

Mathematica 11.3 Integration Test Results

Test results for the 456 problems in "3.1.4 (f x)^m (d+e x^r)^q (a+b log(c x^n))^p.m"

Problem 56: Result more than twice size of optimal antiderivative.

$$\int \frac{x^2 (a + b \operatorname{Log}[c x^n])}{(d + e x)^4} dx$$

Optimal (type 3, 79 leaves, 3 steps):

$$\frac{b d n}{6 e^3 (d + e x)^2} - \frac{2 b n}{3 e^3 (d + e x)} + \frac{x^3 (a + b \operatorname{Log}[c x^n])}{3 d (d + e x)^3} - \frac{b n \operatorname{Log}[d + e x]}{3 d e^3}$$

Result (type 3, 170 leaves):

$$-\frac{1}{6 d e^3 (d + e x)^3} \left(2 a d^3 + 3 b d^3 n + 6 a d^2 e x + 7 b d^2 e n x + 6 a d e^2 x^2 + 4 b d e^2 n x^2 - 2 b n (d + e x)^3 \operatorname{Log}[x] + 2 b d (d^2 + 3 d e x + 3 e^2 x^2) \operatorname{Log}[c x^n] + 2 b d^3 n \operatorname{Log}[d + e x] + 6 b d^2 e n x \operatorname{Log}[d + e x] + 6 b d e^2 n x^2 \operatorname{Log}[d + e x] + 2 b e^3 n x^3 \operatorname{Log}[d + e x] \right)$$

Problem 64: Result more than twice size of optimal antiderivative.

$$\int \frac{x^6 (a + b \operatorname{Log}[c x^n])}{(d + e x)^7} dx$$

Optimal (type 4, 243 leaves, 8 steps):

$$-\frac{x^6 (a + b \operatorname{Log}[c x^n])}{6 e (d + e x)^6} - \frac{x^5 (6 a + b n + 6 b \operatorname{Log}[c x^n])}{30 e^2 (d + e x)^5} - \frac{x^2 (20 a + 19 b n + 20 b \operatorname{Log}[c x^n])}{40 e^5 (d + e x)^2} - \frac{x (20 a + 29 b n + 20 b \operatorname{Log}[c x^n])}{20 e^6 (d + e x)} - \frac{x^4 (30 a + 11 b n + 30 b \operatorname{Log}[c x^n])}{120 e^3 (d + e x)^4} - \frac{x^3 (60 a + 37 b n + 60 b \operatorname{Log}[c x^n])}{180 e^4 (d + e x)^3} + \frac{(20 a + 49 b n + 20 b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{20 e^7} + \frac{b n \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{e^7}$$

Result (type 4, 673 leaves):

$$\frac{1}{360 e^7 (d + e x)^6} \left(882 a d^6 + 812 b d^6 n + 4932 a d^5 e x + 4350 b d^5 e n x + 11250 a d^4 e^2 x^2 + 9399 b d^4 e^2 n x^2 + 13200 a d^3 e^3 x^3 + 10262 b d^3 e^3 n x^3 + 8100 a d^2 e^4 x^4 + 5679 b d^2 e^4 n x^4 + 2160 a d e^5 x^5 + 1278 b d e^5 n x^5 + 882 b d^6 \text{Log}[c x^n] + 4932 b d^5 e x \text{Log}[c x^n] + 11250 b d^4 e^2 x^2 \text{Log}[c x^n] + 13200 b d^3 e^3 x^3 \text{Log}[c x^n] + 8100 b d^2 e^4 x^4 \text{Log}[c x^n] + 2160 b d e^5 x^5 \text{Log}[c x^n] + 360 a d^6 \text{Log}[d + e x] + 882 b d^6 n \text{Log}[d + e x] + 2160 a d^5 e x \text{Log}[d + e x] + 5292 b d^5 e n x \text{Log}[d + e x] + 5400 a d^4 e^2 x^2 \text{Log}[d + e x] + 13230 b d^4 e^2 n x^2 \text{Log}[d + e x] + 7200 a d^3 e^3 x^3 \text{Log}[d + e x] + 17640 b d^3 e^3 n x^3 \text{Log}[d + e x] + 5400 a d^2 e^4 x^4 \text{Log}[d + e x] + 13230 b d^2 e^4 n x^4 \text{Log}[d + e x] + 2160 a d e^5 x^5 \text{Log}[d + e x] + 5292 b d e^5 n x^5 \text{Log}[d + e x] + 360 a e^6 x^6 \text{Log}[d + e x] + 882 b e^6 n x^6 \text{Log}[d + e x] + 360 b d^6 \text{Log}[c x^n] \text{Log}[d + e x] + 2160 b d^5 e x \text{Log}[c x^n] \text{Log}[d + e x] + 5400 b d^4 e^2 x^2 \text{Log}[c x^n] \text{Log}[d + e x] + 7200 b d^3 e^3 x^3 \text{Log}[c x^n] \text{Log}[d + e x] + 5400 b d^2 e^4 x^4 \text{Log}[c x^n] \text{Log}[d + e x] + 2160 b d e^5 x^5 \text{Log}[c x^n] \text{Log}[d + e x] + 360 b e^6 x^6 \text{Log}[c x^n] \text{Log}[d + e x] - 18 b n (d + e x)^6 \text{Log}[x] \left(49 + 20 \text{Log}[d + e x] - 20 \text{Log}\left[1 + \frac{e x}{d}\right] \right) + 360 b n (d + e x)^6 \text{PolyLog}\left[2, -\frac{e x}{d}\right] \right)$$

Problem 65: Result more than twice size of optimal antiderivative.

$$\int \frac{x^5 (a + b \text{Log}[c x^n])}{(d + e x)^7} dx$$

Optimal (type 3, 136 leaves, 3 steps):

$$-\frac{b d^4 n}{30 e^6 (d + e x)^5} + \frac{5 b d^3 n}{24 e^6 (d + e x)^4} - \frac{5 b d^2 n}{9 e^6 (d + e x)^3} + \frac{5 b d n}{6 e^6 (d + e x)^2} - \frac{5 b n}{6 e^6 (d + e x)} + \frac{x^6 (a + b \text{Log}[c x^n])}{6 d (d + e x)^6} - \frac{b n \text{Log}[d + e x]}{6 d e^6}$$

Result (type 3, 335 leaves):

$$\frac{1}{360 d e^6 (d + e x)^6} \left(60 a d^6 + 137 b d^6 n + 360 a d^5 e x + 762 b d^5 e n x + 900 a d^4 e^2 x^2 + 1725 b d^4 e^2 n x^2 + 1200 a d^3 e^3 x^3 + 2000 b d^3 e^3 n x^3 + 900 a d^2 e^4 x^4 + 1200 b d^2 e^4 n x^4 + 360 a d e^5 x^5 + 300 b d e^5 n x^5 - 60 b n (d + e x)^6 \text{Log}[x] + 60 b d (d^5 + 6 d^4 e x + 15 d^3 e^2 x^2 + 20 d^2 e^3 x^3 + 15 d e^4 x^4 + 6 e^5 x^5) \text{Log}[c x^n] + 60 b d^6 n \text{Log}[d + e x] + 360 b d^5 e n x \text{Log}[d + e x] + 900 b d^4 e^2 n x^2 \text{Log}[d + e x] + 1200 b d^3 e^3 n x^3 \text{Log}[d + e x] + 900 b d^2 e^4 n x^4 \text{Log}[d + e x] + 360 b d e^5 n x^5 \text{Log}[d + e x] + 60 b e^6 n x^6 \text{Log}[d + e x] \right)$$

Problem 75: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Log}\left[\frac{x}{c}\right]}{c - x} dx$$

Optimal (type 4, 10 leaves, 1 step):

$$\text{PolyLog}\left[2, 1 - \frac{x}{c}\right]$$

Result (type 4, 27 leaves):

$$-\text{Log}\left[\frac{x}{c}\right] \text{Log}\left[1 - \frac{x}{c}\right] - \text{PolyLog}\left[2, \frac{x}{c}\right]$$

Problem 115: Result more than twice size of optimal antiderivative.

$$\int \frac{x^2 (a + b \text{Log}[c x^n])^2}{(d + e x)^4} dx$$

Optimal (type 4, 161 leaves, 5 steps):

$$\frac{b n x^2 (a + b \text{Log}[c x^n])}{3 d e (d + e x)^2} + \frac{x^3 (a + b \text{Log}[c x^n])^2}{3 d (d + e x)^3} + \frac{b n x (2 a + b n + 2 b \text{Log}[c x^n])}{3 d e^2 (d + e x)} - \frac{b n (2 a + 3 b n + 2 b \text{Log}[c x^n]) \text{Log}\left[1 + \frac{e x}{d}\right]}{3 d e^3} - \frac{2 b^2 n^2 \text{PolyLog}\left[2, -\frac{e x}{d}\right]}{3 d e^3}$$

Result (type 4, 612 leaves):

$$\begin{aligned} & - \frac{(a + b (-n \text{Log}[x] + \text{Log}[c x^n]))^2}{e^3 (d + e x)} + \\ & \frac{a^2 d + 2 a b d (-n \text{Log}[x] + \text{Log}[c x^n]) + b^2 d (-n \text{Log}[x] + \text{Log}[c x^n])^2}{e^3 (d + e x)^2} + \frac{1}{3 e^3 (d + e x)^3} \\ & \left(-a^2 d^2 - 2 a b d^2 (-n \text{Log}[x] + \text{Log}[c x^n]) - b^2 d^2 (-n \text{Log}[x] + \text{Log}[c x^n])^2 \right) + \\ & 2 b n (a + b (-n \text{Log}[x] + \text{Log}[c x^n])) \left(\frac{\frac{x \text{Log}[x]}{d + e x} - \frac{\text{Log}[d + e x]}{e}}{d e^2} - \right. \\ & \left. \frac{e x (2 d + e x) \text{Log}[x] - (d + e x) (-d + (d + e x) \text{Log}[d + e x])}{d e^3 (d + e x)^2} + \frac{1}{6 d e^3 (d + e x)^3} \right. \\ & \left. \left(2 e x (3 d^2 + 3 d e x + e^2 x^2) \text{Log}[x] - (d + e x) (-d (3 d + 2 e x) + 2 (d + e x)^2 \text{Log}[d + e x]) \right) \right) + \\ & b^2 n^2 \left(\frac{1}{d e^3 (d + e x)} \left(\text{Log}[x] \left(e x \text{Log}[x] - 2 (d + e x) \text{Log}\left[1 + \frac{e x}{d}\right] \right) - 2 (d + e x) \text{PolyLog}\left[2, -\frac{e x}{d}\right] \right) - \right. \\ & \frac{1}{d e^3 (d + e x)^2} \left(e x (2 d + e x) \text{Log}[x]^2 + 2 (d + e x)^2 \text{Log}\left[1 + \frac{e x}{d}\right] - 2 (d + e x) \text{Log}[x] \right. \\ & \left. \left(e x + (d + e x) \text{Log}\left[1 + \frac{e x}{d}\right] \right) - 2 (d + e x)^2 \text{PolyLog}\left[2, -\frac{e x}{d}\right] \right) + \frac{1}{3 d e^3 (d + e x)^3} \\ & \left(e x (3 d^2 + 3 d e x + e^2 x^2) \text{Log}[x]^2 + (d + e x)^2 \left(e x + 3 (d + e x) \text{Log}\left[1 + \frac{e x}{d}\right] \right) - (d + e x) \right. \\ & \left. \left. \text{Log}[x] \left(e x (4 d + 3 e x) + 2 (d + e x)^2 \text{Log}\left[1 + \frac{e x}{d}\right] \right) - 2 (d + e x)^3 \text{PolyLog}\left[2, -\frac{e x}{d}\right] \right) \right) \end{aligned}$$

Problem 116: Result more than twice size of optimal antiderivative.

$$\int \frac{x (a + b \operatorname{Log}[c x^n])^2}{(d + e x)^4} dx$$

Optimal (type 4, 210 leaves, 8 steps):

$$\frac{b^2 n^2}{3 d e^2 (d + e x)} - \frac{b n (a + b \operatorname{Log}[c x^n])}{3 e^2 (d + e x)^2} + \frac{b n (a + b \operatorname{Log}[c x^n])}{3 d e^2 (d + e x)} +$$

$$\frac{(a + b \operatorname{Log}[c x^n])^2}{6 d^2 e^2} + \frac{d (a + b \operatorname{Log}[c x^n])^2}{3 e^2 (d + e x)^3} - \frac{(a + b \operatorname{Log}[c x^n])^2}{2 e^2 (d + e x)^2} -$$

$$\frac{b n (a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{3 d^2 e^2} - \frac{b^2 n^2 \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{3 d^2 e^2}$$

Result (type 4, 441 leaves):

$$-\frac{1}{6 d^2 e^2 (d + e x)^3}$$

$$\left(a^2 d^3 + 3 a^2 d^2 e x - 2 a b d^2 e n x + 2 b^2 d^2 e n^2 x - 2 a b d e^2 n x^2 + 4 b^2 d e^2 n^2 x^2 + 2 b^2 e^3 n^2 x^3 + \right.$$

$$b^2 n^2 (d + e x)^3 \operatorname{Log}[x]^2 + 2 a b d^3 \operatorname{Log}[c x^n] + 6 a b d^2 e x \operatorname{Log}[c x^n] - 2 b^2 d^2 e n x \operatorname{Log}[c x^n] -$$

$$2 b^2 d e^2 n x^2 \operatorname{Log}[c x^n] + b^2 d^3 \operatorname{Log}[c x^n]^2 + 3 b^2 d^2 e x \operatorname{Log}[c x^n]^2 + 2 a b d^3 n \operatorname{Log}[d + e x] +$$

$$6 a b d^2 e n x \operatorname{Log}[d + e x] + 6 a b d e^2 n x^2 \operatorname{Log}[d + e x] + 2 a b e^3 n x^3 \operatorname{Log}[d + e x] +$$

$$2 b^2 d^3 n \operatorname{Log}[c x^n] \operatorname{Log}[d + e x] + 6 b^2 d^2 e n x \operatorname{Log}[c x^n] \operatorname{Log}[d + e x] +$$

$$6 b^2 d e^2 n x^2 \operatorname{Log}[c x^n] \operatorname{Log}[d + e x] + 2 b^2 e^3 n x^3 \operatorname{Log}[c x^n] \operatorname{Log}[d + e x] - 2 b n (d + e x)^3 \operatorname{Log}[x]$$

$$\left. \left(a + b \operatorname{Log}[c x^n] + b n \operatorname{Log}[d + e x] - b n \operatorname{Log}\left[1 + \frac{e x}{d}\right] \right) + 2 b^2 n^2 (d + e x)^3 \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right] \right)$$

Problem 121: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b \operatorname{Log}[c x^n])^3}{x (d + e x)} dx$$

Optimal (type 4, 113 leaves, 4 steps):

$$-\frac{\operatorname{Log}\left[1 + \frac{d}{e x}\right] (a + b \operatorname{Log}[c x^n])^3}{d} + \frac{3 b n (a + b \operatorname{Log}[c x^n])^2 \operatorname{PolyLog}\left[2, -\frac{d}{e x}\right]}{d} +$$

$$\frac{6 b^2 n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[3, -\frac{d}{e x}\right]}{d} + \frac{6 b^3 n^3 \operatorname{PolyLog}\left[4, -\frac{d}{e x}\right]}{d}$$

Result (type 4, 243 leaves):

$$\begin{aligned} & \frac{1}{4d} \left(4 \operatorname{Log}[x] (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n])^3 - 4 (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n])^3 \operatorname{Log}[d + e x] + \right. \\ & 6 b n (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n])^2 \left(\operatorname{Log}[x]^2 - 2 \left(\operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{e x}{d}\right] + \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]\right) \right) - \\ & 4 b^2 n^2 (-a + b n \operatorname{Log}[x] - b \operatorname{Log}[c x^n]) \\ & \left. \left(\operatorname{Log}[x]^2 \left(\operatorname{Log}[x] - 3 \operatorname{Log}\left[1 + \frac{e x}{d}\right]\right) - 6 \operatorname{Log}[x] \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right] + 6 \operatorname{PolyLog}\left[3, -\frac{e x}{d}\right] \right) + \right. \\ & b^3 n^3 \left(\operatorname{Log}[x]^4 - 4 \operatorname{Log}[x]^3 \operatorname{Log}\left[1 + \frac{e x}{d}\right] - 12 \operatorname{Log}[x]^2 \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right] + \right. \\ & \left. \left. 24 \operatorname{Log}[x] \operatorname{PolyLog}\left[3, -\frac{e x}{d}\right] - 24 \operatorname{PolyLog}\left[4, -\frac{e x}{d}\right] \right) \right) \end{aligned}$$

Problem 134: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{d+e x} (a+b \operatorname{Log}[c x^n])}{x} dx$$

Optimal (type 4, 211 leaves, 12 steps):

$$\begin{aligned} & -4 b n \sqrt{d+e x} + 4 b \sqrt{d} n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x}}{\sqrt{d}}\right] + 2 b \sqrt{d} n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x}}{\sqrt{d}}\right]^2 + \\ & 2 \sqrt{d+e x} (a+b \operatorname{Log}[c x^n]) - 2 \sqrt{d} \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x}}{\sqrt{d}}\right] (a+b \operatorname{Log}[c x^n]) - \\ & 4 b \sqrt{d} n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x}}{\sqrt{d}}\right] \operatorname{Log}\left[\frac{2 \sqrt{d}}{\sqrt{d}-\sqrt{d+e x}}\right] - 2 b \sqrt{d} n \operatorname{PolyLog}\left[2, 1 - \frac{2 \sqrt{d}}{\sqrt{d}-\sqrt{d+e x}}\right] \end{aligned}$$

Result (type 5, 177 leaves):

$$\begin{aligned} & \frac{1}{\sqrt{1+\frac{d}{e x}}} b n \sqrt{d+e x} \left(-4 \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right\}, \left\{\frac{1}{2}, \frac{1}{2}\right\}, -\frac{d}{e x}\right] + \right. \\ & \left. 2 \sqrt{1+\frac{d}{e x}} \operatorname{Log}[x] - \frac{2 \sqrt{d} \operatorname{ArcSinh}\left[\frac{\sqrt{d}}{\sqrt{e} \sqrt{x}}\right] \operatorname{Log}[x]}{\sqrt{e} \sqrt{x}} \right) + \\ & 2 \sqrt{d+e x} (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n]) - 2 \sqrt{d} \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x}}{\sqrt{d}}\right] (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n]) \end{aligned}$$

Problem 135: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{d+e x} (a+b \operatorname{Log}[c x^n])}{x^2} dx$$

Optimal (type 4, 221 leaves, 11 steps):

$$\begin{aligned}
 & - \frac{b n \sqrt{d+e x}}{x} - \frac{b e n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x}}{\sqrt{d}}\right]}{\sqrt{d}} + \frac{b e n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x}}{\sqrt{d}}\right]^2}{\sqrt{d}} - \\
 & \frac{\sqrt{d+e x} (a+b \operatorname{Log}[c x^n])}{x} - \frac{e \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x}}{\sqrt{d}}\right] (a+b \operatorname{Log}[c x^n])}{\sqrt{d}} - \\
 & \frac{2 b e n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x}}{\sqrt{d}}\right] \operatorname{Log}\left[\frac{2 \sqrt{d}}{\sqrt{d}-\sqrt{d+e x}}\right]}{\sqrt{d}} - \frac{b e n \operatorname{PolyLog}\left[2, 1 - \frac{2 \sqrt{d}}{\sqrt{d}-\sqrt{d+e x}}\right]}{\sqrt{d}}
 \end{aligned}$$

Result (type 5, 193 leaves):

$$\begin{aligned}
 & \frac{1}{\sqrt{d} \sqrt{1 + \frac{d}{e x}} x} \left(-2 b \sqrt{d} n \sqrt{d+e x} \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\}, \left\{\frac{3}{2}, \frac{3}{2}\right\}, -\frac{d}{e x}\right] - \right. \\
 & \left. b \sqrt{e} n \sqrt{x} \sqrt{d+e x} \operatorname{ArcSinh}\left[\frac{\sqrt{d}}{\sqrt{e} \sqrt{x}}\right] (1 + \operatorname{Log}[x]) - \sqrt{1 + \frac{d}{e x}} \right. \\
 & \left. \left(\sqrt{d} \sqrt{d+e x} (a+b n + b \operatorname{Log}[c x^n]) + e x \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x}}{\sqrt{d}}\right] (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n]) \right) \right)
 \end{aligned}$$

Problem 136: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{d+e x} (a+b \operatorname{Log}[c x^n])}{x^3} dx$$

Optimal (type 4, 298 leaves, 16 steps):

$$\begin{aligned}
 & - \frac{b n \sqrt{d+e x}}{4 x^2} - \frac{3 b e n \sqrt{d+e x}}{8 d x} - \frac{b e^2 n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x}}{\sqrt{d}}\right]}{8 d^{3/2}} - \frac{b e^2 n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x}}{\sqrt{d}}\right]^2}{4 d^{3/2}} - \\
 & \frac{\sqrt{d+e x} (a+b \operatorname{Log}[c x^n])}{2 x^2} - \frac{e \sqrt{d+e x} (a+b \operatorname{Log}[c x^n])}{4 d x} + \frac{e^2 \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x}}{\sqrt{d}}\right] (a+b \operatorname{Log}[c x^n])}{4 d^{3/2}} + \\
 & \frac{b e^2 n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x}}{\sqrt{d}}\right] \operatorname{Log}\left[\frac{2 \sqrt{d}}{\sqrt{d}-\sqrt{d+e x}}\right]}{2 d^{3/2}} + \frac{b e^2 n \operatorname{PolyLog}\left[2, 1 - \frac{2 \sqrt{d}}{\sqrt{d}-\sqrt{d+e x}}\right]}{4 d^{3/2}}
 \end{aligned}$$

Result (type 5, 208 leaves):

$$\left(-16 b d^{3/2} n \sqrt{d+ex} \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, \frac{3}{2}, \frac{3}{2}\right\}, \left\{\frac{5}{2}, \frac{5}{2}\right\}, -\frac{d}{ex}\right] + \right. \\ \left. 9 e^2 \sqrt{1 + \frac{d}{ex}} x^2 \operatorname{ArcTanh}\left[\frac{\sqrt{d+ex}}{\sqrt{d}}\right] (a - b n \operatorname{Log}[x] + b \operatorname{Log}[cx^n]) - \right. \\ \left. 9 \sqrt{d+ex} \left(-b e^{3/2} n x^{3/2} \operatorname{ArcSinh}\left[\frac{\sqrt{d}}{\sqrt{e} \sqrt{x}}\right] \operatorname{Log}[x] + \right. \right. \\ \left. \left. \sqrt{d} \sqrt{1 + \frac{d}{ex}} (2d+ex) (a + b \operatorname{Log}[cx^n]) \right) \right) / \left(36 d^{3/2} \sqrt{1 + \frac{d}{ex}} x^2 \right)$$

Problem 141: Result unnecessarily involves higher level functions.

$$\int \frac{(d+ex)^{3/2} (a+b \operatorname{Log}[cx^n])}{x} dx$$

Optimal (type 4, 255 leaves, 18 steps):

$$-\frac{16}{3} b d n \sqrt{d+ex} - \frac{4}{9} b n (d+ex)^{3/2} + \frac{16}{3} b d^{3/2} n \operatorname{ArcTanh}\left[\frac{\sqrt{d+ex}}{\sqrt{d}}\right] + \\ 2 b d^{3/2} n \operatorname{ArcTanh}\left[\frac{\sqrt{d+ex}}{\sqrt{d}}\right]^2 + 2 d \sqrt{d+ex} (a + b \operatorname{Log}[cx^n]) + \\ \frac{2}{3} (d+ex)^{3/2} (a + b \operatorname{Log}[cx^n]) - 2 d^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{d+ex}}{\sqrt{d}}\right] (a + b \operatorname{Log}[cx^n]) - \\ 4 b d^{3/2} n \operatorname{ArcTanh}\left[\frac{\sqrt{d+ex}}{\sqrt{d}}\right] \operatorname{Log}\left[\frac{2 \sqrt{d}}{\sqrt{d} - \sqrt{d+ex}}\right] - 2 b d^{3/2} n \operatorname{PolyLog}\left[2, 1 - \frac{2 \sqrt{d}}{\sqrt{d} - \sqrt{d+ex}}\right]$$

Result (type 5, 272 leaves):

$$\frac{1}{3 \sqrt{1 + \frac{e x}{d}}} b n \sqrt{d + e x} \left(-3 e x \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, 1, 1\right\}, \{2, 2\}, -\frac{e x}{d}\right] + \right. \\ \left. 2 \left(e x \sqrt{1 + \frac{e x}{d}} + d \left(-1 + \sqrt{1 + \frac{e x}{d}} \right) \right) \operatorname{Log}[x] \right) + \frac{1}{\sqrt{1 + \frac{d}{e x}}} \\ b d n \sqrt{d + e x} \left(-4 \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right\}, \left\{\frac{1}{2}, \frac{1}{2}\right\}, -\frac{d}{e x}\right] + \right. \\ \left. 2 \sqrt{1 + \frac{d}{e x}} \operatorname{Log}[x] - \frac{2 \sqrt{d} \operatorname{ArcSinh}\left[\frac{\sqrt{d}}{\sqrt{e} \sqrt{x}}\right] \operatorname{Log}[x]}{\sqrt{e} \sqrt{x}} \right) + \\ \frac{2}{3} \sqrt{d + e x} (4 d + e x) (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n]) - \\ 2 d^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{d + e x}}{\sqrt{d}}\right] (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n])$$

