# Mathematica 11.3 Integration Test Results

on the problems in the test-suite directory "7 Inverse hyperbolic functions\7.4 Inverse hyperbolic cotangent"

## Test results for the 300 problems in "7.4.1 Inverse hyperbolic cotangent functions.m"

### Problem 24: Result unnecessarily involves imaginary or complex numbers.

$$\int x^4 \operatorname{ArcCoth}[a x]^3 \, dx$$

Optimal (type 4, 196 leaves, 22 steps):

$$\frac{x^2}{20 a^3}+\frac{9 x \operatorname{ArcCoth}[a x]}{10 a^4}+\frac{x^3 \operatorname{ArcCoth}[a x]}{10 a^2}-\frac{9 \operatorname{ArcCoth}[a x]^2}{20 a^5}+\frac{3 x^2 \operatorname{ArcCoth}[a x]^2}{10 a^3}+\frac{3 x^4 \operatorname{ArcCoth}[a x]^2}{20 a}+\frac{\operatorname{ArcCoth}[a x]^3}{5 a^5}+\frac{1}{5} x^5 \operatorname{ArcCoth}[a x]^3-\frac{3 \operatorname{ArcCoth}[a x]^2 \operatorname{Log}\left[\frac{2}{1-a x}\right]}{5 a^5}+\frac{\operatorname{Log}\left[1-a^2 x^2\right]}{2 a^5}-\frac{3 \operatorname{ArcCoth}[a x] \operatorname{PolyLog}\left[2,1-\frac{2}{1-a x}\right]}{5 a^5}+\frac{3 \operatorname{PolyLog}\left[3,1-\frac{2}{1-a x}\right]}{10 a^5}$$

Result (type 4, 175 leaves):

$$\frac{1}{40 a^5}\left(-2-i \pi^3+2 a^2 x^2+36 a x \operatorname{ArcCoth}[a x]+4 a^3 x^3 \operatorname{ArcCoth}[a x]-18 \operatorname{ArcCoth}[a x]^2+12 a^2 x^2 \operatorname{ArcCoth}[a x]^2+6 a^4 x^4 \operatorname{ArcCoth}[a x]^2+8 \operatorname{ArcCoth}[a x]^3+8 a^5 x^5 \operatorname{ArcCoth}[a x]^3-24 \operatorname{ArcCoth}[a x]^2 \operatorname{Log}\left[1-e^{2 \operatorname{ArcCoth}[a x]}\right]-40 \operatorname{Log}\left[\frac{1}{a \sqrt{1-\frac{1}{a^2 x^2}}\, x}\right]-24 \operatorname{ArcCoth}[a x] \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcCoth}[a x]}\right]+12 \operatorname{PolyLog}\left[3, e^{2 \operatorname{ArcCoth}[a x]}\right]\right)$$

## Problem 26: Result unnecessarily involves imaginary or complex numbers.

$$\int x^2 \text{ArcCoth}[a x]^3 \, dx$$

Optimal (type 4, 149 leaves, 11 steps):

$$\frac{x \text{ArcCoth}[a x]}{a^2} - \frac{\text{ArcCoth}[a x]^2}{2 a^3} + \frac{x^2 \text{ArcCoth}[a x]^2}{2 a} + \frac{\text{ArcCoth}[a x]^3}{3 a^3} + \frac{1}{3} x^3 \text{ArcCoth}[a x]^3 - \frac{\text{ArcCoth}[a x]^2 \text{Log}\left[\frac{2}{1-a x}\right]}{a^3} + \frac{\text{Log}\left[1-a^2 x^2\right]}{2 a^3} - \frac{\text{ArcCoth}[a x] \text{PolyLog}\left[2, 1-\frac{2}{1-a x}\right]}{a^3} + \frac{\text{PolyLog}\left[3, 1-\frac{2}{1-a x}\right]}{2 a^3}$$

Result (type 4, 140 leaves):

$$\frac{1}{24 a^3} \left(-i \pi^3 + 24 a x \text{ArcCoth}[a x] - 12 \text{ArcCoth}[a x]^2 + 12 a^2 x^2 \text{ArcCoth}[a x]^2 + 8 \text{ArcCoth}[a x]^3 + 8 a^3 x^3 \text{ArcCoth}[a x]^3 - 24 \text{ArcCoth}[a x]^2 \text{Log}\left[1 - e^{2 \text{ArcCoth}[a x]}\right] - 24 \text{Log}\left[\frac{1}{a \sqrt{1-\frac{1}{a^2 x^2}} x}\right] - 24 \text{ArcCoth}[a x] \text{PolyLog}\left[2, e^{2 \text{ArcCoth}[a x]}\right] + 12 \text{PolyLog}\left[3, e^{2 \text{ArcCoth}[a x]}\right]\right)$$

## Problem 34: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcCoth}[c x]^2}{d + e x} \, dx$$

Optimal (type 4, 164 leaves, 1 step):

$$-\frac{\text{ArcCoth}[c x]^2 \text{Log}\left[\frac{2}{1+c x}\right]}{e} + \frac{\text{ArcCoth}[c x]^2 \text{Log}\left[\frac{2 c (d+e x)}{(c d+e) (1+c x)}\right]}{e} + \frac{\text{ArcCoth}[c x] \text{PolyLog}\left[2, 1-\frac{2}{1+c x}\right]}{e} - \frac{\text{ArcCoth}[c x] \text{PolyLog}\left[2, 1-\frac{2 c (d+e x)}{(c d+e) (1+c x)}\right]}{e} + \frac{\text{PolyLog}\left[3, 1-\frac{2}{1+c x}\right]}{2 e} - \frac{\text{PolyLog}\left[3, 1-\frac{2 c (d+e x)}{(c d+e) (1+c x)}\right]}{2 e}$$

Result (type 4, 741 leaves):

$$\frac{1}{24 e^2}\Bigg(-i e \pi^3 + 8 c d \text{ArcCoth}[c x]^3 + 8 e \text{ArcCoth}[c x]^3 - 24 e \text{ArcCoth}[c x]^2 \text{Log}\left[1 - e^{2 \text{ArcCoth}[c x]}\right] - 24 e \text{ArcCoth}[c x] \text{PolyLog}\left[2, e^{2 \text{ArcCoth}[c x]}\right] + 12 e \text{PolyLog}\left[3, e^{2 \text{ArcCoth}[c x]}\right] + \frac{1}{6 c^2 d^2 - 6 e^2} 24 (-c d + e) (c d + e) \Bigg(-2 c d \text{ArcCoth}[c x]^3 + 6 e \text{ArcCoth}[c x]^3 + 4 c d \sqrt{1 - \frac{e^2}{c^2 d^2}} e^{-\text{ArcTanh}\left[\frac{e}{c d}\right]} \text{ArcCoth}[c x]^3 + 6 i e \pi \text{ArcCoth}[c x] \text{Log}\left[\frac{1}{2}\left(e^{-\text{ArcCoth}[c x]} + e^{\text{ArcCoth}[c x]}\right)\right] + 6 e \text{ArcCoth}[c x]^2 \text{Log}\left[1 + \frac{(c d + e) e^{2 \text{ArcCoth}[c x]}}{-c d + e}\right] - 6 e \text{ArcCoth}[c x]^2 \text{Log}\left[1 - e^{\text{ArcCoth}[c x] + \text{ArcTanh}\left[\frac{e}{c d}\right]}\right] - 6 e \text{ArcCoth}[c x]^2 \text{Log}\left[1 + e^{\text{ArcCoth}[c x] + \text{ArcTanh}\left[\frac{e}{c d}\right]}\right] - 6 e \text{ArcCoth}[c x]^2 \text{Log}\left[1 - e^{2\left(\text{ArcCoth}[c x] + \text{ArcTanh}\left[\frac{e}{c d}\right]\right)}\right] - 12 e \text{ArcCoth}[c x] \text{ArcTanh}\left[\frac{e}{c d}\right] \text{Log}\left[\frac{1}{2} i e^{-\text{ArcCoth}[c x] - \text{ArcTanh}\left[\frac{e}{c d}\right]} \left(-1 + e^{2\left(\text{ArcCoth}[c x] + \text{ArcTanh}\left[\frac{e}{c d}\right]\right)}\right)\right] - 6 e \text{ArcCoth}[c x]^2 \text{Log}\left[\frac{1}{2} e^{-\text{ArcCoth}[c x]} \left(c d \left(-1 + e^{2 \text{ArcCoth}[c x]}\right) + e \left(1 + e^{2 \text{ArcCoth}[c x]}\right)\right)\right] - 6 i e \pi \text{ArcCoth}[c x] \text{Log}\left[\frac{1}{\sqrt{1 - \frac{1}{c^2 x^2}}}\right] + 6 e \text{ArcCoth}[c x]^2 \text{Log}\left[\frac{d + e x}{\sqrt{1 - \frac{1}{c^2 x^2}}\, x}\right] + 12 e \text{ArcCoth}[c x] \text{ArcTanh}\left[\frac{e}{c d}\right] \text{Log}\left[i \text{Sinh}\left[\text{ArcCoth}[c x] + \text{ArcTanh}\left[\frac{e}{c d}\right]\right]\right] + 6 e \text{ArcCoth}[c x] \text{PolyLog}\left[2, \frac{(c d + e) e^{2 \text{ArcCoth}[c x]}}{c d - e}\right] - 12 e \text{ArcCoth}[c x] \text{PolyLog}\left[2, -e^{\text{ArcCoth}[c x] + \text{ArcTanh}\left[\frac{e}{c d}\right]}\right] - 12 e \text{ArcCoth}[c x] \text{PolyLog}\left[2, e^{\text{ArcCoth}[c x] + \text{ArcTanh}\left[\frac{e}{c d}\right]}\right] - 6 e \text{ArcCoth}[c x] \text{PolyLog}\left[2, e^{2\left(\text{ArcCoth}[c x] + \text{ArcTanh}\left[\frac{e}{c d}\right]\right)}\right] - 3 e \text{PolyLog}\left[3, \frac{(c d + e) e^{2 \text{ArcCoth}[c x]}}{c d - e}\right] + 12 e \text{PolyLog}\left[3, -e^{\text{ArcCoth}[c x] + \text{ArcTanh}\left[\frac{e}{c d}\right]}\right] + 12 e \text{PolyLog}\left[3, e^{\text{ArcCoth}[c x] + \text{ArcTanh}\left[\frac{e}{c d}\right]}\right] + 3 e \text{PolyLog}\left[3, e^{2\left(\text{ArcCoth}[c x] + \text{ArcTanh}\left[\frac{e}{c d}\right]\right)}\right]\Bigg)\Bigg)$$

## Problem 41: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcCoth}[a x]}{\left(c + d x^2\right)^3} \, dx$$

Optimal (type 4, 657 leaves, 23 steps):

$$\frac{a}{8 c \left(a^2 c+d\right) \left(c+d x^2\right)}+\frac{x \,\text{ArcCoth}[a x]}{4 c \left(c+d x^2\right)^2}+\frac{3 x \,\text{ArcCoth}[a x]}{8 c^2 \left(c+d x^2\right)}+\frac{3 \,\text{ArcCoth}[a x] \,\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]}{8 c^{5/2} \sqrt{d}}+$$

$$\frac{3 i \,\text{Log}\left[\frac{\sqrt{d} (1-a x)}{i a \sqrt{c}+\sqrt{d}}\right] \text{Log}\left[1-\frac{i \sqrt{d} x}{\sqrt{c}}\right]}{32 c^{5/2} \sqrt{d}}-\frac{3 i \,\text{Log}\left[-\frac{\sqrt{d} (1+a x)}{i a \sqrt{c}-\sqrt{d}}\right] \text{Log}\left[1-\frac{i \sqrt{d} x}{\sqrt{c}}\right]}{32 c^{5/2} \sqrt{d}}-\frac{3 i \,\text{Log}\left[-\frac{\sqrt{d} (1-a x)}{i a \sqrt{c}-\sqrt{d}}\right] \text{Log}\left[1+\frac{i \sqrt{d} x}{\sqrt{c}}\right]}{32 c^{5/2} \sqrt{d}}+$$

$$\frac{3 i \,\text{Log}\left[\frac{\sqrt{d} (1+a x)}{i a \sqrt{c}+\sqrt{d}}\right] \text{Log}\left[1+\frac{i \sqrt{d} x}{\sqrt{c}}\right]}{32 c^{5/2} \sqrt{d}}+\frac{a \left(5 a^2 c+3 d\right) \text{Log}\left[1-a^2 x^2\right]}{16 c^2 \left(a^2 c+d\right)^2}-\frac{a \left(5 a^2 c+3 d\right) \text{Log}\left[c+d x^2\right]}{16 c^2 \left(a^2 c+d\right)^2}+$$

$$\frac{3 i \,\text{PolyLog}\left[2,\frac{a \left(\sqrt{c}-i \sqrt{d} x\right)}{a \sqrt{c}-i \sqrt{d}}\right]}{32 c^{5/2} \sqrt{d}}-\frac{3 i \,\text{PolyLog}\left[2,\frac{a \left(\sqrt{c}-i \sqrt{d} x\right)}{a \sqrt{c}+i \sqrt{d}}\right]}{32 c^{5/2} \sqrt{d}}+\frac{3 i \,\text{PolyLog}\left[2,\frac{a \left(\sqrt{c}+i \sqrt{d} x\right)}{a \sqrt{c}-i \sqrt{d}}\right]}{32 c^{5/2} \sqrt{d}}-\frac{3 i \,\text{PolyLog}\left[2,\frac{a \left(\sqrt{c}+i \sqrt{d} x\right)}{a \sqrt{c}+i \sqrt{d}}\right]}{32 c^{5/2} \sqrt{d}}$$

Result (type 4, 1838 leaves):

$$a^5 \left(-\frac{5 \,\text{Log}\left[1+\frac{\left(a^2 c+d\right) \text{Cosh}[2 \,\text{ArcCoth}[a x]]}{-a^2 c+d}\right]}{16 a^2 c \left(a^2 c+d\right)^2}-\frac{3 d \,\text{Log}\left[1+\frac{\left(a^2 c+d\right) \text{Cosh}[2 \,\text{ArcCoth}[a x]]}{-a^2 c+d}\right]}{16 a^4 c^2 \left(a^2 c+d\right)^2}+\right.$$

$$\frac{1}{32 a^2 c \sqrt{a^2 c d} \left(a^2 c+d\right)} 3 \left(-2 i \,\text{ArcCos}\left[-\frac{-a^2 c+d}{a^2 c+d}\right] \text{ArcTan}\left[\frac{a c}{\sqrt{a^2 c d}\, x}\right]+4 \,\text{ArcCoth}[a x] \,\text{ArcTan}\left[\frac{a d x}{\sqrt{a^2 c d}}\right]-\right.$$

$$\left(\text{ArcCos}\left[-\frac{-a^2 c+d}{a^2 c+d}\right]-2 \,\text{ArcTan}\left[\frac{a c}{\sqrt{a^2 c d}\, x}\right]\right) \text{Log}\left[1-\frac{\left(-a^2 c+d-2 i \sqrt{a^2 c d}\right) \left(2 d-\frac{2 i \sqrt{a^2 c d}}{a x}\right)}{\left(a^2 c+d\right) \left(2 d+\frac{2 i \sqrt{a^2 c d}}{a x}\right)}\right]+$$

$$\left(-\text{ArcCos}\left[-\frac{-a^2 c+d}{a^2 c+d}\right]-2 \,\text{ArcTan}\left[\frac{a c}{\sqrt{a^2 c d}\, x}\right]\right) \text{Log}\left[1-\frac{\left(-a^2 c+d+2 i \sqrt{a^2 c d}\right) \left(2 d-\frac{2 i \sqrt{a^2 c d}}{a x}\right)}{\left(a^2 c+d\right) \left(2 d+\frac{2 i \sqrt{a^2 c d}}{a x}\right)}\right]+$$

$$\left(\text{ArcCos}\left[-\frac{-a^2 c+d}{a^2 c+d}\right]+2 i \left(-i \,\text{ArcTan}\left[\frac{a c}{\sqrt{a^2 c d}\, x}\right]-i \,\text{ArcTan}\left[\frac{a d x}{\sqrt{a^2 c d}}\right]\right)\right) \text{Log}\left[\frac{\sqrt{2} \sqrt{a^2 c d}\, e^{-\text{ArcCoth}[a x]}}{\sqrt{a^2 c+d} \sqrt{-a^2 c+d+\left(a^2 c+d\right) \text{Cosh}[2 \,\text{ArcCoth}[a x]]}}\right]+$$

$$\left(\text{ArcCos}\left[-\frac{-a^2 c+d}{a^2 c+d}\right]-2 i \left(-i \text{ArcTan}\left[\frac{a c}{\sqrt{a^2 c d}\, x}\right]-i \text{ArcTan}\left[\frac{a d x}{\sqrt{a^2 c d}}\right]\right)\right) \text{Log}\left[\frac{\sqrt{2} \sqrt{a^2 c d}\, e^{\text{ArcCoth}[a x]}}{\sqrt{a^2 c+d} \sqrt{-a^2 c+d+\left(a^2 c+d\right) \text{Cosh}[2 \text{ArcCoth}[a x]]}}\right]+$$

$$i \left(\text{PolyLog}\left[2,\frac{\left(-a^2 c+d-2 i \sqrt{a^2 c d}\right) \left(2 d-\frac{2 i \sqrt{a^2 c d}}{a x}\right)}{\left(a^2 c+d\right) \left(2 d+\frac{2 i \sqrt{a^2 c d}}{a x}\right)}\right]-\text{PolyLog}\left[2,\frac{\left(-a^2 c+d+2 i \sqrt{a^2 c d}\right) \left(2 d-\frac{2 i \sqrt{a^2 c d}}{a x}\right)}{\left(a^2 c+d\right) \left(2 d+\frac{2 i \sqrt{a^2 c d}}{a x}\right)}\right]\right)\Bigg)+$$

$$\frac{1}{32 a^4 c^2 \sqrt{a^2 c d} \left(a^2 c+d\right)} 3 d \Bigg(-2 i \text{ArcCos}\left[-\frac{-a^2 c+d}{a^2 c+d}\right] \text{ArcTan}\left[\frac{a c}{\sqrt{a^2 c d}\, x}\right]+4 \text{ArcCoth}[a x] \text{ArcTan}\left[\frac{a d x}{\sqrt{a^2 c d}}\right]-$$

$$\left(\text{ArcCos}\left[-\frac{-a^2 c+d}{a^2 c+d}\right]-2 \text{ArcTan}\left[\frac{a c}{\sqrt{a^2 c d}\, x}\right]\right) \text{Log}\left[1-\frac{\left(-a^2 c+d-2 i \sqrt{a^2 c d}\right) \left(2 d-\frac{2 i \sqrt{a^2 c d}}{a x}\right)}{\left(a^2 c+d\right) \left(2 d+\frac{2 i \sqrt{a^2 c d}}{a x}\right)}\right]+$$

$$\left(-\text{ArcCos}\left[-\frac{-a^2 c+d}{a^2 c+d}\right]-2 \text{ArcTan}\left[\frac{a c}{\sqrt{a^2 c d}\, x}\right]\right) \text{Log}\left[1-\frac{\left(-a^2 c+d+2 i \sqrt{a^2 c d}\right) \left(2 d-\frac{2 i \sqrt{a^2 c d}}{a x}\right)}{\left(a^2 c+d\right) \left(2 d+\frac{2 i \sqrt{a^2 c d}}{a x}\right)}\right]+$$

$$\left(\text{ArcCos}\left[-\frac{-a^2 c+d}{a^2 c+d}\right]+2 i \left(-i \text{ArcTan}\left[\frac{a c}{\sqrt{a^2 c d}\, x}\right]-i \text{ArcTan}\left[\frac{a d x}{\sqrt{a^2 c d}}\right]\right)\right) \text{Log}\left[\frac{\sqrt{2} \sqrt{a^2 c d}\, e^{-\text{ArcCoth}[a x]}}{\sqrt{a^2 c+d} \sqrt{-a^2 c+d+\left(a^2 c+d\right) \text{Cosh}[2 \text{ArcCoth}[a x]]}}\right]+$$

$$\left(\text{ArcCos}\left[-\frac{-a^2 c+d}{a^2 c+d}\right]-2 i \left(-i \text{ArcTan}\left[\frac{a c}{\sqrt{a^2 c d}\, x}\right]-i \text{ArcTan}\left[\frac{a d x}{\sqrt{a^2 c d}}\right]\right)\right) \text{Log}\left[\frac{\sqrt{2} \sqrt{a^2 c d}\, e^{\text{ArcCoth}[a x]}}{\sqrt{a^2 c+d} \sqrt{-a^2 c+d+\left(a^2 c+d\right) \text{Cosh}[2 \text{ArcCoth}[a x]]}}\right]+$$

$$i \left(\text{PolyLog}\left[2,\frac{\left(-a^2 c+d-2 i \sqrt{a^2 c d}\right) \left(2 d-\frac{2 i \sqrt{a^2 c d}}{a x}\right)}{\left(a^2 c+d\right) \left(2 d+\frac{2 i \sqrt{a^2 c d}}{a x}\right)}\right]-\text{PolyLog}\left[2,\frac{\left(-a^2 c+d+2 i \sqrt{a^2 c d}\right) \left(2 d-\frac{2 i \sqrt{a^2 c d}}{a x}\right)}{\left(a^2 c+d\right) \left(2 d+\frac{2 i \sqrt{a^2 c d}}{a x}\right)}\right]\right)\Bigg)-$$

$$\frac{d \text{ArcCoth}[a x] \text{Sinh}[2 \text{ArcCoth}[a x]]}{2 a^2 c \left(a^2 c+d\right) \left(-a^2 c+d+a^2 c \text{Cosh}[2 \text{ArcCoth}[a x]]+d \text{Cosh}[2 \text{ArcCoth}[a x]]\right)^2}-$$

$$\left(2 a^2 c d-5 a^4 c^2 \text{ArcCoth}[a x] \text{Sinh}[2 \text{ArcCoth}[a x]]-8 a^2 c d \text{ArcCoth}[a x] \text{Sinh}[2 \text{ArcCoth}[a x]]-3 d^2 \text{ArcCoth}[a x] \text{Sinh}[2 \text{ArcCoth}[a x]]\right)\Big/$$

$$\left(8 a^4 c^2 \left(a^2 c + d\right)^2 \left(-a^2 c + d + a^2 c \,\text{Cosh}[2\,\text{ArcCoth}[a x]] + d\,\text{Cosh}[2\,\text{ArcCoth}[a x]]\right)\right)$$

## Problem 66: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcCoth}[a + b x]}{x}\, dx$$

Optimal (type 4, 92 leaves, 5 steps):

$$-\text{ArcCoth}[a + b x]\,\text{Log}\left[\frac{2}{1 + a + b x}\right] + \text{ArcCoth}[a + b x]\,\text{Log}\left[\frac{2 b x}{(1 - a)\,(1 + a + b x)}\right] + \frac{1}{2}\,\text{PolyLog}\left[2, 1 - \frac{2}{1 + a + b x}\right] - \frac{1}{2}\,\text{PolyLog}\left[2, 1 - \frac{2 b x}{(1 - a)\,(1 + a + b x)}\right]$$

Result (type 4, 259 leaves):

$$(\text{ArcCoth}[a + b x] - \text{ArcTanh}[a + b x])\,\text{Log}[x] + \text{ArcTanh}[a + b x]\left(-\text{Log}\left[\frac{1}{\sqrt{1 - (a + b x)^2}}\right] + \text{Log}[-i\,\text{Sinh}[\text{ArcTanh}[a] - \text{ArcTanh}[a + b x]]]\right) + \frac{1}{8}\left(4\,(\text{ArcTanh}[a] - \text{ArcTanh}[a + b x])^2 - (\pi - 2 i\,\text{ArcTanh}[a + b x])^2 - 8\,(\text{ArcTanh}[a] - \text{ArcTanh}[a + b x])\,\text{Log}\left[1 - e^{2\,\text{ArcTanh}[a] - 2\,\text{ArcTanh}[a + b x]}\right] - 4 i\,(\pi - 2 i\,\text{ArcTanh}[a + b x])\,\text{Log}\left[1 + e^{2\,\text{ArcTanh}[a + b x]}\right] + 4\,(i \pi + 2\,\text{ArcTanh}[a + b x])\,\text{Log}\left[\frac{2}{\sqrt{1 - (a + b x)^2}}\right] + 8\,(\text{ArcTanh}[a] - \text{ArcTanh}[a + b x])\,\text{Log}[-2 i\,\text{Sinh}[\text{ArcTanh}[a] - \text{ArcTanh}[a + b x]]] - 4\,\text{PolyLog}\left[2, e^{2\,\text{ArcTanh}[a] - 2\,\text{ArcTanh}[a + b x]}\right] - 4\,\text{PolyLog}\left[2, -e^{2\,\text{ArcTanh}[a + b x]}\right]\right)$$

## Problem 70: Result more than twice size of optimal antiderivative.

$$\int x^2\,\text{ArcCoth}[a + b x]^2\, dx$$

Optimal (type 4, 204 leaves, 15 steps):

$$\frac{x}{3 b^2}-\frac{2 a (a+b x) \operatorname{ArcCoth}[a+b x]}{b^3}+\frac{(a+b x)^2 \operatorname{ArcCoth}[a+b x]}{3 b^3}+\frac{a \left(3+a^2\right) \operatorname{ArcCoth}[a+b x]^2}{3 b^3}+\frac{\left(1+3 a^2\right) \operatorname{ArcCoth}[a+b x]^2}{3 b^3}+\frac{1}{3} x^3 \operatorname{ArcCoth}[a+b x]^2-\frac{\operatorname{ArcTanh}[a+b x]}{3 b^3}-\frac{2 \left(1+3 a^2\right) \operatorname{ArcCoth}[a+b x] \operatorname{Log}\left[\frac{2}{1-a-b x}\right]}{3 b^3}-\frac{a \operatorname{Log}\left[1-(a+b x)^2\right]}{b^3}-\frac{\left(1+3 a^2\right) \operatorname{PolyLog}\left[2,-\frac{1+a+b x}{1-a-b x}\right]}{3 b^3}$$

Result (type 4, 615 leaves) :

$$-\frac{1}{12 b^3}(a+b x)\sqrt{1-\frac{1}{(a+b x)^2}}\left(1-(a+b x)^2\right)\left(\frac{4 \operatorname{ArcCoth}[a+b x]}{(a+b x)\sqrt{1-\frac{1}{(a+b x)^2}}}+\frac{3 \operatorname{ArcCoth}[a+b x]^2}{(a+b x)\sqrt{1-\frac{1}{(a+b x)^2}}}-\frac{12 a \operatorname{ArcCoth}[a+b x]^2}{(a+b x)\sqrt{1-\frac{1}{(a+b x)^2}}}+\frac{9 a^2 \operatorname{ArcCoth}[a+b x]^2}{(a+b x)\sqrt{1-\frac{1}{(a+b x)^2}}}+\frac{-1+6 a \operatorname{ArcCoth}[a+b x]+3 \operatorname{ArcCoth}[a+b x]^2-3 a^2 \operatorname{ArcCoth}[a+b x]^2}{\sqrt{1-\frac{1}{(a+b x)^2}}}+\operatorname{Cosh}[3 \operatorname{ArcCoth}[a+b x]]-6 a \operatorname{ArcCoth}[a+b x] \operatorname{Cosh}[3 \operatorname{ArcCoth}[a+b x]]+\operatorname{ArcCoth}[a+b x]^2 \operatorname{Cosh}[3 \operatorname{ArcCoth}[a+b x]]+3 a^2 \operatorname{ArcCoth}[a+b x]^2 \operatorname{Cosh}[3 \operatorname{ArcCoth}[a+b x]]+\frac{6 \operatorname{ArcCoth}[a+b x] \operatorname{Log}\left[1-e^{-2 \operatorname{ArcCoth}[a+b x]}\right]}{(a+b x)\sqrt{1-\frac{1}{(a+b x)^2}}}+\frac{18 a^2 \operatorname{ArcCoth}[a+b x] \operatorname{Log}\left[1-e^{-2 \operatorname{ArcCoth}[a+b x]}\right]}{(a+b x)\sqrt{1-\frac{1}{(a+b x)^2}}}-\frac{18 a \operatorname{Log}\left[\frac{1}{(a+b x)\sqrt{1-\frac{1}{(a+b x)^2}}}\right]}{(a+b x)\sqrt{1-\frac{1}{(a+b x)^2}}}+\frac{4\left(1+3 a^2\right) \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcCoth}[a+b x]}\right]}{(a+b x)^3\left(1-\frac{1}{(a+b x)^2}\right)^{3/2}}-\operatorname{ArcCoth}[a+b x]^2 \operatorname{Sinh}[3 \operatorname{ArcCoth}[a+b x]]-3 a^2 \operatorname{ArcCoth}[a+b x]^2 \operatorname{Sinh}[3 \operatorname{ArcCoth}[a+b x]]-2 \operatorname{ArcCoth}[a+b x] \operatorname{Log}\left[1-e^{-2 \operatorname{ArcCoth}[a+b x]}\right] \operatorname{Sinh}[3 \operatorname{ArcCoth}[a+b x]]-6 a^2 \operatorname{ArcCoth}[a+b x] \operatorname{Log}\left[1-e^{-2 \operatorname{ArcCoth}[a+b x]}\right] \operatorname{Sinh}[3 \operatorname{ArcCoth}[a+b x]]+6 a \operatorname{Log}\left[\frac{1}{(a+b x)\sqrt{1-\frac{1}{(a+b x)^2}}}\right] \operatorname{Sinh}[3 \operatorname{ArcCoth}[a+b x]]\right)$$

## Problem 73: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{ArcCoth}[a+b x]^2}{x}\, dx$$

Optimal (type 4, 148 leaves, 2 steps) :

$$-\text{ArcCoth}[a+b x]^2 \text{Log}\left[\frac{2}{1+a+b x}\right]+\text{ArcCoth}[a+b x]^2 \text{Log}\left[\frac{2 b x}{(1-a)(1+a+b x)}\right]+\text{ArcCoth}[a+b x] \text{PolyLog}\left[2,1-\frac{2}{1+a+b x}\right]-\text{ArcCoth}[a+b x] \text{PolyLog}\left[2,1-\frac{2 b x}{(1-a)(1+a+b x)}\right]+\frac{1}{2} \text{PolyLog}\left[3,1-\frac{2}{1+a+b x}\right]-\frac{1}{2} \text{PolyLog}\left[3,1-\frac{2 b x}{(1-a)(1+a+b x)}\right]$$

Result (type 4, 675 leaves) :

$$-\frac{i \pi^3}{24}-\frac{2}{3} \text{ArcCoth}[a+b x]^3-\frac{2}{3} a \text{ArcCoth}[a+b x]^3+\frac{2}{3} \sqrt{1-\frac{1}{a^2}}\, a\, e^{\text{ArcTanh}\left[\frac{1}{a}\right]} \text{ArcCoth}[a+b x]^3-i \pi \text{ArcCoth}[a+b x] \text{Log}\left[\frac{1}{2}\left(e^{-\text{ArcCoth}[a+b x]}+e^{\text{ArcCoth}[a+b x]}\right)\right]-\text{ArcCoth}[a+b x]^2 \text{Log}\left[1-e^{2 \text{ArcCoth}[a+b x]}\right]-\text{ArcCoth}[a+b x]^2 \text{Log}\left[1-\frac{(-1+a)\, e^{2 \text{ArcCoth}[a+b x]}}{1+a}\right]+\text{ArcCoth}[a+b x]^2 \text{Log}\left[1-e^{2 \text{ArcCoth}[a+b x]-2 \text{ArcTanh}\left[\frac{1}{a}\right]}\right]+\text{ArcCoth}[a+b x]^2 \text{Log}\left[1-e^{\text{ArcCoth}[a+b x]-\text{ArcTanh}\left[\frac{1}{a}\right]}\right]+\text{ArcCoth}[a+b x]^2 \text{Log}\left[1+e^{\text{ArcCoth}[a+b x]-\text{ArcTanh}\left[\frac{1}{a}\right]}\right]-2 \text{ArcCoth}[a+b x] \text{ArcTanh}\left[\frac{1}{a}\right] \text{Log}\left[\frac{1}{2} i \left(e^{\text{ArcCoth}[a+b x]-\text{ArcTanh}\left[\frac{1}{a}\right]}-e^{-\text{ArcCoth}[a+b x]+\text{ArcTanh}\left[\frac{1}{a}\right]}\right)\right]+\text{ArcCoth}[a+b x]^2 \text{Log}\left[\frac{1}{2} e^{-\text{ArcCoth}[a+b x]}\left(-1-e^{2 \text{ArcCoth}[a+b x]}+a\left(-1+e^{2 \text{ArcCoth}[a+b x]}\right)\right)\right]+i \pi \text{ArcCoth}[a+b x] \text{Log}\left[\frac{1}{\sqrt{1-\frac{1}{(a+b x)^2}}}\right]-\text{ArcCoth}[a+b x]^2 \text{Log}\left[-\frac{b x}{(a+b x) \sqrt{1-\frac{1}{(a+b x)^2}}}\right]+2 \text{ArcCoth}[a+b x] \text{ArcTanh}\left[\frac{1}{a}\right] \text{Log}\left[i \text{Sinh}\left[\text{ArcCoth}[a+b x]-\text{ArcTanh}\left[\frac{1}{a}\right]\right]\right]-\text{ArcCoth}[a+b x] \text{PolyLog}\left[2, e^{2 \text{ArcCoth}[a+b x]}\right]-\text{ArcCoth}[a+b x] \text{PolyLog}\left[2, \frac{(-1+a)\, e^{2 \text{ArcCoth}[a+b x]}}{1+a}\right]+\text{ArcCoth}[a+b x] \text{PolyLog}\left[2, e^{2 \text{ArcCoth}[a+b x]-2 \text{ArcTanh}\left[\frac{1}{a}\right]}\right]+2 \text{ArcCoth}[a+b x] \text{PolyLog}\left[2,-e^{\text{ArcCoth}[a+b x]-\text{ArcTanh}\left[\frac{1}{a}\right]}\right]+2 \text{ArcCoth}[a+b x] \text{PolyLog}\left[2, e^{\text{ArcCoth}[a+b x]-\text{ArcTanh}\left[\frac{1}{a}\right]}\right]+\frac{1}{2} \text{PolyLog}\left[3, e^{2 \text{ArcCoth}[a+b x]}\right]+\frac{1}{2} \text{PolyLog}\left[3, \frac{(-1+a)\, e^{2 \text{ArcCoth}[a+b x]}}{1+a}\right]-\frac{1}{2} \text{PolyLog}\left[3, e^{2 \text{ArcCoth}[a+b x]-2 \text{ArcTanh}\left[\frac{1}{a}\right]}\right]-2 \text{PolyLog}\left[3,-e^{\text{ArcCoth}[a+b x]-\text{ArcTanh}\left[\frac{1}{a}\right]}\right]-2 \text{PolyLog}\left[3, e^{\text{ArcCoth}[a+b x]-\text{ArcTanh}\left[\frac{1}{a}\right]}\right]$$

## Problem 74: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{ArcCoth}[a+b x]^2}{x^2}\, dx$$

Optimal (type 4, 251 leaves, 17 steps) :

$$-\frac{\text{ArcCoth}[a+b x]^2}{x}+\frac{b\,\text{ArcCoth}[a+b x]\,\text{Log}\left[\frac{2}{1-a-b x}\right]}{1-a}+\frac{b\,\text{ArcCoth}[a+b x]\,\text{Log}\left[\frac{2}{1+a+b x}\right]}{1+a}-\frac{2 b\,\text{ArcCoth}[a+b x]\,\text{Log}\left[\frac{2}{1+a+b x}\right]}{1-a^2}+\frac{2 b\,\text{ArcCoth}[a+b x]\,\text{Log}\left[\frac{2 b x}{(1-a)(1+a+b x)}\right]}{1-a^2}+\frac{b\,\text{PolyLog}\left[2,-\frac{1+a+b x}{1-a-b x}\right]}{2(1-a)}-\frac{b\,\text{PolyLog}\left[2,1-\frac{2}{1+a+b x}\right]}{2(1+a)}+\frac{b\,\text{PolyLog}\left[2,1-\frac{2}{1+a+b x}\right]}{1-a^2}-\frac{b\,\text{PolyLog}\left[2,1-\frac{2 b x}{(1-a)(1+a+b x)}\right]}{1-a^2}$$

Result (type 4, 206 leaves) :

$$\frac{1}{(-1+a^2)\,x}\left(-\left(-1+a^2+\sqrt{1-\frac{1}{a^2}}\,a\,b\,e^{\text{ArcTanh}\left[\frac{1}{a}\right]}x\right)\text{ArcCoth}[a+b x]^2+b x\,\text{ArcCoth}[a+b x]\left(-i\pi+2\,\text{ArcTanh}\left[\frac{1}{a}\right]-2\,\text{Log}\left[1-e^{-2\,\text{ArcCoth}[a+b x]+2\,\text{ArcTanh}\left[\frac{1}{a}\right]}\right]\right)+b x\left(i\pi\left(\text{Log}\left[1+e^{2\,\text{ArcCoth}[a+b x]}\right]-\text{Log}\left[\frac{1}{\sqrt{1-\frac{1}{(a+b x)^2}}}\right]\right)+2\,\text{ArcTanh}\left[\frac{1}{a}\right]\left(\text{Log}\left[1-e^{-2\,\text{ArcCoth}[a+b x]+2\,\text{ArcTanh}\left[\frac{1}{a}\right]}\right]-\text{Log}\left[i\,\text{Sinh}\left[\text{ArcCoth}[a+b x]-\text{ArcTanh}\left[\frac{1}{a}\right]\right]\right]\right)\right)+b x\,\text{PolyLog}\left[2,e^{-2\,\text{ArcCoth}[a+b x]+2\,\text{ArcTanh}\left[\frac{1}{a}\right]}\right]\right)$$

## Problem 75: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\text{ArcCoth}[a+b x]^2}{x^3}\,dx$$

Optimal (type 4, 370 leaves, 21 steps) :

$$-\frac{b\,\text{ArcCoth}[a+b\,x]}{(1-a^2)\,x}-\frac{\text{ArcCoth}[a+b\,x]^2}{2\,x^2}+\frac{b^2\,\text{Log}[x]}{(1-a^2)^2}+\frac{b^2\,\text{ArcCoth}[a+b\,x]\,\text{Log}\left[\frac{2}{1-a-b\,x}\right]}{2\,(1-a)^2}-\frac{b^2\,\text{Log}[1-a-b\,x]}{2\,(1-a)^2\,(1+a)}-$$

$$\frac{b^2\,\text{ArcCoth}[a+b\,x]\,\text{Log}\left[\frac{2}{1+a+b\,x}\right]}{2\,(1+a)^2}-\frac{2\,a\,b^2\,\text{ArcCoth}[a+b\,x]\,\text{Log}\left[\frac{2}{1+a+b\,x}\right]}{(1-a^2)^2}+\frac{2\,a\,b^2\,\text{ArcCoth}[a+b\,x]\,\text{Log}\left[\frac{2\,b\,x}{(1-a)\,(1+a+b\,x)}\right]}{(1-a^2)^2}-\frac{b^2\,\text{Log}[1+a+b\,x]}{2\,(1-a)\,(1+a)^2}+$$

$$\frac{b^2\,\text{PolyLog}\left[2,-\frac{1+a+b\,x}{1-a-b\,x}\right]}{4\,(1-a)^2}+\frac{b^2\,\text{PolyLog}\left[2,1-\frac{2}{1+a+b\,x}\right]}{4\,(1+a)^2}+\frac{a\,b^2\,\text{PolyLog}\left[2,1-\frac{2}{1+a+b\,x}\right]}{(1-a^2)^2}-\frac{a\,b^2\,\text{PolyLog}\left[2,1-\frac{2\,b\,x}{(1-a)\,(1+a+b\,x)}\right]}{(1-a^2)^2}$$

Result (type 4, 291 leaves):

$$\frac{1}{2\,(-1+a^2)^2\,x^2}\left(\left(-1-a^4+b^2\,x^2+a^2\left(2+b^2\left(-1+2\,\sqrt{1-\frac{1}{a^2}}\,e^{\text{ArcTanh}\left[\frac{1}{a}\right]}\right)x^2\right)\right)\text{ArcCoth}[a+b\,x]^2+\right.$$

$$2\,b\,x\,\text{ArcCoth}[a+b\,x]\left(-1+a^2+a\,b\,x+i\,a\,b\,\pi\,x-2\,a\,b\,x\,\text{ArcTanh}\left[\frac{1}{a}\right]+2\,a\,b\,x\,\text{Log}\left[1-e^{-2\,\text{ArcCoth}[a+b\,x]+2\,\text{ArcTanh}\left[\frac{1}{a}\right]}\right]\right)+$$

$$2\,b^2\,x^2\left(-i\,a\,\pi\,\text{Log}\left[1+e^{2\,\text{ArcCoth}[a+b\,x]}\right]+i\,a\,\pi\,\text{Log}\left[\frac{1}{\sqrt{1-\frac{1}{(a+b\,x)^2}}}\right]+\text{Log}\left[-\frac{b\,x}{(a+b\,x)\,\sqrt{1-\frac{1}{(a+b\,x)^2}}}\right]-2\,a\,\text{ArcTanh}\left[\frac{1}{a}\right]\right.$$

$$\left.\left.\left(\text{Log}\left[1-e^{-2\,\text{ArcCoth}[a+b\,x]+2\,\text{ArcTanh}\left[\frac{1}{a}\right]}\right]-\text{Log}\left[i\,\text{Sinh}\left[\text{ArcCoth}[a+b\,x]-\text{ArcTanh}\left[\frac{1}{a}\right]\right]\right]\right)\right)-2\,a\,b^2\,x^2\,\text{PolyLog}\left[2,e^{-2\,\text{ArcCoth}[a+b\,x]+2\,\text{ArcTanh}\left[\frac{1}{a}\right]}\right]\right)$$

## Problem 76: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{\text{ArcCoth}[a+b\,x]}{c+d\,x^2}\,dx$$

Optimal (type 4, 673 leaves, 15 steps):

$$\frac{\text{Log}\left[-\frac{1-a-b\,x}{a+b\,x}\right]\,\text{Log}\left[1+\frac{\left(b^2\,c+a^2\,d\right)\,(1-a-b\,x)}{\left(b^2\,c-b\,\sqrt{-c}\,\sqrt{d}-(1-a)\,a\,d\right)\,(a+b\,x)}\right]}{4\,\sqrt{-c}\,\sqrt{d}}-\frac{\text{Log}\left[-\frac{1-a-b\,x}{a+b\,x}\right]\,\text{Log}\left[1+\frac{\left(b^2\,c+a^2\,d\right)\,(1-a-b\,x)}{\left(b^2\,c+b\,\sqrt{-c}\,\sqrt{d}-(1-a)\,a\,d\right)\,(a+b\,x)}\right]}{4\,\sqrt{-c}\,\sqrt{d}}+$$

$$\frac{\text{Log}\left[\frac{1+a+b\,x}{a+b\,x}\right]\,\text{Log}\left[1-\frac{\left(b^2\,c+a^2\,d\right)\,(1+a+b\,x)}{\left(b^2\,c-b\,\sqrt{-c}\,\sqrt{d}+a\,(1+a)\,d\right)\,(a+b\,x)}\right]}{4\,\sqrt{-c}\,\sqrt{d}}-\frac{\text{Log}\left[\frac{1+a+b\,x}{a+b\,x}\right]\,\text{Log}\left[1-\frac{\left(b^2\,c+a^2\,d\right)\,(1+a+b\,x)}{\left(b^2\,c+b\,\sqrt{-c}\,\sqrt{d}+a\,(1+a)\,d\right)\,(a+b\,x)}\right]}{4\,\sqrt{-c}\,\sqrt{d}}+\frac{\text{PolyLog}\left[2,-\frac{\left(b^2\,c+a^2\,d\right)\,(1-a-b\,x)}{\left(b^2\,c-b\,\sqrt{-c}\,\sqrt{d}-(1-a)\,a\,d\right)\,(a+b\,x)}\right]}{4\,\sqrt{-c}\,\sqrt{d}}-$$

$$\frac{\text{PolyLog}\left[2,-\frac{\left(b^2\,c+a^2\,d\right)\,(1-a-b\,x)}{\left(b^2\,c+b\,\sqrt{-c}\,\sqrt{d}-(1-a)\,a\,d\right)\,(a+b\,x)}\right]}{4\,\sqrt{-c}\,\sqrt{d}}+\frac{\text{PolyLog}\left[2,\frac{\left(b^2\,c+a^2\,d\right)\,(1+a+b\,x)}{\left(b^2\,c-b\,\sqrt{-c}\,\sqrt{d}+a\,(1+a)\,d\right)\,(a+b\,x)}\right]}{4\,\sqrt{-c}\,\sqrt{d}}-\frac{\text{PolyLog}\left[2,\frac{\left(b^2\,c+a^2\,d\right)\,(1+a+b\,x)}{\left(b^2\,c+b\,\sqrt{-c}\,\sqrt{d}+a\,(1+a)\,d\right)\,(a+b\,x)}\right]}{4\,\sqrt{-c}\,\sqrt{d}}$$

Result (type 4, 1450 leaves) :

$$-\frac{1}{4\left(1-a^2\right)\sqrt{c}\,d\,(a+b\,x)^2\left(1-\frac{1}{(a+b\,x)^2}\right)}$$

$$\left(1-(a+b\,x)^2\right)\Bigg(-4\left(-1+a^2\right)\sqrt{d}\,\text{ArcCoth}[a+b\,x]\,\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right]+2\,i\,\sqrt{d}\,\text{ArcTan}\left[\frac{(-1+a)\sqrt{d}}{b\sqrt{c}}\right]\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right]-$$

$$2\,i\,a^2\sqrt{d}\,\text{ArcTan}\left[\frac{(-1+a)\sqrt{d}}{b\sqrt{c}}\right]\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right]-2\,i\,\sqrt{d}\,\text{ArcTan}\left[\frac{(1+a)\sqrt{d}}{b\sqrt{c}}\right]\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right]+$$

$$2\,i\,a^2\sqrt{d}\,\text{ArcTan}\left[\frac{(1+a)\sqrt{d}}{b\sqrt{c}}\right]\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right]-2\,b\sqrt{c}\,\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right]^2+b\sqrt{c}\sqrt{\frac{b^2c+(-1+a)^2d}{b^2c}}\,e^{-i\,\text{ArcTan}\left[\frac{(-1+a)\sqrt{d}}{b\sqrt{c}}\right]}\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right]^2+$$

$$a\,b\sqrt{c}\sqrt{\frac{b^2c+(-1+a)^2d}{b^2c}}\,e^{-i\,\text{ArcTan}\left[\frac{(-1+a)\sqrt{d}}{b\sqrt{c}}\right]}\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right]^2+b\sqrt{c}\sqrt{\frac{b^2c+(1+a)^2d}{b^2c}}\,e^{-i\,\text{ArcTan}\left[\frac{(1+a)\sqrt{d}}{b\sqrt{c}}\right]}\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right]^2-$$

$$a\,b\sqrt{c}\sqrt{\frac{b^2c+(1+a)^2d}{b^2c}}\,e^{-i\,\text{ArcTan}\left[\frac{(1+a)\sqrt{d}}{b\sqrt{c}}\right]}\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right]^2+2\sqrt{d}\,\text{ArcTan}\left[\frac{(-1+a)\sqrt{d}}{b\sqrt{c}}\right]\text{Log}\left[1-e^{-2\,i\left(\text{ArcTan}\left[\frac{(-1+a)\sqrt{d}}{b\sqrt{c}}\right]+\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right]\right)}\right]-$$

$$2\,a^2\sqrt{d}\,\text{ArcTan}\left[\frac{(-1+a)\sqrt{d}}{b\sqrt{c}}\right]\text{Log}\left[1-e^{-2\,i\left(\text{ArcTan}\left[\frac{(-1+a)\sqrt{d}}{b\sqrt{c}}\right]+\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right]\right)}\right]+2\sqrt{d}\,\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right]\text{Log}\left[1-e^{-2\,i\left(\text{ArcTan}\left[\frac{(-1+a)\sqrt{d}}{b\sqrt{c}}\right]+\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right]\right)}\right]-$$

$$2\,a^2\sqrt{d}\,\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right]\text{Log}\left[1-e^{-2\,i\left(\text{ArcTan}\left[\frac{(-1+a)\sqrt{d}}{b\sqrt{c}}\right]+\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right]\right)}\right]-2\sqrt{d}\,\text{ArcTan}\left[\frac{(1+a)\sqrt{d}}{b\sqrt{c}}\right]\text{Log}\left[1-e^{-2\,i\left(\text{ArcTan}\left[\frac{(1+a)\sqrt{d}}{b\sqrt{c}}\right]+\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right]\right)}\right]+$$

$$2\,a^2\sqrt{d}\,\text{ArcTan}\left[\frac{(1+a)\sqrt{d}}{b\sqrt{c}}\right]\text{Log}\left[1-e^{-2\,i\left(\text{ArcTan}\left[\frac{(1+a)\sqrt{d}}{b\sqrt{c}}\right]+\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right]\right)}\right]-2\sqrt{d}\,\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right]\text{Log}\left[1-e^{-2\,i\left(\text{ArcTan}\left[\frac{(1+a)\sqrt{d}}{b\sqrt{c}}\right]+\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right]\right)}\right]+2\,a^2\sqrt{d}$$

$$\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right]\text{Log}\left[1-e^{-2\,i\left(\text{ArcTan}\left[\frac{(1+a)\sqrt{d}}{b\sqrt{c}}\right]+\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right]\right)}\right]-2\sqrt{d}\,\text{ArcTan}\left[\frac{(-1+a)\sqrt{d}}{b\sqrt{c}}\right]\text{Log}\left[-\text{Sin}\left[\text{ArcTan}\left[\frac{(-1+a)\sqrt{d}}{b\sqrt{c}}\right]+\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right]\right]\right]+$$

$$2\,a^2\sqrt{d}\,\text{ArcTan}\left[\frac{(-1+a)\sqrt{d}}{b\sqrt{c}}\right]\text{Log}\left[-\text{Sin}\left[\text{ArcTan}\left[\frac{(-1+a)\sqrt{d}}{b\sqrt{c}}\right]+\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right]\right]\right]+2\sqrt{d}\,\text{ArcTan}\left[\frac{(1+a)\sqrt{d}}{b\sqrt{c}}\right]$$

$$\text{Log}\left[-\text{Sin}\left[\text{ArcTan}\left[\frac{(1+a)\sqrt{d}}{b\sqrt{c}}\right]+\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right]\right]\right]-2\,a^2\sqrt{d}\,\text{ArcTan}\left[\frac{(1+a)\sqrt{d}}{b\sqrt{c}}\right]\text{Log}\left[-\text{Sin}\left[\text{ArcTan}\left[\frac{(1+a)\sqrt{d}}{b\sqrt{c}}\right]+\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right]\right]\right]-$$

$$i\left(-1+a^2\right)\sqrt{d}\,\text{PolyLog}\left[2,e^{-2\,i\left(\text{ArcTan}\left[\frac{(-1+a)\sqrt{d}}{b\sqrt{c}}\right]+\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right]\right)}\right]+i\left(-1+a^2\right)\sqrt{d}\,\text{PolyLog}\left[2,e^{-2\,i\left(\text{ArcTan}\left[\frac{(1+a)\sqrt{d}}{b\sqrt{c}}\right]+\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right]\right)}\right]\Bigg)$$

## Problem 77: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcCoth}[a+b x]}{c+d x} \, dx$$

Optimal (type 4, 120 leaves, 5 steps):

$$-\frac{\text{ArcCoth}[a+b x] \,\text{Log}\left[\frac{2}{1+a+b x}\right]}{d}+\frac{\text{ArcCoth}[a+b x] \,\text{Log}\left[\frac{2 b (c+d x)}{(b c+d-a d) (1+a+b x)}\right]}{d}+\frac{\text{PolyLog}\left[2,1-\frac{2}{1+a+b x}\right]}{2 d}-\frac{\text{PolyLog}\left[2,1-\frac{2 b (c+d x)}{(b c+d-a d) (1+a+b x)}\right]}{2 d}$$

Result (type 4, 330 leaves):

$$\frac{1}{d}\Bigg((\text{ArcCoth}[a+b x]-\text{ArcTanh}[a+b x]) \,\text{Log}[c+d x]+$$
$$\text{ArcTanh}[a+b x]\left(-\text{Log}\left[\frac{1}{\sqrt{1-(a+b x)^2}}\right]+\text{Log}\left[i \,\text{Sinh}\left[\text{ArcTanh}\left[\frac{b c-a d}{d}\right]+\text{ArcTanh}[a+b x]\right]\right]\right)+$$
$$\frac{1}{8}\Bigg(-(\pi-2 i \,\text{ArcTanh}[a+b x])^2+4\left(\text{ArcTanh}\left[\frac{b c-a d}{d}\right]+\text{ArcTanh}[a+b x]\right)^2-4 i (\pi-2 i \,\text{ArcTanh}[a+b x]) \,\text{Log}\left[1+e^{2 \text{ArcTanh}[a+b x]}\right]+$$
$$8\left(\text{ArcTanh}\left[\frac{b c-a d}{d}\right]+\text{ArcTanh}[a+b x]\right) \text{Log}\left[1-e^{-2\left(\text{ArcTanh}\left[\frac{b c-a d}{d}\right]+\text{ArcTanh}[a+b x]\right)}\right]+4 (i \pi+2 \,\text{ArcTanh}[a+b x]) \,\text{Log}\left[\frac{2}{\sqrt{1-(a+b x)^2}}\right]-$$
$$8\left(\text{ArcTanh}\left[\frac{b c-a d}{d}\right]+\text{ArcTanh}[a+b x]\right) \text{Log}\left[2 i \,\text{Sinh}\left[\text{ArcTanh}\left[\frac{b c-a d}{d}\right]+\text{ArcTanh}[a+b x]\right]\right]-$$
$$4 \,\text{PolyLog}\left[2,-e^{2 \text{ArcTanh}[a+b x]}\right]-4 \,\text{PolyLog}\left[2,e^{-2\left(\text{ArcTanh}\left[\frac{b c-a d}{d}\right]+\text{ArcTanh}[a+b x]\right)}\right]\Bigg)\Bigg)$$

## Problem 78: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{ArcCoth}[a+b x]}{c+\frac{d}{x}} \, dx$$

Optimal (type 4, 292 leaves, 37 steps):

$$\frac{(1-a-b x)\,\text{Log}\left[-\frac{1-a-b x}{a+b x}\right]}{2 b c}+\frac{\text{Log}[a+b x]}{2 b c}+\frac{\text{Log}[1+a+b x]}{2 b c}+\frac{(a+b x)\,\text{Log}\left[\frac{1+a+b x}{a+b x}\right]}{2 b c}-\frac{d\,\text{Log}\left[\frac{c (1-a-b x)}{c-a c+b d}\right]\text{Log}[d+c x]}{2 c^2}+$$
$$\frac{d\,\text{Log}\left[-\frac{1-a-b x}{a+b x}\right]\text{Log}[d+c x]}{2 c^2}+\frac{d\,\text{Log}\left[\frac{c (1+a+b x)}{c+a c-b d}\right]\text{Log}[d+c x]}{2 c^2}-\frac{d\,\text{Log}\left[\frac{1+a+b x}{a+b x}\right]\text{Log}[d+c x]}{2 c^2}+\frac{d\,\text{PolyLog}\left[2,-\frac{b (d+c x)}{c+a c-b d}\right]}{2 c^2}-\frac{d\,\text{PolyLog}\left[2,\frac{b (d+c x)}{c-a c+b d}\right]}{2 c^2}$$

Result (type 4, 502 leaves) :

$$\frac{1}{2 b c^3}\Bigg(2 a c^2 \text{ArcCoth}[a+b x]-i b c d \pi\,\text{ArcCoth}[a+b x]+2 b c^2 x\,\text{ArcCoth}[a+b x]+b c d\,\text{ArcCoth}[a+b x]^2+a b c d\,\text{ArcCoth}[a+b x]^2-$$
$$b^2 d^2 \text{ArcCoth}[a+b x]^2-a b c d\sqrt{1-\frac{c^2}{(a c-b d)^2}}\,e^{\text{ArcTanh}\left[\frac{c}{a c-b d}\right]}\text{ArcCoth}[a+b x]^2+b^2 d^2\sqrt{1-\frac{c^2}{(a c-b d)^2}}\,e^{\text{ArcTanh}\left[\frac{c}{a c-b d}\right]}\text{ArcCoth}[a+b x]^2+$$
$$2 b c d\,\text{ArcCoth}[a+b x]\,\text{ArcTanh}\left[\frac{c}{a c-b d}\right]+2 b c d\,\text{ArcCoth}[a+b x]\,\text{Log}\left[1-e^{-2\,\text{ArcCoth}[a+b x]}\right]+i b c d \pi\,\text{Log}\left[1+e^{2\,\text{ArcCoth}[a+b x]}\right]-$$
$$2 b c d\,\text{ArcCoth}[a+b x]\,\text{Log}\left[1-e^{-2\,\text{ArcCoth}[a+b x]+2\,\text{ArcTanh}\left[\frac{c}{a c-b d}\right]}\right]+2 b c d\,\text{ArcTanh}\left[\frac{c}{a c-b d}\right]\text{Log}\left[1-e^{-2\,\text{ArcCoth}[a+b x]+2\,\text{ArcTanh}\left[\frac{c}{a c-b d}\right]}\right]-$$
$$i b c d \pi\,\text{Log}\left[\frac{1}{\sqrt{1-\frac{1}{(a+b x)^2}}}\right]-2 c^2\,\text{Log}\left[\frac{1}{(a+b x)\sqrt{1-\frac{1}{(a+b x)^2}}}\right]-2 b c d\,\text{ArcTanh}\left[\frac{c}{a c-b d}\right]\text{Log}\left[i\,\text{Sinh}\left[\text{ArcCoth}[a+b x]-\text{ArcTanh}\left[\frac{c}{a c-b d}\right]\right]\right]-$$
$$b c d\,\text{PolyLog}\left[2,e^{-2\,\text{ArcCoth}[a+b x]}\right]+b c d\,\text{PolyLog}\left[2,e^{-2\,\text{ArcCoth}[a+b x]+2\,\text{ArcTanh}\left[\frac{c}{a c-b d}\right]}\right]\Bigg)$$

## Problem 79: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcCoth}[a+b x]}{c+\frac{d}{x^2}}\,dx$$

Optimal (type 4, 738 leaves, 57 steps) :

$$\frac{(1-a-b\,x)\,\text{Log}[-1+a+b\,x]}{2\,b\,c}+\frac{x\left(\text{Log}[-1+a+b\,x]-\text{Log}\left[-\frac{1-a-b\,x}{a+b\,x}\right]-\text{Log}[a+b\,x]\right)}{2\,c}-$$

$$\frac{\sqrt{d}\,\text{ArcTan}\left[\frac{\sqrt{c}\,x}{\sqrt{d}}\right]\left(\text{Log}[-1+a+b\,x]-\text{Log}\left[-\frac{1-a-b\,x}{a+b\,x}\right]-\text{Log}[a+b\,x]\right)}{2\,c^{3/2}}+\frac{(1+a+b\,x)\,\text{Log}[1+a+b\,x]}{2\,b\,c}+$$

$$\frac{x\left(\text{Log}[a+b\,x]-\text{Log}[1+a+b\,x]+\text{Log}\left[\frac{1+a+b\,x}{a+b\,x}\right]\right)}{2\,c}-\frac{\sqrt{d}\,\text{ArcTan}\left[\frac{\sqrt{c}\,x}{\sqrt{d}}\right]\left(\text{Log}[a+b\,x]-\text{Log}[1+a+b\,x]+\text{Log}\left[\frac{1+a+b\,x}{a+b\,x}\right]\right)}{2\,c^{3/2}}+$$

$$\frac{\sqrt{d}\,\text{Log}[-1+a+b\,x]\,\text{Log}\left[-\frac{b\left(\sqrt{d}-\sqrt{-c}\,x\right)}{(1-a)\sqrt{-c}-b\sqrt{d}}\right]}{4\,(-c)^{3/2}}-\frac{\sqrt{d}\,\text{Log}[1+a+b\,x]\,\text{Log}\left[\frac{b\left(\sqrt{d}-\sqrt{-c}\,x\right)}{(1+a)\sqrt{-c}+b\sqrt{d}}\right]}{4\,(-c)^{3/2}}+$$

$$\frac{\sqrt{d}\,\text{Log}[1+a+b\,x]\,\text{Log}\left[-\frac{b\left(\sqrt{d}+\sqrt{-c}\,x\right)}{(1+a)\sqrt{-c}-b\sqrt{d}}\right]}{4\,(-c)^{3/2}}-\frac{\sqrt{d}\,\text{Log}[-1+a+b\,x]\,\text{Log}\left[\frac{b\left(\sqrt{d}+\sqrt{-c}\,x\right)}{(1-a)\sqrt{-c}+b\sqrt{d}}\right]}{4\,(-c)^{3/2}}+\frac{\sqrt{d}\,\text{PolyLog}\left[2,\frac{\sqrt{-c}\,(1-a-b\,x)}{(1-a)\sqrt{-c}-b\sqrt{d}}\right]}{4\,(-c)^{3/2}}-$$

$$\frac{\sqrt{d}\,\text{PolyLog}\left[2,\frac{\sqrt{-c}\,(1-a-b\,x)}{(1-a)\sqrt{-c}+b\sqrt{d}}\right]}{4\,(-c)^{3/2}}+\frac{\sqrt{d}\,\text{PolyLog}\left[2,\frac{\sqrt{-c}\,(1+a+b\,x)}{(1+a)\sqrt{-c}-b\sqrt{d}}\right]}{4\,(-c)^{3/2}}-\frac{\sqrt{d}\,\text{PolyLog}\left[2,\frac{\sqrt{-c}\,(1+a+b\,x)}{(1+a)\sqrt{-c}+b\sqrt{d}}\right]}{4\,(-c)^{3/2}}$$

