

Mathematica 11.3 Integration Test Results

Test results for the 189 problems in "4.2.10 (c+d x)^m (a+b cos)^n.m"

Problem 31: Result more than twice size of optimal antiderivative.

$$\int (c + d x) \operatorname{Sec}[a + b x] dx$$

Optimal (type 4, 75 leaves, 5 steps):

$$-\frac{2 i (c + d x) \operatorname{ArcTan}\left[e^{i(a+b x)}\right]}{b} + \frac{i d \operatorname{PolyLog}\left[2, -i e^{i(a+b x)}\right]}{b^2} - \frac{i d \operatorname{PolyLog}\left[2, i e^{i(a+b x)}\right]}{b^2}$$

Result (type 4, 220 leaves):

$$-\frac{c \operatorname{Log}\left[\operatorname{Cos}\left[\frac{a}{2} + \frac{b x}{2}\right] - \operatorname{Sin}\left[\frac{a}{2} + \frac{b x}{2}\right]\right]}{b} + \frac{c \operatorname{Log}\left[\operatorname{Cos}\left[\frac{a}{2} + \frac{b x}{2}\right] + \operatorname{Sin}\left[\frac{a}{2} + \frac{b x}{2}\right]\right]}{b} +$$

$$\frac{1}{b^2} d \left(\left(-a + \frac{\pi}{2} - b x \right) \left(\operatorname{Log}\left[1 - e^{i\left(-a + \frac{\pi}{2} - b x\right)}\right] - \operatorname{Log}\left[1 + e^{i\left(-a + \frac{\pi}{2} - b x\right)}\right] \right) - \left(-a + \frac{\pi}{2} \right) \right.$$

$$\left. \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2}\left(-a + \frac{\pi}{2} - b x\right)\right]\right] + i \left(\operatorname{PolyLog}\left[2, -e^{i\left(-a + \frac{\pi}{2} - b x\right)}\right] - \operatorname{PolyLog}\left[2, e^{i\left(-a + \frac{\pi}{2} - b x\right)}\right] \right) \right)$$

Problem 33: Result more than twice size of optimal antiderivative.

$$\int (c + d x)^3 \operatorname{Sec}[a + b x]^2 dx$$

Optimal (type 4, 114 leaves, 6 steps):

$$-\frac{i (c + d x)^3}{b} + \frac{3 d (c + d x)^2 \operatorname{Log}\left[1 + e^{2 i(a+b x)}\right]}{b^2} -$$

$$\frac{3 i d^2 (c + d x) \operatorname{PolyLog}\left[2, -e^{2 i(a+b x)}\right]}{b^3} + \frac{3 d^3 \operatorname{PolyLog}\left[3, -e^{2 i(a+b x)}\right]}{2 b^4} + \frac{(c + d x)^3 \operatorname{Tan}[a + b x]}{b}$$

Result (type 4, 397 leaves):

$$\begin{aligned}
 & -\frac{1}{4b^4}d^3 e^{-i a} \left(2 i b^2 x^2 \left(2 b e^{2 i a} x + 3 i \left(1 + e^{2 i a} \right) \operatorname{Log}\left[1 + e^{2 i (a+b x)} \right] \right) + \right. \\
 & \quad \left. 6 i b \left(1 + e^{2 i a} \right) x \operatorname{PolyLog}\left[2, -e^{2 i (a+b x)} \right] - 3 \left(1 + e^{2 i a} \right) \operatorname{PolyLog}\left[3, -e^{2 i (a+b x)} \right] \right) \operatorname{Sec}[a] + \\
 & \quad \frac{3 c^2 d \operatorname{Sec}[a] \left(\operatorname{Cos}[a] \operatorname{Log}\left[\operatorname{Cos}[a] \operatorname{Cos}[b x] - \operatorname{Sin}[a] \operatorname{Sin}[b x] \right] + b x \operatorname{Sin}[a] \right)}{b^2 \left(\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2 \right)} + \\
 & \quad \left(3 c d^2 \operatorname{Csc}[a] \left(b^2 e^{-i \operatorname{ArcTan}[\operatorname{Cot}[a]]} x^2 - \frac{1}{\sqrt{1 + \operatorname{Cot}[a]^2}} \operatorname{Cot}[a] \left(i b x \left(-\pi - 2 \operatorname{ArcTan}[\operatorname{Cot}[a]] \right) - \right. \right. \right. \\
 & \quad \left. \left. \left. \pi \operatorname{Log}\left[1 + e^{-2 i b x} \right] - 2 \left(b x - \operatorname{ArcTan}[\operatorname{Cot}[a]] \right) \operatorname{Log}\left[1 - e^{2 i (b x - \operatorname{ArcTan}[\operatorname{Cot}[a])} \right] \right) + \right. \right. \\
 & \quad \left. \left. \left. \pi \operatorname{Log}\left[\operatorname{Cos}[b x] \right] - 2 \operatorname{ArcTan}[\operatorname{Cot}[a]] \operatorname{Log}\left[\operatorname{Sin}[b x - \operatorname{ArcTan}[\operatorname{Cot}[a]] \right] \right] \right) + \right. \right. \\
 & \quad \left. \left. \left. i \operatorname{PolyLog}\left[2, e^{2 i (b x - \operatorname{ArcTan}[\operatorname{Cot}[a])} \right] \right] \right) \operatorname{Sec}[a] \right) / \left(b^3 \sqrt{\operatorname{Csc}[a]^2 \left(\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2 \right)} \right) + \\
 & \quad \frac{\operatorname{Sec}[a] \operatorname{Sec}[a + b x] \left(c^3 \operatorname{Sin}[b x] + 3 c^2 d x \operatorname{Sin}[b x] + 3 c d^2 x^2 \operatorname{Sin}[b x] + d^3 x^3 \operatorname{Sin}[b x] \right)}{b}
 \end{aligned}$$

Problem 34: Result more than twice size of optimal antiderivative.

$$\int (c + d x)^2 \operatorname{Sec}[a + b x]^2 dx$$

Optimal (type 4, 82 leaves, 5 steps):

$$\begin{aligned}
 & -\frac{i (c + d x)^2}{b} + \frac{2 d (c + d x) \operatorname{Log}\left[1 + e^{2 i (a+b x)} \right]}{b^2} - \\
 & \frac{i d^2 \operatorname{PolyLog}\left[2, -e^{2 i (a+b x)} \right]}{b^3} + \frac{(c + d x)^2 \operatorname{Tan}[a + b x]}{b}
 \end{aligned}$$

Result (type 4, 253 leaves):

$$\begin{aligned}
 & \left(2 c d \operatorname{Sec}[a] \left(\operatorname{Cos}[a] \operatorname{Log}\left[\operatorname{Cos}[a] \operatorname{Cos}[b x] - \operatorname{Sin}[a] \operatorname{Sin}[b x] \right] + b x \operatorname{Sin}[a] \right) \right) / \\
 & \quad \left(b^2 \left(\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2 \right) \right) + \\
 & \quad \left(d^2 \operatorname{Csc}[a] \left(b^2 e^{-i \operatorname{ArcTan}[\operatorname{Cot}[a]]} x^2 - \frac{1}{\sqrt{1 + \operatorname{Cot}[a]^2}} \operatorname{Cot}[a] \left(i b x \left(-\pi - 2 \operatorname{ArcTan}[\operatorname{Cot}[a]] \right) - \right. \right. \right. \\
 & \quad \left. \left. \left. \pi \operatorname{Log}\left[1 + e^{-2 i b x} \right] - 2 \left(b x - \operatorname{ArcTan}[\operatorname{Cot}[a]] \right) \operatorname{Log}\left[1 - e^{2 i (b x - \operatorname{ArcTan}[\operatorname{Cot}[a])} \right] \right) + \right. \right. \\
 & \quad \left. \left. \left. \pi \operatorname{Log}\left[\operatorname{Cos}[b x] \right] - 2 \operatorname{ArcTan}[\operatorname{Cot}[a]] \operatorname{Log}\left[\operatorname{Sin}[b x - \operatorname{ArcTan}[\operatorname{Cot}[a]] \right] \right] \right) + \right. \right. \\
 & \quad \left. \left. \left. i \operatorname{PolyLog}\left[2, e^{2 i (b x - \operatorname{ArcTan}[\operatorname{Cot}[a])} \right] \right] \right) \operatorname{Sec}[a] \right) / \left(b^3 \sqrt{\operatorname{Csc}[a]^2 \left(\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2 \right)} \right) + \\
 & \quad \frac{\operatorname{Sec}[a] \operatorname{Sec}[a + b x] \left(c^2 \operatorname{Sin}[b x] + 2 c d x \operatorname{Sin}[b x] + d^2 x^2 \operatorname{Sin}[b x] \right)}{b}
 \end{aligned}$$

