Mathematica 11.3 Integration Test Results

Test results for the 19 problems in "4.1.4.1 (a+b sin)^m (A+B sin+C sin^2).m"

Problem 2: Result more than twice size of optimal antiderivative.

$$\int \text{Sin}[e+f x]^5 \left(6-7\,\text{Sin}[e+f x]^2\right) dx$$

Optimal (type 3, 18 leaves, 1 step):

$$\frac{\text{Cos}[e+f x]\,\text{Sin}[e+f x]^6}{f}$$

Result (type 3, 59 leaves):

$$\frac{5\,\text{Cos}[e+f x]}{64 f}-\frac{9\,\text{Cos}\left[3\,(e+f x)\right]}{64 f}+\frac{5\,\text{Cos}\left[5\,(e+f x)\right]}{64 f}-\frac{\text{Cos}\left[7\,(e+f x)\right]}{64 f}$$

Problem 3: Result more than twice size of optimal antiderivative.

$$\int \text{Sin}[e+f x]^4 \left(5-6\,\text{Sin}[e+f x]^2\right) dx$$

Optimal (type 3, 18 leaves, 1 step):

$$\frac{\text{Cos}[e+f x]\,\text{Sin}[e+f x]^5}{f}$$

Result (type 3, 39 leaves):

$$\frac{24\,e+5\,\text{Sin}\left[2\,(e+f x)\right]-4\,\text{Sin}\left[4\,(e+f x)\right]+\text{Sin}\left[6\,(e+f x)\right]}{32 f}$$

Problem 4: Result more than twice size of optimal antiderivative.

$$\int \text{Sin}[e+f x]^3 \left(4-5\,\text{Sin}[e+f x]^2\right) dx$$

Optimal (type 3, 18 leaves, 1 step):

$$\frac{\text{Cos}[e+f x]\,\text{Sin}[e+f x]^4}{f}$$

Result (type 3, 44 leaves):

$$\frac{\text{Cos}[e+f x]}{8 f}-\frac{3 \text{Cos}[3 (e+f x)]}{16 f}+\frac{\text{Cos}[5 (e+f x)]}{16 f}$$

Problem 6: Result more than twice size of optimal antiderivative.

$$\int \text{Sin}[e+f x] \left(2-3 \text{Sin}[e+f x]^2\right) dx$$

Optimal (type 3, 18 leaves, 1 step):

$$\frac{\text{Cos}[e+f x] \text{Sin}[e+f x]^2}{f}$$

Result (type 3, 51 leaves):

$$-\frac{2 \text{Cos}[e] \text{Cos}[f x]}{f}+\frac{9 \text{Cos}[e+f x]}{4 f}-\frac{\text{Cos}[3 (e+f x)]}{4 f}+\frac{2 \text{Sin}[e] \text{Sin}[f x]}{f}$$

Problem 7: Result more than twice size of optimal antiderivative.

$$\int \left(1-2 \text{Sin}[e+f x]^2\right) dx$$

Optimal (type 3, 16 leaves, 3 steps):

$$\frac{\text{Cos}[e+f x] \text{Sin}[e+f x]}{f}$$

Result (type 3, 33 leaves):

$$\frac{\text{Cos}[2 f x] \text{Sin}[2 e]}{2 f}+\frac{\text{Cos}[2 e] \text{Sin}[2 f x]}{2 f}$$

Problem 8: Result more than twice size of optimal antiderivative.

$$\int -\text{Sin}[e+f x] dx$$

Optimal (type 3, 10 leaves, 1 step):

$$\frac{\text{Cos}[e+f x]}{f}$$

Result (type 3, 22 leaves):

$$\frac{\text{Cos}[e] \text{Cos}[f x]}{f}-\frac{\text{Sin}[e] \text{Sin}[f x]}{f}$$

Problem 10: Result more than twice size of optimal antiderivative.

