Mathematica 11.3 Integration Test Results

Test results for the 51 problems in "4.1.2.3 (g sin)^p (a+b sin)^m (c+d sin)^n.m"

Problem 8: Result more than twice size of optimal antiderivative.

$$\int \text{Csc}[e+f x]^4 \,(a+a \,\text{Sin}[e+f x])^2 \,(c-c \,\text{Sin}[e+f x]) \,dx$$

Optimal (type 3, 61 leaves, 6 steps):

$$\frac{a^2 c \,\text{ArcTanh}[\text{Cos}[e+f x]]}{2 f}-\frac{a^2 c \,\text{Cot}[e+f x]^3}{3 f}-\frac{a^2 c \,\text{Cot}[e+f x] \,\text{Csc}[e+f x]}{2 f}$$

Result (type 3, 172 leaves):

$$a^2 c \left(\frac{\text{Cot}\left[\frac{1}{2}(e+f x)\right]}{6 f}-\frac{\text{Csc}\left[\frac{1}{2}(e+f x)\right]^2}{8 f}-\frac{\text{Cot}\left[\frac{1}{2}(e+f x)\right] \text{Csc}\left[\frac{1}{2}(e+f x)\right]^2}{24 f}+\frac{\text{Log}\left[\text{Cos}\left[\frac{1}{2}(e+f x)\right]\right]}{2 f}-\frac{\text{Log}\left[\text{Sin}\left[\frac{1}{2}(e+f x)\right]\right]}{2 f}+\frac{\text{Sec}\left[\frac{1}{2}(e+f x)\right]^2}{8 f}-\frac{\text{Tan}\left[\frac{1}{2}(e+f x)\right]}{6 f}+\frac{\text{Sec}\left[\frac{1}{2}(e+f x)\right]^2 \text{Tan}\left[\frac{1}{2}(e+f x)\right]}{24 f}\right)$$

Problem 9: Result more than twice size of optimal antiderivative.

$$\int \text{Csc}[e+f x]^5 \,(a+a \,\text{Sin}[e+f x])^2 \,(c-c \,\text{Sin}[e+f x]) \,dx$$

Optimal (type 3, 86 leaves, 11 steps):

$$\frac{a^2 c \,\text{ArcTanh}[\text{Cos}[e+f x]]}{8 f}-\frac{a^2 c \,\text{Cot}[e+f x]^3}{3 f}+\frac{a^2 c \,\text{Cot}[e+f x] \,\text{Csc}[e+f x]}{8 f}-\frac{a^2 c \,\text{Cot}[e+f x] \,\text{Csc}[e+f x]^3}{4 f}$$

Result (type 3, 179 leaves):

$$\frac{a^2 c \,\text{Cot}[e+f x]}{3 f}+\frac{a^2 c \,\text{Csc}\left[\frac{1}{2}(e+f x)\right]^2}{32 f}-\frac{a^2 c \,\text{Csc}\left[\frac{1}{2}(e+f x)\right]^4}{64 f}-\frac{a^2 c \,\text{Cot}[e+f x]\,\text{Csc}[e+f x]^2}{3 f}+\frac{a^2 c \,\text{Log}\left[\text{Cos}\left[\frac{1}{2}(e+f x)\right]\right]}{8 f}-\frac{a^2 c \,\text{Log}\left[\text{Sin}\left[\frac{1}{2}(e+f x)\right]\right]}{8 f}-\frac{a^2 c \,\text{Sec}\left[\frac{1}{2}(e+f x)\right]^2}{32 f}+\frac{a^2 c \,\text{Sec}\left[\frac{1}{2}(e+f x)\right]^4}{64 f}$$

Problem 13: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Csc}[e+f x]\sqrt{a+a\,\text{Sin}[e+f x]}}{c-c\,\text{Sin}[e+f x]}\,dx$$

Optimal (type 3, 69 leaves, 5 steps):

$$-\frac{2\sqrt{a}\,\text{ArcTanh}\left[\frac{\sqrt{a}\,\text{Cos}[e+f x]}{\sqrt{a+a\,\text{Sin}[e+f x]}}\right]}{c f}+\frac{2\,\text{Sec}[e+f x]\sqrt{a+a\,\text{Sin}[e+f x]}}{c f}$$

Result (type 3, 157 leaves):

$$\frac{1}{c f}\text{Sec}[e+f x]\left(2+\text{Cos}\left[\frac{1}{2}(e+f x)\right]\left(-\text{Log}\left[1+\text{Cos}\left[\frac{1}{2}(e+f x)\right]-\text{Sin}\left[\frac{1}{2}(e+f x)\right]\right]+\text{Log}\left[1-\text{Cos}\left[\frac{1}{2}(e+f x)\right]+\text{Sin}\left[\frac{1}{2}(e+f x)\right]\right]\right)+\left(\text{Log}\left[1+\text{Cos}\left[\frac{1}{2}(e+f x)\right]-\text{Sin}\left[\frac{1}{2}(e+f x)\right]\right]-\text{Log}\left[1-\text{Cos}\left[\frac{1}{2}(e+f x)\right]+\text{Sin}\left[\frac{1}{2}(e+f x)\right]\right]\right)\text{Sin}\left[\frac{1}{2}(e+f x)\right]\right)\sqrt{a\,(1+\text{Sin}[e+f x])}$$

Problem 14: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{Csc}[e+f x]}{\sqrt{a+a\,\text{Sin}[e+f x]}\,(c-c\,\text{Sin}[e+f x])}\,dx$$

Optimal (type 3, 120 leaves, 8 steps):

$$-\frac{2\,\text{ArcTanh}\left[\frac{\sqrt{a}\,\text{Cos}[e+f x]}{\sqrt{a+a\,\text{Sin}[e+f x]}}\right]}{\sqrt{a}\,c f}+\frac{\text{ArcTanh}\left[\frac{\sqrt{a}\,\text{Cos}[e+f x]}{\sqrt{2}\sqrt{a+a\,\text{Sin}[e+f x]}}\right]}{\sqrt{2}\sqrt{a}\,c f}+\frac{\text{Sec}[e+f x]\sqrt{a+a\,\text{Sin}[e+f x]}}{a c f}$$

Result (type 3, 234 leaves):

$$\frac{1}{c\, f\, (-1+\text{Sin}[e+f x]) \sqrt{a\,(1+\text{Sin}[e+f x])}}$$
$$\text{Cos}[e+f x] \left(-1+\text{Cos}\left[\frac{1}{2}(e+f x)\right]\right) \left(\text{Log}\left[1+\text{Cos}\left[\frac{1}{2}(e+f x)\right]-\text{Sin}\left[\frac{1}{2}(e+f x)\right]\right]-\right.$$
$$\text{Log}\left[1-\text{Cos}\left[\frac{1}{2}(e+f x)\right]+\text{Sin}\left[\frac{1}{2}(e+f x)\right]\right]\Big)+(1+i)\,(-1)^{3/4}$$
$$\text{ArcTanh}\left[\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\text{Tan}\left[\frac{1}{4}(e+f x)\right]\right)\right]\left(\text{Cos}\left[\frac{1}{2}(e+f x)\right]-\text{Sin}\left[\frac{1}{2}(e+f x)\right]\right)-$$
$$\text{Log}\left[1+\text{Cos}\left[\frac{1}{2}(e+f x)\right]-\text{Sin}\left[\frac{1}{2}(e+f x)\right]\right]\text{Sin}\left[\frac{1}{2}(e+f x)\right]+$$
$$\text{Log}\left[1-\text{Cos}\left[\frac{1}{2}(e+f x)\right]+\text{Sin}\left[\frac{1}{2}(e+f x)\right]\right]\text{Sin}\left[\frac{1}{2}(e+f x)\right]\Big)$$

Problem 15: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{g\,\text{Sin}[e+f x]}\,\sqrt{a+a\,\text{Sin}[e+f x]}}{c-c\,\text{Sin}[e+f x]}\,dx$$

Optimal (type 3, 103 leaves, 6 steps):

$$\frac{2\sqrt{a}\,\sqrt{g}\,\text{ArcTan}\left[\frac{\sqrt{a}\,\sqrt{g}\,\text{Cos}[e+f x]}{\sqrt{g\,\text{Sin}[e+f x]}\,\sqrt{a+a\,\text{Sin}[e+f x]}}\right]}{c\, f}+\frac{2\,\text{Sec}[e+f x]\,\sqrt{g\,\text{Sin}[e+f x]}\,\sqrt{a+a\,\text{Sin}[e+f x]}}{c\, f}$$

Result (type 3, 180 leaves):

$$\Big(2\, e^{i\,(e+f x)}\Big(2\sqrt{-1+e^{2 i\,(e+f x)}}+$$
$$\left(1+i\, e^{i\,(e+f x)}\right)\text{ArcTan}\left[\frac{1}{\sqrt{-1+e^{2 i\,(e+f x)}}}\right]-\left(-i+e^{i\,(e+f x)}\right)\text{Log}\left[e^{i\,(e+f x)}+\sqrt{-1+e^{2 i\,(e+f x)}}\right]\Big)$$
$$\sqrt{g\,\text{Sin}[e+f x]}\,\sqrt{a\,(1+\text{Sin}[e+f x])}\Big)\Big/\left(c\sqrt{-1+e^{2 i\,(e+f x)}}\left(1+e^{2 i\,(e+f x)}\right) f\right)$$

Problem 17: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{g\,\text{Sin}[e+f x]}}{\sqrt{a+a\,\text{Sin}[e+f x]}\,(c-c\,\text{Sin}[e+f x])}\,dx$$

Optimal (type 3, 114 leaves, 6 steps):

$$\frac{\sqrt{g}\,\text{ArcTan}\left[\frac{\sqrt{a}\,\sqrt{g}\,\text{Cos}[e+f x]}{\sqrt{2}\,\sqrt{g\,\text{Sin}[e+f x]}\,\sqrt{a+a\,\text{Sin}[e+f x]}}\right]}{\sqrt{2}\,\sqrt{a}\, c\, f}+\frac{\text{Sec}[e+f x]\,\sqrt{g\,\text{Sin}[e+f x]}\,\sqrt{a+a\,\text{Sin}[e+f x]}}{a\, c\, f}$$

Result (type 4, 232 leaves):

$$\left(\left(\text{Cos}\left[\frac{1}{2}(e+f x)\right]-\text{Sin}\left[\frac{1}{2}(e+f x)\right]\right)\left(\text{Cos}\left[\frac{1}{2}(e+f x)\right]+\text{Sin}\left[\frac{1}{2}(e+f x)\right]\right)\right.$$

$$\sqrt{g\,\text{Sin}[e+f x]}\left(1-\left(\left(\text{EllipticF}\left[\text{ArcSin}\left[\frac{1}{\sqrt{\text{Tan}\left[\frac{1}{4}(e+f x)\right]}}\right],-1\right]+\right.\right.\right.$$

$$\text{EllipticPi}\left[1-\sqrt{2},-\text{ArcSin}\left[\frac{1}{\sqrt{\text{Tan}\left[\frac{1}{4}(e+f x)\right]}}\right],-1\right]+$$

$$\left.\text{EllipticPi}\left[1+\sqrt{2},-\text{ArcSin}\left[\frac{1}{\sqrt{\text{Tan}\left[\frac{1}{4}(e+f x)\right]}}\right],-1\right]\right)$$

$$\left.\text{Sec}\left[\frac{1}{4}(e+f x)\right]^2\left(\text{Cos}\left[\frac{1}{2}(e+f x)\right]-\text{Sin}\left[\frac{1}{2}(e+f x)\right]\right)\right)\Big/$$

$$\left.\left.\left(\sqrt{1-\text{Cot}\left[\frac{1}{4}(e+f x)\right]^2}\ \text{Tan}\left[\frac{1}{4}(e+f x)\right]^{3/2}\right)\right)\right)\Big/$$

$$\left(f\sqrt{a\,(1+\text{Sin}[e+f x])}\ (c-c\,\text{Sin}[e+f x])\right)$$

Problem 18: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\sqrt{g\,\text{Sin}[e+f x]}\ \sqrt{a+a\,\text{Sin}[e+f x]}\ (c-c\,\text{Sin}[e+f x])}\,dx$$

Optimal (type 3, 118 leaves, 6 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{a}\ \sqrt{g}\ \text{Cos}[e+f x]}{\sqrt{2}\ \sqrt{g\,\text{Sin}[e+f x]}\ \sqrt{a+a\,\text{Sin}[e+f x]}}\right]}{\sqrt{2}\ \sqrt{a}\ c\,f\sqrt{g}}+\frac{\text{Sec}[e+f x]\ \sqrt{g\,\text{Sin}[e+f x]}\ \sqrt{a+a\,\text{Sin}[e+f x]}}{a\,c\,f\,g}$$

Result (type 4, 234 leaves):

$$\left(\left(\text{Cos}\left[\frac{1}{2}(e+f x)\right]-\text{Sin}\left[\frac{1}{2}(e+f x)\right]\right)\left(\text{Cos}\left[\frac{1}{2}(e+f x)\right]+\text{Sin}\left[\frac{1}{2}(e+f x)\right]\right)\right.$$

$$\sqrt{g\,\text{Sin}[e+f x]}\left(1+\left(\left(\text{EllipticF}\left[\text{ArcSin}\left[\frac{1}{\sqrt{\text{Tan}\left[\frac{1}{4}(e+f x)\right]}}\right],-1\right]+\right.\right.\right.$$

$$\text{EllipticPi}\left[1-\sqrt{2},-\text{ArcSin}\left[\frac{1}{\sqrt{\text{Tan}\left[\frac{1}{4}(e+f x)\right]}}\right],-1\right]+$$

$$\left.\text{EllipticPi}\left[1+\sqrt{2},-\text{ArcSin}\left[\frac{1}{\sqrt{\text{Tan}\left[\frac{1}{4}(e+f x)\right]}}\right],-1\right]\right)$$

$$\left.\text{Sec}\left[\frac{1}{4}(e+f x)\right]^2\left(\text{Cos}\left[\frac{1}{2}(e+f x)\right]-\text{Sin}\left[\frac{1}{2}(e+f x)\right]\right)\right)\Big/$$

$$\left.\left.\left(\sqrt{1-\text{Cot}\left[\frac{1}{4}(e+f x)\right]^2}\,\text{Tan}\left[\frac{1}{4}(e+f x)\right]^{3/2}\right)\right)\right)\Big/$$

$$\left(f\,g\,\sqrt{a\,(1+\text{Sin}[e+f x])}\,(c-c\,\text{Sin}[e+f x])\right)$$

Problem 20: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\text{Csc}[e+f x]\,\sqrt{a+a\,\text{Sin}[e+f x]}}{\sqrt{c-c\,\text{Sin}[e+f x]}}\,dx$$

Optimal (type 3, 102 leaves, 6 steps):

$$-\frac{a\,\text{Cos}[e+f x]\,\text{Log}[1-\text{Sin}[e+f x]]}{f\sqrt{a+a\,\text{Sin}[e+f x]}\,\sqrt{c-c\,\text{Sin}[e+f x]}}+\frac{1}{c\,f}$$
$$\text{Log}[\text{Sin}[e+f x]]\,\text{Sec}[e+f x]\,\sqrt{a+a\,\text{Sin}[e+f x]}\,\sqrt{c-c\,\text{Sin}[e+f x]}$$

Result (type 3, 144 leaves):

$$\left(\sqrt{2}\,\left(-i+e^{i\,(e+f\,x)}\right)\,\left(2\,\text{ArcTan}\left[e^{i\,(e+f\,x)}\right]+i\,\left(\text{Log}\left[1-e^{2\,i\,(e+f\,x)}\right]-\text{Log}\left[1+e^{2\,i\,(e+f\,x)}\right]\right)\right)\sqrt{a\,\left(1+\text{Sin}[e+f\,x]\right)}\right)\Big/\left(\sqrt{i\,c\,e^{-i\,(e+f\,x)}\,\left(-i+e^{i\,(e+f\,x)}\right)^2}\,\left(i+e^{i\,(e+f\,x)}\right)\,f\right)$$

Problem 21: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\text{Csc}[e+f\,x]\,\sqrt{c-c\,\text{Sin}[e+f\,x]}}{\sqrt{a+a\,\text{Sin}[e+f\,x]}}\,dx$$

Optimal (type 3, 100 leaves, 6 steps):

$$-\frac{c\,\text{Cos}[e+f\,x]\,\text{Log}[1+\text{Sin}[e+f\,x]]}{f\,\sqrt{a+a\,\text{Sin}[e+f\,x]}\,\sqrt{c-c\,\text{Sin}[e+f\,x]}}+\frac{1}{a\,f}\text{Log}[\text{Sin}[e+f\,x]]\,\text{Sec}[e+f\,x]\,\sqrt{a+a\,\text{Sin}[e+f\,x]}\,\sqrt{c-c\,\text{Sin}[e+f\,x]}$$

Result (type 3, 145 leaves):

$$\left(\sqrt{2}\,\left(i+e^{i\,(e+f\,x)}\right)\,\left(2\,\text{ArcTan}\left[e^{i\,(e+f\,x)}\right]-i\,\left(\text{Log}\left[1-e^{2\,i\,(e+f\,x)}\right]-\text{Log}\left[1+e^{2\,i\,(e+f\,x)}\right]\right)\right)\sqrt{c-c\,\text{Sin}[e+f\,x]}\right)\Big/\left(\left(-i+e^{i\,(e+f\,x)}\right)\,\sqrt{-i\,a\,e^{-i\,(e+f\,x)}\,\left(i+e^{i\,(e+f\,x)}\right)^2}\,f\right)$$

Problem 22: Result more than twice size of optimal antiderivative.

$$\int\frac{\text{Csc}[e+f\,x]}{\sqrt{a+a\,\text{Sin}[e+f\,x]}\,\sqrt{c-c\,\text{Sin}[e+f\,x]}}\,dx$$

Optimal (type 3, 46 leaves, 3 steps):

$$\frac{\text{Cos}[e+f\,x]\,\text{Log}[\text{Tan}[e+f\,x]]}{f\,\sqrt{a+a\,\text{Sin}[e+f\,x]}\,\sqrt{c-c\,\text{Sin}[e+f\,x]}}$$

Result (type 3, 96 leaves):

$$-\left(\left(\text{Cos}[e+f\,x]\,\left(\text{Log}\left[\text{Cos}\left[\frac{1}{2}\,(e+f\,x)\right]-\text{Sin}\left[\frac{1}{2}\,(e+f\,x)\right]\right]+\text{Log}\left[\text{Cos}\left[\frac{1}{2}\,(e+f\,x)\right]+\text{Sin}\left[\frac{1}{2}\,(e+f\,x)\right]\right]-\text{Log}[\text{Sin}[e+f\,x]]\right)\right)\Big/\left(f\,\sqrt{a\,\left(1+\text{Sin}[e+f\,x]\right)}\,\sqrt{c-c\,\text{Sin}[e+f\,x]}\right)\right)$$

Problem 23: Result is not expressed in closed-form.

$$\int\frac{\text{Csc}[e+f\,x]\,\sqrt{a+a\,\text{Sin}[e+f\,x]}}{c+d\,\text{Sin}[e+f\,x]}\,dx$$

Optimal (type 3, 105 leaves, 5 steps):

$$-\frac{2\sqrt{a}\,\text{ArcTanh}\left[\frac{\sqrt{a}\,\text{Cos}[e+f x]}{\sqrt{a+a\,\text{Sin}[e+f x]}}\right]}{c f}+\frac{2\sqrt{a}\,\sqrt{d}\,\text{ArcTanh}\left[\frac{\sqrt{a}\,\sqrt{d}\,\text{Cos}[e+f x]}{\sqrt{c+d}\,\sqrt{a+a\,\text{Sin}[e+f x]}}\right]}{c\sqrt{c+d}\,f}$$

Result (type 7, 746 leaves):

$$-\frac{1}{c\sqrt{c+d}\,f\left(\text{Cos}\left[\frac{1}{2}(e+f x)\right]+\text{Sin}\left[\frac{1}{2}(e+f x)\right]\right)}\left(\frac{1}{8}-\frac{i}{8}\right)\left((4+4 i)\sqrt{c+d}\left(\text{Log}\left[1+\text{Cos}\left[\frac{1}{2}(e+f x)\right]-\text{Sin}\left[\frac{1}{2}(e+f x)\right]\right]-\text{Log}\left[1-\text{Cos}\left[\frac{1}{2}(e+f x)\right]+\text{Sin}\left[\frac{1}{2}(e+f x)\right]\right]\right)+\sqrt{d}\,\text{RootSum}\left[-d+2 i c\, e^{i e}\,\#1^2+d\, e^{2 i e}\,\#1^4\,\&,\left((1+i)\, d\sqrt{e^{-i e}}\, f x-(2-2 i)\, d\sqrt{e^{-i e}}\,\text{Log}\left[e^{\frac{i f x}{2}}-\#1\right]-i\sqrt{d}\,\sqrt{c+d}\, f x\,\#1+2\sqrt{d}\,\sqrt{c+d}\,\text{Log}\left[e^{\frac{i f x}{2}}-\#1\right]\#1+\frac{(1-i)\, c f x\,\#1^2}{\sqrt{e^{-i e}}}+\frac{(2+2 i)\, c\,\text{Log}\left[e^{\frac{i f x}{2}}-\#1\right]\#1^2}{\sqrt{e^{-i e}}}-\sqrt{d}\,\sqrt{c+d}\, e^{i e} f x\,\#1^3-2 i\sqrt{d}\,\sqrt{c+d}\, e^{i e}\,\text{Log}\left[e^{\frac{i f x}{2}}-\#1\right]\#1^3\right)\Big/\left(-i d-c\, e^{i e}\,\#1^2\right)\&\right]\left(\text{Cos}\left[\frac{e}{2}\right]+i\,\text{Sin}\left[\frac{e}{2}\right]\right)+\sqrt{d}\,\text{RootSum}\left[-d+2 i c\, e^{i e}\,\#1^2+d\, e^{2 i e}\,\#1^4\,\&,\frac{1}{d-i c\, e^{i e}\,\#1^2}\left((1-i)\, d\sqrt{e^{-i e}}\, f x+(2+2 i)\, d\sqrt{e^{-i e}}\,\text{Log}\left[e^{\frac{i f x}{2}}-\#1\right]+\sqrt{d}\,\sqrt{c+d}\, f x\,\#1+2 i\sqrt{d}\,\sqrt{c+d}\,\text{Log}\left[e^{\frac{i f x}{2}}-\#1\right]\#1-\frac{(1+i)\, c f x\,\#1^2}{\sqrt{e^{-i e}}}+\frac{(2-2 i)\, c\,\text{Log}\left[e^{\frac{i f x}{2}}-\#1\right]\#1^2}{\sqrt{e^{-i e}}}-i\sqrt{d}\,\sqrt{c+d}\, e^{i e} f x\,\#1^3+2\sqrt{d}\,\sqrt{c+d}\, e^{i e}\,\text{Log}\left[e^{\frac{i f x}{2}}-\#1\right]\#1^3\right)\&\right]\left(\text{Cos}\left[\frac{e}{2}\right]+i\,\text{Sin}\left[\frac{e}{2}\right]\right)\right)\sqrt{a\left(1+\text{Sin}[e+f x]\right)}$$

