Mathematica 11.3 Integration Test Results

Test results for the 93 problems in "Welz Problems.m"

Problem 2: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{-2\,\text{Log}\left[-\sqrt{-1+a\,x}\right]+\text{Log}[-1+a\,x]}{2\,\pi\,\sqrt{-1+a\,x}}\,dx$$

Optimal (type 2, 15 leaves, 5 steps):

$$-\frac{2\,\sqrt{1-a\,x}}{a}$$

Result (type 3, 37 leaves):

$$\frac{\sqrt{-1+a\,x}\,\left(-2\,\text{Log}\left[-\sqrt{-1+a\,x}\right]+\text{Log}[-1+a\,x]\right)}{a\,\pi}$$

Problem 6: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{-1+x^2}}{(-i+x)^2}\,dx$$

Optimal (type 3, 64 leaves, 6 steps):

$$\frac{\sqrt{-1+x^2}}{i-x}-\frac{i\,\text{ArcTan}\left[\frac{1-i\,x}{\sqrt{2}\,\sqrt{-1+x^2}}\right]}{\sqrt{2}}+\text{ArcTanh}\left[\frac{x}{\sqrt{-1+x^2}}\right]$$

Result (type 3, 165 leaves):

$$\frac{1}{4}\left(-\frac{4\,\sqrt{-1+x^2}}{-i+x}-2\,i\,\sqrt{2}\,\text{ArcTan}\left[\frac{1}{2}\left(-i+x-\sqrt{2}\,\sqrt{-1+x^2}\right)\right]+4\,\text{ArcTanh}\left[\frac{2\,x}{i-x+\sqrt{-1+x^2}}\right]-\right.$$
$$\left.\sqrt{2}\,\text{Log}[-i+x]+\sqrt{2}\,\text{Log}\left[-i-3\,x+2\,\sqrt{2}\,\sqrt{-1+x^2}\right]+2\,\text{Log}\left[1+2\,i\,x-2\,x^2+2\,i\,\sqrt{-1+x^2}-2\,x\,\sqrt{-1+x^2}\right]\right)$$

Problem 9: Unable to integrate problem.

$$\int \frac{1}{\sqrt{-1+x^2}\,\left(\sqrt{x}+\sqrt{-1+x^2}\right)^2}\,dx$$

Optimal (type 3, 220 leaves, ? steps):

$$\frac{2-4x}{5\left(\sqrt{x}+\sqrt{-1+x^2}\right)}+\frac{1}{25}\sqrt{-110+50\sqrt{5}}\,\text{ArcTan}\left[\frac{1}{2}\sqrt{2+2\sqrt{5}}\,\sqrt{x}\right]-\frac{1}{50}\sqrt{-110+50\sqrt{5}}\,\text{ArcTan}\left[\frac{\sqrt{-2+2\sqrt{5}}\,\sqrt{-1+x^2}}{2-\left(1-\sqrt{5}\right)x}\right]-\frac{1}{25}\sqrt{110+50\sqrt{5}}\,\text{ArcTanh}\left[\frac{1}{2}\sqrt{-2+2\sqrt{5}}\,\sqrt{x}\right]-\frac{1}{50}\sqrt{110+50\sqrt{5}}\,\text{ArcTanh}\left[\frac{\sqrt{2+2\sqrt{5}}\,\sqrt{-1+x^2}}{2-x-\sqrt{5}\,x}\right]$$

Result (type 8, 29 leaves) :

$$\int \frac{1}{\sqrt{-1+x^2}\,\left(\sqrt{x}+\sqrt{-1+x^2}\right)^2}\,dx$$

Problem 10: Unable to integrate problem.

$$\int \frac{\left(\sqrt{x}-\sqrt{-1+x^2}\right)^2}{\left(1+x-x^2\right)^2\sqrt{-1+x^2}}\,dx$$

Optimal (type 3, 220 leaves, ? steps):

$$\frac{2-4x}{5\left(\sqrt{x}+\sqrt{-1+x^2}\right)}+\frac{1}{25}\sqrt{-110+50\sqrt{5}}\,\text{ArcTan}\left[\frac{1}{2}\sqrt{2+2\sqrt{5}}\,\sqrt{x}\right]-\frac{1}{50}\sqrt{-110+50\sqrt{5}}\,\text{ArcTan}\left[\frac{\sqrt{-2+2\sqrt{5}}\,\sqrt{-1+x^2}}{2-\left(1-\sqrt{5}\right)x}\right]-\frac{1}{25}\sqrt{110+50\sqrt{5}}\,\text{ArcTanh}\left[\frac{1}{2}\sqrt{-2+2\sqrt{5}}\,\sqrt{x}\right]-\frac{1}{50}\sqrt{110+50\sqrt{5}}\,\text{ArcTanh}\left[\frac{\sqrt{2+2\sqrt{5}}\,\sqrt{-1+x^2}}{2-x-\sqrt{5}\,x}\right]$$

Result (type 8, 41 leaves) :

$$\int \frac{\left(\sqrt{x}-\sqrt{-1+x^2}\right)^2}{\left(1+x-x^2\right)^2\sqrt{-1+x^2}}\,dx$$

Problem 11: Result more than twice size of optimal antiderivative.

$$\int \left(\frac{1}{\sqrt{2}\,(1+x)^2\sqrt{-i+x^2}}+\frac{1}{\sqrt{2}\,(1+x)^2\sqrt{i+x^2}}\right)dx$$

Optimal (type 3, 138 leaves, 7 steps):

$$-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{-i+x^2}}{\sqrt{2}\,(1+x)}-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{i+x^2}}{\sqrt{2}\,(1+x)}+\frac{\text{ArcTanh}\left[\frac{i+x}{\sqrt{1-i}\,\sqrt{-i+x^2}}\right]}{(1-i)^{3/2}\sqrt{2}}-\frac{\text{ArcTanh}\left[\frac{i-x}{\sqrt{1+i}\,\sqrt{i+x^2}}\right]}{(1+i)^{3/2}\sqrt{2}}$$

Result (type 3, 403 leaves):

$$-\frac{1}{4\sqrt{2}\,(1+x)}\left((2+2i)\sqrt{-i+x^2}+(2-2i)\sqrt{i+x^2}+2\sqrt{1-i}\,(1+x)\,\text{ArcTan}\left[\frac{1+x^2+2i\sqrt{1-i}\,\sqrt{-i+x^2}}{(1-2i)-2ix+x^2}\right]+\right.$$
$$2\sqrt{1+i}\,(1+x)\,\text{ArcTan}\left[\frac{1+x^2-2i\sqrt{1+i}\,\sqrt{i+x^2}}{(1+2i)+2ix+x^2}\right]-i\sqrt{1-i}\,\text{Log}\left[(1+x)^2\right]+i\sqrt{1+i}\,\text{Log}\left[(1+x)^2\right]-i\sqrt{1-i}\,x\,\text{Log}\left[(1+x)^2\right]+$$
$$i\sqrt{1+i}\,x\,\text{Log}\left[(1+x)^2\right]+i\sqrt{1-i}\,\text{Log}\left[i-(2-i)x^2+2\sqrt{1-i}\,x\sqrt{-i+x^2}\right]+i\sqrt{1-i}\,x\,\text{Log}\left[i-(2-i)x^2+2\sqrt{1-i}\,x\sqrt{-i+x^2}\right]-$$
$$\left.i\sqrt{1+i}\,\text{Log}\left[-i-(2+i)x^2+2\sqrt{1+i}\,x\sqrt{i+x^2}\right]-i\sqrt{1+i}\,x\,\text{Log}\left[-i-(2+i)x^2+2\sqrt{1+i}\,x\sqrt{i+x^2}\right]\right)$$

Problem 12: Unable to integrate problem.

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)^2\sqrt{1+x^4}}\,dx$$

Optimal (type 3, 125 leaves, 7 steps):

$$-\frac{\sqrt{1-ix^2}}{2\,(1+x)}-\frac{\sqrt{1+ix^2}}{2\,(1+x)}-\frac{1}{4}(1-i)^{3/2}\,\text{ArcTanh}\left[\frac{1+ix}{\sqrt{1-i}\,\sqrt{1-ix^2}}\right]-\frac{1}{4}(1+i)^{3/2}\,\text{ArcTanh}\left[\frac{1-ix}{\sqrt{1+i}\,\sqrt{1+ix^2}}\right]$$

Result (type 8, 34 leaves):

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)^2\sqrt{1+x^4}}\,dx$$

Problem 13: Unable to integrate problem.

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx$$

Optimal (type 3, 81 leaves, 5 steps):

$$-\frac{1}{2}\sqrt{1-i}\,\text{ArcTanh}\left[\frac{1+i x}{\sqrt{1-i}\,\sqrt{1-i x^2}}\right]-\frac{1}{2}\sqrt{1+i}\,\text{ArcTanh}\left[\frac{1-i x}{\sqrt{1+i}\,\sqrt{1+i x^2}}\right]$$

Result (type 8, 34 leaves):

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx$$

Problem 14: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx$$

Optimal (type 3, 31 leaves, 2 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{2}\,x}{\sqrt{x^2+\sqrt{1+x^4}}}\right]}{\sqrt{2}}$$

Result (type 3, 145 leaves):

$$-\frac{x\left(1+x^4+x^2\sqrt{1+x^4}\right)\left(\text{Log}\left[1-\frac{\sqrt{x^2\left(x^2+\sqrt{1+x^4}\right)}}{\sqrt{2}\,x^2}\right]-\text{Log}\left[1+\frac{\sqrt{x^2\left(x^2+\sqrt{1+x^4}\right)}}{\sqrt{2}\,x^2}\right]\right)}{2\sqrt{2}\,\sqrt{1+x^4}\,\sqrt{x^2+\sqrt{1+x^4}}\,\sqrt{x^2\left(x^2+\sqrt{1+x^4}\right)}}$$

Problem 15: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx$$

Optimal (type 3, 33 leaves, 2 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{2}\,x}{\sqrt{-x^2+\sqrt{1+x^4}}}\right]}{\sqrt{2}}$$

Result (type 3, 162 leaves):

$$\frac{x\left(1+2x^4-2x^2\sqrt{1+x^4}\right)^2\left(1+x^4-x^2\sqrt{1+x^4}\right)\text{ArcSin}\left[x^2-\sqrt{1+x^4}\right]}{\sqrt{2}\sqrt{-x^2+\sqrt{1+x^4}}\sqrt{x^2\left(-x^2+\sqrt{1+x^4}\right)}\left(-4x^2-12x^6-8x^{10}+\sqrt{1+x^4}+8x^4\sqrt{1+x^4}+8x^8\sqrt{1+x^4}\right)}$$

Problem 24: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1-x+3x^2}{\sqrt{1-x+x^2}\left(1+x+x^2\right)^2}\,dx$$

Optimal (type 3, 86 leaves, 6 steps):

$$\frac{(1+x)\sqrt{1-x+x^2}}{1+x+x^2}+\sqrt{2}\,\text{ArcTan}\left[\frac{\sqrt{2}\,(1+x)}{\sqrt{1-x+x^2}}\right]-\frac{\text{ArcTanh}\left[\frac{\sqrt{\frac{2}{3}}\,(1-x)}{\sqrt{1-x+x^2}}\right]}{\sqrt{6}}$$

Result (type 3, 961 leaves):

$$\frac{(1+x)\sqrt{1-x+x^2}}{1+x+x^2}+\frac{1}{4\sqrt{3-3\,i\sqrt{3}}}$$
$$\left(7-i\sqrt{3}\right)\text{ArcTan}\Big[\Big(3\left(-17-64\,i\sqrt{3}+\left(94+32\,i\sqrt{3}\right)x+\left(-103-36\,i\sqrt{3}\right)x^2+14\left(7-2\,i\sqrt{3}\right)x^3+\left(-21-4\,i\sqrt{3}\right)x^4\right)\Big)\Big/$$
$$\Big(\left(96\,i+67\sqrt{3}+\left(84\,i-113\sqrt{3}\right)x^4-52\sqrt{3-3\,i\sqrt{3}}\sqrt{1-x+x^2}+2x\left(132\,i-69\sqrt{3}+26\sqrt{3-3\,i\sqrt{3}}\sqrt{1-x+x^2}\right)+\right.$$
$$\left.x^2\left(-180\,i-59\sqrt{3}+52\sqrt{3-3\,i\sqrt{3}}\sqrt{1-x+x^2}\right)+2x^3\left(138\,i+21\sqrt{3}+52\sqrt{3-3\,i\sqrt{3}}\sqrt{1-x+x^2}\right)\right)\Big)\Big]-\frac{1}{4\sqrt{3+3\,i\sqrt{3}}}$$
$$i\left(-7\,i+\sqrt{3}\right)\text{ArcTan}\Big[\Big(3\left(-17+64\,i\sqrt{3}+\left(94-32\,i\sqrt{3}\right)x+\left(-103+36\,i\sqrt{3}\right)x^2+14\left(7+2\,i\sqrt{3}\right)x^3+\left(-21+4\,i\sqrt{3}\right)x^4\right)\Big)\Big/$$
$$\Big(\left(96\,i-67\sqrt{3}+\left(84\,i+113\sqrt{3}\right)x^4+52\sqrt{3+3\,i\sqrt{3}}\sqrt{1-x+x^2}+x^2\left(-180\,i+59\sqrt{3}-52\sqrt{3+3\,i\sqrt{3}}\sqrt{1-x+x^2}\right)+\right.$$
$$\left.x\left(264\,i+138\sqrt{3}-52\sqrt{3+3\,i\sqrt{3}}\sqrt{1-x+x^2}\right)-2x^3\left(-138\,i+21\sqrt{3}+52\sqrt{3+3\,i\sqrt{3}}\sqrt{1-x+x^2}\right)\right)\Big)\Big]-$$
$$\frac{\left(-7\,i+\sqrt{3}\right)\text{Log}\left[16\left(1+x+x^2\right)^2\right]}{8\sqrt{3+3\,i\sqrt{3}}}-\frac{\left(7\,i+\sqrt{3}\right)\text{Log}\left[16\left(1+x+x^2\right)^2\right]}{8\sqrt{3-3\,i\sqrt{3}}}+\frac{1}{8\sqrt{3-3\,i\sqrt{3}}}$$
$$\left(7\,i+\sqrt{3}\right)\text{Log}\Big[\left(1+x+x^2\right)\left(11\,i+4\sqrt{3}+\left(11\,i+4\sqrt{3}\right)x^2+10\,i\sqrt{1-i\sqrt{3}}\sqrt{1-x+x^2}-x\left(17\,i+4\sqrt{3}+8\,i\sqrt{1-i\sqrt{3}}\sqrt{1-x+x^2}\right)\right)\Big]+$$
$$\frac{1}{8\sqrt{3+3\,i\sqrt{3}}}$$
$$\left(-7\,i+\sqrt{3}\right)\text{Log}\Big[\left(1+x+x^2\right)\left(-11\,i+4\sqrt{3}+\left(-11\,i+4\sqrt{3}\right)x^2-10\,i\sqrt{1+i\sqrt{3}}\sqrt{1-x+x^2}+x\left(17\,i-4\sqrt{3}+8\,i\sqrt{1+i\sqrt{3}}\sqrt{1-x+x^2}\right)\right)\Big]$$

Problem 33: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x\left(1-x^2\right)^{1/3}}\,dx$$

Optimal (type 3, 58 leaves, 5 steps):

$$\frac{1}{2}\sqrt{3}\,\text{ArcTan}\left[\frac{1+2\left(1-x^2\right)^{1/3}}{\sqrt{3}}\right]-\frac{\text{Log}[x]}{2}+\frac{3}{4}\text{Log}\left[1-\left(1-x^2\right)^{1/3}\right]$$

Result (type 5, 41 leaves):

$$-\frac{3\left(\frac{-1+x^2}{x^2}\right)^{1/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},\frac{1}{x^2}\right]}{2\left(1-x^2\right)^{1/3}}$$

Problem 34: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x\left(1-x^2\right)^{2/3}}\,dx$$

Optimal (type 3, 58 leaves, 5 steps):

$$-\frac{1}{2}\sqrt{3}\,\text{ArcTan}\left[\frac{1+2\left(1-x^2\right)^{1/3}}{\sqrt{3}}\right]-\frac{\text{Log}[x]}{2}+\frac{3}{4}\text{Log}\left[1-\left(1-x^2\right)^{1/3}\right]$$

Result (type 5, 41 leaves):

$$-\frac{3\left(\frac{-1+x^2}{x^2}\right)^{2/3}\text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},\frac{1}{x^2}\right]}{4\left(1-x^2\right)^{2/3}}$$

Problem 36: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x\left(1-x^3\right)^{1/3}}\,dx$$

Optimal (type 3, 55 leaves, 5 steps):

$$\frac{\text{ArcTan}\left[\frac{1+2\left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{\text{Log}[x]}{2}+\frac{1}{2}\text{Log}\left[1-\left(1-x^3\right)^{1/3}\right]$$

Result (type 5, 39 leaves):

$$-\frac{\left(\frac{-1+x^3}{x^3}\right)^{1/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},\frac{1}{x^3}\right]}{\left(1-x^3\right)^{1/3}}$$

Problem 37: Unable to integrate problem.

$$\int \frac{1}{(1+x)\left(1-x^3\right)^{1/3}}\,dx$$

Optimal (type 3, 121 leaves, ? steps):

$$-\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{2(1-x)+2^{2/3}\left(1-x^3\right)^{1/3}}{2^{2/3}\sqrt{3}\left(1-x^3\right)^{1/3}}\right]}{2\times 2^{1/3}}-\frac{\text{Log}[1-x]}{4\times 2^{1/3}}-\frac{\text{Log}[1+x]}{2\times 2^{1/3}}+\frac{3\,\text{Log}\left[-1+x+2^{2/3}\left(1-x^3\right)^{1/3}\right]}{4\times 2^{1/3}}$$

Result (type 8, 19 leaves):

$$\int \frac{1}{(1+x)\ (1-x^3)^{1/3}}\, dx$$

Problem 38: Unable to integrate problem.