$$\int \text{Csc}[e+f x]^3 \left(-2+\text{Sin}[e+f x]^2\right) dx$$

Optimal (type 3, 16 leaves, 1 step):

$$\frac{\text{Cot}[e+f x]\,\text{Csc}[e+f x]}{f}$$

Result (type 3, 107 leaves):

$$\frac{\text{Csc}\left[\frac{1}{2}(e+f x)\right]^2}{4 f}-\frac{\text{Log}\left[\text{Cos}\left[\frac{e}{2}+\frac{f x}{2}\right]\right]}{f}+\frac{\text{Log}\left[\text{Cos}\left[\frac{1}{2}(e+f x)\right]\right]}{f}+\frac{\text{Log}\left[\text{Sin}\left[\frac{e}{2}+\frac{f x}{2}\right]\right]}{f}-\frac{\text{Log}\left[\text{Sin}\left[\frac{1}{2}(e+f x)\right]\right]}{f}-\frac{\text{Sec}\left[\frac{1}{2}(e+f x)\right]^2}{4 f}$$

Problem 12: Result more than twice size of optimal antiderivative.

$$\int \text{Csc}[e+f x]^5\left(-4+3\,\text{Sin}[e+f x]^2\right)dx$$

Optimal (type 3, 18 leaves, 1 step):

$$\frac{\text{Cot}[e+f x]\,\text{Csc}[e+f x]^3}{f}$$

Result (type 3, 39 leaves):

$$\frac{\text{Csc}\left[\frac{1}{2}(e+f x)\right]^4}{16 f}-\frac{\text{Sec}\left[\frac{1}{2}(e+f x)\right]^4}{16 f}$$

Problem 13: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int (a+a\,\text{Sin}[e+f x])^m\left(A+C\,\text{Sin}[e+f x]^2\right)dx$$

Optimal (type 5, 171 leaves, 4 steps):

$$\frac{C\,\text{Cos}[e+f x]\,(a+a\,\text{Sin}[e+f x])^m}{f\left(2+3 m+m^2\right)}-\frac{1}{f(1+m)(2+m)}2^{\frac{1}{2}+m}\left(C\left(1+m+m^2\right)+A\left(2+3 m+m^2\right)\right)$$
$$\text{Cos}[e+f x]\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\text{Sin}[e+f x])\right]$$
$$(1+\text{Sin}[e+f x])^{-\frac{1}{2}-m}(a+a\,\text{Sin}[e+f x])^m-\frac{C\,\text{Cos}[e+f x]\,(a+a\,\text{Sin}[e+f x])^{1+m}}{a f(2+m)}$$

Result (type 5, 398 leaves):

$$-\frac{1}{2 f}\left(a\left(1+\text{Sin}[e+f x]\right)\right)^m \left(-\frac{1}{-4+m^2} i\, 2^{-1-2 m}\, C\, e^{-2 i (e+f x)} \left(1+i\, e^{-i (e+f x)}\right)^{-2 m} \left(e^{-\frac{1}{4} i (2 e+\pi+2 f x)} \left(i+e^{i (e+f x)}\right)\right)^{2 m} \left(e^{4 i (e+f x)} (-2+m)\, \text{Hypergeometric2F1}\left[-2-m,-2 m,-1-m,-i\, e^{-i (e+f x)}\right]+(2+m)\, \text{Hypergeometric2F1}\left[2-m,-2 m,3-m,-i\, e^{-i (e+f x)}\right]\right)+\left(4 \sqrt{2}\, A\, \text{Cos}\left[\frac{1}{4}(2 e-\pi+2 f x)\right]^{1+2 m} \text{Hypergeometric2F1}\left[\frac{1}{2},\frac{1}{2}+m,\frac{3}{2}+m,\text{Sin}\left[\frac{1}{4}(2 e+\pi+2 f x)\right]^2\right] \text{Sin}\left[\frac{1}{4}(2 e-\pi+2 f x)\right]\right)\Big/\left((1+2 m) \sqrt{1-\text{Sin}[e+f x]}\right)+\left(2 \sqrt{2}\, C\, \text{Cos}\left[\frac{1}{4}(2 e-\pi+2 f x)\right]^{1+2 m} \text{Hypergeometric2F1}\left[\frac{1}{2},\frac{1}{2}+m,\frac{3}{2}+m,\text{Sin}\left[\frac{1}{4}(2 e+\pi+2 f x)\right]^2\right] \text{Sin}\left[\frac{1}{4}(2 e-\pi+2 f x)\right]\right)\Big/\left((1+2 m) \sqrt{1-\text{Sin}[e+f x]}\right)\right) \text{Sin}\left[\frac{1}{4}(2 e+\pi+2 f x)\right]^{-2 m}$$

