# Mathematica 11.3 Integration Test Results

## Test results for the 123 problems in "1.2.1.5 (a+b x+c x^2)^p (d+e x+f x^2)^q.m"

### Problem 2: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{d+e x+f x^2}\left(a+b x+\frac{b f x^2}{e}\right)}\,dx$$

Optimal (type 3, 82 leaves, 2 steps):

$$-\frac{2 \sqrt{e}\, \text{ArcTanh}\left[\frac{\sqrt{b d-a e}\,(e+2 f x)}{\sqrt{e}\,\sqrt{b e-4 a f}\,\sqrt{d+e x+f x^2}}\right]}{\sqrt{b d-a e}\,\sqrt{b e-4 a f}}$$

Result (type 3, 304 leaves):

$$\frac{1}{\sqrt{b d-a e}\,\sqrt{b e-4 a f}}\sqrt{e}\left(-\text{Log}\left[b e+\sqrt{b}\,\sqrt{e}\,\sqrt{b e-4 a f}+2 b f x\right]+\text{Log}\left[-\sqrt{b}\,\sqrt{e}\,\sqrt{b e-4 a f}+b (e+2 f x)\right]-\text{Log}\left[\sqrt{b}\,\sqrt{e}\,\sqrt{b e-4 a f}\,\left(e^2-4 d f\right)-b e^2 (e+2 f x)+4 a e f (e+2 f x)-4 \sqrt{e}\,\sqrt{b d-a e}\, f \sqrt{b e-4 a f}\,\sqrt{d+x (e+f x)}\right]+\text{Log}\left[\sqrt{b}\,\sqrt{e}\,\sqrt{b e-4 a f}\,\left(e^2-4 d f\right)+b e^2 (e+2 f x)-4 \left(a e f (e+2 f x)+\sqrt{e}\,\sqrt{b d-a e}\, f \sqrt{b e-4 a f}\,\sqrt{d+x (e+f x)}\right)\right]\right)$$

### Problem 3: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{a+b x+c x^2}\left(d+b x+c x^2\right)}\,dx$$

Optimal (type 3, 66 leaves, 2 steps):

$$-\frac{2\, \text{ArcTanh}\left[\frac{\sqrt{a-d}\,(b+2 c x)}{\sqrt{b^2-4 c d}\,\sqrt{a+b x+c x^2}}\right]}{\sqrt{a-d}\,\sqrt{b^2-4 c d}}$$

Result (type 3, 249 leaves):

$$\Big(\text{Log}\big[b-\sqrt{b^2-4cd}+2cx\big]-\text{Log}\big[b+\sqrt{b^2-4cd}+2cx\big]-\text{Log}\Big[-b^3+4bcd+ b^2\left(\sqrt{b^2-4cd}-2cx\right)+4c\left(-a\sqrt{b^2-4cd}+2cdx-\sqrt{a-d}\,\sqrt{b^2-4cd}\,\sqrt{a+x(b+cx)}\right)\Big]+ \text{Log}\Big[b^3-4bcd+b^2\left(\sqrt{b^2-4cd}+2cx\right)- 4c\left(a\sqrt{b^2-4cd}+2cdx+\sqrt{a-d}\,\sqrt{b^2-4cd}\,\sqrt{a+x(b+cx)}\right)\Big]\Big)\Big/\left(\sqrt{a-d}\,\sqrt{b^2-4cd}\right)$$

## Problem 4: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,\left(d+bx+cx^2\right)^2}\,dx$$

Optimal (type 3, 129 leaves, 4 steps):

$$-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)\left(b^2-4cd\right)\left(d+bx+cx^2\right)}+\frac{\left(b^2+4c(a-2d)\right)\text{ArcTanh}\left[\frac{\sqrt{a-d}\,(b+2cx)}{\sqrt{b^2-4cd}\,\sqrt{a+bx+cx^2}}\right]}{(a-d)^{3/2}\left(b^2-4cd\right)^{3/2}}$$

Result (type 3, 339 leaves):

$$\frac{1}{2(a-d)^{3/2}\left(b^2-4cd\right)^{3/2}(d+x(b+cx))}\Big(-2\sqrt{a-d}\,\sqrt{b^2-4cd}\,(b+2cx)\sqrt{a+x(b+cx)}- \left(b^2+4c(a-2d)\right)(d+x(b+cx))\,\text{Log}\big[b-\sqrt{b^2-4cd}+2cx\big]+ \left(b^2+4c(a-2d)\right)(d+x(b+cx))\,\text{Log}\big[b+\sqrt{b^2-4cd}+2cx\big]-\left(b^2+4c(a-2d)\right) (d+x(b+cx))\,\text{Log}\Big[b^2+b\sqrt{b^2-4cd}+2c\left(-2a+\sqrt{b^2-4cd}\,x-2\sqrt{a-d}\,\sqrt{a+x(b+cx)}\right)\Big]+ \left(b^2+4c(a-2d)\right)(d+x(b+cx)) \text{Log}\Big[-b^2+b\sqrt{b^2-4cd}+2c\left(2a+\sqrt{b^2-4cd}\,x+2\sqrt{a-d}\,\sqrt{a+x(b+cx)}\right)\Big]\Big)$$

## Problem 5: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,\left(d+bx+cx^2\right)^3}\,dx$$

Optimal (type 3, 224 leaves, 5 steps):

$$-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)\left(b^2-4cd\right)\left(d+bx+cx^2\right)^2}+\frac{3\left(b^2+4c(a-2d)\right)(b+2cx)\sqrt{a+bx+cx^2}}{4(a-d)^2\left(b^2-4cd\right)^2\left(d+bx+cx^2\right)}- \left(\left(3b^4+8b^2c(a-4d)+16c^2\left(3a^2-8ad+8d^2\right)\right)\text{ArcTanh}\left[\frac{\sqrt{a-d}\,(b+2cx)}{\sqrt{b^2-4cd}\,\sqrt{a+bx+cx^2}}\right]\right)\Big/ \left(4(a-d)^{5/2}\left(b^2-4cd\right)^{5/2}\right)$$

Result (type 3, 486 leaves):

$$\frac{1}{8\,(a-d)^{5/2}\,(b^2-4\,c\,d)^{5/2}\,(d+x\,(b+c\,x))^2}\Big(-2\,\sqrt{a-d}\,\sqrt{b^2-4\,c\,d}\,(b+2\,c\,x)\,\sqrt{a+x\,(b+c\,x)}\,\left(2\,(a-d)\,(b^2-4\,c\,d)-3\,(b^2+4\,c\,(a-2\,d))\,(d+x\,(b+c\,x))\right)+$$
$$(3\,b^4+8\,b^2\,c\,(a-4\,d)+16\,c^2\,(3\,a^2-8\,a\,d+8\,d^2))\,(d+x\,(b+c\,x))^2\,\text{Log}\left[b-\sqrt{b^2-4\,c\,d}+2\,c\,x\right]-$$
$$(3\,b^4+8\,b^2\,c\,(a-4\,d)+16\,c^2\,(3\,a^2-8\,a\,d+8\,d^2))\,(d+x\,(b+c\,x))^2\,\text{Log}\left[b+\sqrt{b^2-4\,c\,d}+2\,c\,x\right]+$$
$$(3\,b^4+8\,b^2\,c\,(a-4\,d)+16\,c^2\,(3\,a^2-8\,a\,d+8\,d^2))\,(d+x\,(b+c\,x))^2\,\text{Log}\left[b^2+b\,\sqrt{b^2-4\,c\,d}+2\,c\,\left(-2\,a+\sqrt{b^2-4\,c\,d}\,x-2\,\sqrt{a-d}\,\sqrt{a+x\,(b+c\,x)}\right)\right]-$$
$$(3\,b^4+8\,b^2\,c\,(a-4\,d)+16\,c^2\,(3\,a^2-8\,a\,d+8\,d^2))\,(d+x\,(b+c\,x))^2\,\text{Log}\left[-b^2+b\,\sqrt{b^2-4\,c\,d}+2\,c\,\left(2\,a+\sqrt{b^2-4\,c\,d}\,x+2\,\sqrt{a-d}\,\sqrt{a+x\,(b+c\,x)}\right)\right]\Big)$$

## Problem 6: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{a+b\,x+c\,x^2}\,(d+b\,x+c\,x^2)^4}\,dx$$

Optimal (type 3, 328 leaves, 6 steps):

$$-\frac{(b+2\,c\,x)\,\sqrt{a+b\,x+c\,x^2}}{3\,(a-d)\,(b^2-4\,c\,d)\,(d+b\,x+c\,x^2)^3}+\frac{5\,(b^2+4\,c\,(a-2\,d))\,(b+2\,c\,x)\,\sqrt{a+b\,x+c\,x^2}}{12\,(a-d)^2\,(b^2-4\,c\,d)^2\,(d+b\,x+c\,x^2)^2}-$$
$$\left((15\,b^4+8\,b^2\,c\,(7\,a-22\,d)+16\,c^2\,(15\,a^2-44\,a\,d+44\,d^2))\,(b+2\,c\,x)\,\sqrt{a+b\,x+c\,x^2}\right)\Big/$$
$$\left(24\,(a-d)^3\,(b^2-4\,c\,d)^3\,(d+b\,x+c\,x^2)\right)+$$
$$\left((b^2+4\,c\,(a-2\,d))\,(5\,b^4-8\,b^2\,c\,(a+4\,d)+16\,c^2\,(5\,a^2-8\,a\,d+8\,d^2))\right.$$
$$\left.\text{ArcTanh}\left[\frac{\sqrt{a-d}\,(b+2\,c\,x)}{\sqrt{b^2-4\,c\,d}\,\sqrt{a+b\,x+c\,x^2}}\right]\right)\Big/\left(8\,(a-d)^{7/2}\,(b^2-4\,c\,d)^{7/2}\right)$$

Result (type 3, 901 leaves):

$$\frac{1}{\sqrt{a+x\,(b+c\,x)}}\left(a+b\,x+c\,x^2\right)\left(-\frac{-b-2\,c\,x}{3\,(a-d)\,\left(-b^2+4\,c\,d\right)\,\left(d+b\,x+c\,x^2\right)^3}+\frac{5\,\left(b^3+4\,a\,b\,c-8\,b\,c\,d+2\,b^2\,c\,x+8\,a\,c^2\,x-16\,c^2\,d\,x\right)}{12\,(a-d)^2\,\left(-b^2+4\,c\,d\right)^2\,\left(d+b\,x+c\,x^2\right)^2}+\left(15\,b^5+56\,a\,b^3\,c+240\,a^2\,b\,c^2-176\,b^3\,c\,d-704\,a\,b\,c^2\,d+704\,b\,c^2\,d^2+30\,b^4\,c\,x+112\,a\,b^2\,c^2\,x+480\,a^2\,c^3\,x-352\,b^2\,c^2\,d\,x-1408\,a\,c^3\,d\,x+1408\,c^3\,d^2\,x\right)\Big/\left(24\,(a-d)^3\,\left(-b^2+4\,c\,d\right)^3\,\left(d+b\,x+c\,x^2\right)\right)\right)+$$

$$\left(\left(b^2+4\,a\,c-8\,c\,d\right)\,\left(5\,b^4-8\,a\,b^2\,c+80\,a^2\,c^2-32\,b^2\,c\,d-128\,a\,c^2\,d+128\,c^2\,d^2\right)\sqrt{a+b\,x+c\,x^2}\;\text{Log}\left[b-\sqrt{b^2-4\,c\,d}+2\,c\,x\right]\right)\Big/\left(16\,\sqrt{a-d}\,(-a+d)^3\,\left(b^2-4\,c\,d\right)^{7/2}\sqrt{a+x\,(b+c\,x)}\right)-$$

$$\left(\left(b^2+4\,a\,c-8\,c\,d\right)\,\left(5\,b^4-8\,a\,b^2\,c+80\,a^2\,c^2-32\,b^2\,c\,d-128\,a\,c^2\,d+128\,c^2\,d^2\right)\sqrt{a+b\,x+c\,x^2}\;\text{Log}\left[b+\sqrt{b^2-4\,c\,d}+2\,c\,x\right]\right)\Big/\left(16\,\sqrt{a-d}\,(-a+d)^3\,\left(b^2-4\,c\,d\right)^{7/2}\sqrt{a+x\,(b+c\,x)}\right)+$$

$$\left(\left(b^2+4\,a\,c-8\,c\,d\right)\,\left(5\,b^4-8\,a\,b^2\,c+80\,a^2\,c^2-32\,b^2\,c\,d-128\,a\,c^2\,d+128\,c^2\,d^2\right)\sqrt{a+b\,x+c\,x^2}\;\text{Log}\left[b^2-4\,a\,c+b\sqrt{b^2-4\,c\,d}+2\,c\sqrt{b^2-4\,c\,d}\,x-4\,c\sqrt{a-d}\,\sqrt{a+b\,x+c\,x^2}\right]\right)\Big/\left(16\,\sqrt{a-d}\,(-a+d)^3\,\left(b^2-4\,c\,d\right)^{7/2}\sqrt{a+x\,(b+c\,x)}\right)-$$

$$\left(\left(b^2+4\,a\,c-8\,c\,d\right)\,\left(5\,b^4-8\,a\,b^2\,c+80\,a^2\,c^2-32\,b^2\,c\,d-128\,a\,c^2\,d+128\,c^2\,d^2\right)\sqrt{a+b\,x+c\,x^2}\;\text{Log}\left[-b^2+4\,a\,c+b\sqrt{b^2-4\,c\,d}+2\,c\sqrt{b^2-4\,c\,d}\,x+4\,c\sqrt{a-d}\,\sqrt{a+b\,x+c\,x^2}\right]\right)\Big/\left(16\,\sqrt{a-d}\,(-a+d)^3\,\left(b^2-4\,c\,d\right)^{7/2}\sqrt{a+x\,(b+c\,x)}\right)$$

## Problem 7: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{\sqrt{d+e\,x+f\,x^2}\,\left(a\,e+b\,e\,x+b\,f\,x^2\right)^2}\,dx$$

Optimal (type 3, 162 leaves, 4 steps):

$$-\frac{b\,(e+2\,f\,x)\,\sqrt{d+e\,x+f\,x^2}}{e\,(b\,d-a\,e)\,(b\,e-4\,a\,f)\,\left(a\,e+b\,e\,x+b\,f\,x^2\right)}-\frac{\left(8\,a\,e\,f-b\,\left(e^2+4\,d\,f\right)\right)\,\text{ArcTanh}\left[\frac{\sqrt{b\,d-a\,e}\,(e+2\,f\,x)}{\sqrt{e}\,\sqrt{b\,e-4\,a\,f}\,\sqrt{d+e\,x+f\,x^2}}\right]}{e^{3/2}\,(b\,d-a\,e)^{3/2}\,(b\,e-4\,a\,f)^{3/2}}$$

Result (type 3, 463 leaves):

$$-\frac{1}{2 e^{3/2} (b d-a e)^{3/2} (b e-4 a f)^{3/2} (a e+b x (e+f x))}$$

$$\Big(2 b \sqrt{e} \sqrt{b d-a e} \sqrt{b e-4 a f} (e+2 f x) \sqrt{d+x (e+f x)}+$$

$$(-8 a e f+b (e^2+4 d f)) (a e+b x (e+f x)) \text{Log}\left[-\sqrt{b} \sqrt{e} \sqrt{b e-4 a f}+b (e+2 f x)\right]-$$

$$(-8 a e f+b (e^2+4 d f)) (a e+b x (e+f x)) \text{Log}\left[\sqrt{b} \sqrt{e} \sqrt{b e-4 a f}+b (e+2 f x)\right]+$$

$$(-8 a e f+b (e^2+4 d f)) (a e+b x (e+f x)) \text{Log}\Big[\sqrt{b} \Big(e^{3/2} \sqrt{b e-4 a f}+$$

$$\sqrt{b} (e^2-4 d f)+2 \sqrt{e} f \sqrt{b e-4 a f} x-4 \sqrt{b d-a e} f \sqrt{d+x (e+f x)}\Big)\Big]-$$

$$(-8 a e f+b (e^2+4 d f)) (a e+b x (e+f x)) \text{Log}\Big[\sqrt{b} \Big(e^{3/2} \sqrt{b e-4 a f}-$$

$$\sqrt{b} (e^2-4 d f)+2 \sqrt{e} f \sqrt{b e-4 a f} x+4 \sqrt{b d-a e} f \sqrt{d+x (e+f x)}\Big)\Big]\Big)$$

## Problem 13: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{a+b x+c x^2} \sqrt{d+f x^2}} \, dx$$

Optimal (type 4, 1077 leaves, 3 steps):

$$-\Bigg(\Bigg(\Big(b^2 d+b \sqrt{b^2-4 a c} \, d-2 a (c d-a f)\Big)^{1/4} \Big(b+\sqrt{b^2-4 a c}+2 c x\Big)^{3/2}$$

$$\sqrt{2 a+\Big(b+\sqrt{b^2-4 a c}\Big) x} \sqrt{\frac{\Big(4 a c-\Big(b+\sqrt{b^2-4 a c}\Big)^2\Big)^2 (d+f x^2)}{\Big(\Big(b+\sqrt{b^2-4 a c}\Big)^2 d+4 a^2 f\Big) \Big(b+\sqrt{b^2-4 a c}+2 c x\Big)^2}}$$

$$\Bigg(1+\Bigg(\sqrt{2 c^2 d-2 a c f+b \Big(b+\sqrt{b^2-4 a c}\Big) f} \Big(2 a+\Big(b+\sqrt{b^2-4 a c}\Big) x\Big)\Bigg)\Big/$$

$$\Bigg(\sqrt{b^2 d+b \sqrt{b^2-4 a c} \, d-2 a (c d-a f)} \Big(b+\sqrt{b^2-4 a c}+2 c x\Big)\Bigg)\Bigg)$$

$$\sqrt{\Bigg(\Bigg(1-\frac{4 \Big(b+\sqrt{b^2-4 a c}\Big) (c d+a f) \Big(2 a+\Big(b+\sqrt{b^2-4 a c}\Big) x\Big)}{\Big(\Big(b+\sqrt{b^2-4 a c}\Big)^2 d+4 a^2 f\Big) \Big(b+\sqrt{b^2-4 a c}+2 c x\Big)}+}$$

$$\frac{\Big(4 c^2 d+\Big(b+\sqrt{b^2-4 a c}\Big)^2 f\Big) \Big(2 a+\Big(b+\sqrt{b^2-4 a c}\Big) x\Big)^2}{\Big(\Big(b+\sqrt{b^2-4 a c}\Big)^2 d+4 a^2 f\Big) \Big(b+\sqrt{b^2-4 a c}+2 c x\Big)^2}\Bigg)\Big/$$

$$\Bigg(1+\Bigg(\sqrt{2 c^2 d-2 a c f+b \Big(b+\sqrt{b^2-4 a c}\Big) f} \Big(2 a+\Big(b+\sqrt{b^2-4 a c}\Big) x\Big)\Bigg)\Big/$$

$$\left.\left.\left(\sqrt{b^2 d+b\sqrt{b^2-4ac}\,d-2a\,(c\,d-a\,f)}\,\left(b+\sqrt{b^2-4ac}+2\,c\,x\right)\right)\right)^2\right)$$

$$\text{EllipticF}\Big[2\,\text{ArcTan}\Big[\left(\left(2\,c^2\,d-2\,a\,c\,f+b\,\left(b+\sqrt{b^2-4ac}\right)f\right)^{1/4}\sqrt{2\,a+\left(b+\sqrt{b^2-4ac}\right)x}\right)\Big/$$

$$\left(\left(b^2 d+b\sqrt{b^2-4ac}\,d-2\,a\,(c\,d-a\,f)\right)^{1/4}\sqrt{b+\sqrt{b^2-4ac}+2\,c\,x}\right)\Big],$$

$$\frac{1}{2}\left(1+\left(\left(b+\sqrt{b^2-4ac}\right)(c\,d+a\,f)\right)\Big/\left(\sqrt{2\,c^2\,d-2\,a\,c\,f+b\,\left(b+\sqrt{b^2-4ac}\right)f}\right.\right.$$

$$\left.\left.\sqrt{b^2 d+b\sqrt{b^2-4ac}\,d-2\,a\,(c\,d-a\,f)}\right)\right)\Big]\Bigg/$$

$$\left(\left(4\,a\,c-\left(b+\sqrt{b^2-4ac}\right)^2\right)\left(2\,c^2\,d-2\,a\,c\,f+b\,\left(b+\sqrt{b^2-4ac}\right)f\right)^{1/4}\sqrt{a+b\,x+c\,x^2}\right.$$

$$\sqrt{d+f\,x^2}\,\sqrt{\left(1-\frac{4\left(b+\sqrt{b^2-4ac}\right)(c\,d+a\,f)\left(2\,a+\left(b+\sqrt{b^2-4ac}\right)x\right)}{\left(\left(b+\sqrt{b^2-4ac}\right)^2 d+4\,a^2\,f\right)\left(b+\sqrt{b^2-4ac}+2\,c\,x\right)}+\right.}$$

$$\left.\left.\frac{\left(4\,c^2\,d+\left(b+\sqrt{b^2-4ac}\right)^2 f\right)\left(2\,a+\left(b+\sqrt{b^2-4ac}\right)x\right)^2}{\left(\left(b+\sqrt{b^2-4ac}\right)^2 d+4\,a^2\,f\right)\left(b+\sqrt{b^2-4ac}+2\,c\,x\right)^2}\right)\right)$$

Result (type 4, 600 leaves) :

$$-\left(\left(2\sqrt{2}\left(-b+\sqrt{b^2-4ac}-2cx\right)\left(-i\sqrt{d}+\sqrt{f}\,x\right)\sqrt{-\frac{c\sqrt{b^2-4ac}\left(i\sqrt{d}+\sqrt{f}\,x\right)}{\left(-2ic\sqrt{d}+\left(b+\sqrt{b^2-4ac}\right)\sqrt{f}\right)\left(-b+\sqrt{b^2-4ac}-2cx\right)}}\right.\right.$$

$$\sqrt{\frac{c\left(-i\sqrt{d}\left(\sqrt{b^2-4ac}+2cx\right)+\sqrt{f}\left(-2a+\sqrt{b^2-4ac}\,x\right)+b\left(-i\sqrt{d}-\sqrt{f}\,x\right)\right)}{\left(2ic\sqrt{d}+\left(b+\sqrt{b^2-4ac}\right)\sqrt{f}\right)\left(-b+\sqrt{b^2-4ac}-2cx\right)}}$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left(-2ic\sqrt{d}+\left(-b+\sqrt{b^2-4ac}\right)\sqrt{f}\right)\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(2ic\sqrt{d}+\left(b+\sqrt{b^2-4ac}\right)\sqrt{f}\right)\left(-b+\sqrt{b^2-4ac}-2cx\right)}}\right],\right.$$

$$\left.\left.\frac{cd-i\sqrt{b^2-4ac}\sqrt{d}\sqrt{f}+af}{cd+i\sqrt{b^2-4ac}\sqrt{d}\sqrt{f}+af}\right]\right)\Bigg/\left(\left(-2ic\sqrt{d}+\left(-b+\sqrt{b^2-4ac}\right)\sqrt{f}\right)\right.$$

$$\sqrt{\frac{ic\sqrt{b^2-4ac}\left(\sqrt{d}+i\sqrt{f}\,x\right)}{\left(2ic\sqrt{d}+\left(b+\sqrt{b^2-4ac}\right)\sqrt{f}\right)\left(-b+\sqrt{b^2-4ac}-2cx\right)}}$$

$$\left.\left.\sqrt{d+fx^2}\sqrt{a+x(b+cx)}\right)\right)$$

## Problem 14: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{\sqrt{-3-4x-x^2}}{3+4x+2x^2}\,dx$$

Optimal (type 3, 98 leaves, 16 steps):

$$-\frac{1}{2}\text{ArcSin}[2+x]-\frac{\text{ArcTan}\left[\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right]}{\sqrt{2}}+\frac{\text{ArcTan}\left[\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right]}{\sqrt{2}}-\frac{1}{2}\text{ArcTanh}\left[\frac{x}{\sqrt{-3-4x-x^2}}\right]$$

