

Mathematica 11.3 Integration Test Results

on the problems in the test-suite directory "6 Hyperbolic functions\6.6 Hyperbolic cosecant"

Test results for the 29 problems in "6.6.1 (c+d x)^m (a+b csch)^n.m"

Problem 3: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int (c + d x) \operatorname{Csch}[a + b x] dx$$

Optimal (type 4, 50 leaves, 5 steps):

$$-\frac{2(c + d x) \operatorname{ArcTanh}\left[e^{a+bx}\right]}{b} - \frac{d \operatorname{PolyLog}\left[2, -e^{a+bx}\right]}{b^2} + \frac{d \operatorname{PolyLog}\left[2, e^{a+bx}\right]}{b^2}$$

Result (type 4, 174 leaves):

$$-\frac{c \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{a}{2} + \frac{bx}{2}\right]\right]}{b} + \frac{c \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{a}{2} + \frac{bx}{2}\right]\right]}{b} + \frac{1}{b^2} d \left(-a \operatorname{Log}\left[\operatorname{Tanh}\left[\frac{1}{2}(a + bx)\right]\right] - \right. \\ \left. i \left((i a + i b x) \left(\operatorname{Log}\left[1 - e^{i(i a + i b x)}\right] - \operatorname{Log}\left[1 + e^{i(i a + i b x)}\right]\right) + i \left(\operatorname{PolyLog}\left[2, -e^{i(i a + i b x)}\right] - \operatorname{PolyLog}\left[2, e^{i(i a + i b x)}\right]\right) \right) \right)$$

Problem 6: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int (c + d x)^2 \operatorname{Csch}[a + b x]^2 dx$$

Optimal (type 4, 74 leaves, 5 steps):

$$-\frac{(c + d x)^2}{b} - \frac{(c + d x)^2 \operatorname{Coth}[a + b x]}{b} + \frac{2 d (c + d x) \operatorname{Log}\left[1 - e^{2(a+bx)}\right]}{b^2} + \frac{d^2 \operatorname{PolyLog}\left[2, e^{2(a+bx)}\right]}{b^3}$$

Result (type 4, 277 leaves):

$$\begin{aligned}
& - \frac{2 c d \operatorname{Csch}[a] \left(-b x \operatorname{Cosh}[a] + \operatorname{Log}[\operatorname{Cosh}[b x] \operatorname{Sinh}[a] + \operatorname{Cosh}[a] \operatorname{Sinh}[b x]] \operatorname{Sinh}[a] \right)}{b^2 \left(-\operatorname{Cosh}[a]^2 + \operatorname{Sinh}[a]^2 \right)} + \\
& \frac{\operatorname{Csch}[a] \operatorname{Csch}[a + b x] \left(c^2 \operatorname{Sinh}[b x] + 2 c d x \operatorname{Sinh}[b x] + d^2 x^2 \operatorname{Sinh}[b x] \right)}{b} + \\
& \left(d^2 \operatorname{Csch}[a] \operatorname{Sech}[a] \left(-b^2 e^{-\operatorname{ArcTanh}[\operatorname{Tanh}[a]]} x^2 + \frac{1}{\sqrt{1 - \operatorname{Tanh}[a]^2}} i \left(-b x \left(-\pi + 2 i \operatorname{ArcTanh}[\operatorname{Tanh}[a]] \right) - \pi \operatorname{Log}[1 + e^{2 b x}] - \right. \right. \right. \\
& \quad \left. \left. \left. 2 \left(i b x + i \operatorname{ArcTanh}[\operatorname{Tanh}[a]] \right) \operatorname{Log}\left[1 - e^{2 i \left(i b x + i \operatorname{ArcTanh}[\operatorname{Tanh}[a]] \right)}\right] + \pi \operatorname{Log}[\operatorname{Cosh}[b x]] + 2 i \operatorname{ArcTanh}[\operatorname{Tanh}[a]] \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{Log}\left[i \operatorname{Sinh}[b x + \operatorname{ArcTanh}[\operatorname{Tanh}[a]]]\right] + i \operatorname{PolyLog}\left[2, e^{2 i \left(i b x + i \operatorname{ArcTanh}[\operatorname{Tanh}[a]] \right)}\right] \right) \operatorname{Tanh}[a] \right) \right) / \left(b^3 \sqrt{\operatorname{Sech}[a]^2 \left(\operatorname{Cosh}[a]^2 - \operatorname{Sinh}[a]^2 \right)} \right)
\end{aligned}$$

Problem 10: Result more than twice size of optimal antiderivative.

$$\int (c + d x)^2 \operatorname{Csch}[a + b x]^3 dx$$

Optimal (type 4, 154 leaves, 9 steps):

$$\begin{aligned}
& \frac{(c + d x)^2 \operatorname{ArcTanh}\left[e^{a + b x}\right]}{b} - \frac{d^2 \operatorname{ArcTanh}[\operatorname{Cosh}[a + b x]]}{b^3} - \frac{d(c + d x) \operatorname{Csch}[a + b x]}{b^2} - \frac{(c + d x)^2 \operatorname{Coth}[a + b x] \operatorname{Csch}[a + b x]}{2 b} + \\
& \frac{d(c + d x) \operatorname{PolyLog}\left[2, -e^{a + b x}\right]}{b^2} - \frac{d(c + d x) \operatorname{PolyLog}\left[2, e^{a + b x}\right]}{b^2} - \frac{d^2 \operatorname{PolyLog}\left[3, -e^{a + b x}\right]}{b^3} + \frac{d^2 \operatorname{PolyLog}\left[3, e^{a + b x}\right]}{b^3}
\end{aligned}$$

Result (type 4, 420 leaves):

$$\begin{aligned}
& - \frac{d(c + d x) \operatorname{Csch}[a]}{b^2} + \frac{\left(-c^2 - 2 c d x - d^2 x^2\right) \operatorname{Csch}\left[\frac{a}{2} + \frac{b x}{2}\right]^2}{8 b} + \\
& \frac{1}{2 b^3} \left(-b^2 c^2 \operatorname{Log}\left[1 - e^{a + b x}\right] + 2 d^2 \operatorname{Log}\left[1 - e^{a + b x}\right] - 2 b^2 c d x \operatorname{Log}\left[1 - e^{a + b x}\right] - b^2 d^2 x^2 \operatorname{Log}\left[1 - e^{a + b x}\right] + \right. \\
& \quad \left. b^2 c^2 \operatorname{Log}\left[1 + e^{a + b x}\right] - 2 d^2 \operatorname{Log}\left[1 + e^{a + b x}\right] + 2 b^2 c d x \operatorname{Log}\left[1 + e^{a + b x}\right] + b^2 d^2 x^2 \operatorname{Log}\left[1 + e^{a + b x}\right] + \right. \\
& \quad \left. 2 b d(c + d x) \operatorname{PolyLog}\left[2, -e^{a + b x}\right] - 2 b d(c + d x) \operatorname{PolyLog}\left[2, e^{a + b x}\right] - 2 d^2 \operatorname{PolyLog}\left[3, -e^{a + b x}\right] + 2 d^2 \operatorname{PolyLog}\left[3, e^{a + b x}\right] \right) + \\
& \frac{\left(-c^2 - 2 c d x - d^2 x^2\right) \operatorname{Sech}\left[\frac{a}{2} + \frac{b x}{2}\right]^2}{8 b} + \frac{\operatorname{Csch}\left[\frac{a}{2}\right] \operatorname{Csch}\left[\frac{a}{2} + \frac{b x}{2}\right] \left(c d \operatorname{Sinh}\left[\frac{b x}{2}\right] + d^2 x \operatorname{Sinh}\left[\frac{b x}{2}\right]\right)}{2 b^2} + \\
& \frac{\operatorname{Sech}\left[\frac{a}{2}\right] \operatorname{Sech}\left[\frac{a}{2} + \frac{b x}{2}\right] \left(c d \operatorname{Sinh}\left[\frac{b x}{2}\right] + d^2 x \operatorname{Sinh}\left[\frac{b x}{2}\right]\right)}{2 b^2}
\end{aligned}$$

Problem 11: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int (c + dx) \operatorname{Csch}[a + bx]^3 dx$$

Optimal (type 4, 92 leaves, 6 steps):

$$\frac{(c + dx) \operatorname{ArcTanh}\left[\frac{e^{a+bx}}{2}\right]}{b} - \frac{d \operatorname{Csch}[a + bx]}{2b^2} - \frac{(c + dx) \operatorname{Coth}[a + bx] \operatorname{Csch}[a + bx]}{2b} + \frac{d \operatorname{PolyLog}\left[2, -e^{a+bx}\right]}{2b^2} - \frac{d \operatorname{PolyLog}\left[2, e^{a+bx}\right]}{2b^2}$$

Result (type 4, 332 leaves):

$$\begin{aligned} & -\frac{dx \operatorname{Csch}\left[\frac{a}{2} + \frac{bx}{2}\right]^2}{8b} - \frac{c \operatorname{Csch}\left[\frac{1}{2}(a + bx)\right]^2}{8b} + \frac{c \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{1}{2}(a + bx)\right]\right]}{2b} - \frac{c \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{1}{2}(a + bx)\right]\right]}{2b} - \frac{1}{2b^2} d \left(-a \operatorname{Log}\left[\operatorname{Tanh}\left[\frac{1}{2}(a + bx)\right]\right] \right) - \\ & \quad i \left((i a + i b x) \left(\operatorname{Log}\left[1 - e^{i(i a + i b x)}\right] - \operatorname{Log}\left[1 + e^{i(i a + i b x)}\right] \right) + i \left(\operatorname{PolyLog}\left[2, -e^{i(i a + i b x)}\right] - \operatorname{PolyLog}\left[2, e^{i(i a + i b x)}\right] \right) \right) - \\ & \frac{dx \operatorname{Sech}\left[\frac{a}{2} + \frac{bx}{2}\right]^2}{8b} - \frac{c \operatorname{Sech}\left[\frac{1}{2}(a + bx)\right]^2}{8b} + \frac{d \operatorname{Csch}\left[\frac{a}{2}\right] \operatorname{Csch}\left[\frac{a}{2} + \frac{bx}{2}\right] \operatorname{Sinh}\left[\frac{bx}{2}\right]}{4b^2} + \frac{d \operatorname{Sech}\left[\frac{a}{2}\right] \operatorname{Sech}\left[\frac{a}{2} + \frac{bx}{2}\right] \operatorname{Sinh}\left[\frac{bx}{2}\right]}{4b^2} \end{aligned}$$

Problem 17: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(e + fx)^3 \operatorname{Cosh}[c + dx]}{a + b \operatorname{Csch}[c + dx]} dx$$

Optimal (type 4, 448 leaves, 17 steps):

$$\begin{aligned} & \frac{b(e + fx)^4}{4a^2 f} - \frac{6f^3 \operatorname{Cosh}[c + dx]}{a d^4} - \frac{3f(e + fx)^2 \operatorname{Cosh}[c + dx]}{a d^2} - \frac{b(e + fx)^3 \operatorname{Log}\left[1 + \frac{a e^{c+dx}}{b - \sqrt{a^2 + b^2}}\right]}{a^2 d} - \\ & \frac{b(e + fx)^3 \operatorname{Log}\left[1 + \frac{a e^{c+dx}}{b + \sqrt{a^2 + b^2}}\right]}{a^2 d} - \frac{3bf(e + fx)^2 \operatorname{PolyLog}\left[2, -\frac{a e^{c+dx}}{b - \sqrt{a^2 + b^2}}\right]}{a^2 d^2} - \frac{3bf(e + fx)^2 \operatorname{PolyLog}\left[2, -\frac{a e^{c+dx}}{b + \sqrt{a^2 + b^2}}\right]}{a^2 d^2} + \\ & \frac{6bf^2(e + fx) \operatorname{PolyLog}\left[3, -\frac{a e^{c+dx}}{b - \sqrt{a^2 + b^2}}\right]}{a^2 d^3} + \frac{6bf^2(e + fx) \operatorname{PolyLog}\left[3, -\frac{a e^{c+dx}}{b + \sqrt{a^2 + b^2}}\right]}{a^2 d^3} - \frac{6bf^3 \operatorname{PolyLog}\left[4, -\frac{a e^{c+dx}}{b - \sqrt{a^2 + b^2}}\right]}{a^2 d^4} - \\ & \frac{6bf^3 \operatorname{PolyLog}\left[4, -\frac{a e^{c+dx}}{b + \sqrt{a^2 + b^2}}\right]}{a^2 d^4} + \frac{6f^2(e + fx) \operatorname{Sinh}[c + dx]}{a d^3} + \frac{(e + fx)^3 \operatorname{Sinh}[c + dx]}{a d} \end{aligned}$$

Result (type 4, 1635 leaves):

$$\begin{aligned}
& \frac{1}{2 a^2 d^3 (a + b \operatorname{Csch}[c + d x])} e^{f^2} \operatorname{Csch}[c + d x] \left(-12 b d x \operatorname{PolyLog}\left[2, -\frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2 + b^2)} e^{2c}}\right] - 12 b d x \operatorname{PolyLog}\left[2, -\frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2 + b^2)} e^{2c}}\right] \right) + \\
& e^{-c} \left(2 b d^3 e^c x^3 - 6 a \operatorname{Cosh}[d x] + 6 a e^{2c} \operatorname{Cosh}[d x] - 6 a d x \operatorname{Cosh}[d x] - 6 a d e^{2c} x \operatorname{Cosh}[d x] - 3 a d^2 x^2 \operatorname{Cosh}[d x] + \right. \\
& 3 a d^2 e^{2c} x^2 \operatorname{Cosh}[d x] - 6 b d^2 e^c x^2 \operatorname{Log}\left[1 + \frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2 + b^2)} e^{2c}}\right] - 6 b d^2 e^c x^2 \operatorname{Log}\left[1 + \frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2 + b^2)} e^{2c}}\right] + \\
& 12 b e^c \operatorname{PolyLog}\left[3, -\frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2 + b^2)} e^{2c}}\right] + 12 b e^c \operatorname{PolyLog}\left[3, -\frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2 + b^2)} e^{2c}}\right] + 6 a \operatorname{Sinh}[d x] + 6 a e^{2c} \operatorname{Sinh}[d x] + \\
& \left. 6 a d x \operatorname{Sinh}[d x] - 6 a d e^{2c} x \operatorname{Sinh}[d x] + 3 a d^2 x^2 \operatorname{Sinh}[d x] + 3 a d^2 e^{2c} x^2 \operatorname{Sinh}[d x] \right) (b + a \operatorname{Sinh}[c + d x]) + \\
& \frac{1}{4 a^2 d^4 (a + b \operatorname{Csch}[c + d x])} f^3 \operatorname{Csch}[c + d x] \left(-12 b d^2 x^2 \operatorname{PolyLog}\left[2, -\frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2 + b^2)} e^{2c}}\right] + \right. \\
& e^{-c} \left(b d^4 e^c x^4 - 12 a \operatorname{Cosh}[d x] - 12 a e^{2c} \operatorname{Cosh}[d x] - 12 a d x \operatorname{Cosh}[d x] + 12 a d e^{2c} x \operatorname{Cosh}[d x] - 6 a d^2 x^2 \operatorname{Cosh}[d x] - 6 a d^2 e^{2c} x^2 \operatorname{Cosh}[d x] - \right. \\
& 2 a d^3 x^3 \operatorname{Cosh}[d x] + 2 a d^3 e^{2c} x^3 \operatorname{Cosh}[d x] - 4 b d^3 e^c x^3 \operatorname{Log}\left[1 + \frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2 + b^2)} e^{2c}}\right] - 4 b d^3 e^c x^3 \operatorname{Log}\left[1 + \frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2 + b^2)} e^{2c}}\right] - \\
& 12 b d^2 e^c x^2 \operatorname{PolyLog}\left[2, -\frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2 + b^2)} e^{2c}}\right] + 24 b d e^c x \operatorname{PolyLog}\left[3, -\frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2 + b^2)} e^{2c}}\right] + \\
& 24 b d e^c x \operatorname{PolyLog}\left[3, -\frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2 + b^2)} e^{2c}}\right] - 24 b e^c \operatorname{PolyLog}\left[4, -\frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2 + b^2)} e^{2c}}\right] - \\
& 24 b e^c \operatorname{PolyLog}\left[4, -\frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2 + b^2)} e^{2c}}\right] + 12 a \operatorname{Sinh}[d x] - 12 a e^{2c} \operatorname{Sinh}[d x] + 12 a d x \operatorname{Sinh}[d x] + 12 a d e^{2c} x \operatorname{Sinh}[d x] + \\
& \left. 6 a d^2 x^2 \operatorname{Sinh}[d x] - 6 a d^2 e^{2c} x^2 \operatorname{Sinh}[d x] + 2 a d^3 x^3 \operatorname{Sinh}[d x] + 2 a d^3 e^{2c} x^3 \operatorname{Sinh}[d x] \right) (b + a \operatorname{Sinh}[c + d x]) + \\
& \frac{e^3 \operatorname{Csch}[c + d x] (b + a \operatorname{Sinh}[c + d x]) \left(-\frac{2 b \operatorname{Log}[b + a \operatorname{Sinh}[c + d x]]}{a^2 d} + \frac{2 \operatorname{Sinh}[c + d x]}{a d} \right)}{2 (a + b \operatorname{Csch}[c + d x])} + \frac{1}{a^2 d^2 (a + b \operatorname{Csch}[c + d x])}
\end{aligned}$$

$$\begin{aligned}
& e^2 \\
& f \\
& \text{Csch}[c + d x] \\
& (b + a \text{Sinh}[c + d x]) \\
& \left(-a \text{Cosh}[c + d x] - b (c + d x) \text{Log}[b + a \text{Sinh}[c + d x]] + b c \text{Log}\left[1 + \frac{a \text{Sinh}[c + d x]}{b}\right] + \right. \\
& i b \left(-\frac{1}{8} i (2 c + i \pi + 2 d x)^2 - 4 i \text{ArcSin}\left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}}\right] \text{ArcTan}\left[\frac{(i a + b) \text{Cot}\left[\frac{1}{4} (2 i c + \pi + 2 i d x)\right]}{\sqrt{a^2 + b^2}}\right] \right) - \\
& \frac{1}{2} \left(-2 i c + \pi - 2 i d x + 4 \text{ArcSin}\left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}}\right] \right) \text{Log}\left[1 + \frac{(-b + \sqrt{a^2 + b^2}) e^{c + d x}}{a}\right] - \\
& \frac{1}{2} \left(-2 i c + \pi - 2 i d x - 4 \text{ArcSin}\left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}}\right] \right) \text{Log}\left[1 - \frac{(b + \sqrt{a^2 + b^2}) e^{c + d x}}{a}\right] + \left(\frac{\pi}{2} - i (c + d x)\right) \text{Log}[b + a \text{Sinh}[c + d x]] + \\
& i \left(\text{PolyLog}\left[2, \frac{(b - \sqrt{a^2 + b^2}) e^{c + d x}}{a}\right] + \text{PolyLog}\left[2, \frac{(b + \sqrt{a^2 + b^2}) e^{c + d x}}{a}\right] \right) + a d x \text{Sinh}[c + d x] \left. \right)
\end{aligned}$$

Problem 18: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(e + f x)^2 \text{Cosh}[c + d x]}{a + b \text{Csch}[c + d x]} dx$$

Optimal (type 4, 330 leaves, 14 steps):

$$\frac{b (e + f x)^3}{3 a^2 f} - \frac{2 f (e + f x) \operatorname{Cosh}[c + d x]}{a d^2} - \frac{b (e + f x)^2 \operatorname{Log}\left[1 + \frac{a e^{c+dx}}{b - \sqrt{a^2 + b^2}}\right]}{a^2 d} - \frac{b (e + f x)^2 \operatorname{Log}\left[1 + \frac{a e^{c+dx}}{b + \sqrt{a^2 + b^2}}\right]}{a^2 d} - \frac{2 b f (e + f x) \operatorname{PolyLog}\left[2, -\frac{a e^{c+dx}}{b - \sqrt{a^2 + b^2}}\right]}{a^2 d^2} - \frac{2 b f (e + f x) \operatorname{PolyLog}\left[2, -\frac{a e^{c+dx}}{b + \sqrt{a^2 + b^2}}\right]}{a^2 d^2} + \frac{2 b f^2 \operatorname{PolyLog}\left[3, -\frac{a e^{c+dx}}{b - \sqrt{a^2 + b^2}}\right]}{a^2 d^3} + \frac{2 b f^2 \operatorname{PolyLog}\left[3, -\frac{a e^{c+dx}}{b + \sqrt{a^2 + b^2}}\right]}{a^2 d^3} + \frac{2 f^2 \operatorname{Sinh}[c + d x]}{a d^3} + \frac{(e + f x)^2 \operatorname{Sinh}[c + d x]}{a d}$$

Result (type 4, 971 leaves):

$$\frac{1}{6 a^2 d^3 (a + b \operatorname{Csch}[c + d x])} f^2 \operatorname{Csch}[c + d x] \left(-12 b d x \operatorname{PolyLog}\left[2, -\frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2 + b^2)} e^{2c}}\right] - 12 b d x \operatorname{PolyLog}\left[2, -\frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2 + b^2)} e^{2c}}\right] \right) +$$

$$e^{-c} \left(2 b d^3 e^c x^3 - 6 a \operatorname{Cosh}[d x] + 6 a e^{2c} \operatorname{Cosh}[d x] - 6 a d x \operatorname{Cosh}[d x] - 6 a d e^{2c} x \operatorname{Cosh}[d x] - 3 a d^2 x^2 \operatorname{Cosh}[d x] + \right.$$

$$3 a d^2 e^{2c} x^2 \operatorname{Cosh}[d x] - 6 b d^2 e^c x^2 \operatorname{Log}\left[1 + \frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2 + b^2)} e^{2c}}\right] - 6 b d^2 e^c x^2 \operatorname{Log}\left[1 + \frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2 + b^2)} e^{2c}}\right] +$$

$$12 b e^c \operatorname{PolyLog}\left[3, -\frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2 + b^2)} e^{2c}}\right] + 12 b e^c \operatorname{PolyLog}\left[3, -\frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2 + b^2)} e^{2c}}\right] + 6 a \operatorname{Sinh}[d x] + 6 a e^{2c} \operatorname{Sinh}[d x] +$$

$$6 a d x \operatorname{Sinh}[d x] - 6 a d e^{2c} x \operatorname{Sinh}[d x] + 3 a d^2 x^2 \operatorname{Sinh}[d x] + 3 a d^2 e^{2c} x^2 \operatorname{Sinh}[d x] \left. \right) (b + a \operatorname{Sinh}[c + d x]) +$$

$$\frac{e^2 \operatorname{Csch}[c + d x] (b + a \operatorname{Sinh}[c + d x]) \left(-\frac{2 b \operatorname{Log}[b + a \operatorname{Sinh}[c + d x]]}{a^2 d} + \frac{2 \operatorname{Sinh}[c + d x]}{a d} \right)}{2 (a + b \operatorname{Csch}[c + d x])} + \frac{1}{a^2 d^2 (a + b \operatorname{Csch}[c + d x])}$$

$$2 e f \operatorname{Csch}[c + d x] (b + a \operatorname{Sinh}[c + d x])$$

$$\left(-a \operatorname{Cosh}[c + d x] - b (c + d x) \operatorname{Log}[b + a \operatorname{Sinh}[c + d x]] + b c \operatorname{Log}\left[1 + \frac{a \operatorname{Sinh}[c + d x]}{b}\right] \right) +$$

$$i b \left(-\frac{1}{8} i (2 c + i \pi + 2 d x)^2 - 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}}\right] \operatorname{ArcTan}\left[\frac{(i a + b) \operatorname{Cot}\left[\frac{1}{4} (2 i c + \pi + 2 i d x)\right]}{\sqrt{a^2 + b^2}}\right] \right) -$$

$$\frac{1}{2} \left(-2 i c + \pi - 2 i d x + 4 \operatorname{ArcSin} \left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}} \right] \right) \operatorname{Log} \left[1 + \frac{(-b + \sqrt{a^2 + b^2}) e^{c+dx}}{a} \right] -$$

$$\frac{1}{2} \left(-2 i c + \pi - 2 i d x - 4 \operatorname{ArcSin} \left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}} \right] \right) \operatorname{Log} \left[1 - \frac{(b + \sqrt{a^2 + b^2}) e^{c+dx}}{a} \right] + \left(\frac{\pi}{2} - i (c + d x) \right) \operatorname{Log} [b + a \operatorname{Sinh} [c + d x]] +$$

$$i \left(\operatorname{PolyLog} \left[2, \frac{(b - \sqrt{a^2 + b^2}) e^{c+dx}}{a} \right] + \operatorname{PolyLog} \left[2, \frac{(b + \sqrt{a^2 + b^2}) e^{c+dx}}{a} \right] \right) + a d x \operatorname{Sinh} [c + d x]$$

Problem 19: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(e + f x) \operatorname{Cosh} [c + d x]}{a + b \operatorname{Csch} [c + d x]} dx$$

Optimal (type 4, 212 leaves, 11 steps):

$$\frac{b (e + f x)^2}{2 a^2 f} - \frac{f \operatorname{Cosh} [c + d x]}{a d^2} - \frac{b (e + f x) \operatorname{Log} \left[1 + \frac{a e^{c+dx}}{b - \sqrt{a^2 + b^2}} \right]}{a^2 d} - \frac{b (e + f x) \operatorname{Log} \left[1 + \frac{a e^{c+dx}}{b + \sqrt{a^2 + b^2}} \right]}{a^2 d}$$

$$\frac{b f \operatorname{PolyLog} \left[2, -\frac{a e^{c+dx}}{b - \sqrt{a^2 + b^2}} \right]}{a^2 d^2} - \frac{b f \operatorname{PolyLog} \left[2, -\frac{a e^{c+dx}}{b + \sqrt{a^2 + b^2}} \right]}{a^2 d^2} + \frac{(e + f x) \operatorname{Sinh} [c + d x]}{a d}$$

Result (type 4, 401 leaves):

$$\begin{aligned}
& - \frac{1}{a^2 d^2 (a + b \operatorname{Csch}[c + d x])} \\
& \operatorname{Csch}[c + d x] (b + a \operatorname{Sinh}[c + d x]) \left(d e (b \operatorname{Log}[b + a \operatorname{Sinh}[c + d x]] - a \operatorname{Sinh}[c + d x]) + \frac{1}{8} f \left(-b (2 c + i \pi + 2 d x)^2 - 32 b \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}}\right] \operatorname{ArcTan}\left[\frac{i a + b}{\sqrt{a^2 + b^2}}\right] \right) \right. \\
& \left. + \frac{(i a + b) \operatorname{Cot}\left[\frac{1}{4} (2 i c + \pi + 2 i d x)\right]}{\sqrt{a^2 + b^2}} + 8 a \operatorname{Cosh}[c + d x] + 4 b \left(2 c + i \pi + 2 d x + 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}}\right] \right) \operatorname{Log}\left[1 + \frac{(-b + \sqrt{a^2 + b^2}) e^{c + d x}}{a}\right] + \right. \\
& \left. 4 b \left(2 c + i \pi + 2 d x - 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}}\right] \right) \operatorname{Log}\left[1 - \frac{(b + \sqrt{a^2 + b^2}) e^{c + d x}}{a}\right] - 4 i b \pi \operatorname{Log}[b + a \operatorname{Sinh}[c + d x]] - \right. \\
& \left. \left. 8 b c \operatorname{Log}\left[1 + \frac{a \operatorname{Sinh}[c + d x]}{b}\right] + 8 b \left(\operatorname{PolyLog}\left[2, \frac{(b - \sqrt{a^2 + b^2}) e^{c + d x}}{a}\right] + \operatorname{PolyLog}\left[2, \frac{(b + \sqrt{a^2 + b^2}) e^{c + d x}}{a}\right] \right) - 8 a d x \operatorname{Sinh}[c + d x] \right) \right)
\end{aligned}$$

Problem 21: Attempted integration timed out after 120 seconds.

$$\int \frac{\operatorname{Cosh}[c + d x]}{(e + f x) (a + b \operatorname{Csch}[c + d x])} dx$$

Optimal (type 9, 34 leaves, 1 step):

$$\operatorname{Unintegrable}\left[\frac{\operatorname{Cosh}[c + d x] \operatorname{Sinh}[c + d x]}{(e + f x) (b + a \operatorname{Sinh}[c + d x])}, x\right]$$

Result (type 1, 1 leaves):

???