Result (type 4, 15460 leaves):

$$-\frac{1}{(a+b\,x)^2\left(1-\frac{1}{(a+b\,x)^2}\right)}$$

$$\left(1-(a+b\,x)^2\right)\left(\frac{(a+b\,x)\,\text{ArcCoth}[a+b\,x]-\text{Log}\left[\frac{1}{(a+b\,x)\sqrt{1-\frac{1}{(a+b\,x)^2}}}\right]}{b\,c}+\frac{1}{c}\,2\,b\,d\left(\frac{\text{ArcCoth}[a+b\,x]\,\text{ArcTan}\left[\frac{-a\,c+\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\sqrt{c}\sqrt{d}}\right]}{2\,b\sqrt{c}\sqrt{d}}+\frac{1}{2\left(a^2\,c+b^2\,d\right)\left(-1+\frac{1}{(a+b\,x)^2}\right)}\right.\right.$$

$$\left(-1+\frac{c\left(a\sqrt{c}-b\sqrt{d}\left(\frac{a\sqrt{c}}{b\sqrt{d}}-\frac{a^2\,c+b^2\,d}{b\sqrt{c}\sqrt{d}\,(a+b\,x)}\right)\right)^2}{\left(a^2\,c+b^2\,d\right)^2}\right)\left(-\frac{\left(a^2\,c+b^2\,d\right)^2\,\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\sqrt{c}\sqrt{d}}\right]^2}{2\left(a^4\,c^2+b^4\,d^2-a^2\,c\left(c-2\,b^2\,d\right)\right)}+\frac{1}{2}\,a^2\sqrt{c}\right.$$

$$\Bigg( \frac{\sqrt{c}\; e^{i\,\text{ArcTan}\left[\frac{-a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]}\,\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]^2}{\left(-a\,c+a^2\,c+b^2\,d\right)\sqrt{1+\frac{\left(-a\,c+a^2\,c+b^2\,d\right)^2}{b^2\,c\,d}}} + \frac{1}{b\,\sqrt{d}\,\left(1+\frac{\left(-a\,c+a^2\,c+b^2\,d\right)^2}{b^2\,c\,d}\right)} \Bigg( -\pi\,\text{Log}\left[1+e^{-2\,i\,\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]}\right] - i\,\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]$$

$$\Bigg(\pi - 2\,\text{ArcTan}\left[\frac{-a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right] - 2\,i\,\text{Log}\left[1-e^{2\,i\,\left(\text{ArcTan}\left[\frac{-a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]+\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]\right)}\right]\Bigg) - 2\,\text{ArcTan}\left[\frac{-a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]$$

$$\text{Log}\left[1-e^{2\,i\,\left(\text{ArcTan}\left[\frac{-a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]+\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]\right)}\right] + \pi\,\text{Log}\left[\frac{1}{\sqrt{\frac{\left(a^2\,c+b^2\,d\right)\left(c+\frac{a^2\,c+b^2\,d}{(a+b\,x)^2}-\frac{2\,a\,c}{a+b\,x}\right)}{b^2\,c\,d}}}\right] + 2\,\text{ArcTan}\left[\frac{-a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]$$

$$\text{Log}\left[\text{Sin}\left[\text{ArcTan}\left[\frac{-a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]+\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]\right]\right] + i\,\text{PolyLog}\left[2,\, e^{2\,i\,\left(\text{ArcTan}\left[\frac{-a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]+\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]\right)}\right]\Bigg)\Bigg) -$$

$$\frac{1}{\left(-a\,c+a^2\,c+b^2\,d\right)\sqrt{1+\frac{\left(-a\,c+a^2\,c+b^2\,d\right)^2}{b^2\,c\,d}}}\,a^3\,c\,\Bigg( e^{i\,\text{ArcTan}\left[\frac{-a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]}\,\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]^2 + \frac{1}{b\,\sqrt{c}\,\sqrt{d}\,\sqrt{1+\frac{\left(-a\,c+a^2\,c+b^2\,d\right)^2}{b^2\,c\,d}}}$$

$$\left(-a\,c+a^2\,c+b^2\,d\right)\Bigg( -\pi\,\text{Log}\left[1+e^{-2\,i\,\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]}\right] - i\,\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]$$

$$\left(\pi - 2\,\text{ArcTan}\left[\frac{-a c + a^2 c + b^2 d}{b\sqrt{c}\,\sqrt{d}}\right] - 2\,i\,\text{Log}\left[1 - e^{2\,i\left(\text{ArcTan}\left[\frac{-a c+a^2 c+b^2 d}{b\sqrt{c}\,\sqrt{d}}\right]+\text{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b\sqrt{c}\,\sqrt{d}}\right]\right)}\right]\right) - 2\,\text{ArcTan}\left[\frac{-a c + a^2 c + b^2 d}{b\sqrt{c}\,\sqrt{d}}\right]$$

$$\text{Log}\left[1 - e^{2\,i\left(\text{ArcTan}\left[\frac{-a c+a^2 c+b^2 d}{b\sqrt{c}\,\sqrt{d}}\right]+\text{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b\sqrt{c}\,\sqrt{d}}\right]\right)}\right] + \pi\,\text{Log}\left[\frac{1}{\sqrt{\frac{\left(a^2 c+b^2 d\right)\left(c+\frac{a^2 c+b^2 d}{(a+b x)^2}-\frac{2 a c}{a+b x}\right)}{b^2 c d}}}\right] + 2\,\text{ArcTan}\left[\frac{-a c + a^2 c + b^2 d}{b\sqrt{c}\,\sqrt{d}}\right]$$

$$\left.\left.\text{Log}\left[\text{Sin}\left[\text{ArcTan}\left[\frac{-a c + a^2 c + b^2 d}{b\sqrt{c}\,\sqrt{d}}\right] + \text{ArcTan}\left[\frac{a c - \frac{a^2 c+b^2 d}{a+b x}}{b\sqrt{c}\,\sqrt{d}}\right]\right]\right] + i\,\text{PolyLog}\left[2, e^{2\,i\left(\text{ArcTan}\left[\frac{-a c+a^2 c+b^2 d}{b\sqrt{c}\,\sqrt{d}}\right]+\text{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b\sqrt{c}\,\sqrt{d}}\right]\right)}\right]\right)\right) +$$

$$\frac{1}{4\left(-a c + a^2 c + b^2 d\right)\sqrt{1+\frac{\left(-a c+a^2 c+b^2 d\right)^2}{b^2 c d}}}\,3\,a^4\,c\left(e^{i\,\text{ArcTan}\left[\frac{-a c+a^2 c+b^2 d}{b\sqrt{c}\,\sqrt{d}}\right]}\,\text{ArcTan}\left[\frac{a c - \frac{a^2 c+b^2 d}{a+b x}}{b\sqrt{c}\,\sqrt{d}}\right]^2 + \frac{1}{b\sqrt{c}\,\sqrt{d}\,\sqrt{1+\frac{\left(-a c+a^2 c+b^2 d\right)^2}{b^2 c d}}}\right.$$

$$\left(-a c + a^2 c + b^2 d\right)\left(-\pi\,\text{Log}\left[1 + e^{-2\,i\,\text{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b\sqrt{c}\,\sqrt{d}}\right]}\right] - i\,\text{ArcTan}\left[\frac{a c - \frac{a^2 c+b^2 d}{a+b x}}{b\sqrt{c}\,\sqrt{d}}\right]\right.$$

$$\left(\pi - 2\,\text{ArcTan}\left[\frac{-a c + a^2 c + b^2 d}{b\sqrt{c}\,\sqrt{d}}\right] - 2\,i\,\text{Log}\left[1 - e^{2\,i\left(\text{ArcTan}\left[\frac{-a c+a^2 c+b^2 d}{b\sqrt{c}\,\sqrt{d}}\right]+\text{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b\sqrt{c}\,\sqrt{d}}\right]\right)}\right]\right) - 2\,\text{ArcTan}\left[\frac{-a c + a^2 c + b^2 d}{b\sqrt{c}\,\sqrt{d}}\right]$$

$$\text{Log}\left[1 - e^{2\,i\left(\text{ArcTan}\left[\frac{-a c+a^2 c+b^2 d}{b\sqrt{c}\,\sqrt{d}}\right]+\text{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b\sqrt{c}\,\sqrt{d}}\right]\right)}\right] + \pi\,\text{Log}\left[\frac{1}{\sqrt{\frac{\left(a^2 c+b^2 d\right)\left(c+\frac{a^2 c+b^2 d}{(a+b x)^2}-\frac{2 a c}{a+b x}\right)}{b^2 c d}}}\right] + 2\,\text{ArcTan}\left[\frac{-a c + a^2 c + b^2 d}{b\sqrt{c}\,\sqrt{d}}\right]$$

$$\text{Log}\left[\text{Sin}\left[\text{ArcTan}\left[\frac{-a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]+\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]\right]\right]+\mathbb{i}\,\text{PolyLog}\left[2,\,e^{2\,\mathbb{i}\left(\text{ArcTan}\left[\frac{-a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]+\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]\right)}\right]\Bigg)\Bigg)+$$

$$\frac{1}{4\,b^2\,d\left(-a\,c+a^2\,c+b^2\,d\right)\sqrt{1+\frac{\left(-a\,c+a^2\,c+b^2\,d\right)^2}{b^2\,c\,d}}}\,a^4\,c^2\left(e^{\mathbb{i}\,\text{ArcTan}\left[\frac{-a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]}\,\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]^2+\right.$$

$$\frac{1}{b\,\sqrt{c}\,\sqrt{d}\,\sqrt{1+\frac{\left(-a\,c+a^2\,c+b^2\,d\right)^2}{b^2\,c\,d}}}\left(-a\,c+a^2\,c+b^2\,d\right)\left(-\pi\,\text{Log}\left[1+e^{-2\,\mathbb{i}\,\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]}\right]-\mathbb{i}\,\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]\right.$$

$$\left(\pi-2\,\text{ArcTan}\left[\frac{-a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]-2\,\mathbb{i}\,\text{Log}\left[1-e^{2\,\mathbb{i}\left(\text{ArcTan}\left[\frac{-a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]+\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]\right)}\right]\right)-2\,\text{ArcTan}\left[\frac{-a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]$$

$$\text{Log}\left[1-e^{2\,\mathbb{i}\left(\text{ArcTan}\left[\frac{-a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]+\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]\right)}\right]+\pi\,\text{Log}\left[\frac{1}{\sqrt{\frac{\left(a^2\,c+b^2\,d\right)\left(c+\frac{a^2\,c+b^2\,d}{(a+b\,x)^2}-\frac{2\,a\,c}{a+b\,x}\right)}{b^2\,c\,d}}}\right]+2\,\text{ArcTan}\left[\frac{-a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]$$

$$\text{Log}\left[\text{Sin}\left[\text{ArcTan}\left[\frac{-a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]+\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]\right]\right]+\mathbb{i}\,\text{PolyLog}\left[2,\,e^{2\,\mathbb{i}\left(\text{ArcTan}\left[\frac{-a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]+\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]\right)}\right]\Bigg)\Bigg)-$$

$$\frac{1}{2\, b^2\, d\, \left(-a\, c + a^2\, c + b^2\, d\right) \sqrt{1 + \frac{\left(-a\, c + a^2\, c + b^2\, d\right)^2}{b^2\, c\, d}}}\, a^5\, c^2 \left( e^{i\, \text{ArcTan}\left[\frac{-a\, c + a^2\, c + b^2\, d}{b \sqrt{c} \sqrt{d}}\right]}\, \text{ArcTan}\left[\frac{a\, c - \frac{a^2\, c + b^2\, d}{a + b\, x}}{b \sqrt{c} \sqrt{d}}\right]^2 + \right.$$

$$\frac{1}{b \sqrt{c} \sqrt{d} \sqrt{1 + \frac{\left(-a\, c + a^2\, c + b^2\, d\right)^2}{b^2\, c\, d}}} \left(-a\, c + a^2\, c + b^2\, d\right) \left( -\pi\, \text{Log}\left[1 + e^{-2\, i\, \text{ArcTan}\left[\frac{a\, c - \frac{a^2\, c + b^2\, d}{a + b\, x}}{b \sqrt{c} \sqrt{d}}\right]}\right] - i\, \text{ArcTan}\left[\frac{a\, c - \frac{a^2\, c + b^2\, d}{a + b\, x}}{b \sqrt{c} \sqrt{d}}\right] \right.$$

$$\left( \pi - 2\, \text{ArcTan}\left[\frac{-a\, c + a^2\, c + b^2\, d}{b \sqrt{c} \sqrt{d}}\right] - 2\, i\, \text{Log}\left[1 - e^{2\, i \left(\text{ArcTan}\left[\frac{-a\, c + a^2\, c + b^2\, d}{b \sqrt{c} \sqrt{d}}\right] + \text{ArcTan}\left[\frac{a\, c - \frac{a^2\, c + b^2\, d}{a + b\, x}}{b \sqrt{c} \sqrt{d}}\right]\right)}\right] \right) - 2\, \text{ArcTan}\left[\frac{-a\, c + a^2\, c + b^2\, d}{b \sqrt{c} \sqrt{d}}\right]$$

$$\text{Log}\left[1 - e^{2\, i \left(\text{ArcTan}\left[\frac{-a\, c + a^2\, c + b^2\, d}{b \sqrt{c} \sqrt{d}}\right] + \text{ArcTan}\left[\frac{a\, c - \frac{a^2\, c + b^2\, d}{a + b\, x}}{b \sqrt{c} \sqrt{d}}\right]\right)}\right] + \pi\, \text{Log}\left[\frac{1}{\sqrt{\frac{\left(a^2\, c + b^2\, d\right) \left(c + \frac{a^2\, c + b^2\, d}{(a + b\, x)^2} - \frac{2\, a\, c}{a + b\, x}\right)}{b^2\, c\, d}}}\right] + 2\, \text{ArcTan}\left[\frac{-a\, c + a^2\, c + b^2\, d}{b \sqrt{c} \sqrt{d}}\right]$$

$$\left. \left. \text{Log}\left[\text{Sin}\left[\text{ArcTan}\left[\frac{-a\, c + a^2\, c + b^2\, d}{b \sqrt{c} \sqrt{d}}\right] + \text{ArcTan}\left[\frac{a\, c - \frac{a^2\, c + b^2\, d}{a + b\, x}}{b \sqrt{c} \sqrt{d}}\right]\right]\right] + i\, \text{PolyLog}\left[2,\, e^{2\, i \left(\text{ArcTan}\left[\frac{-a\, c + a^2\, c + b^2\, d}{b \sqrt{c} \sqrt{d}}\right] + \text{ArcTan}\left[\frac{a\, c - \frac{a^2\, c + b^2\, d}{a + b\, x}}{b \sqrt{c} \sqrt{d}}\right]\right)}\right] \right) \right) +$$

$$\frac{1}{4\, b^2\, d\, \left(-a\, c + a^2\, c + b^2\, d\right) \sqrt{1 + \frac{\left(-a\, c + a^2\, c + b^2\, d\right)^2}{b^2\, c\, d}}}\, a^6\, c^2 \left( e^{i\, \text{ArcTan}\left[\frac{-a\, c + a^2\, c + b^2\, d}{b \sqrt{c} \sqrt{d}}\right]}\, \text{ArcTan}\left[\frac{a\, c - \frac{a^2\, c + b^2\, d}{a + b\, x}}{b \sqrt{c} \sqrt{d}}\right]^2 + \right.$$

$$\frac{1}{b\sqrt{c}\ \sqrt{d}\ \sqrt{1+\frac{\left(-a\,c+a^2\,c+b^2\,d\right)^2}{b^2\,c\,d}}}\left(-a\,c+a^2\,c+b^2\,d\right)\left(-\pi\,\text{Log}\left[1+e^{-2\,i\,\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\sqrt{c}\ \sqrt{d}}\right]}\right]-i\,\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\sqrt{c}\ \sqrt{d}}\right]\right.$$

$$\left(\pi-2\,\text{ArcTan}\left[\frac{-a\,c+a^2\,c+b^2\,d}{b\sqrt{c}\ \sqrt{d}}\right]-2\,i\,\text{Log}\left[1-e^{2\,i\left(\text{ArcTan}\left[\frac{-a\,c+a^2\,c+b^2\,d}{b\sqrt{c}\ \sqrt{d}}\right]+\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\sqrt{c}\ \sqrt{d}}\right]\right)}\right]\right)-2\,\text{ArcTan}\left[\frac{-a\,c+a^2\,c+b^2\,d}{b\sqrt{c}\ \sqrt{d}}\right]$$

$$\text{Log}\left[1-e^{2\,i\left(\text{ArcTan}\left[\frac{-a\,c+a^2\,c+b^2\,d}{b\sqrt{c}\ \sqrt{d}}\right]+\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\sqrt{c}\ \sqrt{d}}\right]\right)}\right]+\pi\,\text{Log}\left[\frac{1}{\sqrt{\frac{\left(a^2\,c+b^2\,d\right)\left(c+\frac{a^2\,c+b^2\,d}{(a+b\,x)^2}-\frac{2\,a\,c}{a+b\,x}\right)}{b^2\,c\,d}}}\right]+2\,\text{ArcTan}\left[\frac{-a\,c+a^2\,c+b^2\,d}{b\sqrt{c}\ \sqrt{d}}\right]$$

$$\left.\left.\text{Log}\left[\text{Sin}\left[\text{ArcTan}\left[\frac{-a\,c+a^2\,c+b^2\,d}{b\sqrt{c}\ \sqrt{d}}\right]+\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\sqrt{c}\ \sqrt{d}}\right]\right]\right]+i\,\text{PolyLog}\left[2,\ e^{2\,i\left(\text{ArcTan}\left[\frac{-a\,c+a^2\,c+b^2\,d}{b\sqrt{c}\ \sqrt{d}}\right]+\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\sqrt{c}\ \sqrt{d}}\right]\right)}\right]\right)\right)+$$

$$\frac{1}{4\left(-a\,c+a^2\,c+b^2\,d\right)\sqrt{1+\frac{\left(-a\,c+a^2\,c+b^2\,d\right)^2}{b^2\,c\,d}}}\,b^2\,d\left(e^{i\,\text{ArcTan}\left[\frac{-a\,c+a^2\,c+b^2\,d}{b\sqrt{c}\ \sqrt{d}}\right]}\,\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\sqrt{c}\ \sqrt{d}}\right]^2+\frac{1}{b\sqrt{c}\ \sqrt{d}\ \sqrt{1+\frac{\left(-a\,c+a^2\,c+b^2\,d\right)^2}{b^2\,c\,d}}}\right.$$

$$\left(-a\,c+a^2\,c+b^2\,d\right)\left(-\pi\,\text{Log}\left[1+e^{-2\,i\,\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\sqrt{c}\ \sqrt{d}}\right]}\right]-i\,\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\sqrt{c}\ \sqrt{d}}\right]\right.$$

$$\left(\pi - 2\,\text{ArcTan}\left[\frac{-a\,c + a^2\,c + b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right] - 2\,i\,\text{Log}\left[1 - e^{2\,i\left(\text{ArcTan}\left[\frac{-a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]+\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]\right)}\right]\right) - 2\,\text{ArcTan}\left[\frac{-a\,c + a^2\,c + b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]$$

$$\text{Log}\left[1 - e^{2\,i\left(\text{ArcTan}\left[\frac{-a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]+\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]\right)}\right] + \pi\,\text{Log}\left[\frac{1}{\sqrt{\frac{\left(a^2\,c+b^2\,d\right)\left(c+\frac{a^2\,c+b^2\,d}{(a+b\,x)^2}-\frac{2\,a\,c}{a+b\,x}\right)}{b^2\,c\,d}}}\right] + 2\,\text{ArcTan}\left[\frac{-a\,c + a^2\,c + b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]$$

$$\text{Log}\left[\text{Sin}\left[\text{ArcTan}\left[\frac{-a\,c + a^2\,c + b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right] + \text{ArcTan}\left[\frac{a\,c - \frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]\right]\right] + i\,\text{PolyLog}\left[2,\, e^{2\,i\left(\text{ArcTan}\left[\frac{-a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]+\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]\right)}\right]\Bigg)\Bigg) -$$

$$\frac{1}{2\left(-a\,c + a^2\,c + b^2\,d\right)\sqrt{1+\frac{\left(-a\,c+a^2\,c+b^2\,d\right)^2}{b^2\,c\,d}}}\,a\,b^2\,d\,\Bigg(e^{i\,\text{ArcTan}\left[\frac{-a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]}\,\text{ArcTan}\left[\frac{a\,c - \frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]^2 + \frac{1}{b\,\sqrt{c}\,\sqrt{d}\,\sqrt{1+\frac{\left(-a\,c+a^2\,c+b^2\,d\right)^2}{b^2\,c\,d}}}$$

$$\left(-a\,c + a^2\,c + b^2\,d\right)\Bigg(-\pi\,\text{Log}\left[1 + e^{-2\,i\,\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]}\right] - i\,\text{ArcTan}\left[\frac{a\,c - \frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]$$

$$\left(\pi - 2\,\text{ArcTan}\left[\frac{-a\,c + a^2\,c + b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right] - 2\,i\,\text{Log}\left[1 - e^{2\,i\left(\text{ArcTan}\left[\frac{-a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]+\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]\right)}\right]\right) - 2\,\text{ArcTan}\left[\frac{-a\,c + a^2\,c + b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]$$

$$\text{Log}\left[1 - e^{2\,i\left(\text{ArcTan}\left[\frac{-a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]+\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]\right)}\right] + \pi\,\text{Log}\left[\frac{1}{\sqrt{\frac{\left(a^2\,c+b^2\,d\right)\left(c+\frac{a^2\,c+b^2\,d}{(a+b\,x)^2}-\frac{2\,a\,c}{a+b\,x}\right)}{b^2\,c\,d}}}\right] + 2\,\text{ArcTan}\left[\frac{-a\,c + a^2\,c + b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]$$

$$\text{Log}\left[\text{Sin}\left[\text{ArcTan}\left[\frac{-a c+a^2 c+b^2 d}{b \sqrt{c} \sqrt{d}}\right]+\text{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b \sqrt{c} \sqrt{d}}\right]\right]\right]+\mathrm{i}\, \text{PolyLog}\left[2, e^{2 \mathrm{i} \left(\text{ArcTan}\left[\frac{-a c+a^2 c+b^2 d}{b \sqrt{c} \sqrt{d}}\right]+\text{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b \sqrt{c} \sqrt{d}}\right]\right)}\right]\Bigg)\Bigg)+$$

$$\frac{1}{4 \left(-a c+a^2 c+b^2 d\right) \sqrt{1+\frac{\left(-a c+a^2 c+b^2 d\right)^2}{b^2 c d}}} 3 a^2 b^2 d \left(e^{\mathrm{i}\, \text{ArcTan}\left[\frac{-a c+a^2 c+b^2 d}{b \sqrt{c} \sqrt{d}}\right]} \text{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b \sqrt{c} \sqrt{d}}\right]^2+\frac{1}{b \sqrt{c} \sqrt{d} \sqrt{1+\frac{\left(-a c+a^2 c+b^2 d\right)^2}{b^2 c d}}}\right.$$

$$\left(-a c+a^2 c+b^2 d\right) \left(-\pi\, \text{Log}\left[1+e^{-2 \mathrm{i}\, \text{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b \sqrt{c} \sqrt{d}}\right]}\right]-\mathrm{i}\, \text{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b \sqrt{c} \sqrt{d}}\right]\right.$$

$$\left(\pi-2\, \text{ArcTan}\left[\frac{-a c+a^2 c+b^2 d}{b \sqrt{c} \sqrt{d}}\right]-2 \mathrm{i}\, \text{Log}\left[1-e^{2 \mathrm{i} \left(\text{ArcTan}\left[\frac{-a c+a^2 c+b^2 d}{b \sqrt{c} \sqrt{d}}\right]+\text{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b \sqrt{c} \sqrt{d}}\right]\right)}\right]\right)-2\, \text{ArcTan}\left[\frac{-a c+a^2 c+b^2 d}{b \sqrt{c} \sqrt{d}}\right]$$

$$\text{Log}\left[1-e^{2 \mathrm{i} \left(\text{ArcTan}\left[\frac{-a c+a^2 c+b^2 d}{b \sqrt{c} \sqrt{d}}\right]+\text{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b \sqrt{c} \sqrt{d}}\right]\right)}\right]+\pi\, \text{Log}\left[\frac{1}{\sqrt{\frac{\left(a^2 c+b^2 d\right) \left(c+\frac{a^2 c+b^2 d}{(a+b x)^2}-\frac{2 a c}{a+b x}\right)}{b^2 c d}}}\right]+2\, \text{ArcTan}\left[\frac{-a c+a^2 c+b^2 d}{b \sqrt{c} \sqrt{d}}\right]$$

$$\text{Log}\left[\text{Sin}\left[\text{ArcTan}\left[\frac{-a c+a^2 c+b^2 d}{b \sqrt{c} \sqrt{d}}\right]+\text{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b \sqrt{c} \sqrt{d}}\right]\right]\right]+\mathrm{i}\, \text{PolyLog}\left[2, e^{2 \mathrm{i} \left(\text{ArcTan}\left[\frac{-a c+a^2 c+b^2 d}{b \sqrt{c} \sqrt{d}}\right]+\text{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b \sqrt{c} \sqrt{d}}\right]\right)}\right]\Bigg)\Bigg)+$$

$$\frac{1}{4 c \left(-a c+a^2 c+b^2 d\right) \sqrt{1+\frac{\left(-a c+a^2 c+b^2 d\right)^2}{b^2 c d}}} b^4 d^2 \left( e^{i \,\text{ArcTan}\left[\frac{-a c+a^2 c+b^2 d}{b \sqrt{c} \sqrt{d}}\right]} \text{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b \sqrt{c} \sqrt{d}}\right]^2 + \frac{1}{b \sqrt{c} \sqrt{d} \sqrt{1+\frac{\left(-a c+a^2 c+b^2 d\right)^2}{b^2 c d}}} \right.$$

$$\left(-a c+a^2 c+b^2 d\right) \left( -\pi \,\text{Log}\left[1+e^{-2 i \,\text{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b \sqrt{c} \sqrt{d}}\right]}\right] - i \,\text{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b \sqrt{c} \sqrt{d}}\right] \right.$$

$$\left(\pi - 2 \,\text{ArcTan}\left[\frac{-a c+a^2 c+b^2 d}{b \sqrt{c} \sqrt{d}}\right] - 2 i \,\text{Log}\left[1-e^{2 i \left(\text{ArcTan}\left[\frac{-a c+a^2 c+b^2 d}{b \sqrt{c} \sqrt{d}}\right]+\text{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b \sqrt{c} \sqrt{d}}\right]\right)}\right]\right) - 2 \,\text{ArcTan}\left[\frac{-a c+a^2 c+b^2 d}{b \sqrt{c} \sqrt{d}}\right]$$

$$\text{Log}\left[1-e^{2 i \left(\text{ArcTan}\left[\frac{-a c+a^2 c+b^2 d}{b \sqrt{c} \sqrt{d}}\right]+\text{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b \sqrt{c} \sqrt{d}}\right]\right)}\right] + \pi \,\text{Log}\left[\frac{1}{\sqrt{\frac{\left(a^2 c+b^2 d\right) \left(c+\frac{a^2 c+b^2 d}{(a+b x)^2}-\frac{2 a c}{a+b x}\right)}{b^2 c d}}}\right] + 2 \,\text{ArcTan}\left[\frac{-a c+a^2 c+b^2 d}{b \sqrt{c} \sqrt{d}}\right]$$

$$\left.\left. \text{Log}\left[\text{Sin}\left[\text{ArcTan}\left[\frac{-a c+a^2 c+b^2 d}{b \sqrt{c} \sqrt{d}}\right]+\text{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b \sqrt{c} \sqrt{d}}\right]\right]\right] + i \,\text{PolyLog}\left[2, e^{2 i \left(\text{ArcTan}\left[\frac{-a c+a^2 c+b^2 d}{b \sqrt{c} \sqrt{d}}\right]+\text{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b \sqrt{c} \sqrt{d}}\right]\right)}\right]\right)\right) +$$

$$\frac{1}{2 \left(a c+a^2 c+b^2 d\right) \sqrt{\frac{b^2 c d+\left(a c+a^2 c+b^2 d\right)^2}{b^2 c d}}} a^2 c \left( e^{-i \,\text{ArcTan}\left[\frac{a c+a^2 c+b^2 d}{b \sqrt{c} \sqrt{d}}\right]} \text{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b \sqrt{c} \sqrt{d}}\right]^2 - \frac{1}{b \sqrt{c} \sqrt{d} \sqrt{1+\frac{\left(a c+a^2 c+b^2 d\right)^2}{b^2 c d}}} \right.$$

$$\left(a\,c+a^2\,c+b^2\,d\right)\left(\mathrm{i}\left(-\pi-2\,\text{ArcTan}\left[\frac{a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]\right)\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]-\pi\,\text{Log}\left[1+e^{-2\,\mathrm{i}\,\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]}\right]-\right.$$

$$2\left(-\text{ArcTan}\left[\frac{a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]+\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]\right)\text{Log}\left[1-e^{2\,\mathrm{i}\left(-\text{ArcTan}\left[\frac{a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]+\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]\right)}\right]+$$

$$\pi\,\text{Log}\left[\frac{1}{\sqrt{\frac{\left(a^2\,c+b^2\,d\right)\left(c+\frac{a^2\,c+b^2\,d}{(a+b\,x)^2}-\frac{2\,a\,c}{a+b\,x}\right)}{b^2\,c\,d}}}\right]-2\,\text{ArcTan}\left[\frac{a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]$$

$$\left.\left.\text{Log}\left[-\text{Sin}\left[\text{ArcTan}\left[\frac{a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]-\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]\right]\right]+\mathrm{i}\,\text{PolyLog}\left[2,\,e^{2\,\mathrm{i}\left(-\text{ArcTan}\left[\frac{a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]+\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]\right)}\right]\right)\right)+$$

$$\frac{1}{\left(a\,c+a^2\,c+b^2\,d\right)\sqrt{\frac{b^2\,c\,d+\left(a\,c+a^2\,c+b^2\,d\right)^2}{b^2\,c\,d}}}\,a^3\,c\left(e^{-\mathrm{i}\,\text{ArcTan}\left[\frac{a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]}\,\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]^2-\frac{1}{b\,\sqrt{c}\,\sqrt{d}\,\sqrt{1+\frac{\left(a\,c+a^2\,c+b^2\,d\right)^2}{b^2\,c\,d}}}\right.$$

$$\left(a\,c+a^2\,c+b^2\,d\right)\left(\mathrm{i}\left(-\pi-2\,\text{ArcTan}\left[\frac{a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]\right)\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]-\pi\,\text{Log}\left[1+e^{-2\,\mathrm{i}\,\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]}\right]-\right.$$

$$2\left(-\text{ArcTan}\left[\frac{a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]+\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]\right)\text{Log}\left[1-e^{2\,\mathrm{i}\left(-\text{ArcTan}\left[\frac{a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]+\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]\right)}\right]+$$

$$\pi \,\text{Log}\left[\frac{1}{\sqrt{\frac{\left(a^2 c+b^2 d\right)\left(c+\frac{a^2 c+b^2 d}{(a+b x)^2}-\frac{2 a c}{a+b x}\right)}{b^2 c d}}}\right]-2\,\text{ArcTan}\left[\frac{a c+a^2 c+b^2 d}{b\sqrt{c}\,\sqrt{d}}\right]$$

$$\text{Log}\left[-\text{Sin}\left[\text{ArcTan}\left[\frac{a c+a^2 c+b^2 d}{b\sqrt{c}\,\sqrt{d}}\right]-\text{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b\sqrt{c}\,\sqrt{d}}\right]\right]\right]+i\,\text{PolyLog}\left[2,\,e^{2 i\left(-\text{ArcTan}\left[\frac{a c+a^2 c+b^2 d}{b\sqrt{c}\sqrt{d}}\right]+\text{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b\sqrt{c}\sqrt{d}}\right]\right)}\right]\Bigg)\Bigg)+$$

$$\frac{1}{4\left(a c+a^2 c+b^2 d\right)\sqrt{\frac{b^2 c d+\left(a c+a^2 c+b^2 d\right)^2}{b^2 c d}}}\,3 a^4 c\left(e^{-i\,\text{ArcTan}\left[\frac{a c+a^2 c+b^2 d}{b\sqrt{c}\sqrt{d}}\right]}\,\text{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b\sqrt{c}\,\sqrt{d}}\right]^2-\frac{1}{b\sqrt{c}\,\sqrt{d}\,\sqrt{1+\frac{\left(a c+a^2 c+b^2 d\right)^2}{b^2 c d}}}\right.$$

$$\left(a c+a^2 c+b^2 d\right)\left(i\left(-\pi-2\,\text{ArcTan}\left[\frac{a c+a^2 c+b^2 d}{b\sqrt{c}\,\sqrt{d}}\right]\right)\text{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b\sqrt{c}\,\sqrt{d}}\right]-\pi\,\text{Log}\left[1+e^{-2 i\,\text{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b\sqrt{c}\sqrt{d}}\right]}\right]-\right.$$

$$2\left(-\text{ArcTan}\left[\frac{a c+a^2 c+b^2 d}{b\sqrt{c}\,\sqrt{d}}\right]+\text{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b\sqrt{c}\,\sqrt{d}}\right]\right)\text{Log}\left[1-e^{2 i\left(-\text{ArcTan}\left[\frac{a c+a^2 c+b^2 d}{b\sqrt{c}\sqrt{d}}\right]+\text{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b\sqrt{c}\sqrt{d}}\right]\right)}\right]+$$

$$\pi\,\text{Log}\left[\frac{1}{\sqrt{\frac{\left(a^2 c+b^2 d\right)\left(c+\frac{a^2 c+b^2 d}{(a+b x)^2}-\frac{2 a c}{a+b x}\right)}{b^2 c d}}}\right]-2\,\text{ArcTan}\left[\frac{a c+a^2 c+b^2 d}{b\sqrt{c}\,\sqrt{d}}\right]$$

$$\left.\left.\mathrm{Log}\left[-\mathrm{Sin}\left[\mathrm{ArcTan}\left[\frac{a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]-\mathrm{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]\right]\right]+\mathrm{i}\,\mathrm{PolyLog}\left[2,\,e^{2\,\mathrm{i}\left(-\mathrm{ArcTan}\left[\frac{a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]+\mathrm{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]\right)}\right]\right)\right)+$$

$$\frac{1}{4\,b^2\,d\,\left(a\,c+a^2\,c+b^2\,d\right)\sqrt{\frac{b^2\,c\,d+\left(a\,c+a^2\,c+b^2\,d\right)^2}{b^2\,c\,d}}}\,a^4\,c^2\left(e^{-\mathrm{i}\,\mathrm{ArcTan}\left[\frac{a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]}\,\mathrm{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]^2-\frac{1}{b\,\sqrt{c}\,\sqrt{d}\,\sqrt{1+\frac{\left(a\,c+a^2\,c+b^2\,d\right)^2}{b^2\,c\,d}}}\right.$$

$$\left(a\,c+a^2\,c+b^2\,d\right)\left(\mathrm{i}\left(-\pi-2\,\mathrm{ArcTan}\left[\frac{a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]\right)\mathrm{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]-\pi\,\mathrm{Log}\left[1+e^{-2\,\mathrm{i}\,\mathrm{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]}\right]-\right.$$

$$2\left(-\mathrm{ArcTan}\left[\frac{a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]+\mathrm{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]\right)\mathrm{Log}\left[1-e^{2\,\mathrm{i}\left(-\mathrm{ArcTan}\left[\frac{a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]+\mathrm{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]\right)}\right]+$$

$$\pi\,\mathrm{Log}\left[\frac{1}{\sqrt{\frac{\left(a^2\,c+b^2\,d\right)\left(c+\frac{a^2\,c+b^2\,d}{(a+b\,x)^2}-\frac{2\,a\,c}{a+b\,x}\right)}{b^2\,c\,d}}}\right]-2\,\mathrm{ArcTan}\left[\frac{a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]$$

$$\left.\left.\mathrm{Log}\left[-\mathrm{Sin}\left[\mathrm{ArcTan}\left[\frac{a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]-\mathrm{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]\right]\right]+\mathrm{i}\,\mathrm{PolyLog}\left[2,\,e^{2\,\mathrm{i}\left(-\mathrm{ArcTan}\left[\frac{a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]+\mathrm{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]\right)}\right]\right)\right)+$$

$$\frac{1}{2 b^2 d \left(a c + a^2 c + b^2 d\right) \sqrt{\frac{b^2 c d + \left(a c + a^2 c + b^2 d\right)^2}{b^2 c d}}} a^5 c^2 \left(e^{-i \operatorname{ArcTan}\left[\frac{a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]} \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]^2 - \frac{1}{b \sqrt{c} \sqrt{d} \sqrt{1 + \frac{\left(a c + a^2 c + b^2 d\right)^2}{b^2 c d}}}\right.$$

$$\left(a c + a^2 c + b^2 d\right) \left(i \left(-\pi - 2 \operatorname{ArcTan}\left[\frac{a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]\right) \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] - \pi \operatorname{Log}\left[1 + e^{-2 i \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]}\right] - \right.$$

$$2 \left(-\operatorname{ArcTan}\left[\frac{a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] + \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]\right) \operatorname{Log}\left[1 - e^{2 i \left(-\operatorname{ArcTan}\left[\frac{a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] + \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]\right)}\right] +$$

$$\pi \operatorname{Log}\left[\frac{1}{\sqrt{\frac{\left(a^2 c + b^2 d\right) \left(c + \frac{a^2 c + b^2 d}{(a + b x)^2} - \frac{2 a c}{a + b x}\right)}{b^2 c d}}}\right] - 2 \operatorname{ArcTan}\left[\frac{a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]$$

$$\left.\left.\operatorname{Log}\left[-\operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] - \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]\right]\right] + i \operatorname{PolyLog}\left[2, e^{2 i \left(-\operatorname{ArcTan}\left[\frac{a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] + \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]\right)}\right]\right)\right) +$$

$$\frac{1}{4 b^2 d \left(a c + a^2 c + b^2 d\right) \sqrt{\frac{b^2 c d + \left(a c + a^2 c + b^2 d\right)^2}{b^2 c d}}} a^6 c^2 \left(e^{-i \operatorname{ArcTan}\left[\frac{a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]} \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]^2 - \frac{1}{b \sqrt{c} \sqrt{d} \sqrt{1 + \frac{\left(a c + a^2 c + b^2 d\right)^2}{b^2 c d}}}\right.$$

$$\left(a c+a^2 c+b^2 d\right)\left(\mathrm{i}\left(-\pi-2\,\mathrm{ArcTan}\left[\frac{a c+a^2 c+b^2 d}{b\sqrt{c}\,\sqrt{d}}\right]\right)\mathrm{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b\sqrt{c}\,\sqrt{d}}\right]-\pi\,\mathrm{Log}\left[1+e^{-2\,\mathrm{i}\,\mathrm{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b\sqrt{c}\,\sqrt{d}}\right]}\right]-\right.$$

$$2\left(-\mathrm{ArcTan}\left[\frac{a c+a^2 c+b^2 d}{b\sqrt{c}\,\sqrt{d}}\right]+\mathrm{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b\sqrt{c}\,\sqrt{d}}\right]\right)\mathrm{Log}\left[1-e^{2\,\mathrm{i}\left(-\mathrm{ArcTan}\left[\frac{a c+a^2 c+b^2 d}{b\sqrt{c}\,\sqrt{d}}\right]+\mathrm{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b\sqrt{c}\,\sqrt{d}}\right]\right)}\right]+$$

$$\pi\,\mathrm{Log}\left[\frac{1}{\sqrt{\frac{\left(a^2 c+b^2 d\right)\left(c+\frac{a^2 c+b^2 d}{(a+b x)^2}-\frac{2 a c}{a+b x}\right)}{b^2 c d}}}\right]-2\,\mathrm{ArcTan}\left[\frac{a c+a^2 c+b^2 d}{b\sqrt{c}\,\sqrt{d}}\right]$$

$$\left.\left.\mathrm{Log}\left[-\mathrm{Sin}\left[\mathrm{ArcTan}\left[\frac{a c+a^2 c+b^2 d}{b\sqrt{c}\,\sqrt{d}}\right]-\mathrm{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b\sqrt{c}\,\sqrt{d}}\right]\right]\right]+\mathrm{i}\,\mathrm{PolyLog}\left[2,\,e^{2\,\mathrm{i}\left(-\mathrm{ArcTan}\left[\frac{a c+a^2 c+b^2 d}{b\sqrt{c}\,\sqrt{d}}\right]+\mathrm{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b\sqrt{c}\,\sqrt{d}}\right]\right)}\right]\right)\right)+$$

$$\frac{1}{4\left(a c+a^2 c+b^2 d\right)\sqrt{\frac{b^2 c d+\left(a c+a^2 c+b^2 d\right)^2}{b^2 c d}}}\,b^2 d\left(e^{-\mathrm{i}\,\mathrm{ArcTan}\left[\frac{a c+a^2 c+b^2 d}{b\sqrt{c}\,\sqrt{d}}\right]}\,\mathrm{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b\sqrt{c}\,\sqrt{d}}\right]^2-\frac{1}{b\sqrt{c}\,\sqrt{d}\,\sqrt{1+\frac{\left(a c+a^2 c+b^2 d\right)^2}{b^2 c d}}}\right.$$

$$\left(a c+a^2 c+b^2 d\right)\left(\mathrm{i}\left(-\pi-2\,\mathrm{ArcTan}\left[\frac{a c+a^2 c+b^2 d}{b\sqrt{c}\,\sqrt{d}}\right]\right)\mathrm{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b\sqrt{c}\,\sqrt{d}}\right]-\pi\,\mathrm{Log}\left[1+e^{-2\,\mathrm{i}\,\mathrm{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b\sqrt{c}\,\sqrt{d}}\right]}\right]-\right.$$

$$2\left(-\mathrm{ArcTan}\left[\frac{a c+a^2 c+b^2 d}{b\sqrt{c}\,\sqrt{d}}\right]+\mathrm{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b\sqrt{c}\,\sqrt{d}}\right]\right)\mathrm{Log}\left[1-e^{2\,\mathrm{i}\left(-\mathrm{ArcTan}\left[\frac{a c+a^2 c+b^2 d}{b\sqrt{c}\,\sqrt{d}}\right]+\mathrm{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b\sqrt{c}\,\sqrt{d}}\right]\right)}\right]+$$

$$\pi \,\mathrm{Log}\left[\frac{1}{\sqrt{\frac{(a^2 c+b^2 d)\left(c+\frac{a^2 c+b^2 d}{(a+b x)^2}-\frac{2 a c}{a+b x}\right)}{b^2 c d}}}\right]-2\,\mathrm{ArcTan}\left[\frac{a c+a^2 c+b^2 d}{b\sqrt{c}\,\sqrt{d}}\right]$$

$$\mathrm{Log}\left[-\mathrm{Sin}\left[\mathrm{ArcTan}\left[\frac{a c+a^2 c+b^2 d}{b\sqrt{c}\,\sqrt{d}}\right]-\mathrm{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b\sqrt{c}\,\sqrt{d}}\right]\right]\right]+\mathrm{i}\,\mathrm{PolyLog}\left[2,\, e^{2\,\mathrm{i}\left(-\mathrm{ArcTan}\left[\frac{a c+a^2 c+b^2 d}{b\sqrt{c}\,\sqrt{d}}\right]+\mathrm{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b\sqrt{c}\,\sqrt{d}}\right]\right)}\right]\Bigg)\Bigg)+$$

$$\frac{1}{2\left(a c+a^2 c+b^2 d\right)\sqrt{\frac{b^2 c d+\left(a c+a^2 c+b^2 d\right)^2}{b^2 c d}}}\,a\,b^2 d\,\Bigg(e^{-\mathrm{i}\,\mathrm{ArcTan}\left[\frac{a c+a^2 c+b^2 d}{b\sqrt{c}\,\sqrt{d}}\right]}\,\mathrm{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b\sqrt{c}\,\sqrt{d}}\right]^2-\frac{1}{b\sqrt{c}\,\sqrt{d}\,\sqrt{1+\frac{\left(a c+a^2 c+b^2 d\right)^2}{b^2 c d}}}$$

$$\left(a c+a^2 c+b^2 d\right)\Bigg(\mathrm{i}\left(-\pi-2\,\mathrm{ArcTan}\left[\frac{a c+a^2 c+b^2 d}{b\sqrt{c}\,\sqrt{d}}\right]\right)\mathrm{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b\sqrt{c}\,\sqrt{d}}\right]-\pi\,\mathrm{Log}\left[1+e^{-2\,\mathrm{i}\,\mathrm{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b\sqrt{c}\,\sqrt{d}}\right]}\right]-$$

$$2\left(-\mathrm{ArcTan}\left[\frac{a c+a^2 c+b^2 d}{b\sqrt{c}\,\sqrt{d}}\right]+\mathrm{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b\sqrt{c}\,\sqrt{d}}\right]\right)\mathrm{Log}\left[1-e^{2\,\mathrm{i}\left(-\mathrm{ArcTan}\left[\frac{a c+a^2 c+b^2 d}{b\sqrt{c}\,\sqrt{d}}\right]+\mathrm{ArcTan}\left[\frac{a c-\frac{a^2 c+b^2 d}{a+b x}}{b\sqrt{c}\,\sqrt{d}}\right]\right)}\right]+$$

$$\pi \,\mathrm{Log}\left[\frac{1}{\sqrt{\frac{(a^2 c+b^2 d)\left(c+\frac{a^2 c+b^2 d}{(a+b x)^2}-\frac{2 a c}{a+b x}\right)}{b^2 c d}}}\right]-2\,\mathrm{ArcTan}\left[\frac{a c+a^2 c+b^2 d}{b\sqrt{c}\,\sqrt{d}}\right]$$

$$\mathrm{Log}\left[-\mathrm{Sin}\left[\mathrm{ArcTan}\left[\frac{a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]-\mathrm{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]\right]\right]+i\,\mathrm{PolyLog}\left[2,\,e^{2\,i\left(-\mathrm{ArcTan}\left[\frac{a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]+\mathrm{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]\right)}\right]\Bigg)\Bigg)+$$

$$\frac{1}{4\,\left(a\,c+a^2\,c+b^2\,d\right)\,\sqrt{\frac{b^2\,c\,d+\left(a\,c+a^2\,c+b^2\,d\right)^2}{b^2\,c\,d}}}\,3\,a^2\,b^2\,d\,\Bigg(e^{-i\,\mathrm{ArcTan}\left[\frac{a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]}\,\mathrm{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]^2-\frac{1}{b\,\sqrt{c}\,\sqrt{d}\,\sqrt{1+\frac{\left(a\,c+a^2\,c+b^2\,d\right)^2}{b^2\,c\,d}}}$$

$$\left(a\,c+a^2\,c+b^2\,d\right)\Bigg(i\left(-\pi-2\,\mathrm{ArcTan}\left[\frac{a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]\right)\,\mathrm{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]-\pi\,\mathrm{Log}\left[1+e^{-2\,i\,\mathrm{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]}\right]-$$

$$2\left(-\mathrm{ArcTan}\left[\frac{a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]+\mathrm{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]\right)\mathrm{Log}\left[1-e^{2\,i\left(-\mathrm{ArcTan}\left[\frac{a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]+\mathrm{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]\right)}\right]+$$

$$\pi\,\mathrm{Log}\left[\frac{1}{\sqrt{\frac{\left(a^2\,c+b^2\,d\right)\left(c+\frac{a^2\,c+b^2\,d}{(a+b\,x)^2}-\frac{2\,a\,c}{a+b\,x}\right)}{b^2\,c\,d}}}\right]-2\,\mathrm{ArcTan}\left[\frac{a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]$$

$$\mathrm{Log}\left[-\mathrm{Sin}\left[\mathrm{ArcTan}\left[\frac{a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]-\mathrm{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]\right]\right]+i\,\mathrm{PolyLog}\left[2,\,e^{2\,i\left(-\mathrm{ArcTan}\left[\frac{a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]+\mathrm{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]\right)}\right]\Bigg)\Bigg)+$$

$$\frac{1}{4\,c\,\left(a\,c+a^2\,c+b^2\,d\right)\sqrt{\frac{b^2\,c\,d+\left(a\,c+a^2\,c+b^2\,d\right)^2}{b^2\,c\,d}}}\;b^4\,d^2\left(e^{-i\,\text{ArcTan}\left[\frac{a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]}\,\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]^2-\frac{1}{b\,\sqrt{c}\,\sqrt{d}\,\sqrt{1+\frac{\left(a\,c+a^2\,c+b^2\,d\right)^2}{b^2\,c\,d}}}\right.$$

$$\left(a\,c+a^2\,c+b^2\,d\right)\left(i\left(-\pi-2\,\text{ArcTan}\left[\frac{a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]\right)\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]-\pi\,\text{Log}\left[1+e^{-2\,i\,\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]}\right]-\right.$$

$$2\left(-\text{ArcTan}\left[\frac{a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]+\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]\right)\text{Log}\left[1-e^{2\,i\left(-\text{ArcTan}\left[\frac{a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]+\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]\right)}\right]+$$

$$\pi\,\text{Log}\left[\frac{1}{\sqrt{\frac{\left(a^2\,c+b^2\,d\right)\left(c+\frac{a^2\,c+b^2\,d}{(a+b\,x)^2}-\frac{2\,a\,c}{a+b\,x}\right)}{b^2\,c\,d}}}\right]-2\,\text{ArcTan}\left[\frac{a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]$$

$$\left.\left.\text{Log}\left[-\text{Sin}\left[\text{ArcTan}\left[\frac{a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]-\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]\right]\right]+i\,\text{PolyLog}\left[2,\,e^{2\,i\left(-\text{ArcTan}\left[\frac{a\,c+a^2\,c+b^2\,d}{b\,\sqrt{c}\,\sqrt{d}}\right]+\text{ArcTan}\left[\frac{a\,c-\frac{a^2\,c+b^2\,d}{a+b\,x}}{b\,\sqrt{c}\,\sqrt{d}}\right]\right)}\right]\right)\right)\Bigg)\Bigg)\Bigg)$$

## Problem 80: Unable to integrate problem.

$$\int\frac{\text{ArcCoth}[a+b\,x]}{c+d\,\sqrt{x}}\,dx$$

Optimal (type 4, 619 leaves, 55 steps):

$$\frac{2\sqrt{1+a}\;\text{ArcTan}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right]}{\sqrt{b}\;d}-\frac{2\sqrt{1-a}\;\text{ArcTanh}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right]}{\sqrt{b}\;d}+\frac{c\,\text{Log}\left[\frac{d\left(\sqrt{-1-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}\,c+\sqrt{-1-a}\,d}\right]\text{Log}\left[c+d\sqrt{x}\right]}{d^2}-\frac{c\,\text{Log}\left[\frac{d\left(\sqrt{1-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}\,c+\sqrt{1-a}\,d}\right]\text{Log}\left[c+d\sqrt{x}\right]}{d^2}+\frac{c\,\text{Log}\left[-\frac{d\left(\sqrt{-1-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{b}\,c-\sqrt{-1-a}\,d}\right]\text{Log}\left[c+d\sqrt{x}\right]}{d^2}-\frac{c\,\text{Log}\left[-\frac{d\left(\sqrt{1-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{b}\,c-\sqrt{1-a}\,d}\right]\text{Log}\left[c+d\sqrt{x}\right]}{d^2}-\frac{\sqrt{x}\;\text{Log}\left[-\frac{1-a-b\,x}{a+b\,x}\right]}{d}+\frac{c\,\text{Log}\left[c+d\sqrt{x}\right]\text{Log}\left[-\frac{1-a-b\,x}{a+b\,x}\right]}{d^2}+\frac{\sqrt{x}\;\text{Log}\left[\frac{1+a+b\,x}{a+b\,x}\right]}{d}-\frac{c\,\text{Log}\left[c+d\sqrt{x}\right]\text{Log}\left[\frac{1+a+b\,x}{a+b\,x}\right]}{d^2}+\frac{c\,\text{PolyLog}\left[2,\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}\,c-\sqrt{-1-a}\,d}\right]}{d^2}+\frac{c\,\text{PolyLog}\left[2,\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}\,c+\sqrt{-1-a}\,d}\right]}{d^2}-\frac{c\,\text{PolyLog}\left[2,\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}\,c-\sqrt{1-a}\,d}\right]}{d^2}-\frac{c\,\text{PolyLog}\left[2,\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}\,c+\sqrt{1-a}\,d}\right]}{d^2}$$

Result (type 8, 20 leaves):

$$\int\frac{\text{ArcCoth}[a+b\,x]}{c+d\sqrt{x}}\,dx$$

## Problem 81: Attempted integration timed out after 120 seconds.

$$\int\frac{\text{ArcCoth}[a+b\,x]}{c+\frac{d}{\sqrt{x}}}\,dx$$

Optimal (type 4, 738 leaves, 65 steps):

$$-\frac{2\sqrt{1+a}\;d\,\text{ArcTan}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right]}{\sqrt{b}\;c^2}+\frac{2\sqrt{1-a}\;d\,\text{ArcTanh}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right]}{\sqrt{b}\;c^2}-\frac{d^2\,\text{Log}\left[\frac{c\left(\sqrt{-1-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{-1-a}\,c+\sqrt{b}\,d}\right]\text{Log}\left[d+c\sqrt{x}\right]}{c^3}+\frac{d^2\,\text{Log}\left[\frac{c\left(\sqrt{1-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{1-a}\,c+\sqrt{b}\,d}\right]\text{Log}\left[d+c\sqrt{x}\right]}{c^3}-\frac{d^2\,\text{Log}\left[\frac{c\left(\sqrt{-1-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{-1-a}\,c-\sqrt{b}\,d}\right]\text{Log}\left[d+c\sqrt{x}\right]}{c^3}+\frac{d^2\,\text{Log}\left[\frac{c\left(\sqrt{1-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{1-a}\,c-\sqrt{b}\,d}\right]\text{Log}\left[d+c\sqrt{x}\right]}{c^3}+\frac{(1-a)\,\text{Log}[1-a-b\,x]}{2\,b\,c}+\frac{d\sqrt{x}\;\text{Log}\left[-\frac{1-a-b\,x}{a+b\,x}\right]}{c^2}-\frac{x\,\text{Log}\left[-\frac{1-a-b\,x}{a+b\,x}\right]}{2\,c}-\frac{d^2\,\text{Log}\left[d+c\sqrt{x}\right]\text{Log}\left[-\frac{1-a-b\,x}{a+b\,x}\right]}{c^3}+\frac{(1+a)\,\text{Log}[1+a+b\,x]}{2\,b\,c}-\frac{d\sqrt{x}\;\text{Log}\left[\frac{1+a+b\,x}{a+b\,x}\right]}{c^2}+\frac{x\,\text{Log}\left[\frac{1+a+b\,x}{a+b\,x}\right]}{2\,c}+\frac{d^2\,\text{Log}\left[d+c\sqrt{x}\right]\text{Log}\left[\frac{1+a+b\,x}{a+b\,x}\right]}{c^3}-\frac{d^2\,\text{PolyLog}\left[2,-\frac{\sqrt{b}\left(d+c\sqrt{x}\right)}{\sqrt{-1-a}\,c-\sqrt{b}\,d}\right]}{c^3}+\frac{d^2\,\text{PolyLog}\left[2,-\frac{\sqrt{b}\left(d+c\sqrt{x}\right)}{\sqrt{1-a}\,c-\sqrt{b}\,d}\right]}{c^3}-\frac{d^2\,\text{PolyLog}\left[2,\frac{\sqrt{b}\left(d+c\sqrt{x}\right)}{\sqrt{-1-a}\,c+\sqrt{b}\,d}\right]}{c^3}+\frac{d^2\,\text{PolyLog}\left[2,\frac{\sqrt{b}\left(d+c\sqrt{x}\right)}{\sqrt{1-a}\,c+\sqrt{b}\,d}\right]}{c^3}$$

Result (type 1, 1 leaves):

???