$$\int \frac{x}{(1+x)\ (1-x^3)^{1/3}}\, dx$$

Optimal (type 3, 145 leaves, 3 steps):

$$\frac{\sqrt{3}\ \text{ArcTan}\left[\frac{1+\frac{2^{1/3}\,(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3}}-\frac{\text{ArcTan}\left[\frac{1-\frac{2x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{\text{Log}\left[(1-x)\ (1+x)^2\right]}{4\times 2^{1/3}}+\frac{1}{2}\text{Log}\left[x+(1-x^3)^{1/3}\right]-\frac{3\,\text{Log}\left[-1+x+2^{2/3}\,(1-x^3)^{1/3}\right]}{4\times 2^{1/3}}$$

Result (type 8, 20 leaves):

$$\int \frac{x}{(1+x)\ (1-x^3)^{1/3}}\, dx$$

Problem 39: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x\ (2-3x+x^2)^{1/3}}\, dx$$

Optimal (type 3, 110 leaves, ? steps):

$$-\frac{\sqrt{3}\ \text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2^{1/3}\,(2-x)}{\sqrt{3}\ (2-3x+x^2)^{1/3}}\right]}{2\times 2^{1/3}}-\frac{\text{Log}[2-x]}{4\times 2^{1/3}}-\frac{\text{Log}[x]}{2\times 2^{1/3}}+\frac{3\,\text{Log}\left[2-x-2^{2/3}\,(2-3x+x^2)^{1/3}\right]}{4\times 2^{1/3}}$$

Result (type 6, 109 leaves):

$$-\left(\left(15\,x\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},\frac{1}{x},\frac{2}{x}\right]\right)\Big/\left(2\ (2-3x+x^2)^{1/3}\left(5\,x\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},\frac{1}{x},\frac{2}{x}\right]+2\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},\frac{4}{3},\frac{8}{3},\frac{1}{x},\frac{2}{x}\right]+\text{AppellF1}\left[\frac{5}{3},\frac{4}{3},\frac{1}{3},\frac{8}{3},\frac{1}{x},\frac{2}{x}\right]\right)\right)\right)$$

Problem 40: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(-5+7x-3x^2+x^3)^{1/3}}\, dx$$

Optimal (type 3, 81 leaves, ? steps):

$$\frac{1}{2}\sqrt{3}\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2\,(-1+x)}{\sqrt{3}\,\left(-5+7\,x-3\,x^2+x^3\right)^{1/3}}\right]+\frac{1}{4}\,\text{Log}[1-x]-\frac{3}{4}\,\text{Log}\left[1-x+\left(-5+7\,x-3\,x^2+x^3\right)^{1/3}\right]$$

Result (type 6, 85 leaves):

$$\frac{1}{4\,\left(-5+7\,x-3\,x^2+x^3\right)^{1/3}}3\,\left((2-i)+i\,x\right)^{1/3}\,\left(i\,(-1+x)\right)^{1/3}\,\left((-1+2\,i)+x\right)\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},-\frac{1}{4}\,i\,\left((-1+2\,i)+x\right),-\frac{1}{2}\,i\,\left((-1+2\,i)+x\right)\right]$$

Problem 41: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(x\,\left(-q+x^2\right)\right)^{1/3}}\,dx$$

Optimal (type 3, 66 leaves, ? steps):

$$\frac{1}{2}\sqrt{3}\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2\,x}{\sqrt{3}\,\left(x\,\left(-q+x^2\right)\right)^{1/3}}\right]+\frac{\text{Log}[x]}{4}-\frac{3}{4}\,\text{Log}\left[-x+\left(x\,\left(-q+x^2\right)\right)^{1/3}\right]$$

Result (type 5, 49 leaves):

$$\frac{3\,x\,\left(\frac{q-x^2}{q}\right)^{1/3}\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},\frac{x^2}{q}\right]}{2\,\left(-q\,x+x^3\right)^{1/3}}$$

Problem 42: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left((-1+x)\,\left(q-2\,x+x^2\right)\right)^{1/3}}\,dx$$

Optimal (type 3, 79 leaves, ? steps):

$$\frac{1}{2}\sqrt{3}\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2\,(-1+x)}{\sqrt{3}\,\left((-1+x)\,\left(q-2\,x+x^2\right)\right)^{1/3}}\right]+\frac{1}{4}\,\text{Log}[1-x]-\frac{3}{4}\,\text{Log}\left[1-x+\left((-1+x)\,\left(q-2\,x+x^2\right)\right)^{1/3}\right]$$

Result (type 5, 61 leaves):

$$\frac{3\,(-1+x)\,\left(\frac{q+(-2+x)\,x}{-1+q}\right)^{1/3}\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{(-1+x)^2}{-1+q}\right]}{2\,\left((-1+x)\,\left(q+(-2+x)\,x\right)\right)^{1/3}}$$

Problem 43: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x\left(\left(-1+x\right)\left(q-2qx+x^2\right)\right)^{1/3}}\,dx$$

Optimal (type 3, 118 leaves, ? steps):

$$\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2q^{1/3}(-1+x)}{\sqrt{3}\left((-1+x)\left(q-2qx+x^2\right)\right)^{1/3}}\right]}{2q^{1/3}}+\frac{\text{Log}[1-x]}{4q^{1/3}}+\frac{\text{Log}[x]}{2q^{1/3}}-\frac{3\,\text{Log}\left[-q^{1/3}(-1+x)+\left((-1+x)\left(q-2qx+x^2\right)\right)^{1/3}\right]}{4q^{1/3}}$$

Result (type 5, 72 leaves) :

$$\frac{3(-1+x)\left(-\frac{q-2qx+x^2}{(-1+q)x^2}\right)^{1/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},\frac{q(-1+x)^2}{(-1+q)x^2}\right]}{2\left((-1+x)\left(q-2qx+x^2\right)\right)^{1/3}}$$

Problem 44: Unable to integrate problem.

$$\int \frac{2-(1+k)x}{\left((1-x)x(1-kx)\right)^{1/3}\left(1-(1+k)x\right)}\,dx$$

Optimal (type 3, 111 leaves, ? steps):

$$\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2k^{1/3}x}{\left((1-x)x(1-kx)\right)^{1/3}}}{\sqrt{3}}\right]}{k^{1/3}}+\frac{\text{Log}[x]}{2k^{1/3}}+\frac{\text{Log}\left[1-(1+k)x\right]}{2k^{1/3}}-\frac{3\,\text{Log}\left[-k^{1/3}x+\left((1-x)x(1-kx)\right)^{1/3}\right]}{2k^{1/3}}$$

Result (type 8, 38 leaves) :

$$\int \frac{2-(1+k)x}{\left((1-x)x(1-kx)\right)^{1/3}\left(1-(1+k)x\right)}\,dx$$

Problem 45: Unable to integrate problem.

$$\int \frac{1-kx}{\left(1+(-2+k)x\right)\left((1-x)x(1-kx)\right)^{2/3}}\,dx$$

Optimal (type 3, 176 leaves, ? steps):

$$-\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2^{1/3}\,(1-k\,x)}{(1-k)^{1/3}\,\left((1-x)\,x\,(1-k\,x)\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\,(1-k)^{1/3}}+\frac{\text{Log}\left[1-(2-k)\,x\right]}{2^{2/3}\,(1-k)^{1/3}}+\frac{\text{Log}\left[1-k\,x\right]}{2\times 2^{2/3}\,(1-k)^{1/3}}-\frac{3\,\text{Log}\left[-1+k\,x+2^{2/3}\,(1-k)^{1/3}\,\left((1-x)\,x\,(1-k\,x)\right)^{1/3}\right]}{2\times 2^{2/3}\,(1-k)^{1/3}}$$

Result (type 8, 35 leaves):

$$\int\frac{1-k\,x}{\left(1+(-2+k)\,x\right)\,\left((1-x)\,x\,(1-k\,x)\right)^{2/3}}\,dx$$

Problem 46: Unable to integrate problem.

$$\int\frac{a+b\,x+c\,x^2}{\left(1-x+x^2\right)\,\left(1-x^3\right)^{1/3}}\,dx$$

Optimal (type 3, 326 leaves, ? steps):

$$-\frac{1}{6}\,c\left(2\sqrt{3}\,\text{ArcTan}\left[\frac{1-\frac{2\,x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]+\text{Log}\left[1+\frac{x^2}{(1-x^3)^{2/3}}-\frac{x}{(1-x^3)^{1/3}}\right]-2\,\text{Log}\left[1+\frac{x}{(1-x^3)^{1/3}}\right]\right)+$$

$$\frac{(a-b-2\,c)\left(-2\sqrt{3}\,\text{ArcTan}\left[\frac{1+2^{2/3}\,(1-x^3)^{1/3}}{\sqrt{3}}\right]-3\,\text{Log}\left[2^{1/3}-(1-x^3)^{1/3}\right]\right)}{12\times 2^{1/3}}+$$

$$\frac{(a+b)\left(2\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2\times 2^{1/3}\,(-1+x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]+\text{Log}\left[3-6\,x+6\,x^2-3\,x^3\right]-3\,\text{Log}\left[-2^{1/3}\,(-1+x)+(1-x^3)^{1/3}\right]\right)}{4\times 2^{1/3}}-$$

$$\frac{(a-b-2\,c)\left(2\sqrt{3}\,\text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3}\,x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]-3\,\text{Log}\left[2^{1/3}\,x+(1-x^3)^{1/3}\right]\right)}{12\times 2^{1/3}}$$

Result (type 8, 34 leaves):

$$\int\frac{a+b\,x+c\,x^2}{\left(1-x+x^2\right)\,\left(1-x^3\right)^{1/3}}\,dx$$

Problem 47: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{(3-2\,x)^{11/2}\,\left(1+x+2\,x^2\right)^5}\,dx$$

Optimal (type 3, 407 leaves, 19 steps):

$$-\frac{19255}{395136\,(3-2x)^{9/2}}-\frac{462025}{30118144\,(3-2x)^{7/2}}-\frac{38491}{8605184\,(3-2x)^{5/2}}-\frac{141045}{120472576\,(3-2x)^{3/2}}-\frac{38225}{240945152\,\sqrt{3-2x}}+\frac{x}{28\,(3-2x)^{9/2}\,(1+x+2x^2)^4}+\frac{23+73x}{1176\,(3-2x)^{9/2}\,(1+x+2x^2)^3}+\frac{1387+3049x}{32928\,(3-2x)^{9/2}\,(1+x+2x^2)^2}+\frac{5\,(3049+4377x)}{153664\,(3-2x)^{9/2}\,(1+x+2x^2)}+\frac{5\sqrt{\frac{1}{2}\left(149046503977+40815066112\sqrt{14}\right)}\,\text{ArcTan}\left[\frac{\sqrt{7+2\sqrt{14}}-2\sqrt{3-2x}}{\sqrt{-7+2\sqrt{14}}}\right]}{3373232128}-\frac{5\sqrt{\frac{1}{2}\left(149046503977+40815066112\sqrt{14}\right)}\,\text{ArcTan}\left[\frac{\sqrt{7+2\sqrt{14}}+2\sqrt{3-2x}}{\sqrt{-7+2\sqrt{14}}}\right]}{3373232128}+\frac{5\sqrt{\frac{1}{2}\left(-149046503977+40815066112\sqrt{14}\right)}\,\text{Log}\left[3+\sqrt{14}-\sqrt{7+2\sqrt{14}}\,\sqrt{3-2x}-2x\right]}{6746464256}-\frac{5\sqrt{\frac{1}{2}\left(-149046503977+40815066112\sqrt{14}\right)}\,\text{Log}\left[3+\sqrt{14}+\sqrt{7+2\sqrt{14}}\,\sqrt{3-2x}-2x\right]}{6746464256}$$

Result (type 3, 206 leaves):

$$\frac{1}{30359089152}\left(-\left(\left(14\left(40289347-429812744x+135202154x^2-1073855156x^3+1627773523x^4-1470758860x^5+2888625656x^6-3106712560x^7+2343370048x^8-2443779648x^9+1873554048x^{10}-677249280x^{11}+88070400x^{12}\right)\right)\Big/\left((3-2x)^{9/2}\,(1+x+2x^2)^4\right)\right)+\frac{45\,i\left(53515\,i+284993\sqrt{7}\right)\text{ArcTan}\left[\frac{\sqrt{6-4x}}{\sqrt{-7-i\sqrt{7}}}\right]}{\sqrt{-\frac{1}{2}\,i\left(-7\,i+\sqrt{7}\right)}}-\frac{45\,i\left(-53515\,i+284993\sqrt{7}\right)\text{ArcTan}\left[\frac{\sqrt{6-4x}}{\sqrt{-7+i\sqrt{7}}}\right]}{\sqrt{\frac{1}{2}\,i\left(7\,i+\sqrt{7}\right)}}\right)$$

Problem 48: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(3-2x)^{21/2}\left(1+x+2x^2\right)^{10}}\,dx$$

Optimal (type 3, 648 leaves, 29 steps):

$$\frac{4718120139975}{351733660450816\,(3-2x)^{19/2}}-\frac{815900548375}{629418129227776\,(3-2x)^{17/2}}-\frac{3029508823715}{1555033025150976\,(3-2x)^{15/2}}-\frac{13515743021825}{13476952884641792\,(3-2x)^{13/2}}-\frac{5846828446875}{14513641568075776\,(3-2x)^{11/2}}-\frac{37283626871975}{261245548225363968\,(3-2x)^{9/2}}-\frac{132355162272575}{2844673747342852096\,(3-2x)^{7/2}}-\frac{11557581705725}{812763927812243456\,(3-2x)^{5/2}}-\frac{46601678385075}{11378694989371408384\,(3-2x)^{3/2}}-\frac{24229218097975}{22757389978742816768\,\sqrt{3-2x}}+\frac{x}{63\,(3-2x)^{19/2}\left(1+x+2x^2\right)^9}+\frac{53+173x}{7056\,(3-2x)^{19/2}\left(1+x+2x^2\right)^8}+\frac{8477+21409x}{691488\,(3-2x)^{19/2}\left(1+x+2x^2\right)^7}+\frac{5\,(21409+47471x)}{6453888\,(3-2x)^{19/2}\left(1+x+2x^2\right)^6}+\frac{41\,(47471+92875x)}{90354432\,(3-2x)^{19/2}\left(1+x+2x^2\right)^5}+\frac{41\,(3436375+5677637x)}{5059848192\,(3-2x)^{19/2}\left(1+x+2x^2\right)^4}+\frac{451\,(811091+998691x)}{10119696384\,(3-2x)^{19/2}\left(1+x+2x^2\right)^3}+\frac{451\,(28962039+14627273x)}{283351498752\,(3-2x)^{19/2}\left(1+x+2x^2\right)^2}+\frac{11275\,(14627273-35058731x)}{3966920982528\,(3-2x)^{19/2}\left(1+x+2x^2\right)}+\frac{11275\sqrt{\frac{1}{2}\left(7+2\sqrt{14}\right)}\left(9756589235+2148932869\sqrt{14}\right)\mathrm{ArcTan}\left[\frac{\sqrt{7+2\sqrt{14}}-2\sqrt{3-2x}}{\sqrt{-7+2\sqrt{14}}}\right]}{318603459702399434752}-\frac{11275\sqrt{\frac{1}{2}\left(7+2\sqrt{14}\right)}\left(9756589235+2148932869\sqrt{14}\right)\mathrm{ArcTan}\left[\frac{\sqrt{7+2\sqrt{14}}+2\sqrt{3-2x}}{\sqrt{-7+2\sqrt{14}}}\right]}{318603459702399434752}+\frac{11275\left(9756589235-2148932869\sqrt{14}\right)\sqrt{\frac{1}{2}\left(-7+2\sqrt{14}\right)}\,\mathrm{Log}\left[3+\sqrt{14}-\sqrt{7+2\sqrt{14}}\,\sqrt{3-2x}-2x\right]}{637206919404798869504}-\frac{11275\left(9756589235-2148932869\sqrt{14}\right)\sqrt{\frac{1}{2}\left(-7+2\sqrt{14}\right)}\,\mathrm{Log}\left[3+\sqrt{14}+\sqrt{7+2\sqrt{14}}\,\sqrt{3-2x}-2x\right]}{637206919404798869504}$$

Result (type 3, 662 leaves):

$$-\frac{47\sqrt{3-2x}-23(3-2x)^{3/2}}{4235364\left(14-7(3-2x)+(3-2x)^2\right)^9}-\frac{44193\sqrt{3-2x}-11993(3-2x)^{3/2}}{948721536\left(14-7(3-2x)+(3-2x)^2\right)^8}+$$

$$\frac{5\left(-1574149\sqrt{3-2x}+340449(3-2x)^{3/2}\right)}{185949421056\left(14-7(3-2x)+(3-2x)^2\right)^7}+\frac{5\left(-37938085\sqrt{3-2x}+5912661(3-2x)^{3/2}\right)}{10413167579136\left(14-7(3-2x)+(3-2x)^2\right)^6}-$$

$$\frac{5\left(107643741\sqrt{3-2x}+38010319(3-2x)^{3/2}\right)}{291568692215808\left(14-7(3-2x)+(3-2x)^2\right)^5}-\frac{-132204145097\sqrt{3-2x}+52802422641(3-2x)^{3/2}}{32655693528170496\left(14-7(3-2x)+(3-2x)^2\right)^4}-$$

$$\frac{-4402987778403\sqrt{3-2x}+1406968826615(3-2x)^{3/2}}{914359418788773888\left(14-7(3-2x)+(3-2x)^2\right)^3}-\frac{11\left(-6489356793153\sqrt{3-2x}+1953387138017(3-2x)^{3/2}\right)}{17068042484057112576\left(14-7(3-2x)+(3-2x)^2\right)^2}-$$

$$\frac{55\left(-4751425354423\sqrt{3-2x}+1410835658499(3-2x)^{3/2}\right)}{68272169936228450304\left(14-7(3-2x)+(3-2x)^2\right)}+\frac{1}{5367029731(3-2x)^{19/2}}+\frac{5}{4802079233(3-2x)^{17/2}}+$$

$$\frac{73}{23727920916(3-2x)^{15/2}}+\frac{165}{25705247659(3-2x)^{13/2}}+\frac{2365}{221460595216(3-2x)^{11/2}}+\frac{30349}{1993145356944(3-2x)^{9/2}}+$$

$$\frac{854095}{43406276662336(3-2x)^{7/2}}+\frac{75933}{3100448333024(3-2x)^{5/2}}+\frac{8519225}{260437659974016(3-2x)^{3/2}}+\frac{891605}{12401793332096\sqrt{3-2x}}-$$

$$\frac{11275\left(-34555708553\,i+2148932869\sqrt{7}\right)\text{ArcTan}\left[\frac{\sqrt{2}\sqrt{3-2x}}{\sqrt{-7-i\sqrt{7}}}\right]}{22757389978742816768\sqrt{14\left(-7-i\sqrt{7}\right)}}-\frac{11275\left(34555708553\,i+2148932869\sqrt{7}\right)\text{ArcTan}\left[\frac{\sqrt{2}\sqrt{3-2x}}{\sqrt{-7+i\sqrt{7}}}\right]}{22757389978742816768\sqrt{14\left(-7+i\sqrt{7}\right)}}$$

