Mathematica 11.3 Integration Test Results

Test results for the 34 problems in "1.1.1.5 P(x) (a+b x)^m (c+d x)^n.m"

Problem 25: Result more than twice size of optimal antiderivative.

$$\int (a+b x)^3 (c+d x)^n \left(A+B x+C x^2+D x^3\right) \, dx$$

Optimal (type 3, 455 leaves, 2 steps):

$$-\frac{(b c-a d)^3 \left(c^2 C d-B c d^2+A d^3-c^3 D\right) (c+d x)^{1+n}}{d^7 (1+n)}-\frac{1}{d^7 (2+n)}(b c-a d)^2 \left(a d \left(2 c C d-B d^2-3 c^2 D\right)-b \left(5 c^2 C d-4 B c d^2+3 A d^3-6 c^3 D\right)\right) (c+d x)^{2+n}-\frac{1}{d^7 (3+n)}(b c-a d) \left(a^2 d^2 (C d-3 c D)-a b d \left(8 c C d-3 B d^2-15 c^2 D\right)+b^2 \left(10 c^2 C d-6 B c d^2+3 A d^3-15 c^3 D\right)\right) (c+d x)^{3+n}+\frac{1}{d^7 (4+n)}\left(a^3 d^3 D+3 a^2 b d^2 (C d-4 c D)-3 a b^2 d \left(4 c C d-B d^2-10 c^2 D\right)+b^3 \left(10 c^2 C d-4 B c d^2+A d^3-20 c^3 D\right)\right) (c+d x)^{4+n}+\frac{1}{d^7 (5+n)}b \left(3 a^2 d^2 D+3 a b d (C d-5 c D)-b^2 \left(5 c C d-B d^2-15 c^2 D\right)\right) (c+d x)^{5+n}+\frac{b^2 (b C d-6 b c D+3 a d D) (c+d x)^{6+n}}{d^7 (6+n)}+\frac{b^3 D (c+d x)^{7+n}}{d^7 (7+n)}$$

Result (type 3, 977 leaves):

$$\begin{aligned}
&\frac{1}{d^7 (1+n) (2+n) (3+n) (4+n) (5+n) (6+n) (7+n)} \\
&(c+d x)^{1+n} \Big(a^3 d^3 (210+107 n+18 n^2+n^3) \big(-6 c^3 D+2 c^2 d (C (4+n)+3 D (1+n) x) - \\
&\quad c d^2 (B (12+7 n+n^2)+(1+n) x (2 C (4+n)+3 D (2+n) x))+d^3 \\
&\quad (A (24+26 n+9 n^2+n^3)+(1+n) x (B (12+7 n+n^2)+(2+n) x (C (4+n)+D (3+n) x)))\big) + \\
&3 a^2 b d^2 (42+13 n+n^2) \big(24 c^4 D-6 c^3 d (C (5+n)+4 D (1+n) x) + \\
&\quad 2 c^2 d^2 (B (20+9 n+n^2)+3 (1+n) x (C (5+n)+2 D (2+n) x)) - \\
&\quad c d^3 (A (60+47 n+12 n^2+n^3)+(1+n) x (2 B (20+9 n+n^2) + \\
&\quad (2+n) x (3 C (5+n)+4 D (3+n) x)))+d^4 (1+n) x (A (60+47 n+12 n^2+n^3) + \\
&\quad (2+n) x (B (20+9 n+n^2)+(3+n) x (C (5+n)+D (4+n) x)))\big) + \\
&3 a b^2 d (7+n) \big(-120 c^5 D+24 c^4 d (C (6+n)+5 D (1+n) x) - \\
&\quad 6 c^3 d^2 (B (30+11 n+n^2)+2 (1+n) x (2 C (6+n)+5 D (2+n) x)) + \\
&\quad 2 c^2 d^3 (A (120+74 n+15 n^2+n^3) + \\
&\quad (1+n) x (3 B (30+11 n+n^2)+2 (2+n) x (3 C (6+n)+5 D (3+n) x))) - \\
&\quad c d^4 (1+n) x (2 A (120+74 n+15 n^2+n^3)+(2+n) x (3 B (30+11 n+n^2) + \\
&\quad (3+n) x (4 C (6+n)+5 D (4+n) x)))+d^5 (2+3 n+n^2) x^2 (A (120+74 n+15 n^2+n^3) + \\
&\quad (3+n) x (B (30+11 n+n^2)+(4+n) x (C (6+n)+D (5+n) x)))\big) + \\
&b^3 \big(720 c^6 D-120 c^5 d (C (7+n)+6 D (1+n) x)+24 c^4 d^2 (B (42+13 n+n^2) + \\
&\quad 5 (1+n) x (C (7+n)+3 D (2+n) x))-6 c^3 d^3 (A (210+107 n+18 n^2+n^3) + \\
&\quad 2 (1+n) x (2 B (42+13 n+n^2)+5 (2+n) x (C (7+n)+2 D (3+n) x))) + \\
&\quad 2 c^2 d^4 (1+n) x (3 A (210+107 n+18 n^2+n^3)+(2+n) x \\
&\quad (6 B (42+13 n+n^2)+5 (3+n) x (2 C (7+n)+3 D (4+n) x))) - \\
&\quad c d^5 (2+3 n+n^2) x^2 (3 A (210+107 n+18 n^2+n^3)+(3+n) x \\
&\quad (4 B (42+13 n+n^2)+(4+n) x (5 C (7+n)+6 D (5+n) x))) + \\
&\quad d^6 (6+11 n+6 n^2+n^3) x^3 (A (210+107 n+18 n^2+n^3) + \\
&\quad (4+n) x (B (42+13 n+n^2)+(5+n) x (C (7+n)+D (6+n) x)))\big)\Big)
\end{aligned}$$

