

Mathematica 11.3 Integration Test Results

on the problems in the test-suite directory "5 Inverse trig functions\5.4 Inverse cotangent"

Test results for the 234 problems in "5.4.1 Inverse cotangent functions.m"

Problem 46: Result more than twice size of optimal antiderivative.

$$\int \frac{x^2 \operatorname{ArcCot}[c x]}{1+x^2} dx$$

Optimal (type 4, 206 leaves, 28 steps):

$$x \operatorname{ArcCot}[c x] - \frac{1}{2} i \operatorname{ArcTan}[x] \operatorname{Log}\left[1 - \frac{i}{c x}\right] + \frac{1}{2} i \operatorname{ArcTan}[x] \operatorname{Log}\left[1 + \frac{i}{c x}\right] + \frac{1}{2} i \operatorname{ArcTan}[x] \operatorname{Log}\left[-\frac{2 i (i - c x)}{(1 - c)(1 - i x)}\right] -$$
$$\frac{1}{2} i \operatorname{ArcTan}[x] \operatorname{Log}\left[-\frac{2 i (i + c x)}{(1 + c)(1 - i x)}\right] + \frac{\operatorname{Log}[1 + c^2 x^2]}{2 c} + \frac{1}{4} \operatorname{PolyLog}\left[2, 1 + \frac{2 i (i - c x)}{(1 - c)(1 - i x)}\right] - \frac{1}{4} \operatorname{PolyLog}\left[2, 1 + \frac{2 i (i + c x)}{(1 + c)(1 - i x)}\right]$$

Result (type 4, 626 leaves):

$$\frac{1}{c} \left(c x \operatorname{ArcCot}[c x] - \operatorname{Log}\left[\frac{1}{c \sqrt{1 + \frac{1}{c^2 x^2}}}\right] + \right. \\
\left. \frac{1}{4} \sqrt{-c^2} \left(2 \operatorname{ArcCos}\left[\frac{1+c^2}{-1+c^2}\right] \operatorname{ArcTanh}\left[\frac{\sqrt{-c^2}}{c x}\right] - 4 \operatorname{ArcCot}[c x] \operatorname{ArcTanh}\left[\frac{c x}{\sqrt{-c^2}}\right] - \left(\operatorname{ArcCos}\left[\frac{1+c^2}{-1+c^2}\right] - 2 i \operatorname{ArcTanh}\left[\frac{\sqrt{-c^2}}{c x}\right] \right) \right. \right. \\
\left. \left. \operatorname{Log}\left[-\frac{2(c^2 + i \sqrt{-c^2})(-i + c x)}{(-1+c^2)(\sqrt{-c^2} - c x)}\right] - \left(\operatorname{ArcCos}\left[\frac{1+c^2}{-1+c^2}\right] + 2 i \operatorname{ArcTanh}\left[\frac{\sqrt{-c^2}}{c x}\right] \right) \operatorname{Log}\left[\frac{2 i(i c^2 + \sqrt{-c^2})(i + c x)}{(-1+c^2)(\sqrt{-c^2} - c x)}\right] + \right. \right. \\
\left. \left. \left(\operatorname{ArcCos}\left[\frac{1+c^2}{-1+c^2}\right] - 2 i \operatorname{ArcTanh}\left[\frac{\sqrt{-c^2}}{c x}\right] + 2 i \operatorname{ArcTanh}\left[\frac{c x}{\sqrt{-c^2}}\right] \right) \operatorname{Log}\left[\frac{\sqrt{2} \sqrt{-c^2} e^{-i \operatorname{ArcCot}[c x]}}{\sqrt{-1+c^2} \sqrt{-1-c^2} + (-1+c^2) \operatorname{Cos}[2 \operatorname{ArcCot}[c x]]}\right] + \right. \right. \\
\left. \left. \left(\operatorname{ArcCos}\left[\frac{1+c^2}{-1+c^2}\right] + 2 i \operatorname{ArcTanh}\left[\frac{\sqrt{-c^2}}{c x}\right] - 2 i \operatorname{ArcTanh}\left[\frac{c x}{\sqrt{-c^2}}\right] \right) \operatorname{Log}\left[\frac{\sqrt{2} \sqrt{-c^2} e^{i \operatorname{ArcCot}[c x]}}{\sqrt{-1+c^2} \sqrt{-1-c^2} + (-1+c^2) \operatorname{Cos}[2 \operatorname{ArcCot}[c x]]}\right] + \right. \right. \\
\left. \left. i \left(-\operatorname{PolyLog}\left[2, \frac{(1+c^2 - 2 i \sqrt{-c^2})(\sqrt{-c^2} + c x)}{(-1+c^2)(\sqrt{-c^2} - c x)}\right] + \operatorname{PolyLog}\left[2, \frac{(1+c^2 + 2 i \sqrt{-c^2})(\sqrt{-c^2} + c x)}{(-1+c^2)(\sqrt{-c^2} - c x)}\right] \right) \right) \right)$$

Problem 48: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{ArcCot}[c x]}{1+x^2} dx$$

Optimal (type 4, 183 leaves, 25 steps):

$$\frac{1}{2} i \operatorname{ArcTan}[x] \operatorname{Log}\left[1 - \frac{i}{c x}\right] - \frac{1}{2} i \operatorname{ArcTan}[x] \operatorname{Log}\left[1 + \frac{i}{c x}\right] - \frac{1}{2} i \operatorname{ArcTan}[x] \operatorname{Log}\left[-\frac{2 i(i - c x)}{(1-c)(1-i x)}\right] + \\
\frac{1}{2} i \operatorname{ArcTan}[x] \operatorname{Log}\left[-\frac{2 i(i + c x)}{(1+c)(1-i x)}\right] - \frac{1}{4} \operatorname{PolyLog}\left[2, 1 + \frac{2 i(i - c x)}{(1-c)(1-i x)}\right] + \frac{1}{4} \operatorname{PolyLog}\left[2, 1 + \frac{2 i(i + c x)}{(1+c)(1-i x)}\right]$$

Result (type 4, 592 leaves):

$$\begin{aligned}
& \frac{1}{4\sqrt{-c^2}} c \left(2 \operatorname{ArcCos}\left[\frac{1+c^2}{-1+c^2}\right] \operatorname{ArcTanh}\left[\frac{\sqrt{-c^2}}{c x}\right] - 4 \operatorname{ArcCot}[c x] \operatorname{ArcTanh}\left[\frac{c x}{\sqrt{-c^2}}\right] - \left(\operatorname{ArcCos}\left[\frac{1+c^2}{-1+c^2}\right] - 2 i \operatorname{ArcTanh}\left[\frac{\sqrt{-c^2}}{c x}\right] \right) \right. \\
& \quad \left. \operatorname{Log}\left[-\frac{2(c^2+i\sqrt{-c^2})(-i+cx)}{(-1+c^2)(\sqrt{-c^2}-cx)}\right] - \left(\operatorname{ArcCos}\left[\frac{1+c^2}{-1+c^2}\right] + 2 i \operatorname{ArcTanh}\left[\frac{\sqrt{-c^2}}{c x}\right] \right) \operatorname{Log}\left[\frac{2i(i c^2+\sqrt{-c^2})(i+cx)}{(-1+c^2)(\sqrt{-c^2}-cx)}\right] + \right. \\
& \quad \left(\operatorname{ArcCos}\left[\frac{1+c^2}{-1+c^2}\right] - 2 i \operatorname{ArcTanh}\left[\frac{\sqrt{-c^2}}{c x}\right] + 2 i \operatorname{ArcTanh}\left[\frac{c x}{\sqrt{-c^2}}\right] \right) \operatorname{Log}\left[\frac{\sqrt{2}\sqrt{-c^2}e^{-i \operatorname{ArcCot}[c x]}}{\sqrt{-1+c^2}\sqrt{-1-c^2+(-1+c^2)\cos[2 \operatorname{ArcCot}[c x]]}}\right] + \\
& \quad \left(\operatorname{ArcCos}\left[\frac{1+c^2}{-1+c^2}\right] + 2 i \operatorname{ArcTanh}\left[\frac{\sqrt{-c^2}}{c x}\right] - 2 i \operatorname{ArcTanh}\left[\frac{c x}{\sqrt{-c^2}}\right] \right) \operatorname{Log}\left[\frac{\sqrt{2}\sqrt{-c^2}e^{i \operatorname{ArcCot}[c x]}}{\sqrt{-1+c^2}\sqrt{-1-c^2+(-1+c^2)\cos[2 \operatorname{ArcCot}[c x]]}}\right] + \\
& \quad \left. i \left(-\operatorname{PolyLog}\left[2, \frac{(1+c^2-2i\sqrt{-c^2})(\sqrt{-c^2}+cx)}{(-1+c^2)(\sqrt{-c^2}-cx)}\right] + \operatorname{PolyLog}\left[2, \frac{(1+c^2+2i\sqrt{-c^2})(\sqrt{-c^2}+cx)}{(-1+c^2)(\sqrt{-c^2}-cx)}\right] \right) \right)
\end{aligned}$$

Problem 50: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{ArcCot}[c x]}{x^2(1+x^2)} dx$$

Optimal (type 4, 212 leaves, 31 steps):

$$\begin{aligned}
& -\frac{\operatorname{ArcCot}[c x]}{x} - \frac{1}{2} i \operatorname{ArcTan}[x] \operatorname{Log}\left[1 - \frac{i}{c x}\right] + \frac{1}{2} i \operatorname{ArcTan}[x] \operatorname{Log}\left[1 + \frac{i}{c x}\right] - c \operatorname{Log}[x] + \frac{1}{2} i \operatorname{ArcTan}[x] \operatorname{Log}\left[-\frac{2i(i-cx)}{(1-c)(1-ix)}\right] - \\
& \frac{1}{2} i \operatorname{ArcTan}[x] \operatorname{Log}\left[-\frac{2i(i+cx)}{(1+c)(1-ix)}\right] + \frac{1}{2} c \operatorname{Log}[1+c^2 x^2] + \frac{1}{4} \operatorname{PolyLog}\left[2, 1 + \frac{2i(i-cx)}{(1-c)(1-ix)}\right] - \frac{1}{4} \operatorname{PolyLog}\left[2, 1 + \frac{2i(i+cx)}{(1+c)(1-ix)}\right]
\end{aligned}$$

Result (type 4, 619 leaves):

$$\begin{aligned}
& -\frac{\text{ArcCot}[c x]}{x} - c \text{Log}\left[\frac{1}{\sqrt{1 + \frac{1}{c^2 x^2}}}\right] - \\
& \frac{1}{4\sqrt{-c^2}} c \left(2 \text{ArcCos}\left[\frac{1+c^2}{-1+c^2}\right] \text{ArcTanh}\left[\frac{\sqrt{-c^2}}{c x}\right] - 4 \text{ArcCot}[c x] \text{ArcTanh}\left[\frac{c x}{\sqrt{-c^2}}\right] - \left(\text{ArcCos}\left[\frac{1+c^2}{-1+c^2}\right] - 2 i \text{ArcTanh}\left[\frac{\sqrt{-c^2}}{c x}\right] \right) \right. \\
& \left. \text{Log}\left[-\frac{2(c^2 + i\sqrt{-c^2})(-i + c x)}{(-1+c^2)(\sqrt{-c^2} - c x)}\right] - \left(\text{ArcCos}\left[\frac{1+c^2}{-1+c^2}\right] + 2 i \text{ArcTanh}\left[\frac{\sqrt{-c^2}}{c x}\right] \right) \text{Log}\left[\frac{2 i(i c^2 + \sqrt{-c^2})(i + c x)}{(-1+c^2)(\sqrt{-c^2} - c x)}\right] + \right. \\
& \left. \left(\text{ArcCos}\left[\frac{1+c^2}{-1+c^2}\right] - 2 i \text{ArcTanh}\left[\frac{\sqrt{-c^2}}{c x}\right] + 2 i \text{ArcTanh}\left[\frac{c x}{\sqrt{-c^2}}\right] \right) \text{Log}\left[\frac{\sqrt{2}\sqrt{-c^2} e^{-i \text{ArcCot}[c x]}}{\sqrt{-1+c^2}\sqrt{-1-c^2} + (-1+c^2)\text{Cos}[2 \text{ArcCot}[c x]]}\right] + \right. \\
& \left. \left(\text{ArcCos}\left[\frac{1+c^2}{-1+c^2}\right] + 2 i \text{ArcTanh}\left[\frac{\sqrt{-c^2}}{c x}\right] - 2 i \text{ArcTanh}\left[\frac{c x}{\sqrt{-c^2}}\right] \right) \text{Log}\left[\frac{\sqrt{2}\sqrt{-c^2} e^{i \text{ArcCot}[c x]}}{\sqrt{-1+c^2}\sqrt{-1-c^2} + (-1+c^2)\text{Cos}[2 \text{ArcCot}[c x]]}\right] + \right. \\
& \left. i \left(-\text{PolyLog}\left[2, \frac{(1+c^2 - 2 i\sqrt{-c^2})(\sqrt{-c^2} + c x)}{(-1+c^2)(\sqrt{-c^2} - c x)}\right] + \text{PolyLog}\left[2, \frac{(1+c^2 + 2 i\sqrt{-c^2})(\sqrt{-c^2} + c x)}{(-1+c^2)(\sqrt{-c^2} - c x)}\right] \right) \right)
\end{aligned}$$

Problem 61: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcCot}[a x]}{(c + d x^2)^{3/2}} dx$$

Optimal (type 3, 66 leaves, 5 steps):

$$\frac{x \text{ArcCot}[a x]}{c \sqrt{c + d x^2}} - \frac{\text{ArcTanh}\left[\frac{a \sqrt{c + d x^2}}{\sqrt{a^2 c - d}}\right]}{c \sqrt{a^2 c - d}}$$

Result (type 3, 169 leaves):

$$\frac{2 x \text{ArcCot}[a x]}{\sqrt{c + d x^2}} + \frac{-\text{Log}\left[\frac{4 a c (a c - i d x + \sqrt{a^2 c - d} \sqrt{c + d x^2})}{\sqrt{a^2 c - d} (i + a x)}\right] - \text{Log}\left[\frac{4 a c (a c + i d x + \sqrt{a^2 c - d} \sqrt{c + d x^2})}{\sqrt{a^2 c - d} (-i + a x)}\right]}{2 c}$$

Problem 62: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{ArcCot}[a x]}{(c + d x^2)^{5/2}} dx$$

Optimal (type 3, 134 leaves, 7 steps):

$$\frac{a}{3 c (a^2 c - d) \sqrt{c + d x^2}} + \frac{x \text{ArcCot}[a x]}{3 c (c + d x^2)^{3/2}} + \frac{2 x \text{ArcCot}[a x]}{3 c^2 \sqrt{c + d x^2}} - \frac{(3 a^2 c - 2 d) \text{ArcTanh}\left[\frac{a \sqrt{c + d x^2}}{\sqrt{a^2 c - d}}\right]}{3 c^2 (a^2 c - d)^{3/2}}$$

Result (type 3, 262 leaves):

$$-\frac{1}{6 c^2} \left(-\frac{2 a c}{(a^2 c - d) \sqrt{c + d x^2}} - \frac{2 x (3 c + 2 d x^2) \text{ArcCot}[a x]}{(c + d x^2)^{3/2}} + \frac{(3 a^2 c - 2 d) \text{Log}\left[\frac{12 a c^2 \sqrt{a^2 c - d} (a c - i d x + \sqrt{a^2 c - d} \sqrt{c + d x^2})}{(3 a^2 c - 2 d) (i + a x)}\right]}{(a^2 c - d)^{3/2}} + \frac{(3 a^2 c - 2 d) \text{Log}\left[\frac{12 a c^2 \sqrt{a^2 c - d} (a c + i d x + \sqrt{a^2 c - d} \sqrt{c + d x^2})}{(3 a^2 c - 2 d) (-i + a x)}\right]}{(a^2 c - d)^{3/2}} \right)$$

Problem 63: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{ArcCot}[a x]}{(c + d x^2)^{7/2}} dx$$

Optimal (type 3, 208 leaves, 8 steps):

$$\frac{a}{15 c (a^2 c - d) (c + d x^2)^{3/2}} + \frac{a (7 a^2 c - 4 d)}{15 c^2 (a^2 c - d)^2 \sqrt{c + d x^2}} + \frac{x \text{ArcCot}[a x]}{5 c (c + d x^2)^{5/2}} + \frac{4 x \text{ArcCot}[a x]}{15 c^2 (c + d x^2)^{3/2}} + \frac{8 x \text{ArcCot}[a x]}{15 c^3 \sqrt{c + d x^2}} - \frac{(15 a^4 c^2 - 20 a^2 c d + 8 d^2) \text{ArcTanh}\left[\frac{a \sqrt{c + d x^2}}{\sqrt{a^2 c - d}}\right]}{15 c^3 (a^2 c - d)^{5/2}}$$

Result (type 3, 345 leaves):

$$\begin{aligned}
& -\frac{1}{30c^3} \left(\frac{2ac(-d(5c+4dx^2) + a^2c(8c+7dx^2))}{(-a^2c+d)^2(c+dx^2)^{3/2}} - \frac{2x(15c^2+20cdx^2+8d^2x^4)\text{ArcCot}[ax]}{(c+dx^2)^{5/2}} + \right. \\
& \left. \frac{(15a^4c^2-20a^2cd+8d^2)\text{Log}\left[\frac{60ac^3(a^2c-d)^{3/2}(ac-idx+\sqrt{a^2c-d}\sqrt{c+dx^2})}{(15a^4c^2-20a^2cd+8d^2)(i+ax)}\right]}{(a^2c-d)^{5/2}} + \frac{(15a^4c^2-20a^2cd+8d^2)\text{Log}\left[\frac{60ac^3(a^2c-d)^{3/2}(ac+idx+\sqrt{a^2c-d}\sqrt{c+dx^2})}{(15a^4c^2-20a^2cd+8d^2)(-i+ax)}\right]}{(a^2c-d)^{5/2}} \right)
\end{aligned}$$

Problem 64: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{ArcCot}[ax]}{(c+dx^2)^{9/2}} dx$$

Optimal (type 3, 293 leaves, 8 steps):

$$\begin{aligned}
& \frac{a}{35c(a^2c-d)(c+dx^2)^{5/2}} + \frac{a(11a^2c-6d)}{105c^2(a^2c-d)^2(c+dx^2)^{3/2}} + \frac{a(19a^4c^2-22a^2cd+8d^2)}{35c^3(a^2c-d)^3\sqrt{c+dx^2}} + \frac{x\text{ArcCot}[ax]}{7c(c+dx^2)^{7/2}} + \\
& \frac{6x\text{ArcCot}[ax]}{35c^2(c+dx^2)^{5/2}} + \frac{8x\text{ArcCot}[ax]}{35c^3(c+dx^2)^{3/2}} + \frac{16x\text{ArcCot}[ax]}{35c^4\sqrt{c+dx^2}} - \frac{(35a^6c^3-70a^4c^2d+56a^2cd^2-16d^3)\text{ArcTanh}\left[\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c-d}}\right]}{35c^4(a^2c-d)^{7/2}}
\end{aligned}$$

Result (type 3, 450 leaves):

$$\begin{aligned}
& \frac{1}{210c^4} \left(\frac{2ac(3c^2(-a^2c+d)^2+c(11a^2c-6d)(a^2c-d)(c+dx^2)+3(19a^4c^2-22a^2cd+8d^2)(c+dx^2)^2)}{(a^2c-d)^3(c+dx^2)^{5/2}} + \right. \\
& \frac{6x(35c^3+70c^2dx^2+56cd^2x^4+16d^3x^6)\text{ArcCot}[ax]}{(c+dx^2)^{7/2}} - \frac{3(35a^6c^3-70a^4c^2d+56a^2cd^2-16d^3)\text{Log}\left[\frac{140ac^4(a^2c-d)^{5/2}(ac-idx+\sqrt{a^2c-d}\sqrt{c+dx^2})}{(35a^6c^3-70a^4c^2d+56a^2cd^2-16d^3)(i+ax)}\right]}{(a^2c-d)^{7/2}} \\
& \left. \frac{3(35a^6c^3-70a^4c^2d+56a^2cd^2-16d^3)\text{Log}\left[\frac{140ac^4(a^2c-d)^{5/2}(ac+idx+\sqrt{a^2c-d}\sqrt{c+dx^2})}{(35a^6c^3-70a^4c^2d+56a^2cd^2-16d^3)(-i+ax)}\right]}{(a^2c-d)^{7/2}} \right)
\end{aligned}$$

Problem 97: Result unnecessarily involves higher level functions.