Problem 22: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(e + f x)^3 \operatorname{Cosh}[c + d x]^2}{a + b \operatorname{Csch}[c + d x]} dx$$

Optimal (type 4, 696 leaves, 24 steps):

$$\begin{aligned} & \frac{3 e f^2 x}{4 a d^2} + \frac{3 f^3 x^2}{8 a d^2} + \frac{(e + f x)^4}{8 a f} + \frac{b^2 (e + f x)^4}{4 a^3 f} - \frac{6 b f^2 (e + f x) \operatorname{Cosh}[c + d x]}{a^2 d^3} - \frac{b (e + f x)^3 \operatorname{Cosh}[c + d x]}{a^2 d} - \frac{3 f^3 \operatorname{Cosh}[c + d x]^2}{8 a d^4} - \\ & \frac{3 f (e + f x)^2 \operatorname{Cosh}[c + d x]^2}{4 a d^2} - \frac{b \sqrt{a^2 + b^2} (e + f x)^3 \operatorname{Log}\left[1 + \frac{a e^{c+dx}}{b - \sqrt{a^2 + b^2}}\right]}{a^3 d} + \frac{b \sqrt{a^2 + b^2} (e + f x)^3 \operatorname{Log}\left[1 + \frac{a e^{c+dx}}{b + \sqrt{a^2 + b^2}}\right]}{a^3 d} - \\ & \frac{3 b \sqrt{a^2 + b^2} f (e + f x)^2 \operatorname{PolyLog}\left[2, -\frac{a e^{c+dx}}{b - \sqrt{a^2 + b^2}}\right]}{a^3 d^2} + \frac{3 b \sqrt{a^2 + b^2} f (e + f x)^2 \operatorname{PolyLog}\left[2, -\frac{a e^{c+dx}}{b + \sqrt{a^2 + b^2}}\right]}{a^3 d^2} + \\ & \frac{6 b \sqrt{a^2 + b^2} f^2 (e + f x) \operatorname{PolyLog}\left[3, -\frac{a e^{c+dx}}{b - \sqrt{a^2 + b^2}}\right]}{a^3 d^3} - \frac{6 b \sqrt{a^2 + b^2} f^2 (e + f x) \operatorname{PolyLog}\left[3, -\frac{a e^{c+dx}}{b + \sqrt{a^2 + b^2}}\right]}{a^3 d^3} - \\ & \frac{6 b \sqrt{a^2 + b^2} f^3 \operatorname{PolyLog}\left[4, -\frac{a e^{c+dx}}{b - \sqrt{a^2 + b^2}}\right]}{a^3 d^4} + \frac{6 b \sqrt{a^2 + b^2} f^3 \operatorname{PolyLog}\left[4, -\frac{a e^{c+dx}}{b + \sqrt{a^2 + b^2}}\right]}{a^3 d^4} + \frac{6 b f^3 \operatorname{Sinh}[c + d x]}{a^2 d^4} + \\ & \frac{3 b f (e + f x)^2 \operatorname{Sinh}[c + d x]}{a^2 d^2} + \frac{3 f^2 (e + f x) \operatorname{Cosh}[c + d x] \operatorname{Sinh}[c + d x]}{4 a d^3} + \frac{(e + f x)^3 \operatorname{Cosh}[c + d x] \operatorname{Sinh}[c + d x]}{2 a d} \end{aligned}$$

Result (type 4, 3560 leaves):

$$\begin{aligned} & e^3 \left(\frac{c}{d} + x - \frac{2 b \operatorname{ArcTan}\left[\frac{a - b \operatorname{Tanh}\left[\frac{1}{2}(c + d x)\right]}{\sqrt{-a^2 - b^2}}\right]}{\sqrt{-a^2 - b^2} d} \right) \operatorname{Csch}[c + d x] (b + a \operatorname{Sinh}[c + d x]) \\ & \frac{1}{4 a (a + b \operatorname{Csch}[c + d x])} + \frac{1}{8 a (a + b \operatorname{Csch}[c + d x])} 3 e^2 f \operatorname{Csch}[c + d x] \\ & \left(x^2 + \frac{1}{d^2} 2 b \left(\frac{i \pi \operatorname{ArcTanh}\left[\frac{-a + b \operatorname{Tanh}\left[\frac{1}{2}(c + d x)\right]}{\sqrt{a^2 + b^2}}\right]}{\sqrt{a^2 + b^2}} + \frac{1}{\sqrt{-a^2 - b^2}} \left(2 \left(c + i \operatorname{ArcCos}\left[-\frac{i b}{a}\right] \right) \operatorname{ArcTan}\left[\frac{(a - i b) \operatorname{Cot}\left[\frac{1}{4}(2 i c + \pi + 2 i d x)\right]}{\sqrt{-a^2 - b^2}}\right]}{(-2 i c + \pi - 2} \right. \right. \\ & \left. \left. i d x) \operatorname{ArcTanh}\left[\frac{(-i a + b) \operatorname{Tan}\left[\frac{1}{4}(2 i c + \pi + 2 i d x)\right]}{\sqrt{-a^2 - b^2}}\right]}{(-2 i c + \pi - 2} \right) - \left(\operatorname{ArcCos}\left[-\frac{i b}{a}\right] - 2 \operatorname{ArcTan}\left[\frac{(a - i b) \operatorname{Cot}\left[\frac{1}{4}(2 i c + \pi + 2 i d x)\right]}{\sqrt{-a^2 - b^2}}\right]} \right) \right) \end{aligned}$$

$$\begin{aligned} & \text{Log} \left[\frac{(a + i b) (a - i b + \sqrt{-a^2 - b^2}) (1 + i \text{Cot} [\frac{1}{4} (2 i c + \pi + 2 i d x)])}{a (a + i b + i \sqrt{-a^2 - b^2} \text{Cot} [\frac{1}{4} (2 i c + \pi + 2 i d x)])} \right] - \left(\text{ArcCos} \left[-\frac{i b}{a} \right] + 2 \right. \\ & \quad \left. \text{ArcTan} \left[\frac{(a - i b) \text{Cot} [\frac{1}{4} (2 i c + \pi + 2 i d x)]}{\sqrt{-a^2 - b^2}} \right] \right) \text{Log} \left[\frac{i (a + i b) (-a + i b + \sqrt{-a^2 - b^2}) (i + \text{Cot} [\frac{1}{4} (2 i c + \pi + 2 i d x)])}{a (a + i b + i \sqrt{-a^2 - b^2} \text{Cot} [\frac{1}{4} (2 i c + \pi + 2 i d x)])} \right] + \\ & \left(\text{ArcCos} \left[-\frac{i b}{a} \right] + 2 \text{ArcTan} \left[\frac{(a - i b) \text{Cot} [\frac{1}{4} (2 i c + \pi + 2 i d x)]}{\sqrt{-a^2 - b^2}} \right] - 2 i \text{ArcTanh} \left[\frac{(-i a + b) \text{Tan} [\frac{1}{4} (2 i c + \pi + 2 i d x)]}{\sqrt{-a^2 - b^2}} \right] \right) \\ & \text{Log} \left[\frac{\sqrt{-a^2 - b^2} e^{\frac{1}{4} (-2 c - i \pi - 2 d x)}}{\sqrt{2} \sqrt{-i a} \sqrt{b + a \text{Sinh} [c + d x]}} \right] + \\ & \left(\text{ArcCos} \left[-\frac{i b}{a} \right] - 2 \text{ArcTan} \left[\frac{(a - i b) \text{Cot} [\frac{1}{4} (2 i c + \pi + 2 i d x)]}{\sqrt{-a^2 - b^2}} \right] + 2 i \text{ArcTanh} \left[\frac{(-i a + b) \text{Tan} [\frac{1}{4} (2 i c + \pi + 2 i d x)]}{\sqrt{-a^2 - b^2}} \right] \right) \\ & \text{Log} \left[\frac{\sqrt{-a^2 - b^2} e^{\frac{1}{4} (2 c + i \pi + 2 d x)}}{\sqrt{2} \sqrt{-i a} \sqrt{b + a \text{Sinh} [c + d x]}} \right] + i \left(\text{PolyLog} \left[2, \frac{(i b + \sqrt{-a^2 - b^2}) (a + i b - i \sqrt{-a^2 - b^2} \text{Cot} [\frac{1}{4} (2 i c + \pi + 2 i d x)])}{a (a + i b + i \sqrt{-a^2 - b^2} \text{Cot} [\frac{1}{4} (2 i c + \pi + 2 i d x)])} \right] \right) - \\ & \left. \left. \left. \left. \text{PolyLog} \left[2, \frac{(b + i \sqrt{-a^2 - b^2}) (i a - b + \sqrt{-a^2 - b^2} \text{Cot} [\frac{1}{4} (2 i c + \pi + 2 i d x)])}{a (a + i b + i \sqrt{-a^2 - b^2} \text{Cot} [\frac{1}{4} (2 i c + \pi + 2 i d x)])} \right] \right] \right) \right) \right) \right) \end{aligned}$$

$$(b + a \text{Sinh} [c + d x]) + \frac{1}{4 a (a + b \text{Csch} [c + d x])} e^{f^2} \text{Csch} [$$

c +
d

$$x] \left(x^3 - \right.$$

$$\left. \frac{1}{d^3 \sqrt{(a^2 + b^2)} e^{2c}} \right)$$

3

b
e^c

$$\left(d^2 x^2 \text{Log} \left[1 + \frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2 + b^2)} e^{2c}} \right] - d^2 x^2 \text{Log} \left[1 + \frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2 + b^2)} e^{2c}} \right] \right) +$$

$$2 d x \operatorname{PolyLog}\left[2, -\frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2+b^2)} e^{2c}}\right] - 2 d x \operatorname{PolyLog}\left[2, -\frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2+b^2)} e^{2c}}\right] -$$

$$2 \operatorname{PolyLog}\left[3, -\frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2+b^2)} e^{2c}}\right] + 2 \operatorname{PolyLog}\left[3, -\frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2+b^2)} e^{2c}}\right] \Bigg) \Bigg)$$

$$(b + a \operatorname{Sinh}[c + d x]) + \frac{1}{16 a (a + b \operatorname{Csch}[c + d x])}$$

f³Csch[
c +
d
x]

$$\left(x^4 - \frac{1}{d^4 \sqrt{(a^2+b^2)} e^{2c}} 4 b e^c \left(d^3 x^3 \operatorname{Log}\left[1 + \frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2+b^2)} e^{2c}}\right] - d^3 x^3 \operatorname{Log}\left[1 + \frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2+b^2)} e^{2c}}\right] \right) + \right.$$

$$3 d^2 x^2 \operatorname{PolyLog}\left[2, -\frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2+b^2)} e^{2c}}\right] - 3 d^2 x^2 \operatorname{PolyLog}\left[2, -\frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2+b^2)} e^{2c}}\right] -$$

$$6 d x \operatorname{PolyLog}\left[3, -\frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2+b^2)} e^{2c}}\right] + 6 d x \operatorname{PolyLog}\left[3, -\frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2+b^2)} e^{2c}}\right] +$$

$$6 \operatorname{PolyLog}\left[4, -\frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2+b^2)} e^{2c}}\right] - 6 \operatorname{PolyLog}\left[4, -\frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2+b^2)} e^{2c}}\right] \Bigg) \Bigg)$$

$$(b + a \operatorname{Sinh}[c + d x]) + \frac{1}{8 a^3 (a + b \operatorname{Csch}[c + d x])} e f^2$$

Csch[
c + d x]

$$\left(2 (a^2 + 4 b^2) x^3 - \frac{1}{d^3 \sqrt{(a^2+b^2)} e^{2c}} 6 b (3 a^2 + 4 b^2) e^c \right.$$

$$\left(d^2 x^2 \operatorname{Log}\left[1 + \frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2+b^2)} e^{2c}}\right] - d^2 x^2 \operatorname{Log}\left[1 + \frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2+b^2)} e^{2c}}\right] + 2 d x \operatorname{PolyLog}\left[2, -\frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2+b^2)} e^{2c}}\right] - \right.$$

$$2 d x \operatorname{PolyLog}\left[2, -\frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2+b^2)} e^{2c}}\right] - 2 \operatorname{PolyLog}\left[3, -\frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2+b^2)} e^{2c}}\right] + 2 \operatorname{PolyLog}\left[3, -\frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2+b^2)} e^{2c}}\right] \Bigg) -$$

$$\frac{24 a b \operatorname{Cosh}[d x] \left((2 + d^2 x^2) \operatorname{Cosh}[c] - 2 d x \operatorname{Sinh}[c] \right)}{d^3} + \frac{3 a^2 \operatorname{Cosh}[2 d x] \left(-2 d x \operatorname{Cosh}[2 c] + (1 + 2 d^2 x^2) \operatorname{Sinh}[2 c] \right)}{d^3} -$$

$$\frac{24 a b \left(-2 d x \operatorname{Cosh}[c] + (2 + d^2 x^2) \operatorname{Sinh}[c] \right) \operatorname{Sinh}[d x]}{d^3} +$$

$$\left. \frac{3 a^2 \left((1 + 2 d^2 x^2) \operatorname{Cosh}[2 c] - 2 d x \operatorname{Sinh}[2 c] \right) \operatorname{Sinh}[2 d x]}{d^3} \right)$$

$$(b + a \operatorname{Sinh}[c + d x]) + \frac{1}{16 a^3 (a + b \operatorname{Csch}[c + d x])}$$

f³Csch[
c + d x]

$$\left((a^2 + 4 b^2) x^4 - \frac{1}{d^4 \sqrt{(a^2 + b^2) e^{2c}}} 4 b (3 a^2 + 4 b^2) e^c \left(d^3 x^3 \operatorname{Log}\left[1 + \frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2 + b^2) e^{2c}}}\right] - d^3 x^3 \operatorname{Log}\left[1 + \frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2 + b^2) e^{2c}}}\right] \right) + \right.$$

$$3 d^2 x^2 \operatorname{PolyLog}\left[2, -\frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2 + b^2) e^{2c}}}\right] - 3 d^2 x^2 \operatorname{PolyLog}\left[2, -\frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2 + b^2) e^{2c}}}\right] - 6 d x \operatorname{PolyLog}\left[3, -\frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2 + b^2) e^{2c}}}\right] +$$

$$6 d x \operatorname{PolyLog}\left[3, -\frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2 + b^2) e^{2c}}}\right] + 6 \operatorname{PolyLog}\left[4, -\frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2 + b^2) e^{2c}}}\right] - 6 \operatorname{PolyLog}\left[4, -\frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2 + b^2) e^{2c}}}\right] \left. - \right.$$

$$\frac{16 a b \operatorname{Cosh}[d x] \left(d x (6 + d^2 x^2) \operatorname{Cosh}[c] - 3 (2 + d^2 x^2) \operatorname{Sinh}[c] \right)}{d^4} + \frac{a^2 \operatorname{Cosh}[2 d x] \left(-3 (1 + 2 d^2 x^2) \operatorname{Cosh}[2 c] + 2 d x (3 + 2 d^2 x^2) \operatorname{Sinh}[2 c] \right)}{d^4} -$$

$$\frac{16 a b \left(-3 (2 + d^2 x^2) \operatorname{Cosh}[c] + d x (6 + d^2 x^2) \operatorname{Sinh}[c] \right) \operatorname{Sinh}[d x]}{d^4} +$$

$$\left. \frac{a^2 \left(2 d x (3 + 2 d^2 x^2) \operatorname{Cosh}[2 c] - 3 (1 + 2 d^2 x^2) \operatorname{Sinh}[2 c] \right) \operatorname{Sinh}[2 d x]}{d^4} \right) (b + a \operatorname{Sinh}[c + d x]) +$$

$$\left(e^3 \operatorname{Csch}[c + d x] (b + a \operatorname{Sinh}[c + d x]) \left((a^2 + 4 b^2) (c + d x) - \frac{2 b (3 a^2 + 4 b^2) \operatorname{ArcTan}\left[\frac{a - b \operatorname{Tanh}\left[\frac{1}{2} (c + d x)\right]}{\sqrt{-a^2 - b^2}}\right]}{\sqrt{-a^2 - b^2}} - 4 a b \operatorname{Cosh}[c + d x] + a^2 \operatorname{Sinh}[2 (c + d x)] \right) \right) /$$

(4

a³

d

(a + b Csch[c + d x])) +

$$\left(\begin{aligned} & 3 e^2 f \operatorname{Csch}[c + d x] (b + a \operatorname{Sinh}[c + d x]) \left((a^2 + 4 b^2) (-c + d x) (c + d x) - 8 a b d x \operatorname{Cosh}[c + d x] - \right. \\ & a^2 \operatorname{Cosh}[2(c + d x)] - 4 b (3 a^2 + 4 b^2) \left(-\frac{c \operatorname{ArcTan}\left[\frac{b + a e^{c+dx}}{\sqrt{-a^2 - b^2}}\right]}{\sqrt{-a^2 - b^2}} + \frac{1}{2 \sqrt{a^2 + b^2}} \right. \\ & \left. \left. \left((c + d x) \left(\operatorname{Log}\left[1 + \frac{a e^{c+dx}}{b - \sqrt{a^2 + b^2}}\right] - \operatorname{Log}\left[1 + \frac{a e^{c+dx}}{b + \sqrt{a^2 + b^2}}\right] \right) + \operatorname{PolyLog}\left[2, \frac{a e^{c+dx}}{-b + \sqrt{a^2 + b^2}}\right] - \operatorname{PolyLog}\left[2, -\frac{a e^{c+dx}}{b + \sqrt{a^2 + b^2}}\right] \right) \right) + \right. \\ & \left. \left. 8 a b \operatorname{Sinh}[c + d x] + 2 a^2 d x \operatorname{Sinh}[2(c + d x)] \right) \right) / (8 a^3 d^2 (a + b \operatorname{Csch}[c + d x])) \end{aligned} \right)$$

Problem 23: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(e + f x)^2 \operatorname{Cosh}[c + d x]^2}{a + b \operatorname{Csch}[c + d x]} dx$$

Optimal (type 4, 510 leaves, 21 steps):

$$\begin{aligned} & \frac{f^2 x}{4 a d^2} + \frac{(e + f x)^3}{6 a f} + \frac{b^2 (e + f x)^3}{3 a^3 f} - \frac{2 b f^2 \operatorname{Cosh}[c + d x]}{a^2 d^3} - \frac{b (e + f x)^2 \operatorname{Cosh}[c + d x]}{a^2 d} - \frac{f (e + f x) \operatorname{Cosh}[c + d x]^2}{2 a d^2} - \\ & \frac{b \sqrt{a^2 + b^2} (e + f x)^2 \operatorname{Log}\left[1 + \frac{a e^{c+dx}}{b - \sqrt{a^2 + b^2}}\right]}{a^3 d} + \frac{b \sqrt{a^2 + b^2} (e + f x)^2 \operatorname{Log}\left[1 + \frac{a e^{c+dx}}{b + \sqrt{a^2 + b^2}}\right]}{a^3 d} - \frac{2 b \sqrt{a^2 + b^2} f (e + f x) \operatorname{PolyLog}\left[2, -\frac{a e^{c+dx}}{b - \sqrt{a^2 + b^2}}\right]}{a^3 d^2} + \\ & \frac{2 b \sqrt{a^2 + b^2} f (e + f x) \operatorname{PolyLog}\left[2, -\frac{a e^{c+dx}}{b + \sqrt{a^2 + b^2}}\right]}{a^3 d^2} + \frac{2 b \sqrt{a^2 + b^2} f^2 \operatorname{PolyLog}\left[3, -\frac{a e^{c+dx}}{b - \sqrt{a^2 + b^2}}\right]}{a^3 d^3} - \frac{2 b \sqrt{a^2 + b^2} f^2 \operatorname{PolyLog}\left[3, -\frac{a e^{c+dx}}{b + \sqrt{a^2 + b^2}}\right]}{a^3 d^3} + \\ & \frac{2 b f (e + f x) \operatorname{Sinh}[c + d x]}{a^2 d^2} + \frac{f^2 \operatorname{Cosh}[c + d x] \operatorname{Sinh}[c + d x]}{4 a d^3} + \frac{(e + f x)^2 \operatorname{Cosh}[c + d x] \operatorname{Sinh}[c + d x]}{2 a d} \end{aligned}$$

Result (type 4, 2497 leaves):

$$\begin{aligned}
& \frac{e^2 \left(\frac{c}{d} + x - \frac{2 b \operatorname{ArcTan} \left[\frac{a-b \operatorname{Tanh} \left[\frac{1}{2} (c+dx) \right]}{\sqrt{-a^2-b^2}} \right]}{\sqrt{-a^2-b^2} d} \right) \operatorname{Csch} [c+dx] (b+a \operatorname{Sinh} [c+dx])}{4 a (a+b \operatorname{Csch} [c+dx])} + \frac{1}{4 a (a+b \operatorname{Csch} [c+dx])} e f \operatorname{Csch} [c+dx] \\
& \left(x^2 + \frac{1}{d^2} 2 b \left(\frac{i \pi \operatorname{ArcTanh} \left[\frac{-a-b \operatorname{Tanh} \left[\frac{1}{2} (c+dx) \right]}{\sqrt{a^2+b^2}} \right]}{\sqrt{a^2+b^2}} + \frac{1}{\sqrt{-a^2-b^2}} \left(2 \left(c+i \operatorname{ArcCos} \left[-\frac{i b}{a} \right] \right) \operatorname{ArcTan} \left[\frac{(a-i b) \operatorname{Cot} \left[\frac{1}{4} (2 i c+\pi+2 i d x) \right]}{\sqrt{-a^2-b^2}} \right]} + (-2 i c+\pi-2 \right. \right. \\
& \quad \left. \left. i d x) \operatorname{ArcTanh} \left[\frac{(-i a+b) \operatorname{Tan} \left[\frac{1}{4} (2 i c+\pi+2 i d x) \right]}{\sqrt{-a^2-b^2}} \right] - \left(\operatorname{ArcCos} \left[-\frac{i b}{a} \right] - 2 \operatorname{ArcTan} \left[\frac{(a-i b) \operatorname{Cot} \left[\frac{1}{4} (2 i c+\pi+2 i d x) \right]}{\sqrt{-a^2-b^2}} \right]} \right) \right) \\
& \quad \operatorname{Log} \left[\frac{(a+i b) (a-i b+\sqrt{-a^2-b^2}) (1+i \operatorname{Cot} \left[\frac{1}{4} (2 i c+\pi+2 i d x) \right])}{a (a+i b+i \sqrt{-a^2-b^2} \operatorname{Cot} \left[\frac{1}{4} (2 i c+\pi+2 i d x) \right])} \right] - \left(\operatorname{ArcCos} \left[-\frac{i b}{a} \right] + 2 \right. \\
& \quad \left. \operatorname{ArcTan} \left[\frac{(a-i b) \operatorname{Cot} \left[\frac{1}{4} (2 i c+\pi+2 i d x) \right]}{\sqrt{-a^2-b^2}} \right] \right) \operatorname{Log} \left[\frac{i (a+i b) (-a+i b+\sqrt{-a^2-b^2}) (i+\operatorname{Cot} \left[\frac{1}{4} (2 i c+\pi+2 i d x) \right])}{a (a+i b+i \sqrt{-a^2-b^2} \operatorname{Cot} \left[\frac{1}{4} (2 i c+\pi+2 i d x) \right])} \right] + \\
& \quad \left(\operatorname{ArcCos} \left[-\frac{i b}{a} \right] + 2 \operatorname{ArcTan} \left[\frac{(a-i b) \operatorname{Cot} \left[\frac{1}{4} (2 i c+\pi+2 i d x) \right]}{\sqrt{-a^2-b^2}} \right] - 2 i \operatorname{ArcTanh} \left[\frac{(-i a+b) \operatorname{Tan} \left[\frac{1}{4} (2 i c+\pi+2 i d x) \right]}{\sqrt{-a^2-b^2}} \right] \right) \\
& \quad \operatorname{Log} \left[\frac{\sqrt{-a^2-b^2} e^{\frac{1}{4} (-2 c-i \pi-2 d x)}}{\sqrt{2} \sqrt{-i a} \sqrt{b+a \operatorname{Sinh} [c+dx]}} \right] + \\
& \quad \left(\operatorname{ArcCos} \left[-\frac{i b}{a} \right] - 2 \operatorname{ArcTan} \left[\frac{(a-i b) \operatorname{Cot} \left[\frac{1}{4} (2 i c+\pi+2 i d x) \right]}{\sqrt{-a^2-b^2}} \right] + 2 i \operatorname{ArcTanh} \left[\frac{(-i a+b) \operatorname{Tan} \left[\frac{1}{4} (2 i c+\pi+2 i d x) \right]}{\sqrt{-a^2-b^2}} \right] \right) \\
& \quad \operatorname{Log} \left[\frac{\sqrt{-a^2-b^2} e^{\frac{1}{4} (2 c+i \pi+2 d x)}}{\sqrt{2} \sqrt{-i a} \sqrt{b+a \operatorname{Sinh} [c+dx]}} \right] + i \left(\operatorname{PolyLog} \left[2, \frac{(i b+\sqrt{-a^2-b^2}) (a+i b-i \sqrt{-a^2-b^2} \operatorname{Cot} \left[\frac{1}{4} (2 i c+\pi+2 i d x) \right])}{a (a+i b+i \sqrt{-a^2-b^2} \operatorname{Cot} \left[\frac{1}{4} (2 i c+\pi+2 i d x) \right])} \right] \right) - \\
& \quad \left. \left. \left. \left. \operatorname{PolyLog} \left[2, \frac{(b+i \sqrt{-a^2-b^2}) (i a-b+\sqrt{-a^2-b^2} \operatorname{Cot} \left[\frac{1}{4} (2 i c+\pi+2 i d x) \right])}{a (a+i b+i \sqrt{-a^2-b^2} \operatorname{Cot} \left[\frac{1}{4} (2 i c+\pi+2 i d x) \right])} \right] \right] \right) \right) \right) \right) \\
& (b+a \operatorname{Sinh} [c+dx]) + \frac{1}{12 a (a+b \operatorname{Csch} [c+dx])} f^2 \operatorname{Csch} [c+dx] \left(x^3 - \frac{1}{d^3 \sqrt{(a^2+b^2)} e^{2 c}} \right)
\end{aligned}$$

3

b

 e^c

$$\left(d^2 x^2 \operatorname{Log}\left[1 + \frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2+b^2) e^{2c}}}\right] - d^2 x^2 \operatorname{Log}\left[1 + \frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2+b^2) e^{2c}}}\right] + \right. \\ \left. 2 d x \operatorname{PolyLog}\left[2, -\frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2+b^2) e^{2c}}}\right] - 2 d x \operatorname{PolyLog}\left[2, -\frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2+b^2) e^{2c}}}\right] - \right. \\ \left. 2 \operatorname{PolyLog}\left[3, -\frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2+b^2) e^{2c}}}\right] + 2 \operatorname{PolyLog}\left[3, -\frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2+b^2) e^{2c}}}\right] \right)$$

$$(b + a \operatorname{Sinh}[c + dx]) + \frac{1}{24 a^3 (a + b \operatorname{Csch}[c + dx])}$$

 f^2

Csch[

c +

$$dx] \left(2 (a^2 + 4 b^2) x^3 - \right.$$

$$\left. \frac{1}{d^3 \sqrt{(a^2 + b^2) e^{2c}}} \right)$$

$$6 b (3 a^2 + 4 b^2) e^c \left(d^2 x^2 \operatorname{Log}\left[1 + \frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2+b^2) e^{2c}}}\right] - d^2 x^2 \operatorname{Log}\left[1 + \frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2+b^2) e^{2c}}}\right] + 2 d x \operatorname{PolyLog}\left[2, -\frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2+b^2) e^{2c}}}\right] - \right.$$

$$\left. 2 d x \operatorname{PolyLog}\left[2, -\frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2+b^2) e^{2c}}}\right] - 2 \operatorname{PolyLog}\left[3, -\frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2+b^2) e^{2c}}}\right] + 2 \operatorname{PolyLog}\left[3, -\frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2+b^2) e^{2c}}}\right] \right) -$$

$$\frac{24 a b \operatorname{Cosh}[dx] \left((2 + d^2 x^2) \operatorname{Cosh}[c] - 2 dx \operatorname{Sinh}[c] \right)}{d^3} + \frac{3 a^2 \operatorname{Cosh}[2 dx] \left(-2 dx \operatorname{Cosh}[2 c] + (1 + 2 d^2 x^2) \operatorname{Sinh}[2 c] \right)}{d^3} -$$

$$\frac{24 a b \left(-2 dx \operatorname{Cosh}[c] + (2 + d^2 x^2) \operatorname{Sinh}[c] \right) \operatorname{Sinh}[dx]}{d^3} +$$

$$\left. \frac{3 a^2 \left((1 + 2 d^2 x^2) \operatorname{Cosh}[2 c] - 2 dx \operatorname{Sinh}[2 c] \right) \operatorname{Sinh}[2 dx]}{d^3} \right) (b +$$

$$a \operatorname{Sinh}[c + dx]) +$$

$$\begin{aligned}
& \left(e^2 \operatorname{Csch}[c + dx] (b + a \operatorname{Sinh}[c + dx]) \left((a^2 + 4b^2)(c + dx) - \frac{2b(3a^2 + 4b^2) \operatorname{ArcTan}\left[\frac{a - b \operatorname{Tanh}\left[\frac{1}{2}(c + dx)\right]}{\sqrt{-a^2 - b^2}}\right]}{\sqrt{-a^2 - b^2}} - \right. \right. \\
& \left. \left. 4ab \operatorname{Cosh}[c + dx] + a^2 \operatorname{Sinh}[2(c + dx)] \right) \right) / (4a^3 d (a + b \operatorname{Csch}[c + dx])) + \\
& \left(e f \operatorname{Csch}[c + dx] (b + a \operatorname{Sinh}[c + dx]) \left((a^2 + 4b^2)(-c + dx)(c + dx) - 8abd x \operatorname{Cosh}[c + dx] - \right. \right. \\
& \left. \left. a^2 \operatorname{Cosh}[2(c + dx)] - 4b(3a^2 + 4b^2) \left(-\frac{c \operatorname{ArcTan}\left[\frac{b + a e^{c+dx}}{\sqrt{-a^2 - b^2}}\right]}{\sqrt{-a^2 - b^2}} + \frac{1}{2\sqrt{a^2 + b^2}} \right) \right) \right. \\
& \left. \left((c + dx) \left(\operatorname{Log}\left[1 + \frac{a e^{c+dx}}{b - \sqrt{a^2 + b^2}}\right] - \operatorname{Log}\left[1 + \frac{a e^{c+dx}}{b + \sqrt{a^2 + b^2}}\right] \right) + \operatorname{PolyLog}\left[2, -\frac{a e^{c+dx}}{-b + \sqrt{a^2 + b^2}}\right] - \operatorname{PolyLog}\left[2, -\frac{a e^{c+dx}}{b + \sqrt{a^2 + b^2}}\right] \right) \right) + \\
& \left. \left. 8ab \operatorname{Sinh}[c + dx] + 2a^2 dx \operatorname{Sinh}[2(c + dx)] \right) \right) / (4a^3 d^2 (a + b \operatorname{Csch}[c + dx]))
\end{aligned}$$

Problem 24: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(e + fx) \operatorname{Cosh}[c + dx]^2}{a + b \operatorname{Csch}[c + dx]} dx$$

Optimal (type 4, 327 leaves, 16 steps):

$$\frac{e x}{2 a} + \frac{b^2 e x}{a^3} + \frac{f x^2}{4 a} + \frac{b^2 f x^2}{2 a^3} - \frac{b (e + f x) \operatorname{Cosh}[c + d x]}{a^2 d} - \frac{f \operatorname{Cosh}[c + d x]^2}{4 a d^2} -$$

$$\frac{b \sqrt{a^2 + b^2} (e + f x) \operatorname{Log}\left[1 + \frac{a e^{c+d x}}{b \sqrt{a^2 + b^2}}\right]}{a^3 d} + \frac{b \sqrt{a^2 + b^2} (e + f x) \operatorname{Log}\left[1 + \frac{a e^{c+d x}}{b \sqrt{a^2 + b^2}}\right]}{a^3 d} - \frac{b \sqrt{a^2 + b^2} f \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x}}{b \sqrt{a^2 + b^2}}\right]}{a^3 d^2} +$$

$$\frac{b \sqrt{a^2 + b^2} f \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x}}{b \sqrt{a^2 + b^2}}\right]}{a^3 d^2} + \frac{b f \operatorname{Sinh}[c + d x]}{a^2 d^2} + \frac{(e + f x) \operatorname{Cosh}[c + d x] \operatorname{Sinh}[c + d x]}{2 a d}$$