## Problem 82: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcCoth}[d+e x]}{a+b x+c x^2} \, dx$$

Optimal (type 4, 335 leaves, 12 steps):

$$\frac{\text{ArcCoth}[d+e x] \text{Log}\left[\frac{2 e \left(b-\sqrt{b^2-4 a c}+2 c x\right)}{\left(2 c (1-d)+\left(b-\sqrt{b^2-4 a c}\right) e\right) (1+d+e x)}\right]}{\sqrt{b^2-4 a c}}-\frac{\text{ArcCoth}[d+e x] \text{Log}\left[\frac{2 e \left(b+\sqrt{b^2-4 a c}+2 c x\right)}{\left(2 c (1-d)+\left(b+\sqrt{b^2-4 a c}\right) e\right) (1+d+e x)}\right]}{\sqrt{b^2-4 a c}}-$$
$$\frac{\text{PolyLog}\left[2,1+\frac{2 \left(2 c d-\left(b-\sqrt{b^2-4 a c}\right) e-2 c (d+e x)\right)}{\left(2 c-2 c d+b e-\sqrt{b^2-4 a c}\, e\right) (1+d+e x)}\right]}{2 \sqrt{b^2-4 a c}}+\frac{\text{PolyLog}\left[2,1+\frac{2 \left(2 c d-\left(b+\sqrt{b^2-4 a c}\right) e-2 c (d+e x)\right)}{\left(2 c (1-d)+\left(b+\sqrt{b^2-4 a c}\right) e\right) (1+d+e x)}\right]}{2 \sqrt{b^2-4 a c}}$$

Result (type 4, 8833 leaves):

$$-\frac{1}{e (d+e x)^2 \left(a+b x+c x^2\right) \left(1-\frac{1}{(d+e x)^2}\right)} \left(a e+b e x+c e x^2\right) \left(1-(d+e x)^2\right)$$
$$\left(-\frac{2 \text{ArcCoth}[d+e x] \text{ArcTanh}\left[\frac{-2 c d+b e+2 c (d+e x)}{\sqrt{b^2-4 a c}\, e}\right]}{\sqrt{b^2-4 a c}}-\frac{1}{c \left(-1+(d+e x)^2\right)} e \left(-1+\frac{\left(2 c d-b e+\sqrt{b^2-4 a c}\, e \left(\frac{b}{\sqrt{b^2-4 a c}}-\frac{2 c d}{\sqrt{b^2-4 a c}\, e}+\frac{2 c (d+e x)}{\sqrt{b^2-4 a c}\, e}\right)\right)^2}{4 c^2}\right)\right.$$
$$\left(\frac{2 c^2 \text{ArcTanh}\left[\frac{-2 c d+b e+2 c (d+e x)}{\sqrt{b^2-4 a c}\, e}\right]^2}{4 c^2 \left(-1+d^2\right)-4 b c d e+b^2 e^2}+\frac{1}{\left(b^2-4 a c\right) (2 c-2 c d+b e) \sqrt{\frac{\left(b^2-4 a c\right) e^2-(2 c (-1+d)-b e)^2}{\left(b^2-4 a c\right) e^2}}}\right.$$

$$2\,a\,c^2\left(-e^{-\text{ArcTanh}\left[\frac{2c(-1+d)-b e}{\sqrt{b^2-4ac}\;e}\right]}\text{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}\;e}\right]^2+\frac{1}{\sqrt{b^2-4ac}\;e\sqrt{1-\frac{(2c(-1+d)-be)^2}{(b^2-4ac)e^2}}}\right.$$

$$i\,(2c(-1+d)-be)\left(-\left(-\pi+2\,i\,\text{ArcTanh}\left[\frac{2c(-1+d)-be}{\sqrt{b^2-4ac}\;e}\right]\right)\text{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}\;e}\right]-\right.$$

$$\pi\,\text{Log}\left[1+e^{2\,\text{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}\;e}\right]}\right]-2\left(i\,\text{ArcTanh}\left[\frac{2c(-1+d)-be}{\sqrt{b^2-4ac}\;e}\right]+i\,\text{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}\;e}\right]\right)$$

$$\text{Log}\left[1-e^{-2\left(\text{ArcTanh}\left[\frac{2c(-1+d)-be}{\sqrt{b^2-4ac}\;e}\right]+\text{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}\;e}\right]\right)}\right]+\pi\,\text{Log}\left[\frac{1}{\sqrt{1-\left(\frac{b}{\sqrt{b^2-4ac}}-\frac{2cd}{\sqrt{b^2-4ac}\;e}+\frac{2c(d+ex)}{\sqrt{b^2-4ac}\;e}\right)^2}}\right]+$$

$$2\,i\,\text{ArcTanh}\left[\frac{2c(-1+d)-be}{\sqrt{b^2-4ac}\;e}\right]\text{Log}\left[i\,\text{Sinh}\left[\text{ArcTanh}\left[\frac{2c(-1+d)-be}{\sqrt{b^2-4ac}\;e}\right]+\text{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}\;e}\right]\right]\right]+$$

$$\left.\left.i\,\text{PolyLog}\left[2,\,e^{-2\left(\text{ArcTanh}\left[\frac{2c(-1+d)-be}{\sqrt{b^2-4ac}\;e}\right]+\text{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}\;e}\right]\right)}\right]\right)\right)+\frac{1}{(b^2-4ac)\,e^2\,(2c-2cd+be)\sqrt{\frac{(b^2-4ac)e^2-(2c(-1+d)-be)^2}{(b^2-4ac)e^2}}}$$

$$2\,c^3\left(-e^{-\text{ArcTanh}\left[\frac{2c(-1+d)-be}{\sqrt{b^2-4ac}\;e}\right]}\text{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}\;e}\right]^2+\frac{1}{\sqrt{b^2-4ac}\;e\sqrt{1-\frac{(2c(-1+d)-be)^2}{(b^2-4ac)e^2}}}\,i\,(2c(-1+d)-be)\right.$$

$$\left(-\left(-\pi+2\,i\,\text{ArcTanh}\left[\frac{2c(-1+d)-be}{\sqrt{b^2-4ac}\,e}\right]\right)\text{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}\,e}\right]-\pi\,\text{Log}\left[1+e^{2\,\text{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}\,e}\right]}\right]-\right.$$

$$2\left(i\,\text{ArcTanh}\left[\frac{2c(-1+d)-be}{\sqrt{b^2-4ac}\,e}\right]+i\,\text{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}\,e}\right]\right)\text{Log}\left[1-e^{-2\left(\text{ArcTanh}\left[\frac{2c(-1+d)-be}{\sqrt{b^2-4ac}\,e}\right]+\text{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}\,e}\right]\right)}\right]+$$

$$\pi\,\text{Log}\left[\frac{1}{\sqrt{1-\left(\frac{b}{\sqrt{b^2-4ac}}-\frac{2cd}{\sqrt{b^2-4ac}\,e}+\frac{2c(d+ex)}{\sqrt{b^2-4ac}\,e}\right)^2}}\right]+2\,i\,\text{ArcTanh}\left[\frac{2c(-1+d)-be}{\sqrt{b^2-4ac}\,e}\right]\text{Log}\left[i\,\text{Sinh}\left[\text{ArcTanh}\left[\frac{2c(-1+d)-be}{\sqrt{b^2-4ac}\,e}\right]+\right.\right.$$

$$\left.\left.\left.\left.\text{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}\,e}\right]\right]\right]+i\,\text{PolyLog}\left[2,e^{-2\left(\text{ArcTanh}\left[\frac{2c(-1+d)-be}{\sqrt{b^2-4ac}\,e}\right]+\text{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}\,e}\right]\right)}\right]\right)\right)-$$

$$\frac{1}{(b^2-4ac)\,e^2\,(2c-2cd+be)\sqrt{\frac{(b^2-4ac)\,e^2-(2c(-1+d)-be)^2}{(b^2-4ac)\,e^2}}}\,4c^3d\left(-e^{-\text{ArcTanh}\left[\frac{2c(-1+d)-be}{\sqrt{b^2-4ac}\,e}\right]}\,\text{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}\,e}\right]^2+\right.$$

$$\frac{1}{\sqrt{b^2-4ac}\,e\sqrt{1-\frac{(2c(-1+d)-be)^2}{(b^2-4ac)\,e^2}}}\,i\,(2c(-1+d)-be)$$

$$\left(-\left(-\pi+2\,i\,\text{ArcTanh}\left[\frac{2c(-1+d)-be}{\sqrt{b^2-4ac}\,e}\right]\right)\text{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}\,e}\right]-\pi\,\text{Log}\left[1+e^{2\,\text{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}\,e}\right]}\right]-\right.$$

$$2\left(i\,\text{ArcTanh}\left[\frac{2c(-1+d)-be}{\sqrt{b^2-4ac}\ e}\right]+i\,\text{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}\ e}\right]\right)\text{Log}\left[1-e^{-2\left(\text{ArcTanh}\left[\frac{2c(-1+d)-be}{\sqrt{b^2-4ac}\ e}\right]+\text{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}\ e}\right]\right)}\right]+$$

$$\pi\,\text{Log}\left[\frac{1}{\sqrt{1-\left(\frac{b}{\sqrt{b^2-4ac}}-\frac{2cd}{\sqrt{b^2-4ac}\ e}+\frac{2c(d+ex)}{\sqrt{b^2-4ac}\ e}\right)^2}}\right]+2\,i\,\text{ArcTanh}\left[\frac{2c(-1+d)-be}{\sqrt{b^2-4ac}\ e}\right]\text{Log}\left[i\,\text{Sinh}\left[\text{ArcTanh}\left[\frac{2c(-1+d)-be}{\sqrt{b^2-4ac}\ e}\right]+\right.\right.$$

$$\left.\left.\left.\text{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}\ e}\right]\right]\right]+i\,\text{PolyLog}\left[2,e^{-2\left(\text{ArcTanh}\left[\frac{2c(-1+d)-be}{\sqrt{b^2-4ac}\ e}\right]+\text{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}\ e}\right]\right)}\right]\right)\Bigg)+$$

$$\frac{1}{(b^2-4ac)\,e^2\,(2c-2cd+be)\sqrt{\frac{(b^2-4ac)e^2-(2c(-1+d)-be)^2}{(b^2-4ac)e^2}}}2c^3d^2\left(-e^{-\text{ArcTanh}\left[\frac{2c(-1+d)-be}{\sqrt{b^2-4ac}\ e}\right]}\text{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}\ e}\right]^2+\right.$$

$$\frac{1}{\sqrt{b^2-4ac}\ e\sqrt{1-\frac{(2c(-1+d)-be)^2}{(b^2-4ac)e^2}}}\,i\,(2c(-1+d)-be)$$

$$\left(-\left(-\pi+2\,i\,\text{ArcTanh}\left[\frac{2c(-1+d)-be}{\sqrt{b^2-4ac}\ e}\right]\right)\text{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}\ e}\right]-\pi\,\text{Log}\left[1+e^{2\,\text{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}\ e}\right]}\right]-\right.$$

$$2\left(i\,\text{ArcTanh}\left[\frac{2c(-1+d)-be}{\sqrt{b^2-4ac}\ e}\right]+i\,\text{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}\ e}\right]\right)\text{Log}\left[1-e^{-2\left(\text{ArcTanh}\left[\frac{2c(-1+d)-be}{\sqrt{b^2-4ac}\ e}\right]+\text{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}\ e}\right]\right)}\right]+$$

$$\pi\,\mathrm{Log}\left[\frac{1}{\sqrt{1-\left(\frac{b}{\sqrt{b^2-4ac}}-\frac{2cd}{\sqrt{b^2-4ac}\;e}+\frac{2c(d+ex)}{\sqrt{b^2-4ac}\;e}\right)^2}}\right]+2\,i\,\mathrm{ArcTanh}\left[\frac{2c(-1+d)-be}{\sqrt{b^2-4ac}\;e}\right]\mathrm{Log}\left[i\,\mathrm{Sinh}\left[\mathrm{ArcTanh}\left[\frac{2c(-1+d)-be}{\sqrt{b^2-4ac}\;e}\right]+\right.\right.$$

$$\left.\left.\left.\mathrm{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}\;e}\right]\right]\right]+i\,\mathrm{PolyLog}\left[2,\,e^{-2\left(\mathrm{ArcTanh}\left[\frac{2c(-1+d)-be}{\sqrt{b^2-4ac}\;e}\right]+\mathrm{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}\;e}\right]\right)}\right]\right)\Bigg)+$$

$$\frac{1}{\left(b^2-4ac\right)e\left(2c-2cd+be\right)\sqrt{\frac{\left(b^2-4ac\right)e^2-(2c(-1+d)-be)^2}{\left(b^2-4ac\right)e^2}}}\,2bc^2\left(-e^{-\mathrm{ArcTanh}\left[\frac{2c(-1+d)-be}{\sqrt{b^2-4ac}\;e}\right]}\mathrm{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}\;e}\right]^2+\right.$$

$$\frac{1}{\sqrt{b^2-4ac}\;e\sqrt{1-\frac{(2c(-1+d)-be)^2}{\left(b^2-4ac\right)e^2}}}\,i\,\left(2c(-1+d)-be\right)$$

$$\left(-\left(-\pi+2\,i\,\mathrm{ArcTanh}\left[\frac{2c(-1+d)-be}{\sqrt{b^2-4ac}\;e}\right]\right)\mathrm{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}\;e}\right]-\pi\,\mathrm{Log}\left[1+e^{2\,\mathrm{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}\;e}\right]}\right]-\right.$$

$$2\left(i\,\mathrm{ArcTanh}\left[\frac{2c(-1+d)-be}{\sqrt{b^2-4ac}\;e}\right]+i\,\mathrm{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}\;e}\right]\right)\mathrm{Log}\left[1-e^{-2\left(\mathrm{ArcTanh}\left[\frac{2c(-1+d)-be}{\sqrt{b^2-4ac}\;e}\right]+\mathrm{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}\;e}\right]\right)}\right]+$$

$$\pi\,\mathrm{Log}\left[\frac{1}{\sqrt{1-\left(\frac{b}{\sqrt{b^2-4ac}}-\frac{2cd}{\sqrt{b^2-4ac}\;e}+\frac{2c(d+ex)}{\sqrt{b^2-4ac}\;e}\right)^2}}\right]+2\,i\,\mathrm{ArcTanh}\left[\frac{2c(-1+d)-be}{\sqrt{b^2-4ac}\;e}\right]\mathrm{Log}\left[i\,\mathrm{Sinh}\left[\mathrm{ArcTanh}\left[\frac{2c(-1+d)-be}{\sqrt{b^2-4ac}\;e}\right]+\right.\right.$$

$$\text{ArcTanh}\left[\frac{-2\,c\,d+b\,e+2\,c\,(d+e\,x)}{\sqrt{b^2-4\,a\,c}\;e}\right]\Big]\Big]+i\,\text{PolyLog}\left[2,\,e^{-2\left(\text{ArcTanh}\left[\frac{2\,c\,(-1+d)-b\,e}{\sqrt{b^2-4\,a\,c}\;e}\right]+\text{ArcTanh}\left[\frac{-2\,c\,d+b\,e+2\,c\,(d+e\,x)}{\sqrt{b^2-4\,a\,c}\;e}\right]\right)}\right]\Bigg)\Bigg)-$$

$$\frac{1}{\left(b^2-4\,a\,c\right)\,e\,\left(2\,c-2\,c\,d+b\,e\right)\,\sqrt{\frac{\left(b^2-4\,a\,c\right)\,e^2-\left(2\,c\,(-1+d)-b\,e\right)^2}{\left(b^2-4\,a\,c\right)\,e^2}}}\;2\,b\,c^2\,d\,\Bigg(-e^{-\text{ArcTanh}\left[\frac{2\,c\,(-1+d)-b\,e}{\sqrt{b^2-4\,a\,c}\;e}\right]}\,\text{ArcTanh}\left[\frac{-2\,c\,d+b\,e+2\,c\,(d+e\,x)}{\sqrt{b^2-4\,a\,c}\;e}\right]^2+$$

$$\frac{1}{\sqrt{b^2-4\,a\,c}\;e\,\sqrt{1-\frac{\left(2\,c\,(-1+d)-b\,e\right)^2}{\left(b^2-4\,a\,c\right)\,e^2}}}\;i\,\left(2\,c\,(-1+d)-b\,e\right)$$

$$\Bigg(-\left(-\pi+2\,i\,\text{ArcTanh}\left[\frac{2\,c\,(-1+d)-b\,e}{\sqrt{b^2-4\,a\,c}\;e}\right]\right)\,\text{ArcTanh}\left[\frac{-2\,c\,d+b\,e+2\,c\,(d+e\,x)}{\sqrt{b^2-4\,a\,c}\;e}\right]-\pi\,\text{Log}\left[1+e^{2\,\text{ArcTanh}\left[\frac{-2\,c\,d+b\,e+2\,c\,(d+e\,x)}{\sqrt{b^2-4\,a\,c}\;e}\right]}\right]-$$

$$2\left(i\,\text{ArcTanh}\left[\frac{2\,c\,(-1+d)-b\,e}{\sqrt{b^2-4\,a\,c}\;e}\right]+i\,\text{ArcTanh}\left[\frac{-2\,c\,d+b\,e+2\,c\,(d+e\,x)}{\sqrt{b^2-4\,a\,c}\;e}\right]\right)\,\text{Log}\left[1-e^{-2\left(\text{ArcTanh}\left[\frac{2\,c\,(-1+d)-b\,e}{\sqrt{b^2-4\,a\,c}\;e}\right]+\text{ArcTanh}\left[\frac{-2\,c\,d+b\,e+2\,c\,(d+e\,x)}{\sqrt{b^2-4\,a\,c}\;e}\right]\right)}\right]+$$

$$\pi\,\text{Log}\left[\frac{1}{\sqrt{1-\left(\frac{b}{\sqrt{b^2-4\,a\,c}}-\frac{2\,c\,d}{\sqrt{b^2-4\,a\,c}\;e}+\frac{2\,c\,(d+e\,x)}{\sqrt{b^2-4\,a\,c}\;e}\right)^2}}\right]+2\,i\,\text{ArcTanh}\left[\frac{2\,c\,(-1+d)-b\,e}{\sqrt{b^2-4\,a\,c}\;e}\right]\,\text{Log}\left[i\,\text{Sinh}\left[\text{ArcTanh}\left[\frac{2\,c\,(-1+d)-b\,e}{\sqrt{b^2-4\,a\,c}\;e}\right]+\right.\right.$$

$$\text{ArcTanh}\left[\frac{-2\,c\,d+b\,e+2\,c\,(d+e\,x)}{\sqrt{b^2-4\,a\,c}\;e}\right]\Big]\Big]+i\,\text{PolyLog}\left[2,\,e^{-2\left(\text{ArcTanh}\left[\frac{2\,c\,(-1+d)-b\,e}{\sqrt{b^2-4\,a\,c}\;e}\right]+\text{ArcTanh}\left[\frac{-2\,c\,d+b\,e+2\,c\,(d+e\,x)}{\sqrt{b^2-4\,a\,c}\;e}\right]\right)}\right]\Bigg)\Bigg)-$$

$$\frac{1}{\left(b^2-4ac\right)\left(-2c-2cd+be\right)\sqrt{\frac{\left(b^2-4ac\right)e^2-\left(2c\left(1+d\right)-be\right)^2}{\left(b^2-4ac\right)e^2}}}\,2ac^2\left(-e^{-\text{ArcTanh}\left[\frac{2c(1+d)-be}{\sqrt{b^2-4ac}\,e}\right]}\,\text{ArcTanh}\left[\frac{-2cd+be+2c\left(d+ex\right)}{\sqrt{b^2-4ac}\,e}\right]^2+\right.$$

$$\frac{1}{\sqrt{b^2-4ac}\,e\sqrt{1-\frac{\left(2c\left(1+d\right)-be\right)^2}{\left(b^2-4ac\right)e^2}}}\,i\left(2c\left(1+d\right)-be\right)\left(-\left(-\pi+2i\,\text{ArcTanh}\left[\frac{2c\left(1+d\right)-be}{\sqrt{b^2-4ac}\,e}\right]\right)\text{ArcTanh}\left[\frac{-2cd+be+2c\left(d+ex\right)}{\sqrt{b^2-4ac}\,e}\right]-\right.$$

$$\pi\,\text{Log}\left[1+e^{2\,\text{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}\,e}\right]}\right]-2\left(i\,\text{ArcTanh}\left[\frac{2c\left(1+d\right)-be}{\sqrt{b^2-4ac}\,e}\right]+i\,\text{ArcTanh}\left[\frac{-2cd+be+2c\left(d+ex\right)}{\sqrt{b^2-4ac}\,e}\right]\right)$$

$$\text{Log}\left[1-e^{-2\left(\text{ArcTanh}\left[\frac{2c(1+d)-be}{\sqrt{b^2-4ac}\,e}\right]+\text{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}\,e}\right]\right)}\right]+\pi\,\text{Log}\left[\frac{1}{\sqrt{1-\left(\frac{b}{\sqrt{b^2-4ac}}-\frac{2cd}{\sqrt{b^2-4ac}\,e}+\frac{2c\left(d+ex\right)}{\sqrt{b^2-4ac}\,e}\right)^2}}\right]+$$

$$2i\,\text{ArcTanh}\left[\frac{2c\left(1+d\right)-be}{\sqrt{b^2-4ac}\,e}\right]\text{Log}\left[i\,\text{Sinh}\left[\text{ArcTanh}\left[\frac{2c\left(1+d\right)-be}{\sqrt{b^2-4ac}\,e}\right]+\text{ArcTanh}\left[\frac{-2cd+be+2c\left(d+ex\right)}{\sqrt{b^2-4ac}\,e}\right]\right]\right]+$$

$$\left.\left.i\,\text{PolyLog}\left[2,\,e^{-2\left(\text{ArcTanh}\left[\frac{2c(1+d)-be}{\sqrt{b^2-4ac}\,e}\right]+\text{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}\,e}\right]\right)}\right]\right)\right)-$$

$$\frac{1}{\left(b^2-4ac\right)e^2\left(-2c-2cd+be\right)\sqrt{\frac{\left(b^2-4ac\right)e^2-\left(2c\left(1+d\right)-be\right)^2}{\left(b^2-4ac\right)e^2}}}\,2c^3\left(-e^{-\text{ArcTanh}\left[\frac{2c(1+d)-be}{\sqrt{b^2-4ac}\,e}\right]}\,\text{ArcTanh}\left[\frac{-2cd+be+2c\left(d+ex\right)}{\sqrt{b^2-4ac}\,e}\right]^2+\right.$$

$$\frac{1}{\sqrt{b^2-4\,a\,c}\;e\sqrt{1-\frac{(2\,c\,(1+d)-b\,e)^2}{(b^2-4\,a\,c)\,e^2}}}\,\mathbb{i}\,(2\,c\,(1+d)-b\,e)\left(-\left(-\pi+2\,\mathbb{i}\,\text{ArcTanh}\left[\frac{2\,c\,(1+d)-b\,e}{\sqrt{b^2-4\,a\,c}\;e}\right]\right)\text{ArcTanh}\left[\frac{-2\,c\,d+b\,e+2\,c\,(d+e\,x)}{\sqrt{b^2-4\,a\,c}\;e}\right]-\right.$$

$$\pi\,\text{Log}\left[1+e^{2\,\text{ArcTanh}\left[\frac{-2\,c\,d+b\,e+2\,c\,(d+e\,x)}{\sqrt{b^2-4\,a\,c}\;e}\right]}\right]-2\left(\mathbb{i}\,\text{ArcTanh}\left[\frac{2\,c\,(1+d)-b\,e}{\sqrt{b^2-4\,a\,c}\;e}\right]+\mathbb{i}\,\text{ArcTanh}\left[\frac{-2\,c\,d+b\,e+2\,c\,(d+e\,x)}{\sqrt{b^2-4\,a\,c}\;e}\right]\right)$$

$$\text{Log}\left[1-e^{-2\left(\text{ArcTanh}\left[\frac{2\,c\,(1+d)-b\,e}{\sqrt{b^2-4\,a\,c}\;e}\right]+\text{ArcTanh}\left[\frac{-2\,c\,d+b\,e+2\,c\,(d+e\,x)}{\sqrt{b^2-4\,a\,c}\;e}\right]\right)}\right]+\pi\,\text{Log}\left[\frac{1}{\sqrt{1-\left(\frac{b}{\sqrt{b^2-4\,a\,c}}-\frac{2\,c\,d}{\sqrt{b^2-4\,a\,c}\;e}+\frac{2\,c\,(d+e\,x)}{\sqrt{b^2-4\,a\,c}\;e}\right)^2}}\right]+$$

$$2\,\mathbb{i}\,\text{ArcTanh}\left[\frac{2\,c\,(1+d)-b\,e}{\sqrt{b^2-4\,a\,c}\;e}\right]\text{Log}\left[\mathbb{i}\,\text{Sinh}\left[\text{ArcTanh}\left[\frac{2\,c\,(1+d)-b\,e}{\sqrt{b^2-4\,a\,c}\;e}\right]+\text{ArcTanh}\left[\frac{-2\,c\,d+b\,e+2\,c\,(d+e\,x)}{\sqrt{b^2-4\,a\,c}\;e}\right]\right]\right]+$$

$$\left.\left.\mathbb{i}\,\text{PolyLog}\left[2,\,e^{-2\left(\text{ArcTanh}\left[\frac{2\,c\,(1+d)-b\,e}{\sqrt{b^2-4\,a\,c}\;e}\right]+\text{ArcTanh}\left[\frac{-2\,c\,d+b\,e+2\,c\,(d+e\,x)}{\sqrt{b^2-4\,a\,c}\;e}\right]\right)}\right]\right)\right)-$$

$$\frac{1}{(b^2-4\,a\,c)\,e^2\,(-2\,c-2\,c\,d+b\,e)\sqrt{\frac{(b^2-4\,a\,c)\,e^2-(2\,c\,(1+d)-b\,e)^2}{(b^2-4\,a\,c)\,e^2}}}\,4\,c^3\,d\left(-e^{-\text{ArcTanh}\left[\frac{2\,c\,(1+d)-b\,e}{\sqrt{b^2-4\,a\,c}\;e}\right]}\,\text{ArcTanh}\left[\frac{-2\,c\,d+b\,e+2\,c\,(d+e\,x)}{\sqrt{b^2-4\,a\,c}\;e}\right]^2+\right.$$

$$\frac{1}{\sqrt{b^2-4\,a\,c}\;e\sqrt{1-\frac{(2\,c\,(1+d)-b\,e)^2}{(b^2-4\,a\,c)\,e^2}}}\,\mathbb{i}\,(2\,c\,(1+d)-b\,e)\left(-\left(-\pi+2\,\mathbb{i}\,\text{ArcTanh}\left[\frac{2\,c\,(1+d)-b\,e}{\sqrt{b^2-4\,a\,c}\;e}\right]\right)\text{ArcTanh}\left[\frac{-2\,c\,d+b\,e+2\,c\,(d+e\,x)}{\sqrt{b^2-4\,a\,c}\;e}\right]-\right.$$

$$\pi \,\text{Log}\Big[1 + e^{2\,\text{ArcTanh}\left[\frac{-2 c d + b e + 2 c (d + e x)}{\sqrt{b^2 - 4 a c}\; e}\right]}\Big] - 2 \left(i\,\text{ArcTanh}\Big[\frac{2 c (1 + d) - b e}{\sqrt{b^2 - 4 a c}\; e}\Big] + i\,\text{ArcTanh}\Big[\frac{-2 c d + b e + 2 c (d + e x)}{\sqrt{b^2 - 4 a c}\; e}\Big]\right)$$

$$\text{Log}\Big[1 - e^{-2\left(\text{ArcTanh}\left[\frac{2 c (1+d) - b e}{\sqrt{b^2-4 a c}\; e}\right] + \text{ArcTanh}\left[\frac{-2 c d + b e + 2 c (d + e x)}{\sqrt{b^2 - 4 a c}\; e}\right]\right)}\Big] + \pi \,\text{Log}\Big[\frac{1}{\sqrt{1 - \left(\frac{b}{\sqrt{b^2-4 a c}} - \frac{2 c d}{\sqrt{b^2-4 a c}\; e} + \frac{2 c (d+e x)}{\sqrt{b^2-4 a c}\; e}\right)^2}}\Big] +$$

$$2\,i\,\text{ArcTanh}\Big[\frac{2 c (1 + d) - b e}{\sqrt{b^2 - 4 a c}\; e}\Big]\,\text{Log}\Big[i\,\text{Sinh}\Big[\text{ArcTanh}\Big[\frac{2 c (1 + d) - b e}{\sqrt{b^2 - 4 a c}\; e}\Big] + \text{ArcTanh}\Big[\frac{-2 c d + b e + 2 c (d + e x)}{\sqrt{b^2 - 4 a c}\; e}\Big]\Big]\Big] +$$

$$\left.\left. i\,\text{PolyLog}\Big[2,\, e^{-2\left(\text{ArcTanh}\left[\frac{2 c (1+d) - b e}{\sqrt{b^2-4 a c}\; e}\right] + \text{ArcTanh}\left[\frac{-2 c d + b e + 2 c (d + e x)}{\sqrt{b^2 - 4 a c}\; e}\right]\right)}\Big]\right)\right) -$$

$$\frac{1}{(b^2 - 4 a c)\, e^2\, (-2 c - 2 c d + b e)\,\sqrt{\frac{(b^2-4 a c)\, e^2 - (2 c (1+d) - b e)^2}{(b^2-4 a c)\, e^2}}}\, 2 c^3 d^2 \left(-e^{-\text{ArcTanh}\left[\frac{2 c (1+d) - b e}{\sqrt{b^2-4 a c}\; e}\right]}\,\text{ArcTanh}\Big[\frac{-2 c d + b e + 2 c (d + e x)}{\sqrt{b^2 - 4 a c}\; e}\Big]^2 + \right.$$

$$\frac{1}{\sqrt{b^2 - 4 a c}\; e\,\sqrt{1 - \frac{(2 c (1+d) - b e)^2}{(b^2-4 a c)\, e^2}}}\, i\,(2 c (1 + d) - b e) \left(-\left(-\pi + 2\,i\,\text{ArcTanh}\Big[\frac{2 c (1 + d) - b e}{\sqrt{b^2 - 4 a c}\; e}\Big]\right)\text{ArcTanh}\Big[\frac{-2 c d + b e + 2 c (d + e x)}{\sqrt{b^2 - 4 a c}\; e}\Big] - \right.$$

$$\pi \,\text{Log}\Big[1 + e^{2\,\text{ArcTanh}\left[\frac{-2 c d + b e + 2 c (d + e x)}{\sqrt{b^2 - 4 a c}\; e}\right]}\Big] - 2 \left(i\,\text{ArcTanh}\Big[\frac{2 c (1 + d) - b e}{\sqrt{b^2 - 4 a c}\; e}\Big] + i\,\text{ArcTanh}\Big[\frac{-2 c d + b e + 2 c (d + e x)}{\sqrt{b^2 - 4 a c}\; e}\Big]\right)$$

$$\text{Log}\Big[1 - e^{-2\left(\text{ArcTanh}\left[\frac{2 c (1+d) - b e}{\sqrt{b^2-4 a c}\; e}\right] + \text{ArcTanh}\left[\frac{-2 c d + b e + 2 c (d + e x)}{\sqrt{b^2 - 4 a c}\; e}\right]\right)}\Big] + \pi \,\text{Log}\Big[\frac{1}{\sqrt{1 - \left(\frac{b}{\sqrt{b^2-4 a c}} - \frac{2 c d}{\sqrt{b^2-4 a c}\; e} + \frac{2 c (d+e x)}{\sqrt{b^2-4 a c}\; e}\right)^2}}\Big] +$$

$$2\,\mathrm{i}\,\mathrm{ArcTanh}\left[\frac{2\,c\,(1+d)-b\,e}{\sqrt{b^2-4\,a\,c}\;e}\right]\mathrm{Log}\left[\mathrm{i}\,\mathrm{Sinh}\left[\mathrm{ArcTanh}\left[\frac{2\,c\,(1+d)-b\,e}{\sqrt{b^2-4\,a\,c}\;e}\right]+\mathrm{ArcTanh}\left[\frac{-2\,c\,d+b\,e+2\,c\,(d+e\,x)}{\sqrt{b^2-4\,a\,c}\;e}\right]\right]\right]+$$

$$\mathrm{i}\,\mathrm{PolyLog}\left[2,\,e^{-2\left(\mathrm{ArcTanh}\left[\frac{2\,c\,(1+d)-b\,e}{\sqrt{b^2-4\,a\,c}\;e}\right]+\mathrm{ArcTanh}\left[\frac{-2\,c\,d+b\,e+2\,c\,(d+e\,x)}{\sqrt{b^2-4\,a\,c}\;e}\right]\right)}\right]\Bigg)\Bigg)+$$

$$\frac{1}{\left(b^2-4\,a\,c\right)e\left(-2\,c-2\,c\,d+b\,e\right)\sqrt{\frac{\left(b^2-4\,a\,c\right)e^2-\left(2\,c\,(1+d)-b\,e\right)^2}{\left(b^2-4\,a\,c\right)e^2}}}\,2\,b\,c^2\Bigg(-e^{-\mathrm{ArcTanh}\left[\frac{2\,c\,(1+d)-b\,e}{\sqrt{b^2-4\,a\,c}\;e}\right]}\,\mathrm{ArcTanh}\left[\frac{-2\,c\,d+b\,e+2\,c\,(d+e\,x)}{\sqrt{b^2-4\,a\,c}\;e}\right]^2+$$

$$\frac{1}{\sqrt{b^2-4\,a\,c}\;e\,\sqrt{1-\frac{\left(2\,c\,(1+d)-b\,e\right)^2}{\left(b^2-4\,a\,c\right)e^2}}}\,\mathrm{i}\,\left(2\,c\,(1+d)-b\,e\right)\Bigg(-\left(-\pi+2\,\mathrm{i}\,\mathrm{ArcTanh}\left[\frac{2\,c\,(1+d)-b\,e}{\sqrt{b^2-4\,a\,c}\;e}\right]\right)\mathrm{ArcTanh}\left[\frac{-2\,c\,d+b\,e+2\,c\,(d+e\,x)}{\sqrt{b^2-4\,a\,c}\;e}\right]-$$

$$\pi\,\mathrm{Log}\left[1+e^{2\,\mathrm{ArcTanh}\left[\frac{-2\,c\,d+b\,e+2\,c\,(d+e\,x)}{\sqrt{b^2-4\,a\,c}\;e}\right]}\right]-2\left(\mathrm{i}\,\mathrm{ArcTanh}\left[\frac{2\,c\,(1+d)-b\,e}{\sqrt{b^2-4\,a\,c}\;e}\right]+\mathrm{i}\,\mathrm{ArcTanh}\left[\frac{-2\,c\,d+b\,e+2\,c\,(d+e\,x)}{\sqrt{b^2-4\,a\,c}\;e}\right]\right)$$

$$\mathrm{Log}\left[1-e^{-2\left(\mathrm{ArcTanh}\left[\frac{2\,c\,(1+d)-b\,e}{\sqrt{b^2-4\,a\,c}\;e}\right]+\mathrm{ArcTanh}\left[\frac{-2\,c\,d+b\,e+2\,c\,(d+e\,x)}{\sqrt{b^2-4\,a\,c}\;e}\right]\right)}\right]+\pi\,\mathrm{Log}\left[\frac{1}{\sqrt{1-\left(\frac{b}{\sqrt{b^2-4\,a\,c}}-\frac{2\,c\,d}{\sqrt{b^2-4\,a\,c}\;e}+\frac{2\,c\,(d+e\,x)}{\sqrt{b^2-4\,a\,c}\;e}\right)^2}}\right]+$$

$$2\,\mathrm{i}\,\mathrm{ArcTanh}\left[\frac{2\,c\,(1+d)-b\,e}{\sqrt{b^2-4\,a\,c}\;e}\right]\mathrm{Log}\left[\mathrm{i}\,\mathrm{Sinh}\left[\mathrm{ArcTanh}\left[\frac{2\,c\,(1+d)-b\,e}{\sqrt{b^2-4\,a\,c}\;e}\right]+\mathrm{ArcTanh}\left[\frac{-2\,c\,d+b\,e+2\,c\,(d+e\,x)}{\sqrt{b^2-4\,a\,c}\;e}\right]\right]\right]+$$

$$\left.\left.\mathrm{i}\,\mathrm{PolyLog}\left[2,\,e^{-2\left(\mathrm{ArcTanh}\left[\frac{2c(1+d)-be}{\sqrt{b^2-4ac}\,e}\right]+\mathrm{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}\,e}\right]\right)}\right]\right)\right)+$$

$$\frac{1}{\left(b^2-4ac\right)e\left(-2c-2cd+be\right)\sqrt{\frac{\left(b^2-4ac\right)e^2-(2c(1+d)-be)^2}{\left(b^2-4ac\right)e^2}}}\,2bc^2d\left(-e^{-\mathrm{ArcTanh}\left[\frac{2c(1+d)-be}{\sqrt{b^2-4ac}\,e}\right]}\,\mathrm{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}\,e}\right]^2+\right.$$

$$\frac{1}{\sqrt{b^2-4ac}\,e\sqrt{1-\frac{(2c(1+d)-be)^2}{\left(b^2-4ac\right)e^2}}}\,\mathrm{i}\left(2c(1+d)-be\right)\left(-\left(-\pi+2\,\mathrm{i}\,\mathrm{ArcTanh}\left[\frac{2c(1+d)-be}{\sqrt{b^2-4ac}\,e}\right]\right)\mathrm{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}\,e}\right]-\right.$$

$$\pi\,\mathrm{Log}\left[1+e^{2\,\mathrm{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}\,e}\right]}\right]-2\left(\mathrm{i}\,\mathrm{ArcTanh}\left[\frac{2c(1+d)-be}{\sqrt{b^2-4ac}\,e}\right]+\mathrm{i}\,\mathrm{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}\,e}\right]\right)$$

$$\mathrm{Log}\left[1-e^{-2\left(\mathrm{ArcTanh}\left[\frac{2c(1+d)-be}{\sqrt{b^2-4ac}\,e}\right]+\mathrm{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}\,e}\right]\right)}\right]+\pi\,\mathrm{Log}\left[\frac{1}{\sqrt{1-\left(\frac{b}{\sqrt{b^2-4ac}}-\frac{2cd}{\sqrt{b^2-4ac}\,e}+\frac{2c(d+ex)}{\sqrt{b^2-4ac}\,e}\right)^2}}\right]+$$

$$2\,\mathrm{i}\,\mathrm{ArcTanh}\left[\frac{2c(1+d)-be}{\sqrt{b^2-4ac}\,e}\right]\mathrm{Log}\left[\mathrm{i}\,\mathrm{Sinh}\left[\mathrm{ArcTanh}\left[\frac{2c(1+d)-be}{\sqrt{b^2-4ac}\,e}\right]+\mathrm{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}\,e}\right]\right]\right]+$$

$$\left.\left.\left.\left.\mathrm{i}\,\mathrm{PolyLog}\left[2,\,e^{-2\left(\mathrm{ArcTanh}\left[\frac{2c(1+d)-be}{\sqrt{b^2-4ac}\,e}\right]+\mathrm{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}\,e}\right]\right)}\right]\right)\right)\right)\right)$$

## Problem 95: Result unnecessarily involves higher level functions.

$$\int \frac{\text{ArcCoth}[a\,x^n]}{x}\,dx$$

Optimal (type 4, 38 leaves, 2 steps):

$$\frac{\text{PolyLog}\left[2,-\frac{x^{-n}}{a}\right]}{2\,n}-\frac{\text{PolyLog}\left[2,\frac{x^{-n}}{a}\right]}{2\,n}$$

Result (type 5, 52 leaves):

$$\frac{a\,x^n\,\text{HypergeometricPFQ}\left[\left\{\frac{1}{2},\frac{1}{2},1\right\},\left\{\frac{3}{2},\frac{3}{2}\right\},a^2\,x^{2\,n}\right]}{n}+\left(\text{ArcCoth}\left[a\,x^n\right]-\text{ArcTanh}\left[a\,x^n\right]\right)\,\text{Log}[x]$$

## Problem 100: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcCoth}[1+x]}{2+2\,x}\,dx$$

Optimal (type 4, 25 leaves, 3 steps):

$$\frac{1}{4}\,\text{PolyLog}\left[2,-\frac{1}{1+x}\right]-\frac{1}{4}\,\text{PolyLog}\left[2,\frac{1}{1+x}\right]$$

Result (type 4, 227 leaves):

$$\frac{1}{16}\left(-\pi^2+4\,i\,\pi\,\text{ArcTanh}[1+x]+8\,\text{ArcTanh}[1+x]^2+8\,\text{ArcTanh}[1+x]\,\text{Log}\left[1-e^{-2\,\text{ArcTanh}[1+x]}\right]-\right.$$
$$4\,i\,\pi\,\text{Log}\left[1+e^{2\,\text{ArcTanh}[1+x]}\right]-8\,\text{ArcTanh}[1+x]\,\text{Log}\left[1+e^{2\,\text{ArcTanh}[1+x]}\right]+8\,\text{ArcCoth}[1+x]\,\text{Log}[1+x]-8\,\text{ArcTanh}[1+x]\,\text{Log}[1+x]-$$
$$8\,\text{ArcTanh}[1+x]\,\text{Log}\left[\frac{1}{\sqrt{-x\,(2+x)}}\right]+4\,i\,\pi\,\text{Log}\left[\frac{2}{\sqrt{-x\,(2+x)}}\right]+8\,\text{ArcTanh}[1+x]\,\text{Log}\left[\frac{2}{\sqrt{-x\,(2+x)}}\right]+$$
$$\left.8\,\text{ArcTanh}[1+x]\,\text{Log}\left[\frac{i\,(1+x)}{\sqrt{-x\,(2+x)}}\right]-8\,\text{ArcTanh}[1+x]\,\text{Log}\left[\frac{2\,i\,(1+x)}{\sqrt{-x\,(2+x)}}\right]-4\,\text{PolyLog}\left[2,e^{-2\,\text{ArcTanh}[1+x]}\right]-4\,\text{PolyLog}\left[2,-e^{2\,\text{ArcTanh}[1+x]}\right]\right)$$

## Problem 101: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcCoth}[a+b\,x]}{\frac{a\,d}{b}+d\,x}\,dx$$

Optimal (type 4, 35 leaves, 3 steps):

$$\frac{\text{PolyLog}\left[2,-\frac{1}{a+b x}\right]}{2 d}-\frac{\text{PolyLog}\left[2,\frac{1}{a+b x}\right]}{2 d}$$

Result (type 4, 291 leaves):

$$-\frac{1}{8 d}\left(\pi^2-4 i \pi \text{ArcTanh}[a+b x]-8 \text{ArcTanh}[a+b x]^2-8 \text{ArcTanh}[a+b x] \text{Log}\left[1-e^{-2 \text{ArcTanh}[a+b x]}\right]+\right.$$
$$4 i \pi \text{Log}\left[1+e^{2 \text{ArcTanh}[a+b x]}\right]+8 \text{ArcTanh}[a+b x] \text{Log}\left[1+e^{2 \text{ArcTanh}[a+b x]}\right]-8 \text{ArcCoth}[a+b x] \text{Log}[a+b x]+$$
$$8 \text{ArcTanh}[a+b x] \text{Log}[a+b x]+8 \text{ArcTanh}[a+b x] \text{Log}\left[\frac{1}{\sqrt{1-(a+b x)^2}}\right]-4 i \pi \text{Log}\left[\frac{2}{\sqrt{1-(a+b x)^2}}\right]-$$
$$8 \text{ArcTanh}[a+b x] \text{Log}\left[\frac{2}{\sqrt{1-(a+b x)^2}}\right]-8 \text{ArcTanh}[a+b x] \text{Log}\left[\frac{i (a+b x)}{\sqrt{1-(a+b x)^2}}\right]+$$
$$\left.8 \text{ArcTanh}[a+b x] \text{Log}\left[\frac{2 i (a+b x)}{\sqrt{1-(a+b x)^2}}\right]+4 \text{PolyLog}\left[2, e^{-2 \text{ArcTanh}[a+b x]}\right]+4 \text{PolyLog}\left[2,-e^{2 \text{ArcTanh}[a+b x]}\right]\right)$$

## Problem 106: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{a+b \text{ArcCoth}[c+d x]}{e+f x} \, dx$$

Optimal (type 4, 130 leaves, 5 steps):

$$-\frac{(a+b \text{ArcCoth}[c+d x]) \text{Log}\left[\frac{2}{1+c+d x}\right]}{f}+\frac{(a+b \text{ArcCoth}[c+d x]) \text{Log}\left[\frac{2 d (e+f x)}{(d e+f-c f) (1+c+d x)}\right]}{f}+$$
$$\frac{b \text{PolyLog}\left[2,1-\frac{2}{1+c+d x}\right]}{2 f}-\frac{b \text{PolyLog}\left[2,1-\frac{2 d (e+f x)}{(d e+f-c f) (1+c+d x)}\right]}{2 f}$$

Result (type 4, 352 leaves):

$$\frac{1}{f}\Bigg(a\,\text{Log}[e+f\,x]+b\,(\text{ArcCoth}[c+d\,x]-\text{ArcTanh}[c+d\,x])\,\text{Log}[e+f\,x]+$$
$$b\,\text{ArcTanh}[c+d\,x]\left(-\text{Log}\left[\frac{1}{\sqrt{1-(c+d\,x)^2}}\right]+\text{Log}\left[i\,\text{Sinh}\left[\text{ArcTanh}\left[\frac{d\,e-c\,f}{f}\right]+\text{ArcTanh}[c+d\,x]\right]\right]\right)-$$
$$\frac{1}{2}\,i\,b\Bigg(-\frac{1}{4}\,i\,(\pi-2\,i\,\text{ArcTanh}[c+d\,x])^2+i\left(\text{ArcTanh}\left[\frac{d\,e-c\,f}{f}\right]+\text{ArcTanh}[c+d\,x]\right)^2+(\pi-2\,i\,\text{ArcTanh}[c+d\,x])\,\text{Log}\left[1+e^{2\,\text{ArcTanh}[c+d\,x]}\right]+$$
$$2\,i\left(\text{ArcTanh}\left[\frac{d\,e-c\,f}{f}\right]+\text{ArcTanh}[c+d\,x]\right)\text{Log}\left[1-e^{-2\left(\text{ArcTanh}\left[\frac{d\,e-c\,f}{f}\right]+\text{ArcTanh}[c+d\,x]\right)}\right]-(\pi-2\,i\,\text{ArcTanh}[c+d\,x])\,\text{Log}\left[\frac{2}{\sqrt{1-(c+d\,x)^2}}\right]-$$
$$2\,i\left(\text{ArcTanh}\left[\frac{d\,e-c\,f}{f}\right]+\text{ArcTanh}[c+d\,x]\right)\text{Log}\left[2\,i\,\text{Sinh}\left[\text{ArcTanh}\left[\frac{d\,e-c\,f}{f}\right]+\text{ArcTanh}[c+d\,x]\right]\right]-$$
$$i\,\text{PolyLog}\left[2,-e^{2\,\text{ArcTanh}[c+d\,x]}\right]-i\,\text{PolyLog}\left[2,e^{-2\left(\text{ArcTanh}\left[\frac{d\,e-c\,f}{f}\right]+\text{ArcTanh}[c+d\,x]\right)}\right]\Bigg)\Bigg)$$

## Problem 109: Result more than twice size of optimal antiderivative.

$$\int (e+f\,x)^2\,(a+b\,\text{ArcCoth}[c+d\,x])^2\,dx$$

Optimal (type 4, 374 leaves, 16 steps):

$$\frac{b^2 f^2 x}{3\,d^2}+\frac{2\,a\,b\,f\,(d\,e-c\,f)\,x}{d^2}+\frac{2\,b^2 f\,(d\,e-c\,f)\,(c+d\,x)\,\text{ArcCoth}[c+d\,x]}{d^3}+\frac{b\,f^2\,(c+d\,x)^2\,(a+b\,\text{ArcCoth}[c+d\,x])}{3\,d^3}-$$
$$\frac{(d\,e-c\,f)\,(d^2 e^2-2\,c\,d\,e\,f+(3+c^2)\,f^2)\,(a+b\,\text{ArcCoth}[c+d\,x])^2}{3\,d^3\,f}+\frac{(3\,d^2 e^2-6\,c\,d\,e\,f+(1+3\,c^2)\,f^2)\,(a+b\,\text{ArcCoth}[c+d\,x])^2}{3\,d^3}+$$
$$\frac{(e+f\,x)^3\,(a+b\,\text{ArcCoth}[c+d\,x])^2}{3\,f}-\frac{b^2 f^2\,\text{ArcTanh}[c+d\,x]}{3\,d^3}-\frac{2\,b\,(3\,d^2 e^2-6\,c\,d\,e\,f+(1+3\,c^2)\,f^2)\,(a+b\,\text{ArcCoth}[c+d\,x])\,\text{Log}\left[\frac{2}{1-c-d\,x}\right]}{3\,d^3}+$$
$$\frac{b^2 f\,(d\,e-c\,f)\,\text{Log}\left[1-(c+d\,x)^2\right]}{d^3}-\frac{b^2\,(3\,d^2 e^2-6\,c\,d\,e\,f+(1+3\,c^2)\,f^2)\,\text{PolyLog}\left[2,-\frac{1+c+d\,x}{1-c-d\,x}\right]}{3\,d^3}$$

Result (type 4, 1054 leaves):

$$a^2 e^2 x+a^2 e\,f\,x^2+\frac{1}{3}\,a^2 f^2 x^3+$$
$$\frac{1}{3}\,a\,b\left(2\,x\,(3\,e^2+3\,e\,f\,x+f^2 x^2)\,\text{ArcCoth}[c+d\,x]+\frac{1}{d^3}\left(d\,f\,x\,(6\,d\,e-4\,c\,f+d\,f\,x)-(-1+c)\,(3\,d^2 e^2-3\,(-1+c)\,d\,e\,f+(-1+c)^2 f^2)\,\text{Log}[1-c-d\,x]+\right.\right.$$

$$(1+c)\left(3 d^2 e^2-3 (1+c) d e f+(1+c)^2 f^2\right) \text{Log}[1+c+d x]\Big)\Big)+\frac{1}{d (c+d x)^2 \left(1-\frac{1}{(c+d x)^2}\right)}$$

$$b^2 e^2 \left(1-(c+d x)^2\right) \left(\text{ArcCoth}[c+d x] \left(\text{ArcCoth}[c+d x]-(c+d x) \text{ArcCoth}[c+d x]+2 \text{Log}\left[1-e^{-2 \text{ArcCoth}[c+d x]}\right]\right)-\text{PolyLog}\left[2, e^{-2 \text{ArcCoth}[c+d x]}\right]\right)-$$

$$\frac{1}{d^2 (c+d x)^2 \left(1-\frac{1}{(c+d x)^2}\right)}$$

$$b^2 e f \left(1-(c+d x)^2\right) \left(2 c \text{ArcCoth}[c+d x]^2+(c+d x)^2 \left(1-\frac{1}{(c+d x)^2}\right) \text{ArcCoth}[c+d x]^2-2 (c+d x) \text{ArcCoth}[c+d x] (-1+c \text{ArcCoth}[c+d x])+\right.$$

$$\left.4 c \text{ArcCoth}[c+d x] \text{Log}\left[1-e^{-2 \text{ArcCoth}[c+d x]}\right]-2 \text{Log}\left[\frac{1}{(c+d x) \sqrt{1-\frac{1}{(c+d x)^2}}}\right]-2 c \text{PolyLog}\left[2, e^{-2 \text{ArcCoth}[c+d x]}\right]\right)-$$

$$\frac{1}{12 d^3} b^2 f^2 (c+d x) \sqrt{1-\frac{1}{(c+d x)^2}} \left(1-(c+d x)^2\right) \left(\frac{4 \text{ArcCoth}[c+d x]}{(c+d x) \sqrt{1-\frac{1}{(c+d x)^2}}}+\frac{3 \text{ArcCoth}[c+d x]^2}{(c+d x) \sqrt{1-\frac{1}{(c+d x)^2}}}-\frac{12 c \text{ArcCoth}[c+d x]^2}{(c+d x) \sqrt{1-\frac{1}{(c+d x)^2}}}+\right.$$

$$\frac{9 c^2 \text{ArcCoth}[c+d x]^2}{(c+d x) \sqrt{1-\frac{1}{(c+d x)^2}}}+\frac{-1+6 c \text{ArcCoth}[c+d x]+3 \text{ArcCoth}[c+d x]^2-3 c^2 \text{ArcCoth}[c+d x]^2}{\sqrt{1-\frac{1}{(c+d x)^2}}}+$$

$$\text{Cosh}[3 \text{ArcCoth}[c+d x]]-6 c \text{ArcCoth}[c+d x] \text{Cosh}[3 \text{ArcCoth}[c+d x]]+\text{ArcCoth}[c+d x]^2 \text{Cosh}[3 \text{ArcCoth}[c+d x]]+$$

$$3 c^2 \text{ArcCoth}[c+d x]^2 \text{Cosh}[3 \text{ArcCoth}[c+d x]]+\frac{6 \text{ArcCoth}[c+d x] \text{Log}\left[1-e^{-2 \text{ArcCoth}[c+d x]}\right]}{(c+d x) \sqrt{1-\frac{1}{(c+d x)^2}}}+\frac{18 c^2 \text{ArcCoth}[c+d x] \text{Log}\left[1-e^{-2 \text{ArcCoth}[c+d x]}\right]}{(c+d x) \sqrt{1-\frac{1}{(c+d x)^2}}}-$$

$$\frac{18 c \text{Log}\left[\frac{1}{(c+d x) \sqrt{1-\frac{1}{(c+d x)^2}}}\right]}{(c+d x) \sqrt{1-\frac{1}{(c+d x)^2}}}+\frac{4 \left(1+3 c^2\right) \text{PolyLog}\left[2, e^{-2 \text{ArcCoth}[c+d x]}\right]}{(c+d x)^3 \left(1-\frac{1}{(c+d x)^2}\right)^{3/2}}-\text{ArcCoth}[c+d x]^2 \text{Sinh}[3 \text{ArcCoth}[c+d x]]-$$

$$3 c^2 \text{ArcCoth}[c+d x]^2 \text{Sinh}[3 \text{ArcCoth}[c+d x]]-2 \text{ArcCoth}[c+d x] \text{Log}\left[1-e^{-2 \text{ArcCoth}[c+d x]}\right] \text{Sinh}[3 \text{ArcCoth}[c+d x]]-$$

$$6 c^2 \text{ArcCoth}[c+d x] \text{Log}\left[1-e^{-2 \text{ArcCoth}[c+d x]}\right] \text{Sinh}[3 \text{ArcCoth}[c+d x]]+6 c \text{Log}\left[\frac{1}{(c+d x) \sqrt{1-\frac{1}{(c+d x)^2}}}\right] \text{Sinh}[3 \text{ArcCoth}[c+d x]]\Bigg)$$

## Problem 112: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(a+b \text{ArcCoth}[c+d x])^2}{e+f x} \, dx$$

Optimal (type 4, 214 leaves, 2 steps):

$$-\frac{(a+b \text{ArcCoth}[c+d x])^2 \text{Log}\left[\frac{2}{1+c+d x}\right]}{f}+\frac{(a+b \text{ArcCoth}[c+d x])^2 \text{Log}\left[\frac{2 d (e+f x)}{(d e+f-c f) (1+c+d x)}\right]}{f}+\frac{b (a+b \text{ArcCoth}[c+d x]) \text{PolyLog}\left[2,1-\frac{2}{1+c+d x}\right]}{f}-\frac{b (a+b \text{ArcCoth}[c+d x]) \text{PolyLog}\left[2,1-\frac{2 d (e+f x)}{(d e+f-c f) (1+c+d x)}\right]}{f}+\frac{b^2 \text{PolyLog}\left[3,1-\frac{2}{1+c+d x}\right]}{2 f}-\frac{b^2 \text{PolyLog}\left[3,1-\frac{2 d (e+f x)}{(d e+f-c f) (1+c+d x)}\right]}{2 f}$$

Result (type 4, 1640 leaves):

$$\frac{a^2 \text{Log}[e+f x]}{f}+2 a b \Bigg(\frac{(\text{ArcCoth}[c+d x]-\text{ArcTanh}[c+d x]) \text{Log}[e+f x]}{f}-$$

$$\frac{1}{f} i \left(i \text{ArcTanh}[c+d x] \left(-\text{Log}\left[\frac{1}{\sqrt{1-(c+d x)^2}}\right]+\text{Log}\left[i \text{Sinh}\left[\text{ArcTanh}\left[\frac{d e-c f}{f}\right]+\text{ArcTanh}[c+d x]\right]\right]\right)+\right.$$

$$\frac{1}{2} \left(-i \left(i \text{ArcTanh}\left[\frac{d e-c f}{f}\right]+i \text{ArcTanh}[c+d x]\right)^2-\frac{1}{4} i (\pi-2 i \text{ArcTanh}[c+d x])^2+\right.$$

$$2 \left(i \text{ArcTanh}\left[\frac{d e-c f}{f}\right]+i \text{ArcTanh}[c+d x]\right) \text{Log}\left[1-e^{2 i \left(i \text{ArcTanh}\left[\frac{d e-c f}{f}\right]+i \text{ArcTanh}[c+d x]\right)}\right]+$$

$$(\pi-2 i \text{ArcTanh}[c+d x]) \text{Log}\left[1-e^{i (\pi-2 i \text{ArcTanh}[c+d x])}\right]-(\pi-2 i \text{ArcTanh}[c+d x]) \text{Log}\left[2 \text{Sin}\left[\frac{1}{2} (\pi-2 i \text{ArcTanh}[c+d x])\right]\right]-$$

$$2 \left(i \text{ArcTanh}\left[\frac{d e-c f}{f}\right]+i \text{ArcTanh}[c+d x]\right) \text{Log}\left[2 i \text{Sinh}\left[\text{ArcTanh}\left[\frac{d e-c f}{f}\right]+\text{ArcTanh}[c+d x]\right]\right]-$$

$$\left.\left.i \text{PolyLog}\left[2,e^{2 i \left(i \text{ArcTanh}\left[\frac{d e-c f}{f}\right]+i \text{ArcTanh}[c+d x]\right)}\right]-i \text{PolyLog}\left[2,e^{i (\pi-2 i \text{ArcTanh}[c+d x])}\right]\right)\right)\Bigg)-$$

$$\frac{1}{d (c+d x)^2 (e+f x) \left(1-\frac{1}{(c+d x)^2}\right)} b^2 (d e-c f+f (c+d x)) \left(1-(c+d x)^2\right)$$

$$\left(-\frac{1}{24 f^2}\left(i f \pi^3 - 8 d e \text{ArcCoth}[c+d x]^3 - 8 f \text{ArcCoth}[c+d x]^3 + 8 c f \text{ArcCoth}[c+d x]^3 + 24 f \text{ArcCoth}[c+d x]^2 \text{Log}\left[1-e^{2 \text{ArcCoth}[c+d x]}\right] + 24 f \text{ArcCoth}[c+d x] \text{PolyLog}\left[2, e^{2 \text{ArcCoth}[c+d x]}\right] - 12 f \text{PolyLog}\left[3, e^{2 \text{ArcCoth}[c+d x]}\right]\right) + \right.$$

$$\frac{1}{6 f^2 (d e + f - c f) (d e - (1+c) f)} (-d e - f + c f) (-d e + f + c f) \left(2 d e \text{ArcCoth}[c+d x]^3 - 6 f \text{ArcCoth}[c+d x]^3 - \right.$$

$$2 c f \text{ArcCoth}[c+d x]^3 - 4 d e\, e^{-\text{ArcTanh}\left[\frac{f}{d e - c f}\right]} \sqrt{\frac{d^2 e^2 - 2 c d e f + (-1+c^2) f^2}{(d e - c f)^2}} \text{ArcCoth}[c+d x]^3 +$$

$$4 c\, e^{-\text{ArcTanh}\left[\frac{f}{d e - c f}\right]} f \sqrt{\frac{d^2 e^2 - 2 c d e f + (-1+c^2) f^2}{(d e - c f)^2}} \text{ArcCoth}[c+d x]^3 + 6 i f \pi \text{ArcCoth}[c+d x] \text{Log}[2] - f \text{ArcCoth}[c+d x]^2 \text{Log}[64] -$$

$$6 i f \pi \text{ArcCoth}[c+d x] \text{Log}\left[e^{-\text{ArcCoth}[c+d x]} + e^{\text{ArcCoth}[c+d x]}\right] + 6 f \text{ArcCoth}[c+d x]^2 \text{Log}\left[1 - e^{\text{ArcCoth}[c+d x] + \text{ArcTanh}\left[\frac{f}{d e - c f}\right]}\right] +$$

$$6 f \text{ArcCoth}[c+d x]^2 \text{Log}\left[1 + e^{\text{ArcCoth}[c+d x] + \text{ArcTanh}\left[\frac{f}{d e - c f}\right]}\right] + 6 f \text{ArcCoth}[c+d x]^2 \text{Log}\left[1 - e^{2\left(\text{ArcCoth}[c+d x] + \text{ArcTanh}\left[\frac{f}{d e - c f}\right]\right)}\right] +$$

$$12 f \text{ArcCoth}[c+d x] \text{ArcTanh}\left[\frac{f}{d e - c f}\right] \text{Log}\left[\frac{1}{2} i\, e^{-\text{ArcCoth}[c+d x] - \text{ArcTanh}\left[\frac{f}{d e - c f}\right]} \left(-1 + e^{2\left(\text{ArcCoth}[c+d x] + \text{ArcTanh}\left[\frac{f}{d e - c f}\right]\right)}\right)\right] +$$

$$6 f \text{ArcCoth}[c+d x]^2 \text{Log}\left[-e^{-\text{ArcCoth}[c+d x]} \left(d e \left(-1 + e^{2 \text{ArcCoth}[c+d x]}\right) + \left(1 + c + e^{2 \text{ArcCoth}[c+d x]} - c\, e^{2 \text{ArcCoth}[c+d x]}\right) f\right)\right] - 6 f \text{ArcCoth}[c+d x]^2$$

$$\text{Log}\left[\frac{-d e \left(-1 + e^{2 \text{ArcCoth}[c+d x]}\right) + \left(-1 - e^{2 \text{ArcCoth}[c+d x]} + c \left(-1 + e^{2 \text{ArcCoth}[c+d x]}\right)\right) f}{d e - (1+c) f}\right] + 6 i f \pi \text{ArcCoth}[c+d x] \text{Log}\left[\frac{1}{\sqrt{1 - \frac{1}{(c+d x)^2}}}\right] -$$

$$6 f \text{ArcCoth}[c+d x]^2 \text{Log}\left[-\frac{f}{\sqrt{1 - \frac{1}{(c+d x)^2}}} - \frac{d e}{(c+d x) \sqrt{1 - \frac{1}{(c+d x)^2}}} + \frac{c f}{(c+d x) \sqrt{1 - \frac{1}{(c+d x)^2}}}\right] - 12 f \text{ArcCoth}[c+d x]$$

$$\text{ArcTanh}\left[\frac{f}{d e - c f}\right] \text{Log}\left[i \text{Sinh}\left[\text{ArcCoth}[c+d x] + \text{ArcTanh}\left[\frac{f}{d e - c f}\right]\right]\right] + 12 f \text{ArcCoth}[c+d x] \text{PolyLog}\left[2, -e^{\text{ArcCoth}[c+d x] + \text{ArcTanh}\left[\frac{f}{d e - c f}\right]}\right] +$$

$$12 f \text{ArcCoth}[c+d x] \text{PolyLog}\left[2, e^{\text{ArcCoth}[c+d x] + \text{ArcTanh}\left[\frac{f}{d e - c f}\right]}\right] + 6 f \text{ArcCoth}[c+d x] \text{PolyLog}\left[2, e^{2\left(\text{ArcCoth}[c+d x] + \text{ArcTanh}\left[\frac{f}{d e - c f}\right]\right)}\right] -$$

$$6 f \text{ArcCoth}[c+d x] \text{PolyLog}\left[2, \frac{e^{2 \text{ArcCoth}[c+d x]} (d e + f - c f)}{d e - (1+c) f}\right] - 12 f \text{PolyLog}\left[3, -e^{\text{ArcCoth}[c+d x] + \text{ArcTanh}\left[\frac{f}{d e - c f}\right]}\right] -$$

$$12\, f\, \text{PolyLog}\left[3,\, e^{\text{ArcCoth}[c+d\,x]+\text{ArcTanh}\left[\frac{f}{d\,e-c\,f}\right]}\right] - 3\, f\, \text{PolyLog}\left[3,\, e^{2\left(\text{ArcCoth}[c+d\,x]+\text{ArcTanh}\left[\frac{f}{d\,e-c\,f}\right]\right)}\right] + 3\, f\, \text{PolyLog}\left[3,\, \frac{e^{2\,\text{ArcCoth}[c+d\,x]}\,(d\,e+f-c\,f)}{d\,e-(1+c)\,f}\right]\Bigg)\Bigg)$$

## Problem 113: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a+b\,\text{ArcCoth}[c+d\,x])^2}{(e+f\,x)^2}\,dx$$

Optimal (type 4, 480 leaves, 24 steps):

$$-\frac{(a+b\,\text{ArcCoth}[c+d\,x])^2}{f\,(e+f\,x)} + \frac{b^2\,d\,\text{ArcCoth}[c+d\,x]\,\text{Log}\left[\frac{2}{1-c-d\,x}\right]}{f\,(d\,e+f-c\,f)} - \frac{a\,b\,d\,\text{Log}[1-c-d\,x]}{f\,(d\,e+f-c\,f)} - \frac{b^2\,d\,\text{ArcCoth}[c+d\,x]\,\text{Log}\left[\frac{2}{1+c+d\,x}\right]}{f\,(d\,e-f-c\,f)} + \frac{2\,b^2\,d\,\text{ArcCoth}[c+d\,x]\,\text{Log}\left[\frac{2}{1+c+d\,x}\right]}{(d\,e+f-c\,f)\,(d\,e-(1+c)\,f)} + \frac{a\,b\,d\,\text{Log}[1+c+d\,x]}{f\,(d\,e-f-c\,f)} + \frac{2\,a\,b\,d\,\text{Log}[e+f\,x]}{f^2-(d\,e-c\,f)^2} - \frac{2\,b^2\,d\,\text{ArcCoth}[c+d\,x]\,\text{Log}\left[\frac{2\,d\,(e+f\,x)}{(d\,e+f-c\,f)\,(1+c+d\,x)}\right]}{(d\,e+f-c\,f)\,(d\,e-(1+c)\,f)} + \frac{b^2\,d\,\text{PolyLog}\left[2,\,-\frac{1+c+d\,x}{1-c-d\,x}\right]}{2\,f\,(d\,e+f-c\,f)} + \frac{b^2\,d\,\text{PolyLog}\left[2,\,1-\frac{2}{1+c+d\,x}\right]}{2\,f\,(d\,e-f-c\,f)} - \frac{b^2\,d\,\text{PolyLog}\left[2,\,1-\frac{2}{1+c+d\,x}\right]}{(d\,e+f-c\,f)\,(d\,e-(1+c)\,f)} + \frac{b^2\,d\,\text{PolyLog}\left[2,\,1-\frac{2\,d\,(e+f\,x)}{(d\,e+f-c\,f)\,(1+c+d\,x)}\right]}{(d\,e+f-c\,f)\,(d\,e-(1+c)\,f)}$$

