{c + d x} + \frac{d e h}{c + d x} - \frac{2 c f h}{c + d x}} \\
& \frac{1}{(c - a d)^2} 2 a b^2 C d f g \left( \frac{\pm c \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \pm \text{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c + d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{\sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c + d x)^2} - \frac{c d f g}{(c + d x)^2} - \frac{c d e h}{(c + d x)^2} + \frac{c^2 f h}{(c + d x)^2} + \frac{d f g}{c + d x} + \frac{d e h}{c + d x} - \frac{2 c f h}{c + d x}}} \right. \\
& \left. \pm a d \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \pm \text{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c + d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right] \right) \\
& - b \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c + d x)^2} - \frac{c d f g}{(c + d x)^2} - \frac{c d e h}{(c + d x)^2} + \frac{c^2 f h}{(c + d x)^2} + \frac{d f g}{c + d x} + \frac{d e h}{c + d x} - \frac{2 c f h}{c + d x}} \\
& \frac{1}{(c - a d)^3} b^4 B c d e h \left( \frac{\pm c \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \pm \text{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c + d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{\sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c + d x)^2} - \frac{c d f g}{(c + d x)^2} - \frac{c d e h}{(c + d x)^2} + \frac{c^2 f h}{(c + d x)^2} + \frac{d f g}{c + d x} + \frac{d e h}{c + d x} - \frac{2 c f h}{c + d x}}} \right.
\end{aligned}$$

$$\left. \begin{aligned} & \frac{\frac{d}{dx} \operatorname{ad} \sqrt{1 - \frac{-de+cf}{f(c+dx)}} \sqrt{1 - \frac{-dg+ch}{h(c+dx)}} \operatorname{EllipticPi}\left[\frac{(bc-ad)h}{b(-dg+ch)}, \frac{\sqrt{\frac{-dg+ch}{h}}}{\sqrt{c+dx}}\right], \frac{(-de+cf)h}{f(-dg+ch)}}{+} \\ & \frac{b \sqrt{-\frac{-dg+ch}{h}} \sqrt{fh + \frac{d^2 eg}{(c+dx)^2} - \frac{cd fg}{(c+dx)^2} - \frac{cd eh}{(c+dx)^2} + \frac{c^2 fh}{(c+dx)^2} + \frac{df g}{c+dx} + \frac{de h}{c+dx} - \frac{2cfh}{c+dx}}}{\sqrt{c+dx}} \end{aligned} \right\}$$

$$\frac{1}{(b c - a d)^3} 2 a b^3 c C d e h \left( \frac{\frac{i c}{f} \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \frac{i}{\text{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right]}, \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{\sqrt{\frac{-d g + c h}{h}}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}} \right)$$

$$\left. \begin{aligned} & \frac{\frac{d}{dx} \operatorname{ad} \sqrt{1 - \frac{-de+cf}{f(c+dx)}} \sqrt{1 - \frac{-dg+ch}{h(c+dx)}} \operatorname{EllipticPi}\left[\frac{(bc-ad)h}{b(-dg+ch)}, \frac{\sqrt{\frac{-dg+ch}{h}}}{\sqrt{c+dx}}\right], \frac{(-de+cf)h}{f(-dg+ch)}}{+} \\ & \frac{b \sqrt{-\frac{-dg+ch}{h}} \sqrt{fh + \frac{d^2 eg}{(c+dx)^2} - \frac{cd fg}{(c+dx)^2} - \frac{cd eh}{(c+dx)^2} + \frac{c^2 fh}{(c+dx)^2} + \frac{df g}{c+dx} + \frac{de h}{c+dx} - \frac{2cfh}{c+dx}}}{\sqrt{c+dx}} \end{aligned} \right\}$$

$$\frac{1}{(b c - a d)^2} 2 b^3 B d e h \left( \frac{\frac{1}{2} c \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \frac{1}{2} \text{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c + d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right] - \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c + d x)^2} - \frac{c d f g}{(c + d x)^2} - \frac{c d e h}{(c + d x)^2} + \frac{c^2 f h}{(c + d x)^2} + \frac{d f g}{c + d x} + \frac{d e h}{c + d x} - \frac{2 c f h}{c + d x}}}{\sqrt{f h + \frac{d^2 e g}{(c + d x)^2} - \frac{c d f g}{(c + d x)^2} - \frac{c d e h}{(c + d x)^2} + \frac{c^2 f h}{(c + d x)^2} + \frac{d f g}{c + d x} + \frac{d e h}{c + d x} - \frac{2 c f h}{c + d x}}}$$

$$\left. \frac{\frac{d}{dx} \operatorname{ad} \sqrt{1 - \frac{-de+cf}{f(c+dx)}} \sqrt{1 - \frac{-dg+ch}{h(c+dx)}} \operatorname{EllipticPi}\left[\frac{(bc-ad)h}{b(-dg+ch)}, \frac{\sqrt{\frac{-dg+ch}{h}}}{\sqrt{c+dx}}\right], \frac{(-de+cf)h}{f(-dg+ch)}}{b \sqrt{-\frac{-dg+ch}{h}} \sqrt{fh + \frac{d^2 eg}{(c+dx)^2} - \frac{cd fg}{(c+dx)^2} - \frac{cd eh}{(c+dx)^2} + \frac{c^2 fh}{(c+dx)^2} + \frac{df g}{c+dx} + \frac{de h}{c+dx} - \frac{2cfh}{c+dx}}} \right] -$$

$$\frac{1}{(b c - a d)^2} 2 a b^2 C d e h \left( \frac{i c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, i \text{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{\sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \right)$$

$$\begin{aligned}
& \left. \frac{\frac{\pm a d}{f(c+d x)} \sqrt{1 - \frac{-d e + c f}{f(c+d x)}} \sqrt{1 - \frac{-d g + c h}{h(c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b(-d g + c h)}, \pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}, \frac{(-d e + c f) h}{f(-d g + c h)}\right]\right]}{b \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \right] + \right. \\
& \frac{1}{(b c - a d)^3} b^4 B c^2 f h \left. \left( \frac{\frac{\pm c}{f(c+d x)} \sqrt{1 - \frac{-d e + c f}{f(c+d x)}} \sqrt{1 - \frac{-d g + c h}{h(c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b(-d g + c h)}, \pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}, \frac{(-d e + c f) h}{f(-d g + c h)}\right]\right]}{\sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} - \right. \right. \\
& \left. \left. \frac{\pm a d}{f(c+d x)} \sqrt{1 - \frac{-d e + c f}{f(c+d x)}} \sqrt{1 - \frac{-d g + c h}{h(c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b(-d g + c h)}, \pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}, \frac{(-d e + c f) h}{f(-d g + c h)}\right]\right]}{b \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \right] - \right. \\
& \frac{1}{(b c - a d)^3} 2 a b^3 c^2 C f h \left. \left( \frac{\frac{\pm c}{f(c+d x)} \sqrt{1 - \frac{-d e + c f}{f(c+d x)}} \sqrt{1 - \frac{-d g + c h}{h(c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b(-d g + c h)}, \pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}, \frac{(-d e + c f) h}{f(-d g + c h)}\right]\right]}{\sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} - \right. \right. \\
& \left. \left. \frac{\pm a d}{f(c+d x)} \sqrt{1 - \frac{-d e + c f}{f(c+d x)}} \sqrt{1 - \frac{-d g + c h}{h(c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b(-d g + c h)}, \pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}, \frac{(-d e + c f) h}{f(-d g + c h)}\right]\right]}{b \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \right] - \right. \\
& \frac{1}{(b c - a d)^2} 2 b^3 B c f h \left. \left( \frac{\frac{\pm c}{f(c+d x)} \sqrt{1 - \frac{-d e + c f}{f(c+d x)}} \sqrt{1 - \frac{-d g + c h}{h(c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b(-d g + c h)}, \pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}, \frac{(-d e + c f) h}{f(-d g + c h)}\right]\right]}{\sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} - \right. \right.
\end{aligned}$$

$$\frac{\frac{1}{2} \operatorname{ad} \sqrt{1 - \frac{-de+cf}{f(c+dx)}} \sqrt{1 - \frac{-dg+ch}{h(c+dx)}} \operatorname{EllipticPi}\left[\frac{(bc-ad)h}{b(-dg+ch)}, \frac{\sqrt{\frac{-dg+ch}{h}}}{\sqrt{c+dx}}\right], \frac{(-de+cf)h}{f(-dg+ch)}}{b \sqrt{-\frac{-dg+ch}{h}} \sqrt{fh + \frac{d^2eg}{(c+dx)^2} - \frac{cdfg}{(c+dx)^2} - \frac{cdeh}{(c+dx)^2} + \frac{c^2fh}{(c+dx)^2} + \frac{dfg}{c+dx} + \frac{deh}{c+dx} - \frac{2cfh}{c+dx}}} +$$

$$\frac{1}{(b c - a d)^2} \frac{4 a b^2 c C f h}{\left(\frac{i c \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \frac{i \text{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c + d x}}\right]}{\sqrt{c + d x}}, \frac{(-d e + c f) h}{f (-d g + c h)}\right] - \sqrt{\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c + d x)^2} - \frac{c d f g}{(c + d x)^2} - \frac{c d e h}{(c + d x)^2} + \frac{c^2 f h}{(c + d x)^2} + \frac{d f g}{c + d x} + \frac{d e h}{c + d x} - \frac{2 c f h}{c + d x}}}{\sqrt{\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c + d x)^2} - \frac{c d f g}{(c + d x)^2} - \frac{c d e h}{(c + d x)^2} + \frac{c^2 f h}{(c + d x)^2} + \frac{d f g}{c + d x} + \frac{d e h}{c + d x} - \frac{2 c f h}{c + d x}}}$$

$$\frac{\frac{1}{2} \operatorname{ad} \sqrt{1 - \frac{-de+cf}{f(c+dx)}} \sqrt{1 - \frac{-dg+ch}{h(c+dx)}} \operatorname{EllipticPi}\left[\frac{(bc-ad)h}{b(-dg+ch)}, \frac{1}{2} \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-dg+ch}{h}}}{\sqrt{c+dx}}\right], \frac{(-de+cf)h}{f(-dg+ch)}\right]}{b \sqrt{-\frac{-dg+ch}{h}} \sqrt{fh + \frac{d^2eg}{(c+dx)^2} - \frac{cdfg}{(c+dx)^2} - \frac{cdhe}{(c+dx)^2} + \frac{c^2fh}{(c+dx)^2} + \frac{dfg}{c+dx} + \frac{deh}{c+dx} - \frac{2cfg}{c+dx}}} -$$

$$\frac{1}{(b c - a d)^2} 2 a b^2 B d f h \left( \frac{i c \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, i \text{ArcSinh}\left[\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{\sqrt{-\frac{-d g + c h}{h}}} \sqrt{f h + \frac{d^2 e g}{(c + d x)^2} - \frac{c d f g}{(c + d x)^2} - \frac{c d e h}{(c + d x)^2} + \frac{c^2 f h}{(c + d x)^2} + \frac{d f g}{c + d x} + \frac{d e h}{c + d x} - \frac{2 c f h}{c + d x}} \right)$$

$$\frac{\frac{1}{2} \operatorname{ad} \sqrt{1 - \frac{-de+cf}{f(c+dx)}} \sqrt{1 - \frac{-dg+ch}{h(c+dx)}} \operatorname{EllipticPi}\left[\frac{(bc-ad)h}{b(-dg+ch)}, \frac{1}{2} \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-dg+ch}{h}}}{\sqrt{c+dx}}\right], \frac{(-de+cf)h}{f(-dg+ch)}\right]}{b \sqrt{-\frac{-dg+ch}{h}} \sqrt{fh + \frac{d^2eg}{(c+dx)^2} - \frac{cdfg}{(c+dx)^2} - \frac{cdeh}{(c+dx)^2} + \frac{c^2fh}{(c+dx)^2} + \frac{dfg}{c+dx} + \frac{deh}{c+dx} - \frac{2cfg}{c+dx}}} +$$

$$\frac{1}{(b c - a d)^2} 2 a^2 b c d f h \left( \frac{i c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, i \text{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{\sqrt{\frac{-d g + c h}{h}}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}} \right)$$

$$\frac{\frac{d}{dx} \operatorname{ad} \sqrt{1 - \frac{-de+cf}{f(c+dx)}} \sqrt{1 - \frac{-dg+ch}{h(c+dx)}} \operatorname{EllipticPi}\left[\frac{(bc-ad)h}{b(-dg+ch)}, \frac{d}{dx} \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-dg+ch}{h}}}{\sqrt{c+dx}}\right], \frac{(-de+cf)h}{f(-dg+ch)}\right]}{b \sqrt{-\frac{-dg+ch}{h}} \sqrt{fh + \frac{d^2 eg}{(c+dx)^2} - \frac{cd fg}{(c+dx)^2} - \frac{cd eh}{(c+dx)^2} + \frac{c^2 fh}{(c+dx)^2} + \frac{df g}{c+dx} + \frac{de h}{c+dx} - \frac{2 cf h}{c+dx}}} +$$

$$\frac{1}{b c - a d} b^2 B f h \left( \frac{\frac{i}{2} c \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \frac{i}{2} \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c + d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{\sqrt{\frac{-d g + c h}{h}}} \sqrt{f h + \frac{d^2 e g}{(c + d x)^2} - \frac{c d f g}{(c + d x)^2} - \frac{c d e h}{(c + d x)^2} + \frac{c^2 f h}{(c + d x)^2} + \frac{d f g}{c + d x} + \frac{d e h}{c + d x} - \frac{2 c f h}{c + d x}} \right)$$

$$\frac{\text{EllipticPi}\left[\frac{(-b c-a d) h}{b (-d g+c h)}, \text{ArcSinh}\left[\frac{\sqrt{\frac{-d g+c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e+c f) h}{f (-d g+c h)}\right]}{\sqrt{1-\frac{-d e+c f}{f (c+d x)}} \sqrt{1-\frac{-d g+c h}{h (c+d x)}}} + b \sqrt{\frac{-d g+c h}{h}} \sqrt{f h+\frac{d^2 e g}{(c+d x)^2}-\frac{c d f g}{(c+d x)^2}-\frac{c d e h}{(c+d x)^2}+\frac{c^2 f h}{(c+d x)^2}+\frac{d f g}{c+d x}+\frac{d e h}{c+d x}-\frac{2 c f h}{c+d x}}$$

$$\frac{1}{b c - a d} \cdot 2 a b C f h \left( \begin{array}{l} \pm c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticPi} \left[ \frac{(b c - a d) h}{b (-d g + c h)}, \pm \text{ArcSinh} \left[ \frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] \\ \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}} \end{array} \right)$$

$$\frac{\text{i a d} \sqrt{1 - \frac{-\text{d e} + \text{c f}}{\text{f} (\text{c} + \text{d x})}} \sqrt{1 - \frac{-\text{d g} + \text{c h}}{\text{h} (\text{c} + \text{d x})}} \text{EllipticPi}\left[\frac{(-\text{b c} - \text{a d}) \text{h}}{\text{b} (-\text{d g} + \text{c h})}, \text{i ArcSinh}\left[\frac{\sqrt{\frac{-\text{d g} + \text{c h}}{\text{h}}}}{\sqrt{\text{c} + \text{d x}}}\right], \frac{(-\text{d e} + \text{c f}) \text{h}}{\text{f} (-\text{d g} + \text{c h})}\right]}{\text{b} \sqrt{\frac{-\text{d g} + \text{c h}}{\text{h}}} \sqrt{\text{f} \text{h} + \frac{\text{d}^2 \text{e} \text{g}}{(\text{c} + \text{d x})^2} - \frac{\text{c} \text{d} \text{f} \text{g}}{(\text{c} + \text{d x})^2} - \frac{\text{c} \text{d} \text{e} \text{h}}{(\text{c} + \text{d x})^2} + \frac{\text{c}^2 \text{f} \text{h}}{(\text{c} + \text{d x})^2} + \frac{\text{d} \text{f} \text{g}}{\text{c} + \text{d x}} + \frac{\text{d} \text{e} \text{h}}{\text{c} + \text{d x}} - \frac{2 \text{c} \text{f} \text{h}}{\text{c} + \text{d x}}}}$$

$$\left( \frac{b^2 B f h - 2 a b C f h + \frac{b^2 B d^2 e g}{(c + d x)^2} - \frac{2 a b C d^2 e g}{(c + d x)^2} - \frac{b^2 B c d f g}{(c + d x)^2} + \frac{2 a b c C d f g}{(c + d x)^2} - \frac{b^2 B c d e h}{(c + d x)^2} + \frac{2 a b c C d e h}{(c + d x)^2} + \frac{b^2 B c^2 f h}{(c + d x)^2} - \frac{2 a b c^2 C f h}{(c + d x)^2} - \frac{2 b^2 C d e g}{c + d x} + \frac{2 b^2 B d f g}{c + d x} - \frac{2 a b C d f g}{c + d x} + \frac{2 b^2 B d e h}{c + d x} - \frac{2 a b C d e h}{c + d x} - \frac{2 b^2 B c f h}{c + d x} + \right)$$

$$\left. \frac{4 a b c C f h}{c + d x} - \frac{2 a b B d f h}{c + d x} + \frac{2 a^2 C d f h}{c + d x} \right) \sqrt{e + \frac{(c + d x) \left(f - \frac{c f}{c+d x}\right)}{d}} \sqrt{g + \frac{(c + d x) \left(h - \frac{c h}{c+d x}\right)}{d}} \Bigg)$$

**Problem 21: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+b x} (a b B - a^2 C + b^2 B x + b^2 C x^2)}{\sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x}} dx$$

Optimal (type 4, 980 leaves, 9 steps):

$$\begin{aligned} & \frac{b (4 b B d f h + C (a d f h - 3 b (d f g + d e h + c f h))) \sqrt{a+b x} \sqrt{e+f x} \sqrt{g+h x}}{4 d f^2 h^2 \sqrt{c+d x}} + \frac{b^2 C \sqrt{a+b x} \sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x}}{2 d f h} - \\ & \left( b \sqrt{d g - c h} \sqrt{f g - e h} (4 b B d f h + C (a d f h - 3 b (d f g + d e h + c f h))) \sqrt{a+b x} \sqrt{-\frac{(d e - c f) (g + h x)}{(f g - e h) (c + d x)}} \right. \\ & \left. \text{EllipticE}[\text{ArcSin}\left[\frac{\sqrt{d g - c h} \sqrt{e + f x}}{\sqrt{f g - e h} \sqrt{c + d x}}, \frac{(b c - a d) (f g - e h)}{(b e - a f) (d g - c h)}\right]] \right) / \left( 4 d^2 f^2 h^2 \sqrt{\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}} \sqrt{g + h x} \right) + \\ & \left( (b e - a f) \sqrt{b g - a h} (a C d f h - b (4 B d f h - C (3 d f g + 3 d e h + c f h))) \sqrt{\frac{(b e - a f) (c + d x)}{(d e - c f) (a + b x)}} \sqrt{g + h x} \right. \\ & \left. \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{b g - a h} \sqrt{e + f x}}{\sqrt{f g - e h} \sqrt{a + b x}}, -\frac{(b c - a d) (f g - e h)}{(d e - c f) (b g - a h)}\right]] \right) / \left( 4 d f^2 h^2 \sqrt{f g - e h} \sqrt{c + d x} \sqrt{-\frac{(b e - a f) (g + h x)}{(f g - e h) (a + b x)}} \right) - \\ & \frac{1}{4 d^2 \sqrt{b c - a d} f^2 h^3 \sqrt{c + d x} \sqrt{e + f x}} \sqrt{-d g + c h} ((a d f h + b (d f g + d e h + c f h)) (4 b B d f h + C (a d f h - 3 b (d f g + d e h + c f h))) + \\ & 4 d f h (2 a^2 C d f h + b^2 C (d e g + c f g + c e h) - a b (4 B d f h - C (d f g + d e h + c f h))) (a + b x) \sqrt{\frac{(b g - a h) (c + d x)}{(d g - c h) (a + b x)}} \\ & \sqrt{\frac{(b g - a h) (e + f x)}{(f g - e h) (a + b x)}} \text{EllipticPi}\left[-\frac{b (d g - c h)}{(b c - a d) h}, \text{ArcSin}\left[\frac{\sqrt{b c - a d} \sqrt{g + h x}}{\sqrt{-d g + c h} \sqrt{a + b x}}, \frac{(b e - a f) (d g - c h)}{(b c - a d) (f g - e h)}\right]\right] \end{aligned}$$

Result (type 4, 21555 leaves): Display of huge result suppressed!

### Problem 22: Result more than twice size of optimal antiderivative.

$$\int \frac{a b B - a^2 C + b^2 B x + b^2 C x^2}{\sqrt{a+b x} \sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x}} dx$$

Optimal (type 4, 734 leaves, 8 steps):

$$\begin{aligned} & \frac{b C \sqrt{d g - c h} \sqrt{f g - e h} \sqrt{a + b x}}{f h \sqrt{c + d x}} \sqrt{-\frac{(d e - c f) (g + h x)}{(f g - e h) (c + d x)}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{d g - c h} \sqrt{e + f x}}{\sqrt{f g - e h} \sqrt{c + d x}}\right], \frac{(b c - a d) (f g - e h)}{(b e - a f) (d g - c h)}\right] \\ & \frac{c (b e - a f) \sqrt{b g - a h}}{f h \sqrt{f g - e h} \sqrt{c + d x}} \sqrt{-\frac{(b e - a f) (c + d x)}{(d e - c f) (a + b x)}} \sqrt{g + h x} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b g - a h} \sqrt{e + f x}}{\sqrt{f g - e h} \sqrt{a + b x}}\right], -\frac{(b c - a d) (f g - e h)}{(d e - c f) (b g - a h)}\right] \\ & \left( \sqrt{-d g + c h} (a C d f h - b (2 B d f h - C (d f g + d e h + c f h))) (a + b x) \sqrt{\frac{(b g - a h) (c + d x)}{(d g - c h) (a + b x)}} \sqrt{\frac{(b g - a h) (e + f x)}{(f g - e h) (a + b x)}} \right. \\ & \left. \text{EllipticPi}\left[-\frac{b (d g - c h)}{(b c - a d) h}, \text{ArcSin}\left[\frac{\sqrt{b c - a d} \sqrt{g + h x}}{\sqrt{-d g + c h} \sqrt{a + b x}}\right], \frac{(b e - a f) (d g - c h)}{(b c - a d) (f g - e h)}\right] \right) / \left( d \sqrt{b c - a d} f h^2 \sqrt{c + d x} \sqrt{e + f x} \right) \end{aligned}$$

Result (type 4, 6667 leaves):

$$\begin{aligned} & \frac{1}{d^2} 2 \left( \frac{b C (c + d x)^{3/2} \left( f + \frac{d e}{c + d x} - \frac{c f}{c + d x} \right) \left( h + \frac{d g}{c + d x} - \frac{c h}{c + d x} \right) \sqrt{a + \frac{(c + d x) (b - \frac{b c}{c + d x})}{d}}}{2 f h \sqrt{e + \frac{(c + d x) (f - \frac{c f}{c + d x})}{d}} \sqrt{g + \frac{(c + d x) (h - \frac{c h}{c + d x})}{d}}} - \right. \\ & \left. d (b g - a h) (d g - c h) (b C f g + b C e h - 2 b B f h + 2 a C f h) \sqrt{c + d x} \sqrt{\left( b - \frac{b c}{c + d x} + \frac{a d}{c + d x} \right) \left( f + \frac{d e}{c + d x} - \frac{c f}{c + d x} \right) \left( h + \frac{d g}{c + d x} - \frac{c h}{c + d x} \right)} \right) \end{aligned}$$

