Mathematica 11.3 Integration Test Results

on the problems in the test-suite directory "7 Inverse hyperbolic functions\7.6 Inverse hyperbolic cosecant"

Test results for the 178 problems in "7.6.1 u (a+b arccsch(c x))^n.m"

Problem 19: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b\,\text{ArcCsch}[c\,x])^2}{x}\,dx$$

Optimal (type 4, 81 leaves, 6 steps):

$$\frac{(a + b\,\text{ArcCsch}[c\,x])^3}{3\,b} - (a + b\,\text{ArcCsch}[c\,x])^2\,\text{Log}\left[1 - e^{2\,\text{ArcCsch}[c\,x]}\right] - b\,(a + b\,\text{ArcCsch}[c\,x])\,\text{PolyLog}\left[2, e^{2\,\text{ArcCsch}[c\,x]}\right] + \frac{1}{2}\,b^2\,\text{PolyLog}\left[3, e^{2\,\text{ArcCsch}[c\,x]}\right]$$

Result (type 4, 121 leaves):

$$a^2\,\text{Log}[c\,x] + a\,b\,\left(-\text{ArcCsch}[c\,x]\,\left(\text{ArcCsch}[c\,x] + 2\,\text{Log}\left[1 - e^{-2\,\text{ArcCsch}[c\,x]}\right]\right) + \text{PolyLog}\left[2, e^{-2\,\text{ArcCsch}[c\,x]}\right]\right) + \frac{1}{24}\,b^2\,\left(-i\,\pi^3 + 8\,\text{ArcCsch}[c\,x]^3 - 24\,\text{ArcCsch}[c\,x]^2\,\text{Log}\left[1 - e^{2\,\text{ArcCsch}[c\,x]}\right] - 24\,\text{ArcCsch}[c\,x]\,\text{PolyLog}\left[2, e^{2\,\text{ArcCsch}[c\,x]}\right] + 12\,\text{PolyLog}\left[3, e^{2\,\text{ArcCsch}[c\,x]}\right]\right)$$

Problem 25: Result more than twice size of optimal antiderivative.

$$\int x^2\,(a + b\,\text{ArcCsch}[c\,x])^3\,dx$$

Optimal (type 4, 194 leaves, 11 steps):

$$\frac{b^2 x \left(a+b\,\text{ArcCsch}[c\,x]\right)}{c^2}+\frac{b\sqrt{1+\frac{1}{c^2 x^2}}\;x^2\left(a+b\,\text{ArcCsch}[c\,x]\right)^2}{2\,c}+\frac{1}{3}x^3\left(a+b\,\text{ArcCsch}[c\,x]\right)^3-$$

$$\frac{b\left(a+b\,\text{ArcCsch}[c\,x]\right)^2\text{ArcTanh}\left[e^{\text{ArcCsch}[c\,x]}\right]}{c^3}+\frac{b^3\,\text{ArcTanh}\left[\sqrt{1+\frac{1}{c^2 x^2}}\right]}{c^3}-\frac{b^2\left(a+b\,\text{ArcCsch}[c\,x]\right)\text{PolyLog}\left[2,-e^{\text{ArcCsch}[c\,x]}\right]}{c^3}+$$

$$\frac{b^2\left(a+b\,\text{ArcCsch}[c\,x]\right)\text{PolyLog}\left[2,e^{\text{ArcCsch}[c\,x]}\right]}{c^3}+\frac{b^3\,\text{PolyLog}\left[3,-e^{\text{ArcCsch}[c\,x]}\right]}{c^3}-\frac{b^3\,\text{PolyLog}\left[3,e^{\text{ArcCsch}[c\,x]}\right]}{c^3}$$

Result (type 4, 548 leaves) :

$$\frac{a^3 x^3}{3}+\frac{a^2 b\,x^2\sqrt{\frac{1+c^2 x^2}{c^2 x^2}}}{2\,c}+a^2 b\,x^3\,\text{ArcCsch}[c\,x]-\frac{a^2 b\,\text{Log}\left[x\left(1+\sqrt{\frac{1+c^2 x^2}{c^2 x^2}}\right)\right]}{2\,c^3}+$$

$$\frac{1}{8\,c^3}a\,b^2\left(8\,\text{PolyLog}\left[2,-e^{-\text{ArcCsch}[c\,x]}\right]+2\,c^3 x^3\left(-2+4\,\text{ArcCsch}[c\,x]^2+2\,\text{Cosh}[2\,\text{ArcCsch}[c\,x]]-\frac{3\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-e^{-\text{ArcCsch}[c\,x]}\right]}{c\,x}+\right.\right.$$

$$\frac{3\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1+e^{-\text{ArcCsch}[c\,x]}\right]}{c\,x}-\frac{4\,\text{PolyLog}\left[2,e^{-\text{ArcCsch}[c\,x]}\right]}{c^3 x^3}+2\,\text{ArcCsch}[c\,x]\,\text{Sinh}[2\,\text{ArcCsch}[c\,x]]+$$

$$\left.\left.\text{ArcCsch}[c\,x]\,\text{Log}\left[1-e^{-\text{ArcCsch}[c\,x]}\right]\text{Sinh}[3\,\text{ArcCsch}[c\,x]]-\text{ArcCsch}[c\,x]\,\text{Log}\left[1+e^{-\text{ArcCsch}[c\,x]}\right]\text{Sinh}[3\,\text{ArcCsch}[c\,x]]\right)\right)+$$

$$\frac{1}{48\,c^3}b^3\left(24\,\text{ArcCsch}[c\,x]\,\text{Coth}\left[\frac{1}{2}\text{ArcCsch}[c\,x]\right]-4\,\text{ArcCsch}[c\,x]^3\,\text{Coth}\left[\frac{1}{2}\text{ArcCsch}[c\,x]\right]+6\,\text{ArcCsch}[c\,x]^2\,\text{Csch}\left[\frac{1}{2}\text{ArcCsch}[c\,x]\right]^2+\right.$$

$$\frac{\text{ArcCsch}[c\,x]^3\,\text{Csch}\left[\frac{1}{2}\text{ArcCsch}[c\,x]\right]^4}{c\,x}+24\,\text{ArcCsch}[c\,x]^2\,\text{Log}\left[1-e^{-\text{ArcCsch}[c\,x]}\right]-24\,\text{ArcCsch}[c\,x]^2\,\text{Log}\left[1+e^{-\text{ArcCsch}[c\,x]}\right]-$$

$$48\,\text{Log}\left[\text{Tanh}\left[\frac{1}{2}\text{ArcCsch}[c\,x]\right]\right]+48\,\text{ArcCsch}[c\,x]\,\text{PolyLog}\left[2,-e^{-\text{ArcCsch}[c\,x]}\right]-48\,\text{ArcCsch}[c\,x]\,\text{PolyLog}\left[2,e^{-\text{ArcCsch}[c\,x]}\right]+$$

$$48\,\text{PolyLog}\left[3,-e^{-\text{ArcCsch}[c\,x]}\right]-48\,\text{PolyLog}\left[3,e^{-\text{ArcCsch}[c\,x]}\right]+6\,\text{ArcCsch}[c\,x]^2\,\text{Sech}\left[\frac{1}{2}\text{ArcCsch}[c\,x]\right]^2+$$

$$\left.16\,c^3 x^3\,\text{ArcCsch}[c\,x]^3\,\text{Sinh}\left[\frac{1}{2}\text{ArcCsch}[c\,x]\right]^4-24\,\text{ArcCsch}[c\,x]\,\text{Tanh}\left[\frac{1}{2}\text{ArcCsch}[c\,x]\right]+4\,\text{ArcCsch}[c\,x]^3\,\text{Tanh}\left[\frac{1}{2}\text{ArcCsch}[c\,x]\right]\right)$$

Problem 27: Result more than twice size of optimal antiderivative.

$$\int (a + b\,\text{ArcCsch}[c\,x])^3\,dx$$

Optimal (type 4, 120 leaves, 9 steps):

$$x\,(a + b\,\text{ArcCsch}[c\,x])^3 + \frac{6\,b\,(a + b\,\text{ArcCsch}[c\,x])^2\,\text{ArcTanh}\left[e^{\text{ArcCsch}[c\,x]}\right]}{c} + \frac{6\,b^2\,(a + b\,\text{ArcCsch}[c\,x])\,\text{PolyLog}\left[2, -e^{\text{ArcCsch}[c\,x]}\right]}{c} - \frac{6\,b^2\,(a + b\,\text{ArcCsch}[c\,x])\,\text{PolyLog}\left[2, e^{\text{ArcCsch}[c\,x]}\right]}{c} - \frac{6\,b^3\,\text{PolyLog}\left[3, -e^{\text{ArcCsch}[c\,x]}\right]}{c} + \frac{6\,b^3\,\text{PolyLog}\left[3, e^{\text{ArcCsch}[c\,x]}\right]}{c}$$

Result (type 4, 246 leaves):

$$a^3\,x + 3\,a^2\,b\,x\,\text{ArcCsch}[c\,x] + \frac{3\,a^2\,b\,\text{Log}\left[c\,x\left(1 + \sqrt{\frac{1+c^2\,x^2}{c^2\,x^2}}\right)\right]}{c} + \frac{1}{c}3\,a\,b^2 \left(\text{ArcCsch}[c\,x]\,\left(c\,x\,\text{ArcCsch}[c\,x] - 2\,\text{Log}\left[1 - e^{-\text{ArcCsch}[c\,x]}\right] + 2\,\text{Log}\left[1 + e^{-\text{ArcCsch}[c\,x]}\right]\right) - 2\,\text{PolyLog}\left[2, -e^{-\text{ArcCsch}[c\,x]}\right] + 2\,\text{PolyLog}\left[2, e^{-\text{ArcCsch}[c\,x]}\right]\right) + \frac{1}{c} b^3\,\left(c\,x\,\text{ArcCsch}[c\,x]^3 - 3\,\text{ArcCsch}[c\,x]^2\,\text{Log}\left[1 - e^{-\text{ArcCsch}[c\,x]}\right] + 3\,\text{ArcCsch}[c\,x]^2\,\text{Log}\left[1 + e^{-\text{ArcCsch}[c\,x]}\right] - 6\,\text{ArcCsch}[c\,x]\,\text{PolyLog}\left[2, -e^{-\text{ArcCsch}[c\,x]}\right] + 6\,\text{ArcCsch}[c\,x]\,\text{PolyLog}\left[2, e^{-\text{ArcCsch}[c\,x]}\right] - 6\,\text{PolyLog}\left[3, -e^{-\text{ArcCsch}[c\,x]}\right] + 6\,\text{PolyLog}\left[3, e^{-\text{ArcCsch}[c\,x]}\right]\right)$$

Problem 28: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b\,\text{ArcCsch}[c\,x])^3}{x}\,dx$$

Optimal (type 4, 110 leaves, 7 steps):

$$\frac{(a + b\,\text{ArcCsch}[c\,x])^4}{4\,b} - (a + b\,\text{ArcCsch}[c\,x])^3\,\text{Log}\left[1 - e^{2\,\text{ArcCsch}[c\,x]}\right] - \frac{3}{2}\,b\,(a + b\,\text{ArcCsch}[c\,x])^2\,\text{PolyLog}\left[2, e^{2\,\text{ArcCsch}[c\,x]}\right] + \frac{3}{2}\,b^2\,(a + b\,\text{ArcCsch}[c\,x])\,\text{PolyLog}\left[3, e^{2\,\text{ArcCsch}[c\,x]}\right] - \frac{3}{4}\,b^3\,\text{PolyLog}\left[4, e^{2\,\text{ArcCsch}[c\,x]}\right]$$

Result (type 4, 213 leaves):

$$a^3 \,\text{Log}[c\,x] + \frac{3}{2} a^2 b \left(-\text{ArcCsch}[c\,x] \left(\text{ArcCsch}[c\,x] + 2\,\text{Log}\left[1 - e^{-2\,\text{ArcCsch}[c\,x]}\right]\right) + \text{PolyLog}\left[2, e^{-2\,\text{ArcCsch}[c\,x]}\right]\right) +$$
$$\frac{1}{8} a\,b^2 \left(-i\,\pi^3 + 8\,\text{ArcCsch}[c\,x]^3 - 24\,\text{ArcCsch}[c\,x]^2 \,\text{Log}\left[1 - e^{2\,\text{ArcCsch}[c\,x]}\right] - 24\,\text{ArcCsch}[c\,x]\,\text{PolyLog}\left[2, e^{2\,\text{ArcCsch}[c\,x]}\right] + 12\,\text{PolyLog}\left[3, e^{2\,\text{ArcCsch}[c\,x]}\right]\right) -$$
$$\frac{1}{64} b^3 \left(\pi^4 - 16\,\text{ArcCsch}[c\,x]^4 + 64\,\text{ArcCsch}[c\,x]^3\,\text{Log}\left[1 - e^{2\,\text{ArcCsch}[c\,x]}\right] + 96\,\text{ArcCsch}[c\,x]^2\,\text{PolyLog}\left[2, e^{2\,\text{ArcCsch}[c\,x]}\right] - 96\,\text{ArcCsch}[c\,x]\,\text{PolyLog}\left[3, e^{2\,\text{ArcCsch}[c\,x]}\right] + 48\,\text{PolyLog}\left[4, e^{2\,\text{ArcCsch}[c\,x]}\right]\right)$$

Problem 48: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{a + b\,\text{ArcCsch}[c\,x]}{d + e\,x}\,dx$$

Optimal (type 4, 215 leaves, 4 steps):

$$\frac{(a + b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1 - \frac{\left(e-\sqrt{c^2 d^2+e^2}\right) e^{\text{ArcCsch}[c\,x]}}{c\,d}\right]}{e} + \frac{(a + b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1 - \frac{\left(e+\sqrt{c^2 d^2+e^2}\right) e^{\text{ArcCsch}[c\,x]}}{c\,d}\right]}{e} -$$
$$\frac{(a + b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1 - e^{2\,\text{ArcCsch}[c\,x]}\right]}{e} + \frac{b\,\text{PolyLog}\left[2, \frac{\left(e-\sqrt{c^2 d^2+e^2}\right) e^{\text{ArcCsch}[c\,x]}}{c\,d}\right]}{e} + \frac{b\,\text{PolyLog}\left[2, \frac{\left(e+\sqrt{c^2 d^2+e^2}\right) e^{\text{ArcCsch}[c\,x]}}{c\,d}\right]}{e} - \frac{b\,\text{PolyLog}\left[2, e^{2\,\text{ArcCsch}[c\,x]}\right]}{2\,e}$$

Result (type 4, 506 leaves):

$$\frac{a\,\text{Log}[d+e\,x]}{e}+\frac{1}{8\,e}b\left(\pi^2-4\,i\,\pi\,\text{ArcCsch}[c\,x]-8\,\text{ArcCsch}[c\,x]^2-32\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\,e}{c\,d}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{(i\,c\,d+e)\,\text{Cot}\left[\frac{1}{4}\,(\pi+2\,i\,\text{ArcCsch}[c\,x])\right]}{\sqrt{c^2\,d^2+e^2}}\right]-\right.$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-e^{-2\,\text{ArcCsch}[c\,x]}\right]+4\,i\,\pi\,\text{Log}\left[1+\frac{\left(-e+\sqrt{c^2\,d^2+e^2}\right)e^{\text{ArcCsch}[c\,x]}}{c\,d}\right]+8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1+\frac{\left(-e+\sqrt{c^2\,d^2+e^2}\right)e^{\text{ArcCsch}[c\,x]}}{c\,d}\right]+$$

$$16\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\,e}{c\,d}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{\left(-e+\sqrt{c^2\,d^2+e^2}\right)e^{\text{ArcCsch}[c\,x]}}{c\,d}\right]+4\,i\,\pi\,\text{Log}\left[1-\frac{\left(e+\sqrt{c^2\,d^2+e^2}\right)e^{\text{ArcCsch}[c\,x]}}{c\,d}\right]+$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-\frac{\left(e+\sqrt{c^2\,d^2+e^2}\right)e^{\text{ArcCsch}[c\,x]}}{c\,d}\right]-16\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\,e}{c\,d}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{\left(e+\sqrt{c^2\,d^2+e^2}\right)e^{\text{ArcCsch}[c\,x]}}{c\,d}\right]-4\,i\,\pi\,\text{Log}\left[e+\frac{d}{x}\right]+$$

$$\left.4\,\text{PolyLog}\left[2,e^{-2\,\text{ArcCsch}[c\,x]}\right]+8\,\text{PolyLog}\left[2,\frac{\left(e-\sqrt{c^2\,d^2+e^2}\right)e^{\text{ArcCsch}[c\,x]}}{c\,d}\right]+8\,\text{PolyLog}\left[2,\frac{\left(e+\sqrt{c^2\,d^2+e^2}\right)e^{\text{ArcCsch}[c\,x]}}{c\,d}\right]\right)$$

Problem 51: Result unnecessarily involves imaginary or complex numbers.

$$\int x^2\,\sqrt{d+e\,x}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)\,dx$$

Optimal (type 4, 918 leaves, 31 steps):

$$-\frac{4\,b\,d\,\sqrt{d+e\,x}\,\left(1+c^2\,x^2\right)}{105\,c^3\,e\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x}+\frac{4\,b\,(d+e\,x)^{3/2}\,\left(1+c^2\,x^2\right)}{35\,c^3\,e\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x}+\frac{2\,d^2\,(d+e\,x)^{3/2}\,(a+b\,\text{ArcCsch}[c\,x])}{3\,e^3}-\frac{4\,d\,(d+e\,x)^{5/2}\,(a+b\,\text{ArcCsch}[c\,x])}{5\,e^3}+$$

$$\frac{2\,(d+e\,x)^{7/2}\,(a+b\,\text{ArcCsch}[c\,x])}{7\,e^3}-\frac{32\,b\,c\,d^2\,\sqrt{d+e\,x}\,\sqrt{1+c^2\,x^2}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],-\frac{2\,\sqrt{-c^2}\,e}{c^2\,d-\sqrt{-c^2}\,e}\right]}{105\,\left(-c^2\right)^{3/2}\,e^2\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\sqrt{\frac{c^2\,(d+e\,x)}{c^2\,d-\sqrt{-c^2}\,e}}}-$$

$$\frac{4\,b\,c\,\left(c^2\,d^2-3\,e^2\right)\,\sqrt{d+e\,x}\,\sqrt{1+c^2\,x^2}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],-\frac{2\,\sqrt{-c^2}\,e}{c^2\,d-\sqrt{-c^2}\,e}\right]}{35\,\left(-c^2\right)^{5/2}\,e^2\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\sqrt{\frac{c^2\,(d+e\,x)}{c^2\,d-\sqrt{-c^2}\,e}}}+$$

$$\frac{32\,b\,c\,d^3\,\sqrt{\frac{c^2\,(d+e\,x)}{c^2\,d-\sqrt{-c^2}\,e}}\,\sqrt{1+c^2\,x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],-\frac{2\,\sqrt{-c^2}\,e}{c^2\,d-\sqrt{-c^2}\,e}\right]}{105\,\left(-c^2\right)^{3/2}\,e^2\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}}-$$

$$\frac{4\,b\,c\,d\,\left(c^2\,d^2+e^2\right)\,\sqrt{\frac{c^2\,(d+e\,x)}{c^2\,d-\sqrt{-c^2}\,e}}\,\sqrt{1+c^2\,x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],-\frac{2\,\sqrt{-c^2}\,e}{c^2\,d-\sqrt{-c^2}\,e}\right]}{105\,\left(-c^2\right)^{5/2}\,e^2\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}}-$$

$$\frac{32\,b\,d^4\,\sqrt{\frac{\sqrt{-c^2}\,(d+e\,x)}{\sqrt{-c^2}\,d+e}}\,\sqrt{1+c^2\,x^2}\,\text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],\frac{2\,e}{\sqrt{-c^2}\,d+e}\right]}{105\,c\,e^3\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}}$$

Result (type 4, 483 leaves) :

$$\frac{1}{105\,e^3}\,2\left(\frac{2\,b\,e^2\,\sqrt{1+\frac{1}{c^2\,x^2}}\;x\,\sqrt{d+e\,x}\,\left(2\,d+3\,e\,x\right)}{c}+a\,\sqrt{d+e\,x}\,\left(8\,d^3-4\,d^2\,e\,x+3\,d\,e^2\,x^2+15\,e^3\,x^3\right)+\right.$$

$$b\,\sqrt{d+e\,x}\,\left(8\,d^3-4\,d^2\,e\,x+3\,d\,e^2\,x^2+15\,e^3\,x^3\right)\,\text{ArcCsch}[c\,x]+\frac{1}{c^4\,\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{1+\frac{1}{c^2\,x^2}}\;x}\,2\,b\,\sqrt{-\frac{e\,\left(-i+c\,x\right)}{c\,d+i\,e}}\,\sqrt{-\frac{e\,\left(i+c\,x\right)}{c\,d-i\,e}}$$

$$\left(\left(-5\,i\,c^3\,d^3+5\,c^2\,d^2\,e-9\,i\,c\,d\,e^2+9\,e^3\right)\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{d+e\,x}\right],\frac{c\,d-i\,e}{c\,d+i\,e}\right]+\left(-4\,i\,c^3\,d^3-5\,c^2\,d^2\,e+8\,i\,c\,d\,e^2-9\,e^3\right)\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{d+e\,x}\right],\frac{c\,d-i\,e}{c\,d+i\,e}\right]+8\,i\,c^3\,d^3\,\text{EllipticPi}\left[1-\frac{i\,e}{c\,d},i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{d+e\,x}\right],\frac{c\,d-i\,e}{c\,d+i\,e}\right]\right)\right)$$

Problem 52: Result unnecessarily involves imaginary or complex numbers.

$$\int x\,\sqrt{d+e\,x}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)\,dx$$

Optimal (type 4, 679 leaves, 24 steps) :

$$\frac{4 b \sqrt{d+e x} \left(1+c^2 x^2\right)}{15 c^3 \sqrt{1+\frac{1}{c^2 x^2}} x}-\frac{2 d (d+e x)^{3/2} (a+b \operatorname{ArcCsch}[c x])}{3 e^2}+$$

$$\frac{2 (d+e x)^{5/2} (a+b \operatorname{ArcCsch}[c x])}{5 e^2}+\frac{8 b c d \sqrt{d+e x} \sqrt{1+c^2 x^2} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2} x}}{\sqrt{2}}\right],-\frac{2 \sqrt{-c^2} e}{c^2 d-\sqrt{-c^2} e}\right]}{15 \left(-c^2\right)^{3/2} e \sqrt{1+\frac{1}{c^2 x^2}} x \sqrt{\frac{c^2 (d+e x)}{c^2 d-\sqrt{-c^2} e}}}-$$

$$\frac{8 b c d^2 \sqrt{\frac{c^2 (d+e x)}{c^2 d-\sqrt{-c^2} e}} \sqrt{1+c^2 x^2} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2} x}}{\sqrt{2}}\right],-\frac{2 \sqrt{-c^2} e}{c^2 d-\sqrt{-c^2} e}\right]}{15 \left(-c^2\right)^{3/2} e \sqrt{1+\frac{1}{c^2 x^2}} x \sqrt{d+e x}}+$$

$$\frac{4 b c \left(c^2 d^2+e^2\right) \sqrt{\frac{c^2 (d+e x)}{c^2 d-\sqrt{-c^2} e}} \sqrt{1+c^2 x^2} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2} x}}{\sqrt{2}}\right],-\frac{2 \sqrt{-c^2} e}{c^2 d-\sqrt{-c^2} e}\right]}{15 \left(-c^2\right)^{5/2} e \sqrt{1+\frac{1}{c^2 x^2}} x \sqrt{d+e x}}+$$

$$\frac{8 b d^3 \sqrt{\frac{\sqrt{-c^2} (d+e x)}{\sqrt{-c^2} d+e}} \sqrt{1+c^2 x^2} \operatorname{EllipticPi}\left[2,\operatorname{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2} x}}{\sqrt{2}}\right],\frac{2 e}{\sqrt{-c^2} d+e}\right]}{15 c e^2 \sqrt{1+\frac{1}{c^2 x^2}} x \sqrt{d+e x}}$$

Result (type 4, 418 leaves) :

$$\frac{1}{15}\left(\frac{4\,b\,\sqrt{1+\frac{1}{c^2 x^2}}\;x\,\sqrt{d+e\,x}}{c}+\frac{2\,a\,\sqrt{d+e\,x}\;\left(-2\,d^2+d\,e\,x+3\,e^2\,x^2\right)}{e^2}+\frac{2\,b\,\sqrt{d+e\,x}\;\left(-2\,d^2+d\,e\,x+3\,e^2\,x^2\right)\,\text{ArcCsch}[c\,x]}{e^2}+\right.$$

$$\left(4\,i\,b\,\sqrt{-\frac{e\,\left(-i+c\,x\right)}{c\,d+i\,e}}\;\sqrt{-\frac{e\,\left(i+c\,x\right)}{c\,d-i\,e}}\;\left(2\,c\,d\,\left(c\,d+i\,e\right)\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\;\sqrt{d+e\,x}\right],\frac{c\,d-i\,e}{c\,d+i\,e}\right]+\right.\right.$$

$$\left(c^2\,d^2-2\,i\,c\,d\,e+e^2\right)\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\;\sqrt{d+e\,x}\right],\frac{c\,d-i\,e}{c\,d+i\,e}\right]-$$

$$\left.\left.\left.2\,c^2\,d^2\,\text{EllipticPi}\left[1-\frac{i\,e}{c\,d},\,i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\;\sqrt{d+e\,x}\right],\frac{c\,d-i\,e}{c\,d+i\,e}\right]\right)\right)\Big/\left(c^3\,\sqrt{-\frac{c}{c\,d-i\,e}}\;e^2\,\sqrt{1+\frac{1}{c^2\,x^2}}\;x\right)\right)$$

Problem 53: Result unnecessarily involves imaginary or complex numbers.

$$\int\sqrt{d+e\,x}\;\left(a+b\,\text{ArcCsch}[c\,x]\right)\,dx$$

Optimal (type 4, 429 leaves, 15 steps) :

$$\frac{2\,(d+e\,x)^{3/2}\,(a+b\,\text{ArcCsch}[c\,x])}{3\,e}+\frac{4\,b\,c\,\sqrt{d+e\,x}\,\sqrt{1+c^2\,x^2}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],-\frac{2\,\sqrt{-c^2}\,e}{c^2\,d-\sqrt{-c^2}\,e}\right]}{3\,(-c^2)^{3/2}\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\sqrt{\frac{d+e\,x}{d+\frac{e}{\sqrt{-c^2}}}}}+$$

$$\frac{4\,b\,c\,d\,\sqrt{\frac{d+e\,x}{d+\frac{e}{\sqrt{-c^2}}}}\,\sqrt{1+c^2\,x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],-\frac{2\,\sqrt{-c^2}\,e}{c^2\,d-\sqrt{-c^2}\,e}\right]}{3\,(-c^2)^{3/2}\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}}-$$

$$\frac{4\,b\,d^2\,\sqrt{\frac{\sqrt{-c^2}\,(d+e\,x)}{\sqrt{-c^2}\,d+e}}\,\sqrt{1+c^2\,x^2}\,\text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],\frac{2\,e}{\sqrt{-c^2}\,d+e}\right]}{3\,c\,e\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}}$$

Result (type 4, 329 leaves):

$$\frac{1}{3\,e}2\left(a\,(d+e\,x)^{3/2}+b\,(d+e\,x)^{3/2}\,\text{ArcCsch}[c\,x]+\frac{1}{c^2\,\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x}\right.$$

$$2\,b\,\sqrt{-\frac{e\,(-i+c\,x)}{c\,d+i\,e}}\,\sqrt{-\frac{e\,(i+c\,x)}{c\,d-i\,e}}\,\left((i\,c\,d-e)\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{d+e\,x}\right],\frac{c\,d-i\,e}{c\,d+i\,e}\right]+(-2\,i\,c\,d+e)\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{d+e\,x}\right],\frac{c\,d-i\,e}{c\,d+i\,e}\right]+i\,c\,d\,\text{EllipticPi}\left[1-\frac{i\,e}{c\,d},i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{d+e\,x}\right],\frac{c\,d-i\,e}{c\,d+i\,e}\right]\right)\right)$$

Problem 56: Result unnecessarily involves imaginary or complex numbers.

$$\int(d+e\,x)^{3/2}\,(a+b\,\text{ArcCsch}[c\,x])\,dx$$

Optimal (type 4, 486 leaves, 22 steps):

$$\frac{4\,b\,e\,\sqrt{d+e\,x}\,\left(1+c^2x^2\right)}{15\,c^3\,\sqrt{1+\frac{1}{c^2x^2}}\,x}+\frac{2\,(d+e\,x)^{5/2}\,(a+b\,\text{ArcCsch}[c\,x])}{5\,e}+\frac{28\,b\,c\,d\,\sqrt{d+e\,x}\,\sqrt{1+c^2x^2}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],-\frac{2\sqrt{-c^2}\,e}{c^2d-\sqrt{-c^2}\,e}\right]}{15\,(-c^2)^{3/2}\,\sqrt{1+\frac{1}{c^2x^2}}\,x\,\sqrt{\frac{d+e\,x}{d+\frac{e}{\sqrt{-c^2}}}}}-$$

$$\frac{4\,b\,c\,\left(2\,c^2d^2-e^2\right)\sqrt{\frac{d+e\,x}{d+\frac{e}{\sqrt{-c^2}}}}\,\sqrt{1+c^2x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],-\frac{2\sqrt{-c^2}\,e}{c^2d-\sqrt{-c^2}\,e}\right]}{15\,(-c^2)^{5/2}\,\sqrt{1+\frac{1}{c^2x^2}}\,x\,\sqrt{d+e\,x}}-$$

$$\frac{4\,b\,d^3\,\sqrt{\frac{\sqrt{-c^2}\,(d+e\,x)}{\sqrt{-c^2}\,d+e}}\,\sqrt{1+c^2x^2}\,\text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],\frac{2\,e}{\sqrt{-c^2}\,d+e}\right]}{5\,c\,e\,\sqrt{1+\frac{1}{c^2x^2}}\,x\,\sqrt{d+e\,x}}$$

Result (type 4, 380 leaves):

$$\frac{1}{15\,e}2\left(\frac{2\,b\,e^2\sqrt{1+\frac{1}{c^2x^2}}\,x\,\sqrt{d+e\,x}}{c}+3\,a\,(d+e\,x)^{5/2}+3\,b\,(d+e\,x)^{5/2}\,\text{ArcCsch}[c\,x]+\frac{1}{c^3\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{1+\frac{1}{c^2x^2}}\,x}\right.$$

$$2\,i\,b\,\sqrt{-\frac{e\,(-i+c\,x)}{c\,d+i\,e}}\,\sqrt{-\frac{e\,(i+c\,x)}{c\,d-i\,e}}\,\left(7\,c\,d\,(c\,d+i\,e)\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{d+e\,x}\right],\frac{c\,d-i\,e}{c\,d+i\,e}\right]+\left(-9\,c^2d^2-7\,i\,c\,d\,e+e^2\right)\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{d+e\,x}\right],\frac{c\,d-i\,e}{c\,d+i\,e}\right]+3\,c^2d^2\,\text{EllipticPi}\left[1-\frac{i\,e}{c\,d},i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{d+e\,x}\right],\frac{c\,d-i\,e}{c\,d+i\,e}\right]\right)\right)$$

Problem 57: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^3 \left(a + b\,\text{ArcCsch}[c\,x]\right)}{\sqrt{d + e\,x}}\,dx$$

Optimal (type 4, 939 leaves, 27 steps) :

$$\frac{4 b \sqrt{d+e x}\,\left(1+c^2 x^2\right)}{35 c^3 e \sqrt{1+\frac{1}{c^2 x^2}}}-\frac{4 b d \sqrt{d+e x}\,\left(1+c^2 x^2\right)}{21 c^3 e^2 \sqrt{1+\frac{1}{c^2 x^2}}\; x}-\frac{2 d^3 \sqrt{d+e x}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{e^4}+$$

$$\frac{2 d^2 \left(d+e x\right)^{3/2} \left(a+b\,\text{ArcCsch}[c\,x]\right)}{e^4}-\frac{6 d \left(d+e x\right)^{5/2} \left(a+b\,\text{ArcCsch}[c\,x]\right)}{5 e^4}+$$

$$\frac{2 \left(d+e x\right)^{7/2} \left(a+b\,\text{ArcCsch}[c\,x]\right)}{7 e^4}+\frac{24 b c d^2 \sqrt{d+e x}\,\sqrt{1+c^2 x^2}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],-\frac{2 \sqrt{-c^2}\,e}{c^2 d-\sqrt{-c^2}\,e}\right]}{35 \left(-c^2\right)^{3/2} e^3 \sqrt{1+\frac{1}{c^2 x^2}}\; x \sqrt{\frac{c^2 (d+e x)}{c^2 d-\sqrt{-c^2}\,e}}}+$$

$$\frac{4 b c \left(2 c^2 d^2+9 e^2\right) \sqrt{d+e x}\,\sqrt{1+c^2 x^2}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],-\frac{2 \sqrt{-c^2}\,e}{c^2 d-\sqrt{-c^2}\,e}\right]}{105 \left(-c^2\right)^{5/2} e^3 \sqrt{1+\frac{1}{c^2 x^2}}\; x \sqrt{\frac{c^2 (d+e x)}{c^2 d-\sqrt{-c^2}\,e}}}-$$

$$\frac{64 b c d^3 \sqrt{\frac{c^2 (d+e x)}{c^2 d-\sqrt{-c^2}\,e}}\,\sqrt{1+c^2 x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],-\frac{2 \sqrt{-c^2}\,e}{c^2 d-\sqrt{-c^2}\,e}\right]}{35 \left(-c^2\right)^{3/2} e^3 \sqrt{1+\frac{1}{c^2 x^2}}\; x \sqrt{d+e x}}-$$

$$\frac{32 b c d \left(c^2 d^2+e^2\right) \sqrt{\frac{c^2 (d+e x)}{c^2 d-\sqrt{-c^2}\,e}}\,\sqrt{1+c^2 x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],-\frac{2 \sqrt{-c^2}\,e}{c^2 d-\sqrt{-c^2}\,e}\right]}{105 \left(-c^2\right)^{5/2} e^3 \sqrt{1+\frac{1}{c^2 x^2}}\; x \sqrt{d+e x}}+$$

$$\frac{64 b d^4 \sqrt{\frac{\sqrt{-c^2}\,(d+e x)}{\sqrt{-c^2}\,d+e}}\,\sqrt{1+c^2 x^2}\,\text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],\frac{2 e}{\sqrt{-c^2}\,d+e}\right]}{35 c e^4 \sqrt{1+\frac{1}{c^2 x^2}}\; x \sqrt{d+e x}}$$

Result (type 4, 485 leaves) :

$$\frac{1}{105\,e^4}\,2\left(\frac{2\,b\,e^2\,\sqrt{1+\frac{1}{c^2\,x^2}}\;x\,\sqrt{d+e\,x}\,\left(-5\,d+3\,e\,x\right)}{c}+3\,a\,\sqrt{d+e\,x}\,\left(-16\,d^3+8\,d^2\,e\,x-6\,d\,e^2\,x^2+5\,e^3\,x^3\right)+\right.$$

$$3\,b\,\sqrt{d+e\,x}\,\left(-16\,d^3+8\,d^2\,e\,x-6\,d\,e^2\,x^2+5\,e^3\,x^3\right)\,\text{ArcCsch}[c\,x]+\frac{1}{c^4\,\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{1+\frac{1}{c^2\,x^2}}\;x}\,2\,b\,\sqrt{-\frac{e\,\left(-i+c\,x\right)}{c\,d+i\,e}}\,\sqrt{-\frac{e\,\left(i+c\,x\right)}{c\,d-i\,e}}$$

$$\left(\left(16\,i\,c^3\,d^3-16\,c^2\,d^2\,e-9\,i\,c\,d\,e^2+9\,e^3\right)\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{d+e\,x}\right],\frac{c\,d-i\,e}{c\,d+i\,e}\right]+\left(24\,i\,c^3\,d^3+16\,c^2\,d^2\,e+i\,c\,d\,e^2-9\,e^3\right)\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{d+e\,x}\right],\frac{c\,d-i\,e}{c\,d+i\,e}\right]-48\,i\,c^3\,d^3\,\text{EllipticPi}\left[1-\frac{i\,e}{c\,d},\,i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{d+e\,x}\right],\frac{c\,d-i\,e}{c\,d+i\,e}\right]\right)\right)$$

Problem 58: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^2\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{\sqrt{d+e\,x}}\,dx$$

Optimal (type 4, 707 leaves, 20 steps):

$$\frac{4\,b\,\sqrt{d+e\,x}\,\left(1+c^2\,x^2\right)}{15\,c^3\,e\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x}+\frac{2\,d^2\,\sqrt{d+e\,x}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{e^3}-\frac{4\,d\,\left(d+e\,x\right)^{3/2}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{3\,e^3}+$$

$$\frac{2\,\left(d+e\,x\right)^{5/2}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{5\,e^3}-\frac{4\,b\,c\,d\,\sqrt{d+e\,x}\,\sqrt{1+c^2\,x^2}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],-\frac{2\,\sqrt{-c^2}\,e}{c^2\,d-\sqrt{-c^2}\,e}\right]}{5\,\left(-c^2\right)^{3/2}\,e^2\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\sqrt{\frac{c^2\,(d+e\,x)}{c^2\,d-\sqrt{-c^2}\,e}}}+$$

$$\frac{32\,b\,c\,d^2\,\sqrt{\frac{c^2\,(d+e\,x)}{c^2\,d-\sqrt{-c^2}\,e}}\,\sqrt{1+c^2\,x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],-\frac{2\,\sqrt{-c^2}\,e}{c^2\,d-\sqrt{-c^2}\,e}\right]}{15\,\left(-c^2\right)^{3/2}\,e^2\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}}+$$

$$\frac{4\,b\,c\,\left(c^2\,d^2+e^2\right)\,\sqrt{\frac{c^2\,(d+e\,x)}{c^2\,d-\sqrt{-c^2}\,e}}\,\sqrt{1+c^2\,x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],-\frac{2\,\sqrt{-c^2}\,e}{c^2\,d-\sqrt{-c^2}\,e}\right]}{15\,\left(-c^2\right)^{5/2}\,e^2\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}}-$$

$$\frac{32\,b\,d^3\,\sqrt{\frac{\sqrt{-c^2}\,(d+e\,x)}{\sqrt{-c^2}\,d+e}}\,\sqrt{1+c^2\,x^2}\,\text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],\frac{2\,e}{\sqrt{-c^2}\,d+e}\right]}{15\,c\,e^3\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}}$$

Result (type 4, 419 leaves) :

$$\frac{1}{15\,e^3}2\left(\frac{2\,b\,e^2\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}}{c}+a\,\sqrt{d+e\,x}\,\left(8\,d^2-4\,d\,e\,x+3\,e^2\,x^2\right)+b\,\sqrt{d+e\,x}\,\left(8\,d^2-4\,d\,e\,x+3\,e^2\,x^2\right)\,\text{ArcCsch}[c\,x]+\right.$$

$$\left(2\,b\,\sqrt{-\frac{e\,(-i+c\,x)}{c\,d+i\,e}}\,\sqrt{-\frac{e\,(i+c\,x)}{c\,d-i\,e}}\,\left(3\,c\,d\,(-i\,c\,d+e)\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{d+e\,x}\right],\frac{c\,d-i\,e}{c\,d+i\,e}\right]+\right.\right.$$

$$\left(-4\,i\,c^2\,d^2-3\,c\,d\,e+i\,e^2\right)\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{d+e\,x}\right],\frac{c\,d-i\,e}{c\,d+i\,e}\right]+$$

$$\left.\left.\left.8\,i\,c^2\,d^2\,\text{EllipticPi}\left[1-\frac{i\,e}{c\,d},\,i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{d+e\,x}\right],\frac{c\,d-i\,e}{c\,d+i\,e}\right]\right)\right)\middle/\left(c^3\,\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\right)\right)$$

Problem 59: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x\,(a+b\,\text{ArcCsch}[c\,x])}{\sqrt{d+e\,x}}\,dx$$

Optimal (type 4, 474 leaves, 14 steps):

$$-\frac{2\,d\,\sqrt{d+e\,x}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{e^2}+\frac{2\,(d+e\,x)^{3/2}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{3\,e^2}+\frac{4\,b\,c\,\sqrt{d+e\,x}\,\sqrt{1+c^2\,x^2}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],-\frac{2\,\sqrt{-c^2}\,e}{c^2\,d-\sqrt{-c^2}\,e}\right]}{3\,(-c^2)^{3/2}\,e\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\sqrt{\frac{c^2\,(d+e\,x)}{c^2\,d-\sqrt{-c^2}\,e}}}-$$

$$\frac{8\,b\,c\,d\,\sqrt{\frac{c^2\,(d+e\,x)}{c^2\,d-\sqrt{-c^2}\,e}}\,\sqrt{1+c^2\,x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],-\frac{2\,\sqrt{-c^2}\,e}{c^2\,d-\sqrt{-c^2}\,e}\right]}{3\,(-c^2)^{3/2}\,e\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}}+$$

$$\frac{8\,b\,d^2\,\sqrt{\frac{\sqrt{-c^2}\,(d+e\,x)}{\sqrt{-c^2}\,d+e}}\,\sqrt{1+c^2\,x^2}\,\text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],\frac{2\,e}{\sqrt{-c^2}\,d+e}\right]}{3\,c\,e^2\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}}$$

Result (type 4, 343 leaves):

$$\frac{1}{3\,e^2}2\left(a\,(-2\,d+e\,x)\,\sqrt{d+e\,x}+b\,(-2\,d+e\,x)\,\sqrt{d+e\,x}\,\text{ArcCsch}[c\,x]+\frac{1}{c^2\,\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x}\right.$$

$$2\,b\,\sqrt{-\frac{e\,(-i+c\,x)}{c\,d+i\,e}}\,\sqrt{-\frac{e\,(i+c\,x)}{c\,d-i\,e}}\,\left((i\,c\,d-e)\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{d+e\,x}\right],\frac{c\,d-i\,e}{c\,d+i\,e}\right]+(i\,c\,d+e)\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{d+e\,x}\right],\frac{c\,d-i\,e}{c\,d+i\,e}\right]-2\,i\,c\,d\,\text{EllipticPi}\left[1-\frac{i\,e}{c\,d},i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{d+e\,x}\right],\frac{c\,d-i\,e}{c\,d+i\,e}\right]\right)\right)$$