Result (type 4, 1663 leaves):

$$e^{\left(\frac{c}{d} + x - \frac{2 b \operatorname{ArcTan}\left[\frac{a - b \operatorname{Tanh}\left[\frac{1}{2}(c + d x)\right]}{\sqrt{-a^2 - b^2}}\right]}{\sqrt{-a^2 - b^2} d}\right)} \operatorname{Csch}[c + d x] (b + a \operatorname{Sinh}[c + d x])$$

$$\frac{1}{4 a (a + b \operatorname{Csch}[c + d x])} + \frac{1}{8 a (a + b \operatorname{Csch}[c + d x])} f \operatorname{Csch}[c + d x]$$

$$\left(x^2 + \frac{1}{d^2} 2 b \left(\frac{i \pi \operatorname{ArcTanh}\left[\frac{-a + b \operatorname{Tanh}\left[\frac{1}{2}(c + d x)\right]}{\sqrt{a^2 + b^2}}\right]}{\sqrt{a^2 + b^2}} + \frac{1}{\sqrt{-a^2 - b^2}} \left(2 \left(c + i \operatorname{ArcCos}\left[-\frac{i b}{a}\right]\right) \operatorname{ArcTan}\left[\frac{(a - i b) \operatorname{Cot}\left[\frac{1}{4}(2 i c + \pi + 2 i d x)\right]}{\sqrt{-a^2 - b^2}}\right]} + (-2 i c + \pi - 2\right.\right.\right.$$

$$\left.\left.\left. i d x\right) \operatorname{ArcTanh}\left[\frac{(-i a + b) \operatorname{Tan}\left[\frac{1}{4}(2 i c + \pi + 2 i d x)\right]}{\sqrt{-a^2 - b^2}}\right] - \left(\operatorname{ArcCos}\left[-\frac{i b}{a}\right] - 2 \operatorname{ArcTan}\left[\frac{(a - i b) \operatorname{Cot}\left[\frac{1}{4}(2 i c + \pi + 2 i d x)\right]}{\sqrt{-a^2 - b^2}}\right]\right)\right)\right]$$

$$\operatorname{Log}\left[\frac{(a + i b) (a - i b + \sqrt{-a^2 - b^2}) \left(1 + i \operatorname{Cot}\left[\frac{1}{4}(2 i c + \pi + 2 i d x)\right]\right)}{a (a + i b + i \sqrt{-a^2 - b^2} \operatorname{Cot}\left[\frac{1}{4}(2 i c + \pi + 2 i d x)\right])}\right] - \left(\operatorname{ArcCos}\left[-\frac{i b}{a}\right] + 2\right.$$

$$\left.\operatorname{ArcTan}\left[\frac{(a - i b) \operatorname{Cot}\left[\frac{1}{4}(2 i c + \pi + 2 i d x)\right]}{\sqrt{-a^2 - b^2}}\right]\right) \operatorname{Log}\left[\frac{i (a + i b) (-a + i b + \sqrt{-a^2 - b^2}) \left(i + \operatorname{Cot}\left[\frac{1}{4}(2 i c + \pi + 2 i d x)\right]\right)}{a (a + i b + i \sqrt{-a^2 - b^2} \operatorname{Cot}\left[\frac{1}{4}(2 i c + \pi + 2 i d x)\right])}\right] +$$

$$\left(\operatorname{ArcCos}\left[-\frac{i b}{a}\right] + 2 \operatorname{ArcTan}\left[\frac{(a - i b) \operatorname{Cot}\left[\frac{1}{4}(2 i c + \pi + 2 i d x)\right]}{\sqrt{-a^2 - b^2}}\right] - 2 i \operatorname{ArcTanh}\left[\frac{(-i a + b) \operatorname{Tan}\left[\frac{1}{4}(2 i c + \pi + 2 i d x)\right]}{\sqrt{-a^2 - b^2}}\right]\right)$$

$$\operatorname{Log}\left[\frac{\sqrt{-a^2 - b^2} e^{\frac{1}{4}(-2 c - i \pi - 2 d x)}}{\sqrt{2} \sqrt{-i a} \sqrt{b + a \operatorname{Sinh}[c + d x]}}\right] +$$

$$\left(\operatorname{ArcCos}\left[-\frac{i b}{a}\right] - 2 \operatorname{ArcTan}\left[\frac{(a - i b) \operatorname{Cot}\left[\frac{1}{4}(2 i c + \pi + 2 i d x)\right]}{\sqrt{-a^2 - b^2}}\right] + 2 i \operatorname{ArcTanh}\left[\frac{(-i a + b) \operatorname{Tan}\left[\frac{1}{4}(2 i c + \pi + 2 i d x)\right]}{\sqrt{-a^2 - b^2}}\right]\right)$$

$$\begin{aligned}
& \left. \left(\text{Log} \left[\frac{\sqrt{-a^2 - b^2} e^{\frac{1}{4}(2c + i\pi + 2dx)}}{\sqrt{2} \sqrt{-ia} \sqrt{b + a \text{Sinh}[c + dx]}} \right] + i \left(\text{PolyLog} \left[2, \frac{(ib + \sqrt{-a^2 - b^2}) (a + ib - i\sqrt{-a^2 - b^2} \text{Cot}[\frac{1}{4}(2ic + \pi + 2id x)])}{a (a + ib + i\sqrt{-a^2 - b^2} \text{Cot}[\frac{1}{4}(2ic + \pi + 2id x)])} \right] \right) - \right. \\
& \left. \left. \left. \left. \left. \text{PolyLog} \left[2, \frac{(b + i\sqrt{-a^2 - b^2}) (ia - b + \sqrt{-a^2 - b^2} \text{Cot}[\frac{1}{4}(2ic + \pi + 2id x)])}{a (a + ib + i\sqrt{-a^2 - b^2} \text{Cot}[\frac{1}{4}(2ic + \pi + 2id x)])} \right] \right] \right) \right) \right) \right) \right) (b + a \text{Sinh}[c + dx]) + \\
& \left(e \text{Csch}[c + dx] (b + a \text{Sinh}[c + dx]) \left((a^2 + 4b^2) (c + dx) - \frac{2b(3a^2 + 4b^2) \text{ArcTan} \left[\frac{a - b \text{Tanh}[\frac{1}{2}(c + dx)]}{\sqrt{-a^2 - b^2}} \right]}{\sqrt{-a^2 - b^2}} - \right. \right. \\
& \left. \left. 4ab \text{Cosh}[c + dx] + \right. \right. \\
& \left. \left. \left. \left. \left. a^2 \text{Sinh}[2(c + dx)] \right] \right) \right) \right) / \\
& (4a^3 d (a + b \text{Csch}[c + dx])) + \left(f \text{Csch}[c + dx] (b + a \text{Sinh}[c + dx]) \right) \\
& \left((a^2 + 4b^2) (-c + dx) (c + dx) - \right. \\
& 8abd x \text{Cosh}[c + dx] - a^2 \text{Cosh}[2(c + dx)] - \\
& 4b(3a^2 + 4b^2) \left(-\frac{c \text{ArcTan} \left[\frac{b + a e^{c + dx}}{\sqrt{-a^2 - b^2}} \right]}{\sqrt{-a^2 - b^2}} + \frac{1}{2\sqrt{a^2 + b^2}} \right) \\
& \left. \left. \left. \left. \left. \left((c + dx) \left(\text{Log} \left[1 + \frac{a e^{c + dx}}{b - \sqrt{a^2 + b^2}} \right] - \text{Log} \left[1 + \frac{a e^{c + dx}}{b + \sqrt{a^2 + b^2}} \right] \right) + \text{PolyLog} \left[2, \frac{a e^{c + dx}}{-b + \sqrt{a^2 + b^2}} \right] - \text{PolyLog} \left[2, -\frac{a e^{c + dx}}{b + \sqrt{a^2 + b^2}} \right] \right) \right) \right) \right) \right) + \right.
\end{aligned}$$

$$\left. \left. \left. \left. 8 a b \operatorname{Sinh}[c+d x]+2 a^2 d x \operatorname{Sinh}[2(c+d x)] \right) \right) \right) \left/ \left(8 a^3 d^2(a+b \operatorname{Csch}[c+d x]) \right) \right)$$

Problem 26: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(e+f x)^3 \operatorname{Cosh}[c+d x]^3}{a+b \operatorname{Csch}[c+d x]} d x$$

Optimal (type 4, 864 leaves, 31 steps):

$$\begin{aligned} & -\frac{3 b f^3 x}{8 a^2 d^3}-\frac{b(e+f x)^3}{4 a^2 d}+\frac{b\left(a^2+b^2\right)(e+f x)^4}{4 a^4 f}-\frac{40 f^3 \operatorname{Cosh}[c+d x]}{9 a d^4}-\frac{6 b^2 f^3 \operatorname{Cosh}[c+d x]}{a^3 d^4}-\frac{2 f(e+f x)^2 \operatorname{Cosh}[c+d x]}{a d^2} \\ & -\frac{3 b^2 f(e+f x)^2 \operatorname{Cosh}[c+d x]}{a^3 d^2}-\frac{2 f^3 \operatorname{Cosh}[c+d x]^3}{27 a d^4}-\frac{f(e+f x)^2 \operatorname{Cosh}[c+d x]^3}{3 a d^2}-\frac{b\left(a^2+b^2\right)(e+f x)^3 \operatorname{Log}\left[1+\frac{a e^{c+d x}}{b-\sqrt{a^2+b^2}}\right]}{a^4 d} \\ & -\frac{b\left(a^2+b^2\right)(e+f x)^3 \operatorname{Log}\left[1+\frac{a e^{c+d x}}{b+\sqrt{a^2+b^2}}\right]}{a^4 d}-\frac{3 b\left(a^2+b^2\right) f(e+f x)^2 \operatorname{PolyLog}\left[2,-\frac{a e^{c+d x}}{b-\sqrt{a^2+b^2}}\right]}{a^4 d^2}-\frac{3 b\left(a^2+b^2\right) f(e+f x)^2 \operatorname{PolyLog}\left[2,-\frac{a e^{c+d x}}{b+\sqrt{a^2+b^2}}\right]}{a^4 d^2} \\ & +\frac{6 b\left(a^2+b^2\right) f^2(e+f x) \operatorname{PolyLog}\left[3,-\frac{a e^{c+d x}}{b-\sqrt{a^2+b^2}}\right]}{a^4 d^3}+\frac{6 b\left(a^2+b^2\right) f^2(e+f x) \operatorname{PolyLog}\left[3,-\frac{a e^{c+d x}}{b+\sqrt{a^2+b^2}}\right]}{a^4 d^3}-\frac{6 b\left(a^2+b^2\right) f^3 \operatorname{PolyLog}\left[4,-\frac{a e^{c+d x}}{b-\sqrt{a^2+b^2}}\right]}{a^4 d^4} \\ & -\frac{6 b\left(a^2+b^2\right) f^3 \operatorname{PolyLog}\left[4,-\frac{a e^{c+d x}}{b+\sqrt{a^2+b^2}}\right]}{a^4 d^4}+\frac{40 f^2(e+f x) \operatorname{Sinh}[c+d x]}{9 a d^3}+\frac{6 b^2 f^2(e+f x) \operatorname{Sinh}[c+d x]}{a^3 d^3}+\frac{2(e+f x)^3 \operatorname{Sinh}[c+d x]}{3 a d} \\ & +\frac{b^2(e+f x)^3 \operatorname{Sinh}[c+d x]}{a^3 d}+\frac{3 b f^3 \operatorname{Cosh}[c+d x] \operatorname{Sinh}[c+d x]}{8 a^2 d^4}+\frac{3 b f(e+f x)^2 \operatorname{Cosh}[c+d x] \operatorname{Sinh}[c+d x]}{4 a^2 d^2} \\ & +\frac{2 f^2(e+f x) \operatorname{Cosh}[c+d x]^2 \operatorname{Sinh}[c+d x]}{9 a d^3}+\frac{(e+f x)^3 \operatorname{Cosh}[c+d x]^2 \operatorname{Sinh}[c+d x]}{3 a d}-\frac{3 b f^2(e+f x) \operatorname{Sinh}[c+d x]^2}{4 a^2 d^3}-\frac{b(e+f x)^3 \operatorname{Sinh}[c+d x]^2}{2 a^2 d} \end{aligned}$$

Result (type 4, 5945 leaves):

$$\begin{aligned} & \frac{1}{4 a^2 d^3(a+b \operatorname{Csch}[c+d x])} e^f \operatorname{Csch}[c+d x] \left(-12 b d x \operatorname{PolyLog}\left[2,-\frac{a e^{2 c+d x}}{b e^c-\sqrt{\left(a^2+b^2\right) e^{2 c}}}\right]-12 b d x \operatorname{PolyLog}\left[2,-\frac{a e^{2 c+d x}}{b e^c+\sqrt{\left(a^2+b^2\right) e^{2 c}}}\right] \right) + \\ & e^{-c} \left(2 b d^3 e^c x^3-6 a \operatorname{Cosh}[d x]+6 a e^{2 c} \operatorname{Cosh}[d x]-6 a d x \operatorname{Cosh}[d x]-6 a d e^{2 c} x \operatorname{Cosh}[d x]-3 a d^2 x^2 \operatorname{Cosh}[d x]+ \right. \end{aligned}$$

$$\begin{aligned}
& 3 a d^2 e^{2c} x^2 \operatorname{Cosh}[d x] - 6 b d^2 e^c x^2 \operatorname{Log}\left[1 + \frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2 + b^2) e^{2c}}}\right] - 6 b d^2 e^c x^2 \operatorname{Log}\left[1 + \frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2 + b^2) e^{2c}}}\right] + \\
& 12 b e^c \operatorname{PolyLog}\left[3, -\frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2 + b^2) e^{2c}}}\right] + 12 b e^c \operatorname{PolyLog}\left[3, -\frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2 + b^2) e^{2c}}}\right] + 6 a \operatorname{Sinh}[d x] + 6 a e^{2c} \operatorname{Sinh}[d x] + \\
& 6 a d x \operatorname{Sinh}[d x] - 6 a d e^{2c} x \operatorname{Sinh}[d x] + 3 a d^2 x^2 \operatorname{Sinh}[d x] + 3 a d^2 e^{2c} x^2 \operatorname{Sinh}[d x] \Bigg) (b + a \operatorname{Sinh}[c + d x]) + \\
& \frac{1}{8 a^2 d^4 (a + b \operatorname{Csch}[c + d x])} f^3 \operatorname{Csch}[c + d x] \left(-12 b d^2 x^2 \operatorname{PolyLog}\left[2, -\frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2 + b^2) e^{2c}}}\right] + \right. \\
& e^{-c} \left(b d^4 e^c x^4 - 12 a \operatorname{Cosh}[d x] - 12 a e^{2c} \operatorname{Cosh}[d x] - 12 a d x \operatorname{Cosh}[d x] + 12 a d e^{2c} x \operatorname{Cosh}[d x] - 6 a d^2 x^2 \operatorname{Cosh}[d x] - 6 a d^2 e^{2c} x^2 \operatorname{Cosh}[d x] - \right. \\
& 2 a d^3 x^3 \operatorname{Cosh}[d x] + 2 a d^3 e^{2c} x^3 \operatorname{Cosh}[d x] - 4 b d^3 e^c x^3 \operatorname{Log}\left[1 + \frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2 + b^2) e^{2c}}}\right] - 4 b d^3 e^c x^3 \operatorname{Log}\left[1 + \frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2 + b^2) e^{2c}}}\right] - \\
& 12 b d^2 e^c x^2 \operatorname{PolyLog}\left[2, -\frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2 + b^2) e^{2c}}}\right] + 24 b d e^c x \operatorname{PolyLog}\left[3, -\frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2 + b^2) e^{2c}}}\right] + \\
& 24 b d e^c x \operatorname{PolyLog}\left[3, -\frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2 + b^2) e^{2c}}}\right] - 24 b e^c \operatorname{PolyLog}\left[4, -\frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2 + b^2) e^{2c}}}\right] - \\
& 24 b e^c \operatorname{PolyLog}\left[4, -\frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2 + b^2) e^{2c}}}\right] + 12 a \operatorname{Sinh}[d x] - 12 a e^{2c} \operatorname{Sinh}[d x] + 12 a d x \operatorname{Sinh}[d x] + 12 a d e^{2c} x \operatorname{Sinh}[d x] + \\
& 6 a d^2 x^2 \operatorname{Sinh}[d x] - 6 a d^2 e^{2c} x^2 \operatorname{Sinh}[d x] + 2 a d^3 x^3 \operatorname{Sinh}[d x] + 2 a d^3 e^{2c} x^3 \operatorname{Sinh}[d x] \Bigg) (b + a \operatorname{Sinh}[c + d x]) + \\
& \frac{1}{144 a^4 d^3 (a + b \operatorname{Csch}[c + d x])} e^{-3c} f^2 \operatorname{Csch}[c + d x] \left(72 a^2 b d^3 e^{3c} x^3 + 144 b^3 d^3 e^{3c} x^3 - 108 a^3 e^{2c} \operatorname{Cosh}[d x] - 432 a b^2 e^{2c} \operatorname{Cosh}[d x] + \right. \\
& 108 a^3 e^{4c} \operatorname{Cosh}[d x] + 432 a b^2 e^{4c} \operatorname{Cosh}[d x] - 108 a^3 d e^{2c} x \operatorname{Cosh}[d x] - 432 a b^2 d e^{2c} x \operatorname{Cosh}[d x] - \\
& 108 a^3 d e^{4c} x \operatorname{Cosh}[d x] - 432 a b^2 d e^{4c} x \operatorname{Cosh}[d x] - 54 a^3 d^2 e^{2c} x^2 \operatorname{Cosh}[d x] - 216 a b^2 d^2 e^{2c} x^2 \operatorname{Cosh}[d x] + \\
& 54 a^3 d^2 e^{4c} x^2 \operatorname{Cosh}[d x] + 216 a b^2 d^2 e^{4c} x^2 \operatorname{Cosh}[d x] - 27 a^2 b e^c \operatorname{Cosh}[2 d x] - 27 a^2 b e^{5c} \operatorname{Cosh}[2 d x] - \\
& 54 a^2 b d e^c x \operatorname{Cosh}[2 d x] + 54 a^2 b d e^{5c} x \operatorname{Cosh}[2 d x] - 54 a^2 b d^2 e^c x^2 \operatorname{Cosh}[2 d x] - 54 a^2 b d^2 e^{5c} x^2 \operatorname{Cosh}[2 d x] - \\
& 4 a^3 \operatorname{Cosh}[3 d x] + 4 a^3 e^{6c} \operatorname{Cosh}[3 d x] - 12 a^3 d x \operatorname{Cosh}[3 d x] - 12 a^3 d e^{6c} x \operatorname{Cosh}[3 d x] - 18 a^3 d^2 x^2 \operatorname{Cosh}[3 d x] +
\end{aligned}$$

$$\begin{aligned}
& 18 a^3 d^2 e^{6c} x^2 \operatorname{Cosh}[3 dx] - 216 a^2 b d^2 e^{3c} x^2 \operatorname{Log}\left[1 + \frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2 + b^2) e^{2c}}}\right] - 432 b^3 d^2 e^{3c} x^2 \operatorname{Log}\left[1 + \frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2 + b^2) e^{2c}}}\right] - \\
& 216 a^2 b d^2 e^{3c} x^2 \operatorname{Log}\left[1 + \frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2 + b^2) e^{2c}}}\right] - 432 b^3 d^2 e^{3c} x^2 \operatorname{Log}\left[1 + \frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2 + b^2) e^{2c}}}\right] - \\
& 432 b (a^2 + 2 b^2) d e^{3c} x \operatorname{PolyLog}\left[2, -\frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2 + b^2) e^{2c}}}\right] - 432 b (a^2 + 2 b^2) d e^{3c} x \operatorname{PolyLog}\left[2, -\frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2 + b^2) e^{2c}}}\right] + \\
& 432 a^2 b e^{3c} \operatorname{PolyLog}\left[3, -\frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2 + b^2) e^{2c}}}\right] + 864 b^3 e^{3c} \operatorname{PolyLog}\left[3, -\frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2 + b^2) e^{2c}}}\right] + \\
& 432 a^2 b e^{3c} \operatorname{PolyLog}\left[3, -\frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2 + b^2) e^{2c}}}\right] + 864 b^3 e^{3c} \operatorname{PolyLog}\left[3, -\frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2 + b^2) e^{2c}}}\right] + 108 a^3 e^{2c} \operatorname{Sinh}[dx] + \\
& 432 a b^2 e^{2c} \operatorname{Sinh}[dx] + 108 a^3 e^{4c} \operatorname{Sinh}[dx] + 432 a b^2 e^{4c} \operatorname{Sinh}[dx] + 108 a^3 d e^{2c} x \operatorname{Sinh}[dx] + 432 a b^2 d e^{2c} x \operatorname{Sinh}[dx] - \\
& 108 a^3 d e^{4c} x \operatorname{Sinh}[dx] - 432 a b^2 d e^{4c} x \operatorname{Sinh}[dx] + 54 a^3 d^2 e^{2c} x^2 \operatorname{Sinh}[dx] + 216 a b^2 d^2 e^{2c} x^2 \operatorname{Sinh}[dx] + \\
& 54 a^3 d^2 e^{4c} x^2 \operatorname{Sinh}[dx] + 216 a b^2 d^2 e^{4c} x^2 \operatorname{Sinh}[dx] + 27 a^2 b e^c \operatorname{Sinh}[2 dx] - 27 a^2 b e^{5c} \operatorname{Sinh}[2 dx] + 54 a^2 b d e^c x \operatorname{Sinh}[2 dx] + \\
& 54 a^2 b d e^{5c} x \operatorname{Sinh}[2 dx] + 54 a^2 b d^2 e^c x^2 \operatorname{Sinh}[2 dx] - 54 a^2 b d^2 e^{5c} x^2 \operatorname{Sinh}[2 dx] + 4 a^3 \operatorname{Sinh}[3 dx] + 4 a^3 e^{6c} \operatorname{Sinh}[3 dx] + \\
& 12 a^3 d x \operatorname{Sinh}[3 dx] - 12 a^3 d e^{6c} x \operatorname{Sinh}[3 dx] + 18 a^3 d^2 x^2 \operatorname{Sinh}[3 dx] + 18 a^3 d^2 e^{6c} x^2 \operatorname{Sinh}[3 dx] \Big) (b + a \operatorname{Sinh}[c + dx]) + \\
& \frac{1}{864 a^4 d^4 (a + b \operatorname{Csch}[c + dx])} e^{-3c} f^3 \operatorname{Csch}[c + dx] \left(108 a^2 b d^4 e^{3c} x^4 + 216 b^3 d^4 e^{3c} x^4 - 648 a^3 e^{2c} \operatorname{Cosh}[dx] - 2592 a b^2 e^{2c} \operatorname{Cosh}[dx] - \right. \\
& 648 a^3 e^{4c} \operatorname{Cosh}[dx] - 2592 a b^2 e^{4c} \operatorname{Cosh}[dx] - 648 a^3 d e^{2c} x \operatorname{Cosh}[dx] - 2592 a b^2 d e^{2c} x \operatorname{Cosh}[dx] + 648 a^3 d e^{4c} x \operatorname{Cosh}[dx] + \\
& 2592 a b^2 d e^{4c} x \operatorname{Cosh}[dx] - 324 a^3 d^2 e^{2c} x^2 \operatorname{Cosh}[dx] - 1296 a b^2 d^2 e^{2c} x^2 \operatorname{Cosh}[dx] - 324 a^3 d^2 e^{4c} x^2 \operatorname{Cosh}[dx] - \\
& 1296 a b^2 d^2 e^{4c} x^2 \operatorname{Cosh}[dx] - 108 a^3 d^3 e^{2c} x^3 \operatorname{Cosh}[dx] - 432 a b^2 d^3 e^{2c} x^3 \operatorname{Cosh}[dx] + 108 a^3 d^3 e^{4c} x^3 \operatorname{Cosh}[dx] + 432 a b^2 d^3 e^{4c} x^3 \operatorname{Cosh}[dx] - \\
& 81 a^2 b e^c \operatorname{Cosh}[2 dx] + 81 a^2 b e^{5c} \operatorname{Cosh}[2 dx] - 162 a^2 b d e^c x \operatorname{Cosh}[2 dx] - 162 a^2 b d e^{5c} x \operatorname{Cosh}[2 dx] - 162 a^2 b d^2 e^c x^2 \operatorname{Cosh}[2 dx] + \\
& 162 a^2 b d^2 e^{5c} x^2 \operatorname{Cosh}[2 dx] - 108 a^2 b d^3 e^c x^3 \operatorname{Cosh}[2 dx] - 108 a^2 b d^3 e^{5c} x^3 \operatorname{Cosh}[2 dx] - 8 a^3 \operatorname{Cosh}[3 dx] - 8 a^3 e^{6c} \operatorname{Cosh}[3 dx] - \\
& 24 a^3 d x \operatorname{Cosh}[3 dx] + 24 a^3 d e^{6c} x \operatorname{Cosh}[3 dx] - 36 a^3 d^2 x^2 \operatorname{Cosh}[3 dx] - 36 a^3 d^2 e^{6c} x^2 \operatorname{Cosh}[3 dx] - 36 a^3 d^3 x^3 \operatorname{Cosh}[3 dx] + \\
& 36 a^3 d^3 e^{6c} x^3 \operatorname{Cosh}[3 dx] - 432 a^2 b d^3 e^{3c} x^3 \operatorname{Log}\left[1 + \frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2 + b^2) e^{2c}}}\right] - 864 b^3 d^3 e^{3c} x^3 \operatorname{Log}\left[1 + \frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2 + b^2) e^{2c}}}\right] - \\
& 432 a^2 b d^3 e^{3c} x^3 \operatorname{Log}\left[1 + \frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2 + b^2) e^{2c}}}\right] - 864 b^3 d^3 e^{3c} x^3 \operatorname{Log}\left[1 + \frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2 + b^2) e^{2c}}}\right] - \\
& 1296 b (a^2 + 2 b^2) d^2 e^{3c} x^2 \operatorname{PolyLog}\left[2, -\frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2 + b^2) e^{2c}}}\right] - 1296 b (a^2 + 2 b^2) d^2 e^{3c} x^2 \operatorname{PolyLog}\left[2, -\frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2 + b^2) e^{2c}}}\right] +
\end{aligned}$$

