

Mathematica 11.3 Integration Test Results

on the problems in the test-suite directory "6 Hyperbolic functions\6.3 Hyperbolic tangent"

Test results for the 77 problems in "6.3.1 (c+d x)^m (a+b tanh)^n.m"

Problem 3: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int (c + d x) \operatorname{Tanh}[e + f x] dx$$

Optimal (type 4, 57 leaves, 4 steps):

$$-\frac{(c + d x)^2}{2 d} + \frac{(c + d x) \operatorname{Log}[1 + e^{2(e+f x)}]}{f} + \frac{d \operatorname{PolyLog}[2, -e^{2(e+f x)}]}{2 f^2}$$

Result (type 4, 211 leaves):

$$\frac{c \operatorname{Log}[\operatorname{Cosh}[e + f x]]}{f} - \left(d \operatorname{Csch}[e] \left(-e^{-\operatorname{ArcTanh}[\operatorname{Coth}[e]]} f^2 x^2 + \frac{1}{\sqrt{1 - \operatorname{Coth}[e]^2}} \right. \right. \\ \left. \left. + i \operatorname{Coth}[e] (-f x (-\pi + 2 i \operatorname{ArcTanh}[\operatorname{Coth}[e]]) - \pi \operatorname{Log}[1 + e^{2 f x}] - 2 (i f x + i \operatorname{ArcTanh}[\operatorname{Coth}[e]]) \operatorname{Log}[1 - e^{2 i (i f x + i \operatorname{ArcTanh}[\operatorname{Coth}[e])}]) \right) + \right. \\ \left. \pi \operatorname{Log}[\operatorname{Cosh}[f x]] + 2 i \operatorname{ArcTanh}[\operatorname{Coth}[e]] \operatorname{Log}[i \operatorname{Sinh}[f x + \operatorname{ArcTanh}[\operatorname{Coth}[e]]]] + i \operatorname{PolyLog}[2, e^{2 i (i f x + i \operatorname{ArcTanh}[\operatorname{Coth}[e])}] \right) \\ \left. \operatorname{Sech}[e] \right) / \left(2 f^2 \sqrt{\operatorname{Csch}[e]^2 (-\operatorname{Cosh}[e]^2 + \operatorname{Sinh}[e]^2)} \right) + \frac{1}{2} d x^2 \operatorname{Tanh}[e]$$

Problem 7: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int (c + d x)^2 \operatorname{Tanh}[e + f x]^2 dx$$

Optimal (type 4, 88 leaves, 6 steps):

$$-\frac{(c+dx)^2}{f} + \frac{(c+dx)^3}{3d} + \frac{2d(c+dx)\operatorname{Log}[1+e^{2(e+fx)}]}{f^2} + \frac{d^2\operatorname{PolyLog}[2, -e^{2(e+fx)}]}{f^3} - \frac{(c+dx)^2\operatorname{Tanh}[e+fx]}{f}$$

Result (type 4, 303 leaves):

$$\frac{1}{3}x(3c^2+3cdx+d^2x^2) + \frac{2cd\operatorname{Sech}[e](\operatorname{Cosh}[e]\operatorname{Log}[\operatorname{Cosh}[e]\operatorname{Cosh}[fx] + \operatorname{Sinh}[e]\operatorname{Sinh}[fx]] - fx\operatorname{Sinh}[e])}{f^2(\operatorname{Cosh}[e]^2 - \operatorname{Sinh}[e]^2)} -$$

$$\left(d^2\operatorname{Csch}[e] \left(-e^{-\operatorname{ArcTanh}[\operatorname{Coth}[e]]} f^2 x^2 + \frac{1}{\sqrt{1 - \operatorname{Coth}[e]^2}} \right. \right.$$

$$\left. \left. + \operatorname{Coth}[e](-fx(-\pi + 2i\operatorname{ArcTanh}[\operatorname{Coth}[e]]) - \pi\operatorname{Log}[1 + e^{2fx}] - 2(ifx + i\operatorname{ArcTanh}[\operatorname{Coth}[e]])\operatorname{Log}[1 - e^{2i(ifx + i\operatorname{ArcTanh}[\operatorname{Coth}[e]])}]) + \right.$$

$$\left. \left. \pi\operatorname{Log}[\operatorname{Cosh}[fx]] + 2i\operatorname{ArcTanh}[\operatorname{Coth}[e]]\operatorname{Log}[i\operatorname{Sinh}[fx + \operatorname{ArcTanh}[\operatorname{Coth}[e]]]] + i\operatorname{PolyLog}[2, e^{2i(ifx + i\operatorname{ArcTanh}[\operatorname{Coth}[e]])}] \right) \operatorname{Sech}[e] \right) /$$

$$\left(f^3 \sqrt{\operatorname{Csch}[e]^2(-\operatorname{Cosh}[e]^2 + \operatorname{Sinh}[e]^2)} \right) + \frac{\operatorname{Sech}[e]\operatorname{Sech}[e+fx](-c^2\operatorname{Sinh}[fx] - 2cdx\operatorname{Sinh}[fx] - d^2x^2\operatorname{Sinh}[fx])}{f}$$

Problem 11: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int (c+dx)^3 \operatorname{Tanh}[e+fx]^3 dx$$

Optimal (type 4, 237 leaves, 13 steps):

$$-\frac{3d(c+dx)^2}{2f^2} + \frac{(c+dx)^3}{2f} - \frac{(c+dx)^4}{4d} + \frac{3d^2(c+dx)\operatorname{Log}[1+e^{2(e+fx)}]}{f^3} +$$

$$\frac{(c+dx)^3\operatorname{Log}[1+e^{2(e+fx)}]}{f} + \frac{3d^3\operatorname{PolyLog}[2, -e^{2(e+fx)}]}{2f^4} + \frac{3d(c+dx)^2\operatorname{PolyLog}[2, -e^{2(e+fx)}]}{2f^2} -$$

$$\frac{3d^2(c+dx)\operatorname{PolyLog}[3, -e^{2(e+fx)}]}{2f^3} + \frac{3d^3\operatorname{PolyLog}[4, -e^{2(e+fx)}]}{4f^4} - \frac{3d(c+dx)^2\operatorname{Tanh}[e+fx]}{2f^2} - \frac{(c+dx)^3\operatorname{Tanh}[e+fx]^2}{2f}$$

Result (type 4, 819 leaves):

$$\begin{aligned}
& \frac{1}{4 f^3} c d^2 e^{-e} \left(-2 f^2 x^2 \left(2 e^{2e} f x - 3 \left(1 + e^{2e} \right) \operatorname{Log} \left[1 + e^{2(e+fx)} \right] \right) + 6 \left(1 + e^{2e} \right) f x \operatorname{PolyLog} \left[2, -e^{2(e+fx)} \right] - 3 \left(1 + e^{2e} \right) \operatorname{PolyLog} \left[3, -e^{2(e+fx)} \right] \right) \operatorname{Sech} [e] + \\
& \frac{1}{4} d^3 e^e \left(-x^4 + \left(1 + e^{-2e} \right) x^4 - \frac{1}{2 f^4} \right. \\
& \quad \left. e^{-2e} \left(1 + e^{2e} \right) \left(2 f^4 x^4 - 4 f^3 x^3 \operatorname{Log} \left[1 + e^{2(e+fx)} \right] - 6 f^2 x^2 \operatorname{PolyLog} \left[2, -e^{2(e+fx)} \right] + 6 f x \operatorname{PolyLog} \left[3, -e^{2(e+fx)} \right] - 3 \operatorname{PolyLog} \left[4, -e^{2(e+fx)} \right] \right) \right) \\
& \operatorname{Sech} [e] + \frac{(c+d x)^3 \operatorname{Sech} [e+fx]^2}{2 f} + \frac{3 c d^2 \operatorname{Sech} [e] \left(\operatorname{Cosh} [e] \operatorname{Log} [\operatorname{Cosh} [e] \operatorname{Cosh} [f x] + \operatorname{Sinh} [e] \operatorname{Sinh} [f x]] - f x \operatorname{Sinh} [e] \right)}{f^3 \left(\operatorname{Cosh} [e]^2 - \operatorname{Sinh} [e]^2 \right)} + \\
& \frac{c^3 \operatorname{Sech} [e] \left(\operatorname{Cosh} [e] \operatorname{Log} [\operatorname{Cosh} [e] \operatorname{Cosh} [f x] + \operatorname{Sinh} [e] \operatorname{Sinh} [f x]] - f x \operatorname{Sinh} [e] \right)}{f \left(\operatorname{Cosh} [e]^2 - \operatorname{Sinh} [e]^2 \right)} - \\
& \left(3 d^3 \operatorname{Csch} [e] \left(-e^{-\operatorname{ArcTanh} [\operatorname{Coth} [e]]} f^2 x^2 + \frac{1}{\sqrt{1 - \operatorname{Coth} [e]^2}} \right. \right. \\
& \quad \left. \left. i \operatorname{Coth} [e] \left(-f x \left(-\pi + 2 i \operatorname{ArcTanh} [\operatorname{Coth} [e]] \right) - \pi \operatorname{Log} \left[1 + e^{2 f x} \right] - 2 \left(i f x + i \operatorname{ArcTanh} [\operatorname{Coth} [e]] \right) \operatorname{Log} \left[1 - e^{2 i \left(i f x + i \operatorname{ArcTanh} [\operatorname{Coth} [e]] \right)} \right] + \right. \right. \\
& \quad \left. \left. \pi \operatorname{Log} [\operatorname{Cosh} [f x]] + 2 i \operatorname{ArcTanh} [\operatorname{Coth} [e]] \operatorname{Log} [i \operatorname{Sinh} [f x + \operatorname{ArcTanh} [\operatorname{Coth} [e]]]] + i \operatorname{PolyLog} \left[2, e^{2 i \left(i f x + i \operatorname{ArcTanh} [\operatorname{Coth} [e]] \right)} \right] \right) \right) \operatorname{Sech} [e] \Bigg) / \\
& \left(2 f^4 \sqrt{\operatorname{Csch} [e]^2 \left(-\operatorname{Cosh} [e]^2 + \operatorname{Sinh} [e]^2 \right)} \right) - \left(3 c^2 d \operatorname{Csch} [e] \left(-e^{-\operatorname{ArcTanh} [\operatorname{Coth} [e]]} f^2 x^2 + \frac{1}{\sqrt{1 - \operatorname{Coth} [e]^2}} \right. \right. \\
& \quad \left. \left. i \operatorname{Coth} [e] \left(-f x \left(-\pi + 2 i \operatorname{ArcTanh} [\operatorname{Coth} [e]] \right) - \pi \operatorname{Log} \left[1 + e^{2 f x} \right] - 2 \left(i f x + i \operatorname{ArcTanh} [\operatorname{Coth} [e]] \right) \operatorname{Log} \left[1 - e^{2 i \left(i f x + i \operatorname{ArcTanh} [\operatorname{Coth} [e]] \right)} \right] + \right. \right. \\
& \quad \left. \left. \pi \operatorname{Log} [\operatorname{Cosh} [f x]] + 2 i \operatorname{ArcTanh} [\operatorname{Coth} [e]] \operatorname{Log} [i \operatorname{Sinh} [f x + \operatorname{ArcTanh} [\operatorname{Coth} [e]]]] + i \operatorname{PolyLog} \left[2, e^{2 i \left(i f x + i \operatorname{ArcTanh} [\operatorname{Coth} [e]] \right)} \right] \right) \right) \operatorname{Sech} [e] \Bigg) / \\
& \left(2 f^2 \sqrt{\operatorname{Csch} [e]^2 \left(-\operatorname{Cosh} [e]^2 + \operatorname{Sinh} [e]^2 \right)} \right) - \frac{3 \operatorname{Sech} [e] \operatorname{Sech} [e+fx] \left(c^2 d \operatorname{Sinh} [f x] + 2 c d^2 x \operatorname{Sinh} [f x] + d^3 x^2 \operatorname{Sinh} [f x] \right)}{2 f^2} + \\
& \frac{1}{4} x \left(4 c^3 + 6 c^2 d x + 4 c d^2 x^2 + d^3 x^3 \right) \operatorname{Tanh} [e]
\end{aligned}$$

Problem 12: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int (c+d x)^2 \operatorname{Tanh} [e+f x]^3 dx$$

Optimal (type 4, 157 leaves, 9 steps):

$$\frac{c dx}{f} + \frac{d^2 x^2}{2f} - \frac{(c+dx)^3}{3d} + \frac{(c+dx)^2 \operatorname{Log}[1+e^{2(e+fx)}]}{f} + \frac{d^2 \operatorname{Log}[\operatorname{Cosh}[e+fx]]}{f^3} +$$

$$\frac{d(c+dx) \operatorname{PolyLog}[2, -e^{2(e+fx)}]}{f^2} - \frac{d^2 \operatorname{PolyLog}[3, -e^{2(e+fx)}]}{2f^3} - \frac{d(c+dx) \operatorname{Tanh}[e+fx]}{f^2} - \frac{(c+dx)^2 \operatorname{Tanh}[e+fx]^2}{2f}$$

Result (type 4, 465 leaves):

$$\frac{1}{12f^3} d^2 e^{-e} \left(-2f^2 x^2 \left(2e^{2e} f x - 3(1+e^{2e}) \operatorname{Log}[1+e^{2(e+fx)}] \right) + 6(1+e^{2e}) f x \operatorname{PolyLog}[2, -e^{2(e+fx)}] - 3(1+e^{2e}) \operatorname{PolyLog}[3, -e^{2(e+fx)}] \right) \operatorname{Sech}[e] +$$

$$\frac{(c+dx)^2 \operatorname{Sech}[e+fx]^2}{2f} + \frac{d^2 \operatorname{Sech}[e] \left(\operatorname{Cosh}[e] \operatorname{Log}[\operatorname{Cosh}[e] \operatorname{Cosh}[fx] + \operatorname{Sinh}[e] \operatorname{Sinh}[fx]] - f x \operatorname{Sinh}[e] \right)}{f^3 \left(\operatorname{Cosh}[e]^2 - \operatorname{Sinh}[e]^2 \right)} +$$

$$\frac{c^2 \operatorname{Sech}[e] \left(\operatorname{Cosh}[e] \operatorname{Log}[\operatorname{Cosh}[e] \operatorname{Cosh}[fx] + \operatorname{Sinh}[e] \operatorname{Sinh}[fx]] - f x \operatorname{Sinh}[e] \right)}{f \left(\operatorname{Cosh}[e]^2 - \operatorname{Sinh}[e]^2 \right)} - \left(c d \operatorname{Csch}[e] \left(-e^{-\operatorname{ArcTanh}[\operatorname{Coth}[e]]} f^2 x^2 + \frac{1}{\sqrt{1-\operatorname{Coth}[e]^2}} \right. \right.$$

$$\left. \left. + i \operatorname{Coth}[e] \left(-f x \left(-\pi + 2i \operatorname{ArcTanh}[\operatorname{Coth}[e]] \right) - \pi \operatorname{Log}[1+e^{2fx}] - 2 \left(i f x + i \operatorname{ArcTanh}[\operatorname{Coth}[e]] \right) \operatorname{Log}[1 - e^{2i(i f x + i \operatorname{ArcTanh}[\operatorname{Coth}[e]])}] \right) + \right.$$

$$\left. \left. \pi \operatorname{Log}[\operatorname{Cosh}[fx]] + 2i \operatorname{ArcTanh}[\operatorname{Coth}[e]] \operatorname{Log}[i \operatorname{Sinh}[fx + \operatorname{ArcTanh}[\operatorname{Coth}[e]]] + i \operatorname{PolyLog}[2, e^{2i(i f x + i \operatorname{ArcTanh}[\operatorname{Coth}[e]])}] \right) \right) \operatorname{Sech}[e] \Bigg) /$$

$$\left(f^2 \sqrt{\operatorname{Csch}[e]^2 \left(-\operatorname{Cosh}[e]^2 + \operatorname{Sinh}[e]^2 \right)} \right) + \frac{\operatorname{Sech}[e] \operatorname{Sech}[e+fx] \left(-c d \operatorname{Sinh}[fx] - d^2 x \operatorname{Sinh}[fx] \right)}{f^2} + \frac{1}{3} x$$

$$\left(3c^2 + 3c d x + d^2 x^2 \right) \operatorname{Tanh}[e]$$

Problem 13: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int (c+dx) \operatorname{Tanh}[e+fx]^3 dx$$

Optimal (type 4, 100 leaves, 7 steps):

$$\frac{dx}{2f} - \frac{(c+dx)^2}{2d} + \frac{(c+dx) \operatorname{Log}[1+e^{2(e+fx)}]}{f} + \frac{d \operatorname{PolyLog}[2, -e^{2(e+fx)}]}{2f^2} - \frac{d \operatorname{Tanh}[e+fx]}{2f^2} - \frac{(c+dx) \operatorname{Tanh}[e+fx]^2}{2f}$$

Result (type 4, 264 leaves):

$$\frac{c \operatorname{Log}[\operatorname{Cosh}[e + f x]]}{f} + \frac{c \operatorname{Sech}[e + f x]^2}{2 f} + \frac{d x \operatorname{Sech}[e + f x]^2}{2 f} -$$

$$\left(d \operatorname{Csch}[e] \left(-e^{-\operatorname{ArcTanh}[\operatorname{Coth}[e]]} f^2 x^2 + \frac{1}{\sqrt{1 - \operatorname{Coth}[e]^2}} i \operatorname{Coth}[e] (-f x (-\pi + 2 i \operatorname{ArcTanh}[\operatorname{Coth}[e]])) - \right. \right.$$

$$\pi \operatorname{Log}[1 + e^{2 f x}] - 2 (i f x + i \operatorname{ArcTanh}[\operatorname{Coth}[e]]) \operatorname{Log}[1 - e^{2 i (i f x + i \operatorname{ArcTanh}[\operatorname{Coth}[e]])}] + \pi \operatorname{Log}[\operatorname{Cosh}[f x]] +$$

$$\left. \left. 2 i \operatorname{ArcTanh}[\operatorname{Coth}[e]] \operatorname{Log}[i \operatorname{Sinh}[f x + \operatorname{ArcTanh}[\operatorname{Coth}[e]]]] + i \operatorname{PolyLog}[2, e^{2 i (i f x + i \operatorname{ArcTanh}[\operatorname{Coth}[e]])}] \right) \operatorname{Sech}[e] \right) /$$

$$\left(2 f^2 \sqrt{\operatorname{Csch}[e]^2 (-\operatorname{Cosh}[e]^2 + \operatorname{Sinh}[e]^2)} \right) - \frac{d \operatorname{Sech}[e] \operatorname{Sech}[e + f x] \operatorname{Sinh}[f x]}{2 f^2} + \frac{1}{2}$$

$$\frac{d}{x^2} \operatorname{Tanh}[e]$$

Problem 16: Attempted integration timed out after 120 seconds.

$$\int (c + d x) (b \operatorname{Tanh}[e + f x])^{5/2} dx$$

Optimal (type 4, 1392 leaves, 44 steps):

$$\begin{aligned}
& \frac{2 b^{5/2} d \operatorname{ArcTan}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right]}{3 f^2} - \frac{(-b)^{5/2} (c+d x) \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right]}{f} - \frac{(-b)^{5/2} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right]^2}{2 f^2} + \frac{2 b^{5/2} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right]}{3 f^2} + \\
& \frac{b^{5/2} (c+d x) \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right]}{f} + \frac{b^{5/2} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right]^2}{2 f^2} - \frac{b^{5/2} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2 \sqrt{b}}{\sqrt{b}-\sqrt{b \operatorname{Tanh}[e+f x]}}\right]}{f^2} + \\
& \frac{b^{5/2} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2 \sqrt{b}}{\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]}}\right]}{f^2} - \frac{b^{5/2} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2 \sqrt{b}(\sqrt{-b}-\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}-\sqrt{b})(\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]})}\right]}{2 f^2} - \\
& \frac{b^{5/2} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2 \sqrt{b}(\sqrt{-b}+\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}+\sqrt{b})(\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]})}\right]}{2 f^2} + \frac{(-b)^{5/2} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right] \operatorname{Log}\left[\frac{2}{1-\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}}\right]}{f^2} - \\
& \frac{(-b)^{5/2} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right] \operatorname{Log}\left[\frac{2(\sqrt{b}-\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}+\sqrt{b})\left(1-\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right)}\right]}{2 f^2} - \frac{(-b)^{5/2} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right] \operatorname{Log}\left[-\frac{2(\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}-\sqrt{b})\left(1-\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right)}\right]}{2 f^2} - \\
& \frac{(-b)^{5/2} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right] \operatorname{Log}\left[\frac{2}{1+\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}}\right]}{f^2} - \frac{b^{5/2} d \operatorname{PolyLog}\left[2, 1-\frac{2 \sqrt{b}}{\sqrt{b}-\sqrt{b \operatorname{Tanh}[e+f x]}}\right]}{2 f^2} - \frac{b^{5/2} d \operatorname{PolyLog}\left[2, 1-\frac{2 \sqrt{b}}{\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]}}\right]}{2 f^2} + \\
& \frac{b^{5/2} d \operatorname{PolyLog}\left[2, 1-\frac{2 \sqrt{b}(\sqrt{-b}-\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}-\sqrt{b})(\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]})}\right]}{4 f^2} + \frac{b^{5/2} d \operatorname{PolyLog}\left[2, 1-\frac{2 \sqrt{b}(\sqrt{-b}+\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}+\sqrt{b})(\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]})}\right]}{4 f^2} + \\
& \frac{(-b)^{5/2} d \operatorname{PolyLog}\left[2, 1-\frac{2}{1-\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}}\right]}{2 f^2} - \frac{(-b)^{5/2} d \operatorname{PolyLog}\left[2, 1-\frac{2(\sqrt{b}-\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}+\sqrt{b})\left(1-\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right)}\right]}{4 f^2} - \frac{(-b)^{5/2} d \operatorname{PolyLog}\left[2, 1+\frac{2(\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}-\sqrt{b})\left(1-\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right)}\right]}{4 f^2} + \\
& \frac{(-b)^{5/2} d \operatorname{PolyLog}\left[2, 1-\frac{2}{1+\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}}\right]}{2 f^2} - \frac{4 b^2 d \sqrt{b \operatorname{Tanh}[e+f x]}}{3 f^2} - \frac{2 b (c+d x) (b \operatorname{Tanh}[e+f x])^{3/2}}{3 f}
\end{aligned}$$

Result (type 1, 1 leaves):

???

Problem 17: Unable to integrate problem.

$$\int (c + d x) (b \operatorname{Tanh}[e + f x])^{3/2} dx$$

Optimal (type 4, 1363 leaves, 43 steps):

$$\begin{aligned}
& - \frac{2 b^{3/2} d \operatorname{ArcTan}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right]}{f^2} - \frac{(-b)^{3/2} (c+d x) \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right]}{f} - \frac{(-b)^{3/2} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right]^2}{2 f^2} + \\
& \frac{2 b^{3/2} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right]}{f^2} + \frac{b^{3/2} (c+d x) \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right]}{f} + \frac{b^{3/2} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right]^2}{2 f^2} - \\
& \frac{b^{3/2} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2 \sqrt{b}}{\sqrt{b}-\sqrt{b \operatorname{Tanh}[e+f x]}}\right]}{f^2} + \frac{b^{3/2} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2 \sqrt{b}}{\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]}}\right]}{f^2} - \\
& \frac{b^{3/2} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2 \sqrt{b}(\sqrt{-b}-\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}-\sqrt{b})(\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]})}\right]}{2 f^2} - \frac{b^{3/2} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2 \sqrt{b}(\sqrt{-b}+\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}+\sqrt{b})(\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]})}\right]}{2 f^2} + \\
& \frac{(-b)^{3/2} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right] \operatorname{Log}\left[\frac{2}{1-\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}}\right]}{f^2} - \frac{(-b)^{3/2} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right] \operatorname{Log}\left[\frac{2(\sqrt{b}-\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}+\sqrt{b})\left(1-\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right)}\right]}{2 f^2} - \\
& \frac{(-b)^{3/2} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right] \operatorname{Log}\left[-\frac{2(\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}-\sqrt{b})\left(1-\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right)}\right]}{2 f^2} - \frac{(-b)^{3/2} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right] \operatorname{Log}\left[\frac{2}{1+\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}}\right]}{f^2} - \\
& \frac{b^{3/2} d \operatorname{PolyLog}\left[2, 1-\frac{2 \sqrt{b}}{\sqrt{b}-\sqrt{b \operatorname{Tanh}[e+f x]}}\right]}{2 f^2} - \frac{b^{3/2} d \operatorname{PolyLog}\left[2, 1-\frac{2 \sqrt{b}}{\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]}}\right]}{2 f^2} + \frac{b^{3/2} d \operatorname{PolyLog}\left[2, 1-\frac{2 \sqrt{b}(\sqrt{-b}-\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}-\sqrt{b})(\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]})}\right]}{4 f^2} + \\
& \frac{b^{3/2} d \operatorname{PolyLog}\left[2, 1-\frac{2 \sqrt{b}(\sqrt{-b}+\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}+\sqrt{b})(\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]})}\right]}{4 f^2} + \frac{(-b)^{3/2} d \operatorname{PolyLog}\left[2, 1-\frac{2}{1-\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}}\right]}{2 f^2} - \frac{(-b)^{3/2} d \operatorname{PolyLog}\left[2, 1-\frac{2(\sqrt{b}-\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}+\sqrt{b})\left(1-\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right)}\right]}{4 f^2} - \\
& \frac{(-b)^{3/2} d \operatorname{PolyLog}\left[2, 1+\frac{2(\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}-\sqrt{b})\left(1-\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right)}\right]}{4 f^2} + \frac{(-b)^{3/2} d \operatorname{PolyLog}\left[2, 1-\frac{2}{1+\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}}\right]}{2 f^2} - \frac{2 b(c+d x) \sqrt{b \operatorname{Tanh}[e+f x]}}{f}
\end{aligned}$$

Result (type 8, 20 leaves):

$$\int (c + d x) (b \operatorname{Tanh}[e + f x])^{3/2} dx$$

Problem 18: Result unnecessarily involves imaginary or complex numbers.

$$\int (c + d x) \sqrt{b \operatorname{Tanh}[e + f x]} dx$$

Optimal (type 4, 1280 leaves, 37 steps):

$$\begin{aligned}
& - \frac{\sqrt{-b} (c + dx) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[e+fx]}{\sqrt{-b}}\right]}{f} - \frac{\sqrt{-b} d \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[e+fx]}{\sqrt{-b}}\right]^2}{2 f^2} + \\
& \frac{\sqrt{b} (c + dx) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[e+fx]}{\sqrt{b}}\right]}{f} + \frac{\sqrt{b} d \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[e+fx]}{\sqrt{b}}\right]^2}{2 f^2} - \frac{\sqrt{b} d \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[e+fx]}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2\sqrt{b}}{\sqrt{b} - \sqrt{b} \operatorname{Tanh}[e+fx]}\right]}{f^2} + \\
& \frac{\sqrt{b} d \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[e+fx]}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2\sqrt{b}}{\sqrt{b} + \sqrt{b} \operatorname{Tanh}[e+fx]}\right]}{f^2} - \frac{\sqrt{b} d \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[e+fx]}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2\sqrt{b} (\sqrt{-b} - \sqrt{b} \operatorname{Tanh}[e+fx])}{(\sqrt{-b} - \sqrt{b}) (\sqrt{b} + \sqrt{b} \operatorname{Tanh}[e+fx])}\right]}{2 f^2} - \\
& \frac{\sqrt{b} d \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[e+fx]}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2\sqrt{b} (\sqrt{-b} + \sqrt{b} \operatorname{Tanh}[e+fx])}{(\sqrt{-b} + \sqrt{b}) (\sqrt{b} + \sqrt{b} \operatorname{Tanh}[e+fx])}\right]}{2 f^2} + \frac{\sqrt{-b} d \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[e+fx]}{\sqrt{-b}}\right] \operatorname{Log}\left[\frac{2}{1 - \frac{\sqrt{b} \operatorname{Tanh}[e+fx]}{\sqrt{-b}}}\right]}{f^2} - \\
& \frac{\sqrt{-b} d \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[e+fx]}{\sqrt{-b}}\right] \operatorname{Log}\left[\frac{2 (\sqrt{b} - \sqrt{b} \operatorname{Tanh}[e+fx])}{(\sqrt{-b} + \sqrt{b}) \left(1 - \frac{\sqrt{b} \operatorname{Tanh}[e+fx]}{\sqrt{-b}}\right)}\right]}{2 f^2} - \frac{\sqrt{-b} d \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[e+fx]}{\sqrt{-b}}\right] \operatorname{Log}\left[-\frac{2 (\sqrt{b} + \sqrt{b} \operatorname{Tanh}[e+fx])}{(\sqrt{-b} - \sqrt{b}) \left(1 - \frac{\sqrt{b} \operatorname{Tanh}[e+fx]}{\sqrt{-b}}\right)}\right]}{2 f^2} - \\
& \frac{\sqrt{-b} d \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[e+fx]}{\sqrt{-b}}\right] \operatorname{Log}\left[\frac{2}{1 + \frac{\sqrt{b} \operatorname{Tanh}[e+fx]}{\sqrt{-b}}}\right]}{f^2} - \frac{\sqrt{b} d \operatorname{PolyLog}\left[2, 1 - \frac{2\sqrt{b}}{\sqrt{b} - \sqrt{b} \operatorname{Tanh}[e+fx]}\right]}{2 f^2} - \frac{\sqrt{b} d \operatorname{PolyLog}\left[2, 1 - \frac{2\sqrt{b}}{\sqrt{b} + \sqrt{b} \operatorname{Tanh}[e+fx]}\right]}{2 f^2} + \\
& \frac{\sqrt{b} d \operatorname{PolyLog}\left[2, 1 - \frac{2\sqrt{b} (\sqrt{-b} - \sqrt{b} \operatorname{Tanh}[e+fx])}{(\sqrt{-b} - \sqrt{b}) (\sqrt{b} + \sqrt{b} \operatorname{Tanh}[e+fx])}\right]}{4 f^2} + \frac{\sqrt{b} d \operatorname{PolyLog}\left[2, 1 - \frac{2\sqrt{b} (\sqrt{-b} + \sqrt{b} \operatorname{Tanh}[e+fx])}{(\sqrt{-b} + \sqrt{b}) (\sqrt{b} + \sqrt{b} \operatorname{Tanh}[e+fx])}\right]}{4 f^2} + \frac{\sqrt{-b} d \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - \frac{\sqrt{b} \operatorname{Tanh}[e+fx]}{\sqrt{-b}}}\right]}{2 f^2} - \\
& \frac{\sqrt{-b} d \operatorname{PolyLog}\left[2, 1 - \frac{2 (\sqrt{b} - \sqrt{b} \operatorname{Tanh}[e+fx])}{(\sqrt{-b} + \sqrt{b}) \left(1 - \frac{\sqrt{b} \operatorname{Tanh}[e+fx]}{\sqrt{-b}}\right)}\right]}{4 f^2} - \frac{\sqrt{-b} d \operatorname{PolyLog}\left[2, 1 + \frac{2 (\sqrt{b} + \sqrt{b} \operatorname{Tanh}[e+fx])}{(\sqrt{-b} - \sqrt{b}) \left(1 - \frac{\sqrt{b} \operatorname{Tanh}[e+fx]}{\sqrt{-b}}\right)}\right]}{4 f^2} + \frac{\sqrt{-b} d \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 + \frac{\sqrt{b} \operatorname{Tanh}[e+fx]}{\sqrt{-b}}}\right]}{2 f^2}
\end{aligned}$$

Result (type 4, 556 leaves):

$$\begin{aligned}
& \frac{1}{8 f^2 \sqrt{\operatorname{Tanh}[e+f x]}} \left(-4 f (c+d x) \left(2 \operatorname{ArcTan}[\sqrt{\operatorname{Tanh}[e+f x]}] + \operatorname{Log}[1-\sqrt{\operatorname{Tanh}[e+f x]}] - \operatorname{Log}[1+\sqrt{\operatorname{Tanh}[e+f x]}] \right) + \right. \\
& d \left(4 i \operatorname{ArcTan}[\sqrt{\operatorname{Tanh}[e+f x]}]^2 - 4 \operatorname{ArcTan}[\sqrt{\operatorname{Tanh}[e+f x]}] \operatorname{Log}\left[1+e^{4 i \operatorname{ArcTan}[\sqrt{\operatorname{Tanh}[e+f x]}]}\right] - \operatorname{Log}\left[1-\sqrt{\operatorname{Tanh}[e+f x]}\right]^2 + \right. \\
& 2 \operatorname{Log}\left[1-\sqrt{\operatorname{Tanh}[e+f x]}\right] \operatorname{Log}\left[\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i+\sqrt{\operatorname{Tanh}[e+f x]}\right)\right] + 2 \operatorname{Log}\left[1-\sqrt{\operatorname{Tanh}[e+f x]}\right] \operatorname{Log}\left[\left(\frac{1}{2}-\frac{i}{2}\right)\left(i+\sqrt{\operatorname{Tanh}[e+f x]}\right)\right] - \\
& 2 \operatorname{Log}\left[1-\sqrt{\operatorname{Tanh}[e+f x]}\right] \operatorname{Log}\left[\frac{1}{2}\left(1+\sqrt{\operatorname{Tanh}[e+f x]}\right)\right] - 2 \operatorname{Log}\left[1-\left(\frac{1}{2}-\frac{i}{2}\right)\left(1+\sqrt{\operatorname{Tanh}[e+f x]}\right)\right] \operatorname{Log}\left[1+\sqrt{\operatorname{Tanh}[e+f x]}\right] + \\
& 2 \operatorname{Log}\left[\frac{1}{2}\left(1-\sqrt{\operatorname{Tanh}[e+f x]}\right)\right] \operatorname{Log}\left[1+\sqrt{\operatorname{Tanh}[e+f x]}\right] - 2 \operatorname{Log}\left[\left(-\frac{1}{2}-\frac{i}{2}\right)\left(i+\sqrt{\operatorname{Tanh}[e+f x]}\right)\right] \operatorname{Log}\left[1+\sqrt{\operatorname{Tanh}[e+f x]}\right] + \\
& \operatorname{Log}\left[1+\sqrt{\operatorname{Tanh}[e+f x]}\right]^2 + i \operatorname{PolyLog}\left[2,-e^{4 i \operatorname{ArcTan}[\sqrt{\operatorname{Tanh}[e+f x]}]}\right] - 2 \operatorname{PolyLog}\left[2,\frac{1}{2}\left(1-\sqrt{\operatorname{Tanh}[e+f x]}\right)\right] + \\
& 2 \operatorname{PolyLog}\left[2,\left(-\frac{1}{2}-\frac{i}{2}\right)\left(-1+\sqrt{\operatorname{Tanh}[e+f x]}\right)\right] + 2 \operatorname{PolyLog}\left[2,\left(-\frac{1}{2}+\frac{i}{2}\right)\left(-1+\sqrt{\operatorname{Tanh}[e+f x]}\right)\right] + 2 \operatorname{PolyLog}\left[2,\frac{1}{2}\left(1+\sqrt{\operatorname{Tanh}[e+f x]}\right)\right] - \\
& \left. \left. 2 \operatorname{PolyLog}\left[2,\left(\frac{1}{2}-\frac{i}{2}\right)\left(1+\sqrt{\operatorname{Tanh}[e+f x]}\right)\right] - 2 \operatorname{PolyLog}\left[2,\left(\frac{1}{2}+\frac{i}{2}\right)\left(1+\sqrt{\operatorname{Tanh}[e+f x]}\right)\right]\right) \right) \sqrt{b \operatorname{Tanh}[e+f x]}
\end{aligned}$$

Problem 19: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{c+d x}{\sqrt{b \operatorname{Tanh}[e+f x]}} dx$$

Optimal (type 4, 1280 leaves, 37 steps):

$$\begin{aligned}
& - \frac{(c + dx) \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+fx]}}{\sqrt{-b}}\right]}{\sqrt{-b} f} - \frac{d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+fx]}}{\sqrt{-b}}\right]^2}{2 \sqrt{-b} f^2} + \\
& \frac{(c + dx) \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+fx]}}{\sqrt{b}}\right]}{\sqrt{b} f} + \frac{d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+fx]}}{\sqrt{b}}\right]^2}{2 \sqrt{b} f^2} - \frac{d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+fx]}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2 \sqrt{b}}{\sqrt{b} - \sqrt{b \operatorname{Tanh}[e+fx]}}\right]}{\sqrt{b} f^2} + \\
& \frac{d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+fx]}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2 \sqrt{b}}{\sqrt{b} + \sqrt{b \operatorname{Tanh}[e+fx]}}\right]}{\sqrt{b} f^2} - \frac{d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+fx]}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2 \sqrt{b} (\sqrt{-b} - \sqrt{b \operatorname{Tanh}[e+fx]})}{(\sqrt{-b} - \sqrt{b}) (\sqrt{b} + \sqrt{b \operatorname{Tanh}[e+fx]})}\right]}{2 \sqrt{b} f^2} - \\
& \frac{d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+fx]}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2 \sqrt{b} (\sqrt{-b} + \sqrt{b \operatorname{Tanh}[e+fx]})}{(\sqrt{-b} + \sqrt{b}) (\sqrt{b} + \sqrt{b \operatorname{Tanh}[e+fx]})}\right]}{2 \sqrt{b} f^2} + \frac{d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+fx]}}{\sqrt{-b}}\right] \operatorname{Log}\left[\frac{2}{1 - \frac{\sqrt{b \operatorname{Tanh}[e+fx]}}{\sqrt{-b}}}\right]}{\sqrt{-b} f^2} - \\
& \frac{d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+fx]}}{\sqrt{-b}}\right] \operatorname{Log}\left[\frac{2 (\sqrt{b} - \sqrt{b \operatorname{Tanh}[e+fx]})}{(\sqrt{-b} + \sqrt{b}) \left(1 - \frac{\sqrt{b \operatorname{Tanh}[e+fx]}}{\sqrt{-b}}\right)}\right]}{2 \sqrt{-b} f^2} - \frac{d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+fx]}}{\sqrt{-b}}\right] \operatorname{Log}\left[-\frac{2 (\sqrt{b} + \sqrt{b \operatorname{Tanh}[e+fx]})}{(\sqrt{-b} - \sqrt{b}) \left(1 - \frac{\sqrt{b \operatorname{Tanh}[e+fx]}}{\sqrt{-b}}\right)}\right]}{2 \sqrt{-b} f^2} - \\
& \frac{d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+fx]}}{\sqrt{-b}}\right] \operatorname{Log}\left[\frac{2}{1 + \frac{\sqrt{b \operatorname{Tanh}[e+fx]}}{\sqrt{-b}}}\right]}{\sqrt{-b} f^2} - \frac{d \operatorname{PolyLog}\left[2, 1 - \frac{2 \sqrt{b}}{\sqrt{b} - \sqrt{b \operatorname{Tanh}[e+fx]}}\right]}{2 \sqrt{b} f^2} - \frac{d \operatorname{PolyLog}\left[2, 1 - \frac{2 \sqrt{b}}{\sqrt{b} + \sqrt{b \operatorname{Tanh}[e+fx]}}\right]}{2 \sqrt{b} f^2} + \\
& \frac{d \operatorname{PolyLog}\left[2, 1 - \frac{2 \sqrt{b} (\sqrt{-b} - \sqrt{b \operatorname{Tanh}[e+fx]})}{(\sqrt{-b} - \sqrt{b}) (\sqrt{b} + \sqrt{b \operatorname{Tanh}[e+fx]})}\right]}{4 \sqrt{b} f^2} + \frac{d \operatorname{PolyLog}\left[2, 1 - \frac{2 \sqrt{b} (\sqrt{-b} + \sqrt{b \operatorname{Tanh}[e+fx]})}{(\sqrt{-b} + \sqrt{b}) (\sqrt{b} + \sqrt{b \operatorname{Tanh}[e+fx]})}\right]}{4 \sqrt{b} f^2} + \frac{d \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - \frac{\sqrt{b \operatorname{Tanh}[e+fx]}}{\sqrt{-b}}}\right]}{2 \sqrt{-b} f^2} - \\
& \frac{d \operatorname{PolyLog}\left[2, 1 - \frac{2 (\sqrt{b} - \sqrt{b \operatorname{Tanh}[e+fx]})}{(\sqrt{-b} + \sqrt{b}) \left(1 - \frac{\sqrt{b \operatorname{Tanh}[e+fx]}}{\sqrt{-b}}\right)}\right]}{4 \sqrt{-b} f^2} - \frac{d \operatorname{PolyLog}\left[2, 1 + \frac{2 (\sqrt{b} + \sqrt{b \operatorname{Tanh}[e+fx]})}{(\sqrt{-b} - \sqrt{b}) \left(1 - \frac{\sqrt{b \operatorname{Tanh}[e+fx]}}{\sqrt{-b}}\right)}\right]}{4 \sqrt{-b} f^2} + \frac{d \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 + \frac{\sqrt{b \operatorname{Tanh}[e+fx]}}{\sqrt{-b}}}\right]}{2 \sqrt{-b} f^2}
\end{aligned}$$

