Mathematica 11.3 Integration Test Results

Test results for the 346 problems in "1.1.2.3 (a+b x^2)^p (c+d x^2)^q.m"

Problem 72: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1+x^2}}{-1+x^2}\, dx$$

Optimal (type 3, 27 leaves, 4 steps):

$$\text{ArcSinh}[x]-\sqrt{2}\ \text{ArcTanh}\left[\frac{\sqrt{2}\ x}{\sqrt{1+x^2}}\right]$$

Result (type 3, 64 leaves):

$$\text{ArcSinh}[x]+\frac{1}{\sqrt{2}}\left(\text{Log}[1-x]-\text{Log}[1+x]+\text{Log}\left[1-x+\sqrt{2}\ \sqrt{1+x^2}\right]-\text{Log}\left[1+x+\sqrt{2}\ \sqrt{1+x^2}\right]\right)$$

Problem 109: Result unnecessarily involves higher level functions.

$$\int \left(a-b x^2\right)^{2/3} \left(3 a+b x^2\right)^3 dx$$

Optimal (type 4, 648 leaves, 8 steps):

$$\frac{18144\, a^3\, x\, \left(a-b\, x^2\right)^{2/3}}{1235}-\frac{23544\, a^2\, x\, \left(a-b\, x^2\right)^{5/3}}{6175}-\frac{378}{475}\, a\, x\, \left(a-b\, x^2\right)^{5/3}\, \left(3\, a+b\, x^2\right)-\frac{3}{25}\, x\, \left(a-b\, x^2\right)^{5/3}\, \left(3\, a+b\, x^2\right)^2-\frac{72576\, a^4\, x}{1235\, \left(\left(1-\sqrt{3}\right)\, a^{1/3}-\left(a-b\, x^2\right)^{1/3}\right)}-\left(36288\times 3^{1/4}\, \sqrt{2+\sqrt{3}}\, a^{13/3}\, \left(a^{1/3}-\left(a-b\, x^2\right)^{1/3}\right)\, \sqrt{\frac{a^{2/3}+a^{1/3}\, \left(a-b\, x^2\right)^{1/3}+\left(a-b\, x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\, a^{1/3}-\left(a-b\, x^2\right)^{1/3}\right)^2}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)\, a^{1/3}-\left(a-b\, x^2\right)^{1/3}}{\left(1-\sqrt{3}\right)\, a^{1/3}-\left(a-b\, x^2\right)^{1/3}}\right],\, -7+4\, \sqrt{3}\right]\right)\Bigg/\left(1235\, b\, x\, \sqrt{-\frac{a^{1/3}\, \left(a^{1/3}-\left(a-b\, x^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right)\, a^{1/3}-\left(a-b\, x^2\right)^{1/3}\right)^2}}\right)+\left(24192\, \sqrt{2}\, 3^{3/4}\, a^{13/3}\, \left(a^{1/3}-\left(a-b\, x^2\right)^{1/3}\right)\, \sqrt{\frac{a^{2/3}+a^{1/3}\, \left(a-b\, x^2\right)^{1/3}+\left(a-b\, x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\, a^{1/3}-\left(a-b\, x^2\right)^{1/3}\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)\, a^{1/3}-\left(a-b\, x^2\right)^{1/3}}{\left(1-\sqrt{3}\right)\, a^{1/3}-\left(a-b\, x^2\right)^{1/3}}\right],\, -7+4\, \sqrt{3}\right]\right)\Bigg/\left(1235\, b\, x\, \sqrt{-\frac{a^{1/3}\, \left(a^{1/3}-\left(a-b\, x^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right)\, a^{1/3}-\left(a-b\, x^2\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 99 leaves):

$$-\left(\left(3\, \left(-15255\, a^4\, x+3390\, a^3\, b\, x^3+8992\, a^2\, b^2\, x^5+2626\, a\, b^3\, x^7+247\, b^4\, x^9-40320\, a^4\, x\, \left(1-\frac{b\, x^2}{a}\right)^{1/3}\, \text{Hypergeometric2F1}\left[\frac{1}{3},\, \frac{1}{2},\, \frac{3}{2},\, \frac{b\, x^2}{a}\right]\right)\right)\Big/\left(6175\, \left(a-b\, x^2\right)^{1/3}\right)\right)$$

Problem 110: Result unnecessarily involves higher level functions.

$$\int \left(a-b\, x^2\right)^{2/3}\, \left(3\, a+b\, x^2\right)^2\, dx$$

Optimal (type 4, 617 leaves, 7 steps):

$$\frac{7776\,a^2\,x\,\left(a-b\,x^2\right)^{2/3}}{1729}-\frac{252}{247}\,a\,x\,\left(a-b\,x^2\right)^{5/3}-\frac{3}{19}\,x\,\left(a-b\,x^2\right)^{5/3}\,\left(3\,a+b\,x^2\right)-\frac{31104\,a^3\,x}{1729\,\left(\left(1-\sqrt{3}\right)\,a^{1/3}-\left(a-b\,x^2\right)^{1/3}\right)}-$$

$$\left(15552\times 3^{1/4}\,\sqrt{2+\sqrt{3}}\,\,a^{10/3}\,\left(a^{1/3}-\left(a-b\,x^2\right)^{1/3}\right)\,\sqrt{\frac{a^{2/3}+a^{1/3}\,\left(a-b\,x^2\right)^{1/3}+\left(a-b\,x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\,a^{1/3}-\left(a-b\,x^2\right)^{1/3}\right)^2}}\right.$$
$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)\,a^{1/3}-\left(a-b\,x^2\right)^{1/3}}{\left(1-\sqrt{3}\right)\,a^{1/3}-\left(a-b\,x^2\right)^{1/3}}\right],\,-7+4\,\sqrt{3}\right]\right)\Bigg/$$
$$\left(1729\,b\,x\,\sqrt{-\frac{a^{1/3}\,\left(a^{1/3}-\left(a-b\,x^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right)\,a^{1/3}-\left(a-b\,x^2\right)^{1/3}\right)^2}}\right)+$$

$$\left(10368\,\sqrt{2}\,\,3^{3/4}\,a^{10/3}\,\left(a^{1/3}-\left(a-b\,x^2\right)^{1/3}\right)\,\sqrt{\frac{a^{2/3}+a^{1/3}\,\left(a-b\,x^2\right)^{1/3}+\left(a-b\,x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\,a^{1/3}-\left(a-b\,x^2\right)^{1/3}\right)^2}}\right.$$
$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)\,a^{1/3}-\left(a-b\,x^2\right)^{1/3}}{\left(1-\sqrt{3}\right)\,a^{1/3}-\left(a-b\,x^2\right)^{1/3}}\right],\,-7+4\,\sqrt{3}\right]\right)\Bigg/$$
$$\left(1729\,b\,x\,\sqrt{-\frac{a^{1/3}\,\left(a^{1/3}-\left(a-b\,x^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right)\,a^{1/3}-\left(a-b\,x^2\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 88 leaves):

$$-\frac{1}{1729\,\left(a-b\,x^2\right)^{1/3}}3\,\left(-1731\,a^3\,x+961\,a^2\,b\,x^3+679\,a\,b^2\,x^5+91\,b^3\,x^7-3456\,a^3\,x\,\left(1-\frac{b\,x^2}{a}\right)^{1/3}\,\text{Hypergeometric2F1}\left[\frac{1}{3},\,\frac{1}{2},\,\frac{3}{2},\,\frac{b\,x^2}{a}\right]\right)$$

Problem 111: Result unnecessarily involves higher level functions.

$$\int\left(a-b\,x^2\right)^{2/3}\,\left(3\,a+b\,x^2\right)\,dx$$

Optimal (type 4, 588 leaves, 6 steps):

$$\frac{18}{13} a x \left(a-b x^2\right)^{2/3}-\frac{3}{13} x \left(a-b x^2\right)^{5/3}-\frac{72 a^2 x}{13 \left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}-$$

$$\left(36\times 3^{1/4} \sqrt{2+\sqrt{3}}\, a^{7/3} \left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right) \sqrt{\frac{a^{2/3}+a^{1/3} \left(a-b x^2\right)^{1/3}+\left(a-b x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}{\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}\right],-7+4 \sqrt{3}\right]\right)\Bigg/$$

$$\left(13 b x \sqrt{-\frac{a^{1/3} \left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right)+$$

$$\left(24 \sqrt{2}\, 3^{3/4} a^{7/3} \left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right) \sqrt{\frac{a^{2/3}+a^{1/3} \left(a-b x^2\right)^{1/3}+\left(a-b x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}{\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}\right],-7+4 \sqrt{3}\right]\right)\Bigg/$$

$$\left(13 b x \sqrt{-\frac{a^{1/3} \left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 76 leaves):

$$-\frac{1}{13 \left(a-b x^2\right)^{1/3}} 3 \left(-5 a^2 x+4 a b x^3+b^2 x^5-8 a^2 x \left(1-\frac{b x^2}{a}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{b x^2}{a}\right]\right)$$

Problem 112: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a-b x^2\right)^{2/3}}{3 a+b x^2}\, dx$$

Optimal (type 4, 740 leaves, 6 steps):

$$\frac{3x}{\left(1-\sqrt{3}\right)a^{1/3}-\left(a-b x^2\right)^{1/3}}+\frac{2^{1/3}a^{1/6}\,\text{ArcTan}\left[\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right]}{\sqrt{3}\sqrt{b}}+\frac{2^{1/3}a^{1/6}\,\text{ArcTan}\left[\frac{\sqrt{3}\,a^{1/6}\left(a^{1/3}-2^{1/3}\left(a-b x^2\right)^{1/3}\right)}{\sqrt{b}\,x}\right]}{\sqrt{3}\sqrt{b}}-$$

$$\frac{2^{1/3}a^{1/6}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x}{\sqrt{a}}\right]}{3\sqrt{b}}+\frac{2^{1/3}a^{1/6}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x}{a^{1/6}\left(a^{1/3}+2^{1/3}\left(a-b x^2\right)^{1/3}\right)}\right]}{\sqrt{b}}+$$

$$\left(3\times 3^{1/4}\sqrt{2+\sqrt{3}}\,a^{1/3}\left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right)\sqrt{\frac{a^{2/3}+a^{1/3}\left(a-b x^2\right)^{1/3}+\left(a-b x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)a^{1/3}-\left(a-b x^2\right)^{1/3}}{\left(1-\sqrt{3}\right)a^{1/3}-\left(a-b x^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(2 b x\sqrt{-\frac{a^{1/3}\left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right)a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right)-$$

$$\left(\sqrt{2}\,3^{3/4}a^{1/3}\left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right)\sqrt{\frac{a^{2/3}+a^{1/3}\left(a-b x^2\right)^{1/3}+\left(a-b x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)a^{1/3}-\left(a-b x^2\right)^{1/3}}{\left(1-\sqrt{3}\right)a^{1/3}-\left(a-b x^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(b x\sqrt{-\frac{a^{1/3}\left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right)a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right)$$

Result (type 6, 162 leaves) :

$$\left(9 a x\left(a-b x^2\right)^{2/3}\text{AppellF1}\left[\frac{1}{2},-\frac{2}{3},1,\frac{3}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]\right)\Big/$$

$$\left(\left(3 a+b x^2\right)\left(9 a\,\text{AppellF1}\left[\frac{1}{2},-\frac{2}{3},1,\frac{3}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]-\right.\right.$$

$$\left.\left.2 b x^2\left(\text{AppellF1}\left[\frac{3}{2},-\frac{2}{3},2,\frac{5}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]+2\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},1,\frac{5}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]\right)\right)\right)$$

Problem 113: Result unnecessarily involves higher level functions.

$$\int\frac{\left(a-b x^2\right)^{2/3}}{\left(3 a+b x^2\right)^2}\,dx$$

Optimal (type 4, 584 leaves, 6 steps) :

$$\frac{x\left(a-b x^{2}\right)^{2/3}}{6 a\left(3 a+b x^{2}\right)}-\frac{x}{6 a\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^{2}\right)^{1/3}\right)}-$$

$$\left(\sqrt{2+\sqrt{3}}\left(a^{1/3}-\left(a-b x^{2}\right)^{1/3}\right)\sqrt{\frac{a^{2/3}+a^{1/3}\left(a-b x^{2}\right)^{1/3}+\left(a-b x^{2}\right)^{2/3}}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^{2}\right)^{1/3}\right)^{2}}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right) a^{1/3}-\left(a-b x^{2}\right)^{1/3}}{\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^{2}\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Bigg/$$

$$\left(4\times 3^{3/4} a^{2/3} b x\sqrt{-\frac{a^{1/3}\left(a^{1/3}-\left(a-b x^{2}\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^{2}\right)^{1/3}\right)^{2}}}\right)+$$

$$\left(\left(a^{1/3}-\left(a-b x^{2}\right)^{1/3}\right)\sqrt{\frac{a^{2/3}+a^{1/3}\left(a-b x^{2}\right)^{1/3}+\left(a-b x^{2}\right)^{2/3}}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^{2}\right)^{1/3}\right)^{2}}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right) a^{1/3}-\left(a-b x^{2}\right)^{1/3}}{\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^{2}\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Bigg/$$

$$\left(3\sqrt{2}\, 3^{1/4} a^{2/3} b x\sqrt{-\frac{a^{1/3}\left(a^{1/3}-\left(a-b x^{2}\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^{2}\right)^{1/3}\right)^{2}}}\right)$$

Result (type 5, 86 leaves):

$$\frac{x\left(a-b x^{2}\right)^{2/3}}{6 a\left(3 a+b x^{2}\right)}+\frac{x\left(\frac{a-b x^{2}}{a}\right)^{1/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{b x^{2}}{a}\right]}{18 a\left(a-b x^{2}\right)^{1/3}}$$

Problem 114: Result unnecessarily involves higher level functions.

$$\int\frac{\left(a-b x^{2}\right)^{2/3}}{\left(3 a+b x^{2}\right)^{3}}\,dx$$

Optimal (type 4, 818 leaves, 8 steps):

$$\frac{x\left(a-b x^2\right)^{2/3}}{12 a\left(3 a+b x^2\right)^2}+\frac{x\left(a-b x^2\right)^{2/3}}{36 a^2\left(3 a+b x^2\right)}-\frac{x}{36 a^2\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}+\frac{\text{ArcTan}\left[\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right]}{72\times 2^{2/3}\sqrt{3}\,a^{11/6}\sqrt{b}}+$$

$$\frac{\text{ArcTan}\left[\frac{\sqrt{3}\,a^{1/6}\left(a^{1/3}-2^{1/3}\left(a-b x^2\right)^{1/3}\right)}{\sqrt{b}\,x}\right]}{72\times 2^{2/3}\sqrt{3}\,a^{11/6}\sqrt{b}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{b}\,x}{\sqrt{a}}\right]}{216\times 2^{2/3}a^{11/6}\sqrt{b}}+\frac{\text{ArcTanh}\left[\frac{\sqrt{b}\,x}{a^{1/6}\left(a^{1/3}+2^{1/3}\left(a-b x^2\right)^{1/3}\right)}\right]}{72\times 2^{2/3}a^{11/6}\sqrt{b}}-$$

$$\left(\sqrt{2+\sqrt{3}}\left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right)\sqrt{\frac{a^{2/3}+a^{1/3}\left(a-b x^2\right)^{1/3}+\left(a-b x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}{\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(24\times 3^{3/4} a^{5/3} b\,x\sqrt{-\frac{a^{1/3}\left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right)+$$

$$\left(\left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right)\sqrt{\frac{a^{2/3}+a^{1/3}\left(a-b x^2\right)^{1/3}+\left(a-b x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}{\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(18\sqrt{2}\,3^{1/4} a^{5/3} b\,x\sqrt{-\frac{a^{1/3}\left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right)$$

Result (type 6, 350 leaves) :

$$\left(x\left(\frac{3\left(a-b x^2\right)\left(6 a+b x^2\right)}{a^2}+\left(54\left(3 a+b x^2\right) \text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]\right)\Big/\right.\right.$$
$$\left(9 a\, \text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]+\right.$$
$$\left.\left.2 b x^2\left(-\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},2,\frac{5}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]+\text{AppellF1}\left[\frac{3}{2},\frac{4}{3},1,\frac{5}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]\right)\right)+$$
$$\left(5 b x^2\left(3 a+b x^2\right) \text{AppellF1}\left[\frac{3}{2},\frac{1}{3},1,\frac{5}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]\right)\Big/$$
$$\left(a\left(15 a\, \text{AppellF1}\left[\frac{3}{2},\frac{1}{3},1,\frac{5}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]+\right.\right.$$
$$2 b x^2\left(-\text{AppellF1}\left[\frac{5}{2},\frac{1}{3},2,\frac{7}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]+\right.$$
$$\left.\left.\left.\left.\left.\text{AppellF1}\left[\frac{5}{2},\frac{4}{3},1,\frac{7}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]\right)\right)\right)\right)\right)\Big/\left(108\left(a-b x^2\right)^{1/3}\left(3 a+b x^2\right)^2\right)$$

Problem 115: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a-b x^2\right)^{2/3}}{\left(3 a+b x^2\right)^4}\, dx$$

Optimal (type 4, 849 leaves, 9 steps):

$$\frac{x\left(a-b x^2\right)^{2/3}}{18\, a\left(3 a+b x^2\right)^3}+\frac{x\left(a-b x^2\right)^{2/3}}{54\, a^2\left(3 a+b x^2\right)^2}+\frac{x\left(a-b x^2\right)^{2/3}}{144\, a^3\left(3 a+b x^2\right)}-$$

$$\frac{x}{144\, a^3\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}+\frac{7\,\text{ArcTan}\left[\frac{\sqrt{3}\,\sqrt{a}}{\sqrt{b}\,x}\right]}{1296\times 2^{2/3}\sqrt{3}\, a^{17/6}\sqrt{b}}+$$

$$\frac{7\,\text{ArcTan}\left[\frac{\sqrt{3}\, a^{1/6}\left(a^{1/3}-2^{1/3}\left(a-b x^2\right)^{1/3}\right)}{\sqrt{b}\,x}\right]}{1296\times 2^{2/3}\sqrt{3}\, a^{17/6}\sqrt{b}}-\frac{7\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x}{\sqrt{a}}\right]}{3888\times 2^{2/3}\, a^{17/6}\sqrt{b}}+\frac{7\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x}{a^{1/6}\left(a^{1/3}+2^{1/3}\left(a-b x^2\right)^{1/3}\right)}\right]}{1296\times 2^{2/3}\, a^{17/6}\sqrt{b}}-$$

$$\left(\sqrt{2+\sqrt{3}}\left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right)\sqrt{\frac{a^{2/3}+a^{1/3}\left(a-b x^2\right)^{1/3}+\left(a-b x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}{\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(96\times 3^{3/4}\, a^{8/3}\, b\, x\sqrt{-\frac{a^{1/3}\left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right)+$$

$$\left(\left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right)\sqrt{\frac{a^{2/3}+a^{1/3}\left(a-b x^2\right)^{1/3}+\left(a-b x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}{\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(72\sqrt{2}\; 3^{1/4}\, a^{8/3}\, b\, x\sqrt{-\frac{a^{1/3}\left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right)$$

Result (type 6, 364 leaves) :

$$\left(x\left(\left(a-b x^2\right)\left(75 a^2+26 a b x^2+3 b^2 x^4\right)+\left(69 a^2\left(3 a+b x^2\right)^2 \text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]\right)\Big/\right.\right.$$
$$\left(9 a \text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]+\right.$$
$$\left.2 b x^2\left(-\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},2,\frac{5}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]+\text{AppellF1}\left[\frac{3}{2},\frac{4}{3},1,\frac{5}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]\right)\right)+$$
$$\left(5 a b\left(3 a x+b x^3\right)^2 \text{AppellF1}\left[\frac{3}{2},\frac{1}{3},1,\frac{5}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]\right)\Big/$$
$$\left(15 a \text{AppellF1}\left[\frac{3}{2},\frac{1}{3},1,\frac{5}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]+2 b x^2\left(-\text{AppellF1}\left[\frac{5}{2},\frac{1}{3},2,\frac{7}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]+\right.\right.$$
$$\left.\left.\left.\left.\text{AppellF1}\left[\frac{5}{2},\frac{4}{3},1,\frac{7}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]\right)\right)\right)\right)\Big/\left(432 a^3\left(a-b x^2\right)^{1/3}\left(3 a+b x^2\right)^3\right)$$

Problem 116: Result unnecessarily involves higher level functions.

$$\int\left(a-b x^2\right)^{5/3}\left(3 a+b x^2\right)^3 dx$$

Optimal (type 4, 668 leaves, 9 steps):

$$\frac{2809728\,a^4\,x\,\left(a-b\,x^2\right)^{2/3}}{267995}+\frac{1404864\,a^3\,x\,\left(a-b\,x^2\right)^{5/3}}{191425}-\frac{33264\,a^2\,x\,\left(a-b\,x^2\right)^{8/3}}{14725}-\frac{432}{775}\,a\,x\,\left(a-b\,x^2\right)^{8/3}\,\left(3\,a+b\,x^2\right)-\frac{3}{31}\,x\,\left(a-b\,x^2\right)^{8/3}\,\left(3\,a+b\,x^2\right)^2-\frac{11238912\,a^5\,x}{267995\,\left(\left(1-\sqrt{3}\right)\,a^{1/3}-\left(a-b\,x^2\right)^{1/3}\right)}-\left(5619456\times 3^{1/4}\,\sqrt{2+\sqrt{3}}\,a^{16/3}\,\left(a^{1/3}-\left(a-b\,x^2\right)^{1/3}\right)\,\sqrt{\frac{a^{2/3}+a^{1/3}\,\left(a-b\,x^2\right)^{1/3}+\left(a-b\,x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\,a^{1/3}-\left(a-b\,x^2\right)^{1/3}\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)\,a^{1/3}-\left(a-b\,x^2\right)^{1/3}}{\left(1-\sqrt{3}\right)\,a^{1/3}-\left(a-b\,x^2\right)^{1/3}}\right],-7+4\,\sqrt{3}\right]\right)\Bigg/\left(267995\,b\,x\,\sqrt{-\frac{a^{1/3}\,\left(a^{1/3}-\left(a-b\,x^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right)\,a^{1/3}-\left(a-b\,x^2\right)^{1/3}\right)^2}}\right)+\left(3746304\,\sqrt{2}\,3^{3/4}\,a^{16/3}\,\left(a^{1/3}-\left(a-b\,x^2\right)^{1/3}\right)\,\sqrt{\frac{a^{2/3}+a^{1/3}\,\left(a-b\,x^2\right)^{1/3}+\left(a-b\,x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\,a^{1/3}-\left(a-b\,x^2\right)^{1/3}\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)\,a^{1/3}-\left(a-b\,x^2\right)^{1/3}}{\left(1-\sqrt{3}\right)\,a^{1/3}-\left(a-b\,x^2\right)^{1/3}}\right],-7+4\,\sqrt{3}\right]\right)\Bigg/\left(267995\,b\,x\,\sqrt{-\frac{a^{1/3}\,\left(a^{1/3}-\left(a-b\,x^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right)\,a^{1/3}-\left(a-b\,x^2\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 110 leaves) :

$$\left(3\,\left(5815935\,a^5\,x-5312355\,a^4\,b\,x^3-1675114\,a^3\,b^2\,x^5+749658\,a^2\,b^3\,x^7+378651\,a\,b^4\,x^9+43225\,b^5\,x^{11}+6243840\,a^5\,x\,\left(1-\frac{b\,x^2}{a}\right)^{1/3}\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{b\,x^2}{a}\right]\right)\right)\Big/\left(1339975\,\left(a-b\,x^2\right)^{1/3}\right)$$

Problem 117: Result unnecessarily involves higher level functions.

$$\int\left(a-b\,x^2\right)^{5/3}\,\left(3\,a+b\,x^2\right)^2\,dx$$

Optimal (type 4, 637 leaves, 8 steps) :

$$\frac{28512\, a^3\, x\, \left(a-b\, x^2\right)^{2/3}}{8645}+\frac{14256\, a^2\, x\, \left(a-b\, x^2\right)^{5/3}}{6175}-\frac{306}{475}\, a\, x\, \left(a-b\, x^2\right)^{8/3}-\frac{3}{25}\, x\, \left(a-b\, x^2\right)^{8/3}\, \left(3\, a+b\, x^2\right)-\frac{114048\, a^4\, x}{8645\, \left(\left(1-\sqrt{3}\right)\, a^{1/3}-\left(a-b\, x^2\right)^{1/3}\right)}-\left(57024\times 3^{1/4}\, \sqrt{2+\sqrt{3}}\, a^{13/3}\, \left(a^{1/3}-\left(a-b\, x^2\right)^{1/3}\right)\, \sqrt{\frac{a^{2/3}+a^{1/3}\, \left(a-b\, x^2\right)^{1/3}+\left(a-b\, x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\, a^{1/3}-\left(a-b\, x^2\right)^{1/3}\right)^2}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)\, a^{1/3}-\left(a-b\, x^2\right)^{1/3}}{\left(1-\sqrt{3}\right)\, a^{1/3}-\left(a-b\, x^2\right)^{1/3}}\right],\, -7+4\, \sqrt{3}\right]\right)\Bigg/\left(8645\, b\, x\, \sqrt{-\frac{a^{1/3}\, \left(a^{1/3}-\left(a-b\, x^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right)\, a^{1/3}-\left(a-b\, x^2\right)^{1/3}\right)^2}}\right)+\left(38016\, \sqrt{2}\, 3^{3/4}\, a^{13/3}\, \left(a^{1/3}-\left(a-b\, x^2\right)^{1/3}\right)\, \sqrt{\frac{a^{2/3}+a^{1/3}\, \left(a-b\, x^2\right)^{1/3}+\left(a-b\, x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\, a^{1/3}-\left(a-b\, x^2\right)^{1/3}\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)\, a^{1/3}-\left(a-b\, x^2\right)^{1/3}}{\left(1-\sqrt{3}\right)\, a^{1/3}-\left(a-b\, x^2\right)^{1/3}}\right],\, -7+4\, \sqrt{3}\right]\right)\Bigg/\left(8645\, b\, x\, \sqrt{-\frac{a^{1/3}\, \left(a^{1/3}-\left(a-b\, x^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right)\, a^{1/3}-\left(a-b\, x^2\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 99 leaves):

$$\left(3\, \left(66315\, a^4\, x-72370\, a^3\, b\, x^3-4956\, a^2\, b^2\, x^5+9282\, a\, b^3\, x^7+1729\, b^4\, x^9+63360\, a^4\, x\, \left(1-\frac{b\, x^2}{a}\right)^{1/3}\, \text{Hypergeometric2F1}\left[\frac{1}{3},\, \frac{1}{2},\, \frac{3}{2},\, \frac{b\, x^2}{a}\right]\right)\right)\Big/\left(43225\, \left(a-b\, x^2\right)^{1/3}\right)$$

Problem 118: Result unnecessarily involves higher level functions.

$$\int \left(a-b\, x^2\right)^{5/3}\, \left(3\, a+b\, x^2\right)\, dx$$

Optimal (type 4, 608 leaves, 7 steps):

$$\frac{1800 a^2 x \left(a-b x^2\right)^{2/3}}{1729}+\frac{180}{247} a x \left(a-b x^2\right)^{5/3}-\frac{3}{19} x \left(a-b x^2\right)^{8/3}-\frac{7200 a^3 x}{1729 \left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}-$$

$$\left(3600 \times 3^{1/4} \sqrt{2+\sqrt{3}}\, a^{10/3} \left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right) \sqrt{\frac{a^{2/3}+a^{1/3} \left(a-b x^2\right)^{1/3}+\left(a-b x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}{\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}\right],-7+4 \sqrt{3}\right]\right) \Big/$$

$$\left(1729\, b\, x \sqrt{-\frac{a^{1/3} \left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right)+$$

$$\left(2400 \sqrt{2}\, 3^{3/4} a^{10/3} \left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right) \sqrt{\frac{a^{2/3}+a^{1/3} \left(a-b x^2\right)^{1/3}+\left(a-b x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}{\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}\right],-7+4 \sqrt{3}\right]\right) \Big/$$

$$\left(1729\, b\, x \sqrt{-\frac{a^{1/3} \left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 88 leaves):

$$\frac{1}{1729 \left(a-b x^2\right)^{1/3}} 3 \left(929 a^3 x-1167 a^2 b x^3+147 a b^2 x^5+\right.$$

$$\left.91 b^3 x^7+800 a^3 x \left(1-\frac{b x^2}{a}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{b x^2}{a}\right]\right)$$

Problem 119: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a-b x^2\right)^{5/3}}{3 a+b x^2} \, dx$$

Optimal (type 4, 765 leaves, 7 steps):

$$-\frac{3}{7} x \left(a-b x^2\right)^{2/3}+\frac{96 a x}{7 \left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}+$$

$$\frac{4\times 2^{1/3} a^{7/6} \text{ArcTan}\left[\frac{\sqrt{3} \sqrt{a}}{\sqrt{b} x}\right]}{\sqrt{3} \sqrt{b}}+\frac{4\times 2^{1/3} a^{7/6} \text{ArcTan}\left[\frac{\sqrt{3} a^{1/6} \left(a^{1/3}-2^{1/3} \left(a-b x^2\right)^{1/3}\right)}{\sqrt{b} x}\right]}{\sqrt{3} \sqrt{b}}-$$

$$\frac{4\times 2^{1/3} a^{7/6} \text{ArcTanh}\left[\frac{\sqrt{b} x}{\sqrt{a}}\right]}{3 \sqrt{b}}+\frac{4\times 2^{1/3} a^{7/6} \text{ArcTanh}\left[\frac{\sqrt{b} x}{a^{1/6} \left(a^{1/3}+2^{1/3} \left(a-b x^2\right)^{1/3}\right)}\right]}{\sqrt{b}}+$$

$$\left(48\times 3^{1/4} \sqrt{2+\sqrt{3}}\, a^{4/3} \left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right) \sqrt{\frac{a^{2/3}+a^{1/3} \left(a-b x^2\right)^{1/3}+\left(a-b x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}{\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}\right],-7+4 \sqrt{3}\right]\right)\Bigg/$$

$$\left(7 b x \sqrt{-\frac{a^{1/3} \left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right)-$$

$$\left(32 \sqrt{2}\, 3^{3/4} a^{4/3} \left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right) \sqrt{\frac{a^{2/3}+a^{1/3} \left(a-b x^2\right)^{1/3}+\left(a-b x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}{\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}\right],-7+4 \sqrt{3}\right]\right)\Bigg/$$

$$\left(7 b x \sqrt{-\frac{a^{1/3} \left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right)$$

Result (type 6, 333 leaves) :

$$\frac{1}{7\left(a-b x^2\right)^{1/3}} x \left(-3 a+3 b x^2+\left(144 a^3 \text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]\right)\Big/\right.$$
$$\left(\left(3 a+b x^2\right)\left(9 a \text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]+\right.\right.$$
$$\left.\left.2 b x^2\left(-\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},2,\frac{5}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]+\text{AppellF1}\left[\frac{3}{2},\frac{4}{3},1,\frac{5}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]\right)\right)\right)-$$
$$\left(160 a^2 b x^2 \text{AppellF1}\left[\frac{3}{2},\frac{1}{3},1,\frac{5}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]\right)\Big/$$
$$\left(\left(3 a+b x^2\right)\left(15 a \text{AppellF1}\left[\frac{3}{2},\frac{1}{3},1,\frac{5}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]+\right.\right.$$
$$\left.\left.\left.2 b x^2\left(-\text{AppellF1}\left[\frac{5}{2},\frac{1}{3},2,\frac{7}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]+\text{AppellF1}\left[\frac{5}{2},\frac{4}{3},1,\frac{7}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]\right)\right)\right)\right)$$

Problem 120: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a-b x^2\right)^{5/3}}{\left(3 a+b x^2\right)^2} \, dx$$

Optimal (type 4, 775 leaves, 7 steps):

$$\frac{2\,x\,\left(a-b\,x^2\right)^{2/3}}{3\,\left(3\,a+b\,x^2\right)}-\frac{11\,x}{3\,\left(\left(1-\sqrt{3}\right)\,a^{1/3}-\left(a-b\,x^2\right)^{1/3}\right)}-$$

$$\frac{2^{1/3}\,a^{1/6}\,\text{ArcTan}\left[\frac{\sqrt{3}\,\sqrt{a}}{\sqrt{b}\,x}\right]}{\sqrt{3}\,\sqrt{b}}-\frac{2^{1/3}\,a^{1/6}\,\text{ArcTan}\left[\frac{\sqrt{3}\,a^{1/6}\,\left(a^{1/3}-2^{1/3}\,\left(a-b\,x^2\right)^{1/3}\right)}{\sqrt{b}\,x}\right]}{\sqrt{3}\,\sqrt{b}}+$$

$$\frac{2^{1/3}\,a^{1/6}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x}{\sqrt{a}}\right]}{3\,\sqrt{b}}-\frac{2^{1/3}\,a^{1/6}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x}{a^{1/6}\,\left(a^{1/3}+2^{1/3}\,\left(a-b\,x^2\right)^{1/3}\right)}\right]}{\sqrt{b}}-$$

$$\left(11\,\sqrt{2+\sqrt{3}}\,a^{1/3}\,\left(a^{1/3}-\left(a-b\,x^2\right)^{1/3}\right)\,\sqrt{\frac{a^{2/3}+a^{1/3}\,\left(a-b\,x^2\right)^{1/3}+\left(a-b\,x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\,a^{1/3}-\left(a-b\,x^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)\,a^{1/3}-\left(a-b\,x^2\right)^{1/3}}{\left(1-\sqrt{3}\right)\,a^{1/3}-\left(a-b\,x^2\right)^{1/3}}\right],\,-7+4\,\sqrt{3}\right]\right)\Bigg/$$

$$\left(2\times 3^{3/4}\,b\,x\,\sqrt{-\frac{a^{1/3}\,\left(a^{1/3}-\left(a-b\,x^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right)\,a^{1/3}-\left(a-b\,x^2\right)^{1/3}\right)^2}}\right)+$$

$$\left(11\,\sqrt{2}\,a^{1/3}\,\left(a^{1/3}-\left(a-b\,x^2\right)^{1/3}\right)\,\sqrt{\frac{a^{2/3}+a^{1/3}\,\left(a-b\,x^2\right)^{1/3}+\left(a-b\,x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\,a^{1/3}-\left(a-b\,x^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)\,a^{1/3}-\left(a-b\,x^2\right)^{1/3}}{\left(1-\sqrt{3}\right)\,a^{1/3}-\left(a-b\,x^2\right)^{1/3}}\right],\,-7+4\,\sqrt{3}\right]\right)\Bigg/$$

$$\left(3\times 3^{1/4}\,b\,x\,\sqrt{-\frac{a^{1/3}\,\left(a^{1/3}-\left(a-b\,x^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right)\,a^{1/3}-\left(a-b\,x^2\right)^{1/3}\right)^2}}\right)$$