Problem 49: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{(3-2x)^{41/2}\left(1+x+2x^2\right)^{20}}\,dx$$

Optimal (type 3, 1058 leaves, 49 steps):

$$-\frac{13056959628363355534285785425}{106924014357253562723941220352(3-2x)^{39/2}}-\frac{3948194343291401740321996415}{202881463139404195937734623232(3-2x)^{37/2}}-$$

$$\frac{304688229262620222736480811}{537361713180043545997243056128(3-2x)^{35/2}}+\frac{2124315846756567455653862925}{1688851098565851144562763890688(3-2x)^{33/2}}+$$

$$\frac{47657515074514118796095929535}{66632852434325399703658138959872(3-2x)^{31/2}}+\frac{34911619993974714062172751985}{124667917457770102671360389021696(3-2x)^{29/2}}+$$

$$\frac{149\,066\,309\,808\,794\,760\,843\,017\,404\,825}{1\,624\,981\,820\,656\,451\,683\,095\,663\,001\,731\,072\,\left(3-2\,x\right)^{27/2}}+\frac{15\,848\,613\,964\,169\,066\,543\,734\,380\,171}{601\,845\,118\,761\,648\,771\,516\,912\,222\,863\,360\,\left(3-2\,x\right)^{25/2}}+$$

$$\frac{11\,155\,168\,222\,970\,774\,232\,376\,891\,145}{1\,685\,166\,332\,532\,616\,560\,247\,354\,224\,017\,408\,\left(3-2\,x\right)^{23/2}}+\frac{14\,011\,818\,498\,091\,020\,272\,474\,956\,375}{10\,110\,997\,995\,195\,699\,361\,484\,125\,344\,104\,448\,\left(3-2\,x\right)^{21/2}}+$$

$$\frac{173\,441\,368\,149\,804\,378\,661\,935\,869\,705}{896\,508\,488\,907\,352\,010\,051\,592\,447\,177\,261\,056\,\left(3-2\,x\right)^{19/2}}-\frac{22\,724\,090\,823\,469\,905\,152\,713\,519\,545}{1\,604\,278\,348\,571\,050\,965\,355\,481\,221\,264\,572\,416\,\left(3-2\,x\right)^{17/2}}-$$

$$\frac{101\,190\,274\,412\,779\,618\,678\,573\,275\,245}{3\,963\,511\,214\,116\,714\,149\,701\,777\,134\,888\,943\,616\,\left(3-2\,x\right)^{15/2}}-\frac{460\,503\,190\,416\,958\,283\,087\,439\,337\,135}{34\,350\,430\,522\,344\,855\,964\,082\,068\,502\,370\,844\,672\,\left(3-2\,x\right)^{13/2}}-$$

$$\frac{2\,211\,619\,588\,790\,911\,794\,826\,342\,607\,495}{406\,920\,484\,649\,315\,986\,036\,049\,119\,181\,931\,544\,576\,\left(3-2\,x\right)^{11/2}}-\frac{143\,401\,467\,550\,777\,247\,627\,940\,437\,025}{73\,985\,542\,663\,511\,997\,461\,099\,839\,851\,260\,280\,832\,\left(3-2\,x\right)^{9/2}}-$$

$$\frac{4\,611\,053\,278\,117\,143\,010\,907\,562\,317\,585}{7\,250\,583\,181\,024\,175\,751\,187\,784\,305\,423\,507\,521\,536\,\left(3-2\,x\right)^{7/2}}-\frac{405\,965\,372\,440\,630\,510\,720\,926\,890\,227}{2\,071\,595\,194\,578\,335\,928\,910\,795\,515\,835\,287\,863\,296\,\left(3-2\,x\right)^{5/2}}-$$

$$\frac{4\,986\,681\,479\,187\,781\,853\,417\,316\,522\,775}{87\,006\,998\,172\,290\,109\,014\,253\,411\,665\,082\,090\,258\,432\,\left(3-2\,x\right)^{3/2}}-\frac{927\,027\,754\,781\,476\,746\,208\,047\,620\,505}{58\,004\,665\,448\,193\,406\,009\,502\,274\,443\,388\,060\,172\,288\,\sqrt{3-2\,x}}+$$

$$\frac{x}{133\,\left(3-2\,x\right)^{39/2}\,\left(1+x+2\,x^2\right)^{19}}+\frac{113+373\,x}{33\,516\,\left(3-2\,x\right)^{39/2}\,\left(1+x+2\,x^2\right)^{18}}+\frac{40\,657+107\,329\,x}{7\,976\,808\,\left(3-2\,x\right)^{39/2}\,\left(1+x+2\,x^2\right)^{17}}+\frac{5\,\left(751\,303+1\,831\,285\,x\right)}{595\,601\,664\,\left(3-2\,x\right)^{39/2}\,\left(1+x+2\,x^2\right)^{16}}+$$

$$\frac{184\,959\,785+429\,411\,497\,x}{25\,015\,269\,888\,\left(3-2\,x\right)^{39/2}\,\left(1+x+2\,x^2\right)^{15}}+\frac{41\,652\,915\,209+92\,630\,823\,167\,x}{4\,902\,992\,898\,048\,\left(3-2\,x\right)^{39/2}\,\left(1+x+2\,x^2\right)^{14}}+\frac{2\,871\,555\,518\,177+6\,100\,156\,355\,517\,x}{297\,448\,235\,814\,912\,\left(3-2\,x\right)^{39/2}\,\left(1+x+2\,x^2\right)^{13}}+$$

$$\frac{77\,559\,130\,805\,859+156\,274\,047\,129\,113\,x}{7\,138\,757\,659\,557\,888\,\left(3-2\,x\right)^{39/2}\,\left(1+x+2\,x^2\right)^{12}}+\frac{5\,\left(2\,656\,658\,801\,194\,921+5\,020\,880\,176\,134\,289\,x\right)}{1\,099\,368\,679\,571\,914\,752\,\left(3-2\,x\right)^{39/2}\,\left(1+x+2\,x^2\right)^{11}}+$$

$$\frac{45\,187\,921\,585\,208\,601+78\,752\,911\,037\,377\,255\,x}{3\,420\,258\,114\,223\,734\,784\,\left(3-2\,x\right)^{39/2}\,\left(1+x+2\,x^2\right)^{10}}+\frac{6\,063\,974\,149\,878\,048\,635+9\,477\,172\,618\,423\,641\,847\,x}{430\,952\,522\,392\,190\,582\,784\,\left(3-2\,x\right)^{39/2}\,\left(1+x+2\,x^2\right)^{9}}+$$

$$\frac{691\,833\,601\,144\,925\,854\,831+919\,498\,192\,874\,055\,581\,221\,x}{48\,266\,682\,507\,925\,345\,271\,808\,\left(3-2\,x\right)^{39/2}\,\left(1+x+2\,x^2\right)^{8}}+\frac{23\,\left(919\,498\,192\,874\,055\,581\,221+908\,287\,136\,092\,467\,468\,517\,x\right)}{1\,576\,711\,628\,592\,227\,945\,545\,728\,\left(3-2\,x\right)^{39/2}\,\left(1+x+2\,x^2\right)^{7}}+$$

$$\frac{115\,\left(908\,287\,136\,092\,467\,468\,517+298\,281\,884\,944\,522\,225\,747\,x\right)}{10\,187\,982\,830\,903\,626\,725\,064\,704\,\left(3-2\,x\right)^{39/2}\,\left(1+x+2\,x^2\right)^{6}}+\frac{23\,\left(2\,599\,313\,568\,802\,265\,110\,081-10\,426\,142\,448\,623\,187\,379\,187\,x\right)}{20\,375\,965\,661\,807\,253\,450\,129\,408\,\left(3-2\,x\right)^{39/2}\,\left(1+x+2\,x^2\right)^{5}}-$$

$$\frac{23\,\left(10\,426\,142\,448\,623\,187\,379\,187+27\,513\,723\,463\,194\,262\,383\,705\,x\right)}{20\,018\,492\,580\,021\,161\,284\,337\,664\,\left(3-2\,x\right)^{39/2}\,\left(1+x+2\,x^2\right)^{4}}-\frac{115\,\left(26\,513\,224\,428\,169\,016\,478\,843+30\,673\,415\,406\,553\,789\,342\,019\,x\right)}{76\,434\,244\,396\,444\,433\,994\,743\,808\,\left(3-2\,x\right)^{39/2}\,\left(1+x+2\,x^2\right)^{3}}-$$

$$\frac{115\,\left(88\,411\,609\,113\,007\,981\,044\,643-5\,712\,269\,536\,245\,152\,162\,963\,x\right)}{125\,891\,696\,652\,967\,303\,050\,166\,272\,\left(3-2\,x\right)^{39/2}\,\left(1+x+2\,x^2\right)^{2}}+\frac{115\,\left(28\,561\,347\,681\,225\,760\,814\,815+965\,934\,812\,839\,019\,490\,346\,107\,x\right)}{195\,831\,528\,126\,838\,026\,966\,925\,312\,\left(3-2\,x\right)^{39/2}\,\left(1+x+2\,x^2\right)}+$$

$$\left(115 \sqrt{\frac{1}{2}\left(7+2\sqrt{14}\right)} \left(30297118912219360725028693061+8061110911143276053983022787\sqrt{14}\right) \text{ArcTan}\left[\frac{\sqrt{7+2\sqrt{14}}-2\sqrt{3-2x}}{\sqrt{-7+2\sqrt{14}}}\right]\right) \Big/$$
$$812065316274707684133031842207432842412032 -$$
$$\left(115 \sqrt{\frac{1}{2}\left(7+2\sqrt{14}\right)} \left(30297118912219360725028693061+8061110911143276053983022787\sqrt{14}\right) \text{ArcTan}\left[\frac{\sqrt{7+2\sqrt{14}}+2\sqrt{3-2x}}{\sqrt{-7+2\sqrt{14}}}\right]\right) \Big/$$
$$812065316274707684133031842207432842412032 + \left(115 \left(30297118912219360725028693061-8061110911143276053983022787\sqrt{14}\right)\right.$$
$$\left.\sqrt{\frac{1}{2}\left(-7+2\sqrt{14}\right)} \text{Log}\left[3+\sqrt{14}-\sqrt{7+2\sqrt{14}}\sqrt{3-2x}-2x\right]\right) \Big/ 1624130632549415368266063684414865684824064 -$$
$$\left(115 \left(30297118912219360725028693061-8061110911143276053983022787\sqrt{14}\right) \sqrt{\frac{1}{2}\left(-7+2\sqrt{14}\right)}\right.$$
$$\left.\text{Log}\left[3+\sqrt{14}+\sqrt{7+2\sqrt{14}}\sqrt{3-2x}-2x\right]\right) \Big/ 1624130632549415368266063684414865684824064$$

Result (type 3, 1242 leaves) :

$$-\frac{393\sqrt{3-2x}+287(3-2x)^{3/2}}{150276832468\left(14-7(3-2x)+(3-2x)^2\right)^{19}}-\frac{-4226921\sqrt{3-2x}+1313129(3-2x)^{3/2}}{75739523563872\left(14-7(3-2x)+(3-2x)^2\right)^{18}}-$$
$$\frac{-3401932701\sqrt{3-2x}+760755809(3-2x)^{3/2}}{36052013216403072\left(14-7(3-2x)+(3-2x)^2\right)^{17}}-\frac{5\left(-146490500023\sqrt{3-2x}+16144709919(3-2x)^{3/2}\right)}{16151301920948576256\left(14-7(3-2x)+(3-2x)^2\right)^{16}}-$$
$$\frac{9745709632283\sqrt{3-2x}-4557912048927(3-2x)^{3/2}}{452236453786560135168\left(14-7(3-2x)+(3-2x)^2\right)^{15}}-\frac{435856117815771\sqrt{3-2x}-123609208162571(3-2x)^{3/2}}{9330352099175345946624\left(14-7(3-2x)+(3-2x)^2\right)^{14}}-$$
$$\frac{127435522656997631\sqrt{3-2x}-31270302414674811(3-2x)^{3/2}}{3396248164099825924571136\left(14-7(3-2x)+(3-2x)^2\right)^{13}}+\frac{5\left(-1540359167602841319\sqrt{3-2x}+342026557757088031(3-2x)^{3/2}\right)}{380379794379180503551967232\left(14-7(3-2x)+(3-2x)^2\right)^{12}}+$$
$$\frac{5\left(-21084628139481190687\sqrt{3-2x}+4158669924550257827(3-2x)^{3/2}\right)}{13017441852087510566000656384\left(14-7(3-2x)+(3-2x)^2\right)^{11}}-$$

$$\frac{1633293973597342712581\sqrt{3-2x}-237080744154193384005(3-2x)^{3/2}}{728976743716900591696036757504\left(14-7(3-2x)+(3-2x)^2\right)^{10}}-$$

$$\frac{7350432513431022017155\sqrt{3-2x}+5131564318471376538977(3-2x)^{3/2}}{61234046472219649702467087630336\left(14-7(3-2x)+(3-2x)^2\right)^{9}}-$$

$$\frac{-113207386492327172550771\sqrt{3-2x}+43421160367342900895387(3-2x)^{3/2}}{279927069587289827211278114881536\left(14-7(3-2x)+(3-2x)^2\right)^{8}}-$$

$$\frac{-22463796720502183624842107\sqrt{3-2x}+7094978194424786431173663(3-2x)^{3/2}}{54865705639108806133410510516781056\left(14-7(3-2x)+(3-2x)^2\right)^{7}}-$$

$$\frac{5\left(-186257412289925530757362143\sqrt{3-2x}+55540178588722046667113711(3-2x)^{3/2}\right)}{3072479515790093143470988588939739136\left(14-7(3-2x)+(3-2x)^2\right)^{6}}-$$

$$\frac{23\left(-255056047077847659080618951\sqrt{3-2x}+74443988473272328189316355(3-2x)^{3/2}\right)}{28676475480707536005729226830104231936\left(14-7(3-2x)+(3-2x)^2\right)^{5}}-$$

$$\frac{23\left(-1110057788286806589656260577\sqrt{3-2x}+321533953909984640923113289(3-2x)^{3/2}\right)}{188927367872896707802451376763039645696\left(14-7(3-2x)+(3-2x)^2\right)^{4}}-$$

$$\frac{23\left(-4820387670797872511726954245\sqrt{3-2x}+1394304490531377203111252689(3-2x)^{3/2}\right)}{1220761453947947958108147357545794633728\left(14-7(3-2x)+(3-2x)^2\right)^{3}}-$$

$$\frac{23\left(-17490402570151108581128226213\sqrt{3-2x}+5072167085782230110284731077(3-2x)^{3/2}\right)}{6214785583735007786732386547505863589888\left(14-7(3-2x)+(3-2x)^2\right)^{2}}-$$

$$\frac{115\left(-82782386138609724168863115877\sqrt{3-2x}+24217623575858523510208130121(3-2x)^{3/2}\right)}{174013996344580218028506823330164180516864\left(14-7(3-2x)+(3-2x)^2\right)}+$$

$$\frac{1}{3111898385606868039(3-2x)^{39/2}}+\frac{10}{2952313853011644037(3-2x)^{37/2}}+\frac{143}{7819642097165976098(3-2x)^{35/2}}+$$

$$\frac{355}{5266289575642392066(3-2x)^{33/2}}+\frac{52865}{277038748585308867472(3-2x)^{31/2}}+\frac{14333}{32395660116830472406(3-2x)^{29/2}}+$$

$$\frac{1478345}{1689042692987850837168(3-2x)^{27/2}}+\frac{475387}{312785683886639043920(3-2x)^{25/2}}+\frac{16575515}{7006399319060714583808(3-2x)^{23/2}}+$$

$$\frac{246866015}{73567192850137503129984\,(3-2x)^{21/2}}+\frac{8192823353}{1863702218870150079292928\,(3-2x)^{19/2}}+\frac{8972680075}{1667523037936450070946304\,(3-2x)^{17/2}}+$$

$$\frac{102495360575}{16479051198430800701116416\,(3-2x)^{15/2}}+\frac{122484655975}{17852305464966700759542784\,(3-2x)^{13/2}}+\frac{10815878546425}{1480368099325700262983624704\,(3-2x)^{11/2}}+$$

$$\frac{769045155125}{100934188590388654294338048\,(3-2x)^{9/2}}+\frac{838467657280275}{105509871806486273289014706176\,(3-2x)^{7/2}}+$$

$$\frac{9270470094105}{1076631344964145645806272512\,(3-2x)^{5/2}}+\frac{320421783064625}{30145677658996078082575630336\,(3-2x)^{3/2}}+\frac{683151246370725}{30145677658996078082575630336\,\sqrt{3-2x}}-$$

$$\left(115\left(-117022014202441653827938545631\,i+8061110911143276053983022787\sqrt{7}\right)\text{ArcTan}\left[\frac{\sqrt{2}\,\sqrt{3-2x}}{\sqrt{-7-i\sqrt{7}}}\right]\right)\Big/$$

$$\left(58004665448193406009502274443388060172288\sqrt{14\left(-7-i\sqrt{7}\right)}\right)-$$

$$\left(115\left(117022014202441653827938545631\,i+8061110911143276053983022787\sqrt{7}\right)\text{ArcTan}\left[\frac{\sqrt{2}\,\sqrt{3-2x}}{\sqrt{-7+i\sqrt{7}}}\right]\right)\Big/$$

$$\left(58004665448193406009502274443388060172288\sqrt{14\left(-7+i\sqrt{7}\right)}\right)$$

Problem 50: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{1}{\left(3-2x+x^2\right)^{11/2}\left(1+x+2x^2\right)^5}\,dx$$

