

Mathematica 11.3 Integration Test Results

on the problems in the test-suite directory "1 Algebraic functions\1.1
Binomial products\1.1.4 Improper"

Test results for the 454 problems in "1.1.4.2 (c x)^m (a x^j+b x^n)^p.m"

Problem 26: Result more than twice size of optimal antiderivative.

$$\int \frac{x^5}{x-x^3} dx$$

Optimal (type 3, 13 leaves, 4 steps):

$$-x - \frac{x^3}{3} + \text{ArcTanh}[x]$$

Result (type 3, 29 leaves):

$$-x - \frac{x^3}{3} - \frac{1}{2} \text{Log}[1-x] + \frac{1}{2} \text{Log}[1+x]$$

Problem 28: Result more than twice size of optimal antiderivative.

$$\int \frac{x^3}{x-x^3} dx$$

Optimal (type 3, 6 leaves, 3 steps):

$$-x + \text{ArcTanh}[x]$$

Result (type 3, 22 leaves):

$$-x - \frac{1}{2} \text{Log}[1-x] + \frac{1}{2} \text{Log}[1+x]$$

Problem 30: Result more than twice size of optimal antiderivative.

$$\int \frac{x}{x - x^3} dx$$

Optimal (type 3, 2 leaves, 2 steps):

$$\text{ArcTanh}[x]$$

Result (type 3, 19 leaves):

$$-\frac{1}{2} \text{Log}[1 - x] + \frac{1}{2} \text{Log}[1 + x]$$

Problem 32: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x(x - x^3)} dx$$

Optimal (type 3, 8 leaves, 3 steps):

$$-\frac{1}{x} + \text{ArcTanh}[x]$$

Result (type 3, 24 leaves):

$$-\frac{1}{x} - \frac{1}{2} \text{Log}[1 - x] + \frac{1}{2} \text{Log}[1 + x]$$

Problem 34: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^3(x - x^3)} dx$$

Optimal (type 3, 15 leaves, 4 steps):

$$-\frac{1}{3x^3} - \frac{1}{x} + \text{ArcTanh}[x]$$

Result (type 3, 31 leaves):

$$-\frac{1}{3x^3} - \frac{1}{x} - \frac{1}{2} \text{Log}[1 - x] + \frac{1}{2} \text{Log}[1 + x]$$

Problem 38: Result unnecessarily involves imaginary or complex numbers.

$$\int x^3 \sqrt{a x + b x^3} dx$$

Optimal (type 4, 163 leaves, 6 steps):

$$-\frac{20 a^2 \sqrt{a x + b x^3}}{231 b^2} + \frac{4 a x^2 \sqrt{a x + b x^3}}{77 b} + \frac{2}{11} x^4 \sqrt{a x + b x^3} + \frac{10 a^{11/4} \sqrt{x} (\sqrt{a} + \sqrt{b} x) \sqrt{\frac{a+b x^2}{(\sqrt{a} + \sqrt{b} x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{231 b^{9/4} \sqrt{a x + b x^3}}$$

Result (type 4, 148 leaves):

$$\left(2 x \left(\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} (-10 a^3 - 4 a^2 b x^2 + 27 a b^2 x^4 + 21 b^3 x^6) + 10 i a^3 \sqrt{1 + \frac{a}{b x^2}} \sqrt{x} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right], -1\right] \right) \right) / \left(231 \sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} b^2 \sqrt{x (a + b x^2)} \right)$$

Problem 39: Result unnecessarily involves imaginary or complex numbers.

$$\int x^2 \sqrt{a x + b x^3} dx$$

Optimal (type 4, 281 leaves, 7 steps):

$$-\frac{4 a^2 x (a + b x^2)}{15 b^{3/2} (\sqrt{a} + \sqrt{b} x) \sqrt{a x + b x^3}} + \frac{4 a x \sqrt{a x + b x^3}}{45 b} + \frac{2}{9} x^3 \sqrt{a x + b x^3} + \frac{4 a^{9/4} \sqrt{x} (\sqrt{a} + \sqrt{b} x) \sqrt{\frac{a+b x^2}{(\sqrt{a} + \sqrt{b} x)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{b^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{15 b^{7/4} \sqrt{a x + b x^3}} - \frac{2 a^{9/4} \sqrt{x} (\sqrt{a} + \sqrt{b} x) \sqrt{\frac{a+b x^2}{(\sqrt{a} + \sqrt{b} x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{15 b^{7/4} \sqrt{a x + b x^3}}$$

Result (type 4, 184 leaves):

$$\left(2x \left(\sqrt{bx} \sqrt{\frac{i\sqrt{b}x}{\sqrt{a}}} (2a^2 + 7abx^2 + 5b^2x^4) - 6a^{5/2} \sqrt{1 + \frac{bx^2}{a}} \operatorname{EllipticE} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{i\sqrt{b}x}{\sqrt{a}}} \right], -1 \right] + \right. \right. \\ \left. \left. 6a^{5/2} \sqrt{1 + \frac{bx^2}{a}} \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{i\sqrt{b}x}{\sqrt{a}}} \right], -1 \right] \right) \right) / \left(45b^{3/2} \sqrt{\frac{i\sqrt{b}x}{\sqrt{a}}} \sqrt{x(a+bx^2)} \right)$$

Problem 40: Result unnecessarily involves imaginary or complex numbers.

$$\int x \sqrt{ax + bx^3} dx$$

Optimal (type 4, 137 leaves, 5 steps):

$$\frac{4a\sqrt{ax+bx^3}}{21b} + \frac{2}{7}x^2\sqrt{ax+bx^3} - \frac{2a^{7/4}\sqrt{x}(\sqrt{a}+\sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} \operatorname{EllipticF} \left[2 \operatorname{ArcTan} \left[\frac{b^{1/4}\sqrt{x}}{a^{1/4}} \right], \frac{1}{2} \right]}{21b^{5/4}\sqrt{ax+bx^3}}$$

Result (type 4, 137 leaves):

$$\frac{2x \left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}} (2a^2 + 5abx^2 + 3b^2x^4) - 2ia^2 \sqrt{1 + \frac{a}{bx^2}} \sqrt{x} \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}} \right], -1 \right] \right)}{21 \sqrt{\frac{i\sqrt{a}}{\sqrt{b}}} b \sqrt{x(a+bx^2)}}$$

Problem 41: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{ax + bx^3} dx$$

Optimal (type 4, 255 leaves, 6 steps):

$$\frac{4 a x (a+b x^2)}{5 \sqrt{b} (\sqrt{a}+\sqrt{b} x) \sqrt{a x+b x^3}}+\frac{2}{5} x \sqrt{a x+b x^3}-\frac{4 a^{5 / 4} \sqrt{x}(\sqrt{a}+\sqrt{b} x) \sqrt{\frac{a+b x^2}{(\sqrt{a}+\sqrt{b} x)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{b^{1 / 4} \sqrt{x}}{a^{1 / 4}}\right], \frac{1}{2}\right]}{5 b^{3 / 4} \sqrt{a x+b x^3}}+\frac{2 a^{5 / 4} \sqrt{x}(\sqrt{a}+\sqrt{b} x) \sqrt{\frac{a+b x^2}{(\sqrt{a}+\sqrt{b} x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{b^{1 / 4} \sqrt{x}}{a^{1 / 4}}\right], \frac{1}{2}\right]}{5 b^{3 / 4} \sqrt{a x+b x^3}}$$

Result (type 4, 170 leaves):

$$\left(2 x\left(\sqrt{b} x \sqrt{\frac{i \sqrt{b} x}{\sqrt{a}}}(a+b x^2)+2 a^{3 / 2} \sqrt{1+\frac{b x^2}{a}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{b} x}{\sqrt{a}}}\right],-1\right]-2 a^{3 / 2} \sqrt{1+\frac{b x^2}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{b} x}{\sqrt{a}}}\right],-1\right]\right)\right) / \left(5 \sqrt{b} \sqrt{\frac{i \sqrt{b} x}{\sqrt{a}}} \sqrt{x(a+b x^2)}\right)$$

Problem 42: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a x+b x^3}}{x} d x$$

Optimal (type 4, 113 leaves, 4 steps):

$$\frac{2}{3} \sqrt{a x+b x^3}+\frac{2 a^{3 / 4} \sqrt{x}(\sqrt{a}+\sqrt{b} x) \sqrt{\frac{a+b x^2}{(\sqrt{a}+\sqrt{b} x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{b^{1 / 4} \sqrt{x}}{a^{1 / 4}}\right], \frac{1}{2}\right]}{3 b^{1 / 4} \sqrt{a x+b x^3}}$$

Result (type 4, 101 leaves):

$$\frac{2}{3} \sqrt{x(a+b x^2)}\left(1+\frac{2 i a \sqrt{1+\frac{a}{b x^2}} \sqrt{x} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}}\right],-1\right]}{\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}}(a+b x^2)}\right)$$

Problem 43: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{ax + bx^3}}{x^2} dx$$

Optimal (type 4, 248 leaves, 6 steps):

$$\frac{4\sqrt{b}x(a+bx^2)}{(\sqrt{a}+\sqrt{b}x)\sqrt{ax+bx^3}} - \frac{2\sqrt{ax+bx^3}}{x} - \frac{4a^{1/4}b^{1/4}\sqrt{x}(\sqrt{a}+\sqrt{b}x)\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{b}x)^2}} \text{EllipticE}\left[2\text{ArcTan}\left[\frac{b^{1/4}\sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{\sqrt{ax+bx^3}} +$$

$$\frac{2a^{1/4}b^{1/4}\sqrt{x}(\sqrt{a}+\sqrt{b}x)\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{b}x)^2}} \text{EllipticF}\left[2\text{ArcTan}\left[\frac{b^{1/4}\sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{\sqrt{ax+bx^3}}$$

Result (type 4, 168 leaves):

$$-\frac{1}{\sqrt{\frac{i\sqrt{b}x}{\sqrt{a}}}\sqrt{x(a+bx^2)}} - 2\left(\sqrt{\frac{i\sqrt{b}x}{\sqrt{a}}}(a+bx^2) - \right.$$

$$\left. 2\sqrt{a}\sqrt{b}x\sqrt{1+\frac{bx^2}{a}} \text{EllipticE}\left[i\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}x}{\sqrt{a}}}\right], -1\right] + 2\sqrt{a}\sqrt{b}x\sqrt{1+\frac{bx^2}{a}} \text{EllipticF}\left[i\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}x}{\sqrt{a}}}\right], -1\right]\right)$$

Problem 44: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{ax + bx^3}}{x^3} dx$$

Optimal (type 4, 116 leaves, 4 steps):

$$-\frac{2\sqrt{ax+bx^3}}{3x^2} + \frac{2b^{3/4}\sqrt{x}(\sqrt{a}+\sqrt{b}x)\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{b}x)^2}} \text{EllipticF}\left[2\text{ArcTan}\left[\frac{b^{1/4}\sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{3a^{1/4}\sqrt{ax+bx^3}}$$

Result (type 4, 104 leaves):

$$\frac{2\sqrt{x(a+bx^2)} \left(-1 + \frac{2ib\sqrt{1+\frac{a}{bx^2}} x^{5/2} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\right], -1\right]}{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}(a+bx^2)} \right)}{3x^2}$$

Problem 45: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{ax+bx^3}}{x^4} dx$$

Optimal (type 4, 283 leaves, 7 steps):

$$\frac{4b^{3/2}x(a+bx^2)}{5a(\sqrt{a}+\sqrt{bx})\sqrt{ax+bx^3}} - \frac{2\sqrt{ax+bx^3}}{5x^3} - \frac{4b\sqrt{ax+bx^3}}{5ax} - \frac{4b^{5/4}\sqrt{x}(\sqrt{a}+\sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4}\sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{5a^{3/4}\sqrt{ax+bx^3}} + \frac{2b^{5/4}\sqrt{x}(\sqrt{a}+\sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4}\sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{5a^{3/4}\sqrt{ax+bx^3}}$$

Result (type 4, 192 leaves):

$$-\left(\left(2 \left(\sqrt{\frac{i\sqrt{b}x}{\sqrt{a}}} (a^2 + 3abx^2 + 2b^2x^4) - 2\sqrt{a}b^{3/2}x^3 \sqrt{1 + \frac{bx^2}{a}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}x}{\sqrt{a}}}\right], -1\right] + 2\sqrt{a}b^{3/2}x^3 \sqrt{1 + \frac{bx^2}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}x}{\sqrt{a}}}\right], -1\right] \right) \right) / \left(5ax^2 \sqrt{\frac{i\sqrt{b}x}{\sqrt{a}}} \sqrt{x(a+bx^2)} \right)$$

Problem 46: Result unnecessarily involves imaginary or complex numbers.

$$\int x^2 (ax+bx^3)^{3/2} dx$$

Optimal (type 4, 186 leaves, 7 steps):

$$\begin{aligned}
& -\frac{8a^3\sqrt{ax+bx^3}}{231b^2} + \frac{8a^2x^2\sqrt{ax+bx^3}}{385b} + \frac{4}{55}ax^4\sqrt{ax+bx^3} + \\
& \frac{2}{15}x^3(ax+bx^3)^{3/2} + \frac{4a^{15/4}\sqrt{x}(\sqrt{a}+\sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} \operatorname{EllipticF}\left[2\operatorname{ArcTan}\left[\frac{b^{1/4}\sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{231b^{9/4}\sqrt{ax+bx^3}}
\end{aligned}$$

Result (type 4, 159 leaves):

$$\begin{aligned}
& \left(2x \left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}} (-20a^4 - 8a^3bx^2 + 131a^2b^2x^4 + 196ab^3x^6 + 77b^4x^8) + 20ia^4\sqrt{1+\frac{a}{bx^2}}\sqrt{x} \operatorname{EllipticF}\left[i\operatorname{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right], -1\right] \right) \right) / \\
& \left(1155\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}b^2\sqrt{x(a+bx^2)} \right)
\end{aligned}$$

Problem 47: Result unnecessarily involves imaginary or complex numbers.

$$\int x(ax+bx^3)^{3/2} dx$$

Optimal (type 4, 304 leaves, 8 steps):

$$\begin{aligned}
& -\frac{8a^3x(a+bx^2)}{65b^{3/2}(\sqrt{a}+\sqrt{bx})\sqrt{ax+bx^3}} + \frac{8a^2x\sqrt{ax+bx^3}}{195b} + \frac{4}{39}ax^3\sqrt{ax+bx^3} + \\
& \frac{2}{13}x^2(ax+bx^3)^{3/2} + \frac{8a^{13/4}\sqrt{x}(\sqrt{a}+\sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} \operatorname{EllipticE}\left[2\operatorname{ArcTan}\left[\frac{b^{1/4}\sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{65b^{7/4}\sqrt{ax+bx^3}} - \\
& \frac{4a^{13/4}\sqrt{x}(\sqrt{a}+\sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} \operatorname{EllipticF}\left[2\operatorname{ArcTan}\left[\frac{b^{1/4}\sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{65b^{7/4}\sqrt{ax+bx^3}}
\end{aligned}$$

Result (type 4, 195 leaves):

$$\left(2x \left(\sqrt{bx} \sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}} (4a^3 + 29a^2bx^2 + 40ab^2x^4 + 15b^3x^6) - 12a^{7/2} \sqrt{1 + \frac{bx^2}{a}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}\right], -1\right] + \right. \right. \\ \left. \left. 12a^{7/2} \sqrt{1 + \frac{bx^2}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}\right], -1\right] \right) \right) / \left(195b^{3/2} \sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}} \sqrt{x(a+bx^2)} \right)$$

Problem 48: Result unnecessarily involves imaginary or complex numbers.

$$\int (ax + bx^3)^{3/2} dx$$

Optimal (type 4, 158 leaves, 6 steps):

$$\frac{8a^2\sqrt{ax+bx^3}}{77b} + \frac{12}{77}ax^2\sqrt{ax+bx^3} + \frac{2}{11}x(ax+bx^3)^{3/2} - \frac{4a^{11/4}\sqrt{x}(\sqrt{a}+\sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4}\sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{77b^{5/4}\sqrt{ax+bx^3}}$$

Result (type 4, 148 leaves):

$$\left(2x \left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}} (4a^3 + 17a^2bx^2 + 20ab^2x^4 + 7b^3x^6) - 4ia^3 \sqrt{1 + \frac{a}{bx^2}} \sqrt{x} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\right], -1\right] \right) \right) / \left(77 \sqrt{\frac{i\sqrt{a}}{\sqrt{b}}} b \sqrt{x(a+bx^2)} \right)$$

Problem 49: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(ax + bx^3)^{3/2}}{x} dx$$

Optimal (type 4, 275 leaves, 7 steps):

$$\frac{8a^2x(a+bx^2)}{15\sqrt{b}(\sqrt{a}+\sqrt{bx})\sqrt{ax+bx^3}} + \frac{4}{15}ax\sqrt{ax+bx^3} + \frac{2}{9}(ax+bx^3)^{3/2} - \frac{8a^{9/4}\sqrt{x}(\sqrt{a}+\sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4}\sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{15b^{3/4}\sqrt{ax+bx^3}} + \frac{4a^{9/4}\sqrt{x}(\sqrt{a}+\sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4}\sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{15b^{3/4}\sqrt{ax+bx^3}}$$

Result (type 4, 184 leaves):

$$\left(2x \left(\sqrt{b} x \sqrt{\frac{i\sqrt{b}x}{\sqrt{a}}} (11a^2 + 16abx^2 + 5b^2x^4) + 12a^{5/2} \sqrt{1 + \frac{bx^2}{a}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}x}{\sqrt{a}}}\right], -1\right] - 12a^{5/2} \sqrt{1 + \frac{bx^2}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}x}{\sqrt{a}}}\right], -1\right] \right) \right) / \left(45\sqrt{b} \sqrt{\frac{i\sqrt{b}x}{\sqrt{a}}} \sqrt{x(a+bx^2)} \right)$$

Problem 50: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(ax + bx^3)^{3/2}}{x^2} dx$$

Optimal (type 4, 134 leaves, 5 steps):

$$\frac{4}{7} a \sqrt{ax + bx^3} + \frac{2(ax + bx^3)^{3/2}}{7x} + \frac{4a^{7/4} \sqrt{x} (\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a} + \sqrt{bx})^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{7b^{1/4} \sqrt{ax + bx^3}}$$

Result (type 4, 113 leaves):

$$\frac{2x \left(3a^2 + 4abx^2 + b^2x^4 + \frac{4ia^2 \sqrt{1 + \frac{a}{bx^2}} \sqrt{x} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right], -1\right]}{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}} \right)}{7\sqrt{x(a+bx^2)}}$$

Problem 51: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(ax + bx^3)^{3/2}}{x^3} dx$$

Optimal (type 4, 274 leaves, 7 steps):

$$\frac{24 a \sqrt{b} x (a + b x^2)}{5 (\sqrt{a} + \sqrt{b} x) \sqrt{a x + b x^3}} + \frac{12}{5} b x \sqrt{a x + b x^3} - \frac{2 (a x + b x^3)^{3/2}}{x^2} - \frac{24 a^{5/4} b^{1/4} \sqrt{x} (\sqrt{a} + \sqrt{b} x) \sqrt{\frac{a + b x^2}{(\sqrt{a} + \sqrt{b} x)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{b^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{5 \sqrt{a x + b x^3}} +$$

$$\frac{12 a^{5/4} b^{1/4} \sqrt{x} (\sqrt{a} + \sqrt{b} x) \sqrt{\frac{a + b x^2}{(\sqrt{a} + \sqrt{b} x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{5 \sqrt{a x + b x^3}}$$

Result (type 4, 183 leaves):

$$\frac{1}{5 \sqrt{\frac{i \sqrt{b} x}{\sqrt{a}} \sqrt{x (a + b x^2)}}} 2 \left(\sqrt{\frac{i \sqrt{b} x}{\sqrt{a}}} (-5 a^2 - 4 a b x^2 + b^2 x^4) + \right.$$

$$\left. 12 a^{3/2} \sqrt{b} x \sqrt{1 + \frac{b x^2}{a}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b} x}{\sqrt{a}}}\right], -1\right] - 12 a^{3/2} \sqrt{b} x \sqrt{1 + \frac{b x^2}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b} x}{\sqrt{a}}}\right], -1\right] \right)$$

Problem 52: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a x + b x^3)^{3/2}}{x^4} dx$$

Optimal (type 4, 134 leaves, 5 steps):

$$\frac{4}{3} b \sqrt{a x + b x^3} - \frac{2 (a x + b x^3)^{3/2}}{3 x^3} + \frac{4 a^{3/4} b^{3/4} \sqrt{x} (\sqrt{a} + \sqrt{b} x) \sqrt{\frac{a + b x^2}{(\sqrt{a} + \sqrt{b} x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{3 \sqrt{a x + b x^3}}$$

Result (type 4, 107 leaves):

$$2 \left(-a^2 + b^2 x^4 + \frac{4 i a b \sqrt{1 + \frac{a}{b x^2}} x^{5/2} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right], -1\right]}{\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}}} \right)$$

$$\frac{\quad}{3 x \sqrt{x (a + b x^2)}}$$

Problem 53: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(ax + bx^3)^{3/2}}{x^5} dx$$

Optimal (type 4, 277 leaves, 7 steps):

$$\frac{24 b^{3/2} x (a + b x^2)}{5 (\sqrt{a} + \sqrt{b} x) \sqrt{a x + b x^3}} - \frac{12 b \sqrt{a x + b x^3}}{5 x} - \frac{2 (a x + b x^3)^{3/2}}{5 x^4} - \frac{24 a^{1/4} b^{5/4} \sqrt{x} (\sqrt{a} + \sqrt{b} x) \sqrt{\frac{a + b x^2}{(\sqrt{a} + \sqrt{b} x)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{b^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{5 \sqrt{a x + b x^3}} +$$

$$\frac{12 a^{1/4} b^{5/4} \sqrt{x} (\sqrt{a} + \sqrt{b} x) \sqrt{\frac{a + b x^2}{(\sqrt{a} + \sqrt{b} x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{5 \sqrt{a x + b x^3}}$$

Result (type 4, 189 leaves):

$$- \left(\left(2 \left(\sqrt{\frac{i \sqrt{b} x}{\sqrt{a}}} (a^2 + 8 a b x^2 + 7 b^2 x^4) - 12 \sqrt{a} b^{3/2} x^3 \sqrt{1 + \frac{b x^2}{a}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b} x}{\sqrt{a}}}\right], -1\right] + \right. \right. \right.$$

$$\left. \left. 12 \sqrt{a} b^{3/2} x^3 \sqrt{1 + \frac{b x^2}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b} x}{\sqrt{a}}}\right], -1\right] \right) \right) / \left(5 x^2 \sqrt{\frac{i \sqrt{b} x}{\sqrt{a}}} \sqrt{x (a + b x^2)} \right)$$

Problem 54: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(ax + bx^3)^{3/2}}{x^6} dx$$

Optimal (type 4, 137 leaves, 5 steps):

$$- \frac{4 b \sqrt{a x + b x^3}}{7 x^2} - \frac{2 (a x + b x^3)^{3/2}}{7 x^5} + \frac{4 b^{7/4} \sqrt{x} (\sqrt{a} + \sqrt{b} x) \sqrt{\frac{a + b x^2}{(\sqrt{a} + \sqrt{b} x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{7 a^{1/4} \sqrt{a x + b x^3}}$$

Result (type 4, 116 leaves):

$$2 \frac{-a^2 - 4 a b x^2 - 3 b^2 x^4 + \frac{4 i b^2 \sqrt{1 + \frac{a}{b x^2}} x^{9/2} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right], -1\right]}{\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}}}}{7 x^3 \sqrt{x (a + b x^2)}}$$

Problem 55: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a x + b x^3)^{3/2}}{x^7} dx$$

Optimal (type 4, 306 leaves, 8 steps):

$$\frac{8 b^{5/2} x (a + b x^2)}{15 a (\sqrt{a} + \sqrt{b} x) \sqrt{a x + b x^3}} - \frac{4 b \sqrt{a x + b x^3}}{15 x^3} - \frac{8 b^2 \sqrt{a x + b x^3}}{15 a x} - \frac{2 (a x + b x^3)^{3/2}}{9 x^6} - \frac{8 b^{9/4} \sqrt{x} (\sqrt{a} + \sqrt{b} x) \sqrt{\frac{a + b x^2}{(\sqrt{a} + \sqrt{b} x)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{15 a^{3/4} \sqrt{a x + b x^3}} + \frac{4 b^{9/4} \sqrt{x} (\sqrt{a} + \sqrt{b} x) \sqrt{\frac{a + b x^2}{(\sqrt{a} + \sqrt{b} x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{15 a^{3/4} \sqrt{a x + b x^3}}$$

Result (type 4, 205 leaves):

$$-\left(2 \left(\sqrt{\frac{i \sqrt{b} x}{\sqrt{a}}} (5 a^3 + 16 a^2 b x^2 + 23 a b^2 x^4 + 12 b^3 x^6) - 12 \sqrt{a} b^{5/2} x^5 \sqrt{1 + \frac{b x^2}{a}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{i \sqrt{b} x}{\sqrt{a}}\right], -1\right] + 12 \sqrt{a} b^{5/2} x^5 \sqrt{1 + \frac{b x^2}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{i \sqrt{b} x}{\sqrt{a}}\right], -1\right]\right) / \left(45 a x^4 \sqrt{\frac{i \sqrt{b} x}{\sqrt{a}}} \sqrt{x (a + b x^2)}\right)$$

Problem 56: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a x + b x^3)^{3/2}}{x^8} dx$$

Optimal (type 4, 163 leaves, 6 steps):

$$-\frac{12 b \sqrt{a x + b x^3}}{77 x^4} - \frac{8 b^2 \sqrt{a x + b x^3}}{77 a x^2} - \frac{2 (a x + b x^3)^{3/2}}{11 x^7} - \frac{4 b^{11/4} \sqrt{x} (\sqrt{a} + \sqrt{b} x) \sqrt{\frac{a + b x^2}{(\sqrt{a} + \sqrt{b} x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{77 a^{5/4} \sqrt{a x + b x^3}}$$

Result (type 4, 150 leaves):

$$- \left(\left(2 \sqrt{\frac{i\sqrt{a}}{\sqrt{b}}} (7a^3 + 20a^2bx^2 + 17ab^2x^4 + 4b^3x^6) + 4ib^3 \sqrt{1 + \frac{a}{bx^2}} x^{13/2} \text{EllipticF}\left[\frac{i\sqrt{a}}{\sqrt{b}} \sqrt{\frac{1}{x}}, -1\right] \right) \right) / \left(77a \sqrt{\frac{i\sqrt{a}}{\sqrt{b}}} x^5 \sqrt{x(a+bx^2)} \right)$$

Problem 57: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4}{\sqrt{ax+bx^3}} dx$$

Optimal (type 4, 140 leaves, 5 steps):

$$-\frac{10a\sqrt{ax+bx^3}}{21b^2} + \frac{2x^2\sqrt{ax+bx^3}}{7b} + \frac{5a^{7/4}\sqrt{x}(\sqrt{a}+\sqrt{b}x)\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{b}x)^2}} \text{EllipticF}\left[2\text{ArcTan}\left[\frac{b^{1/4}\sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{21b^{9/4}\sqrt{ax+bx^3}}$$

Result (type 4, 138 leaves):

$$\frac{-2\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}} x (5a^2 + 2abx^2 - 3b^2x^4) + 10ia^2 \sqrt{1 + \frac{a}{bx^2}} x^{3/2} \text{EllipticF}\left[\frac{i\sqrt{a}}{\sqrt{b}} \sqrt{\frac{1}{x}}, -1\right]}{21\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}} b^2 \sqrt{x(a+bx^2)}}$$

Problem 58: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^3}{\sqrt{ax+bx^3}} dx$$

Optimal (type 4, 258 leaves, 6 steps):

$$\begin{aligned}
& - \frac{6 a x (a + b x^2)}{5 b^{3/2} (\sqrt{a} + \sqrt{b} x) \sqrt{a x + b x^3}} + \frac{2 x \sqrt{a x + b x^3}}{5 b} + \frac{6 a^{5/4} \sqrt{x} (\sqrt{a} + \sqrt{b} x) \sqrt{\frac{a + b x^2}{(\sqrt{a} + \sqrt{b} x)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{5 b^{7/4} \sqrt{a x + b x^3}} \\
& \frac{3 a^{5/4} \sqrt{x} (\sqrt{a} + \sqrt{b} x) \sqrt{\frac{a + b x^2}{(\sqrt{a} + \sqrt{b} x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{5 b^{7/4} \sqrt{a x + b x^3}}
\end{aligned}$$

Result (type 4, 170 leaves):

$$\begin{aligned}
& \left(2 x \left(\sqrt{b} x \sqrt{\frac{i \sqrt{b} x}{\sqrt{a}}} (a + b x^2) - 3 a^{3/2} \sqrt{1 + \frac{b x^2}{a}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{b} x}{\sqrt{a}}}\right], -1\right] + \right. \right. \\
& \left. \left. 3 a^{3/2} \sqrt{1 + \frac{b x^2}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{b} x}{\sqrt{a}}}\right], -1\right] \right) \right) / \left(5 b^{3/2} \sqrt{\frac{i \sqrt{b} x}{\sqrt{a}}} \sqrt{x (a + b x^2)} \right)
\end{aligned}$$

Problem 59: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2}{\sqrt{a x + b x^3}} dx$$

Optimal (type 4, 116 leaves, 4 steps):

$$\frac{2 \sqrt{a x + b x^3}}{3 b} - \frac{a^{3/4} \sqrt{x} (\sqrt{a} + \sqrt{b} x) \sqrt{\frac{a + b x^2}{(\sqrt{a} + \sqrt{b} x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{3 b^{5/4} \sqrt{a x + b x^3}}$$

Result (type 4, 101 leaves):

$$\frac{2 x \left(a + b x^2 - \frac{i a \sqrt{1 + \frac{a}{b x^2}} \sqrt{x} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}}\right], -1\right]}{\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}}} \right)}{3 b \sqrt{x (a + b x^2)}}$$

Problem 60: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{\sqrt{ax + bx^3}} dx$$

Optimal (type 4, 229 leaves, 5 steps):

$$\frac{2x(a + bx^2)}{\sqrt{b}(\sqrt{a} + \sqrt{bx})\sqrt{ax + bx^3}} - \frac{2a^{1/4}\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a} + \sqrt{bx})^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{b^{1/4}\sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{b^{3/4}\sqrt{ax + bx^3}} +$$

$$\frac{a^{1/4}\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a} + \sqrt{bx})^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4}\sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{b^{3/4}\sqrt{ax + bx^3}}$$

Result (type 4, 108 leaves):

$$\frac{2i x^2 \sqrt{1 + \frac{bx^2}{a}} \left(\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i\sqrt{bx}}{a}}\right], -1\right] - \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i\sqrt{bx}}{a}}\right], -1\right] \right)}{\left(\frac{i\sqrt{bx}}{a}\right)^{3/2} \sqrt{x(a + bx^2)}}$$

Problem 61: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{ax + bx^3}} dx$$

Optimal (type 4, 92 leaves, 3 steps):

$$\frac{\sqrt{x}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a} + \sqrt{bx})^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4}\sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{a^{1/4}b^{1/4}\sqrt{ax + bx^3}}$$

Result (type 4, 85 leaves):

$$\frac{2i \sqrt{1 + \frac{a}{bx^2}} x^{3/2} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i\sqrt{a}}{\sqrt{bx}}}\right], -1\right]}{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}} \sqrt{x(a + bx^2)}}$$

Problem 62: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x \sqrt{ax + bx^3}} dx$$

Optimal (type 4, 253 leaves, 6 steps):

$$\frac{2\sqrt{b}x(a+bx^2)}{a(\sqrt{a}+\sqrt{b}x)\sqrt{ax+bx^3}} - \frac{2\sqrt{ax+bx^3}}{ax} - \frac{2b^{1/4}\sqrt{x}(\sqrt{a}+\sqrt{b}x)\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{b}x)^2}} \operatorname{EllipticE}\left[2\operatorname{ArcTan}\left[\frac{b^{1/4}\sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{a^{3/4}\sqrt{ax+bx^3}} +$$

$$\frac{b^{1/4}\sqrt{x}(\sqrt{a}+\sqrt{b}x)\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{b}x)^2}} \operatorname{EllipticF}\left[2\operatorname{ArcTan}\left[\frac{b^{1/4}\sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{a^{3/4}\sqrt{ax+bx^3}}$$

Result (type 4, 170 leaves):

$$-\frac{1}{a\sqrt{\frac{i\sqrt{b}x}{\sqrt{a}}}\sqrt{x(a+bx^2)}} - 2\left(\sqrt{\frac{i\sqrt{b}x}{\sqrt{a}}}(a+bx^2) - \sqrt{a}\sqrt{b}x\sqrt{1+\frac{bx^2}{a}} \operatorname{EllipticE}\left[i\operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}x}{\sqrt{a}}}\right], -1\right] + \sqrt{a}\sqrt{b}x\sqrt{1+\frac{bx^2}{a}} \operatorname{EllipticF}\left[i\operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}x}{\sqrt{a}}}\right], -1\right]\right)$$

Problem 63: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^2 \sqrt{ax + bx^3}} dx$$

Optimal (type 4, 119 leaves, 4 steps):

$$-\frac{2\sqrt{ax+bx^3}}{3ax^2} - \frac{b^{3/4}\sqrt{x}(\sqrt{a}+\sqrt{b}x)\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{b}x)^2}} \operatorname{EllipticF}\left[2\operatorname{ArcTan}\left[\frac{b^{1/4}\sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{3a^{5/4}\sqrt{ax+bx^3}}$$

Result (type 4, 106 leaves):

$$\frac{2 \left(-a - b x^2 - \frac{i b \sqrt{1 + \frac{a}{b x^2}} x^{5/2} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right], -1\right]}{\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}}} \right)}{3 a x \sqrt{x (a + b x^2)}}$$

Problem 64: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^3 \sqrt{a x + b x^3}} dx$$

Optimal (type 4, 286 leaves, 7 steps):

$$\begin{aligned} & -\frac{6 b^{3/2} x (a + b x^2)}{5 a^2 (\sqrt{a} + \sqrt{b} x) \sqrt{a x + b x^3}} - \frac{2 \sqrt{a x + b x^3}}{5 a x^3} + \frac{6 b \sqrt{a x + b x^3}}{5 a^2 x} + \\ & \frac{6 b^{5/4} \sqrt{x} (\sqrt{a} + \sqrt{b} x) \sqrt{\frac{a + b x^2}{(\sqrt{a} + \sqrt{b} x)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{5 a^{7/4} \sqrt{a x + b x^3}} - \frac{3 b^{5/4} \sqrt{x} (\sqrt{a} + \sqrt{b} x) \sqrt{\frac{a + b x^2}{(\sqrt{a} + \sqrt{b} x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{5 a^{7/4} \sqrt{a x + b x^3}} \end{aligned}$$

