Mathematica 11.3 Integration Test Results

Test results for the 99 problems in "4.2.12 (e x)^m (a+b cos(c+d x^n))^p.m"

Problem 3: Result more than twice size of optimal antiderivative.

$$\int x \operatorname{Cos}\left[a+b x^2\right] \, dx$$

Optimal (type 3, 15 leaves, 2 steps):

$$\frac{\operatorname{Sin}\left[a+b x^2\right]}{2 b}$$

Result (type 3, 31 leaves):

$$\frac{\operatorname{Cos}\left[b x^2\right] \operatorname{Sin}[a]}{2 b}+\frac{\operatorname{Cos}[a] \operatorname{Sin}\left[b x^2\right]}{2 b}$$

Problem 90: Result unnecessarily involves imaginary or complex numbers.

$$\int x^2 \operatorname{Cos}\left[a+b \sqrt{c+d x}\right] \, dx$$

Optimal (type 3, 346 leaves, 14 steps):

$$\frac{240 \operatorname{Cos}\left[a+b \sqrt{c+d x}\right]}{b^6 d^3}+\frac{24 c \operatorname{Cos}\left[a+b \sqrt{c+d x}\right]}{b^4 d^3}+\frac{2 c^2 \operatorname{Cos}\left[a+b \sqrt{c+d x}\right]}{b^2 d^3}-\frac{120 (c+d x) \operatorname{Cos}\left[a+b \sqrt{c+d x}\right]}{b^4 d^3}-\frac{12 c (c+d x) \operatorname{Cos}\left[a+b \sqrt{c+d x}\right]}{b^2 d^3}+\frac{10 (c+d x)^2 \operatorname{Cos}\left[a+b \sqrt{c+d x}\right]}{b^2 d^3}+\frac{240 \sqrt{c+d x} \operatorname{Sin}\left[a+b \sqrt{c+d x}\right]}{b^5 d^3}+\frac{24 c \sqrt{c+d x} \operatorname{Sin}\left[a+b \sqrt{c+d x}\right]}{b^3 d^3}+\frac{2 c^2 \sqrt{c+d x} \operatorname{Sin}\left[a+b \sqrt{c+d x}\right]}{b d^3}-\frac{40 (c+d x)^{3/2} \operatorname{Sin}\left[a+b \sqrt{c+d x}\right]}{b^3 d^3}-\frac{4 c (c+d x)^{3/2} \operatorname{Sin}\left[a+b \sqrt{c+d x}\right]}{b d^3}+\frac{2 (c+d x)^{5/2} \operatorname{Sin}\left[a+b \sqrt{c+d x}\right]}{b d^3}$$

Result (type 3, 224 leaves):

$$\frac{1}{b^6 d^3} e^{-i \left(a+b \sqrt{c+d x}\right)} \left(120 + 120 i b \sqrt{c+d x} + i b^5 d^2 x^2 \sqrt{c+d x} - 4 i b^3 \sqrt{c+d x} \left(2 c + 5 d x\right) - 12 b^2 \left(4 c + 5 d x\right) + b^4 d x \left(4 c + 5 d x\right) + e^{2 i \left(a+b \sqrt{c+d x}\right)} \left(120 - 120 i b \sqrt{c+d x} - i b^5 d^2 x^2 \sqrt{c+d x} + 4 i b^3 \sqrt{c+d x} \left(2 c + 5 d x\right) - 12 b^2 \left(4 c + 5 d x\right) + b^4 d x \left(4 c + 5 d x\right)\right)\right)$$

Problem 93: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{Cos}\left[a + b \sqrt{c + d x}\right]}{x} \, dx$$