Result (type 4, 806 leaves):

$$-\frac{a^2}{f\,(e+f\,x)} + \frac{1}{d\,(e+f\,x)^2}2\,a\,b\,\left(1-(c+d\,x)^2\right)\left(\frac{f}{\sqrt{1-\frac{1}{(c+d\,x)^2}}} + \frac{d\,e-c\,f}{(c+d\,x)\sqrt{1-\frac{1}{(c+d\,x)^2}}}\right)^2$$

$$\left(\frac{(-d\,e+c\,f)\,\text{ArcCoth}[c+d\,x]}{f\,(-d\,e-f+c\,f)\,(-d\,e+f+c\,f)} - \frac{\text{ArcCoth}[c+d\,x]}{f\,(c+d\,x)\sqrt{1-\frac{1}{(c+d\,x)^2}}\left(-\frac{f}{\sqrt{1-\frac{1}{(c+d\,x)^2}}} - \frac{d\,e}{(c+d\,x)\sqrt{1-\frac{1}{(c+d\,x)^2}}} + \frac{c\,f}{(c+d\,x)\sqrt{1-\frac{1}{(c+d\,x)^2}}}\right)} + \right.$$

$$\left.\frac{\text{Log}\left[-\frac{f}{\sqrt{1-\frac{1}{(c+d x)^2}}}-\frac{d e}{(c+d x) \sqrt{1-\frac{1}{(c+d x)^2}}}+\frac{c f}{(c+d x) \sqrt{1-\frac{1}{(c+d x)^2}}}\right]}{d^2 e^2-2 c d e f-f^2+c^2 f^2}\right)+\frac{1}{d f (e+f x)^2} b^2 \left(1-(c+d x)^2\right)$$

$$\left(\frac{f}{\sqrt{1-\frac{1}{(c+d x)^2}}}+\frac{d e-c f}{(c+d x) \sqrt{1-\frac{1}{(c+d x)^2}}}\right)^2 \left(\frac{e^{\text{ArcTanh}\left[\frac{f}{-d e+c f}\right]} \text{ArcCoth}[c+d x]^2}{(-d e+c f) \sqrt{1-\frac{f^2}{(d e-c f)^2}}}+\frac{\text{ArcCoth}[c+d x]^2}{(c+d x) \sqrt{1-\frac{1}{(c+d x)^2}} \left(\frac{f}{\sqrt{1-\frac{1}{(c+d x)^2}}}+\frac{d e-c f}{(c+d x) \sqrt{1-\frac{1}{(c+d x)^2}}}\right)}+\right.$$

$$\frac{1}{d^2 e^2-2 c d e f+\left(-1+c^2\right) f^2} f \left(i \pi \text{ArcCoth}[c+d x]+2 \text{ArcCoth}[c+d x] \text{ArcTanh}\left[\frac{f}{d e-c f}\right]-i \pi \text{Log}\left[1+e^{2 \text{ArcCoth}[c+d x]}\right]+2 \text{ArcCoth}[c+d x]\right.$$

$$\text{Log}\left[1-e^{-2 \left(\text{ArcCoth}[c+d x]+\text{ArcTanh}\left[\frac{f}{d e-c f}\right]\right)}\right]-2 \text{ArcTanh}\left[\frac{f}{-d e+c f}\right] \text{Log}\left[1-e^{-2 \left(\text{ArcCoth}[c+d x]+\text{ArcTanh}\left[\frac{f}{d e-c f}\right]\right)}\right]+i \pi \text{Log}\left[\frac{1}{\sqrt{1-\frac{1}{(c+d x)^2}}}\right]+$$

$$\left.\left.2 \text{ArcTanh}\left[\frac{f}{-d e+c f}\right] \text{Log}\left[i \text{Sinh}\left[\text{ArcCoth}[c+d x]+\text{ArcTanh}\left[\frac{f}{d e-c f}\right]\right]\right]-\text{PolyLog}\left[2, e^{-2 \left(\text{ArcCoth}[c+d x]+\text{ArcTanh}\left[\frac{f}{d e-c f}\right]\right)}\right]\right)\right)$$

## Problem 114: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int (e+f x)^2 (a+b \text{ArcCoth}[c+d x])^3 \, dx$$

Optimal (type 4, 546 leaves, 21 steps) :

$$\frac{a b^2 f^2 x}{d^2}+\frac{b^3 f^2 (c+d x) \text{ArcCoth}[c+d x]}{d^3}-\frac{b f^2 (a+b \text{ArcCoth}[c+d x])^2}{2 d^3}+\frac{3 b f (d e-c f) (a+b \text{ArcCoth}[c+d x])^2}{d^3}+$$
$$\frac{3 b f (d e-c f) (c+d x) (a+b \text{ArcCoth}[c+d x])^2}{d^3}+\frac{b f^2 (c+d x)^2 (a+b \text{ArcCoth}[c+d x])^2}{2 d^3}-$$
$$\frac{(d e-c f) \left(d^2 e^2-2 c d e f+\left(3+c^2\right) f^2\right) (a+b \text{ArcCoth}[c+d x])^3}{3 d^3 f}+\frac{\left(3 d^2 e^2-6 c d e f+\left(1+3 c^2\right) f^2\right) (a+b \text{ArcCoth}[c+d x])^3}{3 d^3}+$$
$$\frac{(e+f x)^3 (a+b \text{ArcCoth}[c+d x])^3}{3 f}-\frac{6 b^2 f (d e-c f) (a+b \text{ArcCoth}[c+d x]) \text{Log}\left[\frac{2}{1-c-d x}\right]}{d^3}-$$
$$\frac{b \left(3 d^2 e^2-6 c d e f+\left(1+3 c^2\right) f^2\right) (a+b \text{ArcCoth}[c+d x])^2 \text{Log}\left[\frac{2}{1-c-d x}\right]}{d^3}+\frac{b^3 f^2 \text{Log}\left[1-(c+d x)^2\right]}{2 d^3}-\frac{3 b^3 f (d e-c f) \text{PolyLog}\left[2,-\frac{1+c+d x}{1-c-d x}\right]}{d^3}-$$
$$\frac{b^2 \left(3 d^2 e^2-6 c d e f+\left(1+3 c^2\right) f^2\right) (a+b \text{ArcCoth}[c+d x]) \text{PolyLog}\left[2,1-\frac{2}{1-c-d x}\right]}{d^3}+\frac{b^3 \left(3 d^2 e^2-6 c d e f+\left(1+3 c^2\right) f^2\right) \text{PolyLog}\left[3,1-\frac{2}{1-c-d x}\right]}{2 d^3}$$

Result (type 4, 2594 leaves):

$$\frac{a^2 \left(a d^2 e^2+3 b d e f-2 b c f^2\right) x}{d^2}+\frac{a^2 f (2 a d e+b f) x^2}{2 d}+\frac{1}{3} a^3 f^2 x^3+a^2 b x \left(3 e^2+3 e f x+f^2 x^2\right) \text{ArcCoth}[c+d x]+\frac{1}{2 d^3}$$
$$\left(3 a^2 b d^2 e^2-3 a^2 b c d^2 e^2+3 a^2 b d e f-6 a^2 b c d e f+3 a^2 b c^2 d e f+a^2 b f^2-3 a^2 b c f^2+3 a^2 b c^2 f^2-a^2 b c^3 f^2\right) \text{Log}[1-c-d x]+$$
$$\frac{1}{2 d^3}\left(3 a^2 b d^2 e^2+3 a^2 b c d^2 e^2-3 a^2 b d e f-6 a^2 b c d e f-3 a^2 b c^2 d e f+a^2 b f^2+3 a^2 b c f^2+3 a^2 b c^2 f^2+a^2 b c^3 f^2\right) \text{Log}[1+c+d x]+$$
$$\frac{1}{d (c+d x)^2 \left(1-\frac{1}{(c+d x)^2}\right)} 3 a b^2 e^2 \left(1-(c+d x)^2\right)$$
$$\left(\text{ArcCoth}[c+d x] \left(\text{ArcCoth}[c+d x]-(c+d x) \text{ArcCoth}[c+d x]+2 \text{Log}\left[1-e^{-2 \text{ArcCoth}[c+d x]}\right]\right)-\text{PolyLog}\left[2,e^{-2 \text{ArcCoth}[c+d x]}\right]\right)-$$
$$\frac{1}{d^2 (c+d x)^2 \left(1-\frac{1}{(c+d x)^2}\right)} 3 a b^2 e f \left(1-(c+d x)^2\right) \left(2 c \text{ArcCoth}[c+d x]^2+(c+d x)^2 \left(1-\frac{1}{(c+d x)^2}\right) \text{ArcCoth}[c+d x]^2-2 (c+d x) \text{ArcCoth}[c+d x]\right.$$
$$\left.(-1+c \text{ArcCoth}[c+d x])+4 c \text{ArcCoth}[c+d x] \text{Log}\left[1-e^{-2 \text{ArcCoth}[c+d x]}\right]-2 \text{Log}\left[\frac{1}{(c+d x) \sqrt{1-\frac{1}{(c+d x)^2}}}\right]-2 c \text{PolyLog}\left[2,e^{-2 \text{ArcCoth}[c+d x]}\right]\right)+$$
$$\frac{1}{d (c+d x)^2 \left(1-\frac{1}{(c+d x)^2}\right)} b^3 e^2 \left(1-(c+d x)^2\right) \left(\frac{i \pi^3}{8}-\text{ArcCoth}[c+d x]^3-(c+d x) \text{ArcCoth}[c+d x]^3+3 \text{ArcCoth}[c+d x]^2 \text{Log}\left[1-e^{2 \text{ArcCoth}[c+d x]}\right]+\right.$$

$$3\,\text{ArcCoth}[c+d\,x]\,\text{PolyLog}\left[2,\,e^{2\,\text{ArcCoth}[c+d\,x]}\right]-\frac{3}{2}\,\text{PolyLog}\left[3,\,e^{2\,\text{ArcCoth}[c+d\,x]}\right]\Bigg)-\frac{1}{4\,d^2\,(c+d\,x)^2\left(1-\frac{1}{(c+d\,x)^2}\right)}$$

$$b^3\,e\,f\,\left(1-(c+d\,x)^2\right)\Bigg(\mathrm{i}\,c\,\pi^3-12\,\text{ArcCoth}[c+d\,x]^2+12\,(c+d\,x)\,\text{ArcCoth}[c+d\,x]^2-8\,c\,\text{ArcCoth}[c+d\,x]^3-8\,c\,(c+d\,x)\,\text{ArcCoth}[c+d\,x]^3+$$

$$4\,(c+d\,x)^2\left(1-\frac{1}{(c+d\,x)^2}\right)\text{ArcCoth}[c+d\,x]^3-24\,\text{ArcCoth}[c+d\,x]\,\text{Log}\left[1-e^{-2\,\text{ArcCoth}[c+d\,x]}\right]+24\,c\,\text{ArcCoth}[c+d\,x]^2\,\text{Log}\left[1-e^{2\,\text{ArcCoth}[c+d\,x]}\right]+$$

$$12\,\text{PolyLog}\left[2,\,e^{-2\,\text{ArcCoth}[c+d\,x]}\right]+24\,c\,\text{ArcCoth}[c+d\,x]\,\text{PolyLog}\left[2,\,e^{2\,\text{ArcCoth}[c+d\,x]}\right]-12\,c\,\text{PolyLog}\left[3,\,e^{2\,\text{ArcCoth}[c+d\,x]}\right]\Bigg)-$$

$$\frac{1}{4\,d^3}\,a\,b^2\,f^2\,(c+d\,x)\,\sqrt{1-\frac{1}{(c+d\,x)^2}}\,\left(1-(c+d\,x)^2\right)\Bigg(\frac{4\,\text{ArcCoth}[c+d\,x]}{(c+d\,x)\,\sqrt{1-\frac{1}{(c+d\,x)^2}}}+\frac{3\,\text{ArcCoth}[c+d\,x]^2}{(c+d\,x)\,\sqrt{1-\frac{1}{(c+d\,x)^2}}}-\frac{12\,c\,\text{ArcCoth}[c+d\,x]^2}{(c+d\,x)\,\sqrt{1-\frac{1}{(c+d\,x)^2}}}+$$

$$\frac{9\,c^2\,\text{ArcCoth}[c+d\,x]^2}{(c+d\,x)\,\sqrt{1-\frac{1}{(c+d\,x)^2}}}+\frac{-1+6\,c\,\text{ArcCoth}[c+d\,x]+3\,\text{ArcCoth}[c+d\,x]^2-3\,c^2\,\text{ArcCoth}[c+d\,x]^2}{\sqrt{1-\frac{1}{(c+d\,x)^2}}}+$$

$$\text{Cosh}[3\,\text{ArcCoth}[c+d\,x]]-6\,c\,\text{ArcCoth}[c+d\,x]\,\text{Cosh}[3\,\text{ArcCoth}[c+d\,x]]+\text{ArcCoth}[c+d\,x]^2\,\text{Cosh}[3\,\text{ArcCoth}[c+d\,x]]+$$

$$3\,c^2\,\text{ArcCoth}[c+d\,x]^2\,\text{Cosh}[3\,\text{ArcCoth}[c+d\,x]]+\frac{6\,\text{ArcCoth}[c+d\,x]\,\text{Log}\left[1-e^{-2\,\text{ArcCoth}[c+d\,x]}\right]}{(c+d\,x)\,\sqrt{1-\frac{1}{(c+d\,x)^2}}}+\frac{18\,c^2\,\text{ArcCoth}[c+d\,x]\,\text{Log}\left[1-e^{-2\,\text{ArcCoth}[c+d\,x]}\right]}{(c+d\,x)\,\sqrt{1-\frac{1}{(c+d\,x)^2}}}-$$

$$\frac{18\,c\,\text{Log}\left[\frac{1}{(c+d\,x)\,\sqrt{1-\frac{1}{(c+d\,x)^2}}}\right]}{(c+d\,x)\,\sqrt{1-\frac{1}{(c+d\,x)^2}}}+\frac{4\,\left(1+3\,c^2\right)\,\text{PolyLog}\left[2,\,e^{-2\,\text{ArcCoth}[c+d\,x]}\right]}{(c+d\,x)^3\left(1-\frac{1}{(c+d\,x)^2}\right)^{3/2}}-\text{ArcCoth}[c+d\,x]^2\,\text{Sinh}[3\,\text{ArcCoth}[c+d\,x]]-$$

$$3\,c^2\,\text{ArcCoth}[c+d\,x]^2\,\text{Sinh}[3\,\text{ArcCoth}[c+d\,x]]-2\,\text{ArcCoth}[c+d\,x]\,\text{Log}\left[1-e^{-2\,\text{ArcCoth}[c+d\,x]}\right]\,\text{Sinh}[3\,\text{ArcCoth}[c+d\,x]]-$$

$$6\,c^2\,\text{ArcCoth}[c+d\,x]\,\text{Log}\left[1-e^{-2\,\text{ArcCoth}[c+d\,x]}\right]\,\text{Sinh}[3\,\text{ArcCoth}[c+d\,x]]+6\,c\,\text{Log}\left[\frac{1}{(c+d\,x)\,\sqrt{1-\frac{1}{(c+d\,x)^2}}}\right]\,\text{Sinh}[3\,\text{ArcCoth}[c+d\,x]]\Bigg)+$$

$$\frac{1}{d^3 (c+d x)^2 \left(1-\frac{1}{(c+d x)^2}\right)} b^3 f^2 \left(1-(c+d x)^2\right) \Bigg(3 c \,\text{PolyLog}\left[2, e^{-2 \text{ArcCoth}[c+d x]}\right] + \frac{1}{96} (c+d x)^3 \left(1-\frac{1}{(c+d x)^2}\right)^{3/2} \Bigg(-\frac{3 i \pi^3}{(c+d x) \sqrt{1-\frac{1}{(c+d x)^2}}} -$$

$$\frac{9 i c^2 \pi^3}{(c+d x) \sqrt{1-\frac{1}{(c+d x)^2}}} + \frac{24 \text{ArcCoth}[c+d x]}{\sqrt{1-\frac{1}{(c+d x)^2}}} - \frac{72 c \,\text{ArcCoth}[c+d x]^2}{\sqrt{1-\frac{1}{(c+d x)^2}}} - \frac{48 \text{ArcCoth}[c+d x]^2}{(c+d x) \sqrt{1-\frac{1}{(c+d x)^2}}} + \frac{216 c \,\text{ArcCoth}[c+d x]^2}{(c+d x) \sqrt{1-\frac{1}{(c+d x)^2}}} -$$

$$\frac{24 \text{ArcCoth}[c+d x]^3}{\sqrt{1-\frac{1}{(c+d x)^2}}} + \frac{24 c^2 \text{ArcCoth}[c+d x]^3}{\sqrt{1-\frac{1}{(c+d x)^2}}} + \frac{24 \text{ArcCoth}[c+d x]^3}{(c+d x) \sqrt{1-\frac{1}{(c+d x)^2}}} + \frac{96 c \,\text{ArcCoth}[c+d x]^3}{(c+d x) \sqrt{1-\frac{1}{(c+d x)^2}}} + \frac{72 c^2 \text{ArcCoth}[c+d x]^3}{(c+d x) \sqrt{1-\frac{1}{(c+d x)^2}}} -$$

$$24 \text{ArcCoth}[c+d x] \,\text{Cosh}[3 \text{ArcCoth}[c+d x]] + 72 c \,\text{ArcCoth}[c+d x]^2 \,\text{Cosh}[3 \text{ArcCoth}[c+d x]] - 8 \text{ArcCoth}[c+d x]^3$$

$$\text{Cosh}[3 \text{ArcCoth}[c+d x]] - 24 c^2 \text{ArcCoth}[c+d x]^3 \,\text{Cosh}[3 \text{ArcCoth}[c+d x]] + \frac{432 c \,\text{ArcCoth}[c+d x] \,\text{Log}\left[1-e^{-2 \text{ArcCoth}[c+d x]}\right]}{(c+d x) \sqrt{1-\frac{1}{(c+d x)^2}}} -$$

$$\frac{72 \text{ArcCoth}[c+d x]^2 \,\text{Log}\left[1-e^{2 \text{ArcCoth}[c+d x]}\right]}{(c+d x) \sqrt{1-\frac{1}{(c+d x)^2}}} - \frac{216 c^2 \text{ArcCoth}[c+d x]^2 \,\text{Log}\left[1-e^{2 \text{ArcCoth}[c+d x]}\right]}{(c+d x) \sqrt{1-\frac{1}{(c+d x)^2}}} - \frac{72 \text{Log}\left[\frac{1}{(c+d x) \sqrt{1-\frac{1}{(c+d x)^2}}}\right]}{(c+d x) \sqrt{1-\frac{1}{(c+d x)^2}}} +$$

$$\frac{96 \left(1+3 c^2\right) \text{ArcCoth}[c+d x] \,\text{PolyLog}\left[2, e^{2 \text{ArcCoth}[c+d x]}\right]}{(c+d x)^3 \left(1-\frac{1}{(c+d x)^2}\right)^{3/2}} - \frac{48 \left(1+3 c^2\right) \text{PolyLog}\left[3, e^{2 \text{ArcCoth}[c+d x]}\right]}{(c+d x)^3 \left(1-\frac{1}{(c+d x)^2}\right)^{3/2}} + i \pi^3 \,\text{Sinh}[3 \text{ArcCoth}[c+d x]] +$$

$$3 i c^2 \pi^3 \,\text{Sinh}[3 \text{ArcCoth}[c+d x]] - 72 c \,\text{ArcCoth}[c+d x]^2 \,\text{Sinh}[3 \text{ArcCoth}[c+d x]] - 8 \text{ArcCoth}[c+d x]^3 \,\text{Sinh}[3 \text{ArcCoth}[c+d x]] -$$

$$24 c^2 \text{ArcCoth}[c+d x]^3 \,\text{Sinh}[3 \text{ArcCoth}[c+d x]] - 144 c \,\text{ArcCoth}[c+d x] \,\text{Log}\left[1-e^{-2 \text{ArcCoth}[c+d x]}\right] \text{Sinh}[3 \text{ArcCoth}[c+d x]] +$$

$$24 \text{ArcCoth}[c+d x]^2 \,\text{Log}\left[1-e^{2 \text{ArcCoth}[c+d x]}\right] \text{Sinh}[3 \text{ArcCoth}[c+d x]] +$$

$$72 c^2 \text{ArcCoth}[c+d x]^2 \,\text{Log}\left[1-e^{2 \text{ArcCoth}[c+d x]}\right] \text{Sinh}[3 \text{ArcCoth}[c+d x]] + 24 \text{Log}\left[\frac{1}{(c+d x) \sqrt{1-\frac{1}{(c+d x)^2}}}\right] \text{Sinh}[3 \text{ArcCoth}[c+d x]]\Bigg)\Bigg)$$

## Problem 115: Result unnecessarily involves imaginary or complex numbers.

$$\int (e + f x) (a + b \text{ArcCoth}[c + d x])^3 \, dx$$

Optimal (type 4, 326 leaves, 15 steps):

$$\frac{3 b f (a + b \text{ArcCoth}[c + d x])^2}{2 d^2} + \frac{3 b f (c + d x) (a + b \text{ArcCoth}[c + d x])^2}{2 d^2} + \frac{(d e - c f) (a + b \text{ArcCoth}[c + d x])^3}{d^2} - \frac{\left(d^2 e^2 - 2 c d e f + (1 + c^2) f^2\right) (a + b \text{ArcCoth}[c + d x])^3}{2 d^2 f} + \frac{(e + f x)^2 (a + b \text{ArcCoth}[c + d x])^3}{2 f} - \frac{3 b^2 f (a + b \text{ArcCoth}[c + d x]) \text{Log}\left[\frac{2}{1-c-d x}\right]}{d^2} - \frac{3 b (d e - c f) (a + b \text{ArcCoth}[c + d x])^2 \text{Log}\left[\frac{2}{1-c-d x}\right]}{d^2} - \frac{3 b^3 f \text{PolyLog}\left[2, -\frac{1+c+d x}{1-c-d x}\right]}{2 d^2} - \frac{3 b^2 (d e - c f) (a + b \text{ArcCoth}[c + d x]) \text{PolyLog}\left[2, 1 - \frac{2}{1-c-d x}\right]}{d^2} + \frac{3 b^3 (d e - c f) \text{PolyLog}\left[3, 1 - \frac{2}{1-c-d x}\right]}{2 d^2}$$

Result (type 4, 600 leaves):

$$\frac{1}{4 d^2}\Bigg(2 a^2 (2 a d e+3 b f-2 a c f) (c+d x)+2 a^3 f (c+d x)^2-$$

$$6 a^2 b (c+d x) (c f-d (2 e+f x)) \text{ArcCoth}[c+d x]+3 a^2 b (2 d e+f-2 c f) \text{Log}[1-c-d x]+3 a^2 b (2 d e-(1+2 c) f) \text{Log}[1+c+d x]+$$

$$12 a b^2 f \left((c+d x) \text{ArcCoth}[c+d x]+\frac{1}{2} \left(-1+(c+d x)^2\right) \text{ArcCoth}[c+d x]^2-\text{Log}\left[\frac{1}{(c+d x) \sqrt{1-\frac{1}{(c+d x)^2}}}\right]\right)+$$

$$12 a b^2 d e \left(\text{ArcCoth}[c+d x] \left((-1+c+d x) \text{ArcCoth}[c+d x]-2 \text{Log}\left[1-e^{-2 \text{ArcCoth}[c+d x]}\right]\right)+\text{PolyLog}\left[2, e^{-2 \text{ArcCoth}[c+d x]}\right]\right)-$$
$$12 a b^2 c f \left(\text{ArcCoth}[c+d x] \left((-1+c+d x) \text{ArcCoth}[c+d x]-2 \text{Log}\left[1-e^{-2 \text{ArcCoth}[c+d x]}\right]\right)+\text{PolyLog}\left[2, e^{-2 \text{ArcCoth}[c+d x]}\right]\right)+$$
$$2 b^3 f \left(\text{ArcCoth}[c+d x] \left(3 (-1+c+d x) \text{ArcCoth}[c+d x]+\left(-1+c^2+2 c d x+d^2 x^2\right) \text{ArcCoth}[c+d x]^2-6 \text{Log}\left[1-e^{-2 \text{ArcCoth}[c+d x]}\right]\right)+\right.$$
$$\left.3 \text{PolyLog}\left[2, e^{-2 \text{ArcCoth}[c+d x]}\right]\right)+4 b^3 d e \left(-\frac{i \pi^3}{8}+\text{ArcCoth}[c+d x]^3+(c+d x) \text{ArcCoth}[c+d x]^3-\right.$$
$$\left.3 \text{ArcCoth}[c+d x]^2 \text{Log}\left[1-e^{2 \text{ArcCoth}[c+d x]}\right]-3 \text{ArcCoth}[c+d x] \text{PolyLog}\left[2, e^{2 \text{ArcCoth}[c+d x]}\right]+\frac{3}{2} \text{PolyLog}\left[3, e^{2 \text{ArcCoth}[c+d x]}\right]\right)-$$
$$4 b^3 c f \left(-\frac{i \pi^3}{8}+\text{ArcCoth}[c+d x]^3+(c+d x) \text{ArcCoth}[c+d x]^3-3 \text{ArcCoth}[c+d x]^2 \text{Log}\left[1-e^{2 \text{ArcCoth}[c+d x]}\right]-\right.$$
$$\left.\left.3 \text{ArcCoth}[c+d x] \text{PolyLog}\left[2, e^{2 \text{ArcCoth}[c+d x]}\right]+\frac{3}{2} \text{PolyLog}\left[3, e^{2 \text{ArcCoth}[c+d x]}\right]\right)\right)$$

## Problem 116: Result unnecessarily involves imaginary or complex numbers.

$$\int (a+b \text{ArcCoth}[c+d x])^3 \, dx$$

Optimal (type 4, 132 leaves, 6 steps):

$$\frac{(a+b \text{ArcCoth}[c+d x])^3}{d}+\frac{(c+d x) (a+b \text{ArcCoth}[c+d x])^3}{d}-\frac{3 b (a+b \text{ArcCoth}[c+d x])^2 \text{Log}\left[\frac{2}{1-c-d x}\right]}{d}-$$
$$\frac{3 b^2 (a+b \text{ArcCoth}[c+d x]) \text{PolyLog}\left[2, 1-\frac{2}{1-c-d x}\right]}{d}+\frac{3 b^3 \text{PolyLog}\left[3, 1-\frac{2}{1-c-d x}\right]}{2 d}$$

Result (type 4, 208 leaves):

$$\frac{1}{2d}\left(2a^3(c+dx)+6a^2b(c+dx)\,\text{ArcCoth}[c+dx]+3a^2b\,\text{Log}\left[1-(c+dx)^2\right]+6ab^2\left(\text{ArcCoth}[c+dx]\left((-1+c+dx)\,\text{ArcCoth}[c+dx]-2\,\text{Log}\left[1-e^{-2\,\text{ArcCoth}[c+dx]}\right]\right)+\text{PolyLog}\left[2,e^{-2\,\text{ArcCoth}[c+dx]}\right]\right)+2b^3\left(-\frac{i\pi^3}{8}+\text{ArcCoth}[c+dx]^3+(c+dx)\,\text{ArcCoth}[c+dx]^3-3\,\text{ArcCoth}[c+dx]^2\,\text{Log}\left[1-e^{2\,\text{ArcCoth}[c+dx]}\right]-3\,\text{ArcCoth}[c+dx]\,\text{PolyLog}\left[2,e^{2\,\text{ArcCoth}[c+dx]}\right]+\frac{3}{2}\text{PolyLog}\left[3,e^{2\,\text{ArcCoth}[c+dx]}\right]\right)\right)$$

## Problem 117: Unable to integrate problem.

$$\int\frac{(a+b\,\text{ArcCoth}[c+dx])^3}{e+fx}\,dx$$

Optimal (type 4, 308 leaves, 2 steps):

$$-\frac{(a+b\,\text{ArcCoth}[c+dx])^3\,\text{Log}\left[\frac{2}{1+c+dx}\right]}{f}+\frac{(a+b\,\text{ArcCoth}[c+dx])^3\,\text{Log}\left[\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right]}{f}+\frac{3b(a+b\,\text{ArcCoth}[c+dx])^2\,\text{PolyLog}\left[2,1-\frac{2}{1+c+dx}\right]}{2f}-\frac{3b(a+b\,\text{ArcCoth}[c+dx])^2\,\text{PolyLog}\left[2,1-\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right]}{2f}+\frac{3b^2(a+b\,\text{ArcCoth}[c+dx])\,\text{PolyLog}\left[3,1-\frac{2}{1+c+dx}\right]}{2f}-\frac{3b^2(a+b\,\text{ArcCoth}[c+dx])\,\text{PolyLog}\left[3,1-\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right]}{2f}+\frac{3b^3\,\text{PolyLog}\left[4,1-\frac{2}{1+c+dx}\right]}{4f}-\frac{3b^3\,\text{PolyLog}\left[4,1-\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right]}{4f}$$

Result (type 8, 22 leaves):

$$\int\frac{(a+b\,\text{ArcCoth}[c+dx])^3}{e+fx}\,dx$$

## Problem 118: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{(a+b\,\text{ArcCoth}[c+dx])^3}{(e+fx)^2}\,dx$$

Optimal (type 4, 1089 leaves, 33 steps):

$$-\frac{(a+b\,\text{ArcCoth}[c+d\,x])^3}{f\,(e+f\,x)}+\frac{3\,a\,b^2\,d\,\text{ArcCoth}[c+d\,x]\,\text{Log}\left[\frac{2}{1-c-d\,x}\right]}{f\,(d\,e+f-c\,f)}+\frac{3\,b^3\,d\,\text{ArcCoth}[c+d\,x]^2\,\text{Log}\left[\frac{2}{1-c-d\,x}\right]}{2\,f\,(d\,e+f-c\,f)}-$$

$$\frac{3\,a^2\,b\,d\,\text{Log}[1-c-d\,x]}{2\,f\,(d\,e+f-c\,f)}-\frac{3\,a\,b^2\,d\,\text{ArcCoth}[c+d\,x]\,\text{Log}\left[\frac{2}{1+c+d\,x}\right]}{f\,(d\,e-f-c\,f)}+\frac{6\,a\,b^2\,d\,\text{ArcCoth}[c+d\,x]\,\text{Log}\left[\frac{2}{1+c+d\,x}\right]}{(d\,e+f-c\,f)\,(d\,e-(1+c)\,f)}-$$

$$\frac{3\,b^3\,d\,\text{ArcCoth}[c+d\,x]^2\,\text{Log}\left[\frac{2}{1+c+d\,x}\right]}{2\,f\,(d\,e-f-c\,f)}+\frac{3\,b^3\,d\,\text{ArcCoth}[c+d\,x]^2\,\text{Log}\left[\frac{2}{1+c+d\,x}\right]}{(d\,e+f-c\,f)\,(d\,e-(1+c)\,f)}+\frac{3\,a^2\,b\,d\,\text{Log}[1+c+d\,x]}{2\,f\,(d\,e-f-c\,f)}+$$

$$\frac{3\,a^2\,b\,d\,\text{Log}[e+f\,x]}{f^2-(d\,e-c\,f)^2}-\frac{6\,a\,b^2\,d\,\text{ArcCoth}[c+d\,x]\,\text{Log}\left[\frac{2\,d\,(e+f\,x)}{(d\,e+f-c\,f)\,(1+c+d\,x)}\right]}{(d\,e+f-c\,f)\,(d\,e-(1+c)\,f)}-\frac{3\,b^3\,d\,\text{ArcCoth}[c+d\,x]^2\,\text{Log}\left[\frac{2\,d\,(e+f\,x)}{(d\,e+f-c\,f)\,(1+c+d\,x)}\right]}{(d\,e+f-c\,f)\,(d\,e-(1+c)\,f)}+$$

$$\frac{3\,a\,b^2\,d\,\text{PolyLog}\left[2,-\frac{1+c+d\,x}{1-c-d\,x}\right]}{2\,f\,(d\,e+f-c\,f)}+\frac{3\,b^3\,d\,\text{ArcCoth}[c+d\,x]\,\text{PolyLog}\left[2,1-\frac{2}{1-c-d\,x}\right]}{2\,f\,(d\,e+f-c\,f)}+\frac{3\,a\,b^2\,d\,\text{PolyLog}\left[2,1-\frac{2}{1+c+d\,x}\right]}{2\,f\,(d\,e-f-c\,f)}-$$

$$\frac{3\,a\,b^2\,d\,\text{PolyLog}\left[2,1-\frac{2}{1+c+d\,x}\right]}{(d\,e+f-c\,f)\,(d\,e-(1+c)\,f)}+\frac{3\,b^3\,d\,\text{ArcCoth}[c+d\,x]\,\text{PolyLog}\left[2,1-\frac{2}{1+c+d\,x}\right]}{2\,f\,(d\,e-f-c\,f)}-\frac{3\,b^3\,d\,\text{ArcCoth}[c+d\,x]\,\text{PolyLog}\left[2,1-\frac{2}{1+c+d\,x}\right]}{(d\,e+f-c\,f)\,(d\,e-(1+c)\,f)}+$$

$$\frac{3\,a\,b^2\,d\,\text{PolyLog}\left[2,1-\frac{2\,d\,(e+f\,x)}{(d\,e+f-c\,f)\,(1+c+d\,x)}\right]}{(d\,e+f-c\,f)\,(d\,e-(1+c)\,f)}+\frac{3\,b^3\,d\,\text{ArcCoth}[c+d\,x]\,\text{PolyLog}\left[2,1-\frac{2\,d\,(e+f\,x)}{(d\,e+f-c\,f)\,(1+c+d\,x)}\right]}{(d\,e+f-c\,f)\,(d\,e-(1+c)\,f)}-\frac{3\,b^3\,d\,\text{PolyLog}\left[3,1-\frac{2}{1-c-d\,x}\right]}{4\,f\,(d\,e+f-c\,f)}+$$

$$\frac{3\,b^3\,d\,\text{PolyLog}\left[3,1-\frac{2}{1+c+d\,x}\right]}{4\,f\,(d\,e-f-c\,f)}-\frac{3\,b^3\,d\,\text{PolyLog}\left[3,1-\frac{2}{1+c+d\,x}\right]}{2\,(d\,e+f-c\,f)\,(d\,e-(1+c)\,f)}+\frac{3\,b^3\,d\,\text{PolyLog}\left[3,1-\frac{2\,d\,(e+f\,x)}{(d\,e+f-c\,f)\,(1+c+d\,x)}\right]}{2\,(d\,e+f-c\,f)\,(d\,e-(1+c)\,f)}$$

Result (type 4, 1816 leaves):

$$-\frac{a^3}{f\,(e+f\,x)}-\frac{3\,a^2\,b\,\text{ArcCoth}[c+d\,x]}{f\,(e+f\,x)}+\frac{3\,a^2\,b\,d\,\text{Log}[1-c-d\,x]}{2\,f\,(-d\,e-f+c\,f)}-\frac{3\,a^2\,b\,d\,\text{Log}[1+c+d\,x]}{2\,f\,(-d\,e+f+c\,f)}-$$

$$\frac{3\,a^2\,b\,d\,\text{Log}[e+f\,x]}{d^2\,e^2-2\,c\,d\,e\,f-f^2+c^2\,f^2}+\frac{1}{d\,f\,(e+f\,x)^2}\,3\,a\,b^2\,\left(1-(c+d\,x)^2\right)\left(\frac{f}{\sqrt{1-\frac{1}{(c+d\,x)^2}}}+\frac{d\,e-c\,f}{(c+d\,x)\,\sqrt{1-\frac{1}{(c+d\,x)^2}}}\right)^2$$

$$\left(\frac{e^{\text{ArcTanh}\left[\frac{f}{-d\,e+c\,f}\right]}\,\text{ArcCoth}[c+d\,x]^2}{(-d\,e+c\,f)\,\sqrt{1-\frac{f^2}{(d\,e-c\,f)^2}}}+\frac{\text{ArcCoth}[c+d\,x]^2}{(c+d\,x)\,\sqrt{1-\frac{1}{(c+d\,x)^2}}\,\left(\frac{f}{\sqrt{1-\frac{1}{(c+d\,x)^2}}}+\frac{d\,e-c\,f}{(c+d\,x)\,\sqrt{1-\frac{1}{(c+d\,x)^2}}}\right)}+\right.$$

$$\frac{1}{d^2 e^2 - 2 c d e f + (-1+c^2) f^2} f \Bigg( i \pi \text{ArcCoth}[c+d x] + 2 \text{ArcCoth}[c+d x] \text{ArcTanh}\left[\frac{f}{d e - c f}\right] - i \pi \text{Log}\left[1 + e^{2 \text{ArcCoth}[c+d x]}\right] + 2 \text{ArcCoth}[c+d x]$$

$$\text{Log}\left[1 - e^{-2 \left(\text{ArcCoth}[c+d x] + \text{ArcTanh}\left[\frac{f}{d e - c f}\right]\right)}\right] - 2 \text{ArcTanh}\left[\frac{f}{-d e + c f}\right] \text{Log}\left[1 - e^{-2 \left(\text{ArcCoth}[c+d x] + \text{ArcTanh}\left[\frac{f}{d e - c f}\right]\right)}\right] + i \pi \text{Log}\left[\frac{1}{\sqrt{1 - \frac{1}{(c+d x)^2}}}\right] +$$

$$2 \text{ArcTanh}\left[\frac{f}{-d e + c f}\right] \text{Log}\left[i \text{Sinh}\left[\text{ArcCoth}[c+d x] + \text{ArcTanh}\left[\frac{f}{d e - c f}\right]\right]\right] - \text{PolyLog}\left[2, e^{-2 \left(\text{ArcCoth}[c+d x] + \text{ArcTanh}\left[\frac{f}{d e - c f}\right]\right)}\right]\Bigg)\Bigg) +$$

$$\frac{1}{d (e + f x)^2} b^3 \left(1 - (c + d x)^2\right) \left(\frac{f}{\sqrt{1 - \frac{1}{(c+d x)^2}}} + \frac{d e - c f}{(c + d x) \sqrt{1 - \frac{1}{(c+d x)^2}}}\right)^2$$

$$\Bigg( - \frac{\text{ArcCoth}[c+d x]^3}{f (c + d x) \sqrt{1 - \frac{1}{(c+d x)^2}} \left( - \frac{f}{\sqrt{1 - \frac{1}{(c+d x)^2}}} - \frac{d e}{(c+d x) \sqrt{1 - \frac{1}{(c+d x)^2}}} + \frac{c f}{(c+d x) \sqrt{1 - \frac{1}{(c+d x)^2}}}\right)} + \frac{1}{2 f (d e + f - c f) (d e - (1 + c) f)} \Bigg( 2 d e \text{ArcCoth}[c+d x]^3 -$$

$$6 f \text{ArcCoth}[c+d x]^3 - 2 c f \text{ArcCoth}[c+d x]^3 - 4 d e\, e^{-\text{ArcTanh}\left[\frac{f}{d e - c f}\right]} \sqrt{\frac{d^2 e^2 - 2 c d e f + (-1+c^2) f^2}{(d e - c f)^2}} \text{ArcCoth}[c+d x]^3 +$$

$$4 c\, e^{-\text{ArcTanh}\left[\frac{f}{d e - c f}\right]} f \sqrt{\frac{d^2 e^2 - 2 c d e f + (-1+c^2) f^2}{(d e - c f)^2}} \text{ArcCoth}[c+d x]^3 + 6 i f \pi \text{ArcCoth}[c+d x] \text{Log}[2] - f \text{ArcCoth}[c+d x]^2 \text{Log}[64] -$$

$$6 i f \pi \text{ArcCoth}[c+d x] \text{Log}\left[e^{-\text{ArcCoth}[c+d x]} + e^{\text{ArcCoth}[c+d x]}\right] + 6 f \text{ArcCoth}[c+d x]^2 \text{Log}\left[1 - e^{\text{ArcCoth}[c+d x] + \text{ArcTanh}\left[\frac{f}{d e - c f}\right]}\right] +$$

$$6 f \text{ArcCoth}[c+d x]^2 \text{Log}\left[1 + e^{\text{ArcCoth}[c+d x] + \text{ArcTanh}\left[\frac{f}{d e - c f}\right]}\right] + 6 f \text{ArcCoth}[c+d x]^2 \text{Log}\left[1 - e^{2 \left(\text{ArcCoth}[c+d x] + \text{ArcTanh}\left[\frac{f}{d e - c f}\right]\right)}\right] +$$

$$12\,f\,\text{ArcCoth}[c+d\,x]\,\text{ArcTanh}\left[\frac{f}{d\,e-c\,f}\right]\,\text{Log}\left[\frac{1}{2}\,i\,e^{-\text{ArcCoth}[c+d\,x]-\text{ArcTanh}\left[\frac{f}{d\,e-c\,f}\right]}\left(-1+e^{2\left(\text{ArcCoth}[c+d\,x]+\text{ArcTanh}\left[\frac{f}{d\,e-c\,f}\right]\right)}\right)\right]+$$

$$6\,f\,\text{ArcCoth}[c+d\,x]^2\,\text{Log}\left[-e^{-\text{ArcCoth}[c+d\,x]}\left(d\,e\left(-1+e^{2\,\text{ArcCoth}[c+d\,x]}\right)+\left(1+c+e^{2\,\text{ArcCoth}[c+d\,x]}-c\,e^{2\,\text{ArcCoth}[c+d\,x]}\right)f\right)\right]-$$

$$6\,f\,\text{ArcCoth}[c+d\,x]^2\,\text{Log}\left[\frac{-d\,e\left(-1+e^{2\,\text{ArcCoth}[c+d\,x]}\right)+\left(-1-e^{2\,\text{ArcCoth}[c+d\,x]}+c\left(-1+e^{2\,\text{ArcCoth}[c+d\,x]}\right)\right)f}{d\,e-(1+c)\,f}\right]+$$

$$6\,i\,f\,\pi\,\text{ArcCoth}[c+d\,x]\,\text{Log}\left[\frac{1}{\sqrt{1-\frac{1}{(c+d\,x)^2}}}\right]-6\,f\,\text{ArcCoth}[c+d\,x]^2\,\text{Log}\left[-\frac{f}{\sqrt{1-\frac{1}{(c+d\,x)^2}}}-\frac{d\,e}{(c+d\,x)\sqrt{1-\frac{1}{(c+d\,x)^2}}}+\frac{c\,f}{(c+d\,x)\sqrt{1-\frac{1}{(c+d\,x)^2}}}\right]-$$

$$12\,f\,\text{ArcCoth}[c+d\,x]\,\text{ArcTanh}\left[\frac{f}{d\,e-c\,f}\right]\,\text{Log}\left[i\,\text{Sinh}\left[\text{ArcCoth}[c+d\,x]+\text{ArcTanh}\left[\frac{f}{d\,e-c\,f}\right]\right]\right]+$$

$$12\,f\,\text{ArcCoth}[c+d\,x]\,\text{PolyLog}\left[2,-e^{\text{ArcCoth}[c+d\,x]+\text{ArcTanh}\left[\frac{f}{d\,e-c\,f}\right]}\right]+12\,f\,\text{ArcCoth}[c+d\,x]\,\text{PolyLog}\left[2,e^{\text{ArcCoth}[c+d\,x]+\text{ArcTanh}\left[\frac{f}{d\,e-c\,f}\right]}\right]+$$

$$6\,f\,\text{ArcCoth}[c+d\,x]\,\text{PolyLog}\left[2,e^{2\left(\text{ArcCoth}[c+d\,x]+\text{ArcTanh}\left[\frac{f}{d\,e-c\,f}\right]\right)}\right]-6\,f\,\text{ArcCoth}[c+d\,x]\,\text{PolyLog}\left[2,\frac{e^{2\,\text{ArcCoth}[c+d\,x]}\,(d\,e+f-c\,f)}{d\,e-(1+c)\,f}\right]-$$

$$12\,f\,\text{PolyLog}\left[3,-e^{\text{ArcCoth}[c+d\,x]+\text{ArcTanh}\left[\frac{f}{d\,e-c\,f}\right]}\right]-12\,f\,\text{PolyLog}\left[3,e^{\text{ArcCoth}[c+d\,x]+\text{ArcTanh}\left[\frac{f}{d\,e-c\,f}\right]}\right]-$$

$$\left.\left.3\,f\,\text{PolyLog}\left[3,e^{2\left(\text{ArcCoth}[c+d\,x]+\text{ArcTanh}\left[\frac{f}{d\,e-c\,f}\right]\right)}\right]+3\,f\,\text{PolyLog}\left[3,\frac{e^{2\,\text{ArcCoth}[c+d\,x]}\,(d\,e+f-c\,f)}{d\,e-(1+c)\,f}\right]\right)\right)$$

## Problem 119: Unable to integrate problem.

$$\int (e+f\,x)^m\,(a+b\,\text{ArcCoth}[c+d\,x])\,dx$$

Optimal (type 5, 162 leaves, 6 steps):

$$\frac{(e+f\,x)^{1+m}\,(a+b\,\text{ArcCoth}[c+d\,x])}{f\,(1+m)}+\frac{b\,d\,(e+f\,x)^{2+m}\,\text{Hypergeometric2F1}\left[1,2+m,3+m,\frac{d\,(e+f\,x)}{d\,e-f-c\,f}\right]}{2\,f\,(d\,e-(1+c)\,f)\,(1+m)\,(2+m)}-$$
$$\frac{b\,d\,(e+f\,x)^{2+m}\,\text{Hypergeometric2F1}\left[1,2+m,3+m,\frac{d\,(e+f\,x)}{d\,e+f-c\,f}\right]}{2\,f\,(d\,e+f-c\,f)\,(1+m)\,(2+m)}$$

Result (type 8, 20 leaves):

$$\int (e+f\,x)^m\,(a+b\,\text{ArcCoth}[c+d\,x])\,dx$$

## Problem 123: Unable to integrate problem.

$$\int \frac{\left(a+b\,\text{ArcCoth}\left[\frac{\sqrt{1-c\,x}}{\sqrt{1+c\,x}}\right]\right)^3}{1-c^2\,x^2}\,dx$$

Optimal (type 4, 460 leaves, 9 steps):

$$-\frac{2\left(a+b\,\text{ArcCoth}\left[\frac{\sqrt{1-c\,x}}{\sqrt{1+c\,x}}\right]\right)^3\,\text{ArcCoth}\left[1-\frac{2}{1-\frac{\sqrt{1-c\,x}}{\sqrt{1+c\,x}}}\right]}{c}-\frac{3\,b\left(a+b\,\text{ArcCoth}\left[\frac{\sqrt{1-c\,x}}{\sqrt{1+c\,x}}\right]\right)^2\,\text{PolyLog}\left[2,1-\frac{2}{1+\frac{\sqrt{1-c\,x}}{\sqrt{1+c\,x}}}\right]}{2\,c}+$$
$$\frac{3\,b\left(a+b\,\text{ArcCoth}\left[\frac{\sqrt{1-c\,x}}{\sqrt{1+c\,x}}\right]\right)^2\,\text{PolyLog}\left[2,1-\frac{2\sqrt{1-c\,x}}{\sqrt{1+c\,x}\left(1+\frac{\sqrt{1-c\,x}}{\sqrt{1+c\,x}}\right)}\right]}{2\,c}-\frac{3\,b^2\left(a+b\,\text{ArcCoth}\left[\frac{\sqrt{1-c\,x}}{\sqrt{1+c\,x}}\right]\right)\,\text{PolyLog}\left[3,1-\frac{2}{1+\frac{\sqrt{1-c\,x}}{\sqrt{1+c\,x}}}\right]}{2\,c}+$$
$$\frac{3\,b^2\left(a+b\,\text{ArcCoth}\left[\frac{\sqrt{1-c\,x}}{\sqrt{1+c\,x}}\right]\right)\,\text{PolyLog}\left[3,1-\frac{2\sqrt{1-c\,x}}{\sqrt{1+c\,x}\left(1+\frac{\sqrt{1-c\,x}}{\sqrt{1+c\,x}}\right)}\right]}{2\,c}-\frac{3\,b^3\,\text{PolyLog}\left[4,1-\frac{2}{1+\frac{\sqrt{1-c\,x}}{\sqrt{1+c\,x}}}\right]}{4\,c}+\frac{3\,b^3\,\text{PolyLog}\left[4,1-\frac{2\sqrt{1-c\,x}}{\sqrt{1+c\,x}\left(1+\frac{\sqrt{1-c\,x}}{\sqrt{1+c\,x}}\right)}\right]}{4\,c}$$

Result (type 8, 42 leaves):

$$\int \frac{\left(a+b\,\text{ArcCoth}\left[\frac{\sqrt{1-c\,x}}{\sqrt{1+c\,x}}\right]\right)^3}{1-c^2\,x^2}\,dx$$

## Problem 124: Unable to integrate problem.

$$\int \frac{\left(a+b\,\text{ArcCoth}\left[\frac{\sqrt{1-c\,x}}{\sqrt{1+c\,x}}\right]\right)^2}{1-c^2\,x^2}\,dx$$

Optimal (type 4, 302 leaves, 7 steps):

$$-\frac{2\left(a+b\,\text{ArcCoth}\left[\frac{\sqrt{1-c\,x}}{\sqrt{1+c\,x}}\right]\right)^2\,\text{ArcCoth}\left[1-\frac{2}{1-\frac{\sqrt{1-c\,x}}{\sqrt{1+c\,x}}}\right]}{c}-\frac{b\left(a+b\,\text{ArcCoth}\left[\frac{\sqrt{1-c\,x}}{\sqrt{1+c\,x}}\right]\right)\,\text{PolyLog}\left[2,1-\frac{2}{1+\frac{\sqrt{1-c\,x}}{\sqrt{1+c\,x}}}\right]}{c}+$$
$$\frac{b\left(a+b\,\text{ArcCoth}\left[\frac{\sqrt{1-c\,x}}{\sqrt{1+c\,x}}\right]\right)\,\text{PolyLog}\left[2,1-\frac{2\sqrt{1-c\,x}}{\sqrt{1+c\,x}\left(1+\frac{\sqrt{1-c\,x}}{\sqrt{1+c\,x}}\right)}\right]}{c}-\frac{b^2\,\text{PolyLog}\left[3,1-\frac{2}{1+\frac{\sqrt{1-c\,x}}{\sqrt{1+c\,x}}}\right]}{2\,c}+\frac{b^2\,\text{PolyLog}\left[3,1-\frac{2\sqrt{1-c\,x}}{\sqrt{1+c\,x}\left(1+\frac{\sqrt{1-c\,x}}{\sqrt{1+c\,x}}\right)}\right]}{2\,c}$$

Result (type 8, 42 leaves):

$$\int \frac{\left(a+b\,\text{ArcCoth}\left[\frac{\sqrt{1-c\,x}}{\sqrt{1+c\,x}}\right]\right)^2}{1-c^2\,x^2}\,dx$$

## Problem 139: Result more than twice size of optimal antiderivative.

$$\int x\,\text{ArcCoth}[\text{Tanh}[a+b\,x]]^2\,dx$$

Optimal (type 3, 34 leaves, 3 steps):

$$\frac{x\,\text{ArcCoth}[\text{Tanh}[a+b\,x]]^3}{3\,b}-\frac{\text{ArcCoth}[\text{Tanh}[a+b\,x]]^4}{12\,b^2}$$

Result (type 3, 74 leaves):

$$\frac{1}{12\,b^2}(a+b\,x)\left(-(3\,a-b\,x)\,(a+b\,x)^2+4\left(2\,a^2+a\,b\,x-b^2\,x^2\right)\text{ArcCoth}[\text{Tanh}[a+b\,x]]-6\,(a-b\,x)\,\text{ArcCoth}[\text{Tanh}[a+b\,x]]^2\right)$$

## Problem 150: Result more than twice size of optimal antiderivative.

$$\int x\,\text{ArcCoth}[\text{Tanh}[a+b\,x]]^3\,dx$$

Optimal (type 3, 34 leaves, 3 steps):

$$\frac{x\,\text{ArcCoth}[\text{Tanh}[a+b\,x]]^4}{4\,b}-\frac{\text{ArcCoth}[\text{Tanh}[a+b\,x]]^5}{20\,b^2}$$

Result (type 3, 99 leaves):

$$\frac{1}{20\,b^2}(a+b\,x)\left((4\,a-b\,x)\,(a+b\,x)^3-5\,(3\,a-b\,x)\,(a+b\,x)^2\,\text{ArcCoth}[\text{Tanh}[a+b\,x]]+\right.$$
$$\left.10\left(2\,a^2+a\,b\,x-b^2\,x^2\right)\text{ArcCoth}[\text{Tanh}[a+b\,x]]^2-10\,(a-b\,x)\,\text{ArcCoth}[\text{Tanh}[a+b\,x]]^3\right)$$

## Problem 205: Result more than twice size of optimal antiderivative.

$$\int \text{ArcCoth}[c+d\,\text{Tanh}[a+b\,x]]\,dx$$

Optimal (type 4, 150 leaves, 7 steps):

$$x\,\text{ArcCoth}[c+d\,\text{Tanh}[a+b\,x]]+\frac{1}{2}x\,\text{Log}\left[1+\frac{(1-c-d)\,e^{2a+2bx}}{1-c+d}\right]-\frac{1}{2}x\,\text{Log}\left[1+\frac{(1+c+d)\,e^{2a+2bx}}{1+c-d}\right]+\frac{\text{PolyLog}\left[2,-\frac{(1-c-d)\,e^{2a+2bx}}{1-c+d}\right]}{4b}-\frac{\text{PolyLog}\left[2,-\frac{(1+c+d)\,e^{2a+2bx}}{1+c-d}\right]}{4b}$$

Result (type 4, 366 leaves):

$$x\,\text{ArcCoth}[c+d\,\text{Tanh}[a+b\,x]]+\frac{1}{2b}\left((a+b\,x)\,\text{Log}\left[1-\frac{\sqrt{-1+c+d}\,e^{a+bx}}{\sqrt{1-c+d}}\right]+(a+b\,x)\,\text{Log}\left[1+\frac{\sqrt{-1+c+d}\,e^{a+bx}}{\sqrt{1-c+d}}\right]-(a+b\,x)\,\text{Log}\left[1-\frac{\sqrt{1+c+d}\,e^{a+bx}}{\sqrt{-1-c+d}}\right]-(a+b\,x)\,\text{Log}\left[1+\frac{\sqrt{1+c+d}\,e^{a+bx}}{\sqrt{-1-c+d}}\right]+a\,\text{Log}\left[1+c-d+e^{2(a+bx)}+c\,e^{2(a+bx)}+d\,e^{2(a+bx)}\right]-a\,\text{Log}\left[1+d+e^{2(a+bx)}-d\,e^{2(a+bx)}-c\left(1+e^{2(a+bx)}\right)\right]+\text{PolyLog}\left[2,-\frac{\sqrt{-1+c+d}\,e^{a+bx}}{\sqrt{1-c+d}}\right]+\text{PolyLog}\left[2,\frac{\sqrt{-1+c+d}\,e^{a+bx}}{\sqrt{1-c+d}}\right]-\text{PolyLog}\left[2,-\frac{\sqrt{1+c+d}\,e^{a+bx}}{\sqrt{-1-c+d}}\right]-\text{PolyLog}\left[2,\frac{\sqrt{1+c+d}\,e^{a+bx}}{\sqrt{-1-c+d}}\right]\right)$$

## Problem 210: Result more than twice size of optimal antiderivative.

$$\int \text{ArcCoth}[1+d+d\,\text{Tanh}[a+b\,x]]\,dx$$

Optimal (type 4, 69 leaves, 5 steps):

$$\frac{b\,x^2}{2}+x\,\text{ArcCoth}[1+d+d\,\text{Tanh}[a+b\,x]]-\frac{1}{2}x\,\text{Log}\left[1+(1+d)\,e^{2a+2bx}\right]-\frac{\text{PolyLog}\left[2,-(1+d)\,e^{2a+2bx}\right]}{4b}$$

Result (type 4, 168 leaves):

$$x\,\text{ArcCoth}[1+d+d\,\text{Tanh}[a+b\,x]]-\frac{1}{2b}\left(b\,x\left(-b\,x-\text{Log}\left[e^{-a-bx}+(1+d)\,e^{a+bx}\right]+\text{Log}\left[1-e^{bx}\sqrt{-(1+d)\,e^{2a}}\right]+\text{Log}\left[1+e^{bx}\sqrt{-(1+d)\,e^{2a}}\right]+\text{Log}\left[(2+d)\,\text{Cosh}[a+b\,x]+d\,\text{Sinh}[a+b\,x]\right]\right)+\text{PolyLog}\left[2,-e^{bx}\sqrt{-(1+d)\,e^{2a}}\right]+\text{PolyLog}\left[2,e^{bx}\sqrt{-(1+d)\,e^{2a}}\right]\right)$$

## Problem 215: Result more than twice size of optimal antiderivative.

$$\int \text{ArcCoth}[1-d-d\,\text{Tanh}[a+b\,x]]\,dx$$

Optimal (type 4, 76 leaves, 5 steps):

$$\frac{b\,x^2}{2}+x\,\text{ArcCoth}[1-d-d\,\text{Tanh}[a+b\,x]]-\frac{1}{2}x\,\text{Log}\left[1+(1-d)\,e^{2a+2bx}\right]-\frac{\text{PolyLog}\left[2,-(1-d)\,e^{2a+2bx}\right]}{4b}$$

Result (type 4, 171 leaves):

$$x\,\text{ArcCoth}[1-d-d\,\text{Tanh}[a+b\,x]]-\frac{1}{2\,b}\left(b\,x\left(-b\,x-\text{Log}\left[e^{-a-b\,x}\left(-1+(-1+d)\,e^{2\,(a+b\,x)}\right)\right]+\text{Log}\left[1-e^{b\,x}\sqrt{(-1+d)\,e^{2\,a}}\right]+\text{Log}\left[1+e^{b\,x}\sqrt{(-1+d)\,e^{2\,a}}\right]+\text{Log}\left[(-2+d)\,\text{Cosh}[a+b\,x]+d\,\text{Sinh}[a+b\,x]\right]\right)+\text{PolyLog}\left[2,-e^{b\,x}\sqrt{(-1+d)\,e^{2\,a}}\right]+\text{PolyLog}\left[2,e^{b\,x}\sqrt{(-1+d)\,e^{2\,a}}\right]\right)$$

## Problem 219: Result more than twice size of optimal antiderivative.

$$\int\text{ArcCoth}[c+d\,\text{Coth}[a+b\,x]]\,dx$$

Optimal (type 4, 150 leaves, 7 steps):

$$x\,\text{ArcCoth}[c+d\,\text{Coth}[a+b\,x]]+\frac{1}{2}\,x\,\text{Log}\left[1-\frac{(1-c-d)\,e^{2\,a+2\,b\,x}}{1-c+d}\right]-\frac{1}{2}\,x\,\text{Log}\left[1-\frac{(1+c+d)\,e^{2\,a+2\,b\,x}}{1+c-d}\right]+\frac{\text{PolyLog}\left[2,\frac{(1-c-d)\,e^{2\,a+2\,b\,x}}{1-c+d}\right]}{4\,b}-\frac{\text{PolyLog}\left[2,\frac{(1+c+d)\,e^{2\,a+2\,b\,x}}{1+c-d}\right]}{4\,b}$$

Result (type 4, 369 leaves):

$$x\,\text{ArcCoth}[c+d\,\text{Coth}[a+b\,x]]-\frac{1}{2\,b}\left(-(a+b\,x)\,\text{Log}\left[1-\frac{\sqrt{-1+c+d}\,e^{a+b\,x}}{\sqrt{-1+c-d}}\right]-(a+b\,x)\,\text{Log}\left[1+\frac{\sqrt{-1+c+d}\,e^{a+b\,x}}{\sqrt{-1+c-d}}\right]+(a+b\,x)\,\text{Log}\left[1-\frac{\sqrt{1+c+d}\,e^{a+b\,x}}{\sqrt{1+c-d}}\right]+(a+b\,x)\,\text{Log}\left[1+\frac{\sqrt{1+c+d}\,e^{a+b\,x}}{\sqrt{1+c-d}}\right]+a\,\text{Log}\left[1+d-e^{2\,(a+b\,x)}+d\,e^{2\,(a+b\,x)}+c\left(-1+e^{2\,(a+b\,x)}\right)\right]-a\,\text{Log}\left[1+c-e^{2\,(a+b\,x)}-c\,e^{2\,(a+b\,x)}-d\left(1+e^{2\,(a+b\,x)}\right)\right]-\text{PolyLog}\left[2,-\frac{\sqrt{-1+c+d}\,e^{a+b\,x}}{\sqrt{-1+c-d}}\right]-\text{PolyLog}\left[2,\frac{\sqrt{-1+c+d}\,e^{a+b\,x}}{\sqrt{-1+c-d}}\right]+\text{PolyLog}\left[2,-\frac{\sqrt{1+c+d}\,e^{a+b\,x}}{\sqrt{1+c-d}}\right]+\text{PolyLog}\left[2,\frac{\sqrt{1+c+d}\,e^{a+b\,x}}{\sqrt{1+c-d}}\right]\right)$$

## Problem 224: Result more than twice size of optimal antiderivative.

$$\int\text{ArcCoth}[1+d+d\,\text{Coth}[a+b\,x]]\,dx$$

Optimal (type 4, 69 leaves, 5 steps):

$$\frac{b\,x^2}{2}+x\,\text{ArcCoth}[1+d+d\,\text{Coth}[a+b\,x]]-\frac{1}{2}\,x\,\text{Log}\left[1-(1+d)\,e^{2\,a+2\,b\,x}\right]-\frac{\text{PolyLog}\left[2,(1+d)\,e^{2\,a+2\,b\,x}\right]}{4\,b}$$

Result (type 4, 168 leaves):

$$x\,\text{ArcCoth}[1+d+d\,\text{Coth}[a+b\,x]]-\frac{1}{2\,b}\left(b\,x\left(-b\,x-\text{Log}\left[e^{-a-b\,x}\left(-1+(1+d)\,e^{2\,(a+b\,x)}\right)\right]+\text{Log}\left[1-e^{b\,x}\sqrt{(1+d)\,e^{2\,a}}\right]+\text{Log}\left[1+e^{b\,x}\sqrt{(1+d)\,e^{2\,a}}\right]+\text{Log}[d\,\text{Cosh}[a+b\,x]+(2+d)\,\text{Sinh}[a+b\,x]]\right)+\text{PolyLog}\left[2,-e^{b\,x}\sqrt{(1+d)\,e^{2\,a}}\right]+\text{PolyLog}\left[2,e^{b\,x}\sqrt{(1+d)\,e^{2\,a}}\right]\right)$$

## Problem 229: Result more than twice size of optimal antiderivative.

$$\int \text{ArcCoth}[1-d-d\,\text{Coth}[a+b\,x]]\,dx$$

Optimal (type 4, 76 leaves, 5 steps):

$$\frac{b\,x^2}{2}+x\,\text{ArcCoth}[1-d-d\,\text{Coth}[a+b\,x]]-\frac{1}{2}\,x\,\text{Log}\left[1-(1-d)\,e^{2\,a+2\,b\,x}\right]-\frac{\text{PolyLog}\left[2,(1-d)\,e^{2\,a+2\,b\,x}\right]}{4\,b}$$

Result (type 4, 175 leaves):

$$x\,\text{ArcCoth}[1-d-d\,\text{Coth}[a+b\,x]]-\frac{1}{2\,b}\left(b\,x\left(-b\,x-\text{Log}\left[e^{-a-b\,x}\left(1+(-1+d)\,e^{2\,(a+b\,x)}\right)\right]+\text{Log}\left[1-e^{b\,x}\sqrt{-(-1+d)\,e^{2\,a}}\right]+\text{Log}\left[1+e^{b\,x}\sqrt{-(-1+d)\,e^{2\,a}}\right]+\text{Log}[d\,\text{Cosh}[a+b\,x]+(-2+d)\,\text{Sinh}[a+b\,x]]\right)+\text{PolyLog}\left[2,-e^{b\,x}\sqrt{-(-1+d)\,e^{2\,a}}\right]+\text{PolyLog}\left[2,e^{b\,x}\sqrt{-(-1+d)\,e^{2\,a}}\right]\right)$$

