Mathematica 11.3 Integration Test Results

Test results for the 72 problems in "4.1.1.1 (a+b sin)^n.m"

Problem 4: Result more than twice size of optimal antiderivative.

$$\int \sqrt{a+a\,\text{Sin}[c+d\,x]}\,dx$$

Optimal (type 3, 26 leaves, 1 step):

$$-\frac{2\,a\,\text{Cos}[c+d\,x]}{d\,\sqrt{a+a\,\text{Sin}[c+d\,x]}}$$

Result (type 3, 65 leaves):

$$\frac{2\left(-\text{Cos}\left[\frac{1}{2}(c+d\,x)\right]+\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right)\sqrt{a\,(1+\text{Sin}[c+d\,x])}}{d\left(\text{Cos}\left[\frac{1}{2}(c+d\,x)\right]+\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right)}$$

Problem 5: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{a+a\,\text{Sin}[c+d\,x]}}\,dx$$

Optimal (type 3, 47 leaves, 2 steps):

$$-\frac{\sqrt{2}\,\text{ArcTanh}\left[\frac{\sqrt{a}\,\text{Cos}[c+d\,x]}{\sqrt{2}\,\sqrt{a+a\,\text{Sin}[c+d\,x]}}\right]}{\sqrt{a}\,d}$$

Result (type 3, 73 leaves):

$$\frac{1}{d\sqrt{a\,(1+\text{Sin}[c+d\,x])}}(2+2\,i)\,(-1)^{3/4}\,\text{ArcTanh}\left[\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\text{Tan}\left[\frac{1}{4}(c+d\,x)\right]\right)\right]\left(\text{Cos}\left[\frac{1}{2}(c+d\,x)\right]+\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right)$$

Problem 6: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(a+a\,\text{Sin}[c+d\,x])^{3/2}}\,dx$$

Optimal (type 3, 77 leaves, 3 steps):

$$-\frac{\text{ArcTanh}\left[\frac{\sqrt{a}\ \text{Cos}[c+d\,x]}{\sqrt{2}\ \sqrt{a+a\,\text{Sin}[c+d\,x]}}\right]}{2\sqrt{2}\ a^{3/2}\ d}-\frac{\text{Cos}[c+d\,x]}{2\,d\,\left(a+a\,\text{Sin}[c+d\,x]\right)^{3/2}}$$

Result (type 3, 108 leaves):

$$\left(\left(\text{Cos}\left[\frac{1}{2}(c+d\,x)\right]+\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right)\left(-\text{Cos}\left[\frac{1}{2}(c+d\,x)\right]+\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]+(1+i)\,(-1)^{3/4}\,\text{ArcTanh}\left[\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\text{Tan}\left[\frac{1}{4}(c+d\,x)\right]\right)\right]\left(1+\text{Sin}[c+d\,x]\right)\right)\right)\Big/\left(2\,d\,\left(a\,\left(1+\text{Sin}[c+d\,x]\right)\right)^{3/2}\right)$$

Problem 7: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(a+a\,\text{Sin}[c+d\,x]\right)^{5/2}}\,dx$$

Optimal (type 3, 107 leaves, 4 steps):

$$-\frac{3\,\text{ArcTanh}\left[\frac{\sqrt{a}\ \text{Cos}[c+d\,x]}{\sqrt{2}\ \sqrt{a+a\,\text{Sin}[c+d\,x]}}\right]}{16\sqrt{2}\ a^{5/2}\ d}-\frac{\text{Cos}[c+d\,x]}{4\,d\,\left(a+a\,\text{Sin}[c+d\,x]\right)^{5/2}}-\frac{3\,\text{Cos}[c+d\,x]}{16\,a\,d\,\left(a+a\,\text{Sin}[c+d\,x]\right)^{3/2}}$$