Problem 24: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{\text{Csc}[e+f x]}{\sqrt{a+a\,\text{Sin}[e+f x]}\,\left(c+d\,\text{Sin}[e+f x]\right)}\,dx$$

Optimal (type 3, 165 leaves, 8 steps):

$$-\frac{2\,\text{ArcTanh}\left[\frac{\sqrt{a}\,\text{Cos}[e+f x]}{\sqrt{a+a\,\text{Sin}[e+f x]}}\right]}{\sqrt{a}\,c\,f}+\frac{\sqrt{2}\,\text{ArcTanh}\left[\frac{\sqrt{a}\,\text{Cos}[e+f x]}{\sqrt{2}\,\sqrt{a+a\,\text{Sin}[e+f x]}}\right]}{\sqrt{a}\,(c-d)\,f}-\frac{2\,d^{3/2}\,\text{ArcTanh}\left[\frac{\sqrt{a}\,\sqrt{d}\,\text{Cos}[e+f x]}{\sqrt{c+d}\,\sqrt{a+a\,\text{Sin}[e+f x]}}\right]}{\sqrt{a}\,c\,(c-d)\,\sqrt{c+d}\,f}$$

Result (type 3, 331 leaves):

$$-\frac{1}{c\,(c-d)\,\sqrt{c+d}\,f\,\sqrt{a\,(1+\text{Sin}[e+f x])}}\Bigg(\Bigg((2+2\,i)\,(-1)^{3/4}\,c\,\sqrt{c+d}\,\text{ArcTanh}\left[\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\text{Tan}\left[\frac{1}{4}(e+f x)\right]\right)\right]+$$
$$(c-d)\,\sqrt{c+d}\,\text{Log}\left[1+\text{Cos}\left[\frac{1}{2}(e+f x)\right]-\text{Sin}\left[\frac{1}{2}(e+f x)\right]\right]-$$
$$c\,\sqrt{c+d}\,\text{Log}\left[1-\text{Cos}\left[\frac{1}{2}(e+f x)\right]+\text{Sin}\left[\frac{1}{2}(e+f x)\right]\right]+$$
$$d\,\sqrt{c+d}\,\text{Log}\left[1-\text{Cos}\left[\frac{1}{2}(e+f x)\right]+\text{Sin}\left[\frac{1}{2}(e+f x)\right]\right]+$$
$$d^{3/2}\,\text{Log}\left[\text{Sec}\left[\frac{1}{4}(e+f x)\right]^2\left(\sqrt{c+d}+\sqrt{d}\,\text{Cos}\left[\frac{1}{2}(e+f x)\right]-\sqrt{d}\,\text{Sin}\left[\frac{1}{2}(e+f x)\right]\right)\right]-$$
$$d^{3/2}\,\text{Log}\left[\text{Sec}\left[\frac{1}{4}(e+f x)\right]^2\left(\sqrt{c+d}-\sqrt{d}\,\text{Cos}\left[\frac{1}{2}(e+f x)\right]+\sqrt{d}\,\text{Sin}\left[\frac{1}{2}(e+f x)\right]\right)\right]\Bigg)$$
$$\left(\text{Cos}\left[\frac{1}{2}(e+f x)\right]+\text{Sin}\left[\frac{1}{2}(e+f x)\right]\right)\Bigg)$$

Problem 25: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{\sqrt{g\,\text{Sin}[e+f x]}\,\sqrt{a+a\,\text{Sin}[e+f x]}}{c+d\,\text{Sin}[e+f x]}\,dx$$

Optimal (type 3, 149 leaves, 5 steps):

$$-\frac{2\,\sqrt{a}\,\sqrt{g}\,\text{ArcTan}\left[\frac{\sqrt{a}\,\sqrt{g}\,\text{Cos}[e+f x]}{\sqrt{g\,\text{Sin}[e+f x]}\,\sqrt{a+a\,\text{Sin}[e+f x]}}\right]}{d\,f}+$$
$$\frac{2\,\sqrt{a}\,\sqrt{c}\,\sqrt{g}\,\text{ArcTan}\left[\frac{\sqrt{a}\,\sqrt{c}\,\sqrt{g}\,\text{Cos}[e+f x]}{\sqrt{c+d}\,\sqrt{g\,\text{Sin}[e+f x]}\,\sqrt{a+a\,\text{Sin}[e+f x]}}\right]}{d\,\sqrt{c+d}\,f}$$

Result (type 3, 908 leaves):

$$\Bigg(\left(\frac{1}{2}+\frac{i}{2}\right)\Bigg(-2\,\sqrt{c^2-d^2}\,\sqrt{-c+\sqrt{c^2-d^2}}$$
$$\sqrt{c+\sqrt{c^2-d^2}}\,\text{ArcTan}\left[\frac{1}{\sqrt{-1+\text{Cos}[2\,(e+f x)]+i\,\text{Sin}[2\,(e+f x)]}}\right]+$$

$$\sqrt{2}\,\sqrt{c}\,\left(\sqrt{c+\sqrt{c^2-d^2}}\,\left(-c+d+\sqrt{c^2-d^2}\right)\,\text{Log}\Big[\Big(d^2\,\sqrt{c^2-d^2}\,e^{-i\,e}\,\Big(i\,\sqrt{2}\,d-\sqrt{2}\,c\,e^{i\,(e+f\,x)}+\sqrt{2}\,\sqrt{c^2-d^2}\,e^{i\,(e+f\,x)}-2\,i\,\sqrt{c}\,\sqrt{-c+\sqrt{c^2-d^2}}\,\sqrt{-1+e^{2\,i\,(e+f\,x)}}\Big)\,f\Big)\Big/\left(2\,c^{3/2}\,\sqrt{-c+\sqrt{c^2-d^2}}\,\left(-c+d+\sqrt{c^2-d^2}\right)\,\left(-c+\sqrt{c^2-d^2}+i\,d\,e^{i\,(e+f\,x)}\right)\right)\Big]+i\,\sqrt{-c+\sqrt{c^2-d^2}}\,\left(c-d+\sqrt{c^2-d^2}\right)\,\text{Log}\Big[\Big(i\,d^2\,e^{-i\,e}\,\Big(-i\,\sqrt{2}\,d\,\sqrt{c^2-d^2}+\sqrt{2}\,c^2\,e^{i\,(e+f\,x)}-\sqrt{2}\,d^2\,e^{i\,(e+f\,x)}+\sqrt{2}\,c\,\sqrt{c^2-d^2}\,e^{i\,(e+f\,x)}+2\,\sqrt{c}\,\sqrt{c^2-d^2}\,\sqrt{c+\sqrt{c^2-d^2}}\,\sqrt{-1+e^{2\,i\,(e+f\,x)}}\Big)\,f\Big)\Big/\left(2\,c^{3/2}\,\sqrt{c+\sqrt{c^2-d^2}}\,\left(c-d+\sqrt{c^2-d^2}\right)\,\left(c+\sqrt{c^2-d^2}-i\,d\,e^{i\,(e+f\,x)}\right)\right)\Big]\right)-2\,i\,\sqrt{c^2-d^2}\,\sqrt{-c+\sqrt{c^2-d^2}}\,\sqrt{c+\sqrt{c^2-d^2}}\,\text{Log}\Big[\text{Cos}[e+f\,x]+i\,\text{Sin}[e+f\,x]+\sqrt{-1+\text{Cos}[2\,(e+f\,x)]+i\,\text{Sin}[2\,(e+f\,x)]}\Big]\Big)\left(\text{Cos}\left[\frac{1}{2}\,(e+f\,x)\right]+i\,\text{Sin}\left[\frac{1}{2}\,(e+f\,x)\right]\right)\sqrt{g\,\text{Sin}[e+f\,x]}\,\sqrt{a\,\left(1+\text{Sin}[e+f\,x]\right)}\Bigg)\Bigg/\Bigg(d\,\sqrt{c^2-d^2}\,\sqrt{-c+\sqrt{c^2-d^2}}\,\sqrt{c+\sqrt{c^2-d^2}}\,f\,\left(\text{Cos}\left[\frac{1}{2}\,(e+f\,x)\right]+\text{Sin}\left[\frac{1}{2}\,(e+f\,x)\right]\right)\sqrt{-1+\text{Cos}[2\,(e+f\,x)]+i\,\text{Sin}[2\,(e+f\,x)]}\Bigg)$$

Problem 26: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{a + a \operatorname{Sin}[e + f x]}}{\sqrt{g \operatorname{Sin}[e + f x]} \left(c + d \operatorname{Sin}[e + f x]\right)} \, dx$$

Optimal (type 3, 83 leaves, 2 steps):

$$-\frac{2 \sqrt{a} \operatorname{ArcTan}\left[\frac{\sqrt{a} \sqrt{c} \sqrt{g} \operatorname{Cos}[e+f x]}{\sqrt{c+d} \sqrt{g \operatorname{Sin}[e+f x]} \sqrt{a+a \operatorname{Sin}[e+f x]}}\right]}{\sqrt{c} \sqrt{c+d} f \sqrt{g}}$$

Result (type 3, 748 leaves):

$$\left(\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\sqrt{c+\sqrt{c^2-d^2}}\left(-c+d+\sqrt{c^2-d^2}\right)\right.\right.$$

$$\mathrm{Log}\Big[\left(d\,e^{-i e}\left(\sqrt{2}\,d\sqrt{c^2-d^2}-i\sqrt{2}\,c^2 e^{i(e+fx)}+i\sqrt{2}\,d^2 e^{i(e+fx)}+\right.\right.$$

$$\left.\left.i\sqrt{2}\,c\sqrt{c^2-d^2}\,e^{i(e+fx)}-2\sqrt{c}\sqrt{c^2-d^2}\sqrt{-c+\sqrt{c^2-d^2}}\sqrt{-1+e^{2i(e+fx)}}\right)f\right)\Big/$$

$$\left(\sqrt{c}\sqrt{-c+\sqrt{c^2-d^2}}\left(-c+d+\sqrt{c^2-d^2}\right)\left(-c+\sqrt{c^2-d^2}+i\,d\,e^{i(e+fx)}\right)\right)\Big]-$$

$$\sqrt{-c+\sqrt{c^2-d^2}}\left(c-d+\sqrt{c^2-d^2}\right)$$

$$\mathrm{Log}\Big[\left(d\,e^{-i e}\left(-i\sqrt{2}\,d\sqrt{c^2-d^2}+\sqrt{2}\,c^2 e^{i(e+fx)}-\sqrt{2}\,d^2 e^{i(e+fx)}+\right.\right.$$

$$\left.\left.\sqrt{2}\,c\sqrt{c^2-d^2}\,e^{i(e+fx)}+2\sqrt{c}\sqrt{c^2-d^2}\sqrt{c+\sqrt{c^2-d^2}}\sqrt{-1+e^{2i(e+fx)}}\right)f\right)\Big/$$

$$\left.\left(\sqrt{c}\sqrt{c+\sqrt{c^2-d^2}}\left(c-d+\sqrt{c^2-d^2}\right)\left(c+\sqrt{c^2-d^2}-i\,d\,e^{i(e+fx)}\right)\right)\Big]\right)$$

$$\left(\mathrm{Cos}\left[\frac{1}{2}(e+fx)\right]-i\,\mathrm{Sin}\left[\frac{1}{2}(e+fx)\right]\right)\sqrt{a\,(1+\mathrm{Sin}[e+fx])}$$

$$\left.\sqrt{-1+\mathrm{Cos}[2(e+fx)]+i\,\mathrm{Sin}[2(e+fx)]}\right)\Big/\left(\sqrt{2}\right.$$

$$\sqrt{c}$$

$$\sqrt{c^2-d^2}$$

$$\sqrt{-c+\sqrt{c^2-d^2}}$$

$$\sqrt{c+\sqrt{c^2-d^2}}$$

$$f$$

$$\left(\mathrm{Cos}\left[\frac{1}{2}(e+fx)\right]+\mathrm{Sin}\left[\frac{1}{2}(e+fx)\right]\right)$$

$$\left.\sqrt{g\,\mathrm{Sin}[e+fx]}\right)$$

Problem 27: Result unnecessarily involves higher level functions and more than

twice size of optimal antiderivative.

$$\int \frac{\sqrt{g \operatorname{Sin}[e+f x]}}{\sqrt{a+a \operatorname{Sin}[e+f x]}\,\left(c+d \operatorname{Sin}[e+f x]\right)}\,dx$$

Optimal (type 3, 166 leaves, 5 steps):

$$\frac{\sqrt{2}\,\sqrt{g}\,\operatorname{ArcTan}\left[\frac{\sqrt{a}\,\sqrt{g}\,\operatorname{Cos}[e+f x]}{\sqrt{2}\,\sqrt{g \operatorname{Sin}[e+f x]}\,\sqrt{a+a \operatorname{Sin}[e+f x]}}\right]}{\sqrt{a}\,(c-d)\,f}-\frac{2\,\sqrt{c}\,\sqrt{g}\,\operatorname{ArcTan}\left[\frac{\sqrt{a}\,\sqrt{c}\,\sqrt{g}\,\operatorname{Cos}[e+f x]}{\sqrt{c+d}\,\sqrt{g \operatorname{Sin}[e+f x]}\,\sqrt{a+a \operatorname{Sin}[e+f x]}}\right]}{\sqrt{a}\,(c-d)\,\sqrt{c+d}\,f}$$

Result (type 4, 61 316 leaves): Display of huge result suppressed!

Problem 28: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{g \operatorname{Sin}[e+f x]}\,\sqrt{a+a \operatorname{Sin}[e+f x]}\,\left(c+d \operatorname{Sin}[e+f x]\right)}\,dx$$

Optimal (type 3, 168 leaves, 5 steps):

$$-\frac{\sqrt{2}\,\operatorname{ArcTan}\left[\frac{\sqrt{a}\,\sqrt{g}\,\operatorname{Cos}[e+f x]}{\sqrt{2}\,\sqrt{g \operatorname{Sin}[e+f x]}\,\sqrt{a+a \operatorname{Sin}[e+f x]}}\right]}{\sqrt{a}\,(c-d)\,f\,\sqrt{g}}+\frac{2\,d\,\operatorname{ArcTan}\left[\frac{\sqrt{a}\,\sqrt{c}\,\sqrt{g}\,\operatorname{Cos}[e+f x]}{\sqrt{c+d}\,\sqrt{g \operatorname{Sin}[e+f x]}\,\sqrt{a+a \operatorname{Sin}[e+f x]}}\right]}{\sqrt{a}\,\sqrt{c}\,(c-d)\,\sqrt{c+d}\,f\,\sqrt{g}}$$

Result (type 4, 99 997 leaves): Display of huge result suppressed!

Problem 29: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Csc}[e+f x]\,\sqrt{a+b \operatorname{Sin}[e+f x]}}{c+c \operatorname{Sin}[e+f x]}\,dx$$

Optimal (type 4, 238 leaves, 9 steps):

$$\frac{\operatorname{EllipticE}\left[\frac{1}{2}\left(e-\frac{\pi}{2}+f x\right),\frac{2 b}{a+b}\right]\sqrt{a+b \operatorname{Sin}[e+f x]}}{c\,f\,\sqrt{\frac{a+b \operatorname{Sin}[e+f x]}{a+b}}}-$$

$$\frac{(a-b)\,\operatorname{EllipticF}\left[\frac{1}{2}\left(e-\frac{\pi}{2}+f x\right),\frac{2 b}{a+b}\right]\sqrt{\frac{a+b \operatorname{Sin}[e+f x]}{a+b}}}{c\,f\,\sqrt{a+b \operatorname{Sin}[e+f x]}}+$$

$$\frac{2\,a\,\operatorname{EllipticPi}\left[2,\frac{1}{2}\left(e-\frac{\pi}{2}+f x\right),\frac{2 b}{a+b}\right]\sqrt{\frac{a+b \operatorname{Sin}[e+f x]}{a+b}}}{c\,f\,\sqrt{a+b \operatorname{Sin}[e+f x]}}+\frac{\operatorname{Cos}[e+f x]\,\sqrt{a+b \operatorname{Sin}[e+f x]}}{f\,\left(c+c \operatorname{Sin}[e+f x]\right)}$$

Result (type 4, 611 leaves):

$$-\left(\left(2\,\text{Sin}\left[\frac{1}{2}\,(e+f\,x)\right]\,\left(\text{Cos}\left[\frac{1}{2}\,(e+f\,x)\right]+\text{Sin}\left[\frac{1}{2}\,(e+f\,x)\right]\right)\,\sqrt{a+b\,\text{Sin}[e+f\,x]}\right)\Big/\left(f\,(c+c\,\text{Sin}[e+f\,x])\right)\right)+\frac{1}{4\,f\,(c+c\,\text{Sin}[e+f\,x])}$$

$$\left(\text{Cos}\left[\frac{1}{2}\,(e+f\,x)\right]+\text{Sin}\left[\frac{1}{2}\,(e+f\,x)\right]\right)^2\left(-\frac{4\,b\,\text{EllipticF}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f\,x\right),\frac{2\,b}{a+b}\right]\sqrt{\frac{a+b\,\text{Sin}[e+f\,x]}{a+b}}}{\sqrt{a+b\,\text{Sin}[e+f\,x]}}+\right.$$

$$\frac{2\,(-4\,a-b)\,\text{EllipticPi}\left[2,\frac{1}{2}\left(-e+\frac{\pi}{2}-f\,x\right),\frac{2\,b}{a+b}\right]\sqrt{\frac{a+b\,\text{Sin}[e+f\,x]}{a+b}}}{\sqrt{a+b\,\text{Sin}[e+f\,x]}}+\Big(2\,i\,b\,\text{Cos}[e+f\,x]$$

$$\text{Cos}[2\,(e+f\,x)]\,\Big(2\,a\,(a-b)\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{1}{a+b}}\,\sqrt{a+b\,\text{Sin}[e+f\,x]}\right],\frac{a+b}{a-b}\right]+$$

$$b\,\Big(2\,a\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{1}{a+b}}\,\sqrt{a+b\,\text{Sin}[e+f\,x]}\right],\frac{a+b}{a-b}\right]-$$

$$b\,\text{EllipticPi}\left[\frac{a+b}{a},\,i\,\text{ArcSinh}\left[\sqrt{-\frac{1}{a+b}}\,\sqrt{a+b\,\text{Sin}[e+f\,x]}\right],\frac{a+b}{a-b}\right]\Big)\Big)$$

$$\sqrt{\frac{b-b\,\text{Sin}[e+f\,x]}{a+b}}\,\sqrt{-\frac{b+b\,\text{Sin}[e+f\,x]}{a-b}}\Bigg)\Bigg/\Bigg(a\,\sqrt{-\frac{1}{a+b}}\,\sqrt{1-\text{Sin}[e+f\,x]^2}$$

$$\left(-2\,a^2+b^2+4\,a\,(a+b\,\text{Sin}[e+f\,x])-2\,(a+b\,\text{Sin}[e+f\,x])^2\right)$$

$$\sqrt{-\frac{a^2-b^2-2\,a\,(a+b\,\text{Sin}[e+f\,x])+(a+b\,\text{Sin}[e+f\,x])^2}{b^2}}\Bigg)+$$

$$\left.\frac{2\,\text{Cot}[e+f\,x]\,\sqrt{a+b\,\text{Sin}[e+f\,x]}\,\text{Sin}[2\,(e+f\,x)]}{1-\text{Sin}[e+f\,x]^2}\right)$$

Problem 30: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{\text{Csc}[e+f\,x]}{\sqrt{a+b\,\text{Sin}[e+f\,x]}\,\left(c+c\,\text{Sin}[e+f\,x]\right)}\,dx$$

Optimal (type 4, 246 leaves, 9 steps):

$$\frac{\text{EllipticE}\left[\frac{1}{2}\left(e-\frac{\pi}{2}+f x\right),\frac{2 b}{a+b}\right]\sqrt{a+b\,\text{Sin}[e+f x]}}{(a-b)\,c\,f\sqrt{\frac{a+b\,\text{Sin}[e+f x]}{a+b}}}-\frac{\text{EllipticF}\left[\frac{1}{2}\left(e-\frac{\pi}{2}+f x\right),\frac{2 b}{a+b}\right]\sqrt{\frac{a+b\,\text{Sin}[e+f x]}{a+b}}}{c\,f\sqrt{a+b\,\text{Sin}[e+f x]}}+\frac{2\,\text{EllipticPi}\left[2,\frac{1}{2}\left(e-\frac{\pi}{2}+f x\right),\frac{2 b}{a+b}\right]\sqrt{\frac{a+b\,\text{Sin}[e+f x]}{a+b}}}{c\,f\sqrt{a+b\,\text{Sin}[e+f x]}}+\frac{\text{Cos}[e+f x]\sqrt{a+b\,\text{Sin}[e+f x]}}{(a-b)\,f\,(c+c\,\text{Sin}[e+f x])}$$