Result (type 4, 195 leaves):

$$\begin{aligned} & \left(2 \sqrt{\frac{i \sqrt{b} x}{\sqrt{a}}} (-a^2 + 2 a b x^2 + 3 b^2 x^4) - 6 \sqrt{a} b^{3/2} x^3 \sqrt{1 + \frac{b x^2}{a}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{b} x}{\sqrt{a}}}\right], -1\right] + \right. \\ & \left. 6 \sqrt{a} b^{3/2} x^3 \sqrt{1 + \frac{b x^2}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{b} x}{\sqrt{a}}}\right], -1\right] \right) / \left(5 a^2 x^2 \sqrt{\frac{i \sqrt{b} x}{\sqrt{a}}} \sqrt{x (a + b x^2)} \right) \end{aligned}$$

Problem 65: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^7}{(a x + b x^3)^{3/2}} dx$$

Optimal (type 4, 161 leaves, 6 steps):

$$-\frac{x^5}{b \sqrt{a x + b x^3}} - \frac{15 a \sqrt{a x + b x^3}}{7 b^3} + \frac{9 x^2 \sqrt{a x + b x^3}}{7 b^2} + \frac{15 a^{7/4} \sqrt{x} (\sqrt{a} + \sqrt{b} x) \sqrt{\frac{a + b x^2}{(\sqrt{a} + \sqrt{b} x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{14 b^{13/4} \sqrt{a x + b x^3}}$$

Result (type 4, 137 leaves):

$$\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}} x (-15 a^2 - 6 a b x^2 + 2 b^2 x^4) + 15 i a^2 \sqrt{1 + \frac{a}{b x^2}} x^{3/2} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\right], -1\right]}{7 \sqrt{\frac{i\sqrt{a}}{\sqrt{b}}} b^3 \sqrt{x (a + b x^2)}}$$

Problem 66: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^6}{(a x + b x^3)^{3/2}} dx$$

Optimal (type 4, 279 leaves, 7 steps):

$$\frac{-\frac{x^4}{b \sqrt{a x + b x^3}} - \frac{21 a x (a + b x^2)}{5 b^{5/2} (\sqrt{a} + \sqrt{b} x) \sqrt{a x + b x^3}} + \frac{7 x \sqrt{a x + b x^3}}{5 b^2} + \frac{21 a^{5/4} \sqrt{x} (\sqrt{a} + \sqrt{b} x) \sqrt{\frac{a + b x^2}{(\sqrt{a} + \sqrt{b} x)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{b^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{5 b^{11/4} \sqrt{a x + b x^3}}}{\frac{21 a^{5/4} \sqrt{x} (\sqrt{a} + \sqrt{b} x) \sqrt{\frac{a + b x^2}{(\sqrt{a} + \sqrt{b} x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{10 b^{11/4} \sqrt{a x + b x^3}}}$$

Result (type 4, 173 leaves):

$$\left(x \left(\sqrt{b} x \sqrt{\frac{i\sqrt{b} x}{\sqrt{a}}} (7 a + 2 b x^2) - 21 a^{3/2} \sqrt{1 + \frac{b x^2}{a}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b} x}{\sqrt{a}}}\right], -1\right] + 21 a^{3/2} \sqrt{1 + \frac{b x^2}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b} x}{\sqrt{a}}}\right], -1\right] \right) \right) / \left(5 b^{5/2} \sqrt{\frac{i\sqrt{b} x}{\sqrt{a}}} \sqrt{x (a + b x^2)} \right)$$

Problem 67: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^5}{(a x + b x^3)^{3/2}} dx$$

Optimal (type 4, 137 leaves, 5 steps):

$$-\frac{x^3}{b\sqrt{ax+bx^3}} + \frac{5\sqrt{ax+bx^3}}{3b^2} - \frac{5a^{3/4}\sqrt{x}(\sqrt{a}+\sqrt{b}x)\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{b}x)^2}} \operatorname{EllipticF}\left[2\operatorname{ArcTan}\left[\frac{b^{1/4}\sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{6b^{9/4}\sqrt{ax+bx^3}}$$

Result (type 4, 124 leaves):

$$\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}x(5a+2bx^2) - 5ia\sqrt{1+\frac{a}{bx^2}}x^{3/2} \operatorname{EllipticF}\left[i\operatorname{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right], -1\right]}{3\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}b^2\sqrt{x(a+bx^2)}}$$

Problem 68: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4}{(ax+bx^3)^{3/2}} dx$$

Optimal (type 4, 253 leaves, 6 steps):

$$-\frac{x^2}{b\sqrt{ax+bx^3}} + \frac{3x(a+bx^2)}{b^{3/2}(\sqrt{a}+\sqrt{b}x)\sqrt{ax+bx^3}} - \frac{3a^{1/4}\sqrt{x}(\sqrt{a}+\sqrt{b}x)\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{b}x)^2}} \operatorname{EllipticE}\left[2\operatorname{ArcTan}\left[\frac{b^{1/4}\sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{b^{7/4}\sqrt{ax+bx^3}} +$$

$$\frac{3a^{1/4}\sqrt{x}(\sqrt{a}+\sqrt{b}x)\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{b}x)^2}} \operatorname{EllipticF}\left[2\operatorname{ArcTan}\left[\frac{b^{1/4}\sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{2b^{7/4}\sqrt{ax+bx^3}}$$

Result (type 4, 161 leaves):

$$-\left(\left(x\left(\sqrt{b}x\sqrt{\frac{i\sqrt{b}x}{\sqrt{a}}} - 3\sqrt{a}\sqrt{1+\frac{bx^2}{a}} \operatorname{EllipticE}\left[i\operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}x}{\sqrt{a}}}\right], -1\right] + 3\sqrt{a}\sqrt{1+\frac{bx^2}{a}} \operatorname{EllipticF}\left[i\operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}x}{\sqrt{a}}}\right], -1\right]\right)\right) /$$

$$\left(b^{3/2}\sqrt{\frac{i\sqrt{b}x}{\sqrt{a}}}\sqrt{x(a+bx^2)}\right)$$

Problem 69: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^3}{(ax + bx^3)^{3/2}} dx$$

Optimal (type 4, 115 leaves, 4 steps):

$$-\frac{x}{b\sqrt{ax + bx^3}} + \frac{\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a} + \sqrt{bx})^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4}\sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{2a^{1/4}b^{5/4}\sqrt{ax + bx^3}}$$

Result (type 4, 111 leaves):

$$\frac{-\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}x + i\sqrt{1 + \frac{a}{bx^2}}x^{3/2} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right], -1\right]}{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}b\sqrt{x(a + bx^2)}}$$

Problem 70: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2}{(ax + bx^3)^{3/2}} dx$$

Optimal (type 4, 254 leaves, 6 steps):

$$\frac{x^2}{a\sqrt{ax + bx^3}} - \frac{x(a + bx^2)}{a\sqrt{b}(\sqrt{a} + \sqrt{bx})\sqrt{ax + bx^3}} + \frac{\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a} + \sqrt{bx})^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4}\sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{a^{3/4}b^{3/4}\sqrt{ax + bx^3}} - \frac{\sqrt{x}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a} + \sqrt{bx})^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4}\sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{2a^{3/4}b^{3/4}\sqrt{ax + bx^3}}$$

Result (type 4, 162 leaves):

$$\left(x \left(\sqrt{b} x \sqrt{\frac{i \sqrt{b} x}{\sqrt{a}}} - \sqrt{a} \sqrt{1 + \frac{b x^2}{a}} \operatorname{EllipticE} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{i \sqrt{b} x}{\sqrt{a}}} \right], -1 \right] + \sqrt{a} \sqrt{1 + \frac{b x^2}{a}} \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{i \sqrt{b} x}{\sqrt{a}}} \right], -1 \right] \right) \right) /$$

$$\left(a \sqrt{b} \sqrt{\frac{i \sqrt{b} x}{\sqrt{a}}} \sqrt{x (a + b x^2)} \right)$$

Problem 71: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(a x + b x^3)^{3/2}} dx$$

Optimal (type 4, 114 leaves, 4 steps):

$$\frac{x}{a \sqrt{a x + b x^3}} + \frac{\sqrt{x} (\sqrt{a} + \sqrt{b} x) \sqrt{\frac{a + b x^2}{(\sqrt{a} + \sqrt{b} x)^2}} \operatorname{EllipticF} \left[2 \operatorname{ArcTan} \left[\frac{b^{1/4} \sqrt{x}}{a^{1/4}} \right], \frac{1}{2} \right]}{2 a^{5/4} b^{1/4} \sqrt{a x + b x^3}}$$

Result (type 4, 110 leaves):

$$\frac{\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} x + i \sqrt{1 + \frac{a}{b x^2}} x^{3/2} \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{i \sqrt{a}}{\sqrt{b} x}} \right], -1 \right]}{a \sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} \sqrt{x (a + b x^2)}}$$

Problem 72: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(a x + b x^3)^{3/2}} dx$$

Optimal (type 4, 273 leaves, 7 steps):

$$\frac{1}{a \sqrt{ax+bx^3}} + \frac{3\sqrt{b}x(a+bx^2)}{a^2(\sqrt{a}+\sqrt{b}x)\sqrt{ax+bx^3}} - \frac{3\sqrt{ax+bx^3}}{a^2x} - \frac{3b^{1/4}\sqrt{x}(\sqrt{a}+\sqrt{b}x)\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{b}x)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4}\sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{a^{7/4}\sqrt{ax+bx^3}} +$$

$$\frac{3b^{1/4}\sqrt{x}(\sqrt{a}+\sqrt{b}x)\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{b}x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4}\sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{2a^{7/4}\sqrt{ax+bx^3}}$$

Result (type 4, 174 leaves):

$$\frac{1}{a^2 \sqrt{\frac{i\sqrt{b}x}{\sqrt{a}}}\sqrt{x(a+bx^2)}} \left(-\sqrt{\frac{i\sqrt{b}x}{\sqrt{a}}} (2a+3bx^2) + \right.$$

$$\left. 3\sqrt{a}\sqrt{b}x\sqrt{1+\frac{bx^2}{a}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}x}{\sqrt{a}}}\right], -1\right] - 3\sqrt{a}\sqrt{b}x\sqrt{1+\frac{bx^2}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}x}{\sqrt{a}}}\right], -1\right] \right)$$

Problem 73: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x(ax+bx^3)^{3/2}} dx$$

Optimal (type 4, 139 leaves, 5 steps):

$$\frac{1}{ax\sqrt{ax+bx^3}} - \frac{5\sqrt{ax+bx^3}}{3a^2x^2} - \frac{5b^{3/4}\sqrt{x}(\sqrt{a}+\sqrt{b}x)\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{b}x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4}\sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{6a^{9/4}\sqrt{ax+bx^3}}$$

Result (type 4, 106 leaves):

$$-2a - 5bx^2 - \frac{5ib\sqrt{1+\frac{a}{bx^2}}x^{5/2} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\right], -1\right]}{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}$$

$$\frac{\quad}{3a^2x\sqrt{x(a+bx^2)}}$$

Problem 74: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^2 (ax + bx^3)^{3/2}} dx$$

Optimal (type 4, 306 leaves, 8 steps):

$$\frac{1}{ax^2 \sqrt{ax + bx^3}} - \frac{21 b^{3/2} x (a + bx^2)}{5 a^3 (\sqrt{a} + \sqrt{bx}) \sqrt{ax + bx^3}} - \frac{7 \sqrt{ax + bx^3}}{5 a^2 x^3} +$$

$$\frac{21 b \sqrt{ax + bx^3}}{5 a^3 x} + \frac{21 b^{5/4} \sqrt{x} (\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a} + \sqrt{bx})^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{b^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{5 a^{11/4} \sqrt{ax + bx^3}} -$$

$$\frac{21 b^{5/4} \sqrt{x} (\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a} + \sqrt{bx})^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} \sqrt{x}}{a^{1/4}}\right], \frac{1}{2}\right]}{10 a^{11/4} \sqrt{ax + bx^3}}$$

Result (type 4, 194 leaves):

$$\left(\sqrt{\frac{i \sqrt{bx}}{\sqrt{a}}} (-2 a^2 + 14 a b x^2 + 21 b^2 x^4) - 21 \sqrt{a} b^{3/2} x^3 \sqrt{1 + \frac{bx^2}{a}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{bx}}{\sqrt{a}}}\right], -1\right] + \right.$$

$$\left. 21 \sqrt{a} b^{3/2} x^3 \sqrt{1 + \frac{bx^2}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{bx}}{\sqrt{a}}}\right], -1\right] \right) / \left(5 a^3 x^2 \sqrt{\frac{i \sqrt{bx}}{\sqrt{a}}} \sqrt{x (a + bx^2)} \right)$$

Problem 97: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^3}{\sqrt{ax + bx^4}} dx$$

Optimal (type 4, 224 leaves, 4 steps):

$$\frac{\sqrt{ax + bx^4}}{2 b} - \frac{a^{2/3} x (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{(a^{1/3} + (1 + \sqrt{3}) b^{1/3} x)^2}} \text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3} + (1 - \sqrt{3}) b^{1/3} x}{a^{1/3} + (1 + \sqrt{3}) b^{1/3} x}\right], \frac{1}{4} (2 + \sqrt{3})\right]}{4 \times 3^{1/4} b \sqrt{\frac{b^{1/3} x (a^{1/3} + b^{1/3} x)}{(a^{1/3} + (1 + \sqrt{3}) b^{1/3} x)^2}} \sqrt{ax + bx^4}}$$

Result (type 4, 174 leaves):

$$\frac{1}{6 (-a)^{1/3} b \sqrt{x (a + b x^3)}}$$

$$\left(3 (-a)^{1/3} x (a + b x^3) + i 3^{3/4} a b^{1/3} \sqrt{(-1)^{5/6} \left(-1 + \frac{(-a)^{1/3}}{b^{1/3} x}\right)} x^2 \sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}} + \frac{(-a)^{1/3} x}{b^{1/3}} + x^2}{x^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-a)^{1/3}}{b^{1/3} x}}}{3^{1/4}}}\right], (-1)^{1/3}\right] \right)$$

Problem 98: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{a x + b x^4}} dx$$

Optimal (type 4, 197 leaves, 3 steps):

$$\frac{x (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{(a^{1/3} + (1 + \sqrt{3}) b^{1/3} x)^2}} \text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3} + (1 - \sqrt{3}) b^{1/3} x}{a^{1/3} + (1 + \sqrt{3}) b^{1/3} x}\right], \frac{1}{4} (2 + \sqrt{3})\right]}{3^{1/4} a^{1/3} \sqrt{\frac{b^{1/3} x (a^{1/3} + b^{1/3} x)}{(a^{1/3} + (1 + \sqrt{3}) b^{1/3} x)^2}} \sqrt{a x + b x^4}}$$

Result (type 4, 147 leaves):

$$-\left(\left(2 i b^{1/3} \sqrt{(-1)^{5/6} \left(-1 + \frac{(-a)^{1/3}}{b^{1/3} x}\right)} \sqrt{1 + \frac{(-a)^{2/3}}{b^{2/3} x^2} + \frac{(-a)^{1/3}}{b^{1/3} x}} x^2 \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-a)^{1/3}}{b^{1/3} x}}}{3^{1/4}}}\right], (-1)^{1/3}\right] \right) / \left(3^{1/4} (-a)^{1/3} \sqrt{x (a + b x^3)} \right) \right)$$

Problem 99: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^3 \sqrt{a x + b x^4}} dx$$

Optimal (type 4, 225 leaves, 4 steps):

$$\frac{2 \sqrt{a x + b x^4}}{5 a x^3} - \frac{2 b x (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{(a^{1/3} + (1 + \sqrt{3}) b^{1/3} x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcCos}\left[\frac{a^{1/3} + (1 - \sqrt{3}) b^{1/3} x}{a^{1/3} + (1 + \sqrt{3}) b^{1/3} x}\right], \frac{1}{4} (2 + \sqrt{3})\right]}{5 \times 3^{1/4} a^{4/3} \sqrt{\frac{b^{1/3} x (a^{1/3} + b^{1/3} x)}{(a^{1/3} + (1 + \sqrt{3}) b^{1/3} x)^2}} \sqrt{a x + b x^4}}$$

Result (type 4, 172 leaves):

$$\left(\left(-6 (-a)^{1/3} (a + b x^3) + 4 i 3^{3/4} b^{4/3} \sqrt{(-1)^{5/6} \left(-1 + \frac{(-a)^{1/3}}{b^{1/3} x}\right)} x^4 \sqrt{\frac{(-a)^{2/3} + \frac{(-a)^{1/3} x}{b^{1/3}} + x^2}{x^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i (-a)^{1/3}}{b^{1/3} x}}}{3^{1/4}}\right], (-1)^{1/3}\right] \right) / \right. \\ \left. \left(15 (-a)^{4/3} x^2 \sqrt{x (a + b x^3)} \right) \right)$$

Problem 100: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^5}{\sqrt{a x + b x^4}} dx$$

Optimal (type 4, 503 leaves, 6 steps):

$$\begin{aligned}
& - \frac{5(1+\sqrt{3})ax(a+bx^3)}{8b^{5/3}(a^{1/3}+(1+\sqrt{3})b^{1/3}x)\sqrt{ax+bx^4}} + \frac{x^2\sqrt{ax+bx^4}}{4b} + \\
& \frac{5 \times 3^{1/4} a^{4/3} x (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{(a^{1/3} + (1+\sqrt{3}) b^{1/3} x)^2}} \operatorname{EllipticE}\left[\operatorname{ArcCos}\left[\frac{a^{1/3} + (1-\sqrt{3}) b^{1/3} x}{a^{1/3} + (1+\sqrt{3}) b^{1/3} x}\right], \frac{1}{4}(2+\sqrt{3})\right]}{8b^{5/3} \sqrt{\frac{b^{1/3} x (a^{1/3} + b^{1/3} x)}{(a^{1/3} + (1+\sqrt{3}) b^{1/3} x)^2}} \sqrt{ax+bx^4}} + \\
& \left(\frac{5(1-\sqrt{3})a^{4/3}x(a^{1/3}+b^{1/3}x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{(a^{1/3} + (1+\sqrt{3}) b^{1/3} x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcCos}\left[\frac{a^{1/3} + (1-\sqrt{3}) b^{1/3} x}{a^{1/3} + (1+\sqrt{3}) b^{1/3} x}\right], \frac{1}{4}(2+\sqrt{3})\right]}{16 \times 3^{1/4} b^{5/3} \sqrt{\frac{b^{1/3} x (a^{1/3} + b^{1/3} x)}{(a^{1/3} + (1+\sqrt{3}) b^{1/3} x)^2}} \sqrt{ax+bx^4}} \right) /
\end{aligned}$$

Result (type 4, 355 leaves):

$$\begin{aligned}
& \frac{1}{8b\sqrt{x(a+bx^3)}} \left(5ax \left(-\frac{a^{2/3}}{b^{2/3}} + \frac{a^{1/3}x}{b^{1/3}} - x^2 \right) + 2x^3(a+bx^3) - \frac{1}{2(-1+(-1)^{2/3})b} \right. \\
& \left. 5(-1)^{2/3} a^{4/3} (a^{1/3} + b^{1/3} x)^2 \sqrt{\frac{(1+(-1)^{1/3}) b^{1/3} x (a^{1/3} - (-1)^{1/3} b^{1/3} x)}{(a^{1/3} + b^{1/3} x)^2}} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{a^{1/3} + b^{1/3} x}} \right. \\
& \left. \left((-3-i\sqrt{3}) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(3+i\sqrt{3})} b^{1/3} x}{a^{1/3} + b^{1/3} x}\right], \frac{-i+\sqrt{3}}{i+\sqrt{3}}\right] + (1+i\sqrt{3}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(3+i\sqrt{3})} b^{1/3} x}{a^{1/3} + b^{1/3} x}\right], \frac{-i+\sqrt{3}}{i+\sqrt{3}}\right] \right) \right)
\end{aligned}$$

Problem 101: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2}{\sqrt{ax+bx^4}} dx$$

Optimal (type 4, 474 leaves, 5 steps):

$$\frac{(1 + \sqrt{3}) x (a + b x^3)}{b^{2/3} (a^{1/3} + (1 + \sqrt{3}) b^{1/3} x) \sqrt{a x + b x^4}} - \frac{3^{1/4} a^{1/3} x (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{(a^{1/3} + (1 + \sqrt{3}) b^{1/3} x)^2}} \text{EllipticE}\left[\text{ArcCos}\left[\frac{a^{1/3} + (1 - \sqrt{3}) b^{1/3} x}{a^{1/3} + (1 + \sqrt{3}) b^{1/3} x}\right], \frac{1}{4} (2 + \sqrt{3})\right]}{b^{2/3} \sqrt{\frac{b^{1/3} x (a^{1/3} + b^{1/3} x)}{(a^{1/3} + (1 + \sqrt{3}) b^{1/3} x)^2}} \sqrt{a x + b x^4}} -$$

$$\frac{(1 - \sqrt{3}) a^{1/3} x (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{(a^{1/3} + (1 + \sqrt{3}) b^{1/3} x)^2}} \text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3} + (1 - \sqrt{3}) b^{1/3} x}{a^{1/3} + (1 + \sqrt{3}) b^{1/3} x}\right], \frac{1}{4} (2 + \sqrt{3})\right]}{2 \times 3^{1/4} b^{2/3} \sqrt{\frac{b^{1/3} x (a^{1/3} + b^{1/3} x)}{(a^{1/3} + (1 + \sqrt{3}) b^{1/3} x)^2}} \sqrt{a x + b x^4}}$$

Result (type 4, 333 leaves):

$$\frac{1}{\sqrt{x (a + b x^3)}}$$

$$\left(x \left(\frac{a^{2/3}}{b^{2/3}} - \frac{a^{1/3} x}{b^{1/3}} + x^2 \right) + \frac{1}{2 (-1 + (-1)^{2/3}) b} (-1)^{2/3} a^{1/3} (a^{1/3} + b^{1/3} x)^2 \sqrt{\frac{(1 + (-1)^{1/3}) b^{1/3} x (a^{1/3} - (-1)^{1/3} b^{1/3} x)}{(a^{1/3} + b^{1/3} x)^2}} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{a^{1/3} + b^{1/3} x}} \right.$$

$$\left. \left((-3 - i \sqrt{3}) \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(3 + i \sqrt{3}) b^{1/3} x}}{a^{1/3} + b^{1/3} x}\right], \frac{-i + \sqrt{3}}{i + \sqrt{3}}\right] + (1 + i \sqrt{3}) \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(3 + i \sqrt{3}) b^{1/3} x}}{a^{1/3} + b^{1/3} x}\right], \frac{-i + \sqrt{3}}{i + \sqrt{3}}\right] \right) \right)$$

Problem 102: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x \sqrt{a x + b x^4}} dx$$

Optimal (type 4, 497 leaves, 6 steps):

$$\frac{2(1+\sqrt{3})b^{1/3}x(a+bx^3)}{a(a^{1/3}+(1+\sqrt{3})b^{1/3}x)\sqrt{ax+bx^4}} - \frac{2\sqrt{ax+bx^4}}{ax} -$$

$$\frac{2 \times 3^{1/4} b^{1/3} x (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{(a^{1/3} + (1+\sqrt{3}) b^{1/3} x)^2}} \text{EllipticE}\left[\text{ArcCos}\left[\frac{a^{1/3} + (1-\sqrt{3}) b^{1/3} x}{a^{1/3} + (1+\sqrt{3}) b^{1/3} x}\right], \frac{1}{4}(2+\sqrt{3})\right]}{a^{2/3} \sqrt{\frac{b^{1/3} x (a^{1/3} + b^{1/3} x)}{(a^{1/3} + (1+\sqrt{3}) b^{1/3} x)^2}} \sqrt{ax+bx^4}} -$$

$$\frac{(1-\sqrt{3})b^{1/3}x(a^{1/3}+b^{1/3}x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{(a^{1/3} + (1+\sqrt{3}) b^{1/3} x)^2}} \text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3} + (1-\sqrt{3}) b^{1/3} x}{a^{1/3} + (1+\sqrt{3}) b^{1/3} x}\right], \frac{1}{4}(2+\sqrt{3})\right]}{3^{1/4} a^{2/3} \sqrt{\frac{b^{1/3} x (a^{1/3} + b^{1/3} x)}{(a^{1/3} + (1+\sqrt{3}) b^{1/3} x)^2}} \sqrt{ax+bx^4}}$$

Result (type 4, 334 leaves):

$$\frac{1}{a\sqrt{x(a+bx^3)}} -$$

$$2 \left(-a + a^{2/3} b^{1/3} x - a^{1/3} b^{2/3} x^2 + \frac{1}{2(-1+(-1)^{2/3})} (-1)^{2/3} a^{1/3} (a^{1/3} + b^{1/3} x)^2 \sqrt{\frac{(1+(-1)^{1/3}) b^{1/3} x (a^{1/3} - (-1)^{1/3} b^{1/3} x)}{(a^{1/3} + b^{1/3} x)^2}} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{a^{1/3} + b^{1/3} x}} \right.$$

$$\left. \left((-3 - i\sqrt{3}) \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(3+i\sqrt{3})} b^{1/3} x}{\sqrt{2} \sqrt{a^{1/3} + b^{1/3} x}}\right], \frac{-i + \sqrt{3}}{i + \sqrt{3}}\right] + (1 + i\sqrt{3}) \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(3+i\sqrt{3})} b^{1/3} x}{\sqrt{2} \sqrt{a^{1/3} + b^{1/3} x}}\right], \frac{-i + \sqrt{3}}{i + \sqrt{3}}\right] \right) \right)$$

Problem 131: Result unnecessarily involves higher level functions.

$$\int x^3 \sqrt{bx^{1/3} + ax} dx$$

Optimal (type 4, 301 leaves, 11 steps):

$$\begin{aligned}
& -\frac{884 b^6 \sqrt{b x^{1/3} + a x}}{14421 a^6} + \frac{884 b^5 x^{2/3} \sqrt{b x^{1/3} + a x}}{24035 a^5} - \frac{6188 b^4 x^{4/3} \sqrt{b x^{1/3} + a x}}{216315 a^4} + \frac{476 b^3 x^2 \sqrt{b x^{1/3} + a x}}{19665 a^3} - \frac{28 b^2 x^{8/3} \sqrt{b x^{1/3} + a x}}{1311 a^2} + \\
& \frac{4 b x^{10/3} \sqrt{b x^{1/3} + a x}}{207 a} + \frac{2}{9} x^4 \sqrt{b x^{1/3} + a x} + \frac{442 b^{27/4} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{\frac{b+a x^{2/3}}{(\sqrt{b} + \sqrt{a} x^{1/3})^2}} x^{1/6} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{a^{1/4} x^{1/6}}{b^{1/4}}\right], \frac{1}{2}\right]}{14421 a^{25/4} \sqrt{b x^{1/3} + a x}}
\end{aligned}$$

Result (type 5, 155 leaves):

$$\begin{aligned}
& \frac{1}{216315 a^6 \sqrt{b x^{1/3} + a x}} 2 x^{1/3} \left(-6630 b^7 - 2652 a b^6 x^{2/3} + 884 a^2 b^5 x^{4/3} - 476 a^3 b^4 x^2 + 308 a^4 b^3 x^{8/3} - \right. \\
& \left. 220 a^5 b^2 x^{10/3} + 26125 a^6 b x^4 + 24035 a^7 x^{14/3} - 6630 b^7 \sqrt{1 + \frac{b}{a x^{2/3}}} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -\frac{b}{a x^{2/3}}\right] \right)
\end{aligned}$$

Problem 132: Result unnecessarily involves higher level functions.

$$\int x^2 \sqrt{b x^{1/3} + a x} dx$$

Optimal (type 4, 411 leaves, 11 steps):

$$\begin{aligned}
& \frac{44 b^5 (b + a x^{2/3}) x^{1/3}}{221 a^{9/2} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{b x^{1/3} + a x}} - \frac{44 b^4 x^{1/3} \sqrt{b x^{1/3} + a x}}{663 a^4} + \frac{220 b^3 x \sqrt{b x^{1/3} + a x}}{4641 a^3} - \frac{60 b^2 x^{5/3} \sqrt{b x^{1/3} + a x}}{1547 a^2} + \\
& \frac{4 b x^{7/3} \sqrt{b x^{1/3} + a x}}{119 a} + \frac{2}{7} x^3 \sqrt{b x^{1/3} + a x} - \frac{44 b^{21/4} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{\frac{b+a x^{2/3}}{(\sqrt{b} + \sqrt{a} x^{1/3})^2}} x^{1/6} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{a^{1/4} x^{1/6}}{b^{1/4}}\right], \frac{1}{2}\right]}{221 a^{19/4} \sqrt{b x^{1/3} + a x}} + \\
& \frac{22 b^{21/4} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{\frac{b+a x^{2/3}}{(\sqrt{b} + \sqrt{a} x^{1/3})^2}} x^{1/6} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{a^{1/4} x^{1/6}}{b^{1/4}}\right], \frac{1}{2}\right]}{221 a^{19/4} \sqrt{b x^{1/3} + a x}}
\end{aligned}$$

Result (type 5, 131 leaves):

$$\frac{1}{4641 a^4 \sqrt{b x^{1/3} + a x}}$$

$$2 x^{2/3} \left(-154 b^5 - 44 a b^4 x^{2/3} + 20 a^2 b^3 x^{4/3} - 12 a^3 b^2 x^2 + 741 a^4 b x^{8/3} + 663 a^5 x^{10/3} + 462 b^5 \sqrt{1 + \frac{b}{a x^{2/3}}} \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -\frac{b}{a x^{2/3}} \right] \right)$$

Problem 133: Result unnecessarily involves higher level functions.

$$\int x \sqrt{b x^{1/3} + a x} dx$$

Optimal (type 4, 213 leaves, 8 steps):

$$\frac{12 b^3 \sqrt{b x^{1/3} + a x}}{77 a^3} - \frac{36 b^2 x^{2/3} \sqrt{b x^{1/3} + a x}}{385 a^2} + \frac{4 b x^{4/3} \sqrt{b x^{1/3} + a x}}{55 a} +$$

$$\frac{2}{5} x^2 \sqrt{b x^{1/3} + a x} - \frac{6 b^{15/4} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{\frac{b+a x^{2/3}}{(\sqrt{b} + \sqrt{a} x^{1/3})^2}} x^{1/6} \operatorname{EllipticF} \left[2 \operatorname{ArcTan} \left[\frac{a^{1/4} x^{1/6}}{b^{1/4}} \right], \frac{1}{2} \right]}{77 a^{13/4} \sqrt{b x^{1/3} + a x}}$$

Result (type 5, 118 leaves):

$$\frac{1}{385 a^3 \sqrt{b x^{1/3} + a x}} 2 x^{1/3} \left(30 b^4 + 12 a b^3 x^{2/3} - 4 a^2 b^2 x^{4/3} + 91 a^3 b x^2 + 77 a^4 x^{8/3} + 30 b^4 \sqrt{1 + \frac{b}{a x^{2/3}}} \operatorname{Hypergeometric2F1} \left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -\frac{b}{a x^{2/3}} \right] \right)$$

Problem 134: Result unnecessarily involves higher level functions.

$$\int \sqrt{b x^{1/3} + a x} dx$$

Optimal (type 4, 323 leaves, 8 steps):

$$\begin{aligned}
& - \frac{4 b^2 (b + a x^{2/3}) x^{1/3}}{5 a^{3/2} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{b x^{1/3} + a x}} + \frac{4 b x^{1/3} \sqrt{b x^{1/3} + a x}}{15 a} + \\
& \frac{2}{3} x \sqrt{b x^{1/3} + a x} + \frac{4 b^{9/4} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{\frac{b + a x^{2/3}}{(\sqrt{b} + \sqrt{a} x^{1/3})^2}} x^{1/6} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{a^{1/4} x^{1/6}}{b^{1/4}}\right], \frac{1}{2}\right]}{5 a^{7/4} \sqrt{b x^{1/3} + a x}} - \\
& \frac{2 b^{9/4} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{\frac{b + a x^{2/3}}{(\sqrt{b} + \sqrt{a} x^{1/3})^2}} x^{1/6} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{a^{1/4} x^{1/6}}{b^{1/4}}\right], \frac{1}{2}\right]}{5 a^{7/4} \sqrt{b x^{1/3} + a x}}
\end{aligned}$$

Result (type 5, 94 leaves):

$$\frac{2 x^{2/3} \left(2 b^2 + 7 a b x^{2/3} + 5 a^2 x^{4/3} - 6 b^2 \sqrt{1 + \frac{b}{a x^{2/3}}} \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -\frac{b}{a x^{2/3}}\right] \right)}{15 a \sqrt{b x^{1/3} + a x}}$$

Problem 135: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{b x^{1/3} + a x}}{x} dx$$

Optimal (type 4, 123 leaves, 5 steps):

$$2 \sqrt{b x^{1/3} + a x} + \frac{2 b^{3/4} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{\frac{b + a x^{2/3}}{(\sqrt{b} + \sqrt{a} x^{1/3})^2}} x^{1/6} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{a^{1/4} x^{1/6}}{b^{1/4}}\right], \frac{1}{2}\right]}{a^{1/4} \sqrt{b x^{1/3} + a x}}$$

Result (type 5, 71 leaves):

$$\frac{2 x^{1/3} \left(b + a x^{2/3} - 2 b \sqrt{1 + \frac{b}{a x^{2/3}}} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -\frac{b}{a x^{2/3}}\right] \right)}{\sqrt{b x^{1/3} + a x}}$$

Problem 136: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{b x^{1/3} + a x}}{x^2} dx$$

Optimal (type 4, 325 leaves, 8 steps):

$$\frac{12 a^{3/2} (b + a x^{2/3}) x^{1/3}}{5 b (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{b x^{1/3} + a x}} - \frac{6 \sqrt{b x^{1/3} + a x}}{5 x} - \frac{12 a \sqrt{b x^{1/3} + a x}}{5 b x^{1/3}} -$$

$$\frac{12 a^{5/4} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{\frac{b+a x^{2/3}}{(\sqrt{b} + \sqrt{a} x^{1/3})^2}} x^{1/6} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{a^{1/4} x^{1/6}}{b^{1/4}}\right], \frac{1}{2}\right]}{5 b^{3/4} \sqrt{b x^{1/3} + a x}} +$$

$$\frac{6 a^{5/4} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{\frac{b+a x^{2/3}}{(\sqrt{b} + \sqrt{a} x^{1/3})^2}} x^{1/6} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{a^{1/4} x^{1/6}}{b^{1/4}}\right], \frac{1}{2}\right]}{5 b^{3/4} \sqrt{b x^{1/3} + a x}}$$