## Problem 231: Result more than twice size of optimal antiderivative.

$$\int (e+f\,x)^3\,\text{ArcCoth}[\text{Tan}[a+b\,x]]\,dx$$

Optimal (type 4, 302 leaves, 12 steps):

$$\frac{(e+f\,x)^4\,\text{ArcCoth}[\text{Tan}[a+b\,x]]}{4\,f}+\frac{i\,(e+f\,x)^4\,\text{ArcTan}\left[e^{2\,i\,(a+b\,x)}\right]}{4\,f}-\frac{i\,(e+f\,x)^3\,\text{PolyLog}\left[2,-i\,e^{2\,i\,(a+b\,x)}\right]}{4\,b}+\frac{i\,(e+f\,x)^3\,\text{PolyLog}\left[2,i\,e^{2\,i\,(a+b\,x)}\right]}{4\,b}+\frac{3\,f\,(e+f\,x)^2\,\text{PolyLog}\left[3,-i\,e^{2\,i\,(a+b\,x)}\right]}{8\,b^2}-\frac{3\,f\,(e+f\,x)^2\,\text{PolyLog}\left[3,i\,e^{2\,i\,(a+b\,x)}\right]}{8\,b^2}+\frac{3\,i\,f^2\,(e+f\,x)\,\text{PolyLog}\left[4,-i\,e^{2\,i\,(a+b\,x)}\right]}{8\,b^3}-\frac{3\,i\,f^2\,(e+f\,x)\,\text{PolyLog}\left[4,i\,e^{2\,i\,(a+b\,x)}\right]}{8\,b^3}-\frac{3\,f^3\,\text{PolyLog}\left[5,-i\,e^{2\,i\,(a+b\,x)}\right]}{16\,b^4}+\frac{3\,f^3\,\text{PolyLog}\left[5,i\,e^{2\,i\,(a+b\,x)}\right]}{16\,b^4}$$

Result (type 4, 654 leaves):

$$\frac{1}{4}\,x\,\left(4\,e^3+6\,e^2\,f\,x+4\,e\,f^2\,x^2+f^3\,x^3\right)\,\text{ArcCoth}[\text{Tan}[a+b\,x]]+$$
$$\frac{1}{16\,b^4}\left(-8\,b^4\,e^3\,x\,\text{Log}\left[1-i\,e^{2\,i\,(a+b\,x)}\right]-12\,b^4\,e^2\,f\,x^2\,\text{Log}\left[1-i\,e^{2\,i\,(a+b\,x)}\right]-8\,b^4\,e\,f^2\,x^3\,\text{Log}\left[1-i\,e^{2\,i\,(a+b\,x)}\right]-\right.$$
$$2\,b^4\,f^3\,x^4\,\text{Log}\left[1-i\,e^{2\,i\,(a+b\,x)}\right]+8\,b^4\,e^3\,x\,\text{Log}\left[1+i\,e^{2\,i\,(a+b\,x)}\right]+12\,b^4\,e^2\,f\,x^2\,\text{Log}\left[1+i\,e^{2\,i\,(a+b\,x)}\right]+8\,b^4\,e\,f^2\,x^3\,\text{Log}\left[1+i\,e^{2\,i\,(a+b\,x)}\right]+$$
$$2\,b^4\,f^3\,x^4\,\text{Log}\left[1+i\,e^{2\,i\,(a+b\,x)}\right]-4\,i\,b^3\,(e+f\,x)^3\,\text{PolyLog}\left[2,-i\,e^{2\,i\,(a+b\,x)}\right]+4\,i\,b^3\,(e+f\,x)^3\,\text{PolyLog}\left[2,i\,e^{2\,i\,(a+b\,x)}\right]+$$
$$6\,b^2\,e^2\,f\,\text{PolyLog}\left[3,-i\,e^{2\,i\,(a+b\,x)}\right]+12\,b^2\,e\,f^2\,x\,\text{PolyLog}\left[3,-i\,e^{2\,i\,(a+b\,x)}\right]+6\,b^2\,f^3\,x^2\,\text{PolyLog}\left[3,-i\,e^{2\,i\,(a+b\,x)}\right]-$$
$$6\,b^2\,e^2\,f\,\text{PolyLog}\left[3,i\,e^{2\,i\,(a+b\,x)}\right]-12\,b^2\,e\,f^2\,x\,\text{PolyLog}\left[3,i\,e^{2\,i\,(a+b\,x)}\right]-6\,b^2\,f^3\,x^2\,\text{PolyLog}\left[3,i\,e^{2\,i\,(a+b\,x)}\right]+$$
$$6\,i\,b\,e\,f^2\,\text{PolyLog}\left[4,-i\,e^{2\,i\,(a+b\,x)}\right]+6\,i\,b\,f^3\,x\,\text{PolyLog}\left[4,-i\,e^{2\,i\,(a+b\,x)}\right]-6\,i\,b\,e\,f^2\,\text{PolyLog}\left[4,i\,e^{2\,i\,(a+b\,x)}\right]-$$
$$\left.6\,i\,b\,f^3\,x\,\text{PolyLog}\left[4,i\,e^{2\,i\,(a+b\,x)}\right]-3\,f^3\,\text{PolyLog}\left[5,-i\,e^{2\,i\,(a+b\,x)}\right]+3\,f^3\,\text{PolyLog}\left[5,i\,e^{2\,i\,(a+b\,x)}\right]\right)$$

## Problem 238: Result more than twice size of optimal antiderivative.

$$\int \text{ArcCoth}[c+d\,\text{Tan}[a+b\,x]]\,dx$$

Optimal (type 4, 194 leaves, 7 steps):

$$x\,\text{ArcCoth}[c+d\,\text{Tan}[a+b\,x]]+\frac{1}{2}\,x\,\text{Log}\left[1+\frac{(1-c+i\,d)\,e^{2\,i\,a+2\,i\,b\,x}}{1-c-i\,d}\right]-$$
$$\frac{1}{2}\,x\,\text{Log}\left[1+\frac{(1+c-i\,d)\,e^{2\,i\,a+2\,i\,b\,x}}{1+c+i\,d}\right]-\frac{i\,\text{PolyLog}\left[2,-\frac{(1-c+i\,d)\,e^{2\,i\,a+2\,i\,b\,x}}{1-c-i\,d}\right]}{4\,b}+\frac{i\,\text{PolyLog}\left[2,-\frac{(1+c-i\,d)\,e^{2\,i\,a+2\,i\,b\,x}}{1+c+i\,d}\right]}{4\,b}$$

Result (type 4, 4654 leaves):

$$x\,\text{ArcCoth}[c+d\,\text{Tan}[a+b\,x]]+$$
$$\left(d\left(-a\,\text{Log}\left[-\text{Sec}\left[\frac{1}{2}\,(a+b\,x)\right]^2\,((-1+c)\,\text{Cos}[a+b\,x]+d\,\text{Sin}[a+b\,x])\right]+a\,\text{Log}\left[\text{Sec}\left[\frac{1}{2}\,(a+b\,x)\right]^2\,(\text{Cos}[a+b\,x]+c\,\text{Cos}[a+b\,x]+d\,\text{Sin}[a+b\,x])\right]+\right.\right.$$
$$(a+b\,x)\,\text{Log}\left[\frac{-d+\sqrt{1-2\,c+c^2+d^2}}{-1+c}+\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]\right]+$$
$$i\,\text{Log}\left[\frac{(-1+c)\,\left(1+i\,\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]\right)}{-1+c+i\,d-i\,\sqrt{1-2\,c+c^2+d^2}}\right]\,\text{Log}\left[\frac{-d+\sqrt{1-2\,c+c^2+d^2}}{-1+c}+\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]\right]-i\,\text{Log}\left[-\frac{(-1+c)\,\left(i+\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]\right)}{i-i\,c-d+\sqrt{1-2\,c+c^2+d^2}}\right]$$
$$\text{Log}\left[\frac{-d+\sqrt{1-2\,c+c^2+d^2}}{-1+c}+\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]\right]+(a+b\,x)\,\text{Log}\left[\frac{d+\sqrt{1-2\,c+c^2+d^2}}{1-c}+\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]\right]+$$
$$i\,\text{Log}\left[\frac{(-1+c)\,\left(-i+\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]\right)}{i-i\,c+d+\sqrt{1-2\,c+c^2+d^2}}\right]\,\text{Log}\left[\frac{d+\sqrt{1-2\,c+c^2+d^2}}{1-c}+\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]\right]-i\,\text{Log}\left[\frac{(-1+c)\,\left(i+\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]\right)}{-i+i\,c+d+\sqrt{1-2\,c+c^2+d^2}}\right]$$
$$\text{Log}\left[\frac{d+\sqrt{1-2\,c+c^2+d^2}}{1-c}+\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]\right]-(a+b\,x)\,\text{Log}\left[-\frac{d+\sqrt{1+2\,c+c^2+d^2}}{1+c}+\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]\right]-$$

$$i\,\mathrm{Log}\left[\frac{(1+c)\left(-i+\mathrm{Tan}\left[\frac{1}{2}(a+b\,x)\right]\right)}{-i-i\,c+d+\sqrt{1+2c+c^2+d^2}}\right]\mathrm{Log}\left[-\frac{d+\sqrt{1+2c+c^2+d^2}}{1+c}+\mathrm{Tan}\left[\frac{1}{2}(a+b\,x)\right]\right]+i\,\mathrm{Log}\left[\frac{(1+c)\left(i+\mathrm{Tan}\left[\frac{1}{2}(a+b\,x)\right]\right)}{i+i\,c+d+\sqrt{1+2c+c^2+d^2}}\right]$$

$$\mathrm{Log}\left[-\frac{d+\sqrt{1+2c+c^2+d^2}}{1+c}+\mathrm{Tan}\left[\frac{1}{2}(a+b\,x)\right]\right]-(a+b\,x)\,\mathrm{Log}\left[\frac{-d+\sqrt{1+2c+c^2+d^2}+(1+c)\,\mathrm{Tan}\left[\frac{1}{2}(a+b\,x)\right]}{1+c}\right]+$$

$$i\,\mathrm{Log}\left[\frac{(1+c)\left(1-i\,\mathrm{Tan}\left[\frac{1}{2}(a+b\,x)\right]\right)}{1+c-i\,d+i\sqrt{1+2c+c^2+d^2}}\right]\mathrm{Log}\left[\frac{-d+\sqrt{1+2c+c^2+d^2}+(1+c)\,\mathrm{Tan}\left[\frac{1}{2}(a+b\,x)\right]}{1+c}\right]-$$

$$i\,\mathrm{Log}\left[\frac{(1+c)\left(1+i\,\mathrm{Tan}\left[\frac{1}{2}(a+b\,x)\right]\right)}{1+c+i\,d-i\sqrt{1+2c+c^2+d^2}}\right]\mathrm{Log}\left[\frac{-d+\sqrt{1+2c+c^2+d^2}+(1+c)\,\mathrm{Tan}\left[\frac{1}{2}(a+b\,x)\right]}{1+c}\right]+$$

$$i\,\mathrm{PolyLog}\left[2,\frac{d+\sqrt{1-2c+c^2+d^2}-(-1+c)\,\mathrm{Tan}\left[\frac{1}{2}(a+b\,x)\right]}{i-i\,c+d+\sqrt{1-2c+c^2+d^2}}\right]-i\,\mathrm{PolyLog}\left[2,\frac{d+\sqrt{1-2c+c^2+d^2}-(-1+c)\,\mathrm{Tan}\left[\frac{1}{2}(a+b\,x)\right]}{-i+i\,c+d+\sqrt{1-2c+c^2+d^2}}\right]-$$

$$i\,\mathrm{PolyLog}\left[2,\frac{-d+\sqrt{1-2c+c^2+d^2}+(-1+c)\,\mathrm{Tan}\left[\frac{1}{2}(a+b\,x)\right]}{i-i\,c-d+\sqrt{1-2c+c^2+d^2}}\right]+i\,\mathrm{PolyLog}\left[2,\frac{-d+\sqrt{1-2c+c^2+d^2}+(-1+c)\,\mathrm{Tan}\left[\frac{1}{2}(a+b\,x)\right]}{-i+i\,c-d+\sqrt{1-2c+c^2+d^2}}\right]-$$

$$i\,\mathrm{PolyLog}\left[2,\frac{d+\sqrt{1+2c+c^2+d^2}-(1+c)\,\mathrm{Tan}\left[\frac{1}{2}(a+b\,x)\right]}{-i-i\,c+d+\sqrt{1+2c+c^2+d^2}}\right]+i\,\mathrm{PolyLog}\left[2,\frac{d+\sqrt{1+2c+c^2+d^2}-(1+c)\,\mathrm{Tan}\left[\frac{1}{2}(a+b\,x)\right]}{i+i\,c+d+\sqrt{1+2c+c^2+d^2}}\right]+$$

$$i\,\mathrm{PolyLog}\left[2,\frac{-d+\sqrt{1+2c+c^2+d^2}+(1+c)\,\mathrm{Tan}\left[\frac{1}{2}(a+b\,x)\right]}{-i-i\,c-d+\sqrt{1+2c+c^2+d^2}}\right]-i\,\mathrm{PolyLog}\left[2,\frac{-d+\sqrt{1+2c+c^2+d^2}+(1+c)\,\mathrm{Tan}\left[\frac{1}{2}(a+b\,x)\right]}{i+i\,c-d+\sqrt{1+2c+c^2+d^2}}\right]\Bigg)$$

$$\Big(-\big((2a)/(b(-1+c^2+d^2-\mathrm{Cos}[2(a+b\,x)]+c^2\,\mathrm{Cos}[2(a+b\,x)]-d^2\,\mathrm{Cos}[2(a+b\,x)]+2\,c\,d\,\mathrm{Sin}[2(a+b\,x)]))\big)+$$

$$(2(a+b\,x))/(b(-1+c^2+d^2-\mathrm{Cos}[2(a+b\,x)]+c^2\,\mathrm{Cos}[2(a+b\,x)]-d^2\,\mathrm{Cos}[2(a+b\,x)]+2\,c\,d\,\mathrm{Sin}[2(a+b\,x)]))\Big)\Big/$$

$$\Bigg(\mathrm{Log}\left[\frac{-d+\sqrt{1-2c+c^2+d^2}}{-1+c}+\mathrm{Tan}\left[\frac{1}{2}(a+b\,x)\right]\right]+\mathrm{Log}\left[\frac{d+\sqrt{1-2c+c^2+d^2}}{1-c}+\mathrm{Tan}\left[\frac{1}{2}(a+b\,x)\right]\right]-$$

$$\mathrm{Log}\left[-\frac{d+\sqrt{1+2c+c^2+d^2}}{1+c}+\mathrm{Tan}\left[\frac{1}{2}(a+b\,x)\right]\right]-\mathrm{Log}\left[\frac{-d+\sqrt{1+2c+c^2+d^2}+(1+c)\,\mathrm{Tan}\left[\frac{1}{2}(a+b\,x)\right]}{1+c}\right]+$$

$$\frac{\mathrm{Log}\left[\frac{-d+\sqrt{1+2c+c^2+d^2}+(1+c)\,\mathrm{Tan}\left[\frac{1}{2}(a+b\,x)\right]}{1+c}\right]\mathrm{Sec}\left[\frac{1}{2}(a+b\,x)\right]^2}{2\left(1-i\,\mathrm{Tan}\left[\frac{1}{2}(a+b\,x)\right]\right)}-\frac{\mathrm{Log}\left[\frac{-d+\sqrt{1-2c+c^2+d^2}}{-1+c}+\mathrm{Tan}\left[\frac{1}{2}(a+b\,x)\right]\right]\mathrm{Sec}\left[\frac{1}{2}(a+b\,x)\right]^2}{2\left(1+i\,\mathrm{Tan}\left[\frac{1}{2}(a+b\,x)\right]\right)}+$$

$$\frac{\text{Log}\left[\frac{-d+\sqrt{1+2c+c^2+d^2}+(1+c)\,\text{Tan}\left[\frac{1}{2}(a+b x)\right]}{1+c}\right]\text{Sec}\left[\frac{1}{2}(a+b x)\right]^2}{2\left(1+i\,\text{Tan}\left[\frac{1}{2}(a+b x)\right]\right)}+\frac{i\,\text{Log}\left[\frac{d+\sqrt{1-2c+c^2+d^2}}{1-c}+\text{Tan}\left[\frac{1}{2}(a+b x)\right]\right]\text{Sec}\left[\frac{1}{2}(a+b x)\right]^2}{2\left(-i+\text{Tan}\left[\frac{1}{2}(a+b x)\right]\right)}-$$

$$\frac{i\,\text{Log}\left[-\frac{d+\sqrt{1+2c+c^2+d^2}}{1+c}+\text{Tan}\left[\frac{1}{2}(a+b x)\right]\right]\text{Sec}\left[\frac{1}{2}(a+b x)\right]^2}{2\left(-i+\text{Tan}\left[\frac{1}{2}(a+b x)\right]\right)}-\frac{i\,\text{Log}\left[\frac{-d+\sqrt{1-2c+c^2+d^2}}{-1+c}+\text{Tan}\left[\frac{1}{2}(a+b x)\right]\right]\text{Sec}\left[\frac{1}{2}(a+b x)\right]^2}{2\left(i+\text{Tan}\left[\frac{1}{2}(a+b x)\right]\right)}-$$

$$\frac{i\,\text{Log}\left[\frac{d+\sqrt{1-2c+c^2+d^2}}{1-c}+\text{Tan}\left[\frac{1}{2}(a+b x)\right]\right]\text{Sec}\left[\frac{1}{2}(a+b x)\right]^2}{2\left(i+\text{Tan}\left[\frac{1}{2}(a+b x)\right]\right)}+\frac{i\,\text{Log}\left[-\frac{d+\sqrt{1+2c+c^2+d^2}}{1+c}+\text{Tan}\left[\frac{1}{2}(a+b x)\right]\right]\text{Sec}\left[\frac{1}{2}(a+b x)\right]^2}{2\left(i+\text{Tan}\left[\frac{1}{2}(a+b x)\right]\right)}+$$

$$\frac{(a+b x)\,\text{Sec}\left[\frac{1}{2}(a+b x)\right]^2}{2\left(\frac{-d+\sqrt{1-2c+c^2+d^2}}{-1+c}+\text{Tan}\left[\frac{1}{2}(a+b x)\right]\right)}+\frac{i\,\text{Log}\left[\frac{(-1+c)\left(1+i\,\text{Tan}\left[\frac{1}{2}(a+b x)\right]\right)}{-1+c+i d-i\sqrt{1-2c+c^2+d^2}}\right]\text{Sec}\left[\frac{1}{2}(a+b x)\right]^2}{2\left(\frac{-d+\sqrt{1-2c+c^2+d^2}}{-1+c}+\text{Tan}\left[\frac{1}{2}(a+b x)\right]\right)}-$$

$$\frac{i\,\text{Log}\left[-\frac{(-1+c)\left(i+\text{Tan}\left[\frac{1}{2}(a+b x)\right]\right)}{i-i c-d+\sqrt{1-2c+c^2+d^2}}\right]\text{Sec}\left[\frac{1}{2}(a+b x)\right]^2}{2\left(\frac{-d+\sqrt{1-2c+c^2+d^2}}{-1+c}+\text{Tan}\left[\frac{1}{2}(a+b x)\right]\right)}+\frac{(a+b x)\,\text{Sec}\left[\frac{1}{2}(a+b x)\right]^2}{2\left(\frac{d+\sqrt{1-2c+c^2+d^2}}{1-c}+\text{Tan}\left[\frac{1}{2}(a+b x)\right]\right)}+$$

$$\frac{i\,\text{Log}\left[\frac{(-1+c)\left(-i+\text{Tan}\left[\frac{1}{2}(a+b x)\right]\right)}{i-i c+d+\sqrt{1-2c+c^2+d^2}}\right]\text{Sec}\left[\frac{1}{2}(a+b x)\right]^2}{2\left(\frac{d+\sqrt{1-2c+c^2+d^2}}{1-c}+\text{Tan}\left[\frac{1}{2}(a+b x)\right]\right)}-\frac{i\,\text{Log}\left[\frac{(-1+c)\left(i+\text{Tan}\left[\frac{1}{2}(a+b x)\right]\right)}{-i+i c+d+\sqrt{1-2c+c^2+d^2}}\right]\text{Sec}\left[\frac{1}{2}(a+b x)\right]^2}{2\left(\frac{d+\sqrt{1-2c+c^2+d^2}}{1-c}+\text{Tan}\left[\frac{1}{2}(a+b x)\right]\right)}-$$

$$\frac{(a+b x)\,\text{Sec}\left[\frac{1}{2}(a+b x)\right]^2}{2\left(-\frac{d+\sqrt{1+2c+c^2+d^2}}{1+c}+\text{Tan}\left[\frac{1}{2}(a+b x)\right]\right)}-\frac{i\,\text{Log}\left[\frac{(1+c)\left(-i+\text{Tan}\left[\frac{1}{2}(a+b x)\right]\right)}{-i-i c+d+\sqrt{1+2c+c^2+d^2}}\right]\text{Sec}\left[\frac{1}{2}(a+b x)\right]^2}{2\left(-\frac{d+\sqrt{1+2c+c^2+d^2}}{1+c}+\text{Tan}\left[\frac{1}{2}(a+b x)\right]\right)}+$$

$$\frac{i\,\text{Log}\left[\frac{(1+c)\left(i+\text{Tan}\left[\frac{1}{2}(a+b x)\right]\right)}{i+i c+d+\sqrt{1+2c+c^2+d^2}}\right]\text{Sec}\left[\frac{1}{2}(a+b x)\right]^2}{2\left(-\frac{d+\sqrt{1+2c+c^2+d^2}}{1+c}+\text{Tan}\left[\frac{1}{2}(a+b x)\right]\right)}+\frac{i\,(-1+c)\,\text{Log}\left[1-\frac{d+\sqrt{1-2c+c^2+d^2}-(-1+c)\,\text{Tan}\left[\frac{1}{2}(a+b x)\right]}{i-i c+d+\sqrt{1-2c+c^2+d^2}}\right]\text{Sec}\left[\frac{1}{2}(a+b x)\right]^2}{2\left(d+\sqrt{1-2c+c^2+d^2}-(-1+c)\,\text{Tan}\left[\frac{1}{2}(a+b x)\right]\right)}-$$

$$\frac{i\,(-1+c)\,\mathrm{Log}\left[1-\frac{d+\sqrt{1-2c+c^2+d^2}-(-1+c)\,\mathrm{Tan}\left[\frac{1}{2}(a+b\,x)\right]}{-i+i\,c+d+\sqrt{1-2c+c^2+d^2}}\right]\mathrm{Sec}\left[\frac{1}{2}(a+b\,x)\right]^2}{2\left(d+\sqrt{1-2c+c^2+d^2}-(-1+c)\,\mathrm{Tan}\left[\frac{1}{2}(a+b\,x)\right]\right)}+\frac{i\,(-1+c)\,\mathrm{Log}\left[1-\frac{-d+\sqrt{1-2c+c^2+d^2}+(-1+c)\,\mathrm{Tan}\left[\frac{1}{2}(a+b\,x)\right]}{i-i\,c-d+\sqrt{1-2c+c^2+d^2}}\right]\mathrm{Sec}\left[\frac{1}{2}(a+b\,x)\right]^2}{2\left(-d+\sqrt{1-2c+c^2+d^2}+(-1+c)\,\mathrm{Tan}\left[\frac{1}{2}(a+b\,x)\right]\right)}-$$

$$\frac{i\,(-1+c)\,\mathrm{Log}\left[1-\frac{-d+\sqrt{1-2c+c^2+d^2}+(-1+c)\,\mathrm{Tan}\left[\frac{1}{2}(a+b\,x)\right]}{-i+i\,c-d+\sqrt{1-2c+c^2+d^2}}\right]\mathrm{Sec}\left[\frac{1}{2}(a+b\,x)\right]^2}{2\left(-d+\sqrt{1-2c+c^2+d^2}+(-1+c)\,\mathrm{Tan}\left[\frac{1}{2}(a+b\,x)\right]\right)}-$$

$$\frac{i\,(1+c)\,\mathrm{Log}\left[1-\frac{d+\sqrt{1+2c+c^2+d^2}-(1+c)\,\mathrm{Tan}\left[\frac{1}{2}(a+b\,x)\right]}{-i-i\,c+d+\sqrt{1+2c+c^2+d^2}}\right]\mathrm{Sec}\left[\frac{1}{2}(a+b\,x)\right]^2}{2\left(d+\sqrt{1+2c+c^2+d^2}-(1+c)\,\mathrm{Tan}\left[\frac{1}{2}(a+b\,x)\right]\right)}+\frac{i\,(1+c)\,\mathrm{Log}\left[1-\frac{d+\sqrt{1+2c+c^2+d^2}-(1+c)\,\mathrm{Tan}\left[\frac{1}{2}(a+b\,x)\right]}{i+i\,c+d+\sqrt{1+2c+c^2+d^2}}\right]\mathrm{Sec}\left[\frac{1}{2}(a+b\,x)\right]^2}{2\left(d+\sqrt{1+2c+c^2+d^2}-(1+c)\,\mathrm{Tan}\left[\frac{1}{2}(a+b\,x)\right]\right)}-$$

$$\frac{(1+c)\,(a+b\,x)\,\mathrm{Sec}\left[\frac{1}{2}(a+b\,x)\right]^2}{2\left(-d+\sqrt{1+2c+c^2+d^2}+(1+c)\,\mathrm{Tan}\left[\frac{1}{2}(a+b\,x)\right]\right)}+\frac{i\,(1+c)\,\mathrm{Log}\left[\frac{(1+c)\left(1-i\,\mathrm{Tan}\left[\frac{1}{2}(a+b\,x)\right]\right)}{1+c-i\,d+i\,\sqrt{1+2c+c^2+d^2}}\right]\mathrm{Sec}\left[\frac{1}{2}(a+b\,x)\right]^2}{2\left(-d+\sqrt{1+2c+c^2+d^2}+(1+c)\,\mathrm{Tan}\left[\frac{1}{2}(a+b\,x)\right]\right)}-$$

$$\frac{i\,(1+c)\,\mathrm{Log}\left[\frac{(1+c)\left(1+i\,\mathrm{Tan}\left[\frac{1}{2}(a+b\,x)\right]\right)}{1+c+i\,d-i\,\sqrt{1+2c+c^2+d^2}}\right]\mathrm{Sec}\left[\frac{1}{2}(a+b\,x)\right]^2}{2\left(-d+\sqrt{1+2c+c^2+d^2}+(1+c)\,\mathrm{Tan}\left[\frac{1}{2}(a+b\,x)\right]\right)}-\frac{i\,(1+c)\,\mathrm{Log}\left[1-\frac{-d+\sqrt{1+2c+c^2+d^2}+(1+c)\,\mathrm{Tan}\left[\frac{1}{2}(a+b\,x)\right]}{-i-i\,c-d+\sqrt{1+2c+c^2+d^2}}\right]\mathrm{Sec}\left[\frac{1}{2}(a+b\,x)\right]^2}{2\left(-d+\sqrt{1+2c+c^2+d^2}+(1+c)\,\mathrm{Tan}\left[\frac{1}{2}(a+b\,x)\right]\right)}+$$

$$\frac{i\,(1+c)\,\mathrm{Log}\left[1-\frac{-d+\sqrt{1+2c+c^2+d^2}+(1+c)\,\mathrm{Tan}\left[\frac{1}{2}(a+b\,x)\right]}{i+i\,c-d+\sqrt{1+2c+c^2+d^2}}\right]\mathrm{Sec}\left[\frac{1}{2}(a+b\,x)\right]^2}{2\left(-d+\sqrt{1+2c+c^2+d^2}+(1+c)\,\mathrm{Tan}\left[\frac{1}{2}(a+b\,x)\right]\right)}+\left(a\,\mathrm{Cos}\left[\frac{1}{2}(a+b\,x)\right]^2\right.$$

$$\left(-\mathrm{Sec}\left[\frac{1}{2}(a+b\,x)\right]^2\left(d\,\mathrm{Cos}[a+b\,x]-(-1+c)\,\mathrm{Sin}[a+b\,x]\right)-\mathrm{Sec}\left[\frac{1}{2}(a+b\,x)\right]^2\left((-1+c)\,\mathrm{Cos}[a+b\,x]+d\,\mathrm{Sin}[a+b\,x]\right)\mathrm{Tan}\left[\frac{1}{2}(a+b\,x)\right]\right)\Big)\Big/$$

$$\left((-1+c)\,\mathrm{Cos}[a+b\,x]+d\,\mathrm{Sin}[a+b\,x]\right)+\left(a\,\mathrm{Cos}\left[\frac{1}{2}(a+b\,x)\right]^2\left(\mathrm{Sec}\left[\frac{1}{2}(a+b\,x)\right]^2\left(d\,\mathrm{Cos}[a+b\,x]-\mathrm{Sin}[a+b\,x]-c\,\mathrm{Sin}[a+b\,x]\right)+\right.\right.$$

$$\left.\left.\mathrm{Sec}\left[\frac{1}{2}(a+b\,x)\right]^2\left(\mathrm{Cos}[a+b\,x]+c\,\mathrm{Cos}[a+b\,x]+d\,\mathrm{Sin}[a+b\,x]\right)\mathrm{Tan}\left[\frac{1}{2}(a+b\,x)\right]\right)\right)\Big/\left(\mathrm{Cos}[a+b\,x]+c\,\mathrm{Cos}[a+b\,x]+d\,\mathrm{Sin}[a+b\,x]\right)\Bigg)$$

## Problem 248: Result more than twice size of optimal antiderivative.

$$\int (e+f x)^3 \operatorname{ArcCoth}[\operatorname{Cot}[a+b x]] \, dx$$

Optimal (type 4, 302 leaves, 12 steps):

$$\frac{(e+f x)^4 \operatorname{ArcCoth}[\operatorname{Cot}[a+b x]]}{4 f}+\frac{i (e+f x)^4 \operatorname{ArcTan}\left[e^{2 i (a+b x)}\right]}{4 f}-\frac{i (e+f x)^3 \operatorname{PolyLog}\left[2,-i e^{2 i (a+b x)}\right]}{4 b}+\frac{i (e+f x)^3 \operatorname{PolyLog}\left[2, i e^{2 i (a+b x)}\right]}{4 b}+\frac{3 f (e+f x)^2 \operatorname{PolyLog}\left[3,-i e^{2 i (a+b x)}\right]}{8 b^2}-\frac{3 f (e+f x)^2 \operatorname{PolyLog}\left[3, i e^{2 i (a+b x)}\right]}{8 b^2}+\frac{3 i f^2 (e+f x) \operatorname{PolyLog}\left[4,-i e^{2 i (a+b x)}\right]}{8 b^3}-\frac{3 i f^2 (e+f x) \operatorname{PolyLog}\left[4, i e^{2 i (a+b x)}\right]}{8 b^3}-\frac{3 f^3 \operatorname{PolyLog}\left[5,-i e^{2 i (a+b x)}\right]}{16 b^4}+\frac{3 f^3 \operatorname{PolyLog}\left[5, i e^{2 i (a+b x)}\right]}{16 b^4}$$

Result (type 4, 654 leaves):

$$\frac{1}{4} x \left(4 e^3+6 e^2 f x+4 e f^2 x^2+f^3 x^3\right) \operatorname{ArcCoth}[\operatorname{Cot}[a+b x]]+\frac{1}{16 b^4}\Big(-8 b^4 e^3 x \operatorname{Log}\left[1-i e^{2 i (a+b x)}\right]-12 b^4 e^2 f x^2 \operatorname{Log}\left[1-i e^{2 i (a+b x)}\right]-8 b^4 e f^2 x^3 \operatorname{Log}\left[1-i e^{2 i (a+b x)}\right]-2 b^4 f^3 x^4 \operatorname{Log}\left[1-i e^{2 i (a+b x)}\right]+8 b^4 e^3 x \operatorname{Log}\left[1+i e^{2 i (a+b x)}\right]+12 b^4 e^2 f x^2 \operatorname{Log}\left[1+i e^{2 i (a+b x)}\right]+8 b^4 e f^2 x^3 \operatorname{Log}\left[1+i e^{2 i (a+b x)}\right]+2 b^4 f^3 x^4 \operatorname{Log}\left[1+i e^{2 i (a+b x)}\right]-4 i b^3 (e+f x)^3 \operatorname{PolyLog}\left[2,-i e^{2 i (a+b x)}\right]+4 i b^3 (e+f x)^3 \operatorname{PolyLog}\left[2, i e^{2 i (a+b x)}\right]+6 b^2 e^2 f \operatorname{PolyLog}\left[3,-i e^{2 i (a+b x)}\right]+12 b^2 e f^2 x \operatorname{PolyLog}\left[3,-i e^{2 i (a+b x)}\right]+6 b^2 f^3 x^2 \operatorname{PolyLog}\left[3,-i e^{2 i (a+b x)}\right]-6 b^2 e^2 f \operatorname{PolyLog}\left[3, i e^{2 i (a+b x)}\right]-12 b^2 e f^2 x \operatorname{PolyLog}\left[3, i e^{2 i (a+b x)}\right]-6 b^2 f^3 x^2 \operatorname{PolyLog}\left[3, i e^{2 i (a+b x)}\right]+6 i b e f^2 \operatorname{PolyLog}\left[4,-i e^{2 i (a+b x)}\right]+6 i b f^3 x \operatorname{PolyLog}\left[4,-i e^{2 i (a+b x)}\right]-6 i b e f^2 \operatorname{PolyLog}\left[4, i e^{2 i (a+b x)}\right]-6 i b f^3 x \operatorname{PolyLog}\left[4, i e^{2 i (a+b x)}\right]-3 f^3 \operatorname{PolyLog}\left[5,-i e^{2 i (a+b x)}\right]+3 f^3 \operatorname{PolyLog}\left[5, i e^{2 i (a+b x)}\right]\Big)$$

## Problem 255: Result more than twice size of optimal antiderivative.

$$\int \operatorname{ArcCoth}[c+d \operatorname{Cot}[a+b x]] \, dx$$

Optimal (type 4, 194 leaves, 7 steps):

$$x \operatorname{ArcCoth}[c+d \operatorname{Cot}[a+b x]]+\frac{1}{2} x \operatorname{Log}\left[1-\frac{(1-c-i d) e^{2 i a+2 i b x}}{1-c+i d}\right]-\frac{1}{2} x \operatorname{Log}\left[1-\frac{(1+c+i d) e^{2 i a+2 i b x}}{1+c-i d}\right]-\frac{i \operatorname{PolyLog}\left[2, \frac{(1-c-i d) e^{2 i a+2 i b x}}{1-c+i d}\right]}{4 b}+\frac{i \operatorname{PolyLog}\left[2, \frac{(1+c+i d) e^{2 i a+2 i b x}}{1+c-i d}\right]}{4 b}$$

Result (type 4, 4463 leaves):

$$x \operatorname{ArcCoth}[c+d \operatorname{Cot}[a+b x]]-\Bigg(d \Bigg(a \operatorname{Log}\left[-\operatorname{Sec}\left[\frac{1}{2}(a+b x)\right]^2 (d \operatorname{Cos}[a+b x]+(-1+c) \operatorname{Sin}[a+b x])\right]-a \operatorname{Log}\left[-\operatorname{Sec}\left[\frac{1}{2}(a+b x)\right]^2 (d \operatorname{Cos}[a+b x]+\operatorname{Sin}[a+b x]+c \operatorname{Sin}[a+b x])\right]-$$

$$(a+b\,x)\,\text{Log}\left[-\frac{-1+c+\sqrt{1-2\,c+c^2+d^2}}{d}+\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]\right]-$$

$$i\,\text{Log}\left[\frac{d\,\left(-i+\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]\right)}{-1+c-i\,d+\sqrt{1-2\,c+c^2+d^2}}\right]\,\text{Log}\left[-\frac{-1+c+\sqrt{1-2\,c+c^2+d^2}}{d}+\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]\right]+i\,\text{Log}\left[\frac{d\,\left(i+\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]\right)}{-1+c+i\,d+\sqrt{1-2\,c+c^2+d^2}}\right]$$

$$\text{Log}\left[-\frac{-1+c+\sqrt{1-2\,c+c^2+d^2}}{d}+\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]\right]+(a+b\,x)\,\text{Log}\left[-\frac{1+c+\sqrt{1+2\,c+c^2+d^2}}{d}+\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]\right]+$$

$$i\,\text{Log}\left[\frac{d\,\left(-i+\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]\right)}{1+c-i\,d+\sqrt{1+2\,c+c^2+d^2}}\right]\,\text{Log}\left[-\frac{1+c+\sqrt{1+2\,c+c^2+d^2}}{d}+\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]\right]-i\,\text{Log}\left[\frac{d\,\left(i+\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]\right)}{1+c+i\,d+\sqrt{1+2\,c+c^2+d^2}}\right]$$

$$\text{Log}\left[-\frac{1+c+\sqrt{1+2\,c+c^2+d^2}}{d}+\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]\right]-(a+b\,x)\,\text{Log}\left[\frac{1-c+\sqrt{1-2\,c+c^2+d^2}+d\,\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]}{d}\right]-$$

$$i\,\text{Log}\left[-\frac{d\,\left(-i+\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]\right)}{1-c+i\,d+\sqrt{1-2\,c+c^2+d^2}}\right]\,\text{Log}\left[\frac{1-c+\sqrt{1-2\,c+c^2+d^2}+d\,\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]}{d}\right]+i\,\text{Log}\left[-\frac{d\,\left(i+\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]\right)}{1-c-i\,d+\sqrt{1-2\,c+c^2+d^2}}\right]$$

$$\text{Log}\left[\frac{1-c+\sqrt{1-2\,c+c^2+d^2}+d\,\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]}{d}\right]+(a+b\,x)\,\text{Log}\left[\frac{-1-c+\sqrt{1+2\,c+c^2+d^2}+d\,\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]}{d}\right]+$$

$$i\,\text{Log}\left[-\frac{d\,\left(-i+\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]\right)}{-1-c+i\,d+\sqrt{1+2\,c+c^2+d^2}}\right]\,\text{Log}\left[\frac{-1-c+\sqrt{1+2\,c+c^2+d^2}+d\,\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]}{d}\right]-$$

$$i\,\text{Log}\left[-\frac{d\,\left(i+\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]\right)}{-1-c-i\,d+\sqrt{1+2\,c+c^2+d^2}}\right]\,\text{Log}\left[\frac{-1-c+\sqrt{1+2\,c+c^2+d^2}+d\,\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]}{d}\right]-$$

$$i\,\text{PolyLog}\left[2,\,\frac{-1+c+\sqrt{1-2\,c+c^2+d^2}-d\,\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]}{-1+c-i\,d+\sqrt{1-2\,c+c^2+d^2}}\right]+i\,\text{PolyLog}\left[2,\,\frac{-1+c+\sqrt{1-2\,c+c^2+d^2}-d\,\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]}{-1+c+i\,d+\sqrt{1-2\,c+c^2+d^2}}\right]-$$

$$i\,\text{PolyLog}\left[2,\,\frac{1+c-\sqrt{1+2\,c+c^2+d^2}-d\,\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]}{1+c+i\,d-\sqrt{1+2\,c+c^2+d^2}}\right]+i\,\text{PolyLog}\left[2,\,\frac{1+c+\sqrt{1+2\,c+c^2+d^2}-d\,\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]}{1+c-i\,d+\sqrt{1+2\,c+c^2+d^2}}\right]-$$

$$i\,\text{PolyLog}\left[2,\,\frac{1+c+\sqrt{1+2\,c+c^2+d^2}-d\,\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]}{1+c+i\,d+\sqrt{1+2\,c+c^2+d^2}}\right]+i\,\text{PolyLog}\left[2,\,\frac{1-c+\sqrt{1-2\,c+c^2+d^2}+d\,\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]}{1-c-i\,d+\sqrt{1-2\,c+c^2+d^2}}\right]-$$

$$i\,\text{PolyLog}\left[2,\,\frac{1-c+\sqrt{1-2\,c+c^2+d^2}+d\,\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]}{1-c+i\,d+\sqrt{1-2\,c+c^2+d^2}}\right]+i\,\text{PolyLog}\left[2,\,\frac{-1-c+\sqrt{1+2\,c+c^2+d^2}+d\,\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]}{-1-c+i\,d+\sqrt{1+2\,c+c^2+d^2}}\right]\Bigg)$$

$$\Bigg(\frac{2\,a}{b\,\left(1-c^2-d^2-\text{Cos}[2\,(a+b\,x)]+c^2\,\text{Cos}[2\,(a+b\,x)]-d^2\,\text{Cos}[2\,(a+b\,x)]-2\,c\,d\,\text{Sin}[2\,(a+b\,x)]\right)}-$$

$$\left.\left.\frac{2\,(a+b\,x)}{b\,\left(1-c^2-d^2-\text{Cos}[2\,(a+b\,x)]+c^2\,\text{Cos}[2\,(a+b\,x)]-d^2\,\text{Cos}[2\,(a+b\,x)]-2\,c\,d\,\text{Sin}[2\,(a+b\,x)]\right)}\right)\right)\Big/$$

$$\left(-\text{Log}\left[-\frac{-1+c+\sqrt{1-2\,c+c^2+d^2}}{d}+\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]\right]+\text{Log}\left[-\frac{1+c+\sqrt{1+2\,c+c^2+d^2}}{d}+\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]\right]-\right.$$

$$\text{Log}\left[\frac{1-c+\sqrt{1-2\,c+c^2+d^2}+d\,\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]}{d}\right]+\text{Log}\left[\frac{-1-c+\sqrt{1+2\,c+c^2+d^2}+d\,\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]}{d}\right]-$$

$$\frac{i\,\text{Log}\left[-\frac{-1+c+\sqrt{1-2\,c+c^2+d^2}}{d}+\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]\right]\,\text{Sec}\left[\frac{1}{2}\,(a+b\,x)\right]^2}{2\,\left(-i+\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]\right)}+\frac{i\,\text{Log}\left[-\frac{1+c+\sqrt{1+2\,c+c^2+d^2}}{d}+\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]\right]\,\text{Sec}\left[\frac{1}{2}\,(a+b\,x)\right]^2}{2\,\left(-i+\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]\right)}-$$

$$\frac{i\,\text{Log}\left[\frac{1-c+\sqrt{1-2\,c+c^2+d^2}+d\,\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]}{d}\right]\,\text{Sec}\left[\frac{1}{2}\,(a+b\,x)\right]^2}{2\,\left(-i+\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]\right)}+\frac{i\,\text{Log}\left[\frac{-1-c+\sqrt{1+2\,c+c^2+d^2}+d\,\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]}{d}\right]\,\text{Sec}\left[\frac{1}{2}\,(a+b\,x)\right]^2}{2\,\left(-i+\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]\right)}+$$

$$\frac{i\,\text{Log}\left[-\frac{-1+c+\sqrt{1-2\,c+c^2+d^2}}{d}+\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]\right]\,\text{Sec}\left[\frac{1}{2}\,(a+b\,x)\right]^2}{2\,\left(i+\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]\right)}-\frac{i\,\text{Log}\left[-\frac{1+c+\sqrt{1+2\,c+c^2+d^2}}{d}+\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]\right]\,\text{Sec}\left[\frac{1}{2}\,(a+b\,x)\right]^2}{2\,\left(i+\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]\right)}+$$

$$\frac{i\,\text{Log}\left[\frac{1-c+\sqrt{1-2\,c+c^2+d^2}+d\,\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]}{d}\right]\,\text{Sec}\left[\frac{1}{2}\,(a+b\,x)\right]^2}{2\,\left(i+\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]\right)}-\frac{i\,\text{Log}\left[\frac{-1-c+\sqrt{1+2\,c+c^2+d^2}+d\,\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]}{d}\right]\,\text{Sec}\left[\frac{1}{2}\,(a+b\,x)\right]^2}{2\,\left(i+\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]\right)}-$$

$$\frac{(a+b\,x)\,\text{Sec}\left[\frac{1}{2}\,(a+b\,x)\right]^2}{2\,\left(-\frac{-1+c+\sqrt{1-2\,c+c^2+d^2}}{d}+\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]\right)}-\frac{i\,\text{Log}\left[\frac{d\,\left(-i+\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]\right)}{-1+c-i\,d+\sqrt{1-2\,c+c^2+d^2}}\right]\,\text{Sec}\left[\frac{1}{2}\,(a+b\,x)\right]^2}{2\,\left(-\frac{-1+c+\sqrt{1-2\,c+c^2+d^2}}{d}+\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]\right)}+$$

$$\frac{i\,\text{Log}\left[\frac{d\,\left(i+\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]\right)}{-1+c+i\,d+\sqrt{1-2\,c+c^2+d^2}}\right]\,\text{Sec}\left[\frac{1}{2}\,(a+b\,x)\right]^2}{2\,\left(-\frac{-1+c+\sqrt{1-2\,c+c^2+d^2}}{d}+\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]\right)}+\frac{(a+b\,x)\,\text{Sec}\left[\frac{1}{2}\,(a+b\,x)\right]^2}{2\,\left(-\frac{1+c+\sqrt{1+2\,c+c^2+d^2}}{d}+\text{Tan}\left[\frac{1}{2}\,(a+b\,x)\right]\right)}+$$

$$\frac{\mathrm{i}\,\text{Log}\left[\frac{d\left(-\mathrm{i}+\text{Tan}\left[\frac{1}{2}(a+b x)\right]\right)}{1+c-\mathrm{i}\,d+\sqrt{1+2c+c^2+d^2}}\right]\text{Sec}\left[\frac{1}{2}(a+b x)\right]^2}{2\left(-\frac{1+c+\sqrt{1+2c+c^2+d^2}}{d}+\text{Tan}\left[\frac{1}{2}(a+b x)\right]\right)}-\frac{\mathrm{i}\,\text{Log}\left[\frac{d\left(\mathrm{i}+\text{Tan}\left[\frac{1}{2}(a+b x)\right]\right)}{1+c+\mathrm{i}\,d+\sqrt{1+2c+c^2+d^2}}\right]\text{Sec}\left[\frac{1}{2}(a+b x)\right]^2}{2\left(-\frac{1+c+\sqrt{1+2c+c^2+d^2}}{d}+\text{Tan}\left[\frac{1}{2}(a+b x)\right]\right)}-$$

$$\frac{\mathrm{i}\,d\,\text{Log}\left[1-\frac{-1+c+\sqrt{1-2c+c^2+d^2}-d\,\text{Tan}\left[\frac{1}{2}(a+b x)\right]}{-1+c-\mathrm{i}\,d+\sqrt{1-2c+c^2+d^2}}\right]\text{Sec}\left[\frac{1}{2}(a+b x)\right]^2}{2\left(-1+c+\sqrt{1-2c+c^2+d^2}-d\,\text{Tan}\left[\frac{1}{2}(a+b x)\right]\right)}+\frac{\mathrm{i}\,d\,\text{Log}\left[1-\frac{-1+c+\sqrt{1-2c+c^2+d^2}-d\,\text{Tan}\left[\frac{1}{2}(a+b x)\right]}{-1+c+\mathrm{i}\,d+\sqrt{1-2c+c^2+d^2}}\right]\text{Sec}\left[\frac{1}{2}(a+b x)\right]^2}{2\left(-1+c+\sqrt{1-2c+c^2+d^2}-d\,\text{Tan}\left[\frac{1}{2}(a+b x)\right]\right)}-$$

$$\frac{\mathrm{i}\,d\,\text{Log}\left[1-\frac{1+c-\sqrt{1+2c+c^2+d^2}-d\,\text{Tan}\left[\frac{1}{2}(a+b x)\right]}{1+c+\mathrm{i}\,d-\sqrt{1+2c+c^2+d^2}}\right]\text{Sec}\left[\frac{1}{2}(a+b x)\right]^2}{2\left(1+c-\sqrt{1+2c+c^2+d^2}-d\,\text{Tan}\left[\frac{1}{2}(a+b x)\right]\right)}+\frac{\mathrm{i}\,d\,\text{Log}\left[1-\frac{1+c+\sqrt{1+2c+c^2+d^2}-d\,\text{Tan}\left[\frac{1}{2}(a+b x)\right]}{1+c-\mathrm{i}\,d+\sqrt{1+2c+c^2+d^2}}\right]\text{Sec}\left[\frac{1}{2}(a+b x)\right]^2}{2\left(1+c+\sqrt{1+2c+c^2+d^2}-d\,\text{Tan}\left[\frac{1}{2}(a+b x)\right]\right)}-$$

$$\frac{\mathrm{i}\,d\,\text{Log}\left[1-\frac{1+c+\sqrt{1+2c+c^2+d^2}-d\,\text{Tan}\left[\frac{1}{2}(a+b x)\right]}{1+c+\mathrm{i}\,d+\sqrt{1+2c+c^2+d^2}}\right]\text{Sec}\left[\frac{1}{2}(a+b x)\right]^2}{2\left(1+c+\sqrt{1+2c+c^2+d^2}-d\,\text{Tan}\left[\frac{1}{2}(a+b x)\right]\right)}-\frac{d\,(a+b x)\,\text{Sec}\left[\frac{1}{2}(a+b x)\right]^2}{2\left(1-c+\sqrt{1-2c+c^2+d^2}+d\,\text{Tan}\left[\frac{1}{2}(a+b x)\right]\right)}-$$

$$\frac{\mathrm{i}\,d\,\text{Log}\left[-\frac{d\left(-\mathrm{i}+\text{Tan}\left[\frac{1}{2}(a+b x)\right]\right)}{1-c+\mathrm{i}\,d+\sqrt{1-2c+c^2+d^2}}\right]\text{Sec}\left[\frac{1}{2}(a+b x)\right]^2}{2\left(1-c+\sqrt{1-2c+c^2+d^2}+d\,\text{Tan}\left[\frac{1}{2}(a+b x)\right]\right)}+\frac{\mathrm{i}\,d\,\text{Log}\left[-\frac{d\left(\mathrm{i}+\text{Tan}\left[\frac{1}{2}(a+b x)\right]\right)}{1-c-\mathrm{i}\,d+\sqrt{1-2c+c^2+d^2}}\right]\text{Sec}\left[\frac{1}{2}(a+b x)\right]^2}{2\left(1-c+\sqrt{1-2c+c^2+d^2}+d\,\text{Tan}\left[\frac{1}{2}(a+b x)\right]\right)}-$$

$$\frac{\mathrm{i}\,d\,\text{Log}\left[1-\frac{1-c+\sqrt{1-2c+c^2+d^2}+d\,\text{Tan}\left[\frac{1}{2}(a+b x)\right]}{1-c-\mathrm{i}\,d+\sqrt{1-2c+c^2+d^2}}\right]\text{Sec}\left[\frac{1}{2}(a+b x)\right]^2}{2\left(1-c+\sqrt{1-2c+c^2+d^2}+d\,\text{Tan}\left[\frac{1}{2}(a+b x)\right]\right)}+\frac{\mathrm{i}\,d\,\text{Log}\left[1-\frac{1-c+\sqrt{1-2c+c^2+d^2}+d\,\text{Tan}\left[\frac{1}{2}(a+b x)\right]}{1-c+\mathrm{i}\,d+\sqrt{1-2c+c^2+d^2}}\right]\text{Sec}\left[\frac{1}{2}(a+b x)\right]^2}{2\left(1-c+\sqrt{1-2c+c^2+d^2}+d\,\text{Tan}\left[\frac{1}{2}(a+b x)\right]\right)}+$$

$$\frac{d\,(a+b x)\,\text{Sec}\left[\frac{1}{2}(a+b x)\right]^2}{2\left(-1-c+\sqrt{1+2c+c^2+d^2}+d\,\text{Tan}\left[\frac{1}{2}(a+b x)\right]\right)}+\frac{\mathrm{i}\,d\,\text{Log}\left[-\frac{d\left(-\mathrm{i}+\text{Tan}\left[\frac{1}{2}(a+b x)\right]\right)}{-1-c+\mathrm{i}\,d+\sqrt{1+2c+c^2+d^2}}\right]\text{Sec}\left[\frac{1}{2}(a+b x)\right]^2}{2\left(-1-c+\sqrt{1+2c+c^2+d^2}+d\,\text{Tan}\left[\frac{1}{2}(a+b x)\right]\right)}-$$

$$\frac{\mathrm{i}\,d\,\text{Log}\left[-\frac{d\left(\mathrm{i}+\text{Tan}\left[\frac{1}{2}(a+b x)\right]\right)}{-1-c-\mathrm{i}\,d+\sqrt{1+2c+c^2+d^2}}\right]\text{Sec}\left[\frac{1}{2}(a+b x)\right]^2}{2\left(-1-c+\sqrt{1+2c+c^2+d^2}+d\,\text{Tan}\left[\frac{1}{2}(a+b x)\right]\right)}-\frac{\mathrm{i}\,d\,\text{Log}\left[1-\frac{-1-c+\sqrt{1+2c+c^2+d^2}+d\,\text{Tan}\left[\frac{1}{2}(a+b x)\right]}{-1-c+\mathrm{i}\,d+\sqrt{1+2c+c^2+d^2}}\right]\text{Sec}\left[\frac{1}{2}(a+b x)\right]^2}{2\left(-1-c+\sqrt{1+2c+c^2+d^2}+d\,\text{Tan}\left[\frac{1}{2}(a+b x)\right]\right)}-\left(a\,\text{Cos}\left[\frac{1}{2}(a+b x)\right]^2\right.$$

$$\left.\left(-\text{Sec}\left[\frac{1}{2}(a+b x)\right]^2\left((-1+c)\,\text{Cos}[a+b x]-d\,\text{Sin}[a+b x]\right)-\text{Sec}\left[\frac{1}{2}(a+b x)\right]^2\left(d\,\text{Cos}[a+b x]+(-1+c)\,\text{Sin}[a+b x]\right)\text{Tan}\left[\frac{1}{2}(a+b x)\right]\right)\right)\Big/$$

$$\left(d\,\text{Cos}[a+b x]+(-1+c)\,\text{Sin}[a+b x]\right)+\left(a\,\text{Cos}\left[\frac{1}{2}(a+b x)\right]^2\left(-\text{Sec}\left[\frac{1}{2}(a+b x)\right]^2\left(\text{Cos}[a+b x]+c\,\text{Cos}[a+b x]-d\,\text{Sin}[a+b x]\right)-\right.\right.$$

$$\mathrm{Sec}\left[\frac{1}{2}(a+b x)\right]^2 (d\,\mathrm{Cos}[a+b x]+\mathrm{Sin}[a+b x]+c\,\mathrm{Sin}[a+b x])\,\mathrm{Tan}\left[\frac{1}{2}(a+b x)\right]\Big)\Big) \Big/ (d\,\mathrm{Cos}[a+b x]+\mathrm{Sin}[a+b x]+c\,\mathrm{Sin}[a+b x])\Bigg)$$

## Problem 265: Result unnecessarily involves higher level functions.

$$\int \frac{(a+b\,\mathrm{ArcCoth}[c\,x^n])\,(d+e\,\mathrm{Log}[f\,x^m])}{x}\,dx$$

Optimal (type 4, 160 leaves, 11 steps):

$$a\,d\,\mathrm{Log}[x]+\frac{a\,e\,\mathrm{Log}[f\,x^m]^2}{2\,m}+\frac{b\,d\,\mathrm{PolyLog}\left[2,-\frac{x^{-n}}{c}\right]}{2\,n}+\frac{b\,e\,\mathrm{Log}[f\,x^m]\,\mathrm{PolyLog}\left[2,-\frac{x^{-n}}{c}\right]}{2\,n}-\frac{b\,d\,\mathrm{PolyLog}\left[2,\frac{x^{-n}}{c}\right]}{2\,n}-\frac{b\,e\,\mathrm{Log}[f\,x^m]\,\mathrm{PolyLog}\left[2,\frac{x^{-n}}{c}\right]}{2\,n}+\frac{b\,e\,m\,\mathrm{PolyLog}\left[3,-\frac{x^{-n}}{c}\right]}{2\,n^2}-\frac{b\,e\,m\,\mathrm{PolyLog}\left[3,\frac{x^{-n}}{c}\right]}{2\,n^2}$$

Result (type 5, 131 leaves):

$$-\frac{b\,c\,e\,m\,x^n\,\mathrm{HypergeometricPFQ}\left[\left\{\frac{1}{2},\frac{1}{2},\frac{1}{2},1\right\},\left\{\frac{3}{2},\frac{3}{2},\frac{3}{2}\right\},c^2 x^{2n}\right]}{n^2}+\frac{b\,c\,x^n\,\mathrm{HypergeometricPFQ}\left[\left\{\frac{1}{2},\frac{1}{2},1\right\},\left\{\frac{3}{2},\frac{3}{2}\right\},c^2 x^{2n}\right]\,(d+e\,\mathrm{Log}[f\,x^m])}{n}-\frac{1}{2}\left(a+b\,\mathrm{ArcCoth}[c\,x^n]-b\,\mathrm{ArcTanh}[c\,x^n]\right)\mathrm{Log}[x]\,\left(e\,m\,\mathrm{Log}[x]-2\,(d+e\,\mathrm{Log}[f\,x^m])\right)$$

## Problem 269: Unable to integrate problem.

$$\int \frac{(a+b\,\mathrm{ArcCoth}[c\,x])\,(d+e\,\mathrm{Log}[1-c^2 x^2])}{x}\,dx$$

Optimal (type 4, 381 leaves, 21 steps):

$$-\frac{1}{2} b e \text{Log}\left[1+\frac{1}{c x}\right]^2 \text{Log}\left[-\frac{1}{c x}\right]+\frac{1}{2} b e \text{Log}\left[1-\frac{1}{c x}\right]^2 \text{Log}\left[\frac{1}{c x}\right]+a d \text{Log}[x]-b e \text{Log}\left[\frac{c+\frac{1}{x}}{c}\right] \text{PolyLog}\left[2,\frac{c+\frac{1}{x}}{c}\right]+ b e \text{Log}\left[1-\frac{1}{c x}\right] \text{PolyLog}\left[2,1-\frac{1}{c x}\right]+\frac{1}{2} b d \text{PolyLog}\left[2,-\frac{1}{c x}\right]+\frac{1}{2} b e \text{Log}\left[-c^2 x^2\right] \text{PolyLog}\left[2,-\frac{1}{c x}\right]- \frac{1}{2} b e \left(\text{Log}\left[1-\frac{1}{c x}\right]+\text{Log}\left[1+\frac{1}{c x}\right]+\text{Log}\left[-c^2 x^2\right]-\text{Log}\left[1-c^2 x^2\right]\right) \text{PolyLog}\left[2,-\frac{1}{c x}\right]-\frac{1}{2} b d \text{PolyLog}\left[2,\frac{1}{c x}\right]- \frac{1}{2} b e \text{Log}\left[-c^2 x^2\right] \text{PolyLog}\left[2,\frac{1}{c x}\right]+\frac{1}{2} b e \left(\text{Log}\left[1-\frac{1}{c x}\right]+\text{Log}\left[1+\frac{1}{c x}\right]+\text{Log}\left[-c^2 x^2\right]-\text{Log}\left[1-c^2 x^2\right]\right) \text{PolyLog}\left[2,\frac{1}{c x}\right]- \frac{1}{2} a e \text{PolyLog}\left[2,c^2 x^2\right]+b e \text{PolyLog}\left[3,\frac{c+\frac{1}{x}}{c}\right]-b e \text{PolyLog}\left[3,1-\frac{1}{c x}\right]+b e \text{PolyLog}\left[3,-\frac{1}{c x}\right]-b e \text{PolyLog}\left[3,\frac{1}{c x}\right]$$

Result (type 8, 29 leaves):

$$\int \frac{(a+b \text{ArcCoth}[c x]) \left(d+e \text{Log}\left[1-c^2 x^2\right]\right)}{x} \, dx$$

## Problem 275: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+b \text{ArcCoth}[c x]) \left(d+e \text{Log}\left[1-c^2 x^2\right]\right)}{x^2} \, dx$$

Optimal (type 4, 105 leaves, 6 steps):

$$-\frac{c e (a+b \text{ArcCoth}[c x])^2}{b}-\frac{(a+b \text{ArcCoth}[c x]) \left(d+e \text{Log}\left[1-c^2 x^2\right]\right)}{x}+\frac{1}{2} b c \left(d+e \text{Log}\left[1-c^2 x^2\right]\right) \text{Log}\left[1-\frac{1}{1-c^2 x^2}\right]-\frac{1}{2} b c e \text{PolyLog}\left[2,\frac{1}{1-c^2 x^2}\right]$$

Result (type 4, 332 leaves):

$$-\frac{1}{4 x}\left(4 a d+4 b d \text{ArcCoth}[c x]+4 b c e x \text{ArcCoth}[c x]^2+8 a c e x \text{ArcTanh}[c x]-4 b c d x \text{Log}[x]-b c e x \text{Log}\left[-\frac{1}{c}+x\right]^2- b c e x \text{Log}\left[\frac{1}{c}+x\right]^2-2 b c e x \text{Log}\left[\frac{1}{c}+x\right] \text{Log}\left[\frac{1}{2} (1-c x)\right]+4 b c e x \text{Log}[x] \text{Log}[1-c x]-2 b c e x \text{Log}\left[-\frac{1}{c}+x\right] \text{Log}\left[\frac{1}{2} (1+c x)\right]+ 4 b c e x \text{Log}[x] \text{Log}[1+c x]+4 a e \text{Log}\left[1-c^2 x^2\right]+2 b c d x \text{Log}\left[1-c^2 x^2\right]+4 b e \text{ArcCoth}[c x] \text{Log}\left[1-c^2 x^2\right]- 4 b c e x \text{Log}[x] \text{Log}\left[1-c^2 x^2\right]+2 b c e x \text{Log}\left[-\frac{1}{c}+x\right] \text{Log}\left[1-c^2 x^2\right]+2 b c e x \text{Log}\left[\frac{1}{c}+x\right] \text{Log}\left[1-c^2 x^2\right]+ 4 b c e x \text{PolyLog}[2,-c x]+4 b c e x \text{PolyLog}[2,c x]-2 b c e x \text{PolyLog}\left[2,\frac{1}{2}-\frac{c x}{2}\right]-2 b c e x \text{PolyLog}\left[2,\frac{1}{2} (1+c x)\right]\right)$$