$$\int \frac{\text{ArcCot}[a x^n]}{x} dx$$

Optimal (type 4, 47 leaves, 4 steps):

$$-\frac{i \text{PolyLog}\left[2, -\frac{i x^n}{a}\right]}{2 n} + \frac{i \text{PolyLog}\left[2, \frac{i x^n}{a}\right]}{2 n}$$

Result (type 5, 52 leaves):

$$-\frac{a x^n \text{HypergeometricPFQ}\left[\left\{\frac{1}{2}, \frac{1}{2}, 1\right\}, \left\{\frac{3}{2}, \frac{3}{2}\right\}, -a^2 x^{2 n}\right]}{n} + (\text{ArcCot}[a x^n] + \text{ArcTan}[a x^n]) \text{Log}[x]$$

Problem 103: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcCot}[a + b x]}{x} dx$$

Optimal (type 4, 120 leaves, 5 steps):

$$-\text{ArcCot}[a + b x] \text{Log}\left[\frac{2}{1 - i(a + b x)}\right] + \text{ArcCot}[a + b x] \text{Log}\left[\frac{2 b x}{(i - a)(1 - i(a + b x))}\right] -$$

$$\frac{1}{2} i \text{PolyLog}\left[2, 1 - \frac{2}{1 - i(a + b x)}\right] + \frac{1}{2} i \text{PolyLog}\left[2, 1 - \frac{2 b x}{(i - a)(1 - i(a + b x))}\right]$$

Result (type 4, 256 leaves):

$$(\text{ArcCot}[a + b x] + \text{ArcTan}[a + b x]) \text{Log}[x] + \text{ArcTan}[a + b x] \left(\text{Log}\left[\frac{1}{\sqrt{1 + (a + b x)^2}}\right] - \text{Log}[-\text{Sin}[\text{ArcTan}[a] - \text{ArcTan}[a + b x]]] \right) +$$

$$\frac{1}{2} \left(\frac{1}{4} i (\pi - 2 \text{ArcTan}[a + b x])^2 + i (\text{ArcTan}[a] - \text{ArcTan}[a + b x])^2 - \right.$$

$$(\pi - 2 \text{ArcTan}[a + b x]) \text{Log}\left[1 + e^{-2 i \text{ArcTan}[a + b x]}\right] + 2 (\text{ArcTan}[a] - \text{ArcTan}[a + b x]) \text{Log}\left[1 - e^{2 i (-\text{ArcTan}[a] + \text{ArcTan}[a + b x])}\right] +$$

$$(\pi - 2 \text{ArcTan}[a + b x]) \text{Log}\left[\frac{2}{\sqrt{1 + (a + b x)^2}}\right] + 2 (-\text{ArcTan}[a] + \text{ArcTan}[a + b x]) \text{Log}[-2 \text{Sin}[\text{ArcTan}[a] - \text{ArcTan}[a + b x]]] +$$

$$\left. i \text{PolyLog}\left[2, -e^{-2 i \text{ArcTan}[a + b x]}\right] + i \text{PolyLog}\left[2, e^{2 i (-\text{ArcTan}[a] + \text{ArcTan}[a + b x])}\right] \right)$$

Problem 107: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcCot}[a + b x]}{c + d x^2} dx$$

Optimal (type 4, 642 leaves, 15 steps):

$$\begin{aligned} & - \frac{\text{Log}\left[\frac{i+a+bx}{a+bx}\right] \text{Log}\left[-\frac{b(i\sqrt{c}-\sqrt{d}x)}{(b\sqrt{c}+(1-i)a\sqrt{d})(a+bx)}\right]}{4\sqrt{c}\sqrt{d}} + \frac{\text{Log}\left[-\frac{i-a-bx}{a+bx}\right] \text{Log}\left[\frac{ib(\sqrt{c}+i\sqrt{d}x)}{(b\sqrt{c}-(1+i)a\sqrt{d})(a+bx)}\right]}{4\sqrt{c}\sqrt{d}} - \\ & \frac{\text{Log}\left[-\frac{i-a-bx}{a+bx}\right] \text{Log}\left[\frac{b(i\sqrt{c}+\sqrt{d}x)}{(b\sqrt{c}+(1+i)a\sqrt{d})(a+bx)}\right]}{4\sqrt{c}\sqrt{d}} + \frac{\text{Log}\left[\frac{i+a+bx}{a+bx}\right] \text{Log}\left[-\frac{b(i\sqrt{c}+\sqrt{d}x)}{(b\sqrt{c}+i(i+a)\sqrt{d})(a+bx)}\right]}{4\sqrt{c}\sqrt{d}} + \frac{\text{PolyLog}\left[2, -\frac{(b\sqrt{c}-ia\sqrt{d})(i-a-bx)}{(b\sqrt{c}-(1+i)a\sqrt{d})(a+bx)}\right]}{4\sqrt{c}\sqrt{d}} - \\ & \frac{\text{PolyLog}\left[2, -\frac{(b\sqrt{c}+ia\sqrt{d})(i-a-bx)}{(b\sqrt{c}+(1+i)a\sqrt{d})(a+bx)}\right]}{4\sqrt{c}\sqrt{d}} - \frac{\text{PolyLog}\left[2, \frac{(b\sqrt{c}-ia\sqrt{d})(i+a+bx)}{(b\sqrt{c}+(1-i)a\sqrt{d})(a+bx)}\right]}{4\sqrt{c}\sqrt{d}} + \frac{\text{PolyLog}\left[2, \frac{(b\sqrt{c}+ia\sqrt{d})(i+a+bx)}{(b\sqrt{c}+i(i+a)\sqrt{d})(a+bx)}\right]}{4\sqrt{c}\sqrt{d}} \end{aligned}$$

Result (type 4, 1530 leaves):

$$\begin{aligned} & \frac{1}{4(1+a^2)\sqrt{c}d(a+bx)^2\left(1+\frac{1}{(a+bx)^2}\right)} \left(1+(a+bx)^2\right) \\ & \left(4(1+a^2)\sqrt{d}\text{ArcCot}[a+bx]\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right] + 2\sqrt{d}\text{ArcTan}\left[\frac{(-i+a)\sqrt{d}}{b\sqrt{c}}\right]\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right] + 2a^2\sqrt{d}\text{ArcTan}\left[\frac{(-i+a)\sqrt{d}}{b\sqrt{c}}\right]\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right] - \right. \\ & 2\sqrt{d}\text{ArcTan}\left[\frac{(i+a)\sqrt{d}}{b\sqrt{c}}\right]\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right] - 2a^2\sqrt{d}\text{ArcTan}\left[\frac{(i+a)\sqrt{d}}{b\sqrt{c}}\right]\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right] + 2b\sqrt{c}\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right]^2 - \\ & b\sqrt{c}\sqrt{\frac{b^2c+(-i+a)^2d}{b^2c}}e^{-i\text{ArcTan}\left[\frac{(-i+a)\sqrt{d}}{b\sqrt{c}}\right]}\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right]^2 + ia b\sqrt{c}\sqrt{\frac{b^2c+(-i+a)^2d}{b^2c}}e^{-i\text{ArcTan}\left[\frac{(-i+a)\sqrt{d}}{b\sqrt{c}}\right]}\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right]^2 - \\ & b\sqrt{c}\sqrt{\frac{b^2c+(i+a)^2d}{b^2c}}e^{-i\text{ArcTan}\left[\frac{(i+a)\sqrt{d}}{b\sqrt{c}}\right]}\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right]^2 - ia b\sqrt{c}\sqrt{\frac{b^2c+(i+a)^2d}{b^2c}}e^{-i\text{ArcTan}\left[\frac{(i+a)\sqrt{d}}{b\sqrt{c}}\right]}\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right]^2 - 2i\sqrt{d} \\ & \left.\text{ArcTan}\left[\frac{(-i+a)\sqrt{d}}{b\sqrt{c}}\right]\text{Log}\left[1-e^{-2i\left(\text{ArcTan}\left[\frac{(-i+a)\sqrt{d}}{b\sqrt{c}}\right]+\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right]\right)}\right] - 2ia^2\sqrt{d}\text{ArcTan}\left[\frac{(-i+a)\sqrt{d}}{b\sqrt{c}}\right]\text{Log}\left[1-e^{-2i\left(\text{ArcTan}\left[\frac{(-i+a)\sqrt{d}}{b\sqrt{c}}\right]+\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right]\right)}\right] - \right. \end{aligned}$$

$$\begin{aligned}
& 2 i \sqrt{d} \operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right] \operatorname{Log}\left[1 - e^{-2 i \left(\operatorname{ArcTan}\left[\frac{(-i+a) \sqrt{d}}{b \sqrt{c}}\right] + \operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]\right)}\right] - 2 i a^2 \sqrt{d} \operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right] \operatorname{Log}\left[1 - e^{-2 i \left(\operatorname{ArcTan}\left[\frac{(-i+a) \sqrt{d}}{b \sqrt{c}}\right] + \operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]\right)}\right] + \\
& 2 i \sqrt{d} \operatorname{ArcTan}\left[\frac{(i+a) \sqrt{d}}{b \sqrt{c}}\right] \operatorname{Log}\left[1 - e^{-2 i \left(\operatorname{ArcTan}\left[\frac{(i+a) \sqrt{d}}{b \sqrt{c}}\right] + \operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]\right)}\right] + 2 i a^2 \sqrt{d} \operatorname{ArcTan}\left[\frac{(i+a) \sqrt{d}}{b \sqrt{c}}\right] \operatorname{Log}\left[1 - e^{-2 i \left(\operatorname{ArcTan}\left[\frac{(i+a) \sqrt{d}}{b \sqrt{c}}\right] + \operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]\right)}\right] + \\
& 2 i \sqrt{d} \operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right] \operatorname{Log}\left[1 - e^{-2 i \left(\operatorname{ArcTan}\left[\frac{(i+a) \sqrt{d}}{b \sqrt{c}}\right] + \operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]\right)}\right] + 2 i a^2 \sqrt{d} \operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right] \operatorname{Log}\left[1 - e^{-2 i \left(\operatorname{ArcTan}\left[\frac{(i+a) \sqrt{d}}{b \sqrt{c}}\right] + \operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]\right)}\right] + \\
& 2 i \sqrt{d} \operatorname{ArcTan}\left[\frac{(-i+a) \sqrt{d}}{b \sqrt{c}}\right] \operatorname{Log}\left[-\operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{(-i+a) \sqrt{d}}{b \sqrt{c}}\right] + \operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]\right]\right] + \\
& 2 i a^2 \sqrt{d} \operatorname{ArcTan}\left[\frac{(-i+a) \sqrt{d}}{b \sqrt{c}}\right] \operatorname{Log}\left[-\operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{(-i+a) \sqrt{d}}{b \sqrt{c}}\right] + \operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]\right]\right] - \\
& 2 i \sqrt{d} \operatorname{ArcTan}\left[\frac{(i+a) \sqrt{d}}{b \sqrt{c}}\right] \operatorname{Log}\left[-\operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{(i+a) \sqrt{d}}{b \sqrt{c}}\right] + \operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]\right]\right] - \\
& 2 i a^2 \sqrt{d} \operatorname{ArcTan}\left[\frac{(i+a) \sqrt{d}}{b \sqrt{c}}\right] \operatorname{Log}\left[-\operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{(i+a) \sqrt{d}}{b \sqrt{c}}\right] + \operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]\right]\right] + \\
& \left. \left((1+a^2) \sqrt{d} \operatorname{PolyLog}\left[2, e^{-2 i \left(\operatorname{ArcTan}\left[\frac{(-i+a) \sqrt{d}}{b \sqrt{c}}\right] + \operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]\right)}\right] - (1+a^2) \sqrt{d} \operatorname{PolyLog}\left[2, e^{-2 i \left(\operatorname{ArcTan}\left[\frac{(i+a) \sqrt{d}}{b \sqrt{c}}\right] + \operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]\right)}\right] \right)
\end{aligned}$$

Problem 108: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{ArcCot}[a + b x]}{c + d x} dx$$

Optimal (type 4, 152 leaves, 5 steps):

$$-\frac{\operatorname{ArcCot}[a + b x] \operatorname{Log}\left[\frac{2}{1-i(a+bx)}\right]}{d} + \frac{\operatorname{ArcCot}[a + b x] \operatorname{Log}\left[\frac{2b(c+dx)}{(bc+id-ad)(1-i(a+bx))}\right]}{d} - \frac{i \operatorname{PolyLog}\left[2, 1 - \frac{2}{1-i(a+bx)}\right]}{2d} + \frac{i \operatorname{PolyLog}\left[2, 1 - \frac{2b(c+dx)}{(bc+id-ad)(1-i(a+bx))}\right]}{2d}$$

Result (type 4, 325 leaves):

$$\frac{1}{d} \left((\text{ArcCot}[a + b x] + \text{ArcTan}[a + b x]) \text{Log}[c + d x] + \text{ArcTan}[a + b x] \left(\text{Log}\left[\frac{1}{\sqrt{1 + (a + b x)^2}}\right] - \text{Log}\left[\text{Sin}\left[\text{ArcTan}\left[\frac{b c - a d}{d}\right] + \text{ArcTan}[a + b x]\right]\right] \right) \right) +$$

$$\frac{1}{2} \left(\frac{1}{4} i (\pi - 2 \text{ArcTan}[a + b x])^2 + i \left(\text{ArcTan}\left[\frac{b c - a d}{d}\right] + \text{ArcTan}[a + b x] \right)^2 - (\pi - 2 \text{ArcTan}[a + b x]) \text{Log}\left[1 + e^{-2 i \text{ArcTan}[a + b x]}\right] - \right.$$

$$2 \left(\text{ArcTan}\left[\frac{b c - a d}{d}\right] + \text{ArcTan}[a + b x] \right) \text{Log}\left[1 - e^{2 i \left(\text{ArcTan}\left[\frac{b c - a d}{d}\right] + \text{ArcTan}[a + b x]\right)}\right] + (\pi - 2 \text{ArcTan}[a + b x]) \text{Log}\left[\frac{2}{\sqrt{1 + (a + b x)^2}}\right] +$$

$$2 \left(\text{ArcTan}\left[\frac{b c - a d}{d}\right] + \text{ArcTan}[a + b x] \right) \text{Log}\left[2 \text{Sin}\left[\text{ArcTan}\left[\frac{b c - a d}{d}\right] + \text{ArcTan}[a + b x]\right]\right] +$$

$$\left. i \text{PolyLog}\left[2, -e^{-2 i \text{ArcTan}[a + b x]}\right] + i \text{PolyLog}\left[2, e^{2 i \left(\text{ArcTan}\left[\frac{b c - a d}{d}\right] + \text{ArcTan}[a + b x]\right)}\right] \right)$$

Problem 110: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcCot}[a + b x]}{c + \frac{d}{x^2}} dx$$

Optimal (type 4, 735 leaves, 57 steps):

$$\frac{\text{Log}[i - a - b x]}{2 b c} + \frac{i (a + b x) \text{Log}\left[-\frac{i - a - b x}{a + b x}\right]}{2 b c} - \frac{i \sqrt{d} \text{ArcTan}\left[\frac{\sqrt{c} x}{\sqrt{d}}\right] \text{Log}\left[-\frac{i - a - b x}{a + b x}\right]}{2 c^{3/2}} + \frac{\text{Log}[i + a + b x]}{2 b c} - \frac{i (a + b x) \text{Log}\left[\frac{i + a + b x}{a + b x}\right]}{2 b c} +$$

$$\frac{i \sqrt{d} \text{ArcTan}\left[\frac{\sqrt{c} x}{\sqrt{d}}\right] \text{Log}\left[\frac{i + a + b x}{a + b x}\right]}{2 c^{3/2}} - \frac{\sqrt{d} \text{Log}\left[\frac{-\sqrt{c} (i - a - b x)}{(i - a) \sqrt{c} + i b \sqrt{d}}\right] \text{Log}\left[1 - \frac{i \sqrt{c} x}{\sqrt{d}}\right]}{4 c^{3/2}} + \frac{\sqrt{d} \text{Log}\left[\frac{\sqrt{c} (i + a + b x)}{(i + a) \sqrt{c} - i b \sqrt{d}}\right] \text{Log}\left[1 - \frac{i \sqrt{c} x}{\sqrt{d}}\right]}{4 c^{3/2}} +$$

$$\frac{\sqrt{d} \text{Log}\left[\frac{-\sqrt{c} (i - a - b x)}{(i - a) \sqrt{c} - i b \sqrt{d}}\right] \text{Log}\left[1 + \frac{i \sqrt{c} x}{\sqrt{d}}\right]}{4 c^{3/2}} - \frac{\sqrt{d} \text{Log}\left[\frac{\sqrt{c} (i + a + b x)}{(i + a) \sqrt{c} + i b \sqrt{d}}\right] \text{Log}\left[1 + \frac{i \sqrt{c} x}{\sqrt{d}}\right]}{4 c^{3/2}} - \frac{\sqrt{d} \text{PolyLog}\left[2, \frac{b (\sqrt{d} - i \sqrt{c} x)}{(1 + i a) \sqrt{c} + b \sqrt{d}}\right]}{4 c^{3/2}} +$$

$$\frac{\sqrt{d} \text{PolyLog}\left[2, \frac{b (\sqrt{d} - i \sqrt{c} x)}{i (i + a) \sqrt{c} + b \sqrt{d}}\right]}{4 c^{3/2}} + \frac{\sqrt{d} \text{PolyLog}\left[2, -\frac{b (\sqrt{d} + i \sqrt{c} x)}{(1 + i a) \sqrt{c} - b \sqrt{d}}\right]}{4 c^{3/2}} - \frac{\sqrt{d} \text{PolyLog}\left[2, \frac{b (\sqrt{d} + i \sqrt{c} x)}{(1 - i a) \sqrt{c} + b \sqrt{d}}\right]}{4 c^{3/2}}$$