Problem 142: Result unnecessarily involves higher level functions.

$$\int \frac{(d + e x)^{3/2} (a + b \operatorname{Log}[c x^n])}{x^2} dx$$

Optimal (type 4, 259 leaves, 14 steps):

$$-4 b e n \sqrt{d + e x} - \frac{b d n \sqrt{d + e x}}{x} + 3 b \sqrt{d} e n \operatorname{ArcTanh}\left[\frac{\sqrt{d + e x}}{\sqrt{d}}\right] + \\ 3 b \sqrt{d} e n \operatorname{ArcTanh}\left[\frac{\sqrt{d + e x}}{\sqrt{d}}\right]^2 + 3 e \sqrt{d + e x} (a + b \operatorname{Log}[c x^n]) - \\ \frac{(d + e x)^{3/2} (a + b \operatorname{Log}[c x^n])}{x} - 3 \sqrt{d} e \operatorname{ArcTanh}\left[\frac{\sqrt{d + e x}}{\sqrt{d}}\right] (a + b \operatorname{Log}[c x^n]) - \\ 6 b \sqrt{d} e n \operatorname{ArcTanh}\left[\frac{\sqrt{d + e x}}{\sqrt{d}}\right] \operatorname{Log}\left[\frac{2 \sqrt{d}}{\sqrt{d} - \sqrt{d + e x}}\right] - 3 b \sqrt{d} e n \operatorname{PolyLog}\left[2, 1 - \frac{2 \sqrt{d}}{\sqrt{d} - \sqrt{d + e x}}\right]$$

Result (type 5, 331 leaves):

$$\begin{aligned}
 & - \frac{1}{\sqrt{1 + \frac{d}{e x}} \sqrt{x}} 2 b \sqrt{e} n \sqrt{d+e x} \left(2 \sqrt{e} \sqrt{x} \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right\}, \left\{\frac{1}{2}, \frac{1}{2}\right\}, -\frac{d}{e x}\right] - \right. \\
 & \quad \left. \sqrt{e} \sqrt{1 + \frac{d}{e x}} \sqrt{x} \operatorname{Log}[x] + \sqrt{d} \operatorname{ArcSinh}\left[\frac{\sqrt{d}}{\sqrt{e} \sqrt{x}}\right] \operatorname{Log}[x] \right) - \frac{1}{\sqrt{1 + \frac{d}{e x}} x} \\
 & b \sqrt{d} n \sqrt{d+e x} \left(2 \sqrt{d} \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\}, \left\{\frac{3}{2}, \frac{3}{2}\right\}, -\frac{d}{e x}\right] + \right. \\
 & \quad \left. \left(\sqrt{d} \sqrt{1 + \frac{d}{e x}} + \sqrt{e} \sqrt{x} \operatorname{ArcSinh}\left[\frac{\sqrt{d}}{\sqrt{e} \sqrt{x}}\right] \right) (1 + \operatorname{Log}[x]) \right) - \\
 & \frac{(d - 2 e x) \sqrt{d+e x} (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n])}{x} - \\
 & 3 \sqrt{d} e \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x}}{\sqrt{d}}\right] (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n])
 \end{aligned}$$

Problem 143: Result unnecessarily involves higher level functions.

$$\int \frac{(d+e x)^{3/2} (a+b \operatorname{Log}[c x^n])}{x^3} dx$$

Optimal (type 4, 293 leaves, 16 steps):

$$\begin{aligned}
 & - \frac{b d n \sqrt{d+e x}}{4 x^2} - \frac{11 b e n \sqrt{d+e x}}{8 x} - \frac{9 b e^2 n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x}}{\sqrt{d}}\right]}{8 \sqrt{d}} + \\
 & \frac{3 b e^2 n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x}}{\sqrt{d}}\right]^2}{4 \sqrt{d}} - \frac{3 e \sqrt{d+e x} (a+b \operatorname{Log}[c x^n])}{4 x} - \\
 & \frac{(d+e x)^{3/2} (a+b \operatorname{Log}[c x^n])}{2 x^2} - \frac{3 e^2 \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x}}{\sqrt{d}}\right] (a+b \operatorname{Log}[c x^n])}{4 \sqrt{d}} - \\
 & \frac{3 b e^2 n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x}}{\sqrt{d}}\right] \operatorname{Log}\left[\frac{2 \sqrt{d}}{\sqrt{d}-\sqrt{d+e x}}\right]}{2 \sqrt{d}} - \frac{3 b e^2 n \operatorname{PolyLog}\left[2, 1 - \frac{2 \sqrt{d}}{\sqrt{d}-\sqrt{d+e x}}\right]}{4 \sqrt{d}}
 \end{aligned}$$

Result (type 5, 270 leaves):

$$\frac{1}{36 \sqrt{d} \sqrt{1 + \frac{d}{e x}} x^2} \left(-16 b d^{3/2} n \sqrt{d+e x} \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, \frac{3}{2}, \frac{3}{2}\right\}, \left\{\frac{5}{2}, \frac{5}{2}\right\}, -\frac{d}{e x}\right] - \right.$$

$$9 \left(8 b \sqrt{d} e n x \sqrt{d+e x} \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\}, \left\{\frac{3}{2}, \frac{3}{2}\right\}, -\frac{d}{e x}\right] + \right.$$

$$b e^{3/2} n x^{3/2} \sqrt{d+e x} \operatorname{ArcSinh}\left[\frac{\sqrt{d}}{\sqrt{e} \sqrt{x}}\right] (4+3 \operatorname{Log}[x]) +$$

$$\sqrt{1 + \frac{d}{e x}} \left(3 e^2 x^2 \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x}}{\sqrt{d}}\right] (a-b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n]) + \right.$$

$$\left. \left. \left. \left. \sqrt{d} \sqrt{d+e x} (2 a d+5 a e x+4 b e n x+b (2 d+5 e x) \operatorname{Log}[c x^n]) \right) \right) \right) \right)$$

Problem 148: Result unnecessarily involves higher level functions.

$$\int \frac{a+b \operatorname{Log}[c x^n]}{x \sqrt{d+e x}} dx$$

Optimal (type 4, 152 leaves, 7 steps):

$$\frac{2 b n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x}}{\sqrt{d}}\right]^2}{\sqrt{d}} - \frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x}}{\sqrt{d}}\right] (a+b \operatorname{Log}[c x^n])}{\sqrt{d}} -$$

$$\frac{4 b n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x}}{\sqrt{d}}\right] \operatorname{Log}\left[\frac{2 \sqrt{d}}{\sqrt{d}-\sqrt{d+e x}}\right]}{\sqrt{d}} - \frac{2 b n \operatorname{PolyLog}\left[2, 1-\frac{2 \sqrt{d}}{\sqrt{d}-\sqrt{d+e x}}\right]}{\sqrt{d}}$$

Result (type 5, 132 leaves):

$$\frac{1}{\sqrt{d+e x}} b n \sqrt{1 + \frac{d}{e x}}$$

$$\left(-4 \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\}, \left\{\frac{3}{2}, \frac{3}{2}\right\}, -\frac{d}{e x}\right] - \frac{2 \sqrt{e} \sqrt{x} \operatorname{ArcSinh}\left[\frac{\sqrt{d}}{\sqrt{e} \sqrt{x}}\right] \operatorname{Log}[x]}{\sqrt{d}} \right) -$$

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x}}{\sqrt{d}}\right] (a+b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))}{\sqrt{d}}$$

Problem 149: Result unnecessarily involves higher level functions.

$$\int \frac{a + b \operatorname{Log}[c x^n]}{x^2 \sqrt{d + e x}} dx$$

Optimal (type 4, 226 leaves, 11 steps):

$$\begin{aligned} & -\frac{b n \sqrt{d + e x}}{d x} - \frac{b e n \operatorname{ArcTanh}\left[\frac{\sqrt{d + e x}}{\sqrt{d}}\right]}{d^{3/2}} - \frac{b e n \operatorname{ArcTanh}\left[\frac{\sqrt{d + e x}}{\sqrt{d}}\right]^2}{d^{3/2}} - \\ & \frac{\sqrt{d + e x} (a + b \operatorname{Log}[c x^n])}{d x} + \frac{e \operatorname{ArcTanh}\left[\frac{\sqrt{d + e x}}{\sqrt{d}}\right] (a + b \operatorname{Log}[c x^n])}{d^{3/2}} + \\ & \frac{2 b e n \operatorname{ArcTanh}\left[\frac{\sqrt{d + e x}}{\sqrt{d}}\right] \operatorname{Log}\left[\frac{2 \sqrt{d}}{\sqrt{d} - \sqrt{d + e x}}\right]}{d^{3/2}} + \frac{b e n \operatorname{PolyLog}\left[2, 1 - \frac{2 \sqrt{d}}{\sqrt{d} - \sqrt{d + e x}}\right]}{d^{3/2}} \end{aligned}$$

Result (type 5, 191 leaves):

$$\begin{aligned} & \frac{1}{9 d^{3/2} x \sqrt{d + e x}} \left(2 b d^{3/2} n \sqrt{1 + \frac{d}{e x}} \operatorname{HypergeometricPFQ}\left[\left\{\frac{3}{2}, \frac{3}{2}, \frac{3}{2}\right\}, \left\{\frac{5}{2}, \frac{5}{2}\right\}, -\frac{d}{e x}\right] + \right. \\ & 9 b e^{3/2} n \sqrt{1 + \frac{d}{e x}} x^{3/2} \operatorname{ArcSinh}\left[\frac{\sqrt{d}}{\sqrt{e} \sqrt{x}}\right] (1 + \operatorname{Log}[x]) - 9 \sqrt{d} (d + e x) (a + b n + b \operatorname{Log}[c x^n]) + \\ & \left. 9 e x \sqrt{d + e x} \operatorname{ArcTanh}\left[\frac{\sqrt{d + e x}}{\sqrt{d}}\right] (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n]) \right) \end{aligned}$$

Problem 150: Result unnecessarily involves higher level functions.

$$\int \frac{a + b \operatorname{Log}[c x^n]}{x^3 \sqrt{d + e x}} dx$$

Optimal (type 4, 304 leaves, 16 steps):

$$\begin{aligned} & -\frac{b n \sqrt{d + e x}}{4 d x^2} + \frac{5 b e n \sqrt{d + e x}}{8 d^2 x} + \frac{7 b e^2 n \operatorname{ArcTanh}\left[\frac{\sqrt{d + e x}}{\sqrt{d}}\right]}{8 d^{5/2}} + \\ & \frac{3 b e^2 n \operatorname{ArcTanh}\left[\frac{\sqrt{d + e x}}{\sqrt{d}}\right]^2}{4 d^{5/2}} - \frac{\sqrt{d + e x} (a + b \operatorname{Log}[c x^n])}{2 d x^2} + \\ & \frac{3 e \sqrt{d + e x} (a + b \operatorname{Log}[c x^n])}{4 d^2 x} - \frac{3 e^2 \operatorname{ArcTanh}\left[\frac{\sqrt{d + e x}}{\sqrt{d}}\right] (a + b \operatorname{Log}[c x^n])}{4 d^{5/2}} - \\ & \frac{3 b e^2 n \operatorname{ArcTanh}\left[\frac{\sqrt{d + e x}}{\sqrt{d}}\right] \operatorname{Log}\left[\frac{2 \sqrt{d}}{\sqrt{d} - \sqrt{d + e x}}\right]}{2 d^{5/2}} - \frac{3 b e^2 n \operatorname{PolyLog}\left[2, 1 - \frac{2 \sqrt{d}}{\sqrt{d} - \sqrt{d + e x}}\right]}{4 d^{5/2}} \end{aligned}$$

Result (type 5, 206 leaves):

$$\left(-16 b d^{5/2} n \sqrt{1 + \frac{d}{e x}} \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{2}, \frac{5}{2}, \frac{5}{2}\right\}, \left\{\frac{7}{2}, \frac{7}{2}\right\}, -\frac{d}{e x}\right] - \right. \\ \left. 25 \left(3 b e^{5/2} n \sqrt{1 + \frac{d}{e x}} x^{5/2} \operatorname{ArcSinh}\left[\frac{\sqrt{d}}{\sqrt{e} \sqrt{x}}\right] \operatorname{Log}[x] + \right. \right. \\ \left. \left. \sqrt{d} (2 d^2 - d e x - 3 e^2 x^2) (a + b \operatorname{Log}[c x^n]) + \right. \right. \\ \left. \left. 3 e^2 x^2 \sqrt{d + e x} \operatorname{ArcTanh}\left[\frac{\sqrt{d + e x}}{\sqrt{d}}\right] (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n]) \right) \right) / (100 d^{5/2} x^2 \sqrt{d + e x})$$

Problem 155: Result unnecessarily involves higher level functions.

$$\int \frac{a + b \operatorname{Log}[c x^n]}{x (d + e x)^{3/2}} dx$$

Optimal (type 4, 201 leaves, 11 steps):

$$\frac{4 b n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x}}{\sqrt{d}}\right]}{d^{3/2}} + \frac{2 b n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x}}{\sqrt{d}}\right]^2}{d^{3/2}} + \\ \frac{2 (a + b \operatorname{Log}[c x^n])}{d \sqrt{d + e x}} - \frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x}}{\sqrt{d}}\right] (a + b \operatorname{Log}[c x^n])}{d^{3/2}} - \\ \frac{4 b n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x}}{\sqrt{d}}\right] \operatorname{Log}\left[\frac{2 \sqrt{d}}{\sqrt{d} - \sqrt{d+e x}}\right]}{d^{3/2}} - \frac{2 b n \operatorname{PolyLog}\left[2, 1 - \frac{2 \sqrt{d}}{\sqrt{d} - \sqrt{d+e x}}\right]}{d^{3/2}}$$

Result (type 5, 185 leaves):

$$\left(\left(\left(2 \left(2 b d^{3/2} n \sqrt{1 + \frac{d}{e x}} \operatorname{HypergeometricPFQ}\left[\left\{\frac{3}{2}, \frac{3}{2}, \frac{3}{2}\right\}, \left\{\frac{5}{2}, \frac{5}{2}\right\}, -\frac{d}{e x}\right] + \right. \right. \right. \right. \\ \left. \left. 9 e x \left(b \sqrt{e} n \sqrt{1 + \frac{d}{e x}} \sqrt{x} \operatorname{ArcSinh}\left[\frac{\sqrt{d}}{\sqrt{e} \sqrt{x}}\right] \operatorname{Log}[x] - \sqrt{d} (a + b \operatorname{Log}[c x^n]) + \right. \right. \right. \right. \\ \left. \left. \left. \left. \left. \left. \sqrt{d + e x} \operatorname{ArcTanh}\left[\frac{\sqrt{d + e x}}{\sqrt{d}}\right] (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n]) \right) \right) \right) \right) \right) / (9 d^{3/2} e x \sqrt{d + e x}) \right)$$

Problem 156: Result unnecessarily involves higher level functions.

$$\int \frac{a + b \operatorname{Log}[c x^n]}{x^2 (d + e x)^{3/2}} dx$$

Optimal (type 4, 253 leaves, 15 steps):

$$\begin{aligned}
 & -\frac{bn\sqrt{d+ex}}{d^2x} - \frac{5ben\operatorname{ArcTanh}\left[\frac{\sqrt{d+ex}}{\sqrt{d}}\right]}{d^{5/2}} - \frac{3ben\operatorname{ArcTanh}\left[\frac{\sqrt{d+ex}}{\sqrt{d}}\right]^2}{d^{5/2}} - \\
 & \frac{3e(a+b\operatorname{Log}[cx^n])}{d^2\sqrt{d+ex}} - \frac{a+b\operatorname{Log}[cx^n]}{dx\sqrt{d+ex}} + \frac{3e\operatorname{ArcTanh}\left[\frac{\sqrt{d+ex}}{\sqrt{d}}\right](a+b\operatorname{Log}[cx^n])}{d^{5/2}} + \\
 & \frac{6ben\operatorname{ArcTanh}\left[\frac{\sqrt{d+ex}}{\sqrt{d}}\right]\operatorname{Log}\left[\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right]}{d^{5/2}} + \frac{3ben\operatorname{PolyLog}\left[2, 1-\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+ex}}\right]}{d^{5/2}}
 \end{aligned}$$

Result (type 5, 186 leaves):

$$\begin{aligned}
 & \left(6bd^{5/2}n\sqrt{1+\frac{d}{ex}}\operatorname{HypergeometricPFQ}\left[\left\{\frac{5}{2}, \frac{5}{2}, \frac{5}{2}\right\}, \left\{\frac{7}{2}, \frac{7}{2}\right\}, -\frac{d}{ex}\right] - \right. \\
 & \left. 5\left(2bd^{5/2}n\sqrt{1+\frac{d}{ex}}\operatorname{Hypergeometric2F1}\left[\frac{3}{2}, \frac{5}{2}, \frac{7}{2}, -\frac{d}{ex}\right](1+\operatorname{Log}[x]) + \right. \right. \\
 & \left. \left. 5ex\left(\sqrt{d}(d+3ex) - 3ex\sqrt{d+ex}\operatorname{ArcTanh}\left[\frac{\sqrt{d+ex}}{\sqrt{d}}\right]\right) \right. \right. \\
 & \left. \left. (a-bn\operatorname{Log}[x] + b\operatorname{Log}[cx^n]) \right) \right) / (25d^{5/2}ex^2\sqrt{d+ex})
 \end{aligned}$$

Problem 210: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^5(a+b\operatorname{Log}[cx^n])}{d+ex^2} dx$$

Optimal (type 4, 121 leaves, 6 steps):

$$\begin{aligned}
 & \frac{bdnx^2}{4e^2} - \frac{bnx^4}{16e} - \frac{dx^2(a+b\operatorname{Log}[cx^n])}{2e^2} + \frac{x^4(a+b\operatorname{Log}[cx^n])}{4e} + \\
 & \frac{d^2(a+b\operatorname{Log}[cx^n])\operatorname{Log}\left[1+\frac{ex^2}{d}\right]}{2e^3} + \frac{bd^2n\operatorname{PolyLog}\left[2, -\frac{ex^2}{d}\right]}{4e^3}
 \end{aligned}$$

Result (type 4, 226 leaves):

$$\frac{1}{16 e^3} \left(-8 a d e x^2 + 4 b d e n x^2 + 4 a e^2 x^4 - b e^2 n x^4 - 8 b d e x^2 \operatorname{Log}[c x^n] + \right. \\ \left. 4 b e^2 x^4 \operatorname{Log}[c x^n] + 8 b d^2 n \operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{i \sqrt{e} x}{\sqrt{d}}\right] + 8 b d^2 n \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{i \sqrt{e} x}{\sqrt{d}}\right] + \right. \\ \left. 8 a d^2 \operatorname{Log}[d + e x^2] - 8 b d^2 n \operatorname{Log}[x] \operatorname{Log}[d + e x^2] + 8 b d^2 \operatorname{Log}[c x^n] \operatorname{Log}[d + e x^2] + \right. \\ \left. 8 b d^2 n \operatorname{PolyLog}\left[2, -\frac{i \sqrt{e} x}{\sqrt{d}}\right] + 8 b d^2 n \operatorname{PolyLog}\left[2, \frac{i \sqrt{e} x}{\sqrt{d}}\right] \right)$$

Problem 211: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{x^3 (a + b \operatorname{Log}[c x^n])}{d + e x^2} dx$$

Optimal (type 4, 83 leaves, 5 steps):

$$\frac{b n x^2}{4 e} + \frac{x^2 (a + b \operatorname{Log}[c x^n])}{2 e} - \frac{d (a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x^2}{d}\right]}{2 e^2} - \frac{b d n \operatorname{PolyLog}\left[2, -\frac{e x^2}{d}\right]}{4 e^2}$$

Result (type 4, 174 leaves):

$$-\frac{1}{4 e^2} \left(-2 a e x^2 + b e n x^2 - 2 b e x^2 \operatorname{Log}[c x^n] + 2 b d n \operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{i \sqrt{e} x}{\sqrt{d}}\right] + \right. \\ \left. 2 b d n \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{i \sqrt{e} x}{\sqrt{d}}\right] + 2 a d \operatorname{Log}[d + e x^2] - 2 b d n \operatorname{Log}[x] \operatorname{Log}[d + e x^2] + \right. \\ \left. 2 b d \operatorname{Log}[c x^n] \operatorname{Log}[d + e x^2] + 2 b d n \operatorname{PolyLog}\left[2, -\frac{i \sqrt{e} x}{\sqrt{d}}\right] + 2 b d n \operatorname{PolyLog}\left[2, \frac{i \sqrt{e} x}{\sqrt{d}}\right] \right)$$

Problem 212: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{x (a + b \operatorname{Log}[c x^n])}{d + e x^2} dx$$

Optimal (type 4, 49 leaves, 2 steps):

$$\frac{(a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x^2}{d}\right]}{2 e} + \frac{b n \operatorname{PolyLog}\left[2, -\frac{e x^2}{d}\right]}{4 e}$$

Result (type 4, 111 leaves):

$$\frac{1}{2 e} \left((a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n]) \operatorname{Log}[d + e x^2] + b n \right. \\ \left. \left(\operatorname{Log}[x] \left(\operatorname{Log}\left[1 - \frac{i \sqrt{e} x}{\sqrt{d}}\right] + \operatorname{Log}\left[1 + \frac{i \sqrt{e} x}{\sqrt{d}}\right] \right) + \operatorname{PolyLog}\left[2, -\frac{i \sqrt{e} x}{\sqrt{d}}\right] + \operatorname{PolyLog}\left[2, \frac{i \sqrt{e} x}{\sqrt{d}}\right] \right) \right)$$

Problem 213: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{a + b \operatorname{Log}[c x^n]}{x (d + e x^2)} dx$$

Optimal (type 4, 49 leaves, 2 steps):

$$-\frac{\operatorname{Log}\left[1 + \frac{d}{e x^2}\right] (a + b \operatorname{Log}[c x^n])}{2 d} + \frac{b n \operatorname{PolyLog}\left[2, -\frac{d}{e x^2}\right]}{4 d}$$

Result (type 4, 157 leaves):

$$\begin{aligned} & -\frac{1}{2 d} \left(-2 a \operatorname{Log}[x] + b n \operatorname{Log}[x]^2 - 2 b \operatorname{Log}[x] \operatorname{Log}[c x^n] + b n \operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{i \sqrt{e} x}{\sqrt{d}}\right] + \right. \\ & \quad \left. b n \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{i \sqrt{e} x}{\sqrt{d}}\right] + a \operatorname{Log}[d + e x^2] - b n \operatorname{Log}[x] \operatorname{Log}[d + e x^2] + \right. \\ & \quad \left. b \operatorname{Log}[c x^n] \operatorname{Log}[d + e x^2] + b n \operatorname{PolyLog}\left[2, -\frac{i \sqrt{e} x}{\sqrt{d}}\right] + b n \operatorname{PolyLog}\left[2, \frac{i \sqrt{e} x}{\sqrt{d}}\right] \right) \end{aligned}$$

Problem 214: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{a + b \operatorname{Log}[c x^n]}{x^3 (d + e x^2)} dx$$

Optimal (type 4, 83 leaves, 4 steps):

$$-\frac{b n}{4 d x^2} - \frac{a + b \operatorname{Log}[c x^n]}{2 d x^2} + \frac{e \operatorname{Log}\left[1 + \frac{d}{e x^2}\right] (a + b \operatorname{Log}[c x^n])}{2 d^2} - \frac{b e n \operatorname{PolyLog}\left[2, -\frac{d}{e x^2}\right]}{4 d^2}$$

Result (type 4, 217 leaves):

$$\begin{aligned} & -\frac{a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])}{2 d x^2} - \frac{e \operatorname{Log}[x] (a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))}{d^2} + \\ & \quad \frac{e (a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])) \operatorname{Log}[d + e x^2]}{2 d^2} + \\ & \quad b n \left(-\frac{e \operatorname{Log}[x]^2}{2 d^2} + \frac{-\frac{1}{4 x^2} - \frac{\operatorname{Log}[x]}{2 x^2}}{d} + \frac{e \left(\operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{i \sqrt{e} x}{\sqrt{d}}\right] + \operatorname{PolyLog}\left[2, -\frac{i \sqrt{e} x}{\sqrt{d}}\right] \right)}{2 d^2} + \right. \\ & \quad \left. \frac{e \left(\operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{i \sqrt{e} x}{\sqrt{d}}\right] + \operatorname{PolyLog}\left[2, \frac{i \sqrt{e} x}{\sqrt{d}}\right] \right)}{2 d^2} \right) \end{aligned}$$

Problem 215: Result unnecessarily involves complex numbers and more than

twice size of optimal antiderivative.

$$\int \frac{a + b \operatorname{Log}[c x^n]}{x^5 (d + e x^2)} dx$$

Optimal (type 4, 121 leaves, 6 steps):

$$-\frac{b n}{16 d x^4} + \frac{b e n}{4 d^2 x^2} - \frac{a + b \operatorname{Log}[c x^n]}{4 d x^4} + \frac{e (a + b \operatorname{Log}[c x^n])}{2 d^2 x^2} - \frac{e^2 \operatorname{Log}\left[1 + \frac{d}{e x^2}\right] (a + b \operatorname{Log}[c x^n])}{2 d^3} + \frac{b e^2 n \operatorname{PolyLog}\left[2, -\frac{d}{e x^2}\right]}{4 d^3}$$

