

Mathematica 11.3 Integration Test Results

Test results for the 35 problems in "Bondarenko Problems.m"

Problem 1: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{2} + \cos[z] + \sin[z]} dz$$

Optimal (type 3, 22 leaves, 1 step) :

$$-\frac{1 - \sqrt{2} \sin[z]}{\cos[z] - \sin[z]}$$

Result (type 3, 77 leaves) :

$$\frac{-\left((1+3i) + \sqrt{2}\right) \cos\left[\frac{z}{2}\right] + \left((1+i) - i\sqrt{2}\right) \sin\left[\frac{z}{2}\right]}{\left((1+i) + \sqrt{2}\right) \cos\left[\frac{z}{2}\right] + i\left((-1-i) + \sqrt{2}\right) \sin\left[\frac{z}{2}\right]}$$

Problem 7: Result more than twice size of optimal antiderivative.

$$\int \frac{\log[1+x]}{x \sqrt{1+\sqrt{1+x}}} dx$$

Optimal (type 4, 291 leaves, ? steps):

$$\begin{aligned} & -8 \operatorname{ArcTanh}\left[\sqrt{1+\sqrt{1+x}}\right] - \frac{2 \log[1+x]}{\sqrt{1+\sqrt{1+x}}} - \sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{1+\sqrt{1+x}}}{\sqrt{2}}\right] \log[1+x] + \\ & 2 \sqrt{2} \operatorname{ArcTanh}\left[\frac{1}{\sqrt{2}}\right] \log\left[1 - \sqrt{1+\sqrt{1+x}}\right] - 2 \sqrt{2} \operatorname{ArcTanh}\left[\frac{1}{\sqrt{2}}\right] \log\left[1 + \sqrt{1+\sqrt{1+x}}\right] + \sqrt{2} \operatorname{PolyLog}[2, -\frac{\sqrt{2} \left(1 - \sqrt{1+\sqrt{1+x}}\right)}{2 - \sqrt{2}}] - \\ & \sqrt{2} \operatorname{PolyLog}[2, \frac{\sqrt{2} \left(1 - \sqrt{1+\sqrt{1+x}}\right)}{2 + \sqrt{2}}] - \sqrt{2} \operatorname{PolyLog}[2, -\frac{\sqrt{2} \left(1 + \sqrt{1+\sqrt{1+x}}\right)}{2 - \sqrt{2}}] + \sqrt{2} \operatorname{PolyLog}[2, \frac{\sqrt{2} \left(1 + \sqrt{1+\sqrt{1+x}}\right)}{2 + \sqrt{2}}] \end{aligned}$$

Result (type 4, 816 leaves):

$$\begin{aligned}
& -\frac{4 \left(2 + \text{Log}[1 + \sqrt{1+x}] \right)}{\sqrt{1+\sqrt{1+x}}} - 4 \left(-1 + \frac{1}{\sqrt{1+\sqrt{1+x}}} \right) \left(-1 + \text{Log}\left[-1 + \frac{1}{\sqrt{1+\sqrt{1+x}}} \right] \right) - 4 \left(1 + \frac{1}{\sqrt{1+\sqrt{1+x}}} \right) \left(-1 + \text{Log}\left[1 + \frac{1}{\sqrt{1+\sqrt{1+x}}} \right] \right) + \sqrt{2} \\
& \left(\text{Log}[1+x] - 2 \left(\text{Log}[1 + \sqrt{1+x}] + \text{Log}\left[-1 + \frac{1}{\sqrt{1+\sqrt{1+x}}} \right] + \text{Log}\left[1 + \frac{1}{\sqrt{1+\sqrt{1+x}}} \right] \right) \right) \left(\text{Log}\left[\sqrt{2} - \frac{2}{\sqrt{1+\sqrt{1+x}}} \right] - \text{Log}\left[\sqrt{2} + \frac{2}{\sqrt{1+\sqrt{1+x}}} \right] \right) - \\
& \frac{1}{2\sqrt{1+\sqrt{1+x}}} \left(\text{Log}[1+x] - 2 \left(\text{Log}[1 + \sqrt{1+x}] + \text{Log}\left[-1 + \frac{1}{\sqrt{1+\sqrt{1+x}}} \right] + \text{Log}\left[1 + \frac{1}{\sqrt{1+\sqrt{1+x}}} \right] \right) \right) \\
& \left(4 + \sqrt{2} \sqrt{1+\sqrt{1+x}} \text{Log}\left[\sqrt{2} - \frac{2}{\sqrt{1+\sqrt{1+x}}} \right] - \sqrt{2} \sqrt{1+\sqrt{1+x}} \text{Log}\left[\sqrt{2} + \frac{2}{\sqrt{1+\sqrt{1+x}}} \right] \right) + \\
& \sqrt{2} \left(-\text{Log}\left[1 + \sqrt{1+x} \right] \text{Log}\left[1 + \frac{\sqrt{2}}{\sqrt{1+\sqrt{1+x}}} \right] + 2 \text{PolyLog}\left[2, -\frac{\sqrt{2}}{\sqrt{1+\sqrt{1+x}}} \right] \right) + \\
& \sqrt{2} \left(\text{Log}\left[1 + \sqrt{1+x} \right] \text{Log}\left[1 - \frac{\sqrt{2}}{\sqrt{1+\sqrt{1+x}}} \right] - 2 \text{PolyLog}\left[2, \frac{\sqrt{2}}{\sqrt{1+\sqrt{1+x}}} \right] \right) - \\
& \sqrt{2} \left(\text{Log}\left[-1 + \frac{1}{\sqrt{1+\sqrt{1+x}}} \right] \text{Log}\left[\frac{\sqrt{2} + \frac{2}{\sqrt{1+\sqrt{1+x}}}}{2 + \sqrt{2}} \right] + \text{PolyLog}\left[2, \frac{2 - \frac{2}{\sqrt{1+\sqrt{1+x}}}}{2 + \sqrt{2}} \right] \right) + \\
& \sqrt{2} \left(\text{Log}\left[-1 + \frac{1}{\sqrt{1+\sqrt{1+x}}} \right] \text{Log}\left[1 + \frac{2 - \frac{2}{\sqrt{1+\sqrt{1+x}}}}{-2 + \sqrt{2}} \right] + \text{PolyLog}\left[2, \frac{2 \left(-1 + \frac{1}{\sqrt{1+\sqrt{1+x}}} \right)}{-2 + \sqrt{2}} \right] \right) - \\
& \sqrt{2} \left(\text{Log}\left[1 + \frac{1}{\sqrt{1+\sqrt{1+x}}} \right] \text{Log}\left[\frac{\sqrt{2} + \frac{2}{\sqrt{1+\sqrt{1+x}}}}{-2 + \sqrt{2}} \right] + \text{PolyLog}\left[2, -\frac{2 \left(1 + \frac{1}{\sqrt{1+\sqrt{1+x}}} \right)}{-2 + \sqrt{2}} \right] \right) + \\
& \sqrt{2} \left(\text{Log}\left[1 + \frac{1}{\sqrt{1+\sqrt{1+x}}} \right] \text{Log}\left[1 - \frac{2 \left(1 + \frac{1}{\sqrt{1+\sqrt{1+x}}} \right)}{2 + \sqrt{2}} \right] + \text{PolyLog}\left[2, \frac{2 \left(1 + \frac{1}{\sqrt{1+\sqrt{1+x}}} \right)}{2 + \sqrt{2}} \right] \right)
\end{aligned}$$

Problem 8: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1 + \sqrt{1 + x}} \log[1 + x]}{x} dx$$

Optimal (type 4, 308 leaves, ? steps):

$$\begin{aligned} & -16 \sqrt{1 + \sqrt{1 + x}} + 16 \operatorname{ArcTanh}\left[\sqrt{1 + \sqrt{1 + x}}\right] + 4 \sqrt{1 + \sqrt{1 + x}} \log[1 + x] - 2 \sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{1 + \sqrt{1 + x}}}{\sqrt{2}}\right] \log[1 + x] + \\ & 4 \sqrt{2} \operatorname{ArcTanh}\left[\frac{1}{\sqrt{2}}\right] \log[1 - \sqrt{1 + \sqrt{1 + x}}] - 4 \sqrt{2} \operatorname{ArcTanh}\left[\frac{1}{\sqrt{2}}\right] \log[1 + \sqrt{1 + \sqrt{1 + x}}] + 2 \sqrt{2} \operatorname{PolyLog}[2, -\frac{\sqrt{2} (1 - \sqrt{1 + \sqrt{1 + x}})}{2 - \sqrt{2}}] - \\ & 2 \sqrt{2} \operatorname{PolyLog}[2, \frac{\sqrt{2} (1 - \sqrt{1 + \sqrt{1 + x}})}{2 + \sqrt{2}}] - 2 \sqrt{2} \operatorname{PolyLog}[2, -\frac{\sqrt{2} (1 + \sqrt{1 + \sqrt{1 + x}})}{2 - \sqrt{2}}] + 2 \sqrt{2} \operatorname{PolyLog}[2, \frac{\sqrt{2} (1 + \sqrt{1 + \sqrt{1 + x}})}{2 + \sqrt{2}}] \end{aligned}$$