Problem 38: Result more than twice size of optimal antiderivative.

$$\int (c + d x)^2 \operatorname{Sec}[a + b x]^3 dx$$

Optimal (type 4, 193 leaves, 9 steps):

$$\begin{aligned} & -\frac{i(c+dx)^2 \operatorname{ArcTan}\left[e^{i(a+bx)}\right]}{b} + \frac{d^2 \operatorname{ArcTanh}\left[\operatorname{Sin}[a+bx]\right]}{b^3} + \frac{id(c+dx) \operatorname{PolyLog}\left[2, -ie^{i(a+bx)}\right]}{b^2} \\ & -\frac{id(c+dx) \operatorname{PolyLog}\left[2, ie^{i(a+bx)}\right]}{b^2} - \frac{d^2 \operatorname{PolyLog}\left[3, -ie^{i(a+bx)}\right]}{b^3} + \\ & \frac{d^2 \operatorname{PolyLog}\left[3, ie^{i(a+bx)}\right]}{b^3} - \frac{d(c+dx) \operatorname{Sec}[a+bx]}{b^2} + \frac{(c+dx)^2 \operatorname{Sec}[a+bx] \operatorname{Tan}[a+bx]}{2b} \end{aligned}$$

Result (type 4, 526 leaves):

$$\begin{aligned} & \frac{1}{b^2} \left(-ibc^2 \operatorname{ArcTan}\left[e^{i(a+bx)}\right] - \frac{2id^2 \operatorname{ArcTan}\left[e^{i(a+bx)}\right]}{b} + bcdx \operatorname{Log}\left[1 - ie^{i(a+bx)}\right] + \right. \\ & \left. \frac{1}{2} b d^2 x^2 \operatorname{Log}\left[1 - ie^{i(a+bx)}\right] - bcdx \operatorname{Log}\left[1 + ie^{i(a+bx)}\right] - \frac{1}{2} b d^2 x^2 \operatorname{Log}\left[1 + ie^{i(a+bx)}\right] + \right. \\ & \left. id(c+dx) \operatorname{PolyLog}\left[2, -ie^{i(a+bx)}\right] - id(c+dx) \operatorname{PolyLog}\left[2, ie^{i(a+bx)}\right] - \right. \\ & \left. \frac{d^2 \operatorname{PolyLog}\left[3, -ie^{i(a+bx)}\right]}{b} + \frac{d^2 \operatorname{PolyLog}\left[3, ie^{i(a+bx)}\right]}{b} \right) - \\ & \frac{d(c+dx) \operatorname{Sec}[a]}{b^2} + \frac{c^2 + 2cdx + d^2x^2}{4b \left(\operatorname{Cos}\left[\frac{a}{2} + \frac{bx}{2}\right] - \operatorname{Sin}\left[\frac{a}{2} + \frac{bx}{2}\right] \right)^2} + \\ & \frac{-cd \operatorname{Sin}\left[\frac{bx}{2}\right] - d^2x \operatorname{Sin}\left[\frac{bx}{2}\right]}{b^2 \left(\operatorname{Cos}\left[\frac{a}{2}\right] - \operatorname{Sin}\left[\frac{a}{2}\right] \right) \left(\operatorname{Cos}\left[\frac{a}{2} + \frac{bx}{2}\right] - \operatorname{Sin}\left[\frac{a}{2} + \frac{bx}{2}\right] \right)} + \\ & \frac{-c^2 - 2cdx - d^2x^2}{4b \left(\operatorname{Cos}\left[\frac{a}{2} + \frac{bx}{2}\right] + \operatorname{Sin}\left[\frac{a}{2} + \frac{bx}{2}\right] \right)^2} + \\ & \frac{cd \operatorname{Sin}\left[\frac{bx}{2}\right] + d^2x \operatorname{Sin}\left[\frac{bx}{2}\right]}{b^2 \left(\operatorname{Cos}\left[\frac{a}{2}\right] + \operatorname{Sin}\left[\frac{a}{2}\right] \right) \left(\operatorname{Cos}\left[\frac{a}{2} + \frac{bx}{2}\right] + \operatorname{Sin}\left[\frac{a}{2} + \frac{bx}{2}\right] \right)} \end{aligned}$$

Problem 39: Result more than twice size of optimal antiderivative.

$$\int (c+dx) \operatorname{Sec}[a+bx]^3 dx$$

Optimal (type 4, 117 leaves, 6 steps):

$$\begin{aligned} & -\frac{i(c+dx) \operatorname{ArcTan}\left[e^{i(a+bx)}\right]}{b} + \frac{id \operatorname{PolyLog}\left[2, -ie^{i(a+bx)}\right]}{2b^2} \\ & -\frac{id \operatorname{PolyLog}\left[2, ie^{i(a+bx)}\right]}{2b^2} - \frac{d \operatorname{Sec}[a+bx]}{2b^2} + \frac{(c+dx) \operatorname{Sec}[a+bx] \operatorname{Tan}[a+bx]}{2b} \end{aligned}$$

Result (type 4, 480 leaves):

$$\begin{aligned}
 & - \frac{c \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a+b x)\right]-\operatorname{Sin}\left[\frac{1}{2}(a+b x)\right]\right]}{2 b} + \frac{c \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a+b x)\right]+\operatorname{Sin}\left[\frac{1}{2}(a+b x)\right]\right]}{2 b} + \\
 & \frac{1}{2 b^2} d \left(\left(-a+\frac{\pi}{2}-b x\right) \left(\operatorname{Log}\left[1-e^{i\left(-a+\frac{\pi}{2}-b x\right)}\right]-\operatorname{Log}\left[1+e^{i\left(-a+\frac{\pi}{2}-b x\right)}\right]\right) - \left(-a+\frac{\pi}{2}\right) \right. \\
 & \quad \left. \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2}\left(-a+\frac{\pi}{2}-b x\right)\right]\right] + i \left(\operatorname{PolyLog}\left[2,-e^{i\left(-a+\frac{\pi}{2}-b x\right)}\right]-\operatorname{PolyLog}\left[2,e^{i\left(-a+\frac{\pi}{2}-b x\right)}\right]\right) \right) + \\
 & \frac{d x}{4 b\left(\operatorname{Cos}\left[\frac{a}{2}+\frac{b x}{2}\right]-\operatorname{Sin}\left[\frac{a}{2}+\frac{b x}{2}\right]\right)^2} - \frac{d \operatorname{Sin}\left[\frac{b x}{2}\right]}{2 b^2\left(\operatorname{Cos}\left[\frac{a}{2}\right]-\operatorname{Sin}\left[\frac{a}{2}\right]\right)\left(\operatorname{Cos}\left[\frac{a}{2}+\frac{b x}{2}\right]-\operatorname{Sin}\left[\frac{a}{2}+\frac{b x}{2}\right]\right)} - \\
 & \frac{d x}{4 b\left(\operatorname{Cos}\left[\frac{a}{2}+\frac{b x}{2}\right]+\operatorname{Sin}\left[\frac{a}{2}+\frac{b x}{2}\right]\right)^2} + \\
 & \frac{d \operatorname{Sin}\left[\frac{b x}{2}\right]}{2 b^2\left(\operatorname{Cos}\left[\frac{a}{2}\right]+\operatorname{Sin}\left[\frac{a}{2}\right]\right)\left(\operatorname{Cos}\left[\frac{a}{2}+\frac{b x}{2}\right]+\operatorname{Sin}\left[\frac{a}{2}+\frac{b x}{2}\right]\right)} + \\
 & \frac{c}{4 b\left(\operatorname{Cos}\left[\frac{1}{2}(a+b x)\right]-\operatorname{Sin}\left[\frac{1}{2}(a+b x)\right]\right)^2} - \frac{c}{4 b\left(\operatorname{Cos}\left[\frac{1}{2}(a+b x)\right]+\operatorname{Sin}\left[\frac{1}{2}(a+b x)\right]\right)^2}
 \end{aligned}$$