Result (type 4, 625 leaves):

$$-\left(\left(2\,\text{Sin}\left[\frac{1}{2}\,(e+f\,x)\right]\,\left(\text{Cos}\left[\frac{1}{2}\,(e+f\,x)\right]+\text{Sin}\left[\frac{1}{2}\,(e+f\,x)\right]\right)\,\sqrt{a+b\,\text{Sin}[e+f\,x]}\right)\Big/\left((a-b)\,f\,(c+c\,\text{Sin}[e+f\,x])\right)\right)-\frac{1}{4\,(a-b)\,f\,(c+c\,\text{Sin}[e+f\,x])}$$

$$\left(\text{Cos}\left[\frac{1}{2}\,(e+f\,x)\right]+\text{Sin}\left[\frac{1}{2}\,(e+f\,x)\right]\right)^2\left(\frac{4\,b\,\text{EllipticF}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f\,x\right),\frac{2\,b}{a+b}\right]\sqrt{\frac{a+b\,\text{Sin}[e+f\,x]}{a+b}}}{\sqrt{a+b\,\text{Sin}[e+f\,x]}}-\right.$$

$$\left(2\,(-4\,a+3\,b)\,\text{EllipticPi}\left[2,\frac{1}{2}\left(-e+\frac{\pi}{2}-f\,x\right),\frac{2\,b}{a+b}\right]\sqrt{\frac{a+b\,\text{Sin}[e+f\,x]}{a+b}}\right)\Big/$$

$$\left(\sqrt{a+b\,\text{Sin}[e+f\,x]}\right)-\left(2\,i\,b\,\text{Cos}[e+f\,x]\,\text{Cos}[2\,(e+f\,x)]\right.$$

$$\left(2\,a\,(a-b)\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{1}{a+b}}\,\sqrt{a+b\,\text{Sin}[e+f\,x]}\right],\frac{a+b}{a-b}\right]+\right.$$

$$b\left(2\,a\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{1}{a+b}}\,\sqrt{a+b\,\text{Sin}[e+f\,x]}\right],\frac{a+b}{a-b}\right]-\right.$$

$$\left.\left.b\,\text{EllipticPi}\left[\frac{a+b}{a},i\,\text{ArcSinh}\left[\sqrt{-\frac{1}{a+b}}\,\sqrt{a+b\,\text{Sin}[e+f\,x]}\right],\frac{a+b}{a-b}\right]\right)\right)$$

$$\left.\sqrt{\frac{b-b\,\text{Sin}[e+f\,x]}{a+b}}\,\sqrt{-\frac{b+b\,\text{Sin}[e+f\,x]}{a-b}}\right)\Big/\left(a\,\sqrt{-\frac{1}{a+b}}\,\sqrt{1-\text{Sin}[e+f\,x]^2}\right.$$

$$\left(-2\,a^2+b^2+4\,a\,(a+b\,\text{Sin}[e+f\,x])-2\,(a+b\,\text{Sin}[e+f\,x])^2\right)$$

$$\left.\sqrt{-\frac{a^2-b^2-2\,a\,(a+b\,\text{Sin}[e+f\,x])+(a+b\,\text{Sin}[e+f\,x])^2}{b^2}}\right)-$$

$$\left.\frac{2\,\text{Cot}[e+f\,x]\,\sqrt{a+b\,\text{Sin}[e+f\,x]}\,\text{Sin}[2\,(e+f\,x)]}{1-\text{Sin}[e+f\,x]^2}\right)$$

Problem 31: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{g\,\text{Sin}[e+f\,x]}\,\sqrt{a+b\,\text{Sin}[e+f\,x]}}{c+c\,\text{Sin}[e+f\,x]}\,dx$$

Optimal (type 4, 267 leaves, 3 steps):

$$\frac{1}{\sqrt{a+b}\,c\,f}2\sqrt{g}\,\text{EllipticPi}\Big[\frac{b}{a+b},\,\text{ArcSin}\Big[\frac{\sqrt{a+b}\,\sqrt{g\,\text{Sin}[e+f\,x]}}{\sqrt{g}\,\sqrt{a+b\,\text{Sin}[e+f\,x]}}\Big],\,-\frac{a-b}{a+b}\Big]\,\text{Sec}[e+f\,x]\,\sqrt{\frac{a\,(1-\text{Sin}[e+f\,x])}{a+b\,\text{Sin}[e+f\,x]}}\,\sqrt{\frac{a\,(1+\text{Sin}[e+f\,x])}{a+b\,\text{Sin}[e+f\,x]}}\,(a+b\,\text{Sin}[e+f\,x])+\left(g\,\text{EllipticE}\Big[\text{ArcSin}\Big[\frac{\text{Cos}[e+f\,x]}{1+\text{Sin}[e+f\,x]}\Big],\,-\frac{a-b}{a+b}\Big]\,\sqrt{\frac{\text{Sin}[e+f\,x]}{1+\text{Sin}[e+f\,x]}}\,\sqrt{a+b\,\text{Sin}[e+f\,x]}\right)\Big/\left(c\,f\,\sqrt{g\,\text{Sin}[e+f\,x]}\,\sqrt{\frac{a+b\,\text{Sin}[e+f\,x]}{(a+b)\,(1+\text{Sin}[e+f\,x])}}\right)$$

Result (type 4, 10 621 leaves):

$$\left(2\,\text{Sin}\Big[\frac{1}{2}(e+f\,x)\Big]\left(\text{Cos}\Big[\frac{1}{2}(e+f\,x)\Big]+\text{Sin}\Big[\frac{1}{2}(e+f\,x)\Big]\right)\sqrt{g\,\text{Sin}[e+f\,x]}\,\sqrt{a+b\,\text{Sin}[e+f\,x]}\right)\Big/\left(f\,(c+c\,\text{Sin}[e+f\,x])\right)+$$

$$\left(\left(\text{Cos}\Big[\frac{1}{2}(e+f\,x)\Big]+\text{Sin}\Big[\frac{1}{2}(e+f\,x)\Big]\right)^2\sqrt{g\,\text{Sin}[e+f\,x]}\left(-\frac{a\sqrt{\text{Sin}[e+f\,x]}}{2\sqrt{a+b\,\text{Sin}[e+f\,x]}}+\frac{b\sqrt{\text{Sin}[e+f\,x]}}{2\sqrt{a+b\,\text{Sin}[e+f\,x]}}-\frac{b\,\text{Cos}\big[\frac{3}{2}(e+f\,x)\big]\,\text{Sec}\big[\frac{1}{2}(e+f\,x)\big]\sqrt{\text{Sin}[e+f\,x]}}{2\sqrt{a+b\,\text{Sin}[e+f\,x]}}-\frac{b\,\text{Sec}\big[\frac{1}{2}(e+f\,x)\big]\sqrt{\text{Sin}[e+f\,x]}\,\text{Sin}\big[\frac{3}{2}(e+f\,x)\big]}{2\sqrt{a+b\,\text{Sin}[e+f\,x]}}+\frac{a\sqrt{\text{Sin}[e+f\,x]}\,\text{Tan}\big[\frac{1}{2}(e+f\,x)\big]}{2\sqrt{a+b\,\text{Sin}[e+f\,x]}}-\frac{b\sqrt{\text{Sin}[e+f\,x]}\,\text{Tan}\big[\frac{1}{2}(e+f\,x)\big]}{2\sqrt{a+b\,\text{Sin}[e+f\,x]}}\right)\left(-\sqrt{2}\left(1+\text{Tan}\Big[\frac{1}{2}(e+f\,x)\Big]\right)\right)\right.$$

$$\sqrt{\frac{\text{Tan}\left[\frac{1}{2}(e+f x)\right]}{1+\text{Tan}\left[\frac{1}{2}(e+f x)\right]^2}}\ \sqrt{\frac{a+2 b\, \text{Tan}\left[\frac{1}{2}(e+f x)\right]+a\, \text{Tan}\left[\frac{1}{2}(e+f x)\right]^2}{1+\text{Tan}\left[\frac{1}{2}(e+f x)\right]^2}}\ +$$

$$\frac{1}{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}\ \sqrt{\frac{a+2 b\, \text{Tan}\left[\frac{1}{2}(e+f x)\right]+a\, \text{Tan}\left[\frac{1}{2}(e+f x)\right]^2}{1+\text{Tan}\left[\frac{1}{2}(e+f x)\right]^2}}}\sqrt{2}\ \text{Cot}\left[\frac{1}{2}(e+f x)\right]$$

$$\sqrt{\frac{\text{Tan}\left[\frac{1}{2}(e+f x)\right]}{1+\text{Tan}\left[\frac{1}{2}(e+f x)\right]^2}}\ \left(a\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}+2 b\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}\ \text{Tan}\left[\frac{1}{2}(e+f x)\right]+\right.$$

$$a\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}\ \text{Tan}\left[\frac{1}{2}(e+f x)\right]^2-\mathbb{i}\left(-b+\sqrt{-a^2+b^2}\right)\sqrt{1+\frac{a\, \text{Cot}\left[\frac{1}{2}(e+f x)\right]}{b+\sqrt{-a^2+b^2}}}$$

$$\text{EllipticE}\left[\mathbb{i}\, \text{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\text{Tan}\left[\frac{1}{2}(e+f x)\right]}}\right],\frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}}\right]\text{Tan}\left[\frac{1}{2}(e+f x)\right]^{3/2}$$

$$\sqrt{\frac{a+b\, \text{Tan}\left[\frac{1}{2}(e+f x)\right]-\sqrt{-a^2+b^2}\ \text{Tan}\left[\frac{1}{2}(e+f x)\right]}{b\, \text{Tan}\left[\frac{1}{2}(e+f x)\right]-\sqrt{-a^2+b^2}\ \text{Tan}\left[\frac{1}{2}(e+f x)\right]}}+\mathbb{i}\left(a-b+\sqrt{-a^2+b^2}\right)$$

$$\sqrt{1+\frac{a\, \text{Cot}\left[\frac{1}{2}(e+f x)\right]}{b+\sqrt{-a^2+b^2}}}\ \text{EllipticF}\left[\mathbb{i}\, \text{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\text{Tan}\left[\frac{1}{2}(e+f x)\right]}}\right],\frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}}\right]$$

$$\text{Tan}\left[\frac{1}{2}(e+f x)\right]^{3/2}\sqrt{\frac{a+b\, \text{Tan}\left[\frac{1}{2}(e+f x)\right]-\sqrt{-a^2+b^2}\ \text{Tan}\left[\frac{1}{2}(e+f x)\right]}{b\, \text{Tan}\left[\frac{1}{2}(e+f x)\right]-\sqrt{-a^2+b^2}\ \text{Tan}\left[\frac{1}{2}(e+f x)\right]}}\ +$$

$$2 b\sqrt{1+\frac{a\, \text{Cot}\left[\frac{1}{2}(e+f x)\right]}{b+\sqrt{-a^2+b^2}}}\ \text{EllipticPi}\left[-\frac{\mathbb{i}\left(b+\sqrt{-a^2+b^2}\right)}{a},\right.$$

$$\left.\mathbb{i}\, \text{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\text{Tan}\left[\frac{1}{2}(e+f x)\right]}}\right],\frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}}\right]\text{Tan}\left[\frac{1}{2}(e+f x)\right]^{3/2}$$

$$\sqrt{\frac{a+b\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]-\sqrt{-a^2+b^2}\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]}{b\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]-\sqrt{-a^2+b^2}\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]}}-2\,b\,\sqrt{1+\frac{a\,\text{Cot}\left[\frac{1}{2}(e+f x)\right]}{b+\sqrt{-a^2+b^2}}}$$

$$\text{EllipticPi}\left[\frac{i\left(b+\sqrt{-a^2+b^2}\right)}{a},\,i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\text{Tan}\left[\frac{1}{2}(e+f x)\right]}}\right],\,\frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}}\right]$$

$$\left.\left.\left.\text{Tan}\left[\frac{1}{2}(e+f x)\right]^{3/2}\sqrt{\frac{a+b\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]-\sqrt{-a^2+b^2}\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]}{b\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]-\sqrt{-a^2+b^2}\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]}}\right)\right)\right)\Big/$$

$$\left(f\sqrt{\text{Sin}[e+f x]}\,(c+c\,\text{Sin}[e+f x])\left(-\frac{1}{\sqrt{2}}\text{Sec}\left[\frac{1}{2}(e+f x)\right]^2\sqrt{\frac{\text{Tan}\left[\frac{1}{2}(e+f x)\right]}{1+\text{Tan}\left[\frac{1}{2}(e+f x)\right]^2}}\right.\right.$$

$$\sqrt{\frac{a+2\,b\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]+a\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]^2}{1+\text{Tan}\left[\frac{1}{2}(e+f x)\right]^2}}-$$

$$\frac{1}{\sqrt{2}\,\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}\,\sqrt{\frac{a+2\,b\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]+a\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]^2}{1+\text{Tan}\left[\frac{1}{2}(e+f x)\right]^2}}}\text{Csc}\left[\frac{1}{2}(e+f x)\right]^2$$

$$\sqrt{\frac{\text{Tan}\left[\frac{1}{2}(e+f x)\right]}{1+\text{Tan}\left[\frac{1}{2}(e+f x)\right]^2}}\left(a\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}+2\,b\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]+\right.$$

$$a\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]^2-i\left(-b+\sqrt{-a^2+b^2}\right)\sqrt{1+\frac{a\,\text{Cot}\left[\frac{1}{2}(e+f x)\right]}{b+\sqrt{-a^2+b^2}}}$$

$$\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\text{Tan}\left[\frac{1}{2}(e+f x)\right]}}\right],\,\frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}}\right]\text{Tan}\left[\frac{1}{2}(e+f x)\right]^{3/2}$$

$$\sqrt{\frac{a+b\,\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]-\sqrt{-a^2+b^2}\;\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]}{b\,\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]-\sqrt{-a^2+b^2}\;\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]}}\;+\,\mathrm{i}\,\left(a-b+\sqrt{-a^2+b^2}\right)$$

$$\sqrt{1+\frac{a\,\text{Cot}\left[\frac{1}{2}\,(e+f\,x)\right]}{b+\sqrt{-a^2+b^2}}}\;\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]}}\right],\,\frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}}\right]$$

$$\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]^{3/2}\sqrt{\frac{a+b\,\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]-\sqrt{-a^2+b^2}\;\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]}{b\,\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]-\sqrt{-a^2+b^2}\;\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]}}\;+$$

$$2\,b\,\sqrt{1+\frac{a\,\text{Cot}\left[\frac{1}{2}\,(e+f\,x)\right]}{b+\sqrt{-a^2+b^2}}}\;\text{EllipticPi}\left[-\frac{\mathrm{i}\,\left(b+\sqrt{-a^2+b^2}\right)}{a},\right.$$

$$\left.\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]}}\right],\,\frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}}\right]\,\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]^{3/2}$$

$$\sqrt{\frac{a+b\,\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]-\sqrt{-a^2+b^2}\;\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]}{b\,\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]-\sqrt{-a^2+b^2}\;\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]}}\;-2\,b\,\sqrt{1+\frac{a\,\text{Cot}\left[\frac{1}{2}\,(e+f\,x)\right]}{b+\sqrt{-a^2+b^2}}}$$

$$\text{EllipticPi}\left[\frac{\mathrm{i}\,\left(b+\sqrt{-a^2+b^2}\right)}{a},\,\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]}}\right],\,\frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}}\right]$$

$$\left.\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]^{3/2}\sqrt{\frac{a+b\,\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]-\sqrt{-a^2+b^2}\;\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]}{b\,\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]-\sqrt{-a^2+b^2}\;\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]}}\;\right)-$$

$$\left(\left(1+\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]\right)\sqrt{\frac{a+2\,b\,\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]+a\,\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]^2}{1+\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]^2}}\right.$$

$$\left.\left(-\frac{\text{Sec}\left[\frac{1}{2}\,(e+f\,x)\right]^2\,\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]^2}{\left(1+\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]^2\right)^2}+\frac{\text{Sec}\left[\frac{1}{2}\,(e+f\,x)\right]^2}{2\,\left(1+\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]^2\right)}\right)\right)\Big/$$

$$\left(\sqrt{2}\sqrt{\frac{\text{Tan}\left[\frac{1}{2}(e+f x)\right]}{1+\text{Tan}\left[\frac{1}{2}(e+f x)\right]^2}}\right)+\left(1\Big/\left(\sqrt{2}\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}\right.\right.$$

$$\left.\left.\sqrt{\frac{\text{Tan}\left[\frac{1}{2}(e+f x)\right]}{1+\text{Tan}\left[\frac{1}{2}(e+f x)\right]^2}}\sqrt{\frac{a+2 b\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]+a\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]^2}{1+\text{Tan}\left[\frac{1}{2}(e+f x)\right]^2}}\right)\right)$$

$$\text{Cot}\left[\frac{1}{2}(e+f x)\right]\left(a\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}+2 b\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]+\right.$$

$$a\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]^2-\mathrm{i}\left(-b+\sqrt{-a^2+b^2}\right)\sqrt{1+\frac{a\,\text{Cot}\left[\frac{1}{2}(e+f x)\right]}{b+\sqrt{-a^2+b^2}}}$$

$$\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\text{Tan}\left[\frac{1}{2}(e+f x)\right]}}\right],\frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}}\right]\text{Tan}\left[\frac{1}{2}(e+f x)\right]^{3/2}$$

$$\sqrt{\frac{a+b\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]-\sqrt{-a^2+b^2}\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]}{b\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]-\sqrt{-a^2+b^2}\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]}}+\mathrm{i}\left(a-b+\sqrt{-a^2+b^2}\right)$$

$$\sqrt{1+\frac{a\,\text{Cot}\left[\frac{1}{2}(e+f x)\right]}{b+\sqrt{-a^2+b^2}}}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\text{Tan}\left[\frac{1}{2}(e+f x)\right]}}\right],\frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}}\right]$$

$$\text{Tan}\left[\frac{1}{2}(e+f x)\right]^{3/2}\sqrt{\frac{a+b\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]-\sqrt{-a^2+b^2}\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]}{b\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]-\sqrt{-a^2+b^2}\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]}}+$$

$$2 b\sqrt{1+\frac{a\,\text{Cot}\left[\frac{1}{2}(e+f x)\right]}{b+\sqrt{-a^2+b^2}}}\,\text{EllipticPi}\left[-\frac{\mathrm{i}\left(b+\sqrt{-a^2+b^2}\right)}{a},\right.$$

$$\left.\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\text{Tan}\left[\frac{1}{2}(e+f x)\right]}}\right],\frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}}\right]\text{Tan}\left[\frac{1}{2}(e+f x)\right]^{3/2}$$

$$\sqrt{\frac{a+b\,\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]-\sqrt{-a^2+b^2}\,\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]}{b\,\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]-\sqrt{-a^2+b^2}\,\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]}}-2\,b\,\sqrt{1+\frac{a\,\text{Cot}\left[\frac{1}{2}(e+f\,x)\right]}{b+\sqrt{-a^2+b^2}}}$$

$$\text{EllipticPi}\left[\frac{i\left(b+\sqrt{-a^2+b^2}\right)}{a},\,i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]}}\right],\,\frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}}\right]$$

$$\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]^{3/2}\sqrt{\frac{a+b\,\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]-\sqrt{-a^2+b^2}\,\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]}{b\,\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]-\sqrt{-a^2+b^2}\,\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]}}\;\Bigg)$$

$$\left(-\frac{\text{Sec}\left[\frac{1}{2}(e+f\,x)\right]^2\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]^2}{\left(1+\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]^2\right)^2}+\frac{\text{Sec}\left[\frac{1}{2}(e+f\,x)\right]^2}{2\left(1+\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]^2\right)}\right)-$$

$$\Bigg(\left(1+\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]\right)\sqrt{\frac{\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]}{1+\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]^2}}$$

$$\Bigg(\frac{b\,\text{Sec}\left[\frac{1}{2}(e+f\,x)\right]^2+a\,\text{Sec}\left[\frac{1}{2}(e+f\,x)\right]^2\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]}{1+\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]^2}-$$

$$\left(\text{Sec}\left[\frac{1}{2}(e+f\,x)\right]^2\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]\left(a+2\,b\,\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]+a\,\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]^2\right)\right)\Big/$$

$$\left(1+\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]^2\right)^2\Bigg)\Bigg)\Big/\left(\sqrt{2}\,\sqrt{\frac{a+2\,b\,\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]+a\,\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]^2}{1+\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]^2}}\right)-$$

$$\frac{1}{\sqrt{2}\,\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}\left(\frac{a+2\,b\,\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]+a\,\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]^2}{1+\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]^2}\right)^{3/2}}\,\text{Cot}\left[\frac{1}{2}(e+f\,x)\right]$$

$$\sqrt{\frac{\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]}{1+\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]^2}}\,\Bigg(a\,\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}+2\,b\,\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}\,\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]+$$

$$a\,\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}\,\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]^2-i\left(-b+\sqrt{-a^2+b^2}\right)\sqrt{1+\frac{a\,\text{Cot}\left[\frac{1}{2}(e+f\,x)\right]}{b+\sqrt{-a^2+b^2}}}$$

$$\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\text{Tan}\left[\frac{1}{2}\left(e+f x\right)\right]}}\right],\frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}}\right]\text{Tan}\left[\frac{1}{2}\left(e+f x\right)\right]^{3/2}$$

$$\sqrt{\frac{a+b\,\text{Tan}\left[\frac{1}{2}\left(e+f x\right)\right]-\sqrt{-a^2+b^2}\,\text{Tan}\left[\frac{1}{2}\left(e+f x\right)\right]}{b\,\text{Tan}\left[\frac{1}{2}\left(e+f x\right)\right]-\sqrt{-a^2+b^2}\,\text{Tan}\left[\frac{1}{2}\left(e+f x\right)\right]}}+\mathbb{i}\left(a-b+\sqrt{-a^2+b^2}\right)$$

$$\sqrt{1+\frac{a\,\text{Cot}\left[\frac{1}{2}\left(e+f x\right)\right]}{b+\sqrt{-a^2+b^2}}}\ \text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\text{Tan}\left[\frac{1}{2}\left(e+f x\right)\right]}}\right],\frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}}\right]$$

$$\text{Tan}\left[\frac{1}{2}\left(e+f x\right)\right]^{3/2}\sqrt{\frac{a+b\,\text{Tan}\left[\frac{1}{2}\left(e+f x\right)\right]-\sqrt{-a^2+b^2}\,\text{Tan}\left[\frac{1}{2}\left(e+f x\right)\right]}{b\,\text{Tan}\left[\frac{1}{2}\left(e+f x\right)\right]-\sqrt{-a^2+b^2}\,\text{Tan}\left[\frac{1}{2}\left(e+f x\right)\right]}}+$$

$$2\,b\sqrt{1+\frac{a\,\text{Cot}\left[\frac{1}{2}\left(e+f x\right)\right]}{b+\sqrt{-a^2+b^2}}}\ \text{EllipticPi}\left[-\frac{\mathbb{i}\left(b+\sqrt{-a^2+b^2}\right)}{a},\right.$$

$$\left.\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\text{Tan}\left[\frac{1}{2}\left(e+f x\right)\right]}}\right],\frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}}\right]\text{Tan}\left[\frac{1}{2}\left(e+f x\right)\right]^{3/2}$$

$$\sqrt{\frac{a+b\,\text{Tan}\left[\frac{1}{2}\left(e+f x\right)\right]-\sqrt{-a^2+b^2}\,\text{Tan}\left[\frac{1}{2}\left(e+f x\right)\right]}{b\,\text{Tan}\left[\frac{1}{2}\left(e+f x\right)\right]-\sqrt{-a^2+b^2}\,\text{Tan}\left[\frac{1}{2}\left(e+f x\right)\right]}}-2\,b\sqrt{1+\frac{a\,\text{Cot}\left[\frac{1}{2}\left(e+f x\right)\right]}{b+\sqrt{-a^2+b^2}}}$$

