Mathematica 11.3 Integration Test Results

Test results for the 143 problems in "1.2.1.6 (g+h x)^m (a+b x+c x^2)^p (d+e x+f x^2)^q.m"

Problem 9: Result more than twice size of optimal antiderivative.

$$\int \frac{A + B x}{\left(a + b x + c x^2\right)^{5/2} \left(d - f x^2\right)} \, dx$$

Optimal (type 3, 797 leaves, 7 steps):

$$-\left(\left(2 \left(a B \left(2 c^2 d - b^2 f + 2 a c f\right) + A \left(b^3 f - b c \left(c d + 3 a f\right)\right) + c \left(A b^2 f + b B \left(c d - a f\right) - 2 A c \left(c d + a f\right)\right) x\right)\right) \Big/ \left(3 \left(b^2 - 4 a c\right) \left(b^2 d f - \left(c d + a f\right)^2\right) \left(a + b x + c x^2\right)^{3/2}\right)\right) - \frac{1}{3 \left(b^2 - 4 a c\right)^2 \left(c^2 d^2 + 2 a c d f - f \left(b^2 d - a^2 f\right)\right)^2 \sqrt{a + b x + c x^2}} 2 \left(3 b^6 B d f^2 + 24 a^2 B c^2 f \left(c d + a f\right)^2 - A b^5 f^2 \left(7 c d + 6 a f\right) - b^4 B f \left(7 c^2 d^2 + 14 a c d f - 3 a^2 f^2\right) + A b^3 c f \left(15 c^2 d^2 + 46 a c d f + 43 a^2 f^2\right) + 2 b^2 B c \left(2 c^3 d^3 + 5 a c^2 d^2 f + 4 a^2 c d f^2 - 11 a^3 f^3\right) - 4 A b c^2 \left(2 c^3 d^3 + 9 a c^2 d^2 f + 24 a^2 c d f^2 + 17 a^3 f^3\right) + c \left(3 b^5 B d f^2 - 2 A b^4 f^2 \left(4 c d + 3 a f\right) - 8 A c^2 \left(c d + a f\right)^2 \left(2 c d + 5 a f\right) - b^3 B f \left(17 c^2 d^2 + 10 a c d f - 3 a^2 f^2\right) + 2 A b^2 c f \left(15 c^2 d^2 + 22 a c d f + 19 a^2 f^2\right) + 4 b B c \left(2 c^3 d^3 + 11 a c^2 d^2 f + 4 a^2 c d f^2 - 5 a^3 f^3\right)\right) x\right) - \frac{\left(B \sqrt{d} - A \sqrt{f}\right) f^{3/2} \text{ArcTanh}\left[\frac{b \sqrt{d} - 2 a \sqrt{f} + \left(2 c \sqrt{d} - b \sqrt{f}\right) x}{2 \sqrt{c d - b \sqrt{d} \sqrt{f} + a f} \sqrt{a + b x + c x^2}}\right]}{2 \sqrt{d} \left(c d - b \sqrt{d} \sqrt{f} + a f\right)^{5/2}} + \frac{\left(B \sqrt{d} + A \sqrt{f}\right) f^{3/2} \text{ArcTanh}\left[\frac{b \sqrt{d} + 2 a \sqrt{f} + \left(2 c \sqrt{d} + b \sqrt{f}\right) x}{2 \sqrt{c d + b \sqrt{d} \sqrt{f} + a f} \sqrt{a + b x + c x^2}}\right]}{2 \sqrt{d} \left(c d + b \sqrt{d} \sqrt{f} + a f\right)^{5/2}}$$

Result (type 3, 1847 leaves):

$$\frac{1}{\left(a+x\,(b+c\,x)\right)^{5/2}}\left(a+b\,x+c\,x^2\right)^3\Bigg(-\Big(\Big(2\,\big(-A\,b\,c^2\,d+2\,a\,B\,c^2\,d+A\,b^3\,f-a\,b^2\,B\,f-3\,a\,A\,b\,c\,f+2\,a^2\,B\,c\,f+b\,B\,c^2\,d\,x-2\,A\,c^3\,d\,x+A\,b^2\,c\,f\,x-a\,b\,B\,c\,f\,x-2\,a\,A\,c^2\,f\,x\big)\Big)\Big/\Big(3\,(b^2-4\,a\,c)\,(-c^2\,d^2+b^2\,d\,f-2\,a\,c\,d\,f-a^2\,f^2)\,(a+b\,x+c\,x^2)^2\Big)\Big)-\Big(1\Big/\Big(3\,(b^2-4\,a\,c)^2\,(-c^2\,d^2+b^2\,d\,f-2\,a\,c\,d\,f-a^2\,f^2)^2\,(a+b\,x+c\,x^2)\Big)\Big)\,2\,\big(4\,b^2\,B\,c^4\,d^3-8\,A\,b\,c^5\,d^3-7\,b^4\,B\,c^2\,d^2\,f+15\,A\,b^3\,c^3\,d^2\,f+10\,a\,b^2\,B\,c^3\,d^2\,f-36\,a\,A\,b\,c^4\,d^2\,f+24\,a^2\,B\,c^4\,d^2\,f+3\,b^6\,B\,d\,f^2-7\,A\,b^5\,c\,d\,f^2-14\,a\,b^4\,B\,c\,d\,f^2+46\,a\,A\,b^3\,c^2\,d\,f^2+8\,a^2\,b^2\,B\,c^2\,d\,f^2-96\,a^2\,A\,b\,c^3\,d\,f^2+48\,a^3\,B\,c^3\,d\,f^2-6\,a\,A\,b^5\,f^3+3\,a^2\,b^4\,B\,f^3+43\,a^2\,A\,b^3\,c\,f^3-22\,a^3\,b^2\,B\,c\,f^3-68\,a^3\,A\,b\,c^2\,f^3+24\,a^4\,B\,c^2\,f^3+8\,b\,B\,c^5\,d^3\,x-16\,A\,c^6\,d^3\,x-17\,b^3\,B\,c^3\,d^2\,f\,x+30\,A\,b^2\,c^4\,d^2\,f\,x+44\,a\,b\,B\,c^4\,d^2\,f\,x-72\,a\,A\,c^5\,d^2\,f\,x+3\,b^5\,B\,c\,d\,f^2\,x-8\,A\,b^4\,c^2\,d\,f^2\,x-10\,a\,b^3\,B\,c^2\,d\,f^2\,x+44\,a\,A\,b^2\,c^3\,d\,f^2\,x+16\,a^2\,b\,B\,c^3\,d\,f^2\,x-96\,a^2\,A\,c^4\,d\,f^2\,x-6\,a\,A\,b^4\,c\,f^3\,x+3\,a^2\,b^3\,B\,c\,f^3\,x+38\,a^2\,A\,b^2\,c^2\,f^3\,x-20\,a^3\,b\,B\,c^2\,f^3\,x-40\,a^3\,A\,c^3\,f^3\,x\big)\Bigg)-$$

$$\Big(f\,\big(B\,c^2\,d^{5/2}\,\sqrt{f}-2\,b\,B\,c\,d^2\,f+A\,c^2\,d^2\,f+b^2\,B\,d^{3/2}\,f^{3/2}-2\,A\,b\,c\,d^{3/2}\,f^{3/2}+2\,a\,B\,c\,d^{3/2}\,f^{3/2}+A\,b^2\,d\,f^2-2\,a\,b\,B\,d\,f^2+2\,a\,A\,c\,d\,f^2-2\,a\,A\,b\,\sqrt{d}\,f^{5/2}+a^2\,B\,\sqrt{d}\,f^{5/2}+a^2\,A\,f^3\big)\,(a+b\,x+c\,x^2)^{5/2}\,\mathrm{Log}\big[\sqrt{d}\,\sqrt{f}-f\,x\big]\Big)\Big/\Big(2\,\sqrt{d}\,\sqrt{c\,d+b\,\sqrt{d}\,\sqrt{f}+a\,f}\,(c^2\,d^2-b^2\,d\,f+2\,a\,c\,d\,f+a^2\,f^2)^2\,(a+x\,(b+c\,x))^{5/2}\Big)+$$

$$\Big(f\,\big(-B\,c^2\,d^{5/2}\,\sqrt{f}-2\,b\,B\,c\,d^2\,f+A\,c^2\,d^2\,f-b^2\,B\,d^{3/2}\,f^{3/2}+2\,A\,b\,c\,d^{3/2}\,f^{3/2}-2\,a\,B\,c\,d^{3/2}\,f^{3/2}+A\,b^2\,d\,f^2-2\,a\,b\,B\,d\,f^2+2\,a\,A\,c\,d\,f^2+2\,a\,A\,b\,\sqrt{d}\,f^{5/2}-a^2\,B\,\sqrt{d}\,f^{5/2}+a^2\,A\,f^3\big)\,(a+b\,x+c\,x^2)^{5/2}\,\mathrm{Log}\big[\sqrt{d}\,\sqrt{f}+f\,x\big]\Big)\Big/\Big(2\,\sqrt{d}\,\sqrt{c\,d-b\,\sqrt{d}\,\sqrt{f}+a\,f}\,(c^2\,d^2-b^2\,d\,f+2\,a\,c\,d\,f+a^2\,f^2)^2\,(a+x\,(b+c\,x))^{5/2}\Big)-$$

$$\Big(f\,\big(-B\,c^2\,d^{5/2}\,\sqrt{f}-2\,b\,B\,c\,d^2\,f+A\,c^2\,d^2\,f-b^2\,B\,d^{3/2}\,f^{3/2}+2\,A\,b\,c\,d^{3/2}\,f^{3/2}-2\,a\,B\,c\,d^{3/2}\,f^{3/2}+A\,b^2\,d\,f^2-2\,a\,b\,B\,d\,f^2+2\,a\,A\,c\,d\,f^2+2\,a\,A\,b\,\sqrt{d}\,f^{5/2}-a^2\,B\,\sqrt{d}\,f^{5/2}+a^2\,A\,f^3\big)\,(a+b\,x+c\,x^2)^{5/2}\,\mathrm{Log}\Big[-b\,d+2\,a\,\sqrt{d}\,\sqrt{f}-2\,c\,d\,x+b\,\sqrt{d}\,\sqrt{f}\,x+2\,\sqrt{d}\,\sqrt{c\,d-b\,\sqrt{d}\,\sqrt{f}+a\,f}\,\sqrt{a+b\,x+c\,x^2}\Big]\Big)\Big/\Big(2\,\sqrt{d}\,\sqrt{c\,d-b\,\sqrt{d}\,\sqrt{f}+a\,f}\,(c^2\,d^2-b^2\,d\,f+2\,a\,c\,d\,f+a^2\,f^2)^2\,(a+x\,(b+c\,x))^{5/2}\Big)+$$

$$\Big(f\,\big(B\,c^2\,d^{5/2}\,\sqrt{f}-2\,b\,B\,c\,d^2\,f+A\,c^2\,d^2\,f+b^2\,B\,d^{3/2}\,f^{3/2}-2\,A\,b\,c\,d^{3/2}\,f^{3/2}+2\,a\,B\,c\,d^{3/2}\,f^{3/2}+A\,b^2\,d\,f^2-2\,a\,b\,B\,d\,f^2+2\,a\,A\,c\,d\,f^2-2\,a\,A\,b\,\sqrt{d}\,f^{5/2}+a^2\,B\,\sqrt{d}\,f^{5/2}+a^2\,A\,f^3\big)\,(a+b\,x+c\,x^2)^{5/2}\,\mathrm{Log}\Big[b\,d+2\,a\,\sqrt{d}\,\sqrt{f}+2\,c\,d\,x+b\,\sqrt{d}\,\sqrt{f}\,x+2\,\sqrt{d}\,\sqrt{c\,d+b\,\sqrt{d}\,\sqrt{f}+a\,f}\,\sqrt{a+b\,x+c\,x^2}\Big]\Big)\Big/\Big(2\,\sqrt{d}\,\sqrt{c\,d+b\,\sqrt{d}\,\sqrt{f}+a\,f}\,(c^2\,d^2-b^2\,d\,f+2\,a\,c\,d\,f+a^2\,f^2)^2\,(a+x\,(b+c\,x))^{5/2}\Big)$$

Problem 11: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1+2x}{\left(1+x^2\right)\sqrt{-1+x+x^2}}\,dx$$

Optimal (type 3, 117 leaves, 5 steps):

$$-\sqrt{\frac{1}{2}\left(2+\sqrt{5}\right)}\ \text{ArcTan}\left[\frac{5+2\sqrt{5}-\sqrt{5}\,x}{\sqrt{10\left(2+\sqrt{5}\right)}\ \sqrt{-1+x+x^2}}\right]+$$
$$\sqrt{\frac{1}{2}\left(-2+\sqrt{5}\right)}\ \text{ArcTanh}\left[\frac{5-2\sqrt{5}+\sqrt{5}\,x}{\sqrt{10\left(-2+\sqrt{5}\right)}\ \sqrt{-1+x+x^2}}\right]$$

Result (type 3, 394 leaves):

$$\frac{1}{4}\left(2\sqrt{2-i}\ \text{ArcTan}\left[\left(-8+8\,i\,x^3+\frac{20\sqrt{-1+x+x^2}}{\sqrt{2-i}}+x^2\left(2-(2-4\,i)\sqrt{2-i}\ \sqrt{-1+x+x^2}\right)-2\,i\,x\right.\right.\right.$$
$$\left.\left.\left(1+5\sqrt{2-i}\ \sqrt{-1+x+x^2}\right)\right)\Big/\left((14+5\,i)-(15+14\,i)\,x-(6-5\,i)\,x^2+(5+6\,i)\,x^3\right)\right]+$$
$$2\sqrt{2+i}\ \text{ArcTan}\left[\left(2\left(4\,i+x-4x^3+(2+4\,i)\sqrt{2+i}\ \sqrt{-1+x+x^2}-5\sqrt{2+i}\ x\sqrt{-1+x+x^2}+\right.\right.\right.$$
$$\left.\left.\left.x^2\left(-i+\frac{5\sqrt{-1+x+x^2}}{\sqrt{2+i}}\right)\right)\right)\Big/\left((5+14\,i)-(14+15\,i)\,x+(5-6\,i)\,x^2+(6+5\,i)\,x^3\right)\right]+$$
$$i\left(\left(\sqrt{2-i}+\sqrt{2+i}\right)\text{Log}\left[1+x^2\right]-\sqrt{2-i}\ \text{Log}\left[(3-4\,i)-(8-4\,i)\,x-(13-4\,i)\,x^2+\right.\right.$$
$$\left.4\sqrt{2-i}\ \sqrt{-1+x+x^2}+8\sqrt{2-i}\ x\sqrt{-1+x+x^2}\right]-\sqrt{2+i}\ \text{Log}\left[\right.$$
$$\left.\left.\left.(-3-4\,i)+(8+4\,i)\,x+(13+4\,i)\,x^2+4\sqrt{2+i}\ \sqrt{-1+x+x^2}+8\sqrt{2+i}\ x\sqrt{-1+x+x^2}\right]\right)\right)$$

Problem 12: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a-c+b\,x}{\left(1+x^2\right)\sqrt{a+b\,x+c\,x^2}}\,dx$$

Optimal (type 3, 484 leaves, 5 steps):

$$-\left(\left(\sqrt{\left(a^2+b^2+c\left(c-\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c-\sqrt{a^2+b^2-2ac+c^2}\right)\right)}\right.\right.$$
$$\text{ArcTan}\left[\left(b\sqrt{a^2+b^2-2ac+c^2}-\left(b^2+(a-c)\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)\right)x\right)\Big/\right.$$
$$\left(\sqrt{2}\left(a^2+b^2-2ac+c^2\right)^{1/4}\sqrt{\left(a^2+b^2+c\left(c-\sqrt{a^2+b^2-2ac+c^2}\right)-\right.}\right.$$
$$\left.\left.\left.a\left(2c-\sqrt{a^2+b^2-2ac+c^2}\right)\right)\sqrt{a+bx+cx^2}\right)\right]\right)\Big/\left(\sqrt{2}\left(a^2+b^2-2ac+c^2\right)^{1/4}\right)-$$
$$\left(\sqrt{\left(a^2+b^2+c\left(c+\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c+\sqrt{a^2+b^2-2ac+c^2}\right)\right)}\right.$$
$$\text{ArcTanh}\left[\left(b\sqrt{a^2+b^2-2ac+c^2}+\left(b^2+(a-c)\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\right)x\right)\Big/\right.$$
$$\left(\sqrt{2}\left(a^2+b^2-2ac+c^2\right)^{1/4}\sqrt{\left(a^2+b^2+c\left(c+\sqrt{a^2+b^2-2ac+c^2}\right)-\right.}\right.$$
$$\left.\left.\left.a\left(2c+\sqrt{a^2+b^2-2ac+c^2}\right)\right)\sqrt{a+bx+cx^2}\right)\right]\right)\Big/\left(\sqrt{2}\left(a^2+b^2-2ac+c^2\right)^{1/4}\right)$$