Result (type 4, 16412 leaves):

$$\frac{1}{(a + b x)^2 \left(1 + \frac{1}{(a + b x)^2}\right)}$$

$$(1 + (a + bx)^2) \left(\frac{(a + bx) \operatorname{ArcCot}[a + bx] - \operatorname{Log}\left[\frac{1}{(a+bx)\sqrt{1+\frac{1}{(a+bx)^2}}}\right]}{bc} - \frac{1}{c} 2bd \left(-\frac{\operatorname{ArcCot}[a + bx] \operatorname{ArcTan}\left[\frac{-ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]}{2b\sqrt{c}\sqrt{d}} + \frac{1}{2(a^2c + b^2d)\left(1 + \frac{1}{(a+bx)^2}\right)} \right) \right)$$

$$\left(1 + \frac{c\left(a\sqrt{c} - b\sqrt{d}\left(\frac{a\sqrt{c}}{b\sqrt{d}} - \frac{a^2c + b^2d}{b\sqrt{c}\sqrt{d}(a+bx)}\right)\right)^2}{(a^2c + b^2d)^2} \right) \left(\frac{(a^2c + b^2d)^2 \operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]^2}{2(a^4c^2 + b^4d^2 + a^2c(c + 2b^2d))} - \frac{a^2c e^{\operatorname{ArcTanh}\left[\frac{-iac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]} \operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]^2}{2(-iac + a^2c + b^2d)\sqrt{1 - \frac{(-iac + a^2c + b^2d)^2}{b^2cd}}} \right) -$$

$$\frac{1}{(-iac + a^2c + b^2d)\sqrt{1 - \frac{(-iac + a^2c + b^2d)^2}{b^2cd}}} i a^3 c \left(e^{\operatorname{ArcTanh}\left[\frac{-iac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]} \operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]^2 - \right)$$

$$\frac{1}{b\sqrt{c}\sqrt{d}\sqrt{1 - \frac{(-iac + a^2c + b^2d)^2}{b^2cd}}} (-iac + a^2c + b^2d) \left(\pi \operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] - i\pi \operatorname{Log}\left[1 + e^{-2i \operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]}\right] \right) -$$

$$2i \operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] \operatorname{Log}\left[1 - e^{2\left(i \operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] + \operatorname{ArcTanh}\left[\frac{-iac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]\right)}\right] + i\pi \operatorname{Log}\left[\frac{1}{\sqrt{\frac{(a^2c + b^2d)\left(c + \frac{a^2c + b^2d}{(a+bx)^2} - \frac{2ac}{a+bx}\right)}{b^2cd}}}\right] +$$

$$2 \operatorname{ArcTanh}\left[\frac{-iac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] \left(i \operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] - \operatorname{Log}\left[1 - e^{2\left(i \operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] + \operatorname{ArcTanh}\left[\frac{-iac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]\right)}\right] \right) + \operatorname{Log}\left[\right]$$

$$\left. \left(\text{Sin}\left[\text{ArcTan}\left[\frac{ac - \frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] - i \text{ArcTanh}\left[\frac{-i ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]\right]\right) - \text{PolyLog}\left[2, e^{\left(i \text{ArcTan}\left[\frac{ac - \frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] + \text{ArcTanh}\left[\frac{-i ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]\right)}\right] \right) + \\
 \frac{1}{4(-i ac + a^2c + b^2d)\sqrt{1 - \frac{(-i ac + a^2c + b^2d)^2}{b^2cd}}} \left(3a^4c \left(e^{\text{ArcTanh}\left[\frac{-i ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]} \text{ArcTan}\left[\frac{ac - \frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right]^2 - \frac{1}{b\sqrt{c}\sqrt{d}\sqrt{1 - \frac{(-i ac + a^2c + b^2d)^2}{b^2cd}}} \right. \right. \\
 \left. \left. (-i ac + a^2c + b^2d) \left(\pi \text{ArcTan}\left[\frac{ac - \frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] - i \pi \text{Log}\left[1 + e^{-2i \text{ArcTan}\left[\frac{ac - \frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right]}\right] - \right. \right. \\
 \left. \left. 2i \text{ArcTan}\left[\frac{ac - \frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] \text{Log}\left[1 - e^{2\left(i \text{ArcTan}\left[\frac{ac - \frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] + \text{ArcTanh}\left[\frac{-i ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]\right)}\right] + i \pi \text{Log}\left[\frac{1}{\sqrt{\frac{(a^2c + b^2d)\left(c + \frac{a^2c - b^2d}{(a+bx)^2} - \frac{2ac}{a+bx}\right)}}{b^2cd}}\right] + \right. \right. \\
 \left. \left. 2 \text{ArcTanh}\left[\frac{-i ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] \left(i \text{ArcTan}\left[\frac{ac - \frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] - \text{Log}\left[1 - e^{2\left(i \text{ArcTan}\left[\frac{ac - \frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] + \text{ArcTanh}\left[\frac{-i ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]\right)}\right] + \text{Log}\left[\right. \right. \\
 \left. \left. \text{Sin}\left[\text{ArcTan}\left[\frac{ac - \frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] - i \text{ArcTanh}\left[\frac{-i ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]\right]\right) - \text{PolyLog}\left[2, e^{\left(i \text{ArcTan}\left[\frac{ac - \frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] + \text{ArcTanh}\left[\frac{-i ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]\right)}\right] \right) \right) -$$

$$\frac{1}{4 b^2 d (-i a c + a^2 c + b^2 d) \sqrt{1 - \frac{(-i a c + a^2 c + b^2 d)^2}{b^2 c d}}} a^4 c^2 \left(e^{\text{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]} \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] \right)^2 -$$

$$\frac{1}{b \sqrt{c} \sqrt{d} \sqrt{1 - \frac{(-i a c + a^2 c + b^2 d)^2}{b^2 c d}}} (-i a c + a^2 c + b^2 d) \left(\pi \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] - i \pi \text{Log}\left[1 + e^{-2 i \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]}\right] - \right.$$

$$\left. 2 i \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] \text{Log}\left[1 - e^{2 \left(i \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] + \text{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]\right)}\right] + i \pi \text{Log}\left[\frac{1}{\sqrt{\frac{(a^2 c + b^2 d) \left(c + \frac{a^2 c + b^2 d}{(a + b x)^2} - \frac{2 a c}{a + b x}\right)}{b^2 c d}}}\right] + \right.$$

$$\left. 2 \text{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] \left(i \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] - \text{Log}\left[1 - e^{2 \left(i \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] + \text{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]\right)}\right] + \text{Log}\left[\right.$$

$$\left. \left. \left. \left. \left. \left. \text{Sin}\left[\text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] - i \text{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]\right]\right] \right) - \text{PolyLog}\left[2, e^{2 \left(i \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] + \text{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]\right)}\right] \right) \right) \right) \right) \right) -$$

$$\frac{1}{2 b^2 d (-i a c + a^2 c + b^2 d) \sqrt{1 - \frac{(-i a c + a^2 c + b^2 d)^2}{b^2 c d}}} i a^5 c^2 \left(e^{\text{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]} \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] \right)^2 -$$

$$\begin{aligned}
 & \frac{1}{b\sqrt{c}\sqrt{d}\sqrt{1-\frac{(-i ac+a^2c+b^2d)^2}{b^2cd}}} (-i ac+a^2c+b^2d) \left(\pi \operatorname{ArcTan}\left[\frac{ac-\frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] - i\pi \operatorname{Log}\left[1+e^{-2i \operatorname{ArcTan}\left[\frac{ac-\frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right]}\right] - \right. \\
 & 2i \operatorname{ArcTan}\left[\frac{ac-\frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] \operatorname{Log}\left[1-e^{2\left(i \operatorname{ArcTan}\left[\frac{ac-\frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right]+\operatorname{ArcTanh}\left[\frac{-i ac+a^2c+b^2d}{b\sqrt{c}\sqrt{d}}\right]\right)}\right] + i\pi \operatorname{Log}\left[\frac{1}{\sqrt{\frac{(a^2c+b^2d)\left(c+\frac{a^2c+b^2d}{(a+bx)^2}-\frac{2ac}{a+bx}\right)}{b^2cd}}}\right] + \\
 & 2 \operatorname{ArcTanh}\left[\frac{-i ac+a^2c+b^2d}{b\sqrt{c}\sqrt{d}}\right] \left(i \operatorname{ArcTan}\left[\frac{ac-\frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] - \operatorname{Log}\left[1-e^{2\left(i \operatorname{ArcTan}\left[\frac{ac-\frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right]+\operatorname{ArcTanh}\left[\frac{-i ac+a^2c+b^2d}{b\sqrt{c}\sqrt{d}}\right]\right)}\right] + \operatorname{Log}\left[\right. \\
 & \left. \left. \operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{ac-\frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] - i \operatorname{ArcTanh}\left[\frac{-i ac+a^2c+b^2d}{b\sqrt{c}\sqrt{d}}\right]\right] \right] - \operatorname{PolyLog}\left[2, e^{2\left(i \operatorname{ArcTan}\left[\frac{ac-\frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right]+\operatorname{ArcTanh}\left[\frac{-i ac+a^2c+b^2d}{b\sqrt{c}\sqrt{d}}\right]\right)}\right] \right) \left. \right) + \\
 & \frac{1}{4b^2d(-i ac+a^2c+b^2d)\sqrt{1-\frac{(-i ac+a^2c+b^2d)^2}{b^2cd}}} a^6c^2 \left(e^{\operatorname{ArcTanh}\left[\frac{-i ac+a^2c+b^2d}{b\sqrt{c}\sqrt{d}}\right]} \operatorname{ArcTan}\left[\frac{ac-\frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right]^2 - \right. \\
 & \left. \frac{1}{b\sqrt{c}\sqrt{d}\sqrt{1-\frac{(-i ac+a^2c+b^2d)^2}{b^2cd}}} (-i ac+a^2c+b^2d) \left(\pi \operatorname{ArcTan}\left[\frac{ac-\frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] - i\pi \operatorname{Log}\left[1+e^{-2i \operatorname{ArcTan}\left[\frac{ac-\frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right]}\right] - \right. \right.
 \end{aligned}$$

$$\begin{aligned}
& 2 i \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] \operatorname{Log}\left[1 - e^{2\left(i \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] + \operatorname{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]\right)}\right] + i \pi \operatorname{Log}\left[\frac{1}{\sqrt{\frac{(a^2 c + b^2 d)\left(c + \frac{a^2 c + b^2 d}{(a + b x)^2} - \frac{2 a c}{a + b x}\right)}{b^2 c d}}}\right] + \\
& 2 \operatorname{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] \left(i \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] - \operatorname{Log}\left[1 - e^{2\left(i \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] + \operatorname{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]\right)}\right] + \operatorname{Log}\left[\right. \\
& \left. \operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] - i \operatorname{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]\right]\right] - \operatorname{PolyLog}\left[2, e^{2\left(i \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] + \operatorname{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]\right)}\right] - \left. \right) \\
& \frac{1}{4(-i a c + a^2 c + b^2 d) \sqrt{1 - \frac{(-i a c + a^2 c + b^2 d)^2}{b^2 c d}}} b^2 d \left(e^{\operatorname{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]} \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]^2 - \frac{1}{b \sqrt{c} \sqrt{d} \sqrt{1 - \frac{(-i a c + a^2 c + b^2 d)^2}{b^2 c d}}} \right. \\
& \left. (-i a c + a^2 c + b^2 d) \left(\pi \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] - i \pi \operatorname{Log}\left[1 + e^{-2 i \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]}\right] - \right. \right. \\
& \left. \left. 2 i \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] \operatorname{Log}\left[1 - e^{2\left(i \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] + \operatorname{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]\right)}\right] + i \pi \operatorname{Log}\left[\frac{1}{\sqrt{\frac{(a^2 c + b^2 d)\left(c + \frac{a^2 c + b^2 d}{(a + b x)^2} - \frac{2 a c}{a + b x}\right)}{b^2 c d}}}\right] + \right. \\
& \left. 2 \operatorname{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] \left(i \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] - \operatorname{Log}\left[1 - e^{2\left(i \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] + \operatorname{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]\right)}\right] + \operatorname{Log}\left[\right. \right.
\end{aligned}$$

$$\left. \begin{aligned} & \left(\operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{ac - \frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] - i \operatorname{ArcTanh}\left[\frac{-i ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]\right] \right) - \operatorname{PolyLog}\left[2, e^{\left(i \operatorname{ArcTan}\left[\frac{ac - \frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] + \operatorname{ArcTanh}\left[\frac{-i ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] \right)} \right] \right) \\ & \frac{1}{2(-i ac + a^2c + b^2d) \sqrt{1 - \frac{(-i ac + a^2c + b^2d)^2}{b^2cd}}} i a b^2 d \left(e^{\operatorname{ArcTanh}\left[\frac{-i ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]} \operatorname{ArcTan}\left[\frac{ac - \frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right]^2 - \right. \\ & \frac{1}{b\sqrt{c}\sqrt{d} \sqrt{1 - \frac{(-i ac + a^2c + b^2d)^2}{b^2cd}}} (-i ac + a^2c + b^2d) \left(\pi \operatorname{ArcTan}\left[\frac{ac - \frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] - i \pi \operatorname{Log}\left[1 + e^{-2i \operatorname{ArcTan}\left[\frac{ac - \frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right]} \right] - \right. \\ & \left. 2i \operatorname{ArcTan}\left[\frac{ac - \frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] \operatorname{Log}\left[1 - e^{2\left(i \operatorname{ArcTan}\left[\frac{ac - \frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] + \operatorname{ArcTanh}\left[\frac{-i ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] \right)} \right] + i \pi \operatorname{Log}\left[\frac{1}{\sqrt{\frac{(a^2c+b^2d)\left(c + \frac{a^2c+b^2d}{(a+bx)^2} - \frac{2ac}{a+bx}\right)}}{b^2cd}} \right] + \right. \\ & \left. 2 \operatorname{ArcTanh}\left[\frac{-i ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] \left(i \operatorname{ArcTan}\left[\frac{ac - \frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] - \operatorname{Log}\left[1 - e^{2\left(i \operatorname{ArcTan}\left[\frac{ac - \frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] + \operatorname{ArcTanh}\left[\frac{-i ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] \right)} \right] \right) + \operatorname{Log}\left[\right. \\ & \left. \left. \operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{ac - \frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] - i \operatorname{ArcTanh}\left[\frac{-i ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]\right] \right) - \operatorname{PolyLog}\left[2, e^{\left(i \operatorname{ArcTan}\left[\frac{ac - \frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] + \operatorname{ArcTanh}\left[\frac{-i ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] \right)} \right] \right) \right) + \end{aligned}$$

$$\begin{aligned}
& \frac{1}{4(-i a c + a^2 c + b^2 d) \sqrt{1 - \frac{(-i a c + a^2 c + b^2 d)^2}{b^2 c d}}} - 3 a^2 b^2 d \left(e^{\operatorname{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]} \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]^2 - \right. \\
& \frac{1}{b \sqrt{c} \sqrt{d} \sqrt{1 - \frac{(-i a c + a^2 c + b^2 d)^2}{b^2 c d}}} (-i a c + a^2 c + b^2 d) \left(\pi \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] - i \pi \operatorname{Log}\left[1 + e^{-2 i \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]}\right] - \right. \\
& 2 i \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] \operatorname{Log}\left[1 - e^{2 \left(i \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] + \operatorname{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]\right)}\right] + i \pi \operatorname{Log}\left[\frac{1}{\sqrt{\frac{(a^2 c + b^2 d) \left(c + \frac{a^2 c + b^2 d}{(a + b x)^2} - \frac{2 a c}{a + b x}\right)}{b^2 c d}}}\right] + \\
& 2 \operatorname{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] \left(i \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] - \operatorname{Log}\left[1 - e^{2 \left(i \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] + \operatorname{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]\right)}\right] + \operatorname{Log}\left[\right. \\
& \left. \left. \left. \operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] - i \operatorname{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]\right]\right] - \operatorname{PolyLog}\left[2, e^{2 \left(i \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] + \operatorname{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]\right)}\right] \right) \right) + \\
& \frac{1}{4 c (-i a c + a^2 c + b^2 d) \sqrt{1 - \frac{(-i a c + a^2 c + b^2 d)^2}{b^2 c d}}} b^4 d^2 \left(e^{\operatorname{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]} \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]^2 - \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{b\sqrt{c}\sqrt{d}\sqrt{1-\frac{(-i ac+a^2c+b^2d)^2}{b^2cd}}} (-i ac+a^2c+b^2d) \left(\pi \operatorname{ArcTan}\left[\frac{ac-\frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] - i\pi \operatorname{Log}\left[1+e^{-2i \operatorname{ArcTan}\left[\frac{ac-\frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right]}\right] - \right. \\
& 2i \operatorname{ArcTan}\left[\frac{ac-\frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] \operatorname{Log}\left[1-e^{2\left(i \operatorname{ArcTan}\left[\frac{ac-\frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right]+\operatorname{ArcTanh}\left[\frac{-i ac+a^2c+b^2d}{b\sqrt{c}\sqrt{d}}\right]\right)}\right] + i\pi \operatorname{Log}\left[\frac{1}{\sqrt{\frac{(a^2c+b^2d)\left(c+\frac{a^2c+b^2d}{(a+bx)^2}-\frac{2ac}{a+bx}\right)}{b^2cd}}}\right] + \\
& 2 \operatorname{ArcTanh}\left[\frac{-i ac+a^2c+b^2d}{b\sqrt{c}\sqrt{d}}\right] \left(i \operatorname{ArcTan}\left[\frac{ac-\frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] - \operatorname{Log}\left[1-e^{2\left(i \operatorname{ArcTan}\left[\frac{ac-\frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right]+\operatorname{ArcTanh}\left[\frac{-i ac+a^2c+b^2d}{b\sqrt{c}\sqrt{d}}\right]\right)}\right] \right) + \operatorname{Log}\left[\right. \\
& \left. \left. \operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{ac-\frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] - i \operatorname{ArcTanh}\left[\frac{-i ac+a^2c+b^2d}{b\sqrt{c}\sqrt{d}}\right]\right] \right] - \operatorname{PolyLog}\left[2, e^{2\left(i \operatorname{ArcTan}\left[\frac{ac-\frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right]+\operatorname{ArcTanh}\left[\frac{-i ac+a^2c+b^2d}{b\sqrt{c}\sqrt{d}}\right]\right)}\right] \right) \left. \right) + \\
& \frac{1}{2b\sqrt{d}\left(1-\frac{(-i ac+a^2c+b^2d)^2}{b^2cd}\right)} a^2\sqrt{c} \left(\pi \operatorname{ArcTan}\left[\frac{ac-\frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] - i\pi \operatorname{Log}\left[1+e^{-2i \operatorname{ArcTan}\left[\frac{ac-\frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right]}\right] - 2i \operatorname{ArcTan}\left[\frac{ac-\frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] \right. \\
& \left. \operatorname{Log}\left[1-e^{2\left(i \operatorname{ArcTan}\left[\frac{ac-\frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right]+\operatorname{ArcTanh}\left[\frac{-i ac+a^2c+b^2d}{b\sqrt{c}\sqrt{d}}\right]\right)}\right] + i\pi \operatorname{Log}\left[\frac{1}{\sqrt{\frac{(a^2c+b^2d)\left(c+\frac{a^2c+b^2d}{(a+bx)^2}-\frac{2ac}{a+bx}\right)}{b^2cd}}}\right] + \right. \\
& \left. 2 \operatorname{ArcTanh}\left[\frac{-i ac+a^2c+b^2d}{b\sqrt{c}\sqrt{d}}\right] \left(i \operatorname{ArcTan}\left[\frac{ac-\frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] - \operatorname{Log}\left[1-e^{2\left(i \operatorname{ArcTan}\left[\frac{ac-\frac{a^2c+b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right]+\operatorname{ArcTanh}\left[\frac{-i ac+a^2c+b^2d}{b\sqrt{c}\sqrt{d}}\right]\right)}\right] \right) + \right.
\end{aligned}$$

$$\left. \begin{aligned}
 & \text{Log} \left[\text{Sin} \left[\text{ArcTan} \left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] - i \text{ArcTanh} \left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \right] \right] - \text{PolyLog} \left[2, e^{2 \left(i \text{ArcTan} \left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] + \text{ArcTanh} \left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \right)} \right] \right] - \\
 & \frac{1}{2 (i a c + a^2 c + b^2 d) \sqrt{-\frac{-b^2 c d + (i a c + a^2 c + b^2 d)^2}{b^2 c d}}} a^2 c \left(e^{-\text{ArcTanh} \left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right]} \text{ArcTan} \left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right]^2 + \frac{1}{b \sqrt{c} \sqrt{d} \sqrt{1 - \frac{(i a c + a^2 c + b^2 d)^2}{b^2 c d}}} \right) \\
 & i (i a c + a^2 c + b^2 d) \left(i \text{ArcTan} \left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] \left(-\pi + 2 i \text{ArcTanh} \left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \right) - \pi \text{Log} \left[1 + e^{-2 i \text{ArcTan} \left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right]} \right] - \right. \\
 & 2 \left(\text{ArcTan} \left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] + i \text{ArcTanh} \left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \right) \text{Log} \left[1 - e^{2 i \left(\text{ArcTan} \left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] + i \text{ArcTanh} \left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \right)} \right] \right] + \\
 & \pi \text{Log} \left[\frac{1}{\sqrt{\frac{(a^2 c + b^2 d) \left(c + \frac{a^2 c + b^2 d}{(a + b x)^2} \frac{2 a c}{a + b x} \right)}{b^2 c d}}} \right] + 2 i \text{ArcTanh} \left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \\
 & \left. \left. \left. \text{Log} \left[\text{Sin} \left[\text{ArcTan} \left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] + i \text{ArcTanh} \left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \right] \right] + i \text{PolyLog} \left[2, e^{2 i \left(\text{ArcTan} \left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] + i \text{ArcTanh} \left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \right)} \right] \right] \right] + \right.
 \end{aligned} \right)$$

$$\begin{aligned}
 & \frac{1}{(i a c + a^2 c + b^2 d) \sqrt{-\frac{-b^2 c d + (i a c + a^2 c + b^2 d)^2}{b^2 c d}}} i a^3 c \left(e^{-\text{ArcTanh}\left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]} \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]^2 + \frac{1}{b \sqrt{c} \sqrt{d} \sqrt{1 - \frac{(i a c + a^2 c + b^2 d)^2}{b^2 c d}}} \right. \\
 & \left. i (i a c + a^2 c + b^2 d) \left(i \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] \left(-\pi + 2 i \text{ArcTanh}\left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] \right) - \pi \text{Log}\left[1 + e^{-2 i \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]}\right] - \right. \right. \\
 & \left. \left. 2 \left(\text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] + i \text{ArcTanh}\left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] \right) \text{Log}\left[1 - e^{2 i \left(\text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] + i \text{ArcTanh}\left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] \right)}\right] + \right. \right. \\
 & \left. \left. \pi \text{Log}\left[\frac{1}{\sqrt{\frac{(a^2 c + b^2 d) \left(c + \frac{a^2 c + b^2 d}{(a + b x)^2} - \frac{2 a c}{a + b x} \right)}{b^2 c d}}}\right] + 2 i \text{ArcTanh}\left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] \right. \right. \\
 & \left. \left. \text{Log}\left[\text{Sin}\left[\text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] + i \text{ArcTanh}\left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]\right]\right] + i \text{PolyLog}\left[2, e^{2 i \left(\text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] + i \text{ArcTanh}\left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] \right)}\right] \right) \right) + \\
 & \frac{1}{4 (i a c + a^2 c + b^2 d) \sqrt{-\frac{-b^2 c d + (i a c + a^2 c + b^2 d)^2}{b^2 c d}}} 3 a^4 c \left(e^{-\text{ArcTanh}\left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]} \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]^2 + \right.
 \end{aligned}$$