$$\text{EllipticPi}\left[\frac{\mathbb{i}\left(b+\sqrt{-a^2+b^2}\right)}{a},\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\text{Tan}\left[\frac{1}{2}\left(e+f x\right)\right]}}\right],\frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}}\right]$$

$$\left.\text{Tan}\left[\frac{1}{2}\left(e+f x\right)\right]^{3/2}\sqrt{\frac{a+b\,\text{Tan}\left[\frac{1}{2}\left(e+f x\right)\right]-\sqrt{-a^2+b^2}\,\text{Tan}\left[\frac{1}{2}\left(e+f x\right)\right]}{b\,\text{Tan}\left[\frac{1}{2}\left(e+f x\right)\right]-\sqrt{-a^2+b^2}\,\text{Tan}\left[\frac{1}{2}\left(e+f x\right)\right]}}\right)$$

$$\left(\frac{b\,\text{Sec}\left[\frac{1}{2}\left(e+f x\right)\right]^2+a\,\text{Sec}\left[\frac{1}{2}\left(e+f x\right)\right]^2\text{Tan}\left[\frac{1}{2}\left(e+f x\right)\right]}{1+\text{Tan}\left[\frac{1}{2}\left(e+f x\right)\right]^2}-\right.$$

$$\left(\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\left(a+2b\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]+a\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right)\right)\Big/$$

$$\left.\left(1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right)^2\right)+$$

$$\frac{1}{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}\sqrt{\frac{a+2b\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]+a\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}}}\sqrt{2}\operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]$$

$$\sqrt{\frac{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}}\left(b\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2+a\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}\right.$$

$$\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]+\left(\left(a-b+\sqrt{-a^2+b^2}\right)\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}\right.$$

$$\left.\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2\sqrt{\frac{a+b\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]-\sqrt{-a^2+b^2}\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{b\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]-\sqrt{-a^2+b^2}\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}}\right)\Big/$$

$$\left(4\sqrt{1+\frac{a\operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b-\sqrt{-a^2+b^2}}}\right)-\left(i\,b\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2\right.$$

$$\left.\sqrt{\frac{a+b\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]-\sqrt{-a^2+b^2}\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{b\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]-\sqrt{-a^2+b^2}\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}}\right)\Big/$$

$$\left(2\left(1-i\operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]\right)\sqrt{1+\frac{a\operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b-\sqrt{-a^2+b^2}}}\right)+\left(i\,b\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}\right.$$

$$\left.\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2\sqrt{\frac{a+b\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]-\sqrt{-a^2+b^2}\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{b\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]-\sqrt{-a^2+b^2}\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}}\right)\Big/$$

$$\left(2\left(1+i\operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]\right)\sqrt{1+\frac{a\operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{b-\sqrt{-a^2+b^2}}}\right)-$$

$$\frac{1}{4}\left(-b+\sqrt{-a^2+b^2}\right)\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}\sqrt{1+\frac{a\,\text{Cot}\left[\frac{1}{2}(e+f x)\right]}{b-\sqrt{-a^2+b^2}}}$$

$$\text{Sec}\left[\frac{1}{2}(e+f x)\right]^2\sqrt{\frac{a+b\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]-\sqrt{-a^2+b^2}\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]}{b\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]-\sqrt{-a^2+b^2}\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]}}-$$

$$\frac{3}{4}\,i\left(-b+\sqrt{-a^2+b^2}\right)\sqrt{1+\frac{a\,\text{Cot}\left[\frac{1}{2}(e+f x)\right]}{b+\sqrt{-a^2+b^2}}}$$

$$\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\text{Tan}\left[\frac{1}{2}(e+f x)\right]}}\right],\frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}}\right]\text{Sec}\left[\frac{1}{2}(e+f x)\right]^2$$

$$\sqrt{\text{Tan}\left[\frac{1}{2}(e+f x)\right]}\sqrt{\frac{a+b\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]-\sqrt{-a^2+b^2}\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]}{b\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]-\sqrt{-a^2+b^2}\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]}}+$$

$$\frac{3}{4}\,i\left(a-b+\sqrt{-a^2+b^2}\right)\sqrt{1+\frac{a\,\text{Cot}\left[\frac{1}{2}(e+f x)\right]}{b+\sqrt{-a^2+b^2}}}$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\text{Tan}\left[\frac{1}{2}(e+f x)\right]}}\right],\frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}}\right]\text{Sec}\left[\frac{1}{2}(e+f x)\right]^2$$

$$\sqrt{\text{Tan}\left[\frac{1}{2}(e+f x)\right]}\sqrt{\frac{a+b\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]-\sqrt{-a^2+b^2}\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]}{b\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]-\sqrt{-a^2+b^2}\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]}}+$$

$$\frac{3}{2}\,b\sqrt{1+\frac{a\,\text{Cot}\left[\frac{1}{2}(e+f x)\right]}{b+\sqrt{-a^2+b^2}}}\,\text{EllipticPi}\left[-\frac{i\left(b+\sqrt{-a^2+b^2}\right)}{a},\right.$$

$$\left.i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\text{Tan}\left[\frac{1}{2}(e+f x)\right]}}\right],\frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}}\right]\text{Sec}\left[\frac{1}{2}(e+f x)\right]^2$$

$$\sqrt{\text{Tan}\left[\frac{1}{2}(e+f x)\right]}\sqrt{\frac{a+b\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]-\sqrt{-a^2+b^2}\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]}{b\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]-\sqrt{-a^2+b^2}\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]}}-$$

$$\frac{3}{2} b \sqrt{1+\frac{a \operatorname{Cot}\left[\frac{1}{2}(e+f x)\right]}{b+\sqrt{-a^2+b^2}}} \text{ EllipticPi}\left[\frac{i\left(b+\sqrt{-a^2+b^2}\right)}{a}, i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}}\right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}}\right] \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2 \sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}$$

$$\sqrt{\frac{a+b \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]-\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}{b \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]-\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}}+\left(i a\left(-b+\sqrt{-a^2+b^2}\right)\right.$$

$$\operatorname{Csc}\left[\frac{1}{2}(e+f x)\right]^2 \text{ EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}}\right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}}\right]$$

$$\left.\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^{3/2} \sqrt{\frac{a+b \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]-\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}{b \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]-\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}}\right) \Bigg/$$

$$\left(4\left(b+\sqrt{-a^2+b^2}\right) \sqrt{1+\frac{a \operatorname{Cot}\left[\frac{1}{2}(e+f x)\right]}{b+\sqrt{-a^2+b^2}}}\right)-\left(i a\left(a-b+\sqrt{-a^2+b^2}\right)\right.$$

$$\operatorname{Csc}\left[\frac{1}{2}(e+f x)\right]^2 \text{ EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}}\right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}}\right]$$

$$\left.\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^{3/2} \sqrt{\frac{a+b \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]-\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}{b \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]-\sqrt{-a^2+b^2} \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}}\right) \Bigg/$$

$$\left(4\left(b+\sqrt{-a^2+b^2}\right) \sqrt{1+\frac{a \operatorname{Cot}\left[\frac{1}{2}(e+f x)\right]}{b+\sqrt{-a^2+b^2}}}\right)-\left(a b \operatorname{Csc}\left[\frac{1}{2}(e+f x)\right]^2\right.$$

$$\text{EllipticPi}\left[-\frac{i\left(b+\sqrt{-a^2+b^2}\right)}{a}, i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\text{Tan}\left[\frac{1}{2}(e+f x)\right]}}\right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}}\right]$$

$$\text{Tan}\left[\frac{1}{2}(e+f x)\right]^{3/2}\sqrt{\frac{a+b\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]-\sqrt{-a^2+b^2}\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]}{b\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]-\sqrt{-a^2+b^2}\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]}}\Bigg)\Bigg/$$

$$\left(2\left(b+\sqrt{-a^2+b^2}\right)\sqrt{1+\frac{a\,\text{Cot}\left[\frac{1}{2}(e+f x)\right]}{b+\sqrt{-a^2+b^2}}}\right)+\Bigg(a\,b\,\text{Csc}\left[\frac{1}{2}(e+f x)\right]^2$$

$$\text{EllipticPi}\left[\frac{i\left(b+\sqrt{-a^2+b^2}\right)}{a}, i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\text{Tan}\left[\frac{1}{2}(e+f x)\right]}}\right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}}\right]$$

$$\text{Tan}\left[\frac{1}{2}(e+f x)\right]^{3/2}\sqrt{\frac{a+b\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]-\sqrt{-a^2+b^2}\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]}{b\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]-\sqrt{-a^2+b^2}\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]}}\Bigg)\Bigg/$$

$$\left(2\left(b+\sqrt{-a^2+b^2}\right)\sqrt{1+\frac{a\,\text{Cot}\left[\frac{1}{2}(e+f x)\right]}{b+\sqrt{-a^2+b^2}}}\right)-$$

$$\Bigg(i\left(-b+\sqrt{-a^2+b^2}\right)\sqrt{1+\frac{a\,\text{Cot}\left[\frac{1}{2}(e+f x)\right]}{b+\sqrt{-a^2+b^2}}}$$

$$\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\text{Tan}\left[\frac{1}{2}(e+f x)\right]}}\right], \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}}\right]\text{Tan}\left[\frac{1}{2}(e+f x)\right]^{3/2}$$

$$\left(\frac{\frac{1}{2}b\,\text{Sec}\left[\frac{1}{2}(e+f x)\right]^2-\frac{1}{2}\sqrt{-a^2+b^2}\,\text{Sec}\left[\frac{1}{2}(e+f x)\right]^2}{b\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]-\sqrt{-a^2+b^2}\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]}-\left(\left(\frac{1}{2}b\,\text{Sec}\left[\frac{1}{2}(e+f x)\right]^2-\right.\right.\right.$$

$$\frac{1}{2}\sqrt{-a^2+b^2}\,\text{Sec}\left[\frac{1}{2}(e+fx)\right]^2\Big)\left(a+b\,\text{Tan}\left[\frac{1}{2}(e+fx)\right]-\sqrt{-a^2+b^2}\,\text{Tan}\left[\frac{1}{2}(e+fx)\right]\right)\Big)\Big/\left(b\,\text{Tan}\left[\frac{1}{2}(e+fx)\right]-\sqrt{-a^2+b^2}\,\text{Tan}\left[\frac{1}{2}(e+fx)\right]\right)^2\Bigg)\Bigg/$$

$$\left(2\sqrt{\frac{a+b\,\text{Tan}\left[\frac{1}{2}(e+fx)\right]-\sqrt{-a^2+b^2}\,\text{Tan}\left[\frac{1}{2}(e+fx)\right]}{b\,\text{Tan}\left[\frac{1}{2}(e+fx)\right]-\sqrt{-a^2+b^2}\,\text{Tan}\left[\frac{1}{2}(e+fx)\right]}}\right)+$$

$$\Bigg(i\left(a-b+\sqrt{-a^2+b^2}\right)\sqrt{1+\frac{a\,\text{Cot}\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}}$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\text{Tan}\left[\frac{1}{2}(e+fx)\right]}}\right],\frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}}\right]\text{Tan}\left[\frac{1}{2}(e+fx)\right]^{3/2}$$

$$\left(\frac{\frac{1}{2}b\,\text{Sec}\left[\frac{1}{2}(e+fx)\right]^2-\frac{1}{2}\sqrt{-a^2+b^2}\,\text{Sec}\left[\frac{1}{2}(e+fx)\right]^2}{b\,\text{Tan}\left[\frac{1}{2}(e+fx)\right]-\sqrt{-a^2+b^2}\,\text{Tan}\left[\frac{1}{2}(e+fx)\right]}-\Bigg(\Big(\frac{1}{2}b\,\text{Sec}\left[\frac{1}{2}(e+fx)\right]^2-\right.$$

$$\frac{1}{2}\sqrt{-a^2+b^2}\,\text{Sec}\left[\frac{1}{2}(e+fx)\right]^2\Big)\left(a+b\,\text{Tan}\left[\frac{1}{2}(e+fx)\right]-\sqrt{-a^2+b^2}\,\text{Tan}\left[\frac{1}{2}(e+fx)\right]\right)\Big)\Big/\left(b\,\text{Tan}\left[\frac{1}{2}(e+fx)\right]-\sqrt{-a^2+b^2}\,\text{Tan}\left[\frac{1}{2}(e+fx)\right]\right)^2\Bigg)\Bigg/$$

$$\left(2\sqrt{\frac{a+b\,\text{Tan}\left[\frac{1}{2}(e+fx)\right]-\sqrt{-a^2+b^2}\,\text{Tan}\left[\frac{1}{2}(e+fx)\right]}{b\,\text{Tan}\left[\frac{1}{2}(e+fx)\right]-\sqrt{-a^2+b^2}\,\text{Tan}\left[\frac{1}{2}(e+fx)\right]}}\right)+$$

$$\Bigg(b\sqrt{1+\frac{a\,\text{Cot}\left[\frac{1}{2}(e+fx)\right]}{b+\sqrt{-a^2+b^2}}}\;\text{EllipticPi}\left[-\frac{i\left(b+\sqrt{-a^2+b^2}\right)}{a},\right.$$

$$\mathrm{i}\,\mathrm{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\mathrm{Tan}\left[\frac{1}{2}\left(e+f\,x\right)\right]}}\right],\ \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}}\right]\,\mathrm{Tan}\left[\frac{1}{2}\left(e+f\,x\right)\right]^{3/2}$$

$$\left(\frac{\frac{1}{2}\,b\,\mathrm{Sec}\left[\frac{1}{2}\left(e+f\,x\right)\right]^2-\frac{1}{2}\sqrt{-a^2+b^2}\,\mathrm{Sec}\left[\frac{1}{2}\left(e+f\,x\right)\right]^2}{b\,\mathrm{Tan}\left[\frac{1}{2}\left(e+f\,x\right)\right]-\sqrt{-a^2+b^2}\,\mathrm{Tan}\left[\frac{1}{2}\left(e+f\,x\right)\right]}-\left(\left(\frac{1}{2}\,b\,\mathrm{Sec}\left[\frac{1}{2}\left(e+f\,x\right)\right]^2-\right.\right.\right.$$

$$\left.\frac{1}{2}\sqrt{-a^2+b^2}\,\mathrm{Sec}\left[\frac{1}{2}\left(e+f\,x\right)\right]^2\right)\left(a+b\,\mathrm{Tan}\left[\frac{1}{2}\left(e+f\,x\right)\right]-\sqrt{-a^2+b^2}\,\mathrm{Tan}\left[\right.\right.$$

$$\left.\left.\left.\left.\frac{1}{2}\left(e+f\,x\right)\right]\right)\right)\Big/\left(b\,\mathrm{Tan}\left[\frac{1}{2}\left(e+f\,x\right)\right]-\sqrt{-a^2+b^2}\,\mathrm{Tan}\left[\frac{1}{2}\left(e+f\,x\right)\right]\right)^2\right)\Big/$$

$$\left(\sqrt{\frac{a+b\,\mathrm{Tan}\left[\frac{1}{2}\left(e+f\,x\right)\right]-\sqrt{-a^2+b^2}\,\mathrm{Tan}\left[\frac{1}{2}\left(e+f\,x\right)\right]}{b\,\mathrm{Tan}\left[\frac{1}{2}\left(e+f\,x\right)\right]-\sqrt{-a^2+b^2}\,\mathrm{Tan}\left[\frac{1}{2}\left(e+f\,x\right)\right]}}\right)-$$

$$\left(b\sqrt{1+\frac{a\,\mathrm{Cot}\left[\frac{1}{2}\left(e+f\,x\right)\right]}{b+\sqrt{-a^2+b^2}}}\ \mathrm{EllipticPi}\left[\frac{\mathrm{i}\left(b+\sqrt{-a^2+b^2}\right)}{a},\right.\right.$$

$$\mathrm{i}\,\mathrm{ArcSinh}\left[\frac{\sqrt{\frac{a}{b+\sqrt{-a^2+b^2}}}}{\sqrt{\mathrm{Tan}\left[\frac{1}{2}\left(e+f\,x\right)\right]}}\right],\ \frac{b+\sqrt{-a^2+b^2}}{b-\sqrt{-a^2+b^2}}\right]\,\mathrm{Tan}\left[\frac{1}{2}\left(e+f\,x\right)\right]^{3/2}$$

$$\left(\frac{\frac{1}{2}\,b\,\mathrm{Sec}\left[\frac{1}{2}\left(e+f\,x\right)\right]^2-\frac{1}{2}\sqrt{-a^2+b^2}\,\mathrm{Sec}\left[\frac{1}{2}\left(e+f\,x\right)\right]^2}{b\,\mathrm{Tan}\left[\frac{1}{2}\left(e+f\,x\right)\right]-\sqrt{-a^2+b^2}\,\mathrm{Tan}\left[\frac{1}{2}\left(e+f\,x\right)\right]}-\left(\left(\frac{1}{2}\,b\,\mathrm{Sec}\left[\frac{1}{2}\left(e+f\,x\right)\right]^2-\right.\right.\right.$$

$$\left.\frac{1}{2}\sqrt{-a^2+b^2}\,\mathrm{Sec}\left[\frac{1}{2}\left(e+f\,x\right)\right]^2\right)\left(a+b\,\mathrm{Tan}\left[\frac{1}{2}\left(e+f\,x\right)\right]-\sqrt{-a^2+b^2}\,\mathrm{Tan}\left[\right.\right.$$

$$\left.\left.\left.\left.\frac{1}{2}\left(e+f\,x\right)\right]\right)\right)\Big/\left(b\,\mathrm{Tan}\left[\frac{1}{2}\left(e+f\,x\right)\right]-\sqrt{-a^2+b^2}\,\mathrm{Tan}\left[\frac{1}{2}\left(e+f\,x\right)\right]\right)^2\right)\Big/$$

$$\left.\left.\left.\left(\sqrt{\frac{a+b\,\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]-\sqrt{-a^2+b^2}\,\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]}{b\,\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]-\sqrt{-a^2+b^2}\,\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]}}\right)\right)\right)\right)$$

Problem 32: Attempted integration timed out after 120 seconds.

$$\int \frac{\sqrt{a+b\,\text{Sin}[e+f\,x]}}{\sqrt{g\,\text{Sin}[e+f\,x]}\,(c+c\,\text{Sin}[e+f\,x])}\,dx$$

Optimal (type 4, 116 leaves, 1 step):

$$-\left(\left(\text{EllipticE}\left[\text{ArcSin}\left[\frac{\text{Cos}[e+f\,x]}{1+\text{Sin}[e+f\,x]}\right],-\frac{a-b}{a+b}\right]\sqrt{\frac{\text{Sin}[e+f\,x]}{1+\text{Sin}[e+f\,x]}}\,\sqrt{a+b\,\text{Sin}[e+f\,x]}\right)\Big/\left(c\,f\,\sqrt{g\,\text{Sin}[e+f\,x]}\,\sqrt{\frac{a+b\,\text{Sin}[e+f\,x]}{(a+b)\,(1+\text{Sin}[e+f\,x])}}\right)\right)$$

Result (type 1, 1 leaves):

???

Problem 33: Attempted integration timed out after 120 seconds.

$$\int \frac{\sqrt{g\,\text{Sin}[e+f\,x]}}{\sqrt{a+b\,\text{Sin}[e+f\,x]}\,(c+c\,\text{Sin}[e+f\,x])}\,dx$$

Optimal (type 4, 252 leaves, 3 steps):

$$\left(g\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\text{Cos}[e+f\,x]}{1+\text{Sin}[e+f\,x]}\right],-\frac{a-b}{a+b}\right]\sqrt{\frac{\text{Sin}[e+f\,x]}{1+\text{Sin}[e+f\,x]}}\,\sqrt{a+b\,\text{Sin}[e+f\,x]}\right)\Big/\left((a-b)\,c\,f\,\sqrt{g\,\text{Sin}[e+f\,x]}\,\sqrt{\frac{a+b\,\text{Sin}[e+f\,x]}{(a+b)\,(1+\text{Sin}[e+f\,x])}}\right)-\frac{1}{(a-b)\,c\,f}2\,\sqrt{a+b}\,\sqrt{g}\,\sqrt{\frac{a\,(1-\text{Csc}[e+f\,x])}{a+b}}\,\sqrt{\frac{a\,(1+\text{Csc}[e+f\,x])}{a-b}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{g}\,\sqrt{a+b\,\text{Sin}[e+f\,x]}}{\sqrt{a+b}\,\sqrt{g\,\text{Sin}[e+f\,x]}}\right],-\frac{a+b}{a-b}\right]\text{Tan}[e+f\,x]$$

Result (type 1, 1 leaves):

???

