Mathematica 11.3 Integration Test Results

Test results for the 19 problems in "4.1.9 trig^m (a+b sin^n+c sin^(2 n))^p.m"

Problem 1: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{Sin}[x]^4}{a+b\,\text{Sin}[x]+c\,\text{Sin}[x]^2}\,dx$$

Optimal (type 3, 323 leaves, 12 steps):

$$\frac{x}{2c}+\frac{\left(b^2-a c\right) x}{c^3}-\left(\sqrt{2}\left(b^3-2 a b c-\frac{b^4-4 a b^2 c+2 a^2 c^2}{\sqrt{b^2-4 a c}}\right) \text{ArcTan}\left[\frac{2 c+\left(b-\sqrt{b^2-4 a c}\right) \text{Tan}\left[\frac{x}{2}\right]}{\sqrt{2} \sqrt{b^2-2 c (a+c)-b \sqrt{b^2-4 a c}}}\right]\right) \Bigg/ \left(c^3 \sqrt{b^2-2 c (a+c)-b \sqrt{b^2-4 a c}}\right)-\left(\sqrt{2}\left(b^3-2 a b c+\frac{b^4-4 a b^2 c+2 a^2 c^2}{\sqrt{b^2-4 a c}}\right) \text{ArcTan}\left[\frac{2 c+\left(b+\sqrt{b^2-4 a c}\right) \text{Tan}\left[\frac{x}{2}\right]}{\sqrt{2} \sqrt{b^2-2 c (a+c)+b \sqrt{b^2-4 a c}}}\right]\right) \Bigg/ \left(c^3 \sqrt{b^2-2 c (a+c)+b \sqrt{b^2-4 a c}}\right)+\frac{b\,\text{Cos}[x]}{c^2}-\frac{\text{Cos}[x]\,\text{Sin}[x]}{2 c}$$

Result (type 3, 410 leaves):

$$\frac{1}{4 c^3}\left(4 b^2 x+2 c (-2 a+c) x-\left(4 \left(i b^4-4 i a b^2 c+2 i a^2 c^2+b^3 \sqrt{-b^2+4 a c}-2 a b c \sqrt{-b^2+4 a c}\right) \text{ArcTan}\left[\frac{2 c+\left(b-i \sqrt{-b^2+4 a c}\right) \text{Tan}\left[\frac{x}{2}\right]}{\sqrt{2} \sqrt{b^2-2 c (a+c)-i b \sqrt{-b^2+4 a c}}}\right]\right)\Big/\left(\sqrt{-\frac{b^2}{2}+2 a c} \sqrt{b^2-2 c (a+c)-i b \sqrt{-b^2+4 a c}}\right)-\left(4 \left(-i b^4+4 i a b^2 c-2 i a^2 c^2+b^3 \sqrt{-b^2+4 a c}-2 a b c \sqrt{-b^2+4 a c}\right) \text{ArcTan}\left[\frac{2 c+\left(b+i \sqrt{-b^2+4 a c}\right) \text{Tan}\left[\frac{x}{2}\right]}{\sqrt{2} \sqrt{b^2-2 c (a+c)+i b \sqrt{-b^2+4 a c}}}\right]\right)\Big/\left(\sqrt{-\frac{b^2}{2}+2 a c} \sqrt{b^2-2 c (a+c)+i b \sqrt{-b^2+4 a c}}\right)+4 b c \text{Cos}[x]-c^2 \text{Sin}[2 x]\right)$$

Problem 2: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{Sin}[x]^3}{a+b \text{Sin}[x]+c \text{Sin}[x]^2}\, dx$$

Optimal (type 3, 298 leaves, 10 steps):

$$-\frac{b x}{c^2}+\left(\sqrt{2} b \left(b-\frac{a c}{b}-\frac{b^2}{\sqrt{b^2-4 a c}}+\frac{3 a c}{\sqrt{b^2-4 a c}}\right) \text{ArcTan}\left[\frac{2 c+\left(b-\sqrt{b^2-4 a c}\right) \text{Tan}\left[\frac{x}{2}\right]}{\sqrt{2} \sqrt{b^2-2 c (a+c)-b \sqrt{b^2-4 a c}}}\right]\right)\Big/\left(c^2 \sqrt{b^2-2 c (a+c)-b \sqrt{b^2-4 a c}}\right)+\left(\sqrt{2} b \left(b-\frac{a c}{b}+\frac{b^2}{\sqrt{b^2-4 a c}}-\frac{3 a c}{\sqrt{b^2-4 a c}}\right) \text{ArcTan}\left[\frac{2 c+\left(b+\sqrt{b^2-4 a c}\right) \text{Tan}\left[\frac{x}{2}\right]}{\sqrt{2} \sqrt{b^2-2 c (a+c)+b \sqrt{b^2-4 a c}}}\right]\right)\Big/\left(c^2 \sqrt{b^2-2 c (a+c)+b \sqrt{b^2-4 a c}}\right)-\frac{\text{Cos}[x]}{c}$$