Optimal (type 4, 126 leaves, 8 steps):

$$\text{Cos}\left[a - b \sqrt{c}\right] \text{CosIntegral}\left[b \left(\sqrt{c} + \sqrt{c + d x}\right)\right] + \text{Cos}\left[a + b \sqrt{c}\right] \text{CosIntegral}\left[b \sqrt{c} - b \sqrt{c + d x}\right] - \text{Sin}\left[a - b \sqrt{c}\right] \text{SinIntegral}\left[b \left(\sqrt{c} + \sqrt{c + d x}\right)\right] + \text{Sin}\left[a + b \sqrt{c}\right] \text{SinIntegral}\left[b \sqrt{c} - b \sqrt{c + d x}\right]$$

Result (type 4, 145 leaves):

$$\frac{1}{2} e^{-i \left(a+b \sqrt{c}\right)} \left(\text{ExpIntegralEi}\left[-i b \left(-\sqrt{c} + \sqrt{c + d x}\right)\right] + e^{2 i \left(a+b \sqrt{c}\right)} \text{ExpIntegralEi}\left[i b \left(-\sqrt{c} + \sqrt{c + d x}\right)\right] + e^{2 i b \sqrt{c}} \text{ExpIntegralEi}\left[-i b \left(\sqrt{c} + \sqrt{c + d x}\right)\right] + e^{2 i a} \text{ExpIntegralEi}\left[i b \left(\sqrt{c} + \sqrt{c + d x}\right)\right]\right)$$

Problem 94: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{Cos}\left[a + b \sqrt{c + d x}\right]}{x^2} \, dx$$

Optimal (type 4, 184 leaves, 10 steps):

$$-\frac{\text{Cos}\left[a + b \sqrt{c + d x}\right]}{x} + \frac{b d \,\text{CosIntegral}\left[b \left(\sqrt{c} + \sqrt{c + d x}\right)\right] \text{Sin}\left[a - b \sqrt{c}\right]}{2 \sqrt{c}} - \frac{b d \,\text{CosIntegral}\left[b \sqrt{c} - b \sqrt{c + d x}\right] \text{Sin}\left[a + b \sqrt{c}\right]}{2 \sqrt{c}} + \frac{b d \,\text{Cos}\left[a - b \sqrt{c}\right] \text{SinIntegral}\left[b \left(\sqrt{c} + \sqrt{c + d x}\right)\right]}{2 \sqrt{c}} + \frac{b d \,\text{Cos}\left[a + b \sqrt{c}\right] \text{SinIntegral}\left[b \sqrt{c} - b \sqrt{c + d x}\right]}{2 \sqrt{c}}$$

Result (type 4, 240 leaves):

$$\frac{1}{4\sqrt{c}\,x}\,\mathrm{i}\left(e^{-\mathrm{i}a}\left(2\,\mathrm{i}\sqrt{c}\,e^{-\mathrm{i}b\sqrt{c+dx}}-b\,d\,e^{-\mathrm{i}b\sqrt{c}}\,x\,\mathrm{ExpIntegralEi}\left[-\mathrm{i}b\left(-\sqrt{c}+\sqrt{c+dx}\right)\right]+b\,d\,e^{\mathrm{i}b\sqrt{c}}\,x\,\mathrm{ExpIntegralEi}\left[-\mathrm{i}b\left(\sqrt{c}+\sqrt{c+dx}\right)\right]\right)+e^{\mathrm{i}\left(a-b\sqrt{c}\right)}\left(2\,\mathrm{i}\sqrt{c}\,e^{\mathrm{i}b\left(\sqrt{c}+\sqrt{c+dx}\right)}+b\,d\,e^{2\mathrm{i}b\sqrt{c}}\,x\,\mathrm{ExpIntegralEi}\left[\mathrm{i}b\left(-\sqrt{c}+\sqrt{c+dx}\right)\right]-b\,d\,x\,\mathrm{ExpIntegralEi}\left[\mathrm{i}b\left(\sqrt{c}+\sqrt{c+dx}\right)\right]\right)\right)$$

Problem 95: Result unnecessarily involves imaginary or complex numbers.

