

Mathematica 11.3 Integration Test Results

Test results for the 22 problems in "4.2.1.3 (g tan)^p (a+b cos)^m.m"

Problem 1: Result more than twice size of optimal antiderivative.

$$\int \frac{\tan[x]^4}{a + a \cos[x]} dx$$

Optimal (type 3, 33 leaves, 5 steps):

$$\frac{\text{ArcTanh}[\sin[x]]}{2a} - \frac{\sec[x] \tan[x]}{2a} + \frac{\tan[x]^3}{3a}$$

Result (type 3, 105 leaves):

$$-\frac{1}{24a} \sec[x]^3 \left(9 \cos[x] \left(\log\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] - \log\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right] \right) + 3 \cos[3x] \right. \\ \left. \left(\log\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] - \log\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right] \right) + 2(-3 \sin[x] + 3 \sin[2x] + \sin[3x]) \right)$$

Problem 3: Result more than twice size of optimal antiderivative.

$$\int \frac{\tan[x]^2}{a + a \cos[x]} dx$$

Optimal (type 3, 15 leaves, 4 steps):

$$-\frac{\text{ArcTanh}[\sin[x]]}{a} + \frac{\tan[x]}{a}$$

Result (type 3, 39 leaves):

$$\frac{\log\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] - \log\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right] + \tan[x]}{a}$$

Problem 19: Result more than twice size of optimal antiderivative.

$$\int \sqrt{a + b \cos[x]} \tan[x] dx$$

Optimal (type 3, 37 leaves, 4 steps):

$$2\sqrt{a} \text{ArcTanh}\left[\frac{\sqrt{a + b \cos[x]}}{\sqrt{a}}\right] - 2\sqrt{a + b \cos[x]}$$

Result (type 3, 75 leaves):

$$-\frac{1}{b+a \operatorname{Sec}[x]} \\ 2 \sqrt{a+b \operatorname{Cos}[x]} \left(b+a \operatorname{Sec}[x] - \sqrt{a} \sqrt{b} \operatorname{ArcSinh}\left[\frac{\sqrt{a} \sqrt{\operatorname{Sec}[x]}}{\sqrt{b}}\right] \sqrt{\operatorname{Sec}[x]} \sqrt{1+\frac{a \operatorname{Sec}[x]}{b}} \right)$$

Problem 20: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Tan}[x]}{\sqrt{a+b \operatorname{Cos}[x]}} dx$$

Optimal (type 3, 24 leaves, 3 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Cos}[x]}}{\sqrt{a}}\right]}{\sqrt{a}}$$

Result (type 3, 60 leaves):

$$\frac{2 \sqrt{b} \operatorname{ArcSinh}\left[\frac{\sqrt{a} \sqrt{\operatorname{Sec}[x]}}{\sqrt{b}}\right] \sqrt{\frac{b+a \operatorname{Sec}[x]}{b}}}{\sqrt{a} \sqrt{a+b \operatorname{Cos}[x]} \sqrt{\operatorname{Sec}[x]}}$$

Problem 21: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{e \operatorname{Tan}[c+d x]}}{a+b \operatorname{Cos}[c+d x]} dx$$

Optimal (type 4, 204 leaves, 9 steps):