Result (type 3, 358 leaves):

$$\frac{1}{c^2}\left(-b x+\left(\left(i b^3-3 i a b c+b^2\sqrt{-b^2+4 a c}-a c\sqrt{-b^2+4 a c}\right)\text{ArcTan}\left[\frac{2 c+\left(b-i\sqrt{-b^2+4 a c}\right)\text{Tan}\left[\frac{x}{2}\right]}{\sqrt{2}\sqrt{b^2-2 c (a+c)-i b\sqrt{-b^2+4 a c}}}\right]\right)\Big/\left(\sqrt{-\frac{b^2}{2}+2 a c}\sqrt{b^2-2 c (a+c)-i b\sqrt{-b^2+4 a c}}\right)+\left(\left(-i b^3+3 i a b c+b^2\sqrt{-b^2+4 a c}-a c\sqrt{-b^2+4 a c}\right)\text{ArcTan}\left[\frac{2 c+\left(b+i\sqrt{-b^2+4 a c}\right)\text{Tan}\left[\frac{x}{2}\right]}{\sqrt{2}\sqrt{b^2-2 c (a+c)+i b\sqrt{-b^2+4 a c}}}\right]\right)\Big/\left(\sqrt{-\frac{b^2}{2}+2 a c}\sqrt{b^2-2 c (a+c)+i b\sqrt{-b^2+4 a c}}\right)-c\,\text{Cos}[x]\right)$$

Problem 3: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\text{Sin}[x]^2}{a+b\,\text{Sin}[x]+c\,\text{Sin}[x]^2}\,dx$$

Optimal (type 3, 253 leaves, 9 steps):

$$\frac{x}{c}-\frac{\sqrt{2}\left(b-\frac{b^2-2 a c}{\sqrt{b^2-4 a c}}\right)\text{ArcTan}\left[\frac{2 c+\left(b-\sqrt{b^2-4 a c}\right)\text{Tan}\left[\frac{x}{2}\right]}{\sqrt{2}\sqrt{b^2-2 c (a+c)-b\sqrt{b^2-4 a c}}}\right]}{c\sqrt{b^2-2 c (a+c)-b\sqrt{b^2-4 a c}}}-\frac{\sqrt{2}\left(b+\frac{b^2-2 a c}{\sqrt{b^2-4 a c}}\right)\text{ArcTan}\left[\frac{2 c+\left(b+\sqrt{b^2-4 a c}\right)\text{Tan}\left[\frac{x}{2}\right]}{\sqrt{2}\sqrt{b^2-2 c (a+c)+b\sqrt{b^2-4 a c}}}\right]}{c\sqrt{b^2-2 c (a+c)+b\sqrt{b^2-4 a c}}}$$

Result (type 3, 310 leaves):

$$\frac{1}{c}\left(x-\frac{\left(\mathrm{i}\, b^2-2\,\mathrm{i}\, a\, c+b\sqrt{-b^2+4\,a\,c}\right)\mathrm{ArcTan}\left[\frac{2\,c+\left(b-\mathrm{i}\sqrt{-b^2+4\,a\,c}\right)\mathrm{Tan}\left[\frac{x}{2}\right]}{\sqrt{2}\sqrt{b^2-2\,c\,(a+c)-\mathrm{i}\, b\sqrt{-b^2+4\,a\,c}}}\right]}{\sqrt{-\frac{b^2}{2}+2\,a\,c}\ \sqrt{b^2-2\,c\,(a+c)-\mathrm{i}\, b\sqrt{-b^2+4\,a\,c}}}-\left(\left(-\mathrm{i}\, b^2+2\,\mathrm{i}\, a\, c+b\sqrt{-b^2+4\,a\,c}\right)\mathrm{ArcTan}\left[\frac{2\,c+\left(b+\mathrm{i}\sqrt{-b^2+4\,a\,c}\right)\mathrm{Tan}\left[\frac{x}{2}\right]}{\sqrt{2}\sqrt{b^2-2\,c\,(a+c)+\mathrm{i}\, b\sqrt{-b^2+4\,a\,c}}}\right]\right)\Big/\left(\sqrt{-\frac{b^2}{2}+2\,a\,c}\ \sqrt{b^2-2\,c\,(a+c)+\mathrm{i}\, b\sqrt{-b^2+4\,a\,c}}\right)\right)$$