Result (type 6, 320 leaves) :

$$\left(x\left(6a-6bx^2-\left(27a^2\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right]\right)\Big/\left(9a\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right]+2bx^2\left(-\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},2,\frac{5}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right]+\text{AppellF1}\left[\frac{3}{2},\frac{4}{3},1,\frac{5}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right]\right)\right)+\left(55abx^2\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},1,\frac{5}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right]\right)\Big/\left(15a\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},1,\frac{5}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right]+2bx^2\left(-\text{AppellF1}\left[\frac{5}{2},\frac{1}{3},2,\frac{7}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right]+\text{AppellF1}\left[\frac{5}{2},\frac{4}{3},1,\frac{7}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right]\right)\right)\right)\right)\Big/\left(9\left(a-bx^2\right)^{1/3}\left(3a+bx^2\right)\right)$$

Problem 121: Result unnecessarily involves higher level functions.

$$\int\frac{\left(a-bx^2\right)^{5/3}}{\left(3a+bx^2\right)^3}\,dx$$

Optimal (type 4, 815 leaves, 9 steps):

$$\frac{x\left(a-b x^2\right)^{2/3}}{3\left(3 a+b x^2\right)^2}-\frac{x\left(a-b x^2\right)^{2/3}}{18 a\left(3 a+b x^2\right)}+\frac{x}{18 a\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}+\frac{\text{ArcTan}\left[\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right]}{18\times 2^{2/3}\sqrt{3}\,a^{5/6}\sqrt{b}}+$$

$$\frac{\text{ArcTan}\left[\frac{\sqrt{3}\,a^{1/6}\left(a^{1/3}-2^{1/3}\left(a-b x^2\right)^{1/3}\right)}{\sqrt{b}\,x}\right]}{18\times 2^{2/3}\sqrt{3}\,a^{5/6}\sqrt{b}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{b}\,x}{\sqrt{a}}\right]}{54\times 2^{2/3}a^{5/6}\sqrt{b}}+\frac{\text{ArcTanh}\left[\frac{\sqrt{b}\,x}{a^{1/6}\left(a^{1/3}+2^{1/3}\left(a-b x^2\right)^{1/3}\right)}\right]}{18\times 2^{2/3}a^{5/6}\sqrt{b}}+$$

$$\left(\sqrt{2+\sqrt{3}}\left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right)\sqrt{\frac{a^{2/3}+a^{1/3}\left(a-b x^2\right)^{1/3}+\left(a-b x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)a^{1/3}-\left(a-b x^2\right)^{1/3}}{\left(1-\sqrt{3}\right)a^{1/3}-\left(a-b x^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(12\times 3^{3/4}a^{2/3}\,b\,x\sqrt{-\frac{a^{1/3}\left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right)a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right)-$$

$$\left(\left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right)\sqrt{\frac{a^{2/3}+a^{1/3}\left(a-b x^2\right)^{1/3}+\left(a-b x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)a^{1/3}-\left(a-b x^2\right)^{1/3}}{\left(1-\sqrt{3}\right)a^{1/3}-\left(a-b x^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(9\sqrt{2}\,3^{1/4}a^{2/3}\,b\,x\sqrt{-\frac{a^{1/3}\left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right)a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right)$$

Result (type 6, 346 leaves):

$$\left(x\left(9 a-12 b x^2+\frac{3 b^2 x^4}{a}+\left(27 a\left(3 a+b x^2\right)\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]\right)\Big/\right.\right.$$

$$\left(9 a\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]+\right.$$

$$\left.\left.2 b x^2\left(-\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},2,\frac{5}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]+\text{AppellF1}\left[\frac{3}{2},\frac{4}{3},1,\frac{5}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]\right)\right)-$$

$$\left(5 b x^2\left(3 a+b x^2\right)\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},1,\frac{5}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]\right)\Big/$$

$$\left(15 a\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},1,\frac{5}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]+2 b x^2\left(-\text{AppellF1}\left[\frac{5}{2},\frac{1}{3},2,\frac{7}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]+\right.\right.$$

$$\left.\left.\left.\left.\text{AppellF1}\left[\frac{5}{2},\frac{4}{3},1,\frac{7}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]\right)\right)\right)\right)\Big/\left(54\left(a-b x^2\right)^{1/3}\left(3 a+b x^2\right)^2\right)$$

Problem 122: Result unnecessarily involves higher level functions.

$$\int \frac{\left(3 a+b x^2\right)^4}{\left(a-b x^2\right)^{1/3}} \, dx$$

Optimal (type 4, 659 leaves, 8 steps):

$$-\frac{1552608 a^3 x \left(a-b x^2\right)^{2/3}}{43225}-\frac{36288 a^2 x \left(a-b x^2\right)^{2/3} \left(3 a+b x^2\right)}{6175}-\frac{18}{19} a x \left(a-b x^2\right)^{2/3} \left(3 a+b x^2\right)^2-\frac{3}{25} x \left(a-b x^2\right)^{2/3} \left(3 a+b x^2\right)^3-\frac{3794688 a^4 x}{8645 \left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}-\left(1897344 \times 3^{1/4} \sqrt{2+\sqrt{3}}\, a^{13/3} \left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right) \sqrt{\frac{a^{2/3}+a^{1/3} \left(a-b x^2\right)^{1/3}+\left(a-b x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}{\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}\right],-7+4 \sqrt{3}\right]\right) \Big/ \left(8645 b x \sqrt{-\frac{a^{1/3} \left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right)+\left(1264896 \sqrt{2}\, 3^{3/4} a^{13/3} \left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right) \sqrt{\frac{a^{2/3}+a^{1/3} \left(a-b x^2\right)^{1/3}+\left(a-b x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}{\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}\right],-7+4 \sqrt{3}\right]\right) \Big/ \left(8645 b x \sqrt{-\frac{a^{1/3} \left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 98 leaves):

$$\left(3 x \left(-941085 a^4+727830 a^3 b x^2+184044 a^2 b^2 x^4+27482 a b^3 x^6+1729 b^4 x^8+2108160 a^4 \left(1-\frac{b x^2}{a}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{b x^2}{a}\right]\right)\right) \Big/ \left(43225 \left(a-b x^2\right)^{1/3}\right)$$

Problem 123: Result unnecessarily involves higher level functions.

$$\int \frac{\left(3 a+b x^2\right)^3}{\left(a-b x^2\right)^{1/3}} \, dx$$

Optimal (type 4, 628 leaves, 7 steps):

$$-\frac{15768 a^2 x \left(a-b x^2\right)^{2/3}}{1729}-\frac{324}{247} a x \left(a-b x^2\right)^{2/3} \left(3 a+b x^2\right)-\frac{3}{19} x \left(a-b x^2\right)^{2/3} \left(3 a+b x^2\right)^2-\frac{215136 a^3 x}{1729 \left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}-\left(107568\times 3^{1/4} \sqrt{2+\sqrt{3}}\, a^{10/3} \left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right) \sqrt{\frac{a^{2/3}+a^{1/3} \left(a-b x^2\right)^{1/3}+\left(a-b x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}{\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}\right],-7+4 \sqrt{3}\right]\right)\Big/\left(1729 b x \sqrt{-\frac{a^{1/3} \left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right)+\left(71712 \sqrt{2}\ 3^{3/4} a^{10/3} \left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right) \sqrt{\frac{a^{2/3}+a^{1/3} \left(a-b x^2\right)^{1/3}+\left(a-b x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}{\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}\right],-7+4 \sqrt{3}\right]\right)\Big/\left(1729 b x \sqrt{-\frac{a^{1/3} \left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 88 leaves):

$$\frac{1}{1729 \left(a-b x^2\right)^{1/3}} 3 \left(-8343 a^3 x+7041 a^2 b x^3+1211 a b^2 x^5+91 b^3 x^7+23904 a^3 x \left(1-\frac{b x^2}{a}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{b x^2}{a}\right]\right)$$

Problem 124: Result unnecessarily involves higher level functions.

$$\int \frac{\left(3 a+b x^2\right)^2}{\left(a-b x^2\right)^{1/3}} \, dx$$

Optimal (type 4, 597 leaves, 6 steps):

$$-\frac{198}{91} a x \left(a-b x^2\right)^{2/3}-\frac{3}{13} x \left(a-b x^2\right)^{2/3} \left(3 a+b x^2\right)-\frac{3240 a^2 x}{91 \left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}-$$

$$\left(1620\times 3^{1/4} \sqrt{2+\sqrt{3}}\ a^{7/3} \left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right) \sqrt{\frac{a^{2/3}+a^{1/3} \left(a-b x^2\right)^{1/3}+\left(a-b x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}{\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}\right],-7+4 \sqrt{3}\right]\right)\Bigg/$$

$$\left(91 b x \sqrt{-\frac{a^{1/3} \left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right)+$$

$$\left(1080 \sqrt{2}\ 3^{3/4} a^{7/3} \left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right) \sqrt{\frac{a^{2/3}+a^{1/3} \left(a-b x^2\right)^{1/3}+\left(a-b x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}{\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}\right],-7+4 \sqrt{3}\right]\right)\Bigg/$$

$$\left(91 b x \sqrt{-\frac{a^{1/3} \left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 77 leaves):

$$\frac{1}{91 \left(a-b x^2\right)^{1/3}}$$

$$3 \left(-87 a^2 x+80 a b x^3+7 b^2 x^5+360 a^2 x \left(1-\frac{b x^2}{a}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{b x^2}{a}\right]\right)$$

Problem 125: Result unnecessarily involves higher level functions.

$$\int \frac{3 a+b x^2}{\left(a-b x^2\right)^{1/3}} \, dx$$

Optimal (type 4, 568 leaves, 5 steps):

$$-\frac{3}{7}x\left(a-b x^2\right)^{2/3}-\frac{72 a x}{7\left(\left(1-\sqrt{3}\right)a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}-$$

$$\left(36\times 3^{1/4}\sqrt{2+\sqrt{3}}\,a^{4/3}\left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right)\sqrt{\frac{a^{2/3}+a^{1/3}\left(a-b x^2\right)^{1/3}+\left(a-b x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)a^{1/3}-\left(a-b x^2\right)^{1/3}}{\left(1-\sqrt{3}\right)a^{1/3}-\left(a-b x^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\bigg/$$

$$\left(7 b x\sqrt{-\frac{a^{1/3}\left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right)a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right)+$$

$$\left(24\sqrt{2}\,3^{3/4}a^{4/3}\left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right)\sqrt{\frac{a^{2/3}+a^{1/3}\left(a-b x^2\right)^{1/3}+\left(a-b x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)a^{1/3}-\left(a-b x^2\right)^{1/3}}{\left(1-\sqrt{3}\right)a^{1/3}-\left(a-b x^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\bigg/$$

$$\left(7 b x\sqrt{-\frac{a^{1/3}\left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right)a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 62 leaves):

$$\frac{3 x\left(-a+b x^2+8 a\left(1-\frac{b x^2}{a}\right)^{1/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{b x^2}{a}\right]\right)}{7\left(a-b x^2\right)^{1/3}}$$

Problem 126: Result unnecessarily involves higher level functions.

$$\int\frac{1}{\left(a-b x^2\right)^{1/3}\left(3 a+b x^2\right)}\,dx$$

Optimal (type 3, 204 leaves, 1 step):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right]}{2\times 2^{2/3}\sqrt{3}\,a^{5/6}\sqrt{b}}+\frac{\text{ArcTan}\left[\frac{\sqrt{3}\,a^{1/6}\left(a^{1/3}-2^{1/3}\left(a-b x^2\right)^{1/3}\right)}{\sqrt{b}\,x}\right]}{2\times 2^{2/3}\sqrt{3}\,a^{5/6}\sqrt{b}}-$$

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{b}\,x}{\sqrt{a}}\right]}{6\times 2^{2/3}a^{5/6}\sqrt{b}}+\frac{\text{ArcTanh}\left[\frac{\sqrt{b}\,x}{a^{1/6}\left(a^{1/3}+2^{1/3}\left(a-b x^2\right)^{1/3}\right)}\right]}{2\times 2^{2/3}a^{5/6}\sqrt{b}}$$

Result (type 6, 162 leaves):

$$\left(9\, a\, x\, \text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]\right)\Big/ \left(\left(a-b x^2\right)^{1/3}\left(3 a+b x^2\right)\left(9\, a\, \text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]+ 2\, b\, x^2\left(-\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},2,\frac{5}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]+\text{AppellF1}\left[\frac{3}{2},\frac{4}{3},1,\frac{5}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]\right)\right)\right)$$

Problem 127: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(a-b x^2\right)^{1/3}\left(3 a+b x^2\right)^2}\, dx$$

Optimal (type 4, 787 leaves, 7 steps):

$$\frac{x\left(a-b x^2\right)^{2/3}}{24 a^2\left(3 a+b x^2\right)}-\frac{x}{24 a^2\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}+\frac{\text{ArcTan}\left[\frac{\sqrt{3}\,\sqrt{a}}{\sqrt{b}\,x}\right]}{8\times 2^{2/3}\sqrt{3}\, a^{11/6}\sqrt{b}}+$$

$$\frac{\text{ArcTan}\left[\frac{\sqrt{3}\, a^{1/6}\left(a^{1/3}-2^{1/3}\left(a-b x^2\right)^{1/3}\right)}{\sqrt{b}\, x}\right]}{8\times 2^{2/3}\sqrt{3}\, a^{11/6}\sqrt{b}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{b}\, x}{\sqrt{a}}\right]}{24\times 2^{2/3}\, a^{11/6}\sqrt{b}}+\frac{\text{ArcTanh}\left[\frac{\sqrt{b}\, x}{a^{1/6}\left(a^{1/3}+2^{1/3}\left(a-b x^2\right)^{1/3}\right)}\right]}{8\times 2^{2/3}\, a^{11/6}\sqrt{b}}-$$

$$\left(\sqrt{2+\sqrt{3}}\left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right)\sqrt{\frac{a^{2/3}+a^{1/3}\left(a-b x^2\right)^{1/3}+\left(a-b x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}{\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(16\times 3^{3/4} a^{5/3}\, b\, x\sqrt{-\frac{a^{1/3}\left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right)+$$

$$\left(\left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right)\sqrt{\frac{a^{2/3}+a^{1/3}\left(a-b x^2\right)^{1/3}+\left(a-b x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}{\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(12\sqrt{2}\, 3^{1/4} a^{5/3}\, b\, x\sqrt{-\frac{a^{1/3}\left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right)$$

Result (type 6, 322 leaves):

$$\left(x\left(\left(189\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{b\,x^2}{a},-\frac{b\,x^2}{3\,a}\right]\right)\Big/\left(9\,a\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{b\,x^2}{a},-\frac{b\,x^2}{3\,a}\right]+\right.\right.\right.$$
$$\left.2\,b\,x^2\left(-\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},2,\frac{5}{2},\frac{b\,x^2}{a},-\frac{b\,x^2}{3\,a}\right]+\text{AppellF1}\left[\frac{3}{2},\frac{4}{3},1,\frac{5}{2},\frac{b\,x^2}{a},-\frac{b\,x^2}{3\,a}\right]\right)\right)+$$
$$\frac{1}{a^2}\left(3\,a-3\,b\,x^2+\left(5\,a\,b\,x^2\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},1,\frac{5}{2},\frac{b\,x^2}{a},-\frac{b\,x^2}{3\,a}\right]\right)\Big/\left(15\,a\right.\right.$$
$$\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},1,\frac{5}{2},\frac{b\,x^2}{a},-\frac{b\,x^2}{3\,a}\right]+2\,b\,x^2\left(-\text{AppellF1}\left[\frac{5}{2},\frac{1}{3},2,\frac{7}{2},\frac{b\,x^2}{a},-\frac{b\,x^2}{3\,a}\right]+\right.$$
$$\left.\left.\left.\left.\left.\text{AppellF1}\left[\frac{5}{2},\frac{4}{3},1,\frac{7}{2},\frac{b\,x^2}{a},-\frac{b\,x^2}{3\,a}\right]\right)\right)\right)\right)\right)\Big/\left(72\left(a-b\,x^2\right)^{1/3}\left(3\,a+b\,x^2\right)\right)$$

Problem 128: Result unnecessarily involves higher level functions.

$$\int\frac{1}{\left(a-b\,x^2\right)^{1/3}\left(3\,a+b\,x^2\right)^3}\,dx$$

Optimal (type 4, 818 leaves, 8 steps):

$$\frac{x\left(a-b x^2\right)^{2/3}}{48 a^2\left(3 a+b x^2\right)^2}+\frac{5 x\left(a-b x^2\right)^{2/3}}{288 a^3\left(3 a+b x^2\right)}-\frac{5 x}{288 a^3\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}+$$

$$\frac{5 \operatorname{ArcTan}\left[\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\, x}\right]}{144\times 2^{2/3}\sqrt{3}\, a^{17/6}\sqrt{b}}+\frac{5 \operatorname{ArcTan}\left[\frac{\sqrt{3}\, a^{1/6}\left(a^{1/3}-2^{1/3}\left(a-b x^2\right)^{1/3}\right)}{\sqrt{b}\, x}\right]}{144\times 2^{2/3}\sqrt{3}\, a^{17/6}\sqrt{b}}-$$

$$\frac{5 \operatorname{ArcTanh}\left[\frac{\sqrt{b}\, x}{\sqrt{a}}\right]}{432\times 2^{2/3} a^{17/6}\sqrt{b}}+\frac{5 \operatorname{ArcTanh}\left[\frac{\sqrt{b}\, x}{a^{1/6}\left(a^{1/3}+2^{1/3}\left(a-b x^2\right)^{1/3}\right)}\right]}{144\times 2^{2/3} a^{17/6}\sqrt{b}}-$$

$$\left(5\sqrt{2+\sqrt{3}}\left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right)\sqrt{\frac{a^{2/3}+a^{1/3}\left(a-b x^2\right)^{1/3}+\left(a-b x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\left(1+\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}{\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(192\times 3^{3/4} a^{8/3}\, b\, x\sqrt{-\frac{a^{1/3}\left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right)+$$

$$\left(5\left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right)\sqrt{\frac{a^{2/3}+a^{1/3}\left(a-b x^2\right)^{1/3}+\left(a-b x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\left(1+\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}{\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(144\sqrt{2}\, 3^{1/4} a^{8/3}\, b\, x\sqrt{-\frac{a^{1/3}\left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right)$$

Result (type 6, 352 leaves) :

$$\left(x\left(3\left(a-b x^2\right)\left(21 a+5 b x^2\right)+\left(675 a^2\left(3 a+b x^2\right) \text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]\right)\Big/\right.\right.$$
$$\left(9 a \text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]+\right.$$
$$\left.2 b x^2\left(-\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},2,\frac{5}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]+\text{AppellF1}\left[\frac{3}{2},\frac{4}{3},1,\frac{5}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]\right)\right)+$$
$$\left(25 a b x^2\left(3 a+b x^2\right) \text{AppellF1}\left[\frac{3}{2},\frac{1}{3},1,\frac{5}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]\right)\Big/$$
$$\left(15 a \text{AppellF1}\left[\frac{3}{2},\frac{1}{3},1,\frac{5}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]+2 b x^2\left(-\text{AppellF1}\left[\frac{5}{2},\frac{1}{3},2,\frac{7}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]+\right.\right.$$
$$\left.\left.\left.\left.\text{AppellF1}\left[\frac{5}{2},\frac{4}{3},1,\frac{7}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]\right)\right)\right)\right)\Big/\left(864 a^3\left(a-b x^2\right)^{1/3}\left(3 a+b x^2\right)^2\right)$$

Problem 129: Result unnecessarily involves higher level functions.

$$\int \frac{\left(3 a+b x^2\right)^3}{\left(a-b x^2\right)^{4/3}} \, dx$$

Optimal (type 4, 623 leaves, 7 steps):

$$\frac{2538}{91} a x \left(a-b x^2\right)^{2/3}+\frac{81}{13} x \left(a-b x^2\right)^{2/3} \left(3 a+b x^2\right)+\frac{6 x \left(3 a+b x^2\right)^2}{\left(a-b x^2\right)^{1/3}}+\frac{20088 a^2 x}{91 \left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}+$$
$$\left(10044\times 3^{1/4} \sqrt{2+\sqrt{3}}\ a^{7/3} \left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right) \sqrt{\frac{a^{2/3}+a^{1/3} \left(a-b x^2\right)^{1/3}+\left(a-b x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}{\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}\right],-7+4 \sqrt{3}\right]\right)\Bigg/$$
$$\left(91 b x \sqrt{-\frac{a^{1/3} \left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right)-$$
$$\left(6696 \sqrt{2}\ 3^{3/4} a^{7/3} \left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right) \sqrt{\frac{a^{2/3}+a^{1/3} \left(a-b x^2\right)^{1/3}+\left(a-b x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}{\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}\right],-7+4 \sqrt{3}\right]\right)\Bigg/$$
$$\left(91 b x \sqrt{-\frac{a^{1/3} \left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 76 leaves):

$$-\frac{1}{91 \left(a-b x^2\right)^{1/3}} 3 x \left(-3051 a^2+132 a b x^2+7 b^2 x^4+2232 a^2 \left(1-\frac{b x^2}{a}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{b x^2}{a}\right]\right)$$

Problem 130: Result unnecessarily involves higher level functions.

$$\int \frac{\left(3 a+b x^2\right)^2}{\left(a-b x^2\right)^{4/3}} \, dx$$

Optimal (type 4, 592 leaves, 6 steps):

$$\frac{45}{7} x \left(a-b x^2\right)^{2/3}+\frac{6 x \left(3 a+b x^2\right)}{\left(a-b x^2\right)^{1/3}}+\frac{324 a x}{7 \left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}+$$

$$\left(162\times 3^{1/4} \sqrt{2+\sqrt{3}}\, a^{4/3} \left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right) \sqrt{\frac{a^{2/3}+a^{1/3} \left(a-b x^2\right)^{1/3}+\left(a-b x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}{\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}\right],-7+4 \sqrt{3}\right]\right)\Big/$$

$$\left(7 b x \sqrt{-\frac{a^{1/3} \left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right)-$$

$$\left(108 \sqrt{2}\, 3^{3/4} a^{4/3} \left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right) \sqrt{\frac{a^{2/3}+a^{1/3} \left(a-b x^2\right)^{1/3}+\left(a-b x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}{\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}\right],-7+4 \sqrt{3}\right]\right)\Big/$$

$$\left(7 b x \sqrt{-\frac{a^{1/3} \left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 62 leaves):

$$-\frac{1}{7 \left(a-b x^2\right)^{1/3}} 3 x \left(-57 a+b x^2+36 a \left(1-\frac{b x^2}{a}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{b x^2}{a}\right]\right)$$

Problem 131: Result unnecessarily involves higher level functions.

$$\int \frac{3 a+b x^2}{\left(a-b x^2\right)^{4/3}}\, dx$$

Optimal (type 4, 561 leaves, 5 steps):

$$\frac{6\,x}{\left(a-b\,x^2\right)^{1/3}}+\frac{9\,x}{\left(1-\sqrt{3}\right)a^{1/3}-\left(a-b\,x^2\right)^{1/3}}+$$

$$\left(9\times 3^{1/4}\sqrt{2+\sqrt{3}}\;a^{1/3}\left(a^{1/3}-\left(a-b\,x^2\right)^{1/3}\right)\sqrt{\frac{a^{2/3}+a^{1/3}\left(a-b\,x^2\right)^{1/3}+\left(a-b\,x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)a^{1/3}-\left(a-b\,x^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)a^{1/3}-\left(a-b\,x^2\right)^{1/3}}{\left(1-\sqrt{3}\right)a^{1/3}-\left(a-b\,x^2\right)^{1/3}}\right],\,-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(2\,b\,x\sqrt{-\frac{a^{1/3}\left(a^{1/3}-\left(a-b\,x^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right)a^{1/3}-\left(a-b\,x^2\right)^{1/3}\right)^2}}\right)-$$

$$\left(3\sqrt{2}\;3^{3/4}\,a^{1/3}\left(a^{1/3}-\left(a-b\,x^2\right)^{1/3}\right)\sqrt{\frac{a^{2/3}+a^{1/3}\left(a-b\,x^2\right)^{1/3}+\left(a-b\,x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)a^{1/3}-\left(a-b\,x^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)a^{1/3}-\left(a-b\,x^2\right)^{1/3}}{\left(1-\sqrt{3}\right)a^{1/3}-\left(a-b\,x^2\right)^{1/3}}\right],\,-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(b\,x\sqrt{-\frac{a^{1/3}\left(a^{1/3}-\left(a-b\,x^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right)a^{1/3}-\left(a-b\,x^2\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 51 leaves):

$$-\frac{3\,x\left(-2+\left(1-\frac{b\,x^2}{a}\right)^{1/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\,\frac{1}{2},\,\frac{3}{2},\,\frac{b\,x^2}{a}\right]\right)}{\left(a-b\,x^2\right)^{1/3}}$$

Problem 132: Result unnecessarily involves higher level functions.

$$\int\frac{1}{\left(a-b\,x^2\right)^{4/3}\left(3\,a+b\,x^2\right)}\,dx$$

Optimal (type 4, 776 leaves, 7 steps):

$$\frac{3x}{8a^2\left(a-bx^2\right)^{1/3}}+\frac{3x}{8a^2\left(\left(1-\sqrt{3}\right)a^{1/3}-\left(a-bx^2\right)^{1/3}\right)}+\frac{\text{ArcTan}\left[\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right]}{8\times 2^{2/3}\sqrt{3}\,a^{11/6}\sqrt{b}}+$$

$$\frac{\text{ArcTan}\left[\frac{\sqrt{3}\,a^{1/6}\left(a^{1/3}-2^{1/3}\left(a-bx^2\right)^{1/3}\right)}{\sqrt{b}x}\right]}{8\times 2^{2/3}\sqrt{3}\,a^{11/6}\sqrt{b}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{b}x}{\sqrt{a}}\right]}{24\times 2^{2/3}a^{11/6}\sqrt{b}}+\frac{\text{ArcTanh}\left[\frac{\sqrt{b}x}{a^{1/6}\left(a^{1/3}+2^{1/3}\left(a-bx^2\right)^{1/3}\right)}\right]}{8\times 2^{2/3}a^{11/6}\sqrt{b}}+$$

$$\left(3\times 3^{1/4}\sqrt{2+\sqrt{3}}\left(a^{1/3}-\left(a-bx^2\right)^{1/3}\right)\sqrt{\frac{a^{2/3}+a^{1/3}\left(a-bx^2\right)^{1/3}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)a^{1/3}-\left(a-bx^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)a^{1/3}-\left(a-bx^2\right)^{1/3}}{\left(1-\sqrt{3}\right)a^{1/3}-\left(a-bx^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(16a^{5/3}bx\sqrt{-\frac{a^{1/3}\left(a^{1/3}-\left(a-bx^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right)a^{1/3}-\left(a-bx^2\right)^{1/3}\right)^2}}\right)-$$

$$\left(3^{3/4}\left(a^{1/3}-\left(a-bx^2\right)^{1/3}\right)\sqrt{\frac{a^{2/3}+a^{1/3}\left(a-bx^2\right)^{1/3}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)a^{1/3}-\left(a-bx^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)a^{1/3}-\left(a-bx^2\right)^{1/3}}{\left(1-\sqrt{3}\right)a^{1/3}-\left(a-bx^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(4\sqrt{2}\,a^{5/3}bx\sqrt{-\frac{a^{1/3}\left(a^{1/3}-\left(a-bx^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right)a^{1/3}-\left(a-bx^2\right)^{1/3}\right)^2}}\right)$$

Result (type 6, 325 leaves):

$$\frac{1}{8\left(a-bx^2\right)^{1/3}}x\left(-\left(\left(9\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right]\right)\Big/\right.\right.$$

$$\left(\left(3a+bx^2\right)\left(9a\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right]+2bx^2\right.\right.$$

$$\left.\left.\left.\left(-\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},2,\frac{5}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right]+\text{AppellF1}\left[\frac{3}{2},\frac{4}{3},1,\frac{5}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right]\right)\right)\right)\right)+$$

$$\frac{1}{a^2}\left(3-\left(5abx^2\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},1,\frac{5}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right]\right)\Big/\right.$$

$$\left(\left(3a+bx^2\right)\left(15a\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},1,\frac{5}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right]+2bx^2\right.\right.$$

$$\left.\left.\left.\left.\left(-\text{AppellF1}\left[\frac{5}{2},\frac{1}{3},2,\frac{7}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right]+\text{AppellF1}\left[\frac{5}{2},\frac{4}{3},1,\frac{7}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right]\right)\right)\right)\right)\right)$$

Problem 133: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(a-b x^2\right)^{4/3} \left(3 a+b x^2\right)^2} \, dx$$

Optimal (type 4, 807 leaves, 8 steps):

$$\frac{x}{12 a^3 \left(a-b x^2\right)^{1/3}}+\frac{x}{24 a^2 \left(a-b x^2\right)^{1/3} \left(3 a+b x^2\right)}+\frac{x}{12 a^3 \left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}+\frac{\text{ArcTan}\left[\frac{\sqrt{3} \sqrt{a}}{\sqrt{b} x}\right]}{16\times 2^{2/3} \sqrt{3} a^{17/6} \sqrt{b}}+$$

$$\frac{\text{ArcTan}\left[\frac{\sqrt{3} a^{1/6} \left(a^{1/3}-2^{1/3} \left(a-b x^2\right)^{1/3}\right)}{\sqrt{b} x}\right]}{16\times 2^{2/3} \sqrt{3} a^{17/6} \sqrt{b}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{b} x}{\sqrt{a}}\right]}{48\times 2^{2/3} a^{17/6} \sqrt{b}}+\frac{\text{ArcTanh}\left[\frac{\sqrt{b} x}{a^{1/6} \left(a^{1/3}+2^{1/3} \left(a-b x^2\right)^{1/3}\right)}\right]}{16\times 2^{2/3} a^{17/6} \sqrt{b}}+$$

$$\left(\sqrt{2+\sqrt{3}} \left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right) \sqrt{\frac{a^{2/3}+a^{1/3} \left(a-b x^2\right)^{1/3}+\left(a-b x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}{\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}\right],-7+4 \sqrt{3}\right]\right)\Bigg/$$

$$\left(8\times 3^{3/4} a^{8/3} b x \sqrt{-\frac{a^{1/3} \left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right)-$$

$$\left(\left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right) \sqrt{\frac{a^{2/3}+a^{1/3} \left(a-b x^2\right)^{1/3}+\left(a-b x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}{\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}\right],-7+4 \sqrt{3}\right]\right)\Bigg/$$

$$\left(6 \sqrt{2} 3^{1/4} a^{8/3} b x \sqrt{-\frac{a^{1/3} \left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right)$$

Result (type 6, 323 leaves):

$$\begin{aligned}&\Bigg(x\Bigg(21\,a+6\,b\,x^2+\\
&\quad\left(27\,a^2\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{b\,x^2}{a},-\frac{b\,x^2}{3\,a}\right]\right)\Bigg/\left(9\,a\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{b\,x^2}{a},-\frac{b\,x^2}{3\,a}\right]+\right.\\
&\qquad\left.2\,b\,x^2\left(-\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},2,\frac{5}{2},\frac{b\,x^2}{a},-\frac{b\,x^2}{3\,a}\right]+\text{AppellF1}\left[\frac{3}{2},\frac{4}{3},1,\frac{5}{2},\frac{b\,x^2}{a},-\frac{b\,x^2}{3\,a}\right]\right)\right)-\\
&\quad\left(10\,a\,b\,x^2\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},1,\frac{5}{2},\frac{b\,x^2}{a},-\frac{b\,x^2}{3\,a}\right]\right)\Bigg/\\
&\quad\left(15\,a\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},1,\frac{5}{2},\frac{b\,x^2}{a},-\frac{b\,x^2}{3\,a}\right]+2\,b\,x^2\left(-\text{AppellF1}\left[\frac{5}{2},\frac{1}{3},2,\frac{7}{2},\frac{b\,x^2}{a},-\frac{b\,x^2}{3\,a}\right]+\right.\right.\\
&\qquad\left.\left.\text{AppellF1}\left[\frac{5}{2},\frac{4}{3},1,\frac{7}{2},\frac{b\,x^2}{a},-\frac{b\,x^2}{3\,a}\right]\right)\right)\Bigg)\Bigg)\Bigg/\left(72\,a^3\left(a-b\,x^2\right)^{1/3}\left(3\,a+b\,x^2\right)\right)\end{aligned}$$

Problem 134: Result unnecessarily involves higher level functions.

$$\int\frac{1}{\left(a-b\,x^2\right)^{4/3}\left(3\,a+b\,x^2\right)^3}\,dx$$

Optimal (type 4, 849 leaves, 9 steps):

$$\frac{x}{48 a^2 \left(a-b x^2\right)^{1/3} \left(3 a+b x^2\right)^2}+\frac{17 x}{192 a^3 \left(a-b x^2\right)^{1/3} \left(3 a+b x^2\right)}-$$

$$\frac{19 x \left(a-b x^2\right)^{2/3}}{1152 a^4 \left(3 a+b x^2\right)}+\frac{19 x}{1152 a^4 \left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}+\frac{7 \text{ArcTan}\left[\frac{\sqrt{3} \sqrt{a}}{\sqrt{b} x}\right]}{288\times 2^{2/3} \sqrt{3} a^{23/6} \sqrt{b}}+$$

$$\frac{7 \text{ArcTan}\left[\frac{\sqrt{3} a^{1/6} \left(a^{1/3}-2^{1/3} \left(a-b x^2\right)^{1/3}\right)}{\sqrt{b} x}\right]}{288\times 2^{2/3} \sqrt{3} a^{23/6} \sqrt{b}}-\frac{7 \text{ArcTanh}\left[\frac{\sqrt{b} x}{\sqrt{a}}\right]}{864\times 2^{2/3} a^{23/6} \sqrt{b}}+\frac{7 \text{ArcTanh}\left[\frac{\sqrt{b} x}{a^{1/6} \left(a^{1/3}+2^{1/3} \left(a-b x^2\right)^{1/3}\right)}\right]}{288\times 2^{2/3} a^{23/6} \sqrt{b}}+$$

$$\left(19 \sqrt{2+\sqrt{3}} \left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right) \sqrt{\frac{a^{2/3}+a^{1/3} \left(a-b x^2\right)^{1/3}+\left(a-b x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}{\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}\right],-7+4 \sqrt{3}\right]\right)\Big/$$

$$\left(768\times 3^{3/4} a^{11/3} b x \sqrt{-\frac{a^{1/3} \left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right)-$$

$$\left(19 \left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right) \sqrt{\frac{a^{2/3}+a^{1/3} \left(a-b x^2\right)^{1/3}+\left(a-b x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}{\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}\right],-7+4 \sqrt{3}\right]\right)\Big/$$

$$\left(576 \sqrt{2}\, 3^{1/4} a^{11/3} b x \sqrt{-\frac{a^{1/3} \left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right)$$