Problem 14: Unable to integrate problem.

$$\int (a+b\,\text{Sin}[e+f x])^m \left(A-A\,\text{Sin}[e+f x]^2\right) dx$$

Optimal (type 6, 211 leaves, 7 steps):

$$\left(4 \sqrt{2}\, A\, \text{AppellF1}\left[\frac{1}{2},-\frac{3}{2},-m,\frac{3}{2},\frac{1}{2}(1-\text{Sin}[e+f x]),\frac{b(1-\text{Sin}[e+f x])}{a+b}\right] \text{Cos}[e+f x]\,(a+b\,\text{Sin}[e+f x])^m \left(\frac{a+b\,\text{Sin}[e+f x]}{a+b}\right)^{-m}\right)\Big/\left(f \sqrt{1+\text{Sin}[e+f x]}\right)-\left(4 \sqrt{2}\, A\, \text{AppellF1}\left[\frac{1}{2},-\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}(1-\text{Sin}[e+f x]),\frac{b(1-\text{Sin}[e+f x])}{a+b}\right] \text{Cos}[e+f x]\,(a+b\,\text{Sin}[e+f x])^m \left(\frac{a+b\,\text{Sin}[e+f x]}{a+b}\right)^{-m}\right)\Big/\left(f \sqrt{1+\text{Sin}[e+f x]}\right)$$

Result (type 8, 28 leaves):

$$\int (a+b\,\text{Sin}[e+f x])^m \left(A-A\,\text{Sin}[e+f x]^2\right) dx$$

Problem 15: Unable to integrate problem.

$$\int (a+b\,\text{Sin}[e+f x])^m \left(A+C\,\text{Sin}[e+f x]^2\right) dx$$

Optimal (type 6, 286 leaves, 8 steps):

$$-\frac{C\,\mathrm{Cos}[e+f\,x]\,(a+b\,\mathrm{Sin}[e+f\,x])^{1+m}}{b\,f\,(2+m)}+\left(\sqrt{2}\,a\,(a+b)\,C\,\mathrm{AppellF1}\left[\frac{1}{2},\frac{1}{2},-1-m,\frac{3}{2},\frac{1}{2}(1-\mathrm{Sin}[e+f\,x]),\frac{b\,(1-\mathrm{Sin}[e+f\,x])}{a+b}\right]\right.$$
$$\left.\mathrm{Cos}[e+f\,x]\,(a+b\,\mathrm{Sin}[e+f\,x])^m\left(\frac{a+b\,\mathrm{Sin}[e+f\,x]}{a+b}\right)^{-m}\right)\Big/$$
$$\left(b^2\,f\,(2+m)\,\sqrt{1+\mathrm{Sin}[e+f\,x]}\right)-\left(\sqrt{2}\,\left(a^2\,C+b^2\,(C\,(1+m)+A\,(2+m))\right)\right.$$
$$\mathrm{AppellF1}\left[\frac{1}{2},\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}(1-\mathrm{Sin}[e+f\,x]),\frac{b\,(1-\mathrm{Sin}[e+f\,x])}{a+b}\right]\mathrm{Cos}[e+f\,x]$$
$$\left.(a+b\,\mathrm{Sin}[e+f\,x])^m\left(\frac{a+b\,\mathrm{Sin}[e+f\,x]}{a+b}\right)^{-m}\right)\Big/\left(b^2\,f\,(2+m)\,\sqrt{1+\mathrm{Sin}[e+f\,x]}\right)$$