Problem 34: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{g \operatorname{Sin}[e+f x]} \sqrt{a+b \operatorname{Sin}[e+f x]} (c+c \operatorname{Sin}[e+f x])} \, dx$$

Optimal (type 4, 256 leaves, 3 steps):

$$-\left(\left(\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Cos}[e+f x]}{1+\operatorname{Sin}[e+f x]}\right],-\frac{a-b}{a+b}\right] \sqrt{\frac{\operatorname{Sin}[e+f x]}{1+\operatorname{Sin}[e+f x]}} \sqrt{a+b \operatorname{Sin}[e+f x]}\right) \Big/ \left((a-b) c f \sqrt{g \operatorname{Sin}[e+f x]} \sqrt{\frac{a+b \operatorname{Sin}[e+f x]}{(a+b)(1+\operatorname{Sin}[e+f x])}}\right)\right)+$$
$$\frac{1}{a (a-b) c f \sqrt{g}} 2 b \sqrt{a+b} \sqrt{\frac{a (1-\operatorname{Csc}[e+f x])}{a+b}} \sqrt{\frac{a (1+\operatorname{Csc}[e+f x])}{a-b}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{g} \sqrt{a+b \operatorname{Sin}[e+f x]}}{\sqrt{a+b} \sqrt{g \operatorname{Sin}[e+f x]}}\right],-\frac{a+b}{a-b}\right] \operatorname{Tan}[e+f x]$$

Result (type 4, 1662 leaves):

$$-\left(\left(2 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]+\operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right) \operatorname{Sin}[e+f x] \sqrt{a+b \operatorname{Sin}[e+f x]}\right) \Big/ \left((a-b) f \sqrt{g \operatorname{Sin}[e+f x]} (c+c \operatorname{Sin}[e+f x])\right)\right)+$$
$$\frac{1}{2 (a-b) f \sqrt{g \operatorname{Sin}[e+f x]} (c+c \operatorname{Sin}[e+f x])}\left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]+\operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^2$$
$$\sqrt{\operatorname{Sin}[e+f x]}\left(\left(4 a (a-b) \sqrt{\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2}{-a+b}}\right.\right.$$
$$\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{\operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 (a+b \operatorname{Sin}[e+f x])}{a}}}{\sqrt{2}}\right],-\frac{2 a}{-a+b}\right] \operatorname{Sec}[e+f x]$$
$$\operatorname{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^4 \sqrt{-\frac{(a+b) \operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 \operatorname{Sin}[e+f x]}{a}}$$
$$\left.\sqrt{\frac{\operatorname{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2 (a+b \operatorname{Sin}[e+f x])}{a}}\right) \Big/$$

$$\left(\left(a+b\right)\sqrt{\mathrm{Sin}[e+f\,x]}\ \sqrt{a+b\,\mathrm{Sin}[e+f\,x]}\right)+\frac{2\,a\,\mathrm{ArcTanh}\left[\frac{\sqrt{b}\ \sqrt{\mathrm{Sin}[e+f\,x]}}{\sqrt{a+b\,\mathrm{Sin}[e+f\,x]}}\right]\mathrm{Cos}[e+f\,x]^2}{\sqrt{b}\ \left(1-\mathrm{Sin}[e+f\,x]^2\right)}+$$

$$4\,a^2\left(\left(\sqrt{\frac{\left(a+b\right)\mathrm{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f\,x\right)\right]^2}{-a+b}}\right.\right.$$

$$\mathrm{EllipticF}\left[\mathrm{ArcSin}\left[\frac{\sqrt{\frac{\mathrm{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f\,x\right)\right]^2\left(a+b\,\mathrm{Sin}[e+f\,x]\right)}{a}}}{\sqrt{2}}\right],-\frac{2\,a}{-a+b}\right]\mathrm{Sec}[e+f\,x]$$

$$\mathrm{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f\,x\right)\right]^4\sqrt{-\frac{\left(a+b\right)\mathrm{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f\,x\right)\right]^2\mathrm{Sin}[e+f\,x]}{a}}$$

$$\left.\sqrt{\frac{\mathrm{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f\,x\right)\right]^2\left(a+b\,\mathrm{Sin}[e+f\,x]\right)}{a}}\right)\Big/$$

$$\left(\left(a+b\right)\sqrt{\mathrm{Sin}[e+f\,x]}\ \sqrt{a+b\,\mathrm{Sin}[e+f\,x]}\right)-\left(\sqrt{\frac{\left(a+b\right)\mathrm{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f\,x\right)\right]^2}{-a+b}}\right.$$

$$\mathrm{EllipticPi}\left[-\frac{a}{b},\mathrm{ArcSin}\left[\frac{\sqrt{\frac{\mathrm{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f\,x\right)\right]^2\left(a+b\,\mathrm{Sin}[e+f\,x]\right)}{a}}}{\sqrt{2}}\right],-\frac{2\,a}{-a+b}\right]\mathrm{Sec}[e+f\,x]$$

$$\mathrm{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f\,x\right)\right]^4\sqrt{-\frac{\left(a+b\right)\mathrm{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f\,x\right)\right]^2\mathrm{Sin}[e+f\,x]}{a}}$$

$$\left.\sqrt{\frac{\mathrm{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f\,x\right)\right]^2\left(a+b\,\mathrm{Sin}[e+f\,x]\right)}{a}}\right)\Big/$$

$$\left.\left(b\sqrt{\mathrm{Sin}[e+f\,x]}\ \sqrt{a+b\,\mathrm{Sin}[e+f\,x]}\right)\right)-$$

$$2\,b\left(\frac{\text{Cos}[e+f\,x]\,\sqrt{a+b\,\text{Sin}[e+f\,x]}}{b\,\sqrt{\text{Sin}[e+f\,x]}}+\left(\mathbb{i}\,\text{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f\,x\right)\right]\,\text{Csc}[e+f\,x]\right.\right.$$

$$\left.\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\text{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f\,x\right)\right]}{\sqrt{\text{Sin}[e+f\,x]}}\right],-\frac{2\,a}{-a-b}\right]\sqrt{a+b\,\text{Sin}[e+f\,x]}\right)\Big/$$

$$\left(b\,\sqrt{\text{Cos}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f\,x\right)\right]^2\text{Csc}[e+f\,x]}\,\sqrt{\frac{\text{Csc}[e+f\,x]\,\left(a+b\,\text{Sin}[e+f\,x]\right)}{a+b}}\right)+$$

$$\frac{1}{b}\,2\,a\left(\left(a\,\sqrt{\frac{(a+b)\,\text{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f\,x\right)\right]^2}{-a+b}}\,\text{EllipticF}\right[\right.\right.$$

$$\text{ArcSin}\left[\frac{\sqrt{\frac{\text{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f\,x\right)\right]^2(a+b\,\text{Sin}[e+f\,x])}{a}}}{\sqrt{2}}\right],-\frac{2\,a}{-a+b}\Big]\,\text{Sec}[e+f\,x]$$

$$\text{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f\,x\right)\right]^4\sqrt{-\frac{(a+b)\,\text{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f\,x\right)\right]^2\text{Sin}[e+f\,x]}{a}}$$

$$\left.\sqrt{\frac{\text{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f\,x\right)\right]^2\left(a+b\,\text{Sin}[e+f\,x]\right)}{a}}\right)\Big/$$

$$\left(\left(a+b\right)\sqrt{\text{Sin}[e+f\,x]}\,\sqrt{a+b\,\text{Sin}[e+f\,x]}\right)-\left(a\,\sqrt{\frac{(a+b)\,\text{Cot}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f\,x\right)\right]^2}{-a+b}}\right.$$

$$\text{EllipticPi}\left[-\frac{a}{b},\text{ArcSin}\left[\frac{\sqrt{\frac{\text{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f\,x\right)\right]^2(a+b\,\text{Sin}[e+f\,x])}{a}}}{\sqrt{2}}\right],-\frac{2\,a}{-a+b}\right]\,\text{Sec}[$$

$$e+f\,x]\,\text{Sin}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f\,x\right)\right]^4\sqrt{-\frac{(a+b)\,\text{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f\,x\right)\right]^2\text{Sin}[e+f\,x]}{a}}$$

$$\left.\sqrt{\frac{\mathrm{Csc}\left[\frac{1}{2}\left(-e+\frac{\pi}{2}-f x\right)\right]^2\left(a+b\,\mathrm{Sin}[e+f x]\right)}{a}}\right)\Bigg/$$

$$\left.\left.\left(b\sqrt{\mathrm{Sin}[e+f x]}\;\sqrt{a+b\,\mathrm{Sin}[e+f x]}\right)\right)\right)+$$

$$\left(2\,b\,\mathrm{Cot}[e+f x]\left(-\frac{a\,\mathrm{Log}\left[b\sqrt{\mathrm{Sin}[e+f x]}+\sqrt{b}\;\sqrt{a+b\,\mathrm{Sin}[e+f x]}\right]}{2\,b^{3/2}}+\right.\right.$$

$$\left.\left.\left.\frac{\sqrt{\mathrm{Sin}[e+f x]}\;\sqrt{a+b\,\mathrm{Sin}[e+f x]}}{2\,b}\right)\mathrm{Sin}[2\,(e+f x)]\right)\Big/\left(1-\mathrm{Sin}[e+f x]^2\right)\right)$$

Problem 35: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \mathrm{Csc}[e+f x]\,\sqrt{a+a\,\mathrm{Sin}[e+f x]}\;\sqrt{c+d\,\mathrm{Sin}[e+f x]}\;dx$$

Optimal (type 3, 123 leaves, 5 steps):

$$-\frac{2\sqrt{a}\;\sqrt{d}\;\mathrm{ArcTan}\left[\frac{\sqrt{a}\;\sqrt{d}\;\mathrm{Cos}[e+f x]}{\sqrt{a+a\,\mathrm{Sin}[e+f x]}\;\sqrt{c+d\,\mathrm{Sin}[e+f x]}}\right]}{f}-\frac{2\sqrt{a}\;\sqrt{c}\;\mathrm{ArcTanh}\left[\frac{\sqrt{a}\;\sqrt{c}\;\mathrm{Cos}[e+f x]}{\sqrt{a+a\,\mathrm{Sin}[e+f x]}\;\sqrt{c+d\,\mathrm{Sin}[e+f x]}}\right]}{f}$$

Result (type 3, 567 leaves):

$$-\left(\left(\left(\sqrt{c}\ \text{Log}\left[\left(\left(\frac{1}{2}+\frac{i}{2}\right)e^{-\frac{ie}{2}}\left(-\sqrt{2}\ c\left(-1+e^{i(e+fx)}\right)-i\sqrt{2}\ d\left(1+e^{i(e+fx)}\right)+\right.\right.\right.\right.\right.$$
$$\left.\left.2\,i\sqrt{c}\ \sqrt{2\,c\,e^{i(e+fx)}-i\,d\left(-1+e^{2i(e+fx)}\right)}\right)f\right)\Big/\left(c^{3/2}\left(1+e^{i(e+fx)}\right)\right)\Big]+$$
$$\sqrt{c}\ \text{Log}\Big[\left(\left(\frac{1}{2}+\frac{i}{2}\right)e^{-\frac{ie}{2}}\left(-i\sqrt{2}\ d\left(-1+e^{i(e+fx)}\right)+\sqrt{2}\ c\left(1+e^{i(e+fx)}\right)+\right.\right.$$
$$\left.\left.2\sqrt{c}\ \sqrt{2\,c\,e^{i(e+fx)}-i\,d\left(-1+e^{2i(e+fx)}\right)}\right)f\right)\Big/\left(c^{3/2}\left(-1+e^{i(e+fx)}\right)\right)\Big]-$$
$$i\sqrt{d}\ \left(\text{Log}\Big[\frac{1}{d^{3/2}}2\,e^{-\frac{1}{2}i(e+2fx)}\left((-1)^{3/4}d+(-1)^{1/4}c\,e^{i(e+fx)}+\right.\right.$$
$$\left.i\sqrt{d}\ \sqrt{2\,c\,e^{i(e+fx)}-i\,d\left(-1+e^{2i(e+fx)}\right)}\right)f\Big]-$$
$$\text{Log}\Big[\frac{1}{\sqrt{d}}(1+i)\sqrt{2}\ \left(c-i\,d\,\text{Cos}[e+fx]+d\,\text{Sin}[e+fx]+\right.$$
$$\left.\left.\left.\left.(1-i)\sqrt{d}\ \sqrt{(\text{Cos}[e+fx]+i\,\text{Sin}[e+fx])\,(c+d\,\text{Sin}[e+fx])}\right)\Big]\right)\right)$$
$$\left(\text{Cos}\left[\frac{1}{2}(e+fx)\right]+i\,\text{Sin}\left[\frac{1}{2}(e+fx)\right]\right)\sqrt{a\,(1+\text{Sin}[e+fx])}$$
$$\left.\sqrt{c+d\,\text{Sin}[e+fx]}\right)\Big/$$
$$\left(f\left(\text{Cos}\left[\frac{1}{2}(e+fx)\right]+\text{Sin}\left[\frac{1}{2}(e+fx)\right]\right)\right.$$
$$\left.\left.\sqrt{(\text{Cos}[e+fx]+i\,\text{Sin}[e+fx])\,(c+d\,\text{Sin}[e+fx])}\right)\right)$$

Problem 36: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{\text{Csc}[e+fx]\ \sqrt{a+a\,\text{Sin}[e+fx]}}{\sqrt{c+d\,\text{Sin}[e+fx]}}\,dx$$

Optimal (type 3, 61 leaves, 2 steps):

$$-\frac{2\sqrt{a}\ \text{ArcTanh}\left[\frac{\sqrt{a}\ \sqrt{c}\ \text{Cos}[e+fx]}{\sqrt{a+a\,\text{Sin}[e+fx]}\ \sqrt{c+d\,\text{Sin}[e+fx]}}\right]}{\sqrt{c}\ f}$$

Result (type 3, 367 leaves):

$$-\frac{1}{\sqrt{c}\, f\left(\text{Cos}\left[\frac{1}{2}(e+f x)\right]+\text{Sin}\left[\frac{1}{2}(e+f x)\right]\right)\sqrt{c+d\,\text{Sin}[e+f x]}}$$
$$\left(\text{Log}\left[-\left(\left((1+i)\,e^{\frac{i e}{2}}\left(\sqrt{2}\,c\left(-1+e^{i(e+f x)}\right)+i\sqrt{2}\,d\left(1+e^{i(e+f x)}\right)-\right.\right.\right.\right.$$
$$\left.\left.\left.\left.2\,i\sqrt{c}\sqrt{2\,c\,e^{i(e+f x)}-i\,d\left(-1+e^{2 i(e+f x)}\right)}\right)f\right)\Big/\left(\sqrt{c}\left(1+e^{i(e+f x)}\right)\right)\right)\right]+$$
$$\text{Log}\left[\left((1+i)\,e^{\frac{i e}{2}}\left(-i\sqrt{2}\,d\left(-1+e^{i(e+f x)}\right)+\sqrt{2}\,c\left(1+e^{i(e+f x)}\right)+\right.\right.\right.$$
$$\left.\left.\left.2\sqrt{c}\sqrt{2\,c\,e^{i(e+f x)}-i\,d\left(-1+e^{2 i(e+f x)}\right)}\right)f\right)\Big/\left(\sqrt{c}\left(-1+e^{i(e+f x)}\right)\right)\right]\right)$$
$$\left(\text{Cos}\left[\frac{1}{2}(e+f x)\right]-i\,\text{Sin}\left[\frac{1}{2}(e+f x)\right]\right)\sqrt{a\left(1+\text{Sin}[e+f x]\right)}$$
$$\sqrt{\left(\text{Cos}[e+f x]+i\,\text{Sin}[e+f x]\right)\left(c+d\,\text{Sin}[e+f x]\right)}$$

Problem 37: Humongous result has more than 200000 leaves.

$$\int \frac{\text{Csc}[e+f x]\sqrt{c+d\,\text{Sin}[e+f x]}}{\sqrt{a+a\,\text{Sin}[e+f x]}}\,dx$$

Optimal (type 3, 140 leaves, 5 steps):

$$-\frac{2\sqrt{c}\,\text{ArcTanh}\left[\frac{\sqrt{a}\sqrt{c}\,\text{Cos}[e+f x]}{\sqrt{a+a\,\text{Sin}[e+f x]}\sqrt{c+d\,\text{Sin}[e+f x]}}\right]}{\sqrt{a}\,f}+\frac{\sqrt{2}\sqrt{c-d}\,\text{ArcTanh}\left[\frac{\sqrt{a}\sqrt{c-d}\,\text{Cos}[e+f x]}{\sqrt{2}\sqrt{a+a\,\text{Sin}[e+f x]}\sqrt{c+d\,\text{Sin}[e+f x]}}\right]}{\sqrt{a}\,f}$$

Result (type ?, 472 502 leaves): Display of huge result suppressed!

Problem 38: Humongous result has more than 200000 leaves.

$$\int \frac{\text{Csc}[e+f x]}{\sqrt{a+a\,\text{Sin}[e+f x]}\sqrt{c+d\,\text{Sin}[e+f x]}}\,dx$$

Optimal (type 3, 140 leaves, 5 steps):

$$-\frac{2\,\text{ArcTanh}\left[\frac{\sqrt{a}\sqrt{c}\,\text{Cos}[e+f x]}{\sqrt{a+a\,\text{Sin}[e+f x]}\sqrt{c+d\,\text{Sin}[e+f x]}}\right]}{\sqrt{a}\sqrt{c}\,f}+\frac{\sqrt{2}\,\text{ArcTanh}\left[\frac{\sqrt{a}\sqrt{c-d}\,\text{Cos}[e+f x]}{\sqrt{2}\sqrt{a+a\,\text{Sin}[e+f x]}\sqrt{c+d\,\text{Sin}[e+f x]}}\right]}{\sqrt{a}\sqrt{c-d}\,f}$$

Result (type ?, 309 693 leaves): Display of huge result suppressed!

Problem 40: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{Csc}[e+f x]\sqrt{c+d\,\text{Sin}[e+f x]}}{a+b\,\text{Sin}[e+f x]}\,dx$$

Optimal (type 4, 154 leaves, 5 steps):

$$\frac{2\,c\,\text{EllipticPi}\left[2,\frac{1}{2}\left(e-\frac{\pi}{2}+f\,x\right),\frac{2\,d}{c+d}\right]\sqrt{\frac{c+d\,\text{Sin}[e+f\,x]}{c+d}}}{a\,f\,\sqrt{c+d\,\text{Sin}[e+f\,x]}}-\frac{2\,(b\,c-a\,d)\,\text{EllipticPi}\left[\frac{2\,b}{a+b},\frac{1}{2}\left(e-\frac{\pi}{2}+f\,x\right),\frac{2\,d}{c+d}\right]\sqrt{\frac{c+d\,\text{Sin}[e+f\,x]}{c+d}}}{a\,(a+b)\,f\,\sqrt{c+d\,\text{Sin}[e+f\,x]}}$$

Result (type 4, 179 leaves):

$$\frac{1}{a\sqrt{-\frac{1}{c+d}}\,f}2\,i\left(\text{EllipticPi}\left[\frac{c+d}{c},i\,\text{ArcSinh}\left[\sqrt{-\frac{1}{c+d}}\sqrt{c+d\,\text{Sin}[e+f\,x]}\right],\frac{c+d}{c-d}\right]-\text{EllipticPi}\left[\frac{b\,(c+d)}{b\,c-a\,d},i\,\text{ArcSinh}\left[\sqrt{-\frac{1}{c+d}}\sqrt{c+d\,\text{Sin}[e+f\,x]}\right],\frac{c+d}{c-d}\right]\right)\text{Sec}[e+f\,x]\sqrt{-\frac{d\,(-1+\text{Sin}[e+f\,x])}{c+d}}\sqrt{\frac{d\,(1+\text{Sin}[e+f\,x])}{-c+d}}$$

Problem 41: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\text{Csc}[e+f\,x]}{(a+b\,\text{Sin}[e+f\,x])\sqrt{c+d\,\text{Sin}[e+f\,x]}}\,dx$$

Optimal (type 4, 146 leaves, 5 steps):

$$\frac{2\,\text{EllipticPi}\left[2,\frac{1}{2}\left(e-\frac{\pi}{2}+f\,x\right),\frac{2\,d}{c+d}\right]\sqrt{\frac{c+d\,\text{Sin}[e+f\,x]}{c+d}}}{a\,f\,\sqrt{c+d\,\text{Sin}[e+f\,x]}}-\frac{2\,b\,\text{EllipticPi}\left[\frac{2\,b}{a+b},\frac{1}{2}\left(e-\frac{\pi}{2}+f\,x\right),\frac{2\,d}{c+d}\right]\sqrt{\frac{c+d\,\text{Sin}[e+f\,x]}{c+d}}}{a\,(a+b)\,f\,\sqrt{c+d\,\text{Sin}[e+f\,x]}}$$

Result (type 4, 203 leaves):

$$-\left(\left(2\,i\left(\left(-b\,c+a\,d\right)\text{EllipticPi}\left[\frac{c+d}{c},\,i\,\text{ArcSinh}\left[\sqrt{-\frac{1}{c+d}}\,\sqrt{c+d\,\text{Sin}[e+f\,x]}\right],\,\frac{c+d}{c-d}\right]+b\,c\,\text{EllipticPi}\left[\frac{b\,(c+d)}{b\,c-a\,d},\,i\,\text{ArcSinh}\left[\sqrt{-\frac{1}{c+d}}\,\sqrt{c+d\,\text{Sin}[e+f\,x]}\right],\,\frac{c+d}{c-d}\right]\right)\text{Sec}[e+f\,x]\sqrt{-\frac{d\,(-1+\text{Sin}[e+f\,x])}{c+d}}\,\sqrt{-\frac{d\,(1+\text{Sin}[e+f\,x])}{c-d}}\right)\Big/\left(a\,c\,\sqrt{-\frac{1}{c+d}}\,(b\,c-a\,d)\,f\right)\right)$$

Problem 42: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{\sqrt{g\,\text{Sin}[e+f\,x]}\,\sqrt{a+b\,\text{Sin}[e+f\,x]}}{c+d\,\text{Sin}[e+f\,x]}\,dx$$

Optimal (type 4, 254 leaves, 3 steps):

$$\frac{1}{d\,f}2\sqrt{a+b}\,\sqrt{g}\,\sqrt{\frac{a\,(1-\text{Csc}[e+f\,x])}{a+b}}\,\sqrt{\frac{a\,(1+\text{Csc}[e+f\,x])}{a-b}}\,\text{EllipticPi}\left[\frac{a+b}{b},\,\text{ArcSin}\left[\frac{\sqrt{g}\,\sqrt{a+b\,\text{Sin}[e+f\,x]}}{\sqrt{a+b}\,\sqrt{g\,\text{Sin}[e+f\,x]}}\right],\,-\frac{a+b}{a-b}\right]\text{Tan}[e+f\,x]-\left(2\,(b\,c-a\,d)\,\sqrt{-\text{Cot}[e+f\,x]^2}\,\sqrt{\frac{b+a\,\text{Csc}[e+f\,x]}{a+b}}\,\text{EllipticPi}\left[\frac{2\,c}{c+d},\,\text{ArcSin}\left[\frac{\sqrt{1-\text{Csc}[e+f\,x]}}{\sqrt{2}}\right],\,\frac{2\,a}{a+b}\right]\sqrt{g\,\text{Sin}[e+f\,x]}\,\text{Tan}[e+f\,x]\right)\Big/\left(d\,(c+d)\,f\,\sqrt{a+b\,\text{Sin}[e+f\,x]}\right)$$

Result (type 4, 75 407 leaves): Display of huge result suppressed!