Result (type 3, 182 leaves):

$$\frac{1}{2}\,\mathbb{i}\left(-\sqrt{a-\mathbb{i}\,b-c}\,\text{Log}\left[-\frac{2\,\mathbb{i}\left(2a-2\,\mathbb{i}\,c\,x+b\,(-\mathbb{i}+x)+2\sqrt{a-\mathbb{i}\,b-c}\,\sqrt{a+x\,(b+c\,x)}\right)}{(a-\mathbb{i}\,b-c)^{3/2}\,(\mathbb{i}+x)}\right]+\right.$$
$$\left.\sqrt{a+\mathbb{i}\,b-c}\,\text{Log}\left[\frac{2\,\mathbb{i}\left(2a+2\,\mathbb{i}\,c\,x+b\,(\mathbb{i}+x)+2\sqrt{a+\mathbb{i}\,b-c}\,\sqrt{a+x\,(b+c\,x)}\right)}{(a+\mathbb{i}\,b-c)^{3/2}\,(-\mathbb{i}+x)}\right]\right)$$

Problem 19: Result more than twice size of optimal antiderivative.

$$\int\frac{(A+B\,x)\,\sqrt{a+b\,x+c\,x^2}}{d+e\,x+f\,x^2}\,dx$$

Optimal (type 3, 617 leaves, 9 steps):

$$\frac{B\sqrt{a+b\,x+c\,x^2}}{f}-\frac{\left(2\,B\,c\,e-b\,B\,f-2\,A\,c\,f\right)\,\text{ArcTanh}\left[\frac{b+2\,c\,x}{2\,\sqrt{c}\,\sqrt{a+b\,x+c\,x^2}}\right]}{2\,\sqrt{c}\,f^2}+$$

$$\left(\left(2\,f\,\left(A\,f\,\left(c\,d-a\,f\right)-B\,d\,\left(c\,e-b\,f\right)\right)-\left(e-\sqrt{e^2-4\,d\,f}\right)\,\left(A\,f\,\left(c\,e-b\,f\right)+B\,\left(f\,\left(b\,e-a\,f\right)-c\,\left(e^2-d\,f\right)\right)\right)\right)\right.$$
$$\text{ArcTanh}\left[\left(4\,a\,f-b\,\left(e-\sqrt{e^2-4\,d\,f}\right)+2\,\left(b\,f-c\,\left(e-\sqrt{e^2-4\,d\,f}\right)\right)\,x\right)\Big/\right.$$
$$\left.\left.\left(2\,\sqrt{2}\,\sqrt{c\,e^2-2\,c\,d\,f-b\,e\,f+2\,a\,f^2-\left(c\,e-b\,f\right)\,\sqrt{e^2-4\,d\,f}}\,\sqrt{a+b\,x+c\,x^2}\right)\right]\right)\Big/$$
$$\left(\sqrt{2}\,f^2\,\sqrt{e^2-4\,d\,f}\,\sqrt{c\,e^2-2\,c\,d\,f-b\,e\,f+2\,a\,f^2-\left(c\,e-b\,f\right)\,\sqrt{e^2-4\,d\,f}}\right)-$$

$$\left(\left(2\,f\,\left(A\,f\,\left(c\,d-a\,f\right)-B\,d\,\left(c\,e-b\,f\right)\right)-\left(e+\sqrt{e^2-4\,d\,f}\right)\,\left(A\,f\,\left(c\,e-b\,f\right)+B\,\left(f\,\left(b\,e-a\,f\right)-c\,\left(e^2-d\,f\right)\right)\right)\right)\right.$$
$$\text{ArcTanh}\left[\left(4\,a\,f-b\,\left(e+\sqrt{e^2-4\,d\,f}\right)+2\,\left(b\,f-c\,\left(e+\sqrt{e^2-4\,d\,f}\right)\right)\,x\right)\Big/\right.$$
$$\left.\left.\left(2\,\sqrt{2}\,\sqrt{c\,e^2-2\,c\,d\,f-b\,e\,f+2\,a\,f^2+\left(c\,e-b\,f\right)\,\sqrt{e^2-4\,d\,f}}\,\sqrt{a+b\,x+c\,x^2}\right)\right]\right)\Big/$$
$$\left(\sqrt{2}\,f^2\,\sqrt{e^2-4\,d\,f}\,\sqrt{c\,e^2-2\,c\,d\,f-b\,e\,f+2\,a\,f^2+\left(c\,e-b\,f\right)\,\sqrt{e^2-4\,d\,f}}\right)$$

Result (type 3, 1344 leaves):

$$\frac{1}{2\,f^2}$$
$$\left(2\,B\,f\,\sqrt{a+x\,\left(b+c\,x\right)}+\left(\sqrt{2}\,\left(A\,f\,\left(f\,\left(-b\,e+2\,a\,f+b\,\sqrt{e^2-4\,d\,f}\right)+c\,\left(e^2-2\,d\,f-e\,\sqrt{e^2-4\,d\,f}\right)\right)+\right.\right.\right.$$
$$B\,\left(c\,\left(-e^3+3\,d\,e\,f+e^2\,\sqrt{e^2-4\,d\,f}-d\,f\,\sqrt{e^2-4\,d\,f}\right)+f\,\left(a\,f\,\left(-e+\sqrt{e^2-4\,d\,f}\right)+\right.\right.$$
$$\left.\left.\left.b\,\left(e^2-2\,d\,f-e\,\sqrt{e^2-4\,d\,f}\right)\right)\right)\right)\,\text{Log}\left[-e+\sqrt{e^2-4\,d\,f}-2\,f\,x\right]\Big)\Big/$$
$$\left(\sqrt{e^2-4\,d\,f}\,\sqrt{c\,\left(e^2-2\,d\,f-e\,\sqrt{e^2-4\,d\,f}\right)+f\,\left(2\,a\,f+b\,\left(-e+\sqrt{e^2-4\,d\,f}\right)\right)}\right)+$$
$$\left(\sqrt{2}\,\left(A\,f\,\left(-c\,\left(e^2-2\,d\,f+e\,\sqrt{e^2-4\,d\,f}\right)+f\,\left(-2\,a\,f+b\,\left(e+\sqrt{e^2-4\,d\,f}\right)\right)\right)+\right.\right.$$

$$B\,(c\,(e^3-3def+e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df})+$$
$$f\,(af\,(e+\sqrt{e^2-4df})-b\,(e^2-2df+e\sqrt{e^2-4df}))))\,\mathrm{Log}[e+\sqrt{e^2-4df}+2fx])\Big/$$
$$(\sqrt{e^2-4df}\,\sqrt{c\,(e^2-2df+e\sqrt{e^2-4df})+f\,(2af-b\,(e+\sqrt{e^2-4df}))})-$$
$$\frac{(2Bce-bBf-2Acf)\,\mathrm{Log}[b+2cx+2\sqrt{c}\,\sqrt{a+x\,(b+cx)}]}{\sqrt{c}}-$$
$$(\sqrt{2}\,(Af\,(f\,(-be+2af+b\sqrt{e^2-4df})+c\,(e^2-2df-e\sqrt{e^2-4df}))+$$
$$B\,(c\,(-e^3+3def+e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df})+$$
$$f\,(af\,(-e+\sqrt{e^2-4df})+b\,(e^2-2df-e\sqrt{e^2-4df}))))$$
$$\mathrm{Log}[4af\sqrt{e^2-4df}+2ce^2x-8cdfx-2ce\sqrt{e^2-4df}\,x+$$
$$b\,(e^2-4df-e\sqrt{e^2-4df}+2f\sqrt{e^2-4df}\,x)+2\sqrt{2}\,\sqrt{e^2-4df}$$
$$\sqrt{(f\,(-be+2af+b\sqrt{e^2-4df})+c\,(e^2-2df-e\sqrt{e^2-4df}))}\,\sqrt{a+x\,(b+cx)}\,])\Big/$$
$$(\sqrt{e^2-4df}\,\sqrt{c\,(e^2-2df-e\sqrt{e^2-4df})+f\,(2af+b\,(-e+\sqrt{e^2-4df}))})-$$
$$(\sqrt{2}\,(Af\,(-c\,(e^2-2df+e\sqrt{e^2-4df})+f\,(-2af+b\,(e+\sqrt{e^2-4df})))+$$
$$B\,(c\,(e^3-3def+e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df})+$$
$$f\,(af\,(e+\sqrt{e^2-4df})-b\,(e^2-2df+e\sqrt{e^2-4df}))))$$
$$\mathrm{Log}[4af\sqrt{e^2-4df}-2ce^2x+8cdfx-2ce\sqrt{e^2-4df}\,x+2\sqrt{2}\,\sqrt{e^2-4df}$$
$$\sqrt{(c\,(e^2-2df+e\sqrt{e^2-4df})+f\,(2af-b\,(e+\sqrt{e^2-4df})))}\,\sqrt{a+x\,(b+cx)}-$$
$$b\,(e^2+e\sqrt{e^2-4df}-2f\,(2d+\sqrt{e^2-4df}\,x))])\Big/$$
$$(\sqrt{e^2-4df}\,\sqrt{c\,(e^2-2df+e\sqrt{e^2-4df})+f\,(2af-b\,(e+\sqrt{e^2-4df}))}))$$

Problem 20: Result more than twice size of optimal antiderivative.

$$\int\frac{(A+Bx)\,(a+bx+cx^2)^{3/2}}{d+ex+fx^2}\,dx$$

Optimal (type 3, 1092 leaves, 10 steps):

$$-\frac{1}{8 c f^3}\left(2 A c f (4 c e-5 b f)-B\left(b^2 f^2-2 c f (5 b e-4 a f)+8 c^2\left(e^2-d f\right)\right)+2 c f (2 B c e-b B f-2 A c f) x\right) \sqrt{a+b x+c x^2}+$$

$$\frac{B\left(a+b x+c x^2\right)^{3/2}}{3 f}+\frac{1}{16 c^{3/2} f^4}\left(2 A c f\left(3 b^2 f^2-12 c f (b e-a f)+8 c^2\left(e^2-d f\right)\right)-B\left(b^3 f^3+6 b c f^2 (b e-2 a f)-24 c^2 f\left(b e^2-b d f-a e f\right)+16 c^3\left(e^3-2 d e f\right)\right)\right)$$

$$\text{ArcTanh}\left[\frac{b+2 c x}{2 \sqrt{c} \sqrt{a+b x+c x^2}}\right]-\left(\left(2 c f\left(B d (c e-b f)\left(c e^2-2 c d f-b e f+2 a f^2\right)+A f\left(2 c d f (b e-a f)-f^2\left(b^2 d-a^2 f\right)-c^2 d\left(e^2-d f\right)\right)\right)-c\left(e-\sqrt{e^2-4 d f}\right)\left(A f (c e-b f)\left(f (b e-2 a f)-c\left(e^2-2 d f\right)\right)+B\left(c^2\left(e^4-3 d e^2 f+d^2 f^2\right)-f^2\left(2 a b e f-a^2 f^2-b^2\left(e^2-d f\right)\right)+2 c f\left(a f\left(e^2-d f\right)-b\left(e^3-2 d e f\right)\right)\right)\right)\right)\right.$$

$$\left.\text{ArcTanh}\left[\left(4 a f-b\left(e-\sqrt{e^2-4 d f}\right)+2\left(b f-c\left(e-\sqrt{e^2-4 d f}\right)\right) x\right) \Big/ \left(2 \sqrt{2} \sqrt{c e^2-2 c d f-b e f+2 a f^2-(c e-b f) \sqrt{e^2-4 d f}} \sqrt{a+b x+c x^2}\right)\right]\right) \Big/$$

$$\left(\sqrt{2} c f^4 \sqrt{e^2-4 d f} \sqrt{c e^2-2 c d f-b e f+2 a f^2-(c e-b f) \sqrt{e^2-4 d f}}\right)+$$

$$\left(\left(2 f\left(B d (c e-b f)\left(c e^2-2 c d f-b e f+2 a f^2\right)+A f\left(2 c d f (b e-a f)-f^2\left(b^2 d-a^2 f\right)-c^2 d\left(e^2-d f\right)\right)\right)-\left(e+\sqrt{e^2-4 d f}\right)\left(A f (c e-b f)\left(f (b e-2 a f)-c\left(e^2-2 d f\right)\right)+B\left(c^2\left(e^4-3 d e^2 f+d^2 f^2\right)-f^2\left(2 a b e f-a^2 f^2-b^2\left(e^2-d f\right)\right)+2 c f\left(a f\left(e^2-d f\right)-b\left(e^3-2 d e f\right)\right)\right)\right)\right)\right.$$

$$\left.\text{ArcTanh}\left[\left(4 a f-b\left(e+\sqrt{e^2-4 d f}\right)+2\left(b f-c\left(e+\sqrt{e^2-4 d f}\right)\right) x\right) \Big/ \left(2 \sqrt{2} \sqrt{c e^2-2 c d f-b e f+2 a f^2+(c e-b f) \sqrt{e^2-4 d f}} \sqrt{a+b x+c x^2}\right)\right]\right) \Big/$$

$$\left(\sqrt{2} f^4 \sqrt{e^2-4 d f} \sqrt{c e^2-2 c d f-b e f+2 a f^2+(c e-b f) \sqrt{e^2-4 d f}}\right)$$

Result (type 3, 3733 leaves):

$$\frac{1}{a+b x+c x^2}$$

$$\left(-\frac{1}{24 c f^3}\left(-24 B c^2 e^2+24 B c^2 d f+30 b B c e f+24 A c^2 e f-3 b^2 B f^2-30 A b c f^2-32 a B c f^2\right)+\right.$$

$$\frac{(-6 B c e + 7 b B f + 6 A c f)\, x}{12 f^2} + \frac{B c x^2}{3 f}\Big)\, (a + x\, (b + c x))^{3/2} +$$

$$\Big(\Big(-B c^2 e^5 + 5 B c^2 d e^3 f + 2 b B c e^4 f + A c^2 e^4 f - 5 B c^2 d^2 e f^2 - 8 b B c d e^2 f^2 - 4 A c^2 d e^2 f^2 - b^2 B e^3 f^2 - 2 A b c e^3 f^2 - 2 a B c e^3 f^2 + 4 b B c d^2 f^3 + 2 A c^2 d^2 f^3 + 3 b^2 B d e f^3 + 6 A b c d e f^3 + 6 a B c d e f^3 + A b^2 e^2 f^3 + 2 a b B e^2 f^3 + 2 a A c e^2 f^3 - 2 A b^2 d f^4 - 4 a b B d f^4 - 4 a A c d f^4 - 2 a A b e f^4 - a^2 B e f^4 + 2 a^2 A f^5 + B c^2 e^4 \sqrt{e^2 - 4 d f} - 3 B c^2 d e^2 f \sqrt{e^2 - 4 d f} - 2 b B c e^3 f \sqrt{e^2 - 4 d f} - A c^2 e^3 f \sqrt{e^2 - 4 d f} + B c^2 d^2 f^2 \sqrt{e^2 - 4 d f} + 4 b B c d e f^2 \sqrt{e^2 - 4 d f} + 2 A c^2 d e f^2 \sqrt{e^2 - 4 d f} + b^2 B e^2 f^2 \sqrt{e^2 - 4 d f} + 2 A b c e^2 f^2 \sqrt{e^2 - 4 d f} + 2 a B c e^2 f^2 \sqrt{e^2 - 4 d f} - b^2 B d f^3 \sqrt{e^2 - 4 d f} - 2 A b c d f^3 \sqrt{e^2 - 4 d f} - 2 a B c d f^3 \sqrt{e^2 - 4 d f} - A b^2 e f^3 \sqrt{e^2 - 4 d f} - 2 a b B e f^3 \sqrt{e^2 - 4 d f} - 2 a A c e f^3 \sqrt{e^2 - 4 d f} + 2 a A b f^4 \sqrt{e^2 - 4 d f} + a^2 B f^4 \sqrt{e^2 - 4 d f}\Big)\, (a + x\, (b + c x))^{3/2} \mathrm{Log}\Big[-e + \sqrt{e^2 - 4 d f} - 2 f x\Big]\Big) \Big/ \Big(\sqrt{2}\, f^4 \sqrt{e^2 - 4 d f}\, \sqrt{c e^2 - 2 c d f - b e f + 2 a f^2 - c e \sqrt{e^2 - 4 d f} + b f \sqrt{e^2 - 4 d f}}\, (a + b x + c x^2)^{3/2}\Big) +$$

