Mathematica 11.3 Integration Test Results

Test results for the 143 problems in "1.2.1.1 (a+b x+c x^2)^p.m"

Problem 9: Result more than twice size of optimal antiderivative.

$$\int \sqrt{3x-4x^2}\,dx$$

Optimal (type 3, 35 leaves, 3 steps):

$$-\frac{1}{16}(3-8x)\sqrt{3x-4x^2}-\frac{9}{64}\text{ArcSin}\left[1-\frac{8x}{3}\right]$$

Result (type 3, 72 leaves):

$$\frac{\sqrt{-x(-3+4x)}\left(2\sqrt{x}\sqrt{-3+4x}(-3+8x)-9\,\text{Log}\left[2\sqrt{x}+\sqrt{-3+4x}\right]\right)}{32\sqrt{x}\sqrt{-3+4x}}$$

Problem 11: Result more than twice size of optimal antiderivative.

$$\int \sqrt{5x-9x^2}\,dx$$

Optimal (type 3, 35 leaves, 3 steps):

$$-\frac{1}{36}(5-18x)\sqrt{5x-9x^2}-\frac{25}{216}\text{ArcSin}\left[1-\frac{18x}{5}\right]$$

Result (type 3, 72 leaves):

$$\frac{\sqrt{-x(-5+9x)}\left(3\sqrt{x}\sqrt{-5+9x}(-5+18x)-25\,\text{Log}\left[3\sqrt{x}+\sqrt{-5+9x}\right]\right)}{108\sqrt{x}\sqrt{-5+9x}}$$

Problem 17: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{3\,i\,x+4x^2}}\,dx$$

Optimal (type 3, 16 leaves, 2 steps):

$$\frac{1}{2}\,i\,\text{ArcSin}\left[1-\frac{8\,i\,x}{3}\right]$$

Result (type 3, 50 leaves):

$$\frac{\sqrt{x}\ \sqrt{3\,i+4\,x}\ \text{Log}\left[2\,\sqrt{x}+\sqrt{3\,i+4\,x}\right]}{\sqrt{x\,(3\,i+4\,x)}}$$

Problem 21: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{3\,x-4\,x^2}}\,dx$$

Optimal (type 3, 12 leaves, 2 steps):

$$-\frac{1}{2}\,\text{ArcSin}\left[1-\frac{8\,x}{3}\right]$$

Result (type 3, 45 leaves):

$$\frac{\sqrt{x}\ \sqrt{-3+4\,x}\ \text{Log}\left[2\,\sqrt{x}+\sqrt{-3+4\,x}\right]}{\sqrt{-x\,(-3+4\,x)}}$$

Problem 25: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{b\,x-b^2\,x^2}}\,dx$$

Optimal (type 3, 12 leaves, 2 steps):

$$-\frac{\text{ArcSin}[1-2\,b\,x]}{b}$$

Result (type 3, 58 leaves):

$$\frac{2\,\sqrt{x}\ \sqrt{-1+b\,x}\ \text{Log}\left[b\,\sqrt{x}+\sqrt{b}\ \sqrt{-1+b\,x}\right]}{\sqrt{b}\ \sqrt{-b\,x\,(-1+b\,x)}}$$

Problem 27: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{6\,x-x^2}}\,dx$$

Optimal (type 3, 10 leaves, 2 steps):

$$-\text{ArcSin}\left[1-\frac{x}{3}\right]$$

Result (type 3, 38 leaves):

$$\frac{2\,\sqrt{-6+x}\ \sqrt{x}\ \text{Log}\left[\sqrt{-6+x}+\sqrt{x}\right]}{\sqrt{-(-6+x)\,x}}$$

Problem 28: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{4x+x^2}}\,dx$$

Optimal (type 3, 16 leaves, 2 steps):

$$2\,\text{ArcTanh}\left[\frac{x}{\sqrt{4x+x^2}}\right]$$

Result (type 3, 33 leaves):

$$\frac{2\sqrt{x}\,\sqrt{4+x}\,\text{ArcSinh}\left[\frac{\sqrt{x}}{2}\right]}{\sqrt{x\,(4+x)}}$$

Problem 29: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-2x+x^2}}\,dx$$

Optimal (type 3, 16 leaves, 2 steps):

$$2\,\text{ArcTanh}\left[\frac{x}{\sqrt{-2x+x^2}}\right]$$

Result (type 3, 37 leaves):

$$\frac{2\sqrt{-2+x}\,\sqrt{x}\,\text{Log}\left[\sqrt{-2+x}+\sqrt{x}\right]}{\sqrt{(-2+x)\,x}}$$

Problem 30: Result unnecessarily involves higher level functions.

$$\int \left(b\,x+c\,x^2\right)^{4/3}dx$$

Optimal (type 4, 448 leaves, 6 steps):

$$\frac{3\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3} (b+2\,c\,x) \left(b\,x+c\,x^2\right)^{4/3}}{55\,c \left(-\frac{c\,\left(b\,x+c\,x^2\right)}{b^2}\right)^{4/3}}+$$

$$\frac{3\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{4/3} (b+2\,c\,x) \left(b\,x+c\,x^2\right)^{4/3}}{22\,c \left(-\frac{c\,\left(b\,x+c\,x^2\right)}{b^2}\right)^{4/3}}+\left(2^{1/3}\times 3^{3/4}\sqrt{2-\sqrt{3}}\;b^2 \left(b\,x+c\,x^2\right)^{4/3}\right.$$