$$\begin{aligned}
& 2592 a^2 b d e^{3c} x \operatorname{PolyLog}\left[3, -\frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2+b^2)} e^{2c}}\right] + 5184 b^3 d e^{3c} x \operatorname{PolyLog}\left[3, -\frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2+b^2)} e^{2c}}\right] + \\
& 2592 a^2 b d e^{3c} x \operatorname{PolyLog}\left[3, -\frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2+b^2)} e^{2c}}\right] + 5184 b^3 d e^{3c} x \operatorname{PolyLog}\left[3, -\frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2+b^2)} e^{2c}}\right] - \\
& 2592 a^2 b e^{3c} \operatorname{PolyLog}\left[4, -\frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2+b^2)} e^{2c}}\right] - 5184 b^3 e^{3c} \operatorname{PolyLog}\left[4, -\frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2+b^2)} e^{2c}}\right] - \\
& 2592 a^2 b e^{3c} \operatorname{PolyLog}\left[4, -\frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2+b^2)} e^{2c}}\right] - 5184 b^3 e^{3c} \operatorname{PolyLog}\left[4, -\frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2+b^2)} e^{2c}}\right] + 648 a^3 e^{2c} \operatorname{Sinh}[dx] + \\
& 2592 a b^2 e^{2c} \operatorname{Sinh}[dx] - 648 a^3 e^{4c} \operatorname{Sinh}[dx] - 2592 a b^2 e^{4c} \operatorname{Sinh}[dx] + 648 a^3 d e^{2c} x \operatorname{Sinh}[dx] + 2592 a b^2 d e^{2c} x \operatorname{Sinh}[dx] + \\
& 648 a^3 d e^{4c} x \operatorname{Sinh}[dx] + 2592 a b^2 d e^{4c} x \operatorname{Sinh}[dx] + 324 a^3 d^2 e^{2c} x^2 \operatorname{Sinh}[dx] + 1296 a b^2 d^2 e^{2c} x^2 \operatorname{Sinh}[dx] - \\
& 324 a^3 d^2 e^{4c} x^2 \operatorname{Sinh}[dx] - 1296 a b^2 d^2 e^{4c} x^2 \operatorname{Sinh}[dx] + 108 a^3 d^3 e^{2c} x^3 \operatorname{Sinh}[dx] + 432 a b^2 d^3 e^{2c} x^3 \operatorname{Sinh}[dx] + \\
& 108 a^3 d^3 e^{4c} x^3 \operatorname{Sinh}[dx] + 432 a b^2 d^3 e^{4c} x^3 \operatorname{Sinh}[dx] + 81 a^2 b e^c \operatorname{Sinh}[2dx] + 81 a^2 b e^{5c} \operatorname{Sinh}[2dx] + 162 a^2 b d e^c x \operatorname{Sinh}[2dx] - \\
& 162 a^2 b d e^{5c} x \operatorname{Sinh}[2dx] + 162 a^2 b d^2 e^c x^2 \operatorname{Sinh}[2dx] + 162 a^2 b d^2 e^{5c} x^2 \operatorname{Sinh}[2dx] + 108 a^2 b d^3 e^c x^3 \operatorname{Sinh}[2dx] - \\
& 108 a^2 b d^3 e^{5c} x^3 \operatorname{Sinh}[2dx] + 8 a^3 \operatorname{Sinh}[3dx] - 8 a^3 e^{6c} \operatorname{Sinh}[3dx] + 24 a^3 d x \operatorname{Sinh}[3dx] + 24 a^3 d e^{6c} x \operatorname{Sinh}[3dx] + \\
& 36 a^3 d^2 x^2 \operatorname{Sinh}[3dx] - 36 a^3 d^2 e^{6c} x^2 \operatorname{Sinh}[3dx] + 36 a^3 d^3 x^3 \operatorname{Sinh}[3dx] + 36 a^3 d^3 e^{6c} x^3 \operatorname{Sinh}[3dx] \Big) (b + a \operatorname{Sinh}[c + dx]) + \\
& \frac{e^3 \operatorname{Csch}[c + dx] (b + a \operatorname{Sinh}[c + dx]) \left(-\frac{2b \operatorname{Log}[b + a \operatorname{Sinh}[c + dx]]}{a^2 d} + \frac{2 \operatorname{Sinh}[c + dx]}{a d} \right)}{4 (a + b \operatorname{Csch}[c + dx])} + \\
& \frac{1}{2 a^2 d^2 (a + b \operatorname{Csch}[c + dx])} \\
& 3 e^2 f \operatorname{Csch}[c + dx] (b + a \operatorname{Sinh}[c + dx]) \\
& \left(-a \operatorname{Cosh}[c + dx] - b (c + dx) \operatorname{Log}[b + a \operatorname{Sinh}[c + dx]] + b c \operatorname{Log}\left[1 + \frac{a \operatorname{Sinh}[c + dx]}{b}\right] \right) + \\
& i b \left(-\frac{1}{8} i (2c + i\pi + 2dx)^2 - 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{ib}{a}}}{\sqrt{2}}\right] \operatorname{ArcTan}\left[\frac{(i a + b) \operatorname{Cot}\left[\frac{1}{4} (2i c + \pi + 2i dx)\right]}{\sqrt{a^2 + b^2}}\right] \right) -
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{2} \left(-2 i c + \pi - 2 i d x + 4 \operatorname{ArcSin} \left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}} \right] \right) \operatorname{Log} \left[1 + \frac{(-b + \sqrt{a^2 + b^2}) e^{c+dx}}{a} \right] - \\
& \frac{1}{2} \left(-2 i c + \pi - 2 i d x - 4 \operatorname{ArcSin} \left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}} \right] \right) \operatorname{Log} \left[1 - \frac{(b + \sqrt{a^2 + b^2}) e^{c+dx}}{a} \right] + \left(\frac{\pi}{2} - i (c + d x) \right) \operatorname{Log} [b + a \operatorname{Sinh} [c + d x]] + \\
& i \left(\operatorname{PolyLog} \left[2, \frac{(b - \sqrt{a^2 + b^2}) e^{c+dx}}{a} \right] + \operatorname{PolyLog} \left[2, \frac{(b + \sqrt{a^2 + b^2}) e^{c+dx}}{a} \right] \right) + a d x \operatorname{Sinh} [c + d x] \Bigg) + \\
& \frac{1}{8 (a + b \operatorname{Csch} [c + d x])} e^3 \operatorname{Csch} [c + d x] (b + a \operatorname{Sinh} [c + d x]) \left(-\frac{2 b \operatorname{Cosh} [2 (c + d x)]}{a^2 d} - \frac{4 (a^2 b + 2 b^3) \operatorname{Log} [b + a \operatorname{Sinh} [c + d x]]}{a^4 d} + \right. \\
& \left. \frac{2 (a^2 + 4 b^2) \operatorname{Sinh} [c + d x]}{a^3 d} + \frac{2 \operatorname{Sinh} [3 (c + d x)]}{3 a d} \right) + \\
& \frac{1}{24 a^4 d^2 (a + b \operatorname{Csch} [c + d x])} e^2 f \operatorname{Csch} [c + d x] (b + a \operatorname{Sinh} [c + d x]) \\
& \left(-18 a (a^2 + 4 b^2) \operatorname{Cosh} [c + d x] - 18 a^2 b d x \operatorname{Cosh} [2 (c + d x)] - 2 a^3 \operatorname{Cosh} [3 (c + d x)] + 36 a^2 b c \operatorname{Log} \left[1 + \frac{a \operatorname{Sinh} [c + d x]}{b} \right] + \right. \\
& \left. 72 b^3 c \operatorname{Log} \left[1 + \frac{a \operatorname{Sinh} [c + d x]}{b} \right] - 36 a^2 b \left(-\frac{1}{8} (2 c + i \pi + 2 d x)^2 - 4 \operatorname{ArcSin} \left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}} \right] \operatorname{ArcTan} \left[\frac{(i a + b) \operatorname{Cot} \left[\frac{1}{4} (2 i c + \pi + 2 i d x) \right]}{\sqrt{a^2 + b^2}} \right] \right) + \right. \\
& \left. \frac{1}{2} \left(2 c + i \pi + 2 d x + 4 i \operatorname{ArcSin} \left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}} \right] \right) \operatorname{Log} \left[1 + \frac{(-b + \sqrt{a^2 + b^2}) e^{c+dx}}{a} \right] + \frac{1}{2} \left(2 c + i \pi + 2 d x - 4 i \operatorname{ArcSin} \left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}} \right] \right) \right)
\end{aligned}$$

$$\left. \begin{aligned} & \left(\operatorname{Log}\left[1 - \frac{(b + \sqrt{a^2 + b^2}) e^{c+dx}}{a}\right] - \frac{1}{2} i \pi \operatorname{Log}[b + a \operatorname{Sinh}[c + dx]] + \operatorname{PolyLog}\left[2, \frac{(b - \sqrt{a^2 + b^2}) e^{c+dx}}{a}\right] + \operatorname{PolyLog}\left[2, \frac{(b + \sqrt{a^2 + b^2}) e^{c+dx}}{a}\right] \right) - \\ & 72 b^3 \left(-\frac{1}{8} (2c + i\pi + 2dx)^2 - 4 \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{ib}{a}}}{\sqrt{2}}\right] \operatorname{ArcTan}\left[\frac{(ia + b) \operatorname{Cot}\left[\frac{1}{4}(2ic + \pi + 2idx)\right]}{\sqrt{a^2 + b^2}}\right] + \frac{1}{2} \left(2c + i\pi + 2dx + 4i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{ib}{a}}}{\sqrt{2}}\right] \right) \right) \\ & \operatorname{Log}\left[1 + \frac{(-b + \sqrt{a^2 + b^2}) e^{c+dx}}{a}\right] + \frac{1}{2} \left(2c + i\pi + 2dx - 4i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{ib}{a}}}{\sqrt{2}}\right] \right) \operatorname{Log}\left[1 - \frac{(b + \sqrt{a^2 + b^2}) e^{c+dx}}{a}\right] - \\ & \frac{1}{2} i \pi \operatorname{Log}[b + a \operatorname{Sinh}[c + dx]] + \operatorname{PolyLog}\left[2, \frac{(b - \sqrt{a^2 + b^2}) e^{c+dx}}{a}\right] + \operatorname{PolyLog}\left[2, \frac{(b + \sqrt{a^2 + b^2}) e^{c+dx}}{a}\right] \right) + \\ & \left. 18 a (a^2 + 4 b^2) dx \operatorname{Sinh}[c + dx] + 9 a^2 b \operatorname{Sinh}[2(c + dx)] + 6 a^3 dx \operatorname{Sinh}[3(c + dx)] \right) \end{aligned} \right)$$

Problem 27: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(e + fx)^2 \operatorname{Cosh}[c + dx]^3}{a + b \operatorname{Csch}[c + dx]} dx$$

Optimal (type 4, 636 leaves, 24 steps):

$$\begin{aligned}
& -\frac{b e f x}{2 a^2 d} - \frac{b f^2 x^2}{4 a^2 d} + \frac{b (a^2 + b^2) (e + f x)^3}{3 a^4 f} - \frac{4 f (e + f x) \operatorname{Cosh}[c + d x]}{3 a d^2} - \frac{2 b^2 f (e + f x) \operatorname{Cosh}[c + d x]}{a^3 d^2} - \frac{2 f (e + f x) \operatorname{Cosh}[c + d x]^3}{9 a d^2} \\
& \frac{b (a^2 + b^2) (e + f x)^2 \operatorname{Log}\left[1 + \frac{a e^{c+dx}}{b - \sqrt{a^2 + b^2}}\right]}{a^4 d} - \frac{b (a^2 + b^2) (e + f x)^2 \operatorname{Log}\left[1 + \frac{a e^{c+dx}}{b + \sqrt{a^2 + b^2}}\right]}{a^4 d} - \frac{2 b (a^2 + b^2) f (e + f x) \operatorname{PolyLog}\left[2, -\frac{a e^{c+dx}}{b - \sqrt{a^2 + b^2}}\right]}{a^4 d^2} \\
& \frac{2 b (a^2 + b^2) f (e + f x) \operatorname{PolyLog}\left[2, -\frac{a e^{c+dx}}{b + \sqrt{a^2 + b^2}}\right]}{a^4 d^2} + \frac{2 b (a^2 + b^2) f^2 \operatorname{PolyLog}\left[3, -\frac{a e^{c+dx}}{b - \sqrt{a^2 + b^2}}\right]}{a^4 d^3} + \frac{2 b (a^2 + b^2) f^2 \operatorname{PolyLog}\left[3, -\frac{a e^{c+dx}}{b + \sqrt{a^2 + b^2}}\right]}{a^4 d^3} + \\
& \frac{14 f^2 \operatorname{Sinh}[c + d x]}{9 a d^3} + \frac{2 b^2 f^2 \operatorname{Sinh}[c + d x]}{a^3 d^3} + \frac{2 (e + f x)^2 \operatorname{Sinh}[c + d x]}{3 a d} + \frac{b^2 (e + f x)^2 \operatorname{Sinh}[c + d x]}{a^3 d} + \frac{b f (e + f x) \operatorname{Cosh}[c + d x] \operatorname{Sinh}[c + d x]}{2 a^2 d^2} + \\
& \frac{(e + f x)^2 \operatorname{Cosh}[c + d x]^2 \operatorname{Sinh}[c + d x]}{3 a d} - \frac{b f^2 \operatorname{Sinh}[c + d x]^2}{4 a^2 d^3} - \frac{b (e + f x)^2 \operatorname{Sinh}[c + d x]^2}{2 a^2 d} + \frac{2 f^2 \operatorname{Sinh}[c + d x]^3}{27 a d^3}
\end{aligned}$$

Result (type 4, 3303 leaves):

$$\begin{aligned}
& \frac{1}{12 a^2 d^3 (a + b \operatorname{Csch}[c + d x])} f^2 \operatorname{Csch}[c + d x] \left(-12 b d x \operatorname{PolyLog}\left[2, -\frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2 + b^2) e^{2c}}}\right] - 12 b d x \operatorname{PolyLog}\left[2, -\frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2 + b^2) e^{2c}}}\right] \right) + \\
& e^{-c} \left(2 b d^3 e^c x^3 - 6 a \operatorname{Cosh}[d x] + 6 a e^{2c} \operatorname{Cosh}[d x] - 6 a d x \operatorname{Cosh}[d x] - 6 a d e^{2c} x \operatorname{Cosh}[d x] - 3 a d^2 x^2 \operatorname{Cosh}[d x] + \right. \\
& 3 a d^2 e^{2c} x^2 \operatorname{Cosh}[d x] - 6 b d^2 e^c x^2 \operatorname{Log}\left[1 + \frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2 + b^2) e^{2c}}}\right] - 6 b d^2 e^c x^2 \operatorname{Log}\left[1 + \frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2 + b^2) e^{2c}}}\right] + \\
& 12 b e^c \operatorname{PolyLog}\left[3, -\frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2 + b^2) e^{2c}}}\right] + 12 b e^c \operatorname{PolyLog}\left[3, -\frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2 + b^2) e^{2c}}}\right] + 6 a \operatorname{Sinh}[d x] + 6 a e^{2c} \operatorname{Sinh}[d x] + \\
& \left. 6 a d x \operatorname{Sinh}[d x] - 6 a d e^{2c} x \operatorname{Sinh}[d x] + 3 a d^2 x^2 \operatorname{Sinh}[d x] + 3 a d^2 e^{2c} x^2 \operatorname{Sinh}[d x] \right) (b + a \operatorname{Sinh}[c + d x]) + \\
& \frac{1}{432 a^4 d^3 (a + b \operatorname{Csch}[c + d x])} e^{-3c} f^2 \operatorname{Csch}[c + d x] \left(72 a^2 b d^3 e^{3c} x^3 + 144 b^3 d^3 e^{3c} x^3 - 108 a^3 e^{2c} \operatorname{Cosh}[d x] - 432 a b^2 e^{2c} \operatorname{Cosh}[d x] + \right. \\
& 108 a^3 e^{4c} \operatorname{Cosh}[d x] + 432 a b^2 e^{4c} \operatorname{Cosh}[d x] - 108 a^3 d e^{2c} x \operatorname{Cosh}[d x] - 432 a b^2 d e^{2c} x \operatorname{Cosh}[d x] - 108 a^3 d e^{4c} x \operatorname{Cosh}[d x] - \\
& 432 a b^2 d e^{4c} x \operatorname{Cosh}[d x] - 54 a^3 d^2 e^{2c} x^2 \operatorname{Cosh}[d x] - 216 a b^2 d^2 e^{2c} x^2 \operatorname{Cosh}[d x] + 54 a^3 d^2 e^{4c} x^2 \operatorname{Cosh}[d x] + 216 a b^2 d^2 e^{4c} x^2 \operatorname{Cosh}[d x] - \\
& 27 a^2 b e^c \operatorname{Cosh}[2 d x] - 27 a^2 b e^{5c} \operatorname{Cosh}[2 d x] - 54 a^2 b d e^c x \operatorname{Cosh}[2 d x] + 54 a^2 b d e^{5c} x \operatorname{Cosh}[2 d x] - 54 a^2 b d^2 e^c x^2 \operatorname{Cosh}[2 d x] - \\
& 54 a^2 b d^2 e^{5c} x^2 \operatorname{Cosh}[2 d x] - 4 a^3 \operatorname{Cosh}[3 d x] + 4 a^3 e^{6c} \operatorname{Cosh}[3 d x] - 12 a^3 d x \operatorname{Cosh}[3 d x] - 12 a^3 d e^{6c} x \operatorname{Cosh}[3 d x] - 18 a^3 d^2 x^2 \operatorname{Cosh}[3 d x] + \\
& \left. 18 a^3 d^2 e^{6c} x^2 \operatorname{Cosh}[3 d x] - 216 a^2 b d^2 e^{3c} x^2 \operatorname{Log}\left[1 + \frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2 + b^2) e^{2c}}}\right] - 432 b^3 d^2 e^{3c} x^2 \operatorname{Log}\left[1 + \frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2 + b^2) e^{2c}}}\right] - \right)
\end{aligned}$$

$$\begin{aligned}
& 216 a^2 b d^2 e^{3c} x^2 \operatorname{Log}\left[1 + \frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2 + b^2) e^{2c}}}\right] - 432 b^3 d^2 e^{3c} x^2 \operatorname{Log}\left[1 + \frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2 + b^2) e^{2c}}}\right] - \\
& 432 b (a^2 + 2 b^2) d e^{3c} x \operatorname{PolyLog}\left[2, -\frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2 + b^2) e^{2c}}}\right] - 432 b (a^2 + 2 b^2) d e^{3c} x \operatorname{PolyLog}\left[2, -\frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2 + b^2) e^{2c}}}\right] + \\
& 432 a^2 b e^{3c} \operatorname{PolyLog}\left[3, -\frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2 + b^2) e^{2c}}}\right] + 864 b^3 e^{3c} \operatorname{PolyLog}\left[3, -\frac{a e^{2c+dx}}{b e^c - \sqrt{(a^2 + b^2) e^{2c}}}\right] + \\
& 432 a^2 b e^{3c} \operatorname{PolyLog}\left[3, -\frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2 + b^2) e^{2c}}}\right] + 864 b^3 e^{3c} \operatorname{PolyLog}\left[3, -\frac{a e^{2c+dx}}{b e^c + \sqrt{(a^2 + b^2) e^{2c}}}\right] + 108 a^3 e^{2c} \operatorname{Sinh}[dx] + \\
& 432 a b^2 e^{2c} \operatorname{Sinh}[dx] + 108 a^3 e^{4c} \operatorname{Sinh}[dx] + 432 a b^2 e^{4c} \operatorname{Sinh}[dx] + 108 a^3 d e^{2c} x \operatorname{Sinh}[dx] + 432 a b^2 d e^{2c} x \operatorname{Sinh}[dx] - \\
& 108 a^3 d e^{4c} x \operatorname{Sinh}[dx] - 432 a b^2 d e^{4c} x \operatorname{Sinh}[dx] + 54 a^3 d^2 e^{2c} x^2 \operatorname{Sinh}[dx] + 216 a b^2 d^2 e^{2c} x^2 \operatorname{Sinh}[dx] + \\
& 54 a^3 d^2 e^{4c} x^2 \operatorname{Sinh}[dx] + 216 a b^2 d^2 e^{4c} x^2 \operatorname{Sinh}[dx] + 27 a^2 b e^c \operatorname{Sinh}[2dx] - 27 a^2 b e^{5c} \operatorname{Sinh}[2dx] + 54 a^2 b d e^c x \operatorname{Sinh}[2dx] + \\
& 54 a^2 b d e^{5c} x \operatorname{Sinh}[2dx] + 54 a^2 b d^2 e^c x^2 \operatorname{Sinh}[2dx] - 54 a^2 b d^2 e^{5c} x^2 \operatorname{Sinh}[2dx] + 4 a^3 \operatorname{Sinh}[3dx] + 4 a^3 e^{6c} \operatorname{Sinh}[3dx] + \\
& 12 a^3 d x \operatorname{Sinh}[3dx] - 12 a^3 d e^{6c} x \operatorname{Sinh}[3dx] + 18 a^3 d^2 x^2 \operatorname{Sinh}[3dx] + 18 a^3 d^2 e^{6c} x^2 \operatorname{Sinh}[3dx] \Big) (b + a \operatorname{Sinh}[c + dx]) + \\
& \frac{e^2 \operatorname{Csch}[c + dx] (b + a \operatorname{Sinh}[c + dx]) \left(-\frac{2b \operatorname{Log}[b + a \operatorname{Sinh}[c + dx]]}{a^2 d} + \frac{2 \operatorname{Sinh}[c + dx]}{a d}\right)}{4 (a + b \operatorname{Csch}[c + dx])} + \\
& \frac{1}{a^2 d^2 (a + b \operatorname{Csch}[c + dx])} \\
& e f \operatorname{Csch}[c + dx] (b + a \operatorname{Sinh}[c + dx]) \\
& \left(-a \operatorname{Cosh}[c + dx] - b (c + dx) \operatorname{Log}[b + a \operatorname{Sinh}[c + dx]] + b c \operatorname{Log}\left[1 + \frac{a \operatorname{Sinh}[c + dx]}{b}\right] \right) + \\
& i b \left(-\frac{1}{8} i (2c + i\pi + 2dx)^2 - 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{ib}{a}}}{\sqrt{2}}\right] \operatorname{ArcTan}\left[\frac{(i a + b) \operatorname{Cot}\left[\frac{1}{4} (2i c + \pi + 2i dx)\right]}{\sqrt{a^2 + b^2}}\right] \right) -
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{2} \left(-2 i c + \pi - 2 i d x + 4 \operatorname{ArcSin} \left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}} \right] \right) \operatorname{Log} \left[1 + \frac{(-b + \sqrt{a^2 + b^2}) e^{c+dx}}{a} \right] - \\
& \frac{1}{2} \left(-2 i c + \pi - 2 i d x - 4 \operatorname{ArcSin} \left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}} \right] \right) \operatorname{Log} \left[1 - \frac{(b + \sqrt{a^2 + b^2}) e^{c+dx}}{a} \right] + \left(\frac{\pi}{2} - i (c + d x) \right) \operatorname{Log} [b + a \operatorname{Sinh} [c + d x]] + \\
& i \left(\operatorname{PolyLog} \left[2, \frac{(b - \sqrt{a^2 + b^2}) e^{c+dx}}{a} \right] + \operatorname{PolyLog} \left[2, \frac{(b + \sqrt{a^2 + b^2}) e^{c+dx}}{a} \right] \right) + a d x \operatorname{Sinh} [c + d x] \Bigg) + \\
& \frac{1}{8 (a + b \operatorname{Csch} [c + d x])} e^2 \operatorname{Csch} [c + d x] (b + a \operatorname{Sinh} [c + d x]) \left(-\frac{2 b \operatorname{Cosh} [2 (c + d x)]}{a^2 d} - \frac{4 (a^2 b + 2 b^3) \operatorname{Log} [b + a \operatorname{Sinh} [c + d x]]}{a^4 d} + \right. \\
& \left. \frac{2 (a^2 + 4 b^2) \operatorname{Sinh} [c + d x]}{a^3 d} + \frac{2 \operatorname{Sinh} [3 (c + d x)]}{3 a d} \right) + \\
& \frac{1}{36 a^4 d^2 (a + b \operatorname{Csch} [c + d x])} e f \operatorname{Csch} [c + d x] (b + a \operatorname{Sinh} [c + d x]) \\
& \left(-18 a (a^2 + 4 b^2) \operatorname{Cosh} [c + d x] - 18 a^2 b d x \operatorname{Cosh} [2 (c + d x)] - 2 a^3 \operatorname{Cosh} [3 (c + d x)] + 36 a^2 b c \operatorname{Log} \left[1 + \frac{a \operatorname{Sinh} [c + d x]}{b} \right] + \right. \\
& \left. 72 b^3 c \operatorname{Log} \left[1 + \frac{a \operatorname{Sinh} [c + d x]}{b} \right] - 36 a^2 b \left(-\frac{1}{8} (2 c + i \pi + 2 d x)^2 - 4 \operatorname{ArcSin} \left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}} \right] \operatorname{ArcTan} \left[\frac{(i a + b) \operatorname{Cot} \left[\frac{1}{4} (2 i c + \pi + 2 i d x) \right]}{\sqrt{a^2 + b^2}} \right] \right) + \right. \\
& \left. \frac{1}{2} \left(2 c + i \pi + 2 d x + 4 i \operatorname{ArcSin} \left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}} \right] \right) \operatorname{Log} \left[1 + \frac{(-b + \sqrt{a^2 + b^2}) e^{c+dx}}{a} \right] + \frac{1}{2} \left(2 c + i \pi + 2 d x - 4 i \operatorname{ArcSin} \left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}} \right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \begin{aligned}
& \text{Log}\left[1 - \frac{(b + \sqrt{a^2 + b^2}) e^{c+dx}}{a}\right] - \frac{1}{2} i \pi \text{Log}[b + a \text{Sinh}[c + dx]] + \text{PolyLog}\left[2, \frac{(b - \sqrt{a^2 + b^2}) e^{c+dx}}{a}\right] + \text{PolyLog}\left[2, \frac{(b + \sqrt{a^2 + b^2}) e^{c+dx}}{a}\right] \right) - \\
& 72 b^3 \left(-\frac{1}{8} (2c + i\pi + 2dx)^2 - 4 \text{ArcSin}\left[\frac{\sqrt{1 + \frac{ib}{a}}}{\sqrt{2}}\right] \text{ArcTan}\left[\frac{(ia + b) \text{Cot}\left[\frac{1}{4}(2ic + \pi + 2idx)\right]}{\sqrt{a^2 + b^2}}\right] + \frac{1}{2} \left(2c + i\pi + 2dx + 4i \text{ArcSin}\left[\frac{\sqrt{1 + \frac{ib}{a}}}{\sqrt{2}}\right] \right) \right) \\
& \text{Log}\left[1 + \frac{(-b + \sqrt{a^2 + b^2}) e^{c+dx}}{a}\right] + \frac{1}{2} \left(2c + i\pi + 2dx - 4i \text{ArcSin}\left[\frac{\sqrt{1 + \frac{ib}{a}}}{\sqrt{2}}\right] \right) \text{Log}\left[1 - \frac{(b + \sqrt{a^2 + b^2}) e^{c+dx}}{a}\right] - \\
& \frac{1}{2} i \pi \text{Log}[b + a \text{Sinh}[c + dx]] + \text{PolyLog}\left[2, \frac{(b - \sqrt{a^2 + b^2}) e^{c+dx}}{a}\right] + \text{PolyLog}\left[2, \frac{(b + \sqrt{a^2 + b^2}) e^{c+dx}}{a}\right] \right) + \\
& \left. 18 a (a^2 + 4 b^2) dx \text{Sinh}[c + dx] + 9 a^2 b \text{Sinh}[2(c + dx)] + 6 a^3 dx \text{Sinh}[3(c + dx)] \right)
\end{aligned}
\right)
\end{aligned}$$

Problem 28: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(e + fx) \text{Cosh}[c + dx]^3}{a + b \text{Csch}[c + dx]} dx$$

Optimal (type 4, 400 leaves, 18 steps):

$$\begin{aligned}
& -\frac{b f x}{4 a^2 d} + \frac{b (a^2 + b^2) (e + f x)^2}{2 a^4 f} - \frac{2 f \operatorname{Cosh}[c + d x]}{3 a d^2} - \frac{b^2 f \operatorname{Cosh}[c + d x]}{a^3 d^2} - \frac{f \operatorname{Cosh}[c + d x]^3}{9 a d^2} - \frac{b (a^2 + b^2) (e + f x) \operatorname{Log}\left[1 + \frac{a e^{c+d x}}{b - \sqrt{a^2 + b^2}}\right]}{a^4 d} \\
& \frac{b (a^2 + b^2) (e + f x) \operatorname{Log}\left[1 + \frac{a e^{c+d x}}{b + \sqrt{a^2 + b^2}}\right]}{a^4 d} - \frac{b (a^2 + b^2) f \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x}}{b - \sqrt{a^2 + b^2}}\right]}{a^4 d^2} - \frac{b (a^2 + b^2) f \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x}}{b + \sqrt{a^2 + b^2}}\right]}{a^4 d^2} + \frac{2 (e + f x) \operatorname{Sinh}[c + d x]}{3 a d} \\
& \frac{b^2 (e + f x) \operatorname{Sinh}[c + d x]}{a^3 d} + \frac{b f \operatorname{Cosh}[c + d x] \operatorname{Sinh}[c + d x]}{4 a^2 d^2} + \frac{(e + f x) \operatorname{Cosh}[c + d x]^2 \operatorname{Sinh}[c + d x]}{3 a d} - \frac{b (e + f x) \operatorname{Sinh}[c + d x]^2}{2 a^2 d}
\end{aligned}$$

Result (type 4, 1315 leaves):

$$\begin{aligned}
& \frac{1}{72 a^2 d^2} \left(36 a^2 b c^2 f + 36 b^3 c^2 f + 36 i a^2 b c f \pi + 36 i b^3 c f \pi - 9 a^2 b f \pi^2 - 9 b^3 f \pi^2 + 72 a^2 b c d f x + 72 b^3 c d f x + 36 i a^2 b d f \pi x + \right. \\
& 36 i b^3 d f \pi x + 36 a^2 b d^2 f x^2 + 36 b^3 d^2 f x^2 + 288 a^2 b f \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}}\right] \operatorname{ArcTan}\left[\frac{(i a + b) \operatorname{Cot}\left[\frac{1}{4} (2 i c + \pi + 2 i d x)\right]}{\sqrt{a^2 + b^2}}\right] + \\
& 288 b^3 f \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}}\right] \operatorname{ArcTan}\left[\frac{(i a + b) \operatorname{Cot}\left[\frac{1}{4} (2 i c + \pi + 2 i d x)\right]}{\sqrt{a^2 + b^2}}\right] - 54 a^3 f \operatorname{Cosh}[c + d x] - \\
& 72 a b^2 f \operatorname{Cosh}[c + d x] - 18 a^2 b d e \operatorname{Cosh}[2 (c + d x)] - 18 a^2 b d f x \operatorname{Cosh}[2 (c + d x)] - 2 a^3 f \operatorname{Cosh}[3 (c + d x)] - \\
& 72 a^2 b c f \operatorname{Log}\left[1 + \frac{(-b + \sqrt{a^2 + b^2}) e^{c+d x}}{a}\right] - 72 b^3 c f \operatorname{Log}\left[1 + \frac{(-b + \sqrt{a^2 + b^2}) e^{c+d x}}{a}\right] - 36 i a^2 b f \pi \operatorname{Log}\left[1 + \frac{(-b + \sqrt{a^2 + b^2}) e^{c+d x}}{a}\right] - \\
& 36 i b^3 f \pi \operatorname{Log}\left[1 + \frac{(-b + \sqrt{a^2 + b^2}) e^{c+d x}}{a}\right] - 72 a^2 b d f x \operatorname{Log}\left[1 + \frac{(-b + \sqrt{a^2 + b^2}) e^{c+d x}}{a}\right] - 72 b^3 d f x \operatorname{Log}\left[1 + \frac{(-b + \sqrt{a^2 + b^2}) e^{c+d x}}{a}\right] - \\
& 144 i a^2 b f \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{(-b + \sqrt{a^2 + b^2}) e^{c+d x}}{a}\right] - 144 i b^3 f \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{(-b + \sqrt{a^2 + b^2}) e^{c+d x}}{a}\right] - \\
& 72 a^2 b c f \operatorname{Log}\left[1 - \frac{(b + \sqrt{a^2 + b^2}) e^{c+d x}}{a}\right] - 72 b^3 c f \operatorname{Log}\left[1 - \frac{(b + \sqrt{a^2 + b^2}) e^{c+d x}}{a}\right] - 36 i a^2 b f \pi \operatorname{Log}\left[1 - \frac{(b + \sqrt{a^2 + b^2}) e^{c+d x}}{a}\right] - \\
& 36 i b^3 f \pi \operatorname{Log}\left[1 - \frac{(b + \sqrt{a^2 + b^2}) e^{c+d x}}{a}\right] - 72 a^2 b d f x \operatorname{Log}\left[1 - \frac{(b + \sqrt{a^2 + b^2}) e^{c+d x}}{a}\right] - 72 b^3 d f x \operatorname{Log}\left[1 - \frac{(b + \sqrt{a^2 + b^2}) e^{c+d x}}{a}\right] +
\end{aligned}$$

$$\begin{aligned}
& 144 i a^2 b f \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{(b + \sqrt{a^2 + b^2}) e^{c+dx}}{a}\right] + 144 i b^3 f \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i b}{a}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{(b + \sqrt{a^2 + b^2}) e^{c+dx}}{a}\right] - \\
& 72 a^2 b d e \operatorname{Log}[b + a \operatorname{Sinh}[c + dx]] - 72 b^3 d e \operatorname{Log}[b + a \operatorname{Sinh}[c + dx]] + 36 i a^2 b f \pi \operatorname{Log}[b + a \operatorname{Sinh}[c + dx]] + 36 i b^3 f \pi \operatorname{Log}[b + a \operatorname{Sinh}[c + dx]] + \\
& 72 a^2 b c f \operatorname{Log}\left[1 + \frac{a \operatorname{Sinh}[c + dx]}{b}\right] + 72 b^3 c f \operatorname{Log}\left[1 + \frac{a \operatorname{Sinh}[c + dx]}{b}\right] - 72 b (a^2 + b^2) f \operatorname{PolyLog}\left[2, \frac{(b - \sqrt{a^2 + b^2}) e^{c+dx}}{a}\right] - \\
& 72 b (a^2 + b^2) f \operatorname{PolyLog}\left[2, \frac{(b + \sqrt{a^2 + b^2}) e^{c+dx}}{a}\right] + 54 a^3 d e \operatorname{Sinh}[c + dx] + 72 a b^2 d e \operatorname{Sinh}[c + dx] + 54 a^3 d f x \operatorname{Sinh}[c + dx] + \\
& \left. \begin{aligned}
& 72 a b^2 d f x \operatorname{Sinh}[c + dx] + 9 a^2 b f \operatorname{Sinh}[2(c + dx)] + 6 a^3 d e \operatorname{Sinh}[3(c + dx)] + 6 a^3 d f x \operatorname{Sinh}[3(c + dx)]
\end{aligned} \right)
\end{aligned}$$