$$\Big(\Big(B c^2 e^5 - 5 B c^2 d e^3 f - 2 b B c e^4 f - A c^2 e^4 f + 5 B c^2 d^2 e f^2 + 8 b B c d e^2 f^2 + 4 A c^2 d e^2 f^2 + b^2 B e^3 f^2 + 2 A b c e^3 f^2 + 2 a B c e^3 f^2 - 4 b B c d^2 f^3 - 2 A c^2 d^2 f^3 - 3 b^2 B d e f^3 - 6 A b c d e f^3 - 6 a B c d e f^3 - A b^2 e^2 f^3 - 2 a b B e^2 f^3 - 2 a A c e^2 f^3 + 2 A b^2 d f^4 + 4 a b B d f^4 + 4 a A c d f^4 + 2 a A b e f^4 + a^2 B e f^4 - 2 a^2 A f^5 + B c^2 e^4 \sqrt{e^2 - 4 d f} - 3 B c^2 d e^2 f \sqrt{e^2 - 4 d f} - 2 b B c e^3 f \sqrt{e^2 - 4 d f} - A c^2 e^3 f \sqrt{e^2 - 4 d f} + B c^2 d^2 f^2 \sqrt{e^2 - 4 d f} + 4 b B c d e f^2 \sqrt{e^2 - 4 d f} + 2 A c^2 d e f^2 \sqrt{e^2 - 4 d f} + b^2 B e^2 f^2 \sqrt{e^2 - 4 d f} + 2 A b c e^2 f^2 \sqrt{e^2 - 4 d f} + 2 a B c e^2 f^2 \sqrt{e^2 - 4 d f} - b^2 B d f^3 \sqrt{e^2 - 4 d f} - 2 A b c d f^3 \sqrt{e^2 - 4 d f} - 2 a B c d f^3 \sqrt{e^2 - 4 d f} - A b^2 e f^3 \sqrt{e^2 - 4 d f} - 2 a b B e f^3 \sqrt{e^2 - 4 d f} - 2 a A c e f^3 \sqrt{e^2 - 4 d f} + 2 a A b f^4 \sqrt{e^2 - 4 d f} + a^2 B f^4 \sqrt{e^2 - 4 d f}\Big)\, (a + x\, (b + c x))^{3/2} \mathrm{Log}\Big[e + \sqrt{e^2 - 4 d f} + 2 f x\Big]\Big) \Big/ \Big(\sqrt{2}\, f^4 \sqrt{e^2 - 4 d f}\, \sqrt{c e^2 - 2 c d f - b e f + 2 a f^2 + c e \sqrt{e^2 - 4 d f} - b f \sqrt{e^2 - 4 d f}}\, (a + b x + c x^2)^{3/2}\Big) -$$

$$\Big(\Big(16 B c^3 e^3 - 32 B c^3 d e f - 24 b B c^2 e^2 f - 16 A c^3 e^2 f + 24 b B c^2 d f^2 + 16 A c^3 d f^2 + 6 b^2 B c e f^2 + 24 A b c^2 e f^2 + 24 a B c^2 e f^2 + b^3 B f^3 - 6 A b^2 c f^3 - 12 a b B c f^3 - 24 a A c^2 f^3\Big)\, (a + x\, (b + c x))^{3/2} \mathrm{Log}\Big[b + 2 c x + 2 \sqrt{c}\, \sqrt{a + b x + c x^2}\Big]\Big) \Big/ \Big(16 c^{3/2} f^4\, (a + b x + c x^2)^{3/2}\Big) -$$

$$\Bigg(\Big(B c^2 e^5 - 5 B c^2 d e^3 f - 2 b B c e^4 f - A c^2 e^4 f + 5 B c^2 d^2 e f^2 + 8 b B c d e^2 f^2 + 4 A c^2 d e^2 f^2 + b^2 B e^3 f^2 + 2 A b c e^3 f^2 + 2 a B c e^3 f^2 - 4 b B c d^2 f^3 - 2 A c^2 d^2 f^3 - 3 b^2 B d e f^3 - 6 A b c d e f^3 - 6 a B c d e f^3 - A b^2 e^2 f^3 - 2 a b B e^2 f^3 - 2 a A c e^2 f^3 + 2 A b^2 d f^4 + 4 a b B d f^4 + 4 a A c d f^4 + 2 a A b e f^4 + a^2 B e f^4 - 2 a^2 A f^5 + B c^2 e^4 \sqrt{e^2 - 4 d f} - 3 B c^2 d e^2 f \sqrt{e^2 - 4 d f} - 2 b B c e^3 f \sqrt{e^2 - 4 d f} - A c^2 e^3 f \sqrt{e^2 - 4 d f} + B c^2 d^2 f^2 \sqrt{e^2 - 4 d f} + 4 b B c d e f^2 \sqrt{e^2 - 4 d f} + 2 A c^2 d e f^2 \sqrt{e^2 - 4 d f} + b^2 B e^2 f^2 \sqrt{e^2 - 4 d f} + 2 A b c e^2 f^2 \sqrt{e^2 - 4 d f} + 2 a B c e^2 f^2 \sqrt{e^2 - 4 d f} - b^2 B d f^3 \sqrt{e^2 - 4 d f} - 2 A b c d f^3 \sqrt{e^2 - 4 d f} - 2 a B c d f^3 \sqrt{e^2 - 4 d f} - A b^2 e f^3 \sqrt{e^2 - 4 d f} - 2 a b B e f^3 \sqrt{e^2 - 4 d f} - 2 a A c e f^3 \sqrt{e^2 - 4 d f} + 2 a A b f^4 \sqrt{e^2 - 4 d f} + a^2 B f^4 \sqrt{e^2 - 4 d f}\Big) (a + x (b + c x))^{3/2} \text{Log}\Big[-b e^2 + 4 b d f - b e \sqrt{e^2 - 4 d f} + 4 a f \sqrt{e^2 - 4 d f} - 2 c e^2 x + 8 c d f x - 2 c e \sqrt{e^2 - 4 d f}\, x + 2 b f \sqrt{e^2 - 4 d f}\, x + 2 \sqrt{2} \sqrt{e^2 - 4 d f} \sqrt{c e^2 - 2 c d f - b e f + 2 a f^2 + c e \sqrt{e^2 - 4 d f} - b f \sqrt{e^2 - 4 d f}} \sqrt{a + b x + c x^2}\Big]\Bigg) \Big/ \Bigg(\sqrt{2} f^4 \sqrt{e^2 - 4 d f} \sqrt{c e^2 - 2 c d f - b e f + 2 a f^2 + c e \sqrt{e^2 - 4 d f} - b f \sqrt{e^2 - 4 d f}} (a + b x + c x^2)^{3/2}\Bigg) -$$

$$\Bigg(\Big(-B c^2 e^5 + 5 B c^2 d e^3 f + 2 b B c e^4 f + A c^2 e^4 f - 5 B c^2 d^2 e f^2 - 8 b B c d e^2 f^2 - 4 A c^2 d e^2 f^2 - b^2 B e^3 f^2 - 2 A b c e^3 f^2 - 2 a B c e^3 f^2 + 4 b B c d^2 f^3 + 2 A c^2 d^2 f^3 + 3 b^2 B d e f^3 + 6 A b c d e f^3 + 6 a B c d e f^3 + A b^2 e^2 f^3 + 2 a b B e^2 f^3 + 2 a A c e^2 f^3 - 2 A b^2 d f^4 - 4 a b B d f^4 - 4 a A c d f^4 - 2 a A b e f^4 - a^2 B e f^4 + 2 a^2 A f^5 + B c^2 e^4 \sqrt{e^2 - 4 d f} - 3 B c^2 d e^2 f \sqrt{e^2 - 4 d f} - 2 b B c e^3 f \sqrt{e^2 - 4 d f} - A c^2 e^3 f \sqrt{e^2 - 4 d f} + B c^2 d^2 f^2 \sqrt{e^2 - 4 d f} + 4 b B c d e f^2 \sqrt{e^2 - 4 d f} + 2 A c^2 d e f^2 \sqrt{e^2 - 4 d f} + b^2 B e^2 f^2 \sqrt{e^2 - 4 d f} + 2 A b c e^2 f^2 \sqrt{e^2 - 4 d f} + 2 a B c e^2 f^2 \sqrt{e^2 - 4 d f} - b^2 B d f^3 \sqrt{e^2 - 4 d f} - 2 A b c d f^3 \sqrt{e^2 - 4 d f} - 2 a B c d f^3 \sqrt{e^2 - 4 d f} - A b^2 e f^3 \sqrt{e^2 - 4 d f} - 2 a b B e f^3 \sqrt{e^2 - 4 d f} - 2 a A c e f^3 \sqrt{e^2 - 4 d f} + 2 a A b f^4 \sqrt{e^2 - 4 d f} + a^2 B f^4 \sqrt{e^2 - 4 d f}\Big) (a + x (b + c x))^{3/2} \text{Log}\Big[b e^2 - 4 b d f - b e \sqrt{e^2 - 4 d f} + 4 a f \sqrt{e^2 - 4 d f} + 2 c e^2 x - 8 c d f x - 2 c e \sqrt{e^2 - 4 d f}\, x + 2 b f \sqrt{e^2 - 4 d f}\, x + 2 \sqrt{2} \sqrt{e^2 - 4 d f} \sqrt{c e^2 - 2 c d f - b e f + 2 a f^2 - c e \sqrt{e^2 - 4 d f} + b f \sqrt{e^2 - 4 d f}} \sqrt{a + b x + c x^2}\Big]\Bigg) \Big/$$

$$\left(\sqrt{2}\, f^4 \sqrt{e^2-4 d f} \sqrt{c e^2-2 c d f-b e f+2 a f^2-c e \sqrt{e^2-4 d f}+b f \sqrt{e^2-4 d f}}\, \left(a+b x+c x^2\right)^{3/2}\right)$$

Problem 22: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A+B x}{\left(a+c x^2\right) \sqrt{d+e x+f x^2}} \, dx$$

Optimal (type 3, 780 leaves, 5 steps):

$$\left(\sqrt{a B e+A \left(c d-a f-\sqrt{c^2 d^2+a^2 f^2+a c \left(e^2-2 d f\right)}\right)} \sqrt{-A c e+B \left(c d-a f+\sqrt{c^2 d^2+a^2 f^2+a c \left(e^2-2 d f\right)}\right)}\, \text{ArcTanh}\left[\left(\sqrt{e} \left(a \left(A c e-B \left(c d-a f+\sqrt{c^2 d^2+a^2 f^2+a c \left(e^2-2 d f\right)}\right)\right)-c \left(a B e+A \left(c d-a f-\sqrt{c^2 d^2+a^2 f^2+a c \left(e^2-2 d f\right)}\right)\right) x\right)\right) \Big/ \left(\sqrt{2} \sqrt{a} \sqrt{c} \sqrt{a B e+A \left(c d-a f-\sqrt{c^2 d^2+a^2 f^2+a c \left(e^2-2 d f\right)}\right)} \sqrt{-A c e+B \left(c d-a f+\sqrt{c^2 d^2+a^2 f^2+a c \left(e^2-2 d f\right)}\right)} \sqrt{d+e x+f x^2}\right)\right]\right) \Big/ \left(\sqrt{2} \sqrt{a} \sqrt{c} \sqrt{e} \sqrt{c^2 d^2+a^2 f^2+a c \left(e^2-2 d f\right)}\right) - \left(\sqrt{-A c e+B \left(c d-a f-\sqrt{c^2 d^2+a^2 f^2+a c \left(e^2-2 d f\right)}\right)} \sqrt{a B e+A \left(c d-a f+\sqrt{c^2 d^2+a^2 f^2+a c \left(e^2-2 d f\right)}\right)}\, \text{ArcTanh}\left[\left(\sqrt{e} \left(a \left(A c e-B \left(c d-a f-\sqrt{c^2 d^2+a^2 f^2+a c \left(e^2-2 d f\right)}\right)\right)-c \left(a B e+A \left(c d-a f+\sqrt{c^2 d^2+a^2 f^2+a c \left(e^2-2 d f\right)}\right)\right) x\right)\right) \Big/ \left(\sqrt{2} \sqrt{a} \sqrt{c} \sqrt{-A c e+B \left(c d-a f-\sqrt{c^2 d^2+a^2 f^2+a c \left(e^2-2 d f\right)}\right)} \sqrt{a B e+A \left(c d-a f+\sqrt{c^2 d^2+a^2 f^2+a c \left(e^2-2 d f\right)}\right)} \sqrt{d+e x+f x^2}\right)\right]\right) \Big/ \left(\sqrt{2} \sqrt{a} \sqrt{c} \sqrt{e} \sqrt{c^2 d^2+a^2 f^2+a c \left(e^2-2 d f\right)}\right)$$

Result (type 3, 411 leaves):

$$\frac{1}{2\sqrt{a}\,\sqrt{c}}\left(-\left(\left(\left(\sqrt{a}\,B+i\,A\sqrt{c}\right)\text{Log}\left[-\left(\left(\sqrt{a}\,\sqrt{c}\left(i\sqrt{c}\,(2\,d+e\,x)+\sqrt{a}\,(e+2\,f\,x)+2\,i\sqrt{c\,d-i\sqrt{a}\,\sqrt{c}\,e-a\,f}\,\sqrt{d+x\,(e+f\,x)}\right)\right)\Big/\left(\left(\sqrt{a}\,B+i\,A\sqrt{c}\right)\sqrt{c\,d-i\sqrt{a}\,\sqrt{c}\,e-a\,f}\,\left(\sqrt{a}-i\sqrt{c}\,x\right)\right)\right)\right]\right)\Big/\left(\sqrt{c\,d-i\sqrt{a}\,\sqrt{c}\,e-a\,f}\right)\right)+\left(\left(-\sqrt{a}\,B+i\,A\sqrt{c}\right)\text{Log}\left[\left(i\sqrt{a}\,\sqrt{c}\left(\sqrt{c}\,(2\,d+e\,x)+i\sqrt{a}\,(e+2\,f\,x)+2\sqrt{c\,d+i\sqrt{a}\,\sqrt{c}\,e-a\,f}\,\sqrt{d+x\,(e+f\,x)}\right)\right)\Big/\left(\left(\sqrt{a}\,B-i\,A\sqrt{c}\right)\sqrt{c\,d+i\sqrt{a}\,\sqrt{c}\,e-a\,f}\,\left(\sqrt{a}+i\sqrt{c}\,x\right)\right)\right]\right)\Big/\left(\sqrt{c\,d+i\sqrt{a}\,\sqrt{c}\,e-a\,f}\right)\right)$$

Problem 24: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{A+B\,x}{\left(a+c\,x^2\right)\sqrt{d+f\,x^2}}\,dx$$

Optimal (type 3, 101 leaves, 6 steps):

$$\frac{A\,\text{ArcTan}\left[\frac{\sqrt{c\,d-a\,f}\,x}{\sqrt{a}\,\sqrt{d+f\,x^2}}\right]}{\sqrt{a}\,\sqrt{c\,d-a\,f}}-\frac{B\,\text{ArcTanh}\left[\frac{\sqrt{c}\,\sqrt{d+f\,x^2}}{\sqrt{c\,d-a\,f}}\right]}{\sqrt{c}\,\sqrt{c\,d-a\,f}}$$