Result (type 8, 27 leaves):

$$\int(a+b\,\mathrm{Sin}[e+f\,x])^m\,\left(A+C\,\mathrm{Sin}[e+f\,x]^2\right)\,dx$$

Problem 17: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int(a+a\,\mathrm{Sin}[e+f\,x])^m\,\left(A+B\,\mathrm{Sin}[e+f\,x]+C\,\mathrm{Sin}[e+f\,x]^2\right)\,dx$$

Optimal (type 5, 184 leaves, 4 steps):

$$\frac{(C-B\,(2+m))\,\mathrm{Cos}[e+f\,x]\,(a+a\,\mathrm{Sin}[e+f\,x])^m}{f\,(1+m)\,(2+m)}-$$
$$\frac{1}{f\,(1+m)\,(2+m)}2^{\frac{1}{2}+m}\,\left(B\,m\,(2+m)+C\,\left(1+m+m^2\right)+A\,\left(2+3\,m+m^2\right)\right)$$
$$\mathrm{Cos}[e+f\,x]\,\mathrm{Hypergeometric2F1}\left[\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\mathrm{Sin}[e+f\,x])\right]$$
$$(1+\mathrm{Sin}[e+f\,x])^{-\frac{1}{2}-m}\,(a+a\,\mathrm{Sin}[e+f\,x])^m-\frac{C\,\mathrm{Cos}[e+f\,x]\,(a+a\,\mathrm{Sin}[e+f\,x])^{1+m}}{a\,f\,(2+m)}$$

Result (type 5, 558 leaves):

$$-\frac{1}{2\,f}\left(a\,\left(1+\text{Sin}[e+f\,x]\right)\right)^{m}$$
$$\left(\frac{1}{-1+m^{2}}4^{-m}\,B\,e^{-i\,(e+f\,x)}\,\left(1+i\,e^{-i\,(e+f\,x)}\right)^{-2\,m}\,\left(e^{-\frac{1}{4}\,i\,(2\,e+\pi+2\,f\,x)}\,\left(i+e^{i\,(e+f\,x)}\right)\right)^{2\,m}\right.$$
$$\left(e^{2\,i\,(e+f\,x)}\,\left(-1+m\right)\,\text{Hypergeometric2F1}\left[-1-m,\,-2\,m,\,-m,\,-i\,e^{-i\,(e+f\,x)}\right]-\right.$$
$$\left.\left(1+m\right)\,\text{Hypergeometric2F1}\left[1-m,\,-2\,m,\,2-m,\,-i\,e^{-i\,(e+f\,x)}\right]\right)-\frac{1}{-4+m^{2}}$$
$$i\,2^{-1-2\,m}\,C\,e^{-2\,i\,(e+f\,x)}\,\left(1+i\,e^{-i\,(e+f\,x)}\right)^{-2\,m}\,\left(e^{-\frac{1}{4}\,i\,(2\,e+\pi+2\,f\,x)}\,\left(i+e^{i\,(e+f\,x)}\right)\right)^{2\,m}$$
$$\left(e^{4\,i\,(e+f\,x)}\,\left(-2+m\right)\,\text{Hypergeometric2F1}\left[-2-m,\,-2\,m,\,-1-m,\,-i\,e^{-i\,(e+f\,x)}\right]+\right.$$
$$\left.\left(2+m\right)\,\text{Hypergeometric2F1}\left[2-m,\,-2\,m,\,3-m,\,-i\,e^{-i\,(e+f\,x)}\right]\right)+$$
$$\left(4\,\sqrt{2}\,A\,\text{Cos}\left[\frac{1}{4}\,\left(2\,e-\pi+2\,f\,x\right)\right]^{1+2\,m}\,\text{Hypergeometric2F1}\left[\frac{1}{2},\,\frac{1}{2}+m,\,\frac{3}{2}+m,\right.\right.$$
$$\left.\left.\text{Sin}\left[\frac{1}{4}\,\left(2\,e+\pi+2\,f\,x\right)\right]^{2}\right]\,\text{Sin}\left[\frac{1}{4}\,\left(2\,e-\pi+2\,f\,x\right)\right]\right)\Big/\left(\left(1+2\,m\right)\,\sqrt{1-\text{Sin}[e+f\,x]}\right)+$$
$$\left(2\,\sqrt{2}\,C\,\text{Cos}\left[\frac{1}{4}\,\left(2\,e-\pi+2\,f\,x\right)\right]^{1+2\,m}\,\text{Hypergeometric2F1}\left[\frac{1}{2},\,\frac{1}{2}+m,\,\frac{3}{2}+m,\right.\right.$$
$$\left.\left.\text{Sin}\left[\frac{1}{4}\,\left(2\,e+\pi+2\,f\,x\right)\right]^{2}\right]\,\text{Sin}\left[\frac{1}{4}\,\left(2\,e-\pi+2\,f\,x\right)\right]\right)\Big/$$
$$\left.\left(\left(1+2\,m\right)\,\sqrt{1-\text{Sin}[e+f\,x]}\right)\right)\,\text{Sin}\left[\frac{1}{4}\,\left(2\,e+\pi+2\,f\,x\right)\right]^{-2\,m}$$