Result (type 6, 353 leaves) :

$$\left(819 a^2 x + 420 a b x^3 + 57 b^2 x^5 + \left(999 a^2 x \left(3 a + b x^2\right) \text{AppellF1}\left[\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, \frac{b x^2}{a}, -\frac{b x^2}{3 a}\right]\right) \Big/ \left(9 a \text{AppellF1}\left[\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, \frac{b x^2}{a}, -\frac{b x^2}{3 a}\right] + 2 b x^2 \left(-\text{AppellF1}\left[\frac{3}{2}, \frac{1}{3}, 2, \frac{5}{2}, \frac{b x^2}{a}, -\frac{b x^2}{3 a}\right] + \text{AppellF1}\left[\frac{3}{2}, \frac{4}{3}, 1, \frac{5}{2}, \frac{b x^2}{a}, -\frac{b x^2}{3 a}\right]\right)\right) - \left(95 a b x^3 \left(3 a + b x^2\right) \text{AppellF1}\left[\frac{3}{2}, \frac{1}{3}, 1, \frac{5}{2}, \frac{b x^2}{a}, -\frac{b x^2}{3 a}\right]\right) \Big/ \left(15 a \text{AppellF1}\left[\frac{3}{2}, \frac{1}{3}, 1, \frac{5}{2}, \frac{b x^2}{a}, -\frac{b x^2}{3 a}\right] + 2 b x^2 \left(-\text{AppellF1}\left[\frac{5}{2}, \frac{1}{3}, 2, \frac{7}{2}, \frac{b x^2}{a}, -\frac{b x^2}{3 a}\right] + \text{AppellF1}\left[\frac{5}{2}, \frac{4}{3}, 1, \frac{7}{2}, \frac{b x^2}{a}, -\frac{b x^2}{3 a}\right]\right)\right)\right) \Big/ \left(3456 a^4 \left(a - b x^2\right)^{1/3} \left(3 a + b x^2\right)^2\right)$$

Problem 135: Result unnecessarily involves higher level functions.

$$\int \frac{\left(3 a + b x^2\right)^4}{\left(a - b x^2\right)^{7/3}} \, dx$$

Optimal (type 4, 653 leaves, 8 steps):

$$-\frac{3240}{91} a x \left(a-b x^2\right)^{2/3}-\frac{81}{13} x \left(a-b x^2\right)^{2/3} \left(3 a+b x^2\right)-\frac{9 x \left(3 a+b x^2\right)^2}{2 \left(a-b x^2\right)^{1/3}}+\frac{3 x \left(3 a+b x^2\right)^3}{2 \left(a-b x^2\right)^{4/3}}-\frac{36936 a^2 x}{91 \left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}-$$

$$\left(18468\times 3^{1/4} \sqrt{2+\sqrt{3}}\; a^{7/3} \left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right) \sqrt{\frac{a^{2/3}+a^{1/3} \left(a-b x^2\right)^{1/3}+\left(a-b x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}{\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}\right],-7+4 \sqrt{3}\right]\right)\Big/$$

$$\left(91 b x \sqrt{-\frac{a^{1/3} \left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right)+$$

$$\left(12312 \sqrt{2}\; 3^{3/4} a^{7/3} \left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right) \sqrt{\frac{a^{2/3}+a^{1/3} \left(a-b x^2\right)^{1/3}+\left(a-b x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}{\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}}\right],-7+4 \sqrt{3}\right]\right)\Big/$$

$$\left(91 b x \sqrt{-\frac{a^{1/3} \left(a^{1/3}-\left(a-b x^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right) a^{1/3}-\left(a-b x^2\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 96 leaves):

$$-\frac{1}{91 \left(a-b x^2\right)^{4/3}} 3 \left(1647 a^3 x-4743 a^2 b x^3+177 a b^2 x^5+7 b^3 x^7-4104 a^2 x \left(a-b x^2\right) \left(1-\frac{b x^2}{a}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{b x^2}{a}\right]\right)$$

Problem 136: Result unnecessarily involves higher level functions.

$$\int \frac{\left(3 a+b x^2\right)^3}{\left(a-b x^2\right)^{7/3}}\, dx$$

Optimal (type 4, 596 leaves, 7 steps):

$$-\frac{27}{14}x\left(a-bx^2\right)^{2/3}+\frac{3x\left(3a+bx^2\right)^2}{2\left(a-bx^2\right)^{4/3}}-\frac{324\,a\,x}{7\left(\left(1-\sqrt{3}\right)a^{1/3}-\left(a-bx^2\right)^{1/3}\right)}-$$

$$\left(162\times 3^{1/4}\sqrt{2+\sqrt{3}}\;a^{4/3}\left(a^{1/3}-\left(a-bx^2\right)^{1/3}\right)\sqrt{\frac{a^{2/3}+a^{1/3}\left(a-bx^2\right)^{1/3}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)a^{1/3}-\left(a-bx^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)a^{1/3}-\left(a-bx^2\right)^{1/3}}{\left(1-\sqrt{3}\right)a^{1/3}-\left(a-bx^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Bigg/$$

$$\left(7\,b\,x\sqrt{-\frac{a^{1/3}\left(a^{1/3}-\left(a-bx^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right)a^{1/3}-\left(a-bx^2\right)^{1/3}\right)^2}}\right)+$$

$$\left(108\sqrt{2}\;3^{3/4}\,a^{4/3}\left(a^{1/3}-\left(a-bx^2\right)^{1/3}\right)\sqrt{\frac{a^{2/3}+a^{1/3}\left(a-bx^2\right)^{1/3}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)a^{1/3}-\left(a-bx^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)a^{1/3}-\left(a-bx^2\right)^{1/3}}{\left(1-\sqrt{3}\right)a^{1/3}-\left(a-bx^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Bigg/$$

$$\left(7\,b\,x\sqrt{-\frac{a^{1/3}\left(a^{1/3}-\left(a-bx^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right)a^{1/3}-\left(a-bx^2\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 83 leaves) :

$$\frac{1}{7\left(a-bx^2\right)^{4/3}}$$

$$\left(81a^2x+90abx^3-3b^2x^5+108ax\left(a-bx^2\right)\left(1-\frac{bx^2}{a}\right)^{1/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right]\right)$$

Problem 138: Result unnecessarily involves higher level functions.

$$\int\frac{3a+bx^2}{\left(a-bx^2\right)^{7/3}}\,dx$$

Optimal (type 4, 590 leaves, 6 steps) :

$$\frac{3x}{2\left(a-bx^2\right)^{4/3}}+\frac{9x}{4a\left(a-bx^2\right)^{1/3}}+\frac{9x}{4a\left(\left(1-\sqrt{3}\right)a^{1/3}-\left(a-bx^2\right)^{1/3}\right)}+$$

$$\left(9\times 3^{1/4}\sqrt{2+\sqrt{3}}\left(a^{1/3}-\left(a-bx^2\right)^{1/3}\right)\sqrt{\frac{a^{2/3}+a^{1/3}\left(a-bx^2\right)^{1/3}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)a^{1/3}-\left(a-bx^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)a^{1/3}-\left(a-bx^2\right)^{1/3}}{\left(1-\sqrt{3}\right)a^{1/3}-\left(a-bx^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Bigg/$$

$$\left(8a^{2/3}bx\sqrt{-\frac{a^{1/3}\left(a^{1/3}-\left(a-bx^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right)a^{1/3}-\left(a-bx^2\right)^{1/3}\right)^2}}\right)-$$

$$\left(3\times 3^{3/4}\left(a^{1/3}-\left(a-bx^2\right)^{1/3}\right)\sqrt{\frac{a^{2/3}+a^{1/3}\left(a-bx^2\right)^{1/3}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)a^{1/3}-\left(a-bx^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)a^{1/3}-\left(a-bx^2\right)^{1/3}}{\left(1-\sqrt{3}\right)a^{1/3}-\left(a-bx^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Bigg/$$

$$\left(2\sqrt{2}\,a^{2/3}bx\sqrt{-\frac{a^{1/3}\left(a^{1/3}-\left(a-bx^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right)a^{1/3}-\left(a-bx^2\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 74 leaves):

$$\frac{1}{4a\left(a-bx^2\right)^{4/3}}\left(15ax-9bx^3-3x\left(a-bx^2\right)\left(1-\frac{bx^2}{a}\right)^{1/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right]\right)$$

Problem 139: Result unnecessarily involves higher level functions.

$$\int\frac{1}{\left(a-bx^2\right)^{7/3}\left(3a+bx^2\right)}\,dx$$

Optimal (type 4, 796 leaves, 8 steps):

$$\frac{3x}{32a^2\left(a-bx^2\right)^{4/3}}+\frac{21x}{64a^3\left(a-bx^2\right)^{1/3}}+\frac{21x}{64a^3\left(\left(1-\sqrt{3}\right)a^{1/3}-\left(a-bx^2\right)^{1/3}\right)}+$$

$$\frac{\text{ArcTan}\left[\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right]}{32\times 2^{2/3}\sqrt{3}\,a^{17/6}\sqrt{b}}+\frac{\text{ArcTan}\left[\frac{\sqrt{3}\,a^{1/6}\left(a^{1/3}-2^{1/3}\left(a-bx^2\right)^{1/3}\right)}{\sqrt{b}\,x}\right]}{32\times 2^{2/3}\sqrt{3}\,a^{17/6}\sqrt{b}}-$$

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{b}\,x}{\sqrt{a}}\right]}{96\times 2^{2/3}a^{17/6}\sqrt{b}}+\frac{\text{ArcTanh}\left[\frac{\sqrt{b}\,x}{a^{1/6}\left(a^{1/3}+2^{1/3}\left(a-bx^2\right)^{1/3}\right)}\right]}{32\times 2^{2/3}a^{17/6}\sqrt{b}}+$$

$$\left(21\times 3^{1/4}\sqrt{2+\sqrt{3}}\left(a^{1/3}-\left(a-bx^2\right)^{1/3}\right)\sqrt{\frac{a^{2/3}+a^{1/3}\left(a-bx^2\right)^{1/3}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)a^{1/3}-\left(a-bx^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)a^{1/3}-\left(a-bx^2\right)^{1/3}}{\left(1-\sqrt{3}\right)a^{1/3}-\left(a-bx^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(128a^{8/3}bx\sqrt{-\frac{a^{1/3}\left(a^{1/3}-\left(a-bx^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right)a^{1/3}-\left(a-bx^2\right)^{1/3}\right)^2}}\right)-$$

$$\left(7\times 3^{3/4}\left(a^{1/3}-\left(a-bx^2\right)^{1/3}\right)\sqrt{\frac{a^{2/3}+a^{1/3}\left(a-bx^2\right)^{1/3}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)a^{1/3}-\left(a-bx^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)a^{1/3}-\left(a-bx^2\right)^{1/3}}{\left(1-\sqrt{3}\right)a^{1/3}-\left(a-bx^2\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(32\sqrt{2}\,a^{8/3}bx\sqrt{-\frac{a^{1/3}\left(a^{1/3}-\left(a-bx^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right)a^{1/3}-\left(a-bx^2\right)^{1/3}\right)^2}}\right)$$

Result (type 6, 347 leaves) :

$$\frac{1}{64 a^3 \left(a-b x^2\right)^{1/3}} x \left(\frac{3 \left(9 a-7 b x^2\right)}{a-b x^2} - \left(153 a^2 \text{AppellF1}\left[\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, \frac{b x^2}{a}, -\frac{b x^2}{3 a}\right]\right) \Big/ \left(\left(3 a+b x^2\right) \left(9 a \text{AppellF1}\left[\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, \frac{b x^2}{a}, -\frac{b x^2}{3 a}\right] + 2 b x^2 \left(-\text{AppellF1}\left[\frac{3}{2}, \frac{1}{3}, 2, \frac{5}{2}, \frac{b x^2}{a}, -\frac{b x^2}{3 a}\right] + \text{AppellF1}\left[\frac{3}{2}, \frac{4}{3}, 1, \frac{5}{2}, \frac{b x^2}{a}, -\frac{b x^2}{3 a}\right]\right)\right)\right) - \left(35 a b x^2 \text{AppellF1}\left[\frac{3}{2}, \frac{1}{3}, 1, \frac{5}{2}, \frac{b x^2}{a}, -\frac{b x^2}{3 a}\right]\right) \Big/ \left(\left(3 a+b x^2\right) \left(15 a \text{AppellF1}\left[\frac{3}{2}, \frac{1}{3}, 1, \frac{5}{2}, \frac{b x^2}{a}, -\frac{b x^2}{3 a}\right] + 2 b x^2 \left(-\text{AppellF1}\left[\frac{5}{2}, \frac{1}{3}, 2, \frac{7}{2}, \frac{b x^2}{a}, -\frac{b x^2}{3 a}\right] + \text{AppellF1}\left[\frac{5}{2}, \frac{4}{3}, 1, \frac{7}{2}, \frac{b x^2}{a}, -\frac{b x^2}{3 a}\right]\right)\right)\right)\right)$$

Problem 140: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(a-b x^2\right)^{7/3} \left(3 a+b x^2\right)^2} \, dx$$

Optimal (type 4, 827 leaves, 9 steps):

$$\frac{5\,x}{384\,a^3\,\left(a-b\,x^2\right)^{4/3}}+\frac{79\,x}{768\,a^4\,\left(a-b\,x^2\right)^{1/3}}+\frac{x}{24\,a^2\,\left(a-b\,x^2\right)^{4/3}\,\left(3\,a+b\,x^2\right)}+$$

$$\frac{79\,x}{768\,a^4\,\left(\left(1-\sqrt{3}\right)\,a^{1/3}-\left(a-b\,x^2\right)^{1/3}\right)}+\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{\sqrt{3}\,\sqrt{a}}{\sqrt{b}\,x}\right]}{128\times 2^{2/3}\,a^{23/6}\,\sqrt{b}}+$$

$$\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{\sqrt{3}\,a^{1/6}\,\left(a^{1/3}-2^{1/3}\,\left(a-b\,x^2\right)^{1/3}\right)}{\sqrt{b}\,x}\right]}{128\times 2^{2/3}\,a^{23/6}\,\sqrt{b}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{b}\,x}{\sqrt{a}}\right]}{128\times 2^{2/3}\,a^{23/6}\,\sqrt{b}}+\frac{3\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x}{a^{1/6}\,\left(a^{1/3}+2^{1/3}\,\left(a-b\,x^2\right)^{1/3}\right)}\right]}{128\times 2^{2/3}\,a^{23/6}\,\sqrt{b}}+$$

$$\left(79\,\sqrt{2+\sqrt{3}}\,\left(a^{1/3}-\left(a-b\,x^2\right)^{1/3}\right)\,\sqrt{\frac{a^{2/3}+a^{1/3}\,\left(a-b\,x^2\right)^{1/3}+\left(a-b\,x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\,a^{1/3}-\left(a-b\,x^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)\,a^{1/3}-\left(a-b\,x^2\right)^{1/3}}{\left(1-\sqrt{3}\right)\,a^{1/3}-\left(a-b\,x^2\right)^{1/3}}\right],\,-7+4\,\sqrt{3}\right]\right)\Bigg/$$

$$\left(512\times 3^{3/4}\,a^{11/3}\,b\,x\,\sqrt{-\frac{a^{1/3}\,\left(a^{1/3}-\left(a-b\,x^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right)\,a^{1/3}-\left(a-b\,x^2\right)^{1/3}\right)^2}}\right)-$$

$$\left(79\,\left(a^{1/3}-\left(a-b\,x^2\right)^{1/3}\right)\,\sqrt{\frac{a^{2/3}+a^{1/3}\,\left(a-b\,x^2\right)^{1/3}+\left(a-b\,x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\,a^{1/3}-\left(a-b\,x^2\right)^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)\,a^{1/3}-\left(a-b\,x^2\right)^{1/3}}{\left(1-\sqrt{3}\right)\,a^{1/3}-\left(a-b\,x^2\right)^{1/3}}\right],\,-7+4\,\sqrt{3}\right]\right)\Bigg/$$

$$\left(384\,\sqrt{2}\,3^{1/4}\,a^{11/3}\,b\,x\,\sqrt{-\frac{a^{1/3}\,\left(a^{1/3}-\left(a-b\,x^2\right)^{1/3}\right)}{\left(\left(1-\sqrt{3}\right)\,a^{1/3}-\left(a-b\,x^2\right)^{1/3}\right)^2}}\right)$$

Result (type 6, 346 leaves) :

$$\left(x\left(\frac{897 a^2-444 a b x^2-237 b^2 x^4}{a-b x^2}-\left(1161 a^2 \text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]\right)\Big/\left(9 a \text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]+2 b x^2\left(-\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},2,\frac{5}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]+\text{AppellF1}\left[\frac{3}{2},\frac{4}{3},1,\frac{5}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]\right)\right)-\left(395 a b x^2 \text{AppellF1}\left[\frac{3}{2},\frac{1}{3},1,\frac{5}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]\right)\Big/\left(15 a \text{AppellF1}\left[\frac{3}{2},\frac{1}{3},1,\frac{5}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]+2 b x^2\left(-\text{AppellF1}\left[\frac{5}{2},\frac{1}{3},2,\frac{7}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]+\text{AppellF1}\left[\frac{5}{2},\frac{4}{3},1,\frac{7}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]\right)\right)\right)\right)\Big/\left(2304 a^4\left(a-b x^2\right)^{1/3}\left(3 a+b x^2\right)\right)$$

Problem 141: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(-3 a-b x^2\right)\left(-a+b x^2\right)^{1/3}}\,dx$$

Optimal (type 3, 252 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}\,x}\right]}{2\times 2^{2/3}\sqrt{3}\,(-a)^{1/3}\sqrt{a}\sqrt{b}}-\frac{\text{ArcTan}\left[\frac{\sqrt{3}\sqrt{a}\left((-a)^{1/3}-2^{1/3}\left(-a+b x^2\right)^{1/3}\right)}{(-a)^{1/3}\sqrt{b}\,x}\right]}{2\times 2^{2/3}\sqrt{3}\,(-a)^{1/3}\sqrt{a}\sqrt{b}}+\frac{\text{ArcTanh}\left[\frac{\sqrt{b}\,x}{\sqrt{a}}\right]}{6\times 2^{2/3}(-a)^{1/3}\sqrt{a}\sqrt{b}}-\frac{\text{ArcTanh}\left[\frac{(-a)^{1/3}\sqrt{b}\,x}{\sqrt{a}\left((-a)^{1/3}+2^{1/3}\left(-a+b x^2\right)^{1/3}\right)}\right]}{2\times 2^{2/3}(-a)^{1/3}\sqrt{a}\sqrt{b}}$$

Result (type 6, 163 leaves):

$$-\left(\left(9 a x \text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]\right)\Big/\left(\left(-a+b x^2\right)^{1/3}\left(3 a+b x^2\right)\left(9 a \text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]+2 b x^2\left(-\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},2,\frac{5}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]+\text{AppellF1}\left[\frac{3}{2},\frac{4}{3},1,\frac{5}{2},\frac{b x^2}{a},-\frac{b x^2}{3 a}\right]\right)\right)\right)\right)$$

Problem 142: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(3 a-b x^2\right)\left(a+b x^2\right)^{1/3}}\,dx$$

Optimal (type 3, 202 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{b}\,x}{\sqrt{a}}\right]}{6\times 2^{2/3}\,a^{5/6}\,\sqrt{b}}+\frac{\text{ArcTan}\left[\frac{\sqrt{b}\,x}{a^{1/6}\,\left(a^{1/3}+2^{1/3}\,\left(a+b\,x^2\right)^{1/3}\right)}\right]}{2\times 2^{2/3}\,a^{5/6}\,\sqrt{b}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{3}\,\sqrt{a}}{\sqrt{b}\,x}\right]}{2\times 2^{2/3}\,\sqrt{3}\,a^{5/6}\,\sqrt{b}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{3}\,a^{1/6}\,\left(a^{1/3}-2^{1/3}\,\left(a+b\,x^2\right)^{1/3}\right)}{\sqrt{b}\,x}\right]}{2\times 2^{2/3}\,\sqrt{3}\,a^{5/6}\,\sqrt{b}}$$

Result (type 6, 166 leaves):

$$\left(9\,a\,x\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-\frac{b\,x^2}{a},\frac{b\,x^2}{3\,a}\right]\right)\Big/\left(\left(3\,a-b\,x^2\right)\,\left(a+b\,x^2\right)^{1/3}\,\left(9\,a\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-\frac{b\,x^2}{a},\frac{b\,x^2}{3\,a}\right]+2\,b\,x^2\,\left(\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},2,\frac{5}{2},-\frac{b\,x^2}{a},\frac{b\,x^2}{3\,a}\right]-\text{AppellF1}\left[\frac{3}{2},\frac{4}{3},1,\frac{5}{2},-\frac{b\,x^2}{a},\frac{b\,x^2}{3\,a}\right]\right)\right)\right)$$

Problem 143: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(c-d\,x^2\right)\,\left(c+3\,d\,x^2\right)^{1/3}}\,dx$$

Optimal (type 3, 204 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{3}\,\sqrt{d}\,x}{\sqrt{c}}\right]}{2\times 2^{2/3}\,\sqrt{3}\,c^{5/6}\,\sqrt{d}}+\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{\sqrt{3}\,\sqrt{d}\,x}{c^{1/6}\,\left(c^{1/3}+2^{1/3}\,\left(c+3\,d\,x^2\right)^{1/3}\right)}\right]}{2\times 2^{2/3}\,c^{5/6}\,\sqrt{d}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{c}}{\sqrt{d}\,x}\right]}{2\times 2^{2/3}\,c^{5/6}\,\sqrt{d}}-\frac{\text{ArcTanh}\left[\frac{c^{1/6}\,\left(c^{1/3}-2^{1/3}\,\left(c+3\,d\,x^2\right)^{1/3}\right)}{\sqrt{d}\,x}\right]}{2\times 2^{2/3}\,c^{5/6}\,\sqrt{d}}$$

Result (type 6, 153 leaves):

$$\left(3\,c\,x\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-\frac{3\,d\,x^2}{c},\frac{d\,x^2}{c}\right]\right)\Big/\left(\left(c-d\,x^2\right)\,\left(c+3\,d\,x^2\right)^{1/3}\,\left(3\,c\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-\frac{3\,d\,x^2}{c},\frac{d\,x^2}{c}\right]+2\,d\,x^2\,\left(\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},2,\frac{5}{2},-\frac{3\,d\,x^2}{c},\frac{d\,x^2}{c}\right]-\text{AppellF1}\left[\frac{3}{2},\frac{4}{3},1,\frac{5}{2},-\frac{3\,d\,x^2}{c},\frac{d\,x^2}{c}\right]\right)\right)\right)$$

Problem 144: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(a-b\,x^2\right)^{1/3}\,\left(3\,a+b\,x^2\right)}\,dx$$

Optimal (type 3, 204 leaves, 1 step):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{3}\,\sqrt{a}}{\sqrt{b}\,x}\right]}{2\times 2^{2/3}\,\sqrt{3}\,a^{5/6}\,\sqrt{b}}+\frac{\text{ArcTan}\left[\frac{\sqrt{3}\,a^{1/6}\left(a^{1/3}-2^{1/3}\left(a-b\,x^2\right)^{1/3}\right)}{\sqrt{b}\,x}\right]}{2\times 2^{2/3}\,\sqrt{3}\,a^{5/6}\,\sqrt{b}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{b}\,x}{\sqrt{a}}\right]}{6\times 2^{2/3}\,a^{5/6}\,\sqrt{b}}+\frac{\text{ArcTanh}\left[\frac{\sqrt{b}\,x}{a^{1/6}\left(a^{1/3}+2^{1/3}\left(a-b\,x^2\right)^{1/3}\right)}\right]}{2\times 2^{2/3}\,a^{5/6}\,\sqrt{b}}$$

Result (type 6, 162 leaves):

$$\left(9\,a\,x\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{b\,x^2}{a},-\frac{b\,x^2}{3\,a}\right]\right)\Big/\left(\left(a-b\,x^2\right)^{1/3}\left(3\,a+b\,x^2\right)\left(9\,a\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{b\,x^2}{a},-\frac{b\,x^2}{3\,a}\right]+2\,b\,x^2\left(-\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},2,\frac{5}{2},\frac{b\,x^2}{a},-\frac{b\,x^2}{3\,a}\right]+\text{AppellF1}\left[\frac{3}{2},\frac{4}{3},1,\frac{5}{2},\frac{b\,x^2}{a},-\frac{b\,x^2}{3\,a}\right]\right)\right)\right)$$

Problem 145: Result unnecessarily involves higher level functions.

$$\int\frac{1}{\left(c-3\,d\,x^2\right)^{1/3}\left(c+d\,x^2\right)}\,dx$$

Optimal (type 3, 204 leaves, 1 step):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{c}}{\sqrt{d}\,x}\right]}{2\times 2^{2/3}\,c^{5/6}\,\sqrt{d}}+\frac{\text{ArcTan}\left[\frac{c^{1/6}\left(c^{1/3}-2^{1/3}\left(c-3\,d\,x^2\right)^{1/3}\right)}{\sqrt{d}\,x}\right]}{2\times 2^{2/3}\,c^{5/6}\,\sqrt{d}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{3}\,\sqrt{d}\,x}{\sqrt{c}}\right]}{2\times 2^{2/3}\,\sqrt{3}\,c^{5/6}\,\sqrt{d}}+\frac{\sqrt{3}\,\text{ArcTanh}\left[\frac{\sqrt{3}\,\sqrt{d}\,x}{c^{1/6}\left(c^{1/3}+2^{1/3}\left(c-3\,d\,x^2\right)^{1/3}\right)}\right]}{2\times 2^{2/3}\,c^{5/6}\,\sqrt{d}}$$

Result (type 6, 156 leaves):

$$\left(3\,c\,x\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{3\,d\,x^2}{c},-\frac{d\,x^2}{c}\right]\right)\Big/\left(\left(c-3\,d\,x^2\right)^{1/3}\left(c+d\,x^2\right)\left(3\,c\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{3\,d\,x^2}{c},-\frac{d\,x^2}{c}\right]+2\,d\,x^2\left(-\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},2,\frac{5}{2},\frac{3\,d\,x^2}{c},-\frac{d\,x^2}{c}\right]+\text{AppellF1}\left[\frac{3}{2},\frac{4}{3},1,\frac{5}{2},\frac{3\,d\,x^2}{c},-\frac{d\,x^2}{c}\right]\right)\right)\right)$$

Problem 146: Result unnecessarily involves higher level functions.

$$\int\frac{1}{\left(1-x^2\right)^{1/3}\left(3+x^2\right)}\,dx$$

Optimal (type 3, 113 leaves, 1 step):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{3}}{x}\right]}{2\times 2^{2/3}\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{\sqrt{3}\left(1-2^{1/3}\left(1-x^2\right)^{1/3}\right)}{x}\right]}{2\times 2^{2/3}\sqrt{3}}-\frac{\text{ArcTanh}[x]}{6\times 2^{2/3}}+\frac{\text{ArcTanh}\left[\frac{x}{1+2^{1/3}\left(1-x^2\right)^{1/3}}\right]}{2\times 2^{2/3}}$$

Result (type 6, 118 leaves):

$$-\left(\left(9\,x\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},x^2,-\frac{x^2}{3}\right]\right)\Big/\left(\left(1-x^2\right)^{1/3}\left(3+x^2\right)\left(-9\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},x^2,-\frac{x^2}{3}\right]+2\,x^2\left(\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},2,\frac{5}{2},x^2,-\frac{x^2}{3}\right]-\text{AppellF1}\left[\frac{3}{2},\frac{4}{3},1,\frac{5}{2},x^2,-\frac{x^2}{3}\right]\right)\right)\right)\right)$$

Problem 147: Result unnecessarily involves higher level functions.

$$\int\frac{1}{\left(3-x^2\right)\left(1+x^2\right)^{1/3}}\,dx$$

Optimal (type 3, 109 leaves, 1 step):

$$-\frac{\text{ArcTan}[x]}{6\times 2^{2/3}}+\frac{\text{ArcTan}\left[\frac{x}{1+2^{1/3}\left(1+x^2\right)^{1/3}}\right]}{2\times 2^{2/3}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{3}}{x}\right]}{2\times 2^{2/3}\sqrt{3}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{3}\left(1-2^{1/3}\left(1+x^2\right)^{1/3}\right)}{x}\right]}{2\times 2^{2/3}\sqrt{3}}$$

Result (type 6, 124 leaves):

$$-\left(\left(9\,x\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-x^2,\frac{x^2}{3}\right]\right)\Big/\left(\left(-3+x^2\right)\left(1+x^2\right)^{1/3}\left(9\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-x^2,\frac{x^2}{3}\right]+2\,x^2\left(\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},2,\frac{5}{2},-x^2,\frac{x^2}{3}\right]-\text{AppellF1}\left[\frac{3}{2},\frac{4}{3},1,\frac{5}{2},-x^2,\frac{x^2}{3}\right]\right)\right)\right)\right)$$

Problem 148: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{3-x}{\left(1-x^2\right)^{1/3}\left(3+x^2\right)}\,dx$$

Optimal (type 3, 96 leaves, 1 step):

$$-\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}-\frac{2^{2/3}(1+x)^{2/3}}{\sqrt{3}\,(1-x)^{1/3}}\right]}{2^{2/3}}-\frac{\text{Log}\left[3+x^2\right]}{2\times 2^{2/3}}+\frac{3\,\text{Log}\left[2^{1/3}\left(1-x\right)^{1/3}+\left(1+x\right)^{2/3}\right]}{2\times 2^{2/3}}$$

Result (type 6, 203 leaves):

$$\left(3 x \left(\left(9 \text{AppellF1}\left[\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, x^2, -\frac{x^2}{3}\right]\right) \Big/ \left(9 \text{AppellF1}\left[\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, x^2, -\frac{x^2}{3}\right] + 2 x^2 \left(-\text{AppellF1}\left[\frac{3}{2}, \frac{1}{3}, 2, \frac{5}{2}, x^2, -\frac{x^2}{3}\right] + \text{AppellF1}\left[\frac{3}{2}, \frac{4}{3}, 1, \frac{5}{2}, x^2, -\frac{x^2}{3}\right]\right)\right) + \left(x \text{AppellF1}\left[1, \frac{1}{3}, 1, 2, x^2, -\frac{x^2}{3}\right]\right) \Big/ \left(-6 \text{AppellF1}\left[1, \frac{1}{3}, 1, 2, x^2, -\frac{x^2}{3}\right] + x^2 \left(\text{AppellF1}\left[2, \frac{1}{3}, 2, 3, x^2, -\frac{x^2}{3}\right] - \text{AppellF1}\left[2, \frac{4}{3}, 1, 3, x^2, -\frac{x^2}{3}\right]\right)\right)\right)\right) \Big/ \left(\left(1 - x^2\right)^{1/3} \left(3 + x^2\right)\right)$$

Problem 149: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{3 + x}{\left(1 - x^2\right)^{1/3} \left(3 + x^2\right)} \, dx$$

Optimal (type 3, 95 leaves, 1 step):

$$\frac{\sqrt{3} \text{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2^{2/3} (1-x)^{2/3}}{\sqrt{3} (1+x)^{1/3}}\right]}{2^{2/3}} + \frac{\text{Log}\left[3 + x^2\right]}{2 \times 2^{2/3}} - \frac{3 \text{Log}\left[(1 - x)^{2/3} + 2^{1/3} (1 + x)^{1/3}\right]}{2 \times 2^{2/3}}$$

Result (type 6, 203 leaves):

$$\left(3 x \left(\left(9 \text{AppellF1}\left[\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, x^2, -\frac{x^2}{3}\right]\right) \Big/ \left(9 \text{AppellF1}\left[\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, x^2, -\frac{x^2}{3}\right] + 2 x^2 \left(-\text{AppellF1}\left[\frac{3}{2}, \frac{1}{3}, 2, \frac{5}{2}, x^2, -\frac{x^2}{3}\right] + \text{AppellF1}\left[\frac{3}{2}, \frac{4}{3}, 1, \frac{5}{2}, x^2, -\frac{x^2}{3}\right]\right)\right) + \left(x \text{AppellF1}\left[1, \frac{1}{3}, 1, 2, x^2, -\frac{x^2}{3}\right]\right) \Big/ \left(6 \text{AppellF1}\left[1, \frac{1}{3}, 1, 2, x^2, -\frac{x^2}{3}\right] + x^2 \left(-\text{AppellF1}\left[2, \frac{1}{3}, 2, 3, x^2, -\frac{x^2}{3}\right] + \text{AppellF1}\left[2, \frac{4}{3}, 1, 3, x^2, -\frac{x^2}{3}\right]\right)\right)\right)\right) \Big/ \left(\left(1 - x^2\right)^{1/3} \left(3 + x^2\right)\right)$$

Problem 150: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(a + b x^2\right)^{1/3} \left(\frac{9 a d}{b} + d x^2\right)} \, dx$$

Optimal (type 3, 151 leaves, 1 step):

$$\frac{\sqrt{b} \text{ArcTan}\left[\frac{\sqrt{b} x}{3 \sqrt{a}}\right]}{12 a^{5/6} d} + \frac{\sqrt{b} \text{ArcTan}\left[\frac{\left(a^{1/3} - \left(a + b x^2\right)^{1/3}\right)^2}{3 a^{1/6} \sqrt{b} x}\right]}{12 a^{5/6} d} - \frac{\sqrt{b} \text{ArcTanh}\left[\frac{\sqrt{3} a^{1/6} \left(a^{1/3} - \left(a + b x^2\right)^{1/3}\right)}{\sqrt{b} x}\right]}{4 \sqrt{3} a^{5/6} d}$$