Result (type 4, 276 leaves):

$$\begin{aligned} & -\frac{a - b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])}{4 d x^4} + \frac{e (a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))}{2 d^2 x^2} + \frac{e^2 \operatorname{Log}[x] (a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))}{d^3} - \\ & \frac{e^2 (a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])) \operatorname{Log}[d + e x^2]}{2 d^3} + b n \left(\frac{e^2 \operatorname{Log}[x]^2}{2 d^3} + \frac{-\frac{1}{16 x^4} - \frac{\operatorname{Log}[x]}{4 x^4}}{d} \right) - \\ & \frac{e \left(-\frac{1}{4 x^2} - \frac{\operatorname{Log}[x]}{2 x^2} \right)}{d^2} - \frac{e^2 \left(\operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{i \sqrt{e} x}{\sqrt{d}}\right] + \operatorname{PolyLog}\left[2, -\frac{i \sqrt{e} x}{\sqrt{d}}\right] \right)}{2 d^3} - \\ & \left. \frac{e^2 \left(\operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{i \sqrt{e} x}{\sqrt{d}}\right] + \operatorname{PolyLog}\left[2, \frac{i \sqrt{e} x}{\sqrt{d}}\right] \right)}{2 d^3} \right) \end{aligned}$$

Problem 221: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{x^5 (a + b \operatorname{Log}[c x^n])}{(d + e x^2)^2} dx$$

Optimal (type 4, 129 leaves, 7 steps):

$$-\frac{b n x^2}{4 e^2} + \frac{x^2 (a + b \operatorname{Log}[c x^n])}{2 e^2} + \frac{d x^2 (a + b \operatorname{Log}[c x^n])}{2 e^2 (d + e x^2)} - \frac{b d n \operatorname{Log}[d + e x^2]}{4 e^3} - \frac{d (a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x^2}{d}\right]}{e^3} - \frac{b d n \operatorname{PolyLog}\left[2, -\frac{e x^2}{d}\right]}{2 e^3}$$

Result (type 4, 426 leaves):

$$\begin{aligned}
 & -\frac{1}{4 e^3 (d+e x^2)} \left(2 a d^2 - 2 a d e x^2 + b d e n x^2 - 2 a e^2 x^4 + b e^2 n x^4 - 2 b d^2 n \operatorname{Log}[x] - \right. \\
 & \quad 2 b d e n x^2 \operatorname{Log}[x] + 2 b d^2 \operatorname{Log}[c x^n] - 2 b d e x^2 \operatorname{Log}[c x^n] - 2 b e^2 x^4 \operatorname{Log}[c x^n] + \\
 & \quad 4 b d^2 n \operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{i \sqrt{e} x}{\sqrt{d}}\right] + 4 b d e n x^2 \operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{i \sqrt{e} x}{\sqrt{d}}\right] + \\
 & \quad 4 b d^2 n \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{i \sqrt{e} x}{\sqrt{d}}\right] + 4 b d e n x^2 \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{i \sqrt{e} x}{\sqrt{d}}\right] + \\
 & \quad 4 a d^2 \operatorname{Log}[d+e x^2] + b d^2 n \operatorname{Log}[d+e x^2] + 4 a d e x^2 \operatorname{Log}[d+e x^2] + \\
 & \quad b d e n x^2 \operatorname{Log}[d+e x^2] - 4 b d^2 n \operatorname{Log}[x] \operatorname{Log}[d+e x^2] - 4 b d e n x^2 \operatorname{Log}[x] \operatorname{Log}[d+e x^2] + \\
 & \quad 4 b d^2 \operatorname{Log}[c x^n] \operatorname{Log}[d+e x^2] + 4 b d e x^2 \operatorname{Log}[c x^n] \operatorname{Log}[d+e x^2] + \\
 & \quad \left. 4 b d n (d+e x^2) \operatorname{PolyLog}\left[2, -\frac{i \sqrt{e} x}{\sqrt{d}}\right] + 4 b d n (d+e x^2) \operatorname{PolyLog}\left[2, \frac{i \sqrt{e} x}{\sqrt{d}}\right] \right)
 \end{aligned}$$

Problem 222: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{x^3 (a + b \operatorname{Log}[c x^n])}{(d + e x^2)^2} dx$$

Optimal (type 4, 95 leaves, 6 steps):

$$-\frac{x^2 (a + b \operatorname{Log}[c x^n])}{2 e (d + e x^2)} + \frac{b n \operatorname{Log}[d + e x^2]}{4 e^2} + \frac{(a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x^2}{d}\right]}{2 e^2} + \frac{b n \operatorname{PolyLog}\left[2, -\frac{e x^2}{d}\right]}{4 e^2}$$

Result (type 4, 336 leaves):

$$\begin{aligned}
 & \frac{1}{4 e^2 (d + e x^2)} \\
 & \left(2 a d - 2 b d n \operatorname{Log}[x] - 2 b e n x^2 \operatorname{Log}[x] + 2 b d \operatorname{Log}[c x^n] + 2 b d n \operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{i \sqrt{e} x}{\sqrt{d}}\right] + \right. \\
 & \quad 2 b e n x^2 \operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{i \sqrt{e} x}{\sqrt{d}}\right] + 2 b d n \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{i \sqrt{e} x}{\sqrt{d}}\right] + \\
 & \quad 2 b e n x^2 \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{i \sqrt{e} x}{\sqrt{d}}\right] + 2 a d \operatorname{Log}[d + e x^2] + b d n \operatorname{Log}[d + e x^2] + \\
 & \quad 2 a e x^2 \operatorname{Log}[d + e x^2] + b e n x^2 \operatorname{Log}[d + e x^2] - 2 b d n \operatorname{Log}[x] \operatorname{Log}[d + e x^2] - \\
 & \quad 2 b e n x^2 \operatorname{Log}[x] \operatorname{Log}[d + e x^2] + 2 b d \operatorname{Log}[c x^n] \operatorname{Log}[d + e x^2] + 2 b e x^2 \operatorname{Log}[c x^n] \operatorname{Log}[d + e x^2] + \\
 & \quad \left. 2 b n (d + e x^2) \operatorname{PolyLog}\left[2, -\frac{i \sqrt{e} x}{\sqrt{d}}\right] + 2 b n (d + e x^2) \operatorname{PolyLog}\left[2, \frac{i \sqrt{e} x}{\sqrt{d}}\right] \right)
 \end{aligned}$$

Problem 224: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{a + b \operatorname{Log}[c x^n]}{x (d + e x^2)^2} dx$$

Optimal (type 4, 82 leaves, 3 steps):

$$\frac{a + b \operatorname{Log}[c x^n]}{2 d (d + e x^2)} - \frac{\operatorname{Log}\left[1 + \frac{d}{e x^2}\right] (2 a - b n + 2 b \operatorname{Log}[c x^n])}{4 d^2} + \frac{b n \operatorname{PolyLog}\left[2, -\frac{d}{e x^2}\right]}{4 d^2}$$

Result (type 4, 401 leaves):

$$\begin{aligned} & -\frac{1}{4 d^2 (d + e x^2)} \\ & \left(-2 a d - 4 a d \operatorname{Log}[x] + 2 b d n \operatorname{Log}[x] - 4 a e x^2 \operatorname{Log}[x] + 2 b e n x^2 \operatorname{Log}[x] + 2 b d n \operatorname{Log}[x]^2 + \right. \\ & \quad 2 b e n x^2 \operatorname{Log}[x]^2 - 2 b d \operatorname{Log}[c x^n] - 4 b d \operatorname{Log}[x] \operatorname{Log}[c x^n] - 4 b e x^2 \operatorname{Log}[x] \operatorname{Log}[c x^n] + \\ & \quad 2 b d n \operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{i \sqrt{e} x}{\sqrt{d}}\right] + 2 b e n x^2 \operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{i \sqrt{e} x}{\sqrt{d}}\right] + \\ & \quad 2 b d n \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{i \sqrt{e} x}{\sqrt{d}}\right] + 2 b e n x^2 \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{i \sqrt{e} x}{\sqrt{d}}\right] + 2 a d \operatorname{Log}[d + e x^2] - \\ & \quad b d n \operatorname{Log}[d + e x^2] + 2 a e x^2 \operatorname{Log}[d + e x^2] - b e n x^2 \operatorname{Log}[d + e x^2] - 2 b d n \operatorname{Log}[x] \operatorname{Log}[d + e x^2] - \\ & \quad 2 b e n x^2 \operatorname{Log}[x] \operatorname{Log}[d + e x^2] + 2 b d \operatorname{Log}[c x^n] \operatorname{Log}[d + e x^2] + 2 b e x^2 \operatorname{Log}[c x^n] \operatorname{Log}[d + e x^2] + \\ & \quad \left. 2 b n (d + e x^2) \operatorname{PolyLog}\left[2, -\frac{i \sqrt{e} x}{\sqrt{d}}\right] + 2 b n (d + e x^2) \operatorname{PolyLog}\left[2, \frac{i \sqrt{e} x}{\sqrt{d}}\right] \right) \end{aligned}$$

Problem 225: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{a + b \operatorname{Log}[c x^n]}{x^3 (d + e x^2)^2} dx$$

Optimal (type 4, 126 leaves, 5 steps):

$$\begin{aligned} & -\frac{b n}{2 d^2 x^2} + \frac{a + b \operatorname{Log}[c x^n]}{2 d x^2 (d + e x^2)} - \frac{4 a - b n + 4 b \operatorname{Log}[c x^n]}{4 d^2 x^2} + \\ & \frac{e \operatorname{Log}\left[1 + \frac{d}{e x^2}\right] (4 a - b n + 4 b \operatorname{Log}[c x^n])}{4 d^3} - \frac{b e n \operatorname{PolyLog}\left[2, -\frac{d}{e x^2}\right]}{2 d^3} \end{aligned}$$

Result (type 4, 337 leaves):

$$\begin{aligned}
 & -\frac{1}{4 d^3} \\
 & \left(\frac{2 a d}{x^2} + \frac{b d n}{x^2} + \frac{2 a d e}{d+e x^2} + 8 a e \operatorname{Log}[x] - 2 b e n \operatorname{Log}[x] - \frac{i b \sqrt{d} e n \operatorname{Log}[x]}{-i \sqrt{d} + \sqrt{e} x} + \frac{i b \sqrt{d} e n \operatorname{Log}[x]}{i \sqrt{d} + \sqrt{e} x} - \right. \\
 & \quad \frac{2 b d e n \operatorname{Log}[x]}{d+e x^2} - 4 b e n \operatorname{Log}[x]^2 + \frac{2 b d \operatorname{Log}[c x^n]}{x^2} + \frac{2 b d e \operatorname{Log}[c x^n]}{d+e x^2} + 8 b e \operatorname{Log}[x] \operatorname{Log}[c x^n] - \\
 & \quad 4 b e n \operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{i \sqrt{e} x}{\sqrt{d}}\right] - 4 b e n \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{i \sqrt{e} x}{\sqrt{d}}\right] - 4 a e \operatorname{Log}[d+e x^2] + \\
 & \quad b e n \operatorname{Log}[d+e x^2] + 4 b e n \operatorname{Log}[x] \operatorname{Log}[d+e x^2] - 4 b e \operatorname{Log}[c x^n] \operatorname{Log}[d+e x^2] - \\
 & \quad \left. 4 b e n \operatorname{PolyLog}\left[2, -\frac{i \sqrt{e} x}{\sqrt{d}}\right] - 4 b e n \operatorname{PolyLog}\left[2, \frac{i \sqrt{e} x}{\sqrt{d}}\right] \right)
 \end{aligned}$$

Problem 227: Result more than twice size of optimal antiderivative.

$$\int \frac{x^2 (a + b \operatorname{Log}[c x^n])}{(d + e x^2)^2} dx$$

Optimal (type 4, 164 leaves, 14 steps):

$$\begin{aligned}
 & \frac{b n \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]}{2 \sqrt{d} e^{3/2}} - \frac{x (a + b \operatorname{Log}[c x^n])}{2 e (d + e x^2)} + \frac{\operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] (a + b \operatorname{Log}[c x^n])}{2 \sqrt{d} e^{3/2}} - \\
 & \frac{i b n \operatorname{PolyLog}\left[2, -\frac{i \sqrt{e} x}{\sqrt{d}}\right]}{4 \sqrt{d} e^{3/2}} + \frac{i b n \operatorname{PolyLog}\left[2, \frac{i \sqrt{e} x}{\sqrt{d}}\right]}{4 \sqrt{d} e^{3/2}}
 \end{aligned}$$

Result (type 4, 391 leaves):

$$\begin{aligned}
 & - \frac{x (a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))}{2 e (d + e x^2)} + \frac{\operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] (a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))}{2 \sqrt{d} e^{3/2}} + \\
 & b n \left(\frac{\frac{\operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]}{\sqrt{d} \sqrt{e}} + \frac{i \operatorname{Log}[x]}{\sqrt{d} \sqrt{e}} - \frac{\operatorname{Log}[x]}{\sqrt{e} (-i \sqrt{d} + \sqrt{e} x)} - \frac{i \operatorname{Log}[d+e x^2]}{2 \sqrt{d} \sqrt{e}}}{4 e} + \right. \\
 & \frac{\operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]}{\sqrt{d} \sqrt{e}} - \frac{i \operatorname{Log}[x]}{\sqrt{d} \sqrt{e}} - \frac{\operatorname{Log}[x]}{\sqrt{e} (i \sqrt{d} + \sqrt{e} x)} + \frac{i \operatorname{Log}[d+e x^2]}{2 \sqrt{d} \sqrt{e}}}{4 e} - \\
 & \left. \frac{i \left(\operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{i \sqrt{e} x}{\sqrt{d}}\right] + \operatorname{PolyLog}\left[2, -\frac{i \sqrt{e} x}{\sqrt{d}}\right]\right)}{4 \sqrt{d} e^{3/2}} + \right. \\
 & \left. \frac{i \left(\operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{i \sqrt{e} x}{\sqrt{d}}\right] + \operatorname{PolyLog}\left[2, \frac{i \sqrt{e} x}{\sqrt{d}}\right]\right)}{4 \sqrt{d} e^{3/2}} \right)
 \end{aligned}$$

Problem 228: Result more than twice size of optimal antiderivative.

$$\int \frac{a + b \operatorname{Log}[c x^n]}{(d + e x^2)^2} dx$$

Optimal (type 4, 164 leaves, 7 steps):

$$\begin{aligned}
 & - \frac{b n \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]}{2 d^{3/2} \sqrt{e}} + \frac{x (a + b \operatorname{Log}[c x^n])}{2 d (d + e x^2)} + \frac{\operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] (a + b \operatorname{Log}[c x^n])}{2 d^{3/2} \sqrt{e}} - \\
 & \frac{i b n \operatorname{PolyLog}\left[2, -\frac{i \sqrt{e} x}{\sqrt{d}}\right]}{4 d^{3/2} \sqrt{e}} + \frac{i b n \operatorname{PolyLog}\left[2, \frac{i \sqrt{e} x}{\sqrt{d}}\right]}{4 d^{3/2} \sqrt{e}}
 \end{aligned}$$

Result (type 4, 391 leaves):

$$\begin{aligned}
 & \frac{x (a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))}{2 d (d + e x^2)} + \frac{\operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] (a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))}{2 d^{3/2} \sqrt{e}} + \\
 & b n \left(- \frac{\frac{\operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]}{\sqrt{d} \sqrt{e}} + \frac{i \operatorname{Log}[x]}{\sqrt{d} \sqrt{e}} - \frac{\operatorname{Log}[x]}{\sqrt{e} (-i \sqrt{d} + \sqrt{e} x)} - \frac{i \operatorname{Log}[d+e x^2]}{2 \sqrt{d} \sqrt{e}}}{4 d} - \right. \\
 & \frac{\operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]}{\sqrt{d} \sqrt{e}} - \frac{i \operatorname{Log}[x]}{\sqrt{d} \sqrt{e}} - \frac{\operatorname{Log}[x]}{\sqrt{e} (i \sqrt{d} + \sqrt{e} x)} + \frac{i \operatorname{Log}[d+e x^2]}{2 \sqrt{d} \sqrt{e}}}{4 d} - \\
 & \left. \frac{i \left(\operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{i \sqrt{e} x}{\sqrt{d}}\right] + \operatorname{PolyLog}\left[2, -\frac{i \sqrt{e} x}{\sqrt{d}}\right]\right)}{4 d^{3/2} \sqrt{e}} + \right. \\
 & \left. \frac{i \left(\operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{i \sqrt{e} x}{\sqrt{d}}\right] + \operatorname{PolyLog}\left[2, \frac{i \sqrt{e} x}{\sqrt{d}}\right]\right)}{4 d^{3/2} \sqrt{e}} \right)
 \end{aligned}$$

Problem 229: Result more than twice size of optimal antiderivative.

$$\int \frac{a + b \operatorname{Log}[c x^n]}{x^2 (d + e x^2)^2} dx$$

Optimal (type 4, 183 leaves, 8 steps):

$$\begin{aligned}
 & -\frac{3 b n}{2 d^2 x} + \frac{a + b \operatorname{Log}[c x^n]}{2 d x (d + e x^2)} - \frac{3 a - b n + 3 b \operatorname{Log}[c x^n]}{2 d^2 x} - \frac{\sqrt{e} \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] (3 a - b n + 3 b \operatorname{Log}[c x^n])}{2 d^{5/2}} + \\
 & \frac{3 i b \sqrt{e} n \operatorname{PolyLog}\left[2, -\frac{i \sqrt{e} x}{\sqrt{d}}\right]}{4 d^{5/2}} - \frac{3 i b \sqrt{e} n \operatorname{PolyLog}\left[2, \frac{i \sqrt{e} x}{\sqrt{d}}\right]}{4 d^{5/2}}
 \end{aligned}$$

Result (type 4, 398 leaves):

$$\frac{1}{4 d^{5/2}} \left(-\frac{4 a \sqrt{d}}{x} - \frac{4 b \sqrt{d} n}{x} - \frac{2 a \sqrt{d} e x}{d+e x^2} - \right.$$

$$6 a \sqrt{e} \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] + 2 b \sqrt{e} n \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] + \frac{b \sqrt{d} \sqrt{e} n \operatorname{Log}[x]}{i \sqrt{d} - \sqrt{e} x} -$$

$$\frac{b \sqrt{d} \sqrt{e} n \operatorname{Log}[x]}{i \sqrt{d} + \sqrt{e} x} + \frac{2 b \sqrt{d} e n x \operatorname{Log}[x]}{d+e x^2} + 6 b \sqrt{e} n \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] \operatorname{Log}[x] -$$

$$\frac{4 b \sqrt{d} \operatorname{Log}[c x^n]}{x} - \frac{2 b \sqrt{d} e x \operatorname{Log}[c x^n]}{d+e x^2} - 6 b \sqrt{e} \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] \operatorname{Log}[c x^n] -$$

$$3 i b \sqrt{e} n \operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{i \sqrt{e} x}{\sqrt{d}}\right] + 3 i b \sqrt{e} n \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{i \sqrt{e} x}{\sqrt{d}}\right] +$$

$$\left. 3 i b \sqrt{e} n \operatorname{PolyLog}\left[2, -\frac{i \sqrt{e} x}{\sqrt{d}}\right] - 3 i b \sqrt{e} n \operatorname{PolyLog}\left[2, \frac{i \sqrt{e} x}{\sqrt{d}}\right] \right)$$

Problem 231: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{x^5 (a + b \operatorname{Log}[c x^n])}{(d + e x^2)^3} dx$$

Optimal (type 4, 152 leaves, 10 steps):

$$\frac{b d n}{8 e^3 (d + e x^2)} + \frac{b n \operatorname{Log}[x]}{4 e^3} - \frac{d^2 (a + b \operatorname{Log}[c x^n])}{4 e^3 (d + e x^2)^2} - \frac{x^2 (a + b \operatorname{Log}[c x^n])}{e^2 (d + e x^2)} +$$

$$\frac{3 b n \operatorname{Log}[d + e x^2]}{8 e^3} + \frac{(a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x^2}{d}\right]}{2 e^3} + \frac{b n \operatorname{PolyLog}\left[2, -\frac{e x^2}{d}\right]}{4 e^3}$$

Result (type 4, 553 leaves):

$$\frac{1}{8 e^3 (d+e x^2)^2} \left(6 a d^2 + b d^2 n + 8 a d e x^2 + b d e n x^2 - 6 b d^2 n \operatorname{Log}[x] - 12 b d e n x^2 \operatorname{Log}[x] - \right.$$

$$6 b e^2 n x^4 \operatorname{Log}[x] + 6 b d^2 \operatorname{Log}[c x^n] + 8 b d e x^2 \operatorname{Log}[c x^n] + 4 b d^2 n \operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{i \sqrt{e} x}{\sqrt{d}}\right] +$$

$$8 b d e n x^2 \operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{i \sqrt{e} x}{\sqrt{d}}\right] + 4 b e^2 n x^4 \operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{i \sqrt{e} x}{\sqrt{d}}\right] +$$

$$4 b d^2 n \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{i \sqrt{e} x}{\sqrt{d}}\right] + 8 b d e n x^2 \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{i \sqrt{e} x}{\sqrt{d}}\right] +$$

$$4 b e^2 n x^4 \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{i \sqrt{e} x}{\sqrt{d}}\right] + 4 a d^2 \operatorname{Log}[d+e x^2] + 3 b d^2 n \operatorname{Log}[d+e x^2] +$$

$$8 a d e x^2 \operatorname{Log}[d+e x^2] + 6 b d e n x^2 \operatorname{Log}[d+e x^2] + 4 a e^2 x^4 \operatorname{Log}[d+e x^2] + 3 b e^2 n x^4 \operatorname{Log}[d+e x^2] -$$

$$4 b d^2 n \operatorname{Log}[x] \operatorname{Log}[d+e x^2] - 8 b d e n x^2 \operatorname{Log}[x] \operatorname{Log}[d+e x^2] - 4 b e^2 n x^4 \operatorname{Log}[x] \operatorname{Log}[d+e x^2] +$$

$$4 b d^2 \operatorname{Log}[c x^n] \operatorname{Log}[d+e x^2] + 8 b d e x^2 \operatorname{Log}[c x^n] \operatorname{Log}[d+e x^2] + 4 b e^2 x^4 \operatorname{Log}[c x^n] \operatorname{Log}[d+e x^2] +$$

$$4 b n (d+e x^2)^2 \operatorname{PolyLog}\left[2, -\frac{i \sqrt{e} x}{\sqrt{d}}\right] + 4 b n (d+e x^2)^2 \operatorname{PolyLog}\left[2, \frac{i \sqrt{e} x}{\sqrt{d}}\right] \Big)$$

Problem 234: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{a + b \operatorname{Log}[c x^n]}{x (d + e x^2)^3} dx$$

Optimal (type 4, 115 leaves, 4 steps):

$$\frac{a + b \operatorname{Log}[c x^n]}{4 d (d + e x^2)^2} - \frac{\operatorname{Log}\left[1 + \frac{d}{e x^2}\right] (4 a - 3 b n + 4 b \operatorname{Log}[c x^n])}{8 d^3} +$$

$$\frac{4 a - b n + 4 b \operatorname{Log}[c x^n]}{8 d^2 (d + e x^2)} + \frac{b n \operatorname{PolyLog}\left[2, -\frac{d}{e x^2}\right]}{4 d^3}$$

Result (type 4, 444 leaves):

$$\begin{aligned}
 & -\frac{1}{16 d^3} \left(\frac{b d n}{d - i \sqrt{d} \sqrt{e} x} + \frac{b d n}{d + i \sqrt{d} \sqrt{e} x} - \frac{4 a d^2}{(d + e x^2)^2} - \frac{8 a d}{d + e x^2} - 16 a \operatorname{Log}[x] + 12 b n \operatorname{Log}[x] - \right. \\
 & \frac{b d n \operatorname{Log}[x]}{(\sqrt{d} - i \sqrt{e} x)^2} - \frac{b d n \operatorname{Log}[x]}{(\sqrt{d} + i \sqrt{e} x)^2} + \frac{5 i b \sqrt{d} n \operatorname{Log}[x]}{-i \sqrt{d} + \sqrt{e} x} - \frac{5 i b \sqrt{d} n \operatorname{Log}[x]}{i \sqrt{d} + \sqrt{e} x} + \\
 & \frac{4 b d^2 n \operatorname{Log}[x]}{(d + e x^2)^2} + \frac{8 b d n \operatorname{Log}[x]}{d + e x^2} + 8 b n \operatorname{Log}[x]^2 - \frac{4 b d^2 \operatorname{Log}[c x^n]}{(d + e x^2)^2} - \frac{8 b d \operatorname{Log}[c x^n]}{d + e x^2} - \\
 & 16 b \operatorname{Log}[x] \operatorname{Log}[c x^n] + 8 b n \operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{i \sqrt{e} x}{\sqrt{d}}\right] + 8 b n \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{i \sqrt{e} x}{\sqrt{d}}\right] + \\
 & 8 a \operatorname{Log}[d + e x^2] - 6 b n \operatorname{Log}[d + e x^2] - 8 b n \operatorname{Log}[x] \operatorname{Log}[d + e x^2] + \\
 & \left. 8 b \operatorname{Log}[c x^n] \operatorname{Log}[d + e x^2] + 8 b n \operatorname{PolyLog}\left[2, -\frac{i \sqrt{e} x}{\sqrt{d}}\right] + 8 b n \operatorname{PolyLog}\left[2, \frac{i \sqrt{e} x}{\sqrt{d}}\right] \right)
 \end{aligned}$$