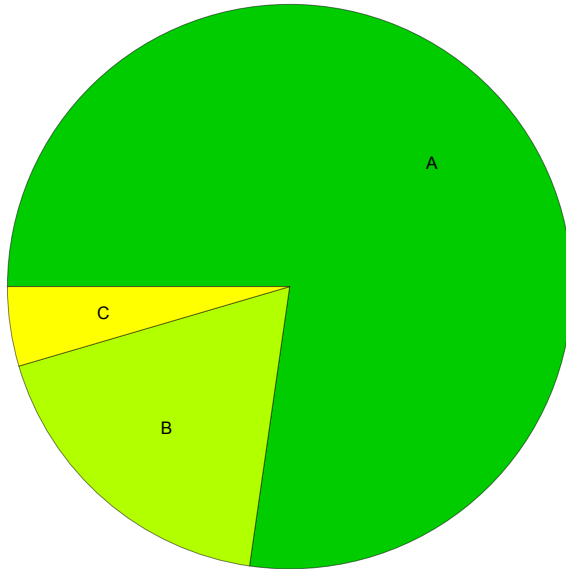
$$-\left(\left(2 \sqrt{2} \sqrt{\operatorname{Cos}[c+d x]} \operatorname{EllipticPi}\left[-\frac{\sqrt{-a+b}}{\sqrt{a+b}}, \operatorname{ArcSin}\left[\frac{\sqrt{\operatorname{Sin}[c+d x]}}{\sqrt{1+\operatorname{Cos}[c+d x]}}\right], -1\right] \sqrt{e \operatorname{Tan}[c+d x]} \right) / \left(\sqrt{-a+b} \sqrt{a+b} d \sqrt{\operatorname{Sin}[c+d x]} \right) \right) + \\ \left(2 \sqrt{2} \sqrt{\operatorname{Cos}[c+d x]} \operatorname{EllipticPi}\left[\frac{\sqrt{-a+b}}{\sqrt{a+b}}, \operatorname{ArcSin}\left[\frac{\sqrt{\operatorname{Sin}[c+d x]}}{\sqrt{1+\operatorname{Cos}[c+d x]}}\right], -1\right] \sqrt{e \operatorname{Tan}[c+d x]} \right) / \left(\sqrt{-a+b} \sqrt{a+b} d \sqrt{\operatorname{Sin}[c+d x]} \right)$$

Result (type 6, 584 leaves):

$$\begin{aligned}
 & \frac{1}{d (a + b \cos [c + d x]) \sqrt{\tan [c + d x]} (1 + \tan [c + d x]^2)^{3/2}} \\
 & 2 \operatorname{Sec}[c + d x]^2 \sqrt{e \tan [c + d x]} \left(b + a \sqrt{1 + \tan [c + d x]^2} \right) \\
 & \left(\left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{a} \sqrt{\tan [c + d x]}}{(a^2 - b^2)^{1/4}} \right] + 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{a} \sqrt{\tan [c + d x]}}{(a^2 - b^2)^{1/4}} \right] \right) + \right. \\
 & \quad \left. \operatorname{Log} \left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\tan [c + d x]} + a \tan [c + d x] \right] - \operatorname{Log} \left[\sqrt{a^2 - b^2} + \right. \right. \\
 & \quad \left. \left. \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\tan [c + d x]} + a \tan [c + d x] \right] \right) / \left(4 \sqrt{2} \sqrt{a} (a^2 - b^2)^{1/4} \right) + \\
 & \left(7 b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan [c + d x]^2, -\frac{a^2 \tan [c + d x]^2}{a^2 - b^2} \right] \tan [c + d x]^{3/2} \right) / \\
 & \left(3 \sqrt{1 + \tan [c + d x]^2} \right. \\
 & \quad \left(-7 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan [c + d x]^2, -\frac{a^2 \tan [c + d x]^2}{a^2 - b^2} \right] + \right. \\
 & \quad 2 \left(2 a^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\tan [c + d x]^2, -\frac{a^2 \tan [c + d x]^2}{a^2 - b^2} \right] + \right. \\
 & \quad \left. \left. (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, -\tan [c + d x]^2, -\frac{a^2 \tan [c + d x]^2}{a^2 - b^2} \right] \right) \right) \\
 & \left. \left. \tan [c + d x]^2 \right) (-b^2 + a^2 (1 + \tan [c + d x]^2)) \right) \Big)
 \end{aligned}$$

Summary of Integration Test Results

22 integration problems



A - 17 optimal antiderivatives

B - 4 more than twice size of optimal antiderivatives

C - 1 unnecessarily complex antiderivatives

D - 0 unable to integrate problems

E - 0 integration timeouts