Result (type 6, 169 leaves):

$$\left(27\,a\,b\,x\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-\frac{b\,x^2}{a},-\frac{b\,x^2}{9\,a}\right]\right)\Big/\left(d\,\left(a+b\,x^2\right)^{1/3}\,\left(9\,a+b\,x^2\right)\,\left(27\,a\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-\frac{b\,x^2}{a},-\frac{b\,x^2}{9\,a}\right]-2\,b\,x^2\,\left(\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},2,\frac{5}{2},-\frac{b\,x^2}{a},-\frac{b\,x^2}{9\,a}\right]+3\,\text{AppellF1}\left[\frac{3}{2},\frac{4}{3},1,\frac{5}{2},-\frac{b\,x^2}{a},-\frac{b\,x^2}{9\,a}\right]\right)\right)\right)$$

Problem 151: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(a-b\,x^2\right)^{1/3}\,\left(-\frac{9\,a\,d}{b}+d\,x^2\right)}\,dx$$

Optimal (type 3, 153 leaves, 1 step):

$$-\frac{\sqrt{b}\,\text{ArcTan}\left[\frac{\sqrt{3}\,a^{1/6}\,\left(a^{1/3}-\left(a-b\,x^2\right)^{1/3}\right)}{\sqrt{b}\,x}\right]}{4\,\sqrt{3}\,a^{5/6}\,d}-\frac{\sqrt{b}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x}{3\,\sqrt{a}}\right]}{12\,a^{5/6}\,d}+\frac{\sqrt{b}\,\text{ArcTanh}\left[\frac{\left(a^{1/3}-\left(a-b\,x^2\right)^{1/3}\right)^2}{3\,a^{1/6}\,\sqrt{b}\,x}\right]}{12\,a^{5/6}\,d}$$

Result (type 6, 167 leaves):

$$-\left(\left(27\,a\,b\,x\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{b\,x^2}{a},\frac{b\,x^2}{9\,a}\right]\right)\Big/\left(d\,\left(a-b\,x^2\right)^{1/3}\,\left(9\,a-b\,x^2\right)\,\left(27\,a\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{b\,x^2}{a},\frac{b\,x^2}{9\,a}\right]+2\,b\,x^2\,\left(\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},2,\frac{5}{2},\frac{b\,x^2}{a},\frac{b\,x^2}{9\,a}\right]+3\,\text{AppellF1}\left[\frac{3}{2},\frac{4}{3},1,\frac{5}{2},\frac{b\,x^2}{a},\frac{b\,x^2}{9\,a}\right]\right)\right)\right)\right)$$

Problem 152: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(-a+b\,x^2\right)^{1/3}\,\left(-\frac{9\,a\,d}{b}+d\,x^2\right)}\,dx$$

Optimal (type 3, 151 leaves, 1 step):

$$\frac{\sqrt{b}\,\text{ArcTan}\left[\frac{\sqrt{3}\,a^{1/6}\,\left(a^{1/3}+\left(-a+b\,x^2\right)^{1/3}\right)}{\sqrt{b}\,x}\right]}{4\,\sqrt{3}\,a^{5/6}\,d}+\frac{\sqrt{b}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x}{3\,\sqrt{a}}\right]}{12\,a^{5/6}\,d}-\frac{\sqrt{b}\,\text{ArcTanh}\left[\frac{\left(a^{1/3}+\left(-a+b\,x^2\right)^{1/3}\right)^2}{3\,a^{1/6}\,\sqrt{b}\,x}\right]}{12\,a^{5/6}\,d}$$

Result (type 6, 168 leaves):

$$-\left(\left(27\,a\,b\,x\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{b\,x^2}{a},\frac{b\,x^2}{9\,a}\right]\right)\Big/\left(d\,\left(9\,a-b\,x^2\right)\,\left(-a+b\,x^2\right)^{1/3}\,\left(27\,a\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{b\,x^2}{a},\frac{b\,x^2}{9\,a}\right]+2\,b\,x^2\,\left(\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},2,\frac{5}{2},\frac{b\,x^2}{a},\frac{b\,x^2}{9\,a}\right]+3\,\text{AppellF1}\left[\frac{3}{2},\frac{4}{3},1,\frac{5}{2},\frac{b\,x^2}{a},\frac{b\,x^2}{9\,a}\right]\right)\right)\right)\right)$$

Problem 153: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(-a-b x^2\right)^{1/3} \left(\frac{9 a d}{b}+d x^2\right)} \, dx$$

Optimal (type 3, 153 leaves, 1 step):

$$-\frac{\sqrt{b}\ \text{ArcTan}\left[\frac{\sqrt{b}\ x}{3 \sqrt{a}}\right]}{12\ a^{5/6}\ d}-\frac{\sqrt{b}\ \text{ArcTan}\left[\frac{\left(a^{1/3}+\left(-a-b x^2\right)^{1/3}\right)^2}{3\ a^{1/6} \sqrt{b}\ x}\right]}{12\ a^{5/6}\ d}+\frac{\sqrt{b}\ \text{ArcTanh}\left[\frac{\sqrt{3}\ a^{1/6} \left(a^{1/3}+\left(-a-b x^2\right)^{1/3}\right)}{\sqrt{b}\ x}\right]}{4 \sqrt{3}\ a^{5/6}\ d}$$

Result (type 6, 172 leaves):

$$\left(27\ a\ b\ x\ \text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-\frac{b x^2}{a},-\frac{b x^2}{9 a}\right]\right) \Big/ \left(d \left(-a-b x^2\right)^{1/3} \left(9 a+b x^2\right) \left(27\ a\ \text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-\frac{b x^2}{a},-\frac{b x^2}{9 a}\right]-2\ b x^2 \left(\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},2,\frac{5}{2},-\frac{b x^2}{a},-\frac{b x^2}{9 a}\right]+3\ \text{AppellF1}\left[\frac{3}{2},\frac{4}{3},1,\frac{5}{2},-\frac{b x^2}{a},-\frac{b x^2}{9 a}\right]\right)\right)\right)$$

Problem 154: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(2+b x^2\right)^{1/3} \left(\frac{18 d}{b}+d x^2\right)} \, dx$$

Optimal (type 3, 151 leaves, 1 step):

$$\frac{\sqrt{b}\ \text{ArcTan}\left[\frac{\sqrt{b}\ x}{3 \sqrt{2}}\right]}{12 \times 2^{5/6}\ d}+\frac{\sqrt{b}\ \text{ArcTan}\left[\frac{\left(2^{1/3}-\left(2+b x^2\right)^{1/3}\right)^2}{3 \times 2^{1/6} \sqrt{b}\ x}\right]}{12 \times 2^{5/6}\ d}-\frac{\sqrt{b}\ \text{ArcTanh}\left[\frac{2^{1/6} \sqrt{3} \left(2^{1/3}-\left(2+b x^2\right)^{1/3}\right)}{\sqrt{b}\ x}\right]}{4 \times 2^{5/6} \sqrt{3}\ d}$$

Result (type 6, 148 leaves):

$$-\left(\left(27\ b\ x\ \text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-\frac{b x^2}{2},-\frac{b x^2}{18}\right]\right) \Big/ \left(d \left(2+b x^2\right)^{1/3} \left(18+b x^2\right) \left(-27\ \text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-\frac{b x^2}{2},-\frac{b x^2}{18}\right]+b x^2 \left(\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},2,\frac{5}{2},-\frac{b x^2}{2},-\frac{b x^2}{18}\right]+3\ \text{AppellF1}\left[\frac{3}{2},\frac{4}{3},1,\frac{5}{2},-\frac{b x^2}{2},-\frac{b x^2}{18}\right]\right)\right)\right)\right)$$

Problem 155: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(-2+b x^2\right)^{1/3} \left(-\frac{18 d}{b}+d x^2\right)} \, dx$$

Optimal (type 3, 147 leaves, 1 step):

$$\frac{\sqrt{b}\,\text{ArcTan}\left[\frac{2^{1/6}\sqrt{3}\left(2^{1/3}+\left(-2+b x^2\right)^{1/3}\right)}{\sqrt{b}\,x}\right]}{4\times 2^{5/6}\sqrt{3}\,d}+\frac{\sqrt{b}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x}{3\sqrt{2}}\right]}{12\times 2^{5/6}\,d}-\frac{\sqrt{b}\,\text{ArcTanh}\left[\frac{\left(2^{1/3}+\left(-2+b x^2\right)^{1/3}\right)^2}{3\times 2^{1/6}\sqrt{b}\,x}\right]}{12\times 2^{5/6}\,d}$$

Result (type 6, 148 leaves):

$$\left(27\,b\,x\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{b x^2}{2},\frac{b x^2}{18}\right]\right)\Big/\left(d\left(-18+b x^2\right)\left(-2+b x^2\right)^{1/3}\left(27\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{b x^2}{2},\frac{b x^2}{18}\right]+b x^2\left(\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},2,\frac{5}{2},\frac{b x^2}{2},\frac{b x^2}{18}\right]+3\,\text{AppellF1}\left[\frac{3}{2},\frac{4}{3},1,\frac{5}{2},\frac{b x^2}{2},\frac{b x^2}{18}\right]\right)\right)\right)$$

Problem 156: Result unnecessarily involves higher level functions.

$$\int\frac{1}{\left(2+3 x^2\right)^{1/3}\left(6 d+d x^2\right)}\,dx$$

Optimal (type 3, 123 leaves, 1 step):

$$\frac{\text{ArcTan}\left[\frac{x}{\sqrt{6}}\right]}{4\times 2^{5/6}\sqrt{3}\,d}+\frac{\text{ArcTan}\left[\frac{\left(2^{1/3}-\left(2+3 x^2\right)^{1/3}\right)^2}{3\times 2^{1/6}\sqrt{3}\,x}\right]}{4\times 2^{5/6}\sqrt{3}\,d}-\frac{\text{ArcTanh}\left[\frac{2^{1/6}\left(2^{1/3}-\left(2+3 x^2\right)^{1/3}\right)}{x}\right]}{4\times 2^{5/6}\,d}$$

Result (type 6, 136 leaves):

$$-\left(\left(9\,x\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-\frac{3 x^2}{2},-\frac{x^2}{6}\right]\right)\Big/\left(d\left(6+x^2\right)\left(2+3 x^2\right)^{1/3}\left(-9\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-\frac{3 x^2}{2},-\frac{x^2}{6}\right]+x^2\left(\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},2,\frac{5}{2},-\frac{3 x^2}{2},-\frac{x^2}{6}\right]+3\,\text{AppellF1}\left[\frac{3}{2},\frac{4}{3},1,\frac{5}{2},-\frac{3 x^2}{2},-\frac{x^2}{6}\right]\right)\right)\right)\right)$$

Problem 157: Result unnecessarily involves higher level functions.

$$\int\frac{1}{\left(2-3 x^2\right)^{1/3}\left(-6 d+d x^2\right)}\,dx$$

Optimal (type 3, 123 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{2^{1/6}\left(2^{1/3}-\left(2-3 x^2\right)^{1/3}\right)}{x}\right]}{4\times 2^{5/6}\,d}-\frac{\text{ArcTanh}\left[\frac{x}{\sqrt{6}}\right]}{4\times 2^{5/6}\sqrt{3}\,d}+\frac{\text{ArcTanh}\left[\frac{\left(2^{1/3}-\left(2-3 x^2\right)^{1/3}\right)^2}{3\times 2^{1/6}\sqrt{3}\,x}\right]}{4\times 2^{5/6}\sqrt{3}\,d}$$

Result (type 6, 136 leaves):

$$\left(9\,x\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{3\,x^2}{2},\frac{x^2}{6}\right]\right)\Big/\left(d\,\left(2-3\,x^2\right)^{1/3}\left(-6+x^2\right)\left(9\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{3\,x^2}{2},\frac{x^2}{6}\right]+x^2\left(\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},2,\frac{5}{2},\frac{3\,x^2}{2},\frac{x^2}{6}\right]+3\,\text{AppellF1}\left[\frac{3}{2},\frac{4}{3},1,\frac{5}{2},\frac{3\,x^2}{2},\frac{x^2}{6}\right]\right)\right)\right)$$

Problem 158: Result unnecessarily involves higher level functions.

$$\int\frac{1}{\left(-2+3\,x^2\right)^{1/3}\left(-6\,d+d\,x^2\right)}\,dx$$

Optimal (type 3, 119 leaves, 1 step):

$$\frac{\text{ArcTan}\left[\frac{2^{1/6}\left(2^{1/3}+\left(-2+3\,x^2\right)^{1/3}\right)}{x}\right]}{4\times2^{5/6}\,d}+\frac{\text{ArcTanh}\left[\frac{x}{\sqrt{6}}\right]}{4\times2^{5/6}\,\sqrt{3}\,d}-\frac{\text{ArcTanh}\left[\frac{\left(2^{1/3}+\left(-2+3\,x^2\right)^{1/3}\right)^2}{3\times2^{1/6}\,\sqrt{3}\,x}\right]}{4\times2^{5/6}\,\sqrt{3}\,d}$$

Result (type 6, 136 leaves):

$$\left(9\,x\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{3\,x^2}{2},\frac{x^2}{6}\right]\right)\Big/\left(d\,\left(-6+x^2\right)\left(-2+3\,x^2\right)^{1/3}\left(9\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{3\,x^2}{2},\frac{x^2}{6}\right]+x^2\left(\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},2,\frac{5}{2},\frac{3\,x^2}{2},\frac{x^2}{6}\right]+3\,\text{AppellF1}\left[\frac{3}{2},\frac{4}{3},1,\frac{5}{2},\frac{3\,x^2}{2},\frac{x^2}{6}\right]\right)\right)\right)$$

Problem 159: Result unnecessarily involves higher level functions.

$$\int\frac{1}{\left(-2-3\,x^2\right)^{1/3}\left(6\,d+d\,x^2\right)}\,dx$$

Optimal (type 3, 119 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{x}{\sqrt{6}}\right]}{4\times2^{5/6}\,\sqrt{3}\,d}-\frac{\text{ArcTan}\left[\frac{\left(2^{1/3}+\left(-2-3\,x^2\right)^{1/3}\right)^2}{3\times2^{1/6}\,\sqrt{3}\,x}\right]}{4\times2^{5/6}\,\sqrt{3}\,d}+\frac{\text{ArcTanh}\left[\frac{2^{1/6}\left(2^{1/3}+\left(-2-3\,x^2\right)^{1/3}\right)}{x}\right]}{4\times2^{5/6}\,d}$$

Result (type 6, 136 leaves):

$$-\left(\left(9\,x\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-\frac{3\,x^2}{2},-\frac{x^2}{6}\right]\right)\Big/\left(d\,\left(-2-3\,x^2\right)^{1/3}\left(6+x^2\right)\left(-9\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-\frac{3\,x^2}{2},-\frac{x^2}{6}\right]+x^2\left(\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},2,\frac{5}{2},-\frac{3\,x^2}{2},-\frac{x^2}{6}\right]+3\,\text{AppellF1}\left[\frac{3}{2},\frac{4}{3},1,\frac{5}{2},-\frac{3\,x^2}{2},-\frac{x^2}{6}\right]\right)\right)\right)\right)$$

Problem 160: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(1+x^2\right)^{1/3}\left(9+x^2\right)}\,dx$$

Optimal (type 3, 70 leaves, 1 step):

$$\frac{1}{12}\,\text{ArcTan}\left[\frac{x}{3}\right]+\frac{1}{12}\,\text{ArcTan}\left[\frac{\left(1-\left(1+x^2\right)^{1/3}\right)^2}{3\,x}\right]-\frac{\text{ArcTanh}\left[\frac{\sqrt{3}\,\left(1-\left(1+x^2\right)^{1/3}\right)}{x}\right]}{4\,\sqrt{3}}$$

Result (type 6, 124 leaves):

$$-\left(\left(27\,x\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-x^2,-\frac{x^2}{9}\right]\right)\middle/\left(\left(1+x^2\right)^{1/3}\left(9+x^2\right)\left(-27\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-x^2,-\frac{x^2}{9}\right]+2\,x^2\left(\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},2,\frac{5}{2},-x^2,-\frac{x^2}{9}\right]+3\,\text{AppellF1}\left[\frac{3}{2},\frac{4}{3},1,\frac{5}{2},-x^2,-\frac{x^2}{9}\right]\right)\right)\right)\right)$$

Problem 161: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(1+b\,x^2\right)^{1/3}\left(9+b\,x^2\right)}\,dx$$

Optimal (type 3, 104 leaves, 1 step):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{b}\,x}{3}\right]}{12\,\sqrt{b}}+\frac{\text{ArcTan}\left[\frac{\left(1-\left(1+b\,x^2\right)^{1/3}\right)^2}{3\,\sqrt{b}\,x}\right]}{12\,\sqrt{b}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{3}\,\left(1-\left(1+b\,x^2\right)^{1/3}\right)}{\sqrt{b}\,x}\right]}{4\,\sqrt{3}\,\sqrt{b}}$$

Result (type 6, 137 leaves):

$$-\left(\left(27\,x\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-b\,x^2,-\frac{b\,x^2}{9}\right]\right)\middle/\left(\left(1+b\,x^2\right)^{1/3}\left(9+b\,x^2\right)\left(-27\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-b\,x^2,-\frac{b\,x^2}{9}\right]+2\,b\,x^2\left(\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},2,\frac{5}{2},-b\,x^2,-\frac{b\,x^2}{9}\right]+3\,\text{AppellF1}\left[\frac{3}{2},\frac{4}{3},1,\frac{5}{2},-b\,x^2,-\frac{b\,x^2}{9}\right]\right)\right)\right)\right)$$

Problem 162: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(1-x^2\right)^{1/3}\left(9-x^2\right)}\,dx$$

Optimal (type 3, 74 leaves, 1 step):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{3}\left(1-\left(1-x^2\right)^{1/3}\right)}{x}\right]}{4\sqrt{3}}+\frac{1}{12}\text{ArcTanh}\left[\frac{x}{3}\right]-\frac{1}{12}\text{ArcTanh}\left[\frac{\left(1-\left(1-x^2\right)^{1/3}\right)^2}{3x}\right]$$

Result (type 6, 125 leaves):

$$\frac{1}{4\left(1-x^2\right)^{1/3}}\left(\left(\frac{-1+x}{-3+x}\right)^{1/3}\left(\frac{1+x}{-3+x}\right)^{1/3}\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},-\frac{4}{-3+x},-\frac{2}{-3+x}\right]-\left(\frac{-1+x}{3+x}\right)^{1/3}\left(\frac{1+x}{3+x}\right)^{1/3}\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},\frac{2}{3+x},\frac{4}{3+x}\right]\right)$$

Problem 163: Result unnecessarily involves imaginary or complex numbers.

$$\int\left(a+b x^2\right)^{3/2}\sqrt{c+d x^2}\,dx$$

Optimal (type 4, 328 leaves, 6 steps):

$$\frac{\left(7ac-\frac{2bc^2}{d}+\frac{3a^2d}{b}\right)x\sqrt{a+bx^2}}{15\sqrt{c+dx^2}}-\frac{2\left(bc-3ad\right)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15d}+\frac{bx\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{5d}+\left(\sqrt{c}\left(2b^2c^2-7abcd-3a^2d^2\right)\sqrt{a+bx^2}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],1-\frac{bc}{ad}\right]\right)\Big/\left(15bd^{3/2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}\sqrt{c+dx^2}\right)-\frac{c^{3/2}\left(bc-9ad\right)\sqrt{a+bx^2}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],1-\frac{bc}{ad}\right]}{15d^{3/2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}\sqrt{c+dx^2}}$$

Result (type 4, 243 leaves):

$$\left(\sqrt{\frac{b}{a}}\,d\,x\left(a+bx^2\right)\left(c+dx^2\right)\left(6ad+b\left(c+3dx^2\right)\right)-i\,c\left(-2b^2c^2+7abcd+3a^2d^2\right)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{b}{a}}\,x\right],\frac{ad}{bc}\right]-2\,i\,c\left(b^2c^2-4abcd+3a^2d^2\right)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{b}{a}}\,x\right],\frac{ad}{bc}\right]\right)\Big/\left(15\sqrt{\frac{b}{a}}\,d^2\sqrt{a+bx^2}\sqrt{c+dx^2}\right)$$

Problem 164: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{a+b x^2}\,\sqrt{c+d x^2}\,dx$$

Optimal (type 4, 249 leaves, 5 steps):

$$\frac{(b c+a d)\,x\,\sqrt{a+b x^2}}{3 b\,\sqrt{c+d x^2}}+\frac{1}{3}\,x\,\sqrt{a+b x^2}\,\sqrt{c+d x^2}-\frac{\sqrt{c}\,(b c+a d)\,\sqrt{a+b x^2}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],1-\frac{b c}{a d}\right]}{3 b\,\sqrt{d}\,\sqrt{\frac{c\,(a+b x^2)}{a\,(c+d x^2)}}\,\sqrt{c+d x^2}}+\frac{2 c^{3/2}\,\sqrt{a+b x^2}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],1-\frac{b c}{a d}\right]}{3\,\sqrt{d}\,\sqrt{\frac{c\,(a+b x^2)}{a\,(c+d x^2)}}\,\sqrt{c+d x^2}}$$

Result (type 4, 198 leaves):

$$\left(\sqrt{\frac{b}{a}}\,d\,x\,(a+b x^2)\,(c+d x^2)-i\,c\,(b c+a d)\,\sqrt{1+\frac{b x^2}{a}}\,\sqrt{1+\frac{d x^2}{c}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{b}{a}}\,x\right],\frac{a d}{b c}\right]-i\,c\,(-b c+a d)\,\sqrt{1+\frac{b x^2}{a}}\,\sqrt{1+\frac{d x^2}{c}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{b}{a}}\,x\right],\frac{a d}{b c}\right]\right)\Big/\left(3\,\sqrt{\frac{b}{a}}\,d\,\sqrt{a+b x^2}\,\sqrt{c+d x^2}\right)$$

Problem 166: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{c+d x^2}}{(a+b x^2)^{3/2}}\,dx$$

Optimal (type 4, 84 leaves, 1 step):

$$\frac{\sqrt{c+d x^2}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{b}\,x}{\sqrt{a}}\right],1-\frac{a d}{b c}\right]}{\sqrt{a}\,\sqrt{b}\,\sqrt{a+b x^2}\,\sqrt{\frac{a\,(c+d x^2)}{c\,(a+b x^2)}}}$$

Result (type 4, 133 leaves):

$$\left(x \left(c+d x^2\right)+\frac{1}{\sqrt{\frac{b}{a}}} i c \sqrt{1+\frac{b x^2}{a}} \sqrt{1+\frac{d x^2}{c}} \left(\text{EllipticE}\left[i \,\text{ArcSinh}\left[\sqrt{\frac{b}{a}}\, x\right], \frac{a d}{b c}\right] - \text{EllipticF}\left[i \,\text{ArcSinh}\left[\sqrt{\frac{b}{a}}\, x\right], \frac{a d}{b c}\right]\right)\right) \Big/ \left(a \sqrt{a+b x^2} \sqrt{c+d x^2}\right)$$

Problem 167: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{c+d x^2}}{\left(a+b x^2\right)^{5/2}} \, dx$$

Optimal (type 4, 237 leaves, 4 steps):

$$\frac{x \sqrt{c+d x^2}}{3 a \left(a+b x^2\right)^{3/2}}+\frac{(2 b c-a d) \sqrt{c+d x^2}\, \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{b}\, x}{\sqrt{a}}\right], 1-\frac{a d}{b c}\right]}{3 a^{3/2} \sqrt{b}\, (b c-a d) \sqrt{a+b x^2} \sqrt{\frac{a \left(c+d x^2\right)}{c \left(a+b x^2\right)}}}-\frac{c^{3/2} \sqrt{d} \sqrt{a+b x^2}\, \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\, x}{\sqrt{c}}\right], 1-\frac{b c}{a d}\right]}{3 a^2 (b c-a d) \sqrt{\frac{c \left(a+b x^2\right)}{a \left(c+d x^2\right)}} \sqrt{c+d x^2}}$$

Result (type 4, 243 leaves):

$$\left(\sqrt{\frac{b}{a}}\, x \left(c+d x^2\right) \left(2 a^2 d-2 b^2 c x^2+a b \left(-3 c+d x^2\right)\right)+i c (-2 b c+a d) \left(a+b x^2\right) \sqrt{1+\frac{b x^2}{a}} \sqrt{1+\frac{d x^2}{c}}\, \text{EllipticE}\left[i \,\text{ArcSinh}\left[\sqrt{\frac{b}{a}}\, x\right], \frac{a d}{b c}\right] - 2 i c (-b c+a d) \left(a+b x^2\right) \sqrt{1+\frac{b x^2}{a}} \sqrt{1+\frac{d x^2}{c}}\, \text{EllipticF}\left[i \,\text{ArcSinh}\left[\sqrt{\frac{b}{a}}\, x\right], \frac{a d}{b c}\right]\right) \Big/ \left(3 a^2 \sqrt{\frac{b}{a}} (-b c+a d) \left(a+b x^2\right)^{3/2} \sqrt{c+d x^2}\right)$$

Problem 168: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{c+d x^2}}{\left(a+b x^2\right)^{7/2}} \, dx$$

Optimal (type 4, 309 leaves, 5 steps):

$$\frac{x \sqrt{c+d x^2}}{5 a \left(a+b x^2\right)^{5/2}}+\frac{(4 b c-3 a d) x \sqrt{c+d x^2}}{15 a^2 (b c-a d) \left(a+b x^2\right)^{3/2}}+\left(\left(8 b^2 c^2-13 a b c d+3 a^2 d^2\right) \sqrt{c+d x^2} \, \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{b} x}{\sqrt{a}}\right], 1-\frac{a d}{b c}\right]\right) \Bigg/ \left(15 a^{5/2} \sqrt{b} (b c-a d)^2 \sqrt{a+b x^2} \sqrt{\frac{a \left(c+d x^2\right)}{c \left(a+b x^2\right)}}\right)-\frac{2 c^{3/2} \sqrt{d} (2 b c-3 a d) \sqrt{a+b x^2} \, \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1-\frac{b c}{a d}\right]}{15 a^3 (b c-a d)^2 \sqrt{\frac{c \left(a+b x^2\right)}{a \left(c+d x^2\right)}} \sqrt{c+d x^2}}$$

Result (type 4, 285 leaves):

$$\frac{1}{15 a^3 \sqrt{\frac{b}{a}} (b c-a d)^2 \left(a+b x^2\right)^{5/2} \sqrt{c+d x^2}}\left(\sqrt{\frac{b}{a}} x \left(c+d x^2\right) \left(3 a^2 (b c-a d)^2+a (-b c+a d) (-4 b c+3 a d) \left(a+b x^2\right)+\left(8 b^2 c^2-13 a b c d+3 a^2 d^2\right) \left(a+b x^2\right)^2\right)+i c \left(a+b x^2\right)^2 \sqrt{1+\frac{b x^2}{a}} \sqrt{1+\frac{d x^2}{c}} \left(\left(8 b^2 c^2-13 a b c d+3 a^2 d^2\right) \text{EllipticE}\left[i \, \text{ArcSinh}\left[\sqrt{\frac{b}{a}} x\right], \frac{a d}{b c}\right]+\left(-8 b^2 c^2+17 a b c d-9 a^2 d^2\right) \text{EllipticF}\left[i \, \text{ArcSinh}\left[\sqrt{\frac{b}{a}} x\right], \frac{a d}{b c}\right]\right)\right)$$

Problem 169: Result unnecessarily involves imaginary or complex numbers.

$$\int \left(a+b x^2\right)^{3/2} \left(c+d x^2\right)^{3/2} \, dx$$

Optimal (type 4, 410 leaves, 7 steps):

$$-\frac{2\,(b\,c+a\,d)\,\left(b^2\,c^2-6\,a\,b\,c\,d+a^2\,d^2\right)\,x\,\sqrt{a+b\,x^2}}{35\,b^2\,d\,\sqrt{c+d\,x^2}}+\frac{1}{35}\left(9\,a\,c+\frac{b\,c^2}{d}-\frac{2\,a^2\,d}{b}\right)\,x\,\sqrt{a+b\,x^2}\,\sqrt{c+d\,x^2}+$$
$$\frac{2\,(4\,b\,c-a\,d)\,x\,\left(a+b\,x^2\right)^{3/2}\,\sqrt{c+d\,x^2}}{35\,b}+\frac{d\,x\,\left(a+b\,x^2\right)^{5/2}\,\sqrt{c+d\,x^2}}{7\,b}+$$
$$\left(2\,\sqrt{c}\,(b\,c+a\,d)\,\left(b^2\,c^2-6\,a\,b\,c\,d+a^2\,d^2\right)\,\sqrt{a+b\,x^2}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],1-\frac{b\,c}{a\,d}\right]\right)\Big/$$
$$\left(35\,b^2\,d^{3/2}\,\sqrt{\frac{c\,\left(a+b\,x^2\right)}{a\,\left(c+d\,x^2\right)}}\,\sqrt{c+d\,x^2}\right)-$$
$$\left(c^{3/2}\,\left(b^2\,c^2-18\,a\,b\,c\,d+a^2\,d^2\right)\,\sqrt{a+b\,x^2}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],1-\frac{b\,c}{a\,d}\right]\right)\Big/$$
$$\left(35\,b\,d^{3/2}\,\sqrt{\frac{c\,\left(a+b\,x^2\right)}{a\,\left(c+d\,x^2\right)}}\,\sqrt{c+d\,x^2}\right)$$

Result (type 4, 302 leaves):

$$\frac{1}{35\,b\,\sqrt{\frac{b}{a}}\,d^2\,\sqrt{a+b\,x^2}\,\sqrt{c+d\,x^2}}$$
$$\left(\sqrt{\frac{b}{a}}\,d\,x\,\left(a+b\,x^2\right)\,\left(c+d\,x^2\right)\,\left(a^2\,d^2+a\,b\,d\,\left(17\,c+8\,d\,x^2\right)+b^2\,\left(c^2+8\,c\,d\,x^2+5\,d^2\,x^4\right)\right)+\right.$$
$$2\,i\,c\,\left(b^3\,c^3-5\,a\,b^2\,c^2\,d-5\,a^2\,b\,c\,d^2+a^3\,d^3\right)\,\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}$$
$$\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{b}{a}}\,x\right],\frac{a\,d}{b\,c}\right]-i\,c\,\left(2\,b^3\,c^3-11\,a\,b^2\,c^2\,d+8\,a^2\,b\,c\,d^2+a^3\,d^3\right)$$
$$\left.\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{b}{a}}\,x\right],\frac{a\,d}{b\,c}\right]\right)$$

Problem 170: Result unnecessarily involves imaginary or complex numbers.

$$\int\sqrt{a+b\,x^2}\,\left(c+d\,x^2\right)^{3/2}\,dx$$

Optimal (type 4, 336 leaves, 6 steps):

$$\frac{\left(3 b^2 c^2+7 a b c d-2 a^2 d^2\right) x \sqrt{a+b x^2}}{15 b^2 \sqrt{c+d x^2}}+\frac{2 (3 b c-a d) x \sqrt{a+b x^2} \sqrt{c+d x^2}}{15 b}+\frac{d x \left(a+b x^2\right)^{3/2} \sqrt{c+d x^2}}{5 b}-\left(\sqrt{c} \left(3 b^2 c^2+7 a b c d-2 a^2 d^2\right) \sqrt{a+b x^2}\ \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\ x}{\sqrt{c}}\right],1-\frac{b c}{a d}\right]\right)\Big/\left(15 b^2 \sqrt{d} \sqrt{\frac{c \left(a+b x^2\right)}{a \left(c+d x^2\right)}} \sqrt{c+d x^2}\right)+\frac{c^{3/2} (9 b c-a d) \sqrt{a+b x^2}\ \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\ x}{\sqrt{c}}\right],1-\frac{b c}{a d}\right]}{15 b \sqrt{d} \sqrt{\frac{c \left(a+b x^2\right)}{a \left(c+d x^2\right)}} \sqrt{c+d x^2}}$$

Result (type 4, 246 leaves):

$$\left(\sqrt{\frac{b}{a}}\ d x \left(a+b x^2\right) \left(c+d x^2\right) \left(6 b c+a d+3 b d x^2\right)+\right.$$
$$i c \left(-3 b^2 c^2-7 a b c d+2 a^2 d^2\right) \sqrt{1+\frac{b x^2}{a}} \sqrt{1+\frac{d x^2}{c}}\ \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{b}{a}}\ x\right],\frac{a d}{b c}\right]-$$
$$\left. i c \left(-3 b^2 c^2+2 a b c d+a^2 d^2\right) \sqrt{1+\frac{b x^2}{a}} \sqrt{1+\frac{d x^2}{c}}\ \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{b}{a}}\ x\right],\frac{a d}{b c}\right]\right)\Big/$$
$$\left(15 b \sqrt{\frac{b}{a}}\ d \sqrt{a+b x^2} \sqrt{c+d x^2}\right)$$

Problem 171: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(c+d x^2\right)^{3/2}}{\sqrt{a+b x^2}}\, dx$$

Optimal (type 4, 273 leaves, 5 steps):

$$\frac{2 d (2 b c - a d) x \sqrt{a + b x^2}}{3 b^2 \sqrt{c + d x^2}} + \frac{d x \sqrt{a + b x^2} \sqrt{c + d x^2}}{3 b} - \frac{2 \sqrt{c} \sqrt{d} (2 b c - a d) \sqrt{a + b x^2} \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1 - \frac{b c}{a d}\right]}{3 b^2 \sqrt{\frac{c (a+b x^2)}{a (c+d x^2)}} \sqrt{c + d x^2}} + \frac{c^{3/2} (3 b c - a d) \sqrt{a + b x^2} \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1 - \frac{b c}{a d}\right]}{3 a b \sqrt{d} \sqrt{\frac{c (a+b x^2)}{a (c+d x^2)}} \sqrt{c + d x^2}}$$

Result (type 4, 199 leaves):

$$\left(\sqrt{\frac{b}{a}} d x (a + b x^2) (c + d x^2) + 2 i c (-2 b c + a d) \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{b}{a}} x\right], \frac{a d}{b c}\right] - i c (-b c + a d) \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{b}{a}} x\right], \frac{a d}{b c}\right]\right) \Big/ \left(3 b \sqrt{\frac{b}{a}} \sqrt{a + b x^2} \sqrt{c + d x^2}\right)$$