Result (type 5, 97 leaves):

$$\frac{6 \left(b^2 + 3 a b x^{2/3} + 2 a^2 x^{4/3} - 2 a^2 \sqrt{1 + \frac{b}{a x^{2/3}}} x^{4/3} \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -\frac{b}{a x^{2/3}}\right] \right)}{5 b x^{2/3} \sqrt{b x^{1/3} + a x}}$$

Problem 137: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{b x^{1/3} + a x}}{x^3} dx$$

Optimal (type 4, 188 leaves, 7 steps):

$$-\frac{6 \sqrt{b x^{1/3} + a x}}{11 x^2} - \frac{12 a \sqrt{b x^{1/3} + a x}}{77 b x^{4/3}} + \frac{20 a^2 \sqrt{b x^{1/3} + a x}}{77 b^2 x^{2/3}} + \frac{10 a^{11/4} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{\frac{b+a x^{2/3}}{(\sqrt{b} + \sqrt{a} x^{1/3})^2}} x^{1/6} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{a^{1/4} x^{1/6}}{b^{1/4}}\right], \frac{1}{2}\right]}{77 b^{9/4} \sqrt{b x^{1/3} + a x}}$$

Result (type 5, 108 leaves):

$$\frac{-42 b^3 - 54 a b^2 x^{2/3} + 8 a^2 b x^{4/3} + 20 a^3 x^2 - 20 a^3 \sqrt{1 + \frac{b}{a x^{2/3}}} x^2 \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -\frac{b}{a x^{2/3}}\right]}{77 b^2 x^{5/3} \sqrt{b x^{1/3} + a x}}$$

Problem 138: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{b x^{1/3} + a x}}{x^4} dx$$

Optimal (type 4, 413 leaves, 11 steps):

$$\begin{aligned} & -\frac{308 a^{9/2} (b + a x^{2/3}) x^{1/3}}{1105 b^4 (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{b x^{1/3} + a x}} - \frac{6 \sqrt{b x^{1/3} + a x}}{17 x^3} - \frac{12 a \sqrt{b x^{1/3} + a x}}{221 b x^{7/3}} + \frac{44 a^2 \sqrt{b x^{1/3} + a x}}{663 b^2 x^{5/3}} - \\ & \frac{308 a^3 \sqrt{b x^{1/3} + a x}}{3315 b^3 x} + \frac{308 a^4 \sqrt{b x^{1/3} + a x}}{1105 b^4 x^{1/3}} + \frac{308 a^{17/4} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{\frac{b+a x^{2/3}}{(\sqrt{b} + \sqrt{a} x^{1/3})^2}} x^{1/6} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{a^{1/4} x^{1/6}}{b^{1/4}}\right], \frac{1}{2}\right]}{1105 b^{15/4} \sqrt{b x^{1/3} + a x}} - \\ & \frac{154 a^{17/4} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{\frac{b+a x^{2/3}}{(\sqrt{b} + \sqrt{a} x^{1/3})^2}} x^{1/6} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{a^{1/4} x^{1/6}}{b^{1/4}}\right], \frac{1}{2}\right]}{1105 b^{15/4} \sqrt{b x^{1/3} + a x}} \end{aligned}$$

Result (type 5, 136 leaves):

$$\begin{aligned} & -\frac{1}{3315 b^4 x^{8/3} \sqrt{b x^{1/3} + a x}} \\ & 2 \left(585 b^5 + 675 a b^4 x^{2/3} - 20 a^2 b^3 x^{4/3} + 44 a^3 b^2 x^2 - 308 a^4 b x^{8/3} - 462 a^5 x^{10/3} + 462 a^5 \sqrt{1 + \frac{b}{a x^{2/3}}} x^{10/3} \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -\frac{b}{a x^{2/3}}\right] \right) \end{aligned}$$

Problem 139: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{b x^{1/3} + a x}}{x^5} dx$$

Optimal (type 4, 276 leaves, 10 steps):

$$\begin{aligned}
& - \frac{6 \sqrt{b x^{1/3} + a x}}{23 x^4} - \frac{12 a \sqrt{b x^{1/3} + a x}}{437 b x^{10/3}} + \frac{68 a^2 \sqrt{b x^{1/3} + a x}}{2185 b^2 x^{8/3}} - \frac{884 a^3 \sqrt{b x^{1/3} + a x}}{24 035 b^3 x^2} + \frac{7956 a^4 \sqrt{b x^{1/3} + a x}}{168 245 b^4 x^{4/3}} - \\
& \frac{2652 a^5 \sqrt{b x^{1/3} + a x}}{33 649 b^5 x^{2/3}} - \frac{1326 a^{23/4} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{\frac{b+a x^{2/3}}{(\sqrt{b} + \sqrt{a} x^{1/3})^2}} x^{1/6} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{a^{1/4} x^{1/6}}{b^{1/4}}\right], \frac{1}{2}\right]}{33 649 b^{21/4} \sqrt{b x^{1/3} + a x}}
\end{aligned}$$

Result (type 5, 145 leaves):

$$\begin{aligned}
& - \left(\left(2 \left(21 945 b^6 + 24 255 a b^5 x^{2/3} - 308 a^2 b^4 x^{4/3} + 476 a^3 b^3 x^2 - 884 a^4 b^2 x^{8/3} + 2652 a^5 b x^{10/3} + \right. \right. \right. \\
& \left. \left. 6630 a^6 x^4 - 6630 a^6 \sqrt{1 + \frac{b}{a x^{2/3}}} x^4 \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -\frac{b}{a x^{2/3}}\right] \right) \right) / \left(168 245 b^5 x^{11/3} \sqrt{b x^{1/3} + a x} \right)
\end{aligned}$$

Problem 140: Result unnecessarily involves higher level functions.

$$\int x^2 (b x^{1/3} + a x)^{3/2} dx$$

Optimal (type 4, 298 leaves, 11 steps):

$$\begin{aligned}
& \frac{1768 b^6 \sqrt{b x^{1/3} + a x}}{100 947 a^5} - \frac{1768 b^5 x^{2/3} \sqrt{b x^{1/3} + a x}}{168 245 a^4} + \frac{1768 b^4 x^{4/3} \sqrt{b x^{1/3} + a x}}{216 315 a^3} - \frac{136 b^3 x^2 \sqrt{b x^{1/3} + a x}}{19 665 a^2} + \frac{8 b^2 x^{8/3} \sqrt{b x^{1/3} + a x}}{1311 a} + \\
& \frac{4}{69} b x^{10/3} \sqrt{b x^{1/3} + a x} + \frac{2}{9} x^3 (b x^{1/3} + a x)^{3/2} - \frac{884 b^{27/4} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{\frac{b+a x^{2/3}}{(\sqrt{b} + \sqrt{a} x^{1/3})^2}} x^{1/6} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{a^{1/4} x^{1/6}}{b^{1/4}}\right], \frac{1}{2}\right]}{100 947 a^{21/4} \sqrt{b x^{1/3} + a x}}
\end{aligned}$$

Result (type 5, 155 leaves):

$$\begin{aligned}
& \frac{1}{1514 205 a^5 \sqrt{b x^{1/3} + a x}} \\
& 2 x^{1/3} \left(13 260 b^7 + 5304 a b^6 x^{2/3} - 1768 a^2 b^5 x^{4/3} + 952 a^3 b^4 x^2 - 616 a^4 b^3 x^{8/3} + 216 755 a^5 b^2 x^{10/3} + 380 380 a^6 b x^4 + 168 245 a^7 x^{14/3} + \right. \\
& \left. 13 260 b^7 \sqrt{1 + \frac{b}{a x^{2/3}}} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -\frac{b}{a x^{2/3}}\right] \right)
\end{aligned}$$

Problem 141: Result unnecessarily involves higher level functions.

$$\int x (b x^{1/3} + a x)^{3/2} dx$$

Optimal (type 4, 408 leaves, 11 steps):

$$\begin{aligned} & -\frac{88 b^5 (b + a x^{2/3}) x^{1/3}}{1105 a^{7/2} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{b x^{1/3} + a x}} + \frac{88 b^4 x^{1/3} \sqrt{b x^{1/3} + a x}}{3315 a^3} - \frac{88 b^3 x \sqrt{b x^{1/3} + a x}}{4641 a^2} + \frac{24 b^2 x^{5/3} \sqrt{b x^{1/3} + a x}}{1547 a} + \\ & \frac{12}{119} b x^{7/3} \sqrt{b x^{1/3} + a x} + \frac{2}{7} x^2 (b x^{1/3} + a x)^{3/2} + \frac{88 b^{21/4} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{\frac{b+a x^{2/3}}{(\sqrt{b} + \sqrt{a} x^{1/3})^2}} x^{1/6} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{a^{1/4} x^{1/6}}{b^{1/4}}\right], \frac{1}{2}\right]}{1105 a^{15/4} \sqrt{b x^{1/3} + a x}} - \\ & \frac{44 b^{21/4} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{\frac{b+a x^{2/3}}{(\sqrt{b} + \sqrt{a} x^{1/3})^2}} x^{1/6} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{a^{1/4} x^{1/6}}{b^{1/4}}\right], \frac{1}{2}\right]}{1105 a^{15/4} \sqrt{b x^{1/3} + a x}} \end{aligned}$$

Result (type 5, 131 leaves):

$$\begin{aligned} & \frac{1}{23205 a^3 \sqrt{b x^{1/3} + a x}} \\ & 2 x^{2/3} \left(308 b^5 + 88 a b^4 x^{2/3} - 40 a^2 b^3 x^{4/3} + 4665 a^3 b^2 x^2 + 7800 a^4 b x^{8/3} + 3315 a^5 x^{10/3} - 924 b^5 \sqrt{1 + \frac{b}{a x^{2/3}}} \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -\frac{b}{a x^{2/3}}\right] \right) \end{aligned}$$

Problem 142: Result unnecessarily involves higher level functions.

$$\int (b x^{1/3} + a x)^{3/2} dx$$

Optimal (type 4, 208 leaves, 8 steps):

$$\begin{aligned} & -\frac{8 b^3 \sqrt{b x^{1/3} + a x}}{77 a^2} + \frac{24 b^2 x^{2/3} \sqrt{b x^{1/3} + a x}}{385 a} + \frac{12}{55} b x^{4/3} \sqrt{b x^{1/3} + a x} + \\ & \frac{2}{5} x (b x^{1/3} + a x)^{3/2} + \frac{4 b^{15/4} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{\frac{b+a x^{2/3}}{(\sqrt{b} + \sqrt{a} x^{1/3})^2}} x^{1/6} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{a^{1/4} x^{1/6}}{b^{1/4}}\right], \frac{1}{2}\right]}{77 a^{9/4} \sqrt{b x^{1/3} + a x}} \end{aligned}$$

Result (type 5, 118 leaves):

$$\frac{1}{385 a^2 \sqrt{b x^{1/3} + a x}} 2 x^{1/3} \left(-20 b^4 - 8 a b^3 x^{2/3} + 131 a^2 b^2 x^{4/3} + 196 a^3 b x^2 + 77 a^4 x^{8/3} - 20 b^4 \sqrt{1 + \frac{b}{a x^{2/3}}} \operatorname{Hypergeometric2F1} \left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -\frac{b}{a x^{2/3}} \right] \right)$$

Problem 143: Result unnecessarily involves higher level functions.

$$\int \frac{(b x^{1/3} + a x)^{3/2}}{x} dx$$

Optimal (type 4, 319 leaves, 8 steps):

$$\frac{8 b^2 (b + a x^{2/3}) x^{1/3}}{5 \sqrt{a} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{b x^{1/3} + a x}} + \frac{4}{5} b x^{1/3} \sqrt{b x^{1/3} + a x} +$$

$$\frac{2}{3} (b x^{1/3} + a x)^{3/2} - \frac{8 b^{9/4} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{\frac{b + a x^{2/3}}{(\sqrt{b} + \sqrt{a} x^{1/3})^2}} x^{1/6} \operatorname{EllipticE} \left[2 \operatorname{ArcTan} \left[\frac{a^{1/4} x^{1/6}}{b^{1/4}} \right], \frac{1}{2} \right]}{5 a^{3/4} \sqrt{b x^{1/3} + a x}} +$$

$$\frac{4 b^{9/4} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{\frac{b + a x^{2/3}}{(\sqrt{b} + \sqrt{a} x^{1/3})^2}} x^{1/6} \operatorname{EllipticF} \left[2 \operatorname{ArcTan} \left[\frac{a^{1/4} x^{1/6}}{b^{1/4}} \right], \frac{1}{2} \right]}{5 a^{3/4} \sqrt{b x^{1/3} + a x}}$$

Result (type 5, 91 leaves):

$$\frac{2 x^{2/3} \left(11 b^2 + 16 a b x^{2/3} + 5 a^2 x^{4/3} + 12 b^2 \sqrt{1 + \frac{b}{a x^{2/3}}} \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -\frac{b}{a x^{2/3}} \right] \right)}{15 \sqrt{b x^{1/3} + a x}}$$

Problem 144: Result unnecessarily involves higher level functions.

$$\int \frac{(b x^{1/3} + a x)^{3/2}}{x^2} dx$$

Optimal (type 4, 144 leaves, 6 steps):

$$4 a \sqrt{b x^{1/3} + a x} - \frac{2 (b x^{1/3} + a x)^{3/2}}{x} + \frac{4 a^{3/4} b^{3/4} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{\frac{b + a x^{2/3}}{(\sqrt{b} + \sqrt{a} x^{1/3})^2}} x^{1/6} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{a^{1/4} x^{1/6}}{b^{1/4}}\right], \frac{1}{2}\right]}{\sqrt{b x^{1/3} + a x}}$$

Result (type 5, 82 leaves):

$$\frac{2 \left(b^2 - a^2 x^{4/3} + 4 a b \sqrt{1 + \frac{b}{a x^{2/3}}} x^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -\frac{b}{a x^{2/3}}\right] \right)}{x^{1/3} \sqrt{b x^{1/3} + a x}}$$

Problem 145: Result unnecessarily involves higher level functions.

$$\int \frac{(b x^{1/3} + a x)^{3/2}}{x^3} dx$$

Optimal (type 4, 350 leaves, 9 steps):

$$\frac{8 a^{5/2} (b + a x^{2/3}) x^{1/3}}{5 b (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{b x^{1/3} + a x}} - \frac{4 a \sqrt{b x^{1/3} + a x}}{5 x} - \frac{8 a^2 \sqrt{b x^{1/3} + a x}}{5 b x^{1/3}} - \frac{2 (b x^{1/3} + a x)^{3/2}}{3 x^2} - \frac{8 a^{9/4} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{\frac{b + a x^{2/3}}{(\sqrt{b} + \sqrt{a} x^{1/3})^2}} x^{1/6} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{a^{1/4} x^{1/6}}{b^{1/4}}\right], \frac{1}{2}\right]}{5 b^{3/4} \sqrt{b x^{1/3} + a x}} + \frac{4 a^{9/4} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{\frac{b + a x^{2/3}}{(\sqrt{b} + \sqrt{a} x^{1/3})^2}} x^{1/6} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{a^{1/4} x^{1/6}}{b^{1/4}}\right], \frac{1}{2}\right]}{5 b^{3/4} \sqrt{b x^{1/3} + a x}}$$

Result (type 5, 108 leaves):

$$\frac{2 \left(5 b^3 + 16 a b^2 x^{2/3} + 23 a^2 b x^{4/3} + 12 a^3 x^2 - 12 a^3 \sqrt{1 + \frac{b}{a x^{2/3}}} x^2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -\frac{b}{a x^{2/3}}\right] \right)}{15 b x^{4/3} \sqrt{b x^{1/3} + a x}}$$

Problem 146: Result unnecessarily involves higher level functions.

$$\int \frac{(b x^{1/3} + a x)^{3/2}}{x^4} dx$$

Optimal (type 4, 213 leaves, 8 steps):

$$-\frac{12 a \sqrt{b x^{1/3} + a x}}{55 x^2} - \frac{24 a^2 \sqrt{b x^{1/3} + a x}}{385 b x^{4/3}} + \frac{8 a^3 \sqrt{b x^{1/3} + a x}}{77 b^2 x^{2/3}} -$$

$$\frac{2 (b x^{1/3} + a x)^{3/2}}{5 x^3} + \frac{4 a^{15/4} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{\frac{b+a x^{2/3}}{(\sqrt{b} + \sqrt{a} x^{1/3})^2}} x^{1/6} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{a^{1/4} x^{1/6}}{b^{1/4}}\right], \frac{1}{2}\right]}{77 b^{9/4} \sqrt{b x^{1/3} + a x}}$$

Result (type 5, 123 leaves):

$$-\frac{1}{385 b^2 x^{7/3} \sqrt{b x^{1/3} + a x}} 2 \left(77 b^4 + 196 a b^3 x^{2/3} + 131 a^2 b^2 x^{4/3} - 8 a^3 b x^2 - 20 a^4 x^{8/3} + 20 a^4 \sqrt{1 + \frac{b}{a x^{2/3}}} x^{8/3} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -\frac{b}{a x^{2/3}}\right] \right)$$

Problem 147: Result unnecessarily involves higher level functions.

$$\int \frac{(b x^{1/3} + a x)^{3/2}}{x^5} dx$$

Optimal (type 4, 438 leaves, 12 steps):

$$-\frac{88 a^{11/2} (b + a x^{2/3}) x^{1/3}}{1105 b^4 (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{b x^{1/3} + a x}} - \frac{12 a \sqrt{b x^{1/3} + a x}}{119 x^3} - \frac{24 a^2 \sqrt{b x^{1/3} + a x}}{1547 b x^{7/3}} + \frac{88 a^3 \sqrt{b x^{1/3} + a x}}{4641 b^2 x^{5/3}} - \frac{88 a^4 \sqrt{b x^{1/3} + a x}}{3315 b^3 x} +$$

$$\frac{88 a^5 \sqrt{b x^{1/3} + a x}}{1105 b^4 x^{1/3}} - \frac{2 (b x^{1/3} + a x)^{3/2}}{7 x^4} + \frac{88 a^{21/4} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{\frac{b+a x^{2/3}}{(\sqrt{b} + \sqrt{a} x^{1/3})^2}} x^{1/6} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{a^{1/4} x^{1/6}}{b^{1/4}}\right], \frac{1}{2}\right]}{1105 b^{15/4} \sqrt{b x^{1/3} + a x}} -$$

$$\frac{44 a^{21/4} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{\frac{b+a x^{2/3}}{(\sqrt{b} + \sqrt{a} x^{1/3})^2}} x^{1/6} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{a^{1/4} x^{1/6}}{b^{1/4}}\right], \frac{1}{2}\right]}{1105 b^{15/4} \sqrt{b x^{1/3} + a x}}$$

Result (type 5, 145 leaves):

$$- \left(\left(2 \left(3315 b^6 + 7800 a b^5 x^{2/3} + 4665 a^2 b^4 x^{4/3} - 40 a^3 b^3 x^2 + 88 a^4 b^2 x^{8/3} - 616 a^5 b x^{10/3} - \right. \right. \right. \\ \left. \left. \left. 924 a^6 x^4 + 924 a^6 \sqrt{1 + \frac{b}{a x^{2/3}}} x^4 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -\frac{b}{a x^{2/3}} \right] \right) \right) / \left(23205 b^4 x^{10/3} \sqrt{b x^{1/3} + a x} \right)$$

Problem 148: Result unnecessarily involves higher level functions.

$$\int \frac{(b x^{1/3} + a x)^{3/2}}{x^6} dx$$

Optimal (type 4, 301 leaves, 11 steps):

$$- \frac{4 a \sqrt{b x^{1/3} + a x}}{69 x^4} - \frac{8 a^2 \sqrt{b x^{1/3} + a x}}{1311 b x^{10/3}} + \frac{136 a^3 \sqrt{b x^{1/3} + a x}}{19665 b^2 x^{8/3}} - \frac{1768 a^4 \sqrt{b x^{1/3} + a x}}{216315 b^3 x^2} + \frac{1768 a^5 \sqrt{b x^{1/3} + a x}}{168245 b^4 x^{4/3}} - \\ \frac{1768 a^6 \sqrt{b x^{1/3} + a x}}{100947 b^5 x^{2/3}} - \frac{2 (b x^{1/3} + a x)^{3/2}}{9 x^5} - \frac{884 a^{27/4} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{\frac{b+a x^{2/3}}{(\sqrt{b} + \sqrt{a} x^{1/3})^2}} x^{1/6} \operatorname{EllipticF} \left[2 \operatorname{ArcTan} \left[\frac{a^{1/4} x^{1/6}}{b^{1/4}} \right], \frac{1}{2} \right]}{100947 b^{21/4} \sqrt{b x^{1/3} + a x}}$$

Result (type 5, 160 leaves):

$$- \left(\left(2 \left(168245 b^7 + 380380 a b^6 x^{2/3} + 216755 a^2 b^5 x^{4/3} - 616 a^3 b^4 x^2 + 952 a^4 b^3 x^{8/3} - 1768 a^5 b^2 x^{10/3} + 5304 a^6 b x^4 + \right. \right. \right. \\ \left. \left. \left. 13260 a^7 x^{14/3} - 13260 a^7 \sqrt{1 + \frac{b}{a x^{2/3}}} x^{14/3} \operatorname{Hypergeometric2F1} \left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -\frac{b}{a x^{2/3}} \right] \right) \right) / \left(1514205 b^5 x^{13/3} \sqrt{b x^{1/3} + a x} \right)$$

Problem 149: Result unnecessarily involves higher level functions.

$$\int \frac{x^4}{\sqrt{b x^{1/3} + a x}} dx$$

Optimal (type 4, 304 leaves, 11 steps):

$$\frac{11050 b^6 \sqrt{b x^{1/3} + a x}}{14421 a^7} - \frac{2210 b^5 x^{2/3} \sqrt{b x^{1/3} + a x}}{4807 a^6} + \frac{15470 b^4 x^{4/3} \sqrt{b x^{1/3} + a x}}{43263 a^5} - \frac{1190 b^3 x^2 \sqrt{b x^{1/3} + a x}}{3933 a^4} + \frac{350 b^2 x^{8/3} \sqrt{b x^{1/3} + a x}}{1311 a^3} -$$

$$\frac{50 b x^{10/3} \sqrt{b x^{1/3} + a x}}{207 a^2} + \frac{2 x^4 \sqrt{b x^{1/3} + a x}}{9 a} - \frac{5525 b^{27/4} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{\frac{b+a x^{2/3}}{(\sqrt{b} + \sqrt{a} x^{1/3})^2}} x^{1/6} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{a^{1/4} x^{1/6}}{b^{1/4}}\right], \frac{1}{2}\right]}{14421 a^{29/4} \sqrt{b x^{1/3} + a x}}$$

Result (type 5, 155 leaves):

$$\frac{1}{43263 a^7 \sqrt{b x^{1/3} + a x}} 2 x^{1/3} \left(16575 b^7 + 6630 a b^6 x^{2/3} - 2210 a^2 b^5 x^{4/3} + 1190 a^3 b^4 x^2 - \right.$$

$$\left. 770 a^4 b^3 x^{8/3} + 550 a^5 b^2 x^{10/3} - 418 a^6 b x^4 + 4807 a^7 x^{14/3} + 16575 b^7 \sqrt{1 + \frac{b}{a x^{2/3}}} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -\frac{b}{a x^{2/3}}\right] \right)$$

Problem 150: Result unnecessarily involves higher level functions.

$$\int \frac{x^3}{\sqrt{b x^{1/3} + a x}} dx$$

Optimal (type 4, 414 leaves, 11 steps):

$$- \frac{418 b^5 (b + a x^{2/3}) x^{1/3}}{221 a^{11/2} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{b x^{1/3} + a x}} + \frac{418 b^4 x^{1/3} \sqrt{b x^{1/3} + a x}}{663 a^5} - \frac{2090 b^3 x \sqrt{b x^{1/3} + a x}}{4641 a^4} + \frac{570 b^2 x^{5/3} \sqrt{b x^{1/3} + a x}}{1547 a^3} -$$

$$\frac{38 b x^{7/3} \sqrt{b x^{1/3} + a x}}{119 a^2} + \frac{2 x^3 \sqrt{b x^{1/3} + a x}}{7 a} + \frac{418 b^{21/4} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{\frac{b+a x^{2/3}}{(\sqrt{b} + \sqrt{a} x^{1/3})^2}} x^{1/6} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{a^{1/4} x^{1/6}}{b^{1/4}}\right], \frac{1}{2}\right]}{221 a^{23/4} \sqrt{b x^{1/3} + a x}} -$$

$$\frac{209 b^{21/4} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{\frac{b+a x^{2/3}}{(\sqrt{b} + \sqrt{a} x^{1/3})^2}} x^{1/6} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{a^{1/4} x^{1/6}}{b^{1/4}}\right], \frac{1}{2}\right]}{221 a^{23/4} \sqrt{b x^{1/3} + a x}}$$

Result (type 5, 131 leaves):

$$\frac{1}{4641 a^5 \sqrt{b x^{1/3} + a x}}$$

$$2 x^{2/3} \left(1463 b^5 + 418 a b^4 x^{2/3} - 190 a^2 b^3 x^{4/3} + 114 a^3 b^2 x^2 - 78 a^4 b x^{8/3} + 663 a^5 x^{10/3} - 4389 b^5 \sqrt{1 + \frac{b}{a x^{2/3}}} \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -\frac{b}{a x^{2/3}} \right] \right)$$

Problem 151: Result unnecessarily involves higher level functions.

$$\int \frac{x^2}{\sqrt{b x^{1/3} + a x}} dx$$

Optimal (type 4, 216 leaves, 8 steps):

$$-\frac{78 b^3 \sqrt{b x^{1/3} + a x}}{77 a^4} + \frac{234 b^2 x^{2/3} \sqrt{b x^{1/3} + a x}}{385 a^3} - \frac{26 b x^{4/3} \sqrt{b x^{1/3} + a x}}{55 a^2} +$$

$$\frac{2 x^2 \sqrt{b x^{1/3} + a x}}{5 a} + \frac{39 b^{15/4} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{\frac{b+a x^{2/3}}{(\sqrt{b} + \sqrt{a} x^{1/3})^2}} x^{1/6} \operatorname{EllipticF} \left[2 \operatorname{ArcTan} \left[\frac{a^{1/4} x^{1/6}}{b^{1/4}} \right], \frac{1}{2} \right]}{77 a^{17/4} \sqrt{b x^{1/3} + a x}}$$

Result (type 5, 118 leaves):

$$\frac{1}{385 a^4 \sqrt{b x^{1/3} + a x}} 2 x^{1/3} \left(-195 b^4 - 78 a b^3 x^{2/3} + 26 a^2 b^2 x^{4/3} - 14 a^3 b x^2 + 77 a^4 x^{8/3} - 195 b^4 \sqrt{1 + \frac{b}{a x^{2/3}}} \operatorname{Hypergeometric2F1} \left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -\frac{b}{a x^{2/3}} \right] \right)$$

Problem 152: Result unnecessarily involves higher level functions.

$$\int \frac{x}{\sqrt{b x^{1/3} + a x}} dx$$

Optimal (type 4, 326 leaves, 8 steps):

$$\frac{14 b^2 (b + a x^{2/3}) x^{1/3}}{5 a^{5/2} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{b x^{1/3} + a x}} - \frac{14 b x^{1/3} \sqrt{b x^{1/3} + a x}}{15 a^2} +$$

$$\frac{2 x \sqrt{b x^{1/3} + a x}}{3 a} - \frac{14 b^{9/4} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{\frac{b + a x^{2/3}}{(\sqrt{b} + \sqrt{a} x^{1/3})^2}} x^{1/6} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{a^{1/4} x^{1/6}}{b^{1/4}}\right], \frac{1}{2}\right]}{5 a^{11/4} \sqrt{b x^{1/3} + a x}} +$$

$$\frac{7 b^{9/4} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{\frac{b + a x^{2/3}}{(\sqrt{b} + \sqrt{a} x^{1/3})^2}} x^{1/6} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{a^{1/4} x^{1/6}}{b^{1/4}}\right], \frac{1}{2}\right]}{5 a^{11/4} \sqrt{b x^{1/3} + a x}}$$

Result (type 5, 94 leaves):

$$\frac{2 x^{2/3} \left(-7 b^2 - 2 a b x^{2/3} + 5 a^2 x^{4/3} + 21 b^2 \sqrt{1 + \frac{b}{a x^{2/3}}} \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -\frac{b}{a x^{2/3}}\right] \right)}{15 a^2 \sqrt{b x^{1/3} + a x}}$$

Problem 153: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\sqrt{b x^{1/3} + a x}} dx$$

Optimal (type 4, 126 leaves, 5 steps):

$$\frac{2 \sqrt{b x^{1/3} + a x}}{a} - \frac{b^{3/4} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{\frac{b + a x^{2/3}}{(\sqrt{b} + \sqrt{a} x^{1/3})^2}} x^{1/6} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{a^{1/4} x^{1/6}}{b^{1/4}}\right], \frac{1}{2}\right]}{a^{5/4} \sqrt{b x^{1/3} + a x}}$$

Result (type 5, 73 leaves):

$$\frac{2 x^{1/3} \left(b + a x^{2/3} + b \sqrt{1 + \frac{b}{a x^{2/3}}} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -\frac{b}{a x^{2/3}}\right] \right)}{a \sqrt{b x^{1/3} + a x}}$$

Problem 154: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x \sqrt{b x^{1/3} + a x}} dx$$

Optimal (type 4, 294 leaves, 7 steps):

$$\frac{6\sqrt{a}(b+ax^{2/3})x^{1/3}}{b(\sqrt{b}+\sqrt{a}x^{1/3})\sqrt{bx^{1/3}+ax}} - \frac{6\sqrt{bx^{1/3}+ax}}{bx^{1/3}} - \frac{6a^{1/4}(\sqrt{b}+\sqrt{a}x^{1/3})\sqrt{\frac{b+ax^{2/3}}{(\sqrt{b}+\sqrt{a}x^{1/3})^2}}x^{1/6}\text{EllipticE}\left[2\text{ArcTan}\left[\frac{a^{1/4}x^{1/6}}{b^{1/4}}\right], \frac{1}{2}\right]}{b^{3/4}\sqrt{bx^{1/3}+ax}} +$$

$$\frac{3a^{1/4}(\sqrt{b}+\sqrt{a}x^{1/3})\sqrt{\frac{b+ax^{2/3}}{(\sqrt{b}+\sqrt{a}x^{1/3})^2}}x^{1/6}\text{EllipticF}\left[2\text{ArcTan}\left[\frac{a^{1/4}x^{1/6}}{b^{1/4}}\right], \frac{1}{2}\right]}{b^{3/4}\sqrt{bx^{1/3}+ax}}$$

Result (type 5, 74 leaves):

$$\frac{6\left(b+ax^{2/3}-a\sqrt{1+\frac{b}{ax^{2/3}}}\right)x^{2/3}\text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -\frac{b}{ax^{2/3}}\right]}{b\sqrt{bx^{1/3}+ax}}$$

Problem 155: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^2\sqrt{bx^{1/3}+ax}} dx$$

Optimal (type 4, 163 leaves, 6 steps):

$$-\frac{6\sqrt{bx^{1/3}+ax}}{7bx^{4/3}} + \frac{10a\sqrt{bx^{1/3}+ax}}{7b^2x^{2/3}} + \frac{5a^{7/4}(\sqrt{b}+\sqrt{a}x^{1/3})\sqrt{\frac{b+ax^{2/3}}{(\sqrt{b}+\sqrt{a}x^{1/3})^2}}x^{1/6}\text{EllipticF}\left[2\text{ArcTan}\left[\frac{a^{1/4}x^{1/6}}{b^{1/4}}\right], \frac{1}{2}\right]}{7b^{9/4}\sqrt{bx^{1/3}+ax}}$$

Result (type 5, 97 leaves):

$$\frac{-6b^2+4abx^{2/3}+10a^2x^{4/3}-10a^2\sqrt{1+\frac{b}{ax^{2/3}}}\left(x^{4/3}\text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -\frac{b}{ax^{2/3}}\right]\right)}{7b^2x\sqrt{bx^{1/3}+ax}}$$

Problem 156: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^3\sqrt{bx^{1/3}+ax}} dx$$

Optimal (type 4, 388 leaves, 10 steps):

$$\begin{aligned}
& - \frac{154 a^{7/2} (b + a x^{2/3}) x^{1/3}}{65 b^4 (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{b x^{1/3} + a x}} - \frac{6 \sqrt{b x^{1/3} + a x}}{13 b x^{7/3}} + \frac{22 a \sqrt{b x^{1/3} + a x}}{39 b^2 x^{5/3}} - \frac{154 a^2 \sqrt{b x^{1/3} + a x}}{195 b^3 x} + \\
& \frac{154 a^3 \sqrt{b x^{1/3} + a x}}{65 b^4 x^{1/3}} + \frac{154 a^{13/4} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{\frac{b+a x^{2/3}}{(\sqrt{b} + \sqrt{a} x^{1/3})^2}} x^{1/6} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{a^{1/4} x^{1/6}}{b^{1/4}}\right], \frac{1}{2}\right]}{65 b^{15/4} \sqrt{b x^{1/3} + a x}} - \\
& \frac{77 a^{13/4} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{\frac{b+a x^{2/3}}{(\sqrt{b} + \sqrt{a} x^{1/3})^2}} x^{1/6} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{a^{1/4} x^{1/6}}{b^{1/4}}\right], \frac{1}{2}\right]}{65 b^{15/4} \sqrt{b x^{1/3} + a x}}
\end{aligned}$$

Result (type 5, 121 leaves):

$$\frac{1}{195 b^4 x^2 \sqrt{b x^{1/3} + a x}} \left(-90 b^4 + 20 a b^3 x^{2/3} - 44 a^2 b^2 x^{4/3} + 308 a^3 b x^2 + 462 a^4 x^{8/3} - 462 a^4 \sqrt{1 + \frac{b}{a x^{2/3}}} x^{8/3} \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -\frac{b}{a x^{2/3}}\right] \right)$$

Problem 157: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^4 \sqrt{b x^{1/3} + a x}} dx$$