Problem 55: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Cos}[a+b x]^2}{(c+d x)^{9/2}} dx$$

Optimal (type 4, 247 leaves, 11 steps):

$$\begin{aligned}
 & - \frac{16 b^2}{105 d^3 (c+d x)^{3/2}} - \frac{2 \operatorname{Cos}[a+b x]^2}{7 d (c+d x)^{7/2}} + \frac{32 b^2 \operatorname{Cos}[a+b x]^2}{105 d^3 (c+d x)^{3/2}} + \\
 & \frac{128 b^{7/2} \sqrt{\pi} \operatorname{Cos}\left[2 a-\frac{2 b c}{d}\right] \operatorname{FresnelC}\left[\frac{2 \sqrt{b} \sqrt{c+d x}}{\sqrt{d} \sqrt{\pi}}\right]}{105 d^{9/2}} - \\
 & \frac{128 b^{7/2} \sqrt{\pi} \operatorname{FresnelS}\left[\frac{2 \sqrt{b} \sqrt{c+d x}}{\sqrt{d} \sqrt{\pi}}\right] \operatorname{Sin}\left[2 a-\frac{2 b c}{d}\right]}{105 d^{9/2}} + \\
 & \frac{8 b \operatorname{Cos}[a+b x] \operatorname{Sin}[a+b x]}{35 d^2 (c+d x)^{5/2}} - \frac{128 b^3 \operatorname{Cos}[a+b x] \operatorname{Sin}[a+b x]}{105 d^4 \sqrt{c+d x}}
 \end{aligned}$$

Result (type 4, 987 leaves):

$$- \frac{1}{7 d (c+d x)^{7/2}} + \frac{1}{2} \operatorname{Cos}[2 a] \left(- \frac{1}{7 d} 32 \sqrt{2} \left(\frac{b}{d}\right)^{7/2} \operatorname{Cos}\left[\frac{b c}{d}\right] \operatorname{Sin}\left[\frac{b c}{d}\right] \right)$$

$$\begin{aligned}
 & \left(\frac{\text{Sin}\left[\frac{2b(c+dx)}{d}\right]}{8\sqrt{2}\left(\frac{b}{d}\right)^{7/2}(c+dx)^{7/2}} + \frac{2}{5} \left(\frac{\text{Cos}\left[\frac{2b(c+dx)}{d}\right]}{4\sqrt{2}\left(\frac{b}{d}\right)^{5/2}(c+dx)^{5/2}} - \frac{2}{3} \left(\frac{\text{Cos}\left[\frac{2b(c+dx)}{d}\right]}{\sqrt{2}\sqrt{\frac{b}{d}}\sqrt{c+dx}} + \right. \right. \right. \\
 & \left. \left. \left. \sqrt{2\pi} \text{FresnelS}\left[\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right] + \frac{\text{Sin}\left[\frac{2b(c+dx)}{d}\right]}{2\sqrt{2}\left(\frac{b}{d}\right)^{3/2}(c+dx)^{3/2}} \right) \right) \right) - \\
 & \frac{1}{7d} 16\sqrt{2}\left(\frac{b}{d}\right)^{7/2} \text{Cos}\left[\frac{2bc}{d}\right] \left(\frac{\text{Cos}\left[\frac{2b(c+dx)}{d}\right]}{8\sqrt{2}\left(\frac{b}{d}\right)^{7/2}(c+dx)^{7/2}} - \right. \\
 & \left. \frac{2}{5} \left(\frac{\text{Sin}\left[\frac{2b(c+dx)}{d}\right]}{4\sqrt{2}\left(\frac{b}{d}\right)^{5/2}(c+dx)^{5/2}} + \frac{2}{3} \left(\frac{\text{Cos}\left[\frac{2b(c+dx)}{d}\right]}{2\sqrt{2}\left(\frac{b}{d}\right)^{3/2}(c+dx)^{3/2}} - \right. \right. \right. \\
 & \left. \left. \left. 2 \left(-\sqrt{2\pi} \text{FresnelC}\left[\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right] + \frac{\text{Sin}\left[\frac{2b(c+dx)}{d}\right]}{\sqrt{2}\sqrt{\frac{b}{d}}\sqrt{c+dx}} \right) \right) \right) \right) - \\
 & 2 \text{Cos}[a] \text{Sin}[a] \left(-\frac{1}{7d} 16\sqrt{2}\left(\frac{b}{d}\right)^{7/2} \left(\text{Cos}\left[\frac{bc}{d}\right] - \text{Sin}\left[\frac{bc}{d}\right] \right) \left(\text{Cos}\left[\frac{bc}{d}\right] + \text{Sin}\left[\frac{bc}{d}\right] \right) \right. \\
 & \left. \left(\frac{\text{Sin}\left[\frac{2b(c+dx)}{d}\right]}{8\sqrt{2}\left(\frac{b}{d}\right)^{7/2}(c+dx)^{7/2}} + \frac{2}{5} \left(\frac{\text{Cos}\left[\frac{2b(c+dx)}{d}\right]}{4\sqrt{2}\left(\frac{b}{d}\right)^{5/2}(c+dx)^{5/2}} - \frac{2}{3} \left(\frac{\text{Cos}\left[\frac{2b(c+dx)}{d}\right]}{\sqrt{2}\sqrt{\frac{b}{d}}\sqrt{c+dx}} + \right. \right. \right. \right. \\
 & \left. \left. \left. \sqrt{2\pi} \text{FresnelS}\left[\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right] + \frac{\text{Sin}\left[\frac{2b(c+dx)}{d}\right]}{2\sqrt{2}\left(\frac{b}{d}\right)^{3/2}(c+dx)^{3/2}} \right) \right) \right) \right) +
 \end{aligned}$$

$$\begin{aligned}
 & \frac{3}{4} \left(-\operatorname{Sin}[a] \left(\frac{1}{5d} 2 \left(\frac{b}{d} \right)^{5/2} \operatorname{Sin}\left[\frac{bc}{d}\right] \left(\frac{\operatorname{Cos}\left[\frac{b(c+dx)}{d}\right]}{\left(\frac{b}{d}\right)^{5/2} (c+dx)^{5/2}} - \right. \right. \right. \\
 & \left. \left. \frac{2}{3} \left(\frac{\operatorname{Cos}\left[\frac{b(c+dx)}{d}\right]}{\sqrt{\frac{b}{d}} \sqrt{c+dx}} + \sqrt{2\pi} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{2}{\pi}} \sqrt{c+dx}\right] + \frac{\operatorname{Sin}\left[\frac{b(c+dx)}{d}\right]}{\left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2}} \right) \right) - \right. \\
 & \left. \frac{1}{5d} 2 \left(\frac{b}{d} \right)^{5/2} \operatorname{Cos}\left[\frac{bc}{d}\right] \left(\frac{\operatorname{Sin}\left[\frac{b(c+dx)}{d}\right]}{\left(\frac{b}{d}\right)^{5/2} (c+dx)^{5/2}} + \right. \right. \\
 & \left. \left. \frac{2}{3} \left(\frac{\operatorname{Cos}\left[\frac{b(c+dx)}{d}\right]}{\left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2}} - 2 \left(-\sqrt{2\pi} \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{2}{\pi}} \sqrt{c+dx}\right] + \frac{\operatorname{Sin}\left[\frac{b(c+dx)}{d}\right]}{\sqrt{\frac{b}{d}} \sqrt{c+dx}} \right) \right) \right) \right) \right) + \\
 & \operatorname{Cos}[a] \left(-\frac{1}{5d} 2 \left(\frac{b}{d} \right)^{5/2} \operatorname{Cos}\left[\frac{bc}{d}\right] \left(\frac{\operatorname{Cos}\left[\frac{b(c+dx)}{d}\right]}{\left(\frac{b}{d}\right)^{5/2} (c+dx)^{5/2}} - \right. \right. \\
 & \left. \left. \frac{2}{3} \left(\frac{\operatorname{Cos}\left[\frac{b(c+dx)}{d}\right]}{\sqrt{\frac{b}{d}} \sqrt{c+dx}} + \sqrt{2\pi} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{2}{\pi}} \sqrt{c+dx}\right] + \frac{\operatorname{Sin}\left[\frac{b(c+dx)}{d}\right]}{\left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2}} \right) \right) - \right. \\
 & \left. \frac{1}{5d} 2 \left(\frac{b}{d} \right)^{5/2} \operatorname{Sin}\left[\frac{bc}{d}\right] \left(\frac{\operatorname{Sin}\left[\frac{b(c+dx)}{d}\right]}{\left(\frac{b}{d}\right)^{5/2} (c+dx)^{5/2}} + \frac{2}{3} \left(\frac{\operatorname{Cos}\left[\frac{b(c+dx)}{d}\right]}{\left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2}} - \right. \right. \right. \\
 & \left. \left. \left. 2 \left(-\sqrt{2\pi} \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{2}{\pi}} \sqrt{c+dx}\right] + \frac{\operatorname{Sin}\left[\frac{b(c+dx)}{d}\right]}{\sqrt{\frac{b}{d}} \sqrt{c+dx}} \right) \right) \right) \right) \right) +
 \end{aligned}$$

$$\int x \sqrt{\cos [a + b x]} \, dx$$

Optimal (type 8, 15 leaves, 0 steps):

$$\text{Int}[x \sqrt{\cos [a + b x]}, x]$$

Result (type 1, 1 leaves):

???