Problem 235: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{a + b \operatorname{Log}[c x^n]}{x^3 (d + e x^2)^3} dx$$

Optimal (type 4, 162 leaves, 6 steps):

$$\begin{aligned}
 & -\frac{3 b n}{4 d^3 x^2} + \frac{a + b \operatorname{Log}[c x^n]}{4 d x^2 (d + e x^2)^2} + \frac{6 a - b n + 6 b \operatorname{Log}[c x^n]}{8 d^2 x^2 (d + e x^2)} - \frac{12 a - 5 b n + 12 b \operatorname{Log}[c x^n]}{8 d^3 x^2} + \\
 & \frac{e \operatorname{Log}\left[1 + \frac{d}{e x^2}\right] (12 a - 5 b n + 12 b \operatorname{Log}[c x^n])}{8 d^4} - \frac{3 b e n \operatorname{PolyLog}\left[2, -\frac{d}{e x^2}\right]}{4 d^4}
 \end{aligned}$$

Result (type 4, 468 leaves):

$$\begin{aligned}
 & \frac{1}{16 d^4} \left(24 b e n \operatorname{PolyLog}\left[2, -\frac{i \sqrt{e} x}{\sqrt{d}}\right] - \right. \\
 & \frac{1}{x^2 (d + e x^2)^2} 2 \left(4 a d^3 + 2 b d^3 n + 18 a d^2 e x^2 + 3 b d^2 e n x^2 + 12 a d e^2 x^4 + b d e^2 n x^4 - \right. \\
 & 12 b e n x^2 (d + e x^2)^2 \operatorname{Log}[x]^2 + 4 b d^3 \operatorname{Log}[c x^n] + 18 b d^2 e x^2 \operatorname{Log}[c x^n] + \\
 & 12 b d e^2 x^4 \operatorname{Log}[c x^n] - 12 a d^2 e x^2 \operatorname{Log}[d + e x^2] + 5 b d^2 e n x^2 \operatorname{Log}[d + e x^2] - \\
 & 24 a d e^2 x^4 \operatorname{Log}[d + e x^2] + 10 b d e^2 n x^4 \operatorname{Log}[d + e x^2] - 12 a e^3 x^6 \operatorname{Log}[d + e x^2] + \\
 & 5 b e^3 n x^6 \operatorname{Log}[d + e x^2] - 12 b d^2 e x^2 \operatorname{Log}[c x^n] \operatorname{Log}[d + e x^2] - \\
 & 24 b d e^2 x^4 \operatorname{Log}[c x^n] \operatorname{Log}[d + e x^2] - 12 b e^3 x^6 \operatorname{Log}[c x^n] \operatorname{Log}[d + e x^2] + \\
 & 2 e x^2 (d + e x^2)^2 \operatorname{Log}[x] \left(12 a - 5 b n + 12 b \operatorname{Log}[c x^n] - 6 b n \operatorname{Log}\left[1 - \frac{i \sqrt{e} x}{\sqrt{d}}\right] - 6 b n \right. \\
 & \left. \left. \operatorname{Log}\left[1 + \frac{i \sqrt{e} x}{\sqrt{d}}\right] + 6 b n \operatorname{Log}[d + e x^2] \right) - 12 b e n x^2 (d + e x^2)^2 \operatorname{PolyLog}\left[2, \frac{i \sqrt{e} x}{\sqrt{d}}\right] \right) \left. \right)
 \end{aligned}$$

Problem 236: Result more than twice size of optimal antiderivative.

$$\int \frac{x^4 (a + b \operatorname{Log}[c x^n])}{(d + e x^2)^3} dx$$

Optimal (type 4, 211 leaves, 24 steps):

$$\begin{aligned} & -\frac{b n x}{8 e^2 (d + e x^2)} + \frac{b n \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]}{2 \sqrt{d} e^{5/2}} + \frac{d x (a + b \operatorname{Log}[c x^n])}{4 e^2 (d + e x^2)^2} - \frac{5 x (a + b \operatorname{Log}[c x^n])}{8 e^2 (d + e x^2)} + \\ & \frac{3 \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] (a + b \operatorname{Log}[c x^n])}{8 \sqrt{d} e^{5/2}} - \frac{3 i b n \operatorname{PolyLog}\left[2, -\frac{i \sqrt{e} x}{\sqrt{d}}\right]}{16 \sqrt{d} e^{5/2}} + \frac{3 i b n \operatorname{PolyLog}\left[2, \frac{i \sqrt{e} x}{\sqrt{d}}\right]}{16 \sqrt{d} e^{5/2}} \end{aligned}$$

Result (type 4, 509 leaves):

$$\begin{aligned} & \frac{1}{16 e^{5/2}} \\ & \left(\frac{b n}{i \sqrt{d} - \sqrt{e} x} - \frac{b n}{i \sqrt{d} + \sqrt{e} x} + \frac{4 a d \sqrt{e} x}{(d + e x^2)^2} - \frac{10 a \sqrt{e} x}{d + e x^2} + \frac{6 a \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]}{\sqrt{d}} + \frac{8 b n \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]}{\sqrt{d}} - \right. \\ & \frac{i b \sqrt{d} n \operatorname{Log}[x]}{(\sqrt{d} - i \sqrt{e} x)^2} + \frac{i b \sqrt{d} n \operatorname{Log}[x]}{(\sqrt{d} + i \sqrt{e} x)^2} - \frac{5 b n \operatorname{Log}[x]}{-i \sqrt{d} + \sqrt{e} x} - \frac{5 b n \operatorname{Log}[x]}{i \sqrt{d} + \sqrt{e} x} - \frac{4 b d \sqrt{e} n x \operatorname{Log}[x]}{(d + e x^2)^2} + \\ & \frac{10 b \sqrt{e} n x \operatorname{Log}[x]}{d + e x^2} - \frac{6 b n \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] \operatorname{Log}[x]}{\sqrt{d}} + \frac{4 b d \sqrt{e} x \operatorname{Log}[c x^n]}{(d + e x^2)^2} - \\ & \frac{10 b \sqrt{e} x \operatorname{Log}[c x^n]}{d + e x^2} + \frac{6 b \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] \operatorname{Log}[c x^n]}{\sqrt{d}} + \frac{3 i b n \operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{i \sqrt{e} x}{\sqrt{d}}\right]}{\sqrt{d}} - \\ & \left. \frac{3 i b n \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{i \sqrt{e} x}{\sqrt{d}}\right]}{\sqrt{d}} - \frac{3 i b n \operatorname{PolyLog}\left[2, -\frac{i \sqrt{e} x}{\sqrt{d}}\right]}{\sqrt{d}} + \frac{3 i b n \operatorname{PolyLog}\left[2, \frac{i \sqrt{e} x}{\sqrt{d}}\right]}{\sqrt{d}} \right) \end{aligned}$$

Problem 237: Result more than twice size of optimal antiderivative.

$$\int \frac{x^2 (a + b \operatorname{Log}[c x^n])}{(d + e x^2)^3} dx$$

Optimal (type 4, 187 leaves, 19 steps):

$$\begin{aligned} & \frac{b n x}{8 d e (d + e x^2)} - \frac{x (a + b \operatorname{Log}[c x^n])}{4 e (d + e x^2)^2} + \frac{x (a + b \operatorname{Log}[c x^n])}{8 d e (d + e x^2)} + \\ & \frac{\operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] (a + b \operatorname{Log}[c x^n])}{8 d^{3/2} e^{3/2}} - \frac{i b n \operatorname{PolyLog}\left[2, -\frac{i \sqrt{e} x}{\sqrt{d}}\right]}{16 d^{3/2} e^{3/2}} + \frac{i b n \operatorname{PolyLog}\left[2, \frac{i \sqrt{e} x}{\sqrt{d}}\right]}{16 d^{3/2} e^{3/2}} \end{aligned}$$

Result (type 4, 589 leaves):

$$\frac{1}{16 d^{3/2} e^{3/2} (d + e x^2)^2} \left(-2 a d^{3/2} \sqrt{e} x + 2 b d^{3/2} \sqrt{e} n x + 2 a \sqrt{d} e^{3/2} x^3 + 2 b \sqrt{d} e^{3/2} n x^3 + 2 a d^2 \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] + 4 a d e x^2 \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] + 2 a e^2 x^4 \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] - 2 b d^2 n \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] \operatorname{Log}[x] - 4 b d e n x^2 \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] \operatorname{Log}[x] - 2 b e^2 n x^4 \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] \operatorname{Log}[x] - 2 b d^{3/2} \sqrt{e} x \operatorname{Log}[c x^n] + 2 b \sqrt{d} e^{3/2} x^3 \operatorname{Log}[c x^n] + 2 b d^2 \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] \operatorname{Log}[c x^n] + 4 b d e x^2 \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] \operatorname{Log}[c x^n] + 2 b e^2 x^4 \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] \operatorname{Log}[c x^n] + i b d^2 n \operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{i \sqrt{e} x}{\sqrt{d}}\right] + 2 i b d e n x^2 \operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{i \sqrt{e} x}{\sqrt{d}}\right] + i b e^2 n x^4 \operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{i \sqrt{e} x}{\sqrt{d}}\right] - i b d^2 n \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{i \sqrt{e} x}{\sqrt{d}}\right] - 2 i b d e n x^2 \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{i \sqrt{e} x}{\sqrt{d}}\right] - i b e^2 n x^4 \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{i \sqrt{e} x}{\sqrt{d}}\right] - i b n (d + e x^2)^2 \operatorname{PolyLog}\left[2, -\frac{i \sqrt{e} x}{\sqrt{d}}\right] + i b n (d + e x^2)^2 \operatorname{PolyLog}\left[2, \frac{i \sqrt{e} x}{\sqrt{d}}\right] \right)$$

Problem 238: Result more than twice size of optimal antiderivative.

$$\int \frac{a + b \operatorname{Log}[c x^n]}{(d + e x^2)^3} dx$$

Optimal (type 4, 210 leaves, 10 steps):

$$-\frac{b n x}{8 d^2 (d + e x^2)} - \frac{b n \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]}{2 d^{5/2} \sqrt{e}} + \frac{x (a + b \operatorname{Log}[c x^n])}{4 d (d + e x^2)^2} + \frac{3 x (a + b \operatorname{Log}[c x^n])}{8 d^2 (d + e x^2)} + \frac{3 \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] (a + b \operatorname{Log}[c x^n])}{8 d^{5/2} \sqrt{e}} - \frac{3 i b n \operatorname{PolyLog}\left[2, -\frac{i \sqrt{e} x}{\sqrt{d}}\right]}{16 d^{5/2} \sqrt{e}} + \frac{3 i b n \operatorname{PolyLog}\left[2, \frac{i \sqrt{e} x}{\sqrt{d}}\right]}{16 d^{5/2} \sqrt{e}}$$

Result (type 4, 533 leaves):

$$\frac{1}{16 d^{5/2}} \left(-\frac{b \sqrt{d} n}{-i \sqrt{d} \sqrt{e+e x}} - \frac{b \sqrt{d} n}{i \sqrt{d} \sqrt{e+e x}} + \frac{4 a d^{3/2} x}{(d+e x^2)^2} + \right. \\ \frac{6 a \sqrt{d} x}{d+e x^2} + \frac{6 a \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]}{\sqrt{e}} - \frac{8 b n \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]}{\sqrt{e}} + \frac{i b d n \operatorname{Log}[x]}{\sqrt{e} (\sqrt{d} + i \sqrt{e} x)^2} + \\ \frac{i b d n \operatorname{Log}[x]}{\sqrt{e} (i \sqrt{d} + \sqrt{e} x)^2} + \frac{3 b \sqrt{d} n \operatorname{Log}[x]}{-i \sqrt{d} \sqrt{e+e x}} + \frac{3 b \sqrt{d} n \operatorname{Log}[x]}{i \sqrt{d} \sqrt{e+e x}} - \frac{4 b d^{3/2} n x \operatorname{Log}[x]}{(d+e x^2)^2} - \\ \frac{6 b \sqrt{d} n x \operatorname{Log}[x]}{d+e x^2} - \frac{6 b n \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] \operatorname{Log}[x]}{\sqrt{e}} + \frac{4 b d^{3/2} x \operatorname{Log}[c x^n]}{(d+e x^2)^2} + \\ \frac{6 b \sqrt{d} x \operatorname{Log}[c x^n]}{d+e x^2} + \frac{6 b \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] \operatorname{Log}[c x^n]}{\sqrt{e}} + \frac{3 i b n \operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{i \sqrt{e} x}{\sqrt{d}}\right]}{\sqrt{e}} - \\ \left. \frac{3 i b n \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{i \sqrt{e} x}{\sqrt{d}}\right]}{\sqrt{e}} - \frac{3 i b n \operatorname{PolyLog}\left[2, -\frac{i \sqrt{e} x}{\sqrt{d}}\right]}{\sqrt{e}} + \frac{3 i b n \operatorname{PolyLog}\left[2, \frac{i \sqrt{e} x}{\sqrt{d}}\right]}{\sqrt{e}} \right)$$

Problem 239: Result more than twice size of optimal antiderivative.

$$\int \frac{a + b \operatorname{Log}[c x^n]}{x^2 (d + e x^2)^3} dx$$

Optimal (type 4, 219 leaves, 9 steps):

$$-\frac{15 b n}{8 d^3 x} + \frac{a + b \operatorname{Log}[c x^n]}{4 d x (d + e x^2)^2} + \frac{5 a - b n + 5 b \operatorname{Log}[c x^n]}{8 d^2 x (d + e x^2)} - \\ \frac{15 a - 8 b n + 15 b \operatorname{Log}[c x^n]}{8 d^3 x} - \frac{\sqrt{e} \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] (15 a - 8 b n + 15 b \operatorname{Log}[c x^n])}{8 d^{7/2}} + \\ \frac{15 i b \sqrt{e} n \operatorname{PolyLog}\left[2, -\frac{i \sqrt{e} x}{\sqrt{d}}\right]}{16 d^{7/2}} - \frac{15 i b \sqrt{e} n \operatorname{PolyLog}\left[2, \frac{i \sqrt{e} x}{\sqrt{d}}\right]}{16 d^{7/2}}$$

Result (type 4, 591 leaves):

$$\frac{1}{16 d^{7/2}} \left(-\frac{16 a \sqrt{d}}{x} - \frac{16 b \sqrt{d} n}{x} + \frac{b \sqrt{d} \sqrt{e} n}{i \sqrt{d} + \sqrt{e} x} + \frac{i b d \sqrt{e} n}{d + i \sqrt{d} \sqrt{e} x} - \frac{4 a d^{3/2} e x}{(d + e x^2)^2} - \frac{14 a \sqrt{d} e x}{d + e x^2} - 30 a \sqrt{e} \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] + 16 b \sqrt{e} n \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] + \frac{i b d \sqrt{e} n \operatorname{Log}[x]}{(\sqrt{d} - i \sqrt{e} x)^2} - \frac{i b d \sqrt{e} n \operatorname{Log}[x]}{(\sqrt{d} + i \sqrt{e} x)^2} - \frac{7 b \sqrt{d} \sqrt{e} n \operatorname{Log}[x]}{-i \sqrt{d} + \sqrt{e} x} - \frac{7 b \sqrt{d} \sqrt{e} n \operatorname{Log}[x]}{i \sqrt{d} + \sqrt{e} x} + \frac{4 b d^{3/2} e n x \operatorname{Log}[x]}{(d + e x^2)^2} + \frac{14 b \sqrt{d} e n x \operatorname{Log}[x]}{d + e x^2} + 30 b \sqrt{e} n \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] \operatorname{Log}[x] - \frac{16 b \sqrt{d} \operatorname{Log}[c x^n]}{x} - \frac{4 b d^{3/2} e x \operatorname{Log}[c x^n]}{(d + e x^2)^2} - \frac{14 b \sqrt{d} e x \operatorname{Log}[c x^n]}{d + e x^2} - 30 b \sqrt{e} \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] \operatorname{Log}[c x^n] - 15 i b \sqrt{e} n \operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{i \sqrt{e} x}{\sqrt{d}}\right] + 15 i b \sqrt{e} n \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{i \sqrt{e} x}{\sqrt{d}}\right] + 15 i b \sqrt{e} n \operatorname{PolyLog}\left[2, -\frac{i \sqrt{e} x}{\sqrt{d}}\right] - 15 i b \sqrt{e} n \operatorname{PolyLog}\left[2, \frac{i \sqrt{e} x}{\sqrt{d}}\right] \right)$$

Problem 240: Result more than twice size of optimal antiderivative.

$$\int \frac{a + b \operatorname{Log}[c x^n]}{x^4 (d + e x^2)^3} dx$$

Optimal (type 4, 260 leaves, 11 steps):

$$-\frac{35 b n}{72 d^3 x^3} + \frac{35 b e n}{8 d^4 x} + \frac{a + b \operatorname{Log}[c x^n]}{4 d x^3 (d + e x^2)^2} + \frac{7 a - b n + 7 b \operatorname{Log}[c x^n]}{8 d^2 x^3 (d + e x^2)} - \frac{35 a - 12 b n + 35 b \operatorname{Log}[c x^n]}{24 d^3 x^3} + \frac{e (35 a - 12 b n + 35 b \operatorname{Log}[c x^n])}{8 d^4 x} + \frac{e^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] (35 a - 12 b n + 35 b \operatorname{Log}[c x^n])}{8 d^{9/2}} - \frac{35 i b e^{3/2} n \operatorname{PolyLog}\left[2, -\frac{i \sqrt{e} x}{\sqrt{d}}\right]}{16 d^{9/2}} + \frac{35 i b e^{3/2} n \operatorname{PolyLog}\left[2, \frac{i \sqrt{e} x}{\sqrt{d}}\right]}{16 d^{9/2}}$$

Result (type 4, 645 leaves):

$$\begin{aligned}
 & \frac{1}{144 d^{9/2}} \left(-\frac{48 a d^{3/2}}{x^3} - \frac{16 b d^{3/2} n}{x^3} + \frac{432 a \sqrt{d} e}{x} + \frac{432 b \sqrt{d} e n}{x} + \frac{9 i b d e^{3/2} n}{d - i \sqrt{d} \sqrt{e} x} - \frac{9 i b d e^{3/2} n}{d + i \sqrt{d} \sqrt{e} x} + \right. \\
 & \frac{36 a d^{3/2} e^2 x}{(d + e x^2)^2} + \frac{198 a \sqrt{d} e^2 x}{d + e x^2} + 630 a e^{3/2} \text{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] - 216 b e^{3/2} n \text{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] - \\
 & \frac{9 i b d e^{3/2} n \text{Log}[x]}{(\sqrt{d} - i \sqrt{e} x)^2} + \frac{9 i b d e^{3/2} n \text{Log}[x]}{(\sqrt{d} + i \sqrt{e} x)^2} + \frac{99 b \sqrt{d} e^{3/2} n \text{Log}[x]}{-i \sqrt{d} + \sqrt{e} x} + \\
 & \frac{99 b \sqrt{d} e^{3/2} n \text{Log}[x]}{i \sqrt{d} + \sqrt{e} x} - \frac{36 b d^{3/2} e^2 n x \text{Log}[x]}{(d + e x^2)^2} - \frac{198 b \sqrt{d} e^2 n x \text{Log}[x]}{d + e x^2} - \\
 & 630 b e^{3/2} n \text{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] \text{Log}[x] - \frac{48 b d^{3/2} \text{Log}[c x^n]}{x^3} + \frac{432 b \sqrt{d} e \text{Log}[c x^n]}{x} + \\
 & \frac{36 b d^{3/2} e^2 x \text{Log}[c x^n]}{(d + e x^2)^2} + \frac{198 b \sqrt{d} e^2 x \text{Log}[c x^n]}{d + e x^2} + 630 b e^{3/2} \text{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] \text{Log}[c x^n] + \\
 & 315 i b e^{3/2} n \text{Log}[x] \text{Log}\left[1 - \frac{i \sqrt{e} x}{\sqrt{d}}\right] - 315 i b e^{3/2} n \text{Log}[x] \text{Log}\left[1 + \frac{i \sqrt{e} x}{\sqrt{d}}\right] - \\
 & \left. 315 i b e^{3/2} n \text{PolyLog}\left[2, -\frac{i \sqrt{e} x}{\sqrt{d}}\right] + 315 i b e^{3/2} n \text{PolyLog}\left[2, \frac{i \sqrt{e} x}{\sqrt{d}}\right] \right)
 \end{aligned}$$

Problem 242: Result more than twice size of optimal antiderivative.

$$\int \frac{x \text{Log}\left[\frac{x^2}{c}\right]}{c - x^2} dx$$

Optimal (type 4, 16 leaves, 2 steps):

$$\frac{1}{2} \text{PolyLog}\left[2, 1 - \frac{x^2}{c}\right]$$

Result (type 4, 37 leaves):

$$-\frac{1}{2} \text{Log}\left[\frac{x^2}{c}\right] \text{Log}\left[1 - \frac{x^2}{c}\right] - \frac{1}{2} \text{PolyLog}\left[2, \frac{x^2}{c}\right]$$

Problem 247: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b \text{Log}[c x^n])^2}{(d + e x^2)^2} dx$$

Optimal (type 4, 509 leaves, 16 steps):

$$\begin{aligned} & \frac{x (a + b \operatorname{Log}[c x^n])^2}{4 (-d)^{3/2} (\sqrt{-d} - \sqrt{e} x)} + \frac{x (a + b \operatorname{Log}[c x^n])^2}{4 (-d)^{3/2} (\sqrt{-d} + \sqrt{e} x)} + \\ & \frac{b n (a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 - \frac{\sqrt{e} x}{\sqrt{-d}}\right]}{2 (-d)^{3/2} \sqrt{e}} - \frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1 - \frac{\sqrt{e} x}{\sqrt{-d}}\right]}{4 (-d)^{3/2} \sqrt{e}} - \\ & \frac{b n (a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{\sqrt{e} x}{\sqrt{-d}}\right]}{2 (-d)^{3/2} \sqrt{e}} + \frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1 + \frac{\sqrt{e} x}{\sqrt{-d}}\right]}{4 (-d)^{3/2} \sqrt{e}} - \\ & \frac{b^2 n^2 \operatorname{PolyLog}\left[2, -\frac{\sqrt{e} x}{\sqrt{-d}}\right]}{2 (-d)^{3/2} \sqrt{e}} + \frac{b n (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{\sqrt{e} x}{\sqrt{-d}}\right]}{2 (-d)^{3/2} \sqrt{e}} + \frac{b^2 n^2 \operatorname{PolyLog}\left[2, \frac{\sqrt{e} x}{\sqrt{-d}}\right]}{2 (-d)^{3/2} \sqrt{e}} - \\ & \frac{b n (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, \frac{\sqrt{e} x}{\sqrt{-d}}\right]}{2 (-d)^{3/2} \sqrt{e}} - \frac{b^2 n^2 \operatorname{PolyLog}\left[3, -\frac{\sqrt{e} x}{\sqrt{-d}}\right]}{2 (-d)^{3/2} \sqrt{e}} + \frac{b^2 n^2 \operatorname{PolyLog}\left[3, \frac{\sqrt{e} x}{\sqrt{-d}}\right]}{2 (-d)^{3/2} \sqrt{e}} \end{aligned}$$

Result (type 4, 666 leaves):

$$\begin{aligned}
 & \frac{1}{4 d^2} \left(\frac{2 d x (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n])^2}{d + e x^2} + \right. \\
 & \frac{2 \sqrt{d} \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n])^2}{\sqrt{e}} + \frac{1}{\sqrt{e} (d + e x^2)} \\
 & 2 b \sqrt{d} n (-a + b n \operatorname{Log}[x] - b \operatorname{Log}[c x^n]) \left(2 d \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] + 2 e x^2 \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] - \right. \\
 & 2 \sqrt{d} \sqrt{e} x \operatorname{Log}[x] - i d \operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{i \sqrt{e} x}{\sqrt{d}}\right] - i e x^2 \operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{i \sqrt{e} x}{\sqrt{d}}\right] + \\
 & i d \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{i \sqrt{e} x}{\sqrt{d}}\right] + i e x^2 \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{i \sqrt{e} x}{\sqrt{d}}\right] + \\
 & \left. i (d + e x^2) \operatorname{PolyLog}\left[2, -\frac{i \sqrt{e} x}{\sqrt{d}}\right] - i (d + e x^2) \operatorname{PolyLog}\left[2, \frac{i \sqrt{e} x}{\sqrt{d}}\right] \right) + \\
 & b^2 n^2 \left(\frac{x \operatorname{Log}[x]^2}{1 - \frac{i \sqrt{e} x}{\sqrt{d}}} + \frac{x \operatorname{Log}[x]^2}{1 + \frac{i \sqrt{e} x}{\sqrt{d}}} - \frac{2 i \sqrt{d} \operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{i \sqrt{e} x}{\sqrt{d}}\right]}{\sqrt{e}} + \frac{i \sqrt{d} \operatorname{Log}[x]^2 \operatorname{Log}\left[1 - \frac{i \sqrt{e} x}{\sqrt{d}}\right]}{\sqrt{e}} + \right. \\
 & \frac{2 i \sqrt{d} \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{i \sqrt{e} x}{\sqrt{d}}\right]}{\sqrt{e}} - \frac{i \sqrt{d} \operatorname{Log}[x]^2 \operatorname{Log}\left[1 + \frac{i \sqrt{e} x}{\sqrt{d}}\right]}{\sqrt{e}} - \\
 & \frac{2 i \sqrt{d} (-1 + \operatorname{Log}[x]) \operatorname{PolyLog}\left[2, -\frac{i \sqrt{e} x}{\sqrt{d}}\right]}{\sqrt{e}} + \frac{2 i \sqrt{d} (-1 + \operatorname{Log}[x]) \operatorname{PolyLog}\left[2, \frac{i \sqrt{e} x}{\sqrt{d}}\right]}{\sqrt{e}} + \\
 & \left. \left. \frac{2 i \sqrt{d} \operatorname{PolyLog}\left[3, -\frac{i \sqrt{e} x}{\sqrt{d}}\right]}{\sqrt{e}} - \frac{2 i \sqrt{d} \operatorname{PolyLog}\left[3, \frac{i \sqrt{e} x}{\sqrt{d}}\right]}{\sqrt{e}} \right) \right)
 \end{aligned}$$