Test results for the 83 problems in "6.6.2 (e x)^m (a+b csch(c+d x^n))^p.m"

Problem 5: Result more than twice size of optimal antiderivative.

$$\int x (a + b \operatorname{Csch}[c + d x^2]) dx$$

Optimal (type 3, 26 leaves, 4 steps):

$$\frac{a x^2}{2} - \frac{b \operatorname{ArcTanh}[\operatorname{Cosh}[c + d x^2]]}{2 d}$$

Result (type 3, 57 leaves):

$$\frac{a x^2}{2} - \frac{b \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{c}{2} + \frac{d x^2}{2}\right]\right]}{2 d} + \frac{b \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{c}{2} + \frac{d x^2}{2}\right]\right]}{2 d}$$

Problem 10: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int x^3 (a + b \operatorname{Csch}[c + d x^2])^2 dx$$

Optimal (type 4, 108 leaves, 10 steps):

$$\frac{a^2 x^4}{4} - \frac{2 a b x^2 \operatorname{ArcTanh}\left[e^{c+d x^2}\right]}{d} - \frac{b^2 x^2 \operatorname{Coth}\left[c+d x^2\right]}{2 d} + \frac{b^2 \operatorname{Log}\left[\operatorname{Sinh}\left[c+d x^2\right]\right]}{2 d^2} - \frac{a b \operatorname{PolyLog}\left[2, -e^{c+d x^2}\right]}{d^2} + \frac{a b \operatorname{PolyLog}\left[2, e^{c+d x^2}\right]}{d^2}$$

Result (type 4, 598 leaves):

$$\frac{b^2 x^2 \operatorname{Coth}\left[c\right] \left(a+b \operatorname{Csch}\left[c+d x^2\right]\right)^2 \operatorname{Sinh}\left[c+d x^2\right]^2}{2 d \left(b+a \operatorname{Sinh}\left[c+d x^2\right]\right)^2} + \frac{x^2 \operatorname{Csch}\left[\frac{c}{2}\right] \left(a+b \operatorname{Csch}\left[c+d x^2\right]\right)^2 \operatorname{Sech}\left[\frac{c}{2}\right] \left(-2 b^2 \operatorname{Cosh}\left[c\right]+a^2 d x^2 \operatorname{Sinh}\left[c\right]\right) \operatorname{Sinh}\left[c+d x^2\right]^2}{8 d \left(b+a \operatorname{Sinh}\left[c+d x^2\right]\right)^2} -$$

$$\frac{\left(b^2 \operatorname{Csch}\left[c\right] \left(a+b \operatorname{Csch}\left[c+d x^2\right]\right)^2 \left(-d x^2 \operatorname{Cosh}\left[c\right]+\operatorname{Log}\left[\operatorname{Cosh}\left[d x^2\right] \operatorname{Sinh}\left[c\right]+\operatorname{Cosh}\left[c\right] \operatorname{Sinh}\left[d x^2\right]\right) \operatorname{Sinh}\left[c\right]\right) \operatorname{Sinh}\left[c+d x^2\right]^2}{\left(2 d^2 \left(-\operatorname{Cosh}\left[c\right]^2+\operatorname{Sinh}\left[c\right]^2\right) \left(b+a \operatorname{Sinh}\left[c+d x^2\right]\right)^2\right)} + \frac{b^2 x^2 \operatorname{Csch}\left[\frac{c}{2}\right] \operatorname{Csch}\left[\frac{c}{2}+\frac{d x^2}{2}\right] \left(a+b \operatorname{Csch}\left[c+d x^2\right]\right)^2 \operatorname{Sinh}\left[\frac{d x^2}{2}\right] \operatorname{Sinh}\left[c+d x^2\right]^2}{4 d \left(b+a \operatorname{Sinh}\left[c+d x^2\right]\right)^2} -$$

$$\frac{b^2 x^2 \left(a+b \operatorname{Csch}\left[c+d x^2\right]\right)^2 \operatorname{Sech}\left[\frac{c}{2}\right] \operatorname{Sech}\left[\frac{c}{2}+\frac{d x^2}{2}\right] \operatorname{Sinh}\left[\frac{d x^2}{2}\right] \operatorname{Sinh}\left[c+d x^2\right]^2}{4 d \left(b+a \operatorname{Sinh}\left[c+d x^2\right]\right)^2} + \frac{1}{d^2 \left(b+a \operatorname{Sinh}\left[c+d x^2\right]\right)^2}$$

$$a b \left(a+b \operatorname{Csch}\left[c+d x^2\right]\right)^2 \operatorname{Sinh}\left[c+d x^2\right]^2 \left(-\frac{2 \operatorname{ArcTan}\left[\frac{\operatorname{Cosh}\left[c\right]+\operatorname{Sinh}\left[c\right] \operatorname{Tanh}\left[\frac{d x^2}{2}\right]}{\sqrt{-\operatorname{Cosh}\left[c\right]^2+\operatorname{Sinh}\left[c\right]^2}}\right] \operatorname{ArcTanh}\left[\operatorname{Tanh}\left[c\right]\right]}{\sqrt{-\operatorname{Cosh}\left[c\right]^2+\operatorname{Sinh}\left[c\right]^2}} - \right.$$

$$\left. \frac{1}{\sqrt{1-\operatorname{Tanh}\left[c\right]^2}} \operatorname{I}\left(\operatorname{I}\left(d x^2+\operatorname{ArcTanh}\left[\operatorname{Tanh}\left[c\right]\right]\right) \left(\operatorname{Log}\left[1-e^{-d x^2-\operatorname{ArcTanh}\left[\operatorname{Tanh}\left[c\right]\right]}\right]-\operatorname{Log}\left[1+e^{-d x^2-\operatorname{ArcTanh}\left[\operatorname{Tanh}\left[c\right]\right]}\right]\right) + \right.$$

$$\left. \operatorname{I}\left(\operatorname{PolyLog}\left[2,-e^{-d x^2-\operatorname{ArcTanh}\left[\operatorname{Tanh}\left[c\right]\right]}\right]-\operatorname{PolyLog}\left[2,e^{-d x^2-\operatorname{ArcTanh}\left[\operatorname{Tanh}\left[c\right]\right]}\right]\right) \operatorname{Sech}\left[c\right] \right)$$

Problem 13: Attempted integration timed out after 120 seconds.

$$\int \frac{\left(a+b \operatorname{Csch}\left[c+d x^2\right]\right)^2}{x} dx$$

Optimal (type 9, 20 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{\left(a+b \operatorname{Csch}\left[c+d x^2\right]\right)^2}{x}, x\right]$$

Result (type 1, 1 leaves):

???

Problem 18: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{x^3}{a + b \operatorname{Csch}[c + d x^2]} dx$$

Optimal (type 4, 225 leaves, 11 steps):

$$\frac{x^4}{4 a} - \frac{b x^2 \operatorname{Log}\left[1 + \frac{a e^{c+d x^2}}{b - \sqrt{a^2 + b^2}}\right]}{2 a \sqrt{a^2 + b^2} d} + \frac{b x^2 \operatorname{Log}\left[1 + \frac{a e^{c-d x^2}}{b + \sqrt{a^2 + b^2}}\right]}{2 a \sqrt{a^2 + b^2} d} - \frac{b \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x^2}}{b - \sqrt{a^2 + b^2}}\right]}{2 a \sqrt{a^2 + b^2} d^2} + \frac{b \operatorname{PolyLog}\left[2, -\frac{a e^{c-d x^2}}{b + \sqrt{a^2 + b^2}}\right]}{2 a \sqrt{a^2 + b^2} d^2}$$

Result (type 4, 1321 leaves):

$$\begin{aligned}
& \frac{x^4 \operatorname{Csch}[c + d x^2] (b + a \operatorname{Sinh}[c + d x^2])}{4 a (a + b \operatorname{Csch}[c + d x^2])} + \frac{1}{2 a d^2 (a + b \operatorname{Csch}[c + d x^2])} \\
& b \operatorname{Csch}[c + d x^2] \left(\frac{i \pi \operatorname{ArcTanh}\left[\frac{-a+b \operatorname{Tanh}\left[\frac{1}{2}(c+d x^2)\right]}{\sqrt{a^2+b^2}}\right]}{\sqrt{a^2+b^2}} + \frac{1}{\sqrt{-a^2-b^2}} \left(2 \left(-i c + \frac{\pi}{2} - i d x^2 \right) \operatorname{ArcTanh}\left[\frac{(-i a+b) \operatorname{Cot}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^2\right)\right]}{\sqrt{-a^2-b^2}}\right]} \right) - \right. \\
& 2 \left(-i c + \operatorname{ArcCos}\left[-\frac{i b}{a}\right] \right) \operatorname{ArcTanh}\left[\frac{(-i a-b) \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^2\right)\right]}{\sqrt{-a^2-b^2}}\right]} + \\
& \left. \left(\operatorname{ArcCos}\left[-\frac{i b}{a}\right] - 2 i \left(\operatorname{ArcTanh}\left[\frac{(-i a+b) \operatorname{Cot}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^2\right)\right]}{\sqrt{-a^2-b^2}}\right]} - \operatorname{ArcTanh}\left[\frac{(-i a-b) \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^2\right)\right]}{\sqrt{-a^2-b^2}}\right]} \right) \right) \right) \\
& \operatorname{Log}\left[\frac{\sqrt{-a^2-b^2} e^{-\frac{1}{2} i \left(-i c + \frac{\pi}{2} - i d x^2\right)}}{\sqrt{2} \sqrt{-i a} \sqrt{b+a \operatorname{Sinh}[c+d x^2]}}\right] + \\
& \left(\operatorname{ArcCos}\left[-\frac{i b}{a}\right] + 2 i \left(\operatorname{ArcTanh}\left[\frac{(-i a+b) \operatorname{Cot}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^2\right)\right]}{\sqrt{-a^2-b^2}}\right]} - \operatorname{ArcTanh}\left[\frac{(-i a-b) \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^2\right)\right]}{\sqrt{-a^2-b^2}}\right]} \right) \right) \\
& \operatorname{Log}\left[\frac{\sqrt{-a^2-b^2} e^{\frac{1}{2} i \left(-i c + \frac{\pi}{2} - i d x^2\right)}}{\sqrt{2} \sqrt{-i a} \sqrt{b+a \operatorname{Sinh}[c+d x^2]}}\right] - \left(\operatorname{ArcCos}\left[-\frac{i b}{a}\right] + 2 i \operatorname{ArcTanh}\left[\frac{(-i a-b) \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^2\right)\right]}{\sqrt{-a^2-b^2}}\right]} \right) \\
& \operatorname{Log}\left[1 - \frac{i \left(b - i \sqrt{-a^2-b^2}\right) \left(-i a + b - \sqrt{-a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^2\right)\right]\right)}{a \left(-i a + b + \sqrt{-a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^2\right)\right]\right)}\right] + \left(-\operatorname{ArcCos}\left[-\frac{i b}{a}\right] + \right. \\
& \left. 2 i \operatorname{ArcTanh}\left[\frac{(-i a-b) \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^2\right)\right]}{\sqrt{-a^2-b^2}}\right]} \right) \operatorname{Log}\left[1 - \frac{i \left(b + i \sqrt{-a^2-b^2}\right) \left(-i a + b - \sqrt{-a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^2\right)\right]\right)}{a \left(-i a + b + \sqrt{-a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^2\right)\right]\right)}\right] + \\
& i \left(\operatorname{PolyLog}\left[2, \frac{i \left(b - i \sqrt{-a^2-b^2}\right) \left(-i a + b - \sqrt{-a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^2\right)\right]\right)}{a \left(-i a + b + \sqrt{-a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^2\right)\right]\right)}\right] - \right. \\
& \left. \operatorname{PolyLog}\left[2, \frac{i \left(b + i \sqrt{-a^2-b^2}\right) \left(-i a + b - \sqrt{-a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^2\right)\right]\right)}{a \left(-i a + b + \sqrt{-a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^2\right)\right]\right)}\right] \right) \right) (b + a \operatorname{Sinh}[c + d x^2])
\end{aligned}$$

Problem 24: Attempted integration timed out after 120 seconds.

$$\int \frac{x^4}{(a + b \operatorname{Csch}[c + d x^2])^2} dx$$

Optimal (type 9, 20 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{x^4}{(a + b \operatorname{Csch}[c + d x^2])^2}, x\right]$$

Result (type 1, 1 leaves):

???

Problem 26: Attempted integration timed out after 120 seconds.

$$\int \frac{x^2}{(a + b \operatorname{Csch}[c + d x^2])^2} dx$$

Optimal (type 9, 20 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{x^2}{(a + b \operatorname{Csch}[c + d x^2])^2}, x\right]$$

Result (type 1, 1 leaves):

???

Problem 28: Attempted integration timed out after 120 seconds.

$$\int \frac{1}{x (a + b \operatorname{Csch}[c + d x^2])^2} dx$$

Optimal (type 9, 20 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{x (a + b \operatorname{Csch}[c + d x^2])^2}, x\right]$$

Result (type 1, 1 leaves):

???

Problem 29: Attempted integration timed out after 120 seconds.

$$\int \frac{1}{x^2 (a + b \operatorname{Csch}[c + d x^2])^2} dx$$

Optimal (type 9, 20 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{x^2 (a + b \operatorname{Csch}[c + d x^2])^2}, x\right]$$

Result (type 1, 1 leaves):

???

Problem 30: Attempted integration timed out after 120 seconds.

$$\int \frac{1}{x^3 (a + b \operatorname{Csch}[c + d x^2])^2} dx$$

Optimal (type 9, 20 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{x^3 (a + b \operatorname{Csch}[c + d x^2])^2}, x\right]$$

Result (type 1, 1 leaves):

???

Problem 38: Result more than twice size of optimal antiderivative.

$$\int x (a + b \operatorname{Csch}[c + d \sqrt{x}])^2 dx$$

Optimal (type 4, 287 leaves, 18 steps):

$$\begin{aligned} & -\frac{2 b^2 x^{3/2}}{d} + \frac{a^2 x^2}{2} - \frac{8 a b x^{3/2} \operatorname{ArcTanh}[e^{c+d \sqrt{x}}]}{d} - \frac{2 b^2 x^{3/2} \operatorname{Coth}[c + d \sqrt{x}]}{d} + \frac{6 b^2 x \operatorname{Log}[1 - e^{2(c+d \sqrt{x})}]}{d^2} - \\ & \frac{12 a b x \operatorname{PolyLog}[2, -e^{c+d \sqrt{x}}]}{d^2} + \frac{12 a b x \operatorname{PolyLog}[2, e^{c+d \sqrt{x}}]}{d^2} + \frac{6 b^2 \sqrt{x} \operatorname{PolyLog}[2, e^{2(c+d \sqrt{x})}]}{d^3} + \frac{24 a b \sqrt{x} \operatorname{PolyLog}[3, -e^{c+d \sqrt{x}}]}{d^3} - \\ & \frac{24 a b \sqrt{x} \operatorname{PolyLog}[3, e^{c+d \sqrt{x}}]}{d^3} - \frac{3 b^2 \operatorname{PolyLog}[3, e^{2(c+d \sqrt{x})}]}{d^4} - \frac{24 a b \operatorname{PolyLog}[4, -e^{c+d \sqrt{x}}]}{d^4} + \frac{24 a b \operatorname{PolyLog}[4, e^{c+d \sqrt{x}}]}{d^4} \end{aligned}$$

Result (type 4, 591 leaves):

$$\frac{a^2 x^2 \left(a + b \operatorname{Csch} [c + d \sqrt{x}] \right)^2 \operatorname{Sinh} [c + d \sqrt{x}]^2}{2 \left(b + a \operatorname{Sinh} [c + d \sqrt{x}] \right)^2} +$$

$$\frac{1}{d^4 \left(b + a \operatorname{Sinh} [c + d \sqrt{x}] \right)^2} b \left(a + b \operatorname{Csch} [c + d \sqrt{x}] \right)^2 \left(-\frac{4 b d^3 e^{2c} x^{3/2}}{-1 + e^{2c}} + 12 b d^2 x \operatorname{Log} [1 - e^{c+d \sqrt{x}}] + 4 a d^3 x^{3/2} \operatorname{Log} [1 - e^{c+d \sqrt{x}}] + \right.$$

$$12 b d^2 x \operatorname{Log} [1 + e^{c+d \sqrt{x}}] - 4 a d^3 x^{3/2} \operatorname{Log} [1 + e^{c+d \sqrt{x}}] - 6 b d^2 x \operatorname{Log} [-1 + e^{2(c+d \sqrt{x})}] - 12 \left(-b d \sqrt{x} + a d^2 x \right) \operatorname{PolyLog} [2, -e^{c+d \sqrt{x}}] +$$

$$12 \left(b d \sqrt{x} + a d^2 x \right) \operatorname{PolyLog} [2, e^{c+d \sqrt{x}}] + 24 a d \sqrt{x} \operatorname{PolyLog} [3, -e^{c+d \sqrt{x}}] - 24 a d \sqrt{x} \operatorname{PolyLog} [3, e^{c+d \sqrt{x}}] -$$

$$\left. 3 b \operatorname{PolyLog} [3, e^{2(c+d \sqrt{x})}] - 24 a \operatorname{PolyLog} [4, -e^{c+d \sqrt{x}}] + 24 a \operatorname{PolyLog} [4, e^{c+d \sqrt{x}}] \right) \operatorname{Sinh} [c + d \sqrt{x}]^2 +$$

$$\frac{b^2 x^{3/2} \operatorname{Csch} \left[\frac{c}{2} \right] \operatorname{Csch} \left[\frac{c}{2} + \frac{d \sqrt{x}}{2} \right] \left(a + b \operatorname{Csch} [c + d \sqrt{x}] \right)^2 \operatorname{Sinh} [c + d \sqrt{x}]^2 \operatorname{Sinh} \left[\frac{d \sqrt{x}}{2} \right]}{d \left(b + a \operatorname{Sinh} [c + d \sqrt{x}] \right)^2} -$$

$$\frac{b^2 x^{3/2} \left(a + b \operatorname{Csch} [c + d \sqrt{x}] \right)^2 \operatorname{Sech} \left[\frac{c}{2} \right] \operatorname{Sech} \left[\frac{c}{2} + \frac{d \sqrt{x}}{2} \right] \operatorname{Sinh} [c + d \sqrt{x}]^2 \operatorname{Sinh} \left[\frac{d \sqrt{x}}{2} \right]}{d \left(b + a \operatorname{Sinh} [c + d \sqrt{x}] \right)^2}$$

Problem 39: Attempted integration timed out after 120 seconds.

$$\int \frac{\left(a + b \operatorname{Csch} [c + d \sqrt{x}] \right)^2}{x} dx$$

Optimal (type 9, 22 leaves, 0 steps):

$$\text{Unintegrable} \left[\frac{\left(a + b \operatorname{Csch} [c + d \sqrt{x}] \right)^2}{x}, x \right]$$

Result (type 1, 1 leaves):

???

Problem 49: Attempted integration timed out after 120 seconds.

$$\int \frac{1}{x \left(a + b \operatorname{Csch} [c + d \sqrt{x}] \right)^2} dx$$

Optimal (type 9, 22 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{x \left(a + b \operatorname{Csch}[c + d \sqrt{x}]\right)^2}, x\right]$$

Result (type 1, 1 leaves):

???

Problem 50: Attempted integration timed out after 120 seconds.

$$\int \frac{1}{x^2 \left(a + b \operatorname{Csch}[c + d \sqrt{x}]\right)^2} dx$$

Optimal (type 9, 22 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{x^2 \left(a + b \operatorname{Csch}[c + d \sqrt{x}]\right)^2}, x\right]$$

Result (type 1, 1 leaves):

???

Problem 57: Result more than twice size of optimal antiderivative.

$$\int \sqrt{x} \left(a + b \operatorname{Csch}[c + d \sqrt{x}]\right)^2 dx$$

Optimal (type 4, 209 leaves, 15 steps):

$$\begin{aligned} & -\frac{2 b^2 x}{d} + \frac{2}{3} a^2 x^{3/2} - \frac{8 a b x \operatorname{ArcTanh}\left[e^{c+d \sqrt{x}}\right]}{d} - \frac{2 b^2 x \operatorname{Coth}[c + d \sqrt{x}]}{d} + \frac{4 b^2 \sqrt{x} \operatorname{Log}\left[1 - e^{2(c+d \sqrt{x})}\right]}{d^2} - \frac{8 a b \sqrt{x} \operatorname{PolyLog}\left[2, -e^{c+d \sqrt{x}}\right]}{d^2} + \\ & \frac{8 a b \sqrt{x} \operatorname{PolyLog}\left[2, e^{c+d \sqrt{x}}\right]}{d^2} + \frac{2 b^2 \operatorname{PolyLog}\left[2, e^{2(c+d \sqrt{x})}\right]}{d^3} + \frac{8 a b \operatorname{PolyLog}\left[3, -e^{c+d \sqrt{x}}\right]}{d^3} - \frac{8 a b \operatorname{PolyLog}\left[3, e^{c+d \sqrt{x}}\right]}{d^3} \end{aligned}$$

Result (type 4, 470 leaves):

$$\frac{2 a^2 x^{3/2} (a + b \operatorname{Csch}[c + d \sqrt{x}])^2 \operatorname{Sinh}[c + d \sqrt{x}]^2}{3 (b + a \operatorname{Sinh}[c + d \sqrt{x}])^2} + \frac{1}{d^3 (b + a \operatorname{Sinh}[c + d \sqrt{x}])^2} 2 b (a + b \operatorname{Csch}[c + d \sqrt{x}])^2$$

$$\left(-\frac{2 b d^2 e^{2c} x}{-1 + e^{2c}} + 2 a d^2 x \operatorname{Log}[1 - e^{c+d \sqrt{x}}] - 2 a d^2 x \operatorname{Log}[1 + e^{c+d \sqrt{x}}] + 2 b d \sqrt{x} \operatorname{Log}[1 - e^{2(c+d \sqrt{x})}] - 4 a d \sqrt{x} \operatorname{PolyLog}[2, -e^{c+d \sqrt{x}}] + \right.$$

$$\left. 4 a d \sqrt{x} \operatorname{PolyLog}[2, e^{c+d \sqrt{x}}] + b \operatorname{PolyLog}[2, e^{2(c+d \sqrt{x})}] + 4 a \operatorname{PolyLog}[3, -e^{c+d \sqrt{x}}] - 4 a \operatorname{PolyLog}[3, e^{c+d \sqrt{x}}] \right) \operatorname{Sinh}[c + d \sqrt{x}]^2 +$$

$$\frac{b^2 x \operatorname{Csch}\left[\frac{c}{2}\right] \operatorname{Csch}\left[\frac{c}{2} + \frac{d \sqrt{x}}{2}\right] (a + b \operatorname{Csch}[c + d \sqrt{x}])^2 \operatorname{Sinh}[c + d \sqrt{x}]^2 \operatorname{Sinh}\left[\frac{d \sqrt{x}}{2}\right]}{d (b + a \operatorname{Sinh}[c + d \sqrt{x}])^2}$$

$$\frac{b^2 x (a + b \operatorname{Csch}[c + d \sqrt{x}])^2 \operatorname{Sech}\left[\frac{c}{2}\right] \operatorname{Sech}\left[\frac{c}{2} + \frac{d \sqrt{x}}{2}\right] \operatorname{Sinh}[c + d \sqrt{x}]^2 \operatorname{Sinh}\left[\frac{d \sqrt{x}}{2}\right]}{d (b + a \operatorname{Sinh}[c + d \sqrt{x}])^2}$$

Problem 69: Attempted integration timed out after 120 seconds.

$$\int \frac{1}{x^{3/2} (a + b \operatorname{Csch}[c + d \sqrt{x}])^2} dx$$

Optimal (type 9, 24 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{x^{3/2} (a + b \operatorname{Csch}[c + d \sqrt{x}])^2}, x\right]$$

Result (type 1, 1 leaves):

???

Problem 70: Attempted integration timed out after 120 seconds.

$$\int \frac{1}{x^{5/2} (a + b \operatorname{Csch}[c + d \sqrt{x}])^2} dx$$

Optimal (type 9, 24 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{x^{5/2} (a + b \operatorname{Csch}[c + d \sqrt{x}])^2}, x\right]$$

Result (type 1, 1 leaves):

???

Problem 74: Unable to integrate problem.

$$\int (e x)^{-1+3 n} (a + b \operatorname{Csch}[c + d x^n]) dx$$

Optimal (type 4, 197 leaves, 11 steps):

$$\frac{a (e x)^{3 n}}{3 e n} - \frac{2 b x^{-n} (e x)^{3 n} \operatorname{ArcTanh}\left[e^{c+d x^n}\right]}{d e n} - \frac{2 b x^{-2 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, -e^{c+d x^n}\right]}{d^2 e n} +$$

$$\frac{2 b x^{-2 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, e^{c+d x^n}\right]}{d^2 e n} + \frac{2 b x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[3, -e^{c+d x^n}\right]}{d^3 e n} - \frac{2 b x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[3, e^{c+d x^n}\right]}{d^3 e n}$$

Result (type 8, 24 leaves):

$$\int (e x)^{-1+3 n} (a + b \operatorname{Csch}[c + d x^n]) dx$$

Problem 76: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int (e x)^{-1+2 n} (a + b \operatorname{Csch}[c + d x^n])^2 dx$$

Optimal (type 4, 198 leaves, 11 steps):

$$\frac{a^2 (e x)^{2 n}}{2 e n} - \frac{4 a b x^{-n} (e x)^{2 n} \operatorname{ArcTanh}\left[e^{c+d x^n}\right]}{d e n} - \frac{b^2 x^{-n} (e x)^{2 n} \operatorname{Coth}[c + d x^n]}{d e n} +$$

$$\frac{b^2 x^{-2 n} (e x)^{2 n} \operatorname{Log}[\operatorname{Sinh}[c + d x^n]]}{d^2 e n} - \frac{2 a b x^{-2 n} (e x)^{2 n} \operatorname{PolyLog}\left[2, -e^{c+d x^n}\right]}{d^2 e n} + \frac{2 a b x^{-2 n} (e x)^{2 n} \operatorname{PolyLog}\left[2, e^{c+d x^n}\right]}{d^2 e n}$$

Result (type 4, 696 leaves):

$$\begin{aligned}
& \frac{b^2 x^{1-n} (e x)^{-1+2n} \operatorname{Coth}[c] (a + b \operatorname{Csch}[c + d x^n])^2 \operatorname{Sinh}[c + d x^n]^2}{d n (b + a \operatorname{Sinh}[c + d x^n])^2} + \\
& \frac{x^{1-n} (e x)^{-1+2n} \operatorname{Csch}\left[\frac{c}{2}\right] (a + b \operatorname{Csch}[c + d x^n])^2 \operatorname{Sech}\left[\frac{c}{2}\right] (-2 b^2 \operatorname{Cosh}[c] + a^2 d x^n \operatorname{Sinh}[c]) \operatorname{Sinh}[c + d x^n]^2}{4 d n (b + a \operatorname{Sinh}[c + d x^n])^2} - \\
& \left(b^2 x^{1-2n} (e x)^{-1+2n} \operatorname{Csch}[c] (a + b \operatorname{Csch}[c + d x^n])^2 (-d x^n \operatorname{Cosh}[c] + \operatorname{Log}[\operatorname{Cosh}[d x^n] \operatorname{Sinh}[c] + \operatorname{Cosh}[c] \operatorname{Sinh}[d x^n]]) \operatorname{Sinh}[c] \right) \operatorname{Sinh}[c + d x^n]^2 \Big/ \\
& \left(d^2 n (-\operatorname{Cosh}[c]^2 + \operatorname{Sinh}[c]^2) (b + a \operatorname{Sinh}[c + d x^n])^2 \right) + \frac{b^2 x^{1-n} (e x)^{-1+2n} \operatorname{Csch}\left[\frac{c}{2}\right] \operatorname{Csch}\left[\frac{c}{2} + \frac{d x^n}{2}\right] (a + b \operatorname{Csch}[c + d x^n])^2 \operatorname{Sinh}\left[\frac{d x^n}{2}\right] \operatorname{Sinh}[c + d x^n]^2}{2 d n (b + a \operatorname{Sinh}[c + d x^n])^2} - \\
& \frac{b^2 x^{1-n} (e x)^{-1+2n} (a + b \operatorname{Csch}[c + d x^n])^2 \operatorname{Sech}\left[\frac{c}{2}\right] \operatorname{Sech}\left[\frac{c}{2} + \frac{d x^n}{2}\right] \operatorname{Sinh}\left[\frac{d x^n}{2}\right] \operatorname{Sinh}[c + d x^n]^2}{2 d n (b + a \operatorname{Sinh}[c + d x^n])^2} + \\
& \left(2 a b x^{1-2n} (e x)^{-1+2n} (a + b \operatorname{Csch}[c + d x^n])^2 \operatorname{Sinh}[c + d x^n]^2 \left(- \frac{2 \operatorname{ArcTan}\left[\frac{\operatorname{Cosh}[c] + \operatorname{Sinh}[c] \operatorname{Tanh}\left[\frac{d x^n}{2}\right]}{\sqrt{-\operatorname{Cosh}[c]^2 + \operatorname{Sinh}[c]^2}}\right] \operatorname{ArcTanh}[\operatorname{Tanh}[c]]}{\sqrt{-\operatorname{Cosh}[c]^2 + \operatorname{Sinh}[c]^2}} - \right. \right. \\
& \left. \frac{1}{\sqrt{1 - \operatorname{Tanh}[c]^2}} \operatorname{i} \left(\operatorname{i} (d x^n + \operatorname{ArcTanh}[\operatorname{Tanh}[c]]) \left(\operatorname{Log}[1 - e^{-d x^n - \operatorname{ArcTanh}[\operatorname{Tanh}[c]}] \right) - \operatorname{Log}[1 + e^{-d x^n - \operatorname{ArcTanh}[\operatorname{Tanh}[c]}] \right) \right) + \\
& \left. \left. \operatorname{i} \left(\operatorname{PolyLog}[2, -e^{-d x^n - \operatorname{ArcTanh}[\operatorname{Tanh}[c]}] \right) - \operatorname{PolyLog}[2, e^{-d x^n - \operatorname{ArcTanh}[\operatorname{Tanh}[c]}] \right) \right) \operatorname{Sech}[c] \right) \Big/ (d^2 n (b + a \operatorname{Sinh}[c + d x^n])^2)
\end{aligned}$$

Problem 77: Attempted integration timed out after 120 seconds.

$$\int (e x)^{-1+3n} (a + b \operatorname{Csch}[c + d x^n])^2 dx$$

Optimal (type 4, 344 leaves, 16 steps):

$$\begin{aligned}
& \frac{a^2 (e x)^{3n}}{3 e n} - \frac{b^2 x^{-n} (e x)^{3n}}{d e n} - \frac{4 a b x^{-n} (e x)^{3n} \operatorname{ArcTanh}[e^{c+d x^n}]}{d e n} - \frac{b^2 x^{-n} (e x)^{3n} \operatorname{Coth}[c + d x^n]}{d e n} + \\
& \frac{2 b^2 x^{-2n} (e x)^{3n} \operatorname{Log}[1 - e^{2(c+d x^n)}]}{d^2 e n} - \frac{4 a b x^{-2n} (e x)^{3n} \operatorname{PolyLog}[2, -e^{c+d x^n}]}{d^2 e n} + \frac{4 a b x^{-2n} (e x)^{3n} \operatorname{PolyLog}[2, e^{c+d x^n}]}{d^2 e n} + \\
& \frac{b^2 x^{-3n} (e x)^{3n} \operatorname{PolyLog}[2, e^{2(c+d x^n)}]}{d^3 e n} + \frac{4 a b x^{-3n} (e x)^{3n} \operatorname{PolyLog}[3, -e^{c+d x^n}]}{d^3 e n} - \frac{4 a b x^{-3n} (e x)^{3n} \operatorname{PolyLog}[3, e^{c+d x^n}]}{d^3 e n}
\end{aligned}$$

Result (type 1, 1 leaves):

???