Result (type 4, 654 leaves):

$$\begin{aligned} & -16 \sqrt{1 + \sqrt{1 + x}} + 4 \sqrt{1 + \sqrt{1 + x}} \log[1 + x] + \sqrt{2} \log[1 + x] \log[\sqrt{2} - \sqrt{1 + \sqrt{1 + x}}] - 8 \log[-1 + \sqrt{1 + \sqrt{1 + x}}] - \\ & 2 \sqrt{2} \log[\sqrt{2} - \sqrt{1 + \sqrt{1 + x}}] \log[-1 + \sqrt{1 + \sqrt{1 + x}}] + 8 \log[1 + \sqrt{1 + \sqrt{1 + x}}] - 2 \sqrt{2} \log[\sqrt{2} - \sqrt{1 + \sqrt{1 + x}}] \log[1 + \sqrt{1 + \sqrt{1 + x}}] - \\ & \sqrt{2} \log[1 + x] \log[\sqrt{2} + \sqrt{1 + \sqrt{1 + x}}] + 2 \sqrt{2} \log[-1 + \sqrt{1 + \sqrt{1 + x}}] \log[\sqrt{2} + \sqrt{1 + \sqrt{1 + x}}] + \\ & 2 \sqrt{2} \log[1 + \sqrt{1 + \sqrt{1 + x}}] \log[\sqrt{2} + \sqrt{1 + \sqrt{1 + x}}] - 2 \sqrt{2} \log[-1 + \sqrt{1 + \sqrt{1 + x}}] \log[(1 - \sqrt{2}) (\sqrt{2} + \sqrt{1 + \sqrt{1 + x}})] - \\ & 2 \sqrt{2} \log[1 + \sqrt{1 + \sqrt{1 + x}}] \log[2 + \sqrt{2} + \sqrt{1 + \sqrt{1 + x}}] + \sqrt{2} \sqrt{1 + \sqrt{1 + x}} + \\ & 2 \sqrt{2} \log[-1 + \sqrt{1 + \sqrt{1 + x}}] \log[1 - (1 + \sqrt{2}) (-1 + \sqrt{1 + \sqrt{1 + x}})] + 2 \sqrt{2} \log[1 + \sqrt{1 + \sqrt{1 + x}}] \log[1 - (-1 + \sqrt{2}) (1 + \sqrt{1 + \sqrt{1 + x}})] - \\ & 2 \sqrt{2} \operatorname{PolyLog}[2, -(-1 + \sqrt{2}) (-1 + \sqrt{1 + \sqrt{1 + x}})] + 2 \sqrt{2} \operatorname{PolyLog}[2, (1 + \sqrt{2}) (-1 + \sqrt{1 + \sqrt{1 + x}})] + \\ & 2 \sqrt{2} \operatorname{PolyLog}[2, (-1 + \sqrt{2}) (1 + \sqrt{1 + \sqrt{1 + x}})] - 2 \sqrt{2} \operatorname{PolyLog}[2, -(1 + \sqrt{2}) (1 + \sqrt{1 + \sqrt{1 + x}})] \end{aligned}$$

Problem 9: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{1 + \sqrt{x + \sqrt{1 + x^2}}} dx$$

Optimal (type 3, 84 leaves, 4 steps):

$$-\frac{1}{2(x + \sqrt{1+x^2})} + \frac{1}{\sqrt{x + \sqrt{1+x^2}}} + \sqrt{x + \sqrt{1+x^2}} + \frac{1}{2} \operatorname{Log}[x + \sqrt{1+x^2}] - 2 \operatorname{Log}[1 + \sqrt{x + \sqrt{1+x^2}}]$$

Result (type 3, 347 leaves):

$$\begin{aligned} & \frac{1}{12} \left(6x - 6\sqrt{1+x^2} + 4(-2x + \sqrt{1+x^2})\sqrt{x + \sqrt{1+x^2}} - 12 \operatorname{Log}[x] + 6 \operatorname{Log}[1 + \sqrt{1+x^2}] + \frac{1}{1+x^2 + x\sqrt{1+x^2}} \right. \\ & \quad \left. 6\sqrt{1+x^2}(x + \sqrt{1+x^2}) \left(2\sqrt{x + \sqrt{1+x^2}} - 2 \operatorname{ArcTan}[\sqrt{x + \sqrt{1+x^2}}] + \operatorname{Log}[1 - \sqrt{x + \sqrt{1+x^2}}] - \operatorname{Log}[1 + \sqrt{x + \sqrt{1+x^2}}] \right) + \right. \\ & \quad \left. \frac{1}{(1+x^2 + x\sqrt{1+x^2})^2} 2(1+x^2)(x + \sqrt{1+x^2})^{3/2} \left(4 + 2x^2 + 2x\sqrt{1+x^2} + 6\sqrt{x + \sqrt{1+x^2}} \operatorname{ArcTan}[\sqrt{x + \sqrt{1+x^2}}] + \right. \right. \\ & \quad \left. \left. 3\sqrt{x + \sqrt{1+x^2}} \operatorname{Log}[1 - \sqrt{x + \sqrt{1+x^2}}] - 3\sqrt{x + \sqrt{1+x^2}} \operatorname{Log}[1 + \sqrt{x + \sqrt{1+x^2}}] \right) \right) \end{aligned}$$

Problem 10: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1+x}}{x + \sqrt{1+\sqrt{1+x}}} dx$$

Optimal (type 3, 41 leaves, 6 steps):

$$2\sqrt{1+x} + \frac{8 \operatorname{Arctanh}\left[\frac{1+2\sqrt{1+\sqrt{1+x}}}{\sqrt{5}}\right]}{\sqrt{5}}$$

Result (type 3, 147 leaves):

$$\begin{aligned} & \frac{1}{5} \left(10\sqrt{1+x} - (-5 + \sqrt{5})\sqrt{2(3 + \sqrt{5})} \operatorname{Arctanh}\left[\sqrt{\frac{2}{3 - \sqrt{5}}} \sqrt{1 + \sqrt{1+x}}\right] + \right. \\ & \quad \left. 2\sqrt{\frac{2}{3 + \sqrt{5}}} (5 + \sqrt{5}) \operatorname{Arctanh}\left[\sqrt{\frac{2}{3 + \sqrt{5}}} \sqrt{1 + \sqrt{1+x}}\right] - 4\sqrt{5} \operatorname{Arctanh}\left[\frac{-1 + 2\sqrt{1+x}}{\sqrt{5}}\right] \right) \end{aligned}$$

Problem 12: Result more than twice size of optimal antiderivative.

$$\int \frac{x}{x + \sqrt{1 - \sqrt{1+x}}} dx$$

Optimal (type 3, 73 leaves, 6 steps):

$$2\sqrt{1+x} - 4\sqrt{1-\sqrt{1+x}} + \left(1-\sqrt{1+x}\right)^2 + \frac{8 \operatorname{Arctanh}\left[\frac{1+2\sqrt{1-\sqrt{1+x}}}{\sqrt{5}}\right]}{\sqrt{5}}$$

Result (type 3, 151 leaves):

$$x - 4\sqrt{1 - \sqrt{1 + x}} + 2(1 + \sqrt{5})\sqrt{\frac{2}{5(3 + \sqrt{5})}} \operatorname{Arctanh}\left[\frac{\sqrt{2 - 2\sqrt{1+x}}}{\sqrt{3+\sqrt{5}}}\right] +$$

$$(-1 + \sqrt{5})\sqrt{\frac{2}{5(3 + \sqrt{5})}} \operatorname{Arctanh}\left[\sqrt{2}\sqrt{\frac{-1 + \sqrt{1+x}}{-3 + \sqrt{5}}}\right] + \frac{4\operatorname{Arctanh}\left[\frac{1+2\sqrt{1+x}}{\sqrt{5}}\right]}{\sqrt{5}}$$

Problem 13: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{x + \sqrt{1+x}}}{\sqrt{1+x} (1+x^2)} dx$$

Optimal (type 3, 365 leaves, 20 steps):

$$-\frac{\frac{i}{2} \operatorname{ArcTan}\left[\frac{2+\sqrt{1-i}-\left(1-2 \sqrt{1-i}\right) \sqrt{1+x}}{2 \sqrt{i+\sqrt{1-i}} \sqrt{x+\sqrt{1+x}}}\right]}{2 \sqrt{\frac{1-i}{i+\sqrt{1-i}}}}+\frac{\frac{i}{2} \operatorname{ArcTan}\left[\frac{2+\sqrt{1+i}-\left(1-2 \sqrt{1+i}\right) \sqrt{1+x}}{2 \sqrt{-i+\sqrt{1+i}} \sqrt{x+\sqrt{1+x}}}\right]}{2 \sqrt{\frac{1+i}{i-\sqrt{1+i}}}}+\frac{\frac{i}{2} \operatorname{ArcTanh}\left[\frac{2-\sqrt{1-i}-\left(1+2 \sqrt{1-i}\right) \sqrt{1+x}}{2 \sqrt{-i+\sqrt{1-i}} \sqrt{x+\sqrt{1+x}}}\right]}{2 \sqrt{\frac{1-i}{i-\sqrt{1-i}}}}-\frac{\frac{i}{2} \operatorname{ArcTanh}\left[\frac{2-\sqrt{1+i}-\left(1+2 \sqrt{1+i}\right) \sqrt{1+x}}{2 \sqrt{i+\sqrt{1+i}} \sqrt{x+\sqrt{1+x}}}\right]}{2 \sqrt{\frac{1+i}{i+\sqrt{1+i}}}}$$

Result (type 3, 2177 leaves) :

$$\frac{1}{2 \sqrt{1 - \frac{x}{2}} \sqrt{\frac{x}{2} - \sqrt{1 - \frac{x}{2}}} \left(-\frac{x}{2} + \sqrt{1 - \frac{x}{2}} \right) \operatorname{ArcTan} \left[\left((-1 - 2 \frac{x}{2}) + (2 - 4 \frac{x}{2}) \sqrt{1 - \frac{x}{2}} - (6 - 6 \frac{x}{2}) \sqrt{1 + x} - (1 - 2 \frac{x}{2}) \sqrt{1 - \frac{x}{2}} \sqrt{1 + x} + 4 \frac{x}{2} (1 + x) + (1 + 3 \frac{x}{2}) \sqrt{1 - \frac{x}{2}} (1 + x) + (4 - 4 \frac{x}{2}) \sqrt{\frac{x}{2} - \sqrt{1 - \frac{x}{2}}} \sqrt{x + \sqrt{1 + x}} - 2 \sqrt{1 - \frac{x}{2}} \sqrt{\frac{x}{2} - \sqrt{1 - \frac{x}{2}}} \sqrt{x + \sqrt{1 + x}} - (2 - 2 \frac{x}{2}) \sqrt{\frac{x}{2} - \sqrt{1 - \frac{x}{2}}} \sqrt{1 + x} \sqrt{x + \sqrt{1 + x}} - \right. \right.}$$