Problem 4: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\mathrm{Sin}[x]}{a+b\,\mathrm{Sin}[x]+c\,\mathrm{Sin}[x]^2}\,dx$$

Optimal (type 3, 226 leaves, 8 steps):

$$\frac{\sqrt{2}\left(1-\frac{b}{\sqrt{b^2-4\,a\,c}}\right)\mathrm{ArcTan}\left[\frac{2\,c+\left(b-\sqrt{b^2-4\,a\,c}\right)\mathrm{Tan}\left[\frac{x}{2}\right]}{\sqrt{2}\sqrt{b^2-2\,c\,(a+c)-b\sqrt{b^2-4\,a\,c}}}\right]}{\sqrt{b^2-2\,c\,(a+c)-b\sqrt{b^2-4\,a\,c}}}+\frac{\sqrt{2}\left(1+\frac{b}{\sqrt{b^2-4\,a\,c}}\right)\mathrm{ArcTan}\left[\frac{2\,c+\left(b+\sqrt{b^2-4\,a\,c}\right)\mathrm{Tan}\left[\frac{x}{2}\right]}{\sqrt{2}\sqrt{b^2-2\,c\,(a+c)+b\sqrt{b^2-4\,a\,c}}}\right]}{\sqrt{b^2-2\,c\,(a+c)+b\sqrt{b^2-4\,a\,c}}}$$

Result (type 3, 268 leaves):

$$\frac{1}{\sqrt{-\frac{b^2}{2}+2\,a\,c}}\left(\frac{\left(i\,b+\sqrt{-b^2+4\,a\,c}\right)\,\text{ArcTan}\left[\frac{2\,c+\left(b-i\,\sqrt{-b^2+4\,a\,c}\right)\,\text{Tan}\left[\frac{x}{2}\right]}{\sqrt{2}\,\sqrt{b^2-2\,c\,(a+c)-i\,b\,\sqrt{-b^2+4\,a\,c}}}\right]}{\sqrt{b^2-2\,c\,(a+c)-i\,b\,\sqrt{-b^2+4\,a\,c}}}+\frac{\left(-i\,b+\sqrt{-b^2+4\,a\,c}\right)\,\text{ArcTan}\left[\frac{2\,c+\left(b+i\,\sqrt{-b^2+4\,a\,c}\right)\,\text{Tan}\left[\frac{x}{2}\right]}{\sqrt{2}\,\sqrt{b^2-2\,c\,(a+c)+i\,b\,\sqrt{-b^2+4\,a\,c}}}\right]}{\sqrt{b^2-2\,c\,(a+c)+i\,b\,\sqrt{-b^2+4\,a\,c}}}\right)$$

Problem 5: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{a+b\,\text{Sin}[x]+c\,\text{Sin}[x]^2}\,dx$$

Optimal (type 3, 221 leaves, 7 steps):

$$\frac{2\,\sqrt{2}\,c\,\text{ArcTan}\left[\frac{2\,c+\left(b-\sqrt{b^2-4\,a\,c}\right)\,\text{Tan}\left[\frac{x}{2}\right]}{\sqrt{2}\,\sqrt{b^2-2\,c\,(a+c)-b\,\sqrt{b^2-4\,a\,c}}}\right]}{\sqrt{b^2-4\,a\,c}\,\sqrt{b^2-2\,c\,(a+c)-b\,\sqrt{b^2-4\,a\,c}}}-\frac{2\,\sqrt{2}\,c\,\text{ArcTan}\left[\frac{2\,c+\left(b+\sqrt{b^2-4\,a\,c}\right)\,\text{Tan}\left[\frac{x}{2}\right]}{\sqrt{2}\,\sqrt{b^2-2\,c\,(a+c)+b\,\sqrt{b^2-4\,a\,c}}}\right]}{\sqrt{b^2-4\,a\,c}\,\sqrt{b^2-2\,c\,(a+c)+b\,\sqrt{b^2-4\,a\,c}}}$$

Result (type 3, 233 leaves):

$$-\frac{1}{\sqrt{-\frac{b^2}{2}+2\,a\,c}}\,2\,i\,c\left(\frac{\text{ArcTan}\left[\frac{2\,c+\left(b-i\,\sqrt{-b^2+4\,a\,c}\right)\,\text{Tan}\left[\frac{x}{2}\right]}{\sqrt{2}\,\sqrt{b^2-2\,c\,(a+c)-i\,b\,\sqrt{-b^2+4\,a\,c}}}\right]}{\sqrt{b^2-2\,c\,(a+c)-i\,b\,\sqrt{-b^2+4\,a\,c}}}-\frac{\text{ArcTan}\left[\frac{2\,c+\left(b+i\,\sqrt{-b^2+4\,a\,c}\right)\,\text{Tan}\left[\frac{x}{2}\right]}{\sqrt{2}\,\sqrt{b^2-2\,c\,(a+c)+i\,b\,\sqrt{-b^2+4\,a\,c}}}\right]}{\sqrt{b^2-2\,c\,(a+c)+i\,b\,\sqrt{-b^2+4\,a\,c}}}\right)$$