Problem 60: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{a+b\,\text{ArcCsch}[c\,x]}{\sqrt{d+e\,x}}\,dx$$

Optimal (type 4, 284 leaves, 9 steps):

$$\frac{2\sqrt{d+e x}\,\left(a+b\,\text{ArcCsch}[c x]\right)}{e}+\frac{4 b c\sqrt{\frac{d+e x}{d+\frac{e}{\sqrt{-c^2}}}}\,\sqrt{1+c^2 x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],-\frac{2\sqrt{-c^2}\,e}{c^2 d-\sqrt{-c^2}\,e}\right]}{\left(-c^2\right)^{3/2}\sqrt{1+\frac{1}{c^2 x^2}}\,x\sqrt{d+e x}}-$$

$$\frac{4 b d\sqrt{\frac{\sqrt{-c^2}\,(d+e x)}{\sqrt{-c^2}\,d+e}}\,\sqrt{1+c^2 x^2}\,\text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],\frac{2 e}{\sqrt{-c^2}\,d+e}\right]}{c e\sqrt{1+\frac{1}{c^2 x^2}}\,x\sqrt{d+e x}}$$

Result (type 4, 250 leaves):

$$\frac{1}{e}2\left(a\sqrt{d+e x}+b\sqrt{d+e x}\,\text{ArcCsch}[c x]-\frac{1}{c\sqrt{-\frac{c}{c d-\mathrm{i} e}}\,\sqrt{1+\frac{1}{c^2 x^2}}\,x}2\,\mathrm{i}\,b\sqrt{-\frac{e\left(-\mathrm{i}+c x\right)}{c d+\mathrm{i} e}}\,\sqrt{-\frac{e\left(\mathrm{i}+c x\right)}{c d-\mathrm{i} e}}\right.$$

$$\left.\left(\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c d-\mathrm{i} e}}\,\sqrt{d+e x}\right],\frac{c d-\mathrm{i} e}{c d+\mathrm{i} e}\right]-\text{EllipticPi}\left[1-\frac{\mathrm{i} e}{c d},\mathrm{i}\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c d-\mathrm{i} e}}\,\sqrt{d+e x}\right],\frac{c d-\mathrm{i} e}{c d+\mathrm{i} e}\right]\right)\right)$$

Problem 63: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^3\left(a+b\,\text{ArcCsch}[c x]\right)}{\left(d+e x\right)^{3/2}}\,dx$$

Optimal (type 4, 731 leaves, 23 steps):

$$\frac{4\,b\,\sqrt{d+e\,x}\,\left(1+c^2\,x^2\right)}{15\,c^3\,e^2\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x}+\frac{2\,d^3\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{e^4\,\sqrt{d+e\,x}}+\frac{6\,d^2\,\sqrt{d+e\,x}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{e^4}-\frac{2\,d\,(d+e\,x)^{3/2}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{e^4}+$$

$$\frac{2\,(d+e\,x)^{5/2}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{5\,e^4}-\frac{32\,b\,c\,d\,\sqrt{d+e\,x}\,\sqrt{1+c^2\,x^2}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],-\frac{2\,\sqrt{-c^2}\,e}{c^2\,d-\sqrt{-c^2}\,e}\right]}{15\,\left(-c^2\right)^{3/2}\,e^3\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\sqrt{\frac{c^2\,(d+e\,x)}{c^2\,d-\sqrt{-c^2}\,e}}}+$$

$$\frac{8\,b\,c\,d^2\,\sqrt{\frac{c^2\,(d+e\,x)}{c^2\,d-\sqrt{-c^2}\,e}}\,\sqrt{1+c^2\,x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],-\frac{2\,\sqrt{-c^2}\,e}{c^2\,d-\sqrt{-c^2}\,e}\right]}{\left(-c^2\right)^{3/2}\,e^3\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}}-$$

$$\frac{4\,b\,c\,\left(2\,c^2\,d^2-e^2\right)\,\sqrt{\frac{c^2\,(d+e\,x)}{c^2\,d-\sqrt{-c^2}\,e}}\,\sqrt{1+c^2\,x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],-\frac{2\,\sqrt{-c^2}\,e}{c^2\,d-\sqrt{-c^2}\,e}\right]}{15\,\left(-c^2\right)^{5/2}\,e^3\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}}-$$

$$\frac{64\,b\,d^3\,\sqrt{\frac{\sqrt{-c^2}\,(d+e\,x)}{\sqrt{-c^2}\,d+e}}\,\sqrt{1+c^2\,x^2}\,\text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],\frac{2\,e}{\sqrt{-c^2}\,d+e}\right]}{5\,c\,e^4\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}}$$

Result (type 4, 441 leaves) :

$$\frac{1}{15\, e^4}2\left(\frac{2\, b\, e^2 \sqrt{1+\frac{1}{c^2 x^2}}\; x \sqrt{d+e\, x}}{c}+\frac{3\, a \left(16\, d^3+8\, d^2\, e\, x-2\, d\, e^2\, x^2+e^3\, x^3\right)}{\sqrt{d+e\, x}}+\frac{3\, b \left(16\, d^3+8\, d^2\, e\, x-2\, d\, e^2\, x^2+e^3\, x^3\right) \text{ArcCsch}[c\, x]}{\sqrt{d+e\, x}}+\frac{1}{c^3 \sqrt{-\frac{c}{c\, d-i\, e}} \sqrt{1+\frac{1}{c^2 x^2}}\; x}\right.$$

$$2\, b \sqrt{-\frac{e\,(-i+c\, x)}{c\, d+i\, e}} \sqrt{-\frac{e\,(i+c\, x)}{c\, d-i\, e}} \left(8\, c\, d\, (-i\, c\, d+e)\, \text{EllipticE}\left[i\, \text{ArcSinh}\left[\sqrt{-\frac{c}{c\, d-i\, e}} \sqrt{d+e\, x}\right], \frac{c\, d-i\, e}{c\, d+i\, e}\right]+\left(-24\, i\, c^2\, d^2-8\, c\, d\, e+i\, e^2\right)\right.$$

$$\left.\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{-\frac{c}{c\, d-i\, e}} \sqrt{d+e\, x}\right], \frac{c\, d-i\, e}{c\, d+i\, e}\right]+48\, i\, c^2\, d^2\, \text{EllipticPi}\left[1-\frac{i\, e}{c\, d}, i\, \text{ArcSinh}\left[\sqrt{-\frac{c}{c\, d-i\, e}} \sqrt{d+e\, x}\right], \frac{c\, d-i\, e}{c\, d+i\, e}\right]\right)\right)$$

Problem 64: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2\, (a+b\, \text{ArcCsch}[c\, x])}{(d+e\, x)^{3/2}}\, dx$$

Optimal (type 4, 499 leaves, 16 steps):

$$-\frac{2 d^2 \left(a+b \,\text{ArcCsch}[c x]\right)}{e^3 \sqrt{d+e x}}-\frac{4 d \sqrt{d+e x} \left(a+b \,\text{ArcCsch}[c x]\right)}{e^3}+$$

$$\frac{2 (d+e x)^{3/2} \left(a+b \,\text{ArcCsch}[c x]\right)}{3 e^3}+\frac{4 b c \sqrt{d+e x} \sqrt{1+c^2 x^2} \,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2} x}}{\sqrt{2}}\right],-\frac{2 \sqrt{-c^2} e}{c^2 d-\sqrt{-c^2} e}\right]}{3 \left(-c^2\right)^{3/2} e^2 \sqrt{1+\frac{1}{c^2 x^2}} x \sqrt{\frac{c^2 (d+e x)}{c^2 d-\sqrt{-c^2} e}}}-$$

$$\frac{20 b c d \sqrt{\frac{c^2 (d+e x)}{c^2 d-\sqrt{-c^2} e}} \sqrt{1+c^2 x^2} \,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2} x}}{\sqrt{2}}\right],-\frac{2 \sqrt{-c^2} e}{c^2 d-\sqrt{-c^2} e}\right]}{3 \left(-c^2\right)^{3/2} e^2 \sqrt{1+\frac{1}{c^2 x^2}} x \sqrt{d+e x}}+$$

$$\frac{32 b d^2 \sqrt{\frac{\sqrt{-c^2} (d+e x)}{\sqrt{-c^2} d+e}} \sqrt{1+c^2 x^2} \,\text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2} x}}{\sqrt{2}}\right],\frac{2 e}{\sqrt{-c^2} d+e}\right]}{3 c e^3 \sqrt{1+\frac{1}{c^2 x^2}} x \sqrt{d+e x}}$$

Result (type 4, 365 leaves) :

$$\frac{1}{3 e^3}2 \left(\frac{a \left(-8 d^2-4 d e x+e^2 x^2\right)}{\sqrt{d+e x}}+\frac{b \left(-8 d^2-4 d e x+e^2 x^2\right) \text{ArcCsch}[c x]}{\sqrt{d+e x}}+\frac{1}{c^2 \sqrt{-\frac{c}{c d-i e}} \sqrt{1+\frac{1}{c^2 x^2}} x}\right.$$

$$2 b \sqrt{-\frac{e (-i+c x)}{c d+i e}} \sqrt{-\frac{e (i+c x)}{c d-i e}} \left((i c d-e) \,\text{EllipticE}\left[i \,\text{ArcSinh}\left[\sqrt{-\frac{c}{c d-i e}} \sqrt{d+e x}\right],\frac{c d-i e}{c d+i e}\right]+(4 i c d+e)\right.$$

$$\left.\left.\text{EllipticF}\left[i \,\text{ArcSinh}\left[\sqrt{-\frac{c}{c d-i e}} \sqrt{d+e x}\right],\frac{c d-i e}{c d+i e}\right]-8 i c d \,\text{EllipticPi}\left[1-\frac{i e}{c d},i \,\text{ArcSinh}\left[\sqrt{-\frac{c}{c d-i e}} \sqrt{d+e x}\right],\frac{c d-i e}{c d+i e}\right]\right)\right)$$

Problem 65: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x\,(a+b\,\text{ArcCsch}[c\,x])}{(d+e\,x)^{3/2}}\,dx$$

Optimal (type 4, 318 leaves, 11 steps):

$$\frac{2\,d\,(a+b\,\text{ArcCsch}[c\,x])}{e^2\,\sqrt{d+e\,x}}+\frac{2\,\sqrt{d+e\,x}\,(a+b\,\text{ArcCsch}[c\,x])}{e^2}+\frac{4\,b\,c\,\sqrt{\frac{c^2\,(d+e\,x)}{c^2\,d-\sqrt{-c^2}\,e}}\,\sqrt{1+c^2\,x^2}\,\text{EllipticF}\Big[\text{ArcSin}\Big[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\Big],-\frac{2\,\sqrt{-c^2}\,e}{c^2\,d-\sqrt{-c^2}\,e}\Big]}{(-c^2)^{3/2}\,e\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}}-$$

$$\frac{8\,b\,d\,\sqrt{\frac{\sqrt{-c^2}\,(d+e\,x)}{\sqrt{-c^2}\,d+e}}\,\sqrt{1+c^2\,x^2}\,\text{EllipticPi}\Big[2,\text{ArcSin}\Big[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\Big],\frac{2\,e}{\sqrt{-c^2}\,d+e}\Big]}{c\,e^2\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}}$$

Result (type 4, 264 leaves):

$$\frac{1}{e^2}2\left(\frac{a\,(2\,d+e\,x)}{\sqrt{d+e\,x}}+\frac{b\,(2\,d+e\,x)\,\text{ArcCsch}[c\,x]}{\sqrt{d+e\,x}}-\frac{1}{c\,\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x}2\,i\,b\,\sqrt{-\frac{e\,(-i+c\,x)}{c\,d+i\,e}}\,\sqrt{-\frac{e\,(i+c\,x)}{c\,d-i\,e}}\right.$$

$$\left.\left(\text{EllipticF}\Big[i\,\text{ArcSinh}\Big[\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{d+e\,x}\Big],\frac{c\,d-i\,e}{c\,d+i\,e}\Big]-2\,\text{EllipticPi}\Big[1-\frac{i\,e}{c\,d},i\,\text{ArcSinh}\Big[\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{d+e\,x}\Big],\frac{c\,d-i\,e}{c\,d+i\,e}\Big]\right)\right)$$

Problem 66: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a+b\,\text{ArcCsch}[c\,x]}{(d+e\,x)^{3/2}}\,dx$$

Optimal (type 4, 149 leaves, 6 steps):

$$-\frac{2\left(a+b\,\text{ArcCsch}[c\,x]\right)}{e\sqrt{d+e\,x}}+\frac{4\,b\sqrt{\frac{\sqrt{-c^2}\,(d+e\,x)}{\sqrt{-c^2}\,d+e}}\,\sqrt{1+c^2\,x^2}\,\text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],\frac{2\,e}{\sqrt{-c^2}\,d+e}\right]}{c\,e\sqrt{1+\frac{1}{c^2\,x^2}}\,x\sqrt{d+e\,x}}$$

Result (type 4, 166 leaves) :

$$\frac{1}{e^2\sqrt{d+e\,x}\,\left(1+c^2\,x^2\right)}\left(-2\,e\left(1+c^2\,x^2\right)\left(a+b\,\text{ArcCsch}[c\,x]\right)+2\,b\,c\left(\mathrm{i}\,c\,d+e\right)\sqrt{2+\frac{2}{c^2\,x^2}}\,x\sqrt{1+\mathrm{i}\,c\,x}\,\sqrt{\frac{c\,e\,(\mathrm{i}+c\,x)\,(d+e\,x)}{(\mathrm{i}\,c\,d+e)^2}}\,\text{EllipticPi}\left[1+\frac{\mathrm{i}\,c\,d}{e},\text{ArcSin}\left[\sqrt{-\frac{e\,(\mathrm{i}+c\,x)}{c\,d-\mathrm{i}\,e}}\right],\frac{\mathrm{i}\,c\,d+e}{2\,e}\right]\right)$$

Problem 69: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^3\left(a+b\,\text{ArcCsch}[c\,x]\right)}{(d+e\,x)^{5/2}}\,dx$$

Optimal (type 4, 777 leaves, 31 steps) :

$$\frac{4\,b\,d^2\,\left(1+c^2\,x^2\right)}{3\,c\,e^2\,\left(c^2\,d^2+e^2\right)\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}}+\frac{2\,d^3\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{3\,e^4\,\left(d+e\,x\right)^{3/2}}-\frac{6\,d^2\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{e^4\,\sqrt{d+e\,x}}-\frac{6\,d\,\sqrt{d+e\,x}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{e^4}+$$

$$\frac{2\,\left(d+e\,x\right)^{3/2}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{3\,e^4}-\frac{8\,b\,\sqrt{-c^2}\,d^2\,\sqrt{d+e\,x}\,\sqrt{1+c^2\,x^2}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],-\frac{2\,\sqrt{-c^2}\,e}{c^2\,d-\sqrt{-c^2}\,e}\right]}{3\,c\,e^3\,\left(c^2\,d^2+e^2\right)\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\sqrt{\frac{c^2\,(d+e\,x)}{c^2\,d-\sqrt{-c^2}\,e}}}+$$

$$\frac{4\,b\,c\,\left(2\,c^2\,d^2+e^2\right)\,\sqrt{d+e\,x}\,\sqrt{1+c^2\,x^2}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],-\frac{2\,\sqrt{-c^2}\,e}{c^2\,d-\sqrt{-c^2}\,e}\right]}{3\,\left(-c^2\right)^{3/2}\,e^3\,\left(c^2\,d^2+e^2\right)\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\sqrt{\frac{c^2\,(d+e\,x)}{c^2\,d-\sqrt{-c^2}\,e}}}-$$

$$\frac{32\,b\,c\,d\,\sqrt{\frac{c^2\,(d+e\,x)}{c^2\,d-\sqrt{-c^2}\,e}}\,\sqrt{1+c^2\,x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],-\frac{2\,\sqrt{-c^2}\,e}{c^2\,d-\sqrt{-c^2}\,e}\right]}{3\,\left(-c^2\right)^{3/2}\,e^3\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}}+$$

$$\frac{64\,b\,d^2\,\sqrt{\frac{\sqrt{-c^2}\,(d+e\,x)}{\sqrt{-c^2}\,d+e}}\,\sqrt{1+c^2\,x^2}\,\text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],\frac{2\,e}{\sqrt{-c^2}\,d+e}\right]}{3\,c\,e^4\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}}$$

Result (type 4, 448 leaves) :

$$\frac{1}{3 e^4} 2 \left(\frac{2 b c d^2 e^2 \sqrt{1+\frac{1}{c^2 x^2}}\, x}{\left(c^2 d^2+e^2\right) \sqrt{d+e x}}+\frac{a \left(-16 d^3-24 d^2 e x-6 d e^2 x^2+e^3 x^3\right)}{(d+e x)^{3/2}}+\right.$$

$$\frac{b \left(-16 d^3-24 d^2 e x-6 d e^2 x^2+e^3 x^3\right) \text{ArcCsch}[c x]}{(d+e x)^{3/2}}-\frac{1}{c^3 \sqrt{1+\frac{1}{c^2 x^2}}\, x} 2 i b \sqrt{-\frac{c}{c d-i e}} \sqrt{-\frac{e (-i+c x)}{c d+i e}} \sqrt{-\frac{e (i+c x)}{c d-i e}}$$

$$\left(e^2 \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{-\frac{c}{c d-i e}} \sqrt{d+e x}\right], \frac{c d-i e}{c d+i e}\right]+\left(8 c^2 d^2-8 i c d e-e^2\right) \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{c}{c d-i e}} \sqrt{d+e x}\right],\right.\right.$$

$$\left.\left.\left.\frac{c d-i e}{c d+i e}\right]-16 c d (c d-i e) \text{EllipticPi}\left[1-\frac{i e}{c d}, i \text{ArcSinh}\left[\sqrt{-\frac{c}{c d-i e}} \sqrt{d+e x}\right], \frac{c d-i e}{c d+i e}\right]\right)\right)$$

Problem 70: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2 (a+b \text{ArcCsch}[c x])}{(d+e x)^{5/2}} \, dx$$

Optimal (type 4, 569 leaves, 25 steps) :

$$-\frac{4 b d \left(1+c^2 x^2\right)}{3 c e \left(c^2 d^2+e^2\right) \sqrt{1+\frac{1}{c^2 x^2}}\, x \sqrt{d+e x}}-\frac{2 d^2 \left(a+b \text{ArcCsch}[c x]\right)}{3 e^3 (d+e x)^{3/2}}+\frac{4 d \left(a+b \text{ArcCsch}[c x]\right)}{e^3 \sqrt{d+e x}}+$$

$$\frac{2 \sqrt{d+e x}\, \left(a+b \text{ArcCsch}[c x]\right)}{e^3}+\frac{4 b \sqrt{-c^2}\, d \sqrt{d+e x}\, \sqrt{1+c^2 x^2}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\, x}}{\sqrt{2}}\right],-\frac{2 \sqrt{-c^2}\, e}{c^2 d-\sqrt{-c^2}\, e}\right]}{3 c e^2 \left(c^2 d^2+e^2\right) \sqrt{1+\frac{1}{c^2 x^2}}\, x \sqrt{\frac{c^2 (d+e x)}{c^2 d-\sqrt{-c^2}\, e}}}+$$

$$\frac{4 b c \sqrt{\frac{c^2 (d+e x)}{c^2 d-\sqrt{-c^2}\, e}}\, \sqrt{1+c^2 x^2}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\, x}}{\sqrt{2}}\right],-\frac{2 \sqrt{-c^2}\, e}{c^2 d-\sqrt{-c^2}\, e}\right]}{\left(-c^2\right)^{3/2} e^2 \sqrt{1+\frac{1}{c^2 x^2}}\, x \sqrt{d+e x}}-$$

$$\frac{32 b d \sqrt{\frac{\sqrt{-c^2}\, (d+e x)}{\sqrt{-c^2}\, d+e}}\, \sqrt{1+c^2 x^2}\, \text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\, x}}{\sqrt{2}}\right],\frac{2 e}{\sqrt{-c^2}\, d+e}\right]}{3 c e^3 \sqrt{1+\frac{1}{c^2 x^2}}\, x \sqrt{d+e x}}$$

Result (type 4, 416 leaves) :

$$\frac{2}{3}\left(-\frac{2\,b\,c\,d\,\sqrt{1+\frac{1}{c^2 x^2}}\;x}{\left(c^2 d^2 e+e^3\right)\sqrt{d+e\,x}}+\frac{a\left(8\,d^2+12\,d\,e\,x+3\,e^2 x^2\right)}{e^3\left(d+e\,x\right)^{3/2}}+\right.$$

$$\frac{b\left(8\,d^2+12\,d\,e\,x+3\,e^2 x^2\right)\text{ArcCsch}[c\,x]}{e^3\left(d+e\,x\right)^{3/2}}-\frac{1}{c^2 e^3\sqrt{1+\frac{1}{c^2 x^2}}\;x}2\,b\sqrt{-\frac{c}{c\,d-\mathrm{i}\,e}}\sqrt{-\frac{e\left(-\mathrm{i}+c\,x\right)}{c\,d+\mathrm{i}\,e}}\sqrt{-\frac{e\left(\mathrm{i}+c\,x\right)}{c\,d-\mathrm{i}\,e}}$$

$$\left(\mathrm{i}\,c\,d\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-\mathrm{i}\,e}}\sqrt{d+e\,x}\right],\frac{c\,d-\mathrm{i}\,e}{c\,d+\mathrm{i}\,e}\right]+\left(-4\,\mathrm{i}\,c\,d-3\,e\right)\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-\mathrm{i}\,e}}\sqrt{d+e\,x}\right],\frac{c\,d-\mathrm{i}\,e}{c\,d+\mathrm{i}\,e}\right]+\right.$$

$$\left.\left.8\left(\mathrm{i}\,c\,d+e\right)\text{EllipticPi}\left[1-\frac{\mathrm{i}\,e}{c\,d},\mathrm{i}\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-\mathrm{i}\,e}}\sqrt{d+e\,x}\right],\frac{c\,d-\mathrm{i}\,e}{c\,d+\mathrm{i}\,e}\right]\right)\right)$$

Problem 71: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x\left(a+b\,\text{ArcCsch}[c\,x]\right)}{\left(d+e\,x\right)^{5/2}}\,dx$$

Optimal (type 4, 393 leaves, 19 steps):

$$\frac{4\,b\,\left(1+c^2\,x^2\right)}{3\,c\,\left(c^2\,d^2+e^2\right)\,\sqrt{1+\frac{1}{c^2\,x^2}}\;x\,\sqrt{d+e\,x}}+\frac{2\,d\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{3\,e^2\,\left(d+e\,x\right)^{3/2}}-$$

$$\frac{2\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{e^2\,\sqrt{d+e\,x}}-\frac{4\,b\,\sqrt{-c^2}\,\sqrt{d+e\,x}\,\sqrt{1+c^2\,x^2}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],-\frac{2\,\sqrt{-c^2}\,e}{c^2\,d-\sqrt{-c^2}\,e}\right]}{3\,c\,e\,\left(c^2\,d^2+e^2\right)\,\sqrt{1+\frac{1}{c^2\,x^2}}\;x\,\sqrt{\frac{c^2\,(d+e\,x)}{c^2\,d-\sqrt{-c^2}\,e}}}+$$

$$\frac{8\,b\,\sqrt{\frac{\sqrt{-c^2}\,(d+e\,x)}{\sqrt{-c^2}\,d+e}}\,\sqrt{1+c^2\,x^2}\;\text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],\frac{2\,e}{\sqrt{-c^2}\,d+e}\right]}{3\,c\,e^2\,\sqrt{1+\frac{1}{c^2\,x^2}}\;x\,\sqrt{d+e\,x}}$$

Result (type 4, 390 leaves):

$$\frac{2}{3}\left(\frac{2\,b\,c\,\sqrt{1+\frac{1}{c^2\,x^2}}\;x}{\left(c^2\,d^2+e^2\right)\,\sqrt{d+e\,x}}-\frac{a\,\left(2\,d+3\,e\,x\right)}{e^2\,\left(d+e\,x\right)^{3/2}}-\frac{b\,\left(2\,d+3\,e\,x\right)\,\text{ArcCsch}[c\,x]}{e^2\,\left(d+e\,x\right)^{3/2}}+\frac{1}{c^2\,d\,e^2\,\sqrt{1+\frac{1}{c^2\,x^2}}\;x}2\,i\,b\,\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{-\frac{e\,\left(-i+c\,x\right)}{c\,d+i\,e}}\,\sqrt{-\frac{e\,\left(i+c\,x\right)}{c\,d-i\,e}}\right.$$

$$\left(c\,d\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{d+e\,x}\right],\frac{c\,d-i\,e}{c\,d+i\,e}\right]-c\,d\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{d+e\,x}\right],\frac{c\,d-i\,e}{c\,d+i\,e}\right]+\right.$$

$$\left.\left.2\,\left(c\,d-i\,e\right)\,\text{EllipticPi}\left[1-\frac{i\,e}{c\,d},i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{d+e\,x}\right],\frac{c\,d-i\,e}{c\,d+i\,e}\right]\right)\right)$$

Problem 72: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{a+b\,\text{ArcCsch}[c\,x]}{\left(d+e\,x\right)^{5/2}}\,dx$$

Optimal (type 4, 369 leaves, 12 steps):

$$-\frac{4\,b\,e\,\left(1+c^2x^2\right)}{3\,c\,d\,\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}\;x\,\sqrt{d+e\,x}}-\frac{2\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{3\,e\,\left(d+e\,x\right)^{3/2}}+\frac{4\,b\,\sqrt{-c^2}\,\sqrt{d+e\,x}\,\sqrt{1+c^2x^2}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],-\frac{2\sqrt{-c^2}\,e}{c^2d-\sqrt{-c^2}\,e}\right]}{3\,c\,d\,\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}\;x\,\sqrt{\frac{d+e\,x}{d+\frac{e}{\sqrt{-c^2}}}}}+$$

$$\frac{4\,b\,\sqrt{\frac{\sqrt{-c^2}\,(d+e\,x)}{\sqrt{-c^2}\,d+e}}\,\sqrt{1+c^2x^2}\;\text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],\frac{2\,e}{\sqrt{-c^2}\,d+e}\right]}{3\,c\,d\,e\,\sqrt{1+\frac{1}{c^2x^2}}\;x\,\sqrt{d+e\,x}}$$

Result (type 4, 375 leaves) :

$$\frac{1}{3\,e}2\left(-\frac{a}{\left(d+e\,x\right)^{3/2}}-\frac{2\,b\,c\,e^2\sqrt{1+\frac{1}{c^2x^2}}\;x}{d\,\left(c^2d^2+e^2\right)\sqrt{d+e\,x}}-\frac{b\,\text{ArcCsch}[c\,x]}{\left(d+e\,x\right)^{3/2}}+\frac{1}{c^2d^2\sqrt{1+\frac{1}{c^2x^2}}\;x}\right.$$

$$2\,b\,\sqrt{-\frac{c}{c\,d-\mathrm{i}\,e}}\,\sqrt{-\frac{e\,\left(-\mathrm{i}+c\,x\right)}{c\,d+\mathrm{i}\,e}}\,\sqrt{-\frac{e\,\left(\mathrm{i}+c\,x\right)}{c\,d-\mathrm{i}\,e}}\left(-\mathrm{i}\,c\,d\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-\mathrm{i}\,e}}\,\sqrt{d+e\,x}\right],\frac{c\,d-\mathrm{i}\,e}{c\,d+\mathrm{i}\,e}\right]+\mathrm{i}\,c\,d\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-\mathrm{i}\,e}}\,\sqrt{d+e\,x}\right],\frac{c\,d-\mathrm{i}\,e}{c\,d+\mathrm{i}\,e}\right]+\left(\mathrm{i}\,c\,d+e\right)\text{EllipticPi}\left[1-\frac{\mathrm{i}\,e}{c\,d},\mathrm{i}\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-\mathrm{i}\,e}}\,\sqrt{d+e\,x}\right],\frac{c\,d-\mathrm{i}\,e}{c\,d+\mathrm{i}\,e}\right]\right)\right)$$

Problem 75: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{a+b\,\text{ArcCsch}[c\,x]}{\left(d+e\,x\right)^{7/2}}\,dx$$

Optimal (type 4, 648 leaves, 19 steps) :

$$-\frac{4\,b\,e\,\left(1+c^2\,x^2\right)}{15\,c\,d\,\left(c^2\,d^2+e^2\right)\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\left(d+e\,x\right)^{3/2}}-\frac{16\,b\,c\,e\,\left(1+c^2\,x^2\right)}{15\,\left(c^2\,d^2+e^2\right)^2\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}}-\frac{4\,b\,e\,\left(1+c^2\,x^2\right)}{5\,c\,d^2\,\left(c^2\,d^2+e^2\right)\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}}-$$

$$\frac{2\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{5\,e\,\left(d+e\,x\right)^{5/2}}-\frac{4\,b\,c\,\left(7\,c^2\,d^2+3\,e^2\right)\,\sqrt{d+e\,x}\,\sqrt{1+c^2\,x^2}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],\frac{2\,\sqrt{-c^2}\,e}{-c^2\,d+\sqrt{-c^2}\,e}\right]}{15\,\sqrt{-c^2}\,d^2\,\left(c^2\,d^2+e^2\right)^2\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\sqrt{\frac{d+e\,x}{d+\frac{e}{\sqrt{-c^2}}}}}-$$

$$\frac{4\,b\,\sqrt{-c^2}\,\sqrt{\frac{d+e\,x}{d+\frac{e}{\sqrt{-c^2}}}}\,\sqrt{1+c^2\,x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],-\frac{2\,\sqrt{-c^2}\,e}{c^2\,d-\sqrt{-c^2}\,e}\right]}{15\,c\,d\,\left(c^2\,d^2+e^2\right)\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}}+$$

$$\frac{4\,b\,\sqrt{\frac{\sqrt{-c^2}\,\left(d+e\,x\right)}{\sqrt{-c^2}\,d+e}}\,\sqrt{1+c^2\,x^2}\,\text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],\frac{2\,e}{\sqrt{-c^2}\,d+e}\right]}{5\,c\,d^2\,e\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}}$$

Result (type 4, 472 leaves) :

$$\frac{1}{15}\left(-\frac{6\,a}{e\,(d+e\,x)^{5/2}}-\frac{4\,b\,c\,e\,\sqrt{1+\frac{1}{c^2 x^2}}\;x\,\left(e^2\,(4\,d+3\,e\,x)+c^2\,d^2\,(8\,d+7\,e\,x)\right)}{d^2\,(c^2\,d^2+e^2)^2\,(d+e\,x)^{3/2}}-\frac{6\,b\,\text{ArcCsch}[c\,x]}{e\,(d+e\,x)^{5/2}}+\right.$$

$$\left(4\,i\,b\,(c\,d+i\,e)\,\sqrt{\frac{e\,(1-i\,c\,x)}{i\,c\,d+e}}\,\sqrt{\frac{e\,(1+i\,c\,x)}{-i\,c\,d+e}}\,\left(c\,d\,(7\,c^2\,d^2+3\,e^2)\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{d+e\,x}\right],\frac{c\,d-i\,e}{c\,d+i\,e}\right]-\right.\right.$$

$$c\,d\,(6\,c^2\,d^2+i\,c\,d\,e+3\,e^2)\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{d+e\,x}\right],\frac{c\,d-i\,e}{c\,d+i\,e}\right]-3\,(c\,d-i\,e)^2\,(c\,d+i\,e)$$

$$\left.\left.\left.\text{EllipticPi}\left[1-\frac{i\,e}{c\,d},\,i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{d+e\,x}\right],\frac{c\,d-i\,e}{c\,d+i\,e}\right]\right)\right)\Bigg/\left(c\,d^3\,\sqrt{-\frac{c}{c\,d-i\,e}}\;e\,(c^2\,d^2+e^2)^2\,\sqrt{1+\frac{1}{c^2\,x^2}}\;x\right)\right)$$

Problem 98: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{x^2\,(a+b\,\text{ArcCsch}[c\,x])}{d+e\,x^2}\,dx$$

Optimal (type 4, 512 leaves, 25 steps):

$$\frac{x\,(a+b\,\text{ArcCsch}[c\,x])}{e}+\frac{b\,\text{ArcTanh}\left[\sqrt{1+\frac{1}{c^2 x^2}}\right]}{c\,e}+\frac{\sqrt{-d}\,(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1-\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2\,e^{3/2}}-$$

$$\frac{\sqrt{-d}\,(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1+\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2\,e^{3/2}}+\frac{\sqrt{-d}\,(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1-\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2\,e^{3/2}}-$$

$$\frac{\sqrt{-d}\,(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1+\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2\,e^{3/2}}-\frac{b\sqrt{-d}\,\text{PolyLog}\left[2,-\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2\,e^{3/2}}+$$

$$\frac{b\sqrt{-d}\,\text{PolyLog}\left[2,\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2\,e^{3/2}}-\frac{b\sqrt{-d}\,\text{PolyLog}\left[2,-\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2\,e^{3/2}}+\frac{b\sqrt{-d}\,\text{PolyLog}\left[2,\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2\,e^{3/2}}$$

Result (type 4, 1239 leaves):

$$\frac{1}{4\,c\,e^{3/2}}\Bigg(4\,a\,c\,\sqrt{e}\,x+4\,b\,c\,\sqrt{e}\,x\,\text{ArcCsch}[c\,x]-4\,a\,c\,\sqrt{d}\,\text{ArcTan}\Big[\frac{\sqrt{e}\,x}{\sqrt{d}}\Big]-$$

$$8\,\mathbb{i}\,b\,c\,\sqrt{d}\,\text{ArcSin}\Big[\frac{\sqrt{1+\frac{\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\Big]\,\text{ArcTan}\Big[\frac{\left(c\,\sqrt{d}-\sqrt{e}\right)\,\text{Cot}\left[\frac{1}{4}\,\left(\pi+2\,\mathbb{i}\,\text{ArcCsch}[c\,x]\right)\right]}{\sqrt{-c^2\,d+e}}\Big]-$$

$$8\,\mathbb{i}\,b\,c\,\sqrt{d}\,\text{ArcSin}\Big[\frac{\sqrt{1-\frac{\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\Big]\,\text{ArcTan}\Big[\frac{\left(c\,\sqrt{d}+\sqrt{e}\right)\,\text{Cot}\left[\frac{1}{4}\,\left(\pi+2\,\mathbb{i}\,\text{ArcCsch}[c\,x]\right)\right]}{\sqrt{-c^2\,d+e}}\Big]+$$

$$b\,c\,\sqrt{d}\,\pi\,\text{Log}\Big[1-\frac{\mathbb{i}\,\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\Big]-2\,\mathbb{i}\,b\,c\,\sqrt{d}\,\text{ArcCsch}[c\,x]\,\text{Log}\Big[1-\frac{\mathbb{i}\,\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\Big]+$$

$$4\,b\,c\,\sqrt{d}\,\text{ArcSin}\Big[\frac{\sqrt{1+\frac{\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\Big]\,\text{Log}\Big[1-\frac{\mathbb{i}\,\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\Big]-$$

$$b\,c\,\sqrt{d}\,\pi\,\text{Log}\Big[1+\frac{\mathbb{i}\,\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\Big]+2\,\mathbb{i}\,b\,c\,\sqrt{d}\,\text{ArcCsch}[c\,x]\,\text{Log}\Big[1+\frac{\mathbb{i}\,\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\Big]-$$

$$4\,b\,c\,\sqrt{d}\,\text{ArcSin}\Big[\frac{\sqrt{1-\frac{\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\Big]\,\text{Log}\Big[1+\frac{\mathbb{i}\,\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\Big]-b\,c\,\sqrt{d}\,\pi\,\text{Log}\Big[1-\frac{\mathbb{i}\,\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\Big]+$$

$$2\,\mathbb{i}\,b\,c\,\sqrt{d}\,\text{ArcCsch}[c\,x]\,\text{Log}\Big[1-\frac{\mathbb{i}\,\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\Big]+4\,b\,c\,\sqrt{d}\,\text{ArcSin}\Big[\frac{\sqrt{1-\frac{\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\Big]\,\text{Log}\Big[1-\frac{\mathbb{i}\,\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\Big]+$$

$$b\,c\,\sqrt{d}\,\pi\,\text{Log}\Big[1+\frac{\mathbb{i}\,\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\Big]-2\,\mathbb{i}\,b\,c\,\sqrt{d}\,\text{ArcCsch}[c\,x]\,\text{Log}\Big[1+\frac{\mathbb{i}\,\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\Big]-$$

$$4\,b\,c\,\sqrt{d}\,\text{ArcSin}\Big[\frac{\sqrt{1+\frac{\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\Big]\,\text{Log}\Big[1+\frac{\mathbb{i}\,\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\Big]+b\,c\,\sqrt{d}\,\pi\,\text{Log}\Big[\sqrt{e}-\frac{\mathbb{i}\,\sqrt{d}}{x}\Big]-$$

$$b\,c\,\sqrt{d}\,\pi\,\text{Log}\Big[\sqrt{e}+\frac{\mathbb{i}\,\sqrt{d}}{x}\Big]+4\,b\,\sqrt{e}\,\text{Log}\Big[\text{Cosh}\Big[\frac{1}{2}\,\text{ArcCsch}[c\,x]\Big]\Big]-4\,b\,\sqrt{e}\,\text{Log}\Big[\text{Sinh}\Big[\frac{1}{2}\,\text{ArcCsch}[c\,x]\Big]\Big]+$$

$$2\,i\,b\,c\,\sqrt{d}\,\text{PolyLog}\left[2,-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]-2\,i\,b\,c\,\sqrt{d}\,\text{PolyLog}\left[2,\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]-$$

$$\left.2\,i\,b\,c\,\sqrt{d}\,\text{PolyLog}\left[2,-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]+2\,i\,b\,c\,\sqrt{d}\,\text{PolyLog}\left[2,\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]\right)$$

Problem 99: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{x\,(a+b\,\text{ArcCsch}[c\,x])}{d+e\,x^2}\,dx$$

Optimal (type 4, 467 leaves, 26 steps):

$$-\frac{(a+b\,\text{ArcCsch}[c\,x])^2}{b\,e}-\frac{(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1-e^{-2\,\text{ArcCsch}[c\,x]}\right]}{e}+\frac{(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2\,e}+$$

$$\frac{(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1+\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2\,e}+\frac{(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2\,e}+$$

$$\frac{(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1+\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2\,e}+\frac{b\,\text{PolyLog}\left[2,e^{-2\,\text{ArcCsch}[c\,x]}\right]}{2\,e}+\frac{b\,\text{PolyLog}\left[2,-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2\,e}+$$

$$\frac{b\,\text{PolyLog}\left[2,\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2\,e}+\frac{b\,\text{PolyLog}\left[2,-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2\,e}+\frac{b\,\text{PolyLog}\left[2,\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2\,e}$$

Result (type 4, 1103 leaves):

$$\frac{1}{8\,e}\left(b\,\pi^2-4\,i\,b\,\pi\,\text{ArcCsch}[c\,x]-8\,b\,\text{ArcCsch}[c\,x]^2+16\,b\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(c\,\sqrt{d}-\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,i\,\text{ArcCsch}[c\,x]\right)\right]}{\sqrt{-c^2 d+e}}\right]-\right.$$

$$16\,b\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(c\,\sqrt{d}+\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,i\,\text{ArcCsch}[c\,x]\right)\right]}{\sqrt{-c^2 d+e}}\right]-8\,b\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-e^{-2\,\text{ArcCsch}[c\,x]}\right]+$$

$$2\,i\,b\,\pi\,\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+4\,b\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,i\,b\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+2\,i\,b\,\pi\,\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,b\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+8\,i\,b\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$2\,i\,b\,\pi\,\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+4\,b\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-$$

$$8\,i\,b\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+2\,i\,b\,\pi\,\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,b\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-8\,i\,b\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-$$

$$2\,i\,b\,\pi\,\text{Log}\left[\sqrt{e}-\frac{i\sqrt{d}}{x}\right]-2\,i\,b\,\pi\,\text{Log}\left[\sqrt{e}+\frac{i\sqrt{d}}{x}\right]+4\,a\,\text{Log}\left[d+e\,x^2\right]+4\,b\,\text{PolyLog}\left[2,\,e^{-2\,\text{ArcCsch}[c\,x]}\right]+$$

$$4\,b\,\text{PolyLog}\left[2,\,-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+4\,b\,\text{PolyLog}\left[2,\,\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$\left.4\,b\,\text{PolyLog}\left[2,\,-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+4\,b\,\text{PolyLog}\left[2,\,\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]\right)$$

Problem 100: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{a+b\,\text{ArcCsch}[c\,x]}{d+e\,x^2}\,dx$$

Optimal (type 4, 477 leaves, 19 steps) :

$$\frac{\left(a+b\,\text{ArcCsch}[c\,x]\right)\,\text{Log}\left[1-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2\,d+e}}\right]}{2\,\sqrt{-d}\,\sqrt{e}}-\frac{\left(a+b\,\text{ArcCsch}[c\,x]\right)\,\text{Log}\left[1+\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2\,d+e}}\right]}{2\,\sqrt{-d}\,\sqrt{e}}+$$

$$\frac{\left(a+b\,\text{ArcCsch}[c\,x]\right)\,\text{Log}\left[1-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2\,d+e}}\right]}{2\,\sqrt{-d}\,\sqrt{e}}-\frac{\left(a+b\,\text{ArcCsch}[c\,x]\right)\,\text{Log}\left[1+\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2\,d+e}}\right]}{2\,\sqrt{-d}\,\sqrt{e}}-\frac{b\,\text{PolyLog}\left[2,-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2\,d+e}}\right]}{2\,\sqrt{-d}\,\sqrt{e}}+$$

$$\frac{b\,\text{PolyLog}\left[2,\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2\,d+e}}\right]}{2\,\sqrt{-d}\,\sqrt{e}}-\frac{b\,\text{PolyLog}\left[2,-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2\,d+e}}\right]}{2\,\sqrt{-d}\,\sqrt{e}}+\frac{b\,\text{PolyLog}\left[2,\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2\,d+e}}\right]}{2\,\sqrt{-d}\,\sqrt{e}}$$

