Mathematica 11.3 Integration Test Results

Test results for the 51 problems in "1.1.2.6 (g x)^m (a+b x^2)^p (c+d x^2)^q (e+f x^2)^r.m"

Problem 28: Result unnecessarily involves higher level functions.

$$\int \frac{(e\,x)^m \left(A+B\,x^2\right)}{\left(a+b\,x^2\right)^2 \left(c+d\,x^2\right)}\,dx$$

Optimal (type 5, 206 leaves, 5 steps):

$$\frac{(A\,b-a\,B)\,(e\,x)^{1+m}}{2\,a\,(b\,c-a\,d)\,e\,\left(a+b\,x^2\right)}+\frac{1}{2\,a^2\,(b\,c-a\,d)^2\,e\,(1+m)}\Big(A\,b\,(b\,c\,(1-m)-a\,d\,(3-m))+a\,B\,(a\,d\,(1-m)+b\,c\,(1+m))\Big)\,(e\,x)^{1+m}\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{2},\frac{3+m}{2},-\frac{b\,x^2}{a}\right]-\frac{d\,(B\,c-A\,d)\,(e\,x)^{1+m}\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{2},\frac{3+m}{2},-\frac{d\,x^2}{c}\right]}{c\,(b\,c-a\,d)^2\,e\,(1+m)}$$

Result (type 6, 377 leaves):

$$\Bigg(a\,c\,x\,(e\,x)^m \Bigg(\Bigg(A\,(3+m)^2\,\text{AppellF1}\left[\frac{1+m}{2},2,1,\frac{3+m}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\Bigg)\Big/\Bigg((1+m)\Bigg(a\,c\,(3+m)\,\text{AppellF1}\left[\frac{1+m}{2},2,1,\frac{3+m}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]-2\,x^2\left(a\,d\,\text{AppellF1}\left[\frac{3+m}{2},2,2,\frac{5+m}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]+2\,b\,c\,\text{AppellF1}\left[\frac{3+m}{2},3,1,\frac{5+m}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\Bigg)\Bigg)+\Bigg(B\,(5+m)\,x^2\,\text{AppellF1}\left[\frac{3+m}{2},2,1,\frac{5+m}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\Bigg)\Big/\Bigg(a\,c\,(5+m)\,\text{AppellF1}\left[\frac{3+m}{2},2,1,\frac{5+m}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]-2\,x^2\left(a\,d\,\text{AppellF1}\left[\frac{5+m}{2},2,2,\frac{7+m}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]+2\,b\,c\,\text{AppellF1}\left[\frac{5+m}{2},3,1,\frac{7+m}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\Bigg)\Bigg)\Bigg)\Big/\left((3+m)\left(a+b\,x^2\right)^2\left(c+d\,x^2\right)\right)$$

Problem 29: Result unnecessarily involves higher level functions.

$$\int \frac{(e\,x)^m\,(A+B\,x^2)}{(a+b\,x^2)^3\,(c+d\,x^2)}\,dx$$

Optimal (type 5, 342 leaves, 6 steps):

$$\frac{(A\,b-a\,B)\,(e\,x)^{1+m}}{4\,a\,(b\,c-a\,d)\,e\,(a+b\,x^2)^2}+\frac{(A\,b\,(b\,c\,(3-m)-a\,d\,(7-m))+a\,B\,(a\,d\,(3-m)+b\,c\,(1+m)))\,(e\,x)^{1+m}}{8\,a^2\,(b\,c-a\,d)^2\,e\,(a+b\,x^2)}+\frac{1}{8\,a^3\,(b\,c-a\,d)^3\,e\,(1+m)}\left(A\,b\,(a^2\,d^2\,(15-8\,m+m^2)-2\,a\,b\,c\,d\,(5-6\,m+m^2)+b^2\,c^2\,(3-4\,m+m^2))+a\,B\,(b^2\,c^2\,(1-m^2)-2\,a\,b\,c\,d\,(3+2\,m-m^2)-a^2\,d^2\,(3-4\,m+m^2))\right)(e\,x)^{1+m}\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{2},\frac{3+m}{2},-\frac{b\,x^2}{a}\right]+\frac{d^2\,(B\,c-A\,d)\,(e\,x)^{1+m}\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{2},\frac{3+m}{2},-\frac{d\,x^2}{c}\right]}{c\,(b\,c-a\,d)^3\,e\,(1+m)}$$