$$\int x^2\,\mathrm{Cos}\left[a+b\,(c+d\,x)^{1/3}\right]\,dx$$

Optimal (type 3, 537 leaves, 20 steps):

$$-\frac{720\,c\,\mathrm{Cos}\left[a+b\,(c+d\,x)^{1/3}\right]}{b^6\,d^3}-\frac{120960\,(c+d\,x)^{1/3}\,\mathrm{Cos}\left[a+b\,(c+d\,x)^{1/3}\right]}{b^8\,d^3}+\frac{6\,c^2\,(c+d\,x)^{1/3}\,\mathrm{Cos}\left[a+b\,(c+d\,x)^{1/3}\right]}{b^2\,d^3}+\frac{360\,c\,(c+d\,x)^{2/3}\,\mathrm{Cos}\left[a+b\,(c+d\,x)^{1/3}\right]}{b^4\,d^3}+\frac{20160\,(c+d\,x)\,\mathrm{Cos}\left[a+b\,(c+d\,x)^{1/3}\right]}{b^6\,d^3}-\frac{30\,c\,(c+d\,x)^{4/3}\,\mathrm{Cos}\left[a+b\,(c+d\,x)^{1/3}\right]}{b^2\,d^3}-\frac{1008\,(c+d\,x)^{5/3}\,\mathrm{Cos}\left[a+b\,(c+d\,x)^{1/3}\right]}{b^4\,d^3}+\frac{24\,(c+d\,x)^{7/3}\,\mathrm{Cos}\left[a+b\,(c+d\,x)^{1/3}\right]}{b^2\,d^3}+\frac{120960\,\mathrm{Sin}\left[a+b\,(c+d\,x)^{1/3}\right]}{b^9\,d^3}-\frac{6\,c^2\,\mathrm{Sin}\left[a+b\,(c+d\,x)^{1/3}\right]}{b^3\,d^3}-\frac{720\,c\,(c+d\,x)^{1/3}\,\mathrm{Sin}\left[a+b\,(c+d\,x)^{1/3}\right]}{b^5\,d^3}-\frac{60480\,(c+d\,x)^{2/3}\,\mathrm{Sin}\left[a+b\,(c+d\,x)^{1/3}\right]}{b^7\,d^3}+\frac{3\,c^2\,(c+d\,x)^{2/3}\,\mathrm{Sin}\left[a+b\,(c+d\,x)^{1/3}\right]}{b\,d^3}+\frac{120\,c\,(c+d\,x)\,\mathrm{Sin}\left[a+b\,(c+d\,x)^{1/3}\right]}{b^3\,d^3}+\frac{5040\,(c+d\,x)^{4/3}\,\mathrm{Sin}\left[a+b\,(c+d\,x)^{1/3}\right]}{b^5\,d^3}-\frac{6\,c\,(c+d\,x)^{5/3}\,\mathrm{Sin}\left[a+b\,(c+d\,x)^{1/3}\right]}{b\,d^3}-\frac{168\,(c+d\,x)^{2}\,\mathrm{Sin}\left[a+b\,(c+d\,x)^{1/3}\right]}{b^3\,d^3}+\frac{3\,(c+d\,x)^{8/3}\,\mathrm{Sin}\left[a+b\,(c+d\,x)^{1/3}\right]}{b\,d^3}$$

Result (type 3, 382 leaves):

$$\frac{1}{2 b^9 d^3} 3 e^{-i \left(a+b (c+d x)^{1/3}\right)} \left(-40320 i \left(-1+e^{2 i \left(a+b (c+d x)^{1/3}\right)}\right)-40320 b \left(1+e^{2 i \left(a+b (c+d x)^{1/3}\right)}\right) (c+d x)^{1/3}+ 20160 i b^2 \left(-1+e^{2 i \left(a+b (c+d x)^{1/3}\right)}\right) (c+d x)^{2/3}-i b^8 d^2 \left(-1+e^{2 i \left(a+b (c+d x)^{1/3}\right)}\right) x^2 (c+d x)^{2/3}+ 2 b^7 d \left(1+e^{2 i \left(a+b (c+d x)^{1/3}\right)}\right) x (c+d x)^{1/3} (3 c+4 d x)- 240 i b^4 \left(-1+e^{2 i \left(a+b (c+d x)^{1/3}\right)}\right) (c+d x)^{1/3} (6 c+7 d x)- 24 b^5 \left(1+e^{2 i \left(a+b (c+d x)^{1/3}\right)}\right) (c+d x)^{2/3} (9 c+14 d x)+240 b^3 \left(1+e^{2 i \left(a+b (c+d x)^{1/3}\right)}\right) (27 c+28 d x)+2 i b^6 \left(-1+e^{2 i \left(a+b (c+d x)^{1/3}\right)}\right) \left(9 c^2+36 c d x+28 d^2 x^2\right)\right)$$