Result (type 3, 982 leaves):

$$\frac{1}{8}\left(-4\,\text{ArcSin}[2+x]+\frac{1}{\sqrt{1-2\,i\,\sqrt{2}}}\,2\,i\,\left(i+2\,\sqrt{2}\right)\,\text{ArcTan}\Big[\Big(60+51\,i\,\sqrt{2}+\left(-16+6\,i\,\sqrt{2}\right)\,x^4+54\,i\,\sqrt{1-2\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}+x\,\left(68+176\,i\,\sqrt{2}+99\,i\,\sqrt{1-2\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)+2\,i\,x^3\,\left(34\,\left(i+\sqrt{2}\right)+9\,\sqrt{1-2\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)+i\,x^2\,\left(44\,i+185\,\sqrt{2}+72\,\sqrt{1-2\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)\Big)\Big/\Big(93\,i+150\,\sqrt{2}+20\,\left(17\,i+22\,\sqrt{2}\right)\,x+\left(493\,i+466\,\sqrt{2}\right)\,x^2+16\,\left(19\,i+13\,\sqrt{2}\right)\,x^3+\left(66\,i+32\,\sqrt{2}\right)\,x^4\Big)\Big]+2\,\sqrt{1+2\,i\,\sqrt{2}}\,\text{ArcTan}\Big[\Big(-60+51\,i\,\sqrt{2}+2\,\left(8+3\,i\,\sqrt{2}\right)\,x^4+54\,i\,\sqrt{1+2\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}+2\,x^3\,\left(34+34\,i\,\sqrt{2}+9\,i\,\sqrt{1+2\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)+x^2\,\left(44+185\,i\,\sqrt{2}+72\,i\,\sqrt{1+2\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)+i\,x\,\left(68\,i+176\,\sqrt{2}+99\,\sqrt{1+2\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)\Big)\Big/\Big(-93\,i+150\,\sqrt{2}+20\,\left(-17\,i+22\,\sqrt{2}\right)\,x+\left(-493\,i+466\,\sqrt{2}\right)\,x^2+16\,\left(-19\,i+13\,\sqrt{2}\right)\,x^3+\left(-66\,i+32\,\sqrt{2}\right)\,x^4\Big)\Big]-\frac{\left(-i+2\,\sqrt{2}\right)\,\text{Log}\left[4\,\left(3+4\,x+2\,x^2\right)^2\right]}{\sqrt{1+2\,i\,\sqrt{2}}}-\frac{\left(i+2\,\sqrt{2}\right)\,\text{Log}\left[4\,\left(3+4\,x+2\,x^2\right)^2\right]}{\sqrt{1-2\,i\,\sqrt{2}}}+\frac{1}{\sqrt{1-2\,i\,\sqrt{2}}}\left(i+2\,\sqrt{2}\right)\,\text{Log}\Big[\left(3+4\,x+2\,x^2\right)\,\Big(3+6\,i\,\sqrt{2}+\left(2+2\,i\,\sqrt{2}\right)\,x^2-2\,\sqrt{2-4\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}+x\,\left(4+8\,i\,\sqrt{2}-2\,\sqrt{2-4\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)\Big)\Big]+\frac{1}{\sqrt{1+2\,i\,\sqrt{2}}}\left(-i+2\,\sqrt{2}\right)\,\text{Log}\Big[\left(3+4\,x+2\,x^2\right)\,\Big(3-6\,i\,\sqrt{2}+\left(2-2\,i\,\sqrt{2}\right)\,x^2-2\,\sqrt{2+4\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}-2\,x\,\left(-2+4\,i\,\sqrt{2}+\sqrt{2+4\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)\Big)\Big]\right)$$

## Problem 62: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{3-x+2x^2}}{2+3x+5x^2}\,dx$$

Optimal (type 3, 174 leaves, 8 steps):

$$-\frac{1}{5}\sqrt{2}\,\text{ArcSinh}\left[\frac{1-4x}{\sqrt{23}}\right]+\frac{1}{5}\sqrt{\frac{11}{31}\left(13+10\sqrt{2}\right)}\,\text{ArcTan}\left[\frac{\sqrt{\frac{11}{62\left(13+10\sqrt{2}\right)}}\left(6+7\sqrt{2}+\left(20+13\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right]-\frac{1}{5}\sqrt{\frac{11}{31}\left(-13+10\sqrt{2}\right)}\,\text{ArcTanh}\left[\frac{\sqrt{\frac{11}{62\left(-13+10\sqrt{2}\right)}}\left(6-7\sqrt{2}+\left(20-13\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right]$$

Result (type 3, 1133 leaves):

$$\frac{1}{5}\sqrt{2}\,\text{ArcSinh}\left[\frac{-1+4x}{\sqrt{23}}\right]-\left(i\left(-13i+\sqrt{31}\right)\text{ArcTan}\left[\left(31\left(7588i+4224\sqrt{31}-27836ix+3872\sqrt{31}x+4347ix^2+2706\sqrt{31}x^2-31860ix^3+2970\sqrt{31}x^3-8675ix^4+1100\sqrt{31}x^4\right)\right)\Big/\left(65472+35044i\sqrt{31}+1083016x-46668i\sqrt{31}x+340318x^2-308889i\sqrt{31}x^2+514910x^3-143180i\sqrt{31}x^3+443300x^4-262775i\sqrt{31}x^4-1000i\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}+2500i\sqrt{682\left(13+i\sqrt{31}\right)}x\sqrt{3-x+2x^2}+3500i\sqrt{682\left(13+i\sqrt{31}\right)}x^2\sqrt{3-x+2x^2}+10000i\sqrt{682\left(13+i\sqrt{31}\right)}x^3\sqrt{3-x+2x^2}\right)\right]\right)\Big/\left(5\sqrt{\frac{62}{11}\left(13+i\sqrt{31}\right)}\right)-$$

$$\left(i\left(13i+\sqrt{31}\right)\text{ArcTanh}\left[\left(-65472i-35044\sqrt{31}-1083016ix+46668\sqrt{31}x-340318ix^2+308889\sqrt{31}x^2-514910ix^3+143180\sqrt{31}x^3-443300ix^4+262775\sqrt{31}x^4-63000\sqrt{22\left(-13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}-72500\sqrt{22\left(-13+i\sqrt{31}\right)}x\sqrt{3-x+2x^2}-124500\right.\right.\right.$$

$$\sqrt{22\left(-13+\mathrm{i}\sqrt{31}\right)}\;x^2\sqrt{3-x+2x^2}+55000\sqrt{22\left(-13+\mathrm{i}\sqrt{31}\right)}\;x^3\sqrt{3-x+2x^2}\Bigg)\Bigg/$$

$$\Big(1764772\,\mathrm{i}+130944\sqrt{31}+2352916\,\mathrm{i}\,x+120032\sqrt{31}\,x+3090243\,\mathrm{i}\,x^2+$$

$$83886\sqrt{31}\,x^2-2052340\,\mathrm{i}\,x^3+92070\sqrt{31}\,x^3+1493925\,\mathrm{i}\,x^4+34100\sqrt{31}\,x^4\Big)\Big]\Bigg)\Bigg/$$

$$\left(5\sqrt{\frac{62}{11}\left(-13+\mathrm{i}\sqrt{31}\right)}\right)-\frac{\left(-13\,\mathrm{i}+\sqrt{31}\right)\mathrm{Log}\left[\left(-3\,\mathrm{i}+\sqrt{31}-10\,\mathrm{i}\,x\right)^2\left(3\,\mathrm{i}+\sqrt{31}+10\,\mathrm{i}\,x\right)^2\right]}{10\sqrt{\frac{62}{11}\left(13+\mathrm{i}\sqrt{31}\right)}}+$$

$$\frac{\mathrm{i}\left(13\,\mathrm{i}+\sqrt{31}\right)\mathrm{Log}\left[\left(-3\,\mathrm{i}+\sqrt{31}-10\,\mathrm{i}\,x\right)^2\left(3\,\mathrm{i}+\sqrt{31}+10\,\mathrm{i}\,x\right)^2\right]}{10\sqrt{\frac{62}{11}\left(-13+\mathrm{i}\sqrt{31}\right)}}-$$

$$\Bigg(\mathrm{i}\left(13\,\mathrm{i}+\sqrt{31}\right)$$

$$\mathrm{Log}\Big[\left(2+3x+5x^2\right)\Big(-142\,\mathrm{i}+66\sqrt{31}+469\,\mathrm{i}\,x-22\sqrt{31}\,x+327\,\mathrm{i}\,x^2+44\sqrt{31}\,x^2+$$

$$\mathrm{i}\sqrt{682\left(-13+\mathrm{i}\sqrt{31}\right)}\sqrt{3-x+2x^2}-4\,\mathrm{i}\sqrt{682\left(-13+\mathrm{i}\sqrt{31}\right)}\;x\sqrt{3-x+2x^2}\Big)\Big]\Bigg)\Bigg/$$

$$\left(10\sqrt{\frac{62}{11}\left(-13+\mathrm{i}\sqrt{31}\right)}\right)+\Bigg(\left(-13\,\mathrm{i}+\sqrt{31}\right)\mathrm{Log}\Big[\left(2+3x+5x^2\right)$$

$$\Big(-1858\,\mathrm{i}+66\sqrt{31}+1041\,\mathrm{i}\,x-22\sqrt{31}\,x-817\,\mathrm{i}\,x^2+44\sqrt{31}\,x^2-63\,\mathrm{i}\sqrt{22\left(13+\mathrm{i}\sqrt{31}\right)}$$

$$\sqrt{3-x+2x^2}+22\,\mathrm{i}\sqrt{22\left(13+\mathrm{i}\sqrt{31}\right)}\;x\sqrt{3-x+2x^2}\Big)\Big]\Bigg)\Bigg/\left(10\sqrt{\frac{62}{11}\left(13+\mathrm{i}\sqrt{31}\right)}\right)$$

## Problem 63: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{\sqrt{3-x+2x^2}}{\left(2+3x+5x^2\right)^2}\,dx$$

Optimal (type 3, 188 leaves, 6 steps):

$$\frac{(3+10x)\sqrt{3-x+2x^2}}{31\,(2+3x+5x^2)}+$$

$$\frac{1}{62}\sqrt{\frac{1}{682}\left(70517+49942\sqrt{2}\right)}\;\text{ArcTan}\left[\frac{\sqrt{\frac{11}{31\left(70517+49942\sqrt{2}\right)}}\left(419+277\sqrt{2}+\left(973+696\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right]-$$

$$\frac{1}{62}\sqrt{\frac{1}{682}\left(-70517+49942\sqrt{2}\right)}$$

$$\text{ArcTanh}\left[\frac{\sqrt{\frac{11}{31\left(-70517+49942\sqrt{2}\right)}}\left(419-277\sqrt{2}+\left(973-696\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right]$$

Result (type 3, 1066 leaves):

$$\frac{(3+10x)\sqrt{3-x+2x^2}}{31\,(2+3x+5x^2)}-$$

$$\Big(i\left(-348\,i+11\sqrt{31}\right)\text{ArcTan}\Big[\Big(31\Big(-5587181+4790313\,i\sqrt{31}+\left(27549757+1169289\,i\sqrt{31}\right)x+$$

$$\left(-32828614+2670822\,i\sqrt{31}\right)x^2+20\left(1416861+85547\,i\sqrt{31}\right)x^3+$$

$$50\,i\left(261413\,i+5324\sqrt{31}\right)x^4\Big)\Big)\Big/\Big(274003389\,i-48486603\sqrt{31}+$$

$$34100\left(7656\,i+7013\sqrt{31}\right)x^4+1248550\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}+$$

$$x^3\left(826454420\,i+92760910\sqrt{31}-12485500\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)+$$

$$x^2\left(95778716\,i+264613118\sqrt{31}-4369925\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)+$$

$$x\left(1344149367\,i+112716791\sqrt{31}-3121375\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)\Big)\Big]\Big)\Big/$$

$$\left(31\sqrt{682\left(13+i\sqrt{31}\right)}\right)+\Big(\left(348-11\,i\sqrt{31}\right)\text{ArcTanh}\Big[$$

$$\Big(11\Big(9\left(23473711\,i+1499997\sqrt{31}\right)+3\left(82253999\,i+1098423\sqrt{31}\right)x+$$

$$\left(273535156\,i+7526862\sqrt{31}\right)x^2+220\left(-1205429\,i+21917\sqrt{31}\right)x^3+$$

$$2200\left(46458\,i+341\sqrt{31}\right)x^4\Big)\Big)\Big/\Big(34100\left(-7656\,i+7013\sqrt{31}\right)x^4+$$

$$x^2\left(-95778716\,i+264613118\sqrt{31}-155444475\sqrt{22\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)+$$

$$x\left(-1\,344\,149\,367\,i+112\,716\,791\sqrt{31}-90\,519\,875\sqrt{22\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2\,x^2}\right)+$$

$$110\,x^3\left(-7\,513\,222\,i+843\,281\sqrt{31}+624\,275\sqrt{22\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2\,x^2}\right)-$$

$$3\left(91\,334\,463\,i+16\,162\,201\sqrt{31}+26\,219\,550\sqrt{22\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2\,x^2}\right)\Bigg)\Bigg]\Bigg)\Bigg/$$

$$\left(31\sqrt{682\,i\left(13\,i+\sqrt{31}\right)}\right)-\frac{\left(-348\,i+11\sqrt{31}\right)\text{Log}\left[400\left(2+3\,x+5\,x^2\right)^2\right]}{62\sqrt{682\left(13+i\sqrt{31}\right)}}+$$

$$\frac{i\left(348\,i+11\sqrt{31}\right)\text{Log}\left[400\left(2+3\,x+5\,x^2\right)^2\right]}{62\sqrt{682\,i\left(13\,i+\sqrt{31}\right)}}+$$

$$\Bigg(\left(-348\,i+11\sqrt{31}\right)$$

$$\text{Log}\Big[\left(2+3\,x+5\,x^2\right)\Big(-1858\,i+66\sqrt{31}+\left(-817\,i+44\sqrt{31}\right)x^2-63\,i\sqrt{286+22\,i\sqrt{31}}$$

$$\sqrt{3-x+2\,x^2}+x\left(1041\,i-22\sqrt{31}+22\,i\sqrt{286+22\,i\sqrt{31}}\sqrt{3-x+2\,x^2}\right)\Big)\Big]\Bigg)\Bigg/$$

$$\left(62\sqrt{682\left(13+i\sqrt{31}\right)}\right)+\Bigg(\left(348-11\,i\sqrt{31}\right)\text{Log}\Big[\left(2+3\,x+5\,x^2\right)$$

$$\Big(-142\,i+66\sqrt{31}+\left(327\,i+44\sqrt{31}\right)x^2+i\sqrt{682\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2\,x^2}+x\Big(469\,i-$$

$$22\sqrt{31}-4\,i\sqrt{682\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2\,x^2}\Big)\Big)\Big]\Bigg)\Bigg/\left(62\sqrt{682\,i\left(13\,i+\sqrt{31}\right)}\right)$$

## Problem 64: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{\sqrt{3-x+2\,x^2}}{\left(2+3\,x+5\,x^2\right)^3}\,dx$$

Optimal (type 3, 223 leaves, 7 steps):

$$\frac{(3+10x)\sqrt{3-x+2x^2}}{62\,(2+3x+5x^2)^2}+\frac{(3464+13665x)\sqrt{3-x+2x^2}}{84568\,(2+3x+5x^2)}+$$
$$\frac{1}{169136}\sqrt{\frac{1}{682}\left(112285869463+79399380740\sqrt{2}\right)}$$
$$\text{ArcTan}\Big[\frac{1}{\sqrt{3-x+2x^2}}\sqrt{\frac{11}{31\left(112285869463+79399380740\sqrt{2}\right)}}$$
$$\left(509587+362788\sqrt{2}+\left(1235163+872375\sqrt{2}\right)x\right)\Big]-$$
$$\frac{1}{169136}\sqrt{\frac{1}{682}\left(-112285869463+79399380740\sqrt{2}\right)}$$
$$\text{ArcTanh}\Big[\frac{1}{\sqrt{3-x+2x^2}}\sqrt{\frac{11}{31\left(-112285869463+79399380740\sqrt{2}\right)}}$$
$$\left(509587-362788\sqrt{2}+\left(1235163-872375\sqrt{2}\right)x\right)\Big]$$

Result (type 3, 1170 leaves):

$$\sqrt{3-x+2x^2}\left(\frac{3+10x}{62\,(2+3x+5x^2)^2}+\frac{3464+13665x}{84568\,(2+3x+5x^2)}\right)-$$
$$\frac{1}{169136\sqrt{682\left(13+i\sqrt{31}\right)}}\,5\,i\left(-174475\,i+6521\sqrt{31}\right)\text{ArcTan}\Big[$$
$$\Big(31\Big(779181710662\,i+621237299826\sqrt{31}-3659080865574\,i\,x+210477093398\sqrt{31}\,x+$$
$$3786698475623\,i\,x^2+345136479754\sqrt{31}\,x^2-3744647381480\,i\,x^3+$$
$$254982903010\sqrt{31}\,x^3+1313174142725\,i\,x^4+46775785100\sqrt{31}\,x^4\Big)\Big)\Big/$$
$$\Big(31886584896738+6160809644426\,i\sqrt{31}+173254405285214\,x-$$
$$13553199916122\,i\sqrt{31}\,x+18159288904922\,x^2-36221356993731\,i\sqrt{31}\,x^2+$$
$$103190181962890\,x^3-13468529326720\,i\sqrt{31}\,x^3+38797325297500\,x^4-$$
$$32372991877825\,i\sqrt{31}\,x^4-158798761480\,i\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}+$$
$$396996903700\,i\sqrt{682\left(13+i\sqrt{31}\right)}\,x\sqrt{3-x+2x^2}+555795665180\,i\sqrt{682\left(13+i\sqrt{31}\right)}$$
$$x^2\sqrt{3-x+2x^2}+1587987614800\,i\sqrt{682\left(13+i\sqrt{31}\right)}\,x^3\sqrt{3-x+2x^2}\Big)\Big]-$$

$$\frac{1}{169136 \sqrt{682 \left(-13+i \sqrt{31}\right)}} 5 i \left(174475 i+6521 \sqrt{31}\right)$$

$$\text{ArcTanh}\Big[\Big(-31886584896738 i-6160809644426 \sqrt{31}-173254405285214 i x+$$
$$13553199916122 \sqrt{31} x-18159288904922 i x^2+36221356993731 \sqrt{31} x^2-$$
$$103190181962890 i x^3+13468529326720 \sqrt{31} x^3-38797325297500 i x^4+$$
$$32372991877825 \sqrt{31} x^4-10004321973240 \sqrt{22 \left(-13+i \sqrt{31}\right)} \sqrt{3-x+2 x^2}-$$
$$11512910207300 \sqrt{22 \left(-13+i \sqrt{31}\right)} x \sqrt{3-x+2 x^2}-$$
$$19770445804260 \sqrt{22 \left(-13+i \sqrt{31}\right)} x^2 \sqrt{3-x+2 x^2}+$$
$$8733931881400 \sqrt{22 \left(-13+i \sqrt{31}\right)} x^3 \sqrt{3-x+2 x^2}\Big)\Big/$$
$$\Big(293442889929478 i+19258356294606 \sqrt{31}+350041661437994 i x+6524789895338$$
$$\sqrt{31} x+394738353028687 i x^2+10699230872374 \sqrt{31} x^2-366664166073320 i x^3+$$
$$7904469993310 \sqrt{31} x^3+153820084388525 i x^4+1450049338100 \sqrt{31} x^4\Big)\Big]-$$

$$\left(5 \left(-174475 i+6521 \sqrt{31}\right) \text{Log}\left[\left(-3 i+\sqrt{31}-10 i x\right)^2 \left(3 i+\sqrt{31}+10 i x\right)^2\right]\right)\Big/$$
$$\left(338272 \sqrt{682 \left(13+i \sqrt{31}\right)}\right)+$$

$$\left(5 i \left(174475 i+6521 \sqrt{31}\right) \text{Log}\left[\left(-3 i+\sqrt{31}-10 i x\right)^2 \left(3 i+\sqrt{31}+10 i x\right)^2\right]\right)\Big/$$
$$\left(338272 \sqrt{682 \left(-13+i \sqrt{31}\right)}\right)-$$

$$\Big(5 i \left(174475 i+6521 \sqrt{31}\right)$$
$$\text{Log}\Big[\left(2+3 x+5 x^2\right) \Big(-142 i+66 \sqrt{31}+469 i x-22 \sqrt{31} x+327 i x^2+44 \sqrt{31} x^2+$$
$$i \sqrt{682 \left(-13+i \sqrt{31}\right)} \sqrt{3-x+2 x^2}-4 i \sqrt{682 \left(-13+i \sqrt{31}\right)} x \sqrt{3-x+2 x^2}\Big)\Big]\Big)\Big/$$
$$\left(338272 \sqrt{682 \left(-13+i \sqrt{31}\right)}\right)+\Big(5 \left(-174475 i+6521 \sqrt{31}\right)$$
$$\text{Log}\Big[\left(2+3 x+5 x^2\right) \Big(-1858 i+66 \sqrt{31}+1041 i x-22 \sqrt{31} x-$$
$$817 i x^2+44 \sqrt{31} x^2-63 i \sqrt{22 \left(13+i \sqrt{31}\right)} \sqrt{3-x+2 x^2}+$$

$$22\, i \sqrt{22\left(13+i\sqrt{31}\right)}\; x\sqrt{3-x+2x^2}\Bigg)\Bigg]\Bigg)\Bigg/\left(338272\sqrt{682\left(13+i\sqrt{31}\right)}\right)$$