$$\left(1-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}\right)\sqrt{\frac{1+2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}+2\times 2^{1/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{2/3}}{\left(1-\sqrt{3}-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}\right)^2}}$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}}{1-\sqrt{3}-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}}\right],\,-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(55\,c\,(b+2\,c\,x)\left(-\frac{c\,\left(b\,x+c\,x^2\right)}{b^2}\right)^{4/3}\sqrt{-\frac{1-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}}{\left(1-\sqrt{3}-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 94 leaves) :

$$\left(3\,x\left(-2\,b^4-b^3\,c\,x+16\,b^2\,c^2\,x^2+25\,b\,c^3\,x^3+10\,c^4\,x^4+\right.\right.$$

$$\left.\left.2\,b^4\left(1+\frac{c\,x}{b}\right)^{2/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{c\,x}{b}\right]\right)\right)\Big/\left(110\,c^2\left(x\,(b+c\,x)\right)^{2/3}\right)$$

Problem 31: Result unnecessarily involves higher level functions.

$$\int \left(b\,x+c\,x^2\right)^{1/3} dx$$

Optimal (type 4, 387 leaves, 5 steps) :

$$\frac{3\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}(b+2 c x)\left(b x+c x^2\right)^{1/3}}{10 c\left(-\frac{c\left(b x+c x^2\right)}{b^2}\right)^{1/3}}+$$

$$\left(3^{3/4}\sqrt{2-\sqrt{3}}\, b^2\left(b x+c x^2\right)^{1/3}\left(1-2^{2/3}\left(-\frac{c x(b+c x)}{b^2}\right)^{1/3}\right)\right.$$

$$\sqrt{\frac{1+2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}+2\times 2^{1/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{2/3}}{\left(1-\sqrt{3}-2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}\right)^2}}$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}}{1-\sqrt{3}-2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(5\times 2^{2/3} c (b+2 c x)\left(-\frac{c\left(b x+c x^2\right)}{b^2}\right)^{1/3}\sqrt{-\frac{1-2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}}{\left(1-\sqrt{3}-2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 70 leaves):

$$\left(3 x\left(b^2+3 b c x+2 c^2 x^2-b^2\left(1+\frac{c x}{b}\right)^{2/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{c x}{b}\right]\right)\right)\Big/$$
$$\left(10 c\left(x(b+c x)\right)^{2/3}\right)$$

Problem 32: Result unnecessarily involves higher level functions.

$$\int\frac{1}{\left(b x+c x^2\right)^{2/3}}\,dx$$

Optimal (type 4, 322 leaves, 4 steps):

$$\left(2^{1/3}\times 3^{3/4}\sqrt{2-\sqrt{3}}\; b^2\left(-\frac{c\left(b\,x+c\,x^2\right)}{b^2}\right)^{2/3}\left(1-2^{2/3}\left(-\frac{c\,x\left(b+c\,x\right)}{b^2}\right)^{1/3}\right)\sqrt{\frac{1+2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}+2\times 2^{1/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{2/3}}{\left(1-\sqrt{3}-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}}{1-\sqrt{3}-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}}\right],\,-7+4\sqrt{3}\right]\right)\Bigg/\left(c\left(b+2\,c\,x\right)\left(b\,x+c\,x^2\right)^{2/3}\sqrt{-\frac{1-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}}{\left(1-\sqrt{3}-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 44 leaves) :

$$\frac{3\,x\left(\frac{b+c\,x}{b}\right)^{2/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\,\frac{2}{3},\,\frac{4}{3},\,-\frac{c\,x}{b}\right]}{\left(x\left(b+c\,x\right)\right)^{2/3}}$$

Problem 33: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(b\,x+c\,x^2\right)^{5/3}}\,dx$$

Optimal (type 4, 384 leaves, 5 steps) :

$$\frac{3\left(b+2 c x\right)\left(-\frac{c\left(b x+c x^2\right)}{b^2}\right)^{5/3}}{2 c\left(-\frac{c x (b+c x)}{b^2}\right)^{2/3}\left(b x+c x^2\right)^{5/3}}+$$

$$\left(2^{1/3}\times 3^{3/4}\sqrt{2-\sqrt{3}}\; b^2\left(-\frac{c\left(b x+c x^2\right)}{b^2}\right)^{5/3}\left(1-2^{2/3}\left(-\frac{c x\left(b+c x\right)}{b^2}\right)^{1/3}\right)\right.$$

$$\sqrt{\frac{1+2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}+2\times 2^{1/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{2/3}}{\left(1-\sqrt{3}-2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}\right)^2}}$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}}{1-\sqrt{3}-2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(c\left(b+2 c x\right)\left(b x+c x^2\right)^{5/3}\sqrt{-\frac{1-2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}}{\left(1-\sqrt{3}-2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 57 leaves):

$$-\frac{3\left(b+2 c x+2 c x\left(1+\frac{c x}{b}\right)^{2/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{c x}{b}\right]\right)}{2 b^2\left(x\left(b+c x\right)\right)^{2/3}}$$

Problem 34: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(b x+c x^2\right)^{8/3}}\, dx$$