Problem 29: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(c+d x)^n (A+B x+C x^2+D x^3)}{a+b x} \, dx$$

Optimal (type 5, 203 leaves, 3 steps):

$$\begin{aligned}
&\frac{\left(a^2 d^2 D-a b d (C d-c D)-b^2 (c C d-B d^2-c^2 D)\right) (c+d x)^{1+n}}{b^3 d^3 (1+n)} + \\
&\frac{(b C d-2 b c D-a d D) (c+d x)^{2+n}}{b^2 d^3 (2+n)}+\frac{D (c+d x)^{3+n}}{b d^3 (3+n)} - \\
&\left(\left(A b^3-a (b^2 B-a b C+a^2 D)\right) (c+d x)^{1+n} \text{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{b (c+d x)}{b c-a d}\right]\right) \Big/ \\
&\left(b^3 (b c-a d) (1+n)\right)
\end{aligned}$$

Result (type 6, 414 leaves):

$$\frac{1}{12}(c+dx)^n\left(\left(18\,a\,B\,c\,x^2\,\text{AppellF1}\left[2,-n,1,3,-\frac{dx}{c},-\frac{bx}{a}\right]\right)\Big/\left((a+bx)\left(3\,a\,c\,\text{AppellF1}\left[2,-n,1,3,-\frac{dx}{c},-\frac{bx}{a}\right]+a\,d\,n\,x\,\text{AppellF1}\left[3,1-n,1,4,-\frac{dx}{c},-\frac{bx}{a}\right]-b\,c\,x\,\text{AppellF1}\left[3,-n,2,4,-\frac{dx}{c},-\frac{bx}{a}\right]\right)\right)+\left(16\,a\,c\,C\,x^3\,\text{AppellF1}\left[3,-n,1,4,-\frac{dx}{c},-\frac{bx}{a}\right]\right)\Big/\left((a+bx)\left(4\,a\,c\,\text{AppellF1}\left[3,-n,1,4,-\frac{dx}{c},-\frac{bx}{a}\right]+a\,d\,n\,x\,\text{AppellF1}\left[4,1-n,1,5,-\frac{dx}{c},-\frac{bx}{a}\right]-b\,c\,x\,\text{AppellF1}\left[4,-n,2,5,-\frac{dx}{c},-\frac{bx}{a}\right]\right)\right)+\left(15\,a\,c\,D\,x^4\,\text{AppellF1}\left[4,-n,1,5,-\frac{dx}{c},-\frac{bx}{a}\right]\right)\Big/\left((a+bx)\left(5\,a\,c\,\text{AppellF1}\left[4,-n,1,5,-\frac{dx}{c},-\frac{bx}{a}\right]+a\,d\,n\,x\,\text{AppellF1}\left[5,1-n,1,6,-\frac{dx}{c},-\frac{bx}{a}\right]-b\,c\,x\,\text{AppellF1}\left[5,-n,2,6,-\frac{dx}{c},-\frac{bx}{a}\right]\right)\right)-\frac{12\,A\,(c+dx)\,\text{Hypergeometric2F1}\left[1,1+n,2+n,\frac{b(c+dx)}{bc-ad}\right]}{(bc-ad)(1+n)}\right)$$

Problem 30: Unable to integrate problem.

$$\int\frac{(c+dx)^n\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^2}\,dx$$