Optimal (type 4, 251 leaves, 9 steps):

$$\begin{aligned}
& - \frac{6 \sqrt{b x^{1/3} + a x}}{19 b x^{10/3}} + \frac{34 a \sqrt{b x^{1/3} + a x}}{95 b^2 x^{8/3}} - \frac{442 a^2 \sqrt{b x^{1/3} + a x}}{1045 b^3 x^2} + \frac{3978 a^3 \sqrt{b x^{1/3} + a x}}{7315 b^4 x^{4/3}} - \\
& \frac{1326 a^4 \sqrt{b x^{1/3} + a x}}{1463 b^5 x^{2/3}} - \frac{663 a^{19/4} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{\frac{b+a x^{2/3}}{(\sqrt{b} + \sqrt{a} x^{1/3})^2}} x^{1/6} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{a^{1/4} x^{1/6}}{b^{1/4}}\right], \frac{1}{2}\right]}{1463 b^{21/4} \sqrt{b x^{1/3} + a x}}
\end{aligned}$$

Result (type 5, 134 leaves):

$$\frac{1}{7315 b^5 x^3 \sqrt{b x^{1/3} + a x}} \left(-2310 b^5 + 308 a b^4 x^{2/3} - 476 a^2 b^3 x^{4/3} + 884 a^3 b^2 x^2 - 2652 a^4 b x^{8/3} - 6630 a^5 x^{10/3} + 6630 a^5 \sqrt{1 + \frac{b}{a x^{2/3}}} x^{10/3} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -\frac{b}{a x^{2/3}}\right] \right)$$

Problem 158: Result unnecessarily involves higher level functions.

$$\int \frac{x^4}{(b x^{1/3} + a x)^{3/2}} dx$$

Optimal (type 4, 437 leaves, 12 steps):

$$\begin{aligned} & -\frac{4807 b^5 (b + a x^{2/3}) x^{1/3}}{221 a^{13/2} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{b x^{1/3} + a x}} - \frac{3 x^4}{a \sqrt{b x^{1/3} + a x}} + \frac{4807 b^4 x^{1/3} \sqrt{b x^{1/3} + a x}}{663 a^6} - \frac{24 035 b^3 x \sqrt{b x^{1/3} + a x}}{4641 a^5} + \frac{6555 b^2 x^{5/3} \sqrt{b x^{1/3} + a x}}{1547 a^4} \\ & \frac{437 b x^{7/3} \sqrt{b x^{1/3} + a x}}{119 a^3} + \frac{23 x^3 \sqrt{b x^{1/3} + a x}}{7 a^2} + \frac{4807 b^{21/4} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{\frac{b+a x^{2/3}}{(\sqrt{b} + \sqrt{a} x^{1/3})^2}} x^{1/6} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{a^{1/4} x^{1/6}}{b^{1/4}}\right], \frac{1}{2}\right]}{221 a^{27/4} \sqrt{b x^{1/3} + a x}} \\ & \frac{4807 b^{21/4} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{\frac{b+a x^{2/3}}{(\sqrt{b} + \sqrt{a} x^{1/3})^2}} x^{1/6} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{a^{1/4} x^{1/6}}{b^{1/4}}\right], \frac{1}{2}\right]}{442 a^{27/4} \sqrt{b x^{1/3} + a x}} \end{aligned}$$

Result (type 5, 131 leaves):

$$\begin{aligned} & \frac{1}{4641 a^6 \sqrt{b x^{1/3} + a x}} x^{2/3} \left(33 649 b^5 + 9614 a b^4 x^{2/3} - 4370 a^2 b^3 x^{4/3} + \right. \\ & \left. 2622 a^3 b^2 x^2 - 1794 a^4 b x^{8/3} + 1326 a^5 x^{10/3} - 100 947 b^5 \sqrt{1 + \frac{b}{a x^{2/3}}} \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -\frac{b}{a x^{2/3}}\right] \right) \end{aligned}$$

Problem 159: Result unnecessarily involves higher level functions.

$$\int \frac{x^3}{(b x^{1/3} + a x)^{3/2}} dx$$

Optimal (type 4, 239 leaves, 9 steps):

$$\begin{aligned}
& -\frac{3x^3}{a\sqrt{bx^{1/3}+ax}} - \frac{663b^3\sqrt{bx^{1/3}+ax}}{77a^5} + \frac{1989b^2x^{2/3}\sqrt{bx^{1/3}+ax}}{385a^4} - \frac{221bx^{4/3}\sqrt{bx^{1/3}+ax}}{55a^3} + \\
& \frac{17x^2\sqrt{bx^{1/3}+ax}}{5a^2} + \frac{663b^{15/4}\left(\sqrt{b} + \sqrt{a}x^{1/3}\right)\sqrt{\frac{b+ax^{2/3}}{(\sqrt{b}+\sqrt{a}x^{1/3})^2}}x^{1/6}\text{EllipticF}\left[2\text{ArcTan}\left[\frac{a^{1/4}x^{1/6}}{b^{1/4}}\right], \frac{1}{2}\right]}{154a^{21/4}\sqrt{bx^{1/3}+ax}}
\end{aligned}$$

Result (type 5, 118 leaves):

$$\begin{aligned}
& \frac{1}{385a^5\sqrt{bx^{1/3}+ax}} \\
& x^{1/3}\left(-3315b^4 - 1326ab^3x^{2/3} + 442a^2b^2x^{4/3} - 238a^3bx^2 + 154a^4x^{8/3} - 3315b^4\sqrt{1 + \frac{b}{ax^{2/3}}}\text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -\frac{b}{ax^{2/3}}\right]\right)
\end{aligned}$$

Problem 160: Result unnecessarily involves higher level functions.

$$\int \frac{x^2}{(bx^{1/3}+ax)^{3/2}} dx$$

Optimal (type 4, 349 leaves, 9 steps):

$$\begin{aligned}
& \frac{77b^2(b+ax^{2/3})x^{1/3}}{5a^{7/2}(\sqrt{b} + \sqrt{a}x^{1/3})\sqrt{bx^{1/3}+ax}} - \frac{3x^2}{a\sqrt{bx^{1/3}+ax}} - \frac{77bx^{1/3}\sqrt{bx^{1/3}+ax}}{15a^3} + \\
& \frac{11x\sqrt{bx^{1/3}+ax}}{3a^2} - \frac{77b^{9/4}\left(\sqrt{b} + \sqrt{a}x^{1/3}\right)\sqrt{\frac{b+ax^{2/3}}{(\sqrt{b}+\sqrt{a}x^{1/3})^2}}x^{1/6}\text{EllipticE}\left[2\text{ArcTan}\left[\frac{a^{1/4}x^{1/6}}{b^{1/4}}\right], \frac{1}{2}\right]}{5a^{15/4}\sqrt{bx^{1/3}+ax}} + \\
& \frac{77b^{9/4}\left(\sqrt{b} + \sqrt{a}x^{1/3}\right)\sqrt{\frac{b+ax^{2/3}}{(\sqrt{b}+\sqrt{a}x^{1/3})^2}}x^{1/6}\text{EllipticF}\left[2\text{ArcTan}\left[\frac{a^{1/4}x^{1/6}}{b^{1/4}}\right], \frac{1}{2}\right]}{10a^{15/4}\sqrt{bx^{1/3}+ax}}
\end{aligned}$$

Result (type 5, 94 leaves):

$$\frac{x^{2/3}\left(-77b^2 - 22abx^{2/3} + 10a^2x^{4/3} + 231b^2\sqrt{1 + \frac{b}{ax^{2/3}}}\text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -\frac{b}{ax^{2/3}}\right]\right)}{15a^3\sqrt{bx^{1/3}+ax}}$$

Problem 161: Result unnecessarily involves higher level functions.

$$\int \frac{x}{(b x^{1/3} + a x)^{3/2}} dx$$

Optimal (type 4, 149 leaves, 6 steps):

$$-\frac{3x}{a\sqrt{bx^{1/3}+ax}} + \frac{5\sqrt{bx^{1/3}+ax}}{a^2} - \frac{5b^{3/4}(\sqrt{b} + \sqrt{a}x^{1/3})\sqrt{\frac{b+ax^{2/3}}{(\sqrt{b} + \sqrt{a}x^{1/3})^2}} x^{1/6} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{a^{1/4}x^{1/6}}{b^{1/4}}\right], \frac{1}{2}\right]}{2a^{9/4}\sqrt{bx^{1/3}+ax}}$$

Result (type 5, 76 leaves):

$$\frac{x^{1/3} \left(5b + 2ax^{2/3} + 5b \sqrt{1 + \frac{b}{ax^{2/3}}} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -\frac{b}{ax^{2/3}}\right] \right)}{a^2 \sqrt{bx^{1/3}+ax}}$$

Problem 162: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(b x^{1/3} + a x)^{3/2}} dx$$

Optimal (type 4, 296 leaves, 7 steps):

$$-\frac{3(b+ax^{2/3})x^{1/3}}{\sqrt{a}b(\sqrt{b} + \sqrt{a}x^{1/3})\sqrt{bx^{1/3}+ax}} + \frac{3x^{2/3}}{b\sqrt{bx^{1/3}+ax}} + \frac{3(\sqrt{b} + \sqrt{a}x^{1/3})\sqrt{\frac{b+ax^{2/3}}{(\sqrt{b} + \sqrt{a}x^{1/3})^2}} x^{1/6} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{a^{1/4}x^{1/6}}{b^{1/4}}\right], \frac{1}{2}\right]}{a^{3/4}b^{3/4}\sqrt{bx^{1/3}+ax}}$$

$$\frac{3(\sqrt{b} + \sqrt{a}x^{1/3})\sqrt{\frac{b+ax^{2/3}}{(\sqrt{b} + \sqrt{a}x^{1/3})^2}} x^{1/6} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{a^{1/4}x^{1/6}}{b^{1/4}}\right], \frac{1}{2}\right]}{2a^{3/4}b^{3/4}\sqrt{bx^{1/3}+ax}}$$

Result (type 5, 65 leaves):

$$-\frac{3x^{2/3} \left(-1 + \sqrt{1 + \frac{b}{ax^{2/3}}} \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -\frac{b}{ax^{2/3}}\right] \right)}{b\sqrt{bx^{1/3}+ax}}$$

Problem 163: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x (b x^{1/3} + a x)^{3/2}} dx$$

Optimal (type 4, 158 leaves, 6 steps):

$$\frac{3}{b x^{1/3} \sqrt{b x^{1/3} + a x}} - \frac{5 \sqrt{b x^{1/3} + a x}}{b^2 x^{2/3}} - \frac{5 a^{3/4} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{\frac{b+a x^{2/3}}{(\sqrt{b} + \sqrt{a} x^{1/3})^2}} x^{1/6} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{a^{1/4} x^{1/6}}{b^{1/4}}\right], \frac{1}{2}\right]}{2 b^{9/4} \sqrt{b x^{1/3} + a x}}$$

Result (type 5, 81 leaves):

$$\frac{-2 b - 5 a x^{2/3} + 5 a \sqrt{1 + \frac{b}{a x^{2/3}}} x^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -\frac{b}{a x^{2/3}}\right]}{b^2 x^{1/3} \sqrt{b x^{1/3} + a x}}$$

Problem 164: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^2 (b x^{1/3} + a x)^{3/2}} dx$$

Optimal (type 4, 383 leaves, 10 steps):

$$\frac{3}{b x^{4/3} \sqrt{b x^{1/3} + a x}} + \frac{77 a^{5/2} (b + a x^{2/3}) x^{1/3}}{5 b^4 (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{b x^{1/3} + a x}} - \frac{11 \sqrt{b x^{1/3} + a x}}{3 b^2 x^{5/3}} + \frac{77 a \sqrt{b x^{1/3} + a x}}{15 b^3 x} - \frac{77 a^2 \sqrt{b x^{1/3} + a x}}{5 b^4 x^{1/3}} - \frac{77 a^{9/4} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{\frac{b+a x^{2/3}}{(\sqrt{b} + \sqrt{a} x^{1/3})^2}} x^{1/6} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{a^{1/4} x^{1/6}}{b^{1/4}}\right], \frac{1}{2}\right]}{5 b^{15/4} \sqrt{b x^{1/3} + a x}} + \frac{77 a^{9/4} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{\frac{b+a x^{2/3}}{(\sqrt{b} + \sqrt{a} x^{1/3})^2}} x^{1/6} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{a^{1/4} x^{1/6}}{b^{1/4}}\right], \frac{1}{2}\right]}{10 b^{15/4} \sqrt{b x^{1/3} + a x}}$$

Result (type 5, 108 leaves):

$$\frac{1}{15 b^4 x^{4/3} \sqrt{b x^{1/3} + a x}} \left(-10 b^3 + 22 a b^2 x^{2/3} - 154 a^2 b x^{4/3} - 231 a^3 x^2 + 231 a^3 \sqrt{1 + \frac{b}{a x^{2/3}}} x^2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -\frac{b}{a x^{2/3}} \right] \right)$$

Problem 165: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^3 (b x^{1/3} + a x)^{3/2}} dx$$

Optimal (type 4, 246 leaves, 9 steps):

$$\frac{3}{b x^{7/3} \sqrt{b x^{1/3} + a x}} - \frac{17 \sqrt{b x^{1/3} + a x}}{5 b^2 x^{8/3}} + \frac{221 a \sqrt{b x^{1/3} + a x}}{55 b^3 x^2} - \frac{1989 a^2 \sqrt{b x^{1/3} + a x}}{385 b^4 x^{4/3}} +$$

$$\frac{663 a^3 \sqrt{b x^{1/3} + a x}}{77 b^5 x^{2/3}} + \frac{663 a^{15/4} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{\frac{b+a x^{2/3}}{(\sqrt{b} + \sqrt{a} x^{1/3})^2}} x^{1/6} \operatorname{EllipticF} \left[2 \operatorname{ArcTan} \left[\frac{a^{1/4} x^{1/6}}{b^{1/4}} \right], \frac{1}{2} \right]}{154 b^{21/4} \sqrt{b x^{1/3} + a x}}$$

Result (type 5, 123 leaves):

$$\frac{1}{385 b^5 x^{7/3} \sqrt{b x^{1/3} + a x}} \left(-154 b^4 + 238 a b^3 x^{2/3} - 442 a^2 b^2 x^{4/3} + 1326 a^3 b x^2 + 3315 a^4 x^{8/3} - 3315 a^4 \sqrt{1 + \frac{b}{a x^{2/3}}} x^{8/3} \operatorname{Hypergeometric2F1} \left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -\frac{b}{a x^{2/3}} \right] \right)$$

Problem 166: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^4 (b x^{1/3} + a x)^{3/2}} dx$$

Optimal (type 4, 471 leaves, 13 steps):

$$\frac{3}{b x^{10/3} \sqrt{b x^{1/3} + a x}} - \frac{4807 a^{11/2} (b + a x^{2/3}) x^{1/3}}{221 b^7 (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{b x^{1/3} + a x}} - \frac{23 \sqrt{b x^{1/3} + a x}}{7 b^2 x^{11/3}} + \frac{437 a \sqrt{b x^{1/3} + a x}}{119 b^3 x^3} - \frac{6555 a^2 \sqrt{b x^{1/3} + a x}}{1547 b^4 x^{7/3}} + \frac{24035 a^3 \sqrt{b x^{1/3} + a x}}{4641 b^5 x^{5/3}} -$$

$$\frac{4807 a^4 \sqrt{b x^{1/3} + a x}}{663 b^6 x} + \frac{4807 a^5 \sqrt{b x^{1/3} + a x}}{221 b^7 x^{1/3}} + \frac{4807 a^{21/4} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{\frac{b+a x^{2/3}}{(\sqrt{b} + \sqrt{a} x^{1/3})^2}} x^{1/6} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{a^{1/4} x^{1/6}}{b^{1/4}}\right], \frac{1}{2}\right]}{221 b^{27/4} \sqrt{b x^{1/3} + a x}} -$$

$$\frac{4807 a^{21/4} (\sqrt{b} + \sqrt{a} x^{1/3}) \sqrt{\frac{b+a x^{2/3}}{(\sqrt{b} + \sqrt{a} x^{1/3})^2}} x^{1/6} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{a^{1/4} x^{1/6}}{b^{1/4}}\right], \frac{1}{2}\right]}{442 b^{27/4} \sqrt{b x^{1/3} + a x}}$$

Result (type 5, 145 leaves):

$$\frac{1}{4641 b^7 x^{10/3} \sqrt{b x^{1/3} + a x}} \left(-1326 b^6 + 1794 a b^5 x^{2/3} - 2622 a^2 b^4 x^{4/3} + 4370 a^3 b^3 x^2 - \right.$$

$$\left. 9614 a^4 b^2 x^{8/3} + 67298 a^5 b x^{10/3} + 100947 a^6 x^4 - 100947 a^6 \sqrt{1 + \frac{b}{a x^{2/3}}} x^4 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -\frac{b}{a x^{2/3}}\right] \right)$$

Problem 281: Result unnecessarily involves higher level functions.

$$\int x^{-3-3n} (a x^2 + b x^3)^n dx$$

Optimal (type 3, 70 leaves, 2 steps):

$$-\frac{x^{-4-3n} (a x^2 + b x^3)^{1+n}}{a(2+n)} + \frac{b x^{-3(1+n)} (a x^2 + b x^3)^{1+n}}{a^2(1+n)(2+n)}$$

Result (type 5, 58 leaves):

$$-\frac{x^{-2-3n} (x^2 (a + b x))^n \left(1 + \frac{bx}{a}\right)^{-n} \text{Hypergeometric2F1}\left[-2-n, -n, -1-n, -\frac{bx}{a}\right]}{2+n}$$

Problem 289: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4}{\sqrt{a x^2 + b x^5}} dx$$

Optimal (type 4, 238 leaves, 3 steps):

$$\frac{2 \sqrt{a x^2 + b x^5}}{5 b} - \frac{4 \sqrt{2 + \sqrt{3}} a x (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 + \sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x}\right], -7 - 4 \sqrt{3}\right]}{5 \times 3^{1/4} b^{4/3} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{((1 + \sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \sqrt{a x^2 + b x^5}}$$

Result (type 4, 165 leaves):

$$\frac{1}{15 (-b)^{4/3} \sqrt{x^2 (a + b x^3)}} \left(-6 (-b)^{1/3} x^2 (a + b x^3) + \right. \\ \left. 4 i 3^{3/4} a^{4/3} x \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1 + \frac{(-b)^{1/3} x}{a^{1/3}} + \frac{(-b)^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right] \right)$$

Problem 290: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{\sqrt{a x^2 + b x^5}} dx$$

Optimal (type 4, 212 leaves, 2 steps):

$$\frac{2 \sqrt{2 + \sqrt{3}} x (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 + \sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x}\right], -7 - 4 \sqrt{3}\right]}{3^{1/4} b^{1/3} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{((1 + \sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \sqrt{a x^2 + b x^5}}$$

Result (type 4, 141 leaves):

$$\left(2 i a^{1/3} x \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3} x}{a^{1/3}} \right)} \sqrt{1 + \frac{(-b)^{1/3} x}{a^{1/3}} + \frac{(-b)^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}} \right], (-1)^{1/3} \right] \right) / \left(3^{1/4} (-b)^{1/3} \sqrt{x^2 (a + b x^3)} \right)$$

Problem 291: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^2 \sqrt{a x^2 + b x^5}} dx$$

Optimal (type 4, 243 leaves, 3 steps):

$$\frac{\sqrt{a x^2 + b x^5}}{2 a x^3} - \frac{\sqrt{2 + \sqrt{3}} b^{2/3} x (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 + \sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7 - 4 \sqrt{3} \right]}{2 \times 3^{1/4} a \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{((1 + \sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \sqrt{a x^2 + b x^5}}$$

Result (type 4, 171 leaves):

$$\left(-3 (-b)^{1/3} (a + b x^3) - i 3^{3/4} a^{1/3} b x^2 \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3} x}{a^{1/3}} \right)} \sqrt{1 + \frac{(-b)^{1/3} x}{a^{1/3}} + \frac{(-b)^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}} \right], (-1)^{1/3} \right] \right) / \left(6 a (-b)^{1/3} x \sqrt{x^2 (a + b x^3)} \right)$$

Problem 292: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^5}{\sqrt{a x^2 + b x^5}} dx$$

Optimal (type 4, 514 leaves, 5 steps):

$$\begin{aligned}
& - \frac{8 a x (a + b x^3)}{7 b^{5/3} \left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right) \sqrt{a x^2 + b x^5}} + \frac{2 x \sqrt{a x^2 + b x^5}}{7 b} + \\
& \left(4 \times 3^{1/4} \sqrt{2 - \sqrt{3}} a^{4/3} x (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7 - 4 \sqrt{3} \right] \right) / \\
& \left(7 b^{5/3} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{a x^2 + b x^5} \right) - \\
& \frac{8 \sqrt{2} a^{4/3} x (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7 - 4 \sqrt{3} \right]}{7 \times 3^{1/4} b^{5/3} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{a x^2 + b x^5}}
\end{aligned}$$

Result (type 4, 228 leaves):

$$\begin{aligned}
& \frac{1}{21 b \sqrt{x^2 (a + b x^3)}} 2 x \left(3 x^2 (a + b x^3) - \frac{1}{(-b)^{2/3}} 4 (-1)^{1/6} 3^{3/4} a^{5/3} \sqrt{\frac{(-1)^{5/6} (-a^{1/3} + (-b)^{1/3} x)}{a^{1/3}}} \sqrt{1 + \frac{(-b)^{1/3} x}{a^{1/3}} + \frac{(-b)^{2/3} x^2}{a^{2/3}}} \right. \\
& \left. \left(-i \sqrt{3} \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - \frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}} \right], (-1)^{1/3} \right] + (-1)^{1/3} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - \frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}} \right], (-1)^{1/3} \right] \right) \right)
\end{aligned}$$

Problem 293: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2}{\sqrt{a x^2 + b x^5}} dx$$

Optimal (type 4, 484 leaves, 4 steps):

$$\frac{2 x (a + b x^3)}{b^{2/3} \left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right) \sqrt{a x^2 + b x^5}} -$$

$$\left(3^{1/4} \sqrt{2 - \sqrt{3}} a^{1/3} x (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7 - 4 \sqrt{3} \right] \right) /$$

$$\left(b^{2/3} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{a x^2 + b x^5} \right) + \frac{2 \sqrt{2} a^{1/3} x (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7 - 4 \sqrt{3} \right]}{3^{1/4} b^{2/3} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{a x^2 + b x^5}}$$

Result (type 4, 202 leaves):

$$\left(2 (-1)^{1/6} a^{2/3} x \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3} x}{a^{1/3}} \right)} \sqrt{1 + \frac{(-b)^{1/3} x}{a^{1/3}} + \frac{(-b)^{2/3} x^2}{a^{2/3}}} \right.$$

$$\left. \left(-i \sqrt{3} \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}} \right], (-1)^{1/3} \right] + (-1)^{1/3} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}} \right], (-1)^{1/3} \right] \right) \right) / \left(3^{1/4} \right.$$

$$\left. (-b)^{2/3} \sqrt{x^2 (a + b x^3)} \right)$$

Problem 294: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x \sqrt{a x^2 + b x^5}} dx$$

Optimal (type 4, 510 leaves, 5 steps):

$$\frac{b^{1/3} x (a + b x^3)}{a \left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right) \sqrt{a x^2 + b x^5}} - \frac{\sqrt{a x^2 + b x^5}}{a x^2} - \left(3^{1/4} \sqrt{2 - \sqrt{3}} b^{1/3} x (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \text{EllipticE} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7 - 4 \sqrt{3} \right] \right) /$$

$$\left(2 a^{2/3} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{a x^2 + b x^5} \right) + \frac{\sqrt{2} b^{1/3} x (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7 - 4 \sqrt{3} \right]}{3^{1/4} a^{2/3} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{a x^2 + b x^5}}$$

Result (type 4, 225 leaves):

$$\frac{1}{3 a \sqrt{x^2 (a + b x^3)}} \left(-3 (a + b x^3) + \frac{1}{(-b)^{2/3}} (-1)^{1/6} 3^{3/4} a^{2/3} b x \sqrt{\frac{(-1)^{5/6} (-a^{1/3} + (-b)^{1/3} x)}{a^{1/3}}} \sqrt{1 + \frac{(-b)^{1/3} x}{a^{1/3}} + \frac{(-b)^{2/3} x^2}{a^{2/3}}} \right.$$

$$\left. \left(-i \sqrt{3} \text{EllipticE} \left[\text{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}} \right], (-1)^{1/3} \right] + (-1)^{1/3} \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}} \right], (-1)^{1/3} \right] \right) \right)$$

Problem 295: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{13/2}}{\sqrt{a x^2 + b x^5}} dx$$

Optimal (type 4, 265 leaves, 5 steps):

$$-\frac{7 a \sqrt{a x^2 + b x^5}}{20 b^2 \sqrt{x}} + \frac{x^{5/2} \sqrt{a x^2 + b x^5}}{5 b} + \frac{7 a^{5/3} x^{3/2} (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{(a^{1/3} + (1 + \sqrt{3}) b^{1/3} x)^2}} \text{EllipticF} \left[\text{ArcCos} \left[\frac{a^{1/3} + (1 - \sqrt{3}) b^{1/3} x}{a^{1/3} + (1 + \sqrt{3}) b^{1/3} x} \right], \frac{1}{4} (2 + \sqrt{3}) \right]}{40 \times 3^{1/4} b^2 \sqrt{\frac{b^{1/3} x (a^{1/3} + b^{1/3} x)}{(a^{1/3} + (1 + \sqrt{3}) b^{1/3} x)^2}} \sqrt{a x^2 + b x^5}}$$

Result (type 4, 194 leaves):

$$\left(x^{3/2} \left(-3 (-a)^{1/3} (7a^2 + 3abx^3 - 4b^2x^6) - 7i 3^{3/4} a^2 b^{1/3} \sqrt{(-1)^{5/6} \left(-1 + \frac{(-a)^{1/3}}{b^{1/3}x} \right)} \right) \right. \\ \left. \sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}} + \frac{(-a)^{1/3}x}{b^{1/3}} + x^2}{x^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-a)^{1/3}}{b^{1/3}x}}}{3^{1/4}}}\right], (-1)^{1/3}\right] \right) \Big/ \left(60 (-a)^{1/3} b^2 \sqrt{x^2 (a + bx^3)} \right)$$

Problem 296: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{11/2}}{\sqrt{ax^2 + bx^5}} dx$$

Optimal (type 4, 525 leaves, 6 steps):

$$-\frac{5(1+\sqrt{3})ax^{3/2}(a+bx^3)}{8b^{5/3}(a^{1/3}+(1+\sqrt{3})b^{1/3}x)\sqrt{ax^2+bx^5}} + \frac{x^{3/2}\sqrt{ax^2+bx^5}}{4b} + \\ \frac{5 \times 3^{1/4} a^{4/3} x^{3/2} (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{(a^{1/3} + (1+\sqrt{3}) b^{1/3} x)^2}} \operatorname{EllipticE}\left[\operatorname{ArcCos}\left[\frac{a^{1/3} + (1-\sqrt{3}) b^{1/3} x}{a^{1/3} + (1+\sqrt{3}) b^{1/3} x}\right], \frac{1}{4}(2+\sqrt{3})\right]}{8b^{5/3} \sqrt{\frac{b^{1/3} x (a^{1/3} + b^{1/3} x)}{(a^{1/3} + (1+\sqrt{3}) b^{1/3} x)^2}} \sqrt{ax^2 + bx^5}} + \\ \left(\frac{5(1-\sqrt{3})a^{4/3}x^{3/2}(a^{1/3}+b^{1/3}x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{(a^{1/3} + (1+\sqrt{3}) b^{1/3} x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcCos}\left[\frac{a^{1/3} + (1-\sqrt{3}) b^{1/3} x}{a^{1/3} + (1+\sqrt{3}) b^{1/3} x}\right], \frac{1}{4}(2+\sqrt{3})\right] \right) \Big/ \\ \left(16 \times 3^{1/4} b^{5/3} \sqrt{\frac{b^{1/3} x (a^{1/3} + b^{1/3} x)}{(a^{1/3} + (1+\sqrt{3}) b^{1/3} x)^2}} \sqrt{ax^2 + bx^5} \right)$$

Result (type 4, 362 leaves):

$$\frac{1}{8 b \sqrt{x^2 (a + b x^3)}} \sqrt{x} \left(5 a x \left(-\frac{a^{2/3}}{b^{2/3}} + \frac{a^{1/3} x}{b^{1/3}} - x^2 \right) + 2 x^3 (a + b x^3) - \frac{1}{2 (-1 + (-1)^{2/3}) b} \right.$$

$$5 (-1)^{2/3} a^{4/3} (a^{1/3} + b^{1/3} x)^2 \sqrt{\frac{(1 + (-1)^{1/3}) b^{1/3} x (a^{1/3} - (-1)^{1/3} b^{1/3} x)}{(a^{1/3} + b^{1/3} x)^2}} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{a^{1/3} + b^{1/3} x}}$$

$$\left. \left((-3 - i \sqrt{3}) \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\frac{\sqrt{\frac{(3+i\sqrt{3}) b^{1/3} x}{a^{1/3} + b^{1/3} x}}}{\sqrt{2}} \right], \frac{-i + \sqrt{3}}{i + \sqrt{3}} \right] + (1 + i \sqrt{3}) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{\frac{(3+i\sqrt{3}) b^{1/3} x}{a^{1/3} + b^{1/3} x}}}{\sqrt{2}} \right], \frac{-i + \sqrt{3}}{i + \sqrt{3}} \right] \right) \right)$$

Problem 298: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{7/2}}{\sqrt{a x^2 + b x^5}} dx$$

Optimal (type 4, 237 leaves, 4 steps):

$$\frac{\sqrt{a x^2 + b x^5}}{2 b \sqrt{x}} - \frac{a^{2/3} x^{3/2} (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{(a^{1/3} + (1 + \sqrt{3}) b^{1/3} x)^2}} \operatorname{EllipticF} \left[\operatorname{ArcCos} \left[\frac{a^{1/3} + (1 - \sqrt{3}) b^{1/3} x}{a^{1/3} + (1 + \sqrt{3}) b^{1/3} x} \right], \frac{1}{4} (2 + \sqrt{3}) \right]}{4 \times 3^{1/4} b \sqrt{\frac{b^{1/3} x (a^{1/3} + b^{1/3} x)}{(a^{1/3} + (1 + \sqrt{3}) b^{1/3} x)^2}} \sqrt{a x^2 + b x^5}}$$

Result (type 4, 178 leaves):

$$\frac{1}{6 (-a)^{1/3} b \sqrt{x^2 (a + b x^3)}}$$

$$x^{3/2} \left(3 (-a)^{1/3} (a + b x^3) + i 3^{3/4} a b^{1/3} \sqrt{(-1)^{5/6} \left(-1 + \frac{(-a)^{1/3}}{b^{1/3} x} \right)} x \sqrt{\frac{(-a)^{2/3} + \frac{(-a)^{1/3} x}{b^{1/3}} + x^2}{x^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{-(-1)^{5/6} - \frac{i (-a)^{1/3}}{b^{1/3} x}}}{3^{1/4}} \right], (-1)^{1/3} \right] \right)$$

Problem 299: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{5/2}}{\sqrt{ax^2 + bx^5}} dx$$

Optimal (type 4, 492 leaves, 5 steps):

$$\frac{(1 + \sqrt{3}) x^{3/2} (a + b x^3)}{b^{2/3} (a^{1/3} + (1 + \sqrt{3}) b^{1/3} x) \sqrt{ax^2 + bx^5}} - \frac{3^{1/4} a^{1/3} x^{3/2} (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{(a^{1/3} + (1 + \sqrt{3}) b^{1/3} x)^2}} \text{EllipticE}\left[\text{ArcCos}\left[\frac{a^{1/3} + (1 - \sqrt{3}) b^{1/3} x}{a^{1/3} + (1 + \sqrt{3}) b^{1/3} x}\right], \frac{1}{4} (2 + \sqrt{3})\right]}{b^{2/3} \sqrt{\frac{b^{1/3} x (a^{1/3} + b^{1/3} x)}{(a^{1/3} + (1 + \sqrt{3}) b^{1/3} x)^2}} \sqrt{ax^2 + bx^5}}$$

$$\left((1 - \sqrt{3}) a^{1/3} x^{3/2} (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{(a^{1/3} + (1 + \sqrt{3}) b^{1/3} x)^2}} \text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3} + (1 - \sqrt{3}) b^{1/3} x}{a^{1/3} + (1 + \sqrt{3}) b^{1/3} x}\right], \frac{1}{4} (2 + \sqrt{3})\right] \right) /$$

$$\left(2 \times 3^{1/4} b^{2/3} \sqrt{\frac{b^{1/3} x (a^{1/3} + b^{1/3} x)}{(a^{1/3} + (1 + \sqrt{3}) b^{1/3} x)^2}} \sqrt{ax^2 + bx^5} \right)$$

Result (type 4, 340 leaves):

$$\frac{1}{\sqrt{x^2 (a + b x^3)}}$$

$$\sqrt{x} \left(x \left(\frac{a^{2/3}}{b^{2/3}} - \frac{a^{1/3} x}{b^{1/3}} + x^2 \right) + \frac{1}{2 (-1 + (-1)^{2/3}) b} (-1)^{2/3} a^{1/3} (a^{1/3} + b^{1/3} x)^2 \sqrt{\frac{(1 + (-1)^{1/3}) b^{1/3} x (a^{1/3} - (-1)^{1/3} b^{1/3} x)}{(a^{1/3} + b^{1/3} x)^2}} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{a^{1/3} + b^{1/3} x}} \right)$$

$$\left((-3 - i\sqrt{3}) \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(3 + i\sqrt{3}) b^{1/3} x}}{a^{1/3} + b^{1/3} x}\right], \frac{-i + \sqrt{3}}{i + \sqrt{3}}\right] + (1 + i\sqrt{3}) \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(3 + i\sqrt{3}) b^{1/3} x}}{a^{1/3} + b^{1/3} x}\right], \frac{-i + \sqrt{3}}{i + \sqrt{3}}\right] \right)$$

Problem 301: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{x}}{\sqrt{a x^2 + b x^5}} dx$$