## Problem 69: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\left(3-x+2x^2\right)^{3/2}}{2+3x+5x^2}\,dx$$

Optimal (type 3, 197 leaves, 9 steps):

$$-\frac{1}{100}(49-20x)\sqrt{3-x+2x^2}-\frac{2203\,\text{ArcSinh}\left[\frac{1-4x}{\sqrt{23}}\right]}{1000\sqrt{2}}+$$
$$\frac{11}{125}\sqrt{\frac{11}{31}\left(247+500\sqrt{2}\right)}\;\text{ArcTan}\left[\frac{\sqrt{\frac{11}{62\left(247+500\sqrt{2}\right)}}\left(8+61\sqrt{2}+\left(130+69\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right]-$$
$$\frac{11}{125}\sqrt{\frac{11}{31}\left(-247+500\sqrt{2}\right)}\;\text{ArcTanh}\left[\frac{\sqrt{\frac{11}{62\left(-247+500\sqrt{2}\right)}}\left(8-61\sqrt{2}+\left(130-69\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right]$$

Result (type 3, 1175 leaves):

$$\left(-\frac{49}{100}+\frac{x}{5}\right)\sqrt{3-x+2x^2}+\frac{2203\,\text{ArcSinh}\left[\frac{-1+4x}{\sqrt{23}}\right]}{1000\sqrt{2}}+$$
$$\Bigg(11\left(69\,i+13\sqrt{31}\right)\text{ArcTan}\Bigg[\Bigg(10827432+603036\,i\sqrt{31}-28693104\,x+2334908\,i\sqrt{31}\,x-$$
$$30301942\,x^2-15923341\,i\sqrt{31}\,x^2-1428790\,x^3-9329420\,i\sqrt{31}\,x^3-$$
$$30587700\,x^4-12631475\,i\sqrt{31}\,x^4+3150000\,i\sqrt{22\left(-13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}+$$
$$3625000\,i\sqrt{22\left(-13+i\sqrt{31}\right)}\;x\sqrt{3-x+2x^2}+6225000\,i\sqrt{22\left(-13+i\sqrt{31}\right)}$$
$$x^2\sqrt{3-x+2x^2}-2750000\,i\sqrt{22\left(-13+i\sqrt{31}\right)}\;x^3\sqrt{3-x+2x^2}\Bigg)\Bigg/$$
$$\Big(82622268\,i+5966136\sqrt{31}+117642204\,i\,x+12374208\sqrt{31}\,x+229312267\,i\,x^2+7834134$$
$$\sqrt{31}\,x^2-63298460\,i\,x^3+6693830\sqrt{31}\,x^3+136148325\,i\,x^4+5762900\sqrt{31}\,x^4\Big)\Bigg]\Bigg)\Bigg/$$
$$\left(125\sqrt{\frac{62}{11}\left(-13+i\sqrt{31}\right)}\right)-\Bigg(11\,i\left(-69\,i+13\sqrt{31}\right)\text{ArcTan}\Bigg[$$
$$\Big(31\Big(560572\,i+192456\sqrt{31}-1391684\,i\,x+399168\sqrt{31}\,x-2195557\,i\,x^2+$$

$$252714\sqrt{31}\,x^2-2861340\,i\,x^3+215930\sqrt{31}\,x^3-2416075\,i\,x^4+185900\sqrt{31}\,x^4\Big)\Big)\Big/$$

$$\Big(-10827432+603036\,i\sqrt{31}+28693104\,x+2334908\,i\sqrt{31}\,x+30301942\,x^2-15923341\,i\sqrt{31}\,x^2+1428790\,x^3-9329420\,i\sqrt{31}\,x^3+30587700\,x^4-12631475\,i\sqrt{31}\,x^4-50000\,i\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}+125000\,i\sqrt{682\left(13+i\sqrt{31}\right)}\,x\sqrt{3-x+2x^2}+175000\,i\sqrt{682\left(13+i\sqrt{31}\right)}\,x^2\sqrt{3-x+2x^2}+500000\,i\sqrt{682\left(13+i\sqrt{31}\right)}\,x^3\sqrt{3-x+2x^2}\Big)\Big]\Big)\Big/\left(125\sqrt{\frac{62}{11}\left(13+i\sqrt{31}\right)}\right)-$$

$$\left(11\left(-69\,i+13\sqrt{31}\right)\text{Log}\left[\left(-3\,i+\sqrt{31}-10\,i\,x\right)^2\left(3\,i+\sqrt{31}+10\,i\,x\right)^2\right]\right)\Big/\left(250\sqrt{\frac{62}{11}\left(13+i\sqrt{31}\right)}\right)+$$

$$\left(11\,i\left(69\,i+13\sqrt{31}\right)\text{Log}\left[\left(-3\,i+\sqrt{31}-10\,i\,x\right)^2\left(3\,i+\sqrt{31}+10\,i\,x\right)^2\right]\right)\Big/\left(250\sqrt{\frac{62}{11}\left(-13+i\sqrt{31}\right)}\right)-$$

$$\Big(11\,i\left(69\,i+13\sqrt{31}\right)\text{Log}\Big[\left(2+3x+5x^2\right)\Big(-142\,i+66\sqrt{31}+469\,i\,x-22\sqrt{31}\,x+327\,i\,x^2+44\sqrt{31}\,x^2+i\sqrt{682\left(-13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}-4\,i\sqrt{682\left(-13+i\sqrt{31}\right)}\,x\sqrt{3-x+2x^2}\Big)\Big]\Big)\Big/$$

$$\left(250\sqrt{\frac{62}{11}\left(-13+i\sqrt{31}\right)}\right)+\Big(11\left(-69\,i+13\sqrt{31}\right)\text{Log}\Big[\left(2+3x+5x^2\right)\Big(-1858\,i+66\sqrt{31}+1041\,i\,x-22\sqrt{31}\,x-817\,i\,x^2+44\sqrt{31}\,x^2-63\,i\sqrt{22\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}+22\,i\sqrt{22\left(13+i\sqrt{31}\right)}\,x\sqrt{3-x+2x^2}\Big)\Big]\Big)\Big/\left(250\sqrt{\frac{62}{11}\left(13+i\sqrt{31}\right)}\right)$$

## Problem 70: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\left(3-x+2 x^2\right)^{3/2}}{\left(2+3 x+5 x^2\right)^2}\, dx$$

Optimal (type 3, 232 leaves, 10 steps):

$$\frac{4}{155} (4-5 x) \sqrt{3-x+2 x^2}+\frac{(3+10 x) \left(3-x+2 x^2\right)^{3/2}}{31 \left(2+3 x+5 x^2\right)}-\frac{2}{25} \sqrt{2}\, \text{ArcSinh}\left[\frac{1-4 x}{\sqrt{23}}\right]+\frac{1}{1550} \sqrt{\frac{11}{31} \left(3169333+2265350 \sqrt{2}\right)}\, \text{ArcTan}\left[\frac{\sqrt{\frac{11}{62 \left(3169333+2265350 \sqrt{2}\right)}} \left(3514+2963 \sqrt{2}+\left(9440+6477 \sqrt{2}\right) x\right)}{\sqrt{3-x+2 x^2}}\right]-\frac{1}{1550} \sqrt{\frac{11}{31} \left(-3169333+2265350 \sqrt{2}\right)}\, \text{ArcTanh}\left[\frac{\sqrt{\frac{11}{62 \left(-3169333+2265350 \sqrt{2}\right)}} \left(3514-2963 \sqrt{2}+\left(9440-6477 \sqrt{2}\right) x\right)}{\sqrt{3-x+2 x^2}}\right]$$

Result (type 3, 1088 leaves):

$$\frac{1}{192200} \left(\frac{13640 (7+13 x) \sqrt{3-x+2 x^2}}{2+3 x+5 x^2}+15376 \sqrt{2}\, \text{ArcSinh}\left[\frac{-1+4 x}{\sqrt{23}}\right]-\frac{1}{\sqrt{\frac{1}{682} \left(13+i \sqrt{31}\right)}} 2 i \left(-6477 i+329 \sqrt{31}\right) \text{ArcTan}\Big[\Big(31 \left(-1332489508+919236384 i \sqrt{31}+\left(5674354076+503954352 i \sqrt{31}\right) x+\left(-3996168827+521299746 i \sqrt{31}\right) x^2+10 \left(589405626+48071177 i \sqrt{31}\right) x^3+25 i \left(25228373 i+4762604 \sqrt{31}\right) x^4\right)\Big) \Big/ \Big(775 \left(93761052 i+66916121 \sqrt{31}\right) x^4+x^3 \left(138879039310 i+24348414380 \sqrt{31}-2265350000 \sqrt{682 \left(13+i \sqrt{31}\right)} \sqrt{3-x+2 x^2}\right)+x^2 \left(44889007438 i+59243175649 \sqrt{31}-792872500 \sqrt{682 \left(13+i \sqrt{31}\right)} \sqrt{3-x+2 x^2}\right)+x \left(251068416456 i+16524047788 \sqrt{31}-566337500 \sqrt{682 \left(13+i \sqrt{31}\right)} \sqrt{3-x+2 x^2}\right)+ \cdots$$

$$4 \left(8\,811\,565\,488\, i - 2\,160\,968\,001 \sqrt{31} + 56\,633\,750 \sqrt{682 \left(13 + i \sqrt{31}\right)} \sqrt{3 - x + 2 x^2}\right)\Big)\Big] +$$

$$\frac{1}{\sqrt{\frac{1}{682}\, i \left(13\, i + \sqrt{31}\right)}} 2 \left(6477 - 329\, i \sqrt{31}\right) \text{ArcTanh}\Big[$$

$$\Big(775 \left(-93\,761\,052\, i + 66\,916\,121 \sqrt{31}\right) x^4 + x^2 \Big(-44\,889\,007\,438\, i + 59\,243\,175\,649 \sqrt{31} -$$

$$28\,203\,607\,500 \sqrt{22\, i \left(13\, i + \sqrt{31}\right)} \sqrt{3 - x + 2 x^2}\Big) + 4 x \Big(-62\,767\,104\,114\, i +$$

$$4\,131\,011\,947 \sqrt{31} - 4\,105\,946\,875 \sqrt{22\, i \left(13\, i + \sqrt{31}\right)} \sqrt{3 - x + 2 x^2}\Big) -$$

$$12 \left(2\,937\,188\,496\, i + 720\,322\,667 \sqrt{31} + 1\,189\,308\,750 \sqrt{22\, i \left(13\, i + \sqrt{31}\right)} \sqrt{3 - x + 2 x^2}\right) +$$

$$10 x^3 \Big(-13\,887\,903\,931\, i + 2\,434\,841\,438 \sqrt{31} +$$

$$1\,245\,942\,500 \sqrt{22\, i \left(13\, i + \sqrt{31}\right)} \sqrt{3 - x + 2 x^2}\Big)\Big) \Big/$$

$$\Big(36 \left(11\,437\,856\,257\, i + 791\,564\,664 \sqrt{31}\right) + 12 \left(42\,786\,843\,863\, i + 1\,301\,882\,076 \sqrt{31}\right) x +$$
$$\left(606\,694\,141\,363\, i + 16\,160\,292\,126 \sqrt{31}\right) x^2 + 10 \left(-50\,595\,065\,594\, i + 1\,490\,206\,487 \sqrt{31}\right) x^3 +$$
$$25 \left(10\,318\,135\,437\, i + 147\,640\,724 \sqrt{31}\right) x^4\Big)\Big] -$$

$$\frac{\left(-6477\, i + 329 \sqrt{31}\right) \text{Log}\left[400 \left(2 + 3 x + 5 x^2\right)^2\right]}{\sqrt{\frac{1}{682} \left(13 + i \sqrt{31}\right)}} +$$

$$\frac{i \left(6477\, i + 329 \sqrt{31}\right) \text{Log}\left[400 \left(2 + 3 x + 5 x^2\right)^2\right]}{\sqrt{\frac{1}{682}\, i \left(13\, i + \sqrt{31}\right)}} +$$

$$\Big(\left(-6477\, i + 329 \sqrt{31}\right)$$

$$\text{Log}\Big[\left(2 + 3 x + 5 x^2\right) \Big(-1858\, i + 66 \sqrt{31} + \left(-817\, i + 44 \sqrt{31}\right) x^2 - 63\, i \sqrt{286 + 22\, i \sqrt{31}}$$

$$\sqrt{3 - x + 2 x^2} + x \left(1041\, i - 22 \sqrt{31} + 22\, i \sqrt{286 + 22\, i \sqrt{31}} \sqrt{3 - x + 2 x^2}\right)\Big)\Big]\Big) \Big/$$

$$\left(\sqrt{\frac{1}{682} \left(13 + i \sqrt{31}\right)}\right) + \Big(\left(6477 - 329\, i \sqrt{31}\right) \text{Log}\Big[\left(2 + 3 x + 5 x^2\right)$$

$$\left(-142\,i+66\sqrt{31}+\left(327\,i+44\sqrt{31}\right)x^2+i\sqrt{682\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2x^2}+x\left(469\,i-22\sqrt{31}-4\,i\sqrt{682\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)\right)\Bigg]\Bigg)\Bigg/\left(\sqrt{\frac{1}{682}\,i\left(13\,i+\sqrt{31}\right)}\right)\Bigg)$$

## Problem 71: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\left(3-x+2x^2\right)^{3/2}}{\left(2+3x+5x^2\right)^3}\,dx$$

Optimal (type 3, 223 leaves, 7 steps):

$$\frac{(3+10x)\left(3-x+2x^2\right)^{3/2}}{62\left(2+3x+5x^2\right)^2}+\frac{3(277+696x)\sqrt{3-x+2x^2}}{3844\left(2+3x+5x^2\right)}+\frac{1}{7688}3\sqrt{\frac{1}{682}\left(366990269+259509026\sqrt{2}\right)}\,\text{ArcTan}\left[\frac{1}{\sqrt{3-x+2x^2}}\sqrt{\frac{11}{31\left(366990269+259509026\sqrt{2}\right)}}\left(29367+20575\sqrt{2}+\left(70517+49942\sqrt{2}\right)x\right)\right]-\frac{1}{7688}3\sqrt{\frac{1}{682}\left(-366990269+259509026\sqrt{2}\right)}\,\text{ArcTanh}\left[\frac{1}{\sqrt{3-x+2x^2}}\sqrt{\frac{11}{31\left(-366990269+259509026\sqrt{2}\right)}}\left(29367-20575\sqrt{2}+\left(70517-49942\sqrt{2}\right)x\right)\right]$$

Result (type 3, 1171 leaves):

$$\sqrt{3-x+2x^2}\left(\frac{11(7+13x)}{310\left(2+3x+5x^2\right)^2}+\frac{3163+11680x}{19220\left(2+3x+5x^2\right)}\right)-\frac{1}{3844\sqrt{682\left(13+i\sqrt{31}\right)}}3\,i\left(-24971\,i+902\sqrt{31}\right)$$

$$\text{ArcTan}\Big[\Big(31\left(31227856109\,i+25278538857\sqrt{31}-148151300773\,i\,x+8050492021\sqrt{31}\,x+158238605196\,i\,x^2+14045028558\sqrt{31}\,x^2-151681537680\,i\,x^3+10089483360\sqrt{31}\,x^3+56810945600\,i\,x^4+1789928800\sqrt{31}\,x^4\right)\Big)\Big/$$

$$\left(1329350472021+251835138467\, i \sqrt{31}+7060303464863\, x-560818641999\, i \sqrt{31}\, x+689282588324\, x^2-1457613959802\, i \sqrt{31}\, x^2+4234217180380\, x^3-535663546990\, i \sqrt{31}\, x^3+1536126024400\, x^4-1305722486200\, i \sqrt{31}\, x^4-6487725650\, i \sqrt{682\left(13+i \sqrt{31}\right)} \sqrt{3-x+2 x^2}+16219314125\, i \sqrt{682\left(13+i \sqrt{31}\right)}\, x \sqrt{3-x+2 x^2}+22707039775\, i \sqrt{682\left(13+i \sqrt{31}\right)}\, x^2 \sqrt{3-x+2 x^2}+64877256500\, i \sqrt{682\left(13+i \sqrt{31}\right)}\, x^3 \sqrt{3-x+2 x^2}\right)\Big]-$$

$$\frac{1}{3844 \sqrt{682\left(-13+i \sqrt{31}\right)}} 3\, i \left(24971\, i+902 \sqrt{31}\right) \text{ArcTanh}\Big[\left(11\left(1091580705511\, i+71239518597 \sqrt{31}+1296309231133\, i\, x+22687750241 \sqrt{31}\, x+1456138041834\, i\, x^2+39581444118 \sqrt{31}\, x^2-1365505300720\, i\, x^3+28433998560 \sqrt{31}\, x^3+562393146150\, i\, x^4+5044344800 \sqrt{31}\, x^4\right)\right) \Big/$$

$$\left(-1329350472021\, i-251835138467 \sqrt{31}-7060303464863\, i\, x+560818641999 \sqrt{31}\, x-689282588324\, i\, x^2+1457613959802 \sqrt{31}\, x^2-4234217180380\, i\, x^3+535663546990 \sqrt{31}\, x^3-1536126024400\, i\, x^4+1305722486200 \sqrt{31}\, x^4-408726715950 \sqrt{22\left(-13+i \sqrt{31}\right)} \sqrt{3-x+2 x^2}-470360109625 \sqrt{22\left(-13+i \sqrt{31}\right)}\, x \sqrt{3-x+2 x^2}-807721843425 \sqrt{22\left(-13+i \sqrt{31}\right)}\, x^2 \sqrt{3-x+2 x^2}+356824910750 \sqrt{22\left(-13+i \sqrt{31}\right)}\, x^3 \sqrt{3-x+2 x^2}\right)\Big]-$$

$$\left(3\left(-24971\, i+902 \sqrt{31}\right) \text{Log}\left[\left(-3\, i+\sqrt{31}-10\, i\, x\right)^2 \left(3\, i+\sqrt{31}+10\, i\, x\right)^2\right]\right) \Big/ \left(7688 \sqrt{682\left(13+i \sqrt{31}\right)}\right)+$$

$$\left(3\, i\left(24971\, i+902 \sqrt{31}\right) \text{Log}\left[\left(-3\, i+\sqrt{31}-10\, i\, x\right)^2 \left(3\, i+\sqrt{31}+10\, i\, x\right)^2\right]\right) \Big/ \left(7688 \sqrt{682\left(-13+i \sqrt{31}\right)}\right)-$$

$$\Big(3\, i\left(24971\, i+902 \sqrt{31}\right)$$

$$\text{Log}\Big[\left(2+3 x+5 x^2\right)\Big(-142\, i+66 \sqrt{31}+469\, i\, x-22 \sqrt{31}\, x+327\, i\, x^2+44 \sqrt{31}\, x^2+$$

$$i \sqrt{682 \left(-13+i \sqrt{31}\right)} \sqrt{3-x+2 x^2}-4 i \sqrt{682 \left(-13+i \sqrt{31}\right)} x \sqrt{3-x+2 x^2}\Big]\Big)\Big/$$
$$\left(7688 \sqrt{682 \left(-13+i \sqrt{31}\right)}\right)+\Bigg(3 \left(-24971 i+902 \sqrt{31}\right) \text{Log}\Big[\Big(2+3 x+5 x^2\Big)$$
$$\Big(-1858 i+66 \sqrt{31}+1041 i x-22 \sqrt{31} x-817 i x^2+44 \sqrt{31} x^2-63 i \sqrt{22 \left(13+i \sqrt{31}\right)}$$
$$\sqrt{3-x+2 x^2}+22 i \sqrt{22 \left(13+i \sqrt{31}\right)} x \sqrt{3-x+2 x^2}\Big)\Big]\Bigg)\Big/\left(7688 \sqrt{682 \left(13+i \sqrt{31}\right)}\right)$$

## Problem 76: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\left(3-x+2 x^2\right)^{5/2}}{2+3 x+5 x^2} \, dx$$

Optimal (type 3, 222 leaves, 10 steps):

$$-\frac{\left(226249-99620 x\right) \sqrt{3-x+2 x^2}}{80000}-\frac{1}{600} \left(103-60 x\right) \left(3-x+2 x^2\right)^{3/2}-$$
$$\frac{7216203 \text{ArcSinh}\left[\frac{1-4 x}{\sqrt{23}}\right]}{800000 \sqrt{2}}-\frac{1}{3125} 121 \sqrt{\frac{11}{31} \left(-15457+25000 \sqrt{2}\right)}$$
$$\text{ArcTan}\left[\frac{\sqrt{\frac{11}{62 \left(-15457+25000 \sqrt{2}\right)}} \left(196-443 \sqrt{2}-\left(690+247 \sqrt{2}\right) x\right)}{\sqrt{3-x+2 x^2}}\right]+\frac{1}{3125}$$
$$121 \sqrt{\frac{11}{31} \left(15457+25000 \sqrt{2}\right)} \text{ArcTanh}\left[\frac{\sqrt{\frac{11}{62 \left(15457+25000 \sqrt{2}\right)}} \left(196+443 \sqrt{2}-\left(690-247 \sqrt{2}\right) x\right)}{\sqrt{3-x+2 x^2}}\right]$$