Problem 172: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(c + d x^2)^{3/2}}{(a + b x^2)^{3/2}} \, dx$$

Optimal (type 4, 267 leaves, 5 steps):

$$-\frac{d (b c - 2 a d) x \sqrt{a + b x^2}}{a b^2 \sqrt{c + d x^2}} + \frac{(b c - a d) x \sqrt{c + d x^2}}{a b \sqrt{a + b x^2}} + \frac{\sqrt{c} \sqrt{d} (b c - 2 a d) \sqrt{a + b x^2} \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1 - \frac{b c}{a d}\right]}{a b^2 \sqrt{\frac{c (a+b x^2)}{a (c+d x^2)}} \sqrt{c + d x^2}} + \frac{c^{3/2} \sqrt{d} \sqrt{a + b x^2} \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1 - \frac{b c}{a d}\right]}{a b \sqrt{\frac{c (a+b x^2)}{a (c+d x^2)}} \sqrt{c + d x^2}}$$

Result (type 4, 191 leaves):

$$\left(-i\, c\, (-b c+2 a d) \sqrt{1+\frac{b x^2}{a}} \sqrt{1+\frac{d x^2}{c}}\, \text{EllipticE}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{b}{a}}\, x\right], \frac{a d}{b c}\right] + (b c-a d) \left(\sqrt{\frac{b}{a}}\, x\, (c+d x^2) - i\, c \sqrt{1+\frac{b x^2}{a}} \sqrt{1+\frac{d x^2}{c}}\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{b}{a}}\, x\right], \frac{a d}{b c}\right]\right)\right) \Bigg/ \left(a^2 \left(\frac{b}{a}\right)^{3/2} \sqrt{a+b x^2} \sqrt{c+d x^2}\right)$$

Problem 173: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(c+d x^2)^{3/2}}{(a+b x^2)^{5/2}}\, dx$$

Optimal (type 4, 229 leaves, 4 steps):

$$\frac{(b c-a d)\, x \sqrt{c+d x^2}}{3 a b\, (a+b x^2)^{3/2}} + \frac{2 (b c+a d) \sqrt{c+d x^2}\, \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{b}\, x}{\sqrt{a}}\right], 1-\frac{a d}{b c}\right]}{3 a^{3/2} b^{3/2} \sqrt{a+b x^2} \sqrt{\frac{a (c+d x^2)}{c (a+b x^2)}}} - \frac{c^{3/2} \sqrt{d} \sqrt{a+b x^2}\, \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\, x}{\sqrt{c}}\right], 1-\frac{b c}{a d}\right]}{3 a^2 b \sqrt{\frac{c (a+b x^2)}{a (c+d x^2)}} \sqrt{c+d x^2}}$$

Result (type 4, 232 leaves):

$$\left(\sqrt{\frac{b}{a}}\, x\, (c+d x^2) \left(a^2 d+2 b^2 c x^2+a b (3 c+2 d x^2)\right) + 2 i\, c\, (b c+a d) (a+b x^2) \sqrt{1+\frac{b x^2}{a}} \sqrt{1+\frac{d x^2}{c}}\, \text{EllipticE}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{b}{a}}\, x\right], \frac{a d}{b c}\right] - i\, c\, (2 b c+a d) (a+b x^2) \sqrt{1+\frac{b x^2}{a}} \sqrt{1+\frac{d x^2}{c}}\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{b}{a}}\, x\right], \frac{a d}{b c}\right]\right) \Bigg/ \left(3 a^3 \left(\frac{b}{a}\right)^{3/2} (a+b x^2)^{3/2} \sqrt{c+d x^2}\right)$$

Problem 174: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(c + d x^2\right)^{3/2}}{\left(a + b x^2\right)^{7/2}} \, dx$$

Optimal (type 4, 315 leaves, 5 steps):

$$\frac{(b c - a d) x \sqrt{c + d x^2}}{5 a b \left(a + b x^2\right)^{5/2}} + \frac{2 (2 b c + a d) x \sqrt{c + d x^2}}{15 a^2 b \left(a + b x^2\right)^{3/2}} + \left(\left(8 b^2 c^2 - 3 a b c d - 2 a^2 d^2\right) \sqrt{c + d x^2} \, \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{b} x}{\sqrt{a}}\right], 1 - \frac{a d}{b c}\right]\right) \Big/ \left(15 a^{5/2} b^{3/2} (b c - a d) \sqrt{a + b x^2} \sqrt{\frac{a \left(c + d x^2\right)}{c \left(a + b x^2\right)}}\right) - \frac{c^{3/2} \sqrt{d} (4 b c - a d) \sqrt{a + b x^2} \, \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1 - \frac{b c}{a d}\right]}{15 a^3 b (b c - a d) \sqrt{\frac{c \left(a + b x^2\right)}{a \left(c + d x^2\right)}} \sqrt{c + d x^2}}$$

Result (type 4, 285 leaves):

$$\left(\sqrt{\frac{b}{a}} x \left(c + d x^2\right) \left(3 a^2 (b c - a d)^2 + 2 a (b c - a d) (2 b c + a d) \left(a + b x^2\right) + \left(8 b^2 c^2 - 3 a b c d - 2 a^2 d^2\right) \left(a + b x^2\right)^2\right) - i c \left(a + b x^2\right)^2 \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \left(\left(-8 b^2 c^2 + 3 a b c d + 2 a^2 d^2\right) \text{EllipticE}\left[i \, \text{ArcSinh}\left[\sqrt{\frac{b}{a}} x\right], \frac{a d}{b c}\right] + \left(8 b^2 c^2 - 7 a b c d - a^2 d^2\right) \text{EllipticF}\left[i \, \text{ArcSinh}\left[\sqrt{\frac{b}{a}} x\right], \frac{a d}{b c}\right]\right)\right) \Big/ \left(15 a^4 \left(\frac{b}{a}\right)^{3/2} (b c - a d) \left(a + b x^2\right)^{5/2} \sqrt{c + d x^2}\right)$$

Problem 175: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{2 + b x^2} \sqrt{3 + d x^2} \, dx$$

Optimal (type 4, 235 leaves, 5 steps):

$$\frac{(3b+2d)\,x\sqrt{2+bx^2}}{3b\sqrt{3+dx^2}}+\frac{1}{3}x\sqrt{2+bx^2}\sqrt{3+dx^2}-\frac{\sqrt{2}\,(3b+2d)\sqrt{2+bx^2}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{3}}\right],1-\frac{3b}{2d}\right]}{3b\sqrt{d}\sqrt{\frac{2+bx^2}{3+dx^2}}\sqrt{3+dx^2}}+\frac{2\sqrt{2}\sqrt{2+bx^2}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{3}}\right],1-\frac{3b}{2d}\right]}{\sqrt{d}\sqrt{\frac{2+bx^2}{3+dx^2}}\sqrt{3+dx^2}}$$

Result (type 4, 127 leaves):

$$\frac{1}{3\sqrt{b}\,d}\left(\sqrt{b}\,d\,x\sqrt{2+bx^2}\sqrt{3+dx^2}-i\sqrt{3}\,(3b+2d)\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{b}\,x}{\sqrt{2}}\right],\frac{2d}{3b}\right]+i\sqrt{3}\,(3b-2d)\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{b}\,x}{\sqrt{2}}\right],\frac{2d}{3b}\right]\right)$$

Problem 188: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{1-x^2}}{\sqrt{1+x^2}}\,dx$$

Optimal (type 4, 13 leaves, 4 steps):

$$-\text{EllipticE}[\text{ArcSin}[x],-1]+2\,\text{EllipticF}[\text{ArcSin}[x],-1]$$

Result (type 4, 12 leaves):

$$-i\,\text{EllipticE}[i\,\text{ArcSinh}[x],-1]$$

Problem 189: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{1-x^2}}{\sqrt{2+3x^2}}\,dx$$

Optimal (type 4, 31 leaves, 3 steps):

$$-\frac{1}{3}\sqrt{2}\,\text{EllipticE}\left[\text{ArcSin}[x],-\frac{3}{2}\right]+\frac{5\,\text{EllipticF}\left[\text{ArcSin}[x],-\frac{3}{2}\right]}{3\sqrt{2}}$$

Result (type 4, 27 leaves):

$$-\frac{i\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{3}{2}}\,x\right],-\frac{2}{3}\right]}{\sqrt{3}}$$

Problem 190: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{4-x^2}}{\sqrt{2+3x^2}}\,dx$$

Optimal (type 4, 35 leaves, 3 steps):

$$-\frac{1}{3}\sqrt{2}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{2}\right],-6\right]+\frac{7}{3}\sqrt{2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{2}\right],-6\right]$$

Result (type 4, 27 leaves):

$$-\frac{2\,i\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{3}{2}}\,x\right],-\frac{1}{6}\right]}{\sqrt{3}}$$

Problem 191: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{1-4x^2}}{\sqrt{2+3x^2}}\,dx$$

Optimal (type 4, 35 leaves, 3 steps):

$$-\frac{2}{3}\sqrt{2}\,\text{EllipticE}\left[\text{ArcSin}[2x],-\frac{3}{8}\right]+\frac{11\,\text{EllipticF}\left[\text{ArcSin}[2x],-\frac{3}{8}\right]}{6\sqrt{2}}$$

Result (type 4, 27 leaves):

$$-\frac{i\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{3}{2}}\,x\right],-\frac{8}{3}\right]}{\sqrt{3}}$$

Problem 192: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{1+x^2}}{\sqrt{2+3x^2}}\,dx$$

Optimal (type 4, 131 leaves, 4 steps):

$$\frac{x\sqrt{2+3x^2}}{3\sqrt{1+x^2}}-\frac{\sqrt{2}\sqrt{2+3x^2}\,\text{EllipticE}\left[\text{ArcTan}[x],-\frac{1}{2}\right]}{3\sqrt{1+x^2}\sqrt{\frac{2+3x^2}{1+x^2}}}+\frac{\sqrt{2+3x^2}\,\text{EllipticF}\left[\text{ArcTan}[x],-\frac{1}{2}\right]}{\sqrt{2}\sqrt{1+x^2}\sqrt{\frac{2+3x^2}{1+x^2}}}$$

Result (type 4, 27 leaves):

$$-\frac{i\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{3}{2}}\,x\right],\frac{2}{3}\right]}{\sqrt{3}}$$

Problem 193: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{4+x^2}}{\sqrt{2+3x^2}}\, dx$$

Optimal (type 4, 136 leaves, 4 steps):

$$\frac{x\sqrt{2+3x^2}}{3\sqrt{4+x^2}} - \frac{\sqrt{2}\sqrt{2+3x^2}\ \text{EllipticE}\left[\text{ArcTan}\left[\frac{x}{2}\right], -5\right]}{3\sqrt{4+x^2}\sqrt{\frac{2+3x^2}{4+x^2}}} + \frac{2\sqrt{2}\sqrt{2+3x^2}\ \text{EllipticF}\left[\text{ArcTan}\left[\frac{x}{2}\right], -5\right]}{\sqrt{4+x^2}\sqrt{\frac{2+3x^2}{4+x^2}}}$$

Result (type 4, 27 leaves):

$$-\frac{2\, i\ \text{EllipticE}\left[i\ \text{ArcSinh}\left[\sqrt{\frac{3}{2}}\ x\right], \frac{1}{6}\right]}{\sqrt{3}}$$

Problem 194: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{1+4x^2}}{\sqrt{2+3x^2}}\, dx$$

Optimal (type 4, 148 leaves, 4 steps):

$$\frac{4x\sqrt{2+3x^2}}{3\sqrt{1+4x^2}} - \frac{2\sqrt{2}\sqrt{2+3x^2}\ \text{EllipticE}\left[\text{ArcTan}[2x], \frac{5}{8}\right]}{3\sqrt{\frac{2+3x^2}{1+4x^2}}\sqrt{1+4x^2}} + \frac{\sqrt{2+3x^2}\ \text{EllipticF}\left[\text{ArcTan}[2x], \frac{5}{8}\right]}{2\sqrt{2}\sqrt{\frac{2+3x^2}{1+4x^2}}\sqrt{1+4x^2}}$$

Result (type 4, 27 leaves):

$$-\frac{i\ \text{EllipticE}\left[i\ \text{ArcSinh}\left[\sqrt{\frac{3}{2}}\ x\right], \frac{8}{3}\right]}{\sqrt{3}}$$

Problem 196: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b x^2\right)^{7/2}}{\sqrt{c+d x^2}}\, dx$$

Optimal (type 4, 423 leaves, 7 steps):

$$-\frac{8\left(b c-2 a d\right)\left(6 b^2 c^2-11 a b c d+11 a^2 d^2\right) x \sqrt{a+b x^2}}{105 d^3 \sqrt{c+d x^2}}+\frac{b\left(24 b^2 c^2-71 a b c d+71 a^2 d^2\right) x \sqrt{a+b x^2}\,\sqrt{c+d x^2}}{105 d^3}-\frac{6 b\left(b c-2 a d\right) x\left(a+b x^2\right)^{3/2}\sqrt{c+d x^2}}{35 d^2}+\frac{b x\left(a+b x^2\right)^{5/2}\sqrt{c+d x^2}}{7 d}+\left(8\sqrt{c}\left(b c-2 a d\right)\left(6 b^2 c^2-11 a b c d+11 a^2 d^2\right)\sqrt{a+b x^2}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],1-\frac{b c}{a d}\right]\right)\Big/\left(105 d^{7/2}\sqrt{\frac{c\left(a+b x^2\right)}{a\left(c+d x^2\right)}}\sqrt{c+d x^2}\right)-\left(\sqrt{c}\left(3 b c-7 a d\right)\left(8 b^2 c^2-11 a b c d+15 a^2 d^2\right)\sqrt{a+b x^2}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],1-\frac{b c}{a d}\right]\right)\Big/\left(105 d^{7/2}\sqrt{\frac{c\left(a+b x^2\right)}{a\left(c+d x^2\right)}}\sqrt{c+d x^2}\right)$$

Result (type 4, 321 leaves):

$$\frac{1}{105\sqrt{\frac{b}{a}}\, d^4\sqrt{a+b x^2}\,\sqrt{c+d x^2}}\left(b\sqrt{\frac{b}{a}}\, d x\left(a+b x^2\right)\left(c+d x^2\right)\left(122 a^2 d^2+a b d\left(-89 c+66 d x^2\right)+3 b^2\left(8 c^2-6 c d x^2+5 d^2 x^4\right)\right)-8 i b c\left(-6 b^3 c^3+23 a b^2 c^2 d-33 a^2 b c d^2+22 a^3 d^3\right)\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{b}{a}}\, x\right],\frac{a d}{b c}\right]-i\left(48 b^4 c^4-208 a b^3 c^3 d+353 a^2 b^2 c^2 d^2-298 a^3 b c d^3+105 a^4 d^4\right)\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{b}{a}}\, x\right],\frac{a d}{b c}\right]\right)$$

Problem 197: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(a+b x^2\right)^{5/2}}{\sqrt{c+d x^2}}\,dx$$

Optimal (type 4, 344 leaves, 6 steps):

$$\frac{\left(8 b^2 c^2-23 a b c d+23 a^2 d^2\right) x \sqrt{a+b x^2}}{15 d^2 \sqrt{c+d x^2}}-\frac{4 b (b c-2 a d) x \sqrt{a+b x^2} \sqrt{c+d x^2}}{15 d^2}+\frac{b x \left(a+b x^2\right)^{3/2} \sqrt{c+d x^2}}{5 d}-$$

$$\left(\sqrt{c} \left(8 b^2 c^2-23 a b c d+23 a^2 d^2\right) \sqrt{a+b x^2} \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1-\frac{b c}{a d}\right]\right) \Big/ \left(15 d^{5/2} \sqrt{\frac{c \left(a+b x^2\right)}{a \left(c+d x^2\right)}} \sqrt{c+d x^2}\right)+$$

$$\left(\sqrt{c} \left(4 b^2 c^2-11 a b c d+15 a^2 d^2\right) \sqrt{a+b x^2} \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1-\frac{b c}{a d}\right]\right) \Big/ \left(15 d^{5/2} \sqrt{\frac{c \left(a+b x^2\right)}{a \left(c+d x^2\right)}} \sqrt{c+d x^2}\right)$$

Result (type 4, 260 leaves) :

$$\left(b \sqrt{\frac{b}{a}} d x \left(a+b x^2\right) \left(c+d x^2\right) \left(-4 b c+11 a d+3 b d x^2\right)-\right.$$

$$i b c \left(8 b^2 c^2-23 a b c d+23 a^2 d^2\right) \sqrt{1+\frac{b x^2}{a}} \sqrt{1+\frac{d x^2}{c}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{b}{a}} x\right], \frac{a d}{b c}\right]-$$

$$i \left(-8 b^3 c^3+27 a b^2 c^2 d-34 a^2 b c d^2+15 a^3 d^3\right) \sqrt{1+\frac{b x^2}{a}} \sqrt{1+\frac{d x^2}{c}}$$

$$\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{b}{a}} x\right], \frac{a d}{b c}\right]\right) \Big/ \left(15 \sqrt{\frac{b}{a}} d^3 \sqrt{a+b x^2} \sqrt{c+d x^2}\right)$$

Problem 198: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b x^2\right)^{3/2}}{\sqrt{c+d x^2}} \, dx$$

Optimal (type 4, 260 leaves, 5 steps) :

$$-\frac{2\left(b c-2 a d\right) x \sqrt{a+b x^2}}{3 d \sqrt{c+d x^2}}+\frac{b x \sqrt{a+b x^2} \sqrt{c+d x^2}}{3 d}+\frac{2 \sqrt{c} \left(b c-2 a d\right) \sqrt{a+b x^2} \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right],1-\frac{b c}{a d}\right]}{3 d^{3/2} \sqrt{\frac{c \left(a+b x^2\right)}{a \left(c+d x^2\right)}} \sqrt{c+d x^2}}-\frac{\sqrt{c} \left(b c-3 a d\right) \sqrt{a+b x^2} \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right],1-\frac{b c}{a d}\right]}{3 d^{3/2} \sqrt{\frac{c \left(a+b x^2\right)}{a \left(c+d x^2\right)}} \sqrt{c+d x^2}}$$

Result (type 4, 216 leaves) :

$$\left(b \sqrt{\frac{b}{a}} d x \left(a+b x^2\right) \left(c+d x^2\right)-2 \mathrm{i} b c \left(-b c+2 a d\right) \sqrt{1+\frac{b x^2}{a}} \sqrt{1+\frac{d x^2}{c}} \text{EllipticE}\left[\mathrm{i} \text{ArcSinh}\left[\sqrt{\frac{b}{a}} x\right],\frac{a d}{b c}\right]-\mathrm{i} \left(2 b^2 c^2-5 a b c d+3 a^2 d^2\right) \sqrt{1+\frac{b x^2}{a}} \sqrt{1+\frac{d x^2}{c}} \text{EllipticF}\left[\mathrm{i} \text{ArcSinh}\left[\sqrt{\frac{b}{a}} x\right],\frac{a d}{b c}\right]\right)\Big/\left(3 \sqrt{\frac{b}{a}} d^2 \sqrt{a+b x^2} \sqrt{c+d x^2}\right)$$

Problem 202: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(a+b x^2\right)^{5/2} \sqrt{c+d x^2}} \, dx$$

Optimal (type 4, 255 leaves, 4 steps) :

$$\frac{b x \sqrt{c+d x^2}}{3 a \left(b c-a d\right) \left(a+b x^2\right)^{3/2}}+\frac{2 \sqrt{b} \left(b c-2 a d\right) \sqrt{c+d x^2} \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{b} x}{\sqrt{a}}\right],1-\frac{a d}{b c}\right]}{3 a^{3/2} \left(b c-a d\right)^2 \sqrt{a+b x^2} \sqrt{\frac{a \left(c+d x^2\right)}{c \left(a+b x^2\right)}}}-\frac{\sqrt{c} \sqrt{d} \left(b c-3 a d\right) \sqrt{a+b x^2} \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right],1-\frac{b c}{a d}\right]}{3 a^2 \left(b c-a d\right)^2 \sqrt{\frac{c \left(a+b x^2\right)}{a \left(c+d x^2\right)}} \sqrt{c+d x^2}}$$

Result (type 4, 261 leaves) :

$$\left(b\sqrt{\frac{b}{a}}\;x\left(c+d x^2\right)\left(-5 a^2 d+2 b^2 c x^2+a b\left(3 c-4 d x^2\right)\right)-\right.$$
$$2\,\mathrm{i}\,b c\left(-b c+2 a d\right)\left(a+b x^2\right)\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}\;\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{b}{a}}\;x\right],\frac{a d}{b c}\right]-$$
$$\mathrm{i}\left(2 b^2 c^2-5 a b c d+3 a^2 d^2\right)\left(a+b x^2\right)\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}$$
$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{b}{a}}\;x\right],\frac{a d}{b c}\right]\right)\Big/\left(3 a^2\sqrt{\frac{b}{a}}\left(b c-a d\right)^2\left(a+b x^2\right)^{3/2}\sqrt{c+d x^2}\right)$$

Problem 203: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(a+b x^2\right)^{7/2}\sqrt{c+d x^2}}\,dx$$

Optimal (type 4, 334 leaves, 5 steps):

$$\frac{b x\sqrt{c+d x^2}}{5 a\left(b c-a d\right)\left(a+b x^2\right)^{5/2}}+\frac{4 b\left(b c-2 a d\right)x\sqrt{c+d x^2}}{15 a^2\left(b c-a d\right)^2\left(a+b x^2\right)^{3/2}}+$$
$$\left(\sqrt{b}\left(8 b^2 c^2-23 a b c d+23 a^2 d^2\right)\sqrt{c+d x^2}\;\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{b}\;x}{\sqrt{a}}\right],1-\frac{a d}{b c}\right]\right)\Big/$$
$$\left(15 a^{5/2}\left(b c-a d\right)^3\sqrt{a+b x^2}\sqrt{\frac{a\left(c+d x^2\right)}{c\left(a+b x^2\right)}}\right)-$$
$$\left(\sqrt{c}\sqrt{d}\left(4 b^2 c^2-11 a b c d+15 a^2 d^2\right)\sqrt{a+b x^2}\;\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\;x}{\sqrt{c}}\right],1-\frac{b c}{a d}\right]\right)\Big/$$
$$\left(15 a^3\left(b c-a d\right)^3\sqrt{\frac{c\left(a+b x^2\right)}{a\left(c+d x^2\right)}}\sqrt{c+d x^2}\right)$$

Result (type 4, 301 leaves):

$$\frac{1}{15\, a^3 \sqrt{\frac{b}{a}}\, (b c - a d)^3\, (a + b x^2)^{5/2} \sqrt{c + d x^2}} \left(b \sqrt{\frac{b}{a}}\, x\, (c + d x^2)\, \left(3 a^2\, (b c - a d)^2 + 4 a\, (b c - 2 a d)\, (b c - a d)\, (a + b x^2) + (8 b^2 c^2 - 23 a b c d + 23 a^2 d^2)\, (a + b x^2)^2\right) + \mathrm{i}\, (a + b x^2)^2 \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \left(b c\, (8 b^2 c^2 - 23 a b c d + 23 a^2 d^2)\, \text{EllipticE}\left[\mathrm{i}\, \text{ArcSinh}\left[\sqrt{\frac{b}{a}}\, x\right], \frac{a d}{b c}\right] + (-8 b^3 c^3 + 27 a b^2 c^2 d - 34 a^2 b c d^2 + 15 a^3 d^3)\, \text{EllipticF}\left[\mathrm{i}\, \text{ArcSinh}\left[\sqrt{\frac{b}{a}}\, x\right], \frac{a d}{b c}\right]\right)\right)$$

Problem 204: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b x^2)^{7/2}}{(c + d x^2)^{3/2}}\, dx$$

Optimal (type 4, 445 leaves, 7 steps):

$$\frac{(48 b^3 c^3 - 128 a b^2 c^2 d + 103 a^2 b c d^2 - 15 a^3 d^3)\, x \sqrt{a + b x^2}}{15 c d^3 \sqrt{c + d x^2}} - \frac{(b c - a d)\, x\, (a + b x^2)^{5/2}}{c d \sqrt{c + d x^2}} - \frac{b\, (24 b^2 c^2 - 43 a b c d + 15 a^2 d^2)\, x \sqrt{a + b x^2} \sqrt{c + d x^2}}{15 c d^3} + \frac{b\, (6 b c - 5 a d)\, x\, (a + b x^2)^{3/2} \sqrt{c + d x^2}}{5 c d^2} - \left((48 b^3 c^3 - 128 a b^2 c^2 d + 103 a^2 b c d^2 - 15 a^3 d^3) \sqrt{a + b x^2}\, \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\, x}{\sqrt{c}}\right], 1 - \frac{b c}{a d}\right]\right) \Big/ \left(15 \sqrt{c}\, d^{7/2} \sqrt{\frac{c\, (a + b x^2)}{a\, (c + d x^2)}} \sqrt{c + d x^2}\right) + \left(b \sqrt{c}\, (24 b^2 c^2 - 61 a b c d + 45 a^2 d^2) \sqrt{a + b x^2}\, \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\, x}{\sqrt{c}}\right], 1 - \frac{b c}{a d}\right]\right) \Big/ \left(15 d^{7/2} \sqrt{\frac{c\, (a + b x^2)}{a\, (c + d x^2)}} \sqrt{c + d x^2}\right)$$

Result (type 4, 318 leaves):

$$\frac{1}{15\sqrt{\frac{b}{a}}\,c\,d^4\,\sqrt{a+b\,x^2}\,\sqrt{c+d\,x^2}}\left(\sqrt{\frac{b}{a}}\,d\,x\,\left(a+b\,x^2\right)\,\left(-45\,a^2\,b\,c\,d^2+15\,a^3\,d^3+a\,b^2\,c\,d\,\left(61\,c+16\,d\,x^2\right)-3\,b^3\,c\,\left(8\,c^2+2\,c\,d\,x^2-d^2\,x^4\right)\right)+\right.$$
$$\mathrm{i}\,b\,c\,\left(-48\,b^3\,c^3+128\,a\,b^2\,c^2\,d-103\,a^2\,b\,c\,d^2+15\,a^3\,d^3\right)\,\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}$$
$$\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{b}{a}}\,x\right],\frac{a\,d}{b\,c}\right]+4\,\mathrm{i}\,b\,c\,\left(12\,b^3\,c^3-38\,a\,b^2\,c^2\,d+41\,a^2\,b\,c\,d^2-15\,a^3\,d^3\right)$$
$$\left.\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{b}{a}}\,x\right],\frac{a\,d}{b\,c}\right]\right)$$

Problem 205: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(a+b\,x^2\right)^{5/2}}{\left(c+d\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 346 leaves, 6 steps):

$$-\frac{\left(8\,b^2\,c^2-13\,a\,b\,c\,d+3\,a^2\,d^2\right)\,x\,\sqrt{a+b\,x^2}}{3\,c\,d^2\,\sqrt{c+d\,x^2}}-$$
$$\frac{\left(b\,c-a\,d\right)\,x\,\left(a+b\,x^2\right)^{3/2}}{c\,d\,\sqrt{c+d\,x^2}}+\frac{b\,\left(4\,b\,c-3\,a\,d\right)\,x\,\sqrt{a+b\,x^2}\,\sqrt{c+d\,x^2}}{3\,c\,d^2}+$$
$$\left(\left(8\,b^2\,c^2-13\,a\,b\,c\,d+3\,a^2\,d^2\right)\,\sqrt{a+b\,x^2}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],1-\frac{b\,c}{a\,d}\right]\right)\Big/$$
$$\left(3\,\sqrt{c}\,d^{5/2}\,\sqrt{\frac{c\,\left(a+b\,x^2\right)}{a\,\left(c+d\,x^2\right)}}\,\sqrt{c+d\,x^2}\right)-$$
$$\frac{2\,b\,\sqrt{c}\,\left(2\,b\,c-3\,a\,d\right)\,\sqrt{a+b\,x^2}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],1-\frac{b\,c}{a\,d}\right]}{3\,d^{5/2}\,\sqrt{\frac{c\,\left(a+b\,x^2\right)}{a\,\left(c+d\,x^2\right)}}\,\sqrt{c+d\,x^2}}$$

Result (type 4, 256 leaves):

$$\left(\sqrt{\frac{b}{a}}\, d\, x\, \left(a+b x^2\right) \left(-6 a b c d+3 a^2 d^2+b^2 c \left(4 c+d x^2\right)\right)+\right.$$
$$i\, b\, c \left(8 b^2 c^2-13 a b c d+3 a^2 d^2\right) \sqrt{1+\frac{b x^2}{a}} \sqrt{1+\frac{d x^2}{c}}\, \text{EllipticE}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{b}{a}}\, x\right],\frac{a d}{b c}\right]-$$
$$\left. i\, b\, c \left(8 b^2 c^2-17 a b c d+9 a^2 d^2\right) \sqrt{1+\frac{b x^2}{a}} \sqrt{1+\frac{d x^2}{c}}\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{b}{a}}\, x\right],\frac{a d}{b c}\right]\right) \Big/$$
$$\left(3 \sqrt{\frac{b}{a}}\, c\, d^3 \sqrt{a+b x^2} \sqrt{c+d x^2}\right)$$

Problem 206: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b x^2\right)^{3/2}}{\left(c+d x^2\right)^{3/2}}\, dx$$

Optimal (type 4, 258 leaves, 5 steps):

$$-\frac{(b c-a d)\, x \sqrt{a+b x^2}}{c\, d \sqrt{c+d x^2}}+\frac{(2 b c-a d)\, x \sqrt{a+b x^2}}{c\, d \sqrt{c+d x^2}}-$$
$$\frac{(2 b c-a d) \sqrt{a+b x^2}\, \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\, x}{\sqrt{c}}\right],1-\frac{b c}{a d}\right]}{\sqrt{c}\, d^{3/2} \sqrt{\frac{c \left(a+b x^2\right)}{a \left(c+d x^2\right)}} \sqrt{c+d x^2}}+$$
$$\frac{b \sqrt{c} \sqrt{a+b x^2}\, \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\, x}{\sqrt{c}}\right],1-\frac{b c}{a d}\right]}{d^{3/2} \sqrt{\frac{c \left(a+b x^2\right)}{a \left(c+d x^2\right)}} \sqrt{c+d x^2}}$$

Result (type 4, 196 leaves):

$$\left(i\, b\, c\, (-2 b c+a d) \sqrt{1+\frac{b x^2}{a}} \sqrt{1+\frac{d x^2}{c}}\, \text{EllipticE}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{b}{a}}\, x\right],\frac{a d}{b c}\right]+(-b c+a d)\right.$$
$$\left.\left(\sqrt{\frac{b}{a}}\, d\, x \left(a+b x^2\right)-2\, i\, b\, c \sqrt{1+\frac{b x^2}{a}} \sqrt{1+\frac{d x^2}{c}}\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{b}{a}}\, x\right],\frac{a d}{b c}\right]\right)\right) \Big/$$
$$\left(\sqrt{\frac{b}{a}}\, c\, d^2 \sqrt{a+b x^2} \sqrt{c+d x^2}\right)$$

Problem 207: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a+b x^2}}{\left(c+d x^2\right)^{3/2}} \, dx$$

Optimal (type 4, 84 leaves, 1 step):

$$\frac{\sqrt{a+b x^2} \, \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d} \, x}{\sqrt{c}}\right], 1-\frac{b c}{a d}\right]}{\sqrt{c} \, \sqrt{d} \, \sqrt{\frac{c \left(a+b x^2\right)}{a \left(c+d x^2\right)}} \, \sqrt{c+d x^2}}$$

Result (type 4, 136 leaves):

$$\left(\frac{x \left(a+b x^2\right)}{c}+\frac{1}{d} i a \sqrt{\frac{b}{a}} \sqrt{1+\frac{b x^2}{a}} \sqrt{1+\frac{d x^2}{c}} \left(\text{EllipticE}\left[i \, \text{ArcSinh}\left[\sqrt{\frac{b}{a}} \, x\right], \frac{a d}{b c}\right]-\text{EllipticF}\left[i \, \text{ArcSinh}\left[\sqrt{\frac{b}{a}} \, x\right], \frac{a d}{b c}\right]\right)\right) \Big/ \left(\sqrt{a+b x^2} \, \sqrt{c+d x^2}\right)$$

Problem 209: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(a+b x^2\right)^{3/2} \left(c+d x^2\right)^{3/2}} \, dx$$

Optimal (type 4, 242 leaves, 4 steps):

$$\frac{b x}{a \left(b c-a d\right) \sqrt{a+b x^2} \, \sqrt{c+d x^2}}+\frac{\sqrt{d} \left(b c+a d\right) \sqrt{a+b x^2} \, \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d} \, x}{\sqrt{c}}\right], 1-\frac{b c}{a d}\right]}{a \sqrt{c} \left(b c-a d\right)^2 \sqrt{\frac{c \left(a+b x^2\right)}{a \left(c+d x^2\right)}} \, \sqrt{c+d x^2}}-\frac{2 b \sqrt{c} \, \sqrt{d} \, \sqrt{a+b x^2} \, \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d} \, x}{\sqrt{c}}\right], 1-\frac{b c}{a d}\right]}{a \left(b c-a d\right)^2 \sqrt{\frac{c \left(a+b x^2\right)}{a \left(c+d x^2\right)}} \, \sqrt{c+d x^2}}$$

Result (type 4, 224 leaves):

$$\left(\sqrt{\frac{b}{a}}\left(\sqrt{\frac{b}{a}}\,x\left(a^2 d^2+a b d^2 x^2+b^2 c\left(c+d x^2\right)\right)+\right.\right.$$
$$i\,b\,c\,(b\,c+a\,d)\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{b}{a}}\,x\right],\frac{a d}{b c}\right]+$$
$$\left.\left.i\,b\,c\,(-b\,c+a\,d)\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{b}{a}}\,x\right],\frac{a d}{b c}\right]\right)\right)\Big/$$
$$\left(b\,c\,(b\,c-a\,d)^2\sqrt{a+b x^2}\sqrt{c+d x^2}\right)$$