Problem 248: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b \operatorname{Log}[c x^n])^3}{(d + e x^2)^2} dx$$

Optimal (type 4, 711 leaves, 20 steps):

$$\begin{aligned}
& \frac{x (a + b \operatorname{Log}[c x^n])^3}{4 (-d)^{3/2} (\sqrt{-d} - \sqrt{e} x)} + \frac{x (a + b \operatorname{Log}[c x^n])^3}{4 (-d)^{3/2} (\sqrt{-d} + \sqrt{e} x)} + \frac{3 b n (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1 - \frac{\sqrt{e} x}{\sqrt{-d}}\right]}{4 (-d)^{3/2} \sqrt{e}} - \\
& \frac{(a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}\left[1 - \frac{\sqrt{e} x}{\sqrt{-d}}\right]}{4 (-d)^{3/2} \sqrt{e}} - \frac{3 b n (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1 + \frac{\sqrt{e} x}{\sqrt{-d}}\right]}{4 (-d)^{3/2} \sqrt{e}} + \\
& \frac{(a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}\left[1 + \frac{\sqrt{e} x}{\sqrt{-d}}\right]}{4 (-d)^{3/2} \sqrt{e}} - \frac{3 b^2 n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{\sqrt{e} x}{\sqrt{-d}}\right]}{2 (-d)^{3/2} \sqrt{e}} + \\
& \frac{3 b n (a + b \operatorname{Log}[c x^n])^2 \operatorname{PolyLog}\left[2, -\frac{\sqrt{e} x}{\sqrt{-d}}\right]}{4 (-d)^{3/2} \sqrt{e}} + \frac{3 b^2 n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, \frac{\sqrt{e} x}{\sqrt{-d}}\right]}{2 (-d)^{3/2} \sqrt{e}} - \\
& \frac{3 b n (a + b \operatorname{Log}[c x^n])^2 \operatorname{PolyLog}\left[2, \frac{\sqrt{e} x}{\sqrt{-d}}\right]}{4 (-d)^{3/2} \sqrt{e}} + \frac{3 b^3 n^3 \operatorname{PolyLog}\left[3, -\frac{\sqrt{e} x}{\sqrt{-d}}\right]}{2 (-d)^{3/2} \sqrt{e}} - \\
& \frac{3 b^2 n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[3, -\frac{\sqrt{e} x}{\sqrt{-d}}\right]}{2 (-d)^{3/2} \sqrt{e}} - \frac{3 b^3 n^3 \operatorname{PolyLog}\left[3, \frac{\sqrt{e} x}{\sqrt{-d}}\right]}{2 (-d)^{3/2} \sqrt{e}} + \\
& \frac{3 b^2 n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[3, \frac{\sqrt{e} x}{\sqrt{-d}}\right]}{2 (-d)^{3/2} \sqrt{e}} + \frac{3 b^3 n^3 \operatorname{PolyLog}\left[4, -\frac{\sqrt{e} x}{\sqrt{-d}}\right]}{2 (-d)^{3/2} \sqrt{e}} - \frac{3 b^3 n^3 \operatorname{PolyLog}\left[4, \frac{\sqrt{e} x}{\sqrt{-d}}\right]}{2 (-d)^{3/2} \sqrt{e}}
\end{aligned}$$

Result (type 4, 1104 leaves):

$$\begin{aligned}
& \frac{1}{4 d^2} \left(\frac{2 d x (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n])^3}{d + e x^2} + \right. \\
& \frac{2 \sqrt{d} \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n])^3}{\sqrt{e}} + \frac{1}{\sqrt{e} (d + e x^2)} \\
& 3 b \sqrt{d} n (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n])^2 \left(-2 d \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] - 2 e x^2 \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] + \right. \\
& 2 \sqrt{d} \sqrt{e} x \operatorname{Log}[x] + i d \operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{i \sqrt{e} x}{\sqrt{d}}\right] + i e x^2 \operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{i \sqrt{e} x}{\sqrt{d}}\right] - \\
& i d \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{i \sqrt{e} x}{\sqrt{d}}\right] - i e x^2 \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{i \sqrt{e} x}{\sqrt{d}}\right] - \\
& \left. i (d + e x^2) \operatorname{PolyLog}\left[2, -\frac{i \sqrt{e} x}{\sqrt{d}}\right] + i (d + e x^2) \operatorname{PolyLog}\left[2, \frac{i \sqrt{e} x}{\sqrt{d}}\right] \right) + \\
& 3 b^2 n^2 (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n]) \left(\frac{x \operatorname{Log}[x]^2}{1 - \frac{i \sqrt{e} x}{\sqrt{d}}} + \frac{x \operatorname{Log}[x]^2}{1 + \frac{i \sqrt{e} x}{\sqrt{d}}} - \frac{2 i \sqrt{d} \operatorname{Log}[x] \operatorname{Log}\left[1 - \frac{i \sqrt{e} x}{\sqrt{d}}\right]}{\sqrt{e}} + \right.
\end{aligned}$$

$$\begin{aligned}
 & \frac{i \sqrt{d} \operatorname{Log}[x]^2 \operatorname{Log}\left[1 - \frac{i \sqrt{e} x}{\sqrt{d}}\right]}{\sqrt{e}} + \frac{2 i \sqrt{d} \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{i \sqrt{e} x}{\sqrt{d}}\right]}{\sqrt{e}} - \\
 & \frac{i \sqrt{d} \operatorname{Log}[x]^2 \operatorname{Log}\left[1 + \frac{i \sqrt{e} x}{\sqrt{d}}\right]}{\sqrt{e}} - \frac{2 i \sqrt{d} (-1 + \operatorname{Log}[x]) \operatorname{PolyLog}\left[2, -\frac{i \sqrt{e} x}{\sqrt{d}}\right]}{\sqrt{e}} + \\
 & \frac{2 i \sqrt{d} (-1 + \operatorname{Log}[x]) \operatorname{PolyLog}\left[2, \frac{i \sqrt{e} x}{\sqrt{d}}\right]}{\sqrt{e}} + \\
 & \left. \frac{2 i \sqrt{d} \operatorname{PolyLog}\left[3, -\frac{i \sqrt{e} x}{\sqrt{d}}\right]}{\sqrt{e}} - \frac{2 i \sqrt{d} \operatorname{PolyLog}\left[3, \frac{i \sqrt{e} x}{\sqrt{d}}\right]}{\sqrt{e}} \right) + \\
 & \frac{1}{\sqrt{e}} b^3 n^3 \left(\frac{\sqrt{e} x \operatorname{Log}[x]^3}{1 - \frac{i \sqrt{e} x}{\sqrt{d}}} + \frac{\sqrt{e} x \operatorname{Log}[x]^3}{1 + \frac{i \sqrt{e} x}{\sqrt{d}}} - 3 i \sqrt{d} \operatorname{Log}[x]^2 \operatorname{Log}\left[1 - \frac{i \sqrt{e} x}{\sqrt{d}}\right] + \right. \\
 & i \sqrt{d} \operatorname{Log}[x]^3 \operatorname{Log}\left[1 - \frac{i \sqrt{e} x}{\sqrt{d}}\right] + 3 i \sqrt{d} \operatorname{Log}[x]^2 \operatorname{Log}\left[1 + \frac{i \sqrt{e} x}{\sqrt{d}}\right] - \\
 & i \sqrt{d} \operatorname{Log}[x]^3 \operatorname{Log}\left[1 + \frac{i \sqrt{e} x}{\sqrt{d}}\right] - 3 i \sqrt{d} (-2 + \operatorname{Log}[x]) \operatorname{Log}[x] \operatorname{PolyLog}\left[2, -\frac{i \sqrt{e} x}{\sqrt{d}}\right] + \\
 & 3 i \sqrt{d} (-2 + \operatorname{Log}[x]) \operatorname{Log}[x] \operatorname{PolyLog}\left[2, \frac{i \sqrt{e} x}{\sqrt{d}}\right] - 6 i \sqrt{d} \operatorname{PolyLog}\left[3, -\frac{i \sqrt{e} x}{\sqrt{d}}\right] + \\
 & 6 i \sqrt{d} \operatorname{Log}[x] \operatorname{PolyLog}\left[3, -\frac{i \sqrt{e} x}{\sqrt{d}}\right] + 6 i \sqrt{d} \operatorname{PolyLog}\left[3, \frac{i \sqrt{e} x}{\sqrt{d}}\right] - 6 i \sqrt{d} \operatorname{Log}[x] \\
 & \left. \left. \operatorname{PolyLog}\left[3, \frac{i \sqrt{e} x}{\sqrt{d}}\right] - 6 i \sqrt{d} \operatorname{PolyLog}\left[4, -\frac{i \sqrt{e} x}{\sqrt{d}}\right] + 6 i \sqrt{d} \operatorname{PolyLog}\left[4, \frac{i \sqrt{e} x}{\sqrt{d}}\right] \right) \right)
 \end{aligned}$$

Problem 254: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{d+e x^2} (a+b \operatorname{Log}[c x^n])}{x} dx$$

Optimal (type 4, 220 leaves, 12 steps):

$$\begin{aligned}
 & -b n \sqrt{d+e x^2} + b \sqrt{d} n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d}}\right] + \frac{1}{2} b \sqrt{d} n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d}}\right]^2 + \\
 & \left(\sqrt{d+e x^2} - \sqrt{d} \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d}}\right] \right) (a+b \operatorname{Log}[c x^n]) - \\
 & b \sqrt{d} n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d}}\right] \operatorname{Log}\left[\frac{2 \sqrt{d}}{\sqrt{d} - \sqrt{d+e x^2}}\right] - \frac{1}{2} b \sqrt{d} n \operatorname{PolyLog}\left[2, 1 - \frac{2 \sqrt{d}}{\sqrt{d} - \sqrt{d+e x^2}}\right]
 \end{aligned}$$

Result (type 5, 203 leaves):

$$\frac{1}{\sqrt{1 + \frac{d}{e x^2}}} b n \sqrt{d + e x^2} \left(-\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right\}, \left\{\frac{1}{2}, \frac{1}{2}\right\}, -\frac{d}{e x^2}\right] + \sqrt{1 + \frac{d}{e x^2}} \text{Log}[x] - \frac{\sqrt{d} \text{ArcSinh}\left[\frac{\sqrt{d}}{\sqrt{e} x}\right] \text{Log}[x]}{\sqrt{e} x} \right) + \sqrt{d + e x^2} (a - b n \text{Log}[x] + b \text{Log}[c x^n]) + \sqrt{d} \text{Log}[x] (a - b n \text{Log}[x] + b \text{Log}[c x^n]) - \sqrt{d} (a - b n \text{Log}[x] + b \text{Log}[c x^n]) \text{Log}[d + \sqrt{d} \sqrt{d + e x^2}]$$

Problem 255: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{d + e x^2} (a + b \text{Log}[c x^n])}{x^3} dx$$

Optimal (type 4, 252 leaves, 14 steps):

$$\frac{b n \sqrt{d + e x^2}}{4 x^2} - \frac{b e n \text{ArcTanh}\left[\frac{\sqrt{d + e x^2}}{\sqrt{d}}\right]}{4 \sqrt{d}} + \frac{b e n \text{ArcTanh}\left[\frac{\sqrt{d + e x^2}}{\sqrt{d}}\right]^2}{4 \sqrt{d}} - \frac{\sqrt{d + e x^2} (a + b \text{Log}[c x^n])}{2 x^2} - \frac{e \text{ArcTanh}\left[\frac{\sqrt{d + e x^2}}{\sqrt{d}}\right] (a + b \text{Log}[c x^n])}{2 \sqrt{d}} - \frac{b e n \text{ArcTanh}\left[\frac{\sqrt{d + e x^2}}{\sqrt{d}}\right] \text{Log}\left[\frac{2 \sqrt{d}}{\sqrt{d} - \sqrt{d + e x^2}}\right]}{2 \sqrt{d}} - \frac{b e n \text{PolyLog}\left[2, 1 - \frac{2 \sqrt{d}}{\sqrt{d} - \sqrt{d + e x^2}}\right]}{4 \sqrt{d}}$$

Result (type 5, 303 leaves):

$$\frac{1}{4\sqrt{d}\sqrt{1+\frac{d}{ex^2}}x^2} \left(-2b\sqrt{d}n\sqrt{d+ex^2} \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\}, \left\{\frac{3}{2}, \frac{3}{2}\right\}, -\frac{d}{ex^2}\right] - \right.$$

$$b\sqrt{e}nx\sqrt{d+ex^2} \operatorname{ArcSinh}\left[\frac{\sqrt{d}}{\sqrt{e}x}\right] (1+2\operatorname{Log}[x]) +$$

$$\sqrt{1+\frac{d}{ex^2}} \left(-2a\sqrt{d}\sqrt{d+ex^2} - b\sqrt{d}n\sqrt{d+ex^2} - 2benx^2\operatorname{Log}[x]^2 - \right.$$

$$2aex^2\operatorname{Log}[d+\sqrt{d}\sqrt{d+ex^2}] + 2ex^2\operatorname{Log}[x] \left(a+b\operatorname{Log}[cx^n] + bn\operatorname{Log}[d+\sqrt{d}\sqrt{d+ex^2}] \right) -$$

$$\left. \left. 2b\operatorname{Log}[cx^n] \left(\sqrt{d}\sqrt{d+ex^2} + ex^2\operatorname{Log}[d+\sqrt{d}\sqrt{d+ex^2}] \right) \right) \right)$$

Problem 256: Result unnecessarily involves higher level functions.

$$\int x^4 \sqrt{d+ex^2} (a+b \operatorname{Log}[cx^n]) dx$$

Optimal (type 4, 469 leaves, 19 steps):

$$\frac{7bd^2nx\sqrt{d+ex^2}}{192e^2} - \frac{5bdnx^3\sqrt{d+ex^2}}{288e} - \frac{1}{36}bnx^5\sqrt{d+ex^2} +$$

$$\frac{5bd^{5/2}n\sqrt{d+ex^2} \operatorname{ArcSinh}\left[\frac{\sqrt{e}x}{\sqrt{d}}\right]}{192e^{5/2}\sqrt{1+\frac{ex^2}{d}}} + \frac{bd^{5/2}n\sqrt{d+ex^2} \operatorname{ArcSinh}\left[\frac{\sqrt{e}x}{\sqrt{d}}\right]^2}{32e^{5/2}\sqrt{1+\frac{ex^2}{d}}} -$$

$$\frac{bd^{5/2}n\sqrt{d+ex^2} \operatorname{ArcSinh}\left[\frac{\sqrt{e}x}{\sqrt{d}}\right] \operatorname{Log}\left[1 - e^{2\operatorname{ArcSinh}\left[\frac{\sqrt{e}x}{\sqrt{d}}\right]}\right]}{16e^{5/2}\sqrt{1+\frac{ex^2}{d}}} - \frac{d^2x\sqrt{d+ex^2} (a+b \operatorname{Log}[cx^n])}{16e^2} +$$

$$\frac{dx^3\sqrt{d+ex^2} (a+b \operatorname{Log}[cx^n])}{24e} + \frac{1}{6}x^5\sqrt{d+ex^2} (a+b \operatorname{Log}[cx^n]) +$$

$$\frac{d^{5/2}\sqrt{d+ex^2} \operatorname{ArcSinh}\left[\frac{\sqrt{e}x}{\sqrt{d}}\right] (a+b \operatorname{Log}[cx^n])}{16e^{5/2}\sqrt{1+\frac{ex^2}{d}}} - \frac{bd^{5/2}n\sqrt{d+ex^2} \operatorname{PolyLog}\left[2, e^{2\operatorname{ArcSinh}\left[\frac{\sqrt{e}x}{\sqrt{d}}\right]}\right]}{32e^{5/2}\sqrt{1+\frac{ex^2}{d}}}$$

Result (type 5, 276 leaves):

$$\frac{1}{1200 e^{5/2} \sqrt{1 + \frac{e x^2}{d}}} \left(-48 b e^{5/2} n x^5 \sqrt{d + e x^2} \operatorname{HypergeometricPFQ} \left[\left\{ -\frac{1}{2}, \frac{5}{2}, \frac{5}{2} \right\}, \left\{ \frac{7}{2}, \frac{7}{2} \right\}, -\frac{e x^2}{d} \right] + \right. \\ \left. 75 b d^{5/2} n \sqrt{d + e x^2} \operatorname{ArcSinh} \left[\frac{\sqrt{e} x}{\sqrt{d}} \right] \operatorname{Log}[x] + 25 \sqrt{1 + \frac{e x^2}{d}} \right. \\ \left. \left(a \sqrt{e} x \sqrt{d + e x^2} (-3 d^2 + 2 d e x^2 + 8 e^2 x^4) + 3 d^3 (a - b n \operatorname{Log}[x]) \operatorname{Log}[e x + \sqrt{e} \sqrt{d + e x^2}] + \right. \right. \\ \left. \left. b \operatorname{Log}[c x^n] \left(\sqrt{e} x \sqrt{d + e x^2} (-3 d^2 + 2 d e x^2 + 8 e^2 x^4) + 3 d^3 \operatorname{Log}[e x + \sqrt{e} \sqrt{d + e x^2}] \right) \right) \right)$$

Problem 257: Result unnecessarily involves higher level functions.

$$\int x^2 \sqrt{d + e x^2} (a + b \operatorname{Log}[c x^n]) dx$$

Optimal (type 4, 409 leaves, 15 steps):

$$-\frac{3 b d n x \sqrt{d + e x^2}}{32 e} - \frac{1}{16} b n x^3 \sqrt{d + e x^2} - \frac{b d^{3/2} n \sqrt{d + e x^2} \operatorname{ArcSinh} \left[\frac{\sqrt{e} x}{\sqrt{d}} \right]}{32 e^{3/2} \sqrt{1 + \frac{e x^2}{d}}} - \\ \frac{b d^{3/2} n \sqrt{d + e x^2} \operatorname{ArcSinh} \left[\frac{\sqrt{e} x}{\sqrt{d}} \right]^2}{16 e^{3/2} \sqrt{1 + \frac{e x^2}{d}}} + \frac{b d^{3/2} n \sqrt{d + e x^2} \operatorname{ArcSinh} \left[\frac{\sqrt{e} x}{\sqrt{d}} \right] \operatorname{Log} \left[1 - e^{2 \operatorname{ArcSinh} \left[\frac{\sqrt{e} x}{\sqrt{d}} \right]} \right]}{8 e^{3/2} \sqrt{1 + \frac{e x^2}{d}}} + \\ \frac{d x \sqrt{d + e x^2} (a + b \operatorname{Log}[c x^n])}{8 e} + \frac{1}{4} x^3 \sqrt{d + e x^2} (a + b \operatorname{Log}[c x^n]) - \\ \frac{d^{3/2} \sqrt{d + e x^2} \operatorname{ArcSinh} \left[\frac{\sqrt{e} x}{\sqrt{d}} \right] (a + b \operatorname{Log}[c x^n])}{8 e^{3/2} \sqrt{1 + \frac{e x^2}{d}}} + \frac{b d^{3/2} n \sqrt{d + e x^2} \operatorname{PolyLog} \left[2, e^{2 \operatorname{ArcSinh} \left[\frac{\sqrt{e} x}{\sqrt{d}} \right]} \right]}{16 e^{3/2} \sqrt{1 + \frac{e x^2}{d}}}$$

Result (type 5, 250 leaves):

$$\frac{1}{72 e^{3/2} \sqrt{1 + \frac{e x^2}{d}}} \left(-8 b e^{3/2} n x^3 \sqrt{d + e x^2} \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, \frac{3}{2}, \frac{3}{2}\right\}, \left\{\frac{5}{2}, \frac{5}{2}\right\}, -\frac{e x^2}{d}\right] - \right.$$

$$9 b d^{3/2} n \sqrt{d + e x^2} \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] \operatorname{Log}[x] +$$

$$9 \sqrt{1 + \frac{e x^2}{d}} \left(a \sqrt{e} x \sqrt{d + e x^2} (d + 2 e x^2) + d^2 (-a + b n \operatorname{Log}[x]) \operatorname{Log}[e x + \sqrt{e} \sqrt{d + e x^2}] + \right.$$

$$\left. \left. b \operatorname{Log}[c x^n] \left(\sqrt{e} x \sqrt{d + e x^2} (d + 2 e x^2) - d^2 \operatorname{Log}[e x + \sqrt{e} \sqrt{d + e x^2}] \right) \right) \right)$$

Problem 258: Result unnecessarily involves higher level functions.

$$\int \sqrt{d + e x^2} (a + b \operatorname{Log}[c x^n]) dx$$

Optimal (type 4, 330 leaves, 11 steps):

$$-\frac{1}{4} b n x \sqrt{d + e x^2} + \frac{b d^{3/2} n \sqrt{1 + \frac{e x^2}{d}} \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]^2}{4 \sqrt{e} \sqrt{d + e x^2}} - \frac{b d n \operatorname{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d + e x^2}}\right]}{4 \sqrt{e}}$$

$$\frac{b d^{3/2} n \sqrt{1 + \frac{e x^2}{d}} \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] \operatorname{Log}\left[1 - e^{2 \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]}\right]}{2 \sqrt{e} \sqrt{d + e x^2}} + \frac{1}{2} x \sqrt{d + e x^2} (a + b \operatorname{Log}[c x^n]) +$$

$$\frac{d^{3/2} \sqrt{1 + \frac{e x^2}{d}} \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] (a + b \operatorname{Log}[c x^n])}{2 \sqrt{e} \sqrt{d + e x^2}} - \frac{b d^{3/2} n \sqrt{1 + \frac{e x^2}{d}} \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]}\right]}{4 \sqrt{e} \sqrt{d + e x^2}}$$

Result (type 5, 237 leaves):

$$\frac{1}{4 \sqrt{e} \sqrt{1 + \frac{e x^2}{d}}} \left(-2 b \sqrt{e} n x \sqrt{d + e x^2} \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\}, \left\{\frac{3}{2}, \frac{3}{2}\right\}, -\frac{e x^2}{d}\right] + \right. \\ \left. b \sqrt{d} n \sqrt{d + e x^2} \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] (-1 + 2 \operatorname{Log}[x]) + \right. \\ \left. \sqrt{1 + \frac{e x^2}{d}} \left(\sqrt{e} (2 a - b n) x \sqrt{d + e x^2} + 2 d (a - b n \operatorname{Log}[x]) \operatorname{Log}\left[ex + \sqrt{e} \sqrt{d + e x^2}\right] + \right. \right. \\ \left. \left. 2 b \operatorname{Log}[c x^n] \left(\sqrt{e} x \sqrt{d + e x^2} + d \operatorname{Log}\left[ex + \sqrt{e} \sqrt{d + e x^2}\right] \right) \right) \right)$$

Problem 259: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{d + e x^2} (a + b \operatorname{Log}[c x^n])}{x^2} dx$$

Optimal (type 4, 345 leaves, 11 steps):

$$-\frac{b n \sqrt{d + e x^2}}{x} + \frac{b \sqrt{e} n \sqrt{d + e x^2} \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]}{\sqrt{d} \sqrt{1 + \frac{e x^2}{d}}} + \frac{b \sqrt{e} n \sqrt{d + e x^2} \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]^2}{2 \sqrt{d} \sqrt{1 + \frac{e x^2}{d}}} - \\ \frac{b \sqrt{e} n \sqrt{d + e x^2} \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] \operatorname{Log}\left[1 - e^{2 \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]}\right]}{\sqrt{d} \sqrt{1 + \frac{e x^2}{d}}} - \frac{\sqrt{d + e x^2} (a + b \operatorname{Log}[c x^n])}{x} + \\ \frac{\sqrt{e} \sqrt{d + e x^2} \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] (a + b \operatorname{Log}[c x^n])}{\sqrt{d} \sqrt{1 + \frac{e x^2}{d}}} - \frac{b \sqrt{e} n \sqrt{d + e x^2} \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]}\right]}{2 \sqrt{d} \sqrt{1 + \frac{e x^2}{d}}}$$