Result (type 3, 1189 leaves):

$$\sqrt{3-x+2 x^2} \left(-\frac{267449}{80000}+\frac{20603 x}{12000}-\frac{133 x^2}{300}+\frac{x^3}{5}\right)+$$
$$\frac{7216203 \text{ArcSinh}\left[\frac{-1+4 x}{\sqrt{23}}\right]}{800000 \sqrt{2}}+\frac{1}{3125 \sqrt{\frac{62}{11} \left(-13+i \sqrt{31}\right)}} 121 \left(247 i+119 \sqrt{31}\right)$$
$$\text{ArcTan}\Big[\Big(910772808-46000516 i \sqrt{31}+727715824 x+277778652 i \sqrt{31} x-1240038998 x^2-$$
$$326488029 i \sqrt{31} x^2+1188688490 x^3-285779980 i \sqrt{31} x^3-1002301300 x^4-$$

$$214\,634\,275\,i\sqrt{31}\,x^4+157\,500\,000\,i\sqrt{22\left(-13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}+$$
$$181\,250\,000\,i\sqrt{22\left(-13+i\sqrt{31}\right)}\,x\sqrt{3-x+2x^2}+311\,250\,000\,i\sqrt{22\left(-13+i\sqrt{31}\right)}$$
$$x^2\sqrt{3-x+2x^2}-137\,500\,000\,i\sqrt{22\left(-13+i\sqrt{31}\right)}\,x^3\sqrt{3-x+2x^2}\Bigg)\Bigg/$$
$$\Big(4\,168\,906\,492\,i+186\,603\,384\sqrt{31}+4\,941\,322\,076\,i\,x+673\,090\,352\sqrt{31}\,x+$$
$$14\,142\,713\,923\,i\,x^2+603\,640\,246\sqrt{31}\,x^2-1\,371\,093\,740\,i\,x^3+$$
$$248\,749\,270\sqrt{31}\,x^3+8\,825\,296\,925\,i\,x^4+482\,890\,100\sqrt{31}\,x^4\Big)\Big]-$$
$$\frac{1}{3125\sqrt{\frac{62}{11}\left(13+i\sqrt{31}\right)}}121\,i\left(-247\,i+119\sqrt{31}\right)\text{ArcTan}\Big[$$
$$\Big(31\,\Big(26\,809\,468\,i+6\,019\,464\sqrt{31}-39\,236\,196\,i\,x+21\,712\,592\sqrt{31}\,x-196\,135\,933\,i\,x^2+19\,472\,266$$
$$\sqrt{31}\,x^2-200\,932\,460\,i\,x^3+8\,024\,170\sqrt{31}\,x^3-185\,896\,675\,i\,x^4+15\,577\,100\sqrt{31}\,x^4\Big)\Big)\Big/$$
$$\Bigg(-910\,772\,808-46\,000\,516\,i\sqrt{31}-727\,715\,824\,x+277\,778\,652\,i\sqrt{31}\,x+1\,240\,038\,998\,x^2-$$
$$326\,488\,029\,i\sqrt{31}\,x^2-1\,188\,688\,490\,x^3-285\,779\,980\,i\sqrt{31}\,x^3+1\,002\,301\,300\,x^4-$$
$$214\,634\,275\,i\sqrt{31}\,x^4-2\,500\,000\,i\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}+$$
$$6\,250\,000\,i\sqrt{682\left(13+i\sqrt{31}\right)}\,x\sqrt{3-x+2x^2}+8\,750\,000\,i\sqrt{682\left(13+i\sqrt{31}\right)}$$
$$x^2\sqrt{3-x+2x^2}+25\,000\,000\,i\sqrt{682\left(13+i\sqrt{31}\right)}\,x^3\sqrt{3-x+2x^2}\Bigg)\Bigg]-$$
$$\left(121\left(-247\,i+119\sqrt{31}\right)\text{Log}\Big[\left(-3\,i+\sqrt{31}-10\,i\,x\right)^2\left(3\,i+\sqrt{31}+10\,i\,x\right)^2\Big]\right)\Big/$$
$$\left(6250\sqrt{\frac{62}{11}\left(13+i\sqrt{31}\right)}\right)+$$
$$\left(121\,i\left(247\,i+119\sqrt{31}\right)\text{Log}\Big[\left(-3\,i+\sqrt{31}-10\,i\,x\right)^2\left(3\,i+\sqrt{31}+10\,i\,x\right)^2\Big]\right)\Big/$$
$$\left(6250\sqrt{\frac{62}{11}\left(-13+i\sqrt{31}\right)}\right)-$$
$$\Bigg(121\,i\left(247\,i+119\sqrt{31}\right)$$
$$\text{Log}\Big[\left(2+3x+5x^2\right)\Big(-142\,i+66\sqrt{31}+469\,i\,x-22\sqrt{31}\,x+327\,i\,x^2+44\sqrt{31}\,x^2+$$

$$\left.\left.\left.i\sqrt{682\left(-13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}-4i\sqrt{682\left(-13+i\sqrt{31}\right)}\,x\sqrt{3-x+2x^2}\right]\right)\right/$$

$$\left(6250\sqrt{\frac{62}{11}\left(-13+i\sqrt{31}\right)}\right)+\left(121\left(-247i+119\sqrt{31}\right)\text{Log}\left[\left(2+3x+5x^2\right)\right.\right.$$

$$\left(-1858i+66\sqrt{31}+1041ix-22\sqrt{31}\,x-817ix^2+44\sqrt{31}\,x^2-63i\sqrt{22\left(13+i\sqrt{31}\right)}\right.$$

$$\left.\left.\left.\sqrt{3-x+2x^2}+22i\sqrt{22\left(13+i\sqrt{31}\right)}\,x\sqrt{3-x+2x^2}\right)\right]\right)\Big/\left(6250\sqrt{\frac{62}{11}\left(13+i\sqrt{31}\right)}\right)$$

## Problem 77: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{\left(3-x+2x^2\right)^{5/2}}{\left(2+3x+5x^2\right)^2}\,dx$$

Optimal (type 3, 255 leaves, 11 steps):

$$-\frac{(1277+2240x)\sqrt{3-x+2x^2}}{7750}+\frac{4}{155}(4-5x)\left(3-x+2x^2\right)^{3/2}+\frac{(3+10x)\left(3-x+2x^2\right)^{5/2}}{31\left(2+3x+5x^2\right)}-$$

$$\frac{4799\,\text{ArcSinh}\left[\frac{1-4x}{\sqrt{23}}\right]}{2500\sqrt{2}}+\frac{1}{38750}11\sqrt{\frac{11}{31}\left(224510383+194487500\sqrt{2}\right)}\,\text{ArcTan}\left[\frac{1}{\sqrt{3-x+2x^2}}\right.$$

$$\left.\sqrt{\frac{11}{62\left(224510383+194487500\sqrt{2}\right)}}\left(21136+33287\sqrt{2}+\left(87710+54423\sqrt{2}\right)x\right)\right]-$$

$$\frac{1}{38750}11\sqrt{\frac{11}{31}\left(-224510383+194487500\sqrt{2}\right)}\,\text{ArcTanh}\left[\frac{1}{\sqrt{3-x+2x^2}}\right.$$

$$\left.\sqrt{\frac{11}{62\left(-224510383+194487500\sqrt{2}\right)}}\left(21136-33287\sqrt{2}+\left(87710-54423\sqrt{2}\right)x\right)\right]$$

Result (type 3, 1110 leaves):

$$\frac{1}{4805000}\left(\frac{620\sqrt{3-x+2x^2}\left(8996+9289x-12555x^2+3100x^3\right)}{2+3x+5x^2}+\right.$$

$$4611839\sqrt{2}\,\text{ArcSinh}\left[\frac{-1+4x}{\sqrt{23}}\right]-\frac{1}{\sqrt{\frac{1}{682}\left(13+i\sqrt{31}\right)}}22\,i\left(-54423\,i+5471\sqrt{31}\right)\text{ArcTan}\Big[$$

$$\left(31\left(-171942569308+82792691784\,i\sqrt{31}+4\left(141772726169+25072968888\,i\sqrt{31}\right)x+7\left(21854082139+8850407478\,i\sqrt{31}\right)x^2+10\left(73391640726+6879711377\,i\sqrt{31}\right)x^3+25\left(14752730827+1317001004\,i\sqrt{31}\right)x^4\right)\right)\Big/$$

$$\left(775\left(13100922252\,i+6966216221\sqrt{31}\right)x^4+x^3\left(7951179150310\,i+3217382742380\sqrt{31}-194487500000\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)+x^2\left(8562978915238\,i+6467393362549\sqrt{31}-68070625000\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)+x\left(19618154755056\,i+442968415588\sqrt{31}-48621875000\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)+4\left(-57356227962\,i-149533752351\sqrt{31}+4862187500\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)\right)\Big]+$$

$$\frac{1}{\sqrt{\frac{1}{682}\,i\left(13\,i+\sqrt{31}\right)}}22\left(54423\,i+5471\sqrt{31}\right)\text{ArcTan}\Big[$$

$$\left(-775\,i\left(-13100922252\,i+6966216221\sqrt{31}\right)x^4-4\,i\,x\left(-4904538688764\,i+110742103897\sqrt{31}-352508593750\sqrt{22\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)+12\left(19118742654+49844584117\,i\sqrt{31}+102105937500\,i\sqrt{22\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)+x^2\left(-8562978915238-6467393362549\,i\sqrt{31}+2421369375000\,i\sqrt{22\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)-10\,i\,x^3\left(-795117915031\,i+321738274238\sqrt{31}+106968125000\sqrt{22\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)\right)\Big/$$

$$\left(36\left(932424454207\,i+71293706814\sqrt{31}\right)+12\left(3879871295413\,i+259087345176\sqrt{31}\right)\right.$$

$$x + \left(67\,464\,554\,574\,163\,i + 1\,920\,538\,422\,726\,\sqrt{31}\right) x^2 + 10\left(-3\,637\,279\,137\,494\,i + 213\,271\,052\,687\,\sqrt{31}\right) x^3 + 25\left(1\,410\,323\,405\,637\,i + 40\,827\,031\,124\,\sqrt{31}\right) x^4\Big)\Big] -$$

$$\frac{11\left(-54\,423\,i + 5471\,\sqrt{31}\right)\,\text{Log}\left[400\left(2+3x+5x^2\right)^2\right]}{\sqrt{\frac{1}{682}\left(13+i\,\sqrt{31}\right)}} + \frac{11\,i\left(54\,423\,i + 5471\,\sqrt{31}\right)\,\text{Log}\left[400\left(2+3x+5x^2\right)^2\right]}{\sqrt{\frac{1}{682}\,i\left(13\,i+\sqrt{31}\right)}} +$$

$$\left(11\left(-54\,423\,i + 5471\,\sqrt{31}\right)\,\text{Log}\Big[\left(2+3x+5x^2\right)\left(-1858\,i + 66\,\sqrt{31} + \left(-817\,i + 44\,\sqrt{31}\right) x^2 - 63\,i\,\sqrt{286 + 22\,i\,\sqrt{31}}\,\sqrt{3 - x + 2x^2} + x\left(1041\,i - 22\,\sqrt{31} + 22\,i\,\sqrt{286 + 22\,i\,\sqrt{31}}\,\sqrt{3 - x + 2x^2}\right)\right)\Big]\right) \Big/ \left(\sqrt{\frac{1}{682}\left(13 + i\,\sqrt{31}\right)}\right) + \left(11\left(54\,423 - 5471\,i\,\sqrt{31}\right)\,\text{Log}\Big[\left(2+3x+5x^2\right)\left(-142\,i + 66\,\sqrt{31} + \left(327\,i + 44\,\sqrt{31}\right) x^2 + i\,\sqrt{682\,i\left(13\,i+\sqrt{31}\right)}\,\sqrt{3-x+2x^2} + x\left(469\,i - 22\,\sqrt{31} - 4\,i\,\sqrt{682\,i\left(13\,i+\sqrt{31}\right)}\,\sqrt{3-x+2x^2}\right)\right)\Big]\right) \Big/ \left(\sqrt{\frac{1}{682}\,i\left(13\,i + \sqrt{31}\right)}\right)\Bigg)$$

## Problem 78: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\left(3 - x + 2x^2\right)^{5/2}}{\left(2 + 3x + 5x^2\right)^3}\,dx$$

Optimal (type 3, 281 leaves, 11 steps):

$$\frac{(11359-12920x)\sqrt{3-x+2x^2}}{48050}+\frac{(3+10x)(3-x+2x^2)^{5/2}}{62(2+3x+5x^2)^2}+\frac{(769+2336x)(3-x+2x^2)^{3/2}}{3844(2+3x+5x^2)}-\frac{4}{125}\sqrt{2}\,\text{ArcSinh}\left[\frac{1-4x}{\sqrt{23}}\right]+\frac{1}{29791000}\sqrt{11\left(1+4\sqrt{2}\right)}\left(2937349+1978861\sqrt{2}\right)\text{ArcTan}\left[\frac{1}{\sqrt{3-x+2x^2}}\sqrt{\frac{11}{62\left(3531015707557+2498852071250\sqrt{2}\right)}}\left(3957722+2937349\sqrt{2}+\left(9832420+6895071\sqrt{2}\right)x\right)\right]-\frac{1}{29791000}\left(2937349-1978861\sqrt{2}\right)\sqrt{11\left(-1+4\sqrt{2}\right)}\,\text{ArcTanh}\left[\frac{1}{\sqrt{3-x+2x^2}}\sqrt{\frac{11}{62\left(-3531015707557+2498852071250\sqrt{2}\right)}}\left(3957722-2937349\sqrt{2}+\left(9832420-6895071\sqrt{2}\right)x\right)\right]$$

Result (type 3, 1203 leaves):

$$\sqrt{3-x+2x^2}\left(\frac{121(61+69x)}{7750(2+3x+5x^2)^2}+\frac{11(35579+97155x)}{480500(2+3x+5x^2)}\right)+\frac{4}{125}\sqrt{2}\,\text{ArcSinh}\left[\frac{-1+4x}{\sqrt{23}}\right]-\frac{1}{961000\sqrt{\frac{62}{11}\left(13+i\sqrt{31}\right)}}i\left(-6895071i+280267\sqrt{31}\right)\text{ArcTan}\Big[\Big(31\Big(1286646864280132i+987421307406336\sqrt{31}-5888947864615004ix+386335744679808\sqrt{31}x+5595672650742083ix^2+549395637070434\sqrt{31}x^2-6029547074679540ix^3+433781845112330\sqrt{31}x^3+1742846817367925ix^4+86404550417900\sqrt{31}x^4\Big)\Big)\Big/\Big(47470658398910208+9672976872245316i\sqrt{31}+274205806118598024x-20598732824854252i\sqrt{31}x+33816025817929102x^2-59172316611299521i\sqrt{31}x^2+160404448215022990x^3-22636449983151020i\sqrt{31}x^3+65896915460933700x^4-52587956640176975i\sqrt{31}x^4-249885207125000i\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}+624713017812500i\sqrt{682\left(13+i\sqrt{31}\right)}x\sqrt{3-x+2x^2}+874598224937500i\sqrt{682\left(13+i\sqrt{31}\right)}x^2\sqrt{3-x+2x^2}+2498852071250000i\sqrt{682\left(13+i\sqrt{31}\right)}x^3\sqrt{3-x+2x^2}\Big)\Big]-$$

$$\frac{1}{961000\sqrt{\frac{62}{11}\left(-13+i\sqrt{31}\right)}}\, i\left(6895071\, i+280267\sqrt{31}\right)$$

$$\text{ArcTanh}\Big[\Big(-47470658398910208\, i-9672976872245316\sqrt{31}-274205806118598024\, i\, x+20598732824854252\sqrt{31}\, x-33816025817929102\, i\, x^2+59172316611299521\sqrt{31}\, x^2-160404448215022990\, i\, x^3+22636449983151020\sqrt{31}\, x^3-65896915460933700\, i\, x^4+52587956640176975\sqrt{31}\, x^4-15742768048875000\sqrt{22\left(-13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}-18116677516562500\sqrt{22\left(-13+i\sqrt{31}\right)}\, x\sqrt{3-x+2x^2}-31110708287062500\sqrt{22\left(-13+i\sqrt{31}\right)}\, x^2\sqrt{3-x+2x^2}+13743686391875000\sqrt{22\left(-13+i\sqrt{31}\right)}\, x^3\sqrt{3-x+2x^2}\Big)\Big/\Big(459884361457315908\, i+30610060529596416\sqrt{31}+554886342419315124\, i\, x+11976408085074048\sqrt{31}\, x+632413940805120427\, i\, x^2+17031264749183454\sqrt{31}\, x^2-572735070344934260\, i\, x^3+13447237198482230\sqrt{31}\, x^3+252081127389719325\, i\, x^4+2678541062954900\sqrt{31}\, x^4\Big)\Big]-$$

$$\left(\left(-6895071\, i+280267\sqrt{31}\right)\text{Log}\left[\left(-3\, i+\sqrt{31}-10\, i\, x\right)^2\left(3\, i+\sqrt{31}+10\, i\, x\right)^2\right]\right)\Big/\left(1922000\sqrt{\frac{62}{11}\left(13+i\sqrt{31}\right)}\right)+$$

$$\left(i\left(6895071\, i+280267\sqrt{31}\right)\text{Log}\left[\left(-3\, i+\sqrt{31}-10\, i\, x\right)^2\left(3\, i+\sqrt{31}+10\, i\, x\right)^2\right]\right)\Big/\left(1922000\sqrt{\frac{62}{11}\left(-13+i\sqrt{31}\right)}\right)-$$

$$\Big(i\left(6895071\, i+280267\sqrt{31}\right)\text{Log}\Big[\left(2+3x+5x^2\right)\Big(-142\, i+66\sqrt{31}+469\, i\, x-22\sqrt{31}\, x+327\, i\, x^2+44\sqrt{31}\, x^2+i\sqrt{682\left(-13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}-4\, i\sqrt{682\left(-13+i\sqrt{31}\right)}\, x\sqrt{3-x+2x^2}\Big)\Big]\Big)\Big/$$

$$\left(1922000\sqrt{\frac{62}{11}\left(-13+i\sqrt{31}\right)}\right)+\Big(\left(-6895071\, i+280267\sqrt{31}\right)$$

$$\text{Log}\Big[\left(2+3x+5x^2\right)\Big(-1858\,i+66\sqrt{31}+1041\,i\,x-22\sqrt{31}\,x-817\,i\,x^2+44\sqrt{31}\,x^2-63\,i\sqrt{22\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}+22\,i\sqrt{22\left(13+i\sqrt{31}\right)}\,x\sqrt{3-x+2x^2}\Big)\Big]\Big)\Big/\left(1922000\sqrt{\frac{62}{11}\left(13+i\sqrt{31}\right)}\right)$$

## Problem 83: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx$$

Optimal (type 3, 148 leaves, 5 steps):

$$\sqrt{\frac{1}{682}\left(13+10\sqrt{2}\right)}\ \text{ArcTan}\Big[\frac{\sqrt{\frac{11}{31\left(13+10\sqrt{2}\right)}}\left(7+3\sqrt{2}+\left(13+10\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\Big]-\sqrt{\frac{1}{682}\left(-13+10\sqrt{2}\right)}\ \text{ArcTanh}\Big[\frac{\sqrt{\frac{11}{31\left(-13+10\sqrt{2}\right)}}\left(7-3\sqrt{2}+\left(13-10\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\Big]$$

Result (type 3, 874 leaves):

$$\frac{1}{4\sqrt{341}}\left(2\,i\sqrt{-13+i\sqrt{31}}\,\text{ArcTan}\left[\left(31\left(-7+11\,i\sqrt{31}+50\,x-100\,x^2\right)\left(3-x+2\,x^2\right)\right)\Big/\left(3069\,i-363\sqrt{31}+1100\sqrt{31}\,x^4+10\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2\,x^2}+x^3\left(110\left(62\,i+\sqrt{31}\right)-100\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2\,x^2}\right)+x^2\left(22\left(-62\,i+49\sqrt{31}\right)-35\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2\,x^2}\right)+x\left(9207\,i+1111\sqrt{31}-25\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2\,x^2}\right)\right)\right]-2\sqrt{13+i\sqrt{31}}\,\text{ArcTan}\left[\left(11\left(-1759+93\,i\sqrt{31}+\left(-1797-31\,i\sqrt{31}\right)x+\left(-1906+62\,i\sqrt{31}\right)x^2+2200\,x^3-550\,x^4\right)\right)\Big/\left(1100\sqrt{31}\,x^4+x^2\left(22\left(62\,i+49\sqrt{31}\right)-1245\sqrt{22\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2\,x^2}\right)+x\left(-9207\,i+1111\sqrt{31}-725\sqrt{22\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2\,x^2}\right)+110\,x^3\left(-62\,i+\sqrt{31}+5\sqrt{22\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2\,x^2}\right)-3\left(1023\,i+121\sqrt{31}+210\sqrt{22\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2\,x^2}\right)\right)\right]+\sqrt{-13+i\sqrt{31}}\,\text{Log}\left[400\left(2+3\,x+5\,x^2\right)^2\right]+i\sqrt{13+i\sqrt{31}}\,\text{Log}\left[400\left(2+3\,x+5\,x^2\right)^2\right]-\sqrt{-13+i\sqrt{31}}\,\text{Log}\left[\left(2+3\,x+5\,x^2\right)\left(-1858\,i+66\sqrt{31}+\left(-817\,i+44\sqrt{31}\right)x^2-63\,i\sqrt{286+22\,i\sqrt{31}}\sqrt{3-x+2\,x^2}+x\left(1041\,i-22\sqrt{31}+22\,i\sqrt{286+22\,i\sqrt{31}}\sqrt{3-x+2\,x^2}\right)\right)\right]-i\sqrt{13+i\sqrt{31}}\,\text{Log}\left[\left(2+3\,x+5\,x^2\right)\left(-142\,i+66\sqrt{31}+\left(327\,i+44\sqrt{31}\right)x^2+i\sqrt{682\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2\,x^2}+x\left(469\,i-22\sqrt{31}-4\,i\sqrt{682\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2\,x^2}\right)\right)\right]\right)$$

## Problem 84: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{1}{\sqrt{3-x+2\,x^2}\left(2+3\,x+5\,x^2\right)^2}\,dx$$

Optimal (type 3, 188 leaves, 6 steps):

$$\frac{(4+65x)\sqrt{3-x+2x^2}}{682\left(2+3x+5x^2\right)}+\frac{1}{1364}\sqrt{\frac{1}{682}\left(2343727+1678700\sqrt{2}\right)}\,\text{ArcTan}\left[\frac{\sqrt{\frac{11}{31\left(2343727+1678700\sqrt{2}\right)}}\left(2119+1816\sqrt{2}+\left(5751+3935\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right]-\frac{1}{1364}\sqrt{\frac{1}{682}\left(-2343727+1678700\sqrt{2}\right)}\,\text{ArcTanh}\left[\frac{\sqrt{\frac{11}{31\left(-2343727+1678700\sqrt{2}\right)}}\left(2119-1816\sqrt{2}+\left(5751-3935\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right]$$