Optimal (type 5, 220 leaves, 4 steps):

$$\frac{(bCd-bcD-2adD)(c+dx)^{1+n}}{b^3d^2(1+n)}-\frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)(c+dx)^{1+n}}{(bc-ad)(a+bx)}+\frac{D(c+dx)^{2+n}}{b^2d^2(2+n)}+\left(\left(a^3dD(3+n)-b^3(Bc+Adn)+ab^2(2cC+Bd(1+n))-a^2b(3cD+Cd(2+n))\right)(c+dx)^{1+n}\,\text{Hypergeometric2F1}\left[1,1+n,2+n,\frac{b(c+dx)}{bc-ad}\right]\right)\Big/\left(b^3(bc-ad)^2(1+n)\right)$$

Result (type 8, 32 leaves):

$$\int\frac{(c+dx)^n\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^2}\,dx$$

Problem 31: Unable to integrate problem.

$$\int\frac{(c+dx)^n\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^3}\,dx$$

Optimal (type 5, 329 leaves, 4 steps):

$$\frac{D\,(c+d\,x)^{1+n}}{b^3\,d\,(1+n)}-\frac{\left(A\,b^3-a\,\left(b^2\,B-a\,b\,C+a^2\,D\right)\right)\,(c+d\,x)^{1+n}}{2\,b^3\,(b\,c-a\,d)\,(a+b\,x)^2}-\left(\left(b^3\,(2\,B\,c-A\,d\,(1-n))-a^3\,d\,D\,(5+n)-a\,b^2\,(4\,c\,C+B\,d\,(1+n))+a^2\,b\,(6\,c\,D+C\,d\,(3+n))\right)\,(c+d\,x)^{1+n}\right)\Big/\left(2\,b^3\,(b\,c-a\,d)^2\,(a+b\,x)\right)-\left(\left(b^3\,\left(2\,c^2\,C+2\,B\,c\,d\,n-A\,d^2\,(1-n)\,n\right)-a^3\,d^2\,D\,\left(6+5\,n+n^2\right)+a^2\,b\,d\,(2+n)\,(6\,c\,D+C\,d\,(1+n))-a\,b^2\,\left(6\,c^2\,D+4\,c\,C\,d\,(1+n)+B\,d^2\,n\,(1+n)\right)\right)\,(c+d\,x)^{1+n}\,\text{Hypergeometric2F1}\left[1,1+n,2+n,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]\right)\Big/\left(2\,b^3\,(b\,c-a\,d)^3\,(1+n)\right)$$

Result (type 8, 32 leaves):

$$\int\frac{(c+d\,x)^n\,\left(A+B\,x+C\,x^2+D\,x^3\right)}{(a+b\,x)^3}\,dx$$

Problem 32: Result unnecessarily involves higher level functions.

$$\int(a+b\,x)^m\,(A+B\,x)\,(c+d\,x)^n\,dx$$

Optimal (type 5, 141 leaves, 3 steps):

$$\frac{B\,(a+b\,x)^{1+m}\,(c+d\,x)^{1+n}}{b\,d\,(2+m+n)}+\left((A\,b\,d\,(2+m+n)-B\,(b\,c\,(1+m)+a\,d\,(1+n)))\,(a+b\,x)^{1+m}\,(c+d\,x)^n\,\left(\frac{b\,(c+d\,x)}{b\,c-a\,d}\right)^{-n}\text{Hypergeometric2F1}\left[1+m,-n,2+m,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]\right)\Big/\left(b^2\,d\,(1+m)\,(2+m+n)\right)$$

Result (type 6, 202 leaves):

$$(a+b\,x)^m\,(c+d\,x)^n\left(\left(3\,a\,B\,c\,x^2\,\text{AppellF1}\left[2,-m,-n,3,-\frac{b\,x}{a},-\frac{d\,x}{c}\right]\right)\Big/\left(6\,a\,c\,\text{AppellF1}\left[2,-m,-n,3,-\frac{b\,x}{a},-\frac{d\,x}{c}\right]+2\,b\,c\,m\,x\,\text{AppellF1}\left[3,1-m,-n,4,-\frac{b\,x}{a},-\frac{d\,x}{c}\right]+2\,a\,d\,n\,x\,\text{AppellF1}\left[3,-m,1-n,4,-\frac{b\,x}{a},-\frac{d\,x}{c}\right]\right)+\frac{1}{d\,(1+n)}A\left(\frac{d\,(a+b\,x)}{-b\,c+a\,d}\right)^{-m}(c+d\,x)\,\text{Hypergeometric2F1}\left[-m,1+n,2+n,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]\right)$$