$$\begin{aligned}
& \sqrt{a + \frac{(c+d x) \left(b - \frac{b c}{c+d x}\right)}{d}} \left( d e \sqrt{-\frac{(b c - a d) (-d g + c h) \left(-\frac{b}{b c - a d} + \frac{1}{c+d x}\right)}{-b d g + a d h}} \left( -\frac{f}{-d e + c f} + \frac{1}{c + d x} \right) \right. \\
& \left. \sqrt{\frac{-\frac{h}{-d g + c h} + \frac{1}{c+d x}}{\frac{f}{-d e + c f} - \frac{h}{-d g + c h}}} \left( (-b d g + a d h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(h + \frac{d g}{c+d x} - \frac{c h}{c+d x}\right)}{d (-f g + e h)}}\right], \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)}] \right. \right. \\
& \left. \left. \frac{b \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(h + \frac{d g}{c+d x} - \frac{c h}{c+d x}\right)}{d (-f g + e h)}}\right], \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)}]}{b c - a d} \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-d e + c f} + \frac{1}{c+d x}}{\frac{f}{-d e + c f} + \frac{h}{-d g + c h}}} \sqrt{\left(b + \frac{-b c + a d}{c + d x}\right) \left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} - c f \sqrt{-\frac{(b c - a d) (-d g + c h) \left(-\frac{b}{b c - a d} + \frac{1}{c+d x}\right)}{-b d g + a d h}} \right. \\
& \left. \left( -\frac{f}{-d e + c f} + \frac{1}{c + d x} \right) \sqrt{\frac{-\frac{h}{-d g + c h} + \frac{1}{c+d x}}{\frac{f}{-d e + c f} - \frac{h}{-d g + c h}}} \left( (-b d g + a d h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(h + \frac{d g}{c+d x} - \frac{c h}{c+d x}\right)}{d (-f g + e h)}}\right], \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)}] \right. \right. \\
& \left. \left. \frac{b \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(h + \frac{d g}{c+d x} - \frac{c h}{c+d x}\right)}{d (-f g + e h)}}\right], \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)}]}{b c - a d} \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-d e + c f} + \frac{1}{c+d x}}{\frac{f}{-d e + c f} + \frac{h}{-d g + c h}}} \sqrt{\left(b + \frac{-b c + a d}{c + d x}\right) \left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} + f \sqrt{\frac{-\frac{b}{b c - a d} + \frac{1}{c+d x}}{-\frac{b}{b c - a d} + \frac{h}{-d g + c h}}} \sqrt{\frac{-\frac{f}{-d e + c f} + \frac{1}{c+d x}}{-\frac{f}{-d e + c f} + \frac{h}{-d g + c h}}} \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{h}{-d g + c h} + \frac{1}{c + d x} \right) \text{EllipticF}[\text{ArcSin}\left[ \sqrt{\frac{(-d e + c f) \left( -h - \frac{d g}{c+d x} + \frac{c h}{c+d x} \right)}{d (-f g + e h)}} \right], \frac{(b c - a d) \left( -f g + e h \right)}{(-d e + c f) \left( -b g + a h \right)}] \Bigg] / \\
& \left( \sqrt{\frac{-\frac{h}{-d g + c h} + \frac{1}{c+d x}}{\frac{f}{-d e+c f} - \frac{h}{-d g+c h}}} \sqrt{\left( b + \frac{-b c + a d}{c + d x} \right) \left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) \Bigg] / \\
& \left( 2 f h^2 (f g - e h) \left( b - \frac{b c}{c + d x} + \frac{a d}{c + d x} \right) \sqrt{e + \frac{(c + d x) \left( f - \frac{c f}{c+d x} \right)}{d}} \sqrt{g + \frac{(c + d x) \left( h - \frac{c h}{c+d x} \right)}{d}} \right) + \\
& \left( d (b e - a f) (d e - c f) (b C f g + b C e h - 2 b B f h + 2 a C f h) \sqrt{c + d x} \right. \\
& \left. \sqrt{\left( b - \frac{b c}{c + d x} + \frac{a d}{c + d x} \right) \left( f + \frac{d e}{c + d x} - \frac{c f}{c + d x} \right) \left( h + \frac{d g}{c + d x} - \frac{c h}{c + d x} \right)} \sqrt{a + \frac{(c + d x) \left( b - \frac{b c}{c+d x} \right)}{d}} \right. \\
& \left( d g \sqrt{-\frac{(b c - a d) \left( -d g + c h \right) \left( -\frac{b}{b c - a d} + \frac{1}{c+d x} \right)}{-b d g + a d h}} \left( -\frac{f}{-d e + c f} + \frac{1}{c + d x} \right) \sqrt{\frac{-\frac{h}{-d g + c h} + \frac{1}{c+d x}}{\frac{f}{-d e+c f} - \frac{h}{-d g+c h}}} \right. \\
& \left. \left( -b d g + a d h \right) \text{EllipticE}[\text{ArcSin}\left[ \sqrt{\frac{(d e - c f) \left( h + \frac{d g}{c+d x} - \frac{c h}{c+d x} \right)}{d (-f g + e h)}} \right], \frac{(b c - a d) \left( -f g + e h \right)}{(-d e + c f) \left( -b g + a h \right)}] \right. \\
& \left. \left. \left. \left. \frac{b \text{EllipticF}[\text{ArcSin}\left[ \sqrt{\frac{(d e - c f) \left( h + \frac{d g}{c+d x} - \frac{c h}{c+d x} \right)}{d (-f g + e h)}} \right], \frac{(b c - a d) \left( -f g + e h \right)}{(-d e + c f) \left( -b g + a h \right)}]}{b c - a d} \right) \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{\frac{-\frac{f}{-de+cf} + \frac{1}{c+dx}}{-\frac{f}{-de+cf} + \frac{h}{-dg+ch}}} \sqrt{\left( b + \frac{-bc+ad}{c+dx} \right) \left( f + \frac{de-cf}{c+dx} \right) \left( h + \frac{dg-ch}{c+dx} \right)} \right) - \left( ch \sqrt{-\frac{(bc-ad)(-dg+ch)\left(-\frac{b}{bc-ad} + \frac{1}{c+dx}\right)}{-bdg+adh}} \right. \\
& \left( -\frac{f}{-de+cf} + \frac{1}{c+dx} \right) \sqrt{\frac{-\frac{h}{-dg+ch} + \frac{1}{c+dx}}{\frac{f}{-de+cf} - \frac{h}{-dg+ch}}} \left( -bdg+adh \right) \text{EllipticE} \left[ \text{ArcSin} \left[ \sqrt{\frac{(de-cf)\left(h+\frac{dg}{c+dx}-\frac{ch}{c+dx}\right)}{d(-fg+eh)}} \right], \frac{(bc-ad)(-fg+eh)}{(-de+cf)(-bg+ah)} \right] \\
& \left. \left. \frac{b \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(de-cf)\left(h+\frac{dg}{c+dx}-\frac{ch}{c+dx}\right)}{d(-fg+eh)}} \right], \frac{(bc-ad)(-fg+eh)}{(-de+cf)(-bg+ah)} \right]}{bc-ad} \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-de+cf} + \frac{1}{c+dx}}{-\frac{f}{-de+cf} + \frac{h}{-dg+ch}}} \sqrt{\left( b + \frac{-bc+ad}{c+dx} \right) \left( f + \frac{de-cf}{c+dx} \right) \left( h + \frac{dg-ch}{c+dx} \right)} + \left( h \sqrt{\frac{-\frac{b}{bc-ad} + \frac{1}{c+dx}}{-\frac{b}{bc-ad} + \frac{h}{-dg+ch}}} \sqrt{\frac{-\frac{f}{-de+cf} + \frac{1}{c+dx}}{-\frac{f}{-de+cf} + \frac{h}{-dg+ch}}} \right. \right. \\
& \left. \left( -\frac{h}{-dg+ch} + \frac{1}{c+dx} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-de+cf)\left(-h-\frac{dg}{c+dx}+\frac{ch}{c+dx}\right)}{d(-fg+eh)}} \right], \frac{(bc-ad)(-fg+eh)}{(-de+cf)(-bg+ah)} \right] \right) / \\
& \left( \sqrt{\frac{-\frac{h}{-dg+ch} + \frac{1}{c+dx}}{\frac{f}{-de+cf} - \frac{h}{-dg+ch}}} \sqrt{\left( b + \frac{-bc+ad}{c+dx} \right) \left( f + \frac{de-cf}{c+dx} \right) \left( h + \frac{dg-ch}{c+dx} \right)} \right) \right) / \\
& \left( 2f^2h(fg-eh) \left( b - \frac{bc}{c+dx} + \frac{ad}{c+dx} \right) \sqrt{e + \frac{(c+dx)\left(f-\frac{cf}{c+dx}\right)}{d}} \sqrt{g + \frac{(c+dx)\left(h-\frac{ch}{c+dx}\right)}{d}} \right. + \\
& \left. \frac{1}{2f^2h^2 \left( b - \frac{bc}{c+dx} + \frac{ad}{c+dx} \right) \sqrt{e + \frac{(c+dx)\left(f-\frac{cf}{c+dx}\right)}{d}} \sqrt{g + \frac{(c+dx)\left(h-\frac{ch}{c+dx}\right)}{d}}} \right. \\
& \left. b(bCd\,fg + bCd\,eh + bcC\,fh - 2bB\,df\,h + aCd\,fh) \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{c + d x} \sqrt{\left(b - \frac{b c}{c + d x} + \frac{a d}{c + d x}\right) \left(f + \frac{d e}{c + d x} - \frac{c f}{c + d x}\right) \left(h + \frac{d g}{c + d x} - \frac{c h}{c + d x}\right)} \\
& \sqrt{a + \frac{(c + d x) \left(b - \frac{b c}{c + d x}\right)}{d}} \\
& \left( \left( d^2 e g \sqrt{-\frac{(b c - a d) (-d g + c h) \left(-\frac{b}{b c - a d} + \frac{1}{c + d x}\right)}{-b d g + a d h}} \left(-\frac{f}{-d e + c f} + \frac{1}{c + d x}\right) \sqrt{\frac{-\frac{h}{-d g + c h} + \frac{1}{c + d x}}{\frac{f}{-d e + c f} - \frac{h}{-d g + c h}}} \right. \right. \\
& \left. \left. \left( (-b d g + a d h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(h + \frac{d g}{c + d x} - \frac{c h}{c + d x}\right)}{d (-f g + e h)}}\right], \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)}] \right. \right. \right. \\
& \left. \left. \left. \left( b c - a d \right) (-d g + c h) \right) \right. \right. \\
& \left. \left. \left. b \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(h + \frac{d g}{c + d x} - \frac{c h}{c + d x}\right)}{d (-f g + e h)}}\right], \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)}] \right) \right. \right) / \\
& \left( \left( \sqrt{\frac{-\frac{f}{-d e + c f} + \frac{1}{c + d x}}{-\frac{f}{-d e + c f} + \frac{h}{-d g + c h}}} \sqrt{\left(b + \frac{-b c + a d}{c + d x}\right) \left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) - \left( c d f g \sqrt{-\frac{(b c - a d) (-d g + c h) \left(-\frac{b}{b c - a d} + \frac{1}{c + d x}\right)}{-b d g + a d h}} \right. \right. \\
& \left. \left. \left( -\frac{f}{-d e + c f} + \frac{1}{c + d x} \right) \sqrt{\frac{-\frac{h}{-d g + c h} + \frac{1}{c + d x}}{\frac{f}{-d e + c f} - \frac{h}{-d g + c h}}} \left( (-b d g + a d h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(h + \frac{d g}{c + d x} - \frac{c h}{c + d x}\right)}{d (-f g + e h)}}\right], \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)}] \right. \right. \right. \\
& \left. \left. \left. \left( b c - a d \right) (-d g + c h) \right) \right. \right. \\
& \left. \left. \left. b \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(d e - c f) \left(h + \frac{d g}{c + d x} - \frac{c h}{c + d x}\right)}{d (-f g + e h)}}\right], \frac{(b c - a d) (-f g + e h)}{(-d e + c f) (-b g + a h)}] \right) \right. \right) /
\end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{\frac{-\frac{f}{-de+cf} + \frac{1}{c+dx}}{-\frac{f}{-de+cf} + \frac{h}{-dg+ch}}} \sqrt{\left(b + \frac{-bc+ad}{c+dx}\right) \left(f + \frac{de-cf}{c+dx}\right) \left(h + \frac{dg-ch}{c+dx}\right)} - cdeh \sqrt{-\frac{(bc-ad)(-dg+ch)\left(-\frac{b}{bc-ad} + \frac{1}{c+dx}\right)}{-bdg+adh}} \right. \\
& \left. \left( -\frac{f}{-de+cf} + \frac{1}{c+dx} \right) \sqrt{\frac{-\frac{h}{-dg+ch} + \frac{1}{c+dx}}{\frac{f}{-de+cf} - \frac{h}{-dg+ch}}} \frac{(-bdg+adh) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(de-cf)(h+\frac{dg-ch}{c+dx}-\frac{ch}{c+dx})}{d(-fg+eh)}}\right], \frac{(bc-ad)(-fg+eh)}{(-de+cf)(-bg+ah)}]}{(bc-ad)(-dg+ch)} - \right. \\
& \left. b \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(de-cf)(h+\frac{dg-ch}{c+dx}-\frac{ch}{c+dx})}{d(-fg+eh)}}\right], \frac{(bc-ad)(-fg+eh)}{(-de+cf)(-bg+ah)}] \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-de+cf} + \frac{1}{c+dx}}{-\frac{f}{-de+cf} + \frac{h}{-dg+ch}}} \sqrt{\left(b + \frac{-bc+ad}{c+dx}\right) \left(f + \frac{de-cf}{c+dx}\right) \left(h + \frac{dg-ch}{c+dx}\right)} + c^2 fh \sqrt{-\frac{(bc-ad)(-dg+ch)\left(-\frac{b}{bc-ad} + \frac{1}{c+dx}\right)}{-bdg+adh}} \right. \\
& \left. \left( -\frac{f}{-de+cf} + \frac{1}{c+dx} \right) \sqrt{\frac{-\frac{h}{-dg+ch} + \frac{1}{c+dx}}{\frac{f}{-de+cf} - \frac{h}{-dg+ch}}} \frac{(-bdg+adh) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(de-cf)(h+\frac{dg-ch}{c+dx}-\frac{ch}{c+dx})}{d(-fg+eh)}}\right], \frac{(bc-ad)(-fg+eh)}{(-de+cf)(-bg+ah)}]}{(bc-ad)(-dg+ch)} - \right. \\
& \left. b \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(de-cf)(h+\frac{dg-ch}{c+dx}-\frac{ch}{c+dx})}{d(-fg+eh)}}\right], \frac{(bc-ad)(-fg+eh)}{(-de+cf)(-bg+ah)}] \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-de+cf} + \frac{1}{c+dx}}{-\frac{f}{-de+cf} + \frac{h}{-dg+ch}}} \sqrt{\left(b + \frac{-bc+ad}{c+dx}\right) \left(f + \frac{de-cf}{c+dx}\right) \left(h + \frac{dg-ch}{c+dx}\right)} + dfg \sqrt{\frac{-\frac{b}{bc-ad} + \frac{1}{c+dx}}{-\frac{b}{bc-ad} + \frac{h}{-dg+ch}}} \sqrt{\frac{-\frac{f}{-de+cf} + \frac{1}{c+dx}}{-\frac{f}{-de+cf} + \frac{h}{-dg+ch}}} \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{h}{-dg + ch} + \frac{1}{c + dx} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-de + cf) \left( -h - \frac{dg}{c+dx} + \frac{ch}{c+dx} \right)}{d (-fg + eh)}} \right], \frac{(bc - ad) \left( -fg + eh \right)}{(-de + cf) \left( -bg + ah \right)} \right] \Bigg] \\
& \left( \sqrt{\frac{-\frac{h}{-dg+ch} + \frac{1}{c+dx}}{\frac{f}{-de+cf} - \frac{h}{-dg+ch}}} \sqrt{\left( b + \frac{-bc+ad}{c+dx} \right) \left( f + \frac{de-cf}{c+dx} \right) \left( h + \frac{dg-ch}{c+dx} \right)} \right) + \\
& \left( deh \sqrt{\frac{-\frac{b}{bc-ad} + \frac{1}{c+dx}}{-\frac{b}{bc-ad} + \frac{h}{-dg+ch}}} \sqrt{\frac{-\frac{f}{-de+cf} + \frac{1}{c+dx}}{-\frac{f}{-de+cf} + \frac{h}{-dg+ch}}} \left( -\frac{h}{-dg+ch} + \frac{1}{c+dx} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-de + cf) \left( -h - \frac{dg}{c+dx} + \frac{ch}{c+dx} \right)}{d (-fg + eh)}} \right], \right. \right. \\
& \left. \left. \frac{(bc - ad) \left( -fg + eh \right)}{(-de + cf) \left( -bg + ah \right)} \right] \right) \Bigg] \Bigg/ \left( \sqrt{\frac{-\frac{h}{-dg+ch} + \frac{1}{c+dx}}{\frac{f}{-de+cf} - \frac{h}{-dg+ch}}} \sqrt{\left( b + \frac{-bc+ad}{c+dx} \right) \left( f + \frac{de-cf}{c+dx} \right) \left( h + \frac{dg-ch}{c+dx} \right)} \right) - \\
& \left( 2cfh \sqrt{\frac{-\frac{b}{bc-ad} + \frac{1}{c+dx}}{-\frac{b}{bc-ad} + \frac{h}{-dg+ch}}} \sqrt{\frac{-\frac{f}{-de+cf} + \frac{1}{c+dx}}{-\frac{f}{-de+cf} + \frac{h}{-dg+ch}}} \left( -\frac{h}{-dg+ch} + \frac{1}{c+dx} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-de + cf) \left( -h - \frac{dg}{c+dx} + \frac{ch}{c+dx} \right)}{d (-fg + eh)}} \right], \right. \right. \\
& \left. \left. \frac{(bc - ad) \left( -fg + eh \right)}{(-de + cf) \left( -bg + ah \right)} \right] \right) \Bigg] \Bigg/ \left( \sqrt{\frac{-\frac{h}{-dg+ch} + \frac{1}{c+dx}}{\frac{f}{-de+cf} - \frac{h}{-dg+ch}}} \sqrt{\left( b + \frac{-bc+ad}{c+dx} \right) \left( f + \frac{de-cf}{c+dx} \right) \left( h + \frac{dg-ch}{c+dx} \right)} \right) - \\
& \left( f (-dg + ch) \left( -\frac{f}{-de+cf} + \frac{h}{-dg+ch} \right) \sqrt{\frac{-\frac{b}{bc-ad} + \frac{1}{c+dx}}{-\frac{b}{bc-ad} + \frac{h}{-dg+ch}}} \sqrt{-\frac{\left( -\frac{f}{-de+cf} + \frac{1}{c+dx} \right) \left( -\frac{h}{-dg+ch} + \frac{1}{c+dx} \right)}{\left( -\frac{f}{-de+cf} + \frac{h}{-dg+ch} \right)^2}} \text{EllipticPi} \left[ -\frac{-dfg + deh}{(-de + cf) h}, \right. \right. \\
& \left. \left. \text{ArcSin} \left[ \sqrt{\frac{(-de + cf) \left( -h - \frac{dg}{c+dx} + \frac{ch}{c+dx} \right)}{d (-fg + eh)}} \right], \frac{(bc - ad) \left( -fg + eh \right)}{(-de + cf) \left( -bg + ah \right)} \right] \right) \Bigg/ \left( \sqrt{\left( b + \frac{-bc+ad}{c+dx} \right) \left( f + \frac{de-cf}{c+dx} \right) \left( h + \frac{dg-ch}{c+dx} \right)} \right) \right)
\end{aligned}$$

**Problem 24: Result more than twice size of optimal antiderivative.**

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a + bx)^{5/2} \sqrt{c + dx} \sqrt{e + fx} \sqrt{g + hx}} dx$$

Optimal (type 4, 616 leaves, 8 steps):

$$\begin{aligned}
 & \frac{2 b (b B - 2 a C) d \sqrt{a + b x} \sqrt{e + f x} \sqrt{g + h x}}{(b c - a d) (b e - a f) (b g - a h) \sqrt{c + d x}} - \frac{2 b^2 (b B - 2 a C) \sqrt{c + d x} \sqrt{e + f x} \sqrt{g + h x}}{(b c - a d) (b e - a f) (b g - a h) \sqrt{a + b x}} - \\
 & \left( 2 b (b B - 2 a C) \sqrt{d g - c h} \sqrt{f g - e h} \sqrt{a + b x} \sqrt{-\frac{(d e - c f) (g + h x)}{(f g - e h) (c + d x)}} \text{EllipticE}[\text{ArcSin}\left[\frac{\sqrt{d g - c h} \sqrt{e + f x}}{\sqrt{f g - e h} \sqrt{c + d x}}\right], \frac{(b c - a d) (f g - e h)}{(b e - a f) (d g - c h)}] \right) / \\
 & \left( (b c - a d) (b e - a f) (b g - a h) \sqrt{\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}} \sqrt{g + h x} \right) + \\
 & \frac{2 (b c C - b B d + a C d) \sqrt{\frac{(b e - a f) (c + d x)}{(d e - c f) (a + b x)}} \sqrt{g + h x} \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{b g - a h} \sqrt{e + f x}}{\sqrt{f g - e h} \sqrt{a + b x}}\right], -\frac{(b c - a d) (f g - e h)}{(d e - c f) (b g - a h)}]}{(b c - a d) \sqrt{b g - a h} \sqrt{f g - e h} \sqrt{c + d x} \sqrt{-\frac{(b e - a f) (g + h x)}{(f g - e h) (a + b x)}}}
 \end{aligned}$$

Result (type 4, 1753 leaves):

$$\begin{aligned}
 & -\frac{2 b^2 (b B - 2 a C) \sqrt{c + d x} \sqrt{e + f x} \sqrt{g + h x}}{(b c - a d) (b e - a f) (b g - a h) \sqrt{a + b x}} + \\
 & \frac{1}{b (-b c + a d) (-b e + a f) (-b g + a h)} 2 \sqrt{\frac{(-b B + 2 a C) (a + b x)^{5/2} \left(d + \frac{b c}{a + b x} - \frac{a d}{a + b x}\right) \left(f + \frac{b e}{a + b x} - \frac{a f}{a + b x}\right) \left(h + \frac{b g}{a + b x} - \frac{a h}{a + b x}\right)}{\sqrt{c + \frac{(a + b x) (d - \frac{a d}{a + b x})}{b}} \sqrt{e + \frac{(a + b x) (f - \frac{a f}{a + b x})}{b}} \sqrt{g + \frac{(a + b x) (h - \frac{a h}{a + b x})}{b}}} - \\
 & \frac{1}{\sqrt{c + \frac{(a + b x) (d - \frac{a d}{a + b x})}{b}} \sqrt{e + \frac{(a + b x) (f - \frac{a f}{a + b x})}{b}} \sqrt{g + \frac{(a + b x) (h - \frac{a h}{a + b x})}{b}}} (b c - a d) (b e - a f) (b g - a h) (a + b x)^{3/2} \\
 & \sqrt{\left(d + \frac{b c}{a + b x} - \frac{a d}{a + b x}\right) \left(f + \frac{b e}{a + b x} - \frac{a f}{a + b x}\right) \left(h + \frac{b g}{a + b x} - \frac{a h}{a + b x}\right)} \left(-\sqrt{b B} \sqrt{\frac{(b c - a d) (b g - a h) \left(-\frac{d}{-b c + a d} + \frac{1}{a + b x}\right)}{b d g - b c h}}\right)
 \end{aligned}$$

$$\begin{aligned}
& \left( -\frac{f}{-b e + a f} + \frac{1}{a + b x} \right) \sqrt{\frac{-\frac{h}{-b g + a h} + \frac{1}{a+b x}}{\frac{f}{-b e + a f} - \frac{h}{-b g + a h}}} \left( -\frac{(b d g - b c h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e - a f) \left(h + \frac{b g}{a+b x} - \frac{a h}{a+b x}\right)}{b (-f g + e h)}}, \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}}\right]}{(b c - a d) (b g - a h)} - \right. \\
& \left. \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e - a f) \left(h + \frac{b g}{a+b x} - \frac{a h}{a+b x}\right)}{b (-f g + e h)}}, \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}}\right]}{-b c + a d} \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-b e + a f} + \frac{1}{a+b x}}{-\frac{f}{-b e + a f} + \frac{h}{-b g + a h}}} \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right) \left(h + \frac{b g - a h}{a + b x}\right)} \right) + 2 a C \sqrt{\frac{(b c - a d) (b g - a h) \left(-\frac{d}{-b c + a d} + \frac{1}{a+b x}\right)}{b d g - b c h}} \\
& \left( -\frac{f}{-b e + a f} + \frac{1}{a + b x} \right) \sqrt{\frac{-\frac{h}{-b g + a h} + \frac{1}{a+b x}}{\frac{f}{-b e + a f} - \frac{h}{-b g + a h}}} \left( -\frac{(b d g - b c h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e - a f) \left(h + \frac{b g}{a+b x} - \frac{a h}{a+b x}\right)}{b (-f g + e h)}}, \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}}\right]}{(b c - a d) (b g - a h)} - \right. \\
& \left. \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e - a f) \left(h + \frac{b g}{a+b x} - \frac{a h}{a+b x}\right)}{b (-f g + e h)}}, \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}}\right]}{-b c + a d} \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-b e + a f} + \frac{1}{a+b x}}{-\frac{f}{-b e + a f} + \frac{h}{-b g + a h}}} \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right) \left(h + \frac{b g - a h}{a + b x}\right)} - \left(C \sqrt{\frac{-\frac{d}{-b c + a d} + \frac{1}{a+b x}}{-\frac{d}{-b c + a d} + \frac{h}{-b g + a h}}} \sqrt{\frac{-\frac{f}{-b e + a f} + \frac{1}{a+b x}}{-\frac{f}{-b e + a f} + \frac{h}{-b g + a h}}} \right. \right. \\
& \left. \left. \left( -\frac{h}{-b g + a h} + \frac{1}{a + b x} \right) \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(-b e + a f) \left(-h - \frac{b g}{a+b x} + \frac{a h}{a+b x}\right)}{b (-f g + e h)}}, \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}}\right] \right) \right)
\end{aligned}$$

$$\left( \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+bx}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \sqrt{\left( d + \frac{bc-ad}{a+bx} \right) \left( f + \frac{be-af}{a+bx} \right) \left( h + \frac{bg-ah}{a+bx} \right)} \right)$$

**Problem 25:** Result more than twice size of optimal antiderivative.

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^{7/2} \sqrt{c+dx} \sqrt{e+fx} \sqrt{g+hx}} dx$$

Optimal (type 4, 1128 leaves, 9 steps):

$$\begin{aligned}
& \left( 2 b d (9 a^3 C d f h - b^3 (2 B d e g - c (3 C e g - 2 B f g - 2 B e h))) + \right. \\
& \quad a b^2 (C (d e g + c f g + c e h) + 4 B (d f g + d e h + c f h)) - a^2 b (6 B d f h + 5 C (d f g + d e h + c f h))) \sqrt{a+b x} \sqrt{e+f x} \sqrt{g+h x} \Big) / \\
& \quad \left( 3 (b c - a d)^2 (b e - a f)^2 (b g - a h)^2 \sqrt{c+d x} \right) - \frac{2 b^2 (b B - 2 a C) \sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x}}{3 (b c - a d) (b e - a f) (b g - a h) (a+b x)^{3/2}} - \\
& \left( 2 b^2 (9 a^3 C d f h - b^3 (2 B d e g - c (3 C e g - 2 B f g - 2 B e h))) + a b^2 (C (d e g + c f g + c e h) + 4 B (d f g + d e h + c f h)) - \right. \\
& \quad a^2 b (6 B d f h + 5 C (d f g + d e h + c f h))) \sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x} \Big) / \left( 3 (b c - a d)^2 (b e - a f)^2 (b g - a h)^2 \sqrt{a+b x} \right) - \\
& \left( 2 b \sqrt{d g - c h} \sqrt{f g - e h} (9 a^3 C d f h - b^3 (2 B d e g - c (3 C e g - 2 B f g - 2 B e h))) + a b^2 (C (d e g + c f g + c e h) + 4 B (d f g + d e h + c f h)) - \right. \\
& \quad a^2 b (6 B d f h + 5 C (d f g + d e h + c f h))) \sqrt{a+b x} \sqrt{- \frac{(d e - c f) (g+h x)}{(f g - e h) (c+d x)}} \\
& \quad \text{EllipticE} \left[ \text{ArcSin} \left[ \frac{\sqrt{d g - c h} \sqrt{e+f x}}{\sqrt{f g - e h} \sqrt{c+d x}} \right], \frac{(b c - a d) (f g - e h)}{(b e - a f) (d g - c h)} \right] / \left( 3 (b c - a d)^2 (b e - a f)^2 (b g - a h)^2 \sqrt{\frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}} \sqrt{g+h x} \right) - \\
& \left( 2 (3 a^3 C d^2 f h - b^3 (2 B d^2 e g - B c^2 f h - c d (3 C e g - B f g - B e h))) - 3 a^2 b d (B d f h + C (d f g + d e h - c f h)) + \right. \\
& \quad a b^2 (3 B d^2 (f g + e h) + C (d^2 e g - c d f g - c d e h - 2 c^2 f h))) \\
& \quad \sqrt{\frac{(b e - a f) (c+d x)}{(d e - c f) (a+b x)}} \sqrt{g+h x} \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\sqrt{b g - a h} \sqrt{e+f x}}{\sqrt{f g - e h} \sqrt{a+b x}} \right], - \frac{(b c - a d) (f g - e h)}{(d e - c f) (b g - a h)} \right] / \\
& \quad \left. \left( 3 (b c - a d)^2 (b e - a f) (b g - a h)^{3/2} \sqrt{f g - e h} \sqrt{c+d x} \sqrt{- \frac{(b e - a f) (g+h x)}{(f g - e h) (a+b x)}} \right) \right)
\end{aligned}$$

Result (type 4, 10 645 leaves):

$$\begin{aligned}
& \sqrt{a+b x} \sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x} \\
& \left( - \frac{2 b^2 (b B - 2 a C)}{3 (b c - a d) (b e - a f) (b g - a h) (a+b x)^2} - (2 b^2 (3 b^3 c C e g - 2 b^3 B d e g + a b^2 C d e g - 2 b^3 B c f g + a b^2 c C f g + 4 a b^2 B d f g - \right. \\
& \quad 5 a^2 b C d f g - 2 b^3 B c e h + a b^2 c C e h + 4 a b^2 B d e h - 5 a^2 b C d e h + 4 a b^2 B c f h - 5 a^2 b c C f h - 6 a^2 b B d f h + 9 a^3 C d f h) \Big) / \\
& \quad \left. \left( 3 (b c - a d)^2 (b e - a f)^2 (b g - a h)^2 (a+b x) \right) \right) - \frac{1}{3 b (-b c + a d)^2 (-b e + a f)^2 (-b g + a h)^2}
\end{aligned}$$

$$\begin{aligned}
& 2 \left( \left( -3 b^3 c C e g + 2 b^3 B d e g - a b^2 C d e g + 2 b^3 B c f g - a b^2 c C f g - 4 a b^2 B d f g + 5 a^2 b C d f g + 2 b^3 B c e h - a b^2 c C e h - 4 a b^2 B d e h + \right. \right. \\
& \quad \left. \left. 5 a^2 b C d e h - 4 a b^2 B c f h + 5 a^2 b c C f h + 6 a^2 b B d f h - 9 a^3 C d f h \right) (a + b x)^{5/2} \left( d + \frac{b c}{a + b x} - \frac{a d}{a + b x} \right) \left( f + \frac{b e}{a + b x} - \frac{a f}{a + b x} \right) \right. \\
& \quad \left( h + \frac{b g}{a + b x} - \frac{a h}{a + b x} \right) \Bigg) \Bigg/ \left( \sqrt{c + \frac{(a + b x) (d - \frac{a d}{a + b x})}{b}} \sqrt{e + \frac{(a + b x) (f - \frac{a f}{a + b x})}{b}} \sqrt{g + \frac{(a + b x) (h - \frac{a h}{a + b x})}{b}} \right) - \\
& \frac{1}{\sqrt{c + \frac{(a + b x) (d - \frac{a d}{a + b x})}{b}} \sqrt{e + \frac{(a + b x) (f - \frac{a f}{a + b x})}{b}} \sqrt{g + \frac{(a + b x) (h - \frac{a h}{a + b x})}{b}}} (b c - a d) (b e - a f) (b g - a h) (a + b x)^{3/2} \\
& \sqrt{\left( d + \frac{b c}{a + b x} - \frac{a d}{a + b x} \right) \left( f + \frac{b e}{a + b x} - \frac{a f}{a + b x} \right) \left( h + \frac{b g}{a + b x} - \frac{a h}{a + b x} \right)} \left( - \left( 3 b^3 c C e g \sqrt{\frac{(b c - a d) (b g - a h) \left( -\frac{d}{-b c + a d} + \frac{1}{a + b x} \right)}{b d g - b c h}} \right. \right. \\
& \left. \left. \left( -\frac{f}{-b e + a f} + \frac{1}{a + b x} \right) \sqrt{\frac{-\frac{h}{-b g + a h} + \frac{1}{a + b x}}{\frac{f}{-b e + a f} - \frac{h}{-b g + a h}}} \right. \right. \\
& \left. \left. - \frac{(b d g - b c h) \text{EllipticE}[\text{ArcSin}\left[\sqrt{\frac{(b e - a f) \left( h + \frac{b g}{a + b x} - \frac{a h}{a + b x} \right)}{b (-f g + e h)}}, \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}\right]}]{(b c - a d) (b g - a h)} \right. \right. \\
& \left. \left. \left. \frac{d \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{(b e - a f) \left( h + \frac{b g}{a + b x} - \frac{a h}{a + b x} \right)}{b (-f g + e h)}}, \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}\right]}]{-b c + a d} \right) \right) \right) \Bigg/ \\
& \left( \sqrt{\frac{-\frac{f}{-b e + a f} + \frac{1}{a + b x}}{\frac{f}{-b e + a f} + \frac{h}{-b g + a h}}} \sqrt{\left( d + \frac{b c - a d}{a + b x} \right) \left( f + \frac{b e - a f}{a + b x} \right) \left( h + \frac{b g - a h}{a + b x} \right)} \right) + \left( 2 b^3 B d e g \sqrt{\frac{(b c - a d) (b g - a h) \left( -\frac{d}{-b c + a d} + \frac{1}{a + b x} \right)}{b d g - b c h}} \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{f}{-b e + a f} + \frac{1}{a + b x} \right) \sqrt{\frac{-\frac{h}{-b g + a h} + \frac{1}{a + b x}}{\frac{f}{-b e + a f} - \frac{h}{-b g + a h}}} \left( -\frac{(b d g - b c h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e - a f) \left(h + \frac{b g}{a + b x} - \frac{a h}{a + b x}\right)}{b (-f g + e h)}}, \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}\right]}]{(b c - a d) (b g - a h)} \right. \\
& \left. \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e - a f) \left(h + \frac{b g}{a + b x} - \frac{a h}{a + b x}\right)}{b (-f g + e h)}}, \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}\right]}]{-b c + a d} \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-b e + a f} + \frac{1}{a + b x}}{\frac{f}{-b e + a f} + \frac{h}{-b g + a h}}} \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right) \left(h + \frac{b g - a h}{a + b x}\right)} \right) - \left( a b^2 c d e g \sqrt{\frac{(b c - a d) (b g - a h) \left(-\frac{d}{-b c + a d} + \frac{1}{a + b x}\right)}{b d g - b c h}} \right. \\
& \left. \left( -\frac{f}{-b e + a f} + \frac{1}{a + b x} \right) \sqrt{\frac{-\frac{h}{-b g + a h} + \frac{1}{a + b x}}{\frac{f}{-b e + a f} - \frac{h}{-b g + a h}}} \left( -\frac{(b d g - b c h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e - a f) \left(h + \frac{b g}{a + b x} - \frac{a h}{a + b x}\right)}{b (-f g + e h)}}, \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}\right]}]{(b c - a d) (b g - a h)} \right. \right. \\
& \left. \left. \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e - a f) \left(h + \frac{b g}{a + b x} - \frac{a h}{a + b x}\right)}{b (-f g + e h)}}, \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}\right]}]{-b c + a d} \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-b e + a f} + \frac{1}{a + b x}}{\frac{f}{-b e + a f} + \frac{h}{-b g + a h}}} \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right) \left(h + \frac{b g - a h}{a + b x}\right)} \right) + \left( 2 b^3 B c f g \sqrt{\frac{(b c - a d) (b g - a h) \left(-\frac{d}{-b c + a d} + \frac{1}{a + b x}\right)}{b d g - b c h}} \right. \\
& \left. \left( -\frac{f}{-b e + a f} + \frac{1}{a + b x} \right) \sqrt{\frac{-\frac{h}{-b g + a h} + \frac{1}{a + b x}}{\frac{f}{-b e + a f} - \frac{h}{-b g + a h}}} \left( -\frac{(b d g - b c h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e - a f) \left(h + \frac{b g}{a + b x} - \frac{a h}{a + b x}\right)}{b (-f g + e h)}}, \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}\right]}]{(b c - a d) (b g - a h)} \right. \right. 
\end{aligned}$$