Result (type 4, 556 leaves):

$$\begin{aligned}
& \frac{1}{8 f^2 \sqrt{b \operatorname{Tanh}[e+f x]}} \left(4 f (c+d x) \left(2 \operatorname{ArcTan}[\sqrt{\operatorname{Tanh}[e+f x]}] - \operatorname{Log}[1 - \sqrt{\operatorname{Tanh}[e+f x]}] + \operatorname{Log}[1 + \sqrt{\operatorname{Tanh}[e+f x]}] \right) + \right. \\
& d \left(-4 i \operatorname{ArcTan}[\sqrt{\operatorname{Tanh}[e+f x]}]^2 + 4 \operatorname{ArcTan}[\sqrt{\operatorname{Tanh}[e+f x]}] \operatorname{Log}[1 + e^{4 i \operatorname{ArcTan}[\sqrt{\operatorname{Tanh}[e+f x]}]}] - \operatorname{Log}[1 - \sqrt{\operatorname{Tanh}[e+f x]}]^2 + \right. \\
& 2 \operatorname{Log}[1 - \sqrt{\operatorname{Tanh}[e+f x]}] \operatorname{Log}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-i + \sqrt{\operatorname{Tanh}[e+f x]})\right] + 2 \operatorname{Log}[1 - \sqrt{\operatorname{Tanh}[e+f x]}] \operatorname{Log}\left[\left(\frac{1}{2} - \frac{i}{2}\right) (i + \sqrt{\operatorname{Tanh}[e+f x]})\right] - \\
& 2 \operatorname{Log}[1 - \sqrt{\operatorname{Tanh}[e+f x]}] \operatorname{Log}\left[\frac{1}{2} (1 + \sqrt{\operatorname{Tanh}[e+f x]})\right] - 2 \operatorname{Log}\left[1 - \left(\frac{1}{2} - \frac{i}{2}\right) (1 + \sqrt{\operatorname{Tanh}[e+f x]})\right] \operatorname{Log}[1 + \sqrt{\operatorname{Tanh}[e+f x]}] + \\
& 2 \operatorname{Log}\left[\frac{1}{2} (1 - \sqrt{\operatorname{Tanh}[e+f x]})\right] \operatorname{Log}[1 + \sqrt{\operatorname{Tanh}[e+f x]}] - 2 \operatorname{Log}\left[\left(-\frac{1}{2} - \frac{i}{2}\right) (i + \sqrt{\operatorname{Tanh}[e+f x]})\right] \operatorname{Log}[1 + \sqrt{\operatorname{Tanh}[e+f x]}] + \\
& \operatorname{Log}[1 + \sqrt{\operatorname{Tanh}[e+f x]}]^2 - i \operatorname{PolyLog}\left[2, -e^{4 i \operatorname{ArcTan}[\sqrt{\operatorname{Tanh}[e+f x]}]}\right] - 2 \operatorname{PolyLog}\left[2, \frac{1}{2} (1 - \sqrt{\operatorname{Tanh}[e+f x]})\right] + \\
& 2 \operatorname{PolyLog}\left[2, \left(-\frac{1}{2} - \frac{i}{2}\right) (-1 + \sqrt{\operatorname{Tanh}[e+f x]})\right] + 2 \operatorname{PolyLog}\left[2, \left(-\frac{1}{2} + \frac{i}{2}\right) (-1 + \sqrt{\operatorname{Tanh}[e+f x]})\right] + 2 \operatorname{PolyLog}\left[2, \frac{1}{2} (1 + \sqrt{\operatorname{Tanh}[e+f x]})\right] - \\
& 2 \operatorname{PolyLog}\left[2, \left(\frac{1}{2} - \frac{i}{2}\right) (1 + \sqrt{\operatorname{Tanh}[e+f x]})\right] - 2 \operatorname{PolyLog}\left[2, \left(\frac{1}{2} + \frac{i}{2}\right) (1 + \sqrt{\operatorname{Tanh}[e+f x]})\right] \right) \sqrt{\operatorname{Tanh}[e+f x]}
\end{aligned}$$

Problem 20: Unable to integrate problem.

$$\int \frac{c + d x}{(b \operatorname{Tanh}[e+f x])^{3/2}} dx$$

Optimal (type 4, 1365 leaves, 43 steps):

$$\begin{aligned}
& \frac{2 d \operatorname{ArcTan}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right]}{b^{3/2} f^2} - \frac{(c+d x) \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right]}{(-b)^{3/2} f} - \frac{d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right]^2}{2(-b)^{3/2} f^2} + \frac{2 d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right]}{b^{3/2} f^2} + \\
& \frac{(c+d x) \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right]}{b^{3/2} f} + \frac{d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right]^2}{2 b^{3/2} f^2} - \frac{d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2 \sqrt{b}}{\sqrt{b}-\sqrt{b \operatorname{Tanh}[e+f x]}}\right]}{b^{3/2} f^2} + \\
& \frac{d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2 \sqrt{b}}{\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]}}\right]}{b^{3/2} f^2} - \frac{d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2 \sqrt{b}\left(\sqrt{-b}-\sqrt{b \operatorname{Tanh}[e+f x]}\right)}{\left(\sqrt{-b}-\sqrt{b}\right)\left(\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]}\right)}\right]}{2 b^{3/2} f^2} - \\
& \frac{d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2 \sqrt{b}\left(\sqrt{-b}+\sqrt{b \operatorname{Tanh}[e+f x]}\right)}{\left(\sqrt{-b}+\sqrt{b}\right)\left(\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]}\right)}\right]}{2 b^{3/2} f^2} + \frac{d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right] \operatorname{Log}\left[\frac{2}{1-\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}}\right]}{(-b)^{3/2} f^2} - \\
& \frac{d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right] \operatorname{Log}\left[\frac{2\left(\sqrt{b}-\sqrt{b \operatorname{Tanh}[e+f x]}\right)}{\left(\sqrt{-b}+\sqrt{b}\right)\left(1-\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right)}\right]}{2(-b)^{3/2} f^2} - \frac{d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right] \operatorname{Log}\left[-\frac{2\left(\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]}\right)}{\left(\sqrt{-b}-\sqrt{b}\right)\left(1-\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right)}\right]}{2(-b)^{3/2} f^2} - \\
& \frac{d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right] \operatorname{Log}\left[\frac{2}{1+\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}}\right]}{(-b)^{3/2} f^2} - \frac{d \operatorname{PolyLog}\left[2, 1-\frac{2 \sqrt{b}}{\sqrt{b}-\sqrt{b \operatorname{Tanh}[e+f x]}}\right]}{2 b^{3/2} f^2} - \frac{d \operatorname{PolyLog}\left[2, 1-\frac{2 \sqrt{b}}{\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]}}\right]}{2 b^{3/2} f^2} + \\
& \frac{d \operatorname{PolyLog}\left[2, 1-\frac{2 \sqrt{b}\left(\sqrt{-b}-\sqrt{b \operatorname{Tanh}[e+f x]}\right)}{\left(\sqrt{-b}-\sqrt{b}\right)\left(\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]}\right)}\right]}{4 b^{3/2} f^2} + \frac{d \operatorname{PolyLog}\left[2, 1-\frac{2 \sqrt{b}\left(\sqrt{-b}+\sqrt{b \operatorname{Tanh}[e+f x]}\right)}{\left(\sqrt{-b}+\sqrt{b}\right)\left(\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]}\right)}\right]}{4 b^{3/2} f^2} + \frac{d \operatorname{PolyLog}\left[2, 1-\frac{2}{1-\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}}\right]}{2(-b)^{3/2} f^2} - \\
& \frac{d \operatorname{PolyLog}\left[2, 1-\frac{2\left(\sqrt{b}-\sqrt{b \operatorname{Tanh}[e+f x]}\right)}{\left(\sqrt{-b}+\sqrt{b}\right)\left(1-\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right)}\right]}{4(-b)^{3/2} f^2} - \frac{d \operatorname{PolyLog}\left[2, 1+\frac{2\left(\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]}\right)}{\left(\sqrt{-b}-\sqrt{b}\right)\left(1-\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right)}\right]}{4(-b)^{3/2} f^2} + \frac{d \operatorname{PolyLog}\left[2, 1-\frac{2}{1+\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}}\right]}{2(-b)^{3/2} f^2} - \frac{2(c+d x)}{b f \sqrt{b \operatorname{Tanh}[e+f x]}}
\end{aligned}$$

Result (type 8, 20 leaves):

$$\int \frac{c+d x}{(b \operatorname{Tanh}[e+f x])^{3/2}} dx$$

Problem 22: Attempted integration timed out after 120 seconds.

$$\int (c + d x)^2 \sqrt{b \operatorname{Tanh}[e + f x]} \, dx$$

Optimal (type 9, 22 leaves, 0 steps):

$$\text{Unintegrable}[(c + d x)^2 \sqrt{b \operatorname{Tanh}[e + f x]}, x]$$

Result (type 1, 1 leaves):

???

Problem 23: Attempted integration timed out after 120 seconds.

$$\int \frac{(c + d x)^2}{\sqrt{b \operatorname{Tanh}[e + f x]}} \, dx$$

Optimal (type 9, 22 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(c + d x)^2}{\sqrt{b \operatorname{Tanh}[e + f x]}}, x\right]$$

Result (type 1, 1 leaves):

???

Problem 50: Result more than twice size of optimal antiderivative.

$$\int \frac{(c + d x)^m}{a + a \operatorname{Tanh}[e + f x]} \, dx$$

Optimal (type 4, 89 leaves, 2 steps):

$$\frac{(c + d x)^{1+m}}{2 a d (1+m)} - \frac{2^{-2-m} e^{-2e + \frac{2cf}{d}} (c + d x)^m \left(\frac{f(c+dx)}{d}\right)^{-m} \operatorname{Gamma}\left[1+m, \frac{2f(c+dx)}{d}\right]}{a f}$$

Result (type 4, 186 leaves):

$$\left(2^{-2-m} (c+dx)^m \left(-\frac{f(c+dx)}{d} \right)^m \left(-\frac{f^2(c+dx)^2}{d^2} \right)^{-m} \operatorname{Sech}[e+fx] \right. \\ \left. \left(d(1+m) \operatorname{Gamma}\left[1+m, \frac{2f(c+dx)}{d}\right] \left(-\operatorname{Cosh}\left[e-\frac{cf}{d}\right] + \operatorname{Sinh}\left[e-\frac{cf}{d}\right] \right) + 2^{1+m} f \left(f \left(\frac{c}{d} + x \right) \right)^m (c+dx) \left(\operatorname{Cosh}\left[e-\frac{cf}{d}\right] + \operatorname{Sinh}\left[e-\frac{cf}{d}\right] \right) \right) \right. \\ \left. \left(\operatorname{Cosh}\left[f \left(\frac{c}{d} + x \right)\right] + \operatorname{Sinh}\left[f \left(\frac{c}{d} + x \right)\right] \right) \right) / (adf(1+m)(1+\operatorname{Tanh}[e+fx]))$$

Problem 51: Attempted integration timed out after 120 seconds.

$$\int \frac{(c+dx)^m}{(a+a \operatorname{Tanh}[e+fx])^2} dx$$

Optimal (type 4, 153 leaves, 4 steps):

$$\frac{(c+dx)^{1+m}}{4a^2d(1+m)} - \frac{2^{-2-m} e^{-2e+\frac{2cf}{d}} (c+dx)^m \left(\frac{f(c+dx)}{d} \right)^{-m} \operatorname{Gamma}\left[1+m, \frac{2f(c+dx)}{d}\right]}{a^2f} - \frac{4^{-2-m} e^{-4e+\frac{4cf}{d}} (c+dx)^m \left(\frac{f(c+dx)}{d} \right)^{-m} \operatorname{Gamma}\left[1+m, \frac{4f(c+dx)}{d}\right]}{a^2f}$$

Result (type 1, 1 leaves):

???

Problem 52: Attempted integration timed out after 120 seconds.

$$\int \frac{(c+dx)^m}{(a+a \operatorname{Tanh}[e+fx])^3} dx$$

Optimal (type 4, 224 leaves, 5 steps):

$$\frac{(c+dx)^{1+m}}{8a^3d(1+m)} - \frac{3 \times 2^{-4-m} e^{-2e+\frac{2cf}{d}} (c+dx)^m \left(\frac{f(c+dx)}{d} \right)^{-m} \operatorname{Gamma}\left[1+m, \frac{2f(c+dx)}{d}\right]}{a^3f} - \\ \frac{3 \times 2^{-5-2m} e^{-4e+\frac{4cf}{d}} (c+dx)^m \left(\frac{f(c+dx)}{d} \right)^{-m} \operatorname{Gamma}\left[1+m, \frac{4f(c+dx)}{d}\right]}{a^3f} - \frac{2^{-4-m} \times 3^{-1-m} e^{-6e+\frac{6cf}{d}} (c+dx)^m \left(\frac{f(c+dx)}{d} \right)^{-m} \operatorname{Gamma}\left[1+m, \frac{6f(c+dx)}{d}\right]}{a^3f}$$

Result (type 1, 1 leaves):

???

Problem 55: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int (c + d x) (a + b \operatorname{Tanh}[e + f x]) dx$$

Optimal (type 4, 75 leaves, 6 steps):

$$\frac{a (c + d x)^2}{2 d} - \frac{b (c + d x)^2}{2 d} + \frac{b (c + d x) \operatorname{Log}[1 + e^{2 (e + f x)}]}{f} + \frac{b d \operatorname{PolyLog}[2, -e^{2 (e + f x)}]}{2 f^2}$$

Result (type 4, 227 leaves):

$$a c x + \frac{1}{2} a d x^2 + \frac{b c \operatorname{Log}[\operatorname{Cosh}[e + f x]]}{f} - \left(b d \operatorname{Csch}[e] \left(-e^{-\operatorname{ArcTanh}[\operatorname{Coth}[e]]} f^2 x^2 + \frac{1}{\sqrt{1 - \operatorname{Coth}[e]^2}} i \operatorname{Coth}[e] (-f x (-\pi + 2 i \operatorname{ArcTanh}[\operatorname{Coth}[e]])) - \pi \operatorname{Log}[1 + e^{2 f x}] - 2 (i f x + i \operatorname{ArcTanh}[\operatorname{Coth}[e]]) \operatorname{Log}[1 - e^{2 i (i f x + i \operatorname{ArcTanh}[\operatorname{Coth}[e]])}] + \pi \operatorname{Log}[\operatorname{Cosh}[f x]] + 2 i \operatorname{ArcTanh}[\operatorname{Coth}[e]] \operatorname{Log}[i \operatorname{Sinh}[f x + \operatorname{ArcTanh}[\operatorname{Coth}[e]]]] + i \operatorname{PolyLog}[2, e^{2 i (i f x + i \operatorname{ArcTanh}[\operatorname{Coth}[e]])}] \right) \operatorname{Sech}[e] \right) / \left(2 f^2 \sqrt{\operatorname{Csch}[e]^2 (-\operatorname{Cosh}[e]^2 + \operatorname{Sinh}[e]^2)} \right) + \frac{1}{2} b d x^2 \operatorname{Tanh}[e]$$

Problem 58: Result more than twice size of optimal antiderivative.

$$\int (c + d x)^3 (a + b \operatorname{Tanh}[e + f x])^2 dx$$

Optimal (type 4, 277 leaves, 15 steps):

$$-\frac{b^2 (c + d x)^3}{f} + \frac{a^2 (c + d x)^4}{4 d} - \frac{a b (c + d x)^4}{2 d} + \frac{b^2 (c + d x)^4}{4 d} + \frac{3 b^2 d (c + d x)^2 \operatorname{Log}[1 + e^{2 (e + f x)}]}{f^2} + \frac{2 a b (c + d x)^3 \operatorname{Log}[1 + e^{2 (e + f x)}]}{f} + \frac{3 b^2 d^2 (c + d x) \operatorname{PolyLog}[2, -e^{2 (e + f x)}]}{f^3} + \frac{3 a b d (c + d x)^2 \operatorname{PolyLog}[2, -e^{2 (e + f x)}]}{f^2} - \frac{3 b^2 d^3 \operatorname{PolyLog}[3, -e^{2 (e + f x)}]}{2 f^4} - \frac{3 a b d^2 (c + d x) \operatorname{PolyLog}[3, -e^{2 (e + f x)}]}{f^3} + \frac{3 a b d^3 \operatorname{PolyLog}[4, -e^{2 (e + f x)}]}{2 f^4} - \frac{b^2 (c + d x)^3 \operatorname{Tanh}[e + f x]}{f}$$

Result (type 4, 1062 leaves):

$$\begin{aligned}
& \frac{1}{2(1+e^{2e})f} \\
& b e^{2e} \left(-12 b c^2 d x - 8 a c^3 f x - 12 b c d^2 x^2 - 12 a c^2 d f x^2 - 4 b d^3 x^3 - 8 a c d^2 f x^3 - 2 a d^3 f x^4 + 4 a c^3 \operatorname{Log}[1+e^{2(e+fx)}] + 4 a c^3 e^{-2e} \operatorname{Log}[1+e^{2(e+fx)}] + \right. \\
& \frac{6 b c^2 d \operatorname{Log}[1+e^{2(e+fx)}]}{f} + \frac{6 b c^2 d e^{-2e} \operatorname{Log}[1+e^{2(e+fx)}]}{f} + 12 a c^2 d x \operatorname{Log}[1+e^{2(e+fx)}] + 12 a c^2 d e^{-2e} x \operatorname{Log}[1+e^{2(e+fx)}] + \\
& \frac{12 b c d^2 x \operatorname{Log}[1+e^{2(e+fx)}]}{f} + \frac{12 b c d^2 e^{-2e} x \operatorname{Log}[1+e^{2(e+fx)}]}{f} + 12 a c d^2 x^2 \operatorname{Log}[1+e^{2(e+fx)}] + \\
& 12 a c d^2 e^{-2e} x^2 \operatorname{Log}[1+e^{2(e+fx)}] + \frac{6 b d^3 x^2 \operatorname{Log}[1+e^{2(e+fx)}]}{f} + \frac{6 b d^3 e^{-2e} x^2 \operatorname{Log}[1+e^{2(e+fx)}]}{f} + 4 a d^3 x^3 \operatorname{Log}[1+e^{2(e+fx)}] + \\
& 4 a d^3 e^{-2e} x^3 \operatorname{Log}[1+e^{2(e+fx)}] + \frac{6 d e^{-2e} (1+e^{2e}) (c+dx) (bd+af(c+dx)) \operatorname{PolyLog}[2, -e^{2(e+fx)}]}{f^2} - \\
& \left. \frac{3 d^2 e^{-2e} (1+e^{2e}) (bd+2af(c+dx)) \operatorname{PolyLog}[3, -e^{2(e+fx)}]}{f^3} + \frac{3 a d^3 \operatorname{PolyLog}[4, -e^{2(e+fx)}]}{f^3} + \frac{3 a d^3 e^{-2e} \operatorname{PolyLog}[4, -e^{2(e+fx)}]}{f^3} \right) + \\
& \frac{1}{8f} \operatorname{Sech}[e] \operatorname{Sech}[e+fx] (4 a^2 c^3 f x \operatorname{Cosh}[fx] + 4 b^2 c^3 f x \operatorname{Cosh}[fx] + 6 a^2 c^2 d f x^2 \operatorname{Cosh}[fx] + 6 b^2 c^2 d f x^2 \operatorname{Cosh}[fx] + \\
& 4 a^2 c d^2 f x^3 \operatorname{Cosh}[fx] + 4 b^2 c d^2 f x^3 \operatorname{Cosh}[fx] + a^2 d^3 f x^4 \operatorname{Cosh}[fx] + b^2 d^3 f x^4 \operatorname{Cosh}[fx] + 4 a^2 c^3 f x \operatorname{Cosh}[2e+fx] + \\
& 4 b^2 c^3 f x \operatorname{Cosh}[2e+fx] + 6 a^2 c^2 d f x^2 \operatorname{Cosh}[2e+fx] + 6 b^2 c^2 d f x^2 \operatorname{Cosh}[2e+fx] + 4 a^2 c d^2 f x^3 \operatorname{Cosh}[2e+fx] + \\
& 4 b^2 c d^2 f x^3 \operatorname{Cosh}[2e+fx] + a^2 d^3 f x^4 \operatorname{Cosh}[2e+fx] + b^2 d^3 f x^4 \operatorname{Cosh}[2e+fx] - 8 b^2 c^3 \operatorname{Sinh}[fx] - 24 b^2 c^2 d x \operatorname{Sinh}[fx] - \\
& 8 a b c^3 f x \operatorname{Sinh}[fx] - 24 b^2 c d^2 x^2 \operatorname{Sinh}[fx] - 12 a b c^2 d f x^2 \operatorname{Sinh}[fx] - 8 b^2 d^3 x^3 \operatorname{Sinh}[fx] - 8 a b c d^2 f x^3 \operatorname{Sinh}[fx] - \\
& 2 a b d^3 f x^4 \operatorname{Sinh}[fx] + 8 a b c^3 f x \operatorname{Sinh}[2e+fx] + 12 a b c^2 d f x^2 \operatorname{Sinh}[2e+fx] + 8 a b c d^2 f x^3 \operatorname{Sinh}[2e+fx] + 2 a b d^3 f x^4 \operatorname{Sinh}[2e+fx])
\end{aligned}$$

Problem 63: Result more than twice size of optimal antiderivative.

$$\int (c+dx)^3 (a+b \operatorname{Tanh}[e+fx])^3 dx$$

Optimal (type 4, 566 leaves, 28 steps):

$$\begin{aligned}
& - \frac{3 b^3 d (c + d x)^2}{2 f^2} - \frac{3 a b^2 (c + d x)^3}{f} + \frac{b^3 (c + d x)^3}{2 f} + \frac{a^3 (c + d x)^4}{4 d} - \frac{3 a^2 b (c + d x)^4}{4 d} + \frac{3 a b^2 (c + d x)^4}{4 d} - \\
& \frac{b^3 (c + d x)^4}{4 d} + \frac{3 b^3 d^2 (c + d x) \operatorname{Log}[1 + e^{2(e+f x)}]}{f^3} + \frac{9 a b^2 d (c + d x)^2 \operatorname{Log}[1 + e^{2(e+f x)}]}{f^2} + \frac{3 a^2 b (c + d x)^3 \operatorname{Log}[1 + e^{2(e+f x)}]}{f} + \\
& \frac{b^3 (c + d x)^3 \operatorname{Log}[1 + e^{2(e+f x)}]}{f} + \frac{3 b^3 d^3 \operatorname{PolyLog}[2, -e^{2(e+f x)}]}{2 f^4} + \frac{9 a b^2 d^2 (c + d x) \operatorname{PolyLog}[2, -e^{2(e+f x)}]}{f^3} + \\
& \frac{9 a^2 b d (c + d x)^2 \operatorname{PolyLog}[2, -e^{2(e+f x)}]}{2 f^2} + \frac{3 b^3 d (c + d x)^2 \operatorname{PolyLog}[2, -e^{2(e+f x)}]}{2 f^2} - \frac{9 a b^2 d^3 \operatorname{PolyLog}[3, -e^{2(e+f x)}]}{2 f^4} - \\
& \frac{9 a^2 b d^2 (c + d x) \operatorname{PolyLog}[3, -e^{2(e+f x)}]}{2 f^3} - \frac{3 b^3 d^2 (c + d x) \operatorname{PolyLog}[3, -e^{2(e+f x)}]}{2 f^3} + \frac{9 a^2 b d^3 \operatorname{PolyLog}[4, -e^{2(e+f x)}]}{4 f^4} + \\
& \frac{3 b^3 d^3 \operatorname{PolyLog}[4, -e^{2(e+f x)}]}{4 f^4} - \frac{3 b^3 d (c + d x)^2 \operatorname{Tanh}[e + f x]}{2 f^2} - \frac{3 a b^2 (c + d x)^3 \operatorname{Tanh}[e + f x]}{f} - \frac{b^3 (c + d x)^3 \operatorname{Tanh}[e + f x]^2}{2 f}
\end{aligned}$$

Result (type 4, 2010 leaves):

$$\begin{aligned}
& \frac{1}{4 (1 + e^{2e}) f^2} \\
& b e^{2e} \left(-24 b^2 c d^2 x - 72 a b c^2 d f x - 24 a^2 c^3 f^2 x - 8 b^2 c^3 f^2 x - 12 b^2 d^3 x^2 - 72 a b c d^2 f x^2 - 36 a^2 c^2 d f^2 x^2 - 12 b^2 c^2 d f^2 x^2 - 24 a b d^3 f x^3 - \right. \\
& 24 a^2 c d^2 f^2 x^3 - 8 b^2 c d^2 f^2 x^3 - 6 a^2 d^3 f^2 x^4 - 2 b^2 d^3 f^2 x^4 + 36 a b c^2 d \operatorname{Log}[1 + e^{2(e+f x)}] + 36 a b c^2 d e^{-2e} \operatorname{Log}[1 + e^{2(e+f x)}] + \\
& \frac{12 b^2 c d^2 \operatorname{Log}[1 + e^{2(e+f x)}]}{f} + \frac{12 b^2 c d^2 e^{-2e} \operatorname{Log}[1 + e^{2(e+f x)}]}{f} + 12 a^2 c^3 f \operatorname{Log}[1 + e^{2(e+f x)}] + 4 b^2 c^3 f \operatorname{Log}[1 + e^{2(e+f x)}] + \\
& 12 a^2 c^3 e^{-2e} f \operatorname{Log}[1 + e^{2(e+f x)}] + 4 b^2 c^3 e^{-2e} f \operatorname{Log}[1 + e^{2(e+f x)}] + 72 a b c d^2 x \operatorname{Log}[1 + e^{2(e+f x)}] + 72 a b c d^2 e^{-2e} x \operatorname{Log}[1 + e^{2(e+f x)}] + \\
& \frac{12 b^2 d^3 x \operatorname{Log}[1 + e^{2(e+f x)}]}{f} + \frac{12 b^2 d^3 e^{-2e} x \operatorname{Log}[1 + e^{2(e+f x)}]}{f} + 36 a^2 c^2 d f x \operatorname{Log}[1 + e^{2(e+f x)}] + 12 b^2 c^2 d f x \operatorname{Log}[1 + e^{2(e+f x)}] + \\
& 36 a^2 c^2 d e^{-2e} f x \operatorname{Log}[1 + e^{2(e+f x)}] + 12 b^2 c^2 d e^{-2e} f x \operatorname{Log}[1 + e^{2(e+f x)}] + 36 a b d^3 x^2 \operatorname{Log}[1 + e^{2(e+f x)}] + 36 a b d^3 e^{-2e} x^2 \operatorname{Log}[1 + e^{2(e+f x)}] + \\
& 36 a^2 c d^2 f x^2 \operatorname{Log}[1 + e^{2(e+f x)}] + 12 b^2 c d^2 f x^2 \operatorname{Log}[1 + e^{2(e+f x)}] + 36 a^2 c d^2 e^{-2e} f x^2 \operatorname{Log}[1 + e^{2(e+f x)}] + 12 b^2 c d^2 e^{-2e} f x^2 \operatorname{Log}[1 + e^{2(e+f x)}] + \\
& 12 a^2 d^3 f x^3 \operatorname{Log}[1 + e^{2(e+f x)}] + 4 b^2 d^3 f x^3 \operatorname{Log}[1 + e^{2(e+f x)}] + 12 a^2 d^3 e^{-2e} f x^3 \operatorname{Log}[1 + e^{2(e+f x)}] + 4 b^2 d^3 e^{-2e} f x^3 \operatorname{Log}[1 + e^{2(e+f x)}] + \\
& \frac{1}{f^2} 6 d e^{-2e} (1 + e^{2e}) (6 a b d f (c + d x) + 3 a^2 f^2 (c + d x)^2 + b^2 (d^2 + c^2 f^2 + 2 c d f^2 x + d^2 f^2 x^2)) \operatorname{PolyLog}[2, -e^{2(e+f x)}] - \\
& \frac{6 d^2 e^{-2e} (1 + e^{2e}) (3 a b d + 3 a^2 f (c + d x) + b^2 f (c + d x)) \operatorname{PolyLog}[3, -e^{2(e+f x)}]}{f^2} + \frac{9 a^2 d^3 \operatorname{PolyLog}[4, -e^{2(e+f x)}]}{f^2} + \\
& \left. \frac{3 b^2 d^3 \operatorname{PolyLog}[4, -e^{2(e+f x)}]}{f^2} + \frac{9 a^2 d^3 e^{-2e} \operatorname{PolyLog}[4, -e^{2(e+f x)}]}{f^2} + \frac{3 b^2 d^3 e^{-2e} \operatorname{PolyLog}[4, -e^{2(e+f x)}]}{f^2} \right) + \\
& \frac{(b^3 c^3 + 3 b^3 c^2 d x + 3 b^3 c d^2 x^2 + b^3 d^3 x^3) \operatorname{Sech}[e + f x]^2}{2 f} + \\
& (3 x^2 (a^3 c^2 d - 3 a^2 b c^2 d + 3 a b^2 c^2 d - b^3 c^2 d + a^3 c^2 d \operatorname{Cosh}[2 e] + 3 a^2 b c^2 d \operatorname{Cosh}[2 e] + 3 a b^2 c^2 d \operatorname{Cosh}[2 e] + b^3 c^2 d \operatorname{Cosh}[2 e] +
\end{aligned}$$

$$\begin{aligned}
& \left(a^3 c^2 d \operatorname{Sinh}[2e] + 3 a^2 b c^2 d \operatorname{Sinh}[2e] + 3 a b^2 c^2 d \operatorname{Sinh}[2e] + b^3 c^2 d \operatorname{Sinh}[2e] \right) / \left(2 \left(1 + \operatorname{Cosh}[2e] + \operatorname{Sinh}[2e] \right) \right) + \\
& \frac{1}{1 + \operatorname{Cosh}[2e] + \operatorname{Sinh}[2e]} x^3 \left(a^3 c d^2 - 3 a^2 b c d^2 + 3 a b^2 c d^2 - b^3 c d^2 + a^3 c d^2 \operatorname{Cosh}[2e] + 3 a^2 b c d^2 \operatorname{Cosh}[2e] + 3 a b^2 c d^2 \operatorname{Cosh}[2e] + \right. \\
& \left. b^3 c d^2 \operatorname{Cosh}[2e] + a^3 c d^2 \operatorname{Sinh}[2e] + 3 a^2 b c d^2 \operatorname{Sinh}[2e] + 3 a b^2 c d^2 \operatorname{Sinh}[2e] + b^3 c d^2 \operatorname{Sinh}[2e] \right) + \\
& \left(x^4 \left(a^3 d^3 - 3 a^2 b d^3 + 3 a b^2 d^3 - b^3 d^3 + a^3 d^3 \operatorname{Cosh}[2e] + 3 a^2 b d^3 \operatorname{Cosh}[2e] + 3 a b^2 d^3 \operatorname{Cosh}[2e] + b^3 d^3 \operatorname{Cosh}[2e] + \right. \right. \\
& \left. \left. a^3 d^3 \operatorname{Sinh}[2e] + 3 a^2 b d^3 \operatorname{Sinh}[2e] + 3 a b^2 d^3 \operatorname{Sinh}[2e] + b^3 d^3 \operatorname{Sinh}[2e] \right) \right) / \left(4 \left(1 + \operatorname{Cosh}[2e] + \operatorname{Sinh}[2e] \right) \right) + \\
& x \left(a^3 c^3 + 3 a b^2 c^3 - \frac{3 a^2 b c^3}{1 + \operatorname{Cosh}[2e] + \operatorname{Sinh}[2e]} + \frac{3 a^2 b c^3 \operatorname{Cosh}[2e] + 3 a^2 b c^3 \operatorname{Sinh}[2e]}{1 + \operatorname{Cosh}[2e] + \operatorname{Sinh}[2e]} + \right. \\
& \left. \frac{2 b^3 c^3 \operatorname{Cosh}[2e] + 2 b^3 c^3 \operatorname{Sinh}[2e]}{\left(1 + \operatorname{Cosh}[2e] + \operatorname{Sinh}[2e] \right) \left(1 - \operatorname{Cosh}[2e] + \operatorname{Cosh}[4e] - \operatorname{Sinh}[2e] + \operatorname{Sinh}[4e] \right)} + \right. \\
& \left. \frac{-2 b^3 c^3 \operatorname{Cosh}[4e] - 2 b^3 c^3 \operatorname{Sinh}[4e]}{\left(1 + \operatorname{Cosh}[2e] + \operatorname{Sinh}[2e] \right) \left(1 - \operatorname{Cosh}[2e] + \operatorname{Cosh}[4e] - \operatorname{Sinh}[2e] + \operatorname{Sinh}[4e] \right)} - \right. \\
& \left. \frac{b^3 c^3}{1 + \operatorname{Cosh}[6e] + \operatorname{Sinh}[6e]} + \frac{b^3 c^3 \operatorname{Cosh}[6e] + b^3 c^3 \operatorname{Sinh}[6e]}{1 + \operatorname{Cosh}[6e] + \operatorname{Sinh}[6e]} \right) - \frac{1}{2 f^2} \\
& 3 \operatorname{Sech}[e] \operatorname{Sech}[e + f x] \left(b^3 c^2 d \operatorname{Sinh}[f x] + 2 a b^2 c^3 f \operatorname{Sinh}[f x] + 2 b^3 c d^2 x \operatorname{Sinh}[f x] + 6 a b^2 c^2 d f x \operatorname{Sinh}[f x] + \right. \\
& \left. b^3 d^3 x^2 \operatorname{Sinh}[f x] + 6 a b^2 c d^2 f x^2 \operatorname{Sinh}[f x] + 2 a b^2 d^3 f x^3 \operatorname{Sinh}[f x] \right)
\end{aligned}$$

Problem 64: Result more than twice size of optimal antiderivative.

$$\int (c + d x)^2 (a + b \operatorname{Tanh}[e + f x])^3 dx$$

Optimal (type 4, 405 leaves, 22 steps):

$$\begin{aligned}
& \frac{b^3 c d x}{f} + \frac{b^3 d^2 x^2}{2 f} - \frac{3 a b^2 (c + d x)^2}{f} + \frac{a^3 (c + d x)^3}{3 d} - \frac{a^2 b (c + d x)^3}{d} + \frac{a b^2 (c + d x)^3}{d} - \frac{b^3 (c + d x)^3}{3 d} + \frac{6 a b^2 d (c + d x) \operatorname{Log}\left[1 + e^{2(e + f x)}\right]}{f^2} + \\
& \frac{3 a^2 b (c + d x)^2 \operatorname{Log}\left[1 + e^{2(e + f x)}\right]}{f} + \frac{b^3 (c + d x)^2 \operatorname{Log}\left[1 + e^{2(e + f x)}\right]}{f} + \frac{b^3 d^2 \operatorname{Log}\left[\operatorname{Cosh}[e + f x]\right]}{f^3} + \frac{3 a b^2 d^2 \operatorname{PolyLog}\left[2, -e^{2(e + f x)}\right]}{f^3} + \\
& \frac{3 a^2 b d (c + d x) \operatorname{PolyLog}\left[2, -e^{2(e + f x)}\right]}{f^2} + \frac{b^3 d (c + d x) \operatorname{PolyLog}\left[2, -e^{2(e + f x)}\right]}{f^2} - \frac{3 a^2 b d^2 \operatorname{PolyLog}\left[3, -e^{2(e + f x)}\right]}{2 f^3} - \\
& \frac{b^3 d^2 \operatorname{PolyLog}\left[3, -e^{2(e + f x)}\right]}{2 f^3} - \frac{b^3 d (c + d x) \operatorname{Tanh}[e + f x]}{f^2} - \frac{3 a b^2 (c + d x)^2 \operatorname{Tanh}[e + f x]}{f} - \frac{b^3 (c + d x)^2 \operatorname{Tanh}[e + f x]^2}{2 f}
\end{aligned}$$

Result (type 4, 1142 leaves):

$$\frac{1}{6 f^3} b \left(-\frac{4 e^{2e} f x (9 a b d f (2 c + d x) + 3 a^2 f^2 (3 c^2 + 3 c d x + d^2 x^2) + b^2 (3 c^2 f^2 + 3 c d f^2 x + d^2 (3 + f^2 x^2)))}{1 + e^{2e}} + \right. \\ \left. 6 (6 a b d f (c + d x) + 3 a^2 f^2 (c + d x)^2 + b^2 (c^2 f^2 + 2 c d f^2 x + d^2 (1 + f^2 x^2))) \operatorname{Log}[1 + e^{2(e+fx)}] + \right. \\ \left. 6 d (3 a b d + 3 a^2 f (c + d x) + b^2 f (c + d x)) \operatorname{PolyLog}[2, -e^{2(e+fx)}] - 3 (3 a^2 + b^2) d^2 \operatorname{PolyLog}[3, -e^{2(e+fx)}] \right) + \\ \frac{1}{12 f^2} \operatorname{Sech}[e] \operatorname{Sech}[e + f x]^2 (6 b^3 c^2 f \operatorname{Cosh}[e] + 12 b^3 c d f x \operatorname{Cosh}[e] + 6 a^3 c^2 f^2 x \operatorname{Cosh}[e] + 18 a b^2 c^2 f^2 x \operatorname{Cosh}[e] + 6 b^3 d^2 f x^2 \operatorname{Cosh}[e] + \\ 6 a^3 c d f^2 x^2 \operatorname{Cosh}[e] + 18 a b^2 c d f^2 x^2 \operatorname{Cosh}[e] + 2 a^3 d^2 f^2 x^3 \operatorname{Cosh}[e] + 6 a b^2 d^2 f^2 x^3 \operatorname{Cosh}[e] + 3 a^3 c^2 f^2 x \operatorname{Cosh}[e + 2 f x] + \\ 9 a b^2 c^2 f^2 x \operatorname{Cosh}[e + 2 f x] + 3 a^3 c d f^2 x^2 \operatorname{Cosh}[e + 2 f x] + 9 a b^2 c d f^2 x^2 \operatorname{Cosh}[e + 2 f x] + a^3 d^2 f^2 x^3 \operatorname{Cosh}[e + 2 f x] + \\ 3 a b^2 d^2 f^2 x^3 \operatorname{Cosh}[e + 2 f x] + 3 a^3 c^2 f^2 x \operatorname{Cosh}[3 e + 2 f x] + 9 a b^2 c^2 f^2 x \operatorname{Cosh}[3 e + 2 f x] + 3 a^3 c d f^2 x^2 \operatorname{Cosh}[3 e + 2 f x] + \\ 9 a b^2 c d f^2 x^2 \operatorname{Cosh}[3 e + 2 f x] + a^3 d^2 f^2 x^3 \operatorname{Cosh}[3 e + 2 f x] + 3 a b^2 d^2 f^2 x^3 \operatorname{Cosh}[3 e + 2 f x] + 6 b^3 c d \operatorname{Sinh}[e] + 18 a b^2 c^2 f \operatorname{Sinh}[e] + \\ 6 b^3 d^2 x \operatorname{Sinh}[e] + 36 a b^2 c d f x \operatorname{Sinh}[e] + 18 a^2 b c^2 f^2 x \operatorname{Sinh}[e] + 6 b^3 c^2 f^2 x \operatorname{Sinh}[e] + 18 a b^2 d^2 f x^2 \operatorname{Sinh}[e] + \\ 18 a^2 b c d f^2 x^2 \operatorname{Sinh}[e] + 6 b^3 c d f^2 x^2 \operatorname{Sinh}[e] + 6 a^2 b d^2 f^2 x^3 \operatorname{Sinh}[e] + 2 b^3 d^2 f^2 x^3 \operatorname{Sinh}[e] - 6 b^3 c d \operatorname{Sinh}[e + 2 f x] - \\ 18 a b^2 c^2 f \operatorname{Sinh}[e + 2 f x] - 6 b^3 d^2 x \operatorname{Sinh}[e + 2 f x] - 36 a b^2 c d f x \operatorname{Sinh}[e + 2 f x] - 9 a^2 b c^2 f^2 x \operatorname{Sinh}[e + 2 f x] - \\ 3 b^3 c^2 f^2 x \operatorname{Sinh}[e + 2 f x] - 18 a b^2 d^2 f x^2 \operatorname{Sinh}[e + 2 f x] - 9 a^2 b c d f^2 x^2 \operatorname{Sinh}[e + 2 f x] - 3 b^3 c d f^2 x^2 \operatorname{Sinh}[e + 2 f x] - \\ 3 a^2 b d^2 f^2 x^3 \operatorname{Sinh}[e + 2 f x] - b^3 d^2 f^2 x^3 \operatorname{Sinh}[e + 2 f x] + 9 a^2 b c^2 f^2 x \operatorname{Sinh}[3 e + 2 f x] + 3 b^3 c^2 f^2 x \operatorname{Sinh}[3 e + 2 f x] + \\ 9 a^2 b c d f^2 x^2 \operatorname{Sinh}[3 e + 2 f x] + 3 b^3 c d f^2 x^2 \operatorname{Sinh}[3 e + 2 f x] + 3 a^2 b d^2 f^2 x^3 \operatorname{Sinh}[3 e + 2 f x] + b^3 d^2 f^2 x^3 \operatorname{Sinh}[3 e + 2 f x])$$

Problem 73: Result more than twice size of optimal antiderivative.