Result (type 3, 282 leaves):

$$\left(-\left(\sqrt{a}\,B+i\,A\sqrt{c}\right)\text{Log}\left[\frac{2\sqrt{a}\,\sqrt{c}\left(\sqrt{c}\,d-i\sqrt{a}\,f\,x+\sqrt{c\,d-a\,f}\,\sqrt{d+f\,x^2}\right)}{\left(\sqrt{a}\,B+i\,A\sqrt{c}\right)\sqrt{c\,d-a\,f}\,\left(i\sqrt{a}+\sqrt{c}\,x\right)}\right]+\left(-\sqrt{a}\,B+i\,A\sqrt{c}\right)\text{Log}\left[\frac{2\,i\sqrt{a}\,\sqrt{c}\left(\sqrt{c}\,d+i\sqrt{a}\,f\,x+\sqrt{c\,d-a\,f}\,\sqrt{d+f\,x^2}\right)}{\left(\sqrt{a}\,B-i\,A\sqrt{c}\right)\sqrt{c\,d-a\,f}\,\left(\sqrt{a}+i\sqrt{c}\,x\right)}\right]\right)\Big/\left(2\sqrt{a}\,\sqrt{c}\,\sqrt{c\,d-a\,f}\right)$$

Problem 27: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{2+x}{\left(2+4\,x-3\,x^2\right)\left(1+3\,x-2\,x^2\right)^{5/2}}\,dx$$

Optimal (type 3, 193 leaves, 7 steps):

$$-\frac{2\,(15+14\,x)}{51\,(1+3\,x-2\,x^2)^{3/2}}-\frac{2\,(291+4814\,x)}{867\,\sqrt{1+3\,x-2\,x^2}}+$$
$$\frac{9}{2}\sqrt{\frac{1}{5}\left(-53+17\,\sqrt{10}\right)}\;\text{ArcTan}\Big[\frac{3\left(4-\sqrt{10}\right)+\left(1+4\,\sqrt{10}\right)x}{2\,\sqrt{1+\sqrt{10}}\;\sqrt{1+3\,x-2\,x^2}}\Big]+$$
$$\frac{9}{2}\sqrt{\frac{1}{5}\left(53+17\,\sqrt{10}\right)}\;\text{ArcTanh}\Big[\frac{3\left(4+\sqrt{10}\right)+\left(1-4\,\sqrt{10}\right)x}{2\,\sqrt{-1+\sqrt{10}}\;\sqrt{1+3\,x-2\,x^2}}\Big]$$

Result (type 3, 304 leaves):

$$-\frac{2\,(546+5925\,x+13\,860\,x^2-9628\,x^3)}{867\,(1+3\,x-2\,x^2)^{3/2}}-\frac{27\left(-4+\sqrt{10}\right)\text{ArcTan}\Big[\frac{12-3\sqrt{10}+x+4\sqrt{10}\,x}{2\sqrt{1+\sqrt{10}}\,\sqrt{1+3\,x-2\,x^2}}\Big]}{2\,\sqrt{10\left(1+\sqrt{10}\right)}}-$$
$$\frac{27\left(4+\sqrt{10}\right)\text{Log}\left[2+\sqrt{10}-3\,x\right]}{2\,\sqrt{10\left(-1+\sqrt{10}\right)}}-\frac{27\,i\left(-4+\sqrt{10}\right)\text{Log}\Big[\left(-2+\sqrt{10}+3\,x\right)^2\Big]}{4\,\sqrt{10\left(1+\sqrt{10}\right)}}+$$
$$\frac{27\,i\left(-4+\sqrt{10}\right)\text{Log}\left[14-4\,\sqrt{10}+6\left(-2+\sqrt{10}\right)x+9\,x^2\right]}{4\,\sqrt{10\left(1+\sqrt{10}\right)}}+$$
$$\left(27\left(4+\sqrt{10}\right)\text{Log}\left[30+12\,\sqrt{10}-40\,x+\sqrt{10}\,x+2\,\sqrt{10\left(-1+\sqrt{10}\right)}\,\sqrt{1+3\,x-2\,x^2}\,\right]\right)\Big/$$
$$\left(2\,\sqrt{10\left(-1+\sqrt{10}\right)}\right)$$

Problem 31: Result more than twice size of optimal antiderivative.

$$\int\frac{1+x}{(4+2\,x+x^2)\,\sqrt{5+2\,x+x^2}}\,dx$$

Optimal (type 3, 15 leaves, 2 steps):

$$-\text{ArcTanh}\left[\sqrt{5+2\,x+x^2}\,\right]$$

Result (type 3, 41 leaves):

$$\frac{1}{2}\,\text{Log}\left[1-\sqrt{5+2\,x+x^2}\,\right]-\frac{1}{2}\,\text{Log}\left[1+\sqrt{5+2\,x+x^2}\,\right]$$

Problem 32: Result more than twice size of optimal antiderivative.

$$\int \frac{4+x}{\left(4+2x+x^2\right)\sqrt{5+2x+x^2}}\,dx$$

Optimal (type 3, 44 leaves, 5 steps):

$$\sqrt{3}\,\text{ArcTan}\left[\frac{1+x}{\sqrt{3}\,\sqrt{5+2x+x^2}}\right]-\text{ArcTanh}\left[\sqrt{5+2x+x^2}\right]$$

Result (type 3, 109 leaves):

$$\frac{1}{2}\left(2\sqrt{3}\,\text{ArcTan}\left[\frac{\sqrt{3}\left(4+x^2+\sqrt{5+2x+x^2}+x\left(2+\sqrt{5+2x+x^2}\right)\right)}{11+4x+2x^2}\right]+\right.$$
$$\left.\text{Log}\left[\left(4+2x+x^2\right)^2\right]-\text{Log}\left[\left(4+2x+x^2\right)\left(6+2x+x^2+2\sqrt{5+2x+x^2}\right)\right]\right)$$

Problem 34: Result more than twice size of optimal antiderivative.

$$\int \frac{x}{\left(3+x+x^2\right)\sqrt{5+x+x^2}}\,dx$$

Optimal (type 3, 56 leaves, 5 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{\frac{2}{11}}\,(1+2x)}{\sqrt{5+x+x^2}}\right]}{\sqrt{22}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{5+x+x^2}}{\sqrt{2}}\right]}{\sqrt{2}}$$

Result (type 3, 126 leaves):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{11}\,\left(-3+7x+7x^2\right)}{-57-19x^2+12\sqrt{2}\,\sqrt{5+x+x^2}+x\left(-19+24\sqrt{2}\,\sqrt{5+x+x^2}\right)}\right]}{\sqrt{22}}+\frac{1}{2\sqrt{2}}$$
$$\left(\text{Log}[16]+\text{Log}\left[\left(3+x+x^2\right)^2\right]-\text{Log}\left[\left(3+x+x^2\right)\left(7+x+x^2+2\sqrt{2}\,\sqrt{5+x+x^2}\right)\right]\right)$$

Problem 35: Result more than twice size of optimal antiderivative.

$$\int \frac{A+Bx}{\sqrt{d+ex+fx^2}\,\left(ae+bex+bfx^2\right)^2}\,dx$$

Optimal (type 3, 249 leaves, 6 steps):

$$-\frac{\left(\left(A\,b-2\,a\,B\right)\,e-b\,\left(B\,e-2\,A\,f\right)\,x\right)\,\sqrt{d+e\,x+f\,x^2}}{e\,\left(b\,d-a\,e\right)\,\left(b\,e-4\,a\,f\right)\,\left(a\,e+b\,e\,x+b\,f\,x^2\right)}+\frac{\left(B\,e-2\,A\,f\right)\,\left(8\,a\,e\,f-b\,\left(e^2+4\,d\,f\right)\right)\,\text{ArcTanh}\left[\frac{\sqrt{b\,d-a\,e}\,\left(e+2\,f\,x\right)}{\sqrt{e}\,\sqrt{b\,e-4\,a\,f}\,\sqrt{d+e\,x+f\,x^2}}\right]}{2\,e^{3/2}\,\left(b\,d-a\,e\right)^{3/2}\,f\,\left(b\,e-4\,a\,f\right)^{3/2}}+\frac{B\,\text{ArcTanh}\left[\frac{\sqrt{b}\,\sqrt{d+e\,x+f\,x^2}}{\sqrt{b\,d-a\,e}}\right]}{2\,\sqrt{b}\,\left(b\,d-a\,e\right)^{3/2}\,f}$$

Result (type 3, 767 leaves):

$$-\frac{1}{4\,b\,e^{3/2}\,\left(b\,d-a\,e\right)^{3/2}\,f\,\left(b\,e-4\,a\,f\right)^{3/2}\,\left(a\,e+b\,x\,\left(e+f\,x\right)\right)}$$
$$\Big(4\,b\,\sqrt{e}\,\sqrt{b\,d-a\,e}\,f\,\sqrt{b\,e-4\,a\,f}\,\sqrt{d+x\,\left(e+f\,x\right)}\,\left(-B\,e\,\left(2\,a+b\,x\right)+A\,b\,\left(e+2\,f\,x\right)\right)-$$
$$\left(-b^{3/2}\,B\,e^{5/2}\,\sqrt{b\,e-4\,a\,f}+4\,a\,\sqrt{b}\,B\,e^{3/2}\,f\,\sqrt{b\,e-4\,a\,f}-8\,a\,b\,e\,f\,\left(B\,e-2\,A\,f\right)+\right.$$
$$\left.b^2\,\left(B\,e-2\,A\,f\right)\,\left(e^2+4\,d\,f\right)\right)\,\left(a\,e+b\,x\,\left(e+f\,x\right)\right)\,\text{Log}\left[-\sqrt{b}\,\sqrt{e}\,\sqrt{b\,e-4\,a\,f}+b\,\left(e+2\,f\,x\right)\right]+$$
$$\left(b^{3/2}\,B\,e^{5/2}\,\sqrt{b\,e-4\,a\,f}-4\,a\,\sqrt{b}\,B\,e^{3/2}\,f\,\sqrt{b\,e-4\,a\,f}-8\,a\,b\,e\,f\,\left(B\,e-2\,A\,f\right)+\right.$$
$$\left.b^2\,\left(B\,e-2\,A\,f\right)\,\left(e^2+4\,d\,f\right)\right)\,\left(a\,e+b\,x\,\left(e+f\,x\right)\right)\,\text{Log}\left[\sqrt{b}\,\sqrt{e}\,\sqrt{b\,e-4\,a\,f}+b\,\left(e+2\,f\,x\right)\right]-$$
$$\left(b^{3/2}\,B\,e^{5/2}\,\sqrt{b\,e-4\,a\,f}-4\,a\,\sqrt{b}\,B\,e^{3/2}\,f\,\sqrt{b\,e-4\,a\,f}-8\,a\,b\,e\,f\,\left(B\,e-2\,A\,f\right)+\right.$$
$$\left.b^2\,\left(B\,e-2\,A\,f\right)\,\left(e^2+4\,d\,f\right)\right)\,\left(a\,e+b\,x\,\left(e+f\,x\right)\right)\,\text{Log}\Big[\sqrt{b}\,\Big(e^{3/2}\,\sqrt{b\,e-4\,a\,f}+$$
$$\sqrt{b}\,\left(e^2-4\,d\,f\right)+2\,\sqrt{e}\,f\,\sqrt{b\,e-4\,a\,f}\,x-4\,\sqrt{b\,d-a\,e}\,f\,\sqrt{d+x\,\left(e+f\,x\right)}\Big)\Big]+$$
$$\left(-b^{3/2}\,B\,e^{5/2}\,\sqrt{b\,e-4\,a\,f}+4\,a\,\sqrt{b}\,B\,e^{3/2}\,f\,\sqrt{b\,e-4\,a\,f}-8\,a\,b\,e\,f\,\left(B\,e-2\,A\,f\right)+\right.$$
$$\left.b^2\,\left(B\,e-2\,A\,f\right)\,\left(e^2+4\,d\,f\right)\right)\,\left(a\,e+b\,x\,\left(e+f\,x\right)\right)\,\text{Log}\Big[\sqrt{b}\,\Big(e^{3/2}\,\sqrt{b\,e-4\,a\,f}-$$
$$\sqrt{b}\,\left(e^2-4\,d\,f\right)+2\,\sqrt{e}\,f\,\sqrt{b\,e-4\,a\,f}\,x+4\,\sqrt{b\,d-a\,e}\,f\,\sqrt{d+x\,\left(e+f\,x\right)}\Big)\Big]\Big)$$

Problem 37: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{3+2\,x}{\sqrt{-3-4\,x-x^2}\,\left(3+4\,x+2\,x^2\right)}\,dx$$

Optimal (type 3, 17 leaves, 2 steps):

$$\text{ArcTanh}\left[\frac{x}{\sqrt{-3-4\,x-x^2}}\right]$$

Result (type 3, 873 leaves):

$$\frac{1}{4}\left(-\frac{1}{\sqrt{1-2\,i\sqrt{2}}}2\,i\left(-i+\sqrt{2}\right)\text{ArcTan}\Big[\Big((2+x)\left(12-6\,i\sqrt{2}+\left(8+6\,i\sqrt{2}\right)x^3+9\,i\sqrt{1-2\,i\sqrt{2}}\sqrt{-3-4x-x^2}+x^2\left(36+8\,i\sqrt{2}+6\,i\sqrt{1-2\,i\sqrt{2}}\sqrt{-3-4x-x^2}\right)+x\left(40-5\,i\sqrt{2}+12\,i\sqrt{1-2\,i\sqrt{2}}\sqrt{-3-4x-x^2}\right)\right)\Big)\Big/\left(21\,i+6\sqrt{2}+4\left(7\,i+8\sqrt{2}\right)x+\left(19\,i+58\sqrt{2}\right)x^2+8\left(2\,i+5\sqrt{2}\right)x^3+\left(6\,i+8\sqrt{2}\right)x^4\right)\Big]+\frac{1}{\sqrt{1+2\,i\sqrt{2}}}2\left(i+\sqrt{2}\right)\text{ArcTanh}\Big[\Big((2+x)\left(12\,i-6\sqrt{2}+\left(8\,i+6\sqrt{2}\right)x^3+9\sqrt{1+2\,i\sqrt{2}}\sqrt{-3-4x-x^2}+x^2\left(36\,i+8\sqrt{2}+6\sqrt{1+2\,i\sqrt{2}}\sqrt{-3-4x-x^2}\right)+x\left(40\,i-5\sqrt{2}+12\sqrt{1+2\,i\sqrt{2}}\sqrt{-3-4x-x^2}\right)\right)\Big)\Big/\left(-21\,i+6\sqrt{2}+4\left(-7\,i+8\sqrt{2}\right)x+\left(-19\,i+58\sqrt{2}\right)x^2+8\left(-2\,i+5\sqrt{2}\right)x^3+\left(-6\,i+8\sqrt{2}\right)x^4\right)\Big]+\frac{\left(-i+\sqrt{2}\right)\text{Log}\left[4\left(3+4x+2x^2\right)^2\right]}{\sqrt{1-2\,i\sqrt{2}}}+\frac{\left(i+\sqrt{2}\right)\text{Log}\left[4\left(3+4x+2x^2\right)^2\right]}{\sqrt{1+2\,i\sqrt{2}}}-\frac{1}{\sqrt{1-2\,i\sqrt{2}}}\left(-i+\sqrt{2}\right)\text{Log}\Big[\left(3+4x+2x^2\right)\left(3+6\,i\sqrt{2}+\left(2+2\,i\sqrt{2}\right)x^2-2\sqrt{2-4\,i\sqrt{2}}\sqrt{-3-4x-x^2}+x\left(4+8\,i\sqrt{2}-2\sqrt{2-4\,i\sqrt{2}}\sqrt{-3-4x-x^2}\right)\right)\Big]-\frac{1}{\sqrt{1+2\,i\sqrt{2}}}\left(i+\sqrt{2}\right)\text{Log}\Big[\left(3+4x+2x^2\right)\left(3-6\,i\sqrt{2}+\left(2-2\,i\sqrt{2}\right)x^2-2\sqrt{2+4\,i\sqrt{2}}\sqrt{-3-4x-x^2}-2x\left(-2+4\,i\sqrt{2}+\sqrt{2+4\,i\sqrt{2}}\sqrt{-3-4x-x^2}\right)\right)\Big]\right)$$

Problem 38: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{3+4x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx$$

Optimal (type 3, 86 leaves, 13 steps):

$$\sqrt{2}\,\text{ArcTan}\Big[\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\Big]-\sqrt{2}\,\text{ArcTan}\Big[\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\Big]+\text{ArcTanh}\Big[\frac{x}{\sqrt{-3-4x-x^2}}\Big]$$