Result (type 6, 377 leaves):

$$\left(a\,c\,x\,(e\,x)^m\left(\left(A\,(3+m)^2\,\text{AppellF1}\left[\frac{1+m}{2},3,1,\frac{3+m}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\Big/\left((1+m)\left(a\,c\,(3+m)\,\text{AppellF1}\left[\frac{1+m}{2},3,1,\frac{3+m}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]-2\,x^2\left(a\,d\,\text{AppellF1}\left[\frac{3+m}{2},3,2,\frac{5+m}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]+3\,b\,c\,\text{AppellF1}\left[\frac{3+m}{2},4,1,\frac{5+m}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\right)\right)+\left(B\,(5+m)\,x^2\,\text{AppellF1}\left[\frac{3+m}{2},3,1,\frac{5+m}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\Big/\left(a\,c\,(5+m)\,\text{AppellF1}\left[\frac{3+m}{2},3,1,\frac{5+m}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]-2\,x^2\left(a\,d\,\text{AppellF1}\left[\frac{5+m}{2},3,2,\frac{7+m}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]+3\,b\,c\,\text{AppellF1}\left[\frac{5+m}{2},4,1,\frac{7+m}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\right)\right)\right)\Big/\left((3+m)\,(a+b\,x^2)^3\,(c+d\,x^2)\right)$$

Problem 34: Result unnecessarily involves higher level functions.

$$\int \frac{(e\,x)^m\,(A+B\,x^2)}{(a+b\,x^2)\,(c+d\,x^2)^2}\,dx$$

Optimal (type 5, 205 leaves, 5 steps):

$$\frac{(B c - A d)\,(e x)^{1+m}}{2 c\,(b c - a d)\,e\,(c + d x^2)} + \frac{b\,(A b - a B)\,(e x)^{1+m}\,\text{Hypergeometric2F1}\left[1, \frac{1+m}{2}, \frac{3+m}{2}, -\frac{b x^2}{a}\right]}{a\,(b c - a d)^2\,e\,(1 + m)} +$$
$$\Big((b c\,(B c\,(1 - m) - A d\,(3 - m)) + a d\,(A d\,(1 - m) + B c\,(1 + m)))\,(e x)^{1+m}$$
$$\text{Hypergeometric2F1}\left[1, \frac{1+m}{2}, \frac{3+m}{2}, -\frac{d x^2}{c}\right]\Big) \Big/ \left(2 c^2\,(b c - a d)^2\,e\,(1 + m)\right)$$

Result (type 6, 377 leaves):

$$\Big(a c x\,(e x)^m$$
$$\Big(\Big(A\,(3 + m)^2\,\text{AppellF1}\left[\frac{1+m}{2}, 2, 1, \frac{3+m}{2}, -\frac{d x^2}{c}, -\frac{b x^2}{a}\right]\Big) \Big/ \Big((1 + m)\,\Big(a c\,(3 + m)\,\text{AppellF1}\left[\frac{1+m}{2},\right.$$
$$\left.2, 1, \frac{3+m}{2}, -\frac{d x^2}{c}, -\frac{b x^2}{a}\right] - 2 x^2\,\Big(b c\,\text{AppellF1}\left[\frac{3+m}{2}, 2, 2, \frac{5+m}{2}, -\frac{d x^2}{c}, -\frac{b x^2}{a}\right] +$$
$$2 a d\,\text{AppellF1}\left[\frac{3+m}{2}, 3, 1, \frac{5+m}{2}, -\frac{d x^2}{c}, -\frac{b x^2}{a}\right]\Big)\Big)\Big) +$$
$$\Big(B\,(5 + m)\,x^2\,\text{AppellF1}\left[\frac{3+m}{2}, 2, 1, \frac{5+m}{2}, -\frac{d x^2}{c}, -\frac{b x^2}{a}\right]\Big) \Big/$$
$$\Big(a c\,(5 + m)\,\text{AppellF1}\left[\frac{3+m}{2}, 2, 1, \frac{5+m}{2}, -\frac{d x^2}{c}, -\frac{b x^2}{a}\right] -$$
$$2 x^2\,\Big(b c\,\text{AppellF1}\left[\frac{5+m}{2}, 2, 2, \frac{7+m}{2}, -\frac{d x^2}{c}, -\frac{b x^2}{a}\right] +$$
$$2 a d\,\text{AppellF1}\left[\frac{5+m}{2}, 3, 1, \frac{7+m}{2}, -\frac{d x^2}{c}, -\frac{b x^2}{a}\right]\Big)\Big)\Big)\Big) \Big/ \left((3 + m)\,(a + b x^2)\,(c + d x^2)^2\right)$$