$$\int \frac{(c + d x)^3}{(a + b \operatorname{Tanh}[e + f x])^2} dx$$

Optimal (type 4, 642 leaves, 28 steps):

$$-\frac{2 b^2 (c + d x)^3}{(a^2 - b^2)^2 f} + \frac{2 b^2 (c + d x)^3}{(a - b) (a + b)^2 (a - b + (a + b) e^{2e+2fx}) f} + \frac{(c + d x)^4}{4 (a - b)^2 d} + \frac{3 b^2 d (c + d x)^2 \operatorname{Log}\left[1 + \frac{(a+b) e^{2e+2fx}}{a-b}\right]}{(a^2 - b^2)^2 f^2} - \frac{2 b (c + d x)^3 \operatorname{Log}\left[1 + \frac{(a+b) e^{2e+2fx}}{a-b}\right]}{(a - b)^2 (a + b) f} + \\ \frac{2 b^2 (c + d x)^3 \operatorname{Log}\left[1 + \frac{(a+b) e^{2e+2fx}}{a-b}\right]}{(a^2 - b^2)^2 f} + \frac{3 b^2 d^2 (c + d x) \operatorname{PolyLog}\left[2, -\frac{(a+b) e^{2e+2fx}}{a-b}\right]}{(a^2 - b^2)^2 f^3} - \frac{3 b d (c + d x)^2 \operatorname{PolyLog}\left[2, -\frac{(a+b) e^{2e+2fx}}{a-b}\right]}{(a - b)^2 (a + b) f^2} + \\ \frac{3 b^2 d (c + d x)^2 \operatorname{PolyLog}\left[2, -\frac{(a+b) e^{2e+2fx}}{a-b}\right]}{(a^2 - b^2)^2 f^2} - \frac{3 b^2 d^3 \operatorname{PolyLog}\left[3, -\frac{(a+b) e^{2e+2fx}}{a-b}\right]}{2 (a^2 - b^2)^2 f^4} + \frac{3 b d^2 (c + d x) \operatorname{PolyLog}\left[3, -\frac{(a+b) e^{2e+2fx}}{a-b}\right]}{(a - b)^2 (a + b) f^3} - \\ \frac{3 b^2 d^2 (c + d x) \operatorname{PolyLog}\left[3, -\frac{(a+b) e^{2e+2fx}}{a-b}\right]}{(a^2 - b^2)^2 f^3} - \frac{3 b d^3 \operatorname{PolyLog}\left[4, -\frac{(a+b) e^{2e+2fx}}{a-b}\right]}{2 (a - b)^2 (a + b) f^4} + \frac{3 b^2 d^3 \operatorname{PolyLog}\left[4, -\frac{(a+b) e^{2e+2fx}}{a-b}\right]}{2 (a^2 - b^2)^2 f^4}$$

Result (type 4, 2119 leaves):

$$\begin{aligned}
& - \frac{1}{2(a-b)^2(a+b)^2(b(-1+e^{2e})+a(1+e^{2e}))f^4} \\
& b \left(12abc^2de^{2e}f^3x + 12b^2c^2de^{2e}f^3x - 8a^2c^3e^{2e}f^4x - 8abc^3e^{2e}f^4x + 12abc d^2e^{2e}f^3x^2 + 12b^2cd^2e^{2e}f^3x^2 - \right. \\
& 12a^2c^2de^{2e}f^4x^2 - 12abc^2de^{2e}f^4x^2 + 4abd^3e^{2e}f^3x^3 + 4b^2d^3e^{2e}f^3x^3 - 8a^2cd^2e^{2e}f^4x^3 - 8abc d^2e^{2e}f^4x^3 - \\
& 2a^2d^3e^{2e}f^4x^4 - 2abd^3e^{2e}f^4x^4 - 12abc d^2f^2x \operatorname{Log}\left[1 + \frac{(a+b)e^{2(e+fx)}}{a-b}\right] + 12b^2cd^2f^2x \operatorname{Log}\left[1 + \frac{(a+b)e^{2(e+fx)}}{a-b}\right] - \\
& 12abc d^2e^{2e}f^2x \operatorname{Log}\left[1 + \frac{(a+b)e^{2(e+fx)}}{a-b}\right] - 12b^2cd^2e^{2e}f^2x \operatorname{Log}\left[1 + \frac{(a+b)e^{2(e+fx)}}{a-b}\right] + 12a^2c^2df^3x \operatorname{Log}\left[1 + \frac{(a+b)e^{2(e+fx)}}{a-b}\right] - \\
& 12abc^2df^3x \operatorname{Log}\left[1 + \frac{(a+b)e^{2(e+fx)}}{a-b}\right] + 12a^2c^2de^{2e}f^3x \operatorname{Log}\left[1 + \frac{(a+b)e^{2(e+fx)}}{a-b}\right] + 12abc^2de^{2e}f^3x \operatorname{Log}\left[1 + \frac{(a+b)e^{2(e+fx)}}{a-b}\right] - \\
& 6abd^3f^2x^2 \operatorname{Log}\left[1 + \frac{(a+b)e^{2(e+fx)}}{a-b}\right] + 6b^2d^3f^2x^2 \operatorname{Log}\left[1 + \frac{(a+b)e^{2(e+fx)}}{a-b}\right] - 6abd^3e^{2e}f^2x^2 \operatorname{Log}\left[1 + \frac{(a+b)e^{2(e+fx)}}{a-b}\right] - \\
& 6b^2d^3e^{2e}f^2x^2 \operatorname{Log}\left[1 + \frac{(a+b)e^{2(e+fx)}}{a-b}\right] + 12a^2cd^2f^3x^2 \operatorname{Log}\left[1 + \frac{(a+b)e^{2(e+fx)}}{a-b}\right] - 12abc d^2f^3x^2 \operatorname{Log}\left[1 + \frac{(a+b)e^{2(e+fx)}}{a-b}\right] + \\
& 12a^2cd^2e^{2e}f^3x^2 \operatorname{Log}\left[1 + \frac{(a+b)e^{2(e+fx)}}{a-b}\right] + 12abc d^2e^{2e}f^3x^2 \operatorname{Log}\left[1 + \frac{(a+b)e^{2(e+fx)}}{a-b}\right] + 4a^2d^3f^3x^3 \operatorname{Log}\left[1 + \frac{(a+b)e^{2(e+fx)}}{a-b}\right] - \\
& 4abd^3f^3x^3 \operatorname{Log}\left[1 + \frac{(a+b)e^{2(e+fx)}}{a-b}\right] + 4a^2d^3e^{2e}f^3x^3 \operatorname{Log}\left[1 + \frac{(a+b)e^{2(e+fx)}}{a-b}\right] + 4abd^3e^{2e}f^3x^3 \operatorname{Log}\left[1 + \frac{(a+b)e^{2(e+fx)}}{a-b}\right] - \\
& 6abc^2df^2 \operatorname{Log}\left[b(-1+e^{2(e+fx)})+a(1+e^{2(e+fx)})\right] + 6b^2c^2df^2 \operatorname{Log}\left[b(-1+e^{2(e+fx)})+a(1+e^{2(e+fx)})\right] - \\
& 6abc^2de^{2e}f^2 \operatorname{Log}\left[b(-1+e^{2(e+fx)})+a(1+e^{2(e+fx)})\right] - 6b^2c^2de^{2e}f^2 \operatorname{Log}\left[b(-1+e^{2(e+fx)})+a(1+e^{2(e+fx)})\right] + \\
& 4a^2c^3f^3 \operatorname{Log}\left[b(-1+e^{2(e+fx)})+a(1+e^{2(e+fx)})\right] - 4abc^3f^3 \operatorname{Log}\left[b(-1+e^{2(e+fx)})+a(1+e^{2(e+fx)})\right] + \\
& 4a^2c^3e^{2e}f^3 \operatorname{Log}\left[b(-1+e^{2(e+fx)})+a(1+e^{2(e+fx)})\right] + 4abc^3e^{2e}f^3 \operatorname{Log}\left[b(-1+e^{2(e+fx)})+a(1+e^{2(e+fx)})\right] + \\
& 6d(b(-1+e^{2e})+a(1+e^{2e}))f(c+dx)(-bd+af(c+dx)) \operatorname{PolyLog}\left[2, -\frac{(a+b)e^{2(e+fx)}}{a-b}\right] - \\
& 3d^2(b(-1+e^{2e})+a(1+e^{2e}))(-bd+2af(c+dx)) \operatorname{PolyLog}\left[3, -\frac{(a+b)e^{2(e+fx)}}{a-b}\right] + 3a^2d^3 \operatorname{PolyLog}\left[4, -\frac{(a+b)e^{2(e+fx)}}{a-b}\right] - \\
& 3abd^3 \operatorname{PolyLog}\left[4, -\frac{(a+b)e^{2(e+fx)}}{a-b}\right] + 3a^2d^3e^{2e} \operatorname{PolyLog}\left[4, -\frac{(a+b)e^{2(e+fx)}}{a-b}\right] + 3abd^3e^{2e} \operatorname{PolyLog}\left[4, -\frac{(a+b)e^{2(e+fx)}}{a-b}\right] \Big) + \\
& (4a^2c^3fx \operatorname{Cosh}[fx] + 4b^2c^3fx \operatorname{Cosh}[fx] + 6a^2c^2dfx^2 \operatorname{Cosh}[fx] + 6b^2c^2dfx^2 \operatorname{Cosh}[fx] + 4a^2cd^2fx^3 \operatorname{Cosh}[fx] + \\
& 4b^2cd^2fx^3 \operatorname{Cosh}[fx] + a^2d^3fx^4 \operatorname{Cosh}[fx] + b^2d^3fx^4 \operatorname{Cosh}[fx] + 4a^2c^3fx \operatorname{Cosh}[2e+fx] - \\
& 4b^2c^3fx \operatorname{Cosh}[2e+fx] + 6a^2c^2dfx^2 \operatorname{Cosh}[2e+fx] - 6b^2c^2dfx^2 \operatorname{Cosh}[2e+fx] + \\
& 4a^2cd^2fx^3 \operatorname{Cosh}[2e+fx] - 4b^2cd^2fx^3 \operatorname{Cosh}[2e+fx] + a^2d^3fx^4 \operatorname{Cosh}[2e+fx] - b^2d^3fx^4 \operatorname{Cosh}[2e+fx] - \\
& 8b^2c^3 \operatorname{Sinh}[fx] - 24b^2c^2dx \operatorname{Sinh}[fx] + 8abc^3fx \operatorname{Sinh}[fx] - 24b^2cd^2x^2 \operatorname{Sinh}[fx] + \\
& 12abc^2dfx^2 \operatorname{Sinh}[fx] - 8b^2d^3x^3 \operatorname{Sinh}[fx] + 8abc d^2fx^3 \operatorname{Sinh}[fx] + 2abd^3fx^4 \operatorname{Sinh}[fx]) / \\
& (8(a-b)(a+b)f(a \operatorname{Cosh}[e] + b \operatorname{Sinh}[e])(a \operatorname{Cosh}[e+fx] + b \operatorname{Sinh}[e+fx]))
\end{aligned}$$

Problem 75: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{c + d x}{(a + b \operatorname{Tanh}[e + f x])^2} dx$$

Optimal (type 4, 196 leaves, 5 steps):

$$-\frac{(c + d x)^2}{2 (a^2 - b^2) d} + \frac{(b d - 2 a c f - 2 a d f x)^2}{4 a (a - b) (a + b)^2 d f^2} + \frac{b (b d - 2 a c f - 2 a d f x) \operatorname{Log}\left[1 + \frac{(a-b) e^{-2(e+fx)}}{a+b}\right]}{(a^2 - b^2)^2 f^2} +$$

$$\frac{a b d \operatorname{PolyLog}\left[2, -\frac{(a-b) e^{-2(e+fx)}}{a+b}\right]}{(a^2 - b^2)^2 f^2} + \frac{b (c + d x)}{(a^2 - b^2) f (a + b \operatorname{Tanh}[e + f x])}$$

Result (type 4, 751 leaves):

$$\begin{aligned}
& \frac{(e + f x) (-2 d e + 2 c f + d (e + f x)) \operatorname{Sech}[e + f x]^2 (a \operatorname{Cosh}[e + f x] + b \operatorname{Sinh}[e + f x])^2}{2 (a - b) (a + b) f^2 (a + b \operatorname{Tanh}[e + f x])^2} + \\
& \left(\frac{b^2 d (-b (e + f x) + a \operatorname{Log}[a \operatorname{Cosh}[e + f x] + b \operatorname{Sinh}[e + f x]]) \operatorname{Sech}[e + f x]^2 (a \operatorname{Cosh}[e + f x] + b \operatorname{Sinh}[e + f x])^2}{(a (a - b) (a + b) (a^2 - b^2) f^2 (a + b \operatorname{Tanh}[e + f x])^2)} + \right. \\
& \left(\frac{2 b d e (-b (e + f x) + a \operatorname{Log}[a \operatorname{Cosh}[e + f x] + b \operatorname{Sinh}[e + f x]]) \operatorname{Sech}[e + f x]^2 (a \operatorname{Cosh}[e + f x] + b \operatorname{Sinh}[e + f x])^2}{((a - b) (a + b) (a^2 - b^2) f^2 (a + b \operatorname{Tanh}[e + f x])^2)} - \right. \\
& \left. \frac{2 b c (-b (e + f x) + a \operatorname{Log}[a \operatorname{Cosh}[e + f x] + b \operatorname{Sinh}[e + f x]]) \operatorname{Sech}[e + f x]^2 (a \operatorname{Cosh}[e + f x] + b \operatorname{Sinh}[e + f x])^2}{((a - b) (a + b) (a^2 - b^2) f (a + b \operatorname{Tanh}[e + f x])^2)} + \right. \\
& \left. d \left(-e^{-\operatorname{ArcTanh}\left[\frac{a}{b}\right]} (e + f x)^2 + \frac{1}{\sqrt{1 - \frac{a^2}{b^2}}} b \right) \right. \\
& \left. i a \left(-(e + f x) \left(-\pi + 2 i \operatorname{ArcTanh}\left[\frac{a}{b}\right] \right) - \pi \operatorname{Log}\left[1 + e^{2(e + f x)}\right] - 2 \left(i (e + f x) + i \operatorname{ArcTanh}\left[\frac{a}{b}\right] \right) \operatorname{Log}\left[1 - e^{2 i \left(i (e + f x) + i \operatorname{ArcTanh}\left[\frac{a}{b}\right] \right)}\right] + \right. \right. \\
& \left. \left. \pi \operatorname{Log}\left[\operatorname{Cosh}[e + f x]\right] + 2 i \operatorname{ArcTanh}\left[\frac{a}{b}\right] \operatorname{Log}\left[i \operatorname{Sinh}\left[e + f x + \operatorname{ArcTanh}\left[\frac{a}{b}\right]\right]\right] + i \operatorname{PolyLog}\left[2, e^{2 i \left(i (e + f x) + i \operatorname{ArcTanh}\left[\frac{a}{b}\right] \right)}\right] \right) \right. \\
& \left. \operatorname{Sech}[e + f x]^2 (a \operatorname{Cosh}[e + f x] + b \operatorname{Sinh}[e + f x])^2 \right) / \left((a - b) (a + b) \sqrt{\frac{-a^2 + b^2}{b^2}} f^2 (a + b \operatorname{Tanh}[e + f x])^2 \right) + \\
& \frac{(\operatorname{Sech}[e + f x]^2 (a \operatorname{Cosh}[e + f x] + b \operatorname{Sinh}[e + f x]) (b^2 d e \operatorname{Sinh}[e + f x] - b^2 c f \operatorname{Sinh}[e + f x] - b^2 d (e + f x) \operatorname{Sinh}[e + f x]))}{(a (a - b) (a + b) f^2 (a + b \operatorname{Tanh}[e + f x])^2)}
\end{aligned}$$

Problem 76: Attempted integration timed out after 120 seconds.

$$\int \frac{1}{(c + d x) (a + b \operatorname{Tanh}[e + f x])^2} dx$$

Optimal (type 9, 22 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{(c + d x) (a + b \operatorname{Tanh}[e + f x])^2}, x\right]$$

Result (type 1, 1 leaves):

???

Problem 77: Attempted integration timed out after 120 seconds.

$$\int \frac{1}{(c + d x)^2 (a + b \operatorname{Tanh}[e + f x])^2} dx$$

Optimal (type 9, 22 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{(c + d x)^2 (a + b \operatorname{Tanh}[e + f x])^2}, x\right]$$

Result (type 1, 1 leaves):

???

Test results for the 247 problems in "6.3.2 Hyperbolic tangent functions.m"

Problem 41: Result more than twice size of optimal antiderivative.

$$\int (a + a \operatorname{Tanh}[c + d x])^5 dx$$

Optimal (type 3, 100 leaves, 5 steps):

$$16 a^5 x + \frac{16 a^5 \operatorname{Log}[\operatorname{Cosh}[c + d x]]}{d} - \frac{8 a^5 \operatorname{Tanh}[c + d x]}{d} - \frac{2 a^2 (a + a \operatorname{Tanh}[c + d x])^3}{3 d} - \frac{a (a + a \operatorname{Tanh}[c + d x])^4}{4 d} - \frac{2 a (a^2 + a^2 \operatorname{Tanh}[c + d x])^2}{d}$$

Result (type 3, 202 leaves):

$$\frac{1}{12 d} a^5 \operatorname{Sech}[c] \operatorname{Sech}[c + d x]^4$$

$$(18 \operatorname{Cosh}[3 c + 2 d x] + 48 d x \operatorname{Cosh}[3 c + 2 d x] + 12 d x \operatorname{Cosh}[3 c + 4 d x] + 12 d x \operatorname{Cosh}[5 c + 4 d x] + 48 \operatorname{Cosh}[3 c + 2 d x] \operatorname{Log}[\operatorname{Cosh}[c + d x]] + 12 \operatorname{Cosh}[3 c + 4 d x] \operatorname{Log}[\operatorname{Cosh}[c + d x]] + 12 \operatorname{Cosh}[5 c + 4 d x] \operatorname{Log}[\operatorname{Cosh}[c + d x]] + 6 \operatorname{Cosh}[c + 2 d x] (3 + 8 d x + 8 \operatorname{Log}[\operatorname{Cosh}[c + d x]]) + \operatorname{Cosh}[c] (33 + 72 d x + 72 \operatorname{Log}[\operatorname{Cosh}[c + d x]]) + 75 \operatorname{Sinh}[c] - 70 \operatorname{Sinh}[c + 2 d x] + 30 \operatorname{Sinh}[3 c + 2 d x] - 25 \operatorname{Sinh}[3 c + 4 d x])$$

Problem 42: Result more than twice size of optimal antiderivative.

$$\int (a + a \operatorname{Tanh}[c + d x])^4 dx$$

Optimal (type 3, 77 leaves, 4 steps):

$$8 a^4 x + \frac{8 a^4 \operatorname{Log}[\operatorname{Cosh}[c + d x]]}{d} - \frac{4 a^4 \operatorname{Tanh}[c + d x]}{d} - \frac{a (a + a \operatorname{Tanh}[c + d x])^3}{3 d} - \frac{(a^2 + a^2 \operatorname{Tanh}[c + d x])^2}{d}$$

Result (type 3, 178 leaves):

$$\frac{1}{6 d (\operatorname{Cosh}[d x] + \operatorname{Sinh}[d x])^4} a^4 \operatorname{Sech}[c] \operatorname{Sech}[c + d x]^3 (\operatorname{Cosh}[4 d x] + \operatorname{Sinh}[4 d x]) \\ (6 d x \operatorname{Cosh}[2 c + 3 d x] + 6 d x \operatorname{Cosh}[4 c + 3 d x] + 6 \operatorname{Cosh}[2 c + 3 d x] \operatorname{Log}[\operatorname{Cosh}[c + d x]] + 6 \operatorname{Cosh}[4 c + 3 d x] \operatorname{Log}[\operatorname{Cosh}[c + d x]] + 6 \operatorname{Cosh}[d x] \\ (1 + 3 d x + 3 \operatorname{Log}[\operatorname{Cosh}[c + d x]]) + 6 \operatorname{Cosh}[2 c + d x] (1 + 3 d x + 3 \operatorname{Log}[\operatorname{Cosh}[c + d x]]) - 21 \operatorname{Sinh}[d x] + 12 \operatorname{Sinh}[2 c + d x] - 11 \operatorname{Sinh}[2 c + 3 d x])$$

Problem 57: Result more than twice size of optimal antiderivative.

$$\int (a + b \operatorname{Tanh}[c + d x])^5 dx$$

Optimal (type 3, 142 leaves, 5 steps):

$$a (a^4 + 10 a^2 b^2 + 5 b^4) x + \frac{b (5 a^4 + 10 a^2 b^2 + b^4) \operatorname{Log}[\operatorname{Cosh}[c + d x]]}{d} - \frac{4 a b^2 (a^2 + b^2) \operatorname{Tanh}[c + d x]}{d} - \\ \frac{b (3 a^2 + b^2) (a + b \operatorname{Tanh}[c + d x])^2}{2 d} - \frac{2 a b (a + b \operatorname{Tanh}[c + d x])^3}{3 d} - \frac{b (a + b \operatorname{Tanh}[c + d x])^4}{4 d}$$

Result (type 3, 366 leaves):

$$-\frac{b^5 \operatorname{Cosh}[c + d x] (a + b \operatorname{Tanh}[c + d x])^5}{4 d (a \operatorname{Cosh}[c + d x] + b \operatorname{Sinh}[c + d x])^5} + \frac{b^3 (5 a^2 + b^2) \operatorname{Cosh}[c + d x]^3 (a + b \operatorname{Tanh}[c + d x])^5}{d (a \operatorname{Cosh}[c + d x] + b \operatorname{Sinh}[c + d x])^5} + \\ \frac{a (a^4 + 10 a^2 b^2 + 5 b^4) (c + d x) \operatorname{Cosh}[c + d x]^5 (a + b \operatorname{Tanh}[c + d x])^5}{d (a \operatorname{Cosh}[c + d x] + b \operatorname{Sinh}[c + d x])^5} + \frac{(5 a^4 b + 10 a^2 b^3 + b^5) \operatorname{Cosh}[c + d x]^5 \operatorname{Log}[\operatorname{Cosh}[c + d x]] (a + b \operatorname{Tanh}[c + d x])^5}{d (a \operatorname{Cosh}[c + d x] + b \operatorname{Sinh}[c + d x])^5} + \\ \frac{5 a b^4 \operatorname{Cosh}[c + d x]^2 \operatorname{Sinh}[c + d x] (a + b \operatorname{Tanh}[c + d x])^5}{3 d (a \operatorname{Cosh}[c + d x] + b \operatorname{Sinh}[c + d x])^5} - \frac{10 \operatorname{Cosh}[c + d x]^4 (3 a^3 b^2 \operatorname{Sinh}[c + d x] + 2 a b^4 \operatorname{Sinh}[c + d x]) (a + b \operatorname{Tanh}[c + d x])^5}{3 d (a \operatorname{Cosh}[c + d x] + b \operatorname{Sinh}[c + d x])^5}$$

Problem 73: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Csch}[x]}{1 + \operatorname{Tanh}[x]} dx$$

Optimal (type 3, 12 leaves, 8 steps):

$$-\operatorname{ArcTanh}[\operatorname{Cosh}[x]] + \operatorname{Cosh}[x] - \operatorname{Sinh}[x]$$

Result (type 3, 49 leaves):

$$\frac{\cosh[x] - \log\left[\cosh\left[\frac{x}{2}\right]\right] + \log\left[\sinh\left[\frac{x}{2}\right]\right] - \left(\log\left[\cosh\left[\frac{x}{2}\right]\right] - \log\left[\sinh\left[\frac{x}{2}\right]\right] + \sinh[x]\right) \tanh[x]}{1 + \tanh[x]}$$

Problem 75: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{csch}[x]^3}{1 + \tanh[x]} dx$$

Optimal (type 3, 18 leaves, 8 steps):

$$-\frac{1}{2} \operatorname{ArcTanh}[\cosh[x]] + \operatorname{csch}[x] - \frac{1}{2} \operatorname{Coth}[x] \operatorname{csch}[x]$$

Result (type 3, 59 leaves):

$$\frac{1}{8} \left(4 \operatorname{Coth}\left[\frac{x}{2}\right] - \operatorname{csch}\left[\frac{x}{2}\right]^2 - 4 \log\left[\cosh\left[\frac{x}{2}\right]\right] + 4 \log\left[\sinh\left[\frac{x}{2}\right]\right] - \operatorname{sech}\left[\frac{x}{2}\right]^2 - 4 \tanh\left[\frac{x}{2}\right] \right)$$

Problem 77: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{csch}[x]^5}{1 + \tanh[x]} dx$$

Optimal (type 3, 34 leaves, 9 steps):

$$\frac{1}{8} \operatorname{ArcTanh}[\cosh[x]] - \frac{1}{8} \operatorname{Coth}[x] \operatorname{csch}[x] + \frac{\operatorname{csch}[x]^3}{3} - \frac{1}{4} \operatorname{Coth}[x] \operatorname{csch}[x]^3$$

Result (type 3, 69 leaves):

$$\frac{1}{192} \operatorname{csch}[x]^4 \left(-42 \cosh[x] - 6 \cosh[3x] + 2 \sinh[x] \left(32 - 9 \left(\log\left[\cosh\left[\frac{x}{2}\right]\right] - \log\left[\sinh\left[\frac{x}{2}\right]\right] \right) \sinh[x] + 3 \left(\log\left[\cosh\left[\frac{x}{2}\right]\right] - \log\left[\sinh\left[\frac{x}{2}\right]\right] \right) \sinh[3x] \right)$$

Problem 79: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{csch}[x]^7}{1 + \tanh[x]} dx$$

Optimal (type 3, 44 leaves, 10 steps):

$$-\frac{1}{16} \text{ArcTanh}[\text{Cosh}[x]] + \frac{1}{16} \text{Coth}[x] \text{Csch}[x] - \frac{1}{24} \text{Coth}[x] \text{Csch}[x]^3 + \frac{\text{Csch}[x]^5}{5} - \frac{1}{6} \text{Coth}[x] \text{Csch}[x]^5$$

Result (type 3, 124 leaves):

$$\frac{1}{1920} \left(72 \text{Coth}\left[\frac{x}{2}\right] + 30 \text{Csch}\left[\frac{x}{2}\right]^2 - 120 \text{Log}\left[\text{Cosh}\left[\frac{x}{2}\right]\right] + 120 \text{Log}\left[\text{Sinh}\left[\frac{x}{2}\right]\right] + 30 \text{Sech}\left[\frac{x}{2}\right]^2 - 5 \text{Sech}\left[\frac{x}{2}\right]^6 - \right. \\ \left. 288 \text{Csch}[x]^3 \text{Sinh}\left[\frac{x}{2}\right]^4 - 384 \text{Csch}[x]^5 \text{Sinh}\left[\frac{x}{2}\right]^6 - 18 \text{Csch}\left[\frac{x}{2}\right]^4 \text{Sinh}[x] + \text{Csch}\left[\frac{x}{2}\right]^6 (-5 + 6 \text{Sinh}[x]) - 72 \text{Tanh}\left[\frac{x}{2}\right] \right)$$

Problem 144: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{x \text{Sech}[c + dx]^2}{a + b \text{Tanh}[c + dx]^2} dx$$

Optimal (type 4, 231 leaves, 9 steps):

$$\frac{x \text{Log}\left[1 + \frac{(a+b) e^{2c+2dx}}{a-2\sqrt{-a}\sqrt{b}-b}\right]}{2\sqrt{-a}\sqrt{b}d} - \frac{x \text{Log}\left[1 + \frac{(a+b) e^{2c+2dx}}{a+2\sqrt{-a}\sqrt{b}-b}\right]}{2\sqrt{-a}\sqrt{b}d} + \frac{\text{PolyLog}\left[2, -\frac{(a+b) e^{2c+2dx}}{a-2\sqrt{-a}\sqrt{b}-b}\right]}{4\sqrt{-a}\sqrt{b}d^2} - \frac{\text{PolyLog}\left[2, -\frac{(a+b) e^{2c+2dx}}{a+2\sqrt{-a}\sqrt{b}-b}\right]}{4\sqrt{-a}\sqrt{b}d^2}$$

Result (type 4, 690 leaves):

$$\begin{aligned}
& \left(2 \operatorname{ArcTan} \left[\frac{(\operatorname{Cosh}[c+dx] + \operatorname{Sinh}[c+dx]) (a \operatorname{Cosh}[c+dx] + b \operatorname{Sinh}[c+dx])}{\sqrt{a} \sqrt{b}} \right] - (c+dx) \operatorname{Log} \left[1 - \frac{\operatorname{Cosh}[c+dx] + \operatorname{Sinh}[c+dx]}{\sqrt{-\frac{\sqrt{a}-i\sqrt{b}}{\sqrt{a}+i\sqrt{b}}}} \right] - \right. \\
& (c+dx) \operatorname{Log} \left[1 + \frac{\operatorname{Cosh}[c+dx] + \operatorname{Sinh}[c+dx]}{\sqrt{-\frac{\sqrt{a}-i\sqrt{b}}{\sqrt{a}+i\sqrt{b}}}} \right] + (c+dx) \operatorname{Log} \left[1 - \frac{\operatorname{Cosh}[c+dx] + \operatorname{Sinh}[c+dx]}{\sqrt{-\frac{\sqrt{a}+i\sqrt{b}}{\sqrt{a}-i\sqrt{b}}}} \right] + \\
& (c+dx) \operatorname{Log} \left[1 + \frac{\operatorname{Cosh}[c+dx] + \operatorname{Sinh}[c+dx]}{\sqrt{-\frac{\sqrt{a}+i\sqrt{b}}{\sqrt{a}-i\sqrt{b}}}} \right] - \operatorname{PolyLog} \left[2, -\frac{\operatorname{Cosh}[c+dx] + \operatorname{Sinh}[c+dx]}{\sqrt{-\frac{\sqrt{a}-i\sqrt{b}}{\sqrt{a}+i\sqrt{b}}}} \right] - \\
& \left. \left. \left. \operatorname{PolyLog} \left[2, \frac{\operatorname{Cosh}[c+dx] + \operatorname{Sinh}[c+dx]}{\sqrt{-\frac{\sqrt{a}-i\sqrt{b}}{\sqrt{a}+i\sqrt{b}}}} \right] + \operatorname{PolyLog} \left[2, -\frac{\operatorname{Cosh}[c+dx] + \operatorname{Sinh}[c+dx]}{\sqrt{-\frac{\sqrt{a}+i\sqrt{b}}{\sqrt{a}-i\sqrt{b}}}} \right] + \operatorname{PolyLog} \left[2, \frac{\operatorname{Cosh}[c+dx] + \operatorname{Sinh}[c+dx]}{\sqrt{-\frac{\sqrt{a}+i\sqrt{b}}{\sqrt{a}-i\sqrt{b}}}} \right] \right) \right) / \\
& \left(\left(-\sqrt{\frac{-\sqrt{a}+i\sqrt{b}}{\sqrt{a}+i\sqrt{b}}} + \sqrt{\frac{\sqrt{a}+i\sqrt{b}}{\sqrt{a}-i\sqrt{b}}} \right) \left(\sqrt{\frac{-\sqrt{a}+i\sqrt{b}}{\sqrt{a}+i\sqrt{b}}} + \sqrt{\frac{\sqrt{a}+i\sqrt{b}}{\sqrt{a}-i\sqrt{b}}} \right) (a+b) d^2 \right)
\end{aligned}$$

Problem 145: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2 \operatorname{Sech}[c+dx]^2}{a+b \operatorname{Tanh}[c+dx]^2} dx$$

Optimal (type 4, 351 leaves, 11 steps):

$$\begin{aligned}
& \frac{x^2 \operatorname{Log} \left[1 + \frac{(a+b) e^{2c+2dx}}{a-2\sqrt{-a}\sqrt{b}-b} \right]}{2\sqrt{-a}\sqrt{b}d} - \frac{x^2 \operatorname{Log} \left[1 + \frac{(a+b) e^{2c+2dx}}{a+2\sqrt{-a}\sqrt{b}-b} \right]}{2\sqrt{-a}\sqrt{b}d} + \frac{x \operatorname{PolyLog} \left[2, -\frac{(a+b) e^{2c+2dx}}{a-2\sqrt{-a}\sqrt{b}-b} \right]}{2\sqrt{-a}\sqrt{b}d^2} - \\
& \frac{x \operatorname{PolyLog} \left[2, -\frac{(a+b) e^{2c+2dx}}{a+2\sqrt{-a}\sqrt{b}-b} \right]}{2\sqrt{-a}\sqrt{b}d^2} - \frac{\operatorname{PolyLog} \left[3, -\frac{(a+b) e^{2c+2dx}}{a-2\sqrt{-a}\sqrt{b}-b} \right]}{4\sqrt{-a}\sqrt{b}d^3} + \frac{\operatorname{PolyLog} \left[3, -\frac{(a+b) e^{2c+2dx}}{a+2\sqrt{-a}\sqrt{b}-b} \right]}{4\sqrt{-a}\sqrt{b}d^3}
\end{aligned}$$

Result (type 4, 316 leaves):

$$\frac{1}{4\sqrt{a}\sqrt{b}d^3}i\left(2d^2x^2\operatorname{Log}\left[1+\frac{(\sqrt{a}-i\sqrt{b})e^{2(c+dx)}}{\sqrt{a}+i\sqrt{b}}\right]-2d^2x^2\operatorname{Log}\left[1+\frac{(\sqrt{a}+i\sqrt{b})e^{2(c+dx)}}{\sqrt{a}-i\sqrt{b}}\right]+2dx\operatorname{PolyLog}\left[2,-\frac{(\sqrt{a}-i\sqrt{b})e^{2(c+dx)}}{\sqrt{a}+i\sqrt{b}}\right]-2dx\operatorname{PolyLog}\left[2,-\frac{(\sqrt{a}+i\sqrt{b})e^{2(c+dx)}}{\sqrt{a}-i\sqrt{b}}\right]-\operatorname{PolyLog}\left[3,-\frac{(\sqrt{a}-i\sqrt{b})e^{2(c+dx)}}{\sqrt{a}+i\sqrt{b}}\right]+\operatorname{PolyLog}\left[3,-\frac{(\sqrt{a}+i\sqrt{b})e^{2(c+dx)}}{\sqrt{a}-i\sqrt{b}}\right]\right)$$

Problem 146: Result more than twice size of optimal antiderivative.

$$\int x^3 \operatorname{Tanh}[a + 2 \operatorname{Log}[x]] dx$$

Optimal (type 3, 29 leaves, 4 steps):

$$\frac{x^4}{4} - \frac{1}{2} e^{-2a} \operatorname{Log}[1 + e^{2a} x^4]$$

Result (type 3, 64 leaves):

$$\frac{x^4}{4} - \frac{1}{2} \operatorname{Cosh}[2a] \operatorname{Log}[\operatorname{Cosh}[a] + x^4 \operatorname{Cosh}[a] - \operatorname{Sinh}[a] + x^4 \operatorname{Sinh}[a]] + \frac{1}{2} \operatorname{Log}[\operatorname{Cosh}[a] + x^4 \operatorname{Cosh}[a] - \operatorname{Sinh}[a] + x^4 \operatorname{Sinh}[a]] \operatorname{Sinh}[2a]$$

Problem 147: Result is not expressed in closed-form.

$$\int x^2 \operatorname{Tanh}[a + 2 \operatorname{Log}[x]] dx$$

Optimal (type 3, 151 leaves, 11 steps):

$$\frac{x^3}{3} + \frac{e^{-3a/2} \operatorname{ArcTan}[1 - \sqrt{2} e^{a/2} x]}{\sqrt{2}} - \frac{e^{-3a/2} \operatorname{ArcTan}[1 + \sqrt{2} e^{a/2} x]}{\sqrt{2}} - \frac{e^{-3a/2} \operatorname{Log}[1 - \sqrt{2} e^{a/2} x + e^a x^2]}{2\sqrt{2}} + \frac{e^{-3a/2} \operatorname{Log}[1 + \sqrt{2} e^{a/2} x + e^a x^2]}{2\sqrt{2}}$$

Result (type 7, 64 leaves):

$$\frac{1}{6} \left(2x^3 + 3 \operatorname{RootSum}[\operatorname{Cosh}[a] - \operatorname{Sinh}[a] + \operatorname{Cosh}[a] \#1^4 + \operatorname{Sinh}[a] \#1^4 \&, \frac{\operatorname{Log}[x] - \operatorname{Log}[x - \#1]}{\#1} \&] (\operatorname{Cosh}[2a] - \operatorname{Sinh}[2a]) \right)$$

Problem 149: Result is not expressed in closed-form.

$$\int \operatorname{Tanh}[a + 2 \operatorname{Log}[x]] dx$$

Optimal (type 3, 145 leaves, 11 steps):

$$x + \frac{e^{-a/2} \operatorname{ArcTan}[1 - \sqrt{2} e^{a/2} x]}{\sqrt{2}} - \frac{e^{-a/2} \operatorname{ArcTan}[1 + \sqrt{2} e^{a/2} x]}{\sqrt{2}} + \frac{e^{-a/2} \operatorname{Log}[1 - \sqrt{2} e^{a/2} x + e^a x^2]}{2\sqrt{2}} - \frac{e^{-a/2} \operatorname{Log}[1 + \sqrt{2} e^{a/2} x + e^a x^2]}{2\sqrt{2}}$$

Result (type 7, 58 leaves):

$$x + \frac{1}{2} \operatorname{RootSum}[\operatorname{Cosh}[a] - \operatorname{Sinh}[a] + \operatorname{Cosh}[a] \#1^4 + \operatorname{Sinh}[a] \#1^4 \&, \frac{\operatorname{Log}[x] - \operatorname{Log}[x - \#1]}{\#1^3} \&] (\operatorname{Cosh}[2a] - \operatorname{Sinh}[2a])$$

Problem 151: Result is not expressed in closed-form.

$$\int \frac{\operatorname{Tanh}[a + 2 \operatorname{Log}[x]]}{x^2} dx$$

Optimal (type 3, 147 leaves, 11 steps):

$$\frac{1}{x} - \frac{e^{a/2} \operatorname{ArcTan}[1 - \sqrt{2} e^{a/2} x]}{\sqrt{2}} + \frac{e^{a/2} \operatorname{ArcTan}[1 + \sqrt{2} e^{a/2} x]}{\sqrt{2}} + \frac{e^{a/2} \operatorname{Log}[1 - \sqrt{2} e^{a/2} x + e^a x^2]}{2\sqrt{2}} - \frac{e^{a/2} \operatorname{Log}[1 + \sqrt{2} e^{a/2} x + e^a x^2]}{2\sqrt{2}}$$

Result (type 7, 59 leaves):

$$\frac{2 - x \operatorname{RootSum}[\operatorname{Cosh}[a] + \operatorname{Sinh}[a] + \operatorname{Cosh}[a] \#1^4 - \operatorname{Sinh}[a] \#1^4 \&, \frac{\operatorname{Log}[x] + \operatorname{Log}[\frac{1}{x} - \#1]}{\#1^3} \&] (\operatorname{Cosh}[a] + \operatorname{Sinh}[a])^2}{2x}$$

Problem 154: Result unnecessarily involves imaginary or complex numbers.

$$\int x^2 \operatorname{Tanh}[a + 2 \operatorname{Log}[x]]^2 dx$$

Optimal (type 3, 173 leaves, 12 steps):

$$\frac{x^3}{3} + \frac{x^3}{1 + e^{2a} x^4} + \frac{3 e^{-3a/2} \operatorname{ArcTan}[1 - \sqrt{2} e^{a/2} x]}{2\sqrt{2}} - \frac{3 e^{-3a/2} \operatorname{ArcTan}[1 + \sqrt{2} e^{a/2} x]}{2\sqrt{2}} - \frac{3 e^{-3a/2} \operatorname{Log}[1 - \sqrt{2} e^{a/2} x + e^a x^2]}{4\sqrt{2}} + \frac{3 e^{-3a/2} \operatorname{Log}[1 + \sqrt{2} e^{a/2} x + e^a x^2]}{4\sqrt{2}}$$

Result (type 3, 174 leaves):

$$\frac{1}{12} \left(4x^3 + \frac{12x^3}{1 + e^{2a} x^4} + 9(-1)^{3/4} e^{-3a/2} \operatorname{Log}[(-1)^{1/4} e^{-3a/2} - e^{-a} x] + 9(-1)^{1/4} e^{-3a/2} \operatorname{Log}[(-1)^{3/4} e^{-3a/2} - e^{-a} x] - 9(-1)^{3/4} e^{-3a/2} \operatorname{Log}[(-1)^{1/4} e^{-3a/2} + e^{-a} x] - 9(-1)^{1/4} e^{-3a/2} \operatorname{Log}[(-1)^{3/4} e^{-3a/2} + e^{-a} x] \right)$$

Problem 156: Result unnecessarily involves imaginary or complex numbers.