$$\begin{aligned}
& \frac{4 \sqrt{1 - \frac{x}{2}} \sqrt{\frac{x}{2} - \sqrt{1 - \frac{x}{2}}} \sqrt{1 + x} \sqrt{x + \sqrt{1 + x}}}{\left(1 - (4 - 2 \frac{x}{2}) \sqrt{1 - \frac{x}{2}} - (2 - 2 \frac{x}{2}) \sqrt{1 + x} + \frac{4 \sqrt{1 + x}}{\sqrt{1 - \frac{x}{2}}} + (6 - 4 \frac{x}{2}) (1 + x) + 8 \sqrt{1 - \frac{x}{2}} (1 + x)\right)} + \\
& \frac{1}{2 \sqrt{1 - \frac{x}{2}}} \frac{i}{\sqrt{\frac{x}{2} + \sqrt{1 - \frac{x}{2}}}} \operatorname{ArcTan}\left[\left((1 + 2 \frac{x}{2}) + (2 - 4 \frac{x}{2}) \sqrt{1 - \frac{x}{2}} + (6 - 6 \frac{x}{2}) \sqrt{1 + x} - (1 - 2 \frac{x}{2}) \sqrt{1 - \frac{x}{2}} \sqrt{1 + x} - 4 \frac{x}{2} (1 + x) + (1 + 3 \frac{x}{2}) \sqrt{1 - \frac{x}{2}} (1 + x) - (4 - 4 \frac{x}{2}) \sqrt{\frac{x}{2} + \sqrt{1 - \frac{x}{2}}} \sqrt{x + \sqrt{1 + x}} - 2 \sqrt{1 - \frac{x}{2}} \sqrt{\frac{x}{2} + \sqrt{1 - \frac{x}{2}}} \sqrt{x + \sqrt{1 + x}} + (2 - 2 \frac{x}{2}) \sqrt{\frac{x}{2} + \sqrt{1 - \frac{x}{2}}} \sqrt{1 + x} \sqrt{x + \sqrt{1 + x}} - 4 \sqrt{1 - \frac{x}{2}} \sqrt{\frac{x}{2} + \sqrt{1 - \frac{x}{2}}} \sqrt{1 + x} \sqrt{x + \sqrt{1 + x}}\right)\right] - \\
& \frac{1}{2 \sqrt{1 + \frac{x}{2}} \sqrt{\frac{x}{2} - \sqrt{1 + \frac{x}{2}}}} \frac{i}{\sqrt{\frac{x}{2} + \sqrt{1 + \frac{x}{2}}}} \operatorname{ArcTan}\left[\left((-2 - \frac{x}{2}) + (4 - 2 \frac{x}{2}) \sqrt{1 + \frac{x}{2}} + (6 - 6 \frac{x}{2}) \sqrt{1 + x} - (2 - \frac{x}{2}) \sqrt{1 + \frac{x}{2}} \sqrt{1 + x} + 4 (1 + x) - (3 + \frac{x}{2}) \sqrt{1 + \frac{x}{2}} (1 + x) + 2 \frac{x}{2} \sqrt{1 + \frac{x}{2}} \sqrt{\frac{x}{2} - \sqrt{1 + \frac{x}{2}}} \sqrt{x + \sqrt{1 + x}} + 4 \frac{x}{2} \sqrt{1 + \frac{x}{2}} \sqrt{\frac{x}{2} - \sqrt{1 + \frac{x}{2}}} \sqrt{1 + x} \sqrt{x + \sqrt{1 + x}}\right)\right] - \\
& \frac{1}{2 \sqrt{1 + \frac{x}{2}}} \sqrt{\frac{x}{2} + \sqrt{1 + \frac{x}{2}}} \operatorname{ArcTan}\left[\left((2 + \frac{x}{2}) + (4 - 2 \frac{x}{2}) \sqrt{1 + \frac{x}{2}} - (6 - 6 \frac{x}{2}) \sqrt{1 + x} - (2 - \frac{x}{2}) \sqrt{1 + \frac{x}{2}} \sqrt{1 + x} - 4 (1 + x) - (3 + \frac{x}{2}) \sqrt{1 + \frac{x}{2}} (1 + x) + 2 \frac{x}{2} \sqrt{1 + \frac{x}{2}} \sqrt{\frac{x}{2} + \sqrt{1 + \frac{x}{2}}} \sqrt{x + \sqrt{1 + x}} + 4 \frac{x}{2} \sqrt{1 + \frac{x}{2}} \sqrt{\frac{x}{2} + \sqrt{1 + \frac{x}{2}}} \sqrt{1 + x} \sqrt{x + \sqrt{1 + x}}\right)\right] - \\
& \frac{\frac{i}{2} \sqrt{\frac{x}{2} + \sqrt{1 - \frac{x}{2}}} \operatorname{Log}\left[\left(\sqrt{1 - \frac{x}{2}} - \sqrt{1 + x}\right)^2\right]}{4 \sqrt{1 - \frac{x}{2}} \sqrt{\frac{x}{2} - \sqrt{1 - \frac{x}{2}}}} - \frac{\frac{i}{2} \sqrt{\frac{x}{2} + \sqrt{1 + \frac{x}{2}}} \operatorname{Log}\left[\left(\sqrt{1 + \frac{x}{2}} - \sqrt{1 + x}\right)^2\right]}{4 \sqrt{1 + \frac{x}{2}}} - \\
& \frac{\frac{i}{2} \left(-\frac{x}{2} + \sqrt{1 + \frac{x}{2}}\right) \operatorname{Log}\left[\left(\sqrt{1 + \frac{x}{2}} + \sqrt{1 + x}\right)^2\right]}{4 \sqrt{1 - \frac{x}{2}}} + \\
& \frac{\frac{i}{2} \sqrt{1 - \frac{x}{2}} \sqrt{\frac{x}{2} - \sqrt{1 - \frac{x}{2}}} \operatorname{Log}\left[\left(3 + 5 \frac{x}{2}\right) + \frac{4}{\sqrt{1 - \frac{x}{2}}}\right] - 8 \sqrt{1 + x} + (3 - 7 \frac{x}{2}) \sqrt{1 - \frac{x}{2}} \sqrt{1 + x} - (8 - 5 \frac{x}{2}) (1 + x) - \frac{4 (1 + x)}{\sqrt{1 - \frac{x}{2}}} - 2 (1 - \frac{x}{2})^{3/2} \sqrt{\frac{x}{2} - \sqrt{1 - \frac{x}{2}}} \sqrt{x + \sqrt{1 + x}} - 4 (1 - \frac{x}{2})^{3/2} \sqrt{\frac{x}{2} - \sqrt{1 - \frac{x}{2}}} \sqrt{1 + x} \sqrt{x + \sqrt{1 + x}}}{4 \sqrt{1 - \frac{x}{2}} \sqrt{\frac{x}{2} - \sqrt{1 - \frac{x}{2}}}}
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{4 \sqrt{1 - \frac{i}{x}}} \sqrt{\frac{i}{x} + \sqrt{1 - \frac{i}{x}}} \operatorname{Log} \left[(-3 - 5 \frac{i}{x}) + \frac{4}{\sqrt{1 - \frac{i}{x}}} + 8 \sqrt{1 + x} + (3 - 7 \frac{i}{x}) \sqrt{1 - \frac{i}{x}} \sqrt{1 + x} + (8 - 5 \frac{i}{x}) (1 + x) - \frac{4 (1 + x)}{\sqrt{1 - \frac{i}{x}}} - \right. \\
& \left. 2 (1 - \frac{i}{x})^{3/2} \sqrt{\frac{i}{x} + \sqrt{1 - \frac{i}{x}}} \sqrt{x + \sqrt{1 + x}} - 4 (1 - \frac{i}{x})^{3/2} \sqrt{\frac{i}{x} + \sqrt{1 - \frac{i}{x}}} \sqrt{1 + x} \sqrt{x + \sqrt{1 + x}} \right] + \frac{1}{4 \sqrt{1 + \frac{i}{x}} \sqrt{\frac{i}{x} - \sqrt{1 + \frac{i}{x}}}} \\
& \frac{i}{x} \left(-\frac{i}{x} + \sqrt{1 + \frac{i}{x}} \right) \operatorname{Log} \left[(-5 + 5 \frac{i}{x}) - (6 - 2 \frac{i}{x}) \sqrt{1 + \frac{i}{x}} + (1 + 3 \frac{i}{x}) \sqrt{1 + \frac{i}{x}} \sqrt{1 + x} - 5 (1 + x) + (6 - 2 \frac{i}{x}) \sqrt{1 + \frac{i}{x}} (1 + x) + \right. \\
& \left. 8 \sqrt{\frac{i}{x} - \sqrt{1 + \frac{i}{x}}} \sqrt{x + \sqrt{1 + x}} + \frac{4 \sqrt{\frac{i}{x} - \sqrt{1 + \frac{i}{x}}} \sqrt{x + \sqrt{1 + x}}}{\sqrt{1 + \frac{i}{x}}} - 4 \sqrt{\frac{i}{x} - \sqrt{1 + \frac{i}{x}}} \sqrt{1 + x} \sqrt{x + \sqrt{1 + x}} + \frac{8 \sqrt{\frac{i}{x} - \sqrt{1 + \frac{i}{x}}} \sqrt{1 + x} \sqrt{x + \sqrt{1 + x}}}{\sqrt{1 + \frac{i}{x}}} \right] + \\
& \frac{1}{4 \sqrt{1 + \frac{i}{x}}} \frac{i}{x} \sqrt{\frac{i}{x} + \sqrt{1 + \frac{i}{x}}} \operatorname{Log} \left[(5 - 5 \frac{i}{x}) - (6 - 2 \frac{i}{x}) \sqrt{1 + \frac{i}{x}} + (1 + 3 \frac{i}{x}) \sqrt{1 + \frac{i}{x}} \sqrt{1 + x} + 5 (1 + x) + (6 - 2 \frac{i}{x}) \sqrt{1 + \frac{i}{x}} (1 + x) - \right. \\
& \left. 8 \sqrt{\frac{i}{x} + \sqrt{1 + \frac{i}{x}}} \sqrt{x + \sqrt{1 + x}} + \frac{4 \sqrt{\frac{i}{x} + \sqrt{1 + \frac{i}{x}}} \sqrt{x + \sqrt{1 + x}}}{\sqrt{1 + \frac{i}{x}}} + 4 \sqrt{\frac{i}{x} + \sqrt{1 + \frac{i}{x}}} \sqrt{1 + x} \sqrt{x + \sqrt{1 + x}} + \frac{8 \sqrt{\frac{i}{x} + \sqrt{1 + \frac{i}{x}}} \sqrt{1 + x} \sqrt{x + \sqrt{1 + x}}}{\sqrt{1 + \frac{i}{x}}} \right]
\end{aligned}$$

Problem 14: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{x + \sqrt{1 + x}}}{1 + x^2} dx$$