Problem 86: Attempted integration timed out after 120 seconds.

$$\int \frac{x}{\cos [a + b x]^{3/2}} \, dx$$

Optimal (type 8, 55 leaves, 1 step):

$$\frac{4 \sqrt{\cos [a + b x]}}{b^2} + \frac{2 x \sin [a + b x]}{b \sqrt{\cos [a + b x]}} - \text{Int}[x \sqrt{\cos [a + b x]}, x]$$

Result (type 1, 1 leaves):

???

Problem 129: Result more than twice size of optimal antiderivative.

$$\int \frac{(c + d x)^2}{a + a \cos [e + f x]} \, dx$$

Optimal (type 4, 101 leaves, 6 steps):

$$-\frac{i (c + d x)^2}{a f} + \frac{4 d (c + d x) \text{Log}[1 + e^{i (e + f x)}]}{a f^2} - \frac{4 i d^2 \text{PolyLog}[2, -e^{i (e + f x)}]}{a f^3} + \frac{(c + d x)^2 \text{Tan}\left[\frac{e}{2} + \frac{f x}{2}\right]}{a f}$$

Result (type 4, 454 leaves):

$$\left(8 c d \cos \left[\frac{e}{2} + \frac{f x}{2} \right]^2 \sec \left[\frac{e}{2} \right] \left(\cos \left[\frac{e}{2} \right] \log \left[\cos \left[\frac{e}{2} \right] \cos \left[\frac{f x}{2} \right] - \sin \left[\frac{e}{2} \right] \sin \left[\frac{f x}{2} \right] \right) + \frac{1}{2} f x \sin \left[\frac{e}{2} \right] \right) / \right. \\ \left. \left(f^2 (a + a \cos [e + f x]) \left(\cos \left[\frac{e}{2} \right]^2 + \sin \left[\frac{e}{2} \right]^2 \right) \right) + \right. \\ \left. \left(8 d^2 \cos \left[\frac{e}{2} + \frac{f x}{2} \right]^2 \csc \left[\frac{e}{2} \right] \left(\frac{1}{4} e^{-i \operatorname{ArcTan} \left[\cot \left[\frac{e}{2} \right] \right]} f^2 x^2 - \frac{1}{\sqrt{1 + \cot \left[\frac{e}{2} \right]^2}} \right. \right. \right. \right. \\ \left. \left. \cot \left[\frac{e}{2} \right] \left(\frac{1}{2} i f x \left(-\pi - 2 \operatorname{ArcTan} \left[\cot \left[\frac{e}{2} \right] \right] \right) - \pi \log \left[1 + e^{-i f x} \right] - 2 \left(\frac{f x}{2} - \operatorname{ArcTan} \left[\cot \left[\frac{e}{2} \right] \right] \right) \right. \right. \right. \right. \\ \left. \left. \log \left[1 - e^{2 i \left(\frac{f x}{2} - \operatorname{ArcTan} \left[\cot \left[\frac{e}{2} \right] \right]} \right]} \right) + \pi \log \left[\cos \left[\frac{f x}{2} \right] \right] - 2 \operatorname{ArcTan} \left[\cot \left[\frac{e}{2} \right] \right] \right. \right. \right. \\ \left. \left. \log \left[\sin \left[\frac{f x}{2} - \operatorname{ArcTan} \left[\cot \left[\frac{e}{2} \right] \right] \right] \right] + i \operatorname{PolyLog} \left[2, e^{2 i \left(\frac{f x}{2} - \operatorname{ArcTan} \left[\cot \left[\frac{e}{2} \right] \right]} \right]} \right] \right) \right) \sec \left[\frac{e}{2} \right] \right) / \\ \left(f^3 (a + a \cos [e + f x]) \sqrt{\csc \left[\frac{e}{2} \right]^2 \left(\cos \left[\frac{e}{2} \right]^2 + \sin \left[\frac{e}{2} \right]^2 \right)} \right) + \\ \frac{2 \cos \left[\frac{e}{2} + \frac{f x}{2} \right] \sec \left[\frac{e}{2} \right] \left(c^2 \sin \left[\frac{f x}{2} \right] + 2 c d x \sin \left[\frac{f x}{2} \right] + d^2 x^2 \sin \left[\frac{f x}{2} \right] \right)}{f (a + a \cos [e + f x])}$$

Problem 133: Result more than twice size of optimal antiderivative.

$$\int \frac{(c + d x)^3}{(a + a \cos [e + f x])^2} dx$$

Optimal (type 4, 271 leaves, 10 steps):

$$-\frac{i(c + d x)^3}{3 a^2 f} + \frac{2 d (c + d x)^2 \log [1 + e^{i(e + f x)}]}{a^2 f^2} + \frac{4 d^3 \log [\cos [\frac{e}{2} + \frac{f x}{2}]]}{a^2 f^4} - \\ \frac{4 i d^2 (c + d x) \operatorname{PolyLog} [2, -e^{i(e + f x)}]}{a^2 f^3} + \frac{4 d^3 \operatorname{PolyLog} [3, -e^{i(e + f x)}]}{a^2 f^4} - \frac{d (c + d x)^2 \sec [\frac{e}{2} + \frac{f x}{2}]^2}{2 a^2 f^2} + \\ \frac{2 d^2 (c + d x) \tan [\frac{e}{2} + \frac{f x}{2}]}{a^2 f^3} + \frac{(c + d x)^3 \tan [\frac{e}{2} + \frac{f x}{2}]}{3 a^2 f} + \frac{(c + d x)^3 \sec [\frac{e}{2} + \frac{f x}{2}]^2 \tan [\frac{e}{2} + \frac{f x}{2}]}{6 a^2 f}$$

Result (type 4, 1016 leaves):

$$-\left(\left(4 d^3 e^{-\frac{i e}{2}} \cos \left[\frac{e}{2} + \frac{f x}{2} \right]^4 \left(i f^2 x^2 \left(e^{i e} f x + 3 i \left(1 + e^{i e} \right) \log [1 + e^{i(e + f x)}] \right) + \right. \right. \right. \\ \left. \left. 6 i \left(1 + e^{i e} \right) f x \operatorname{PolyLog} [2, -e^{i(e + f x)}] - 6 \left(1 + e^{i e} \right) \operatorname{PolyLog} [3, -e^{i(e + f x)}] \right) \sec \left[\frac{e}{2} \right] \right) / \\ \left(3 f^4 (a + a \cos [e + f x])^2 \right) \right) + \left(16 d^3 \cos \left[\frac{e}{2} + \frac{f x}{2} \right]^4 \sec \left[\frac{e}{2} \right] \right)$$