$$\begin{aligned}
& \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) \left(-f g+e h\right)}{(-b e+a f) \left(-d g+c h\right)}]}{-b c+a d} \\
& -\frac{\frac{f}{-b e+a f}+\frac{1}{a+b x}}{-\frac{f}{-b e+a f}+\frac{h}{-b g+a h}} \sqrt{\left(d+\frac{b c-a d}{a+b x}\right)\left(f+\frac{b e-a f}{a+b x}\right)\left(h+\frac{b g-a h}{a+b x}\right)}-\left(a b^2 c C f g \sqrt{\frac{(b c-a d) \left(b g-a h\right) \left(-\frac{d}{-b c+a d}+\frac{1}{a+b x}\right)}{b d g-b c h}}\right. \\
& \left.\left(-\frac{f}{-b e+a f}+\frac{1}{a+b x}\right) \sqrt{\frac{-\frac{h}{-b g+a h}+\frac{1}{a+b x}}{\frac{f}{-b e+a f}-\frac{h}{-b g+a h}}\left(-\frac{(b d g-b c h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) \left(-f g+e h\right)}{(-b e+a f) \left(-d g+c h\right)}}{(b c-a d) \left(b g-a h\right)}\right.\right. \\
& \left.\left.d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) \left(-f g+e h\right)}{(-b e+a f) \left(-d g+c h\right)}]\right)\right] \\
& -\frac{\frac{f}{-b e+a f}+\frac{1}{a+b x}}{-\frac{f}{-b e+a f}+\frac{h}{-b g+a h}} \sqrt{\left(d+\frac{b c-a d}{a+b x}\right)\left(f+\frac{b e-a f}{a+b x}\right)\left(h+\frac{b g-a h}{a+b x}\right)}-\left(4 a b^2 B d f g \sqrt{\frac{(b c-a d) \left(b g-a h\right) \left(-\frac{d}{-b c+a d}+\frac{1}{a+b x}\right)}{b d g-b c h}}\right. \\
& \left.\left(-\frac{f}{-b e+a f}+\frac{1}{a+b x}\right) \sqrt{\frac{-\frac{h}{-b g+a h}+\frac{1}{a+b x}}{\frac{f}{-b e+a f}-\frac{h}{-b g+a h}}\left(-\frac{(b d g-b c h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) \left(-f g+e h\right)}{(-b e+a f) \left(-d g+c h\right)}}{(b c-a d) \left(b g-a h\right)}\right.\right. \\
& \left.\left.d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) \left(-f g+e h\right)}{(-b e+a f) \left(-d g+c h\right)}]\right)\right]
\end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+bx}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+bx}\right) \left(f + \frac{be-af}{a+bx}\right) \left(h + \frac{bg-ah}{a+bx}\right)} \right) + \left( 5a^2 b C d f g \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+bx}\right)}{bdg-bch}} \right. \\
& \left( -\frac{f}{-be+af} + \frac{1}{a+bx} \right) \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+bx}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \left( -\frac{(bdg-bch) \operatorname{EllipticE}[\operatorname{ArcSin}\left(\sqrt{\frac{(be-af)\left(h+\frac{bg}{a+bx}-\frac{ah}{a+bx}\right)}{b(-fg+eh)}}\right), \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{(bc-ad)(bg-ah)} \right. \\
& \left. \left. d \operatorname{EllipticF}[\operatorname{ArcSin}\left(\sqrt{\frac{(be-af)\left(h+\frac{bg}{a+bx}-\frac{ah}{a+bx}\right)}{b(-fg+eh)}}\right), \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+bx}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+bx}\right) \left(f + \frac{be-af}{a+bx}\right) \left(h + \frac{bg-ah}{a+bx}\right)} \right) + \left( 2b^3 B c e h \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+bx}\right)}{bdg-bch}} \right. \\
& \left( -\frac{f}{-be+af} + \frac{1}{a+bx} \right) \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+bx}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \left( -\frac{(bdg-bch) \operatorname{EllipticE}[\operatorname{ArcSin}\left(\sqrt{\frac{(be-af)\left(h+\frac{bg}{a+bx}-\frac{ah}{a+bx}\right)}{b(-fg+eh)}}\right), \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{(bc-ad)(bg-ah)} \right. \\
& \left. \left. d \operatorname{EllipticF}[\operatorname{ArcSin}\left(\sqrt{\frac{(be-af)\left(h+\frac{bg}{a+bx}-\frac{ah}{a+bx}\right)}{b(-fg+eh)}}\right), \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+bx}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+bx}\right) \left(f + \frac{be-af}{a+bx}\right) \left(h + \frac{bg-ah}{a+bx}\right)} \right) - \left( a b^2 c C e h \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+bx}\right)}{bdg-bch}} \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{f}{-be+af} + \frac{1}{a+bx} \right) \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+bx}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \left( -\frac{(bdg-bch) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{(bc-ad)(bg-ah)} \right. \\
& \left. \left. - \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{-bc+ad} \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+bx}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+bx}\right) \left(f + \frac{be-af}{a+bx}\right) \left(h + \frac{bg-ah}{a+bx}\right)} - \left( 4ab^2Bdeh \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+bx}\right)}{bdg-bch}} \right. \right. \\
& \left. \left. - \frac{(bdg-bch) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{(bc-ad)(bg-ah)} \right. \right. \\
& \left. \left. - \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{-bc+ad} \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+bx}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+bx}\right) \left(f + \frac{be-af}{a+bx}\right) \left(h + \frac{bg-ah}{a+bx}\right)} + \left( 5a^2bCdeh \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+bx}\right)}{bdg-bch}} \right. \right. \\
& \left. \left. - \frac{(bdg-bch) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{(bc-ad)(bg-ah)} \right. \right. 
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) (-f g+e h)}{(-b e+a f) (-d g+c h)}}}{-b c+a d}\right\} / \\
& \left( \sqrt{\frac{-\frac{f}{-b e+a f}+\frac{1}{a+b x}}{-\frac{f}{-b e+a f}+\frac{h}{-b g+a h}}} \sqrt{\left(d+\frac{b c-a d}{a+b x}\right)\left(f+\frac{b e-a f}{a+b x}\right)\left(h+\frac{b g-a h}{a+b x}\right)} - \left(4 a b^2 B c f h \sqrt{\frac{(b c-a d) (b g-a h) \left(-\frac{d}{-b c+a d}+\frac{1}{a+b x}\right)}{b d g-b c h}} \right. \right. \\
& \left. \left. \left(-\frac{f}{-b e+a f}+\frac{1}{a+b x}\right) \sqrt{\frac{-\frac{h}{-b g+a h}+\frac{1}{a+b x}}{\frac{f}{-b e+a f}-\frac{h}{-b g+a h}}} \left(-\frac{(b d g-b c h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) (-f g+e h)}{(-b e+a f) (-d g+c h)}}}{(b c-a d) (b g-a h)} - \right. \right. \right. \\
& \left. \left. \left.d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) (-f g+e h)}{(-b e+a f) (-d g+c h)}}}{-b c+a d}\right)\right\} / \\
& \left( \sqrt{\frac{-\frac{f}{-b e+a f}+\frac{1}{a+b x}}{-\frac{f}{-b e+a f}+\frac{h}{-b g+a h}}} \sqrt{\left(d+\frac{b c-a d}{a+b x}\right)\left(f+\frac{b e-a f}{a+b x}\right)\left(h+\frac{b g-a h}{a+b x}\right)} + \left(5 a^2 b c c f h \sqrt{\frac{(b c-a d) (b g-a h) \left(-\frac{d}{-b c+a d}+\frac{1}{a+b x}\right)}{b d g-b c h}} \right. \right. \\
& \left. \left. \left(-\frac{f}{-b e+a f}+\frac{1}{a+b x}\right) \sqrt{\frac{-\frac{h}{-b g+a h}+\frac{1}{a+b x}}{\frac{f}{-b e+a f}-\frac{h}{-b g+a h}}} \left(-\frac{(b d g-b c h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) (-f g+e h)}{(-b e+a f) (-d g+c h)}}}{(b c-a d) (b g-a h)} - \right. \right. \right. \\
& \left. \left. \left.d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) (-f g+e h)}{(-b e+a f) (-d g+c h)}}}{-b c+a d}\right)\right\}
\end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) + \left( 6 a^2 b B d f h \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+b x}\right)}{bdg-bch}} \right. \\
& \left( -\frac{f}{-be+af} + \frac{1}{a+b x} \right) \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+b x}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \left( -\frac{(bdg-bch) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)\left(h+\frac{bg}{a+b x}-\frac{ah}{a+b x}\right)}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{(bc-ad)(bg-ah)} \right. \\
& \left. \left. \left. d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)\left(h+\frac{bg}{a+b x}-\frac{ah}{a+b x}\right)}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] \right) \right) / \right. \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) - \left( 9 a^3 C d f h \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+b x}\right)}{bdg-bch}} \right. \\
& \left( -\frac{f}{-be+af} + \frac{1}{a+b x} \right) \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+b x}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \left( -\frac{(bdg-bch) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)\left(h+\frac{bg}{a+b x}-\frac{ah}{a+b x}\right)}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{(bc-ad)(bg-ah)} \right. \\
& \left. \left. \left. d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)\left(h+\frac{bg}{a+b x}-\frac{ah}{a+b x}\right)}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] \right) \right) / \right. \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) + \left( b^2 B d f g \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+b x}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{h}{-bg + ah} + \frac{1}{a + bx} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-be + af) \left( -h - \frac{bg}{a+bx} + \frac{ah}{a+bx} \right)}{b (-fg + eh)}} \right], \frac{(-bc + ad) (-fg + eh)}{(-be + af) (-dg + ch)} \right] \Bigg] / \\
& \left( \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+bx}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \sqrt{\left( d + \frac{bc - ad}{a + bx} \right) \left( f + \frac{be - af}{a + bx} \right) \left( h + \frac{bg - ah}{a + bx} \right)} \right) - \left( 2abcdfg \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+bx}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+bx}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \right. \\
& \left. \left( -\frac{h}{-bg + ah} + \frac{1}{a + bx} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-be + af) \left( -h - \frac{bg}{a+bx} + \frac{ah}{a+bx} \right)}{b (-fg + eh)}} \right], \frac{(-bc + ad) (-fg + eh)}{(-be + af) (-dg + ch)} \right] \right] / \\
& \left( \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+bx}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \sqrt{\left( d + \frac{bc - ad}{a + bx} \right) \left( f + \frac{be - af}{a + bx} \right) \left( h + \frac{bg - ah}{a + bx} \right)} \right) + \left( b^2 B d e h \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+bx}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+bx}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \right. \\
& \left. \left( -\frac{h}{-bg + ah} + \frac{1}{a + bx} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-be + af) \left( -h - \frac{bg}{a+bx} + \frac{ah}{a+bx} \right)}{b (-fg + eh)}} \right], \frac{(-bc + ad) (-fg + eh)}{(-be + af) (-dg + ch)} \right] \right] / \\
& \left( \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+bx}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \sqrt{\left( d + \frac{bc - ad}{a + bx} \right) \left( f + \frac{be - af}{a + bx} \right) \left( h + \frac{bg - ah}{a + bx} \right)} \right) - \left( 2abcdeh \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+bx}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+bx}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \right. \\
& \left. \left( -\frac{h}{-bg + ah} + \frac{1}{a + bx} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-be + af) \left( -h - \frac{bg}{a+bx} + \frac{ah}{a+bx} \right)}{b (-fg + eh)}} \right], \frac{(-bc + ad) (-fg + eh)}{(-be + af) (-dg + ch)} \right] \right] / \\
& \left( \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+bx}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \sqrt{\left( d + \frac{bc - ad}{a + bx} \right) \left( f + \frac{be - af}{a + bx} \right) \left( h + \frac{bg - ah}{a + bx} \right)} \right) + \left( b^2 B c f h \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+bx}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+bx}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \right. \\
& \left. \left( -\frac{h}{-bg + ah} + \frac{1}{a + bx} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-be + af) \left( -h - \frac{bg}{a+bx} + \frac{ah}{a+bx} \right)}{b (-fg + eh)}} \right], \frac{(-bc + ad) (-fg + eh)}{(-be + af) (-dg + ch)} \right] \right]
\end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{\frac{-\frac{h}{bg+ah} + \frac{1}{a+b x}}{\frac{f}{-be+af} - \frac{h}{bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) - \left( 2abcCfh \sqrt{\frac{-\frac{d}{bc+ad} + \frac{1}{a+b x}}{-\frac{d}{bc+ad} + \frac{h}{bg+ah}}} \sqrt{\frac{-\frac{f}{be+af} + \frac{1}{a+b x}}{-\frac{f}{be+af} + \frac{h}{bg+ah}}} \right. \\
& \left. \left( -\frac{h}{bg+ah} + \frac{1}{a+b x} \right) \text{EllipticF}[\text{ArcSin}\left[ \sqrt{\frac{(-be+af)(-h - \frac{bg}{a+b x} + \frac{ah}{a+b x})}{b(-fg+eh)}} \right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] \right) / \\
& \left( \sqrt{\frac{-\frac{h}{bg+ah} + \frac{1}{a+b x}}{\frac{f}{-be+af} - \frac{h}{bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) - \left( 3abBdfh \sqrt{\frac{-\frac{d}{bc+ad} + \frac{1}{a+b x}}{-\frac{d}{bc+ad} + \frac{h}{bg+ah}}} \sqrt{\frac{-\frac{f}{be+af} + \frac{1}{a+b x}}{-\frac{f}{be+af} + \frac{h}{bg+ah}}} \right. \\
& \left. \left( -\frac{h}{bg+ah} + \frac{1}{a+b x} \right) \text{EllipticF}[\text{ArcSin}\left[ \sqrt{\frac{(-be+af)(-h - \frac{bg}{a+b x} + \frac{ah}{a+b x})}{b(-fg+eh)}} \right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] \right) / \\
& \left( \sqrt{\frac{-\frac{h}{bg+ah} + \frac{1}{a+b x}}{\frac{f}{-be+af} - \frac{h}{bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) + \left( 6a^2Cdfh \sqrt{\frac{-\frac{d}{bc+ad} + \frac{1}{a+b x}}{-\frac{d}{bc+ad} + \frac{h}{bg+ah}}} \sqrt{\frac{-\frac{f}{be+af} + \frac{1}{a+b x}}{-\frac{f}{be+af} + \frac{h}{bg+ah}}} \right. \\
& \left. \left( -\frac{h}{bg+ah} + \frac{1}{a+b x} \right) \text{EllipticF}[\text{ArcSin}\left[ \sqrt{\frac{(-be+af)(-h - \frac{bg}{a+b x} + \frac{ah}{a+b x})}{b(-fg+eh)}} \right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] \right) / \\
& \left( \sqrt{\frac{-\frac{h}{bg+ah} + \frac{1}{a+b x}}{\frac{f}{-be+af} - \frac{h}{bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right)
\end{aligned}$$

**Problem 26:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(a+b x)^2 (A+C x^2)}{\sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x}} dx$$

Optimal (type 4, 1097 leaves, 9 steps):

$$\begin{aligned}
& \frac{1}{105 d^3 f^3 h^3} 2 (4 C (2 a d f h - 3 b (d f g + d e h + c f h)) (a d f h - 2 b (d f g + d e h + c f h)) + \\
& 5 b d f h (7 A b d f h - C (5 b (d e g + c f g + c e h) + 2 a (d f g + d e h + c f h)))) \sqrt{c + d x} \sqrt{e + f x} \sqrt{g + h x} + \\
& \frac{4 C (2 a d f h - 3 b (d f g + d e h + c f h)) (a + b x) \sqrt{c + d x} \sqrt{e + f x} \sqrt{g + h x}}{35 d^2 f^2 h^2} + \frac{2 C (a + b x)^2 \sqrt{c + d x} \sqrt{e + f x} \sqrt{g + h x}}{7 d f h} - \\
& \frac{1}{105 d^4 f^{7/2} h^4 \sqrt{e + f x} \sqrt{\frac{d (g + h x)}{d g - c h}}} 4 \sqrt{-d e + c f} (35 a^2 C d^2 f^2 h^2 (d f g + d e h + c f h)) - \\
& 7 a b d f h (15 A d^2 f^2 h^2 + C (8 c^2 f^2 h^2 + 7 c d f h (f g + e h) + d^2 (8 f^2 g^2 + 7 e f g h + 8 e^2 h^2))) + b^2 (35 A d^2 f^2 h^2 (d f g + d e h + c f h) + \\
& 2 C (12 c^3 f^3 h^3 + 10 c^2 d f^2 h^2 (f g + e h) + c d^2 f h (10 f^2 g^2 + 9 e f g h + 10 e^2 h^2)) + 2 d^3 (6 f^3 g^3 + 5 e f^2 g^2 h + 5 e^2 f g h^2 + 6 e^3 h^3))) \\
& \sqrt{\frac{d (e + f x)}{d e - c f}} \sqrt{g + h x} \text{EllipticE}[\text{ArcSin}\left[\frac{\sqrt{f} \sqrt{c + d x}}{\sqrt{-d e + c f}}\right], \frac{(d e - c f) h}{f (d g - c h)}] + \frac{1}{105 d^4 f^{7/2} h^4 \sqrt{e + f x} \sqrt{g + h x}} \\
& 2 \sqrt{-d e + c f} (35 a^2 d^2 f^2 h^2 (3 A d f h^2 + C (c h (f g - e h) + d g (2 f g + e h))) - \\
& 14 a b d f h (15 A d^2 f^2 g h^2 + C (4 c^2 f h^2 (f g - e h) + c d h (3 f^2 g^2 + e f g h - 4 e^2 h^2)) + d^2 g (8 f^2 g^2 + 3 e f g h + 4 e^2 h^2)) + \\
& b^2 (35 A d^2 f^2 h^2 (c h (f g - e h) + d g (2 f g + e h)) + C (24 c^3 f^2 h^3 (f g - e h) + c^2 d f h^2 (17 f^2 g^2 + 6 e f g h - 23 e^2 h^2) + \\
& 2 c d^2 h (8 f^3 g^3 + e f^2 g^2 h + 3 e^2 f g h^2 - 12 e^3 h^3)) + d^3 g (48 f^3 g^3 + 16 e f^2 g^2 h + 17 e^2 f g h^2 + 24 e^3 h^3))) \\
& \sqrt{\frac{d (e + f x)}{d e - c f}} \sqrt{\frac{d (g + h x)}{d g - c h}} \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{f} \sqrt{c + d x}}{\sqrt{-d e + c f}}\right], \frac{(d e - c f) h}{f (d g - c h)}]
\end{aligned}$$

Result (type 4, 18383 leaves):

$$\begin{aligned}
& \sqrt{c + d x} \sqrt{e + f x} \sqrt{g + h x} \\
& \left( \frac{1}{105 d^3 f^3 h^3} 2 (24 b^2 C d^2 f^2 g^2 + 23 b^2 C d^2 e f g h + 23 b^2 c C d f^2 g h - 56 a b C d^2 f^2 g h + 24 b^2 C d^2 e^2 h^2 + 23 b^2 c C d e f h^2 - 56 a b C d^2 e f h^2 + \right. \\
& 24 b^2 c^2 C f^2 h^2 - 56 a b c C d f^2 h^2 + 35 A b^2 d^2 f^2 h^2 + 35 a^2 C d^2 f^2 h^2) - \frac{4 b C (3 b d f g + 3 b d e h + 3 b c f h - 7 a d f h) x}{35 d^2 f^2 h^2} + \frac{2 b^2 C x^2}{7 d f h} \Big) + \frac{1}{105 d^5 f^3 h^3} \\
& \left( \frac{1}{f h \sqrt{e + \frac{(c+d x)(f - \frac{c f}{c+d x})}{d}}} \sqrt{g + \frac{(c+d x)(h - \frac{c h}{c+d x})}{d}} 4 (-24 b^2 C d^3 f^3 g^3 - 20 b^2 C d^3 e f^2 g^2 h - 20 b^2 c C d^2 f^3 g^2 h + 56 a b C d^3 f^3 g^2 h - 20 b^2 C d^3 e^2 f g h^2 - \right. \\
& 18 b^2 c C d^2 e f^2 g h^2 + 49 a b C d^3 e f^2 g h^2 - 20 b^2 c^2 C d f^3 g h^2 + 49 a b c C d^2 f^3 g h^2 - 35 A b^2 d^3 f^3 g h^2 - 35 a^2 C d^3 f^3 g h^2 - 24 b^2 C d^3 e^3 h^3 - \\
& 20 b^2 c C d^2 e^2 f h^3 + 56 a b C d^3 e^2 f h^3 - 20 b^2 c^2 C d e f^2 h^3 + 49 a b c C d^2 e f^2 h^3 - 35 A b^2 d^3 e f^2 h^3 - 35 a^2 C d^3 e f^2 h^3 - 24 b^2 c^3 C f^3 h^3 + \\
& 56 a b c^2 C d f^3 h^3 - 35 A b^2 c d^2 f^3 h^3 - 35 a^2 c C d^2 f^3 h^3 + 105 a A b d^3 f^3 h^3) (c + d x)^{3/2} \left( f + \frac{d e}{c + d x} - \frac{c f}{c + d x} \right) \left( h + \frac{d g}{c + d x} - \frac{c h}{c + d x} \right) +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{f h \sqrt{e + \frac{(c+d x) \left(f - \frac{c f}{c+d x}\right)}{d}} \sqrt{g + \frac{(c+d x) \left(h - \frac{c h}{c+d x}\right)}{d}}} 2 (c+d x) \sqrt{\left(f + \frac{d e}{c+d x} - \frac{c f}{c+d x}\right) \left(h + \frac{d g}{c+d x} - \frac{c h}{c+d x}\right)} \left(48 i b^2 C d^5 e f^3 g^4 h \sqrt{1 - \frac{-d e + c f}{f (c+d x)}}\right. \\
& \left. \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \left(\text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\frac{-d e+c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g+c h)}{(-d e+c f) h}\right] - \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\frac{-d e+c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g+c h)}{(-d e+c f) h}\right]\right)\right) / \\
& \left(\sqrt{-\frac{-d e+c f}{f}} (-d g+c h) \sqrt{\left(f + \frac{d e-c f}{c+d x}\right) \left(h + \frac{d g-c h}{c+d x}\right)} - 48 i b^2 c C d^4 f^4 g^4 h \sqrt{1 - \frac{-d e+c f}{f (c+d x)}} \sqrt{1 - \frac{-d g+c h}{h (c+d x)}}\right. \\
& \left. \left(\text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\frac{-d e+c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g+c h)}{(-d e+c f) h}\right] - \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\frac{-d e+c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g+c h)}{(-d e+c f) h}\right]\right)\right) / \\
& \left(\sqrt{-\frac{-d e+c f}{f}} (-d g+c h) \sqrt{\left(f + \frac{d e-c f}{c+d x}\right) \left(h + \frac{d g-c h}{c+d x}\right)} + 40 i b^2 C d^5 e^2 f^2 g^3 h^2 \sqrt{1 - \frac{-d e+c f}{f (c+d x)}} \sqrt{1 - \frac{-d g+c h}{h (c+d x)}}\right. \\
& \left. \left(\text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\frac{-d e+c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g+c h)}{(-d e+c f) h}\right] - \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\frac{-d e+c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g+c h)}{(-d e+c f) h}\right]\right)\right) / \\
& \left(\sqrt{-\frac{-d e+c f}{f}} (-d g+c h) \sqrt{\left(f + \frac{d e-c f}{c+d x}\right) \left(h + \frac{d g-c h}{c+d x}\right)} - 48 i b^2 c C d^4 e f^3 g^3 h^2 \sqrt{1 - \frac{-d e+c f}{f (c+d x)}} \sqrt{1 - \frac{-d g+c h}{h (c+d x)}}\right)
\end{aligned}$$

$$\left( \text{EllipticE} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) - \left( 112 \text{i a b C d}^5 e f^3 g^3 h^2 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right.$$

$$\left( \text{EllipticE} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) + \left( 8 \text{i b}^2 c^2 C d^3 f^4 g^3 h^2 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) + \left( 112 \text{i a b c C d}^4 f^4 g^3 h^2 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) + \left( 40 \pm b^2 C d^5 e^3 f g^2 h^3 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) - \left( 44 \frac{i b^2 c C d^4 e^2 f^2 g^2 h^3}{\sqrt{1 - \frac{d e + c f}{f (c + d x)}}} \sqrt{1 - \frac{d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) - \left( 98 i a b C d^5 e^2 f^2 g^2 h^3 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) + \left( 4 \pm b^2 c^2 C d^3 e f^3 g^2 h^3 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) + \left( 112 \text{i} a b c C d^4 e f^3 g^2 h^3 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right.$$

$$\left( \text{EllipticE}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) + \left( 70 \text{i} A b^2 d^5 e f^3 g^2 h^3 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right.$$

$$\left( \text{EllipticE}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) + \left( 70 \text{i} a^2 C d^5 e f^3 g^2 h^3 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF}\left[ \text{i} \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) - \left( 14 \pm a b c^2 C d^3 f^4 g^2 h^3 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) - \left( 70 \pm A b^2 c C d^4 f^4 g^2 h^3 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) - \left( 70 \pm a^2 c C d^4 f^4 g^2 h^3 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) + \left( 48 \pm b^2 C d^5 e^4 g h^4 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) - \left( 48 \pm b^2 c C d^4 e^3 f g h^4 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right.$$

$$\left. \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) - \left( 112 \pm a b C d^5 e^3 f g h^4 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right.$$

$$\left. \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) + \left( 4 \pm b^2 c^2 C d^3 e^2 f^2 g h^4 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right.$$

$$\left. \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) + \left( 112 \text{I} a b c C d^4 e^2 f^2 g h^4 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right.$$

$$\left. \left( \text{EllipticE} \left[ \text{I} \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{I} \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) + \left( 70 \text{I} A b^2 d^5 e^2 f^2 g h^4 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right.$$

$$\left. \left( \text{EllipticE} \left[ \text{I} \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{I} \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) + \left( 70 \text{I} a^2 C d^5 e^2 f^2 g h^4 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left. \left( \text{EllipticE} \left[ \text{I} \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{I} \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) + \left( 4 \text{I} b^2 c^3 C d^2 e f^3 g h^4 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) - \left( 14 \text{i a b c}^2 \text{C d}^3 \text{e f}^3 \text{g h}^4 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right.$$

$$\left( \text{EllipticE} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) - \left( 70 \text{i A b}^2 \text{c d}^4 \text{e f}^3 \text{g h}^4 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) - \left( 70 \text{i a}^2 \text{c C d}^4 \text{e f}^3 \text{g h}^4 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) - \left( 210 \text{Integrate } a A b d^5 e f^3 g h^4 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) - \left( 8 \text{Integrate } b^2 c^4 C d f^4 g h^4 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) + \left( 14 \text{Integrate } a b c^3 C d^2 f^4 g h^4 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) + \left( 210 \text{Integrate } a A b c d^4 f^4 g h^4 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) - \left( 48 i b^2 c C d^4 e^4 h^5 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right.$$

$$\left( \text{EllipticE} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) + \left( 8 i b^2 c^2 C d^3 e^3 f h^5 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) + \left( 112 i a b c C d^4 e^3 f h^5 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) - \left( 14 \pm a b c^2 C d^3 e^2 f^2 h^5 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) - \left( 70 \pm A b^2 c C d^4 e^2 f^2 h^5 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) - \left( 70 \pm a^2 c C d^4 e^2 f^2 h^5 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) - \left( 8 \pm b^2 c^4 C d e f^3 h^5 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) + \left( 14 \pm a b c^3 C d^2 e f^3 h^5 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right.$$

$$\left. \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) + \left( 210 \pm a A b c d^4 e f^3 h^5 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right.$$

$$\left. \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) + \left( 48 \pm b^2 c^5 C f^4 h^5 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right.$$

$$\left. \text{EllipticE}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF}\left[ \pm \text{ArcSinh}\left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) - \left( 112 \pm a b c^4 C d f^4 h^5 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) + \left( 70 \pm A b^2 c^3 d^2 f^4 h^5 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) + \left( 70 \pm a^2 c^3 C d^2 f^4 h^5 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) - \left( 210 \pm a A b c^2 d^3 f^4 h^5 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\begin{aligned}
& \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) / \\
& \left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} - \right. \\
& \left. \frac{24 \pm b^2 C d^4 e f^3 g^3 h \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right]}{\sqrt{-\frac{-d e + c f}{f}} \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)}} + \right. \\
& \left. \frac{24 \pm b^2 c C d^3 f^4 g^3 h \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right]}{\sqrt{-\frac{-d e + c f}{f}} \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)}} - \right. \\
& \left. \frac{23 \pm b^2 C d^4 e^2 f^2 g^2 h^2 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right]}{\sqrt{-\frac{-d e + c f}{f}} \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)}} + \right. \\
& \left. \frac{6 \pm b^2 c C d^3 e f^3 g^2 h^2 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right]}{\sqrt{-\frac{-d e + c f}{f}} \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)}} + \right. \\
& \left. \frac{56 \pm a b C d^4 e f^3 g^2 h^2 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right]}{\sqrt{-\frac{-d e + c f}{f}} \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)}} + \right.
\end{aligned}$$

$$\frac{17 \pm b^2 c^2 C d^2 f^4 g^2 h^2 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}] \sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}}}{\sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}}$$

$$\frac{56 \pm a b c C d^3 f^4 g^2 h^2 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}] \sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}}}{\sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}}$$

$$\frac{24 \pm b^2 C d^4 e^3 f g h^3 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}] \sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}}}{\sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}}$$

$$\frac{6 \pm b^2 c C d^3 e^2 f^2 g h^3 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}] \sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}}}{\sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}}$$

$$\frac{56 \pm a b c C d^4 e^2 f^2 g h^3 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}] \sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}}}{\sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}}$$

$$\frac{2 \pm b^2 c^2 C d^2 e f^3 g h^3 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}] \sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}}}{\sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}}$$

$$\frac{14 \mathfrak{i} a b c C d^3 e f^3 g h^3 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[\mathfrak{i} \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}\right]}{\sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}}$$

$$\frac{35 \mathfrak{i} A b^2 d^4 e f^3 g h^3 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[\mathfrak{i} \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}\right]}{\sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}}$$

$$\frac{35 \mathfrak{i} a^2 C d^4 e f^3 g h^3 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[\mathfrak{i} \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}\right]}{\sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}} +$$

$$\frac{16 \mathfrak{i} b^2 c^3 C d f^4 g h^3 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[\mathfrak{i} \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}\right]}{\sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}}$$

$$\frac{42 \mathfrak{i} a b c^2 C d^2 f^4 g h^3 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[\mathfrak{i} \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}\right]}{\sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}} +$$

$$\frac{35 \mathfrak{i} A b^2 c d^3 f^4 g h^3 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[\mathfrak{i} \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}\right]}{\sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}} +$$

$$\frac{35 \pm a^2 c C d^3 f^4 g h^3 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}] + \sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}}}{}$$

$$\frac{24 \pm b^2 c C d^3 e^3 f h^4 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}] + \sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}}}{}$$

$$\frac{17 \pm b^2 c^2 C d^2 e^2 f^2 h^4 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}] - \sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}}}{}$$

$$\frac{56 \pm a b c C d^3 e^2 f^2 h^4 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}] + \sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}}}{}$$

$$\frac{16 \pm b^2 c^3 C d e f^3 h^4 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}] - \sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}}}{}$$

$$\frac{42 \pm a b c^2 C d^2 e f^3 h^4 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}] + \sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}}}{}$$

$$\frac{35 \pm A b^2 c d^3 e f^3 h^4 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}] + \sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}}}{}$$

$$\frac{35 \pm a^2 c C d^3 e f^3 h^4 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}] + \sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}}}{}$$

$$\frac{48 \pm b^2 c^4 C f^4 h^4 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}] - \sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}}}{}$$

$$\frac{112 \pm a b c^3 C d f^4 h^4 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}] + \sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}}}{}$$

$$\frac{70 \pm A b^2 c^2 d^2 f^4 h^4 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}] + \sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}}}{}$$

$$\frac{70 \pm a^2 c^2 C d^2 f^4 h^4 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}] - \sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}}}{}$$

$$\begin{aligned}
& \frac{210 \pm a A b c d^3 f^4 h^4 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}] }{+} \\
& \sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)} \\
& \left. \frac{105 \pm a^2 A d^4 f^4 h^4 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}]}{\sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}}\right)
\end{aligned}$$