Result (type 3, 976 leaves):

$$\frac{1}{4}\Bigg(2\sqrt{1-2\,i\sqrt{2}}\,\text{ArcTan}\Big[\Big(60+51\,i\sqrt{2}+\big(-16+6\,i\sqrt{2}\big)x^4+54\,i\sqrt{1-2\,i\sqrt{2}}\sqrt{-3-4x-x^2}+x\Big(68+176\,i\sqrt{2}+99\,i\sqrt{1-2\,i\sqrt{2}}\sqrt{-3-4x-x^2}\Big)+2\,i\,x^3\Big(34\big(i+\sqrt{2}\big)+9\sqrt{1-2\,i\sqrt{2}}\sqrt{-3-4x-x^2}\Big)+i\,x^2\Big(44\,i+185\sqrt{2}+72\sqrt{1-2\,i\sqrt{2}}\sqrt{-3-4x-x^2}\Big)\Big)\Big/\Big(93\,i+150\sqrt{2}+20\big(17\,i+22\sqrt{2}\big)x+\big(493\,i+466\sqrt{2}\big)x^2+16\big(19\,i+13\sqrt{2}\big)x^3+\big(66\,i+32\sqrt{2}\big)x^4\Big)\Big]-\frac{1}{\sqrt{1+2\,i\sqrt{2}}}2\,i\big(-i+2\sqrt{2}\big)\text{ArcTan}\Big[\Big(-60+51\,i\sqrt{2}+2\big(8+3\,i\sqrt{2}\big)x^4+54\,i\sqrt{1+2\,i\sqrt{2}}\sqrt{-3-4x-x^2}+2x^3\Big(34+34\,i\sqrt{2}+9\,i\sqrt{1+2\,i\sqrt{2}}\sqrt{-3-4x-x^2}\Big)+x^2\Big(44+185\,i\sqrt{2}+72\,i\sqrt{1+2\,i\sqrt{2}}\sqrt{-3-4x-x^2}\Big)+i\,x\Big(68\,i+176\sqrt{2}+99\sqrt{1+2\,i\sqrt{2}}\sqrt{-3-4x-x^2}\Big)\Big)\Big/\Big(-93\,i+150\sqrt{2}+20\big(-17\,i+22\sqrt{2}\big)x+\big(-493\,i+466\sqrt{2}\big)x^2+16\big(-19\,i+13\sqrt{2}\big)x^3+\big(-66\,i+32\sqrt{2}\big)x^4\Big)\Big]+\frac{\big(-i+2\sqrt{2}\big)\text{Log}\big[4\big(3+4x+2x^2\big)^2\big]}{\sqrt{1+2\,i\sqrt{2}}}+\frac{\big(i+2\sqrt{2}\big)\text{Log}\big[4\big(3+4x+2x^2\big)^2\big]}{\sqrt{1-2\,i\sqrt{2}}}-\frac{1}{\sqrt{1-2\,i\sqrt{2}}}\big(i+2\sqrt{2}\big)\text{Log}\Big[\big(3+4x+2x^2\big)\Big(3+6\,i\sqrt{2}+\big(2+2\,i\sqrt{2}\big)x^2-2\sqrt{2-4\,i\sqrt{2}}\sqrt{-3-4x-x^2}+x\Big(4+8\,i\sqrt{2}-2\sqrt{2-4\,i\sqrt{2}}\sqrt{-3-4x-x^2}\Big)\Big)\Big]-\frac{1}{\sqrt{1+2\,i\sqrt{2}}}\big(-i+2\sqrt{2}\big)\text{Log}\Big[\big(3+4x+2x^2\big)\Big(3-6\,i\sqrt{2}+\big(2-2\,i\sqrt{2}\big)x^2-2\sqrt{2+4\,i\sqrt{2}}\sqrt{-3-4x-x^2}-2x\Big(-2+4\,i\sqrt{2}+\sqrt{2+4\,i\sqrt{2}}\sqrt{-3-4x-x^2}\Big)\Big)\Big]\Bigg)$$

Problem 59: Result more than twice size of optimal antiderivative.

$$\int \frac{x \left(a + c x^2\right)^{3/2}}{d + e x + f x^2} \, dx$$

Optimal (type 3, 553 leaves, 10 steps):

$$\frac{\left(2 \left(a f^2 + c \left(e^2 - d f\right)\right) - c e f x\right) \sqrt{a + c x^2}}{2 f^3} + \frac{\left(a + c x^2\right)^{3/2}}{3 f} - \frac{\sqrt{c}\, e \left(3 a f^2 + 2 c \left(e^2 - 2 d f\right)\right) \text{ArcTanh}\left[\frac{\sqrt{c}\, x}{\sqrt{a + c x^2}}\right]}{2 f^4} - \left(\left(2 c d e f \left(2 a f^2 + c \left(e^2 - 2 d f\right)\right) - \left(e - \sqrt{e^2 - 4 d f}\right) \left(a^2 f^4 + 2 a c f^2 \left(e^2 - d f\right) + c^2 \left(e^4 - 3 d e^2 f + d^2 f^2\right)\right)\right) \text{ArcTanh}\left[\frac{2 a f - c \left(e - \sqrt{e^2 - 4 d f}\right) x}{\sqrt{2} \sqrt{2 a f^2 + c \left(e^2 - 2 d f - e \sqrt{e^2 - 4 d f}\right)} \sqrt{a + c x^2}}\right]\right) \Big/ \left(\sqrt{2}\, f^4 \sqrt{e^2 - 4 d f} \sqrt{2 a f^2 + c \left(e^2 - 2 d f - e \sqrt{e^2 - 4 d f}\right)}\right) + \left(\left(2 c d e f \left(2 a f^2 + c \left(e^2 - 2 d f\right)\right) - \left(e + \sqrt{e^2 - 4 d f}\right) \left(a^2 f^4 + 2 a c f^2 \left(e^2 - d f\right) + c^2 \left(e^4 - 3 d e^2 f + d^2 f^2\right)\right)\right) \text{ArcTanh}\left[\frac{2 a f - c \left(e + \sqrt{e^2 - 4 d f}\right) x}{\sqrt{2} \sqrt{2 a f^2 + c \left(e^2 - 2 d f + e \sqrt{e^2 - 4 d f}\right)} \sqrt{a + c x^2}}\right]\right) \Big/ \left(\sqrt{2}\, f^4 \sqrt{e^2 - 4 d f} \sqrt{2 a f^2 + c \left(e^2 - 2 d f + e \sqrt{e^2 - 4 d f}\right)}\right)$$

Result (type 3, 1176 leaves):

$$\frac{1}{6 f^4}\left(f \sqrt{a+c x^2} \left(8 a f^2+c \left(6 e^2-3 e f x+2 f \left(-3 d+f x^2\right)\right)\right)+\right.$$

$$\left(3 \sqrt{2} \left(a^2 f^4 \left(-e+\sqrt{e^2-4 d f}\right)-2 a c f^2 \left(e^3-3 d e f-e^2 \sqrt{e^2-4 d f}+d f \sqrt{e^2-4 d f}\right)+\right.\right.$$
$$\left.\left.c^2 \left(-e^5+5 d e^3 f-5 d^2 e f^2+e^4 \sqrt{e^2-4 d f}-3 d e^2 f \sqrt{e^2-4 d f}+d^2 f^2 \sqrt{e^2-4 d f}\right)\right)\right.$$
$$\left.\text{Log}\left[-e+\sqrt{e^2-4 d f}-2 f x\right]\right)\Big/\left(\sqrt{e^2-4 d f} \sqrt{2 a f^2+c \left(e^2-2 d f-e \sqrt{e^2-4 d f}\right)}\right)+$$

$$\left(3 \sqrt{2} \left(a^2 f^4 \left(e+\sqrt{e^2-4 d f}\right)+2 a c f^2 \left(e^3-3 d e f+e^2 \sqrt{e^2-4 d f}-d f \sqrt{e^2-4 d f}\right)+\right.\right.$$
$$\left.\left.c^2 \left(e^5-5 d e^3 f+5 d^2 e f^2+e^4 \sqrt{e^2-4 d f}-3 d e^2 f \sqrt{e^2-4 d f}+d^2 f^2 \sqrt{e^2-4 d f}\right)\right)\right.$$
$$\left.\text{Log}\left[e+\sqrt{e^2-4 d f}+2 f x\right]\right)\Big/\left(\sqrt{e^2-4 d f} \sqrt{2 a f^2+c \left(e^2-2 d f+e \sqrt{e^2-4 d f}\right)}\right)-$$

$$3 \sqrt{c}\, e \left(3 a f^2+2 c \left(e^2-2 d f\right)\right) \text{Log}\left[c x+\sqrt{c} \sqrt{a+c x^2}\right]-$$

$$\frac{1}{\sqrt{e^2-4 d f} \sqrt{2 a f^2+c \left(e^2-2 d f-e \sqrt{e^2-4 d f}\right)}}$$
$$3 \sqrt{2} \left(a^2 f^4 \left(-e+\sqrt{e^2-4 d f}\right)-2 a c f^2 \left(e^3-3 d e f-e^2 \sqrt{e^2-4 d f}+d f \sqrt{e^2-4 d f}\right)+\right.$$
$$\left.c^2 \left(-e^5+5 d e^3 f-5 d^2 e f^2+e^4 \sqrt{e^2-4 d f}-3 d e^2 f \sqrt{e^2-4 d f}+d^2 f^2 \sqrt{e^2-4 d f}\right)\right)$$
$$\text{Log}\left[2 a f \sqrt{e^2-4 d f}+c \left(e^2-4 d f-e \sqrt{e^2-4 d f}\right) x+\right.$$
$$\left.\sqrt{2} \sqrt{e^2-4 d f} \sqrt{2 a f^2+c \left(e^2-2 d f-e \sqrt{e^2-4 d f}\right)} \sqrt{a+c x^2}\right]-$$

$$\frac{1}{\sqrt{e^2-4 d f} \sqrt{2 a f^2+c \left(e^2-2 d f+e \sqrt{e^2-4 d f}\right)}}$$
$$3 \sqrt{2} \left(a^2 f^4 \left(e+\sqrt{e^2-4 d f}\right)+2 a c f^2 \left(e^3-3 d e f+e^2 \sqrt{e^2-4 d f}-d f \sqrt{e^2-4 d f}\right)+\right.$$
$$\left.c^2 \left(e^5-5 d e^3 f+5 d^2 e f^2+e^4 \sqrt{e^2-4 d f}-3 d e^2 f \sqrt{e^2-4 d f}+d^2 f^2 \sqrt{e^2-4 d f}\right)\right)$$
$$\text{Log}\left[2 a f \sqrt{e^2-4 d f}-c \left(e^2-4 d f+e \sqrt{e^2-4 d f}\right) x+\right.$$
$$\left.\left.\sqrt{2} \sqrt{e^2-4 d f} \sqrt{2 a f^2+c \left(e^2-2 d f+e \sqrt{e^2-4 d f}\right)} \sqrt{a+c x^2}\right]\right)$$

Problem 93: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(x+x^2\right)^{3/2}}{1+x^2}\,dx$$

Optimal (type 3, 130 leaves, 10 steps):

$$\frac{1}{4}(5+2x)\sqrt{x+x^2}+\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left[\frac{1+\sqrt{2}-x}{\sqrt{2\left(1+\sqrt{2}\right)}\,\sqrt{x+x^2}}\right]-\sqrt{-1+\sqrt{2}}\,\text{ArcTanh}\left[\frac{1-\sqrt{2}-x}{\sqrt{2\left(-1+\sqrt{2}\right)}\,\sqrt{x+x^2}}\right]-\frac{5}{4}\text{ArcTanh}\left[\frac{x}{\sqrt{x+x^2}}\right]$$

Result (type 3, 124 leaves):

$$\frac{1}{4\sqrt{x(1+x)}}\sqrt{x}\,\sqrt{1+x}\left(5\sqrt{x}\,\sqrt{1+x}+2x^{3/2}\sqrt{1+x}-5\,\text{ArcSinh}\left[\sqrt{x}\right]-4\sqrt{2-2\,i}\,\text{ArcTan}\left[(1-i)^{3/2}\sqrt{\frac{x}{2+2x}}\right]-4\sqrt{2+2\,i}\,\text{ArcTan}\left[(1+i)^{3/2}\sqrt{\frac{x}{2+2x}}\right]\right)$$

Problem 110: Result more than twice size of optimal antiderivative.

$$\int \frac{x\sqrt{a+b\,x+c\,x^2}}{d+e\,x+f\,x^2}\,dx$$

Optimal (type 3, 549 leaves, 9 steps):

$$\frac{\sqrt{a+b x+c x^2}}{f}-\frac{(2 c e-b f) \operatorname{ArcTanh}\left[\frac{b+2 c x}{2 \sqrt{c} \sqrt{a+b x+c x^2}}\right]}{2 \sqrt{c} f^2}-$$

$$\left(\left(2 d f (c e-b f)+\left(e-\sqrt{e^2-4 d f}\right)\left(f (b e-a f)-c\left(e^2-d f\right)\right)\right)\right.$$

$$\operatorname{ArcTanh}\left[\left(4 a f-b\left(e-\sqrt{e^2-4 d f}\right)+2\left(b f-c\left(e-\sqrt{e^2-4 d f}\right)\right) x\right) \Big/\right.$$

$$\left.\left.\left(2 \sqrt{2} \sqrt{c e^2-2 c d f-b e f+2 a f^2-(c e-b f) \sqrt{e^2-4 d f}} \sqrt{a+b x+c x^2}\right)\right]\right) \Big/$$

$$\left(\sqrt{2} f^2 \sqrt{e^2-4 d f} \sqrt{c e^2-2 c d f-b e f+2 a f^2-(c e-b f) \sqrt{e^2-4 d f}}\right)+$$

$$\left(\left(2 d f (c e-b f)+\left(e+\sqrt{e^2-4 d f}\right)\left(f (b e-a f)-c\left(e^2-d f\right)\right)\right)\right.$$

$$\operatorname{ArcTanh}\left[\left(4 a f-b\left(e+\sqrt{e^2-4 d f}\right)+2\left(b f-c\left(e+\sqrt{e^2-4 d f}\right)\right) x\right) \Big/\right.$$

$$\left.\left.\left(2 \sqrt{2} \sqrt{c e^2-2 c d f-b e f+2 a f^2+(c e-b f) \sqrt{e^2-4 d f}} \sqrt{a+b x+c x^2}\right)\right]\right) \Big/$$

$$\left(\sqrt{2} f^2 \sqrt{e^2-4 d f} \sqrt{c e^2-2 c d f-b e f+2 a f^2+(c e-b f) \sqrt{e^2-4 d f}}\right)$$