Problem 79: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(e x)^{-1+2 n}}{a+b \operatorname{Csch}\left[c+d x^n\right]} dx$$

Optimal (type 4, 291 leaves, 12 steps):

$$\frac{(e x)^{2 n}}{2 a e n} - \frac{b x^{-n} (e x)^{2 n} \operatorname{Log}\left[1 + \frac{a e^{c+d x^n}}{b - \sqrt{a^2 + b^2}}\right]}{a \sqrt{a^2 + b^2} d e n} + \frac{b x^{-n} (e x)^{2 n} \operatorname{Log}\left[1 + \frac{a e^{c+d x^n}}{b + \sqrt{a^2 + b^2}}\right]}{a \sqrt{a^2 + b^2} d e n} -$$

$$\frac{b x^{-2 n} (e x)^{2 n} \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x^n}}{b - \sqrt{a^2 + b^2}}\right]}{a \sqrt{a^2 + b^2} d^2 e n} + \frac{b x^{-2 n} (e x)^{2 n} \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x^n}}{b + \sqrt{a^2 + b^2}}\right]}{a \sqrt{a^2 + b^2} d^2 e n}$$

Result (type 4, 1347 leaves):

$$\begin{aligned}
& \frac{x (e x)^{-1+2 n} \operatorname{Csch}[c+d x^n] (b+a \operatorname{Sinh}[c+d x^n])}{2 a n (a+b \operatorname{Csch}[c+d x^n])} + \\
& \frac{1}{a d^2 n (a+b \operatorname{Csch}[c+d x^n])} b x^{1-2 n} (e x)^{-1+2 n} \operatorname{Csch}[c+d x^n] \left(\frac{i \pi \operatorname{ArcTanh}\left[\frac{-a+b \operatorname{Tanh}\left[\frac{1}{2}(c+d x^n)\right]}{\sqrt{a^2+b^2}}\right]}{\sqrt{a^2+b^2}} + \frac{1}{\sqrt{-a^2-b^2}} \left(2 \left(-i c + \frac{\pi}{2} - i d x^n \right) \right. \right. \\
& \left. \left. \operatorname{ArcTanh}\left[\frac{(-i a+b) \operatorname{Cot}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right]}{\sqrt{-a^2-b^2}}\right] - 2 \left(-i c + \operatorname{ArcCos}\left[-\frac{i b}{a}\right] \right) \operatorname{ArcTanh}\left[\frac{(-i a-b) \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right]}{\sqrt{-a^2-b^2}}\right]} \right) + \right. \\
& \left. \left(\operatorname{ArcCos}\left[-\frac{i b}{a}\right] - 2 i \left(\operatorname{ArcTanh}\left[\frac{(-i a+b) \operatorname{Cot}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right]}{\sqrt{-a^2-b^2}}\right] - \operatorname{ArcTanh}\left[\frac{(-i a-b) \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right]}{\sqrt{-a^2-b^2}}\right] \right) \right) \right) \\
& \operatorname{Log}\left[\frac{\sqrt{-a^2-b^2} e^{-\frac{1}{2} i\left(-i c + \frac{\pi}{2} - i d x^n\right)}}{\sqrt{2} \sqrt{-i a} \sqrt{b+a \operatorname{Sinh}[c+d x^n]}}\right] + \\
& \left(\operatorname{ArcCos}\left[-\frac{i b}{a}\right] + 2 i \left(\operatorname{ArcTanh}\left[\frac{(-i a+b) \operatorname{Cot}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right]}{\sqrt{-a^2-b^2}}\right] - \operatorname{ArcTanh}\left[\frac{(-i a-b) \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right]}{\sqrt{-a^2-b^2}}\right] \right) \right) \\
& \operatorname{Log}\left[\frac{\sqrt{-a^2-b^2} e^{\frac{1}{2} i\left(-i c + \frac{\pi}{2} - i d x^n\right)}}{\sqrt{2} \sqrt{-i a} \sqrt{b+a \operatorname{Sinh}[c+d x^n]}}\right] - \left(\operatorname{ArcCos}\left[-\frac{i b}{a}\right] + 2 i \operatorname{ArcTanh}\left[\frac{(-i a-b) \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right]}{\sqrt{-a^2-b^2}}\right] \right) \\
& \operatorname{Log}\left[1 - \frac{i (b - i \sqrt{-a^2-b^2}) (-i a + b - \sqrt{-a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right])}{a (-i a + b + \sqrt{-a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right])}\right] + \left(-\operatorname{ArcCos}\left[-\frac{i b}{a}\right] + \right. \\
& \left. 2 i \operatorname{ArcTanh}\left[\frac{(-i a-b) \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right]}{\sqrt{-a^2-b^2}}\right] \right) \operatorname{Log}\left[1 - \frac{i (b + i \sqrt{-a^2-b^2}) (-i a + b - \sqrt{-a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right])}{a (-i a + b + \sqrt{-a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right])}\right] + \\
& i \left(\operatorname{PolyLog}\left[2, \frac{i (b - i \sqrt{-a^2-b^2}) (-i a + b - \sqrt{-a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right])}{a (-i a + b + \sqrt{-a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right])}\right] - \right. \\
& \left. \operatorname{PolyLog}\left[2, \frac{i (b + i \sqrt{-a^2-b^2}) (-i a + b - \sqrt{-a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right])}{a (-i a + b + \sqrt{-a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right])}\right] \right) \right) (b+a \operatorname{Sinh}[c+d x^n])
\end{aligned}$$

Problem 80: Unable to integrate problem.

$$\int \frac{(e x)^{-1+3 n}}{a+b \operatorname{Csch}[c+d x^n]} dx$$

Optimal (type 4, 428 leaves, 14 steps):

$$\frac{(e x)^{3 n}}{3 a e n} - \frac{b x^{-n} (e x)^{3 n} \operatorname{Log}\left[1 + \frac{a e^{c+d x^n}}{b-\sqrt{a^2+b^2}}\right]}{a \sqrt{a^2+b^2} d e n} + \frac{b x^{-n} (e x)^{3 n} \operatorname{Log}\left[1 + \frac{a e^{c+d x^n}}{b+\sqrt{a^2+b^2}}\right]}{a \sqrt{a^2+b^2} d e n} - \frac{2 b x^{-2 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x^n}}{b-\sqrt{a^2+b^2}}\right]}{a \sqrt{a^2+b^2} d^2 e n} +$$

$$\frac{2 b x^{-2 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x^n}}{b+\sqrt{a^2+b^2}}\right]}{a \sqrt{a^2+b^2} d^2 e n} + \frac{2 b x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[3, -\frac{a e^{c+d x^n}}{b-\sqrt{a^2+b^2}}\right]}{a \sqrt{a^2+b^2} d^3 e n} - \frac{2 b x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[3, -\frac{a e^{c+d x^n}}{b+\sqrt{a^2+b^2}}\right]}{a \sqrt{a^2+b^2} d^3 e n}$$

Result (type 8, 26 leaves):

$$\int \frac{(e x)^{-1+3 n}}{a+b \operatorname{Csch}[c+d x^n]} dx$$

Problem 82: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(e x)^{-1+2 n}}{(a+b \operatorname{Csch}[c+d x^n])^2} dx$$

Optimal (type 4, 681 leaves, 23 steps):

$$\frac{(e x)^{2 n}}{2 a^2 e n} + \frac{b^3 x^{-n} (e x)^{2 n} \operatorname{Log}\left[1 + \frac{a e^{c+d x^n}}{b-\sqrt{a^2+b^2}}\right]}{a^2 (a^2+b^2)^{3/2} d e n} - \frac{2 b x^{-n} (e x)^{2 n} \operatorname{Log}\left[1 + \frac{a e^{c+d x^n}}{b-\sqrt{a^2+b^2}}\right]}{a^2 \sqrt{a^2+b^2} d e n} - \frac{b^3 x^{-n} (e x)^{2 n} \operatorname{Log}\left[1 + \frac{a e^{c+d x^n}}{b+\sqrt{a^2+b^2}}\right]}{a^2 (a^2+b^2)^{3/2} d e n} + \frac{2 b x^{-n} (e x)^{2 n} \operatorname{Log}\left[1 + \frac{a e^{c+d x^n}}{b+\sqrt{a^2+b^2}}\right]}{a^2 \sqrt{a^2+b^2} d e n} +$$

$$\frac{b^2 x^{-2 n} (e x)^{2 n} \operatorname{Log}[b+a \operatorname{Sinh}[c+d x^n]]}{a^2 (a^2+b^2) d^2 e n} + \frac{b^3 x^{-2 n} (e x)^{2 n} \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x^n}}{b-\sqrt{a^2+b^2}}\right]}{a^2 (a^2+b^2)^{3/2} d^2 e n} - \frac{2 b x^{-2 n} (e x)^{2 n} \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x^n}}{b+\sqrt{a^2+b^2}}\right]}{a^2 \sqrt{a^2+b^2} d^2 e n} -$$

$$\frac{b^3 x^{-2 n} (e x)^{2 n} \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x^n}}{b+\sqrt{a^2+b^2}}\right]}{a^2 (a^2+b^2)^{3/2} d^2 e n} + \frac{2 b x^{-2 n} (e x)^{2 n} \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x^n}}{b+\sqrt{a^2+b^2}}\right]}{a^2 \sqrt{a^2+b^2} d^2 e n} - \frac{b^2 x^{-n} (e x)^{2 n} \operatorname{Cosh}[c+d x^n]}{a (a^2+b^2) d e n (b+a \operatorname{Sinh}[c+d x^n])}$$

Result (type 4, 3256 leaves):

$$\frac{b^2 x^{1-n} (e x)^{-1+2 n} \operatorname{Csch}\left[\frac{c}{2}\right] \operatorname{Csch}[c+d x^n]^2 \operatorname{Sech}\left[\frac{c}{2}\right] (b \operatorname{Cosh}[c]+a \operatorname{Sinh}[d x^n]) (b+a \operatorname{Sinh}[c+d x^n])}{2 a^2 (a^2+b^2) d n (a+b \operatorname{Csch}[c+d x^n])^2} +$$

$$\begin{aligned}
& \frac{b^2 x^{1-n} (e x)^{-1+2n} \operatorname{Coth}[c] \operatorname{Csch}[c+d x^n]^2 (b+a \operatorname{Sinh}[c+d x^n])^2}{a^2 (a^2+b^2) d n (a+b \operatorname{Csch}[c+d x^n])^2} - \\
& \frac{2 b^3 x^{1-2n} (e x)^{-1+2n} \operatorname{ArcTan}\left[\frac{a-b \operatorname{Tanh}\left[\frac{1}{2}(c+d x^n)\right]}{\sqrt{-a^2-b^2}}\right] \operatorname{Coth}[c] \operatorname{Csch}[c+d x^n]^2 (b+a \operatorname{Sinh}[c+d x^n])^2}{a^2 \sqrt{-a^2-b^2} (a^2+b^2) d^2 n (a+b \operatorname{Csch}[c+d x^n])^2} + \\
& \frac{1}{(a^2+b^2) d^2 n (a+b \operatorname{Csch}[c+d x^n])^2} 2 b x^{1-2n} (e x)^{-1+2n} \operatorname{Csch}[c+d x^n]^2 \\
& \left(\frac{i \pi \operatorname{ArcTanh}\left[\frac{-a+b \operatorname{Tanh}\left[\frac{1}{2}(c+d x^n)\right]}{\sqrt{a^2+b^2}}\right]}{\sqrt{a^2+b^2}} + \frac{1}{\sqrt{-a^2-b^2}} \left(2 \left(-i c + \frac{\pi}{2} - i d x^n\right) \operatorname{ArcTanh}\left[\frac{(-i a+b) \operatorname{Cot}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right]}{\sqrt{-a^2-b^2}}\right]} \right) - \right. \\
& 2 \left(-i c + \operatorname{ArcCos}\left[-\frac{i b}{a}\right]\right) \operatorname{ArcTanh}\left[\frac{(-i a-b) \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right]}{\sqrt{-a^2-b^2}}\right] + \\
& \left. \left(\operatorname{ArcCos}\left[-\frac{i b}{a}\right] - 2 i \left(\operatorname{ArcTanh}\left[\frac{(-i a+b) \operatorname{Cot}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right]}{\sqrt{-a^2-b^2}}\right]} - \operatorname{ArcTanh}\left[\frac{(-i a-b) \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right]}{\sqrt{-a^2-b^2}}\right]} \right) \right) \\
& \operatorname{Log}\left[\frac{\sqrt{-a^2-b^2} e^{-\frac{1}{2} i \left(-i c + \frac{\pi}{2} - i d x^n\right)}}{\sqrt{2} \sqrt{-i a} \sqrt{b+a \operatorname{Sinh}[c+d x^n]}}\right] + \\
& \left(\operatorname{ArcCos}\left[-\frac{i b}{a}\right] + 2 i \left(\operatorname{ArcTanh}\left[\frac{(-i a+b) \operatorname{Cot}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right]}{\sqrt{-a^2-b^2}}\right]} - \operatorname{ArcTanh}\left[\frac{(-i a-b) \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right]}{\sqrt{-a^2-b^2}}\right]} \right) \right) \\
& \operatorname{Log}\left[\frac{\sqrt{-a^2-b^2} e^{\frac{1}{2} i \left(-i c + \frac{\pi}{2} - i d x^n\right)}}{\sqrt{2} \sqrt{-i a} \sqrt{b+a \operatorname{Sinh}[c+d x^n]}}\right] - \left(\operatorname{ArcCos}\left[-\frac{i b}{a}\right] + 2 i \operatorname{ArcTanh}\left[\frac{(-i a-b) \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right]}{\sqrt{-a^2-b^2}}\right]} \right) \\
& \operatorname{Log}\left[1 - \frac{i (b-i \sqrt{-a^2-b^2}) \left(-i a+b - \sqrt{-a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right]\right)}{a \left(-i a+b + \sqrt{-a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right]\right)}\right] + \left(-\operatorname{ArcCos}\left[-\frac{i b}{a}\right] + \right. \\
& \left. 2 i \operatorname{ArcTanh}\left[\frac{(-i a-b) \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right]}{\sqrt{-a^2-b^2}}\right]\right) \operatorname{Log}\left[1 - \frac{i (b+i \sqrt{-a^2-b^2}) \left(-i a+b - \sqrt{-a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right]\right)}{a \left(-i a+b + \sqrt{-a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right]\right)}\right] + \\
& i \left(\operatorname{PolyLog}\left[2, \frac{i (b-i \sqrt{-a^2-b^2}) \left(-i a+b - \sqrt{-a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right]\right)}{a \left(-i a+b + \sqrt{-a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right]\right)}\right] - \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \text{PolyLog}\left[2, \frac{i(b + i\sqrt{-a^2 - b^2})(-ia + b - \sqrt{-a^2 - b^2} \tan[\frac{1}{2}(-ic + \frac{\pi}{2} - id x^n)])}{a(-ia + b + \sqrt{-a^2 - b^2} \tan[\frac{1}{2}(-ic + \frac{\pi}{2} - id x^n)])}\right]\right) \left. \right) (b + a \sinh[c + dx^n])^2 + \\
& \frac{1}{a^2(a^2 + b^2)d^2n(a + b \operatorname{Csch}[c + dx^n])^2} b^3 x^{1-2n} (ex)^{-1+2n} \operatorname{Csch}[c + dx^n]^2 \left(\frac{i\pi \operatorname{ArcTanh}\left[\frac{-a+b \operatorname{Tanh}\left[\frac{1}{2}(c+dx^n)\right]}{\sqrt{a^2+b^2}}\right]}{\sqrt{a^2+b^2}} + \right. \\
& \frac{1}{\sqrt{-a^2-b^2}} \left(2(-ic + \frac{\pi}{2} - id x^n) \operatorname{ArcTanh}\left[\frac{(-ia+b) \operatorname{Cot}\left[\frac{1}{2}(-ic + \frac{\pi}{2} - id x^n)\right]}{\sqrt{-a^2-b^2}}\right] - \right. \\
& 2\left(-ic + \operatorname{ArcCos}\left[-\frac{ib}{a}\right]\right) \operatorname{ArcTanh}\left[\frac{(-ia-b) \tan\left[\frac{1}{2}(-ic + \frac{\pi}{2} - id x^n)\right]}{\sqrt{-a^2-b^2}}\right] + \\
& \left. \left(\operatorname{ArcCos}\left[-\frac{ib}{a}\right] - 2i \left(\operatorname{ArcTanh}\left[\frac{(-ia+b) \operatorname{Cot}\left[\frac{1}{2}(-ic + \frac{\pi}{2} - id x^n)\right]}{\sqrt{-a^2-b^2}}\right] - \operatorname{ArcTanh}\left[\frac{(-ia-b) \tan\left[\frac{1}{2}(-ic + \frac{\pi}{2} - id x^n)\right]}{\sqrt{-a^2-b^2}}\right] \right) \right) \right) \\
& \operatorname{Log}\left[\frac{\sqrt{-a^2-b^2} e^{-\frac{1}{2}i(-ic + \frac{\pi}{2} - id x^n)}}{\sqrt{2} \sqrt{-ia} \sqrt{b+a \sinh[c + dx^n]}}\right] + \\
& \left(\operatorname{ArcCos}\left[-\frac{ib}{a}\right] + 2i \left(\operatorname{ArcTanh}\left[\frac{(-ia+b) \operatorname{Cot}\left[\frac{1}{2}(-ic + \frac{\pi}{2} - id x^n)\right]}{\sqrt{-a^2-b^2}}\right] - \operatorname{ArcTanh}\left[\frac{(-ia-b) \tan\left[\frac{1}{2}(-ic + \frac{\pi}{2} - id x^n)\right]}{\sqrt{-a^2-b^2}}\right] \right) \right) \\
& \operatorname{Log}\left[\frac{\sqrt{-a^2-b^2} e^{\frac{1}{2}i(-ic + \frac{\pi}{2} - id x^n)}}{\sqrt{2} \sqrt{-ia} \sqrt{b+a \sinh[c + dx^n]}}\right] - \left(\operatorname{ArcCos}\left[-\frac{ib}{a}\right] + 2i \operatorname{ArcTanh}\left[\frac{(-ia-b) \tan\left[\frac{1}{2}(-ic + \frac{\pi}{2} - id x^n)\right]}{\sqrt{-a^2-b^2}}\right] \right) \\
& \operatorname{Log}\left[1 - \frac{i(b - i\sqrt{-a^2 - b^2})(-ia + b - \sqrt{-a^2 - b^2} \tan[\frac{1}{2}(-ic + \frac{\pi}{2} - id x^n)])}{a(-ia + b + \sqrt{-a^2 - b^2} \tan[\frac{1}{2}(-ic + \frac{\pi}{2} - id x^n)])}\right] + \left(-\operatorname{ArcCos}\left[-\frac{ib}{a}\right] + \right. \\
& \left. 2i \operatorname{ArcTanh}\left[\frac{(-ia-b) \tan\left[\frac{1}{2}(-ic + \frac{\pi}{2} - id x^n)\right]}{\sqrt{-a^2-b^2}}\right] \right) \operatorname{Log}\left[1 - \frac{i(b + i\sqrt{-a^2 - b^2})(-ia + b - \sqrt{-a^2 - b^2} \tan[\frac{1}{2}(-ic + \frac{\pi}{2} - id x^n)])}{a(-ia + b + \sqrt{-a^2 - b^2} \tan[\frac{1}{2}(-ic + \frac{\pi}{2} - id x^n)])}\right] + \\
& i \left(\operatorname{PolyLog}\left[2, \frac{i(b - i\sqrt{-a^2 - b^2})(-ia + b - \sqrt{-a^2 - b^2} \tan[\frac{1}{2}(-ic + \frac{\pi}{2} - id x^n)])}{a(-ia + b + \sqrt{-a^2 - b^2} \tan[\frac{1}{2}(-ic + \frac{\pi}{2} - id x^n)])}\right] - \right.
\end{aligned}$$

$$\begin{aligned}
 & \left. \left(\text{PolyLog}\left[2, \frac{i \left(b + i \sqrt{-a^2 - b^2} \right) \left(-i a + b - \sqrt{-a^2 - b^2} \operatorname{Tan}\left[\frac{1}{2} \left(-i c + \frac{\pi}{2} - i d x^n \right) \right] \right)}{a \left(-i a + b + \sqrt{-a^2 - b^2} \operatorname{Tan}\left[\frac{1}{2} \left(-i c + \frac{\pi}{2} - i d x^n \right) \right] \right)} \right] \right) \right) \left(b + a \operatorname{Sinh}[c + d x^n] \right)^2 + \\
 & \left(x^{1-n} (e x)^{-1+2n} \operatorname{Csch}\left[\frac{c}{2}\right] \operatorname{Csch}[c + d x^n]^2 \operatorname{Sech}\left[\frac{c}{2}\right] \left(-2 b^2 \operatorname{Cosh}[c] + a^2 d x^n \operatorname{Sinh}[c] + b^2 d x^n \operatorname{Sinh}[c] \right) \right. \\
 & \quad \left. (b + a \operatorname{Sinh}[c + d x^n])^2 \right) / \\
 & \left(4 a^2 (a^2 + b^2) d n (a + b \operatorname{Csch}[c + d x^n])^2 \right) - \left(b^2 x^{1-2n} (e x)^{-1+2n} \operatorname{Csch}[c] \right. \\
 & \quad \left. \operatorname{Csch}[c + d x^n]^2 \right. \\
 & \quad \left. \left(-a d x^n \operatorname{Cosh}[c] + a \operatorname{Log}\left[b + a \operatorname{Cosh}[d x^n] \operatorname{Sinh}[c] + a \operatorname{Cosh}[c] \operatorname{Sinh}[d x^n] \right] \operatorname{Sinh}[c] + \frac{2 a b \operatorname{ArcTan}\left[\frac{a \operatorname{Cosh}[c] + (-b + a \operatorname{Sinh}[c]) \operatorname{Tanh}\left[\frac{d x^n}{2}\right]}{\sqrt{-b^2 - a^2 \operatorname{Cosh}[c]^2 + a^2 \operatorname{Sinh}[c]^2}} \right]}{\sqrt{-b^2 - a^2 \operatorname{Cosh}[c]^2 + a^2 \operatorname{Sinh}[c]^2}} \right] \operatorname{Cosh}[c] \right) \right. \\
 & \quad \left. (b + a \operatorname{Sinh}[c + d x^n])^2 \right) / \\
 & \left(a (a^2 + b^2) d^2 n (a + b \operatorname{Csch}[c + d x^n])^2 \left(-a^2 \operatorname{Cosh}[c]^2 + a^2 \operatorname{Sinh}[c]^2 \right) \right)
 \end{aligned}$$

Problem 83: Attempted integration timed out after 120 seconds.

$$\int \frac{(e x)^{-1+3n}}{(a + b \operatorname{Csch}[c + d x^n])^2} dx$$

Optimal (type 4, 1218 leaves, 32 steps):

$$\begin{aligned}
& \frac{(e x)^{3 n}}{3 a^2 e n} - \frac{b^2 x^{-n} (e x)^{3 n}}{a^2 (a^2 + b^2) d e n} + \frac{2 b^2 x^{-2 n} (e x)^{3 n} \operatorname{Log}\left[1 + \frac{a e^{c+d x^n}}{b-\sqrt{a^2+b^2}}\right]}{a^2 (a^2 + b^2) d^2 e n} + \frac{b^3 x^{-n} (e x)^{3 n} \operatorname{Log}\left[1 + \frac{a e^{c+d x^n}}{b-\sqrt{a^2+b^2}}\right]}{a^2 (a^2 + b^2)^{3/2} d e n} - \\
& \frac{2 b x^{-n} (e x)^{3 n} \operatorname{Log}\left[1 + \frac{a e^{c+d x^n}}{b-\sqrt{a^2+b^2}}\right]}{a^2 \sqrt{a^2 + b^2} d e n} + \frac{2 b^2 x^{-2 n} (e x)^{3 n} \operatorname{Log}\left[1 + \frac{a e^{c+d x^n}}{b+\sqrt{a^2+b^2}}\right]}{a^2 (a^2 + b^2) d^2 e n} - \frac{b^3 x^{-n} (e x)^{3 n} \operatorname{Log}\left[1 + \frac{a e^{c+d x^n}}{b+\sqrt{a^2+b^2}}\right]}{a^2 (a^2 + b^2)^{3/2} d e n} + \\
& \frac{2 b x^{-n} (e x)^{3 n} \operatorname{Log}\left[1 + \frac{a e^{c+d x^n}}{b+\sqrt{a^2+b^2}}\right]}{a^2 \sqrt{a^2 + b^2} d e n} + \frac{2 b^2 x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x^n}}{b-\sqrt{a^2+b^2}}\right]}{a^2 (a^2 + b^2) d^3 e n} + \frac{2 b^3 x^{-2 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x^n}}{b-\sqrt{a^2+b^2}}\right]}{a^2 (a^2 + b^2)^{3/2} d^2 e n} - \\
& \frac{4 b x^{-2 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x^n}}{b-\sqrt{a^2+b^2}}\right]}{a^2 \sqrt{a^2 + b^2} d^2 e n} + \frac{2 b^2 x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x^n}}{b+\sqrt{a^2+b^2}}\right]}{a^2 (a^2 + b^2) d^3 e n} - \frac{2 b^3 x^{-2 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x^n}}{b+\sqrt{a^2+b^2}}\right]}{a^2 (a^2 + b^2)^{3/2} d^2 e n} + \\
& \frac{4 b x^{-2 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x^n}}{b+\sqrt{a^2+b^2}}\right]}{a^2 \sqrt{a^2 + b^2} d^2 e n} - \frac{2 b^3 x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[3, -\frac{a e^{c+d x^n}}{b-\sqrt{a^2+b^2}}\right]}{a^2 (a^2 + b^2)^{3/2} d^3 e n} + \frac{4 b x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[3, -\frac{a e^{c+d x^n}}{b-\sqrt{a^2+b^2}}\right]}{a^2 \sqrt{a^2 + b^2} d^3 e n} + \\
& \frac{2 b^3 x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[3, -\frac{a e^{c+d x^n}}{b+\sqrt{a^2+b^2}}\right]}{a^2 (a^2 + b^2)^{3/2} d^3 e n} - \frac{4 b x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[3, -\frac{a e^{c+d x^n}}{b+\sqrt{a^2+b^2}}\right]}{a^2 \sqrt{a^2 + b^2} d^3 e n} - \frac{b^2 x^{-n} (e x)^{3 n} \operatorname{Cosh}[c + d x^n]}{a (a^2 + b^2) d e n (b + a \operatorname{Sinh}[c + d x^n])}
\end{aligned}$$

Result (type 1, 1 leaves):

???