**Problem 27: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + b x) (A + C x^2)}{\sqrt{c + d x} \sqrt{e + f x} \sqrt{g + h x}} dx$$

Optimal (type 4, 611 leaves, 8 steps):

$$\begin{aligned}
& \frac{4 C (a d f h - 2 b (d f g + d e h + c f h)) \sqrt{c + d x} \sqrt{e + f x} \sqrt{g + h x}}{15 d^2 f^2 h^2} + \frac{2 C (a + b x) \sqrt{c + d x} \sqrt{e + f x} \sqrt{g + h x}}{5 d f h} - \\
& \left( 2 \sqrt{-d e + c f} (10 a C d f h (d f g + d e h + c f h) - b (15 A d^2 f^2 h^2 + C (8 c^2 f^2 h^2 + 7 c d f h (f g + e h) + d^2 (8 f^2 g^2 + 7 e f g h + 8 e^2 h^2)))) \right. \\
& \left. \sqrt{\frac{d (e + f x)}{d e - c f}} \sqrt{g + h x} \operatorname{EllipticE}[\operatorname{ArcSin}\left[\frac{\sqrt{f} \sqrt{c + d x}}{\sqrt{-d e + c f}}\right], \frac{(d e - c f) h}{f (d g - c h)}] \right) / \left( 15 d^3 f^{5/2} h^3 \sqrt{e + f x} \sqrt{\frac{d (g + h x)}{d g - c h}} \right) + \\
& \left( 2 \sqrt{-d e + c f} (5 a d f h (3 A d f h^2 + C (c h (f g - e h) + d g (2 f g + e h))) - \right. \\
& \left. b (15 A d^2 f^2 g h^2 + C (4 c^2 f h^2 (f g - e h) + c d h (3 f^2 g^2 + e f g h - 4 e^2 h^2) + d^2 g (8 f^2 g^2 + 3 e f g h + 4 e^2 h^2))) \right) \\
& \left. \sqrt{\frac{d (e + f x)}{d e - c f}} \sqrt{\frac{d (g + h x)}{d g - c h}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{\sqrt{f} \sqrt{c + d x}}{\sqrt{-d e + c f}}\right], \frac{(d e - c f) h}{f (d g - c h)}] \right) / \left( 15 d^3 f^{5/2} h^3 \sqrt{e + f x} \sqrt{g + h x} \right)
\end{aligned}$$

Result (type 4, 8828 leaves):

$$\begin{aligned}
& \sqrt{c + d x} \sqrt{e + f x} \left( \frac{2 c (-4 b d f g - 4 b d e h - 4 b c f h + 5 a d f h)}{15 d^2 f^2 h^2} + \frac{2 b c x}{5 d f h} \right) \sqrt{g + h x} - \frac{1}{15 d^4 f^2 h^2} \\
& 2 \left( \left( -8 b c d^2 f^2 g^2 - 7 b c d^2 e f g h - 7 b c C d f^2 g h + 10 a C d^2 f^2 g h - 8 b C d^2 e^2 h^2 - 7 b c C d e f h^2 + 10 a C d^2 e f h^2 - 8 b c^2 C f^2 h^2 + 10 a c C d f^2 h^2 - \right. \right. \\
& \quad \left. \left. 15 A b d^2 f^2 h^2 \right) (c + d x)^{3/2} \left( f + \frac{d e}{c + d x} - \frac{c f}{c + d x} \right) \left( h + \frac{d g}{c + d x} - \frac{c h}{c + d x} \right) \right) / \left( f h \sqrt{e + \frac{(c + d x) \left( f - \frac{c f}{c + d x} \right)}{d}} \sqrt{g + \frac{(c + d x) \left( h - \frac{c h}{c + d x} \right)}{d}} \right) + \\
& \frac{1}{f h \sqrt{e + \frac{(c + d x) \left( f - \frac{c f}{c + d x} \right)}{d}} \sqrt{g + \frac{(c + d x) \left( h - \frac{c h}{c + d x} \right)}{d}}} (c + d x) \sqrt{\left( f + \frac{d e}{c + d x} - \frac{c f}{c + d x} \right) \left( h + \frac{d g}{c + d x} - \frac{c h}{c + d x} \right)} \left( \left( 8 \pm b C d^4 e f^2 g^3 h \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \right. \right. \\
& \quad \left. \left. \text{EllipticE}[\pm \text{ArcSinh}[\frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}}, \frac{f (-d g + c h)}{(-d e + c f) h}], \frac{f (-d g + c h)}{(-d e + c f) h}] - \text{EllipticF}[\pm \text{ArcSinh}[\frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}}, \frac{f (-d g + c h)}{(-d e + c f) h}] \right) \right) / \\
& \left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} - \left( 8 \pm b c C d^3 f^3 g^3 h \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right. \right. \\
& \quad \left. \left. \text{EllipticE}[\pm \text{ArcSinh}[\frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}}, \frac{f (-d g + c h)}{(-d e + c f) h}], \frac{f (-d g + c h)}{(-d e + c f) h}] - \text{EllipticF}[\pm \text{ArcSinh}[\frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}}, \frac{f (-d g + c h)}{(-d e + c f) h}] \right) \right) /
\end{aligned}$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) + \left( 7 \frac{i b c d^4 e^2 f g^2 h^2}{\sqrt{1 - \frac{-d e + c f}{f (c + d x)}}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE}\left[\frac{i \text{ArcSinh}\left[\frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}\right] - \text{EllipticF}\left[\frac{i \text{ArcSinh}\left[\frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}\right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) - \left( 8 \frac{i b c C d^3 e f^2 g^2 h^2}{\sqrt{1 - \frac{-d e + c f}{f (c + d x)}}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE}\left[\frac{i \text{ArcSinh}\left[\frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}\right] - \text{EllipticF}\left[\frac{i \text{ArcSinh}\left[\frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}\right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) - \left( 10 \frac{i a c d^4 e f^2 g^2 h^2}{\sqrt{1 - \frac{-d e + c f}{f (c + d x)}}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE}\left[\frac{i \text{ArcSinh}\left[\frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}\right] - \text{EllipticF}\left[\frac{i \text{ArcSinh}\left[\frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}\right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) + \left( \frac{i b c^2 C d^2 f^3 g^2 h^2}{\sqrt{1 - \frac{-d e + c f}{f (c + d x)}}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) + \left( 10 \text{i} a c C d^3 f^3 g^2 h^2 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right.$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) + \left( 8 \text{i} b C d^4 e^3 g h^3 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) - \left( 8 \text{i} b c C d^3 e^2 f g h^3 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) - \left( 10 \text{I} a C d^4 e^2 f g h^3 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{I} \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{I} \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) + \left( \text{I} b c^2 C d^2 e f^2 g h^3 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{I} \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{I} \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) + \left( 10 \text{I} a c C d^3 e f^2 g h^3 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{I} \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{I} \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) + \left( 15 \text{I} A b d^4 e f^2 g h^3 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) - \left( i b c^3 C d f^3 g h^3 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) - \left( 15 i A b c d^3 f^3 g h^3 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} \right) - \left( 8 i b c C d^3 e^3 h^4 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) + \left( \frac{i b c^2 C d^2 e^2 f h^4}{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) + \left( 10 i a c C d^3 e^2 f h^4 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) - \left( \frac{i b c^3 C d e f^2 h^4}{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) - \left( 15 i A b c d^3 e f^2 h^4 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right)$$

$$\left( \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} + \left( 8 i b c^4 C f^3 h^4 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right. \right.$$

$$\left. \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} - \left( 10 i a c^3 C d f^3 h^4 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right. \right.$$

$$\left. \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left( f + \frac{d e - c f}{c + d x} \right) \left( h + \frac{d g - c h}{c + d x} \right)} + \left( 15 i A b c^2 d^2 f^3 h^4 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \right. \right.$$

$$\left. \text{EllipticE} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] - \text{EllipticF} \left[ \text{i} \operatorname{ArcSinh} \left[ \frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}} \right], \frac{f (-d g + c h)}{(-d e + c f) h} \right] \right) /$$

$$\begin{aligned}
& \left( \sqrt{-\frac{-d e + c f}{f}} (-d g + c h) \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)} \right) - \\
& \frac{4 i b C d^3 e f^2 g^2 h \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \operatorname{EllipticF}\left[\text{i} \operatorname{ArcSinh}\left[\frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}\right]}{\sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)}} + \\
& \frac{4 i b c C d^2 f^3 g^2 h \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \operatorname{EllipticF}\left[\text{i} \operatorname{ArcSinh}\left[\frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}\right]}{\sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)}} - \\
& \frac{4 i b C d^3 e^2 f g h^2 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \operatorname{EllipticF}\left[\text{i} \operatorname{ArcSinh}\left[\frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}\right]}{\sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)}} + \\
& \frac{\text{i} b c C d^2 e f^2 g h^2 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \operatorname{EllipticF}\left[\text{i} \operatorname{ArcSinh}\left[\frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}\right]}{\sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)}} + \\
& \frac{5 i a C d^3 e f^2 g h^2 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \operatorname{EllipticF}\left[\text{i} \operatorname{ArcSinh}\left[\frac{\sqrt{-\frac{-d e + c f}{f}}}{\sqrt{c + d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}\right]}{\sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)}}
\end{aligned}$$

$$\frac{3 i b c^2 C d f^3 g h^2 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \operatorname{EllipticF}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c + d x}}, \frac{f (-d g + c h)}{(-d e + c f) h}\right]}{\sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)}}$$

$$\frac{5 i a c C d^2 f^3 g h^2 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \operatorname{EllipticF}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c + d x}}, \frac{f (-d g + c h)}{(-d e + c f) h}\right]}{\sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)}} +$$

$$\frac{4 i b c C d^2 e^2 f h^3 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \operatorname{EllipticF}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c + d x}}, \frac{f (-d g + c h)}{(-d e + c f) h}\right]}{\sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)}} +$$

$$\frac{3 i b c^2 C d e f^2 h^3 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \operatorname{EllipticF}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c + d x}}, \frac{f (-d g + c h)}{(-d e + c f) h}\right]}{\sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)}} -$$

$$\frac{5 i a c C d^2 e f^2 h^3 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \operatorname{EllipticF}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c + d x}}, \frac{f (-d g + c h)}{(-d e + c f) h}\right]}{\sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)}} +$$

$$\frac{8 i b c^3 C f^3 h^3 \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \operatorname{EllipticF}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c + d x}}, \frac{f (-d g + c h)}{(-d e + c f) h}\right]}{\sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c + d x}\right) \left(h + \frac{d g - c h}{c + d x}\right)}} -$$

$$\begin{aligned}
& \frac{10 \pm a c^2 C d f^3 h^3 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}\right]}{\sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}} + \\
& \frac{15 \pm A b c d^2 f^3 h^3 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}\right]}{\sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}} - \\
& \left. \frac{15 \pm a A d^3 f^3 h^3 \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d e + c f}{f}}}{\sqrt{c+d x}}\right], \frac{f (-d g + c h)}{(-d e + c f) h}\right]}{\sqrt{-\frac{-d e + c f}{f}} \sqrt{\left(f + \frac{d e - c f}{c+d x}\right) \left(h + \frac{d g - c h}{c+d x}\right)}} \right)
\end{aligned}$$

Problem 28: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A + C x^2}{\sqrt{c + d x} \sqrt{e + f x} \sqrt{g + h x}} dx$$

Optimal (type 4, 368 leaves, 7 steps):

$$\begin{aligned}
& \frac{2 C \sqrt{c + d x} \sqrt{e + f x} \sqrt{g + h x}}{3 d f h} - \frac{4 C \sqrt{-d e + c f} (d f g + d e h + c f h) \sqrt{\frac{d (e + f x)}{d e - c f}} \sqrt{g + h x} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{f} \sqrt{c+d x}}{\sqrt{-d e+c f}}\right], \frac{(d e-c f) h}{f (d g-c h)}\right]}{3 d^2 f^{3/2} h^2 \sqrt{e + f x} \sqrt{\frac{d (g+h x)}{d g-c h}}} + \\
& \left( 2 \sqrt{-d e + c f} (3 A d f h^2 + C (c h (f g - e h) + d g (2 f g + e h))) \sqrt{\frac{d (e + f x)}{d e - c f}} \sqrt{\frac{d (g + h x)}{d g - c h}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{f} \sqrt{c+d x}}{\sqrt{-d e+c f}}\right], \frac{(d e-c f) h}{f (d g-c h)}\right] \right) /
\end{aligned}$$

Result (type 4, 390 leaves):

$$\begin{aligned}
& \left( \sqrt{c + d x} \right. \\
& \left. \left( 2 C d^2 f h (e + f x) (g + h x) - \frac{4 C d^2 (d f g + d e h + c f h) (e + f x) (g + h x)}{c + d x} - 4 \pm C \sqrt{-c + \frac{d e}{f}} f h (d f g + d e h + c f h) \sqrt{c + d x} \sqrt{\frac{d (e + f x)}{f (c + d x)}} \right. \right. \\
& \left. \left. \sqrt{\frac{d (g + h x)}{h (c + d x)}} \operatorname{EllipticE}[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{-c + \frac{d e}{f}}}{\sqrt{c + d x}}\right], \frac{d f g - c f h}{d e h - c f h}] + \frac{1}{\sqrt{-c + \frac{d e}{f}}} 2 \pm d h (3 A d f^2 h + c C f (-f g + e h) + C d e (f g + 2 e h)) \right. \right. \\
& \left. \left. \sqrt{c + d x} \sqrt{\frac{d (e + f x)}{f (c + d x)}} \sqrt{\frac{d (g + h x)}{h (c + d x)}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\frac{\sqrt{-c + \frac{d e}{f}}}{\sqrt{c + d x}}\right], \frac{d f g - c f h}{d e h - c f h}] \right) \right) \Bigg) \Bigg) / (3 d^3 f^2 h^2 \sqrt{e + f x} \sqrt{g + h x})
\end{aligned}$$

Problem 29: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{A + C x^2}{(a + b x) \sqrt{c + d x} \sqrt{e + f x} \sqrt{g + h x}} dx$$

Optimal (type 4, 465 leaves, 11 steps):

$$\frac{2 C \sqrt{-d e + c f} \sqrt{\frac{d (e+f x)}{d e - c f}} \sqrt{g+h x} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{f} \sqrt{c+d x}}{\sqrt{-d e+c f}}\right], \frac{(d e-c f) h}{f (d g-c h)}\right]}{b d \sqrt{f} h \sqrt{e+f x} \sqrt{\frac{d (g+h x)}{d g-c h}}}$$

$$\frac{2 C \sqrt{-d e + c f} (b g + a h) \sqrt{\frac{d (e+f x)}{d e - c f}} \sqrt{\frac{d (g+h x)}{d g-c h}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{f} \sqrt{c+d x}}{\sqrt{-d e+c f}}\right], \frac{(d e-c f) h}{f (d g-c h)}\right]}{b^2 d \sqrt{f} h \sqrt{e+f x} \sqrt{g+h x}}$$

$$\frac{2 \left(A + \frac{a^2 C}{b^2}\right) \sqrt{-d e + c f} \sqrt{\frac{d (e+f x)}{d e - c f}} \sqrt{\frac{d (g+h x)}{d g-c h}} \operatorname{EllipticPi}\left[-\frac{b (d e-c f)}{(b c-a d) f}, \operatorname{ArcSin}\left[\frac{\sqrt{f} \sqrt{c+d x}}{\sqrt{-d e+c f}}\right], \frac{(d e-c f) h}{f (d g-c h)}\right]}{(b c-a d) \sqrt{f} \sqrt{e+f x} \sqrt{g+h x}}$$

Result (type 4, 13075 leaves):

$$\begin{aligned} & -\frac{1}{d^2} 2 \left( -\frac{C (c+d x)^{3/2} \left(f + \frac{d e}{c+d x} - \frac{c f}{c+d x}\right) \left(h + \frac{d g}{c+d x} - \frac{c h}{c+d x}\right)}{b f h \sqrt{e + \frac{(c+d x) \left(f - \frac{c f}{c+d x}\right)}{d}} \sqrt{g + \frac{(c+d x) \left(h - \frac{c h}{c+d x}\right)}{d}}} + \left( c+d x \right) \left(b - \frac{b c}{c+d x} + \frac{a d}{c+d x}\right) \sqrt{f + \frac{d e}{c+d x} - \frac{c f}{c+d x}} \right. \\ & \quad \left. \sqrt{h + \frac{d g}{c+d x} - \frac{c h}{c+d x}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}} \right. \\ & \quad \left. - \frac{d (b C e g + a C f g - a C e h + A b f h)}{b f (b g - a h) \sqrt{f + \frac{d e}{c+d x} - \frac{c f}{c+d x}} \sqrt{h + \frac{d g}{c+d x} - \frac{c h}{c+d x}}} - \frac{a C d^3 e g \sqrt{h + \frac{d g}{c+d x} - \frac{c h}{c+d x}}}{b (b c - a d) f h (d g - c h) \sqrt{f + \frac{d e}{c+d x} - \frac{c f}{c+d x}}} - \right. \\ & \quad \left. \frac{c^3 C \sqrt{h + \frac{d g}{c+d x} - \frac{c h}{c+d x}}}{(b c - a d) (-d g + c h) \sqrt{f + \frac{d e}{c+d x} - \frac{c f}{c+d x}}} + \frac{a c^2 C d \sqrt{h + \frac{d g}{c+d x} - \frac{c h}{c+d x}}}{b (b c - a d) (-d g + c h) \sqrt{f + \frac{d e}{c+d x} - \frac{c f}{c+d x}}} + \frac{c^2 C d e \sqrt{h + \frac{d g}{c+d x} - \frac{c h}{c+d x}}}{(b c - a d) f (-d g + c h) \sqrt{f + \frac{d e}{c+d x} - \frac{c f}{c+d x}}} \right) \end{aligned}$$

$$\begin{aligned}
& \frac{a c C d^2 e \sqrt{h + \frac{d g}{c+d x} - \frac{c h}{c+d x}}}{b (b c - a d) f (-d g + c h) \sqrt{f + \frac{d e}{c+d x} - \frac{c f}{c+d x}}} + \frac{c^2 C d g \sqrt{h + \frac{d g}{c+d x} - \frac{c h}{c+d x}}}{(b c - a d) h (-d g + c h) \sqrt{f + \frac{d e}{c+d x} - \frac{c f}{c+d x}}} - \frac{a c C d^2 g \sqrt{h + \frac{d g}{c+d x} - \frac{c h}{c+d x}}}{b (b c - a d) h (-d g + c h) \sqrt{f + \frac{d e}{c+d x} - \frac{c f}{c+d x}}} - \\
& \left. \frac{c C d^2 e g \sqrt{h + \frac{d g}{c+d x} - \frac{c h}{c+d x}}}{(b c - a d) f h (-d g + c h) \sqrt{f + \frac{d e}{c+d x} - \frac{c f}{c+d x}}} + \frac{(A b^2 + a^2 C) d \sqrt{h + \frac{d g}{c+d x} - \frac{c h}{c+d x}}}{b (b g - a h) \left(b - \frac{b c}{c+d x} + \frac{a d}{c+d x}\right) \sqrt{f + \frac{d e}{c+d x} - \frac{c f}{c+d x}}} \right\} \left( \pm C d^2 e f g \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \right) \\
& \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c+d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c+d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] \right) / \\
& \left( (-d e + c f) \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}} \right) - \left( \pm c C d f^2 g \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \right. \\
& \left. \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c+d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c+d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] \right) \right) / \\
& \left( (-d e + c f) \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}} \right) - \left( \pm c C d e f h \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \right. \\
& \left. \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c+d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c+d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left( (-d e + c f) \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2} + \frac{d f g + d e h - 2 c f h}{c + d x}} \right) + \left( \pm c^2 C f^2 h \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \right. \\
& \left. \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \left( \text{EllipticE} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] - \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] \right) \right) / \\
& \left( (-d e + c f) \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2} + \frac{d f g + d e h - 2 c f h}{c + d x}} \right) - \\
& \frac{\pm b^2 c C d^2 e g \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right]}{(b c - a d)^2 \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2} + \frac{d f g + d e h - 2 c f h}{c + d x}}} + \\
& \frac{\pm a b C d^3 e g \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right]}{(b c - a d)^2 \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2} + \frac{d f g + d e h - 2 c f h}{c + d x}}} + \\
& \frac{\pm b C d^2 e g \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right]}{(b c - a d) \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2} + \frac{d f g + d e h - 2 c f h}{c + d x}}} + \\
& \frac{\pm b^2 c^2 C d f g \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \text{EllipticF} \left[ \pm \text{ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right]}{(b c - a d)^2 \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2} + \frac{d f g + d e h - 2 c f h}{c + d x}}}
\end{aligned}$$

$$\frac{\text{i } a b c C d^2 f g \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticF}[\text{i ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}]}{(b c - a d)^2 \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}} -$$

$$\frac{\text{i } b c C d f g \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticF}[\text{i ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}]}{(b c - a d) \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}} +$$

$$\frac{\text{i } b^2 c^2 C d e h \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticF}[\text{i ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}]}{(b c - a d)^2 \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}} -$$

$$\frac{\text{i } a b c C d^2 e h \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticF}[\text{i ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}]}{(b c - a d)^2 \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}} -$$

$$\frac{\text{i } b c C d e h \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticF}[\text{i ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}]}{(b c - a d) \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}} -$$

$$\frac{\text{i } b^2 c^3 C f h \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticF}[\text{i ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}]}{(b c - a d)^2 \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}} +$$

$$\frac{i a b c^2 C d f h \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{(b c - a d)^2 \sqrt{-\frac{d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}} +$$

$$\frac{2 i b c^2 C f h \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{(b c - a d) \sqrt{-\frac{d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}} +$$

$$\frac{i A b d^2 f h \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{(b c - a d) \sqrt{-\frac{d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}} + \frac{1}{(b c - a d)^3}$$

$$b^3 c C d^2 e g \left( \frac{i c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{\sqrt{-\frac{d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} - \right.$$

$$\left. \frac{i a d \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{b \sqrt{-\frac{d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \right) -$$

$$\frac{1}{(b c - a d)^3} a b^2 C d^3 e g \left( \frac{i c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{\sqrt{-\frac{d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} - \right)$$

$$\begin{aligned}
& \left. \frac{\frac{1}{2} a d \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}, \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{b \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \right] - \right. \\
& \frac{1}{(b c - a d)^2} b^2 C d^2 e g \left( \frac{\frac{1}{2} c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}, \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{\sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \right. - \\
& \left. \frac{\frac{1}{2} a d \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}, \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{b \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \right] - \\
& \frac{1}{(b c - a d)^3} b^3 c^2 C d f g \left( \frac{\frac{1}{2} c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}, \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{\sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \right. - \\
& \left. \frac{\frac{1}{2} a d \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}, \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{b \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \right] + \\
& \frac{1}{(b c - a d)^3} a b^2 c C d^2 f g \left( \frac{\frac{1}{2} c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}, \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{\sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \right. - 
\end{aligned}$$

$$\frac{\frac{1}{2} \operatorname{ad} \sqrt{1 - \frac{-de+cf}{f(c+dx)}} \sqrt{1 - \frac{-dg+ch}{h(c+dx)}} \operatorname{EllipticPi}\left[\frac{(bc-ad)h}{b(-dg+ch)}, \frac{1}{2} \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-dg+ch}{h}}}{\sqrt{c+dx}}\right], \frac{(-de+cf)h}{f(-dg+ch)}\right]}{b \sqrt{-\frac{-dg+ch}{h}} \sqrt{f h + \frac{d^2 e g}{(c+dx)^2} - \frac{c d f g}{(c+dx)^2} - \frac{c d e h}{(c+dx)^2} + \frac{c^2 f h}{(c+dx)^2} + \frac{d f g}{c+dx} + \frac{d e h}{c+dx} - \frac{2 c f h}{c+dx}}}$$

$$\frac{1}{(b c - a d)^2} b^2 c C d f g \left( \frac{i c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, i \text{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{\sqrt{\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \right)$$

$$\frac{\frac{1}{2} \operatorname{ad} \sqrt{1 - \frac{-de+cf}{f(c+dx)}} \sqrt{1 - \frac{-dg+ch}{h(c+dx)}} \operatorname{EllipticPi}\left[\frac{(bc-ad)h}{b(-dg+ch)}, \frac{1}{2} \operatorname{ArcSinh}\left(\frac{\sqrt{\frac{-dg+ch}{h}}}{\sqrt{c+dx}}\right), \frac{(-de+cf)h}{f(-dg+ch)}\right]}{b \sqrt{-\frac{-dg+ch}{h}} \sqrt{fh + \frac{d^2 eg}{(c+dx)^2} - \frac{cd fg}{(c+dx)^2} - \frac{cd eh}{(c+dx)^2} + \frac{c^2 fh}{(c+dx)^2} + \frac{df g}{c+dx} + \frac{de h}{c+dx} - \frac{2cfh}{c+dx}}}$$

$$\frac{1}{(b c - a d)^3} b^3 c^2 C d e h \left( \frac{\frac{i}{2} c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \frac{i}{2} \text{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{\sqrt{\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \right)$$

$$\frac{\text{d} \operatorname{ad} \sqrt{1 - \frac{-\text{d e} + \text{c f}}{\text{f} (\text{c} + \text{d x})}} \sqrt{1 - \frac{-\text{d g} + \text{c h}}{\text{h} (\text{c} + \text{d x})}} \operatorname{EllipticPi}\left[\frac{(\text{b c} - \text{a d}) \text{h}}{\text{b} (-\text{d g} + \text{c h})}, \text{d} \operatorname{ArcSinh}\left[\frac{\sqrt{-\frac{-\text{d g} + \text{c h}}{\text{h}}}}{\sqrt{\text{c} + \text{d x}}}\right], \frac{(-\text{d e} + \text{c f}) \text{h}}{\text{f} (-\text{d g} + \text{c h})}\right]}{\text{b} \sqrt{-\frac{-\text{d g} + \text{c h}}{\text{h}}} \sqrt{\text{f} \text{h} + \frac{\text{d}^2 \text{e g}}{(\text{c} + \text{d x})^2} - \frac{\text{c d f g}}{(\text{c} + \text{d x})^2} - \frac{\text{c d e h}}{(\text{c} + \text{d x})^2} + \frac{\text{c}^2 \text{f h}}{(\text{c} + \text{d x})^2} + \frac{\text{d f g}}{\text{c} + \text{d x}} + \frac{\text{d e h}}{\text{c} + \text{d x}} - \frac{2 \text{c f h}}{\text{c} + \text{d x}}}}$$

$$\frac{1}{(b c - a d)^3} a b^2 c C d^2 e h \left( \frac{\pm c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \pm \text{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{\sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \right)$$

$$\begin{aligned}
& \frac{\pm a d}{c-a d} \sqrt{1 - \frac{-d e+c f}{f(c+d x)}} \sqrt{1 - \frac{-d g+c h}{h(c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c-a d) h}{b(-d g+c h)}, \pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g+c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e+c f) h}{f(-d g+c h)}\right] \\
& + b \sqrt{-\frac{-d g+c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}} \\
& \frac{1}{(c-a d)^2} b^2 c C d e h \left( \begin{array}{l} \frac{\pm c}{c-a d} \sqrt{1 - \frac{-d e+c f}{f(c+d x)}} \sqrt{1 - \frac{-d g+c h}{h(c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c-a d) h}{b(-d g+c h)}, \pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g+c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e+c f) h}{f(-d g+c h)}\right] \\
\sqrt{-\frac{-d g+c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}} \end{array} \right) \\
& \frac{\pm a d}{c-a d} \sqrt{1 - \frac{-d e+c f}{f(c+d x)}} \sqrt{1 - \frac{-d g+c h}{h(c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c-a d) h}{b(-d g+c h)}, \pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g+c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e+c f) h}{f(-d g+c h)}\right] \\
& + b \sqrt{-\frac{-d g+c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}} \\
& \frac{1}{(c-a d)^3} b^3 c^3 C f h \left( \begin{array}{l} \frac{\pm c}{c-a d} \sqrt{1 - \frac{-d e+c f}{f(c+d x)}} \sqrt{1 - \frac{-d g+c h}{h(c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c-a d) h}{b(-d g+c h)}, \pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g+c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e+c f) h}{f(-d g+c h)}\right] \\
\sqrt{-\frac{-d g+c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}} \end{array} \right) \\
& \frac{\pm a d}{c-a d} \sqrt{1 - \frac{-d e+c f}{f(c+d x)}} \sqrt{1 - \frac{-d g+c h}{h(c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c-a d) h}{b(-d g+c h)}, \pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g+c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e+c f) h}{f(-d g+c h)}\right] \\
& - b \sqrt{-\frac{-d g+c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}} \\
& \frac{1}{(c-a d)^3} a b^2 c^2 C d f h \left( \begin{array}{l} \frac{\pm c}{c-a d} \sqrt{1 - \frac{-d e+c f}{f(c+d x)}} \sqrt{1 - \frac{-d g+c h}{h(c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c-a d) h}{b(-d g+c h)}, \pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g+c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e+c f) h}{f(-d g+c h)}\right] \\
\sqrt{-\frac{-d g+c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}} \end{array} \right)
\end{aligned}$$

$$\frac{\text{i } \text{ad} \sqrt{1 - \frac{-\text{de}+\text{cf}}{\text{f} (\text{c}+\text{d} x)}} \sqrt{1 - \frac{-\text{dg}+\text{ch}}{\text{h} (\text{c}+\text{d} x)}} \text{EllipticPi}\left[\frac{(\text{b} \text{c}-\text{a} \text{d}) \text{h}}{\text{b} (-\text{dg}+\text{ch})}, \text{i } \text{ArcSinh}\left[\frac{\sqrt{\frac{-\text{dg}+\text{ch}}{\text{h}}}}{\sqrt{\text{c}+\text{d} x}}\right], \frac{(-\text{de}+\text{cf}) \text{h}}{\text{f} (-\text{dg}+\text{ch})}\right]}{\text{b} \sqrt{-\frac{\text{dg}+\text{ch}}{\text{h}}} \sqrt{\text{f} \text{h} + \frac{\text{d}^2 \text{e} \text{g}}{(\text{c}+\text{d} x)^2} - \frac{\text{c} \text{d} \text{f} \text{g}}{(\text{c}+\text{d} x)^2} - \frac{\text{c} \text{d} \text{e} \text{h}}{(\text{c}+\text{d} x)^2} + \frac{\text{c}^2 \text{f} \text{h}}{(\text{c}+\text{d} x)^2} + \frac{\text{d} \text{f} \text{g}}{\text{c}+\text{d} x} + \frac{\text{d} \text{e} \text{h}}{\text{c}+\text{d} x} - \frac{2 \text{c} \text{f} \text{h}}{\text{c}+\text{d} x}}}$$

$$\frac{1}{(b c - a d)^2} 2 b^2 c^2 C f h \left( \frac{\frac{i}{2} c \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \frac{i}{2} \text{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c + d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{\sqrt{\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c + d x)^2} - \frac{c d f g}{(c + d x)^2} - \frac{c d e h}{(c + d x)^2} + \frac{c^2 f h}{(c + d x)^2} + \frac{d f g}{c + d x} + \frac{d e h}{c + d x} - \frac{2 c f h}{c + d x}}} \right)$$