Result (type 5, 183 leaves):

$$\frac{1}{x \sqrt{1 + \frac{e x^2}{d}}}$$

$$b n \sqrt{d + e x^2} \left(-\text{HypergeometricPFQ} \left[\left\{ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2} \right\}, \left\{ \frac{1}{2}, \frac{1}{2} \right\}, -\frac{e x^2}{d} \right] - \sqrt{1 + \frac{e x^2}{d}} \text{Log}[x] + \frac{\sqrt{e} x \text{ArcSinh} \left[\frac{\sqrt{e} x}{\sqrt{d}} \right] \text{Log}[x]}{\sqrt{d}} \right) - \frac{\sqrt{d + e x^2} (a - b n \text{Log}[x] + b \text{Log}[c x^n])}{x} + \sqrt{e} (a - b n \text{Log}[x] + b \text{Log}[c x^n]) \text{Log}[e x + \sqrt{e} \sqrt{d + e x^2}]$$

Problem 266: Result unnecessarily involves higher level functions.

$$\int \frac{(d + e x^2)^{3/2} (a + b \text{Log}[c x^n])}{x} dx$$

Optimal (type 4, 260 leaves, 17 steps):

$$-\frac{4}{3} b d n \sqrt{d + e x^2} - \frac{1}{9} b n (d + e x^2)^{3/2} + \frac{4}{3} b d^{3/2} n \text{ArcTanh} \left[\frac{\sqrt{d + e x^2}}{\sqrt{d}} \right] + \frac{1}{2} b d^{3/2} n \text{ArcTanh} \left[\frac{\sqrt{d + e x^2}}{\sqrt{d}} \right]^2 + \frac{1}{3} \left(3 d \sqrt{d + e x^2} + (d + e x^2)^{3/2} - 3 d^{3/2} \text{ArcTanh} \left[\frac{\sqrt{d + e x^2}}{\sqrt{d}} \right] \right) (a + b \text{Log}[c x^n]) - b d^{3/2} n \text{ArcTanh} \left[\frac{\sqrt{d + e x^2}}{\sqrt{d}} \right] \text{Log} \left[\frac{2 \sqrt{d}}{\sqrt{d} - \sqrt{d + e x^2}} \right] - \frac{1}{2} b d^{3/2} n \text{PolyLog} \left[2, 1 - \frac{2 \sqrt{d}}{\sqrt{d} - \sqrt{d + e x^2}} \right]$$

Result (type 5, 315 leaves):

$$\frac{1}{12 \sqrt{1 + \frac{e x^2}{d}}} b n \sqrt{d + e x^2} \left(-3 e x^2 \text{HypergeometricPFQ} \left[\left\{ -\frac{1}{2}, 1, 1 \right\}, \{2, 2\}, -\frac{e x^2}{d} \right] + \right.$$

$$\left. 4 \left(e x^2 \sqrt{1 + \frac{e x^2}{d}} + d \left(-1 + \sqrt{1 + \frac{e x^2}{d}} \right) \right) \text{Log}[x] \right) + \frac{1}{\sqrt{1 + \frac{d}{e x^2}}}$$

$$b d n \sqrt{d + e x^2} \left(-\text{HypergeometricPFQ} \left[\left\{ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2} \right\}, \left\{ \frac{1}{2}, \frac{1}{2} \right\}, -\frac{d}{e x^2} \right] + \right.$$

$$\left. \sqrt{1 + \frac{d}{e x^2}} \text{Log}[x] - \frac{\sqrt{d} \text{ArcSinh} \left[\frac{\sqrt{d}}{\sqrt{e} x} \right] \text{Log}[x]}{\sqrt{e} x} \right) +$$

$$\frac{1}{3} \sqrt{d + e x^2} (4 d + e x^2) (a - b n \text{Log}[x] + b \text{Log}[c x^n]) +$$

$$d^{3/2} \text{Log}[x] (a - b n \text{Log}[x] + b \text{Log}[c x^n]) -$$

$$d^{3/2} (a - b n \text{Log}[x] + b \text{Log}[c x^n]) \text{Log}[d + \sqrt{d} \sqrt{d + e x^2}]$$

Problem 267: Result unnecessarily involves higher level functions.

$$\int \frac{(d + e x^2)^{3/2} (a + b \text{Log}[c x^n])}{x^3} dx$$

Optimal (type 4, 295 leaves, 18 steps):

$$-b e n \sqrt{d + e x^2} - \frac{b d n \sqrt{d + e x^2}}{4 x^2} + \frac{3}{4} b \sqrt{d} e n \text{ArcTanh} \left[\frac{\sqrt{d + e x^2}}{\sqrt{d}} \right] +$$

$$\frac{3}{4} b \sqrt{d} e n \text{ArcTanh} \left[\frac{\sqrt{d + e x^2}}{\sqrt{d}} \right]^2 + \frac{3}{2} e \sqrt{d + e x^2} (a + b \text{Log}[c x^n]) -$$

$$\frac{(d + e x^2)^{3/2} (a + b \text{Log}[c x^n])}{2 x^2} - \frac{3}{2} \sqrt{d} e \text{ArcTanh} \left[\frac{\sqrt{d + e x^2}}{\sqrt{d}} \right] (a + b \text{Log}[c x^n]) -$$

$$\frac{3}{2} b \sqrt{d} e n \text{ArcTanh} \left[\frac{\sqrt{d + e x^2}}{\sqrt{d}} \right] \text{Log} \left[\frac{2 \sqrt{d}}{\sqrt{d} - \sqrt{d + e x^2}} \right] -$$

$$\frac{3}{4} b \sqrt{d} e n \text{PolyLog} \left[2, 1 - \frac{2 \sqrt{d}}{\sqrt{d} - \sqrt{d + e x^2}} \right]$$

Result (type 5, 349 leaves):

$$\begin{aligned}
 & \frac{1}{\sqrt{1 + \frac{d}{e x^2}}} b e n \sqrt{d + e x^2} \left(-\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right\}, \left\{\frac{1}{2}, \frac{1}{2}\right\}, -\frac{d}{e x^2}\right] + \right. \\
 & \left. \sqrt{1 + \frac{d}{e x^2}} \text{Log}[x] - \frac{\sqrt{d} \text{ArcSinh}\left[\frac{\sqrt{d}}{\sqrt{e} x}\right] \text{Log}[x]}{\sqrt{e} x} \right) - \frac{1}{4 \sqrt{1 + \frac{d}{e x^2}} x^2} \\
 & b \sqrt{d} n \sqrt{d + e x^2} \left(2 \sqrt{d} \text{HypergeometricPFQ}\left[\left\{\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\}, \left\{\frac{3}{2}, \frac{3}{2}\right\}, -\frac{d}{e x^2}\right] + \right. \\
 & \left. \left(\sqrt{d} \sqrt{1 + \frac{d}{e x^2}} + \sqrt{e} x \text{ArcSinh}\left[\frac{\sqrt{d}}{\sqrt{e} x}\right] \right) (1 + 2 \text{Log}[x]) \right) - \\
 & \frac{(d - 2 e x^2) \sqrt{d + e x^2} (a - b n \text{Log}[x] + b \text{Log}[c x^n])}{2 x^2} + \\
 & \frac{3}{2} \sqrt{d} e \text{Log}[x] (a - b n \text{Log}[x] + b \text{Log}[c x^n]) - \\
 & \frac{3}{2} \sqrt{d} e (a - b n \text{Log}[x] + b \text{Log}[c x^n]) \text{Log}\left[d + \sqrt{d} \sqrt{d + e x^2}\right]
 \end{aligned}$$

Problem 268: Result unnecessarily involves higher level functions.

$$\int x^2 (d + e x^2)^{3/2} (a + b \text{Log}[c x^n]) dx$$

Optimal (type 4, 464 leaves, 19 steps):

$$\begin{aligned}
 & -\frac{11 b d^2 n x \sqrt{d+e x^2}}{192 e} - \frac{23}{288} b d n x^3 \sqrt{d+e x^2} - \frac{1}{36} b e n x^5 \sqrt{d+e x^2} - \\
 & \frac{b d^{5/2} n \sqrt{d+e x^2} \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]}{192 e^{3/2} \sqrt{1+\frac{e x^2}{d}}} - \frac{b d^{5/2} n \sqrt{d+e x^2} \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]^2}{32 e^{3/2} \sqrt{1+\frac{e x^2}{d}}} + \\
 & \frac{b d^{5/2} n \sqrt{d+e x^2} \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] \operatorname{Log}\left[1-e^{2 \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]}\right]}{16 e^{3/2} \sqrt{1+\frac{e x^2}{d}}} + \frac{d^2 x \sqrt{d+e x^2} (a+b \operatorname{Log}[c x^n])}{16 e} + \\
 & \frac{1}{8} d x^3 \sqrt{d+e x^2} (a+b \operatorname{Log}[c x^n]) + \frac{1}{6} x^3 (d+e x^2)^{3/2} (a+b \operatorname{Log}[c x^n]) - \\
 & \frac{d^{5/2} \sqrt{d+e x^2} \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] (a+b \operatorname{Log}[c x^n])}{16 e^{3/2} \sqrt{1+\frac{e x^2}{d}}} + \frac{b d^{5/2} n \sqrt{d+e x^2} \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]}\right]}{32 e^{3/2} \sqrt{1+\frac{e x^2}{d}}}
 \end{aligned}$$

Result (type 5, 331 leaves):

$$\begin{aligned}
 & \frac{1}{3600 e^{3/2} \sqrt{1+\frac{e x^2}{d}}} \\
 & \left(-400 b d e^{3/2} n x^3 \sqrt{d+e x^2} \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, \frac{3}{2}, \frac{3}{2}\right\}, \left\{\frac{5}{2}, \frac{5}{2}\right\}, -\frac{e x^2}{d}\right] - \right. \\
 & 144 b e^{5/2} n x^5 \sqrt{d+e x^2} \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, \frac{5}{2}, \frac{5}{2}\right\}, \left\{\frac{7}{2}, \frac{7}{2}\right\}, -\frac{e x^2}{d}\right] - \\
 & 75 \left(3 b d^{5/2} n \sqrt{d+e x^2} \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] \operatorname{Log}[x] + \sqrt{1+\frac{e x^2}{d}} \left(-a \sqrt{e} x \sqrt{d+e x^2} \right. \right. \\
 & \left. \left. (3 d^2 + 14 d e x^2 + 8 e^2 x^4) + 3 d^3 (a - b n \operatorname{Log}[x]) \operatorname{Log}\left[ex + \sqrt{e} \sqrt{d+e x^2}\right] - \right. \right. \\
 & \left. \left. b \operatorname{Log}[c x^n] \left(\sqrt{e} x \sqrt{d+e x^2} (3 d^2 + 14 d e x^2 + 8 e^2 x^4) - 3 d^3 \operatorname{Log}\left[ex + \sqrt{e} \sqrt{d+e x^2}\right] \right) \right) \right) \left. \right)
 \end{aligned}$$

Problem 269: Result unnecessarily involves higher level functions.

$$\int (d+e x^2)^{3/2} (a+b \operatorname{Log}[c x^n]) dx$$

Optimal (type 4, 378 leaves, 16 steps):

$$\begin{aligned}
 & -\frac{9}{32} b d n x \sqrt{d+e x^2} - \frac{1}{16} b n x (d+e x^2)^{3/2} + \frac{3 b d^{5/2} n \sqrt{1+\frac{e x^2}{d}} \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]^2}{16 \sqrt{e} \sqrt{d+e x^2}} - \\
 & \frac{9 b d^2 n \operatorname{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}}\right]}{32 \sqrt{e}} - \frac{3 b d^{5/2} n \sqrt{1+\frac{e x^2}{d}} \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] \operatorname{Log}\left[1-e^{2 \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]}\right]}{8 \sqrt{e} \sqrt{d+e x^2}} + \\
 & \frac{3}{8} d x \sqrt{d+e x^2} (a+b \operatorname{Log}[c x^n]) + \frac{1}{4} x (d+e x^2)^{3/2} (a+b \operatorname{Log}[c x^n]) + \\
 & \frac{3 d^{5/2} \sqrt{1+\frac{e x^2}{d}} \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] (a+b \operatorname{Log}[c x^n])}{8 \sqrt{e} \sqrt{d+e x^2}} - \frac{3 b d^{5/2} n \sqrt{1+\frac{e x^2}{d}} \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]}\right]}{16 \sqrt{e} \sqrt{d+e x^2}}
 \end{aligned}$$

Result (type 5, 314 leaves):

$$\begin{aligned}
 & \frac{1}{72 \sqrt{e} \sqrt{1+\frac{e x^2}{d}}} \left(-8 b e^{3/2} n x^3 \sqrt{d+e x^2} \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, \frac{3}{2}, \frac{3}{2}\right\}, \left\{\frac{5}{2}, \frac{5}{2}\right\}, -\frac{e x^2}{d}\right] + \right. \\
 & \left. 9 \left(-4 b d \sqrt{e} n x \sqrt{d+e x^2} \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\}, \left\{\frac{3}{2}, \frac{3}{2}\right\}, -\frac{e x^2}{d}\right] + \right. \right. \\
 & \left. \left. b d^{3/2} n \sqrt{d+e x^2} \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] (-2+3 \operatorname{Log}[x]) + \sqrt{1+\frac{e x^2}{d}} \right. \right. \\
 & \left. \left. \left(\sqrt{e} x \sqrt{d+e x^2} (5 a d-2 b d n+2 a e x^2) + 3 d^2 (a-b n \operatorname{Log}[x]) \operatorname{Log}\left[e x+\sqrt{e} \sqrt{d+e x^2}\right] + \right. \right. \right. \\
 & \left. \left. \left. b \operatorname{Log}[c x^n] \left(\sqrt{e} x \sqrt{d+e x^2} (5 d+2 e x^2) + 3 d^2 \operatorname{Log}\left[e x+\sqrt{e} \sqrt{d+e x^2}\right] \right) \right) \right) \right)
 \end{aligned}$$

Problem 270: Result unnecessarily involves higher level functions.

$$\int \frac{(d+e x^2)^{3/2} (a+b \operatorname{Log}[c x^n])}{x^2} dx$$

Optimal (type 4, 400 leaves, 14 steps):

$$\begin{aligned}
 & -\frac{b d n \sqrt{d+e x^2}}{x} - \frac{1}{4} b e n x \sqrt{d+e x^2} + \\
 & \frac{3 b \sqrt{d} \sqrt{e} n \sqrt{d+e x^2} \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]}{4 \sqrt{1+\frac{e x^2}{d}}} + \frac{3 b \sqrt{d} \sqrt{e} n \sqrt{d+e x^2} \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]^2}{4 \sqrt{1+\frac{e x^2}{d}}} - \\
 & \frac{3 b \sqrt{d} \sqrt{e} n \sqrt{d+e x^2} \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] \operatorname{Log}\left[1-e^{2 \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]}\right]}{2 \sqrt{1+\frac{e x^2}{d}}} + \frac{3}{2} e x \sqrt{d+e x^2} (a+b \operatorname{Log}[c x^n]) - \\
 & \frac{(d+e x^2)^{3/2} (a+b \operatorname{Log}[c x^n])}{x} + \frac{3 \sqrt{d} \sqrt{e} \sqrt{d+e x^2} \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] (a+b \operatorname{Log}[c x^n])}{2 \sqrt{1+\frac{e x^2}{d}}} - \\
 & \frac{3 b \sqrt{d} \sqrt{e} n \sqrt{d+e x^2} \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]}\right]}{4 \sqrt{1+\frac{e x^2}{d}}}
 \end{aligned}$$

Result (type 5, 329 leaves):

$$\begin{aligned}
 & -\frac{1}{x \sqrt{1+\frac{e x^2}{d}}} b \sqrt{d} n \sqrt{d+e x^2} \left(\sqrt{d} \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right\}, \left\{\frac{1}{2}, \frac{1}{2}\right\}, -\frac{e x^2}{d}\right] + \right. \\
 & \left. \left(\sqrt{d} \sqrt{1+\frac{e x^2}{d}} - \sqrt{e} x \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] \right) \operatorname{Log}[x] \right) + \frac{1}{4 \sqrt{1+\frac{e x^2}{d}}} \\
 & b \sqrt{e} n \sqrt{d+e x^2} \left(-2 \sqrt{e} x \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\}, \left\{\frac{3}{2}, \frac{3}{2}\right\}, -\frac{e x^2}{d}\right] + \right. \\
 & \left. \left(\sqrt{e} x \sqrt{1+\frac{e x^2}{d}} + \sqrt{d} \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] \right) (-1+2 \operatorname{Log}[x]) \right) - \\
 & \frac{(2 d-e x^2) \sqrt{d+e x^2} (a-b n \operatorname{Log}[x]+b \operatorname{Log}[c x^n])}{2 x} + \\
 & \frac{3}{2} d \sqrt{e} (a-b n \operatorname{Log}[x]+b \operatorname{Log}[c x^n]) \operatorname{Log}\left[e x+\sqrt{e} \sqrt{d+e x^2}\right]
 \end{aligned}$$

Problem 271: Result unnecessarily involves higher level functions.

$$\int \frac{(d+ex^2)^{3/2} (a+b \operatorname{Log}[cx^n])}{x^4} dx$$

Optimal (type 4, 400 leaves, 13 steps):

$$\begin{aligned} & -\frac{4ben\sqrt{d+ex^2}}{3x} - \frac{bn(d+ex^2)^{3/2}}{9x^3} + \frac{4be^{3/2}n\sqrt{d+ex^2} \operatorname{ArcSinh}\left[\frac{\sqrt{e}x}{\sqrt{d}}\right]}{3\sqrt{d}\sqrt{1+\frac{ex^2}{d}}} + \\ & \frac{be^{3/2}n\sqrt{d+ex^2} \operatorname{ArcSinh}\left[\frac{\sqrt{e}x}{\sqrt{d}}\right]^2}{2\sqrt{d}\sqrt{1+\frac{ex^2}{d}}} - \frac{be^{3/2}n\sqrt{d+ex^2} \operatorname{ArcSinh}\left[\frac{\sqrt{e}x}{\sqrt{d}}\right] \operatorname{Log}\left[1-e^{2\operatorname{ArcSinh}\left[\frac{\sqrt{e}x}{\sqrt{d}}\right]}\right]}{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}} - \\ & \frac{e\sqrt{d+ex^2}(a+b \operatorname{Log}[cx^n])}{x} - \frac{(d+ex^2)^{3/2}(a+b \operatorname{Log}[cx^n])}{3x^3} + \\ & \frac{e^{3/2}\sqrt{d+ex^2} \operatorname{ArcSinh}\left[\frac{\sqrt{e}x}{\sqrt{d}}\right](a+b \operatorname{Log}[cx^n])}{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}} - \frac{be^{3/2}n\sqrt{d+ex^2} \operatorname{PolyLog}\left[2, e^{2\operatorname{ArcSinh}\left[\frac{\sqrt{e}x}{\sqrt{d}}\right]}\right]}{2\sqrt{d}\sqrt{1+\frac{ex^2}{d}}} \end{aligned}$$

Result (type 5, 269 leaves):

$$\begin{aligned} & \frac{1}{9x^3\sqrt{1+\frac{ex^2}{d}}} \\ & bdn\sqrt{d+ex^2} \left(-\operatorname{Hypergeometric2F1}\left[-\frac{3}{2}, -\frac{3}{2}, -\frac{1}{2}, -\frac{ex^2}{d}\right] - 3\left(1+\frac{ex^2}{d}\right)^{3/2} \operatorname{Log}[x] \right) + \frac{1}{x\sqrt{1+\frac{ex^2}{d}}} \\ & ben\sqrt{d+ex^2} \left(-\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right\}, \left\{\frac{1}{2}, \frac{1}{2}\right\}, -\frac{ex^2}{d}\right] - \sqrt{1+\frac{ex^2}{d}} \operatorname{Log}[x] + \right. \\ & \left. \frac{\sqrt{e}x \operatorname{ArcSinh}\left[\frac{\sqrt{e}x}{\sqrt{d}}\right] \operatorname{Log}[x]}{\sqrt{d}} \right) - \frac{\sqrt{d+ex^2}(d+4ex^2)(a-bn \operatorname{Log}[x] + b \operatorname{Log}[cx^n])}{3x^3} + \\ & e^{3/2}(a-bn \operatorname{Log}[x] + b \operatorname{Log}[cx^n]) \operatorname{Log}\left[ex + \sqrt{e}\sqrt{d+ex^2}\right] \end{aligned}$$

Problem 279: Result unnecessarily involves higher level functions.

$$\int \frac{a + b \operatorname{Log}[c x^n]}{x \sqrt{d + e x^2}} dx$$

Optimal (type 4, 166 leaves, 8 steps):

$$\frac{b n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d}}\right]^2}{2 \sqrt{d}} - \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d}}\right] (a + b \operatorname{Log}[c x^n])}{\sqrt{d}} -$$

$$\frac{b n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d}}\right] \operatorname{Log}\left[\frac{2 \sqrt{d}}{\sqrt{d}-\sqrt{d+e x^2}}\right]}{\sqrt{d}} - \frac{b n \operatorname{PolyLog}\left[2, 1 - \frac{2 \sqrt{d}}{\sqrt{d}-\sqrt{d+e x^2}}\right]}{2 \sqrt{d}}$$

Result (type 5, 162 leaves):

$$\frac{1}{\sqrt{d + e x^2}} b n \sqrt{1 + \frac{d}{e x^2}}$$

$$\left(-\operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\}, \left\{\frac{3}{2}, \frac{3}{2}\right\}, -\frac{d}{e x^2}\right] - \frac{\sqrt{e} x \operatorname{ArcSinh}\left[\frac{\sqrt{d}}{\sqrt{e} x}\right] \operatorname{Log}[x]}{\sqrt{d}} \right) -$$

$$\frac{\operatorname{Log}[x] (-a - b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))}{\sqrt{d}} + \frac{(-a - b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])) \operatorname{Log}[d + \sqrt{d} \sqrt{d + e x^2}]}{\sqrt{d}}$$

Problem 280: Result unnecessarily involves higher level functions.

$$\int \frac{a + b \operatorname{Log}[c x^n]}{x^3 \sqrt{d + e x^2}} dx$$

Optimal (type 4, 258 leaves, 14 steps):

$$-\frac{b n \sqrt{d + e x^2}}{4 d x^2} - \frac{b e n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d}}\right]}{4 d^{3/2}} - \frac{b e n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d}}\right]^2}{4 d^{3/2}} -$$

$$\frac{\sqrt{d + e x^2} (a + b \operatorname{Log}[c x^n])}{2 d x^2} + \frac{e \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d}}\right] (a + b \operatorname{Log}[c x^n])}{2 d^{3/2}} +$$

$$\frac{b e n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d}}\right] \operatorname{Log}\left[\frac{2 \sqrt{d}}{\sqrt{d}-\sqrt{d+e x^2}}\right]}{2 d^{3/2}} + \frac{b e n \operatorname{PolyLog}\left[2, 1 - \frac{2 \sqrt{d}}{\sqrt{d}-\sqrt{d+e x^2}}\right]}{4 d^{3/2}}$$

Result (type 5, 229 leaves):

$$\frac{1}{36 d^{3/2}} \left(\frac{1}{x^2 \sqrt{d+ex^2}} b n \sqrt{1 + \frac{d}{ex^2}} \left(2 d^{3/2} \text{HypergeometricPFQ} \left[\left\{ \frac{3}{2}, \frac{3}{2}, \frac{3}{2} \right\}, \left\{ \frac{5}{2}, \frac{5}{2} \right\}, -\frac{d}{ex^2} \right] + \right. \right. \\ \left. \left. 9 ex^2 \left(-\sqrt{d} \sqrt{1 + \frac{d}{ex^2}} + \sqrt{e} x \text{ArcSinh} \left[\frac{\sqrt{d}}{\sqrt{e} x} \right] \right) (1 + 2 \text{Log}[x]) \right) - \right. \\ \left. \frac{18 \sqrt{d} \sqrt{d+ex^2} (a - b n \text{Log}[x] + b \text{Log}[cx^n])}{x^2} - 18 e \text{Log}[x] (a - b n \text{Log}[x] + b \text{Log}[cx^n]) + \right. \\ \left. 18 e (a - b n \text{Log}[x] + b \text{Log}[cx^n]) \text{Log} [d + \sqrt{d} \sqrt{d+ex^2}] \right)$$

Problem 281: Result unnecessarily involves higher level functions.

$$\int \frac{x^2 (a + b \text{Log}[cx^n])}{\sqrt{d+ex^2}} dx$$

Optimal (type 4, 359 leaves, 12 steps):

$$-\frac{bnx\sqrt{d+ex^2}}{4e} - \frac{bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\text{ArcSinh}\left[\frac{\sqrt{e}x}{\sqrt{d}}\right]}{4e^{3/2}\sqrt{d+ex^2}} - \frac{bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\text{ArcSinh}\left[\frac{\sqrt{e}x}{\sqrt{d}}\right]^2}{4e^{3/2}\sqrt{d+ex^2}} + \\ \frac{bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\text{ArcSinh}\left[\frac{\sqrt{e}x}{\sqrt{d}}\right]\text{Log}\left[1-e^{2\text{ArcSinh}\left[\frac{\sqrt{e}x}{\sqrt{d}}\right]}\right]}{2e^{3/2}\sqrt{d+ex^2}} + \frac{x\sqrt{d+ex^2}(a+b\text{Log}[cx^n])}{2e} - \\ \frac{d^{3/2}\sqrt{1+\frac{ex^2}{d}}\text{ArcSinh}\left[\frac{\sqrt{e}x}{\sqrt{d}}\right](a+b\text{Log}[cx^n])}{2e^{3/2}\sqrt{d+ex^2}} + \frac{bd^{3/2}n\sqrt{1+\frac{ex^2}{d}}\text{PolyLog}\left[2, e^{2\text{ArcSinh}\left[\frac{\sqrt{e}x}{\sqrt{d}}\right]}\right]}{4e^{3/2}\sqrt{d+ex^2}}$$