Problem 6: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{Csc}[x]}{a+b\,\text{Sin}[x]+c\,\text{Sin}[x]^2}\,dx$$

Optimal (type 3, 244 leaves, 10 steps):

$$-\frac{\sqrt{2}\,c\left(1+\frac{b}{\sqrt{b^2-4\,a\,c}}\right)\text{ArcTan}\left[\frac{2\,c+\left(b-\sqrt{b^2-4\,a\,c}\right)\text{Tan}\left[\frac{x}{2}\right]}{\sqrt{2}\,\sqrt{b^2-2\,c\,(a+c)-b\,\sqrt{b^2-4\,a\,c}}}\right]}{a\,\sqrt{b^2-2\,c\,(a+c)-b\,\sqrt{b^2-4\,a\,c}}}-\frac{\sqrt{2}\,c\left(1-\frac{b}{\sqrt{b^2-4\,a\,c}}\right)\text{ArcTan}\left[\frac{2\,c+\left(b+\sqrt{b^2-4\,a\,c}\right)\text{Tan}\left[\frac{x}{2}\right]}{\sqrt{2}\,\sqrt{b^2-2\,c\,(a+c)+b\,\sqrt{b^2-4\,a\,c}}}\right]}{a\,\sqrt{b^2-2\,c\,(a+c)+b\,\sqrt{b^2-4\,a\,c}}}-\frac{\text{ArcTanh}[\text{Cos}[x]]}{a}$$

Result (type 3, 306 leaves):

$$-\frac{1}{a}\left(\frac{c\left(-\mathrm{i}\,b+\sqrt{-b^2+4\,a\,c}\right)\text{ArcTan}\left[\frac{2\,c+\left(b-\mathrm{i}\,\sqrt{-b^2+4\,a\,c}\right)\text{Tan}\left[\frac{x}{2}\right]}{\sqrt{2}\,\sqrt{b^2-2\,c\,(a+c)-\mathrm{i}\,b\,\sqrt{-b^2+4\,a\,c}}}\right]}{\sqrt{-\frac{b^2}{2}+2\,a\,c}\,\sqrt{b^2-2\,c\,(a+c)-\mathrm{i}\,b\,\sqrt{-b^2+4\,a\,c}}}+\frac{c\left(\mathrm{i}\,b+\sqrt{-b^2+4\,a\,c}\right)\text{ArcTan}\left[\frac{2\,c+\left(b+\mathrm{i}\,\sqrt{-b^2+4\,a\,c}\right)\text{Tan}\left[\frac{x}{2}\right]}{\sqrt{2}\,\sqrt{b^2-2\,c\,(a+c)+\mathrm{i}\,b\,\sqrt{-b^2+4\,a\,c}}}\right]}{\sqrt{-\frac{b^2}{2}+2\,a\,c}\,\sqrt{b^2-2\,c\,(a+c)+\mathrm{i}\,b\,\sqrt{-b^2+4\,a\,c}}}+\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right]-\text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right]\right)$$

Problem 7: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\text{Csc}[x]^2}{a+b\,\text{Sin}[x]+c\,\text{Sin}[x]^2}\,dx$$

Optimal (type 3, 271 leaves, 12 steps):

$$\frac{\sqrt{2}\,b\,c\left(1+\frac{b^2-2\,a\,c}{b\,\sqrt{b^2-4\,a\,c}}\right)\text{ArcTan}\left[\frac{2\,c+\left(b-\sqrt{b^2-4\,a\,c}\right)\text{Tan}\left[\frac{x}{2}\right]}{\sqrt{2}\,\sqrt{b^2-2\,c\,(a+c)-b\,\sqrt{b^2-4\,a\,c}}}\right]}{a^2\,\sqrt{b^2-2\,c\,(a+c)-b\,\sqrt{b^2-4\,a\,c}}}+\frac{\sqrt{2}\,b\,c\left(1-\frac{b^2-2\,a\,c}{b\,\sqrt{b^2-4\,a\,c}}\right)\text{ArcTan}\left[\frac{2\,c+\left(b+\sqrt{b^2-4\,a\,c}\right)\text{Tan}\left[\frac{x}{2}\right]}{\sqrt{2}\,\sqrt{b^2-2\,c\,(a+c)+b\,\sqrt{b^2-4\,a\,c}}}\right]}{a^2\,\sqrt{b^2-2\,c\,(a+c)+b\,\sqrt{b^2-4\,a\,c}}}+\frac{b\,\text{ArcTanh}[\text{Cos}[x]]}{a^2}-\frac{\text{Cot}[x]}{a}$$