Problem 98: Result is not expressed in closed-form.

$$\int \frac{\text{Cos}\left[a+b (c+d x)^{1/3}\right]}{x} \, dx$$

Optimal (type 4, 234 leaves, 11 steps):

$$\text{Cos}\left[a+b c^{1/3}\right] \text{CosIntegral}\left[b c^{1/3}-b (c+d x)^{1/3}\right]+ \text{Cos}\left[a+(-1)^{2/3} b c^{1/3}\right] \text{CosIntegral}\left[(-1)^{2/3} b c^{1/3}-b (c+d x)^{1/3}\right]+ \text{Cos}\left[a-(-1)^{1/3} b c^{1/3}\right] \text{CosIntegral}\left[(-1)^{1/3} b c^{1/3}+b (c+d x)^{1/3}\right]+ \text{Sin}\left[a+b c^{1/3}\right] \text{SinIntegral}\left[b c^{1/3}-b (c+d x)^{1/3}\right]+ \text{Sin}\left[a+(-1)^{2/3} b c^{1/3}\right] \text{SinIntegral}\left[(-1)^{2/3} b c^{1/3}-b (c+d x)^{1/3}\right]- \text{Sin}\left[a-(-1)^{1/3} b c^{1/3}\right] \text{SinIntegral}\left[(-1)^{1/3} b c^{1/3}+b (c+d x)^{1/3}\right]$$

Result (type 7, 243 leaves):

$$\frac{1}{2}\left(\text{RootSum}\left[c-\#1^3\&, \text{Cos}[a+b \#1] \text{CosIntegral}\left[b \left((c+d x)^{1/3}-\#1\right)\right]- i \text{CosIntegral}\left[b \left((c+d x)^{1/3}-\#1\right)\right] \text{Sin}[a+b \#1]-i \text{Cos}[a+b \#1] \text{SinIntegral}\left[b \left((c+d x)^{1/3}-\#1\right)\right]-\text{Sin}[a+b \#1] \text{SinIntegral}\left[b \left((c+d x)^{1/3}-\#1\right)\right]\&\right]+ \text{RootSum}\left[c-\#1^3\&, \text{Cos}[a+b \#1] \text{CosIntegral}\left[b \left((c+d x)^{1/3}-\#1\right)\right]+ i \text{CosIntegral}\left[b \left((c+d x)^{1/3}-\#1\right)\right] \text{Sin}[a+b \#1]+i \text{Cos}[a+b \#1] \text{SinIntegral}\left[b \left((c+d x)^{1/3}-\#1\right)\right]-\text{Sin}[a+b \#1] \text{SinIntegral}\left[b \left((c+d x)^{1/3}-\#1\right)\right]\&\right]\right)$$

Problem 99: Result is not expressed in closed-form.

$$\int \frac{\text{Cos}\left[a+b (c+d x)^{1/3}\right]}{x^2} \, dx$$

Optimal (type 4, 332 leaves, 13 steps):