Result (type 5, 205 leaves):

$$\frac{1}{36 e^2} \left(\frac{1}{\sqrt{d+ex^2}} b n \sqrt{1 + \frac{ex^2}{d}} \left(2 e^2 x^3 \text{HypergeometricPFQ} \left[\left\{ \frac{3}{2}, \frac{3}{2}, \frac{3}{2} \right\}, \left\{ \frac{5}{2}, \frac{5}{2} \right\}, -\frac{ex^2}{d} \right] + \right. \right. \\ \left. \left. 9 d \sqrt{e} \left(\sqrt{e} x \sqrt{1 + \frac{ex^2}{d}} - \sqrt{d} \text{ArcSinh} \left[\frac{\sqrt{e} x}{\sqrt{d}} \right] \right) (-1 + 2 \text{Log}[x]) \right) + \right. \\ \left. 18 e x \sqrt{d+ex^2} (a - b n \text{Log}[x] + b \text{Log}[cx^n]) - \right. \\ \left. 18 d \sqrt{e} (a - b n \text{Log}[x] + b \text{Log}[cx^n]) \text{Log} [e x + \sqrt{e} \sqrt{d+ex^2}] \right)$$

Problem 290: Result unnecessarily involves higher level functions.

$$\int \frac{a + b \operatorname{Log}[c x^n]}{x (d + e x^2)^{3/2}} dx$$

Optimal (type 4, 209 leaves, 11 steps):

$$\frac{b n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d}}\right]}{d^{3/2}} + \frac{b n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d}}\right]^2}{2 d^{3/2}} + \left(\frac{1}{d \sqrt{d+e x^2}} - \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d}}\right]}{d^{3/2}}\right) (a + b \operatorname{Log}[c x^n]) - \frac{b n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d}}\right] \operatorname{Log}\left[\frac{2 \sqrt{d}}{\sqrt{d}-\sqrt{d+e x^2}}\right]}{d^{3/2}} - \frac{b n \operatorname{PolyLog}\left[2, 1 - \frac{2 \sqrt{d}}{\sqrt{d}-\sqrt{d+e x^2}}\right]}{2 d^{3/2}}$$

Result (type 5, 241 leaves):

$$\left(-b d^{3/2} n \sqrt{1 + \frac{d}{e x^2}} \operatorname{HypergeometricPFQ}\left[\left\{\frac{3}{2}, \frac{3}{2}, \frac{3}{2}\right\}, \left\{\frac{5}{2}, \frac{5}{2}\right\}, -\frac{d}{e x^2}\right] + 9 e x^2 \left(-b \sqrt{e} n \sqrt{1 + \frac{d}{e x^2}} x \operatorname{ArcSinh}\left[\frac{\sqrt{d}}{\sqrt{e} x}\right] \operatorname{Log}[x] - b n \sqrt{d+e x^2} \operatorname{Log}[x]^2 + \sqrt{d+e x^2} \operatorname{Log}[x] (a + b \operatorname{Log}[c x^n] + b n \operatorname{Log}[d + \sqrt{d} \sqrt{d+e x^2}]) + (a + b \operatorname{Log}[c x^n]) (\sqrt{d} - \sqrt{d+e x^2} \operatorname{Log}[d + \sqrt{d} \sqrt{d+e x^2}])\right)\right) / (9 d^{3/2} e x^2 \sqrt{d+e x^2})$$

Problem 291: Result unnecessarily involves higher level functions.

$$\int \frac{a + b \operatorname{Log}[c x^n]}{x^3 (d + e x^2)^{3/2}} dx$$

Optimal (type 4, 287 leaves, 12 steps):

$$\begin{aligned}
 & - \frac{b n \sqrt{d+e x^2}}{4 d^2 x^2} - \frac{5 b e n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d}}\right]}{4 d^{5/2}} - \frac{3 b e n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d}}\right]^2}{4 d^{5/2}} \\
 & \frac{3 e (a+b \operatorname{Log}[c x^n])}{2 d^2 \sqrt{d+e x^2}} - \frac{a+b \operatorname{Log}[c x^n]}{2 d x^2 \sqrt{d+e x^2}} + \frac{3 e \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d}}\right] (a+b \operatorname{Log}[c x^n])}{2 d^{5/2}} + \\
 & \frac{3 b e n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d}}\right] \operatorname{Log}\left[\frac{2 \sqrt{d}}{\sqrt{d}-\sqrt{d+e x^2}}\right]}{2 d^{5/2}} + \frac{3 b e n \operatorname{PolyLog}\left[2, 1 - \frac{2 \sqrt{d}}{\sqrt{d}-\sqrt{d+e x^2}}\right]}{4 d^{5/2}}
 \end{aligned}$$

Result (type 5, 218 leaves):

$$\begin{aligned}
 & \left(3 b d^{5/2} n \sqrt{1 + \frac{d}{e x^2}} \operatorname{HypergeometricPFQ}\left[\left\{\frac{5}{2}, \frac{5}{2}, \frac{5}{2}\right\}, \left\{\frac{7}{2}, \frac{7}{2}\right\}, -\frac{d}{e x^2}\right] - \right. \\
 & 5 b d^{5/2} n \sqrt{1 + \frac{d}{e x^2}} \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, \frac{5}{2}, \frac{7}{2}, -\frac{d}{e x^2}\right] (1 + 2 \operatorname{Log}[x]) - \\
 & 25 e x^2 (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n]) \left(\sqrt{d} (d + 3 e x^2) + 3 e x^2 \sqrt{d+e x^2} \operatorname{Log}[x] - \right. \\
 & \left. \left. 3 e x^2 \sqrt{d+e x^2} \operatorname{Log}\left[d + \sqrt{d} \sqrt{d+e x^2}\right] \right) \right) / \left(50 d^{5/2} e x^4 \sqrt{d+e x^2} \right)
 \end{aligned}$$

Problem 292: Result unnecessarily involves higher level functions.

$$\int \frac{x^2 (a + b \operatorname{Log}[c x^n])}{(d + e x^2)^{3/2}} dx$$

Optimal (type 4, 328 leaves, 11 steps):

$$\begin{aligned}
 & \frac{b \sqrt{d} n \sqrt{1 + \frac{e x^2}{d}} \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]}{e^{3/2} \sqrt{d+e x^2}} + \frac{b \sqrt{d} n \sqrt{1 + \frac{e x^2}{d}} \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]^2}{2 e^{3/2} \sqrt{d+e x^2}} \\
 & \frac{b \sqrt{d} n \sqrt{1 + \frac{e x^2}{d}} \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] \operatorname{Log}\left[1 - e^{2 \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]}\right]}{e^{3/2} \sqrt{d+e x^2}} - \frac{x (a + b \operatorname{Log}[c x^n])}{e \sqrt{d+e x^2}} + \\
 & \frac{\sqrt{d} \sqrt{1 + \frac{e x^2}{d}} \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] (a + b \operatorname{Log}[c x^n])}{e^{3/2} \sqrt{d+e x^2}} - \frac{b \sqrt{d} n \sqrt{1 + \frac{e x^2}{d}} \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]}\right]}{2 e^{3/2} \sqrt{d+e x^2}}
 \end{aligned}$$

Result (type 5, 217 leaves):

$$\begin{aligned}
 & - \left(\left(b n \sqrt{1 + \frac{e x^2}{d}} \left(e^{3/2} x^3 (d + e x^2) \text{HypergeometricPFQ} \left[\left\{ \frac{3}{2}, \frac{3}{2}, \frac{3}{2} \right\}, \left\{ \frac{5}{2}, \frac{5}{2} \right\}, -\frac{e x^2}{d} \right] + 9 d^2 \sqrt{e} x \right. \right. \right. \\
 & \quad \left. \left. \left. \sqrt{1 + \frac{e x^2}{d}} \text{Log}[x] - 9 d^{3/2} (d + e x^2) \text{ArcSinh} \left[\frac{\sqrt{e} x}{\sqrt{d}} \right] \text{Log}[x] \right) \right) / \left(9 d e^{3/2} (d + e x^2)^{3/2} \right) \right) - \\
 & \quad \frac{x (a - b n \text{Log}[x] + b \text{Log}[c x^n])}{e \sqrt{d + e x^2}} + \frac{(a - b n \text{Log}[x] + b \text{Log}[c x^n]) \text{Log}[e x + \sqrt{e} \sqrt{d + e x^2}]}{e^{3/2}}
 \end{aligned}$$

Problem 301: Result unnecessarily involves higher level functions.

$$\int \frac{a + b \text{Log}[c x^n]}{x (d + e x^2)^{5/2}} dx$$

Optimal (type 4, 251 leaves, 15 steps):

$$\begin{aligned}
 & - \frac{b n}{3 d^2 \sqrt{d + e x^2}} + \frac{4 b n \text{ArcTanh} \left[\frac{\sqrt{d + e x^2}}{\sqrt{d}} \right]}{3 d^{5/2}} + \frac{b n \text{ArcTanh} \left[\frac{\sqrt{d + e x^2}}{\sqrt{d}} \right]^2}{2 d^{5/2}} + \\
 & \frac{1}{3} \left(\frac{1}{d (d + e x^2)^{3/2}} + \frac{3}{d^2 \sqrt{d + e x^2}} - \frac{3 \text{ArcTanh} \left[\frac{\sqrt{d + e x^2}}{\sqrt{d}} \right]}{d^{5/2}} \right) (a + b \text{Log}[c x^n]) - \\
 & \frac{b n \text{ArcTanh} \left[\frac{\sqrt{d + e x^2}}{\sqrt{d}} \right] \text{Log} \left[\frac{2 \sqrt{d}}{\sqrt{d} - \sqrt{d + e x^2}} \right]}{d^{5/2}} - \frac{b n \text{PolyLog} \left[2, 1 - \frac{2 \sqrt{d}}{\sqrt{d} - \sqrt{d + e x^2}} \right]}{2 d^{5/2}}
 \end{aligned}$$

Result (type 5, 273 leaves):

$$\begin{aligned}
 & \left(b n \sqrt{1 + \frac{d}{e x^2}} \left(-3 d^{5/2} (d + e x^2)^2 \text{HypergeometricPFQ} \left[\left\{ \frac{5}{2}, \frac{5}{2}, \frac{5}{2} \right\}, \left\{ \frac{7}{2}, \frac{7}{2} \right\}, -\frac{d}{e x^2} \right] + 25 \sqrt{d} e^3 \right. \right. \\
 & \quad \left. \left. \left. \sqrt{1 + \frac{d}{e x^2}} x^6 (4 d + 3 e x^2) \text{Log}[x] - 75 e^{5/2} x^5 (d + e x^2)^2 \text{ArcSinh} \left[\frac{\sqrt{d}}{\sqrt{e} x} \right] \text{Log}[x] \right) \right) / \right. \\
 & \quad \left. \left(75 d^{5/2} e^2 x^4 (d + e x^2)^{5/2} \right) + \frac{(4 d + 3 e x^2) (a - b n \text{Log}[x] + b \text{Log}[c x^n])}{3 d^2 (d + e x^2)^{3/2}} + \right. \\
 & \quad \left. \frac{\text{Log}[x] (a - b n \text{Log}[x] + b \text{Log}[c x^n])}{d^{5/2}} - \right. \\
 & \quad \left. \frac{(a - b n \text{Log}[x] + b \text{Log}[c x^n]) \text{Log}[d + \sqrt{d} \sqrt{d + e x^2}]}{d^{5/2}} \right)
 \end{aligned}$$

Problem 302: Result unnecessarily involves higher level functions.

$$\int \frac{a + b \operatorname{Log}[c x^n]}{x^3 (d + e x^2)^{5/2}} dx$$

Optimal (type 4, 337 leaves, 14 steps):

$$\begin{aligned} & \frac{b e n}{3 d^3 \sqrt{d + e x^2}} - \frac{b n \sqrt{d + e x^2}}{4 d^3 x^2} - \frac{31 b e n \operatorname{ArcTanh}\left[\frac{\sqrt{d + e x^2}}{\sqrt{d}}\right]}{12 d^{7/2}} - \\ & \frac{5 b e n \operatorname{ArcTanh}\left[\frac{\sqrt{d + e x^2}}{\sqrt{d}}\right]^2}{4 d^{7/2}} - \frac{5 e (a + b \operatorname{Log}[c x^n])}{6 d^2 (d + e x^2)^{3/2}} - \frac{a + b \operatorname{Log}[c x^n]}{2 d x^2 (d + e x^2)^{3/2}} - \\ & \frac{5 e (a + b \operatorname{Log}[c x^n])}{2 d^3 \sqrt{d + e x^2}} + \frac{5 e \operatorname{ArcTanh}\left[\frac{\sqrt{d + e x^2}}{\sqrt{d}}\right] (a + b \operatorname{Log}[c x^n])}{2 d^{7/2}} + \\ & \frac{5 b e n \operatorname{ArcTanh}\left[\frac{\sqrt{d + e x^2}}{\sqrt{d}}\right] \operatorname{Log}\left[\frac{2 \sqrt{d}}{\sqrt{d} - \sqrt{d + e x^2}}\right]}{2 d^{7/2}} + \frac{5 b e n \operatorname{PolyLog}\left[2, 1 - \frac{2 \sqrt{d}}{\sqrt{d} - \sqrt{d + e x^2}}\right]}{4 d^{7/2}} \end{aligned}$$

Result (type 5, 227 leaves):

$$\begin{aligned} & \left(b n \sqrt{1 + \frac{d}{e x^2}} \left(5 \operatorname{HypergeometricPFQ}\left[\left\{\frac{7}{2}, \frac{7}{2}, \frac{7}{2}\right\}, \left\{\frac{9}{2}, \frac{9}{2}\right\}, -\frac{d}{e x^2}\right] - \right. \right. \\ & \quad \left. \left. 7 \operatorname{Hypergeometric2F1}\left[\frac{5}{2}, \frac{7}{2}, \frac{9}{2}, -\frac{d}{e x^2}\right] (1 + 2 \operatorname{Log}[x]) \right) \right) / \\ & \left(98 e^2 x^6 \sqrt{d + e x^2} \right) - \frac{(3 d^2 + 20 d e x^2 + 15 e^2 x^4) (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n])}{6 d^3 x^2 (d + e x^2)^{3/2}} - \\ & \frac{5 e \operatorname{Log}[x] (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n])}{2 d^{7/2}} + \\ & \frac{5 e (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[d + \sqrt{d} \sqrt{d + e x^2}\right]}{2 d^{7/2}} \end{aligned}$$

Problem 303: Result unnecessarily involves higher level functions.

$$\int \frac{x^6 (a + b \operatorname{Log}[c x^n])}{(d + e x^2)^{5/2}} dx$$

Optimal (type 4, 443 leaves, 24 steps):

$$\frac{b d n x}{3 e^3 \sqrt{d+e x^2}} - \frac{b n x \sqrt{d+e x^2}}{4 e^3} - \frac{31 b d^{3/2} n \sqrt{1+\frac{e x^2}{d}} \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]}{12 e^{7/2} \sqrt{d+e x^2}} -$$

$$\frac{5 b d^{3/2} n \sqrt{1+\frac{e x^2}{d}} \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]^2}{4 e^{7/2} \sqrt{d+e x^2}} + \frac{5 b d^{3/2} n \sqrt{1+\frac{e x^2}{d}} \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] \operatorname{Log}\left[1-e^{2 \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]}\right]}{2 e^{7/2} \sqrt{d+e x^2}} -$$

$$\frac{x^5 (a+b \operatorname{Log}[c x^n])}{3 e (d+e x^2)^{3/2}} - \frac{5 x^3 (a+b \operatorname{Log}[c x^n])}{3 e^2 \sqrt{d+e x^2}} + \frac{5 x \sqrt{d+e x^2} (a+b \operatorname{Log}[c x^n])}{2 e^3} -$$

$$\frac{5 d^{3/2} \sqrt{1+\frac{e x^2}{d}} \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] (a+b \operatorname{Log}[c x^n])}{2 e^{7/2} \sqrt{d+e x^2}} + \frac{5 b d^{3/2} n \sqrt{1+\frac{e x^2}{d}} \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]}\right]}{4 e^{7/2} \sqrt{d+e x^2}}$$

Result (type 5, 199 leaves):

$$\frac{1}{98 d^2 \sqrt{d+e x^2}} b n x^7 \sqrt{1+\frac{e x^2}{d}} \left(5 \operatorname{HypergeometricPFQ}\left[\left\{\frac{7}{2}, \frac{7}{2}, \frac{7}{2}\right\}, \left\{\frac{9}{2}, \frac{9}{2}\right\}, -\frac{e x^2}{d}\right] + \right.$$

$$\left. 7 \operatorname{Hypergeometric2F1}\left[\frac{5}{2}, \frac{7}{2}, \frac{9}{2}, -\frac{e x^2}{d}\right] (-1+2 \operatorname{Log}[x]) \right) +$$

$$\frac{x (15 d^2 + 20 d e x^2 + 3 e^2 x^4) (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n])}{6 e^3 (d+e x^2)^{3/2}} -$$

$$\frac{5 d (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[e x + \sqrt{e} \sqrt{d+e x^2}\right]}{2 e^{7/2}}$$

Problem 304: Result unnecessarily involves higher level functions.

$$\int \frac{x^4 (a+b \operatorname{Log}[c x^n])}{(d+e x^2)^{5/2}} dx$$

Optimal (type 4, 383 leaves, 13 steps):

$$\begin{aligned}
 & - \frac{b n x}{3 e^2 \sqrt{d+e x^2}} + \frac{4 b \sqrt{d} n \sqrt{1+\frac{e x^2}{d}} \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]}{3 e^{5/2} \sqrt{d+e x^2}} + \frac{b \sqrt{d} n \sqrt{1+\frac{e x^2}{d}} \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]^2}{2 e^{5/2} \sqrt{d+e x^2}} \\
 & \frac{b \sqrt{d} n \sqrt{1+\frac{e x^2}{d}} \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] \operatorname{Log}\left[1-e^{2 \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]}\right]}{e^{5/2} \sqrt{d+e x^2}} - \frac{x^3 (a+b \operatorname{Log}[c x^n])}{3 e (d+e x^2)^{3/2}} \\
 & \frac{x (a+b \operatorname{Log}[c x^n])}{e^2 \sqrt{d+e x^2}} + \frac{\sqrt{d} \sqrt{1+\frac{e x^2}{d}} \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] (a+b \operatorname{Log}[c x^n])}{e^{5/2} \sqrt{d+e x^2}} \\
 & \frac{b \sqrt{d} n \sqrt{1+\frac{e x^2}{d}} \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]}\right]}{2 e^{5/2} \sqrt{d+e x^2}}
 \end{aligned}$$

Result (type 5, 244 leaves):

$$\begin{aligned}
 & - \left(\left(b n \sqrt{1+\frac{e x^2}{d}} \left(3 e^{5/2} x^5 (d+e x^2)^2 \operatorname{HypergeometricPFQ}\left[\left\{\frac{5}{2}, \frac{5}{2}, \frac{5}{2}\right\}, \left\{\frac{7}{2}, \frac{7}{2}\right\}, -\frac{e x^2}{d}\right] + \right. \right. \right. \\
 & \left. \left. \left. 25 d^3 \sqrt{e} x (3 d+4 e x^2) \sqrt{1+\frac{e x^2}{d}} \operatorname{Log}[x] - 75 d^{5/2} (d+e x^2)^2 \operatorname{ArcSinh}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] \operatorname{Log}[x] \right) \right) \right) / \\
 & \left(75 d^2 e^{5/2} (d+e x^2)^{5/2} \right) - \frac{x (3 d+4 e x^2) (a-b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n])}{3 e^2 (d+e x^2)^{3/2}} + \\
 & \frac{(a-b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[e x + \sqrt{e} \sqrt{d+e x^2}\right]}{e^{5/2}}
 \end{aligned}$$

Problem 312: Result unnecessarily involves higher level functions.

$$\int \frac{a+b \operatorname{Log}[c x^n]}{x^3 \sqrt{d-e x} \sqrt{d+e x}} dx$$

Optimal (type 4, 489 leaves, 13 steps):

$$\begin{aligned}
 & - \frac{b n (d^2 - e^2 x^2)}{4 d^2 x^2 \sqrt{d - e x} \sqrt{d + e x}} + \frac{b e^2 n \sqrt{1 - \frac{e^2 x^2}{d^2}} \operatorname{ArcTanh}\left[\sqrt{1 - \frac{e^2 x^2}{d^2}}\right]}{4 d^2 \sqrt{d - e x} \sqrt{d + e x}} + \\
 & \frac{b e^2 n \sqrt{1 - \frac{e^2 x^2}{d^2}} \operatorname{ArcTanh}\left[\sqrt{1 - \frac{e^2 x^2}{d^2}}\right]^2}{4 d^2 \sqrt{d - e x} \sqrt{d + e x}} - \frac{(d^2 - e^2 x^2) (a + b \operatorname{Log}[c x^n])}{2 d^2 x^2 \sqrt{d - e x} \sqrt{d + e x}} - \\
 & \frac{e^2 \sqrt{1 - \frac{e^2 x^2}{d^2}} \operatorname{ArcTanh}\left[\sqrt{1 - \frac{e^2 x^2}{d^2}}\right] (a + b \operatorname{Log}[c x^n])}{2 d^2 \sqrt{d - e x} \sqrt{d + e x}} - \\
 & \frac{b e^2 n \sqrt{1 - \frac{e^2 x^2}{d^2}} \operatorname{ArcTanh}\left[\sqrt{1 - \frac{e^2 x^2}{d^2}}\right] \operatorname{Log}\left[\frac{2}{1 - \sqrt{1 - \frac{e^2 x^2}{d^2}}}\right]}{2 d^2 \sqrt{d - e x} \sqrt{d + e x}} - \frac{b e^2 n \sqrt{1 - \frac{e^2 x^2}{d^2}} \operatorname{PolyLog}\left[2, -\frac{1 + \sqrt{1 - \frac{e^2 x^2}{d^2}}}{1 - \sqrt{1 - \frac{e^2 x^2}{d^2}}}\right]}{4 d^2 \sqrt{d - e x} \sqrt{d + e x}}
 \end{aligned}$$

Result (type 5, 255 leaves):

$$\begin{aligned}
 & \frac{1}{36 d^3} \left(\left(b n (-d^2 + e^2 x^2) \left(2 d^3 \operatorname{HypergeometricPFQ}\left[\left\{\frac{3}{2}, \frac{3}{2}, \frac{3}{2}\right\}, \left\{\frac{5}{2}, \frac{5}{2}\right\}, \frac{d^2}{e^2 x^2}\right] + \right. \right. \right. \\
 & \left. \left. \left. 9 e^2 x^2 \left(d \sqrt{1 - \frac{d^2}{e^2 x^2}} - e x \operatorname{ArcSin}\left[\frac{d}{e x}\right] \right) (1 + 2 \operatorname{Log}[x]) \right) \right) \right) / \\
 & \left(e^2 \sqrt{1 - \frac{d^2}{e^2 x^2}} x^4 \sqrt{d - e x} \sqrt{d + e x} \right) - \frac{18 d \sqrt{d - e x} \sqrt{d + e x} (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n])}{x^2} + \\
 & 18 e^2 \operatorname{Log}[x] (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n]) - \\
 & \left. 18 e^2 (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[d + \sqrt{d - e x} \sqrt{d + e x}\right] \right)
 \end{aligned}$$

Problem 348: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Log}\left[\frac{a}{x}\right]}{a x - x^2} dx$$

Optimal (type 4, 14 leaves, 4 steps):

$$\frac{\operatorname{PolyLog}\left[2, 1 - \frac{a}{x}\right]}{a}$$

Result (type 4, 61 leaves):

$$\frac{1}{2 a} \left(2 \operatorname{Log} \left[\frac{a}{x} \right] (\operatorname{Log}[x] - \operatorname{Log}[-a+x]) + \right. \\ \left. \operatorname{Log}[x] \left(\operatorname{Log}[x] - 2 \operatorname{Log}[-a+x] + 2 \operatorname{Log} \left[1 - \frac{x}{a} \right] \right) + 2 \operatorname{PolyLog} \left[2, \frac{x}{a} \right] \right)$$

Problem 349: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Log} \left[\frac{a}{x^2} \right]}{a x - x^3} dx$$