Problem 35: Result unnecessarily involves higher level functions.

$$\int \frac{(e x)^m\,(A + B x^2)}{(a + b x^2)^2\,(c + d x^2)^2}\,dx$$

Optimal (type 5, 304 leaves, 6 steps):

$$\frac{d\,(A b c - 2 a B c + a A d)\,(e x)^{1+m}}{2 a c\,(b c - a d)^2\,e\,(c + d x^2)} + \frac{(A b - a B)\,(e x)^{1+m}}{2 a\,(b c - a d)\,e\,(a + b x^2)\,(c + d x^2)} +$$
$$\Big(b\,(A b\,(b c\,(1 - m) - a d\,(5 - m)) + a B\,(a d\,(3 - m) + b c\,(1 + m)))\,(e x)^{1+m}$$
$$\text{Hypergeometric2F1}\left[1, \frac{1+m}{2}, \frac{3+m}{2}, -\frac{b x^2}{a}\right]\Big) \Big/ \left(2 a^2\,(b c - a d)^3\,e\,(1 + m)\right) -$$
$$\Big(d\,(b c\,(B c\,(3 - m) - A d\,(5 - m)) + a d\,(A d\,(1 - m) + B c\,(1 + m)))\,(e x)^{1+m}$$
$$\text{Hypergeometric2F1}\left[1, \frac{1+m}{2}, \frac{3+m}{2}, -\frac{d x^2}{c}\right]\Big) \Big/ \left(2 c^2\,(b c - a d)^3\,e\,(1 + m)\right)$$

Result (type 6, 375 leaves):

$$\left(a c x (e x)^m \left(\left(A (3+m)^2 \text{AppellF1}\left[\frac{1+m}{2}, 2, 2, \frac{3+m}{2}, -\frac{b x^2}{a}, -\frac{d x^2}{c}\right]\right) \Big/ \left((1+m) \left(a c (3+m) \text{AppellF1}\left[\frac{1+m}{2}, 2, 2, \frac{3+m}{2}, -\frac{b x^2}{a}, -\frac{d x^2}{c}\right] - 4 x^2 \left(a d \text{AppellF1}\left[\frac{3+m}{2}, 2, 3, \frac{5+m}{2}, -\frac{b x^2}{a}, -\frac{d x^2}{c}\right] + b c \text{AppellF1}\left[\frac{3+m}{2}, 3, 2, \frac{5+m}{2}, -\frac{b x^2}{a}, -\frac{d x^2}{c}\right]\right)\right)\right) + \left(B (5+m) x^2 \text{AppellF1}\left[\frac{3+m}{2}, 2, 2, \frac{5+m}{2}, -\frac{b x^2}{a}, -\frac{d x^2}{c}\right]\right) \Big/ \left(a c (5+m) \text{AppellF1}\left[\frac{3+m}{2}, 2, 2, \frac{5+m}{2}, -\frac{b x^2}{a}, -\frac{d x^2}{c}\right] - 4 x^2 \left(a d \text{AppellF1}\left[\frac{5+m}{2}, 2, 3, \frac{7+m}{2}, -\frac{b x^2}{a}, -\frac{d x^2}{c}\right] + b c \text{AppellF1}\left[\frac{5+m}{2}, 3, 2, \frac{7+m}{2}, -\frac{b x^2}{a}, -\frac{d x^2}{c}\right]\right)\right)\right)\right) \Big/ \left((3+m) \left(a + b x^2\right)^2 \left(c + d x^2\right)^2\right)$$