$$\int \operatorname{Tanh}[a + 2 \operatorname{Log}[x]]^2 dx$$

Optimal (type 3, 165 leaves, 13 steps):

$$x + \frac{x}{1 + e^{2a} x^4} + \frac{e^{-a/2} \operatorname{ArcTan}[1 - \sqrt{2} e^{a/2} x]}{2\sqrt{2}} - \frac{e^{-a/2} \operatorname{ArcTan}[1 + \sqrt{2} e^{a/2} x]}{2\sqrt{2}} + \frac{e^{-a/2} \operatorname{Log}[1 - \sqrt{2} e^{a/2} x + e^a x^2]}{4\sqrt{2}} - \frac{e^{-a/2} \operatorname{Log}[1 + \sqrt{2} e^{a/2} x + e^a x^2]}{4\sqrt{2}}$$

Result (type 3, 146 leaves):

$$\frac{1}{4} \left(4x + \frac{4x}{1 + e^{2a} x^4} + (-1)^{1/4} e^{-a/2} \operatorname{Log}[(-1)^{1/4} e^{-a/2} - x] + \right. \\ \left. (-1)^{3/4} e^{-a/2} \operatorname{Log}[(-1)^{3/4} e^{-a/2} - x] - (-1)^{1/4} e^{-a/2} \operatorname{Log}[(-1)^{1/4} e^{-a/2} + x] - (-1)^{3/4} e^{-a/2} \operatorname{Log}[(-1)^{3/4} e^{-a/2} + x] \right)$$

Problem 158: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Tanh}[a + 2 \operatorname{Log}[x]]^2}{x^2} dx$$

Optimal (type 3, 190 leaves, 12 steps):

$$-\frac{1}{x(1 + e^{2a} x^4)} - \frac{2e^{2a} x^3}{1 + e^{2a} x^4} + \frac{e^{a/2} \operatorname{ArcTan}[1 - \sqrt{2} e^{a/2} x]}{2\sqrt{2}} - \\ \frac{e^{a/2} \operatorname{ArcTan}[1 + \sqrt{2} e^{a/2} x]}{2\sqrt{2}} - \frac{e^{a/2} \operatorname{Log}[1 - \sqrt{2} e^{a/2} x + e^a x^2]}{4\sqrt{2}} + \frac{e^{a/2} \operatorname{Log}[1 + \sqrt{2} e^{a/2} x + e^a x^2]}{4\sqrt{2}}$$

Result (type 3, 181 leaves):

$$\frac{1}{4} \left(-\frac{4}{x} - \frac{4}{\frac{e^{-2a}}{x^3} + x} + (-1)^{3/4} e^{a/2} \operatorname{Log}\left[\frac{e^{-2a} \left((-1)^{1/4} - e^{a/2} x \right)}{x^4}\right] + \right. \\ \left. (-1)^{1/4} e^{a/2} \operatorname{Log}\left[\frac{e^{-2a} \left((-1)^{3/4} - e^{a/2} x \right)}{x^4}\right] - (-1)^{3/4} e^{a/2} \operatorname{Log}\left[\frac{e^{-2a} \left((-1)^{1/4} + e^{a/2} x \right)}{x^4}\right] - (-1)^{1/4} e^{a/2} \operatorname{Log}\left[\frac{e^{-2a} \left((-1)^{3/4} + e^{a/2} x \right)}{x^4}\right] \right)$$

Problem 161: Result more than twice size of optimal antiderivative.

$$\int (e x)^m \operatorname{Tanh}[a + 2 \operatorname{Log}[x]]^2 dx$$

Optimal (type 5, 79 leaves, 4 steps):

$$\frac{(e x)^{1+m}}{e(1+m)} + \frac{(e x)^{1+m}}{e(1+e^{2a}x^4)} - \frac{(e x)^{1+m} \text{Hypergeometric2F1}\left[1, \frac{1+m}{4}, \frac{5+m}{4}, -e^{2a}x^4\right]}{e}$$

Result (type 5, 168 leaves):

$$\frac{1}{(\text{Cosh}[a] - \text{Sinh}[a])^2} x (e x)^m \left(\frac{1}{(5+m)(9+m)} x^4 (\text{Cosh}[a] + \text{Sinh}[a]) \left(-2(9+m) \text{Hypergeometric2F1}\left[2, \frac{5+m}{4}, \frac{9+m}{4}, -x^4 (\text{Cosh}[2a] + \text{Sinh}[2a])\right] (\text{Cosh}[a] - \text{Sinh}[a]) + (5+m) x^4 \text{Hypergeometric2F1}\left[2, \frac{9+m}{4}, \frac{13+m}{4}, -x^4 (\text{Cosh}[2a] + \text{Sinh}[2a])\right] (\text{Cosh}[a] + \text{Sinh}[a]) \right) + \frac{\text{Hypergeometric2F1}\left[2, \frac{1+m}{4}, \frac{5+m}{4}, -x^4 (\text{Cosh}[2a] + \text{Sinh}[2a])\right] (\text{Cosh}[2a] - \text{Sinh}[2a])}{1+m} \right)$$

Problem 163: Result more than twice size of optimal antiderivative.

$$\int \text{Tanh}[a + b \text{Log}[x]]^p dx$$

Optimal (type 6, 79 leaves, 3 steps):

$$x(1 - e^{2a}x^{2b})^{-p} (-1 + e^{2a}x^{2b})^p \text{AppellF1}\left[\frac{1}{2b}, -p, p, \frac{1}{2}\left(2 + \frac{1}{b}\right), e^{2a}x^{2b}, -e^{2a}x^{2b}\right]$$

Result (type 6, 259 leaves):

$$\left((1+2b)x \left(\frac{-1 + e^{2a}x^{2b}}{1 + e^{2a}x^{2b}} \right)^p \text{AppellF1}\left[\frac{1}{2b}, -p, p, 1 + \frac{1}{2b}, e^{2a}x^{2b}, -e^{2a}x^{2b}\right] \right) / \left(-2b e^{2a} p x^{2b} \text{AppellF1}\left[1 + \frac{1}{2b}, 1 - p, p, 2 + \frac{1}{2b}, e^{2a}x^{2b}, -e^{2a}x^{2b}\right] - 2b e^{2a} p x^{2b} \text{AppellF1}\left[1 + \frac{1}{2b}, -p, 1 + p, 2 + \frac{1}{2b}, e^{2a}x^{2b}, -e^{2a}x^{2b}\right] + (1+2b) \text{AppellF1}\left[\frac{1}{2b}, -p, p, 1 + \frac{1}{2b}, e^{2a}x^{2b}, -e^{2a}x^{2b}\right] \right)$$

Problem 164: Result more than twice size of optimal antiderivative.

$$\int (e x)^m \text{Tanh}[a + b \text{Log}[x]]^p dx$$

Optimal (type 6, 99 leaves, 3 steps):

$$\frac{(e x)^{1+m} (1 - e^{2a} x^{2b})^{-p} (-1 + e^{2a} x^{2b})^p \text{AppellF1}\left[\frac{1+m}{2b}, -p, p, 1 + \frac{1+m}{2b}, e^{2a} x^{2b}, -e^{2a} x^{2b}\right]}{e(1+m)}$$

Result (type 6, 287 leaves):

$$\left((1+2b+m) x (e x)^m \left(\frac{-1 + e^{2a} x^{2b}}{1 + e^{2a} x^{2b}} \right)^p \text{AppellF1}\left[\frac{1+m}{2b}, -p, p, 1 + \frac{1+m}{2b}, e^{2a} x^{2b}, -e^{2a} x^{2b}\right] \right) /$$

$$\left((1+m) \left((1+2b+m) \text{AppellF1}\left[\frac{1+m}{2b}, -p, p, \frac{1+2b+m}{2b}, e^{2a} x^{2b}, -e^{2a} x^{2b}\right] - \right. \right.$$

$$\left. \left. 2b e^{2a} p x^{2b} \left(\text{AppellF1}\left[\frac{1+2b+m}{2b}, 1-p, p, \frac{1+4b+m}{2b}, e^{2a} x^{2b}, -e^{2a} x^{2b}\right] + \text{AppellF1}\left[\frac{1+2b+m}{2b}, -p, 1+p, \frac{1+4b+m}{2b}, e^{2a} x^{2b}, -e^{2a} x^{2b}\right] \right) \right) \right)$$

Problem 166: Result unnecessarily involves higher level functions.

$$\int \text{Tanh}\left[a + \frac{\text{Log}[x]}{4}\right]^p dx$$

Optimal (type 5, 106 leaves, 4 steps):

$$e^{-4a} (-1 + e^{2a} \sqrt{x})^{1+p} (1 + e^{2a} \sqrt{x})^{1-p} - \frac{2^{1-p} e^{-4a} p (-1 + e^{2a} \sqrt{x})^{1+p} \text{Hypergeometric2F1}\left[p, 1+p, 2+p, \frac{1}{2} (1 - e^{2a} \sqrt{x})\right]}{1+p}$$

Result (type 6, 176 leaves):

$$- \left(\left(3 \left(\frac{-1 + e^{2a} \sqrt{x}}{1 + e^{2a} \sqrt{x}} \right)^p x \text{AppellF1}\left[2, -p, p, 3, e^{2a} \sqrt{x}, -e^{2a} \sqrt{x}\right] \right) / \left(-3 \text{AppellF1}\left[2, -p, p, 3, e^{2a} \sqrt{x}, -e^{2a} \sqrt{x}\right] + \right. \right.$$

$$\left. \left. e^{2a} p \sqrt{x} \left(\text{AppellF1}\left[3, 1-p, p, 4, e^{2a} \sqrt{x}, -e^{2a} \sqrt{x}\right] + \text{AppellF1}\left[3, -p, 1+p, 4, e^{2a} \sqrt{x}, -e^{2a} \sqrt{x}\right] \right) \right) \right)$$

Problem 167: Result unnecessarily involves higher level functions.

$$\int \text{Tanh}\left[a + \frac{\text{Log}[x]}{6}\right]^p dx$$

Optimal (type 5, 158 leaves, 5 steps):

$$-e^{-6a} p (-1 + e^{2a} x^{1/3})^{1+p} (1 + e^{2a} x^{1/3})^{1-p} + e^{-4a} (-1 + e^{2a} x^{1/3})^{1+p} (1 + e^{2a} x^{1/3})^{1-p} x^{1/3} +$$

$$\frac{2^{-p} e^{-6a} (1+2p^2) (-1 + e^{2a} x^{1/3})^{1+p} \text{Hypergeometric2F1}\left[p, 1+p, 2+p, \frac{1}{2} (1 - e^{2a} x^{1/3})\right]}{1+p}$$

Result (type 6, 177 leaves):

$$\left(4 \left(\frac{-1 + e^{2a} x^{1/3}}{1 + e^{2a} x^{1/3}} \right)^p \times \text{AppellF1}\left[3, -p, p, 4, e^{2a} x^{1/3}, -e^{2a} x^{1/3}\right] \right) / \left(4 \text{AppellF1}\left[3, -p, p, 4, e^{2a} x^{1/3}, -e^{2a} x^{1/3}\right] - e^{2a} p x^{1/3} \left(\text{AppellF1}\left[4, 1-p, p, 5, e^{2a} x^{1/3}, -e^{2a} x^{1/3}\right] + \text{AppellF1}\left[4, -p, 1+p, 5, e^{2a} x^{1/3}, -e^{2a} x^{1/3}\right] \right) \right)$$

Problem 168: Result unnecessarily involves higher level functions.

$$\int \text{Tanh}\left[a + \frac{\text{Log}[x]}{8}\right]^p dx$$

Optimal (type 5, 190 leaves, 5 steps):

$$\frac{1}{3} e^{-12a} (-1 + e^{2a} x^{1/4})^{1+p} (1 + e^{2a} x^{1/4})^{1-p} (e^{4a} (3 + 2p^2) - 2e^{6a} p x^{1/4}) + e^{-4a} (-1 + e^{2a} x^{1/4})^{1+p} (1 + e^{2a} x^{1/4})^{1-p} \sqrt{x} - \frac{2^{2-p} e^{-8a} p (2 + p^2) (-1 + e^{2a} x^{1/4})^{1+p} \text{Hypergeometric2F1}\left[p, 1+p, 2+p, \frac{1}{2} (1 - e^{2a} x^{1/4})\right]}{3 (1+p)}$$

Result (type 6, 177 leaves):

$$\left(5 \left(\frac{-1 + e^{2a} x^{1/4}}{1 + e^{2a} x^{1/4}} \right)^p \times \text{AppellF1}\left[4, -p, p, 5, e^{2a} x^{1/4}, -e^{2a} x^{1/4}\right] \right) / \left(5 \text{AppellF1}\left[4, -p, p, 5, e^{2a} x^{1/4}, -e^{2a} x^{1/4}\right] - e^{2a} p x^{1/4} \left(\text{AppellF1}\left[5, 1-p, p, 6, e^{2a} x^{1/4}, -e^{2a} x^{1/4}\right] + \text{AppellF1}\left[5, -p, 1+p, 6, e^{2a} x^{1/4}, -e^{2a} x^{1/4}\right] \right) \right)$$

Problem 169: Result more than twice size of optimal antiderivative.

$$\int \text{Tanh}[a + \text{Log}[x]]^p dx$$

Optimal (type 6, 61 leaves, 3 steps):

$$x (1 - e^{2a} x^2)^{-p} (-1 + e^{2a} x^2)^p \text{AppellF1}\left[\frac{1}{2}, -p, p, \frac{3}{2}, e^{2a} x^2, -e^{2a} x^2\right]$$

Result (type 6, 171 leaves):

$$\left(3 x \left(\frac{-1 + e^{2a} x^2}{1 + e^{2a} x^2} \right)^p \text{AppellF1}\left[\frac{1}{2}, -p, p, \frac{3}{2}, e^{2a} x^2, -e^{2a} x^2\right] \right) / \left(3 \text{AppellF1}\left[\frac{1}{2}, -p, p, \frac{3}{2}, e^{2a} x^2, -e^{2a} x^2\right] - 2 e^{2a} p x^2 \left(\text{AppellF1}\left[\frac{3}{2}, 1-p, p, \frac{5}{2}, e^{2a} x^2, -e^{2a} x^2\right] + \text{AppellF1}\left[\frac{3}{2}, -p, 1+p, \frac{5}{2}, e^{2a} x^2, -e^{2a} x^2\right] \right) \right)$$

Problem 170: Result more than twice size of optimal antiderivative.

$$\int \text{Tanh}[a + 2 \text{Log}[x]]^p dx$$

Optimal (type 6, 61 leaves, 3 steps):

$$x (1 - e^{2a} x^4)^{-p} (-1 + e^{2a} x^4)^p \text{AppellF1}\left[\frac{1}{4}, -p, p, \frac{5}{4}, e^{2a} x^4, -e^{2a} x^4\right]$$

Result (type 6, 171 leaves):

$$\left(5x \left(\frac{-1 + e^{2a} x^4}{1 + e^{2a} x^4}\right)^p \text{AppellF1}\left[\frac{1}{4}, -p, p, \frac{5}{4}, e^{2a} x^4, -e^{2a} x^4\right]\right) / \left(5 \text{AppellF1}\left[\frac{1}{4}, -p, p, \frac{5}{4}, e^{2a} x^4, -e^{2a} x^4\right] - 4 e^{2a} p x^4 \left(\text{AppellF1}\left[\frac{5}{4}, 1 - p, p, \frac{9}{4}, e^{2a} x^4, -e^{2a} x^4\right] + \text{AppellF1}\left[\frac{5}{4}, -p, 1 + p, \frac{9}{4}, e^{2a} x^4, -e^{2a} x^4\right]\right)\right)$$

Problem 171: Result more than twice size of optimal antiderivative.

$$\int \text{Tanh}[a + 3 \text{Log}[x]]^p dx$$

Optimal (type 6, 61 leaves, 3 steps):

$$x (1 - e^{2a} x^6)^{-p} (-1 + e^{2a} x^6)^p \text{AppellF1}\left[\frac{1}{6}, -p, p, \frac{7}{6}, e^{2a} x^6, -e^{2a} x^6\right]$$

Result (type 6, 171 leaves):

$$\left(7x \left(\frac{-1 + e^{2a} x^6}{1 + e^{2a} x^6}\right)^p \text{AppellF1}\left[\frac{1}{6}, -p, p, \frac{7}{6}, e^{2a} x^6, -e^{2a} x^6\right]\right) / \left(7 \text{AppellF1}\left[\frac{1}{6}, -p, p, \frac{7}{6}, e^{2a} x^6, -e^{2a} x^6\right] - 6 e^{2a} p x^6 \left(\text{AppellF1}\left[\frac{7}{6}, 1 - p, p, \frac{13}{6}, e^{2a} x^6, -e^{2a} x^6\right] + \text{AppellF1}\left[\frac{7}{6}, -p, 1 + p, \frac{13}{6}, e^{2a} x^6, -e^{2a} x^6\right]\right)\right)$$

Problem 172: Result more than twice size of optimal antiderivative.

$$\int x^3 \text{Tanh}[d (a + b \text{Log}[c x^n])] dx$$

Optimal (type 5, 59 leaves, 4 steps):

$$\frac{x^4}{4} - \frac{1}{2} x^4 \text{Hypergeometric2F1}\left[1, \frac{2}{bdn}, 1 + \frac{2}{bdn}, -e^{2ad} (c x^n)^{2bd}\right]$$

Result (type 5, 127 leaves):

$$\frac{1}{8 + 4 b d n} x^4 \left(2 e^{2d (a + b \text{Log}[c x^n])} \text{Hypergeometric2F1}\left[1, 1 + \frac{2}{bdn}, 2 + \frac{2}{bdn}, -e^{2d (a + b \text{Log}[c x^n])}\right] - (2 + b d n) \text{Hypergeometric2F1}\left[1, \frac{2}{bdn}, 1 + \frac{2}{bdn}, -e^{2d (a + b \text{Log}[c x^n])}\right]\right)$$

Problem 173: Result more than twice size of optimal antiderivative.

$$\int x^2 \operatorname{Tanh}[d(a + b \operatorname{Log}[c x^n])] dx$$

Optimal (type 5, 63 leaves, 4 steps):

$$\frac{x^3}{3} - \frac{2}{3} x^3 \operatorname{Hypergeometric2F1}\left[1, \frac{3}{2 b d n}, 1 + \frac{3}{2 b d n}, -e^{2 a d} (c x^n)^{2 b d}\right]$$

Result (type 5, 136 leaves):

$$\frac{1}{9 + 6 b d n} x^3 \left(3 e^{2 d (a + b \operatorname{Log}[c x^n])} \operatorname{Hypergeometric2F1}\left[1, 1 + \frac{3}{2 b d n}, 2 + \frac{3}{2 b d n}, -e^{2 d (a + b \operatorname{Log}[c x^n])}\right] - (3 + 2 b d n) \operatorname{Hypergeometric2F1}\left[1, \frac{3}{2 b d n}, 1 + \frac{3}{2 b d n}, -e^{2 d (a + b \operatorname{Log}[c x^n])}\right] \right)$$

Problem 174: Result more than twice size of optimal antiderivative.

$$\int x \operatorname{Tanh}[d(a + b \operatorname{Log}[c x^n])] dx$$

Optimal (type 5, 55 leaves, 4 steps):

$$\frac{x^2}{2} - x^2 \operatorname{Hypergeometric2F1}\left[1, \frac{1}{b d n}, 1 + \frac{1}{b d n}, -e^{2 a d} (c x^n)^{2 b d}\right]$$

Result (type 5, 124 leaves):

$$\frac{1}{2 + 2 b d n} x^2 \left(e^{2 a d} (c x^n)^{2 b d} \operatorname{Hypergeometric2F1}\left[1, 1 + \frac{1}{b d n}, 2 + \frac{1}{b d n}, -e^{2 a d} (c x^n)^{2 b d}\right] - (1 + b d n) \operatorname{Hypergeometric2F1}\left[1, \frac{1}{b d n}, 1 + \frac{1}{b d n}, -e^{2 d (a + b \operatorname{Log}[c x^n])}\right] \right)$$

Problem 175: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Tanh}[d(a + b \operatorname{Log}[c x^n])] dx$$

Optimal (type 5, 53 leaves, 4 steps):

$$x - 2 x \operatorname{Hypergeometric2F1}\left[1, \frac{1}{2 b d n}, 1 + \frac{1}{2 b d n}, -e^{2 a d} (c x^n)^{2 b d}\right]$$

Result (type 5, 129 leaves):

$$\frac{e^{2ad} x (c x^n)^{2bd} \text{Hypergeometric2F1}\left[1, 1 + \frac{1}{2bdn}, 2 + \frac{1}{2bdn}, -e^{2ad} (c x^n)^{2bd}\right]}{1 + 2bdn} - x \text{Hypergeometric2F1}\left[1, \frac{1}{2bdn}, 1 + \frac{1}{2bdn}, -e^{2ad} (c x^n)^{2bd}\right]$$

Problem 177: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Tanh}\left[d (a + b \text{Log}[c x^n])\right]}{x^2} dx$$

Optimal (type 5, 59 leaves, 4 steps):

$$-\frac{1}{x} + \frac{2 \text{Hypergeometric2F1}\left[1, -\frac{1}{2bdn}, 1 - \frac{1}{2bdn}, -e^{2ad} (c x^n)^{2bd}\right]}{x}$$

Result (type 5, 126 leaves):

$$\frac{1}{x} \left(\frac{e^{2d(a+b \text{Log}[c x^n])} \text{Hypergeometric2F1}\left[1, 1 - \frac{1}{2bdn}, 2 - \frac{1}{2bdn}, -e^{2d(a+b \text{Log}[c x^n])}\right]}{-1 + 2bdn} + \text{Hypergeometric2F1}\left[1, -\frac{1}{2bdn}, 1 - \frac{1}{2bdn}, -e^{2d(a+b \text{Log}[c x^n])}\right] \right)$$

Problem 178: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Tanh}\left[d (a + b \text{Log}[c x^n])\right]}{x^3} dx$$

Optimal (type 5, 56 leaves, 4 steps):

$$-\frac{1}{2x^2} + \frac{\text{Hypergeometric2F1}\left[1, -\frac{1}{bdn}, 1 - \frac{1}{bdn}, -e^{2ad} (c x^n)^{2bd}\right]}{x^2}$$

Result (type 5, 120 leaves):

$$\frac{1}{2x^2} \left(\frac{e^{2d(a+b \text{Log}[c x^n])} \text{Hypergeometric2F1}\left[1, 1 - \frac{1}{bdn}, 2 - \frac{1}{bdn}, -e^{2d(a+b \text{Log}[c x^n])}\right]}{-1 + bdn} + \text{Hypergeometric2F1}\left[1, -\frac{1}{bdn}, 1 - \frac{1}{bdn}, -e^{2d(a+b \text{Log}[c x^n])}\right] \right)$$

Problem 192: Result more than twice size of optimal antiderivative.

$$\int \text{Tanh}\left[d (a + b \text{Log}[c x^n])\right]^p dx$$

Optimal (type 6, 115 leaves, 4 steps):

$$x \left(1 - e^{2ad} (c x^n)^{2bd}\right)^{-p} \left(-1 + e^{2ad} (c x^n)^{2bd}\right)^p \text{AppellF1}\left[\frac{1}{2bdn}, -p, p, 1 + \frac{1}{2bdn}, e^{2ad} (c x^n)^{2bd}, -e^{2ad} (c x^n)^{2bd}\right]$$

Result (type 6, 387 leaves):

$$\left((1 + 2 b d n) x \left(\frac{-1 + e^{2 a d} (c x^n)^{2 b d}}{1 + e^{2 a d} (c x^n)^{2 b d}} \right)^p \text{AppellF1} \left[\frac{1}{2 b d n}, -p, p, 1 + \frac{1}{2 b d n}, e^{2 a d} (c x^n)^{2 b d}, -e^{2 a d} (c x^n)^{2 b d} \right] \right) /$$

$$\left(-2 b d e^{2 a d} n p (c x^n)^{2 b d} \text{AppellF1} \left[1 + \frac{1}{2 b d n}, 1 - p, p, 2 + \frac{1}{2 b d n}, e^{2 a d} (c x^n)^{2 b d}, -e^{2 a d} (c x^n)^{2 b d} \right] - \right.$$

$$2 b d e^{2 a d} n p (c x^n)^{2 b d} \text{AppellF1} \left[1 + \frac{1}{2 b d n}, -p, 1 + p, 2 + \frac{1}{2 b d n}, e^{2 a d} (c x^n)^{2 b d}, -e^{2 a d} (c x^n)^{2 b d} \right] +$$

$$\left. (1 + 2 b d n) \text{AppellF1} \left[\frac{1}{2 b d n}, -p, p, 1 + \frac{1}{2 b d n}, e^{2 a d} (c x^n)^{2 b d}, -e^{2 a d} (c x^n)^{2 b d} \right] \right)$$

Problem 193: Result more than twice size of optimal antiderivative.

$$\int (e x)^m \text{Tanh} [d (a + b \text{Log} [c x^n])]^p dx$$

Optimal (type 6, 135 leaves, 4 steps):

$$\frac{1}{e (1 + m)} (e x)^{1+m} (1 - e^{2 a d} (c x^n)^{2 b d})^{-p} (-1 + e^{2 a d} (c x^n)^{2 b d})^p \text{AppellF1} \left[\frac{1+m}{2 b d n}, -p, p, 1 + \frac{1+m}{2 b d n}, e^{2 a d} (c x^n)^{2 b d}, -e^{2 a d} (c x^n)^{2 b d} \right]$$

Result (type 6, 417 leaves):

$$\left((1 + m + 2 b d n) x (e x)^m \left(\frac{-1 + e^{2 a d} (c x^n)^{2 b d}}{1 + e^{2 a d} (c x^n)^{2 b d}} \right)^p \text{AppellF1} \left[\frac{1+m}{2 b d n}, -p, p, 1 + \frac{1+m}{2 b d n}, e^{2 a d} (c x^n)^{2 b d}, -e^{2 a d} (c x^n)^{2 b d} \right] \right) /$$

$$\left((1 + m) \left((1 + m + 2 b d n) \text{AppellF1} \left[\frac{1+m}{2 b d n}, -p, p, \frac{1+m+2 b d n}{2 b d n}, e^{2 a d} (c x^n)^{2 b d}, -e^{2 a d} (c x^n)^{2 b d} \right] - \right.$$

$$2 b d e^{2 a d} n p (c x^n)^{2 b d} \left(\text{AppellF1} \left[\frac{1+m+2 b d n}{2 b d n}, 1-p, p, \frac{1+m+4 b d n}{2 b d n}, e^{2 a d} (c x^n)^{2 b d}, -e^{2 a d} (c x^n)^{2 b d} \right] + \right.$$

$$\left. \left. \left. \text{AppellF1} \left[\frac{1+m+2 b d n}{2 b d n}, -p, 1+p, \frac{1+m+4 b d n}{2 b d n}, e^{2 a d} (c x^n)^{2 b d}, -e^{2 a d} (c x^n)^{2 b d} \right] \right) \right) \right)$$

Problem 200: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Tanh} [x]^5}{\sqrt{a + b \text{Tanh} [x]^2 + c \text{Tanh} [x]^4}} dx$$

Optimal (type 3, 135 leaves, 8 steps):

$$\frac{(b-2c) \operatorname{ArcTanh}\left[\frac{b+2c \operatorname{Tanh}[x]^2}{2\sqrt{c} \sqrt{a+b \operatorname{Tanh}[x]^2+c \operatorname{Tanh}[x]^4}}\right]}{4c^{3/2}} + \frac{\operatorname{ArcTanh}\left[\frac{2a+b+(b+2c) \operatorname{Tanh}[x]^2}{2\sqrt{a+b+c} \sqrt{a+b \operatorname{Tanh}[x]^2+c \operatorname{Tanh}[x]^4}}\right]}{2\sqrt{a+b+c}} - \frac{\sqrt{a+b \operatorname{Tanh}[x]^2+c \operatorname{Tanh}[x]^4}}{2c}$$

Result (type 3, 42734 leaves): Display of huge result suppressed!

Problem 201: Attempted integration timed out after 120 seconds.

$$\int \frac{\operatorname{Tanh}[x]^3}{\sqrt{a+b \operatorname{Tanh}[x]^2+c \operatorname{Tanh}[x]^4}} dx$$

Optimal (type 3, 105 leaves, 7 steps):

$$-\frac{\operatorname{ArcTanh}\left[\frac{b+2c \operatorname{Tanh}[x]^2}{2\sqrt{c} \sqrt{a+b \operatorname{Tanh}[x]^2+c \operatorname{Tanh}[x]^4}}\right]}{2\sqrt{c}} + \frac{\operatorname{ArcTanh}\left[\frac{2a+b+(b+2c) \operatorname{Tanh}[x]^2}{2\sqrt{a+b+c} \sqrt{a+b \operatorname{Tanh}[x]^2+c \operatorname{Tanh}[x]^4}}\right]}{2\sqrt{a+b+c}}$$

Result (type 1, 1 leaves):

???

Problem 202: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Tanh}[x]}{\sqrt{a+b \operatorname{Tanh}[x]^2+c \operatorname{Tanh}[x]^4}} dx$$

Optimal (type 3, 58 leaves, 4 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{2a+b+(b+2c) \operatorname{Tanh}[x]^2}{2\sqrt{a+b+c} \sqrt{a+b \operatorname{Tanh}[x]^2+c \operatorname{Tanh}[x]^4}}\right]}{2\sqrt{a+b+c}}$$

Result (type 3, 59564 leaves): Display of huge result suppressed!

Problem 203: Unable to integrate problem.

$$\int \frac{\operatorname{Coth}[x]}{\sqrt{a+b \operatorname{Tanh}[x]^2+c \operatorname{Tanh}[x]^4}} dx$$

Optimal (type 3, 106 leaves, 8 steps):

$$-\frac{\text{ArcTanh}\left[\frac{2a+b \text{Tanh}[x]^2}{2\sqrt{a}\sqrt{a+b \text{Tanh}[x]^2+c \text{Tanh}[x]^4}}\right]}{2\sqrt{a}} + \frac{\text{ArcTanh}\left[\frac{2a+b+(b+2c) \text{Tanh}[x]^2}{2\sqrt{a+b+c}\sqrt{a+b \text{Tanh}[x]^2+c \text{Tanh}[x]^4}}\right]}{2\sqrt{a+b+c}}$$

Result (type 8, 23 leaves):

$$\int \frac{\text{Coth}[x]}{\sqrt{a+b \text{Tanh}[x]^2+c \text{Tanh}[x]^4}} dx$$

Problem 204: Unable to integrate problem.

$$\int \frac{\text{Coth}[x]^3}{\sqrt{a+b \text{Tanh}[x]^2+c \text{Tanh}[x]^4}} dx$$

Optimal (type 3, 183 leaves, 11 steps):

$$-\frac{\text{ArcTanh}\left[\frac{2a+b \text{Tanh}[x]^2}{2\sqrt{a}\sqrt{a+b \text{Tanh}[x]^2+c \text{Tanh}[x]^4}}\right]}{2\sqrt{a}} + \frac{b \text{ArcTanh}\left[\frac{2a+b \text{Tanh}[x]^2}{2\sqrt{a}\sqrt{a+b \text{Tanh}[x]^2+c \text{Tanh}[x]^4}}\right]}{4a^{3/2}} +$$

$$\frac{\text{ArcTanh}\left[\frac{2a+b+(b+2c) \text{Tanh}[x]^2}{2\sqrt{a+b+c}\sqrt{a+b \text{Tanh}[x]^2+c \text{Tanh}[x]^4}}\right]}{2\sqrt{a+b+c}} - \frac{\text{Coth}[x]^2 \sqrt{a+b \text{Tanh}[x]^2+c \text{Tanh}[x]^4}}{2a}$$

Result (type 8, 25 leaves):

$$\int \frac{\text{Coth}[x]^3}{\sqrt{a+b \text{Tanh}[x]^2+c \text{Tanh}[x]^4}} dx$$

Problem 205: Result more than twice size of optimal antiderivative.

$$\int \text{Tanh}[x] \sqrt{a+b \text{Tanh}[x]^2+c \text{Tanh}[x]^4} dx$$

Optimal (type 3, 132 leaves, 8 steps):

$$-\frac{(b+2c) \text{ArcTanh}\left[\frac{b+2c \text{Tanh}[x]^2}{2\sqrt{c}\sqrt{a+b \text{Tanh}[x]^2+c \text{Tanh}[x]^4}}\right]}{4\sqrt{c}} + \frac{1}{2} \sqrt{a+b+c} \text{ArcTanh}\left[\frac{2a+b+(b+2c) \text{Tanh}[x]^2}{2\sqrt{a+b+c}\sqrt{a+b \text{Tanh}[x]^2+c \text{Tanh}[x]^4}}\right] - \frac{1}{2} \sqrt{a+b \text{Tanh}[x]^2+c \text{Tanh}[x]^4}$$

Result (type 3, 178715 leaves): Display of huge result suppressed!

Problem 214: Result is not expressed in closed-form.

$$\int e^x \operatorname{Tanh}[2x]^2 dx$$

Optimal (type 3, 113 leaves, 13 steps):

$$e^x + \frac{e^x}{1 + e^{4x}} + \frac{\operatorname{ArcTan}[1 - \sqrt{2} e^x]}{2\sqrt{2}} - \frac{\operatorname{ArcTan}[1 + \sqrt{2} e^x]}{2\sqrt{2}} + \frac{\operatorname{Log}[1 - \sqrt{2} e^x + e^{2x}]}{4\sqrt{2}} - \frac{\operatorname{Log}[1 + \sqrt{2} e^x + e^{2x}]}{4\sqrt{2}}$$

Result (type 7, 48 leaves):

$$e^x + \frac{e^x}{1 + e^{4x}} + \frac{1}{4} \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{x - \operatorname{Log}[e^x - \#1]}{\#1^3} \&\right]$$

Problem 215: Result is not expressed in closed-form.

$$\int e^x \operatorname{Tanh}[2x] dx$$

Optimal (type 3, 95 leaves, 11 steps):

$$e^x + \frac{\operatorname{ArcTan}[1 - \sqrt{2} e^x]}{\sqrt{2}} - \frac{\operatorname{ArcTan}[1 + \sqrt{2} e^x]}{\sqrt{2}} + \frac{\operatorname{Log}[1 - \sqrt{2} e^x + e^{2x}]}{2\sqrt{2}} - \frac{\operatorname{Log}[1 + \sqrt{2} e^x + e^{2x}]}{2\sqrt{2}}$$

Result (type 7, 35 leaves):

$$e^x + \frac{1}{2} \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{x - \operatorname{Log}[e^x - \#1]}{\#1^3} \&\right]$$

Problem 218: Result is not expressed in closed-form.

$$\int e^x \operatorname{Tanh}[3x]^2 dx$$

Optimal (type 3, 113 leaves, 14 steps):

$$e^x + \frac{2e^x}{3(1 + e^{6x})} - \frac{2\operatorname{ArcTan}[e^x]}{9} + \frac{1}{9}\operatorname{ArcTan}[\sqrt{3} - 2e^x] - \frac{1}{9}\operatorname{ArcTan}[\sqrt{3} + 2e^x] + \frac{\operatorname{Log}[1 - \sqrt{3} e^x + e^{2x}]}{6\sqrt{3}} - \frac{\operatorname{Log}[1 + \sqrt{3} e^x + e^{2x}]}{6\sqrt{3}}$$

Result (type 7, 97 leaves):

$$e^x + \frac{2e^x}{3(1 + e^{6x})} - \frac{2\operatorname{ArcTan}[e^x]}{9} - \frac{1}{9} \operatorname{RootSum}\left[1 - \#1^2 + \#1^4 \&, \frac{-2x + 2\operatorname{Log}[e^x - \#1] + x\#1^2 - \operatorname{Log}[e^x - \#1]\#1^2}{-\#1 + 2\#1^3} \&\right]$$

Problem 219: Result is not expressed in closed-form.

$$\int e^x \operatorname{Tanh}[3x] dx$$

Optimal (type 3, 97 leaves, 12 steps):

$$e^x - \frac{2 \operatorname{ArcTan}[e^x]}{3} + \frac{1}{3} \operatorname{ArcTan}[\sqrt{3} - 2e^x] - \frac{1}{3} \operatorname{ArcTan}[\sqrt{3} + 2e^x] + \frac{\operatorname{Log}[1 - \sqrt{3}e^x + e^{2x}]}{2\sqrt{3}} - \frac{\operatorname{Log}[1 + \sqrt{3}e^x + e^{2x}]}{2\sqrt{3}}$$

Result (type 7, 81 leaves):

$$e^x - \frac{2 \operatorname{ArcTan}[e^x]}{3} - \frac{1}{3} \operatorname{RootSum}\left[1 - \#1^2 + \#1^4 \&, \frac{-2x + 2 \operatorname{Log}[e^x - \#1] + x \#1^2 - \operatorname{Log}[e^x - \#1] \#1^2}{-\#1 + 2 \#1^3} \&\right]$$

Problem 222: Result is not expressed in closed-form.

$$\int e^x \operatorname{Tanh}[4x]^2 dx$$

Optimal (type 3, 382 leaves, 23 steps):

$$e^x + \frac{e^x}{2(1 + e^{8x})} + \frac{\operatorname{ArcTan}\left[\frac{\sqrt{2-\sqrt{2}} - 2e^x}{\sqrt{2+\sqrt{2}}}\right]}{8\sqrt{2(2-\sqrt{2})}} + \frac{\operatorname{ArcTan}\left[\frac{\sqrt{2+\sqrt{2}} - 2e^x}{\sqrt{2-\sqrt{2}}}\right]}{8\sqrt{2(2+\sqrt{2})}} - \frac{\operatorname{ArcTan}\left[\frac{\sqrt{2-\sqrt{2}} + 2e^x}{\sqrt{2+\sqrt{2}}}\right]}{8\sqrt{2(2-\sqrt{2})}} -$$

$$\frac{\operatorname{ArcTan}\left[\frac{\sqrt{2+\sqrt{2}} + 2e^x}{\sqrt{2-\sqrt{2}}}\right]}{8\sqrt{2(2+\sqrt{2})}} + \frac{1}{32} \sqrt{2-\sqrt{2}} \operatorname{Log}[1 - \sqrt{2-\sqrt{2}}e^x + e^{2x}] - \frac{1}{32} \sqrt{2-\sqrt{2}} \operatorname{Log}[1 + \sqrt{2-\sqrt{2}}e^x + e^{2x}] +$$

$$\frac{1}{32} \sqrt{2+\sqrt{2}} \operatorname{Log}[1 - \sqrt{2+\sqrt{2}}e^x + e^{2x}] - \frac{1}{32} \sqrt{2+\sqrt{2}} \operatorname{Log}[1 + \sqrt{2+\sqrt{2}}e^x + e^{2x}]$$

Result (type 7, 51 leaves):

$$e^x + \frac{e^x}{2(1 + e^{8x})} + \frac{1}{16} \operatorname{RootSum}\left[1 + \#1^8 \&, \frac{x - \operatorname{Log}[e^x - \#1]}{\#1^7} \&\right]$$

Problem 223: Result is not expressed in closed-form.

$$\int e^x \operatorname{Tanh}[4x] dx$$

Optimal (type 3, 366 leaves, 21 steps):

$$e^x + \frac{\operatorname{ArcTan}\left[\frac{\sqrt{2-\sqrt{2}}-2e^x}{\sqrt{2+\sqrt{2}}}\right]}{2\sqrt{2(2-\sqrt{2})}} + \frac{\operatorname{ArcTan}\left[\frac{\sqrt{2+\sqrt{2}}-2e^x}{\sqrt{2-\sqrt{2}}}\right]}{2\sqrt{2(2+\sqrt{2})}} - \frac{\operatorname{ArcTan}\left[\frac{\sqrt{2-\sqrt{2}}+2e^x}{\sqrt{2+\sqrt{2}}}\right]}{2\sqrt{2(2-\sqrt{2})}} - \frac{\operatorname{ArcTan}\left[\frac{\sqrt{2+\sqrt{2}}+2e^x}{\sqrt{2-\sqrt{2}}}\right]}{2\sqrt{2(2+\sqrt{2})}} + \frac{1}{8}\sqrt{2-\sqrt{2}} \operatorname{Log}\left[1-\sqrt{2-\sqrt{2}}e^x+e^{2x}\right] - \frac{1}{8}\sqrt{2-\sqrt{2}} \operatorname{Log}\left[1+\sqrt{2-\sqrt{2}}e^x+e^{2x}\right] + \frac{1}{8}\sqrt{2+\sqrt{2}} \operatorname{Log}\left[1-\sqrt{2+\sqrt{2}}e^x+e^{2x}\right] - \frac{1}{8}\sqrt{2+\sqrt{2}} \operatorname{Log}\left[1+\sqrt{2+\sqrt{2}}e^x+e^{2x}\right]$$

Result (type 7, 35 leaves):

$$e^x + \frac{1}{4} \operatorname{RootSum}\left[1 + \#1^8 \&, \frac{x - \operatorname{Log}[e^x - \#1]}{\#1^7} \&\right]$$

Problem 224: Result is not expressed in closed-form.

$$\int e^x \operatorname{Coth}[4x] dx$$

Optimal (type 3, 116 leaves, 15 steps):

$$e^x - \frac{\operatorname{ArcTan}[e^x]}{2} + \frac{\operatorname{ArcTan}[1-\sqrt{2}e^x]}{2\sqrt{2}} - \frac{\operatorname{ArcTan}[1+\sqrt{2}e^x]}{2\sqrt{2}} - \frac{\operatorname{ArcTanh}[e^x]}{2} + \frac{\operatorname{Log}[1-\sqrt{2}e^x+e^{2x}]}{4\sqrt{2}} - \frac{\operatorname{Log}[1+\sqrt{2}e^x+e^{2x}]}{4\sqrt{2}}$$