Result (type 4, 1055 leaves) :

$$\frac{1}{4\sqrt{d}\sqrt{e}}\left(4\,a\,\text{ArcTan}\left[\frac{\sqrt{e}\,x}{\sqrt{d}}\right]+8\,i\,b\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(c\sqrt{d}-\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,i\,\text{ArcCsch}[c\,x]\right)\right]}{\sqrt{-c^2 d+e}}\right]+\right.$$

$$8\,i\,b\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(c\sqrt{d}+\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,i\,\text{ArcCsch}[c\,x]\right)\right]}{\sqrt{-c^2 d+e}}\right]-b\,\pi\,\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$2\,i\,b\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-4\,b\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$b\,\pi\,\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-2\,i\,b\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,b\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+b\,\pi\,\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-$$

$$2\,i\,b\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-4\,b\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-$$

$$b\,\pi\,\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+2\,i\,b\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,b\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-b\,\pi\,\text{Log}\left[\sqrt{e}-\frac{i\sqrt{d}}{x}\right]+b\,\pi\,\text{Log}\left[\sqrt{e}+\frac{i\sqrt{d}}{x}\right]-$$

$$2\,i\,b\,\text{PolyLog}\left[2,-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+2\,i\,b\,\text{PolyLog}\left[2,\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$\left.2\,i\,b\,\text{PolyLog}\left[2,-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-2\,i\,b\,\text{PolyLog}\left[2,\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]\right)$$

Problem 101: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{a+b\,\text{ArcCsch}[c\,x]}{x\,(d+e\,x^2)}\,dx$$

Optimal (type 4, 425 leaves, 19 steps):

$$\frac{(a+b\,\text{ArcCsch}[c\,x])^2}{2\,b\,d}-\frac{(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1-\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2\,d+e}}\right]}{2\,d}-\frac{(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1+\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2\,d+e}}\right]}{2\,d}-$$
$$\frac{(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1-\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2\,d+e}}\right]}{2\,d}-\frac{(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1+\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2\,d+e}}\right]}{2\,d}-\frac{b\,\text{PolyLog}\left[2,-\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2\,d+e}}\right]}{2\,d}-$$
$$\frac{b\,\text{PolyLog}\left[2,\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2\,d+e}}\right]}{2\,d}-\frac{b\,\text{PolyLog}\left[2,-\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2\,d+e}}\right]}{2\,d}-\frac{b\,\text{PolyLog}\left[2,\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2\,d+e}}\right]}{2\,d}$$

Result (type 4, 1075 leaves):

$$-\frac{1}{8\,d}\left(b\,\pi^2-4\,i\,b\,\pi\,\text{ArcCsch}[c\,x]-4\,b\,\text{ArcCsch}[c\,x]^2+16\,b\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(c\,\sqrt{d}-\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,i\,\text{ArcCsch}[c\,x]\right)\right]}{\sqrt{-c^2\,d+e}}\right]-\right.$$

$$16\,b\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(c\,\sqrt{d}+\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,i\,\text{ArcCsch}[c\,x]\right)\right]}{\sqrt{-c^2\,d+e}}\right]+2\,i\,b\,\pi\,\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$4\,b\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]+8\,i\,b\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$2\,i\,b\,\pi\,\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]+4\,b\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$8\,i\,b\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]+2\,i\,b\,\pi\,\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$4\,b\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]-8\,i\,b\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$2\,i\,b\,\pi\,\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]+4\,b\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]-$$

$$8\,i\,b\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]-2\,i\,b\,\pi\,\text{Log}\left[\sqrt{e}-\frac{i\,\sqrt{d}}{x}\right]-2\,i\,b\,\pi\,\text{Log}\left[\sqrt{e}+\frac{i\,\sqrt{d}}{x}\right]-8\,a\,\text{Log}[x]+$$

$$4\,a\,\text{Log}\left[d+e\,x^2\right]+4\,b\,\text{PolyLog}\left[2,-\frac{i\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]+4\,b\,\text{PolyLog}\left[2,\frac{i\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$\left.4\,b\,\text{PolyLog}\left[2,-\frac{i\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]+4\,b\,\text{PolyLog}\left[2,\frac{i\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]\right)$$

Problem 102: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{a + b\,\text{ArcCsch}[c\,x]}{x^2\,(d + e\,x^2)}\,dx$$

Optimal (type 4, 518 leaves, 24 steps):

$$\frac{b\,c\,\sqrt{1+\frac{1}{c^2 x^2}}}{d} - \frac{a}{d\,x} - \frac{b\,\text{ArcCsch}[c\,x]}{d\,x} + \frac{\sqrt{e}\,(a + b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1 - \frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2\,(-d)^{3/2}} - \frac{\sqrt{e}\,(a + b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1 + \frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2\,(-d)^{3/2}} +$$

$$\frac{\sqrt{e}\,(a + b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1 - \frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2\,(-d)^{3/2}} - \frac{\sqrt{e}\,(a + b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1 + \frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2\,(-d)^{3/2}} - \frac{b\sqrt{e}\,\text{PolyLog}\left[2, -\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2\,(-d)^{3/2}} +$$

$$\frac{b\sqrt{e}\,\text{PolyLog}\left[2, \frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2\,(-d)^{3/2}} - \frac{b\sqrt{e}\,\text{PolyLog}\left[2, -\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2\,(-d)^{3/2}} + \frac{b\sqrt{e}\,\text{PolyLog}\left[2, \frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2\,(-d)^{3/2}}$$

Result (type 4, 1211 leaves):

$$-\frac{a}{d\,x} - \frac{a\sqrt{e}\,\text{ArcTan}\left[\frac{\sqrt{e}\,x}{\sqrt{d}}\right]}{d^{3/2}} + b\left(\frac{c\sqrt{1+\frac{1}{c^2 x^2}} - \frac{\text{ArcCsch}[c\,x]}{x}}{d} -\right.$$

$$\frac{1}{16\,d^{3/2}}\,i\,\sqrt{e}\left(\pi^2 - 4\,i\,\pi\,\text{ArcCsch}[c\,x] - 8\,\text{ArcCsch}[c\,x]^2 + 32\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(c\sqrt{d}-\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi + 2\,i\,\text{ArcCsch}[c\,x]\right)\right]}{\sqrt{-c^2 d+e}}\right] -\right.$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1 - e^{-2\,\text{ArcCsch}[c\,x]}\right] + 4\,i\,\pi\,\text{Log}\left[1 - \frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right] +$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1 - \frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right] + 16\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1 - \frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right] +$$

$$4\,i\,\pi\,\text{Log}\left[1 + \frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right] + 8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1 + \frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right] -$$

$$16\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-4\,i\,\pi\,\text{Log}\left[\sqrt{e}+\frac{i\sqrt{d}}{x}\right]+4\,\text{PolyLog}\left[2,e^{-2\,\text{ArcCsch}[c\,x]}\right]+$$

$$\left.8\,\text{PolyLog}\left[2,\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+8\,\text{PolyLog}\left[2,-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]\right)+$$

$$\frac{1}{16\,d^{3/2}}\,i\sqrt{e}\left(\pi^2-4\,i\,\pi\,\text{ArcCsch}[c\,x]-8\,\text{ArcCsch}[c\,x]^2-32\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(c\sqrt{d}+\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,i\,\text{ArcCsch}[c\,x]\right)\right]}{\sqrt{-c^2 d+e}}\right]-\right.$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-e^{-2\,\text{ArcCsch}[c\,x]}\right]+4\,i\,\pi\,\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+16\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,i\,\pi\,\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-$$

$$16\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-4\,i\,\pi\,\text{Log}\left[\sqrt{e}-\frac{i\sqrt{d}}{x}\right]+4\,\text{PolyLog}\left[2,e^{-2\,\text{ArcCsch}[c\,x]}\right]+$$

$$\left.\left.8\,\text{PolyLog}\left[2,-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+8\,\text{PolyLog}\left[2,\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]\right)\right)$$

Problem 103: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{x^5\left(a+b\,\text{ArcCsch}[c\,x]\right)}{\left(d+e\,x^2\right)^2}\,dx$$

Optimal (type 4, 591 leaves, 31 steps) :

$$\frac{b\sqrt{1+\frac{1}{c^2 x^2}}\,x}{2\,c\,e^2}+\frac{d\,(a+b\,\text{ArcCsch}[c\,x])}{2\,e^2\left(e+\frac{d}{x^2}\right)}+\frac{x^2\,(a+b\,\text{ArcCsch}[c\,x])}{2\,e^2}+\frac{2\,d\,(a+b\,\text{ArcCsch}[c\,x])^2}{b\,e^3}-\frac{b\,d\,\text{ArcTan}\left[\frac{\sqrt{c^2 d-e}}{c\,\sqrt{e}\,\sqrt{1+\frac{1}{c^2 x^2}}\,x}\right]}{2\,\sqrt{c^2 d-e}\,e^{5/2}}+$$
$$\frac{2\,d\,(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1-e^{-2\,\text{ArcCsch}[c\,x]}\right]}{e^3}-\frac{d\,(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{e^3}-\frac{d\,(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1+\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{e^3}-$$
$$\frac{d\,(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{e^3}-\frac{d\,(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1+\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{e^3}-\frac{b\,d\,\text{PolyLog}\left[2,\,e^{-2\,\text{ArcCsch}[c\,x]}\right]}{e^3}-$$
$$\frac{b\,d\,\text{PolyLog}\left[2,\,-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{e^3}-\frac{b\,d\,\text{PolyLog}\left[2,\,\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{e^3}-\frac{b\,d\,\text{PolyLog}\left[2,\,-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{e^3}-\frac{b\,d\,\text{PolyLog}\left[2,\,\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{e^3}$$

Result (type 4, 1554 leaves) :

$$\frac{a\,x^2}{2\,e^2}-\frac{a\,d^2}{2\,e^3\,(d+e\,x^2)}-\frac{a\,d\,\text{Log}\left[d+e\,x^2\right]}{e^3}+b\left(\frac{x\left(\sqrt{1+\frac{1}{c^2 x^2}}+c\,x\,\text{ArcCsch}[c\,x]\right)}{2\,c\,e^2}+\right.$$

$$\frac{i\, d^{3/2}\left(-\frac{\mathrm{ArcCsch}[c\, x]}{i\sqrt{d}\,\sqrt{e}+e\, x}-\frac{i\left(\frac{\mathrm{ArcSinh}\left[\frac{1}{c\, x}\right]}{\sqrt{e}}-\frac{\mathrm{Log}\left[\frac{2\sqrt{d}\,\sqrt{e}\,\left(i\sqrt{e}+c\left(c\sqrt{d}+i\sqrt{-c^2 d+e}\,\sqrt{1+\frac{1}{c^2 x^2}}\right)x\right)}{\sqrt{-c^2 d+e}\,\left(i\sqrt{d}+\sqrt{e}\, x\right)}\right]}{\sqrt{-c^2 d+e}}\right)}{\sqrt{d}}\right)}{4\, e^{5/2}}-\frac{i\, d^{3/2}\left(-\frac{\mathrm{ArcCsch}[c\, x]}{-i\sqrt{d}\,\sqrt{e}+e\, x}+\frac{i\left(\frac{\mathrm{ArcSinh}\left[\frac{1}{c\, x}\right]}{\sqrt{e}}-\frac{\mathrm{Log}\left[-\frac{2\sqrt{d}\,\sqrt{e}\,\left(\sqrt{e}+c\left(i\, c\sqrt{d}+\sqrt{-c^2 d+e}\,\sqrt{1+\frac{1}{c^2 x^2}}\right)x\right)}{\sqrt{-c^2 d+e}\,\left(\sqrt{d}+i\sqrt{e}\, x\right)}\right]}{\sqrt{-c^2 d+e}}\right)}{\sqrt{d}}\right)}{4\, e^{5/2}}-$$

$$\frac{1}{8\, e^3} d\left(\pi^2-4\, i\,\pi\,\mathrm{ArcCsch}[c\, x]-8\,\mathrm{ArcCsch}[c\, x]^2+32\,\mathrm{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\mathrm{ArcTan}\left[\frac{\left(c\sqrt{d}-\sqrt{e}\right)\mathrm{Cot}\left[\frac{1}{4}\left(\pi+2\, i\,\mathrm{ArcCsch}[c\, x]\right)\right]}{\sqrt{-c^2 d+e}}\right]-\right.$$

$$8\,\mathrm{ArcCsch}[c\, x]\,\mathrm{Log}\left[1-e^{-2\,\mathrm{ArcCsch}[c\, x]}\right]+4\, i\,\pi\,\mathrm{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\mathrm{ArcCsch}[c\, x]}}{c\sqrt{d}}\right]+$$

$$8\,\mathrm{ArcCsch}[c\, x]\,\mathrm{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\mathrm{ArcCsch}[c\, x]}}{c\sqrt{d}}\right]+16\, i\,\mathrm{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\mathrm{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\mathrm{ArcCsch}[c\, x]}}{c\sqrt{d}}\right]+$$

$$4\, i\,\pi\,\mathrm{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\mathrm{ArcCsch}[c\, x]}}{c\sqrt{d}}\right]+8\,\mathrm{ArcCsch}[c\, x]\,\mathrm{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\mathrm{ArcCsch}[c\, x]}}{c\sqrt{d}}\right]-$$

$$16\, i\,\mathrm{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\mathrm{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\mathrm{ArcCsch}[c\, x]}}{c\sqrt{d}}\right]-4\, i\,\pi\,\mathrm{Log}\left[\sqrt{e}+\frac{i\sqrt{d}}{x}\right]+4\,\mathrm{PolyLog}\left[2,\, e^{-2\,\mathrm{ArcCsch}[c\, x]}\right]+$$

$$\left.8\,\mathrm{PolyLog}\left[2,\,\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\mathrm{ArcCsch}[c\, x]}}{c\sqrt{d}}\right]+8\,\mathrm{PolyLog}\left[2,\,-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\mathrm{ArcCsch}[c\, x]}}{c\sqrt{d}}\right]\right)-$$

$$\frac{1}{8 e^3} d \Bigg(\pi^2 - 4 i \pi \text{ArcCsch}[c x] - 8 \text{ArcCsch}[c x]^2 - 32 \text{ArcSin}\Big[\frac{\sqrt{1 - \frac{\sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\Big] \text{ArcTan}\Big[\frac{\left(c \sqrt{d} + \sqrt{e}\right) \text{Cot}\left[\frac{1}{4} \left(\pi + 2 i \text{ArcCsch}[c x]\right)\right]}{\sqrt{-c^2 d + e}}\Big] -$$

$$8 \text{ArcCsch}[c x] \text{Log}\left[1 - e^{-2 \text{ArcCsch}[c x]}\right] + 4 i \pi \text{Log}\Big[1 + \frac{i \left(-\sqrt{e} + \sqrt{-c^2 d + e}\right) e^{\text{ArcCsch}[c x]}}{c \sqrt{d}}\Big] +$$

$$8 \text{ArcCsch}[c x] \text{Log}\Big[1 + \frac{i \left(-\sqrt{e} + \sqrt{-c^2 d + e}\right) e^{\text{ArcCsch}[c x]}}{c \sqrt{d}}\Big] + 16 i \text{ArcSin}\Big[\frac{\sqrt{1 - \frac{\sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\Big] \text{Log}\Big[1 + \frac{i \left(-\sqrt{e} + \sqrt{-c^2 d + e}\right) e^{\text{ArcCsch}[c x]}}{c \sqrt{d}}\Big] +$$

$$4 i \pi \text{Log}\Big[1 - \frac{i \left(\sqrt{e} + \sqrt{-c^2 d + e}\right) e^{\text{ArcCsch}[c x]}}{c \sqrt{d}}\Big] + 8 \text{ArcCsch}[c x] \text{Log}\Big[1 - \frac{i \left(\sqrt{e} + \sqrt{-c^2 d + e}\right) e^{\text{ArcCsch}[c x]}}{c \sqrt{d}}\Big] -$$

$$16 i \text{ArcSin}\Big[\frac{\sqrt{1 - \frac{\sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\Big] \text{Log}\Big[1 - \frac{i \left(\sqrt{e} + \sqrt{-c^2 d + e}\right) e^{\text{ArcCsch}[c x]}}{c \sqrt{d}}\Big] - 4 i \pi \text{Log}\Big[\sqrt{e} - \frac{i \sqrt{d}}{x}\Big] + 4 \text{PolyLog}\left[2, e^{-2 \text{ArcCsch}[c x]}\right] +$$

$$8 \text{PolyLog}\Big[2, -\frac{i \left(-\sqrt{e} + \sqrt{-c^2 d + e}\right) e^{\text{ArcCsch}[c x]}}{c \sqrt{d}}\Big] + 8 \text{PolyLog}\Big[2, \frac{i \left(\sqrt{e} + \sqrt{-c^2 d + e}\right) e^{\text{ArcCsch}[c x]}}{c \sqrt{d}}\Big]\Bigg)$$

Problem 104: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{x^3 \left(a+b\,\text{ArcCsch}[c\,x]\right)}{\left(d+e\,x^2\right)^2}\,dx$$

Optimal (type 4, 553 leaves, 29 steps):

$$-\frac{a+b\,\text{ArcCsch}[c\,x]}{2\,e\left(e+\frac{d}{x^2}\right)}-\frac{\left(a+b\,\text{ArcCsch}[c\,x]\right)^2}{b\,e^2}+\frac{b\,\text{ArcTan}\left[\frac{\sqrt{c^2 d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2 x^2}}\,x}\right]}{2\sqrt{c^2 d-e}\;e^{3/2}}-\frac{\left(a+b\,\text{ArcCsch}[c\,x]\right)\text{Log}\left[1-e^{-2\,\text{ArcCsch}[c\,x]}\right]}{e^2}+$$
$$\frac{\left(a+b\,\text{ArcCsch}[c\,x]\right)\text{Log}\left[1-\frac{c\sqrt{-d}\;e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2\,e^2}+\frac{\left(a+b\,\text{ArcCsch}[c\,x]\right)\text{Log}\left[1+\frac{c\sqrt{-d}\;e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2\,e^2}+$$
$$\frac{\left(a+b\,\text{ArcCsch}[c\,x]\right)\text{Log}\left[1-\frac{c\sqrt{-d}\;e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2\,e^2}+\frac{\left(a+b\,\text{ArcCsch}[c\,x]\right)\text{Log}\left[1+\frac{c\sqrt{-d}\;e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2\,e^2}+\frac{b\,\text{PolyLog}\left[2,\,e^{-2\,\text{ArcCsch}[c\,x]}\right]}{2\,e^2}+$$
$$\frac{b\,\text{PolyLog}\left[2,-\frac{c\sqrt{-d}\;e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2\,e^2}+\frac{b\,\text{PolyLog}\left[2,\frac{c\sqrt{-d}\;e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2\,e^2}+\frac{b\,\text{PolyLog}\left[2,-\frac{c\sqrt{-d}\;e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2\,e^2}+\frac{b\,\text{PolyLog}\left[2,\frac{c\sqrt{-d}\;e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2\,e^2}$$

Result (type 4, 1410 leaves):

$$\frac{1}{8\,e^2}\left(b\,\pi^2+\frac{4\,a\,d}{d+e\,x^2}-4\,i\,b\,\pi\,\text{ArcCsch}[c\,x]+\frac{2\,b\sqrt{d}\;\text{ArcCsch}[c\,x]}{\sqrt{d}-i\sqrt{e}\;x}+\frac{2\,b\sqrt{d}\;\text{ArcCsch}[c\,x]}{\sqrt{d}+i\sqrt{e}\;x}-\right.$$

$$8\,b\,\text{ArcCsch}[c\,x]^2-4\,b\,\text{ArcSinh}\left[\frac{1}{c\,x}\right]+16\,b\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(c\sqrt{d}-\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,i\,\text{ArcCsch}[c\,x]\right)\right]}{\sqrt{-c^2 d+e}}\right]-$$

$$16\,b\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(c\sqrt{d}+\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,i\,\text{ArcCsch}[c\,x]\right)\right]}{\sqrt{-c^2 d+e}}\right]-8\,b\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-e^{-2\,\text{ArcCsch}[c\,x]}\right]+$$

$$2\,i\,b\,\pi\,\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+4\,b\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,i\,b\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+2\,i\,b\,\pi\,\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,b\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+8\,i\,b\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$2\,i\,b\,\pi\,\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+4\,b\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-$$

$$8\,i\,b\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+2\,i\,b\,\pi\,\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,b\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-8\,i\,b\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-$$

$$2\,i\,b\,\pi\,\text{Log}\left[\sqrt{e}-\frac{i\sqrt{d}}{x}\right]-2\,i\,b\,\pi\,\text{Log}\left[\sqrt{e}+\frac{i\sqrt{d}}{x}\right]+\frac{2\,b\sqrt{e}\,\text{Log}\left[\frac{2\sqrt{d}\sqrt{e}\left(i\sqrt{e}+c\left(c\sqrt{d}+i\sqrt{-c^2 d+e}\sqrt{1+\frac{1}{c^2x^2}}\right)x\right)}{\sqrt{-c^2 d+e}\left(i\sqrt{d}+\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2 d+e}}+$$

$$\frac{2\,b\sqrt{e}\,\text{Log}\left[-\frac{2\sqrt{d}\sqrt{e}\left(\sqrt{e}+c\left(i\,c\sqrt{d}+\sqrt{-c^2 d+e}\sqrt{1+\frac{1}{c^2x^2}}\right)x\right)}{\sqrt{-c^2 d+e}\left(\sqrt{d}+i\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2 d+e}}+4\,a\,\text{Log}\left[d+e\,x^2\right]+4\,b\,\text{PolyLog}\left[2,\,e^{-2\,\text{ArcCsch}[c\,x]}\right]+$$

$$4\,b\,\text{PolyLog}\left[2,\,-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+4\,b\,\text{PolyLog}\left[2,\,\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,b\,\text{PolyLog}\left[2,-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+4\,b\,\text{PolyLog}\left[2,\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]\Bigg)$$

Problem 105: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x\,(a+b\,\text{ArcCsch}[c\,x])}{\left(d+e\,x^2\right)^2}\,dx$$

Optimal (type 3, 139 leaves, 7 steps):

$$-\frac{a+b\,\text{ArcCsch}[c\,x]}{2\,e\,\left(d+e\,x^2\right)}+\frac{b\,c\,x\,\text{ArcTan}\left[\sqrt{-1-c^2 x^2}\right]}{2\,d\,e\,\sqrt{-c^2 x^2}}+\frac{b\,c\,x\,\text{ArcTanh}\left[\frac{\sqrt{e}\,\sqrt{-1-c^2 x^2}}{\sqrt{c^2 d-e}}\right]}{2\,d\,\sqrt{c^2 d-e}\,\sqrt{e}\,\sqrt{-c^2 x^2}}$$

Result (type 3, 271 leaves):

$$-\frac{1}{4\,e}\left(\frac{2\,a}{d+e\,x^2}+\frac{2\,b\,\text{ArcCsch}[c\,x]}{d+e\,x^2}-\frac{2\,b\,\text{ArcSinh}\left[\frac{1}{c\,x}\right]}{d}+\right.$$

$$\left.\frac{b\,\sqrt{e}\,\text{Log}\left[-\frac{4\left(i\,d\,e+c\,d\,\sqrt{e}\left(c\,\sqrt{d}+i\,\sqrt{-c^2 d+e}\,\sqrt{1+\frac{1}{c^2 x^2}}\right)x\right)}{b\,\sqrt{-c^2 d+e}\,\left(\sqrt{d}-i\,\sqrt{e}\,x\right)}\right]}{d\,\sqrt{-c^2 d+e}}+\frac{b\,\sqrt{e}\,\text{Log}\left[\frac{4\,i\left(d\,e+c\,d\,\sqrt{e}\left(i\,c\,\sqrt{d}+\sqrt{-c^2 d+e}\,\sqrt{1+\frac{1}{c^2 x^2}}\right)x\right)}{b\,\sqrt{-c^2 d+e}\,\left(\sqrt{d}+i\,\sqrt{e}\,x\right)}\right]}{d\,\sqrt{-c^2 d+e}}\right)$$

Problem 106: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{a+b\,\text{ArcCsch}[c\,x]}{x\,\left(d+e\,x^2\right)^2}\,dx$$

Optimal (type 4, 515 leaves, 24 steps):

$$-\frac{e\,(a+b\,\text{ArcCsch}[c\,x])}{2\,d^2\left(e+\frac{d}{x^2}\right)}+\frac{(a+b\,\text{ArcCsch}[c\,x])^2}{2\,b\,d^2}+\frac{b\sqrt{e}\,\text{ArcTan}\left[\frac{\sqrt{c^2 d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}\,x}\right]}{2\,d^2\sqrt{c^2 d-e}}-\frac{(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1-\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2\,d^2}-$$

$$\frac{(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1+\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2\,d^2}-\frac{(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1-\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2\,d^2}-\frac{(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1+\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2\,d^2}-$$

$$\frac{b\,\text{PolyLog}\left[2,-\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2\,d^2}-\frac{b\,\text{PolyLog}\left[2,\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2\,d^2}-\frac{b\,\text{PolyLog}\left[2,-\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2\,d^2}-\frac{b\,\text{PolyLog}\left[2,\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2\,d^2}$$

Result (type 4, 1382 leaves):

$$-\frac{1}{8\,d^2}\left(b\,\pi^2-\frac{4\,a\,d}{d+e\,x^2}-4\,i\,b\,\pi\,\text{ArcCsch}[c\,x]-\frac{2\,b\sqrt{d}\,\text{ArcCsch}[c\,x]}{\sqrt{d}-i\sqrt{e}\,x}-\frac{2\,b\sqrt{d}\,\text{ArcCsch}[c\,x]}{\sqrt{d}+i\sqrt{e}\,x}-\right.$$

$$4\,b\,\text{ArcCsch}[c\,x]^2+4\,b\,\text{ArcSinh}\left[\frac{1}{c\,x}\right]+16\,b\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(c\sqrt{d}-\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,i\,\text{ArcCsch}[c\,x]\right)\right]}{\sqrt{-c^2 d+e}}\right]-$$

$$16\,b\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(c\sqrt{d}+\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,i\,\text{ArcCsch}[c\,x]\right)\right]}{\sqrt{-c^2 d+e}}\right]+2\,i\,b\,\pi\,\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,b\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+8\,i\,b\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$2\,i\,b\,\pi\,\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+4\,b\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,i\,b\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+2\,i\,b\,\pi\,\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,b\,\text{ArcCsch}[c\,x]\,\text{Log}\Big[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\Big]-8\,i\,b\,\text{ArcSin}\Big[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\Big]\,\text{Log}\Big[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\Big]+$$

$$2\,i\,b\,\pi\,\text{Log}\Big[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\Big]+4\,b\,\text{ArcCsch}[c\,x]\,\text{Log}\Big[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\Big]-$$

$$8\,i\,b\,\text{ArcSin}\Big[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\Big]\,\text{Log}\Big[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\Big]-2\,i\,b\,\pi\,\text{Log}\Big[\sqrt{e}-\frac{i\sqrt{d}}{x}\Big]-2\,i\,b\,\pi\,\text{Log}\Big[\sqrt{e}+\frac{i\sqrt{d}}{x}\Big]-$$

$$8\,a\,\text{Log}[x]-\frac{2\,b\sqrt{e}\,\text{Log}\Big[\frac{2\sqrt{d}\sqrt{e}\left(i\sqrt{e}+c\left(c\sqrt{d}+i\sqrt{-c^2 d+e}\sqrt{1+\frac{1}{c^2x^2}}\right)x\right)}{\sqrt{-c^2 d+e}\left(i\sqrt{d}+\sqrt{e}\,x\right)}\Big]}{\sqrt{-c^2 d+e}}-\frac{2\,b\sqrt{e}\,\text{Log}\Big[-\frac{2\sqrt{d}\sqrt{e}\left(\sqrt{e}+c\left(i\,c\sqrt{d}+\sqrt{-c^2 d+e}\sqrt{1+\frac{1}{c^2x^2}}\right)x\right)}{\sqrt{-c^2 d+e}\left(\sqrt{d}+i\sqrt{e}\,x\right)}\Big]}{\sqrt{-c^2 d+e}}+$$

$$4\,a\,\text{Log}\big[d+e\,x^2\big]+4\,b\,\text{PolyLog}\Big[2,-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\Big]+4\,b\,\text{PolyLog}\Big[2,\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\Big]+$$

$$\left.4\,b\,\text{PolyLog}\Big[2,-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\Big]+4\,b\,\text{PolyLog}\Big[2,\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\Big]\right)$$

Problem 107: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{x^4\left(a+b\,\text{ArcCsch}[c\,x]\right)}{\left(d+e\,x^2\right)^2}\,dx$$

Optimal (type 4, 756 leaves, 51 steps):

$$-\frac{d\left(a+b\,\text{ArcCsch}[c\,x]\right)}{4\,e^2\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)}+\frac{d\left(a+b\,\text{ArcCsch}[c\,x]\right)}{4\,e^2\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)}+\frac{x\left(a+b\,\text{ArcCsch}[c\,x]\right)}{e^2}+\frac{b\,\text{ArcTanh}\left[\sqrt{1+\frac{1}{c^2\,x^2}}\right]}{c\,e^2}+\frac{b\,\sqrt{d}\,\text{ArcTanh}\left[\frac{c^2\,d-\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\,\sqrt{d}\,\sqrt{c^2\,d-e}\,\sqrt{1+\frac{1}{c^2\,x^2}}}\right]}{4\,\sqrt{c^2\,d-e}\,e^2}+$$

$$\frac{b\,\sqrt{d}\,\text{ArcTanh}\left[\frac{c^2\,d+\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\,\sqrt{d}\,\sqrt{c^2\,d-e}\,\sqrt{1+\frac{1}{c^2\,x^2}}}\right]}{4\,\sqrt{c^2\,d-e}\,e^2}+\frac{3\,\sqrt{-d}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)\,\text{Log}\left[1-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2\,d+e}}\right]}{4\,e^{5/2}}-\frac{3\,\sqrt{-d}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)\,\text{Log}\left[1+\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2\,d+e}}\right]}{4\,e^{5/2}}+$$

$$\frac{3\,\sqrt{-d}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)\,\text{Log}\left[1-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2\,d+e}}\right]}{4\,e^{5/2}}-\frac{3\,\sqrt{-d}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)\,\text{Log}\left[1+\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2\,d+e}}\right]}{4\,e^{5/2}}-\frac{3\,b\,\sqrt{-d}\,\text{PolyLog}\left[2,-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2\,d+e}}\right]}{4\,e^{5/2}}+$$

$$\frac{3\,b\,\sqrt{-d}\,\text{PolyLog}\left[2,\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2\,d+e}}\right]}{4\,e^{5/2}}-\frac{3\,b\,\sqrt{-d}\,\text{PolyLog}\left[2,-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2\,d+e}}\right]}{4\,e^{5/2}}+\frac{3\,b\,\sqrt{-d}\,\text{PolyLog}\left[2,\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2\,d+e}}\right]}{4\,e^{5/2}}$$

Result (type 4, 1593 leaves) :

$$\frac{a\,x}{e^2}+\frac{a\,d\,x}{2\,e^2\left(d+e\,x^2\right)}-\frac{3\,a\,\sqrt{d}\,\text{ArcTan}\left[\frac{\sqrt{e}\,x}{\sqrt{d}}\right]}{2\,e^{5/2}}+$$

$$b\left(-\frac{d\left(-\frac{\text{ArcCsch}[c\,x]}{i\sqrt{d}\,\sqrt{e}+e\,x}-\frac{i\left(\frac{\text{ArcSinh}\left[\frac{1}{c\,x}\right]}{\sqrt{e}}-\frac{\text{Log}\left[\frac{2\sqrt{d}\,\sqrt{e}\left(i\sqrt{e}+c\left(c\sqrt{d}+i\sqrt{-c^2\,d+e}\,\sqrt{1+\frac{1}{c^2\,x^2}}\right)x\right)}{\sqrt{-c^2\,d+e}\left(i\sqrt{d}+\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2\,d+e}}\right)}{\sqrt{d}}\right)}{4\,e^2}-\frac{d\left(-\frac{\text{ArcCsch}[c\,x]}{-i\sqrt{d}\,\sqrt{e}+e\,x}+\frac{i\left(\frac{\text{ArcSinh}\left[\frac{1}{c\,x}\right]}{\sqrt{e}}-\frac{\text{Log}\left[-\frac{2\sqrt{d}\,\sqrt{e}\left(\sqrt{e}+c\left(i\,c\sqrt{d}+\sqrt{-c^2\,d+e}\,\sqrt{1+\frac{1}{c^2\,x^2}}\right)x\right)}{\sqrt{-c^2\,d+e}\left(\sqrt{d}+i\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2\,d+e}}\right)}{\sqrt{d}}\right)}{4\,e^2}-\right.$$

$$\frac{1}{32\,e^{5/2}}\,3\,i\sqrt{d}\left(\pi^2-4\,i\,\pi\,\text{ArcCsch}[c\,x]-8\,\text{ArcCsch}[c\,x]^2+32\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(c\sqrt{d}-\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,i\,\text{ArcCsch}[c\,x]\right)\right]}{\sqrt{-c^2\,d+e}}\right]-\right.$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-e^{-2\,\text{ArcCsch}[c\,x]}\right]+4\,i\,\pi\,\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+16\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,i\,\pi\,\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-$$

$$16\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-4\,i\,\pi\,\text{Log}\left[\sqrt{e}+\frac{i\sqrt{d}}{x}\right]+4\,\text{PolyLog}\left[2,\,e^{-2\,\text{ArcCsch}[c\,x]}\right]+$$

$$8\,\text{PolyLog}\left[2,\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c x]}}{c\sqrt{d}}\right]+8\,\text{PolyLog}\left[2,-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c x]}}{c\sqrt{d}}\right]\Bigg)+$$

$$\frac{1}{32\,e^{5/2}}3\,i\sqrt{d}\left(\pi^2-4\,i\,\pi\,\text{ArcCsch}[c x]-8\,\text{ArcCsch}[c x]^2-32\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(c\sqrt{d}+\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,i\,\text{ArcCsch}[c x]\right)\right]}{\sqrt{-c^2 d+e}}\right]-\right.$$

$$8\,\text{ArcCsch}[c x]\,\text{Log}\left[1-e^{-2\,\text{ArcCsch}[c x]}\right]+4\,i\,\pi\,\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c x]}}{c\sqrt{d}}\right]+$$

$$8\,\text{ArcCsch}[c x]\,\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c x]}}{c\sqrt{d}}\right]+16\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c x]}}{c\sqrt{d}}\right]+$$

$$4\,i\,\pi\,\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c x]}}{c\sqrt{d}}\right]+8\,\text{ArcCsch}[c x]\,\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c x]}}{c\sqrt{d}}\right]-$$

$$16\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c x]}}{c\sqrt{d}}\right]-4\,i\,\pi\,\text{Log}\left[\sqrt{e}-\frac{i\sqrt{d}}{x}\right]+4\,\text{PolyLog}\left[2,e^{-2\,\text{ArcCsch}[c x]}\right]+$$

$$\left.8\,\text{PolyLog}\left[2,-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c x]}}{c\sqrt{d}}\right]+8\,\text{PolyLog}\left[2,\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c x]}}{c\sqrt{d}}\right]\right)+\frac{1}{c\,e^2}$$

$$\left(\frac{1}{2}\,\text{ArcCsch}[c\,x]\,\text{Coth}\left[\frac{1}{2}\,\text{ArcCsch}[c\,x]\right]+\text{Log}\left[\text{Cosh}\left[\frac{1}{2}\,\text{ArcCsch}[c\,x]\right]\right]-\text{Log}\left[\text{Sinh}\left[\frac{1}{2}\,\text{ArcCsch}[c\,x]\right]\right]-\frac{1}{2}\,\text{ArcCsch}[c\,x]\,\text{Tanh}\left[\frac{1}{2}\,\text{ArcCsch}[c\,x]\right]\right)\Bigg)$$

Problem 108: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{x^2\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{\left(d+e\,x^2\right)^2}\,dx$$

Optimal (type 4, 719 leaves, 27 steps) :

$$\frac{a+b\,\text{ArcCsch}[c\,x]}{4\,e\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)}-\frac{a+b\,\text{ArcCsch}[c\,x]}{4\,e\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)}-\frac{b\,\text{ArcTanh}\left[\frac{c^2\,d-\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\,\sqrt{d}\,\sqrt{c^2\,d-e}\,\sqrt{1+\frac{1}{c^2\,x^2}}}\right]}{4\,\sqrt{d}\,\sqrt{c^2\,d-e}\,e}-\frac{b\,\text{ArcTanh}\left[\frac{c^2\,d+\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\,\sqrt{d}\,\sqrt{c^2\,d-e}\,\sqrt{1+\frac{1}{c^2\,x^2}}}\right]}{4\,\sqrt{d}\,\sqrt{c^2\,d-e}\,e}+$$

$$\frac{(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2\,d+e}}\right]}{4\,\sqrt{-d}\,e^{3/2}}-\frac{(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1+\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2\,d+e}}\right]}{4\,\sqrt{-d}\,e^{3/2}}+$$

$$\frac{(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2\,d+e}}\right]}{4\,\sqrt{-d}\,e^{3/2}}-\frac{(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1+\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2\,d+e}}\right]}{4\,\sqrt{-d}\,e^{3/2}}-\frac{b\,\text{PolyLog}\left[2,-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2\,d+e}}\right]}{4\,\sqrt{-d}\,e^{3/2}}+$$

$$\frac{b\,\text{PolyLog}\left[2,\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2\,d+e}}\right]}{4\,\sqrt{-d}\,e^{3/2}}-\frac{b\,\text{PolyLog}\left[2,-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2\,d+e}}\right]}{4\,\sqrt{-d}\,e^{3/2}}+\frac{b\,\text{PolyLog}\left[2,\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2\,d+e}}\right]}{4\,\sqrt{-d}\,e^{3/2}}$$

Result (type 4, 1442 leaves):

$$\frac{1}{8\,e^{3/2}}\left(-\frac{4\,a\,\sqrt{e}\,x}{d+e\,x^2}+\frac{4\,a\,\text{ArcTan}\left[\frac{\sqrt{e}\,x}{\sqrt{d}}\right]}{\sqrt{d}}+b\left(\frac{2\,\text{ArcCsch}[c\,x]}{i\,\sqrt{d}-\sqrt{e}\,x}-\frac{2\,\text{ArcCsch}[c\,x]}{i\,\sqrt{d}+\sqrt{e}\,x}+\right.\right.$$

$$\frac{8\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]\,\text{ArcTan}\left[\frac{\left(c\,\sqrt{d}-\sqrt{e}\right)\,\text{Cot}\left[\frac{1}{4}\,(\pi+2\,i\,\text{ArcCsch}[c\,x])\right]}{\sqrt{-c^2\,d+e}}\right]}{\sqrt{d}}+\frac{8\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]\,\text{ArcTan}\left[\frac{\left(c\,\sqrt{d}+\sqrt{e}\right)\,\text{Cot}\left[\frac{1}{4}\,(\pi+2\,i\,\text{ArcCsch}[c\,x])\right]}{\sqrt{-c^2\,d+e}}\right]}{\sqrt{d}}-$$

$$\frac{\pi\,\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]}{\sqrt{d}}+\frac{2\,i\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]}{\sqrt{d}}-$$

$$\frac{4\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]\,\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]}{\sqrt{d}}+\frac{\pi\,\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]}{\sqrt{d}}-$$

$$\frac{2\,\mathrm{i}\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1+\frac{\mathrm{i}\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]}{\sqrt{d}}+\frac{4\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{\mathrm{i}\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]}{\sqrt{d}}+$$

$$\frac{\pi\,\text{Log}\left[1-\frac{\mathrm{i}\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]}{\sqrt{d}}-\frac{2\,\mathrm{i}\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-\frac{\mathrm{i}\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]}{\sqrt{d}}-$$

$$\frac{4\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{\mathrm{i}\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]}{\sqrt{d}}-\frac{\pi\,\text{Log}\left[1+\frac{\mathrm{i}\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]}{\sqrt{d}}+$$

$$\frac{2\,\mathrm{i}\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1+\frac{\mathrm{i}\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]}{\sqrt{d}}+\frac{4\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{\mathrm{i}\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]}{\sqrt{d}}-$$

$$\frac{\pi\,\text{Log}\left[\sqrt{e}-\frac{\mathrm{i}\sqrt{d}}{x}\right]}{\sqrt{d}}+\frac{\pi\,\text{Log}\left[\sqrt{e}+\frac{\mathrm{i}\sqrt{d}}{x}\right]}{\sqrt{d}}+\frac{2\,\mathrm{i}\sqrt{e}\,\text{Log}\left[\frac{2\sqrt{d}\sqrt{e}\left(\mathrm{i}\sqrt{e}+c\left(c\sqrt{d}+\mathrm{i}\sqrt{-c^2 d+e}\sqrt{1+\frac{1}{c^2 x^2}}\right)x\right)}{\sqrt{-c^2 d+e}\left(\mathrm{i}\sqrt{d}+\sqrt{e}\,x\right)}\right]}{\sqrt{d}\sqrt{-c^2 d+e}}-$$

$$\frac{2\,\mathrm{i}\sqrt{e}\,\text{Log}\left[-\frac{2\sqrt{d}\sqrt{e}\left(\sqrt{e}+c\left(\mathrm{i}\,c\sqrt{d}+\sqrt{-c^2 d+e}\sqrt{1+\frac{1}{c^2 x^2}}\right)x\right)}{\sqrt{-c^2 d+e}\left(\sqrt{d}+\mathrm{i}\sqrt{e}\,x\right)}\right]}{\sqrt{d}\sqrt{-c^2 d+e}}-\frac{2\,\mathrm{i}\,\text{PolyLog}\left[2,-\frac{\mathrm{i}\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]}{\sqrt{d}}+$$

$$\left.\left.\frac{2\,\mathrm{i}\,\text{PolyLog}\left[2,\frac{\mathrm{i}\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]}{\sqrt{d}}+\frac{2\,\mathrm{i}\,\text{PolyLog}\left[2,-\frac{\mathrm{i}\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]}{\sqrt{d}}-\frac{2\,\mathrm{i}\,\text{PolyLog}\left[2,\frac{\mathrm{i}\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]}{\sqrt{d}}\right)\right)$$