$$\begin{aligned}
& \frac{1}{b\sqrt{c}\sqrt{d}\sqrt{1-\frac{(i a c+a^2 c+b^2 d)^2}{b^2 c d}}} i (i a c+a^2 c+b^2 d) \left(i \operatorname{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b\sqrt{c}\sqrt{d}}\right] \left(-\pi+2 i \operatorname{ArcTanh}\left[\frac{i a c+a^2 c+b^2 d}{b\sqrt{c}\sqrt{d}}\right]\right) - \right. \\
& \pi \operatorname{Log}\left[1+e^{-2 i \operatorname{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b\sqrt{c}\sqrt{d}}\right]}\right] - 2 \left(\operatorname{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b\sqrt{c}\sqrt{d}}\right] + i \operatorname{ArcTanh}\left[\frac{i a c+a^2 c+b^2 d}{b\sqrt{c}\sqrt{d}}\right] \right) \\
& \operatorname{Log}\left[1-e^{2 i \left(\operatorname{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b\sqrt{c}\sqrt{d}}\right]+i \operatorname{ArcTanh}\left[\frac{i a c+a^2 c+b^2 d}{b\sqrt{c}\sqrt{d}}\right]\right)}\right] + \pi \operatorname{Log}\left[\frac{1}{\sqrt{\frac{(a^2 c+b^2 d)\left(c+\frac{a^2 c+b^2 d}{(a+b x)^2}-\frac{2 a c}{a+b x}\right)}{b^2 c d}}}\right] + 2 i \operatorname{ArcTanh}\left[\frac{i a c+a^2 c+b^2 d}{b\sqrt{c}\sqrt{d}}\right] \\
& \left. \operatorname{Log}\left[\operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b\sqrt{c}\sqrt{d}}\right]+i \operatorname{ArcTanh}\left[\frac{i a c+a^2 c+b^2 d}{b\sqrt{c}\sqrt{d}}\right]\right]\right] + i \operatorname{PolyLog}\left[2, e^{2 i \left(\operatorname{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b\sqrt{c}\sqrt{d}}\right]+i \operatorname{ArcTanh}\left[\frac{i a c+a^2 c+b^2 d}{b\sqrt{c}\sqrt{d}}\right]\right)}\right] \right) - \left. \right) \\
& \frac{1}{4 b^2 d (i a c+a^2 c+b^2 d) \sqrt{-\frac{-b^2 c d+(i a c+a^2 c+b^2 d)^2}{b^2 c d}}} a^4 c^2 \left(e^{-\operatorname{ArcTanh}\left[\frac{i a c+a^2 c+b^2 d}{b\sqrt{c}\sqrt{d}}\right]} \operatorname{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b\sqrt{c}\sqrt{d}}\right]^2 + \right. \\
& \left. \frac{1}{b\sqrt{c}\sqrt{d}\sqrt{1-\frac{(i a c+a^2 c+b^2 d)^2}{b^2 c d}}} i (i a c+a^2 c+b^2 d) \left(i \operatorname{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b\sqrt{c}\sqrt{d}}\right] \left(-\pi+2 i \operatorname{ArcTanh}\left[\frac{i a c+a^2 c+b^2 d}{b\sqrt{c}\sqrt{d}}\right]\right) - \right. \right.
\end{aligned}$$

$$\pi \operatorname{Log}\left[1 + e^{-2i \operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]}\right] - 2 \left(\operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] + i \operatorname{ArcTanh}\left[\frac{i ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] \right)$$

$$\operatorname{Log}\left[1 - e^{2i \left(\operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] + i \operatorname{ArcTanh}\left[\frac{i ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] \right)}\right] + \pi \operatorname{Log}\left[\frac{1}{\sqrt{\frac{(a^2c + b^2d)\left(c + \frac{a^2c + b^2d}{(a-bx)^2} - \frac{2ac}{a-bx}\right)}{b^2cd}}}\right] + 2i \operatorname{ArcTanh}\left[\frac{i ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]$$

$$\left. \operatorname{Log}\left[\operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] + i \operatorname{ArcTanh}\left[\frac{i ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]\right]\right] + i \operatorname{PolyLog}\left[2, e^{2i \left(\operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] + i \operatorname{ArcTanh}\left[\frac{i ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] \right)}\right] \right) +$$

$$\frac{1}{2b^2d(i ac + a^2c + b^2d)\sqrt{-\frac{-b^2cd + (i ac + a^2c + b^2d)^2}{b^2cd}}} i a^5 c^2 \left(e^{-\operatorname{ArcTanh}\left[\frac{i ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]} \operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]^2 + \right.$$

$$\left. \frac{1}{b\sqrt{c}\sqrt{d}\sqrt{1 - \frac{(i ac + a^2c + b^2d)^2}{b^2cd}}} i (i ac + a^2c + b^2d) \left(i \operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] \left(-\pi + 2i \operatorname{ArcTanh}\left[\frac{i ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] \right) - \right.$$

$$\pi \operatorname{Log}\left[1 + e^{-2i \operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]}\right] - 2 \left(\operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] + i \operatorname{ArcTanh}\left[\frac{i ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] \right)$$

$$\operatorname{Log}\left[1 - e^{2i \left(\operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] + i \operatorname{ArcTanh}\left[\frac{i ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] \right)}\right] + \pi \operatorname{Log}\left[\frac{1}{\sqrt{\frac{(a^2c + b^2d)\left(c + \frac{a^2c + b^2d}{(a-bx)^2} - \frac{2ac}{a-bx}\right)}{b^2cd}}}\right] + 2i \operatorname{ArcTanh}\left[\frac{i ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]$$

$$\begin{aligned}
 & \frac{1}{4 (i a c + a^2 c + b^2 d) \sqrt{-\frac{-b^2 c d + (i a c + a^2 c + b^2 d)^2}{b^2 c d}}} b^2 d \left(e^{-\text{ArcTanh}\left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]} \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]^2 + \frac{1}{b \sqrt{c} \sqrt{d} \sqrt{1 - \frac{(i a c + a^2 c + b^2 d)^2}{b^2 c d}}} \right. \\
 & \left. i (i a c + a^2 c + b^2 d) \left(i \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] \left(-\pi + 2 i \text{ArcTanh}\left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] \right) - \pi \text{Log}\left[1 + e^{-2 i \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]}\right] - \right. \right. \\
 & \left. \left. 2 \left(\text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] + i \text{ArcTanh}\left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] \right) \text{Log}\left[1 - e^{2 i \left(\text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] + i \text{ArcTanh}\left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] \right)}\right] + \right. \right. \\
 & \left. \left. \pi \text{Log}\left[\frac{1}{\sqrt{\frac{(a^2 c + b^2 d) \left(c + \frac{a^2 c + b^2 d}{(a + b x)^2} - \frac{2 a c}{a + b x} \right)}{b^2 c d}}}\right] + 2 i \text{ArcTanh}\left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] \right. \right. \\
 & \left. \left. \text{Log}\left[\text{Sin}\left[\text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] + i \text{ArcTanh}\left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]\right]\right] + i \text{PolyLog}\left[2, e^{2 i \left(\text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] + i \text{ArcTanh}\left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] \right)}\right] \right) \right) + \\
 & \frac{1}{2 (i a c + a^2 c + b^2 d) \sqrt{-\frac{-b^2 c d + (i a c + a^2 c + b^2 d)^2}{b^2 c d}}} i a b^2 d \left(e^{-\text{ArcTanh}\left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]} \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]^2 + \right.
 \end{aligned}$$

$$\begin{aligned}
& \frac{1}{b\sqrt{c}\sqrt{d}\sqrt{1-\frac{(i a c+a^2 c+b^2 d)^2}{b^2 c d}}} i (i a c+a^2 c+b^2 d) \left(i \operatorname{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b\sqrt{c}\sqrt{d}}\right] \left(-\pi+2 i \operatorname{ArcTanh}\left[\frac{i a c+a^2 c+b^2 d}{b\sqrt{c}\sqrt{d}}\right]\right) - \right. \\
& \pi \operatorname{Log}\left[1+e^{-2 i \operatorname{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b\sqrt{c}\sqrt{d}}\right]}\right] - 2 \left(\operatorname{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b\sqrt{c}\sqrt{d}}\right] + i \operatorname{ArcTanh}\left[\frac{i a c+a^2 c+b^2 d}{b\sqrt{c}\sqrt{d}}\right] \right) \\
& \operatorname{Log}\left[1-e^{2 i \left(\operatorname{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b\sqrt{c}\sqrt{d}}\right]+i \operatorname{ArcTanh}\left[\frac{i a c+a^2 c+b^2 d}{b\sqrt{c}\sqrt{d}}\right]\right)}\right] + \pi \operatorname{Log}\left[\frac{1}{\sqrt{\frac{(a^2 c+b^2 d)\left(c+\frac{a^2 c+b^2 d}{(a+b x)^2}-\frac{2 a c}{a+b x}\right)}{b^2 c d}}}\right] + 2 i \operatorname{ArcTanh}\left[\frac{i a c+a^2 c+b^2 d}{b\sqrt{c}\sqrt{d}}\right] \\
& \left. \operatorname{Log}\left[\operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b\sqrt{c}\sqrt{d}}\right]+i \operatorname{ArcTanh}\left[\frac{i a c+a^2 c+b^2 d}{b\sqrt{c}\sqrt{d}}\right]\right]\right] + i \operatorname{PolyLog}\left[2, e^{2 i \left(\operatorname{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b\sqrt{c}\sqrt{d}}\right]+i \operatorname{ArcTanh}\left[\frac{i a c+a^2 c+b^2 d}{b\sqrt{c}\sqrt{d}}\right]\right)}\right] \right) + \\
& \frac{1}{4(i a c+a^2 c+b^2 d)\sqrt{-\frac{-b^2 c d+(i a c+a^2 c+b^2 d)^2}{b^2 c d}}} 3 a^2 b^2 d \left(e^{-\operatorname{ArcTanh}\left[\frac{i a c+a^2 c+b^2 d}{b\sqrt{c}\sqrt{d}}\right]} \operatorname{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b\sqrt{c}\sqrt{d}}\right]^2 + \right. \\
& \left. \frac{1}{b\sqrt{c}\sqrt{d}\sqrt{1-\frac{(i a c+a^2 c+b^2 d)^2}{b^2 c d}}} i (i a c+a^2 c+b^2 d) \left(i \operatorname{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b\sqrt{c}\sqrt{d}}\right] \left(-\pi+2 i \operatorname{ArcTanh}\left[\frac{i a c+a^2 c+b^2 d}{b\sqrt{c}\sqrt{d}}\right]\right) - \right.
\end{aligned}$$

$$\pi \operatorname{Log}\left[1 + e^{-2i \operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]}\right] - 2 \left(\operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] + i \operatorname{ArcTanh}\left[\frac{i ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] \right)$$

$$\operatorname{Log}\left[1 - e^{2i \left(\operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] + i \operatorname{ArcTanh}\left[\frac{i ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] \right)}\right] + \pi \operatorname{Log}\left[\frac{1}{\sqrt{\frac{(a^2c + b^2d)\left(c + \frac{a^2c + b^2d}{(a-bx)^2} - \frac{2ac}{a-bx}\right)}{b^2cd}}}\right] + 2i \operatorname{ArcTanh}\left[\frac{i ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]$$

$$\left. \operatorname{Log}\left[\operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] + i \operatorname{ArcTanh}\left[\frac{i ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]\right]\right] + i \operatorname{PolyLog}\left[2, e^{2i \left(\operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] + i \operatorname{ArcTanh}\left[\frac{i ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] \right)}\right] \right) +$$

$$\frac{1}{4c(i ac + a^2c + b^2d)\sqrt{-\frac{-b^2cd + (i ac + a^2c + b^2d)^2}{b^2cd}}} b^4 d^2 \left(e^{-\operatorname{ArcTanh}\left[\frac{i ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]} \operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]^2 + \right.$$

$$\left. \frac{1}{b\sqrt{c}\sqrt{d}\sqrt{1 - \frac{(i ac + a^2c + b^2d)^2}{b^2cd}}} i (i ac + a^2c + b^2d) \left(i \operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] \left(-\pi + 2i \operatorname{ArcTanh}\left[\frac{i ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] \right) - \right.$$

$$\pi \operatorname{Log}\left[1 + e^{-2i \operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]}\right] - 2 \left(\operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] + i \operatorname{ArcTanh}\left[\frac{i ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] \right)$$

$$\operatorname{Log}\left[1 - e^{2i \left(\operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] + i \operatorname{ArcTanh}\left[\frac{i ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] \right)}\right] + \pi \operatorname{Log}\left[\frac{1}{\sqrt{\frac{(a^2c + b^2d)\left(c + \frac{a^2c + b^2d}{(a-bx)^2} - \frac{2ac}{a-bx}\right)}{b^2cd}}}\right] + 2i \operatorname{ArcTanh}\left[\frac{i ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]$$

$$\text{Log}\left[\text{Sin}\left[\text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] + i \text{ArcTanh}\left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]\right]\right] + i \text{PolyLog}\left[2, e^{2 i \left(\text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] + i \text{ArcTanh}\left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]\right)}\right]\right)$$

Problem 111: Result is not expressed in closed-form.

$$\int \frac{\text{ArcCot}[a + b x]}{c + d \sqrt{x}} dx$$

Optimal (type 4, 693 leaves, 55 steps):

$$\begin{aligned} & -\frac{2 i \sqrt{i+a} \text{ArcTan}\left[\frac{\sqrt{b} \sqrt{x}}{\sqrt{i+a}}\right]}{\sqrt{b} d} + \frac{2 i \sqrt{i-a} \text{ArcTanh}\left[\frac{\sqrt{b} \sqrt{x}}{\sqrt{i-a}}\right]}{\sqrt{b} d} - \frac{i c \text{Log}\left[\frac{d(\sqrt{-i-a}-\sqrt{b} \sqrt{x})}{\sqrt{b} c+\sqrt{-i-a} d}\right] \text{Log}[c+d \sqrt{x}]}{d^2} + \\ & \frac{i c \text{Log}\left[\frac{d(\sqrt{-i-a}-\sqrt{b} \sqrt{x})}{\sqrt{b} c+\sqrt{-i-a} d}\right] \text{Log}[c+d \sqrt{x}]}{d^2} - \frac{i c \text{Log}\left[-\frac{d(\sqrt{-i-a}+\sqrt{b} \sqrt{x})}{\sqrt{b} c-\sqrt{-i-a} d}\right] \text{Log}[c+d \sqrt{x}]}{d^2} + \frac{i c \text{Log}\left[-\frac{d(\sqrt{-i-a}+\sqrt{b} \sqrt{x})}{\sqrt{b} c-\sqrt{-i-a} d}\right] \text{Log}[c+d \sqrt{x}]}{d^2} + \\ & \frac{i \sqrt{x} \text{Log}\left[-\frac{i-a-b x}{a+b x}\right]}{d} - \frac{i c \text{Log}[c+d \sqrt{x}] \text{Log}\left[-\frac{i-a-b x}{a+b x}\right]}{d^2} - \frac{i \sqrt{x} \text{Log}\left[\frac{i+a+b x}{a+b x}\right]}{d} + \frac{i c \text{Log}[c+d \sqrt{x}] \text{Log}\left[\frac{i+a+b x}{a+b x}\right]}{d^2} - \\ & \frac{i c \text{PolyLog}\left[2, \frac{\sqrt{b}(c+d \sqrt{x})}{\sqrt{b} c-\sqrt{-i-a} d}\right]}{d^2} - \frac{i c \text{PolyLog}\left[2, \frac{\sqrt{b}(c+d \sqrt{x})}{\sqrt{b} c+\sqrt{-i-a} d}\right]}{d^2} + \frac{i c \text{PolyLog}\left[2, \frac{\sqrt{b}(c+d \sqrt{x})}{\sqrt{b} c-\sqrt{-i-a} d}\right]}{d^2} + \frac{i c \text{PolyLog}\left[2, \frac{\sqrt{b}(c+d \sqrt{x})}{\sqrt{b} c+\sqrt{-i-a} d}\right]}{d^2} \end{aligned}$$

Result (type 7, 313 leaves):

$$\begin{aligned} & \frac{1}{2 d^2} \left(4 \text{ArcCot}[a + b x] \left(d \sqrt{x} - c \text{Log}[c + d \sqrt{x}] \right) + \frac{1}{\sqrt{b}} \right. \\ & d \left(\frac{4(1+i a) \text{ArcTan}\left[\frac{\sqrt{b} \sqrt{x}}{\sqrt{-i+a}}\right]}{\sqrt{-i+a}} + \frac{4(1-i a) \text{ArcTan}\left[\frac{\sqrt{b} \sqrt{x}}{\sqrt{i+a}}\right]}{\sqrt{i+a}} - \sqrt{b} c d \text{RootSum}\left[b^2 c^4 + 2 a b c^2 d^2 + d^4 + a^2 d^4 - 4 b^2 c^3 \#1 - 4 a b c d^2 \#1 + \right. \right. \\ & \left. \left. 6 b^2 c^2 \#1^2 + 2 a b d^2 \#1^2 - 4 b^2 c \#1^3 + b^2 \#1^4 \&, \frac{-\text{Log}[c + d \sqrt{x}]^2 + 2 \text{Log}[c + d \sqrt{x}] \text{Log}\left[1 - \frac{c+d \sqrt{x}}{\#1}\right] + 2 \text{PolyLog}\left[2, \frac{c+d \sqrt{x}}{\#1}\right]}{b c^2 + a d^2 - 2 b c \#1 + b \#1^2} \& \right) \end{aligned}$$

Problem 112: Unable to integrate problem.

$$\int \frac{\text{ArcCot}[a + b x]}{c + \frac{d}{\sqrt{x}}} dx$$

Optimal (type 4, 830 leaves, 65 steps):

$$\begin{aligned} & \frac{2 i \sqrt{i+a} d \text{ArcTan}\left[\frac{\sqrt{b} \sqrt{x}}{\sqrt{i+a}}\right]}{\sqrt{b} c^2} - \frac{2 i \sqrt{i-a} d \text{ArcTanh}\left[\frac{\sqrt{b} \sqrt{x}}{\sqrt{i-a}}\right]}{\sqrt{b} c^2} + \frac{i d^2 \text{Log}\left[\frac{c(\sqrt{-i-a}-\sqrt{b} \sqrt{x})}{\sqrt{-i-a} c+\sqrt{b} d}\right] \text{Log}[d+c \sqrt{x}]}{c^3} - \\ & \frac{i d^2 \text{Log}\left[\frac{c(\sqrt{-i-a}-\sqrt{b} \sqrt{x})}{\sqrt{-i-a} c+\sqrt{b} d}\right] \text{Log}[d+c \sqrt{x}]}{c^3} + \frac{i d^2 \text{Log}\left[\frac{c(\sqrt{-i-a}+\sqrt{b} \sqrt{x})}{\sqrt{-i-a} c-\sqrt{b} d}\right] \text{Log}[d+c \sqrt{x}]}{c^3} - \frac{i d^2 \text{Log}\left[\frac{c(\sqrt{i-a}+\sqrt{b} \sqrt{x})}{\sqrt{i-a} c-\sqrt{b} d}\right] \text{Log}[d+c \sqrt{x}]}{c^3} + \\ & \frac{(1+i a) \text{Log}[i-a-b x]}{2 b c} - \frac{i d \sqrt{x} \text{Log}\left[-\frac{i-a-b x}{a+b x}\right]}{c^2} + \frac{i x \text{Log}\left[-\frac{i-a-b x}{a+b x}\right]}{2 c} + \frac{i d^2 \text{Log}[d+c \sqrt{x}] \text{Log}\left[-\frac{i-a-b x}{a+b x}\right]}{c^3} + \\ & \frac{(1-i a) \text{Log}[i+a+b x]}{2 b c} + \frac{i d \sqrt{x} \text{Log}\left[\frac{i+a+b x}{a+b x}\right]}{c^2} - \frac{i x \text{Log}\left[\frac{i+a+b x}{a+b x}\right]}{2 c} - \frac{i d^2 \text{Log}[d+c \sqrt{x}] \text{Log}\left[\frac{i+a+b x}{a+b x}\right]}{c^3} + \\ & \frac{i d^2 \text{PolyLog}\left[2, -\frac{\sqrt{b}(d+c \sqrt{x})}{\sqrt{-i-a} c-\sqrt{b} d}\right]}{c^3} - \frac{i d^2 \text{PolyLog}\left[2, -\frac{\sqrt{b}(d+c \sqrt{x})}{\sqrt{-i-a} c-\sqrt{b} d}\right]}{c^3} + \frac{i d^2 \text{PolyLog}\left[2, \frac{\sqrt{b}(d+c \sqrt{x})}{\sqrt{-i-a} c+\sqrt{b} d}\right]}{c^3} - \frac{i d^2 \text{PolyLog}\left[2, \frac{\sqrt{b}(d+c \sqrt{x})}{\sqrt{i-a} c+\sqrt{b} d}\right]}{c^3} \end{aligned}$$

Result (type 8, 20 leaves):

$$\int \frac{\text{ArcCot}[a + b x]}{c + \frac{d}{\sqrt{x}}} dx$$

Problem 113: Attempted integration timed out after 120 seconds.

$$\int \frac{\text{ArcCot}[d + e x]}{a + b x + c x^2} dx$$

Optimal (type 4, 367 leaves, 12 steps):

$$\frac{\text{ArcCot}[d + e x] \text{Log}\left[\frac{2 e (b - \sqrt{b^2 - 4 a c} + 2 c x)}{(2 c (i - d) + (b - \sqrt{b^2 - 4 a c}) e) (1 - i (d + e x))}\right]}{\sqrt{b^2 - 4 a c}} - \frac{\text{ArcCot}[d + e x] \text{Log}\left[\frac{2 e (b + \sqrt{b^2 - 4 a c} + 2 c x)}{(2 c (i - d) + (b + \sqrt{b^2 - 4 a c}) e) (1 - i (d + e x))}\right]}{\sqrt{b^2 - 4 a c}} +$$

$$\frac{i \text{PolyLog}\left[2, 1 + \frac{2 (2 c d - (b - \sqrt{b^2 - 4 a c}) e - 2 c (d + e x))}{(2 i c - 2 c d + b e - \sqrt{b^2 - 4 a c}) e} (1 - i (d + e x))\right]}{2 \sqrt{b^2 - 4 a c}} - \frac{i \text{PolyLog}\left[2, 1 + \frac{2 (2 c d - (b + \sqrt{b^2 - 4 a c}) e - 2 c (d + e x))}{(2 c (i - d) + (b + \sqrt{b^2 - 4 a c}) e) (1 - i (d + e x))}\right]}{2 \sqrt{b^2 - 4 a c}}$$

Result (type 1, 1 leaves):

???