Result (type 3, 196 leaves):

$$\frac{1}{16\,d\,\left(a\,\left(1+\text{Sin}[c+d\,x]\right)\right)^{5/2}}\left(\text{Cos}\left[\frac{1}{2}(c+d\,x)\right]+\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right)\left(8\,\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]-4\left(\text{Cos}\left[\frac{1}{2}(c+d\,x)\right]+\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right)+6\,\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\left(\text{Cos}\left[\frac{1}{2}(c+d\,x)\right]+\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right)^2-3\left(\text{Cos}\left[\frac{1}{2}(c+d\,x)\right]+\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right)^3+(3+3\,i)\,(-1)^{3/4}\,\text{ArcTanh}\left[\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\text{Tan}\left[\frac{1}{4}(c+d\,x)\right]\right)\right]\left(\text{Cos}\left[\frac{1}{2}(c+d\,x)\right]+\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right)^4\right)$$

Problem 8: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\left(a+a\,\text{Sin}[c+d\,x]\right)^{4/3}\,dx$$

Optimal (type 5, 67 leaves, 2 steps):

$$-\left(\left(2\times 2^{5/6}\,a\,\text{Cos}[c+d\,x]\,\text{Hypergeometric2F1}\left[-\frac{5}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}\left(1-\text{Sin}[c+d\,x]\right)\right]\left(a+a\,\text{Sin}[c+d\,x]\right)^{1/3}\right)\Big/\left(d\,\left(1+\text{Sin}[c+d\,x]\right)^{5/6}\right)\right)$$

Result (type 5, 314 leaves):

$$\frac{1}{2\,d\left(\text{Cos}\left[\frac{1}{2}\,(c+d\,x)\right]+\text{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]\right)^3}$$

$$\left(-\frac{3}{2}\,(-5+\text{Cos}[c+d\,x])\left(\text{Cos}\left[\frac{1}{2}\,(c+d\,x)\right]+\text{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]\right)-\right.$$

$$\frac{1}{8\left(1+i\,e^{-i\,(c+d\,x)}\right)^{2/3}\sqrt{1-\text{Sin}[c+d\,x]}}$$

$$3\,(-1)^{3/4}\,e^{-\frac{3}{2}\,i\,(c+d\,x)}\left(i+e^{i\,(c+d\,x)}\right)\left(-20\,e^{i\,(c+d\,x)}\sqrt{\text{Cos}\left[\frac{1}{4}\,(2\,c+\pi+2\,d\,x)\right]^2}\right.$$

$$\text{Hypergeometric2F1}\left[-\frac{1}{3},\frac{1}{3},\frac{2}{3},-i\,e^{-i\,(c+d\,x)}\right]+2\left(1+i\,e^{-i\,(c+d\,x)}\right)^{2/3}\left(1+e^{2\,i\,(c+d\,x)}\right)$$

$$\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{5}{6},\frac{11}{6},\text{Sin}\left[\frac{1}{4}\,(2\,c+\pi+2\,d\,x)\right]^2\right]-5\,i\,\text{Hypergeometric2F1}\Big[$$

$$\left.\left.\frac{1}{3},\frac{2}{3},\frac{5}{3},-i\,e^{-i\,(c+d\,x)}\right]\sqrt{2-2\,\text{Sin}[c+d\,x]}\right)\right)\left(a\,(1+\text{Sin}[c+d\,x])\right)^{4/3}$$

Problem 10: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int (a+a\,\text{Sin}[c+d\,x])^{1/3}\,dx$$

Optimal (type 5, 66 leaves, 2 steps):

$$-\left(\left(2^{5/6}\,\text{Cos}[c+d\,x]\,\text{Hypergeometric2F1}\left[\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}\,(1-\text{Sin}[c+d\,x])\right]\,(a+a\,\text{Sin}[c+d\,x])^{1/3}\right)\Big/\left(d\,(1+\text{Sin}[c+d\,x])^{5/6}\right)\right)$$

Result (type 5, 546 leaves):

$$\frac{3\left(a\left(1+\text{Sin}[c+d\,x]\right)\right)^{1/3}}{d}+\frac{1}{d\left(\text{Cos}\left[\frac{1}{2}(c+d\,x)\right]+\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right)}2\sqrt{2}\left(1+\text{Sin}[c+d\,x]\right)^{1/6}\left(a\left(1+\text{Sin}[c+d\,x]\right)\right)^{1/3}$$