Problem 109: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{a+b\,\text{ArcCsch}[c\,x]}{\left(d+e\,x^2\right)^2}\,dx$$

Optimal (type 4, 713 leaves, 47 steps):

$$-\frac{a+b\,\text{ArcCsch}[c\,x]}{4\,d\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)}+\frac{a+b\,\text{ArcCsch}[c\,x]}{4\,d\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)}+\frac{b\,\text{ArcTanh}\left[\frac{c^2\,d-\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\,\sqrt{d}\,\sqrt{c^2\,d-e}\,\sqrt{1+\frac{1}{c^2\,x^2}}}\right]}{4\,d^{3/2}\,\sqrt{c^2\,d-e}}+\frac{b\,\text{ArcTanh}\left[\frac{c^2\,d+\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\,\sqrt{d}\,\sqrt{c^2\,d-e}\,\sqrt{1+\frac{1}{c^2\,x^2}}}\right]}{4\,d^{3/2}\,\sqrt{c^2\,d-e}}-$$

$$\frac{\left(a+b\,\text{ArcCsch}[c\,x]\right)\,\text{Log}\left[1-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2\,d+e}}\right]}{4\,(-d)^{3/2}\,\sqrt{e}}+\frac{\left(a+b\,\text{ArcCsch}[c\,x]\right)\,\text{Log}\left[1+\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2\,d+e}}\right]}{4\,(-d)^{3/2}\,\sqrt{e}}-$$

$$\frac{\left(a+b\,\text{ArcCsch}[c\,x]\right)\,\text{Log}\left[1-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2\,d+e}}\right]}{4\,(-d)^{3/2}\,\sqrt{e}}+\frac{\left(a+b\,\text{ArcCsch}[c\,x]\right)\,\text{Log}\left[1+\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2\,d+e}}\right]}{4\,(-d)^{3/2}\,\sqrt{e}}+\frac{b\,\text{PolyLog}\left[2,-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2\,d+e}}\right]}{4\,(-d)^{3/2}\,\sqrt{e}}-$$

$$\frac{b\,\text{PolyLog}\left[2,\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2\,d+e}}\right]}{4\,(-d)^{3/2}\,\sqrt{e}}+\frac{b\,\text{PolyLog}\left[2,-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2\,d+e}}\right]}{4\,(-d)^{3/2}\,\sqrt{e}}-\frac{b\,\text{PolyLog}\left[2,\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2\,d+e}}\right]}{4\,(-d)^{3/2}\,\sqrt{e}}$$

Result (type 4, 1520 leaves):

$$\frac{a\,x}{2\,d\left(d+e\,x^2\right)}+\frac{a\,\text{ArcTan}\left[\frac{\sqrt{e}\,x}{\sqrt{d}}\right]}{2\,d^{3/2}\,\sqrt{e}}+$$

$$b\left(-\frac{-\frac{\text{ArcCsch}[c\,x]}{i\,\sqrt{d}\,\sqrt{e}+e\,x}-\frac{i\left(\frac{\text{ArcSinh}\left[\frac{1}{c\,x}\right]}{\sqrt{e}}-\frac{\text{Log}\left[\frac{2\,\sqrt{d}\,\sqrt{e}\left(i\,\sqrt{e}+c\left(c\,\sqrt{d}+i\,\sqrt{-c^2\,d+e}\,\sqrt{1+\frac{1}{c^2\,x^2}}\right)x\right)}{\sqrt{-c^2\,d+e}\left(i\,\sqrt{d}+\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2\,d+e}}\right)}{\sqrt{d}}}{4\,d}-\frac{-\frac{\text{ArcCsch}[c\,x]}{-i\,\sqrt{d}\,\sqrt{e}+e\,x}+\frac{i\left(\frac{\text{ArcSinh}\left[\frac{1}{c\,x}\right]}{\sqrt{e}}-\frac{\text{Log}\left[-\frac{2\,\sqrt{d}\,\sqrt{e}\left(\sqrt{e}+c\left(i\,c\,\sqrt{d}+\sqrt{-c^2\,d+e}\,\sqrt{1+\frac{1}{c^2\,x^2}}\right)x\right)}{\sqrt{-c^2\,d+e}\left(\sqrt{d}+i\,\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2\,d+e}}\right)}{\sqrt{d}}}{4\,d}+\right.$$

$$\frac{1}{32\, d^{3/2}\sqrt{e}}\, i \left(\pi^2 - 4\, i\, \pi\, \text{ArcCsch}[c\, x] - 8\, \text{ArcCsch}[c\, x]^2 + 32\, \text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right] \text{ArcTan}\left[\frac{\left(c\sqrt{d}-\sqrt{e}\right) \text{Cot}\left[\frac{1}{4}\left(\pi + 2\, i\, \text{ArcCsch}[c\, x]\right)\right]}{\sqrt{-c^2 d + e}}\right] -\right.$$

$$8\, \text{ArcCsch}[c\, x]\, \text{Log}\left[1 - e^{-2\, \text{ArcCsch}[c\, x]}\right] + 4\, i\, \pi\, \text{Log}\left[1 - \frac{i\left(-\sqrt{e}+\sqrt{-c^2 d + e}\right) e^{\text{ArcCsch}[c\, x]}}{c\sqrt{d}}\right] +$$

$$8\, \text{ArcCsch}[c\, x]\, \text{Log}\left[1 - \frac{i\left(-\sqrt{e}+\sqrt{-c^2 d + e}\right) e^{\text{ArcCsch}[c\, x]}}{c\sqrt{d}}\right] + 16\, i\, \text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right] \text{Log}\left[1 - \frac{i\left(-\sqrt{e}+\sqrt{-c^2 d + e}\right) e^{\text{ArcCsch}[c\, x]}}{c\sqrt{d}}\right] +$$

$$4\, i\, \pi\, \text{Log}\left[1 + \frac{i\left(\sqrt{e}+\sqrt{-c^2 d + e}\right) e^{\text{ArcCsch}[c\, x]}}{c\sqrt{d}}\right] + 8\, \text{ArcCsch}[c\, x]\, \text{Log}\left[1 + \frac{i\left(\sqrt{e}+\sqrt{-c^2 d + e}\right) e^{\text{ArcCsch}[c\, x]}}{c\sqrt{d}}\right] -$$

$$16\, i\, \text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right] \text{Log}\left[1 + \frac{i\left(\sqrt{e}+\sqrt{-c^2 d + e}\right) e^{\text{ArcCsch}[c\, x]}}{c\sqrt{d}}\right] - 4\, i\, \pi\, \text{Log}\left[\sqrt{e} + \frac{i\sqrt{d}}{x}\right] + 4\, \text{PolyLog}\left[2, e^{-2\, \text{ArcCsch}[c\, x]}\right] +$$

$$\left. 8\, \text{PolyLog}\left[2, \frac{i\left(-\sqrt{e}+\sqrt{-c^2 d + e}\right) e^{\text{ArcCsch}[c\, x]}}{c\sqrt{d}}\right] + 8\, \text{PolyLog}\left[2, -\frac{i\left(\sqrt{e}+\sqrt{-c^2 d + e}\right) e^{\text{ArcCsch}[c\, x]}}{c\sqrt{d}}\right]\right) -$$

$$\frac{1}{32\, d^{3/2}\sqrt{e}}\, i \left(\pi^2 - 4\, i\, \pi\, \text{ArcCsch}[c\, x] - 8\, \text{ArcCsch}[c\, x]^2 - 32\, \text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right] \text{ArcTan}\left[\frac{\left(c\sqrt{d}+\sqrt{e}\right) \text{Cot}\left[\frac{1}{4}\left(\pi + 2\, i\, \text{ArcCsch}[c\, x]\right)\right]}{\sqrt{-c^2 d + e}}\right] -\right.$$

$$8\, \text{ArcCsch}[c\, x]\, \text{Log}\left[1 - e^{-2\, \text{ArcCsch}[c\, x]}\right] + 4\, i\, \pi\, \text{Log}\left[1 + \frac{i\left(-\sqrt{e}+\sqrt{-c^2 d + e}\right) e^{\text{ArcCsch}[c\, x]}}{c\sqrt{d}}\right] +$$

$$8\, \text{ArcCsch}[c\, x]\, \text{Log}\left[1 + \frac{i\left(-\sqrt{e}+\sqrt{-c^2 d + e}\right) e^{\text{ArcCsch}[c\, x]}}{c\sqrt{d}}\right] + 16\, i\, \text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right] \text{Log}\left[1 + \frac{i\left(-\sqrt{e}+\sqrt{-c^2 d + e}\right) e^{\text{ArcCsch}[c\, x]}}{c\sqrt{d}}\right] +$$

$$4\, i\, \pi\, \text{Log}\left[1 - \frac{i\left(\sqrt{e}+\sqrt{-c^2 d + e}\right) e^{\text{ArcCsch}[c\, x]}}{c\sqrt{d}}\right] + 8\, \text{ArcCsch}[c\, x]\, \text{Log}\left[1 - \frac{i\left(\sqrt{e}+\sqrt{-c^2 d + e}\right) e^{\text{ArcCsch}[c\, x]}}{c\sqrt{d}}\right] -$$

$$16\,i\,\mathrm{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\mathrm{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\mathrm{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-4\,i\,\pi\,\mathrm{Log}\left[\sqrt{e}-\frac{i\sqrt{d}}{x}\right]+4\,\mathrm{PolyLog}\left[2,\,e^{-2\,\mathrm{ArcCsch}[c\,x]}\right]+$$

$$\left.\left.8\,\mathrm{PolyLog}\left[2,\,-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\mathrm{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+8\,\mathrm{PolyLog}\left[2,\,\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\mathrm{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]\right)\right)$$

Problem 110: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{a+b\,\mathrm{ArcCsch}[c\,x]}{x^2\left(d+e\,x^2\right)^2}\,dx$$

Optimal (type 4, 758 leaves, 50 steps) :

$$\frac{b c \sqrt{1+\frac{1}{c^2 x^2}}}{d^2}-\frac{a}{d^2 x}-\frac{b \operatorname{ArcCsch}[c x]}{d^2 x}+\frac{e (a+b \operatorname{ArcCsch}[c x])}{4 d^2 \left(\sqrt{-d} \sqrt{e}-\frac{d}{x}\right)}-\frac{e (a+b \operatorname{ArcCsch}[c x])}{4 d^2 \left(\sqrt{-d} \sqrt{e}+\frac{d}{x}\right)}-\frac{b e \operatorname{ArcTanh}\left[\frac{c^2 d-\frac{\sqrt{-d} \sqrt{e}}{x}}{c \sqrt{d} \sqrt{c^2 d-e} \sqrt{1+\frac{1}{c^2 x^2}}}\right]}{4 d^{5/2} \sqrt{c^2 d-e}}-$$

$$\frac{b e \operatorname{ArcTanh}\left[\frac{c^2 d+\frac{\sqrt{-d} \sqrt{e}}{x}}{c \sqrt{d} \sqrt{c^2 d-e} \sqrt{1+\frac{1}{c^2 x^2}}}\right]}{4 d^{5/2} \sqrt{c^2 d-e}}-\frac{3 \sqrt{e} (a+b \operatorname{ArcCsch}[c x]) \operatorname{Log}\left[1-\frac{c \sqrt{-d}\, e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{4 (-d)^{5/2}}+\frac{3 \sqrt{e} (a+b \operatorname{ArcCsch}[c x]) \operatorname{Log}\left[1+\frac{c \sqrt{-d}\, e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{4 (-d)^{5/2}}-$$

$$\frac{3 \sqrt{e} (a+b \operatorname{ArcCsch}[c x]) \operatorname{Log}\left[1-\frac{c \sqrt{-d}\, e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{4 (-d)^{5/2}}+\frac{3 \sqrt{e} (a+b \operatorname{ArcCsch}[c x]) \operatorname{Log}\left[1+\frac{c \sqrt{-d}\, e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{4 (-d)^{5/2}}+\frac{3 b \sqrt{e} \operatorname{PolyLog}\left[2,-\frac{c \sqrt{-d}\, e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{4 (-d)^{5/2}}-$$

$$\frac{3 b \sqrt{e} \operatorname{PolyLog}\left[2,\frac{c \sqrt{-d}\, e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{4 (-d)^{5/2}}+\frac{3 b \sqrt{e} \operatorname{PolyLog}\left[2,-\frac{c \sqrt{-d}\, e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{4 (-d)^{5/2}}-\frac{3 b \sqrt{e} \operatorname{PolyLog}\left[2,\frac{c \sqrt{-d}\, e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{4 (-d)^{5/2}}$$

Result (type 4, 1487 leaves):

$$\frac{1}{8 d^{5/2}}$$

$$\left(-\frac{8 a \sqrt{d}}{x}-\frac{4 a \sqrt{d}\, e x}{d+e x^2}-12 a \sqrt{e} \operatorname{ArcTan}\left[\frac{\sqrt{e}\, x}{\sqrt{d}}\right]+b \left(8 c \sqrt{d} \sqrt{1+\frac{1}{c^2 x^2}}-\frac{8 \sqrt{d} \operatorname{ArcCsch}[c x]}{x}-\frac{2 \sqrt{d}\, e \operatorname{ArcCsch}[c x]}{-i \sqrt{d} \sqrt{e}+e x}-\frac{2 \sqrt{d}\, e \operatorname{ArcCsch}[c x]}{i \sqrt{d} \sqrt{e}+e x}-\right.\right.$$

$$24 i \sqrt{e} \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{ArcTan}\left[\frac{\left(c \sqrt{d}-\sqrt{e}\right) \operatorname{Cot}\left[\frac{1}{4} (\pi+2 i \operatorname{ArcCsch}[c x])\right]}{\sqrt{-c^2 d+e}}\right]-$$

$$24 i \sqrt{e} \operatorname{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{ArcTan}\left[\frac{\left(c \sqrt{d}+\sqrt{e}\right) \operatorname{Cot}\left[\frac{1}{4} (\pi+2 i \operatorname{ArcCsch}[c x])\right]}{\sqrt{-c^2 d+e}}\right]+3 \sqrt{e}\, \pi \operatorname{Log}\left[1-\frac{i \left(-\sqrt{e}+\sqrt{-c^2 d+e}\right) e^{\operatorname{ArcCsch}[c x]}}{c \sqrt{d}}\right]-$$

$$6 i \sqrt{e} \operatorname{ArcCsch}[c x] \operatorname{Log}\left[1-\frac{i \left(-\sqrt{e}+\sqrt{-c^2 d+e}\right) e^{\operatorname{ArcCsch}[c x]}}{c \sqrt{d}}\right]+12 \sqrt{e} \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1-\frac{i \left(-\sqrt{e}+\sqrt{-c^2 d+e}\right) e^{\operatorname{ArcCsch}[c x]}}{c \sqrt{d}}\right]-$$

$$3\sqrt{e}\,\pi\,\mathrm{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\mathrm{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+6\,i\sqrt{e}\,\mathrm{ArcCsch}[c\,x]\,\mathrm{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\mathrm{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-$$

$$12\sqrt{e}\,\mathrm{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\mathrm{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\mathrm{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-3\sqrt{e}\,\pi\,\mathrm{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\mathrm{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$6\,i\sqrt{e}\,\mathrm{ArcCsch}[c\,x]\,\mathrm{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\mathrm{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+12\sqrt{e}\,\mathrm{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\mathrm{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\mathrm{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$3\sqrt{e}\,\pi\,\mathrm{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\mathrm{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-6\,i\sqrt{e}\,\mathrm{ArcCsch}[c\,x]\,\mathrm{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\mathrm{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-$$

$$12\sqrt{e}\,\mathrm{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\mathrm{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\mathrm{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+3\sqrt{e}\,\pi\,\mathrm{Log}\left[\sqrt{e}-\frac{i\sqrt{d}}{x}\right]-3\sqrt{e}\,\pi\,\mathrm{Log}\left[\sqrt{e}+\frac{i\sqrt{d}}{x}\right]+$$

$$\frac{2\,i\,e\,\mathrm{Log}\left[\frac{2\sqrt{d}\sqrt{e}\left(i\sqrt{e}+c\left(c\sqrt{d}+i\sqrt{-c^2 d+e}\sqrt{1+\frac{1}{c^2 x^2}}\right)x\right)}{\sqrt{-c^2 d+e}\left(i\sqrt{d}+\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2 d+e}}-\frac{2\,i\,e\,\mathrm{Log}\left[-\frac{2\sqrt{d}\sqrt{e}\left(\sqrt{e}+c\left(i\,c\sqrt{d}+\sqrt{-c^2 d+e}\sqrt{1+\frac{1}{c^2 x^2}}\right)x\right)}{\sqrt{-c^2 d+e}\left(\sqrt{d}+i\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2 d+e}}+$$

$$6\,i\sqrt{e}\,\mathrm{PolyLog}\left[2,-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\mathrm{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-6\,i\sqrt{e}\,\mathrm{PolyLog}\left[2,\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\mathrm{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-$$

$$\left.\left.6\,i\sqrt{e}\,\mathrm{PolyLog}\left[2,-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\mathrm{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+6\,i\sqrt{e}\,\mathrm{PolyLog}\left[2,\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\mathrm{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]\right)\right)$$

Problem 111: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{x^5\left(a+b\,\mathrm{ArcCsch}[c\,x]\right)}{\left(d+e\,x^2\right)^3}\,dx$$

Optimal (type 4, 694 leaves, 33 steps):

$$\frac{b\,c\,d\,\sqrt{1+\frac{1}{c^2 x^2}}}{8\left(c^2 d-e\right) e^2\left(e+\frac{d}{x^2}\right) x}-\frac{a+b\,\text{ArcCsch}[c\,x]}{4\,e\left(e+\frac{d}{x^2}\right)^2}-\frac{a+b\,\text{ArcCsch}[c\,x]}{2\,e^2\left(e+\frac{d}{x^2}\right)}-\frac{\left(a+b\,\text{ArcCsch}[c\,x]\right)^2}{b\,e^3}+$$

$$\frac{b\left(c^2 d-2\,e\right)\text{ArcTan}\left[\frac{\sqrt{c^2 d-e}}{c\sqrt{e}\,\sqrt{1+\frac{1}{c^2 x^2}}\,x}\right]}{8\left(c^2 d-e\right)^{3/2} e^{5/2}}+\frac{b\,\text{ArcTan}\left[\frac{\sqrt{c^2 d-e}}{c\sqrt{e}\,\sqrt{1+\frac{1}{c^2 x^2}}\,x}\right]}{2\sqrt{c^2 d-e}\;e^{5/2}}-\frac{\left(a+b\,\text{ArcCsch}[c\,x]\right)\text{Log}\left[1-e^{-2\,\text{ArcCsch}[c\,x]}\right]}{e^3}+$$

$$\frac{\left(a+b\,\text{ArcCsch}[c\,x]\right)\text{Log}\left[1-\frac{c\sqrt{-d}\;e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2\,e^3}+\frac{\left(a+b\,\text{ArcCsch}[c\,x]\right)\text{Log}\left[1+\frac{c\sqrt{-d}\;e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2\,e^3}+$$

$$\frac{\left(a+b\,\text{ArcCsch}[c\,x]\right)\text{Log}\left[1-\frac{c\sqrt{-d}\;e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2\,e^3}+\frac{\left(a+b\,\text{ArcCsch}[c\,x]\right)\text{Log}\left[1+\frac{c\sqrt{-d}\;e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2\,e^3}+\frac{b\,\text{PolyLog}\left[2,\,e^{-2\,\text{ArcCsch}[c\,x]}\right]}{2\,e^3}+$$

$$\frac{b\,\text{PolyLog}\left[2,\,-\frac{c\sqrt{-d}\;e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2\,e^3}+\frac{b\,\text{PolyLog}\left[2,\,\frac{c\sqrt{-d}\;e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2\,e^3}+\frac{b\,\text{PolyLog}\left[2,\,-\frac{c\sqrt{-d}\;e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2\,e^3}+\frac{b\,\text{PolyLog}\left[2,\,\frac{c\sqrt{-d}\;e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2\,e^3}$$

Result (type 4, 2023 leaves) :

$$-\frac{a\,d^2}{4\,e^3\left(d+e\,x^2\right)^2}+\frac{a\,d}{e^3\left(d+e\,x^2\right)}+\frac{a\,\text{Log}\left[d+e\,x^2\right]}{2\,e^3}+b\left(-\frac{1}{16\,e^{5/2}}d\left(\frac{i\,c\sqrt{e}\,\sqrt{1+\frac{1}{c^2 x^2}}\,x}{\sqrt{d}\left(c^2 d-e\right)\left(-i\sqrt{d}+\sqrt{e}\,x\right)}-\right.\right.$$

$$\left.\frac{\text{ArcCsch}[c\,x]}{\sqrt{e}\left(-i\sqrt{d}+\sqrt{e}\,x\right)^2}-\frac{\text{ArcSinh}\left[\frac{1}{c\,x}\right]}{d\sqrt{e}}+\frac{i\left(2c^2d-e\right)\text{Log}\left[\frac{4d\sqrt{c^2d-e}\,\sqrt{e}\left(\sqrt{e}+i\,c\left(c\sqrt{d}-\sqrt{c^2d-e}\,\sqrt{1+\frac{1}{c^2x^2}}\right)x\right)}{\left(2c^2d-e\right)\left(\sqrt{d}+i\sqrt{e}\,x\right)}\right]}{d\left(c^2d-e\right)^{3/2}}\right)-\frac{1}{16\,e^{5/2}}$$

$$d\left(-\frac{i\,c\sqrt{e}\,\sqrt{1+\frac{1}{c^2x^2}}\,x}{\sqrt{d}\left(c^2d-e\right)\left(i\sqrt{d}+\sqrt{e}\,x\right)}-\frac{\text{ArcCsch}[c\,x]}{\sqrt{e}\left(i\sqrt{d}+\sqrt{e}\,x\right)^2}-\frac{\text{ArcSinh}\left[\frac{1}{c\,x}\right]}{d\sqrt{e}}+\frac{i\left(2c^2d-e\right)\text{Log}\left[\frac{4\,i\,d\sqrt{c^2d-e}\,\sqrt{e}\left(i\sqrt{e}+c\left(c\sqrt{d}+\sqrt{c^2d-e}\,\sqrt{1+\frac{1}{c^2x^2}}\right)x\right)}{\left(2c^2d-e\right)\left(\sqrt{d}-i\sqrt{e}\,x\right)}\right]}{d\left(c^2d-e\right)^{3/2}}\right)-$$

$$\frac{7\,i\sqrt{d}\left(-\frac{\text{ArcCsch}[c\,x]}{i\sqrt{d}\,\sqrt{e}+e\,x}-\frac{i\left(\frac{\text{ArcSinh}\left[\frac{1}{c\,x}\right]}{\sqrt{e}}-\frac{\text{Log}\left[\frac{2\sqrt{d}\,\sqrt{e}\left(i\sqrt{e}+c\left(c\sqrt{d}+i\sqrt{-c^2d+e}\,\sqrt{1+\frac{1}{c^2x^2}}\right)x\right)}{\sqrt{-c^2d+e}\left(i\sqrt{d}+\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2d+e}}\right)}{\sqrt{d}}\right)}{16\,e^{5/2}}+\frac{7\,i\sqrt{d}\left(-\frac{\text{ArcCsch}[c\,x]}{-i\sqrt{d}\,\sqrt{e}+e\,x}+\frac{i\left(\frac{\text{ArcSinh}\left[\frac{1}{c\,x}\right]}{\sqrt{e}}-\frac{\text{Log}\left[-\frac{2\sqrt{d}\,\sqrt{e}\left(\sqrt{e}+c\left(i\,c\sqrt{d}+\sqrt{-c^2d+e}\,\sqrt{1+\frac{1}{c^2x^2}}\right)x\right)}{\sqrt{-c^2d+e}\left(\sqrt{d}+i\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2d+e}}\right)}{\sqrt{d}}\right)}{16\,e^{5/2}}+$$

$$\frac{1}{16\,e^3}\left(\pi^2-4\,i\,\pi\,\text{ArcCsch}[c\,x]-8\,\text{ArcCsch}[c\,x]^2+32\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(c\sqrt{d}-\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,i\,\text{ArcCsch}[c\,x]\right)\right]}{\sqrt{-c^2d+e}}\right]-\right.$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-e^{-2\,\text{ArcCsch}[c\,x]}\right]+4\,i\,\pi\,\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+16\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,i\,\pi\,\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-$$

$$16\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-4\,i\,\pi\,\text{Log}\left[\sqrt{e}+\frac{i\sqrt{d}}{x}\right]+4\,\text{PolyLog}\left[2,\,e^{-2\,\text{ArcCsch}[c\,x]}\right]+$$

$$\left.8\,\text{PolyLog}\left[2,\,\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+8\,\text{PolyLog}\left[2,\,-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]\right)+$$

$$\frac{1}{16\,e^3}\left(\pi^2-4\,i\,\pi\,\text{ArcCsch}[c\,x]-8\,\text{ArcCsch}[c\,x]^2-32\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(c\sqrt{d}+\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,i\,\text{ArcCsch}[c\,x]\right)\right]}{\sqrt{-c^2 d+e}}\right]-\right.$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-e^{-2\,\text{ArcCsch}[c\,x]}\right]+4\,i\,\pi\,\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+16\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,i\,\pi\,\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-$$

$$16\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-4\,i\,\pi\,\text{Log}\left[\sqrt{e}-\frac{i\sqrt{d}}{x}\right]+4\,\text{PolyLog}\left[2,\,e^{-2\,\text{ArcCsch}[c\,x]}\right]+$$

$$8\,\text{PolyLog}\left[2,-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+8\,\text{PolyLog}\left[2,\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]\Bigg)\Bigg)$$

Problem 112: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{x^3\left(a+b\,\text{ArcCsch}[c\,x]\right)}{\left(d+e\,x^2\right)^3}\,dx$$

Optimal (type 3, 167 leaves, 6 steps):

$$-\frac{b\,c\,x\sqrt{-1-c^2x^2}}{8\left(c^2d-e\right)e\sqrt{-c^2x^2}\left(d+e\,x^2\right)}+\frac{x^4\left(a+b\,\text{ArcCsch}[c\,x]\right)}{4\,d\left(d+e\,x^2\right)^2}+\frac{b\,c\left(c^2d-2\,e\right)x\,\text{ArcTanh}\left[\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{\sqrt{c^2d-e}}\right]}{8\,d\left(c^2d-e\right)^{3/2}e^{3/2}\sqrt{-c^2x^2}}$$

Result (type 3, 375 leaves):

$$-\frac{1}{16 e^2}\left(-\frac{4 a d}{\left(d+e x^2\right)^2}+\frac{8 a}{d+e x^2}-\frac{2 b c e \sqrt{1+\frac{1}{c^2 x^2}}\, x}{\left(-c^2 d+e\right)\left(d+e x^2\right)}+\frac{4 b \left(d+2 e x^2\right) \text{ArcCsch}[c x]}{\left(d+e x^2\right)^2}-\right.$$

$$\frac{4 b \,\text{ArcSinh}\left[\frac{1}{c x}\right]}{d}+\frac{b \sqrt{e}\left(-c^2 d+2 e\right) \text{Log}\left[\frac{16 d e^{3/2} \sqrt{-c^2 d+e}\left(\sqrt{e}+c\left(-i c \sqrt{d}+\sqrt{-c^2 d+e} \sqrt{1+\frac{1}{c^2 x^2}}\right) x\right)}{b\left(-c^2 d+2 e\right)\left(i \sqrt{d}+\sqrt{e}\, x\right)}\right]}{d\left(-c^2 d+e\right)^{3/2}}+$$

$$\left.\frac{b \sqrt{e}\left(-c^2 d+2 e\right) \text{Log}\left[-\frac{16 i d e^{3/2} \sqrt{-c^2 d+e}\left(\sqrt{e}+c\left(i c \sqrt{d}+\sqrt{-c^2 d+e} \sqrt{1+\frac{1}{c^2 x^2}}\right) x\right)}{b\left(c^2 d-2 e\right)\left(\sqrt{d}+i \sqrt{e}\, x\right)}\right]}{d\left(-c^2 d+e\right)^{3/2}}\right)$$

Problem 113: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x\left(a+b \,\text{ArcCsch}[c x]\right)}{\left(d+e x^2\right)^3}\, dx$$

Optimal (type 3, 205 leaves, 8 steps) :

$$\frac{b c x \sqrt{-1-c^2 x^2}}{8 d\left(c^2 d-e\right) \sqrt{-c^2 x^2}\left(d+e x^2\right)}-\frac{a+b \,\text{ArcCsch}[c x]}{4 e\left(d+e x^2\right)^2}+\frac{b c x \,\text{ArcTan}\left[\sqrt{-1-c^2 x^2}\right]}{4 d^2 e \sqrt{-c^2 x^2}}+\frac{b c\left(3 c^2 d-2 e\right) x \,\text{ArcTanh}\left[\frac{\sqrt{e} \sqrt{-1-c^2 x^2}}{\sqrt{c^2 d-e}}\right]}{8 d^2\left(c^2 d-e\right)^{3/2} \sqrt{e} \sqrt{-c^2 x^2}}$$

Result (type 3, 368 leaves) :

$$\frac{1}{16}\left(-\frac{4\,a}{e\,(d+e\,x^2)^2}+\frac{2\,b\,c\,\sqrt{1+\frac{1}{c^2 x^2}}\,x}{d\,(c^2 d-e)\,(d+e\,x^2)}-\frac{4\,b\,\text{ArcCsch}[c\,x]}{e\,(d+e\,x^2)^2}+\frac{4\,b\,\text{ArcSinh}\left[\frac{1}{c\,x}\right]}{d^2\,e}+\right.$$

$$\left.\frac{b\,(3\,c^2 d-2\,e)\,\text{Log}\left[\frac{16\,d^2\sqrt{e}\,\sqrt{-c^2 d+e}\,\left(\sqrt{e}+c\left(-i\,c\,\sqrt{d}+\sqrt{-c^2 d+e}\,\sqrt{1+\frac{1}{c^2 x^2}}\right)x\right)}{b\,(-3\,c^2 d+2\,e)\,(i\,\sqrt{d}+\sqrt{e}\,x)}\right]}{d^2\sqrt{e}\,(-c^2 d+e)^{3/2}}+\frac{b\,(3\,c^2 d-2\,e)\,\text{Log}\left[-\frac{16\,i\,d^2\sqrt{e}\,\sqrt{-c^2 d+e}\,\left(\sqrt{e}+c\left(i\,c\,\sqrt{d}+\sqrt{-c^2 d+e}\,\sqrt{1+\frac{1}{c^2 x^2}}\right)x\right)}{b\,(3\,c^2 d-2\,e)\,(\sqrt{d}+i\,\sqrt{e}\,x)}\right]}{d^2\sqrt{e}\,(-c^2 d+e)^{3/2}}\right)$$

Problem 114: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{a+b\,\text{ArcCsch}[c\,x]}{x\,(d+e\,x^2)^3}\,dx$$

Optimal (type 4, 657 leaves, 28 steps):

$$-\frac{b\,c\,e\,\sqrt{1+\frac{1}{c^2 x^2}}}{8\,d^2\,(c^2 d-e)\,\left(e+\frac{d}{x^2}\right)x}+\frac{e^2\,(a+b\,\text{ArcCsch}[c\,x])}{4\,d^3\left(e+\frac{d}{x^2}\right)^2}-\frac{e\,(a+b\,\text{ArcCsch}[c\,x])}{d^3\left(e+\frac{d}{x^2}\right)}+\frac{(a+b\,\text{ArcCsch}[c\,x])^2}{2\,b\,d^3}-$$

$$\frac{b\,(c^2 d-2\,e)\,\sqrt{e}\,\text{ArcTan}\left[\frac{\sqrt{c^2 d-e}}{c\,\sqrt{e}\,\sqrt{1+\frac{1}{c^2 x^2}}\,x}\right]}{8\,d^3\,(c^2 d-e)^{3/2}}+\frac{b\,\sqrt{e}\,\text{ArcTan}\left[\frac{\sqrt{c^2 d-e}}{c\,\sqrt{e}\,\sqrt{1+\frac{1}{c^2 x^2}}\,x}\right]}{d^3\,\sqrt{c^2 d-e}}-\frac{(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2\,d^3}-$$

$$\frac{(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1+\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2\,d^3}-\frac{(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2\,d^3}-\frac{(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1+\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2\,d^3}-$$

$$\frac{b\,\text{PolyLog}\left[2,-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2\,d^3}-\frac{b\,\text{PolyLog}\left[2,\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2\,d^3}-\frac{b\,\text{PolyLog}\left[2,-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2\,d^3}-\frac{b\,\text{PolyLog}\left[2,\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2\,d^3}$$

Result (type 4, 2077 leaves):

$$\frac{a}{4\,d\,\left(d+e\,x^2\right)^2}+\frac{a}{2\,d^2\,\left(d+e\,x^2\right)}+\frac{a\,\text{Log}[x]}{d^3}-\frac{a\,\text{Log}\left[d+e\,x^2\right]}{2\,d^3}+b\,\Bigg(\frac{1}{16\,d^2}\sqrt{e}\,\Bigg(\frac{i\,c\,\sqrt{e}\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x}{\sqrt{d}\,\left(c^2\,d-e\right)\,\left(-i\,\sqrt{d}+\sqrt{e}\,x\right)}-$$

$$\frac{\text{ArcCsch}[c\,x]}{\sqrt{e}\,\left(-i\,\sqrt{d}+\sqrt{e}\,x\right)^2}-\frac{\text{ArcSinh}\left[\frac{1}{c\,x}\right]}{d\,\sqrt{e}}+\frac{i\,\left(2\,c^2\,d-e\right)\,\text{Log}\left[\frac{4\,d\,\sqrt{c^2\,d-e}\,\sqrt{e}\,\left(\sqrt{e}+i\,c\,\left(c\,\sqrt{d}-\sqrt{c^2\,d-e}\,\sqrt{1+\frac{1}{c^2\,x^2}}\right)\,x\right)}{\left(2\,c^2\,d-e\right)\,\left(\sqrt{d}+i\,\sqrt{e}\,x\right)}\right]}{d\,\left(c^2\,d-e\right)^{3/2}}\Bigg)+\frac{1}{16\,d^2}\sqrt{e}$$

$$\Bigg(-\frac{i\,c\,\sqrt{e}\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x}{\sqrt{d}\,\left(c^2\,d-e\right)\,\left(i\,\sqrt{d}+\sqrt{e}\,x\right)}-\frac{\text{ArcCsch}[c\,x]}{\sqrt{e}\,\left(i\,\sqrt{d}+\sqrt{e}\,x\right)^2}-\frac{\text{ArcSinh}\left[\frac{1}{c\,x}\right]}{d\,\sqrt{e}}+\frac{i\,\left(2\,c^2\,d-e\right)\,\text{Log}\left[\frac{4\,i\,d\,\sqrt{c^2\,d-e}\,\sqrt{e}\,\left(i\,\sqrt{e}+c\,\left(c\,\sqrt{d}+\sqrt{c^2\,d-e}\,\sqrt{1+\frac{1}{c^2\,x^2}}\right)\,x\right)}{\left(2\,c^2\,d-e\right)\,\left(\sqrt{d}-i\,\sqrt{e}\,x\right)}\right]}{d\,\left(c^2\,d-e\right)^{3/2}}\Bigg)-$$

$$\frac{5\,i\sqrt{e}\left(-\frac{\text{ArcCsch}[c\,x]}{i\sqrt{d}\sqrt{e}+e\,x}-\frac{i\left(\frac{\text{ArcSinh}\left[\frac{1}{c\,x}\right]}{\sqrt{e}}-\frac{\text{Log}\left[\frac{2\sqrt{d}\sqrt{e}\left(i\sqrt{e}+c\left(c\sqrt{d}+i\sqrt{-c^2 d+e}\sqrt{1+\frac{1}{c^2 x^2}}\right)x\right)}{\sqrt{-c^2 d+e}\left(i\sqrt{d}+\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2 d+e}}\right)}{\sqrt{d}}\right)}{16\,d^{5/2}}+\frac{5\,i\sqrt{e}\left(-\frac{\text{ArcCsch}[c\,x]}{-i\sqrt{d}\sqrt{e}+e\,x}+\frac{i\left(\frac{\text{ArcSinh}\left[\frac{1}{c\,x}\right]}{\sqrt{e}}-\frac{\text{Log}\left[-\frac{2\sqrt{d}\sqrt{e}\left(\sqrt{e}+c\left(i\,c\sqrt{d}+\sqrt{-c^2 d+e}\sqrt{1+\frac{1}{c^2 x^2}}\right)x\right)}{\sqrt{-c^2 d+e}\left(\sqrt{d}+i\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2 d+e}}\right)}{\sqrt{d}}\right)}{16\,d^{5/2}}+$$

$$\frac{-\text{ArcCsch}[c\,x]\left(\text{ArcCsch}[c\,x]+2\,\text{Log}\left[1-e^{-2\,\text{ArcCsch}[c\,x]}\right]\right)+\text{PolyLog}\left[2,e^{-2\,\text{ArcCsch}[c\,x]}\right]}{2\,d^3}-$$

$$\frac{1}{16\,d^3}\left(\pi^2-4\,i\,\pi\,\text{ArcCsch}[c\,x]-8\,\text{ArcCsch}[c\,x]^2+32\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(c\sqrt{d}-\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,i\,\text{ArcCsch}[c\,x]\right)\right]}{\sqrt{-c^2 d+e}}\right]-\right.$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-e^{-2\,\text{ArcCsch}[c\,x]}\right]+4\,i\,\pi\,\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+16\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,i\,\pi\,\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-$$

$$16\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-4\,i\,\pi\,\text{Log}\left[\sqrt{e}+\frac{i\sqrt{d}}{x}\right]+4\,\text{PolyLog}\left[2,e^{-2\,\text{ArcCsch}[c\,x]}\right]+$$

$$\left.8\,\text{PolyLog}\left[2,\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+8\,\text{PolyLog}\left[2,-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]\right)-$$

$$\frac{1}{16\,d^3}\Bigg(\pi^2-4\,i\,\pi\,\text{ArcCsch}[c\,x]-8\,\text{ArcCsch}[c\,x]^2-32\,\text{ArcSin}\Big[\frac{\sqrt{1-\frac{\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\Big]\,\text{ArcTan}\Big[\frac{\left(c\,\sqrt{d}+\sqrt{e}\right)\,\text{Cot}\left[\frac{1}{4}\left(\pi+2\,i\,\text{ArcCsch}[c\,x]\right)\right]}{\sqrt{-c^2\,d+e}}\Big]-$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-e^{-2\,\text{ArcCsch}[c\,x]}\right]+4\,i\,\pi\,\text{Log}\Big[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\Big]+$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\Big[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\Big]+16\,i\,\text{ArcSin}\Big[\frac{\sqrt{1-\frac{\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\Big]\,\text{Log}\Big[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\Big]+$$

$$4\,i\,\pi\,\text{Log}\Big[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\Big]+8\,\text{ArcCsch}[c\,x]\,\text{Log}\Big[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\Big]-$$

$$16\,i\,\text{ArcSin}\Big[\frac{\sqrt{1-\frac{\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\Big]\,\text{Log}\Big[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\Big]-4\,i\,\pi\,\text{Log}\Big[\sqrt{e}-\frac{i\,\sqrt{d}}{x}\Big]+4\,\text{PolyLog}\left[2,\,e^{-2\,\text{ArcCsch}[c\,x]}\right]+$$

$$8\,\text{PolyLog}\Big[2,\,-\frac{i\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\Big]+8\,\text{PolyLog}\Big[2,\,\frac{i\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\Big]\Bigg)$$

Problem 115: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4 \left(a+b\,\text{ArcCsch}[c\,x]\right)}{\left(d+e\,x^2\right)^3}\,dx$$

Optimal (type 4, 1106 leaves, 35 steps):

$$-\frac{b\,c\,\sqrt{-d}\,\sqrt{1+\frac{1}{c^2 x^2}}}{16\left(c^2 d-e\right)e^{3/2}\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)}-\frac{b\,c\,\sqrt{-d}\,\sqrt{1+\frac{1}{c^2 x^2}}}{16\left(c^2 d-e\right)e^{3/2}\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)}+\frac{\sqrt{-d}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{16\,e^{3/2}\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)^2}+\frac{3\left(a+b\,\text{ArcCsch}[c\,x]\right)}{16\,e^2\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)}-$$

$$\frac{\sqrt{-d}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{16\,e^{3/2}\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)^2}-\frac{3\left(a+b\,\text{ArcCsch}[c\,x]\right)}{16\,e^2\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)}-\frac{3\,b\,\text{ArcTanh}\left[\frac{c^2 d-\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\,\sqrt{d}\,\sqrt{c^2 d-e}\,\sqrt{1+\frac{1}{c^2 x^2}}}\right]}{16\,\sqrt{d}\,\sqrt{c^2 d-e}\,e^2}+\frac{b\,\text{ArcTanh}\left[\frac{c^2 d-\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\,\sqrt{d}\,\sqrt{c^2 d-e}\,\sqrt{1+\frac{1}{c^2 x^2}}}\right]}{16\,\sqrt{d}\,\left(c^2 d-e\right)^{3/2}e}-$$

$$\frac{3\,b\,\text{ArcTanh}\left[\frac{c^2 d+\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\,\sqrt{d}\,\sqrt{c^2 d-e}\,\sqrt{1+\frac{1}{c^2 x^2}}}\right]}{16\,\sqrt{d}\,\sqrt{c^2 d-e}\,e^2}+\frac{b\,\text{ArcTanh}\left[\frac{c^2 d+\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\,\sqrt{d}\,\sqrt{c^2 d-e}\,\sqrt{1+\frac{1}{c^2 x^2}}}\right]}{16\,\sqrt{d}\,\left(c^2 d-e\right)^{3/2}e}+\frac{3\left(a+b\,\text{ArcCsch}[c\,x]\right)\text{Log}\left[1-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{16\,\sqrt{-d}\,e^{5/2}}-$$

$$\frac{3\left(a+b\,\text{ArcCsch}[c\,x]\right)\text{Log}\left[1+\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{16\,\sqrt{-d}\,e^{5/2}}+\frac{3\left(a+b\,\text{ArcCsch}[c\,x]\right)\text{Log}\left[1-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{16\,\sqrt{-d}\,e^{5/2}}-\frac{3\left(a+b\,\text{ArcCsch}[c\,x]\right)\text{Log}\left[1+\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{16\,\sqrt{-d}\,e^{5/2}}-$$

$$\frac{3\,b\,\text{PolyLog}\left[2,-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{16\,\sqrt{-d}\,e^{5/2}}+\frac{3\,b\,\text{PolyLog}\left[2,\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{16\,\sqrt{-d}\,e^{5/2}}-\frac{3\,b\,\text{PolyLog}\left[2,-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{16\,\sqrt{-d}\,e^{5/2}}+\frac{3\,b\,\text{PolyLog}\left[2,\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{16\,\sqrt{-d}\,e^{5/2}}$$