Result (type 7, 59 leaves):

$$\frac{1}{4} \left(4e^x - 2\operatorname{ArcTan}[e^x] + \operatorname{Log}[1-e^x] - \operatorname{Log}[1+e^x] + \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{x - \operatorname{Log}[e^x - \#1]}{\#1^3} \&\right] \right)$$

Problem 225: Result is not expressed in closed-form.

$$\int e^x \operatorname{Coth}[4x]^2 dx$$

Optimal (type 3, 134 leaves, 17 steps):

$$e^x + \frac{e^x}{2(1-e^{8x})} - \frac{\operatorname{ArcTan}[e^x]}{8} + \frac{\operatorname{ArcTan}[1-\sqrt{2}e^x]}{8\sqrt{2}} - \frac{\operatorname{ArcTan}[1+\sqrt{2}e^x]}{8\sqrt{2}} - \frac{\operatorname{ArcTanh}[e^x]}{8} + \frac{\operatorname{Log}[1-\sqrt{2}e^x+e^{2x}]}{16\sqrt{2}} - \frac{\operatorname{Log}[1+\sqrt{2}e^x+e^{2x}]}{16\sqrt{2}}$$

Result (type 7, 73 leaves):

$$\frac{1}{16} \left(16 e^x - \frac{8 e^x}{-1 + e^{8x}} - 2 \operatorname{ArcTan}[e^x] + \operatorname{Log}[1 - e^x] - \operatorname{Log}[1 + e^x] + \operatorname{RootSum}\left[1 + \#1^4, \frac{x - \operatorname{Log}[e^x - \#1]}{\#1^3} \& \right] \right)$$

Problem 226: Result is not expressed in closed-form.

$$\int \frac{e^x}{a - \operatorname{Tanh}[2x]} dx$$

Optimal (type 3, 107 leaves, 5 steps):

$$-\frac{e^x}{1-a} + \frac{\operatorname{ArcTan}\left[\frac{(1-a)^{1/4} e^x}{(1+a)^{1/4}}\right]}{(1-a)\sqrt{1+a}(1-a^2)^{1/4}} + \frac{\operatorname{ArcTanh}\left[\frac{(1-a)^{1/4} e^x}{(1+a)^{1/4}}\right]}{(1-a)\sqrt{1+a}(1-a^2)^{1/4}}$$

Result (type 7, 54 leaves):

$$\frac{2(-1+a)e^x + \operatorname{RootSum}\left[1+a - \#1^4 + a\#1^4, \frac{x - \operatorname{Log}[e^x - \#1]}{\#1^3} \& \right]}{2(-1+a)^2}$$

Problem 227: Result is not expressed in closed-form.

$$\int \frac{e^x}{(a - \operatorname{Tanh}[2x])^2} dx$$

Optimal (type 3, 152 leaves, 7 steps):

$$\frac{e^x}{(1-a)^2} + \frac{e^x}{(1-a)^2(1+a)(1+a+(-1+a)e^{4x})} - \frac{(1+4a)\operatorname{ArcTan}\left[\frac{(1-a)^{1/4} e^x}{(1+a)^{1/4}}\right]}{2(1-a)^2(1+a)^{3/2}(1-a^2)^{1/4}} - \frac{(1+4a)\operatorname{ArcTanh}\left[\frac{(1-a)^{1/4} e^x}{(1+a)^{1/4}}\right]}{2(1-a)^2(1+a)^{3/2}(1-a^2)^{1/4}}$$

Result (type 7, 107 leaves):

$$\frac{4(-1+a)e^x(2+2a-e^{4x}+a^2(1+e^{4x}))}{1+a-e^{4x}+ae^{4x}} + \frac{(1+4a)\operatorname{RootSum}\left[1+a - \#1^4 + a\#1^4, \frac{x - \operatorname{Log}[e^x - \#1]}{\#1^3} \& \right]}{4(-1+a)^3(1+a)}$$

Problem 230: Result more than twice size of optimal antiderivative.

$$\int e^{c(a+bx)} \operatorname{Tanh}[d+ex] dx$$

Optimal (type 5, 67 leaves, 4 steps):

$$\frac{e^{c(a+bx)}}{bc} - \frac{2e^{c(a+bx)} \operatorname{Hypergeometric2F1}\left[1, \frac{bc}{2e}, 1 + \frac{bc}{2e}, -e^{2(d+ex)}\right]}{bc}$$

Result (type 5, 141 leaves):

$$\frac{1}{bc(b c + 2 e) (1 + e^{2d})} e^{c(a+bx)} \left(2bc e^{2(d+ex)} \operatorname{Hypergeometric2F1}\left[1, 1 + \frac{bc}{2e}, 2 + \frac{bc}{2e}, -e^{2(d+ex)}\right] - (bc + 2e) \left(1 - e^{2d} + 2e^{2d} \operatorname{Hypergeometric2F1}\left[1, \frac{bc}{2e}, 1 + \frac{bc}{2e}, -e^{2(d+ex)}\right] \right) \right)$$

Problem 231: Result more than twice size of optimal antiderivative.

$$\int e^{c(a+bx)} \operatorname{Coth}[d+ex] dx$$

Optimal (type 5, 65 leaves, 4 steps):

$$\frac{e^{c(a+bx)}}{bc} - \frac{2e^{c(a+bx)} \operatorname{Hypergeometric2F1}\left[1, \frac{bc}{2e}, 1 + \frac{bc}{2e}, e^{2(d+ex)}\right]}{bc}$$

Result (type 5, 134 leaves):

$$\frac{1}{bc(b c + 2 e) (-1 + e^{2d})} e^{c(a+bx)} \left(2bc e^{2(d+ex)} \operatorname{Hypergeometric2F1}\left[1, 1 + \frac{bc}{2e}, 2 + \frac{bc}{2e}, e^{2(d+ex)}\right] + (bc + 2e) \left(1 + e^{2d} - 2e^{2d} \operatorname{Hypergeometric2F1}\left[1, \frac{bc}{2e}, 1 + \frac{bc}{2e}, e^{2(d+ex)}\right] \right) \right)$$

Test results for the 263 problems in "6.3.7 (d hyper)^m (a+b (c tanh)^n)^p.m"

Problem 7: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Csch}[c+dx]^3 (a+b \operatorname{Tanh}[c+dx]^2) dx$$

Optimal (type 3, 51 leaves, 4 steps):

$$\frac{(a-2b) \operatorname{ArcTanh}[\operatorname{Cosh}[c+dx]]}{2d} - \frac{a \operatorname{Coth}[c+dx] \operatorname{Csch}[c+dx]}{2d} + \frac{b \operatorname{Sech}[c+dx]}{d}$$

Result (type 3, 123 leaves):

$$-\frac{a \operatorname{Csch}\left[\frac{1}{2}(c+dx)\right]^2}{8d} + \frac{a \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right]\right]}{2d} - \frac{b \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right]\right]}{d} -$$

$$\frac{a \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right]\right]}{2d} + \frac{b \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right]\right]}{d} - \frac{a \operatorname{Sech}\left[\frac{1}{2}(c+dx)\right]^2}{8d} + \frac{b \operatorname{Sech}[c+dx]}{d}$$

Problem 26: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Sinh}[c+dx]^3}{a+b \operatorname{Tanh}[c+dx]^2} dx$$

Optimal (type 3, 75 leaves, 4 steps):

$$\frac{a \sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Sech}[c+dx]}{\sqrt{a+b}}\right]}{(a+b)^{5/2} d} - \frac{a \operatorname{Cosh}[c+dx]}{(a+b)^2 d} + \frac{\operatorname{Cosh}[c+dx]^3}{3(a+b)d}$$

Result (type 3, 135 leaves):

$$\frac{1}{12(a+b)^{5/2} d} \left(12 i a \sqrt{b} \left(\operatorname{ArcTan}\left[\frac{-i \sqrt{a+b} - \sqrt{a} \operatorname{Tanh}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{b}}\right] + \operatorname{ArcTan}\left[\frac{-i \sqrt{a+b} + \sqrt{a} \operatorname{Tanh}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{b}}\right] \right) - \right.$$

$$\left. 3(3a-b) \sqrt{a+b} \operatorname{Cosh}[c+dx] + (a+b)^{3/2} \operatorname{Cosh}[3(c+dx)] \right)$$

Problem 28: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sinh}[c+dx]}{a+b \operatorname{Tanh}[c+dx]^2} dx$$

Optimal (type 3, 53 leaves, 3 steps):

$$-\frac{\sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Sech}[c+dx]}{\sqrt{a+b}}\right]}{(a+b)^{3/2} d} + \frac{\operatorname{Cosh}[c+dx]}{(a+b)d}$$

Result (type 3, 107 leaves):

$$\frac{-i \sqrt{b} \left(\operatorname{ArcTan}\left[\frac{-i \sqrt{a+b} - \sqrt{a} \operatorname{Tanh}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{b}}\right] + \operatorname{ArcTan}\left[\frac{-i \sqrt{a+b} + \sqrt{a} \operatorname{Tanh}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{b}}\right] \right) + \sqrt{a+b} \operatorname{Cosh}[c+dx]}{(a+b)^{3/2} d}$$

Problem 29: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Csch}[c + d x]}{a + b \text{Tanh}[c + d x]^2} dx$$

Optimal (type 3, 55 leaves, 4 steps):

$$-\frac{\text{ArcTanh}[\text{Cosh}[c + d x]]}{a d} + \frac{\sqrt{b} \text{ArcTanh}\left[\frac{\sqrt{b} \text{Sech}[c + d x]}{\sqrt{a + b}}\right]}{a \sqrt{a + b} d}$$

Result (type 3, 135 leaves):

$$\frac{1}{a d} \left(\frac{i \sqrt{b} \text{ArcTan}\left[\frac{-i \sqrt{a + b} - \sqrt{a} \text{Tanh}\left[\frac{1}{2}(c + d x)\right]}{\sqrt{b}}\right]}{\sqrt{a + b}} + \frac{i \sqrt{b} \text{ArcTan}\left[\frac{-i \sqrt{a + b} + \sqrt{a} \text{Tanh}\left[\frac{1}{2}(c + d x)\right]}{\sqrt{b}}\right]}{\sqrt{a + b}} - \text{Log}\left[\text{Cosh}\left[\frac{1}{2}(c + d x)\right]\right] + \text{Log}\left[\text{Sinh}\left[\frac{1}{2}(c + d x)\right]\right] \right)$$

Problem 31: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Csch}[c + d x]^3}{a + b \text{Tanh}[c + d x]^2} dx$$

Optimal (type 3, 85 leaves, 5 steps):

$$\frac{(a + 2 b) \text{ArcTanh}[\text{Cosh}[c + d x]]}{2 a^2 d} - \frac{\sqrt{b} \sqrt{a + b} \text{ArcTanh}\left[\frac{\sqrt{b} \text{Sech}[c + d x]}{\sqrt{a + b}}\right]}{a^2 d} - \frac{\text{Coth}[c + d x] \text{Csch}[c + d x]}{2 a d}$$

Result (type 3, 198 leaves):

$$-\frac{1}{8 a^2 d} \left(8 i \sqrt{b} \sqrt{a + b} \text{ArcTan}\left[\frac{-i \sqrt{a + b} - \sqrt{a} \text{Tanh}\left[\frac{1}{2}(c + d x)\right]}{\sqrt{b}}\right] + 8 i \sqrt{b} \sqrt{a + b} \text{ArcTan}\left[\frac{-i \sqrt{a + b} + \sqrt{a} \text{Tanh}\left[\frac{1}{2}(c + d x)\right]}{\sqrt{b}}\right] + a \text{Csch}\left[\frac{1}{2}(c + d x)\right]^2 - 4 a \text{Log}\left[\text{Cosh}\left[\frac{1}{2}(c + d x)\right]\right] - 8 b \text{Log}\left[\text{Cosh}\left[\frac{1}{2}(c + d x)\right]\right] + 4 a \text{Log}\left[\text{Sinh}\left[\frac{1}{2}(c + d x)\right]\right] + 8 b \text{Log}\left[\text{Sinh}\left[\frac{1}{2}(c + d x)\right]\right] + a \text{Sech}\left[\frac{1}{2}(c + d x)\right]^2 \right)$$

Problem 34: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{Sinh}[c + d x]^3}{(a + b \text{Tanh}[c + d x]^2)^2} dx$$

Optimal (type 3, 124 leaves, 5 steps):

$$\frac{(3a - 2b) \sqrt{b} \text{ArcTanh}\left[\frac{\sqrt{b} \text{Sech}[c + d x]}{\sqrt{a + b}}\right]}{2(a + b)^{7/2} d} - \frac{(a - b) \text{Cosh}[c + d x]}{(a + b)^3 d} + \frac{\text{Cosh}[c + d x]^3}{3(a + b)^2 d} + \frac{ab \text{Sech}[c + d x]}{2(a + b)^3 d (a + b - b \text{Sech}[c + d x]^2)}$$

Result (type 3, 160 leaves):

$$\frac{1}{12d} \left(\frac{6i(3a - 2b)\sqrt{b} \left(\text{ArcTan}\left[\frac{-i\sqrt{a+b} - \sqrt{a} \text{Tanh}\left[\frac{1}{2}(c + d x)\right]}{\sqrt{b}}\right] + \text{ArcTan}\left[\frac{-i\sqrt{a+b} + \sqrt{a} \text{Tanh}\left[\frac{1}{2}(c + d x)\right]}{\sqrt{b}}\right] \right)}{(a + b)^{7/2}} + \frac{3 \text{Cosh}[c + d x] \left(5b + a \left(-3 + \frac{4b}{a - b + (a + b) \text{Cosh}[2(c + d x)]} \right) \right)}{(a + b)^3} + \frac{\text{Cosh}[3(c + d x)]}{(a + b)^2} \right)$$

Problem 36: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{Sinh}[c + d x]}{(a + b \text{Tanh}[c + d x]^2)^2} dx$$

Optimal (type 3, 92 leaves, 4 steps):

$$-\frac{3\sqrt{b} \text{ArcTanh}\left[\frac{\sqrt{b} \text{Sech}[c + d x]}{\sqrt{a + b}}\right]}{2(a + b)^{5/2} d} + \frac{3 \text{Cosh}[c + d x]}{2(a + b)^2 d} - \frac{\text{Cosh}[c + d x]}{2(a + b) d (a + b - b \text{Sech}[c + d x]^2)}$$

Result (type 3, 133 leaves):

$$-\frac{3i\sqrt{b} \left(\text{ArcTan}\left[\frac{-i\sqrt{a+b} - \sqrt{a} \text{Tanh}\left[\frac{1}{2}(c + d x)\right]}{\sqrt{b}}\right] + \text{ArcTan}\left[\frac{-i\sqrt{a+b} + \sqrt{a} \text{Tanh}\left[\frac{1}{2}(c + d x)\right]}{\sqrt{b}}\right] \right)}{(a + b)^{5/2}} + \frac{2 \text{Cosh}[c + d x] \left(1 - \frac{b}{a - b + (a + b) \text{Cosh}[2(c + d x)]} \right)}{(a + b)^2}$$

2 d

Problem 37: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{Csch}[c + d x]}{(a + b \text{Tanh}[c + d x]^2)^2} dx$$

Optimal (type 3, 103 leaves, 5 steps):

$$-\frac{\text{ArcTanh}[\text{Cosh}[c + d x]]}{a^2 d} + \frac{\sqrt{b} (3 a + 2 b) \text{ArcTanh}\left[\frac{\sqrt{b} \text{Sech}[c + d x]}{\sqrt{a + b}}\right]}{2 a^2 (a + b)^{3/2} d} + \frac{b \text{Sech}[c + d x]}{2 a (a + b) d (a + b - b \text{Sech}[c + d x]^2)}$$

Result (type 3, 188 leaves):

$$\frac{1}{2 a^2 d} \left(\frac{i \sqrt{b} (3 a + 2 b) \text{ArcTan}\left[\frac{-i \sqrt{a + b} - \sqrt{a} \text{Tanh}\left[\frac{1}{2} (c + d x)\right]}{\sqrt{b}}\right]}{(a + b)^{3/2}} + \frac{i \sqrt{b} (3 a + 2 b) \text{ArcTan}\left[\frac{-i \sqrt{a + b} + \sqrt{a} \text{Tanh}\left[\frac{1}{2} (c + d x)\right]}{\sqrt{b}}\right]}{(a + b)^{3/2}} + \right. \\ \left. \frac{2 a b \text{Cosh}[c + d x]}{(a + b) (a - b + (a + b) \text{Cosh}[2 (c + d x)])} - 2 \text{Log}\left[\text{Cosh}\left[\frac{1}{2} (c + d x)\right]\right] + 2 \text{Log}\left[\text{Sinh}\left[\frac{1}{2} (c + d x)\right]\right] \right)$$

Problem 39: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Csch}[c + d x]^3}{(a + b \text{Tanh}[c + d x]^2)^2} dx$$

Optimal (type 3, 141 leaves, 6 steps):

$$\frac{(a + 4 b) \text{ArcTanh}[\text{Cosh}[c + d x]]}{2 a^3 d} - \frac{\sqrt{b} (3 a + 4 b) \text{ArcTanh}\left[\frac{\sqrt{b} \text{Sech}[c + d x]}{\sqrt{a + b}}\right]}{2 a^3 \sqrt{a + b} d} - \frac{\text{Coth}[c + d x] \text{Csch}[c + d x]}{2 a d (a + b - b \text{Sech}[c + d x]^2)} - \frac{b \text{Sech}[c + d x]}{a^2 d (a + b - b \text{Sech}[c + d x]^2)}$$

Result (type 3, 314 leaves):

$$\begin{aligned}
& \frac{i \sqrt{b} (3a + 4b) \operatorname{ArcTan} \left[\frac{\operatorname{Sech} \left[\frac{1}{2} (c + dx) \right] \left(-i \sqrt{a+b} \operatorname{Cosh} \left[\frac{1}{2} (c + dx) \right] - \sqrt{a} \operatorname{Sinh} \left[\frac{1}{2} (c + dx) \right] \right)}{\sqrt{b}} \right]}{2 a^3 \sqrt{a+b} d} \\
& \frac{i \sqrt{b} (3a + 4b) \operatorname{ArcTan} \left[\frac{\operatorname{Sech} \left[\frac{1}{2} (c + dx) \right] \left(-i \sqrt{a+b} \operatorname{Cosh} \left[\frac{1}{2} (c + dx) \right] + \sqrt{a} \operatorname{Sinh} \left[\frac{1}{2} (c + dx) \right] \right)}{\sqrt{b}} \right]}{2 a^3 \sqrt{a+b} d} - \frac{b \operatorname{Cosh} [c + dx]}{a^2 d (a - b + a \operatorname{Cosh} [2 (c + dx)] + b \operatorname{Cosh} [2 (c + dx)])} \\
& \frac{\operatorname{Csch} \left[\frac{1}{2} (c + dx) \right]^2}{8 a^2 d} + \frac{(a + 4b) \operatorname{Log} \left[\operatorname{Cosh} \left[\frac{1}{2} (c + dx) \right] \right]}{2 a^3 d} + \frac{(-a - 4b) \operatorname{Log} \left[\operatorname{Sinh} \left[\frac{1}{2} (c + dx) \right] \right]}{2 a^3 d} - \frac{\operatorname{Sech} \left[\frac{1}{2} (c + dx) \right]^2}{8 a^2 d}
\end{aligned}$$

Problem 42: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Sinh} [c + dx]^3}{(a + b \operatorname{Tanh} [c + dx]^2)^3} dx$$

Optimal (type 3, 166 leaves, 6 steps):

$$\begin{aligned}
& \frac{5 (3a - 4b) \sqrt{b} \operatorname{ArcTanh} \left[\frac{\sqrt{b} \operatorname{Sech} [c + dx]}{\sqrt{a+b}} \right]}{8 (a + b)^{9/2} d} - \frac{(a - 2b) \operatorname{Cosh} [c + dx]}{(a + b)^4 d} + \\
& \frac{\operatorname{Cosh} [c + dx]^3}{3 (a + b)^3 d} + \frac{a b \operatorname{Sech} [c + dx]}{4 (a + b)^3 d (a + b - b \operatorname{Sech} [c + dx]^2)^2} + \frac{(7a - 4b) b \operatorname{Sech} [c + dx]}{8 (a + b)^4 d (a + b - b \operatorname{Sech} [c + dx]^2)}
\end{aligned}$$

Result (type 3, 227 leaves):

$$\begin{aligned}
& \frac{1}{24 d} \left(\frac{15 i (3a - 4b) \sqrt{b} \left(\operatorname{ArcTan} \left[\frac{-i \sqrt{a+b} - \sqrt{a} \operatorname{Tanh} \left[\frac{1}{2} (c + dx) \right]}{\sqrt{b}} \right] + \operatorname{ArcTan} \left[\frac{-i \sqrt{a+b} + \sqrt{a} \operatorname{Tanh} \left[\frac{1}{2} (c + dx) \right]}{\sqrt{b}} \right] \right)}{(a + b)^{9/2}} - \right. \\
& \left. \left(6 \operatorname{Cosh} [c + dx] \left(3a^3 - 24a^2 b + 30a b^2 - 13b^3 + (6a^3 - 27a^2 b - 11a b^2 + 22b^3) \operatorname{Cosh} [2 (c + dx)] + 3(a - 3b) (a + b)^2 \operatorname{Cosh} [2 (c + dx)]^2 \right) \right) / \right. \\
& \left. \left((a + b)^4 (a - b + (a + b) \operatorname{Cosh} [2 (c + dx)])^2 + \frac{2 \operatorname{Cosh} [3 (c + dx)]}{(a + b)^3} \right) \right)
\end{aligned}$$

Problem 44: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Sinh} [c + dx]}{(a + b \operatorname{Tanh} [c + dx]^2)^3} dx$$

Optimal (type 3, 126 leaves, 5 steps):

$$-\frac{15\sqrt{b}\operatorname{ArcTanh}\left[\frac{\sqrt{b}\operatorname{Sech}[c+dx]}{\sqrt{a+b}}\right]}{8(a+b)^{7/2}d} + \frac{15\operatorname{Cosh}[c+dx]}{8(a+b)^3d} - \frac{\operatorname{Cosh}[c+dx]}{4(a+b)d(a+b-b\operatorname{Sech}[c+dx])^2} - \frac{5\operatorname{Cosh}[c+dx]}{8(a+b)^2d(a+b-b\operatorname{Sech}[c+dx])^2}$$

Result (type 3, 157 leaves):

$$\frac{1}{8d} \left(\frac{15i\sqrt{b} \left(\operatorname{ArcTan}\left[\frac{-i\sqrt{a+b}-\sqrt{a}\operatorname{Tanh}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{b}}\right] + \operatorname{ArcTan}\left[\frac{-i\sqrt{a+b}+\sqrt{a}\operatorname{Tanh}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{b}}\right] \right)}{(a+b)^{7/2}} + \frac{2\operatorname{Cosh}[c+dx] \left(4 - \frac{4b^2}{(a-b+(a+b)\operatorname{Cosh}[2(c+dx)])^2} - \frac{9b}{a-b+(a+b)\operatorname{Cosh}[2(c+dx)]} \right)}{(a+b)^3} \right)$$

Problem 45: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Csch}[c+dx]}{(a+b\operatorname{Tanh}[c+dx])^3} dx$$

Optimal (type 3, 156 leaves, 6 steps):

$$-\frac{\operatorname{ArcTanh}[\operatorname{Cosh}[c+dx]]}{a^3d} + \frac{\sqrt{b}(15a^2+20ab+8b^2)\operatorname{ArcTanh}\left[\frac{\sqrt{b}\operatorname{Sech}[c+dx]}{\sqrt{a+b}}\right]}{8a^3(a+b)^{5/2}d} + \frac{b\operatorname{Sech}[c+dx]}{4a(a+b)d(a+b-b\operatorname{Sech}[c+dx])^2} + \frac{b(7a+4b)\operatorname{Sech}[c+dx]}{8a^2(a+b)^2d(a+b-b\operatorname{Sech}[c+dx])^2}$$

Result (type 3, 249 leaves):

$$\frac{1}{8a^3d} \left(\frac{i\sqrt{b}(15a^2+20ab+8b^2)\operatorname{ArcTan}\left[\frac{-i\sqrt{a+b}-\sqrt{a}\operatorname{Tanh}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{b}}\right]}{(a+b)^{5/2}} + \frac{i\sqrt{b}(15a^2+20ab+8b^2)\operatorname{ArcTan}\left[\frac{-i\sqrt{a+b}+\sqrt{a}\operatorname{Tanh}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{b}}\right]}{(a+b)^{5/2}} + \frac{8a^2b^2\operatorname{Cosh}[c+dx]}{(a+b)^2(a-b+(a+b)\operatorname{Cosh}[2(c+dx)])^2} + \frac{2ab(9a+4b)\operatorname{Cosh}[c+dx]}{(a+b)^2(a-b+(a+b)\operatorname{Cosh}[2(c+dx)])} - 8\operatorname{Log}\left[\operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right]\right] + 8\operatorname{Log}\left[\operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right]\right] \right)$$

Problem 47: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Csch}[c + d x]^3}{(a + b \text{Tanh}[c + d x]^2)^3} dx$$

Optimal (type 3, 196 leaves, 7 steps):

$$\frac{(a + 6 b) \text{ArcTanh}[\text{Cosh}[c + d x]]}{2 a^4 d} - \frac{\sqrt{b} (15 a^2 + 40 a b + 24 b^2) \text{ArcTanh}\left[\frac{\sqrt{b} \text{Sech}[c + d x]}{\sqrt{a + b}}\right]}{8 a^4 (a + b)^{3/2} d} - \frac{\text{Coth}[c + d x] \text{Csch}[c + d x]}{2 a d (a + b - b \text{Sech}[c + d x]^2)^2} - \frac{3 b \text{Sech}[c + d x]}{4 a^2 d (a + b - b \text{Sech}[c + d x]^2)^2} - \frac{b (11 a + 12 b) \text{Sech}[c + d x]}{8 a^3 (a + b) d (a + b - b \text{Sech}[c + d x]^2)^2}$$

Result (type 3, 401 leaves):

$$\frac{i \sqrt{b} (15 a^2 + 40 a b + 24 b^2) \text{ArcTan}\left[\frac{\text{Sech}\left[\frac{1}{2}(c + d x)\right] \left(-i \sqrt{a + b} \text{Cosh}\left[\frac{1}{2}(c + d x)\right] - \sqrt{a} \text{Sinh}\left[\frac{1}{2}(c + d x)\right]\right)}{\sqrt{b}}\right]}{8 a^4 (a + b)^{3/2} d} - \frac{i \sqrt{b} (15 a^2 + 40 a b + 24 b^2) \text{ArcTan}\left[\frac{\text{Sech}\left[\frac{1}{2}(c + d x)\right] \left(-i \sqrt{a + b} \text{Cosh}\left[\frac{1}{2}(c + d x)\right] + \sqrt{a} \text{Sinh}\left[\frac{1}{2}(c + d x)\right]\right)}{\sqrt{b}}\right]}{8 a^4 (a + b)^{3/2} d} - \frac{b^2 \text{Cosh}[c + d x]}{a^2 (a + b) d (a - b + a \text{Cosh}[2(c + d x)] + b \text{Cosh}[2(c + d x)])^2} + \frac{-9 a b \text{Cosh}[c + d x] - 8 b^2 \text{Cosh}[c + d x]}{4 a^3 (a + b) d (a - b + a \text{Cosh}[2(c + d x)] + b \text{Cosh}[2(c + d x)])} - \frac{\text{Csch}\left[\frac{1}{2}(c + d x)\right]^2}{8 a^3 d} + \frac{(a + 6 b) \text{Log}\left[\text{Cosh}\left[\frac{1}{2}(c + d x)\right]\right]}{2 a^4 d} + \frac{(-a - 6 b) \text{Log}\left[\text{Sinh}\left[\frac{1}{2}(c + d x)\right]\right]}{2 a^4 d} - \frac{\text{Sech}\left[\frac{1}{2}(c + d x)\right]^2}{8 a^3 d}$$

Problem 73: Result is not expressed in closed-form.

$$\int \frac{\text{Sinh}[c + d x]^4}{a + b \text{Tanh}[c + d x]^3} dx$$

Optimal (type 3, 491 leaves, 11 steps):

$$\begin{aligned}
& - \frac{a^{2/3} b^{1/3} (a^2 + 3 a^{4/3} b^{2/3} - b^2) \operatorname{ArcTan}\left[\frac{a^{1/3} - 2 b^{1/3} \operatorname{Tanh}[c+dx]}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} (a^{4/3} + a^{2/3} b^{2/3} + b^{4/3})^3 d} - \frac{3 a (a - 5 b) \operatorname{Log}[1 - \operatorname{Tanh}[c + dx]]}{16 (a + b)^3 d} + \\
& \frac{3 a (a + 5 b) \operatorname{Log}[1 + \operatorname{Tanh}[c + dx]]}{16 (a - b)^3 d} - \frac{a^{2/3} b^{1/3} (a^4 + 7 a^2 b^2 + b^4 + 3 a^{2/3} b^{4/3} (2 a^2 + b^2)) \operatorname{Log}[a^{1/3} + b^{1/3} \operatorname{Tanh}[c + dx]]}{3 (a^2 - b^2)^3 d} + \\
& \frac{a^{2/3} b^{1/3} (a^4 + 7 a^2 b^2 + b^4 + 3 a^{2/3} b^{4/3} (2 a^2 + b^2)) \operatorname{Log}[a^{2/3} - a^{1/3} b^{1/3} \operatorname{Tanh}[c + dx] + b^{2/3} \operatorname{Tanh}[c + dx]^2]}{6 (a^2 - b^2)^3 d} - \frac{a^2 b (a^2 + 2 b^2) \operatorname{Log}[a + b \operatorname{Tanh}[c + dx]^3]}{(a^2 - b^2)^3 d} + \\
& \frac{1}{16 (a + b) d (1 - \operatorname{Tanh}[c + dx])^2} - \frac{5 a - b}{16 (a + b)^2 d (1 - \operatorname{Tanh}[c + dx])} - \frac{1}{16 (a - b) d (1 + \operatorname{Tanh}[c + dx])^2} + \frac{5 a + b}{16 (a - b)^2 d (1 + \operatorname{Tanh}[c + dx])}
\end{aligned}$$

Result (type 7, 645 leaves):

$$\begin{aligned}
& \frac{1}{96 (a - b)^2 (a + b)^3 d} \\
& \left(-32 a b \operatorname{RootSum}\left[a - b + 3 a \#1 + 3 b \#1 + 3 a \#1^2 - 3 b \#1^2 + a \#1^3 + b \#1^3 \&, \frac{1}{a - b + 2 a \#1 + 2 b \#1 + a \#1^2 - b \#1^2} \left(-6 a^3 c - 12 a b^2 c - 6 a^3 d x - \right. \right. \\
& \quad 12 a b^2 d x + 3 a^3 \operatorname{Log}\left[e^{2(c+dx)} - \#1 \right] + 6 a b^2 \operatorname{Log}\left[e^{2(c+dx)} - \#1 \right] - 8 a^3 c \#1 + 4 a^2 b c \#1 + 8 a b^2 c \#1 - 4 b^3 c \#1 - 8 a^3 d x \#1 + \\
& \quad 4 a^2 b d x \#1 + 8 a b^2 d x \#1 - 4 b^3 d x \#1 + 4 a^3 \operatorname{Log}\left[e^{2(c+dx)} - \#1 \right] \#1 - 2 a^2 b \operatorname{Log}\left[e^{2(c+dx)} - \#1 \right] \#1 - 4 a b^2 \operatorname{Log}\left[e^{2(c+dx)} - \#1 \right] \#1 + \\
& \quad 2 b^3 \operatorname{Log}\left[e^{2(c+dx)} - \#1 \right] \#1 - 10 a^3 c \#1^2 + 20 a^2 b c \#1^2 - 20 a b^2 c \#1^2 + 4 b^3 c \#1^2 - 10 a^3 d x \#1^2 + 20 a^2 b d x \#1^2 - 20 a b^2 d x \#1^2 + \\
& \quad \left. \left. 4 b^3 d x \#1^2 + 5 a^3 \operatorname{Log}\left[e^{2(c+dx)} - \#1 \right] \#1^2 - 10 a^2 b \operatorname{Log}\left[e^{2(c+dx)} - \#1 \right] \#1^2 + 10 a b^2 \operatorname{Log}\left[e^{2(c+dx)} - \#1 \right] \#1^2 - 2 b^3 \operatorname{Log}\left[e^{2(c+dx)} - \#1 \right] \#1^2 \right) \& \right] + \\
& \quad 3 \left(4 b (5 a^3 + 5 a^2 b + a b^2 + b^3) \operatorname{Cosh}[2(c + dx)] - (a - b) b (a + b)^2 \operatorname{Cosh}[4(c + dx)] - 8 a (a^3 + a^2 b + 2 a b^2 + 2 b^3) \operatorname{Sinh}[2(c + dx)] + \right. \\
& \quad \left. a (a - b) \left(12 (a^2 - 6 a b + 5 b^2) (c + dx) + (a + b)^2 \operatorname{Sinh}[4(c + dx)] \right) \right) \right)
\end{aligned}$$

Problem 75: Result is not expressed in closed-form.

$$\int \frac{\operatorname{Sinh}[c + dx]^2}{a + b \operatorname{Tanh}[c + dx]^3} dx$$

Optimal (type 3, 384 leaves, 11 steps):

$$\frac{a^{2/3} b^{1/3} (a^2 - 3 a^{2/3} b^{4/3} + 2 b^2) \operatorname{ArcTan}\left[\frac{a^{1/3} - 2 b^{1/3} \operatorname{Tanh}[c + d x]}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} (a^2 - b^2)^2 d} + \frac{(a - 2 b) \operatorname{Log}[1 - \operatorname{Tanh}[c + d x]]}{4 (a + b)^2 d} -$$

$$\frac{(a + 2 b) \operatorname{Log}[1 + \operatorname{Tanh}[c + d x]]}{4 (a - b)^2 d} + \frac{a^{2/3} b^{1/3} (a^2 + 3 a^{2/3} b^{4/3} + 2 b^2) \operatorname{Log}[a^{1/3} + b^{1/3} \operatorname{Tanh}[c + d x]]}{3 (a^2 - b^2)^2 d} -$$

$$\frac{a^{2/3} b^{1/3} (a^2 + 3 a^{2/3} b^{4/3} + 2 b^2) \operatorname{Log}[a^{2/3} - a^{1/3} b^{1/3} \operatorname{Tanh}[c + d x] + b^{2/3} \operatorname{Tanh}[c + d x]^2]}{6 (a^2 - b^2)^2 d} +$$

$$\frac{b (2 a^2 + b^2) \operatorname{Log}[a + b \operatorname{Tanh}[c + d x]^3]}{3 (a^2 - b^2)^2 d} + \frac{1}{4 (a + b) d (1 - \operatorname{Tanh}[c + d x])} - \frac{1}{4 (a - b) d (1 + \operatorname{Tanh}[c + d x])}$$

Result (type 7, 423 leaves):

$$- \frac{1}{12 (a - b) (a + b)^2 d} \left(6 (a^2 - 3 a b + 2 b^2) (c + d x) + 3 b (a + b) \operatorname{Cosh}[2 (c + d x)] + \right.$$

$$4 b \operatorname{RootSum}\left[a - b + 3 a \#1 + 3 b \#1 + 3 a \#1^2 - 3 b \#1^2 + a \#1^3 + b \#1^3 \&, \frac{1}{a - b + 2 a \#1 + 2 b \#1 + a \#1^2 - b \#1^2} \right.$$

$$\left(4 a^2 c + 2 b^2 c + 4 a^2 d x + 2 b^2 d x - 2 a^2 \operatorname{Log}\left[e^{2 (c + d x)} - \#1 \right] - b^2 \operatorname{Log}\left[e^{2 (c + d x)} - \#1 \right] + 4 a^2 c \#1 - 4 b^2 c \#1 + 4 a^2 d x \#1 - 4 b^2 d x \#1 - \right.$$

$$2 a^2 \operatorname{Log}\left[e^{2 (c + d x)} - \#1 \right] \#1 + 2 b^2 \operatorname{Log}\left[e^{2 (c + d x)} - \#1 \right] \#1 + 8 a^2 c \#1^2 - 8 a b c \#1^2 + 2 b^2 c \#1^2 + 8 a^2 d x \#1^2 - 8 a b d x \#1^2 + 2 b^2 d x \#1^2 -$$

$$\left. \left. 4 a^2 \operatorname{Log}\left[e^{2 (c + d x)} - \#1 \right] \#1^2 + 4 a b \operatorname{Log}\left[e^{2 (c + d x)} - \#1 \right] \#1^2 - b^2 \operatorname{Log}\left[e^{2 (c + d x)} - \#1 \right] \#1^2 \right] \& \right) - 3 a (a + b) \operatorname{Sinh}[2 (c + d x)] \left. \right)$$

Problem 78: Result is not expressed in closed-form.

$$\int \frac{\operatorname{Csch}[c + d x]^2}{a + b \operatorname{Tanh}[c + d x]^3} dx$$

Optimal (type 3, 157 leaves, 8 steps):

$$\frac{b^{1/3} \operatorname{ArcTan}\left[\frac{a^{1/3} - 2 b^{1/3} \operatorname{Tanh}[c + d x]}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} a^{4/3} d} - \frac{\operatorname{Coth}[c + d x]}{a d} + \frac{b^{1/3} \operatorname{Log}[a^{1/3} + b^{1/3} \operatorname{Tanh}[c + d x]]}{3 a^{4/3} d} - \frac{b^{1/3} \operatorname{Log}[a^{2/3} - a^{1/3} b^{1/3} \operatorname{Tanh}[c + d x] + b^{2/3} \operatorname{Tanh}[c + d x]^2]}{6 a^{4/3} d}$$

Result (type 7, 190 leaves):

$$- \frac{1}{3 a d} \left(3 \operatorname{Coth}[c + d x] + 2 b \operatorname{RootSum}\left[a - b + 3 a \#1 + 3 b \#1 + 3 a \#1^2 - 3 b \#1^2 + a \#1^3 + b \#1^3 \&, \right.$$

$$\left. \left(-c - d x - \operatorname{Log}[-\operatorname{Cosh}[c + d x] - \operatorname{Sinh}[c + d x]] + \operatorname{Cosh}[c + d x] \#1 - \operatorname{Sinh}[c + d x] \#1 + c \#1 + d x \#1 + \right. \right.$$

$$\left. \left. \operatorname{Log}[-\operatorname{Cosh}[c + d x] - \operatorname{Sinh}[c + d x] + \operatorname{Cosh}[c + d x] \#1 - \operatorname{Sinh}[c + d x] \#1] \#1 \right) / (a + b + 2 a \#1 - 2 b \#1 + a \#1^2 + b \#1^2) \& \right) \left. \right)$$

Problem 80: Result is not expressed in closed-form.

$$\int \frac{\text{Csch}[c + d x]^4}{a + b \text{Tanh}[c + d x]^3} dx$$

Optimal (type 3, 215 leaves, 12 steps):

$$\begin{aligned} & - \frac{b^{1/3} \text{ArcTan}\left[\frac{a^{1/3} - 2b^{1/3} \text{Tanh}[c + d x]}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} a^{4/3} d} + \frac{\text{Coth}[c + d x]}{a d} - \frac{\text{Coth}[c + d x]^3}{3 a d} - \frac{b \text{Log}[\text{Tanh}[c + d x]]}{a^2 d} \\ & - \frac{b^{1/3} \text{Log}[a^{1/3} + b^{1/3} \text{Tanh}[c + d x]]}{3 a^{4/3} d} + \frac{b^{1/3} \text{Log}[a^{2/3} - a^{1/3} b^{1/3} \text{Tanh}[c + d x] + b^{2/3} \text{Tanh}[c + d x]^2]}{6 a^{4/3} d} + \frac{b \text{Log}[a + b \text{Tanh}[c + d x]^3]}{3 a^2 d} \end{aligned}$$

Result (type 7, 322 leaves):

$$\begin{aligned} & \frac{1}{3 a^2 d} \left(-a \text{Coth}[c + d x] \left(-2 + \text{Csch}[c + d x]^2 \right) + 3 b \left(c + d x - \text{Log}[\text{Sinh}[c + d x]] \right) + \right. \\ & \left. b \text{RootSum}\left[a - b + 3 a \#1 + 3 b \#1 + 3 a \#1^2 - 3 b \#1^2 + a \#1^3 + b \#1^3 \&, \left(-2 a c + 2 b c - 2 a d x + 2 b d x + a \text{Log}\left[e^{2(c+dx)} - \#1 \right] - b \text{Log}\left[e^{2(c+dx)} - \#1 \right] - \right. \right. \right. \\ & \left. \left. 8 a c \#1 - 4 b c \#1 - 8 a d x \#1 - 4 b d x \#1 + 4 a \text{Log}\left[e^{2(c+dx)} - \#1 \right] \#1 + 2 b \text{Log}\left[e^{2(c+dx)} - \#1 \right] \#1 + 2 a c \#1^2 + 2 b c \#1^2 + \right. \right. \\ & \left. \left. 2 a d x \#1^2 + 2 b d x \#1^2 - a \text{Log}\left[e^{2(c+dx)} - \#1 \right] \#1^2 - b \text{Log}\left[e^{2(c+dx)} - \#1 \right] \#1^2 \right) / \left(a - b + 2 a \#1 + 2 b \#1 + a \#1^2 - b \#1^2 \right) \& \right) \end{aligned}$$