## Problem 276: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+b\,\text{ArcCoth}[c\,x])\,(d+e\,\text{Log}[1-c^2 x^2])}{x^4}\,dx$$

Optimal (type 4, 197 leaves, 15 steps):

$$\frac{2 c^2 e\,(a+b\,\text{ArcCoth}[c\,x])}{3 x}-\frac{c^3 e\,(a+b\,\text{ArcCoth}[c\,x])^2}{3 b}-b c^3 e\,\text{Log}[x]+\frac{1}{3} b c^3 e\,\text{Log}[1-c^2 x^2]-\frac{b c\,(1-c^2 x^2)\,(d+e\,\text{Log}[1-c^2 x^2])}{6 x^2}-\frac{(a+b\,\text{ArcCoth}[c\,x])\,(d+e\,\text{Log}[1-c^2 x^2])}{3 x^3}+\frac{1}{6} b c^3\,(d+e\,\text{Log}[1-c^2 x^2])\,\text{Log}\left[1-\frac{1}{1-c^2 x^2}\right]-\frac{1}{6} b c^3 e\,\text{PolyLog}\left[2,\frac{1}{1-c^2 x^2}\right]$$

Result (type 4, 457 leaves):

$$\frac{1}{6}\left(-\frac{2 a d}{x^3}-\frac{b c d}{x^2}+\frac{4 a c^2 e}{x}-\frac{2 b d\,\text{ArcCoth}[c\,x]}{x^3}+\frac{4 b c^2 e\,\text{ArcCoth}[c\,x]}{x}-2 b c^3 e\,\text{ArcCoth}[c\,x]^2-4 a c^3 e\,\text{ArcTanh}[c\,x]-4 b c^3 e\,\text{Log}\left[\frac{1}{\sqrt{1-\frac{1}{c^2 x^2}}}\right]+\right.$$
$$2 b c^3 d\,\text{Log}[x]-2 b c^3 e\,\text{Log}[x]+\frac{1}{2} b c^3 e\,\text{Log}\left[-\frac{1}{c}+x\right]^2+\frac{1}{2} b c^3 e\,\text{Log}\left[\frac{1}{c}+x\right]^2+b c^3 e\,\text{Log}\left[\frac{1}{c}+x\right]\text{Log}\left[\frac{1}{2}(1-c\,x)\right]-2 b c^3 e\,\text{Log}[x]\,\text{Log}[1-c\,x]+$$
$$b c^3 e\,\text{Log}\left[-\frac{1}{c}+x\right]\text{Log}\left[\frac{1}{2}(1+c\,x)\right]-2 b c^3 e\,\text{Log}[x]\,\text{Log}[1+c\,x]-b c^3 d\,\text{Log}[1-c^2 x^2]+b c^3 e\,\text{Log}[1-c^2 x^2]-\frac{2 a e\,\text{Log}[1-c^2 x^2]}{x^3}-$$
$$\frac{b c e\,\text{Log}[1-c^2 x^2]}{x^2}-\frac{2 b e\,\text{ArcCoth}[c\,x]\,\text{Log}[1-c^2 x^2]}{x^3}+2 b c^3 e\,\text{Log}[x]\,\text{Log}[1-c^2 x^2]-b c^3 e\,\text{Log}\left[-\frac{1}{c}+x\right]\text{Log}[1-c^2 x^2]-$$
$$\left.b c^3 e\,\text{Log}\left[\frac{1}{c}+x\right]\text{Log}[1-c^2 x^2]-2 b c^3 e\,\text{PolyLog}[2,-c\,x]-2 b c^3 e\,\text{PolyLog}[2,c\,x]+b c^3 e\,\text{PolyLog}\left[2,\frac{1}{2}-\frac{c\,x}{2}\right]+b c^3 e\,\text{PolyLog}\left[2,\frac{1}{2}(1+c\,x)\right]\right)$$

## Problem 277: Unable to integrate problem.

$$\int \frac{(a+b\,\text{ArcCoth}[c\,x])\,(d+e\,\text{Log}[1-c^2 x^2])}{x^6}\,dx$$

Optimal (type 4, 256 leaves, 24 steps):

$$\frac{7 b c^3 e}{60 x^2}+\frac{2 c^2 e (a+b \text{ArcCoth}[c x])}{15 x^3}+\frac{2 c^4 e (a+b \text{ArcCoth}[c x])}{5 x}-\frac{c^5 e (a+b \text{ArcCoth}[c x])^2}{5 b}-$$
$$\frac{5}{6} b c^5 e \text{Log}[x]+\frac{19}{60} b c^5 e \text{Log}[1-c^2 x^2]-\frac{b c (d+e \text{Log}[1-c^2 x^2])}{20 x^4}-\frac{b c^3 (1-c^2 x^2) (d+e \text{Log}[1-c^2 x^2])}{10 x^2}-$$
$$\frac{(a+b \text{ArcCoth}[c x]) (d+e \text{Log}[1-c^2 x^2])}{5 x^5}+\frac{1}{10} b c^5 (d+e \text{Log}[1-c^2 x^2]) \text{Log}\left[1-\frac{1}{1-c^2 x^2}\right]-\frac{1}{10} b c^5 e \text{PolyLog}\left[2,\frac{1}{1-c^2 x^2}\right]$$

Result (type 8, 29 leaves):

$$\int \frac{(a+b \text{ArcCoth}[c x]) (d+e \text{Log}[1-c^2 x^2])}{x^6} \, dx$$

## Problem 278: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int x (a+b \text{ArcCoth}[c x]) (d+e \text{Log}[f+g x^2]) \, dx$$

Optimal (type 4, 512 leaves, 22 steps):

$$\frac{b (d-e) x}{2 c}-\frac{b e x}{c}+\frac{1}{2} d x^2 (a+b \text{ArcCoth}[c x])-\frac{1}{2} e x^2 (a+b \text{ArcCoth}[c x])+\frac{b e \sqrt{f} \text{ArcTan}\left[\frac{\sqrt{g} x}{\sqrt{f}}\right]}{c \sqrt{g}}-\frac{b (d-e) \text{ArcTanh}[c x]}{2 c^2}-$$
$$\frac{b e (c^2 f+g) \text{ArcTanh}[c x] \text{Log}\left[\frac{2}{1+c x}\right]}{c^2 g}+\frac{b e (c^2 f+g) \text{ArcTanh}[c x] \text{Log}\left[\frac{2 c (\sqrt{-f}-\sqrt{g} x)}{(c \sqrt{-f}-\sqrt{g}) (1+c x)}\right]}{2 c^2 g}+\frac{b e (c^2 f+g) \text{ArcTanh}[c x] \text{Log}\left[\frac{2 c (\sqrt{-f}+\sqrt{g} x)}{(c \sqrt{-f}+\sqrt{g}) (1+c x)}\right]}{2 c^2 g}+$$
$$\frac{b e x \text{Log}[f+g x^2]}{2 c}+\frac{e (f+g x^2) (a+b \text{ArcCoth}[c x]) \text{Log}[f+g x^2]}{2 g}-\frac{b e (c^2 f+g) \text{ArcTanh}[c x] \text{Log}[f+g x^2]}{2 c^2 g}+$$
$$\frac{b e (c^2 f+g) \text{PolyLog}\left[2,1-\frac{2}{1+c x}\right]}{2 c^2 g}-\frac{b e (c^2 f+g) \text{PolyLog}\left[2,1-\frac{2 c (\sqrt{-f}-\sqrt{g} x)}{(c \sqrt{-f}-\sqrt{g}) (1+c x)}\right]}{4 c^2 g}-\frac{b e (c^2 f+g) \text{PolyLog}\left[2,1-\frac{2 c (\sqrt{-f}+\sqrt{g} x)}{(c \sqrt{-f}+\sqrt{g}) (1+c x)}\right]}{4 c^2 g}$$

Result (type 4, 1128 leaves):

$$\frac{1}{4 c^2 g}\Bigg(2 b c d g x-6 b c e g x+2 a c^2 d g x^2-2 a c^2 e g x^2-2 b d g \text{ArcCoth}[c x]+2 b e g \text{ArcCoth}[c x]+2 b c^2 d g x^2 \text{ArcCoth}[c x]-$$
$$2 b c^2 e g x^2 \text{ArcCoth}[c x]+4 b c e \sqrt{f} \sqrt{g} \text{ArcTan}\left[\frac{\sqrt{g} x}{\sqrt{f}}\right]-4 i b c^2 e f \text{ArcSin}\left[\sqrt{\frac{g}{c^2 f+g}}\right] \text{ArcTanh}\left[\frac{c f}{\sqrt{-c^2 f g} x}\right]-$$

$$4\,\mathrm{i}\,b\,e\,g\,\text{ArcSin}\left[\sqrt{\frac{g}{c^2 f+g}}\right]\text{ArcTanh}\left[\frac{c\,f}{\sqrt{-c^2 f g}\;x}\right]-4\,b\,c^2\,e\,f\,\text{ArcCoth}[c\,x]\,\text{Log}\left[1-e^{-2\,\text{ArcCoth}[c\,x]}\right]-4\,b\,e\,g\,\text{ArcCoth}[c\,x]\,\text{Log}\left[1-e^{-2\,\text{ArcCoth}[c\,x]}\right]+$$

$$2\,b\,c^2\,e\,f\,\text{ArcCoth}[c\,x]\,\text{Log}\left[\frac{e^{-2\,\text{ArcCoth}[c\,x]}\left(c^2\left(-1+e^{2\,\text{ArcCoth}[c\,x]}\right)f+g+e^{2\,\text{ArcCoth}[c\,x]}\,g-2\sqrt{-c^2 f g}\right)}{c^2 f+g}\right]+$$

$$2\,b\,e\,g\,\text{ArcCoth}[c\,x]\,\text{Log}\left[\frac{e^{-2\,\text{ArcCoth}[c\,x]}\left(c^2\left(-1+e^{2\,\text{ArcCoth}[c\,x]}\right)f+g+e^{2\,\text{ArcCoth}[c\,x]}\,g-2\sqrt{-c^2 f g}\right)}{c^2 f+g}\right]-$$

$$2\,\mathrm{i}\,b\,c^2\,e\,f\,\text{ArcSin}\left[\sqrt{\frac{g}{c^2 f+g}}\right]\text{Log}\left[\frac{e^{-2\,\text{ArcCoth}[c\,x]}\left(c^2\left(-1+e^{2\,\text{ArcCoth}[c\,x]}\right)f+g+e^{2\,\text{ArcCoth}[c\,x]}\,g-2\sqrt{-c^2 f g}\right)}{c^2 f+g}\right]-$$

$$2\,\mathrm{i}\,b\,e\,g\,\text{ArcSin}\left[\sqrt{\frac{g}{c^2 f+g}}\right]\text{Log}\left[\frac{e^{-2\,\text{ArcCoth}[c\,x]}\left(c^2\left(-1+e^{2\,\text{ArcCoth}[c\,x]}\right)f+g+e^{2\,\text{ArcCoth}[c\,x]}\,g-2\sqrt{-c^2 f g}\right)}{c^2 f+g}\right]+$$

$$2\,b\,c^2\,e\,f\,\text{ArcCoth}[c\,x]\,\text{Log}\left[\frac{e^{-2\,\text{ArcCoth}[c\,x]}\left(c^2\left(-1+e^{2\,\text{ArcCoth}[c\,x]}\right)f+g+e^{2\,\text{ArcCoth}[c\,x]}\,g+2\sqrt{-c^2 f g}\right)}{c^2 f+g}\right]+$$

$$2\,b\,e\,g\,\text{ArcCoth}[c\,x]\,\text{Log}\left[\frac{e^{-2\,\text{ArcCoth}[c\,x]}\left(c^2\left(-1+e^{2\,\text{ArcCoth}[c\,x]}\right)f+g+e^{2\,\text{ArcCoth}[c\,x]}\,g+2\sqrt{-c^2 f g}\right)}{c^2 f+g}\right]+$$

$$2\,\mathrm{i}\,b\,c^2\,e\,f\,\text{ArcSin}\left[\sqrt{\frac{g}{c^2 f+g}}\right]\text{Log}\left[\frac{e^{-2\,\text{ArcCoth}[c\,x]}\left(c^2\left(-1+e^{2\,\text{ArcCoth}[c\,x]}\right)f+g+e^{2\,\text{ArcCoth}[c\,x]}\,g+2\sqrt{-c^2 f g}\right)}{c^2 f+g}\right]+$$

$$2\,\mathrm{i}\,b\,e\,g\,\text{ArcSin}\left[\sqrt{\frac{g}{c^2 f+g}}\right]\text{Log}\left[\frac{e^{-2\,\text{ArcCoth}[c\,x]}\left(c^2\left(-1+e^{2\,\text{ArcCoth}[c\,x]}\right)f+g+e^{2\,\text{ArcCoth}[c\,x]}\,g+2\sqrt{-c^2 f g}\right)}{c^2 f+g}\right]+2\,a\,c^2\,e\,f\,\text{Log}\left[f+g\,x^2\right]+$$

$$2\,b\,c\,e\,g\,x\,\text{Log}\left[f+g\,x^2\right]+2\,a\,c^2\,e\,g\,x^2\,\text{Log}\left[f+g\,x^2\right]-2\,b\,e\,g\,\text{ArcCoth}[c\,x]\,\text{Log}\left[f+g\,x^2\right]+2\,b\,c^2\,e\,g\,x^2\,\text{ArcCoth}[c\,x]\,\text{Log}\left[f+g\,x^2\right]+$$

$$2\,b\,e\left(c^2 f+g\right)\text{PolyLog}\left[2,\,e^{-2\,\text{ArcCoth}[c\,x]}\right]-b\,e\left(c^2 f+g\right)\text{PolyLog}\left[2,\,\frac{e^{-2\,\text{ArcCoth}[c\,x]}\left(c^2 f-g+2\sqrt{-c^2 f g}\right)}{c^2 f+g}\right]-$$

$$b\,c^2\,e\,f\,\text{PolyLog}\left[2,\,-\frac{e^{-2\,\text{ArcCoth}[c\,x]}\left(-c^2 f+g+2\sqrt{-c^2 f g}\right)}{c^2 f+g}\right]-b\,e\,g\,\text{PolyLog}\left[2,\,-\frac{e^{-2\,\text{ArcCoth}[c\,x]}\left(-c^2 f+g+2\sqrt{-c^2 f g}\right)}{c^2 f+g}\right]\Bigg)$$

## Problem 279: Result more than twice size of optimal antiderivative.

$$\int (a+b\,\text{ArcCoth}[c\,x])\,(d+e\,\text{Log}[f+g\,x^2])\,dx$$

Optimal (type 4, 546 leaves, 38 steps):

$$-2aex-2bex\,\text{ArcCoth}[cx]+\frac{2ae\sqrt{f}\,\text{ArcTan}\left[\frac{\sqrt{g}\,x}{\sqrt{f}}\right]}{\sqrt{g}}-\frac{be\sqrt{f}\,\text{ArcTan}\left[\frac{\sqrt{g}\,x}{\sqrt{f}}\right]\text{Log}\left[1-\frac{1}{cx}\right]}{\sqrt{g}}+\frac{be\sqrt{f}\,\text{ArcTan}\left[\frac{\sqrt{g}\,x}{\sqrt{f}}\right]\text{Log}\left[1+\frac{1}{cx}\right]}{\sqrt{g}}+$$

$$\frac{be\sqrt{f}\,\text{ArcTan}\left[\frac{\sqrt{g}\,x}{\sqrt{f}}\right]\text{Log}\left[-\frac{2\sqrt{f}\sqrt{g}\,(1-cx)}{\left(i c\sqrt{f}-\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}\,x\right)}\right]}{\sqrt{g}}-\frac{be\sqrt{f}\,\text{ArcTan}\left[\frac{\sqrt{g}\,x}{\sqrt{f}}\right]\text{Log}\left[\frac{2\sqrt{f}\sqrt{g}\,(1+cx)}{\left(i c\sqrt{f}+\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}\,x\right)}\right]}{\sqrt{g}}-$$

$$\frac{be\,\text{Log}[1-c^2x^2]}{c}+x\,(a+b\,\text{ArcCoth}[cx])\,(d+e\,\text{Log}[f+gx^2])+\frac{b\,\text{Log}\left[\frac{g(1-c^2x^2)}{c^2f+g}\right](d+e\,\text{Log}[f+gx^2])}{2c}+$$

$$\frac{be\,\text{PolyLog}\left[2,\frac{c^2(f+gx^2)}{c^2f+g}\right]}{2c}-\frac{i be\sqrt{f}\,\text{PolyLog}\left[2,1+\frac{2\sqrt{f}\sqrt{g}\,(1-cx)}{\left(i c\sqrt{f}-\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}\,x\right)}\right]}{2\sqrt{g}}+\frac{i be\sqrt{f}\,\text{PolyLog}\left[2,1-\frac{2\sqrt{f}\sqrt{g}\,(1+cx)}{\left(i c\sqrt{f}+\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}\,x\right)}\right]}{2\sqrt{g}}$$

Result (type 4, 1287 leaves):

$$adx-2aex+bdx\,\text{ArcCoth}[cx]+\frac{2ae\sqrt{f}\,\text{ArcTan}\left[\frac{\sqrt{g}\,x}{\sqrt{f}}\right]}{\sqrt{g}}+\frac{bd\,\text{Log}[1-c^2x^2]}{2c}+aex\,\text{Log}[f+gx^2]+$$

$$be\left(x\,\text{ArcCoth}[cx]+\frac{\text{Log}[1-c^2x^2]}{2c}\right)\text{Log}[f+gx^2]+\frac{1}{2c}be\left(-4cx\,\text{ArcCoth}[cx]+4\,\text{Log}\left[\frac{1}{c\sqrt{1-\frac{1}{c^2x^2}}\,x}\right]+\right.$$

$$\frac{1}{g}\sqrt{c^2fg}\left(-2i\,\text{ArcCos}\left[\frac{c^2f-g}{c^2f+g}\right]\text{ArcTan}\left[\frac{\sqrt{c^2fg}}{cgx}\right]+4\,\text{ArcCoth}[cx]\,\text{ArcTan}\left[\frac{cgx}{\sqrt{c^2fg}}\right]-\left(\text{ArcCos}\left[\frac{c^2f-g}{c^2f+g}\right]+2\,\text{ArcTan}\left[\frac{\sqrt{c^2fg}}{cgx}\right]\right)\right.$$

$$\text{Log}\left[\frac{2ig\left(ic^2f+\sqrt{c^2fg}\right)\left(-1+\frac{1}{cx}\right)}{(c^2f+g)\left(g+\frac{i\sqrt{c^2fg}}{cx}\right)}\right]-\left(\text{ArcCos}\left[\frac{c^2f-g}{c^2f+g}\right]-2\,\text{ArcTan}\left[\frac{\sqrt{c^2fg}}{cgx}\right]\right)\text{Log}\left[\frac{2g\left(c^2f+i\sqrt{c^2fg}\right)\left(1+\frac{1}{cx}\right)}{(c^2f+g)\left(g+\frac{i\sqrt{c^2fg}}{cx}\right)}\right]+$$

$$\left(\text{ArcCos}\left[\frac{c^2 f-g}{c^2 f+g}\right]+2\left(\text{ArcTan}\left[\frac{\sqrt{c^2 f g}}{c g x}\right]+\text{ArcTan}\left[\frac{c g x}{\sqrt{c^2 f g}}\right]\right)\right)\text{Log}\left[\frac{\sqrt{2}\, e^{-\text{ArcCoth}[c x]}\sqrt{c^2 f g}}{\sqrt{c^2 f+g}\sqrt{-c^2 f+g+\left(c^2 f+g\right)\text{Cosh}[2\,\text{ArcCoth}[c x]]}}\right]+$$

$$\left(\text{ArcCos}\left[\frac{c^2 f-g}{c^2 f+g}\right]-2\left(\text{ArcTan}\left[\frac{\sqrt{c^2 f g}}{c g x}\right]+\text{ArcTan}\left[\frac{c g x}{\sqrt{c^2 f g}}\right]\right)\right)\text{Log}\left[\frac{\sqrt{2}\, e^{\text{ArcCoth}[c x]}\sqrt{c^2 f g}}{\sqrt{c^2 f+g}\sqrt{-c^2 f+g+\left(c^2 f+g\right)\text{Cosh}[2\,\text{ArcCoth}[c x]]}}\right]+$$

$$i\left(-\text{PolyLog}\left[2,\frac{\left(-c^2 f+g+2 i\sqrt{c^2 f g}\right)\left(g-\frac{i\sqrt{c^2 f g}}{c x}\right)}{\left(c^2 f+g\right)\left(g+\frac{i\sqrt{c^2 f g}}{c x}\right)}\right]+\text{PolyLog}\left[2,\frac{\left(c^2 f-g+2 i\sqrt{c^2 f g}\right)\left(i g+\frac{\sqrt{c^2 f g}}{c x}\right)}{\left(c^2 f+g\right)\left(-i g+\frac{\sqrt{c^2 f g}}{c x}\right)}\right]\right)\Bigg)\Bigg)-$$

$$\frac{1}{c} b e g\left(\frac{\left(-\text{Log}\left[-\frac{1}{c}+x\right]-\text{Log}\left[\frac{1}{c}+x\right]+\text{Log}\left[1-c^2 x^2\right]\right)\text{Log}\left[f+g x^2\right]}{2 g}+\frac{\text{Log}\left[-\frac{1}{c}+x\right]\text{Log}\left[1-\frac{\sqrt{g}\left(-\frac{1}{c}+x\right)}{-i\sqrt{f}-\frac{\sqrt{g}}{c}}\right]+\text{PolyLog}\left[2,\frac{\sqrt{g}\left(-\frac{1}{c}+x\right)}{-i\sqrt{f}-\frac{\sqrt{g}}{c}}\right]}{2 g}+\right.$$

$$\frac{\text{Log}\left[-\frac{1}{c}+x\right]\text{Log}\left[1-\frac{\sqrt{g}\left(-\frac{1}{c}+x\right)}{i\sqrt{f}-\frac{\sqrt{g}}{c}}\right]+\text{PolyLog}\left[2,\frac{\sqrt{g}\left(-\frac{1}{c}+x\right)}{i\sqrt{f}-\frac{\sqrt{g}}{c}}\right]}{2 g}+$$

$$\left.\frac{\text{Log}\left[\frac{1}{c}+x\right]\text{Log}\left[1-\frac{\sqrt{g}\left(\frac{1}{c}+x\right)}{-i\sqrt{f}+\frac{\sqrt{g}}{c}}\right]+\text{PolyLog}\left[2,\frac{\sqrt{g}\left(\frac{1}{c}+x\right)}{-i\sqrt{f}+\frac{\sqrt{g}}{c}}\right]}{2 g}+\frac{\text{Log}\left[\frac{1}{c}+x\right]\text{Log}\left[1-\frac{\sqrt{g}\left(\frac{1}{c}+x\right)}{i\sqrt{f}+\frac{\sqrt{g}}{c}}\right]+\text{PolyLog}\left[2,\frac{\sqrt{g}\left(\frac{1}{c}+x\right)}{i\sqrt{f}+\frac{\sqrt{g}}{c}}\right]}{2 g}\right)$$

## Problem 281: Result more than twice size of optimal antiderivative.

$$\int\frac{\left(a+b\,\text{ArcCoth}[c x]\right)\left(d+e\,\text{Log}\left[f+g x^2\right]\right)}{x^2}\,dx$$

Optimal (type 4, 560 leaves, 38 steps) :

$$\frac{2\,a\,e\,\sqrt{g}\,\text{ArcTan}\left[\frac{\sqrt{g}\,x}{\sqrt{f}}\right]}{\sqrt{f}}-\frac{b\,e\,\sqrt{g}\,\text{ArcTan}\left[\frac{\sqrt{g}\,x}{\sqrt{f}}\right]\text{Log}\left[1-\frac{1}{c\,x}\right]}{\sqrt{f}}+\frac{b\,e\,\sqrt{g}\,\text{ArcTan}\left[\frac{\sqrt{g}\,x}{\sqrt{f}}\right]\text{Log}\left[1+\frac{1}{c\,x}\right]}{\sqrt{f}}+$$

$$\frac{b\,e\,\sqrt{g}\,\text{ArcTan}\left[\frac{\sqrt{g}\,x}{\sqrt{f}}\right]\text{Log}\left[-\frac{2\sqrt{f}\,\sqrt{g}\,(1-c\,x)}{\left(i\,c\,\sqrt{f}-\sqrt{g}\right)\left(\sqrt{f}-i\,\sqrt{g}\,x\right)}\right]}{\sqrt{f}}-\frac{b\,e\,\sqrt{g}\,\text{ArcTan}\left[\frac{\sqrt{g}\,x}{\sqrt{f}}\right]\text{Log}\left[\frac{2\sqrt{f}\,\sqrt{g}\,(1+c\,x)}{\left(i\,c\,\sqrt{f}+\sqrt{g}\right)\left(\sqrt{f}-i\,\sqrt{g}\,x\right)}\right]}{\sqrt{f}}-$$

$$\frac{(a+b\,\text{ArcCoth}[c\,x])\left(d+e\,\text{Log}\left[f+g\,x^2\right]\right)}{x}+\frac{1}{2}\,b\,c\,\text{Log}\left[-\frac{g\,x^2}{f}\right]\left(d+e\,\text{Log}\left[f+g\,x^2\right]\right)-$$

$$\frac{1}{2}\,b\,c\,\text{Log}\left[\frac{g\left(1-c^2\,x^2\right)}{c^2\,f+g}\right]\left(d+e\,\text{Log}\left[f+g\,x^2\right]\right)-\frac{1}{2}\,b\,c\,e\,\text{PolyLog}\left[2,\frac{c^2\left(f+g\,x^2\right)}{c^2\,f+g}\right]+\frac{1}{2}\,b\,c\,e\,\text{PolyLog}\left[2,1+\frac{g\,x^2}{f}\right]-$$

$$\frac{i\,b\,e\,\sqrt{g}\,\text{PolyLog}\left[2,1+\frac{2\sqrt{f}\,\sqrt{g}\,(1-c\,x)}{\left(i\,c\,\sqrt{f}-\sqrt{g}\right)\left(\sqrt{f}-i\,\sqrt{g}\,x\right)}\right]}{2\sqrt{f}}+\frac{i\,b\,e\,\sqrt{g}\,\text{PolyLog}\left[2,1-\frac{2\sqrt{f}\,\sqrt{g}\,(1+c\,x)}{\left(i\,c\,\sqrt{f}+\sqrt{g}\right)\left(\sqrt{f}-i\,\sqrt{g}\,x\right)}\right]}{2\sqrt{f}}$$

Result (type 4, 1236 leaves) :

$$-\frac{a\,d}{x}-\frac{b\,d\,\text{ArcCoth}[c\,x]}{x}+b\,c\,d\,\text{Log}[x]-\frac{1}{2}\,b\,c\,d\,\text{Log}\left[1-c^2\,x^2\right]+$$

$$a\,e\left(\frac{2\,\sqrt{g}\,\,\text{ArcTan}\left[\frac{\sqrt{g}\,x}{\sqrt{f}}\right]}{\sqrt{f}}-\frac{\text{Log}\left[f+g\,x^2\right]}{x}\right)+\frac{1}{2}\,b\,e\,\Bigg(-\frac{\left(2\,\text{ArcCoth}[c\,x]+c\,x\,\left(-2\,\text{Log}[x]+\text{Log}\left[1-c^2\,x^2\right]\right)\right)\,\text{Log}\left[f+g\,x^2\right]}{x}-$$

$$2\,c\,\Bigg(\text{Log}[x]\,\left(\text{Log}\left[1-\frac{i\,\sqrt{g}\,x}{\sqrt{f}}\right]+\text{Log}\left[1+\frac{i\,\sqrt{g}\,x}{\sqrt{f}}\right]\right)+\text{PolyLog}\left[2,-\frac{i\,\sqrt{g}\,x}{\sqrt{f}}\right]+\text{PolyLog}\left[2,\frac{i\,\sqrt{g}\,x}{\sqrt{f}}\right]\Bigg)+$$

$$c\,\Bigg(\text{Log}\left[-\frac{1}{c}+x\right]\,\text{Log}\left[\frac{c\,\left(\sqrt{f}-i\,\sqrt{g}\,x\right)}{c\,\sqrt{f}-i\,\sqrt{g}}\right]+\text{Log}\left[\frac{1}{c}+x\right]\,\text{Log}\left[\frac{c\,\left(\sqrt{f}-i\,\sqrt{g}\,x\right)}{c\,\sqrt{f}+i\,\sqrt{g}}\right]+\text{Log}\left[-\frac{1}{c}+x\right]\,\text{Log}\left[\frac{c\,\left(\sqrt{f}+i\,\sqrt{g}\,x\right)}{c\,\sqrt{f}+i\,\sqrt{g}}\right]-$$

$$\left(\text{Log}\left[-\frac{1}{c}+x\right]+\text{Log}\left[\frac{1}{c}+x\right]-\text{Log}\left[1-c^2\,x^2\right]\right)\,\text{Log}\left[f+g\,x^2\right]+\text{Log}\left[\frac{1}{c}+x\right]\,\text{Log}\left[1-\frac{\sqrt{g}\,\,(1+c\,x)}{i\,c\,\sqrt{f}+\sqrt{g}}\right]+\text{PolyLog}\left[2,\frac{c\,\sqrt{g}\,\,\left(\frac{1}{c}+x\right)}{i\,c\,\sqrt{f}+\sqrt{g}}\right]+$$

$$\text{PolyLog}\left[2,\frac{i\,\sqrt{g}\,\,(-1+c\,x)}{c\,\sqrt{f}-i\,\sqrt{g}}\right]+\text{PolyLog}\left[2,-\frac{i\,\sqrt{g}\,\,(-1+c\,x)}{c\,\sqrt{f}+i\,\sqrt{g}}\right]+\text{PolyLog}\left[2,\frac{i\,\sqrt{g}\,\,(1+c\,x)}{c\,\sqrt{f}+i\,\sqrt{g}}\right]\Bigg)-$$

$$\frac{1}{\sqrt{c^2\,f\,g}}\,c\,g\,\Bigg(2\,i\,\text{ArcCos}\left[\frac{c^2\,f-g}{c^2\,f+g}\right]\,\text{ArcTan}\left[\frac{c\,f}{\sqrt{c^2\,f\,g}\,\,x}\right]-4\,\text{ArcCoth}[c\,x]\,\text{ArcTan}\left[\frac{c\,g\,x}{\sqrt{c^2\,f\,g}}\right]+\left(\text{ArcCos}\left[\frac{c^2\,f-g}{c^2\,f+g}\right]+2\,\text{ArcTan}\left[\frac{c\,f}{\sqrt{c^2\,f\,g}\,\,x}\right]\right)$$

$$\text{Log}\left[\frac{2\,g\,\left(c^2\,f-i\,\sqrt{c^2\,f\,g}\right)\,\,(-1+c\,x)}{\left(c^2\,f+g\right)\,\,\left(i\,\sqrt{c^2\,f\,g}\,+c\,g\,x\right)}\right]+\left(\text{ArcCos}\left[\frac{c^2\,f-g}{c^2\,f+g}\right]-2\,\text{ArcTan}\left[\frac{c\,f}{\sqrt{c^2\,f\,g}\,\,x}\right]\right)\,\text{Log}\left[\frac{2\,g\,\left(c^2\,f+i\,\sqrt{c^2\,f\,g}\right)\,\,(1+c\,x)}{\left(c^2\,f+g\right)\,\,\left(i\,\sqrt{c^2\,f\,g}\,+c\,g\,x\right)}\right]-$$

$$\left(\text{ArcCos}\left[\frac{c^2\,f-g}{c^2\,f+g}\right]+2\,\left(\text{ArcTan}\left[\frac{c\,f}{\sqrt{c^2\,f\,g}\,\,x}\right]+\text{ArcTan}\left[\frac{c\,g\,x}{\sqrt{c^2\,f\,g}}\right]\right)\right)\,\text{Log}\left[\frac{\sqrt{2}\,\,e^{-\text{ArcCoth}[c\,x]}\,\sqrt{c^2\,f\,g}}{\sqrt{c^2\,f+g}\,\,\sqrt{-c^2\,f+g+\left(c^2\,f+g\right)\,\text{Cosh}[2\,\text{ArcCoth}[c\,x]]}}\right]-$$

$$\left(\text{ArcCos}\left[\frac{c^2\,f-g}{c^2\,f+g}\right]-2\,\left(\text{ArcTan}\left[\frac{c\,f}{\sqrt{c^2\,f\,g}\,\,x}\right]+\text{ArcTan}\left[\frac{c\,g\,x}{\sqrt{c^2\,f\,g}}\right]\right)\right)\,\text{Log}\left[\frac{\sqrt{2}\,\,e^{\text{ArcCoth}[c\,x]}\,\sqrt{c^2\,f\,g}}{\sqrt{c^2\,f+g}\,\,\sqrt{-c^2\,f+g+\left(c^2\,f+g\right)\,\text{Cosh}[2\,\text{ArcCoth}[c\,x]]}}\right]+$$

$$i\,\left(\text{PolyLog}\left[2,\frac{\left(c^2\,f-g-2\,i\,\sqrt{c^2\,f\,g}\right)\,\,\left(\sqrt{c^2\,f\,g}\,+i\,c\,g\,x\right)}{\left(c^2\,f+g\right)\,\,\left(\sqrt{c^2\,f\,g}\,-i\,c\,g\,x\right)}\right]-\text{PolyLog}\left[2,\frac{\left(c^2\,f-g+2\,i\,\sqrt{c^2\,f\,g}\right)\,\,\left(\sqrt{c^2\,f\,g}\,+i\,c\,g\,x\right)}{\left(c^2\,f+g\right)\,\,\left(\sqrt{c^2\,f\,g}\,-i\,c\,g\,x\right)}\right]\right)\Bigg)\Bigg)$$

## Problem 282: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b \operatorname{ArcCoth}[c x]) (d + e \operatorname{Log}[f + g x^2])}{x^3} \, dx$$

Optimal (type 4, 712 leaves, 32 steps):

$$\frac{b c e \sqrt{g} \operatorname{ArcTan}\left[\frac{\sqrt{g} x}{\sqrt{f}}\right]}{\sqrt{f}} + \frac{a e g \operatorname{Log}[x]}{f} + \frac{b e g \operatorname{ArcCoth}[c x] \operatorname{Log}\left[\frac{2}{1+c x}\right]}{f} + b c^2 e \operatorname{ArcTanh}[c x] \operatorname{Log}\left[\frac{2}{1 + c x}\right] -$$

$$\frac{b e g \operatorname{ArcCoth}[c x] \operatorname{Log}\left[\frac{2 c \left(\sqrt{-f} - \sqrt{g} x\right)}{\left(c \sqrt{-f} - \sqrt{g}\right) (1+c x)}\right]}{2 f} - \frac{1}{2} b c^2 e \operatorname{ArcTanh}[c x] \operatorname{Log}\left[\frac{2 c \left(\sqrt{-f} - \sqrt{g} x\right)}{\left(c \sqrt{-f} - \sqrt{g}\right) (1 + c x)}\right] -$$

$$\frac{b e g \operatorname{ArcCoth}[c x] \operatorname{Log}\left[\frac{2 c \left(\sqrt{-f} + \sqrt{g} x\right)}{\left(c \sqrt{-f} + \sqrt{g}\right) (1+c x)}\right]}{2 f} - \frac{1}{2} b c^2 e \operatorname{ArcTanh}[c x] \operatorname{Log}\left[\frac{2 c \left(\sqrt{-f} + \sqrt{g} x\right)}{\left(c \sqrt{-f} + \sqrt{g}\right) (1 + c x)}\right] - \frac{a e g \operatorname{Log}[f + g x^2]}{2 f} -$$

$$\frac{b c (d + e \operatorname{Log}[f + g x^2])}{2 x} - \frac{(a + b \operatorname{ArcCoth}[c x]) (d + e \operatorname{Log}[f + g x^2])}{2 x^2} + \frac{1}{2} b c^2 \operatorname{ArcTanh}[c x] (d + e \operatorname{Log}[f + g x^2]) + \frac{b e g \operatorname{PolyLog}\left[2, -\frac{1}{c x}\right]}{2 f} -$$

$$\frac{b e g \operatorname{PolyLog}\left[2, \frac{1}{c x}\right]}{2 f} - \frac{1}{2} b c^2 e \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 + c x}\right] - \frac{b e g \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+c x}\right]}{2 f} + \frac{1}{4} b c^2 e \operatorname{PolyLog}\left[2, 1 - \frac{2 c \left(\sqrt{-f} - \sqrt{g} x\right)}{\left(c \sqrt{-f} - \sqrt{g}\right) (1 + c x)}\right] +$$

$$\frac{b e g \operatorname{PolyLog}\left[2, 1 - \frac{2 c \left(\sqrt{-f} - \sqrt{g} x\right)}{\left(c \sqrt{-f} - \sqrt{g}\right) (1+c x)}\right]}{4 f} + \frac{1}{4} b c^2 e \operatorname{PolyLog}\left[2, 1 - \frac{2 c \left(\sqrt{-f} + \sqrt{g} x\right)}{\left(c \sqrt{-f} + \sqrt{g}\right) (1 + c x)}\right] + \frac{b e g \operatorname{PolyLog}\left[2, 1 - \frac{2 c \left(\sqrt{-f} + \sqrt{g} x\right)}{\left(c \sqrt{-f} + \sqrt{g}\right) (1+c x)}\right]}{4 f}$$

Result (type 4, 1193 leaves):

$$\frac{1}{4 f x^2} \left(-2 a d f - 2 b c d f x - 2 b d f \operatorname{ArcCoth}[c x] + 2 b c^2 d f x^2 \operatorname{ArcCoth}[c x] + 4 b c e \sqrt{f} \sqrt{g} x^2 \operatorname{ArcTan}\left[\frac{\sqrt{g} x}{\sqrt{f}}\right] + \right.$$

$$4 i b c^2 e f x^2 \operatorname{ArcSin}\left[\sqrt{\frac{g}{c^2 f + g}}\right] \operatorname{ArcTanh}\left[\frac{c f}{\sqrt{-c^2 f g} x}\right] + 4 i b e g x^2 \operatorname{ArcSin}\left[\sqrt{\frac{g}{c^2 f + g}}\right] \operatorname{ArcTanh}\left[\frac{c f}{\sqrt{-c^2 f g} x}\right] +$$

$$4 b c^2 e f x^2 \operatorname{ArcCoth}[c x] \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcCoth}[c x]}\right] + 4 b e g x^2 \operatorname{ArcCoth}[c x] \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcCoth}[c x]}\right] -$$

$$2\,b\,c^2\,e\,f\,x^2\,\text{ArcCoth}[c\,x]\,\text{Log}\Big[\frac{e^{-2\,\text{ArcCoth}[c\,x]}\left(c^2\left(-1+e^{2\,\text{ArcCoth}[c\,x]}\right)f+g+e^{2\,\text{ArcCoth}[c\,x]}\,g-2\sqrt{-c^2\,f\,g}\right)}{c^2\,f+g}\Big]-$$

$$2\,b\,e\,g\,x^2\,\text{ArcCoth}[c\,x]\,\text{Log}\Big[\frac{e^{-2\,\text{ArcCoth}[c\,x]}\left(c^2\left(-1+e^{2\,\text{ArcCoth}[c\,x]}\right)f+g+e^{2\,\text{ArcCoth}[c\,x]}\,g-2\sqrt{-c^2\,f\,g}\right)}{c^2\,f+g}\Big]+$$

$$2\,i\,b\,c^2\,e\,f\,x^2\,\text{ArcSin}\Big[\sqrt{\frac{g}{c^2\,f+g}}\Big]\,\text{Log}\Big[\frac{e^{-2\,\text{ArcCoth}[c\,x]}\left(c^2\left(-1+e^{2\,\text{ArcCoth}[c\,x]}\right)f+g+e^{2\,\text{ArcCoth}[c\,x]}\,g-2\sqrt{-c^2\,f\,g}\right)}{c^2\,f+g}\Big]+$$

$$2\,i\,b\,e\,g\,x^2\,\text{ArcSin}\Big[\sqrt{\frac{g}{c^2\,f+g}}\Big]\,\text{Log}\Big[\frac{e^{-2\,\text{ArcCoth}[c\,x]}\left(c^2\left(-1+e^{2\,\text{ArcCoth}[c\,x]}\right)f+g+e^{2\,\text{ArcCoth}[c\,x]}\,g-2\sqrt{-c^2\,f\,g}\right)}{c^2\,f+g}\Big]-$$

$$2\,b\,c^2\,e\,f\,x^2\,\text{ArcCoth}[c\,x]\,\text{Log}\Big[\frac{e^{-2\,\text{ArcCoth}[c\,x]}\left(c^2\left(-1+e^{2\,\text{ArcCoth}[c\,x]}\right)f+g+e^{2\,\text{ArcCoth}[c\,x]}\,g+2\sqrt{-c^2\,f\,g}\right)}{c^2\,f+g}\Big]-$$

$$2\,b\,e\,g\,x^2\,\text{ArcCoth}[c\,x]\,\text{Log}\Big[\frac{e^{-2\,\text{ArcCoth}[c\,x]}\left(c^2\left(-1+e^{2\,\text{ArcCoth}[c\,x]}\right)f+g+e^{2\,\text{ArcCoth}[c\,x]}\,g+2\sqrt{-c^2\,f\,g}\right)}{c^2\,f+g}\Big]-$$

$$2\,i\,b\,c^2\,e\,f\,x^2\,\text{ArcSin}\Big[\sqrt{\frac{g}{c^2\,f+g}}\Big]\,\text{Log}\Big[\frac{e^{-2\,\text{ArcCoth}[c\,x]}\left(c^2\left(-1+e^{2\,\text{ArcCoth}[c\,x]}\right)f+g+e^{2\,\text{ArcCoth}[c\,x]}\,g+2\sqrt{-c^2\,f\,g}\right)}{c^2\,f+g}\Big]-$$

$$2\,i\,b\,e\,g\,x^2\,\text{ArcSin}\Big[\sqrt{\frac{g}{c^2\,f+g}}\Big]\,\text{Log}\Big[\frac{e^{-2\,\text{ArcCoth}[c\,x]}\left(c^2\left(-1+e^{2\,\text{ArcCoth}[c\,x]}\right)f+g+e^{2\,\text{ArcCoth}[c\,x]}\,g+2\sqrt{-c^2\,f\,g}\right)}{c^2\,f+g}\Big]+$$

$$4\,a\,e\,g\,x^2\,\text{Log}[x]-2\,a\,e\,f\,\text{Log}\big[f+g\,x^2\big]-2\,b\,c\,e\,f\,x\,\text{Log}\big[f+g\,x^2\big]-2\,a\,e\,g\,x^2\,\text{Log}\big[f+g\,x^2\big]-2\,b\,e\,f\,\text{ArcCoth}[c\,x]\,\text{Log}\big[f+g\,x^2\big]+$$

$$2\,b\,c^2\,e\,f\,x^2\,\text{ArcCoth}[c\,x]\,\text{Log}\big[f+g\,x^2\big]-2\,b\,e\,g\,x^2\,\text{PolyLog}\big[2,-e^{-2\,\text{ArcCoth}[c\,x]}\big]-2\,b\,c^2\,e\,f\,x^2\,\text{PolyLog}\big[2,e^{-2\,\text{ArcCoth}[c\,x]}\big]+$$

$$b\,c^2\,e\,f\,x^2\,\text{PolyLog}\Big[2,\frac{e^{-2\,\text{ArcCoth}[c\,x]}\left(c^2\,f-g+2\sqrt{-c^2\,f\,g}\right)}{c^2\,f+g}\Big]+b\,e\,g\,x^2\,\text{PolyLog}\Big[2,\frac{e^{-2\,\text{ArcCoth}[c\,x]}\left(c^2\,f-g+2\sqrt{-c^2\,f\,g}\right)}{c^2\,f+g}\Big]+$$

$$b\,c^2\,e\,f\,x^2\,\text{PolyLog}\Big[2,-\frac{e^{-2\,\text{ArcCoth}[c\,x]}\left(-c^2\,f+g+2\sqrt{-c^2\,f\,g}\right)}{c^2\,f+g}\Big]+b\,e\,g\,x^2\,\text{PolyLog}\Big[2,-\frac{e^{-2\,\text{ArcCoth}[c\,x]}\left(-c^2\,f+g+2\sqrt{-c^2\,f\,g}\right)}{c^2\,f+g}\Big]\Bigg)$$

# Test results for the 935 problems in "7.4.2 Exponentials of inverse hyperbolic cotangent

# functions.m"

## Problem 14: Result more than twice size of optimal antiderivative.

$$\int \frac{e^{2\,\text{ArcCoth}[a\,x]}}{x}\,dx$$

Optimal (type 3, 14 leaves, 4 steps):

$$-\text{Log}[x]+2\,\text{Log}[1-a\,x]$$

Result (type 3, 29 leaves):

$$-\text{Log}\left[1-e^{2\,\text{ArcCoth}[a\,x]}\right]-\text{Log}\left[1+e^{2\,\text{ArcCoth}[a\,x]}\right]$$

## Problem 46: Result more than twice size of optimal antiderivative.

$$\int \frac{e^{-2\,\text{ArcCoth}[a\,x]}}{x}\,dx$$

Optimal (type 3, 13 leaves, 4 steps):

$$-\text{Log}[x]+2\,\text{Log}[1+a\,x]$$

Result (type 3, 29 leaves):

$$-\text{Log}\left[1-e^{-2\,\text{ArcCoth}[a\,x]}\right]-\text{Log}\left[1+e^{-2\,\text{ArcCoth}[a\,x]}\right]$$

## Problem 64: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{1}{2}\text{ArcCoth}[a\,x]}}{x}\,dx$$

Optimal (type 3, 291 leaves, 17 steps):

$$-\sqrt{2}\,\text{ArcTan}\left[1-\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]+\sqrt{2}\,\text{ArcTan}\left[1+\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]+$$

$$2\,\text{ArcTan}\left[\frac{\left(1+\frac{1}{a\,x}\right)^{1/4}}{\left(1-\frac{1}{a\,x}\right)^{1/4}}\right]+2\,\text{ArcTanh}\left[\frac{\left(1+\frac{1}{a\,x}\right)^{1/4}}{\left(1-\frac{1}{a\,x}\right)^{1/4}}\right]+\frac{\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a\,x}}}{\sqrt{1+\frac{1}{a\,x}}}-\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{\sqrt{2}}-\frac{\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a\,x}}}{\sqrt{1+\frac{1}{a\,x}}}+\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{\sqrt{2}}$$

Result (type 7, 87 leaves):

$$2\,\text{ArcTan}\left[e^{\frac{1}{2}\text{ArcCoth}[a\,x]}\right]-\text{Log}\left[1-e^{\frac{1}{2}\text{ArcCoth}[a\,x]}\right]+\text{Log}\left[1+e^{\frac{1}{2}\text{ArcCoth}[a\,x]}\right]-\frac{1}{2}\,\text{RootSum}\left[1+\#1^4\,\&,\ \frac{-\text{ArcCoth}[a\,x]+2\,\text{Log}\left[e^{\frac{1}{2}\text{ArcCoth}[a\,x]}-\#1\right]}{\#1^3}\,\&\right]$$

## Problem 65: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{1}{2}\text{ArcCoth}[a\,x]}}{x^2}\,dx$$

Optimal (type 3, 267 leaves, 13 steps):

$$a\left(1-\frac{1}{a\,x}\right)^{3/4}\left(1+\frac{1}{a\,x}\right)^{1/4}-\frac{a\,\text{ArcTan}\left[1-\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{\sqrt{2}}+\frac{a\,\text{ArcTan}\left[1+\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{\sqrt{2}}+\frac{a\,\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a\,x}}}{\sqrt{1+\frac{1}{a\,x}}}-\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{2\sqrt{2}}-\frac{a\,\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a\,x}}}{\sqrt{1+\frac{1}{a\,x}}}+\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{2\sqrt{2}}$$

Result (type 7, 70 leaves):

$$a\left(\frac{2\,e^{\frac{1}{2}\text{ArcCoth}[a\,x]}}{1+e^{2\,\text{ArcCoth}[a\,x]}}-\frac{1}{4}\,\text{RootSum}\left[1+\#1^4\,\&,\ \frac{-\text{ArcCoth}[a\,x]+2\,\text{Log}\left[e^{\frac{1}{2}\text{ArcCoth}[a\,x]}-\#1\right]}{\#1^3}\,\&\right]\right)$$

## Problem 66: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{1}{2}\text{ArcCoth}[a\,x]}}{x^3}\,dx$$

Optimal (type 3, 319 leaves, 14 steps):

$$\frac{1}{4}a^2\left(1-\frac{1}{a\,x}\right)^{3/4}\left(1+\frac{1}{a\,x}\right)^{1/4}+\frac{1}{2}a^2\left(1-\frac{1}{a\,x}\right)^{3/4}\left(1+\frac{1}{a\,x}\right)^{5/4}-\frac{a^2\,\text{ArcTan}\left[1-\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{4\sqrt{2}}+$$

$$\frac{a^2\,\text{ArcTan}\left[1+\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{4\sqrt{2}}+\frac{a^2\,\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a\,x}}}{\sqrt{1+\frac{1}{a\,x}}}-\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{8\sqrt{2}}-\frac{a^2\,\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a\,x}}}{\sqrt{1+\frac{1}{a\,x}}}+\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{8\sqrt{2}}$$

Result (type 7, 85 leaves):

$$\frac{1}{16} a^2 \left( \frac{8 e^{\frac{1}{2} \text{ArcCoth}[a x]} \left(1 + 5 e^{2 \text{ArcCoth}[a x]}\right)}{\left(1 + e^{2 \text{ArcCoth}[a x]}\right)^2} - \text{RootSum}\left[1 + \#1^4 \&, \frac{-\text{ArcCoth}[a x] + 2 \text{Log}\left[e^{\frac{1}{2} \text{ArcCoth}[a x]} - \#1\right]}{\#1^3} \&\right] \right)$$

## Problem 67: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{1}{2} \text{ArcCoth}[a x]}}{x^4} \, dx$$

Optimal (type 3, 356 leaves, 15 steps):

$$\frac{3}{8} a^3 \left(1 - \frac{1}{a x}\right)^{3/4} \left(1 + \frac{1}{a x}\right)^{1/4} + \frac{1}{12} a^3 \left(1 - \frac{1}{a x}\right)^{3/4} \left(1 + \frac{1}{a x}\right)^{5/4} + \frac{a^2 \left(1 - \frac{1}{a x}\right)^{3/4} \left(1 + \frac{1}{a x}\right)^{5/4}}{3 x} - \frac{3 a^3 \text{ArcTan}\left[1 - \frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{8 \sqrt{2}} +$$

$$\frac{3 a^3 \text{ArcTan}\left[1 + \frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{8 \sqrt{2}} + \frac{3 a^3 \text{Log}\left[1 + \frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}} - \frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{16 \sqrt{2}} - \frac{3 a^3 \text{Log}\left[1 + \frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}} + \frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{16 \sqrt{2}}$$

Result (type 7, 93 leaves):

$$\frac{1}{96} a^3 \left( \frac{8 e^{\frac{1}{2} \text{ArcCoth}[a x]} \left(9 + 6 e^{2 \text{ArcCoth}[a x]} + 29 e^{4 \text{ArcCoth}[a x]}\right)}{\left(1 + e^{2 \text{ArcCoth}[a x]}\right)^3} + 9 \,\text{RootSum}\left[1 + \#1^4 \&, \frac{\text{ArcCoth}[a x] - 2 \text{Log}\left[e^{\frac{1}{2} \text{ArcCoth}[a x]} - \#1\right]}{\#1^3} \&\right] \right)$$

## Problem 73: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{3}{2} \text{ArcCoth}[a x]}}{x} \, dx$$

Optimal (type 3, 291 leaves, 17 steps):

$$-\sqrt{2}\,\text{ArcTan}\left[1-\frac{\sqrt{2}\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]+\sqrt{2}\,\text{ArcTan}\left[1+\frac{\sqrt{2}\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]-$$

$$2\,\text{ArcTan}\left[\frac{\left(1+\frac{1}{a x}\right)^{1/4}}{\left(1-\frac{1}{a x}\right)^{1/4}}\right]+2\,\text{ArcTanh}\left[\frac{\left(1+\frac{1}{a x}\right)^{1/4}}{\left(1-\frac{1}{a x}\right)^{1/4}}\right]-\frac{\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}-\frac{\sqrt{2}\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{\sqrt{2}}+\frac{\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}+\frac{\sqrt{2}\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{\sqrt{2}}$$

Result (type 7, 87 leaves) :

$$-2\,\text{ArcTan}\left[e^{\frac{1}{2}\text{ArcCoth}[a x]}\right]-\text{Log}\left[1-e^{\frac{1}{2}\text{ArcCoth}[a x]}\right]+\text{Log}\left[1+e^{\frac{1}{2}\text{ArcCoth}[a x]}\right]-\frac{1}{2}\,\text{RootSum}\left[1+\#1^4\,\&,\ \frac{-\text{ArcCoth}[a x]+2\,\text{Log}\left[e^{\frac{1}{2}\text{ArcCoth}[a x]}-\#1\right]}{\#1}\,\&\right]$$

## Problem 74: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{3}{2}\text{ArcCoth}[a x]}}{x^2}\,dx$$

Optimal (type 3, 268 leaves, 13 steps) :

$$a\left(1-\frac{1}{a x}\right)^{1/4}\left(1+\frac{1}{a x}\right)^{3/4}-\frac{3\,a\,\text{ArcTan}\left[1-\frac{\sqrt{2}\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{\sqrt{2}}+$$

$$\frac{3\,a\,\text{ArcTan}\left[1+\frac{\sqrt{2}\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{\sqrt{2}}-\frac{3\,a\,\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}-\frac{\sqrt{2}\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{2\sqrt{2}}+\frac{3\,a\,\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}+\frac{\sqrt{2}\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{2\sqrt{2}}$$

Result (type 7, 68 leaves) :

$$a\left(\frac{2\,e^{\frac{3}{2}\text{ArcCoth}[a x]}}{1+e^{2\,\text{ArcCoth}[a x]}}+\frac{3}{4}\,\text{RootSum}\left[1+\#1^4\,\&,\ \frac{\text{ArcCoth}[a x]-2\,\text{Log}\left[e^{\frac{1}{2}\text{ArcCoth}[a x]}-\#1\right]}{\#1}\,\&\right]\right)$$

## Problem 75: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{3}{2}\text{ArcCoth}[a x]}}{x^3}\,dx$$

Optimal (type 3, 319 leaves, 14 steps):

$$\frac{3}{4} a^2 \left(1-\frac{1}{a x}\right)^{1/4} \left(1+\frac{1}{a x}\right)^{3/4}+\frac{1}{2} a^2 \left(1-\frac{1}{a x}\right)^{1/4} \left(1+\frac{1}{a x}\right)^{7/4}-\frac{9 a^2 \text{ArcTan}\left[1-\frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{4 \sqrt{2}}+\frac{9 a^2 \text{ArcTan}\left[1+\frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{4 \sqrt{2}}-\frac{9 a^2 \text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}-\frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{8 \sqrt{2}}+\frac{9 a^2 \text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}+\frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{8 \sqrt{2}}$$

Result (type 7, 84 leaves):

$$a^2 \left(\frac{e^{\frac{3}{2} \text{ArcCoth}[a x]} \left(3+7 e^{2 \text{ArcCoth}[a x]}\right)}{2 \left(1+e^{2 \text{ArcCoth}[a x]}\right)^2}+\frac{9}{16} \text{RootSum}\left[1+\#1^4 \&, \frac{\text{ArcCoth}[a x]-2 \text{Log}\left[e^{\frac{1}{2} \text{ArcCoth}[a x]}-\#1\right]}{\#1} \&\right]\right)$$

## Problem 76: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{3}{2} \text{ArcCoth}[a x]}}{x^4} \, dx$$

Optimal (type 3, 356 leaves, 15 steps):

$$\frac{17}{24} a^3 \left(1-\frac{1}{a x}\right)^{1/4} \left(1+\frac{1}{a x}\right)^{3/4}+\frac{1}{4} a^3 \left(1-\frac{1}{a x}\right)^{1/4} \left(1+\frac{1}{a x}\right)^{7/4}+\frac{a^2 \left(1-\frac{1}{a x}\right)^{1/4} \left(1+\frac{1}{a x}\right)^{7/4}}{3 x}-\frac{17 a^3 \text{ArcTan}\left[1-\frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{8 \sqrt{2}}+\frac{17 a^3 \text{ArcTan}\left[1+\frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{8 \sqrt{2}}-\frac{17 a^3 \text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}-\frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{16 \sqrt{2}}+\frac{17 a^3 \text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}+\frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{16 \sqrt{2}}$$

Result (type 7, 93 leaves):

$$\frac{1}{96} a^3 \left(\frac{8 e^{\frac{3}{2} \text{ArcCoth}[a x]} \left(17+30 e^{2 \text{ArcCoth}[a x]}+45 e^{4 \text{ArcCoth}[a x]}\right)}{\left(1+e^{2 \text{ArcCoth}[a x]}\right)^3}+51 \text{RootSum}\left[1+\#1^4 \&, \frac{\text{ArcCoth}[a x]-2 \text{Log}\left[e^{\frac{1}{2} \text{ArcCoth}[a x]}-\#1\right]}{\#1} \&\right]\right)$$

## Problem 82: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{5}{2}\text{ArcCoth}[a x]}}{x}\,dx$$

Optimal (type 3, 320 leaves, 19 steps):

$$-\frac{8\left(1+\frac{1}{a x}\right)^{1/4}}{\left(1-\frac{1}{a x}\right)^{1/4}}+\sqrt{2}\,\text{ArcTan}\left[1-\frac{\sqrt{2}\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]-\sqrt{2}\,\text{ArcTan}\left[1+\frac{\sqrt{2}\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]+$$
$$2\,\text{ArcTan}\left[\frac{\left(1+\frac{1}{a x}\right)^{1/4}}{\left(1-\frac{1}{a x}\right)^{1/4}}\right]+2\,\text{ArcTanh}\left[\frac{\left(1+\frac{1}{a x}\right)^{1/4}}{\left(1-\frac{1}{a x}\right)^{1/4}}\right]-\frac{\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}-\frac{\sqrt{2}\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{\sqrt{2}}+\frac{\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}+\frac{\sqrt{2}\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{\sqrt{2}}$$

Result (type 7, 97 leaves):

$$-8\,e^{\frac{1}{2}\text{ArcCoth}[a x]}+2\,\text{ArcTan}\left[e^{\frac{1}{2}\text{ArcCoth}[a x]}\right]-\text{Log}\left[1-e^{\frac{1}{2}\text{ArcCoth}[a x]}\right]+$$
$$\text{Log}\left[1+e^{\frac{1}{2}\text{ArcCoth}[a x]}\right]-\frac{1}{2}\,\text{RootSum}\left[1+\#1^4\,\&,\ \frac{\text{ArcCoth}[a x]-2\,\text{Log}\left[e^{\frac{1}{2}\text{ArcCoth}[a x]}-\#1\right]}{\#1^3}\,\&\right]$$

## Problem 83: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{5}{2}\text{ArcCoth}[a x]}}{x^2}\,dx$$

Optimal (type 3, 299 leaves, 14 steps):

$$-5\,a\left(1-\frac{1}{a x}\right)^{3/4}\left(1+\frac{1}{a x}\right)^{1/4}-\frac{4\,a\left(1+\frac{1}{a x}\right)^{5/4}}{\left(1-\frac{1}{a x}\right)^{1/4}}+\frac{5\,a\,\text{ArcTan}\left[1-\frac{\sqrt{2}\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{\sqrt{2}}-$$
$$\frac{5\,a\,\text{ArcTan}\left[1+\frac{\sqrt{2}\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{\sqrt{2}}-\frac{5\,a\,\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}-\frac{\sqrt{2}\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{2\sqrt{2}}+\frac{5\,a\,\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}+\frac{\sqrt{2}\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{2\sqrt{2}}$$

Result (type 7, 80 leaves):

$$a\left(-8\,e^{\frac{1}{2}\text{ArcCoth}[a\,x]}-\frac{2\,e^{\frac{1}{2}\text{ArcCoth}[a\,x]}}{1+e^{2\,\text{ArcCoth}[a\,x]}}-\frac{5}{4}\,\text{RootSum}\left[1+\#1^4\,\&,\,\frac{\text{ArcCoth}[a\,x]-2\,\text{Log}\left[e^{\frac{1}{2}\text{ArcCoth}[a\,x]}-\#1\right]}{\#1^3}\,\&\right]\right)$$

## Problem 84: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{5}{2}\text{ArcCoth}[a\,x]}}{x^3}\,dx$$

Optimal (type 3, 351 leaves, 15 steps):

$$-\frac{25}{4}\,a^2\left(1-\frac{1}{a\,x}\right)^{3/4}\left(1+\frac{1}{a\,x}\right)^{1/4}-\frac{5}{2}\,a^2\left(1-\frac{1}{a\,x}\right)^{3/4}\left(1+\frac{1}{a\,x}\right)^{5/4}-\frac{2\,a^2\left(1+\frac{1}{a\,x}\right)^{9/4}}{\left(1-\frac{1}{a\,x}\right)^{1/4}}+\frac{25\,a^2\,\text{ArcTan}\left[1-\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{4\,\sqrt{2}}-$$

$$\frac{25\,a^2\,\text{ArcTan}\left[1+\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{4\,\sqrt{2}}-\frac{25\,a^2\,\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a\,x}}}{\sqrt{1+\frac{1}{a\,x}}}-\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{8\,\sqrt{2}}+\frac{25\,a^2\,\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a\,x}}}{\sqrt{1+\frac{1}{a\,x}}}+\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{8\,\sqrt{2}}$$

Result (type 7, 94 leaves):

$$a^2\left(-\frac{e^{\frac{1}{2}\text{ArcCoth}[a\,x]}\left(25+45\,e^{2\,\text{ArcCoth}[a\,x]}+16\,e^{4\,\text{ArcCoth}[a\,x]}\right)}{2\left(1+e^{2\,\text{ArcCoth}[a\,x]}\right)^2}-\frac{25}{16}\,\text{RootSum}\left[1+\#1^4\,\&,\,\frac{\text{ArcCoth}[a\,x]-2\,\text{Log}\left[e^{\frac{1}{2}\text{ArcCoth}[a\,x]}-\#1\right]}{\#1^3}\,\&\right]\right)$$