Result (type 4, 2045 leaves):

$$\frac{a\,d\,x}{4\,e^2\,\left(d+e\,x^2\right)^2}-\frac{5\,a\,x}{8\,e^2\,\left(d+e\,x^2\right)}+\frac{3\,a\,\mathrm{ArcTan}\left[\frac{\sqrt{e}\,x}{\sqrt{d}}\right]}{8\,\sqrt{d}\,e^{5/2}}+b\left(\frac{1}{16\,e^2}\,i\,\sqrt{d}\,\left(\frac{i\,c\,\sqrt{e}\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x}{\sqrt{d}\,\left(c^2\,d-e\right)\,\left(-i\,\sqrt{d}+\sqrt{e}\,x\right)}-\right.\right.$$

$$\left.\frac{\mathrm{ArcCsch}[c\,x]}{\sqrt{e}\,\left(-i\,\sqrt{d}+\sqrt{e}\,x\right)^2}-\frac{\mathrm{ArcSinh}\left[\frac{1}{c\,x}\right]}{d\,\sqrt{e}}+\frac{i\,\left(2\,c^2\,d-e\right)\,\mathrm{Log}\left[\frac{4\,d\,\sqrt{c^2\,d-e}\,\sqrt{e}\,\left(\sqrt{e}+i\,c\,\left(c\,\sqrt{d}-\sqrt{c^2\,d-e}\,\sqrt{1+\frac{1}{c^2\,x^2}}\right)\,x\right)}{\left(2\,c^2\,d-e\right)\,\left(\sqrt{d}+i\,\sqrt{e}\,x\right)}\right]}{d\,\left(c^2\,d-e\right)^{3/2}}\right)-\frac{1}{16\,e^2}\,i\,\sqrt{d}$$

$$\left(-\frac{i\,c\,\sqrt{e}\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x}{\sqrt{d}\,\left(c^2\,d-e\right)\,\left(i\,\sqrt{d}+\sqrt{e}\,x\right)}-\frac{\mathrm{ArcCsch}[c\,x]}{\sqrt{e}\,\left(i\,\sqrt{d}+\sqrt{e}\,x\right)^2}-\frac{\mathrm{ArcSinh}\left[\frac{1}{c\,x}\right]}{d\,\sqrt{e}}+\frac{i\,\left(2\,c^2\,d-e\right)\,\mathrm{Log}\left[\frac{4\,i\,d\,\sqrt{c^2\,d-e}\,\sqrt{e}\,\left(i\,\sqrt{e}+c\,\left(c\,\sqrt{d}+\sqrt{c^2\,d-e}\,\sqrt{1+\frac{1}{c^2\,x^2}}\right)\,x\right)}{\left(2\,c^2\,d-e\right)\,\left(\sqrt{d}-i\,\sqrt{e}\,x\right)}\right]}{d\,\left(c^2\,d-e\right)^{3/2}}\right)+$$

$$\frac{5\left(-\frac{\mathrm{ArcCsch}[c\,x]}{\mathrm{i}\sqrt{d}\sqrt{e}+e\,x}-\frac{\mathrm{i}\left(\frac{\mathrm{ArcSinh}\left[\frac{1}{c\,x}\right]}{\sqrt{e}}-\frac{\mathrm{Log}\left[\frac{2\sqrt{d}\sqrt{e}\left(\mathrm{i}\sqrt{e}+c\left(c\sqrt{d}+\mathrm{i}\sqrt{-c^2 d+e}\sqrt{1+\frac{1}{c^2 x^2}}\right)x\right)}{\sqrt{-c^2 d+e}\left(\mathrm{i}\sqrt{d}+\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2 d+e}}\right)}{\sqrt{d}}\right)}{16\,e^2}+\frac{5\left(-\frac{\mathrm{ArcCsch}[c\,x]}{-\mathrm{i}\sqrt{d}\sqrt{e}+e\,x}+\frac{\mathrm{i}\left(\frac{\mathrm{ArcSinh}\left[\frac{1}{c\,x}\right]}{\sqrt{e}}-\frac{\mathrm{Log}\left[-\frac{2\sqrt{d}\sqrt{e}\left(\sqrt{e}+c\left(\mathrm{i}\,c\sqrt{d}+\sqrt{-c^2 d+e}\sqrt{1+\frac{1}{c^2 x^2}}\right)x\right)}{\sqrt{-c^2 d+e}\left(\sqrt{d}+\mathrm{i}\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2 d+e}}\right)}{\sqrt{d}}\right)}{16\,e^2}+$$

$$\frac{1}{128\sqrt{d}\,e^{5/2}}3\,\mathrm{i}\left(\pi^2-4\,\mathrm{i}\,\pi\,\mathrm{ArcCsch}[c\,x]-8\,\mathrm{ArcCsch}[c\,x]^2+32\,\mathrm{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\mathrm{ArcTan}\left[\frac{\left(c\sqrt{d}-\sqrt{e}\right)\mathrm{Cot}\left[\frac{1}{4}\left(\pi+2\,\mathrm{i}\,\mathrm{ArcCsch}[c\,x]\right)\right]}{\sqrt{-c^2 d+e}}\right]-\right.$$

$$8\,\mathrm{ArcCsch}[c\,x]\,\mathrm{Log}\left[1-e^{-2\,\mathrm{ArcCsch}[c\,x]}\right]+4\,\mathrm{i}\,\pi\,\mathrm{Log}\left[1-\frac{\mathrm{i}\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\mathrm{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,\mathrm{ArcCsch}[c\,x]\,\mathrm{Log}\left[1-\frac{\mathrm{i}\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\mathrm{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+16\,\mathrm{i}\,\mathrm{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\mathrm{Log}\left[1-\frac{\mathrm{i}\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\mathrm{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,\mathrm{i}\,\pi\,\mathrm{Log}\left[1+\frac{\mathrm{i}\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\mathrm{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+8\,\mathrm{ArcCsch}[c\,x]\,\mathrm{Log}\left[1+\frac{\mathrm{i}\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\mathrm{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-$$

$$16\,\mathrm{i}\,\mathrm{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\mathrm{Log}\left[1+\frac{\mathrm{i}\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\mathrm{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-4\,\mathrm{i}\,\pi\,\mathrm{Log}\left[\sqrt{e}+\frac{\mathrm{i}\sqrt{d}}{x}\right]+4\,\mathrm{PolyLog}\left[2,e^{-2\,\mathrm{ArcCsch}[c\,x]}\right]+$$

$$\left.8\,\mathrm{PolyLog}\left[2,\frac{\mathrm{i}\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\mathrm{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+8\,\mathrm{PolyLog}\left[2,-\frac{\mathrm{i}\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\mathrm{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]\right)-$$

$$\frac{1}{128\sqrt{d}\,e^{5/2}}3\,i\left(\pi^2-4\,i\,\pi\,\text{ArcCsch}[c\,x]-8\,\text{ArcCsch}[c\,x]^2-32\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(c\sqrt{d}+\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,i\,\text{ArcCsch}[c\,x]\right)\right]}{\sqrt{-c^2 d+e}}\right]-\right.$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-e^{-2\,\text{ArcCsch}[c\,x]}\right]+4\,i\,\pi\,\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+16\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,i\,\pi\,\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-$$

$$16\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-4\,i\,\pi\,\text{Log}\left[\sqrt{e}-\frac{i\sqrt{d}}{x}\right]+4\,\text{PolyLog}\left[2,e^{-2\,\text{ArcCsch}[c\,x]}\right]+$$

$$\left.8\,\text{PolyLog}\left[2,-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+8\,\text{PolyLog}\left[2,\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]\right)$$

Problem 116: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2 \left(a+b\,\text{ArcCsch}[c\,x]\right)}{\left(d+e\,x^2\right)^3}\,dx$$

Optimal (type 4, 1106 leaves, 63 steps):

$$-\frac{b\,c\,\sqrt{1+\frac{1}{c^2 x^2}}}{16\sqrt{-d}\,\left(c^2 d-e\right)\sqrt{e}\,\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)}-\frac{b\,c\,\sqrt{1+\frac{1}{c^2 x^2}}}{16\sqrt{-d}\,\left(c^2 d-e\right)\sqrt{e}\,\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)}+\frac{a+b\,\text{ArcCsch}[c\,x]}{16\sqrt{-d}\,\sqrt{e}\,\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)^2}+\frac{a+b\,\text{ArcCsch}[c\,x]}{16\,d\,e\,\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)}-$$

$$\frac{a+b\,\text{ArcCsch}[c\,x]}{16\sqrt{-d}\,\sqrt{e}\,\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)^2}-\frac{a+b\,\text{ArcCsch}[c\,x]}{16\,d\,e\,\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)}-\frac{b\,\text{ArcTanh}\left[\frac{c^2 d-\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\sqrt{d}\,\sqrt{c^2 d-e}\,\sqrt{1+\frac{1}{c^2 x^2}}}\right]}{16\,d^{3/2}\left(c^2 d-e\right)^{3/2}}-\frac{b\,\text{ArcTanh}\left[\frac{c^2 d-\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\sqrt{d}\,\sqrt{c^2 d-e}\,\sqrt{1+\frac{1}{c^2 x^2}}}\right]}{16\,d^{3/2}\sqrt{c^2 d-e}\;e}-$$

$$\frac{b\,\text{ArcTanh}\left[\frac{c^2 d+\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\sqrt{d}\,\sqrt{c^2 d-e}\,\sqrt{1+\frac{1}{c^2 x^2}}}\right]}{16\,d^{3/2}\left(c^2 d-e\right)^{3/2}}-\frac{b\,\text{ArcTanh}\left[\frac{c^2 d+\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\sqrt{d}\,\sqrt{c^2 d-e}\,\sqrt{1+\frac{1}{c^2 x^2}}}\right]}{16\,d^{3/2}\sqrt{c^2 d-e}\;e}-\frac{\left(a+b\,\text{ArcCsch}[c\,x]\right)\,\text{Log}\left[1-\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{16\,(-d)^{3/2}\,e^{3/2}}+$$

$$\frac{\left(a+b\,\text{ArcCsch}[c\,x]\right)\,\text{Log}\left[1+\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{16\,(-d)^{3/2}\,e^{3/2}}-\frac{\left(a+b\,\text{ArcCsch}[c\,x]\right)\,\text{Log}\left[1-\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{16\,(-d)^{3/2}\,e^{3/2}}+\frac{\left(a+b\,\text{ArcCsch}[c\,x]\right)\,\text{Log}\left[1+\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{16\,(-d)^{3/2}\,e^{3/2}}+$$

$$\frac{b\,\text{PolyLog}\left[2,-\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{16\,(-d)^{3/2}\,e^{3/2}}-\frac{b\,\text{PolyLog}\left[2,\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{16\,(-d)^{3/2}\,e^{3/2}}+\frac{b\,\text{PolyLog}\left[2,-\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{16\,(-d)^{3/2}\,e^{3/2}}-\frac{b\,\text{PolyLog}\left[2,\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{16\,(-d)^{3/2}\,e^{3/2}}$$

Result (type 4, 2053 leaves):

$$-\frac{a\,x}{4\,e\,\left(d+e\,x^2\right)^2}+\frac{a\,x}{8\,d\,e\,\left(d+e\,x^2\right)}+\frac{a\,\mathrm{ArcTan}\left[\frac{\sqrt{e}\,x}{\sqrt{d}}\right]}{8\,d^{3/2}\,e^{3/2}}+b\,\Bigg(-\frac{1}{16\,\sqrt{d}\,e}\,i\,\Bigg(\frac{i\,c\,\sqrt{e}\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x}{\sqrt{d}\,\left(c^2\,d-e\right)\,\left(-i\,\sqrt{d}+\sqrt{e}\,x\right)}-$$

$$\frac{\mathrm{ArcCsch}[c\,x]}{\sqrt{e}\,\left(-i\,\sqrt{d}+\sqrt{e}\,x\right)^2}-\frac{\mathrm{ArcSinh}\left[\frac{1}{c\,x}\right]}{d\,\sqrt{e}}+\frac{i\,\left(2\,c^2\,d-e\right)\,\mathrm{Log}\left[\frac{4\,d\,\sqrt{c^2\,d-e}\,\sqrt{e}\,\left(\sqrt{e}+i\,c\,\left(c\,\sqrt{d}-\sqrt{c^2\,d-e}\,\sqrt{1+\frac{1}{c^2\,x^2}}\right)\,x\right)}{\left(2\,c^2\,d-e\right)\,\left(\sqrt{d}+i\,\sqrt{e}\,x\right)}\right]}{d\,\left(c^2\,d-e\right)^{3/2}}\Bigg)+\frac{1}{16\,\sqrt{d}\,e}$$

$$i\,\Bigg(-\frac{i\,c\,\sqrt{e}\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x}{\sqrt{d}\,\left(c^2\,d-e\right)\,\left(i\,\sqrt{d}+\sqrt{e}\,x\right)}-\frac{\mathrm{ArcCsch}[c\,x]}{\sqrt{e}\,\left(i\,\sqrt{d}+\sqrt{e}\,x\right)^2}-\frac{\mathrm{ArcSinh}\left[\frac{1}{c\,x}\right]}{d\,\sqrt{e}}+\frac{i\,\left(2\,c^2\,d-e\right)\,\mathrm{Log}\left[\frac{4\,i\,d\,\sqrt{c^2\,d-e}\,\sqrt{e}\,\left(i\,\sqrt{e}+c\,\left(c\,\sqrt{d}+\sqrt{c^2\,d-e}\,\sqrt{1+\frac{1}{c^2\,x^2}}\right)\,x\right)}{\left(2\,c^2\,d-e\right)\,\left(\sqrt{d}-i\,\sqrt{e}\,x\right)}\right]}{d\,\left(c^2\,d-e\right)^{3/2}}\Bigg)-$$

$$\frac{-\frac{\mathrm{ArcCsch}[c\,x]}{i\,\sqrt{d}\,\sqrt{e}+e\,x}-\frac{i\,\left(\frac{\mathrm{ArcSinh}\left[\frac{1}{c\,x}\right]}{\sqrt{e}}-\frac{\mathrm{Log}\left[\frac{2\,\sqrt{d}\,\sqrt{e}\,\left(i\,\sqrt{e}+c\,\left(c\,\sqrt{d}+i\,\sqrt{-c^2\,d+e}\,\sqrt{1+\frac{1}{c^2\,x^2}}\right)\,x\right)}{\sqrt{-c^2\,d+e}\,\left(i\,\sqrt{d}+\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2\,d+e}}\right)}{\sqrt{d}}}{16\,d\,e}-\frac{-\frac{\mathrm{ArcCsch}[c\,x]}{-i\,\sqrt{d}\,\sqrt{e}+e\,x}+\frac{i\,\left(\frac{\mathrm{ArcSinh}\left[\frac{1}{c\,x}\right]}{\sqrt{e}}-\frac{\mathrm{Log}\left[-\frac{2\,\sqrt{d}\,\sqrt{e}\,\left(\sqrt{e}+c\,\left(i\,c\,\sqrt{d}+\sqrt{-c^2\,d+e}\,\sqrt{1+\frac{1}{c^2\,x^2}}\right)\,x\right)}{\sqrt{-c^2\,d+e}\,\left(\sqrt{d}+i\,\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2\,d+e}}\right)}{\sqrt{d}}}{16\,d\,e}+$$

$$\frac{1}{128\,d^{3/2}\,e^{3/2}}\,i\,\Bigg(\pi^2-4\,i\,\pi\,\mathrm{ArcCsch}[c\,x]-8\,\mathrm{ArcCsch}[c\,x]^2+32\,\mathrm{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]\,\mathrm{ArcTan}\left[\frac{\left(c\,\sqrt{d}-\sqrt{e}\right)\,\mathrm{Cot}\left[\frac{1}{4}\,\left(\pi+2\,i\,\mathrm{ArcCsch}[c\,x]\right)\right]}{\sqrt{-c^2\,d+e}}\right]-$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-e^{-2\,\text{ArcCsch}[c\,x]}\right]+4\,i\,\pi\,\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+16\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,i\,\pi\,\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-$$

$$16\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-4\,i\,\pi\,\text{Log}\left[\sqrt{e}+\frac{i\sqrt{d}}{x}\right]+4\,\text{PolyLog}\left[2,\,e^{-2\,\text{ArcCsch}[c\,x]}\right]+$$

$$\left.8\,\text{PolyLog}\left[2,\,\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+8\,\text{PolyLog}\left[2,\,-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]\right)-$$

$$\frac{1}{128\,d^{3/2}\,e^{3/2}}\,i\left(\pi^2-4\,i\,\pi\,\text{ArcCsch}[c\,x]-8\,\text{ArcCsch}[c\,x]^2-32\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(c\sqrt{d}+\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,i\,\text{ArcCsch}[c\,x]\right)\right]}{\sqrt{-c^2 d+e}}\right]-\right.$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-e^{-2\,\text{ArcCsch}[c\,x]}\right]+4\,i\,\pi\,\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+16\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,i\,\pi\,\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-$$

$$16\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-4\,i\,\pi\,\text{Log}\left[\sqrt{e}-\frac{i\sqrt{d}}{x}\right]+4\,\text{PolyLog}\left[2,\,e^{-2\,\text{ArcCsch}[c\,x]}\right]+$$

$$8\,\text{PolyLog}\left[2,-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+8\,\text{PolyLog}\left[2,\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]\Bigg)\Bigg)$$

Problem 117: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{a+b\,\text{ArcCsch}[c\,x]}{\left(d+e\,x^2\right)^3}\,dx$$

Optimal (type 4, 1096 leaves, 81 steps):

$$-\frac{b c \sqrt{e} \sqrt{1+\frac{1}{c^2 x^2}}}{16 (-d)^{3/2} \left(c^2 d-e\right) \left(\sqrt{-d} \sqrt{e}-\frac{d}{x}\right)}-\frac{b c \sqrt{e} \sqrt{1+\frac{1}{c^2 x^2}}}{16 (-d)^{3/2} \left(c^2 d-e\right) \left(\sqrt{-d} \sqrt{e}+\frac{d}{x}\right)}+\frac{\sqrt{e} (a+b \text{ArcCsch}[c x])}{16 (-d)^{3/2} \left(\sqrt{-d} \sqrt{e}-\frac{d}{x}\right)^2}-\frac{5 (a+b \text{ArcCsch}[c x])}{16 d^2 \left(\sqrt{-d} \sqrt{e}-\frac{d}{x}\right)}-$$

$$\frac{\sqrt{e} (a+b \text{ArcCsch}[c x])}{16 (-d)^{3/2} \left(\sqrt{-d} \sqrt{e}+\frac{d}{x}\right)^2}+\frac{5 (a+b \text{ArcCsch}[c x])}{16 d^2 \left(\sqrt{-d} \sqrt{e}+\frac{d}{x}\right)}+\frac{5 b \text{ArcTanh}\left[\frac{c^2 d-\frac{\sqrt{-d} \sqrt{e}}{x}}{c \sqrt{d} \sqrt{c^2 d-e} \sqrt{1+\frac{1}{c^2 x^2}}}\right]}{16 d^{5/2} \sqrt{c^2 d-e}}+\frac{b e \text{ArcTanh}\left[\frac{c^2 d-\frac{\sqrt{-d} \sqrt{e}}{x}}{c \sqrt{d} \sqrt{c^2 d-e} \sqrt{1+\frac{1}{c^2 x^2}}}\right]}{16 d^{5/2} \left(c^2 d-e\right)^{3/2}}+$$

$$\frac{5 b \text{ArcTanh}\left[\frac{c^2 d+\frac{\sqrt{-d} \sqrt{e}}{x}}{c \sqrt{d} \sqrt{c^2 d-e} \sqrt{1+\frac{1}{c^2 x^2}}}\right]}{16 d^{5/2} \sqrt{c^2 d-e}}+\frac{b e \text{ArcTanh}\left[\frac{c^2 d+\frac{\sqrt{-d} \sqrt{e}}{x}}{c \sqrt{d} \sqrt{c^2 d-e} \sqrt{1+\frac{1}{c^2 x^2}}}\right]}{16 d^{5/2} \left(c^2 d-e\right)^{3/2}}+\frac{3 (a+b \text{ArcCsch}[c x]) \text{Log}\left[1-\frac{c \sqrt{-d}\, e^{\text{ArcCsch}[c x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{16 (-d)^{5/2} \sqrt{e}}-$$

$$\frac{3 (a+b \text{ArcCsch}[c x]) \text{Log}\left[1+\frac{c \sqrt{-d}\, e^{\text{ArcCsch}[c x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{16 (-d)^{5/2} \sqrt{e}}+\frac{3 (a+b \text{ArcCsch}[c x]) \text{Log}\left[1-\frac{c \sqrt{-d}\, e^{\text{ArcCsch}[c x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{16 (-d)^{5/2} \sqrt{e}}-\frac{3 (a+b \text{ArcCsch}[c x]) \text{Log}\left[1+\frac{c \sqrt{-d}\, e^{\text{ArcCsch}[c x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{16 (-d)^{5/2} \sqrt{e}}-$$

$$\frac{3 b \text{PolyLog}\left[2,-\frac{c \sqrt{-d}\, e^{\text{ArcCsch}[c x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{16 (-d)^{5/2} \sqrt{e}}+\frac{3 b \text{PolyLog}\left[2,\frac{c \sqrt{-d}\, e^{\text{ArcCsch}[c x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{16 (-d)^{5/2} \sqrt{e}}-\frac{3 b \text{PolyLog}\left[2,-\frac{c \sqrt{-d}\, e^{\text{ArcCsch}[c x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{16 (-d)^{5/2} \sqrt{e}}+\frac{3 b \text{PolyLog}\left[2,\frac{c \sqrt{-d}\, e^{\text{ArcCsch}[c x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{16 (-d)^{5/2} \sqrt{e}}$$

Result (type 4, 2038 leaves) :

$$\frac{a x}{4 d \left(d+e x^2\right)^2}+\frac{3 a x}{8 d^2 \left(d+e x^2\right)}+\frac{3 a \operatorname{ArcTan}\left[\frac{\sqrt{e}\, x}{\sqrt{d}}\right]}{8 d^{5/2} \sqrt{e}}+b \left(\frac{1}{16 d^{3/2}}\right.$$

$$i \left(\frac{i c \sqrt{e} \sqrt{1+\frac{1}{c^2 x^2}}\, x}{\sqrt{d} \left(c^2 d-e\right) \left(-i \sqrt{d}+\sqrt{e}\, x\right)}-\frac{\operatorname{ArcCsch}[c x]}{\sqrt{e} \left(-i \sqrt{d}+\sqrt{e}\, x\right)^2}-\frac{\operatorname{ArcSinh}\left[\frac{1}{c x}\right]}{d \sqrt{e}}+\frac{i \left(2 c^2 d-e\right) \operatorname{Log}\left[\frac{4 d \sqrt{c^2 d-e} \sqrt{e} \left(\sqrt{e}+i c \left(c \sqrt{d}-\sqrt{c^2 d-e} \sqrt{1+\frac{1}{c^2 x^2}}\right) x\right)}{\left(2 c^2 d-e\right) \left(\sqrt{d}+i \sqrt{e}\, x\right)}\right]}{d \left(c^2 d-e\right)^{3/2}}\right)-$$

$$\frac{1}{16 d^{3/2}}$$

$$i \left(-\frac{i c \sqrt{e} \sqrt{1+\frac{1}{c^2 x^2}}\, x}{\sqrt{d} \left(c^2 d-e\right) \left(i \sqrt{d}+\sqrt{e}\, x\right)}-\frac{\operatorname{ArcCsch}[c x]}{\sqrt{e} \left(i \sqrt{d}+\sqrt{e}\, x\right)^2}-\frac{\operatorname{ArcSinh}\left[\frac{1}{c x}\right]}{d \sqrt{e}}+\frac{i \left(2 c^2 d-e\right) \operatorname{Log}\left[\frac{4 i d \sqrt{c^2 d-e} \sqrt{e} \left(i \sqrt{e}+c \left(c \sqrt{d}+\sqrt{c^2 d-e} \sqrt{1+\frac{1}{c^2 x^2}}\right) x\right)}{\left(2 c^2 d-e\right) \left(\sqrt{d}-i \sqrt{e}\, x\right)}\right]}{d \left(c^2 d-e\right)^{3/2}}\right)-$$

$$\frac{3\left(-\frac{\text{ArcCsch}[c\,x]}{i\sqrt{d}\,\sqrt{e}+e\,x}-\frac{i\left(\frac{\text{ArcSinh}\left[\frac{1}{c\,x}\right]}{\sqrt{e}}-\frac{\text{Log}\left[\frac{2\sqrt{d}\,\sqrt{e}\left(i\sqrt{e}+c\left(c\sqrt{d}+i\sqrt{-c^2\,d+e}\,\sqrt{1+\frac{1}{c^2\,x^2}}\right)x\right)}{\sqrt{-c^2\,d+e}\left(i\sqrt{d}+\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2\,d+e}}\right)}{\sqrt{d}}\right)}{16\,d^2}-\frac{3\left(-\frac{\text{ArcCsch}[c\,x]}{-i\sqrt{d}\,\sqrt{e}+e\,x}+\frac{i\left(\frac{\text{ArcSinh}\left[\frac{1}{c\,x}\right]}{\sqrt{e}}-\frac{\text{Log}\left[-\frac{2\sqrt{d}\,\sqrt{e}\left(\sqrt{e}+c\left(i\,c\sqrt{d}+\sqrt{-c^2\,d+e}\,\sqrt{1+\frac{1}{c^2\,x^2}}\right)x\right)}{\sqrt{-c^2\,d+e}\left(\sqrt{d}+i\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2\,d+e}}\right)}{\sqrt{d}}\right)}{16\,d^2}+$$

$$\frac{1}{128\,d^{5/2}\sqrt{e}}\,3\,i\left(\pi^2-4\,i\,\pi\,\text{ArcCsch}[c\,x]-8\,\text{ArcCsch}[c\,x]^2+32\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(c\sqrt{d}-\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,i\,\text{ArcCsch}[c\,x]\right)\right]}{\sqrt{-c^2\,d+e}}\right]-\right.$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-e^{-2\,\text{ArcCsch}[c\,x]}\right]+4\,i\,\pi\,\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+16\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,i\,\pi\,\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-$$

$$16\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-4\,i\,\pi\,\text{Log}\left[\sqrt{e}+\frac{i\sqrt{d}}{x}\right]+4\,\text{PolyLog}\left[2,\,e^{-2\,\text{ArcCsch}[c\,x]}\right]+$$

$$\left.8\,\text{PolyLog}\left[2,\,\frac{i\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+8\,\text{PolyLog}\left[2,\,-\frac{i\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]\right)-$$

$$\frac{1}{128\, d^{5/2}\, \sqrt{e}} 3\, i \left(\pi^2 - 4\, i\, \pi\, \text{ArcCsch}[c\, x] - 8\, \text{ArcCsch}[c\, x]^2 - 32\, \text{ArcSin}\left[\frac{\sqrt{1 - \frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right] \text{ArcTan}\left[\frac{\left(c\sqrt{d} + \sqrt{e}\right) \text{Cot}\left[\frac{1}{4}\left(\pi + 2\, i\, \text{ArcCsch}[c\, x]\right)\right]}{\sqrt{-c^2 d + e}}\right] - \right.$$

$$8\, \text{ArcCsch}[c\, x]\, \text{Log}\left[1 - e^{-2\, \text{ArcCsch}[c\, x]}\right] + 4\, i\, \pi\, \text{Log}\left[1 + \frac{i\left(-\sqrt{e} + \sqrt{-c^2 d + e}\right) e^{\text{ArcCsch}[c\, x]}}{c\sqrt{d}}\right] +$$

$$8\, \text{ArcCsch}[c\, x]\, \text{Log}\left[1 + \frac{i\left(-\sqrt{e} + \sqrt{-c^2 d + e}\right) e^{\text{ArcCsch}[c\, x]}}{c\sqrt{d}}\right] + 16\, i\, \text{ArcSin}\left[\frac{\sqrt{1 - \frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right] \text{Log}\left[1 + \frac{i\left(-\sqrt{e} + \sqrt{-c^2 d + e}\right) e^{\text{ArcCsch}[c\, x]}}{c\sqrt{d}}\right] +$$

$$4\, i\, \pi\, \text{Log}\left[1 - \frac{i\left(\sqrt{e} + \sqrt{-c^2 d + e}\right) e^{\text{ArcCsch}[c\, x]}}{c\sqrt{d}}\right] + 8\, \text{ArcCsch}[c\, x]\, \text{Log}\left[1 - \frac{i\left(\sqrt{e} + \sqrt{-c^2 d + e}\right) e^{\text{ArcCsch}[c\, x]}}{c\sqrt{d}}\right] -$$

$$16\, i\, \text{ArcSin}\left[\frac{\sqrt{1 - \frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right] \text{Log}\left[1 - \frac{i\left(\sqrt{e} + \sqrt{-c^2 d + e}\right) e^{\text{ArcCsch}[c\, x]}}{c\sqrt{d}}\right] - 4\, i\, \pi\, \text{Log}\left[\sqrt{e} - \frac{i\sqrt{d}}{x}\right] + 4\, \text{PolyLog}\left[2, e^{-2\, \text{ArcCsch}[c\, x]}\right] +$$

$$\left. 8\, \text{PolyLog}\left[2, -\frac{i\left(-\sqrt{e} + \sqrt{-c^2 d + e}\right) e^{\text{ArcCsch}[c\, x]}}{c\sqrt{d}}\right] + 8\, \text{PolyLog}\left[2, \frac{i\left(\sqrt{e} + \sqrt{-c^2 d + e}\right) e^{\text{ArcCsch}[c\, x]}}{c\sqrt{d}}\right] \right)$$

Problem 118: Result unnecessarily involves higher level functions.

$$\int x^5 \sqrt{d+e x^2}\,(a+b\,\text{ArcCsch}[c x])\,dx$$

Optimal (type 3, 413 leaves, 12 steps):

$$-\frac{b\left(23 c^4 d^2-12 c^2 d e-75 e^2\right) x \sqrt{-1-c^2 x^2}\,\sqrt{d+e x^2}}{1680 c^5 e^2 \sqrt{-c^2 x^2}}-\frac{b\left(29 c^2 d+25 e\right) x \sqrt{-1-c^2 x^2}\,\left(d+e x^2\right)^{3/2}}{840 c^3 e^2 \sqrt{-c^2 x^2}}+\frac{b x \sqrt{-1-c^2 x^2}\,\left(d+e x^2\right)^{5/2}}{42 c e^2 \sqrt{-c^2 x^2}}+\frac{d^2\left(d+e x^2\right)^{3/2}(a+b\,\text{ArcCsch}[c x])}{3 e^3}-\frac{2 d\left(d+e x^2\right)^{5/2}(a+b\,\text{ArcCsch}[c x])}{5 e^3}+\frac{\left(d+e x^2\right)^{7/2}(a+b\,\text{ArcCsch}[c x])}{7 e^3}+\frac{b\left(105 c^6 d^3+35 c^4 d^2 e+63 c^2 d e^2-75 e^3\right) x\,\text{ArcTan}\left[\frac{\sqrt{e}\,\sqrt{-1-c^2 x^2}}{c\sqrt{d+e x^2}}\right]}{1680 c^6 e^{5/2} \sqrt{-c^2 x^2}}+\frac{8 b c d^{7/2} x\,\text{ArcTan}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d}\,\sqrt{-1-c^2 x^2}}\right]}{105 e^3 \sqrt{-c^2 x^2}}$$

Result (type 6, 713 leaves):

$$\left(b\,d\,\sqrt{1+\frac{1}{c^2 x^2}}\;x^3\left(-\left(105 c^6 d^3+35 c^4 d^2 e+63 c^2 d e^2-75 e^3\right) \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2 x^2,-\frac{e x^2}{d}\right]\right.\right.$$
$$\left(c^2 d\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]+e\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]\right)+4\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]$$
$$\left(\left(35 c^6 d^2 e^2 x^2+63 c^4 d e^3 x^2-75 c^2 e^4 x^2+c^8 d^3\left(-128 d+105 e x^2\right)\right) \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2 x^2,-\frac{e x^2}{d}\right]+\right.$$
$$\left.\left.\left.32 c^8 d^3 x^2\left(e\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-c^2 x^2,-\frac{e x^2}{d}\right]+c^2 d\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-c^2 x^2,-\frac{e x^2}{d}\right]\right)\right)\right)\right)\Big/$$
$$\left(840 c^5 e^2\left(1+c^2 x^2\right)\sqrt{d+e x^2}\left(-4 c^2 e x^2\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]+c^2 d\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]+\right.\right.$$
$$\left.e\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]\right)$$
$$\left.\left(-4 d\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2 x^2,-\frac{e x^2}{d}\right]+x^2\left(e\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-c^2 x^2,-\frac{e x^2}{d}\right]+c^2 d\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-c^2 x^2,-\frac{e x^2}{d}\right]\right)\right)\right)+$$
$$\frac{1}{1680 c^5 e^3}\sqrt{d+e x^2}\left(16 a c^5\left(8 d^3-4 d^2 e x^2+3 d e^2 x^4+15 e^3 x^6\right)+b e\sqrt{1+\frac{1}{c^2 x^2}}\;x\left(75 e^2-2 c^2 e\left(19 d+25 e x^2\right)+c^4\left(-41 d^2+22 d e x^2+40 e^2 x^4\right)\right)+\right.$$
$$\left.16 b c^5\left(8 d^3-4 d^2 e x^2+3 d e^2 x^4+15 e^3 x^6\right) \text{ArcCsch}[c\,x]\right)$$

Problem 119: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int x^3\sqrt{d+e x^2}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)\,dx$$

Optimal (type 3, 302 leaves, 11 steps):

$$\frac{b\left(c^2 d-9 e\right) x\sqrt{-1-c^2 x^2}\,\sqrt{d+e x^2}}{120 c^3 e\sqrt{-c^2 x^2}}+\frac{b x\sqrt{-1-c^2 x^2}\,\left(d+e x^2\right)^{3/2}}{20 c e\sqrt{-c^2 x^2}}-\frac{d\left(d+e x^2\right)^{3/2}\left(a+b\,\text{ArcCsch}[c\,x]\right)}{3 e^2}+$$
$$\frac{\left(d+e x^2\right)^{5/2}\left(a+b\,\text{ArcCsch}[c\,x]\right)}{5 e^2}-\frac{b\left(15 c^4 d^2+10 c^2 d e-9 e^2\right) x\,\text{ArcTan}\left[\frac{\sqrt{e}\,\sqrt{-1-c^2 x^2}}{c\sqrt{d+e x^2}}\right]}{120 c^4 e^{3/2}\sqrt{-c^2 x^2}}-\frac{2 b c d^{5/2} x\,\text{ArcTan}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d}\,\sqrt{-1-c^2 x^2}}\right]}{15 e^2\sqrt{-c^2 x^2}}$$

Result (type 6, 635 leaves):

$$-\Bigg(\Bigg(b\,d\,\sqrt{1+\frac{1}{c^2\,x^2}}\;x^3\,\Bigg(-\left(15\,c^4\,d^2+10\,c^2\,d\,e-9\,e^2\right)\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2\,x^2,-\frac{e\,x^2}{d}\right]$$

$$\left(c^2\,d\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\right]+e\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\right]\right)+$$

$$4\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\right]\Bigg(\left(10\,c^4\,d\,e^2\,x^2-9\,c^2\,e^3\,x^2+c^6\,d^2\left(-16\,d+15\,e\,x^2\right)\right)\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2\,x^2,-\frac{e\,x^2}{d}\right]+$$

$$4\,c^6\,d^2\,x^2\left(e\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-c^2\,x^2,-\frac{e\,x^2}{d}\right]+c^2\,d\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-c^2\,x^2,-\frac{e\,x^2}{d}\right]\right)\Bigg)\Bigg)\Bigg)\Bigg/$$

$$\Bigg(60\,c^3\,e\left(1+c^2\,x^2\right)\sqrt{d+e\,x^2}\,\left(-4\,c^2\,e\,x^2\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\right]+c^2\,d\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\right]+\right.$$

$$\left.e\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\right]\right)$$

$$\left(-4\,d\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2\,x^2,-\frac{e\,x^2}{d}\right]+x^2\left(e\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-c^2\,x^2,-\frac{e\,x^2}{d}\right]+c^2\,d\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-c^2\,x^2,-\frac{e\,x^2}{d}\right]\right)\right)\Bigg)+$$

$$\frac{1}{120\,c^3\,e^2}\sqrt{d+e\,x^2}\,\Bigg(8\,a\,c^3\left(-2\,d^2+d\,e\,x^2+3\,e^2\,x^4\right)+b\,e\,\sqrt{1+\frac{1}{c^2\,x^2}}\;x\left(-9\,e+c^2\left(7\,d+6\,e\,x^2\right)\right)+$$

$$8\,b\,c^3\left(-2\,d^2+d\,e\,x^2+3\,e^2\,x^4\right)\text{ArcCsch}[c\,x]\Bigg)$$

Problem 120: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int x\,\sqrt{d+e\,x^2}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)\,dx$$

Optimal (type 3, 203 leaves, 9 steps):

$$\frac{b\,x\,\sqrt{-1-c^2\,x^2}\,\sqrt{d+e\,x^2}}{6\,c\,\sqrt{-c^2\,x^2}}+\frac{\left(d+e\,x^2\right)^{3/2}\left(a+b\,\text{ArcCsch}[c\,x]\right)}{3\,e}+\frac{b\left(3\,c^2\,d-e\right)x\,\text{ArcTan}\left[\frac{\sqrt{e}\,\sqrt{-1-c^2\,x^2}}{c\,\sqrt{d+e\,x^2}}\right]}{6\,c^2\,\sqrt{e}\,\sqrt{-c^2\,x^2}}+\frac{b\,c\,d^{3/2}\,x\,\text{ArcTan}\left[\frac{\sqrt{d+e\,x^2}}{\sqrt{d}\,\sqrt{-1-c^2\,x^2}}\right]}{3\,e\,\sqrt{-c^2\,x^2}}$$

Result (type 6, 556 leaves):

$$\left(b\,d\,\sqrt{1+\frac{1}{c^2 x^2}}\;x^3\right.$$

$$\left(-\left(3 c^2 d-e\right) \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2 x^2,-\frac{e x^2}{d}\right] \left(c^2 d\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]+e\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]\right)+\right.$$

$$2\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right] \left(-2 \left(c^2 e^2 x^2+c^4 d \left(2 d-3 e x^2\right)\right) \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2 x^2,-\frac{e x^2}{d}\right]+\right.$$

$$\left.\left.\left.c^4 d x^2 \left(e\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-c^2 x^2,-\frac{e x^2}{d}\right]+c^2 d\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-c^2 x^2,-\frac{e x^2}{d}\right]\right)\right)\right)\right) \Big/ \left(3 c \left(1+c^2 x^2\right) \sqrt{d+e x^2}\right.$$

$$\left(-4 c^2 e x^2\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]+c^2 d\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]+e\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]\right)$$

$$\left.\left(-4 d\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2 x^2,-\frac{e x^2}{d}\right]+x^2 \left(e\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-c^2 x^2,-\frac{e x^2}{d}\right]+c^2 d\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-c^2 x^2,-\frac{e x^2}{d}\right]\right)\right)\right)+$$

$$\frac{\sqrt{d+e x^2}\left(b\,e\sqrt{1+\frac{1}{c^2 x^2}}\;x+2 a c \left(d+e x^2\right)+2 b c \left(d+e x^2\right) \text{ArcCsch}[c x]\right)}{6 c e}$$

Problem 126: Unable to integrate problem.

$$\int \frac{\sqrt{d+e x^2}\,\left(a+b\,\text{ArcCsch}[c x]\right)}{x^4}\,dx$$

Optimal (type 4, 389 leaves, 8 steps):

$$-\frac{2 b c^3 \left(c^2 d-2 e\right) x^2 \sqrt{d+e x^2}}{9 d \sqrt{-c^2 x^2}\,\sqrt{-1-c^2 x^2}}-\frac{2 b c \left(c^2 d-2 e\right) \sqrt{-1-c^2 x^2}\,\sqrt{d+e x^2}}{9 d \sqrt{-c^2 x^2}}+\frac{b c \sqrt{-1-c^2 x^2}\,\sqrt{d+e x^2}}{9 x^2 \sqrt{-c^2 x^2}}-\frac{\left(d+e x^2\right)^{3/2} \left(a+b\,\text{ArcCsch}[c x]\right)}{3 d x^3}+$$

$$\frac{2 b c^2 \left(c^2 d-2 e\right) x \sqrt{d+e x^2}\;\text{EllipticE}\left[\text{ArcTan}[c x],1-\frac{e}{c^2 d}\right]}{9 d \sqrt{-c^2 x^2}\,\sqrt{-1-c^2 x^2}\,\sqrt{\frac{d+e x^2}{d \left(1+c^2 x^2\right)}}}-\frac{b \left(c^2 d-3 e\right) e x \sqrt{d+e x^2}\;\text{EllipticF}\left[\text{ArcTan}[c x],1-\frac{e}{c^2 d}\right]}{9 d^2 \sqrt{-c^2 x^2}\,\sqrt{-1-c^2 x^2}\,\sqrt{\frac{d+e x^2}{d \left(1+c^2 x^2\right)}}}$$

Result (type 8, 25 leaves):

$$\int \frac{\sqrt{d+e x^2}\,\left(a+b\,\text{ArcCsch}[c x]\right)}{x^4}\,dx$$

Problem 127: Unable to integrate problem.

$$\int \frac{\sqrt{d+e\,x^2}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{x^6}\,dx$$