Problem 43: Result more than twice size of optimal antiderivative.

$$\int\frac{\sqrt{a+b\,\text{Sin}[e+f\,x]}}{\sqrt{g\,\text{Sin}[e+f\,x]}\,(c+d\,\text{Sin}[e+f\,x])}\,dx$$

Optimal (type 4, 250 leaves, 3 steps):

$$-\frac{1}{c\,f\,\sqrt{g}}2\,\sqrt{a+b}\,\sqrt{\frac{a\,(1-\mathrm{Csc}[e+f\,x])}{a+b}}\,\sqrt{\frac{a\,(1+\mathrm{Csc}[e+f\,x])}{a-b}}\,\mathrm{EllipticF}\Big[\mathrm{ArcSin}\Big[\frac{\sqrt{g}\,\sqrt{a+b\,\mathrm{Sin}[e+f\,x]}}{\sqrt{a+b}\,\sqrt{g\,\mathrm{Sin}[e+f\,x]}}\Big],\,-\frac{a+b}{a-b}\Big]\,\mathrm{Tan}[e+f\,x]+\left(2\,(b\,c-a\,d)\,\sqrt{-\mathrm{Cot}[e+f\,x]^2}\,\sqrt{\frac{b+a\,\mathrm{Csc}[e+f\,x]}{a+b}}\,\mathrm{EllipticPi}\Big[\frac{2\,c}{c+d},\,\mathrm{ArcSin}\Big[\frac{\sqrt{1-\mathrm{Csc}[e+f\,x]}}{\sqrt{2}}\Big],\,\frac{2\,a}{a+b}\Big]\,\sqrt{g\,\mathrm{Sin}[e+f\,x]}\,\mathrm{Tan}[e+f\,x]\right)\Big/\left(c\,(c+d)\,f\,g\,\sqrt{a+b\,\mathrm{Sin}[e+f\,x]}\right)$$

Result (type 4, 45 019 leaves): Display of huge result suppressed!

Problem 44: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{g\,\mathrm{Sin}[e+f\,x]}}{\sqrt{a+b\,\mathrm{Sin}[e+f\,x]}\,(c+d\,\mathrm{Sin}[e+f\,x])}\,dx$$

Optimal (type 4, 114 leaves, 1 step):

$$\left(2\,\sqrt{-\mathrm{Cot}[e+f\,x]^2}\,\sqrt{\frac{b+a\,\mathrm{Csc}[e+f\,x]}{a+b}}\,\mathrm{EllipticPi}\Big[\frac{2\,c}{c+d},\,\mathrm{ArcSin}\Big[\frac{\sqrt{1-\mathrm{Csc}[e+f\,x]}}{\sqrt{2}}\Big],\,\frac{2\,a}{a+b}\Big]\,\sqrt{g\,\mathrm{Sin}[e+f\,x]}\,\mathrm{Tan}[e+f\,x]\right)\Big/\left((c+d)\,f\,\sqrt{a+b\,\mathrm{Sin}[e+f\,x]}\right)$$

Result (type 4, 3427 leaves):

$$\Bigg(a\,\sqrt{-a^2+b^2}$$

$$\Bigg(\left(a\,c+\left(b+\sqrt{-a^2+b^2}\right)\left(-d+\sqrt{-c^2+d^2}\right)\right)\,\mathrm{EllipticPi}\Big[\frac{2\,\sqrt{-a^2+b^2}\,c}{b\,c+\sqrt{-a^2+b^2}\,c-a\,d+a\,\sqrt{-c^2+d^2}},$$

$$\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{-a^2+b^2}+a\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]}{\sqrt{-a^2+b^2}}}}{\sqrt{2}}\right],\ \frac{2\sqrt{-a^2+b^2}}{b+\sqrt{-a^2+b^2}}\Big]+$$

$$\left(-a\,c+\left(b+\sqrt{-a^2+b^2}\right)\left(d+\sqrt{-c^2+d^2}\right)\right)\text{EllipticPi}\left[\frac{2\sqrt{-a^2+b^2}\,c}{b\,c+\sqrt{-a^2+b^2}\,c-a\left(d+\sqrt{-c^2+d^2}\right)},\right.$$

$$\left.\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{-a^2+b^2}+a\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]}{\sqrt{-a^2+b^2}}}}{\sqrt{2}}\right],\ \frac{2\sqrt{-a^2+b^2}}{b+\sqrt{-a^2+b^2}}\right]\Bigg)$$

$$\left.\sqrt{\text{Sin}[e+f x]}\ \sqrt{g\,\text{Sin}[e+f x]}\ \sqrt{\frac{a\,\text{Sec}\left[\frac{1}{2}(e+f x)\right]^2 (a+b\,\text{Sin}[e+f x])}{a^2-b^2}}\right)\Big/$$

$$\Bigg(\left(b+\sqrt{-a^2+b^2}\right)^2 (b\,c-a\,d)\sqrt{-c^2+d^2}\ f\,(a+b\,\text{Sin}[e+f x])$$

$$(c+d\,\text{Sin}[e+f x])\sqrt{-\frac{a\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]}{b+\sqrt{-a^2+b^2}}}$$

$$\Bigg(\Bigg(a^2\sqrt{-a^2+b^2}\ \Bigg(\left(a\,c+\left(b+\sqrt{-a^2+b^2}\right)\left(-d+\sqrt{-c^2+d^2}\right)\right)$$

$$\text{EllipticPi}\left[\frac{2\sqrt{-a^2+b^2}\,c}{b\,c+\sqrt{-a^2+b^2}\,c-a\,d+a\sqrt{-c^2+d^2}},\ \text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{-a^2+b^2}+a\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]}{\sqrt{-a^2+b^2}}}}{\sqrt{2}}\right],\right.$$

$$\left.\frac{2\sqrt{-a^2+b^2}}{b+\sqrt{-a^2+b^2}}\right]+\left(-a\,c+\left(b+\sqrt{-a^2+b^2}\right)\left(d+\sqrt{-c^2+d^2}\right)\right)$$

$$\text{EllipticPi}\left[\frac{2\sqrt{-a^2+b^2}\ c}{b\ c+\sqrt{-a^2+b^2}\ c-a\left(d+\sqrt{-c^2+d^2}\right)},\right.$$
$$\left.\left.\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{-a^2+b^2}+a\,\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]}{\sqrt{-a^2+b^2}}}}{\sqrt{2}}\right],\ \frac{2\sqrt{-a^2+b^2}}{b+\sqrt{-a^2+b^2}}\right]\right)$$

$$\left.\text{Sec}\left[\frac{1}{2}(e+f\,x)\right]^2\sqrt{\text{Sin}[e+f\,x]}\ \sqrt{\frac{a\,\text{Sec}\left[\frac{1}{2}(e+f\,x)\right]^2\left(a+b\,\text{Sin}[e+f\,x]\right)}{a^2-b^2}}\right)\Big/$$

$$\left(4\left(b+\sqrt{-a^2+b^2}\right)^3\left(b\ c-a\ d\right)\sqrt{-c^2+d^2}\ \sqrt{a+b\,\text{Sin}[e+f\,x]}\ \left(-\frac{a\,\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]}{b+\sqrt{-a^2+b^2}}\right)^{3/2}\right)-$$

$$\left(a\ b\sqrt{-a^2+b^2}\ \text{Cos}[e+f\,x]\left(\left(a\ c+\left(b+\sqrt{-a^2+b^2}\right)\left(-d+\sqrt{-c^2+d^2}\right)\right)\right.\right.$$

$$\text{EllipticPi}\left[\frac{2\sqrt{-a^2+b^2}\ c}{b\ c+\sqrt{-a^2+b^2}\ c-a\ d+a\sqrt{-c^2+d^2}},\ \text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{-a^2+b^2}+a\,\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]}{\sqrt{-a^2+b^2}}}}{\sqrt{2}}\right],\right.$$
$$\left.\frac{2\sqrt{-a^2+b^2}}{b+\sqrt{-a^2+b^2}}\right]+\left(-a\ c+\left(b+\sqrt{-a^2+b^2}\right)\left(d+\sqrt{-c^2+d^2}\right)\right)$$

$$\text{EllipticPi}\left[\frac{2\sqrt{-a^2+b^2}\ c}{b\ c+\sqrt{-a^2+b^2}\ c-a\left(d+\sqrt{-c^2+d^2}\right)},\ \text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{-a^2+b^2}+a\,\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]}{\sqrt{-a^2+b^2}}}}{\sqrt{2}}\right],\right.$$

$$\left.\left.\frac{2\sqrt{-a^2+b^2}}{b+\sqrt{-a^2+b^2}}\right]\right)\sqrt{\text{Sin}[e+f\,x]}\ \sqrt{\frac{a\,\text{Sec}\left[\frac{1}{2}(e+f\,x)\right]^2\left(a+b\,\text{Sin}[e+f\,x]\right)}{a^2-b^2}}\right)\Big/$$

$$\left(2\left(b+\sqrt{-a^2+b^2}\right)^2 (b c-a d)\sqrt{-c^2+d^2}\,(a+b\,\text{Sin}[e+f x])^{3/2}\sqrt{-\frac{a\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]}{b+\sqrt{-a^2+b^2}}}\right)+$$

$$\left(a\sqrt{-a^2+b^2}\,\text{Cos}[e+f x]\left(\left(a c+\left(b+\sqrt{-a^2+b^2}\right)\left(-d+\sqrt{-c^2+d^2}\right)\right)\right.\right.$$

$$\text{EllipticPi}\left[\frac{2\sqrt{-a^2+b^2}\,c}{b c+\sqrt{-a^2+b^2}\,c-a d+a\sqrt{-c^2+d^2}},\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{-a^2+b^2}+a\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]}{\sqrt{-a^2+b^2}}}}{\sqrt{2}}\right],\right.$$

$$\left.\frac{2\sqrt{-a^2+b^2}}{b+\sqrt{-a^2+b^2}}\right]+\left(-a c+\left(b+\sqrt{-a^2+b^2}\right)\left(d+\sqrt{-c^2+d^2}\right)\right)$$

$$\text{EllipticPi}\left[\frac{2\sqrt{-a^2+b^2}\,c}{b c+\sqrt{-a^2+b^2}\,c-a\left(d+\sqrt{-c^2+d^2}\right)},\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{-a^2+b^2}+a\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]}{\sqrt{-a^2+b^2}}}}{\sqrt{2}}\right],\right.$$

$$\left.\left.\left.\frac{2\sqrt{-a^2+b^2}}{b+\sqrt{-a^2+b^2}}\right]\right)\sqrt{\frac{a\,\text{Sec}\left[\frac{1}{2}(e+f x)\right]^2(a+b\,\text{Sin}[e+f x])}{a^2-b^2}}\right)\Big/$$

$$\left(2\left(b+\sqrt{-a^2+b^2}\right)^2 (b c-a d)\sqrt{-c^2+d^2}\,\sqrt{\text{Sin}[e+f x]}\,\sqrt{a+b\,\text{Sin}[e+f x]}\right.$$

$$\left.\sqrt{-\frac{a\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]}{b+\sqrt{-a^2+b^2}}}\right)+$$

$$\left(a\sqrt{-a^2+b^2}\left(\left(a c+\left(b+\sqrt{-a^2+b^2}\right)\left(-d+\sqrt{-c^2+d^2}\right)\right)\right.\right.$$

$$\text{EllipticPi}\left[\frac{2\sqrt{-a^2+b^2}\ c}{b\,c+\sqrt{-a^2+b^2}\ c-a\,d+a\sqrt{-c^2+d^2}},\ \text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{-a^2+b^2}+a\,\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]}{\sqrt{-a^2+b^2}}}}{\sqrt{2}}\right],\right.$$

$$\left.\frac{2\sqrt{-a^2+b^2}}{b+\sqrt{-a^2+b^2}}\right]+\left(-a\,c+\left(b+\sqrt{-a^2+b^2}\right)\left(d+\sqrt{-c^2+d^2}\right)\right)$$

$$\text{EllipticPi}\left[\frac{2\sqrt{-a^2+b^2}\ c}{b\,c+\sqrt{-a^2+b^2}\ c-a\left(d+\sqrt{-c^2+d^2}\right)},\ \text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{-a^2+b^2}+a\,\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]}{\sqrt{-a^2+b^2}}}}{\sqrt{2}}\right],\right.$$

$$\left.\left.\frac{2\sqrt{-a^2+b^2}}{b+\sqrt{-a^2+b^2}}\right]\right)\sqrt{\text{Sin}[e+f\,x]}\ \left(\frac{a\,b\,\text{Cos}[e+f\,x]\ \text{Sec}\left[\frac{1}{2}(e+f\,x)\right]^2}{a^2-b^2}+\right.$$

$$\left.\left.\frac{a\,\text{Sec}\left[\frac{1}{2}(e+f\,x)\right]^2\left(a+b\,\text{Sin}[e+f\,x]\right)\ \text{Tan}\left[\frac{1}{2}(e+f\,x)\right]}{a^2-b^2}\right)\right)\Big/$$

$$\left(2\left(b+\sqrt{-a^2+b^2}\right)^2\left(b\,c-a\,d\right)\sqrt{-c^2+d^2}\ \sqrt{a+b\,\text{Sin}[e+f\,x]}\right.$$

$$\left.\sqrt{\frac{a\,\text{Sec}\left[\frac{1}{2}(e+f\,x)\right]^2\left(a+b\,\text{Sin}[e+f\,x]\right)}{a^2-b^2}}\ \sqrt{-\frac{a\,\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]}{b+\sqrt{-a^2+b^2}}}\right)+$$

$$\left(a\sqrt{-a^2+b^2}\ \sqrt{\text{Sin}[e+f\,x]}\ \sqrt{\frac{a\,\text{Sec}\left[\frac{1}{2}(e+f\,x)\right]^2\left(a+b\,\text{Sin}[e+f\,x]\right)}{a^2-b^2}}\right.$$

$$\left(\left(a\left(a\,c+\left(b+\sqrt{-a^2+b^2}\right)\left(-d+\sqrt{-c^2+d^2}\right)\right)\text{Sec}\left[\frac{1}{2}(e+f\,x)\right]^2\right)\Big/\right.$$

$$\left(4\sqrt{2}\,\sqrt{-a^2+b^2}\,\sqrt{\frac{b+\sqrt{-a^2+b^2}+a\,\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]}{\sqrt{-a^2+b^2}}}\right.$$

$$\sqrt{1-\frac{b+\sqrt{-a^2+b^2}+a\,\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]}{2\,\sqrt{-a^2+b^2}}}\,\sqrt{1-\frac{b+\sqrt{-a^2+b^2}+a\,\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]}{b+\sqrt{-a^2+b^2}}}$$

$$\left.\left(1-\frac{c\left(b+\sqrt{-a^2+b^2}+a\,\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]\right)}{b\,c+\sqrt{-a^2+b^2}\,c-a\,d+a\,\sqrt{-c^2+d^2}}\right)\right)+$$

$$\left(a\left(-a\,c+\left(b+\sqrt{-a^2+b^2}\right)\left(d+\sqrt{-c^2+d^2}\right)\right)\text{Sec}\left[\frac{1}{2}\,(e+f\,x)\right]^2\right)\Big/$$

$$\left(4\sqrt{2}\,\sqrt{-a^2+b^2}\,\sqrt{\frac{b+\sqrt{-a^2+b^2}+a\,\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]}{\sqrt{-a^2+b^2}}}\right.$$

$$\sqrt{1-\frac{b+\sqrt{-a^2+b^2}+a\,\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]}{2\,\sqrt{-a^2+b^2}}}\,\sqrt{1-\frac{b+\sqrt{-a^2+b^2}+a\,\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]}{b+\sqrt{-a^2+b^2}}}$$

$$\left.\left.\left.\left(1-\frac{c\left(b+\sqrt{-a^2+b^2}+a\,\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]\right)}{b\,c+\sqrt{-a^2+b^2}\,c-a\left(d+\sqrt{-c^2+d^2}\right)}\right)\right)\right)\right)\Big/$$

$$\left(\left(b+\sqrt{-a^2+b^2}\right)^2(b\,c-a\,d)\,\sqrt{-c^2+d^2}\,\sqrt{a+b\,\text{Sin}[e+f\,x]}\,\sqrt{-\frac{a\,\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]}{b+\sqrt{-a^2+b^2}}}\right)\Bigg)\Bigg)$$

Problem 45: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{\sqrt{g\,\text{Sin}[e+f\,x]}\,\sqrt{a+b\,\text{Sin}[e+f\,x]}\,(c+d\,\text{Sin}[e+f\,x])}\,dx$$

Optimal (type 4, 246 leaves, 3 steps):

$$-\frac{1}{a\,c\,f\,\sqrt{g}}2\,\sqrt{a+b}\,\sqrt{\frac{a\,(1-\text{Csc}[e+f\,x])}{a+b}}\,\sqrt{\frac{a\,(1+\text{Csc}[e+f\,x])}{a-b}}$$

$$\text{EllipticF}\Big[\text{ArcSin}\Big[\frac{\sqrt{g}\,\sqrt{a+b\,\text{Sin}[e+f\,x]}}{\sqrt{a+b}\,\sqrt{g\,\text{Sin}[e+f\,x]}}\Big],\,-\frac{a+b}{a-b}\Big]\,\text{Tan}[e+f\,x]-$$

$$\Bigg(2\,d\,\sqrt{-\text{Cot}[e+f\,x]^2}\,\sqrt{\frac{b+a\,\text{Csc}[e+f\,x]}{a+b}}\,\text{EllipticPi}\Big[\frac{2\,c}{c+d},\,\text{ArcSin}\Big[\frac{\sqrt{1-\text{Csc}[e+f\,x]}}{\sqrt{2}}\Big],\,\frac{2\,a}{a+b}\Big]$$

$$\sqrt{g\,\text{Sin}[e+f\,x]}\,\text{Tan}[e+f\,x]\Bigg)\Big/\Big(c\,(c+d)\,f\,g\,\sqrt{a+b\,\text{Sin}[e+f\,x]}\Big)$$

Result (type 4, 4935 leaves):

$$-\Bigg(\Bigg(4\,\sqrt{-a^2+b^2}\,\text{Cos}\Big[\frac{1}{2}\,(e+f\,x)\Big]^4$$

$$\Bigg(-2\,\Big(b+\sqrt{-a^2+b^2}\Big)\,(b\,c-a\,d)\,\sqrt{-c^2+d^2}\,\text{EllipticF}\Bigg[\text{ArcSin}\Bigg[\frac{\sqrt{\frac{b+\sqrt{-a^2+b^2}+a\,\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]}{\sqrt{-a^2+b^2}}}}{\sqrt{2}}\Bigg],$$

$$\frac{2\,\sqrt{-a^2+b^2}}{b+\sqrt{-a^2+b^2}}\Bigg]-a\,d\,\Bigg(\Big(a\,c+\Big(b+\sqrt{-a^2+b^2}\Big)\,\Big(-d+\sqrt{-c^2+d^2}\Big)\Big)$$

$$\text{EllipticPi}\Bigg[\frac{2\,\sqrt{-a^2+b^2}\,c}{b\,c+\sqrt{-a^2+b^2}\,c-a\,d+a\,\sqrt{-c^2+d^2}},\,\text{ArcSin}\Bigg[\frac{\sqrt{\frac{b+\sqrt{-a^2+b^2}+a\,\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]}{\sqrt{-a^2+b^2}}}}{\sqrt{2}}\Bigg],$$

$$\frac{2\,\sqrt{-a^2+b^2}}{b+\sqrt{-a^2+b^2}}\Bigg]+\Big(-a\,c+\Big(b+\sqrt{-a^2+b^2}\Big)\,\Big(d+\sqrt{-c^2+d^2}\Big)\Big)$$

$$\text{EllipticPi}\Bigg[\frac{2\,\sqrt{-a^2+b^2}\,c}{b\,c+\sqrt{-a^2+b^2}\,c-a\,\Big(d+\sqrt{-c^2+d^2}\Big)},$$

$$\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{-a^2+b^2}+a\,\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]}{\sqrt{-a^2+b^2}}}}{\sqrt{2}}\right],\ \frac{2\,\sqrt{-a^2+b^2}}{b+\sqrt{-a^2+b^2}}\Bigg]\Bigg)\Bigg)$$

$$\sqrt{\frac{a\,\text{Sec}\left[\frac{1}{2}\,(e+f\,x)\right]^2\,(a+b\,\text{Sin}[e+f\,x])}{a^2-b^2}}\,\left(-\frac{a\,\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]}{b+\sqrt{-a^2+b^2}}\right)^{3/2}\Bigg)\Bigg/$$

$$\Bigg(a^2\,c\,(-b\,c+a\,d)\,\sqrt{-c^2+d^2}\,f\,\text{Sin}[e+f\,x]^{3/2}\Bigg)$$

$$\sqrt{g\,\text{Sin}[e+f\,x]}\,(a+b\,\text{Sin}[e+f\,x])\,(c+d\,\text{Sin}[e+f\,x])$$

$$\Bigg(\Bigg(3\,\sqrt{-a^2+b^2}\,\text{Cos}\left[\frac{1}{2}\,(e+f\,x)\right]^2\,\Bigg(-2\,\left(b+\sqrt{-a^2+b^2}\right)\,(b\,c-a\,d)\,\sqrt{-c^2+d^2}\,\text{EllipticF}\Big[$$

$$\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{-a^2+b^2}+a\,\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]}{\sqrt{-a^2+b^2}}}}{\sqrt{2}}\right],\ \frac{2\,\sqrt{-a^2+b^2}}{b+\sqrt{-a^2+b^2}}\Big]-a\,d\,\Bigg(a\,c+\left(b+\sqrt{-a^2+b^2}\right)$$

$$\left(-d+\sqrt{-c^2+d^2}\right)\Bigg)\,\text{EllipticPi}\Big[\frac{2\,\sqrt{-a^2+b^2}\,c}{b\,c+\sqrt{-a^2+b^2}\,c-a\,d+a\,\sqrt{-c^2+d^2}},$$

$$\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{-a^2+b^2}+a\,\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]}{\sqrt{-a^2+b^2}}}}{\sqrt{2}}\right],\ \frac{2\,\sqrt{-a^2+b^2}}{b+\sqrt{-a^2+b^2}}\Big]+\Bigg(-a\,c+\left(b+\sqrt{-a^2+b^2}\right)$$

$$\left(d+\sqrt{-c^2+d^2}\right)\Big)\ \text{EllipticPi}\Big[\frac{2\sqrt{-a^2+b^2}\ c}{b\,c+\sqrt{-a^2+b^2}\ c-a\left(d+\sqrt{-c^2+d^2}\right)},$$

$$\text{ArcSin}\Big[\frac{\sqrt{\frac{b+\sqrt{-a^2+b^2}+a\,\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]}{\sqrt{-a^2+b^2}}}}{\sqrt{2}}\Big],\ \frac{2\sqrt{-a^2+b^2}}{b+\sqrt{-a^2+b^2}}\Big]\Big)\Big)$$

$$\sqrt{\frac{a\,\text{Sec}\left[\frac{1}{2}(e+f\,x)\right]^2\left(a+b\,\text{Sin}[e+f\,x]\right)}{a^2-b^2}}\ \sqrt{-\frac{a\,\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]}{b+\sqrt{-a^2+b^2}}}\Bigg)\Big/$$