Result (type 3, 1063 leaves):

$$\frac{1}{1860496}\left(\frac{2728\,(4+65x)\sqrt{3-x+2x^2}}{2+3x+5x^2}-\right.$$
$$\left(10\,i\left(-787\,i+41\sqrt{31}\right)\text{ArcTan}\left[\left(31\left(-802246+546546\,i\sqrt{31}+10\left(338727+31031\,i\sqrt{31}\right)x+\left(-2284079+311146\,i\sqrt{31}\right)x^2+\left(3529208+291346\,i\sqrt{31}\right)x^3+\left(-299597+73964\,i\sqrt{31}\right)x^4\right)\right)\Big/\left(20294274\,i-5110826\sqrt{31}+31\left(1419748\,i+1001071\sqrt{31}\right)x^4+134296\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}+x^3\left(81775210\,i+14709760\sqrt{31}-1342960\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)+x^2\left(27657146\,i+35512659\sqrt{31}-470036\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)+x\left(148907198\,i+9626874\sqrt{31}-335740\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)\right)\right]\right)\Big/$$
$$\left(\sqrt{\frac{1}{682}\left(13+i\sqrt{31}\right)}\right)+\left(10\left(787-41\,i\sqrt{31}\right)\text{ArcTanh}\left[\left(31\left(-1419748\,i+1001071\sqrt{31}\right)x^4+\right.\right.\right.$$
$$x^2\left(-27657146\,i+35512659\sqrt{31}-16719852\sqrt{22\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)+$$

$$2\,x\left(-74\,453\,599\,\mathrm{i}+4\,813\,437\sqrt{31}-4\,868\,230\sqrt{22\,\mathrm{i}\left(13\,\mathrm{i}+\sqrt{31}\right)}\,\sqrt{3-x+2\,x^2}\right)-$$

$$14\left(1\,449\,591\,\mathrm{i}+365\,059\sqrt{31}+604\,332\sqrt{22\,\mathrm{i}\left(13\,\mathrm{i}+\sqrt{31}\right)}\,\sqrt{3-x+2\,x^2}\right)+$$

$$10\,x^3\left(-8\,177\,521\,\mathrm{i}+1\,470\,976\sqrt{31}+738\,628\sqrt{22\,\mathrm{i}\left(13\,\mathrm{i}+\sqrt{31}\right)}\,\sqrt{3-x+2\,x^2}\right)\Bigg)\Bigg)\Bigg/$$

$$\Big(98\left(2\,486\,963\,\mathrm{i}+172\,887\sqrt{31}\right)+70\left(4\,358\,663\,\mathrm{i}+137\,423\sqrt{31}\right)x+$$

$$\left(362\,298\,151\,\mathrm{i}+9\,645\,526\sqrt{31}\right)x^2+\left(-298\,854\,392\,\mathrm{i}+9\,031\,726\sqrt{31}\right)x^3+$$

$$\left(155\,225\,093\,\mathrm{i}+2\,292\,884\sqrt{31}\right)x^4\Big)\Big]\Bigg)\Bigg/$$

$$\left(\sqrt{\frac{1}{682}\,\mathrm{i}\left(13\,\mathrm{i}+\sqrt{31}\right)}\right)-\frac{5\left(-787\,\mathrm{i}+41\sqrt{31}\right)\mathrm{Log}\left[400\left(2+3\,x+5\,x^2\right)^2\right]}{\sqrt{\frac{1}{682}\left(13+\mathrm{i}\sqrt{31}\right)}}+$$

$$\frac{5\,\mathrm{i}\left(787\,\mathrm{i}+41\sqrt{31}\right)\mathrm{Log}\left[400\left(2+3\,x+5\,x^2\right)^2\right]}{\sqrt{\frac{1}{682}\,\mathrm{i}\left(13\,\mathrm{i}+\sqrt{31}\right)}}+$$

$$\Bigg(5\left(-787\,\mathrm{i}+41\sqrt{31}\right)$$

$$\mathrm{Log}\Big[\left(2+3\,x+5\,x^2\right)\Big(-1858\,\mathrm{i}+66\sqrt{31}+\left(-817\,\mathrm{i}+44\sqrt{31}\right)x^2-63\,\mathrm{i}\sqrt{286+22\,\mathrm{i}\sqrt{31}}$$

$$\sqrt{3-x+2\,x^2}+x\left(1041\,\mathrm{i}-22\sqrt{31}+22\,\mathrm{i}\sqrt{286+22\,\mathrm{i}\sqrt{31}}\,\sqrt{3-x+2\,x^2}\right)\Big)\Big]\Bigg)\Bigg/$$

$$\left(\sqrt{\frac{1}{682}\left(13+\mathrm{i}\sqrt{31}\right)}\right)+\Bigg(5\left(787-41\,\mathrm{i}\sqrt{31}\right)\mathrm{Log}\Big[\left(2+3\,x+5\,x^2\right)$$

$$\Big(-142\,\mathrm{i}+66\sqrt{31}+\left(327\,\mathrm{i}+44\sqrt{31}\right)x^2+\mathrm{i}\sqrt{682\,\mathrm{i}\left(13\,\mathrm{i}+\sqrt{31}\right)}\,\sqrt{3-x+2\,x^2}+x\Big(469\,\mathrm{i}-$$

$$22\sqrt{31}-4\,\mathrm{i}\sqrt{682\,\mathrm{i}\left(13\,\mathrm{i}+\sqrt{31}\right)}\,\sqrt{3-x+2\,x^2}\Big)\Big)\Big]\Bigg)\Bigg/\left(\sqrt{\frac{1}{682}\,\mathrm{i}\left(13\,\mathrm{i}+\sqrt{31}\right)}\right)$$

## Problem 85: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{1}{\sqrt{3-x+2\,x^2}\,\left(2+3\,x+5\,x^2\right)^3}\,dx$$

Optimal (type 3, 223 leaves, 7 steps):

$$\frac{(4+65x)\sqrt{3-x+2x^2}}{1364\left(2+3x+5x^2\right)^2}+\frac{(26794+86265x)\sqrt{3-x+2x^2}}{1860496\left(2+3x+5x^2\right)}+\frac{1}{3720992}25\sqrt{\frac{1}{682}\left(6414867847+4536374600\sqrt{2}\right)}\,\text{ArcTan}\left[\frac{1}{\sqrt{3-x+2x^2}}\sqrt{\frac{11}{31\left(6414867847+4536374600\sqrt{2}\right)}}\left(123161+85754\sqrt{2}+\left(294669+208915\sqrt{2}\right)x\right)\right]-\frac{1}{3720992}25\sqrt{\frac{1}{682}\left(-6414867847+4536374600\sqrt{2}\right)}\,\text{ArcTanh}\left[\frac{1}{\sqrt{3-x+2x^2}}\sqrt{\frac{11}{31\left(-6414867847+4536374600\sqrt{2}\right)}}\left(123161-85754\sqrt{2}+\left(294669-208915\sqrt{2}\right)x\right)\right]$$

Result (type 3, 1170 leaves):

$$\sqrt{3-x+2x^2}\left(\frac{4+65x}{1364\left(2+3x+5x^2\right)^2}+\frac{26794+86265x}{1860496\left(2+3x+5x^2\right)}\right)-\frac{1}{3720992\sqrt{682\left(13+i\sqrt{31}\right)}}125\,i\left(-41783\,i+1489\sqrt{31}\right)\text{ArcTan}\Big[\Big(31\Big(1733669734\,i+1411781250\sqrt{31}-8257920150\,i\,x+438440750\sqrt{31}\,x+8927431079\,i\,x^2+784505986\sqrt{31}\,x^2-8456927744\,i\,x^3+557246338\sqrt{31}\,x^3+3245899757\,i\,x^4+97553324\sqrt{31}\,x^4\Big)\Big)\Big/\Big(74935517250+14089391258\,i\sqrt{31}+394528763486\,x-31523713098\,i\sqrt{31}\,x+37412913890\,x^2-81049798431\,i\sqrt{31}\,x^2+237240959890\,x^3-29645645200\,i\sqrt{31}\,x^3+84861105868\,x^4-72669503461\,i\sqrt{31}\,x^4-362909968\,i\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}+907274920\,i\sqrt{682\left(13+i\sqrt{31}\right)}\,x\sqrt{3-x+2x^2}+1270184888\,i\sqrt{682\left(13+i\sqrt{31}\right)}\,x^2\sqrt{3-x+2x^2}+3629099680\,i\sqrt{682\left(13+i\sqrt{31}\right)}\,x^3\sqrt{3-x+2x^2}\Big)\Big]-\frac{1}{3720992\sqrt{682\left(-13+i\sqrt{31}\right)}}125\,i\left(41783\,i+1489\sqrt{31}\right)$$

$$\text{ArcTanh}\Big[\Big(-74\,935\,517\,250\,i - 14\,089\,391\,258\sqrt{31} - 394\,528\,763\,486\,i\,x + 31\,523\,713\,098\sqrt{31}\,x - 37\,412\,913\,890\,i\,x^2 + 81\,049\,798\,431\sqrt{31}\,x^2 - 237\,240\,959\,890\,i\,x^3 + 29\,645\,645\,200\sqrt{31}\,x^3 - 84\,861\,105\,868\,i\,x^4 + 72\,669\,503\,461\sqrt{31}\,x^4 - 22\,863\,327\,984\sqrt{22\left(-13+i\sqrt{31}\right)}\sqrt{3-x+2x^2} - 26\,310\,972\,680\sqrt{22\left(-13+i\sqrt{31}\right)}\,x\sqrt{3-x+2x^2} - 45\,182\,291\,016\sqrt{22\left(-13+i\sqrt{31}\right)}\,x^2\sqrt{3-x+2x^2} + 19\,960\,048\,240\sqrt{22\left(-13+i\sqrt{31}\right)}\,x^3\sqrt{3-x+2x^2}\Big)\Big/\Big(672\,076\,174\,246\,i + 43\,765\,218\,750\sqrt{31} + 796\,731\,376\,970\,i\,x + 13\,591\,663\,250\sqrt{31}\,x + 893\,634\,283\,351\,i\,x^2 + 24\,319\,685\,566\sqrt{31}\,x^2 - 841\,081\,542\,656\,i\,x^3 + 17\,274\,636\,478\sqrt{31}\,x^3 + 343\,941\,818\,333\,i\,x^4 + 3\,024\,153\,044\sqrt{31}\,x^4\Big)\Big] -$$

$$\left(125\left(-41\,783\,i+1489\sqrt{31}\right)\text{Log}\left[\left(-3\,i+\sqrt{31}-10\,i\,x\right)^2\left(3\,i+\sqrt{31}+10\,i\,x\right)^2\right]\right)\Big/\left(7\,441\,984\sqrt{682\left(13+i\sqrt{31}\right)}\right) +$$

$$\left(125\,i\left(41\,783\,i+1489\sqrt{31}\right)\text{Log}\left[\left(-3\,i+\sqrt{31}-10\,i\,x\right)^2\left(3\,i+\sqrt{31}+10\,i\,x\right)^2\right]\right)\Big/\left(7\,441\,984\sqrt{682\left(-13+i\sqrt{31}\right)}\right) -$$

$$\Big(125\,i\left(41\,783\,i+1489\sqrt{31}\right)\text{Log}\Big[\left(2+3x+5x^2\right)\Big(-142\,i+66\sqrt{31}+469\,i\,x-22\sqrt{31}\,x+327\,i\,x^2+44\sqrt{31}\,x^2 + i\sqrt{682\left(-13+i\sqrt{31}\right)}\sqrt{3-x+2x^2} - 4\,i\sqrt{682\left(-13+i\sqrt{31}\right)}\,x\sqrt{3-x+2x^2}\Big)\Big]\Big)\Big/\left(7\,441\,984\sqrt{682\left(-13+i\sqrt{31}\right)}\right) +$$

$$\Big(125\left(-41\,783\,i+1489\sqrt{31}\right)\text{Log}\Big[\left(2+3x+5x^2\right)\Big(-1858\,i+66\sqrt{31}+1041\,i\,x-22\sqrt{31}\,x - 817\,i\,x^2+44\sqrt{31}\,x^2-63\,i\sqrt{22\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2} + 22\,i\sqrt{22\left(13+i\sqrt{31}\right)}\,x\sqrt{3-x+2x^2}\Big)\Big]\Big)\Big/\left(7\,441\,984\sqrt{682\left(13+i\sqrt{31}\right)}\right)$$

## Problem 90: Result unnecessarily involves complex numbers and more than

## twice size of optimal antiderivative.

$$\int \frac{1}{\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)}\,dx$$

Optimal (type 3, 176 leaves, 6 steps):

$$\frac{13-6x}{253\sqrt{3-x+2x^2}}+$$

$$\frac{1}{22}\sqrt{\frac{1}{682}\left(247+500\sqrt{2}\right)}\ \text{ArcTan}\left[\frac{\sqrt{\frac{11}{31\left(247+500\sqrt{2}\right)}}\left(61+4\sqrt{2}+\left(69+65\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right]-$$

$$\frac{1}{22}\sqrt{\frac{1}{682}\left(-247+500\sqrt{2}\right)}\ \text{ArcTanh}\left[\frac{\sqrt{\frac{11}{31\left(-247+500\sqrt{2}\right)}}\left(61-4\sqrt{2}+\left(69-65\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right]$$

Result (type 3, 1044 leaves):

$$\frac{13-6x}{253\sqrt{3-x+2x^2}}+$$

$$\left(5\,i\left(13\,i+\sqrt{31}\right)\text{ArcTan}\left[\left(31\left(-74-66\,i\sqrt{31}+14\left(3+11\,i\sqrt{31}\right)x+7\left(185-22\,i\sqrt{31}\right)x^2+\right.\right.\right.\right.$$

$$\left.\left.\left(-1160+110\,i\sqrt{31}\right)x^3+\left(797-44\,i\sqrt{31}\right)x^4\right)\right)\Big/$$

$$\left(-14322\,i+602\sqrt{31}+\left(17732\,i+4439\sqrt{31}\right)x^4-40\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}+\right.$$

$$10x^3\left(-1705\,i+512\sqrt{31}+40\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)+$$

$$2x\left(-3751\,i-2133\sqrt{31}+50\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)+$$

$$\left.\left.\left.x^2\left(21142\,i+6405\sqrt{31}+140\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)\right)\right]\right)\Big/$$

$$\left(22\sqrt{682\left(13+i\sqrt{31}\right)}\right)-\left(5\left(-13\,i+\sqrt{31}\right)\text{ArcTan}\left[\left(\left(17732+4439\,i\sqrt{31}\right)x^4+\right.\right.\right.$$

$$10\,i\,x^3\left(1705\,i+512\sqrt{31}-220\sqrt{22\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)+$$

$$x\left(-7502-4266\,i\sqrt{31}+2900\,i\sqrt{22\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)+$$

$$x^2\left(21142+6405\,i\sqrt{31}+4980\,i\sqrt{22\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)+$$

$$14\,i\left(1023\,i+43\sqrt{31}+180\sqrt{22\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)\Big)\Big/$$

$$\left(82\,294\,i+2046\sqrt{31}+\left(58\,298\,i-4774\sqrt{31}\right)x+\left(88\,855\,i+4774\sqrt{31}\right)x^2-\right.$$

$$\left.10\left(8564\,i+341\sqrt{31}\right)x^3+\left(24\,293\,i+1364\sqrt{31}\right)x^4\right)\Big]\Big)\Big/$$

$$\left(22\sqrt{682\,i\left(13\,i+\sqrt{31}\right)}\right)-\frac{5\,i\left(-13\,i+\sqrt{31}\right)\text{Log}\left[400\left(2+3x+5x^2\right)^2\right]}{44\sqrt{682\,i\left(13\,i+\sqrt{31}\right)}}+$$

$$\frac{5\left(13\,i+\sqrt{31}\right)\text{Log}\left[400\left(2+3x+5x^2\right)^2\right]}{44\sqrt{682\left(13+i\sqrt{31}\right)}}-$$

$$\Big(5\left(13\,i+\sqrt{31}\right)$$

$$\text{Log}\Big[\left(2+3x+5x^2\right)\Big(-1858\,i+66\sqrt{31}+\left(-817\,i+44\sqrt{31}\right)x^2-63\,i\sqrt{286+22\,i\sqrt{31}}$$

$$\sqrt{3-x+2x^2}+x\left(1041\,i-22\sqrt{31}+22\,i\sqrt{286+22\,i\sqrt{31}}\sqrt{3-x+2x^2}\right)\Big)\Big]\Big)\Big/$$

$$\left(44\sqrt{682\left(13+i\sqrt{31}\right)}\right)+\Big(5\left(13+i\sqrt{31}\right)\text{Log}\Big[\left(2+3x+5x^2\right)$$

$$\Big(-142\,i+66\sqrt{31}+\left(327\,i+44\sqrt{31}\right)x^2+i\sqrt{682\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2x^2}+x\Big(469\,i-$$

$$22\sqrt{31}-4\,i\sqrt{682\,i\left(13\,i+\sqrt{31}\right)}\sqrt{3-x+2x^2}\Big)\Big)\Big]\Big)\Big/\left(44\sqrt{682\,i\left(13\,i+\sqrt{31}\right)}\right)$$

## Problem 91: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{1}{\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^2}\,dx$$

Optimal (type 3, 211 leaves, 7 steps):

$$-\frac{6315-2306\,x}{345092\,\sqrt{3-x+2\,x^2}}+\frac{4+65\,x}{682\,\sqrt{3-x+2\,x^2}\,\left(2+3\,x+5\,x^2\right)}+$$

$$\frac{1}{30008}\sqrt{\frac{1}{682}\left(129694447+103775000\,\sqrt{2}\right)}\,\text{ArcTan}\Big[\frac{1}{\sqrt{3-x+2\,x^2}}$$

$$\sqrt{\frac{11}{31\left(129694447+103775000\,\sqrt{2}\right)}}\,\left(12611+16454\,\sqrt{2}+\left(45519+29065\,\sqrt{2}\right)\,x\right)\Big]-$$

$$\frac{1}{30008}\sqrt{\frac{1}{682}\left(-129694447+103775000\,\sqrt{2}\right)}\,\text{ArcTanh}\Big[\frac{1}{\sqrt{3-x+2\,x^2}}$$

$$\sqrt{\frac{11}{31\left(-129694447+103775000\,\sqrt{2}\right)}}\,\left(12611-16454\,\sqrt{2}+\left(45519-29065\,\sqrt{2}\right)\,x\right)\Big]$$

Result (type 3, 1170 leaves):

$$\sqrt{3-x+2\,x^2}\,\left(\frac{-31-14\,x}{5566\,\left(3-x+2\,x^2\right)}+\frac{-98+345\,x}{15004\,\left(2+3\,x+5\,x^2\right)}\right)-$$

$$\frac{1}{30008\,\sqrt{682\,\left(13+i\,\sqrt{31}\right)}}\,5\,i\,\left(-5813\,i+499\,\sqrt{31}\right)\,\text{ArcTan}\Big[$$

$$\Big(31\,\Big(67211446\,i+35267826\,\sqrt{31}-236270118\,i\,x+36393566\,\sqrt{31}\,x-2553985\,i\,x^2+23896114\,\sqrt{31}\,x^2-282686240\,i\,x^3+26621650\,\sqrt{31}\,x^3-104765803\,i\,x^4+10956044\,\sqrt{31}\,x^4\Big)\Big)\Big/$$

$$\Big(294638322+278507402\,i\,\sqrt{31}+8796989102\,x-311643066\,i\,\sqrt{31}\,x+3166163858\,x^2-2655130695\,i\,\sqrt{31}\,x^2+3951866050\,x^3-1267524880\,i\,\sqrt{31}\,x^3+3956537068\,x^4-2241477661\,i\,\sqrt{31}\,x^4-8302000\,i\,\sqrt{682\,\left(13+i\,\sqrt{31}\right)}\,\sqrt{3-x+2\,x^2}+20755000\,i\,\sqrt{682\,\left(13+i\,\sqrt{31}\right)}\,x\,\sqrt{3-x+2\,x^2}+29057000\,i\,\sqrt{682\,\left(13+i\,\sqrt{31}\right)}\,x^2\,\sqrt{3-x+2\,x^2}+83020000\,i\,\sqrt{682\,\left(13+i\,\sqrt{31}\right)}\,x^3\,\sqrt{3-x+2\,x^2}\Big)\Big]-$$

$$\frac{1}{30008\,\sqrt{682\,\left(-13+i\,\sqrt{31}\right)}}\,5\,i\,\left(5813\,i+499\,\sqrt{31}\right)\,\text{ArcTanh}\Big[$$

$$\Big(-294638322\,i-278507402\,\sqrt{31}-8796989102\,i\,x+311643066\,\sqrt{31}\,x-3166163858\,i\,x^2+2655130695\,\sqrt{31}\,x^2-3951866050\,i\,x^3+1267524880\,\sqrt{31}\,x^3-3956537068\,i\,x^4+$$

$$2\,241\,477\,661\,\sqrt{31}\,x^4-523\,026\,000\,\sqrt{22\left(-13+i\sqrt{31}\right)}\,\sqrt{3-x+2x^2}-601\,895\,000\,\sqrt{22\left(-13+i\sqrt{31}\right)}\,x\sqrt{3-x+2x^2}-1\,033\,599\,000\,\sqrt{22\left(-13+i\sqrt{31}\right)}\,x^2\sqrt{3-x+2x^2}+456\,610\,000\,\sqrt{22\left(-13+i\sqrt{31}\right)}\,x^3\sqrt{3-x+2x^2}\Big)\Big/$$

$$\Big(14\,520\,445\,174\,i+1\,093\,302\,606\,\sqrt{31}+19\,694\,353\,658\,i\,x+1\,128\,200\,546\,\sqrt{31}\,x+26\,853\,123\,535\,i\,x^2+740\,779\,534\,\sqrt{31}\,x^2-16\,474\,806\,560\,i\,x^3+825\,271\,150\,\sqrt{31}\,x^3+13\,417\,689\,893\,i\,x^4+339\,637\,364\,\sqrt{31}\,x^4\Big)\Big]-$$

$$\left(5\left(-5813\,i+499\sqrt{31}\right)\text{Log}\left[\left(-3\,i+\sqrt{31}-10\,i\,x\right)^2\left(3\,i+\sqrt{31}+10\,i\,x\right)^2\right]\right)\Big/\left(60\,016\,\sqrt{682\left(13+i\sqrt{31}\right)}\right)+$$

$$\left(5\,i\left(5813\,i+499\sqrt{31}\right)\text{Log}\left[\left(-3\,i+\sqrt{31}-10\,i\,x\right)^2\left(3\,i+\sqrt{31}+10\,i\,x\right)^2\right]\right)\Big/\left(60\,016\,\sqrt{682\left(-13+i\sqrt{31}\right)}\right)-$$

$$\Big(5\,i\left(5813\,i+499\sqrt{31}\right)\text{Log}\Big[\left(2+3x+5x^2\right)\Big(-142\,i+66\sqrt{31}+469\,i\,x-22\sqrt{31}\,x+327\,i\,x^2+44\sqrt{31}\,x^2+i\sqrt{682\left(-13+i\sqrt{31}\right)}\,\sqrt{3-x+2x^2}-4\,i\sqrt{682\left(-13+i\sqrt{31}\right)}\,x\sqrt{3-x+2x^2}\Big)\Big]\Big)\Big/$$

$$\left(60\,016\,\sqrt{682\left(-13+i\sqrt{31}\right)}\right)+\Big(5\left(-5813\,i+499\sqrt{31}\right)\text{Log}\Big[\left(2+3x+5x^2\right)\Big(-1858\,i+66\sqrt{31}+1041\,i\,x-22\sqrt{31}\,x-817\,i\,x^2+44\sqrt{31}\,x^2-63\,i\sqrt{22\left(13+i\sqrt{31}\right)}\,\sqrt{3-x+2x^2}+22\,i\sqrt{22\left(13+i\sqrt{31}\right)}\,x\sqrt{3-x+2x^2}\Big)\Big]\Big)\Big/\left(60\,016\,\sqrt{682\left(13+i\sqrt{31}\right)}\right)$$

## Problem 92: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{1}{\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^3}\,dx$$