Optimal (type 4, 527 leaves, 9 steps):

$$\frac{b\,c^3\left(24\,c^4\,d^2-19\,c^2\,d\,e-31\,e^2\right)x^2\sqrt{d+e\,x^2}}{225\,d^2\sqrt{-c^2\,x^2}\,\sqrt{-1-c^2\,x^2}}+\frac{b\,c\left(24\,c^4\,d^2-19\,c^2\,d\,e-31\,e^2\right)\sqrt{-1-c^2\,x^2}\,\sqrt{d+e\,x^2}}{225\,d^2\sqrt{-c^2\,x^2}}-$$
$$\frac{b\,c\left(12\,c^2\,d+e\right)\sqrt{-1-c^2\,x^2}\,\sqrt{d+e\,x^2}}{225\,d\,x^2\sqrt{-c^2\,x^2}}+\frac{b\,c\sqrt{-1-c^2\,x^2}\left(d+e\,x^2\right)^{3/2}}{25\,d\,x^4\sqrt{-c^2\,x^2}}-\frac{\left(d+e\,x^2\right)^{3/2}\left(a+b\,\text{ArcCsch}[c\,x]\right)}{5\,d\,x^5}+$$
$$\frac{2\,e\left(d+e\,x^2\right)^{3/2}\left(a+b\,\text{ArcCsch}[c\,x]\right)}{15\,d^2\,x^3}-\frac{b\,c^2\left(24\,c^4\,d^2-19\,c^2\,d\,e-31\,e^2\right)x\sqrt{d+e\,x^2}\;\text{EllipticE}\left[\text{ArcTan}[c\,x],1-\frac{e}{c^2\,d}\right]}{225\,d^2\sqrt{-c^2\,x^2}\,\sqrt{-1-c^2\,x^2}\,\sqrt{\frac{d+e\,x^2}{d\left(1+c^2\,x^2\right)}}}+$$
$$\frac{2\,b\,e\left(6\,c^4\,d^2-4\,c^2\,d\,e-15\,e^2\right)x\sqrt{d+e\,x^2}\;\text{EllipticF}\left[\text{ArcTan}[c\,x],1-\frac{e}{c^2\,d}\right]}{225\,d^3\sqrt{-c^2\,x^2}\,\sqrt{-1-c^2\,x^2}\,\sqrt{\frac{d+e\,x^2}{d\left(1+c^2\,x^2\right)}}}$$

Result (type 8, 25 leaves):

$$\int \frac{\sqrt{d+e\,x^2}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{x^6}\,dx$$

Problem 128: Result unnecessarily involves higher level functions.

$$\int x^3\left(d+e\,x^2\right)^{3/2}\left(a+b\,\text{ArcCsch}[c\,x]\right)dx$$

Optimal (type 3, 384 leaves, 12 steps):

$$-\frac{b\left(3c^4d^2+38c^2de-25e^2\right)x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{560c^5e\sqrt{-c^2x^2}}+\frac{b\left(13c^2d-25e\right)x\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}{840c^3e\sqrt{-c^2x^2}}+\frac{bx\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{5/2}}{42ce\sqrt{-c^2x^2}}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\,\text{ArcCsch}[cx]\right)}{5e^2}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\,\text{ArcCsch}[cx]\right)}{7e^2}-\frac{b\left(35c^6d^3+35c^4d^2e-63c^2de^2+25e^3\right)x\,\text{ArcTan}\left[\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{c\sqrt{d+ex^2}}\right]}{560c^6e^{3/2}\sqrt{-c^2x^2}}-\frac{2bcd^{7/2}x\,\text{ArcTan}\left[\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right]}{35e^2\sqrt{-c^2x^2}}$$

Result (type 6, 687 leaves):

$$-\left(\left(b\,d\sqrt{1+\frac{1}{c^2x^2}}\,x^3\left(-\left(35c^6d^3+35c^4d^2e-63c^2de^2+25e^3\right)\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2x^2,-\frac{ex^2}{d}\right]\right.\right.\right.$$
$$\left(c^2d\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-\frac{1}{c^2x^2},-\frac{d}{ex^2}\right]+e\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-\frac{1}{c^2x^2},-\frac{d}{ex^2}\right]\right)+4\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2x^2},-\frac{d}{ex^2}\right]$$
$$\left(\left(35c^6d^2e^2x^2-63c^4de^3x^2+25c^2e^4x^2+c^8d^3\left(-32d+35ex^2\right)\right)\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2x^2,-\frac{ex^2}{d}\right]+\right.$$
$$\left.\left.\left.8c^8d^3x^2\left(e\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-c^2x^2,-\frac{ex^2}{d}\right]+c^2d\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-c^2x^2,-\frac{ex^2}{d}\right]\right)\right)\right)\right)\Big/$$
$$\left(280c^5e\left(1+c^2x^2\right)\sqrt{d+ex^2}\left(-4c^2ex^2\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2x^2},-\frac{d}{ex^2}\right]+c^2d\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-\frac{1}{c^2x^2},-\frac{d}{ex^2}\right]+\right.\right.$$
$$\left.e\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-\frac{1}{c^2x^2},-\frac{d}{ex^2}\right]\right)$$
$$\left.\left(-4d\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2x^2,-\frac{ex^2}{d}\right]+x^2\left(e\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-c^2x^2,-\frac{ex^2}{d}\right]+c^2d\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-c^2x^2,-\frac{ex^2}{d}\right]\right)\right)\right)\right)+$$
$$\frac{1}{1680c^5e^2}\sqrt{d+ex^2}\left(-48ac^5\left(2d-5ex^2\right)\left(d+ex^2\right)^2+be\sqrt{1+\frac{1}{c^2x^2}}\,x\left(75e^2-2c^2e\left(82d+25ex^2\right)+c^4\left(57d^2+106dex^2+40e^2x^4\right)\right)-\right.$$
$$\left.48bc^5\left(2d-5ex^2\right)\left(d+ex^2\right)^2\text{ArcCsch}[cx]\right)$$

Problem 129: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int x \left(d+e x^2\right)^{3/2} \left(a+b\,\text{ArcCsch}[c\,x]\right) \, dx$$

Optimal (type 3, 270 leaves, 10 steps):

$$\frac{b\left(7 c^2 d-3 e\right) x \sqrt{-1-c^2 x^2} \sqrt{d+e x^2}}{40 c^3 \sqrt{-c^2 x^2}}+\frac{b x \sqrt{-1-c^2 x^2} \left(d+e x^2\right)^{3/2}}{20 c \sqrt{-c^2 x^2}}+$$
$$\frac{\left(d+e x^2\right)^{5/2} \left(a+b\,\text{ArcCsch}[c\,x]\right)}{5 e}+\frac{b\left(15 c^4 d^2-10 c^2 d e+3 e^2\right) x\,\text{ArcTan}\left[\frac{\sqrt{e} \sqrt{-1-c^2 x^2}}{c \sqrt{d+e x^2}}\right]}{40 c^4 \sqrt{e} \sqrt{-c^2 x^2}}+\frac{b c d^{5/2} x\,\text{ArcTan}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d} \sqrt{-1-c^2 x^2}}\right]}{5 e \sqrt{-c^2 x^2}}$$

Result (type 6, 610 leaves):

$$\left(b d \sqrt{1+\frac{1}{c^2 x^2}}\, x^3 \left(-\left(15 c^4 d^2-10 c^2 d e+3 e^2\right) \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2 x^2,-\frac{e x^2}{d}\right]\right.\right.$$
$$\left(c^2 d\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]+e\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]\right)+$$
$$4\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right] \left(\left(-10 c^4 d e^2 x^2+3 c^2 e^3 x^2+c^6 d^2 \left(-8 d+15 e x^2\right)\right) \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2 x^2,-\frac{e x^2}{d}\right]+\right.$$
$$\left.\left.\left.2 c^6 d^2 x^2 \left(e\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-c^2 x^2,-\frac{e x^2}{d}\right]+c^2 d\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-c^2 x^2,-\frac{e x^2}{d}\right]\right)\right)\right)\right)\Big/$$
$$\left(20 c^3 \left(1+c^2 x^2\right) \sqrt{d+e x^2} \left(-4 c^2 e x^2\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]+c^2 d\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]+\right.\right.$$
$$\left.e\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]\right)$$
$$\left.\left(-4 d\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2 x^2,-\frac{e x^2}{d}\right]+x^2 \left(e\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-c^2 x^2,-\frac{e x^2}{d}\right]+c^2 d\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-c^2 x^2,-\frac{e x^2}{d}\right]\right)\right)\right)+$$
$$\frac{\sqrt{d+e x^2} \left(8 a c^3 \left(d+e x^2\right)^2+b e \sqrt{1+\frac{1}{c^2 x^2}}\, x \left(-3 e+c^2 \left(9 d+2 e x^2\right)\right)+8 b c^3 \left(d+e x^2\right)^2 \text{ArcCsch}[c\,x]\right)}{40 c^3 e}$$

Problem 136: Unable to integrate problem.

$$\int \frac{\left(d+e x^2\right)^{3/2} \left(a+b\,\text{ArcCsch}[c\,x]\right)}{x^6} \, dx$$

Optimal (type 4, 492 leaves, 9 steps):

$$\frac{b c^3 \left(8 c^4 d^2-23 c^2 d e+23 e^2\right) x^2 \sqrt{d+e x^2}}{75 d \sqrt{-c^2 x^2} \sqrt{-1-c^2 x^2}}+\frac{b c \left(8 c^4 d^2-23 c^2 d e+23 e^2\right) \sqrt{-1-c^2 x^2} \sqrt{d+e x^2}}{75 d \sqrt{-c^2 x^2}}-\frac{4 b c \left(c^2 d-2 e\right) \sqrt{-1-c^2 x^2} \sqrt{d+e x^2}}{75 x^2 \sqrt{-c^2 x^2}}+$$

$$\frac{b c \sqrt{-1-c^2 x^2} \left(d+e x^2\right)^{3/2}}{25 x^4 \sqrt{-c^2 x^2}}-\frac{\left(d+e x^2\right)^{5/2} \left(a+b \text{ArcCsch}[c x]\right)}{5 d x^5}-\frac{b c^2 \left(8 c^4 d^2-23 c^2 d e+23 e^2\right) x \sqrt{d+e x^2} \text{ EllipticE}\left[\text{ArcTan}[c x],1-\frac{e}{c^2 d}\right]}{75 d \sqrt{-c^2 x^2} \sqrt{-1-c^2 x^2} \sqrt{\frac{d+e x^2}{d \left(1+c^2 x^2\right)}}}+$$

$$\frac{b e \left(4 c^4 d^2-11 c^2 d e+15 e^2\right) x \sqrt{d+e x^2} \text{ EllipticF}\left[\text{ArcTan}[c x],1-\frac{e}{c^2 d}\right]}{75 d^2 \sqrt{-c^2 x^2} \sqrt{-1-c^2 x^2} \sqrt{\frac{d+e x^2}{d \left(1+c^2 x^2\right)}}}$$

Result (type 8, 25 leaves):

$$\int \frac{\left(d+e x^2\right)^{3/2} \left(a+b \text{ArcCsch}[c x]\right)}{x^6} \, dx$$

Problem 137: Unable to integrate problem.

$$\int \frac{\left(d+e x^2\right)^{3/2} \left(a+b \text{ArcCsch}[c x]\right)}{x^8} \, dx$$

Optimal (type 4, 643 leaves, 10 steps):

$$-\frac{b c^3 \left(240 c^6 d^3-528 c^4 d^2 e+193 c^2 d e^2+247 e^3\right) x^2 \sqrt{d+e x^2}}{3675 d^2 \sqrt{-c^2 x^2} \sqrt{-1-c^2 x^2}}-\frac{b c \left(240 c^6 d^3-528 c^4 d^2 e+193 c^2 d e^2+247 e^3\right) \sqrt{-1-c^2 x^2} \sqrt{d+e x^2}}{3675 d^2 \sqrt{-c^2 x^2}}+$$

$$\frac{b c \left(120 c^4 d^2-159 c^2 d e-37 e^2\right) \sqrt{-1-c^2 x^2} \sqrt{d+e x^2}}{3675 d x^2 \sqrt{-c^2 x^2}}-\frac{b c \left(30 c^2 d-11 e\right) \sqrt{-1-c^2 x^2} \left(d+e x^2\right)^{3/2}}{1225 d x^4 \sqrt{-c^2 x^2}}+$$

$$\frac{b c \sqrt{-1-c^2 x^2} \left(d+e x^2\right)^{5/2}}{49 d x^6 \sqrt{-c^2 x^2}}-\frac{\left(d+e x^2\right)^{5/2} \left(a+b \text{ArcCsch}[c x]\right)}{7 d x^7}+\frac{2 e \left(d+e x^2\right)^{5/2} \left(a+b \text{ArcCsch}[c x]\right)}{35 d^2 x^5}+$$

$$\frac{b c^2 \left(240 c^6 d^3-528 c^4 d^2 e+193 c^2 d e^2+247 e^3\right) x \sqrt{d+e x^2} \text{ EllipticE}\left[\text{ArcTan}[c x],1-\frac{e}{c^2 d}\right]}{3675 d^2 \sqrt{-c^2 x^2} \sqrt{-1-c^2 x^2} \sqrt{\frac{d+e x^2}{d \left(1+c^2 x^2\right)}}}-$$

$$\frac{b e \left(120 c^6 d^3-249 c^4 d^2 e+71 c^2 d e^2+210 e^3\right) x \sqrt{d+e x^2} \text{ EllipticF}\left[\text{ArcTan}[c x],1-\frac{e}{c^2 d}\right]}{3675 d^3 \sqrt{-c^2 x^2} \sqrt{-1-c^2 x^2} \sqrt{\frac{d+e x^2}{d \left(1+c^2 x^2\right)}}}$$

Result (type 8, 25 leaves):

$$\int \frac{\left(d+e x^2\right)^{3/2} (a+b \text{ArcCsch}[c x])}{x^8} \, dx$$

Problem 138: Result unnecessarily involves higher level functions.

$$\int \frac{x^5 (a+b \text{ArcCsch}[c x])}{\sqrt{d+e x^2}} \, dx$$

Optimal (type 3, 329 leaves, 11 steps):

$$-\frac{b \left(19 c^2 d+9 e\right) x \sqrt{-1-c^2 x^2} \sqrt{d+e x^2}}{120 c^3 e^2 \sqrt{-c^2 x^2}}+\frac{b x \sqrt{-1-c^2 x^2} \left(d+e x^2\right)^{3/2}}{20 c e^2 \sqrt{-c^2 x^2}}+\frac{d^2 \sqrt{d+e x^2} (a+b \text{ArcCsch}[c x])}{e^3}-\frac{2 d \left(d+e x^2\right)^{3/2} (a+b \text{ArcCsch}[c x])}{3 e^3}+$$
$$\frac{\left(d+e x^2\right)^{5/2} (a+b \text{ArcCsch}[c x])}{5 e^3}+\frac{b \left(45 c^4 d^2+10 c^2 d e+9 e^2\right) x \text{ArcTan}\left[\frac{\sqrt{e} \sqrt{-1-c^2 x^2}}{c \sqrt{d+e x^2}}\right]}{120 c^4 e^{5/2} \sqrt{-c^2 x^2}}+\frac{8 b c d^{5/2} x \text{ArcTan}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d} \sqrt{-1-c^2 x^2}}\right]}{15 e^3 \sqrt{-c^2 x^2}}$$

Result (type 6, 637 leaves):

$$\Bigg(b d \sqrt{1+\frac{1}{c^2 x^2}} x^3 \Bigg(-\left(45 c^4 d^2+10 c^2 d e+9 e^2\right) \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2 x^2,-\frac{e x^2}{d}\right]$$
$$\left(c^2 d \text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]+e \text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]\right)+$$
$$4 \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right] \Bigg(\left(10 c^4 d e^2 x^2+9 c^2 e^3 x^2+c^6 d^2 \left(-64 d+45 e x^2\right)\right) \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2 x^2,-\frac{e x^2}{d}\right]+$$
$$16 c^6 d^2 x^2 \left(e \text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-c^2 x^2,-\frac{e x^2}{d}\right]+c^2 d \text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-c^2 x^2,-\frac{e x^2}{d}\right]\right)\Bigg)\Bigg)\Bigg) \Bigg/$$
$$\Bigg(60 c^3 e^2 \left(1+c^2 x^2\right) \sqrt{d+e x^2} \left(-4 c^2 e x^2 \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]+c^2 d \text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]+\right.$$
$$\left. e \text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]\right)$$
$$\left(-4 d \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2 x^2,-\frac{e x^2}{d}\right]+x^2 \left(e \text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-c^2 x^2,-\frac{e x^2}{d}\right]+c^2 d \text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-c^2 x^2,-\frac{e x^2}{d}\right]\right)\right)\Bigg)+$$
$$\frac{1}{120 c^3 e^3} \sqrt{d+e x^2} \left(8 a c^3 \left(8 d^2-4 d e x^2+3 e^2 x^4\right)+b e \sqrt{1+\frac{1}{c^2 x^2}} x \left(-9 e+c^2 \left(-13 d+6 e x^2\right)\right)+8 b c^3 \left(8 d^2-4 d e x^2+3 e^2 x^4\right) \text{ArcCsch}[c x]\right)$$

Problem 139: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{x^3 \left(a+b\,\mathrm{ArcCsch}[c\,x]\right)}{\sqrt{d+e\,x^2}}\,dx$$

Optimal (type 3, 229 leaves, 10 steps):

$$\frac{b\,x\sqrt{-1-c^2x^2}\,\sqrt{d+e\,x^2}}{6\,c\,e\sqrt{-c^2x^2}}-\frac{d\sqrt{d+e\,x^2}\,\left(a+b\,\mathrm{ArcCsch}[c\,x]\right)}{e^2}+$$
$$\frac{\left(d+e\,x^2\right)^{3/2}\left(a+b\,\mathrm{ArcCsch}[c\,x]\right)}{3\,e^2}-\frac{b\left(3\,c^2d+e\right)x\,\mathrm{ArcTan}\left[\frac{\sqrt{e}\,\sqrt{-1-c^2x^2}}{c\sqrt{d+e\,x^2}}\right]}{6\,c^2e^{3/2}\sqrt{-c^2x^2}}-\frac{2\,b\,c\,d^{3/2}\,x\,\mathrm{ArcTan}\left[\frac{\sqrt{d+e\,x^2}}{\sqrt{d}\,\sqrt{-1-c^2x^2}}\right]}{3\,e^2\sqrt{-c^2x^2}}$$

Result (type 6, 560 leaves):

$$-\left(\left(b\,d\sqrt{1+\frac{1}{c^2x^2}}\;x^3\left(-\left(3\,c^2d+e\right)\mathrm{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2x^2,-\frac{e\,x^2}{d}\right]\right.\right.\right.$$
$$\left(c^2d\,\mathrm{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-\frac{1}{c^2x^2},-\frac{d}{e\,x^2}\right]+e\,\mathrm{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-\frac{1}{c^2x^2},-\frac{d}{e\,x^2}\right]\right)+$$
$$4\,\mathrm{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2x^2},-\frac{d}{e\,x^2}\right]\left(\left(c^2e^2x^2+c^4d\left(-4\,d+3\,e\,x^2\right)\right)\mathrm{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2x^2,-\frac{e\,x^2}{d}\right]+\right.$$
$$\left.\left.\left.c^4d\,x^2\left(e\,\mathrm{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-c^2x^2,-\frac{e\,x^2}{d}\right]+c^2d\,\mathrm{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-c^2x^2,-\frac{e\,x^2}{d}\right]\right)\right)\right)\right)\Big/$$
$$\left(3\,c\,e\left(1+c^2x^2\right)\sqrt{d+e\,x^2}\left(-4\,c^2e\,x^2\,\mathrm{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2x^2},-\frac{d}{e\,x^2}\right]+c^2d\,\mathrm{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-\frac{1}{c^2x^2},-\frac{d}{e\,x^2}\right]+\right.\right.$$
$$\left.e\,\mathrm{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-\frac{1}{c^2x^2},-\frac{d}{e\,x^2}\right]\right)$$
$$\left.\left(-4\,d\,\mathrm{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2x^2,-\frac{e\,x^2}{d}\right]+x^2\left(e\,\mathrm{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-c^2x^2,-\frac{e\,x^2}{d}\right]+c^2d\,\mathrm{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-c^2x^2,-\frac{e\,x^2}{d}\right]\right)\right)\right)\right)+$$
$$\frac{\sqrt{d+e\,x^2}\left(-4\,a\,c\,d+b\,e\sqrt{1+\frac{1}{c^2x^2}}\;x+2\,a\,c\,e\,x^2+2\,b\,c\left(-2\,d+e\,x^2\right)\mathrm{ArcCsch}[c\,x]\right)}{6\,c\,e^2}$$

Problem 140: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{x\,(a+b\,\text{ArcCsch}[c\,x])}{\sqrt{d+e\,x^2}}\,dx$$

Optimal (type 3, 135 leaves, 9 steps):

$$\frac{\sqrt{d+e\,x^2}\,(a+b\,\text{ArcCsch}[c\,x])}{e}+\frac{b\,x\,\text{ArcTan}\left[\frac{\sqrt{e}\,\sqrt{-1-c^2\,x^2}}{c\,\sqrt{d+e\,x^2}}\right]}{\sqrt{e}\,\sqrt{-c^2\,x^2}}+\frac{b\,c\,\sqrt{d}\,x\,\text{ArcTan}\left[\frac{\sqrt{d+e\,x^2}}{\sqrt{d}\,\sqrt{-1-c^2\,x^2}}\right]}{e\,\sqrt{-c^2\,x^2}}$$

Result (type 6, 271 leaves):

$$\left(3\,b\,(c^2\,d-e)\,\sqrt{1+\frac{1}{c^2\,x^2}}\,\sqrt{d+e\,x^2}\,\text{AppellF1}\left[\frac{1}{2},-\frac{1}{2},1,\frac{3}{2},\frac{e\,(1+c^2\,x^2)}{-c^2\,d+e},1+c^2\,x^2\right]\right)\Big/$$
$$\left(c\,e\,x\left(3\,(c^2\,d-e)\,\text{AppellF1}\left[\frac{1}{2},-\frac{1}{2},1,\frac{3}{2},\frac{e\,(1+c^2\,x^2)}{-c^2\,d+e},1+c^2\,x^2\right]+(1+c^2\,x^2)\left(2\,(c^2\,d-e)\,\text{AppellF1}\left[\frac{3}{2},-\frac{1}{2},2,\frac{5}{2},\frac{e\,(1+c^2\,x^2)}{-c^2\,d+e},1+c^2\,x^2\right]+\right.\right.\right.$$
$$\left.\left.\left.e\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},\frac{e\,(1+c^2\,x^2)}{-c^2\,d+e},1+c^2\,x^2\right]\right)\right)\right)+\frac{\sqrt{d+e\,x^2}\,(a+b\,\text{ArcCsch}[c\,x])}{e}$$

Problem 146: Unable to integrate problem.

$$\int \frac{a+b\,\text{ArcCsch}[c\,x]}{x^4\,\sqrt{d+e\,x^2}}\,dx$$

Optimal (type 4, 425 leaves, 8 steps):

$$-\frac{b\,c^3\,(2\,c^2\,d+5\,e)\,x^2\,\sqrt{d+e\,x^2}}{9\,d^2\,\sqrt{-c^2\,x^2}\,\sqrt{-1-c^2\,x^2}}-\frac{b\,c\,(2\,c^2\,d+5\,e)\,\sqrt{-1-c^2\,x^2}\,\sqrt{d+e\,x^2}}{9\,d^2\,\sqrt{-c^2\,x^2}}+$$
$$\frac{b\,c\,\sqrt{-1-c^2\,x^2}\,\sqrt{d+e\,x^2}}{9\,d\,x^2\,\sqrt{-c^2\,x^2}}-\frac{\sqrt{d+e\,x^2}\,(a+b\,\text{ArcCsch}[c\,x])}{3\,d\,x^3}+\frac{2\,e\,\sqrt{d+e\,x^2}\,(a+b\,\text{ArcCsch}[c\,x])}{3\,d^2\,x}+$$
$$\frac{b\,c^2\,(2\,c^2\,d+5\,e)\,x\,\sqrt{d+e\,x^2}\,\text{EllipticE}\left[\text{ArcTan}[c\,x],1-\frac{e}{c^2\,d}\right]}{9\,d^2\,\sqrt{-c^2\,x^2}\,\sqrt{-1-c^2\,x^2}\,\sqrt{\frac{d+e\,x^2}{d\,(1+c^2\,x^2)}}}-\frac{b\,e\,(c^2\,d+6\,e)\,x\,\sqrt{d+e\,x^2}\,\text{EllipticF}\left[\text{ArcTan}[c\,x],1-\frac{e}{c^2\,d}\right]}{9\,d^3\,\sqrt{-c^2\,x^2}\,\sqrt{-1-c^2\,x^2}\,\sqrt{\frac{d+e\,x^2}{d\,(1+c^2\,x^2)}}}$$

Result (type 8, 25 leaves):

$$\int \frac{a+b\,\text{ArcCsch}[c\,x]}{x^4\,\sqrt{d+e\,x^2}}\,dx$$

Problem 147: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{x^5\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{\left(d+e\,x^2\right)^{3/2}}\,dx$$

Optimal (type 3, 256 leaves, 10 steps):

$$\frac{b\,x\,\sqrt{-1-c^2\,x^2}\,\sqrt{d+e\,x^2}}{6\,c\,e^2\,\sqrt{-c^2\,x^2}}-\frac{d^2\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{e^3\,\sqrt{d+e\,x^2}}-\frac{2\,d\,\sqrt{d+e\,x^2}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{e^3}+\frac{\left(d+e\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{3\,e^3}-\frac{b\,\left(9\,c^2\,d+e\right)\,x\,\text{ArcTan}\left[\frac{\sqrt{e}\,\sqrt{-1-c^2\,x^2}}{c\,\sqrt{d+e\,x^2}}\right]}{6\,c^2\,e^{5/2}\,\sqrt{-c^2\,x^2}}-\frac{8\,b\,c\,d^{3/2}\,x\,\text{ArcTan}\left[\frac{\sqrt{d+e\,x^2}}{\sqrt{d}\,\sqrt{-1-c^2\,x^2}}\right]}{3\,e^3\,\sqrt{-c^2\,x^2}}$$

Result (type 6, 592 leaves):

$$-\left(\left(b\,d\,\sqrt{1+\frac{1}{c^2 x^2}}\,x^3 \left(-\left(9 c^2 d+e\right) \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2 x^2,-\frac{e x^2}{d}\right] \left(c^2 d\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]+e\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]\right)+4\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right] \left(\left(c^2 e^2 x^2+c^4 d \left(-16 d+9 e x^2\right)\right) \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2 x^2,-\frac{e x^2}{d}\right]+4 c^4 d x^2 \left(e\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-c^2 x^2,-\frac{e x^2}{d}\right]+c^2 d\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-c^2 x^2,-\frac{e x^2}{d}\right]\right)\right)\right)\right)\Big/\left(3 c e^2 \left(1+c^2 x^2\right) \sqrt{d+e x^2} \left(-4 c^2 e x^2\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]+c^2 d\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]+e\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]\right) \left(-4 d\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2 x^2,-\frac{e x^2}{d}\right]+x^2 \left(e\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-c^2 x^2,-\frac{e x^2}{d}\right]+c^2 d\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-c^2 x^2,-\frac{e x^2}{d}\right]\right)\right)\right)\right)+$$

$$\frac{b e \sqrt{1+\frac{1}{c^2 x^2}}\,x \left(d+e x^2\right)-2 a c \left(8 d^2+4 d e x^2-e^2 x^4\right)-2 b c \left(8 d^2+4 d e x^2-e^2 x^4\right) \text{ArcCsch}[c x]}{6 c e^3 \sqrt{d+e x^2}}$$

Problem 148: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{x^3 \left(a+b\,\text{ArcCsch}[c x]\right)}{\left(d+e x^2\right)^{3/2}}\,dx$$

Optimal (type 3, 160 leaves, 9 steps):

$$\frac{d \left(a+b\,\text{ArcCsch}[c x]\right)}{e^2 \sqrt{d+e x^2}}+\frac{\sqrt{d+e x^2}\,\left(a+b\,\text{ArcCsch}[c x]\right)}{e^2}+\frac{b\,x\,\text{ArcTan}\left[\frac{\sqrt{e}\,\sqrt{-1-c^2 x^2}}{c \sqrt{d+e x^2}}\right]}{e^{3/2} \sqrt{-c^2 x^2}}+\frac{2 b c \sqrt{d}\,x\,\text{ArcTan}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d}\,\sqrt{-1-c^2 x^2}}\right]}{e^2 \sqrt{-c^2 x^2}}$$

Result (type 6, 334 leaves):

$$\frac{1}{e\left(1+c^2 x^2\right)\sqrt{d+e x^2}} 2 b c d \sqrt{1+\frac{1}{c^2 x^2}}\, x^3 \left(-\left(\left(2 c^2 \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]\right)\Big/\left(4 c^2 e x^2 \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]-c^2 d \text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]-e \text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]\right)\right)+\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2 x^2,-\frac{e x^2}{d}\right]\Big/\left(4 d \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2 x^2,-\frac{e x^2}{d}\right]-x^2\left(e \text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-c^2 x^2,-\frac{e x^2}{d}\right]+c^2 d \text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-c^2 x^2,-\frac{e x^2}{d}\right]\right)\right)\right)+\frac{\left(2 d+e x^2\right)\left(a+b \text{ArcCsch}[c x]\right)}{e^2\sqrt{d+e x^2}}$$

Problem 149: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{x\left(a+b \text{ArcCsch}[c x]\right)}{\left(d+e x^2\right)^{3/2}}\, dx$$

Optimal (type 3, 82 leaves, 4 steps):

$$-\frac{a+b \text{ArcCsch}[c x]}{e\sqrt{d+e x^2}}-\frac{b c x \text{ArcTan}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d}\sqrt{-1-c^2 x^2}}\right]}{\sqrt{d}\, e\sqrt{-c^2 x^2}}$$

Result (type 6, 192 leaves):

$$-\left(\left(2 b c^3\sqrt{1+\frac{1}{c^2 x^2}}\, x^3 \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]\right)\Big/\left(\left(1+c^2 x^2\right)\sqrt{d+e x^2}\left(-4 c^2 e x^2 \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]+c^2 d \text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]+e \text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]\right)\right)\right)-\frac{a+b \text{ArcCsch}[c x]}{e\sqrt{d+e x^2}}$$

Problem 155: Unable to integrate problem.

$$\int \frac{a+b \text{ArcCsch}[c x]}{x^2\left(d+e x^2\right)^{3/2}}\, dx$$

Optimal (type 4, 321 leaves, 7 steps):

$$\frac{b c^3 x^2 \sqrt{d+e x^2}}{d^2 \sqrt{-c^2 x^2} \sqrt{-1-c^2 x^2}}+\frac{b c \sqrt{-1-c^2 x^2} \sqrt{d+e x^2}}{d^2 \sqrt{-c^2 x^2}}-\frac{a+b \text{ArcCsch}[c x]}{d x \sqrt{d+e x^2}}-\frac{2 e x (a+b \text{ArcCsch}[c x])}{d^2 \sqrt{d+e x^2}}-$$

$$\frac{b c^2 x \sqrt{d+e x^2} \text{ EllipticE}\left[\text{ArcTan}[c x], 1-\frac{e}{c^2 d}\right]}{d^2 \sqrt{-c^2 x^2} \sqrt{-1-c^2 x^2} \sqrt{\frac{d+e x^2}{d (1+c^2 x^2)}}}+\frac{2 b e x \sqrt{d+e x^2} \text{ EllipticF}\left[\text{ArcTan}[c x], 1-\frac{e}{c^2 d}\right]}{d^3 \sqrt{-c^2 x^2} \sqrt{-1-c^2 x^2} \sqrt{\frac{d+e x^2}{d (1+c^2 x^2)}}}$$

Result (type 8, 25 leaves) :

$$\int \frac{a+b \text{ArcCsch}[c x]}{x^2 (d+e x^2)^{3/2}} \, dx$$

Problem 156: Result unnecessarily involves higher level functions.

$$\int \frac{x^5 (a+b \text{ArcCsch}[c x])}{(d+e x^2)^{5/2}} \, dx$$

Optimal (type 3, 251 leaves, 10 steps) :

$$\frac{b c d x \sqrt{-1-c^2 x^2}}{3 (c^2 d-e) e^2 \sqrt{-c^2 x^2} \sqrt{d+e x^2}}-\frac{d^2 (a+b \text{ArcCsch}[c x])}{3 e^3 (d+e x^2)^{3/2}}+\frac{2 d (a+b \text{ArcCsch}[c x])}{e^3 \sqrt{d+e x^2}}+$$

$$\frac{\sqrt{d+e x^2} (a+b \text{ArcCsch}[c x])}{e^3}+\frac{b x \text{ ArcTan}\left[\frac{\sqrt{e} \sqrt{-1-c^2 x^2}}{c \sqrt{d+e x^2}}\right]}{e^{5/2} \sqrt{-c^2 x^2}}+\frac{8 b c \sqrt{d} x \text{ ArcTan}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d} \sqrt{-1-c^2 x^2}}\right]}{3 e^3 \sqrt{-c^2 x^2}}$$

Result (type 6, 428 leaves) :

$$\frac{1}{3\,e^2\,\left(1+c^2\,x^2\right)\,\sqrt{d+e\,x^2}}$$

$$2\,b\,c\,d\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x^3\,\Bigg(-\Bigg(\Bigg(8\,c^2\,\text{AppellF1}\Big[1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\Big]\Bigg)\Big/\Bigg(4\,c^2\,e\,x^2\,\text{AppellF1}\Big[1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\Big]-c^2\,d$$

$$\text{AppellF1}\Big[2,\frac{1}{2},\frac{3}{2},3,-\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\Big]-e\,\text{AppellF1}\Big[2,\frac{3}{2},\frac{1}{2},3,-\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\Big]\Bigg)\Bigg)-\Bigg(3\,\text{AppellF1}\Big[1,\frac{1}{2},\frac{1}{2},2,-c^2\,x^2,-\frac{e\,x^2}{d}\Big]\Bigg)\Big/$$

$$\Bigg(-4\,d\,\text{AppellF1}\Big[1,\frac{1}{2},\frac{1}{2},2,-c^2\,x^2,-\frac{e\,x^2}{d}\Big]+x^2\,\Big(e\,\text{AppellF1}\Big[2,\frac{1}{2},\frac{3}{2},3,-c^2\,x^2,-\frac{e\,x^2}{d}\Big]+c^2\,d\,\text{AppellF1}\Big[2,\frac{3}{2},\frac{1}{2},3,-c^2\,x^2,-\frac{e\,x^2}{d}\Big]\Big)\Bigg)\Bigg)+$$

$$\Bigg(b\,c\,d\,e\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\left(d+e\,x^2\right)+a\,\left(c^2\,d-e\right)\,\left(8\,d^2+12\,d\,e\,x^2+3\,e^2\,x^4\right)+b\,\left(c^2\,d-e\right)\,\left(8\,d^2+12\,d\,e\,x^2+3\,e^2\,x^4\right)\,\text{ArcCsch}[c\,x]\Bigg)\Big/$$

$$\left(3\,\left(c^2\,d-e\right)\,e^3\,\left(d+e\,x^2\right)^{3/2}\right)$$

Problem 157: Result unnecessarily involves higher level functions.

$$\int\frac{x^3\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{\left(d+e\,x^2\right)^{5/2}}\,dx$$

Optimal (type 3, 169 leaves, 7 steps):

$$-\frac{b\,c\,x\,\sqrt{-1-c^2\,x^2}}{3\,\left(c^2\,d-e\right)\,e\,\sqrt{-c^2\,x^2}\,\sqrt{d+e\,x^2}}+\frac{d\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{3\,e^2\,\left(d+e\,x^2\right)^{3/2}}-\frac{a+b\,\text{ArcCsch}[c\,x]}{e^2\,\sqrt{d+e\,x^2}}-\frac{2\,b\,c\,x\,\text{ArcTan}\Big[\frac{\sqrt{d+e\,x^2}}{\sqrt{d}\,\sqrt{-1-c^2\,x^2}}\Big]}{3\,\sqrt{d}\,e^2\,\sqrt{-c^2\,x^2}}$$

Result (type 6, 273 leaves):

$$-\Bigg(\Bigg(4\,b\,c^3\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x^3\,\text{AppellF1}\Big[1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\Big]\Bigg)\Big/\Bigg(3\,e\,\left(1+c^2\,x^2\right)\,\sqrt{d+e\,x^2}\,\Big(-4\,c^2\,e\,x^2\,\text{AppellF1}\Big[1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\Big]+$$

$$c^2\,d\,\text{AppellF1}\Big[2,\frac{1}{2},\frac{3}{2},3,-\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\Big]+e\,\text{AppellF1}\Big[2,\frac{3}{2},\frac{1}{2},3,-\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\Big]\Big)\Bigg)\Bigg)+$$

$$\frac{b\,c\,e\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\left(d+e\,x^2\right)+a\,\left(c^2\,d-e\right)\,\left(2\,d+3\,e\,x^2\right)+b\,\left(c^2\,d-e\right)\,\left(2\,d+3\,e\,x^2\right)\,\text{ArcCsch}[c\,x]}{3\,e^2\,\left(-c^2\,d+e\right)\,\left(d+e\,x^2\right)^{3/2}}$$

Problem 158: Result unnecessarily involves higher level functions.

$$\int \frac{x\,(a+b\,\text{ArcCsch}[c\,x])}{(d+e\,x^2)^{5/2}}\,dx$$

Optimal (type 3, 144 leaves, 5 steps):

$$\frac{b\,c\,x\,\sqrt{-1-c^2x^2}}{3\,d\,(c^2d-e)\,\sqrt{-c^2x^2}\,\sqrt{d+e\,x^2}}-\frac{a+b\,\text{ArcCsch}[c\,x]}{3\,e\,(d+e\,x^2)^{3/2}}-\frac{b\,c\,x\,\text{ArcTan}\left[\frac{\sqrt{d+e\,x^2}}{\sqrt{d}\,\sqrt{-1-c^2x^2}}\right]}{3\,d^{3/2}\,e\,\sqrt{-c^2x^2}}$$

Result (type 6, 257 leaves):

$$-\left(\left(2\,b\,c^3\,\sqrt{1+\frac{1}{c^2x^2}}\,x^3\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2x^2},-\frac{d}{e\,x^2}\right]\right)\Big/\right.$$
$$\left(3\,d\,(1+c^2x^2)\,\sqrt{d+e\,x^2}\,\left(-4\,c^2e\,x^2\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2x^2},-\frac{d}{e\,x^2}\right]+c^2d\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-\frac{1}{c^2x^2},-\frac{d}{e\,x^2}\right]+\right.\right.$$
$$\left.\left.\left.e\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-\frac{1}{c^2x^2},-\frac{d}{e\,x^2}\right]\right)\right)\right)+\frac{a\,d\,(-c^2d+e)+b\,c\,e\,\sqrt{1+\frac{1}{c^2x^2}}\,x\,(d+e\,x^2)+b\,d\,(-c^2d+e)\,\text{ArcCsch}[c\,x]}{3\,d\,(c^2d-e)\,e\,(d+e\,x^2)^{3/2}}$$

Problem 164: Unable to integrate problem.

$$\int \frac{a+b\,\text{ArcCsch}[c\,x]}{(d+e\,x^2)^{5/2}}\,dx$$

Optimal (type 4, 278 leaves, 5 steps):

$$\frac{x\,(a+b\,\text{ArcCsch}[c\,x])}{3\,d\,(d+e\,x^2)^{3/2}}+\frac{2\,x\,(a+b\,\text{ArcCsch}[c\,x])}{3\,d^2\,\sqrt{d+e\,x^2}}-$$
$$\frac{b\,c\,\sqrt{e}\,x\,\sqrt{-1-c^2x^2}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{e}\,x}{\sqrt{d}}\right],1-\frac{c^2d}{e}\right]}{3\,d^{3/2}\,(c^2d-e)\,\sqrt{-c^2x^2}\,\sqrt{\frac{d\,(1+c^2x^2)}{d+e\,x^2}}\,\sqrt{d+e\,x^2}}-\frac{b\,(3\,c^2d-2\,e)\,x\,\sqrt{d+e\,x^2}\,\text{EllipticF}\left[\text{ArcTan}[c\,x],1-\frac{e}{c^2d}\right]}{3\,d^3\,(c^2d-e)\,\sqrt{-c^2x^2}\,\sqrt{-1-c^2x^2}\,\sqrt{\frac{d+e\,x^2}{d\,(1+c^2x^2)}}}$$

Result (type 8, 22 leaves):

$$\int \frac{a + b\,\text{ArcCsch}[c\,x]}{\left(d + e\,x^2\right)^{5/2}}\,dx$$

Test results for the 71 problems in "7.6.2 Inverse hyperbolic cosecant functions.m"

Problem 4: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcCsch}[a + b\,x]}{x}\,dx$$

Optimal (type 4, 162 leaves, 14 steps):

$$\text{ArcCsch}[a + b\,x]\,\text{Log}\left[1 - \frac{a\,e^{\text{ArcCsch}[a+b\,x]}}{1 - \sqrt{1 + a^2}}\right] + \text{ArcCsch}[a + b\,x]\,\text{Log}\left[1 - \frac{a\,e^{\text{ArcCsch}[a+b\,x]}}{1 + \sqrt{1 + a^2}}\right] - \text{ArcCsch}[a + b\,x]\,\text{Log}\left[1 - e^{2\,\text{ArcCsch}[a+b\,x]}\right] + \text{PolyLog}\left[2, \frac{a\,e^{\text{ArcCsch}[a+b\,x]}}{1 - \sqrt{1 + a^2}}\right] + \text{PolyLog}\left[2, \frac{a\,e^{\text{ArcCsch}[a+b\,x]}}{1 + \sqrt{1 + a^2}}\right] - \frac{1}{2}\,\text{PolyLog}\left[2, e^{2\,\text{ArcCsch}[a+b\,x]}\right]$$