Problem 33: Result unnecessarily involves higher level functions.

$$\int(a+b\,x)^m\,(c+d\,x)^n\,\left(A+B\,x+C\,x^2\right)\,dx$$

Optimal (type 5, 268 leaves, 4 steps):

$$-\left(\left(\left(a C d (4+m+2 n)+b (c C (2+m)-B d (3+m+n))\right) (a+b x)^{1+m} (c+d x)^{1+n}\right)\Big/\left(b^2 d^2 (2+m+n) (3+m+n)\right)\right)+\frac{C (a+b x)^{2+m} (c+d x)^{1+n}}{b^2 d (3+m+n)}-\left(\left(d (2+m+n) \left(a b c C (2+m)+a^2 C d (1+n)-A b^2 d (3+m+n)\right)-(b c (1+m)+a d (1+n)) (a C d (4+m+2 n)+b (c C (2+m)-B d (3+m+n)))\right) (a+b x)^{1+m} (c+d x)^n \left(\frac{b (c+d x)}{b c-a d}\right)^{-n} \text{Hypergeometric2F1}\left[1+m,-n,2+m,-\frac{d (a+b x)}{b c-a d}\right]\right)\Big/\left(b^3 d^2 (1+m) (2+m+n) (3+m+n)\right)$$

Result (type 6, 327 leaves):

$$\frac{1}{3} (a+b x)^m (c+d x)^n \left(\left(9 a B c x^2 \text{AppellF1}\left[2,-m,-n,3,-\frac{b x}{a},-\frac{d x}{c}\right]\right)\Big/\left(6 a c \text{AppellF1}\left[2,-m,-n,3,-\frac{b x}{a},-\frac{d x}{c}\right]+2 b c m x \text{AppellF1}\left[3,1-m,-n,4,-\frac{b x}{a},-\frac{d x}{c}\right]+2 a d n x \text{AppellF1}\left[3,-m,1-n,4,-\frac{b x}{a},-\frac{d x}{c}\right]\right)+\left(4 a c C x^3 \text{AppellF1}\left[3,-m,-n,4,-\frac{b x}{a},-\frac{d x}{c}\right]\right)\Big/\left(4 a c \text{AppellF1}\left[3,-m,-n,4,-\frac{b x}{a},-\frac{d x}{c}\right]+b c m x \text{AppellF1}\left[4,1-m,-n,5,-\frac{b x}{a},-\frac{d x}{c}\right]+a d n x \text{AppellF1}\left[4,-m,1-n,5,-\frac{b x}{a},-\frac{d x}{c}\right]\right)+\frac{1}{d (1+n)} 3 A \left(\frac{d (a+b x)}{-b c+a d}\right)^{-m} (c+d x) \text{Hypergeometric2F1}\left[-m,1+n,2+n,\frac{b (c+d x)}{b c-a d}\right]\right)$$

Problem 34: Result unnecessarily involves higher level functions.

$$\int (a+b x)^m (c+d x)^n \left(A+B x+C x^2+D x^3\right) \, dx$$

Optimal (type 5, 610 leaves, 5 steps):