Problem 126: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcCot}[1 + x]}{2 + 2 x} dx$$

Optimal (type 4, 35 leaves, 5 steps):

$$-\frac{1}{4} i \text{PolyLog}\left[2, -\frac{i}{1 + x}\right] + \frac{1}{4} i \text{PolyLog}\left[2, \frac{i}{1 + x}\right]$$

Result (type 4, 157 leaves):

$$\frac{1}{16} (i \pi^2 - 4 i \pi \text{ArcTan}[1 + x] + 8 i \text{ArcTan}[1 + x]^2 + \pi \text{Log}[16] - 4 \pi \text{Log}[1 + e^{-2 i \text{ArcTan}[1 + x]}] +$$

$$8 \text{ArcTan}[1 + x] \text{Log}[1 + e^{-2 i \text{ArcTan}[1 + x]}] - 8 \text{ArcTan}[1 + x] \text{Log}[1 - e^{2 i \text{ArcTan}[1 + x]}] + 8 \text{ArcCot}[1 + x] \text{Log}[1 + x] +$$

$$8 \text{ArcTan}[1 + x] \text{Log}[1 + x] - 2 \pi \text{Log}[2 + 2 x + x^2] + 4 i \text{PolyLog}[2, -e^{-2 i \text{ArcTan}[1 + x]}] + 4 i \text{PolyLog}[2, e^{2 i \text{ArcTan}[1 + x]}])$$

Problem 127: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcCot}[a + b x]}{\frac{a d}{b} + d x} dx$$

Optimal (type 4, 45 leaves, 5 steps):

$$-\frac{i \text{PolyLog}\left[2, -\frac{i}{a + b x}\right]}{2 d} + \frac{i \text{PolyLog}\left[2, \frac{i}{a + b x}\right]}{2 d}$$

Result (type 4, 195 leaves):

$$\frac{1}{8d} \left(i \pi^2 - 4 i \pi \operatorname{ArcTan}[a + b x] + 8 i \operatorname{ArcTan}[a + b x]^2 + \pi \operatorname{Log}[16] - 4 \pi \operatorname{Log}[1 + e^{-2 i \operatorname{ArcTan}[a + b x]}] + \right. \\ \left. 8 \operatorname{ArcTan}[a + b x] \operatorname{Log}[1 + e^{-2 i \operatorname{ArcTan}[a + b x]}] - 8 \operatorname{ArcTan}[a + b x] \operatorname{Log}[1 - e^{2 i \operatorname{ArcTan}[a + b x]}] + 8 \operatorname{ArcCot}[a + b x] \operatorname{Log}[a + b x] + \right. \\ \left. 8 \operatorname{ArcTan}[a + b x] \operatorname{Log}[a + b x] - 2 \pi \operatorname{Log}[1 + a^2 + 2 a b x + b^2 x^2] + 4 i \operatorname{PolyLog}[2, -e^{-2 i \operatorname{ArcTan}[a + b x]}] + 4 i \operatorname{PolyLog}[2, e^{2 i \operatorname{ArcTan}[a + b x]}] \right)$$

Problem 133: Result more than twice size of optimal antiderivative.

$$\int \frac{a + b \operatorname{ArcCot}[c + d x]}{e + f x} dx$$

Optimal (type 4, 162 leaves, 5 steps):

$$-\frac{(a + b \operatorname{ArcCot}[c + d x]) \operatorname{Log}\left[\frac{2}{1 - i(c + d x)}\right]}{f} + \frac{(a + b \operatorname{ArcCot}[c + d x]) \operatorname{Log}\left[\frac{2d(e + f x)}{(de + i f - c f)(1 - i(c + d x))}\right]}{f} - \\ \frac{i b \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - i(c + d x)}\right]}{2f} + \frac{i b \operatorname{PolyLog}\left[2, 1 - \frac{2d(e + f x)}{(de + i f - c f)(1 - i(c + d x))}\right]}{2f}$$

Result (type 4, 336 leaves):

$$\frac{1}{f} \left(a \operatorname{Log}[e + f x] + \right. \\ \left. b \left((\operatorname{ArcCot}[c + d x] + \operatorname{ArcTan}[c + d x]) \operatorname{Log}[e + f x] + \operatorname{ArcTan}[c + d x] \left(\operatorname{Log}\left[\frac{1}{\sqrt{1 + (c + d x)^2}}\right] - \operatorname{Log}\left[\operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{de - cf}{f}\right] + \operatorname{ArcTan}[c + d x]\right]\right] \right) \right) + \right. \\ \left. \frac{1}{2} \left(\frac{1}{4} i (\pi - 2 \operatorname{ArcTan}[c + d x])^2 + i \left(\operatorname{ArcTan}\left[\frac{de - cf}{f}\right] + \operatorname{ArcTan}[c + d x] \right)^2 - (\pi - 2 \operatorname{ArcTan}[c + d x]) \operatorname{Log}[1 + e^{-2 i \operatorname{ArcTan}[c + d x]}] - \right. \right. \\ \left. \left. 2 \left(\operatorname{ArcTan}\left[\frac{de - cf}{f}\right] + \operatorname{ArcTan}[c + d x] \right) \operatorname{Log}\left[1 - e^{2 i \left(\operatorname{ArcTan}\left[\frac{de - cf}{f}\right] + \operatorname{ArcTan}[c + d x]\right)}\right] + (\pi - 2 \operatorname{ArcTan}[c + d x]) \operatorname{Log}\left[\frac{2}{\sqrt{1 + (c + d x)^2}}\right] + \right. \right. \\ \left. \left. 2 \left(\operatorname{ArcTan}\left[\frac{de - cf}{f}\right] + \operatorname{ArcTan}[c + d x] \right) \operatorname{Log}\left[2 \operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{de - cf}{f}\right] + \operatorname{ArcTan}[c + d x]\right]\right] + \right. \right. \\ \left. \left. i \operatorname{PolyLog}\left[2, -e^{-2 i \operatorname{ArcTan}[c + d x]}\right] + i \operatorname{PolyLog}\left[2, e^{2 i \left(\operatorname{ArcTan}\left[\frac{de - cf}{f}\right] + \operatorname{ArcTan}[c + d x]\right)}\right] \right) \right) \right)$$

Problem 139: Attempted integration timed out after 120 seconds.

$$\int \frac{(a + b \operatorname{ArcCot}[c + d x])^2}{e + f x} dx$$

Optimal (type 4, 261 leaves, 2 steps):

$$\begin{aligned} & - \frac{(a + b \operatorname{ArcCot}[c + d x])^2 \operatorname{Log}\left[\frac{2}{1-i(c+dx)}\right]}{f} + \frac{(a + b \operatorname{ArcCot}[c + d x])^2 \operatorname{Log}\left[\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right]}{f} \\ & - \frac{i b (a + b \operatorname{ArcCot}[c + d x]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1-i(c+dx)}\right]}{f} + \frac{i b (a + b \operatorname{ArcCot}[c + d x]) \operatorname{PolyLog}\left[2, 1 - \frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right]}{f} \\ & - \frac{b^2 \operatorname{PolyLog}\left[3, 1 - \frac{2}{1-i(c+dx)}\right]}{2f} + \frac{b^2 \operatorname{PolyLog}\left[3, 1 - \frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right]}{2f} \end{aligned}$$

Result (type 1, 1 leaves):

???

Problem 140: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b \operatorname{ArcCot}[c + d x])^2}{(e + f x)^2} dx$$

Optimal (type 4, 567 leaves, 25 steps):

$$\begin{aligned} & \frac{i b^2 d \operatorname{ArcCot}[c + d x]^2}{d^2 e^2 - 2 c d e f + (1 + c^2) f^2} + \frac{b^2 d (d e - c f) \operatorname{ArcCot}[c + d x]^2}{f (d^2 e^2 - 2 c d e f + (1 + c^2) f^2)} - \frac{(a + b \operatorname{ArcCot}[c + d x])^2}{f (e + f x)} - \frac{2 a b d (d e - c f) \operatorname{ArcTan}[c + d x]}{f (f^2 + (d e - c f)^2)} - \frac{2 a b d \operatorname{Log}[e + f x]}{f^2 + (d e - c f)^2} \\ & - \frac{2 b^2 d \operatorname{ArcCot}[c + d x] \operatorname{Log}\left[\frac{2}{1-i(c+dx)}\right]}{d^2 e^2 - 2 c d e f + (1 + c^2) f^2} - \frac{2 b^2 d \operatorname{ArcCot}[c + d x] \operatorname{Log}\left[\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right]}{d^2 e^2 - 2 c d e f + (1 + c^2) f^2} - \frac{2 b^2 d \operatorname{ArcCot}[c + d x] \operatorname{Log}\left[\frac{2}{1+i(c+dx)}\right]}{d^2 e^2 - 2 c d e f + (1 + c^2) f^2} \\ & + \frac{a b d \operatorname{Log}[1 + (c + d x)^2]}{f^2 + (d e - c f)^2} + \frac{i b^2 d \operatorname{PolyLog}\left[2, 1 - \frac{2}{1-i(c+dx)}\right]}{d^2 e^2 - 2 c d e f + (1 + c^2) f^2} - \frac{i b^2 d \operatorname{PolyLog}\left[2, 1 - \frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right]}{d^2 e^2 - 2 c d e f + (1 + c^2) f^2} + \frac{i b^2 d \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+i(c+dx)}\right]}{d^2 e^2 - 2 c d e f + (1 + c^2) f^2} \end{aligned}$$

Result (type 4, 1188 leaves):

$$\begin{aligned}
& -\frac{a^2}{f(e+fx)} - \frac{1}{df(e+fx)^2} 2ab(1+(c+dx)^2) \left(\frac{f}{\sqrt{1+\frac{1}{(c+dx)^2}}} + \frac{de-cf}{(c+dx)\sqrt{1+\frac{1}{(c+dx)^2}}} \right)^2 \left(\frac{\text{ArcCot}[c+dx]}{(c+dx)\sqrt{1+\frac{1}{(c+dx)^2}} \left(\frac{f}{\sqrt{1+\frac{1}{(c+dx)^2}}} + \frac{de-cf}{(c+dx)\sqrt{1+\frac{1}{(c+dx)^2}}} \right)} \right) + \\
& \left. \frac{-de \text{ArcCot}[c+dx] + cf \text{ArcCot}[c+dx] + f \text{Log}\left[-\frac{f}{\sqrt{1+\frac{1}{(c+dx)^2}}} - \frac{de}{(c+dx)\sqrt{1+\frac{1}{(c+dx)^2}}} + \frac{cf}{(c+dx)\sqrt{1+\frac{1}{(c+dx)^2}}}\right]}{d^2e^2 - 2cdef + (1+c^2)f^2} \right) -
\end{aligned}$$

$$\frac{1}{d(e+fx)^2} b^2 (1+(c+dx)^2) \left(\frac{f}{\sqrt{1+\frac{1}{(c+dx)^2}}} + \frac{de-cf}{(c+dx)\sqrt{1+\frac{1}{(c+dx)^2}}} \right)^2$$

$$\left(-\frac{\text{ArcCot}[c+dx]^2}{f(c+dx)\sqrt{1+\frac{1}{(c+dx)^2}} \left(-\frac{f}{\sqrt{1+\frac{1}{(c+dx)^2}}} - \frac{de}{(c+dx)\sqrt{1+\frac{1}{(c+dx)^2}}} + \frac{cf}{(c+dx)\sqrt{1+\frac{1}{(c+dx)^2}}} \right)} + \frac{1}{f} 2 \left(\frac{de \text{ArcCot}[c+dx]^2}{2(d^2e^2 - 2cdef + f^2 + c^2f^2)} - \right. \right.$$

$$\left. \frac{if \text{ArcCot}[c+dx]^2}{2(d^2e^2 - 2cdef + f^2 + c^2f^2)} - \frac{cf \text{ArcCot}[c+dx]^2}{2(d^2e^2 - 2cdef + f^2 + c^2f^2)} - \text{ArcCot}[c+dx] \left(2(de - if - cf) \text{ArcCot}[c+dx] + 2if \text{ArcTan}\left[\right. \right. \right.$$

$$\left. \left. \left. \frac{1}{c+dx} \right] - f \text{Log}\left[\frac{f}{\sqrt{1+\frac{1}{(c+dx)^2}}} + \frac{de}{(c+dx)\sqrt{1+\frac{1}{(c+dx)^2}}} - \frac{cf}{(c+dx)\sqrt{1+\frac{1}{(c+dx)^2}}} \right] \right] \right) \Big/ (2(d^2e^2 - 2cdef + (1+c^2)f^2)) -$$

$$\begin{aligned}
& \frac{1}{2(d^2 e^2 - 2c d e f + (1+c^2) f^2)} f \left(-i \pi \operatorname{ArcCot}[c + d x] + c \operatorname{ArcCot}[c + d x]^2 - \frac{d e \operatorname{ArcCot}[c + d x]^2}{f} - c e^{i \operatorname{ArcTan}\left[\frac{f}{d e - c f}\right]} \right. \\
& \sqrt{\frac{d^2 e^2 - 2c d e f + (1+c^2) f^2}{(d e - c f)^2}} \operatorname{ArcCot}[c + d x]^2 + \frac{d e e^{i \operatorname{ArcTan}\left[\frac{f}{d e - c f}\right]} \sqrt{\frac{d^2 e^2 - 2c d e f + (1+c^2) f^2}{(d e - c f)^2}} \operatorname{ArcCot}[c + d x]^2}{f} - i \operatorname{ArcTan}\left[\frac{1}{c + d x}\right]^2 - \\
& \pi \operatorname{Log}\left[1 + e^{-2i \operatorname{ArcCot}[c + d x]}\right] - 2 \operatorname{ArcCot}[c + d x] \operatorname{Log}\left[1 - e^{2i \left(\operatorname{ArcCot}[c + d x] + \operatorname{ArcTan}\left[\frac{f}{d e - c f}\right]}\right)}\right] + 2 \operatorname{ArcTan}\left[\frac{f}{-d e + c f}\right] \operatorname{Log}\left[1 - \right. \\
& \left. e^{2i \left(\operatorname{ArcCot}[c + d x] + \operatorname{ArcTan}\left[\frac{f}{d e - c f}\right]}\right)}\right] + \pi \operatorname{Log}\left[\frac{1}{\sqrt{1 + \frac{1}{(c + d x)^2}}}\right] + \operatorname{ArcCot}[c + d x] \operatorname{Log}\left[\left(\frac{f}{\sqrt{1 + \frac{1}{(c + d x)^2}}} + \frac{d e - c f}{(c + d x) \sqrt{1 + \frac{1}{(c + d x)^2}}}\right)^2\right] + 2 \operatorname{ArcTan}\left[\right. \\
& \left. \frac{f}{d e - c f}\right] \left(i \operatorname{ArcCot}[c + d x] + \operatorname{Log}\left[\operatorname{Sin}\left[\operatorname{ArcCot}[c + d x] + \operatorname{ArcTan}\left[\frac{f}{d e - c f}\right]\right]\right] \right) + i \operatorname{PolyLog}\left[2, e^{2i \left(\operatorname{ArcCot}[c + d x] + \operatorname{ArcTan}\left[\frac{f}{d e - c f}\right]}\right)}\right] \left. \right) \left. \right) \left. \right)
\end{aligned}$$

Problem 141: Result more than twice size of optimal antiderivative.

$$\int (e + f x)^2 (a + b \operatorname{ArcCot}[c + d x])^3 dx$$

Optimal (type 4, 565 leaves, 21 steps):

$$\begin{aligned}
& \frac{a b^2 f^2 x}{d^2} + \frac{b^3 f^2 (c + d x) \operatorname{ArcCot}[c + d x]}{d^3} + \frac{b f^2 (a + b \operatorname{ArcCot}[c + d x])^2}{2 d^3} + \frac{3 i b f (d e - c f) (a + b \operatorname{ArcCot}[c + d x])^2}{d^3} + \\
& \frac{3 b f (d e - c f) (c + d x) (a + b \operatorname{ArcCot}[c + d x])^2}{d^3} + \frac{b f^2 (c + d x)^2 (a + b \operatorname{ArcCot}[c + d x])^2}{2 d^3} + \\
& \frac{i (3 d^2 e^2 - 6 c d e f - (1 - 3 c^2) f^2) (a + b \operatorname{ArcCot}[c + d x])^3}{3 d^3} - \frac{(d e - c f) (d^2 e^2 - 2 c d e f - (3 - c^2) f^2) (a + b \operatorname{ArcCot}[c + d x])^3}{3 d^3 f} + \\
& \frac{(e + f x)^3 (a + b \operatorname{ArcCot}[c + d x])^3}{3 f} - \frac{6 b^2 f (d e - c f) (a + b \operatorname{ArcCot}[c + d x]) \operatorname{Log}\left[\frac{2}{1+i(c+dx)}\right]}{d^3} - \\
& \frac{b (3 d^2 e^2 - 6 c d e f - (1 - 3 c^2) f^2) (a + b \operatorname{ArcCot}[c + d x])^2 \operatorname{Log}\left[\frac{2}{1+i(c+dx)}\right]}{d^3} + \frac{b^3 f^2 \operatorname{Log}\left[1 + (c + d x)^2\right]}{2 d^3} + \\
& \frac{3 i b^3 f (d e - c f) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+i(c+dx)}\right]}{d^3} + \frac{i b^2 (3 d^2 e^2 - 6 c d e f - (1 - 3 c^2) f^2) (a + b \operatorname{ArcCot}[c + d x]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+i(c+dx)}\right]}{d^3} - \\
& \frac{b^3 (3 d^2 e^2 - 6 c d e f - (1 - 3 c^2) f^2) \operatorname{PolyLog}\left[3, 1 - \frac{2}{1+i(c+dx)}\right]}{2 d^3}
\end{aligned}$$