Optimal (type 3, 337 leaves, 22 steps):

$$\begin{aligned}
& \frac{1}{2} \frac{i}{x} \sqrt{\frac{i}{x} + \sqrt{1 - \frac{i}{x}}} \operatorname{ArcTan} \left[\frac{2 + \sqrt{1 - \frac{i}{x}} - (1 - 2 \sqrt{1 - \frac{i}{x}}) \sqrt{1 + x}}{2 \sqrt{\frac{i}{x} + \sqrt{1 - \frac{i}{x}}} \sqrt{x + \sqrt{1 + x}}} \right] - \frac{1}{2} \frac{i}{x} \sqrt{-\frac{i}{x} + \sqrt{1 + \frac{i}{x}}} \operatorname{ArcTan} \left[\frac{2 + \sqrt{1 + \frac{i}{x}} - (1 - 2 \sqrt{1 + \frac{i}{x}}) \sqrt{1 + x}}{2 \sqrt{-\frac{i}{x} + \sqrt{1 + \frac{i}{x}}} \sqrt{x + \sqrt{1 + x}}} \right] + \\
& \frac{1}{2} \frac{i}{x} \sqrt{-\frac{i}{x} + \sqrt{1 - \frac{i}{x}}} \operatorname{ArcTanh} \left[\frac{2 - \sqrt{1 - \frac{i}{x}} - (1 + 2 \sqrt{1 - \frac{i}{x}}) \sqrt{1 + x}}{2 \sqrt{-\frac{i}{x} + \sqrt{1 - \frac{i}{x}}} \sqrt{x + \sqrt{1 + x}}} \right] - \frac{1}{2} \frac{i}{x} \sqrt{\frac{i}{x} + \sqrt{1 + \frac{i}{x}}} \operatorname{ArcTanh} \left[\frac{2 - \sqrt{1 + \frac{i}{x}} - (1 + 2 \sqrt{1 + \frac{i}{x}}) \sqrt{1 + x}}{2 \sqrt{\frac{i}{x} + \sqrt{1 + \frac{i}{x}}} \sqrt{x + \sqrt{1 + x}}} \right]
\end{aligned}$$

Result (type 3, 2581 leaves):

$$\begin{aligned}
& \frac{1}{2 \sqrt{1 - \frac{i}{x}} \sqrt{\frac{i}{x} - \sqrt{1 - \frac{i}{x}}}} \left((1 + \frac{i}{x}) + \sqrt{1 - \frac{i}{x}} \right) \\
& \operatorname{ArcTan} \left[\left((2 - 3 \frac{i}{x}) + (3 - \frac{i}{x}) \sqrt{1 - \frac{i}{x}} - 8 \sqrt{1 + x} - 5 \sqrt{1 - \frac{i}{x}} \sqrt{1 + x} + (2 + 5 \frac{i}{x}) (1 + x) + 5 \frac{i}{x} \sqrt{1 - \frac{i}{x}} (1 + x) + 4 \sqrt{\frac{i}{x} - \sqrt{1 - \frac{i}{x}}} \sqrt{x + \sqrt{1 + x}} + \right. \right. \\
& \left. \left. 2 \sqrt{1 - \frac{i}{x}} \sqrt{\frac{i}{x} - \sqrt{1 - \frac{i}{x}}} \sqrt{x + \sqrt{1 + x}} - (6 + 2 \frac{i}{x}) \sqrt{\frac{i}{x} - \sqrt{1 - \frac{i}{x}}} \sqrt{1 + x} \sqrt{x + \sqrt{1 + x}} - \frac{8 \sqrt{\frac{i}{x} - \sqrt{1 - \frac{i}{x}}} \sqrt{1 + x} \sqrt{x + \sqrt{1 + x}}}{\sqrt{1 - \frac{i}{x}}} \right) \right] /
\end{aligned}$$

$$\begin{aligned}
& \frac{\left((-4 + 7 \text{i}) - (6 - 2 \text{i}) \sqrt{1 - \text{i}} + (4 - 2 \text{i}) \sqrt{1 + x} + (6 - 2 \text{i}) \sqrt{1 - \text{i}} \sqrt{1 + x} + (10 + \text{i}) (1 + x) + (8 + 4 \text{i}) \sqrt{1 - \text{i}} (1 + x) \right) \left[\frac{1}{2 \sqrt{1 - \text{i}} \sqrt{\text{i} + \sqrt{1 - \text{i}}}} \left((-1 - \text{i}) + \sqrt{1 - \text{i}} \right) \right] + \\
& \quad \text{ArcTan} \left[\left((-2 + 3 \text{i}) + (3 - \text{i}) \sqrt{1 - \text{i}} + 8 \sqrt{1 + x} - 5 \sqrt{1 - \text{i}} \sqrt{1 + x} - (2 + 5 \text{i}) (1 + x) + 5 \text{i} \sqrt{1 - \text{i}} (1 + x) - 4 \sqrt{\text{i} + \sqrt{1 - \text{i}}} \sqrt{x + \sqrt{1 + x}} + \right. \right. \\
& \quad \left. \left. 2 \sqrt{1 - \text{i}} \sqrt{\text{i} + \sqrt{1 - \text{i}}} \sqrt{x + \sqrt{1 + x}} + (6 + 2 \text{i}) \sqrt{\text{i} + \sqrt{1 - \text{i}}} \sqrt{1 + x} \sqrt{x + \sqrt{1 + x}} - \frac{8 \sqrt{\text{i} + \sqrt{1 - \text{i}}} \sqrt{1 + x} \sqrt{x + \sqrt{1 + x}}}{\sqrt{1 - \text{i}}} \right) \right] / \\
& \quad \left((4 - 7 \text{i}) - (6 - 2 \text{i}) \sqrt{1 - \text{i}} - (4 - 2 \text{i}) \sqrt{1 + x} + (6 - 2 \text{i}) \sqrt{1 - \text{i}} \sqrt{1 + x} - (10 + \text{i}) (1 + x) + (8 + 4 \text{i}) \sqrt{1 - \text{i}} (1 + x) \right) \left[\frac{1}{2 \sqrt{1 + \text{i}} \sqrt{\text{i} - \sqrt{1 + \text{i}}}} \text{i} \left((-1 + \text{i}) + \sqrt{1 + \text{i}} \right) \text{ArcTan} \left[\left((1 + 8 \text{i}) - 5 (1 + \text{i})^{3/2} - (16 + 8 \text{i}) \sqrt{1 + x} + (10 + 5 \text{i}) \sqrt{1 + \text{i}} \sqrt{1 + x} + \right. \right. \right. \\
& \quad \left. \left. \left. (9 - 8 \text{i}) (1 + x) - (5 - 10 \text{i}) \sqrt{1 + \text{i}} (1 + x) - 4 \sqrt{\text{i} - \sqrt{1 + \text{i}}} \sqrt{x + \sqrt{1 + x}} + (4 - 2 \text{i}) \sqrt{1 + \text{i}} \sqrt{\text{i} - \sqrt{1 + \text{i}}} \sqrt{x + \sqrt{1 + x}} - \right. \right. \right. \\
& \quad \left. \left. \left. 8 \sqrt{\text{i} - \sqrt{1 + \text{i}}} \sqrt{1 + x} \sqrt{x + \sqrt{1 + x}} + (8 - 4 \text{i}) \sqrt{1 + \text{i}} \sqrt{\text{i} - \sqrt{1 + \text{i}}} \sqrt{1 + x} \sqrt{x + \sqrt{1 + x}} \right) \right] / \\
& \quad \left((9 + 20 \text{i}) - 12 (1 + \text{i})^{3/2} - (14 + 20 \text{i}) \sqrt{1 + x} + (22 + 12 \text{i}) \sqrt{1 + \text{i}} \sqrt{1 + x} + (6 - 15 \text{i}) (1 + x) + (2 + 12 \text{i}) \sqrt{1 + \text{i}} (1 + x) \right) \left[\frac{1}{2 \sqrt{1 + \text{i}} \sqrt{\text{i} + \sqrt{1 + \text{i}}}} \text{i} \left((1 - \text{i}) + \sqrt{1 + \text{i}} \right) \text{ArcTan} \left[\left((-1 - 8 \text{i}) - 5 (1 + \text{i})^{3/2} + (16 + 8 \text{i}) \sqrt{1 + x} + (10 + 5 \text{i}) \sqrt{1 + \text{i}} \sqrt{1 + x} - \right. \right. \right. \\
& \quad \left. \left. \left. (9 - 8 \text{i}) (1 + x) - (5 - 10 \text{i}) \sqrt{1 + \text{i}} (1 + x) + 4 \sqrt{\text{i} + \sqrt{1 + \text{i}}} \sqrt{x + \sqrt{1 + x}} + (4 - 2 \text{i}) \sqrt{1 + \text{i}} \sqrt{\text{i} + \sqrt{1 + \text{i}}} \sqrt{x + \sqrt{1 + x}} + \right. \right. \right. \\
& \quad \left. \left. \left. 8 \sqrt{\text{i} + \sqrt{1 + \text{i}}} \sqrt{1 + x} \sqrt{x + \sqrt{1 + x}} + (8 - 4 \text{i}) \sqrt{1 + \text{i}} \sqrt{\text{i} + \sqrt{1 + \text{i}}} \sqrt{1 + x} \sqrt{x + \sqrt{1 + x}} \right) \right] / \\
& \quad \left((-9 - 20 \text{i}) - 12 (1 + \text{i})^{3/2} + (14 + 20 \text{i}) \sqrt{1 + x} + (22 + 12 \text{i}) \sqrt{1 + \text{i}} \sqrt{1 + x} - (6 - 15 \text{i}) (1 + x) + (2 + 12 \text{i}) \sqrt{1 + \text{i}} (1 + x) \right) \left[\frac{\text{i} \left((1 + \text{i}) + \sqrt{1 - \text{i}} \right) \text{Log} \left[\left(\sqrt{1 - \text{i}} - \sqrt{1 + x} \right)^2 \right]}{4 \sqrt{1 - \text{i}} \sqrt{\text{i} - \sqrt{1 - \text{i}}}} + \frac{\left((1 - \text{i}) + \sqrt{1 + \text{i}} \right) \text{Log} \left[\left(\sqrt{1 + \text{i}} - \sqrt{1 + x} \right)^2 \right]}{4 \sqrt{1 + \text{i}} \sqrt{\text{i} + \sqrt{1 + \text{i}}}} \right. \\
& \quad \left. + \frac{\frac{\text{i} \left((-1 - \text{i}) + \sqrt{1 - \text{i}} \right) \text{Log} \left[\left(\sqrt{1 - \text{i}} + \sqrt{1 + x} \right)^2 \right]}{4 \sqrt{1 - \text{i}} \sqrt{\text{i} + \sqrt{1 - \text{i}}}} - \frac{\left((-1 + \text{i}) + \sqrt{1 + \text{i}} \right) \text{Log} \left[\left(\sqrt{1 + \text{i}} + \sqrt{1 + x} \right)^2 \right]}{4 \sqrt{1 + \text{i}} \sqrt{\text{i} - \sqrt{1 + \text{i}}}} \right. \\
& \quad \left. - \frac{1}{4 \sqrt{1 - \text{i}} \sqrt{\text{i} - \sqrt{1 - \text{i}}}} \right]
\end{aligned}$$