Optimal (type 3, 246 leaves, 8 steps):

$$-\frac{4353943-6508666\,x}{941410976\,\sqrt{3-x+2\,x^2}}+\frac{4+65\,x}{1364\,\sqrt{3-x+2\,x^2}\,\left(2+3\,x+5\,x^2\right)^2}+$$
$$\frac{5\,(7318+17315\,x)}{1860496\,\sqrt{3-x+2\,x^2}\,\left(2+3\,x+5\,x^2\right)}+\frac{1}{81861824}$$
$$3\,\sqrt{\frac{1}{682}\left(13874275807943+9819738650000\,\sqrt{2}\right)}$$
$$\text{ArcTan}\Big[\frac{1}{\sqrt{3-x+2\,x^2}}\sqrt{\frac{11}{31\left(13874275807943+9819738650000\,\sqrt{2}\right)}}$$
$$\left(5538393+4123702\,\sqrt{2}+\left(13785797+9662095\,\sqrt{2}\right)x\right)\Big]-$$
$$\frac{1}{81861824}3\,\sqrt{\frac{1}{682}\left(-13874275807943+9819738650000\,\sqrt{2}\right)}$$
$$\text{ArcTanh}\Big[\frac{1}{\sqrt{3-x+2\,x^2}}\sqrt{\frac{11}{31\left(-13874275807943+9819738650000\,\sqrt{2}\right)}}$$
$$\left(5538393-4123702\,\sqrt{2}+\left(13785797-9662095\,\sqrt{2}\right)x\right)\Big]$$

Result (type 3, 1191 leaves):

$$\sqrt{3-x+2\,x^2}\left(\frac{-11+90\,x}{122452\,\left(3-x+2\,x^2\right)}+\frac{-98+345\,x}{30008\,\left(2+3\,x+5\,x^2\right)^2}+\frac{231418+632255\,x}{40930912\,\left(2+3\,x+5\,x^2\right)}\right)-$$
$$\frac{1}{81861824\,\sqrt{682\left(13+i\,\sqrt{31}\right)}}15\,i\,\left(-1932419\,i+79037\,\sqrt{31}\right)\text{ArcTan}\Big[$$
$$\Big(31\,\Big(4059546477574\,i+3106527877794\,\sqrt{31}-18544569435542\,i\,x+1227936189854\,\sqrt{31}\,x+$$
$$17501774027535\,i\,x^2+1728828684066\,\sqrt{31}\,x^2-18989790004560\,i\,x^3+$$
$$1371533012850\,\sqrt{31}\,x^3+5399410180693\,i\,x^4+274861284236\,\sqrt{31}\,x^4\Big)\Big)\Big/$$
$$\Big(148573472722818+30402744893338\,i\,\sqrt{31}+862374952340638\,x-$$
$$64577765937354\,i\,\sqrt{31}\,x+107573401361602\,x^2-186540875521455\,i\,\sqrt{31}\,x^2+$$
$$503769328622450\,x^3-71509340960720\,i\,\sqrt{31}\,x^3+208327267086092\,x^4-$$
$$165714245597909\,i\,\sqrt{31}\,x^4-785579092000\,i\,\sqrt{682\left(13+i\,\sqrt{31}\right)}\,\sqrt{3-x+2\,x^2}+$$
$$1963947730000\,i\,\sqrt{682\left(13+i\,\sqrt{31}\right)}\,x\,\sqrt{3-x+2\,x^2}+$$
$$2749526822000\,i\,\sqrt{682\left(13+i\,\sqrt{31}\right)}\,x^2\,\sqrt{3-x+2\,x^2}+$$

$$7\,855\,790\,920\,000\, i \sqrt{682\left(13+i\sqrt{31}\right)}\, x^3 \sqrt{3-x+2x^2}\Big)\Big] -$$

$$\frac{1}{81\,861\,824\sqrt{682\left(-13+i\sqrt{31}\right)}}\, 15\, i \left(1\,932\,419\, i+79\,037\sqrt{31}\right)$$

$$\text{ArcTanh}\Big[\Big(-148\,573\,472\,722\,818\, i-30\,402\,744\,893\,338\sqrt{31}-862\,374\,952\,340\,638\, i\, x+$$
$$64\,577\,765\,937\,354\sqrt{31}\, x-107\,573\,401\,361\,602\, i\, x^2+186\,540\,875\,521\,455\sqrt{31}\, x^2-$$
$$503\,769\,328\,622\,450\, i\, x^3+71\,509\,340\,960\,720\sqrt{31}\, x^3-208\,327\,267\,086\,092\, i\, x^4+$$
$$165\,714\,245\,597\,909\sqrt{31}\, x^4-49\,491\,482\,796\,000\sqrt{22\left(-13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}-$$
$$56\,954\,484\,170\,000\sqrt{22\left(-13+i\sqrt{31}\right)}\, x\sqrt{3-x+2x^2}-$$
$$97\,804\,596\,954\,000\sqrt{22\left(-13+i\sqrt{31}\right)}\, x^2\sqrt{3-x+2x^2}+$$
$$43\,206\,850\,060\,000\sqrt{22\left(-13+i\sqrt{31}\right)}\, x^3\sqrt{3-x+2x^2}\Big)\Big/$$
$$\Big(1\,445\,312\,243\,195\,206\, i+96\,302\,364\,211\,614\sqrt{31}+1\,745\,394\,499\,581\,802\, i\, x+38\,066\,021\,885\,474$$
$$\sqrt{31}\, x+1\,990\,937\,576\,846\,415\, i\, x^2+53\,593\,689\,206\,046\sqrt{31}\, x^2-1\,799\,476\,949\,538\,640\, i\, x^3+$$
$$42\,517\,523\,398\,350\sqrt{31}\, x^3+794\,952\,672\,098\,517\, i\, x^4+8\,520\,699\,811\,316\sqrt{31}\, x^4\Big)\Big] -$$

$$\left(15\left(-1\,932\,419\, i+79\,037\sqrt{31}\right)\text{Log}\left[\left(-3\, i+\sqrt{31}-10\, i\, x\right)^2\left(3\, i+\sqrt{31}+10\, i\, x\right)^2\right]\right)\Big/$$
$$\left(163\,723\,648\sqrt{682\left(13+i\sqrt{31}\right)}\right)+$$

$$\left(15\, i\left(1\,932\,419\, i+79\,037\sqrt{31}\right)\text{Log}\left[\left(-3\, i+\sqrt{31}-10\, i\, x\right)^2\left(3\, i+\sqrt{31}+10\, i\, x\right)^2\right]\right)\Big/$$
$$\left(163\,723\,648\sqrt{682\left(-13+i\sqrt{31}\right)}\right)-$$

$$\Big(15\, i\left(1\,932\,419\, i+79\,037\sqrt{31}\right)$$
$$\text{Log}\Big[\left(2+3x+5x^2\right)\Big(-142\, i+66\sqrt{31}+469\, i\, x-22\sqrt{31}\, x+327\, i\, x^2+44\sqrt{31}\, x^2+$$
$$i\sqrt{682\left(-13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}-4\, i\sqrt{682\left(-13+i\sqrt{31}\right)}\, x\sqrt{3-x+2x^2}\Big)\Big]\Big)\Big/$$
$$\left(163\,723\,648\sqrt{682\left(-13+i\sqrt{31}\right)}\right)+\Big(15\left(-1\,932\,419\, i+79\,037\sqrt{31}\right)$$

$$\text{Log}\left[\left(2+3x+5x^2\right)\left(-1858\,i+66\sqrt{31}+1041\,i\,x-22\sqrt{31}\,x-817\,i\,x^2+44\sqrt{31}\,x^2-63\,i\sqrt{22\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}+22\,i\sqrt{22\left(13+i\sqrt{31}\right)}\,x\sqrt{3-x+2x^2}\right)\right]\Bigg/\left(163\,723\,648\sqrt{682\left(13+i\sqrt{31}\right)}\right)$$

## Problem 97: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{1}{\left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)}\,dx$$

Optimal (type 3, 199 leaves, 7 steps):

$$\frac{13-6x}{759\left(3-x+2x^2\right)^{3/2}}+\frac{3603-658x}{128\,018\sqrt{3-x+2x^2}}+\frac{1}{484}\sqrt{\frac{1}{682}\left(-15\,457+25\,000\sqrt{2}\right)}\,\text{ArcTan}\left[\frac{\sqrt{\frac{11}{31\left(-15\,457+25\,000\sqrt{2}\right)}}\left(443-98\sqrt{2}+\left(247+345\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right]-\frac{1}{484}\sqrt{\frac{1}{682}\left(15\,457+25\,000\sqrt{2}\right)}\,\text{ArcTanh}\left[\frac{\sqrt{\frac{11}{31\left(15\,457+25\,000\sqrt{2}\right)}}\left(443+98\sqrt{2}+\left(247-345\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right]$$

Result (type 3, 1080 leaves):

$$\frac{13-6x}{759\left(3-x+2x^2\right)^{3/2}}+\frac{3603-658x}{128\,018\sqrt{3-x+2x^2}}+\Bigg(5\,i\left(69\,i+13\sqrt{31}\right)\text{ArcTan}\Big[\Big(31\Big(-3626-594\,i\sqrt{31}+\left(24\,058+5346\,i\sqrt{31}\right)x+\left(-10\,465-13\,266\,i\sqrt{31}\right)x^2+\left(-106\,560+7150\,i\sqrt{31}\right)x^3+\left(-17\,707-7436\,i\sqrt{31}\right)x^4\Big)\Big)\Big/\Big(-186\left(2013\,i+167\sqrt{31}\right)+\left(1\,223\,508\,i+526\,291\sqrt{31}\right)x^4-2000\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}+10x^3\left(18\,755\,i+37\,528\sqrt{31}+2000\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)+2x\left(661\,881\,i-36\,077\sqrt{31}+2500\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)+x^2\left(1\,185\,998\,i+657\,545\sqrt{31}+7000\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}\right)\Big)\Big]\Bigg)\Bigg/$$

$$\left(484 \sqrt{682 \left(13+i \sqrt{31}\right)}\right)-\left(5 \left(-69 i+13 \sqrt{31}\right) \mathrm{ArcTan}\left[\left(\left(1\,223\,508+526\,291 i \sqrt{31}\right) x^4+\right.\right.\right.$$

$$10 x^3 \left(18\,755+37\,528 i \sqrt{31}-11\,000 i \sqrt{22 i \left(13 i+\sqrt{31}\right)} \sqrt{3-x+2 x^2}\right)+$$

$$6 \left(-62\,403-5177 i \sqrt{31}+21\,000 i \sqrt{22 i \left(13 i+\sqrt{31}\right)} \sqrt{3-x+2 x^2}\right)+$$

$$2 x \left(661\,881-36\,077 i \sqrt{31}+72\,500 i \sqrt{22 i \left(13 i+\sqrt{31}\right)} \sqrt{3-x+2 x^2}\right)+$$

$$\left.x^2 \left(1\,185\,998+657\,545 i \sqrt{31}+249\,000 i \sqrt{22 i \left(13 i+\sqrt{31}\right)} \sqrt{3-x+2 x^2}\right)\right)\Big/$$

$$\left(4\,112\,406 i+18\,414 \sqrt{31}-6 \left(-372\,367 i+27\,621 \sqrt{31}\right) x+\left(6\,774\,415 i+411\,246 \sqrt{31}\right) x^2-\right.$$

$$\left.\left.\left.10 \left(277\,664 i+22\,165 \sqrt{31}\right) x^3+\left(2\,998\,917 i+230\,516 \sqrt{31}\right) x^4\right)\right]\right)\Big/$$

$$\left(484 \sqrt{682 i \left(13 i+\sqrt{31}\right)}\right)-\frac{5 i \left(-69 i+13 \sqrt{31}\right) \mathrm{Log}\left[400 \left(2+3 x+5 x^2\right)^2\right]}{968 \sqrt{682 i \left(13 i+\sqrt{31}\right)}}+$$

$$\frac{5 \left(69 i+13 \sqrt{31}\right) \mathrm{Log}\left[400 \left(2+3 x+5 x^2\right)^2\right]}{968 \sqrt{682 \left(13+i \sqrt{31}\right)}}-$$

$$\left(5 \left(69 i+13 \sqrt{31}\right)\right.$$

$$\mathrm{Log}\left[\left(2+3 x+5 x^2\right) \left(-1858 i+66 \sqrt{31}+\left(-817 i+44 \sqrt{31}\right) x^2-63 i \sqrt{286+22 i \sqrt{31}}\right.\right.$$

$$\left.\left.\left.\sqrt{3-x+2 x^2}+x \left(1041 i-22 \sqrt{31}+22 i \sqrt{286+22 i \sqrt{31}} \sqrt{3-x+2 x^2}\right)\right)\right]\right)\Big/$$

$$\left(968 \sqrt{682 \left(13+i \sqrt{31}\right)}\right)+\left(5 \left(69+13 i \sqrt{31}\right) \mathrm{Log}\left[\left(2+3 x+5 x^2\right)\right.\right.$$

$$\left(-142 i+66 \sqrt{31}+\left(327 i+44 \sqrt{31}\right) x^2+i \sqrt{682 i \left(13 i+\sqrt{31}\right)} \sqrt{3-x+2 x^2}+x \left(469 i-\right.\right.$$

$$\left.\left.\left.\left.22 \sqrt{31}-4 i \sqrt{682 i \left(13 i+\sqrt{31}\right)} \sqrt{3-x+2 x^2}\right)\right)\right]\right)\Big/\left(968 \sqrt{682 i \left(13 i+\sqrt{31}\right)}\right)$$

## Problem 98: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)^2}\,dx$$

Optimal (type 3, 234 leaves, 8 steps):

$$-\frac{15101-8654x}{1035276\left(3-x+2x^2\right)^{3/2}}-\frac{3133427+1352542x}{523849656\sqrt{3-x+2x^2}}+\frac{4+65x}{682\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)}+\frac{1}{660176}625\sqrt{\frac{1}{682}\left(30463+23600\sqrt{2}\right)}\,\text{ArcTan}\left[\frac{\sqrt{\frac{11}{31\left(30463+23600\sqrt{2}\right)}}\left(203+242\sqrt{2}+\left(687+445\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right]-\frac{1}{660176}625\sqrt{\frac{1}{682}\left(-30463+23600\sqrt{2}\right)}\,\text{ArcTanh}\left[\frac{\sqrt{\frac{11}{31\left(-30463+23600\sqrt{2}\right)}}\left(203-242\sqrt{2}+\left(687-445\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right]$$

Result (type 3, 1191 leaves):

$$\sqrt{3-x+2x^2}\left(\frac{-31-14x}{16698\left(3-x+2x^2\right)^2}+\frac{-10769-17230x}{4224594\left(3-x+2x^2\right)}+\frac{-1474+1235x}{330088\left(2+3x+5x^2\right)}\right)-$$
$$\left(3125\,i\left(-89\,i+7\sqrt{31}\right)\text{ArcTan}\left[\left(31\left(14518\,i+7986\sqrt{31}-52806\,i\,x+7502\sqrt{31}\,x+6503\,i\,x^2+5170\sqrt{31}\,x^2-60944\,i\,x^3+5698\sqrt{31}\,x^3-17827\,i\,x^4+2156\sqrt{31}\,x^4\right)\right)\Big/\left(112530+65642\,i\sqrt{31}+2037134x-84762\,i\sqrt{31}\,x+658130x^2-587559\,i\sqrt{31}\,x^2+958210x^3-274000\,i\sqrt{31}\,x^3+849772x^4-499069\,i\sqrt{31}\,x^4-1888\,i\sqrt{682\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}+4720\,i\sqrt{682\left(13+i\sqrt{31}\right)}\,x\sqrt{3-x+2x^2}+6608\,i\sqrt{682\left(13+i\sqrt{31}\right)}\,x^2\sqrt{3-x+2x^2}+18880\,i\sqrt{682\left(13+i\sqrt{31}\right)}\,x^3\sqrt{3-x+2x^2}\right)\right]\right)\Big/$$
$$\left(660176\sqrt{682\left(13+i\sqrt{31}\right)}\right)-\left(3125\,i\left(89\,i+7\sqrt{31}\right)\right.$$
$$\text{ArcTanh}\left[\left(-112530\,i-65642\sqrt{31}-2037134\,i\,x+84762\sqrt{31}\,x-658130\,i\,x^2+587559\sqrt{31}\,x^2-958210\,i\,x^3+274000\sqrt{31}\,x^3-849772\,i\,x^4+499069\sqrt{31}\,x^4-118944\sqrt{22\left(-13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}-136880\sqrt{22\left(-13+i\sqrt{31}\right)}\,x\sqrt{3-x+2x^2}-235056\right.\right.$$

$$\left.\sqrt{22\left(-13+i\sqrt{31}\right)}\,x^2\sqrt{3-x+2x^2}+103840\sqrt{22\left(-13+i\sqrt{31}\right)}\,x^3\sqrt{3-x+2x^2}\right)\Big/$$

$$\left(3325942\,i+247566\sqrt{31}+4450106\,i\,x+232562\sqrt{31}\,x+5887207\,i\,x^2+\right.$$

$$\left.\left.\left.160270\sqrt{31}\,x^2-3850256\,i\,x^3+176638\sqrt{31}\,x^3+2865437\,i\,x^4+66836\sqrt{31}\,x^4\right)\right]\right)\Big/$$

$$\left(660176\sqrt{682\left(-13+i\sqrt{31}\right)}\right)-\left(3125\left(-89\,i+7\sqrt{31}\right)\right.$$

$$\left.\text{Log}\left[\left(-3\,i+\sqrt{31}-10\,i\,x\right)^2\left(3\,i+\sqrt{31}+10\,i\,x\right)^2\right]\right)\Big/$$

$$\left(1320352\sqrt{682\left(13+i\sqrt{31}\right)}\right)+$$

$$\left(3125\,i\left(89\,i+7\sqrt{31}\right)\text{Log}\left[\left(-3\,i+\sqrt{31}-10\,i\,x\right)^2\left(3\,i+\sqrt{31}+10\,i\,x\right)^2\right]\right)\Big/$$

$$\left(1320352\sqrt{682\left(-13+i\sqrt{31}\right)}\right)-$$

$$\left(3125\,i\left(89\,i+7\sqrt{31}\right)\right.$$

$$\text{Log}\left[\left(2+3x+5x^2\right)\left(-142\,i+66\sqrt{31}+469\,i\,x-22\sqrt{31}\,x+327\,i\,x^2+44\sqrt{31}\,x^2+\right.\right.$$

$$\left.\left.\left.i\sqrt{682\left(-13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}-4\,i\sqrt{682\left(-13+i\sqrt{31}\right)}\,x\sqrt{3-x+2x^2}\right)\right]\right)\Big/$$

$$\left(1320352\sqrt{682\left(-13+i\sqrt{31}\right)}\right)+\left(3125\left(-89\,i+7\sqrt{31}\right)\right.$$

$$\text{Log}\left[\left(2+3x+5x^2\right)\left(-1858\,i+66\sqrt{31}+1041\,i\,x-22\sqrt{31}\,x-\right.\right.$$

$$817\,i\,x^2+44\sqrt{31}\,x^2-63\,i\sqrt{22\left(13+i\sqrt{31}\right)}\sqrt{3-x+2x^2}+$$

$$\left.\left.\left.22\,i\sqrt{22\left(13+i\sqrt{31}\right)}\,x\sqrt{3-x+2x^2}\right)\right]\right)\Big/\left(1320352\sqrt{682\left(13+i\sqrt{31}\right)}\right)$$

## Problem 99: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{1}{\left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)^3}\,dx$$

Optimal (type 3, 269 leaves, 9 steps):

$$-\frac{12\,280\,939-19\,536\,786\,x}{2\,824\,232\,928\,\left(3-x+2\,x^2\right)^{3/2}}-\frac{1\,134\,826\,571-1\,504\,660\,754\,x}{476\,353\,953\,856\,\sqrt{3-x+2\,x^2}}+\frac{4+65\,x}{1364\,\left(3-x+2\,x^2\right)^{3/2}\,\left(2+3\,x+5\,x^2\right)^2}+\frac{46\,386+86\,885\,x}{1\,860\,496\,\left(3-x+2\,x^2\right)^{3/2}\,\left(2+3\,x+5\,x^2\right)}+$$

$$\frac{1}{1\,800\,960\,128}35\,\sqrt{\frac{1}{682}\left(2\,243\,059\,557\,247+2\,011\,748\,500\,000\,\sqrt{2}\right)}\,\text{ArcTan}\Big[\frac{1}{\sqrt{3-x+2\,x^2}}\sqrt{\frac{11}{31\,\left(2\,243\,059\,557\,247+2\,011\,748\,500\,000\,\sqrt{2}\right)}}\left(1\,432\,939+2\,428\,746\,\sqrt{2}+\left(6\,290\,431+3\,861\,685\,\sqrt{2}\right)x\right)\Big]-$$

$$\frac{1}{1\,800\,960\,128}35\,\sqrt{\frac{1}{682}\left(-2\,243\,059\,557\,247+2\,011\,748\,500\,000\,\sqrt{2}\right)}\,\text{ArcTanh}\Big[\frac{1}{\sqrt{3-x+2\,x^2}}\sqrt{\frac{11}{31\,\left(-2\,243\,059\,557\,247+2\,011\,748\,500\,000\,\sqrt{2}\right)}}\left(1\,432\,939-2\,428\,746\,\sqrt{2}+\left(6\,290\,431-3\,861\,685\,\sqrt{2}\right)x\right)\Big]$$