Problem 18: Unable to integrate problem.

$$\int\left(a+b\,\text{Sin}[e+f\,x]\right)^{m}\,\left(A+\left(A+C\right)\,\text{Sin}[e+f\,x]+C\,\text{Sin}[e+f\,x]^{2}\right)\,dx$$

Optimal (type 6, 215 leaves, 7 steps):

$$-\left(\left(4\,\sqrt{2}\,C\,\text{AppellF1}\left[\frac{1}{2},\,-\frac{3}{2},\,-m,\,\frac{3}{2},\,\frac{1}{2}\,\left(1-\text{Sin}[e+f\,x]\right),\,\frac{b\,\left(1-\text{Sin}[e+f\,x]\right)}{a+b}\right]\right.\right.$$
$$\left.\left.\text{Cos}[e+f\,x]\,\left(a+b\,\text{Sin}[e+f\,x]\right)^{m}\,\left(\frac{a+b\,\text{Sin}[e+f\,x]}{a+b}\right)^{-m}\right)\Big/\left(f\,\sqrt{1+\text{Sin}[e+f\,x]}\right)\right)-$$
$$\left(2\,\sqrt{2}\,\left(A-C\right)\,\text{AppellF1}\left[\frac{1}{2},\,-\frac{1}{2},\,-m,\,\frac{3}{2},\,\frac{1}{2}\,\left(1-\text{Sin}[e+f\,x]\right),\,\frac{b\,\left(1-\text{Sin}[e+f\,x]\right)}{a+b}\right]\right.$$
$$\left.\text{Cos}[e+f\,x]\,\left(a+b\,\text{Sin}[e+f\,x]\right)^{m}\,\left(\frac{a+b\,\text{Sin}[e+f\,x]}{a+b}\right)^{-m}\right)\Big/\left(f\,\sqrt{1+\text{Sin}[e+f\,x]}\right)$$

Result (type 8, 37 leaves):

$$\int\left(a+b\,\text{Sin}[e+f\,x]\right)^{m}\,\left(A+\left(A+C\right)\,\text{Sin}[e+f\,x]+C\,\text{Sin}[e+f\,x]^{2}\right)\,dx$$

Problem 19: Unable to integrate problem.