Problem 210: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(a+b x^2\right)^{5/2}\left(c+d x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 323 leaves, 5 steps):

$$\frac{b x}{3 a (b c-a d)\left(a+b x^2\right)^{3/2}\sqrt{c+d x^2}}+\frac{2 b (b c-3 a d) x}{3 a^2 (b c-a d)^2\sqrt{a+b x^2}\sqrt{c+d x^2}}+$$
$$\left(\sqrt{d}\left(2 b^2 c^2-7 a b c d-3 a^2 d^2\right)\sqrt{a+b x^2}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],1-\frac{b c}{a d}\right]\right)\Big/$$
$$\left(3 a^2\sqrt{c}\,(b c-a d)^3\sqrt{\frac{c\left(a+b x^2\right)}{a\left(c+d x^2\right)}}\sqrt{c+d x^2}\right)-$$
$$\frac{b\sqrt{c}\sqrt{d}\,(b c-9 a d)\sqrt{a+b x^2}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],1-\frac{b c}{a d}\right]}{3 a^2 (b c-a d)^3\sqrt{\frac{c\left(a+b x^2\right)}{a\left(c+d x^2\right)}}\sqrt{c+d x^2}}$$

Result (type 4, 337 leaves):

$$\frac{1}{3 a^2 \sqrt{\frac{b}{a}}\, c\, (-b c+a d)^3 \left(a+b x^2\right)^{3/2} \sqrt{c+d x^2}}$$
$$\left(\sqrt{\frac{b}{a}}\, x \left(3 a^4 d^3+6 a^3 b d^3 x^2-2 b^4 c^2 x^2 \left(c+d x^2\right)+a^2 b^2 d \left(8 c^2+8 c d x^2+3 d^2 x^4\right)+\right.\right.$$
$$\left.a b^3 c \left(-3 c^2+4 c d x^2+7 d^2 x^4\right)\right)+\mathrm{i}\, b c \left(-2 b^2 c^2+7 a b c d+3 a^2 d^2\right) \left(a+b x^2\right) \sqrt{1+\frac{b x^2}{a}}$$
$$\sqrt{1+\frac{d x^2}{c}}\, \text{EllipticE}\left[\mathrm{i}\, \text{ArcSinh}\left[\sqrt{\frac{b}{a}}\, x\right], \frac{a d}{b c}\right]+2\, \mathrm{i}\, b c \left(b^2 c^2-4 a b c d+3 a^2 d^2\right)$$
$$\left.\left(a+b x^2\right) \sqrt{1+\frac{b x^2}{a}} \sqrt{1+\frac{d x^2}{c}}\, \text{EllipticF}\left[\mathrm{i}\, \text{ArcSinh}\left[\sqrt{\frac{b}{a}}\, x\right], \frac{a d}{b c}\right]\right)$$

Problem 216: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{1-x^2}\, \sqrt{2+4 x^2}}\, dx$$

Optimal (type 4, 10 leaves, 1 step):

$$\frac{\text{EllipticF}[\text{ArcSin}[x], -2]}{\sqrt{2}}$$

Result (type 4, 58 leaves):

$$-\frac{\mathrm{i}\, \sqrt{1-x^2}\, \sqrt{1+2 x^2}\, \text{EllipticF}\left[\mathrm{i}\, \text{ArcSinh}\left[\sqrt{2}\, x\right], -\frac{1}{2}\right]}{2 \sqrt{1+x^2-2 x^4}}$$

Problem 219: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{1-x^2}\, \sqrt{2+x^2}}\, dx$$

Optimal (type 4, 12 leaves, 1 step):

$$\frac{\text{EllipticF}\left[\text{ArcSin}[x], -\frac{1}{2}\right]}{\sqrt{2}}$$

Result (type 4, 18 leaves):

$$-\mathrm{i}\, \text{EllipticF}\left[\mathrm{i}\, \text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right], -2\right]$$

Problem 221: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{2-2x^2}\,\sqrt{1-x^2}}\,dx$$

Optimal (type 3, 8 leaves, 2 steps):

$$\frac{\text{ArcTanh}[x]}{\sqrt{2}}$$

Result (type 3, 26 leaves):

$$-\frac{\frac{1}{2}\,\text{Log}[1-x]-\frac{1}{2}\,\text{Log}[1+x]}{\sqrt{2}}$$

Problem 225: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{1+x^2}\,\sqrt{2+5x^2}}\,dx$$

Optimal (type 4, 51 leaves, 1 step):

$$\frac{\sqrt{2+5x^2}\;\text{EllipticF}\left[\text{ArcTan}[x],-\frac{3}{2}\right]}{\sqrt{2}\,\sqrt{1+x^2}\,\sqrt{\frac{2+5x^2}{1+x^2}}}$$

Result (type 4, 19 leaves):

$$-\frac{i\;\text{EllipticF}\left[i\,\text{ArcSinh}[x],\frac{5}{2}\right]}{\sqrt{2}}$$

Problem 226: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{1+x^2}\,\sqrt{2+4x^2}}\,dx$$

Optimal (type 4, 49 leaves, 1 step):

$$\frac{\sqrt{1+2x^2}\;\text{EllipticF}[\text{ArcTan}[x],-1]}{\sqrt{2}\,\sqrt{1+x^2}\,\sqrt{\frac{1+2x^2}{1+x^2}}}$$

Result (type 4, 17 leaves):

$$-\frac{i\;\text{EllipticF}[i\,\text{ArcSinh}[x],2]}{\sqrt{2}}$$

Problem 227: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{1+x^2}\,\sqrt{2+3\,x^2}}\,dx$$

Optimal (type 4, 51 leaves, 1 step):

$$\frac{\sqrt{2+3\,x^2}\,\text{EllipticF}\left[\text{ArcTan}[x],-\frac{1}{2}\right]}{\sqrt{2}\,\sqrt{1+x^2}\,\sqrt{\frac{2+3\,x^2}{1+x^2}}}$$

Result (type 4, 19 leaves):

$$-\frac{i\,\text{EllipticF}\left[i\,\text{ArcSinh}[x],\frac{3}{2}\right]}{\sqrt{2}}$$

Problem 229: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{1+x^2}\,\sqrt{2+x^2}}\,dx$$

Optimal (type 4, 47 leaves, 1 step):

$$\frac{\sqrt{2+x^2}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2}\,\sqrt{1+x^2}\,\sqrt{\frac{2+x^2}{1+x^2}}}$$

Result (type 4, 19 leaves):

$$-\frac{i\,\text{EllipticF}\left[i\,\text{ArcSinh}[x],\frac{1}{2}\right]}{\sqrt{2}}$$

Problem 230: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2-x^2}\,\sqrt{1+x^2}}\,dx$$

Optimal (type 4, 10 leaves, 1 step):

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]$$

Result (type 4, 19 leaves):

$$-\frac{i\,\text{EllipticF}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]}{\sqrt{2}}$$

Problem 240: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{2-x^2}\,\sqrt{-1+x^2}}\,dx$$

Optimal (type 4, 12 leaves, 1 step):

$$-\text{EllipticF}\left[\text{ArcCos}\left[\frac{x}{\sqrt{2}}\right],2\right]$$

Result (type 4, 47 leaves):

$$\frac{\sqrt{1-x^2}\,\sqrt{1-\frac{x^2}{2}}\,\text{EllipticF}\left[\text{ArcSin}[x],\frac{1}{2}\right]}{\sqrt{-2+3x^2-x^4}}$$

Problem 245: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-1-x^2}\,\sqrt{2+5x^2}}\,dx$$

Optimal (type 4, 53 leaves, 1 step):

$$\frac{\sqrt{2+5x^2}\,\text{EllipticF}\left[\text{ArcTan}[x],-\frac{3}{2}\right]}{\sqrt{2}\,\sqrt{-1-x^2}\,\sqrt{\frac{2+5x^2}{1+x^2}}}$$

Result (type 4, 39 leaves):

$$-\frac{i\,\sqrt{1+x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}[x],\frac{5}{2}\right]}{\sqrt{2}\,\sqrt{-1-x^2}}$$

Problem 246: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-1-x^2}\,\sqrt{2+4x^2}}\,dx$$

Optimal (type 4, 51 leaves, 1 step):

$$\frac{\sqrt{1+2x^2}\,\text{EllipticF}[\text{ArcTan}[x],-1]}{\sqrt{2}\,\sqrt{-1-x^2}\,\sqrt{\frac{1+2x^2}{1+x^2}}}$$

Result (type 4, 37 leaves):

$$-\frac{i\,\sqrt{1+x^2}\,\text{EllipticF}[i\,\text{ArcSinh}[x],2]}{\sqrt{2}\,\sqrt{-1-x^2}}$$

Problem 247: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-1-x^2}\ \sqrt{2+3x^2}}\, dx$$

Optimal (type 4, 53 leaves, 1 step):

$$\frac{\sqrt{2+3x^2}\ \text{EllipticF}\left[\text{ArcTan}[x], -\frac{1}{2}\right]}{\sqrt{2}\ \sqrt{-1-x^2}\ \sqrt{\frac{2+3x^2}{1+x^2}}}$$

Result (type 4, 39 leaves):

$$-\frac{i\sqrt{1+x^2}\ \text{EllipticF}\left[i\,\text{ArcSinh}[x], \frac{3}{2}\right]}{\sqrt{2}\ \sqrt{-1-x^2}}$$

Problem 249: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-1-x^2}\ \sqrt{2+x^2}}\, dx$$

Optimal (type 4, 49 leaves, 1 step):

$$\frac{\sqrt{2+x^2}\ \text{EllipticF}\left[\text{ArcTan}[x], \frac{1}{2}\right]}{\sqrt{2}\ \sqrt{-1-x^2}\ \sqrt{\frac{2+x^2}{1+x^2}}}$$

Result (type 4, 53 leaves):

$$-\frac{i\sqrt{1+x^2}\ \sqrt{2+x^2}\ \text{EllipticF}\left[i\,\text{ArcSinh}[x], \frac{1}{2}\right]}{\sqrt{2}\ \sqrt{-\left(1+x^2\right)\left(2+x^2\right)}}$$

Problem 250: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-1-x^2}\ \sqrt{2-x^2}}\, dx$$

Optimal (type 4, 31 leaves, 2 steps):

$$\frac{\sqrt{1+x^2}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right], -2\right]}{\sqrt{-1-x^2}}$$

Result (type 4, 39 leaves):

$$-\frac{i\sqrt{1+x^2}\ \text{EllipticF}\left[i\,\text{ArcSinh}[x], -\frac{1}{2}\right]}{\sqrt{2}\ \sqrt{-1-x^2}}$$

Problem 289: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{4+x^2}\ \sqrt{c+d\,x^2}}\,dx$$

Optimal (type 4, 61 leaves, 1 step):

$$\frac{\sqrt{c+d\,x^2}\ \text{EllipticF}\left[\text{ArcTan}\left[\frac{x}{2}\right],1-\frac{4\,d}{c}\right]}{c\,\sqrt{4+x^2}\ \sqrt{\frac{c+d\,x^2}{c\,\left(4+x^2\right)}}}$$

Result (type 4, 47 leaves):

$$-\frac{i\,\sqrt{\frac{c+d\,x^2}{c}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{2}\right],\frac{4\,d}{c}\right]}{\sqrt{c+d\,x^2}}$$

Problem 290: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{1-x^2}\ \sqrt{-1+2\,x^2}}\,dx$$

Optimal (type 4, 6 leaves, 1 step):

$$-\text{EllipticF}[\text{ArcCos}[x],2]$$

Result (type 4, 27 leaves):

$$\frac{\sqrt{1-2\,x^2}\ \text{EllipticF}[\text{ArcSin}[x],2]}{\sqrt{-1+2\,x^2}}$$

Problem 298: Result unnecessarily involves higher level functions.

$$\int \frac{\left(1-2\,x^2\right)^m}{\sqrt{1-x^2}}\,dx$$

Optimal (type 5, 62 leaves, ? steps):

$$-\frac{2^{-2-m}\,\sqrt{x^2}\ \left(2-4\,x^2\right)^{1+m}\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\left(1-2\,x^2\right)^2\right]}{(1+m)\,x}$$

Result (type 6, 122 leaves):

$$\left(3\,x\,\left(1-2\,x^2\right)^m\,\text{AppellF1}\left[\frac{1}{2},-m,\frac{1}{2},\frac{3}{2},2\,x^2,x^2\right]\right)\Big/$$
$$\left(\sqrt{1-x^2}\ \left(3\,\text{AppellF1}\left[\frac{1}{2},-m,\frac{1}{2},\frac{3}{2},2\,x^2,x^2\right]+\right.\right.$$
$$\left.\left.x^2\,\left(-4\,m\,\text{AppellF1}\left[\frac{3}{2},1-m,\frac{1}{2},\frac{5}{2},2\,x^2,x^2\right]+\text{AppellF1}\left[\frac{3}{2},-m,\frac{3}{2},\frac{5}{2},2\,x^2,x^2\right]\right)\right)\right)$$

Problem 301: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(2+3 x^2\right)^{1/4} \left(4+3 x^2\right)} \, dx$$

Optimal (type 3, 129 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{2\times 2^{3/4}+2\times 2^{1/4} \sqrt{2+3 x^2}}{2 \sqrt{3}\, x \left(2+3 x^2\right)^{1/4}}\right]}{2\times 2^{3/4} \sqrt{3}}-\frac{\text{ArcTanh}\left[\frac{2\times 2^{3/4}-2\times 2^{1/4} \sqrt{2+3 x^2}}{2 \sqrt{3}\, x \left(2+3 x^2\right)^{1/4}}\right]}{2\times 2^{3/4} \sqrt{3}}$$

Result (type 6, 135 leaves):

$$-\left(\left(4 x \,\text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-\frac{3 x^2}{2},-\frac{3 x^2}{4}\right]\right)\Big/\left(\left(2+3 x^2\right)^{1/4} \left(4+3 x^2\right) \left(-4 \,\text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-\frac{3 x^2}{2},-\frac{3 x^2}{4}\right]+x^2 \left(2 \,\text{AppellF1}\left[\frac{3}{2},\frac{1}{4},2,\frac{5}{2},-\frac{3 x^2}{2},-\frac{3 x^2}{4}\right]+\text{AppellF1}\left[\frac{3}{2},\frac{5}{4},1,\frac{5}{2},-\frac{3 x^2}{2},-\frac{3 x^2}{4}\right]\right)\right)\right)\right)$$

Problem 302: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(2-3 x^2\right)^{1/4} \left(4-3 x^2\right)} \, dx$$

Optimal (type 3, 120 leaves, 1 step):

$$\frac{\text{ArcTan}\left[\frac{2-\sqrt{2} \sqrt{2-3 x^2}}{2^{1/4} \sqrt{3}\, x \left(2-3 x^2\right)^{1/4}}\right]}{2\times 2^{3/4} \sqrt{3}}+\frac{\text{ArcTanh}\left[\frac{2+\sqrt{2} \sqrt{2-3 x^2}}{2^{1/4} \sqrt{3}\, x \left(2-3 x^2\right)^{1/4}}\right]}{2\times 2^{3/4} \sqrt{3}}$$

Result (type 6, 135 leaves):

$$-\left(\left(4 x \,\text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},\frac{3 x^2}{2},\frac{3 x^2}{4}\right]\right)\Big/\left(\left(2-3 x^2\right)^{1/4} \left(-4+3 x^2\right) \left(4 \,\text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},\frac{3 x^2}{2},\frac{3 x^2}{4}\right]+x^2 \left(2 \,\text{AppellF1}\left[\frac{3}{2},\frac{1}{4},2,\frac{5}{2},\frac{3 x^2}{2},\frac{3 x^2}{4}\right]+\text{AppellF1}\left[\frac{3}{2},\frac{5}{4},1,\frac{5}{2},\frac{3 x^2}{2},\frac{3 x^2}{4}\right]\right)\right)\right)\right)$$

Problem 303: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(2+b x^2\right)^{1/4} \left(4+b x^2\right)} \, dx$$

Optimal (type 3, 129 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{2\times 2^{3/4}+2\times 2^{1/4}\sqrt{2+b x^2}}{2\sqrt{b}\, x\,(2+b x^2)^{1/4}}\right]}{2\times 2^{3/4}\sqrt{b}}-\frac{\text{ArcTanh}\left[\frac{2\times 2^{3/4}-2\times 2^{1/4}\sqrt{2+b x^2}}{2\sqrt{b}\, x\,(2+b x^2)^{1/4}}\right]}{2\times 2^{3/4}\sqrt{b}}$$

Result (type 6, 144 leaves):

$$-\left(\left(12\, x\, \text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-\frac{b x^2}{2},-\frac{b x^2}{4}\right]\right)\Big/\left(\left(2+b x^2\right)^{1/4}\left(4+b x^2\right)\left(-12\, \text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-\frac{b x^2}{2},-\frac{b x^2}{4}\right]+b x^2\left(2\, \text{AppellF1}\left[\frac{3}{2},\frac{1}{4},2,\frac{5}{2},-\frac{b x^2}{2},-\frac{b x^2}{4}\right]+\text{AppellF1}\left[\frac{3}{2},\frac{5}{4},1,\frac{5}{2},-\frac{b x^2}{2},-\frac{b x^2}{4}\right]\right)\right)\right)\right)$$

Problem 304: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(2-b x^2\right)^{1/4}\left(4-b x^2\right)}\,dx$$

Optimal (type 3, 124 leaves, 1 step):

$$\frac{\text{ArcTan}\left[\frac{2-\sqrt{2}\sqrt{2-b x^2}}{2^{1/4}\sqrt{b}\, x\,(2-b x^2)^{1/4}}\right]}{2\times 2^{3/4}\sqrt{b}}+\frac{\text{ArcTanh}\left[\frac{2+\sqrt{2}\sqrt{2-b x^2}}{2^{1/4}\sqrt{b}\, x\,(2-b x^2)^{1/4}}\right]}{2\times 2^{3/4}\sqrt{b}}$$

Result (type 6, 145 leaves):

$$-\left(\left(12\, x\, \text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},\frac{b x^2}{2},\frac{b x^2}{4}\right]\right)\Big/\left(\left(2-b x^2\right)^{1/4}\left(-4+b x^2\right)\left(12\, \text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},\frac{b x^2}{2},\frac{b x^2}{4}\right]+b x^2\left(2\, \text{AppellF1}\left[\frac{3}{2},\frac{1}{4},2,\frac{5}{2},\frac{b x^2}{2},\frac{b x^2}{4}\right]+\text{AppellF1}\left[\frac{3}{2},\frac{5}{4},1,\frac{5}{2},\frac{b x^2}{2},\frac{b x^2}{4}\right]\right)\right)\right)\right)$$

Problem 305: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(a+3 x^2\right)^{1/4}\left(2 a+3 x^2\right)}\,dx$$

Optimal (type 3, 120 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{a^{3/4}\left(1+\frac{\sqrt{a+3 x^2}}{\sqrt{a}}\right)}{\sqrt{3}\, x\,(a+3 x^2)^{1/4}}\right]}{2\sqrt{3}\, a^{3/4}}-\frac{\text{ArcTanh}\left[\frac{a^{3/4}\left(1-\frac{\sqrt{a+3 x^2}}{\sqrt{a}}\right)}{\sqrt{3}\, x\,(a+3 x^2)^{1/4}}\right]}{2\sqrt{3}\, a^{3/4}}$$

Result (type 6, 155 leaves):

$$-\left(\left(2\,a\,x\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-\frac{3x^2}{a},-\frac{3x^2}{2a}\right]\right)\Big/\left(\left(a+3x^2\right)^{1/4}\left(2a+3x^2\right)\left(-2\,a\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-\frac{3x^2}{a},-\frac{3x^2}{2a}\right]+x^2\left(2\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{4},2,\frac{5}{2},-\frac{3x^2}{a},-\frac{3x^2}{2a}\right]+\text{AppellF1}\left[\frac{3}{2},\frac{5}{4},1,\frac{5}{2},-\frac{3x^2}{a},-\frac{3x^2}{2a}\right]\right)\right)\right)\right)$$

Problem 306: Result unnecessarily involves higher level functions.

$$\int\frac{1}{\left(a-3x^2\right)^{1/4}\left(2a-3x^2\right)}\,dx$$

Optimal (type 3, 120 leaves, 1 step):

$$\frac{\text{ArcTan}\left[\frac{a^{3/4}\left(1-\frac{\sqrt{a-3x^2}}{\sqrt{a}}\right)}{\sqrt{3}\,x\left(a-3x^2\right)^{1/4}}\right]}{2\sqrt{3}\,a^{3/4}}+\frac{\text{ArcTanh}\left[\frac{a^{3/4}\left(1+\frac{\sqrt{a-3x^2}}{\sqrt{a}}\right)}{\sqrt{3}\,x\left(a-3x^2\right)^{1/4}}\right]}{2\sqrt{3}\,a^{3/4}}$$

Result (type 6, 155 leaves):

$$-\left(\left(2\,a\,x\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},\frac{3x^2}{a},\frac{3x^2}{2a}\right]\right)\Big/\left(\left(a-3x^2\right)^{1/4}\left(-2a+3x^2\right)\left(2\,a\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},\frac{3x^2}{a},\frac{3x^2}{2a}\right]+x^2\left(2\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{4},2,\frac{5}{2},\frac{3x^2}{a},\frac{3x^2}{2a}\right]+\text{AppellF1}\left[\frac{3}{2},\frac{5}{4},1,\frac{5}{2},\frac{3x^2}{a},\frac{3x^2}{2a}\right]\right)\right)\right)\right)$$

Problem 307: Result unnecessarily involves higher level functions.

$$\int\frac{1}{\left(a+b\,x^2\right)^{1/4}\left(2a+b\,x^2\right)}\,dx$$

Optimal (type 3, 120 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{a^{3/4}\left(1+\frac{\sqrt{a+b\,x^2}}{\sqrt{a}}\right)}{\sqrt{b}\,x\left(a+b\,x^2\right)^{1/4}}\right]}{2\,a^{3/4}\sqrt{b}}-\frac{\text{ArcTanh}\left[\frac{a^{3/4}\left(1-\frac{\sqrt{a+b\,x^2}}{\sqrt{a}}\right)}{\sqrt{b}\,x\left(a+b\,x^2\right)^{1/4}}\right]}{2\,a^{3/4}\sqrt{b}}$$

Result (type 6, 165 leaves):

$$\left(6\,a\,x\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-\frac{b\,x^2}{a},-\frac{b\,x^2}{2a}\right]\right)\Big/\left(\left(a+b\,x^2\right)^{1/4}\left(2a+b\,x^2\right)\left(6\,a\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-\frac{b\,x^2}{a},-\frac{b\,x^2}{2a}\right]-b\,x^2\left(2\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{4},2,\frac{5}{2},-\frac{b\,x^2}{a},-\frac{b\,x^2}{2a}\right]+\text{AppellF1}\left[\frac{3}{2},\frac{5}{4},1,\frac{5}{2},-\frac{b\,x^2}{a},-\frac{b\,x^2}{2a}\right]\right)\right)\right)$$

Problem 308: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(a-b x^2\right)^{1/4} \left(2 a-b x^2\right)} \, dx$$

Optimal (type 3, 124 leaves, 1 step):

$$\frac{\text{ArcTan}\left[\frac{a^{3/4} \left(1-\frac{\sqrt{a-b x^2}}{\sqrt{a}}\right)}{\sqrt{b}\, x \left(a-b x^2\right)^{1/4}}\right]}{2 a^{3/4} \sqrt{b}}+\frac{\text{ArcTanh}\left[\frac{a^{3/4} \left(1+\frac{\sqrt{a-b x^2}}{\sqrt{a}}\right)}{\sqrt{b}\, x \left(a-b x^2\right)^{1/4}}\right]}{2 a^{3/4} \sqrt{b}}$$

Result (type 6, 162 leaves):

$$\left(6 a x \,\text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},\frac{b x^2}{a},\frac{b x^2}{2 a}\right]\right) \Big/ \left(\left(a-b x^2\right)^{1/4} \left(2 a-b x^2\right) \left(6 a \,\text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},\frac{b x^2}{a},\frac{b x^2}{2 a}\right]+ b x^2 \left(2 \,\text{AppellF1}\left[\frac{3}{2},\frac{1}{4},2,\frac{5}{2},\frac{b x^2}{a},\frac{b x^2}{2 a}\right]+\text{AppellF1}\left[\frac{3}{2},\frac{5}{4},1,\frac{5}{2},\frac{b x^2}{a},\frac{b x^2}{2 a}\right]\right)\right)\right)$$

Problem 309: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(-2+3 x^2\right) \left(-1+3 x^2\right)^{1/4}} \, dx$$

Optimal (type 3, 61 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{\frac{3}{2}}\, x}{\left(-1+3 x^2\right)^{1/4}}\right]}{2 \sqrt{6}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{\frac{3}{2}}\, x}{\left(-1+3 x^2\right)^{1/4}}\right]}{2 \sqrt{6}}$$

Result (type 6, 127 leaves):

$$\left(2 x \,\text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},3 x^2,\frac{3 x^2}{2}\right]\right) \Big/ \left(\left(-2+3 x^2\right) \left(-1+3 x^2\right)^{1/4} \left(2 \,\text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},3 x^2,\frac{3 x^2}{2}\right]+ x^2 \left(2 \,\text{AppellF1}\left[\frac{3}{2},\frac{1}{4},2,\frac{5}{2},3 x^2,\frac{3 x^2}{2}\right]+\text{AppellF1}\left[\frac{3}{2},\frac{5}{4},1,\frac{5}{2},3 x^2,\frac{3 x^2}{2}\right]\right)\right)\right)$$

Problem 310: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(-2-3 x^2\right) \left(-1-3 x^2\right)^{1/4}} \, dx$$

Optimal (type 3, 61 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{\frac{3}{2}}\,x}{\left(-1-3x^2\right)^{1/4}}\right]}{2\sqrt{6}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{\frac{3}{2}}\,x}{\left(-1-3x^2\right)^{1/4}}\right]}{2\sqrt{6}}$$

Result (type 6, 127 leaves):

$$\left(2\,x\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-3x^2,-\frac{3x^2}{2}\right]\right)\Big/\left(\left(-1-3x^2\right)^{1/4}\left(2+3x^2\right)\left(-2\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-3x^2,-\frac{3x^2}{2}\right]+x^2\left(2\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{4},2,\frac{5}{2},-3x^2,-\frac{3x^2}{2}\right]+\text{AppellF1}\left[\frac{3}{2},\frac{5}{4},1,\frac{5}{2},-3x^2,-\frac{3x^2}{2}\right]\right)\right)\right)$$

Problem 311: Result unnecessarily involves higher level functions.

$$\int\frac{1}{\left(-2+b x^2\right)\left(-1+b x^2\right)^{1/4}}\,dx$$

Optimal (type 3, 77 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{b}\,x}{\sqrt{2}\left(-1+b x^2\right)^{1/4}}\right]}{2\sqrt{2}\sqrt{b}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{b}\,x}{\sqrt{2}\left(-1+b x^2\right)^{1/4}}\right]}{2\sqrt{2}\sqrt{b}}$$

Result (type 6, 132 leaves):

$$\left(6\,x\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},b x^2,\frac{b x^2}{2}\right]\right)\Big/\left(\left(-2+b x^2\right)\left(-1+b x^2\right)^{1/4}\left(6\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},b x^2,\frac{b x^2}{2}\right]+b x^2\left(2\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{4},2,\frac{5}{2},b x^2,\frac{b x^2}{2}\right]+\text{AppellF1}\left[\frac{3}{2},\frac{5}{4},1,\frac{5}{2},b x^2,\frac{b x^2}{2}\right]\right)\right)\right)$$

Problem 312: Result unnecessarily involves higher level functions.

$$\int\frac{1}{\left(-2-b x^2\right)\left(-1-b x^2\right)^{1/4}}\,dx$$

Optimal (type 3, 79 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{b}\,x}{\sqrt{2}\left(-1-b x^2\right)^{1/4}}\right]}{2\sqrt{2}\sqrt{b}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{b}\,x}{\sqrt{2}\left(-1-b x^2\right)^{1/4}}\right]}{2\sqrt{2}\sqrt{b}}$$

Result (type 6, 137 leaves):

$$\left(6\,x\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-b\,x^2,-\frac{b\,x^2}{2}\right]\right)\Big/\left(\left(-1-b\,x^2\right)^{1/4}\left(2+b\,x^2\right)\left(-6\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-b\,x^2,-\frac{b\,x^2}{2}\right]+b\,x^2\left(2\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{4},2,\frac{5}{2},-b\,x^2,-\frac{b\,x^2}{2}\right]+\text{AppellF1}\left[\frac{3}{2},\frac{5}{4},1,\frac{5}{2},-b\,x^2,-\frac{b\,x^2}{2}\right]\right)\right)\right)$$

Problem 313: Result unnecessarily involves higher level functions.

$$\int\frac{1}{\left(-2\,a+3\,x^2\right)\left(-a+3\,x^2\right)^{1/4}}\,dx$$

Optimal (type 3, 85 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{\frac{3}{2}}\,x}{a^{1/4}\left(-a+3\,x^2\right)^{1/4}}\right]}{2\sqrt{6}\,a^{3/4}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{\frac{3}{2}}\,x}{a^{1/4}\left(-a+3\,x^2\right)^{1/4}}\right]}{2\sqrt{6}\,a^{3/4}}$$

Result (type 6, 157 leaves):

$$\left(2\,a\,x\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},\frac{3\,x^2}{a},\frac{3\,x^2}{2\,a}\right]\right)\Big/\left(\left(-2\,a+3\,x^2\right)\left(-a+3\,x^2\right)^{1/4}\left(2\,a\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},\frac{3\,x^2}{a},\frac{3\,x^2}{2\,a}\right]+x^2\left(2\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{4},2,\frac{5}{2},\frac{3\,x^2}{a},\frac{3\,x^2}{2\,a}\right]+\text{AppellF1}\left[\frac{3}{2},\frac{5}{4},1,\frac{5}{2},\frac{3\,x^2}{a},\frac{3\,x^2}{2\,a}\right]\right)\right)\right)$$

Problem 314: Result unnecessarily involves higher level functions.

$$\int\frac{1}{\left(-2\,a-3\,x^2\right)\left(-a-3\,x^2\right)^{1/4}}\,dx$$

Optimal (type 3, 85 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{\frac{3}{2}}\,x}{a^{1/4}\left(-a-3\,x^2\right)^{1/4}}\right]}{2\sqrt{6}\,a^{3/4}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{\frac{3}{2}}\,x}{a^{1/4}\left(-a-3\,x^2\right)^{1/4}}\right]}{2\sqrt{6}\,a^{3/4}}$$

Result (type 6, 157 leaves):

$$\left(2\,a\,x\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-\frac{3\,x^2}{a},-\frac{3\,x^2}{2\,a}\right]\right)\Big/\left(\left(-a-3\,x^2\right)^{1/4}\left(2\,a+3\,x^2\right)\left(-2\,a\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-\frac{3\,x^2}{a},-\frac{3\,x^2}{2\,a}\right]+x^2\left(2\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{4},2,\frac{5}{2},-\frac{3\,x^2}{a},-\frac{3\,x^2}{2\,a}\right]+\text{AppellF1}\left[\frac{3}{2},\frac{5}{4},1,\frac{5}{2},-\frac{3\,x^2}{a},-\frac{3\,x^2}{2\,a}\right]\right)\right)\right)$$

Problem 315: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(-2 a + b x^2\right) \left(-a + b x^2\right)^{1/4}} \, dx$$

Optimal (type 3, 101 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{b}\, x}{\sqrt{2}\, a^{1/4} \left(-a+b x^2\right)^{1/4}}\right]}{2 \sqrt{2}\, a^{3/4} \sqrt{b}} - \frac{\text{ArcTanh}\left[\frac{\sqrt{b}\, x}{\sqrt{2}\, a^{1/4} \left(-a+b x^2\right)^{1/4}}\right]}{2 \sqrt{2}\, a^{3/4} \sqrt{b}}$$

Result (type 6, 163 leaves):

$$-\left(\left(6 a x \,\text{AppellF1}\left[\frac{1}{2}, \frac{1}{4}, 1, \frac{3}{2}, \frac{b x^2}{a}, \frac{b x^2}{2 a}\right]\right) \Big/ \left(\left(2 a - b x^2\right) \left(-a + b x^2\right)^{1/4} \left(6 a \,\text{AppellF1}\left[\frac{1}{2}, \frac{1}{4}, 1, \frac{3}{2}, \frac{b x^2}{a}, \frac{b x^2}{2 a}\right] + b x^2 \left(2 \,\text{AppellF1}\left[\frac{3}{2}, \frac{1}{4}, 2, \frac{5}{2}, \frac{b x^2}{a}, \frac{b x^2}{2 a}\right] + \text{AppellF1}\left[\frac{3}{2}, \frac{5}{4}, 1, \frac{5}{2}, \frac{b x^2}{a}, \frac{b x^2}{2 a}\right]\right)\right)\right)\right)$$

Problem 316: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(-2 a - b x^2\right) \left(-a - b x^2\right)^{1/4}} \, dx$$

Optimal (type 3, 103 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{b}\, x}{\sqrt{2}\, a^{1/4} \left(-a-b x^2\right)^{1/4}}\right]}{2 \sqrt{2}\, a^{3/4} \sqrt{b}} - \frac{\text{ArcTanh}\left[\frac{\sqrt{b}\, x}{\sqrt{2}\, a^{1/4} \left(-a-b x^2\right)^{1/4}}\right]}{2 \sqrt{2}\, a^{3/4} \sqrt{b}}$$

Result (type 6, 168 leaves):

$$-\left(\left(6 a x \,\text{AppellF1}\left[\frac{1}{2}, \frac{1}{4}, 1, \frac{3}{2}, -\frac{b x^2}{a}, -\frac{b x^2}{2 a}\right]\right) \Big/ \left(\left(-a - b x^2\right)^{1/4} \left(2 a + b x^2\right) \left(6 a \,\text{AppellF1}\left[\frac{1}{2}, \frac{1}{4}, 1, \frac{3}{2}, -\frac{b x^2}{a}, -\frac{b x^2}{2 a}\right] - b x^2 \left(2 \,\text{AppellF1}\left[\frac{3}{2}, \frac{1}{4}, 2, \frac{5}{2}, -\frac{b x^2}{a}, -\frac{b x^2}{2 a}\right] + \text{AppellF1}\left[\frac{3}{2}, \frac{5}{4}, 1, \frac{5}{2}, -\frac{b x^2}{a}, -\frac{b x^2}{2 a}\right]\right)\right)\right)\right)$$

Problem 317: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(2 - x^2\right) \left(-1 + x^2\right)^{1/4}} \, dx$$