Optimal (type 4, 203 leaves, 3 steps):

$$x^{3/2} (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{(a^{1/3} + (1 + \sqrt{3}) b^{1/3} x)^2}} \text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3} + (1 - \sqrt{3}) b^{1/3} x}{a^{1/3} + (1 + \sqrt{3}) b^{1/3} x}\right], \frac{1}{4} (2 + \sqrt{3})\right]$$

$$3^{1/4} a^{1/3} \sqrt{\frac{b^{1/3} x (a^{1/3} + b^{1/3} x)}{(a^{1/3} + (1 + \sqrt{3}) b^{1/3} x)^2}} \sqrt{a x^2 + b x^5}$$

Result (type 4, 151 leaves):

$$- \left(\left(2 i b^{1/3} \sqrt{(-1)^{5/6} \left(-1 + \frac{(-a)^{1/3}}{b^{1/3} x}\right)} \sqrt{1 + \frac{(-a)^{2/3}}{b^{2/3} x^2} + \frac{(-a)^{1/3}}{b^{1/3} x}} x^{5/2} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-a)^{1/3}}{b^{1/3} x}}}{3^{1/4}}}\right], (-1)^{1/3}\right] \right) / \right. \\ \left. \left(3^{1/4} (-a)^{1/3} \sqrt{x^2 (a + b x^3)} \right) \right)$$

Problem 302: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{x} \sqrt{a x^2 + b x^5}} dx$$

Optimal (type 4, 519 leaves, 6 steps):

$$\frac{2(1+\sqrt{3})b^{1/3}x^{3/2}(a+bx^3)}{a(a^{1/3}+(1+\sqrt{3})b^{1/3}x)\sqrt{ax^2+bx^5}} - \frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}}$$

$$\frac{2 \times 3^{1/4} b^{1/3} x^{3/2} (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{(a^{1/3} + (1+\sqrt{3}) b^{1/3} x)^2}} \text{EllipticE}\left[\text{ArcCos}\left[\frac{a^{1/3} + (1-\sqrt{3}) b^{1/3} x}{a^{1/3} + (1+\sqrt{3}) b^{1/3} x}\right], \frac{1}{4}(2+\sqrt{3})\right]}{a^{2/3} \sqrt{\frac{b^{1/3} x (a^{1/3} + b^{1/3} x)}{(a^{1/3} + (1+\sqrt{3}) b^{1/3} x)^2}} \sqrt{ax^2+bx^5}}$$

$$\left((1-\sqrt{3}) b^{1/3} x^{3/2} (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{(a^{1/3} + (1+\sqrt{3}) b^{1/3} x)^2}} \text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3} + (1-\sqrt{3}) b^{1/3} x}{a^{1/3} + (1+\sqrt{3}) b^{1/3} x}\right], \frac{1}{4}(2+\sqrt{3})\right] \right) /$$

$$\left(3^{1/4} a^{2/3} \sqrt{\frac{b^{1/3} x (a^{1/3} + b^{1/3} x)}{(a^{1/3} + (1+\sqrt{3}) b^{1/3} x)^2}} \sqrt{ax^2+bx^5} \right)$$

Result (type 4, 341 leaves):

$$\frac{1}{a\sqrt{x^2(a+bx^3)}} 2\sqrt{x}$$

$$\left(-a + a^{2/3} b^{1/3} x - a^{1/3} b^{2/3} x^2 + \frac{1}{2(-1+(-1)^{2/3})} (-1)^{2/3} a^{1/3} (a^{1/3} + b^{1/3} x)^2 \sqrt{\frac{(1+(-1)^{1/3}) b^{1/3} x (a^{1/3} - (-1)^{1/3} b^{1/3} x)}{(a^{1/3} + b^{1/3} x)^2}} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{a^{1/3} + b^{1/3} x}} \right.$$

$$\left. \left((-3 - i\sqrt{3}) \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(3+i\sqrt{3})} b^{1/3} x}{a^{1/3} + b^{1/3} x}\right], \frac{-i+\sqrt{3}}{i+\sqrt{3}}\right] + (1+i\sqrt{3}) \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(3+i\sqrt{3})} b^{1/3} x}{a^{1/3} + b^{1/3} x}\right], \frac{-i+\sqrt{3}}{i+\sqrt{3}}\right] \right) \right)$$

Problem 304: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^{5/2} \sqrt{ax^2+bx^5}} dx$$

Optimal (type 4, 235 leaves, 4 steps):

$$\frac{2 \sqrt{a x^2 + b x^5}}{5 a x^{7/2}} - \frac{2 b x^{3/2} (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{(a^{1/3} + (1 + \sqrt{3}) b^{1/3} x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcCos}\left[\frac{a^{1/3} + (1 - \sqrt{3}) b^{1/3} x}{a^{1/3} + (1 + \sqrt{3}) b^{1/3} x}\right], \frac{1}{4} (2 + \sqrt{3})\right]}{5 \times 3^{1/4} a^{4/3} \sqrt{\frac{b^{1/3} x (a^{1/3} + b^{1/3} x)}{(a^{1/3} + (1 + \sqrt{3}) b^{1/3} x)^2}} \sqrt{a x^2 + b x^5}}$$

Result (type 4, 176 leaves):

$$\left(6 (-a)^{1/3} (a + b x^3) - 4 i 3^{3/4} b^{4/3} \sqrt{(-1)^{5/6} \left(-1 + \frac{(-a)^{1/3}}{b^{1/3} x}\right)} x^4 \sqrt{\frac{(-a)^{2/3} + \frac{(-a)^{1/3} x}{b^{1/3}} + x^2}{x^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i (-a)^{1/3}}{b^{1/3} x}}}{3^{1/4}}\right], (-1)^{1/3}\right] \right) /$$

$$\left(15 (-a)^{4/3} x^{3/2} \sqrt{x^2 (a + b x^3)} \right)$$

Problem 305: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^{7/2} \sqrt{a x^2 + b x^5}} dx$$

Optimal (type 4, 555 leaves, 7 steps):

$$-\frac{8 (1 + \sqrt{3}) b^{4/3} x^{3/2} (a + b x^3)}{7 a^2 (a^{1/3} + (1 + \sqrt{3}) b^{1/3} x) \sqrt{a x^2 + b x^5}} - \frac{2 \sqrt{a x^2 + b x^5}}{7 a x^{9/2}} + \frac{8 b \sqrt{a x^2 + b x^5}}{7 a^2 x^{3/2}} +$$

$$\frac{8 \times 3^{1/4} b^{4/3} x^{3/2} (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{(a^{1/3} + (1 + \sqrt{3}) b^{1/3} x)^2}} \operatorname{EllipticE}\left[\operatorname{ArcCos}\left[\frac{a^{1/3} + (1 - \sqrt{3}) b^{1/3} x}{a^{1/3} + (1 + \sqrt{3}) b^{1/3} x}\right], \frac{1}{4} (2 + \sqrt{3})\right]}{7 a^{5/3} \sqrt{\frac{b^{1/3} x (a^{1/3} + b^{1/3} x)}{(a^{1/3} + (1 + \sqrt{3}) b^{1/3} x)^2}} \sqrt{a x^2 + b x^5}} +$$

$$\left(4 (1 - \sqrt{3}) b^{4/3} x^{3/2} (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{(a^{1/3} + (1 + \sqrt{3}) b^{1/3} x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcCos}\left[\frac{a^{1/3} + (1 - \sqrt{3}) b^{1/3} x}{a^{1/3} + (1 + \sqrt{3}) b^{1/3} x}\right], \frac{1}{4} (2 + \sqrt{3})\right] \right) /$$

$$\left(7 \times 3^{1/4} a^{5/3} \sqrt{\frac{b^{1/3} x (a^{1/3} + b^{1/3} x)}{(a^{1/3} + (1 + \sqrt{3}) b^{1/3} x)^2}} \sqrt{a x^2 + b x^5} \right)$$

Result (type 4, 369 leaves):

$$\frac{1}{7 a^2 \sqrt{x^2 (a + b x^3)}} 2 \sqrt{x} \left(-4 b^{4/3} x (a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2) + \frac{(a + b x^3) (-a + 4 b x^3)}{x^3} - \right.$$

$$\frac{1}{-1 + (-1)^{2/3}} 2 (-1)^{2/3} a^{1/3} b (a^{1/3} + b^{1/3} x)^2 \sqrt{\frac{(1 + (-1)^{1/3}) b^{1/3} x (a^{1/3} - (-1)^{1/3} b^{1/3} x)}{(a^{1/3} + b^{1/3} x)^2}} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{a^{1/3} + b^{1/3} x}}$$

$$\left. \left((-3 - i \sqrt{3}) \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\frac{\sqrt{(3 + i \sqrt{3}) b^{1/3} x}}{a^{1/3} + b^{1/3} x}}{\sqrt{2}} \right], \frac{-i + \sqrt{3}}{i + \sqrt{3}} \right] + (1 + i \sqrt{3}) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{(3 + i \sqrt{3}) b^{1/3} x}}{a^{1/3} + b^{1/3} x}}{\sqrt{2}} \right], \frac{-i + \sqrt{3}}{i + \sqrt{3}} \right] \right) \right)$$

Problem 307: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^{11/2} \sqrt{a x^2 + b x^5}} dx$$

Optimal (type 4, 265 leaves, 5 steps):

$$-\frac{2 \sqrt{a x^2 + b x^5}}{11 a x^{13/2}} + \frac{16 b \sqrt{a x^2 + b x^5}}{55 a^2 x^{7/2}} + \frac{16 b^2 x^{3/2} (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{(a^{1/3} + (1 + \sqrt{3}) b^{1/3} x)^2}} \operatorname{EllipticF} \left[\operatorname{ArcCos} \left[\frac{a^{1/3} + (1 - \sqrt{3}) b^{1/3} x}{a^{1/3} + (1 + \sqrt{3}) b^{1/3} x} \right], \frac{1}{4} (2 + \sqrt{3}) \right]}{55 \times 3^{1/4} a^{7/3} \sqrt{\frac{b^{1/3} x (a^{1/3} + b^{1/3} x)}{(a^{1/3} + (1 + \sqrt{3}) b^{1/3} x)^2}} \sqrt{a x^2 + b x^5}}$$

Result (type 4, 190 leaves):

$$\left(6 (-a)^{1/3} (-5a^2 + 3abx^3 + 8b^2x^6) - 32i 3^{3/4} b^{7/3} \sqrt{(-1)^{5/6} \left(-1 + \frac{(-a)^{1/3}}{b^{1/3}x}\right)} x^7 \right. \\ \left. \sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}} + \frac{(-a)^{1/3}x}{b^{1/3}} + x^2}{x^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i(-a)^{1/3}}{b^{1/3}x}}}{3^{1/4}}}\right], (-1)^{1/3}\right] \right) / \left(165 (-a)^{7/3} x^{9/2} \sqrt{x^2 (a + bx^3)}\right)$$

Problem 328: Result more than twice size of optimal antiderivative.

$$\int (ax + bx^{14})^{12} dx$$

Optimal (type 1, 16 leaves, 2 steps):

$$\frac{(a + bx^{13})^{13}}{169b}$$

Result (type 1, 160 leaves):

$$\frac{a^{12}x^{13}}{13} + \frac{6}{13}a^{11}bx^{26} + \frac{22}{13}a^{10}b^2x^{39} + \frac{55}{13}a^9b^3x^{52} + \frac{99}{13}a^8b^4x^{65} + \frac{132}{13}a^7b^5x^{78} + \\ \frac{132}{13}a^6b^6x^{91} + \frac{99}{13}a^5b^7x^{104} + \frac{55}{13}a^4b^8x^{117} + \frac{22}{13}a^3b^9x^{130} + \frac{6}{13}a^2b^{10}x^{143} + \frac{1}{13}ab^{11}x^{156} + \frac{b^{12}x^{169}}{169}$$

Problem 329: Result more than twice size of optimal antiderivative.

$$\int x^{12} (ax + bx^{26})^{12} dx$$

Optimal (type 1, 16 leaves, 2 steps):

$$\frac{(a + bx^{25})^{13}}{325b}$$

Result (type 1, 160 leaves):

$$\frac{a^{12}x^{25}}{25} + \frac{6}{25}a^{11}bx^{50} + \frac{22}{25}a^{10}b^2x^{75} + \frac{11}{5}a^9b^3x^{100} + \frac{99}{25}a^8b^4x^{125} + \frac{132}{25}a^7b^5x^{150} + \\ \frac{132}{25}a^6b^6x^{175} + \frac{99}{25}a^5b^7x^{200} + \frac{11}{5}a^4b^8x^{225} + \frac{22}{25}a^3b^9x^{250} + \frac{6}{25}a^2b^{10}x^{275} + \frac{1}{25}ab^{11}x^{300} + \frac{b^{12}x^{325}}{325}$$

Problem 330: Result more than twice size of optimal antiderivative.

$$\int x^{24} (a x + b x^{38})^{12} dx$$

Optimal (type 1, 16 leaves, 2 steps):

$$\frac{(a + b x^{37})^{13}}{481 b}$$

Result (type 1, 160 leaves):

$$\frac{a^{12} x^{37}}{37} + \frac{6}{37} a^{11} b x^{74} + \frac{22}{37} a^{10} b^2 x^{111} + \frac{55}{37} a^9 b^3 x^{148} + \frac{99}{37} a^8 b^4 x^{185} + \frac{132}{37} a^7 b^5 x^{222} +$$

$$\frac{132}{37} a^6 b^6 x^{259} + \frac{99}{37} a^5 b^7 x^{296} + \frac{55}{37} a^4 b^8 x^{333} + \frac{22}{37} a^3 b^9 x^{370} + \frac{6}{37} a^2 b^{10} x^{407} + \frac{1}{37} a b^{11} x^{444} + \frac{b^{12} x^{481}}{481}$$

Problem 332: Result more than twice size of optimal antiderivative.

$$\int (a x + b x^{14})^{12} dx$$

Optimal (type 1, 16 leaves, 2 steps):

$$\frac{(a + b x^{13})^{13}}{169 b}$$

Result (type 1, 160 leaves):

$$\frac{a^{12} x^{13}}{13} + \frac{6}{13} a^{11} b x^{26} + \frac{22}{13} a^{10} b^2 x^{39} + \frac{55}{13} a^9 b^3 x^{52} + \frac{99}{13} a^8 b^4 x^{65} + \frac{132}{13} a^7 b^5 x^{78} +$$

$$\frac{132}{13} a^6 b^6 x^{91} + \frac{99}{13} a^5 b^7 x^{104} + \frac{55}{13} a^4 b^8 x^{117} + \frac{22}{13} a^3 b^9 x^{130} + \frac{6}{13} a^2 b^{10} x^{143} + \frac{1}{13} a b^{11} x^{156} + \frac{b^{12} x^{169}}{169}$$

Problem 333: Result more than twice size of optimal antiderivative.

$$\int (a x^2 + b x^{27})^{12} dx$$

Optimal (type 1, 16 leaves, 2 steps):

$$\frac{(a + b x^{25})^{13}}{325 b}$$

Result (type 1, 160 leaves):

$$\frac{a^{12} x^{25}}{25} + \frac{6}{25} a^{11} b x^{50} + \frac{22}{25} a^{10} b^2 x^{75} + \frac{11}{5} a^9 b^3 x^{100} + \frac{99}{25} a^8 b^4 x^{125} + \frac{132}{25} a^7 b^5 x^{150} +$$

$$\frac{132}{25} a^6 b^6 x^{175} + \frac{99}{25} a^5 b^7 x^{200} + \frac{11}{5} a^4 b^8 x^{225} + \frac{22}{25} a^3 b^9 x^{250} + \frac{6}{25} a^2 b^{10} x^{275} + \frac{1}{25} a b^{11} x^{300} + \frac{b^{12} x^{325}}{325}$$

Problem 334: Result more than twice size of optimal antiderivative.

$$\int (a x^3 + b x^{40})^{12} dx$$

Optimal (type 1, 16 leaves, 2 steps):

$$\frac{(a + b x^{37})^{13}}{481 b}$$

Result (type 1, 160 leaves):

$$\frac{a^{12} x^{37}}{37} + \frac{6}{37} a^{11} b x^{74} + \frac{22}{37} a^{10} b^2 x^{111} + \frac{55}{37} a^9 b^3 x^{148} + \frac{99}{37} a^8 b^4 x^{185} + \frac{132}{37} a^7 b^5 x^{222} +$$

$$\frac{132}{37} a^6 b^6 x^{259} + \frac{99}{37} a^5 b^7 x^{296} + \frac{55}{37} a^4 b^8 x^{333} + \frac{22}{37} a^3 b^9 x^{370} + \frac{6}{37} a^2 b^{10} x^{407} + \frac{1}{37} a b^{11} x^{444} + \frac{b^{12} x^{481}}{481}$$

Problem 335: Result more than twice size of optimal antiderivative.

$$\int (a x^m + b x^{1+13m})^{12} dx$$

Optimal (type 3, 27 leaves, 2 steps):

$$\frac{(a + b x^{1+12m})^{13}}{13 b (1 + 12 m)}$$

Result (type 3, 193 leaves):

$$\frac{1}{13 + 156 m} x^{1+12m} (13 a^{12} + 78 a^{11} b x^{1+12m} + 286 a^{10} b^2 x^{2+24m} + 715 a^9 b^3 x^{3+36m} + 1287 a^8 b^4 x^{4+48m} + 1716 a^7 b^5 x^{5+60m} +$$

$$1716 a^6 b^6 x^{6+72m} + 1287 a^5 b^7 x^{7+84m} + 715 a^4 b^8 x^{8+96m} + 286 a^3 b^9 x^{9+108m} + 78 a^2 b^{10} x^{10+120m} + 13 a b^{11} x^{11+132m} + b^{12} x^{12+144m})$$

Problem 336: Result more than twice size of optimal antiderivative.

$$\int (a x^m + b x^{1+6m})^5 dx$$

Optimal (type 3, 27 leaves, 2 steps):

$$\frac{(a + b x^{1+5m})^6}{6b(1+5m)}$$

Result (type 3, 88 leaves):

$$\frac{x^{1+5m} (6a^5 + 15a^4 b x^{1+5m} + 20a^3 b^2 x^{2+10m} + 15a^2 b^3 x^{3+15m} + 6a b^4 x^{4+20m} + b^5 x^{5+25m})}{6 + 30m}$$

Problem 348: Result more than twice size of optimal antiderivative.

$$\int x^p (a x^n + b x^{1+13n+p})^{12} dx$$

Optimal (type 3, 29 leaves, 2 steps):

$$\frac{(a + b x^{1+12n+p})^{13}}{13b(1+12n+p)}$$

Result (type 3, 232 leaves):

$$\frac{1}{13(1+12n+p)} x^{1+12n+p} (13a^{12} + 78a^{11} b x^{1+12n+p} + 286a^3 b^9 x^{9(1+12n+p)} + 78a^2 b^{10} x^{10(1+12n+p)} + 13a b^{11} x^{11(1+12n+p)} + b^{12} x^{12(1+12n+p)} + 286a^{10} b^2 x^{2+24n+2p} + 715a^9 b^3 x^{3+36n+3p} + 1287a^8 b^4 x^{4+48n+4p} + 1716a^7 b^5 x^{5+60n+5p} + 1716a^6 b^6 x^{6+72n+6p} + 1287a^5 b^7 x^{7+84n+7p} + 715a^4 b^8 x^{8+96n+8p})$$

Problem 349: Result more than twice size of optimal antiderivative.

$$\int x^{12} (a + b x^{13})^{12} dx$$

Optimal (type 1, 16 leaves, 1 step):

$$\frac{(a + b x^{13})^{13}}{169b}$$

Result (type 1, 160 leaves):

$$\frac{a^{12} x^{13}}{13} + \frac{6}{13} a^{11} b x^{26} + \frac{22}{13} a^{10} b^2 x^{39} + \frac{55}{13} a^9 b^3 x^{52} + \frac{99}{13} a^8 b^4 x^{65} + \frac{132}{13} a^7 b^5 x^{78} + \frac{132}{13} a^6 b^6 x^{91} + \frac{99}{13} a^5 b^7 x^{104} + \frac{55}{13} a^4 b^8 x^{117} + \frac{22}{13} a^3 b^9 x^{130} + \frac{6}{13} a^2 b^{10} x^{143} + \frac{1}{13} a b^{11} x^{156} + \frac{b^{12} x^{169}}{169}$$

Problem 350: Result more than twice size of optimal antiderivative.

$$\int x^{12} (a x + b x^{26})^{12} dx$$

Optimal (type 1, 16 leaves, 2 steps):

$$\frac{(a + b x^{25})^{13}}{325 b}$$

Result (type 1, 160 leaves):

$$\begin{aligned} & \frac{a^{12} x^{25}}{25} + \frac{6}{25} a^{11} b x^{50} + \frac{22}{25} a^{10} b^2 x^{75} + \frac{11}{5} a^9 b^3 x^{100} + \frac{99}{25} a^8 b^4 x^{125} + \frac{132}{25} a^7 b^5 x^{150} + \\ & \frac{132}{25} a^6 b^6 x^{175} + \frac{99}{25} a^5 b^7 x^{200} + \frac{11}{5} a^4 b^8 x^{225} + \frac{22}{25} a^3 b^9 x^{250} + \frac{6}{25} a^2 b^{10} x^{275} + \frac{1}{25} a b^{11} x^{300} + \frac{b^{12} x^{325}}{325} \end{aligned}$$

Problem 351: Result more than twice size of optimal antiderivative.

$$\int x^{12} (a x^2 + b x^{39})^{12} dx$$

Optimal (type 1, 16 leaves, 2 steps):

$$\frac{(a + b x^{37})^{13}}{481 b}$$

Result (type 1, 160 leaves):

$$\begin{aligned} & \frac{a^{12} x^{37}}{37} + \frac{6}{37} a^{11} b x^{74} + \frac{22}{37} a^{10} b^2 x^{111} + \frac{55}{37} a^9 b^3 x^{148} + \frac{99}{37} a^8 b^4 x^{185} + \frac{132}{37} a^7 b^5 x^{222} + \\ & \frac{132}{37} a^6 b^6 x^{259} + \frac{99}{37} a^5 b^7 x^{296} + \frac{55}{37} a^4 b^8 x^{333} + \frac{22}{37} a^3 b^9 x^{370} + \frac{6}{37} a^2 b^{10} x^{407} + \frac{1}{37} a b^{11} x^{444} + \frac{b^{12} x^{481}}{481} \end{aligned}$$

Problem 352: Result more than twice size of optimal antiderivative.

$$\int x^{24} (a + b x^{25})^{12} dx$$

Optimal (type 1, 16 leaves, 1 step):

$$\frac{(a + b x^{25})^{13}}{325 b}$$

Result (type 1, 160 leaves):

$$\frac{a^{12} x^{25}}{25} + \frac{6}{25} a^{11} b x^{50} + \frac{22}{25} a^{10} b^2 x^{75} + \frac{11}{5} a^9 b^3 x^{100} + \frac{99}{25} a^8 b^4 x^{125} + \frac{132}{25} a^7 b^5 x^{150} +$$

$$\frac{132}{25} a^6 b^6 x^{175} + \frac{99}{25} a^5 b^7 x^{200} + \frac{11}{5} a^4 b^8 x^{225} + \frac{22}{25} a^3 b^9 x^{250} + \frac{6}{25} a^2 b^{10} x^{275} + \frac{1}{25} a b^{11} x^{300} + \frac{b^{12} x^{325}}{325}$$

Problem 353: Result more than twice size of optimal antiderivative.

$$\int x^{24} (a x + b x^{38})^{12} dx$$

Optimal (type 1, 16 leaves, 2 steps):

$$\frac{(a + b x^{37})^{13}}{481 b}$$

Result (type 1, 160 leaves):

$$\frac{a^{12} x^{37}}{37} + \frac{6}{37} a^{11} b x^{74} + \frac{22}{37} a^{10} b^2 x^{111} + \frac{55}{37} a^9 b^3 x^{148} + \frac{99}{37} a^8 b^4 x^{185} + \frac{132}{37} a^7 b^5 x^{222} +$$

$$\frac{132}{37} a^6 b^6 x^{259} + \frac{99}{37} a^5 b^7 x^{296} + \frac{55}{37} a^4 b^8 x^{333} + \frac{22}{37} a^3 b^9 x^{370} + \frac{6}{37} a^2 b^{10} x^{407} + \frac{1}{37} a b^{11} x^{444} + \frac{b^{12} x^{481}}{481}$$

Problem 354: Result more than twice size of optimal antiderivative.

$$\int x^{36} (a + b x^{37})^{12} dx$$

Optimal (type 1, 16 leaves, 1 step):

$$\frac{(a + b x^{37})^{13}}{481 b}$$

Result (type 1, 160 leaves):

$$\frac{a^{12} x^{37}}{37} + \frac{6}{37} a^{11} b x^{74} + \frac{22}{37} a^{10} b^2 x^{111} + \frac{55}{37} a^9 b^3 x^{148} + \frac{99}{37} a^8 b^4 x^{185} + \frac{132}{37} a^7 b^5 x^{222} +$$

$$\frac{132}{37} a^6 b^6 x^{259} + \frac{99}{37} a^5 b^7 x^{296} + \frac{55}{37} a^4 b^8 x^{333} + \frac{22}{37} a^3 b^9 x^{370} + \frac{6}{37} a^2 b^{10} x^{407} + \frac{1}{37} a b^{11} x^{444} + \frac{b^{12} x^{481}}{481}$$

Problem 378: Unable to integrate problem.

$$\int \sqrt{c x} \left(\frac{a}{x} + b x^n \right)^{3/2} dx$$

Optimal (type 3, 117 leaves, 5 steps):

$$\frac{2 a \sqrt{c x} \sqrt{\frac{a}{x} + b x^n}}{1+n} + \frac{2 (c x)^{3/2} \left(\frac{a}{x} + b x^n\right)^{3/2}}{3 c (1+n)} - \frac{2 a^{3/2} c \sqrt{x} \operatorname{ArcTanh}\left[\frac{\sqrt{a}}{\sqrt{x} \sqrt{\frac{a}{x} + b x^n}}\right]}{(1+n) \sqrt{c x}}$$

Result (type 8, 25 leaves):

$$\int \sqrt{c x} \left(\frac{a}{x} + b x^n\right)^{3/2} dx$$

Problem 380: Unable to integrate problem.

$$\int (c x)^{7/2} \left(\frac{a}{x^3} + b x^n\right)^{3/2} dx$$

Optimal (type 3, 122 leaves, 5 steps):

$$\frac{2 a c^2 (c x)^{3/2} \sqrt{\frac{a}{x^3} + b x^n}}{3+n} + \frac{2 (c x)^{9/2} \left(\frac{a}{x^3} + b x^n\right)^{3/2}}{3 c (3+n)} - \frac{2 a^{3/2} c^4 \sqrt{x} \operatorname{ArcTanh}\left[\frac{\sqrt{a}}{x^{3/2} \sqrt{\frac{a}{x^3} + b x^n}}\right]}{(3+n) \sqrt{c x}}$$

Result (type 8, 25 leaves):

$$\int (c x)^{7/2} \left(\frac{a}{x^3} + b x^n\right)^{3/2} dx$$

Problem 392: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{a x^2 + b x^n}} dx$$

Optimal (type 3, 37 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{a} x}{\sqrt{a x^2 + b x^n}}\right]}{\sqrt{a} (2-n)}$$

Result (type 3, 78 leaves):

$$-\frac{2 \sqrt{b} x^{n/2} \sqrt{1 + \frac{a x^{2-n}}{b}} \operatorname{ArcSinh}\left[\frac{\sqrt{a} x^{1-n/2}}{\sqrt{b}}\right]}{\sqrt{a} (-2+n) \sqrt{a x^2 + b x^n}}$$

Problem 396: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{c^2 x^2 \sqrt{\frac{a}{x^2} + b x^n}} dx$$

Optimal (type 3, 40 leaves, 3 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{a}}{x \sqrt{\frac{a}{x^2} + b x^n}}\right]}{\sqrt{a} c^2 (2+n)}$$

Result (type 3, 81 leaves):

$$\frac{2 \sqrt{a + b x^{2+n}} \left(\operatorname{Log}\left[x^{\frac{2+n}{2}}\right] - \operatorname{Log}\left[a + \sqrt{a} \sqrt{a + b x^{2+n}}\right] \right)}{\sqrt{a} c^2 (2+n) x \sqrt{\frac{a}{x^2} + b x^n}}$$

Problem 409: Unable to integrate problem.

$$\int \frac{1}{\sqrt{\frac{a+b x^5}{x^3}}} dx$$

Optimal (type 3, 32 leaves, 3 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{b} x}{\sqrt{\frac{a}{x^3} + b x^2}}\right]}{5 \sqrt{b}}$$

Result (type 8, 17 leaves):

$$\int \frac{1}{\sqrt{\frac{a+b x^5}{x^3}}} dx$$

Problem 410: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{x^{2-n} (a + b x^n)}} dx$$

Optimal (type 3, 37 leaves, 3 steps):

$$\frac{2 \operatorname{ArcTanh} \left[\frac{\sqrt{b} x}{\sqrt{b x^2 + a x^{2-n}}} \right]}{\sqrt{b} n}$$

Result (type 3, 76 leaves):

$$\frac{2 x^{\frac{2-n}{2}} \sqrt{a + b x^n} \operatorname{ArcTanh} \left[\frac{\sqrt{b} x^{n/2}}{\sqrt{a + b x^n}} \right]}{\sqrt{b} n \sqrt{x^{2-n} (a + b x^n)}}$$

Problem 413: Unable to integrate problem.

$$\int \frac{1}{\sqrt{\frac{a - b x^5}{x^3}}} dx$$

Optimal (type 3, 33 leaves, 3 steps):

$$\frac{2 \operatorname{ArcTan} \left[\frac{\sqrt{b} x}{\sqrt{\frac{a}{x^3} - b x^2}} \right]}{5 \sqrt{b}}$$

Result (type 8, 18 leaves):

$$\int \frac{1}{\sqrt{\frac{a - b x^5}{x^3}}} dx$$

Problem 415: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{x^n (a + b x^{2-n})}} dx$$

Optimal (type 3, 37 leaves, 3 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{b} x}{\sqrt{b x^2 + a x^n}}\right]}{\sqrt{b} (2 - n)}$$

Result (type 3, 78 leaves):

$$\frac{2 \sqrt{a} x^{n/2} \sqrt{1 + \frac{b x^{2-n}}{a}} \operatorname{ArcSinh}\left[\frac{\sqrt{b} x^{1/n}}{\sqrt{a}}\right]}{\sqrt{b} (-2 + n) \sqrt{b x^2 + a x^n}}$$

Problem 416: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{x^2 (b + a x^{-2+n})}} dx$$

Optimal (type 3, 37 leaves, 3 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{b} x}{\sqrt{b x^2 + a x^n}}\right]}{\sqrt{b} (2 - n)}$$

Result (type 3, 78 leaves):

$$\frac{2 \sqrt{a} x^{n/2} \sqrt{1 + \frac{b x^{2-n}}{a}} \operatorname{ArcSinh}\left[\frac{\sqrt{b} x^{1/n}}{\sqrt{a}}\right]}{\sqrt{b} (-2 + n) \sqrt{b x^2 + a x^n}}$$

Problem 417: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{x (b x + a x^{-1+n})}} dx$$

Optimal (type 3, 37 leaves, 3 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{b} x}{\sqrt{b x^2 + a x^n}}\right]}{\sqrt{b} (2 - n)}$$

Result (type 3, 78 leaves):

$$\frac{2\sqrt{a} x^{n/2} \sqrt{1 + \frac{bx^{2-n}}{a}} \operatorname{ArcSinh}\left[\frac{\sqrt{b} x^{1-\frac{n}{2}}}{\sqrt{a}}\right]}{\sqrt{b} (-2+n) \sqrt{bx^2 + ax^n}}$$

Problem 418: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{x^n (a - bx^{2-n})}} dx$$

Optimal (type 3, 38 leaves, 3 steps):

$$\frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{b} x}{\sqrt{-bx^2 + ax^n}}\right]}{\sqrt{b} (2-n)}$$

Result (type 3, 80 leaves):

$$\frac{2\sqrt{a} x^{n/2} \sqrt{1 - \frac{bx^{2-n}}{a}} \operatorname{ArcSin}\left[\frac{\sqrt{b} x^{1-\frac{n}{2}}}{\sqrt{a}}\right]}{\sqrt{b} (-2+n) \sqrt{-bx^2 + ax^n}}$$

Problem 419: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{x^2 (-b + ax^{-2+n})}} dx$$

Optimal (type 3, 38 leaves, 3 steps):

$$\frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{b} x}{\sqrt{-bx^2 + ax^n}}\right]}{\sqrt{b} (2-n)}$$

Result (type 3, 80 leaves):

$$\frac{2\sqrt{a} x^{n/2} \sqrt{1 - \frac{bx^{2-n}}{a}} \operatorname{ArcSin}\left[\frac{\sqrt{b} x^{1-\frac{n}{2}}}{\sqrt{a}}\right]}{\sqrt{b} (-2+n) \sqrt{-bx^2 + ax^n}}$$

Problem 420: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{x(-bx + ax^{-1+n})}} dx$$

Optimal (type 3, 38 leaves, 3 steps):

$$\frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{b} x}{\sqrt{-bx^2 + ax^n}}\right]}{\sqrt{b}(2-n)}$$

Result (type 3, 80 leaves):

$$\frac{2\sqrt{a} x^{n/2} \sqrt{1 - \frac{bx^{2-n}}{a}} \operatorname{ArcSin}\left[\frac{\sqrt{b} x^{1-\frac{n}{2}}}{\sqrt{a}}\right]}{\sqrt{b}(-2+n) \sqrt{-bx^2 + ax^n}}$$

Problem 421: Result more than twice size of optimal antiderivative.

$$\int (cx)^m (ax^j + bx^n)^{3/2} dx$$

Optimal (type 5, 107 leaves, 3 steps):

$$\frac{2bx^{1+n} (cx)^m \sqrt{ax^j + bx^n} \operatorname{Hypergeometric2F1}\left[-\frac{3}{2}, \frac{1+m+\frac{3n}{2}}{j-n}, 1 + \frac{1+m+\frac{3n}{2}}{j-n}, -\frac{ax^{j-n}}{b}\right]}{(2+2m+3n) \sqrt{1 + \frac{ax^{j-n}}{b}}}$$

Result (type 5, 218 leaves):

$$\left(2 (cx)^m \left((2+4j+2m-n) x^{-m} (ax^j + bx^n) (a(2-j+2m+4n) x^{1+j+m} + b(2+2j+2m+n) x^{1+m+n}) + \right. \right. \\ \left. \left. 3a^2 (j-n)^2 x^{1+2j} \sqrt{1 + \frac{ax^{j-n}}{b}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+4j+2m-n}{2j-2n}, \frac{2+6j+2m-3n}{2j-2n}, -\frac{ax^{j-n}}{b}\right] \right) \right) / \\ \left((2+4j+2m-n) (2+2j+2m+n) (2+2m+3n) \sqrt{ax^j + bx^n} \right)$$