Result (type 4, 2336 leaves):

$$\begin{aligned}
& \frac{a^2 (a d^2 e^2 + 3 b d e f - 2 b c f^2) x}{d^2} + \frac{a^2 f (2 a d e + b f) x^2}{2 d} + \frac{1}{3} a^3 f^2 x^3 + \\
& a^2 b x (3 e^2 + 3 e f x + f^2 x^2) \operatorname{ArcCot}[c + d x] + \frac{(-3 a^2 b c d^2 e^2 - 3 a^2 b d e f + 3 a^2 b c^2 d e f + 3 a^2 b c f^2 - a^2 b c^3 f^2) \operatorname{ArcTan}[c + d x]}{d^3} + \\
& \frac{(3 a^2 b d^2 e^2 - 6 a^2 b c d e f - a^2 b f^2 + 3 a^2 b c^2 f^2) \operatorname{Log}\left[1 + c^2 + 2 c d x + d^2 x^2\right]}{2 d^3} + \frac{1}{4 d (c + d x)^2 \left(1 + \frac{1}{(c + d x)^2}\right) \left(\frac{1}{\sqrt{1 + \frac{1}{(c + d x)^2}}} - \frac{c}{(c + d x) \sqrt{1 + \frac{1}{(c + d x)^2}}}\right)^2} \\
& a b^2 f^2 x^2 \left(1 + (c + d x)^2\right) \left((c + d x) (1 - 6 c \operatorname{ArcCot}[c + d x] + 3 \operatorname{ArcCot}[c + d x]^2 + 3 c^2 \operatorname{ArcCot}[c + d x]^2) - \right. \\
& \left. (c + d x) \sqrt{1 + \frac{1}{(c + d x)^2}} (1 - 6 c \operatorname{ArcCot}[c + d x] - \operatorname{ArcCot}[c + d x]^2 + 3 c^2 \operatorname{ArcCot}[c + d x]^2) \operatorname{Cos}[3 \operatorname{ArcCot}[c + d x]] - \right.
\end{aligned}$$

$$\begin{aligned}
& 2 \left(-2 \operatorname{ArcCot}[c + d x] + i \operatorname{ArcCot}[c + d x]^2 + 6 c \operatorname{ArcCot}[c + d x]^2 - 3 i c^2 \operatorname{ArcCot}[c + d x]^2 + \right. \\
& \quad \left. 2(-1 + 3 c^2) \operatorname{ArcCot}[c + d x] \operatorname{Log}\left[1 - e^{2 i \operatorname{ArcCot}[c + d x]}\right] - 6 c \operatorname{Log}\left[\frac{1}{(c + d x) \sqrt{1 + \frac{1}{(c + d x)^2}}}\right] + \operatorname{Cos}[2 \operatorname{ArcCot}[c + d x]] \right. \\
& \quad \left. \left(i(-1 + 3 c^2) \operatorname{ArcCot}[c + d x]^2 + (2 - 6 c^2) \operatorname{ArcCot}[c + d x] \operatorname{Log}\left[1 - e^{2 i \operatorname{ArcCot}[c + d x]}\right] + 6 c \operatorname{Log}\left[\frac{1}{(c + d x) \sqrt{1 + \frac{1}{(c + d x)^2}}}\right] \right) \right] + \\
& \quad \left. \frac{4 i(-1 + 3 c^2) \operatorname{PolyLog}\left[2, e^{2 i \operatorname{ArcCot}[c + d x]}\right]}{(c + d x)^2 \left(1 + \frac{1}{(c + d x)^2}\right)} \right) - \frac{1}{d(c + d x)^2 \left(1 + \frac{1}{(c + d x)^2}\right)} 3 a b^2 e^2 \left(1 + (c + d x)^2\right) \\
& \quad \left(- (c + d x) \operatorname{ArcCot}[c + d x]^2 + 2 \operatorname{ArcCot}[c + d x] \operatorname{Log}\left[1 - e^{2 i \operatorname{ArcCot}[c + d x]}\right] - i \left(\operatorname{ArcCot}[c + d x]^2 + \operatorname{PolyLog}\left[2, e^{2 i \operatorname{ArcCot}[c + d x]}\right] \right) \right) + \\
& \quad \frac{1}{(c + d x)^2 \left(1 + \frac{1}{(c + d x)^2}\right)} \\
& 6 a b^2 e f \left(1 + (c + d x)^2\right) \\
& \quad \left(\frac{(c + d x) \operatorname{ArcCot}[c + d x]}{d^2} - \frac{c(c + d x) \operatorname{ArcCot}[c + d x]^2}{d^2} + \frac{(c + d x)^2 \left(1 + \frac{1}{(c + d x)^2}\right) \operatorname{ArcCot}[c + d x]^2}{2 d^2} - \frac{\operatorname{Log}\left[\frac{1}{(c + d x) \sqrt{1 + \frac{1}{(c + d x)^2}}}\right]}{d^2} + \right. \\
& \quad \left. \frac{2 c \left(\operatorname{ArcCot}[c + d x] \operatorname{Log}\left[1 - e^{2 i \operatorname{ArcCot}[c + d x]}\right] - \frac{1}{2} i \left(\operatorname{ArcCot}[c + d x]^2 + \operatorname{PolyLog}\left[2, e^{2 i \operatorname{ArcCot}[c + d x]}\right] \right) \right)}{d^2} \right) - \frac{1}{d(c + d x)^2 \left(1 + \frac{1}{(c + d x)^2}\right)} \\
& b^3 e^2 \left(1 + (c + d x)^2\right) \left(-\frac{i \pi^3}{8} + i \operatorname{ArcCot}[c + d x]^3 - (c + d x) \operatorname{ArcCot}[c + d x]^3 + 3 \operatorname{ArcCot}[c + d x]^2 \operatorname{Log}\left[1 - e^{-2 i \operatorname{ArcCot}[c + d x]}\right] + \right.
\end{aligned}$$

$$\begin{aligned}
& 3 \operatorname{Im} \operatorname{ArcCot}[c+dx] \operatorname{PolyLog}\left[2, e^{-2i \operatorname{ArcCot}[c+dx]}\right] + \frac{3}{2} \operatorname{PolyLog}\left[3, e^{-2i \operatorname{ArcCot}[c+dx]}\right] \Bigg) + \frac{1}{4d^2(c+dx)^2 \left(1 + \frac{1}{(c+dx)^2}\right)} \\
& b^3 e f \left(1 + (c+dx)^2\right) \left(-i c \pi^3 + 12 \operatorname{Im} \operatorname{ArcCot}[c+dx]^2 + 12(c+dx) \operatorname{ArcCot}[c+dx]^2 + 8 \operatorname{Im} c \operatorname{ArcCot}[c+dx]^3 - 8c(c+dx) \operatorname{ArcCot}[c+dx]^3 + \right. \\
& 4(c+dx)^2 \left(1 + \frac{1}{(c+dx)^2}\right) \operatorname{ArcCot}[c+dx]^3 + 24c \operatorname{ArcCot}[c+dx]^2 \operatorname{Log}\left[1 - e^{-2i \operatorname{ArcCot}[c+dx]}\right] - 24 \operatorname{ArcCot}[c+dx] \operatorname{Log}\left[1 - e^{-2i \operatorname{ArcCot}[c+dx]}\right] + \\
& \left. 24 \operatorname{Im} c \operatorname{ArcCot}[c+dx] \operatorname{PolyLog}\left[2, e^{-2i \operatorname{ArcCot}[c+dx]}\right] + 12 \operatorname{Im} \operatorname{PolyLog}\left[2, e^{2i \operatorname{ArcCot}[c+dx]}\right] + 12c \operatorname{PolyLog}\left[3, e^{-2i \operatorname{ArcCot}[c+dx]}\right] \right) - \\
& \frac{1}{d^3(c+dx)^2 \left(1 + \frac{1}{(c+dx)^2}\right)} b^3 f^2 \left(1 + (c+dx)^2\right) \left(i(-1+3c^2) \operatorname{ArcCot}[c+dx] \operatorname{PolyLog}\left[2, e^{-2i \operatorname{ArcCot}[c+dx]}\right] + \right. \\
& \left. \frac{1}{96}(c+dx)^3 \left(1 + \frac{1}{(c+dx)^2}\right)^{3/2} \left(\frac{3 \operatorname{Im} \pi^3}{(c+dx) \sqrt{1 + \frac{1}{(c+dx)^2}}} - \frac{9 \operatorname{Im} c^2 \pi^3}{(c+dx) \sqrt{1 + \frac{1}{(c+dx)^2}}} - \frac{24 \operatorname{ArcCot}[c+dx]}{\sqrt{1 + \frac{1}{(c+dx)^2}}} + \right. \right. \\
& \left. \frac{72c \operatorname{ArcCot}[c+dx]^2}{\sqrt{1 + \frac{1}{(c+dx)^2}}} - \frac{48 \operatorname{ArcCot}[c+dx]^2}{(c+dx) \sqrt{1 + \frac{1}{(c+dx)^2}}} + \frac{216 \operatorname{Im} c \operatorname{ArcCot}[c+dx]^2}{(c+dx) \sqrt{1 + \frac{1}{(c+dx)^2}}} - \frac{24 \operatorname{ArcCot}[c+dx]^3}{\sqrt{1 + \frac{1}{(c+dx)^2}}} - \frac{24c^2 \operatorname{ArcCot}[c+dx]^3}{\sqrt{1 + \frac{1}{(c+dx)^2}}} - \right. \\
& \left. \frac{24 \operatorname{Im} \operatorname{ArcCot}[c+dx]^3}{(c+dx) \sqrt{1 + \frac{1}{(c+dx)^2}}} + \frac{96c \operatorname{ArcCot}[c+dx]^3}{(c+dx) \sqrt{1 + \frac{1}{(c+dx)^2}}} + \frac{72 \operatorname{Im} c^2 \operatorname{ArcCot}[c+dx]^3}{(c+dx) \sqrt{1 + \frac{1}{(c+dx)^2}}} + 24 \operatorname{ArcCot}[c+dx] \operatorname{Cos}\left[3 \operatorname{ArcCot}[c+dx]\right] - \right. \\
& \left. \frac{72c \operatorname{ArcCot}[c+dx]^2 \operatorname{Cos}\left[3 \operatorname{ArcCot}[c+dx]\right] - 8 \operatorname{ArcCot}[c+dx]^3 \operatorname{Cos}\left[3 \operatorname{ArcCot}[c+dx]\right] + 24c^2 \operatorname{ArcCot}[c+dx]^3 \operatorname{Cos}\left[3 \operatorname{ArcCot}[c+dx]\right]}{72 \operatorname{ArcCot}[c+dx]^2 \operatorname{Log}\left[1 - e^{-2i \operatorname{ArcCot}[c+dx]}\right]} + \frac{216c^2 \operatorname{ArcCot}[c+dx]^2 \operatorname{Log}\left[1 - e^{-2i \operatorname{ArcCot}[c+dx]}\right]}{(c+dx) \sqrt{1 + \frac{1}{(c+dx)^2}}} - \right. \\
& \left. \frac{432c \operatorname{ArcCot}[c+dx] \operatorname{Log}\left[1 - e^{-2i \operatorname{ArcCot}[c+dx]}\right]}{(c+dx) \sqrt{1 + \frac{1}{(c+dx)^2}}} + \frac{72 \operatorname{Log}\left[\frac{1}{(c+dx) \sqrt{1 + \frac{1}{(c+dx)^2}}}\right]}{(c+dx) \sqrt{1 + \frac{1}{(c+dx)^2}}} + \frac{288 \operatorname{Im} c \operatorname{PolyLog}\left[2, e^{2i \operatorname{ArcCot}[c+dx]}\right]}{(c+dx)^3 \left(1 + \frac{1}{(c+dx)^2}\right)^{3/2}} + \right.
\end{aligned}$$

$$\frac{48 (-1 + 3 c^2) \text{PolyLog}\left[3, e^{-2 i \text{ArcCot}[c+dx]}\right]}{(c+dx)^3 \left(1 + \frac{1}{(c+dx)^2}\right)^{3/2}} - i \pi^3 \text{Sin}\left[3 \text{ArcCot}[c+dx]\right] + 3 i c^2 \pi^3 \text{Sin}\left[3 \text{ArcCot}[c+dx]\right] - 72 i c \text{ArcCot}[c+dx]^2$$

$$\text{Sin}\left[3 \text{ArcCot}[c+dx]\right] + 8 i \text{ArcCot}[c+dx]^3 \text{Sin}\left[3 \text{ArcCot}[c+dx]\right] - 24 i c^2 \text{ArcCot}[c+dx]^3 \text{Sin}\left[3 \text{ArcCot}[c+dx]\right] +$$

$$24 \text{ArcCot}[c+dx]^2 \text{Log}\left[1 - e^{-2 i \text{ArcCot}[c+dx]}\right] \text{Sin}\left[3 \text{ArcCot}[c+dx]\right] - 72 c^2 \text{ArcCot}[c+dx]^2 \text{Log}\left[1 - e^{-2 i \text{ArcCot}[c+dx]}\right] \text{Sin}\left[3 \text{ArcCot}[c+dx]\right] +$$

$$144 c \text{ArcCot}[c+dx] \text{Log}\left[1 - e^{2 i \text{ArcCot}[c+dx]}\right] \text{Sin}\left[3 \text{ArcCot}[c+dx]\right] - 24 \text{Log}\left[\frac{1}{(c+dx) \sqrt{1 + \frac{1}{(c+dx)^2}}}\right] \text{Sin}\left[3 \text{ArcCot}[c+dx]\right] \left. \right)$$

Problem 144: Attempted integration timed out after 120 seconds.

$$\int \frac{(a + b \text{ArcCot}[c + dx])^3}{e + fx} dx$$

Optimal (type 4, 372 leaves, 2 steps):

$$-\frac{(a + b \text{ArcCot}[c + dx])^3 \text{Log}\left[\frac{2}{1-i(c+dx)}\right]}{f} + \frac{(a + b \text{ArcCot}[c + dx])^3 \text{Log}\left[\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right]}{f} -$$

$$\frac{3 i b (a + b \text{ArcCot}[c + dx])^2 \text{PolyLog}\left[2, 1 - \frac{2}{1-i(c+dx)}\right]}{2 f} + \frac{3 i b (a + b \text{ArcCot}[c + dx])^2 \text{PolyLog}\left[2, 1 - \frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right]}{2 f} -$$

$$\frac{3 b^2 (a + b \text{ArcCot}[c + dx]) \text{PolyLog}\left[3, 1 - \frac{2}{1-i(c+dx)}\right]}{2 f} + \frac{3 b^2 (a + b \text{ArcCot}[c + dx]) \text{PolyLog}\left[3, 1 - \frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right]}{2 f} +$$

$$\frac{3 i b^3 \text{PolyLog}\left[4, 1 - \frac{2}{1-i(c+dx)}\right]}{4 f} - \frac{3 i b^3 \text{PolyLog}\left[4, 1 - \frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right]}{4 f}$$

Result (type 1, 1 leaves):

???

Problem 145: Attempted integration timed out after 120 seconds.

$$\int \frac{(a + b \text{ArcCot}[c + dx])^3}{(e + fx)^2} dx$$

Optimal (type 4, 1233 leaves, 35 steps):

$$\begin{aligned}
& \frac{3 i a b^2 d \operatorname{ArcCot}[c+d x]^2}{d^2 e^2-2 c d e f+(1+c^2) f^2}+\frac{3 a b^2 d(d e-c f) \operatorname{ArcCot}[c+d x]^2}{f\left(d^2 e^2-2 c d e f+(1+c^2) f^2\right)}+\frac{i b^3 d \operatorname{ArcCot}[c+d x]^3}{d^2 e^2-2 c d e f+(1+c^2) f^2}+ \\
& \frac{b^3 d(d e-c f) \operatorname{ArcCot}[c+d x]^3}{f\left(d^2 e^2-2 c d e f+(1+c^2) f^2\right)}-\frac{(a+b \operatorname{ArcCot}[c+d x])^3}{f(e+f x)}-\frac{3 a^2 b d(d e-c f) \operatorname{ArcTan}[c+d x]}{f\left(f^2+(d e-c f)^2\right)}-\frac{3 a^2 b d \operatorname{Log}[e+f x]}{f^2+(d e-c f)^2}+ \\
& \frac{6 a b^2 d \operatorname{ArcCot}[c+d x] \operatorname{Log}\left[\frac{2}{1-i(c+d x)}\right]}{d^2 e^2-2 c d e f+(1+c^2) f^2}+\frac{3 b^3 d \operatorname{ArcCot}[c+d x]^2 \operatorname{Log}\left[\frac{2}{1-i(c+d x)}\right]}{d^2 e^2-2 c d e f+(1+c^2) f^2}-\frac{6 a b^2 d \operatorname{ArcCot}[c+d x] \operatorname{Log}\left[\frac{2 d(e+f x)}{(d e+i f-c f)(1-i(c+d x))}\right]}{d^2 e^2-2 c d e f+(1+c^2) f^2}- \\
& \frac{3 b^3 d \operatorname{ArcCot}[c+d x]^2 \operatorname{Log}\left[\frac{2 d(e+f x)}{(d e+i f-c f)(1-i(c+d x))}\right]}{d^2 e^2-2 c d e f+(1+c^2) f^2}-\frac{6 a b^2 d \operatorname{ArcCot}[c+d x] \operatorname{Log}\left[\frac{2}{1+i(c+d x)}\right]}{d^2 e^2-2 c d e f+(1+c^2) f^2}-\frac{3 b^3 d \operatorname{ArcCot}[c+d x]^2 \operatorname{Log}\left[\frac{2}{1+i(c+d x)}\right]}{d^2 e^2-2 c d e f+(1+c^2) f^2}+ \\
& \frac{3 a^2 b d \operatorname{Log}\left[1+(c+d x)^2\right]}{2\left(f^2+(d e-c f)^2\right)}+\frac{3 i a b^2 d \operatorname{PolyLog}\left[2, 1-\frac{2}{1-i(c+d x)}\right]}{d^2 e^2-2 c d e f+(1+c^2) f^2}+\frac{3 i b^3 d \operatorname{ArcCot}[c+d x] \operatorname{PolyLog}\left[2, 1-\frac{2}{1-i(c+d x)}\right]}{d^2 e^2-2 c d e f+(1+c^2) f^2}- \\
& \frac{3 i a b^2 d \operatorname{PolyLog}\left[2, 1-\frac{2 d(e+f x)}{(d e+i f-c f)(1-i(c+d x))}\right]}{d^2 e^2-2 c d e f+(1+c^2) f^2}-\frac{3 i b^3 d \operatorname{ArcCot}[c+d x] \operatorname{PolyLog}\left[2, 1-\frac{2 d(e+f x)}{(d e+i f-c f)(1-i(c+d x))}\right]}{d^2 e^2-2 c d e f+(1+c^2) f^2}+ \\
& \frac{3 i a b^2 d \operatorname{PolyLog}\left[2, 1-\frac{2}{1+i(c+d x)}\right]}{d^2 e^2-2 c d e f+(1+c^2) f^2}+\frac{3 i b^3 d \operatorname{ArcCot}[c+d x] \operatorname{PolyLog}\left[2, 1-\frac{2}{1+i(c+d x)}\right]}{d^2 e^2-2 c d e f+(1+c^2) f^2}+ \\
& \frac{3 b^3 d \operatorname{PolyLog}\left[3, 1-\frac{2}{1-i(c+d x)}\right]}{2\left(d^2 e^2-2 c d e f+(1+c^2) f^2\right)}-\frac{3 b^3 d \operatorname{PolyLog}\left[3, 1-\frac{2 d(e+f x)}{(d e+i f-c f)(1-i(c+d x))}\right]}{2\left(d^2 e^2-2 c d e f+(1+c^2) f^2\right)}-\frac{3 b^3 d \operatorname{PolyLog}\left[3, 1-\frac{2}{1+i(c+d x)}\right]}{2\left(d^2 e^2-2 c d e f+(1+c^2) f^2\right)}
\end{aligned}$$

Result (type 1, 1 leaves):

???