## Problem 85: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{5}{2}\text{ArcCoth}[a\,x]}}{x^4}\,dx$$

Optimal (type 3, 385 leaves, 16 steps):

$$-\frac{55}{8}a^3\left(1-\frac{1}{a x}\right)^{3/4}\left(1+\frac{1}{a x}\right)^{1/4}-\frac{11}{4}a^3\left(1-\frac{1}{a x}\right)^{3/4}\left(1+\frac{1}{a x}\right)^{5/4}-\frac{2a^3\left(1+\frac{1}{a x}\right)^{9/4}}{\left(1-\frac{1}{a x}\right)^{1/4}}-\frac{1}{3}a^3\left(1-\frac{1}{a x}\right)^{3/4}\left(1+\frac{1}{a x}\right)^{9/4}+$$

$$\frac{55a^3\,\text{ArcTan}\left[1-\frac{\sqrt{2}\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{8\sqrt{2}}-\frac{55a^3\,\text{ArcTan}\left[1+\frac{\sqrt{2}\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{8\sqrt{2}}-\frac{55a^3\,\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}-\frac{\sqrt{2}\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{16\sqrt{2}}+\frac{55a^3\,\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}+\frac{\sqrt{2}\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{16\sqrt{2}}$$

Result (type 7, 104 leaves) :

$$a^3\left(-\frac{e^{\frac{1}{2}\text{ArcCoth}[a x]}\left(165+462\,e^{2\,\text{ArcCoth}[a x]}+425\,e^{4\,\text{ArcCoth}[a x]}+96\,e^{6\,\text{ArcCoth}[a x]}\right)}{12\left(1+e^{2\,\text{ArcCoth}[a x]}\right)^3}-\frac{55}{32}\text{RootSum}\left[1+\#1^4\,\&,\ \frac{\text{ArcCoth}[a x]-2\,\text{Log}\left[e^{\frac{1}{2}\text{ArcCoth}[a x]}-\#1\right]}{\#1^3}\,\&\right]\right)$$

## Problem 91: Result is not expressed in closed-form.

$$\int\frac{e^{-\frac{1}{2}\text{ArcCoth}[a x]}}{x}\,dx$$

Optimal (type 3, 291 leaves, 17 steps) :

$$\sqrt{2}\,\text{ArcTan}\left[1-\frac{\sqrt{2}\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]-\sqrt{2}\,\text{ArcTan}\left[1+\frac{\sqrt{2}\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]-$$

$$2\,\text{ArcTan}\left[\frac{\left(1+\frac{1}{a x}\right)^{1/4}}{\left(1-\frac{1}{a x}\right)^{1/4}}\right]+2\,\text{ArcTanh}\left[\frac{\left(1+\frac{1}{a x}\right)^{1/4}}{\left(1-\frac{1}{a x}\right)^{1/4}}\right]+\frac{\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}-\frac{\sqrt{2}\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{\sqrt{2}}-\frac{\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}+\frac{\sqrt{2}\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{\sqrt{2}}$$

Result (type 7, 85 leaves) :

$$2\,\text{ArcTan}\left[e^{-\frac{1}{2}\text{ArcCoth}[a x]}\right]-\text{Log}\left[1-e^{-\frac{1}{2}\text{ArcCoth}[a x]}\right]+\text{Log}\left[1+e^{-\frac{1}{2}\text{ArcCoth}[a x]}\right]-\frac{1}{2}\text{RootSum}\left[1+\#1^4\,\&,\ \frac{\text{ArcCoth}[a x]+2\,\text{Log}\left[e^{-\frac{1}{2}\text{ArcCoth}[a x]}-\#1\right]}{\#1^3}\,\&\right]$$

## Problem 92: Result is not expressed in closed-form.

$$\int\frac{e^{-\frac{1}{2}\text{ArcCoth}[a x]}}{x^2}\,dx$$

Optimal (type 3, 268 leaves, 13 steps):

$$-a\left(1-\frac{1}{a x}\right)^{1/4}\left(1+\frac{1}{a x}\right)^{3/4}-\frac{a\,\text{ArcTan}\left[1-\frac{\sqrt{2}\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{\sqrt{2}}+\frac{a\,\text{ArcTan}\left[1+\frac{\sqrt{2}\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{\sqrt{2}}-\frac{a\,\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}-\frac{\sqrt{2}\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{2\sqrt{2}}+\frac{a\,\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}+\frac{\sqrt{2}\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{2\sqrt{2}}$$

Result (type 7, 70 leaves):

$$a\left(-\frac{2\,e^{-\frac{1}{2}\text{ArcCoth}[a x]}}{1+e^{-2\,\text{ArcCoth}[a x]}}-\frac{1}{4}\,\text{RootSum}\left[1+\#1^4\,\&,\ \frac{-\text{ArcCoth}[a x]-2\,\text{Log}\left[e^{-\frac{1}{2}\text{ArcCoth}[a x]}-\#1\right]}{\#1^3}\,\&\right]\right)$$

## Problem 93: Result is not expressed in closed-form.

$$\int \frac{e^{-\frac{1}{2}\text{ArcCoth}[a x]}}{x^3}\,dx$$

Optimal (type 3, 319 leaves, 14 steps):

$$\frac{1}{4}a^2\left(1-\frac{1}{a x}\right)^{1/4}\left(1+\frac{1}{a x}\right)^{3/4}+\frac{1}{2}a^2\left(1-\frac{1}{a x}\right)^{5/4}\left(1+\frac{1}{a x}\right)^{3/4}+\frac{a^2\,\text{ArcTan}\left[1-\frac{\sqrt{2}\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{4\sqrt{2}}-\frac{a^2\,\text{ArcTan}\left[1+\frac{\sqrt{2}\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{4\sqrt{2}}+\frac{a^2\,\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}-\frac{\sqrt{2}\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{8\sqrt{2}}-\frac{a^2\,\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}+\frac{\sqrt{2}\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{8\sqrt{2}}$$

Result (type 7, 81 leaves):

$$\frac{1}{16}a^2\left(\frac{8\,e^{\frac{3}{2}\text{ArcCoth}[a x]}\left(5+e^{2\,\text{ArcCoth}[a x]}\right)}{\left(1+e^{2\,\text{ArcCoth}[a x]}\right)^2}-\text{RootSum}\left[1+\#1^4\,\&,\ \frac{\text{ArcCoth}[a x]+2\,\text{Log}\left[e^{-\frac{1}{2}\text{ArcCoth}[a x]}-\#1\right]}{\#1^3}\,\&\right]\right)$$

## Problem 94: Result is not expressed in closed-form.

$$\int \frac{e^{-\frac{1}{2}\text{ArcCoth}[a x]}}{x^4}\,dx$$

Optimal (type 3, 356 leaves, 15 steps):

$$-\frac{3}{8}a^3\left(1-\frac{1}{a x}\right)^{1/4}\left(1+\frac{1}{a x}\right)^{3/4}-\frac{1}{12}a^3\left(1-\frac{1}{a x}\right)^{5/4}\left(1+\frac{1}{a x}\right)^{3/4}+\frac{a^2\left(1-\frac{1}{a x}\right)^{5/4}\left(1+\frac{1}{a x}\right)^{3/4}}{3 x}-\frac{3 a^3\,\text{ArcTan}\left[1-\frac{\sqrt{2}\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{8\sqrt{2}}+\frac{3 a^3\,\text{ArcTan}\left[1+\frac{\sqrt{2}\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{8\sqrt{2}}-\frac{3 a^3\,\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}-\frac{\sqrt{2}\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{16\sqrt{2}}+\frac{3 a^3\,\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}+\frac{\sqrt{2}\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{16\sqrt{2}}$$

Result (type 7, 93 leaves):

$$\frac{1}{96}a^3\left(-\frac{8 e^{\frac{3}{2}\text{ArcCoth}[a x]}\left(29+6 e^{2\,\text{ArcCoth}[a x]}+9 e^{4\,\text{ArcCoth}[a x]}\right)}{\left(1+e^{2\,\text{ArcCoth}[a x]}\right)^3}+9\,\text{RootSum}\left[1+\#1^4\&,\frac{\text{ArcCoth}[a x]+2\,\text{Log}\left[e^{-\frac{1}{2}\text{ArcCoth}[a x]}-\#1\right]}{\#1^3}\&\right]\right)$$

## Problem 100: Result is not expressed in closed-form.

$$\int \frac{e^{-\frac{3}{2}\text{ArcCoth}[a x]}}{x}\,dx$$

Optimal (type 3, 291 leaves, 17 steps):

$$\sqrt{2}\,\text{ArcTan}\left[1-\frac{\sqrt{2}\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]-\sqrt{2}\,\text{ArcTan}\left[1+\frac{\sqrt{2}\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]+2\,\text{ArcTan}\left[\frac{\left(1+\frac{1}{a x}\right)^{1/4}}{\left(1-\frac{1}{a x}\right)^{1/4}}\right]+2\,\text{ArcTanh}\left[\frac{\left(1+\frac{1}{a x}\right)^{1/4}}{\left(1-\frac{1}{a x}\right)^{1/4}}\right]-\frac{\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}-\frac{\sqrt{2}\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{\sqrt{2}}+\frac{\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}+\frac{\sqrt{2}\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{\sqrt{2}}$$

Result (type 7, 85 leaves):

$$-2\,\text{ArcTan}\left[e^{-\frac{1}{2}\text{ArcCoth}[a\,x]}\right]-\text{Log}\left[1-e^{-\frac{1}{2}\text{ArcCoth}[a\,x]}\right]+\text{Log}\left[1+e^{-\frac{1}{2}\text{ArcCoth}[a\,x]}\right]-\frac{1}{2}\,\text{RootSum}\left[1+\#1^4\,\&,\,\frac{\text{ArcCoth}[a\,x]+2\,\text{Log}\left[e^{-\frac{1}{2}\text{ArcCoth}[a\,x]}-\#1\right]}{\#1}\,\&\right]$$

## Problem 101: Result is not expressed in closed-form.

$$\int \frac{e^{-\frac{3}{2}\text{ArcCoth}[a\,x]}}{x^2}\,dx$$

Optimal (type 3, 269 leaves, 13 steps):

$$-a\left(1-\frac{1}{a\,x}\right)^{3/4}\left(1+\frac{1}{a\,x}\right)^{1/4}-\frac{3\,a\,\text{ArcTan}\left[1-\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{\sqrt{2}}+\frac{3\,a\,\text{ArcTan}\left[1+\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{\sqrt{2}}+\frac{3\,a\,\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a\,x}}}{\sqrt{1+\frac{1}{a\,x}}}-\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{2\,\sqrt{2}}-\frac{3\,a\,\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a\,x}}}{\sqrt{1+\frac{1}{a\,x}}}+\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{2\,\sqrt{2}}$$

Result (type 7, 68 leaves):

$$a\left(-\frac{2\,e^{-\frac{3}{2}\text{ArcCoth}[a\,x]}}{1+e^{-2\,\text{ArcCoth}[a\,x]}}+\frac{3}{4}\,\text{RootSum}\left[1+\#1^4\,\&,\,\frac{\text{ArcCoth}[a\,x]+2\,\text{Log}\left[e^{-\frac{1}{2}\text{ArcCoth}[a\,x]}-\#1\right]}{\#1}\,\&\right]\right)$$

## Problem 102: Result is not expressed in closed-form.

$$\int \frac{e^{-\frac{3}{2}\text{ArcCoth}[a\,x]}}{x^3}\,dx$$

Optimal (type 3, 319 leaves, 14 steps):

$$\frac{3}{4} a^2 \left(1-\frac{1}{a x}\right)^{3/4} \left(1+\frac{1}{a x}\right)^{1/4} + \frac{1}{2} a^2 \left(1-\frac{1}{a x}\right)^{7/4} \left(1+\frac{1}{a x}\right)^{1/4} + \frac{9 a^2 \text{ArcTan}\left[1-\frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{4 \sqrt{2}} - \frac{9 a^2 \text{ArcTan}\left[1+\frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{4 \sqrt{2}} - \frac{9 a^2 \text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}-\frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{8 \sqrt{2}} + \frac{9 a^2 \text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}+\frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{8 \sqrt{2}}$$

Result (type 7, 84 leaves):

$$a^2 \left(\frac{e^{\frac{1}{2} \text{ArcCoth}[a x]} \left(7+3 e^{2 \text{ArcCoth}[a x]}\right)}{2 \left(1+e^{2 \text{ArcCoth}[a x]}\right)^2} - \frac{9}{16} \text{RootSum}\left[1+\#1^4 \&, \frac{\text{ArcCoth}[a x]+2 \text{Log}\left[e^{-\frac{1}{2} \text{ArcCoth}[a x]}-\#1\right]}{\#1} \&\right]\right)$$

## Problem 103: Result is not expressed in closed-form.

$$\int \frac{e^{-\frac{3}{2} \text{ArcCoth}[a x]}}{x^4} \, dx$$

Optimal (type 3, 356 leaves, 15 steps):

$$-\frac{17}{24} a^3 \left(1-\frac{1}{a x}\right)^{3/4} \left(1+\frac{1}{a x}\right)^{1/4} - \frac{1}{4} a^3 \left(1-\frac{1}{a x}\right)^{7/4} \left(1+\frac{1}{a x}\right)^{1/4} + \frac{a^2 \left(1-\frac{1}{a x}\right)^{7/4} \left(1+\frac{1}{a x}\right)^{1/4}}{3 x} - \frac{17 a^3 \text{ArcTan}\left[1-\frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{8 \sqrt{2}} + \frac{17 a^3 \text{ArcTan}\left[1+\frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{8 \sqrt{2}} + \frac{17 a^3 \text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}-\frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{16 \sqrt{2}} - \frac{17 a^3 \text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a x}}}{\sqrt{1+\frac{1}{a x}}}+\frac{\sqrt{2} \left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}\right]}{16 \sqrt{2}}$$

Result (type 7, 93 leaves):

$$\frac{1}{96} a^3 \left(-\frac{8 e^{\frac{1}{2} \text{ArcCoth}[a x]} \left(45+30 e^{2 \text{ArcCoth}[a x]}+17 e^{4 \text{ArcCoth}[a x]}\right)}{\left(1+e^{2 \text{ArcCoth}[a x]}\right)^3} + 51 \text{RootSum}\left[1+\#1^4 \&, \frac{\text{ArcCoth}[a x]+2 \text{Log}\left[e^{-\frac{1}{2} \text{ArcCoth}[a x]}-\#1\right]}{\#1} \&\right]\right)$$

## Problem 109: Result is not expressed in closed-form.

$$\int \frac{e^{-\frac{5}{2}\text{ArcCoth}[a\,x]}}{x}\,dx$$

Optimal (type 3, 320 leaves, 19 steps):

$$-\frac{8\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}-\sqrt{2}\,\text{ArcTan}\left[1-\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]+\sqrt{2}\,\text{ArcTan}\left[1+\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]-$$

$$2\,\text{ArcTan}\left[\frac{\left(1+\frac{1}{a\,x}\right)^{1/4}}{\left(1-\frac{1}{a\,x}\right)^{1/4}}\right]+2\,\text{ArcTanh}\left[\frac{\left(1+\frac{1}{a\,x}\right)^{1/4}}{\left(1-\frac{1}{a\,x}\right)^{1/4}}\right]-\frac{\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a\,x}}}{\sqrt{1+\frac{1}{a\,x}}}-\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{\sqrt{2}}+\frac{\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a\,x}}}{\sqrt{1+\frac{1}{a\,x}}}+\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{\sqrt{2}}$$

Result (type 7, 99 leaves):

$$-8\,e^{-\frac{1}{2}\text{ArcCoth}[a\,x]}+2\,\text{ArcTan}\left[e^{-\frac{1}{2}\text{ArcCoth}[a\,x]}\right]-\text{Log}\left[1-e^{-\frac{1}{2}\text{ArcCoth}[a\,x]}\right]+$$

$$\text{Log}\left[1+e^{-\frac{1}{2}\text{ArcCoth}[a\,x]}\right]-\frac{1}{2}\,\text{RootSum}\left[1+\#1^4\,\&,\ \frac{-\text{ArcCoth}[a\,x]-2\,\text{Log}\left[e^{-\frac{1}{2}\text{ArcCoth}[a\,x]}-\#1\right]}{\#1^3}\,\&\right]$$

## Problem 110: Result is not expressed in closed-form.

$$\int \frac{e^{-\frac{5}{2}\text{ArcCoth}[a\,x]}}{x^2}\,dx$$

Optimal (type 3, 299 leaves, 14 steps):

$$\frac{4\,a\left(1-\frac{1}{a\,x}\right)^{5/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}+5\,a\left(1-\frac{1}{a\,x}\right)^{1/4}\left(1+\frac{1}{a\,x}\right)^{3/4}+\frac{5\,a\,\text{ArcTan}\left[1-\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{\sqrt{2}}-$$

$$\frac{5\,a\,\text{ArcTan}\left[1+\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{\sqrt{2}}+\frac{5\,a\,\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a\,x}}}{\sqrt{1+\frac{1}{a\,x}}}-\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{2\sqrt{2}}-\frac{5\,a\,\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a\,x}}}{\sqrt{1+\frac{1}{a\,x}}}+\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{2\sqrt{2}}$$

Result (type 7, 80 leaves):

$$a\left(8\,e^{-\frac{1}{2}\text{ArcCoth}[a\,x]}+\frac{2\,e^{-\frac{1}{2}\text{ArcCoth}[a\,x]}}{1+e^{-2\,\text{ArcCoth}[a\,x]}}-\frac{5}{4}\,\text{RootSum}\left[1+\#1^4\,\&,\,\frac{\text{ArcCoth}[a\,x]+2\,\text{Log}\left[e^{-\frac{1}{2}\text{ArcCoth}[a\,x]}-\#1\right]}{\#1^3}\,\&\right]\right)$$

## Problem 111: Result is not expressed in closed-form.

$$\int\frac{e^{-\frac{5}{2}\text{ArcCoth}[a\,x]}}{x^3}\,dx$$

Optimal (type 3, 351 leaves, 15 steps):

$$-\frac{2\,a^2\left(1-\frac{1}{a\,x}\right)^{9/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}-\frac{25}{4}\,a^2\left(1-\frac{1}{a\,x}\right)^{1/4}\left(1+\frac{1}{a\,x}\right)^{3/4}-\frac{5}{2}\,a^2\left(1-\frac{1}{a\,x}\right)^{5/4}\left(1+\frac{1}{a\,x}\right)^{3/4}-\frac{25\,a^2\,\text{ArcTan}\left[1-\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{4\sqrt{2}}+$$

$$\frac{25\,a^2\,\text{ArcTan}\left[1+\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{4\sqrt{2}}-\frac{25\,a^2\,\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a\,x}}}{\sqrt{1+\frac{1}{a\,x}}}-\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{8\sqrt{2}}+\frac{25\,a^2\,\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{a\,x}}}{\sqrt{1+\frac{1}{a\,x}}}+\frac{\sqrt{2}\left(1-\frac{1}{a\,x}\right)^{1/4}}{\left(1+\frac{1}{a\,x}\right)^{1/4}}\right]}{8\sqrt{2}}$$

Result (type 7, 94 leaves):

$$a^2\left(-\frac{e^{-\frac{1}{2}\text{ArcCoth}[a\,x]}\left(16+45\,e^{2\,\text{ArcCoth}[a\,x]}+25\,e^{4\,\text{ArcCoth}[a\,x]}\right)}{2\left(1+e^{2\,\text{ArcCoth}[a\,x]}\right)^2}+\frac{25}{16}\,\text{RootSum}\left[1+\#1^4\,\&,\,\frac{\text{ArcCoth}[a\,x]+2\,\text{Log}\left[e^{-\frac{1}{2}\text{ArcCoth}[a\,x]}-\#1\right]}{\#1^3}\,\&\right]\right)$$

## Problem 112: Result is not expressed in closed-form.

$$\int\frac{e^{-\frac{5}{2}\text{ArcCoth}[a\,x]}}{x^4}\,dx$$

Optimal (type 3, 385 leaves, 16 steps):

$$\frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\left(1+\frac{1}{ax}\right)^{1/4}}+\frac{55}{8}a^3\left(1-\frac{1}{ax}\right)^{1/4}\left(1+\frac{1}{ax}\right)^{3/4}+\frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}+\frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{9/4}\left(1+\frac{1}{ax}\right)^{3/4}+$$

$$\frac{55a^3\,\text{ArcTan}\left[1-\frac{\sqrt{2}\left(1-\frac{1}{ax}\right)^{1/4}}{\left(1+\frac{1}{ax}\right)^{1/4}}\right]}{8\sqrt{2}}-\frac{55a^3\,\text{ArcTan}\left[1+\frac{\sqrt{2}\left(1-\frac{1}{ax}\right)^{1/4}}{\left(1+\frac{1}{ax}\right)^{1/4}}\right]}{8\sqrt{2}}+\frac{55a^3\,\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\left(1-\frac{1}{ax}\right)^{1/4}}{\left(1+\frac{1}{ax}\right)^{1/4}}\right]}{16\sqrt{2}}-\frac{55a^3\,\text{Log}\left[1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\left(1-\frac{1}{ax}\right)^{1/4}}{\left(1+\frac{1}{ax}\right)^{1/4}}\right]}{16\sqrt{2}}$$

Result (type 7, 104 leaves):

$$a^3\left(\frac{e^{-\frac{1}{2}\text{ArcCoth}[a\,x]}\left(96+425\,e^{2\,\text{ArcCoth}[a\,x]}+462\,e^{4\,\text{ArcCoth}[a\,x]}+165\,e^{6\,\text{ArcCoth}[a\,x]}\right)}{12\left(1+e^{2\,\text{ArcCoth}[a\,x]}\right)^3}-\frac{55}{32}\text{RootSum}\left[1+\#1^4\,\&,\frac{\text{ArcCoth}[a\,x]+2\,\text{Log}\left[e^{-\frac{1}{2}\text{ArcCoth}[a\,x]}-\#1\right]}{\#1^3}\,\&\right]\right)$$

## Problem 116: Result is not expressed in closed-form.

$$\int\frac{e^{\frac{\text{ArcCoth}[x]}{3}}}{x}\,dx$$

Optimal (type 3, 402 leaves, 25 steps):

$$-\sqrt{3}\,\text{ArcTan}\left[\frac{1-\frac{2\left(1+\frac{1}{x}\right)^{1/6}}{\left(\frac{-1+x}{x}\right)^{1/6}}}{\sqrt{3}}\right]+\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2\left(1+\frac{1}{x}\right)^{1/6}}{\left(\frac{-1+x}{x}\right)^{1/6}}}{\sqrt{3}}\right]-\text{ArcTan}\left[\sqrt{3}-\frac{2\left(\frac{-1+x}{x}\right)^{1/6}}{\left(1+\frac{1}{x}\right)^{1/6}}\right]+$$

$$\text{ArcTan}\left[\sqrt{3}+\frac{2\left(\frac{-1+x}{x}\right)^{1/6}}{\left(1+\frac{1}{x}\right)^{1/6}}\right]+2\,\text{ArcTan}\left[\frac{\left(\frac{-1+x}{x}\right)^{1/6}}{\left(1+\frac{1}{x}\right)^{1/6}}\right]+2\,\text{ArcTanh}\left[\frac{\left(1+\frac{1}{x}\right)^{1/6}}{\left(\frac{-1+x}{x}\right)^{1/6}}\right]-\frac{1}{2}\text{Log}\left[1+\frac{\left(1+\frac{1}{x}\right)^{1/3}}{\left(\frac{-1+x}{x}\right)^{1/3}}-\frac{\left(1+\frac{1}{x}\right)^{1/6}}{\left(\frac{-1+x}{x}\right)^{1/6}}\right]+$$

$$\frac{1}{2}\text{Log}\left[1+\frac{\left(1+\frac{1}{x}\right)^{1/3}}{\left(\frac{-1+x}{x}\right)^{1/3}}+\frac{\left(1+\frac{1}{x}\right)^{1/6}}{\left(\frac{-1+x}{x}\right)^{1/6}}\right]+\frac{1}{2}\sqrt{3}\,\text{Log}\left[1-\frac{\sqrt{3}\left(\frac{-1+x}{x}\right)^{1/6}}{\left(1+\frac{1}{x}\right)^{1/6}}+\frac{\left(\frac{-1+x}{x}\right)^{1/3}}{\left(1+\frac{1}{x}\right)^{1/3}}\right]-\frac{1}{2}\sqrt{3}\,\text{Log}\left[1+\frac{\sqrt{3}\left(\frac{-1+x}{x}\right)^{1/6}}{\left(1+\frac{1}{x}\right)^{1/6}}+\frac{\left(\frac{-1+x}{x}\right)^{1/3}}{\left(1+\frac{1}{x}\right)^{1/3}}\right]$$

Result (type 7, 218 leaves):

$$-2\,\text{ArcTan}\left[e^{\frac{\text{ArcCoth}[x]}{3}}\right]+\sqrt{3}\,\text{ArcTan}\left[\frac{-1+2\,e^{\frac{\text{ArcCoth}[x]}{3}}}{\sqrt{3}}\right]+\sqrt{3}\,\text{ArcTan}\left[\frac{1+2\,e^{\frac{\text{ArcCoth}[x]}{3}}}{\sqrt{3}}\right]-$$
$$\text{Log}\left[1-e^{\frac{\text{ArcCoth}[x]}{3}}\right]+\text{Log}\left[1+e^{\frac{\text{ArcCoth}[x]}{3}}\right]-\frac{1}{2}\text{Log}\left[1-e^{\frac{\text{ArcCoth}[x]}{3}}+e^{\frac{2\,\text{ArcCoth}[x]}{3}}\right]+\frac{1}{2}\text{Log}\left[1+e^{\frac{\text{ArcCoth}[x]}{3}}+e^{\frac{2\,\text{ArcCoth}[x]}{3}}\right]+$$
$$\frac{1}{3}\text{RootSum}\left[1-\#1^2+\#1^4\,\&,\ \frac{2\,\text{ArcCoth}[x]-6\,\text{Log}\left[e^{\frac{\text{ArcCoth}[x]}{3}}-\#1\right]-\text{ArcCoth}[x]\,\#1^2+3\,\text{Log}\left[e^{\frac{\text{ArcCoth}[x]}{3}}-\#1\right]\#1^2}{-\#1+2\,\#1^3}\,\&\right]$$

## Problem 117: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{\text{ArcCoth}[x]}{3}}}{x^2}\,dx$$

Optimal (type 3, 233 leaves, 14 steps):

$$\left(1+\frac{1}{x}\right)^{1/6}\left(\frac{-1+x}{x}\right)^{5/6}-\frac{1}{3}\text{ArcTan}\left[\sqrt{3}-\frac{2\left(\frac{-1+x}{x}\right)^{1/6}}{\left(1+\frac{1}{x}\right)^{1/6}}\right]+\frac{1}{3}\text{ArcTan}\left[\sqrt{3}+\frac{2\left(\frac{-1+x}{x}\right)^{1/6}}{\left(1+\frac{1}{x}\right)^{1/6}}\right]+$$
$$\frac{2}{3}\text{ArcTan}\left[\frac{\left(\frac{-1+x}{x}\right)^{1/6}}{\left(1+\frac{1}{x}\right)^{1/6}}\right]+\frac{\text{Log}\left[1-\frac{\sqrt{3}\left(\frac{-1+x}{x}\right)^{1/6}}{\left(1+\frac{1}{x}\right)^{1/6}}+\frac{\left(\frac{-1+x}{x}\right)^{1/3}}{\left(1+\frac{1}{x}\right)^{1/3}}\right]}{2\sqrt{3}}-\frac{\text{Log}\left[1+\frac{\sqrt{3}\left(\frac{-1+x}{x}\right)^{1/6}}{\left(1+\frac{1}{x}\right)^{1/6}}+\frac{\left(\frac{-1+x}{x}\right)^{1/3}}{\left(1+\frac{1}{x}\right)^{1/3}}\right]}{2\sqrt{3}}$$

Result (type 7, 116 leaves):

$$\frac{2\,e^{\frac{\text{ArcCoth}[x]}{3}}}{1+e^{2\,\text{ArcCoth}[x]}}-\frac{2}{3}\text{ArcTan}\left[e^{\frac{\text{ArcCoth}[x]}{3}}\right]+\frac{1}{9}\text{RootSum}\left[1-\#1^2+\#1^4\,\&,\ \frac{2\,\text{ArcCoth}[x]-6\,\text{Log}\left[e^{\frac{\text{ArcCoth}[x]}{3}}-\#1\right]-\text{ArcCoth}[x]\,\#1^2+3\,\text{Log}\left[e^{\frac{\text{ArcCoth}[x]}{3}}-\#1\right]\#1^2}{-\#1+2\,\#1^3}\,\&\right]$$

## Problem 118: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{\text{ArcCoth}[x]}{3}}}{x^3}\,dx$$

Optimal (type 3, 260 leaves, 15 steps):

$$\frac{1}{6}\left(1+\frac{1}{x}\right)^{1/6}\left(\frac{-1+x}{x}\right)^{5/6}+\frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}-\frac{1}{18}\,\text{ArcTan}\left[\sqrt{3}-\frac{2\left(\frac{-1+x}{x}\right)^{1/6}}{\left(1+\frac{1}{x}\right)^{1/6}}\right]+$$

$$\frac{1}{18}\,\text{ArcTan}\left[\sqrt{3}+\frac{2\left(\frac{-1+x}{x}\right)^{1/6}}{\left(1+\frac{1}{x}\right)^{1/6}}\right]+\frac{1}{9}\,\text{ArcTan}\left[\frac{\left(\frac{-1+x}{x}\right)^{1/6}}{\left(1+\frac{1}{x}\right)^{1/6}}\right]+\frac{\text{Log}\left[1-\frac{\sqrt{3}\left(\frac{-1+x}{x}\right)^{1/6}}{\left(1+\frac{1}{x}\right)^{1/6}}+\frac{\left(\frac{-1+x}{x}\right)^{1/3}}{\left(1+\frac{1}{x}\right)^{1/3}}\right]}{12\sqrt{3}}-\frac{\text{Log}\left[1+\frac{\sqrt{3}\left(\frac{-1+x}{x}\right)^{1/6}}{\left(1+\frac{1}{x}\right)^{1/6}}+\frac{\left(\frac{-1+x}{x}\right)^{1/3}}{\left(1+\frac{1}{x}\right)^{1/3}}\right]}{12\sqrt{3}}$$

Result (type 7, 124 leaves):

$$\frac{1}{54}\left(\frac{18\,e^{\frac{\text{ArcCoth}[x]}{3}}\left(1+7\,e^{2\,\text{ArcCoth}[x]}\right)}{\left(1+e^{2\,\text{ArcCoth}[x]}\right)^{2}}-6\,\text{ArcTan}\left[e^{\frac{\text{ArcCoth}[x]}{3}}\right]+\right.$$

$$\left.\text{RootSum}\left[1-\#1^{2}+\#1^{4}\,\&,\,\frac{2\,\text{ArcCoth}[x]-6\,\text{Log}\left[e^{\frac{\text{ArcCoth}[x]}{3}}-\#1\right]-\text{ArcCoth}[x]\,\#1^{2}+3\,\text{Log}\left[e^{\frac{\text{ArcCoth}[x]}{3}}-\#1\right]\#1^{2}}{-\#1+2\,\#1^{3}}\,\&\right]\right)$$

## Problem 119: Result is not expressed in closed-form.

$$\int\frac{e^{\frac{\text{ArcCoth}[x]}{3}}}{x^{4}}\,dx$$

Optimal (type 3, 287 leaves, 16 steps):

$$\frac{19}{54}\left(1+\frac{1}{x}\right)^{1/6}\left(\frac{-1+x}{x}\right)^{5/6}+\frac{1}{18}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}+\frac{\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}}{3\,x}-\frac{19}{162}\,\text{ArcTan}\left[\sqrt{3}-\frac{2\left(\frac{-1+x}{x}\right)^{1/6}}{\left(1+\frac{1}{x}\right)^{1/6}}\right]+$$

$$\frac{19}{162}\,\text{ArcTan}\left[\sqrt{3}+\frac{2\left(\frac{-1+x}{x}\right)^{1/6}}{\left(1+\frac{1}{x}\right)^{1/6}}\right]+\frac{19}{81}\,\text{ArcTan}\left[\frac{\left(\frac{-1+x}{x}\right)^{1/6}}{\left(1+\frac{1}{x}\right)^{1/6}}\right]+\frac{19\,\text{Log}\left[1-\frac{\sqrt{3}\left(\frac{-1+x}{x}\right)^{1/6}}{\left(1+\frac{1}{x}\right)^{1/6}}+\frac{\left(\frac{-1+x}{x}\right)^{1/3}}{\left(1+\frac{1}{x}\right)^{1/3}}\right]}{108\sqrt{3}}-\frac{19\,\text{Log}\left[1+\frac{\sqrt{3}\left(\frac{-1+x}{x}\right)^{1/6}}{\left(1+\frac{1}{x}\right)^{1/6}}+\frac{\left(\frac{-1+x}{x}\right)^{1/3}}{\left(1+\frac{1}{x}\right)^{1/3}}\right]}{108\sqrt{3}}$$

Result (type 7, 133 leaves):

$$\frac{1}{486}\left(\frac{18\,e^{\frac{\text{ArcCoth}[x]}{3}}\left(19+8\,e^{2\,\text{ArcCoth}[x]}+61\,e^{4\,\text{ArcCoth}[x]}\right)}{\left(1+e^{2\,\text{ArcCoth}[x]}\right)^{3}}-114\,\text{ArcTan}\left[e^{\frac{\text{ArcCoth}[x]}{3}}\right]-\right.$$

$$\left.19\,\text{RootSum}\left[1-\#1^{2}+\#1^{4}\,\&,\,\frac{-2\,\text{ArcCoth}[x]+6\,\text{Log}\left[e^{\frac{\text{ArcCoth}[x]}{3}}-\#1\right]+\text{ArcCoth}[x]\,\#1^{2}-3\,\text{Log}\left[e^{\frac{\text{ArcCoth}[x]}{3}}-\#1\right]\#1^{2}}{-\#1+2\,\#1^{3}}\,\&\right]\right)$$

## Problem 123: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{2 \text{ArcCoth}[x]}{3}}}{x} \, dx$$

Optimal (type 3, 155 leaves, 4 steps):

$$-\sqrt{3} \text{ ArcTan}\left[\frac{1}{\sqrt{3}}-\frac{2 \left(\frac{-1+x}{x}\right)^{1/3}}{\sqrt{3} \left(1+\frac{1}{x}\right)^{1/3}}\right]-\sqrt{3} \text{ ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2 \left(\frac{-1+x}{x}\right)^{1/3}}{\sqrt{3} \left(1+\frac{1}{x}\right)^{1/3}}\right]-\frac{3}{2} \text{Log}\left[\left(1+\frac{1}{x}\right)^{1/3}-\left(\frac{-1+x}{x}\right)^{1/3}\right]-\frac{3}{2} \text{Log}\left[1+\frac{\left(\frac{-1+x}{x}\right)^{1/3}}{\left(1+\frac{1}{x}\right)^{1/3}}\right]-\frac{1}{2} \text{Log}\left[1+\frac{1}{x}\right]-\frac{\text{Log}[x]}{2}$$

Result (type 7, 217 leaves):

$$\frac{1}{6} \left(4 \text{ ArcCoth}[x]+3 \left(2 \sqrt{3} \text{ ArcTan}\left[\frac{-1+2 e^{\frac{\text{ArcCoth}[x]}{3}}}{\sqrt{3}}\right]-2 \sqrt{3} \text{ ArcTan}\left[\frac{1+2 e^{\frac{\text{ArcCoth}[x]}{3}}}{\sqrt{3}}\right]-2 \text{ Log}\left[1-e^{\frac{\text{ArcCoth}[x]}{3}}\right]-2 \text{ Log}\left[1+e^{\frac{\text{ArcCoth}[x]}{3}}\right]-2 \text{ Log}\left[1+e^{\frac{2 \text{ArcCoth}[x]}{3}}\right]+\text{Log}\left[1-e^{\frac{\text{ArcCoth}[x]}{3}}+e^{\frac{2 \text{ArcCoth}[x]}{3}}\right]+\text{Log}\left[1+e^{\frac{\text{ArcCoth}[x]}{3}}+e^{\frac{2 \text{ArcCoth}[x]}{3}}\right]\right)+2 \text{ RootSum}\left[1-\#1^2+\#1^4\&,\frac{\text{ArcCoth}[x]-3 \text{ Log}\left[e^{\frac{\text{ArcCoth}[x]}{3}}-\#1\right]+\text{ArcCoth}[x] \#1^2-3 \text{ Log}\left[e^{\frac{\text{ArcCoth}[x]}{3}}-\#1\right] \#1^2}{-2+\#1^2}\&\right]\right)$$

## Problem 124: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{2 \text{ArcCoth}[x]}{3}}}{x^2} \, dx$$

Optimal (type 3, 99 leaves, 3 steps):

$$\left(1+\frac{1}{x}\right)^{1/3} \left(\frac{-1+x}{x}\right)^{2/3}-\frac{2 \text{ ArcTan}\left[\frac{1}{\sqrt{3}}-\frac{2 \left(\frac{-1+x}{x}\right)^{1/3}}{\sqrt{3} \left(1+\frac{1}{x}\right)^{1/3}}\right]}{\sqrt{3}}-\text{Log}\left[1+\frac{\left(\frac{-1+x}{x}\right)^{1/3}}{\left(1+\frac{1}{x}\right)^{1/3}}\right]-\frac{1}{3} \text{Log}\left[1+\frac{1}{x}\right]$$

Result (type 7, 112 leaves):

$$\frac{2}{9}\left(\frac{9\,e^{\frac{2\,\text{ArcCoth}[x]}{3}}}{1+e^{2\,\text{ArcCoth}[x]}}+2\,\text{ArcCoth}[x]-3\,\text{Log}\left[1+e^{\frac{2\,\text{ArcCoth}[x]}{3}}\right]+\right.$$

$$\left.\text{RootSum}\left[1-\#1^2+\#1^4\,\&,\,\frac{\text{ArcCoth}[x]-3\,\text{Log}\left[e^{\frac{\text{ArcCoth}[x]}{3}}-\#1\right]+\text{ArcCoth}[x]\,\#1^2-3\,\text{Log}\left[e^{\frac{\text{ArcCoth}[x]}{3}}-\#1\right]\#1^2}{-2+\#1^2}\,\&\right]\right)$$

## Problem 125: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{2\,\text{ArcCoth}[x]}{3}}}{x^3}\,dx$$

Optimal (type 3, 130 leaves, 4 steps):

$$\frac{1}{3}\left(1+\frac{1}{x}\right)^{1/3}\left(\frac{-1+x}{x}\right)^{2/3}+\frac{1}{2}\left(1+\frac{1}{x}\right)^{4/3}\left(\frac{-1+x}{x}\right)^{2/3}-\frac{2\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}-\frac{2\left(\frac{-1+x}{x}\right)^{1/3}}{\sqrt{3}\left(1+\frac{1}{x}\right)^{1/3}}\right]}{3\sqrt{3}}-\frac{1}{3}\,\text{Log}\left[1+\frac{\left(\frac{-1+x}{x}\right)^{1/3}}{\left(1+\frac{1}{x}\right)^{1/3}}\right]-\frac{1}{9}\,\text{Log}\left[1+\frac{1}{x}\right]$$

Result (type 7, 134 leaves):

$$-\frac{2}{27}\left(\frac{27\,e^{\frac{2\,\text{ArcCoth}[x]}{3}}}{\left(1+e^{2\,\text{ArcCoth}[x]}\right)^2}-\frac{36\,e^{\frac{2\,\text{ArcCoth}[x]}{3}}}{1+e^{2\,\text{ArcCoth}[x]}}-2\,\text{ArcCoth}[x]+3\,\text{Log}\left[1+e^{\frac{2\,\text{ArcCoth}[x]}{3}}\right]-\right.$$

$$\left.\text{RootSum}\left[1-\#1^2+\#1^4\,\&,\,\frac{\text{ArcCoth}[x]-3\,\text{Log}\left[e^{\frac{\text{ArcCoth}[x]}{3}}-\#1\right]+\text{ArcCoth}[x]\,\#1^2-3\,\text{Log}\left[e^{\frac{\text{ArcCoth}[x]}{3}}-\#1\right]\#1^2}{-2+\#1^2}\,\&\right]\right)$$

## Problem 126: Result is not expressed in closed-form.

$$\int e^{\frac{1}{4}\,\text{ArcCoth}[a\,x]}\,x^2\,dx$$

Optimal (type 3, 429 leaves, 19 steps):

$$\frac{37\left(1-\frac{1}{a x}\right)^{7/8}\left(1+\frac{1}{a x}\right)^{1/8} x}{96 a^2}+\frac{3\left(1-\frac{1}{a x}\right)^{7/8}\left(1+\frac{1}{a x}\right)^{1/8} x^2}{8 a}+\frac{1}{3}\left(1-\frac{1}{a x}\right)^{7/8}\left(1+\frac{1}{a x}\right)^{1/8} x^3-\frac{11\,\text{ArcTan}\left[1-\frac{\sqrt{2}\left(1+\frac{1}{a x}\right)^{1/8}}{\left(1-\frac{1}{a x}\right)^{1/8}}\right]}{64\sqrt{2}\, a^3}+\frac{11\,\text{ArcTan}\left[1+\frac{\sqrt{2}\left(1+\frac{1}{a x}\right)^{1/8}}{\left(1-\frac{1}{a x}\right)^{1/8}}\right]}{64\sqrt{2}\, a^3}+$$

$$\frac{11\,\text{ArcTan}\left[\frac{\left(1+\frac{1}{a x}\right)^{1/8}}{\left(1-\frac{1}{a x}\right)^{1/8}}\right]}{64 a^3}+\frac{11\,\text{ArcTanh}\left[\frac{\left(1+\frac{1}{a x}\right)^{1/8}}{\left(1-\frac{1}{a x}\right)^{1/8}}\right]}{64 a^3}-\frac{11\,\text{Log}\left[1-\frac{\sqrt{2}\left(1+\frac{1}{a x}\right)^{1/8}}{\left(1-\frac{1}{a x}\right)^{1/8}}+\frac{\left(1+\frac{1}{a x}\right)^{1/4}}{\left(1-\frac{1}{a x}\right)^{1/4}}\right]}{128\sqrt{2}\, a^3}+\frac{11\,\text{Log}\left[1+\frac{\sqrt{2}\left(1+\frac{1}{a x}\right)^{1/8}}{\left(1-\frac{1}{a x}\right)^{1/8}}+\frac{\left(1+\frac{1}{a x}\right)^{1/4}}{\left(1-\frac{1}{a x}\right)^{1/4}}\right]}{128\sqrt{2}\, a^3}$$

Result (type 7, 167 leaves):

$$\frac{1}{1536 a^3}\left(-4\left(-\frac{1024\, e^{\frac{1}{4}\text{ArcCoth}[a x]}}{\left(-1+e^{2\,\text{ArcCoth}[a x]}\right)^3}-\frac{1600\, e^{\frac{1}{4}\text{ArcCoth}[a x]}}{\left(-1+e^{2\,\text{ArcCoth}[a x]}\right)^2}-\frac{840\, e^{\frac{1}{4}\text{ArcCoth}[a x]}}{-1+e^{2\,\text{ArcCoth}[a x]}}-66\,\text{ArcTan}\left[e^{\frac{1}{4}\text{ArcCoth}[a x]}\right]+\right.\right.$$

$$\left.\left.33\,\text{Log}\left[1-e^{\frac{1}{4}\text{ArcCoth}[a x]}\right]-33\,\text{Log}\left[1+e^{\frac{1}{4}\text{ArcCoth}[a x]}\right]\right)-33\,\text{RootSum}\left[1+\#1^4\,\&,\ \frac{\text{ArcCoth}[a x]-4\,\text{Log}\left[e^{\frac{1}{4}\text{ArcCoth}[a x]}-\#1\right]}{\#1^3}\,\&\right]\right)$$

## Problem 127: Result is not expressed in closed-form.

$$\int e^{\frac{1}{4}\text{ArcCoth}[a x]}\, x\, dx$$

Optimal (type 3, 392 leaves, 17 steps):

$$\frac{\left(1-\frac{1}{a x}\right)^{7/8}\left(1+\frac{1}{a x}\right)^{1/8} x}{8 a}+\frac{1}{2}\left(1-\frac{1}{a x}\right)^{7/8}\left(1+\frac{1}{a x}\right)^{9/8} x^2-\frac{\text{ArcTan}\left[1-\frac{\sqrt{2}\left(1+\frac{1}{a x}\right)^{1/8}}{\left(1-\frac{1}{a x}\right)^{1/8}}\right]}{16\sqrt{2}\, a^2}+\frac{\text{ArcTan}\left[1+\frac{\sqrt{2}\left(1+\frac{1}{a x}\right)^{1/8}}{\left(1-\frac{1}{a x}\right)^{1/8}}\right]}{16\sqrt{2}\, a^2}+$$

$$\frac{\text{ArcTan}\left[\frac{\left(1+\frac{1}{a x}\right)^{1/8}}{\left(1-\frac{1}{a x}\right)^{1/8}}\right]}{16 a^2}+\frac{\text{ArcTanh}\left[\frac{\left(1+\frac{1}{a x}\right)^{1/8}}{\left(1-\frac{1}{a x}\right)^{1/8}}\right]}{16 a^2}-\frac{\text{Log}\left[1-\frac{\sqrt{2}\left(1+\frac{1}{a x}\right)^{1/8}}{\left(1-\frac{1}{a x}\right)^{1/8}}+\frac{\left(1+\frac{1}{a x}\right)^{1/4}}{\left(1-\frac{1}{a x}\right)^{1/4}}\right]}{32\sqrt{2}\, a^2}+\frac{\text{Log}\left[1+\frac{\sqrt{2}\left(1+\frac{1}{a x}\right)^{1/8}}{\left(1-\frac{1}{a x}\right)^{1/8}}+\frac{\left(1+\frac{1}{a x}\right)^{1/4}}{\left(1-\frac{1}{a x}\right)^{1/4}}\right]}{32\sqrt{2}\, a^2}$$

Result (type 7, 141 leaves):

$$\frac{1}{128 a^2}\left(-4\left(-\frac{64\, e^{\frac{1}{4}\text{ArcCoth}[a x]}}{\left(-1+e^{2\,\text{ArcCoth}[a x]}\right)^2}-\frac{72\, e^{\frac{1}{4}\text{ArcCoth}[a x]}}{-1+e^{2\,\text{ArcCoth}[a x]}}-2\,\text{ArcTan}\left[e^{\frac{1}{4}\text{ArcCoth}[a x]}\right]+\text{Log}\left[1-e^{\frac{1}{4}\text{ArcCoth}[a x]}\right]-\text{Log}\left[1+e^{\frac{1}{4}\text{ArcCoth}[a x]}\right]\right)-\right.$$

$$\left.\text{RootSum}\left[1+\#1^4\,\&,\ \frac{\text{ArcCoth}[a x]-4\,\text{Log}\left[e^{\frac{1}{4}\text{ArcCoth}[a x]}-\#1\right]}{\#1^3}\,\&\right]\right)$$

## Problem 128: Result is not expressed in closed-form.

$$\int e^{\frac{1}{4}\text{ArcCoth}[a x]}\,dx$$

Optimal (type 3, 352 leaves, 16 steps):

$$\left(1-\frac{1}{a x}\right)^{7/8}\left(1+\frac{1}{a x}\right)^{1/8} x-\frac{\text{ArcTan}\left[1-\frac{\sqrt{2}\left(1+\frac{1}{a x}\right)^{1/8}}{\left(1-\frac{1}{a x}\right)^{1/8}}\right]}{2\sqrt{2}\, a}+\frac{\text{ArcTan}\left[1+\frac{\sqrt{2}\left(1+\frac{1}{a x}\right)^{1/8}}{\left(1-\frac{1}{a x}\right)^{1/8}}\right]}{2\sqrt{2}\, a}+$$

$$\frac{\text{ArcTan}\left[\frac{\left(1+\frac{1}{a x}\right)^{1/8}}{\left(1-\frac{1}{a x}\right)^{1/8}}\right]}{2 a}+\frac{\text{ArcTanh}\left[\frac{\left(1+\frac{1}{a x}\right)^{1/8}}{\left(1-\frac{1}{a x}\right)^{1/8}}\right]}{2 a}-\frac{\text{Log}\left[1-\frac{\sqrt{2}\left(1+\frac{1}{a x}\right)^{1/8}}{\left(1-\frac{1}{a x}\right)^{1/8}}+\frac{\left(1+\frac{1}{a x}\right)^{1/4}}{\left(1-\frac{1}{a x}\right)^{1/4}}\right]}{4\sqrt{2}\, a}+\frac{\text{Log}\left[1+\frac{\sqrt{2}\left(1+\frac{1}{a x}\right)^{1/8}}{\left(1-\frac{1}{a x}\right)^{1/8}}+\frac{\left(1+\frac{1}{a x}\right)^{1/4}}{\left(1-\frac{1}{a x}\right)^{1/4}}\right]}{4\sqrt{2}\, a}$$

Result (type 7, 117 leaves):

$$\frac{1}{16 a}\left(-4\left(-\frac{8 e^{\frac{1}{4}\text{ArcCoth}[a x]}}{-1+e^{2\,\text{ArcCoth}[a x]}}-2\,\text{ArcTan}\left[e^{\frac{1}{4}\text{ArcCoth}[a x]}\right]+\text{Log}\left[1-e^{\frac{1}{4}\text{ArcCoth}[a x]}\right]-\text{Log}\left[1+e^{\frac{1}{4}\text{ArcCoth}[a x]}\right]\right)-\right.$$

$$\left.\text{RootSum}\left[1+\#1^4\,\&,\ \frac{\text{ArcCoth}[a x]-4\,\text{Log}\left[e^{\frac{1}{4}\text{ArcCoth}[a x]}-\#1\right]}{\#1^3}\,\&\right]\right)$$

## Problem 129: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{1}{4}\text{ArcCoth}[a x]}}{x}\,dx$$

Optimal (type 3, 919 leaves, 39 steps):

$$-\sqrt{2+\sqrt{2}}\;\text{ArcTan}\left[\frac{\sqrt{2-\sqrt{2}}-\frac{2\left(1-\frac{1}{a x}\right)^{1/8}}{\left(1+\frac{1}{a x}\right)^{1/8}}}{\sqrt{2+\sqrt{2}}}\right]-\sqrt{2-\sqrt{2}}\;\text{ArcTan}\left[\frac{\sqrt{2+\sqrt{2}}-\frac{2\left(1-\frac{1}{a x}\right)^{1/8}}{\left(1+\frac{1}{a x}\right)^{1/8}}}{\sqrt{2-\sqrt{2}}}\right]+\sqrt{2+\sqrt{2}}\;\text{ArcTan}\left[\frac{\sqrt{2-\sqrt{2}}+\frac{2\left(1-\frac{1}{a x}\right)^{1/8}}{\left(1+\frac{1}{a x}\right)^{1/8}}}{\sqrt{2+\sqrt{2}}}\right]+$$

$$\sqrt{2-\sqrt{2}}\;\text{ArcTan}\left[\frac{\sqrt{2+\sqrt{2}}+\frac{2\left(1-\frac{1}{a x}\right)^{1/8}}{\left(1+\frac{1}{a x}\right)^{1/8}}}{\sqrt{2-\sqrt{2}}}\right]-\sqrt{2}\;\text{ArcTan}\left[1-\frac{\sqrt{2}\left(1+\frac{1}{a x}\right)^{1/8}}{\left(1-\frac{1}{a x}\right)^{1/8}}\right]+\sqrt{2}\;\text{ArcTan}\left[1+\frac{\sqrt{2}\left(1+\frac{1}{a x}\right)^{1/8}}{\left(1-\frac{1}{a x}\right)^{1/8}}\right]+$$

$$2\,\text{ArcTan}\left[\frac{\left(1+\frac{1}{a x}\right)^{1/8}}{\left(1-\frac{1}{a x}\right)^{1/8}}\right]+2\,\text{ArcTanh}\left[\frac{\left(1+\frac{1}{a x}\right)^{1/8}}{\left(1-\frac{1}{a x}\right)^{1/8}}\right]+\frac{1}{2}\sqrt{2-\sqrt{2}}\;\text{Log}\left[1+\frac{\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}-\frac{\sqrt{2-\sqrt{2}}\left(1-\frac{1}{a x}\right)^{1/8}}{\left(1+\frac{1}{a x}\right)^{1/8}}\right]-$$

$$\frac{1}{2}\sqrt{2-\sqrt{2}}\;\text{Log}\left[1+\frac{\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}+\frac{\sqrt{2-\sqrt{2}}\left(1-\frac{1}{a x}\right)^{1/8}}{\left(1+\frac{1}{a x}\right)^{1/8}}\right]+\frac{1}{2}\sqrt{2+\sqrt{2}}\;\text{Log}\left[1+\frac{\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}-\frac{\sqrt{2+\sqrt{2}}\left(1-\frac{1}{a x}\right)^{1/8}}{\left(1+\frac{1}{a x}\right)^{1/8}}\right]-$$

$$\frac{1}{2}\sqrt{2+\sqrt{2}}\;\text{Log}\left[1+\frac{\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}+\frac{\sqrt{2+\sqrt{2}}\left(1-\frac{1}{a x}\right)^{1/8}}{\left(1+\frac{1}{a x}\right)^{1/8}}\right]-\frac{\text{Log}\left[1-\frac{\sqrt{2}\left(1+\frac{1}{a x}\right)^{1/8}}{\left(1-\frac{1}{a x}\right)^{1/8}}+\frac{\left(1+\frac{1}{a x}\right)^{1/4}}{\left(1-\frac{1}{a x}\right)^{1/4}}\right]}{\sqrt{2}}+\frac{\text{Log}\left[1+\frac{\sqrt{2}\left(1+\frac{1}{a x}\right)^{1/8}}{\left(1-\frac{1}{a x}\right)^{1/8}}+\frac{\left(1+\frac{1}{a x}\right)^{1/4}}{\left(1-\frac{1}{a x}\right)^{1/4}}\right]}{\sqrt{2}}$$

Result (type 7, 128 leaves):

$$2\,\text{ArcTan}\left[e^{\frac{1}{4}\text{ArcCoth}[a x]}\right]-\text{Log}\left[1-e^{\frac{1}{4}\text{ArcCoth}[a x]}\right]+\text{Log}\left[1+e^{\frac{1}{4}\text{ArcCoth}[a x]}\right]-$$

$$\frac{1}{4}\text{RootSum}\left[1+\#1^4\,\&,\;\frac{\text{ArcCoth}[a x]-4\,\text{Log}\left[e^{\frac{1}{4}\text{ArcCoth}[a x]}-\#1\right]}{\#1^3}\,\&\right]-\frac{1}{4}\text{RootSum}\left[1+\#1^8\,\&,\;\frac{-\text{ArcCoth}[a x]+4\,\text{Log}\left[e^{\frac{1}{4}\text{ArcCoth}[a x]}-\#1\right]}{\#1^7}\,\&\right]$$

## Problem 130: Result is not expressed in closed-form.

$$\int\frac{e^{\frac{1}{4}\text{ArcCoth}[a x]}}{x^2}\,dx$$

Optimal (type 3, 676 leaves, 25 steps):

$$a\left(1-\frac{1}{a x}\right)^{7/8}\left(1+\frac{1}{a x}\right)^{1/8}-\frac{1}{4}\sqrt{2+\sqrt{2}}\ a\,\text{ArcTan}\left[\frac{\sqrt{2-\sqrt{2}}-\frac{2\left(1-\frac{1}{a x}\right)^{1/8}}{\left(1+\frac{1}{a x}\right)^{1/8}}}{\sqrt{2+\sqrt{2}}}\right]-\frac{1}{4}\sqrt{2-\sqrt{2}}\ a\,\text{ArcTan}\left[\frac{\sqrt{2+\sqrt{2}}-\frac{2\left(1-\frac{1}{a x}\right)^{1/8}}{\left(1+\frac{1}{a x}\right)^{1/8}}}{\sqrt{2-\sqrt{2}}}\right]+$$

$$\frac{1}{4}\sqrt{2+\sqrt{2}}\ a\,\text{ArcTan}\left[\frac{\sqrt{2-\sqrt{2}}+\frac{2\left(1-\frac{1}{a x}\right)^{1/8}}{\left(1+\frac{1}{a x}\right)^{1/8}}}{\sqrt{2+\sqrt{2}}}\right]+\frac{1}{4}\sqrt{2-\sqrt{2}}\ a\,\text{ArcTan}\left[\frac{\sqrt{2+\sqrt{2}}+\frac{2\left(1-\frac{1}{a x}\right)^{1/8}}{\left(1+\frac{1}{a x}\right)^{1/8}}}{\sqrt{2-\sqrt{2}}}\right]+$$

$$\frac{1}{8}\sqrt{2-\sqrt{2}}\ a\,\text{Log}\left[1+\frac{\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}-\frac{\sqrt{2-\sqrt{2}}\left(1-\frac{1}{a x}\right)^{1/8}}{\left(1+\frac{1}{a x}\right)^{1/8}}\right]-\frac{1}{8}\sqrt{2-\sqrt{2}}\ a\,\text{Log}\left[1+\frac{\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}+\frac{\sqrt{2-\sqrt{2}}\left(1-\frac{1}{a x}\right)^{1/8}}{\left(1+\frac{1}{a x}\right)^{1/8}}\right]+$$

$$\frac{1}{8}\sqrt{2+\sqrt{2}}\ a\,\text{Log}\left[1+\frac{\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}-\frac{\sqrt{2+\sqrt{2}}\left(1-\frac{1}{a x}\right)^{1/8}}{\left(1+\frac{1}{a x}\right)^{1/8}}\right]-\frac{1}{8}\sqrt{2+\sqrt{2}}\ a\,\text{Log}\left[1+\frac{\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}+\frac{\sqrt{2+\sqrt{2}}\left(1-\frac{1}{a x}\right)^{1/8}}{\left(1+\frac{1}{a x}\right)^{1/8}}\right]$$

Result (type 7, 70 leaves):

$$a\left(\frac{2\,e^{\frac{1}{4}\text{ArcCoth}[a x]}}{1+e^{2\,\text{ArcCoth}[a x]}}-\frac{1}{16}\,\text{RootSum}\left[1+\#1^8\,\&,\ \frac{-\text{ArcCoth}[a x]+4\,\text{Log}\left[e^{\frac{1}{4}\text{ArcCoth}[a x]}-\#1\right]}{\#1^7}\,\&\right]\right)$$

## Problem 131: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{1}{4}\text{ArcCoth}[a x]}}{x^3}\,dx$$

Optimal (type 3, 731 leaves, 26 steps):

$$\frac{1}{8}a^2\left(1-\frac{1}{a x}\right)^{7/8}\left(1+\frac{1}{a x}\right)^{1/8}+\frac{1}{2}a^2\left(1-\frac{1}{a x}\right)^{7/8}\left(1+\frac{1}{a x}\right)^{9/8}-\frac{1}{32}\sqrt{2+\sqrt{2}}\ a^2\,\text{ArcTan}\left[\frac{\sqrt{2-\sqrt{2}}-\frac{2\left(1-\frac{1}{a x}\right)^{1/8}}{\left(1+\frac{1}{a x}\right)^{1/8}}}{\sqrt{2+\sqrt{2}}}\right]-$$

$$\frac{1}{32}\sqrt{2-\sqrt{2}}\ a^2\,\text{ArcTan}\left[\frac{\sqrt{2+\sqrt{2}}-\frac{2\left(1-\frac{1}{a x}\right)^{1/8}}{\left(1+\frac{1}{a x}\right)^{1/8}}}{\sqrt{2-\sqrt{2}}}\right]+\frac{1}{32}\sqrt{2+\sqrt{2}}\ a^2\,\text{ArcTan}\left[\frac{\sqrt{2-\sqrt{2}}+\frac{2\left(1-\frac{1}{a x}\right)^{1/8}}{\left(1+\frac{1}{a x}\right)^{1/8}}}{\sqrt{2+\sqrt{2}}}\right]+\frac{1}{32}\sqrt{2-\sqrt{2}}\ a^2\,\text{ArcTan}\left[\frac{\sqrt{2+\sqrt{2}}+\frac{2\left(1-\frac{1}{a x}\right)^{1/8}}{\left(1+\frac{1}{a x}\right)^{1/8}}}{\sqrt{2-\sqrt{2}}}\right]+$$

$$\frac{1}{64}\sqrt{2-\sqrt{2}}\ a^2\,\text{Log}\left[1+\frac{\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}-\frac{\sqrt{2-\sqrt{2}}\left(1-\frac{1}{a x}\right)^{1/8}}{\left(1+\frac{1}{a x}\right)^{1/8}}\right]-\frac{1}{64}\sqrt{2-\sqrt{2}}\ a^2\,\text{Log}\left[1+\frac{\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}+\frac{\sqrt{2-\sqrt{2}}\left(1-\frac{1}{a x}\right)^{1/8}}{\left(1+\frac{1}{a x}\right)^{1/8}}\right]+$$

$$\frac{1}{64}\sqrt{2+\sqrt{2}}\ a^2\,\text{Log}\left[1+\frac{\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}-\frac{\sqrt{2+\sqrt{2}}\left(1-\frac{1}{a x}\right)^{1/8}}{\left(1+\frac{1}{a x}\right)^{1/8}}\right]-\frac{1}{64}\sqrt{2+\sqrt{2}}\ a^2\,\text{Log}\left[1+\frac{\left(1-\frac{1}{a x}\right)^{1/4}}{\left(1+\frac{1}{a x}\right)^{1/4}}+\frac{\sqrt{2+\sqrt{2}}\left(1-\frac{1}{a x}\right)^{1/8}}{\left(1+\frac{1}{a x}\right)^{1/8}}\right]$$