Optimal (type 3, 378 leaves, 14 steps) :

$$-\frac{3450497-2004270 x}{123480000\left(3-2 x+x^2\right)^{9/2}}-\frac{4878869-2578034 x}{411600000\left(3-2 x+x^2\right)^{7/2}}-\frac{30316369-15043110 x}{6860000000\left(3-2 x+x^2\right)^{5/2}}-\frac{63043297-29625922 x}{41160000000\left(3-2 x+x^2\right)^{3/2}}-\frac{31(7434109-3088870 x)}{411600000000 \sqrt{3-2 x+x^2}}-\frac{1-10 x}{280\left(3-2 x+x^2\right)^{9/2}\left(1+x+2 x^2\right)^4}+\frac{28+67 x}{1050\left(3-2 x+x^2\right)^{9/2}\left(1+x+2 x^2\right)^3}+\frac{5485+8878 x}{117600\left(3-2 x+x^2\right)^{9/2}\left(1+x+2 x^2\right)^2}+\frac{3(8822+8233 x)}{343000\left(3-2 x+x^2\right)^{9/2}\left(1+x+2 x^2\right)}+\frac{1}{137200000000} \sqrt{\frac{1}{70}\left(151363871237318045+110320475741093888 \sqrt{2}\right)} \text{ArcTan}\left[\frac{1}{\sqrt{3-2 x+x^2}} \sqrt{\frac{5}{7\left(151363871237318045+110320475741093888 \sqrt{2}\right)}}\left(308108167+312239803 \sqrt{2}+\left(932587773+620347970 \sqrt{2}\right) x\right)\right]-\frac{1}{137200000000} \sqrt{\frac{1}{70}\left(-151363871237318045+110320475741093888 \sqrt{2}\right)} \text{ArcTanh}\left[\frac{1}{\sqrt{3-2 x+x^2}} \sqrt{\frac{5}{7\left(-151363871237318045+110320475741093888 \sqrt{2}\right)}}\left(308108167-312239803 \sqrt{2}+\left(932587773-620347970 \sqrt{2}\right) x\right)\right]$$

Result (type 3, 1236 leaves) :

$$\sqrt{3-2 x+x^2}\left(\frac{1}{225000\left(3-2 x+x^2\right)^5}+\frac{1+2 x}{350000\left(3-2 x+x^2\right)^4}+\frac{3(-38+45 x)}{8750000\left(3-2 x+x^2\right)^3}+\frac{-2003+1198 x}{52500000\left(3-2 x+x^2\right)^2}+\frac{-97229+29420 x}{1050000000\left(3-2 x+x^2\right)}+\frac{-797-1998 x}{28000000\left(1+x+2 x^2\right)^4}+\frac{-14087-5995 x}{105000000\left(1+x+2 x^2\right)^3}+\frac{-795589+1892994 x}{11760000000\left(1+x+2 x^2\right)^2}+\frac{3035369+14037055 x}{34300000000\left(1+x+2 x^2\right)}\right)+\frac{1}{68600000000 \sqrt{70\left(-5+\mathrm{i} \sqrt{7}\right)}}\left(310173985 \mathrm{i}+44900803 \sqrt{7}\right) \text{ArcTan}\Big[\Big(9627448535205165+357977536529228045 \mathrm{i} \sqrt{7}-2892591314086740000 x+36106220736881480 \mathrm{i} \sqrt{7} x+464983088285203040 x^2-1038569725622524380 \mathrm{i} \sqrt{7} x^2+12836598046940220 x^3+328748064746064540 \mathrm{i} \sqrt{7} x^3-487447134867348425 x^4-428071291440525685 \mathrm{i} \sqrt{7} x^4+358541546158555136 \mathrm{i} \sqrt{10\left(-5+\mathrm{i} \sqrt{7}\right)} \sqrt{3-2 x+x^2}+220640951482187776 \mathrm{i} \sqrt{10\left(-5+\mathrm{i} \sqrt{7}\right)} x \sqrt{3-2 x+x^2}+579182497640742912 \mathrm{i} \sqrt{10\left(-5+\mathrm{i} \sqrt{7}\right)} x^2 \sqrt{3-2 x+x^2}-275801189352734720 \mathrm{i} \sqrt{10\left(-5+\mathrm{i} \sqrt{7}\right)} x^3 \sqrt{3-2 x+x^2}\Big)\Big/\Big(4321741285513437647 \mathrm{i}+827387564543169945 \sqrt{7}+3694994885631086104 \mathrm{i} x+285423303382928480 \sqrt{7} x+5471192788852131980 \mathrm{i} x^2-70525532316488480 \sqrt{7} x^2-6268363351511187532 \mathrm{i} x^3+$$

$$137\,879\,256\,656\,321\,740\,\sqrt{7}\,x^3+2\,092\,254\,277\,956\,040\,633\,i\,x^4+70\,562\,873\,851\,568\,315\,\sqrt{7}\,x^4\Big)\Big]-$$

$$\frac{1}{68\,600\,000\,000\,\sqrt{70\left(5+i\sqrt{7}\right)}}\,i\left(-310\,173\,985\,i+44\,900\,803\,\sqrt{7}\right)\text{ArcTan}\Big[\Big(35\,\Big(15\,210\,275\,631\,276\,955\,i+23\,639\,644\,701\,233\,427\,\sqrt{7}-$$

$$80\,355\,173\,705\,781\,000\,i\,x+8\,154\,951\,525\,226\,528\,\sqrt{7}\,x+32\,801\,021\,588\,957\,180\,i\,x^2-2\,015\,015\,209\,042\,528\,\sqrt{7}\,x^2-$$

$$22\,632\,774\,169\,109\,180\,i\,x^3+3\,939\,407\,333\,037\,764\,\sqrt{7}\,x^3-9\,346\,476\,174\,243\,955\,i\,x^4+2\,016\,082\,110\,044\,809\,\sqrt{7}\,x^4\Big)\Big)\Big/$$

$$\Big(-9\,627\,448\,535\,205\,165+357\,977\,536\,529\,228\,045\,i\,\sqrt{7}+2\,892\,591\,314\,086\,740\,000\,x+36\,106\,220\,736\,881\,480\,i\,\sqrt{7}\,x-$$

$$464\,983\,088\,285\,203\,040\,x^2-1\,038\,569\,725\,622\,524\,380\,i\,\sqrt{7}\,x^2-12\,836\,598\,046\,940\,220\,x^3+328\,748\,064\,746\,064\,540\,i\,\sqrt{7}\,x^3+$$

$$487\,447\,134\,867\,348\,425\,x^4-428\,071\,291\,440\,525\,685\,i\,\sqrt{7}\,x^4-27\,580\,118\,935\,273\,472\,i\,\sqrt{70\left(5+i\sqrt{7}\right)}\,\sqrt{3-2x+x^2}-$$

$$27\,580\,118\,935\,273\,472\,i\,\sqrt{70\left(5+i\sqrt{7}\right)}\,x^2\sqrt{3-2x+x^2}+55\,160\,237\,870\,546\,944\,i\,\sqrt{70\left(5+i\sqrt{7}\right)}\,x^3\sqrt{3-2x+x^2}\Big)\Big]-$$

$$\frac{\left(-310\,173\,985\,i+44\,900\,803\,\sqrt{7}\right)\text{Log}\Big[\left(-i+\sqrt{7}-4\,i\,x\right)^2\left(i+\sqrt{7}+4\,i\,x\right)^2\Big]}{137\,200\,000\,000\,\sqrt{70\left(5+i\sqrt{7}\right)}}+$$

$$\frac{i\left(310\,173\,985\,i+44\,900\,803\,\sqrt{7}\right)\text{Log}\Big[\left(-i+\sqrt{7}-4\,i\,x\right)^2\left(i+\sqrt{7}+4\,i\,x\right)^2\Big]}{137\,200\,000\,000\,\sqrt{70\left(-5+i\sqrt{7}\right)}}-$$

$$\Big(i\left(310\,173\,985\,i+44\,900\,803\,\sqrt{7}\right)\text{Log}\Big[\left(1+x+2x^2\right)$$

$$\Big(-13\,i+15\sqrt{7}+22\,i\,x-10\sqrt{7}\,x+9\,i\,x^2+5\sqrt{7}\,x^2+i\,\sqrt{70\left(-5+i\sqrt{7}\right)}\,\sqrt{3-2x+x^2}-i\,\sqrt{70\left(-5+i\sqrt{7}\right)}\,x\sqrt{3-2x+x^2}\Big)\Big]\Big)\Big/$$

$$\Big(137\,200\,000\,000\,\sqrt{70\left(-5+i\sqrt{7}\right)}\Big)+\Big(\left(-310\,173\,985\,i+44\,900\,803\,\sqrt{7}\right)\text{Log}\Big[\left(1+x+2x^2\right)\Big(-163\,i+15\sqrt{7}+122\,i\,x-10\sqrt{7}\,x-$$

$$41\,i\,x^2+5\sqrt{7}\,x^2-13\,i\,\sqrt{10\left(5+i\sqrt{7}\right)}\,\sqrt{3-2x+x^2}+5\,i\,\sqrt{10\left(5+i\sqrt{7}\right)}\,x\sqrt{3-2x+x^2}\Big)\Big]\Big)\Big/\Big(137\,200\,000\,000\,\sqrt{70\left(5+i\sqrt{7}\right)}\Big)$$

Problem 51: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(3-2x+x^2\right)^{21/2}\left(1+x+2x^2\right)^{10}}\,dx$$

Optimal (type 3, 638 leaves, 24 steps):

$$\frac{37\,358\,055\,634\,422\,583-14\,024\,622\,879\,097\,678\,x}{1\,840\,124\,479\,200\,000\,000\,\left(3-2\,x+x^2\right)^{19/2}}+\frac{476\,849\,951\,294\,984\,711-125\,181\,871\,472\,148\,210\,x}{104\,273\,720\,488\,000\,000\,000\,\left(3-2\,x+x^2\right)^{17/2}}+$$
$$\frac{7\,851\,758\,375\,483\,333\,511+1\,942\,164\,996\,204\,584\,234\,x}{15\,641\,058\,073\,200\,000\,000\,000\,\left(3-2\,x+x^2\right)^{15/2}}-\frac{11\,\left(7\,502\,325\,106\,308\,201\,089-7\,813\,986\,379\,726\,516\,886\,x\right)}{406\,667\,509\,903\,200\,000\,000\,000\,\left(3-2\,x+x^2\right)^{13/2}}-$$
$$\frac{3\,\left(69\,053\,268\,515\,296\,359\,011-44\,840\,736\,195\,018\,286\,006\,x\right)}{1\,147\,010\,925\,368\,000\,000\,000\,000\,\left(3-2\,x+x^2\right)^{11/2}}-\frac{838\,519\,439\,380\,295\,335\,657-466\,189\,390\,555\,853\,643\,870\,x}{9\,384\,634\,843\,920\,000\,000\,000\,000\,\left(3-2\,x+x^2\right)^{9/2}}-$$
$$\frac{1\,117\,646\,664\,729\,238\,460\,189-568\,839\,749\,685\,437\,871\,554\,x}{31\,282\,116\,146\,400\,000\,000\,000\,000\,\left(3-2\,x+x^2\right)^{7/2}}-\frac{6\,551\,405\,511\,565\,449\,301\,689-3\,127\,298\,559\,983\,309\,301\,910\,x}{521\,368\,602\,440\,000\,000\,000\,000\,000\,\left(3-2\,x+x^2\right)^{5/2}}-$$
$$\frac{4\,179\,039\,782\,398\,459\,850\,819-1\,886\,993\,445\,589\,652\,402\,694\,x}{1\,042\,737\,204\,880\,000\,000\,000\,000\,000\,\left(3-2\,x+x^2\right)^{3/2}}-\frac{12\,105\,495\,874\,518\,671\,061\,833-5\,117\,656\,435\,043\,679\,338\,190\,x}{10\,427\,372\,048\,800\,000\,000\,000\,000\,000\,\sqrt{3-2\,x+x^2}}-$$
$$\frac{1-10\,x}{630\,\left(3-2\,x+x^2\right)^{19/2}\left(1+x+2\,x^2\right)^9}+\frac{887+2218\,x}{88\,200\,\left(3-2\,x+x^2\right)^{19/2}\left(1+x+2\,x^2\right)^8}+\frac{14\,453+29\,371\,x}{1\,080\,450\,\left(3-2\,x+x^2\right)^{19/2}\left(1+x+2\,x^2\right)^7}+$$
$$\frac{8\,837\,931+17\,459\,234\,x}{605\,052\,000\,\left(3-2\,x+x^2\right)^{19/2}\left(1+x+2\,x^2\right)^6}+\frac{447\,940\,041+813\,432\,205\,x}{26\,471\,025\,000\,\left(3-2\,x+x^2\right)^{19/2}\left(1+x+2\,x^2\right)^5}+$$
$$\frac{592\,729\,157\,441+911\,061\,463\,974\,x}{29\,647\,548\,000\,000\,\left(3-2\,x+x^2\right)^{19/2}\left(1+x+2\,x^2\right)^4}+\frac{277\,010\,166\,219+310\,705\,340\,015\,x}{12\,353\,145\,000\,000\,\left(3-2\,x+x^2\right)^{19/2}\left(1+x+2\,x^2\right)^3}+$$
$$\frac{5\,488\,221\,294\,349+1\,384\,103\,301\,166\,x}{276\,710\,448\,000\,000\,\left(3-2\,x+x^2\right)^{19/2}\left(1+x+2\,x^2\right)^2}-\frac{37\,857\,197\,792\,117+146\,548\,895\,467\,025\,x}{2\,421\,216\,420\,000\,000\,\left(3-2\,x+x^2\right)^{19/2}\left(1+x+2\,x^2\right)}+\frac{1}{32\,282\,885\,600\,000\,000\,000\,000\,000}$$
$$\sqrt{\left(\frac{1}{70}\left(81\,042\,225\,921\,274\,689\,605\,478\,944\,797\,800\,854\,846\,405+57\,305\,922\,523\,001\,707\,126\,026\,363\,878\,666\,500\,308\,992\,\sqrt{2}\right)\right)}$$
$$\text{ArcTan}\Big[\frac{1}{\sqrt{3-2\,x+x^2}}\sqrt{\left(5\Big/\left(7\left(81\,042\,225\,921\,274\,689\,605\,478\,944\,797\,800\,854\,846\,405+57\,305\,922\,523\,001\,707\,126\,026\,363\,878\,666\,500\,308\,992\,\sqrt{2}\right)\right)\right)}$$
$$\Big(272\,944\,589\,523\,248\,381\,749+191\,941\,026\,386\,645\,109\,841\,\sqrt{2}+$$
$$\left(656\,826\,642\,296\,538\,601\,431+464\,885\,615\,909\,893\,491\,590\,\sqrt{2}\right)x\Big)\Big]-\frac{1}{32\,282\,885\,600\,000\,000\,000\,000\,000}$$
$$\sqrt{\left(\frac{1}{70}\left(-81\,042\,225\,921\,274\,689\,605\,478\,944\,797\,800\,854\,846\,405+57\,305\,922\,523\,001\,707\,126\,026\,363\,878\,666\,500\,308\,992\,\sqrt{2}\right)\right)}\,\text{ArcTanh}\Big[$$
$$\frac{1}{\sqrt{3-2\,x+x^2}}\sqrt{\left(5\Big/\left(7\left(-81\,042\,225\,921\,274\,689\,605\,478\,944\,797\,800\,854\,846\,405+57\,305\,922\,523\,001\,707\,126\,026\,363\,878\,666\,500\,308\,992\,\sqrt{2}\right)\right)\right)}$$
$$\left(272\,944\,589\,523\,248\,381\,749-191\,941\,026\,386\,645\,109\,841\,\sqrt{2}+\left(656\,826\,642\,296\,538\,601\,431-464\,885\,615\,909\,893\,491\,590\,\sqrt{2}\right)x\right)\Big]$$