$$\frac{\frac{1}{2} \operatorname{ad} \sqrt{1 - \frac{-de+cf}{f(c+dx)}} \sqrt{1 - \frac{-dg+ch}{h(c+dx)}} \operatorname{EllipticPi}\left[\frac{(bc-ad)h}{b(-dg+ch)}, \frac{1}{2} \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-dg+ch}{h}}}{\sqrt{c+dx}}\right], \frac{(-de+cf)h}{f(-dg+ch)}\right]}{b \sqrt{-\frac{-dg+ch}{h}} \sqrt{fh + \frac{d^2 eg}{(c+dx)^2} - \frac{cd fg}{(c+dx)^2} - \frac{cd eh}{(c+dx)^2} + \frac{c^2 fh}{(c+dx)^2} + \frac{df g}{c+dx} + \frac{de h}{c+dx} - \frac{2cfh}{c+dx}}}$$

$$\frac{1}{(b c - a d)^2} A b^2 d^2 f h \left( \frac{i c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, i \text{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right]\right], \frac{(-d e + c f) h}{f (-d g + c h)}}{\sqrt{-\frac{d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}}\right)$$

$$\frac{\text{i} \operatorname{ad} \sqrt{1 - \frac{-de+cf}{f(c+dx)}} \sqrt{1 - \frac{-dg+ch}{h(c+dx)}} \operatorname{EllipticPi}\left[\frac{(bc-ad)h}{b(-dg+ch)}, \text{i} \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-dg+ch}{h}}}{\sqrt{c+dx}}\right], \frac{(-de+cf)h}{f(-dg+ch)}\right]}{b \sqrt{-\frac{-dg+ch}{h}} \sqrt{fh + \frac{d^2 eg}{(c+dx)^2} - \frac{cd fg}{(c+dx)^2} - \frac{cdeh}{(c+dx)^2} + \frac{c^2 fh}{(c+dx)^2} + \frac{dfg}{c+dx} + \frac{deh}{c+dx} - \frac{2cfh}{c+dx}}} +$$

$$\frac{1}{b c - a d} b c C f h \left( \frac{\frac{i}{2} c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \frac{i}{2} \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{\sqrt{\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \right)$$

$$\begin{aligned}
& \frac{i \operatorname{ad} \sqrt{1 - \frac{-de+cf}{f(c+dx)}} \sqrt{1 - \frac{-dg+ch}{h(c+dx)}} \operatorname{EllipticPi}\left[\frac{(bc-ad)h}{b(-dg+ch)}, i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-dg+ch}{h}}}{\sqrt{c+dx}}\right], \frac{(-de+cf)h}{f(-dg+ch)}\right]}{+} \\
& b \sqrt{-\frac{-dg+ch}{h}} \sqrt{f h + \frac{d^2 e g}{(c+dx)^2} - \frac{c d f g}{(c+dx)^2} - \frac{c d e h}{(c+dx)^2} + \frac{c^2 f h}{(c+dx)^2} + \frac{d f g}{c+dx} + \frac{d e h}{c+dx} - \frac{2 c f h}{c+dx}} \\
& \frac{1}{b c - a d} a C d f h \left( \frac{i c \sqrt{1 - \frac{-de+cf}{f(c+dx)}} \sqrt{1 - \frac{-dg+ch}{h(c+dx)}} \operatorname{EllipticPi}\left[\frac{(bc-ad)h}{b(-dg+ch)}, i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-dg+ch}{h}}}{\sqrt{c+dx}}\right], \frac{(-de+cf)h}{f(-dg+ch)}\right]}{-} \right. \\
& \left. b \sqrt{-\frac{-dg+ch}{h}} \sqrt{f h + \frac{d^2 e g}{(c+dx)^2} - \frac{c d f g}{(c+dx)^2} - \frac{c d e h}{(c+dx)^2} + \frac{c^2 f h}{(c+dx)^2} + \frac{d f g}{c+dx} + \frac{d e h}{c+dx} - \frac{2 c f h}{c+dx}} \right. \\
& \left. \frac{i \operatorname{ad} \sqrt{1 - \frac{-de+cf}{f(c+dx)}} \sqrt{1 - \frac{-dg+ch}{h(c+dx)}} \operatorname{EllipticPi}\left[\frac{(bc-ad)h}{b(-dg+ch)}, i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-dg+ch}{h}}}{\sqrt{c+dx}}\right], \frac{(-de+cf)h}{f(-dg+ch)}\right]}{+} \right) \\
& \left( -b c C f h - a C d f h - \frac{b c C d^2 e g}{(c+d x)^2} + \frac{a C d^3 e g}{(c+d x)^2} + \frac{b c^2 C d f g}{(c+d x)^2} - \frac{a c C d^2 f g}{(c+d x)^2} + \frac{b c^2 C d e h}{(c+d x)^2} - \frac{a c C d^2 e h}{(c+d x)^2} - \frac{b c^3 C f h}{(c+d x)^2} + \frac{a c^2 C d f h}{(c+d x)^2} + \right. \\
& \left. \frac{b C d^2 e g}{c+d x} - \frac{b c C d f g}{c+d x} - \frac{b c C d e h}{c+d x} + \frac{2 b c^2 C f h}{c+d x} + \frac{A b d^2 f h}{c+d x} \right) \sqrt{e + \frac{(c+d x) \left(f - \frac{c f}{c+d x}\right)}{d}} \sqrt{g + \frac{(c+d x) \left(h - \frac{c h}{c+d x}\right)}{d}}
\end{aligned}$$

Problem 30: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{A + C x^2}{(a + b x)^2 \sqrt{c + d x} \sqrt{e + f x} \sqrt{g + h x}} dx$$

Optimal (type 4, 738 leaves, 12 steps):

$$\begin{aligned}
& - \frac{(A b^2 + a^2 C) \sqrt{c + d x} \sqrt{e + f x} \sqrt{g + h x}}{(b c - a d) (b e - a f) (b g - a h) (a + b x)} + \frac{\left(A b + \frac{a^2 c}{b}\right) \sqrt{f} \sqrt{-d e + c f} \sqrt{\frac{d(e+f x)}{d e - c f}} \sqrt{g + h x} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{f} \sqrt{c+d x}}{\sqrt{-d e+c f}}\right], \frac{(d e-c f) h}{f(d g-c h)}\right]}{(b c - a d) (b e - a f) (b g - a h) \sqrt{e + f x} \sqrt{\frac{d(g+h x)}{d g - c h}}} + \\
& \left( \sqrt{-d e + c f} (a^2 C d f - 2 a b C (d e + c f) + b^2 (2 c C e - A d f)) \sqrt{\frac{d(e+f x)}{d e - c f}} \sqrt{\frac{d(g+h x)}{d g - c h}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{f} \sqrt{c+d x}}{\sqrt{-d e+c f}}\right], \frac{(d e-c f) h}{f(d g-c h)}\right] \right) / \\
& \left( b^2 d (b c - a d) \sqrt{f} (b e - a f) \sqrt{e + f x} \sqrt{g + h x} \right) - \\
& \left( \sqrt{-d e + c f} (a^4 C d f h - A b^4 (d e g + c f g + c e h) - 2 a^3 b C (d f g + d e h + c f h) - 2 a b^3 (2 c C e g - A d f g - A d e h - A c f h) - \right. \\
& \left. 3 a^2 b^2 (A d f h - C (d e g + c f g + c e h))) \sqrt{\frac{d(e+f x)}{d e - c f}} \sqrt{\frac{d(g+h x)}{d g - c h}} \right. \\
& \left. \operatorname{EllipticPi}\left[-\frac{b(d e-c f)}{(b c-a d)f}, \operatorname{ArcSin}\left[\frac{\sqrt{f} \sqrt{c+d x}}{\sqrt{-d e+c f}}\right], \frac{(d e-c f) h}{f(d g-c h)}\right]\right) / \left( b^2 (b c - a d)^2 \sqrt{f} (b e - a f) (b g - a h) \sqrt{e + f x} \sqrt{g + h x} \right)
\end{aligned}$$

Result (type 4, 17743 leaves):

$$\begin{aligned}
& \frac{(-A b^2 - a^2 C) \sqrt{c + d x} \sqrt{e + f x} \sqrt{g + h x}}{(b c - a d) (b e - a f) (b g - a h) (a + b x)} - \\
& \frac{1}{d (-b c + a d) (-b e + a f) (-b g + a h)} \left( \frac{(A b^2 + a^2 C) (c + d x)^{3/2} \left(f + \frac{d e}{c+d x} - \frac{c f}{c+d x}\right) \left(h + \frac{d g}{c+d x} - \frac{c h}{c+d x}\right)}{b \sqrt{e + \frac{(c+d x)(f - \frac{c f}{c+d x})}{d}} \sqrt{g + \frac{(c+d x)(h - \frac{c h}{c+d x})}{d}}} + \left( c + d x \right) \left( -b + \frac{b c}{c+d x} - \frac{a d}{c+d x} \right) \right. \\
& \left. \sqrt{f + \frac{d e}{c+d x} - \frac{c f}{c+d x}} \sqrt{h + \frac{d g}{c+d x} - \frac{c h}{c+d x}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}} \right. \\
& \left. - \frac{(b c - a d) (2 b^2 C e g^2 - 2 a b C f g^2 + A b^2 f g h + a^2 C f g h + A b^2 e h^2 - a^2 C e h^2 - 2 a A b f h^2)}{b (b g - a h) \sqrt{f + \frac{d e}{c+d x} - \frac{c f}{c+d x}} \sqrt{h + \frac{d g}{c+d x} - \frac{c h}{c+d x}}} - \frac{(A b^2 + a^2 C) (d e - c f) \sqrt{h + \frac{d g}{c+d x} - \frac{c h}{c+d x}}}{b \sqrt{f + \frac{d e}{c+d x} - \frac{c f}{c+d x}}} \right)
\end{aligned}$$

$$\begin{aligned}
& \left( (-4 a b^3 c C e g - A b^4 d e g + 3 a^2 b^2 C d e g - A b^4 c f g + 3 a^2 b^2 c C f g + 2 a A b^3 d f g - 2 a^3 b C d f g - A b^4 c e h + 3 a^2 b^2 c C e h + \right. \\
& \quad \left. 2 a A b^3 d e h - 2 a^3 b C d e h + 2 a A b^3 c f h - 2 a^3 b c C f h - 3 a^2 A b^2 d f h + a^4 C d f h) \sqrt{h + \frac{d g}{c + d x} - \frac{c h}{c + d x}} \right) / \\
& \left. \left( b (b g - a h) \left( b - \frac{b c}{c + d x} + \frac{a d}{c + d x} \right) \sqrt{f + \frac{d e}{c + d x} - \frac{c f}{c + d x}} \right) \right) \left( \begin{array}{l} \pm A b^2 d^2 e f g \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \\ \text{EllipticE}[\pm \text{ArcSinh}\left(\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}}\right), \frac{(-d e + c f) h}{f (-d g + c h)}] - \text{EllipticF}[\pm \text{ArcSinh}\left(\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}}\right), \frac{(-d e + c f) h}{f (-d g + c h)}] \end{array} \right) \right) / \\
& \left( (b c - a d) (-d e + c f) \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2} + \frac{d f g + d e h - 2 c f h}{c + d x}} \right) + \left( \begin{array}{l} \pm a^2 C d^2 e f g \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \\ \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \left( \text{EllipticE}[\pm \text{ArcSinh}\left(\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}}\right), \frac{(-d e + c f) h}{f (-d g + c h)}] - \text{EllipticF}[\pm \text{ArcSinh}\left(\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}}\right), \frac{(-d e + c f) h}{f (-d g + c h)}] \right) \end{array} \right) \right) / \\
& \left( (b c - a d) (-d e + c f) \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2} + \frac{d f g + d e h - 2 c f h}{c + d x}} \right) - \left( \begin{array}{l} \pm A b^2 c d f^2 g \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \end{array} \right)
\end{aligned}$$

$$\sqrt{1 - \frac{-d g + c h}{h(c + d x)}} \left( \text{EllipticE} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] - \text{EllipticF} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] \right)$$

$$\left( (b c - a d) (-d e + c f) \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2}} + \frac{d f g + d e h - 2 c f h}{c + d x} \right) - \left( i a^2 c C d f^2 g \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \right)$$

$$\sqrt{1 - \frac{-d g + c h}{h(c + d x)}} \left( \text{EllipticE} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] - \text{EllipticF} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] \right)$$

$$\left( (b c - a d) (-d e + c f) \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2}} + \frac{d f g + d e h - 2 c f h}{c + d x} \right) - \left( i A b^2 c d e f h \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \right)$$

$$\sqrt{1 - \frac{-d g + c h}{h(c + d x)}} \left( \text{EllipticE} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] - \text{EllipticF} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] \right)$$

$$\left( (b c - a d) (-d e + c f) \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2}} + \frac{d f g + d e h - 2 c f h}{c + d x} \right) - \left( i a^2 c C d e f h \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \right)$$

$$\sqrt{1 - \frac{-d g + c h}{h(c + d x)}} \left( \text{EllipticE} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] - \text{EllipticF} \left[ \text{i ArcSinh} \left[ \frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}} \right], \frac{(-d e + c f) h}{f (-d g + c h)} \right] \right)$$

$$\left( (b c - a d) (-d e + c f) \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2} + \frac{d f g + d e h - 2 c f h}{c + d x}} \right) + \left( i A b^2 c^2 f^2 h \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \right)$$

$$\sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \left( \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right] - \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]\right)$$

$$\left( (b c - a d) (-d e + c f) \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2} + \frac{d f g + d e h - 2 c f h}{c + d x}} \right) + \left( i a^2 c^2 C f^2 h \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \right)$$

$$\sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \left( \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right] - \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]\right)$$

$$\left( (b c - a d) (-d e + c f) \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2} + \frac{d f g + d e h - 2 c f h}{c + d x}} \right) -$$

$$\frac{i A b^3 d^2 e g \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{(b c - a d)^2 \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2} + \frac{d f g + d e h - 2 c f h}{c + d x}}} -$$

$$\frac{i a^2 b C d^2 e g \sqrt{1 - \frac{-d e + c f}{f (c + d x)}} \sqrt{1 - \frac{-d g + c h}{h (c + d x)}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\frac{-d g + c h}{h}}}{\sqrt{c + d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{(b c - a d)^2 \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c + d x)^2} + \frac{d f g + d e h - 2 c f h}{c + d x}}} -$$

$$\frac{2 i b^2 c C e g \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{(b c - a d) \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}}$$

$$\frac{2 i a b C d e g \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{(b c - a d) \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}}$$

$$\frac{i A b^3 c d f g \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{(b c - a d)^2 \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}}$$

$$\frac{i a^2 b c C d f g \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{(b c - a d)^2 \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}}$$

$$\frac{2 i a b c C f g \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{(b c - a d) \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}}$$

$$\frac{2 i A b^2 d f g \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{(b c - a d) \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}}$$

$$\frac{i A b^3 c d e h \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{(b c - a d)^2 \sqrt{-\frac{d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}} +$$

$$\frac{i a^2 b c C d e h \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{(b c - a d)^2 \sqrt{-\frac{d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}} +$$

$$\frac{2 i a b c C e h \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{(b c - a d) \sqrt{-\frac{d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}} -$$

$$\frac{2 i A b^2 d e h \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{(b c - a d) \sqrt{-\frac{d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}} -$$

$$\frac{i A b^3 c^2 f h \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{(b c - a d)^2 \sqrt{-\frac{d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}} -$$

$$\frac{i a^2 b c^2 C f h \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{(b c - a d)^2 \sqrt{-\frac{d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}} +$$

$$\frac{2 i A b^2 c f h \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{(b c - a d) \sqrt{-\frac{d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}} +$$

$$\frac{2 i a A b d f h \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{(b c - a d) \sqrt{-\frac{d g + c h}{h}} \sqrt{f h + \frac{d^2 e g - c d f g - c d e h + c^2 f h}{(c+d x)^2} + \frac{d f g + d e h - 2 c f h}{c+d x}}} + \frac{1}{(b c - a d)^3}$$

$$A b^4 d^2 e g \left( \frac{i c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{\sqrt{-\frac{d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} - \right.$$

$$\left. \frac{i a d \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{b \sqrt{-\frac{d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \right) +$$

$$\frac{1}{(b c - a d)^3} a^2 b^2 C d^2 e g \left( \frac{i c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{\sqrt{-\frac{d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} - \right.$$

$$\left. \frac{i a d \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{b \sqrt{-\frac{d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \right) +$$

$$\frac{1}{(b c - a d)^2} 2 b^3 c c e g \left( \frac{\frac{1}{2} c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \frac{1}{2} \text{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{\sqrt{-\frac{d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \right)$$

$$\frac{\text{i a d} \sqrt{1 - \frac{-\text{d e} + \text{c f}}{\text{f} (\text{c} + \text{d x})}} \sqrt{1 - \frac{-\text{d g} + \text{c h}}{\text{h} (\text{c} + \text{d x})}} \text{EllipticPi}\left[\frac{(\text{b c} - \text{a d}) \text{h}}{\text{b} (-\text{d g} + \text{c h})}, \text{i ArcSinh}\left[\frac{\sqrt{\frac{-\text{d g} + \text{c h}}{\text{h}}}}{\sqrt{\text{c} + \text{d x}}}\right], \frac{(-\text{d e} + \text{c f}) \text{h}}{\text{f} (-\text{d g} + \text{c h})}\right]}{b \sqrt{-\frac{-\text{d g} + \text{c h}}{\text{h}}} \sqrt{\text{f} \text{h} + \frac{\text{d}^2 \text{e g}}{(\text{c} + \text{d x})^2} - \frac{\text{c d f g}}{(\text{c} + \text{d x})^2} - \frac{\text{c d e h}}{(\text{c} + \text{d x})^2} + \frac{\text{c}^2 \text{f h}}{(\text{c} + \text{d x})^2} + \frac{\text{d f g}}{\text{c} + \text{d x}} + \frac{\text{d e h}}{\text{c} + \text{d x}} - \frac{2 \text{c f h}}{\text{c} + \text{d x}}}}$$

$$\frac{1}{(b c - a d)^2} 2 a b^2 C d e g \left( \begin{array}{l} i c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \frac{i}{\sqrt{c+d x}}\right], \frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}, \frac{(-d e + c f) h}{f (-d g + c h)} \\ \sqrt{\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}} \end{array} \right)$$

$$\frac{\pm ad \sqrt{1 - \frac{-de+cf}{f(c+dx)}} \sqrt{1 - \frac{-dg+ch}{h(c+dx)}} \operatorname{EllipticPi}\left[\frac{(bc-ad)h}{b(-dg+ch)}, \pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-dg+ch}{h}}}{\sqrt{c+dx}}\right], \frac{(-de+cf)h}{f(-dg+ch)}\right]}{b \sqrt{-\frac{-dg+ch}{h}} \sqrt{fh + \frac{d^2 eg}{(c+dx)^2} - \frac{cd fg}{(c+dx)^2} - \frac{cd eh}{(c+dx)^2} + \frac{c^2 fh}{(c+dx)^2} + \frac{df g}{c+dx} + \frac{de h}{c+dx} - \frac{2cfh}{c+dx}}}$$

$$\frac{1}{b c - a d} \cdot 2 b^2 C e g \left( \frac{\pm c \sqrt{1 - \frac{-d e + c f}{f(c+d x)}} \sqrt{1 - \frac{-d g + c h}{h(c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{\sqrt{\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}}} \right)$$

$$\frac{\frac{1}{2} \operatorname{ad} \sqrt{1 - \frac{-de+cf}{f(c+dx)}} \sqrt{1 - \frac{-dg+ch}{h(c+dx)}} \operatorname{EllipticPi}\left[\frac{(bc-ad)h}{b(-dg+ch)}, \frac{1}{2} \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-dg+ch}{h}}}{\sqrt{c+dx}}\right], \frac{(-de+cf)h}{f(-dg+ch)}\right]}{b \sqrt{\frac{-dg+ch}{h}} \sqrt{fh + \frac{d^2 eg}{(c+dx)^2} - \frac{c df g}{(c+dx)^2} - \frac{c de h}{(c+dx)^2} + \frac{c^2 fh}{(c+dx)^2} + \frac{df g}{c+dx} + \frac{de h}{c+dx} - \frac{2 cf h}{c+dx}}}$$

$$\frac{1}{(b c - a d)^3} A b^4 c d f g \left( \begin{array}{l} \frac{\pm c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \pm \text{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}, \frac{(-d e + c f) h}{f (-d g + c h)}\right]\right]}{\sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \\ \\ \frac{\pm a d \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \pm \text{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}, \frac{(-d e + c f) h}{f (-d g + c h)}\right]\right]}{b \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \end{array} \right) -$$

$$\frac{1}{(b c - a d)^3} a^2 b^2 c C d f g \left( \begin{array}{l} \frac{\pm c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \pm \text{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}, \frac{(-d e + c f) h}{f (-d g + c h)}\right]\right]}{\sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \\ \\ \frac{\pm a d \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \pm \text{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}, \frac{(-d e + c f) h}{f (-d g + c h)}\right]\right]}{b \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \end{array} \right) -$$

$$\frac{1}{(b c - a d)^2} 2 a b^2 c C f g \left( \begin{array}{l} \frac{\pm c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \pm \text{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}, \frac{(-d e + c f) h}{f (-d g + c h)}\right]\right]}{\sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \\ \\ \frac{\pm a d \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \text{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \pm \text{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}, \frac{(-d e + c f) h}{f (-d g + c h)}\right]\right]}{b \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \end{array} \right) +$$

$$\begin{aligned}
& \frac{1}{(b c - a d)^2} 2 A b^3 d f g \left( \begin{array}{l} \frac{\pm c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}, \frac{(-d e + c f) h}{f (-d g + c h)}\right]\right]}{\sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \\ \frac{\pm a d \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}, \frac{(-d e + c f) h}{f (-d g + c h)}\right]\right]}{b \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \end{array} \right) + \\
& \frac{1}{b c - a d} 2 a b c f g \left( \begin{array}{l} \frac{\pm c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}, \frac{(-d e + c f) h}{f (-d g + c h)}\right]\right]}{\sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \\ \frac{\pm a d \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}, \frac{(-d e + c f) h}{f (-d g + c h)}\right]\right]}{b \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \end{array} \right) - \\
& \frac{1}{(b c - a d)^3} A b^4 c d e h \left( \begin{array}{l} \frac{\pm c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}, \frac{(-d e + c f) h}{f (-d g + c h)}\right]\right]}{\sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \\ \frac{\pm a d \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \pm \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}, \frac{(-d e + c f) h}{f (-d g + c h)}\right]\right]}{b \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \end{array} \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{(b c - a d)^3} a^2 b^2 c C d e h \left( \begin{array}{l} \frac{i c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{\sqrt{-\frac{-d g + c h}{h}}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}} \\ \frac{i a d \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{b \sqrt{-\frac{-d g + c h}{h}}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}} \end{array} \right) - \\
& \frac{1}{(b c - a d)^2} 2 a b^2 c C e h \left( \begin{array}{l} \frac{i c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{\sqrt{-\frac{-d g + c h}{h}}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}} \\ \frac{i a d \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{b \sqrt{-\frac{-d g + c h}{h}}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}} \end{array} \right) + \\
& \frac{1}{(b c - a d)^2} 2 A b^3 d e h \left( \begin{array}{l} \frac{i c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{\sqrt{-\frac{-d g + c h}{h}}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}} \\ \frac{i a d \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{b \sqrt{-\frac{-d g + c h}{h}}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}} \end{array} \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{b c - a d} \frac{1}{2 a b C e h} \left( \begin{array}{l} \frac{i c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, i \operatorname{ArcSinh}\left[\frac{\sqrt{-d g + c h}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{\sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \\ \\ \frac{i a d \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, i \operatorname{ArcSinh}\left[\frac{\sqrt{-d g + c h}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{b \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \end{array} \right) + \\
& \frac{1}{(b c - a d)^3} a b^4 c^2 f h \left( \begin{array}{l} \frac{i c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, i \operatorname{ArcSinh}\left[\frac{\sqrt{-d g + c h}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{\sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \\ \\ \frac{i a d \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, i \operatorname{ArcSinh}\left[\frac{\sqrt{-d g + c h}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{b \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \end{array} \right) - \\
& \frac{1}{(b c - a d)^3} a^2 b^2 c^2 C f h \left( \begin{array}{l} \frac{i c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, i \operatorname{ArcSinh}\left[\frac{\sqrt{-d g + c h}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{\sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \\ \\ \frac{i a d \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, i \operatorname{ArcSinh}\left[\frac{\sqrt{-d g + c h}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f (-d g + c h)}\right]}{b \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \end{array} \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{(b c - a d)^2} 2 A b^3 c f h \left( \frac{\frac{i}{2} c \sqrt{1 - \frac{-d e + c f}{f(c+d x)}} \sqrt{1 - \frac{-d g + c h}{h(c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b(-d g + c h)}, \frac{i}{2} \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f(-d g + c h)}\right]}{\sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \right. \\
& \left. - \frac{\frac{i}{2} a d \sqrt{1 - \frac{-d e + c f}{f(c+d x)}} \sqrt{1 - \frac{-d g + c h}{h(c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b(-d g + c h)}, \frac{i}{2} \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f(-d g + c h)}\right]}{b \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \right) \\
& \frac{1}{(b c - a d)^2} 2 a A b^2 d f h \left( \frac{\frac{i}{2} c \sqrt{1 - \frac{-d e + c f}{f(c+d x)}} \sqrt{1 - \frac{-d g + c h}{h(c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b(-d g + c h)}, \frac{i}{2} \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f(-d g + c h)}\right]}{\sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \right. \\
& \left. + \frac{\frac{i}{2} a d \sqrt{1 - \frac{-d e + c f}{f(c+d x)}} \sqrt{1 - \frac{-d g + c h}{h(c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b(-d g + c h)}, \frac{i}{2} \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f(-d g + c h)}\right]}{b \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \right) \\
& \frac{1}{b c - a d} A b^2 f h \left( \frac{\frac{i}{2} c \sqrt{1 - \frac{-d e + c f}{f(c+d x)}} \sqrt{1 - \frac{-d g + c h}{h(c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b(-d g + c h)}, \frac{i}{2} \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f(-d g + c h)}\right]}{\sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \right. \\
& \left. - \frac{\frac{i}{2} a d \sqrt{1 - \frac{-d e + c f}{f(c+d x)}} \sqrt{1 - \frac{-d g + c h}{h(c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b(-d g + c h)}, \frac{i}{2} \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{-d g + c h}{h}}}{\sqrt{c+d x}}\right], \frac{(-d e + c f) h}{f(-d g + c h)}\right]}{b \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{b c - a d} a^2 C f h \left( \frac{\frac{i c \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \frac{i \operatorname{ArcSinh}\left[\frac{\sqrt{-d g + c h}}{\sqrt{c+d x}}\right]}{f (-d g + c h)}\right]}{\sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} - \right. \\
& \left. \frac{i a d \sqrt{1 - \frac{-d e + c f}{f (c+d x)}} \sqrt{1 - \frac{-d g + c h}{h (c+d x)}} \operatorname{EllipticPi}\left[\frac{(b c - a d) h}{b (-d g + c h)}, \frac{i \operatorname{ArcSinh}\left[\frac{\sqrt{-d g + c h}}{\sqrt{c+d x}}\right]}{f (-d g + c h)}\right]}{b \sqrt{-\frac{-d g + c h}{h}} \sqrt{f h + \frac{d^2 e g}{(c+d x)^2} - \frac{c d f g}{(c+d x)^2} - \frac{c d e h}{(c+d x)^2} + \frac{c^2 f h}{(c+d x)^2} + \frac{d f g}{c+d x} + \frac{d e h}{c+d x} - \frac{2 c f h}{c+d x}}} \right] \right) / \\
& \left( \left( -2 b^2 C e g + 2 a b C f g + 2 a b C e h + A b^2 f h - a^2 C f h + \frac{A b^2 d^2 e g}{(c+d x)^2} + \frac{a^2 C d^2 e g}{(c+d x)^2} - \frac{A b^2 c d f g}{(c+d x)^2} - \frac{a^2 c C d f g}{(c+d x)^2} - \frac{A b^2 c d e h}{(c+d x)^2} - \right. \right. \\
& \left. \left. \frac{a^2 c C d e h}{(c+d x)^2} + \frac{A b^2 c^2 f h}{(c+d x)^2} + \frac{a^2 c^2 C f h}{(c+d x)^2} + \frac{2 b^2 c C e g}{c+d x} + \frac{2 a b C d e g}{c+d x} - \frac{2 a b c C f g}{c+d x} + \frac{2 A b^2 d f g}{c+d x} - \frac{2 a b c C e h}{c+d x} + \right. \right. \\
& \left. \left. \frac{2 A b^2 d e h}{c+d x} - \frac{2 A b^2 c f h}{c+d x} - \frac{2 a A b d f h}{c+d x} \right) \sqrt{e + \frac{(c+d x) \left(f - \frac{c f}{c+d x}\right)}{d}} \sqrt{g + \frac{(c+d x) \left(h - \frac{c h}{c+d x}\right)}{d}} \right)
\end{aligned}$$

Problem 31: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+b x)^{3/2} (A+C x^2)}{\sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x}} dx$$

Optimal (type 4, 1395 leaves, 10 steps):

$$\begin{aligned}
& \frac{1}{24 b d^2 f^3 h^3 \sqrt{c+d x}} (C (3 a d f h - 5 b (d f g + d e h + c f h)) (a d f h - 3 b (d f g + d e h + c f h)) + \\
& 8 b d f h (3 A b d f h - C (2 b (d e g + c f g + c e h) + a (d f g + d e h + c f h)))) \sqrt{a+b x} \sqrt{e+f x} \sqrt{g+h x} + \\
& \frac{C (3 a d f h - 5 b (d f g + d e h + c f h)) \sqrt{a+b x} \sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x}}{12 d^2 f^2 h^2} + \frac{C (a+b x)^{3/2} \sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x}}{3 d f h} - \\
& \left( \sqrt{d g - c h} \sqrt{f g - e h} (C (3 a d f h - 5 b (d f g + d e h + c f h)) (a d f h - 3 b (d f g + d e h + c f h)) + \right. \\
& \left. 8 b d f h (3 A b d f h - C (2 b (d e g + c f g + c e h) + a (d f g + d e h + c f h)))) \sqrt{a+b x} \sqrt{-\frac{(d e - c f) (g+h x)}{(f g - e h) (c+d x)}} \right. \\
& \text{EllipticE} \left[ \text{ArcSin} \left[ \frac{\sqrt{d g - c h} \sqrt{e+f x}}{\sqrt{f g - e h} \sqrt{c+d x}}, \frac{(b c - a d) (f g - e h)}{(b e - a f) (d g - c h)} \right] \right] / \left( 24 b d^3 f^3 h^3 \sqrt{\frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}} \sqrt{g+h x} \right) + \left( (b e - a f) \sqrt{b g - a h} \right. \\
& (3 a^2 C d^2 f^2 h^2 + 6 a b C d f h (c f h + 2 d (f g + e h)) - b^2 (24 A d^2 f^2 h^2 + C (5 c^2 f^2 h^2 + 4 c d f h (f g + e h) + d^2 (15 f^2 g^2 + 14 e f g h + 15 e^2 h^2)))) \\
& \left. \sqrt{\frac{(b e - a f) (c+d x)}{(d e - c f) (a+b x)}} \sqrt{g+h x} \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\sqrt{b g - a h} \sqrt{e+f x}}{\sqrt{f g - e h} \sqrt{a+b x}}, -\frac{(b c - a d) (f g - e h)}{(d e - c f) (b g - a h)} \right] \right] \right) / \\
& \left( 24 b^2 d^2 f^3 h^3 \sqrt{f g - e h} \sqrt{c+d x} \sqrt{-\frac{(b e - a f) (g+h x)}{(f g - e h) (a+b x)}} - \frac{1}{24 b^2 d^3 \sqrt{b c - a d} f^3 h^4 \sqrt{c+d x} \sqrt{e+f x}} \right. \\
& \sqrt{-d g + c h} (4 b d f h (C (b (d e g + c f g + c e h) + a (d f g + d e h + c f h)) (3 a d f h - 5 b (d f g + d e h + c f h)) + \\
& 2 d f h (3 b^2 c C e g + 2 a^2 C (d f g + d e h + c f h) - a b (12 A d f h - 5 C (d e g + c f g + c e h)))) + \\
& (a d f h + b (d f g + d e h + c f h)) (C (3 a d f h - 5 b (d f g + d e h + c f h)) (a d f h - 3 b (d f g + d e h + c f h)) + \\
& 8 b d f h (3 A b d f h - C (2 b (d e g + c f g + c e h) + a (d f g + d e h + c f h)))) (a+b x) \sqrt{\frac{(b g - a h) (c+d x)}{(d g - c h) (a+b x)}} \\
& \left. \sqrt{\frac{(b g - a h) (e+f x)}{(f g - e h) (a+b x)}} \text{EllipticPi} \left[ -\frac{b (d g - c h)}{(b c - a d) h}, \text{ArcSin} \left[ \frac{\sqrt{b c - a d} \sqrt{g+h x}}{\sqrt{-d g + c h} \sqrt{a+b x}}, \frac{(b e - a f) (d g - c h)}{(b c - a d) (f g - e h)} \right] \right] \right)
\end{aligned}$$

Result (type 4, 38310 leaves): Display of huge result suppressed!