Problem 104: Result more than twice size of optimal antiderivative.

$$\int \text{Sech}[c + d x]^4 (a + b \text{Tanh}[c + d x]^2)^3 dx$$

Optimal (type 3, 102 leaves, 3 steps):

$$\frac{a^3 \text{Tanh}[c + d x]}{d} - \frac{a^2 (a - 3 b) \text{Tanh}[c + d x]^3}{3 d} - \frac{3 a (a - b) b \text{Tanh}[c + d x]^5}{5 d} - \frac{(3 a - b) b^2 \text{Tanh}[c + d x]^7}{7 d} - \frac{b^3 \text{Tanh}[c + d x]^9}{9 d}$$

Result (type 3, 218 leaves):

$$\begin{aligned} & \frac{1}{20160 d} \left(5775 a^3 - 1071 a^2 b + 621 a b^2 - 725 b^3 + \right. \\ & \left. 10 (903 a^3 - 63 a^2 b - 27 a b^2 + 107 b^3) \text{Cosh}[2(c + d x)] + 8 (525 a^3 + 126 a^2 b - 81 a b^2 - 50 b^3) \text{Cosh}[4(c + d x)] + \right. \\ & \left. 1050 a^3 \text{Cosh}[6(c + d x)] + 630 a^2 b \text{Cosh}[6(c + d x)] + 270 a b^2 \text{Cosh}[6(c + d x)] + 50 b^3 \text{Cosh}[6(c + d x)] + 105 a^3 \text{Cosh}[8(c + d x)] + \right. \\ & \left. 63 a^2 b \text{Cosh}[8(c + d x)] + 27 a b^2 \text{Cosh}[8(c + d x)] + 5 b^3 \text{Cosh}[8(c + d x)] \right) \text{Sech}[c + d x]^8 \text{Tanh}[c + d x] \end{aligned}$$

Problem 133: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Sech}[c + d x]^7}{(a + b \text{Tanh}[c + d x]^2)^3} dx$$

Optimal (type 3, 156 leaves, 6 steps):

$$-\frac{\text{ArcTan}[\text{Sinh}[c + d x]]}{b^3 d} + \frac{\sqrt{a+b} (8 a^2 - 4 a b + 3 b^2) \text{ArcTan}\left[\frac{\sqrt{a+b} \text{Sinh}[c+d x]}{\sqrt{a}}\right]}{8 a^{5/2} b^3 d} +$$

$$\frac{(a+b) \text{Sinh}[c + d x]}{4 a b d (a + (a+b) \text{Sinh}[c + d x])^2} - \frac{(4 a - 3 b) (a+b) \text{Sinh}[c + d x]}{8 a^2 b^2 d (a + (a+b) \text{Sinh}[c + d x])^2}$$

Result (type 3, 317 leaves):

$$-\frac{1}{32 b^3 d} \left(\frac{2 \sqrt{a+b} (8 a^2 - 4 a b + 3 b^2) \text{ArcTan}\left[\frac{\sqrt{a} \text{Csch}[c+d x]}{\sqrt{a+b}}\right]}{a^{5/2}} + \frac{2 (8 a^3 + 4 a^2 b - a b^2 + 3 b^3) \text{ArcTan}\left[\frac{\sqrt{a} \text{Csch}[c+d x]}{\sqrt{a+b}}\right]}{a^{5/2} \sqrt{a+b}} + \right.$$

$$64 \text{ArcTan}\left[\text{Tanh}\left[\frac{1}{2} (c + d x)\right]\right] + \frac{i \sqrt{a+b} (8 a^2 - 4 a b + 3 b^2) \text{Log}[a - b + (a+b) \text{Cosh}[2 (c + d x)]]}{a^{5/2}} -$$

$$\left. \frac{i (8 a^3 + 4 a^2 b - a b^2 + 3 b^3) \text{Log}[a - b + (a+b) \text{Cosh}[2 (c + d x)]]}{a^{5/2} \sqrt{a+b}} - \frac{32 b^2 (a+b) \text{Sinh}[c + d x]}{a (a - b + (a+b) \text{Cosh}[2 (c + d x)])^2} + \frac{8 b (4 a^2 + a b - 3 b^2) \text{Sinh}[c + d x]}{a^2 (a - b + (a+b) \text{Cosh}[2 (c + d x)])} \right)$$

Problem 144: Result more than twice size of optimal antiderivative.

$$\int \text{Tanh}[c + d x]^4 (a + b \text{Tanh}[c + d x]^2)^2 dx$$

Optimal (type 3, 83 leaves, 4 steps):

$$(a+b)^2 x - \frac{(a+b)^2 \text{Tanh}[c + d x]}{d} - \frac{(a+b)^2 \text{Tanh}[c + d x]^3}{3 d} - \frac{b (2 a + b) \text{Tanh}[c + d x]^5}{5 d} - \frac{b^2 \text{Tanh}[c + d x]^7}{7 d}$$

Result (type 3, 205 leaves):

$$a^2 x + 2 a b x + b^2 x - \frac{4 a^2 \text{Tanh}[c + d x]}{3 d} - \frac{46 a b \text{Tanh}[c + d x]}{15 d} - \frac{176 b^2 \text{Tanh}[c + d x]}{105 d} +$$

$$\frac{a^2 \text{Sech}[c + d x]^2 \text{Tanh}[c + d x]}{3 d} + \frac{22 a b \text{Sech}[c + d x]^2 \text{Tanh}[c + d x]}{15 d} + \frac{122 b^2 \text{Sech}[c + d x]^2 \text{Tanh}[c + d x]}{105 d} -$$

$$\frac{2 a b \text{Sech}[c + d x]^4 \text{Tanh}[c + d x]}{5 d} - \frac{22 b^2 \text{Sech}[c + d x]^4 \text{Tanh}[c + d x]}{35 d} + \frac{b^2 \text{Sech}[c + d x]^6 \text{Tanh}[c + d x]}{7 d}$$

Problem 146: Result more than twice size of optimal antiderivative.

$$\int \text{Tanh}[c + d x]^2 (a + b \text{Tanh}[c + d x]^2)^2 dx$$

Optimal (type 3, 63 leaves, 4 steps):

$$(a+b)^2 x - \frac{(a+b)^2 \operatorname{Tanh}[c+dx]}{d} - \frac{b(2a+b) \operatorname{Tanh}[c+dx]^3}{3d} - \frac{b^2 \operatorname{Tanh}[c+dx]^5}{5d}$$

Result (type 3, 132 leaves):

$$a^2 x + 2 a b x + b^2 x - \frac{a^2 \operatorname{Tanh}[c+dx]}{d} - \frac{8 a b \operatorname{Tanh}[c+dx]}{3 d} - \frac{23 b^2 \operatorname{Tanh}[c+dx]}{15 d} + \frac{2 a b \operatorname{Sech}[c+dx]^2 \operatorname{Tanh}[c+dx]}{3 d} + \frac{11 b^2 \operatorname{Sech}[c+dx]^2 \operatorname{Tanh}[c+dx]}{15 d} - \frac{b^2 \operatorname{Sech}[c+dx]^4 \operatorname{Tanh}[c+dx]}{5 d}$$

Problem 154: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Coth}[c+dx]^6 (a+b \operatorname{Tanh}[c+dx]^2)^2 dx$$

Optimal (type 3, 63 leaves, 4 steps):

$$(a+b)^2 x - \frac{(a+b)^2 \operatorname{Coth}[c+dx]}{d} - \frac{a(a+2b) \operatorname{Coth}[c+dx]^3}{3d} - \frac{a^2 \operatorname{Coth}[c+dx]^5}{5d}$$

Result (type 3, 132 leaves):

$$a^2 x + 2 a b x + b^2 x - \frac{23 a^2 \operatorname{Coth}[c+dx]}{15 d} - \frac{8 a b \operatorname{Coth}[c+dx]}{3 d} - \frac{b^2 \operatorname{Coth}[c+dx]}{d} - \frac{11 a^2 \operatorname{Coth}[c+dx] \operatorname{Csch}[c+dx]^2}{15 d} - \frac{2 a b \operatorname{Coth}[c+dx] \operatorname{Csch}[c+dx]^2}{3 d} - \frac{a^2 \operatorname{Coth}[c+dx] \operatorname{Csch}[c+dx]^4}{5 d}$$

Problem 156: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Tanh}[c+dx]^4 (a+b \operatorname{Tanh}[c+dx]^2)^3 dx$$

Optimal (type 3, 114 leaves, 4 steps):

$$(a+b)^3 x - \frac{(a+b)^3 \operatorname{Tanh}[c+dx]}{d} - \frac{(a+b)^3 \operatorname{Tanh}[c+dx]^3}{3d} - \frac{b(3a^2+3ab+b^2) \operatorname{Tanh}[c+dx]^5}{5d} - \frac{b^2(3a+b) \operatorname{Tanh}[c+dx]^7}{7d} - \frac{b^3 \operatorname{Tanh}[c+dx]^9}{9d}$$

Result (type 3, 640 leaves):

$$\frac{1}{80640d} \operatorname{Sech}[c+dx]^9$$

$$\begin{aligned} & (39690a^3(c+dx)\operatorname{Cosh}[c+dx] + 119070a^2b(c+dx)\operatorname{Cosh}[c+dx] + 119070ab^2(c+dx)\operatorname{Cosh}[c+dx] + 39690b^3(c+dx)\operatorname{Cosh}[c+dx] + \\ & 26460a^3(c+dx)\operatorname{Cosh}[3(c+dx)] + 79380a^2b(c+dx)\operatorname{Cosh}[3(c+dx)] + 79380ab^2(c+dx)\operatorname{Cosh}[3(c+dx)] + \\ & 26460b^3(c+dx)\operatorname{Cosh}[3(c+dx)] + 11340a^3(c+dx)\operatorname{Cosh}[5(c+dx)] + 34020a^2b(c+dx)\operatorname{Cosh}[5(c+dx)] + \\ & 34020ab^2(c+dx)\operatorname{Cosh}[5(c+dx)] + 11340b^3(c+dx)\operatorname{Cosh}[5(c+dx)] + 2835a^3(c+dx)\operatorname{Cosh}[7(c+dx)] + \\ & 8505a^2b(c+dx)\operatorname{Cosh}[7(c+dx)] + 8505ab^2(c+dx)\operatorname{Cosh}[7(c+dx)] + 2835b^3(c+dx)\operatorname{Cosh}[7(c+dx)] + \\ & 315a^3(c+dx)\operatorname{Cosh}[9(c+dx)] + 945a^2b(c+dx)\operatorname{Cosh}[9(c+dx)] + 945ab^2(c+dx)\operatorname{Cosh}[9(c+dx)] + 315b^3(c+dx)\operatorname{Cosh}[9(c+dx)] - \\ & 3780a^3\operatorname{Sinh}[c+dx] - 12474a^2b\operatorname{Sinh}[c+dx] - 10584ab^2\operatorname{Sinh}[c+dx] - 7938b^3\operatorname{Sinh}[c+dx] - 7980a^3\operatorname{Sinh}[3(c+dx)] - \\ & 24696a^2b\operatorname{Sinh}[3(c+dx)] - 24696ab^2\operatorname{Sinh}[3(c+dx)] - 5292b^3\operatorname{Sinh}[3(c+dx)] - 6300a^3\operatorname{Sinh}[5(c+dx)] - 18144a^2b\operatorname{Sinh}[5(c+dx)] - \\ & 19224ab^2\operatorname{Sinh}[5(c+dx)] - 7668b^3\operatorname{Sinh}[5(c+dx)] - 2520a^3\operatorname{Sinh}[7(c+dx)] - 7371a^2b\operatorname{Sinh}[7(c+dx)] - 6696ab^2\operatorname{Sinh}[7(c+dx)] - \\ & 1917b^3\operatorname{Sinh}[7(c+dx)] - 420a^3\operatorname{Sinh}[9(c+dx)] - 1449a^2b\operatorname{Sinh}[9(c+dx)] - 1584ab^2\operatorname{Sinh}[9(c+dx)] - 563b^3\operatorname{Sinh}[9(c+dx)] \end{aligned}$$

Problem 202: Result more than twice size of optimal antiderivative.

$$\int \sqrt{1 - \operatorname{Tanh}[x]^2} dx$$

Optimal (type 3, 3 leaves, 3 steps):

$$\operatorname{ArcSin}[\operatorname{Tanh}[x]]$$

Result (type 3, 19 leaves):

$$2 \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{x}{2}\right]\right] \operatorname{Cosh}[x] \sqrt{\operatorname{Sech}[x]^2}$$

Problem 208: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Tanh}[x]^5 \sqrt{a + b \operatorname{Tanh}[x]^2} dx$$

Optimal (type 3, 87 leaves, 7 steps):

$$\sqrt{a+b} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Tanh}[x]^2}}{\sqrt{a+b}}\right] - \sqrt{a+b \operatorname{Tanh}[x]^2} + \frac{(a-b)(a+b \operatorname{Tanh}[x]^2)^{3/2}}{3b^2} - \frac{(a+b \operatorname{Tanh}[x]^2)^{5/2}}{5b^2}$$

Result (type 3, 184 leaves):

$$\frac{1}{15\sqrt{2}} \sqrt{(a-b+(a+b)\operatorname{Cosh}[2x])\operatorname{Sech}[x]^2} \left(-23 + \frac{2a^2}{b^2} - \frac{6a}{b} - \right.$$

$$\left. \left(15\sqrt{2}\sqrt{a+b}\operatorname{Cosh}[x] \left(\operatorname{Log}\left[-\operatorname{Sech}\left[\frac{x}{2}\right]^2\right] - \operatorname{Log}\left[a+b + \frac{\sqrt{a+b}\sqrt{(a-b+(a+b)\operatorname{Cosh}[2x])\operatorname{Sech}\left[\frac{x}{2}\right]^4}}{\sqrt{2}} + (a+b)\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right]\right) \operatorname{Sech}\left[\frac{x}{2}\right]^2 \right) / \right.$$

$$\left. \left(\sqrt{(a-b+(a+b)\operatorname{Cosh}[2x])\operatorname{Sech}\left[\frac{x}{2}\right]^4} + \left(11 + \frac{a}{b}\right)\operatorname{Sech}[x]^2 - 3\operatorname{Sech}[x]^4 \right) \right)$$

Problem 209: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \operatorname{Tanh}[x]^4 \sqrt{a+b\operatorname{Tanh}[x]^2} dx$$

Optimal (type 3, 121 leaves, 8 steps):

$$\frac{(a^2 - 4ab - 8b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{b}\operatorname{Tanh}[x]}{\sqrt{a+b\operatorname{Tanh}[x]^2}}\right]}{8b^{3/2}} + \sqrt{a+b} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b}\operatorname{Tanh}[x]}{\sqrt{a+b\operatorname{Tanh}[x]^2}}\right] - \frac{(a+4b)\operatorname{Tanh}[x]\sqrt{a+b\operatorname{Tanh}[x]^2}}{8b} - \frac{1}{4}\operatorname{Tanh}[x]^3\sqrt{a+b\operatorname{Tanh}[x]^2}$$

Result (type 4, 580 leaves):

$$\begin{aligned}
& \frac{1}{4b} \left(- \left(\left(b (a^2 - 4b^2) \sqrt{\frac{a - b + (a+b) \operatorname{Cosh}[2x]}{1 + \operatorname{Cosh}[2x]}} \sqrt{-\frac{a \operatorname{Coth}[x]^2}{b}} \sqrt{-\frac{a (1 + \operatorname{Cosh}[2x]) \operatorname{Csch}[x]^2}{b}} \sqrt{\frac{(a - b + (a+b) \operatorname{Cosh}[2x]) \operatorname{Csch}[x]^2}{b}} \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{Csch}[2x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a-b+(a+b) \operatorname{Cosh}[2x]) \operatorname{Csch}[x]^2}{b}}}{\sqrt{2}}}\right], 1\right] \operatorname{Sinh}[x]^4 \right) / (a (a - b + (a+b) \operatorname{Cosh}[2x])) \right) - \right. \\
& \quad \frac{1}{\sqrt{a - b + (a+b) \operatorname{Cosh}[2x]}} 4 i b (4 a b + 4 b^2) \sqrt{1 + \operatorname{Cosh}[2x]} \sqrt{\frac{a - b + (a+b) \operatorname{Cosh}[2x]}{1 + \operatorname{Cosh}[2x]}} \\
& \quad \left(- \left(\left(i \sqrt{-\frac{a \operatorname{Coth}[x]^2}{b}} \sqrt{-\frac{a (1 + \operatorname{Cosh}[2x]) \operatorname{Csch}[x]^2}{b}} \sqrt{\frac{(a - b + (a+b) \operatorname{Cosh}[2x]) \operatorname{Csch}[x]^2}{b}} \operatorname{Csch}[2x] \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a-b+(a+b) \operatorname{Cosh}[2x]) \operatorname{Csch}[x]^2}{b}}}{\sqrt{2}}}\right], 1\right] \operatorname{Sinh}[x]^4 \right) / \left(4 a \sqrt{1 + \operatorname{Cosh}[2x]} \sqrt{a - b + (a+b) \operatorname{Cosh}[2x]} \right) \right) + \\
& \quad \left(i \sqrt{-\frac{a \operatorname{Coth}[x]^2}{b}} \sqrt{-\frac{a (1 + \operatorname{Cosh}[2x]) \operatorname{Csch}[x]^2}{b}} \sqrt{\frac{(a - b + (a+b) \operatorname{Cosh}[2x]) \operatorname{Csch}[x]^2}{b}} \operatorname{Csch}[2x] \right. \\
& \quad \left. \left. \left. \operatorname{EllipticPi}\left[\frac{b}{a+b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a-b+(a+b) \operatorname{Cosh}[2x]) \operatorname{Csch}[x]^2}{b}}}{\sqrt{2}}}\right], 1\right] \operatorname{Sinh}[x]^4 \right) / \left(2 (a+b) \sqrt{1 + \operatorname{Cosh}[2x]} \sqrt{a - b + (a+b) \operatorname{Cosh}[2x]} \right) \right) \right) + \\
& \quad \sqrt{\frac{a - b + a \operatorname{Cosh}[2x] + b \operatorname{Cosh}[2x]}{1 + \operatorname{Cosh}[2x]}} \left(\frac{\operatorname{Sech}[x] (-a \operatorname{Sinh}[x] - 6 b \operatorname{Sinh}[x])}{8 b} + \frac{1}{4} \operatorname{Sech}[x]^2 \operatorname{Tanh}[x] \right)
\end{aligned}$$

Problem 210: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Tanh}[x]^3 \sqrt{a + b \operatorname{Tanh}[x]^2} dx$$

Optimal (type 3, 63 leaves, 6 steps):

$$\sqrt{a + b} \operatorname{ArcTanh}\left[\frac{\sqrt{a + b \operatorname{Tanh}[x]^2}}{\sqrt{a + b}}\right] - \sqrt{a + b \operatorname{Tanh}[x]^2} - \frac{(a + b \operatorname{Tanh}[x]^2)^{3/2}}{3b}$$

Result (type 3, 310 leaves):

$$\begin{aligned} & \sqrt{\frac{a - b + a \operatorname{Cosh}[2x] + b \operatorname{Cosh}[2x]}{1 + \operatorname{Cosh}[2x]}} \left(-\frac{a + 4b}{3b} + \frac{\operatorname{Sech}[x]^2}{3} \right) + \left(\sqrt{a + b} (1 + \operatorname{Cosh}[x]) \sqrt{\frac{1 + \operatorname{Cosh}[2x]}{(1 + \operatorname{Cosh}[x])^2}} \sqrt{\frac{a - b + (a + b) \operatorname{Cosh}[2x]}{1 + \operatorname{Cosh}[2x]}} \right. \\ & \left. \left(\operatorname{Log}\left[-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right] - \operatorname{Log}\left[a + b + a \operatorname{Tanh}\left[\frac{x}{2}\right]^2 + b \operatorname{Tanh}\left[\frac{x}{2}\right]^2 + \sqrt{a + b} \sqrt{4b \operatorname{Tanh}\left[\frac{x}{2}\right]^2 + a \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2}\right] \right) \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) \right. \\ & \left. \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) \sqrt{\frac{4b \operatorname{Tanh}\left[\frac{x}{2}\right]^2 + a \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2}{\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2}} \right) / \left(\sqrt{a - b + (a + b) \operatorname{Cosh}[2x]} \sqrt{\left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2} \sqrt{4b \operatorname{Tanh}\left[\frac{x}{2}\right]^2 + a \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2} \right) \end{aligned}$$

Problem 211: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \operatorname{Tanh}[x]^2 \sqrt{a + b \operatorname{Tanh}[x]^2} dx$$

Optimal (type 3, 85 leaves, 7 steps):

$$-\frac{(a + 2b) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[x]}{\sqrt{a + b \operatorname{Tanh}[x]^2}}\right]}{2\sqrt{b}} + \sqrt{a + b} \operatorname{ArcTanh}\left[\frac{\sqrt{a + b} \operatorname{Tanh}[x]}{\sqrt{a + b \operatorname{Tanh}[x]^2}}\right] - \frac{1}{2} \operatorname{Tanh}[x] \sqrt{a + b \operatorname{Tanh}[x]^2}$$

Result (type 4, 531 leaves):

$$\left(b^2 \sqrt{\frac{a - b + (a + b) \operatorname{Cosh}[2x]}{1 + \operatorname{Cosh}[2x]}} \sqrt{-\frac{a \operatorname{Coth}[x]^2}{b}} \sqrt{-\frac{a (1 + \operatorname{Cosh}[2x]) \operatorname{Csch}[x]^2}{b}} \sqrt{\frac{(a - b + (a + b) \operatorname{Cosh}[2x]) \operatorname{Csch}[x]^2}{b}} \right)$$

$$\begin{aligned}
& \left. \text{Csch}[2x] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(a-b+(a+b)\text{Cosh}[2x])\text{Csch}[x]^2}}{b}\right], 1\right] \text{Sinh}[x]^4\right) / (a(a-b+(a+b)\text{Cosh}[2x])) - \\
& \frac{1}{\sqrt{a-b+(a+b)\text{Cosh}[2x]}} 4 i b (a+b) \sqrt{1+\text{Cosh}[2x]} \sqrt{\frac{a-b+(a+b)\text{Cosh}[2x]}{1+\text{Cosh}[2x]}} \\
& \left(- \left(\left(i \sqrt{-\frac{a\text{Coth}[x]^2}{b}} \sqrt{-\frac{a(1+\text{Cosh}[2x])\text{Csch}[x]^2}{b}} \sqrt{\frac{(a-b+(a+b)\text{Cosh}[2x])\text{Csch}[x]^2}{b}} \text{CsCh}[2x] \right. \right. \right. \\
& \left. \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(a-b+(a+b)\text{Cosh}[2x])\text{Csch}[x]^2}}{b}\right], 1\right] \text{Sinh}[x]^4\right) / \left(4a\sqrt{1+\text{Cosh}[2x]}\sqrt{a-b+(a+b)\text{Cosh}[2x]}\right) \right) + \right. \\
& \left(i \sqrt{-\frac{a\text{Coth}[x]^2}{b}} \sqrt{-\frac{a(1+\text{Cosh}[2x])\text{Csch}[x]^2}{b}} \sqrt{\frac{(a-b+(a+b)\text{Cosh}[2x])\text{Csch}[x]^2}{b}} \text{CsCh}[2x] \right. \\
& \left. \left. \left. \text{EllipticPi}\left[\frac{b}{a+b}, \text{ArcSin}\left[\frac{\sqrt{(a-b+(a+b)\text{Cosh}[2x])\text{Csch}[x]^2}}{b}\right], 1\right] \text{Sinh}[x]^4\right) / \right. \right. \\
& \left. \left. \left. \left(2(a+b)\sqrt{1+\text{Cosh}[2x]}\sqrt{a-b+(a+b)\text{Cosh}[2x]}\right) - \frac{1}{2} \sqrt{\frac{a-b+a\text{Cosh}[2x]+b\text{Cosh}[2x]}{1+\text{Cosh}[2x]}} \text{Tanh}[x] \right) \right)
\end{aligned}$$

Problem 212: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Tanh}[x] \sqrt{a + b \operatorname{Tanh}[x]^2} \, dx$$

Optimal (type 3, 44 leaves, 5 steps):

$$\sqrt{a + b} \operatorname{ArcTanh}\left[\frac{\sqrt{a + b \operatorname{Tanh}[x]^2}}{\sqrt{a + b}}\right] - \sqrt{a + b \operatorname{Tanh}[x]^2}$$

Result (type 3, 214 leaves):

$$\begin{aligned} & - \left(\left(\left(\sqrt{\frac{a - b + a \operatorname{Cosh}[2x] + b \operatorname{Cosh}[2x]}{3 + 4 \operatorname{Cosh}[x] + \operatorname{Cosh}[2x]}} + \operatorname{Cosh}[x] \left(\sqrt{\frac{a - b + a \operatorname{Cosh}[2x] + b \operatorname{Cosh}[2x]}{3 + 4 \operatorname{Cosh}[x] + \operatorname{Cosh}[2x]}} + \right. \right. \right. \\ & \left. \left. \left. \sqrt{a + b} \operatorname{Log}\left[-\operatorname{Sech}\left[\frac{x}{2}\right]^2\right] - \sqrt{a + b} \operatorname{Log}\left[a + b + \frac{\sqrt{a + b} \sqrt{(a - b + (a + b) \operatorname{Cosh}[2x]) \operatorname{Sech}\left[\frac{x}{2}\right]^4}}{\sqrt{2}} + (a + b) \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right] \right) \right) \right) \\ & \left. \operatorname{Sech}\left[\frac{x}{2}\right]^2 \sqrt{(a - b + (a + b) \operatorname{Cosh}[2x]) \operatorname{Sech}[x]^2} \right) / \left(\sqrt{(a - b + (a + b) \operatorname{Cosh}[2x]) \operatorname{Sech}\left[\frac{x}{2}\right]^4} \right) \end{aligned}$$

Problem 213: Result more than twice size of optimal antiderivative.

$$\int \sqrt{a + b \operatorname{Tanh}[x]^2} \, dx$$

Optimal (type 3, 60 leaves, 6 steps):

$$-\sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[x]}{\sqrt{a + b \operatorname{Tanh}[x]^2}}\right] + \sqrt{a + b} \operatorname{ArcTanh}\left[\frac{\sqrt{a + b} \operatorname{Tanh}[x]}{\sqrt{a + b \operatorname{Tanh}[x]^2}}\right]$$

Result (type 3, 137 leaves):

$$\begin{aligned} & \frac{1}{2} \left(-\sqrt{a + b} \operatorname{Log}[1 - \operatorname{Tanh}[x]] + \sqrt{a + b} \operatorname{Log}[1 + \operatorname{Tanh}[x]] - 2\sqrt{b} \operatorname{Log}[b \operatorname{Tanh}[x] + \sqrt{b} \sqrt{a + b \operatorname{Tanh}[x]^2}] - \right. \\ & \left. \sqrt{a + b} \operatorname{Log}[a - b \operatorname{Tanh}[x] + \sqrt{a + b} \sqrt{a + b \operatorname{Tanh}[x]^2}] + \sqrt{a + b} \operatorname{Log}[a + b \operatorname{Tanh}[x] + \sqrt{a + b} \sqrt{a + b \operatorname{Tanh}[x]^2}] \right) \end{aligned}$$

Problem 214: Result more than twice size of optimal antiderivative.

$$\int \coth [x] \sqrt{a + b \operatorname{Tanh} [x]^2} dx$$

Optimal (type 3, 56 leaves, 7 steps):

$$-\sqrt{a} \operatorname{ArcTanh} \left[\frac{\sqrt{a + b \operatorname{Tanh} [x]^2}}{\sqrt{a}} \right] + \sqrt{a + b} \operatorname{ArcTanh} \left[\frac{\sqrt{a + b \operatorname{Tanh} [x]^2}}{\sqrt{a + b}} \right]$$

Result (type 3, 124 leaves):

$$-\left(\left(\operatorname{Cosh} [x] \left(\sqrt{a} \operatorname{ArcTanh} \left[\frac{\sqrt{2} \sqrt{a} \operatorname{Cosh} [x]}{\sqrt{a - b + (a + b) \operatorname{Cosh} [2x]}} \right] - \sqrt{a + b} \operatorname{Log} \left[\sqrt{2} \sqrt{a + b} \operatorname{Cosh} [x] + \sqrt{a - b + (a + b) \operatorname{Cosh} [2x]} \right] \right) \right. \right. \\ \left. \left. \sqrt{(a - b + (a + b) \operatorname{Cosh} [2x]) \operatorname{Sech} [x]^2} \right) / \left(\sqrt{a - b + (a + b) \operatorname{Cosh} [2x]} \right) \right)$$

Problem 215: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \coth [x]^2 \sqrt{a + b \operatorname{Tanh} [x]^2} dx$$

Optimal (type 3, 48 leaves, 5 steps):

$$\sqrt{a + b} \operatorname{ArcTanh} \left[\frac{\sqrt{a + b} \operatorname{Tanh} [x]}{\sqrt{a + b \operatorname{Tanh} [x]^2}} \right] - \coth [x] \sqrt{a + b \operatorname{Tanh} [x]^2}$$

Result (type 4, 192 leaves):

$$\begin{aligned}
& - \left(\left(\left((a - b + (a + b) \operatorname{Cosh}[2x]) \operatorname{Csch}[x]^2 - \sqrt{2} (a + b) \sqrt{\frac{(a - b + (a + b) \operatorname{Cosh}[2x]) \operatorname{Csch}[x]^2}{b}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a - b + (a + b) \operatorname{Cosh}[2x]) \operatorname{Csch}[x]^2}}{b}\right], 1\right] + \right. \right. \right. \\
& \left. \left. \left. \sqrt{2} a \sqrt{\frac{(a - b + (a + b) \operatorname{Cosh}[2x]) \operatorname{Csch}[x]^2}{b}} \operatorname{EllipticPi}\left[\frac{b}{a + b}, \operatorname{ArcSin}\left[\frac{\sqrt{(a - b + (a + b) \operatorname{Cosh}[2x]) \operatorname{Csch}[x]^2}}{b}\right], 1\right] \right) \right) \right) \\
& \left. \operatorname{Tanh}[x] \right) / \left(\sqrt{2} \sqrt{(a - b + (a + b) \operatorname{Cosh}[2x]) \operatorname{Sech}[x]^2} \right)
\end{aligned}$$

Problem 216: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Coth}[x]^3 \sqrt{a + b \operatorname{Tanh}[x]^2} dx$$

Optimal (type 3, 83 leaves, 8 steps):

$$-\frac{(2a + b) \operatorname{ArcTanh}\left[\frac{\sqrt{a + b \operatorname{Tanh}[x]^2}}{\sqrt{a}}\right]}{2\sqrt{a}} + \sqrt{a + b} \operatorname{ArcTanh}\left[\frac{\sqrt{a + b \operatorname{Tanh}[x]^2}}{\sqrt{a + b}}\right] - \frac{1}{2} \operatorname{Coth}[x]^2 \sqrt{a + b \operatorname{Tanh}[x]^2}$$

Result (type 3, 864 leaves):

$$\begin{aligned}
& \sqrt{\frac{a - b + a \operatorname{Cosh}[2x] + b \operatorname{Cosh}[2x]}{1 + \operatorname{Cosh}[2x]}} \left(-\frac{1}{2} - \frac{\operatorname{Csch}[x]^2}{2} \right) + \\
& \frac{1}{2} \left(\left((3a + b) (1 + \operatorname{Cosh}[x]) \sqrt{\frac{1 + \operatorname{Cosh}[2x]}{(1 + \operatorname{Cosh}[x])^2}} \sqrt{\frac{a - b + (a + b) \operatorname{Cosh}[2x]}{1 + \operatorname{Cosh}[2x]}} \left(-\operatorname{Log}\left[\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right] + \right. \right. \right. \\
& \left. \left. \left. \operatorname{Log}\left[a + 2b + a \operatorname{Tanh}\left[\frac{x}{2}\right]^2 + \sqrt{a} \sqrt{4b \operatorname{Tanh}\left[\frac{x}{2}\right]^2 + a \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2}\right] + \operatorname{Log}\left[a + a \operatorname{Tanh}\left[\frac{x}{2}\right]^2 + 2b \operatorname{Tanh}\left[\frac{x}{2}\right]^2 + \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \sqrt{a} \sqrt{4 b \operatorname{Tanh}\left[\frac{x}{2}\right]^2 + a \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2} \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) \sqrt{\frac{4 b \operatorname{Tanh}\left[\frac{x}{2}\right]^2 + a \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2}{\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2}} \right) / \\
& \left(4 \sqrt{a} \sqrt{a - b + (a + b) \operatorname{Cosh}[2 x]} \sqrt{\left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2} \sqrt{4 b \operatorname{Tanh}\left[\frac{x}{2}\right]^2 + a \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2} \right) + \frac{1}{\sqrt{a - b + (a + b) \operatorname{Cosh}[2 x]}} \\
& 3 (a + b) \sqrt{1 + \operatorname{Cosh}[2 x]} \sqrt{\frac{a - b + (a + b) \operatorname{Cosh}[2 x]}{1 + \operatorname{Cosh}[2 x]}} \left(\left(4 \operatorname{Cosh}[x]^2 \sqrt{-2 b + a (1 + \operatorname{Cosh}[2 x]) + b (1 + \operatorname{Cosh}[2 x])} \right. \right. \\
& \left. \left. \operatorname{Coth}[x] \left[-\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{1 + \operatorname{Cosh}[2 x]}}{\sqrt{b (-1 + \operatorname{Cosh}[2 x]) + a (1 + \operatorname{Cosh}[2 x])}}\right]}{\sqrt{a}} + \frac{1}{\sqrt{a + b}} \operatorname{Log}\left[a \sqrt{1 + \operatorname{Cosh}[2 x]} + b \sqrt{1 + \operatorname{Cosh}[2 x]} + \sqrt{a + b}\right] \right. \right. \right. \\
& \left. \left. \left. \sqrt{b (-1 + \operatorname{Cosh}[2 x]) + a (1 + \operatorname{Cosh}[2 x])} \right] \operatorname{Sinh}[2 x] \right) / \left(3 (1 + \operatorname{Cosh}[2 x])^2 \sqrt{a - b + (a + b) \operatorname{Cosh}[2 x]} \right) - \\
& \left((1 + \operatorname{Cosh}[x]) \sqrt{\frac{1 + \operatorname{Cosh}[2 x]}{(1 + \operatorname{Cosh}[x])^2}} \left(-\operatorname{Log}\left[\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right] + \operatorname{Log}\left[a + 2 b + a \operatorname{Tanh}\left[\frac{x}{2}\right]^2 + \sqrt{a} \sqrt{4 b \operatorname{Tanh}\left[\frac{x}{2}\right]^2 + a \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2}\right] \right) + \right. \\
& \left. \operatorname{Log}\left[a + a \operatorname{Tanh}\left[\frac{x}{2}\right]^2 + 2 b \operatorname{Tanh}\left[\frac{x}{2}\right]^2 + \sqrt{a} \sqrt{4 b \operatorname{Tanh}\left[\frac{x}{2}\right]^2 + a \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2}\right] \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) \right. \\
& \left. \left. \left. \sqrt{\frac{4 b \operatorname{Tanh}\left[\frac{x}{2}\right]^2 + a \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2}{\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2}} \right) / \left(4 \sqrt{a} \sqrt{1 + \operatorname{Cosh}[2 x]} \sqrt{\left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2} \sqrt{4 b \operatorname{Tanh}\left[\frac{x}{2}\right]^2 + a \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2} \right) \right) \right)
\end{aligned}$$

Problem 217: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \operatorname{Coth}[x]^4 \sqrt{a + b \operatorname{Tanh}[x]^2} \, dx$$

Optimal (type 3, 78 leaves, 6 steps):

$$\sqrt{a + b} \operatorname{ArcTanh}\left[\frac{\sqrt{a + b} \operatorname{Tanh}[x]}{\sqrt{a + b \operatorname{Tanh}[x]^2}}\right] - \frac{(3a + b) \operatorname{Coth}[x] \sqrt{a + b \operatorname{Tanh}[x]^2}}{3a} - \frac{1}{3} \operatorname{Coth}[x]^3 \sqrt{a + b \operatorname{Tanh}[x]^2}$$

Result (type 4, 558 leaves):

$$\begin{aligned}
& \sqrt{\frac{a-b+a \operatorname{Cosh}[2x]+b \operatorname{Cosh}[2x]}{1+\operatorname{Cosh}[2x]}} \left(\frac{(-4a \operatorname{Cosh}[x]-b \operatorname{Cosh}[x]) \operatorname{Csch}[x]}{3a} - \frac{1}{3} \operatorname{Coth}[x] \operatorname{Csch}[x]^2 \right) + \\
& (a+b) \left(- \left(\left(b \sqrt{\frac{a-b+(a+b) \operatorname{Cosh}[2x]}{1+\operatorname{Cosh}[2x]}} \sqrt{-\frac{a \operatorname{Coth}[x]^2}{b}} \sqrt{-\frac{a(1+\operatorname{Cosh}[2x]) \operatorname{Csch}[x]^2}{b}} \sqrt{\frac{(a-b+(a+b) \operatorname{Cosh}[2x]) \operatorname{Csch}[x]^2}{b}} \right. \right. \right. \\
& \left. \left. \left. \operatorname{Csch}[2x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a-b+(a+b) \operatorname{Cosh}[2x]) \operatorname{Csch}[x]^2}{b}}}{\sqrt{2}}}\right], 1\right] \operatorname{Sinh}[x]^4 \right) / \left(a(a-b+(a+b) \operatorname{Cosh}[2x]) \right) \right) - \\
& \frac{1}{\sqrt{a-b+(a+b) \operatorname{Cosh}[2x]}} 4i b \sqrt{1+\operatorname{Cosh}[2x]} \sqrt{\frac{a-b+(a+b) \operatorname{Cosh}[2x]}{1+\operatorname{Cosh}[2x]}} \\
& \left(- \left(\left(i \sqrt{-\frac{a \operatorname{Coth}[x]^2}{b}} \sqrt{-\frac{a(1+\operatorname{Cosh}[2x]) \operatorname{Csch}[x]^2}{b}} \sqrt{\frac{(a-b+(a+b) \operatorname{Cosh}[2x]) \operatorname{Csch}[x]^2}{b}} \operatorname{Csch}[2x] \right. \right. \right. \\
& \left. \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a-b+(a+b) \operatorname{Cosh}[2x]) \operatorname{Csch}[x]^2}{b}}}{\sqrt{2}}}\right], 1\right] \operatorname{Sinh}[x]^4 \right) / \left(4a \sqrt{1+\operatorname{Cosh}[2x]} \sqrt{a-b+(a+b) \operatorname{Cosh}[2x]} \right) \right) + \\
& \left(i \sqrt{-\frac{a \operatorname{Coth}[x]^2}{b}} \sqrt{-\frac{a(1+\operatorname{Cosh}[2x]) \operatorname{Csch}[x]^2}{b}} \sqrt{\frac{(a-b+(a+b) \operatorname{Cosh}[2x]) \operatorname{Csch}[x]^2}{b}} \operatorname{Csch}[2x] \right. \\
& \left. \left. \left. \operatorname{EllipticPi}\left[\frac{b}{a+b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a-b+(a+b) \operatorname{Cosh}[2x]) \operatorname{Csch}[x]^2}{b}}}{\sqrt{2}}}\right], 1\right] \operatorname{Sinh}[x]^4 \right) / \left(2(a+b) \sqrt{1+\operatorname{Cosh}[2x]} \sqrt{a-b+(a+b) \operatorname{Cosh}[2x]} \right) \right) \right)
\end{aligned}$$

Problem 218: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Coth}[x]^5 \sqrt{a + b \operatorname{Tanh}[x]^2} dx$$

Optimal (type 3, 121 leaves, 9 steps):

$$-\frac{(8a^2 + 4ab - b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Tanh}[x]^2}}{\sqrt{a}}\right]}{8a^{3/2}} + \sqrt{a+b} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Tanh}[x]^2}}{\sqrt{a+b}}\right] - \frac{(4a+b) \operatorname{Coth}[x]^2 \sqrt{a+b \operatorname{Tanh}[x]^2}}{8a} - \frac{1}{4} \operatorname{Coth}[x]^4 \sqrt{a+b \operatorname{Tanh}[x]^2}$$