Result (type 3, 1431 leaves):

$$\sqrt{3-2\,x+x^2}\,\left(\frac{1-x}{11\,875\,000\,000\,\left(3-2\,x+x^2\right)^{10}}+\frac{265-113\,x}{403\,750\,000\,000\,\left(3-2\,x+x^2\right)^9}+\frac{82\,361-4841\,x}{60\,562\,500\,000\,000\,\left(3-2\,x+x^2\right)^8}+\right.$$

$$\frac{1062937+1642511x}{1574625000000000\left(3-2x+x^2\right)^7}+\frac{7\left(-678331+833371x\right)}{2220625000000000\left(3-2x+x^2\right)^6}+\frac{7\left(-73161291+43964675x\right)}{90843750000000000\left(3-2x+x^2\right)^5}+$$

$$\frac{-1340879383+430593031x}{181687500000000000\left(3-2x+x^2\right)^4}-\frac{11\left(1626125723+112950205x\right)}{3028125000000000000\left(3-2x+x^2\right)^3}-\frac{11\left(3311570647+15286717673x\right)}{36337500000000000000\left(3-2x+x^2\right)^2}-$$

$$\frac{11\left(-411521923277+484788625685x\right)}{363375000000000000000\left(3-2x+x^2\right)}+\frac{251943+221770x}{6300000000000\left(1+x+2x^2\right)^9}-\frac{73\left(-888423+1604678x\right)}{882000000000000\left(1+x+2x^2\right)^8}+$$

$$\frac{-2596903794-4965311863x}{10804500000000000\left(1+x+2x^2\right)^7}+\frac{-539608494637-334647150510x}{1210104000000000000\left(1+x+2x^2\right)^6}+\frac{-40800462989458+56711874696335x}{264710250000000000000\left(1+x+2x^2\right)^5}+$$

$$\frac{42018358198215561+129196597088670934x}{296475480000000000000000\left(1+x+2x^2\right)^4}+\frac{62819559864314747+169630389653846945x}{370594350000000000000000\left(1+x+2x^2\right)^3}+$$

$$\left.\frac{1082422109196374795+4797048907791526114x}{8301313440000000000000000\left(1+x+2x^2\right)^2}+\frac{65571203144429922747+367152793968978953465x}{363182463000000000000000000\left(1+x+2x^2\right)}\right)+$$

$$\frac{1}{16141442800000000000000000\sqrt{70\left(-5+i\sqrt{7}\right)}}\left(232442807954946745795\,i+21634177831191924841\sqrt{7}\right)$$

$$\text{ArcTan}\Big[\Big(-135063738860435016899586558948733259113515+188630894626466690216855285995045889396405\,i\sqrt{7}-$$
$$1506241361872688008559268776761430483700000\,x-105711500937472192718115651350352447938680\,i\sqrt{7}\,x+$$
$$491153540508443587025809789813541985707360\,x^2-460764064177139993399975100872663310399420\,i\sqrt{7}\,x^2-$$
$$180084985147246689199448745264977678818020\,x^3+197868296377913870863837680953446009396860\,i\sqrt{7}\,x^3-$$
$$176004816500761880926774485599831047775825\,x^4-207342833228459577163557043035558264835165\,i\sqrt{7}\,x^4+$$
$$186244248199755548159585682605666126004224\,i\sqrt{10\left(-5+i\sqrt{7}\right)}\sqrt{3-2x+x^2}+$$
$$114611845046003414252052727757333000617984\,i\sqrt{10\left(-5+i\sqrt{7}\right)}\,x\sqrt{3-2x+x^2}+$$
$$300856093245758962411638410362999126622208\,i\sqrt{10\left(-5+i\sqrt{7}\right)}\,x^2\sqrt{3-2x+x^2}-$$
$$143264806307504267815065909696666250772480\,i\sqrt{10\left(-5+i\sqrt{7}\right)}\,x^3\sqrt{3-2x+x^2}\Big)\Big/$$
$$\Big(2368773290838836979864678493023884746594823\,i+423642940259238735473942663180025956729505\sqrt{7}+$$
$$1890613486065620301760074218556745311646936\,i\,x+6150574559311228258394328777942059796320\sqrt{7}\,x+$$
$$2511300259855822962340893027852239157667820\,i\,x^2-2027867550801106189867763431094227596320\sqrt{7}\,x^2-$$
$$3134217746230760357128318797499380812303788\,i\,x^3+63430431602720043279192866968369397935660\sqrt{7}\,x^3+$$

$$944\,749\,064\,886\,626\,467\,328\,385\,369\,190\,460\,703\,669\,697\,i\,x^4 + 16\,381\,317\,765\,107\,264\,789\,462\,917\,221\,030\,750\,634\,835\,\sqrt{7}\,x^4\Big)\Big] -$$

$$\frac{1}{16\,141\,442\,800\,000\,000\,000\,000\,000\,\sqrt{70\left(5+i\sqrt{7}\right)}}\,i\left(-232\,442\,807\,954\,946\,745\,795\,i + 21\,634\,177\,831\,191\,924\,841\,\sqrt{7}\right)$$

$$\text{ArcTan}\Big[\Big(35\,\big(4\,362\,494\,290\,663\,946\,676\,585\,186\,218\,212\,607\,628\,595\,i + 12\,104\,084\,007\,406\,821\,013\,541\,218\,948\,000\,741\,620\,843\,\sqrt{7} -$$
$$40\,919\,031\,596\,617\,332\,707\,196\,094\,500\,783\,237\,405\,000\,i\,x + 175\,730\,701\,694\,606\,521\,668\,409\,393\,655\,487\,422\,752\,\sqrt{7}\,x +$$
$$26\,487\,288\,329\,265\,127\,577\,733\,965\,853\,364\,310\,310\,620\,i\,x^2 - 57\,939\,072\,880\,031\,605\,424\,793\,240\,888\,406\,502\,752\,\sqrt{7}\,x^2 -$$
$$15\,238\,894\,149\,752\,825\,683\,924\,814\,021\,007\,863\,070\,620\,i\,x^3 + 1\,812\,298\,045\,792\,001\,236\,548\,367\,627\,667\,697\,083\,876\,\sqrt{7}\,x^3 -$$
$$795\,837\,271\,959\,975\,808\,913\,244\,203\,765\,619\,963\,595\,i\,x^4 + 468\,037\,650\,431\,636\,136\,841\,797\,634\,886\,592\,875\,281\,\sqrt{7}\,x^4\big)\Big)\Big/$$
$$\Big(135\,063\,738\,860\,435\,016\,899\,586\,558\,948\,733\,259\,113\,515 + 188\,630\,894\,626\,466\,690\,216\,855\,285\,995\,045\,889\,396\,405\,i\,\sqrt{7} +$$
$$1\,506\,241\,361\,872\,688\,008\,559\,268\,776\,761\,430\,483\,700\,000\,x - 105\,711\,500\,937\,472\,192\,718\,115\,651\,350\,352\,447\,938\,680\,i\,\sqrt{7}\,x -$$
$$491\,153\,540\,508\,443\,587\,025\,809\,789\,813\,541\,985\,707\,360\,x^2 - 460\,764\,064\,177\,139\,993\,399\,975\,100\,872\,663\,310\,399\,420\,i\,\sqrt{7}\,x^2 +$$
$$180\,084\,985\,147\,246\,689\,199\,448\,745\,264\,977\,678\,818\,020\,x^3 + 197\,868\,296\,377\,913\,870\,863\,837\,680\,953\,446\,009\,396\,860\,i\,\sqrt{7}\,x^3 +$$
$$176\,004\,816\,500\,761\,880\,926\,774\,485\,599\,831\,047\,775\,825\,x^4 - 207\,342\,833\,228\,459\,577\,163\,557\,043\,035\,558\,264\,835\,165\,i\,\sqrt{7}\,x^4 -$$
$$14\,326\,480\,630\,750\,426\,781\,506\,590\,969\,666\,625\,077\,248\,i\,\sqrt{70\left(5+i\sqrt{7}\right)}\,\sqrt{3-2x+x^2} - 14\,326\,480\,630\,750\,426\,781\,506\,590\,969\,666\,625\,077\,248$$
$$i\,\sqrt{70\left(5+i\sqrt{7}\right)}\,x^2\sqrt{3-2x+x^2} + 28\,652\,961\,261\,500\,853\,563\,013\,181\,939\,333\,250\,154\,496\,i\,\sqrt{70\left(5+i\sqrt{7}\right)}\,x^3\sqrt{3-2x+x^2}\Big)\Big] -$$

$$\Big(\left(-232\,442\,807\,954\,946\,745\,795\,i + 21\,634\,177\,831\,191\,924\,841\,\sqrt{7}\right)\text{Log}\Big[\left(-i+\sqrt{7}-4\,i\,x\right)^2\left(i+\sqrt{7}+4\,i\,x\right)^2\Big]\Big)\Big/$$
$$\left(32\,282\,885\,600\,000\,000\,000\,000\,000\,\sqrt{70\left(5+i\sqrt{7}\right)}\right) +$$

$$\Big(i\left(232\,442\,807\,954\,946\,745\,795\,i + 21\,634\,177\,831\,191\,924\,841\,\sqrt{7}\right)\text{Log}\Big[\left(-i+\sqrt{7}-4\,i\,x\right)^2\left(i+\sqrt{7}+4\,i\,x\right)^2\Big]\Big)\Big/$$
$$\left(32\,282\,885\,600\,000\,000\,000\,000\,000\,\sqrt{70\left(-5+i\sqrt{7}\right)}\right) -$$

$$\Big(i\left(232\,442\,807\,954\,946\,745\,795\,i + 21\,634\,177\,831\,191\,924\,841\,\sqrt{7}\right)\text{Log}\Big[\left(1+x+2x^2\right)$$
$$\left(-13\,i + 15\sqrt{7} + 22\,i\,x - 10\sqrt{7}\,x + 9\,i\,x^2 + 5\sqrt{7}\,x^2 + i\,\sqrt{70\left(-5+i\sqrt{7}\right)}\,\sqrt{3-2x+x^2} - i\,\sqrt{70\left(-5+i\sqrt{7}\right)}\,x\sqrt{3-2x+x^2}\right)\Big]\Big)\Big/$$
$$\left(32\,282\,885\,600\,000\,000\,000\,000\,000\,\sqrt{70\left(-5+i\sqrt{7}\right)}\right) + \Big(\left(-232\,442\,807\,954\,946\,745\,795\,i + 21\,634\,177\,831\,191\,924\,841\,\sqrt{7}\right)$$

$$\text{Log}\left[\left(1+x+2x^2\right)\left(-163\,i+15\sqrt{7}+122\,i\,x-10\sqrt{7}\,x-41\,i\,x^2+5\sqrt{7}\,x^2-13\,i\sqrt{10\left(5+i\sqrt{7}\right)}\sqrt{3-2x+x^2}+5\,i\sqrt{10\left(5+i\sqrt{7}\right)}\,x\sqrt{3-2x+x^2}\right)\right]\Big/\left(32\,282\,885\,600\,000\,000\,000\,000\,000\sqrt{70\left(5+i\sqrt{7}\right)}\right)$$

Problem 52: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{-a-\sqrt{1+a^2}+x}{\left(-a+\sqrt{1+a^2}+x\right)\sqrt{(-a+x)\left(1+x^2\right)}}\,dx$$

Optimal (type 3, 66 leaves, ? steps):

$$-\sqrt{2}\sqrt{a+\sqrt{1+a^2}}\,\text{ArcTan}\left[\frac{\sqrt{2}\sqrt{-a+\sqrt{1+a^2}}\,(-a+x)}{\sqrt{(-a+x)\left(1+x^2\right)}}\right]$$

Result (type 4, 213 leaves):

$$\left(2\sqrt{\frac{a-x}{i+a}}\left(-\left(-i-a+\sqrt{1+a^2}\right)\sqrt{1+i\,x}\,(i+x)\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-i\,x}}{\sqrt{2}}\right],\frac{2\,i}{i+a}\right]+2\,i\sqrt{1+a^2}\sqrt{1-i\,x}\sqrt{1+x^2}\,\text{EllipticPi}\left[\frac{2\,i}{i+a-\sqrt{1+a^2}},\text{ArcSin}\left[\frac{\sqrt{1-i\,x}}{\sqrt{2}}\right],\frac{2\,i}{i+a}\right]\right)\right)\Big/\left(\left(i+a-\sqrt{1+a^2}\right)\sqrt{1-i\,x}\sqrt{(-a+x)\left(1+x^2\right)}\right)$$

Problem 53: Result unnecessarily involves higher level functions.

$$\int\frac{a+b\,x}{\left(1-x^2\right)^{1/3}\left(3+x^2\right)}\,dx$$

Optimal (type 3, 198 leaves, 7 steps):

$$\frac{a\,\text{ArcTan}\left[\frac{\sqrt{3}}{x}\right]}{2\times2^{2/3}\sqrt{3}}+\frac{\sqrt{3}\,b\,\text{ArcTan}\left[\frac{1+\left(2-2x^2\right)^{1/3}}{\sqrt{3}}\right]}{2\times2^{2/3}}+\frac{a\,\text{ArcTan}\left[\frac{\sqrt{3}\left(1-2^{1/3}\left(1-x^2\right)^{1/3}\right)}{x}\right]}{2\times2^{2/3}\sqrt{3}}-\frac{a\,\text{ArcTanh}[x]}{6\times2^{2/3}}+\frac{a\,\text{ArcTanh}\left[\frac{x}{1+2^{1/3}\left(1-x^2\right)^{1/3}}\right]}{2\times2^{2/3}}-\frac{b\,\text{Log}\left[3+x^2\right]}{4\times2^{2/3}}+\frac{3\,b\,\text{Log}\left[2^{2/3}-\left(1-x^2\right)^{1/3}\right]}{4\times2^{2/3}}$$

Result (type 6, 205 leaves):

$$\frac{1}{\left(1-x^2\right)^{1/3}\left(3+x^2\right)}3x\left(\left(3\,a\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},x^2,-\frac{x^2}{3}\right]\right)\Big/\left(9\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},x^2,-\frac{x^2}{3}\right]+2x^2\left(-\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},2,\frac{5}{2},x^2,-\frac{x^2}{3}\right]+\text{AppellF1}\left[\frac{3}{2},\frac{4}{3},1,\frac{5}{2},x^2,-\frac{x^2}{3}\right]\right)\right)+\left(b\,x\,\text{AppellF1}\left[1,\frac{1}{3},1,2,x^2,-\frac{x^2}{3}\right]\right)\Big/\left(6\,\text{AppellF1}\left[1,\frac{1}{3},1,2,x^2,-\frac{x^2}{3}\right]+x^2\left(-\text{AppellF1}\left[2,\frac{1}{3},2,3,x^2,-\frac{x^2}{3}\right]+\text{AppellF1}\left[2,\frac{4}{3},1,3,x^2,-\frac{x^2}{3}\right]\right)\right)\right)$$

Problem 54: Result unnecessarily involves higher level functions.

$$\int\frac{a+b\,x}{\left(3-x^2\right)\left(1+x^2\right)^{1/3}}\,dx$$

Optimal (type 3, 198 leaves, 7 steps):

$$-\frac{a\,\text{ArcTan}[x]}{6\times 2^{2/3}}+\frac{a\,\text{ArcTan}\left[\frac{x}{1+2^{1/3}\left(1+x^2\right)^{1/3}}\right]}{2\times 2^{2/3}}-\frac{\sqrt{3}\,b\,\text{ArcTan}\left[\frac{1+2^{1/3}\left(1+x^2\right)^{1/3}}{\sqrt{3}}\right]}{2\times 2^{2/3}}-\frac{a\,\text{ArcTanh}\left[\frac{\sqrt{3}}{x}\right]}{2\times 2^{2/3}\sqrt{3}}-\frac{a\,\text{ArcTanh}\left[\frac{\sqrt{3}\left(1-2^{1/3}\left(1+x^2\right)^{1/3}\right)}{x}\right]}{2\times 2^{2/3}\sqrt{3}}+\frac{b\,\text{Log}\left[3-x^2\right]}{4\times 2^{2/3}}-\frac{3\,b\,\text{Log}\left[2^{2/3}-\left(1+x^2\right)^{1/3}\right]}{4\times 2^{2/3}}$$

Result (type 6, 220 leaves):

$$\frac{1}{\left(-3+x^2\right)\left(1+x^2\right)^{1/3}}3x\left(-\left(\left(3\,a\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-x^2,\frac{x^2}{3}\right]\right)\Big/\left(9\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-x^2,\frac{x^2}{3}\right]+2x^2\left(\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},2,\frac{5}{2},-x^2,\frac{x^2}{3}\right]-\text{AppellF1}\left[\frac{3}{2},\frac{4}{3},1,\frac{5}{2},-x^2,\frac{x^2}{3}\right]\right)\right)\right)-\left(b\,x\,\text{AppellF1}\left[1,\frac{1}{3},1,2,-x^2,\frac{x^2}{3}\right]\right)\Big/\left(6\,\text{AppellF1}\left[1,\frac{1}{3},1,2,-x^2,\frac{x^2}{3}\right]+x^2\left(\text{AppellF1}\left[2,\frac{1}{3},2,3,-x^2,\frac{x^2}{3}\right]-\text{AppellF1}\left[2,\frac{4}{3},1,3,-x^2,\frac{x^2}{3}\right]\right)\right)\right)$$

Problem 55: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{1}{x\left(4-6x+3x^2\right)^{1/3}}\,dx$$

Optimal (type 3, 88 leaves, ? steps):

$$\frac{\text{ArcTan}\left[\frac{-2+x-2\times 2^{1/3}\left(4-6x+3x^2\right)^{1/3}}{\sqrt{3}\,(-2+x)}\right]}{2^{2/3}\sqrt{3}}+\frac{\text{Log}\left[\frac{-4+2x+2\times 2^{1/3}\left(4-6x+3x^2\right)^{1/3}}{x}\right]}{2\times 2^{2/3}}$$

Result (type 6, 273 leaves):

$$-\left(\left(15\,x\left(-3-i\sqrt{3}+3x\right)\left(-3+i\sqrt{3}+3x\right)\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},\frac{3-i\sqrt{3}}{3x},\frac{3+i\sqrt{3}}{3x}\right]\right)\Big/\right.$$
$$\left(2\left(4-6x+3x^2\right)^{4/3}\left(15\,x\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},\frac{3-i\sqrt{3}}{3x},\frac{3+i\sqrt{3}}{3x}\right]+\right.\right.$$
$$\left.\left.\left.\left(3+i\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},\frac{4}{3},\frac{8}{3},\frac{3-i\sqrt{3}}{3x},\frac{3+i\sqrt{3}}{3x}\right]+\left(3-i\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{4}{3},\frac{1}{3},\frac{8}{3},\frac{3-i\sqrt{3}}{3x},\frac{3+i\sqrt{3}}{3x}\right]\right)\right)\right)$$

Problem 56: Result unnecessarily involves higher level functions.

$$\int x\left(1-x^3\right)^{1/3}dx$$

Optimal (type 3, 107 leaves, 8 steps):

$$\frac{1}{3}x^2\left(1-x^3\right)^{1/3}-\frac{\text{ArcTan}\left[\frac{1-\frac{2x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3\sqrt{3}}+\frac{1}{18}\text{Log}\left[1+\frac{x^2}{\left(1-x^3\right)^{2/3}}-\frac{x}{\left(1-x^3\right)^{1/3}}\right]-\frac{1}{9}\text{Log}\left[1+\frac{x}{\left(1-x^3\right)^{1/3}}\right]$$

Result (type 5, 34 leaves):

$$\frac{1}{6}x^2\left(2\left(1-x^3\right)^{1/3}+\text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},x^3\right]\right)$$

Problem 57: Result unnecessarily involves higher level functions.

$$\int\frac{\left(1-x^3\right)^{1/3}}{x}dx$$

Optimal (type 3, 67 leaves, 6 steps):

$$\left(1-x^3\right)^{1/3}-\frac{\text{ArcTan}\left[\frac{1+2\left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{\text{Log}[x]}{2}+\frac{1}{2}\text{Log}\left[1-\left(1-x^3\right)^{1/3}\right]$$

Result (type 5, 48 leaves):

$$\frac{2-2x^3-\left(1-\frac{1}{x^3}\right)^{2/3}\text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},\frac{1}{x^3}\right]}{2\left(1-x^3\right)^{2/3}}$$

Problem 58: Unable to integrate problem.

$$\int \frac{\left(1-x^3\right)^{1/3}}{1+x}\,dx$$

Optimal (type 1, 1 leaves, ? steps):

$$0$$

Result (type 8, 19 leaves) :

$$\int \frac{\left(1-x^3\right)^{1/3}}{1+x}\,dx$$

Problem 59: Unable to integrate problem.