Optimal (type 4, 448 leaves, 6 steps):

$$\frac{3\left(b+2\,c\,x\right)\left(-\frac{c\left(b\,x+c\,x^2\right)}{b^2}\right)^{8/3}}{5\,c\left(-\frac{c\,x\left(b+c\,x\right)}{b^2}\right)^{5/3}\left(b\,x+c\,x^2\right)^{8/3}}+\frac{21\left(b+2\,c\,x\right)\left(-\frac{c\left(b\,x+c\,x^2\right)}{b^2}\right)^{8/3}}{5\,c\left(-\frac{c\,x\left(b+c\,x\right)}{b^2}\right)^{2/3}\left(b\,x+c\,x^2\right)^{8/3}}+$$

$$\left(14\times 2^{1/3}\times 3^{3/4}\sqrt{2-\sqrt{3}}\;b^2\left(-\frac{c\left(b\,x+c\,x^2\right)}{b^2}\right)^{8/3}\right.$$

$$\left(1-2^{2/3}\left(-\frac{c\,x\left(b+c\,x\right)}{b^2}\right)^{1/3}\right)\sqrt{\frac{1+2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}+2\times 2^{1/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{2/3}}{\left(1-\sqrt{3}-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}\right)^2}}$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}}{1-\sqrt{3}-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}}\right],\,-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(5\,c\left(b+2\,c\,x\right)\left(b\,x+c\,x^2\right)^{8/3}\sqrt{-\frac{1-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}}{\left(1-\sqrt{3}-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 90 leaves) :

$$\left(-3\,b^3+15\,b^2\,c\,x+63\,b\,c^2\,x^2+42\,c^3\,x^3+\right.$$

$$\left.42\,c^2\,x^2\left(b+c\,x\right)\left(1+\frac{c\,x}{b}\right)^{2/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{c\,x}{b}\right]\right)\Big/\left(5\,b^4\left(x\left(b+c\,x\right)\right)^{5/3}\right)$$

Problem 35: Result unnecessarily involves higher level functions.

$$\int\left(b\,x+c\,x^2\right)^{5/3}\,dx$$

Optimal (type 4, 842 leaves, 8 steps) :

$$\frac{15\left(-\frac{c x (b+c x)}{b^2}\right)^{2/3}(b+2 c x)\left(b x+c x^2\right)^{5/3}}{364 c\left(-\frac{c\left(b x+c x^2\right)}{b^2}\right)^{5/3}}+\frac{3\left(-\frac{c x (b+c x)}{b^2}\right)^{5/3}(b+2 c x)\left(b x+c x^2\right)^{5/3}}{26 c\left(-\frac{c\left(b x+c x^2\right)}{b^2}\right)^{5/3}}-$$

$$\frac{15 (b+2 c x)\left(b x+c x^2\right)^{5/3}}{182\times 2^{1/3} c\left(-\frac{c\left(b x+c x^2\right)}{b^2}\right)^{5/3}\left(1-\sqrt{3}-2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}\right)}-$$

$$\left(15\times 3^{1/4}\sqrt{2+\sqrt{3}}\, b^2\left(b x+c x^2\right)^{5/3}\left(1-2^{2/3}\left(-\frac{c x(b+c x)}{b^2}\right)^{1/3}\right)\right.$$

$$\sqrt{\frac{1+2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}+2\times 2^{1/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{2/3}}{\left(1-\sqrt{3}-2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}\right)^2}}$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}}{1-\sqrt{3}-2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(364\times 2^{1/3} c (b+2 c x)\left(-\frac{c\left(b x+c x^2\right)}{b^2}\right)^{5/3}\sqrt{-\frac{1-2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}}{\left(1-\sqrt{3}-2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}\right)^2}}\right)+$$

$$\left(5\times 3^{3/4} b^2\left(b x+c x^2\right)^{5/3}\left(1-2^{2/3}\left(-\frac{c x(b+c x)}{b^2}\right)^{1/3}\right)\right.$$

$$\sqrt{\frac{1+2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}+2\times 2^{1/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{2/3}}{\left(1-\sqrt{3}-2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}\right)^2}}$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}}{1-\sqrt{3}-2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(91\times 2^{5/6} c (b+2 c x)\left(-\frac{c\left(b x+c x^2\right)}{b^2}\right)^{5/3}\sqrt{-\frac{1-2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}}{\left(1-\sqrt{3}-2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 94 leaves) :

$$\left(3 x \left(-5 b^4 - b^3 c x + 46 b^2 c^2 x^2 + 70 b c^3 x^3 + 28 c^4 x^4 + 5 b^4 \left(1 + \frac{c x}{b}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{c x}{b}\right]\right)\right) \Big/ \left(364 c^2 \left(x \left(b + c x\right)\right)^{1/3}\right)$$

Problem 36: Result unnecessarily involves higher level functions.

$$\int \left(b x + c x^2\right)^{2/3} \, dx$$

Optimal (type 4, 781 leaves, 7 steps):

$$\frac{3\left(-\frac{c x (b+c x)}{b^2}\right)^{2/3}(b+2 c x)\left(b x+c x^2\right)^{2/3}}{14 c\left(-\frac{c\left(b x+c x^2\right)}{b^2}\right)^{2/3}}-$$

$$\frac{3(b+2 c x)\left(b x+c x^2\right)^{2/3}}{7\times 2^{1/3} c\left(-\frac{c\left(b x+c x^2\right)}{b^2}\right)^{2/3}\left(1-\sqrt{3}-2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}\right)}-$$

$$\left(3\times 3^{1/4}\sqrt{2+\sqrt{3}}\, b^2\left(b x+c x^2\right)^{2/3}\left(1-2^{2/3}\left(-\frac{c x(b+c x)}{b^2}\right)^{1/3}\right)\right.$$