Result (type 4, 428 leaves):

$$\frac{1}{8}\left(\pi^2-4\,i\,\pi\,\text{ArcCsch}[a+b\,x]-8\,\text{ArcCsch}[a+b\,x]^2-\right.$$

$$32\,i\,\text{ArcSin}\left[\frac{\sqrt{\frac{-i+a}{a}}}{\sqrt{2}}\right]\text{ArcTanh}\left[\frac{(i+a)\,\text{Cot}\left[\frac{1}{4}\left(\pi+2\,i\,\text{ArcCsch}[a+b\,x]\right)\right]}{\sqrt{1+a^2}}\right]-8\,\text{ArcCsch}[a+b\,x]\,\text{Log}\left[1-e^{-2\,\text{ArcCsch}[a+b\,x]}\right]+$$

$$4\,i\,\pi\,\text{Log}\left[1-\frac{\left(-1+\sqrt{1+a^2}\right)e^{\text{ArcCsch}[a+b\,x]}}{a}\right]+8\,\text{ArcCsch}[a+b\,x]\,\text{Log}\left[1-\frac{\left(-1+\sqrt{1+a^2}\right)e^{\text{ArcCsch}[a+b\,x]}}{a}\right]-$$

$$16\,i\,\text{ArcSin}\left[\frac{\sqrt{\frac{-i+a}{a}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{\left(-1+\sqrt{1+a^2}\right)e^{\text{ArcCsch}[a+b\,x]}}{a}\right]+4\,i\,\pi\,\text{Log}\left[1+\frac{\left(1+\sqrt{1+a^2}\right)e^{\text{ArcCsch}[a+b\,x]}}{a}\right]+$$

$$8\,\text{ArcCsch}[a+b\,x]\,\text{Log}\left[1+\frac{\left(1+\sqrt{1+a^2}\right)e^{\text{ArcCsch}[a+b\,x]}}{a}\right]+16\,i\,\text{ArcSin}\left[\frac{\sqrt{\frac{-i+a}{a}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{\left(1+\sqrt{1+a^2}\right)e^{\text{ArcCsch}[a+b\,x]}}{a}\right]-4\,i\,\pi\,\text{Log}\left[-\frac{b\,x}{a+b\,x}\right]+$$

$$\left.4\,\text{PolyLog}\left[2,\,e^{-2\,\text{ArcCsch}[a+b\,x]}\right]+8\,\text{PolyLog}\left[2,\,\frac{\left(-1+\sqrt{1+a^2}\right)e^{\text{ArcCsch}[a+b\,x]}}{a}\right]+8\,\text{PolyLog}\left[2,\,-\frac{\left(1+\sqrt{1+a^2}\right)e^{\text{ArcCsch}[a+b\,x]}}{a}\right]\right)$$

Problem 5: Result more than twice size of optimal antiderivative.

$$\int\frac{\text{ArcCsch}[a+b\,x]}{x^2}\,dx$$

Optimal (type 3, 63 leaves, 6 steps):

$$-\frac{b\,\text{ArcCsch}[a+b\,x]}{a}-\frac{\text{ArcCsch}[a+b\,x]}{x}+\frac{2\,b\,\text{ArcTanh}\left[\frac{a+\text{Tanh}\left[\frac{1}{2}\text{ArcCsch}[a+b\,x]\right]}{\sqrt{1+a^2}}\right]}{a\,\sqrt{1+a^2}}$$

Result (type 3, 141 leaves):

$$-\frac{\text{ArcCsch}[a+b x]}{x}-\frac{1}{a\sqrt{1+a^2}}$$

$$b\left(\sqrt{1+a^2}\,\text{ArcSinh}\left[\frac{1}{a+b x}\right]+\text{Log}[x]-\text{Log}\left[1+a^2+a b x+a\sqrt{1+a^2}\sqrt{\frac{1+a^2+2 a b x+b^2 x^2}{(a+b x)^2}}+\sqrt{1+a^2}\,b x\sqrt{\frac{1+a^2+2 a b x+b^2 x^2}{(a+b x)^2}}\right]\right)$$

Problem 7: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int(e+f x)^3\,(a+b\,\text{ArcCsch}[c+d x])^2\,dx$$

Optimal (type 4, 501 leaves, 20 steps):

$$\frac{b^2 f^2 (d e-c f) x}{d^3}+\frac{b^2 f^3 (c+d x)^2}{12 d^4}-\frac{b f^3 (c+d x)\sqrt{1+\frac{1}{(c+d x)^2}}\,(a+b\,\text{ArcCsch}[c+d x])}{3 d^4}+$$
$$\frac{3 b f (d e-c f)^2 (c+d x)\sqrt{1+\frac{1}{(c+d x)^2}}\,(a+b\,\text{ArcCsch}[c+d x])}{d^4}+\frac{b f^2 (d e-c f)(c+d x)^2\sqrt{1+\frac{1}{(c+d x)^2}}\,(a+b\,\text{ArcCsch}[c+d x])}{d^4}+$$
$$\frac{b f^3 (c+d x)^3\sqrt{1+\frac{1}{(c+d x)^2}}\,(a+b\,\text{ArcCsch}[c+d x])}{6 d^4}-\frac{(d e-c f)^4 (a+b\,\text{ArcCsch}[c+d x])^2}{4 d^4 f}+\frac{(e+f x)^4 (a+b\,\text{ArcCsch}[c+d x])^2}{4 f}-$$
$$\frac{2 b f^2 (d e-c f)(a+b\,\text{ArcCsch}[c+d x])\,\text{ArcTanh}\left[e^{\text{ArcCsch}[c+d x]}\right]}{d^4}+\frac{4 b (d e-c f)^3 (a+b\,\text{ArcCsch}[c+d x])\,\text{ArcTanh}\left[e^{\text{ArcCsch}[c+d x]}\right]}{d^4}-$$
$$\frac{b^2 f^3\,\text{Log}[c+d x]}{3 d^4}+\frac{3 b^2 f (d e-c f)^2\,\text{Log}[c+d x]}{d^4}-\frac{b^2 f^2 (d e-c f)\,\text{PolyLog}\left[2,-e^{\text{ArcCsch}[c+d x]}\right]}{d^4}+$$
$$\frac{2 b^2 (d e-c f)^3\,\text{PolyLog}\left[2,-e^{\text{ArcCsch}[c+d x]}\right]}{d^4}+\frac{b^2 f^2 (d e-c f)\,\text{PolyLog}\left[2,e^{\text{ArcCsch}[c+d x]}\right]}{d^4}-\frac{2 b^2 (d e-c f)^3\,\text{PolyLog}\left[2,e^{\text{ArcCsch}[c+d x]}\right]}{d^4}$$

Result (type 4, 1429 leaves):

$$a^2 e^3 x+\frac{3}{2} a^2 e^2 f x^2+a^2 e f^2 x^3+\frac{1}{4} a^2 f^3 x^4+\frac{1}{6} a b \Bigg(3 x \left(4 e^3+6 e^2 f x+4 e f^2 x^2+f^3 x^3\right)\text{ArcCsch}[c+d x]+$$
$$\frac{1}{d^4}\Bigg(f (c+d x)\sqrt{\frac{1+c^2+2 c d x+d^2 x^2}{(c+d x)^2}}\left(\left(-2+13 c^2\right) f^2-2 c d f (15 e+2 f x)+d^2\left(18 e^2+6 e f x+f^2 x^2\right)\right)-$$
$$3 c\left(-4 d^3 e^3+6 c d^2 e^2 f-4 c^2 d e f^2+c^3 f^3\right)\text{ArcSinh}\left[\frac{1}{c+d x}\right]+$$

$$6\left(2 d^3 e^3-6 c d^2 e^2 f+\left(-1+6 c^2\right) d e f^2+c\left(1-2 c^2\right) f^3\right) \text{Log}\left[(c+d x)\left(1+\sqrt{\frac{1+c^2+2 c d x+d^2 x^2}{(c+d x)^2}}\right)\right]\Bigg)\Bigg)-$$

$$\frac{1}{d} b^2 e^3\left(-\text{ArcCsch}[c+d x]\left((c+d x)\,\text{ArcCsch}[c+d x]-2 \text{Log}\left[1-e^{-\text{ArcCsch}[c+d x]}\right]+2 \text{Log}\left[1+e^{-\text{ArcCsch}[c+d x]}\right]\right)+\right.$$

$$\left.2 \text{PolyLog}\left[2,-e^{-\text{ArcCsch}[c+d x]}\right]-2 \text{PolyLog}\left[2, e^{-\text{ArcCsch}[c+d x]}\right]\right)-\frac{1}{(c+d x)\left(-1+\frac{c}{c+d x}\right)}$$

$$3 b^2 d e^2 f x\left(\frac{(c+d x) \sqrt{1+\frac{1}{(c+d x)^2}}\, \text{ArcCsch}[c+d x]}{d^2}+\frac{(c+d x)^2 \text{ArcCsch}[c+d x]^2}{2 d^2}-\frac{c\, \text{ArcCsch}[c+d x]^2 \text{Coth}\left[\frac{1}{2} \text{ArcCsch}[c+d x]\right]}{2 d^2}-\frac{\text{Log}\left[\frac{1}{c+d x}\right]}{d^2}-\right.$$

$$\frac{1}{d^2} 2 i c\left(i\, \text{ArcCsch}[c+d x]\left(\text{Log}\left[1-e^{-\text{ArcCsch}[c+d x]}\right]-\text{Log}\left[1+e^{-\text{ArcCsch}[c+d x]}\right]\right)+i\left(\text{PolyLog}\left[2,-e^{-\text{ArcCsch}[c+d x]}\right]-\text{PolyLog}\left[2, e^{-\text{ArcCsch}[c+d x]}\right]\right)\right)+$$

$$\left.\frac{c\, \text{ArcCsch}[c+d x]^2 \text{Tanh}\left[\frac{1}{2} \text{ArcCsch}[c+d x]\right]}{2 d^2}\right)-$$

$$\frac{1}{8 d^3} b^2 e f^2\left(2\left(-2+12 c\, \text{ArcCsch}[c+d x]+\text{ArcCsch}[c+d x]^2-6 c^2 \text{ArcCsch}[c+d x]^2\right) \text{Coth}\left[\frac{1}{2} \text{ArcCsch}[c+d x]\right]+\right.$$

$$2 \text{ArcCsch}[c+d x]\left(-1+3 c\, \text{ArcCsch}[c+d x]\right) \text{Csch}\left[\frac{1}{2} \text{ArcCsch}[c+d x]\right]^2-\frac{\text{ArcCsch}[c+d x]^2 \text{Csch}\left[\frac{1}{2} \text{ArcCsch}[c+d x]\right]^4}{2(c+d x)}-48 c\, \text{Log}\left[\frac{1}{c+d x}\right]+$$

$$8\left(-1+6 c^2\right)\left(\text{ArcCsch}[c+d x]\left(\text{Log}\left[1-e^{-\text{ArcCsch}[c+d x]}\right]-\text{Log}\left[1+e^{-\text{ArcCsch}[c+d x]}\right]\right)+\text{PolyLog}\left[2,-e^{-\text{ArcCsch}[c+d x]}\right]-\text{PolyLog}\left[2, e^{-\text{ArcCsch}[c+d x]}\right]\right)-$$

$$2 \text{ArcCsch}[c+d x]\left(1+3 c\, \text{ArcCsch}[c+d x]\right) \text{Sech}\left[\frac{1}{2} \text{ArcCsch}[c+d x]\right]^2-8(c+d x)^3 \text{ArcCsch}[c+d x]^2 \text{Sinh}\left[\frac{1}{2} \text{ArcCsch}[c+d x]\right]^4+$$

$$\left.2\left(2+12 c\, \text{ArcCsch}[c+d x]-\text{ArcCsch}[c+d x]^2+6 c^2 \text{ArcCsch}[c+d x]^2\right) \text{Tanh}\left[\frac{1}{2} \text{ArcCsch}[c+d x]\right]\right)-\frac{1}{192 d (c+d x)^3\left(-1+\frac{c}{c+d x}\right)^3}$$

$$b^2 f^3 x^3\left(-16\left(2 \text{ArcCsch}[c+d x]-18 c^2 \text{ArcCsch}[c+d x]+6 c^3 \text{ArcCsch}[c+d x]^2-3 c\left(-2+\text{ArcCsch}[c+d x]^2\right)\right) \text{Coth}\left[\frac{1}{2} \text{ArcCsch}[c+d x]\right]+\right.$$

$$2\left(2-24 c\, \text{ArcCsch}[c+d x]-3 \text{ArcCsch}[c+d x]^2+36 c^2 \text{ArcCsch}[c+d x]^2\right) \text{Csch}\left[\frac{1}{2} \text{ArcCsch}[c+d x]\right]^2+3 \text{ArcCsch}[c+d x]^2$$

$$\text{Csch}\left[\frac{1}{2}\text{ArcCsch}[c+d x]\right]^4-\frac{2\,\text{ArcCsch}[c+d x]\,\left(-1+6\,c\,\text{ArcCsch}[c+d x]\right)\,\text{Csch}\left[\frac{1}{2}\text{ArcCsch}[c+d x]\right]^4}{c+d x}-64\,\left(-1+9\,c^2\right)\,\text{Log}\left[\frac{1}{c+d x}\right]+192\,c$$
$$\left(-1+2\,c^2\right)\,\left(\text{ArcCsch}[c+d x]\,\left(\text{Log}\left[1-e^{-\text{ArcCsch}[c+d x]}\right]-\text{Log}\left[1+e^{-\text{ArcCsch}[c+d x]}\right]\right)+\text{PolyLog}\left[2,-e^{-\text{ArcCsch}[c+d x]}\right]-\text{PolyLog}\left[2,e^{-\text{ArcCsch}[c+d x]}\right]\right)-$$
$$2\,\left(2+24\,c\,\text{ArcCsch}[c+d x]-3\,\text{ArcCsch}[c+d x]^2+36\,c^2\,\text{ArcCsch}[c+d x]^2\right)\,\text{Sech}\left[\frac{1}{2}\text{ArcCsch}[c+d x]\right]^2+$$
$$3\,\text{ArcCsch}[c+d x]^2\,\text{Sech}\left[\frac{1}{2}\text{ArcCsch}[c+d x]\right]^4-32\,(c+d x)^3\,\text{ArcCsch}[c+d x]\,\left(1+6\,c\,\text{ArcCsch}[c+d x]\right)\,\text{Sinh}\left[\frac{1}{2}\text{ArcCsch}[c+d x]\right]^4+$$
$$\left.16\,\left(-2\,\text{ArcCsch}[c+d x]+18\,c^2\,\text{ArcCsch}[c+d x]+6\,c^3\,\text{ArcCsch}[c+d x]^2-3\,c\,\left(-2+\text{ArcCsch}[c+d x]^2\right)\right)\,\text{Tanh}\left[\frac{1}{2}\text{ArcCsch}[c+d x]\right]\right)$$

Problem 8: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int (e+f x)^2\,\left(a+b\,\text{ArcCsch}[c+d x]\right)^2\,dx$$

Optimal (type 4, 351 leaves, 17 steps):

$$\frac{b^2 f^2 x}{3 d^2}+\frac{2\,b\,f\,(d e-c f)\,(c+d x)\,\sqrt{1+\frac{1}{(c+d x)^2}}\,\left(a+b\,\text{ArcCsch}[c+d x]\right)}{d^3}+\frac{b\,f^2\,(c+d x)^2\,\sqrt{1+\frac{1}{(c+d x)^2}}\,\left(a+b\,\text{ArcCsch}[c+d x]\right)}{3 d^3}-$$
$$\frac{(d e-c f)^3\,\left(a+b\,\text{ArcCsch}[c+d x]\right)^2}{3 d^3 f}+\frac{(e+f x)^3\,\left(a+b\,\text{ArcCsch}[c+d x]\right)^2}{3 f}-\frac{2\,b\,f^2\,\left(a+b\,\text{ArcCsch}[c+d x]\right)\,\text{ArcTanh}\left[e^{\text{ArcCsch}[c+d x]}\right]}{3 d^3}+$$
$$\frac{4\,b\,(d e-c f)^2\,\left(a+b\,\text{ArcCsch}[c+d x]\right)\,\text{ArcTanh}\left[e^{\text{ArcCsch}[c+d x]}\right]}{d^3}+\frac{2\,b^2\,f\,(d e-c f)\,\text{Log}[c+d x]}{d^3}-\frac{b^2 f^2\,\text{PolyLog}\left[2,-e^{\text{ArcCsch}[c+d x]}\right]}{3 d^3}+$$
$$\frac{2\,b^2\,(d e-c f)^2\,\text{PolyLog}\left[2,-e^{\text{ArcCsch}[c+d x]}\right]}{d^3}+\frac{b^2 f^2\,\text{PolyLog}\left[2,e^{\text{ArcCsch}[c+d x]}\right]}{3 d^3}-\frac{2\,b^2\,(d e-c f)^2\,\text{PolyLog}\left[2,e^{\text{ArcCsch}[c+d x]}\right]}{d^3}$$

Result (type 4, 864 leaves):

$$a^2 e^2 x + a^2 e f x^2 + \frac{1}{3} a^2 f^2 x^3 +$$

$$\frac{1}{3} a b \left(2 x \left(3 e^2 + 3 e f x + f^2 x^2\right) \text{ArcCsch}[c + d x] + \frac{1}{d^3} \left(-f (c + d x) \sqrt{\frac{1 + c^2 + 2 c d x + d^2 x^2}{(c + d x)^2}} \left(5 c f - d (6 e + f x)\right) + 2 c \left(3 d^2 e^2 - 3 c d e f + c^2 f^2\right) \right.\right.$$

$$\left.\left. \text{ArcSinh}\left[\frac{1}{c + d x}\right] + \left(6 d^2 e^2 - 12 c d e f + \left(-1 + 6 c^2\right) f^2\right) \text{Log}\left[(c + d x) \left(1 + \sqrt{\frac{1 + c^2 + 2 c d x + d^2 x^2}{(c + d x)^2}}\right)\right]\right)\right) -$$

$$\frac{1}{d} b^2 e^2 \left(-\text{ArcCsch}[c + d x] \left((c + d x) \text{ArcCsch}[c + d x] - 2 \text{Log}\left[1 - e^{-\text{ArcCsch}[c+d x]}\right] + 2 \text{Log}\left[1 + e^{-\text{ArcCsch}[c+d x]}\right]\right) + \right.$$

$$\left. 2 \text{PolyLog}\left[2, -e^{-\text{ArcCsch}[c+d x]}\right] - 2 \text{PolyLog}\left[2, e^{-\text{ArcCsch}[c+d x]}\right]\right) - \frac{1}{(c + d x) \left(-1 + \frac{c}{c+d x}\right)}$$

$$2 b^2 d e f x \left(\frac{(c + d x) \sqrt{1 + \frac{1}{(c+d x)^2}} \text{ArcCsch}[c + d x]}{d^2} + \frac{(c + d x)^2 \text{ArcCsch}[c + d x]^2}{2 d^2} - \frac{c \text{ArcCsch}[c + d x]^2 \text{Coth}\left[\frac{1}{2} \text{ArcCsch}[c + d x]\right]}{2 d^2} - \frac{\text{Log}\left[\frac{1}{c+d x}\right]}{d^2} - \right.$$

$$\frac{1}{d^2} 2 i c \left(i \text{ArcCsch}[c + d x] \left(\text{Log}\left[1 - e^{-\text{ArcCsch}[c+d x]}\right] - \text{Log}\left[1 + e^{-\text{ArcCsch}[c+d x]}\right]\right) + i \left(\text{PolyLog}\left[2, -e^{-\text{ArcCsch}[c+d x]}\right] - \text{PolyLog}\left[2, e^{-\text{ArcCsch}[c+d x]}\right]\right)\right) +$$

$$\left. \frac{c \text{ArcCsch}[c + d x]^2 \text{Tanh}\left[\frac{1}{2} \text{ArcCsch}[c + d x]\right]}{2 d^2} \right) -$$

$$\frac{1}{24 d^3} b^2 f^2 \left(2 \left(-2 + 12 c \text{ArcCsch}[c + d x] + \text{ArcCsch}[c + d x]^2 - 6 c^2 \text{ArcCsch}[c + d x]^2\right) \text{Coth}\left[\frac{1}{2} \text{ArcCsch}[c + d x]\right] + \right.$$

$$2 \text{ArcCsch}[c + d x] \left(-1 + 3 c \text{ArcCsch}[c + d x]\right) \text{Csch}\left[\frac{1}{2} \text{ArcCsch}[c + d x]\right]^2 - \frac{\text{ArcCsch}[c + d x]^2 \text{Csch}\left[\frac{1}{2} \text{ArcCsch}[c + d x]\right]^4}{2 (c + d x)} - 48 c \text{Log}\left[\frac{1}{c + d x}\right] +$$

$$8 \left(-1 + 6 c^2\right) \left(\text{ArcCsch}[c + d x] \left(\text{Log}\left[1 - e^{-\text{ArcCsch}[c+d x]}\right] - \text{Log}\left[1 + e^{-\text{ArcCsch}[c+d x]}\right]\right) + \text{PolyLog}\left[2, -e^{-\text{ArcCsch}[c+d x]}\right] - \text{PolyLog}\left[2, e^{-\text{ArcCsch}[c+d x]}\right]\right) -$$

$$2 \text{ArcCsch}[c + d x] \left(1 + 3 c \text{ArcCsch}[c + d x]\right) \text{Sech}\left[\frac{1}{2} \text{ArcCsch}[c + d x]\right]^2 - 8 (c + d x)^3 \text{ArcCsch}[c + d x]^2 \text{Sinh}\left[\frac{1}{2} \text{ArcCsch}[c + d x]\right]^4 +$$

$$\left. 2 \left(2 + 12 c \text{ArcCsch}[c + d x] - \text{ArcCsch}[c + d x]^2 + 6 c^2 \text{ArcCsch}[c + d x]^2\right) \text{Tanh}\left[\frac{1}{2} \text{ArcCsch}[c + d x]\right]\right)$$

Problem 9: Result more than twice size of optimal antiderivative.

$$\int (e+f x) (a+b \text{ArcCsch}[c+d x])^2 \, dx$$

Optimal (type 4, 194 leaves, 11 steps):

$$\frac{b f (c+d x) \sqrt{1+\frac{1}{(c+d x)^2}} (a+b \text{ArcCsch}[c+d x])}{d^2}-\frac{(d e-c f)^2 (a+b \text{ArcCsch}[c+d x])^2}{2 d^2 f}+\frac{(e+f x)^2 (a+b \text{ArcCsch}[c+d x])^2}{2 f}+\frac{4 b (d e-c f) (a+b \text{ArcCsch}[c+d x]) \text{ArcTanh}\left[e^{\text{ArcCsch}[c+d x]}\right]}{d^2}+\frac{b^2 f \text{Log}[c+d x]}{d^2}+\frac{2 b^2 (d e-c f) \text{PolyLog}\left[2,-e^{\text{ArcCsch}[c+d x]}\right]}{d^2}-\frac{2 b^2 (d e-c f) \text{PolyLog}\left[2,e^{\text{ArcCsch}[c+d x]}\right]}{d^2}$$

Result (type 4, 427 leaves):

$$\frac{1}{2 d^2}\left(2 a^2 (d e-c f) (c+d x)+a^2 f (c+d x)^2+2 a b f (c+d x)\left(\sqrt{1+\frac{1}{(c+d x)^2}}+(c+d x) \text{ArcCsch}[c+d x]\right)+\right.$$
$$2 b^2 f\left((c+d x) \sqrt{1+\frac{1}{(c+d x)^2}} \text{ArcCsch}[c+d x]+\frac{1}{2}(c+d x)^2 \text{ArcCsch}[c+d x]^2-\text{Log}\left[\frac{1}{c+d x}\right]\right)+$$
$$4 a b d e\left((c+d x) \text{ArcCsch}[c+d x]+\text{Log}\left[\frac{\text{Csch}\left[\frac{1}{2} \text{ArcCsch}[c+d x]\right]}{2 (c+d x)}\right]-\text{Log}\left[\text{Sinh}\left[\frac{1}{2} \text{ArcCsch}[c+d x]\right]\right]\right)-$$
$$4 a b c f\left((c+d x) \text{ArcCsch}[c+d x]+\text{Log}\left[\frac{\text{Csch}\left[\frac{1}{2} \text{ArcCsch}[c+d x]\right]}{2 (c+d x)}\right]-\text{Log}\left[\text{Sinh}\left[\frac{1}{2} \text{ArcCsch}[c+d x]\right]\right]\right)+$$
$$2 b^2 d e\left(\text{ArcCsch}[c+d x]\left((c+d x) \text{ArcCsch}[c+d x]-2 \text{Log}\left[1-e^{-\text{ArcCsch}[c+d x]}\right]+2 \text{Log}\left[1+e^{-\text{ArcCsch}[c+d x]}\right]\right)-2 \text{PolyLog}\left[2,-e^{-\text{ArcCsch}[c+d x]}\right]+\right.$$
$$\left.2 \text{PolyLog}\left[2,e^{-\text{ArcCsch}[c+d x]}\right]\right)-2 b^2 c f\left(\text{ArcCsch}[c+d x]\left((c+d x) \text{ArcCsch}[c+d x]-2 \text{Log}\left[1-e^{-\text{ArcCsch}[c+d x]}\right]+2 \text{Log}\left[1+e^{-\text{ArcCsch}[c+d x]}\right]\right)-\right.$$
$$\left.\left.2 \text{PolyLog}\left[2,-e^{-\text{ArcCsch}[c+d x]}\right]+2 \text{PolyLog}\left[2,e^{-\text{ArcCsch}[c+d x]}\right]\right)\right)$$

Problem 10: Result more than twice size of optimal antiderivative.

$$\int (a+b \text{ArcCsch}[c+d x])^2 \, dx$$

Optimal (type 4, 85 leaves, 8 steps):

$$\frac{(c+d x)\,(a+b\,\text{ArcCsch}[c+d x])^2}{d}+\frac{4 b\,(a+b\,\text{ArcCsch}[c+d x])\,\text{ArcTanh}\left[e^{\text{ArcCsch}[c+d x]}\right]}{d}+\frac{2 b^2\,\text{PolyLog}\left[2,-e^{\text{ArcCsch}[c+d x]}\right]}{d}-\frac{2 b^2\,\text{PolyLog}\left[2,e^{\text{ArcCsch}[c+d x]}\right]}{d}$$

Result (type 4, 176 leaves):

$$\frac{1}{d}\left(a^2 c+a^2 d x+2 a b\,(c+d x)\,\text{ArcCsch}[c+d x]+b^2 c\,\text{ArcCsch}[c+d x]^2+b^2 d x\,\text{ArcCsch}[c+d x]^2-2 b^2\,\text{ArcCsch}[c+d x]\,\text{Log}\left[1-e^{-\text{ArcCsch}[c+d x]}\right]+2 b^2\,\text{ArcCsch}[c+d x]\,\text{Log}\left[1+e^{-\text{ArcCsch}[c+d x]}\right]+2 a b\,\text{Log}\left[\text{Cosh}\left[\frac{1}{2}\text{ArcCsch}[c+d x]\right]\right]-2 a b\,\text{Log}\left[\text{Sinh}\left[\frac{1}{2}\text{ArcCsch}[c+d x]\right]\right]-2 b^2\,\text{PolyLog}\left[2,-e^{-\text{ArcCsch}[c+d x]}\right]+2 b^2\,\text{PolyLog}\left[2,e^{-\text{ArcCsch}[c+d x]}\right]\right)$$

Problem 11: Unable to integrate problem.

$$\int \frac{(a+b\,\text{ArcCsch}[c+d x])^2}{e+f x}\,dx$$

Optimal (type 4, 475 leaves, 17 steps):

$$-\frac{(a+b\,\text{ArcCsch}[c+d x])^2\,\text{Log}\left[1-e^{2\,\text{ArcCsch}[c+d x]}\right]}{f}+\frac{(a+b\,\text{ArcCsch}[c+d x])^2\,\text{Log}\left[1+\frac{e^{\text{ArcCsch}[c+d x]}\,(d e-c f)}{f-\sqrt{d^2 e^2-2 c d e f+(1+c^2) f^2}}\right]}{f}+\frac{(a+b\,\text{ArcCsch}[c+d x])^2\,\text{Log}\left[1+\frac{e^{\text{ArcCsch}[c+d x]}\,(d e-c f)}{f+\sqrt{d^2 e^2-2 c d e f+(1+c^2) f^2}}\right]}{f}-\frac{b\,(a+b\,\text{ArcCsch}[c+d x])\,\text{PolyLog}\left[2,e^{2\,\text{ArcCsch}[c+d x]}\right]}{f}+\frac{2 b\,(a+b\,\text{ArcCsch}[c+d x])\,\text{PolyLog}\left[2,-\frac{e^{\text{ArcCsch}[c+d x]}\,(d e-c f)}{f-\sqrt{d^2 e^2-2 c d e f+(1+c^2) f^2}}\right]}{f}+\frac{2 b\,(a+b\,\text{ArcCsch}[c+d x])\,\text{PolyLog}\left[2,-\frac{e^{\text{ArcCsch}[c+d x]}\,(d e-c f)}{f+\sqrt{d^2 e^2-2 c d e f+(1+c^2) f^2}}\right]}{f}+\frac{b^2\,\text{PolyLog}\left[3,e^{2\,\text{ArcCsch}[c+d x]}\right]}{2 f}-\frac{2 b^2\,\text{PolyLog}\left[3,-\frac{e^{\text{ArcCsch}[c+d x]}\,(d e-c f)}{f-\sqrt{d^2 e^2-2 c d e f+(1+c^2) f^2}}\right]}{f}-\frac{2 b^2\,\text{PolyLog}\left[3,-\frac{e^{\text{ArcCsch}[c+d x]}\,(d e-c f)}{f+\sqrt{d^2 e^2-2 c d e f+(1+c^2) f^2}}\right]}{f}$$

Result (type 8, 22 leaves):

$$\int \frac{(a+b\,\text{ArcCsch}[c+d x])^2}{e+f x}\,dx$$

Problem 12: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(a+b\,\text{ArcCsch}[c+d\,x])^2}{(e+f\,x)^2}\,dx$$

Optimal (type 4, 448 leaves, 12 steps):

$$\frac{d\,(a+b\,\text{ArcCsch}[c+d\,x])^2}{f\,(d\,e-c\,f)}-\frac{(a+b\,\text{ArcCsch}[c+d\,x])^2}{f\,(e+f\,x)}-\frac{2\,b\,d\,(a+b\,\text{ArcCsch}[c+d\,x])\,\text{Log}\left[1+\frac{e^{\text{ArcCsch}[c+d\,x]}\,(d\,e-c\,f)}{f-\sqrt{d^2\,e^2-2\,c\,d\,e\,f+(1+c^2)\,f^2}}\right]}{(d\,e-c\,f)\,\sqrt{d^2\,e^2-2\,c\,d\,e\,f+(1+c^2)\,f^2}}+\frac{2\,b\,d\,(a+b\,\text{ArcCsch}[c+d\,x])\,\text{Log}\left[1+\frac{e^{\text{ArcCsch}[c+d\,x]}\,(d\,e-c\,f)}{f+\sqrt{d^2\,e^2-2\,c\,d\,e\,f+(1+c^2)\,f^2}}\right]}{(d\,e-c\,f)\,\sqrt{d^2\,e^2-2\,c\,d\,e\,f+(1+c^2)\,f^2}}-\frac{2\,b^2\,d\,\text{PolyLog}\left[2,-\frac{e^{\text{ArcCsch}[c+d\,x]}\,(d\,e-c\,f)}{f-\sqrt{d^2\,e^2-2\,c\,d\,e\,f+(1+c^2)\,f^2}}\right]}{(d\,e-c\,f)\,\sqrt{d^2\,e^2-2\,c\,d\,e\,f+(1+c^2)\,f^2}}+\frac{2\,b^2\,d\,\text{PolyLog}\left[2,-\frac{e^{\text{ArcCsch}[c+d\,x]}\,(d\,e-c\,f)}{f+\sqrt{d^2\,e^2-2\,c\,d\,e\,f+(1+c^2)\,f^2}}\right]}{(d\,e-c\,f)\,\sqrt{d^2\,e^2-2\,c\,d\,e\,f+(1+c^2)\,f^2}}$$

Result (type 4, 2061 leaves):

$$-\frac{a^2}{f\,(e+f\,x)}-\frac{2\,a\,b\,(c+d\,x)^2\left(f+\frac{d\,e-c\,f}{c+d\,x}\right)^2\left(\frac{\text{ArcCsch}[c+d\,x]}{f+\frac{d\,e}{c+d\,x}-\frac{c\,f}{c+d\,x}}-\frac{2\,\text{ArcTan}\left[\frac{d\,e-c\,f-f\,\text{Tanh}\left[\frac{1}{2}\,\text{ArcCsch}[c+d\,x]\right]}{\sqrt{-d^2\,e^2+2\,c\,d\,e\,f-(1+c^2)\,f^2}}\right]}{\sqrt{-d^2\,e^2+2\,c\,d\,e\,f-(1+c^2)\,f^2}}\right)}{d\,(-d\,e+c\,f)\,(e+f\,x)^2}-$$

$$\frac{1}{d\,(e+f\,x)^2}\,b^2\,(c+d\,x)^2\left(f+\frac{d\,e-c\,f}{c+d\,x}\right)^2\left(\frac{\text{ArcCsch}[c+d\,x]^2}{(-d\,e+c\,f)\left(f+\frac{d\,e}{c+d\,x}-\frac{c\,f}{c+d\,x}\right)}+\frac{1}{d\,e-c\,f}\,2\left(-\frac{i\,\pi\,\text{ArcTanh}\left[\frac{-d\,e+c\,f+f\,\text{Tanh}\left[\frac{1}{2}\,\text{ArcCsch}[c+d\,x]\right]}{\sqrt{f^2+(d\,e-c\,f)^2}}\right]}{\sqrt{f^2+(d\,e-c\,f)^2}}-\right.\right.$$

$$\frac{1}{\sqrt{-d^2\,e^2+2\,c\,d\,e\,f-f^2-c^2\,f^2}}\left(2\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d\,x]\right)\text{ArcTanh}\left[\frac{(f-i\,(d\,e-c\,f))\,\text{Cot}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d\,x]\right)\right]}{\sqrt{-d^2\,e^2+2\,c\,d\,e\,f-f^2-c^2\,f^2}}\right]-\right.$$

$$2\,\text{ArcCos}\left[-\frac{i\,f}{d\,e-c\,f}\right]\text{ArcTanh}\left[\frac{(-f-i\,(d\,e-c\,f))\,\text{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d\,x]\right)\right]}{\sqrt{-d^2\,e^2+2\,c\,d\,e\,f-f^2-c^2\,f^2}}\right]+$$

$$\left(\text{ArcCos}\left[-\frac{i\,f}{d\,e-c\,f}\right]-2\,i\left(\text{ArcTanh}\left[\frac{\left(f-i\left(d\,e-c\,f\right)\right)\text{Cot}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d\,x]\right)\right]}{\sqrt{-d^2\,e^2+2\,c\,d\,e\,f-f^2-c^2\,f^2}}\right]-\right.\right.$$

$$\left.\left.\text{ArcTanh}\left[\frac{\left(-f-i\left(d\,e-c\,f\right)\right)\text{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d\,x]\right)\right]}{\sqrt{-d^2\,e^2+2\,c\,d\,e\,f-f^2-c^2\,f^2}}\right]\right)\right)\text{Log}\left[\frac{e^{-\frac{1}{2}i\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d\,x]\right)}\sqrt{-d^2\,e^2+2\,c\,d\,e\,f-f^2-c^2\,f^2}}{\sqrt{2}\sqrt{-i\left(d\,e-c\,f\right)}\sqrt{f+\frac{d\,e-c\,f}{c+d\,x}}}\right]+$$

$$\left(\text{ArcCos}\left[-\frac{i\,f}{d\,e-c\,f}\right]+2\,i\left(\text{ArcTanh}\left[\frac{\left(f-i\left(d\,e-c\,f\right)\right)\text{Cot}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d\,x]\right)\right]}{\sqrt{-d^2\,e^2+2\,c\,d\,e\,f-f^2-c^2\,f^2}}\right]-\text{ArcTanh}\left[\right.\right.\right.$$

$$\left.\left.\left.\frac{\left(-f-i\left(d\,e-c\,f\right)\right)\text{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d\,x]\right)\right]}{\sqrt{-d^2\,e^2+2\,c\,d\,e\,f-f^2-c^2\,f^2}}\right]\right)\right)\text{Log}\left[\frac{e^{\frac{1}{2}i\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d\,x]\right)}\sqrt{-d^2\,e^2+2\,c\,d\,e\,f-f^2-c^2\,f^2}}{\sqrt{2}\sqrt{-i\left(d\,e-c\,f\right)}\sqrt{f+\frac{d\,e-c\,f}{c+d\,x}}}\right]-$$

$$\left(\text{ArcCos}\left[-\frac{i\,f}{d\,e-c\,f}\right]+2\,i\,\text{ArcTanh}\left[\frac{\left(-f-i\left(d\,e-c\,f\right)\right)\text{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d\,x]\right)\right]}{\sqrt{-d^2\,e^2+2\,c\,d\,e\,f-f^2-c^2\,f^2}}\right]\right)\text{Log}\left[\right.$$

$$1-\left(i\left(f-i\sqrt{-d^2\,e^2+2\,c\,d\,e\,f-f^2-c^2\,f^2}\right)\left(f-i\left(d\,e-c\,f\right)-\sqrt{-d^2\,e^2+2\,c\,d\,e\,f-f^2-c^2\,f^2}\;\text{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d\,x]\right)\right]\right)\right)\Big/$$

$$\left.\left(\left(d\,e-c\,f\right)\left(f-i\left(d\,e-c\,f\right)+\sqrt{-d^2\,e^2+2\,c\,d\,e\,f-f^2-c^2\,f^2}\;\text{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d\,x]\right)\right]\right)\right)\right]+$$

$$\left(-\text{ArcCos}\left[-\frac{i\,f}{d\,e-c\,f}\right]+2\,i\,\text{ArcTanh}\left[\frac{\left(-f-i\left(d\,e-c\,f\right)\right)\text{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d\,x]\right)\right]}{\sqrt{-d^2\,e^2+2\,c\,d\,e\,f-f^2-c^2\,f^2}}\right]\right)\text{Log}\left[\right.$$

$$1-\left(i\left(f+i\sqrt{-d^2\,e^2+2\,c\,d\,e\,f-f^2-c^2\,f^2}\right)\left(f-i\left(d\,e-c\,f\right)-\sqrt{-d^2\,e^2+2\,c\,d\,e\,f-f^2-c^2\,f^2}\;\text{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d\,x]\right)\right]\right)\right)\Big/$$

$$\left.\left(\left(d\,e-c\,f\right)\left(f-i\left(d\,e-c\,f\right)+\sqrt{-d^2\,e^2+2\,c\,d\,e\,f-f^2-c^2\,f^2}\;\text{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d\,x]\right)\right]\right)\right)\right]+i\left(\text{PolyLog}\left[2,\right.\right.$$

$$\left(i\left(f-i\sqrt{-d^2\,e^2+2\,c\,d\,e\,f-f^2-c^2\,f^2}\right)\left(f-i\left(d\,e-c\,f\right)-\sqrt{-d^2\,e^2+2\,c\,d\,e\,f-f^2-c^2\,f^2}\;\text{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d\,x]\right)\right]\right)\right)\Big/$$

$$\left.\left(\left(d\,e-c\,f\right)\left(f-i\left(d\,e-c\,f\right)+\sqrt{-d^2\,e^2+2\,c\,d\,e\,f-f^2-c^2\,f^2}\;\text{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d\,x]\right)\right]\right)\right)\right]-\text{PolyLog}\left[2,\right.$$

$$\left(i\left(f+i\sqrt{-d^2\,e^2+2\,c\,d\,e\,f-f^2-c^2\,f^2}\right)\left(f-i\left(d\,e-c\,f\right)-\sqrt{-d^2\,e^2+2\,c\,d\,e\,f-f^2-c^2\,f^2}\;\text{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d\,x]\right)\right]\right)\right)\Big/$$

$$\left((d e-c f)\left(f-i (d e-c f)+\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}\ \text{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\ \text{ArcCsch}[c+d x]\right)\right]\right)\right)\Bigg)\Bigg)\Bigg)$$

Problem 13: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(a+b\ \text{ArcCsch}[c+d x])^2}{(e+f x)^3}\, dx$$