Result (type 3, 1112 leaves) :

$$
\begin{aligned}
&\frac{1}{2 f^2}\Bigg(2 f \sqrt{a+x (b+c x)}+\Bigg(\sqrt{2} \left(c \left(-e^3+3 d e f+e^2 \sqrt{e^2-4 d f}-d f \sqrt{e^2-4 d f}\right)+\right.\\
&\qquad \left.f \left(a f \left(-e+\sqrt{e^2-4 d f}\right)+b \left(e^2-2 d f-e \sqrt{e^2-4 d f}\right)\right)\right) \operatorname{Log}\left[-e+\sqrt{e^2-4 d f}-2 f x\right]\Bigg)\Bigg/\\
&\quad \left(\sqrt{e^2-4 d f} \sqrt{c \left(e^2-2 d f-e \sqrt{e^2-4 d f}\right)+f \left(2 a f+b \left(-e+\sqrt{e^2-4 d f}\right)\right)}\right)+\\
&\quad \Bigg(\sqrt{2} \left(c \left(e^3-3 d e f+e^2 \sqrt{e^2-4 d f}-d f \sqrt{e^2-4 d f}\right)+\right.\\
&\qquad \left.f \left(a f \left(e+\sqrt{e^2-4 d f}\right)-b \left(e^2-2 d f+e \sqrt{e^2-4 d f}\right)\right)\right) \operatorname{Log}\left[e+\sqrt{e^2-4 d f}+2 f x\right]\Bigg)\Bigg/\\
&\quad \left(\sqrt{e^2-4 d f} \sqrt{c \left(e^2-2 d f+e \sqrt{e^2-4 d f}\right)+f \left(2 a f-b \left(e+\sqrt{e^2-4 d f}\right)\right)}\right)-\\
&\quad \frac{(2 c e-b f) \operatorname{Log}\left[b+2 c x+2 \sqrt{c} \sqrt{a+x (b+c x)}\right]}{\sqrt{c}}-\\
&\quad \Bigg(\sqrt{2} \left(c \left(-e^3+3 d e f+e^2 \sqrt{e^2-4 d f}-d f \sqrt{e^2-4 d f}\right)+\right.\\
&\qquad \left.f \left(a f \left(-e+\sqrt{e^2-4 d f}\right)+b \left(e^2-2 d f-e \sqrt{e^2-4 d f}\right)\right)\right)\\
&\quad \operatorname{Log}\Big[4 a f \sqrt{e^2-4 d f}+2 c e^2 x-8 c d f x-2 c e \sqrt{e^2-4 d f}\, x+\\
&\qquad b \left(e^2-4 d f-e \sqrt{e^2-4 d f}+2 f \sqrt{e^2-4 d f}\, x\right)+2 \sqrt{2} \sqrt{e^2-4 d f}\\
&\qquad \sqrt{\left(f \left(-b e+2 a f+b \sqrt{e^2-4 d f}\right)+c \left(e^2-2 d f-e \sqrt{e^2-4 d f}\right)\right)} \sqrt{a+x (b+c x)}\Big]\Bigg)\Bigg/\\
&\quad \left(\sqrt{e^2-4 d f} \sqrt{c \left(e^2-2 d f-e \sqrt{e^2-4 d f}\right)+f \left(2 a f+b \left(-e+\sqrt{e^2-4 d f}\right)\right)}\right)-\\
&\quad \Bigg(\sqrt{2} \left(c \left(e^3-3 d e f+e^2 \sqrt{e^2-4 d f}-d f \sqrt{e^2-4 d f}\right)+\right.\\
&\qquad \left.f \left(a f \left(e+\sqrt{e^2-4 d f}\right)-b \left(e^2-2 d f+e \sqrt{e^2-4 d f}\right)\right)\right)\\
&\quad \operatorname{Log}\Big[4 a f \sqrt{e^2-4 d f}-2 c e^2 x+8 c d f x-2 c e \sqrt{e^2-4 d f}\, x+\\
&\qquad 2 \sqrt{2} \sqrt{e^2-4 d f} \sqrt{\left(c \left(e^2-2 d f+e \sqrt{e^2-4 d f}\right)+f \left(2 a f-b \left(e+\sqrt{e^2-4 d f}\right)\right)\right)}\\
&\qquad \sqrt{a+x (b+c x)}-b \left(e^2+e \sqrt{e^2-4 d f}-2 f \left(2 d+\sqrt{e^2-4 d f}\, x\right)\right)\Big]\Bigg)\Bigg/\\
&\quad \left(\sqrt{e^2-4 d f} \sqrt{c \left(e^2-2 d f+e \sqrt{e^2-4 d f}\right)+f \left(2 a f-b \left(e+\sqrt{e^2-4 d f}\right)\right)}\right)\Bigg)
\end{aligned}
$$

Problem 118: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x \sqrt{a + b x + c x^2} \left(d + e x + f x^2\right)} \, dx$$

Optimal (type 3, 451 leaves, 9 steps):

$$-\frac{\text{ArcTanh}\left[\frac{2 a+b x}{2 \sqrt{a} \sqrt{a+b x+c x^2}}\right]}{\sqrt{a} d} + \left(f \left(e+\sqrt{e^2-4 d f}\right) \text{ArcTanh}\left[\left(4 a f-b \left(e-\sqrt{e^2-4 d f}\right)+2 \left(b f-c \left(e-\sqrt{e^2-4 d f}\right)\right) x\right) \Big/ \left(2 \sqrt{2} \sqrt{c e^2-2 c d f-b e f+2 a f^2-(c e-b f) \sqrt{e^2-4 d f}} \sqrt{a+b x+c x^2}\right)\right]\right) \Big/ \left(\sqrt{2} d \sqrt{e^2-4 d f} \sqrt{c e^2-2 c d f-b e f+2 a f^2-(c e-b f) \sqrt{e^2-4 d f}}\right) - \left(f \left(e-\sqrt{e^2-4 d f}\right) \text{ArcTanh}\left[\left(4 a f-b \left(e+\sqrt{e^2-4 d f}\right)+2 \left(b f-c \left(e+\sqrt{e^2-4 d f}\right)\right) x\right) \Big/ \left(2 \sqrt{2} \sqrt{c e^2-2 c d f-b e f+2 a f^2+(c e-b f) \sqrt{e^2-4 d f}} \sqrt{a+b x+c x^2}\right)\right]\right) \Big/ \left(\sqrt{2} d \sqrt{e^2-4 d f} \sqrt{c e^2-2 c d f-b e f+2 a f^2+(c e-b f) \sqrt{e^2-4 d f}}\right)$$

Result (type 3, 994 leaves):

$$\frac{\sqrt{a+b x+c x^2}\ \mathrm{Log}[x]}{\sqrt{a}\ d\sqrt{a+x\ (b+c x)}}-$$

$$\left(f\left(e+\sqrt{e^2-4 d f}\right)\sqrt{a+b x+c x^2}\ \mathrm{Log}\left[-e+\sqrt{e^2-4 d f}-2 f x\right]\right)\Big/\left(\sqrt{2}\ d\sqrt{e^2-4 d f}\sqrt{c e^2-2 c d f-b e f+2 a f^2-c e\sqrt{e^2-4 d f}+b f\sqrt{e^2-4 d f}}\ \sqrt{a+x\ (b+c x)}\right)-$$

$$\left(f\left(-e+\sqrt{e^2-4 d f}\right)\sqrt{a+b x+c x^2}\ \mathrm{Log}\left[e+\sqrt{e^2-4 d f}+2 f x\right]\right)\Big/\left(\sqrt{2}\ d\sqrt{e^2-4 d f}\sqrt{c e^2-2 c d f-b e f+2 a f^2+c e\sqrt{e^2-4 d f}-b f\sqrt{e^2-4 d f}}\ \sqrt{a+x\ (b+c x)}\right)-$$

$$\frac{\sqrt{a+b x+c x^2}\ \mathrm{Log}\left[2 a+b x+2\sqrt{a}\ \sqrt{a+b x+c x^2}\right]}{\sqrt{a}\ d\sqrt{a+x\ (b+c x)}}+$$

$$\left(f\left(-e+\sqrt{e^2-4 d f}\right)\sqrt{a+b x+c x^2}\ \mathrm{Log}\left[-b e^2+4 b d f-b e\sqrt{e^2-4 d f}+4 a f\sqrt{e^2-4 d f}-2 c e^2 x+8 c d f x-2 c e\sqrt{e^2-4 d f}\ x+2 b f\sqrt{e^2-4 d f}\ x+2\sqrt{2}\ \sqrt{e^2-4 d f}\sqrt{c e^2-2 c d f-b e f+2 a f^2+c e\sqrt{e^2-4 d f}-b f\sqrt{e^2-4 d f}}\ \sqrt{a+b x+c x^2}\right]\right)\Big/\left(\sqrt{2}\ d\sqrt{e^2-4 d f}\sqrt{c e^2-2 c d f-b e f+2 a f^2+c e\sqrt{e^2-4 d f}-b f\sqrt{e^2-4 d f}}\ \sqrt{a+x\ (b+c x)}\right)+$$

$$\left(f\left(e+\sqrt{e^2-4 d f}\right)\sqrt{a+b x+c x^2}\ \mathrm{Log}\left[b e^2-4 b d f-b e\sqrt{e^2-4 d f}+4 a f\sqrt{e^2-4 d f}+2 c e^2 x-8 c d f x-2 c e\sqrt{e^2-4 d f}\ x+2 b f\sqrt{e^2-4 d f}\ x+2\sqrt{2}\ \sqrt{e^2-4 d f}\sqrt{c e^2-2 c d f-b e f+2 a f^2-c e\sqrt{e^2-4 d f}+b f\sqrt{e^2-4 d f}}\ \sqrt{a+b x+c x^2}\right]\right)\Big/\left(\sqrt{2}\ d\sqrt{e^2-4 d f}\sqrt{c e^2-2 c d f-b e f+2 a f^2-c e\sqrt{e^2-4 d f}+b f\sqrt{e^2-4 d f}}\ \sqrt{a+x\ (b+c x)}\right)$$

Problem 126: Result unnecessarily involves complex numbers and more than

twice size of optimal antiderivative.

$$\int \frac{x^4}{\sqrt{-3-4x-x^2}\,\left(3+4x+2x^2\right)}\,dx$$

Optimal (type 3, 140 leaves, 24 steps):

$$\frac{5}{2}\sqrt{-3-4x-x^2}-\frac{1}{4}x\sqrt{-3-4x-x^2}+\frac{11}{2}\text{ArcSin}[2+x]+\frac{\text{ArcTan}\left[\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right]}{2\sqrt{2}}-\frac{\text{ArcTan}\left[\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right]}{2\sqrt{2}}-\frac{5}{4}\text{ArcTanh}\left[\frac{x}{\sqrt{-3-4x-x^2}}\right]$$

Result (type 3, 1000 leaves):

$$\frac{1}{16}\left(-4\,(-10+x)\,\sqrt{-3-4x-x^2}+88\,\text{ArcSin}[2+x]+\frac{1}{\sqrt{1-2\,i\,\sqrt{2}}}\right.$$

$$2\left(7+4\,i\,\sqrt{2}\right)\text{ArcTan}\Big[\Big(132-471\,i\,\sqrt{2}+\left(224+78\,i\,\sqrt{2}\right)x^4+486\,i\,\sqrt{1-2\,i\,\sqrt{2}}\,\sqrt{-3-4x-x^2}+$$

$$2x^3\left(638+10\,i\,\sqrt{2}+81\,i\,\sqrt{1-2\,i\,\sqrt{2}}\,\sqrt{-3-4x-x^2}\right)+$$

$$x^2\left(2236-727\,i\,\sqrt{2}+648\,i\,\sqrt{1-2\,i\,\sqrt{2}}\,\sqrt{-3-4x-x^2}\right)+$$

$$x\left(1316-1168\,i\,\sqrt{2}+891\,i\,\sqrt{1-2\,i\,\sqrt{2}}\,\sqrt{-3-4x-x^2}\right)\Big)\Big/\Big(885\,i+6\sqrt{2}+$$

$$4\left(349\,i+26\sqrt{2}\right)x+\left(685\,i+514\sqrt{2}\right)x^2+16\left(13\,i+34\sqrt{2}\right)x^3+2\left(33\,i+64\sqrt{2}\right)x^4\Big)\Big]-$$

$$\frac{1}{\sqrt{1+2\,i\,\sqrt{2}}}2\left(7\,i+4\sqrt{2}\right)\text{ArcTanh}\Big[\Big(132\,i-471\sqrt{2}+\left(224\,i+78\sqrt{2}\right)x^4+$$

$$486\sqrt{1+2\,i\,\sqrt{2}}\,\sqrt{-3-4x-x^2}+2x^3\left(638\,i+10\sqrt{2}+81\sqrt{1+2\,i\,\sqrt{2}}\,\sqrt{-3-4x-x^2}\right)+$$

$$x^2\left(2236\,i-727\sqrt{2}+648\sqrt{1+2\,i\,\sqrt{2}}\,\sqrt{-3-4x-x^2}\right)+$$

$$x\left(1316\,i-1168\sqrt{2}+891\sqrt{1+2\,i\,\sqrt{2}}\,\sqrt{-3-4x-x^2}\right)\Big)\Big/$$

$$\Big(-885\,i+6\sqrt{2}+4\left(-349\,i+26\sqrt{2}\right)x+\left(-685\,i+514\sqrt{2}\right)x^2+$$

$$16\left(-13\,i+34\sqrt{2}\right)x^3+2\left(-33\,i+64\sqrt{2}\right)x^4\Big)\Big]-$$

$$\frac{\left(-7\,i+4\sqrt{2}\right)\text{Log}\left[4\left(3+4x+2x^2\right)^2\right]}{\sqrt{1-2\,i\,\sqrt{2}}}-\frac{\left(7\,i+4\sqrt{2}\right)\text{Log}\left[4\left(3+4x+2x^2\right)^2\right]}{\sqrt{1+2\,i\,\sqrt{2}}}+$$

$$\frac{1}{\sqrt{1-2\,i\,\sqrt{2}}}$$

$$\left(-7\,i+4\sqrt{2}\right)$$

$$\text{Log}\Big[\left(3+4x+2x^2\right)\Big(3+6\,i\,\sqrt{2}+\left(2+2\,i\,\sqrt{2}\right)x^2-2\sqrt{2-4\,i\,\sqrt{2}}\,\sqrt{-3-4x-x^2}+$$

$$x\left(4+8\,i\,\sqrt{2}-2\sqrt{2-4\,i\,\sqrt{2}}\,\sqrt{-3-4x-x^2}\right)\Big)\Big]+$$

$$\frac{1}{\sqrt{1+2\,i\,\sqrt{2}}}\left(7\,i+4\sqrt{2}\right)\text{Log}\Big[\left(3+4x+2x^2\right)\Big(3-6\,i\,\sqrt{2}+\left(2-2\,i\,\sqrt{2}\right)x^2-$$

$$\left.2\sqrt{2+4\,i\,\sqrt{2}}\,\sqrt{-3-4x-x^2}-2x\left(-2+4\,i\,\sqrt{2}+\sqrt{2+4\,i\,\sqrt{2}}\,\sqrt{-3-4x-x^2}\right)\Big)\Big]\right)$$

Problem 127: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{x^3}{\sqrt{-3-4x-x^2}\,\left(3+4x+2x^2\right)}\,dx$$

Optimal (type 3, 115 leaves, 20 steps):

$$-\frac{1}{2}\sqrt{-3-4x-x^2}-2\,\text{ArcSin}[2+x]+\frac{\text{ArcTan}\left[\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right]}{2\sqrt{2}}-\frac{\text{ArcTan}\left[\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right]}{2\sqrt{2}}+\text{ArcTanh}\left[\frac{x}{\sqrt{-3-4x-x^2}}\right]$$

Result (type 3, 1001 leaves):

$$\frac{1}{16}\left(-8\sqrt{-3-4x-x^2}-32\,\text{ArcSin}[2+x]-\frac{1}{\sqrt{1-2\,\text{i}\sqrt{2}}}\right.$$

$$2\,\text{i}\left(-2\,\text{i}+5\sqrt{2}\right)\text{ArcTan}\Big[\Big(\left(40+66\,\text{i}\sqrt{2}\right)x^4+6\left(56-\text{i}\sqrt{2}+27\,\text{i}\sqrt{1-2\,\text{i}\sqrt{2}}\sqrt{-3-4x-x^2}\right)+$$

$$x^3\left(332+316\,\text{i}\sqrt{2}+54\,\text{i}\sqrt{1-2\,\text{i}\sqrt{2}}\sqrt{-3-4x-x^2}\right)+$$

$$x^2\left(920+469\,\text{i}\sqrt{2}+216\,\text{i}\sqrt{1-2\,\text{i}\sqrt{2}}\sqrt{-3-4x-x^2}\right)+$$

$$x\left(964+208\,\text{i}\sqrt{2}+297\,\text{i}\sqrt{1-2\,\text{i}\sqrt{2}}\sqrt{-3-4x-x^2}\right)\Big)\Big/$$

$$\Big(132\,\text{i}+192\sqrt{2}+4\left(71\,\text{i}+184\sqrt{2}\right)x+\left(455\,\text{i}+1004\sqrt{2}\right)x^2+$$

$$56\left(7\,\text{i}+10\sqrt{2}\right)x^3+2\left(57\,\text{i}+50\sqrt{2}\right)x^4\Big)\Big]+\frac{1}{\sqrt{1+2\,\text{i}\sqrt{2}}}$$

$$2\left(2\,\text{i}+5\sqrt{2}\right)\text{ArcTanh}\Big[\Big(\left(40\,\text{i}+66\sqrt{2}\right)x^4-6\left(-56\,\text{i}+\sqrt{2}-27\sqrt{1+2\,\text{i}\sqrt{2}}\sqrt{-3-4x-x^2}\right)+$$

$$x^3\left(332\,\text{i}+316\sqrt{2}+54\sqrt{1+2\,\text{i}\sqrt{2}}\sqrt{-3-4x-x^2}\right)+$$

$$x^2\left(920\,\text{i}+469\sqrt{2}+216\sqrt{1+2\,\text{i}\sqrt{2}}\sqrt{-3-4x-x^2}\right)+$$

$$x\left(964\,\text{i}+208\sqrt{2}+297\sqrt{1+2\,\text{i}\sqrt{2}}\sqrt{-3-4x-x^2}\right)\Big)\Big/$$

$$\Big(-132\,\text{i}+192\sqrt{2}+4\left(-71\,\text{i}+184\sqrt{2}\right)x+\left(-455\,\text{i}+1004\sqrt{2}\right)x^2+$$

$$56\left(-7\,\text{i}+10\sqrt{2}\right)x^3+2\left(-57\,\text{i}+50\sqrt{2}\right)x^4\Big)\Big]+$$

$$\frac{\left(-2\,\text{i}+5\sqrt{2}\right)\text{Log}\left[4\left(3+4x+2x^2\right)^2\right]}{\sqrt{1-2\,\text{i}\sqrt{2}}}+\frac{\left(2\,\text{i}+5\sqrt{2}\right)\text{Log}\left[4\left(3+4x+2x^2\right)^2\right]}{\sqrt{1+2\,\text{i}\sqrt{2}}}-$$

$$\frac{1}{\sqrt{1-2\,\text{i}\sqrt{2}}}$$

$$\left(-2\,\text{i}+5\sqrt{2}\right)$$

$$\text{Log}\Big[\left(3+4x+2x^2\right)\left(3+6\,\text{i}\sqrt{2}+\left(2+2\,\text{i}\sqrt{2}\right)x^2-2\sqrt{2-4\,\text{i}\sqrt{2}}\sqrt{-3-4x-x^2}+\right.$$

$$\left.x\left(4+8\,\text{i}\sqrt{2}-2\sqrt{2-4\,\text{i}\sqrt{2}}\sqrt{-3-4x-x^2}\right)\right)\Big]-$$

$$\frac{1}{\sqrt{1+2\,\text{i}\sqrt{2}}}\left(2\,\text{i}+5\sqrt{2}\right)\text{Log}\Big[\left(3+4x+2x^2\right)\left(3-6\,\text{i}\sqrt{2}+\left(2-2\,\text{i}\sqrt{2}\right)x^2-\right.$$

$$\left.\left.2\sqrt{2+4\,\text{i}\sqrt{2}}\sqrt{-3-4x-x^2}-2x\left(-2+4\,\text{i}\sqrt{2}+\sqrt{2+4\,\text{i}\sqrt{2}}\sqrt{-3-4x-x^2}\right)\right)\Big]\right)$$