$$\int \frac{\left(1-x^3\right)^{1/3}}{1-x+x^2}\,dx$$

Optimal (type 3, 280 leaves, ? steps):

$$\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2\times 2^{1/3}(-1+x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}}+\frac{\text{ArcTan}\left[\frac{1-\frac{2x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3}x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1+2^{2/3}(1-x^3)^{1/3}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}}-\frac{\text{Log}\left[-3(-1+x)\left(1-x+x^2\right)\right]}{2\times 2^{2/3}}+$$
$$\frac{\text{Log}\left[2^{1/3}-\left(1-x^3\right)^{1/3}\right]}{2\times 2^{2/3}}+\frac{3\,\text{Log}\left[-2^{1/3}(-1+x)+\left(1-x^3\right)^{1/3}\right]}{2\times 2^{2/3}}+\frac{1}{2}\text{Log}\left[x+\left(1-x^3\right)^{1/3}\right]-\frac{\text{Log}\left[2^{1/3}x+\left(1-x^3\right)^{1/3}\right]}{2\times 2^{2/3}}$$

Result (type 8, 24 leaves) :

$$\int \frac{\left(1-x^3\right)^{1/3}}{1-x+x^2}\,dx$$

Problem 61: Result is not expressed in closed-form.

$$\int \frac{3+12x+20x^2}{9+24x-12x^2+80x^3+320x^4}\,dx$$

Optimal (type 3, 59 leaves, 1 step) :

$$-\frac{\text{ArcTan}\left[\frac{7-40 x}{5 \sqrt{11}}\right]}{2 \sqrt{11}}+\frac{\text{ArcTan}\left[\frac{57+30 x-40 x^2+800 x^3}{6 \sqrt{11}}\right]}{2 \sqrt{11}}$$

Result (type 7, 86 leaves):

$$\frac{1}{8} \text{RootSum}\left[9+24 \#1-12 \#1^2+80 \#1^3+320 \#1^4 \&, \frac{3 \text{Log}[x-\#1]+12 \text{Log}[x-\#1] \#1+20 \text{Log}[x-\#1] \#1^2}{3-3 \#1+30 \#1^2+160 \#1^3} \&\right]$$

Problem 62: Result is not expressed in closed-form.

$$\int -\frac{84+576 x+400 x^2-2560 x^3}{9+24 x-12 x^2+80 x^3+320 x^4} \, dx$$

Optimal (type 3, 78 leaves, 2 steps):

$$2 \sqrt{11} \text{ArcTan}\left[\frac{7-40 x}{5 \sqrt{11}}\right]-2 \sqrt{11} \text{ArcTan}\left[\frac{57+30 x-40 x^2+800 x^3}{6 \sqrt{11}}\right]+2 \text{Log}\left[9+24 x-12 x^2+80 x^3+320 x^4\right]$$

Result (type 7, 99 leaves):

$$\frac{1}{2} \text{RootSum}\left[9+24 \#1-12 \#1^2+80 \#1^3+320 \#1^4 \&, \frac{-21 \text{Log}[x-\#1]-144 \text{Log}[x-\#1] \#1-100 \text{Log}[x-\#1] \#1^2+640 \text{Log}[x-\#1] \#1^3}{3-3 \#1+30 \#1^2+160 \#1^3} \&\right]$$

Problem 63: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1-x^4}}{1+x^4} \, dx$$

Optimal (type 3, 49 leaves, 1 step):

$$\frac{1}{2} \text{ArcTan}\left[\frac{x \left(1+x^2\right)}{\sqrt{1-x^4}}\right]+\frac{1}{2} \text{ArcTanh}\left[\frac{x \left(1-x^2\right)}{\sqrt{1-x^4}}\right]$$

Result (type 6, 110 leaves):

$$-\left(\left(5 x \sqrt{1-x^4} \text{AppellF1}\left[\frac{1}{4},-\frac{1}{2},1,\frac{5}{4},x^4,-x^4\right]\right) \Big/ \left(\left(1+x^4\right) \left(-5 \text{AppellF1}\left[\frac{1}{4},-\frac{1}{2},1,\frac{5}{4},x^4,-x^4\right]+2 x^4 \left(2 \text{AppellF1}\left[\frac{5}{4},-\frac{1}{2},2,\frac{9}{4},x^4,-x^4\right]+\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},x^4,-x^4\right]\right)\right)\right)\right)$$

Problem 64: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1+x^4}}{1-x^4}\,dx$$

Optimal (type 3, 53 leaves, 4 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right]}{2\sqrt{2}}+\frac{\text{ArcTanh}\left[\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right]}{2\sqrt{2}}$$

Result (type 6, 108 leaves):

$$-\left(\left(5\,x\,\sqrt{1+x^4}\;\text{AppellF1}\left[\frac{1}{4},-\frac{1}{2},1,\frac{5}{4},-x^4,x^4\right]\right)\Big/\left(\left(-1+x^4\right)\left(5\,\text{AppellF1}\left[\frac{1}{4},-\frac{1}{2},1,\frac{5}{4},-x^4,x^4\right]+2\,x^4\left(2\,\text{AppellF1}\left[\frac{5}{4},-\frac{1}{2},2,\frac{9}{4},-x^4,x^4\right]+\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-x^4,x^4\right]\right)\right)\right)\right)$$

Problem 65: Unable to integrate problem.

$$\int \frac{\sqrt{1+p\,x^2+x^4}}{1-x^4}\,dx$$

Optimal (type 3, 75 leaves, 4 steps):

$$\frac{1}{4}\sqrt{2-p}\;\text{ArcTan}\left[\frac{\sqrt{2-p}\;x}{\sqrt{1+p\,x^2+x^4}}\right]+\frac{1}{4}\sqrt{2+p}\;\text{ArcTanh}\left[\frac{\sqrt{2+p}\;x}{\sqrt{1+p\,x^2+x^4}}\right]$$

Result (type 8, 26 leaves):

$$\int \frac{\sqrt{1+p\,x^2+x^4}}{1-x^4}\,dx$$

Problem 66: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{1+p\,x^2-x^4}}{1+x^4}\,dx$$

Optimal (type 3, 171 leaves, 1 step):

$$-\frac{\sqrt{p+\sqrt{4+p^2}}\ \text{ArcTan}\left[\frac{\sqrt{p+\sqrt{4+p^2}}\ x\left(p-\sqrt{4+p^2}-2x^2\right)}{2\sqrt{2}\ \sqrt{1+p x^2-x^4}}\right]}{2\sqrt{2}}+\frac{\sqrt{-p+\sqrt{4+p^2}}\ \text{ArcTanh}\left[\frac{\sqrt{-p+\sqrt{4+p^2}}\ x\left(p+\sqrt{4+p^2}-2x^2\right)}{2\sqrt{2}\ \sqrt{1+p x^2-x^4}}\right]}{2\sqrt{2}}$$

Result (type 4, 322 leaves):

$$\left(\sqrt{2+\frac{4x^2}{-p+\sqrt{4+p^2}}}\ \sqrt{1-\frac{2x^2}{p+\sqrt{4+p^2}}}\ \left(2\,\mathrm{i}\ \text{EllipticF}\left[\mathrm{i}\ \text{ArcSinh}\left[\sqrt{2}\ \sqrt{\frac{1}{-p+\sqrt{4+p^2}}}\ x\right],\frac{p-\sqrt{4+p^2}}{p+\sqrt{4+p^2}}\right]-\right.\right.$$

$$\left(2\,\mathrm{i}+p\right)\text{EllipticPi}\left[\frac{1}{2}\,\mathrm{i}\left(p-\sqrt{4+p^2}\right),\mathrm{i}\ \text{ArcSinh}\left[\sqrt{2}\ \sqrt{\frac{1}{-p+\sqrt{4+p^2}}}\ x\right],\frac{p-\sqrt{4+p^2}}{p+\sqrt{4+p^2}}\right]+$$

$$\left.\left.\left(-2\,\mathrm{i}+p\right)\text{EllipticPi}\left[\frac{1}{2}\,\mathrm{i}\left(-p+\sqrt{4+p^2}\right),\mathrm{i}\ \text{ArcSinh}\left[\sqrt{2}\ \sqrt{\frac{1}{-p+\sqrt{4+p^2}}}\ x\right],\frac{p-\sqrt{4+p^2}}{p+\sqrt{4+p^2}}\right]\right)\right)\Bigg/\left(4\sqrt{\frac{1}{-p+\sqrt{4+p^2}}}\ \sqrt{1+p x^2-x^4}\right)$$

Problem 67: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{a+b x}{\left(2-x^2\right)\left(-1+x^2\right)^{1/4}}\,dx$$

Optimal (type 3, 80 leaves, 7 steps):

$$\frac{a\ \text{ArcTan}\left[\frac{x}{\sqrt{2}\ \left(-1+x^2\right)^{1/4}}\right]}{2\sqrt{2}}-b\ \text{ArcTan}\left[\left(-1+x^2\right)^{1/4}\right]+\frac{a\ \text{ArcTanh}\left[\frac{x}{\sqrt{2}\ \left(-1+x^2\right)^{1/4}}\right]}{2\sqrt{2}}+b\ \text{ArcTanh}\left[\left(-1+x^2\right)^{1/4}\right]$$

Result (type 6, 203 leaves):

$$\frac{1}{\left(-2+x^2\right)\left(-1+x^2\right)^{1/4}}2x\left(-\left(\left(3\,a\ \text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},x^2,\frac{x^2}{2}\right]\right)\Big/\right.\right.$$

$$\left.\left.\left(6\ \text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},x^2,\frac{x^2}{2}\right]+x^2\left(2\ \text{AppellF1}\left[\frac{3}{2},\frac{1}{4},2,\frac{5}{2},x^2,\frac{x^2}{2}\right]+\text{AppellF1}\left[\frac{3}{2},\frac{5}{4},1,\frac{5}{2},x^2,\frac{x^2}{2}\right]\right)\right)\right)-$$

$$\left.\frac{2\,b\,x\ \text{AppellF1}\left[1,\frac{1}{4},1,2,x^2,\frac{x^2}{2}\right]}{8\ \text{AppellF1}\left[1,\frac{1}{4},1,2,x^2,\frac{x^2}{2}\right]+x^2\left(2\ \text{AppellF1}\left[2,\frac{1}{4},2,3,x^2,\frac{x^2}{2}\right]+\text{AppellF1}\left[2,\frac{5}{4},1,3,x^2,\frac{x^2}{2}\right]\right)}\right)$$

Problem 68: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{a+b\,x}{\left(-1-x^2\right)^{1/4}\left(2+x^2\right)}\,dx$$

Optimal (type 3, 88 leaves, 7 steps):

$$\frac{a\,\text{ArcTan}\left[\frac{x}{\sqrt{2}\left(-1-x^2\right)^{1/4}}\right]}{2\sqrt{2}}+b\,\text{ArcTan}\left[\left(-1-x^2\right)^{1/4}\right]+\frac{a\,\text{ArcTanh}\left[\frac{x}{\sqrt{2}\left(-1-x^2\right)^{1/4}}\right]}{2\sqrt{2}}-b\,\text{ArcTanh}\left[\left(-1-x^2\right)^{1/4}\right]$$

Result (type 6, 221 leaves):

$$\frac{1}{\left(-1-x^2\right)^{1/4}\left(2+x^2\right)}2\,x\left(-\left(\left(3\,a\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-x^2,-\frac{x^2}{2}\right]\right)\Big/\left(-6\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-x^2,-\frac{x^2}{2}\right]+x^2\left(2\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{4},2,\frac{5}{2},-x^2,-\frac{x^2}{2}\right]+\text{AppellF1}\left[\frac{3}{2},\frac{5}{4},1,\frac{5}{2},-x^2,-\frac{x^2}{2}\right]\right)\right)\right)-\left(2\,b\,x\,\text{AppellF1}\left[1,\frac{1}{4},1,2,-x^2,-\frac{x^2}{2}\right]\right)\Big/\left(-8\,\text{AppellF1}\left[1,\frac{1}{4},1,2,-x^2,-\frac{x^2}{2}\right]+x^2\left(2\,\text{AppellF1}\left[2,\frac{1}{4},2,3,-x^2,-\frac{x^2}{2}\right]+\text{AppellF1}\left[2,\frac{5}{4},1,3,-x^2,-\frac{x^2}{2}\right]\right)\right)\right)$$

Problem 69: Result unnecessarily involves higher level functions.

$$\int \frac{a+b\,x}{\left(1-x^2\right)^{1/4}\left(2-x^2\right)}\,dx$$

Optimal (type 3, 149 leaves, 3 steps):

$$\frac{b\,\text{ArcTan}\left[\frac{1-\sqrt{1-x^2}}{\sqrt{2}\left(1-x^2\right)^{1/4}}\right]}{\sqrt{2}}+\frac{1}{2}\,a\,\text{ArcTan}\left[\frac{1-\sqrt{1-x^2}}{x\left(1-x^2\right)^{1/4}}\right]+\frac{b\,\text{ArcTanh}\left[\frac{1+\sqrt{1-x^2}}{\sqrt{2}\left(1-x^2\right)^{1/4}}\right]}{\sqrt{2}}+\frac{1}{2}\,a\,\text{ArcTanh}\left[\frac{1+\sqrt{1-x^2}}{x\left(1-x^2\right)^{1/4}}\right]$$

Result (type 6, 205 leaves):

$$\frac{1}{\left(1-x^2\right)^{1/4}\left(-2+x^2\right)}2\,x\left(-\left(\left(3\,a\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},x^2,\frac{x^2}{2}\right]\right)\Big/\left(6\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},x^2,\frac{x^2}{2}\right]+x^2\left(2\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{4},2,\frac{5}{2},x^2,\frac{x^2}{2}\right]+\text{AppellF1}\left[\frac{3}{2},\frac{5}{4},1,\frac{5}{2},x^2,\frac{x^2}{2}\right]\right)\right)\right)-\frac{2\,b\,x\,\text{AppellF1}\left[1,\frac{1}{4},1,2,x^2,\frac{x^2}{2}\right]}{8\,\text{AppellF1}\left[1,\frac{1}{4},1,2,x^2,\frac{x^2}{2}\right]+x^2\left(2\,\text{AppellF1}\left[2,\frac{1}{4},2,3,x^2,\frac{x^2}{2}\right]+\text{AppellF1}\left[2,\frac{5}{4},1,3,x^2,\frac{x^2}{2}\right]\right)}\right)$$

Problem 70: Result unnecessarily involves higher level functions.

$$\int \frac{a+b\,x}{\left(1+x^2\right)^{1/4}\,\left(2+x^2\right)}\,dx$$

Optimal (type 3, 135 leaves, 3 steps):

$$-\frac{b\,\text{ArcTan}\left[\frac{1-\sqrt{1+x^2}}{\sqrt{2}\,\left(1+x^2\right)^{1/4}}\right]}{\sqrt{2}}-\frac{1}{2}\,a\,\text{ArcTan}\left[\frac{1+\sqrt{1+x^2}}{x\,\left(1+x^2\right)^{1/4}}\right]-\frac{1}{2}\,a\,\text{ArcTanh}\left[\frac{1-\sqrt{1+x^2}}{x\,\left(1+x^2\right)^{1/4}}\right]-\frac{b\,\text{ArcTanh}\left[\frac{1+\sqrt{1+x^2}}{\sqrt{2}\,\left(1+x^2\right)^{1/4}}\right]}{\sqrt{2}}$$

Result (type 6, 219 leaves):

$$\frac{1}{\left(1+x^2\right)^{1/4}\,\left(2+x^2\right)}2\,x\,\left(-\left(\left(3\,a\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-x^2,-\frac{x^2}{2}\right]\right)\Big/\left(-6\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-x^2,-\frac{x^2}{2}\right]+x^2\,\left(2\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{4},2,\frac{5}{2},-x^2,-\frac{x^2}{2}\right]+\text{AppellF1}\left[\frac{3}{2},\frac{5}{4},1,\frac{5}{2},-x^2,-\frac{x^2}{2}\right]\right)\right)\right)-\left(2\,b\,x\,\text{AppellF1}\left[1,\frac{1}{4},1,2,-x^2,-\frac{x^2}{2}\right]\right)\Big/\left(-8\,\text{AppellF1}\left[1,\frac{1}{4},1,2,-x^2,-\frac{x^2}{2}\right]+x^2\,\left(2\,\text{AppellF1}\left[2,\frac{1}{4},2,3,-x^2,-\frac{x^2}{2}\right]+\text{AppellF1}\left[2,\frac{5}{4},1,3,-x^2,-\frac{x^2}{2}\right]\right)\right)\right)$$

Problem 71: Result unnecessarily involves higher level functions.

$$\int \frac{x}{\sqrt{1-x^3}\,\left(4-x^3\right)}\,dx$$

Optimal (type 3, 127 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{3}\,\left(1-2^{1/3}\,x\right)}{\sqrt{1-x^3}}\right]}{3\times2^{2/3}\,\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{\sqrt{1-x^3}}{\sqrt{3}}\right]}{3\times2^{2/3}\,\sqrt{3}}-\frac{\text{ArcTanh}\left[\frac{1+2^{1/3}\,x}{\sqrt{1-x^3}}\right]}{3\times2^{2/3}}+\frac{\text{ArcTanh}\left[\sqrt{1-x^3}\right]}{9\times2^{2/3}}$$

Result (type 6, 120 leaves):

$$-\left(\left(10\,x^2\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},x^3,\frac{x^3}{4}\right]\right)\Big/\left(\sqrt{1-x^3}\,\left(-4+x^3\right)\,\left(20\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},x^3,\frac{x^3}{4}\right]+3\,x^3\,\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},x^3,\frac{x^3}{4}\right]+2\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},x^3,\frac{x^3}{4}\right]\right)\right)\right)\right)$$

Problem 72: Result unnecessarily involves higher level functions.

$$\int \frac{x}{\left(4-d x^3\right) \sqrt{-1+d x^3}} \, dx$$

Optimal (type 3, 157 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{1+2^{1/3} d^{1/3} x}{\sqrt{-1+d x^3}}\right]}{3\times 2^{2/3} d^{2/3}}-\frac{\text{ArcTan}\left[\sqrt{-1+d x^3}\right]}{9\times 2^{2/3} d^{2/3}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{3}\left(1-2^{1/3} d^{1/3} x\right)}{\sqrt{-1+d x^3}}\right]}{3\times 2^{2/3} \sqrt{3}\, d^{2/3}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{-1+d x^3}}{\sqrt{3}}\right]}{3\times 2^{2/3} \sqrt{3}\, d^{2/3}}$$