Optimal (type 4, 1024 leaves, 23 steps):

$$-\frac{b d^2 f \sqrt{1+\frac{1}{(c+d x)^2}}\ (a+b\ \text{ArcCsch}[c+d x])}{(d e-c f)\left(d^2 e^2-2 c d e f+\left(1+c^2\right) f^2\right)\left(f+\frac{d e-c f}{c+d x}\right)}+\frac{d^2 (a+b\ \text{ArcCsch}[c+d x])^2}{2 f (d e-c f)^2}-\frac{(a+b\ \text{ArcCsch}[c+d x])^2}{2 f (e+f x)^2}+$$

$$\frac{b d^2 f^2 (a+b\ \text{ArcCsch}[c+d x])\ \text{Log}\left[1+\frac{e^{\text{ArcCsch}[c+d x]} (d e-c f)}{f-\sqrt{d^2 e^2-2 c d e f+\left(1+c^2\right) f^2}}\right]}{(d e-c f)^2\left(d^2 e^2-2 c d e f+\left(1+c^2\right) f^2\right)^{3/2}}-\frac{2 b d^2 (a+b\ \text{ArcCsch}[c+d x])\ \text{Log}\left[1+\frac{e^{\text{ArcCsch}[c+d x]} (d e-c f)}{f-\sqrt{d^2 e^2-2 c d e f+\left(1+c^2\right) f^2}}\right]}{(d e-c f)^2 \sqrt{d^2 e^2-2 c d e f+\left(1+c^2\right) f^2}}-$$

$$\frac{b d^2 f^2 (a+b\ \text{ArcCsch}[c+d x])\ \text{Log}\left[1+\frac{e^{\text{ArcCsch}[c+d x]} (d e-c f)}{f+\sqrt{d^2 e^2-2 c d e f+\left(1+c^2\right) f^2}}\right]}{(d e-c f)^2\left(d^2 e^2-2 c d e f+\left(1+c^2\right) f^2\right)^{3/2}}+\frac{2 b d^2 (a+b\ \text{ArcCsch}[c+d x])\ \text{Log}\left[1+\frac{e^{\text{ArcCsch}[c+d x]} (d e-c f)}{f+\sqrt{d^2 e^2-2 c d e f+\left(1+c^2\right) f^2}}\right]}{(d e-c f)^2 \sqrt{d^2 e^2-2 c d e f+\left(1+c^2\right) f^2}}+$$

$$\frac{b^2 d^2 f\ \text{Log}\left[f+\frac{d e-c f}{c+d x}\right]}{(d e-c f)^2\left(d^2 e^2-2 c d e f+\left(1+c^2\right) f^2\right)}+\frac{b^2 d^2 f^2\ \text{PolyLog}\left[2,-\frac{e^{\text{ArcCsch}[c+d x]} (d e-c f)}{f-\sqrt{d^2 e^2-2 c d e f+\left(1+c^2\right) f^2}}\right]}{(d e-c f)^2\left(d^2 e^2-2 c d e f+\left(1+c^2\right) f^2\right)^{3/2}}-\frac{2 b^2 d^2\ \text{PolyLog}\left[2,-\frac{e^{\text{ArcCsch}[c+d x]} (d e-c f)}{f-\sqrt{d^2 e^2-2 c d e f+\left(1+c^2\right) f^2}}\right]}{(d e-c f)^2 \sqrt{d^2 e^2-2 c d e f+\left(1+c^2\right) f^2}}-$$

$$\frac{b^2 d^2 f^2\ \text{PolyLog}\left[2,-\frac{e^{\text{ArcCsch}[c+d x]} (d e-c f)}{f+\sqrt{d^2 e^2-2 c d e f+\left(1+c^2\right) f^2}}\right]}{(d e-c f)^2\left(d^2 e^2-2 c d e f+\left(1+c^2\right) f^2\right)^{3/2}}+\frac{2 b^2 d^2\ \text{PolyLog}\left[2,-\frac{e^{\text{ArcCsch}[c+d x]} (d e-c f)}{f+\sqrt{d^2 e^2-2 c d e f+\left(1+c^2\right) f^2}}\right]}{(d e-c f)^2 \sqrt{d^2 e^2-2 c d e f+\left(1+c^2\right) f^2}}$$

Result (type 4, 8348 leaves):

$$-\frac{a^2}{2 f (e+f x)^2}-\frac{1}{d (d e-c f)^2 (e+f x)^3} a b (d e-c f+f (c+d x))^3$$

$$\left(\frac{\frac{f\,(d\,e-c\,f)\,\sqrt{1+\frac{1}{(c+d\,x)^2}}}{d^2\,e^2-2\,c\,d\,e\,f+(1+c^2)\,f^2}-2\,\text{ArcCsch}[c+d\,x]}{f+\frac{d\,e-c\,f}{c+d\,x}}+\frac{f\,\text{ArcCsch}[c+d\,x]}{\left(f+\frac{d\,e-c\,f}{c+d\,x}\right)^2}-\frac{2\,\left(2\,d^2\,e^2-4\,c\,d\,e\,f+(1+2\,c^2)\,f^2\right)\,\text{ArcTan}\left[\frac{d\,e-c\,f-f\,\text{Tanh}\left[\frac{1}{2}\,\text{ArcCsch}[c+d\,x]\right]}{\sqrt{-d^2\,e^2+2\,c\,d\,e\,f-(1+c^2)\,f^2}}\right]}{\left(-d^2\,e^2+2\,c\,d\,e\,f-(1+c^2)\,f^2\right)^{3/2}}\right)-$$

$$\frac{1}{d\,(e+f\,x)^3}\,b^2\,\left(d\,e-c\,f+f\,(c+d\,x)\right)^3\,\left(\frac{f\,(c+d\,x)^3\,\left(f+\frac{d\,e}{c+d\,x}-\frac{c\,f}{c+d\,x}\right)^3\,\text{ArcCsch}[c+d\,x]^2}{2\,(d\,e-c\,f)^2\,\left(-f-\frac{d\,e}{c+d\,x}+\frac{c\,f}{c+d\,x}\right)^2\,\left(d\,e-c\,f+f\,(c+d\,x)\right)^3}+\right.$$

$$\left((c+d\,x)^3\,\left(f+\frac{d\,e}{c+d\,x}-\frac{c\,f}{c+d\,x}\right)^3\,\left(-d\,e\,f\,\sqrt{1+\frac{1}{(c+d\,x)^2}}\,\text{ArcCsch}[c+d\,x]+c\,f^2\,\sqrt{1+\frac{1}{(c+d\,x)^2}}\,\text{ArcCsch}[c+d\,x]+\right.\right.$$

$$\left.\left.d^2\,e^2\,\text{ArcCsch}[c+d\,x]^2-2\,c\,d\,e\,f\,\text{ArcCsch}[c+d\,x]^2+f^2\,\text{ArcCsch}[c+d\,x]^2+c^2\,f^2\,\text{ArcCsch}[c+d\,x]^2\right)\right)\Big/$$

$$\left((d\,e-c\,f)^2\,\left(d^2\,e^2-2\,c\,d\,e\,f+f^2+c^2\,f^2\right)\,\left(-f-\frac{d\,e}{c+d\,x}+\frac{c\,f}{c+d\,x}\right)\,\left(d\,e-c\,f+f\,(c+d\,x)\right)^3\right)+$$

$$\frac{d\,e\,f\,(c+d\,x)^3\,\left(f+\frac{d\,e}{c+d\,x}-\frac{c\,f}{c+d\,x}\right)^3\,\text{Log}\left[1+\frac{d\,e-c\,f}{f\,(c+d\,x)}\right]}{(d\,e-c\,f)^2\,(-d\,e+c\,f)\,\left(d^2\,e^2-2\,c\,d\,e\,f+f^2+c^2\,f^2\right)\,\left(d\,e-c\,f+f\,(c+d\,x)\right)^3}-$$

$$\frac{c\,f^2\,(c+d\,x)^3\,\left(f+\frac{d\,e}{c+d\,x}-\frac{c\,f}{c+d\,x}\right)^3\,\text{Log}\left[1+\frac{d\,e-c\,f}{f\,(c+d\,x)}\right]}{(d\,e-c\,f)^2\,(-d\,e+c\,f)\,\left(d^2\,e^2-2\,c\,d\,e\,f+f^2+c^2\,f^2\right)\,\left(d\,e-c\,f+f\,(c+d\,x)\right)^3}+\frac{1}{(d\,e-c\,f)^2\,\left(d^2\,e^2-2\,c\,d\,e\,f+f^2+c^2\,f^2\right)\,\left(d\,e-c\,f+f\,(c+d\,x)\right)^3}$$

$$2\,d^2\,e^2\,(c+d\,x)^3\,\left(f+\frac{d\,e}{c+d\,x}-\frac{c\,f}{c+d\,x}\right)^3\,\left(-\frac{i\,\pi\,\text{ArcTanh}\left[\frac{-d\,e+c\,f+f\,\text{Tanh}\left[\frac{1}{2}\,\text{ArcCsch}[c+d\,x]\right]}{\sqrt{f^2+(d\,e-c\,f)^2}}\right]}{\sqrt{f^2+(d\,e-c\,f)^2}}-\right.$$

$$\frac{1}{\sqrt{-d^2\,e^2+2\,c\,d\,e\,f-f^2-c^2\,f^2}}\left(2\,\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d\,x]\right)\,\text{ArcTanh}\left[\frac{\left(f-i\,(d\,e-c\,f)\right)\,\text{Cot}\left[\frac{1}{2}\,\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d\,x]\right)\right]}{\sqrt{-d^2\,e^2+2\,c\,d\,e\,f-f^2-c^2\,f^2}}\right]-\right.$$

$$2\,\text{ArcCos}\left[-\frac{i\,f}{d\,e-c\,f}\right]\,\text{ArcTanh}\left[\frac{\left(-f-i\,(d\,e-c\,f)\right)\,\text{Tan}\left[\frac{1}{2}\,\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d\,x]\right)\right]}{\sqrt{-d^2\,e^2+2\,c\,d\,e\,f-f^2-c^2\,f^2}}\right]+$$

$$\left(\text{ArcCos}\left[-\frac{i f}{d e-c f}\right]-2 i\left(\text{ArcTanh}\left[\frac{(f-i(d e-c f))\,\text{Cot}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d x]\right)\right]}{\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}}\right]-\right.\right.$$

$$\left.\left.\text{ArcTanh}\left[\frac{(-f-i(d e-c f))\,\text{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d x]\right)\right]}{\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}}\right]\right)\right)\text{Log}\left[\frac{e^{-\frac{1}{2} i\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d x]\right)}\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}}{\sqrt{2}\sqrt{-i(d e-c f)}\sqrt{f+\frac{d e-c f}{c+d x}}}\right]+$$

$$\left(\text{ArcCos}\left[-\frac{i f}{d e-c f}\right]+2 i\left(\text{ArcTanh}\left[\frac{(f-i(d e-c f))\,\text{Cot}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d x]\right)\right]}{\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}}\right]-\text{ArcTanh}\left[\right.\right.\right.$$

$$\left.\left.\left.\frac{(-f-i(d e-c f))\,\text{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d x]\right)\right]}{\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}}\right]\right)\right)\text{Log}\left[\frac{e^{\frac{1}{2} i\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d x]\right)}\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}}{\sqrt{2}\sqrt{-i(d e-c f)}\sqrt{f+\frac{d e-c f}{c+d x}}}\right]-$$

$$\left(\text{ArcCos}\left[-\frac{i f}{d e-c f}\right]+2 i\,\text{ArcTanh}\left[\frac{(-f-i(d e-c f))\,\text{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d x]\right)\right]}{\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}}\right]\right)\text{Log}\Big[$$

$$1-\left(i\left(f-i\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}\right)\left(f-i(d e-c f)-\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}\,\text{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d x]\right)\right]\right)\right)\Big/$$

$$\left((d e-c f)\left(f-i(d e-c f)+\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}\,\text{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d x]\right)\right]\right)\right)\Big]+$$

$$\left(-\text{ArcCos}\left[-\frac{i f}{d e-c f}\right]+2 i\,\text{ArcTanh}\left[\frac{(-f-i(d e-c f))\,\text{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d x]\right)\right]}{\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}}\right]\right)\text{Log}\Big[$$

$$1-\left(i\left(f+i\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}\right)\left(f-i(d e-c f)-\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}\,\text{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d x]\right)\right]\right)\right)\Big/$$

$$\left((d e-c f)\left(f-i(d e-c f)+\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}\,\text{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d x]\right)\right]\right)\right)\Big]+i\Big(\text{PolyLog}\Big[2,$$

$$\left(i\left(f-i\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}\right)\left(f-i(d e-c f)-\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}\,\text{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d x]\right)\right]\right)\right)\Big/$$

$$\left((d e-c f)\left(f-i(d e-c f)+\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}\,\text{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d x]\right)\right]\right)\right)\Big]-\text{PolyLog}\Big[2,$$

$$\left(i\left(f+i\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}\right)\left(f-i(d e-c f)-\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}\,\text{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d x]\right)\right]\right)\right)\Big/$$

$$\left(\left(d e-c f\right)\left(f-\mathbb{i}\left(d e-c f\right)+\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}\ \mathrm{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-\mathbb{i}\,\mathrm{ArcCsch}[c+d x]\right)\right]\right)\right)\Bigg)\Bigg)\Bigg) -$$

$$\frac{1}{\left(d e-c f\right)^2\left(d^2 e^2-2 c d e f+f^2+c^2 f^2\right)\left(d e-c f+f\left(c+d x\right)\right)^3}\,4 c d e f\left(c+d x\right)^3\left(f+\frac{d e}{c+d x}-\frac{c f}{c+d x}\right)^3$$

$$\left(-\frac{\mathbb{i}\,\pi\,\mathrm{ArcTanh}\left[\frac{-d e+c f+f\,\mathrm{Tanh}\left[\frac{1}{2}\mathrm{ArcCsch}[c+d x]\right]}{\sqrt{f^2+(d e-c f)^2}}\right]}{\sqrt{f^2+\left(d e-c f\right)^2}}-\right.$$

$$\frac{1}{\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}}\left(2\left(\frac{\pi}{2}-\mathbb{i}\,\mathrm{ArcCsch}[c+d x]\right)\mathrm{ArcTanh}\left[\frac{\left(f-\mathbb{i}\left(d e-c f\right)\right)\mathrm{Cot}\left[\frac{1}{2}\left(\frac{\pi}{2}-\mathbb{i}\,\mathrm{ArcCsch}[c+d x]\right)\right]}{\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}}\right]-\right.$$

$$2\,\mathrm{ArcCos}\left[-\frac{\mathbb{i} f}{d e-c f}\right]\mathrm{ArcTanh}\left[\frac{\left(-f-\mathbb{i}\left(d e-c f\right)\right)\mathrm{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-\mathbb{i}\,\mathrm{ArcCsch}[c+d x]\right)\right]}{\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}}\right]+$$

$$\left(\mathrm{ArcCos}\left[-\frac{\mathbb{i} f}{d e-c f}\right]-2\,\mathbb{i}\left(\mathrm{ArcTanh}\left[\frac{\left(f-\mathbb{i}\left(d e-c f\right)\right)\mathrm{Cot}\left[\frac{1}{2}\left(\frac{\pi}{2}-\mathbb{i}\,\mathrm{ArcCsch}[c+d x]\right)\right]}{\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}}\right]-\right.\right.$$

$$\left.\left.\mathrm{ArcTanh}\left[\frac{\left(-f-\mathbb{i}\left(d e-c f\right)\right)\mathrm{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-\mathbb{i}\,\mathrm{ArcCsch}[c+d x]\right)\right]}{\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}}\right]\right)\right)\mathrm{Log}\left[\frac{e^{-\frac{1}{2}\mathbb{i}\left(\frac{\pi}{2}-\mathbb{i}\,\mathrm{ArcCsch}[c+d x]\right)}\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}}{\sqrt{2}\sqrt{-\mathbb{i}\left(d e-c f\right)}\sqrt{f+\frac{d e-c f}{c+d x}}}\right]+$$

$$\left(\mathrm{ArcCos}\left[-\frac{\mathbb{i} f}{d e-c f}\right]+2\,\mathbb{i}\left(\mathrm{ArcTanh}\left[\frac{\left(f-\mathbb{i}\left(d e-c f\right)\right)\mathrm{Cot}\left[\frac{1}{2}\left(\frac{\pi}{2}-\mathbb{i}\,\mathrm{ArcCsch}[c+d x]\right)\right]}{\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}}\right]-\mathrm{ArcTanh}\left[\right.\right.\right.$$

$$\left.\left.\left.\frac{\left(-f-\mathbb{i}\left(d e-c f\right)\right)\mathrm{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-\mathbb{i}\,\mathrm{ArcCsch}[c+d x]\right)\right]}{\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}}\right]\right)\right)\mathrm{Log}\left[\frac{e^{\frac{1}{2}\mathbb{i}\left(\frac{\pi}{2}-\mathbb{i}\,\mathrm{ArcCsch}[c+d x]\right)}\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}}{\sqrt{2}\sqrt{-\mathbb{i}\left(d e-c f\right)}\sqrt{f+\frac{d e-c f}{c+d x}}}\right]-$$

$$\left(\mathrm{ArcCos}\left[-\frac{\mathbb{i} f}{d e-c f}\right]+2\,\mathbb{i}\,\mathrm{ArcTanh}\left[\frac{\left(-f-\mathbb{i}\left(d e-c f\right)\right)\mathrm{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-\mathbb{i}\,\mathrm{ArcCsch}[c+d x]\right)\right]}{\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}}\right]\right)\mathrm{Log}\left[\right.$$

$$1-\left(\mathrm{i}\left(f-\mathrm{i}\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}\right)\left(f-\mathrm{i}(d e-c f)-\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}\,\mathrm{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-\mathrm{i}\,\mathrm{ArcCsch}[c+d x]\right)\right]\right)\right)\Big/$$

$$\left((d e-c f)\left(f-\mathrm{i}(d e-c f)+\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}\,\mathrm{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-\mathrm{i}\,\mathrm{ArcCsch}[c+d x]\right)\right]\right)\right)\Big]+$$

$$\left(-\mathrm{ArcCos}\left[-\frac{\mathrm{i} f}{d e-c f}\right]+2\mathrm{i}\,\mathrm{ArcTanh}\left[\frac{(-f-\mathrm{i}(d e-c f))\,\mathrm{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-\mathrm{i}\,\mathrm{ArcCsch}[c+d x]\right)\right]}{\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}}\right]\right)\mathrm{Log}\Big[$$

$$1-\left(\mathrm{i}\left(f+\mathrm{i}\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}\right)\left(f-\mathrm{i}(d e-c f)-\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}\,\mathrm{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-\mathrm{i}\,\mathrm{ArcCsch}[c+d x]\right)\right]\right)\right)\Big/$$

$$\left((d e-c f)\left(f-\mathrm{i}(d e-c f)+\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}\,\mathrm{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-\mathrm{i}\,\mathrm{ArcCsch}[c+d x]\right)\right]\right)\right)\Big]+\mathrm{i}\Big(\mathrm{PolyLog}\Big[2,$$

$$\left(\mathrm{i}\left(f-\mathrm{i}\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}\right)\left(f-\mathrm{i}(d e-c f)-\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}\,\mathrm{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-\mathrm{i}\,\mathrm{ArcCsch}[c+d x]\right)\right]\right)\right)\Big/$$

$$\left((d e-c f)\left(f-\mathrm{i}(d e-c f)+\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}\,\mathrm{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-\mathrm{i}\,\mathrm{ArcCsch}[c+d x]\right)\right]\right)\right)\Big]-\mathrm{PolyLog}\Big[2,$$

$$\left(\mathrm{i}\left(f+\mathrm{i}\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}\right)\left(f-\mathrm{i}(d e-c f)-\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}\,\mathrm{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-\mathrm{i}\,\mathrm{ArcCsch}[c+d x]\right)\right]\right)\right)\Big/$$

$$\left((d e-c f)\left(f-\mathrm{i}(d e-c f)+\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}\,\mathrm{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-\mathrm{i}\,\mathrm{ArcCsch}[c+d x]\right)\right]\right)\right)\Big]\Big)\Big)\Big)+$$

$$\frac{1}{(d e-c f)^2\left(d^2 e^2-2 c d e f+f^2+c^2 f^2\right)\left(d e-c f+f(c+d x)\right)^3}f^2(c+d x)^3\left(f+\frac{d e}{c+d x}-\frac{c f}{c+d x}\right)^3$$

$$\left(-\frac{\mathrm{i}\pi\,\mathrm{ArcTanh}\left[\frac{-d e+c f+f\,\mathrm{Tanh}\left[\frac{1}{2}\mathrm{ArcCsch}[c+d x]\right]}{\sqrt{f^2+(d e-c f)^2}}\right]}{\sqrt{f^2+(d e-c f)^2}}-\right.$$

$$\frac{1}{\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}}\left(2\left(\frac{\pi}{2}-\mathrm{i}\,\mathrm{ArcCsch}[c+d x]\right)\mathrm{ArcTanh}\left[\frac{(f-\mathrm{i}(d e-c f))\,\mathrm{Cot}\left[\frac{1}{2}\left(\frac{\pi}{2}-\mathrm{i}\,\mathrm{ArcCsch}[c+d x]\right)\right]}{\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}}\right]-\right.$$

$$2\,\mathrm{ArcCos}\left[-\frac{\mathrm{i} f}{d e-c f}\right]\mathrm{ArcTanh}\left[\frac{(-f-\mathrm{i}(d e-c f))\,\mathrm{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-\mathrm{i}\,\mathrm{ArcCsch}[c+d x]\right)\right]}{\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}}\right]+$$

$$\left(\text{ArcCos}\left[-\frac{i f}{d e-c f}\right]-2 i\left(\text{ArcTanh}\left[\frac{(f-i(d e-c f))\,\text{Cot}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d x]\right)\right]}{\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}}\right]-\right.\right.$$
$$\left.\left.\text{ArcTanh}\left[\frac{(-f-i(d e-c f))\,\text{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d x]\right)\right]}{\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}}\right]\right)\right)\text{Log}\left[\frac{e^{-\frac{1}{2} i\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d x]\right)}\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}}{\sqrt{2}\sqrt{-i(d e-c f)}\sqrt{f+\frac{d e-c f}{c+d x}}}\right]+$$

$$\left(\text{ArcCos}\left[-\frac{i f}{d e-c f}\right]+2 i\left(\text{ArcTanh}\left[\frac{(f-i(d e-c f))\,\text{Cot}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d x]\right)\right]}{\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}}\right]-\text{ArcTanh}\left[\right.\right.\right.$$
$$\left.\left.\left.\frac{(-f-i(d e-c f))\,\text{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d x]\right)\right]}{\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}}\right]\right)\right)\text{Log}\left[\frac{e^{\frac{1}{2} i\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d x]\right)}\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}}{\sqrt{2}\sqrt{-i(d e-c f)}\sqrt{f+\frac{d e-c f}{c+d x}}}\right]-$$

$$\left(\text{ArcCos}\left[-\frac{i f}{d e-c f}\right]+2 i\,\text{ArcTanh}\left[\frac{(-f-i(d e-c f))\,\text{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d x]\right)\right]}{\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}}\right]\right)\text{Log}\left[\right.$$
$$1-\left(i\left(f-i\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}\right)\left(f-i(d e-c f)-\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}\,\text{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d x]\right)\right]\right)\right)\Big/$$
$$\left.\left((d e-c f)\left(f-i(d e-c f)+\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}\,\text{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d x]\right)\right]\right)\right)\right]+$$

$$\left(-\text{ArcCos}\left[-\frac{i f}{d e-c f}\right]+2 i\,\text{ArcTanh}\left[\frac{(-f-i(d e-c f))\,\text{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d x]\right)\right]}{\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}}\right]\right)\text{Log}\left[\right.$$
$$1-\left(i\left(f+i\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}\right)\left(f-i(d e-c f)-\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}\,\text{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d x]\right)\right]\right)\right)\Big/$$
$$\left.\left((d e-c f)\left(f-i(d e-c f)+\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}\,\text{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d x]\right)\right]\right)\right)\right]+i\left(\text{PolyLog}\left[2,\right.\right.$$
$$\left(i\left(f-i\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}\right)\left(f-i(d e-c f)-\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}\,\text{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d x]\right)\right]\right)\right)\Big/$$
$$\left.\left((d e-c f)\left(f-i(d e-c f)+\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}\,\text{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d x]\right)\right]\right)\right)\right]-\text{PolyLog}\left[2,\right.$$
$$\left(i\left(f+i\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}\right)\left(f-i(d e-c f)-\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}\,\text{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\text{ArcCsch}[c+d x]\right)\right]\right)\right)\Big/$$

$$\left(\left(d\,e-c\,f\right)\left(f-i\left(d\,e-c\,f\right)+\sqrt{-d^2e^2+2\,c\,d\,e\,f-f^2-c^2f^2}\;\mathrm{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\mathrm{ArcCsch}[c+d\,x]\right)\right]\right)\right)\Bigg)\Bigg)\Bigg)+$$

$$\frac{1}{\left(d\,e-c\,f\right)^2\left(d^2e^2-2\,c\,d\,e\,f+f^2+c^2f^2\right)\left(d\,e-c\,f+f\left(c+d\,x\right)\right)^3}\,2\,c^2f^2\left(c+d\,x\right)^3\left(f+\frac{d\,e}{c+d\,x}-\frac{c\,f}{c+d\,x}\right)^3$$

$$\left(-\frac{i\,\pi\,\mathrm{ArcTanh}\left[\frac{-d\,e+c\,f+f\,\mathrm{Tanh}\left[\frac{1}{2}\mathrm{ArcCsch}[c+d\,x]\right]}{\sqrt{f^2+(d\,e-c\,f)^2}}\right]}{\sqrt{f^2+\left(d\,e-c\,f\right)^2}}-\right.$$

$$\frac{1}{\sqrt{-d^2e^2+2\,c\,d\,e\,f-f^2-c^2f^2}}\left(2\left(\frac{\pi}{2}-i\,\mathrm{ArcCsch}[c+d\,x]\right)\mathrm{ArcTanh}\left[\frac{\left(f-i\left(d\,e-c\,f\right)\right)\mathrm{Cot}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\mathrm{ArcCsch}[c+d\,x]\right)\right]}{\sqrt{-d^2e^2+2\,c\,d\,e\,f-f^2-c^2f^2}}\right]-\right.$$

$$2\,\mathrm{ArcCos}\left[-\frac{i\,f}{d\,e-c\,f}\right]\mathrm{ArcTanh}\left[\frac{\left(-f-i\left(d\,e-c\,f\right)\right)\mathrm{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\mathrm{ArcCsch}[c+d\,x]\right)\right]}{\sqrt{-d^2e^2+2\,c\,d\,e\,f-f^2-c^2f^2}}\right]+$$

$$\left(\mathrm{ArcCos}\left[-\frac{i\,f}{d\,e-c\,f}\right]-2\,i\left(\mathrm{ArcTanh}\left[\frac{\left(f-i\left(d\,e-c\,f\right)\right)\mathrm{Cot}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\mathrm{ArcCsch}[c+d\,x]\right)\right]}{\sqrt{-d^2e^2+2\,c\,d\,e\,f-f^2-c^2f^2}}\right]-\right.\right.$$

$$\left.\left.\mathrm{ArcTanh}\left[\frac{\left(-f-i\left(d\,e-c\,f\right)\right)\mathrm{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\mathrm{ArcCsch}[c+d\,x]\right)\right]}{\sqrt{-d^2e^2+2\,c\,d\,e\,f-f^2-c^2f^2}}\right]\right)\right)\mathrm{Log}\left[\frac{e^{-\frac{1}{2}i\left(\frac{\pi}{2}-i\,\mathrm{ArcCsch}[c+d\,x]\right)}\sqrt{-d^2e^2+2\,c\,d\,e\,f-f^2-c^2f^2}}{\sqrt{2}\,\sqrt{-i\left(d\,e-c\,f\right)}\,\sqrt{f+\frac{d\,e-c\,f}{c+d\,x}}}\right]+$$

$$\left(\mathrm{ArcCos}\left[-\frac{i\,f}{d\,e-c\,f}\right]+2\,i\left(\mathrm{ArcTanh}\left[\frac{\left(f-i\left(d\,e-c\,f\right)\right)\mathrm{Cot}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\mathrm{ArcCsch}[c+d\,x]\right)\right]}{\sqrt{-d^2e^2+2\,c\,d\,e\,f-f^2-c^2f^2}}\right]-\mathrm{ArcTanh}\left[\right.\right.\right.$$

$$\left.\left.\left.\frac{\left(-f-i\left(d\,e-c\,f\right)\right)\mathrm{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\mathrm{ArcCsch}[c+d\,x]\right)\right]}{\sqrt{-d^2e^2+2\,c\,d\,e\,f-f^2-c^2f^2}}\right]\right)\right)\mathrm{Log}\left[\frac{e^{\frac{1}{2}i\left(\frac{\pi}{2}-i\,\mathrm{ArcCsch}[c+d\,x]\right)}\sqrt{-d^2e^2+2\,c\,d\,e\,f-f^2-c^2f^2}}{\sqrt{2}\,\sqrt{-i\left(d\,e-c\,f\right)}\,\sqrt{f+\frac{d\,e-c\,f}{c+d\,x}}}\right]-$$

$$\left(\mathrm{ArcCos}\left[-\frac{i\,f}{d\,e-c\,f}\right]+2\,i\,\mathrm{ArcTanh}\left[\frac{\left(-f-i\left(d\,e-c\,f\right)\right)\mathrm{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\mathrm{ArcCsch}[c+d\,x]\right)\right]}{\sqrt{-d^2e^2+2\,c\,d\,e\,f-f^2-c^2f^2}}\right]\right)\mathrm{Log}\Big[$$

$$1-\left(i\left(f-i\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}\right)\left(f-i\,(d e-c f)-\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}\ \mathrm{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\mathrm{ArcCsch}[c+d x]\right)\right]\right)\right)\Big/$$

$$\left((d e-c f)\left(f-i\,(d e-c f)+\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}\ \mathrm{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\mathrm{ArcCsch}[c+d x]\right)\right]\right)\right)\Bigg]+$$

$$\left(-\mathrm{ArcCos}\left[-\frac{i f}{d e-c f}\right]+2\,i\,\mathrm{ArcTanh}\left[\frac{(-f-i\,(d e-c f))\ \mathrm{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\mathrm{ArcCsch}[c+d x]\right)\right]}{\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}}\right]\right)\mathrm{Log}\Bigg[$$

$$1-\left(i\left(f+i\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}\right)\left(f-i\,(d e-c f)-\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}\ \mathrm{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\mathrm{ArcCsch}[c+d x]\right)\right]\right)\right)\Big/$$

$$\left((d e-c f)\left(f-i\,(d e-c f)+\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}\ \mathrm{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\mathrm{ArcCsch}[c+d x]\right)\right]\right)\right)\Bigg]+i\,\Bigg(\mathrm{PolyLog}\Big[2,$$

$$\left(i\left(f-i\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}\right)\left(f-i\,(d e-c f)-\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}\ \mathrm{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\mathrm{ArcCsch}[c+d x]\right)\right]\right)\right)\Big/$$

$$\left((d e-c f)\left(f-i\,(d e-c f)+\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}\ \mathrm{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\mathrm{ArcCsch}[c+d x]\right)\right]\right)\right)\Big]-\mathrm{PolyLog}\Big[2,$$

$$\left(i\left(f+i\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}\right)\left(f-i\,(d e-c f)-\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}\ \mathrm{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\mathrm{ArcCsch}[c+d x]\right)\right]\right)\right)\Big/$$

$$\left((d e-c f)\left(f-i\,(d e-c f)+\sqrt{-d^2 e^2+2 c d e f-f^2-c^2 f^2}\ \mathrm{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2}-i\,\mathrm{ArcCsch}[c+d x]\right)\right]\right)\right)\Big]\Bigg)\Bigg)\Bigg)\Bigg)$$

Problem 23: Result unnecessarily involves higher level functions.

$$\int \frac{\mathrm{ArcCsch}[a\,x^n]}{x}\,dx$$

Optimal (type 4, 61 leaves, 7 steps):

$$\frac{\mathrm{ArcCsch}[a\,x^n]^2}{2\,n}-\frac{\mathrm{ArcCsch}[a\,x^n]\ \mathrm{Log}\left[1-e^{2\,\mathrm{ArcCsch}[a\,x^n]}\right]}{n}-\frac{\mathrm{PolyLog}\left[2,\ e^{2\,\mathrm{ArcCsch}[a\,x^n]}\right]}{2\,n}$$

Result (type 5, 64 leaves):

$$-\frac{x^{-n}\ \mathrm{HypergeometricPFQ}\left[\left\{\frac{1}{2},\frac{1}{2},\frac{1}{2}\right\},\left\{\frac{3}{2},\frac{3}{2}\right\},-\frac{x^{-2n}}{a^2}\right]}{a\,n}+\left(\mathrm{ArcCsch}[a\,x^n]-\mathrm{ArcSinh}\left[\frac{x^{-n}}{a}\right]\right)\mathrm{Log}[x]$$

Problem 25: Result more than twice size of optimal antiderivative.

$$\int \text{ArcCsch}\left[c\, e^{a+b x}\right] dx$$

Optimal (type 4, 77 leaves, 7 steps):

$$\frac{\text{ArcCsch}\left[c\, e^{a+b x}\right]^2}{2 b}-\frac{\text{ArcCsch}\left[c\, e^{a+b x}\right] \text{Log}\left[1-e^{2 \text{ArcCsch}\left[c\, e^{a+b x}\right]}\right]}{b}-\frac{\text{PolyLog}\left[2,\, e^{2 \text{ArcCsch}\left[c\, e^{a+b x}\right]}\right]}{2 b}$$

Result (type 4, 236 leaves):

$$x\, \text{ArcCsch}\left[c\, e^{a+b x}\right]+\frac{1}{8 b c \sqrt{1+\frac{e^{-2 (a+b x)}}{c^2}}} e^{-a-b x} \sqrt{1+c^2 e^{2 (a+b x)}} \left(\text{Log}\left[-c^2 e^{2 (a+b x)}\right]^2+\text{ArcTanh}\left[\sqrt{1+c^2 e^{2 (a+b x)}}\right] \left(-8 b x+4 \text{Log}\left[-c^2 e^{2 (a+b x)}\right]\right)-\right.$$
$$\left.4 \text{Log}\left[-c^2 e^{2 (a+b x)}\right] \text{Log}\left[\frac{1}{2} \left(1+\sqrt{1+c^2 e^{2 (a+b x)}}\right)\right]+2 \text{Log}\left[\frac{1}{2} \left(1+\sqrt{1+c^2 e^{2 (a+b x)}}\right)\right]^2-4 \text{PolyLog}\left[2,\, \frac{1}{2} \left(1-\sqrt{1+c^2 e^{2 (a+b x)}}\right)\right]\right)$$

Problem 38: Result unnecessarily involves higher level functions.

$$\int e^{\text{ArcCsch}\left[a x^2\right]} x^4\, dx$$

Optimal (type 4, 202 leaves, 8 steps):

$$-\frac{2 \sqrt{1+\frac{1}{a^2 x^4}}}{5 a^2 \left(a+\frac{1}{x^2}\right) x}+\frac{2 \sqrt{1+\frac{1}{a^2 x^4}}\, x}{5 a^2}+\frac{x^3}{3 a}+\frac{1}{5} \sqrt{1+\frac{1}{a^2 x^4}}\, x^5+$$
$$\frac{2 \sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)^2}} \left(a+\frac{1}{x^2}\right) \text{EllipticE}\left[2 \text{ArcCot}\left[\sqrt{a}\, x\right],\, \frac{1}{2}\right]}{5 a^{7/2} \sqrt{1+\frac{1}{a^2 x^4}}}-\frac{\sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)^2}} \left(a+\frac{1}{x^2}\right) \text{EllipticF}\left[2 \text{ArcCot}\left[\sqrt{a}\, x\right],\, \frac{1}{2}\right]}{5 a^{7/2} \sqrt{1+\frac{1}{a^2 x^4}}}$$

Result (type 5, 126 leaves):

$$-\frac{1}{15 a \left(a x^2\right)^{3/2}} 2 \sqrt{2}\, e^{-\text{ArcCsch}\left[a x^2\right]} \left(\frac{e^{\text{ArcCsch}\left[a x^2\right]}}{-1+e^{2 \text{ArcCsch}\left[a x^2\right]}}\right)^{5/2} x^3$$
$$\left(-1-2 e^{2 \text{ArcCsch}\left[a x^2\right]}-3 e^{4 \text{ArcCsch}\left[a x^2\right]}+\left(1-e^{2 \text{ArcCsch}\left[a x^2\right]}\right)^{5/2} \text{Hypergeometric2F1}\left[\frac{1}{2},\, \frac{3}{4},\, \frac{7}{4},\, e^{2 \text{ArcCsch}\left[a x^2\right]}\right]\right)$$

Problem 40: Result unnecessarily involves higher level functions.

$$\int e^{\text{ArcCsch}[a x^2]} x^2 \, dx$$

Optimal (type 4, 86 leaves, 5 steps):

$$\frac{x}{a}+\frac{1}{3}\sqrt{1+\frac{1}{a^2 x^4}}\, x^3-\frac{\sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)^2}}\left(a+\frac{1}{x^2}\right)\text{EllipticF}\left[2\,\text{ArcCot}\left[\sqrt{a}\,x\right],\frac{1}{2}\right]}{3\,a^{5/2}\sqrt{1+\frac{1}{a^2 x^4}}}$$

Result (type 5, 113 leaves):

$$-\frac{1}{3\,a\sqrt{a x^2}}2\sqrt{2}\,e^{-\text{ArcCsch}[a x^2]}\left(\frac{e^{\text{ArcCsch}[a x^2]}}{-1+e^{2\,\text{ArcCsch}[a x^2]}}\right)^{3/2}x\left(1-2\,e^{2\,\text{ArcCsch}[a x^2]}-\left(1-e^{2\,\text{ArcCsch}[a x^2]}\right)^{3/2}\text{Hypergeometric2F1}\left[\frac{1}{4},\frac{1}{2},\frac{5}{4},e^{2\,\text{ArcCsch}[a x^2]}\right]\right)$$

Problem 42: Result unnecessarily involves higher level functions.

$$\int e^{\text{ArcCsch}[a x^2]} \, dx$$

Optimal (type 4, 165 leaves, 7 steps):

$$-\frac{1}{a\,x}-\frac{2\sqrt{1+\frac{1}{a^2 x^4}}}{\left(a+\frac{1}{x^2}\right)x}+\sqrt{1+\frac{1}{a^2 x^4}}\,x+\frac{2\sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)^2}}\left(a+\frac{1}{x^2}\right)\text{EllipticE}\left[2\,\text{ArcCot}\left[\sqrt{a}\,x\right],\frac{1}{2}\right]}{a^{3/2}\sqrt{1+\frac{1}{a^2 x^4}}}-\frac{\sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)^2}}\left(a+\frac{1}{x^2}\right)\text{EllipticF}\left[2\,\text{ArcCot}\left[\sqrt{a}\,x\right],\frac{1}{2}\right]}{a^{3/2}\sqrt{1+\frac{1}{a^2 x^4}}}$$

Result (type 5, 96 leaves):

$$\frac{1}{3\sqrt{a x^2}}\sqrt{2}\,e^{\text{ArcCsch}[a x^2]}\sqrt{\frac{e^{\text{ArcCsch}[a x^2]}}{-1+e^{2\,\text{ArcCsch}[a x^2]}}}\,x\left(3-2\sqrt{1-e^{2\,\text{ArcCsch}[a x^2]}}\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{3}{4},\frac{7}{4},e^{2\,\text{ArcCsch}[a x^2]}\right]\right)$$

Problem 44: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{\text{ArcCsch}[a x^2]}}{x^2} \, dx$$

Optimal (type 4, 91 leaves, 5 steps):

$$-\frac{1}{3\,a\,x^3}-\frac{\sqrt{1+\frac{1}{a^2 x^4}}}{3\,x}-\frac{\sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)^2}}\,\left(a+\frac{1}{x^2}\right)\,\text{EllipticF}\left[2\,\text{ArcCot}\left[\sqrt{a}\,x\right],\frac{1}{2}\right]}{3\,\sqrt{a}\,\sqrt{1+\frac{1}{a^2 x^4}}}$$

Result (type 4, 95 leaves):

$$-\frac{\frac{1}{a}+\sqrt{1+\frac{1}{a^2 x^4}}\,x^2+\frac{2\,(-1)^{1/4}\,\sqrt{1+\frac{1}{a^2 x^4}}\,x^2\,\left(a\,x^2\right)^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[(-1)^{1/4}\,\sqrt{a\,x^2}\right],-1\right]}{\sqrt{1+a^2 x^4}}}{3\,x^3}$$

Problem 46: Result unnecessarily involves higher level functions.

$$\int \frac{e^{\text{ArcCsch}\left[a\,x^2\right]}}{x^4}\,dx$$

Optimal (type 4, 181 leaves, 7 steps):

$$-\frac{1}{5\,a\,x^5}-\frac{\sqrt{1+\frac{1}{a^2 x^4}}}{5\,x^3}-\frac{2\,a^2\,\sqrt{1+\frac{1}{a^2 x^4}}}{5\,\left(a+\frac{1}{x^2}\right)\,x}+$$

$$\frac{2\,\sqrt{a}\,\sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)^2}}\,\left(a+\frac{1}{x^2}\right)\,\text{EllipticE}\left[2\,\text{ArcCot}\left[\sqrt{a}\,x\right],\frac{1}{2}\right]}{5\,\sqrt{1+\frac{1}{a^2 x^4}}}-\frac{\sqrt{a}\,\sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)^2}}\,\left(a+\frac{1}{x^2}\right)\,\text{EllipticF}\left[2\,\text{ArcCot}\left[\sqrt{a}\,x\right],\frac{1}{2}\right]}{5\,\sqrt{1+\frac{1}{a^2 x^4}}}$$

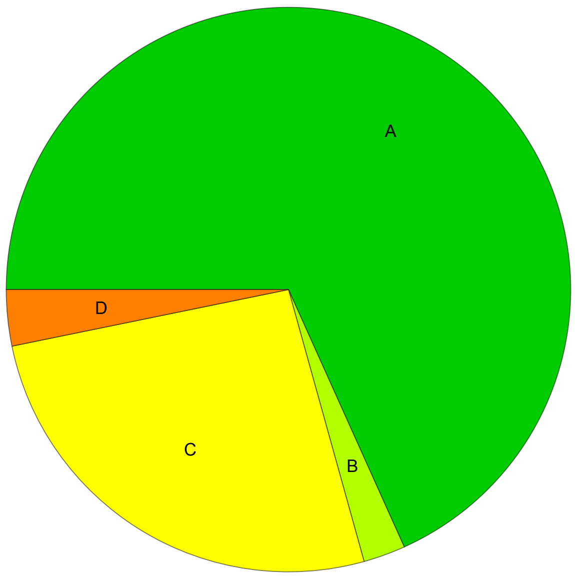
Result (type 5, 119 leaves):

$$-\frac{1}{10\,x^3}e^{-\text{ArcCsch}\left[a\,x^2\right]}\,\sqrt{\frac{e^{\text{ArcCsch}\left[a\,x^2\right]}}{-2+2\,e^{2\,\text{ArcCsch}\left[a\,x^2\right]}}}\,\left(a\,x^2\right)^{3/2}$$

$$\left(-3+2\,e^{2\,\text{ArcCsch}\left[a\,x^2\right]}+e^{4\,\text{ArcCsch}\left[a\,x^2\right]}+8\,\sqrt{1-e^{2\,\text{ArcCsch}\left[a\,x^2\right]}}\;\text{Hypergeometric2F1}\left[-\frac{1}{4},\frac{1}{2},\frac{3}{4},e^{2\,\text{ArcCsch}\left[a\,x^2\right]}\right]\right)$$

Summary of Integration Test Results

249 integration problems



A - 170 optimal antiderivatives

B - 6 more than twice size of optimal antiderivatives

C - 65 unnecessarily complex antiderivatives

D - 8 unable to integrate problems

E - 0 integration timeouts