Problem 146: Unable to integrate problem.

$$\int (e+f x)^m (a+b \operatorname{ArcCot}[c+d x]) dx$$

Optimal (type 5, 177 leaves, 6 steps):

$$\begin{aligned}
& \frac{(e+f x)^{1+m} (a+b \operatorname{ArcCot}[c+d x])}{f(1+m)}+\frac{i b d(e+f x)^{2+m} \operatorname{Hypergeometric2F1}\left[1, 2+m, 3+m, \frac{d(e+f x)}{d e+i f-c f}\right]}{2 f(d e+(i-c) f)(1+m)(2+m)}- \\
& \frac{i b d(e+f x)^{2+m} \operatorname{Hypergeometric2F1}\left[1, 2+m, 3+m, \frac{d(e+f x)}{d e-(i+c) f}\right]}{2 f(d e-(i+c) f)(1+m)(2+m)}
\end{aligned}$$

Result (type 8, 20 leaves):

$$\int (e + f x)^m (a + b \operatorname{ArcCot}[c + d x]) dx$$

Problem 152: Unable to integrate problem.

$$\int \frac{\left(a + b \operatorname{ArcCot}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^3}{1 - c^2 x^2} dx$$

Optimal (type 4, 488 leaves, 9 steps):

$$\begin{aligned} & - \frac{2 \left(a + b \operatorname{ArcCot}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^3 \operatorname{ArcCoth}\left[1 - \frac{2}{1 + \frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right] - 3 i b \left(a + b \operatorname{ArcCot}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^2 \operatorname{PolyLog}\left[2, 1 - \frac{2i}{i + \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right]}{c} + \frac{3 i b \left(a + b \operatorname{ArcCot}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^2 \operatorname{PolyLog}\left[2, 1 - \frac{2\sqrt{1-cx}}{\sqrt{1+cx} \left(i + \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right] - 3 b^2 \left(a + b \operatorname{ArcCot}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right) \operatorname{PolyLog}\left[3, 1 - \frac{2i}{i + \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right]}{2c} \\ & + \frac{3 b^2 \left(a + b \operatorname{ArcCot}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right) \operatorname{PolyLog}\left[3, 1 - \frac{2\sqrt{1-cx}}{\sqrt{1+cx} \left(i + \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right] - 3 i b^3 \operatorname{PolyLog}\left[4, 1 - \frac{2i}{i + \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right] - 3 i b^3 \operatorname{PolyLog}\left[4, 1 - \frac{2\sqrt{1-cx}}{\sqrt{1+cx} \left(i + \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right]}{2c} - \frac{3 i b^3 \operatorname{PolyLog}\left[4, 1 - \frac{2i}{i + \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right]}{4c} + \frac{3 i b^3 \operatorname{PolyLog}\left[4, 1 - \frac{2\sqrt{1-cx}}{\sqrt{1+cx} \left(i + \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right]}{4c} \end{aligned}$$

Result (type 8, 42 leaves):

$$\int \frac{\left(a + b \operatorname{ArcCot}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^3}{1 - c^2 x^2} dx$$

Problem 153: Unable to integrate problem.

$$\int \frac{\left(a + b \operatorname{ArcCot}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^2}{1 - c^2 x^2} dx$$

Optimal (type 4, 321 leaves, 7 steps):

$$\frac{2 \left(a + b \operatorname{ArcCot} \left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}} \right] \right)^2 \operatorname{ArcCoth} \left[1 - \frac{2}{1 + \frac{i\sqrt{1-cx}}{\sqrt{1+cx}}} \right] + i b \left(a + b \operatorname{ArcCot} \left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}} \right] \right) \operatorname{PolyLog} \left[2, 1 - \frac{2i}{i + \frac{\sqrt{1-cx}}{\sqrt{1+cx}}} \right]}{c} + \frac{i b \left(a + b \operatorname{ArcCot} \left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}} \right] \right) \operatorname{PolyLog} \left[2, 1 - \frac{2\sqrt{1-cx}}{\sqrt{1+cx} \left(i + \frac{\sqrt{1-cx}}{\sqrt{1+cx}} \right)} \right] + b^2 \operatorname{PolyLog} \left[3, 1 - \frac{2i}{i + \frac{\sqrt{1-cx}}{\sqrt{1+cx}}} \right] + b^2 \operatorname{PolyLog} \left[3, 1 - \frac{2\sqrt{1-cx}}{\sqrt{1+cx} \left(i + \frac{\sqrt{1-cx}}{\sqrt{1+cx}} \right)} \right]}{2c}$$

Result (type 8, 42 leaves):

$$\int \frac{\left(a + b \operatorname{ArcCot} \left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}} \right] \right)^2}{1 - c^2 x^2} dx$$

Problem 160: Result more than twice size of optimal antiderivative.

$$\int \operatorname{ArcCot} [c + d \operatorname{Tan} [a + b x]] dx$$

Optimal (type 4, 198 leaves, 7 steps):

$$x \operatorname{ArcCot} [c + d \operatorname{Tan} [a + b x]] - \frac{1}{2} i x \operatorname{Log} \left[1 + \frac{(1 + i c + d) e^{2ia + 2ibx}}{1 + i c - d} \right] + \frac{1}{2} i x \operatorname{Log} \left[1 + \frac{(c + i(1 - d)) e^{2ia + 2ibx}}{c + i(1 + d)} \right] - \frac{\operatorname{PolyLog} \left[2, -\frac{(1 + i c + d) e^{2ia + 2ibx}}{1 + i c - d} \right]}{4b} + \frac{\operatorname{PolyLog} \left[2, -\frac{(c + i(1 - d)) e^{2ia + 2ibx}}{c + i(1 + d)} \right]}{4b}$$

Result (type 4, 418 leaves):

$$x \operatorname{ArcCot} [c + d \operatorname{Tan} [a + b x]] - \frac{1}{4b} \left(2a \operatorname{ArcTan} \left[\frac{c(1 + e^{2i(a+bx)})}{1 + d + e^{2i(a+bx)} - d e^{2i(a+bx)}} \right] + 2a \operatorname{ArcTan} \left[\frac{c(1 + e^{2i(a+bx)})}{1 + e^{2i(a+bx)} + d(-1 + e^{2i(a+bx)})} \right] + 2i(a+bx) \operatorname{Log} \left[1 + \frac{(c - i(1 + d)) e^{2i(a+bx)}}{c + i(-1 + d)} \right] - 2i(a+bx) \operatorname{Log} \left[1 + \frac{(i + c - id) e^{2i(a+bx)}}{c + i(1 + d)} \right] + i a \operatorname{Log} \left[e^{-4i(a+bx)} \left(c^2 (1 + e^{2i(a+bx)})^2 + (1 + d + e^{2i(a+bx)} - d e^{2i(a+bx)})^2 \right) \right] + \operatorname{PolyLog} \left[2, -\frac{(c - i(1 + d)) e^{2i(a+bx)}}{c + i(-1 + d)} \right] - \operatorname{PolyLog} \left[2, -\frac{(i + c - id) e^{2i(a+bx)}}{c + i(1 + d)} \right] \right)$$

Problem 173: Result more than twice size of optimal antiderivative.

$$\int \text{ArcCot}[c + d \text{Cot}[a + b x]] dx$$

Optimal (type 4, 198 leaves, 7 steps):

$$\begin{aligned} & x \text{ArcCot}[c + d \text{Cot}[a + b x]] - \frac{1}{2} i x \text{Log}\left[1 - \frac{(1 + i c - d) e^{2 i a + 2 i b x}}{1 + i c + d}\right] + \\ & \frac{1}{2} i x \text{Log}\left[1 - \frac{(c + i(1 + d)) e^{2 i a + 2 i b x}}{c + i(1 - d)}\right] - \frac{\text{PolyLog}\left[2, \frac{(1 + i c - d) e^{2 i a + 2 i b x}}{1 + i c + d}\right]}{4 b} + \frac{\text{PolyLog}\left[2, \frac{(c + i(1 + d)) e^{2 i a + 2 i b x}}{c + i(1 - d)}\right]}{4 b} \end{aligned}$$

Result (type 4, 416 leaves):

$$\begin{aligned} & x \text{ArcCot}[c + d \text{Cot}[a + b x]] - \\ & \frac{1}{4 b} \left(2 a \text{ArcTan}\left[\frac{c(-1 + e^{-2 i(a+b x)})}{-1 + d + e^{-2 i(a+b x)} + d e^{-2 i(a+b x)}}\right] + 2 a \text{ArcTan}\left[\frac{c(-1 + e^{2 i(a+b x)})}{-1 + d + e^{2 i(a+b x)} + d e^{2 i(a+b x)}}\right] + 2 i(a + b x) \text{Log}\left[1 - \frac{(c + i(-1 + d)) e^{2 i(a+b x)}}{c - i(1 + d)}\right] \right) - \\ & 2 i(a + b x) \text{Log}\left[1 - \frac{(c + i(1 + d)) e^{2 i(a+b x)}}{i + c - i d}\right] - i a \text{Log}\left[e^{-4 i(a+b x)} \left(c^2(-1 + e^{2 i(a+b x)})^2 + (1 + d - e^{2 i(a+b x)} + d e^{2 i(a+b x)})^2\right) + \right. \\ & \left. i a \text{Log}\left[e^{-4 i(a+b x)} \left(c^2(-1 + e^{2 i(a+b x)})^2 + (-1 + d + e^{2 i(a+b x)} + d e^{2 i(a+b x)})^2\right)\right] + \right. \\ & \left. \text{PolyLog}\left[2, \frac{(c + i(-1 + d)) e^{2 i(a+b x)}}{c - i(1 + d)}\right] - \text{PolyLog}\left[2, \frac{(c + i(1 + d)) e^{2 i(a+b x)}}{i + c - i d}\right] \right) \end{aligned}$$

Problem 183: Result more than twice size of optimal antiderivative.

$$\int (e + f x)^3 \text{ArcCot}[\text{Tanh}[a + b x]] dx$$

Optimal (type 4, 299 leaves, 12 steps):

$$\begin{aligned} & \frac{(e + f x)^4 \text{ArcCot}[\text{Tanh}[a + b x]]}{4 f} + \frac{(e + f x)^4 \text{ArcTan}[e^{2 a + 2 b x}]}{4 f} - \frac{i(e + f x)^3 \text{PolyLog}[2, -i e^{2 a + 2 b x}]}{4 b} + \\ & \frac{i(e + f x)^3 \text{PolyLog}[2, i e^{2 a + 2 b x}]}{4 b} + \frac{3 i f (e + f x)^2 \text{PolyLog}[3, -i e^{2 a + 2 b x}]}{8 b^2} - \frac{3 i f (e + f x)^2 \text{PolyLog}[3, i e^{2 a + 2 b x}]}{8 b^2} - \\ & \frac{3 i f^2 (e + f x) \text{PolyLog}[4, -i e^{2 a + 2 b x}]}{8 b^3} + \frac{3 i f^2 (e + f x) \text{PolyLog}[4, i e^{2 a + 2 b x}]}{8 b^3} + \frac{3 i f^3 \text{PolyLog}[5, -i e^{2 a + 2 b x}]}{16 b^4} - \frac{3 i f^3 \text{PolyLog}[5, i e^{2 a + 2 b x}]}{16 b^4} \end{aligned}$$

Result (type 4, 600 leaves):

$$\frac{1}{4} x (4 e^3 + 6 e^2 f x + 4 e f^2 x^2 + f^3 x^3) \operatorname{ArcCot}[\operatorname{Tanh}[a + b x]] +$$

$$\frac{1}{16 b^4} i \left(8 b^4 e^3 x \operatorname{Log}\left[1 - i e^{2(a+bx)}\right] + 12 b^4 e^2 f x^2 \operatorname{Log}\left[1 - i e^{2(a+bx)}\right] + 8 b^4 e f^2 x^3 \operatorname{Log}\left[1 - i e^{2(a+bx)}\right] + 2 b^4 f^3 x^4 \operatorname{Log}\left[1 - i e^{2(a+bx)}\right] - \right.$$

$$8 b^4 e^3 x \operatorname{Log}\left[1 + i e^{2(a+bx)}\right] - 12 b^4 e^2 f x^2 \operatorname{Log}\left[1 + i e^{2(a+bx)}\right] - 8 b^4 e f^2 x^3 \operatorname{Log}\left[1 + i e^{2(a+bx)}\right] - 2 b^4 f^3 x^4 \operatorname{Log}\left[1 + i e^{2(a+bx)}\right] -$$

$$4 b^3 (e + f x)^3 \operatorname{PolyLog}\left[2, -i e^{2(a+bx)}\right] + 4 b^3 (e + f x)^3 \operatorname{PolyLog}\left[2, i e^{2(a+bx)}\right] + 6 b^2 e^2 f \operatorname{PolyLog}\left[3, -i e^{2(a+bx)}\right] +$$

$$12 b^2 e f^2 x \operatorname{PolyLog}\left[3, -i e^{2(a+bx)}\right] + 6 b^2 f^3 x^2 \operatorname{PolyLog}\left[3, -i e^{2(a+bx)}\right] - 6 b^2 e^2 f \operatorname{PolyLog}\left[3, i e^{2(a+bx)}\right] -$$

$$12 b^2 e f^2 x \operatorname{PolyLog}\left[3, i e^{2(a+bx)}\right] - 6 b^2 f^3 x^2 \operatorname{PolyLog}\left[3, i e^{2(a+bx)}\right] - 6 b e f^2 \operatorname{PolyLog}\left[4, -i e^{2(a+bx)}\right] - 6 b f^3 x \operatorname{PolyLog}\left[4, -i e^{2(a+bx)}\right] +$$

$$6 b e f^2 \operatorname{PolyLog}\left[4, i e^{2(a+bx)}\right] + 6 b f^3 x \operatorname{PolyLog}\left[4, i e^{2(a+bx)}\right] + 3 f^3 \operatorname{PolyLog}\left[5, -i e^{2(a+bx)}\right] - 3 f^3 \operatorname{PolyLog}\left[5, i e^{2(a+bx)}\right] \left. \right)$$

Problem 190: Result more than twice size of optimal antiderivative.

$$\int \operatorname{ArcCot}[c + d \operatorname{Tanh}[a + b x]] dx$$

Optimal (type 4, 174 leaves, 7 steps):

$$x \operatorname{ArcCot}[c + d \operatorname{Tanh}[a + b x]] - \frac{1}{2} i x \operatorname{Log}\left[1 + \frac{(i - c - d) e^{2a+2bx}}{i - c + d}\right] +$$

$$\frac{1}{2} i x \operatorname{Log}\left[1 + \frac{(i + c + d) e^{2a+2bx}}{i + c - d}\right] - \frac{i \operatorname{PolyLog}\left[2, -\frac{(i-c-d) e^{2a+2bx}}{i-c+d}\right]}{4 b} + \frac{i \operatorname{PolyLog}\left[2, -\frac{(i+c+d) e^{2a+2bx}}{i+c-d}\right]}{4 b}$$

Result (type 4, 365 leaves):

$$x \operatorname{ArcCot}[c + d \operatorname{Tanh}[a + b x]] -$$

$$\frac{1}{2 b} i \left(2 i a \operatorname{ArcTan}\left[\frac{1 + e^{2(a+bx)}}{c - d + c e^{2(a+bx)} + d e^{2(a+bx)}}\right] + (a + b x) \operatorname{Log}\left[1 - \frac{\sqrt{-i + c + d} e^{a+bx}}{\sqrt{i - c + d}}\right] + (a + b x) \operatorname{Log}\left[1 + \frac{\sqrt{-i + c + d} e^{a+bx}}{\sqrt{i - c + d}}\right] - \right.$$

$$(a + b x) \operatorname{Log}\left[1 - \frac{\sqrt{i + c + d} e^{a+bx}}{\sqrt{-i - c + d}}\right] - (a + b x) \operatorname{Log}\left[1 + \frac{\sqrt{i + c + d} e^{a+bx}}{\sqrt{-i - c + d}}\right] + \operatorname{PolyLog}\left[2, -\frac{\sqrt{-i + c + d} e^{a+bx}}{\sqrt{i - c + d}}\right] +$$

$$\left. \operatorname{PolyLog}\left[2, \frac{\sqrt{-i + c + d} e^{a+bx}}{\sqrt{i - c + d}}\right] - \operatorname{PolyLog}\left[2, -\frac{\sqrt{i + c + d} e^{a+bx}}{\sqrt{-i - c + d}}\right] - \operatorname{PolyLog}\left[2, \frac{\sqrt{i + c + d} e^{a+bx}}{\sqrt{-i - c + d}}\right] \right)$$

Problem 200: Result more than twice size of optimal antiderivative.

$$\int (e + f x)^3 \operatorname{ArcCot}[\operatorname{Coth}[a + b x]] dx$$

Optimal (type 4, 299 leaves, 12 steps):

$$\frac{(e+fx)^4 \operatorname{ArcCot}[\operatorname{Coth}[a+bx]]}{4f} - \frac{(e+fx)^4 \operatorname{ArcTan}[e^{2a+2bx}]}{4f} + \frac{i(e+fx)^3 \operatorname{PolyLog}[2, -ie^{2a+2bx}]}{4b} -$$

$$\frac{i(e+fx)^3 \operatorname{PolyLog}[2, ie^{2a+2bx}]}{4b} - \frac{3if(e+fx)^2 \operatorname{PolyLog}[3, -ie^{2a+2bx}]}{8b^2} + \frac{3if(e+fx)^2 \operatorname{PolyLog}[3, ie^{2a+2bx}]}{8b^2} +$$

$$\frac{3if^2(e+fx) \operatorname{PolyLog}[4, -ie^{2a+2bx}]}{8b^3} - \frac{3if^2(e+fx) \operatorname{PolyLog}[4, ie^{2a+2bx}]}{8b^3} - \frac{3if^3 \operatorname{PolyLog}[5, -ie^{2a+2bx}]}{16b^4} + \frac{3if^3 \operatorname{PolyLog}[5, ie^{2a+2bx}]}{16b^4}$$

Result (type 4, 600 leaves):

$$\frac{1}{4} x (4e^3 + 6e^2 f x + 4e f^2 x^2 + f^3 x^3) \operatorname{ArcCot}[\operatorname{Coth}[a+bx]] -$$

$$\frac{1}{16b^4} i \left(8b^4 e^3 x \operatorname{Log}[1 - ie^{2(a+bx)}] + 12b^4 e^2 f x^2 \operatorname{Log}[1 - ie^{2(a+bx)}] + 8b^4 e f^2 x^3 \operatorname{Log}[1 - ie^{2(a+bx)}] + 2b^4 f^3 x^4 \operatorname{Log}[1 - ie^{2(a+bx)}] - \right.$$

$$8b^4 e^3 x \operatorname{Log}[1 + ie^{2(a+bx)}] - 12b^4 e^2 f x^2 \operatorname{Log}[1 + ie^{2(a+bx)}] - 8b^4 e f^2 x^3 \operatorname{Log}[1 + ie^{2(a+bx)}] - 2b^4 f^3 x^4 \operatorname{Log}[1 + ie^{2(a+bx)}] -$$

$$4b^3 (e+fx)^3 \operatorname{PolyLog}[2, -ie^{2(a+bx)}] + 4b^3 (e+fx)^3 \operatorname{PolyLog}[2, ie^{2(a+bx)}] + 6b^2 e^2 f \operatorname{PolyLog}[3, -ie^{2(a+bx)}] +$$

$$12b^2 e f^2 x \operatorname{PolyLog}[3, -ie^{2(a+bx)}] + 6b^2 f^3 x^2 \operatorname{PolyLog}[3, -ie^{2(a+bx)}] - 6b^2 e^2 f \operatorname{PolyLog}[3, ie^{2(a+bx)}] -$$

$$12b^2 e f^2 x \operatorname{PolyLog}[3, ie^{2(a+bx)}] - 6b^2 f^3 x^2 \operatorname{PolyLog}[3, ie^{2(a+bx)}] - 6b e f^2 \operatorname{PolyLog}[4, -ie^{2(a+bx)}] - 6b f^3 x \operatorname{PolyLog}[4, -ie^{2(a+bx)}] +$$

$$6b e f^2 \operatorname{PolyLog}[4, ie^{2(a+bx)}] + 6b f^3 x \operatorname{PolyLog}[4, ie^{2(a+bx)}] + 3f^3 \operatorname{PolyLog}[5, -ie^{2(a+bx)}] - 3f^3 \operatorname{PolyLog}[5, ie^{2(a+bx)}] \left. \right)$$

Problem 207: Result more than twice size of optimal antiderivative.

$$\int \operatorname{ArcCot}[c + d \operatorname{Coth}[a+bx]] dx$$

Optimal (type 4, 174 leaves, 7 steps):

$$x \operatorname{ArcCot}[c + d \operatorname{Coth}[a+bx]] - \frac{1}{2} i x \operatorname{Log}\left[1 - \frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right] +$$

$$\frac{1}{2} i x \operatorname{Log}\left[1 - \frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right] - \frac{i \operatorname{PolyLog}\left[2, \frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right]}{4b} + \frac{i \operatorname{PolyLog}\left[2, \frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right]}{4b}$$