Result (type 6, 135 leaves):

$$-\left(\left(10 x^2 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},d x^3,\frac{d x^3}{4}\right]\right)\Big/\left(\left(-4+d x^3\right)\sqrt{-1+d x^3}\right.\right.$$
$$\left.\left.\left(20 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},d x^3,\frac{d x^3}{4}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},d x^3,\frac{d x^3}{4}\right]+2 \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},d x^3,\frac{d x^3}{4}\right]\right)\right)\right)\right)$$

Problem 73: Result unnecessarily involves higher level functions.

$$\int \frac{x}{\sqrt{-1+x^3}\left(8+x^3\right)} \, dx$$

Optimal (type 3, 74 leaves, 8 steps):

$$\frac{1}{18}\text{ArcTan}\left[\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right]+\frac{1}{18}\text{ArcTan}\left[\frac{1}{3}\sqrt{-1+x^3}\right]-\frac{\text{ArcTanh}\left[\frac{\sqrt{3}\,(1-x)}{\sqrt{-1+x^3}}\right]}{6\sqrt{3}}$$

Result (type 6, 118 leaves):

$$-\left(\left(20 x^2 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},x^3,-\frac{x^3}{8}\right]\right)\Big/\right.$$
$$\left.\left(\sqrt{-1+x^3}\left(8+x^3\right)\left(-40 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},x^3,-\frac{x^3}{8}\right]+3 x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},x^3,-\frac{x^3}{8}\right]-4 \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},x^3,-\frac{x^3}{8}\right]\right)\right)\right)\right)$$

Problem 74: Result unnecessarily involves higher level functions.

$$\int \frac{x}{\left(8-d x^3\right)\sqrt{1+d x^3}} \, dx$$

Optimal (type 3, 103 leaves, 8 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{3}\,\left(1+d^{1/3}\,x\right)}{\sqrt{1+d\,x^3}}\right]}{6\,\sqrt{3}\,d^{2/3}}+\frac{\text{ArcTanh}\left[\frac{\left(1+d^{1/3}\,x\right)^2}{3\,\sqrt{1+d\,x^3}}\right]}{18\,d^{2/3}}-\frac{\text{ArcTanh}\left[\frac{1}{3}\,\sqrt{1+d\,x^3}\right]}{18\,d^{2/3}}$$

Result (type 6, 139 leaves):

$$-\left(\left(20\,x^2\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-d\,x^3,\frac{d\,x^3}{8}\right]\right)\Big/\left(\left(-8+d\,x^3\right)\sqrt{1+d\,x^3}\right.\right.$$
$$\left.\left.\left(40\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-d\,x^3,\frac{d\,x^3}{8}\right]+3\,d\,x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-d\,x^3,\frac{d\,x^3}{8}\right]-4\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-d\,x^3,\frac{d\,x^3}{8}\right]\right)\right)\right)\right)$$

Problem 75: Result unnecessarily involves higher level functions.

$$\int\frac{1}{\left(1-3\,x^2\right)^{1/3}\,\left(3-x^2\right)}\,dx$$

Optimal (type 3, 81 leaves, 1 step):

$$\frac{1}{4}\,\text{ArcTan}\left[\frac{1-\left(1-3\,x^2\right)^{1/3}}{x}\right]+\frac{\text{ArcTanh}\left[\frac{x}{\sqrt{3}}\right]}{4\,\sqrt{3}}-\frac{\text{ArcTanh}\left[\frac{\left(1-\left(1-3\,x^2\right)^{1/3}\right)^2}{3\,\sqrt{3}\,x}\right]}{4\,\sqrt{3}}$$

Result (type 6, 126 leaves):

$$-\left(\left(9\,x\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},3\,x^2,\frac{x^2}{3}\right]\right)\Big/\right.$$
$$\left.\left(\left(1-3\,x^2\right)^{1/3}\left(-3+x^2\right)\left(9\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},3\,x^2,\frac{x^2}{3}\right]+2\,x^2\left(\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},2,\frac{5}{2},3\,x^2,\frac{x^2}{3}\right]+3\,\text{AppellF1}\left[\frac{3}{2},\frac{4}{3},1,\frac{5}{2},3\,x^2,\frac{x^2}{3}\right]\right)\right)\right)\right)$$

Problem 76: Result unnecessarily involves higher level functions.

$$\int\frac{1}{\left(3+x^2\right)\,\left(1+3\,x^2\right)^{1/3}}\,dx$$

Optimal (type 3, 81 leaves, 1 step):

$$\frac{\text{ArcTan}\left[\frac{x}{\sqrt{3}}\right]}{4\,\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{\left(1-\left(1+3\,x^2\right)^{1/3}\right)^2}{3\,\sqrt{3}\,x}\right]}{4\,\sqrt{3}}-\frac{1}{4}\,\text{ArcTanh}\left[\frac{1-\left(1+3\,x^2\right)^{1/3}}{x}\right]$$

Result (type 6, 126 leaves):

$$-\left(\left(9\,x\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-3\,x^2,-\frac{x^2}{3}\right]\right)\Big/\left(\left(3+x^2\right)\left(1+3\,x^2\right)^{1/3}\left(-9\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-3\,x^2,-\frac{x^2}{3}\right]+2\,x^2\left(\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},2,\frac{5}{2},-3\,x^2,-\frac{x^2}{3}\right]+3\,\text{AppellF1}\left[\frac{3}{2},\frac{4}{3},1,\frac{5}{2},-3\,x^2,-\frac{x^2}{3}\right]\right)\right)\right)\right)$$

Problem 77: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(1-x^2\right)^{1/3}\left(3+x^2\right)}\,dx$$

Optimal (type 3, 113 leaves, 1 step):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{3}}{x}\right]}{2\times 2^{2/3}\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{\sqrt{3}\left(1-2^{1/3}\left(1-x^2\right)^{1/3}\right)}{x}\right]}{2\times 2^{2/3}\sqrt{3}}-\frac{\text{ArcTanh}[x]}{6\times 2^{2/3}}+\frac{\text{ArcTanh}\left[\frac{x}{1+2^{1/3}\left(1-x^2\right)^{1/3}}\right]}{2\times 2^{2/3}}$$

Result (type 6, 118 leaves):

$$-\left(\left(9\,x\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},x^2,-\frac{x^2}{3}\right]\right)\Big/\left(\left(1-x^2\right)^{1/3}\left(3+x^2\right)\left(-9\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},x^2,-\frac{x^2}{3}\right]+2\,x^2\left(\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},2,\frac{5}{2},x^2,-\frac{x^2}{3}\right]-\text{AppellF1}\left[\frac{3}{2},\frac{4}{3},1,\frac{5}{2},x^2,-\frac{x^2}{3}\right]\right)\right)\right)\right)$$

Problem 78: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(3-x^2\right)\left(1+x^2\right)^{1/3}}\,dx$$

Optimal (type 3, 109 leaves, 1 step):

$$-\frac{\text{ArcTan}[x]}{6\times 2^{2/3}}+\frac{\text{ArcTan}\left[\frac{x}{1+2^{1/3}\left(1+x^2\right)^{1/3}}\right]}{2\times 2^{2/3}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{3}}{x}\right]}{2\times 2^{2/3}\sqrt{3}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{3}\left(1-2^{1/3}\left(1+x^2\right)^{1/3}\right)}{x}\right]}{2\times 2^{2/3}\sqrt{3}}$$

Result (type 6, 124 leaves):

$$-\left(\left(9\,x\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-x^2,\frac{x^2}{3}\right]\right)\Big/\left(\left(-3+x^2\right)\left(1+x^2\right)^{1/3}\left(9\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-x^2,\frac{x^2}{3}\right]+2\,x^2\left(\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},2,\frac{5}{2},-x^2,\frac{x^2}{3}\right]-\text{AppellF1}\left[\frac{3}{2},\frac{4}{3},1,\frac{5}{2},-x^2,\frac{x^2}{3}\right]\right)\right)\right)\right)$$

Problem 79: Result unnecessarily involves higher level functions.

$$\int \frac{a+x}{(-a+x)\sqrt{a^2 x-\left(1+a^2\right) x^2+x^3}}\,dx$$

Optimal (type 3, 87 leaves, 4 steps):

$$-\frac{2\sqrt{x}\,\sqrt{a^2-\left(1+a^2\right) x+x^2}\;\text{ArcTan}\left[\frac{(1-a)\sqrt{x}}{\sqrt{a^2-\left(1+a^2\right) x+x^2}}\right]}{(1-a)\sqrt{a^2 x-\left(1+a^2\right) x^2+x^3}}$$

Result (type 4, 159 leaves):

$$-\left(\left(2\,i\,\left(a^2-x\right)^{3/2}\sqrt{\frac{-1+x}{-a^2+x}}\sqrt{\frac{x}{-a^2+x}}\left((1+a)\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-a^2}}{\sqrt{a^2-x}}\right],1-\frac{1}{a^2}\right]-2\,\text{EllipticPi}\left[\frac{-1+a}{a},i\,\text{ArcSinh}\left[\frac{\sqrt{-a^2}}{\sqrt{a^2-x}}\right],1-\frac{1}{a^2}\right]\right)\right)\Big/\left((-1+a)\sqrt{-a^2}\sqrt{(-1+x)\,x\,\left(-a^2+x\right)}\right)\right)$$

Problem 80: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{-2+a+x}{(-a+x)\sqrt{(2-a)\,a\,x+\left(-1-2a+a^2\right) x^2+x^3}}\,dx$$

Optimal (type 1, 1 leaves, ? steps):

$$0$$

Result (type 4, 100 leaves):

$$-\left(\left(2\,i\sqrt{1+\frac{1}{-1+x}}\sqrt{1+\frac{(-1+a)^2}{-1+x}}\,(-1+x)^{3/2}\left(\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{1}{\sqrt{-1+x}}\right],(-1+a)^2\right]-2\,\text{EllipticPi}\left[1-a,i\,\text{ArcSinh}\left[\frac{1}{\sqrt{-1+x}}\right],(-1+a)^2\right]\right)\right)\Big/\left(\sqrt{(-1+x)\,x\,\left(-2a+a^2+x\right)}\right)\right)$$

Problem 81: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{-a+(-1+2a)\,x}{(-a+x)\sqrt{a^2 x-(-1+2a+a^2)\,x^2+(-1+2a)\,x^3}}\,dx$$

Optimal (type 3, 46 leaves, ? steps):

$$\text{Log}\left[\frac{-a^2+2ax+x^2-2\left(x+\sqrt{(1-x)\,x\,(a^2+x-2ax)}\right)}{(a-x)^2}\right]$$

Result (type 4, 133 leaves):

$$\left(2\,i\,(-1+x)^{3/2}\sqrt{\frac{x}{-1+x}}\sqrt{-\frac{a^2+x-2ax}{(-1+2a)\,(-1+x)}}\left(-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{1}{\sqrt{-1+x}}\right],-\frac{(-1+a)^2}{-1+2a}\right]+2\,a\,\text{EllipticPi}\left[1-a,\,i\,\text{ArcSinh}\left[\frac{1}{\sqrt{-1+x}}\right],-\frac{(-1+a)^2}{-1+2a}\right]\right)\right)\Big/\left(\sqrt{-(-1+x)\,x\,(a^2+x-2ax)}\right)$$

Problem 82: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1-2^{1/3}x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx$$

Optimal (type 3, 32 leaves, 2 steps):

$$\frac{2\,\text{ArcTan}\left[\frac{\sqrt{3}\,(1+2^{1/3}x)}{\sqrt{1+x^3}}\right]}{\sqrt{3}}$$

Result (type 4, 323 leaves):

$$-\left(\left(2\sqrt{\frac{2}{3}}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\left(\sqrt{-i+\sqrt{3}+2ix}\left(6i+3i\,2^{1/3}-2\sqrt{3}+2^{1/3}\sqrt{3}+\left(-3i\,2^{1/3}+4\sqrt{3}+2^{1/3}\sqrt{3}\right)x\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\,3^{1/4}}\right],\frac{2\sqrt{3}}{3i+\sqrt{3}}\right]-6i\sqrt{3}\sqrt{i+\sqrt{3}-2ix}\sqrt{1-x+x^2}\,\text{EllipticPi}\left[\frac{2\sqrt{3}}{i+2i\,2^{2/3}+\sqrt{3}},\text{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\,3^{1/4}}\right],\frac{2\sqrt{3}}{3i+\sqrt{3}}\right]\right)\right)\Big/\left(\left(1+2\times 2^{2/3}-i\sqrt{3}\right)\sqrt{i+\sqrt{3}-2ix}\sqrt{1+x^3}\right)\right)$$

Problem 83: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{1+x}{(-2+x)\sqrt{1+x^3}}\,dx$$

Optimal (type 3, 23 leaves, 2 steps):

$$-\frac{2}{3}\text{ArcTanh}\left[\frac{(1+x)^2}{3\sqrt{1+x^3}}\right]$$

Result (type 4, 262 leaves):

$$\left(2\sqrt{6}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\left(\sqrt{-i+\sqrt{3}+2ix}\left(1+i\sqrt{3}+x-i\sqrt{3}\,x\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\,3^{1/4}}\right],\frac{2\sqrt{3}}{3i+\sqrt{3}}\right]-2\sqrt{3}\sqrt{i+\sqrt{3}-2ix}\sqrt{1-x+x^2}\,\text{EllipticPi}\left[\frac{2\sqrt{3}}{-3i+\sqrt{3}},\text{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\,3^{1/4}}\right],\frac{2\sqrt{3}}{3i+\sqrt{3}}\right]\right)\right)\Big/\left(\left(-3i+\sqrt{3}\right)\sqrt{i+\sqrt{3}-2ix}\sqrt{1+x^3}\right)$$

Problem 84: Result unnecessarily involves higher level functions.

$$\int\frac{x}{\sqrt{1+x^3}\left(10+6\sqrt{3}+x^3\right)}\,dx$$

Optimal (type 3, 218 leaves, 1 step):

$$-\frac{\left(2-\sqrt{3}\right)\text{ArcTan}\left[\frac{3^{1/4}\left(1+\sqrt{3}\right)(1+x)}{\sqrt{2}\sqrt{1+x^3}}\right]}{2\sqrt{2}\,3^{3/4}}-\frac{\left(2-\sqrt{3}\right)\text{ArcTan}\left[\frac{\left(1-\sqrt{3}\right)\sqrt{1+x^3}}{\sqrt{2}\,3^{3/4}}\right]}{3\sqrt{2}\,3^{3/4}}-\frac{\left(2-\sqrt{3}\right)\text{ArcTanh}\left[\frac{3^{1/4}\left(1+\sqrt{3}-2x\right)}{\sqrt{2}\sqrt{1+x^3}}\right]}{3\sqrt{2}\,3^{1/4}}-\frac{\left(2-\sqrt{3}\right)\text{ArcTanh}\left[\frac{3^{1/4}\left(1-\sqrt{3}\right)(1+x)}{\sqrt{2}\sqrt{1+x^3}}\right]}{6\sqrt{2}\,3^{1/4}}$$

Result (type 6, 206 leaves) :

$$-\left(\left(10\left(26+15\sqrt{3}\right)x^2\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-x^3,-\frac{x^3}{10+6\sqrt{3}}\right]\right)\Big/\left(\left(5+3\sqrt{3}\right)\sqrt{1+x^3}\left(10+6\sqrt{3}+x^3\right)\left(-10\left(5+3\sqrt{3}\right)\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-x^3,-\frac{x^3}{10+6\sqrt{3}}\right]+3x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-x^3,-\frac{x^3}{10+6\sqrt{3}}\right]+\left(5+3\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-x^3,-\frac{x^3}{10+6\sqrt{3}}\right]\right)\right)\right)\right)$$

Problem 85: Result unnecessarily involves higher level functions.

$$\int\frac{x}{\sqrt{1+x^3}\left(10-6\sqrt{3}+x^3\right)}\,dx$$

Optimal (type 3, 210 leaves, 1 step) :

$$-\frac{\left(2+\sqrt{3}\right)\text{ArcTan}\left[\frac{3^{1/4}\left(1-\sqrt{3}-2x\right)}{\sqrt{2}\sqrt{1+x^3}}\right]}{3\sqrt{2}\,3^{1/4}}-\frac{\left(2+\sqrt{3}\right)\text{ArcTan}\left[\frac{3^{1/4}\left(1+\sqrt{3}\right)(1+x)}{\sqrt{2}\sqrt{1+x^3}}\right]}{6\sqrt{2}\,3^{1/4}}+\frac{\left(2+\sqrt{3}\right)\text{ArcTanh}\left[\frac{3^{1/4}\left(1-\sqrt{3}\right)(1+x)}{\sqrt{2}\sqrt{1+x^3}}\right]}{2\sqrt{2}\,3^{3/4}}+\frac{\left(2+\sqrt{3}\right)\text{ArcTanh}\left[\frac{\left(1+\sqrt{3}\right)\sqrt{1+x^3}}{\sqrt{2}\,3^{3/4}}\right]}{3\sqrt{2}\,3^{3/4}}$$

Result (type 6, 207 leaves) :

$$\left(10\left(26-15\sqrt{3}\right)x^2\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-x^3,\frac{1}{4}\left(5+3\sqrt{3}\right)x^3\right]\right)\Big/\left(\left(-5+3\sqrt{3}\right)\left(-10+6\sqrt{3}-x^3\right)\sqrt{1+x^3}\left(\left(50-30\sqrt{3}\right)\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-x^3,\frac{1}{4}\left(5+3\sqrt{3}\right)x^3\right]-3x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-x^3,\frac{1}{4}\left(5+3\sqrt{3}\right)x^3\right]+\left(5-3\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-x^3,\frac{1}{4}\left(5+3\sqrt{3}\right)x^3\right]\right)\right)\right)$$

Problem 86: Result unnecessarily involves higher level functions.