$$\left(-\left(\left(\text{i}\,\text{Cos}\left[\frac{\pi}{4}+\frac{1}{2}(-c-d\,x)\right]\right)^{1/3}\left(-\left(\left(3\,\text{i}\left(e^{-\text{i}\left(\frac{\pi}{4}+\frac{1}{2}(-c-d\,x)\right)}+e^{\text{i}\left(\frac{\pi}{4}+\frac{1}{2}(-c-d\,x)\right)}\right)^{2/3}\text{Hypergeometric2F1}\left[-\frac{1}{3},\frac{1}{3},\frac{2}{3},-e^{2\,\text{i}\left(\frac{\pi}{4}+\frac{1}{2}(-c-d\,x)\right)}\right]\right)\Big/\left(2^{2/3}\left(1+e^{2\,\text{i}\left(\frac{\pi}{4}+\frac{1}{2}(-c-d\,x)\right)}\right)^{2/3}\right)\right)-\left(3\,\text{i}\,e^{\text{i}\left(\frac{\pi}{4}+\frac{1}{2}(-c-d\,x)\right)}\left(1+e^{2\,\text{i}\left(\frac{\pi}{4}+\frac{1}{2}(-c-d\,x)\right)}\right)^{1/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},-e^{2\,\text{i}\left(\frac{\pi}{4}+\frac{1}{2}(-c-d\,x)\right)}\right]\right)\Big/\left(2\times 2^{2/3}\left(e^{-\text{i}\left(\frac{\pi}{4}+\frac{1}{2}(-c-d\,x)\right)}+e^{\text{i}\left(\frac{\pi}{4}+\frac{1}{2}(-c-d\,x)\right)}\right)^{1/3}\right)\right)\right)\Big/\left(2\left(1+\text{Cos}\left[2\left(\frac{\pi}{4}+\frac{1}{2}(-c-d\,x)\right)\right]\right)^{1/6}\right)\right)+\left(3\,\text{Cos}\left[\frac{\pi}{4}+\frac{1}{2}(-c-d\,x)\right]^2\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{5}{6},\frac{11}{6},\text{Cos}\left[\frac{\pi}{4}+\frac{1}{2}(-c-d\,x)\right]^2\right]\text{Sin}\left[\frac{\pi}{4}+\frac{1}{2}(-c-d\,x)\right]\right)\Big/\left(5\left(1+\text{Cos}\left[2\left(\frac{\pi}{4}+\frac{1}{2}(-c-d\,x)\right)\right]\right)^{1/6}\sqrt{\text{Sin}\left[\frac{\pi}{4}+\frac{1}{2}(-c-d\,x)\right]^2}\right)\right)$$

Problem 12: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{1}{\left(a+a\,\text{Sin}[c+d\,x]\right)^{2/3}}\,dx$$

Optimal (type 5, 66 leaves, 2 steps):

$$-\left(\left(\text{Cos}[c+d\,x]\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{7}{6},\frac{3}{2},\frac{1}{2}\left(1-\text{Sin}[c+d\,x]\right)\right]\left(1+\text{Sin}[c+d\,x]\right)^{1/6}\right)\Big/\left(2^{1/6}\,d\left(a+a\,\text{Sin}[c+d\,x]\right)^{2/3}\right)\right)$$

Result (type 5, 604 leaves):

$$\left(2 \left(\text{Cos}\left[\frac{1}{2} (c+d x)\right]+\text{Sin}\left[\frac{1}{2} (c+d x)\right]\right)^2 \left(-3+\frac{3\, \text{Sin}\left[\frac{1}{2} (c+d x)\right]}{\text{Cos}\left[\frac{1}{2} (c+d x)\right]+\text{Sin}\left[\frac{1}{2} (c+d x)\right]}\right)\right) \Big/$$

$$\left(d \left(a \left(1+\text{Sin}[c+d x]\right)\right)^{2/3}\right) -$$

$$\frac{1}{d \left(a \left(1+\text{Sin}[c+d x]\right)\right)^{2/3}}\, 2 \sqrt{2} \left(\text{Cos}\left[\frac{1}{2} (c+d x)\right]+\text{Sin}\left[\frac{1}{2} (c+d x)\right]\right) \left(1+\text{Sin}[c+d x]\right)^{1/6}$$