Problem 128: Result unnecessarily involves complex numbers and more than

twice size of optimal antiderivative.

$$\int \frac{x^2}{\sqrt{-3-4x-x^2}\,\left(3+4x+2x^2\right)}\,dx$$

Optimal (type 3, 98 leaves, 16 steps):

$$\frac{1}{2}\,\text{ArcSin}[2+x]-\frac{\text{ArcTan}\left[\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right]}{\sqrt{2}}+\frac{\text{ArcTan}\left[\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right]}{\sqrt{2}}-\frac{1}{2}\,\text{ArcTanh}\left[\frac{x}{\sqrt{-3-4x-x^2}}\right]$$

Result (type 3, 982 leaves):

$$\frac{1}{8}\left(4\,\text{ArcSin}[2+x]+\frac{1}{\sqrt{1-2\,i\,\sqrt{2}}}\,2\,i\left(i+2\sqrt{2}\right)\text{ArcTan}\Big[\Big(60+51\,i\,\sqrt{2}+\left(-16+6\,i\,\sqrt{2}\right)x^4+54\,i\,\sqrt{1-2\,i\,\sqrt{2}}\,\sqrt{-3-4x-x^2}+x\left(68+176\,i\,\sqrt{2}+99\,i\,\sqrt{1-2\,i\,\sqrt{2}}\,\sqrt{-3-4x-x^2}\right)+2\,i\,x^3\left(34\left(i+\sqrt{2}\right)+9\sqrt{1-2\,i\,\sqrt{2}}\,\sqrt{-3-4x-x^2}\right)+i\,x^2\left(44\,i+185\sqrt{2}+72\sqrt{1-2\,i\,\sqrt{2}}\,\sqrt{-3-4x-x^2}\right)\Big)\Big/\Big(93\,i+150\sqrt{2}+20\left(17\,i+22\sqrt{2}\right)x+\left(493\,i+466\sqrt{2}\right)x^2+16\left(19\,i+13\sqrt{2}\right)x^3+\left(66\,i+32\sqrt{2}\right)x^4\Big)\Big]+2\sqrt{1+2\,i\,\sqrt{2}}\,\text{ArcTan}\Big[\Big(-60+51\,i\,\sqrt{2}+2\left(8+3\,i\,\sqrt{2}\right)x^4+54\,i\,\sqrt{1+2\,i\,\sqrt{2}}\,\sqrt{-3-4x-x^2}+2x^3\left(34+34\,i\,\sqrt{2}+9\,i\,\sqrt{1+2\,i\,\sqrt{2}}\,\sqrt{-3-4x-x^2}\right)+x^2\left(44+185\,i\,\sqrt{2}+72\,i\,\sqrt{1+2\,i\,\sqrt{2}}\,\sqrt{-3-4x-x^2}\right)+i\,x\left(68\,i+176\sqrt{2}+99\sqrt{1+2\,i\,\sqrt{2}}\,\sqrt{-3-4x-x^2}\right)\Big)\Big/\Big(-93\,i+150\sqrt{2}+20\left(-17\,i+22\sqrt{2}\right)x+\left(-493\,i+466\sqrt{2}\right)x^2+16\left(-19\,i+13\sqrt{2}\right)x^3+\left(-66\,i+32\sqrt{2}\right)x^4\Big)\Big]-\frac{\left(-i+2\sqrt{2}\right)\text{Log}\left[4\left(3+4x+2x^2\right)^2\right]}{\sqrt{1+2\,i\,\sqrt{2}}}-\frac{\left(i+2\sqrt{2}\right)\text{Log}\left[4\left(3+4x+2x^2\right)^2\right]}{\sqrt{1-2\,i\,\sqrt{2}}}+\frac{1}{\sqrt{1-2\,i\,\sqrt{2}}}\left(i+2\sqrt{2}\right)\text{Log}\Big[\left(3+4x+2x^2\right)\Big(3+6\,i\,\sqrt{2}+\left(2+2\,i\,\sqrt{2}\right)x^2-2\sqrt{2-4\,i\,\sqrt{2}}\,\sqrt{-3-4x-x^2}+x\left(4+8\,i\,\sqrt{2}-2\sqrt{2-4\,i\,\sqrt{2}}\,\sqrt{-3-4x-x^2}\right)\Big)\Big]+\frac{1}{\sqrt{1+2\,i\,\sqrt{2}}}\left(-i+2\sqrt{2}\right)\text{Log}\Big[\left(3+4x+2x^2\right)\Big(3-6\,i\,\sqrt{2}+\left(2-2\,i\,\sqrt{2}\right)x^2-2\sqrt{2+4\,i\,\sqrt{2}}\,\sqrt{-3-4x-x^2}-2x\left(-2+4\,i\,\sqrt{2}+\sqrt{2+4\,i\,\sqrt{2}}\,\sqrt{-3-4x-x^2}\right)\Big)\Big]\right)$$

Problem 129: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{x}{\sqrt{-3-4x-x^2}\,\left(3+4x+2x^2\right)}\,dx$$

Optimal (type 3, 68 leaves, 6 steps):

$$-\frac{\text{ArcTan}\left[\frac{1-\frac{3\sqrt{-1-x}}{\sqrt{3+x}}}{\sqrt{2}}\right]}{\sqrt{2}}+\frac{\text{ArcTan}\left[\frac{1+\frac{3\sqrt{-1-x}}{\sqrt{3+x}}}{\sqrt{2}}\right]}{\sqrt{2}}$$

Result (type 3, 814 leaves):

$$\frac{1}{8}\left(\frac{1}{\sqrt{1+2\,i\,\sqrt{2}}}\,2\left(2+i\,\sqrt{2}\right)\text{ArcTan}\Big[\left((2+x)\left(3\left(5+4\,i\,\sqrt{2}\right)+16\left(2+i\,\sqrt{2}\right)x+2\left(9+2\,i\,\sqrt{2}\right)x^2\right)\right)\Big/\left(12\,i-6\,\sqrt{2}+\left(8\,i+6\,\sqrt{2}\right)x^3-9\,\sqrt{1+2\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}+x\left(40\,i-5\,\sqrt{2}-12\,\sqrt{1+2\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)+x^2\left(36\,i+8\,\sqrt{2}-6\,\sqrt{1+2\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)\right)\Big]-\frac{1}{\sqrt{1-2\,i\,\sqrt{2}}}\,2\left(2\,i+\sqrt{2}\right)\text{ArcTanh}\Big[\left((2+x)\left(3\left(5\,i+4\,\sqrt{2}\right)+16\left(2\,i+\sqrt{2}\right)x+2\left(9\,i+2\,\sqrt{2}\right)x^2\right)\right)\Big/\left(-5\left(8\,i+\sqrt{2}\right)x+\left(-8\,i+6\,\sqrt{2}\right)x^3-12\,\sqrt{1-2\,i\,\sqrt{2}}\,x\,\sqrt{-3-4\,x-x^2}+x^2\left(-36\,i+8\,\sqrt{2}-6\,\sqrt{1-2\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)-3\left(4\,i+2\,\sqrt{2}+3\,\sqrt{1-2\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)\right)\Big]+\frac{\left(-2\,i+\sqrt{2}\right)\text{Log}\left[4\left(3+4\,x+2\,x^2\right)^2\right]}{\sqrt{1+2\,i\,\sqrt{2}}}+\frac{\left(2\,i+\sqrt{2}\right)\text{Log}\left[4\left(3+4\,x+2\,x^2\right)^2\right]}{\sqrt{1-2\,i\,\sqrt{2}}}-\frac{1}{\sqrt{1-2\,i\,\sqrt{2}}}\left(2\,i+\sqrt{2}\right)\text{Log}\Big[\left(3+4\,x+2\,x^2\right)\left(3+6\,i\,\sqrt{2}+\left(2+2\,i\,\sqrt{2}\right)x^2-2\,\sqrt{2-4\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}+x\left(4+8\,i\,\sqrt{2}-2\,\sqrt{2-4\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)\right)\Big]-\frac{1}{\sqrt{1+2\,i\,\sqrt{2}}}\left(-2\,i+\sqrt{2}\right)\text{Log}\Big[\left(3+4\,x+2\,x^2\right)\left(3-6\,i\,\sqrt{2}+\left(2-2\,i\,\sqrt{2}\right)x^2-2\,\sqrt{2+4\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}-2\,x\left(-2+4\,i\,\sqrt{2}+\sqrt{2+4\,i\,\sqrt{2}}\,\sqrt{-3-4\,x-x^2}\right)\right)\Big]\right)$$

Problem 130: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-3-4\,x-x^2}\,\left(3+4\,x+2\,x^2\right)}\,dx$$

Optimal (type 3, 95 leaves, 10 steps):

$$-\frac{1}{3}\sqrt{2}\,\text{ArcTan}\left[\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right]+\frac{1}{3}\sqrt{2}\,\text{ArcTan}\left[\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right]+\frac{1}{3}\text{ArcTanh}\left[\frac{x}{\sqrt{-3-4x-x^2}}\right]$$

Result (type 3, 800 leaves):

$$\begin{aligned}
&\frac{1}{12}\Bigg(-2\sqrt{1-2i\sqrt{2}}\,\text{ArcTan}\Big[\Big(\left(3+4x+x^2\right)\left(7+2i\sqrt{2}+8x+2x^2\right)\Big)\Big/ \\
&\quad\Big(2\sqrt{2}\,x^4+x\left(28i+16\sqrt{2}-11\sqrt{1+2i\sqrt{2}}\sqrt{-3-4x-x^2}\right)+ \\
&\quad x^2\left(20i+23\sqrt{2}-8\sqrt{1+2i\sqrt{2}}\sqrt{-3-4x-x^2}\right)+3\Big(4i+\sqrt{2}- \\
&\qquad 2\sqrt{1+2i\sqrt{2}}\sqrt{-3-4x-x^2}\Big)+2x^3\left(2i+6\sqrt{2}-\sqrt{1+2i\sqrt{2}}\sqrt{-3-4x-x^2}\right)\Big)\Big]+ \\
&\quad 2i\sqrt{1+2i\sqrt{2}}\,\text{ArcTanh}\Big[\Big(\left(7i+2\sqrt{2}+8ix+2ix^2\right)\left(3+4x+x^2\right)\Big)\Big/ \\
&\quad\Big(2\sqrt{2}\,x^4+x\left(-28i+16\sqrt{2}-11\sqrt{1-2i\sqrt{2}}\sqrt{-3-4x-x^2}\right)+ \\
&\quad x^2\left(-20i+23\sqrt{2}-8\sqrt{1-2i\sqrt{2}}\sqrt{-3-4x-x^2}\right)+ \\
&\quad 3\left(-4i+\sqrt{2}-2\sqrt{1-2i\sqrt{2}}\sqrt{-3-4x-x^2}\right)+ \\
&\quad 2x^3\left(-2i+6\sqrt{2}-\sqrt{1-2i\sqrt{2}}\sqrt{-3-4x-x^2}\right)\Big)\Big]+ \\
&\quad i\Bigg(\left(\sqrt{1-2i\sqrt{2}}-\sqrt{1+2i\sqrt{2}}\right)\text{Log}\left[4\left(3+4x+2x^2\right)^2\right]+ \\
&\quad \sqrt{1+2i\sqrt{2}}\,\text{Log}\Big[\left(3+4x+2x^2\right)\Big(3+6i\sqrt{2}+\left(2+2i\sqrt{2}\right)x^2- \\
&\qquad 2\sqrt{2-4i\sqrt{2}}\sqrt{-3-4x-x^2}+x\left(4+8i\sqrt{2}-2\sqrt{2-4i\sqrt{2}}\sqrt{-3-4x-x^2}\right)\Big)\Big]- \\
&\quad \sqrt{1-2i\sqrt{2}}\,\text{Log}\Big[\left(3+4x+2x^2\right)\Big(3-6i\sqrt{2}+\left(2-2i\sqrt{2}\right)x^2- \\
&\qquad 2\sqrt{2+4i\sqrt{2}}\sqrt{-3-4x-x^2}-2x\left(-2+4i\sqrt{2}+\sqrt{2+4i\sqrt{2}}\sqrt{-3-4x-x^2}\right)\Big)\Big]\Bigg)\Bigg)
\end{aligned}$$

Problem 131: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{x\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx$$

Optimal (type 3, 130 leaves, 17 steps):

$$-\frac{\text{ArcTan}\left[\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right]}{3\sqrt{3}}+\frac{1}{9}\sqrt{2}\,\text{ArcTan}\left[\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right]-\frac{1}{9}\sqrt{2}\,\text{ArcTan}\left[\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right]-\frac{4}{9}\,\text{ArcTanh}\left[\frac{x}{\sqrt{-3-4x-x^2}}\right]$$