Problem 36: Result unnecessarily involves higher level functions.

$$\int \frac{(e x)^m \left(A + B x^2\right)}{\left(a + b x^2\right)^3 \left(c + d x^2\right)^2} \, dx$$

Optimal (type 5, 491 leaves, 7 steps):

$$-\left(\left(d \left(A \left(4 a^2 d^2 - b^2 c^2 (3-m) + a b c d (11-m)\right) - a B c \left(a d (11-m) + b c (1+m)\right)\right) (e x)^{1+m}\right) \Big/ \left(8 a^2 c (b c - a d)^3 e \left(c + d x^2\right)\right)\right) + \frac{(A b - a B) (e x)^{1+m}}{4 a (b c - a d) e \left(a + b x^2\right)^2 \left(c + d x^2\right)} + \left(\left(A b \left(b c (3-m) - a d (9-m)\right) + a B \left(a d (5-m) + b c (1+m)\right)\right) (e x)^{1+m}\right) \Big/ \left(8 a^2 (b c - a d)^2 e \left(a + b x^2\right) \left(c + d x^2\right)\right) + \left(b \left(a B \left(b^2 c^2 \left(1 - m^2\right) - 2 a b c d \left(5 + 4 m - m^2\right) - a^2 d^2 \left(15 - 8 m + m^2\right)\right) + A b \left(a^2 d^2 \left(35 - 12 m + m^2\right) - 2 a b c d \left(7 - 8 m + m^2\right) + b^2 c^2 \left(3 - 4 m + m^2\right)\right)\right) (e x)^{1+m} \text{Hypergeometric2F1}\left[1, \frac{1+m}{2}, \frac{3+m}{2}, -\frac{b x^2}{a}\right]\right) \Big/ \left(8 a^3 (b c - a d)^4 e (1+m)\right) + \left(d^2 \left(b c \left(B c (5-m) - A d (7-m)\right) + a d \left(A d (1-m) + B c (1+m)\right)\right) (e x)^{1+m} \text{Hypergeometric2F1}\left[1, \frac{1+m}{2}, \frac{3+m}{2}, -\frac{d x^2}{c}\right]\right) \Big/ \left(2 c^2 (b c - a d)^4 e (1+m)\right)$$

Result (type 6, 379 leaves):

$$\left(a\,c\,x\,(e\,x)^m \left(\left(A\,(3+m)^2\,\text{AppellF1}\left[\frac{1+m}{2},3,2,\frac{3+m}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\Big/\left((1+m)\left(a\,c\,(3+m)\,\text{AppellF1}\left[\frac{1+m}{2},3,2,\frac{3+m}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]-2\,x^2\left(2\,a\,d\,\text{AppellF1}\left[\frac{3+m}{2},3,3,\frac{5+m}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]+3\,b\,c\,\text{AppellF1}\left[\frac{3+m}{2},4,2,\frac{5+m}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\right)\right)+\left(B\,(5+m)\,x^2\,\text{AppellF1}\left[\frac{3+m}{2},3,2,\frac{5+m}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\Big/\left(a\,c\,(5+m)\,\text{AppellF1}\left[\frac{3+m}{2},3,2,\frac{5+m}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]-2\,x^2\left(2\,a\,d\,\text{AppellF1}\left[\frac{5+m}{2},3,3,\frac{7+m}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]+3\,b\,c\,\text{AppellF1}\left[\frac{5+m}{2},4,2,\frac{7+m}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\right)\right)\right)\Big/\left((3+m)\,(a+b\,x^2)^3\,(c+d\,x^2)^2\right)$$