$$\begin{aligned}
& \frac{i}{4\sqrt{1-i}} \frac{1}{\sqrt{i+\sqrt{1-i}}} \left[\left((1+i) + \sqrt{1-i} \right) \operatorname{Log} \left[(5+17i) + 14i\sqrt{1-i} - (10+22i)\sqrt{1+x} + (5-19i)\sqrt{1-i}\sqrt{1+x} - (25+2i)(1+x) - \right. \right. \\
& \quad \left. \left. (15+9i)\sqrt{1-i}(1+x) - (4-4i)\sqrt{i-\sqrt{1-i}}\sqrt{x+\sqrt{1+x}} - (6-2i)\sqrt{1-i}\sqrt{i-\sqrt{1-i}}\sqrt{x+\sqrt{1+x}} - \right. \right. \\
& \quad \left. \left. (8-8i)\sqrt{i-\sqrt{1-i}}\sqrt{1+x}\sqrt{x+\sqrt{1+x}} - (12-4i)\sqrt{1-i}\sqrt{i-\sqrt{1-i}}\sqrt{1+x}\sqrt{x+\sqrt{1+x}} \right] - \right. \\
& \quad \left. \frac{1}{4\sqrt{1+i}} \frac{1}{\sqrt{i-\sqrt{1+i}}} \left[(-1-i) + \sqrt{1+i} \right] \operatorname{Log} \left[(-5-17i) + 14i\sqrt{1-i} + (10+22i)\sqrt{1+x} + (5-19i)\sqrt{1-i}\sqrt{1+x} + \right. \right. \\
& \quad \left. \left. (25+2i)(1+x) - (15+9i)\sqrt{1-i}(1+x) + (4-4i)\sqrt{i+\sqrt{1-i}}\sqrt{x+\sqrt{1+x}} - (6-2i)\sqrt{1-i}\sqrt{i+\sqrt{1-i}}\sqrt{x+\sqrt{1+x}} + \right. \right. \\
& \quad \left. \left. (8-8i)\sqrt{i+\sqrt{1-i}}\sqrt{1+x}\sqrt{x+\sqrt{1+x}} - (12-4i)\sqrt{1-i}\sqrt{i+\sqrt{1-i}}\sqrt{1+x}\sqrt{x+\sqrt{1+x}} \right] - \right. \\
& \quad \left. \frac{1}{4\sqrt{1+i}} \frac{1}{\sqrt{i-\sqrt{1+i}}} \left[(-1+i) + \sqrt{1+i} \right] \operatorname{Log} \left[(-3+5i) - (2+4i)\sqrt{1+i} + (2-2i)\sqrt{1+x} - (1-3i)\sqrt{1+i}\sqrt{1+x} - \right. \right. \\
& \quad \left. \left. (8+7i)(1+x) + (9+3i)\sqrt{1+i}(1+x) + (4+4i)\sqrt{i-\sqrt{1+i}}\sqrt{x+\sqrt{1+x}} - 2(1+i)^{3/2}\sqrt{i-\sqrt{1+i}}\sqrt{x+\sqrt{1+x}} - \right. \right. \\
& \quad \left. \left. (8+4i)\sqrt{i-\sqrt{1+i}}\sqrt{1+x}\sqrt{x+\sqrt{1+x}} + 8\sqrt{1+i}\sqrt{i-\sqrt{1+i}}\sqrt{1+x}\sqrt{x+\sqrt{1+x}} \right] - \right. \\
& \quad \left. \frac{1}{4\sqrt{1+i}} \frac{1}{\sqrt{i+\sqrt{1+i}}} \left[(1-i) + \sqrt{1+i} \right] \operatorname{Log} \left[(3-5i) - (2+4i)\sqrt{1+i} - (2-2i)\sqrt{1+x} - (1-3i)\sqrt{1+i}\sqrt{1+x} + \right. \right. \\
& \quad \left. \left. (8+7i)(1+x) + (9+3i)\sqrt{1+i}(1+x) - (4+4i)\sqrt{i+\sqrt{1+i}}\sqrt{x+\sqrt{1+x}} - 2(1+i)^{3/2}\sqrt{i+\sqrt{1+i}}\sqrt{x+\sqrt{1+x}} + \right. \right. \\
& \quad \left. \left. (8+4i)\sqrt{i+\sqrt{1+i}}\sqrt{1+x}\sqrt{x+\sqrt{1+x}} + 8\sqrt{1+i}\sqrt{i+\sqrt{1+i}}\sqrt{1+x}\sqrt{x+\sqrt{1+x}} \right] \right]
\end{aligned}$$

Problem 15: Unable to integrate problem.

$$\int \sqrt{1+\sqrt{x} + \sqrt{1+2\sqrt{x}+2x}} \, dx$$

Optimal (type 2, 77 leaves, 2 steps):

$$\frac{2\sqrt{1+\sqrt{x} + \sqrt{1+2\sqrt{x}+2x}}}{15\sqrt{x}} \left(2 + \sqrt{x} + 6x^{3/2} - (2 - \sqrt{x})\sqrt{1+2\sqrt{x}+2x} \right)$$

Result (type 8, 29 leaves):

$$\int \sqrt{1+\sqrt{x} + \sqrt{1+2\sqrt{x}+2x}} \, dx$$

Problem 16: Unable to integrate problem.

$$\int \sqrt{\sqrt{2} + \sqrt{x} + \sqrt{2 + 2\sqrt{2}\sqrt{x} + 2x}} dx$$

Optimal (type 2, 118 leaves, 3 steps):

$$\frac{1}{15\sqrt{x}} 2\sqrt{2} \sqrt{\sqrt{2} + \sqrt{x} + \sqrt{2}\sqrt{1 + \sqrt{2}\sqrt{x} + x}} \left(4 + \sqrt{2}\sqrt{x} + 3\sqrt{2}x^{3/2} - \sqrt{2}(2\sqrt{2} - \sqrt{x})\sqrt{1 + \sqrt{2}\sqrt{x} + x} \right)$$

Result (type 8, 38 leaves):

$$\int \sqrt{\sqrt{2} + \sqrt{x} + \sqrt{2 + 2\sqrt{2}\sqrt{x} + 2x}} dx$$

Problem 18: Unable to integrate problem.

$$\int \sqrt{\sqrt{1 + \frac{1}{x}} + \frac{1}{x}} dx$$

Optimal (type 3, 96 leaves, 7 steps):

$$\sqrt{\sqrt{1 + \frac{1}{x}} + \frac{1}{x}} x + \frac{1}{4} \text{ArcTan}\left[\frac{3 + \sqrt{1 + \frac{1}{x}}}{2\sqrt{\sqrt{1 + \frac{1}{x}} + \frac{1}{x}}}\right] - \frac{3}{4} \text{ArcTanh}\left[\frac{1 - 3\sqrt{1 + \frac{1}{x}}}{2\sqrt{\sqrt{1 + \frac{1}{x}} + \frac{1}{x}}}\right]$$

Result (type 8, 19 leaves):

$$\int \sqrt{\sqrt{1 + \frac{1}{x}} + \frac{1}{x}} dx$$

Problem 19: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1 + e^{-x}}}{-e^{-x} + e^x} dx$$

Optimal (type 3, 25 leaves, 6 steps):

$$-\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{1+e^{-x}}}{\sqrt{2}}\right]$$

Result (type 3, 112 leaves):

$$\frac{e^{x/2} \sqrt{1+e^{-x}} \left(\operatorname{Log}[1-e^{x/2}] - \operatorname{Log}[1+e^{x/2}] + \operatorname{Log}[1-e^{x/2}+\sqrt{2} \sqrt{1+e^x}] - \operatorname{Log}[1+e^{x/2}+\sqrt{2} \sqrt{1+e^x}]\right)}{\sqrt{2} \sqrt{1+e^x}}$$

Problem 20: Result more than twice size of optimal antiderivative.

$$\int \sqrt{1+e^{-x}} \operatorname{Csch}[x] dx$$

Optimal (type 3, 25 leaves, 7 steps):

$$-2 \sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{1+e^{-x}}}{\sqrt{2}}\right]$$

Result (type 3, 126 leaves):

$$\frac{1}{\sqrt{1+e^x}} \sqrt{2} e^{x/2} \sqrt{1+e^{-x}} \left(\operatorname{Log}[1-e^{-x/2}] + \operatorname{Log}[1+e^{-x/2}] - \operatorname{Log}[e^{-x/2} \left(-1+e^{x/2}+\sqrt{2} \sqrt{1+e^x}\right)] - \operatorname{Log}[e^{-x/2} \left(1+e^{x/2}+\sqrt{2} \sqrt{1+e^x}\right)]\right)$$

Problem 21: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{(\operatorname{Cos}[x]+\operatorname{Cos}[3 x])^5} dx$$

Optimal (type 3, 108 leaves, ? steps):

$$\begin{aligned} & -\frac{523}{256} \operatorname{ArcTanh}[\operatorname{Sin}[x]] + \frac{1483 \operatorname{ArcTanh}[\sqrt{2} \operatorname{Sin}[x]]}{512 \sqrt{2}} + \frac{\operatorname{Sin}[x]}{32 (1-2 \operatorname{Sin}[x]^2)^4} - \\ & \frac{17 \operatorname{Sin}[x]}{192 (1-2 \operatorname{Sin}[x]^2)^3} + \frac{203 \operatorname{Sin}[x]}{768 (1-2 \operatorname{Sin}[x]^2)^2} - \frac{437 \operatorname{Sin}[x]}{512 (1-2 \operatorname{Sin}[x]^2)} - \frac{43}{256} \operatorname{Sec}[x] \operatorname{Tan}[x] - \frac{1}{128} \operatorname{Sec}[x]^3 \operatorname{Tan}[x] \end{aligned}$$