Result (type 3, 959 leaves) :

$$\frac{1}{36}\left(-4\sqrt{3}\,\mathrm{ArcTan}\left[\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right]+\frac{1}{\sqrt{1-2\,\mathrm{i}\sqrt{2}}}\right.$$

$$6\left(2+\mathrm{i}\sqrt{2}\right)\mathrm{ArcTan}\left[\left(\left(8+2\,\mathrm{i}\sqrt{2}\right)x^4-18\,\mathrm{i}\left(\sqrt{2}-\sqrt{1-2\,\mathrm{i}\sqrt{2}}\sqrt{-3-4x-x^2}\right)+\right.\right.$$

$$x^3\left(44-4\,\mathrm{i}\sqrt{2}+6\,\mathrm{i}\sqrt{1-2\,\mathrm{i}\sqrt{2}}\sqrt{-3-4x-x^2}\right)+$$

$$x^2\left(72-35\,\mathrm{i}\sqrt{2}+24\,\mathrm{i}\sqrt{1-2\,\mathrm{i}\sqrt{2}}\sqrt{-3-4x-x^2}\right)+$$

$$\left.x\left(36-48\,\mathrm{i}\sqrt{2}+33\,\mathrm{i}\sqrt{1-2\,\mathrm{i}\sqrt{2}}\sqrt{-3-4x-x^2}\right)\right)\Big/$$

$$\left.\left(36\,\mathrm{i}+60\,\mathrm{i}\,x+\left(31\,\mathrm{i}+12\sqrt{2}\right)x^2+8\left(\mathrm{i}+2\sqrt{2}\right)x^3+\left(2\,\mathrm{i}+4\sqrt{2}\right)x^4\right)\right]-\frac{1}{\sqrt{1+2\,\mathrm{i}\sqrt{2}}}$$

$$6\left(2\,\mathrm{i}+\sqrt{2}\right)\mathrm{ArcTanh}\left[\left(2\left(4\,\mathrm{i}+\sqrt{2}\right)x^4-18\left(\sqrt{2}-\sqrt{1+2\,\mathrm{i}\sqrt{2}}\sqrt{-3-4x-x^2}\right)+\right.\right.$$

$$x^3\left(44\,\mathrm{i}-4\sqrt{2}+6\sqrt{1+2\,\mathrm{i}\sqrt{2}}\sqrt{-3-4x-x^2}\right)+$$

$$x^2\left(72\,\mathrm{i}-35\sqrt{2}+24\sqrt{1+2\,\mathrm{i}\sqrt{2}}\sqrt{-3-4x-x^2}\right)+$$

$$\left.x\left(36\,\mathrm{i}-48\sqrt{2}+33\sqrt{1+2\,\mathrm{i}\sqrt{2}}\sqrt{-3-4x-x^2}\right)\right)\Big/$$

$$\left.\left(-36\,\mathrm{i}-60\,\mathrm{i}\,x+\left(-31\,\mathrm{i}+12\sqrt{2}\right)x^2+8\left(-\mathrm{i}+2\sqrt{2}\right)x^3+\left(-2\,\mathrm{i}+4\sqrt{2}\right)x^4\right)\right]-$$

$$\frac{3\left(-2\,\mathrm{i}+\sqrt{2}\right)\mathrm{Log}\left[4\left(3+4x+2x^2\right)^2\right]}{\sqrt{1-2\,\mathrm{i}\sqrt{2}}}-\frac{3\left(2\,\mathrm{i}+\sqrt{2}\right)\mathrm{Log}\left[4\left(3+4x+2x^2\right)^2\right]}{\sqrt{1+2\,\mathrm{i}\sqrt{2}}}+$$

$$\frac{1}{\sqrt{1-2\,\mathrm{i}\sqrt{2}}}$$

$$3\left(-2\,\mathrm{i}+\sqrt{2}\right)\mathrm{Log}\left[\left(3+4x+2x^2\right)\left(3+6\,\mathrm{i}\sqrt{2}+\left(2+2\,\mathrm{i}\sqrt{2}\right)x^2-\right.\right.$$

$$\left.\left.2\sqrt{2-4\,\mathrm{i}\sqrt{2}}\sqrt{-3-4x-x^2}+x\left(4+8\,\mathrm{i}\sqrt{2}-2\sqrt{2-4\,\mathrm{i}\sqrt{2}}\sqrt{-3-4x-x^2}\right)\right)\right]+$$

$$\frac{1}{\sqrt{1+2\,\mathrm{i}\sqrt{2}}}3\left(2\,\mathrm{i}+\sqrt{2}\right)\mathrm{Log}\left[\left(3+4x+2x^2\right)\left(3-6\,\mathrm{i}\sqrt{2}+\left(2-2\,\mathrm{i}\sqrt{2}\right)x^2-\right.\right.$$

$$\left.\left.\left.2\sqrt{2+4\,\mathrm{i}\sqrt{2}}\sqrt{-3-4x-x^2}-2x\left(-2+4\,\mathrm{i}\sqrt{2}+\sqrt{2+4\,\mathrm{i}\sqrt{2}}\sqrt{-3-4x-x^2}\right)\right)\right]\right)$$

Problem 132: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{1}{x^2\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx$$

Optimal (type 3, 151 leaves, 20 steps):

$$\frac{\sqrt{-3-4x-x^2}}{9x}+\frac{2\,\text{ArcTan}\left[\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right]}{3\sqrt{3}}+\frac{2}{27}\sqrt{2}\,\text{ArcTan}\left[\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right]-\frac{2}{27}\sqrt{2}\,\text{ArcTan}\left[\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right]+\frac{10}{27}\,\text{ArcTanh}\left[\frac{x}{\sqrt{-3-4x-x^2}}\right]$$

Result (type 3, 1039 leaves):

$$\frac{1}{18}\left(\frac{2\sqrt{-3-4x-x^2}}{x}+4\sqrt{3}\,\text{ArcTan}\left[\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right]-\frac{1}{\sqrt{1-2i\sqrt{2}}}\right.$$

$$2i\left(-i+2\sqrt{2}\right)\text{ArcTan}\left[\left(2\left(8+11i\sqrt{2}\right)x^4+9\left(12-i\sqrt{2}+6i\sqrt{1-2i\sqrt{2}}\sqrt{-3-4x-x^2}\right)+\right.\right.$$
$$2x^3\left(62+50i\sqrt{2}+9i\sqrt{1-2i\sqrt{2}}\sqrt{-3-4x-x^2}\right)+$$
$$x^2\left(324+137i\sqrt{2}+72i\sqrt{1-2i\sqrt{2}}\sqrt{-3-4x-x^2}\right)+$$
$$\left.x\left(324+48i\sqrt{2}+99i\sqrt{1-2i\sqrt{2}}\sqrt{-3-4x-x^2}\right)\right)\Big/$$
$$\left(9\left(5i+6\sqrt{2}\right)+12\left(7i+18\sqrt{2}\right)x+\left(125i+306\sqrt{2}\right)x^2+\right.$$
$$\left.\left.16\left(7i+11\sqrt{2}\right)x^3+\left(34i+32\sqrt{2}\right)x^4\right)\right]+\frac{1}{\sqrt{1+2i\sqrt{2}}}$$

$$2\left(i+2\sqrt{2}\right)\text{ArcTanh}\left[\left(2\left(8i+11\sqrt{2}\right)x^4-9\left(-12i+\sqrt{2}-6\sqrt{1+2i\sqrt{2}}\sqrt{-3-4x-x^2}\right)+\right.\right.$$
$$2x^3\left(62i+50\sqrt{2}+9\sqrt{1+2i\sqrt{2}}\sqrt{-3-4x-x^2}\right)+$$
$$x^2\left(324i+137\sqrt{2}+72\sqrt{1+2i\sqrt{2}}\sqrt{-3-4x-x^2}\right)+$$
$$\left.x\left(324i+48\sqrt{2}+99\sqrt{1+2i\sqrt{2}}\sqrt{-3-4x-x^2}\right)\right)\Big/\left(9\left(-5i+6\sqrt{2}\right)+\right.$$
$$\left.\left.12\left(-7i+18\sqrt{2}\right)x+\left(-125i+306\sqrt{2}\right)x^2+16\left(-7i+11\sqrt{2}\right)x^3+\left(-34i+32\sqrt{2}\right)x^4\right)\right]+$$

$$\frac{\left(-i+2\sqrt{2}\right)\text{Log}\left[4\left(3+4x+2x^2\right)^2\right]}{\sqrt{1-2i\sqrt{2}}}+\frac{\left(i+2\sqrt{2}\right)\text{Log}\left[4\left(3+4x+2x^2\right)^2\right]}{\sqrt{1+2i\sqrt{2}}}-$$

$$\frac{1}{\sqrt{1-2i\sqrt{2}}}\left(-i+2\sqrt{2}\right)$$
$$\text{Log}\left[\left(3+4x+2x^2\right)\left(3+6i\sqrt{2}+\left(2+2i\sqrt{2}\right)x^2-2\sqrt{2-4i\sqrt{2}}\sqrt{-3-4x-x^2}+\right.\right.$$
$$\left.\left.x\left(4+8i\sqrt{2}-2\sqrt{2-4i\sqrt{2}}\sqrt{-3-4x-x^2}\right)\right)\right]-$$

$$\frac{1}{\sqrt{1+2i\sqrt{2}}}\left(i+2\sqrt{2}\right)\text{Log}\left[\left(3+4x+2x^2\right)\left(3-6i\sqrt{2}+\left(2-2i\sqrt{2}\right)x^2-\right.\right.$$
$$\left.\left.\left.2\sqrt{2+4i\sqrt{2}}\sqrt{-3-4x-x^2}-2x\left(-2+4i\sqrt{2}+\sqrt{2+4i\sqrt{2}}\sqrt{-3-4x-x^2}\right)\right)\right]\right)$$

Problem 142: Result unnecessarily involves higher level functions.

$$\int \frac{g+h x}{\left(-\frac{c g^2}{h^2}+9 c x^2\right)^{1/3} \left(g^2+3 h^2 x^2\right)} \, dx$$

Optimal (type 3, 242 leaves, 2 steps):

$$\frac{\left(1-\frac{9 h^2 x^2}{g^2}\right)^{1/3} \text{ArcTan}\left[\frac{1}{\sqrt{3}}-\frac{2^{2/3} \left(1-\frac{3 h x}{g}\right)^{2/3}}{\sqrt{3} \left(1+\frac{3 h x}{g}\right)^{1/3}}\right]}{2^{2/3} \sqrt{3} h \left(-\frac{c g^2}{h^2}+9 c x^2\right)^{1/3}}+\frac{\left(1-\frac{9 h^2 x^2}{g^2}\right)^{1/3} \text{Log}\left[g^2+3 h^2 x^2\right]}{6\times 2^{2/3} h \left(-\frac{c g^2}{h^2}+9 c x^2\right)^{1/3}}-\frac{\left(1-\frac{9 h^2 x^2}{g^2}\right)^{1/3} \text{Log}\left[\left(1-\frac{3 h x}{g}\right)^{2/3}+2^{1/3} \left(1+\frac{3 h x}{g}\right)^{1/3}\right]}{2\times 2^{2/3} h \left(-\frac{c g^2}{h^2}+9 c x^2\right)^{1/3}}$$

Result (type 6, 331 leaves):

$$\Bigg(g^2 x \Bigg(\left(g \,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{9 h^2 x^2}{g^2},-\frac{3 h^2 x^2}{g^2}\right]\right) \Big/ \left(g^2 \,\text{AppellF1}\left[\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{9 h^2 x^2}{g^2},-\frac{3 h^2 x^2}{g^2}\right]+2 h^2 x^2 \left(-\text{AppellF1}\left[\frac{3}{2},\frac{1}{3},2,\frac{5}{2},\frac{9 h^2 x^2}{g^2},-\frac{3 h^2 x^2}{g^2}\right]+\text{AppellF1}\left[\frac{3}{2},\frac{4}{3},1,\frac{5}{2},\frac{9 h^2 x^2}{g^2},-\frac{3 h^2 x^2}{g^2}\right]\right)\right) - \left(h x \,\text{AppellF1}\left[1,\frac{1}{3},1,2,\frac{9 h^2 x^2}{g^2},-\frac{3 h^2 x^2}{g^2}\right]\right) \Big/ \left(-2 g^2 \,\text{AppellF1}\left[1,\frac{1}{3},1,2,\frac{9 h^2 x^2}{g^2},-\frac{3 h^2 x^2}{g^2}\right]+3 h^2 x^2 \left(\text{AppellF1}\left[2,\frac{1}{3},2,3,\frac{9 h^2 x^2}{g^2},-\frac{3 h^2 x^2}{g^2}\right]-\text{AppellF1}\left[2,\frac{4}{3},1,3,\frac{9 h^2 x^2}{g^2},-\frac{3 h^2 x^2}{g^2}\right]\right)\right)\Bigg)\Bigg) \Big/ \left(\left(c \left(-\frac{g^2}{h^2}+9 x^2\right)\right)^{1/3} \left(g^2+3 h^2 x^2\right)\right)$$

Problem 143: Unable to integrate problem.

$$\int \left((g+h x) \Big/ \left(\left(\frac{-c^2 g^2+b c g h+2 b^2 h^2}{9 c h^2}+b x+c x^2\right)^{1/3} \left(\frac{f \left(b^2-\frac{-c^2 g^2+b c g h+2 b^2 h^2}{3 h^2}\right)}{c^2}+\frac{b f x}{c}+f x^2\right)\right)\right) dx$$

Optimal (type 3, 488 leaves, 2 steps):

$$\left(3\times 3^{1/6}\,h\left(\frac{c\,h^2\left(\frac{(c\,g-2\,b\,h)\,(c\,g+b\,h)}{c\,h^2}-9\,b\,x-9\,c\,x^2\right)}{(2\,c\,g-b\,h)^2}\right)^{1/3}\text{ArcTan}\left[\frac{1}{\sqrt{3}}-\frac{2^{2/3}\left(1-\frac{3\,h\,(b+2\,c\,x)}{2\,c\,g-b\,h}\right)^{2/3}}{\sqrt{3}\left(1+\frac{3\,h\,(b+2\,c\,x)}{2\,c\,g-b\,h}\right)^{1/3}}\right]\right)\Bigg/$$

$$\left(f\left(-\frac{(c\,g-2\,b\,h)\,(c\,g+b\,h)}{c\,h^2}+9\,b\,x+9\,c\,x^2\right)^{1/3}\right)+$$

$$\left(3^{2/3}\,h\left(\frac{c\,h^2\left(\frac{(c\,g-2\,b\,h)\,(c\,g+b\,h)}{c\,h^2}-9\,b\,x-9\,c\,x^2\right)}{(2\,c\,g-b\,h)^2}\right)^{1/3}\text{Log}\left[\frac{f\left(c^2\,g^2-b\,c\,g\,h+b^2\,h^2\right)}{3\,c^2\,h^2}+\frac{b\,f\,x}{c}+f\,x^2\right]\right)\Bigg/$$

$$\left(2\,f\left(-\frac{(c\,g-2\,b\,h)\,(c\,g+b\,h)}{c\,h^2}+9\,b\,x+9\,c\,x^2\right)^{1/3}\right)-$$

$$\left(3\times 3^{2/3}\,h\left(\frac{c\,h^2\left(\frac{(c\,g-2\,b\,h)\,(c\,g+b\,h)}{c\,h^2}-9\,b\,x-9\,c\,x^2\right)}{(2\,c\,g-b\,h)^2}\right)^{1/3}\right.$$

$$\left.\text{Log}\left[\left(1-\frac{3\,h\,(b+2\,c\,x)}{2\,c\,g-b\,h}\right)^{2/3}+2^{1/3}\left(1+\frac{3\,h\,(b+2\,c\,x)}{2\,c\,g-b\,h}\right)^{1/3}\right]\right)\Bigg/$$

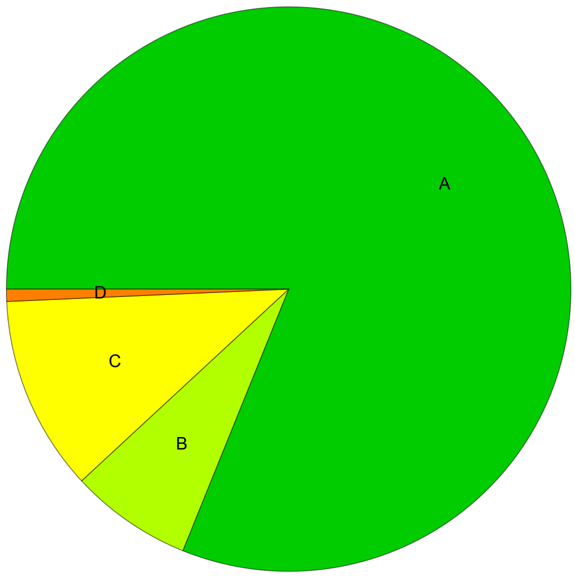
$$\left(2\,f\left(-\frac{(c\,g-2\,b\,h)\,(c\,g+b\,h)}{c\,h^2}+9\,b\,x+9\,c\,x^2\right)^{1/3}\right)$$

Result (type 8, 106 leaves) :

$$\int\left((g+h\,x)\Bigg/\left(\left(\frac{-c^2\,g^2+b\,c\,g\,h+2\,b^2\,h^2}{9\,c\,h^2}+b\,x+c\,x^2\right)^{1/3}\left(\frac{f\left(b^2-\frac{-c^2\,g^2+b\,c\,g\,h+2\,b^2\,h^2}{3\,h^2}\right)}{c^2}+\frac{b\,f\,x}{c}+f\,x^2\right)\right)\right)dx$$

Summary of Integration Test Results

143 integration problems



A - 116 optimal antiderivatives

B - 10 more than twice size of optimal antiderivatives

C - 16 unnecessarily complex antiderivatives

D - 1 unable to integrate problems

E - 0 integration timeouts