$$\begin{aligned}
 & \left(\cos\left[\frac{e}{2}\right] \log\left[\cos\left[\frac{e}{2}\right] \cos\left[\frac{f x}{2}\right] - \sin\left[\frac{e}{2}\right] \sin\left[\frac{f x}{2}\right]\right] + \frac{1}{2} f x \sin\left[\frac{e}{2}\right] \right) / \\
 & \left(f^4 (a + a \cos[e + f x])^2 \left(\cos\left[\frac{e}{2}\right]^2 + \sin\left[\frac{e}{2}\right]^2 \right) \right) + \\
 & \left(8 c^2 d \cos\left[\frac{e}{2} + \frac{f x}{2}\right]^4 \sec\left[\frac{e}{2}\right] \right. \\
 & \quad \left. \left(\cos\left[\frac{e}{2}\right] \log\left[\cos\left[\frac{e}{2}\right] \cos\left[\frac{f x}{2}\right] - \sin\left[\frac{e}{2}\right] \sin\left[\frac{f x}{2}\right]\right] + \frac{1}{2} f x \sin\left[\frac{e}{2}\right] \right) \right) / \\
 & \left(f^2 (a + a \cos[e + f x])^2 \left(\cos\left[\frac{e}{2}\right]^2 + \sin\left[\frac{e}{2}\right]^2 \right) \right) + \\
 & \left(16 c d^2 \cos\left[\frac{e}{2} + \frac{f x}{2}\right]^4 \csc\left[\frac{e}{2}\right] \left(\frac{1}{4} e^{-i \operatorname{ArcTan}\left[\cot\left[\frac{e}{2}\right]\right]} f^2 x^2 - \frac{1}{\sqrt{1 + \cot\left[\frac{e}{2}\right]^2}} \right. \right. \\
 & \quad \left. \left. \cot\left[\frac{e}{2}\right] \left(\frac{1}{2} i f x \left(-\pi - 2 \operatorname{ArcTan}\left[\cot\left[\frac{e}{2}\right]\right] \right) - \pi \log\left[1 + e^{-i f x}\right] - 2 \left(\frac{f x}{2} - \operatorname{ArcTan}\left[\cot\left[\frac{e}{2}\right]\right] \right) \right) \right. \right. \\
 & \quad \left. \left. \log\left[1 - e^{2 i \left(\frac{f x}{2} - \operatorname{ArcTan}\left[\cot\left[\frac{e}{2}\right]\right] \right)} \right] + \pi \log\left[\cos\left[\frac{f x}{2}\right]\right] - 2 \operatorname{ArcTan}\left[\cot\left[\frac{e}{2}\right]\right] \right) \right) \sec\left[\frac{e}{2}\right] \right) / \\
 & \left(f^3 (a + a \cos[e + f x])^2 \sqrt{\csc\left[\frac{e}{2}\right]^2 \left(\cos\left[\frac{e}{2}\right]^2 + \sin\left[\frac{e}{2}\right]^2 \right)} \right) + \\
 & \frac{1}{3 f^3 (a + a \cos[e + f x])^2} \\
 & \cos\left[\frac{e}{2} + \frac{f x}{2}\right] \sec\left[\frac{e}{2}\right] \\
 & \left(-3 c^2 d f \cos\left[\frac{f x}{2}\right] - 6 c d^2 f x \cos\left[\frac{f x}{2}\right] - 3 d^3 f x^2 \cos\left[\frac{f x}{2}\right] - \right. \\
 & \quad 3 c^2 d f \cos\left[e + \frac{f x}{2}\right] - 6 c d^2 f x \cos\left[e + \frac{f x}{2}\right] - 3 d^3 f x^2 \cos\left[e + \frac{f x}{2}\right] + \\
 & \quad 12 c d^2 \sin\left[\frac{f x}{2}\right] + 3 c^3 f^2 \sin\left[\frac{f x}{2}\right] + 12 d^3 x \sin\left[\frac{f x}{2}\right] + 9 c^2 d f^2 x \sin\left[\frac{f x}{2}\right] + \\
 & \quad 9 c d^2 f^2 x^2 \sin\left[\frac{f x}{2}\right] + 3 d^3 f^2 x^3 \sin\left[\frac{f x}{2}\right] - 6 c d^2 \sin\left[e + \frac{f x}{2}\right] - 6 d^3 x \sin\left[e + \frac{f x}{2}\right] + \\
 & \quad 6 c d^2 \sin\left[e + \frac{3 f x}{2}\right] + c^3 f^2 \sin\left[e + \frac{3 f x}{2}\right] + 6 d^3 x \sin\left[e + \frac{3 f x}{2}\right] + \\
 & \quad \left. 3 c^2 d f^2 x \sin\left[e + \frac{3 f x}{2}\right] + 3 c d^2 f^2 x^2 \sin\left[e + \frac{3 f x}{2}\right] + d^3 f^2 x^3 \sin\left[e + \frac{3 f x}{2}\right] \right)
 \end{aligned}$$

Problem 134: Result more than twice size of optimal antiderivative.

$$\int \frac{(c + d x)^2}{(a + a \cos [e + f x])^2} dx$$

Optimal (type 4, 212 leaves, 9 steps):

$$\begin{aligned} & -\frac{i (c + d x)^2}{3 a^2 f} + \frac{4 d (c + d x) \operatorname{Log}[1 + e^{i (e + f x)}]}{3 a^2 f^2} - \\ & \frac{4 i d^2 \operatorname{PolyLog}[2, -e^{i (e + f x)}]}{3 a^2 f^3} - \frac{d (c + d x) \operatorname{Sec}\left[\frac{e}{2} + \frac{f x}{2}\right]^2}{3 a^2 f^2} + \frac{2 d^2 \operatorname{Tan}\left[\frac{e}{2} + \frac{f x}{2}\right]}{3 a^2 f^3} + \\ & \frac{(c + d x)^2 \operatorname{Tan}\left[\frac{e}{2} + \frac{f x}{2}\right]}{3 a^2 f} + \frac{(c + d x)^2 \operatorname{Sec}\left[\frac{e}{2} + \frac{f x}{2}\right]^2 \operatorname{Tan}\left[\frac{e}{2} + \frac{f x}{2}\right]}{6 a^2 f} \end{aligned}$$

Result (type 4, 619 leaves):

$$\begin{aligned} & \left(16 c d \cos\left[\frac{e}{2} + \frac{f x}{2}\right]^4 \operatorname{Sec}\left[\frac{e}{2}\right] \right. \\ & \quad \left. \left(\cos\left[\frac{e}{2}\right] \operatorname{Log}\left[\cos\left[\frac{e}{2}\right] \cos\left[\frac{f x}{2}\right] - \sin\left[\frac{e}{2}\right] \sin\left[\frac{f x}{2}\right]\right] + \frac{1}{2} f x \sin\left[\frac{e}{2}\right] \right) \right) / \\ & \quad \left(3 f^2 (a + a \cos [e + f x])^2 \left(\cos\left[\frac{e}{2}\right]^2 + \sin\left[\frac{e}{2}\right]^2 \right) \right) + \\ & \quad \left(16 d^2 \cos\left[\frac{e}{2} + \frac{f x}{2}\right]^4 \operatorname{Csc}\left[\frac{e}{2}\right] \left(\frac{1}{4} e^{-i \operatorname{ArcTan}\left[\operatorname{Cot}\left[\frac{e}{2}\right]\right]} f^2 x^2 - \frac{1}{\sqrt{1 + \operatorname{Cot}\left[\frac{e}{2}\right]^2}} \right. \right. \right. \\ & \quad \left. \left. \operatorname{Cot}\left[\frac{e}{2}\right] \left(\frac{1}{2} i f x \left(-\pi - 2 \operatorname{ArcTan}\left[\operatorname{Cot}\left[\frac{e}{2}\right]\right] \right) - \pi \operatorname{Log}\left[1 + e^{-i f x}\right] - 2 \left(\frac{f x}{2} - \operatorname{ArcTan}\left[\operatorname{Cot}\left[\frac{e}{2}\right]\right] \right) \right. \right. \right. \\ & \quad \left. \left. \operatorname{Log}\left[1 - e^{2 i \left(\frac{f x}{2} - \operatorname{ArcTan}\left[\operatorname{Cot}\left[\frac{e}{2}\right]\right] \right)}\right] + \pi \operatorname{Log}\left[\cos\left[\frac{f x}{2}\right]\right] - 2 \operatorname{ArcTan}\left[\operatorname{Cot}\left[\frac{e}{2}\right]\right] \right) \right) \right. \\ & \quad \left. \left. \operatorname{Log}\left[\sin\left[\frac{f x}{2} - \operatorname{ArcTan}\left[\operatorname{Cot}\left[\frac{e}{2}\right]\right]\right] \right] + i \operatorname{PolyLog}\left[2, e^{2 i \left(\frac{f x}{2} - \operatorname{ArcTan}\left[\operatorname{Cot}\left[\frac{e}{2}\right]\right] \right)}\right] \right) \right) \operatorname{Sec}\left[\frac{e}{2}\right] \right) / \\ & \quad \left(3 f^3 (a + a \cos [e + f x])^2 \sqrt{\operatorname{Csc}\left[\frac{e}{2}\right]^2 \left(\cos\left[\frac{e}{2}\right]^2 + \sin\left[\frac{e}{2}\right]^2 \right)} \right) + \\ & \quad \left(\cos\left[\frac{e}{2} + \frac{f x}{2}\right] \operatorname{Sec}\left[\frac{e}{2}\right] \left(-2 c d f \cos\left[\frac{f x}{2}\right] - 2 d^2 f x \cos\left[\frac{f x}{2}\right] - 2 c d f \cos\left[e + \frac{f x}{2}\right] - \right. \right. \\ & \quad \left. \left. 2 d^2 f x \cos\left[e + \frac{f x}{2}\right] + 4 d^2 \sin\left[\frac{f x}{2}\right] + 3 c^2 f^2 \sin\left[\frac{f x}{2}\right] + 6 c d f^2 x \sin\left[\frac{f x}{2}\right] + \right. \right. \\ & \quad \left. \left. 3 d^2 f^2 x^2 \sin\left[\frac{f x}{2}\right] - 2 d^2 \sin\left[e + \frac{f x}{2}\right] + 2 d^2 \sin\left[e + \frac{3 f x}{2}\right] + c^2 f^2 \sin\left[e + \frac{3 f x}{2}\right] + \right. \right. \\ & \quad \left. \left. 2 c d f^2 x \sin\left[e + \frac{3 f x}{2}\right] + d^2 f^2 x^2 \sin\left[e + \frac{3 f x}{2}\right] \right) \right) / \left(3 f^3 (a + a \cos [e + f x])^2 \right) \end{aligned}$$