Problem 437: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a x^{1/3} + b x^{2/3})^{1/3}} dx$$

Optimal (type 4, 988 leaves, 11 steps):

$$\frac{45 a^2 (a + 2 b x^{1/3}) \left(-\frac{b (a x^{1/3} + b x^{2/3})}{a^2} \right)^{1/3}}{14 \times 2^{1/3} b^3 \left(1 - \sqrt{3} - 2^{2/3} \left(-\frac{b (a + b x^{1/3}) x^{1/3}}{a^2} \right)^{1/3} \right) (a x^{1/3} + b x^{2/3})^{1/3}} - \frac{45 a (a + b x^{1/3}) x^{1/3}}{28 b^2 (a x^{1/3} + b x^{2/3})^{1/3}} + \frac{9 (a + b x^{1/3}) x^{2/3}}{7 b (a x^{1/3} + b x^{2/3})^{1/3}}$$

$$\left(45 \times 3^{1/4} \sqrt{2 + \sqrt{3}} a^4 \left(1 - 2^{2/3} \left(-\frac{b (a + b x^{1/3}) x^{1/3}}{a^2} \right)^{1/3} \right) \sqrt{\frac{1 + 2^{2/3} \left(-\frac{b (a + b x^{1/3}) x^{1/3}}{a^2} \right)^{1/3} + 2 \times 2^{1/3} \left(-\frac{b (a + b x^{1/3}) x^{1/3}}{a^2} \right)^{2/3}}{\left(1 - \sqrt{3} - 2^{2/3} \left(-\frac{b (a + b x^{1/3}) x^{1/3}}{a^2} \right)^{1/3} \right)^2}} \right)$$

$$\left(-\frac{b (a x^{1/3} + b x^{2/3})}{a^2} \right)^{1/3} \text{EllipticE} \left[\text{ArcSin} \left[\frac{1 + \sqrt{3} - 2^{2/3} \left(-\frac{b (a + b x^{1/3}) x^{1/3}}{a^2} \right)^{1/3}}{1 - \sqrt{3} - 2^{2/3} \left(-\frac{b (a + b x^{1/3}) x^{1/3}}{a^2} \right)^{1/3}}, -7 + 4 \sqrt{3} \right], -7 + 4 \sqrt{3} \right] /$$

$$\left(28 \times 2^{1/3} b^3 \sqrt{\frac{1 - 2^{2/3} \left(-\frac{b (a + b x^{1/3}) x^{1/3}}{a^2} \right)^{1/3}}{\left(1 - \sqrt{3} - 2^{2/3} \left(-\frac{b (a + b x^{1/3}) x^{1/3}}{a^2} \right)^{1/3} \right)^2}} (a + 2 b x^{1/3}) (a x^{1/3} + b x^{2/3})^{1/3}} + \right)$$

$$\left(15 \times 3^{3/4} a^4 \left(1 - 2^{2/3} \left(-\frac{b (a + b x^{1/3}) x^{1/3}}{a^2} \right)^{1/3} \right) \sqrt{\frac{1 + 2^{2/3} \left(-\frac{b (a + b x^{1/3}) x^{1/3}}{a^2} \right)^{1/3} + 2 \times 2^{1/3} \left(-\frac{b (a + b x^{1/3}) x^{1/3}}{a^2} \right)^{2/3}}{\left(1 - \sqrt{3} - 2^{2/3} \left(-\frac{b (a + b x^{1/3}) x^{1/3}}{a^2} \right)^{1/3} \right)^2}} \right)$$

$$\left(-\frac{b(a x^{1/3} + b x^{2/3})}{a^2} \right)^{1/3} \text{EllipticF} \left[\text{ArcSin} \left[\frac{1 + \sqrt{3} - 2^{2/3} \left(-\frac{b(a+b x^{1/3}) x^{1/3}}{a^2} \right)^{1/3}}{1 - \sqrt{3} - 2^{2/3} \left(-\frac{b(a+b x^{1/3}) x^{1/3}}{a^2} \right)^{1/3}} \right], -7 + 4\sqrt{3} \right] \Big/$$

$$\left(7 \times 2^{5/6} b^3 \sqrt{-\frac{1 - 2^{2/3} \left(-\frac{b(a+b x^{1/3}) x^{1/3}}{a^2} \right)^{1/3}}{\left(1 - \sqrt{3} - 2^{2/3} \left(-\frac{b(a+b x^{1/3}) x^{1/3}}{a^2} \right)^{1/3} \right)^2}} (a + 2 b x^{1/3}) (a x^{1/3} + b x^{2/3})^{1/3}} \right)$$

Result (type 5, 99 leaves):

$$\frac{9 \left(-5 a^2 x^{1/3} - a b x^{2/3} + 4 b^2 x + 5 a^2 \left(1 + \frac{b x^{1/3}}{a} \right)^{1/3} x^{1/3} \text{Hypergeometric2F1} \left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{b x^{1/3}}{a} \right] \right)}{28 b^2 \left((a + b x^{1/3}) x^{1/3} \right)^{1/3}}$$

Problem 438: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a x^{1/3} + b x^{2/3})^{2/3}} dx$$

Optimal (type 4, 487 leaves, 9 steps):

$$\begin{aligned}
& -\frac{18 a (a+b x^{1/3}) x^{1/3}}{5 b^2 (a x^{1/3}+b x^{2/3})^{2/3}}+\frac{9(a+b x^{1/3}) x^{2/3}}{5 b(a x^{1/3}+b x^{2/3})^{2/3}}+ \\
& \left(6 \times 2^{1/3} \times 3^{3/4} \sqrt{2-\sqrt{3}} a^4\left(1-2^{2/3}\left(-\frac{b(a+b x^{1/3}) x^{1/3}}{a^2}\right)^{1/3}\right) \sqrt{\frac{1+2^{2/3}\left(-\frac{b(a+b x^{1/3}) x^{1/3}}{a^2}\right)^{1/3}+2 \times 2^{1/3}\left(-\frac{b(a+b x^{1/3}) x^{1/3}}{a^2}\right)^{2/3}}{\left(1-\sqrt{3}-2^{2/3}\left(-\frac{b(a+b x^{1/3}) x^{1/3}}{a^2}\right)^{1/3}\right)^2}}}\right. \\
& \left.\left(-\frac{b(a x^{1/3}+b x^{2/3})}{a^2}\right)^{2/3} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}-2^{2/3}\left(-\frac{b(a+b x^{1/3}) x^{1/3}}{a^2}\right)^{1/3}}{1-\sqrt{3}-2^{2/3}\left(-\frac{b(a+b x^{1/3}) x^{1/3}}{a^2}\right)^{1/3}}\right],-7+4 \sqrt{3}\right]\right) / \\
& \left(5 b^3 \sqrt{-\frac{1-2^{2/3}\left(-\frac{b(a+b x^{1/3}) x^{1/3}}{a^2}\right)^{1/3}}{\left(1-\sqrt{3}-2^{2/3}\left(-\frac{b(a+b x^{1/3}) x^{1/3}}{a^2}\right)^{1/3}\right)^2}}(a+2 b x^{1/3})(a x^{1/3}+b x^{2/3})^{2/3}}\right)
\end{aligned}$$

Result (type 5, 98 leaves):

$$\frac{9\left(-2 a^2 x^{1/3}-a b x^{2/3}+b^2 x+2 a^2\left(1+\frac{b x^{1/3}}{a}\right)^{2/3} x^{1/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3},-\frac{b x^{1/3}}{a}\right]\right)}{5 b^2\left((a+b x^{1/3}) x^{1/3}\right)^{2/3}}$$

Problem 453: Result more than twice size of optimal antiderivative.

$$\int x^{-1+n-p(1+q)}(a x^n+b x^p)^q dx$$

Optimal (type 3, 39 leaves, 1 step):

$$\frac{x^{-p(1+q)}(a x^n+b x^p)^{1+q}}{a(n-p)(1+q)}$$

Result (type 3, 100 leaves):

$$\frac{x^{-p(1+q)}\left(1+\frac{a x^{n-p}}{b}\right)^{-q}(a x^n+b x^p)^q\left(a x^n\left(1+\frac{a x^{n-p}}{b}\right)^q+b x^p\left(-1+\left(1+\frac{a x^{n-p}}{b}\right)^q\right)\right)}{a(n-p)(1+q)}$$

Test results for the 298 problems in "1.1.4.3 (e x)^m (a x^j+b x^k)^p (c+d x^n)^q.m"

Problem 220: Result unnecessarily involves imaginary or complex numbers.

$$\int x^{5/2} (A + B x^2) \sqrt{b x^2 + c x^4} dx$$

Optimal (type 4, 243 leaves, 7 steps):

$$\frac{4 b^2 (3 b B - 5 A c) \sqrt{b x^2 + c x^4}}{231 c^3 \sqrt{x}} - \frac{4 b (3 b B - 5 A c) x^{3/2} \sqrt{b x^2 + c x^4}}{385 c^2} - \frac{2 (3 b B - 5 A c) x^{7/2} \sqrt{b x^2 + c x^4}}{55 c} +$$

$$\frac{2 B x^{3/2} (b x^2 + c x^4)^{3/2}}{15 c} - \frac{2 b^{11/4} (3 b B - 5 A c) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b+c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{231 c^{13/4} \sqrt{b x^2 + c x^4}}$$

Result (type 4, 177 leaves):

$$\frac{1}{1155 c^3} 2 \sqrt{x^2 (b + c x^2)} \left(\frac{30 b^3 B + 2 b c^2 x^2 (15 A + 7 B x^2) - 2 b^2 c (25 A + 9 B x^2) + 7 c^3 x^4 (15 A + 11 B x^2)}{\sqrt{x}} + \right.$$

$$\left. \frac{10 i b^3 (-3 b B + 5 A c) \sqrt{1 + \frac{b}{c x^2}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]}{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} (b + c x^2)} \right)$$

Problem 221: Result unnecessarily involves imaginary or complex numbers.

$$\int x^{3/2} (A + B x^2) \sqrt{b x^2 + c x^4} dx$$

Optimal (type 4, 369 leaves, 8 steps):

$$\frac{4 b^2 (7 b B - 13 A c) x^{3/2} (b + c x^2)}{195 c^{5/2} (\sqrt{b} + \sqrt{c} x) \sqrt{b x^2 + c x^4}} - \frac{4 b (7 b B - 13 A c) \sqrt{x} \sqrt{b x^2 + c x^4}}{585 c^2} - \frac{2 (7 b B - 13 A c) x^{5/2} \sqrt{b x^2 + c x^4}}{117 c} +$$

$$\frac{2 B \sqrt{x} (b x^2 + c x^4)^{3/2}}{13 c} - \frac{4 b^{9/4} (7 b B - 13 A c) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b+c x^2}{(\sqrt{b}+\sqrt{c} x)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{195 c^{11/4} \sqrt{b x^2 + c x^4}} +$$

$$\frac{2 b^{9/4} (7 b B - 13 A c) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b+c x^2}{(\sqrt{b}+\sqrt{c} x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{195 c^{11/4} \sqrt{b x^2 + c x^4}}$$

Result (type 4, 273 leaves):

$$\frac{1}{195 x} \sqrt{x^2 (b + c x^2)} \left(\frac{2 x^{3/2} (-14 b^2 B + 2 b c (13 A + 5 B x^2) + 5 c^2 x^2 (13 A + 9 B x^2))}{3 c^2} + \frac{1}{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}} c^3 \sqrt{x} (b + c x^2)}} - 4 b^2 (7 b B - 13 A c) \left(\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} (b + c x^2) - \right. \right.$$

$$\left. \left. \sqrt{b} \sqrt{c} \sqrt{1 + \frac{b}{c x^2}} x^{3/2} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right] + \sqrt{b} \sqrt{c} \sqrt{1 + \frac{b}{c x^2}} x^{3/2} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right] \right) \right)$$

Problem 222: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{x} (A + B x^2) \sqrt{b x^2 + c x^4} dx$$

Optimal (type 4, 204 leaves, 6 steps):

$$- \frac{4 b (5 b B - 11 A c) \sqrt{b x^2 + c x^4}}{231 c^2 \sqrt{x}} - \frac{2 (5 b B - 11 A c) x^{3/2} \sqrt{b x^2 + c x^4}}{77 c} +$$

$$\frac{2 B (b x^2 + c x^4)^{3/2}}{11 c \sqrt{x}} + \frac{2 b^{7/4} (5 b B - 11 A c) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b+c x^2}{(\sqrt{b}+\sqrt{c} x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{231 c^{9/4} \sqrt{b x^2 + c x^4}}$$

Result (type 4, 159 leaves):

$$\frac{1}{231} \sqrt{x^2 (b + c x^2)} \left(\frac{-20 b^2 B + 4 b c (11 A + 3 B x^2) + 6 c^2 x^2 (11 A + 7 B x^2)}{c^2 \sqrt{x}} + \frac{4 i b^2 (5 b B - 11 A c) \sqrt{1 + \frac{b}{c x^2}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]}{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} c^2 (b + c x^2)} \right)$$

Problem 223: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(A + B x^2) \sqrt{b x^2 + c x^4}}{\sqrt{x}} dx$$

Optimal (type 4, 326 leaves, 7 steps):

$$\begin{aligned} & -\frac{4 b (b B - 3 A c) x^{3/2} (b + c x^2)}{15 c^{3/2} (\sqrt{b} + \sqrt{c} x) \sqrt{b x^2 + c x^4}} - \frac{2 (b B - 3 A c) \sqrt{x} \sqrt{b x^2 + c x^4}}{15 c} + \\ & \frac{2 B (b x^2 + c x^4)^{3/2}}{9 c x^{3/2}} + \frac{4 b^{5/4} (b B - 3 A c) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{15 c^{7/4} \sqrt{b x^2 + c x^4}} - \\ & \frac{2 b^{5/4} (b B - 3 A c) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{15 c^{7/4} \sqrt{b x^2 + c x^4}} \end{aligned}$$

Result (type 4, 247 leaves):

$$\begin{aligned} & \frac{1}{15 x} \sqrt{x^2 (b + c x^2)} \left(\frac{2 x^{3/2} (2 b B + 9 A c + 5 B c x^2)}{3 c} - \frac{1}{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} c^2 \sqrt{x} (b + c x^2)} - 4 b (b B - 3 A c) \left(\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} (b + c x^2) - \right. \right. \\ & \left. \left. \sqrt{b} \sqrt{c} \sqrt{1 + \frac{b}{c x^2}} x^{3/2} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right] + \sqrt{b} \sqrt{c} \sqrt{1 + \frac{b}{c x^2}} x^{3/2} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right] \right) \right) \end{aligned}$$

Problem 224: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(A + B x^2) \sqrt{b x^2 + c x^4}}{x^{3/2}} dx$$

Optimal (type 4, 165 leaves, 5 steps):

$$-\frac{2 (b B - 7 A c) \sqrt{b x^2 + c x^4}}{21 c \sqrt{x}} + \frac{2 B (b x^2 + c x^4)^{3/2}}{7 c x^{5/2}} - \frac{2 b^{3/4} (b B - 7 A c) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b+c x^2}{(\sqrt{b}+\sqrt{c} x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{21 c^{5/4} \sqrt{b x^2 + c x^4}}$$

Result (type 4, 134 leaves):

$$\frac{1}{21} \sqrt{x^2 (b + c x^2)} \left(\frac{2 (2 b B + 7 A c + 3 B c x^2)}{c \sqrt{x}} - \frac{4 i b (b B - 7 A c) \sqrt{1 + \frac{b}{c x^2}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]}{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} c (b + c x^2)} \right)$$

Problem 225: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(A + B x^2) \sqrt{b x^2 + c x^4}}{x^{5/2}} dx$$

Optimal (type 4, 323 leaves, 7 steps):

$$\frac{4 (b B + 5 A c) x^{3/2} (b + c x^2)}{5 \sqrt{c} (\sqrt{b} + \sqrt{c} x) \sqrt{b x^2 + c x^4}} + \frac{2 (b B + 5 A c) \sqrt{x} \sqrt{b x^2 + c x^4}}{5 b} - \frac{2 A (b x^2 + c x^4)^{3/2}}{b x^{7/2}} - \frac{4 b^{1/4} (b B + 5 A c) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b+c x^2}{(\sqrt{b}+\sqrt{c} x)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{5 c^{3/4} \sqrt{b x^2 + c x^4}} + \frac{2 b^{1/4} (b B + 5 A c) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b+c x^2}{(\sqrt{b}+\sqrt{c} x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{5 c^{3/4} \sqrt{b x^2 + c x^4}}$$

Result (type 4, 218 leaves):

$$\frac{1}{5x} \sqrt{x^2 (b + cx^2)} \left(\frac{2(2bB + 5Ac + Bcx^2)}{c\sqrt{x}} + \frac{4i \sqrt{\frac{i\sqrt{b}}{\sqrt{c}}} (bB + 5Ac) \sqrt{1 + \frac{b}{cx^2}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]}{b + cx^2} \right. \\ \left. \frac{4i \sqrt{\frac{i\sqrt{b}}{\sqrt{c}}} (bB + 5Ac) \sqrt{1 + \frac{b}{cx^2}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]}{b + cx^2} \right)$$

Problem 226: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(A + Bx^2) \sqrt{bx^2 + cx^4}}{x^{7/2}} dx$$

Optimal (type 4, 163 leaves, 5 steps):

$$\frac{2(bB + Ac) \sqrt{bx^2 + cx^4}}{3b\sqrt{x}} - \frac{2A(bx^2 + cx^4)^{3/2}}{3bx^{9/2}} + \frac{2(bB + Ac)x(\sqrt{b} + \sqrt{c}x) \sqrt{\frac{b+cx^2}{(\sqrt{b} + \sqrt{c}x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{3b^{1/4}c^{1/4}\sqrt{bx^2 + cx^4}}$$

Result (type 4, 119 leaves):

$$\frac{1}{3} \sqrt{x^2 (b + cx^2)} \left(\frac{2(-A + Bx^2)}{x^{5/2}} + \frac{4i(bB + Ac) \sqrt{1 + \frac{b}{cx^2}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]}{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}} (b + cx^2)} \right)$$

Problem 227: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(A + Bx^2) \sqrt{bx^2 + cx^4}}{x^{9/2}} dx$$

Optimal (type 4, 328 leaves, 7 steps):

$$\frac{4\sqrt{c}(5bB+Ac)x^{3/2}(b+cx^2)}{5b(\sqrt{b}+\sqrt{c}x)\sqrt{bx^2+cx^4}} - \frac{2(5bB+Ac)\sqrt{bx^2+cx^4}}{5bx^{3/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{5bx^{11/2}} -$$

$$\frac{4c^{1/4}(5bB+Ac)x(\sqrt{b}+\sqrt{c}x)\sqrt{\frac{b+cx^2}{(\sqrt{b}+\sqrt{c}x)^2}}\text{EllipticE}\left[2\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{5b^{3/4}\sqrt{bx^2+cx^4}} +$$

$$\frac{2c^{1/4}(5bB+Ac)x(\sqrt{b}+\sqrt{c}x)\sqrt{\frac{b+cx^2}{(\sqrt{b}+\sqrt{c}x)^2}}\text{EllipticF}\left[2\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{5b^{3/4}\sqrt{bx^2+cx^4}}$$

Result (type 4, 219 leaves):

$$\left(2 \left(\sqrt{b} \sqrt{\frac{i\sqrt{b}}{\sqrt{c}}} (-A + 5Bx^2)(b+cx^2) - 2\sqrt{c}(5bB+Ac) \sqrt{1 + \frac{b}{cx^2}} x^{7/2} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right] + \right. \right.$$

$$\left. \left. 2\sqrt{c}(5bB+Ac) \sqrt{1 + \frac{b}{cx^2}} x^{7/2} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right] \right) \right) / \left(5\sqrt{b} \sqrt{\frac{i\sqrt{b}}{\sqrt{c}}} x^{3/2} \sqrt{x^2(b+cx^2)} \right)$$

Problem 228: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{11/2}} dx$$

Optimal (type 4, 167 leaves, 5 steps):

$$-\frac{2(7bB-Ac)\sqrt{bx^2+cx^4}}{21bx^{5/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{7bx^{13/2}} + \frac{2c^{3/4}(7bB-Ac)x(\sqrt{b}+\sqrt{c}x)\sqrt{\frac{b+cx^2}{(\sqrt{b}+\sqrt{c}x)^2}}\text{EllipticF}\left[2\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{21b^{5/4}\sqrt{bx^2+cx^4}}$$

Result (type 4, 138 leaves):

$$\frac{1}{21} \sqrt{x^2 (b + c x^2)} \left(-\frac{2 (3 A b + 7 b B x^2 + 2 A c x^2)}{b x^{9/2}} + \frac{4 i c (7 b B - A c) \sqrt{1 + \frac{b}{c x^2}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]}{b \sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} (b + c x^2)} \right)$$

Problem 229: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(A + B x^2) \sqrt{b x^2 + c x^4}}{x^{13/2}} dx$$

Optimal (type 4, 369 leaves, 8 steps):

$$\frac{4 c^{3/2} (3 b B - A c) x^{3/2} (b + c x^2)}{15 b^2 (\sqrt{b} + \sqrt{c} x) \sqrt{b x^2 + c x^4}} - \frac{2 (3 b B - A c) \sqrt{b x^2 + c x^4}}{15 b x^{7/2}} - \frac{4 c (3 b B - A c) \sqrt{b x^2 + c x^4}}{15 b^2 x^{3/2}} - \frac{2 A (b x^2 + c x^4)^{3/2}}{9 b x^{15/2}} - \frac{4 c^{5/4} (3 b B - A c) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{15 b^{7/4} \sqrt{b x^2 + c x^4}} + \frac{2 c^{5/4} (3 b B - A c) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{15 b^{7/4} \sqrt{b x^2 + c x^4}}$$

Result (type 4, 241 leaves):

$$-\left(\left(2 \left(\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} (b + c x^2) (9 b B x^2 (b + 2 c x^2) + A (5 b^2 + 2 b c x^2 - 6 c^2 x^4)) - 6 \sqrt{b} c^{3/2} (3 b B - A c) x^5 \sqrt{1 + \frac{c x^2}{b}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}}\right], -1\right] + 6 \sqrt{b} c^{3/2} (3 b B - A c) x^5 \sqrt{1 + \frac{c x^2}{b}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}}\right], -1\right] \right) \right) / \left(45 b^2 x^{7/2} \sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} \sqrt{x^2 (b + c x^2)} \right)$$

Problem 230: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(A + B x^2) \sqrt{b x^2 + c x^4}}{x^{15/2}} dx$$

Optimal (type 4, 204 leaves, 6 steps):

$$\frac{2 (11 b B - 5 A c) \sqrt{b x^2 + c x^4}}{77 b x^{9/2}} - \frac{4 c (11 b B - 5 A c) \sqrt{b x^2 + c x^4}}{231 b^2 x^{5/2}} - \frac{2 A (b x^2 + c x^4)^{3/2}}{11 b x^{17/2}} - \frac{2 c^{7/4} (11 b B - 5 A c) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b+c x^2}{(\sqrt{b}+\sqrt{c} x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{231 b^{9/4} \sqrt{b x^2 + c x^4}}$$

Result (type 4, 158 leaves):

$$\frac{1}{231 b^2}$$

$$2 \sqrt{x^2 (b + c x^2)} \left(\frac{-11 b B x^2 (3 b + 2 c x^2) + A (-21 b^2 - 6 b c x^2 + 10 c^2 x^4)}{x^{13/2}} + \frac{2 i c^2 (-11 b B + 5 A c) \sqrt{1 + \frac{b}{c x^2}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]}{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} (b + c x^2)} \right)$$

Problem 231: Result unnecessarily involves imaginary or complex numbers.

$$\int x^{7/2} (A + B x^2) (b x^2 + c x^4)^{3/2} dx$$

Optimal (type 4, 486 leaves, 11 steps):

$$\frac{88 b^5 (3 b B - 5 A c) x^{3/2} (b + c x^2)}{16575 c^{9/2} (\sqrt{b} + \sqrt{c} x) \sqrt{b x^2 + c x^4}} - \frac{88 b^4 (3 b B - 5 A c) \sqrt{x} \sqrt{b x^2 + c x^4}}{49725 c^4} + \frac{88 b^3 (3 b B - 5 A c) x^{5/2} \sqrt{b x^2 + c x^4}}{69615 c^3} -$$

$$\frac{8 b^2 (3 b B - 5 A c) x^{9/2} \sqrt{b x^2 + c x^4}}{7735 c^2} - \frac{4 b (3 b B - 5 A c) x^{13/2} \sqrt{b x^2 + c x^4}}{595 c} - \frac{2 (3 b B - 5 A c) x^{9/2} (b x^2 + c x^4)^{3/2}}{105 c} +$$

$$\frac{2 B x^{5/2} (b x^2 + c x^4)^{5/2}}{25 c} - \frac{88 b^{21/4} (3 b B - 5 A c) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b+c x^2}{(\sqrt{b}+\sqrt{c} x)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{16575 c^{19/4} \sqrt{b x^2 + c x^4}} +$$

$$\frac{44 b^{21/4} (3 b B - 5 A c) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b+c x^2}{(\sqrt{b}+\sqrt{c} x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{16575 c^{19/4} \sqrt{b x^2 + c x^4}}$$

Result (type 4, 332 leaves):

$$\frac{1}{348075 c^5 x^3 (b + c x^2)^2}$$

$$2 (x^2 (b + c x^2))^{3/2} \left(\frac{1}{\sqrt{x}} (b + c x^2) (2772 b^6 B - 924 b^5 c (5 A + B x^2) + 220 b^4 c^2 x^2 (7 A + 3 B x^2) + 36 b^2 c^4 x^6 (25 A + 13 B x^2) + 663 c^6 x^{10} (25 A + 21 B x^2) -$$

$$20 b^3 c^3 x^4 (55 A + 27 B x^2) + 39 b c^5 x^8 (575 A + 459 B x^2)) + 924 i b^5 \sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} c (3 b B - 5 A c) \sqrt{1 + \frac{b}{c x^2}} x$$

$$\text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right] - 924 i b^5 \sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} c (3 b B - 5 A c) \sqrt{1 + \frac{b}{c x^2}} x \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right] \right)$$

Problem 232: Result unnecessarily involves imaginary or complex numbers.

$$\int x^{5/2} (A + B x^2) (b x^2 + c x^4)^{3/2} dx$$

Optimal (type 4, 321 leaves, 9 steps):

$$\begin{aligned}
& - \frac{24 b^4 (13 b B - 23 A c) \sqrt{b x^2 + c x^4}}{33\,649 c^4 \sqrt{x}} + \frac{72 b^3 (13 b B - 23 A c) x^{3/2} \sqrt{b x^2 + c x^4}}{168\,245 c^3} - \\
& \frac{8 b^2 (13 b B - 23 A c) x^{7/2} \sqrt{b x^2 + c x^4}}{24\,035 c^2} - \frac{4 b (13 b B - 23 A c) x^{11/2} \sqrt{b x^2 + c x^4}}{2185 c} - \frac{2 (13 b B - 23 A c) x^{7/2} (b x^2 + c x^4)^{3/2}}{437 c} + \\
& \frac{2 B x^{3/2} (b x^2 + c x^4)^{5/2}}{23 c} + \frac{12 b^{19/4} (13 b B - 23 A c) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b+c x^2}{(\sqrt{b}+\sqrt{c} x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{33\,649 c^{17/4} \sqrt{b x^2 + c x^4}}
\end{aligned}$$

Result (type 4, 219 leaves):

$$\frac{1}{168\,245 c^4}$$

$$\left(\begin{aligned}
& 2 \sqrt{x^2 (b + c x^2)} \left(\frac{1}{\sqrt{x}} (-780 b^5 B + 28 b^2 c^3 x^4 (23 A + 11 B x^2) + 385 c^5 x^8 (23 A + 19 B x^2) + 12 b^4 c (115 A + 39 B x^2) - 4 b^3 c^2 x^2 (207 A + 91 B x^2) + \right. \right. \\
& \left. \left. 77 b c^4 x^6 (161 A + 125 B x^2) + \frac{60 i b^5 (13 b B - 23 A c) \sqrt{1 + \frac{b}{c x^2}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]}{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} (b + c x^2)} \right) \right)
\end{aligned} \right)$$

Problem 233: Result unnecessarily involves imaginary or complex numbers.

$$\int x^{3/2} (A + B x^2) (b x^2 + c x^4)^{3/2} dx$$

Optimal (type 4, 447 leaves, 10 steps):

$$\begin{aligned}
& - \frac{8 b^4 (11 b B - 21 A c) x^{3/2} (b + c x^2)}{3315 c^{7/2} (\sqrt{b} + \sqrt{c} x) \sqrt{b x^2 + c x^4}} + \frac{8 b^3 (11 b B - 21 A c) \sqrt{x} \sqrt{b x^2 + c x^4}}{9945 c^3} - \\
& \frac{8 b^2 (11 b B - 21 A c) x^{5/2} \sqrt{b x^2 + c x^4}}{13923 c^2} - \frac{4 b (11 b B - 21 A c) x^{9/2} \sqrt{b x^2 + c x^4}}{1547 c} - \frac{2 (11 b B - 21 A c) x^{5/2} (b x^2 + c x^4)^{3/2}}{357 c} + \\
& \frac{2 B \sqrt{x} (b x^2 + c x^4)^{5/2}}{21 c} + \frac{8 b^{17/4} (11 b B - 21 A c) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b+c x^2}{(\sqrt{b}+\sqrt{c} x)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{3315 c^{15/4} \sqrt{b x^2 + c x^4}} - \\
& \frac{4 b^{17/4} (11 b B - 21 A c) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b+c x^2}{(\sqrt{b}+\sqrt{c} x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{3315 c^{15/4} \sqrt{b x^2 + c x^4}}
\end{aligned}$$

Result (type 4, 313 leaves):

$$\begin{aligned}
& \frac{1}{69615 c^4 x^3 (b + c x^2)^2} 2 (x^2 (b + c x^2))^{3/2} \\
& \left(c x^{3/2} (b + c x^2) (28 b^3 (11 b B - 21 A c) - 20 b^2 c (11 b B - 21 A c) x^2 + 45 b c^2 (4 b B + 133 A c) x^4 + 195 c^3 (23 b B + 21 A c) x^6 + 3315 B c^4 x^8) - \right. \\
& \left. \frac{1}{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}} \sqrt{x}}} 84 b^4 (11 b B - 21 A c) \left(\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} (b + c x^2) - \right. \right. \\
& \left. \left. \sqrt{b} \sqrt{c} \sqrt{1 + \frac{b}{c x^2}} x^{3/2} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right] + \sqrt{b} \sqrt{c} \sqrt{1 + \frac{b}{c x^2}} x^{3/2} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right] \right) \right)
\end{aligned}$$

Problem 234: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{x} (A + B x^2) (b x^2 + c x^4)^{3/2} dx$$

Optimal (type 4, 282 leaves, 8 steps):

$$\frac{8 b^3 (9 b B - 19 A c) \sqrt{b x^2 + c x^4}}{4389 c^3 \sqrt{x}} - \frac{8 b^2 (9 b B - 19 A c) x^{3/2} \sqrt{b x^2 + c x^4}}{7315 c^2} - \frac{4 b (9 b B - 19 A c) x^{7/2} \sqrt{b x^2 + c x^4}}{1045 c} -$$

$$\frac{2 (9 b B - 19 A c) x^{3/2} (b x^2 + c x^4)^{3/2}}{285 c} + \frac{2 B (b x^2 + c x^4)^{5/2}}{19 c \sqrt{x}} - \frac{4 b^{15/4} (9 b B - 19 A c) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b+c x^2}{(\sqrt{b}+\sqrt{c} x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{4389 c^{13/4} \sqrt{b x^2 + c x^4}}$$

Result (type 4, 198 leaves):

$$\frac{1}{21945 c^3} 2 \sqrt{x^2 (b + c x^2)} \left(\frac{1}{\sqrt{x}} (180 b^4 B + 12 b^2 c^2 x^2 (19 A + 7 B x^2) + 77 c^4 x^6 (19 A + 15 B x^2) - 4 b^3 c (95 A + 27 B x^2) + 7 b c^3 x^4 (323 A + 231 B x^2)) + \right.$$

$$\left. \frac{20 i b^4 (-9 b B + 19 A c) \sqrt{1 + \frac{b}{c x^2}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]}{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} (b + c x^2)} \right)$$

Problem 235: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(A + B x^2) (b x^2 + c x^4)^{3/2}}{\sqrt{x}} dx$$

Optimal (type 4, 408 leaves, 9 steps):

$$\frac{8 b^3 (7 b B - 17 A c) x^{3/2} (b + c x^2)}{1105 c^{5/2} (\sqrt{b} + \sqrt{c} x) \sqrt{b x^2 + c x^4}} - \frac{8 b^2 (7 b B - 17 A c) \sqrt{x} \sqrt{b x^2 + c x^4}}{3315 c^2} - \frac{4 b (7 b B - 17 A c) x^{5/2} \sqrt{b x^2 + c x^4}}{663 c} -$$

$$\frac{2 (7 b B - 17 A c) \sqrt{x} (b x^2 + c x^4)^{3/2}}{221 c} + \frac{2 B (b x^2 + c x^4)^{5/2}}{17 c x^{3/2}} - \frac{8 b^{13/4} (7 b B - 17 A c) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{1105 c^{11/4} \sqrt{b x^2 + c x^4}} +$$

$$\frac{4 b^{13/4} (7 b B - 17 A c) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{1105 c^{11/4} \sqrt{b x^2 + c x^4}}$$

Result (type 4, 303 leaves):

$$\frac{1}{1105 x^3} (x^2 (b + c x^2))^{3/2} \left(\frac{2 x^{3/2} (-28 b^3 B + 4 b^2 c (17 A + 5 B x^2) + 15 c^3 x^4 (17 A + 13 B x^2) + 5 b c^2 x^2 (85 A + 57 B x^2))}{3 c^2 (b + c x^2)} + \right.$$

$$\left. \left(8 b^3 (7 b B - 17 A c) \left(\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} (b + c x^2) - \sqrt{b} \sqrt{c} \sqrt{1 + \frac{b}{c x^2}} x^{3/2} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right] + \right. \right. \right.$$

$$\left. \left. \left. \sqrt{b} \sqrt{c} \sqrt{1 + \frac{b}{c x^2}} x^{3/2} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right] \right) \right) / \left(\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} c^3 \sqrt{x} (b + c x^2)^2 \right) \right)$$