$$\sqrt{\frac{1+2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}+2\times 2^{1/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{2/3}}{\left(1-\sqrt{3}-2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}\right)^2}}$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}}{1-\sqrt{3}-2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(14\times 2^{1/3} c (b+2 c x)\left(-\frac{c\left(b x+c x^2\right)}{b^2}\right)^{2/3}\sqrt{-\frac{1-2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}}{\left(1-\sqrt{3}-2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}\right)^2}}\right)+$$

$$\left(2^{1/6}\times 3^{3/4} b^2\left(b x+c x^2\right)^{2/3}\left(1-2^{2/3}\left(-\frac{c x(b+c x)}{b^2}\right)^{1/3}\right)\right.$$

$$\sqrt{\frac{1+2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}+2\times 2^{1/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{2/3}}{\left(1-\sqrt{3}-2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}\right)^2}}$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}}{1-\sqrt{3}-2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(7 c (b+2 c x)\left(-\frac{c\left(b x+c x^2\right)}{b^2}\right)^{2/3}\sqrt{-\frac{1-2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}}{\left(1-\sqrt{3}-2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 70 leaves) :

$$\left(3 x\left(b^2+3 b c x+2 c^2 x^2-b^2\left(1+\frac{c x}{b}\right)^{1/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{c x}{b}\right]\right)\right)\Big/$$

$$\left(14 c\left(x(b+c x)\right)^{1/3}\right)$$

Problem 37: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(b\,x+c\,x^2\right)^{1/3}}\,dx$$

Optimal (type 4, 715 leaves, 6 steps):

$$-\frac{3\left(b+2\,c\,x\right)\left(-\frac{c\left(b\,x+c\,x^2\right)}{b^2}\right)^{1/3}}{2^{1/3}\,c\left(b\,x+c\,x^2\right)^{1/3}\left(1-\sqrt{3}-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}\right)}-$$

$$\left(3\times 3^{1/4}\sqrt{2+\sqrt{3}}\;b^2\left(-\frac{c\left(b\,x+c\,x^2\right)}{b^2}\right)^{1/3}\left(1-2^{2/3}\left(-\frac{c\,x\left(b+c\,x\right)}{b^2}\right)^{1/3}\right)\right.$$

$$\sqrt{\frac{1+2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}+2\times 2^{1/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{2/3}}{\left(1-\sqrt{3}-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}\right)^2}}$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}}{1-\sqrt{3}-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Bigg/$$

$$\left(2\times 2^{1/3}\,c\left(b+2\,c\,x\right)\left(b\,x+c\,x^2\right)^{1/3}\sqrt{-\frac{1-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}}{\left(1-\sqrt{3}-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}\right)^2}}\right)+$$

$$\left(2^{1/6}\times 3^{3/4}\,b^2\left(-\frac{c\left(b\,x+c\,x^2\right)}{b^2}\right)^{1/3}\left(1-2^{2/3}\left(-\frac{c\,x\left(b+c\,x\right)}{b^2}\right)^{1/3}\right)\right.$$

$$\sqrt{\frac{1+2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}+2\times 2^{1/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{2/3}}{\left(1-\sqrt{3}-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}\right)^2}}$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}}{1-\sqrt{3}-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}}\right],-7+4\sqrt{3}\right]\right)\Bigg/$$

$$\left(c\left(b+2\,c\,x\right)\left(b\,x+c\,x^2\right)^{1/3}\sqrt{-\frac{1-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}}{\left(1-\sqrt{3}-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 46 leaves):

$$\frac{3\,x\,\left(\frac{b+c\,x}{b}\right)^{1/3}\,\text{Hypergeometric2F1}\left[\frac{1}{3},\,\frac{2}{3},\,\frac{5}{3},\,-\frac{c\,x}{b}\right]}{2\,\left(x\,\left(b+c\,x\right)\right)^{1/3}}$$

Problem 38: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(b\,x+c\,x^2\right)^{4/3}}\,dx$$

Optimal (type 4, 773 leaves, 7 steps) :

$$\frac{3\left(b+2 c x\right)\left(-\frac{c\left(b x+c x^2\right)}{b^2}\right)^{4/3}}{c\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}\left(b x+c x^2\right)^{4/3}}+$$

$$\frac{3\times 2^{2/3}\left(b+2 c x\right)\left(-\frac{c\left(b x+c x^2\right)}{b^2}\right)^{4/3}}{c\left(b x+c x^2\right)^{4/3}\left(1-\sqrt{3}-2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}\right)}+\Bigg(3\times 3^{1/4}\sqrt{2+\sqrt{3}}\ b^2\left(-\frac{c\left(b x+c x^2\right)}{b^2}\right)^{4/3}$$

$$\left(1-2^{2/3}\left(-\frac{c x\left(b+c x\right)}{b^2}\right)^{1/3}\right)\sqrt{\frac{1+2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}+2\times 2^{1/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{2/3}}{\left(1-\sqrt{3}-2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}\right)^2}}$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}}{1-\sqrt{3}-2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}}\right],-7+4\sqrt{3}\right]\Bigg)\Bigg/$$

$$\left(2^{1/3} c\left(b+2 c x\right)\left(b x+c x^2\right)^{4/3}\sqrt{-\frac{1-2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}}{\left(1-\sqrt{3}-2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}\right)^2}}\right)-$$

$$\Bigg(2\times 2^{1/6}\times 3^{3/4} b^2\left(-\frac{c\left(b x+c x^2\right)}{b^2}\right)^{4/3}\left(1-2^{2/3}\left(-\frac{c x\left(b+c x\right)}{b^2}\right)^{1/3}\right)$$