### Problem 32: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{a+b x} (A+C x^2)}{\sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x}} dx$$

Optimal (type 4, 937 leaves, 9 steps):

$$\begin{aligned}
& \frac{C (a d f h - 3 b (d f g + d e h + c f h)) \sqrt{a+b x} \sqrt{e+f x} \sqrt{g+h x}}{4 b d f^2 h^2 \sqrt{c+d x}} + \frac{C \sqrt{a+b x} \sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x}}{2 d f h} - \\
& \left( C \sqrt{d g - c h} \sqrt{f g - e h} (a d f h - 3 b (d f g + d e h + c f h)) \sqrt{a+b x} \sqrt{-\frac{(d e - c f) (g + h x)}{(f g - e h) (c + d x)}} \right. \\
& \quad \left. \text{EllipticE} \left[ \text{ArcSin} \left[ \frac{\sqrt{d g - c h} \sqrt{e + f x}}{\sqrt{f g - e h} \sqrt{c + d x}} \right], \frac{(b c - a d) (f g - e h)}{(b e - a f) (d g - c h)} \right] \right) / \left( 4 b d^2 f^2 h^2 \sqrt{\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}} \sqrt{g + h x} \right) + \\
& \left( C (b e - a f) \sqrt{b g - a h} (a d f h + b (c f h + 3 d (f g + e h))) \sqrt{\frac{(b e - a f) (c + d x)}{(d e - c f) (a + b x)}} \sqrt{g + h x} \right. \\
& \quad \left. \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\sqrt{b g - a h} \sqrt{e + f x}}{\sqrt{f g - e h} \sqrt{a + b x}} \right], -\frac{(b c - a d) (f g - e h)}{(d e - c f) (b g - a h)} \right] \right) / \left( 4 b^2 d f^2 h^2 \sqrt{f g - e h} \sqrt{c + d x} \sqrt{-\frac{(b e - a f) (g + h x)}{(f g - e h) (a + b x)}} \right) - \\
& \left( \sqrt{-d g + c h} (C (a d f h - 3 b (d f g + d e h + c f h)) (a d f h + b (d f g + d e h + c f h)) - \right. \\
& \quad \left. 4 b d f h (2 A b d f h - C (b (d e g + c f g + c e h) + a (d f g + d e h + c f h))) (a + b x) \sqrt{\frac{(b g - a h) (c + d x)}{(d g - c h) (a + b x)}} \sqrt{\frac{(b g - a h) (e + f x)}{(f g - e h) (a + b x)}} \right. \\
& \quad \left. \text{EllipticPi} \left[ -\frac{b (d g - c h)}{(b c - a d) h}, \text{ArcSin} \left[ \frac{\sqrt{b c - a d} \sqrt{g + h x}}{\sqrt{-d g + c h} \sqrt{a + b x}} \right], \frac{(b e - a f) (d g - c h)}{(b c - a d) (f g - e h)} \right] \right) / \left( 4 b^2 d^2 \sqrt{b c - a d} f^2 h^3 \sqrt{c + d x} \sqrt{e + f x} \right)
\end{aligned}$$

Result (type 4, 16 659 leaves):

$$\frac{C \sqrt{a+b x} \sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x}}{2 d f h} +$$

$$\begin{aligned}
& \frac{1}{2 b^3 d f h} \left( \left( C (-3 b d f g - 3 b d e h - 3 b c f h + a d f h) (a + b x)^{5/2} \left( d + \frac{b c}{a + b x} - \frac{a d}{a + b x} \right) \left( f + \frac{b e}{a + b x} - \frac{a f}{a + b x} \right) \left( h + \frac{b g}{a + b x} - \frac{a h}{a + b x} \right) \right) \right. \\
& \left. \left( 2 d f h \sqrt{c + \frac{(a + b x) (d - \frac{a d}{a + b x})}{b}} \sqrt{e + \frac{(a + b x) (f - \frac{a f}{a + b x})}{b}} \sqrt{g + \frac{(a + b x) (h - \frac{a h}{a + b x})}{b}} \right) + \right. \\
& \left. \frac{1}{2 d f h \sqrt{c + \frac{(a + b x) (d - \frac{a d}{a + b x})}{b}} \sqrt{e + \frac{(a + b x) (f - \frac{a f}{a + b x})}{b}} \sqrt{g + \frac{(a + b x) (h - \frac{a h}{a + b x})}{b}}} \right. \\
& \left. (a + b x)^{3/2} \sqrt{\left( d + \frac{b c}{a + b x} - \frac{a d}{a + b x} \right) \left( f + \frac{b e}{a + b x} - \frac{a f}{a + b x} \right) \left( h + \frac{b g}{a + b x} - \frac{a h}{a + b x} \right)} \right) \left( 3 b^4 c C d e f g^2 \sqrt{\frac{(b c - a d) (b g - a h) \left( -\frac{d}{-b c + a d} + \frac{1}{a + b x} \right)}{b d g - b c h}} \right. \\
& \left. \left( -\frac{f}{-b e + a f} + \frac{1}{a + b x} \right) \sqrt{\frac{-\frac{h}{-b g + a h} + \frac{1}{a + b x}}{\frac{f}{-b e + a f} - \frac{h}{-b g + a h}}} \right. \\
& \left. - \frac{(b d g - b c h) \text{EllipticE}[\text{ArcSin}\left[\sqrt{\frac{(b e - a f) (h + \frac{b g}{a + b x} - \frac{a h}{a + b x})}{b (-f g + e h)}}\right], \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}]}{(b c - a d) (b g - a h)} - \right. \\
& \left. \left. d \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{(b e - a f) (h + \frac{b g}{a + b x} - \frac{a h}{a + b x})}{b (-f g + e h)}}\right], \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}] \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-b e + a f} + \frac{1}{a + b x}}{\frac{f}{-b e + a f} - \frac{h}{-b g + a h}}} \sqrt{\left( d + \frac{b c - a d}{a + b x} \right) \left( f + \frac{b e - a f}{a + b x} \right) \left( h + \frac{b g - a h}{a + b x} \right)} \right) - \left( 3 a b^3 C d^2 e f g^2 \sqrt{\frac{(b c - a d) (b g - a h) \left( -\frac{d}{-b c + a d} + \frac{1}{a + b x} \right)}{b d g - b c h}} \right. \\
& \left. \left( -\frac{f}{-b e + a f} + \frac{1}{a + b x} \right) \sqrt{\frac{-\frac{h}{-b g + a h} + \frac{1}{a + b x}}{\frac{f}{-b e + a f} - \frac{h}{-b g + a h}}} \right. \\
& \left. - \frac{(b d g - b c h) \text{EllipticE}[\text{ArcSin}\left[\sqrt{\frac{(b e - a f) (h + \frac{b g}{a + b x} - \frac{a h}{a + b x})}{b (-f g + e h)}}\right], \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}]}{(b c - a d) (b g - a h)} \right)
\end{aligned}$$

$$\left. \frac{d \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(b e - a f) (h + \frac{b g}{a+b x} - \frac{a h}{a+b x})}{b (-f g + e h)}} \right], \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)} \right]}{-b c + a d} \right\}$$

$$\left( \sqrt{\frac{-\frac{f}{-b e + a f} + \frac{1}{a + b x}}{-\frac{f}{-b e + a f} + \frac{h}{-b g + a h}}} \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right) \left(h + \frac{b g - a h}{a + b x}\right)} \right) - \left( 3 a b^3 c C d f^2 g^2 \sqrt{\frac{(b c - a d) (b g - a h) \left(-\frac{d}{-b c + a d} + \frac{1}{a + b x}\right)}{b d g - b c h}} \right)$$

$$\left( -\frac{f}{-b e + a f} + \frac{1}{a + b x} \right) \sqrt{\frac{-\frac{h}{-b g + a h} + \frac{1}{a + b x}}{\frac{f}{-b e + a f} - \frac{h}{-b g + a h}}} \left( -\frac{(b d g - b c h) \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(b e - a f) (h + \frac{b g}{a+b x} - \frac{a h}{a+b x})}{b (-f g + e h)}} \right], \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)} \right]}{(b c - a d) (b g - a h)} \right) -$$

$$\left. \frac{d \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(b e - a f) (h + \frac{b g}{a+b x} - \frac{a h}{a+b x})}{b (-f g + e h)}} \right], \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)} \right]}{-b c + a d} \right\}$$

$$\left( \sqrt{\frac{-\frac{f}{-b e + a f} + \frac{1}{a + b x}}{-\frac{f}{-b e + a f} + \frac{h}{-b g + a h}}} \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right) \left(h + \frac{b g - a h}{a + b x}\right)} \right) + \left( 3 a^2 b^2 C d^2 f^2 g^2 \sqrt{\frac{(b c - a d) (b g - a h) \left(-\frac{d}{-b c + a d} + \frac{1}{a + b x}\right)}{b d g - b c h}} \right)$$

$$\left( -\frac{f}{-b e + a f} + \frac{1}{a + b x} \right) \sqrt{\frac{-\frac{h}{-b g + a h} + \frac{1}{a + b x}}{\frac{f}{-b e + a f} - \frac{h}{-b g + a h}}} \left( -\frac{(b d g - b c h) \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(b e - a f) (h + \frac{b g}{a+b x} - \frac{a h}{a+b x})}{b (-f g + e h)}} \right], \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)} \right]}{(b c - a d) (b g - a h)} \right) -$$

$$\left. \frac{d \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(b e - a f) (h + \frac{b g}{a+b x} - \frac{a h}{a+b x})}{b (-f g + e h)}} \right], \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)} \right]}{-b c + a d} \right\}$$

$$\begin{aligned}
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) + \left( 3 b^4 c C d e^2 g h \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+b x}\right)}{b d g - b c h}} \right. \\
& \left. \left( -\frac{f}{-be+af} + \frac{1}{a+b x} \right) \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+b x}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \left( -\frac{(bdg-bch)}{(bc-ad)(bg-ah)} \text{EllipticE}[\text{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg-ah}{a+b x})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] \right. \right. \\
& \left. \left. - \frac{d \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg-ah}{a+b x})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{-bc+ad} \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) - \left( 3 a b^3 C d^2 e^2 g h \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+b x}\right)}{b d g - b c h}} \right. \\
& \left. \left( -\frac{f}{-be+af} + \frac{1}{a+b x} \right) \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+b x}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \left( -\frac{(bdg-bch)}{(bc-ad)(bg-ah)} \text{EllipticE}[\text{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg-ah}{a+b x})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] \right. \right. \\
& \left. \left. - \frac{d \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg-ah}{a+b x})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{-bc+ad} \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) + \left( 3 b^4 c^2 C e f g h \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+b x}\right)}{b d g - b c h}} \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{f}{-be+af} + \frac{1}{a+bx} \right) \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+bx}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \left( -\frac{(bdg-bch) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{(bc-ad)(bg-ah)} \right. \\
& \left. \left. - \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{-bc+ad} \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+bx}}{\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+bx}\right) \left(f + \frac{be-af}{a+bx}\right) \left(h + \frac{bg-ah}{a+bx}\right)} \right) - \left( 10ab^3cCddefgh \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+bx}\right)}{bdg-bch}} \right. \\
& \left. - \frac{(bdg-bch) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{(bc-ad)(bg-ah)} \right. \\
& \left. - \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{-bc+ad} \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+bx}}{\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+bx}\right) \left(f + \frac{be-af}{a+bx}\right) \left(h + \frac{bg-ah}{a+bx}\right)} \right) + \left( 7a^2b^2Cd^2efgh \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+bx}\right)}{bdg-bch}} \right. \\
& \left. - \frac{(bdg-bch) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{(bc-ad)(bg-ah)} \right. \\
& \left. - \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{-bc+ad} \right)
\end{aligned}$$

$$\left. \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}, \frac{(-b c+a d) \left(-f g+e h\right)}{(-b e+a f) \left(-d g+c h\right)}\right], -b c+a d]}{-b c+a d}\right\}$$

$$\left( \sqrt{\frac{-\frac{f}{-b e+a f}+\frac{1}{a+b x}}{-\frac{f}{-b e+a f}+\frac{h}{-b g+a h}}} \sqrt{\left(d+\frac{b c-a d}{a+b x}\right) \left(f+\frac{b e-a f}{a+b x}\right) \left(h+\frac{b g-a h}{a+b x}\right)} \right) - \left( 3 a b^3 c^2 C f^2 g h \sqrt{\frac{(b c-a d) (b g-a h) \left(-\frac{d}{-b c+a d}+\frac{1}{a+b x}\right)}{b d g-b c h}} \right)$$

$$\left( -\frac{f}{-b e+a f}+\frac{1}{a+b x} \right) \sqrt{\frac{-\frac{h}{-b g+a h}+\frac{1}{a+b x}}{\frac{f}{-b e+a f}-\frac{h}{-b g+a h}}} \left( -\frac{(b d g-b c h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}, \frac{(-b c+a d) \left(-f g+e h\right)}{(-b e+a f) \left(-d g+c h\right)}\right], (b c-a d) (b g-a h)}{(b c-a d) (b g-a h)} \right) -$$

$$\left. \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}, \frac{(-b c+a d) \left(-f g+e h\right)}{(-b e+a f) \left(-d g+c h\right)}\right], -b c+a d]}{-b c+a d}\right\}$$

$$\left( \sqrt{\frac{-\frac{f}{-b e+a f}+\frac{1}{a+b x}}{-\frac{f}{-b e+a f}+\frac{h}{-b g+a h}}} \sqrt{\left(d+\frac{b c-a d}{a+b x}\right) \left(f+\frac{b e-a f}{a+b x}\right) \left(h+\frac{b g-a h}{a+b x}\right)} \right) + \left( 7 a^2 b^2 c C d f^2 g h \sqrt{\frac{(b c-a d) (b g-a h) \left(-\frac{d}{-b c+a d}+\frac{1}{a+b x}\right)}{b d g-b c h}} \right)$$

$$\left( -\frac{f}{-b e+a f}+\frac{1}{a+b x} \right) \sqrt{\frac{-\frac{h}{-b g+a h}+\frac{1}{a+b x}}{\frac{f}{-b e+a f}-\frac{h}{-b g+a h}}} \left( -\frac{(b d g-b c h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}, \frac{(-b c+a d) \left(-f g+e h\right)}{(-b e+a f) \left(-d g+c h\right)}\right], (b c-a d) (b g-a h)}{(b c-a d) (b g-a h)} \right) -$$

$$\left. \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}, \frac{(-b c+a d) \left(-f g+e h\right)}{(-b e+a f) \left(-d g+c h\right)}\right], -b c+a d]}{-b c+a d}\right\}$$

$$\begin{aligned}
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) - \left( 4 a^3 b C d^2 f^2 g h \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+b x}\right)}{bdg-bch}} \right. \\
& \left. \left( -\frac{f}{-be+af} + \frac{1}{a+b x} \right) \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+b x}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \left( -\frac{(bdg-bch)}{(bc-ad)(bg-ah)} \text{EllipticE}[\text{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg-ah}{a+b x})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] \right. \right. \\
& \left. \left. \left. \frac{d \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg-ah}{a+b x})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{-bc+ad} \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) - \left( 3 a b^3 c C d e^2 h^2 \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+b x}\right)}{bdg-bch}} \right. \\
& \left. \left( -\frac{f}{-be+af} + \frac{1}{a+b x} \right) \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+b x}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \left( -\frac{(bdg-bch)}{(bc-ad)(bg-ah)} \text{EllipticE}[\text{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg-ah}{a+b x})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] \right. \right. \\
& \left. \left. \left. \frac{d \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg-ah}{a+b x})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{-bc+ad} \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) + \left( 3 a^2 b^2 C d^2 e^2 h^2 \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+b x}\right)}{bdg-bch}} \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{f}{-be+af} + \frac{1}{a+bx} \right) \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+bx}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \left( -\frac{(bdg-bch) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{(bc-ad)(bg-ah)} \right. \\
& \left. \left. - \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{-bc+ad} \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+bx}}{\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+bx}\right) \left(f + \frac{be-af}{a+bx}\right) \left(h + \frac{bg-ah}{a+bx}\right)} - \left(3ab^3c^2cef h^2 \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+bx}\right)}{bdg-bch}} \right. \right. \\
& \left. \left. - \frac{(bdg-bch) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{(bc-ad)(bg-ah)} \right. \right. \\
& \left. \left. - \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{-bc+ad} \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+bx}}{\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+bx}\right) \left(f + \frac{be-af}{a+bx}\right) \left(h + \frac{bg-ah}{a+bx}\right)} + \left(7a^2b^2cCdefh^2 \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+bx}\right)}{bdg-bch}} \right. \right. \\
& \left. \left. - \frac{(bdg-bch) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{(bc-ad)(bg-ah)} \right. \right. \\
& \left. \left. - \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{-bc+ad} \right) \right)
\end{aligned}$$

$$\left. \frac{d \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(b e - a f) (h + \frac{b g}{a+b x} - \frac{a h}{a+b x})}{b (-f g + e h)}} \right], \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)} \right]}{-b c + a d} \right\}$$

$$\left( \sqrt{\frac{-\frac{f}{-b e + a f} + \frac{1}{a + b x}}{-\frac{f}{-b e + a f} + \frac{h}{-b g + a h}}} \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right) \left(h + \frac{b g - a h}{a + b x}\right)} \right) - \left( 4 a^3 b C d^2 e f h^2 \sqrt{\frac{(b c - a d) (b g - a h) \left(-\frac{d}{-b c + a d} + \frac{1}{a + b x}\right)}{b d g - b c h}} \right.$$

$$\left( -\frac{f}{-b e + a f} + \frac{1}{a + b x} \right) \sqrt{\frac{-\frac{h}{-b g + a h} + \frac{1}{a + b x}}{\frac{f}{-b e + a f} - \frac{h}{-b g + a h}}} \left( -\frac{(b d g - b c h) \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(b e - a f) (h + \frac{b g}{a+b x} - \frac{a h}{a+b x})}{b (-f g + e h)}} \right], \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)} \right]}{(b c - a d) (b g - a h)} \right) -$$

$$\left. \frac{d \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(b e - a f) (h + \frac{b g}{a+b x} - \frac{a h}{a+b x})}{b (-f g + e h)}} \right], \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)} \right]}{-b c + a d} \right\}$$

$$\left( \sqrt{\frac{-\frac{f}{-b e + a f} + \frac{1}{a + b x}}{-\frac{f}{-b e + a f} + \frac{h}{-b g + a h}}} \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right) \left(h + \frac{b g - a h}{a + b x}\right)} \right) + \left( 3 a^2 b^2 c^2 C f^2 h^2 \sqrt{\frac{(b c - a d) (b g - a h) \left(-\frac{d}{-b c + a d} + \frac{1}{a + b x}\right)}{b d g - b c h}} \right)$$

$$\left( -\frac{f}{-b e + a f} + \frac{1}{a + b x} \right) \sqrt{\frac{-\frac{h}{-b g + a h} + \frac{1}{a + b x}}{\frac{f}{-b e + a f} - \frac{h}{-b g + a h}}} \left( -\frac{(b d g - b c h) \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(b e - a f) (h + \frac{b g}{a+b x} - \frac{a h}{a+b x})}{b (-f g + e h)}} \right], \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)} \right]}{(b c - a d) (b g - a h)} \right) -$$

$$\left. \frac{d \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(b e - a f) (h + \frac{b g}{a+b x} - \frac{a h}{a+b x})}{b (-f g + e h)}} \right], \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)} \right]}{-b c + a d} \right\}$$

$$\begin{aligned}
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) - \left( 4 a^3 b c C d f^2 h^2 \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+b x}\right)}{bdg-bch}} \right. \\
& \left. \left( -\frac{f}{-be+af} + \frac{1}{a+b x} \right) \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+b x}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \left( -\frac{(bdg-bch)}{(bc-ad)(bg-ah)} \text{EllipticE}[\text{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg-ah}{a+b x})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] \right. \right. \\
& \left. \left. \left. \frac{d \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg-ah}{a+b x})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{-bc+ad} \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) + \left( a^4 C d^2 f^2 h^2 \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+b x}\right)}{bdg-bch}} \right. \\
& \left. \left( -\frac{f}{-be+af} + \frac{1}{a+b x} \right) \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+b x}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \left( -\frac{(bdg-bch)}{(bc-ad)(bg-ah)} \text{EllipticE}[\text{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg-ah}{a+b x})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] \right. \right. \\
& \left. \left. \left. \frac{d \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg-ah}{a+b x})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{-bc+ad} \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) + \left( 2 b^3 c C d e f g h \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+b x}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \left( -\frac{h}{-bg+ah} + \frac{1}{a+b x} \right) \text{EllipticF}[\text{ArcSin}\left[ \sqrt{\frac{(-be+af)(-h-\frac{bg}{a+b x}+\frac{ah}{a+b x})}{b(-fg+eh)}} \right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] / \\
& \left( \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+b x}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) - \left( 2ab^2Cd^2efgh \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+b x}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \right. \\
& \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \left( -\frac{h}{-bg+ah} + \frac{1}{a+b x} \right) \text{EllipticF}[\text{ArcSin}\left[ \sqrt{\frac{(-be+af)(-h-\frac{bg}{a+b x}+\frac{ah}{a+b x})}{b(-fg+eh)}} \right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] / \\
& \left( \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+b x}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) - \left( 2ab^2CcCd^2f^2gh \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+b x}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \right. \\
& \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \left( -\frac{h}{-bg+ah} + \frac{1}{a+b x} \right) \text{EllipticF}[\text{ArcSin}\left[ \sqrt{\frac{(-be+af)(-h-\frac{bg}{a+b x}+\frac{ah}{a+b x})}{b(-fg+eh)}} \right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] / \\
& \left( \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+b x}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) + \left( 2a^2bCd^2f^2gh \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+b x}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \right. \\
& \left( -\frac{h}{-bg+ah} + \frac{1}{a+b x} \right) \text{EllipticF}[\text{ArcSin}\left[ \sqrt{\frac{(-be+af)(-h-\frac{bg}{a+b x}+\frac{ah}{a+b x})}{b(-fg+eh)}} \right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] / \\
& \left( \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+b x}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) - \left( 2ab^2CcCd^2efh^2 \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+b x}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \right. \\
& \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \left( -\frac{h}{-bg+ah} + \frac{1}{a+b x} \right) \text{EllipticF}[\text{ArcSin}\left[ \sqrt{\frac{(-be+af)(-h-\frac{bg}{a+b x}+\frac{ah}{a+b x})}{b(-fg+eh)}} \right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] /
\end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+b x}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) + \left( 2a^2 b C d^2 e f h^2 \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+b x}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \right. \\
& \left. \left( -\frac{h}{-bg+ah} + \frac{1}{a+b x} \right) \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{(-be+af) (-h - \frac{bg}{a+b x} + \frac{ah}{a+b x})}{b (-fg+eh)}}\right], \frac{(-bc+ad) (-fg+eh)}{(-be+af) (-dg+ch)}] \right) / \\
& \left( \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+b x}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) + \left( 2a^2 b c C d f^2 h^2 \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+b x}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \right. \\
& \left. \left( -\frac{h}{-bg+ah} + \frac{1}{a+b x} \right) \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{(-be+af) (-h - \frac{bg}{a+b x} + \frac{ah}{a+b x})}{b (-fg+eh)}}\right], \frac{(-bc+ad) (-fg+eh)}{(-be+af) (-dg+ch)}] \right) / \\
& \left( \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+b x}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) - \\
& \left( 2a^3 C d^2 f^2 h^2 \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+b x}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \left( -\frac{h}{-bg+ah} + \frac{1}{a+b x} \right) \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{(-be+af) (-h - \frac{bg}{a+b x} + \frac{ah}{a+b x})}{b (-fg+eh)}}\right], \right. \\
& \left. \left. \frac{(-bc+ad) (-fg+eh)}{(-be+af) (-dg+ch)} \right) \right) / \left( \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+b x}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) + \\
& \left( 2b^2 C d^2 e f g (-bg+ah) \left( -\frac{f}{-be+af} + \frac{h}{-bg+ah} \right) \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+b x}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{-\frac{\left( -\frac{f}{-be+af} + \frac{1}{a+b x} \right) \left( -\frac{h}{-bg+ah} + \frac{1}{a+b x} \right)}{\left( -\frac{f}{-be+af} + \frac{h}{-bg+ah} \right)^2}} \right. \\
& \left. \text{EllipticPi}\left[ -\frac{-bf g + be h}{(-be+af) h}, \text{ArcSin}\left[\sqrt{\frac{(-be+af) (-h - \frac{bg}{a+b x} + \frac{ah}{a+b x})}{b (-fg+eh)}}\right], \frac{(-bc+ad) (-fg+eh)}{(-be+af) (-dg+ch)} \right] \right)
\end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{\left( d + \frac{b c - a d}{a + b x} \right) \left( f + \frac{b e - a f}{a + b x} \right) \left( h + \frac{b g - a h}{a + b x} \right)} \right) + \left( 2 b^2 c d f^2 g (-b g + a h) \left( -\frac{f}{-b e + a f} + \frac{h}{-b g + a h} \right) \right. \\
& \quad \left. \sqrt{\frac{-\frac{d}{-b c+a d} + \frac{1}{a+b x}}{-\frac{d}{-b c+a d} + \frac{h}{-b g+a h}}} \sqrt{-\frac{\left( -\frac{f}{-b e+a f} + \frac{1}{a+b x} \right) \left( -\frac{h}{-b g+a h} + \frac{1}{a+b x} \right)}{\left( -\frac{f}{-b e+a f} + \frac{h}{-b g+a h} \right)^2}} \operatorname{EllipticPi}\left[ -\frac{-b f g + b e h}{(-b e + a f) h}, \right. \right. \\
& \quad \left. \left. \operatorname{ArcSin}\left[ \sqrt{\frac{(-b e + a f) \left( -h - \frac{b g}{a+b x} + \frac{a h}{a+b x} \right)}{b (-f g + e h)}}, \frac{(-b c + a d) \left( -f g + e h \right)}{(-b e + a f) \left( -d g + c h \right)} \right] \right] \right) / \left( \sqrt{\left( d + \frac{b c - a d}{a + b x} \right) \left( f + \frac{b e - a f}{a + b x} \right) \left( h + \frac{b g - a h}{a + b x} \right)} \right) - \\
& \quad \left( 2 a b c d^2 f^2 g (-b g + a h) \left( -\frac{f}{-b e + a f} + \frac{h}{-b g + a h} \right) \sqrt{\frac{-\frac{d}{-b c+a d} + \frac{1}{a+b x}}{-\frac{d}{-b c+a d} + \frac{h}{-b g+a h}}} \sqrt{-\frac{\left( -\frac{f}{-b e+a f} + \frac{1}{a+b x} \right) \left( -\frac{h}{-b g+a h} + \frac{1}{a+b x} \right)}{\left( -\frac{f}{-b e+a f} + \frac{h}{-b g+a h} \right)^2}} \right. \\
& \quad \left. \operatorname{EllipticPi}\left[ -\frac{-b f g + b e h}{(-b e + a f) h}, \operatorname{ArcSin}\left[ \sqrt{\frac{(-b e + a f) \left( -h - \frac{b g}{a+b x} + \frac{a h}{a+b x} \right)}{b (-f g + e h)}}, \frac{(-b c + a d) \left( -f g + e h \right)}{(-b e + a f) \left( -d g + c h \right)} \right] \right] \right) / \\
& \quad \left( \sqrt{\left( d + \frac{b c - a d}{a + b x} \right) \left( f + \frac{b e - a f}{a + b x} \right) \left( h + \frac{b g - a h}{a + b x} \right)} \right) + \left( 3 b^2 c d^2 f^2 g^2 (-b g + a h) \left( -\frac{f}{-b e + a f} + \frac{h}{-b g + a h} \right) \sqrt{\frac{-\frac{d}{-b c+a d} + \frac{1}{a+b x}}{-\frac{d}{-b c+a d} + \frac{h}{-b g+a h}}} \right. \\
& \quad \left. \sqrt{-\frac{\left( -\frac{f}{-b e+a f} + \frac{1}{a+b x} \right) \left( -\frac{h}{-b g+a h} + \frac{1}{a+b x} \right)}{\left( -\frac{f}{-b e+a f} + \frac{h}{-b g+a h} \right)^2}} \operatorname{EllipticPi}\left[ -\frac{-b f g + b e h}{(-b e + a f) h}, \operatorname{ArcSin}\left[ \sqrt{\frac{(-b e + a f) \left( -h - \frac{b g}{a+b x} + \frac{a h}{a+b x} \right)}{b (-f g + e h)}}, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{(-b c + a d) \left( -f g + e h \right)}{(-b e + a f) \left( -d g + c h \right)} \right] \right) / \left( h \sqrt{\left( d + \frac{b c - a d}{a + b x} \right) \left( f + \frac{b e - a f}{a + b x} \right) \left( h + \frac{b g - a h}{a + b x} \right)} \right) + \right. \\
& \quad \left( 3 b^2 c d^2 e^2 h (-b g + a h) \left( -\frac{f}{-b e + a f} + \frac{h}{-b g + a h} \right) \sqrt{\frac{-\frac{d}{-b c+a d} + \frac{1}{a+b x}}{-\frac{d}{-b c+a d} + \frac{h}{-b g+a h}}} \sqrt{-\frac{\left( -\frac{f}{-b e+a f} + \frac{1}{a+b x} \right) \left( -\frac{h}{-b g+a h} + \frac{1}{a+b x} \right)}{\left( -\frac{f}{-b e+a f} + \frac{h}{-b g+a h} \right)^2}} \right)
\end{aligned}$$