Result (type 3, 911 leaves):

$$\begin{aligned} & \sqrt{\frac{a-b+a \operatorname{Cosh}[2x]+b \operatorname{Cosh}[2x]}{1+\operatorname{Cosh}[2x]}} \left(-\frac{6a+b}{8a} + \frac{(-8a-b) \operatorname{Csch}[x]^2}{8a} - \frac{\operatorname{Csch}[x]^4}{4} \right) + \\ & \frac{1}{4a} \left(\left((6a^2+2ab-b^2)(1+\operatorname{Cosh}[x]) \sqrt{\frac{1+\operatorname{Cosh}[2x]}{(1+\operatorname{Cosh}[x])^2}} \sqrt{\frac{a-b+(a+b) \operatorname{Cosh}[2x]}{1+\operatorname{Cosh}[2x]}} \right. \right. \\ & \left. \left(-\operatorname{Log}\left[\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right] + \operatorname{Log}\left[a+2b+a \operatorname{Tanh}\left[\frac{x}{2}\right]^2 + \sqrt{a} \sqrt{4b \operatorname{Tanh}\left[\frac{x}{2}\right]^2 + a \left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2}\right] + \operatorname{Log}\left[a+a \operatorname{Tanh}\left[\frac{x}{2}\right]^2 + 2b \operatorname{Tanh}\left[\frac{x}{2}\right]^2 + \right. \right. \right. \\ & \left. \left. \left. \sqrt{a} \sqrt{4b \operatorname{Tanh}\left[\frac{x}{2}\right]^2 + a \left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2}\right] \right) \left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) \left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) \sqrt{\frac{4b \operatorname{Tanh}\left[\frac{x}{2}\right]^2 + a \left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2}{\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2}} \right) \right) / \\ & \left(4\sqrt{a} \sqrt{a-b+(a+b) \operatorname{Cosh}[2x]} \sqrt{\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2} \sqrt{4b \operatorname{Tanh}\left[\frac{x}{2}\right]^2 + a \left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2} \right) + \frac{1}{\sqrt{a-b+(a+b) \operatorname{Cosh}[2x]}} \\ & 3(2a^2+2ab) \sqrt{1+\operatorname{Cosh}[2x]} \sqrt{\frac{a-b+(a+b) \operatorname{Cosh}[2x]}{1+\operatorname{Cosh}[2x]}} \left(\left(4 \operatorname{Cosh}[x]^2 \sqrt{-2b+a(1+\operatorname{Cosh}[2x])} + b(1+\operatorname{Cosh}[2x]) \right) \right. \\ & \left. \operatorname{Coth}[x] \left(-\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{1+\operatorname{Cosh}[2x]}}{\sqrt{b(-1+\operatorname{Cosh}[2x])}+a(1+\operatorname{Cosh}[2x])}}\right]}{\sqrt{a}} + \frac{1}{\sqrt{a+b}} \operatorname{Log}\left[a \sqrt{1+\operatorname{Cosh}[2x]} + b \sqrt{1+\operatorname{Cosh}[2x]} + \sqrt{a+b} \right] \right) \right) \end{aligned}$$

$$\left. \sqrt{b(-1 + \cosh[2x]) + a(1 + \cosh[2x])} \right) \sinh[2x] \left/ \left(3(1 + \cosh[2x])^2 \sqrt{a - b + (a + b) \cosh[2x]} \right) - \right.$$

$$\left((1 + \cosh[x]) \sqrt{\frac{1 + \cosh[2x]}{(1 + \cosh[x])^2}} \left(-\log\left[\tanh\left[\frac{x}{2}\right]^2\right] + \log\left[a + 2b + a \tanh\left[\frac{x}{2}\right]^2\right] + \sqrt{a} \sqrt{4b \tanh\left[\frac{x}{2}\right]^2 + a(1 + \tanh\left[\frac{x}{2}\right]^2)^2} \right] + \right.$$

$$\left. \log\left[a + a \tanh\left[\frac{x}{2}\right]^2 + 2b \tanh\left[\frac{x}{2}\right]^2 + \sqrt{a} \sqrt{4b \tanh\left[\frac{x}{2}\right]^2 + a(1 + \tanh\left[\frac{x}{2}\right]^2)^2} \right] \left(-1 + \tanh\left[\frac{x}{2}\right]^2 \right) \left(1 + \tanh\left[\frac{x}{2}\right]^2 \right) \right.$$

$$\left. \left. \sqrt{\frac{4b \tanh\left[\frac{x}{2}\right]^2 + a(1 + \tanh\left[\frac{x}{2}\right]^2)^2}{(-1 + \tanh\left[\frac{x}{2}\right]^2)^2}} \right/ \left(4\sqrt{a} \sqrt{1 + \cosh[2x]} \sqrt{(1 + \tanh\left[\frac{x}{2}\right]^2)^2} \sqrt{4b \tanh\left[\frac{x}{2}\right]^2 + a(1 + \tanh\left[\frac{x}{2}\right]^2)^2} \right) \right) \right)$$

Problem 219: Result more than twice size of optimal antiderivative.

$$\int \tanh[x]^3 (a + b \tanh[x]^2)^{3/2} dx$$

Optimal (type 3, 82 leaves, 7 steps):

$$(a + b)^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a + b \tanh[x]^2}}{\sqrt{a + b}}\right] - (a + b) \sqrt{a + b \tanh[x]^2} - \frac{1}{3} (a + b \tanh[x]^2)^{3/2} - \frac{(a + b \tanh[x]^2)^{5/2}}{5b}$$

Result (type 3, 184 leaves):

$$\frac{1}{15\sqrt{2}} \sqrt{(a-b+(a+b)\cosh[2x])\operatorname{sech}[x]^2} \left(-26a - \frac{3a^2}{b} - 23b - \right.$$

$$\left. \left(15\sqrt{2}(a+b)^{3/2}\cosh[x] \left(\operatorname{Log}\left[-\operatorname{sech}\left[\frac{x}{2}\right]^2\right] - \operatorname{Log}\left[a+b + \frac{\sqrt{a+b}\sqrt{(a-b+(a+b)\cosh[2x])\operatorname{sech}\left[\frac{x}{2}\right]^4}}{\sqrt{2}}}\right] + (a+b)\operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right) \operatorname{sech}\left[\frac{x}{2}\right]^2 \right) / \right.$$

$$\left. \left(\sqrt{(a-b+(a+b)\cosh[2x])\operatorname{sech}\left[\frac{x}{2}\right]^4} + (6a+11b)\operatorname{sech}[x]^2 - 3b\operatorname{sech}[x]^4 \right) \right)$$

Problem 220: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \operatorname{Tanh}[x]^2 (a+b\operatorname{Tanh}[x]^2)^{3/2} dx$$

Optimal (type 3, 123 leaves, 8 steps):

$$-\frac{(3a^2+12ab+8b^2)\operatorname{ArcTanh}\left[\frac{\sqrt{b}\operatorname{Tanh}[x]}{\sqrt{a+b\operatorname{Tanh}[x]^2}}\right]}{8\sqrt{b}} + (a+b)^{3/2}\operatorname{ArcTanh}\left[\frac{\sqrt{a+b}\operatorname{Tanh}[x]}{\sqrt{a+b\operatorname{Tanh}[x]^2}}\right] -$$

$$\frac{1}{8}(5a+4b)\operatorname{Tanh}[x]\sqrt{a+b\operatorname{Tanh}[x]^2} - \frac{1}{4}b\operatorname{Tanh}[x]^3\sqrt{a+b\operatorname{Tanh}[x]^2}$$

Result (type 4, 584 leaves):

$$\begin{aligned}
& \frac{1}{4} \left(- \left(\left(b (a^2 - 4 a b - 4 b^2) \sqrt{\frac{a - b + (a + b) \operatorname{Cosh}[2 x]}{1 + \operatorname{Cosh}[2 x]}} \sqrt{-\frac{a \operatorname{Coth}[x]^2}{b}} \sqrt{-\frac{a (1 + \operatorname{Cosh}[2 x]) \operatorname{Csch}[x]^2}{b}} \sqrt{\frac{(a - b + (a + b) \operatorname{Cosh}[2 x]) \operatorname{Csch}[x]^2}{b}} \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{Csch}[2 x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a - b + (a + b) \operatorname{Cosh}[2 x]) \operatorname{Csch}[x]^2}{b}}}{\sqrt{2}}}\right], 1\right] \operatorname{Sinh}[x]^4 \right) / (a (a - b + (a + b) \operatorname{Cosh}[2 x])) \right) - \right. \\
& \quad \frac{1}{\sqrt{a - b + (a + b) \operatorname{Cosh}[2 x]}} 4 i b (4 a^2 + 8 a b + 4 b^2) \sqrt{1 + \operatorname{Cosh}[2 x]} \sqrt{\frac{a - b + (a + b) \operatorname{Cosh}[2 x]}{1 + \operatorname{Cosh}[2 x]}} \\
& \quad \left(- \left(\left(i \sqrt{-\frac{a \operatorname{Coth}[x]^2}{b}} \sqrt{-\frac{a (1 + \operatorname{Cosh}[2 x]) \operatorname{Csch}[x]^2}{b}} \sqrt{\frac{(a - b + (a + b) \operatorname{Cosh}[2 x]) \operatorname{Csch}[x]^2}{b}} \operatorname{Csch}[2 x] \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a - b + (a + b) \operatorname{Cosh}[2 x]) \operatorname{Csch}[x]^2}{b}}}{\sqrt{2}}}\right], 1\right] \operatorname{Sinh}[x]^4 \right) / \left(4 a \sqrt{1 + \operatorname{Cosh}[2 x]} \sqrt{a - b + (a + b) \operatorname{Cosh}[2 x]} \right) \right) + \right. \\
& \quad \left(i \sqrt{-\frac{a \operatorname{Coth}[x]^2}{b}} \sqrt{-\frac{a (1 + \operatorname{Cosh}[2 x]) \operatorname{Csch}[x]^2}{b}} \sqrt{\frac{(a - b + (a + b) \operatorname{Cosh}[2 x]) \operatorname{Csch}[x]^2}{b}} \operatorname{Csch}[2 x] \right. \\
& \quad \left. \left. \left. \operatorname{EllipticPi}\left[\frac{b}{a + b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a - b + (a + b) \operatorname{Cosh}[2 x]) \operatorname{Csch}[x]^2}{b}}}{\sqrt{2}}}\right], 1\right] \operatorname{Sinh}[x]^4 \right) / \left(2 (a + b) \sqrt{1 + \operatorname{Cosh}[2 x]} \sqrt{a - b + (a + b) \operatorname{Cosh}[2 x]} \right) \right) \right) + \\
& \quad \sqrt{\frac{a - b + a \operatorname{Cosh}[2 x] + b \operatorname{Cosh}[2 x]}{1 + \operatorname{Cosh}[2 x]}} \left(\frac{1}{8} \operatorname{Sech}[x] (-5 a \operatorname{Sinh}[x] - 6 b \operatorname{Sinh}[x]) + \frac{1}{4} b \operatorname{Sech}[x]^2 \operatorname{Tanh}[x] \right)
\end{aligned}$$

Problem 221: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Tanh}[x] (a + b \operatorname{Tanh}[x]^2)^{3/2} dx$$

Optimal (type 3, 63 leaves, 6 steps):

$$(a + b)^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a + b \operatorname{Tanh}[x]^2}}{\sqrt{a + b}}\right] - (a + b) \sqrt{a + b \operatorname{Tanh}[x]^2} - \frac{1}{3} (a + b \operatorname{Tanh}[x]^2)^{3/2}$$

Result (type 3, 164 leaves):

$$\frac{1}{\sqrt{2}} \sqrt{(a - b + (a + b) \operatorname{Cosh}[2x]) \operatorname{Sech}[x]^2} \left(-\frac{4}{3} (a + b) - \left(\sqrt{2} (a + b)^{3/2} \operatorname{Cosh}[x] \left(\operatorname{Log}\left[-\operatorname{Sech}\left[\frac{x}{2}\right]^2\right] - \operatorname{Log}\left[a + b + \frac{\sqrt{a + b} \sqrt{(a - b + (a + b) \operatorname{Cosh}[2x]) \operatorname{Sech}\left[\frac{x}{2}\right]^4}}{\sqrt{2}} + (a + b) \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right] \operatorname{Sech}\left[\frac{x}{2}\right]^2 \right) / \left(\sqrt{(a - b + (a + b) \operatorname{Cosh}[2x]) \operatorname{Sech}\left[\frac{x}{2}\right]^4} + \frac{1}{3} b \operatorname{Sech}[x]^2 \right) \right)$$

Problem 223: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Coth}[x] (a + b \operatorname{Tanh}[x]^2)^{3/2} dx$$

Optimal (type 3, 71 leaves, 8 steps):

$$-a^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a + b \operatorname{Tanh}[x]^2}}{\sqrt{a}}\right] + (a + b)^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a + b \operatorname{Tanh}[x]^2}}{\sqrt{a + b}}\right] - b \sqrt{a + b \operatorname{Tanh}[x]^2}$$

Result (type 3, 872 leaves):

$$\begin{aligned}
& -b \sqrt{\frac{a - b + a \operatorname{Cosh}[2x] + b \operatorname{Cosh}[2x]}{1 + \operatorname{Cosh}[2x]}} + \\
& \frac{1}{2} \left(\left((3a^2 - 2ab - b^2) (1 + \operatorname{Cosh}[x]) \sqrt{\frac{1 + \operatorname{Cosh}[2x]}{(1 + \operatorname{Cosh}[x])^2}} \sqrt{\frac{a - b + (a + b) \operatorname{Cosh}[2x]}{1 + \operatorname{Cosh}[2x]}} \left(-\operatorname{Log}\left[\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right] + \operatorname{Log}\left[a + 2b + a \operatorname{Tanh}\left[\frac{x}{2}\right]^2 + \right. \right. \right. \right. \\
& \left. \left. \left. \sqrt{a} \sqrt{4b \operatorname{Tanh}\left[\frac{x}{2}\right]^2 + a \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2} \right] + \operatorname{Log}\left[a + a \operatorname{Tanh}\left[\frac{x}{2}\right]^2 + 2b \operatorname{Tanh}\left[\frac{x}{2}\right]^2 + \sqrt{a} \sqrt{4b \operatorname{Tanh}\left[\frac{x}{2}\right]^2 + a \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2} \right] \right) \right. \\
& \left. \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right) \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right) \sqrt{\frac{4b \operatorname{Tanh}\left[\frac{x}{2}\right]^2 + a \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2}{\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2}} \right) \right) / \\
& \left(4\sqrt{a} \sqrt{a - b + (a + b) \operatorname{Cosh}[2x]} \sqrt{\left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2} \sqrt{4b \operatorname{Tanh}\left[\frac{x}{2}\right]^2 + a \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2} \right) + \\
& \frac{1}{\sqrt{a - b + (a + b) \operatorname{Cosh}[2x]}} 3(a^2 + 2ab + b^2) \sqrt{1 + \operatorname{Cosh}[2x]} \sqrt{\frac{a - b + (a + b) \operatorname{Cosh}[2x]}{1 + \operatorname{Cosh}[2x]}} \\
& \left(\left(4 \operatorname{Cosh}[x]^2 \sqrt{-2b + a(1 + \operatorname{Cosh}[2x]) + b(1 + \operatorname{Cosh}[2x])} \operatorname{Coth}[x] \left(-\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{1 + \operatorname{Cosh}[2x]}}{\sqrt{b(-1 + \operatorname{Cosh}[2x]) + a(1 + \operatorname{Cosh}[2x])}}\right]}{\sqrt{a}} + \right. \right. \right. \\
& \left. \left. \left. \frac{1}{\sqrt{a + b}} \operatorname{Log}\left[a \sqrt{1 + \operatorname{Cosh}[2x]} + b \sqrt{1 + \operatorname{Cosh}[2x]} + \sqrt{a + b} \sqrt{b(-1 + \operatorname{Cosh}[2x]) + a(1 + \operatorname{Cosh}[2x])}\right] \right) \right) \right) \\
& \left. \operatorname{Sinh}[2x] \right) / \left(3(1 + \operatorname{Cosh}[2x])^2 \sqrt{a - b + (a + b) \operatorname{Cosh}[2x]} \right) -
\end{aligned}$$

$$\left((1 + \text{Cosh}[x]) \sqrt{\frac{1 + \text{Cosh}[2x]}{(1 + \text{Cosh}[x])^2}} \left(-\text{Log}\left[\text{Tanh}\left[\frac{x}{2}\right]^2\right] + \text{Log}\left[a + 2b + a \text{Tanh}\left[\frac{x}{2}\right]^2 + \sqrt{a} \sqrt{4b \text{Tanh}\left[\frac{x}{2}\right]^2 + a \left(1 + \text{Tanh}\left[\frac{x}{2}\right]^2\right)^2}\right] + \right. \right. \\ \left. \left. \text{Log}\left[a + a \text{Tanh}\left[\frac{x}{2}\right]^2 + 2b \text{Tanh}\left[\frac{x}{2}\right]^2 + \sqrt{a} \sqrt{4b \text{Tanh}\left[\frac{x}{2}\right]^2 + a \left(1 + \text{Tanh}\left[\frac{x}{2}\right]^2\right)^2}\right] \right) \left(-1 + \text{Tanh}\left[\frac{x}{2}\right]^2\right) \left(1 + \text{Tanh}\left[\frac{x}{2}\right]^2\right) \right. \\ \left. \left. \sqrt{\frac{4b \text{Tanh}\left[\frac{x}{2}\right]^2 + a \left(1 + \text{Tanh}\left[\frac{x}{2}\right]^2\right)^2}{\left(-1 + \text{Tanh}\left[\frac{x}{2}\right]^2\right)^2}} \right) \right) / \left(4\sqrt{a} \sqrt{1 + \text{Cosh}[2x]} \sqrt{\left(1 + \text{Tanh}\left[\frac{x}{2}\right]^2\right)^2} \sqrt{4b \text{Tanh}\left[\frac{x}{2}\right]^2 + a \left(1 + \text{Tanh}\left[\frac{x}{2}\right]^2\right)^2} \right) \right)$$

Problem 224: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \text{Coth}[x]^2 (a + b \text{Tanh}[x]^2)^{3/2} dx$$

Optimal (type 3, 77 leaves, 7 steps):

$$-b^{3/2} \text{ArcTanh}\left[\frac{\sqrt{b} \text{Tanh}[x]}{\sqrt{a + b \text{Tanh}[x]^2}}\right] + (a + b)^{3/2} \text{ArcTanh}\left[\frac{\sqrt{a + b} \text{Tanh}[x]}{\sqrt{a + b \text{Tanh}[x]^2}}\right] - a \text{Coth}[x] \sqrt{a + b \text{Tanh}[x]^2}$$

Result (type 4, 197 leaves):

$$- \left(\left(a \left((a - b + (a + b) \text{Cosh}[2x]) \text{Csch}[x]^2 - \sqrt{2} (a + 2b) \sqrt{\frac{(a - b + (a + b) \text{Cosh}[2x]) \text{Csch}[x]^2}{b}} \right. \right. \right. \\ \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{(a - b + (a + b) \text{Cosh}[2x]) \text{Csch}[x]^2}{b}}}{\sqrt{2}}}\right], 1\right] + \sqrt{2} (a + b) \sqrt{\frac{(a - b + (a + b) \text{Cosh}[2x]) \text{Csch}[x]^2}{b}} \right. \right. \\ \left. \left. \text{EllipticPi}\left[\frac{b}{a + b}, \text{ArcSin}\left[\frac{\sqrt{\frac{(a - b + (a + b) \text{Cosh}[2x]) \text{Csch}[x]^2}{b}}}{\sqrt{2}}}\right], 1\right] \text{Tanh}[x] \right) / \left(\sqrt{2} \sqrt{(a - b + (a + b) \text{Cosh}[2x]) \text{Sech}[x]^2} \right)$$

Problem 229: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Tanh}[x]^5}{\sqrt{a + b \operatorname{Tanh}[x]^2}} dx$$

Optimal (type 3, 70 leaves, 6 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Tanh}[x]^2}}{\sqrt{a+b}}\right]}{\sqrt{a+b}} + \frac{(a-b) \sqrt{a+b \operatorname{Tanh}[x]^2}}{b^2} - \frac{(a+b \operatorname{Tanh}[x]^2)^{3/2}}{3 b^2}$$

Result (type 3, 313 leaves):

$$\begin{aligned} & \sqrt{\frac{a-b+a \operatorname{Cosh}[2x]+b \operatorname{Cosh}[2x]}{1+\operatorname{Cosh}[2x]}} \left(\frac{2(a-2b)}{3b^2} + \frac{\operatorname{Sech}[x]^2}{3b} \right) + \left((1+\operatorname{Cosh}[x]) \sqrt{\frac{1+\operatorname{Cosh}[2x]}{(1+\operatorname{Cosh}[x])^2}} \sqrt{\frac{a-b+(a+b) \operatorname{Cosh}[2x]}{1+\operatorname{Cosh}[2x]}} \right. \\ & \left. \left(\operatorname{Log}\left[-1+\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right] - \operatorname{Log}\left[a+b+a \operatorname{Tanh}\left[\frac{x}{2}\right]^2+b \operatorname{Tanh}\left[\frac{x}{2}\right]^2+\sqrt{a+b} \sqrt{4b \operatorname{Tanh}\left[\frac{x}{2}\right]^2+a\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2}\right] \right) \right. \\ & \left. \left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right) \left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right) \sqrt{\frac{4b \operatorname{Tanh}\left[\frac{x}{2}\right]^2+a\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2}{\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2}} \right) \right) / \\ & \left(\sqrt{a+b} \sqrt{a-b+(a+b) \operatorname{Cosh}[2x]} \sqrt{\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2} \sqrt{4b \operatorname{Tanh}\left[\frac{x}{2}\right]^2+a\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2} \right) \end{aligned}$$

Problem 230: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Tanh}[x]^4}{\sqrt{a + b \operatorname{Tanh}[x]^2}} dx$$

Optimal (type 3, 88 leaves, 7 steps):

$$\frac{(a-2b) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[x]}{\sqrt{a+b \operatorname{Tanh}[x]^2}}\right]}{2 b^{3/2}} + \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b} \operatorname{Tanh}[x]}{\sqrt{a+b \operatorname{Tanh}[x]^2}}\right]}{\sqrt{a+b}} - \frac{\operatorname{Tanh}[x] \sqrt{a+b \operatorname{Tanh}[x]^2}}{2b}$$

Result (type 4, 542 leaves):

$$\begin{aligned}
& \frac{1}{b} \left(\left(\left((a-b) b \sqrt{\frac{a-b+(a+b) \operatorname{Cosh}[2x]}{1+\operatorname{Cosh}[2x]}} \sqrt{-\frac{a \operatorname{Coth}[x]^2}{b}} \sqrt{-\frac{a(1+\operatorname{Cosh}[2x]) \operatorname{Csch}[x]^2}{b}} \sqrt{\frac{(a-b+(a+b) \operatorname{Cosh}[2x]) \operatorname{Csch}[x]^2}{b}} \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{Csch}[2x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a-b+(a+b) \operatorname{Cosh}[2x]) \operatorname{Csch}[x]^2}{b}}}{\sqrt{2}}}\right], 1\right] \operatorname{Sinh}[x]^4 \right) / (a(a-b+(a+b) \operatorname{Cosh}[2x])) \right) - \right. \\
& \quad \frac{1}{\sqrt{a-b+(a+b) \operatorname{Cosh}[2x]}} 4 i b^2 \sqrt{1+\operatorname{Cosh}[2x]} \sqrt{\frac{a-b+(a+b) \operatorname{Cosh}[2x]}{1+\operatorname{Cosh}[2x]}} \\
& \quad \left(\left(\left(i \sqrt{-\frac{a \operatorname{Coth}[x]^2}{b}} \sqrt{-\frac{a(1+\operatorname{Cosh}[2x]) \operatorname{Csch}[x]^2}{b}} \sqrt{\frac{(a-b+(a+b) \operatorname{Cosh}[2x]) \operatorname{Csch}[x]^2}{b}} \operatorname{Csch}[2x] \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a-b+(a+b) \operatorname{Cosh}[2x]) \operatorname{Csch}[x]^2}{b}}}{\sqrt{2}}}\right], 1\right] \operatorname{Sinh}[x]^4 \right) / \left(4 a \sqrt{1+\operatorname{Cosh}[2x]} \sqrt{a-b+(a+b) \operatorname{Cosh}[2x]} \right) \right) + \\
& \quad \left(i \sqrt{-\frac{a \operatorname{Coth}[x]^2}{b}} \sqrt{-\frac{a(1+\operatorname{Cosh}[2x]) \operatorname{Csch}[x]^2}{b}} \sqrt{\frac{(a-b+(a+b) \operatorname{Cosh}[2x]) \operatorname{Csch}[x]^2}{b}} \operatorname{Csch}[2x] \right. \\
& \quad \left. \operatorname{EllipticPi}\left[\frac{b}{a+b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a-b+(a+b) \operatorname{Cosh}[2x]) \operatorname{Csch}[x]^2}{b}}}{\sqrt{2}}}\right], 1\right] \operatorname{Sinh}[x]^4 \right) /
\end{aligned}$$

$$\left(2 (a + b) \sqrt{1 + \text{Cosh}[2x]} \sqrt{a - b + (a + b) \text{Cosh}[2x]} \right) \left| - \frac{\sqrt{\frac{a - b + a \text{Cosh}[2x] + b \text{Cosh}[2x]}{1 + \text{Cosh}[2x]}} \text{Tanh}[x]}{2b} \right.$$

Problem 231: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Tanh}[x]^3}{\sqrt{a + b \text{Tanh}[x]^2}} dx$$

Optimal (type 3, 47 leaves, 5 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b \text{Tanh}[x]^2}}{\sqrt{a+b}}\right]}{\sqrt{a+b}} - \frac{\sqrt{a+b \text{Tanh}[x]^2}}{b}$$

Result (type 3, 227 leaves):

$$\begin{aligned} & - \left(\left(\text{Sech}\left[\frac{x}{2}\right]^2 \left(4b \text{Cosh}[x] \text{Log}\left[-\text{Sech}\left[\frac{x}{2}\right]^2\right] - 4b \text{Cosh}[x] \text{Log}\left[a + b + \frac{\sqrt{a+b} \sqrt{(a-b+(a+b)\text{Cosh}[2x]) \text{Sech}\left[\frac{x}{2}\right]^4}}{\sqrt{2}}}\right] + (a+b) \text{Tanh}\left[\frac{x}{2}\right]^2 \right) \right. \right. \\ & \left. \left. + \sqrt{2} \sqrt{a+b} \sqrt{(a-b+(a+b)\text{Cosh}[2x]) \text{Sech}\left[\frac{x}{2}\right]^4} + \sqrt{2} \sqrt{a+b} \text{Cosh}[x] \sqrt{(a-b+(a+b)\text{Cosh}[2x]) \text{Sech}\left[\frac{x}{2}\right]^4} \right) \right) \\ & \left. \left. \sqrt{(a-b+(a+b)\text{Cosh}[2x]) \text{Sech}[x]^2} \right) / \left(4b \sqrt{a+b} \sqrt{(a-b+(a+b)\text{Cosh}[2x]) \text{Sech}\left[\frac{x}{2}\right]^4} \right) \right) \end{aligned}$$

Problem 232: Result unnecessarily involves higher level functions.

$$\int \frac{\text{Tanh}[x]^2}{\sqrt{a + b \text{Tanh}[x]^2}} dx$$

Optimal (type 3, 60 leaves, 6 steps):

$$-\frac{\text{ArcTanh}\left[\frac{\sqrt{b} \tanh[x]}{\sqrt{a+b} \tanh[x]^2}\right]}{\sqrt{b}} + \frac{\text{ArcTanh}\left[\frac{\sqrt{a+b} \tanh[x]}{\sqrt{a+b} \tanh[x]^2}\right]}{\sqrt{a+b}}$$

Result (type 4, 101 leaves):

$$-\left(\left(a \coth[x] \text{EllipticPi}\left[\frac{b}{a+b}, \text{ArcSin}\left[\frac{\sqrt{(a-b+(a+b) \cosh[2x]) \text{Csch}[x]^2}}{b}}\right], 1\right] \sqrt{(a-b+(a+b) \cosh[2x]) \text{Sech}[x]^2} \right) / \right. \\ \left. \left(b (a+b) \sqrt{\frac{(a-b+(a+b) \cosh[2x]) \text{Csch}[x]^2}{b}} \right) \right)$$

Problem 233: Result more than twice size of optimal antiderivative.

$$\int \frac{\tanh[x]}{\sqrt{a+b \tanh[x]^2}} dx$$

Optimal (type 3, 29 leaves, 4 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b \tanh[x]^2}}{\sqrt{a+b}}\right]}{\sqrt{a+b}}$$

Result (type 3, 136 leaves):

$$-\left(\left(\cosh[x] \left(\text{Log}\left[-\text{Sech}\left[\frac{x}{2}\right]^2\right] - \text{Log}\left[a+b + \frac{\sqrt{a+b} \sqrt{(a-b+(a+b) \cosh[2x]) \text{Sech}\left[\frac{x}{2}\right]^4}}{\sqrt{2}}}\right] + (a+b) \tanh\left[\frac{x}{2}\right]^2 \right) \right) \right. \\ \left. \text{Sech}\left[\frac{x}{2}\right]^2 \sqrt{(a-b+(a+b) \cosh[2x]) \text{Sech}[x]^2} \right) / \left(\sqrt{a+b} \sqrt{(a-b+(a+b) \cosh[2x]) \text{Sech}\left[\frac{x}{2}\right]^4} \right) \right)$$

Problem 234: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{a + b \operatorname{Tanh}[x]^2}} dx$$

Optimal (type 3, 31 leaves, 3 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b} \operatorname{Tanh}[x]}{\sqrt{a+b \operatorname{Tanh}[x]^2}}\right]}{\sqrt{a+b}}$$

Result (type 3, 83 leaves):

$$\frac{1}{2\sqrt{a+b}} \left(-\operatorname{Log}[1 - \operatorname{Tanh}[x]] + \operatorname{Log}[1 + \operatorname{Tanh}[x]] - \operatorname{Log}[a - b \operatorname{Tanh}[x] + \sqrt{a+b} \sqrt{a+b \operatorname{Tanh}[x]^2}] + \operatorname{Log}[a + b \operatorname{Tanh}[x] + \sqrt{a+b} \sqrt{a+b \operatorname{Tanh}[x]^2}] \right)$$

Problem 235: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Coth}[x]}{\sqrt{a + b \operatorname{Tanh}[x]^2}} dx$$

Optimal (type 3, 56 leaves, 7 steps):

$$-\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Tanh}[x]^2}}{\sqrt{a}}\right]}{\sqrt{a}} + \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Tanh}[x]^2}}{\sqrt{a+b}}\right]}{\sqrt{a+b}}$$

Result (type 3, 161 leaves):

$$\left(\sqrt{\operatorname{Cosh}[x]^2} \left(-\sqrt{a+b} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{1 + \operatorname{Cosh}[2x]}}{\sqrt{a-b + (a+b) \operatorname{Cosh}[2x]}} \right] + \sqrt{a} \operatorname{Log}\left[a \sqrt{1 + \operatorname{Cosh}[2x]} + b \sqrt{1 + \operatorname{Cosh}[2x]} + \sqrt{a+b} \sqrt{a-b + (a+b) \operatorname{Cosh}[2x]} \right] \right) \right. \\ \left. \sqrt{(a-b + (a+b) \operatorname{Cosh}[2x]) \operatorname{Sech}[x]^2} \right) / \left(\sqrt{a} \sqrt{a+b} \sqrt{a-b + (a+b) \operatorname{Cosh}[2x]} \right)$$

Problem 236: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Coth}[x]^2}{\sqrt{a + b \operatorname{Tanh}[x]^2}} dx$$

Optimal (type 3, 51 leaves, 5 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b} \operatorname{Tanh}[x]}{\sqrt{a+b \operatorname{Tanh}[x]^2}}\right]}{\sqrt{a+b}} - \frac{\operatorname{Coth}[x] \sqrt{a+b \operatorname{Tanh}[x]^2}}{a}$$

Result (type 4, 206 leaves):

$$- \left(\left(\left((a+b) (a-b + (a+b) \operatorname{Cosh}[2x]) \operatorname{Csch}[x]^2 - \right. \right. \right. \\ \left. \left. \sqrt{2} a (a+b) \sqrt{\frac{(a-b + (a+b) \operatorname{Cosh}[2x]) \operatorname{Csch}[x]^2}{b}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a-b + (a+b) \operatorname{Cosh}[2x]) \operatorname{Csch}[x]^2}}{b}}{\sqrt{2}}\right], 1\right] + \right. \right. \\ \left. \left. \sqrt{2} a^2 \sqrt{\frac{(a-b + (a+b) \operatorname{Cosh}[2x]) \operatorname{Csch}[x]^2}{b}} \operatorname{EllipticPi}\left[\frac{b}{a+b}, \operatorname{ArcSin}\left[\frac{\sqrt{(a-b + (a+b) \operatorname{Cosh}[2x]) \operatorname{Csch}[x]^2}}{b}}{\sqrt{2}}\right], 1\right] \right) \right) \\ \left. \operatorname{Tanh}[x] \right) / \left(\sqrt{2} a (a+b) \sqrt{(a-b + (a+b) \operatorname{Cosh}[2x]) \operatorname{Sech}[x]^2} \right)$$

Problem 237: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Coth}[x]^3}{\sqrt{a + b \operatorname{Tanh}[x]^2}} dx$$

Optimal (type 3, 88 leaves, 8 steps):

$$-\frac{(2a-b) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Tanh}[x]^2}}{\sqrt{a}}\right]}{2a^{3/2}} + \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Tanh}[x]^2}}{\sqrt{a+b}}\right]}{\sqrt{a+b}} - \frac{\operatorname{Coth}[x]^2 \sqrt{a+b \operatorname{Tanh}[x]^2}}{2a}$$

Result (type 3, 874 leaves):

$$\begin{aligned} & \sqrt{\frac{a-b+a \operatorname{Cosh}[2x]+b \operatorname{Cosh}[2x]}{1+\operatorname{Cosh}[2x]}} \left(-\frac{1}{2a}-\frac{\operatorname{Csch}[x]^2}{2a}\right) + \\ & \frac{1}{2a} \left((3a-2b)(1+\operatorname{Cosh}[x]) \sqrt{\frac{1+\operatorname{Cosh}[2x]}{(1+\operatorname{Cosh}[x])^2}} \sqrt{\frac{a-b+(a+b) \operatorname{Cosh}[2x]}{1+\operatorname{Cosh}[2x]}} \left(-\operatorname{Log}\left[\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right] + \operatorname{Log}\left[a+2b+a \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right] + \right. \right. \\ & \quad \left. \left. \sqrt{a} \sqrt{4b \operatorname{Tanh}\left[\frac{x}{2}\right]^2+a\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2}\right] + \operatorname{Log}\left[a+a \operatorname{Tanh}\left[\frac{x}{2}\right]^2+2b \operatorname{Tanh}\left[\frac{x}{2}\right]^2+\sqrt{a} \sqrt{4b \operatorname{Tanh}\left[\frac{x}{2}\right]^2+a\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2}\right] \right) \\ & \quad \left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) \sqrt{\frac{4b \operatorname{Tanh}\left[\frac{x}{2}\right]^2+a\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2}{\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2}} \right) / \left(4\sqrt{a} \sqrt{a-b+(a+b) \operatorname{Cosh}[2x]}\right. \\ & \quad \left. \sqrt{\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2} \sqrt{4b \operatorname{Tanh}\left[\frac{x}{2}\right]^2+a\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2}\right) + \\ & \quad \frac{1}{\sqrt{a-b+(a+b) \operatorname{Cosh}[2x]}} 3a \sqrt{1+\operatorname{Cosh}[2x]} \sqrt{\frac{a-b+(a+b) \operatorname{Cosh}[2x]}{1+\operatorname{Cosh}[2x]}} \left(4 \operatorname{Cosh}[x]^2 \sqrt{-2b+a(1+\operatorname{Cosh}[2x])+b(1+\operatorname{Cosh}[2x])}\right. \\ & \quad \left. \operatorname{Coth}[x] \left(-\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{1+\operatorname{Cosh}[2x]}}{\sqrt{b(-1+\operatorname{Cosh}[2x])+a(1+\operatorname{Cosh}[2x])}}\right]}{\sqrt{a}} + \frac{1}{\sqrt{a+b}} \operatorname{Log}\left[a \sqrt{1+\operatorname{Cosh}[2x]}+b \sqrt{1+\operatorname{Cosh}[2x]}+\sqrt{a+b}\right] \right. \right. \\ & \quad \left. \left. \sqrt{b(-1+\operatorname{Cosh}[2x])+a(1+\operatorname{Cosh}[2x])}\right) \operatorname{Sinh}[2x] \right) / \left(3(1+\operatorname{Cosh}[2x])^2 \sqrt{a-b+(a+b) \operatorname{Cosh}[2x]}\right) - \end{aligned}$$

$$\left((1 + \text{Cosh}[x]) \sqrt{\frac{1 + \text{Cosh}[2x]}{(1 + \text{Cosh}[x])^2}} \left(-\text{Log}\left[\text{Tanh}\left[\frac{x}{2}\right]^2\right] + \text{Log}\left[a + 2b + a \text{Tanh}\left[\frac{x}{2}\right]^2 + \sqrt{a} \sqrt{4b \text{Tanh}\left[\frac{x}{2}\right]^2 + a \left(1 + \text{Tanh}\left[\frac{x}{2}\right]^2\right)^2}\right] + \right. \right. \\ \left. \left. \text{Log}\left[a + a \text{Tanh}\left[\frac{x}{2}\right]^2 + 2b \text{Tanh}\left[\frac{x}{2}\right]^2 + \sqrt{a} \sqrt{4b \text{Tanh}\left[\frac{x}{2}\right]^2 + a \left(1 + \text{Tanh}\left[\frac{x}{2}\right]^2\right)^2}\right] \right) \left(-1 + \text{Tanh}\left[\frac{x}{2}\right]^2\right) \left(1 + \text{Tanh}\left[\frac{x}{2}\right]^2\right) \right. \\ \left. \sqrt{\frac{4b \text{Tanh}\left[\frac{x}{2}\right]^2 + a \left(1 + \text{Tanh}\left[\frac{x}{2}\right]^2\right)^2}{\left(-1 + \text{Tanh}\left[\frac{x}{2}\right]^2\right)^2}} \right) / \left(4\sqrt{a} \sqrt{1 + \text{Cosh}[2x]} \sqrt{\left(1 + \text{Tanh}\left[\frac{x}{2}\right]^2\right)^2} \sqrt{4b \text{Tanh}\left[\frac{x}{2}\right]^2 + a \left(1 + \text{Tanh}\left[\frac{x}{2}\right]^2\right)^2} \right) \right)$$

Problem 238: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Tanh}[x]^5}{(a + b \text{Tanh}[x]^2)^{3/2}} dx$$

Optimal (type 3, 72 leaves, 6 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b \text{Tanh}[x]^2}}{\sqrt{a+b}}\right]}{(a+b)^{3/2}} - \frac{a^2}{b^2 (a+b) \sqrt{a+b \text{Tanh}[x]^2}} - \frac{\sqrt{a+b \text{Tanh}[x]^2}}{b^2}$$

Result (type 3, 200 leaves):

$$\frac{1}{\sqrt{2}} \left(\frac{-2a^2 + b^2 - (2a^2 + 2ab + b^2) \text{Cosh}[2x]}{b^2 (a+b) (a-b + (a+b) \text{Cosh}[2x])} - \right. \\ \left. \left(\sqrt{2} \text{Cosh}[x] \left(\text{Log}\left[-\text{Sech}\left[\frac{x}{2}\right]^2\right] - \text{Log}\left[a+b + \frac{\sqrt{a+b} \sqrt{(a-b + (a+b) \text{Cosh}[2x]) \text{Sech}\left[\frac{x}{2}\right]^4}}{\sqrt{2}}}\right] + (a+b) \text{Tanh}\left[\frac{x}{2}\right]^2 \right) \text{Sech}\left[\frac{x}{2}\right]^2 \right) / \right. \\ \left. \left((a+b)^{3/2} \sqrt{(a-b + (a+b) \text{Cosh}[2x]) \text{Sech}\left[\frac{x}{2}\right]^4} \right) \sqrt{(a-b + (a+b) \text{Cosh}[2x]) \text{Sech}[x]^2} \right)$$

Problem 239: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Tanh}[x]^4}{(a + b \operatorname{Tanh}[x]^2)^{3/2}} dx$$

Optimal (type 3, 84 leaves, 7 steps):

$$-\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[x]}{\sqrt{a+b \operatorname{Tanh}[x]^2}}\right]}{b^{3/2}} + \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b} \operatorname{Tanh}[x]}{\sqrt{a+b \operatorname{Tanh}[x]^2}}\right]}{(a+b)^{3/2}} + \frac{a \operatorname{Tanh}[x]}{b(a+b)\sqrt{a+b \operatorname{Tanh}[x]^2}}$$