$$\Bigg(-\Bigg(\left(\mathbb{i}\, \text{Cos}\left[\frac{\pi}{4}+\frac{1}{2} (-c-d x)\right]\right)^{1/3} \Bigg(-\Bigg(\Bigg(3\, \mathbb{i} \left(e^{-\mathbb{i} \left(\frac{\pi}{4}+\frac{1}{2} (-c-d x)\right)}+e^{\mathbb{i} \left(\frac{\pi}{4}+\frac{1}{2} (-c-d x)\right)}\right)^{2/3} \text{Hypergeometric2F1}\Big[$$

$$-\frac{1}{3}, \frac{1}{3}, \frac{2}{3}, -e^{2 \mathbb{i} \left(\frac{\pi}{4}+\frac{1}{2} (-c-d x)\right)}\Big]\Bigg) \Big/ \left(2^{2/3} \left(1+e^{2 \mathbb{i} \left(\frac{\pi}{4}+\frac{1}{2} (-c-d x)\right)}\right)^{2/3}\right)\Bigg) -$$

$$\Bigg(3\, \mathbb{i}\, e^{\mathbb{i} \left(\frac{\pi}{4}+\frac{1}{2} (-c-d x)\right)} \left(1+e^{2 \mathbb{i} \left(\frac{\pi}{4}+\frac{1}{2} (-c-d x)\right)}\right)^{1/3} \text{Hypergeometric2F1}\Big[\frac{1}{3}, \frac{2}{3}, \frac{5}{3},$$

$$-e^{2 \mathbb{i} \left(\frac{\pi}{4}+\frac{1}{2} (-c-d x)\right)}\Big]\Bigg) \Big/ \left(2 \times 2^{2/3} \left(e^{-\mathbb{i} \left(\frac{\pi}{4}+\frac{1}{2} (-c-d x)\right)}+e^{\mathbb{i} \left(\frac{\pi}{4}+\frac{1}{2} (-c-d x)\right)}\right)^{1/3}\right)\Bigg)\Bigg) \Big/$$

$$\left(2 \left(1+\text{Cos}\left[2 \left(\frac{\pi}{4}+\frac{1}{2} (-c-d x)\right)\right]\right)^{1/6}\right)\Bigg) + \Bigg(3\, \text{Cos}\left[\frac{\pi}{4}+\frac{1}{2} (-c-d x)\right]^2$$

$$\text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \text{Cos}\left[\frac{\pi}{4}+\frac{1}{2} (-c-d x)\right]^2\right] \text{Sin}\left[\frac{\pi}{4}+\frac{1}{2} (-c-d x)\right]\Bigg) \Big/$$

$$\left(5 \left(1+\text{Cos}\left[2 \left(\frac{\pi}{4}+\frac{1}{2} (-c-d x)\right)\right]\right)^{1/6} \sqrt{\text{Sin}\left[\frac{\pi}{4}+\frac{1}{2} (-c-d x)\right]^2}\right)\Bigg)$$

Problem 58: Result more than twice size of optimal antiderivative.

$$\int \left(a+b\, \text{Sin}[c+d x]\right)^{4/3} \, dx$$

Optimal (type 6, 109 leaves, 3 steps):

$$-\Bigg(\Bigg(\sqrt{2}\, (a+b)\, \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -\frac{4}{3}, \frac{3}{2}, \frac{1}{2} \left(1-\text{Sin}[c+d x]\right), \frac{b \left(1-\text{Sin}[c+d x]\right)}{a+b}\right]$$

$$\text{Cos}[c+d x] \left(a+b\, \text{Sin}[c+d x]\right)^{1/3}\Bigg) \Big/ \left(d \sqrt{1+\text{Sin}[c+d x]} \left(\frac{a+b\, \text{Sin}[c+d x]}{a+b}\right)^{1/3}\right)\Bigg)$$