Result (type 3, 388 leaves):

$$\left(\mathrm{Csc}[x]^2\left(-2a-c+c\,\mathrm{Cos}[2x]-2b\,\mathrm{Sin}[x]\right)\left(-\left(\left(2c\left(-i\,b^2+2\,i\,a\,c+b\sqrt{-b^2+4ac}\right)\mathrm{ArcTan}\left[\frac{2c+\left(b-i\sqrt{-b^2+4ac}\right)\mathrm{Tan}\left[\frac{x}{2}\right]}{\sqrt{2}\sqrt{b^2-2c(a+c)-i\,b\sqrt{-b^2+4ac}}}\right]\right)\Big/\left(\sqrt{-\frac{b^2}{2}+2ac}\sqrt{b^2-2c(a+c)-i\,b\sqrt{-b^2+4ac}}\right)\right)+\left(2\,i\,c\left(-b^2+2ac+i\,b\sqrt{-b^2+4ac}\right)\mathrm{ArcTan}\left[\frac{2c+\left(b+i\sqrt{-b^2+4ac}\right)\mathrm{Tan}\left[\frac{x}{2}\right]}{\sqrt{2}\sqrt{b^2-2c(a+c)+i\,b\sqrt{-b^2+4ac}}}\right]\right)\Big/\left(\sqrt{-\frac{b^2}{2}+2ac}\sqrt{b^2-2c(a+c)+i\,b\sqrt{-b^2+4ac}}\right)+a\,\mathrm{Cot}\left[\frac{x}{2}\right]-2b\,\mathrm{Log}\left[\mathrm{Cos}\left[\frac{x}{2}\right]\right]+2b\,\mathrm{Log}\left[\mathrm{Sin}\left[\frac{x}{2}\right]\right]-a\,\mathrm{Tan}\left[\frac{x}{2}\right]\right)\right)\Big/\left(4a^2\left(c+b\,\mathrm{Csc}[x]+a\,\mathrm{Csc}[x]^2\right)\right)$$

Problem 8: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\mathrm{Csc}[x]^3}{a+b\,\mathrm{Sin}[x]+c\,\mathrm{Sin}[x]^2}\,dx$$

Optimal (type 3, 331 leaves, 14 steps):

$$-\left(\left(\sqrt{2}\,c\left(b^3-3abc+\sqrt{b^2-4ac}\left(b^2-ac\right)\right)\mathrm{ArcTan}\left[\frac{2c+\left(b-\sqrt{b^2-4ac}\right)\mathrm{Tan}\left[\frac{x}{2}\right]}{\sqrt{2}\sqrt{b^2-2c(a+c)-b\sqrt{b^2-4ac}}}\right]\right)\Big/\left(a^3\sqrt{b^2-4ac}\sqrt{b^2-2c(a+c)-b\sqrt{b^2-4ac}}\right)\right)+\left(\sqrt{2}\,c\left(b^3-3abc-\sqrt{b^2-4ac}\left(b^2-ac\right)\right)\mathrm{ArcTan}\left[\frac{2c+\left(b+\sqrt{b^2-4ac}\right)\mathrm{Tan}\left[\frac{x}{2}\right]}{\sqrt{2}\sqrt{b^2-2c(a+c)+b\sqrt{b^2-4ac}}}\right]\right)\Big/\left(a^3\sqrt{b^2-4ac}\sqrt{b^2-2c(a+c)+b\sqrt{b^2-4ac}}\right)-\frac{\mathrm{ArcTanh}[\mathrm{Cos}[x]]}{2a}-\frac{\left(b^2-ac\right)\mathrm{ArcTanh}[\mathrm{Cos}[x]]}{a^3}+\frac{b\,\mathrm{Cot}[x]}{a^2}-\frac{\mathrm{Cot}[x]\,\mathrm{Csc}[x]}{2a}$$