Problem 41: Result unnecessarily involves higher level functions.

$$\int\frac{(e\,x)^m\,(A+B\,x^2)}{(a+b\,x^2)\,(c+d\,x^2)^3}\,dx$$

Optimal (type 5, 333 leaves, 6 steps):

$$\frac{(B\,c-A\,d)\,(e\,x)^{1+m}}{4\,c\,(b\,c-a\,d)\,e\,(c+d\,x^2)^2}+\left((b\,c\,(B\,c\,(3-m)-A\,d\,(7-m))+a\,d\,(A\,d\,(3-m)+B\,c\,(1+m)))\,(e\,x)^{1+m}\right)\Big/\left(8\,c^2\,(b\,c-a\,d)^2\,e\,(c+d\,x^2)\right)+\frac{b^2\,(A\,b-a\,B)\,(e\,x)^{1+m}\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{2},\frac{3+m}{2},-\frac{b\,x^2}{a}\right]}{a\,(b\,c-a\,d)^3\,e\,(1+m)}+\left(\left(b^2\,c^2\,(B\,c\,(1-m)-A\,d\,(5-m))\,(3-m)-a^2\,d^2\,(1-m)\,(A\,d\,(3-m)+B\,c\,(1+m))+2\,a\,b\,c\,d\,\left(B\,c\,(3+2\,m-m^2)+A\,d\,(5-6\,m+m^2)\right)\right)\,(e\,x)^{1+m}\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{2},\frac{3+m}{2},-\frac{d\,x^2}{c}\right]\right)\Big/\left(8\,c^3\,(b\,c-a\,d)^3\,e\,(1+m)\right)$$

Result (type 6, 377 leaves):

$$\left(a\,c\,x\,(e\,x)^m\left(\left(A\,(3+m)^2\,\text{AppellF1}\left[\frac{1+m}{2},3,1,\frac{3+m}{2},-\frac{d\,x^2}{c},-\frac{b\,x^2}{a}\right]\right)\bigg/\left((1+m)\left(a\,c\,(3+m)\,\text{AppellF1}\left[\frac{1+m}{2},3,1,\frac{3+m}{2},-\frac{d\,x^2}{c},-\frac{b\,x^2}{a}\right]-2\,x^2\left(b\,c\,\text{AppellF1}\left[\frac{3+m}{2},3,2,\frac{5+m}{2},-\frac{d\,x^2}{c},-\frac{b\,x^2}{a}\right]+3\,a\,d\,\text{AppellF1}\left[\frac{3+m}{2},4,1,\frac{5+m}{2},-\frac{d\,x^2}{c},-\frac{b\,x^2}{a}\right]\right)\right)\right)+\left(B\,(5+m)\,x^2\,\text{AppellF1}\left[\frac{3+m}{2},3,1,\frac{5+m}{2},-\frac{d\,x^2}{c},-\frac{b\,x^2}{a}\right]\right)\bigg/\left(a\,c\,(5+m)\,\text{AppellF1}\left[\frac{3+m}{2},3,1,\frac{5+m}{2},-\frac{d\,x^2}{c},-\frac{b\,x^2}{a}\right]-2\,x^2\left(b\,c\,\text{AppellF1}\left[\frac{5+m}{2},3,2,\frac{7+m}{2},-\frac{d\,x^2}{c},-\frac{b\,x^2}{a}\right]+3\,a\,d\,\text{AppellF1}\left[\frac{5+m}{2},4,1,\frac{7+m}{2},-\frac{d\,x^2}{c},-\frac{b\,x^2}{a}\right]\right)\right)\right)\right)\bigg/\left((3+m)\,(a+b\,x^2)\,(c+d\,x^2)^3\right)$$

Problem 42: Result unnecessarily involves higher level functions.