Result (type 3, 1218 leaves):

$$\sqrt{3-x+2\,x^2}\left(\frac{-11+90\,x}{367\,356\,\left(3-x+2\,x^2\right)^2}+\frac{-39\,095+53\,754\,x}{61\,960\,712\,\left(3-x+2\,x^2\right)}+\frac{-1474+1235\,x}{660\,176\,\left(2+3\,x+5\,x^2\right)^2}+\frac{157\,362+468\,895\,x}{81\,861\,824\,\left(2+3\,x+5\,x^2\right)}\right)+$$

$$\frac{1}{1\,800\,960\,128\,\sqrt{682\,\left(-13+i\,\sqrt{31}\right)}}175\,\left(772\,337\,i+81\,951\,\sqrt{31}\right)\,\text{ArcTan}\Big[\Big(4\,655\,364\,448\,878+4\,766\,043\,812\,202\,i\,\sqrt{31}-158\,699\,364\,373\,902\,x-2\,787\,485\,821\,466\,i\,\sqrt{31}\,x-74\,012\,991\,583\,058\,x^2-54\,042\,219\,198\,695\,i\,\sqrt{31}\,x^2-61\,598\,686\,386\,050\,x^3-27\,260\,449\,836\,880\,i\,\sqrt{31}\,x^3-86\,332\,728\,860\,268\,x^4-44\,936\,737\,584\,061\,i\,\sqrt{31}\,x^4+10\,139\,212\,440\,000\,i\,\sqrt{22\,\left(-13+i\,\sqrt{31}\right)}\,\sqrt{3-x+2\,x^2}+11\,668\,141\,300\,000\,i\,\sqrt{22\,\left(-13+i\,\sqrt{31}\right)}\,x\,\sqrt{3-x+2\,x^2}+20\,037\,015\,060\,000\,i\,\sqrt{22\,\left(-13+i\,\sqrt{31}\right)}\,x^2\,\sqrt{3-x+2\,x^2}-8\,851\,693\,400\,000\,i\,\sqrt{22\,\left(-13+i\,\sqrt{31}\right)}\,x^3\,\sqrt{3-x+2\,x^2}\Big)\Big/$$

$$\left(276\,508\,696\,366\,774\,i + 21\,211\,104\,525\,006\,\sqrt{31} + 386\,113\,686\,180\,858\,i\,x + 27\,073\,970\,836\,946\,\sqrt{31}\,x + 572\,257\,780\,896\,535\,i\,x^2 + 16\,500\,157\,269\,134\,\sqrt{31}\,x^2 - 293\,982\,300\,056\,560\,i\,x^3 + 18\,182\,603\,589\,150\,\sqrt{31}\,x^3 + 303\,413\,457\,358\,093\,i\,x^4 + 9\,160\,578\,170\,964\,\sqrt{31}\,x^4\right)\Big] -$$

$$\frac{1}{1\,800\,960\,128\,\sqrt{682\left(13+i\sqrt{31}\right)}}\,175\,i\left(-772\,337\,i + 81\,951\,\sqrt{31}\right)\,\text{ArcTan}\Big[$$

$$\left(31\left(1\,463\,582\,697\,846\,i + 684\,229\,178\,226\,\sqrt{31} - 4\,719\,782\,741\,318\,i\,x + 873\,353\,897\,966\,\sqrt{31}\,x - 1\,716\,989\,286\,985\,i\,x^2 + 532\,263\,137\,714\,\sqrt{31}\,x^2 - 6\,299\,191\,456\,240\,i\,x^3 + 586\,535\,599\,650\,\sqrt{31}\,x^3 - 3\,427\,809\,818\,003\,i\,x^4 + 295\,502\,521\,644\,\sqrt{31}\,x^4\right)\right)\Big/$$

$$\Big(-4\,655\,364\,448\,878 + 4\,766\,043\,812\,202\,i\,\sqrt{31} + 158\,699\,364\,373\,902\,x - 2\,787\,485\,821\,466\,i\,\sqrt{31}\,x + 74\,012\,991\,583\,058\,x^2 - 54\,042\,219\,198\,695\,i\,\sqrt{31}\,x^2 + 61\,598\,686\,386\,050\,x^3 - 27\,260\,449\,836\,880\,i\,\sqrt{31}\,x^3 + 86\,332\,728\,860\,268\,x^4 - 44\,936\,737\,584\,061\,i\,\sqrt{31}\,x^4 - 160\,939\,880\,000\,i\,\sqrt{682\left(13+i\sqrt{31}\right)}\,\sqrt{3-x+2x^2} + 402\,349\,700\,000\,i\,\sqrt{682\left(13+i\sqrt{31}\right)}\,x\,\sqrt{3-x+2x^2} + 563\,289\,580\,000\,i\,\sqrt{682\left(13+i\sqrt{31}\right)}\,x^2\,\sqrt{3-x+2x^2} + 1\,609\,398\,800\,000\,i\,\sqrt{682\left(13+i\sqrt{31}\right)}\,x^3\,\sqrt{3-x+2x^2}\Big)\Big] -$$

$$\left(175\left(-772\,337\,i + 81\,951\,\sqrt{31}\right)\text{Log}\Big[\left(-3\,i + \sqrt{31} - 10\,i\,x\right)^2\left(3\,i + \sqrt{31} + 10\,i\,x\right)^2\Big]\right)\Big/\left(3\,601\,920\,256\,\sqrt{682\left(13+i\sqrt{31}\right)}\right) +$$

$$\left(175\,i\left(772\,337\,i + 81\,951\,\sqrt{31}\right)\text{Log}\Big[\left(-3\,i + \sqrt{31} - 10\,i\,x\right)^2\left(3\,i + \sqrt{31} + 10\,i\,x\right)^2\Big]\right)\Big/\left(3\,601\,920\,256\,\sqrt{682\left(-13+i\sqrt{31}\right)}\right) -$$

$$\Big(175\,i\left(772\,337\,i + 81\,951\,\sqrt{31}\right)\,\text{Log}\Big[\left(2+3x+5x^2\right)\Big(-142\,i + 66\,\sqrt{31} + 469\,i\,x - 22\,\sqrt{31}\,x + 327\,i\,x^2 + 44\,\sqrt{31}\,x^2 + i\,\sqrt{682\left(-13+i\sqrt{31}\right)}\,\sqrt{3-x+2x^2} - 4\,i\,\sqrt{682\left(-13+i\sqrt{31}\right)}\,x\,\sqrt{3-x+2x^2}\Big)\Big]\Big)\Big/$$

$$\left(3\,601\,920\,256\,\sqrt{682\left(-13+i\sqrt{31}\right)}\right) + \Big(175\left(-772\,337\,i + 81\,951\,\sqrt{31}\right)$$

$$\text{Log}\Big[\left(2+3x+5x^2\right)\Big(-1858\,i + 66\,\sqrt{31} + 1041\,i\,x - 22\,\sqrt{31}\,x - 817\,i\,x^2 + 44\,\sqrt{31}\,x^2 -$$

$$63\,i\,\sqrt{22\left(13+i\sqrt{31}\right)}\,\sqrt{3-x+2x^2}+22\,i\,\sqrt{22\left(13+i\sqrt{31}\right)}\,x\sqrt{3-x+2x^2}\Big)\Big]\Big)\Big/$$

$$\left(3\,601\,920\,256\sqrt{682\left(13+i\sqrt{31}\right)}\right)$$

## Problem 106: Result more than twice size of optimal antiderivative.

$$\int\frac{\left(a+b\,x+c\,x^2\right)^{3/2}}{d+e\,x+f\,x^2}\,dx$$

Optimal (type 3, 679 leaves, 9 steps):

$$-\frac{\left(4\,c\,e-5\,b\,f-2\,c\,f\,x\right)\sqrt{a+b\,x+c\,x^2}}{4\,f^2}+$$

$$\frac{\left(3\,b^2\,f^2-12\,c\,f\left(b\,e-a\,f\right)+8\,c^2\left(e^2-d\,f\right)\right)\text{ArcTanh}\left[\frac{b+2\,c\,x}{2\sqrt{c}\,\sqrt{a+b\,x+c\,x^2}}\right]}{8\sqrt{c}\,f^3}+$$

$$\Bigg(\Bigg(\left(c\,e-b\,f\right)\left(e-\sqrt{e^2-4\,d\,f}\right)\left(f\left(b\,e-2\,a\,f\right)-c\left(e^2-2\,d\,f\right)\right)-$$

$$2\,f\left(2\,c\,d\,f\left(b\,e-a\,f\right)-f^2\left(b^2\,d-a^2\,f\right)-c^2\,d\left(e^2-d\,f\right)\right)\Bigg)$$

$$\text{ArcTanh}\Bigg[\left(4\,a\,f-b\left(e-\sqrt{e^2-4\,d\,f}\right)+2\left(b\,f-c\left(e-\sqrt{e^2-4\,d\,f}\right)\right)x\right)\Big/$$

$$\left(2\sqrt{2}\sqrt{c\,e^2-2\,c\,d\,f-b\,e\,f+2\,a\,f^2-\left(c\,e-b\,f\right)\sqrt{e^2-4\,d\,f}}\,\sqrt{a+b\,x+c\,x^2}\right)\Bigg]\Bigg)\Big/$$

$$\left(\sqrt{2}\,f^3\sqrt{e^2-4\,d\,f}\,\sqrt{c\,e^2-2\,c\,d\,f-b\,e\,f+2\,a\,f^2-\left(c\,e-b\,f\right)\sqrt{e^2-4\,d\,f}}\right)-$$

$$\Bigg(\Bigg(\left(c\,e-b\,f\right)\left(e+\sqrt{e^2-4\,d\,f}\right)\left(f\left(b\,e-2\,a\,f\right)-c\left(e^2-2\,d\,f\right)\right)-$$

$$2\,f\left(2\,c\,d\,f\left(b\,e-a\,f\right)-f^2\left(b^2\,d-a^2\,f\right)-c^2\,d\left(e^2-d\,f\right)\right)\Bigg)$$

$$\text{ArcTanh}\Bigg[\left(4\,a\,f-b\left(e+\sqrt{e^2-4\,d\,f}\right)+2\left(b\,f-c\left(e+\sqrt{e^2-4\,d\,f}\right)\right)x\right)\Big/$$

$$\left(2\sqrt{2}\sqrt{c\,e^2-2\,c\,d\,f-b\,e\,f+2\,a\,f^2+\left(c\,e-b\,f\right)\sqrt{e^2-4\,d\,f}}\,\sqrt{a+b\,x+c\,x^2}\right)\Bigg]\Bigg)\Big/$$

$$\left(\sqrt{2}\,f^3\sqrt{e^2-4\,d\,f}\,\sqrt{c\,e^2-2\,c\,d\,f-b\,e\,f+2\,a\,f^2+\left(c\,e-b\,f\right)\sqrt{e^2-4\,d\,f}}\right)$$

Result (type 3, 1934 leaves):

$$\frac{\left(\frac{-4 c e+5 b f}{4 f^2}+\frac{c x}{2 f}\right) \left(a+x (b+c x)\right)^{3/2}}{a+b x+c x^2}-$$

$$\left(\left(-c^2 e^4+4 c^2 d e^2 f+2 b c e^3 f-2 c^2 d^2 f^2-6 b c d e f^2-b^2 e^2 f^2-2 a c e^2 f^2+2 b^2 d f^3+4 a c d f^3+2 a b e f^3-2 a^2 f^4+c^2 e^3 \sqrt{e^2-4 d f}-2 c^2 d e f \sqrt{e^2-4 d f}-2 b c e^2 f \sqrt{e^2-4 d f}+2 b c d f^2 \sqrt{e^2-4 d f}+b^2 e f^2 \sqrt{e^2-4 d f}+2 a c e f^2 \sqrt{e^2-4 d f}-2 a b f^3 \sqrt{e^2-4 d f}\right) \left(a+x (b+c x)\right)^{3/2} \text{Log}\left[-e+\sqrt{e^2-4 d f}-2 f x\right]\right) \Big/ \left(\sqrt{2} f^3 \sqrt{e^2-4 d f} \sqrt{c e^2-2 c d f-b e f+2 a f^2-c e \sqrt{e^2-4 d f}+b f \sqrt{e^2-4 d f}} \left(a+b x+c x^2\right)^{3/2}\right)-$$

$$\left(\left(c^2 e^4-4 c^2 d e^2 f-2 b c e^3 f+2 c^2 d^2 f^2+6 b c d e f^2+b^2 e^2 f^2+2 a c e^2 f^2-2 b^2 d f^3-4 a c d f^3-2 a b e f^3+2 a^2 f^4+c^2 e^3 \sqrt{e^2-4 d f}-2 c^2 d e f \sqrt{e^2-4 d f}-2 b c e^2 f \sqrt{e^2-4 d f}+2 b c d f^2 \sqrt{e^2-4 d f}+b^2 e f^2 \sqrt{e^2-4 d f}+2 a c e f^2 \sqrt{e^2-4 d f}-2 a b f^3 \sqrt{e^2-4 d f}\right) \left(a+x (b+c x)\right)^{3/2} \text{Log}\left[e+\sqrt{e^2-4 d f}+2 f x\right]\right) \Big/ \left(\sqrt{2} f^3 \sqrt{e^2-4 d f} \sqrt{c e^2-2 c d f-b e f+2 a f^2+c e \sqrt{e^2-4 d f}-b f \sqrt{e^2-4 d f}} \left(a+b x+c x^2\right)^{3/2}\right)+$$

$$\left(\left(8 c^2 e^2-8 c^2 d f-12 b c e f+3 b^2 f^2+12 a c f^2\right) \left(a+x (b+c x)\right)^{3/2} \text{Log}\left[b+2 c x+2 \sqrt{c} \sqrt{a+b x+c x^2}\right]\right) \Big/ \left(8 \sqrt{c} f^3 \left(a+b x+c x^2\right)^{3/2}\right)+$$

$$\left(\left(c^2 e^4-4 c^2 d e^2 f-2 b c e^3 f+2 c^2 d^2 f^2+6 b c d e f^2+b^2 e^2 f^2+2 a c e^2 f^2-2 b^2 d f^3-4 a c d f^3-2 a b e f^3+2 a^2 f^4+c^2 e^3 \sqrt{e^2-4 d f}-2 c^2 d e f \sqrt{e^2-4 d f}-2 b c e^2 f \sqrt{e^2-4 d f}+2 b c d f^2 \sqrt{e^2-4 d f}+b^2 e f^2 \sqrt{e^2-4 d f}+2 a c e f^2 \sqrt{e^2-4 d f}-2 a b f^3 \sqrt{e^2-4 d f}\right) \left(a+x (b+c x)\right)^{3/2} \text{Log}\left[-b e^2+4 b d f-b e \sqrt{e^2-4 d f}+4 a f \sqrt{e^2-4 d f}-2 c e^2 x+8 c d f x-2 c e \sqrt{e^2-4 d f} x+2 b f \sqrt{e^2-4 d f} x+2 \sqrt{2} \sqrt{e^2-4 d f} \sqrt{c e^2-2 c d f-b e f+2 a f^2+c e \sqrt{e^2-4 d f}-b f \sqrt{e^2-4 d f}} \sqrt{a+b x+c x^2}\right]\right) \Big/$$

$$\left(\sqrt{2} f^3 \sqrt{e^2-4 d f} \sqrt{c e^2-2 c d f-b e f+2 a f^2+c e \sqrt{e^2-4 d f}-b f \sqrt{e^2-4 d f}}\right.$$

$$\left.\left(a+b\,x+c\,x^2\right)^{3/2}\right)+$$

$$\left(\left(-c^2 e^4+4 c^2 d e^2 f+2 b c e^3 f-2 c^2 d^2 f^2-6 b c d e f^2-b^2 e^2 f^2-2 a c e^2 f^2+2 b^2 d f^3+\right.\right.$$

$$4 a c d f^3+2 a b e f^3-2 a^2 f^4+c^2 e^3 \sqrt{e^2-4 d f}-2 c^2 d e f \sqrt{e^2-4 d f}-2 b c e^2 f \sqrt{e^2-4 d f}+$$

$$\left.2 b c d f^2 \sqrt{e^2-4 d f}+b^2 e f^2 \sqrt{e^2-4 d f}+2 a c e f^2 \sqrt{e^2-4 d f}-2 a b f^3 \sqrt{e^2-4 d f}\right)$$

$$\left(a+x\,(b+c\,x)\right)^{3/2} \text{Log}\left[b e^2-4 b d f-b e \sqrt{e^2-4 d f}+4 a f \sqrt{e^2-4 d f}+2 c e^2 x-\right.$$

$$8 c d f x-2 c e \sqrt{e^2-4 d f}\,x+2 b f \sqrt{e^2-4 d f}\,x+2 \sqrt{2} \sqrt{e^2-4 d f}$$

$$\left.\left.\sqrt{c e^2-2 c d f-b e f+2 a f^2-c e \sqrt{e^2-4 d f}+b f \sqrt{e^2-4 d f}}\,\sqrt{a+b x+c x^2}\right]\right)\Big/$$

$$\left(\sqrt{2}\, f^3 \sqrt{e^2-4 d f} \sqrt{c e^2-2 c d f-b e f+2 a f^2-c e \sqrt{e^2-4 d f}+b f \sqrt{e^2-4 d f}}\right.$$

$$\left.\left(a+b\,x+c\,x^2\right)^{3/2}\right)$$

## Problem 107: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(a+b\,x+c\,x^2\right)^{3/2}}{\left(d+e\,x+f\,x^2\right)^2}\,dx$$

Optimal (type 3, 704 leaves, 10 steps):

$$-\frac{\left(c e-2 b f-2 c f x\right) \sqrt{a+b x+c x^2}}{f \left(e^2-4 d f\right)}-\frac{\left(e+2 f x\right) \left(a+b x+c x^2\right)^{3/2}}{\left(e^2-4 d f\right) \left(d+e x+f x^2\right)}+\frac{c^{3/2} \text{ArcTanh}\left[\frac{b+2 c x}{2 \sqrt{c} \sqrt{a+b x+c x^2}}\right]}{f^2}-\left(\left(\left(c e-b f\right) \left(f \left(b e-2 a f\right)+2 c \left(e^2-5 d f\right)\right) \left(e-\sqrt{e^2-4 d f}\right)-2 f \left(2 c^2 d \left(e^2-4 d f\right)+f \left(2 b^2 d f+4 a f \left(c d+a f\right)-b e \left(c d+3 a f\right)\right)\right)\right) \text{ArcTanh}\left[\left(4 a f-b \left(e-\sqrt{e^2-4 d f}\right)+2 \left(b f-c \left(e-\sqrt{e^2-4 d f}\right)\right) x\right) \Big/ \left(2 \sqrt{2} \sqrt{c e^2-2 c d f-b e f+2 a f^2-\left(c e-b f\right) \sqrt{e^2-4 d f}} \sqrt{a+b x+c x^2}\right)\right]\right) \Big/ \left(2 \sqrt{2} f^2 \left(e^2-4 d f\right)^{3/2} \sqrt{c e^2-2 c d f-b e f+2 a f^2-\left(c e-b f\right) \sqrt{e^2-4 d f}}\right)+\left(\left(\left(c e-b f\right) \left(f \left(b e-2 a f\right)+2 c \left(e^2-5 d f\right)\right) \left(e+\sqrt{e^2-4 d f}\right)-2 f \left(2 c^2 d \left(e^2-4 d f\right)+f \left(2 b^2 d f+4 a f \left(c d+a f\right)-b e \left(c d+3 a f\right)\right)\right)\right) \text{ArcTanh}\left[\left(4 a f-b \left(e+\sqrt{e^2-4 d f}\right)+2 \left(b f-c \left(e+\sqrt{e^2-4 d f}\right)\right) x\right) \Big/ \left(2 \sqrt{2} \sqrt{c e^2-2 c d f-b e f+2 a f^2+\left(c e-b f\right) \sqrt{e^2-4 d f}} \sqrt{a+b x+c x^2}\right)\right]\right) \Big/ \left(2 \sqrt{2} f^2 \left(e^2-4 d f\right)^{3/2} \sqrt{c e^2-2 c d f-b e f+2 a f^2+\left(c e-b f\right) \sqrt{e^2-4 d f}}\right)$$

Result (type 3, 1844 leaves):

$$\left(\left(c d e-2 b d f+a e f+c e^2 x-2 c d f x-b e f x+2 a f^2 x\right) \left(a+x \left(b+c x\right)\right)^{3/2}\right) \Big/ \left(f \left(-e^2+4 d f\right) \left(a+b x+c x^2\right) \left(d+e x+f x^2\right)\right)-\left(\left(-2 c^2 e^4+14 c^2 d e^2 f+b c e^3 f-16 c^2 d^2 f^2-12 b c d e f^2+b^2 e^2 f^2+2 a c e^2 f^2+4 b^2 d f^3+8 a c d f^3-8 a b e f^3+8 a^2 f^4+2 c^2 e^3 \sqrt{e^2-4 d f}-10 c^2 d e f \sqrt{e^2-4 d f}-b c e^2 f \sqrt{e^2-4 d f}+10 b c d f^2 \sqrt{e^2-4 d f}-b^2 e f^2 \sqrt{e^2-4 d f}-2 a c e f^2 \sqrt{e^2-4 d f}+2 a b f^3 \sqrt{e^2-4 d f}\right) \left(a+x \left(b+c x\right)\right)^{3/2} \text{Log}\left[-e+\sqrt{e^2-4 d f}-2 f x\right]\right) \Big/ \left(2 \sqrt{2} f^2 \left(e^2-4 d f\right)^{3/2} \sqrt{c e^2-2 c d f-b e f+2 a f^2-c e \sqrt{e^2-4 d f}+b f \sqrt{e^2-4 d f}} \left(a+b x+c x^2\right)^{3/2}\right)-$$

$$\Bigg(\Big(2 c^2 e^4 - 14 c^2 d e^2 f - b c e^3 f + 16 c^2 d^2 f^2 + 12 b c d e f^2 - b^2 e^2 f^2 - 2 a c e^2 f^2 - 4 b^2 d f^3 - 8 a c d f^3 + 8 a b e f^3 - 8 a^2 f^4 + 2 c^2 e^3 \sqrt{e^2 - 4 d f} - 10 c^2 d e f \sqrt{e^2 - 4 d f} - b c e^2 f \sqrt{e^2 - 4 d f} + 10 b c d f^2 \sqrt{e^2 - 4 d f} - b^2 e f^2 \sqrt{e^2 - 4 d f} - 2 a c e f^2 \sqrt{e^2 - 4 d f} + 2 a b f^3 \sqrt{e^2 - 4 d f}\Big) (a + x (b + c x))^{3/2} \operatorname{Log}\left[e + \sqrt{e^2 - 4 d f} + 2 f x\right]\Bigg) \Bigg/ \Bigg(2 \sqrt{2} f^2 (e^2 - 4 d f)^{3/2} \sqrt{c e^2 - 2 c d f - b e f + 2 a f^2 + c e \sqrt{e^2 - 4 d f} - b f \sqrt{e^2 - 4 d f}} (a + b x + c x^2)^{3/2}\Bigg) +$$

$$\frac{c^{3/2} (a + x (b + c x))^{3/2} \operatorname{Log}\left[b + 2 c x + 2 \sqrt{c} \sqrt{a + b x + c x^2}\right]}{f^2 (a + b x + c x^2)^{3/2}} +$$

$$\Bigg(\Big(2 c^2 e^4 - 14 c^2 d e^2 f - b c e^3 f + 16 c^2 d^2 f^2 + 12 b c d e f^2 - b^2 e^2 f^2 - 2 a c e^2 f^2 - 4 b^2 d f^3 - 8 a c d f^3 + 8 a b e f^3 - 8 a^2 f^4 + 2 c^2 e^3 \sqrt{e^2 - 4 d f} - 10 c^2 d e f \sqrt{e^2 - 4 d f} - b c e^2 f \sqrt{e^2 - 4 d f} + 10 b c d f^2 \sqrt{e^2 - 4 d f} - b^2 e f^2 \sqrt{e^2 - 4 d f} - 2 a c e f^2 \sqrt{e^2 - 4 d f} + 2 a b f^3 \sqrt{e^2 - 4 d f}\Big) (a + x (b + c x))^{3/2} \operatorname{Log}\Big[b e - 4 a f + b \sqrt{e^2 - 4 d f} + 2 c e x - 2 b f x + 2 c \sqrt{e^2 - 4 d f}\, x - 2 \sqrt{2} \sqrt{c e^2 - 2 c d f - b e f + 2 a f^2 + c e \sqrt{e^2 - 4 d f} - b f \sqrt{e^2 - 4 d f}} \sqrt{a + b x + c x^2}\Big]\Bigg) \Bigg/ \Bigg(2 \sqrt{2} f^2 (e^2 - 4 d f)^{3/2} \sqrt{c e^2 - 2 c d f - b e f + 2 a f^2 + c e \sqrt{e^2 - 4 d f} - b f \sqrt{e^2 - 4 d f}} (a + b x + c x^2)^{3/2}\Bigg) +$$