Result (type 3, 478 leaves):

$$\begin{aligned}
& - \frac{1483 \operatorname{ArcTan} \left[\frac{\cos \left[\frac{x}{2} \right] - \sin \left[\frac{x}{2} \right] - \sqrt{2} \sin \left[\frac{x}{2} \right]}{-\cos \left[\frac{x}{2} \right] + \sqrt{2} \cos \left[\frac{x}{2} \right] - \sin \left[\frac{x}{2} \right]} \right] + \left(\frac{1483}{2048} + \frac{1483 i}{2048} \right) \left((-1 - i) + \sqrt{2} \right) \operatorname{ArcTan} \left[\frac{\cos \left[\frac{x}{2} \right] + \sin \left[\frac{x}{2} \right] - \sqrt{2} \sin \left[\frac{x}{2} \right]}{\cos \left[\frac{x}{2} \right] + \sqrt{2} \cos \left[\frac{x}{2} \right] - \sin \left[\frac{x}{2} \right]} \right] \\
& - \frac{1024 \sqrt{2}}{(-1 + i) + \sqrt{2}} + \\
& \frac{523}{256} \log \left[\cos \left[\frac{x}{2} \right] - \sin \left[\frac{x}{2} \right] \right] - \frac{523}{256} \log \left[\cos \left[\frac{x}{2} \right] + \sin \left[\frac{x}{2} \right] \right] + \frac{1483 \log \left[\sqrt{2} + 2 \sin[x] \right]}{1024 \sqrt{2}} - \frac{1483 \log \left[2 - \sqrt{2} \cos[x] - \sqrt{2} \sin[x] \right]}{2048 \sqrt{2}} + \\
& \left(\frac{1483}{4096} - \frac{1483 i}{4096} \right) \left((-1 - i) + \sqrt{2} \right) \log \left[2 + \sqrt{2} \cos[x] - \sqrt{2} \sin[x] \right] - \frac{1}{512 \left(\cos \left[\frac{x}{2} \right] - \sin \left[\frac{x}{2} \right] \right)^4} - \frac{43}{512 \left(\cos \left[\frac{x}{2} \right] - \sin \left[\frac{x}{2} \right] \right)^2} + \\
& \frac{1}{512 \left(\cos \left[\frac{x}{2} \right] + \sin \left[\frac{x}{2} \right] \right)^4} + \frac{43}{512 \left(\cos \left[\frac{x}{2} \right] + \sin \left[\frac{x}{2} \right] \right)^2} - \frac{17}{768 \left(\cos[x] - \sin[x] \right)^3} - \frac{437}{1024 \left(\cos[x] - \sin[x] \right)} + \frac{\sin[x]}{128 \left(\cos[x] - \sin[x] \right)^4} + \\
& \frac{83 \sin[x]}{512 \left(\cos[x] - \sin[x] \right)^2} + \frac{\sin[x]}{128 \left(\cos[x] + \sin[x] \right)^4} + \frac{17}{768 \left(\cos[x] + \sin[x] \right)^3} + \frac{83 \sin[x]}{512 \left(\cos[x] + \sin[x] \right)^2} + \frac{437}{1024 \left(\cos[x] + \sin[x] \right)}
\end{aligned}$$

Problem 24: Result more than twice size of optimal antiderivative.

$$\int \frac{\tanh[x]}{\sqrt{e^x + e^{2x}}} dx$$

Optimal (type 3, 110 leaves, ? steps):

$$2 e^{-x} \sqrt{e^x + e^{2x}} - \frac{\operatorname{ArcTan} \left[\frac{i - (1 - 2i) e^x}{2 \sqrt{1+i} \sqrt{e^x + e^{2x}}} \right]}{\sqrt{1+i}} + \frac{\operatorname{ArcTan} \left[\frac{i + (1+2i) e^x}{2 \sqrt{1-i} \sqrt{e^x + e^{2x}}} \right]}{\sqrt{1-i}}$$

Result (type 3, 444 leaves):

$$\begin{aligned}
& \frac{1}{2 \sqrt{e^x (1 + e^x)}} \\
& \left(4 + 4 e^x + (1 + i)^{3/2} e^{x/2} \sqrt{1 + e^x} \log \left[(-1)^{1/4} - e^{-x/2} \right] + (1 - i)^{3/2} e^{x/2} \sqrt{1 + e^x} \log \left[-(-1)^{3/4} - e^{-x/2} \right] + (1 + i)^{3/2} e^{x/2} \sqrt{1 + e^x} \log \left[(-1)^{1/4} + e^{-x/2} \right] + \right. \\
& (1 - i)^{3/2} e^{x/2} \sqrt{1 + e^x} \log \left[-(-1)^{3/4} + e^{-x/2} \right] - (1 - i)^{3/2} e^{x/2} \sqrt{1 + e^x} \log \left[e^{-x/2} \left(-(-1)^{3/4} + e^{x/2} + \sqrt{1-i} \sqrt{1 + e^x} \right) \right] - \\
& (1 - i)^{3/2} e^{x/2} \sqrt{1 + e^x} \log \left[e^{-x/2} \left((-1)^{3/4} + e^{x/2} + \sqrt{1-i} \sqrt{1 + e^x} \right) \right] - (1 + i)^{3/2} e^{x/2} \sqrt{1 + e^x} \log \left[e^{-x/2} \left(-(-1)^{1/4} + e^{x/2} + \sqrt{1+i} \sqrt{1 + e^x} \right) \right] - \\
& \left. (1 + i)^{3/2} e^{x/2} \sqrt{1 + e^x} \log \left[e^{-x/2} \left((-1)^{1/4} + e^{x/2} + \sqrt{1+i} \sqrt{1 + e^x} \right) \right] \right)
\end{aligned}$$

Problem 26: Unable to integrate problem.

$$\int \text{Log}[x^2 + \sqrt{1 - x^2}] \, dx$$

Optimal (type 3, 185 leaves, ? steps):

$$\begin{aligned} & -2x - \text{ArcSin}[x] + \sqrt{\frac{1}{2}(1+\sqrt{5})} \text{ArcTan}\left[\sqrt{\frac{2}{1+\sqrt{5}}}x\right] + \sqrt{\frac{1}{2}(1+\sqrt{5})} \text{ArcTan}\left[\frac{\sqrt{\frac{1}{2}(1+\sqrt{5})}x}{\sqrt{1-x^2}}\right] + \\ & \sqrt{\frac{1}{2}(-1+\sqrt{5})} \text{ArcTanh}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}x\right] - \sqrt{\frac{1}{2}(-1+\sqrt{5})} \text{ArcTanh}\left[\frac{\sqrt{\frac{1}{2}(-1+\sqrt{5})}x}{\sqrt{1-x^2}}\right] + x \text{Log}[x^2 + \sqrt{1-x^2}] \end{aligned}$$

Result (type 8, 18 leaves):

$$\int \text{Log}[x^2 + \sqrt{1 - x^2}] \, dx$$

Problem 27: Unable to integrate problem.

$$\int \frac{\text{Log}[1+e^x]}{1+e^{2x}} \, dx$$

Optimal (type 4, 102 leaves, 12 steps):

$$\begin{aligned} & -\frac{1}{2} \text{Log}\left(\left(\frac{1}{2}-\frac{i}{2}\right) \left(\frac{i}{2}-e^x\right)\right) \text{Log}[1+e^x] - \frac{1}{2} \text{Log}\left(\left(-\frac{1}{2}-\frac{i}{2}\right) \left(\frac{i}{2}+e^x\right)\right) \text{Log}[1+e^x] - \\ & \text{PolyLog}[2, -e^x] - \frac{1}{2} \text{PolyLog}[2, \left(\frac{1}{2}-\frac{i}{2}\right) (1+e^x)] - \frac{1}{2} \text{PolyLog}[2, \left(\frac{1}{2}+\frac{i}{2}\right) (1+e^x)] \end{aligned}$$

Result (type 8, 18 leaves):

$$\int \frac{\text{Log}[1+e^x]}{1+e^{2x}} \, dx$$

Problem 28: Unable to integrate problem.

$$\int \cosh[x] \text{Log}[\cosh[x]^2]^2 \, dx$$

Optimal (type 4, 159 leaves, 13 steps):

$$\begin{aligned}
& -8 \sqrt{2} \operatorname{ArcTan}\left[\frac{\operatorname{Sinh}[x]}{\sqrt{2}}\right] + 4 i \sqrt{2} \operatorname{ArcTan}\left[\frac{\operatorname{Sinh}[x]}{\sqrt{2}}\right]^2 + 8 \sqrt{2} \operatorname{ArcTan}\left[\frac{\operatorname{Sinh}[x]}{\sqrt{2}}\right] \operatorname{Log}\left[\frac{2 \sqrt{2}}{\sqrt{2}+i \operatorname{Sinh}[x]}\right] + 4 \sqrt{2} \operatorname{ArcTan}\left[\frac{\operatorname{Sinh}[x]}{\sqrt{2}}\right] \operatorname{Log}\left[2+\operatorname{Sinh}[x]^2\right] + \\
& 4 i \sqrt{2} \operatorname{PolyLog}\left[2,1-\frac{2 \sqrt{2}}{\sqrt{2}+i \operatorname{Sinh}[x]}\right] + 8 \operatorname{Sinh}[x]-4 \operatorname{Log}\left[2+\operatorname{Sinh}[x]^2\right] \operatorname{Sinh}[x]+\operatorname{Log}\left[2+\operatorname{Sinh}[x]^2\right]^2 \operatorname{Sinh}[x]
\end{aligned}$$

Result (type 8, 14 leaves) :

$$\int \operatorname{Cosh}[x] \operatorname{Log}\left[1+\operatorname{Cosh}[x]^2\right]^2 dx$$

Problem 29: Unable to integrate problem.

$$\int \operatorname{Cosh}[x] \operatorname{Log}\left[\operatorname{Cosh}[x]^2+\operatorname{Sinh}[x]\right]^2 dx$$