$$\int\frac{(e\,x)^m\,(A+B\,x^2)}{(a+b\,x^2)^2\,(c+d\,x^2)^3}\,dx$$

Optimal (type 5, 452 leaves, 7 steps):

$$\frac{d\,(2\,A\,b\,c-3\,a\,B\,c+a\,A\,d)\,(e\,x)^{1+m}}{4\,a\,c\,(b\,c-a\,d)^2\,e\,(c+d\,x^2)^2}+\frac{(A\,b-a\,B)\,(e\,x)^{1+m}}{2\,a\,(b\,c-a\,d)\,e\,(a+b\,x^2)\,(c+d\,x^2)^2}+\left(d\,\left(A\,\left(4\,b^2\,c^2-a^2\,d^2\,(3-m)+a\,b\,c\,d\,(11-m)\right)-a\,B\,c\,\left(b\,c\,(11-m)+a\,d\,(1+m)\right)\right)\,(e\,x)^{1+m}\right)\bigg/\left(8\,a\,c^2\,(b\,c-a\,d)^3\,e\,(c+d\,x^2)\right)+\left(b^2\,\left(A\,b\,\left(b\,c\,(1-m)-a\,d\,(7-m)\right)+a\,B\,\left(a\,d\,(5-m)+b\,c\,(1+m)\right)\right)\,(e\,x)^{1+m}\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{2},\frac{3+m}{2},-\frac{b\,x^2}{a}\right]\right)\bigg/\left(2\,a^2\,(b\,c-a\,d)^4\,e\,(1+m)\right)-\left(d\,\left(b^2\,c^2\,\left(B\,c\,(3-m)-A\,d\,(7-m)\right)\,(5-m)-a^2\,d^2\,(1-m)\,\left(A\,d\,(3-m)+B\,c\,(1+m)\right)+2\,a\,b\,c\,d\,\left(B\,c\,(5+4\,m-m^2)+A\,d\,(7-8\,m+m^2)\right)\right)\,(e\,x)^{1+m}\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{2},\frac{3+m}{2},-\frac{d\,x^2}{c}\right]\right)\bigg/\left(8\,c^3\,(b\,c-a\,d)^4\,e\,(1+m)\right)$$

Result (type 6, 379 leaves):

$$\left(a\,c\,x\,(e\,x)^m\right.$$
$$\left(\left(\left(A\,(3+m)^2\,\text{AppellF1}\left[\frac{1+m}{2},2,3,\frac{3+m}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\Big/\left((1+m)\left(a\,c\,(3+m)\,\text{AppellF1}\left[\frac{1+m}{2},2,3,\frac{3+m}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]-2\,x^2\left(3\,a\,d\,\text{AppellF1}\left[\frac{3+m}{2},2,4,\frac{5+m}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]+2\,b\,c\,\text{AppellF1}\left[\frac{3+m}{2},3,3,\frac{5+m}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\right)\right)\right)+$$
$$\left(B\,(5+m)\,x^2\,\text{AppellF1}\left[\frac{3+m}{2},2,3,\frac{5+m}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\Big/\left(a\,c\,(5+m)\,\text{AppellF1}\left[\frac{3+m}{2},2,3,\frac{5+m}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]-2\,x^2\left(3\,a\,d\,\text{AppellF1}\left[\frac{5+m}{2},2,4,\frac{7+m}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]+2\,b\,c\,\text{AppellF1}\left[\frac{5+m}{2},3,3,\frac{7+m}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\right)\Big)\Big)\Big/\left((3+m)\,(a+b\,x^2)^2\,(c+d\,x^2)^3\right)$$

Problem 43: Result unnecessarily involves higher level functions.