$$\left(a\left(b+\sqrt{-a^2+b^2}\right)c\left(-b\,c+a\,d\right)\sqrt{-c^2+d^2}\ \text{Sin}[e+f\,x]^{3/2}\sqrt{a+b\,\text{Sin}[e+f\,x]}\right)+$$

$$\left(1\Big/\left(a^2\,c\left(-b\,c+a\,d\right)\sqrt{-c^2+d^2}\ \text{Sin}[e+f\,x]^{3/2}\left(a+b\,\text{Sin}[e+f\,x]\right)^{3/2}\right)\right)$$

$$2\,b\sqrt{-a^2+b^2}\ \text{Cos}\left[\frac{1}{2}(e+f\,x)\right]^4\text{Cos}[e+f\,x]$$

$$\Bigg(-2\left(b+\sqrt{-a^2+b^2}\right)\left(b\,c-a\,d\right)\sqrt{-c^2+d^2}\ \text{EllipticF}\Big[\text{ArcSin}\Big[\frac{\sqrt{\frac{b+\sqrt{-a^2+b^2}+a\,\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]}{\sqrt{-a^2+b^2}}}}{\sqrt{2}}\Big],$$

$$\frac{2\sqrt{-a^2+b^2}}{b+\sqrt{-a^2+b^2}}\Big]-a\,d\Bigg(\left(a\,c+\left(b+\sqrt{-a^2+b^2}\right)\left(-d+\sqrt{-c^2+d^2}\right)\right)\text{EllipticPi}\Big[$$

$$\frac{2\sqrt{-a^2+b^2}\ c}{b\,c+\sqrt{-a^2+b^2}\ c-a\,d+a\sqrt{-c^2+d^2}},\ \text{ArcSin}\Big[\frac{\sqrt{\frac{b+\sqrt{-a^2+b^2}+a\,\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]}{\sqrt{-a^2+b^2}}}}{\sqrt{2}}\Big],$$

$$\frac{2\sqrt{-a^2+b^2}}{b+\sqrt{-a^2+b^2}}\Big]+\left(-a\,c+\left(b+\sqrt{-a^2+b^2}\right)\left(d+\sqrt{-c^2+d^2}\right)\right)$$

$$\text{EllipticPi}\left[\frac{2\sqrt{-a^2+b^2}\ c}{b\,c+\sqrt{-a^2+b^2}\ c-a\left(d+\sqrt{-c^2+d^2}\right)},\right.$$

$$\left.\left.\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{-a^2+b^2}+a\,\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]}{\sqrt{-a^2+b^2}}}}{\sqrt{2}}\right],\ \frac{2\sqrt{-a^2+b^2}}{b+\sqrt{-a^2+b^2}}\right]\right)\right)$$

$$\sqrt{\frac{a\,\text{Sec}\left[\frac{1}{2}(e+f\,x)\right]^2\left(a+b\,\text{Sin}[e+f\,x]\right)}{a^2-b^2}}\left(-\frac{a\,\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]}{b+\sqrt{-a^2+b^2}}\right)^{3/2}+$$

$$\frac{1}{a^2\,c\left(-b\,c+a\,d\right)\sqrt{-c^2+d^2}\ \text{Sin}[e+f\,x]^{5/2}\sqrt{a+b\,\text{Sin}[e+f\,x]}}$$

$$6\sqrt{-a^2+b^2}\ \text{Cos}\left[\frac{1}{2}(e+f\,x)\right]^4\text{Cos}[e+f\,x]$$

$$\left(-2\left(b+\sqrt{-a^2+b^2}\right)\left(b\,c-a\,d\right)\sqrt{-c^2+d^2}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{-a^2+b^2}+a\,\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]}{\sqrt{-a^2+b^2}}}}{\sqrt{2}}\right],\right.\right.$$

$$\left.\frac{2\sqrt{-a^2+b^2}}{b+\sqrt{-a^2+b^2}}\right]-a\,d\left(\left(a\,c+\left(b+\sqrt{-a^2+b^2}\right)\left(-d+\sqrt{-c^2+d^2}\right)\right)\text{EllipticPi}\left[\right.\right.$$

$$\frac{2\sqrt{-a^2+b^2}\ c}{b\,c+\sqrt{-a^2+b^2}\ c-a\,d+a\sqrt{-c^2+d^2}},\ \text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{-a^2+b^2}+a\,\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]}{\sqrt{-a^2+b^2}}}}{\sqrt{2}}\right],$$

$$\left.\frac{2\sqrt{-a^2+b^2}}{b+\sqrt{-a^2+b^2}}\right]+\left(-a\,c+\left(b+\sqrt{-a^2+b^2}\right)\left(d+\sqrt{-c^2+d^2}\right)\right)$$

$$\text{EllipticPi}\left[\frac{2\sqrt{-a^2+b^2}\ c}{b\,c+\sqrt{-a^2+b^2}\ c-a\left(d+\sqrt{-c^2+d^2}\right)},\right.$$

$$\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{-a^2+b^2}+a\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]}{\sqrt{-a^2+b^2}}}}{\sqrt{2}}\right],\ \frac{2\sqrt{-a^2+b^2}}{b+\sqrt{-a^2+b^2}}\Big]\Bigg)\Bigg)$$

$$\sqrt{\frac{a\,\text{Sec}\left[\frac{1}{2}(e+f x)\right]^2(a+b\,\text{Sin}[e+f x])}{a^2-b^2}}\left(-\frac{a\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]}{b+\sqrt{-a^2+b^2}}\right)^{3/2}+$$

$$\frac{1}{a^2 c(-b c+a d)\sqrt{-c^2+d^2}\,\text{Sin}[e+f x]^{3/2}\sqrt{a+b\,\text{Sin}[e+f x]}}$$

$$8\sqrt{-a^2+b^2}\,\text{Cos}\left[\frac{1}{2}(e+f x)\right]^3$$

$$\Bigg(-2\left(b+\sqrt{-a^2+b^2}\right)(b c-a d)\sqrt{-c^2+d^2}\,\text{EllipticF}\Big[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{-a^2+b^2}+a\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]}{\sqrt{-a^2+b^2}}}}{\sqrt{2}}\right],$$

$$\frac{2\sqrt{-a^2+b^2}}{b+\sqrt{-a^2+b^2}}\Big]-a d\Bigg(\left(a c+\left(b+\sqrt{-a^2+b^2}\right)\left(-d+\sqrt{-c^2+d^2}\right)\right)\text{EllipticPi}\Big[$$

$$\frac{2\sqrt{-a^2+b^2}\,c}{b c+\sqrt{-a^2+b^2}\,c-a d+a\sqrt{-c^2+d^2}},\ \text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{-a^2+b^2}+a\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]}{\sqrt{-a^2+b^2}}}}{\sqrt{2}}\right],$$

$$\frac{2\sqrt{-a^2+b^2}}{b+\sqrt{-a^2+b^2}}\Big]+\left(-a c+\left(b+\sqrt{-a^2+b^2}\right)\left(d+\sqrt{-c^2+d^2}\right)\right)$$

$$\text{EllipticPi}\Big[\frac{2\sqrt{-a^2+b^2}\,c}{b c+\sqrt{-a^2+b^2}\,c-a\left(d+\sqrt{-c^2+d^2}\right)},$$

$$\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{-a^2+b^2}+a\,\text{Tan}\left[\frac{1}{2}(e+f x)\right]}{\sqrt{-a^2+b^2}}}}{\sqrt{2}}\right],\ \frac{2\sqrt{-a^2+b^2}}{b+\sqrt{-a^2+b^2}}\Big]\Bigg)\,\text{Sin}\left[\frac{1}{2}(e+f x)\right]$$

$$\sqrt{\frac{a\,\text{Sec}\left[\frac{1}{2}\left(e+f\,x\right)\right]^2\left(a+b\,\text{Sin}\left[e+f\,x\right]\right)}{a^2-b^2}}\left(-\frac{a\,\text{Tan}\left[\frac{1}{2}\left(e+f\,x\right)\right]}{b+\sqrt{-a^2+b^2}}\right)^{3/2}-$$

$$\left(2\sqrt{-a^2+b^2}\,\text{Cos}\left[\frac{1}{2}\left(e+f\,x\right)\right]^4\left(-2\left(b+\sqrt{-a^2+b^2}\right)\left(b\,c-a\,d\right)\sqrt{-c^2+d^2}\right.\right.$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{-a^2+b^2}+a\,\text{Tan}\left[\frac{1}{2}\left(e+f\,x\right)\right]}{\sqrt{-a^2+b^2}}}}{\sqrt{2}}\right],\frac{2\sqrt{-a^2+b^2}}{b+\sqrt{-a^2+b^2}}\right]-$$

$$a\,d\left(\left(a\,c+\left(b+\sqrt{-a^2+b^2}\right)\left(-d+\sqrt{-c^2+d^2}\right)\right)\text{EllipticPi}\left[\right.\right.$$

$$\frac{2\sqrt{-a^2+b^2}\,c}{b\,c+\sqrt{-a^2+b^2}\,c-a\,d+a\sqrt{-c^2+d^2}},\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{-a^2+b^2}+a\,\text{Tan}\left[\frac{1}{2}\left(e+f\,x\right)\right]}{\sqrt{-a^2+b^2}}}}{\sqrt{2}}\right],$$

$$\left.\frac{2\sqrt{-a^2+b^2}}{b+\sqrt{-a^2+b^2}}\right]+\left(-a\,c+\left(b+\sqrt{-a^2+b^2}\right)\left(d+\sqrt{-c^2+d^2}\right)\right)\text{EllipticPi}\left[\right.$$

$$\frac{2\sqrt{-a^2+b^2}\,c}{b\,c+\sqrt{-a^2+b^2}\,c-a\left(d+\sqrt{-c^2+d^2}\right)},\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{-a^2+b^2}+a\,\text{Tan}\left[\frac{1}{2}\left(e+f\,x\right)\right]}{\sqrt{-a^2+b^2}}}}{\sqrt{2}}\right],$$

$$\left.\left.\left.\frac{2\sqrt{-a^2+b^2}}{b+\sqrt{-a^2+b^2}}\right]\right)\right)\left(-\frac{a\,\text{Tan}\left[\frac{1}{2}\left(e+f\,x\right)\right]}{b+\sqrt{-a^2+b^2}}\right)^{3/2}\left(\frac{a\,b\,\text{Cos}\left[e+f\,x\right]\,\text{Sec}\left[\frac{1}{2}\left(e+f\,x\right)\right]^2}{a^2-b^2}+\right.$$

$$\left.\left.\frac{a\,\mathrm{Sec}\left[\frac{1}{2}\,(e+f\,x)\right]^2\,(a+b\,\mathrm{Sin}[e+f\,x])\,\mathrm{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]}{a^2-b^2}\right)\right)\Big/$$

$$\left(a^2\,c\,(-b\,c+a\,d)\,\sqrt{-c^2+d^2}\,\mathrm{Sin}[e+f\,x]^{3/2}\,\sqrt{a+b\,\mathrm{Sin}[e+f\,x]}\right.$$

$$\left.\sqrt{\frac{a\,\mathrm{Sec}\left[\frac{1}{2}\,(e+f\,x)\right]^2\,(a+b\,\mathrm{Sin}[e+f\,x])}{a^2-b^2}}\right)-$$

$$\frac{1}{a^2\,c\,(-b\,c+a\,d)\,\sqrt{-c^2+d^2}\,\mathrm{Sin}[e+f\,x]^{3/2}\,\sqrt{a+b\,\mathrm{Sin}[e+f\,x]}}$$

$$4\,\sqrt{-a^2+b^2}\,\mathrm{Cos}\left[\frac{1}{2}\,(e+f\,x)\right]^4$$

$$\sqrt{\frac{a\,\mathrm{Sec}\left[\frac{1}{2}\,(e+f\,x)\right]^2\,(a+b\,\mathrm{Sin}[e+f\,x])}{a^2-b^2}}\,\left(-\frac{a\,\mathrm{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]}{b+\sqrt{-a^2+b^2}}\right)^{3/2}$$

$$\left(-\left(\left(a\,\left(b+\sqrt{-a^2+b^2}\right)\,(b\,c-a\,d)\,\sqrt{-c^2+d^2}\,\mathrm{Sec}\left[\frac{1}{2}\,(e+f\,x)\right]^2\right)\Big/\left(2\,\sqrt{2}\,\sqrt{-a^2+b^2}\right.\right.\right.$$

$$\sqrt{\frac{b+\sqrt{-a^2+b^2}+a\,\mathrm{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]}{\sqrt{-a^2+b^2}}}\,\sqrt{1-\frac{b+\sqrt{-a^2+b^2}+a\,\mathrm{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]}{2\,\sqrt{-a^2+b^2}}}$$

$$\left.\left.\sqrt{1-\frac{b+\sqrt{-a^2+b^2}+a\,\mathrm{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]}{b+\sqrt{-a^2+b^2}}}\right)\right)-a\,d$$

$$\left(\left(a\,\left(a\,c+\left(b+\sqrt{-a^2+b^2}\right)\,\left(-d+\sqrt{-c^2+d^2}\right)\right)\,\mathrm{Sec}\left[\frac{1}{2}\,(e+f\,x)\right]^2\right)\Big/\left(4\,\sqrt{2}\,\sqrt{-a^2+b^2}\right.\right.$$

$$\sqrt{\frac{b+\sqrt{-a^2+b^2}+a\,\mathrm{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]}{\sqrt{-a^2+b^2}}}\,\sqrt{1-\frac{b+\sqrt{-a^2+b^2}+a\,\mathrm{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]}{2\,\sqrt{-a^2+b^2}}}$$

$$\sqrt{1-\frac{b+\sqrt{-a^2+b^2}+a\,\mathrm{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]}{b+\sqrt{-a^2+b^2}}}$$

$$\left(1-\frac{c\left(b+\sqrt{-a^2+b^2}+a\,\mathrm{Tan}\left[\frac{1}{2}(e+f\,x)\right]\right)}{b\,c+\sqrt{-a^2+b^2}\;c-a\,d+a\,\sqrt{-c^2+d^2}}\right)\Bigg)\Bigg)+\Bigg(a\,\Bigg(-a\,c+$$

$$\left(b+\sqrt{-a^2+b^2}\right)\left(d+\sqrt{-c^2+d^2}\right)\Bigg)\,\mathrm{Sec}\left[\frac{1}{2}(e+f\,x)\right]^2\Bigg)\Bigg/\Bigg(4\sqrt{2}\,\sqrt{-a^2+b^2}$$

$$\sqrt{\frac{b+\sqrt{-a^2+b^2}+a\,\mathrm{Tan}\left[\frac{1}{2}(e+f\,x)\right]}{\sqrt{-a^2+b^2}}}\;\sqrt{1-\frac{b+\sqrt{-a^2+b^2}+a\,\mathrm{Tan}\left[\frac{1}{2}(e+f\,x)\right]}{2\sqrt{-a^2+b^2}}}$$

$$\sqrt{1-\frac{b+\sqrt{-a^2+b^2}+a\,\mathrm{Tan}\left[\frac{1}{2}(e+f\,x)\right]}{b+\sqrt{-a^2+b^2}}}$$

$$\left(1-\frac{c\left(b+\sqrt{-a^2+b^2}+a\,\mathrm{Tan}\left[\frac{1}{2}(e+f\,x)\right]\right)}{b\,c+\sqrt{-a^2+b^2}\;c-a\left(d+\sqrt{-c^2+d^2}\right)}\right)\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)$$

Problem 46: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{\sqrt{g\,\mathrm{Sin}[e+f\,x]}\;\sqrt{c+d\,\mathrm{Sin}[e+f\,x]}}{a+b\,\mathrm{Sin}[e+f\,x]}\,dx$$

Optimal (type 4, 254 leaves, 3 steps):

$$\frac{1}{b\,f}2\sqrt{c+d}\,\sqrt{g}\,\sqrt{\frac{c\,(1-\mathrm{Csc}[e+f\,x])}{c+d}}\,\sqrt{\frac{c\,(1+\mathrm{Csc}[e+f\,x])}{c-d}}$$

$$\mathrm{EllipticPi}\left[\frac{c+d}{d},\,\mathrm{ArcSin}\left[\frac{\sqrt{g}\,\sqrt{c+d\,\mathrm{Sin}[e+f\,x]}}{\sqrt{c+d}\,\sqrt{g\,\mathrm{Sin}[e+f\,x]}}\right],\,-\frac{c+d}{c-d}\right]\mathrm{Tan}[e+f\,x]+$$

$$\Bigg(2\,(b\,c-a\,d)\,\sqrt{-\mathrm{Cot}[e+f\,x]^2}\,\sqrt{\frac{d+c\,\mathrm{Csc}[e+f\,x]}{c+d}}\,\mathrm{EllipticPi}\left[\frac{2\,a}{a+b},\,\mathrm{ArcSin}\left[\frac{\sqrt{1-\mathrm{Csc}[e+f\,x]}}{\sqrt{2}}\right],\right.$$

$$\left.\frac{2\,c}{c+d}\right]\sqrt{g\,\mathrm{Sin}[e+f\,x]}\;\mathrm{Tan}[e+f\,x]\Bigg)\Bigg/\left(b\,(a+b)\,f\,\sqrt{c+d\,\mathrm{Sin}[e+f\,x]}\right)$$

Result (type 4, 75413 leaves): Display of huge result suppressed!

Problem 47: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{g\,\text{Sin}[e+f\,x]}}{(a+b\,\text{Sin}[e+f\,x])\,\sqrt{c+d\,\text{Sin}[e+f\,x]}}\,dx$$

Optimal (type 4, 114 leaves, 1 step):

$$\left(2\sqrt{-\text{Cot}[e+f\,x]^2}\,\sqrt{\frac{d+c\,\text{Csc}[e+f\,x]}{c+d}}\,\text{EllipticPi}\left[\frac{2\,a}{a+b},\,\text{ArcSin}\left[\frac{\sqrt{1-\text{Csc}[e+f\,x]}}{\sqrt{2}}\right],\,\frac{2\,c}{c+d}\right]\sqrt{g\,\text{Sin}[e+f\,x]}\,\text{Tan}[e+f\,x]\right)\Big/\left((a+b)\,f\,\sqrt{c+d\,\text{Sin}[e+f\,x]}\right)$$

Result (type 4, 3429 leaves):

$$-\left(\left(c\,\sqrt{-c^2+d^2}\,\left(\left(-a\,c+\left(b+\sqrt{-a^2+b^2}\right)\left(d+\sqrt{-c^2+d^2}\right)\right)\right.\right.\right.$$

$$\text{EllipticPi}\left[\frac{2\,a\,\sqrt{-c^2+d^2}}{-b\,c-\sqrt{-a^2+b^2}\,c+a\left(d+\sqrt{-c^2+d^2}\right)},\,\text{ArcSin}\left[\frac{\sqrt{\frac{d+\sqrt{-c^2+d^2}+c\,\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]}{\sqrt{-c^2+d^2}}}}{\sqrt{2}}\right],\right.$$

$$\left.\frac{2\sqrt{-c^2+d^2}}{d+\sqrt{-c^2+d^2}}\right]+\left(a\,c+\left(-b+\sqrt{-a^2+b^2}\right)\left(d+\sqrt{-c^2+d^2}\right)\right)$$

$$\text{EllipticPi}\left[\frac{2\,a\,\sqrt{-c^2+d^2}}{-b\,c+\sqrt{-a^2+b^2}\,c+a\left(d+\sqrt{-c^2+d^2}\right)},\right.$$

$$\left.\left.\text{ArcSin}\left[\frac{\sqrt{\frac{d+\sqrt{-c^2+d^2}+c\,\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]}{\sqrt{-c^2+d^2}}}}{\sqrt{2}}\right],\,\frac{2\sqrt{-c^2+d^2}}{d+\sqrt{-c^2+d^2}}\right]\right)$$

$$\left.\sqrt{\text{Sin}[e+f\,x]}\,\sqrt{g\,\text{Sin}[e+f\,x]}\,\sqrt{\frac{c\,\text{Sec}\left[\frac{1}{2}(e+f\,x)\right]^2\,(c+d\,\text{Sin}[e+f\,x])}{c^2-d^2}}\right)\Big/$$

$$\left(\sqrt{-a^2+b^2}\,(b\,c-a\,d)\,\left(d+\sqrt{-c^2+d^2}\right)^2 f\,(a+b\,\mathrm{Sin}[e+f\,x])\right.$$

$$(c+d\,\mathrm{Sin}[e+f\,x])\,\sqrt{-\frac{c\,\mathrm{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]}{d+\sqrt{-c^2+d^2}}}$$

$$\left(-\left(\left(c^2\,\sqrt{-c^2+d^2}\,\left(\left(-a\,c+\left(b+\sqrt{-a^2+b^2}\right)\,\left(d+\sqrt{-c^2+d^2}\right)\right)\,\mathrm{EllipticPi}\left[\right.\right.\right.\right.\right.$$

$$\frac{2\,a\,\sqrt{-c^2+d^2}}{-b\,c-\sqrt{-a^2+b^2}\,c+a\,\left(d+\sqrt{-c^2+d^2}\right)},\,\mathrm{ArcSin}\left[\frac{\sqrt{\frac{d+\sqrt{-c^2+d^2}+c\,\mathrm{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]}{\sqrt{-c^2+d^2}}}}{\sqrt{2}}\right],$$

$$\left.\frac{2\,\sqrt{-c^2+d^2}}{d+\sqrt{-c^2+d^2}}\right]+\left(a\,c+\left(-b+\sqrt{-a^2+b^2}\right)\,\left(d+\sqrt{-c^2+d^2}\right)\right)\,\mathrm{EllipticPi}\left[\right.$$

$$\frac{2\,a\,\sqrt{-c^2+d^2}}{-b\,c+\sqrt{-a^2+b^2}\,c+a\,\left(d+\sqrt{-c^2+d^2}\right)},\,\mathrm{ArcSin}\left[\frac{\sqrt{\frac{d+\sqrt{-c^2+d^2}+c\,\mathrm{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]}{\sqrt{-c^2+d^2}}}}{\sqrt{2}}\right],$$

$$\left.\left.\frac{2\,\sqrt{-c^2+d^2}}{d+\sqrt{-c^2+d^2}}\right]\right)\,\mathrm{Sec}\left[\frac{1}{2}\,(e+f\,x)\right]^2\,\sqrt{\mathrm{Sin}[e+f\,x]}$$