Problem 139: Result more than twice size of optimal antiderivative.

$$\int \frac{(c+dx)^2}{a-a \cos[ex+fx]} dx$$

Optimal (type 4, 102 leaves, 6 steps):

$$-\frac{i(c+dx)^2}{af} - \frac{(c+dx)^2 \operatorname{Cot}\left[\frac{e}{2} + \frac{fx}{2}\right]}{af} + \frac{4d(c+dx) \operatorname{Log}[1 - e^{i(ex+fx)}]}{af^2} - \frac{4i d^2 \operatorname{PolyLog}[2, e^{i(ex+fx)}]}{af^3}$$

Result (type 4, 447 leaves):

$$\frac{2 \operatorname{Csc}\left[\frac{e}{2}\right] \left(c^2 \operatorname{Sin}\left[\frac{fx}{2}\right] + 2cdx \operatorname{Sin}\left[\frac{fx}{2}\right] + d^2 x^2 \operatorname{Sin}\left[\frac{fx}{2}\right]\right) \operatorname{Sin}\left[\frac{e}{2} + \frac{fx}{2}\right]}{f(a-a \cos[ex+fx])} +$$

$$\left(8cd \operatorname{Csc}\left[\frac{e}{2}\right] \left(-\frac{1}{2}fx \operatorname{Cos}\left[\frac{e}{2}\right] + \operatorname{Log}\left[\operatorname{Cos}\left[\frac{fx}{2}\right] \operatorname{Sin}\left[\frac{e}{2}\right] + \operatorname{Cos}\left[\frac{e}{2}\right] \operatorname{Sin}\left[\frac{fx}{2}\right]\right] \operatorname{Sin}\left[\frac{e}{2}\right]\right) \right.$$

$$\left. \operatorname{Sin}\left[\frac{e}{2} + \frac{fx}{2}\right]^2\right) / \left(f^2(a-a \cos[ex+fx]) \left(\operatorname{Cos}\left[\frac{e}{2}\right]^2 + \operatorname{Sin}\left[\frac{e}{2}\right]^2\right)\right) -$$

$$\left(8d^2 \operatorname{Csc}\left[\frac{e}{2}\right] \operatorname{Sec}\left[\frac{e}{2}\right] \operatorname{Sin}\left[\frac{e}{2} + \frac{fx}{2}\right]^2 \left(\frac{1}{4} e^{i \operatorname{ArcTan}\left[\operatorname{Tan}\left[\frac{e}{2}\right]\right]} f^2 x^2 + \frac{1}{\sqrt{1 + \operatorname{Tan}\left[\frac{e}{2}\right]^2}}\right) \right.$$

$$\left(\frac{1}{2} i f x \left(-\pi + 2 \operatorname{ArcTan}\left[\operatorname{Tan}\left[\frac{e}{2}\right]\right]\right) - \pi \operatorname{Log}\left[1 + e^{-ifx}\right] - 2\left(\frac{fx}{2} + \operatorname{ArcTan}\left[\operatorname{Tan}\left[\frac{e}{2}\right]\right]\right) \right.$$

$$\left. \operatorname{Log}\left[1 - e^{2i\left(\frac{fx}{2} + \operatorname{ArcTan}\left[\operatorname{Tan}\left[\frac{e}{2}\right]\right]\right)}\right] + \pi \operatorname{Log}\left[\operatorname{Cos}\left[\frac{fx}{2}\right]\right] + 2 \operatorname{ArcTan}\left[\operatorname{Tan}\left[\frac{e}{2}\right]\right] \right.$$

$$\left. \left. \operatorname{Log}\left[\operatorname{Sin}\left[\frac{fx}{2} + \operatorname{ArcTan}\left[\operatorname{Tan}\left[\frac{e}{2}\right]\right]\right]\right] + i \operatorname{PolyLog}\left[2, e^{2i\left(\frac{fx}{2} + \operatorname{ArcTan}\left[\operatorname{Tan}\left[\frac{e}{2}\right]\right]\right)}\right]\right) \operatorname{Tan}\left[\frac{e}{2}\right] \right) /$$

$$\left(f^3(a-a \cos[ex+fx]) \sqrt{\operatorname{Sec}\left[\frac{e}{2}\right]^2 \left(\operatorname{Cos}\left[\frac{e}{2}\right]^2 + \operatorname{Sin}\left[\frac{e}{2}\right]^2\right)}\right)$$

Problem 172: Result more than twice size of optimal antiderivative.

$$\int \frac{x}{\sqrt{a+a \cos[c+dx]}} dx$$

Optimal (type 4, 156 leaves, 6 steps):

$$\begin{aligned}
 & - \frac{4 i x \operatorname{ArcTan}\left[e^{\frac{1}{2} i (c+dx)}\right] \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]}{d \sqrt{a+a \operatorname{Cos}[c+dx]}} + \\
 & \frac{4 i \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{PolyLog}\left[2, -i e^{\frac{1}{2} i (c+dx)}\right]}{d^2 \sqrt{a+a \operatorname{Cos}[c+dx]}} - \frac{4 i \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{PolyLog}\left[2, i e^{\frac{1}{2} i (c+dx)}\right]}{d^2 \sqrt{a+a \operatorname{Cos}[c+dx]}}
 \end{aligned}$$

Result (type 4, 333 leaves):

$$\begin{aligned}
 & - \frac{1}{d^2 \sqrt{a(1+\operatorname{Cos}[c+dx])}} \\
 & 2 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \left(dx \operatorname{Log}\left[1 - \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]\right] + 2 i \operatorname{Log}\left[1 - \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]\right] \right) \\
 & \operatorname{Log}\left[\left(\frac{1}{2} + \frac{i}{2}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]\right)\right] - 2 i \operatorname{Log}\left[1 - \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]\right] \\
 & \operatorname{Log}\left[\left(\frac{1}{2} - \frac{i}{2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]\right)\right] - dx \operatorname{Log}\left[1 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]\right] - \\
 & 2 i \operatorname{Log}\left[\frac{1}{2} \left((1+i) - (1-i) \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]\right)\right] \operatorname{Log}\left[1 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]\right] + \\
 & 2 i \operatorname{Log}\left[\left(-\frac{1}{2} - \frac{i}{2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]\right)\right] \operatorname{Log}\left[1 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]\right] + \\
 & 2 i \operatorname{PolyLog}\left[2, \left(-\frac{1}{2} - \frac{i}{2}\right) \left(-1 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]\right)\right] - \\
 & 2 i \operatorname{PolyLog}\left[2, \left(-\frac{1}{2} + \frac{i}{2}\right) \left(-1 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]\right)\right] - \\
 & 2 i \operatorname{PolyLog}\left[2, \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]\right)\right] + \\
 & 2 i \operatorname{PolyLog}\left[2, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]\right)\right]
 \end{aligned}$$