Result (type 6, 244 leaves):

$$-\frac{1}{16\,b\,d}3\,\text{Sec}[c+d\,x]\,(a+b\,\text{Sin}[c+d\,x])^{1/3}\left(4\,b^2\,\text{Cos}[c+d\,x]^2+4\,(a^2-b^2)\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},\frac{a+b\,\text{Sin}[c+d\,x]}{a-b},\frac{a+b\,\text{Sin}[c+d\,x]}{a+b}\right]\sqrt{-\frac{b\,(-1+\text{Sin}[c+d\,x])}{a+b}}\sqrt{\frac{b\,(1+\text{Sin}[c+d\,x])}{-a+b}}-5\,a\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},\frac{1}{2},\frac{7}{3},\frac{a+b\,\text{Sin}[c+d\,x]}{a-b},\frac{a+b\,\text{Sin}[c+d\,x]}{a+b}\right]\sqrt{-\frac{b\,(-1+\text{Sin}[c+d\,x])}{a+b}}\sqrt{\frac{b\,(1+\text{Sin}[c+d\,x])}{-a+b}}\,(a+b\,\text{Sin}[c+d\,x])\right)$$

Problem 63: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{(a+b\,\text{Sin}[c+d\,x])^{4/3}}\,dx$$

Optimal (type 6, 111 leaves, 3 steps):

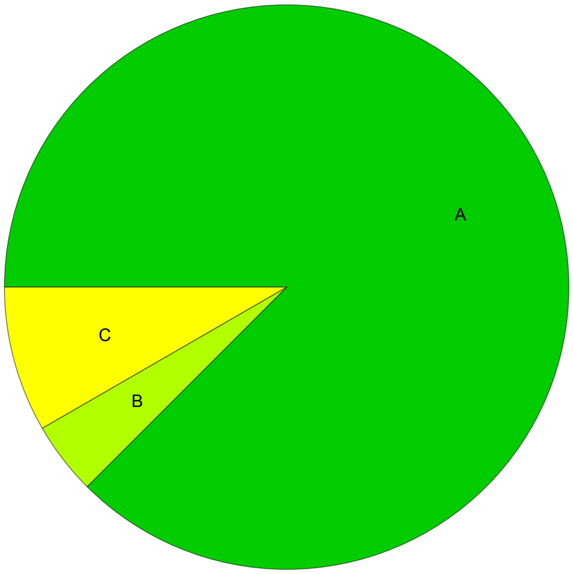
$$-\frac{\sqrt{2}\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{2},\frac{4}{3},\frac{3}{2},\frac{1}{2}\,(1-\text{Sin}[c+d\,x]),\frac{b\,(1-\text{Sin}[c+d\,x])}{a+b}\right]\text{Cos}[c+d\,x]\left(\frac{a+b\,\text{Sin}[c+d\,x]}{a+b}\right)^{1/3}}{(a+b)\,d\,\sqrt{1+\text{Sin}[c+d\,x]}\,(a+b\,\text{Sin}[c+d\,x])^{1/3}}$$

Result (type 6, 262 leaves):

$$-\left(\left(3\,\text{Sec}[c+d\,x]\left(5\,a\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},\frac{a+b\,\text{Sin}[c+d\,x]}{a-b},\frac{a+b\,\text{Sin}[c+d\,x]}{a+b}\right]\sqrt{-\frac{b\,(-1+\text{Sin}[c+d\,x])}{a+b}}\sqrt{-\frac{b\,(1+\text{Sin}[c+d\,x])}{a-b}}\,(a+b\,\text{Sin}[c+d\,x])-2\left(5\,b^2\,\text{Cos}[c+d\,x]^2+2\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},\frac{1}{2},\frac{8}{3},\frac{a+b\,\text{Sin}[c+d\,x]}{a-b},\frac{a+b\,\text{Sin}[c+d\,x]}{a+b}\right]\sqrt{-\frac{b\,(-1+\text{Sin}[c+d\,x])}{a+b}}\sqrt{\frac{b\,(1+\text{Sin}[c+d\,x])}{-a+b}}\,(a+b\,\text{Sin}[c+d\,x])^2\right)\right)\right)\Big/\left(10\,b\,(a^2-b^2)\,d\,(a+b\,\text{Sin}[c+d\,x])^{1/3}\right)\right)$$

Summary of Integration Test Results

72 integration problems



A - 63 optimal antiderivatives

B - 3 more than twice size of optimal antiderivatives

C - 6 unnecessarily complex antiderivatives

D - 0 unable to integrate problems

E - 0 integration timeouts