$$\int\frac{(e\,x)^m\,(A+B\,x^2)}{(a+b\,x^2)^3\,(c+d\,x^2)^3}\,dx$$

Optimal (type 5, 665 leaves, 8 steps):

$$-\left(\left(d\left(A\left(2\,a^2\,d^2-b^2\,c^2\,(3-m)+a\,b\,c\,d\,(13-m)\right)-a\,B\,c\left(a\,d\,(11-m)+b\,c\,(1+m)\right)\right)(e\,x)^{1+m}\right)\Big/\left(8\,a^2\,c\,(b\,c-a\,d)^3\,e\,(c+d\,x^2)^2\right)\right)+\frac{(A\,b-a\,B)\,(e\,x)^{1+m}}{4\,a\,(b\,c-a\,d)\,e\,(a+b\,x^2)^2\,(c+d\,x^2)^2}+$$
$$\left(\left(A\,b\left(b\,c\,(3-m)-a\,d\,(11-m)\right)+a\,B\left(a\,d\,(7-m)+b\,c\,(1+m)\right)\right)(e\,x)^{1+m}\right)\Big/\left(8\,a^2\,(b\,c-a\,d)^2\,e\,(a+b\,x^2)\,(c+d\,x^2)^2\right)+$$
$$\left(d\left(A\,(b\,c+a\,d)\left(b^2\,c^2\,(3-m)+a^2\,d^2\,(3-m)-2\,a\,b\,c\,d\,(9-m)\right)+a\,B\,c\left(2\,a\,b\,c\,d\,(11-m)+b^2\,c^2\,(1+m)+a^2\,d^2\,(1+m)\right)\right)(e\,x)^{1+m}\right)\Big/\left(8\,a^2\,c^2\,(b\,c-a\,d)^4\,e\,(c+d\,x^2)\right)+$$
$$\left(b^2\left(a\,B\left(b^2\,c^2\,(1-m^2)-2\,a\,b\,c\,d\,(7+6\,m-m^2)-a^2\,d^2\,(35-12\,m+m^2)\right)+A\,b\left(a^2\,d^2\,(63-16\,m+m^2)-2\,a\,b\,c\,d\,(9-10\,m+m^2)+b^2\,c^2\,(3-4\,m+m^2)\right)\right)(e\,x)^{1+m}\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{2},\frac{3+m}{2},-\frac{b\,x^2}{a}\right]\right)\Big/\left(8\,a^3\,(b\,c-a\,d)^5\,e\,(1+m)\right)+$$
$$\left(d^2\left(b^2\,c^2\left(B\,c\,(5-m)-A\,d\,(9-m)\right)(7-m)-a^2\,d^2\,(1-m)\left(A\,d\,(3-m)+B\,c\,(1+m)\right)+2\,a\,b\,c\,d\left(B\,c\,(7+6\,m-m^2)+A\,d\,(9-10\,m+m^2)\right)\right)(e\,x)^{1+m}\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{2},\frac{3+m}{2},-\frac{d\,x^2}{c}\right]\right)\Big/\left(8\,c^3\,(b\,c-a\,d)^5\,e\,(1+m)\right)$$

Result (type 6, 375 leaves):

$$\left(a\,c\,x\,(e\,x)^m\left(\left(A\,(3+m)^2\,\text{AppellF1}\left[\frac{1+m}{2},3,3,\frac{3+m}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\Big/\left((1+m)\left(a\,c\,(3+m)\,\text{AppellF1}\left[\frac{1+m}{2},3,3,\frac{3+m}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]-6\,x^2\left(a\,d\,\text{AppellF1}\left[\frac{3+m}{2},3,4,\frac{5+m}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{3+m}{2},4,3,\frac{5+m}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\right)\right)+\left(B\,(5+m)\,x^2\,\text{AppellF1}\left[\frac{3+m}{2},3,3,\frac{5+m}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\Big/\left(a\,c\,(5+m)\,\text{AppellF1}\left[\frac{3+m}{2},3,3,\frac{5+m}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]-6\,x^2\left(a\,d\,\text{AppellF1}\left[\frac{5+m}{2},3,4,\frac{7+m}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{5+m}{2},4,3,\frac{7+m}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\right)\right)\right)\Big/\left((3+m)\,(a+b\,x^2)^3\,(c+d\,x^2)^3\right)$$

Problem 47: Result more than twice size of optimal antiderivative.