$$\Bigg(\Big(-2 c^2 e^4 + 14 c^2 d e^2 f + b c e^3 f - 16 c^2 d^2 f^2 - 12 b c d e f^2 + b^2 e^2 f^2 + 2 a c e^2 f^2 + 4 b^2 d f^3 + 8 a c d f^3 - 8 a b e f^3 + 8 a^2 f^4 + 2 c^2 e^3 \sqrt{e^2 - 4 d f} - 10 c^2 d e f \sqrt{e^2 - 4 d f} - b c e^2 f \sqrt{e^2 - 4 d f} + 10 b c d f^2 \sqrt{e^2 - 4 d f} - b^2 e f^2 \sqrt{e^2 - 4 d f} - 2 a c e f^2 \sqrt{e^2 - 4 d f} + 2 a b f^3 \sqrt{e^2 - 4 d f}\Big) (a + x (b + c x))^{3/2} \operatorname{Log}\Big[-b e + 4 a f + b \sqrt{e^2 - 4 d f} - 2 c e x + 2 b f x + 2 c \sqrt{e^2 - 4 d f}\, x + 2 \sqrt{2} \sqrt{c e^2 - 2 c d f - b e f + 2 a f^2 - c e \sqrt{e^2 - 4 d f} + b f \sqrt{e^2 - 4 d f}} \sqrt{a + b x + c x^2}\Big]\Bigg) \Bigg/ \Bigg(2 \sqrt{2} f^2 (e^2 - 4 d f)^{3/2} \sqrt{c e^2 - 2 c d f - b e f + 2 a f^2 - c e \sqrt{e^2 - 4 d f} + b f \sqrt{e^2 - 4 d f}}$$

$$\left.(a+bx+cx^2)^{3/2}\right)$$

## Problem 108: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex+fx^2)^3}\,dx$$

Optimal (type 3, 671 leaves, 7 steps):

$$-\frac{(e+2fx)\,(a+bx+cx^2)^{3/2}}{2\,(e^2-4df)\,(d+ex+fx^2)^2}+\left(3\left(4cde+4aef-b\left(e^2+4df\right)+2\left(ce^2-2bef+4af^2\right)x\right)\sqrt{a+bx+cx^2}\right)\Big/\left(4\left(e^2-4df\right)^2\left(d+ex+fx^2\right)\right)-\left(3\left(2\,(2cd-be+2af)\,(ce-bf)\left(e-\sqrt{e^2-4df}\right)-f\left(4be\,(cd+3af)-b^2\left(e^2+4df\right)-4a\left(ce^2+4af^2\right)\right)\right)\text{ArcTanh}\left[\left(4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x\right)\Big/\left(2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\,\sqrt{a+bx+cx^2}\right)\right]\right)\Big/\left(4\sqrt{2}\left(e^2-4df\right)^{5/2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\right)+\left(3\left(2\,(2cd-be+2af)\,(ce-bf)\left(e+\sqrt{e^2-4df}\right)-f\left(4be\,(cd+3af)-b^2\left(e^2+4df\right)-4a\left(ce^2+4af^2\right)\right)\right)\text{ArcTanh}\left[\left(4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x\right)\Big/\left(2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}\,\sqrt{a+bx+cx^2}\right)\right]\right)\Big/\left(4\sqrt{2}\left(e^2-4df\right)^{5/2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}\right)$$

Result (type 3, 1621 leaves):

$$\frac{1}{a+bx+cx^2}\left(a+x\,(b+cx)\right)^{3/2}$$

$$\left(\frac{c\,d\,e-2\,b\,d\,f+a\,e\,f+c\,e^2\,x-2\,c\,d\,f\,x-b\,e\,f\,x+2\,a\,f^2\,x}{2\,f\,\left(-e^2+4\,d\,f\right)\,\left(d+e\,x+f\,x^2\right)^2}+\left(2\,c\,e^3+4\,c\,d\,e\,f-7\,b\,e^2\,f+4\,b\,d\,f^2+12\,a\,e\,f^2+2\,c\,e^2\,f\,x+16\,c\,d\,f^2\,x-12\,b\,e\,f^2\,x+24\,a\,f^3\,x\right)\Big/\left(4\,f\,\left(-e^2+4\,d\,f\right)^2\,\left(d+e\,x+f\,x^2\right)\right)\right)+$$

$$\left(3\,\left(4\,c^2\,d\,e^2-2\,b\,c\,e^3-8\,b\,c\,d\,e\,f+3\,b^2\,e^2\,f+8\,a\,c\,e^2\,f+4\,b^2\,d\,f^2-16\,a\,b\,e\,f^2+16\,a^2\,f^3-4\,c^2\,d\,e\,\sqrt{e^2-4\,d\,f}+2\,b\,c\,e^2\,\sqrt{e^2-4\,d\,f}+4\,b\,c\,d\,f\,\sqrt{e^2-4\,d\,f}-2\,b^2\,e\,f\,\sqrt{e^2-4\,d\,f}-4\,a\,c\,e\,f\,\sqrt{e^2-4\,d\,f}+4\,a\,b\,f^2\,\sqrt{e^2-4\,d\,f}\right)\,\left(a+x\,\left(b+c\,x\right)\right)^{3/2}\,\text{Log}\left[-e+\sqrt{e^2-4\,d\,f}-2\,f\,x\right]\right)\Big/$$

$$\left(4\,\sqrt{2}\,\left(e^2-4\,d\,f\right)^{5/2}\,\sqrt{c\,e^2-2\,c\,d\,f-b\,e\,f+2\,a\,f^2-c\,e\,\sqrt{e^2-4\,d\,f}+b\,f\,\sqrt{e^2-4\,d\,f}}\,\left(a+b\,x+c\,x^2\right)^{3/2}\right)+$$

$$\left(3\,\left(-4\,c^2\,d\,e^2+2\,b\,c\,e^3+8\,b\,c\,d\,e\,f-3\,b^2\,e^2\,f-8\,a\,c\,e^2\,f-4\,b^2\,d\,f^2+16\,a\,b\,e\,f^2-16\,a^2\,f^3-4\,c^2\,d\,e\,\sqrt{e^2-4\,d\,f}+2\,b\,c\,e^2\,\sqrt{e^2-4\,d\,f}+4\,b\,c\,d\,f\,\sqrt{e^2-4\,d\,f}-2\,b^2\,e\,f\,\sqrt{e^2-4\,d\,f}-4\,a\,c\,e\,f\,\sqrt{e^2-4\,d\,f}+4\,a\,b\,f^2\,\sqrt{e^2-4\,d\,f}\right)\,\left(a+x\,\left(b+c\,x\right)\right)^{3/2}\,\text{Log}\left[e+\sqrt{e^2-4\,d\,f}+2\,f\,x\right]\right)\Big/$$

$$\left(4\,\sqrt{2}\,\left(e^2-4\,d\,f\right)^{5/2}\,\sqrt{c\,e^2-2\,c\,d\,f-b\,e\,f+2\,a\,f^2+c\,e\,\sqrt{e^2-4\,d\,f}-b\,f\,\sqrt{e^2-4\,d\,f}}\,\left(a+b\,x+c\,x^2\right)^{3/2}\right)-$$

$$\left(3\,\left(-4\,c^2\,d\,e^2+2\,b\,c\,e^3+8\,b\,c\,d\,e\,f-3\,b^2\,e^2\,f-8\,a\,c\,e^2\,f-4\,b^2\,d\,f^2+16\,a\,b\,e\,f^2-16\,a^2\,f^3-4\,c^2\,d\,e\,\sqrt{e^2-4\,d\,f}+2\,b\,c\,e^2\,\sqrt{e^2-4\,d\,f}+4\,b\,c\,d\,f\,\sqrt{e^2-4\,d\,f}-2\,b^2\,e\,f\,\sqrt{e^2-4\,d\,f}-4\,a\,c\,e\,f\,\sqrt{e^2-4\,d\,f}+4\,a\,b\,f^2\,\sqrt{e^2-4\,d\,f}\right)\,\left(a+x\,\left(b+c\,x\right)\right)^{3/2}\,\text{Log}\left[b\,e-4\,a\,f+b\,\sqrt{e^2-4\,d\,f}+2\,c\,e\,x-2\,b\,f\,x+2\,c\,\sqrt{e^2-4\,d\,f}\,x-2\,\sqrt{2}\,\sqrt{c\,e^2-2\,c\,d\,f-b\,e\,f+2\,a\,f^2+c\,e\,\sqrt{e^2-4\,d\,f}-b\,f\,\sqrt{e^2-4\,d\,f}}\,\sqrt{a+b\,x+c\,x^2}\right]\right)\Big/$$

$$\left(4\,\sqrt{2}\,\left(e^2-4\,d\,f\right)^{5/2}\,\sqrt{c\,e^2-2\,c\,d\,f-b\,e\,f+2\,a\,f^2+c\,e\,\sqrt{e^2-4\,d\,f}-b\,f\,\sqrt{e^2-4\,d\,f}}\,\left(a+b\,x+c\,x^2\right)^{3/2}\right)-$$

$$\left(3\left(4c^2de^2-2bce^3-8bcdef+3b^2e^2f+8ace^2f+4b^2df^2-16abef^2+16a^2f^3-4c^2de\sqrt{e^2-4df}+2bce^2\sqrt{e^2-4df}+4bcdf\sqrt{e^2-4df}-2b^2ef\sqrt{e^2-4df}-4acef\sqrt{e^2-4df}+4abf^2\sqrt{e^2-4df}\right)(a+x(b+cx))^{3/2}\,\text{Log}\left[-be+4af+b\sqrt{e^2-4df}-2cex+2bfx+2c\sqrt{e^2-4df}\,x+2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-ce\sqrt{e^2-4df}+bf\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}\right]\right)\Big/\left(4\sqrt{2}\left(e^2-4df\right)^{5/2}\sqrt{ce^2-2cdf-bef+2af^2-ce\sqrt{e^2-4df}+bf\sqrt{e^2-4df}}\left(a+bx+cx^2\right)^{3/2}\right)$$

## Problem 113: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)^2}\,dx$$

Optimal (type 3, 789 leaves, 6 steps):

$$\left(\left(f\left(b e^2-2 b d f-a e f\right)-c\left(e^3-3 d e f\right)+f\left(f\left(b e-2 a f\right)-c\left(e^2-2 d f\right)\right) x\right) \sqrt{a+b x+c x^2}\right) \Big/ \left(\left(e^2-4 d f\right)\left(\left(c d-a f\right)^2-\left(b d-a e\right)\left(c e-b f\right)\right)\left(d+e x+f x^2\right)\right)+$$

$$\left(\left(f\left(2 c d-b e+2 a f\right)\left(c e-b f\right)\left(e-\sqrt{e^2-4 d f}\right)-2 f\left(2 c^2 d\left(e^2-4 d f\right)+f\left(3 a b e f-4 a^2 f^2+b^2\left(e^2-6 d f\right)\right)-c\left(4 a f\left(e^2-3 d f\right)+b\left(e^3-5 d e f\right)\right)\right)\right) \text{ArcTanh}\left[\left(4 a f-b\left(e-\sqrt{e^2-4 d f}\right)+2\left(b f-c\left(e-\sqrt{e^2-4 d f}\right)\right) x\right) \Big/ \left(2 \sqrt{2} \sqrt{c e^2-2 c d f-b e f+2 a f^2-\left(c e-b f\right) \sqrt{e^2-4 d f}} \sqrt{a+b x+c x^2}\right)\right]\right) \Big/ \left(2 \sqrt{2}\left(e^2-4 d f\right)^{3/2}\left(\left(c d-a f\right)^2-\left(b d-a e\right)\left(c e-b f\right)\right) \sqrt{c e^2-2 c d f-b e f+2 a f^2-\left(c e-b f\right) \sqrt{e^2-4 d f}}\right)-$$

$$\left(\left(f\left(2 c d-b e+2 a f\right)\left(c e-b f\right)\left(e+\sqrt{e^2-4 d f}\right)-2 f\left(2 c^2 d\left(e^2-4 d f\right)+f\left(3 a b e f-4 a^2 f^2+b^2\left(e^2-6 d f\right)\right)-c\left(4 a f\left(e^2-3 d f\right)+b\left(e^3-5 d e f\right)\right)\right)\right) \text{ArcTanh}\left[\left(4 a f-b\left(e+\sqrt{e^2-4 d f}\right)+2\left(b f-c\left(e+\sqrt{e^2-4 d f}\right)\right) x\right) \Big/ \left(2 \sqrt{2} \sqrt{c e^2-2 c d f-b e f+2 a f^2+\left(c e-b f\right) \sqrt{e^2-4 d f}} \sqrt{a+b x+c x^2}\right)\right]\right) \Big/ \left(2 \sqrt{2}\left(e^2-4 d f\right)^{3/2}\left(\left(c d-a f\right)^2-\left(b d-a e\right)\left(c e-b f\right)\right) \sqrt{c e^2-2 c d f-b e f+2 a f^2+\left(c e-b f\right) \sqrt{e^2-4 d f}}\right)$$

Result (type 3, 1836 leaves):

$$\left(\left(-c e^3+3 c d e f+b e^2 f-2 b d f^2-a e f^2-c e^2 f x+2 c d f^2 x+b e f^2 x-2 a f^3 x\right)\left(a+b x+c x^2\right)\right) \Big/ \left(\left(e^2-4 d f\right)\left(c^2 d^2-b c d e+a c e^2+b^2 d f-2 a c d f-a b e f+a^2 f^2\right)\left(d+e x+f x^2\right) \sqrt{a+x\left(b+c x\right)}\right)-$$

$$\left(f\left(-2 c^2 d e^2+b c e^3+16 c^2 d^2 f-12 b c d e f-b^2 e^2 f+10 a c e^2 f+12 b^2 d f^2-24 a c d f^2-8 a b e f^2+8 a^2 f^3-2 c^2 d e \sqrt{e^2-4 d f}+b c e^2 \sqrt{e^2-4 d f}+2 b c d f \sqrt{e^2-4 d f}-b^2 e f \sqrt{e^2-4 d f}-\right.\right.$$

$$2 a c e f \sqrt{e^2-4 d f}+2 a b f^2 \sqrt{e^2-4 d f}\Big) \sqrt{a+b x+c x^2}\, \text{Log}\Big[-e+\sqrt{e^2-4 d f}-2 f x\Big]\Big) \Big/$$

$$\Big(2 \sqrt{2} \left(e^2-4 d f\right)^{3/2} \left(c^2 d^2-b c d e+a c e^2+b^2 d f-2 a c d f-a b e f+a^2 f^2\right)$$

$$\sqrt{c e^2-2 c d f-b e f+2 a f^2-c e \sqrt{e^2-4 d f}+b f \sqrt{e^2-4 d f}}\, \sqrt{a+x (b+c x)}\Big)-$$

$$\Big(f \Big(2 c^2 d e^2-b c e^3-16 c^2 d^2 f+12 b c d e f+b^2 e^2 f-10 a c e^2 f-12 b^2 d f^2+24 a c d f^2+8 a b e f^2-$$

$$8 a^2 f^3-2 c^2 d e \sqrt{e^2-4 d f}+b c e^2 \sqrt{e^2-4 d f}+2 b c d f \sqrt{e^2-4 d f}-b^2 e f \sqrt{e^2-4 d f}-$$

$$2 a c e f \sqrt{e^2-4 d f}+2 a b f^2 \sqrt{e^2-4 d f}\Big) \sqrt{a+b x+c x^2}\, \text{Log}\Big[e+\sqrt{e^2-4 d f}+2 f x\Big]\Big) \Big/$$

$$\Big(2 \sqrt{2} \left(e^2-4 d f\right)^{3/2} \left(c^2 d^2-b c d e+a c e^2+b^2 d f-2 a c d f-a b e f+a^2 f^2\right)$$

$$\sqrt{c e^2-2 c d f-b e f+2 a f^2+c e \sqrt{e^2-4 d f}-b f \sqrt{e^2-4 d f}}\, \sqrt{a+x (b+c x)}\Big)+$$

$$\Big(f \Big(2 c^2 d e^2-b c e^3-16 c^2 d^2 f+12 b c d e f+b^2 e^2 f-10 a c e^2 f-12 b^2 d f^2+$$

$$24 a c d f^2+8 a b e f^2-8 a^2 f^3-2 c^2 d e \sqrt{e^2-4 d f}+b c e^2 \sqrt{e^2-4 d f}+$$

$$2 b c d f \sqrt{e^2-4 d f}-b^2 e f \sqrt{e^2-4 d f}-2 a c e f \sqrt{e^2-4 d f}+2 a b f^2 \sqrt{e^2-4 d f}\Big)$$

$$\sqrt{a+b x+c x^2}\, \text{Log}\Big[b e-4 a f+b \sqrt{e^2-4 d f}+2 c e x-2 b f x+2 c \sqrt{e^2-4 d f}\, x-$$

$$2 \sqrt{2} \sqrt{\left(c e^2-2 c d f-b e f+2 a f^2+c e \sqrt{e^2-4 d f}-b f \sqrt{e^2-4 d f}\right) \sqrt{a+b x+c x^2}}\Big]\Big) \Big/$$

$$\Big(2 \sqrt{2} \left(e^2-4 d f\right)^{3/2} \left(c^2 d^2-b c d e+a c e^2+b^2 d f-2 a c d f-a b e f+a^2 f^2\right)$$

$$\sqrt{c e^2-2 c d f-b e f+2 a f^2+c e \sqrt{e^2-4 d f}-b f \sqrt{e^2-4 d f}}\, \sqrt{a+x (b+c x)}\Big)+$$

$$\Big(f \Big(-2 c^2 d e^2+b c e^3+16 c^2 d^2 f-12 b c d e f-b^2 e^2 f+10 a c e^2 f+12 b^2 d f^2-$$

$$24 a c d f^2-8 a b e f^2+8 a^2 f^3-2 c^2 d e \sqrt{e^2-4 d f}+b c e^2 \sqrt{e^2-4 d f}+$$

$$2 b c d f \sqrt{e^2-4 d f}-b^2 e f \sqrt{e^2-4 d f}-2 a c e f \sqrt{e^2-4 d f}+2 a b f^2 \sqrt{e^2-4 d f}\Big)$$

$$\sqrt{a+b x+c x^2}\, \text{Log}\Big[-b e+4 a f+b \sqrt{e^2-4 d f}-2 c e x+2 b f x+2 c \sqrt{e^2-4 d f}\, x+$$

$$2 \sqrt{2} \sqrt{\left(c e^2-2 c d f-b e f+2 a f^2-c e \sqrt{e^2-4 d f}+b f \sqrt{e^2-4 d f}\right) \sqrt{a+b x+c x^2}}\Big]\Big) \Big/$$

$$\left(2\sqrt{2}\,\left(e^2-4\,d\,f\right)^{3/2}\,\left(c^2\,d^2-b\,c\,d\,e+a\,c\,e^2+b^2\,d\,f-2\,a\,c\,d\,f-a\,b\,e\,f+a^2\,f^2\right)\right.$$
$$\left.\sqrt{c\,e^2-2\,c\,d\,f-b\,e\,f+2\,a\,f^2-c\,e\,\sqrt{e^2-4\,d\,f}+b\,f\,\sqrt{e^2-4\,d\,f}}\,\sqrt{a+x\,(b+c\,x)}\right)$$

## Problem 121: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-7+2\,x+5\,x^2}\,\left(8+12\,x+5\,x^2\right)}\,dx$$

Optimal (type 3, 51 leaves, 5 steps):

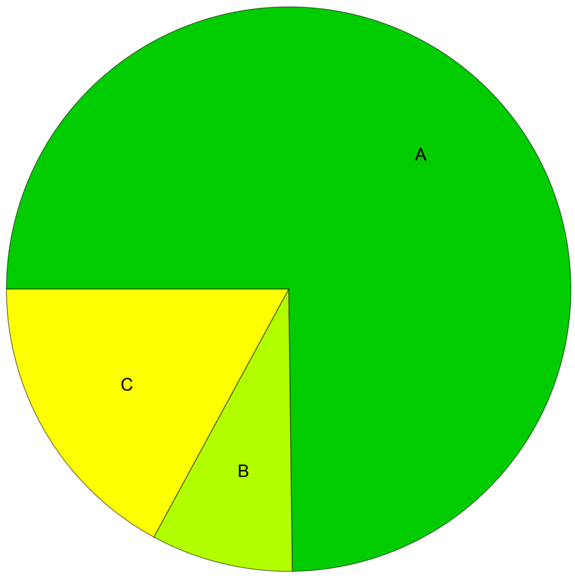
$$\frac{1}{10}\,\text{ArcTan}\left[\frac{5\,(2+x)}{2\,\sqrt{-7+2\,x+5\,x^2}}\right]+\frac{1}{5}\,\text{ArcTanh}\left[\frac{5\,(1+x)}{\sqrt{-7+2\,x+5\,x^2}}\right]$$

Result (type 3, 193 leaves):

$$\left(\frac{1}{20}+\frac{i}{10}\right)\text{ArcTan}\left[\frac{5\,(2+x)}{2\,\sqrt{-7+2\,x+5\,x^2}}\right]-\left(\frac{1}{20}-\frac{i}{10}\right)\text{ArcTan}\left[\frac{2\,\sqrt{-7+2\,x+5\,x^2}}{5\,(2+x)}\right]-$$
$$\frac{1}{20}\,i\,\text{Log}\left[\left((2+2\,i)+(1+2\,i)\,x\right)\,\left((2+2\,i)+(2+i)\,x\right)\right]-$$
$$\left(\frac{1}{20}-\frac{i}{40}\right)\text{Log}\left[9+26\,x+15\,x^2-5\,\sqrt{-7+2\,x+5\,x^2}-5\,x\,\sqrt{-7+2\,x+5\,x^2}\right]+$$
$$\left(\frac{1}{20}+\frac{i}{40}\right)\text{Log}\left[9+26\,x+15\,x^2+5\,\sqrt{-7+2\,x+5\,x^2}+5\,x\,\sqrt{-7+2\,x+5\,x^2}\right]$$

# Summary of Integration Test Results

123 integration problems




A - 92 optimal antiderivatives

B - 10 more than twice size of optimal antiderivatives

C - 21 unnecessarily complex antiderivatives

D - 0 unable to integrate problems

E - 0 integration timeouts