$$\begin{aligned}
& \text{EllipticPi}\left[-\frac{-b f g + b e h}{(-b e + a f) h}, \text{ArcSin}\left[\sqrt{\frac{(-b e + a f) \left(-h - \frac{b g}{a+b x} + \frac{a h}{a+b x}\right)}{b (-f g + e h)}}\right], \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}\right] / \\
& \left(\sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right) \left(h + \frac{b g - a h}{a+b x}\right)} + \left(2 b^2 c C d e f h (-b g + a h) \left(-\frac{f}{-b e + a f} + \frac{h}{-b g + a h}\right)\right.\right. \\
& \left.\left. - \frac{d}{-b c + a d} + \frac{1}{a+b x}\right) \sqrt{-\frac{\left(-\frac{f}{-b e + a f} + \frac{1}{a+b x}\right) \left(-\frac{h}{-b g + a h} + \frac{1}{a+b x}\right)}{\left(-\frac{f}{-b e + a f} + \frac{h}{-b g + a h}\right)^2}} \text{EllipticPi}\left[-\frac{-b f g + b e h}{(-b e + a f) h},\right.\right. \\
& \left.\left. \text{ArcSin}\left[\sqrt{\frac{(-b e + a f) \left(-h - \frac{b g}{a+b x} + \frac{a h}{a+b x}\right)}{b (-f g + e h)}}\right], \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}\right] / \left(\sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right) \left(h + \frac{b g - a h}{a+b x}\right)} - \right. \right. \\
& \left.\left. 2 a b C d^2 e f h (-b g + a h) \left(-\frac{f}{-b e + a f} + \frac{h}{-b g + a h}\right) \sqrt{\frac{-\frac{d}{-b c + a d} + \frac{1}{a+b x}}{-\frac{d}{-b c + a d} + \frac{h}{-b g + a h}}} \sqrt{-\frac{\left(-\frac{f}{-b e + a f} + \frac{1}{a+b x}\right) \left(-\frac{h}{-b g + a h} + \frac{1}{a+b x}\right)}{\left(-\frac{f}{-b e + a f} + \frac{h}{-b g + a h}\right)^2}} \right. \right. \\
& \left.\left. \text{EllipticPi}\left[-\frac{-b f g + b e h}{(-b e + a f) h}, \text{ArcSin}\left[\sqrt{\frac{(-b e + a f) \left(-h - \frac{b g}{a+b x} + \frac{a h}{a+b x}\right)}{b (-f g + e h)}}\right], \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}\right]\right) / \right. \right. \\
& \left.\left. \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right) \left(h + \frac{b g - a h}{a+b x}\right)} + \left(3 b^2 c^2 f^2 h (-b g + a h) \left(-\frac{f}{-b e + a f} + \frac{h}{-b g + a h}\right) \sqrt{\frac{-\frac{d}{-b c + a d} + \frac{1}{a+b x}}{-\frac{d}{-b c + a d} + \frac{h}{-b g + a h}}\right.\right. \right. \\
& \left.\left.\left. - \frac{\left(-\frac{f}{-b e + a f} + \frac{1}{a+b x}\right) \left(-\frac{h}{-b g + a h} + \frac{1}{a+b x}\right)}{\left(-\frac{f}{-b e + a f} + \frac{h}{-b g + a h}\right)^2}} \text{EllipticPi}\left[-\frac{-b f g + b e h}{(-b e + a f) h}, \text{ArcSin}\left[\sqrt{\frac{(-b e + a f) \left(-h - \frac{b g}{a+b x} + \frac{a h}{a+b x}\right)}{b (-f g + e h)}}\right], \right.\right. \right. \\
& \left.\left.\left. \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}\right]\right) / \left(\sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right) \left(h + \frac{b g - a h}{a+b x}\right)} - \right. \right. \right. 
\end{aligned}$$

$$\begin{aligned}
& \left( 2 a b c C d f^2 h (-b g + a h) \left( -\frac{f}{-b e + a f} + \frac{h}{-b g + a h} \right) \sqrt{\frac{-\frac{d}{-b c + a d} + \frac{1}{a+b x}}{-\frac{d}{-b c + a d} + \frac{h}{-b g + a h}}} \sqrt{-\frac{\left( -\frac{f}{-b e + a f} + \frac{1}{a+b x} \right) \left( -\frac{h}{-b g + a h} + \frac{1}{a+b x} \right)}{\left( -\frac{f}{-b e + a f} + \frac{h}{-b g + a h} \right)^2}} \right. \\
& \left. \text{EllipticPi}\left[ -\frac{-b f g + b e h}{(-b e + a f) h}, \text{ArcSin}\left[ \sqrt{\frac{(-b e + a f) \left( -h - \frac{b g}{a+b x} + \frac{a h}{a+b x} \right)}{b (-f g + e h)}}, \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)} \right] \right] \right) / \\
& \left( \sqrt{\left( d + \frac{b c - a d}{a + b x} \right) \left( f + \frac{b e - a f}{a + b x} \right) \left( h + \frac{b g - a h}{a + b x} \right)} + \left( 8 A b^2 d^2 f^2 h (-b g + a h) \left( -\frac{f}{-b e + a f} + \frac{h}{-b g + a h} \right) \sqrt{\frac{-\frac{d}{-b c + a d} + \frac{1}{a+b x}}{-\frac{d}{-b c + a d} + \frac{h}{-b g + a h}}} \right. \right. \\
& \left. \left. \sqrt{-\frac{\left( -\frac{f}{-b e + a f} + \frac{1}{a+b x} \right) \left( -\frac{h}{-b g + a h} + \frac{1}{a+b x} \right)}{\left( -\frac{f}{-b e + a f} + \frac{h}{-b g + a h} \right)^2}} \text{EllipticPi}\left[ -\frac{-b f g + b e h}{(-b e + a f) h}, \text{ArcSin}\left[ \sqrt{\frac{(-b e + a f) \left( -h - \frac{b g}{a+b x} + \frac{a h}{a+b x} \right)}{b (-f g + e h)}}, \right. \right. \right. \\
& \left. \left. \left. \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)} \right] \right) \right) / \left( \sqrt{\left( d + \frac{b c - a d}{a + b x} \right) \left( f + \frac{b e - a f}{a + b x} \right) \left( h + \frac{b g - a h}{a + b x} \right)} - \right. \\
& \left. \left( a^2 C d^2 f^2 h (-b g + a h) \left( -\frac{f}{-b e + a f} + \frac{h}{-b g + a h} \right) \sqrt{\frac{-\frac{d}{-b c + a d} + \frac{1}{a+b x}}{-\frac{d}{-b c + a d} + \frac{h}{-b g + a h}}} \sqrt{-\frac{\left( -\frac{f}{-b e + a f} + \frac{1}{a+b x} \right) \left( -\frac{h}{-b g + a h} + \frac{1}{a+b x} \right)}{\left( -\frac{f}{-b e + a f} + \frac{h}{-b g + a h} \right)^2}} \text{EllipticPi}\left[ -\frac{-b f g + b e h}{(-b e + a f) h}, \right. \right. \right. \\
& \left. \left. \left. \text{ArcSin}\left[ \sqrt{\frac{(-b e + a f) \left( -h - \frac{b g}{a+b x} + \frac{a h}{a+b x} \right)}{b (-f g + e h)}}, \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)} \right] \right) / \left( \sqrt{\left( d + \frac{b c - a d}{a + b x} \right) \left( f + \frac{b e - a f}{a + b x} \right) \left( h + \frac{b g - a h}{a + b x} \right)} \right) \right)
\end{aligned}$$

Problem 33: Result more than twice size of optimal antiderivative.

$$\int \frac{A + C x^2}{\sqrt{a + b x} \sqrt{c + d x} \sqrt{e + f x} \sqrt{g + h x}} dx$$

Optimal (type 4, 757 leaves, 8 steps):

$$\begin{aligned}
& \frac{C \sqrt{d g - c h} \sqrt{f g - e h} \sqrt{a + b x}}{b f h \sqrt{c + d x}} \sqrt{-\frac{(d e - c f) (g + h x)}{(f g - e h) (c + d x)}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{d g - c h} \sqrt{e + f x}}{\sqrt{f g - e h} \sqrt{c + d x}}\right], \frac{(b c - a d) (f g - e h)}{(b e - a f) (d g - c h)}\right] \\
& + \frac{b d f h \sqrt{\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}} \sqrt{g + h x}}{+} \\
& \left( (a^2 C f h + a b C (f g + e h) - b^2 (C e g - 2 A f h)) \sqrt{\frac{(b e - a f) (c + d x)}{(d e - c f) (a + b x)}} \sqrt{g + h x} \right. \\
& \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{b g - a h} \sqrt{e + f x}}{\sqrt{f g - e h} \sqrt{a + b x}}\right], -\frac{(b c - a d) (f g - e h)}{(d e - c f) (b g - a h)}\right] \right) / \left( b^2 f h \sqrt{b g - a h} \sqrt{f g - e h} \sqrt{c + d x} \sqrt{-\frac{(b e - a f) (g + h x)}{(f g - e h) (a + b x)}} \right) - \\
& \left( C \sqrt{-d g + c h} (a d f h + b (d f g + d e h + c f h)) (a + b x) \sqrt{\frac{(b g - a h) (c + d x)}{(d g - c h) (a + b x)}} \sqrt{\frac{(b g - a h) (e + f x)}{(f g - e h) (a + b x)}} \right. \\
& \left. \operatorname{EllipticPi}\left[-\frac{b (d g - c h)}{(b c - a d) h}, \operatorname{ArcSin}\left[\frac{\sqrt{b c - a d} \sqrt{g + h x}}{\sqrt{-d g + c h} \sqrt{a + b x}}\right], \frac{(b e - a f) (d g - c h)}{(b c - a d) (f g - e h)}\right] \right) / \left( b^2 d \sqrt{b c - a d} f h^2 \sqrt{c + d x} \sqrt{e + f x} \right)
\end{aligned}$$

Result (type 4, 6207 leaves):

$$\begin{aligned}
& -\frac{1}{b^3} 2 \left( -\frac{C (a + b x)^{5/2} \left(d + \frac{b c}{a + b x} - \frac{a d}{a + b x}\right) \left(f + \frac{b e}{a + b x} - \frac{a f}{a + b x}\right) \left(h + \frac{b g}{a + b x} - \frac{a h}{a + b x}\right)}{2 d f h \sqrt{c + \frac{(a + b x) \left(d - \frac{a d}{a + b x}\right)}{b}} \sqrt{e + \frac{(a + b x) \left(f - \frac{a f}{a + b x}\right)}{b}} \sqrt{g + \frac{(a + b x) \left(h - \frac{a h}{a + b x}\right)}{b}}} + \frac{1}{2 d f h \sqrt{c + \frac{(a + b x) \left(d - \frac{a d}{a + b x}\right)}{b}} \sqrt{e + \frac{(a + b x) \left(f - \frac{a f}{a + b x}\right)}{b}} \sqrt{g + \frac{(a + b x) \left(h - \frac{a h}{a + b x}\right)}{b}}} \right. \\
& \left. (a + b x)^{3/2} \sqrt{\left(d + \frac{b c}{a + b x} - \frac{a d}{a + b x}\right) \left(f + \frac{b e}{a + b x} - \frac{a f}{a + b x}\right) \left(h + \frac{b g}{a + b x} - \frac{a h}{a + b x}\right)} \left(b^3 c C e g \sqrt{\frac{(b c - a d) (b g - a h) \left(-\frac{d}{-b c + a d} + \frac{1}{a + b x}\right)}{b d g - b c h}}\right. \right. \\
& \left. \left. \left(-\frac{f}{-b e + a f} + \frac{1}{a + b x}\right) \sqrt{\frac{-\frac{h}{-b g + a h} + \frac{1}{a + b x}}{\frac{f}{-b e + a f} - \frac{h}{-b g + a h}}} \right. \right. \\
& \left. \left. - \frac{(b d g - b c h) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(b e - a f) \left(h + \frac{b g}{a + b x} - \frac{a h}{a + b x}\right)}{b (-f g + e h)}}\right], \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}\right]}{(b c - a d) (b g - a h)} \right. \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) (-f g+e h)}{(-b e+a f) (-d g+c h)}}}{-b c+a d}\right\} / \\
& \left( \sqrt{\frac{-\frac{f}{-b e+a f}+\frac{1}{a+b x}}{-\frac{f}{-b e+a f}+\frac{h}{-b g+a h}}} \sqrt{\left(d+\frac{b c-a d}{a+b x}\right)\left(f+\frac{b e-a f}{a+b x}\right)\left(h+\frac{b g-a h}{a+b x}\right)} - \left(a b^2 C d e g \sqrt{\frac{(b c-a d) (b g-a h) \left(-\frac{d}{-b c+a d}+\frac{1}{a+b x}\right)}{b d g-b c h}}\right.\right. \\
& \left.\left. \left(-\frac{f}{-b e+a f}+\frac{1}{a+b x}\right) \sqrt{\frac{-\frac{h}{-b g+a h}+\frac{1}{a+b x}}{\frac{f}{-b e+a f}-\frac{h}{-b g+a h}}} \left(-\frac{(b d g-b c h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) (-f g+e h)}{(-b e+a f) (-d g+c h)}}}{(b c-a d) (b g-a h)} - \right.\right. \right. \\
& \left.\left.\left. d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) (-f g+e h)}{(-b e+a f) (-d g+c h)}}}{-b c+a d}\right)\right\} / \\
& \left( \sqrt{\frac{-\frac{f}{-b e+a f}+\frac{1}{a+b x}}{-\frac{f}{-b e+a f}+\frac{h}{-b g+a h}}} \sqrt{\left(d+\frac{b c-a d}{a+b x}\right)\left(f+\frac{b e-a f}{a+b x}\right)\left(h+\frac{b g-a h}{a+b x}\right)} - \left(a b^2 c C f g \sqrt{\frac{(b c-a d) (b g-a h) \left(-\frac{d}{-b c+a d}+\frac{1}{a+b x}\right)}{b d g-b c h}}\right.\right. \\
& \left.\left. \left(-\frac{f}{-b e+a f}+\frac{1}{a+b x}\right) \sqrt{\frac{-\frac{h}{-b g+a h}+\frac{1}{a+b x}}{\frac{f}{-b e+a f}-\frac{h}{-b g+a h}}} \left(-\frac{(b d g-b c h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) (-f g+e h)}{(-b e+a f) (-d g+c h)}}}{(b c-a d) (b g-a h)} - \right.\right. \right. \\
& \left.\left.\left. d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) (-f g+e h)}{(-b e+a f) (-d g+c h)}}}{-b c+a d}\right)\right\}
\end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) + \left( a^2 b c d f g \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+b x}\right)}{bdg-bch}} \right. \\
& \left( -\frac{f}{-be+af} + \frac{1}{a+b x} \right) \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+b x}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \left( -\frac{(bdg-bch) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+b x}-\frac{ah}{a+b x})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{(bc-ad)(bg-ah)} \right. \\
& \left. \left. \left. d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+b x}-\frac{ah}{a+b x})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) - \left( a b^2 c c e h \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+b x}\right)}{bdg-bch}} \right. \\
& \left( -\frac{f}{-be+af} + \frac{1}{a+b x} \right) \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+b x}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \left( -\frac{(bdg-bch) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+b x}-\frac{ah}{a+b x})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{(bc-ad)(bg-ah)} \right. \\
& \left. \left. \left. d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+b x}-\frac{ah}{a+b x})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) + \left( a^2 b c d e h \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+b x}\right)}{bdg-bch}} \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{f}{-b e + a f} + \frac{1}{a + b x} \right) \sqrt{\frac{-\frac{h}{-b g + a h} + \frac{1}{a+b x}}{\frac{f}{-b e + a f} - \frac{h}{-b g + a h}}} \left( -\frac{(b d g - b c h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e - a f) \left(h + \frac{b g}{a+b x} - \frac{a h}{a+b x}\right)}{b (-f g + e h)}}, \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}}\right]}{(b c - a d) (b g - a h)} \right. \\
& \left. \left. - \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e - a f) \left(h + \frac{b g}{a+b x} - \frac{a h}{a+b x}\right)}{b (-f g + e h)}}, \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}}\right]}{-b c + a d} \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-b e + a f} + \frac{1}{a+b x}}{\frac{f}{-b e + a f} + \frac{h}{-b g + a h}}} \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right) \left(h + \frac{b g - a h}{a + b x}\right)} + a^2 b c C f h \sqrt{\frac{(b c - a d) (b g - a h) \left(-\frac{d}{-b c + a d} + \frac{1}{a+b x}\right)}{b d g - b c h}} \right. \\
& \left. - \left( -\frac{f}{-b e + a f} + \frac{1}{a + b x} \right) \sqrt{\frac{-\frac{h}{-b g + a h} + \frac{1}{a+b x}}{\frac{f}{-b e + a f} - \frac{h}{-b g + a h}}} \left( -\frac{(b d g - b c h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e - a f) \left(h + \frac{b g}{a+b x} - \frac{a h}{a+b x}\right)}{b (-f g + e h)}}, \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}}\right]}{(b c - a d) (b g - a h)} \right. \right. \\
& \left. \left. - \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e - a f) \left(h + \frac{b g}{a+b x} - \frac{a h}{a+b x}\right)}{b (-f g + e h)}}, \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}}\right]}{-b c + a d} \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-b e + a f} + \frac{1}{a+b x}}{\frac{f}{-b e + a f} + \frac{h}{-b g + a h}}} \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right) \left(h + \frac{b g - a h}{a + b x}\right)} - a^3 C d f h \sqrt{\frac{(b c - a d) (b g - a h) \left(-\frac{d}{-b c + a d} + \frac{1}{a+b x}\right)}{b d g - b c h}} \right. \\
& \left. - \left( -\frac{f}{-b e + a f} + \frac{1}{a + b x} \right) \sqrt{\frac{-\frac{h}{-b g + a h} + \frac{1}{a+b x}}{\frac{f}{-b e + a f} - \frac{h}{-b g + a h}}} \left( -\frac{(b d g - b c h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e - a f) \left(h + \frac{b g}{a+b x} - \frac{a h}{a+b x}\right)}{b (-f g + e h)}}, \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}}\right]}{(b c - a d) (b g - a h)} \right. \right. \right)
\end{aligned}$$

$$\begin{aligned}
& \left. d \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(b e - a f) \left( h + \frac{b g}{a+b x} - \frac{a h}{a+b x} \right)}{b (-f g + e h)}} \right], \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)} \right] \right\} / \\
& \left( \sqrt{\frac{-\frac{f}{-b e + a f} + \frac{1}{a+b x}}{-\frac{f}{-b e + a f} + \frac{h}{-b g + a h}}} \sqrt{\left( d + \frac{b c - a d}{a+b x} \right) \left( f + \frac{b e - a f}{a+b x} \right) \left( h + \frac{b g - a h}{a+b x} \right)} + \left( 2 A b^2 d f h \sqrt{\frac{-\frac{d}{-b c + a d} + \frac{1}{a+b x}}{-\frac{d}{-b c + a d} + \frac{h}{-b g + a h}}} \sqrt{\frac{-\frac{f}{-b e + a f} + \frac{1}{a+b x}}{-\frac{f}{-b e + a f} + \frac{h}{-b g + a h}}} \right. \right. \\
& \left. \left. \left( -\frac{h}{-b g + a h} + \frac{1}{a+b x} \right) \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(-b e + a f) \left( -h - \frac{b g}{a+b x} + \frac{a h}{a+b x} \right)}{b (-f g + e h)}}, \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)} \right] \right\} / \right. \right. \\
& \left. \left. \left( \sqrt{\frac{-\frac{h}{-b g + a h} + \frac{1}{a+b x}}{\frac{f}{-b e + a f} - \frac{h}{-b g + a h}}} \sqrt{\left( d + \frac{b c - a d}{a+b x} \right) \left( f + \frac{b e - a f}{a+b x} \right) \left( h + \frac{b g - a h}{a+b x} \right)} \right) + \right. \right. \\
& \left. \left. \left( 2 a^2 C d f h \sqrt{\frac{-\frac{d}{-b c + a d} + \frac{1}{a+b x}}{-\frac{d}{-b c + a d} + \frac{h}{-b g + a h}}} \sqrt{\frac{-\frac{f}{-b e + a f} + \frac{1}{a+b x}}{-\frac{f}{-b e + a f} + \frac{h}{-b g + a h}}} \left( -\frac{h}{-b g + a h} + \frac{1}{a+b x} \right) \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(-b e + a f) \left( -h - \frac{b g}{a+b x} + \frac{a h}{a+b x} \right)}{b (-f g + e h)}}, \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left( -b c + a d \right) \left( -f g + e h \right) \right) \right] \right) \right\} / \left( \sqrt{\frac{-\frac{h}{-b g + a h} + \frac{1}{a+b x}}{\frac{f}{-b e + a f} - \frac{h}{-b g + a h}}} \sqrt{\left( d + \frac{b c - a d}{a+b x} \right) \left( f + \frac{b e - a f}{a+b x} \right) \left( h + \frac{b g - a h}{a+b x} \right)} \right) + \right. \right. \\
& \left. \left. \left( b C d e (-b g + a h) \left( -\frac{f}{-b e + a f} + \frac{h}{-b g + a h} \right) \sqrt{\frac{-\frac{d}{-b c + a d} + \frac{1}{a+b x}}{-\frac{d}{-b c + a d} + \frac{h}{-b g + a h}}} \sqrt{-\frac{\left( -\frac{f}{-b e + a f} + \frac{1}{a+b x} \right) \left( -\frac{h}{-b g + a h} + \frac{1}{a+b x} \right)}{\left( -\frac{f}{-b e + a f} + \frac{h}{-b g + a h} \right)^2}} \operatorname{EllipticPi} \left[ -\frac{-b f g + b e h}{(-b e + a f) h}, \right. \right. \right. \right. \\
& \left. \left. \left. \left. \operatorname{ArcSin} \left[ \sqrt{\frac{(-b e + a f) \left( -h - \frac{b g}{a+b x} + \frac{a h}{a+b x} \right)}{b (-f g + e h)}}, \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)} \right] \right) \right\} / \left( \sqrt{\left( d + \frac{b c - a d}{a+b x} \right) \left( f + \frac{b e - a f}{a+b x} \right) \left( h + \frac{b g - a h}{a+b x} \right)} \right) + \right. \right. \\
& \left. \left. \left. \left. \left( b c C f (-b g + a h) \left( -\frac{f}{-b e + a f} + \frac{h}{-b g + a h} \right) \sqrt{\frac{-\frac{d}{-b c + a d} + \frac{1}{a+b x}}{-\frac{d}{-b c + a d} + \frac{h}{-b g + a h}}} \sqrt{-\frac{\left( -\frac{f}{-b e + a f} + \frac{1}{a+b x} \right) \left( -\frac{h}{-b g + a h} + \frac{1}{a+b x} \right)}{\left( -\frac{f}{-b e + a f} + \frac{h}{-b g + a h} \right)^2}} \operatorname{EllipticPi} \left[ -\frac{-b f g + b e h}{(-b e + a f) h}, \right. \right. \right. \right. \right)
\end{aligned}$$

$$\begin{aligned}
& \text{ArcSin}\left[\sqrt{\frac{(-b e + a f) \left(-h - \frac{b g}{a+b x} + \frac{a h}{a+b x}\right)}{b (-f g + e h)}}\right], \frac{(-b c + a d) \left(-f g + e h\right)}{(-b e + a f) \left(-d g + c h\right)}\Bigg] \Bigg/ \left(\sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right) \left(h + \frac{b g - a h}{a+b x}\right)}\right) + \\
& \left(a C d f (-b g + a h) \left(-\frac{f}{-b e + a f} + \frac{h}{-b g + a h}\right) \sqrt{\frac{-\frac{d}{-b c+a d} + \frac{1}{a+b x}}{-\frac{d}{-b c+a d} + \frac{h}{-b g+a h}}} \sqrt{-\frac{\left(-\frac{f}{-b e+a f} + \frac{1}{a+b x}\right) \left(-\frac{h}{-b g+a h} + \frac{1}{a+b x}\right)}{\left(-\frac{f}{-b e+a f} + \frac{h}{-b g+a h}\right)^2}} \text{EllipticPi}\left[-\frac{-b f g + b e h}{(-b e + a f) h},\right.\right. \\
& \text{ArcSin}\left[\sqrt{\frac{(-b e + a f) \left(-h - \frac{b g}{a+b x} + \frac{a h}{a+b x}\right)}{b (-f g + e h)}}\right], \frac{(-b c + a d) \left(-f g + e h\right)}{(-b e + a f) \left(-d g + c h\right)}\Bigg] \Bigg/ \left(\sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right) \left(h + \frac{b g - a h}{a+b x}\right)}\right) + \\
& \left(b C d f g (-b g + a h) \left(-\frac{f}{-b e + a f} + \frac{h}{-b g + a h}\right) \sqrt{\frac{-\frac{d}{-b c+a d} + \frac{1}{a+b x}}{-\frac{d}{-b c+a d} + \frac{h}{-b g+a h}}} \sqrt{-\frac{\left(-\frac{f}{-b e+a f} + \frac{1}{a+b x}\right) \left(-\frac{h}{-b g+a h} + \frac{1}{a+b x}\right)}{\left(-\frac{f}{-b e+a f} + \frac{h}{-b g+a h}\right)^2}} \text{EllipticPi}\left[-\frac{-b f g + b e h}{(-b e + a f) h},\right.\right. \\
& \text{ArcSin}\left[\sqrt{\frac{(-b e + a f) \left(-h - \frac{b g}{a+b x} + \frac{a h}{a+b x}\right)}{b (-f g + e h)}}\right], \frac{(-b c + a d) \left(-f g + e h\right)}{(-b e + a f) \left(-d g + c h\right)}\Bigg] \Bigg/ \left(h \sqrt{\left(d + \frac{b c - a d}{a+b x}\right) \left(f + \frac{b e - a f}{a+b x}\right) \left(h + \frac{b g - a h}{a+b x}\right)}\right)\Bigg)
\end{aligned}$$

**Problem 34: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + C x^2}{(a + b x)^{3/2} \sqrt{c + d x} \sqrt{e + f x} \sqrt{g + h x}} dx$$

Optimal (type 4, 867 leaves, 9 steps):

$$\begin{aligned}
& \frac{2 (A b^2 + a^2 C) d \sqrt{a+b x} \sqrt{e+f x} \sqrt{g+h x}}{b (b c - a d) (b e - a f) (b g - a h) \sqrt{c+d x}} - \frac{2 (A b^2 + a^2 C) \sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x}}{(b c - a d) (b e - a f) (b g - a h) \sqrt{a+b x}} - \\
& \left( 2 (A b^2 + a^2 C) \sqrt{d g - c h} \sqrt{f g - e h} \sqrt{a+b x} \sqrt{-\frac{(d e - c f) (g+h x)}{(f g - e h) (c+d x)}} \text{EllipticE}[\text{ArcSin}\left[\frac{\sqrt{d g - c h} \sqrt{e+f x}}{\sqrt{f g - e h} \sqrt{c+d x}}\right], \frac{(b c - a d) (f g - e h)}{(b e - a f) (d g - c h)}] \right) / \\
& \left( b (b c - a d) (b e - a f) (b g - a h) \sqrt{\frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}} \sqrt{g+h x} \right) - \\
& \frac{2 (2 a b c C + A b^2 d - a^2 C d) \sqrt{\frac{(b e - a f) (c+d x)}{(d e - c f) (a+b x)}} \sqrt{g+h x} \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{b g - a h} \sqrt{e+f x}}{\sqrt{f g - e h} \sqrt{a+b x}}\right], -\frac{(b c - a d) (f g - e h)}{(d e - c f) (b g - a h)}]}{b^2 (b c - a d) \sqrt{b g - a h} \sqrt{f g - e h} \sqrt{c+d x} \sqrt{-\frac{(b e - a f) (g+h x)}{(f g - e h) (a+b x)}}} + \\
& \left( 2 C \sqrt{-d g + c h} (a+b x) \sqrt{\frac{(b g - a h) (c+d x)}{(d g - c h) (a+b x)}} \sqrt{\frac{(b g - a h) (e+f x)}{(f g - e h) (a+b x)}} \right. \\
& \left. \text{EllipticPi}\left[-\frac{b (d g - c h)}{(b c - a d) h}, \text{ArcSin}\left[\frac{\sqrt{b c - a d} \sqrt{g+h x}}{\sqrt{-d g + c h} \sqrt{a+b x}}\right], \frac{(b e - a f) (d g - c h)}{(b c - a d) (f g - e h)}\right]\right) / \left(b^2 \sqrt{b c - a d} h \sqrt{c+d x} \sqrt{e+f x}\right)
\end{aligned}$$