Test results for the 175 problems in "6.6.3 Hyperbolic cosecant functions.m"

Problem 1: Result more than twice size of optimal antiderivative.

$$\int \operatorname{CsCh}[a + b x] dx$$

Optimal (type 3, 12 leaves, 1 step):

$$\frac{\operatorname{ArcTanh}[\operatorname{Cosh}[a + b x]]}{b}$$

Result (type 3, 38 leaves):

$$-\frac{\text{Log}\left[\text{Cosh}\left[\frac{a}{2} + \frac{bx}{2}\right]\right]}{b} + \frac{\text{Log}\left[\text{Sinh}\left[\frac{a}{2} + \frac{bx}{2}\right]\right]}{b}$$

Problem 3: Result more than twice size of optimal antiderivative.

$$\int \text{Csch}[a + bx]^3 dx$$

Optimal (type 3, 34 leaves, 2 steps):

$$\frac{\text{ArcTanh}[\text{Cosh}[a + bx]]}{2b} - \frac{\text{Coth}[a + bx] \text{Csch}[a + bx]}{2b}$$

Result (type 3, 75 leaves):

$$-\frac{\text{Csch}\left[\frac{1}{2}(a + bx)\right]^2}{8b} + \frac{\text{Log}\left[\text{Cosh}\left[\frac{1}{2}(a + bx)\right]\right]}{2b} - \frac{\text{Log}\left[\text{Sinh}\left[\frac{1}{2}(a + bx)\right]\right]}{2b} - \frac{\text{Sech}\left[\frac{1}{2}(a + bx)\right]^2}{8b}$$

Problem 5: Result more than twice size of optimal antiderivative.

$$\int \text{Csch}[a + bx]^5 dx$$

Optimal (type 3, 55 leaves, 3 steps):

$$-\frac{3 \text{ArcTanh}[\text{Cosh}[a + bx]]}{8b} + \frac{3 \text{Coth}[a + bx] \text{Csch}[a + bx]}{8b} - \frac{\text{Coth}[a + bx] \text{Csch}[a + bx]^3}{4b}$$

Result (type 3, 113 leaves):

$$\frac{3 \text{Csch}\left[\frac{1}{2}(a + bx)\right]^2}{32b} - \frac{\text{Csch}\left[\frac{1}{2}(a + bx)\right]^4}{64b} - \frac{3 \text{Log}\left[\text{Cosh}\left[\frac{1}{2}(a + bx)\right]\right]}{8b} + \frac{3 \text{Log}\left[\text{Sinh}\left[\frac{1}{2}(a + bx)\right]\right]}{8b} + \frac{3 \text{Sech}\left[\frac{1}{2}(a + bx)\right]^2}{32b} + \frac{\text{Sech}\left[\frac{1}{2}(a + bx)\right]^4}{64b}$$

Problem 23: Result more than twice size of optimal antiderivative.

$$\int (-\text{Csch}[x]^2)^{3/2} dx$$

Optimal (type 3, 24 leaves, 3 steps):

$$\frac{1}{2} \text{ArcSin}[\text{Coth}[x]] + \frac{1}{2} \text{Coth}[x] \sqrt{-\text{Csch}[x]^2}$$

Result (type 3, 51 leaves):

$$\frac{1}{8} \sqrt{-\operatorname{Csch}[x]^2} \left(\operatorname{Csch}\left[\frac{x}{2}\right]^2 - 4 \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{x}{2}\right]\right] + 4 \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{x}{2}\right]\right] + \operatorname{Sech}\left[\frac{x}{2}\right]^2 \right) \operatorname{Sinh}[x]$$

Problem 24: Result more than twice size of optimal antiderivative.

$$\int \sqrt{-\operatorname{Csch}[x]^2} \, dx$$

Optimal (type 3, 3 leaves, 2 steps):

$$\operatorname{ArcSin}[\operatorname{Coth}[x]]$$

Result (type 3, 30 leaves):

$$\sqrt{-\operatorname{Csch}[x]^2} \left(-\operatorname{Log}\left[\operatorname{Cosh}\left[\frac{x}{2}\right]\right] + \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{x}{2}\right]\right] \right) \operatorname{Sinh}[x]$$

Problem 54: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{a + i a \operatorname{Csch}[c + d x]}} \, dx$$

Optimal (type 3, 91 leaves, 5 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Coth}[c + d x]}{\sqrt{a + i a \operatorname{Csch}[c + d x]}}\right]}{\sqrt{a} d} - \frac{\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Coth}[c + d x]}{\sqrt{2} \sqrt{a + i a \operatorname{Csch}[c + d x]}}\right]}{\sqrt{a} d}$$

Result (type 3, 254 leaves):

$$\left(\sqrt{a} \operatorname{Coth}[c + d x] \left(\sqrt{2} \operatorname{ArcTan}\left[\frac{\sqrt{2} \sqrt{a}}{\sqrt{i a (i + \operatorname{Csch}[c + d x])}}\right] - i \left(\operatorname{Log}\left[-\frac{2 a (-2 i \sqrt{a} + \sqrt{i a (i + \operatorname{Csch}[c + d x])} + i \sqrt{a + i a \operatorname{Csch}[c + d x]})}{-\sqrt{a} + \sqrt{a + i a \operatorname{Csch}[c + d x]}}\right] \right) + \operatorname{Log}\left[\frac{2 i a (2 \sqrt{a} + i \sqrt{i a (i + \operatorname{Csch}[c + d x])} + \sqrt{a + i a \operatorname{Csch}[c + d x]})}{\sqrt{a} + \sqrt{a + i a \operatorname{Csch}[c + d x]}}\right] \right) \right) / \left(d \sqrt{i a (i + \operatorname{Csch}[c + d x])} \sqrt{a + i a \operatorname{Csch}[c + d x]} \right)$$

Problem 55: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + i a \operatorname{Csch}[c + d x])^{3/2}} \, dx$$

Optimal (type 3, 123 leaves, 6 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Coth}[c+dx]}{\sqrt{a+i a \operatorname{Csch}[c+dx]}}\right]}{a^{3/2} d} - \frac{5 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Coth}[c+dx]}{\sqrt{2} \sqrt{a+i a \operatorname{Csch}[c+dx]}}\right]}{2 \sqrt{2} a^{3/2} d} - \frac{\operatorname{Coth}[c+dx]}{2 d (a+i a \operatorname{Csch}[c+dx])^{3/2}}$$

Result (type 3, 380 leaves):

$$\left(i \left(\left(a^{3/2} \operatorname{Coth}[c+dx] \left(-4 i \sqrt{2} \operatorname{ArcTan}\left[\frac{\sqrt{2} \sqrt{a}}{\sqrt{i a (i + \operatorname{Csch}[c+dx])}}\right] + \sqrt{2} \operatorname{Log}\left[-\frac{2(-i \sqrt{2} \sqrt{a} + \sqrt{i a (i + \operatorname{Csch}[c+dx])})}{\sqrt{a+i a \operatorname{Csch}[c+dx]}}\right] \right) - 4 \left(\operatorname{Log}\left[-\frac{2 a(-2 i \sqrt{a} + \sqrt{i a (i + \operatorname{Csch}[c+dx])} + i \sqrt{a+i a \operatorname{Csch}[c+dx]})}{-\sqrt{a} + \sqrt{a+i a \operatorname{Csch}[c+dx]}}\right] + \operatorname{Log}\left[\frac{2 i a(2 \sqrt{a} + i \sqrt{i a (i + \operatorname{Csch}[c+dx])} + \sqrt{a+i a \operatorname{Csch}[c+dx]})}{\sqrt{a} + \sqrt{a+i a \operatorname{Csch}[c+dx]}}\right] \right) \right) \right) / \left(\sqrt{i a (i + \operatorname{Csch}[c+dx])} + \frac{2 a \left(\operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right] + i \operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right] \right)}{\operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right] - i \operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right]} \right) / \left(4 a^2 d \sqrt{a+i a \operatorname{Csch}[c+dx]} \right)$$

Problem 57: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{a-i a \operatorname{Csch}[c+dx]}} dx$$

Optimal (type 3, 91 leaves, 5 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Coth}[c+dx]}{\sqrt{a-i a \operatorname{Csch}[c+dx]}}\right]}{\sqrt{a} d} - \frac{\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Coth}[c+dx]}{\sqrt{2} \sqrt{a-i a \operatorname{Csch}[c+dx]}}\right]}{\sqrt{a} d}$$

Result (type 3, 253 leaves):

$$\left(\sqrt{a} \operatorname{Coth}[c+dx] \left(\sqrt{2} \operatorname{ArcTan}\left[\frac{\sqrt{2} \sqrt{a}}{\sqrt{-i a (-i + \operatorname{Csch}[c+dx])}}\right] - i \left(\operatorname{Log}\left[-\frac{2 a(-2 i \sqrt{a} + \sqrt{-i a (-i + \operatorname{Csch}[c+dx])} + i \sqrt{a-i a \operatorname{Csch}[c+dx]})}{-\sqrt{a} + \sqrt{a-i a \operatorname{Csch}[c+dx]}}\right] + \operatorname{Log}\left[\frac{2 i a(2 \sqrt{a} + i \sqrt{-i a (-i + \operatorname{Csch}[c+dx])} + \sqrt{a-i a \operatorname{Csch}[c+dx]})}{\sqrt{a} + \sqrt{a-i a \operatorname{Csch}[c+dx]}}\right] \right) \right) \right) / \left(d \sqrt{a(-1-i \operatorname{Csch}[c+dx])} \sqrt{a-i a \operatorname{Csch}[c+dx]} \right)$$

Problem 60: Result more than twice size of optimal antiderivative.

$$\int \sqrt{-3 + 3 i \operatorname{Csch}[x]} \, dx$$

Optimal (type 3, 23 leaves, 2 steps):

$$-2 \sqrt{3} \operatorname{ArcTan}\left[\frac{\operatorname{Coth}[x]}{\sqrt{-1 + i \operatorname{Csch}[x]}}\right]$$

Result (type 3, 67 leaves):

$$\frac{\sqrt{3} \operatorname{Coth}[x] \left(\operatorname{Log}\left[1 - \sqrt{1 + i \operatorname{Csch}[x]}\right] - \operatorname{Log}\left[1 + \sqrt{1 + i \operatorname{Csch}[x]}\right] \right)}{\sqrt{-1 + i \operatorname{Csch}[x]} \sqrt{1 + i \operatorname{Csch}[x]}}$$

Problem 61: Result more than twice size of optimal antiderivative.

$$\int \sqrt{-3 - 3 i \operatorname{Csch}[x]} \, dx$$

Optimal (type 3, 23 leaves, 2 steps):

$$-2 \sqrt{3} \operatorname{ArcTan}\left[\frac{\operatorname{Coth}[x]}{\sqrt{-1 - i \operatorname{Csch}[x]}}\right]$$

Result (type 3, 67 leaves):

$$\frac{\sqrt{3} \operatorname{Coth}[x] \left(\operatorname{Log}\left[1 - \sqrt{1 - i \operatorname{Csch}[x]}\right] - \operatorname{Log}\left[1 + \sqrt{1 - i \operatorname{Csch}[x]}\right] \right)}{\sqrt{-1 - i \operatorname{Csch}[x]} \sqrt{1 - i \operatorname{Csch}[x]}}$$

Problem 67: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Csch}[x]^2}{i + \operatorname{Csch}[x]} \, dx$$

Optimal (type 3, 17 leaves, 3 steps):

$$-\operatorname{ArcTanh}[\operatorname{Cosh}[x]] + \frac{\operatorname{Coth}[x]}{i + \operatorname{Csch}[x]}$$

Result (type 3, 46 leaves):

$$-\operatorname{Log}\left[\operatorname{Cosh}\left[\frac{x}{2}\right]\right] + \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{x}{2}\right]\right] - \frac{2 i \operatorname{Sinh}\left[\frac{x}{2}\right]}{\operatorname{Cosh}\left[\frac{x}{2}\right] + i \operatorname{Sinh}\left[\frac{x}{2}\right]}$$

Problem 68: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Csch}[x]^3}{i + \text{Csch}[x]} dx$$

Optimal (type 3, 26 leaves, 4 steps):

$$i \text{ArcTanh}[\text{Cosh}[x]] - \text{Coth}[x] - \frac{i \text{Coth}[x]}{i + \text{Csch}[x]}$$

Result (type 3, 70 leaves):

$$-\frac{1}{2} \text{Coth}\left[\frac{x}{2}\right] + i \text{Log}\left[\text{Cosh}\left[\frac{x}{2}\right]\right] - i \text{Log}\left[\text{Sinh}\left[\frac{x}{2}\right]\right] - \frac{2 \text{Sinh}\left[\frac{x}{2}\right]}{\text{Cosh}\left[\frac{x}{2}\right] + i \text{Sinh}\left[\frac{x}{2}\right]} - \frac{1}{2} \text{Tanh}\left[\frac{x}{2}\right]$$

Problem 69: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Csch}[x]^4}{i + \text{Csch}[x]} dx$$

Optimal (type 3, 37 leaves, 6 steps):

$$\frac{3}{2} \text{ArcTanh}[\text{Cosh}[x]] + 2 i \text{Coth}[x] - \frac{3}{2} \text{Coth}[x] \text{Csch}[x] + \frac{\text{Coth}[x] \text{Csch}[x]^2}{i + \text{Csch}[x]}$$

Result (type 3, 90 leaves):

$$\frac{1}{8} \left(4 i \text{Coth}\left[\frac{x}{2}\right] - \text{Csch}\left[\frac{x}{2}\right]^2 + 12 \text{Log}\left[\text{Cosh}\left[\frac{x}{2}\right]\right] - 12 \text{Log}\left[\text{Sinh}\left[\frac{x}{2}\right]\right] - \text{Sech}\left[\frac{x}{2}\right]^2 + \frac{16 \text{Sinh}\left[\frac{x}{2}\right]}{-i \text{Cosh}\left[\frac{x}{2}\right] + \text{Sinh}\left[\frac{x}{2}\right]} + 4 i \text{Tanh}\left[\frac{x}{2}\right] \right)$$

Problem 70: Result more than twice size of optimal antiderivative.

$$\int (a + b \text{Csch}[c + d x])^4 dx$$

Optimal (type 3, 109 leaves, 6 steps):

$$a^4 x - \frac{2 a b (2 a^2 - b^2) \text{ArcTanh}[\text{Cosh}[c + d x]]}{d} - \frac{b^2 (17 a^2 - 2 b^2) \text{Coth}[c + d x]}{3 d} - \frac{4 a b^3 \text{Coth}[c + d x] \text{Csch}[c + d x]}{3 d} - \frac{b^2 \text{Coth}[c + d x] (a + b \text{Csch}[c + d x])^2}{3 d}$$

Result (type 3, 567 leaves):

$$\begin{aligned}
& \frac{a^4 (c + dx) (a + b \operatorname{Csch}[c + dx])^4 \operatorname{Sinh}[c + dx]^4}{d (b + a \operatorname{Sinh}[c + dx])^4} + \frac{1}{3 d (b + a \operatorname{Sinh}[c + dx])^4} \\
& \left(-9 a^2 b^2 \operatorname{Cosh}\left[\frac{1}{2}(c + dx)\right] + b^4 \operatorname{Cosh}\left[\frac{1}{2}(c + dx)\right] \right) \operatorname{Csch}\left[\frac{1}{2}(c + dx)\right] (a + b \operatorname{Csch}[c + dx])^4 \operatorname{Sinh}[c + dx]^4 - \\
& \frac{a b^3 \operatorname{Csch}\left[\frac{1}{2}(c + dx)\right]^2 (a + b \operatorname{Csch}[c + dx])^4 \operatorname{Sinh}[c + dx]^4}{2 d (b + a \operatorname{Sinh}[c + dx])^4} - \frac{b^4 \operatorname{Coth}\left[\frac{1}{2}(c + dx)\right] \operatorname{Csch}\left[\frac{1}{2}(c + dx)\right]^2 (a + b \operatorname{Csch}[c + dx])^4 \operatorname{Sinh}[c + dx]^4}{24 d (b + a \operatorname{Sinh}[c + dx])^4} + \\
& \frac{2 (-2 a^3 b + a b^3) (a + b \operatorname{Csch}[c + dx])^4 \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{1}{2}(c + dx)\right]\right] \operatorname{Sinh}[c + dx]^4}{d (b + a \operatorname{Sinh}[c + dx])^4} - \\
& \frac{2 (-2 a^3 b + a b^3) (a + b \operatorname{Csch}[c + dx])^4 \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{1}{2}(c + dx)\right]\right] \operatorname{Sinh}[c + dx]^4}{d (b + a \operatorname{Sinh}[c + dx])^4} - \frac{a b^3 (a + b \operatorname{Csch}[c + dx])^4 \operatorname{Sech}\left[\frac{1}{2}(c + dx)\right]^2 \operatorname{Sinh}[c + dx]^4}{2 d (b + a \operatorname{Sinh}[c + dx])^4} + \\
& \frac{1}{3 d (b + a \operatorname{Sinh}[c + dx])^4} (a + b \operatorname{Csch}[c + dx])^4 \operatorname{Sech}\left[\frac{1}{2}(c + dx)\right] \left(-9 a^2 b^2 \operatorname{Sinh}\left[\frac{1}{2}(c + dx)\right] + b^4 \operatorname{Sinh}\left[\frac{1}{2}(c + dx)\right] \right) \operatorname{Sinh}[c + dx]^4 + \\
& \frac{b^4 (a + b \operatorname{Csch}[c + dx])^4 \operatorname{Sech}\left[\frac{1}{2}(c + dx)\right]^2 \operatorname{Sinh}[c + dx]^4 \operatorname{Tanh}\left[\frac{1}{2}(c + dx)\right]}{24 d (b + a \operatorname{Sinh}[c + dx])^4}
\end{aligned}$$

Problem 71: Result more than twice size of optimal antiderivative.

$$\int (a + b \operatorname{Csch}[c + dx])^3 dx$$

Optimal (type 3, 75 leaves, 5 steps):

$$a^3 x - \frac{b (6 a^2 - b^2) \operatorname{ArcTanh}[\operatorname{Cosh}[c + dx]]}{2 d} - \frac{5 a b^2 \operatorname{Coth}[c + dx]}{2 d} - \frac{b^2 \operatorname{Coth}[c + dx] (a + b \operatorname{Csch}[c + dx])}{2 d}$$

Result (type 3, 151 leaves):

$$\begin{aligned}
& -\frac{1}{8 d} \left(-8 a^3 c - 8 a^3 dx + 12 a b^2 \operatorname{Coth}\left[\frac{1}{2}(c + dx)\right] + b^3 \operatorname{Csch}\left[\frac{1}{2}(c + dx)\right]^2 + 24 a^2 b \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{1}{2}(c + dx)\right]\right] - 4 b^3 \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{1}{2}(c + dx)\right]\right] - \right. \\
& \left. 24 a^2 b \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{1}{2}(c + dx)\right]\right] + 4 b^3 \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{1}{2}(c + dx)\right]\right] + b^3 \operatorname{Sech}\left[\frac{1}{2}(c + dx)\right]^2 + 12 a b^2 \operatorname{Tanh}\left[\frac{1}{2}(c + dx)\right] \right)
\end{aligned}$$

Problem 72: Result more than twice size of optimal antiderivative.

$$\int (a + b \operatorname{Csch}[c + dx])^2 dx$$

Optimal (type 3, 34 leaves, 4 steps):

$$a^2 x - \frac{2 a b \operatorname{ArcTanh}[\operatorname{Cosh}[c + d x]]}{d} - \frac{b^2 \operatorname{Coth}[c + d x]}{d}$$

Result (type 3, 75 leaves):

$$-\frac{1}{2d} \left(b^2 \operatorname{Coth}\left[\frac{1}{2}(c + d x)\right] - 2 a \left(a c + a d x - 2 b \operatorname{Log}[\operatorname{Cosh}\left[\frac{1}{2}(c + d x)\right]] + 2 b \operatorname{Log}[\operatorname{Sinh}\left[\frac{1}{2}(c + d x)\right]] \right) + b^2 \operatorname{Tanh}\left[\frac{1}{2}(c + d x)\right] \right)$$

Problem 73: Result more than twice size of optimal antiderivative.

$$\int (a + b \operatorname{Csch}[c + d x]) dx$$

Optimal (type 3, 17 leaves, 2 steps):

$$a x - \frac{b \operatorname{ArcTanh}[\operatorname{Cosh}[c + d x]]}{d}$$

Result (type 3, 43 leaves):

$$a x - \frac{b \operatorname{Log}[\operatorname{Cosh}[\frac{c}{2} + \frac{d x}{2}]]}{d} + \frac{b \operatorname{Log}[\operatorname{Sinh}[\frac{c}{2} + \frac{d x}{2}]]}{d}$$

Problem 89: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sech}[x]^2}{i + \operatorname{Csch}[x]} dx$$

Optimal (type 3, 19 leaves, 6 steps):

$$-\frac{1}{3} \operatorname{Sech}[x]^3 - \frac{1}{3} i \operatorname{Tanh}[x]^3$$

Result (type 3, 64 leaves):

$$\frac{-3 + \operatorname{Cosh}[x] + \operatorname{Cosh}[2 x] - 2 i \operatorname{Sinh}[x] + i \operatorname{Cosh}[x] \operatorname{Sinh}[x]}{6 \left(\operatorname{Cosh}\left[\frac{x}{2}\right] - i \operatorname{Sinh}\left[\frac{x}{2}\right] \right) \left(\operatorname{Cosh}\left[\frac{x}{2}\right] + i \operatorname{Sinh}\left[\frac{x}{2}\right] \right)^3}$$

Problem 91: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sech}[x]^4}{i + \operatorname{Csch}[x]} dx$$

Optimal (type 3, 29 leaves, 7 steps):

$$-\frac{1}{5} \operatorname{Sech}[x]^5 - \frac{1}{3} i \operatorname{Tanh}[x]^3 + \frac{1}{5} i \operatorname{Tanh}[x]^5$$

Result (type 3, 96 leaves):

$$\left(-240 + 54 \operatorname{Cosh}[x] + 32 \operatorname{Cosh}[2x] + 18 \operatorname{Cosh}[3x] + 16 \operatorname{Cosh}[4x] - 96 i \operatorname{Sinh}[x] + 18 i \operatorname{Sinh}[2x] - 32 i \operatorname{Sinh}[3x] + 9 i \operatorname{Sinh}[4x] \right) /$$

$$\left(960 \left(\operatorname{Cosh}\left[\frac{x}{2}\right] - i \operatorname{Sinh}\left[\frac{x}{2}\right] \right)^3 \left(\operatorname{Cosh}\left[\frac{x}{2}\right] + i \operatorname{Sinh}\left[\frac{x}{2}\right] \right)^5 \right)$$

Problem 103: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Tanh}[x]^4}{i + \operatorname{Csch}[x]} dx$$

Optimal (type 3, 52 leaves, 5 steps):

$$-i x + \frac{1}{15} (15 i - 8 \operatorname{Csch}[x]) \operatorname{Tanh}[x] + \frac{1}{15} (5 i - 4 \operatorname{Csch}[x]) \operatorname{Tanh}[x]^3 + \frac{1}{5} (i - \operatorname{Csch}[x]) \operatorname{Tanh}[x]^5$$

Result (type 3, 126 leaves):

$$\left(-200 + 6 (89 - 120 i x) \operatorname{Cosh}[x] - 128 \operatorname{Cosh}[2x] + 178 \operatorname{Cosh}[3x] - 240 i x \operatorname{Cosh}[3x] - 184 \operatorname{Cosh}[4x] + 64 i \operatorname{Sinh}[x] + 178 i \operatorname{Sinh}[2x] + \right.$$

$$\left. 240 x \operatorname{Sinh}[2x] + 128 i \operatorname{Sinh}[3x] + 89 i \operatorname{Sinh}[4x] + 120 x \operatorname{Sinh}[4x] \right) / \left(960 \left(\operatorname{Cosh}\left[\frac{x}{2}\right] - i \operatorname{Sinh}\left[\frac{x}{2}\right] \right)^3 \left(\operatorname{Cosh}\left[\frac{x}{2}\right] + i \operatorname{Sinh}\left[\frac{x}{2}\right] \right)^5 \right)$$

Problem 109: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Coth}[x]^3}{i + \operatorname{Csch}[x]} dx$$

Optimal (type 3, 12 leaves, 3 steps):

$$-\operatorname{Csch}[x] - i \operatorname{Log}[\operatorname{Sinh}[x]]$$

Result (type 3, 28 leaves):

$$-\frac{1}{2} \operatorname{Coth}\left[\frac{x}{2}\right] - i \operatorname{Log}[\operatorname{Sinh}[x]] + \frac{1}{2} \operatorname{Tanh}\left[\frac{x}{2}\right]$$

Problem 110: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Coth}[x]^4}{i + \operatorname{Csch}[x]} dx$$

Optimal (type 3, 27 leaves, 4 steps):

$$-i x - \frac{1}{2} \operatorname{ArcTanh}[\operatorname{Cosh}[x]] + \frac{1}{2} \operatorname{Coth}[x] (2i - \operatorname{Csch}[x])$$

Result (type 3, 76 leaves):

$$-i x + \frac{1}{2} i \operatorname{Coth}\left[\frac{x}{2}\right] - \frac{1}{8} \operatorname{Csch}\left[\frac{x}{2}\right]^2 - \frac{1}{2} \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{x}{2}\right]\right] + \frac{1}{2} \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{x}{2}\right]\right] - \frac{1}{8} \operatorname{Sech}\left[\frac{x}{2}\right]^2 + \frac{1}{2} i \operatorname{Tanh}\left[\frac{x}{2}\right]$$

Problem 111: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Coth}[x]^5}{i + \operatorname{Csch}[x]} dx$$

Optimal (type 3, 30 leaves, 3 steps):

$$-\operatorname{Csch}[x] + \frac{1}{2} i \operatorname{Csch}[x]^2 - \frac{\operatorname{Csch}[x]^3}{3} - i \operatorname{Log}[\operatorname{Sinh}[x]]$$

Result (type 3, 92 leaves):

$$-\frac{5}{12} \operatorname{Coth}\left[\frac{x}{2}\right] + \frac{1}{8} i \operatorname{Csch}\left[\frac{x}{2}\right]^2 - \frac{1}{24} \operatorname{Coth}\left[\frac{x}{2}\right] \operatorname{Csch}\left[\frac{x}{2}\right]^2 - i \operatorname{Log}[\operatorname{Sinh}[x]] - \frac{1}{8} i \operatorname{Sech}\left[\frac{x}{2}\right]^2 + \frac{5}{12} \operatorname{Tanh}\left[\frac{x}{2}\right] - \frac{1}{24} \operatorname{Sech}\left[\frac{x}{2}\right]^2 \operatorname{Tanh}\left[\frac{x}{2}\right]$$

Problem 112: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Coth}[x]^6}{i + \operatorname{Csch}[x]} dx$$

Optimal (type 3, 43 leaves, 5 steps):

$$-i x - \frac{3}{8} \operatorname{ArcTanh}[\operatorname{Cosh}[x]] + \frac{1}{12} \operatorname{Coth}[x]^3 (4i - 3 \operatorname{Csch}[x]) + \frac{1}{8} \operatorname{Coth}[x] (8i - 3 \operatorname{Csch}[x])$$

Result (type 3, 140 leaves):

$$-i x + \frac{2}{3} i \operatorname{Coth}\left[\frac{x}{2}\right] - \frac{5}{32} \operatorname{Csch}\left[\frac{x}{2}\right]^2 + \frac{1}{24} i \operatorname{Coth}\left[\frac{x}{2}\right] \operatorname{Csch}\left[\frac{x}{2}\right]^2 - \frac{1}{64} \operatorname{Csch}\left[\frac{x}{2}\right]^4 - \frac{3}{8} \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{x}{2}\right]\right] + \frac{3}{8} \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{x}{2}\right]\right] - \frac{5}{32} \operatorname{Sech}\left[\frac{x}{2}\right]^2 + \frac{1}{64} \operatorname{Sech}\left[\frac{x}{2}\right]^4 + \frac{2}{3} i \operatorname{Tanh}\left[\frac{x}{2}\right] - \frac{1}{24} i \operatorname{Sech}\left[\frac{x}{2}\right]^2 \operatorname{Tanh}\left[\frac{x}{2}\right]$$

Problem 122: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Coth}[x]^5}{a + b \operatorname{Csch}[x]} dx$$

Optimal (type 3, 70 leaves, 3 steps):

$$-\frac{(a^2 + 2b^2) \operatorname{Csch}[x]}{b^3} + \frac{a \operatorname{Csch}[x]^2}{2b^2} - \frac{\operatorname{Csch}[x]^3}{3b} + \frac{(a^2 + b^2)^2 \operatorname{Log}[a + b \operatorname{Csch}[x]]}{ab^4} + \frac{\operatorname{Log}[\operatorname{Sinh}[x]]}{a}$$

Result (type 3, 180 leaves):

$$\frac{1}{48ab^4} \left(-4ab(6a^2 + 11b^2) \operatorname{Coth}\left[\frac{x}{2}\right] + 6a^2b^2 \operatorname{Csch}\left[\frac{x}{2}\right]^2 - 48a^4 \operatorname{Log}[\operatorname{Sinh}[x]] - \right. \\ \left. 96a^2b^2 \operatorname{Log}[\operatorname{Sinh}[x]] + 48a^4 \operatorname{Log}[b + a \operatorname{Sinh}[x]] + 96a^2b^2 \operatorname{Log}[b + a \operatorname{Sinh}[x]] + 48b^4 \operatorname{Log}[b + a \operatorname{Sinh}[x]] - \right. \\ \left. 6a^2b^2 \operatorname{Sech}\left[\frac{x}{2}\right]^2 - 16ab^3 \operatorname{Csch}[x]^3 \operatorname{Sinh}\left[\frac{x}{2}\right]^4 - ab^3 \operatorname{Csch}\left[\frac{x}{2}\right]^4 \operatorname{Sinh}[x] + 24a^3b \operatorname{Tanh}\left[\frac{x}{2}\right] + 44ab^3 \operatorname{Tanh}\left[\frac{x}{2}\right] \right)$$

Problem 124: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Coth}[x]^7}{a + b \operatorname{Csch}[x]} dx$$

Optimal (type 3, 119 leaves, 3 steps):

$$-\frac{(a^4 + 3a^2b^2 + 3b^4) \operatorname{Csch}[x]}{b^5} + \frac{a(a^2 + 3b^2) \operatorname{Csch}[x]^2}{2b^4} - \\ \frac{(a^2 + 3b^2) \operatorname{Csch}[x]^3}{3b^3} + \frac{a \operatorname{Csch}[x]^4}{4b^2} - \frac{\operatorname{Csch}[x]^5}{5b} + \frac{(a^2 + b^2)^3 \operatorname{Log}[a + b \operatorname{Csch}[x]]}{ab^6} + \frac{\operatorname{Log}[\operatorname{Sinh}[x]]}{a}$$