Problem 236: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(A + B x^2) (b x^2 + c x^4)^{3/2}}{x^{3/2}} dx$$

Optimal (type 4, 239 leaves, 7 steps):

$$\begin{aligned}
& - \frac{8 b^2 (b B - 3 A c) \sqrt{b x^2 + c x^4}}{231 c^2 \sqrt{x}} - \frac{4 b (b B - 3 A c) x^{3/2} \sqrt{b x^2 + c x^4}}{77 c} - \frac{2 (b B - 3 A c) (b x^2 + c x^4)^{3/2}}{33 c \sqrt{x}} + \\
& \frac{2 B (b x^2 + c x^4)^{5/2}}{15 c x^{5/2}} + \frac{4 b^{11/4} (b B - 3 A c) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b+c x^2}{(\sqrt{b}+\sqrt{c} x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{231 c^{9/4} \sqrt{b x^2 + c x^4}}
\end{aligned}$$

Result (type 4, 174 leaves):

$$\frac{1}{1155 c^2} 2 \sqrt{x^2 (b + c x^2)} \left(\frac{-20 b^3 B + 12 b^2 c (5 A + B x^2) + 7 c^3 x^4 (15 A + 11 B x^2) + b c^2 x^2 (195 A + 119 B x^2)}{\sqrt{x}} + \right.$$

$$\left. \frac{20 i b^3 (b B - 3 A c) \sqrt{1 + \frac{b}{c x^2}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]}{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} (b + c x^2)} \right)$$

Problem 237: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(A + B x^2) (b x^2 + c x^4)^{3/2}}{x^{5/2}} dx$$

Optimal (type 4, 369 leaves, 8 steps):

$$\begin{aligned}
& - \frac{8 b^2 (3 b B - 13 A c) x^{3/2} (b + c x^2)}{195 c^{3/2} (\sqrt{b} + \sqrt{c} x) \sqrt{b x^2 + c x^4}} - \frac{4 b (3 b B - 13 A c) \sqrt{x} \sqrt{b x^2 + c x^4}}{195 c} - \frac{2 (3 b B - 13 A c) (b x^2 + c x^4)^{3/2}}{117 c x^{3/2}} + \\
& \frac{2 B (b x^2 + c x^4)^{5/2}}{13 c x^{7/2}} + \frac{8 b^{9/4} (3 b B - 13 A c) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b+c x^2}{(\sqrt{b}+\sqrt{c} x)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{195 c^{7/4} \sqrt{b x^2 + c x^4}} - \\
& \frac{4 b^{9/4} (3 b B - 13 A c) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b+c x^2}{(\sqrt{b}+\sqrt{c} x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{195 c^{7/4} \sqrt{b x^2 + c x^4}}
\end{aligned}$$

Result (type 4, 281 leaves):

$$\begin{aligned}
& \frac{1}{195 x^3} \\
& (x^2 (b + c x^2))^{3/2} \left(\frac{2 x^{3/2} (12 b^2 B + 5 c^2 x^2 (13 A + 9 B x^2) + b c (143 A + 75 B x^2))}{3 c (b + c x^2)} - \left(8 b^2 (3 b B - 13 A c) \left(\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} (b + c x^2) - \sqrt{b} \sqrt{c} \sqrt{1 + \frac{b}{c x^2}} x^{3/2} \right. \right. \right. \\
& \left. \left. \left. \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right] + \sqrt{b} \sqrt{c} \sqrt{1 + \frac{b}{c x^2}} x^{3/2} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right] \right) \right) \left/ \left(\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} c^2 \sqrt{x} (b + c x^2)^2 \right) \right)
\end{aligned}$$

Problem 238: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(A + B x^2) (b x^2 + c x^4)^{3/2}}{x^{7/2}} dx$$

Optimal (type 4, 201 leaves, 6 steps):

$$\begin{aligned}
& - \frac{4 b (b B - 11 A c) \sqrt{b x^2 + c x^4}}{77 c \sqrt{x}} - \frac{2 (b B - 11 A c) (b x^2 + c x^4)^{3/2}}{77 c x^{5/2}} + \\
& \frac{2 B (b x^2 + c x^4)^{5/2}}{11 c x^{9/2}} - \frac{4 b^{7/4} (b B - 11 A c) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b+c x^2}{(\sqrt{b}+\sqrt{c} x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{77 c^{5/4} \sqrt{b x^2 + c x^4}}
\end{aligned}$$

Result (type 4, 153 leaves):

$$\frac{2 \sqrt{x^2 (b + c x^2)} \left(\frac{4 b^2 B + c^2 x^2 (11 A + 7 B x^2) + b c (33 A + 13 B x^2)}{\sqrt{x}} - \frac{4 i b^2 (b B - 11 A c) \sqrt{1 + \frac{b}{c x^2}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]}{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} (b + c x^2)} \right)}{77 c}$$

Problem 239: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(A + B x^2) (b x^2 + c x^4)^{3/2}}{x^{9/2}} dx$$

Optimal (type 4, 356 leaves, 8 steps):

$$\frac{8 b (b B + 9 A c) x^{3/2} (b + c x^2)}{15 \sqrt{c} (\sqrt{b} + \sqrt{c} x) \sqrt{b x^2 + c x^4}} + \frac{4}{15} (b B + 9 A c) \sqrt{x} \sqrt{b x^2 + c x^4} + \frac{2 (b B + 9 A c) (b x^2 + c x^4)^{3/2}}{9 b x^{3/2}} -$$

$$\frac{2 A (b x^2 + c x^4)^{5/2}}{b x^{11/2}} - \frac{8 b^{5/4} (b B + 9 A c) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{15 c^{3/4} \sqrt{b x^2 + c x^4}} +$$

$$\frac{4 b^{5/4} (b B + 9 A c) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{15 c^{3/4} \sqrt{b x^2 + c x^4}}$$

Result (type 4, 249 leaves):

$$\left(2 \sqrt{x} \left(\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} (b + c x^2) (12 b^2 B + c^2 x^2 (9 A + 5 B x^2) + b c (63 A + 11 B x^2)) - \right. \right.$$

$$12 b^{3/2} \sqrt{c} (b B + 9 A c) \sqrt{1 + \frac{b}{c x^2}} x^{3/2} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right] +$$

$$\left. \left. 12 b^{3/2} \sqrt{c} (b B + 9 A c) \sqrt{1 + \frac{b}{c x^2}} x^{3/2} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right] \right) \right) / \left(45 \sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} c \sqrt{x^2 (b + c x^2)} \right)$$

Problem 240: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(A + Bx^2)(bx^2 + cx^4)^{3/2}}{x^{11/2}} dx$$

Optimal (type 4, 200 leaves, 6 steps):

$$\frac{4(3bB + 7Ac)\sqrt{bx^2 + cx^4}}{21\sqrt{x}} + \frac{2(3bB + 7Ac)(bx^2 + cx^4)^{3/2}}{21bx^{5/2}} - \frac{2A(bx^2 + cx^4)^{5/2}}{3bx^{13/2}} +$$

$$\frac{4b^{3/4}(3bB + 7Ac)x(\sqrt{b} + \sqrt{c}x)\sqrt{\frac{b+cx^2}{(\sqrt{b} + \sqrt{c}x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{21c^{1/4}\sqrt{bx^2 + cx^4}}$$

Result (type 4, 138 leaves):

$$\frac{2}{21}\sqrt{x^2(b+cx^2)} \left(\frac{-7Ab + 9bBx^2 + 7Acx^2 + 3Bcx^4}{x^{5/2}} + \frac{4ib(3bB + 7Ac)\sqrt{1 + \frac{b}{cx^2}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]}{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}(b+cx^2)} \right)$$

Problem 241: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(A + Bx^2)(bx^2 + cx^4)^{3/2}}{x^{13/2}} dx$$

Optimal (type 4, 354 leaves, 8 steps):

$$\frac{24\sqrt{c}(bB + Ac)x^{3/2}(b+cx^2)}{5(\sqrt{b} + \sqrt{c}x)\sqrt{bx^2 + cx^4}} + \frac{12c(bB + Ac)\sqrt{x}\sqrt{bx^2 + cx^4}}{5b} - \frac{2(bB + Ac)(bx^2 + cx^4)^{3/2}}{bx^{7/2}} -$$

$$\frac{2A(bx^2 + cx^4)^{5/2}}{5bx^{15/2}} - \frac{24b^{1/4}c^{1/4}(bB + Ac)x(\sqrt{b} + \sqrt{c}x)\sqrt{\frac{b+cx^2}{(\sqrt{b} + \sqrt{c}x)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{5\sqrt{bx^2 + cx^4}} +$$

$$\frac{12b^{1/4}c^{1/4}(bB + Ac)x(\sqrt{b} + \sqrt{c}x)\sqrt{\frac{b+cx^2}{(\sqrt{b} + \sqrt{c}x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{5\sqrt{bx^2 + cx^4}}$$

Result (type 4, 232 leaves):

$$\left(2 \left(\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}} (b + c x^2) (-A b + 7 b B x^2 + 5 A c x^2 + B c x^4) - 12 \sqrt{b} \sqrt{c} (b B + A c) \sqrt{1 + \frac{b}{c x^2}} x^{7/2} \text{EllipticE} \left[i \text{ArcSinh} \left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}} \right], -1 \right] + \right. \right. \\ \left. \left. 12 \sqrt{b} \sqrt{c} (b B + A c) \sqrt{1 + \frac{b}{c x^2}} x^{7/2} \text{EllipticF} \left[i \text{ArcSinh} \left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}} \right], -1 \right] \right) \right) / \left(5 \sqrt{\frac{i\sqrt{b}}{\sqrt{c}}} x^{3/2} \sqrt{x^2 (b + c x^2)} \right)$$

Problem 242: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(A + B x^2) (b x^2 + c x^4)^{3/2}}{x^{15/2}} dx$$

Optimal (type 4, 204 leaves, 6 steps):

$$\frac{4 c (7 b B + 3 A c) \sqrt{b x^2 + c x^4}}{21 b \sqrt{x}} - \frac{2 (7 b B + 3 A c) (b x^2 + c x^4)^{3/2}}{21 b x^{9/2}} - \frac{2 A (b x^2 + c x^4)^{5/2}}{7 b x^{17/2}} + \\ \frac{4 c^{3/4} (7 b B + 3 A c) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b+c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \text{EllipticF} \left[2 \text{ArcTan} \left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}} \right], \frac{1}{2} \right]}{21 b^{1/4} \sqrt{b x^2 + c x^4}}$$

Result (type 4, 139 leaves):

$$\frac{2}{21} \sqrt{x^2 (b + c x^2)} \left(\frac{7 B x^2 (-b + c x^2) - 3 A (b + 3 c x^2)}{x^{9/2}} + \frac{4 i c (7 b B + 3 A c) \sqrt{1 + \frac{b}{c x^2}} \text{EllipticF} \left[i \text{ArcSinh} \left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}} \right], -1 \right]}{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}} (b + c x^2)} \right)$$

Problem 243: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(A + B x^2) (b x^2 + c x^4)^{3/2}}{x^{17/2}} dx$$

Optimal (type 4, 364 leaves, 8 steps):

$$\frac{8 c^{3/2} (9 b B + A c) x^{3/2} (b + c x^2)}{15 b (\sqrt{b} + \sqrt{c} x) \sqrt{b x^2 + c x^4}} - \frac{4 c (9 b B + A c) \sqrt{b x^2 + c x^4}}{15 b x^{3/2}} - \frac{2 (9 b B + A c) (b x^2 + c x^4)^{3/2}}{45 b x^{11/2}} -$$

$$\frac{2 A (b x^2 + c x^4)^{5/2}}{9 b x^{19/2}} - \frac{8 c^{5/4} (9 b B + A c) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b+c x^2}{(\sqrt{b}+\sqrt{c} x)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{15 b^{3/4} \sqrt{b x^2 + c x^4}} +$$

$$\frac{4 c^{5/4} (9 b B + A c) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b+c x^2}{(\sqrt{b}+\sqrt{c} x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{15 b^{3/4} \sqrt{b x^2 + c x^4}}$$

Result (type 4, 236 leaves):

$$- \left(\left(2 \sqrt{b} \sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} (b + c x^2) (9 B x^2 (b - 5 c x^2) + A (5 b + 11 c x^2)) + 12 c^{3/2} (9 b B + A c) \sqrt{1 + \frac{b}{c x^2}} x^{11/2} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right] - \right. \right.$$

$$\left. \left. 12 c^{3/2} (9 b B + A c) \sqrt{1 + \frac{b}{c x^2}} x^{11/2} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right] \right) \right) / \left(45 \sqrt{b} \sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} x^{7/2} \sqrt{x^2 (b + c x^2)} \right)$$

Problem 244: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{13/2} (A + B x^2)}{\sqrt{b x^2 + c x^4}} dx$$

Optimal (type 4, 243 leaves, 7 steps):

$$- \frac{2 b^2 (13 b B - 15 A c) \sqrt{b x^2 + c x^4}}{77 c^4 \sqrt{x}} + \frac{6 b (13 b B - 15 A c) x^{3/2} \sqrt{b x^2 + c x^4}}{385 c^3} - \frac{2 (13 b B - 15 A c) x^{7/2} \sqrt{b x^2 + c x^4}}{165 c^2} +$$

$$\frac{2 B x^{11/2} \sqrt{b x^2 + c x^4}}{15 c} + \frac{b^{11/4} (13 b B - 15 A c) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b+c x^2}{(\sqrt{b}+\sqrt{c} x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{77 c^{17/4} \sqrt{b x^2 + c x^4}}$$

Result (type 4, 196 leaves):

$$\left(-2 \sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} x^{3/2} (b + c x^2) (195 b^3 B - 7 c^3 x^4 (15 A + 11 B x^2) - 9 b^2 c (25 A + 13 B x^2) + b c^2 x^2 (135 A + 91 B x^2)) - \right. \\ \left. 30 i b^3 (-13 b B + 15 A c) \sqrt{1 + \frac{b}{c x^2}} x^2 \text{EllipticF}\left[\text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right] \right) / \left(1155 \sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} c^4 \sqrt{x^2 (b + c x^2)} \right)$$

Problem 245: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{11/2} (A + B x^2)}{\sqrt{b x^2 + c x^4}} dx$$

Optimal (type 4, 369 leaves, 8 steps):

$$\begin{aligned} & - \frac{14 b^2 (11 b B - 13 A c) x^{3/2} (b + c x^2)}{195 c^{7/2} (\sqrt{b} + \sqrt{c} x) \sqrt{b x^2 + c x^4}} + \frac{14 b (11 b B - 13 A c) \sqrt{x} \sqrt{b x^2 + c x^4}}{585 c^3} - \frac{2 (11 b B - 13 A c) x^{5/2} \sqrt{b x^2 + c x^4}}{117 c^2} + \\ & \frac{2 B x^{9/2} \sqrt{b x^2 + c x^4}}{13 c} + \frac{14 b^{9/4} (11 b B - 13 A c) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b+c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{195 c^{15/4} \sqrt{b x^2 + c x^4}} - \\ & \frac{7 b^{9/4} (11 b B - 13 A c) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b+c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{195 c^{15/4} \sqrt{b x^2 + c x^4}} \end{aligned}$$

Result (type 4, 264 leaves):

$$\frac{1}{585 c^4 \sqrt{x^2 (b + c x^2)}}$$

$$2 x \left(c x^{3/2} (b + c x^2) (77 b^2 B + 5 c^2 x^2 (13 A + 9 B x^2) - b c (91 A + 55 B x^2)) - \frac{1}{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}} \sqrt{x}}} 21 b^2 (11 b B - 13 A c) \left(\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} (b + c x^2) - \sqrt{b} \sqrt{c} \sqrt{1 + \frac{b}{c x^2}} x^{3/2} \text{EllipticE} \left[i \text{ArcSinh} \left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}} \right], -1 \right] + \sqrt{b} \sqrt{c} \sqrt{1 + \frac{b}{c x^2}} x^{3/2} \text{EllipticF} \left[i \text{ArcSinh} \left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}} \right], -1 \right] \right) \right)$$

Problem 246: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{9/2} (A + B x^2)}{\sqrt{b x^2 + c x^4}} dx$$

Optimal (type 4, 204 leaves, 6 steps):

$$\frac{10 b (9 b B - 11 A c) \sqrt{b x^2 + c x^4}}{231 c^3 \sqrt{x}} - \frac{2 (9 b B - 11 A c) x^{3/2} \sqrt{b x^2 + c x^4}}{77 c^2} +$$

$$\frac{2 B x^{7/2} \sqrt{b x^2 + c x^4}}{11 c} - \frac{5 b^{7/4} (9 b B - 11 A c) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \text{EllipticF} \left[2 \text{ArcTan} \left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}} \right], \frac{1}{2} \right]}{231 c^{13/4} \sqrt{b x^2 + c x^4}}$$

Result (type 4, 176 leaves):

$$\left(2 \sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} x^{3/2} (b + c x^2) (45 b^2 B + 3 c^2 x^2 (11 A + 7 B x^2) - b c (55 A + 27 B x^2)) + \right.$$

$$\left. 10 i b^2 (-9 b B + 11 A c) \sqrt{1 + \frac{b}{c x^2}} x^2 \text{EllipticF} \left[i \text{ArcSinh} \left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}} \right], -1 \right] \right) / \left(231 \sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} c^3 \sqrt{x^2 (b + c x^2)} \right)$$

Problem 247: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{7/2} (A + B x^2)}{\sqrt{b x^2 + c x^4}} dx$$

Optimal (type 4, 330 leaves, 7 steps):

$$\frac{2 b (7 b B - 9 A c) x^{3/2} (b + c x^2)}{15 c^{5/2} (\sqrt{b} + \sqrt{c} x) \sqrt{b x^2 + c x^4}} - \frac{2 (7 b B - 9 A c) \sqrt{x} \sqrt{b x^2 + c x^4}}{45 c^2} +$$

$$\frac{2 B x^{5/2} \sqrt{b x^2 + c x^4}}{9 c} - \frac{2 b^{5/4} (7 b B - 9 A c) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b+c x^2}{(\sqrt{b}+\sqrt{c} x)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{15 c^{11/4} \sqrt{b x^2 + c x^4}} +$$

$$\frac{b^{5/4} (7 b B - 9 A c) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b+c x^2}{(\sqrt{b}+\sqrt{c} x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{15 c^{11/4} \sqrt{b x^2 + c x^4}}$$

Result (type 4, 237 leaves):

$$\frac{1}{45 c^3 \sqrt{x^2 (b + c x^2)}} \left(2 \sqrt{x} (b + c x^2) (21 b^2 B + c^2 x^2 (9 A + 5 B x^2) - b c (27 A + 7 B x^2)) + \right.$$

$$6 i b \sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} c (7 b B - 9 A c) \sqrt{1 + \frac{b}{c x^2}} x^2 \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right] -$$

$$\left. 6 i b \sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} c (7 b B - 9 A c) \sqrt{1 + \frac{b}{c x^2}} x^2 \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right] \right)$$

Problem 248: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{5/2} (A + B x^2)}{\sqrt{b x^2 + c x^4}} dx$$

Optimal (type 4, 167 leaves, 5 steps):

$$-\frac{2(5bB-7Ac)\sqrt{bx^2+cx^4}}{21c^2\sqrt{x}} + \frac{2Bx^{3/2}\sqrt{bx^2+cx^4}}{7c} + \frac{b^{3/4}(5bB-7Ac)x(\sqrt{b}+\sqrt{c}x)\sqrt{\frac{b+cx^2}{(\sqrt{b}+\sqrt{c}x)^2}} \operatorname{EllipticF}\left[2\operatorname{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{21c^{9/4}\sqrt{bx^2+cx^4}}$$

Result (type 4, 151 leaves):

$$\left(-2\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}x^{3/2}(b+cx^2)(5bB-7Ac-3Bcx^2) - 2ib(-5bB+7Ac)\sqrt{1+\frac{b}{cx^2}}x^2 \operatorname{EllipticF}\left[i\operatorname{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right] \right) /$$

$$\left(21\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}c^2\sqrt{x^2(b+cx^2)} \right)$$

Problem 249: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{3/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}} dx$$

Optimal (type 4, 293 leaves, 6 steps):

$$-\frac{2(3bB-5Ac)x^{3/2}(b+cx^2)}{5c^{3/2}(\sqrt{b}+\sqrt{c}x)\sqrt{bx^2+cx^4}} + \frac{2B\sqrt{x}\sqrt{bx^2+cx^4}}{5c} + \frac{2b^{1/4}(3bB-5Ac)x(\sqrt{b}+\sqrt{c}x)\sqrt{\frac{b+cx^2}{(\sqrt{b}+\sqrt{c}x)^2}} \operatorname{EllipticE}\left[2\operatorname{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{5c^{7/4}\sqrt{bx^2+cx^4}}$$

$$\frac{b^{1/4}(3bB-5Ac)x(\sqrt{b}+\sqrt{c}x)\sqrt{\frac{b+cx^2}{(\sqrt{b}+\sqrt{c}x)^2}} \operatorname{EllipticF}\left[2\operatorname{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{5c^{7/4}\sqrt{bx^2+cx^4}}$$

Result (type 4, 209 leaves):

$$\frac{1}{5c\sqrt{x^2(b+cx^2)}} 2x \left(\frac{(b+cx^2)(-3bB+5Ac+Bcx^2)}{c\sqrt{x}} - i\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}} (3bB-5Ac) \sqrt{1+\frac{b}{cx^2}} \times \text{EllipticE}\left[i\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right] + \right. \\ \left. i\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}} (3bB-5Ac) \sqrt{1+\frac{b}{cx^2}} \times \text{EllipticF}\left[i\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right] \right)$$

Problem 250: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{x}(A+Bx^2)}{\sqrt{bx^2+cx^4}} dx$$

Optimal (type 4, 130 leaves, 4 steps):

$$\frac{2B\sqrt{bx^2+cx^4}}{3c\sqrt{x}} - \frac{(bB-3Ac)x(\sqrt{b}+\sqrt{c}x)\sqrt{\frac{b+cx^2}{(\sqrt{b}+\sqrt{c}x)^2}} \text{EllipticF}\left[2\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{3b^{1/4}c^{5/4}\sqrt{bx^2+cx^4}}$$

Result (type 4, 134 leaves):

$$\frac{2Bx^{3/2}(b+cx^2)}{3c\sqrt{x^2(b+cx^2)}} - \frac{2i(bB-3Ac)\sqrt{1+\frac{b}{cx^2}}x^2 \text{EllipticF}\left[i\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}c\sqrt{x^2(b+cx^2)}}$$

Problem 251: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A+Bx^2}{\sqrt{x}\sqrt{bx^2+cx^4}} dx$$

Optimal (type 4, 281 leaves, 6 steps):

$$\frac{2 (b B + A c) x^{3/2} (b + c x^2)}{b \sqrt{c} (\sqrt{b} + \sqrt{c} x) \sqrt{b x^2 + c x^4}} - \frac{2 A \sqrt{b x^2 + c x^4}}{b x^{3/2}} - \frac{2 (b B + A c) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{b^{3/4} c^{3/4} \sqrt{b x^2 + c x^4}} +$$

$$\frac{(b B + A c) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{b^{3/4} c^{3/4} \sqrt{b x^2 + c x^4}}$$

Result (type 4, 191 leaves):

$$- \left(\left(2 i x^{3/2} \left(A \sqrt{c} \sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} (b + c x^2) - \sqrt{b} (b B + A c) x \sqrt{1 + \frac{c x^2}{b}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}}\right], -1\right] + \right. \right. \right.$$

$$\left. \left. \left. \sqrt{b} (b B + A c) x \sqrt{1 + \frac{c x^2}{b}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}}\right], -1\right] \right) \right) / \left(b^{3/2} \left(\frac{i \sqrt{c} x}{\sqrt{b}} \right)^{3/2} \sqrt{x^2 (b + c x^2)} \right) \right)$$

Problem 252: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A + B x^2}{x^{3/2} \sqrt{b x^2 + c x^4}} dx$$

Optimal (type 4, 131 leaves, 4 steps):

$$- \frac{2 A \sqrt{b x^2 + c x^4}}{3 b x^{5/2}} + \frac{(3 b B - A c) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{3 b^{5/4} c^{1/4} \sqrt{b x^2 + c x^4}}$$

Result (type 4, 119 leaves):

$$2 \left(-A (b + c x^2) + \frac{i (3 b B - A c) \sqrt{1 + \frac{b}{c x^2}} x^{5/2} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}\right], -1\right]}{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}} \right)$$

$$\frac{\hspace{10em}}{3 b \sqrt{x} \sqrt{x^2 (b + c x^2)}}$$

Problem 253: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A + B x^2}{x^{5/2} \sqrt{b x^2 + c x^4}} dx$$

Optimal (type 4, 332 leaves, 7 steps):

$$\frac{2 \sqrt{c} (5 b B - 3 A c) x^{3/2} (b + c x^2)}{5 b^2 (\sqrt{b} + \sqrt{c} x) \sqrt{b x^2 + c x^4}} - \frac{2 A \sqrt{b x^2 + c x^4}}{5 b x^{7/2}} - \frac{2 (5 b B - 3 A c) \sqrt{b x^2 + c x^4}}{5 b^2 x^{3/2}} -$$

$$\frac{2 c^{1/4} (5 b B - 3 A c) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{5 b^{7/4} \sqrt{b x^2 + c x^4}} +$$

$$\frac{c^{1/4} (5 b B - 3 A c) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{5 b^{7/4} \sqrt{b x^2 + c x^4}}$$

Result (type 4, 222 leaves):

$$\left(2 \sqrt{b} \sqrt{c} (5 b B - 3 A c) x^3 \sqrt{1 + \frac{c x^2}{b}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}}\right], -1\right] - \right.$$

$$\left. 2 \left(\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} (b + c x^2) (5 b B x^2 + A (b - 3 c x^2)) + \sqrt{b} \sqrt{c} (5 b B - 3 A c) x^3 \sqrt{1 + \frac{c x^2}{b}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}}\right], -1\right] \right) \right) / \left(5 b^2 x^{3/2} \sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} \sqrt{x^2 (b + c x^2)} \right)$$

Problem 254: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A + B x^2}{x^{7/2} \sqrt{b x^2 + c x^4}} dx$$

Optimal (type 4, 167 leaves, 5 steps):

$$\frac{-\frac{2A\sqrt{bx^2+cx^4}}{7bx^{9/2}} - \frac{2(7bB-5Ac)\sqrt{bx^2+cx^4}}{21b^2x^{5/2}} - \frac{c^{3/4}(7bB-5Ac)x(\sqrt{b}+\sqrt{c}x)\sqrt{\frac{b+cx^2}{(\sqrt{b}+\sqrt{c}x)^2}} \operatorname{EllipticF}\left[2\operatorname{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{21b^{9/4}\sqrt{bx^2+cx^4}}$$

Result (type 4, 156 leaves):

$$\left(-2\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}(b+cx^2)(3Ab+7bBx^2-5Acx^2) + 2ic(-7bB+5Ac)\sqrt{1+\frac{b}{cx^2}}x^{9/2}\operatorname{EllipticF}\left[i\operatorname{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right] \right) /$$

$$\left(21b^2\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}x^{5/2}\sqrt{x^2(b+cx^2)} \right)$$

Problem 255: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A+Bx^2}{x^{9/2}\sqrt{bx^2+cx^4}} dx$$

Optimal (type 4, 369 leaves, 8 steps):

$$\frac{-\frac{2c^{3/2}(9bB-7Ac)x^{3/2}(b+cx^2)}{15b^3(\sqrt{b}+\sqrt{c}x)\sqrt{bx^2+cx^4}} - \frac{2A\sqrt{bx^2+cx^4}}{9bx^{11/2}} - \frac{2(9bB-7Ac)\sqrt{bx^2+cx^4}}{45b^2x^{7/2}} + \frac{2c(9bB-7Ac)\sqrt{bx^2+cx^4}}{15b^3x^{3/2}} + \frac{2c^{5/4}(9bB-7Ac)x(\sqrt{b}+\sqrt{c}x)\sqrt{\frac{b+cx^2}{(\sqrt{b}+\sqrt{c}x)^2}} \operatorname{EllipticE}\left[2\operatorname{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{15b^{11/4}\sqrt{bx^2+cx^4}} - \frac{c^{5/4}(9bB-7Ac)x(\sqrt{b}+\sqrt{c}x)\sqrt{\frac{b+cx^2}{(\sqrt{b}+\sqrt{c}x)^2}} \operatorname{EllipticF}\left[2\operatorname{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{15b^{11/4}\sqrt{bx^2+cx^4}}$$

Result (type 4, 242 leaves):

$$\left(2 \sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} (b + c x^2) (-9 b B x^2 (b - 3 c x^2) + A (-5 b^2 + 7 b c x^2 - 21 c^2 x^4)) - \right. \\ \left. 6 \sqrt{b} c^{3/2} (9 b B - 7 A c) x^5 \sqrt{1 + \frac{c x^2}{b}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}}\right], -1\right] + \right. \\ \left. 6 \sqrt{b} c^{3/2} (9 b B - 7 A c) x^5 \sqrt{1 + \frac{c x^2}{b}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}}\right], -1\right] \right) / \left(45 b^3 x^{7/2} \sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} \sqrt{x^2 (b + c x^2)} \right)$$

Problem 256: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A + B x^2}{x^{11/2} \sqrt{b x^2 + c x^4}} dx$$

Optimal (type 4, 204 leaves, 6 steps):

$$\frac{-\frac{2 A \sqrt{b x^2 + c x^4}}{11 b x^{13/2}} - \frac{2 (11 b B - 9 A c) \sqrt{b x^2 + c x^4}}{77 b^2 x^{9/2}} + \frac{10 c (11 b B - 9 A c) \sqrt{b x^2 + c x^4}}{231 b^3 x^{5/2}} + \frac{5 c^{7/4} (11 b B - 9 A c) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{231 b^{13/4} \sqrt{b x^2 + c x^4}}$$

Result (type 4, 181 leaves):

$$\left(-2 \sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} (b + c x^2) (11 b B x^2 (3 b - 5 c x^2) + 3 A (7 b^2 - 9 b c x^2 + 15 c^2 x^4)) - \right. \\ \left. 10 i c^2 (-11 b B + 9 A c) \sqrt{1 + \frac{b}{c x^2}} x^{13/2} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}\right], -1\right] \right) / \left(231 b^3 \sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} x^{9/2} \sqrt{x^2 (b + c x^2)} \right)$$

Problem 257: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{17/2} (A + B x^2)}{(b x^2 + c x^4)^{3/2}} dx$$

Optimal (type 4, 251 leaves, 7 steps):

$$\begin{aligned} & -\frac{(bB - Ac) x^{15/2}}{bc \sqrt{bx^2 + cx^4}} + \frac{15b(13bB - 11Ac) \sqrt{bx^2 + cx^4}}{77c^4 \sqrt{x}} - \frac{9(13bB - 11Ac) x^{3/2} \sqrt{bx^2 + cx^4}}{77c^3} + \\ & \frac{(13bB - 11Ac) x^{7/2} \sqrt{bx^2 + cx^4}}{11bc^2} - \frac{15b^{7/4}(13bB - 11Ac) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b+cx^2}{(\sqrt{b} + \sqrt{c} x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{154c^{17/4} \sqrt{bx^2 + cx^4}} \end{aligned}$$

Result (type 4, 189 leaves):

$$\begin{aligned} & \left(\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}} x^{3/2} (195b^3B + 2c^3x^4(11A + 7Bx^2) - 2bc^2x^2(33A + 13Bx^2) + b^2(-165Ac + 78Bcx^2)) + \right. \\ & \left. 15ib^2(-13bB + 11Ac) \sqrt{1 + \frac{b}{cx^2}} x^2 \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right] \right) / \left(77 \sqrt{\frac{i\sqrt{b}}{\sqrt{c}}} c^4 \sqrt{x^2(b + cx^2)} \right) \end{aligned}$$

Problem 258: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{15/2} (A + B x^2)}{(b x^2 + c x^4)^{3/2}} dx$$

Optimal (type 4, 377 leaves, 8 steps):

$$\begin{aligned}
& - \frac{(bB - Ac) x^{13/2}}{bc \sqrt{bx^2 + cx^4}} + \frac{7b(11bB - 9Ac) x^{3/2} (b + cx^2)}{15c^{7/2} (\sqrt{b} + \sqrt{c}x) \sqrt{bx^2 + cx^4}} - \frac{7(11bB - 9Ac) \sqrt{x} \sqrt{bx^2 + cx^4}}{45c^3} + \\
& \frac{(11bB - 9Ac) x^{5/2} \sqrt{bx^2 + cx^4}}{9bc^2} - \frac{7b^{5/4} (11bB - 9Ac) x (\sqrt{b} + \sqrt{c}x) \sqrt{\frac{b+cx^2}{(\sqrt{b} + \sqrt{c}x)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{15c^{15/4} \sqrt{bx^2 + cx^4}} + \\
& \frac{7b^{5/4} (11bB - 9Ac) x (\sqrt{b} + \sqrt{c}x) \sqrt{\frac{b+cx^2}{(\sqrt{b} + \sqrt{c}x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{30c^{15/4} \sqrt{bx^2 + cx^4}}
\end{aligned}$$

Result (type 4, 263 leaves):

$$\begin{aligned}
& \left(\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}} \sqrt{x} (231b^3B - 7b^2c(27A - 22Bx^2) + 2c^3x^4(9A + 5Bx^2) - 2bc^2x^2(63A + 11Bx^2)) + \right. \\
& 21b^{3/2} \sqrt{c} (-11bB + 9Ac) \sqrt{1 + \frac{b}{cx^2}} x^2 \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right] - \\
& \left. 21b^{3/2} \sqrt{c} (-11bB + 9Ac) \sqrt{1 + \frac{b}{cx^2}} x^2 \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right] \right) / \left(45 \sqrt{\frac{i\sqrt{b}}{\sqrt{c}}} c^4 \sqrt{x^2(b + cx^2)} \right)
\end{aligned}$$

Problem 259: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{13/2} (A + Bx^2)}{(bx^2 + cx^4)^{3/2}} dx$$