$$\sqrt{\frac{1+2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}+2\times 2^{1/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{2/3}}{\left(1-\sqrt{3}-2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}\right)^2}}$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}}{1-\sqrt{3}-2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}}\right],-7+4\sqrt{3}\right]\Bigg)\Bigg/$$

$$\left(c\left(b+2 c x\right)\left(b x+c x^2\right)^{4/3}\sqrt{-\frac{1-2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}}{\left(1-\sqrt{3}-2^{2/3}\left(-\frac{c x (b+c x)}{b^2}\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 57 leaves):

$$\frac{-3\left(b+2 c x\right)+3 c x\left(1+\frac{c x}{b}\right)^{1/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{c x}{b}\right]}{b^2\left(x\left(b+c x\right)\right)^{1/3}}$$

Problem 39: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(b\,x + c\,x^2\right)^{7/3}}\,dx$$

Optimal (type 4, 838 leaves, 8 steps) :

$$\frac{3\,(b+2\,c\,x)\left(-\frac{c\,(b\,x+c\,x^2)}{b^2}\right)^{7/3}}{4\,c\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{4/3}(b\,x+c\,x^2)^{7/3}}+\frac{15\,(b+2\,c\,x)\left(-\frac{c\,(b\,x+c\,x^2)}{b^2}\right)^{7/3}}{2\,c\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}(b\,x+c\,x^2)^{7/3}}+$$

$$\frac{15\,(b+2\,c\,x)\left(-\frac{c\,(b\,x+c\,x^2)}{b^2}\right)^{7/3}}{2^{1/3}\,c\,(b\,x+c\,x^2)^{7/3}\left(1-\sqrt{3}-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}\right)}+$$

$$\left(15\times 3^{1/4}\sqrt{2+\sqrt{3}}\;b^2\left(-\frac{c\,(b\,x+c\,x^2)}{b^2}\right)^{7/3}\left(1-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}\right)\right.$$

$$\sqrt{\frac{1+2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}+2\times 2^{1/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{2/3}}{\left(1-\sqrt{3}-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}\right)^2}}$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}}{1-\sqrt{3}-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}}\right],\,-7+4\sqrt{3}\right]\right)\Bigg/$$

$$\left(2\times 2^{1/3}\,c\,(b+2\,c\,x)\,(b\,x+c\,x^2)^{7/3}\sqrt{-\frac{1-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}}{\left(1-\sqrt{3}-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}\right)^2}}\right)-$$

$$\left(5\times 2^{1/6}\times 3^{3/4}\,b^2\left(-\frac{c\,(b\,x+c\,x^2)}{b^2}\right)^{7/3}\left(1-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}\right)\right.$$

$$\sqrt{\frac{1+2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}+2\times 2^{1/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{2/3}}{\left(1-\sqrt{3}-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}\right)^2}}$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}}{1-\sqrt{3}-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}}\right],\,-7+4\sqrt{3}\right]\right)\Bigg/$$

$$\left(c\,(b+2\,c\,x)\,(b\,x+c\,x^2)^{7/3}\sqrt{-\frac{1-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}}{\left(1-\sqrt{3}-2^{2/3}\left(-\frac{c\,x\,(b+c\,x)}{b^2}\right)^{1/3}\right)^2}}\right)$$

Result (type 5, 90 leaves) :

$$\left(-3 b^3+24 b^2 c x+90 b c^2 x^2+60 c^3 x^3-30 c^2 x^2 (b+c x) \left(1+\frac{c x}{b}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{c x}{b}\right]\right)\Big/\left(4 b^4 (x (b+c x))^{4/3}\right)$$

Problem 40: Result unnecessarily involves higher level functions.

$$\int \left(b x+c x^2\right)^{5/4} dx$$

Optimal (type 4, 119 leaves, 5 steps):

$$-\frac{5 b^2 (b+2 c x) \left(b x+c x^2\right)^{1/4}}{84 c^2}+\frac{(b+2 c x) \left(b x+c x^2\right)^{5/4}}{7 c}+\frac{5 b^5 \left(-\frac{c \left(b x+c x^2\right)}{b^2}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcSin}\left[1+\frac{2 c x}{b}\right],2\right]}{84 \sqrt{2} c^3 \left(b x+c x^2\right)^{3/4}}$$

Result (type 5, 94 leaves):

$$\left(x \left(-5 b^4-3 b^3 c x+38 b^2 c^2 x^2+60 b c^3 x^3+24 c^4 x^4+5 b^4 \left(1+\frac{c x}{b}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{5}{4},-\frac{c x}{b}\right]\right)\right)\Big/\left(84 c^2 (x (b+c x))^{3/4}\right)$$

Problem 41: Result unnecessarily involves higher level functions.

$$\int \left(b x+c x^2\right)^{3/4} dx$$

Optimal (type 4, 90 leaves, 4 steps):

$$\frac{(b+2 c x) \left(b x+c x^2\right)^{3/4}}{5 c}-\frac{3 b^3 \left(-\frac{c \left(b x+c x^2\right)}{b^2}\right)^{1/4} \text{EllipticE}\left[\frac{1}{2} \text{ArcSin}\left[1+\frac{2 c x}{b}\right],2\right]}{10 \sqrt{2} c^2 \left(b x+c x^2\right)^{1/4}}$$