Optimal (type 3, 53 leaves, 1 step):

$$\frac{\text{ArcTan}\left[\frac{x}{\sqrt{2}\,\left(-1+x^2\right)^{1/4}}\right]}{2\sqrt{2}}+\frac{\text{ArcTanh}\left[\frac{x}{\sqrt{2}\,\left(-1+x^2\right)^{1/4}}\right]}{2\sqrt{2}}$$

Result (type 6, 115 leaves):

$$-\left(\left(6\,x\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},x^2,\frac{x^2}{2}\right]\right)\Big/\left(\left(-2+x^2\right)\left(-1+x^2\right)^{1/4}\left(6\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},x^2,\frac{x^2}{2}\right]+x^2\left(2\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{4},2,\frac{5}{2},x^2,\frac{x^2}{2}\right]+\text{AppellF1}\left[\frac{3}{2},\frac{5}{4},1,\frac{5}{2},x^2,\frac{x^2}{2}\right]\right)\right)\right)\right)$$

Problem 318: Result unnecessarily involves higher level functions.

$$\int\frac{\left(a+b\,x^2\right)^{7/4}}{c+d\,x^2}\,dx$$

Optimal (type 4, 362 leaves, 13 steps):

$$\frac{6\,a\,b\,x}{5\,d\,\left(a+b\,x^2\right)^{1/4}}-\frac{2\,b\,\left(b\,c-a\,d\right)\,x}{d^2\,\left(a+b\,x^2\right)^{1/4}}+\frac{2\,b\,x\,\left(a+b\,x^2\right)^{3/4}}{5\,d}-\frac{6\,a^{3/2}\,\sqrt{b}\,\left(1+\frac{b\,x^2}{a}\right)^{1/4}\,\text{EllipticE}\left[\frac{1}{2}\,\text{ArcTan}\left[\frac{\sqrt{b}\,x}{\sqrt{a}}\right],2\right]}{5\,d\,\left(a+b\,x^2\right)^{1/4}}+\frac{2\,\sqrt{a}\,\sqrt{b}\,\left(b\,c-a\,d\right)\,\left(1+\frac{b\,x^2}{a}\right)^{1/4}\,\text{EllipticE}\left[\frac{1}{2}\,\text{ArcTan}\left[\frac{\sqrt{b}\,x}{\sqrt{a}}\right],2\right]}{d^2\,\left(a+b\,x^2\right)^{1/4}}+\frac{1}{d^{5/2}\,x}a^{1/4}\,\left(-b\,c+a\,d\right)^{3/2}\,\sqrt{-\frac{b\,x^2}{a}}\,\text{EllipticPi}\left[-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{-b\,c+a\,d}},\text{ArcSin}\left[\frac{\left(a+b\,x^2\right)^{1/4}}{a^{1/4}}\right],-1\right]-\frac{1}{d^{5/2}\,x}a^{1/4}\,\left(-b\,c+a\,d\right)^{3/2}\,\sqrt{-\frac{b\,x^2}{a}}\,\text{EllipticPi}\left[\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{-b\,c+a\,d}},\text{ArcSin}\left[\frac{\left(a+b\,x^2\right)^{1/4}}{a^{1/4}}\right],-1\right]$$

Result (type 6, 431 leaves):

$$\left(2 x \left(-\left(\left(9 a^2 c (-2 b c+5 a d) \text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]\right)\Big/\left(-6 a c \text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]+x^2 \left(4 a d \text{AppellF1}\left[\frac{3}{2},\frac{1}{4},2,\frac{5}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]+b c \text{AppellF1}\left[\frac{3}{2},\frac{5}{4},1,\frac{5}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]\right)\right)\right)+\left(b \left(-5 a c \left(6 a c+b c x^2+14 a d x^2+6 b d x^4\right) \text{AppellF1}\left[\frac{3}{2},\frac{1}{4},1,\frac{5}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]+3 x^2 \left(a+b x^2\right) \left(c+d x^2\right) \left(4 a d \text{AppellF1}\left[\frac{5}{2},\frac{1}{4},2,\frac{7}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]+b c \text{AppellF1}\left[\frac{5}{2},\frac{5}{4},1,\frac{7}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]\right)\right)\right)\Big/\left(-10 a c \text{AppellF1}\left[\frac{3}{2},\frac{1}{4},1,\frac{5}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]+x^2 \left(4 a d \text{AppellF1}\left[\frac{5}{2},\frac{1}{4},2,\frac{7}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]+b c \text{AppellF1}\left[\frac{5}{2},\frac{5}{4},1,\frac{7}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]\right)\right)\right)\right)\Big/\left(15 d \left(a+b x^2\right)^{1/4} \left(c+d x^2\right)\right)$$

Problem 319: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^2\right)^{5/4}}{c+d x^2} \, dx$$

Optimal (type 4, 302 leaves, 12 steps):

$$\frac{2 b x \left(a+b x^2\right)^{1/4}}{3 d}+\frac{2 a^{3/2} \sqrt{b} \left(1+\frac{b x^2}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x}{\sqrt{a}}\right],2\right]}{3 d \left(a+b x^2\right)^{3/4}}-\frac{2 \sqrt{a} \sqrt{b} (b c-a d) \left(1+\frac{b x^2}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x}{\sqrt{a}}\right],2\right]}{d^2 \left(a+b x^2\right)^{3/4}}+\frac{1}{d^2 x}a^{1/4} (b c-a d) \sqrt{-\frac{b x^2}{a}} \text{EllipticPi}\left[-\frac{\sqrt{a} \sqrt{d}}{\sqrt{-b c+a d}},\text{ArcSin}\left[\frac{\left(a+b x^2\right)^{1/4}}{a^{1/4}}\right],-1\right]+\frac{1}{d^2 x}a^{1/4} (b c-a d) \sqrt{-\frac{b x^2}{a}} \text{EllipticPi}\left[\frac{\sqrt{a} \sqrt{d}}{\sqrt{-b c+a d}},\text{ArcSin}\left[\frac{\left(a+b x^2\right)^{1/4}}{a^{1/4}}\right],-1\right]$$

Result (type 6, 435 leaves):

$$\left(2 x \left(-\left(\left(9 a^2 c (-2 b c+3 a d) \text{AppellF1}\left[\frac{1}{2},\frac{3}{4},1,\frac{3}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]\right)\Big/\right.\right.\right.$$
$$\left(-6 a c \text{AppellF1}\left[\frac{1}{2},\frac{3}{4},1,\frac{3}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]+x^2 \left(4 a d \text{AppellF1}\left[\frac{3}{2},\frac{3}{4},2,\right.\right.\right.$$
$$\left.\left.\left.\left.\frac{5}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]+3 b c \text{AppellF1}\left[\frac{3}{2},\frac{7}{4},1,\frac{5}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]\right)\right)\right)+$$
$$\left(b \left(-5 a c \left(6 a c+3 b c x^2+10 a d x^2+6 b d x^4\right) \text{AppellF1}\left[\frac{3}{2},\frac{3}{4},1,\frac{5}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]+\right.\right.$$
$$3 x^2 \left(a+b x^2\right) \left(c+d x^2\right) \left(4 a d \text{AppellF1}\left[\frac{5}{2},\frac{3}{4},2,\frac{7}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]+\right.$$
$$\left.\left.\left.3 b c \text{AppellF1}\left[\frac{5}{2},\frac{7}{4},1,\frac{7}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]\right)\right)\right)\Big/\left(-10 a c\right.$$
$$\text{AppellF1}\left[\frac{3}{2},\frac{3}{4},1,\frac{5}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]+x^2 \left(4 a d \text{AppellF1}\left[\frac{5}{2},\frac{3}{4},2,\frac{7}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]+\right.$$
$$\left.\left.\left.\left.3 b c \text{AppellF1}\left[\frac{5}{2},\frac{7}{4},1,\frac{7}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]\right)\right)\right)\right)\Big/\left(9 d \left(a+b x^2\right)^{3/4} \left(c+d x^2\right)\right)$$

Problem 320: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^2\right)^{3/4}}{c+d x^2} \, dx$$

Optimal (type 4, 244 leaves, 8 steps):

$$\frac{2 b x}{d \left(a+b x^2\right)^{1/4}}-\frac{2 \sqrt{a} \sqrt{b} \left(1+\frac{b x^2}{a}\right)^{1/4} \text{EllipticE}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x}{\sqrt{a}}\right],2\right]}{d \left(a+b x^2\right)^{1/4}}+\frac{1}{d^{3/2} x}$$
$$a^{1/4} \sqrt{-b c+a d} \sqrt{-\frac{b x^2}{a}} \text{EllipticPi}\left[-\frac{\sqrt{a} \sqrt{d}}{\sqrt{-b c+a d}},\text{ArcSin}\left[\frac{\left(a+b x^2\right)^{1/4}}{a^{1/4}}\right],-1\right]-$$
$$\frac{1}{d^{3/2} x} a^{1/4} \sqrt{-b c+a d} \sqrt{-\frac{b x^2}{a}} \text{EllipticPi}\left[\frac{\sqrt{a} \sqrt{d}}{\sqrt{-b c+a d}},\text{ArcSin}\left[\frac{\left(a+b x^2\right)^{1/4}}{a^{1/4}}\right],-1\right]$$

Result (type 6, 161 leaves):

$$\left(6 a c x \left(a+b x^2\right)^{3/4} \text{AppellF1}\left[\frac{1}{2},-\frac{3}{4},1,\frac{3}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]\right)\Big/$$
$$\left(\left(c+d x^2\right) \left(6 a c \text{AppellF1}\left[\frac{1}{2},-\frac{3}{4},1,\frac{3}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]+x^2 \left(-4 a d\right.\right.\right.$$
$$\left.\left.\left.\text{AppellF1}\left[\frac{3}{2},-\frac{3}{4},2,\frac{5}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]+3 b c \text{AppellF1}\left[\frac{3}{2},\frac{1}{4},1,\frac{5}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]\right)\right)\right)$$

Problem 321: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^2\right)^{1/4}}{c+d x^2} \, dx$$

Optimal (type 4, 199 leaves, 8 steps):

$$\frac{2 \sqrt{a} \sqrt{b} \left(1+\frac{b x^2}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x}{\sqrt{a}}\right], 2\right]}{d \left(a+b x^2\right)^{3/4}} - \frac{a^{1/4} \sqrt{-\frac{b x^2}{a}} \text{EllipticPi}\left[-\frac{\sqrt{a} \sqrt{d}}{\sqrt{-b c+a d}}, \text{ArcSin}\left[\frac{\left(a+b x^2\right)^{1/4}}{a^{1/4}}\right], -1\right]}{d x} - \frac{a^{1/4} \sqrt{-\frac{b x^2}{a}} \text{EllipticPi}\left[\frac{\sqrt{a} \sqrt{d}}{\sqrt{-b c+a d}}, \text{ArcSin}\left[\frac{\left(a+b x^2\right)^{1/4}}{a^{1/4}}\right], -1\right]}{d x}$$

Result (type 6, 160 leaves):

$$\left(6 a c x \left(a+b x^2\right)^{1/4} \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{4}, 1, \frac{3}{2}, -\frac{b x^2}{a}, -\frac{d x^2}{c}\right]\right) \Big/ \left(\left(c+d x^2\right) \left(6 a c \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{4}, 1, \frac{3}{2}, -\frac{b x^2}{a}, -\frac{d x^2}{c}\right] + x^2 \left(-4 a d \text{AppellF1}\left[\frac{3}{2}, -\frac{1}{4}, 2, \frac{5}{2}, -\frac{b x^2}{a}, -\frac{d x^2}{c}\right] + b c \text{AppellF1}\left[\frac{3}{2}, \frac{3}{4}, 1, \frac{5}{2}, -\frac{b x^2}{a}, -\frac{d x^2}{c}\right]\right)\right)\right)$$

Problem 322: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(a+b x^2\right)^{1/4} \left(c+d x^2\right)} \, dx$$

Optimal (type 4, 167 leaves, 4 steps):

$$\frac{a^{1/4} \sqrt{-\frac{b x^2}{a}} \text{EllipticPi}\left[-\frac{\sqrt{a} \sqrt{d}}{\sqrt{-b c+a d}}, \text{ArcSin}\left[\frac{\left(a+b x^2\right)^{1/4}}{a^{1/4}}\right], -1\right]}{\sqrt{d} \sqrt{-b c+a d} \, x} - \frac{a^{1/4} \sqrt{-\frac{b x^2}{a}} \text{EllipticPi}\left[\frac{\sqrt{a} \sqrt{d}}{\sqrt{-b c+a d}}, \text{ArcSin}\left[\frac{\left(a+b x^2\right)^{1/4}}{a^{1/4}}\right], -1\right]}{\sqrt{d} \sqrt{-b c+a d} \, x}$$

Result (type 6, 160 leaves):

$$-\left(\left(6\,a\,c\,x\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\Bigg/\left(\left(a+b\,x^2\right)^{1/4}\left(c+d\,x^2\right)\left(-6\,a\,c\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]+x^2\left(4\,a\,d\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{4},2,\frac{5}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{3}{2},\frac{5}{4},1,\frac{5}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\right)\right)\right)$$

Problem 323: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(a+b\,x^2\right)^{3/4}\left(c+d\,x^2\right)}\,dx$$

Optimal (type 4, 152 leaves, 5 steps):

$$\frac{a^{1/4}\sqrt{-\frac{b\,x^2}{a}}\ \text{EllipticPi}\left[-\frac{\sqrt{a}\ \sqrt{d}}{\sqrt{-b\,c+a\,d}},\text{ArcSin}\left[\frac{\left(a+b\,x^2\right)^{1/4}}{a^{1/4}}\right],-1\right]}{\left(b\,c-a\,d\right)\,x}+\frac{a^{1/4}\sqrt{-\frac{b\,x^2}{a}}\ \text{EllipticPi}\left[\frac{\sqrt{a}\ \sqrt{d}}{\sqrt{-b\,c+a\,d}},\text{ArcSin}\left[\frac{\left(a+b\,x^2\right)^{1/4}}{a^{1/4}}\right],-1\right]}{\left(b\,c-a\,d\right)\,x}$$

Result (type 6, 161 leaves):

$$-\left(\left(6\,a\,c\,x\,\text{AppellF1}\left[\frac{1}{2},\frac{3}{4},1,\frac{3}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\Bigg/\left(\left(a+b\,x^2\right)^{3/4}\left(c+d\,x^2\right)\left(-6\,a\,c\,\text{AppellF1}\left[\frac{1}{2},\frac{3}{4},1,\frac{3}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]+x^2\left(4\,a\,d\,\text{AppellF1}\left[\frac{3}{2},\frac{3}{4},2,\frac{5}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]+3\,b\,c\,\text{AppellF1}\left[\frac{3}{2},\frac{7}{4},1,\frac{5}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\right)\right)\right)$$

Problem 324: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(a+b\,x^2\right)^{5/4}\left(c+d\,x^2\right)}\,dx$$

Optimal (type 4, 233 leaves, 7 steps):

$$\frac{2\sqrt{b}\left(1+\frac{b x^2}{a}\right)^{1/4}\text{EllipticE}\left[\frac{1}{2}\text{ArcTan}\left[\frac{\sqrt{b}\,x}{\sqrt{a}}\right],2\right]}{\sqrt{a}\,(b c-a d)\left(a+b x^2\right)^{1/4}}+\frac{a^{1/4}\sqrt{d}\sqrt{-\frac{b x^2}{a}}\,\text{EllipticPi}\left[-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{-b c+a d}},\text{ArcSin}\left[\frac{(a+b x^2)^{1/4}}{a^{1/4}}\right],-1\right]}{(-b c+a d)^{3/2}\,x}-\frac{a^{1/4}\sqrt{d}\sqrt{-\frac{b x^2}{a}}\,\text{EllipticPi}\left[\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{-b c+a d}},\text{ArcSin}\left[\frac{(a+b x^2)^{1/4}}{a^{1/4}}\right],-1\right]}{(-b c+a d)^{3/2}\,x}$$

Result (type 6, 339 leaves):

$$\left(2x\left(-\frac{3b}{a}-\left(9c\,(b c+a d)\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]\right)\Big/\left(\left(c+d x^2\right)\left(-6 a c\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]+x^2\left(4 a d\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{4},2,\frac{5}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]+b c\,\text{AppellF1}\left[\frac{3}{2},\frac{5}{4},1,\frac{5}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]\right)\right)\right)-\left(5 b c d x^2\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{4},1,\frac{5}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]\right)\Big/\left(\left(c+d x^2\right)\left(-10 a c\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{4},1,\frac{5}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]+x^2\left(4 a d\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{4},2,\frac{7}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]+b c\,\text{AppellF1}\left[\frac{5}{2},\frac{5}{4},1,\frac{7}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]\right)\right)\right)\right)\right)\Big/\left(3\,(-b c+a d)\left(a+b x^2\right)^{1/4}\right)$$

Problem 325: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(a+b x^2\right)^{7/4}\left(c+d x^2\right)}\,dx$$

Optimal (type 4, 254 leaves, 9 steps):

$$\frac{2 b x}{3 a\,(b c-a d)\left(a+b x^2\right)^{3/4}}+\frac{2\sqrt{b}\left(1+\frac{b x^2}{a}\right)^{3/4}\text{EllipticF}\left[\frac{1}{2}\text{ArcTan}\left[\frac{\sqrt{b}\,x}{\sqrt{a}}\right],2\right]}{3\sqrt{a}\,(b c-a d)\left(a+b x^2\right)^{3/4}}-\frac{a^{1/4} d\sqrt{-\frac{b x^2}{a}}\,\text{EllipticPi}\left[-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{-b c+a d}},\text{ArcSin}\left[\frac{(a+b x^2)^{1/4}}{a^{1/4}}\right],-1\right]}{(b c-a d)^2\,x}-\frac{a^{1/4} d\sqrt{-\frac{b x^2}{a}}\,\text{EllipticPi}\left[\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{-b c+a d}},\text{ArcSin}\left[\frac{(a+b x^2)^{1/4}}{a^{1/4}}\right],-1\right]}{(b c-a d)^2\,x}$$

Result (type 6, 342 leaves):

$$\left(2 x \left(-\frac{3 b}{a}+\left(9 c (b c-3 a d) \text{AppellF1}\left[\frac{1}{2},\frac{3}{4},1,\frac{3}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]\right)\Big/\left(\left(c+d x^2\right) \left(-6 a c \text{AppellF1}\left[\frac{1}{2},\frac{3}{4},1,\frac{3}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]+x^2 \left(4 a d \text{AppellF1}\left[\frac{3}{2},\frac{3}{4},2,\frac{5}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]+3 b c \text{AppellF1}\left[\frac{3}{2},\frac{7}{4},1,\frac{5}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]\right)\right)\right)+\left(5 b c d x^2 \text{AppellF1}\left[\frac{3}{2},\frac{3}{4},1,\frac{5}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]\right)\Big/\left(\left(c+d x^2\right) \left(-10 a c \text{AppellF1}\left[\frac{3}{2},\frac{3}{4},1,\frac{5}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]+x^2 \left(4 a d \text{AppellF1}\left[\frac{5}{2},\frac{3}{4},2,\frac{7}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]+3 b c \text{AppellF1}\left[\frac{5}{2},\frac{7}{4},1,\frac{7}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]\right)\right)\right)\right)\right)\Big/\left(9 (-b c+a d) \left(a+b x^2\right)^{3/4}\right)$$

Problem 326: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(a+b x^2\right)^{9/4} \left(c+d x^2\right)} \, dx$$

Optimal (type 4, 274 leaves, 10 steps):

$$\frac{2 b x}{5 a (b c-a d) \left(a+b x^2\right)^{5/4}}+\frac{2 \sqrt{b} (3 b c-8 a d) \left(1+\frac{b x^2}{a}\right)^{1/4} \text{EllipticE}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x}{\sqrt{a}}\right],2\right]}{5 a^{3/2} (b c-a d)^2 \left(a+b x^2\right)^{1/4}}+\frac{a^{1/4} d^{3/2} \sqrt{-\frac{b x^2}{a}} \text{EllipticPi}\left[-\frac{\sqrt{a} \sqrt{d}}{\sqrt{-b c+a d}},\text{ArcSin}\left[\frac{\left(a+b x^2\right)^{1/4}}{a^{1/4}}\right],-1\right]}{(-b c+a d)^{5/2} x}-\frac{a^{1/4} d^{3/2} \sqrt{-\frac{b x^2}{a}} \text{EllipticPi}\left[\frac{\sqrt{a} \sqrt{d}}{\sqrt{-b c+a d}},\text{ArcSin}\left[\frac{\left(a+b x^2\right)^{1/4}}{a^{1/4}}\right],-1\right]}{(-b c+a d)^{5/2} x}$$

Result (type 6, 404 leaves):

$$\left(2 x \left(\frac{3 b \left(-9 a^2 d+3 b^2 c x^2+4 a b \left(c-2 d x^2\right)\right)}{a+b x^2}+\left(9 a c \left(-3 b^2 c^2+8 a b c d+5 a^2 d^2\right) \text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]\right)\Big/\left(\left(c+d x^2\right) \left(6 a c \text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]-x^2 \left(4 a d \text{AppellF1}\left[\frac{3}{2},\frac{1}{4},2,\frac{5}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]+b c \text{AppellF1}\left[\frac{3}{2},\frac{5}{4},1,\frac{5}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]\right)\right)\right)-\left(5 a b c d \left(-3 b c+8 a d\right) x^2 \text{AppellF1}\left[\frac{3}{2},\frac{1}{4},1,\frac{5}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]\right)\Big/\left(\left(c+d x^2\right) \left(-10 a c \text{AppellF1}\left[\frac{3}{2},\frac{1}{4},1,\frac{5}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]+x^2 \left(4 a d \text{AppellF1}\left[\frac{5}{2},\frac{1}{4},2,\frac{7}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]+b c \text{AppellF1}\left[\frac{5}{2},\frac{5}{4},1,\frac{7}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]\right)\right)\right)\right)\right)\Big/\left(15 a^2 \left(b c-a d\right)^2 \left(a+b x^2\right)^{1/4}\right)$$

Problem 327: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(a+b x^2\right)^{11/4} \left(c+d x^2\right)} \, dx$$

Optimal (type 4, 304 leaves, 10 steps):

$$\frac{2 b x}{7 a \left(b c-a d\right) \left(a+b x^2\right)^{7/4}}+\frac{2 b \left(5 b c-12 a d\right) x}{21 a^2 \left(b c-a d\right)^2 \left(a+b x^2\right)^{3/4}}+\frac{2 \sqrt{b} \left(5 b c-12 a d\right) \left(1+\frac{b x^2}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x}{\sqrt{a}}\right],2\right]}{21 a^{3/2} \left(b c-a d\right)^2 \left(a+b x^2\right)^{3/4}}+\frac{a^{1/4} d^2 \sqrt{-\frac{b x^2}{a}} \text{EllipticPi}\left[-\frac{\sqrt{a} \sqrt{d}}{\sqrt{-b c+a d}},\text{ArcSin}\left[\frac{\left(a+b x^2\right)^{1/4}}{a^{1/4}}\right],-1\right]}{\left(b c-a d\right)^3 x}+\frac{a^{1/4} d^2 \sqrt{-\frac{b x^2}{a}} \text{EllipticPi}\left[\frac{\sqrt{a} \sqrt{d}}{\sqrt{-b c+a d}},\text{ArcSin}\left[\frac{\left(a+b x^2\right)^{1/4}}{a^{1/4}}\right],-1\right]}{\left(b c-a d\right)^3 x}$$

Result (type 6, 408 leaves):

$$\left(2\,x\left(\frac{3\,b\left(-15\,a^2\,d+5\,b^2\,c\,x^2+4\,a\,b\left(2\,c-3\,d\,x^2\right)\right)}{a+b\,x^2}+\left(9\,a\,c\left(5\,b^2\,c^2-12\,a\,b\,c\,d+21\,a^2\,d^2\right)\text{AppellF1}\left[\frac{1}{2},\frac{3}{4},1,\frac{3}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\Big/\left(\left(c+d\,x^2\right)\left(6\,a\,c\,\text{AppellF1}\left[\frac{1}{2},\frac{3}{4},1,\frac{3}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]-x^2\left(4\,a\,d\,\text{AppellF1}\left[\frac{3}{2},\frac{3}{4},2,\frac{5}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]+3\,b\,c\,\text{AppellF1}\left[\frac{3}{2},\frac{7}{4},1,\frac{5}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\right)\right)+\left(5\,a\,b\,c\,d\left(-5\,b\,c+12\,a\,d\right)x^2\,\text{AppellF1}\left[\frac{3}{2},\frac{3}{4},1,\frac{5}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\Big/\left(\left(c+d\,x^2\right)\left(-10\,a\,c\,\text{AppellF1}\left[\frac{3}{2},\frac{3}{4},1,\frac{5}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]+x^2\left(4\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{4},2,\frac{7}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]+3\,b\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{7}{4},1,\frac{7}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\right)\right)\right)\right)\Big/\left(63\,a^2\left(b\,c-a\,d\right)^2\left(a+b\,x^2\right)^{3/4}\right)$$

Problem 328: Result unnecessarily involves higher level functions.

$$\int\frac{\left(a+b\,x^2\right)^{7/4}}{\left(c+d\,x^2\right)^2}\,dx$$

Optimal (type 4, 340 leaves, 9 steps):

$$\frac{b\left(5\,b\,c-a\,d\right)x}{2\,c\,d^2\left(a+b\,x^2\right)^{1/4}}-\frac{\left(b\,c-a\,d\right)x\left(a+b\,x^2\right)^{3/4}}{2\,c\,d\left(c+d\,x^2\right)}-\frac{\sqrt{a}\,\sqrt{b}\left(5\,b\,c-a\,d\right)\left(1+\frac{b\,x^2}{a}\right)^{1/4}\text{EllipticE}\left[\frac{1}{2}\text{ArcTan}\left[\frac{\sqrt{b}\,x}{\sqrt{a}}\right],2\right]}{2\,c\,d^2\left(a+b\,x^2\right)^{1/4}}+\frac{1}{4\,c\,d^{5/2}\,x}a^{1/4}\sqrt{-b\,c+a\,d}\left(5\,b\,c+2\,a\,d\right)\sqrt{-\frac{b\,x^2}{a}}\,\text{EllipticPi}\left[-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{-b\,c+a\,d}},\text{ArcSin}\left[\frac{\left(a+b\,x^2\right)^{1/4}}{a^{1/4}}\right],-1\right]-\frac{1}{4\,c\,d^{5/2}\,x}a^{1/4}\sqrt{-b\,c+a\,d}\left(5\,b\,c+2\,a\,d\right)\sqrt{-\frac{b\,x^2}{a}}\,\text{EllipticPi}\left[\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{-b\,c+a\,d}},\text{ArcSin}\left[\frac{\left(a+b\,x^2\right)^{1/4}}{a^{1/4}}\right],-1\right]$$

Result (type 6, 436 leaves):

$$\left(x\left(-\left(\left(18 a^2 (b c+a d) \text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]\right)\Big/\left(-6 a c \text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]+x^2\left(4 a d \text{AppellF1}\left[\frac{3}{2},\frac{1}{4},2,\frac{5}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]+b c \text{AppellF1}\left[\frac{3}{2},\frac{5}{4},1,\frac{5}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]\right)\right)\right)+\left(5 a c\left(6 a^2 d-b^2 c x^2+a b\left(-6 c+5 d x^2\right)\right) \text{AppellF1}\left[\frac{3}{2},\frac{1}{4},1,\frac{5}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]+3 (b c-a d) x^2\left(a+b x^2\right)\left(4 a d \text{AppellF1}\left[\frac{5}{2},\frac{1}{4},2,\frac{7}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]+b c \text{AppellF1}\left[\frac{5}{2},\frac{5}{4},1,\frac{7}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]\right)\right)\Big/\left(c\left(10 a c \text{AppellF1}\left[\frac{3}{2},\frac{1}{4},1,\frac{5}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]-x^2\left(4 a d \text{AppellF1}\left[\frac{5}{2},\frac{1}{4},2,\frac{7}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]+b c \text{AppellF1}\left[\frac{5}{2},\frac{5}{4},1,\frac{7}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]\right)\right)\right)\right)\right)\Big/\left(6 d\left(a+b x^2\right)^{1/4}\left(c+d x^2\right)\right)$$

Problem 329: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^2\right)^{5/4}}{\left(c+d x^2\right)^2} \, dx$$

Optimal (type 4, 279 leaves, 9 steps):

$$-\frac{(b c-a d) x \left(a+b x^2\right)^{1/4}}{2 c d \left(c+d x^2\right)}+\frac{\sqrt{a} \sqrt{b} (3 b c+a d) \left(1+\frac{b x^2}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x}{\sqrt{a}}\right],2\right]}{2 c d^2 \left(a+b x^2\right)^{3/4}}-\frac{1}{4 c d^2 x} a^{1/4} (3 b c+2 a d) \sqrt{-\frac{b x^2}{a}} \text{EllipticPi}\left[-\frac{\sqrt{a} \sqrt{d}}{\sqrt{-b c+a d}},\text{ArcSin}\left[\frac{\left(a+b x^2\right)^{1/4}}{a^{1/4}}\right],-1\right]-\frac{1}{4 c d^2 x} a^{1/4} (3 b c+2 a d) \sqrt{-\frac{b x^2}{a}} \text{EllipticPi}\left[\frac{\sqrt{a} \sqrt{d}}{\sqrt{-b c+a d}},\text{ArcSin}\left[\frac{\left(a+b x^2\right)^{1/4}}{a^{1/4}}\right],-1\right]$$

Result (type 6, 439 leaves):

$$\left(x\left(-\left(\left(18\,a^2\,(b\,c+a\,d)\,\text{AppellF1}\left[\frac{1}{2},\frac{3}{4},1,\frac{3}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\Big/\right.\right.\right.$$
$$\left(-6\,a\,c\,\text{AppellF1}\left[\frac{1}{2},\frac{3}{4},1,\frac{3}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]+x^2\left(4\,a\,d\,\text{AppellF1}\left[\frac{3}{2},\frac{3}{4},2,\right.\right.\right.$$
$$\left.\left.\left.\left.\frac{5}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]+3\,b\,c\,\text{AppellF1}\left[\frac{3}{2},\frac{7}{4},1,\frac{5}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\right)\right)+$$
$$\left(5\,a\,c\,\left(6\,a^2\,d-3\,b^2\,c\,x^2+a\,b\,\left(-6\,c+7\,d\,x^2\right)\right)\,\text{AppellF1}\left[\frac{3}{2},\frac{3}{4},1,\frac{5}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]+\right.$$
$$3\,(b\,c-a\,d)\,x^2\,\left(a+b\,x^2\right)\,\left(4\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{4},2,\frac{7}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]+\right.$$
$$\left.\left.3\,b\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{7}{4},1,\frac{7}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\right)\Big/$$
$$\left(c\,\left(10\,a\,c\,\text{AppellF1}\left[\frac{3}{2},\frac{3}{4},1,\frac{5}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]-\right.\right.$$
$$x^2\,\left(4\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{4},2,\frac{7}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]+\right.$$
$$\left.\left.\left.\left.\left.3\,b\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{7}{4},1,\frac{7}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\right)\right)\right)\right)\Big/\left(6\,d\,\left(a+b\,x^2\right)^{3/4}\,\left(c+d\,x^2\right)\right)$$

Problem 330: Result unnecessarily involves higher level functions.

$$\int\frac{\left(a+b\,x^2\right)^{3/4}}{\left(c+d\,x^2\right)^2}\,dx$$

Optimal (type 4, 309 leaves, 9 steps):

$$-\frac{b\,x}{2\,c\,d\,\left(a+b\,x^2\right)^{1/4}}+\frac{x\,\left(a+b\,x^2\right)^{3/4}}{2\,c\,\left(c+d\,x^2\right)}+\frac{\sqrt{a}\,\sqrt{b}\,\left(1+\frac{b\,x^2}{a}\right)^{1/4}\,\text{EllipticE}\left[\frac{1}{2}\,\text{ArcTan}\left[\frac{\sqrt{b}\,x}{\sqrt{a}}\right],2\right]}{2\,c\,d\,\left(a+b\,x^2\right)^{1/4}}+$$
$$\left(a^{1/4}\,(b\,c+2\,a\,d)\,\sqrt{-\frac{b\,x^2}{a}}\,\text{EllipticPi}\left[-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{-b\,c+a\,d}},\text{ArcSin}\left[\frac{\left(a+b\,x^2\right)^{1/4}}{a^{1/4}}\right],-1\right]\right)\Big/$$
$$\left(4\,c\,d^{3/2}\,\sqrt{-b\,c+a\,d}\,x\right)-$$
$$\left(a^{1/4}\,(b\,c+2\,a\,d)\,\sqrt{-\frac{b\,x^2}{a}}\,\text{EllipticPi}\left[\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{-b\,c+a\,d}},\text{ArcSin}\left[\frac{\left(a+b\,x^2\right)^{1/4}}{a^{1/4}}\right],-1\right]\right)\Big/$$
$$\left(4\,c\,d^{3/2}\,\sqrt{-b\,c+a\,d}\,x\right)$$

Result (type 6, 320 leaves):

$$\left(x\left(\frac{3\left(a+b x^2\right)}{c}-\left(18 a^2 \text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]\right)\Big/\left(-6 a c \text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]+x^2\left(4 a d \text{AppellF1}\left[\frac{3}{2},\frac{1}{4},2,\frac{5}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]+b c \text{AppellF1}\left[\frac{3}{2},\frac{5}{4},1,\frac{5}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]\right)\right)+\left(5 a b x^2 \text{AppellF1}\left[\frac{3}{2},\frac{1}{4},1,\frac{5}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]\right)\Big/\left(-10 a c \text{AppellF1}\left[\frac{3}{2},\frac{1}{4},1,\frac{5}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]+x^2\left(4 a d \text{AppellF1}\left[\frac{5}{2},\frac{1}{4},2,\frac{7}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]+b c \text{AppellF1}\left[\frac{5}{2},\frac{5}{4},1,\frac{7}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]\right)\right)\right)\right)\Big/\left(6\left(a+b x^2\right)^{1/4}\left(c+d x^2\right)\right)$$

Problem 331: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^2\right)^{1/4}}{\left(c+d x^2\right)^2} \, dx$$