Optimal (type 4, 214 leaves, 6 steps):

$$\frac{-(bB - Ac) x^{11/2}}{bc \sqrt{bx^2 + cx^4}} - \frac{5(9bB - 7Ac) \sqrt{bx^2 + cx^4}}{21c^3 \sqrt{x}} + \frac{(9bB - 7Ac) x^{3/2} \sqrt{bx^2 + cx^4}}{7bc^2} +$$

$$\frac{5b^{3/4} (9bB - 7Ac) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b+cx^2}{(\sqrt{b} + \sqrt{c} x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{42c^{13/4} \sqrt{bx^2 + cx^4}}$$

Result (type 4, 165 leaves):

$$\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}} x^{3/2} (-45b^2B + bc(35A - 18Bx^2) + 2c^2x^2(7A + 3Bx^2)) - 5ib(-9bB + 7Ac) \sqrt{1 + \frac{b}{cx^2}} x^2 \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right] \right) /$$

$$\left(21 \sqrt{\frac{i\sqrt{b}}{\sqrt{c}}} c^3 \sqrt{x^2(b + cx^2)} \right)$$

Problem 260: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{11/2} (A + Bx^2)}{(bx^2 + cx^4)^{3/2}} dx$$

Optimal (type 4, 340 leaves, 7 steps):

$$\frac{-(bB - Ac) x^{9/2}}{bc \sqrt{bx^2 + cx^4}} - \frac{3(7bB - 5Ac) x^{3/2} (b + cx^2)}{5c^{5/2} (\sqrt{b} + \sqrt{c} x) \sqrt{bx^2 + cx^4}} + \frac{(7bB - 5Ac) \sqrt{x} \sqrt{bx^2 + cx^4}}{5bc^2} +$$

$$\frac{3b^{1/4} (7bB - 5Ac) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b+cx^2}{(\sqrt{b} + \sqrt{c} x)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{5c^{11/4} \sqrt{bx^2 + cx^4}} -$$

$$\frac{3b^{1/4} (7bB - 5Ac) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b+cx^2}{(\sqrt{b} + \sqrt{c} x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{10c^{11/4} \sqrt{bx^2 + cx^4}}$$

Result (type 4, 240 leaves):

$$\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}} \sqrt{x} \left(-21 b^2 B + b c (15 A - 14 B x^2) + 2 c^2 x^2 (5 A + B x^2) \right) - 3 \sqrt{b} \sqrt{c} (-7 b B + 5 A c) \sqrt{1 + \frac{b}{c x^2}} x^2 \text{EllipticE} \left[i \text{ArcSinh} \left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}} \right], -1 \right] + \right. \\ \left. 3 \sqrt{b} \sqrt{c} (-7 b B + 5 A c) \sqrt{1 + \frac{b}{c x^2}} x^2 \text{EllipticF} \left[i \text{ArcSinh} \left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}} \right], -1 \right] \right) / \left(5 \sqrt{\frac{i\sqrt{b}}{\sqrt{c}}} c^3 \sqrt{x^2 (b + c x^2)} \right)$$

Problem 261: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{9/2} (A + B x^2)}{(b x^2 + c x^4)^{3/2}} dx$$

Optimal (type 4, 178 leaves, 5 steps):

$$-\frac{(bB - Ac) x^{7/2}}{bc \sqrt{bx^2 + cx^4}} + \frac{(5bB - 3Ac) \sqrt{bx^2 + cx^4}}{3bc^2 \sqrt{x}} - \frac{(5bB - 3Ac) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b+cx^2}{(\sqrt{b} + \sqrt{c} x)^2}} \text{EllipticF} \left[2 \text{ArcTan} \left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}} \right], \frac{1}{2} \right]}{6 b^{1/4} c^{9/4} \sqrt{bx^2 + cx^4}}$$

Result (type 4, 142 leaves):

$$\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}} x^{3/2} (5bB - 3Ac + 2Bcx^2) + i(-5bB + 3Ac) \sqrt{1 + \frac{b}{cx^2}} x^2 \text{EllipticF} \left[i \text{ArcSinh} \left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}} \right], -1 \right]}{3 \sqrt{\frac{i\sqrt{b}}{\sqrt{c}}} c^2 \sqrt{x^2 (b + cx^2)}}$$

Problem 262: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{7/2} (A + B x^2)}{(b x^2 + c x^4)^{3/2}} dx$$

Optimal (type 4, 299 leaves, 6 steps):

$$\begin{aligned}
& - \frac{(bB - Ac) x^{5/2}}{bc \sqrt{bx^2 + cx^4}} + \frac{(3bB - Ac) x^{3/2} (b + cx^2)}{bc^{3/2} (\sqrt{b} + \sqrt{c} x) \sqrt{bx^2 + cx^4}} - \frac{(3bB - Ac) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b+cx^2}{(\sqrt{b} + \sqrt{c} x)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{b^{3/4} c^{7/4} \sqrt{bx^2 + cx^4}} + \\
& \frac{(3bB - Ac) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b+cx^2}{(\sqrt{b} + \sqrt{c} x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{2 b^{3/4} c^{7/4} \sqrt{bx^2 + cx^4}}
\end{aligned}$$

Result (type 4, 213 leaves):

$$\begin{aligned}
& \left(i \left(\sqrt{b} \sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} \sqrt{x} (3bB - Ac + 2Bcx^2) + \sqrt{c} (-3bB + Ac) \sqrt{1 + \frac{b}{cx^2}} x^2 \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right] - \right. \right. \\
& \left. \left. \sqrt{c} (-3bB + Ac) \sqrt{1 + \frac{b}{cx^2}} x^2 \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right] \right) \right) / \left(\left(\frac{i \sqrt{b}}{\sqrt{c}} \right)^{3/2} c^{5/2} \sqrt{x^2 (b + cx^2)} \right)
\end{aligned}$$

Problem 263: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{5/2} (A + Bx^2)}{(bx^2 + cx^4)^{3/2}} dx$$

Optimal (type 4, 137 leaves, 4 steps):

$$- \frac{(bB - Ac) x^{3/2}}{bc \sqrt{bx^2 + cx^4}} + \frac{(bB + Ac) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b+cx^2}{(\sqrt{b} + \sqrt{c} x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{2 b^{5/4} c^{5/4} \sqrt{bx^2 + cx^4}}$$

Result (type 4, 132 leaves):

$$\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} (-bB + Ac) x^{3/2} + i (bB + Ac) \sqrt{1 + \frac{b}{cx^2}} x^2 \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]}{b \sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} c \sqrt{x^2 (b + cx^2)}}$$

Problem 264: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{3/2} (A + B x^2)}{(b x^2 + c x^4)^{3/2}} dx$$

Optimal (type 4, 318 leaves, 7 steps):

$$\begin{aligned} & -\frac{2 A \sqrt{x}}{b \sqrt{b x^2 + c x^4}} + \frac{(b B - 3 A c) x^{5/2}}{b^2 \sqrt{b x^2 + c x^4}} - \frac{(b B - 3 A c) x^{3/2} (b + c x^2)}{b^2 \sqrt{c} (\sqrt{b} + \sqrt{c} x) \sqrt{b x^2 + c x^4}} + \\ & \frac{(b B - 3 A c) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b+c x^2}{(\sqrt{b}+\sqrt{c} x)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{b^{7/4} c^{3/4} \sqrt{b x^2 + c x^4}} - \\ & \frac{(b B - 3 A c) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b+c x^2}{(\sqrt{b}+\sqrt{c} x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{2 b^{7/4} c^{3/4} \sqrt{b x^2 + c x^4}} \end{aligned}$$

Result (type 4, 203 leaves):

$$\begin{aligned} & \left(i x^{3/2} \left(\sqrt{c} \sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} (-2 A b + b B x^2 - 3 A c x^2) - \sqrt{b} (b B - 3 A c) x \sqrt{1 + \frac{c x^2}{b}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}}\right], -1\right] + \right. \right. \\ & \left. \left. \sqrt{b} (b B - 3 A c) x \sqrt{1 + \frac{c x^2}{b}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}}\right], -1\right] \right) \right) / \left(b^{5/2} \left(\frac{i \sqrt{c} x}{\sqrt{b}} \right)^{3/2} \sqrt{x^2 (b + c x^2)} \right) \end{aligned}$$

Problem 265: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{x} (A + B x^2)}{(b x^2 + c x^4)^{3/2}} dx$$

Optimal (type 4, 167 leaves, 5 steps):

$$\begin{aligned} & -\frac{2 A}{3 b \sqrt{x} \sqrt{b x^2 + c x^4}} + \frac{(3 b B - 5 A c) x^{3/2}}{3 b^2 \sqrt{b x^2 + c x^4}} + \frac{(3 b B - 5 A c) x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b+c x^2}{(\sqrt{b}+\sqrt{c} x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{6 b^{9/4} c^{1/4} \sqrt{b x^2 + c x^4}} \end{aligned}$$

Result (type 4, 147 leaves):

$$\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}} (-2Ab + 3bBx^2 - 5Acx^2) - i(-3bB + 5Ac) \sqrt{1 + \frac{b}{cx^2}} x^{5/2} \text{EllipticF}\left[\frac{i\sqrt{b}}{\sqrt{c}} \text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]}{3b^2 \sqrt{\frac{i\sqrt{b}}{\sqrt{c}}} \sqrt{x} \sqrt{x^2(b+cx^2)}}$$

Problem 266: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A + Bx^2}{\sqrt{x} (bx^2 + cx^4)^{3/2}} dx$$

Optimal (type 4, 368 leaves, 8 steps):

$$\begin{aligned} & -\frac{2A}{5bx^{3/2}\sqrt{bx^2+cx^4}} + \frac{(5bB-7Ac)\sqrt{x}}{5b^2\sqrt{bx^2+cx^4}} + \frac{3\sqrt{c}(5bB-7Ac)x^{3/2}(b+cx^2)}{5b^3(\sqrt{b}+\sqrt{c}x)\sqrt{bx^2+cx^4}} \\ & -\frac{3(5bB-7Ac)\sqrt{bx^2+cx^4}}{5b^3x^{3/2}} - \frac{3c^{1/4}(5bB-7Ac)x(\sqrt{b}+\sqrt{c}x)\sqrt{\frac{b+cx^2}{(\sqrt{b}+\sqrt{c}x)^2}} \text{EllipticE}\left[2\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{5b^{11/4}\sqrt{bx^2+cx^4}} + \\ & \frac{3c^{1/4}(5bB-7Ac)x(\sqrt{b}+\sqrt{c}x)\sqrt{\frac{b+cx^2}{(\sqrt{b}+\sqrt{c}x)^2}} \text{EllipticF}\left[2\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{10b^{11/4}\sqrt{bx^2+cx^4}} \end{aligned}$$

Result (type 4, 236 leaves):

$$\begin{aligned} & \left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}} (-5bBx^2(2b+3cx^2) + A(-2b^2+14bcx^2+21c^2x^4)) + 3\sqrt{b}\sqrt{c}(5bB-7Ac)x^3 \sqrt{1+\frac{cx^2}{b}} \text{EllipticE}\left[i\text{ArcSinh}\left[\frac{i\sqrt{c}x}{\sqrt{b}}\right], -1\right] - \right. \\ & \left. 3\sqrt{b}\sqrt{c}(5bB-7Ac)x^3 \sqrt{1+\frac{cx^2}{b}} \text{EllipticF}\left[i\text{ArcSinh}\left[\frac{i\sqrt{c}x}{\sqrt{b}}\right], -1\right] \right) / \left(5b^3x^{3/2} \sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}} \sqrt{x^2(b+cx^2)} \right) \end{aligned}$$

Problem 267: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A + Bx^2}{x^{3/2} (bx^2 + cx^4)^{3/2}} dx$$

Optimal (type 4, 203 leaves, 6 steps):

$$-\frac{2A}{7bx^{5/2}\sqrt{bx^2+cx^4}} + \frac{7bB-9Ac}{7b^2\sqrt{x}\sqrt{bx^2+cx^4}} - \frac{5(7bB-9Ac)\sqrt{bx^2+cx^4}}{21b^3x^{5/2}} -$$

$$\frac{5c^{3/4}(7bB-9Ac)x(\sqrt{b}+\sqrt{c}x)\sqrt{\frac{b+cx^2}{(\sqrt{b}+\sqrt{c}x)^2}} \operatorname{EllipticF}\left[2\operatorname{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{42b^{13/4}\sqrt{bx^2+cx^4}}$$

Result (type 4, 170 leaves):

$$\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}} (-7bBx^2(2b+5cx^2) + A(-6b^2+18bcx^2+45c^2x^4)) + 5ic(-7bB+9Ac) \sqrt{1+\frac{b}{cx^2}} x^{9/2} \operatorname{EllipticF}\left[i\operatorname{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right] \right) /$$

$$\left(21b^3 \sqrt{\frac{i\sqrt{b}}{\sqrt{c}}} x^{5/2} \sqrt{x^2(b+cx^2)} \right)$$

Problem 268: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A+Bx^2}{x^{5/2}(bx^2+cx^4)^{3/2}} dx$$

Optimal (type 4, 405 leaves, 9 steps):

$$-\frac{2A}{9bx^{7/2}\sqrt{bx^2+cx^4}} + \frac{9bB-11Ac}{9b^2x^{3/2}\sqrt{bx^2+cx^4}} - \frac{7c^{3/2}(9bB-11Ac)x^{3/2}(b+cx^2)}{15b^4(\sqrt{b}+\sqrt{c}x)\sqrt{bx^2+cx^4}} - \frac{7(9bB-11Ac)\sqrt{bx^2+cx^4}}{45b^3x^{7/2}} +$$

$$\frac{7c(9bB-11Ac)\sqrt{bx^2+cx^4}}{15b^4x^{3/2}} + \frac{7c^{5/4}(9bB-11Ac)x(\sqrt{b}+\sqrt{c}x)\sqrt{\frac{b+cx^2}{(\sqrt{b}+\sqrt{c}x)^2}} \operatorname{EllipticE}\left[2\operatorname{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{15b^{15/4}\sqrt{bx^2+cx^4}} -$$

$$\frac{7c^{5/4}(9bB-11Ac)x(\sqrt{b}+\sqrt{c}x)\sqrt{\frac{b+cx^2}{(\sqrt{b}+\sqrt{c}x)^2}} \operatorname{EllipticF}\left[2\operatorname{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{30b^{15/4}\sqrt{bx^2+cx^4}}$$

Result (type 4, 259 leaves):

$$\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}} (9bBx^2(-2b^2 + 14bcx^2 + 21c^2x^4) - A(10b^3 - 22b^2cx^2 + 154bc^2x^4 + 231c^3x^6)) - \right. \\ \left. 21\sqrt{b}c^{3/2}(9bB - 11Ac)x^5\sqrt{1 + \frac{cx^2}{b}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right], -1\right] + \right. \\ \left. 21\sqrt{b}c^{3/2}(9bB - 11Ac)x^5\sqrt{1 + \frac{cx^2}{b}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right], -1\right] \right) / \left(45b^4x^{7/2}\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\sqrt{x^2(b + cx^2)} \right)$$

Problem 276: Result more than twice size of optimal antiderivative.

$$\int (ex)^m (c + dx^n)^q (ax^j + bx^{j+n})^p dx$$

Optimal (type 6, 113 leaves, 4 steps):

$$\frac{1}{1+m+jp} x (ex)^m \left(1 + \frac{bx^n}{a}\right)^{-p} (c + dx^n)^q \left(1 + \frac{dx^n}{c}\right)^{-q} (ax^j + bx^{j+n})^p \text{AppellF1}\left[\frac{1+m+jp}{n}, -p, -q, \frac{1+m+n+jp}{n}, -\frac{bx^n}{a}, -\frac{dx^n}{c}\right]$$

Result (type 6, 267 leaves):

$$\left(ac(1+m+n+jp)x(ex)^m(x^j(a+bx^n))^p(c+dx^n)^q \text{AppellF1}\left[\frac{1+m+jp}{n}, -p, -q, \frac{1+m+n+jp}{n}, -\frac{bx^n}{a}, -\frac{dx^n}{c}\right] \right) / \\ \left((1+m+jp) \left(ac(1+m+n+jp) \text{AppellF1}\left[\frac{1+m+jp}{n}, -p, -q, \frac{1+m+n+jp}{n}, -\frac{bx^n}{a}, -\frac{dx^n}{c}\right] + \right. \right. \\ \left. \left. nx^n \left(bc^p \text{AppellF1}\left[\frac{1+m+n+jp}{n}, 1-p, -q, \frac{1+m+2n+jp}{n}, -\frac{bx^n}{a}, -\frac{dx^n}{c}\right] + \right. \right. \right. \\ \left. \left. \left. adq \text{AppellF1}\left[\frac{1+m+n+jp}{n}, -p, 1-q, \frac{1+m+2n+jp}{n}, -\frac{bx^n}{a}, -\frac{dx^n}{c}\right] \right) \right) \right)$$

Problem 277: Result more than twice size of optimal antiderivative.

$$\int (ex)^{7/4} (c + dx^n)^q (ax^j + bx^{j+n})^{5/3} dx$$

Optimal (type 6, 129 leaves, 4 steps):

$$\frac{1}{(33+20j)\left(1 + \frac{bx^n}{a}\right)^{2/3}} 12aex^{2+j}(ex)^{3/4}(c+dx^n)^q \left(1 + \frac{dx^n}{c}\right)^{-q} (ax^j + bx^{j+n})^{2/3} \text{AppellF1}\left[\frac{33+20j}{12n}, -\frac{5}{3}, -q, \frac{33+20j+12n}{12n}, -\frac{bx^n}{a}, -\frac{dx^n}{c}\right]$$

Result (type 6, 580 leaves):

$$\frac{1}{33 + 20j + 12n}$$

$$12 a c x^{1+j} (e x)^{7/4} (x^j (a + b x^n))^{2/3} (c + d x^n)^q \left(\left(a (33 + 20j + 12n)^2 \text{AppellF1} \left[\frac{33 + 20j}{12n}, -\frac{2}{3}, -q, \frac{\frac{11}{4} + \frac{5j}{3} + n}{n}, -\frac{b x^n}{a}, -\frac{d x^n}{c} \right] \right) / \left((33 + 20j) \right. \right. \\ \left. \left. \left(a c (33 + 20j + 12n) \text{AppellF1} \left[\frac{33 + 20j}{12n}, -\frac{2}{3}, -q, \frac{33 + 20j + 12n}{12n}, -\frac{b x^n}{a}, -\frac{d x^n}{c} \right] + 4 n x^n \left(3 a d q \text{AppellF1} \left[\frac{33 + 20j + 12n}{12n}, -\frac{2}{3}, 1 - q, \right. \right. \right. \right. \right. \\ \left. \left. \left. \frac{33 + 20j + 24n}{12n}, -\frac{b x^n}{a}, -\frac{d x^n}{c} \right] + 2 b c \text{AppellF1} \left[\frac{33 + 20j + 12n}{12n}, \frac{1}{3}, -q, \frac{33 + 20j + 24n}{12n}, -\frac{b x^n}{a}, -\frac{d x^n}{c} \right] \right) \right) \right) + \\ \left(b (33 + 20j + 24n) x^n \text{AppellF1} \left[\frac{33 + 20j + 12n}{12n}, -\frac{2}{3}, -q, \frac{33 + 20j + 24n}{12n}, -\frac{b x^n}{a}, -\frac{d x^n}{c} \right] \right) / \\ \left(a c (33 + 20j + 24n) \text{AppellF1} \left[\frac{33 + 20j + 12n}{12n}, -\frac{2}{3}, -q, \frac{33 + 20j + 24n}{12n}, -\frac{b x^n}{a}, -\frac{d x^n}{c} \right] + \right. \\ \left. 4 n x^n \left(3 a d q \text{AppellF1} \left[\frac{33 + 20j + 24n}{12n}, -\frac{2}{3}, 1 - q, \frac{33 + 20j + 36n}{12n}, -\frac{b x^n}{a}, -\frac{d x^n}{c} \right] + \right. \right. \\ \left. \left. 2 b c \text{AppellF1} \left[\frac{33 + 20j + 24n}{12n}, \frac{1}{3}, -q, \frac{33 + 20j + 36n}{12n}, -\frac{b x^n}{a}, -\frac{d x^n}{c} \right] \right) \right) \right)$$

Problem 282: Unable to integrate problem.

$$\int \frac{a x^m + b x^n}{c x^m + d x^n} dx$$

Optimal (type 5, 54 leaves, 4 steps):

$$\frac{a x}{c} + \frac{(b c - a d) x \text{Hypergeometric2F1} \left[1, \frac{1}{m-n}, 1 + \frac{1}{m-n}, -\frac{c x^{m-n}}{d} \right]}{c d}$$

Result (type 8, 27 leaves):

$$\int \frac{a x^m + b x^n}{c x^m + d x^n} dx$$

Problem 284: Unable to integrate problem.

$$\int \frac{\left(a + \frac{b}{x} \right)^n x^m}{c + d x} dx$$

Optimal (type 6, 64 leaves, 4 steps):

$$\frac{\left(a + \frac{b}{x}\right)^n \left(1 + \frac{b}{ax}\right)^{-n} x^m \operatorname{AppellF1}\left[-m, -n, 1, 1-m, -\frac{b}{ax}, -\frac{c}{dx}\right]}{d m}$$

Result (type 8, 22 leaves):

$$\int \frac{\left(a + \frac{b}{x}\right)^n x^m}{c + dx} dx$$

Problem 285: Unable to integrate problem.

$$\int \frac{\left(a + \frac{b}{x}\right)^n x^2}{c + dx} dx$$

Optimal (type 5, 195 leaves, 7 steps):

$$\begin{aligned} & -\frac{(2ac + bd(1-n))\left(a + \frac{b}{x}\right)^{1+n} x}{2a^2 d^2} + \frac{\left(a + \frac{b}{x}\right)^{1+n} x^2}{2ad} - \frac{c^3 \left(a + \frac{b}{x}\right)^{1+n} \operatorname{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{c\left(a + \frac{b}{x}\right)}{ac-bd}\right]}{d^3 (ac-bd)(1+n)} + \\ & \frac{(2a^2 c^2 - 2abcdn - b^2 d^2(1-n)n)\left(a + \frac{b}{x}\right)^{1+n} \operatorname{Hypergeometric2F1}\left[1, 1+n, 2+n, 1 + \frac{b}{ax}\right]}{2a^3 d^3 (1+n)} \end{aligned}$$

Result (type 8, 22 leaves):

$$\int \frac{\left(a + \frac{b}{x}\right)^n x^2}{c + dx} dx$$

Problem 286: Unable to integrate problem.

$$\int \frac{\left(a + \frac{b}{x}\right)^n x}{c + dx} dx$$

Optimal (type 5, 131 leaves, 6 steps):

$$\frac{\left(a + \frac{b}{x}\right)^{1+n} x}{ad} + \frac{c^2 \left(a + \frac{b}{x}\right)^{1+n} \operatorname{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{c\left(a + \frac{b}{x}\right)}{ac-bd}\right]}{d^2 (ac-bd)(1+n)} - \frac{(ac-bdn)\left(a + \frac{b}{x}\right)^{1+n} \operatorname{Hypergeometric2F1}\left[1, 1+n, 2+n, 1 + \frac{b}{ax}\right]}{a^2 d^2 (1+n)}$$

Result (type 8, 20 leaves):

$$\int \frac{\left(a + \frac{b}{x}\right)^n x}{c + d x} dx$$

Problem 287: Unable to integrate problem.

$$\int \frac{\left(a + \frac{b}{x}\right)^n}{c + d x} dx$$

Optimal (type 5, 101 leaves, 5 steps):

$$-\frac{c \left(a + \frac{b}{x}\right)^{1+n} \text{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{c \left(a + \frac{b}{x}\right)}{a c - b d}\right]}{d (a c - b d) (1+n)} + \frac{\left(a + \frac{b}{x}\right)^{1+n} \text{Hypergeometric2F1}\left[1, 1+n, 2+n, 1 + \frac{b}{a x}\right]}{a d (1+n)}$$

Result (type 8, 19 leaves):

$$\int \frac{\left(a + \frac{b}{x}\right)^n}{c + d x} dx$$

Problem 288: Unable to integrate problem.

$$\int \frac{\left(a + \frac{b}{x}\right)^n}{x (c + d x)} dx$$

Optimal (type 5, 54 leaves, 3 steps):

$$\frac{\left(a + \frac{b}{x}\right)^{1+n} \text{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{c \left(a + \frac{b}{x}\right)}{a c - b d}\right]}{(a c - b d) (1+n)}$$

Result (type 8, 22 leaves):

$$\int \frac{\left(a + \frac{b}{x}\right)^n}{x (c + d x)} dx$$

Problem 289: Unable to integrate problem.

$$\int \frac{\left(a + \frac{b}{x}\right)^n}{x^2 (c + d x)} dx$$

Optimal (type 5, 84 leaves, 4 steps):

$$\frac{\left(a + \frac{b}{x}\right)^{1+n}}{b c (1+n)} - \frac{d \left(a + \frac{b}{x}\right)^{1+n} \text{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{c \left(a + \frac{b}{x}\right)}{a c - b d}\right]}{c (a c - b d) (1+n)}$$

Result (type 8, 22 leaves):

$$\int \frac{\left(a + \frac{b}{x}\right)^n}{x^2 (c + d x)} dx$$

Problem 290: Unable to integrate problem.

$$\int \frac{\left(a + \frac{b}{x}\right)^n}{x^3 (c + d x)} dx$$

Optimal (type 5, 115 leaves, 5 steps):

$$\frac{(a c + b d) \left(a + \frac{b}{x}\right)^{1+n}}{b^2 c^2 (1+n)} - \frac{\left(a + \frac{b}{x}\right)^{2+n}}{b^2 c (2+n)} + \frac{d^2 \left(a + \frac{b}{x}\right)^{1+n} \text{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{c \left(a + \frac{b}{x}\right)}{a c - b d}\right]}{c^2 (a c - b d) (1+n)}$$

Result (type 8, 22 leaves):

$$\int \frac{\left(a + \frac{b}{x}\right)^n}{x^3 (c + d x)} dx$$

Problem 291: Unable to integrate problem.

$$\int \frac{\left(a + \frac{b}{x}\right)^n}{x^5 (c + d x)} dx$$

Optimal (type 5, 207 leaves, 5 steps):

$$\frac{(a c + b d) (a^2 c^2 + b^2 d^2) \left(a + \frac{b}{x}\right)^{1+n}}{b^4 c^4 (1+n)} - \frac{(3 a^2 c^2 + 2 a b c d + b^2 d^2) \left(a + \frac{b}{x}\right)^{2+n}}{b^4 c^3 (2+n)} +$$

$$\frac{(3 a c + b d) \left(a + \frac{b}{x}\right)^{3+n}}{b^4 c^2 (3+n)} - \frac{\left(a + \frac{b}{x}\right)^{4+n}}{b^4 c (4+n)} + \frac{d^4 \left(a + \frac{b}{x}\right)^{1+n} \text{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{c \left(a + \frac{b}{x}\right)}{a c - b d}\right]}{c^4 (a c - b d) (1+n)}$$

Result (type 8, 22 leaves):

$$\int \frac{\left(a + \frac{b}{x}\right)^n}{x^5 (c + d x)} dx$$

Problem 292: Unable to integrate problem.

$$\int \frac{\left(a + \frac{b}{x}\right)^n x^m}{(c + d x)^2} dx$$

Optimal (type 6, 73 leaves, 4 steps):

$$\frac{\left(a + \frac{b}{x}\right)^n \left(1 + \frac{b}{a x}\right)^{-n} x^{-1+m} \text{AppellF1}\left[1 - m, -n, 2, 2 - m, -\frac{b}{a x}, -\frac{c}{d x}\right]}{d^2 (1 - m)}$$

Result (type 8, 22 leaves):

$$\int \frac{\left(a + \frac{b}{x}\right)^n x^m}{(c + d x)^2} dx$$

Problem 293: Unable to integrate problem.

$$\int \frac{\left(a + \frac{b}{x}\right)^n x^2}{(c + d x)^2} dx$$

Optimal (type 5, 202 leaves, 7 steps):

$$\frac{c (2 a c - b d) \left(a + \frac{b}{x}\right)^{1+n}}{a d^2 (a c - b d) \left(d + \frac{c}{x}\right)} + \frac{\left(a + \frac{b}{x}\right)^{1+n} x}{a d \left(d + \frac{c}{x}\right)} + \frac{c^2 (2 a c - b d (2 - n)) \left(a + \frac{b}{x}\right)^{1+n} \text{Hypergeometric2F1}\left[1, 1 + n, 2 + n, \frac{c \left(a + \frac{b}{x}\right)}{a c - b d}\right]}{d^3 (a c - b d)^2 (1 + n)}$$

$$\frac{(2 a c - b d n) \left(a + \frac{b}{x}\right)^{1+n} \text{Hypergeometric2F1}\left[1, 1 + n, 2 + n, 1 + \frac{b}{a x}\right]}{a^2 d^3 (1 + n)}$$

Result (type 8, 22 leaves):

$$\int \frac{\left(a + \frac{b}{x}\right)^n x^2}{(c + d x)^2} dx$$

Problem 294: Unable to integrate problem.

$$\int \frac{\left(a + \frac{b}{x}\right)^n x}{(c + d x)^2} dx$$

Optimal (type 5, 150 leaves, 6 steps):

$$\frac{c \left(a + \frac{b}{x}\right)^{1+n}}{d (a c - b d) \left(d + \frac{c}{x}\right)} - \frac{c (a c - b d) (1 - n) \left(a + \frac{b}{x}\right)^{1+n} \text{Hypergeometric2F1}\left[1, 1 + n, 2 + n, \frac{c \left(a + \frac{b}{x}\right)}{a c - b d}\right]}{d^2 (a c - b d)^2 (1 + n)} + \frac{\left(a + \frac{b}{x}\right)^{1+n} \text{Hypergeometric2F1}\left[1, 1 + n, 2 + n, 1 + \frac{b}{a x}\right]}{a d^2 (1 + n)}$$

Result (type 8, 20 leaves):

$$\int \frac{\left(a + \frac{b}{x}\right)^n x}{(c + d x)^2} dx$$

Problem 295: Unable to integrate problem.

$$\int \frac{\left(a + \frac{b}{x}\right)^n}{(c + d x)^2} dx$$

Optimal (type 5, 56 leaves, 3 steps):

$$- \frac{b \left(a + \frac{b}{x}\right)^{1+n} \text{Hypergeometric2F1}\left[2, 1 + n, 2 + n, \frac{c \left(a + \frac{b}{x}\right)}{a c - b d}\right]}{(a c - b d)^2 (1 + n)}$$

Result (type 8, 19 leaves):

$$\int \frac{\left(a + \frac{b}{x}\right)^n}{(c + d x)^2} dx$$

Problem 296: Unable to integrate problem.

$$\int \frac{\left(a + \frac{b}{x}\right)^n}{x (c + d x)^2} dx$$

Optimal (type 5, 105 leaves, 4 steps):

$$-\frac{d \left(a + \frac{b}{x}\right)^{1+n}}{c (a c - b d) \left(d + \frac{c}{x}\right)} + \frac{(a c - b d (1+n)) \left(a + \frac{b}{x}\right)^{1+n} \text{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{c \left(a + \frac{b}{x}\right)}{a c - b d}\right]}{c (a c - b d)^2 (1+n)}$$

Result (type 8, 22 leaves):

$$\int \frac{\left(a + \frac{b}{x}\right)^n}{x (c + d x)^2} dx$$

Problem 297: Unable to integrate problem.

$$\int \frac{\left(a + \frac{b}{x}\right)^n}{x^2 (c + d x)^2} dx$$

Optimal (type 5, 133 leaves, 5 steps):

$$-\frac{\left(a + \frac{b}{x}\right)^{1+n}}{b c^2 (1+n)} + \frac{d^2 \left(a + \frac{b}{x}\right)^{1+n}}{c^2 (a c - b d) \left(d + \frac{c}{x}\right)} - \frac{d (2 a c - b d (2+n)) \left(a + \frac{b}{x}\right)^{1+n} \text{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{c \left(a + \frac{b}{x}\right)}{a c - b d}\right]}{c^2 (a c - b d)^2 (1+n)}$$

Result (type 8, 22 leaves):

$$\int \frac{\left(a + \frac{b}{x}\right)^n}{x^2 (c + d x)^2} dx$$

Problem 298: Unable to integrate problem.

$$\int \frac{\left(a + \frac{b}{x}\right)^n}{x^3 (c + d x)^2} dx$$

Optimal (type 5, 217 leaves, 5 steps):

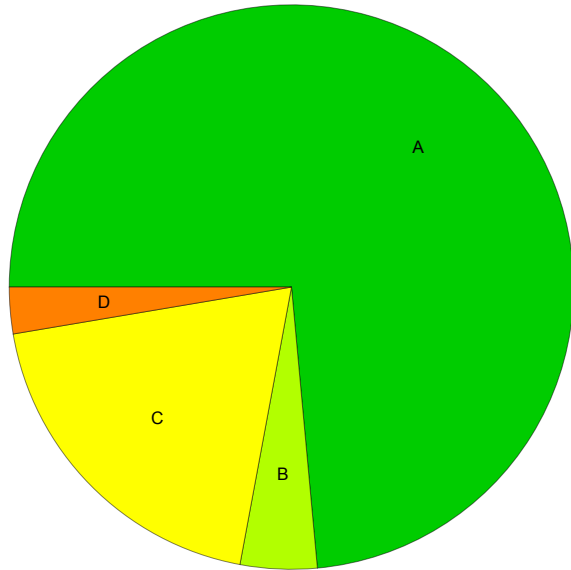
$$-\frac{\left(a + \frac{b}{x}\right)^{1+n} \left(d (b d (2+n) (a c + b d (3+n)) - a c (a c + b d (5+3n))) - \frac{c (a c - b d) (a c + b d (3+n))}{x}\right)}{b^2 c^3 (a c - b d) (1+n) (2+n) \left(d + \frac{c}{x}\right)} - \frac{\left(a + \frac{b}{x}\right)^{1+n}}{b c (2+n) \left(d + \frac{c}{x}\right) x^2} + \frac{d^2 (3 a c - b d (3+n)) \left(a + \frac{b}{x}\right)^{1+n} \text{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{c \left(a + \frac{b}{x}\right)}{a c - b d}\right]}{c^3 (a c - b d)^2 (1+n)}$$

Result (type 8, 22 leaves):

$$\int \frac{\left(a + \frac{b}{x}\right)^n}{x^3 (c + dx)^2} dx$$

Summary of Integration Test Results

752 integration problems



A - 553 optimal antiderivatives

B - 33 more than twice size of optimal antiderivatives

C - 146 unnecessarily complex antiderivatives

D - 20 unable to integrate problems

E - 0 integration timeouts