Optimal (type 4, 395 leaves, 28 steps) :

$$\begin{aligned}
& -4 \sqrt{3} \operatorname{ArcTan}\left[\frac{1+2 \operatorname{Sinh}[x]}{\sqrt{3}}\right]-\frac{1}{2} \left(1-i \sqrt{3}\right) \operatorname{Log}\left[1-i \sqrt{3}+2 \operatorname{Sinh}[x]\right]^2-\left(1+i \sqrt{3}\right) \operatorname{Log}\left[\frac{i \left(1-i \sqrt{3}+2 \operatorname{Sinh}[x]\right)}{2 \sqrt{3}}\right] \operatorname{Log}\left[1+i \sqrt{3}+2 \operatorname{Sinh}[x]\right]- \\
& \frac{1}{2} \left(1+i \sqrt{3}\right) \operatorname{Log}\left[1+i \sqrt{3}+2 \operatorname{Sinh}[x]\right]^2-\left(1-i \sqrt{3}\right) \operatorname{Log}\left[1-i \sqrt{3}+2 \operatorname{Sinh}[x]\right] \operatorname{Log}\left[-\frac{i \left(1+i \sqrt{3}+2 \operatorname{Sinh}[x]\right)}{2 \sqrt{3}}\right]- \\
& 2 \operatorname{Log}\left[1+\operatorname{Sinh}[x]+\operatorname{Sinh}[x]^2\right]+\left(1-i \sqrt{3}\right) \operatorname{Log}\left[1-i \sqrt{3}+2 \operatorname{Sinh}[x]\right] \operatorname{Log}\left[1+\operatorname{Sinh}[x]+\operatorname{Sinh}[x]^2\right]+ \\
& \left(1+i \sqrt{3}\right) \operatorname{Log}\left[1+i \sqrt{3}+2 \operatorname{Sinh}[x]\right] \operatorname{Log}\left[1+\operatorname{Sinh}[x]+\operatorname{Sinh}[x]^2\right]-\left(1+i \sqrt{3}\right) \operatorname{PolyLog}\left[2,-\frac{i-\sqrt{3}+2 i \operatorname{Sinh}[x]}{2 \sqrt{3}}\right]- \\
& \left(1-i \sqrt{3}\right) \operatorname{PolyLog}\left[2,\frac{i+\sqrt{3}+2 i \operatorname{Sinh}[x]}{2 \sqrt{3}}\right]+8 \operatorname{Sinh}[x]-4 \operatorname{Log}\left[1+\operatorname{Sinh}[x]+\operatorname{Sinh}[x]^2\right] \operatorname{Sinh}[x]+\operatorname{Log}\left[1+\operatorname{Sinh}[x]+\operatorname{Sinh}[x]^2\right]^2 \operatorname{Sinh}[x]
\end{aligned}$$

Result (type 8, 15 leaves) :

$$\int \operatorname{Cosh}[x] \operatorname{Log}\left[\operatorname{Cosh}[x]^2+\operatorname{Sinh}[x]\right]^2 dx$$

Problem 31: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Log}\left[x+\sqrt{1+x}\right]^2}{(1+x)^2} dx$$

Optimal (type 4, 555 leaves, 35 steps) :

$$\begin{aligned}
& \text{Log}[1+x] + \frac{2 \text{Log}[x+\sqrt{1+x}]}{\sqrt{1+x}} - 6 \text{Log}[\sqrt{1+x}] \text{Log}[x+\sqrt{1+x}] - \frac{\text{Log}[x+\sqrt{1+x}]^2}{1+x} - (1+\sqrt{5}) \text{Log}[1-\sqrt{5}+2\sqrt{1+x}] + \\
& 6 \text{Log}\left[\frac{1}{2}(-1+\sqrt{5})\right] \text{Log}[1-\sqrt{5}+2\sqrt{1+x}] + (3+\sqrt{5}) \text{Log}[x+\sqrt{1+x}] \text{Log}[1-\sqrt{5}+2\sqrt{1+x}] - \\
& \frac{1}{2}(3+\sqrt{5}) \text{Log}[1-\sqrt{5}+2\sqrt{1+x}]^2 - (1-\sqrt{5}) \text{Log}[1+\sqrt{5}+2\sqrt{1+x}] + (3-\sqrt{5}) \text{Log}[x+\sqrt{1+x}] \text{Log}[1+\sqrt{5}+2\sqrt{1+x}] - \\
& (3-\sqrt{5}) \text{Log}\left[-\frac{1-\sqrt{5}+2\sqrt{1+x}}{2\sqrt{5}}\right] \text{Log}[1+\sqrt{5}+2\sqrt{1+x}] - \frac{1}{2}(3-\sqrt{5}) \text{Log}[1+\sqrt{5}+2\sqrt{1+x}]^2 - \\
& (3+\sqrt{5}) \text{Log}[1-\sqrt{5}+2\sqrt{1+x}] \text{Log}\left[\frac{1+\sqrt{5}+2\sqrt{1+x}}{2\sqrt{5}}\right] + 6 \text{Log}[\sqrt{1+x}] \text{Log}\left[1+\frac{2\sqrt{1+x}}{1+\sqrt{5}}\right] + 6 \text{PolyLog}[2, -\frac{2\sqrt{1+x}}{1+\sqrt{5}}] - \\
& (3+\sqrt{5}) \text{PolyLog}[2, -\frac{1-\sqrt{5}+2\sqrt{1+x}}{2\sqrt{5}}] - (3-\sqrt{5}) \text{PolyLog}[2, \frac{1+\sqrt{5}+2\sqrt{1+x}}{2\sqrt{5}}] - 6 \text{PolyLog}[2, 1+\frac{2\sqrt{1+x}}{1-\sqrt{5}}]
\end{aligned}$$

Result (type 4, 1283 leaves):

$$\begin{aligned}
& \text{Log}[1+x] - \text{Log}[-1+\sqrt{5}-2\sqrt{1+x}] - \sqrt{5} \text{ Log}[-1+\sqrt{5}-2\sqrt{1+x}] + \frac{\text{Log}[100] \text{ Log}\left[\frac{1}{2}-\frac{\sqrt{5}}{2}+\sqrt{1+x}\right]}{\sqrt{5}} - 6 \text{ Log}\left[\frac{2 \sqrt{1+x}}{-1+\sqrt{5}}\right] \text{ Log}\left[\frac{1}{2}-\frac{\sqrt{5}}{2}+\sqrt{1+x}\right] + \\
& 3 \text{ Log}[1+x] \text{ Log}\left[\frac{1}{2}-\frac{\sqrt{5}}{2}+\sqrt{1+x}\right] - 3 \text{ Log}[-1+\sqrt{5}-2\sqrt{1+x}] \text{ Log}\left[\frac{1}{2}-\frac{\sqrt{5}}{2}+\sqrt{1+x}\right] - \sqrt{5} \text{ Log}[-1+\sqrt{5}-2\sqrt{1+x}] \text{ Log}\left[\frac{1}{2}-\frac{\sqrt{5}}{2}+\sqrt{1+x}\right] + \\
& \frac{3}{2} \text{ Log}\left[\frac{1}{2}-\frac{\sqrt{5}}{2}+\sqrt{1+x}\right]^2 + \frac{1}{2} \sqrt{5} \text{ Log}\left[\frac{1}{2}-\frac{\sqrt{5}}{2}+\sqrt{1+x}\right]^2 + \frac{\text{Log}[8] \text{ Log}\left[\frac{1}{2}(1+\sqrt{5})+\sqrt{1+x}\right]}{2 \sqrt{5}} - \\
& 3 \text{ Log}[-1+\sqrt{5}-2\sqrt{1+x}] \text{ Log}\left[\frac{1}{2}(1+\sqrt{5})+\sqrt{1+x}\right] - \sqrt{5} \text{ Log}[-1+\sqrt{5}-2\sqrt{1+x}] \text{ Log}\left[\frac{1}{2}(1+\sqrt{5})+\sqrt{1+x}\right] + \\
& \frac{3}{2} \text{ Log}\left[\frac{1}{2}(1+\sqrt{5})+\sqrt{1+x}\right]^2 - \frac{\text{Log}\left[\frac{1}{2}(1+\sqrt{5})+\sqrt{1+x}\right]^2}{\sqrt{5}} + \frac{2 \text{ Log}[x+\sqrt{1+x}]}{\sqrt{1+x}} - 3 \text{ Log}[1+x] \text{ Log}[x+\sqrt{1+x}] + \\
& 3 \text{ Log}[-1+\sqrt{5}-2\sqrt{1+x}] \text{ Log}[x+\sqrt{1+x}] + \sqrt{5} \text{ Log}[-1+\sqrt{5}-2\sqrt{1+x}] \text{ Log}[x+\sqrt{1+x}] - \frac{\text{Log}[x+\sqrt{1+x}]^2}{1+x} - \text{Log}[1+\sqrt{5}+2\sqrt{1+x}] + \\
& \sqrt{5} \text{ Log}[1+\sqrt{5}+2\sqrt{1+x}] - 3 \text{ Log}\left[\frac{1}{2}-\frac{\sqrt{5}}{2}+\sqrt{1+x}\right] \text{ Log}[1+\sqrt{5}+2\sqrt{1+x}] + \sqrt{5} \text{ Log}\left[\frac{1}{2}-\frac{\sqrt{5}}{2}+\sqrt{1+x}\right] \text{ Log}[1+\sqrt{5}+2\sqrt{1+x}] - \\
& 3 \text{ Log}\left[\frac{1}{2}(1+\sqrt{5})+\sqrt{1+x}\right] \text{ Log}[1+\sqrt{5}+2\sqrt{1+x}] + \frac{7 \text{ Log}\left[\frac{1}{2}(1+\sqrt{5})+\sqrt{1+x}\right] \text{ Log}[1+\sqrt{5}+2\sqrt{1+x}]}{2 \sqrt{5}} + \\
& 3 \text{ Log}[x+\sqrt{1+x}] \text{ Log}[1+\sqrt{5}+2\sqrt{1+x}] - \sqrt{5} \text{ Log}[x+\sqrt{1+x}] \text{ Log}[1+\sqrt{5}+2\sqrt{1+x}] + 3 \text{ Log}\left[\frac{1}{2}-\frac{\sqrt{5}}{2}+\sqrt{1+x}\right] \text{ Log}\left[\frac{1+\sqrt{5}+2\sqrt{1+x}}{2 \sqrt{5}}\right] - \\
& \frac{3 \text{ Log}\left[\frac{1}{2}-\frac{\sqrt{5}}{2}+\sqrt{1+x}\right] \text{ Log}\left[\frac{1+\sqrt{5}+2\sqrt{1+x}}{2 \sqrt{5}}\right]}{\sqrt{5}} + 3 \text{ Log}\left[\frac{1}{2}(1+\sqrt{5})+\sqrt{1+x}\right] \text{ Log}\left[\frac{1}{10}(5-\sqrt{5}-2\sqrt{5}\sqrt{1+x})\right] + \\
& \sqrt{5} \text{ Log}\left[\frac{1}{2}(1+\sqrt{5})+\sqrt{1+x}\right] \text{ Log}\left[\frac{1}{10}(5-\sqrt{5}-2\sqrt{5}\sqrt{1+x})\right] - \frac{2 \text{ Log}\left[\frac{1}{2}-\frac{\sqrt{5}}{2}+\sqrt{1+x}\right] \text{ Log}[5+\sqrt{5}+2\sqrt{5}\sqrt{1+x}]}{\sqrt{5}} + \\
& 3 \text{ Log}[1+x] \text{ Log}\left[1+\frac{2 \sqrt{1+x}}{1+\sqrt{5}}\right] + 6 \text{ PolyLog}[2, -\frac{2 \sqrt{1+x}}{1+\sqrt{5}}] - (-3+\sqrt{5}) \text{ PolyLog}[2, \frac{-1+\sqrt{5}-2\sqrt{1+x}}{2 \sqrt{5}}] - \\
& 6 \text{ PolyLog}[2, \frac{-1+\sqrt{5}-2\sqrt{1+x}}{-1+\sqrt{5}}] + 3 \text{ PolyLog}[2, \frac{1+\sqrt{5}+2\sqrt{1+x}}{2 \sqrt{5}}] + \sqrt{5} \text{ PolyLog}[2, \frac{1+\sqrt{5}+2\sqrt{1+x}}{2 \sqrt{5}}]
\end{aligned}$$