Result (type 5, 70 leaves):

$$\left(x \left(b^2+3 b c x+2 c^2 x^2-b^2 \left(1+\frac{c x}{b}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{7}{4},-\frac{c x}{b}\right]\right)\right)\Big/\left(5 c (x (b+c x))^{1/4}\right)$$

Problem 42: Result unnecessarily involves higher level functions.

$$\int \left(b x+c x^2\right)^{1/4} dx$$

Optimal (type 4, 90 leaves, 4 steps):

$$\frac{(b+2 c x) \left(b x+c x^2\right)^{1/4}}{3 c}-\frac{b^3 \left(-\frac{c \left(b x+c x^2\right)}{b^2}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcSin}\left[1+\frac{2 c x}{b}\right],2\right]}{3 \sqrt{2} c^2 \left(b x+c x^2\right)^{3/4}}$$

Result (type 5, 70 leaves):

$$\left(x\left(b^2+3\,b\,c\,x+2\,c^2\,x^2-b^2\left(1+\frac{c\,x}{b}\right)^{3/4}\text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{5}{4},-\frac{c\,x}{b}\right]\right)\right)\Big/\left(3\,c\,\left(x\,\left(b+c\,x\right)\right)^{3/4}\right)$$

Problem 43: Result unnecessarily involves higher level functions.

$$\int\frac{1}{\left(b\,x+c\,x^2\right)^{1/4}}\,dx$$

Optimal (type 4, 58 leaves, 3 steps):

$$\frac{\sqrt{2}\,b\left(-\frac{c\,(b\,x+c\,x^2)}{b^2}\right)^{1/4}\text{EllipticE}\left[\frac{1}{2}\,\text{ArcSin}\left[1+\frac{2\,c\,x}{b}\right],2\right]}{c\,\left(b\,x+c\,x^2\right)^{1/4}}$$

Result (type 5, 46 leaves):

$$\frac{4\,x\left(\frac{b+c\,x}{b}\right)^{1/4}\text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{7}{4},-\frac{c\,x}{b}\right]}{3\,\left(x\,\left(b+c\,x\right)\right)^{1/4}}$$

Problem 44: Result unnecessarily involves higher level functions.

$$\int\frac{1}{\left(b\,x+c\,x^2\right)^{3/4}}\,dx$$

Optimal (type 4, 59 leaves, 3 steps):

$$\frac{2\,\sqrt{2}\,b\left(-\frac{c\,(b\,x+c\,x^2)}{b^2}\right)^{3/4}\text{EllipticF}\left[\frac{1}{2}\,\text{ArcSin}\left[1+\frac{2\,c\,x}{b}\right],2\right]}{c\,\left(b\,x+c\,x^2\right)^{3/4}}$$

Result (type 5, 44 leaves):

$$\frac{4\,x\left(\frac{b+c\,x}{b}\right)^{3/4}\text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{5}{4},-\frac{c\,x}{b}\right]}{\left(x\,\left(b+c\,x\right)\right)^{3/4}}$$

Problem 45: Result unnecessarily involves higher level functions.

$$\int\frac{1}{\left(b\,x+c\,x^2\right)^{5/4}}\,dx$$

Optimal (type 4, 83 leaves, 4 steps):

$$-\frac{4\,\left(b+2\,c\,x\right)}{b^2\,\left(b\,x+c\,x^2\right)^{1/4}}+\frac{4\,\sqrt{2}\,\left(-\frac{c\,(b\,x+c\,x^2)}{b^2}\right)^{1/4}\text{EllipticE}\left[\frac{1}{2}\,\text{ArcSin}\left[1+\frac{2\,c\,x}{b}\right],2\right]}{b\,\left(b\,x+c\,x^2\right)^{1/4}}$$

Result (type 5, 59 leaves):

$$-\frac{4\left(3\,b+6\,c\,x-4\,c\,x\left(1+\frac{c\,x}{b}\right)^{1/4}\text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{7}{4},-\frac{c\,x}{b}\right]\right)}{3\,b^2\,\left(x\,\left(b+c\,x\right)\right)^{1/4}}$$

Problem 46: Result unnecessarily involves higher level functions.

$$\int\frac{1}{\left(b\,x+c\,x^2\right)^{9/4}}\,dx$$

Optimal (type 4, 115 leaves, 5 steps):

$$-\frac{4\,\left(b+2\,c\,x\right)}{5\,b^2\,\left(b\,x+c\,x^2\right)^{5/4}}+\frac{48\,c\,\left(b+2\,c\,x\right)}{5\,b^4\,\left(b\,x+c\,x^2\right)^{1/4}}-\frac{48\,\sqrt{2}\,c\,\left(-\frac{c\,\left(b\,x+c\,x^2\right)}{b^2}\right)^{1/4}\text{EllipticE}\left[\frac{1}{2}\,\text{ArcSin}\left[1+\frac{2\,c\,x}{b}\right],2\right]}{5\,b^3\,\left(b\,x+c\,x^2\right)^{1/4}}$$

Result (type 5, 90 leaves):

$$\left(-4\,b^3+40\,b^2\,c\,x+144\,b\,c^2\,x^2+96\,c^3\,x^3-64\,c^2\,x^2\,\left(b+c\,x\right)\,\left(1+\frac{c\,x}{b}\right)^{1/4}\text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{7}{4},-\frac{c\,x}{b}\right]\right)\Big/\left(5\,b^4\,\left(x\,\left(b+c\,x\right)\right)^{5/4}\right)$$

Problem 47: Result unnecessarily involves higher level functions.