Optimal (type 4, 278 leaves, 9 steps):

$$\frac{x\left(a+b x^2\right)^{1/4}}{2 c\left(c+d x^2\right)}+\frac{\sqrt{a}\sqrt{b}\left(1+\frac{b x^2}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x}{\sqrt{a}}\right],2\right]}{2 c d\left(a+b x^2\right)^{3/4}}-\frac{1}{4 c d(b c-a d) x} a^{1/4}(b c-2 a d)\sqrt{-\frac{b x^2}{a}} \text{EllipticPi}\left[-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-b c+a d}},\text{ArcSin}\left[\frac{\left(a+b x^2\right)^{1/4}}{a^{1/4}}\right],-1\right]-\frac{1}{4 c d(b c-a d) x} a^{1/4}(b c-2 a d)\sqrt{-\frac{b x^2}{a}} \text{EllipticPi}\left[\frac{\sqrt{a}\sqrt{d}}{\sqrt{-b c+a d}},\text{ArcSin}\left[\frac{\left(a+b x^2\right)^{1/4}}{a^{1/4}}\right],-1\right]$$

Result (type 6, 322 leaves):

$$\left(x\left(\frac{3\left(a+b x^2\right)}{c}-\left(18 a^2 \text{AppellF1}\left[\frac{1}{2},\frac{3}{4},1,\frac{3}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]\right)\Big/\left(-6 a c \text{AppellF1}\left[\frac{1}{2},\frac{3}{4},1,\frac{3}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]+x^2\left(4 a d \text{AppellF1}\left[\frac{3}{2},\frac{3}{4},2,\frac{5}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]+3 b c \text{AppellF1}\left[\frac{3}{2},\frac{7}{4},1,\frac{5}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]\right)\right)-\left(5 a b x^2 \text{AppellF1}\left[\frac{3}{2},\frac{3}{4},1,\frac{5}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]\right)\Big/\left(-10 a c \text{AppellF1}\left[\frac{3}{2},\frac{3}{4},1,\frac{5}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]+x^2\left(4 a d \text{AppellF1}\left[\frac{5}{2},\frac{3}{4},2,\frac{7}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]+3 b c \text{AppellF1}\left[\frac{5}{2},\frac{7}{4},1,\frac{7}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]\right)\right)\right)\right)\Big/\left(6\left(a+b x^2\right)^{3/4}\left(c+d x^2\right)\right)$$

Problem 332: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(a+b x^2\right)^{1/4} \left(c+d x^2\right)^2} \, dx$$

Optimal (type 4, 336 leaves, 9 steps):

$$\frac{b x}{2 c (b c-a d) \left(a+b x^2\right)^{1/4}}-\frac{d x \left(a+b x^2\right)^{3/4}}{2 c (b c-a d) \left(c+d x^2\right)}-\frac{\sqrt{a} \sqrt{b} \left(1+\frac{b x^2}{a}\right)^{1/4} \text{EllipticE}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x}{\sqrt{a}}\right],2\right]}{2 c (b c-a d) \left(a+b x^2\right)^{1/4}}-\left(a^{1/4} (3 b c-2 a d) \sqrt{-\frac{b x^2}{a}} \text{EllipticPi}\left[-\frac{\sqrt{a} \sqrt{d}}{\sqrt{-b c+a d}},\text{ArcSin}\left[\frac{\left(a+b x^2\right)^{1/4}}{a^{1/4}}\right],-1\right]\right)\Big/\left(4 c \sqrt{d} (-b c+a d)^{3/2} x\right)+\left(a^{1/4} (3 b c-2 a d) \sqrt{-\frac{b x^2}{a}} \text{EllipticPi}\left[\frac{\sqrt{a} \sqrt{d}}{\sqrt{-b c+a d}},\text{ArcSin}\left[\frac{\left(a+b x^2\right)^{1/4}}{a^{1/4}}\right],-1\right]\right)\Big/\left(4 c \sqrt{d} (-b c+a d)^{3/2} x\right)$$

Result (type 6, 358 leaves):

$$\left(x\left(-\frac{3 d \left(a+b x^2\right)}{c (b c-a d)}+\left(18 a (-2 b c+a d) \text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]\right)\Big/\left((b c-a d)\left(-6 a c \text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]+x^2\left(4 a d \text{AppellF1}\left[\frac{3}{2},\frac{1}{4},2,\frac{5}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]+b c \text{AppellF1}\left[\frac{3}{2},\frac{5}{4},1,\frac{5}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]\right)\right)\right)+\left(5 a b d x^2 \text{AppellF1}\left[\frac{3}{2},\frac{1}{4},1,\frac{5}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]\right)\Big/\left((-b c+a d)\left(-10 a c \text{AppellF1}\left[\frac{3}{2},\frac{1}{4},1,\frac{5}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]+x^2\left(4 a d \text{AppellF1}\left[\frac{5}{2},\frac{1}{4},2,\frac{7}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]+b c \text{AppellF1}\left[\frac{5}{2},\frac{5}{4},1,\frac{7}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]\right)\right)\right)\right)\right)\Big/\left(6 \left(a+b x^2\right)^{1/4} \left(c+d x^2\right)\right)$$

Problem 333: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(a+b x^2\right)^{3/4} \left(c+d x^2\right)^2} \, dx$$

Optimal (type 4, 292 leaves, 9 steps):

$$-\frac{d\,x\,\left(a+b\,x^2\right)^{1/4}}{2\,c\,(b\,c-a\,d)\,\left(c+d\,x^2\right)}-\frac{\sqrt{a}\,\sqrt{b}\,\left(1+\frac{b\,x^2}{a}\right)^{3/4}\,\text{EllipticF}\left[\frac{1}{2}\,\text{ArcTan}\left[\frac{\sqrt{b}\,x}{\sqrt{a}}\right],2\right]}{2\,c\,(b\,c-a\,d)\,\left(a+b\,x^2\right)^{3/4}}+\frac{1}{4\,c\,(b\,c-a\,d)^2\,x}$$

$$a^{1/4}\,(5\,b\,c-2\,a\,d)\,\sqrt{-\frac{b\,x^2}{a}}\,\text{EllipticPi}\left[-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{-b\,c+a\,d}},\text{ArcSin}\left[\frac{\left(a+b\,x^2\right)^{1/4}}{a^{1/4}}\right],-1\right]+$$

$$\frac{1}{4\,c\,(b\,c-a\,d)^2\,x}a^{1/4}\,(5\,b\,c-2\,a\,d)\,\sqrt{-\frac{b\,x^2}{a}}\,\text{EllipticPi}\left[\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{-b\,c+a\,d}},\text{ArcSin}\left[\frac{\left(a+b\,x^2\right)^{1/4}}{a^{1/4}}\right],-1\right]$$

Result (type 6, 340 leaves):

$$\left(x\left(-\frac{3\,d\,\left(a+b\,x^2\right)}{c}+\left(18\,a\,(-2\,b\,c+a\,d)\,\text{AppellF1}\left[\frac{1}{2},\frac{3}{4},1,\frac{3}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\middle/\right.\right.$$

$$\left(-6\,a\,c\,\text{AppellF1}\left[\frac{1}{2},\frac{3}{4},1,\frac{3}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]+x^2\left(4\,a\,d\,\text{AppellF1}\left[\frac{3}{2},\frac{3}{4},2,\right.\right.\right.$$

$$\left.\left.\left.\frac{5}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]+3\,b\,c\,\text{AppellF1}\left[\frac{3}{2},\frac{7}{4},1,\frac{5}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\right)+$$

$$\left(5\,a\,b\,d\,x^2\,\text{AppellF1}\left[\frac{3}{2},\frac{3}{4},1,\frac{5}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\middle/\left(-10\,a\,c\,\text{AppellF1}\left[\frac{3}{2},\frac{3}{4},\right.\right.$$

$$1,\frac{5}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]+x^2\left(4\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{4},2,\frac{7}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]+\right.$$

$$\left.\left.\left.\left.3\,b\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{7}{4},1,\frac{7}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\right)\right)\right)\middle/\left(6\,(b\,c-a\,d)\,\left(a+b\,x^2\right)^{3/4}\,\left(c+d\,x^2\right)\right)$$

Problem 334: Result unnecessarily involves higher level functions.

$$\int\frac{1}{\left(a+b\,x^2\right)^{5/4}\,\left(c+d\,x^2\right)^2}\,dx$$

Optimal (type 4, 314 leaves, 10 steps):

$$-\frac{d\,x}{2\,c\,(b\,c-a\,d)\,\left(a+b\,x^2\right)^{1/4}\,\left(c+d\,x^2\right)}+\frac{\sqrt{b}\,(4\,b\,c+a\,d)\,\left(1+\frac{b\,x^2}{a}\right)^{1/4}\,\text{EllipticE}\left[\frac{1}{2}\,\text{ArcTan}\left[\frac{\sqrt{b}\,x}{\sqrt{a}}\right],\,2\right]}{2\,\sqrt{a}\,c\,(b\,c-a\,d)^2\,\left(a+b\,x^2\right)^{1/4}}-$$

$$\left(a^{1/4}\,\sqrt{d}\,(7\,b\,c-2\,a\,d)\,\sqrt{-\frac{b\,x^2}{a}}\,\text{EllipticPi}\left[-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{-b\,c+a\,d}},\,\text{ArcSin}\left[\frac{\left(a+b\,x^2\right)^{1/4}}{a^{1/4}}\right],\,-1\right]\right)\Big/$$
$$\left(4\,c\,(-b\,c+a\,d)^{5/2}\,x\right)+$$
$$\left(a^{1/4}\,\sqrt{d}\,(7\,b\,c-2\,a\,d)\,\sqrt{-\frac{b\,x^2}{a}}\,\text{EllipticPi}\left[\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{-b\,c+a\,d}},\,\text{ArcSin}\left[\frac{\left(a+b\,x^2\right)^{1/4}}{a^{1/4}}\right],\,-1\right]\right)\Big/$$
$$\left(4\,c\,(-b\,c+a\,d)^{5/2}\,x\right)$$

Result (type 6, 480 leaves):

$$\left(x\left(\left(18\,\left(2\,b^2\,c^2+4\,a\,b\,c\,d-a^2\,d^2\right)\,\text{AppellF1}\left[\frac{1}{2},\,\frac{1}{4},\,1,\,\frac{3}{2},\,-\frac{b\,x^2}{a},\,-\frac{d\,x^2}{c}\right]\right)\Big/\right.\right.$$
$$\left(-6\,a\,c\,\text{AppellF1}\left[\frac{1}{2},\,\frac{1}{4},\,1,\,\frac{3}{2},\,-\frac{b\,x^2}{a},\,-\frac{d\,x^2}{c}\right]+x^2\,\left(4\,a\,d\right.\right.$$
$$\left.\left.\text{AppellF1}\left[\frac{3}{2},\,\frac{1}{4},\,2,\,\frac{5}{2},\,-\frac{b\,x^2}{a},\,-\frac{d\,x^2}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{3}{2},\,\frac{5}{4},\,1,\,\frac{5}{2},\,-\frac{b\,x^2}{a},\,-\frac{d\,x^2}{c}\right]\right)\right)+$$
$$\left(5\,a\,c\,\left(6\,a^2\,d^2+5\,a\,b\,d^2\,x^2+4\,b^2\,c\,\left(6\,c+5\,d\,x^2\right)\right)\,\text{AppellF1}\left[\frac{3}{2},\,\frac{1}{4},\,1,\,\frac{5}{2},\,-\frac{b\,x^2}{a},\,-\frac{d\,x^2}{c}\right]-\right.$$
$$3\,x^2\,\left(a^2\,d^2+a\,b\,d^2\,x^2+4\,b^2\,c\,\left(c+d\,x^2\right)\right)\,\left(4\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\,\frac{1}{4},\,2,\,\frac{7}{2},\,-\frac{b\,x^2}{a},\,-\frac{d\,x^2}{c}\right]+\right.$$
$$\left.\left.b\,c\,\text{AppellF1}\left[\frac{5}{2},\,\frac{5}{4},\,1,\,\frac{7}{2},\,-\frac{b\,x^2}{a},\,-\frac{d\,x^2}{c}\right]\right)\right)\Big/$$
$$\left(a\,c\,\left(10\,a\,c\,\text{AppellF1}\left[\frac{3}{2},\,\frac{1}{4},\,1,\,\frac{5}{2},\,-\frac{b\,x^2}{a},\,-\frac{d\,x^2}{c}\right]-\right.\right.$$
$$x^2\,\left(4\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\,\frac{1}{4},\,2,\,\frac{7}{2},\,-\frac{b\,x^2}{a},\,-\frac{d\,x^2}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{2},\,\frac{5}{4},\,1,\right.\right.$$
$$\left.\left.\left.\left.\left.\left.\frac{7}{2},\,-\frac{b\,x^2}{a},\,-\frac{d\,x^2}{c}\right]\right)\right)\right)\right)\right)\Big/\left(6\,(b\,c-a\,d)^2\,\left(a+b\,x^2\right)^{1/4}\,\left(c+d\,x^2\right)\right)$$

Problem 335: Result unnecessarily involves higher level functions.

$$\int\frac{1}{\left(a+b\,x^2\right)^{7/4}\,\left(c+d\,x^2\right)^2}\,dx$$

Optimal (type 4, 345 leaves, 10 steps):

$$\frac{b\,(4\,b\,c+3\,a\,d)\,x}{6\,a\,c\,(b\,c-a\,d)^2\,\left(a+b\,x^2\right)^{3/4}}-\frac{d\,x}{2\,c\,(b\,c-a\,d)\,\left(a+b\,x^2\right)^{3/4}\,\left(c+d\,x^2\right)}+$$
$$\frac{\sqrt{b}\,(4\,b\,c+3\,a\,d)\,\left(1+\frac{b\,x^2}{a}\right)^{3/4}\,\text{EllipticF}\left[\frac{1}{2}\,\text{ArcTan}\left[\frac{\sqrt{b}\,x}{\sqrt{a}}\right],2\right]}{6\,\sqrt{a}\,c\,(b\,c-a\,d)^2\,\left(a+b\,x^2\right)^{3/4}}-\frac{1}{4\,c\,(b\,c-a\,d)^3\,x}$$
$$a^{1/4}\,d\,(9\,b\,c-2\,a\,d)\,\sqrt{-\frac{b\,x^2}{a}}\,\text{EllipticPi}\left[-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{-b\,c+a\,d}},\text{ArcSin}\left[\frac{\left(a+b\,x^2\right)^{1/4}}{a^{1/4}}\right],-1\right]-$$
$$\frac{1}{4\,c\,(b\,c-a\,d)^3\,x}a^{1/4}\,d\,(9\,b\,c-2\,a\,d)\,\sqrt{-\frac{b\,x^2}{a}}\,\text{EllipticPi}\left[\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{-b\,c+a\,d}},\text{ArcSin}\left[\frac{\left(a+b\,x^2\right)^{1/4}}{a^{1/4}}\right],-1\right]$$

Result (type 6, 485 leaves) :

$$\left(x\left(-\left(\left(18\left(2\,b^2\,c^2-12\,a\,b\,c\,d+3\,a^2\,d^2\right)\text{AppellF1}\left[\frac{1}{2},\frac{3}{4},1,\frac{3}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\Big/\right.\right.\right.$$
$$\left(-6\,a\,c\,\text{AppellF1}\left[\frac{1}{2},\frac{3}{4},1,\frac{3}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]+x^2\left(4\,a\,d\,\text{AppellF1}\left[\frac{3}{2},\frac{3}{4},2,\right.\right.\right.$$
$$\left.\left.\left.\left.\frac{5}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]+3\,b\,c\,\text{AppellF1}\left[\frac{3}{2},\frac{7}{4},1,\frac{5}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\right)\right)+$$
$$\left(5\,a\,c\,\left(18\,a^2\,d^2+21\,a\,b\,d^2\,x^2+4\,b^2\,c\,\left(6\,c+7\,d\,x^2\right)\right)\text{AppellF1}\left[\frac{3}{2},\frac{3}{4},1,\frac{5}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]-\right.$$
$$3\,x^2\,\left(3\,a^2\,d^2+3\,a\,b\,d^2\,x^2+4\,b^2\,c\,\left(c+d\,x^2\right)\right)\left(4\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{4},2,\frac{7}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]+\right.$$
$$\left.\left.3\,b\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{7}{4},1,\frac{7}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\right)\Big/$$
$$\left(a\,c\,\left(10\,a\,c\,\text{AppellF1}\left[\frac{3}{2},\frac{3}{4},1,\frac{5}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]-\right.\right.$$
$$x^2\left(4\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{4},2,\frac{7}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]+3\,b\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{7}{4},1,\right.\right.$$
$$\left.\left.\left.\left.\left.\frac{7}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\right)\right)\right)\Big/\left(18\,(b\,c-a\,d)^2\,\left(a+b\,x^2\right)^{3/4}\,\left(c+d\,x^2\right)\right)$$

Problem 336: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(a+b\,x^2\right)^{9/4}\,\left(c+d\,x^2\right)^2}\,dx$$

Optimal (type 4, 371 leaves, 11 steps) :

$$\frac{b\,(4\,b\,c+5\,a\,d)\,x}{10\,a\,c\,(b\,c-a\,d)^2\,(a+b\,x^2)^{5/4}}-\frac{d\,x}{2\,c\,(b\,c-a\,d)\,(a+b\,x^2)^{5/4}\,(c+d\,x^2)}+$$

$$\left(\sqrt{b}\,\left(12\,b^2\,c^2-52\,a\,b\,c\,d-5\,a^2\,d^2\right)\,\left(1+\frac{b\,x^2}{a}\right)^{1/4}\,\text{EllipticE}\left[\frac{1}{2}\,\text{ArcTan}\left[\frac{\sqrt{b}\,x}{\sqrt{a}}\right],2\right]\right)\Big/$$

$$\left(10\,a^{3/2}\,c\,(b\,c-a\,d)^3\,(a+b\,x^2)^{1/4}\right)-$$

$$\left(a^{1/4}\,d^{3/2}\,(11\,b\,c-2\,a\,d)\,\sqrt{-\frac{b\,x^2}{a}}\,\text{EllipticPi}\left[-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{-b\,c+a\,d}},\text{ArcSin}\left[\frac{(a+b\,x^2)^{1/4}}{a^{1/4}}\right],-1\right]\right)\Big/$$

$$\left(4\,c\,(-b\,c+a\,d)^{7/2}\,x\right)+$$

$$\left(a^{1/4}\,d^{3/2}\,(11\,b\,c-2\,a\,d)\,\sqrt{-\frac{b\,x^2}{a}}\,\text{EllipticPi}\left[\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{-b\,c+a\,d}},\text{ArcSin}\left[\frac{(a+b\,x^2)^{1/4}}{a^{1/4}}\right],-1\right]\right)\Big/$$

$$\left(4\,c\,(-b\,c+a\,d)^{7/2}\,x\right)$$

Result (type 6, 634 leaves):

$$\frac{1}{30\,a^2\,(b\,c-a\,d)^3\,(a+b\,x^2)^{1/4}\,(c+d\,x^2)}$$

$$x\,\Bigg(\left(18\,a\,\left(6\,b^3\,c^3-26\,a\,b^2\,c^2\,d-30\,a^2\,b\,c\,d^2+5\,a^3\,d^3\right)\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\Big/$$

$$\left(-6\,a\,c\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]+x^2\,\left(4\,a\,d\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{4},2,\frac{5}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{3}{2},\frac{5}{4},1,\frac{5}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\right)+$$

$$\Bigg(-5\,a\,c\,\Big(30\,a^4\,d^3+55\,a^3\,b\,d^3\,x^2-12\,b^4\,c^2\,x^2\,(6\,c+5\,d\,x^2)+a^2\,b^2\,d\,\left(336\,c^2+284\,c\,d\,x^2+25\,d^2\,x^4\right)+$$

$$4\,a\,b^3\,c\,\left(-24\,c^2+57\,c\,d\,x^2+65\,d^2\,x^4\right)\Big)\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{4},1,\frac{5}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]+$$

$$3\,x^2\,\Big(5\,a^4\,d^3+10\,a^3\,b\,d^3\,x^2-12\,b^4\,c^2\,x^2\,(c+d\,x^2)+a^2\,b^2\,d\,\left(56\,c^2+56\,c\,d\,x^2+5\,d^2\,x^4\right)+$$

$$4\,a\,b^3\,c\,\left(-4\,c^2+9\,c\,d\,x^2+13\,d^2\,x^4\right)\Big)\,\left(4\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{4},2,\frac{7}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]+\right.$$

$$\left.b\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{5}{4},1,\frac{7}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\Bigg)\Big/$$

$$\Bigg(c\,(a+b\,x^2)\,\Bigg(10\,a\,c\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{4},1,\frac{5}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]-x^2\,\Bigg(4\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\right.$$

$$\left.\frac{1}{4},2,\frac{7}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{5}{4},1,\frac{7}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\Bigg)\Bigg)\Bigg)\Bigg)$$

Problem 337: Result unnecessarily involves higher level functions.

$$\int\frac{1}{(a+b\,x^2)^{11/4}\,(c+d\,x^2)^2}\,dx$$

Optimal (type 4, 419 leaves, 11 steps):

$$\frac{b\,(4\,b\,c+7\,a\,d)\,x}{14\,a\,c\,(b\,c-a\,d)^2\,(a+b\,x^2)^{7/4}}+\frac{b\,(20\,b^2\,c^2-76\,a\,b\,c\,d-21\,a^2\,d^2)\,x}{42\,a^2\,c\,(b\,c-a\,d)^3\,(a+b\,x^2)^{3/4}}-\frac{d\,x}{2\,c\,(b\,c-a\,d)\,(a+b\,x^2)^{7/4}\,(c+d\,x^2)}+\left(\sqrt{b}\,(20\,b^2\,c^2-76\,a\,b\,c\,d-21\,a^2\,d^2)\left(1+\frac{b\,x^2}{a}\right)^{3/4}\text{EllipticF}\left[\frac{1}{2}\text{ArcTan}\left[\frac{\sqrt{b}\,x}{\sqrt{a}}\right],2\right]\right)\Big/\left(42\,a^{3/2}\,c\,(b\,c-a\,d)^3\,(a+b\,x^2)^{3/4}\right)+\frac{1}{4\,c\,(b\,c-a\,d)^4\,x}a^{1/4}\,d^2\,(13\,b\,c-2\,a\,d)\sqrt{-\frac{b\,x^2}{a}}\,\text{EllipticPi}\left[-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{-b\,c+a\,d}},\text{ArcSin}\left[\frac{(a+b\,x^2)^{1/4}}{a^{1/4}}\right],-1\right]+\frac{1}{4\,c\,(b\,c-a\,d)^4\,x}a^{1/4}\,d^2\,(13\,b\,c-2\,a\,d)\sqrt{-\frac{b\,x^2}{a}}\,\text{EllipticPi}\left[\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{-b\,c+a\,d}},\text{ArcSin}\left[\frac{(a+b\,x^2)^{1/4}}{a^{1/4}}\right],-1\right]$$

Result (type 6, 637 leaves):

$$\frac{1}{126\,a^2\,(b\,c-a\,d)^3\,(a+b\,x^2)^{3/4}\,(c+d\,x^2)}\,x\left(\left(18\,a\,(-10\,b^3\,c^3+38\,a\,b^2\,c^2\,d-126\,a^2\,b\,c\,d^2+21\,a^3\,d^3)\,\text{AppellF1}\left[\frac{1}{2},\frac{3}{4},1,\frac{3}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\Big/\left(-6\,a\,c\,\text{AppellF1}\left[\frac{1}{2},\frac{3}{4},1,\frac{3}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]+x^2\left(4\,a\,d\,\text{AppellF1}\left[\frac{3}{2},\frac{3}{4},2,\frac{5}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]+3\,b\,c\,\text{AppellF1}\left[\frac{3}{2},\frac{7}{4},1,\frac{5}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\right)+\left(-5\,a\,c\,(126\,a^4\,d^3+273\,a^3\,b\,d^3\,x^2-20\,b^4\,c^2\,x^2\,(6\,c+7\,d\,x^2)+4\,a\,b^3\,c\,(-48\,c^2+61\,c\,d\,x^2+133\,d^2\,x^4)+a^2\,b^2\,d\,(528\,c^2+604\,c\,d\,x^2+147\,d^2\,x^4))\,\text{AppellF1}\left[\frac{3}{2},\frac{3}{4},1,\frac{5}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]+3\,x^2\,(21\,a^4\,d^3+42\,a^3\,b\,d^3\,x^2-20\,b^4\,c^2\,x^2\,(c+d\,x^2)+4\,a\,b^3\,c\,(-8\,c^2+11\,c\,d\,x^2+19\,d^2\,x^4)+a^2\,b^2\,d\,(88\,c^2+88\,c\,d\,x^2+21\,d^2\,x^4))\left(4\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{4},2,\frac{7}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]+3\,b\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{7}{4},1,\frac{7}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\right)\Big/\left(c\,(a+b\,x^2)\left(10\,a\,c\,\text{AppellF1}\left[\frac{3}{2},\frac{3}{4},1,\frac{5}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]-x^2\left(4\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{4},2,\frac{7}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]+3\,b\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{7}{4},1,\frac{7}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\right)\right)\right)$$

Problem 338: Result more than twice size of optimal antiderivative.

$$\int \left(a+b x^2\right)^p \left(c+d x^2\right)^q \, dx$$

Optimal (type 6, 79 leaves, 3 steps):

$$x \left(a+b x^2\right)^p \left(1+\frac{b x^2}{a}\right)^{-p} \left(c+d x^2\right)^q \left(1+\frac{d x^2}{c}\right)^{-q} \text{AppellF1}\left[\frac{1}{2},-p,-q,\frac{3}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]$$

Result (type 6, 172 leaves):

$$\left(3 a c x \left(a+b x^2\right)^p \left(c+d x^2\right)^q \text{AppellF1}\left[\frac{1}{2},-p,-q,\frac{3}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]\right) \Big/ \left(3 a c \text{AppellF1}\left[\frac{1}{2},-p,-q,\frac{3}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]+ 2 x^2 \left(b c p \text{AppellF1}\left[\frac{3}{2},1-p,-q,\frac{5}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]+ a d q \text{AppellF1}\left[\frac{3}{2},-p,1-q,\frac{5}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]\right)\right)$$

Problem 343: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(a+b x^2\right)^p}{c+d x^2} \, dx$$

Optimal (type 6, 57 leaves, 2 steps):

$$\frac{x \left(a+b x^2\right)^p \left(1+\frac{b x^2}{a}\right)^{-p} \text{AppellF1}\left[\frac{1}{2},-p,1,\frac{3}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]}{c}$$

Result (type 6, 162 leaves):

$$-\left(\left(3 a c x \left(a+b x^2\right)^p \text{AppellF1}\left[\frac{1}{2},-p,1,\frac{3}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]\right) \Big/ \left(\left(c+d x^2\right) \left(-3 a c \text{AppellF1}\left[\frac{1}{2},-p,1,\frac{3}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]+2 x^2 \left(-b c p \text{AppellF1}\left[\frac{3}{2}, 1-p,1,\frac{5}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]+a d \text{AppellF1}\left[\frac{3}{2},-p,2,\frac{5}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]\right)\right)\right)\right)$$

Problem 344: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(a+b x^2\right)^p}{\left(c+d x^2\right)^2} \, dx$$

Optimal (type 6, 57 leaves, 2 steps):

$$\frac{x \left(a+b x^2\right)^p \left(1+\frac{b x^2}{a}\right)^{-p} \text{AppellF1}\left[\frac{1}{2},-p,2,\frac{3}{2},-\frac{b x^2}{a},-\frac{d x^2}{c}\right]}{c^2}$$

Result (type 6, 162 leaves):

$$-\left(\left(3\,a\,c\,x\,\left(a+b\,x^2\right)^p\,\text{AppellF1}\left[\frac{1}{2},-p,2,\frac{3}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\Big/\left(\left(c+d\,x^2\right)^2\,\left(-3\,a\,c\,\text{AppellF1}\left[\frac{1}{2},-p,2,\frac{3}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]-2\,x^2\,\left(b\,c\,p\,\text{AppellF1}\left[\frac{3}{2},1-p,2,\frac{5}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]-2\,a\,d\,\text{AppellF1}\left[\frac{3}{2},-p,3,\frac{5}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\right)\right)\right)$$

Problem 345: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(a+b\,x^2\right)^p}{\left(c+d\,x^2\right)^3}\,dx$$

Optimal (type 6, 57 leaves, 2 steps):

$$\frac{x\,\left(a+b\,x^2\right)^p\,\left(1+\frac{b\,x^2}{a}\right)^{-p}\,\text{AppellF1}\left[\frac{1}{2},-p,3,\frac{3}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]}{c^3}$$

Result (type 6, 162 leaves):

$$-\left(\left(3\,a\,c\,x\,\left(a+b\,x^2\right)^p\,\text{AppellF1}\left[\frac{1}{2},-p,3,\frac{3}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\Big/\left(\left(c+d\,x^2\right)^3\,\left(-3\,a\,c\,\text{AppellF1}\left[\frac{1}{2},-p,3,\frac{3}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]-2\,x^2\,\left(b\,c\,p\,\text{AppellF1}\left[\frac{3}{2},1-p,3,\frac{5}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]-3\,a\,d\,\text{AppellF1}\left[\frac{3}{2},-p,4,\frac{5}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\right)\right)\right)$$

Problem 346: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \left(a+b\,x^2\right)^{-1-\frac{b\,c}{2\,b\,c-2\,a\,d}}\,\left(c+d\,x^2\right)^{-1+\frac{a\,d}{2\,b\,c-2\,a\,d}}\,dx$$

Optimal (type 3, 53 leaves, 1 step):

$$\frac{x\,\left(a+b\,x^2\right)^{-\frac{b\,c}{2\,b\,c-2\,a\,d}}\,\left(c+d\,x^2\right)^{\frac{a\,d}{2\,b\,c-2\,a\,d}}}{a\,c}$$

Result (type 6, 594 leaves):

$$3\,a\,c\,x\,\left(a+b\,x^2\right)^{\frac{b\,c}{-2\,b\,c+2\,a\,d}}\,\left(c+d\,x^2\right)^{\frac{a\,d}{2\,b\,c-2\,a\,d}}$$

$$\left(\left(d\,\text{AppellF1}\left[\frac{1}{2},\,\frac{b\,c}{2\,b\,c-2\,a\,d},\,1+\frac{a\,d}{-2\,b\,c+2\,a\,d},\,\frac{3}{2},\,-\frac{b\,x^2}{a},\,-\frac{d\,x^2}{c}\right]\right)\Big/\right.$$

$$\left(\left(c+d\,x^2\right)\,\left(3\,a\,c\,\left(-b\,c+a\,d\right)\,\text{AppellF1}\left[\frac{1}{2},\,\frac{b\,c}{2\,b\,c-2\,a\,d},\,1+\frac{a\,d}{-2\,b\,c+2\,a\,d},\,\frac{3}{2},\,-\frac{b\,x^2}{a},\,-\frac{d\,x^2}{c}\right]+\right.\right.$$

$$x^2\,\left(a\,d\,\left(2\,b\,c-3\,a\,d\right)\,\text{AppellF1}\left[\frac{3}{2},\,\frac{b\,c}{2\,b\,c-2\,a\,d},\,2+\frac{a\,d}{-2\,b\,c+2\,a\,d},\,\frac{5}{2},\,-\frac{b\,x^2}{a},\,-\frac{d\,x^2}{c}\right]+\right.$$

$$\left.\left.\left.b^2\,c^2\,\text{AppellF1}\left[\frac{3}{2},\,1+\frac{b\,c}{2\,b\,c-2\,a\,d},\,1+\frac{a\,d}{-2\,b\,c+2\,a\,d},\,\frac{5}{2},\,-\frac{b\,x^2}{a},\,-\frac{d\,x^2}{c}\right]\right)\right)\right)+$$

$$\left(b\,\text{AppellF1}\left[\frac{1}{2},\,1+\frac{b\,c}{2\,b\,c-2\,a\,d},\,\frac{a\,d}{-2\,b\,c+2\,a\,d},\,\frac{3}{2},\,-\frac{b\,x^2}{a},\,-\frac{d\,x^2}{c}\right]\right)\Big/$$

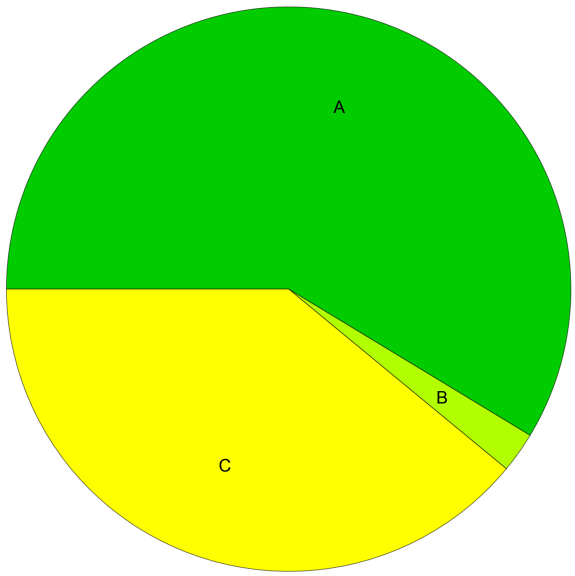
$$\left(\left(a+b\,x^2\right)\,\left(3\,a\,c\,\left(b\,c-a\,d\right)\,\text{AppellF1}\left[\frac{1}{2},\,1+\frac{b\,c}{2\,b\,c-2\,a\,d},\,\frac{a\,d}{-2\,b\,c+2\,a\,d},\,\frac{3}{2},\,-\frac{b\,x^2}{a},\,-\frac{d\,x^2}{c}\right]+\right.\right.$$

$$x^2\,\left(a^2\,d^2\,\text{AppellF1}\left[\frac{3}{2},\,1+\frac{b\,c}{2\,b\,c-2\,a\,d},\,1+\frac{a\,d}{-2\,b\,c+2\,a\,d},\,\frac{5}{2},\,-\frac{b\,x^2}{a},\,-\frac{d\,x^2}{c}\right]+\right.$$

$$\left.\left.\left.\left.b\,c\,\left(-3\,b\,c+2\,a\,d\right)\,\text{AppellF1}\left[\frac{3}{2},\,2+\frac{b\,c}{2\,b\,c-2\,a\,d},\,\frac{a\,d}{-2\,b\,c+2\,a\,d},\,\frac{5}{2},\,-\frac{b\,x^2}{a},\,-\frac{d\,x^2}{c}\right]\right)\right)\right)\right)$$

Summary of Integration Test Results

346 integration problems



A - 203 optimal antiderivatives

B - 8 more than twice size of optimal antiderivatives

C - 135 unnecessarily complex antiderivatives

D - 0 unable to integrate problems

E - 0 integration timeouts