Problem 180: Result more than twice size of optimal antiderivative.

$$\int \frac{x^3}{(a+a \operatorname{Cos}[x])^{3/2}} dx$$

Optimal (type 4, 423 leaves, 16 steps):

$$\begin{aligned}
 & -\frac{3x^2}{a\sqrt{a+a\cos[x]}} - \frac{24ix\operatorname{ArcTan}\left[e^{\frac{ix}{2}}\right]\operatorname{Cos}\left[\frac{x}{2}\right]}{a\sqrt{a+a\cos[x]}} - \frac{ix^3\operatorname{ArcTan}\left[e^{\frac{ix}{2}}\right]\operatorname{Cos}\left[\frac{x}{2}\right]}{a\sqrt{a+a\cos[x]}} + \\
 & \frac{24i\operatorname{Cos}\left[\frac{x}{2}\right]\operatorname{PolyLog}\left[2,-ie^{\frac{ix}{2}}\right]}{a\sqrt{a+a\cos[x]}} + \frac{3ix^2\operatorname{Cos}\left[\frac{x}{2}\right]\operatorname{PolyLog}\left[2,-ie^{\frac{ix}{2}}\right]}{a\sqrt{a+a\cos[x]}} - \\
 & \frac{24i\operatorname{Cos}\left[\frac{x}{2}\right]\operatorname{PolyLog}\left[2,ie^{\frac{ix}{2}}\right]}{a\sqrt{a+a\cos[x]}} - \frac{3ix^2\operatorname{Cos}\left[\frac{x}{2}\right]\operatorname{PolyLog}\left[2,ie^{\frac{ix}{2}}\right]}{a\sqrt{a+a\cos[x]}} - \\
 & \frac{12x\operatorname{Cos}\left[\frac{x}{2}\right]\operatorname{PolyLog}\left[3,-ie^{\frac{ix}{2}}\right]}{a\sqrt{a+a\cos[x]}} + \frac{12x\operatorname{Cos}\left[\frac{x}{2}\right]\operatorname{PolyLog}\left[3,ie^{\frac{ix}{2}}\right]}{a\sqrt{a+a\cos[x]}} - \\
 & \frac{24i\operatorname{Cos}\left[\frac{x}{2}\right]\operatorname{PolyLog}\left[4,-ie^{\frac{ix}{2}}\right]}{a\sqrt{a+a\cos[x]}} + \frac{24i\operatorname{Cos}\left[\frac{x}{2}\right]\operatorname{PolyLog}\left[4,ie^{\frac{ix}{2}}\right]}{a\sqrt{a+a\cos[x]}} + \frac{x^3\tan\left[\frac{x}{2}\right]}{2a\sqrt{a+a\cos[x]}}
 \end{aligned}$$

Result (type 4, 1391 leaves):

$$\begin{aligned}
 & -\frac{6x^2\operatorname{Cos}\left[\frac{x}{2}\right]^3}{\left(a(1+\operatorname{Cos}[x])\right)^{3/2}} + \left(48\operatorname{Cos}\left[\frac{x}{2}\right]^3\right. \\
 & \quad \left.\left(\frac{1}{2}x\left(\operatorname{Log}\left[1-ie^{\frac{ix}{2}}\right]-\operatorname{Log}\left[1+ie^{\frac{ix}{2}}\right]\right)+i\left(\operatorname{PolyLog}\left[2,-ie^{\frac{ix}{2}}\right]-\operatorname{PolyLog}\left[2,ie^{\frac{ix}{2}}\right]\right)\right)\right)/ \\
 & \quad \left(a(1+\operatorname{Cos}[x])\right)^{3/2} + \frac{1}{\left(a(1+\operatorname{Cos}[x])\right)^{3/2}} \\
 & 8\operatorname{Cos}\left[\frac{x}{2}\right]^3\left(\frac{1}{8}\pi^3\operatorname{Log}\left[\operatorname{Cot}\left[\frac{1}{2}\left(\frac{\pi}{2}-\frac{x}{2}\right)\right]\right]+\frac{3}{4}\pi^2\left(\left(\frac{\pi}{2}-\frac{x}{2}\right)\left(\operatorname{Log}\left[1-e^{i\left(\frac{\pi-x}{2}\right)}\right]-\operatorname{Log}\left[1+e^{i\left(\frac{\pi-x}{2}\right)}\right]\right)\right)+\right. \\
 & \quad \left. i\left(\operatorname{PolyLog}\left[2,-e^{i\left(\frac{\pi-x}{2}\right)}\right]-\operatorname{PolyLog}\left[2,e^{i\left(\frac{\pi-x}{2}\right)}\right]\right)\right)- \\
 & \quad \frac{3}{2}\pi\left(\left(\frac{\pi}{2}-\frac{x}{2}\right)^2\left(\operatorname{Log}\left[1-e^{i\left(\frac{\pi-x}{2}\right)}\right]-\operatorname{Log}\left[1+e^{i\left(\frac{\pi-x}{2}\right)}\right]\right)+\right. \\
 & \quad \left.2i\left(\frac{\pi}{2}-\frac{x}{2}\right)\left(\operatorname{PolyLog}\left[2,-e^{i\left(\frac{\pi-x}{2}\right)}\right]-\operatorname{PolyLog}\left[2,e^{i\left(\frac{\pi-x}{2}\right)}\right]\right)\right)+ \\
 & \quad 2\left(-\operatorname{PolyLog}\left[3,-e^{i\left(\frac{\pi-x}{2}\right)}\right]+\operatorname{PolyLog}\left[3,e^{i\left(\frac{\pi-x}{2}\right)}\right]\right)+8\left(\frac{1}{4}i\left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\frac{x}{2}\right)\right)^4+\right. \\
 & \quad \left.\frac{1}{64}i\left(\frac{\pi}{2}-\frac{x}{2}\right)^4-\frac{1}{8}\pi^3\left(i\left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\frac{x}{2}\right)\right)-\operatorname{Log}\left[1+e^{2i\left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\frac{x}{2}\right)\right)}\right]\right)-\right. \\
 & \quad \left.\left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\frac{x}{2}\right)\right)^3\operatorname{Log}\left[1+e^{2i\left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\frac{x}{2}\right)\right)}\right]-\frac{1}{8}\left(\frac{\pi}{2}-\frac{x}{2}\right)^3\operatorname{Log}\left[1+e^{i\left(\frac{\pi-x}{2}\right)}\right]+\right. \\
 & \quad \left.\frac{3}{4}\pi^2\left(\frac{1}{2}i\left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\frac{x}{2}\right)\right)^2-\left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\frac{x}{2}\right)\right)\operatorname{Log}\left[1+e^{2i\left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\frac{x}{2}\right)\right)}\right]+\right. \\
 & \quad \left.\frac{1}{2}i\operatorname{PolyLog}\left[2,-e^{2i\left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\frac{x}{2}\right)\right)}\right]+\frac{3}{2}i\left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\frac{x}{2}\right)\right)^2\right. \\
 & \quad \left.\operatorname{PolyLog}\left[2,-e^{2i\left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\frac{x}{2}\right)\right)}\right]+\frac{3}{8}i\left(\frac{\pi}{2}-\frac{x}{2}\right)^2\operatorname{PolyLog}\left[2,-e^{i\left(\frac{\pi-x}{2}\right)}\right]-\right. \\
 & \quad \left.\frac{3}{2}\pi\left(\frac{1}{3}i\left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\frac{x}{2}\right)\right)^3-\left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\frac{x}{2}\right)\right)^2\operatorname{Log}\left[1+e^{2i\left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\frac{x}{2}\right)\right)}\right]+\right.
 \end{aligned}$$