Result (type 3, 481 leaves):

$$\frac{1}{16\,a^3\,\left(c+b\,\mathrm{Csc}[x]+a\,\mathrm{Csc}[x]^2\right)}\mathrm{Csc}[x]^2\,\left(-2\,a-c+c\,\mathrm{Cos}[2\,x]-2\,b\,\mathrm{Sin}[x]\right)$$

$$\left(\left(8\,c\,\left(-i\,b^3+3\,i\,a\,b\,c+b^2\,\sqrt{-b^2+4\,a\,c}-a\,c\,\sqrt{-b^2+4\,a\,c}\right)\right.\right.$$

$$\left.\mathrm{ArcTan}\left[\frac{2\,c+\left(b-i\,\sqrt{-b^2+4\,a\,c}\right)\,\mathrm{Tan}\left[\frac{x}{2}\right]}{\sqrt{2}\,\sqrt{b^2-2\,c\,(a+c)-i\,b\,\sqrt{-b^2+4\,a\,c}}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{b^2}{2}+2\,a\,c}\,\sqrt{b^2-2\,c\,(a+c)-i\,b\,\sqrt{-b^2+4\,a\,c}}\right)+$$

$$\left(8\,c\,\left(i\,b^3-3\,i\,a\,b\,c+b^2\,\sqrt{-b^2+4\,a\,c}-a\,c\,\sqrt{-b^2+4\,a\,c}\right)\right.$$

$$\left.\mathrm{ArcTan}\left[\frac{2\,c+\left(b+i\,\sqrt{-b^2+4\,a\,c}\right)\,\mathrm{Tan}\left[\frac{x}{2}\right]}{\sqrt{2}\,\sqrt{b^2-2\,c\,(a+c)+i\,b\,\sqrt{-b^2+4\,a\,c}}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{b^2}{2}+2\,a\,c}\,\sqrt{b^2-2\,c\,(a+c)+i\,b\,\sqrt{-b^2+4\,a\,c}}\right)-4\,a\,b\,\mathrm{Cot}\left[\frac{x}{2}\right]+a^2\,\mathrm{Csc}\left[\frac{x}{2}\right]^2+$$

$$\left.4\,\left(a^2+2\,b^2-2\,a\,c\right)\,\mathrm{Log}\left[\mathrm{Cos}\left[\frac{x}{2}\right]\right]-4\,\left(a^2+2\,b^2-2\,a\,c\right)\,\mathrm{Log}\left[\mathrm{Sin}\left[\frac{x}{2}\right]\right]-a^2\,\mathrm{Sec}\left[\frac{x}{2}\right]^2+4\,a\,b\,\mathrm{Tan}\left[\frac{x}{2}\right]\right)$$

Problem 10: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\mathrm{Cos}[x]^2}{a+b\,\mathrm{Sin}[x]+c\,\mathrm{Sin}[x]^2}\,dx$$

Optimal (type 3, 230 leaves, 9 steps):

$$-\frac{x}{c}-\frac{1}{c\,\sqrt{b^2-4\,a\,c}}$$

$$\sqrt{2}\,\sqrt{b^2-2\,c\,(a+c)-b\,\sqrt{b^2-4\,a\,c}}\,\mathrm{ArcTan}\left[\frac{2\,c+\left(b-\sqrt{b^2-4\,a\,c}\right)\,\mathrm{Tan}\left[\frac{x}{2}\right]}{\sqrt{2}\,\sqrt{b^2-2\,c\,(a+c)-b\,\sqrt{b^2-4\,a\,c}}}\right]+$$

$$\frac{1}{c\,\sqrt{b^2-4\,a\,c}}\sqrt{2}\,\sqrt{b^2-2\,c\,(a+c)+b\,\sqrt{b^2-4\,a\,c}}\,\mathrm{ArcTan}\left[\frac{2\,c+\left(b+\sqrt{b^2-4\,a\,c}\right)\,\mathrm{Tan}\left[\frac{x}{2}\right]}{\sqrt{2}\,\sqrt{b^2-2\,c\,(a+c)+b\,\sqrt{b^2-4\,a\,c}}}\right]$$

Result (type 3, 314 leaves):

$$\frac{1}{c}\left(-x+\left(\left(i\,b^2-2\,i\,c\,(a+c)+b\,\sqrt{-b^2+4\,a\,c}\right)\,\text{ArcTan}\left[\frac{2\,c+\left(b-i\,\sqrt{-b^2+4\,a\,c}\right)\,\text{Tan}\left[\frac{x}{2}\right]}{\sqrt{2}\,\sqrt{b^2-2\,c\,(a+c)-i\,b\,\sqrt{-b^2+4\,a\,c}}}\right]\right)\Big/\left(\sqrt{-\frac{b^2}{2}+2\,a\,c}\,\sqrt{b^2-2\,c\,(a+c)-i\,b\,\sqrt{-b^2+4\,a\,c}}\right)+\left(\left(-i\,b^2+2\,i\,c\,(a+c)+b\,\sqrt{-b^2+4\,a\,c}\right)\,\text{ArcTan}\left[\frac{2\,c+\left(b+i\,\sqrt{-b^2+4\,a\,c}\right)\,\text{Tan}\left[\frac{x}{2}\right]}{\sqrt{2}\,\sqrt{b^2-2\,c\,(a+c)+i\,b\,\sqrt{-b^2+4\,a\,c}}}\right]\right)\Big/\left(\sqrt{-\frac{b^2}{2}+2\,a\,c}\,\sqrt{b^2-2\,c\,(a+c)+i\,b\,\sqrt{-b^2+4\,a\,c}}\right)\right)$$