Optimal (type 4, 17 leaves, 4 steps):

$$\frac{\operatorname{PolyLog} \left[2, 1 - \frac{a}{x^2} \right]}{2 a}$$

Result (type 4, 104 leaves):

$$\frac{1}{2 a} \left(2 \operatorname{Log} \left[\frac{a}{x^2} \right] \operatorname{Log}[x] + 2 \operatorname{Log}[x]^2 + 2 \operatorname{Log}[x] \operatorname{Log} \left[1 - \frac{x}{\sqrt{a}} \right] + 2 \operatorname{Log}[x] \operatorname{Log} \left[1 + \frac{x}{\sqrt{a}} \right] - \right. \\ \left. \operatorname{Log} \left[\frac{a}{x^2} \right] \operatorname{Log}[-a+x^2] - 2 \operatorname{Log}[x] \operatorname{Log}[-a+x^2] + 2 \operatorname{PolyLog} \left[2, -\frac{x}{\sqrt{a}} \right] + 2 \operatorname{PolyLog} \left[2, \frac{x}{\sqrt{a}} \right] \right)$$

Problem 350: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Log} [a x^{1-n}]}{a x - x^n} dx$$

Optimal (type 4, 26 leaves, 3 steps):

$$\frac{\operatorname{PolyLog} [2, 1 - a x^{1-n}]}{a (1-n)}$$

Result (type 4, 103 leaves):

$$\frac{1}{2 a (-1+n)} \left((-1+n^2) \operatorname{Log}[x]^2 + 2 \operatorname{Log}[x] \left(n \operatorname{Log} [a x^{1-n}] + (-1+n) \left(\operatorname{Log} \left[1 - \frac{x^{-1+n}}{a} \right] - \operatorname{Log} [-a x + x^n] \right) \right) - \right. \\ \left. 2 \operatorname{Log} [a x^{1-n}] \operatorname{Log} [-a x + x^n] + 2 \operatorname{PolyLog} \left[2, \frac{x^{-1+n}}{a} \right] \right)$$

Problem 363: Result more than twice size of optimal antiderivative.

$$\int \frac{(f x)^{-1+m} (a+b \operatorname{Log} [c x^n])^2}{d+e x^n} dx$$

Optimal (type 4, 129 leaves, 4 steps):

$$\frac{x^{1-m} (f x)^{-1+m} (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1 + \frac{e x^m}{d}\right]}{e m} + \frac{2 b n x^{1-m} (f x)^{-1+m} (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{e x^m}{d}\right]}{e m^2} - \frac{2 b^2 n^2 x^{1-m} (f x)^{-1+m} \operatorname{PolyLog}\left[3, -\frac{e x^m}{d}\right]}{e m^3}$$

Result (type 4, 502 leaves):

$$\begin{aligned} & \frac{1}{3 e f m^3} x^{-m} (f x)^m \left(3 a^2 m^3 \operatorname{Log}[x] - 6 a b m^3 n \operatorname{Log}[x]^2 + 4 b^2 m^3 n^2 \operatorname{Log}[x]^3 + 6 a b m^3 \operatorname{Log}[x] \operatorname{Log}[c x^n] - \right. \\ & 6 b^2 m^3 n \operatorname{Log}[x]^2 \operatorname{Log}[c x^n] + 3 b^2 m^3 \operatorname{Log}[x] \operatorname{Log}[c x^n]^2 + 3 b^2 m^2 n^2 \operatorname{Log}[x]^2 \operatorname{Log}\left[1 + \frac{d x^{-m}}{e}\right] + \\ & 3 a^2 m^2 \operatorname{Log}[d - d x^m] - 6 a b m^2 n \operatorname{Log}[x] \operatorname{Log}[d - d x^m] + 3 b^2 m^2 n^2 \operatorname{Log}[x]^2 \operatorname{Log}[d - d x^m] + \\ & 6 a b m^2 \operatorname{Log}[c x^n] \operatorname{Log}[d - d x^m] - 6 b^2 m^2 n \operatorname{Log}[x] \operatorname{Log}[c x^n] \operatorname{Log}[d - d x^m] + \\ & 3 b^2 m^2 \operatorname{Log}[c x^n]^2 \operatorname{Log}[d - d x^m] + 6 a b m^2 n \operatorname{Log}[x] \operatorname{Log}[d + e x^m] - \\ & 6 b^2 m^2 n^2 \operatorname{Log}[x]^2 \operatorname{Log}[d + e x^m] - 6 a b m n \operatorname{Log}\left[-\frac{e x^m}{d}\right] \operatorname{Log}[d + e x^m] + \\ & 6 b^2 m n^2 \operatorname{Log}[x] \operatorname{Log}\left[-\frac{e x^m}{d}\right] \operatorname{Log}[d + e x^m] + 6 b^2 m^2 n \operatorname{Log}[x] \operatorname{Log}[c x^n] \operatorname{Log}[d + e x^m] - \\ & 6 b^2 m n \operatorname{Log}\left[-\frac{e x^m}{d}\right] \operatorname{Log}[c x^n] \operatorname{Log}[d + e x^m] - 6 b^2 m n^2 \operatorname{Log}[x] \operatorname{PolyLog}\left[2, -\frac{d x^{-m}}{e}\right] - \\ & \left. 6 b m n (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, 1 + \frac{e x^m}{d}\right] - 6 b^2 n^2 \operatorname{PolyLog}\left[3, -\frac{d x^{-m}}{e}\right] \right) \end{aligned}$$

Problem 430: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b \operatorname{Log}[c x^n])^2}{x (d + e x^r)} dx$$

Optimal (type 4, 94 leaves, 3 steps):

$$-\frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1 + \frac{d x^{-r}}{e}\right]}{d r} + \frac{2 b n (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{d x^{-r}}{e}\right]}{d r^2} + \frac{2 b^2 n^2 \operatorname{PolyLog}\left[3, -\frac{d x^{-r}}{e}\right]}{d r^3}$$

Result (type 4, 270 leaves):

$$\begin{aligned} & -\frac{1}{d r^3} \left(a^2 r^2 \operatorname{Log}[d - d x^r] - \right. \\ & 2 a b r^2 (n \operatorname{Log}[x] - \operatorname{Log}[c x^n]) \operatorname{Log}[d - d x^r] + b^2 r^2 (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])^2 \operatorname{Log}[d - d x^r] - \\ & 2 a b n r \left(\frac{1}{2} r^2 \operatorname{Log}[x]^2 + \left(-r \operatorname{Log}[x] + \operatorname{Log}\left[-\frac{e x^r}{d}\right] \right) \operatorname{Log}[d + e x^r] + \operatorname{PolyLog}\left[2, 1 + \frac{e x^r}{d}\right] \right) + \\ & 2 b^2 n r (n \operatorname{Log}[x] - \operatorname{Log}[c x^n]) \\ & \left(\frac{1}{2} r^2 \operatorname{Log}[x]^2 + \left(-r \operatorname{Log}[x] + \operatorname{Log}\left[-\frac{e x^r}{d}\right] \right) \operatorname{Log}[d + e x^r] + \operatorname{PolyLog}\left[2, 1 + \frac{e x^r}{d}\right] \right) + \\ & \left. b^2 n^2 \left(r^2 \operatorname{Log}[x]^2 \operatorname{Log}\left[1 + \frac{d x^{-r}}{e}\right] - 2 r \operatorname{Log}[x] \operatorname{PolyLog}\left[2, -\frac{d x^{-r}}{e}\right] - 2 \operatorname{PolyLog}\left[3, -\frac{d x^{-r}}{e}\right] \right) \right) \end{aligned}$$

Problem 431: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b \operatorname{Log}[c x^n])^2}{x (d + e x^r)^2} dx$$

Optimal (type 4, 182 leaves, 7 steps):

$$\frac{(a + b \operatorname{Log}[c x^n])^2}{d r (d + e x^r)} + \frac{2 b n (a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{d x^{-r}}{e}\right]}{d^2 r^2} - \frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1 + \frac{d x^{-r}}{e}\right]}{d^2 r}$$

$$\frac{2 b^2 n^2 \operatorname{PolyLog}\left[2, -\frac{d x^{-r}}{e}\right]}{d^2 r^3} + \frac{2 b n (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{d x^{-r}}{e}\right]}{d^2 r^2} + \frac{2 b^2 n^2 \operatorname{PolyLog}\left[3, -\frac{d x^{-r}}{e}\right]}{d^2 r^3}$$

Result (type 4, 397 leaves):

$$\frac{1}{d^2 r^3} \left(\frac{d r^2 (a + b \operatorname{Log}[c x^n])^2}{d + e x^r} + 2 a b n r \operatorname{Log}[d - d x^r] - \right.$$

$$a^2 r^2 \operatorname{Log}[d - d x^r] + 2 a b r^2 (n \operatorname{Log}[x] - \operatorname{Log}[c x^n]) \operatorname{Log}[d - d x^r] +$$

$$2 b^2 n r (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]) \operatorname{Log}[d - d x^r] - b^2 r^2 (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])^2 \operatorname{Log}[d - d x^r] -$$

$$2 b^2 n^2 \left(\frac{1}{2} r^2 \operatorname{Log}[x]^2 + \left(-r \operatorname{Log}[x] + \operatorname{Log}\left[-\frac{e x^r}{d}\right] \right) \operatorname{Log}[d + e x^r] + \operatorname{PolyLog}\left[2, 1 + \frac{e x^r}{d}\right] \right) +$$

$$2 a b n r \left(\frac{1}{2} r^2 \operatorname{Log}[x]^2 + \left(-r \operatorname{Log}[x] + \operatorname{Log}\left[-\frac{e x^r}{d}\right] \right) \operatorname{Log}[d + e x^r] + \operatorname{PolyLog}\left[2, 1 + \frac{e x^r}{d}\right] \right) +$$

$$2 b^2 n r (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])$$

$$\left(\frac{1}{2} r^2 \operatorname{Log}[x]^2 + \left(-r \operatorname{Log}[x] + \operatorname{Log}\left[-\frac{e x^r}{d}\right] \right) \operatorname{Log}[d + e x^r] + \operatorname{PolyLog}\left[2, 1 + \frac{e x^r}{d}\right] \right) -$$

$$\left. b^2 n^2 \left(r^2 \operatorname{Log}[x]^2 \operatorname{Log}\left[1 + \frac{d x^{-r}}{e}\right] - 2 r \operatorname{Log}[x] \operatorname{PolyLog}\left[2, -\frac{d x^{-r}}{e}\right] - 2 \operatorname{PolyLog}\left[3, -\frac{d x^{-r}}{e}\right] \right) \right)$$

Problem 433: Unable to integrate problem.

$$\int \frac{(d + e x^r)^{5/2} (a + b \operatorname{Log}[c x^n])}{x} dx$$

Optimal (type 4, 327 leaves, 23 steps):

$$\begin{aligned}
 & -\frac{92 b d^2 n \sqrt{d+e x^r}}{15 r^2} - \frac{32 b d n (d+e x^r)^{3/2}}{45 r^2} - \frac{4 b n (d+e x^r)^{5/2}}{25 r^2} + \\
 & \frac{92 b d^{5/2} n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^r}}{\sqrt{d}}\right]}{15 r^2} + \frac{2 b d^{5/2} n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^r}}{\sqrt{d}}\right]^2}{r^2} + \frac{2}{15} \\
 & \left(\frac{15 d^2 \sqrt{d+e x^r}}{r} + \frac{5 d (d+e x^r)^{3/2}}{r} + \frac{3 (d+e x^r)^{5/2}}{r} - \frac{15 d^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^r}}{\sqrt{d}}\right]}{r} \right) (a+b \operatorname{Log}[c x^n]) - \\
 & \frac{4 b d^{5/2} n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^r}}{\sqrt{d}}\right] \operatorname{Log}\left[\frac{2 \sqrt{d}}{\sqrt{d}-\sqrt{d+e x^r}}\right]}{r^2} - \frac{2 b d^{5/2} n \operatorname{PolyLog}\left[2, 1 - \frac{2 \sqrt{d}}{\sqrt{d}-\sqrt{d+e x^r}}\right]}{r^2}
 \end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{(d+e x^r)^{5/2} (a+b \operatorname{Log}[c x^n])}{x} dx$$

Problem 434: Unable to integrate problem.

$$\int \frac{(d+e x^r)^{3/2} (a+b \operatorname{Log}[c x^n])}{x} dx$$

Optimal (type 4, 284 leaves, 17 steps):

$$\begin{aligned}
 & -\frac{16 b d n \sqrt{d+e x^r}}{3 r^2} - \frac{4 b n (d+e x^r)^{3/2}}{9 r^2} + \frac{16 b d^{3/2} n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^r}}{\sqrt{d}}\right]}{3 r^2} + \frac{2 b d^{3/2} n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^r}}{\sqrt{d}}\right]^2}{r^2} + \\
 & \frac{2}{3} \left(\frac{3 d \sqrt{d+e x^r}}{r} + \frac{(d+e x^r)^{3/2}}{r} - \frac{3 d^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^r}}{\sqrt{d}}\right]}{r} \right) (a+b \operatorname{Log}[c x^n]) - \\
 & \frac{4 b d^{3/2} n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^r}}{\sqrt{d}}\right] \operatorname{Log}\left[\frac{2 \sqrt{d}}{\sqrt{d}-\sqrt{d+e x^r}}\right]}{r^2} - \frac{2 b d^{3/2} n \operatorname{PolyLog}\left[2, 1 - \frac{2 \sqrt{d}}{\sqrt{d}-\sqrt{d+e x^r}}\right]}{r^2}
 \end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{(d+e x^r)^{3/2} (a+b \operatorname{Log}[c x^n])}{x} dx$$

Problem 435: Unable to integrate problem.

$$\int \frac{\sqrt{d+e x^r} (a+b \operatorname{Log}[c x^n])}{x} dx$$

Optimal (type 4, 240 leaves, 12 steps):

$$\begin{aligned}
 & - \frac{4 b n \sqrt{d+e x^r}}{r^2} + \frac{4 b \sqrt{d} n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^r}}{\sqrt{d}}\right]}{r^2} + \frac{2 b \sqrt{d} n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^r}}{\sqrt{d}}\right]^2}{r^2} + \\
 & 2 \left(\frac{\sqrt{d+e x^r}}{r} - \frac{\sqrt{d} \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^r}}{\sqrt{d}}\right]}{r} \right) (a+b \operatorname{Log}[c x^n]) - \\
 & \frac{4 b \sqrt{d} n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^r}}{\sqrt{d}}\right] \operatorname{Log}\left[\frac{2 \sqrt{d}}{\sqrt{d}-\sqrt{d+e x^r}}\right]}{r^2} - \frac{2 b \sqrt{d} n \operatorname{PolyLog}\left[2, 1 - \frac{2 \sqrt{d}}{\sqrt{d}-\sqrt{d+e x^r}}\right]}{r^2}
 \end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{\sqrt{d+e x^r} (a+b \operatorname{Log}[c x^n])}{x} dx$$

Problem 436: Unable to integrate problem.

$$\int \frac{a+b \operatorname{Log}[c x^n]}{x \sqrt{d+e x^r}} dx$$

Optimal (type 4, 174 leaves, 8 steps):

$$\begin{aligned}
 & \frac{2 b n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^r}}{\sqrt{d}}\right]^2}{\sqrt{d} r^2} - \frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^r}}{\sqrt{d}}\right] (a+b \operatorname{Log}[c x^n])}{\sqrt{d} r} - \\
 & \frac{4 b n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^r}}{\sqrt{d}}\right] \operatorname{Log}\left[\frac{2 \sqrt{d}}{\sqrt{d}-\sqrt{d+e x^r}}\right]}{\sqrt{d} r^2} - \frac{2 b n \operatorname{PolyLog}\left[2, 1 - \frac{2 \sqrt{d}}{\sqrt{d}-\sqrt{d+e x^r}}\right]}{\sqrt{d} r^2}
 \end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{a+b \operatorname{Log}[c x^n]}{x \sqrt{d+e x^r}} dx$$

Problem 437: Unable to integrate problem.

$$\int \frac{a+b \operatorname{Log}[c x^n]}{x (d+e x^r)^{3/2}} dx$$

Optimal (type 4, 225 leaves, 11 steps):

$$\frac{4 b n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^r}}{\sqrt{d}}\right]}{d^{3/2} r^2} + \frac{2 b n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^r}}{\sqrt{d}}\right]^2}{d^{3/2} r^2} +$$

$$2 \left(\frac{1}{d r \sqrt{d+e x^r}} - \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^r}}{\sqrt{d}}\right]}{d^{3/2} r} \right) (a + b \operatorname{Log}[c x^n]) -$$

$$\frac{4 b n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^r}}{\sqrt{d}}\right] \operatorname{Log}\left[\frac{2 \sqrt{d}}{\sqrt{d}-\sqrt{d+e x^r}}\right]}{d^{3/2} r^2} - \frac{2 b n \operatorname{PolyLog}\left[2, 1 - \frac{2 \sqrt{d}}{\sqrt{d}-\sqrt{d+e x^r}}\right]}{d^{3/2} r^2}$$

Result (type 8, 27 leaves):

$$\int \frac{a + b \operatorname{Log}[c x^n]}{x (d + e x^r)^{3/2}} dx$$

Problem 438: Unable to integrate problem.

$$\int \frac{a + b \operatorname{Log}[c x^n]}{x (d + e x^r)^{5/2}} dx$$

Optimal (type 4, 271 leaves, 15 steps):

$$-\frac{4 b n}{3 d^2 r^2 \sqrt{d+e x^r}} + \frac{16 b n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^r}}{\sqrt{d}}\right]}{3 d^{5/2} r^2} + \frac{2 b n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^r}}{\sqrt{d}}\right]^2}{d^{5/2} r^2} +$$

$$\frac{2}{3} \left(\frac{1}{d r (d + e x^r)^{3/2}} + \frac{3}{d^2 r \sqrt{d+e x^r}} - \frac{3 \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^r}}{\sqrt{d}}\right]}{d^{5/2} r} \right) (a + b \operatorname{Log}[c x^n]) -$$

$$\frac{4 b n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^r}}{\sqrt{d}}\right] \operatorname{Log}\left[\frac{2 \sqrt{d}}{\sqrt{d}-\sqrt{d+e x^r}}\right]}{d^{5/2} r^2} - \frac{2 b n \operatorname{PolyLog}\left[2, 1 - \frac{2 \sqrt{d}}{\sqrt{d}-\sqrt{d+e x^r}}\right]}{d^{5/2} r^2}$$

Result (type 8, 27 leaves):

$$\int \frac{a + b \operatorname{Log}[c x^n]}{x (d + e x^r)^{5/2}} dx$$

Problem 439: Unable to integrate problem.

$$\int \frac{a + b \operatorname{Log}[c x^n]}{x (d + e x^r)^{7/2}} dx$$

Optimal (type 4, 314 leaves, 20 steps):

$$\begin{aligned}
 & -\frac{4 b n}{15 d^2 r^2 (d+e x^r)^{3/2}} - \frac{32 b n}{15 d^3 r^2 \sqrt{d+e x^r}} + \frac{92 b n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^r}}{\sqrt{d}}\right]}{15 d^{7/2} r^2} + \frac{2 b n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^r}}{\sqrt{d}}\right]^2}{d^{7/2} r^2} + \\
 & \frac{2}{15} \left(\frac{3}{d r (d+e x^r)^{5/2}} + \frac{5}{d^2 r (d+e x^r)^{3/2}} + \frac{15}{d^3 r \sqrt{d+e x^r}} - \frac{15 \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^r}}{\sqrt{d}}\right]}{d^{7/2} r} \right) (a+b \operatorname{Log}[c x^n]) - \\
 & \frac{4 b n \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^r}}{\sqrt{d}}\right] \operatorname{Log}\left[\frac{2 \sqrt{d}}{\sqrt{d}-\sqrt{d+e x^r}}\right]}{d^{7/2} r^2} - \frac{2 b n \operatorname{PolyLog}\left[2, 1 - \frac{2 \sqrt{d}}{\sqrt{d}-\sqrt{d+e x^r}}\right]}{d^{7/2} r^2}
 \end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{a+b \operatorname{Log}[c x^n]}{x (d+e x^r)^{7/2}} dx$$

Problem 456: Result more than twice size of optimal antiderivative.

$$\int \frac{(f+g x) (a+b \operatorname{Log}[c x^n])^3}{(d+e x)^3} dx$$

Optimal (type 4, 295 leaves, 11 steps):

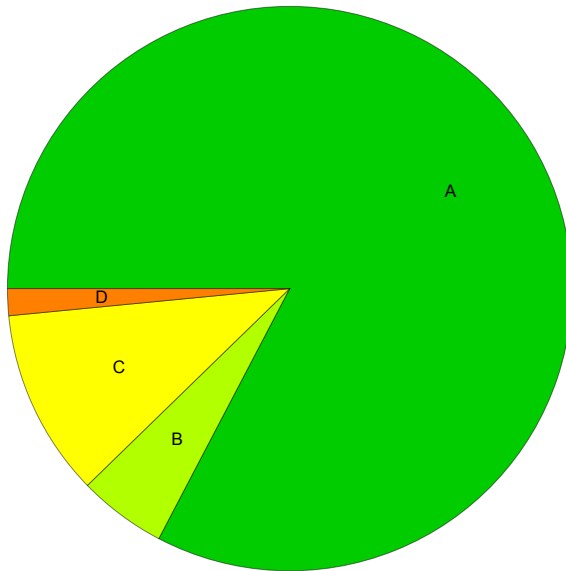
$$\begin{aligned}
 & -\frac{3 b (e f-d g) n x (a+b \operatorname{Log}[c x^n])^2}{2 d^2 e (d+e x)} + \frac{f^2 (a+b \operatorname{Log}[c x^n])^3}{2 d^2 (e f-d g)} - \\
 & \frac{(f+g x)^2 (a+b \operatorname{Log}[c x^n])^3}{2 (e f-d g) (d+e x)^2} + \frac{3 b^2 (e f-d g) n^2 (a+b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1+\frac{e x}{d}\right]}{d^2 e^2} - \\
 & \frac{3 b (e f+d g) n (a+b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1+\frac{e x}{d}\right]}{2 d^2 e^2} + \frac{3 b^3 (e f-d g) n^3 \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d^2 e^2} - \\
 & \frac{3 b^2 (e f+d g) n^2 (a+b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d^2 e^2} + \frac{3 b^3 (e f+d g) n^3 \operatorname{PolyLog}\left[3, -\frac{e x}{d}\right]}{d^2 e^2}
 \end{aligned}$$

Result (type 4, 674 leaves):

$$\begin{aligned}
& - \frac{1}{2 d^2 e^2 (d+e x)^2} \left(-3 b (e f + d g) n (d+e x)^2 \operatorname{Log}[x] (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n])^2 + \right. \\
& \quad 3 b d^2 n (d g + e (f + 2 g x)) \operatorname{Log}[x] (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n])^2 - \\
& \quad d^2 (-e f + d g) (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n])^3 + d (d+e x) (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n])^2 \\
& \quad (2 a d g - 3 b e f n + 3 b d g n + 2 b d g (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])) + \\
& \quad 3 b (e f + d g) n (d+e x)^2 (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[d+e x] + \\
& \quad 3 b^2 d g n^2 (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n]) \left(-e^2 x^2 \operatorname{Log}[x]^2 + 2 (d+e x)^2 \operatorname{Log}\left[1 + \frac{e x}{d}\right] + \right. \\
& \quad \quad \left. 2 (d+e x) \operatorname{Log}[x] \left(-e x + (d+e x) \operatorname{Log}\left[1 + \frac{e x}{d}\right]\right) + 2 (d+e x)^2 \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right] \right) + \\
& \quad 3 b^2 e f n^2 (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n]) \left(-e x (2 d+e x) \operatorname{Log}[x]^2 - 2 (d+e x)^2 \operatorname{Log}\left[1 + \frac{e x}{d}\right] + \right. \\
& \quad \quad \left. 2 (d+e x) \operatorname{Log}[x] \left(e x + (d+e x) \operatorname{Log}\left[1 + \frac{e x}{d}\right]\right) + 2 (d+e x)^2 \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right] \right) + \\
& \quad b^3 e f n^3 \left(-e x (2 d+e x) \operatorname{Log}[x]^3 + 3 (d+e x) \operatorname{Log}[x]^2 \left(e x + (d+e x) \operatorname{Log}\left[1 + \frac{e x}{d}\right] \right) - \right. \\
& \quad \quad 6 (d+e x)^2 \operatorname{Log}[x] \left(\operatorname{Log}\left[1 + \frac{e x}{d}\right] - \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right] \right) - \\
& \quad \quad \left. 6 (d+e x)^2 \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right] - 6 (d+e x)^2 \operatorname{PolyLog}\left[3, -\frac{e x}{d}\right] \right) + \\
& \quad b^3 d g n^3 \left(\operatorname{Log}[x] \left(-e^2 x^2 \operatorname{Log}[x]^2 + 6 (d+e x)^2 \operatorname{Log}\left[1 + \frac{e x}{d}\right] + \right. \right. \\
& \quad \quad \left. \left. 3 (d+e x) \operatorname{Log}[x] \left(-e x + (d+e x) \operatorname{Log}\left[1 + \frac{e x}{d}\right] \right) \right) + \right. \\
& \quad \quad \left. \left. 6 (d+e x)^2 (1 + \operatorname{Log}[x]) \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right] - 6 (d+e x)^2 \operatorname{PolyLog}\left[3, -\frac{e x}{d}\right] \right) \right)
\end{aligned}$$

Summary of Integration Test Results

456 integration problems



A - 377 optimal antiderivatives

B - 23 more than twice size of optimal antiderivatives

C - 49 unnecessarily complex antiderivatives

D - 7 unable to integrate problems

E - 0 integration timeouts