$$\int\frac{1}{\left(b\,x+c\,x^2\right)^{13/4}}\,dx$$

Optimal (type 4, 146 leaves, 6 steps):

$$-\frac{4\,\left(b+2\,c\,x\right)}{9\,b^2\,\left(b\,x+c\,x^2\right)^{9/4}}+\frac{112\,c\,\left(b+2\,c\,x\right)}{45\,b^4\,\left(b\,x+c\,x^2\right)^{5/4}}-\frac{448\,c^2\,\left(b+2\,c\,x\right)}{15\,b^6\,\left(b\,x+c\,x^2\right)^{1/4}}+\frac{448\,\sqrt{2}\,c^2\,\left(-\frac{c\,\left(b\,x+c\,x^2\right)}{b^2}\right)^{1/4}\text{EllipticE}\left[\frac{1}{2}\,\text{ArcSin}\left[1+\frac{2\,c\,x}{b}\right],2\right]}{15\,b^5\,\left(b\,x+c\,x^2\right)^{1/4}}$$

Result (type 5, 114 leaves):

$$-\left(\left(4\,\left(5\,b^5-18\,b^4\,c\,x+252\,b^3\,c^2\,x^2+1288\,b^2\,c^3\,x^3+1680\,b\,c^4\,x^4+672\,c^5\,x^5-448\,c^3\,x^3\,\left(b+c\,x\right)^2\left(1+\frac{c\,x}{b}\right)^{1/4}\text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{7}{4},-\frac{c\,x}{b}\right]\right)\right)\Big/\left(45\,b^6\,\left(x\,\left(b+c\,x\right)\right)^{9/4}\right)\right)$$

Problem 83: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{3+4\,x+x^2}\,dx$$

Optimal (type 3, 6 leaves, 3 steps):

$$-\text{ArcTanh}\left[2+x\right]$$

Result (type 3, 17 leaves):

$$\frac{1}{2}\,\text{Log}[1+x]-\frac{1}{2}\,\text{Log}[3+x]$$

Problem 102: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{1+x^2+2\,x\,\text{Cos}\left[\frac{\pi}{7}\right]}\,dx$$

Optimal (type 3, 23 leaves, 2 steps):

$$\text{ArcTan}\left[\text{Cot}\left[\frac{\pi}{7}\right]+x\,\text{Csc}\left[\frac{\pi}{7}\right]\right]\,\text{Csc}\left[\frac{\pi}{7}\right]$$

Result (type 3, 56 leaves):

$$\frac{2\,\text{ArcTan}\left[\frac{(-1)^{1/7}-(-1)^{6/7}+2\,x}{\sqrt{2-(-1)^{2/7}+(-1)^{5/7}}}\right]}{\sqrt{2-(-1)^{2/7}+(-1)^{5/7}}}$$

Problem 133: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \left(3+4\,x+5\,x^2\right)^p\,dx$$

Optimal (type 5, 37 leaves, 2 steps):

$$5^{-1-p}\times 11^p\,(2+5\,x)\,\text{Hypergeometric2F1}\left[\frac{1}{2},\,-p,\,\frac{3}{2},\,-\frac{1}{11}\,(2+5\,x)^2\right]$$

Result (type 5, 93 leaves):

$$\frac{1}{5\,(1+p)}11^{p/2}\left(-2\,i+\sqrt{11}-5\,i\,x\right)^{-p}\left(2-i\,\sqrt{11}+5\,x\right)$$
$$\left(6+8\,x+10\,x^2\right)^p\,\text{Hypergeometric2F1}\left[-p,\,1+p,\,2+p,\,\frac{2\,i+\sqrt{11}+5\,i\,x}{2\,\sqrt{11}}\right]$$

Problem 134: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \left(3+4\,x+4\,x^2\right)^p\,dx$$

Optimal (type 5, 32 leaves, 2 steps):

$$2^{-1+p}\,(1+2\,x)\,\text{Hypergeometric2F1}\left[\frac{1}{2},\,-p,\,\frac{3}{2},\,-\frac{1}{2}\,(1+2\,x)^2\right]$$

Result (type 5, 94 leaves):

$$\frac{1}{1+p}2^{-1+\frac{3p}{2}}\left(-i+\sqrt{2}-2\,i\,x\right)^{-p}\left(1-i\sqrt{2}+2\,x\right)\left(3+4\,x+4\,x^2\right)^p$$
$$\text{Hypergeometric2F1}\left[-p,\,1+p,\,2+p,\,\frac{1}{4}\left(2+i\sqrt{2}+2\,i\sqrt{2}\,x\right)\right]$$

Problem 135: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\left(3+4\,x+3\,x^2\right)^p\,dx$$

Optimal (type 5, 37 leaves, 2 steps):

$$3^{-1-p}\times 5^p\,(2+3\,x)\,\text{Hypergeometric2F1}\left[\frac{1}{2},\,-p,\,\frac{3}{2},\,-\frac{1}{5}\,(2+3\,x)^2\right]$$

Result (type 5, 93 leaves):

$$\frac{1}{3\,(1+p)}5^{p/2}\left(-2\,i+\sqrt{5}-3\,i\,x\right)^{-p}\left(2-i\sqrt{5}+3\,x\right)$$
$$\left(6+8\,x+6\,x^2\right)^p\,\text{Hypergeometric2F1}\left[-p,\,1+p,\,2+p,\,\frac{2\,i+\sqrt{5}+3\,i\,x}{2\sqrt{5}}\right]$$

Problem 136: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\left(3+4\,x+2\,x^2\right)^p\,dx$$