$$\int\frac{x}{\sqrt{-1+x^3}\left(-10-6\sqrt{3}+x^3\right)}\,dx$$

Optimal (type 3, 222 leaves, 1 step) :

$$\frac{\left(2-\sqrt{3}\right) \text{ArcTan}\left[\frac{3^{1/4} \left(1-\sqrt{3}\right) (1-x)}{\sqrt{2} \sqrt{-1+x^3}}\right]}{6 \sqrt{2}\, 3^{1/4}}+\frac{\left(2-\sqrt{3}\right) \text{ArcTan}\left[\frac{3^{1/4} \left(1+\sqrt{3}+2 x\right)}{\sqrt{2} \sqrt{-1+x^3}}\right]}{3 \sqrt{2}\, 3^{1/4}}+\frac{\left(2-\sqrt{3}\right) \text{ArcTanh}\left[\frac{3^{1/4} \left(1+\sqrt{3}\right) (1-x)}{\sqrt{2} \sqrt{-1+x^3}}\right]}{2 \sqrt{2}\, 3^{3/4}}-\frac{\left(2-\sqrt{3}\right) \text{ArcTanh}\left[\frac{\left(1-\sqrt{3}\right) \sqrt{-1+x^3}}{\sqrt{2}\, 3^{3/4}}\right]}{3 \sqrt{2}\, 3^{3/4}}$$

Result (type 6, 196 leaves):

$$-\left(\left(10 \left(26+15 \sqrt{3}\right) x^2 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},x^3,\frac{x^3}{10+6 \sqrt{3}}\right]\right)\Big/\right.$$
$$\left(\left(5+3 \sqrt{3}\right) \left(10+6 \sqrt{3}-x^3\right) \sqrt{-1+x^3} \left(10 \left(5+3 \sqrt{3}\right) \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},x^3,\frac{x^3}{10+6 \sqrt{3}}\right]+\right.\right.$$
$$\left.\left.\left.3 x^3 \left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},x^3,\frac{x^3}{10+6 \sqrt{3}}\right]+\left(5+3 \sqrt{3}\right) \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},x^3,\frac{x^3}{10+6 \sqrt{3}}\right]\right)\right)\right)\right)$$

Problem 87: Result unnecessarily involves higher level functions.

$$\int \frac{x}{\sqrt{-1+x^3} \left(-10+6 \sqrt{3}+x^3\right)} \, dx$$

Optimal (type 3, 214 leaves, 1 step):

$$-\frac{\left(2+\sqrt{3}\right) \text{ArcTan}\left[\frac{3^{1/4} \left(1-\sqrt{3}\right) (1-x)}{\sqrt{2} \sqrt{-1+x^3}}\right]}{2 \sqrt{2}\, 3^{3/4}}+\frac{\left(2+\sqrt{3}\right) \text{ArcTan}\left[\frac{\left(1+\sqrt{3}\right) \sqrt{-1+x^3}}{\sqrt{2}\, 3^{3/4}}\right]}{3 \sqrt{2}\, 3^{3/4}}+\frac{\left(2+\sqrt{3}\right) \text{ArcTanh}\left[\frac{3^{1/4} \left(1+\sqrt{3}\right) (1-x)}{\sqrt{2} \sqrt{-1+x^3}}\right]}{6 \sqrt{2}\, 3^{1/4}}+\frac{\left(2+\sqrt{3}\right) \text{ArcTanh}\left[\frac{3^{1/4} \left(1-\sqrt{3}+2 x\right)}{\sqrt{2} \sqrt{-1+x^3}}\right]}{3 \sqrt{2}\, 3^{1/4}}$$

Result (type 6, 198 leaves):

$$\left(10 \left(26-15 \sqrt{3}\right) x^2 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},x^3,-\frac{1}{4} \left(5+3 \sqrt{3}\right) x^3\right]\right)\Big/$$
$$\left(\left(-5+3 \sqrt{3}\right) \sqrt{-1+x^3} \left(-10+6 \sqrt{3}+x^3\right) \left(10 \left(-5+3 \sqrt{3}\right) \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},x^3,-\frac{1}{4} \left(5+3 \sqrt{3}\right) x^3\right]-\right.\right.$$
$$\left.\left.3 x^3 \left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},x^3,-\frac{1}{4} \left(5+3 \sqrt{3}\right) x^3\right]+\left(5-3 \sqrt{3}\right) \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},x^3,-\frac{1}{4} \left(5+3 \sqrt{3}\right) x^3\right]\right)\right)\right)$$

Problem 88: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right) \sqrt{-4+4 \sqrt{3} x^2+x^4}} \, dx$$

Optimal (type 3, 65 leaves, 2 steps):

$$\frac{1}{3}\sqrt{-3+2\sqrt{3}}\ \text{ArcTanh}\left[\frac{\left(1-\sqrt{3}+x\right)^2}{\sqrt{3\left(-3+2\sqrt{3}\right)}\ \sqrt{-4+4\sqrt{3}\ x^2+x^4}}\right]$$

Result (type 4, 685 leaves) :

$$\left(\left(-1+\sqrt{3}+x\right)^2 \sqrt{2\left(1+\sqrt{3}\right)-2\left(2+\sqrt{3}\right)x+\left(-1+\sqrt{3}\right)x^2-x^3}\ \sqrt{\frac{1+\sqrt{3}-\frac{4}{-1+\sqrt{3}+x}}{3+\sqrt{3}+i\sqrt{2\left(2+\sqrt{3}\right)}}}\right.$$

$$\left(\left(i\left(-1+\sqrt{3}+i\sqrt{2\left(2+\sqrt{3}\right)}\right)+\frac{2\left(2\,i\sqrt{3}-\sqrt{2\left(2+\sqrt{3}\right)}+\sqrt{6\left(2+\sqrt{3}\right)}\right)}{-1+\sqrt{3}+x}\right)\sqrt{\sqrt{2\left(2+\sqrt{3}\right)}+i\left(1-\sqrt{3}+\frac{8}{-1+\sqrt{3}+x}\right)}\right.$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\sqrt{2\left(2+\sqrt{3}\right)}-i\left(1-\sqrt{3}+\frac{8}{-1+\sqrt{3}+x}\right)}}{2^{3/4}\left(2+\sqrt{3}\right)^{1/4}}\right],\ \frac{2\,i\sqrt{2\left(2+\sqrt{3}\right)}}{3+\sqrt{3}+i\sqrt{2\left(2+\sqrt{3}\right)}}\right]+$$

$$2\sqrt{6}\ \sqrt{\frac{4+2\sqrt{3}+x^2}{\left(-1+\sqrt{3}+x\right)^2}}\ \sqrt{\sqrt{2\left(2+\sqrt{3}\right)}-i\left(1-\sqrt{3}+\frac{8}{-1+\sqrt{3}+x}\right)}$$

$$\left.\left.\text{EllipticPi}\left[\frac{2\sqrt{2\left(2+\sqrt{3}\right)}}{\sqrt{2\left(2+\sqrt{3}\right)}+i\left(3+\sqrt{3}\right)},\ \text{ArcSin}\left[\frac{\sqrt{\sqrt{2\left(2+\sqrt{3}\right)}-i\left(1-\sqrt{3}+\frac{8}{-1+\sqrt{3}+x}\right)}}{2^{3/4}\left(2+\sqrt{3}\right)^{1/4}}\right],\ \frac{2\,i\sqrt{2\left(2+\sqrt{3}\right)}}{3+\sqrt{3}+i\sqrt{2\left(2+\sqrt{3}\right)}}\right]\right)\right)\Bigg/$$

$$\left(\left(\sqrt{2\left(2+\sqrt{3}\right)}+i\left(3+\sqrt{3}\right)\right)\sqrt{1+\sqrt{3}-\left(2+\sqrt{3}\right)x+\frac{1}{2}\left(-1+\sqrt{3}\right)x^2-\frac{x^3}{2}}\ \sqrt{-4+4\sqrt{3}\,x^2+x^4}\ \sqrt{\sqrt{2\left(2+\sqrt{3}\right)}-i\left(1-\sqrt{3}+\frac{8}{-1+\sqrt{3}+x}\right)}\right)$$

Problem 89: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}\,x^2+x^4}}\,dx$$

Optimal (type 3, 63 leaves, 2 steps):

$$-\frac{1}{3}\sqrt{3+2\sqrt{3}}\,\text{ArcTan}\left[\frac{\left(1+\sqrt{3}+x\right)^2}{\sqrt{3\left(3+2\sqrt{3}\right)}\,\sqrt{-4-4\sqrt{3}\,x^2+x^4}}\right]$$

Result (type 4, 1137 leaves):

$$-\left(\left(\left(-1-\sqrt{3}+x\right)^2\sqrt{\frac{-1+\sqrt{3}+\frac{4}{-1-\sqrt{3}+x}}{-3+\sqrt{3}-i\sqrt{4-2\sqrt{3}}}}\,\sqrt{-24+16\sqrt{3}+\left(20-8\sqrt{3}\right)\left(1-\sqrt{3}+x\right)+\left(-2+4\sqrt{3}\right)\left(1-\sqrt{3}+x\right)^2+\left(1-\sqrt{3}+x\right)^3}\right.\right.$$

$$\left(\left(i\sqrt{\sqrt{4-2\sqrt{3}}+i\left(1+\sqrt{3}\right)+\frac{8\,i}{-1-\sqrt{3}+x}}+i\sqrt{3}\sqrt{\sqrt{4-2\sqrt{3}}+i\left(1+\sqrt{3}\right)+\frac{8\,i}{-1-\sqrt{3}+x}}+\right.\right.$$

$$\sqrt{-2\,i+2\,i\sqrt{3}-2\sqrt{12-6\sqrt{3}}+4\sqrt{4-2\sqrt{3}}-\frac{16\,i\left(-2+\sqrt{3}\right)}{-1-\sqrt{3}+x}}+\frac{1}{-1-\sqrt{3}+x}$$

$$2\left(2\,i\sqrt{3}\sqrt{\sqrt{4-2\sqrt{3}}+i\left(1+\sqrt{3}\right)+\frac{8\,i}{-1-\sqrt{3}+x}}+\sqrt{6}\sqrt{-i+i\sqrt{3}-\sqrt{12-6\sqrt{3}}+2\sqrt{4-2\sqrt{3}}-\frac{8\,i\left(-2+\sqrt{3}\right)}{-1-\sqrt{3}+x}}+\right.$$

$$\left.\left.\left.\sqrt{-2\,i+2\,i\sqrt{3}-2\sqrt{12-6\sqrt{3}}+4\sqrt{4-2\sqrt{3}}-\frac{16\,i\left(-2+\sqrt{3}\right)}{-1-\sqrt{3}+x}}\right)\right)\right)$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\sqrt{4-2\sqrt{3}}-i\left(1+\sqrt{3}\right)-\frac{8\,i}{-1-\sqrt{3}+x}}}{2^{3/4}\left(2-\sqrt{3}\right)^{1/4}}\right],\ \frac{2\sqrt{4-2\sqrt{3}}}{\sqrt{4-2\sqrt{3}}+i\left(-3+\sqrt{3}\right)}\right]+$$

$$2\sqrt{6}\,\sqrt{\sqrt{4-2\sqrt{3}}-i\left(1+\sqrt{3}\right)-\frac{8\,i}{-1-\sqrt{3}+x}}\,\sqrt{1+\frac{8}{\left(-1-\sqrt{3}+x\right)^2}+\frac{2\left(1+\sqrt{3}\right)}{-1-\sqrt{3}+x}}$$

$$\text{EllipticPi}\left[\frac{2\sqrt{4-2\sqrt{3}}}{\sqrt{4-2\sqrt{3}}-i\left(-3+\sqrt{3}\right)},\,\text{ArcSin}\left[\frac{\sqrt{\sqrt{4-2\sqrt{3}}-i\left(1+\sqrt{3}\right)-\frac{8\,i}{-1-\sqrt{3}+x}}}{2^{3/4}\left(2-\sqrt{3}\right)^{1/4}}\right],\,\frac{2\sqrt{4-2\sqrt{3}}}{\sqrt{4-2\sqrt{3}}+i\left(-3+\sqrt{3}\right)}\right]\Bigg)\Bigg)\Big/$$

$$\Bigg(\left(\sqrt{4-2\sqrt{3}}-i\left(-3+\sqrt{3}\right)\right)\sqrt{\sqrt{4-2\sqrt{3}}-i\left(1+\sqrt{3}\right)-\frac{8\,i}{-1-\sqrt{3}+x}}$$

$$\sqrt{8\left(1+\sqrt{3}\right)+4\left(3+\sqrt{3}\right)\left(-1-\sqrt{3}+x\right)+2\left(1+\sqrt{3}\right)\left(-1-\sqrt{3}+x\right)^2+\frac{1}{2}\left(-1-\sqrt{3}+x\right)^3}$$

$$\sqrt{\Big(48-32\sqrt{3}-64\left(1-\sqrt{3}+x\right)+32\sqrt{3}\left(1-\sqrt{3}+x\right)+24\left(1-\sqrt{3}+x\right)^2-}$$

$$\left.\left.16\sqrt{3}\left(1-\sqrt{3}+x\right)^2-4\left(1-\sqrt{3}+x\right)^3+4\sqrt{3}\left(1-\sqrt{3}+x\right)^3+\left(1-\sqrt{3}+x\right)^4\right)\right)$$

Problem 90: Unable to integrate problem.

$$\int\frac{-1+x}{(1+x)\left(2+x^3\right)^{1/3}}\,dx$$

Optimal (type 3, 53 leaves, 1 step):

$$\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2(2+x)}{\left(2+x^3\right)^{1/3}}}{\sqrt{3}}\right]+\text{Log}[1+x]-\frac{3}{2}\,\text{Log}\left[2+x-\left(2+x^3\right)^{1/3}\right]$$

Result (type 8, 20 leaves):

$$\int\frac{-1+x}{(1+x)\left(2+x^3\right)^{1/3}}\,dx$$

Problem 91: Unable to integrate problem.

$$\int \frac{1}{(1+x)\ (2+x^3)^{1/3}}\, dx$$

Optimal (type 3, 108 leaves, 3 steps):

$$\frac{\text{ArcTan}\left[\frac{1+\frac{2x}{(2+x^3)^{1/3}}}{\sqrt{3}}\right]}{2\sqrt{3}}-\frac{1}{2}\sqrt{3}\ \text{ArcTan}\left[\frac{1+\frac{2(2+x)}{(2+x^3)^{1/3}}}{\sqrt{3}}\right]-\frac{1}{2}\text{Log}[1+x]+\frac{3}{4}\text{Log}\left[2+x-(2+x^3)^{1/3}\right]-\frac{1}{4}\text{Log}\left[-x+(2+x^3)^{1/3}\right]$$

Result (type 8, 17 leaves):

$$\int \frac{1}{(1+x)\ (2+x^3)^{1/3}}\, dx$$

Problem 92: Unable to integrate problem.

$$\int \frac{1+x}{(1-x+x^2)\ (1-x^3)^{1/3}}\, dx$$

Optimal (type 3, 135 leaves, ? steps):

$$\frac{\sqrt{3}\ \text{ArcTan}\left[\frac{1-\frac{2\ 2^{1/3}(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{2\times 2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{2^{1/3}}$$

Result (type 8, 27 leaves):

$$\int \frac{1+x}{(1-x+x^2)\ (1-x^3)^{1/3}}\, dx$$

Problem 93: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(1+x)^2}{(1-x^3)^{1/3}\ (1+x^3)}\, dx$$

Optimal (type 3, 135 leaves, ? steps):

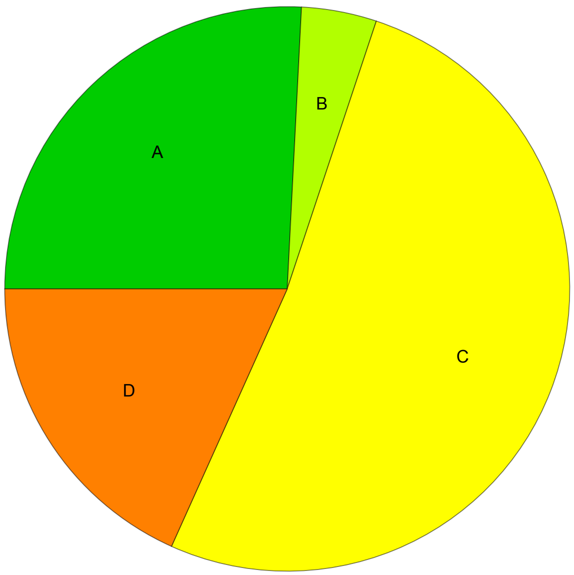
$$\frac{\sqrt{3}\ \text{ArcTan}\left[\frac{1-\frac{2\ 2^{1/3}(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{2\times 2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{2^{1/3}}$$

Result (type 6, 315 leaves) :

$$-\left(\left(5\,x^2\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right]\right)\Big/\left(\left(1-x^3\right)^{1/3}\left(1+x^3\right)\left(-5\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right]+x^3\left(3\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},2,\frac{8}{3},x^3,-x^3\right]-\text{AppellF1}\left[\frac{5}{3},\frac{4}{3},1,\frac{8}{3},x^3,-x^3\right]\right)\right)\right)\right)-\left(2\,x^3\,\text{AppellF1}\left[1,\frac{1}{3},1,2,x^3,-x^3\right]\right)\Big/\left(\left(1-x^3\right)^{1/3}\left(1+x^3\right)\left(-6\,\text{AppellF1}\left[1,\frac{1}{3},1,2,x^3,-x^3\right]+x^3\left(3\,\text{AppellF1}\left[2,\frac{1}{3},2,3,x^3,-x^3\right]-\text{AppellF1}\left[2,\frac{4}{3},1,3,x^3,-x^3\right]\right)\right)\right)+\frac{2\sqrt{3}\,\text{ArcTan}\left[\frac{-1+\frac{2\times 2^{1/3}\,x}{\left(-1+x^3\right)^{1/3}}}{\sqrt{3}}\right]-\text{Log}\left[1+\frac{2^{2/3}\,x^2}{\left(-1+x^3\right)^{2/3}}-\frac{2^{1/3}\,x}{\left(-1+x^3\right)^{1/3}}\right]+2\,\text{Log}\left[1+\frac{2^{1/3}\,x}{\left(-1+x^3\right)^{1/3}}\right]}{6\times 2^{1/3}}$$

Summary of Integration Test Results

93 integration problems



A - 24 optimal antiderivatives

B - 4 more than twice size of optimal antiderivatives

C - 48 unnecessarily complex antiderivatives

D - 17 unable to integrate problems

E - 0 integration timeouts