$$\left.\sqrt{\frac{c\,\mathrm{Sec}\left[\frac{1}{2}\,(e+f\,x)\right]^2\,(c+d\,\mathrm{Sin}[e+f\,x])}{c^2-d^2}}\right)\Big/\left(4\,\sqrt{-a^2+b^2}\,(b\,c-a\,d)\right.$$

$$\left.\left.\left(d+\sqrt{-c^2+d^2}\right)^3 \sqrt{c+d\,\text{Sin}[e+f\,x]} \left(-\frac{c\,\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]}{d+\sqrt{-c^2+d^2}}\right)^{3/2}\right)\right) +$$

$$\left(c\,d\,\sqrt{-c^2+d^2}\,\text{Cos}[e+f\,x]\left(\left(-a\,c+\left(b+\sqrt{-a^2+b^2}\right)\left(d+\sqrt{-c^2+d^2}\right)\right)\text{EllipticPi}\left[\right.\right.\right.$$

$$\frac{2\,a\,\sqrt{-c^2+d^2}}{-b\,c-\sqrt{-a^2+b^2}\,c+a\left(d+\sqrt{-c^2+d^2}\right)},\ \text{ArcSin}\left[\frac{\sqrt{\frac{d+\sqrt{-c^2+d^2}+c\,\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]}{\sqrt{-c^2+d^2}}}}{\sqrt{2}}\right],$$

$$\left.\frac{2\,\sqrt{-c^2+d^2}}{d+\sqrt{-c^2+d^2}}\right]+\left(a\,c+\left(-b+\sqrt{-a^2+b^2}\right)\left(d+\sqrt{-c^2+d^2}\right)\right)\text{EllipticPi}\left[\right.$$

$$\frac{2\,a\,\sqrt{-c^2+d^2}}{-b\,c+\sqrt{-a^2+b^2}\,c+a\left(d+\sqrt{-c^2+d^2}\right)},\ \text{ArcSin}\left[\frac{\sqrt{\frac{d+\sqrt{-c^2+d^2}+c\,\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]}{\sqrt{-c^2+d^2}}}}{\sqrt{2}}\right],$$

$$\left.\left.\frac{2\,\sqrt{-c^2+d^2}}{d+\sqrt{-c^2+d^2}}\right]\right)\sqrt{\text{Sin}[e+f\,x]}\,\sqrt{\frac{c\,\text{Sec}\left[\frac{1}{2}\,(e+f\,x)\right]^2\,(c+d\,\text{Sin}[e+f\,x])}{c^2-d^2}}\right)\Big/$$

$$\left(2\,\sqrt{-a^2+b^2}\,(b\,c-a\,d)\left(d+\sqrt{-c^2+d^2}\right)^2\,(c+d\,\text{Sin}[e+f\,x])^{3/2}\,\sqrt{-\frac{c\,\text{Tan}\left[\frac{1}{2}\,(e+f\,x)\right]}{d+\sqrt{-c^2+d^2}}}\right)-$$

$$\left(c\,\sqrt{-c^2+d^2}\,\text{Cos}[e+f\,x]\left(\left(-a\,c+\left(b+\sqrt{-a^2+b^2}\right)\left(d+\sqrt{-c^2+d^2}\right)\right)\text{EllipticPi}\left[\right.\right.\right.$$

$$\frac{2\,a\,\sqrt{-c^2+d^2}}{-b\,c-\sqrt{-a^2+b^2}\,c+a\left(d+\sqrt{-c^2+d^2}\right)},\ \text{ArcSin}\Big[\frac{\sqrt{\frac{d+\sqrt{-c^2+d^2}+c\,\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]}{\sqrt{-c^2+d^2}}}}{\sqrt{2}}\Big],$$

$$\frac{2\,\sqrt{-c^2+d^2}}{d+\sqrt{-c^2+d^2}}\Big]+\left(a\,c+\left(-b+\sqrt{-a^2+b^2}\right)\left(d+\sqrt{-c^2+d^2}\right)\right)\text{EllipticPi}\Big[$$

$$\frac{2\,a\,\sqrt{-c^2+d^2}}{-b\,c+\sqrt{-a^2+b^2}\,c+a\left(d+\sqrt{-c^2+d^2}\right)},\ \text{ArcSin}\Big[\frac{\sqrt{\frac{d+\sqrt{-c^2+d^2}+c\,\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]}{\sqrt{-c^2+d^2}}}}{\sqrt{2}}\Big],$$

$$\frac{2\,\sqrt{-c^2+d^2}}{d+\sqrt{-c^2+d^2}}\Big]\left.\right)\sqrt{\frac{c\,\text{Sec}\left[\frac{1}{2}(e+f\,x)\right]^2\left(c+d\,\text{Sin}[e+f\,x]\right)}{c^2-d^2}}\left.\right)\Big/$$

$$\left(2\,\sqrt{-a^2+b^2}\,(b\,c-a\,d)\left(d+\sqrt{-c^2+d^2}\right)^2\sqrt{\text{Sin}[e+f\,x]}\,\sqrt{c+d\,\text{Sin}[e+f\,x]}\right.$$

$$\left.\sqrt{-\frac{c\,\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]}{d+\sqrt{-c^2+d^2}}}\right)-$$

$$\left(c\,\sqrt{-c^2+d^2}\left(\left(-a\,c+\left(b+\sqrt{-a^2+b^2}\right)\left(d+\sqrt{-c^2+d^2}\right)\right)\text{EllipticPi}\Big[\right.\right.$$

$$\frac{2\,a\,\sqrt{-c^2+d^2}}{-b\,c-\sqrt{-a^2+b^2}\,c+a\left(d+\sqrt{-c^2+d^2}\right)},\ \text{ArcSin}\Big[\frac{\sqrt{\frac{d+\sqrt{-c^2+d^2}+c\,\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]}{\sqrt{-c^2+d^2}}}}{\sqrt{2}}\Big],$$

$$\frac{2\,\sqrt{-c^2+d^2}}{d+\sqrt{-c^2+d^2}}\Big]+\left(a\,c+\left(-b+\sqrt{-a^2+b^2}\right)\left(d+\sqrt{-c^2+d^2}\right)\right)\text{EllipticPi}\Big[$$

$$\frac{2\,a\,\sqrt{-c^2+d^2}}{-b\,c+\sqrt{-a^2+b^2}\;c+a\left(d+\sqrt{-c^2+d^2}\right)},\ \text{ArcSin}\left[\frac{\sqrt{\frac{d+\sqrt{-c^2+d^2}+c\,\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]}{\sqrt{-c^2+d^2}}}}{\sqrt{2}}\right],$$

$$\left.\left.\frac{2\sqrt{-c^2+d^2}}{d+\sqrt{-c^2+d^2}}\right]\right)\sqrt{\text{Sin}[e+f\,x]}\left(\frac{c\,d\,\text{Cos}[e+f\,x]\,\text{Sec}\left[\frac{1}{2}(e+f\,x)\right]^2}{c^2-d^2}+\right.$$

$$\left.\left.\frac{c\,\text{Sec}\left[\frac{1}{2}(e+f\,x)\right]^2\left(c+d\,\text{Sin}[e+f\,x]\right)\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]}{c^2-d^2}\right)\right)\Bigg/$$

$$\left(2\sqrt{-a^2+b^2}\,(b\,c-a\,d)\left(d+\sqrt{-c^2+d^2}\right)^2\sqrt{c+d\,\text{Sin}[e+f\,x]}\right.$$

$$\left.\sqrt{\frac{c\,\text{Sec}\left[\frac{1}{2}(e+f\,x)\right]^2\left(c+d\,\text{Sin}[e+f\,x]\right)}{c^2-d^2}}\sqrt{-\frac{c\,\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]}{d+\sqrt{-c^2+d^2}}}\right)-$$

$$\left(c\sqrt{-c^2+d^2}\,\sqrt{\text{Sin}[e+f\,x]}\,\sqrt{\frac{c\,\text{Sec}\left[\frac{1}{2}(e+f\,x)\right]^2\left(c+d\,\text{Sin}[e+f\,x]\right)}{c^2-d^2}}\right.$$

$$\left(\left(c\left(-a\,c+\left(b+\sqrt{-a^2+b^2}\right)\left(d+\sqrt{-c^2+d^2}\right)\right)\text{Sec}\left[\frac{1}{2}(e+f\,x)\right]^2\right)\Bigg/\right.$$

$$\left(4\sqrt{2}\,\sqrt{-c^2+d^2}\,\sqrt{\frac{d+\sqrt{-c^2+d^2}+c\,\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]}{\sqrt{-c^2+d^2}}}\right.$$

$$\sqrt{1-\frac{d+\sqrt{-c^2+d^2}+c\,\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]}{2\sqrt{-c^2+d^2}}}\,\sqrt{1-\frac{d+\sqrt{-c^2+d^2}+c\,\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]}{d+\sqrt{-c^2+d^2}}}$$

$$\left(1-\frac{a\left(d+\sqrt{-c^2+d^2}+c\,\text{Tan}\left[\frac{1}{2}\left(e+f\,x\right)\right]\right)}{-b\,c-\sqrt{-a^2+b^2}\,c+a\left(d+\sqrt{-c^2+d^2}\right)}\right)\Bigg)\Bigg)+$$

$$\left(c\left(a\,c+\left(-b+\sqrt{-a^2+b^2}\right)\left(d+\sqrt{-c^2+d^2}\right)\right)\text{Sec}\left[\frac{1}{2}\left(e+f\,x\right)\right]^2\right)\Big/$$

$$\left(4\sqrt{2}\sqrt{-c^2+d^2}\sqrt{\frac{d+\sqrt{-c^2+d^2}+c\,\text{Tan}\left[\frac{1}{2}\left(e+f\,x\right)\right]}{\sqrt{-c^2+d^2}}}\right.$$

$$\sqrt{1-\frac{d+\sqrt{-c^2+d^2}+c\,\text{Tan}\left[\frac{1}{2}\left(e+f\,x\right)\right]}{2\sqrt{-c^2+d^2}}}\sqrt{1-\frac{d+\sqrt{-c^2+d^2}+c\,\text{Tan}\left[\frac{1}{2}\left(e+f\,x\right)\right]}{d+\sqrt{-c^2+d^2}}}$$

$$\left.\left(1-\frac{a\left(d+\sqrt{-c^2+d^2}+c\,\text{Tan}\left[\frac{1}{2}\left(e+f\,x\right)\right]\right)}{-b\,c+\sqrt{-a^2+b^2}\,c+a\left(d+\sqrt{-c^2+d^2}\right)}\right)\right)\Bigg)\Bigg)\Big/$$

$$\left(\sqrt{-a^2+b^2}\left(b\,c-a\,d\right)\left(d+\sqrt{-c^2+d^2}\right)^2\sqrt{c+d\,\text{Sin}[e+f\,x]}\sqrt{-\frac{c\,\text{Tan}\left[\frac{1}{2}\left(e+f\,x\right)\right]}{d+\sqrt{-c^2+d^2}}}\right)\Bigg)\Bigg)\Bigg)$$

Problem 48: Unable to integrate problem.

$$\int \text{Csc}[e+f\,x]\sqrt{a+b\,\text{Sin}[e+f\,x]}\sqrt{c+d\,\text{Sin}[e+f\,x]}\,dx$$

Optimal (type 4, 391 leaves, 3 steps):

$$-\frac{1}{\sqrt{a+b}\,f}2\sqrt{c+d}\,\text{EllipticPi}\left[\frac{a\left(c+d\right)}{\left(a+b\right)c},\text{ArcSin}\left[\frac{\sqrt{a+b}\sqrt{c+d\,\text{Sin}[e+f\,x]}}{\sqrt{c+d}\sqrt{a+b\,\text{Sin}[e+f\,x]}}\right],\frac{\left(a-b\right)\left(c+d\right)}{\left(a+b\right)\left(c-d\right)}\right]$$

$$\text{Sec}[e+f\,x]\sqrt{-\frac{\left(b\,c-a\,d\right)\left(1-\text{Sin}[e+f\,x]\right)}{\left(c+d\right)\left(a+b\,\text{Sin}[e+f\,x]\right)}}$$

$$\sqrt{\frac{\left(b\,c-a\,d\right)\left(1+\text{Sin}[e+f\,x]\right)}{\left(c-d\right)\left(a+b\,\text{Sin}[e+f\,x]\right)}}\left(a+b\,\text{Sin}[e+f\,x]\right)+\frac{1}{\sqrt{a+b}\,f}$$

$$2\sqrt{c+d}\,\text{EllipticPi}\left[\frac{b\left(c+d\right)}{\left(a+b\right)d},\text{ArcSin}\left[\frac{\sqrt{a+b}\sqrt{c+d\,\text{Sin}[e+f\,x]}}{\sqrt{c+d}\sqrt{a+b\,\text{Sin}[e+f\,x]}}\right],\frac{\left(a-b\right)\left(c+d\right)}{\left(a+b\right)\left(c-d\right)}\right]$$

$$\text{Sec}[e+f\,x]\sqrt{-\frac{\left(b\,c-a\,d\right)\left(1-\text{Sin}[e+f\,x]\right)}{\left(c+d\right)\left(a+b\,\text{Sin}[e+f\,x]\right)}}\sqrt{\frac{\left(b\,c-a\,d\right)\left(1+\text{Sin}[e+f\,x]\right)}{\left(c-d\right)\left(a+b\,\text{Sin}[e+f\,x]\right)}}\left(a+b\,\text{Sin}[e+f\,x]\right)$$

Result (type 8, 37 leaves):

$$\int \text{Csc}[e+fx]\,\sqrt{a+b\,\text{Sin}[e+fx]}\,\sqrt{c+d\,\text{Sin}[e+fx]}\,dx$$

Problem 49: Unable to integrate problem.

$$\int \frac{\text{Csc}[e+fx]\,\sqrt{a+b\,\text{Sin}[e+fx]}}{\sqrt{c+d\,\text{Sin}[e+fx]}}\,dx$$

Optimal (type 4, 198 leaves, 1 step):

$$-\frac{1}{\sqrt{a+b}\;c\,f}2\sqrt{c+d}\;\text{EllipticPi}\left[\frac{a\,(c+d)}{(a+b)\,c},\text{ArcSin}\left[\frac{\sqrt{a+b}\,\sqrt{c+d\,\text{Sin}[e+fx]}}{\sqrt{c+d}\,\sqrt{a+b\,\text{Sin}[e+fx]}}\right],\frac{(a-b)\,(c+d)}{(a+b)\,(c-d)}\right]$$
$$\text{Sec}[e+fx]\,\sqrt{-\frac{(b\,c-a\,d)\,(1-\text{Sin}[e+fx])}{(c+d)\,(a+b\,\text{Sin}[e+fx])}}\,\sqrt{\frac{(b\,c-a\,d)\,(1+\text{Sin}[e+fx])}{(c-d)\,(a+b\,\text{Sin}[e+fx])}}\,(a+b\,\text{Sin}[e+fx])$$

Result (type 8, 37 leaves):

$$\int \frac{\text{Csc}[e+fx]\,\sqrt{a+b\,\text{Sin}[e+fx]}}{\sqrt{c+d\,\text{Sin}[e+fx]}}\,dx$$

Problem 50: Unable to integrate problem.

$$\int \frac{\text{Csc}[e+fx]}{\sqrt{a+b\,\text{Sin}[e+fx]}\,\sqrt{c+d\,\text{Sin}[e+fx]}}\,dx$$

Optimal (type 4, 398 leaves, 3 steps):

$$-\frac{1}{a\sqrt{a+b}\;c\,f}2\sqrt{c+d}$$
$$\text{EllipticPi}\left[\frac{a\,(c+d)}{(a+b)\,c},\text{ArcSin}\left[\frac{\sqrt{a+b}\,\sqrt{c+d\,\text{Sin}[e+fx]}}{\sqrt{c+d}\,\sqrt{a+b\,\text{Sin}[e+fx]}}\right],\frac{(a-b)\,(c+d)}{(a+b)\,(c-d)}\right]\text{Sec}[e+fx]$$
$$\sqrt{-\frac{(b\,c-a\,d)\,(1-\text{Sin}[e+fx])}{(c+d)\,(a+b\,\text{Sin}[e+fx])}}\,\sqrt{\frac{(b\,c-a\,d)\,(1+\text{Sin}[e+fx])}{(c-d)\,(a+b\,\text{Sin}[e+fx])}}\,(a+b\,\text{Sin}[e+fx])-$$
$$\left(2\,b\sqrt{a+b}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c+d}\,\sqrt{a+b\,\text{Sin}[e+fx]}}{\sqrt{a+b}\,\sqrt{c+d\,\text{Sin}[e+fx]}}\right],\frac{(a+b)\,(c-d)}{(a-b)\,(c+d)}\right]\right.$$
$$\text{Sec}[e+fx]\,\sqrt{\frac{(b\,c-a\,d)\,(1-\text{Sin}[e+fx])}{(a+b)\,(c+d\,\text{Sin}[e+fx])}}\,\sqrt{-\frac{(b\,c-a\,d)\,(1+\text{Sin}[e+fx])}{(a-b)\,(c+d\,\text{Sin}[e+fx])}}$$
$$\left.(c+d\,\text{Sin}[e+fx])\right)\Big/\left(a\sqrt{c+d}\,(b\,c-a\,d)\,f\right)$$

Result (type 8, 37 leaves):

$$\int \frac{\text{Csc}[e+f x]}{\sqrt{a+b\,\text{Sin}[e+f x]}\,\sqrt{c+d\,\text{Sin}[e+f x]}}\,dx$$

Problem 51: Result more than twice size of optimal antiderivative.

$$\int (a+a\,\text{Sin}[e+f x])^m\,(A+B\,\text{Sin}[e+f x])^p\,(c-c\,\text{Sin}[e+f x])^n\,dx$$

Optimal (type 6, 157 leaves, 4 steps):

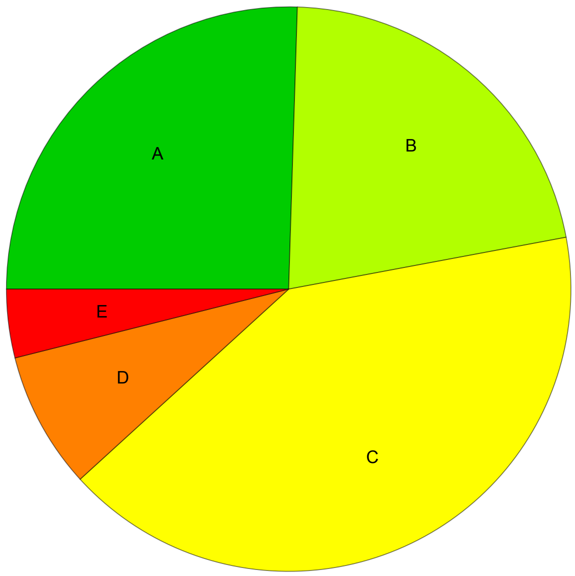
$$\frac{1}{a f\,(1+2 m)} 2^{\frac{1}{2}+n}\,\text{AppellF1}\left[\frac{1}{2}+m,\ \frac{1}{2}-n,\ -p,\ \frac{3}{2}+m,\ \frac{1}{2}\,(1+\text{Sin}[e+f x]),\ -\frac{B\,(1+\text{Sin}[e+f x])}{A-B}\right]$$
$$\text{Sec}[e+f x]\,(1-\text{Sin}[e+f x])^{\frac{1}{2}-n}\,(a+a\,\text{Sin}[e+f x])^{1+m}$$
$$(A+B\,\text{Sin}[e+f x])^p\left(\frac{A+B\,\text{Sin}[e+f x]}{A-B}\right)^{-p}(c-c\,\text{Sin}[e+f x])^n$$

Result (type 6, 417 leaves):

$$\Bigg(2\,(A+B)\,(3+2 n)$$
$$\text{AppellF1}\left[\frac{1}{2}+n,\ \frac{1}{2}-m,\ -p,\ \frac{3}{2}+n,\ \text{Cos}\left[\frac{1}{4}\,(2 e+\pi+2 f x)\right]^2,\ \frac{2 B\,\text{Sin}\left[\frac{1}{4}\,(2 e-\pi+2 f x)\right]^2}{A+B}\right]$$
$$\left(\text{Cos}\left[\frac{1}{4}\,(2 e-\pi+2 f x)\right]^2\right)^{-\frac{1}{2}+m}\,\text{Cot}\left[\frac{1}{4}\,(2 e+\pi+2 f x)\right]\,(a\,(1+\text{Sin}[e+f x]))^m$$
$$(A+B\,\text{Sin}[e+f x])^p\,(c-c\,\text{Sin}[e+f x])^n\left(\text{Sin}\left[\frac{1}{4}\,(2 e+\pi+2 f x)\right]^2\right)^{\frac{1}{2}-m}\Bigg)\Bigg/$$
$$\Bigg(f\,(1+2 n)\,\Bigg(-(A+B)\,(3+2 n)\,\text{AppellF1}\left[\frac{1}{2}+n,\ \frac{1}{2}-m,\ -p,\ \frac{3}{2}+n,\right.$$
$$\left.\text{Cos}\left[\frac{1}{4}\,(2 e+\pi+2 f x)\right]^2,\ \frac{2 B\,\text{Sin}\left[\frac{1}{4}\,(2 e-\pi+2 f x)\right]^2}{A+B}\right]+$$
$$\Bigg(4 B p\,\text{AppellF1}\left[\frac{3}{2}+n,\ \frac{1}{2}-m,\ 1-p,\ \frac{5}{2}+n,\ \text{Cos}\left[\frac{1}{4}\,(2 e+\pi+2 f x)\right]^2,\right.$$
$$\left.\frac{2 B\,\text{Sin}\left[\frac{1}{4}\,(2 e-\pi+2 f x)\right]^2}{A+B}\right]+(A+B)\,(-1+2 m)\,\text{AppellF1}\left[\frac{3}{2}+n,\ \frac{3}{2}-m,\ -p,\ \frac{5}{2}+n,\right.$$
$$\left.\text{Cos}\left[\frac{1}{4}\,(2 e+\pi+2 f x)\right]^2,\ \frac{2 B\,\text{Sin}\left[\frac{1}{4}\,(2 e-\pi+2 f x)\right]^2}{A+B}\right]\Bigg)\,\text{Cos}\left[\frac{1}{4}\,(2 e+\pi+2 f x)\right]^2\Bigg)\Bigg)$$

Summary of Integration Test Results

51 integration problems



A - 13 optimal antiderivatives

B - 11 more than twice size of optimal antiderivatives

C - 21 unnecessarily complex antiderivatives

D - 4 unable to integrate problems

E - 2 integration timeouts