Problem 33: Result more than twice size of optimal antiderivative.

$$\int \text{ArcTan}[2 \tan[x]] \, dx$$

Optimal (type 4, 80 leaves, 7 steps):

$$x \operatorname{ArcTan}[2 \operatorname{Tan}[x]] + \frac{1}{2} i x \operatorname{Log}\left[1 - 3 e^{2 i x}\right] - \frac{1}{2} i x \operatorname{Log}\left[1 - \frac{1}{3} e^{2 i x}\right] - \frac{1}{4} \operatorname{PolyLog}\left[2, \frac{1}{3} e^{2 i x}\right] + \frac{1}{4} \operatorname{PolyLog}\left[2, 3 e^{2 i x}\right]$$

Result (type 4, 262 leaves):

$$x \operatorname{ArcTan}[2 \operatorname{Tan}[x]] -$$

$$\begin{aligned} & \frac{1}{4} i \left(4 i x \operatorname{ArcTan}\left[\frac{\operatorname{Cot}[x]}{2}\right] + 2 i \operatorname{ArcCos}\left[\frac{5}{3}\right] \operatorname{ArcTan}[2 \operatorname{Tan}[x]] + \left(\operatorname{ArcCos}\left[\frac{5}{3}\right] + 2 \operatorname{ArcTan}\left[\frac{\operatorname{Cot}[x]}{2}\right] + 2 \operatorname{ArcTan}[2 \operatorname{Tan}[x]]\right) \operatorname{Log}\left[\frac{2 i \sqrt{\frac{2}{3}} e^{-i x}}{\sqrt{-5 + 3 \cos[2 x]}}\right] + \right. \\ & \left(\operatorname{ArcCos}\left[\frac{5}{3}\right] - 2 \operatorname{ArcTan}\left[\frac{\operatorname{Cot}[x]}{2}\right] - 2 \operatorname{ArcTan}[2 \operatorname{Tan}[x]] \right) \operatorname{Log}\left[\frac{2 i \sqrt{\frac{2}{3}} e^{i x}}{\sqrt{-5 + 3 \cos[2 x]}}\right] - \left(\operatorname{ArcCos}\left[\frac{5}{3}\right] - 2 \operatorname{ArcTan}[2 \operatorname{Tan}[x]]\right) \operatorname{Log}\left[\frac{4 i - 4 \operatorname{Tan}[x]}{i + 2 \operatorname{Tan}[x]}\right] - \\ & \left. \left(\operatorname{ArcCos}\left[\frac{5}{3}\right] + 2 \operatorname{ArcTan}[2 \operatorname{Tan}[x]] \right) \operatorname{Log}\left[\frac{4 (i + \operatorname{Tan}[x])}{3 i + 6 \operatorname{Tan}[x]}\right] + i \left(-\operatorname{PolyLog}\left[2, \frac{-3 i + 6 \operatorname{Tan}[x]}{i + 2 \operatorname{Tan}[x]}\right] + \operatorname{PolyLog}\left[2, \frac{-i + 2 \operatorname{Tan}[x]}{3 i + 6 \operatorname{Tan}[x]}\right]\right) \right) \end{aligned}$$

Problem 35: Result more than twice size of optimal antiderivative.

$$\int \sqrt{1+x^2} \operatorname{ArcTan}[x]^2 dx$$

Optimal (type 4, 121 leaves, 10 steps):

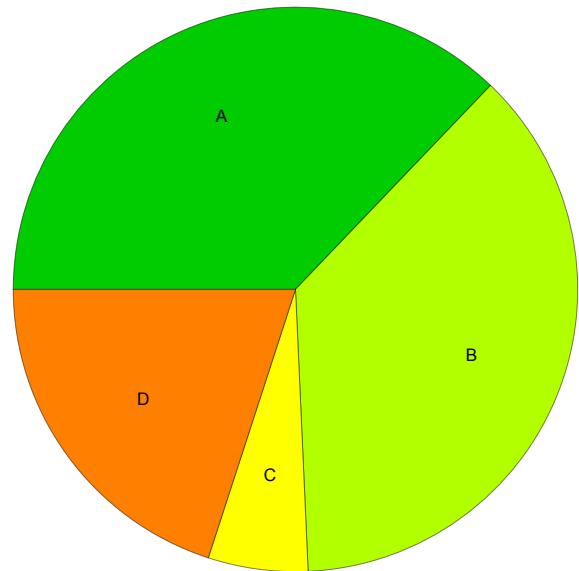
$$\begin{aligned} & \operatorname{ArcSinh}[x] - \sqrt{1+x^2} \operatorname{ArcTan}[x] + \frac{1}{2} x \sqrt{1+x^2} \operatorname{ArcTan}[x]^2 - i \operatorname{ArcTan}\left[e^{i \operatorname{ArcTan}[x]}\right] \operatorname{ArcTan}[x]^2 + \\ & i \operatorname{ArcTan}[x] \operatorname{PolyLog}\left[2, -i e^{i \operatorname{ArcTan}[x]}\right] - i \operatorname{ArcTan}[x] \operatorname{PolyLog}\left[2, i e^{i \operatorname{ArcTan}[x]}\right] - \operatorname{PolyLog}\left[3, -i e^{i \operatorname{ArcTan}[x]}\right] + \operatorname{PolyLog}\left[3, i e^{i \operatorname{ArcTan}[x]}\right] \end{aligned}$$

Result (type 4, 405 leaves):

$$\begin{aligned}
& \frac{1}{2} \left(\sqrt{1+x^2} \operatorname{ArcTan}[x] (-2+x \operatorname{ArcTan}[x]) - \pi \operatorname{ArcTan}[x] \operatorname{Log}[2] + \operatorname{ArcTan}[x]^2 \operatorname{Log}\left[1 - i e^{i \operatorname{ArcTan}[x]}\right] - \operatorname{ArcTan}[x]^2 \operatorname{Log}\left[1 + i e^{i \operatorname{ArcTan}[x]}\right] + \right. \\
& \quad \pi \operatorname{ArcTan}[x] \operatorname{Log}\left(\left(-\frac{1}{2} - \frac{i}{2}\right) e^{-\frac{1}{2} i \operatorname{ArcTan}[x]} (-i + e^{i \operatorname{ArcTan}[x]})\right) - \operatorname{ArcTan}[x]^2 \operatorname{Log}\left(\left(\frac{1}{2} + \frac{i}{2}\right) e^{-\frac{1}{2} i \operatorname{ArcTan}[x]} (-i + e^{i \operatorname{ArcTan}[x]})\right) + \\
& \quad \pi \operatorname{ArcTan}[x] \operatorname{Log}\left[\frac{1}{2} e^{-\frac{1}{2} i \operatorname{ArcTan}[x]} ((1+i) + (1-i) e^{i \operatorname{ArcTan}[x]})\right] + \operatorname{ArcTan}[x]^2 \operatorname{Log}\left[\frac{1}{2} e^{-\frac{1}{2} i \operatorname{ArcTan}[x]} ((1+i) + (1-i) e^{i \operatorname{ArcTan}[x]})\right] - \\
& \quad \pi \operatorname{ArcTan}[x] \operatorname{Log}\left[-\cos\left[\frac{1}{4} (\pi + 2 \operatorname{ArcTan}[x])\right]\right] - 2 \operatorname{Log}\left[\cos\left[\frac{\operatorname{ArcTan}[x]}{2}\right] - \sin\left[\frac{\operatorname{ArcTan}[x]}{2}\right]\right] + \\
& \quad \operatorname{ArcTan}[x]^2 \operatorname{Log}\left[\cos\left[\frac{\operatorname{ArcTan}[x]}{2}\right] - \sin\left[\frac{\operatorname{ArcTan}[x]}{2}\right]\right] + 2 \operatorname{Log}\left[\cos\left[\frac{\operatorname{ArcTan}[x]}{2}\right] + \sin\left[\frac{\operatorname{ArcTan}[x]}{2}\right]\right] - \\
& \quad \operatorname{ArcTan}[x]^2 \operatorname{Log}\left[\cos\left[\frac{\operatorname{ArcTan}[x]}{2}\right] + \sin\left[\frac{\operatorname{ArcTan}[x]}{2}\right]\right] - \pi \operatorname{ArcTan}[x] \operatorname{Log}\left[\sin\left[\frac{1}{4} (\pi + 2 \operatorname{ArcTan}[x])\right]\right] + \\
& \quad \left. 2 i \operatorname{ArcTan}[x] \operatorname{PolyLog}\left[2, -i e^{i \operatorname{ArcTan}[x]}\right] - 2 i \operatorname{ArcTan}[x] \operatorname{PolyLog}\left[2, i e^{i \operatorname{ArcTan}[x]}\right] - 2 \operatorname{PolyLog}\left[3, -i e^{i \operatorname{ArcTan}[x]}\right] + 2 \operatorname{PolyLog}\left[3, i e^{i \operatorname{ArcTan}[x]}\right] \right)
\end{aligned}$$

Summary of Integration Test Results

35 integration problems



A - 13 optimal antiderivatives

B - 13 more than twice size of optimal antiderivatives

C - 2 unnecessarily complex antiderivatives

D - 7 unable to integrate problems

E - 0 integration timeouts