$$\int\frac{(e\,x)^m\,(a+b\,x^2)^p\,(A+B\,x^2)}{c+d\,x^2}\,dx$$

Optimal (type 6, 162 leaves, 6 steps):

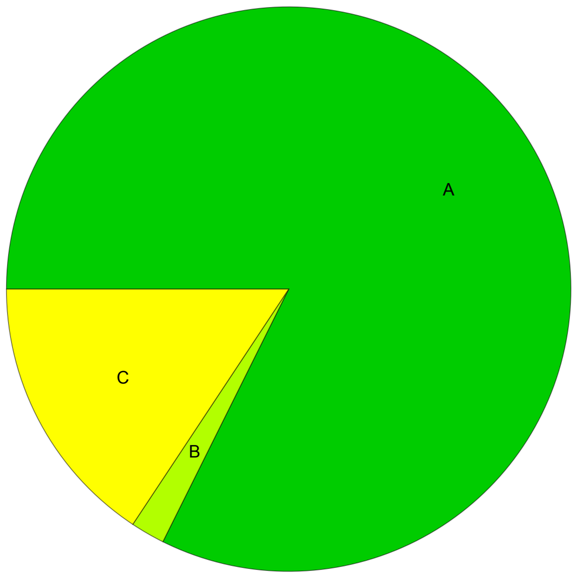
$$-\frac{(B\,c-A\,d)\,(e\,x)^{1+m}\,(a+b\,x^2)^p\left(1+\frac{b\,x^2}{a}\right)^{-p}\text{AppellF1}\left[\frac{1+m}{2},-p,1,\frac{3+m}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]}{c\,d\,e\,(1+m)}+\frac{B\,(e\,x)^{1+m}\,(a+b\,x^2)^p\left(1+\frac{b\,x^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left[\frac{1+m}{2},-p,\frac{3+m}{2},-\frac{b\,x^2}{a}\right]}{d\,e\,(1+m)}$$

Result (type 6, 446 leaves):

$$\left(x\,(e\,x)^m\,\left(a+b\,x^2\right)^p\,\left(-a\,B\,c^2\,(3+m)\,\text{AppellF1}\left[\frac{1+m}{2},-p,1,\frac{3+m}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]+\right.\right.$$
$$a\,A\,c\,d\,(3+m)\,\text{AppellF1}\left[\frac{1+m}{2},-p,1,\frac{3+m}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]+$$
$$B\left(1+\frac{b\,x^2}{a}\right)^{-p}\left(c+d\,x^2\right)\left(a\,c\,(3+m)\,\text{AppellF1}\left[\frac{1+m}{2},-p,1,\frac{3+m}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]+\right.$$
$$2\,x^2\left(b\,c\,p\,\text{AppellF1}\left[\frac{3+m}{2},1-p,1,\frac{5+m}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]-a\,d\,\text{AppellF1}\left[\frac{3+m}{2},-p,\right.\right.$$
$$\left.\left.\left.\left.\left.2,\frac{5+m}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\right)\text{Hypergeometric2F1}\left[\frac{1+m}{2},-p,\frac{3+m}{2},-\frac{b\,x^2}{a}\right]\right)\right)\Big/$$
$$\left(d\,(1+m)\,\left(c+d\,x^2\right)\,\left(a\,c\,(3+m)\,\text{AppellF1}\left[\frac{1+m}{2},-p,1,\frac{3+m}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]+\right.\right.$$
$$2\,x^2\left(b\,c\,p\,\text{AppellF1}\left[\frac{3+m}{2},1-p,1,\frac{5+m}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]-\right.$$
$$\left.\left.\left.a\,d\,\text{AppellF1}\left[\frac{3+m}{2},-p,2,\frac{5+m}{2},-\frac{b\,x^2}{a},-\frac{d\,x^2}{c}\right]\right)\right)\right)$$

Summary of Integration Test Results

51 integration problems



A - 42 optimal antiderivatives

B - 1 more than twice size of optimal antiderivatives

C - 8 unnecessarily complex antiderivatives

D - 0 unable to integrate problems

E - 0 integration timeouts