Result (type 4, 365 leaves):

$$x \operatorname{ArcCot}[c + d \operatorname{Coth}[a+bx]] -$$

$$\frac{1}{2b} i \left(2ia \operatorname{ArcTan}\left[\frac{-1 + e^{2(a+bx)}}{-c+d+ce^{2(a+bx)}+de^{2(a+bx)}}\right] + (a+bx) \operatorname{Log}\left[1 - \frac{\sqrt{-i+c+d} e^{a+bx}}{\sqrt{-i+c-d}}\right] + (a+bx) \operatorname{Log}\left[1 + \frac{\sqrt{-i+c+d} e^{a+bx}}{\sqrt{-i+c-d}}\right] - \right.$$

$$(a+bx) \operatorname{Log}\left[1 - \frac{\sqrt{i+c+d} e^{a+bx}}{\sqrt{i+c-d}}\right] - (a+bx) \operatorname{Log}\left[1 + \frac{\sqrt{i+c+d} e^{a+bx}}{\sqrt{i+c-d}}\right] + \operatorname{PolyLog}\left[2, -\frac{\sqrt{-i+c+d} e^{a+bx}}{\sqrt{-i+c-d}}\right] +$$

$$\left. \operatorname{PolyLog}\left[2, \frac{\sqrt{-i+c+d} e^{a+bx}}{\sqrt{-i+c-d}}\right] - \operatorname{PolyLog}\left[2, -\frac{\sqrt{i+c+d} e^{a+bx}}{\sqrt{i+c-d}}\right] - \operatorname{PolyLog}\left[2, \frac{\sqrt{i+c+d} e^{a+bx}}{\sqrt{i+c-d}}\right] \right)$$

Problem 217: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b \operatorname{ArcCot}[c x^n]) (d + e \operatorname{Log}[f x^m])}{x} dx$$

Optimal (type 4, 187 leaves, 13 steps):

$$a d \operatorname{Log}[x] + \frac{a e \operatorname{Log}[f x^m]^2}{2 m} - \frac{i b d \operatorname{PolyLog}\left[2, -\frac{i x^n}{c}\right]}{2 n} - \frac{i b e \operatorname{Log}[f x^m] \operatorname{PolyLog}\left[2, -\frac{i x^n}{c}\right]}{2 n} +$$

$$\frac{i b d \operatorname{PolyLog}\left[2, \frac{i x^n}{c}\right]}{2 n} + \frac{i b e \operatorname{Log}[f x^m] \operatorname{PolyLog}\left[2, \frac{i x^n}{c}\right]}{2 n} - \frac{i b e m \operatorname{PolyLog}\left[3, -\frac{i x^n}{c}\right]}{2 n^2} + \frac{i b e m \operatorname{PolyLog}\left[3, \frac{i x^n}{c}\right]}{2 n^2}$$

Result (type 5, 132 leaves):

$$\frac{b c e m x^n \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 1\right\}, \left\{\frac{3}{2}, \frac{3}{2}, \frac{3}{2}\right\}, -c^2 x^{2 n}\right]}{n^2} - \frac{b c x^n \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{2}, \frac{1}{2}, 1\right\}, \left\{\frac{3}{2}, \frac{3}{2}\right\}, -c^2 x^{2 n}\right] (d + e \operatorname{Log}[f x^m])}{n}$$

$$\frac{1}{2} (a + b \operatorname{ArcCot}[c x^n] + b \operatorname{ArcTan}[c x^n]) \operatorname{Log}[x] (e m \operatorname{Log}[x] - 2 (d + e \operatorname{Log}[f x^m]))$$

Problem 224: Attempted integration timed out after 120 seconds.

$$\int \operatorname{ArcCot}[a + b f^{c+d x}] dx$$

Optimal (type 4, 196 leaves, 6 steps):

$$-\frac{\operatorname{ArcCot}[a + b f^{c+d x}] \operatorname{Log}\left[\frac{2}{1-i(a+b f^{c+d x})}\right]}{d \operatorname{Log}[f]} + \frac{\operatorname{ArcCot}[a + b f^{c+d x}] \operatorname{Log}\left[\frac{2 b f^{c+d x}}{(i-a)(1-i(a+b f^{c+d x}))}\right]}{d \operatorname{Log}[f]}$$

$$\frac{i \operatorname{PolyLog}\left[2, 1 - \frac{2}{1-i(a+b f^{c+d x})}\right]}{2 d \operatorname{Log}[f]} + \frac{i \operatorname{PolyLog}\left[2, 1 - \frac{2 b f^{c+d x}}{(i-a)(1-i(a+b f^{c+d x}))}\right]}{2 d \operatorname{Log}[f]}$$

Result (type 1, 1 leaves):

???

Problem 225: Unable to integrate problem.

$$\int x \operatorname{ArcCot}[a + b f^{c+d x}] dx$$

Optimal (type 4, 250 leaves, 25 steps):

$$-\frac{1}{4} i x^2 \operatorname{Log}\left[1 - \frac{b f^{c+dx}}{i-a}\right] + \frac{1}{4} i x^2 \operatorname{Log}\left[1 + \frac{b f^{c+dx}}{i+a}\right] + \frac{1}{4} i x^2 \operatorname{Log}\left[1 - \frac{i}{a+b f^{c+dx}}\right] - \frac{1}{4} i x^2 \operatorname{Log}\left[1 + \frac{i}{a+b f^{c+dx}}\right] -$$

$$\frac{i x \operatorname{PolyLog}\left[2, \frac{b f^{c+dx}}{i-a}\right]}{2 d \operatorname{Log}[f]} + \frac{i x \operatorname{PolyLog}\left[2, -\frac{b f^{c+dx}}{i+a}\right]}{2 d \operatorname{Log}[f]} + \frac{i \operatorname{PolyLog}\left[3, \frac{b f^{c+dx}}{i-a}\right]}{2 d^2 \operatorname{Log}[f]^2} - \frac{i \operatorname{PolyLog}\left[3, -\frac{b f^{c+dx}}{i+a}\right]}{2 d^2 \operatorname{Log}[f]^2}$$

Result (type 8, 16 leaves):

$$\int x \operatorname{ArcCot}[a + b f^{c+dx}] dx$$

Problem 226: Unable to integrate problem.

$$\int x^2 \operatorname{ArcCot}[a + b f^{c+dx}] dx$$

Optimal (type 4, 313 leaves, 29 steps):

$$-\frac{1}{6} i x^3 \operatorname{Log}\left[1 - \frac{b f^{c+dx}}{i-a}\right] + \frac{1}{6} i x^3 \operatorname{Log}\left[1 + \frac{b f^{c+dx}}{i+a}\right] + \frac{1}{6} i x^3 \operatorname{Log}\left[1 - \frac{i}{a+b f^{c+dx}}\right] - \frac{1}{6} i x^3 \operatorname{Log}\left[1 + \frac{i}{a+b f^{c+dx}}\right] - \frac{i x^2 \operatorname{PolyLog}\left[2, \frac{b f^{c+dx}}{i-a}\right]}{2 d \operatorname{Log}[f]} +$$

$$\frac{i x^2 \operatorname{PolyLog}\left[2, -\frac{b f^{c+dx}}{i+a}\right]}{2 d \operatorname{Log}[f]} + \frac{i x \operatorname{PolyLog}\left[3, \frac{b f^{c+dx}}{i-a}\right]}{d^2 \operatorname{Log}[f]^2} - \frac{i x \operatorname{PolyLog}\left[3, -\frac{b f^{c+dx}}{i+a}\right]}{d^2 \operatorname{Log}[f]^2} - \frac{i \operatorname{PolyLog}\left[4, \frac{b f^{c+dx}}{i-a}\right]}{d^3 \operatorname{Log}[f]^3} + \frac{i \operatorname{PolyLog}\left[4, -\frac{b f^{c+dx}}{i+a}\right]}{d^3 \operatorname{Log}[f]^3}$$

Result (type 8, 18 leaves):

$$\int x^2 \operatorname{ArcCot}[a + b f^{c+dx}] dx$$

Problem 230: Result is not expressed in closed-form.

$$\int e^{c(a+bx)} \operatorname{ArcCot}[\operatorname{Cosh}[a c + b c x]] dx$$

Optimal (type 3, 103 leaves, 8 steps):

$$\frac{e^{a c + b c x} \operatorname{ArcCot}[\operatorname{Cosh}[c(a+bx)]]}{b c} + \frac{(1 - \sqrt{2}) \operatorname{Log}[3 - 2\sqrt{2} + e^{2c(a+bx)}]}{2 b c} + \frac{(1 + \sqrt{2}) \operatorname{Log}[3 + 2\sqrt{2} + e^{2c(a+bx)}]}{2 b c}$$

Result (type 7, 146 leaves):

$$\frac{1}{2 b c} \left(4 c (a + b x) + 2 e^{c(a+bx)} \operatorname{ArcCot}\left[\frac{1}{2} e^{-c(a+bx)} (1 + e^{2c(a+bx)})\right] + \right.$$

$$\left. \operatorname{RootSum}\left[1 + 6 \#1^2 + \#1^4 \&, \frac{-a c - b c x + \operatorname{Log}[e^{c(a+bx)} - \#1] - 7 a c \#1^2 - 7 b c x \#1^2 + 7 \operatorname{Log}[e^{c(a+bx)} - \#1] \#1^2}{1 + 3 \#1^2} \&\right] \right)$$

Problem 231: Result is not expressed in closed-form.

$$\int e^{c(a+bx)} \operatorname{ArcCot}[\operatorname{Tanh}[ac+bcx]] dx$$

Optimal (type 3, 180 leaves, 13 steps):

$$\frac{e^{ac+bcx} \operatorname{ArcCot}[\operatorname{Tanh}[c(a+bx)]]}{bc} - \frac{\operatorname{ArcTan}[1 - \sqrt{2} e^{ac+bcx}]}{\sqrt{2} bc} +$$

$$\frac{\operatorname{ArcTan}[1 + \sqrt{2} e^{ac+bcx}]}{\sqrt{2} bc} + \frac{\operatorname{Log}[1 + e^{2c(a+bx)} - \sqrt{2} e^{ac+bcx}]}{2\sqrt{2} bc} - \frac{\operatorname{Log}[1 + e^{2c(a+bx)} + \sqrt{2} e^{ac+bcx}]}{2\sqrt{2} bc}$$

Result (type 7, 89 leaves):

$$\frac{2 e^{c(a+bx)} \operatorname{ArcCot}\left[\frac{-1+e^{2c(a+bx)}}{1+e^{2c(a+bx)}}\right] + \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{-ac-bcx + \operatorname{Log}\left[e^{c(a+bx)} - \#1\right]}{\#1} \&\right]}{2bc}$$

Problem 232: Result is not expressed in closed-form.

$$\int e^{c(a+bx)} \operatorname{ArcCot}[\operatorname{Coth}[ac+bcx]] dx$$

Optimal (type 3, 180 leaves, 13 steps):

$$\frac{e^{ac+bcx} \operatorname{ArcCot}[\operatorname{Coth}[c(a+bx)]]}{bc} + \frac{\operatorname{ArcTan}[1 - \sqrt{2} e^{ac+bcx}]}{\sqrt{2} bc} -$$

$$\frac{\operatorname{ArcTan}[1 + \sqrt{2} e^{ac+bcx}]}{\sqrt{2} bc} - \frac{\operatorname{Log}[1 + e^{2c(a+bx)} - \sqrt{2} e^{ac+bcx}]}{2\sqrt{2} bc} + \frac{\operatorname{Log}[1 + e^{2c(a+bx)} + \sqrt{2} e^{ac+bcx}]}{2\sqrt{2} bc}$$

Result (type 7, 89 leaves):

$$\frac{2 e^{c(a+bx)} \operatorname{ArcCot}\left[\frac{1+e^{2c(a+bx)}}{-1+e^{2c(a+bx)}}\right] + \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{ac+bcx - \operatorname{Log}\left[e^{c(a+bx)} - \#1\right]}{\#1} \&\right]}{2bc}$$

Problem 233: Result is not expressed in closed-form.

$$\int e^{c(a+bx)} \operatorname{ArcCot}[\operatorname{Sech}[ac+bcx]] dx$$

Optimal (type 3, 103 leaves, 8 steps):

$$\frac{e^{a c+b c x} \operatorname{ArcCot}\left[\operatorname{Sech}\left[c(a+b x)\right]\right]}{b c}-\frac{\left(1-\sqrt{2}\right) \operatorname{Log}\left[3-2 \sqrt{2}+e^{2 c(a+b x)}\right]}{2 b c}-\frac{\left(1+\sqrt{2}\right) \operatorname{Log}\left[3+2 \sqrt{2}+e^{2 c(a+b x)}\right]}{2 b c}$$

Result (type 7, 145 leaves):

$$\frac{1}{2 b c}\left(-4 c(a+b x)+2 e^{c(a+b x)} \operatorname{ArcCot}\left[\frac{2 e^{c(a+b x)}}{1+e^{2 c(a+b x)}}\right]+\right. \\ \left.\operatorname{RootSum}\left[1+6 \#1^2+\#1^4 \&, \frac{a c+b c x-\operatorname{Log}\left[e^{c(a+b x)}-\#1\right]+7 a c \#1^2+7 b c x \#1^2-7 \operatorname{Log}\left[e^{c(a+b x)}-\#1\right] \#1^2}{1+3 \#1^2} \&\right]\right)$$

Test results for the 12 problems in "5.4.2 Exponentials of inverse cotangent.m"

Problem 8: Unable to integrate problem.

$$\int \frac{e^{n \operatorname{ArcCot}[a x]}}{\left(c+a^2 c x^2\right)^{1 / 3}} d x$$

Optimal (type 5, 147 leaves, 3 steps):

$$\frac{1}{\left(c+a^2 c x^2\right)^{1 / 3}}\left(1+\frac{1}{a^2 x^2}\right)^{1 / 3}\left(\frac{a-\frac{i}{x}}{a+\frac{i}{x}}\right)^{\frac{1}{6}(2-3 i n)}\left(1-\frac{i}{a x}\right)^{\frac{1}{6}(-2+3 i n)}\left(1+\frac{i}{a x}\right)^{\frac{1}{6}(4-3 i n)} \times \operatorname{Hypergeometric2F1}\left[-\frac{1}{3}, \frac{1}{6}(2-3 i n), \frac{2}{3}, \frac{2 i}{\left(a+\frac{i}{x}\right) x}\right]$$

Result (type 8, 25 leaves):

$$\int \frac{e^{n \operatorname{ArcCot}[a x]}}{\left(c+a^2 c x^2\right)^{1 / 3}} d x$$

Problem 9: Unable to integrate problem.

$$\int \frac{e^{n \operatorname{ArcCot}[a x]}}{\left(c+a^2 c x^2\right)^{2 / 3}} d x$$

Optimal (type 5, 147 leaves, 3 steps):

$$-\frac{1}{\left(c+a^2 c x^2\right)^{2 / 3}}\left(1+\frac{1}{a^2 x^2}\right)^{2 / 3}\left(\frac{a-\frac{i}{x}}{a+\frac{i}{x}}\right)^{\frac{1}{6}(4-3 i n)}\left(1-\frac{i}{a x}\right)^{\frac{1}{6}(-4+3 i n)}\left(1+\frac{i}{a x}\right)^{\frac{1}{6}(2-3 i n)} \times \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{6}(4-3 i n), \frac{4}{3}, \frac{2 i}{\left(a+\frac{i}{x}\right) x}\right]$$

Result (type 8, 25 leaves):

$$\int \frac{e^{n \operatorname{ArcCot}[a x]}}{(c + a^2 c x^2)^{2/3}} dx$$

Problem 10: Unable to integrate problem.

$$\int \frac{e^{n \operatorname{ArcCot}[a x]}}{(c + a^2 c x^2)^{4/3}} dx$$

Optimal (type 5, 207 leaves, 4 steps):

$$\frac{3 e^{n \operatorname{ArcCot}[a x]} (3 n - 2 a x)}{a c (4 + 9 n^2) (c + a^2 c x^2)^{1/3}} - \left(6 \left(1 + \frac{1}{a^2 x^2} \right)^{1/3} \left(\frac{a - \frac{i}{x}}{a + \frac{i}{x}} \right)^{\frac{1}{6} (2-3 i n)} \left(1 - \frac{i}{a x} \right)^{\frac{1}{6} (-2+3 i n)} \left(1 + \frac{i}{a x} \right)^{\frac{1}{6} (4-3 i n)} \times \operatorname{Hypergeometric2F1} \left[-\frac{1}{3}, \frac{1}{6} (2-3 i n), \frac{2}{3}, \frac{2 i}{\left(a + \frac{i}{x} \right) x} \right] \right) / (c (4 + 9 n^2) (c + a^2 c x^2)^{1/3})$$

Result (type 8, 25 leaves):

$$\int \frac{e^{n \operatorname{ArcCot}[a x]}}{(c + a^2 c x^2)^{4/3}} dx$$

Problem 11: Unable to integrate problem.

$$\int \frac{e^{n \operatorname{ArcCot}[a x]}}{(c + a^2 c x^2)^{5/3}} dx$$

Optimal (type 5, 207 leaves, 4 steps):

$$\frac{3 e^{n \operatorname{ArcCot}[a x]} (3 n - 4 a x)}{a c (16 + 9 n^2) (c + a^2 c x^2)^{2/3}} - \left(12 \left(1 + \frac{1}{a^2 x^2} \right)^{2/3} \left(\frac{a - \frac{i}{x}}{a + \frac{i}{x}} \right)^{\frac{1}{6} (4-3 i n)} \left(1 - \frac{i}{a x} \right)^{\frac{1}{6} (-4+3 i n)} \left(1 + \frac{i}{a x} \right)^{\frac{1}{6} (2-3 i n)} \times \operatorname{Hypergeometric2F1} \left[\frac{1}{3}, \frac{1}{6} (4-3 i n), \frac{4}{3}, \frac{2 i}{\left(a + \frac{i}{x} \right) x} \right] \right) / (c (16 + 9 n^2) (c + a^2 c x^2)^{2/3})$$

Result (type 8, 25 leaves):

$$\int \frac{e^{n \operatorname{ArcCot}[a x]}}{(c + a^2 c x^2)^{5/3}} dx$$

Problem 12: Unable to integrate problem.

$$\int \frac{e^{n \operatorname{ArcCot}[a x]}}{(c + a^2 c x^2)^{7/3}} dx$$

Optimal (type 5, 272 leaves, 5 steps):

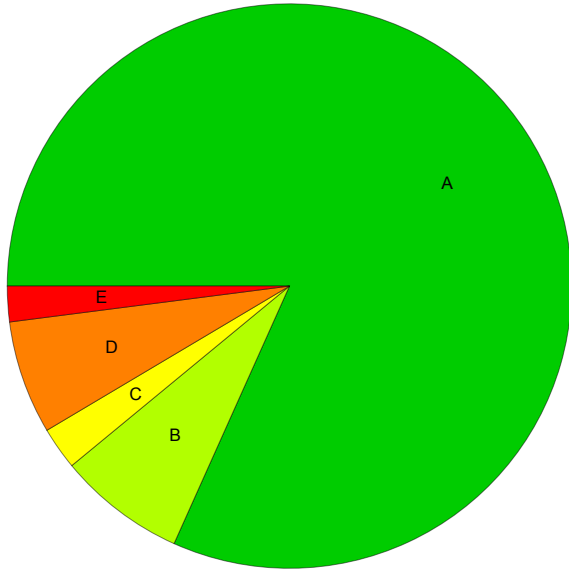
$$\begin{aligned} & - \frac{3 e^{n \operatorname{ArcCot}[a x]} (3 n - 8 a x)}{a c (64 + 9 n^2) (c + a^2 c x^2)^{4/3}} - \frac{120 e^{n \operatorname{ArcCot}[a x]} (3 n - 2 a x)}{a c^2 (4 + 9 n^2) (64 + 9 n^2) (c + a^2 c x^2)^{1/3}} - \\ & \left(240 \left(1 + \frac{1}{a^2 x^2} \right)^{1/3} \left(\frac{a - \frac{i}{x}}{a + \frac{i}{x}} \right)^{\frac{1}{6} (2-3 i n)} \left(1 - \frac{i}{a x} \right)^{\frac{1}{6} (-2+3 i n)} \left(1 + \frac{i}{a x} \right)^{\frac{1}{6} (4-3 i n)} \right. \\ & \quad \left. \times \operatorname{Hypergeometric2F1} \left[-\frac{1}{3}, \frac{1}{6} (2-3 i n), \frac{2}{3}, \frac{2 i}{\left(a + \frac{i}{x} \right) x} \right] \right) / \\ & (c^2 (4 + 9 n^2) (64 + 9 n^2) (c + a^2 c x^2)^{1/3}) \end{aligned}$$

Result (type 8, 25 leaves):

$$\int \frac{e^{n \operatorname{ArcCot}[a x]}}{(c + a^2 c x^2)^{7/3}} dx$$

Summary of Integration Test Results

246 integration problems



A - 201 optimal antiderivatives

B - 18 more than twice size of optimal antiderivatives

C - 6 unnecessarily complex antiderivatives

D - 16 unable to integrate problems

E - 5 integration timeouts