$$\int\left(a+b\,\text{Sin}[e+f\,x]\right)^{m}\,\left(A+B\,\text{Sin}[e+f\,x]+C\,\text{Sin}[e+f\,x]^{2}\right)\,dx$$

Optimal (type 6, 304 leaves, 8 steps):

$$-\frac{C\,\mathrm{Cos}[e+f\,x]\,\left(a+b\,\mathrm{Sin}[e+f\,x]\right)^{1+m}}{b\,f\,(2+m)}+$$

$$\left(\sqrt{2}\,(a+b)\,\left(a\,C-b\,B\,(2+m)\right)\,\mathrm{AppellF1}\left[\frac{1}{2},\frac{1}{2},-1-m,\frac{3}{2},\frac{1}{2}\left(1-\mathrm{Sin}[e+f\,x]\right),\right.\right.$$

$$\left.\left.\frac{b\,\left(1-\mathrm{Sin}[e+f\,x]\right)}{a+b}\right]\,\mathrm{Cos}[e+f\,x]\,\left(a+b\,\mathrm{Sin}[e+f\,x]\right)^{m}\,\left(\frac{a+b\,\mathrm{Sin}[e+f\,x]}{a+b}\right)^{-m}\right)\Big/$$

$$\left(b^2\,f\,(2+m)\,\sqrt{1+\mathrm{Sin}[e+f\,x]}\right)-\left(\sqrt{2}\,\left(a^2\,C+b^2\,C\,(1+m)+A\,b^2\,(2+m)-a\,b\,B\,(2+m)\right)\right.$$

$$\mathrm{AppellF1}\left[\frac{1}{2},\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}\left(1-\mathrm{Sin}[e+f\,x]\right),\frac{b\,\left(1-\mathrm{Sin}[e+f\,x]\right)}{a+b}\right]\,\mathrm{Cos}[e+f\,x]$$

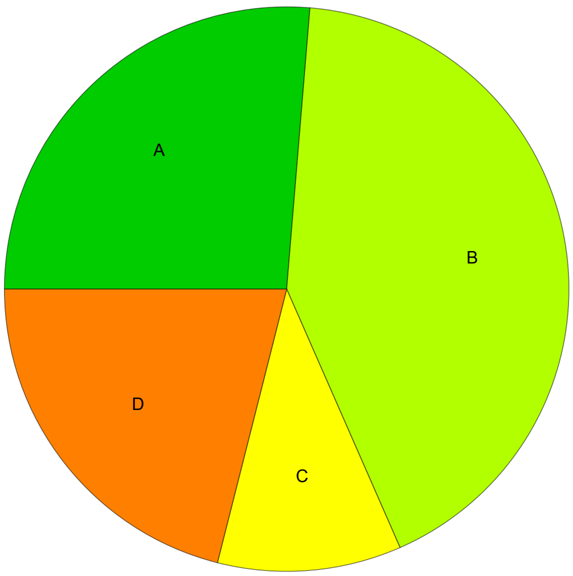
$$\left.\left(a+b\,\mathrm{Sin}[e+f\,x]\right)^{m}\,\left(\frac{a+b\,\mathrm{Sin}[e+f\,x]}{a+b}\right)^{-m}\right)\Big/\left(b^2\,f\,(2+m)\,\sqrt{1+\mathrm{Sin}[e+f\,x]}\right)$$

Result (type 8, 35 leaves):

$$\int\left(a+b\,\mathrm{Sin}[e+f\,x]\right)^{m}\,\left(A+B\,\mathrm{Sin}[e+f\,x]+C\,\mathrm{Sin}[e+f\,x]^{2}\right)\,dx$$

Summary of Integration Test Results

19 integration problems



A - 5 optimal antiderivatives

B - 8 more than twice size of optimal antiderivatives

C - 2 unnecessarily complex antiderivatives

D - 4 unable to integrate problems

E - 0 integration timeouts