Result (type 7, 85 leaves):

$$\frac{1}{128}a^2\left(\frac{32\,e^{\frac{1}{4}\text{ArcCoth}[a x]}\left(1+9\,e^{2\,\text{ArcCoth}[a x]}\right)}{\left(1+e^{2\,\text{ArcCoth}[a x]}\right)^2}-\text{RootSum}\left[1+\#1^8\,\&,\ \frac{-\text{ArcCoth}[a x]+4\,\text{Log}\left[e^{\frac{1}{4}\text{ArcCoth}[a x]}-\#1\right]}{\#1^7}\,\&\right]\right)$$

## Problem 133: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int e^{3\,\text{ArcCoth}[a x]}\,x^m\,dx$$

Optimal (type 5, 151 leaves, 9 steps):

$$-\frac{3\,x^{1+m}\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{1}{2}(-1-m),\frac{1-m}{2},\frac{1}{a^2x^2}\right]}{1+m}-\frac{x^m\,\text{Hypergeometric2F1}\left[\frac{1}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{a^2x^2}\right]}{a\,m}+$$

$$\frac{4\,x^{1+m}\,\text{Hypergeometric2F1}\left[\frac{3}{2},\frac{1}{2}(-1-m),\frac{1-m}{2},\frac{1}{a^2x^2}\right]}{1+m}+\frac{4\,x^m\,\text{Hypergeometric2F1}\left[\frac{3}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{a^2x^2}\right]}{a\,m}$$

Result (type 6, 381 leaves):

$$\frac{1}{1+m} x^{1+m} \left(\left(4\,(1+m)^2 \sqrt{1-\frac{1}{a^2 x^2}} \sqrt{\frac{1+a x}{a^2}} \,\text{AppellF1}\left[m,-\frac{1}{2},\frac{3}{2},1+m,-a x,a x\right]\right) \Big/ \left(m\,(-1+a x)^{3/2} \sqrt{-\frac{1}{a^2}+x^2} \left(2\,(1+m)\,\text{AppellF1}\left[m,-\frac{1}{2},\frac{3}{2},1+m,-a x,a x\right]+a x \left(3\,\text{AppellF1}\left[1+m,-\frac{1}{2},\frac{5}{2},2+m,-a x,a x\right]+\text{AppellF1}\left[1+m,\frac{1}{2},\frac{3}{2},2+m,-a x,a x\right]\right)\right)\right)+\text{Hypergeometric2F1}\left[-\frac{1}{2},-\frac{1}{2}-\frac{m}{2},\frac{1}{2}-\frac{m}{2},\frac{1}{a^2 x^2}\right]-\left(6\,(1+m)^2 \sqrt{1-\frac{1}{a^2 x^2}} \sqrt{1-a x} \sqrt{\frac{1+a x}{a^2}} \sqrt{1-a^2 x^2} \,\text{AppellF1}\left[m,-\frac{1}{2},\frac{1}{2},1+m,-a x,a x\right]\right) \Big/ \left(m\,(-1+a x)^{3/2} \sqrt{1+a x} \sqrt{-\frac{1}{a^2}+x^2} \left(2\,(1+m)\,\text{AppellF1}\left[m,-\frac{1}{2},\frac{1}{2},1+m,-a x,a x\right]+a x \left(\text{AppellF1}\left[1+m,-\frac{1}{2},\frac{3}{2},2+m,-a x,a x\right]+\text{HypergeometricPFQ}\left[\left\{\frac{1}{2},\frac{1}{2}+\frac{m}{2}\right\},\left\{\frac{3}{2}+\frac{m}{2}\right\},a^2 x^2\right]\right)\right)\right)\right)$$

## Problem 135: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int e^{\text{ArcCoth}[a x]}\, x^m \, dx$$

Optimal (type 5, 74 leaves, 4 steps):

$$\frac{x^{1+m}\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{1}{2}(-1-m),\frac{1-m}{2},\frac{1}{a^2 x^2}\right]}{1+m}+\frac{x^m\,\text{Hypergeometric2F1}\left[\frac{1}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{a^2 x^2}\right]}{a\,m}$$

Result (type 6, 232 leaves):

$$\frac{1}{1+m}x^{1+m}\left(\text{Hypergeometric2F1}\left[-\frac{1}{2},-\frac{1}{2}-\frac{m}{2},\frac{1}{2}-\frac{m}{2},\frac{1}{a^2 x^2}\right]-\left(2\,(1+m)^2\sqrt{1-\frac{1}{a^2 x^2}}\,\sqrt{1-a\,x}\,\sqrt{\frac{1+a\,x}{a^2}}\,\sqrt{1-a^2 x^2}\,\text{AppellF1}\left[m,-\frac{1}{2},\frac{1}{2},1+m,-a\,x,a\,x\right]\right)\Big/\left(m\,(-1+a\,x)^{3/2}\sqrt{1+a\,x}\,\sqrt{-\frac{1}{a^2}+x^2}\left(2\,(1+m)\,\text{AppellF1}\left[m,-\frac{1}{2},\frac{1}{2},1+m,-a\,x,a\,x\right]+a\,x\left(\text{AppellF1}\left[1+m,-\frac{1}{2},\frac{3}{2},2+m,-a\,x,a\,x\right]+\text{HypergeometricPFQ}\left[\left\{\frac{1}{2},\frac{1}{2}+\frac{m}{2}\right\},\left\{\frac{3}{2}+\frac{m}{2}\right\},a^2 x^2\right]\right)\right)\right)\right)$$

## Problem 136: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int e^{-\text{ArcCoth}[a\,x]}\,x^m\,dx$$

Optimal (type 5, 75 leaves, 4 steps):

$$\frac{x^{1+m}\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{1}{2}(-1-m),\frac{1-m}{2},\frac{1}{a^2 x^2}\right]}{1+m}-\frac{x^m\,\text{Hypergeometric2F1}\left[\frac{1}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{a^2 x^2}\right]}{a\,m}$$

Result (type 6, 199 leaves):

$$\frac{1}{1+m}x^{1+m}\left(\text{Hypergeometric2F1}\left[-\frac{1}{2},-\frac{1}{2}-\frac{m}{2},\frac{1}{2}-\frac{m}{2},\frac{1}{a^2 x^2}\right]+\left(2\,(1+m)^2\sqrt{1-\frac{1}{a^2 x^2}}\,\sqrt{\frac{-1+a\,x}{a^2}}\,\text{AppellF1}\left[m,-\frac{1}{2},\frac{1}{2},1+m,a\,x,-a\,x\right]\right)\Big/\left(m\sqrt{1+a\,x}\,\sqrt{-\frac{1}{a^2}+x^2}\left(-2\,(1+m)\,\text{AppellF1}\left[m,-\frac{1}{2},\frac{1}{2},1+m,a\,x,-a\,x\right]+a\,x\left(\text{AppellF1}\left[1+m,-\frac{1}{2},\frac{3}{2},2+m,a\,x,-a\,x\right]+\text{HypergeometricPFQ}\left[\left\{\frac{1}{2},\frac{1}{2}+\frac{m}{2}\right\},\left\{\frac{3}{2}+\frac{m}{2}\right\},a^2 x^2\right]\right)\right)\right)\right)$$

## Problem 138: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int e^{-3\,\text{ArcCoth}[a\,x]}\,x^m\,dx$$

Optimal (type 5, 150 leaves, 9 steps):

$$-\frac{3\,x^{1+m}\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{1}{2}(-1-m),\frac{1-m}{2},\frac{1}{a^2 x^2}\right]}{1+m}+\frac{x^m\,\text{Hypergeometric2F1}\left[\frac{1}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{a^2 x^2}\right]}{a\,m}+\frac{4\,x^{1+m}\,\text{Hypergeometric2F1}\left[\frac{3}{2},\frac{1}{2}(-1-m),\frac{1-m}{2},\frac{1}{a^2 x^2}\right]}{1+m}-\frac{4\,x^m\,\text{Hypergeometric2F1}\left[\frac{3}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{a^2 x^2}\right]}{a\,m}$$

Result (type 6, 349 leaves):

$$\frac{1}{1+m}x^{1+m}\left(\left(4\,(1+m)^2\sqrt{1-\frac{1}{a^2 x^2}}\sqrt{\frac{-1+a\,x}{a^2}}\,\text{AppellF1}\left[m,-\frac{1}{2},\frac{3}{2},1+m,a\,x,-a\,x\right]\right)\Big/\left(m\,(1+a\,x)^{3/2}\sqrt{-\frac{1}{a^2}+x^2}\left(2\,(1+m)\,\text{AppellF1}\left[m,-\frac{1}{2},\frac{3}{2},1+m,a\,x,-a\,x\right]-a\,x\left(3\,\text{AppellF1}\left[1+m,-\frac{1}{2},\frac{5}{2},2+m,a\,x,-a\,x\right]+\text{AppellF1}\left[1+m,\frac{1}{2},\frac{3}{2},2+m,a\,x,-a\,x\right]\right)\right)\right)+\text{Hypergeometric2F1}\left[-\frac{1}{2},-\frac{1}{2}-\frac{m}{2},\frac{1}{2}-\frac{m}{2},\frac{1}{a^2 x^2}\right]+\left(6\,(1+m)^2\sqrt{1-\frac{1}{a^2 x^2}}\sqrt{\frac{-1+a\,x}{a^2}}\,\text{AppellF1}\left[m,-\frac{1}{2},\frac{1}{2},1+m,a\,x,-a\,x\right]\right)\Big/\left(m\sqrt{1+a\,x}\sqrt{-\frac{1}{a^2}+x^2}\left(-2\,(1+m)\,\text{AppellF1}\left[m,-\frac{1}{2},\frac{1}{2},1+m,a\,x,-a\,x\right]+a\,x\left(\text{AppellF1}\left[1+m,-\frac{1}{2},\frac{3}{2},2+m,a\,x,-a\,x\right]+\text{HypergeometricPFQ}\left[\left\{\frac{1}{2},\frac{1}{2}+\frac{m}{2}\right\},\left\{\frac{3}{2}+\frac{m}{2}\right\},a^2 x^2\right]\right)\right)\right)\right)$$

## Problem 139: Unable to integrate problem.

$$\int e^{\frac{5}{2}\,\text{ArcCoth}[a\,x]}\,x^m\,dx$$

Optimal (type 6, 41 leaves, 2 steps):

$$\frac{x^{1+m}\,\text{AppellF1}\left[-1-m,\frac{5}{4},-\frac{5}{4},-m,\frac{1}{a x},-\frac{1}{a x}\right]}{1+m}$$

Result (type 8, 16 leaves) :

$$\int e^{\frac{5}{2}\text{ArcCoth}[a x]}\,x^m\,dx$$

## Problem 140: Unable to integrate problem.

$$\int e^{\frac{3}{2}\text{ArcCoth}[a x]}\,x^m\,dx$$

Optimal (type 6, 41 leaves, 2 steps) :

$$\frac{x^{1+m}\,\text{AppellF1}\left[-1-m,\frac{3}{4},-\frac{3}{4},-m,\frac{1}{a x},-\frac{1}{a x}\right]}{1+m}$$

Result (type 8, 16 leaves) :

$$\int e^{\frac{3}{2}\text{ArcCoth}[a x]}\,x^m\,dx$$

## Problem 141: Unable to integrate problem.

$$\int e^{\frac{1}{2}\text{ArcCoth}[a x]}\,x^m\,dx$$

Optimal (type 6, 41 leaves, 2 steps) :

$$\frac{x^{1+m}\,\text{AppellF1}\left[-1-m,\frac{1}{4},-\frac{1}{4},-m,\frac{1}{a x},-\frac{1}{a x}\right]}{1+m}$$

Result (type 8, 16 leaves) :

$$\int e^{\frac{1}{2}\text{ArcCoth}[a x]}\,x^m\,dx$$

## Problem 142: Unable to integrate problem.

$$\int e^{-\frac{1}{2}\text{ArcCoth}[a x]}\,x^m\,dx$$

Optimal (type 6, 41 leaves, 2 steps) :

$$\frac{x^{1+m}\,\text{AppellF1}\left[-1-m,-\frac{1}{4},\frac{1}{4},-m,\frac{1}{a x},-\frac{1}{a x}\right]}{1+m}$$

Result (type 8, 16 leaves):

$$\int e^{-\frac{1}{2}\text{ArcCoth}[a x]}\,x^m\,dx$$

## Problem 143: Unable to integrate problem.

$$\int e^{-\frac{3}{2}\text{ArcCoth}[a x]}\,x^m\,dx$$

Optimal (type 6, 41 leaves, 2 steps):

$$\frac{x^{1+m}\,\text{AppellF1}\left[-1-m,-\frac{3}{4},\frac{3}{4},-m,\frac{1}{a x},-\frac{1}{a x}\right]}{1+m}$$

Result (type 8, 16 leaves):

$$\int e^{-\frac{3}{2}\text{ArcCoth}[a x]}\,x^m\,dx$$

## Problem 144: Unable to integrate problem.

$$\int e^{-\frac{5}{2}\text{ArcCoth}[a x]}\,x^m\,dx$$

Optimal (type 6, 41 leaves, 2 steps):

$$\frac{x^{1+m}\,\text{AppellF1}\left[-1-m,-\frac{5}{4},\frac{5}{4},-m,\frac{1}{a x},-\frac{1}{a x}\right]}{1+m}$$

Result (type 8, 16 leaves):

$$\int e^{-\frac{5}{2}\text{ArcCoth}[a x]}\,x^m\,dx$$

## Problem 145: Unable to integrate problem.

$$\int e^{\frac{2\,\text{ArcCoth}[x]}{3}}\,x^m\,dx$$

Optimal (type 6, 34 leaves, 2 steps):

$$\frac{x^{1+m}\,\text{AppellF1}\left[-1-m,\frac{1}{3},-\frac{1}{3},-m,\frac{1}{x},-\frac{1}{x}\right]}{1+m}$$

Result (type 8, 14 leaves):

$$\int e^{\frac{2\,\text{ArcCoth}[x]}{3}}\,x^m\,dx$$

## Problem 146: Unable to integrate problem.

$$\int e^{\frac{\text{ArcCoth}[x]}{3}}\,x^m\,dx$$

Optimal (type 6, 34 leaves, 2 steps):

$$\frac{x^{1+m}\,\text{AppellF1}\left[-1-m,\frac{1}{6},-\frac{1}{6},-m,\frac{1}{x},-\frac{1}{x}\right]}{1+m}$$

Result (type 8, 14 leaves):

$$\int e^{\frac{\text{ArcCoth}[x]}{3}}\,x^m\,dx$$

## Problem 147: Unable to integrate problem.

$$\int e^{\frac{1}{4}\text{ArcCoth}[a\,x]}\,x^m\,dx$$

Optimal (type 6, 41 leaves, 2 steps):

$$\frac{x^{1+m}\,\text{AppellF1}\left[-1-m,\frac{1}{8},-\frac{1}{8},-m,\frac{1}{a\,x},-\frac{1}{a\,x}\right]}{1+m}$$

Result (type 8, 16 leaves):

$$\int e^{\frac{1}{4}\text{ArcCoth}[a\,x]}\,x^m\,dx$$

## Problem 148: Unable to integrate problem.

$$\int e^{n\,\text{ArcCoth}[a\,x]}\,x^m\,dx$$

Optimal (type 6, 45 leaves, 2 steps):

$$\frac{x^{1+m}\,\text{AppellF1}\left[-1-m,\frac{n}{2},-\frac{n}{2},-m,\frac{1}{a\,x},-\frac{1}{a\,x}\right]}{1+m}$$

Result (type 8, 14 leaves) :

$$\int e^{n\,\text{ArcCoth}[a\,x]}\,x^m\,dx$$

## Problem 211: Unable to integrate problem.

$$\int \frac{e^{-2\,\text{ArcCoth}[a\,x]}}{c-a\,c\,x}\,dx$$

Optimal (type 3, 14 leaves, 3 steps) :

$$-\frac{\text{Log}[1+a\,x]}{a\,c}$$

Result (type 8, 20 leaves) :

$$\int \frac{e^{-2\,\text{ArcCoth}[a\,x]}}{c-a\,c\,x}\,dx$$

## Problem 212: Unable to integrate problem.

$$\int \frac{e^{-2\,\text{ArcCoth}[a\,x]}}{(c-a\,c\,x)^2}\,dx$$

Optimal (type 3, 12 leaves, 4 steps) :

$$-\frac{\text{ArcTanh}[a\,x]}{a\,c^2}$$

Result (type 8, 20 leaves) :

$$\int \frac{e^{-2\,\text{ArcCoth}[a\,x]}}{(c-a\,c\,x)^2}\,dx$$

## Problem 295: Unable to integrate problem.

$$\int e^{\text{ArcCoth}[a\,x]}\,x^m\,\sqrt{c-a\,c\,x}\,dx$$

Optimal (type 5, 65 leaves, 3 steps) :

$$\frac{2\,x^{1+m}\,\sqrt{c-a\,c\,x}\;\text{Hypergeometric2F1}\left[-\frac{1}{2},\,-\frac{3}{2}-m,\,-\frac{1}{2}-m,\,-\frac{1}{a\,x}\right]}{(3+2\,m)\,\sqrt{1-\frac{1}{a\,x}}}$$

Result (type 8, 23 leaves):

$$\int e^{\text{ArcCoth}[a\,x]}\,x^m\,\sqrt{c-a\,c\,x}\;dx$$

## Problem 335: Unable to integrate problem.

$$\int e^{-\text{ArcCoth}[a\,x]}\,x^m\,\sqrt{c-a\,c\,x}\;dx$$

Optimal (type 5, 131 leaves, 4 steps):

$$\frac{2\,\sqrt{1+\frac{1}{a\,x}}\;x^{1+m}\,\sqrt{c-a\,c\,x}}{(3+2\,m)\,\sqrt{1-\frac{1}{a\,x}}}-\frac{2\,(5+4\,m)\,x^m\,\sqrt{c-a\,c\,x}\;\text{Hypergeometric2F1}\left[\frac{1}{2},\,-\frac{1}{2}-m,\,\frac{1}{2}-m,\,-\frac{1}{a\,x}\right]}{a\,(1+2\,m)\,(3+2\,m)\,\sqrt{1-\frac{1}{a\,x}}}$$

Result (type 8, 25 leaves):

$$\int e^{-\text{ArcCoth}[a\,x]}\,x^m\,\sqrt{c-a\,c\,x}\;dx$$

## Problem 359: Unable to integrate problem.

$$\int e^{n\,\text{ArcCoth}[a\,x]}\,(c-a\,c\,x)^{2+\frac{n}{2}}\;dx$$

Optimal (type 3, 278 leaves, 6 steps):

$$-\frac{\left(56+14\,n+n^2\right)\left(1-\frac{1}{a\,x}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{a\,x}\right)^{\frac{2+n}{2}}(c-a\,c\,x)^{\frac{4+n}{2}}}{a\,(4+n)\,(6+n)}+\frac{2\,\left(56+14\,n+n^2\right)\left(1-\frac{1}{a\,x}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{a\,x}\right)^{\frac{2+n}{2}}(c-a\,c\,x)^{\frac{4+n}{2}}}{a^2\,(6+n)\,\left(8+6\,n+n^2\right)\,x}+$$

$$\frac{(8+n)\left(1-\frac{1}{a\,x}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{a\,x}\right)^{\frac{2+n}{2}}x\,(c-a\,c\,x)^{\frac{4+n}{2}}}{6+n}-\frac{\left(a-\frac{1}{x}\right)\left(1-\frac{1}{a\,x}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{a\,x}\right)^{\frac{2+n}{2}}x\,(c-a\,c\,x)^{\frac{4+n}{2}}}{a}$$

Result (type 8, 26 leaves):

$$\int e^{n\,\text{ArcCoth}[a\,x]}\,(c-a\,c\,x)^{2+\frac{n}{2}}\;dx$$

## Problem 360: Unable to integrate problem.

$$\int e^{n\,\text{ArcCoth}[a\,x]}\,(c-a\,c\,x)^{1+\frac{n}{2}}\,dx$$

Optimal (type 3, 127 leaves, 4 steps):

$$-\frac{2\,(6+n)\left(1-\frac{1}{a\,x}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{a\,x}\right)^{\frac{2+n}{2}}(c-a\,c\,x)^{\frac{2+n}{2}}}{a\,(2+n)\,(4+n)}+\frac{2\left(1-\frac{1}{a\,x}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{a\,x}\right)^{\frac{2+n}{2}}x\,(c-a\,c\,x)^{\frac{2+n}{2}}}{4+n}$$

Result (type 8, 26 leaves):

$$\int e^{n\,\text{ArcCoth}[a\,x]}\,(c-a\,c\,x)^{1+\frac{n}{2}}\,dx$$

## Problem 362: Unable to integrate problem.

$$\int e^{n\,\text{ArcCoth}[a\,x]}\,(c-a\,c\,x)^{-1+\frac{n}{2}}\,dx$$

Optimal (type 5, 80 leaves, 3 steps):

$$\frac{2\left(1-\frac{1}{a\,x}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{a\,x}\right)^{n/2}x\,(c-a\,c\,x)^{\frac{1}{2}(-2+n)}\,\text{Hypergeometric2F1}\left[1,-\frac{n}{2},1-\frac{n}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right]}{n}$$

Result (type 8, 26 leaves):

$$\int e^{n\,\text{ArcCoth}[a\,x]}\,(c-a\,c\,x)^{-1+\frac{n}{2}}\,dx$$

## Problem 363: Unable to integrate problem.

$$\int e^{n\,\text{ArcCoth}[a\,x]}\,(c-a\,c\,x)^{-2+\frac{n}{2}}\,dx$$

Optimal (type 5, 88 leaves, 3 steps):

$$-\frac{2\left(1-\frac{1}{a\,x}\right)^{2-\frac{n}{2}}\left(1+\frac{1}{a\,x}\right)^{\frac{1}{2}(-2+n)}x\,(c-a\,c\,x)^{\frac{1}{2}(-4+n)}\,\text{Hypergeometric2F1}\left[2,1-\frac{n}{2},2-\frac{n}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right]}{2-n}$$

Result (type 8, 26 leaves):

$$\int e^{n\,\text{ArcCoth}[a\,x]}\,(c-a\,c\,x)^{-2+\frac{n}{2}}\,dx$$

## Problem 364: Unable to integrate problem.

$$\int e^{n\,\text{ArcCoth}[a\,x]}\,(c-a\,c\,x)^p\,dx$$

Optimal (type 5, 104 leaves, 3 steps):

$$\frac{\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(n-2p)}\left(1-\frac{1}{a\,x}\right)^{-n/2}\left(1+\frac{1}{a\,x}\right)^{\frac{2+n}{2}}x\,(c-a\,c\,x)^p\,\text{Hypergeometric2F1}\left[\frac{1}{2}(n-2p),\,-1-p,\,-p,\,\frac{2}{\left(a+\frac{1}{x}\right)x}\right]}{1+p}$$

Result (type 8, 20 leaves):

$$\int e^{n\,\text{ArcCoth}[a\,x]}\,(c-a\,c\,x)^p\,dx$$

## Problem 365: Result more than twice size of optimal antiderivative.

$$\int e^{n\,\text{ArcCoth}[a\,x]}\,(c-a\,c\,x)^3\,dx$$

Optimal (type 5, 81 leaves, 3 steps):

$$-\frac{32\,c^3\left(1-\frac{1}{a\,x}\right)^{4-\frac{n}{2}}\left(1+\frac{1}{a\,x}\right)^{\frac{1}{2}(-8+n)}\text{Hypergeometric2F1}\left[5,\,4-\frac{n}{2},\,5-\frac{n}{2},\,\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right]}{a\,(8-n)}$$

Result (type 5, 190 leaves):

$$-\frac{1}{24\,a\,(2+n)}c^3\,e^{n\,\text{ArcCoth}[a\,x]}\left(e^{2\,\text{ArcCoth}[a\,x]}\,n\left(-48+44\,n-12\,n^2+n^3\right)\text{Hypergeometric2F1}\left[1,\,1+\frac{n}{2},\,2+\frac{n}{2},\,e^{2\,\text{ArcCoth}[a\,x]}\right]+\right.$$
$$(2+n)\left(a\,n^3\,x+n^2\left(-1-12\,a\,x+a^2\,x^2\right)+2\,n\left(6+21\,a\,x-6\,a^2\,x^2+a^3\,x^3\right)+\right.$$
$$\left.\left.6\left(-7-4\,a\,x+6\,a^2\,x^2-4\,a^3\,x^3+a^4\,x^4\right)+\left(-48+44\,n-12\,n^2+n^3\right)\text{Hypergeometric2F1}\left[1,\,\frac{n}{2},\,1+\frac{n}{2},\,e^{2\,\text{ArcCoth}[a\,x]}\right]\right)\right)$$

## Problem 372: Unable to integrate problem.

$$\int e^{n\,\text{ArcCoth}[a\,x]}\,(c-a\,c\,x)^{5/2}\,dx$$

Optimal (type 5, 98 leaves, 3 steps):

$$\frac{2}{7}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(-5+n)}\left(1-\frac{1}{a x}\right)^{-n/2}\left(1+\frac{1}{a x}\right)^{\frac{2+n}{2}} x\,(c-a c x)^{5/2}\,\text{Hypergeometric2F1}\left[-\frac{7}{2},\frac{1}{2}(-5+n),-\frac{5}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right]$$

Result (type 8, 22 leaves):

$$\int e^{n\,\text{ArcCoth}[a x]}\,(c-a c x)^{5/2}\,dx$$

## Problem 373: Unable to integrate problem.

$$\int e^{n\,\text{ArcCoth}[a x]}\,(c-a c x)^{3/2}\,dx$$

Optimal (type 5, 98 leaves, 3 steps):

$$\frac{2}{5}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(-3+n)}\left(1-\frac{1}{a x}\right)^{-n/2}\left(1+\frac{1}{a x}\right)^{\frac{2+n}{2}} x\,(c-a c x)^{3/2}\,\text{Hypergeometric2F1}\left[-\frac{5}{2},\frac{1}{2}(-3+n),-\frac{3}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right]$$

Result (type 8, 22 leaves):

$$\int e^{n\,\text{ArcCoth}[a x]}\,(c-a c x)^{3/2}\,dx$$

## Problem 374: Unable to integrate problem.

$$\int e^{n\,\text{ArcCoth}[a x]}\,\sqrt{c-a c x}\,dx$$

Optimal (type 5, 98 leaves, 3 steps):

$$\frac{2}{3}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(-1+n)}\left(1-\frac{1}{a x}\right)^{-n/2}\left(1+\frac{1}{a x}\right)^{\frac{2+n}{2}} x\sqrt{c-a c x}\,\text{Hypergeometric2F1}\left[-\frac{3}{2},\frac{1}{2}(-1+n),-\frac{1}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right]$$

Result (type 8, 22 leaves):

$$\int e^{n\,\text{ArcCoth}[a x]}\,\sqrt{c-a c x}\,dx$$

## Problem 375: Unable to integrate problem.

$$\int \frac{e^{n\,\text{ArcCoth}[a x]}}{\sqrt{c-a c x}}\,dx$$

Optimal (type 5, 96 leaves, 3 steps):

$$\frac{2\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1+n}{2}}\left(1-\frac{1}{a x}\right)^{-n/2}\left(1+\frac{1}{a x}\right)^{\frac{2+n}{2}} x \,\text{Hypergeometric2F1}\left[-\frac{1}{2},\frac{1+n}{2},\frac{1}{2},\frac{2}{\left(a+\frac{1}{x}\right) x}\right]}{\sqrt{c-a c x}}$$

Result (type 8, 22 leaves):

$$\int \frac{e^{n \,\text{ArcCoth}[a x]}}{\sqrt{c-a c x}}\, dx$$

## Problem 376: Unable to integrate problem.

$$\int \frac{e^{n \,\text{ArcCoth}[a x]}}{(c-a c x)^{3/2}}\, dx$$

Optimal (type 5, 96 leaves, 3 steps):

$$-\frac{2\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3+n}{2}}\left(1-\frac{1}{a x}\right)^{-n/2}\left(1+\frac{1}{a x}\right)^{\frac{2+n}{2}} x \,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{3+n}{2},\frac{3}{2},\frac{2}{\left(a+\frac{1}{x}\right) x}\right]}{(c-a c x)^{3/2}}$$

Result (type 8, 22 leaves):

$$\int \frac{e^{n \,\text{ArcCoth}[a x]}}{(c-a c x)^{3/2}}\, dx$$

## Problem 377: Unable to integrate problem.

$$\int \frac{e^{n \,\text{ArcCoth}[a x]}}{(c-a c x)^{5/2}}\, dx$$

Optimal (type 5, 167 leaves, 4 steps):

$$-\frac{a\left(1-\frac{1}{a x}\right)^{\frac{2-n}{2}}\left(1+\frac{1}{a x}\right)^{\frac{2+n}{2}} x^2}{(3+n)\,(c-a c x)^{5/2}}+\frac{a\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3+n}{2}}\left(1-\frac{1}{a x}\right)^{\frac{2-n}{2}}\left(1+\frac{1}{a x}\right)^{\frac{2+n}{2}} x^2 \,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{3+n}{2},\frac{3}{2},\frac{2}{\left(a+\frac{1}{x}\right) x}\right]}{(3+n)\,(c-a c x)^{5/2}}$$

Result (type 8, 22 leaves):

$$\int \frac{e^{n \,\text{ArcCoth}[a x]}}{(c-a c x)^{5/2}}\, dx$$

## Problem 378: Unable to integrate problem.

$$\int \frac{e^{n\,\text{ArcCoth}[a\,x]}}{(c-a\,c\,x)^{7/2}}\,dx$$

Optimal (type 5, 245 leaves, 5 steps):

$$-\frac{a\left(1-\frac{1}{a\,x}\right)^{\frac{2-n}{2}}\left(1+\frac{1}{a\,x}\right)^{\frac{2+n}{2}}x^2}{(5+n)\,(c-a\,c\,x)^{7/2}}+\frac{3\,a^2\left(1-\frac{1}{a\,x}\right)^{\frac{4-n}{2}}\left(1+\frac{1}{a\,x}\right)^{\frac{2+n}{2}}x^3}{2\left(15+8\,n+n^2\right)\,(c-a\,c\,x)^{7/2}}-\frac{3\,a^2\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3+n}{2}}\left(1-\frac{1}{a\,x}\right)^{\frac{4-n}{2}}\left(1+\frac{1}{a\,x}\right)^{\frac{2+n}{2}}x^3\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{3+n}{2},\frac{3}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right]}{2\left(15+8\,n+n^2\right)\,(c-a\,c\,x)^{7/2}}$$

Result (type 8, 22 leaves):

$$\int \frac{e^{n\,\text{ArcCoth}[a\,x]}}{(c-a\,c\,x)^{7/2}}\,dx$$

## Problem 426: Result more than twice size of optimal antiderivative.

$$\int \frac{e^{-2\,\text{ArcCoth}[a\,x]}}{\left(c-\frac{c}{a\,x}\right)^2}\,dx$$

Optimal (type 3, 18 leaves, 6 steps):

$$\frac{x}{c^2}-\frac{\text{ArcTanh}[a\,x]}{a\,c^2}$$

Result (type 3, 39 leaves):

$$\frac{x}{c^2}+\frac{\text{Log}[1-a\,x]}{2\,a\,c^2}-\frac{\text{Log}[1+a\,x]}{2\,a\,c^2}$$

## Problem 545: Attempted integration timed out after 120 seconds.

$$\int e^{n\,\text{ArcCoth}[a\,x]}\left(c-\frac{c}{a\,x}\right)^{3/2}dx$$

Optimal (type 6, 111 leaves, 3 steps):

$$-\frac{2^{\frac{5}{2}-\frac{n}{2}}\left(1+\frac{1}{a\,x}\right)^{\frac{2+n}{2}}\left(c-\frac{c}{a\,x}\right)^{3/2}\text{AppellF1}\left[\frac{2+n}{2},\frac{1}{2}\,(-3+n),2,\frac{4+n}{2},\frac{a+\frac{1}{x}}{2\,a},1+\frac{1}{a\,x}\right]}{a\,(2+n)\left(1-\frac{1}{a\,x}\right)^{3/2}}$$

Result (type 1, 1 leaves):

???

## Problem 546: Attempted integration timed out after 120 seconds.

$$\int e^{n\,\text{ArcCoth}[a\,x]}\sqrt{c-\frac{c}{a\,x}}\,dx$$

Optimal (type 6, 111 leaves, 3 steps):

$$-\frac{2^{\frac{3}{2}-\frac{n}{2}}\left(1+\frac{1}{a\,x}\right)^{\frac{2+n}{2}}\sqrt{c-\frac{c}{a\,x}}\,\text{AppellF1}\left[\frac{2+n}{2},\frac{1}{2}\left(-1+n\right),2,\frac{4+n}{2},\frac{a+\frac{1}{x}}{2\,a},1+\frac{1}{a\,x}\right]}{a\left(2+n\right)\sqrt{1-\frac{1}{a\,x}}}$$

Result (type 1, 1 leaves):

???

## Problem 547: Attempted integration timed out after 120 seconds.

$$\int \frac{e^{n\,\text{ArcCoth}[a\,x]}}{\sqrt{c-\frac{c}{a\,x}}}\,dx$$

Optimal (type 6, 111 leaves, 3 steps):

$$-\frac{2^{\frac{1}{2}-\frac{n}{2}}\sqrt{1-\frac{1}{a\,x}}\left(1+\frac{1}{a\,x}\right)^{\frac{2+n}{2}}\text{AppellF1}\left[\frac{2+n}{2},\frac{1+n}{2},2,\frac{4+n}{2},\frac{a+\frac{1}{x}}{2\,a},1+\frac{1}{a\,x}\right]}{a\left(2+n\right)\sqrt{c-\frac{c}{a\,x}}}$$

Result (type 1, 1 leaves):

???

## Problem 548: Attempted integration timed out after 120 seconds.

$$\int \frac{e^{n\,\text{ArcCoth}[a\,x]}}{\left(c-\frac{c}{a\,x}\right)^{3/2}}\,dx$$

Optimal (type 6, 111 leaves, 3 steps):

$$-\frac{2^{-\frac{1}{2}-\frac{n}{2}} \left(1-\frac{1}{a x}\right)^{3/2} \left(1+\frac{1}{a x}\right)^{\frac{2+n}{2}} \text{AppellF1}\left[\frac{2+n}{2},\frac{3+n}{2},2,\frac{4+n}{2},\frac{a+\frac{1}{x}}{2 a},1+\frac{1}{a x}\right]}{a (2+n) \left(c-\frac{c}{a x}\right)^{3/2}}$$

Result (type 1, 1 leaves):

???

## Problem 549: Unable to integrate problem.

$$\int e^{n \text{ArcCoth}[a x]} \left(c-\frac{c}{a x}\right)^p dx$$

Optimal (type 6, 110 leaves, 3 steps):

$$-\frac{2^{1-\frac{n}{2}+p} \left(1-\frac{1}{a x}\right)^{-p} \left(1+\frac{1}{a x}\right)^{\frac{2+n}{2}} \left(c-\frac{c}{a x}\right)^p \text{AppellF1}\left[\frac{2+n}{2},\frac{1}{2} (n-2 p),2,\frac{4+n}{2},\frac{a+\frac{1}{x}}{2 a},1+\frac{1}{a x}\right]}{a (2+n)}$$

Result (type 8, 24 leaves):

$$\int e^{n \text{ArcCoth}[a x]} \left(c-\frac{c}{a x}\right)^p dx$$

## Problem 550: Unable to integrate problem.

$$\int e^{2 p \text{ArcCoth}[a x]} \left(c-\frac{c}{a x}\right)^p dx$$

Optimal (type 5, 67 leaves, 3 steps):

$$-\frac{\left(1-\frac{1}{a x}\right)^{-p} \left(1+\frac{1}{a x}\right)^{1+p} \left(c-\frac{c}{a x}\right)^p \text{Hypergeometric2F1}\left[2,1+p,2+p,1+\frac{1}{a x}\right]}{a (1+p)}$$

Result (type 8, 25 leaves):

$$\int e^{2 p \text{ArcCoth}[a x]} \left(c-\frac{c}{a x}\right)^p dx$$

## Problem 551: Unable to integrate problem.

$$\int e^{-2 p \text{ArcCoth}[a x]} \left(c-\frac{c}{a x}\right)^p dx$$

Optimal (type 6, 93 leaves, 3 steps):

$$-\frac{4^p \left(1-\frac{1}{a x}\right)^{-p} \left(1+\frac{1}{a x}\right)^{1-p} \left(c-\frac{c}{a x}\right)^p \text{AppellF1}\left[1-p,-2 p,2,2-p,\frac{a+\frac{1}{x}}{2 a},1+\frac{1}{a x}\right]}{a (1-p)}$$

Result (type 8, 25 leaves) :

$$\int e^{-2 p \text{ArcCoth}[a x]} \left(c-\frac{c}{a x}\right)^p dx$$

## Problem 552: Unable to integrate problem.

$$\int e^{2 \text{ArcCoth}[a x]} \left(c-\frac{c}{a x}\right)^p dx$$

Optimal (type 5, 57 leaves, 7 steps) :

$$\left(c-\frac{c}{a x}\right)^p x+\frac{(2-p) \left(c-\frac{c}{a x}\right)^p \text{Hypergeometric2F1}\left[1,p,1+p,1-\frac{1}{a x}\right]}{a p}$$

Result (type 8, 24 leaves) :

$$\int e^{2 \text{ArcCoth}[a x]} \left(c-\frac{c}{a x}\right)^p dx$$

## Problem 553: Unable to integrate problem.

$$\int e^{\text{ArcCoth}[a x]} \left(c-\frac{c}{a x}\right)^p dx$$

Optimal (type 6, 90 leaves, 3 steps) :

$$-\frac{2^{\frac{1}{2}+p} \left(1-\frac{1}{a x}\right)^{-p} \left(1+\frac{1}{a x}\right)^{3/2} \left(c-\frac{c}{a x}\right)^p \text{AppellF1}\left[\frac{3}{2},\frac{1}{2}-p,2,\frac{5}{2},\frac{a+\frac{1}{x}}{2 a},1+\frac{1}{a x}\right]}{3 a}$$

Result (type 8, 22 leaves) :

$$\int e^{\text{ArcCoth}[a x]} \left(c-\frac{c}{a x}\right)^p dx$$

## Problem 554: Unable to integrate problem.

$$\int e^{-\text{ArcCoth}[a x]} \left(c-\frac{c}{a x}\right)^p dx$$

Optimal (type 6, 88 leaves, 3 steps) :

$$-\frac{2^{\frac{3}{2}+p}\left(1-\frac{1}{a x}\right)^{-p}\sqrt{1+\frac{1}{a x}}\left(c-\frac{c}{a x}\right)^{p}\text{AppellF1}\left[\frac{1}{2},-\frac{1}{2}-p,2,\frac{3}{2},\frac{a+\frac{1}{x}}{2 a},1+\frac{1}{a x}\right]}{a}$$

Result (type 8, 24 leaves):

$$\int e^{-\text{ArcCoth}[a x]}\left(c-\frac{c}{a x}\right)^{p}dx$$

## Problem 555: Unable to integrate problem.

$$\int e^{-2\,\text{ArcCoth}[a x]}\left(c-\frac{c}{a x}\right)^{p}dx$$

Optimal (type 5, 114 leaves, 9 steps):

$$\frac{\left(c-\frac{c}{a x}\right)^{2+p}x}{c^2}+\frac{\left(c-\frac{c}{a x}\right)^{2+p}\text{Hypergeometric2F1}\left[1,2+p,3+p,\frac{a-\frac{1}{x}}{2 a}\right]}{2 a c^2(2+p)}-\frac{\left(c-\frac{c}{a x}\right)^{2+p}\text{Hypergeometric2F1}\left[1,2+p,3+p,1-\frac{1}{a x}\right]}{a c^2}$$

Result (type 8, 24 leaves):

$$\int e^{-2\,\text{ArcCoth}[a x]}\left(c-\frac{c}{a x}\right)^{p}dx$$

## Problem 569: Result unnecessarily involves higher level functions.

$$\int\frac{e^{2\,\text{ArcCoth}[a x]}}{c-a^2 c x^2}dx$$

Optimal (type 1, 16 leaves, 3 steps):

$$-\frac{1}{a c(1-a x)}$$

Result (type 3, 18 leaves):

$$\frac{e^{2\,\text{ArcCoth}[a x]}}{2 a c}$$

## Problem 584: Result more than twice size of optimal antiderivative.

$$\int e^{4\,\text{ArcCoth}[a x]}\left(c-a^2 c x^2\right)^2dx$$

Optimal (type 1, 17 leaves, 3 steps):

$$\frac{c^2 (1 + a x)^5}{5 a}$$

Result (type 1, 49 leaves):

$$c^2 x + 2 a c^2 x^2 + 2 a^2 c^2 x^3 + a^3 c^2 x^4 + \frac{1}{5} a^4 c^2 x^5$$

## Problem 586: Result unnecessarily involves higher level functions.

$$\int \frac{e^{4 \text{ArcCoth}[a x]}}{c - a^2 c x^2} \, dx$$

Optimal (type 1, 13 leaves, 3 steps):

$$\frac{x}{c (1 - a x)^2}$$

Result (type 3, 18 leaves):

$$\frac{e^{4 \text{ArcCoth}[a x]}}{4 a c}$$

## Problem 602: Result unnecessarily involves higher level functions.

$$\int \frac{e^{-2 \text{ArcCoth}[a x]}}{c - a^2 c x^2} \, dx$$

Optimal (type 1, 14 leaves, 3 steps):

$$\frac{1}{a c (1 + a x)}$$

Result (type 3, 18 leaves):

$$-\frac{e^{-2 \text{ArcCoth}[a x]}}{2 a c}$$

## Problem 647: Unable to integrate problem.

$$\int \frac{e^{-\text{ArcCoth}[a x]}}{\sqrt{c - a^2 c x^2}} \, dx$$

Optimal (type 3, 37 leaves, 3 steps):

$$\frac{\sqrt{1-\frac{1}{a^2 x^2}}\ x \operatorname{Log}[1+a x]}{\sqrt{c-a^2 c x^2}}$$

Result (type 8, 26 leaves):

$$\int \frac{e^{-\operatorname{ArcCoth}[a x]}}{\sqrt{c-a^2 c x^2}}\, dx$$

## Problem 730: Unable to integrate problem.

$$\int e^{3 \operatorname{ArcCoth}[a x]}\, x^m \sqrt{c-a^2 c x^2}\, dx$$

Optimal (type 5, 136 leaves, 5 steps):

$$\frac{3 x^m \sqrt{c-a^2 c x^2}}{a (1+m) \sqrt{1-\frac{1}{a^2 x^2}}}+\frac{x^{1+m} \sqrt{c-a^2 c x^2}}{(2+m) \sqrt{1-\frac{1}{a^2 x^2}}}-\frac{4 x^m \sqrt{c-a^2 c x^2}\ \text{Hypergeometric2F1}[1, 1+m, 2+m, a x]}{a (1+m) \sqrt{1-\frac{1}{a^2 x^2}}}$$

Result (type 8, 29 leaves):

$$\int e^{3 \operatorname{ArcCoth}[a x]}\, x^m \sqrt{c-a^2 c x^2}\, dx$$

## Problem 731: Result unnecessarily involves higher level functions.

$$\int e^{2 \operatorname{ArcCoth}[a x]}\, x^m \sqrt{c-a^2 c x^2}\, dx$$

Optimal (type 5, 172 leaves, 8 steps):

$$\frac{x^{1+m} \sqrt{c-a^2 c x^2}}{2+m}-\frac{c (3+2 m) x^{1+m} \sqrt{1-a^2 x^2}\ \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, a^2 x^2\right]}{(1+m) (2+m) \sqrt{c-a^2 c x^2}}-\frac{2 a c x^{2+m} \sqrt{1-a^2 x^2}\ \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, a^2 x^2\right]}{(2+m) \sqrt{c-a^2 c x^2}}$$

Result (type 6, 192 leaves):

$$\frac{1}{1+m}x^{1+m}\left(\frac{\sqrt{c-a^2 c x^2}\ \text{Hypergeometric2F1}\left[-\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},a^2 x^2\right]}{\sqrt{1-a^2 x^2}}+\left(4\ (2+m)\ \sqrt{-c\ (1+a x)}\ \text{AppellF1}\left[1+m,\frac{1}{2},-\frac{1}{2},2+m,a x,-a x\right]\right)\Big/\left(\sqrt{-1+a x}\ \left(2\ (2+m)\ \text{AppellF1}\left[1+m,\frac{1}{2},-\frac{1}{2},2+m,a x,-a x\right]+a x\left(\text{AppellF1}\left[2+m,\frac{3}{2},-\frac{1}{2},3+m,a x,-a x\right]+\text{HypergeometricPFQ}\left[\left\{\frac{1}{2},1+\frac{m}{2}\right\},\left\{2+\frac{m}{2}\right\},a^2 x^2\right]\right)\right)\right)\right)$$

## Problem 734: Result unnecessarily involves higher level functions.

$$\int e^{-2\,\text{ArcCoth}[a x]}\ x^m\ \sqrt{c-a^2 c x^2}\ dx$$

Optimal (type 5, 172 leaves, 8 steps):

$$\frac{x^{1+m}\sqrt{c-a^2 c x^2}}{2+m}-\frac{c\ (3+2 m)\ x^{1+m}\sqrt{1-a^2 x^2}\ \text{Hypergeometric2F1}\left[\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},a^2 x^2\right]}{(1+m)\ (2+m)\ \sqrt{c-a^2 c x^2}}+\frac{2 a c x^{2+m}\sqrt{1-a^2 x^2}\ \text{Hypergeometric2F1}\left[\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},a^2 x^2\right]}{(2+m)\ \sqrt{c-a^2 c x^2}}$$

Result (type 6, 191 leaves):

$$\frac{1}{1+m}x^{1+m}\left(\frac{\sqrt{c-a^2 c x^2}\ \text{Hypergeometric2F1}\left[-\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},a^2 x^2\right]}{\sqrt{1-a^2 x^2}}+\left(4\ (2+m)\ \sqrt{c-a c x}\ \text{AppellF1}\left[1+m,\frac{1}{2},-\frac{1}{2},2+m,-a x,a x\right]\right)\Big/\left(\sqrt{1+a x}\ \left(-2\ (2+m)\ \text{AppellF1}\left[1+m,\frac{1}{2},-\frac{1}{2},2+m,-a x,a x\right]+a x\left(\text{AppellF1}\left[2+m,\frac{3}{2},-\frac{1}{2},3+m,-a x,a x\right]+\text{HypergeometricPFQ}\left[\left\{\frac{1}{2},1+\frac{m}{2}\right\},\left\{2+\frac{m}{2}\right\},a^2 x^2\right]\right)\right)\right)\right)$$

## Problem 735: Unable to integrate problem.

$$\int e^{-3\,\text{ArcCoth}[a x]}\ x^m\ \sqrt{c-a^2 c x^2}\ dx$$

Optimal (type 5, 137 leaves, 5 steps):

$$-\frac{3 x^m\sqrt{c-a^2 c x^2}}{a\ (1+m)\ \sqrt{1-\frac{1}{a^2 x^2}}}+\frac{x^{1+m}\sqrt{c-a^2 c x^2}}{(2+m)\ \sqrt{1-\frac{1}{a^2 x^2}}}+\frac{4 x^m\sqrt{c-a^2 c x^2}\ \text{Hypergeometric2F1}[1,1+m,2+m,-a x]}{a\ (1+m)\ \sqrt{1-\frac{1}{a^2 x^2}}}$$

Result (type 8, 29 leaves):

$$\int e^{-3\,\text{ArcCoth}[a\,x]}\,x^m\,\sqrt{c-a^2\,c\,x^2}\,dx$$

## Problem 736: Result more than twice size of optimal antiderivative.

$$\int e^{n\,\text{ArcCoth}[a\,x]}\,\left(c-a^2\,c\,x^2\right)^3\,dx$$

Optimal (type 5, 81 leaves, 3 steps):

$$-\frac{256\,c^3\,\left(1-\frac{1}{a\,x}\right)^{4-\frac{n}{2}}\,\left(1+\frac{1}{a\,x}\right)^{\frac{1}{2}\,(-8+n)}\,\text{Hypergeometric2F1}\left[8,\,4-\frac{n}{2},\,5-\frac{n}{2},\,\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right]}{a\,(8-n)}$$

Result (type 5, 267 leaves):

$$-\frac{1}{5040\,a}c^3\,e^{n\,\text{ArcCoth}[a\,x]}\,\left(-912\,n+58\,n^3-n^5-5040\,a\,x+912\,a\,n^2\,x-58\,a\,n^4\,x+a\,n^6\,x+1368\,a^2\,n\,x^2-64\,a^2\,n^3\,x^2+a^2\,n^5\,x^2+5040\,a^3\,x^3-152\,a^3\,n^2\,x^3+2\,a^3\,n^4\,x^3-576\,a^4\,n\,x^4+6\,a^4\,n^3\,x^4-3024\,a^5\,x^5+24\,a^5\,n^2\,x^5+120\,a^6\,n\,x^6+720\,a^7\,x^7+e^{2\,\text{ArcCoth}[a\,x]}\,n\,\left(-1152+576\,n+104\,n^2-52\,n^3-2\,n^4+n^5\right)\,\text{Hypergeometric2F1}\left[1,\,1+\frac{n}{2},\,2+\frac{n}{2},\,e^{2\,\text{ArcCoth}[a\,x]}\right]+\left(-2304+784\,n^2-56\,n^4+n^6\right)\,\text{Hypergeometric2F1}\left[1,\,\frac{n}{2},\,1+\frac{n}{2},\,e^{2\,\text{ArcCoth}[a\,x]}\right]\right)$$

## Problem 737: Result more than twice size of optimal antiderivative.

$$\int e^{n\,\text{ArcCoth}[a\,x]}\,\left(c-a^2\,c\,x^2\right)^2\,dx$$

Optimal (type 5, 81 leaves, 3 steps):

$$\frac{64\,c^2\,\left(1-\frac{1}{a\,x}\right)^{3-\frac{n}{2}}\,\left(1+\frac{1}{a\,x}\right)^{\frac{1}{2}\,(-6+n)}\,\text{Hypergeometric2F1}\left[6,\,3-\frac{n}{2},\,4-\frac{n}{2},\,\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right]}{a\,(6-n)}$$

Result (type 5, 179 leaves):

$$\frac{1}{120\,a}c^2\,e^{n\,\text{ArcCoth}[a\,x]}\,\left(22\,n-n^3+120\,a\,x-22\,a\,n^2\,x+a\,n^4\,x-28\,a^2\,n\,x^2+a^2\,n^3\,x^2-80\,a^3\,x^3+2\,a^3\,n^2\,x^3+6\,a^4\,n\,x^4+24\,a^5\,x^5+e^{2\,\text{ArcCoth}[a\,x]}\,n\,\left(32-16\,n-2\,n^2+n^3\right)\,\text{Hypergeometric2F1}\left[1,\,1+\frac{n}{2},\,2+\frac{n}{2},\,e^{2\,\text{ArcCoth}[a\,x]}\right]+\left(64-20\,n^2+n^4\right)\,\text{Hypergeometric2F1}\left[1,\,\frac{n}{2},\,1+\frac{n}{2},\,e^{2\,\text{ArcCoth}[a\,x]}\right]\right)$$

## Problem 744: Result more than twice size of optimal antiderivative.

$$\int e^{n \text{ArcCoth}[a x]} \left(c - a^2 c x^2\right)^{3/2} dx$$

Optimal (type 5, 116 leaves, 3 steps):

$$\frac{32 \left(1 - \frac{1}{a x}\right)^{\frac{5-n}{2}} \left(1 + \frac{1}{a x}\right)^{\frac{1}{2}(-5+n)} \left(c - a^2 c x^2\right)^{3/2} \text{Hypergeometric2F1}\left[5, \frac{5-n}{2}, \frac{7-n}{2}, \frac{a - \frac{1}{x}}{a + \frac{1}{x}}\right]}{a^4 (5-n) \left(1 - \frac{1}{a^2 x^2}\right)^{3/2} x^3}$$

Result (type 5, 280 leaves):

$$\frac{1}{192 a \left(c - a^2 c x^2\right)^{3/2}}$$

$$c^2 \left(96 a^3 c \left(1 - \frac{1}{a^2 x^2}\right)^{3/2} x^3 \left(a e^{n \text{ArcCoth}[a x]} \sqrt{1 - \frac{1}{a^2 x^2}}\, x (n + a x) + 2 e^{(1+n) \text{ArcCoth}[a x]} (-1+n) \text{Hypergeometric2F1}\left[1, \frac{1+n}{2}, \frac{3+n}{2}, e^{2 \text{ArcCoth}[a x]}\right]\right) - \right.$$

$$c \left(-1 + a^2 x^2\right)$$

$$\left(2 e^{n \text{ArcCoth}[a x]} \left(-1 + a^2 x^2\right)^2 \left(-a \left(-21 + n^2\right) x + 2 n \left(1 - n^2 + \left(3 + n^2\right) \text{Cosh}[2 \text{ArcCoth}[a x]]\right) + a \left(3 + n^2\right) \sqrt{1 - \frac{1}{a^2 x^2}}\, x \text{Cosh}[3 \text{ArcCoth}[a x]]\right) + \right.$$

$$\left.\left. 16 a e^{(1+n) \text{ArcCoth}[a x]} \left(-3 + 3 n - n^2 + n^3\right) \sqrt{1 - \frac{1}{a^2 x^2}}\, x \text{Hypergeometric2F1}\left[1, \frac{1+n}{2}, \frac{3+n}{2}, e^{2 \text{ArcCoth}[a x]}\right]\right)\right)$$

## Problem 762: Unable to integrate problem.

$$\int e^{n \text{ArcCoth}[a x]} \left(c - a^2 c x^2\right)^p dx$$

Optimal (type 5, 127 leaves, 3 steps):

$$\frac{1}{1+2p} \left(1 - \frac{1}{a^2 x^2}\right)^{-p} \left(\frac{a - \frac{1}{x}}{a + \frac{1}{x}}\right)^{\frac{1}{2}(n-2p)} \left(1 - \frac{1}{a x}\right)^{-\frac{n}{2}+p} \left(1 + \frac{1}{a x}\right)^{1+\frac{n}{2}+p} x \left(c - a^2 c x^2\right)^p \text{Hypergeometric2F1}\left[-1-2p, \frac{1}{2}(n-2p), -2p, \frac{2}{\left(a + \frac{1}{x}\right) x}\right]$$

Result (type 8, 24 leaves):

$$\int e^{n \text{ArcCoth}[a x]} \left(c - a^2 c x^2\right)^p dx$$

# Problem 765: Result more than twice size of optimal antiderivative.

$$\int e^{4\,\text{ArcCoth}[a\,x]}\,\left(c-a^2\,c\,x^2\right)^p\,dx$$

Optimal (type 5, 63 leaves, 4 steps):

$$\frac{2^{2+p}\,c\,(1+a\,x)^{1-p}\,\left(c-a^2\,c\,x^2\right)^{-1+p}\,\text{Hypergeometric2F1}\left[-2-p,\,-1+p,\,p,\,\frac{1}{2}\,(1-a\,x)\right]}{a\,(1-p)}$$

Result (type 5, 159 leaves):

$$\frac{1}{a\,(1+p)}\left(-\left(-1+a\,x\right)^2\right)^{-p}\,(-2+2\,a\,x)^p\,\left(1-a^2\,x^2\right)^{-p}\,\left(c-a^2\,c\,x^2\right)^p\,\left(a\,(1+p)\,x\,\left(\frac{1}{2}-\frac{a\,x}{2}\right)^p\,\text{Hypergeometric2F1}\left[\frac{1}{2},\,-p,\,\frac{3}{2},\,a^2\,x^2\right]-(1+a\,x)\,\left(1-a^2\,x^2\right)^p\,\left(2\,\text{Hypergeometric2F1}\left[1-p,\,1+p,\,2+p,\,\frac{1}{2}\,(1+a\,x)\right]-\text{Hypergeometric2F1}\left[2-p,\,1+p,\,2+p,\,\frac{1}{2}\,(1+a\,x)\right]\right)\right)$$

# Problem 767: Result more than twice size of optimal antiderivative.

$$\int e^{2\,\text{ArcCoth}[a\,x]}\,\left(c-a^2\,c\,x^2\right)^p\,dx$$

Optimal (type 5, 54 leaves, 4 steps):

$$\frac{2^{1+p}\,(1+a\,x)^{-p}\,\left(c-a^2\,c\,x^2\right)^p\,\text{Hypergeometric2F1}\left[-1-p,\,p,\,1+p,\,\frac{1}{2}\,(1-a\,x)\right]}{a\,p}$$

Result (type 5, 133 leaves):

$$\frac{1}{a\,(1+p)}\left(-\left(-1+a\,x\right)^2\right)^{-p}\,(-2+2\,a\,x)^p\,\left(1-a^2\,x^2\right)^{-p}\,\left(c-a^2\,c\,x^2\right)^p\,\left(a\,(1+p)\,x\,\left(\frac{1}{2}-\frac{a\,x}{2}\right)^p\,\text{Hypergeometric2F1}\left[\frac{1}{2},\,-p,\,\frac{3}{2},\,a^2\,x^2\right]-(1+a\,x)\,\left(1-a^2\,x^2\right)^p\,\text{Hypergeometric2F1}\left[1-p,\,1+p,\,2+p,\,\frac{1}{2}\,(1+a\,x)\right]\right)$$

# Problem 770: Result more than twice size of optimal antiderivative.

$$\int e^{-2\,\text{ArcCoth}[a\,x]}\,\left(c-a^2\,c\,x^2\right)^p\,dx$$

Optimal (type 5, 55 leaves, 4 steps):

$$-\frac{2^{1+p}\,(1-a\,x)^{-p}\,\left(c-a^2\,c\,x^2\right)^p\,\text{Hypergeometric2F1}\left[-1-p,\,p,\,1+p,\,\frac{1}{2}\,(1+a\,x)\right]}{a\,p}$$

Result (type 5, 125 leaves):

$$\frac{1}{a\,(1+p)}2^{p}\,(1+a\,x)^{-p}\,(1-a^2\,x^2)^{-p}\,(c-a^2\,c\,x^2)^{p}$$
$$\left(a\,(1+p)\,x\left(\frac{1}{2}+\frac{a\,x}{2}\right)^{p}\text{Hypergeometric2F1}\left[\frac{1}{2},-p,\frac{3}{2},a^2\,x^2\right]-(-1+a\,x)\,(1-a^2\,x^2)^{p}\,\text{Hypergeometric2F1}\left[1-p,1+p,2+p,\frac{1}{2}-\frac{a\,x}{2}\right]\right)$$

## Problem 933: Unable to integrate problem.

$$\int e^{n\,\text{ArcCoth}[a\,x]}\left(c-\frac{c}{a^2\,x^2}\right)^{p}dx$$

Optimal (type 6, 116 leaves, 3 steps):

$$-\frac{2^{1-\frac{n}{2}+p}\left(1-\frac{1}{a^2\,x^2}\right)^{-p}\left(c-\frac{c}{a^2\,x^2}\right)^{p}\left(1+\frac{1}{a\,x}\right)^{1+\frac{n}{2}+p}\text{AppellF1}\left[1+\frac{n}{2}+p,\frac{1}{2}\,(n-2\,p),2,2+\frac{n}{2}+p,\frac{a+\frac{1}{x}}{2\,a},1+\frac{1}{a\,x}\right]}{a\,(2+n+2\,p)}$$

Result (type 8, 24 leaves):

$$\int e^{n\,\text{ArcCoth}[a\,x]}\left(c-\frac{c}{a^2\,x^2}\right)^{p}dx$$

## Problem 934: Unable to integrate problem.

$$\int e^{-2\,p\,\text{ArcCoth}[a\,x]}\left(c-\frac{c}{a^2\,x^2}\right)^{p}dx$$

Optimal (type 5, 76 leaves, 3 steps):

$$\frac{\left(1-\frac{1}{a^2\,x^2}\right)^{-p}\left(c-\frac{c}{a^2\,x^2}\right)^{p}\left(1-\frac{1}{a\,x}\right)^{1+2\,p}\text{Hypergeometric2F1}\left[2,1+2\,p,2\,(1+p),1-\frac{1}{a\,x}\right]}{a\,(1+2\,p)}$$

Result (type 8, 25 leaves):

$$\int e^{-2\,p\,\text{ArcCoth}[a\,x]}\left(c-\frac{c}{a^2\,x^2}\right)^{p}dx$$

## Problem 935: Unable to integrate problem.

$$\int e^{2\,p\,\text{ArcCoth}[a\,x]}\left(c-\frac{c}{a^2\,x^2}\right)^{p}dx$$

Optimal (type 5, 75 leaves, 3 steps):

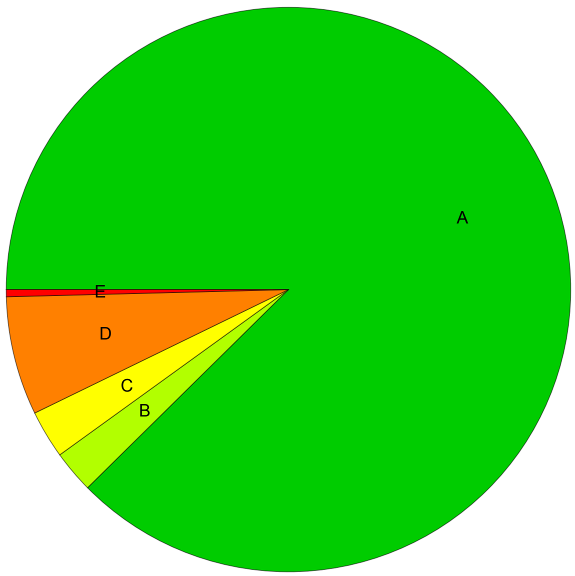
$$-\frac{\left(1-\frac{1}{a^2 x^2}\right)^{-p} \left(c-\frac{c}{a^2 x^2}\right)^p \left(1+\frac{1}{a x}\right)^{1+2 p} \text{Hypergeometric2F1}\left[2,1+2 p,2 (1+p),1+\frac{1}{a x}\right]}{a (1+2 p)}$$

Result (type 8, 25 leaves) :

$$\int e^{2 p \,\text{ArcCoth}[a x]} \left(c-\frac{c}{a^2 x^2}\right)^p dx$$

# Summary of Integration Test Results

1235 integration problems




A - 1082 optimal antiderivatives

B - 30 more than twice size of optimal antiderivatives

C - 34 unnecessarily complex antiderivatives

D - 84 unable to integrate problems

E - 5 integration timeouts