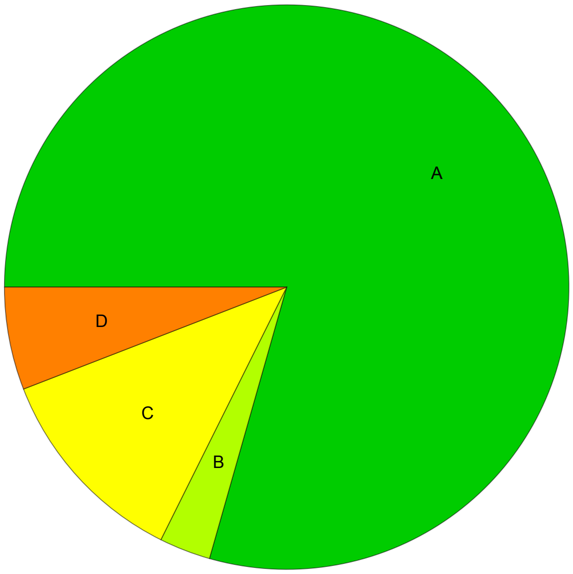
$$\Big(\big(a^2 d^2 D \left(m^2 + m (8 + 3 n) + 3 (6 + 5 n + n^2)\right) + b^2 \left(c^2 D (6 + 5 m + m^2) - c C d (2 + m) (4 + m + n) + B d^2 (12 + m^2 + 7 n + n^2 + m (7 + 2 n))\right) + a b d \left(c D (2 + m) (6 + m + 3 n) - C d (m^2 + m (8 + 3 n) + 2 (8 + 6 n + n^2))\right)\big) (a + b x)^{1+m} (c + d x)^{1+n}\Big) \Big/ \left(b^3 d^3 (2 + m + n) (3 + m + n) (4 + m + n)\right) - \Big(\big(a d D (9 + 2 m + 3 n) + b (c D (3 + m) - C d (4 + m + n))\big) (a + b x)^{2+m} (c + d x)^{1+n}\Big) \Big/ \left(b^3 d^2 (3 + m + n) (4 + m + n)\right) + \frac{D (a + b x)^{3+m} (c + d x)^{1+n}}{b^3 d (4 + m + n)} + \frac{1}{b^4 d^3 (1 + m) (2 + m + n) (3 + m + n) (4 + m + n)} \Big(d (2 + m + n) \big(a^3 d^2 D (1 + n) (6 + m + 2 n) + a b^2 c (2 + m) (c D (3 + m) - C d (4 + m + n)) + A b^3 d^2 (12 + m^2 + 7 n + n^2 + m (7 + 2 n)) - a^2 b d (C d (1 + n) (4 + m + n) - c D (2 + m) (6 + m + 3 n))\big) - (b c (1 + m) + a d (1 + n)) \big(a^2 d^2 D (m^2 + m (8 + 3 n) + 3 (6 + 5 n + n^2)) + b^2 (c^2 D (6 + 5 m + m^2) - c C d (2 + m) (4 + m + n) + B d^2 (12 + m^2 + 7 n + n^2 + m (7 + 2 n))) + a b d (c D (2 + m) (6 + m + 3 n) - C d (m^2 + m (8 + 3 n) + 2 (8 + 6 n + n^2)))\big)\Big) (a + b x)^{1+m} (c + d x)^n \left(\frac{b (c + d x)}{b c - a d}\right)^{-n} \text{Hypergeometric2F1}\left[1 + m, -n, 2 + m, -\frac{d (a + b x)}{b c - a d}\right]$$

Result (type 6, 446 leaves) :

$$\frac{1}{12} (a + b x)^m (c + d x)^n \Bigg(\left(18 a B c x^2 \text{AppellF1}\left[2, -m, -n, 3, -\frac{b x}{a}, -\frac{d x}{c}\right]\right) \Big/ \left(3 a c\, \text{AppellF1}\left[2, -m, -n, 3, -\frac{b x}{a}, -\frac{d x}{c}\right] + b c m x\, \text{AppellF1}\left[3, 1 - m, -n, 4, -\frac{b x}{a}, -\frac{d x}{c}\right] + a d n x\, \text{AppellF1}\left[3, -m, 1 - n, 4, -\frac{b x}{a}, -\frac{d x}{c}\right]\right) + \left(16 a c C x^3 \text{AppellF1}\left[3, -m, -n, 4, -\frac{b x}{a}, -\frac{d x}{c}\right]\right) \Big/ \left(4 a c\, \text{AppellF1}\left[3, -m, -n, 4, -\frac{b x}{a}, -\frac{d x}{c}\right] + b c m x\, \text{AppellF1}\left[4, 1 - m, -n, 5, -\frac{b x}{a}, -\frac{d x}{c}\right] + a d n x\, \text{AppellF1}\left[4, -m, 1 - n, 5, -\frac{b x}{a}, -\frac{d x}{c}\right]\right) + \left(15 a c D x^4 \text{AppellF1}\left[4, -m, -n, 5, -\frac{b x}{a}, -\frac{d x}{c}\right]\right) \Big/ \left(5 a c\, \text{AppellF1}\left[4, -m, -n, 5, -\frac{b x}{a}, -\frac{d x}{c}\right] + b c m x\, \text{AppellF1}\left[5, 1 - m, -n, 6, -\frac{b x}{a}, -\frac{d x}{c}\right] + a d n x\, \text{AppellF1}\left[5, -m, 1 - n, 6, -\frac{b x}{a}, -\frac{d x}{c}\right]\right) + \frac{1}{d (1 + n)} 12 A \left(\frac{d (a + b x)}{-b c + a d}\right)^{-m} (c + d x)\, \text{Hypergeometric2F1}\left[-m, 1 + n, 2 + n, \frac{b (c + d x)}{b c - a d}\right]\Bigg)$$

Summary of Integration Test Results

34 integration problems



A - 27 optimal antiderivatives

B - 1 more than twice size of optimal antiderivatives

C - 4 unnecessarily complex antiderivatives

D - 2 unable to integrate problems

E - 0 integration timeouts