Problem 13: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\text{Sec}[x]^2}{a+b\,\text{Sin}[x]+c\,\text{Sin}[x]^2}\,dx$$

Optimal (type 3, 324 leaves, 11 steps):

$$-\frac{\sqrt{2}\,b\,c\,\left(1+\frac{b^2-2\,c\,(a+c)}{b\,\sqrt{b^2-4\,a\,c}}\right)\,\text{ArcTan}\left[\frac{2\,c+\left(b-\sqrt{b^2-4\,a\,c}\right)\,\text{Tan}\left[\frac{x}{2}\right]}{\sqrt{2}\,\sqrt{b^2-2\,c\,(a+c)-b\,\sqrt{b^2-4\,a\,c}}}\right]}{(a-b+c)\,(a+b+c)\,\sqrt{b^2-2\,c\,(a+c)-b\,\sqrt{b^2-4\,a\,c}}}-\frac{\sqrt{2}\,b\,c\,\left(1-\frac{b^2-2\,c\,(a+c)}{b\,\sqrt{b^2-4\,a\,c}}\right)\,\text{ArcTan}\left[\frac{2\,c+\left(b+\sqrt{b^2-4\,a\,c}\right)\,\text{Tan}\left[\frac{x}{2}\right]}{\sqrt{2}\,\sqrt{b^2-2\,c\,(a+c)+b\,\sqrt{b^2-4\,a\,c}}}\right]}{(a-b+c)\,(a+b+c)\,\sqrt{b^2-2\,c\,(a+c)+b\,\sqrt{b^2-4\,a\,c}}}+\frac{\text{Cos}[x]}{2\,(a+b+c)\,(1-\text{Sin}[x])}-\frac{\text{Cos}[x]}{2\,(a-b+c)\,(1+\text{Sin}[x])}$$

Result (type 3, 407 leaves):

$$-\left(\left(c\left(-i b^2+2 i c (a+c)+b\sqrt{-b^2+4 a c}\right)\text{ArcTan}\left[\frac{2 c+\left(b-i\sqrt{-b^2+4 a c}\right)\text{Tan}\left[\frac{x}{2}\right]}{\sqrt{2}\sqrt{b^2-2 c (a+c)-i b\sqrt{-b^2+4 a c}}}\right]\right)\right/$$

$$\left(\sqrt{-\frac{b^2}{2}+2 a c}\left(a^2-b^2+2 a c+c^2\right)\sqrt{b^2-2 c (a+c)-i b\sqrt{-b^2+4 a c}}\right)\Bigg)-$$

$$\left(c\left(i b^2-2 i c (a+c)+b\sqrt{-b^2+4 a c}\right)\text{ArcTan}\left[\frac{2 c+\left(b+i\sqrt{-b^2+4 a c}\right)\text{Tan}\left[\frac{x}{2}\right]}{\sqrt{2}\sqrt{b^2-2 c (a+c)+i b\sqrt{-b^2+4 a c}}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{b^2}{2}+2 a c}\left(a^2-b^2+2 a c+c^2\right)\sqrt{b^2-2 c (a+c)+i b\sqrt{-b^2+4 a c}}\right)+$$

$$\frac{\text{Sin}\left[\frac{x}{2}\right]}{(a+b+c)\left(\text{Cos}\left[\frac{x}{2}\right]-\text{Sin}\left[\frac{x}{2}\right]\right)}+\frac{\text{Sin}\left[\frac{x}{2}\right]}{(a-b+c)\left(\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]\right)}$$

Problem 14: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Sec}[x]^3}{a+b\,\text{Sin}[x]+c\,\text{Sin}[x]^2}\,dx$$