Result (type 3, 344 leaves):

$$\frac{1}{960ab^6} \left(-4ab(120a^4 + 340a^2b^2 + 309b^4) \operatorname{Coth}\left[\frac{x}{2}\right] + 30a^2b^2(4a^2 + 11b^2) \operatorname{Csch}\left[\frac{x}{2}\right]^2 - \right. \\ \left. 960a^6 \operatorname{Log}[\operatorname{Sinh}[x]] - 2880a^4b^2 \operatorname{Log}[\operatorname{Sinh}[x]] - 2880a^2b^4 \operatorname{Log}[\operatorname{Sinh}[x]] + 960a^6 \operatorname{Log}[b + a \operatorname{Sinh}[x]] + \right. \\ \left. 2880a^4b^2 \operatorname{Log}[b + a \operatorname{Sinh}[x]] + 2880a^2b^4 \operatorname{Log}[b + a \operatorname{Sinh}[x]] + 960b^6 \operatorname{Log}[b + a \operatorname{Sinh}[x]] - 120a^4b^2 \operatorname{Sech}\left[\frac{x}{2}\right]^2 - \right. \\ \left. 330a^2b^4 \operatorname{Sech}\left[\frac{x}{2}\right]^2 + 15a^2b^4 \operatorname{Sech}\left[\frac{x}{2}\right]^4 - 320a^3b^3 \operatorname{Csch}[x]^3 \operatorname{Sinh}\left[\frac{x}{2}\right]^4 - 816ab^5 \operatorname{Csch}[x]^3 \operatorname{Sinh}\left[\frac{x}{2}\right]^4 - 3ab^5 \operatorname{Csch}\left[\frac{x}{2}\right]^6 \operatorname{Sinh}[x] - \right. \\ \left. ab^3 \operatorname{Csch}\left[\frac{x}{2}\right]^4 (-15ab + 20a^2 \operatorname{Sinh}[x] + 51b^2 \operatorname{Sinh}[x]) + 480a^5b \operatorname{Tanh}\left[\frac{x}{2}\right] + 1360a^3b^3 \operatorname{Tanh}\left[\frac{x}{2}\right] + 1236ab^5 \operatorname{Tanh}\left[\frac{x}{2}\right] + 6ab^5 \operatorname{Sech}\left[\frac{x}{2}\right]^4 \operatorname{Tanh}\left[\frac{x}{2}\right] \right)$$

Problem 132: Result unnecessarily involves higher level functions.

$$\int \frac{x^5}{\sqrt{\operatorname{Csch}[2 \operatorname{Log}[cx]]}} dx$$

Optimal (type 4, 81 leaves, 6 steps):

$$-\frac{2x^2}{21c^4\sqrt{\text{Csch}[2\text{Log}[cx]]}} + \frac{x^6}{7\sqrt{\text{Csch}[2\text{Log}[cx]]}} + \frac{2\text{EllipticF}[\text{ArcCsc}[cx], -1]}{21c^7\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\text{Csch}[2\text{Log}[cx]]}}$$

Result (type 5, 81 leaves):

$$\frac{\sqrt{\frac{c^2x^2}{-2+2c^4x^4}} \left(2 - 5c^4x^4 + 3c^8x^8 - 2\sqrt{1-c^4x^4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, c^4x^4\right] \right)}{21c^6}$$

Problem 134: Result unnecessarily involves higher level functions.

$$\int \frac{x^3}{\sqrt{\text{Csch}[2\text{Log}[cx]]}} dx$$

Optimal (type 4, 119 leaves, 9 steps):

$$-\frac{2}{5c^4\sqrt{\text{Csch}[2\text{Log}[cx]]}} + \frac{x^4}{5\sqrt{\text{Csch}[2\text{Log}[cx]]}} - \frac{2\text{EllipticE}[\text{ArcCsc}[cx], -1]}{5c^5\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\text{Csch}[2\text{Log}[cx]]}} + \frac{2\text{EllipticF}[\text{ArcCsc}[cx], -1]}{5c^5\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\text{Csch}[2\text{Log}[cx]]}}$$

Result (type 5, 76 leaves):

$$\frac{x^2\sqrt{\frac{c^2x^2}{-2+2c^4x^4}} \left(-3 + 3c^4x^4 - 2\sqrt{1-c^4x^4} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, c^4x^4\right] \right)}{15c^2}$$

Problem 136: Result unnecessarily involves higher level functions.

$$\int \frac{x}{\sqrt{\text{Csch}[2\text{Log}[cx]]}} dx$$

Optimal (type 4, 60 leaves, 5 steps):

$$\frac{x^2}{3\sqrt{\text{Csch}[2\text{Log}[cx]]}} + \frac{2\text{EllipticF}[\text{ArcCsc}[cx], -1]}{3c^3\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\text{Csch}[2\text{Log}[cx]]}}$$

Result (type 5, 72 leaves):

$$\frac{\sqrt{\frac{c^2 x^2}{-2+2c^4 x^4}} \left(-1 + c^4 x^4 - 2 \sqrt{1 - c^4 x^4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, c^4 x^4\right] \right)}{3 c^2}$$

Problem 140: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{\operatorname{Csch}[2 \operatorname{Log}[c x]]}}{x^3} dx$$

Optimal (type 4, 74 leaves, 7 steps):

$$-c^3 \sqrt{1 - \frac{1}{c^4 x^4}} x \sqrt{\operatorname{Csch}[2 \operatorname{Log}[c x]]} \operatorname{EllipticE}[\operatorname{ArcCsc}[c x], -1] + c^3 \sqrt{1 - \frac{1}{c^4 x^4}} x \sqrt{\operatorname{Csch}[2 \operatorname{Log}[c x]]} \operatorname{EllipticF}[\operatorname{ArcCsc}[c x], -1]$$

Result (type 4, 56 leaves):

$$c^2 \sqrt{\operatorname{Csch}[2 \operatorname{Log}[c x]]} \left(-\operatorname{EllipticE}\left[\frac{\pi}{4} - i \operatorname{Log}[c x], 2\right] \sqrt{i \operatorname{Sinh}[2 \operatorname{Log}[c x]]} + \operatorname{Sinh}[2 \operatorname{Log}[c x]] \right)$$

Problem 142: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{\operatorname{Csch}[2 \operatorname{Log}[c x]]}}{x^5} dx$$

Optimal (type 4, 64 leaves, 5 steps):

$$\frac{1}{3} \left(c^4 - \frac{1}{x^4} \right) \sqrt{\operatorname{Csch}[2 \operatorname{Log}[c x]]} - \frac{1}{3} c^5 \sqrt{1 - \frac{1}{c^4 x^4}} x \sqrt{\operatorname{Csch}[2 \operatorname{Log}[c x]]} \operatorname{EllipticF}[\operatorname{ArcCsc}[c x], -1]$$

Result (type 5, 81 leaves):

$$\frac{\sqrt{2} \sqrt{\frac{c^2 x^2}{-1+c^4 x^4}} \left(-1 + c^4 x^4 + c^4 x^4 \sqrt{1 - c^4 x^4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, c^4 x^4\right] \right)}{3 x^4}$$

Problem 144: Result unnecessarily involves higher level functions.

$$\int \frac{x^7}{\operatorname{Csch}[2 \operatorname{Log}[c x]]^{3/2}} dx$$

Optimal (type 4, 118 leaves, 7 steps):

$$\frac{4}{77 c^4 \left(c^4 - \frac{1}{x^4}\right) \operatorname{Csch}[2 \operatorname{Log}[c x]]^{3/2}} - \frac{6 x^4}{77 \left(c^4 - \frac{1}{x^4}\right) \operatorname{Csch}[2 \operatorname{Log}[c x]]^{3/2}} + \frac{x^8}{11 \operatorname{Csch}[2 \operatorname{Log}[c x]]^{3/2}} - \frac{4 \operatorname{EllipticF}[\operatorname{ArcCsc}[c x], -1]}{77 c^{11} \left(1 - \frac{1}{c^4 x^4}\right)^{3/2} x^3 \operatorname{Csch}[2 \operatorname{Log}[c x]]^{3/2}}$$

Result (type 5, 89 leaves):

$$\frac{\sqrt{\frac{c^2 x^2}{-2+2 c^4 x^4}} \left(-4 + 17 c^4 x^4 - 20 c^8 x^8 + 7 c^{12} x^{12} + 4 \sqrt{1 - c^4 x^4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, c^4 x^4\right]\right)}{154 c^8}$$

Problem 146: Result unnecessarily involves higher level functions.

$$\int \frac{x^5}{\operatorname{Csch}[2 \operatorname{Log}[c x]]^{3/2}} dx$$

Optimal (type 4, 162 leaves, 10 steps):

$$\frac{4}{15 c^4 \left(c^4 - \frac{1}{x^4}\right) x^2 \operatorname{Csch}[2 \operatorname{Log}[c x]]^{3/2}} - \frac{2 x^2}{15 \left(c^4 - \frac{1}{x^4}\right) \operatorname{Csch}[2 \operatorname{Log}[c x]]^{3/2}} + \frac{x^6}{9 \operatorname{Csch}[2 \operatorname{Log}[c x]]^{3/2}} + \frac{4 \operatorname{EllipticE}[\operatorname{ArcCsc}[c x], -1]}{15 c^9 \left(1 - \frac{1}{c^4 x^4}\right)^{3/2} x^3 \operatorname{Csch}[2 \operatorname{Log}[c x]]^{3/2}} - \frac{4 \operatorname{EllipticF}[\operatorname{ArcCsc}[c x], -1]}{15 c^9 \left(1 - \frac{1}{c^4 x^4}\right)^{3/2} x^3 \operatorname{Csch}[2 \operatorname{Log}[c x]]^{3/2}}$$

Result (type 5, 84 leaves):

$$\frac{x^2 \sqrt{\frac{c^2 x^2}{-2+2 c^4 x^4}} \left(11 - 16 c^4 x^4 + 5 c^8 x^8 + 4 \sqrt{1 - c^4 x^4} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, c^4 x^4\right]\right)}{90 c^4}$$

Problem 148: Result unnecessarily involves higher level functions.

$$\int \frac{x^3}{\operatorname{Csch}[2 \operatorname{Log}[c x]]^{3/2}} dx$$

Optimal (type 4, 86 leaves, 6 steps):

$$-\frac{2}{7 \left(c^4 - \frac{1}{x^4}\right) \operatorname{Csch}[2 \operatorname{Log}[c x]]^{3/2}} + \frac{x^4}{7 \operatorname{Csch}[2 \operatorname{Log}[c x]]^{3/2}} - \frac{4 \operatorname{EllipticF}[\operatorname{ArcCsc}[c x], -1]}{7 c^7 \left(1 - \frac{1}{c^4 x^4}\right)^{3/2} x^3 \operatorname{Csch}[2 \operatorname{Log}[c x]]^{3/2}}$$

Result (type 5, 80 leaves):

$$\frac{\sqrt{\frac{c^2 x^2}{-2+2 c^4 x^4}} \left(3 - 4 c^4 x^4 + c^8 x^8 + 4 \sqrt{1 - c^4 x^4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, c^4 x^4\right]\right)}{14 c^4}$$

Problem 150: Result unnecessarily involves higher level functions.

$$\int \frac{x}{\operatorname{Csch}[2 \operatorname{Log}[c x]]^{3/2}} dx$$

Optimal (type 4, 130 leaves, 9 steps):

$$-\frac{6}{5 \left(c^4 - \frac{1}{x^4}\right) x^2 \operatorname{Csch}[2 \operatorname{Log}[c x]]^{3/2}} + \frac{x^2}{5 \operatorname{Csch}[2 \operatorname{Log}[c x]]^{3/2}} - \frac{12 \operatorname{EllipticE}[\operatorname{ArcCsc}[c x], -1]}{5 c^5 \left(1 - \frac{1}{c^4 x^4}\right)^{3/2} x^3 \operatorname{Csch}[2 \operatorname{Log}[c x]]^{3/2}} + \frac{12 \operatorname{EllipticF}[\operatorname{ArcCsc}[c x], -1]}{5 c^5 \left(1 - \frac{1}{c^4 x^4}\right)^{3/2} x^3 \operatorname{Csch}[2 \operatorname{Log}[c x]]^{3/2}}$$

Result (type 5, 83 leaves):

$$\frac{\sqrt{\frac{c^2 x^2}{-2+2 c^4 x^4}} \left(7 - 8 c^4 x^4 + c^8 x^8 - 12 \sqrt{1 - c^4 x^4} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, c^4 x^4\right]\right)}{10 c^4 x^2}$$

Problem 154: Result unnecessarily involves higher level functions.

$$\int \frac{\operatorname{Csch}[2 \operatorname{Log}[c x]]^{3/2}}{x^3} dx$$

Optimal (type 4, 69 leaves, 5 steps):

$$-\frac{1}{2} \left(c^4 - \frac{1}{x^4}\right) x^2 \operatorname{Csch}[2 \operatorname{Log}[c x]]^{3/2} + \frac{1}{2} c^5 \left(1 - \frac{1}{c^4 x^4}\right)^{3/2} x^3 \operatorname{Csch}[2 \operatorname{Log}[c x]]^{3/2} \operatorname{EllipticF}[\operatorname{ArcCsc}[c x], -1]$$

Result (type 5, 66 leaves):

$$-\sqrt{2} c^2 \sqrt{\frac{c^2 x^2}{-1+c^4 x^4}} \left(1 + \sqrt{1 - c^4 x^4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, c^4 x^4\right]\right)$$

Problem 159: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Csch}[a + b \operatorname{Log}[c x^n]]^4 dx$$

Optimal (type 5, 68 leaves, 4 steps):

$$\frac{16 e^{4a} x (c x^n)^{4b} \text{Hypergeometric2F1}\left[4, \frac{1}{2}\left(4 + \frac{1}{bn}\right), \frac{1}{2}\left(6 + \frac{1}{bn}\right), e^{2a} (c x^n)^{2b}\right]}{1 + 4 b n}$$

Result (type 5, 488 leaves):

$$\begin{aligned} & -\frac{1}{6 b^3 n^3} (-1 + 4 b^2 n^2) x \text{Csch}\left[a + b (-n \text{Log}[x] + \text{Log}[c x^n])\right] \text{Csch}\left[a + b n \text{Log}[x] + b (-n \text{Log}[x] + \text{Log}[c x^n])\right] \text{Sinh}[b n \text{Log}[x]] + \\ & \frac{1}{3 b n} x \text{Csch}\left[a + b (-n \text{Log}[x] + \text{Log}[c x^n])\right] \text{Csch}\left[a + b n \text{Log}[x] + b (-n \text{Log}[x] + \text{Log}[c x^n])\right]^3 \text{Sinh}[b n \text{Log}[x]] - \\ & \frac{1}{6 b^2 n^2} x \text{Csch}\left[a + b (-n \text{Log}[x] + \text{Log}[c x^n])\right] \text{Csch}\left[a + b n \text{Log}[x] + b (-n \text{Log}[x] + \text{Log}[c x^n])\right]^2 \\ & \left(2 b n \text{Cosh}\left[a + b (-n \text{Log}[x] + \text{Log}[c x^n])\right] + \text{Sinh}\left[a + b (-n \text{Log}[x] + \text{Log}[c x^n])\right]\right) + \\ & \frac{1}{6 b^3 n^3 (1 + 2 b n)} e^{-\frac{a+b(-n \text{Log}[x] + \text{Log}[c x^n])}{bn}} (-1 + 4 b^2 n^2) \text{Csch}\left[a + b (-n \text{Log}[x] + \text{Log}[c x^n])\right] \\ & \left(e^{\left(2 + \frac{1}{bn}\right)(a+b \text{Log}[c x^n])} \text{Hypergeometric2F1}\left[1, 1 + \frac{1}{2 b n}, 2 + \frac{1}{2 b n}, e^{2(a+b \text{Log}[c x^n])}\right] \text{Sinh}\left[a + b (-n \text{Log}[x] + \text{Log}[c x^n])\right] + \right. \\ & \left. e^{\frac{a}{bn} + \frac{-n \text{Log}[x] + \text{Log}[c x^n]}{n}} (1 + 2 b n) x \left(\text{Cosh}\left[a + b (-n \text{Log}[x] + \text{Log}[c x^n])\right] + \right. \right. \\ & \left. \left. \text{Hypergeometric2F1}\left[1, \frac{1}{2 b n}, 1 + \frac{1}{2 b n}, e^{2(a+b n \text{Log}[x] + b(-n \text{Log}[x] + \text{Log}[c x^n])}\right)\right] \text{Sinh}\left[a + b (-n \text{Log}[x] + \text{Log}[c x^n])\right] \right) \right) \end{aligned}$$

Problem 161: Result more than twice size of optimal antiderivative.

$$\int \text{Csch}\left[a + 2 \text{Log}[c \sqrt{x}]\right]^3 dx$$

Optimal (type 1, 26 leaves, 3 steps):

$$-\frac{2 c^6 e^{-a}}{\left(c^4 - \frac{e^{-2a}}{x^2}\right)^2}$$

Result (type 1, 62 leaves):

$$\frac{2 (\text{Cosh}[a] - \text{Sinh}[a]) (-2 c^4 x^2 + \text{Cosh}[a]^2 - 2 \text{Cosh}[a] \text{Sinh}[a] + \text{Sinh}[a]^2)}{c^2 ((-1 + c^4 x^2) \text{Cosh}[a] + (1 + c^4 x^2) \text{Sinh}[a])^2}$$

Problem 162: Result more than twice size of optimal antiderivative.

$$\int \text{Csch}\left[a + 2 \text{Log}\left[\frac{c}{\sqrt{x}}\right]\right]^3 dx$$

Optimal (type 1, 26 leaves, 4 steps):

$$\frac{2 c^2 e^{-3 a}}{\left(e^{-2 a} - \frac{c^4}{x^2}\right)^2}$$

Result (type 1, 65 leaves):

$$-\frac{2 c^6 \left((c^4 - 2 x^2) \operatorname{Cosh}[a] + (c^4 + 2 x^2) \operatorname{Sinh}[a] \right) \left(\operatorname{Cosh}[2 a] + \operatorname{Sinh}[2 a] \right)}{\left((-c^4 + x^2) \operatorname{Cosh}[a] - (c^4 + x^2) \operatorname{Sinh}[a] \right)^2}$$

Problem 164: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Csch}\left[a - \frac{\operatorname{Log}[c x^n]}{n(-2+p)}\right]^p dx$$

Optimal (type 3, 66 leaves, 3 steps):

$$\frac{(2-p) x \left(1 - e^{-2 a} (c x^n)^{-\frac{2}{n(2-p)}} \right) \operatorname{Csch}\left[a + \frac{\operatorname{Log}[c x^n]}{n(2-p)}\right]^p}{2(1-p)}$$

Result (type 3, 140 leaves):

$$\frac{2^{-1+p} e^{-\frac{2 a p}{-2+p}} (-2+p) x \left(e^{\frac{2 a p}{-2+p}} - e^{\frac{4 a}{-2+p}} (c x^n)^{\frac{2}{n(-2+p)}} \right) \left(-\frac{e^{\frac{a(2+p)}{-2+p}} (c x^n)^{\frac{1}{n(-2+p)}}}{-e^{-2+p} + e^{-2+p} (c x^n)^{\frac{2}{n(-2+p)}}} \right)^p}{-1+p}$$

Problem 165: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Csch}[a + b \operatorname{Log}[c x^n]]}{x} dx$$

Optimal (type 3, 20 leaves, 2 steps):

$$-\frac{\operatorname{ArcTanh}[\operatorname{Cosh}[a + b \operatorname{Log}[c x^n]]]}{b n}$$

Result (type 3, 54 leaves):

$$-\frac{\operatorname{Log}\left[\operatorname{Cosh}\left[\frac{a}{2} + \frac{1}{2} b \operatorname{Log}[c x^n]\right]\right]}{b n} + \frac{\operatorname{Log}\left[\operatorname{Sinh}\left[\frac{a}{2} + \frac{1}{2} b \operatorname{Log}[c x^n]\right]\right]}{b n}$$

Test results for the 27 problems in "6.6.7 (d hyper)^m (a+b (c csch)^n)^p.m"

Problem 2: Result more than twice size of optimal antiderivative.

$$\int (a + b \operatorname{Csch}[c + d x]^2)^3 dx$$

Optimal (type 3, 74 leaves, 4 steps):

$$a^3 x - \frac{b (3 a^2 - 3 a b + b^2) \operatorname{Coth}[c + d x]}{d} - \frac{(3 a - 2 b) b^2 \operatorname{Coth}[c + d x]^3}{3 d} - \frac{b^3 \operatorname{Coth}[c + d x]^5}{5 d}$$

Result (type 3, 266 leaves):

$$\begin{aligned} & - \frac{8 b^3 \operatorname{Cosh}[c + d x] (a + b \operatorname{Csch}[c + d x]^2)^3 \operatorname{Sinh}[c + d x]}{5 d (-a + 2 b + a \operatorname{Cosh}[2 (c + d x)])^3} - \frac{8 (15 a b^2 \operatorname{Cosh}[c + d x] - 4 b^3 \operatorname{Cosh}[c + d x]) (a + b \operatorname{Csch}[c + d x]^2)^3 \operatorname{Sinh}[c + d x]^3}{15 d (-a + 2 b + a \operatorname{Cosh}[2 (c + d x)])^3} \\ & + \frac{8 (45 a^2 b \operatorname{Cosh}[c + d x] - 30 a b^2 \operatorname{Cosh}[c + d x] + 8 b^3 \operatorname{Cosh}[c + d x]) (a + b \operatorname{Csch}[c + d x]^2)^3 \operatorname{Sinh}[c + d x]^5}{15 d (-a + 2 b + a \operatorname{Cosh}[2 (c + d x)])^3} \\ & + \frac{8 a^3 (c + d x) (a + b \operatorname{Csch}[c + d x]^2)^3 \operatorname{Sinh}[c + d x]^6}{d (-a + 2 b + a \operatorname{Cosh}[2 (c + d x)])^3} \end{aligned}$$

Problem 9: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int (a + b \operatorname{Csch}[c + d x]^2)^{5/2} dx$$

Optimal (type 3, 174 leaves, 8 steps):

$$\begin{aligned} & \frac{a^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Coth}[c + d x]}{\sqrt{a - b + b \operatorname{Coth}[c + d x]^2}}\right]}{d} - \frac{\sqrt{b} (15 a^2 - 10 a b + 3 b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Coth}[c + d x]}{\sqrt{a - b + b \operatorname{Coth}[c + d x]^2}}\right]}{8 d} \\ & - \frac{(7 a - 3 b) b \operatorname{Coth}[c + d x] \sqrt{a - b + b \operatorname{Coth}[c + d x]^2}}{8 d} - \frac{b \operatorname{Coth}[c + d x] (a - b + b \operatorname{Coth}[c + d x]^2)^{3/2}}{4 d} \end{aligned}$$

Result (type 3, 391 leaves):

$$\begin{aligned}
& - \left(\left((-4a^3 + 15a^2b - 10ab^2 + 3b^3) \operatorname{ArcTanh} \left[\frac{\sqrt{2} \sqrt{b} \operatorname{Cosh}[c+dx]}{\sqrt{-a+2b-a \operatorname{Cos} \left[2 \left(\frac{\pi}{2} - i(c+dx) \right) \right]}} \right] (a+b \operatorname{Csch}[c+dx]^2)^{5/2} \operatorname{Sinh}[c+dx]^5 \right) / \right. \\
& \left. \left(\sqrt{2} \sqrt{b} d (-a+2b+a \operatorname{Cosh}[2(c+dx)])^{5/2} \right) \right) + \\
& \left((a+b \operatorname{Csch}[c+dx]^2)^{5/2} \left(-\frac{3}{2} (3ab \operatorname{Cosh}[c+dx] - b^2 \operatorname{Cosh}[c+dx]) \operatorname{Csch}[c+dx]^2 - b^2 \operatorname{Coth}[c+dx] \operatorname{Csch}[c+dx]^3 \right) \operatorname{Sinh}[c+dx]^5 \right) / \\
& \left(d (-a+2b+a \operatorname{Cosh}[2(c+dx)])^2 \right) + \\
& \left(4a^3 (a+b \operatorname{Csch}[c+dx]^2)^{5/2} \left(-\frac{\operatorname{ArcTanh} \left[\frac{\sqrt{2} \sqrt{b} \operatorname{Cosh}[c+dx]}{\sqrt{-a+2b+a \operatorname{Cosh}[2(c+dx)]}} \right]}{\sqrt{2} \sqrt{b}} + \frac{\sqrt{2} \operatorname{Log} \left[\sqrt{2} \sqrt{a} \operatorname{Cosh}[c+dx] + \sqrt{-a+2b+a \operatorname{Cosh}[2(c+dx)]} \right]}{\sqrt{a}} \right) \right) \\
& \left. \operatorname{Sinh}[c+dx]^5 \right) / \left(d (-a+2b+a \operatorname{Cosh}[2(c+dx)])^{5/2} \right)
\end{aligned}$$

Problem 12: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{a+b \operatorname{Csch}[c+dx]^2}} dx$$

Optimal (type 3, 38 leaves, 3 steps):

$$\frac{\operatorname{ArcTanh} \left[\frac{\sqrt{a} \operatorname{Coth}[c+dx]}{\sqrt{a+b \operatorname{Csch}[c+dx]^2}} \right]}{\sqrt{a} d}$$

Result (type 3, 97 leaves):

$$\left(\sqrt{-a+2b+a \operatorname{Cosh}[2(c+dx)]} \operatorname{Csch}[c+dx] \operatorname{Log} \left[\sqrt{2} \sqrt{a} \operatorname{Cosh}[c+dx] + \sqrt{-a+2b+a \operatorname{Cosh}[2(c+dx)]} \right] \right) / \left(\sqrt{2} \sqrt{a} d \sqrt{a+b \operatorname{Csch}[c+dx]^2} \right)$$

Problem 20: Result more than twice size of optimal antiderivative.

$$\int \sqrt{1 - \operatorname{Csch}[x]^2} \, dx$$

Optimal (type 3, 26 leaves, 5 steps):

$$\operatorname{ArcSin}\left[\frac{\operatorname{Coth}[x]}{\sqrt{2}}\right] + \operatorname{ArcTanh}\left[\frac{\operatorname{Coth}[x]}{\sqrt{2 - \operatorname{Coth}[x]^2}}\right]$$

Result (type 3, 65 leaves):

$$\frac{\sqrt{2 - 2 \operatorname{Csch}[x]^2} \left(\operatorname{ArcTan}\left[\frac{\sqrt{2} \operatorname{Cosh}[x]}{\sqrt{-3 + \operatorname{Cosh}[2x]}}\right] + \operatorname{Log}\left[\sqrt{2} \operatorname{Cosh}[x] + \sqrt{-3 + \operatorname{Cosh}[2x]}\right] \right) \operatorname{Sinh}[x]}{\sqrt{-3 + \operatorname{Cosh}[2x]}}$$

Problem 21: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{1 - \operatorname{Csch}[x]^2}} \, dx$$

Optimal (type 3, 16 leaves, 3 steps):

$$\operatorname{ArcTanh}\left[\frac{\operatorname{Coth}[x]}{\sqrt{2 - \operatorname{Coth}[x]^2}}\right]$$

Result (type 3, 45 leaves):

$$\frac{\sqrt{-3 + \operatorname{Cosh}[2x]} \operatorname{Csch}[x] \operatorname{Log}\left[\sqrt{2} \operatorname{Cosh}[x] + \sqrt{-3 + \operatorname{Cosh}[2x]}\right]}{\sqrt{2 - 2 \operatorname{Csch}[x]^2}}$$

Problem 23: Result more than twice size of optimal antiderivative.

$$\int \sqrt{-1 + \operatorname{Csch}[x]^2} \, dx$$

Optimal (type 3, 33 leaves, 6 steps):

$$-\operatorname{ArcTan}\left[\frac{\operatorname{Coth}[x]}{\sqrt{-2 + \operatorname{Coth}[x]^2}}\right] - \operatorname{ArcTanh}\left[\frac{\operatorname{Coth}[x]}{\sqrt{-2 + \operatorname{Coth}[x]^2}}\right]$$

Result (type 3, 68 leaves):

$$\frac{\sqrt{2} \sqrt{-1 + \text{Csch}[x]^2} \left(\text{ArcTan} \left[\frac{\sqrt{2} \text{Cosh}[x]}{\sqrt{-3 + \text{Cosh}[2x]}} \right] + \text{Log} \left[\sqrt{2} \text{Cosh}[x] + \sqrt{-3 + \text{Cosh}[2x]} \right] \right) \text{Sinh}[x]}{\sqrt{-3 + \text{Cosh}[2x]}}$$

Problem 24: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-1 + \text{Csch}[x]^2}} dx$$

Optimal (type 3, 14 leaves, 3 steps):

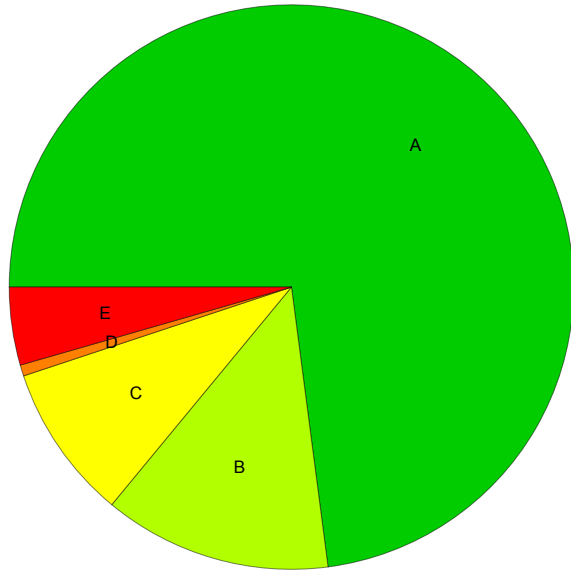
$$\text{ArcTan} \left[\frac{\text{Coth}[x]}{\sqrt{-2 + \text{Coth}[x]^2}} \right]$$

Result (type 3, 48 leaves):

$$\frac{\sqrt{-3 + \text{Cosh}[2x]} \text{Csch}[x] \text{Log} \left[\sqrt{2} \text{Cosh}[x] + \sqrt{-3 + \text{Cosh}[2x]} \right]}{\sqrt{2} \sqrt{-1 + \text{Csch}[x]^2}}$$

Summary of Integration Test Results

314 integration problems



A - 229 optimal antiderivatives

B - 41 more than twice size of optimal antiderivatives

C - 28 unnecessarily complex antiderivatives

D - 2 unable to integrate problems

E - 14 integration timeouts