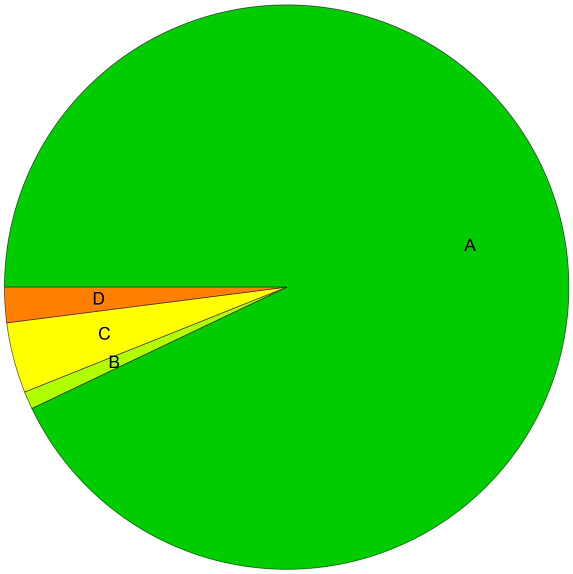
$$-\frac{\text{Cos}\left[a+b\,(c+d\,x)^{1/3}\right]}{x}-\frac{b\,d\,\text{CosIntegral}\left[b\,c^{1/3}-b\,(c+d\,x)^{1/3}\right]\,\text{Sin}\left[a+b\,c^{1/3}\right]}{3\,c^{2/3}}+$$
$$\frac{1}{3\,c^{2/3}}(-1)^{1/3}\,b\,d\,\text{CosIntegral}\left[(-1)^{1/3}\,b\,c^{1/3}+b\,(c+d\,x)^{1/3}\right]\,\text{Sin}\left[a-(-1)^{1/3}\,b\,c^{1/3}\right]-$$
$$\frac{1}{3\,c^{2/3}}(-1)^{2/3}\,b\,d\,\text{CosIntegral}\left[(-1)^{2/3}\,b\,c^{1/3}-b\,(c+d\,x)^{1/3}\right]\,\text{Sin}\left[a+(-1)^{2/3}\,b\,c^{1/3}\right]+$$
$$\frac{b\,d\,\text{Cos}\left[a+b\,c^{1/3}\right]\,\text{SinIntegral}\left[b\,c^{1/3}-b\,(c+d\,x)^{1/3}\right]}{3\,c^{2/3}}+\frac{1}{3\,c^{2/3}}$$
$$(-1)^{2/3}\,b\,d\,\text{Cos}\left[a+(-1)^{2/3}\,b\,c^{1/3}\right]\,\text{SinIntegral}\left[(-1)^{2/3}\,b\,c^{1/3}-b\,(c+d\,x)^{1/3}\right]+$$
$$\frac{1}{3\,c^{2/3}}(-1)^{1/3}\,b\,d\,\text{Cos}\left[a-(-1)^{1/3}\,b\,c^{1/3}\right]\,\text{SinIntegral}\left[(-1)^{1/3}\,b\,c^{1/3}+b\,(c+d\,x)^{1/3}\right]$$

Result (type 7, 138 leaves):

$$-\frac{\text{Cos}\left[a+b\,(c+d\,x)^{1/3}\right]}{x}-$$
$$\frac{1}{6}\,\mathrm{i}\,b\,d\,\text{RootSum}\left[c-\#1^3\,\&,\,\frac{e^{-\mathrm{i}\,a-\mathrm{i}\,b\,\#1}\,\text{ExpIntegralEi}\left[-\mathrm{i}\,b\left((c+d\,x)^{1/3}-\#1\right)\right]}{\#1^2}\,\&\right]+$$
$$\frac{1}{6}\,\mathrm{i}\,b\,d\,\text{RootSum}\left[c-\#1^3\,\&,\,\frac{e^{\mathrm{i}\,a+\mathrm{i}\,b\,\#1}\,\text{ExpIntegralEi}\left[\mathrm{i}\,b\left((c+d\,x)^{1/3}-\#1\right)\right]}{\#1^2}\,\&\right]$$

Summary of Integration Test Results

99 integration problems



A - 92 optimal antiderivatives

B - 1 more than twice size of optimal antiderivatives

C - 4 unnecessarily complex antiderivatives

D - 2 unable to integrate problems

E - 0 integration timeouts