Optimal (type 3, 206 leaves, 10 steps):

$$-\frac{\left(b^4+2 c^2 (a+c)^2-2 b^2 c (2 a+c)\right)\text{ArcTanh}\left[\frac{b+2 c\,\text{Sin}[x]}{\sqrt{b^2-4 a c}}\right]}{\sqrt{b^2-4 a c}\left(a^2-b^2+2 a c+c^2\right)^2}-$$

$$\frac{(a+2 b+3 c)\,\text{Log}[1-\text{Sin}[x]]}{4 (a+b+c)^2}+\frac{(a-2 b+3 c)\,\text{Log}[1+\text{Sin}[x]]}{4 (a-b+c)^2}+$$

$$\frac{b\left(b^2-2 c (a+c)\right)\text{Log}\left[a+b\,\text{Sin}[x]+c\,\text{Sin}[x]^2\right]}{2\left(a^2-b^2+2 a c+c^2\right)^2}-\frac{\text{Sec}[x]^2\left(b-(a+c)\,\text{Sin}[x]\right)}{2 (a-b+c)(a+b+c)}$$

Result (type 3, 481 leaves):

$$\frac{1}{4}\left(-\frac{8\,i\,b^3\,x}{(a-b+c)^2\,(a+b+c)^2}+\frac{16\,i\,b\,c\,(a+c)\,x}{(a-b+c)^2\,(a+b+c)^2}+\frac{2\,i\,(a-2\,b+3\,c)\,\text{ArcTan}[\text{Cot}[x]]}{(a-b+c)^2}-\right.$$

$$\frac{2\,i\,(a+2\,b+3\,c)\,\text{ArcTan}[\text{Cot}[x]]}{(a+b+c)^2}-\frac{4\,b^4\,\text{ArcTan}\left[\frac{\sqrt{-b^2+4\,a\,c}}{b+2\,c\,\text{Sin}[x]}\right]}{\sqrt{-b^2+4\,a\,c}\,\left(a^2-b^2+2\,a\,c+c^2\right)^2}-$$

$$\frac{8\,c^2\,(a+c)^2\,\text{ArcTan}\left[\frac{\sqrt{-b^2+4\,a\,c}}{b+2\,c\,\text{Sin}[x]}\right]}{\sqrt{-b^2+4\,a\,c}\,\left(a^2-b^2+2\,a\,c+c^2\right)^2}+\frac{8\,b^2\,c\,(2\,a+c)\,\text{ArcTan}\left[\frac{\sqrt{-b^2+4\,a\,c}}{b+2\,c\,\text{Sin}[x]}\right]}{\sqrt{-b^2+4\,a\,c}\,\left(a^2-b^2+2\,a\,c+c^2\right)^2}+$$

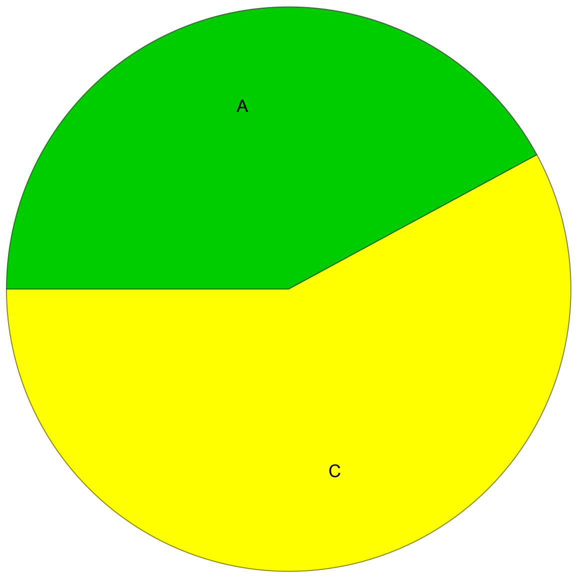
$$\frac{(a-2\,b+3\,c)\,\text{Log}\left[\left(\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]\right)^2\right]}{(a-b+c)^2}-\frac{(a+2\,b+3\,c)\,\text{Log}[1-\text{Sin}[x]]}{(a+b+c)^2}+$$

$$\frac{2\,b^3\,\text{Log}[2\,a+c-c\,\text{Cos}[2\,x]+2\,b\,\text{Sin}[x]]}{\left(a^2-b^2+2\,a\,c+c^2\right)^2}-\frac{4\,b\,c\,(a+c)\,\text{Log}[2\,a+c-c\,\text{Cos}[2\,x]+2\,b\,\text{Sin}[x]]}{\left(a^2-b^2+2\,a\,c+c^2\right)^2}+$$

$$\left.\frac{1}{(a+b+c)\,\left(\text{Cos}\left[\frac{x}{2}\right]-\text{Sin}\left[\frac{x}{2}\right]\right)^2}-\frac{1}{(a-b+c)\,\left(\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]\right)^2}\right)$$

Summary of Integration Test Results

19 integration problems



A - 8 optimal antiderivatives

B - 0 more than twice size of optimal antiderivatives

C - 11 unnecessarily complex antiderivatives

D - 0 unable to integrate problems

E - 0 integration timeouts