Result (type 4, 2103 leaves):

$$\begin{aligned}
& -\frac{2 (A b^2 + a^2 C) \sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x}}{(b c - a d) (b e - a f) (b g - a h) \sqrt{a+b x}} + \\
& \frac{1}{b^3 (-b c + a d) (-b e + a f) (-b g + a h)} 2 \left( \frac{(-A b^2 - a^2 C) (a+b x)^{5/2} \left(d + \frac{b c}{a+b x} - \frac{a d}{a+b x}\right) \left(f + \frac{b e}{a+b x} - \frac{a f}{a+b x}\right) \left(h + \frac{b g}{a+b x} - \frac{a h}{a+b x}\right)}{\sqrt{c + \frac{(a+b x) (d - \frac{a d}{a+b x})}{b}}} \sqrt{e + \frac{(a+b x) (f - \frac{a f}{a+b x})}{b}} \sqrt{g + \frac{(a+b x) (h - \frac{a h}{a+b x})}{b}} + \right. \\
& \left. \frac{1}{\sqrt{c + \frac{(a+b x) (d - \frac{a d}{a+b x})}{b}} \sqrt{e + \frac{(a+b x) (f - \frac{a f}{a+b x})}{b}} \sqrt{g + \frac{(a+b x) (h - \frac{a h}{a+b x})}{b}}} (b c - a d) (b e - a f) (b g - a h) (a+b x)^{3/2} \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\left(d + \frac{bc}{a+bx} - \frac{ad}{a+bx}\right) \left(f + \frac{be}{a+bx} - \frac{af}{a+bx}\right) \left(h + \frac{bg}{a+bx} - \frac{ah}{a+bx}\right)} \left( A b^2 \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+bx}\right)}{bdg-bch}} \right. \\
& \left( -\frac{f}{-be+af} + \frac{1}{a+bx} \right) \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+bx}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \left( -\frac{(bdg-bch) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-ad)(bg-ah)}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{(bc-ad)(bg-ah)} \right. \\
& \left. \left. d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-ad)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+bx}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+bx}\right) \left(f + \frac{be-ad}{a+bx}\right) \left(h + \frac{bg-ah}{a+bx}\right)} + \left(a^2 C \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+bx}\right)}{bdg-bch}} \right. \right. \\
& \left. \left( -\frac{f}{-be+af} + \frac{1}{a+bx} \right) \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+bx}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \left( -\frac{(bdg-bch) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-ad)(bg-ah)}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{(bc-ad)(bg-ah)} \right. \right. \\
& \left. \left. d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-ad)(h+\frac{bg}{a+bx}-\frac{ah}{a+bx})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+bx}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+bx}\right) \left(f + \frac{be-ad}{a+bx}\right) \left(h + \frac{bg-ah}{a+bx}\right)} - \left( 2 a C \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+bx}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+bx}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{h}{-bg + ah} + \frac{1}{a + bx} \right) \operatorname{EllipticF}[\operatorname{ArcSin}\left[ \sqrt{\frac{(-be + af) \left( -h - \frac{bg}{a+bx} + \frac{ah}{a+bx} \right)}{b (-fg + eh)}} \right], \frac{(-bc + ad) (-fg + eh)}{(-be + af) (-dg + ch)}] \Bigg] \\
& \left( \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+bx}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \sqrt{\left( d + \frac{bc - ad}{a + bx} \right) \left( f + \frac{be - af}{a + bx} \right) \left( h + \frac{bg - ah}{a + bx} \right)} \right) - \\
& \left( C (-bg + ah) \left( -\frac{f}{-be + af} + \frac{h}{-bg + ah} \right) \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+bx}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{-\frac{\left( -\frac{f}{-be+af} + \frac{1}{a+bx} \right) \left( -\frac{h}{-bg+ah} + \frac{1}{a+bx} \right)}{\left( -\frac{f}{-be+af} + \frac{h}{-bg+ah} \right)^2}} \operatorname{EllipticPi}\left[ -\frac{-bf g + be h}{(-be + af) h}, \right. \right. \\
& \left. \left. \operatorname{ArcSin}\left[ \sqrt{\frac{(-be + af) \left( -h - \frac{bg}{a+bx} + \frac{ah}{a+bx} \right)}{b (-fg + eh)}} \right], \frac{(-bc + ad) (-fg + eh)}{(-be + af) (-dg + ch)} \right] \Bigg] \Bigg) / \left( h \sqrt{\left( d + \frac{bc - ad}{a + bx} \right) \left( f + \frac{be - af}{a + bx} \right) \left( h + \frac{bg - ah}{a + bx} \right)} \right)
\end{aligned}$$

**Problem 35: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + C x^2}{(a + b x)^{5/2} \sqrt{c + d x} \sqrt{e + f x} \sqrt{g + h x}} dx$$

Optimal (type 4, 1070 leaves, 8 steps):

$$\begin{aligned}
& - \left( \left( 4 d (A b^3 (d e g + c f g + c e h) + a^3 C (d f g + d e h + c f h) + a^2 b (3 A d f h - 2 C (d e g + c f g + c e h)) - a b^2 (2 A d (f g + e h) - c (3 C e g - 2 A f h))) \right. \right. \\
& \quad \left. \left. \sqrt{a+b x} \sqrt{e+f x} \sqrt{g+h x} \right) / \left( 3 (b c - a d)^2 (b e - a f)^2 (b g - a h)^2 \sqrt{c+d x} \right) - \frac{2 (A b^2 + a^2 C) \sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x}}{3 (b c - a d) (b e - a f) (b g - a h) (a+b x)^{3/2}} + \right. \\
& \left( 4 b (A b^3 (d e g + c f g + c e h) + a^3 C (d f g + d e h + c f h) + a^2 b (3 A d f h - 2 C (d e g + c f g + c e h)) - a b^2 (2 A d (f g + e h) - c (3 C e g - 2 A f h))) \right. \\
& \quad \left. \sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x} \right) / \left( 3 (b c - a d)^2 (b e - a f)^2 (b g - a h)^2 \sqrt{a+b x} \right) + \left( 4 \sqrt{d g - c h} \sqrt{f g - e h} \right. \\
& \left. (A b^3 (d e g + c f g + c e h) + a^3 C (d f g + d e h + c f h) + a^2 b (3 A d f h - 2 C (d e g + c f g + c e h)) - a b^2 (2 A d (f g + e h) - c (3 C e g - 2 A f h))) \right. \\
& \quad \left. \sqrt{a+b x} \sqrt{- \frac{(d e - c f) (g+h x)}{(f g - e h) (c+d x)}} \text{EllipticE}[\text{ArcSin}\left[ \frac{\sqrt{d g - c h} \sqrt{e+f x}}{\sqrt{f g - e h} \sqrt{c+d x}} \right], \frac{(b c - a d) (f g - e h)}{(b e - a f) (d g - c h)}] \right) / \\
& \left( 3 (b c - a d)^2 (b e - a f)^2 (b g - a h)^2 \sqrt{\frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}} \sqrt{g+h x} \right) - \\
& \left( 2 (3 a b (c^2 C + A d^2) (f g + e h) - b^2 (2 A d^2 e g + A c d (f g + e h) + c^2 (3 C e g - A f h)) - a^2 (3 A d^2 f h - C (d^2 e g - c d f g - c d e h - 2 c^2 f h))) \right. \\
& \quad \left. \sqrt{\frac{(b e - a f) (c+d x)}{(d e - c f) (a+b x)}} \sqrt{g+h x} \text{EllipticF}[\text{ArcSin}\left[ \frac{\sqrt{b g - a h} \sqrt{e+f x}}{\sqrt{f g - e h} \sqrt{a+b x}} \right], - \frac{(b c - a d) (f g - e h)}{(d e - c f) (b g - a h)}] \right) / \\
& \left( 3 (b c - a d)^2 (b e - a f) (b g - a h)^{3/2} \sqrt{f g - e h} \sqrt{c+d x} \sqrt{- \frac{(b e - a f) (g+h x)}{(f g - e h) (a+b x)}} \right)
\end{aligned}$$

Result (type 4, 11160 leaves):

$$\begin{aligned}
& \sqrt{a+b x} \sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x} \\
& \left( - \frac{2 (A b^2 + a^2 C)}{3 (b c - a d) (b e - a f) (b g - a h) (a+b x)^2} + \left( 4 b (3 a b^2 c C e g + A b^3 d e g - 2 a^2 b C d e g + A b^3 c f g - 2 a^2 b c C f g - \right. \right. \\
& \quad \left. \left. 2 a A b^2 d f g + a^3 C d f g + A b^3 c e h - 2 a^2 b c C e h - 2 a A b^2 d e h + a^3 C d e h - 2 a A b^2 c f h + a^3 c C f h + 3 a^2 A b d f h) \right) / \right. \\
& \left. \left( 3 (b c - a d)^2 (b e - a f)^2 (b g - a h)^2 (a+b x) \right) \right) + \frac{1}{3 b^2 (-b c + a d)^2 (-b e + a f)^2 (-b g + a h)^2}
\end{aligned}$$

$$\begin{aligned}
& 2 \left( - \left( 2 \left( 3 a b^2 c C e g + A b^3 d e g - 2 a^2 b C d e g + A b^3 c f g - 2 a^2 b c C f g - 2 a A b^2 d f g + a^3 C d f g + A b^3 c e h - 2 a^2 b c C e h - 2 a A b^2 d e h + \right. \right. \right. \\
& \quad \left. \left. \left. a^3 C d e h - 2 a A b^2 c f h + a^3 c C f h + 3 a^2 A b d f h \right) (a + b x)^{5/2} \left( d + \frac{b c}{a + b x} - \frac{a d}{a + b x} \right) \left( f + \frac{b e}{a + b x} - \frac{a f}{a + b x} \right) \left( h + \frac{b g}{a + b x} - \frac{a h}{a + b x} \right) \right) / \\
& \quad \left( \sqrt{c + \frac{(a + b x) (d - \frac{a d}{a + b x})}{b}} \sqrt{e + \frac{(a + b x) (f - \frac{a f}{a + b x})}{b}} \sqrt{g + \frac{(a + b x) (h - \frac{a h}{a + b x})}{b}} \right) + \\
& \quad \frac{1}{\sqrt{c + \frac{(a + b x) (d - \frac{a d}{a + b x})}{b}} \sqrt{e + \frac{(a + b x) (f - \frac{a f}{a + b x})}{b}} \sqrt{g + \frac{(a + b x) (h - \frac{a h}{a + b x})}{b}}} (b c - a d) (b e - a f) (b g - a h) (a + b x)^{3/2} \\
& \quad \sqrt{\left( d + \frac{b c}{a + b x} - \frac{a d}{a + b x} \right) \left( f + \frac{b e}{a + b x} - \frac{a f}{a + b x} \right) \left( h + \frac{b g}{a + b x} - \frac{a h}{a + b x} \right)} \left( 6 a b^2 c C e g \sqrt{\frac{(b c - a d) (b g - a h) \left( -\frac{d}{-b c + a d} + \frac{1}{a + b x} \right)}{b d g - b c h}} \right. \\
& \quad \left( -\frac{f}{-b e + a f} + \frac{1}{a + b x} \right) \sqrt{\frac{-\frac{h}{-b g + a h} + \frac{1}{a + b x}}{\frac{f}{-b e + a f} - \frac{h}{-b g + a h}}} \left( -\frac{(b d g - b c h) \text{EllipticE}[\text{ArcSin}\left[\sqrt{\frac{(b e - a f) (h + \frac{b g}{a + b x} - \frac{a h}{a + b x})}{b (-f g + e h)}}\right], \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}]}{(b c - a d) (b g - a h)} \right. \\
& \quad \left. \left. \frac{d \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{(b e - a f) (h + \frac{b g}{a + b x} - \frac{a h}{a + b x})}{b (-f g + e h)}}\right], \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}]}{-b c + a d} \right) \right) / \\
& \quad \left( \sqrt{\frac{-\frac{f}{-b e + a f} + \frac{1}{a + b x}}{-\frac{f}{-b e + a f} + \frac{h}{-b g + a h}}} \sqrt{\left( d + \frac{b c - a d}{a + b x} \right) \left( f + \frac{b e - a f}{a + b x} \right) \left( h + \frac{b g - a h}{a + b x} \right)} + \left( 2 A b^3 d e g \sqrt{\frac{(b c - a d) (b g - a h) \left( -\frac{d}{-b c + a d} + \frac{1}{a + b x} \right)}{b d g - b c h}} \right. \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{f}{-b e + a f} + \frac{1}{a + b x} \right) \sqrt{\frac{-\frac{h}{-b g + a h} + \frac{1}{a + b x}}{\frac{f}{-b e + a f} - \frac{h}{-b g + a h}}} \left( -\frac{(b d g - b c h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e - a f) \left(h + \frac{b g}{a + b x} - \frac{a h}{a + b x}}{b (-f g + e h)}}\right], \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}]}]{(b c - a d) (b g - a h)} \right. \right. \\
& \left. \left. \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e - a f) \left(h + \frac{b g}{a + b x} - \frac{a h}{a + b x}\right)}{b (-f g + e h)}}, \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}\right]]}{-b c + a d} \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-b e + a f} + \frac{1}{a + b x}}{\frac{f}{-b e + a f} + \frac{h}{-b g + a h}}} \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right) \left(h + \frac{b g - a h}{a + b x}\right)} \right) - \left( 4 a^2 b c d e g \sqrt{\frac{(b c - a d) (b g - a h) \left(-\frac{d}{-b c + a d} + \frac{1}{a + b x}\right)}{b d g - b c h}} \right. \\
& \left. \left( -\frac{f}{-b e + a f} + \frac{1}{a + b x} \right) \sqrt{\frac{-\frac{h}{-b g + a h} + \frac{1}{a + b x}}{\frac{f}{-b e + a f} - \frac{h}{-b g + a h}}} \left( -\frac{(b d g - b c h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e - a f) \left(h + \frac{b g}{a + b x} - \frac{a h}{a + b x}\right)}{b (-f g + e h)}}, \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}\right]]}{(b c - a d) (b g - a h)} \right. \right. \\
& \left. \left. \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e - a f) \left(h + \frac{b g}{a + b x} - \frac{a h}{a + b x}\right)}{b (-f g + e h)}}, \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}\right]]}{-b c + a d} \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-b e + a f} + \frac{1}{a + b x}}{\frac{f}{-b e + a f} + \frac{h}{-b g + a h}}} \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right) \left(h + \frac{b g - a h}{a + b x}\right)} \right) + \left( 2 A b^3 c f g \sqrt{\frac{(b c - a d) (b g - a h) \left(-\frac{d}{-b c + a d} + \frac{1}{a + b x}\right)}{b d g - b c h}} \right. \\
& \left. \left( -\frac{f}{-b e + a f} + \frac{1}{a + b x} \right) \sqrt{\frac{-\frac{h}{-b g + a h} + \frac{1}{a + b x}}{\frac{f}{-b e + a f} - \frac{h}{-b g + a h}}} \left( -\frac{(b d g - b c h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e - a f) \left(h + \frac{b g}{a + b x} - \frac{a h}{a + b x}\right)}{b (-f g + e h)}}, \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}\right]]}{(b c - a d) (b g - a h)} \right. \right. 
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) (-f g+e h)}{(-b e+a f) (-d g+c h)}}}{-b c+a d}\right\} / \\
& \left( \sqrt{\frac{-\frac{f}{-b e+a f}+\frac{1}{a+b x}}{-\frac{f}{-b e+a f}+\frac{h}{-b g+a h}}} \sqrt{\left(d+\frac{b c-a d}{a+b x}\right)\left(f+\frac{b e-a f}{a+b x}\right)\left(h+\frac{b g-a h}{a+b x}\right)} - \left(4 a^2 b c C f g \sqrt{\frac{(b c-a d) (b g-a h) \left(-\frac{d}{-b c+a d}+\frac{1}{a+b x}\right)}{b d g-b c h}} \right.\right. \\
& \left. \left. \left(-\frac{f}{-b e+a f}+\frac{1}{a+b x}\right) \sqrt{\frac{-\frac{h}{-b g+a h}+\frac{1}{a+b x}}{\frac{f}{-b e+a f}-\frac{h}{-b g+a h}}} \left(-\frac{(b d g-b c h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) (-f g+e h)}{(-b e+a f) (-d g+c h)}}}{(b c-a d) (b g-a h)} - \right.\right. \right. \\
& \left. \left. \left.d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) (-f g+e h)}{(-b e+a f) (-d g+c h)}}\right]\right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-b e+a f}+\frac{1}{a+b x}}{-\frac{f}{-b e+a f}+\frac{h}{-b g+a h}}} \sqrt{\left(d+\frac{b c-a d}{a+b x}\right)\left(f+\frac{b e-a f}{a+b x}\right)\left(h+\frac{b g-a h}{a+b x}\right)} - \left(4 a A b^2 d f g \sqrt{\frac{(b c-a d) (b g-a h) \left(-\frac{d}{-b c+a d}+\frac{1}{a+b x}\right)}{b d g-b c h}} \right.\right. \\
& \left. \left. \left(-\frac{f}{-b e+a f}+\frac{1}{a+b x}\right) \sqrt{\frac{-\frac{h}{-b g+a h}+\frac{1}{a+b x}}{\frac{f}{-b e+a f}-\frac{h}{-b g+a h}}} \left(-\frac{(b d g-b c h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) (-f g+e h)}{(-b e+a f) (-d g+c h)}}}{(b c-a d) (b g-a h)} - \right.\right. \right. \\
& \left. \left. \left.d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) (-f g+e h)}{(-b e+a f) (-d g+c h)}}\right]\right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) + \left( 2 a^3 C d f g \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+b x}\right)}{bdg-bch}} \right. \\
& \left( -\frac{f}{-be+af} + \frac{1}{a+b x} \right) \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+b x}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \left( -\frac{(bdg-bch) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+b x}-\frac{ah}{a+b x})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{(bc-ad)(bg-ah)} \right. \\
& \left. \left. d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+b x}-\frac{ah}{a+b x})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) + \left( 2 A b^3 c e h \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+b x}\right)}{bdg-bch}} \right. \\
& \left( -\frac{f}{-be+af} + \frac{1}{a+b x} \right) \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+b x}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \left( -\frac{(bdg-bch) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+b x}-\frac{ah}{a+b x})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{(bc-ad)(bg-ah)} \right. \\
& \left. \left. d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)(h+\frac{bg}{a+b x}-\frac{ah}{a+b x})}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) - \left( 4 a^2 b c c e h \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+b x}\right)}{bdg-bch}} \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{f}{-b e + a f} + \frac{1}{a + b x} \right) \sqrt{\frac{-\frac{h}{-b g + a h} + \frac{1}{a + b x}}{\frac{f}{-b e + a f} - \frac{h}{-b g + a h}}} \left( -\frac{(b d g - b c h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e - a f) \left(h + \frac{b g}{a + b x} - \frac{a h}{a + b x}\right)}{b (-f g + e h)}}, \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}}\right]}{(b c - a d) (b g - a h)} \right. \\
& \left. \left. - \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e - a f) \left(h + \frac{b g}{a + b x} - \frac{a h}{a + b x}\right)}{b (-f g + e h)}}, \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}}\right]}{-b c + a d} \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-b e + a f} + \frac{1}{a + b x}}{\frac{f}{-b e + a f} + \frac{h}{-b g + a h}}} \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right) \left(h + \frac{b g - a h}{a + b x}\right)} - \left(4 a A b^2 d e h \sqrt{\frac{(b c - a d) (b g - a h) \left(-\frac{d}{-b c + a d} + \frac{1}{a + b x}\right)}{b d g - b c h}} \right. \right. \\
& \left. \left. - \left( -\frac{f}{-b e + a f} + \frac{1}{a + b x} \right) \sqrt{\frac{-\frac{h}{-b g + a h} + \frac{1}{a + b x}}{\frac{f}{-b e + a f} - \frac{h}{-b g + a h}}} \left( -\frac{(b d g - b c h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e - a f) \left(h + \frac{b g}{a + b x} - \frac{a h}{a + b x}\right)}{b (-f g + e h)}}, \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}}\right]}{(b c - a d) (b g - a h)} \right. \right. \\
& \left. \left. - \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e - a f) \left(h + \frac{b g}{a + b x} - \frac{a h}{a + b x}\right)}{b (-f g + e h)}}, \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}}\right]}{-b c + a d} \right) \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-b e + a f} + \frac{1}{a + b x}}{\frac{f}{-b e + a f} + \frac{h}{-b g + a h}}} \sqrt{\left(d + \frac{b c - a d}{a + b x}\right) \left(f + \frac{b e - a f}{a + b x}\right) \left(h + \frac{b g - a h}{a + b x}\right)} + \left(2 a^3 C d e h \sqrt{\frac{(b c - a d) (b g - a h) \left(-\frac{d}{-b c + a d} + \frac{1}{a + b x}\right)}{b d g - b c h}} \right. \right. \\
& \left. \left. - \left( -\frac{f}{-b e + a f} + \frac{1}{a + b x} \right) \sqrt{\frac{-\frac{h}{-b g + a h} + \frac{1}{a + b x}}{\frac{f}{-b e + a f} - \frac{h}{-b g + a h}}} \left( -\frac{(b d g - b c h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e - a f) \left(h + \frac{b g}{a + b x} - \frac{a h}{a + b x}\right)}{b (-f g + e h)}}, \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}}\right]}{(b c - a d) (b g - a h)} \right. \right. \\
& \left. \left. - \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e - a f) \left(h + \frac{b g}{a + b x} - \frac{a h}{a + b x}\right)}{b (-f g + e h)}}, \frac{(-b c + a d) (-f g + e h)}{(-b e + a f) (-d g + c h)}}\right]}{-b c + a d} \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) (-f g+e h)}{(-b e+a f) (-d g+c h)}}}{-b c+a d}\right\} / \\
& \left( \sqrt{\frac{-\frac{f}{-b e+a f}+\frac{1}{a+b x}}{-\frac{f}{-b e+a f}+\frac{h}{-b g+a h}}} \sqrt{\left(d+\frac{b c-a d}{a+b x}\right)\left(f+\frac{b e-a f}{a+b x}\right)\left(h+\frac{b g-a h}{a+b x}\right)} - \left(4 a A b^2 c f h \sqrt{\frac{(b c-a d) (b g-a h) \left(-\frac{d}{-b c+a d}+\frac{1}{a+b x}\right)}{b d g-b c h}} \right. \right. \\
& \left. \left. \left(-\frac{f}{-b e+a f}+\frac{1}{a+b x}\right) \sqrt{\frac{-\frac{h}{-b g+a h}+\frac{1}{a+b x}}{\frac{f}{-b e+a f}-\frac{h}{-b g+a h}}} \left(-\frac{(b d g-b c h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) (-f g+e h)}{(-b e+a f) (-d g+c h)}}}{(b c-a d) (b g-a h)} - \right. \right. \right. \\
& \left. \left. \left.d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) (-f g+e h)}{(-b e+a f) (-d g+c h)}}}{-b c+a d}\right)\right\} / \\
& \left( \sqrt{\frac{-\frac{f}{-b e+a f}+\frac{1}{a+b x}}{-\frac{f}{-b e+a f}+\frac{h}{-b g+a h}}} \sqrt{\left(d+\frac{b c-a d}{a+b x}\right)\left(f+\frac{b e-a f}{a+b x}\right)\left(h+\frac{b g-a h}{a+b x}\right)} + \left(2 a^3 c C f h \sqrt{\frac{(b c-a d) (b g-a h) \left(-\frac{d}{-b c+a d}+\frac{1}{a+b x}\right)}{b d g-b c h}} \right. \right. \\
& \left. \left. \left(-\frac{f}{-b e+a f}+\frac{1}{a+b x}\right) \sqrt{\frac{-\frac{h}{-b g+a h}+\frac{1}{a+b x}}{\frac{f}{-b e+a f}-\frac{h}{-b g+a h}}} \left(-\frac{(b d g-b c h) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) (-f g+e h)}{(-b e+a f) (-d g+c h)}}}{(b c-a d) (b g-a h)} - \right. \right. \right. \\
& \left. \left. \left.d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(b e-a f) \left(h+\frac{b g}{a+b x}-\frac{a h}{a+b x}\right)}{b (-f g+e h)}}\right], \frac{(-b c+a d) (-f g+e h)}{(-b e+a f) (-d g+c h)}}}{-b c+a d}\right)\right\}
\end{aligned}$$

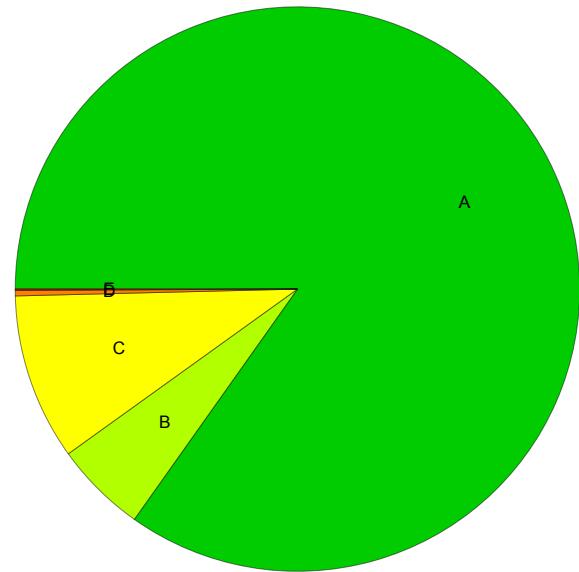
$$\begin{aligned}
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) + \left( 6 a^2 A b d f h \sqrt{\frac{(bc-ad)(bg-ah)\left(-\frac{d}{-bc+ad} + \frac{1}{a+b x}\right)}{bdg-bch}} \right. \\
& \left( -\frac{f}{-be+af} + \frac{1}{a+b x} \right) \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+b x}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \left( -\frac{(bdg-bch) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)\left(h+\frac{bg-ah}{a+b x}\right)}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{(bc-ad)(bg-ah)} \right. \\
& \left. \left. \left. \frac{d \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(be-af)\left(h+\frac{bg-ah}{a+b x}\right)}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}]}{-bc+ad} \right) \right) / \\
& \left( \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) - \left( 3 b^2 c C e g \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+b x}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \right. \\
& \left( -\frac{h}{-bg+ah} + \frac{1}{a+b x} \right) \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(-be+af)\left(-h-\frac{bg}{a+b x} + \frac{ah}{a+b x}\right)}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] \right) / \\
& \left( \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+b x}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) + \left( 3 a b C d e g \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+b x}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \right. \\
& \left( -\frac{h}{-bg+ah} + \frac{1}{a+b x} \right) \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(-be+af)\left(-h-\frac{bg}{a+b x} + \frac{ah}{a+b x}\right)}{b(-fg+eh)}}\right], \frac{(-bc+ad)(-fg+eh)}{(-be+af)(-dg+ch)}] \right) / \\
& \left( \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+b x}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) + \left( 3 a b c C f g \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+b x}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{h}{-bg + ah} + \frac{1}{a + bx} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-be + af) \left( -h - \frac{bg}{a+bx} + \frac{ah}{a+bx} \right)}{b (-fg + eh)}} \right], \frac{(-bc + ad) (-fg + eh)}{(-be + af) (-dg + ch)} \right] \Bigg] / \\
& \left( \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+bx}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \sqrt{\left( d + \frac{bc - ad}{a + bx} \right) \left( f + \frac{be - af}{a + bx} \right) \left( h + \frac{bg - ah}{a + bx} \right)} \right) + \left( Ab^2 df g \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+bx}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+bx}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \right. \\
& \left. \left( -\frac{h}{-bg + ah} + \frac{1}{a + bx} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-be + af) \left( -h - \frac{bg}{a+bx} + \frac{ah}{a+bx} \right)}{b (-fg + eh)}} \right], \frac{(-bc + ad) (-fg + eh)}{(-be + af) (-dg + ch)} \right] \right] / \\
& \left( \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+bx}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \sqrt{\left( d + \frac{bc - ad}{a + bx} \right) \left( f + \frac{be - af}{a + bx} \right) \left( h + \frac{bg - ah}{a + bx} \right)} \right) - \left( 2a^2 C df g \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+bx}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+bx}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \right. \\
& \left. \left( -\frac{h}{-bg + ah} + \frac{1}{a + bx} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-be + af) \left( -h - \frac{bg}{a+bx} + \frac{ah}{a+bx} \right)}{b (-fg + eh)}} \right], \frac{(-bc + ad) (-fg + eh)}{(-be + af) (-dg + ch)} \right] \right] / \\
& \left( \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+bx}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \sqrt{\left( d + \frac{bc - ad}{a + bx} \right) \left( f + \frac{be - af}{a + bx} \right) \left( h + \frac{bg - ah}{a + bx} \right)} \right) + \left( 3abcbeh \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+bx}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+bx}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \right. \\
& \left. \left( -\frac{h}{-bg + ah} + \frac{1}{a + bx} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-be + af) \left( -h - \frac{bg}{a+bx} + \frac{ah}{a+bx} \right)}{b (-fg + eh)}} \right], \frac{(-bc + ad) (-fg + eh)}{(-be + af) (-dg + ch)} \right] \right] / \\
& \left( \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+bx}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \sqrt{\left( d + \frac{bc - ad}{a + bx} \right) \left( f + \frac{be - af}{a + bx} \right) \left( h + \frac{bg - ah}{a + bx} \right)} \right) + \left( Ab^2 deh \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+bx}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+bx}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \right. \\
& \left. \left( -\frac{h}{-bg + ah} + \frac{1}{a + bx} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-be + af) \left( -h - \frac{bg}{a+bx} + \frac{ah}{a+bx} \right)}{b (-fg + eh)}} \right], \frac{(-bc + ad) (-fg + eh)}{(-be + af) (-dg + ch)} \right] \right]
\end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+b x}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) - \left( 2 a^2 C d e h \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+b x}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \right. \\
& \left. \left( -\frac{h}{-bg+ah} + \frac{1}{a+b x} \right) \text{EllipticF}[\text{ArcSin}\left[ \sqrt{\frac{(-be+af) (-h - \frac{bg}{a+b x} + \frac{ah}{a+b x})}{b (-fg+eh)}} \right], \frac{(-bc+ad) (-fg+eh)}{(-be+af) (-dg+ch)}] \right) / \\
& \left( \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+b x}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) + \left( A b^2 c f h \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+b x}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \right. \\
& \left. \left( -\frac{h}{-bg+ah} + \frac{1}{a+b x} \right) \text{EllipticF}[\text{ArcSin}\left[ \sqrt{\frac{(-be+af) (-h - \frac{bg}{a+b x} + \frac{ah}{a+b x})}{b (-fg+eh)}} \right], \frac{(-bc+ad) (-fg+eh)}{(-be+af) (-dg+ch)}] \right) / \\
& \left( \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+b x}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) - \left( 2 a^2 c C f h \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+b x}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \right. \\
& \left. \left( -\frac{h}{-bg+ah} + \frac{1}{a+b x} \right) \text{EllipticF}[\text{ArcSin}\left[ \sqrt{\frac{(-be+af) (-h - \frac{bg}{a+b x} + \frac{ah}{a+b x})}{b (-fg+eh)}} \right], \frac{(-bc+ad) (-fg+eh)}{(-be+af) (-dg+ch)}] \right) / \\
& \left( \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+b x}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right) - \left( 3 a A b d f h \sqrt{\frac{-\frac{d}{-bc+ad} + \frac{1}{a+b x}}{-\frac{d}{-bc+ad} + \frac{h}{-bg+ah}}} \sqrt{\frac{-\frac{f}{-be+af} + \frac{1}{a+b x}}{-\frac{f}{-be+af} + \frac{h}{-bg+ah}}} \right. \\
& \left. \left( -\frac{h}{-bg+ah} + \frac{1}{a+b x} \right) \text{EllipticF}[\text{ArcSin}\left[ \sqrt{\frac{(-be+af) (-h - \frac{bg}{a+b x} + \frac{ah}{a+b x})}{b (-fg+eh)}} \right], \frac{(-bc+ad) (-fg+eh)}{(-be+af) (-dg+ch)}] \right) / \\
& \left( \sqrt{\frac{-\frac{h}{-bg+ah} + \frac{1}{a+b x}}{\frac{f}{-be+af} - \frac{h}{-bg+ah}}} \sqrt{\left(d + \frac{bc-ad}{a+b x}\right) \left(f + \frac{be-af}{a+b x}\right) \left(h + \frac{bg-ah}{a+b x}\right)} \right)
\end{aligned}$$

## Summary of Integration Test Results

5424 integration problems



A - 4600 optimal antiderivatives

B - 286 more than twice size of optimal antiderivatives

C - 517 unnecessarily complex antiderivatives

D - 17 unable to integrate problems

E - 4 integration timeouts