Result (type 4, 188 leaves):

$$-\left(\left(a \left(-2a - 2b + \sqrt{2} (a+b) \sqrt{\frac{(a-b+(a+b)\operatorname{Cosh}[2x])\operatorname{Csch}[x]^2}{b}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(a-b+(a+b)\operatorname{Cosh}[2x])\operatorname{Csch}[x]^2}}{b}}{\sqrt{2}}\right], 1 \right] + \right. \right. \right. \\ \left. \left. \left. \sqrt{2} b \sqrt{\frac{(a-b+(a+b)\operatorname{Cosh}[2x])\operatorname{Csch}[x]^2}{b}} \operatorname{EllipticPi}\left[\frac{b}{a+b}, \operatorname{ArcSin}\left[\frac{\sqrt{(a-b+(a+b)\operatorname{Cosh}[2x])\operatorname{Csch}[x]^2}}{b}}{\sqrt{2}}\right], 1 \right] \right) \right) \right) \\ \left. \operatorname{Tanh}[x] \right) / \left(\sqrt{2} b (a+b)^2 \sqrt{(a-b+(a+b)\operatorname{Cosh}[2x])\operatorname{Sech}[x]^2} \right)$$

Problem 240: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Tanh}[x]^3}{(a + b \operatorname{Tanh}[x]^2)^{3/2}} dx$$

Optimal (type 3, 52 leaves, 5 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Tanh}[x]^2}}{\sqrt{a+b}}\right]}{(a+b)^{3/2}} + \frac{a}{b(a+b)\sqrt{a+b \operatorname{Tanh}[x]^2}}$$

Result (type 3, 178 leaves):

$$\frac{1}{\sqrt{2}} \left(\frac{2 a \operatorname{Cosh}[x]^2}{b (a+b) (a-b+(a+b) \operatorname{Cosh}[2x])} - \left(\sqrt{2} \operatorname{Cosh}[x] \left(\operatorname{Log}\left[-\operatorname{Sech}\left[\frac{x}{2}\right]^2\right] - \operatorname{Log}\left[a+b + \frac{\sqrt{a+b} \sqrt{(a-b+(a+b) \operatorname{Cosh}[2x]) \operatorname{Sech}\left[\frac{x}{2}\right]^4}}{\sqrt{2}} + (a+b) \operatorname{Tanh}\left[\frac{x}{2}\right]^2}\right] \operatorname{Sech}\left[\frac{x}{2}\right]^2 \right) \right) \right) \left((a+b)^{3/2} \sqrt{(a-b+(a+b) \operatorname{Cosh}[2x]) \operatorname{Sech}\left[\frac{x}{2}\right]^4} \right) \sqrt{(a-b+(a+b) \operatorname{Cosh}[2x]) \operatorname{Sech}[x]^2}$$

Problem 241: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Tanh}[x]^2}{(a+b \operatorname{Tanh}[x]^2)^{3/2}} dx$$

Optimal (type 3, 53 leaves, 4 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b} \operatorname{Tanh}[x]}{\sqrt{a+b \operatorname{Tanh}[x]^2}}\right]}{(a+b)^{3/2}} - \frac{\operatorname{Tanh}[x]}{(a+b) \sqrt{a+b \operatorname{Tanh}[x]^2}}$$

Result (type 4, 182 leaves):

$$\left(\left(\sqrt{2} (a+b) \sqrt{\frac{(a-b+(a+b) \operatorname{Cosh}[2x]) \operatorname{Csch}[x]^2}{b}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a-b+(a+b) \operatorname{Cosh}[2x]) \operatorname{Csch}[x]^2}{b}}}{\sqrt{2}}}\right], 1\right] - \right. \right. \\ \left. \left. 2 \left(a + b + \frac{a \sqrt{\frac{(a-b+(a+b) \operatorname{Cosh}[2x]) \operatorname{Csch}[x]^2}{b}} \operatorname{EllipticPi}\left[\frac{-b}{a+b}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{(a-b+(a+b) \operatorname{Cosh}[2x]) \operatorname{Csch}[x]^2}{b}}}{\sqrt{2}}}\right], 1\right]}{\sqrt{2}} \right) \right) \right) \\ \left. \operatorname{Tanh}[x] \right) / \left(\sqrt{2} (a+b)^2 \sqrt{(a-b+(a+b) \operatorname{Cosh}[2x]) \operatorname{Sech}[x]^2} \right)$$

Problem 242: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Tanh}[x]}{(a+b \operatorname{Tanh}[x]^2)^{3/2}} dx$$

Optimal (type 3, 49 leaves, 5 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Tanh}[x]^2}}{\sqrt{a+b}}\right]}{(a+b)^{3/2}} - \frac{1}{(a+b) \sqrt{a+b \operatorname{Tanh}[x]^2}}$$

Result (type 3, 174 leaves):

$$\frac{1}{\sqrt{2}} \left(-\frac{2 \operatorname{Cosh}[x]^2}{(a+b)(a-b+(a+b)\operatorname{Cosh}[2x])} - \left(\sqrt{2} \operatorname{Cosh}[x] \left(\operatorname{Log}\left[-\operatorname{Sech}\left[\frac{x}{2}\right]^2\right] - \operatorname{Log}\left[a+b + \frac{\sqrt{a+b} \sqrt{(a-b+(a+b)\operatorname{Cosh}[2x]) \operatorname{Sech}\left[\frac{x}{2}\right]^4}}{\sqrt{2}}}\right] + (a+b) \operatorname{Tanh}\left[\frac{x}{2}\right]^2 \right) \operatorname{Sech}\left[\frac{x}{2}\right]^2 \right) / \left((a+b)^{3/2} \sqrt{(a-b+(a+b)\operatorname{Cosh}[2x]) \operatorname{Sech}\left[\frac{x}{2}\right]^4} \right) \sqrt{(a-b+(a+b)\operatorname{Cosh}[2x]) \operatorname{Sech}[x]^2} \right)$$

Problem 244: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Coth}[x]}{(a+b \operatorname{Tanh}[x]^2)^{3/2}} dx$$

Optimal (type 3, 78 leaves, 8 steps):

$$-\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Tanh}[x]^2}}{\sqrt{a}}\right]}{a^{3/2}} + \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Tanh}[x]^2}}{\sqrt{a+b}}\right]}{(a+b)^{3/2}} + \frac{b}{a(a+b)\sqrt{a+b \operatorname{Tanh}[x]^2}}$$

Result (type 3, 903 leaves):

$$\sqrt{\frac{a-b+a \operatorname{Cosh}[2x]+b \operatorname{Cosh}[2x]}{1+\operatorname{Cosh}[2x]}} \left(\frac{b}{a(a+b)^2} + \frac{2b^2}{a(a+b)^2(a-b+a \operatorname{Cosh}[2x]+b \operatorname{Cosh}[2x])} \right) + \frac{1}{2a(a+b)} \left(\left((3a+4b)(1+\operatorname{Cosh}[x]) \sqrt{\frac{1+\operatorname{Cosh}[2x]}{(1+\operatorname{Cosh}[x])^2}} \sqrt{\frac{a-b+(a+b)\operatorname{Cosh}[2x]}{1+\operatorname{Cosh}[2x]}} \right. \right. \\ \left. \left. - \operatorname{Log}\left[\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right] + \operatorname{Log}\left[a+2b+a \operatorname{Tanh}\left[\frac{x}{2}\right]^2 + \sqrt{a} \sqrt{4b \operatorname{Tanh}\left[\frac{x}{2}\right]^2 + a\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2}\right] \right) + \right)$$

$$\begin{aligned}
& \left. \left(\text{Log} \left[a + a \text{Tanh} \left[\frac{x}{2} \right]^2 + 2 b \text{Tanh} \left[\frac{x}{2} \right]^2 + \sqrt{a} \sqrt{4 b \text{Tanh} \left[\frac{x}{2} \right]^2 + a \left(1 + \text{Tanh} \left[\frac{x}{2} \right]^2 \right)^2} \right] \right) \left(-1 + \text{Tanh} \left[\frac{x}{2} \right]^2 \right) \left(1 + \text{Tanh} \left[\frac{x}{2} \right]^2 \right) \right. \\
& \left. \sqrt{\frac{4 b \text{Tanh} \left[\frac{x}{2} \right]^2 + a \left(1 + \text{Tanh} \left[\frac{x}{2} \right]^2 \right)^2}{\left(-1 + \text{Tanh} \left[\frac{x}{2} \right]^2 \right)^2}} \right) / \left(4 \sqrt{a} \sqrt{a - b + (a + b) \text{Cosh} [2 x]} \sqrt{\left(1 + \text{Tanh} \left[\frac{x}{2} \right]^2 \right)^2} \sqrt{4 b \text{Tanh} \left[\frac{x}{2} \right]^2 + a \left(1 + \text{Tanh} \left[\frac{x}{2} \right]^2 \right)^2} \right) + \\
& \frac{1}{\sqrt{a - b + (a + b) \text{Cosh} [2 x]}} 3 a \sqrt{1 + \text{Cosh} [2 x]} \sqrt{\frac{a - b + (a + b) \text{Cosh} [2 x]}{1 + \text{Cosh} [2 x]}} \left(\left(4 \text{Cosh} [x]^2 \sqrt{-2 b + a (1 + \text{Cosh} [2 x])} + b (1 + \text{Cosh} [2 x]) \right) \right. \\
& \left. \text{Coth} [x] \left(- \frac{\text{ArcTanh} \left[\frac{\sqrt{a} \sqrt{1 + \text{Cosh} [2 x]}}{\sqrt{b (-1 + \text{Cosh} [2 x]) + a (1 + \text{Cosh} [2 x])}} \right]}{\sqrt{a}} + \frac{1}{\sqrt{a + b}} \text{Log} [a \sqrt{1 + \text{Cosh} [2 x]} + b \sqrt{1 + \text{Cosh} [2 x]} + \sqrt{a + b}] \right. \right. \\
& \left. \left. \sqrt{b (-1 + \text{Cosh} [2 x]) + a (1 + \text{Cosh} [2 x])} \right) \text{Sinh} [2 x] \right) / \left(3 (1 + \text{Cosh} [2 x])^2 \sqrt{a - b + (a + b) \text{Cosh} [2 x]} \right) - \\
& \left((1 + \text{Cosh} [x]) \sqrt{\frac{1 + \text{Cosh} [2 x]}{(1 + \text{Cosh} [x])^2}} \left(-\text{Log} \left[\text{Tanh} \left[\frac{x}{2} \right]^2 \right] + \text{Log} \left[a + 2 b + a \text{Tanh} \left[\frac{x}{2} \right]^2 + \sqrt{a} \sqrt{4 b \text{Tanh} \left[\frac{x}{2} \right]^2 + a \left(1 + \text{Tanh} \left[\frac{x}{2} \right]^2 \right)^2} \right] \right) + \right. \\
& \left. \text{Log} \left[a + a \text{Tanh} \left[\frac{x}{2} \right]^2 + 2 b \text{Tanh} \left[\frac{x}{2} \right]^2 + \sqrt{a} \sqrt{4 b \text{Tanh} \left[\frac{x}{2} \right]^2 + a \left(1 + \text{Tanh} \left[\frac{x}{2} \right]^2 \right)^2} \right] \right) \left(-1 + \text{Tanh} \left[\frac{x}{2} \right]^2 \right) \left(1 + \text{Tanh} \left[\frac{x}{2} \right]^2 \right) \right. \\
& \left. \left. \sqrt{\frac{4 b \text{Tanh} \left[\frac{x}{2} \right]^2 + a \left(1 + \text{Tanh} \left[\frac{x}{2} \right]^2 \right)^2}{\left(-1 + \text{Tanh} \left[\frac{x}{2} \right]^2 \right)^2}} \right) / \left(4 \sqrt{a} \sqrt{1 + \text{Cosh} [2 x]} \sqrt{\left(1 + \text{Tanh} \left[\frac{x}{2} \right]^2 \right)^2} \sqrt{4 b \text{Tanh} \left[\frac{x}{2} \right]^2 + a \left(1 + \text{Tanh} \left[\frac{x}{2} \right]^2 \right)^2} \right) \right) \right)
\end{aligned}$$

Problem 245: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\text{Coth}[x]^2}{(a + b \text{Tanh}[x]^2)^{3/2}} dx$$

Optimal (type 3, 85 leaves, 6 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b} \text{Tanh}[x]}{\sqrt{a+b \text{Tanh}[x]^2}}\right]}{(a+b)^{3/2}} + \frac{b \text{Coth}[x]}{a(a+b)\sqrt{a+b \text{Tanh}[x]^2}} - \frac{(a+2b) \text{Coth}[x] \sqrt{a+b \text{Tanh}[x]^2}}{a^2(a+b)}$$

Result (type 4, 230 leaves):

$$- \left(\left((a+b) (a^2 - 2b^2 + (a^2 + 2ab + 2b^2) \text{Cosh}[2x]) \text{Csch}[x]^2 - \right. \right. \\ \left. \sqrt{2} a^2 (a+b) \sqrt{\frac{(a-b + (a+b) \text{Cosh}[2x]) \text{Csch}[x]^2}{b}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(a-b + (a+b) \text{Cosh}[2x]) \text{Csch}[x]^2}}{b}\right], 1\right] + \right. \\ \left. \left. \sqrt{2} a^3 \sqrt{\frac{(a-b + (a+b) \text{Cosh}[2x]) \text{Csch}[x]^2}{b}} \text{EllipticPi}\left[\frac{b}{a+b}, \text{ArcSin}\left[\frac{\sqrt{(a-b + (a+b) \text{Cosh}[2x]) \text{Csch}[x]^2}}{b}\right], 1\right] \right) \right) \\ \left. \text{Sech}[x]^2 \text{Sinh}[2x] \right) / \left(2 \sqrt{2} a^2 (a+b)^2 \sqrt{(a-b + (a+b) \text{Cosh}[2x]) \text{Sech}[x]^2} \right)$$

Problem 246: Result unnecessarily involves higher level functions.

$$\int \frac{\text{Tanh}[x]^6}{(a + b \text{Tanh}[x]^2)^{5/2}} dx$$

Optimal (type 3, 118 leaves, 8 steps):

$$-\frac{\text{ArcTanh}\left[\frac{\sqrt{b}\text{Tanh}[x]}{\sqrt{a+b\text{Tanh}[x]^2}}\right]}{b^{5/2}} + \frac{\text{ArcTanh}\left[\frac{\sqrt{a+b}\text{Tanh}[x]}{\sqrt{a+b\text{Tanh}[x]^2}}\right]}{(a+b)^{5/2}} + \frac{a\text{Tanh}[x]^3}{3b(a+b)(a+b\text{Tanh}[x]^2)^{3/2}} + \frac{a(a+2b)\text{Tanh}[x]}{b^2(a+b)^2\sqrt{a+b\text{Tanh}[x]^2}}$$

Result (type 4, 231 leaves):

$$\frac{1}{3\sqrt{2}b^2(a+b)^3}$$

$$\sqrt{(a-b+(a+b)\text{Cosh}[2x])\text{Sech}[x]^2} \left(- \left(\left(3\sqrt{2}a\text{Coth}[x] \left((a^2+3ab+2b^2)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(a-b+(a+b)\text{Cosh}[2x])\text{Csch}[x]^2}}{b}\right], 1\right] + \right. \right. \right. \right.$$

$$\left. \left. \left. b^2\text{EllipticPi}\left[\frac{b}{a+b}, \text{ArcSin}\left[\frac{\sqrt{(a-b+(a+b)\text{Cosh}[2x])\text{Csch}[x]^2}}{\sqrt{2}}\right], 1\right] \right) \right) \right) /$$

$$\left(b\sqrt{\frac{(a-b+(a+b)\text{Cosh}[2x])\text{Csch}[x]^2}{b}} \right) + \frac{a(a+b)(3a^2+2ab-7b^2+(3a^2+10ab+7b^2)\text{Cosh}[2x])\text{Sinh}[2x]}{(a-b+(a+b)\text{Cosh}[2x])^2}$$

Problem 247: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Tanh}[x]^5}{(a+b\text{Tanh}[x]^2)^{5/2}} dx$$

Optimal (type 3, 84 leaves, 6 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b\text{Tanh}[x]^2}}{\sqrt{a+b}}\right]}{(a+b)^{5/2}} - \frac{a^2}{3b^2(a+b)(a+b\text{Tanh}[x]^2)^{3/2}} + \frac{a(a+2b)}{b^2(a+b)^2\sqrt{a+b\text{Tanh}[x]^2}}$$

Result (type 3, 376 leaves):

$$\sqrt{\frac{a - b + a \operatorname{Cosh}[2x] + b \operatorname{Cosh}[2x]}{1 + \operatorname{Cosh}[2x]}} \left(\frac{2a(a+3b)}{3b^2(a+b)^3} - \frac{4a^2}{3(a+b)^3(a-b+a\operatorname{Cosh}[2x]+b\operatorname{Cosh}[2x])^2} + \frac{2a(a+6b)}{3b(a+b)^3(a-b+a\operatorname{Cosh}[2x]+b\operatorname{Cosh}[2x])} \right) +$$

$$\left((1 + \operatorname{Cosh}[x]) \sqrt{\frac{1 + \operatorname{Cosh}[2x]}{(1 + \operatorname{Cosh}[x])^2}} \sqrt{\frac{a - b + (a+b)\operatorname{Cosh}[2x]}{1 + \operatorname{Cosh}[2x]}} \right.$$

$$\left. \left(\operatorname{Log}\left[-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right] - \operatorname{Log}\left[a + b + a \operatorname{Tanh}\left[\frac{x}{2}\right]^2 + b \operatorname{Tanh}\left[\frac{x}{2}\right]^2 + \sqrt{a+b} \sqrt{4b \operatorname{Tanh}\left[\frac{x}{2}\right]^2 + a\left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2}\right] \right) \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) \right.$$

$$\left. \sqrt{\frac{4b \operatorname{Tanh}\left[\frac{x}{2}\right]^2 + a\left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2}{\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2}} \right) / \left((a+b)^{5/2} \sqrt{a-b+(a+b)\operatorname{Cosh}[2x]} \sqrt{\left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2} \sqrt{4b \operatorname{Tanh}\left[\frac{x}{2}\right]^2 + a\left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2} \right)$$

Problem 248: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Tanh}[x]^4}{(a+b\operatorname{Tanh}[x]^2)^{5/2}} dx$$

Optimal (type 3, 90 leaves, 6 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b}\operatorname{Tanh}[x]}{\sqrt{a+b\operatorname{Tanh}[x]^2}}\right]}{(a+b)^{5/2}} + \frac{a\operatorname{Tanh}[x]}{3b(a+b)(a+b\operatorname{Tanh}[x]^2)^{3/2}} - \frac{(a+4b)\operatorname{Tanh}[x]}{3b(a+b)^2\sqrt{a+b\operatorname{Tanh}[x]^2}}$$

Result (type 4, 595 leaves):

$$\begin{aligned}
& \frac{1}{(a+b)^2} \\
& \left(- \left(\left(b \sqrt{\frac{a-b+(a+b)\operatorname{Cosh}[2x]}{1+\operatorname{Cosh}[2x]}} \sqrt{-\frac{a\operatorname{Coth}[x]^2}{b}} \sqrt{-\frac{a(1+\operatorname{Cosh}[2x])\operatorname{Csch}[x]^2}{b}} \sqrt{\frac{(a-b+(a+b)\operatorname{Cosh}[2x])\operatorname{Csch}[x]^2}{b}} \operatorname{Csch}[2x] \operatorname{EllipticF} \left[\right. \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{ArcSin} \left[\frac{\sqrt{\frac{(a-b+(a+b)\operatorname{Cosh}[2x])\operatorname{Csch}[x]^2}{b}}}{\sqrt{2}}}, 1 \right] \operatorname{Sinh}[x]^4 \right) / (a(a-b+(a+b)\operatorname{Cosh}[2x])) \right) - \frac{1}{\sqrt{a-b+(a+b)\operatorname{Cosh}[2x]}} 4 i b \sqrt{1+\operatorname{Cosh}[2x]} \right. \\
& \quad \left. \sqrt{\frac{a-b+(a+b)\operatorname{Cosh}[2x]}{1+\operatorname{Cosh}[2x]}} \left(- \left(\left(i \sqrt{-\frac{a\operatorname{Coth}[x]^2}{b}} \sqrt{-\frac{a(1+\operatorname{Cosh}[2x])\operatorname{Csch}[x]^2}{b}} \sqrt{\frac{(a-b+(a+b)\operatorname{Cosh}[2x])\operatorname{Csch}[x]^2}{b}} \right. \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{Csch}[2x] \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{\frac{(a-b+(a+b)\operatorname{Cosh}[2x])\operatorname{Csch}[x]^2}{b}}}{\sqrt{2}}}, 1 \right] \operatorname{Sinh}[x]^4 \right) / (4 a \sqrt{1+\operatorname{Cosh}[2x]} \sqrt{a-b+(a+b)\operatorname{Cosh}[2x]}) \right) \right) + \right. \\
& \quad \left(i \sqrt{-\frac{a\operatorname{Coth}[x]^2}{b}} \sqrt{-\frac{a(1+\operatorname{Cosh}[2x])\operatorname{Csch}[x]^2}{b}} \sqrt{\frac{(a-b+(a+b)\operatorname{Cosh}[2x])\operatorname{Csch}[x]^2}{b}} \operatorname{Csch}[2x] \right. \\
& \quad \left. \left. \left. \operatorname{EllipticPi} \left[\frac{b}{a+b}, \operatorname{ArcSin} \left[\frac{\sqrt{\frac{(a-b+(a+b)\operatorname{Cosh}[2x])\operatorname{Csch}[x]^2}{b}}}{\sqrt{2}}}, 1 \right] \operatorname{Sinh}[x]^4 \right) / (2(a+b)\sqrt{1+\operatorname{Cosh}[2x]}\sqrt{a-b+(a+b)\operatorname{Cosh}[2x]}) \right) \right) \right) + \\
& \quad \left. \sqrt{\frac{a-b+a\operatorname{Cosh}[2x]+b\operatorname{Cosh}[2x]}{1+\operatorname{Cosh}[2x]}} \left(\frac{2 a \operatorname{Sinh}[2 x]}{3(a+b)^2(a-b+a\operatorname{Cosh}[2x]+b\operatorname{Cosh}[2x])^2} - \frac{4 \operatorname{Sinh}[2 x]}{3(a+b)^2(a-b+a\operatorname{Cosh}[2x]+b\operatorname{Cosh}[2x])^2} \right) \right)
\end{aligned}$$

Problem 249: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Tanh}[x]^3}{(a + b \operatorname{Tanh}[x]^2)^{5/2}} dx$$

Optimal (type 3, 74 leaves, 6 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Tanh}[x]^2}}{\sqrt{a+b}}\right]}{(a+b)^{5/2}} + \frac{a}{3b(a+b)(a+b \operatorname{Tanh}[x]^2)^{3/2}} - \frac{1}{(a+b)^2 \sqrt{a+b \operatorname{Tanh}[x]^2}}$$

Result (type 3, 372 leaves):

$$\begin{aligned} & \sqrt{\frac{a-b+a \operatorname{Cosh}[2x]+b \operatorname{Cosh}[2x]}{1+\operatorname{Cosh}[2x]}} \left(\frac{a-3b}{3b(a+b)^3} + \frac{4ab}{3(a+b)^3(a-b+a \operatorname{Cosh}[2x]+b \operatorname{Cosh}[2x])^2} + \frac{2(2a-3b)}{3(a+b)^3(a-b+a \operatorname{Cosh}[2x]+b \operatorname{Cosh}[2x])} \right) + \\ & \left((1+\operatorname{Cosh}[x]) \sqrt{\frac{1+\operatorname{Cosh}[2x]}{(1+\operatorname{Cosh}[x])^2}} \sqrt{\frac{a-b+(a+b) \operatorname{Cosh}[2x]}{1+\operatorname{Cosh}[2x]}} \right. \\ & \left. \left(\operatorname{Log}\left[-1+\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right] - \operatorname{Log}\left[a+b+a \operatorname{Tanh}\left[\frac{x}{2}\right]^2+b \operatorname{Tanh}\left[\frac{x}{2}\right]^2+\sqrt{a+b} \sqrt{4b \operatorname{Tanh}\left[\frac{x}{2}\right]^2+a\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2}\right] \right) \left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) \left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) \right. \\ & \left. \sqrt{\frac{4b \operatorname{Tanh}\left[\frac{x}{2}\right]^2+a\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2}{\left(-1+\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2}} \right) / \left((a+b)^{5/2} \sqrt{a-b+(a+b) \operatorname{Cosh}[2x]} \sqrt{\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2} \sqrt{4b \operatorname{Tanh}\left[\frac{x}{2}\right]^2+a\left(1+\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2} \right) \end{aligned}$$

Problem 250: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Tanh}[x]^2}{(a + b \operatorname{Tanh}[x]^2)^{5/2}} dx$$

Optimal (type 3, 88 leaves, 6 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Tanh}[x]^2}}{\sqrt{a+b \operatorname{Tanh}[x]^2}}\right]}{(a+b)^{5/2}} - \frac{\operatorname{Tanh}[x]}{3(a+b)(a+b \operatorname{Tanh}[x]^2)^{3/2}} - \frac{(2a-b) \operatorname{Tanh}[x]}{3a(a+b)^2 \sqrt{a+b \operatorname{Tanh}[x]^2}}$$

Result (type 4, 608 leaves):

$$\begin{aligned}
& \frac{1}{(a+b)^2} \\
& \left(- \left(\left(b \sqrt{\frac{a-b+(a+b)\operatorname{Cosh}[2x]}{1+\operatorname{Cosh}[2x]}} \sqrt{-\frac{a\operatorname{Coth}[x]^2}{b}} \sqrt{-\frac{a(1+\operatorname{Cosh}[2x])\operatorname{Csch}[x]^2}{b}} \sqrt{\frac{(a-b+(a+b)\operatorname{Cosh}[2x])\operatorname{Csch}[x]^2}{b}} \operatorname{Csch}[2x] \operatorname{EllipticF} \left[\right. \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{ArcSin} \left[\frac{\sqrt{\frac{(a-b+(a+b)\operatorname{Cosh}[2x])\operatorname{Csch}[x]^2}{b}}}{\sqrt{2}}}, 1 \right] \operatorname{Sinh}[x]^4 \right) / (a(a-b+(a+b)\operatorname{Cosh}[2x])) \right) - \frac{1}{\sqrt{a-b+(a+b)\operatorname{Cosh}[2x]}} 4 i b \sqrt{1+\operatorname{Cosh}[2x]} \right. \\
& \quad \left. \sqrt{\frac{a-b+(a+b)\operatorname{Cosh}[2x]}{1+\operatorname{Cosh}[2x]}} \left(- \left(\left(i \left(\left(\sqrt{-\frac{a\operatorname{Coth}[x]^2}{b}} \sqrt{-\frac{a(1+\operatorname{Cosh}[2x])\operatorname{Csch}[x]^2}{b}} \sqrt{\frac{(a-b+(a+b)\operatorname{Cosh}[2x])\operatorname{Csch}[x]^2}{b}} \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{Csch}[2x] \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{\frac{(a-b+(a+b)\operatorname{Cosh}[2x])\operatorname{Csch}[x]^2}{b}}}{\sqrt{2}}}, 1 \right] \operatorname{Sinh}[x]^4 \right) / \left(4 a \sqrt{1+\operatorname{Cosh}[2x]} \sqrt{a-b+(a+b)\operatorname{Cosh}[2x]} \right) \right) \right) + \right. \\
& \quad \left(i \sqrt{-\frac{a\operatorname{Coth}[x]^2}{b}} \sqrt{-\frac{a(1+\operatorname{Cosh}[2x])\operatorname{Csch}[x]^2}{b}} \sqrt{\frac{(a-b+(a+b)\operatorname{Cosh}[2x])\operatorname{Csch}[x]^2}{b}} \operatorname{Csch}[2x] \right. \\
& \quad \left. \left. \left. \operatorname{EllipticPi} \left[\frac{b}{a+b}, \operatorname{ArcSin} \left[\frac{\sqrt{\frac{(a-b+(a+b)\operatorname{Cosh}[2x])\operatorname{Csch}[x]^2}{b}}}{\sqrt{2}}}, 1 \right] \operatorname{Sinh}[x]^4 \right) / \left(2(a+b)\sqrt{1+\operatorname{Cosh}[2x]}\sqrt{a-b+(a+b)\operatorname{Cosh}[2x]} \right) \right) \right) \right) + \\
& \quad \sqrt{\frac{a-b+a\operatorname{Cosh}[2x]+b\operatorname{Cosh}[2x]}{1+\operatorname{Cosh}[2x]}} \left(-\frac{2b\operatorname{Sinh}[2x]}{3(a+b)^2(a-b+a\operatorname{Cosh}[2x]+b\operatorname{Cosh}[2x])^2} + \frac{-3a\operatorname{Sinh}[2x]+b\operatorname{Sinh}[2x]}{3a(a+b)^2(a-b+a\operatorname{Cosh}[2x]+b\operatorname{Cosh}[2x])} \right)
\end{aligned}$$

Problem 251: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Tanh}[x]}{(a + b \text{Tanh}[x]^2)^{5/2}} dx$$

Optimal (type 3, 70 leaves, 6 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b \text{Tanh}[x]^2}}{\sqrt{a+b}}\right]}{(a+b)^{5/2}} - \frac{1}{3(a+b)(a+b \text{Tanh}[x]^2)^{3/2}} - \frac{1}{(a+b)^2 \sqrt{a+b \text{Tanh}[x]^2}}$$

Result (type 3, 359 leaves):

$$\begin{aligned} & \sqrt{\frac{a-b+a \text{Cosh}[2x]+b \text{Cosh}[2x]}{1+\text{Cosh}[2x]}} \left(-\frac{4}{3(a+b)^3} - \frac{4b^2}{3(a+b)^3(a-b+a \text{Cosh}[2x]+b \text{Cosh}[2x])^2} - \frac{10b}{3(a+b)^3(a-b+a \text{Cosh}[2x]+b \text{Cosh}[2x])} \right) + \\ & \left((1+\text{Cosh}[x]) \sqrt{\frac{1+\text{Cosh}[2x]}{(1+\text{Cosh}[x])^2}} \sqrt{\frac{a-b+(a+b) \text{Cosh}[2x]}{1+\text{Cosh}[2x]}} \right. \\ & \left. \left(\text{Log}\left[-1+\text{Tanh}\left[\frac{x}{2}\right]^2\right] - \text{Log}\left[a+b+a \text{Tanh}\left[\frac{x}{2}\right]^2+b \text{Tanh}\left[\frac{x}{2}\right]^2+\sqrt{a+b} \sqrt{4b \text{Tanh}\left[\frac{x}{2}\right]^2+a\left(1+\text{Tanh}\left[\frac{x}{2}\right]^2\right)^2}\right] \right) \left(-1+\text{Tanh}\left[\frac{x}{2}\right]^2\right) \left(1+\text{Tanh}\left[\frac{x}{2}\right]^2\right) \right. \\ & \left. \sqrt{\frac{4b \text{Tanh}\left[\frac{x}{2}\right]^2+a\left(1+\text{Tanh}\left[\frac{x}{2}\right]^2\right)^2}{\left(-1+\text{Tanh}\left[\frac{x}{2}\right]^2\right)^2}} \right) / \left((a+b)^{5/2} \sqrt{a-b+(a+b) \text{Cosh}[2x]} \sqrt{\left(1+\text{Tanh}\left[\frac{x}{2}\right]^2\right)^2} \sqrt{4b \text{Tanh}\left[\frac{x}{2}\right]^2+a\left(1+\text{Tanh}\left[\frac{x}{2}\right]^2\right)^2} \right) \end{aligned}$$

Problem 253: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Coth}[x]}{(a + b \text{Tanh}[x]^2)^{5/2}} dx$$

Optimal (type 3, 108 leaves, 9 steps):

$$-\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b \text{Tanh}[x]^2}}{\sqrt{a}}\right]}{a^{5/2}} + \frac{\text{ArcTanh}\left[\frac{\sqrt{a+b \text{Tanh}[x]^2}}{\sqrt{a+b}}\right]}{(a+b)^{5/2}} + \frac{b}{3a(a+b)(a+b \text{Tanh}[x]^2)^{3/2}} + \frac{b(2a+b)}{a^2(a+b)^2 \sqrt{a+b \text{Tanh}[x]^2}}$$

Result (type 3, 966 leaves):

$$\begin{aligned}
& \sqrt{\frac{a - b + a \operatorname{Cosh}[2x] + b \operatorname{Cosh}[2x]}{1 + \operatorname{Cosh}[2x]}} \\
& \left(\frac{b(7a + 3b)}{3a^2(a+b)^3} + \frac{4b^3}{3a(a+b)^3(a-b+a\operatorname{Cosh}[2x]+b\operatorname{Cosh}[2x])^2} + \frac{2b^2(8a+3b)}{3a^2(a+b)^3(a-b+a\operatorname{Cosh}[2x]+b\operatorname{Cosh}[2x])} \right) + \\
& \frac{1}{2a^2(a+b)^2} \left(\left((3a^2 + 8ab + 4b^2)(1 + \operatorname{Cosh}[x]) \sqrt{\frac{1 + \operatorname{Cosh}[2x]}{(1 + \operatorname{Cosh}[x])^2}} \sqrt{\frac{a - b + (a+b)\operatorname{Cosh}[2x]}{1 + \operatorname{Cosh}[2x]}} \right. \right. \\
& \left. \left(-\operatorname{Log}\left[\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right] + \operatorname{Log}\left[a + 2b + a\operatorname{Tanh}\left[\frac{x}{2}\right]^2 + \sqrt{a} \sqrt{4b\operatorname{Tanh}\left[\frac{x}{2}\right]^2 + a\left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2}\right] \right) + \right. \\
& \left. \operatorname{Log}\left[a + a\operatorname{Tanh}\left[\frac{x}{2}\right]^2 + 2b\operatorname{Tanh}\left[\frac{x}{2}\right]^2 + \sqrt{a} \sqrt{4b\operatorname{Tanh}\left[\frac{x}{2}\right]^2 + a\left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2}\right] \right) \left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) \left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right) \right. \\
& \left. \sqrt{\frac{4b\operatorname{Tanh}\left[\frac{x}{2}\right]^2 + a\left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2}{\left(-1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2}} \right) / \left(4\sqrt{a} \sqrt{a - b + (a+b)\operatorname{Cosh}[2x]} \sqrt{\left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2} \sqrt{4b\operatorname{Tanh}\left[\frac{x}{2}\right]^2 + a\left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2} \right) + \\
& \frac{1}{\sqrt{a - b + (a+b)\operatorname{Cosh}[2x]}} \frac{3a^2\sqrt{1 + \operatorname{Cosh}[2x]}}{\sqrt{\frac{a - b + (a+b)\operatorname{Cosh}[2x]}{1 + \operatorname{Cosh}[2x]}}} \left(\left(4\operatorname{Cosh}[x]^2 \sqrt{-2b + a(1 + \operatorname{Cosh}[2x]) + b(1 + \operatorname{Cosh}[2x])} \right. \right. \\
& \left. \left. \operatorname{Coth}[x] \left(-\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a}\sqrt{1 + \operatorname{Cosh}[2x]}}{\sqrt{b(-1 + \operatorname{Cosh}[2x]) + a(1 + \operatorname{Cosh}[2x])}}\right]}{\sqrt{a}} + \frac{1}{\sqrt{a+b}} \operatorname{Log}\left[a\sqrt{1 + \operatorname{Cosh}[2x]} + b\sqrt{1 + \operatorname{Cosh}[2x]} + \sqrt{a+b}\right] \right. \right. \right. \\
& \left. \left. \left. \sqrt{b(-1 + \operatorname{Cosh}[2x]) + a(1 + \operatorname{Cosh}[2x])} \right) \operatorname{Sinh}[2x] \right) / \left(3(1 + \operatorname{Cosh}[2x])^2 \sqrt{a - b + (a+b)\operatorname{Cosh}[2x]} \right) - \\
& \left((1 + \operatorname{Cosh}[x]) \sqrt{\frac{1 + \operatorname{Cosh}[2x]}{(1 + \operatorname{Cosh}[x])^2}} \left(-\operatorname{Log}\left[\operatorname{Tanh}\left[\frac{x}{2}\right]^2\right] + \operatorname{Log}\left[a + 2b + a\operatorname{Tanh}\left[\frac{x}{2}\right]^2 + \sqrt{a} \sqrt{4b\operatorname{Tanh}\left[\frac{x}{2}\right]^2 + a\left(1 + \operatorname{Tanh}\left[\frac{x}{2}\right]^2\right)^2}\right] \right) + \right.
\end{aligned}$$

Problem 259: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Tanh}[x] (a + b \operatorname{Tanh}[x]^4)^{3/2} dx$$

Optimal (type 3, 124 leaves, 9 steps):

$$-\frac{1}{4} \sqrt{b} (3a + 2b) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[x]^2}{\sqrt{a + b \operatorname{Tanh}[x]^4}}\right] +$$

$$\frac{1}{2} (a + b)^{3/2} \operatorname{ArcTanh}\left[\frac{a + b \operatorname{Tanh}[x]^2}{\sqrt{a + b} \sqrt{a + b \operatorname{Tanh}[x]^4}}\right] - \frac{1}{4} (2(a + b) + b \operatorname{Tanh}[x]^2) \sqrt{a + b \operatorname{Tanh}[x]^4} - \frac{1}{6} (a + b \operatorname{Tanh}[x]^4)^{3/2}$$

Result (type 3, 62021 leaves): Display of huge result suppressed!

Problem 260: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Tanh}[x] \sqrt{a + b \operatorname{Tanh}[x]^4} dx$$

Optimal (type 3, 89 leaves, 8 steps):

$$-\frac{1}{2} \sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[x]^2}{\sqrt{a + b \operatorname{Tanh}[x]^4}}\right] + \frac{1}{2} \sqrt{a + b} \operatorname{ArcTanh}\left[\frac{a + b \operatorname{Tanh}[x]^2}{\sqrt{a + b} \sqrt{a + b \operatorname{Tanh}[x]^4}}\right] - \frac{1}{2} \sqrt{a + b \operatorname{Tanh}[x]^4}$$

Result (type 3, 31650 leaves): Display of huge result suppressed!

Problem 261: Unable to integrate problem.

$$\int \frac{\operatorname{Tanh}[x]}{\sqrt{a + b \operatorname{Tanh}[x]^4}} dx$$

Optimal (type 3, 40 leaves, 4 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{a + b \operatorname{Tanh}[x]^2}{\sqrt{a + b} \sqrt{a + b \operatorname{Tanh}[x]^4}}\right]}{2 \sqrt{a + b}}$$

Result (type 8, 17 leaves):

$$\int \frac{\operatorname{Tanh}[x]}{\sqrt{a + b \operatorname{Tanh}[x]^4}} dx$$

Problem 262: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Tanh}[x]}{(a + b \text{Tanh}[x]^4)^{3/2}} dx$$

Optimal (type 3, 74 leaves, 6 steps):

$$\frac{\text{ArcTanh}\left[\frac{a+b \text{Tanh}[x]^2}{\sqrt{a+b} \sqrt{a+b \text{Tanh}[x]^4}}\right]}{2 (a+b)^{3/2}} - \frac{a - b \text{Tanh}[x]^2}{2 a (a+b) \sqrt{a+b \text{Tanh}[x]^4}}$$

Result (type 3, 33271 leaves): Display of huge result suppressed!

Problem 263: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Tanh}[x]}{(a + b \text{Tanh}[x]^4)^{5/2}} dx$$

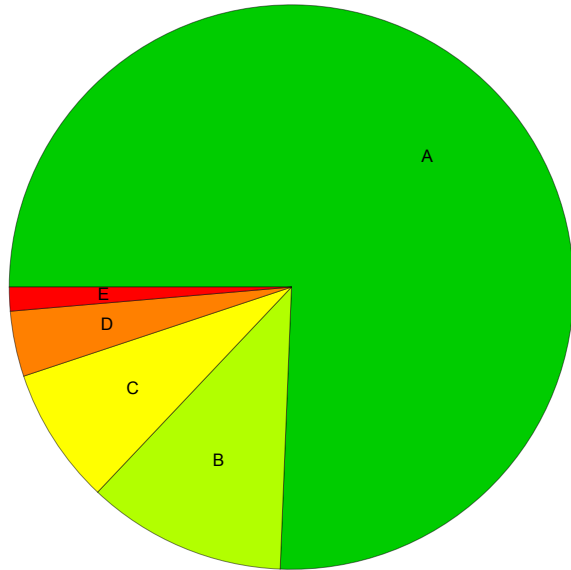
Optimal (type 3, 118 leaves, 7 steps):

$$\frac{\text{ArcTanh}\left[\frac{a+b \text{Tanh}[x]^2}{\sqrt{a+b} \sqrt{a+b \text{Tanh}[x]^4}}\right]}{2 (a+b)^{5/2}} - \frac{a - b \text{Tanh}[x]^2}{6 a (a+b) (a+b \text{Tanh}[x]^4)^{3/2}} - \frac{3 a^2 - b (5 a + 2 b) \text{Tanh}[x]^2}{6 a^2 (a+b)^2 \sqrt{a+b \text{Tanh}[x]^4}}$$

Result (type 3, 41215 leaves): Display of huge result suppressed!

Summary of Integration Test Results

587 integration problems



A - 444 optimal antiderivatives

B - 67 more than twice size of optimal antiderivatives

C - 46 unnecessarily complex antiderivatives

D - 22 unable to integrate problems

E - 8 integration timeouts