$$\begin{aligned}
 & \left(\frac{i}{2} \left(\frac{\pi}{2} + \frac{1}{2} \left(-\frac{\pi}{2} + \frac{x}{2} \right) \right) \text{PolyLog}\left[2, -e^{2i \left(\frac{\pi}{2} + \frac{1}{2} \left(-\frac{\pi}{2} + \frac{x}{2} \right) \right)}\right] - \frac{1}{2} \text{PolyLog}\left[3, -e^{2i \left(\frac{\pi}{2} + \frac{1}{2} \left(-\frac{\pi}{2} + \frac{x}{2} \right) \right)}\right] \right) - \\
 & \frac{3}{2} \left(\frac{\pi}{2} + \frac{1}{2} \left(-\frac{\pi}{2} + \frac{x}{2} \right) \right) \text{PolyLog}\left[3, -e^{2i \left(\frac{\pi}{2} + \frac{1}{2} \left(-\frac{\pi}{2} + \frac{x}{2} \right) \right)}\right] - \frac{3}{4} \left(\frac{\pi}{2} - \frac{x}{2} \right) \text{PolyLog}\left[3, -e^{i \left(\frac{\pi}{2} - \frac{x}{2} \right)}\right] - \\
 & \frac{3}{4} i \text{PolyLog}\left[4, -e^{2i \left(\frac{\pi}{2} + \frac{1}{2} \left(-\frac{\pi}{2} + \frac{x}{2} \right) \right)}\right] - \frac{3}{4} i \text{PolyLog}\left[4, -e^{i \left(\frac{\pi}{2} - \frac{x}{2} \right)}\right] \right) + \\
 & \frac{x^3 \text{Cos}\left[\frac{x}{2}\right]^3}{2 \left(a \left(1 + \text{Cos}[x] \right) \right)^{3/2} \left(\text{Cos}\left[\frac{x}{4}\right] - \text{Sin}\left[\frac{x}{4}\right] \right)^2} - \\
 & \frac{6 x^2 \text{Cos}\left[\frac{x}{2}\right]^3 \text{Sin}\left[\frac{x}{4}\right]}{\left(a \left(1 + \text{Cos}[x] \right) \right)^{3/2} \left(\text{Cos}\left[\frac{x}{4}\right] - \text{Sin}\left[\frac{x}{4}\right] \right)} - \\
 & \frac{x^3 \text{Cos}\left[\frac{x}{2}\right]^3}{2 \left(a \left(1 + \text{Cos}[x] \right) \right)^{3/2} \left(\text{Cos}\left[\frac{x}{4}\right] + \text{Sin}\left[\frac{x}{4}\right] \right)^2} + \\
 & \frac{6 x^2 \text{Cos}\left[\frac{x}{2}\right]^3 \text{Sin}\left[\frac{x}{4}\right]}{\left(a \left(1 + \text{Cos}[x] \right) \right)^{3/2} \left(\text{Cos}\left[\frac{x}{4}\right] + \text{Sin}\left[\frac{x}{4}\right] \right)}
 \end{aligned}$$

Problem 187: Result more than twice size of optimal antiderivative.

$$\int \frac{x}{a + b \text{Cos}[c + dx]} dx$$

Optimal (type 4, 214 leaves, 8 steps):

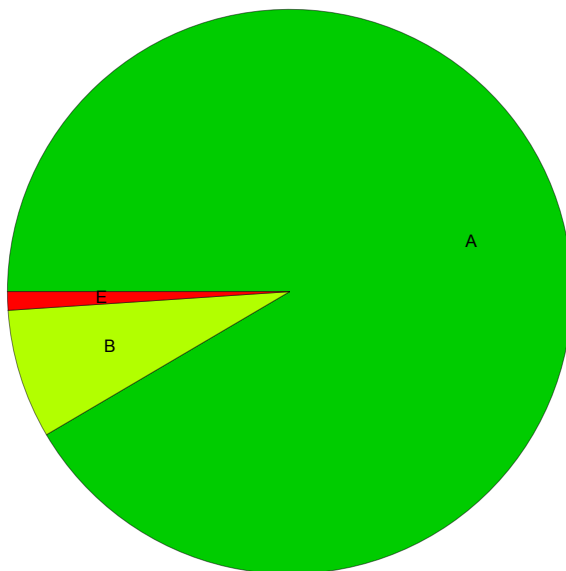
$$- \frac{i x \text{Log}\left[1 + \frac{b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{\sqrt{a^2 - b^2} d} + \frac{i x \text{Log}\left[1 + \frac{b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{\sqrt{a^2 - b^2} d} - \frac{\text{PolyLog}\left[2, -\frac{b e^{i(c+dx)}}{a - \sqrt{a^2 - b^2}}\right]}{\sqrt{a^2 - b^2} d^2} + \frac{\text{PolyLog}\left[2, -\frac{b e^{i(c+dx)}}{a + \sqrt{a^2 - b^2}}\right]}{\sqrt{a^2 - b^2} d^2}$$

Result (type 4, 756 leaves):

$$\begin{aligned}
 & \frac{1}{\sqrt{-a^2+b^2} d^2} \left(2 (c+d x) \operatorname{ArcTanh} \left[\frac{(a+b) \operatorname{Cot} \left[\frac{1}{2} (c+d x) \right]}{\sqrt{-a^2+b^2}} \right] - \right. \\
 & 2 \left(c + \operatorname{ArcCos} \left[-\frac{a}{b} \right] \right) \operatorname{ArcTanh} \left[\frac{(-a+b) \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right]}{\sqrt{-a^2+b^2}} \right] + \\
 & \left. \left(\operatorname{ArcCos} \left[-\frac{a}{b} \right] - 2 i \operatorname{ArcTanh} \left[\frac{(a+b) \operatorname{Cot} \left[\frac{1}{2} (c+d x) \right]}{\sqrt{-a^2+b^2}} \right] + 2 i \operatorname{ArcTanh} \left[\frac{(-a+b) \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right]}{\sqrt{-a^2+b^2}} \right] \right) \right. \\
 & \left. \operatorname{Log} \left[\frac{\sqrt{-a^2+b^2} e^{-\frac{1}{2} i (c+d x)}}{\sqrt{2} \sqrt{b} \sqrt{a+b \operatorname{Cos} [c+d x]}} \right] + \right. \\
 & \left. \left(\operatorname{ArcCos} \left[-\frac{a}{b} \right] + 2 i \left(\operatorname{ArcTanh} \left[\frac{(a+b) \operatorname{Cot} \left[\frac{1}{2} (c+d x) \right]}{\sqrt{-a^2+b^2}} \right] - \operatorname{ArcTanh} \left[\frac{(-a+b) \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right]}{\sqrt{-a^2+b^2}} \right] \right) \right) \right. \\
 & \left. \operatorname{Log} \left[\frac{\sqrt{-a^2+b^2} e^{\frac{1}{2} i (c+d x)}}{\sqrt{2} \sqrt{b} \sqrt{a+b \operatorname{Cos} [c+d x]}} \right] - \left(\operatorname{ArcCos} \left[-\frac{a}{b} \right] - 2 i \operatorname{ArcTanh} \left[\frac{(-a+b) \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right]}{\sqrt{-a^2+b^2}} \right] \right) \right. \\
 & \left. \operatorname{Log} \left[\frac{(a+b) (-a+b-i \sqrt{-a^2+b^2}) \left(1+i \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right] \right)}{b (a+b+\sqrt{-a^2+b^2} \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right])} \right] - \right. \\
 & \left. \left(\operatorname{ArcCos} \left[-\frac{a}{b} \right] + 2 i \operatorname{ArcTanh} \left[\frac{(-a+b) \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right]}{\sqrt{-a^2+b^2}} \right] \right) \right. \\
 & \left. \operatorname{Log} \left[\frac{(a+b) (i a-i b+\sqrt{-a^2+b^2}) (i+\operatorname{Tan} \left[\frac{1}{2} (c+d x) \right])}{b (a+b+\sqrt{-a^2+b^2} \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right])} \right] + \right. \\
 & i \left(\operatorname{PolyLog} \left[2, \frac{(a-i \sqrt{-a^2+b^2}) (a+b-\sqrt{-a^2+b^2} \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right])}{b (a+b+\sqrt{-a^2+b^2} \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right])} \right] - \right. \\
 & \left. \left. \operatorname{PolyLog} \left[2, \frac{(a+i \sqrt{-a^2+b^2}) (a+b-\sqrt{-a^2+b^2} \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right])}{b (a+b+\sqrt{-a^2+b^2} \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right])} \right] \right) \right)
 \end{aligned}$$

Summary of Integration Test Results

189 integration problems



A - 173 optimal antiderivatives

B - 14 more than twice size of optimal antiderivatives

C - 0 unnecessarily complex antiderivatives

D - 0 unable to integrate problems

E - 2 integration timeouts