Optimal (type 5, 21 leaves, 2 steps):

$$(1+x)\,\text{Hypergeometric2F1}\left[\frac{1}{2},\,-p,\,\frac{3}{2},\,-2\,(1+x)^2\right]$$

Result (type 5, 92 leaves):

$$\frac{1}{1+p}2^{-1+\frac{3p}{2}}\left(-2\,i+\sqrt{2}-2\,i\,x\right)^{-p}\left(2-i\sqrt{2}+2\,x\right)$$
$$\left(3+4\,x+2\,x^2\right)^p\,\text{Hypergeometric2F1}\left[-p,\,1+p,\,2+p,\,\frac{2\,i+\sqrt{2}+2\,i\,x}{2\sqrt{2}}\right]$$

Problem 139: Result more than twice size of optimal antiderivative.

$$\int\left(3+4\,x-x^2\right)^p\,dx$$

Optimal (type 5, 31 leaves, 2 steps):

$-7^p (2-x)$ Hypergeometric2F1$\left[\frac{1}{2}, -p, \frac{3}{2}, \frac{1}{7}(2-x)^2\right]$

Result (type 5, 83 leaves):

$$-\frac{1}{1+p}\left(2+\sqrt{7}-x\right)\left(3+4x-x^2\right)^p\left(1+\frac{-2-\sqrt{7}+x}{2\sqrt{7}}\right)^{-p}\text{Hypergeometric2F1}\left[-p, 1+p, 2+p, -\frac{-2-\sqrt{7}+x}{2\sqrt{7}}\right]$$

Problem 140: Result more than twice size of optimal antiderivative.

$$\int\left(3+4x-2x^2\right)^p\,dx$$

Optimal (type 5, 31 leaves, 2 steps):

$-5^p (1-x)$ Hypergeometric2F1$\left[\frac{1}{2}, -p, \frac{3}{2}, \frac{2}{5}(1-x)^2\right]$

Result (type 5, 86 leaves):

$$-\frac{1}{1+p}2^{-1+\frac{3p}{2}}\times 5^{p/2}\left(2+\sqrt{10}-2x\right)\left(-2+\sqrt{10}+2x\right)^{-p}\left(3+4x-2x^2\right)^p\text{Hypergeometric2F1}\left[-p, 1+p, 2+p, \frac{1}{2}+\frac{1}{\sqrt{10}}-\frac{x}{\sqrt{10}}\right]$$

Problem 141: Result more than twice size of optimal antiderivative.

$$\int\left(3+4x-3x^2\right)^p\,dx$$

Optimal (type 5, 38 leaves, 2 steps):

$-3^{-1-p}\times 13^p (2-3x)$ Hypergeometric2F1$\left[\frac{1}{2}, -p, \frac{3}{2}, \frac{1}{13}(2-3x)^2\right]$

Result (type 5, 81 leaves):

$$-\frac{1}{3(1+p)}13^{p/2}\left(2+\sqrt{13}-3x\right)\left(-2+\sqrt{13}+3x\right)^{-p}\left(6+8x-6x^2\right)^p\text{Hypergeometric2F1}\left[-p, 1+p, 2+p, \frac{2+\sqrt{13}-3x}{2\sqrt{13}}\right]$$

Problem 143: Result more than twice size of optimal antiderivative.

$$\int\left(3+4x-5x^2\right)^p\,dx$$

Optimal (type 5, 38 leaves, 2 steps):

$$-5^{-1-p}\times 19^{p}\,(2-5x)\,\text{Hypergeometric2F1}\left[\frac{1}{2},-p,\frac{3}{2},\frac{1}{19}(2-5x)^{2}\right]$$

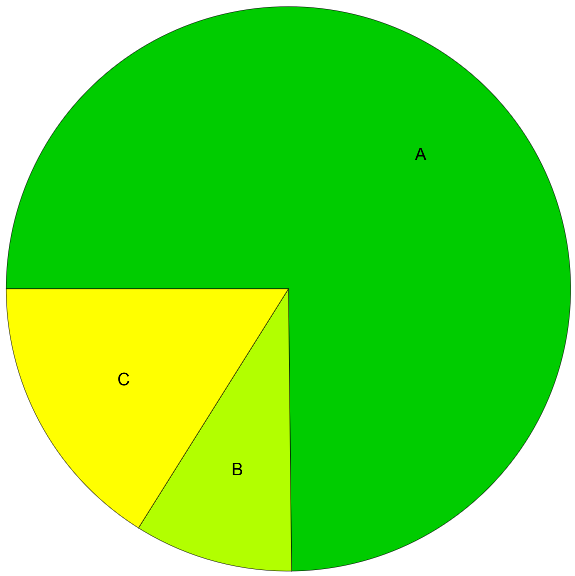
Result (type 5, 81 leaves) :

$$-\frac{1}{5(1+p)}19^{p/2}\left(2+\sqrt{19}-5x\right)\left(-2+\sqrt{19}+5x\right)^{-p}$$

$$\left(6+8x-10x^{2}\right)^{p}\,\text{Hypergeometric2F1}\left[-p,1+p,2+p,\frac{2+\sqrt{19}-5x}{2\sqrt{19}}\right]$$

Summary of Integration Test Results

143 integration problems



A - 107 optimal antiderivatives

B - 13 more than twice size of optimal antiderivatives

C - 23 unnecessarily complex antiderivatives

D - 0 unable to integrate problems

E - 0 integration timeouts