

# Mathematica 11.3 Integration Test Results

Test results for the 98 problems in "4.2.7 (d trig)<sup>m</sup> (a+b (c cos)<sup>n</sup>)<sup>p.m"</sup>

Problem 6: Result more than twice size of optimal antiderivative.

$$\int \frac{\sin[x]}{a - a \cos[x]^2} dx$$

Optimal (type 3, 8 leaves, 2 steps):

$$-\frac{\operatorname{ArcTanh}[\cos[x]]}{a}$$

Result (type 3, 21 leaves):

$$-\frac{\log[\cos[\frac{x}{2}]] + \log[\sin[\frac{x}{2}]]}{a}$$

Problem 7: Result more than twice size of optimal antiderivative.

$$\int \frac{\csc[x]}{a - a \cos[x]^2} dx$$

Optimal (type 3, 22 leaves, 3 steps):

$$-\frac{\operatorname{ArcTanh}[\cos[x]]}{2a} - \frac{\cot[x] \csc[x]}{2a}$$

Result (type 3, 51 leaves):

$$-\frac{\frac{1}{8} \csc[\frac{x}{2}]^2 - \frac{1}{2} \log[\cos[\frac{x}{2}]] + \frac{1}{2} \log[\sin[\frac{x}{2}]] + \frac{1}{8} \sec[\frac{x}{2}]^2}{a}$$

Problem 9: Result more than twice size of optimal antiderivative.

$$\int \frac{\csc[x]^3}{a - a \cos[x]^2} dx$$

Optimal (type 3, 35 leaves, 4 steps):

$$-\frac{3 \operatorname{ArcTanh}[\cos[x]]}{8a} - \frac{3 \cot[x] \csc[x]}{8a} - \frac{\cot[x] \csc[x]^3}{4a}$$

Result (type 3, 75 leaves):

$$\frac{-\frac{3}{32} \csc\left(\frac{x}{2}\right)^2 - \frac{1}{64} \csc\left(\frac{x}{2}\right)^4 - \frac{3}{8} \log[\cos\left(\frac{x}{2}\right)] + \frac{3}{8} \log[\sin\left(\frac{x}{2}\right)] + \frac{3}{32} \sec\left(\frac{x}{2}\right)^2 + \frac{1}{64} \sec\left(\frac{x}{2}\right)^4}{a}$$

**Problem 11:** Result more than twice size of optimal antiderivative.

$$\int \frac{\sin[x]^5}{a + b \cos[x]^2} dx$$

Optimal (type 3, 54 leaves, 4 steps):

$$-\frac{(a+b)^2 \operatorname{ArcTan}\left[\frac{\sqrt{b} \cos [x]}{\sqrt{a}}\right]}{\sqrt{a} b^{5/2}} + \frac{(a+2 b) \cos [x]}{b^2} - \frac{\cos [x]^3}{3 b}$$

Result (type 3, 116 leaves):

$$\begin{aligned} & \frac{1}{12 b^{5/2}} \left( -\frac{12 (a+b)^2 \operatorname{ArcTan}\left[\frac{\sqrt{b}-\sqrt{a+b} \tan\left[\frac{x}{2}\right]}{\sqrt{a}}\right]}{\sqrt{a}} - \right. \\ & \left. \frac{12 (a+b)^2 \operatorname{ArcTan}\left[\frac{\sqrt{b}+\sqrt{a+b} \tan\left[\frac{x}{2}\right]}{\sqrt{a}}\right]}{\sqrt{a}} + 3 \sqrt{b} (4 a+7 b) \cos [x] - b^{3/2} \cos [3 x] \right) \end{aligned}$$

**Problem 12:** Result more than twice size of optimal antiderivative.

$$\int \frac{\sin[x]^3}{a + b \cos[x]^2} dx$$

Optimal (type 3, 36 leaves, 3 steps):

$$-\frac{(a+b) \operatorname{ArcTan}\left[\frac{\sqrt{b} \cos [x]}{\sqrt{a}}\right]}{\sqrt{a} b^{3/2}} + \frac{\cos [x]}{b}$$

Result (type 3, 90 leaves):

$$\begin{aligned} & \frac{1}{\sqrt{a} b^{3/2}} \\ & \left( - (a+b) \operatorname{ArcTan}\left[\frac{\sqrt{b}-\sqrt{a+b} \tan\left[\frac{x}{2}\right]}{\sqrt{a}}\right] - (a+b) \operatorname{ArcTan}\left[\frac{\sqrt{b}+\sqrt{a+b} \tan\left[\frac{x}{2}\right]}{\sqrt{a}}\right] + \sqrt{a} \sqrt{b} \cos [x] \right) \end{aligned}$$

**Problem 15:** Result more than twice size of optimal antiderivative.

$$\int \frac{\csc [x]^3}{a + b \cos[x]^2} dx$$

Optimal (type 3, 62 leaves, 5 steps):

$$-\frac{b^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \cos [x]}{\sqrt{a}}\right]}{\sqrt{a} (a+b)^2}-\frac{(a+3 b) \operatorname{ArcTanh}[\cos [x]]}{2 (a+b)^2}-\frac{\cot [x] \csc [x]}{2 (a+b)}$$

Result (type 3, 140 leaves) :

$$\frac{1}{8 \sqrt{a} (a+b)^2} \left( -8 b^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{b}-\sqrt{a+b} \tan \left[\frac{x}{2}\right]}{\sqrt{a}}\right]-8 b^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{b}+\sqrt{a+b} \tan \left[\frac{x}{2}\right]}{\sqrt{a}}\right]+ \right. \\ \left. \sqrt{a} \left(- (a+b) \csc \left[\frac{x}{2}\right]^2-4 (a+3 b) \left(\log [\cos \left[\frac{x}{2}\right]]-\log [\sin \left[\frac{x}{2}\right]]\right)+(a+b) \sec \left[\frac{x}{2}\right]^2\right)\right)$$

Problem 16: Result more than twice size of optimal antiderivative.

$$\int \frac{\csc [x]^5}{a+b \cos [x]^2} dx$$

Optimal (type 3, 94 leaves, 6 steps) :

$$-\frac{b^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \cos [x]}{\sqrt{a}}\right]}{\sqrt{a} (a+b)^3}-\frac{(3 a^2+10 a b+15 b^2) \operatorname{ArcTanh}[\cos [x]]}{8 (a+b)^3}- \\ \frac{(3 a+7 b) \cot [x] \csc [x]}{8 (a+b)^2}-\frac{\cot [x] \csc [x]^3}{4 (a+b)}$$

Result (type 3, 204 leaves) :

$$\frac{1}{64 \sqrt{a} (a+b)^3} \left( -64 b^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{b}-\sqrt{a+b} \tan \left[\frac{x}{2}\right]}{\sqrt{a}}\right]-64 b^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{b}+\sqrt{a+b} \tan \left[\frac{x}{2}\right]}{\sqrt{a}}\right]+ \right. \\ \left. \sqrt{a} \left(-2 (3 a^2+10 a b+7 b^2) \csc \left[\frac{x}{2}\right]^2-(a+b)^2 \csc \left[\frac{x}{2}\right]^4-8 (3 a^2+10 a b+15 b^2) \right. \right. \\ \left. \left. \left(\log [\cos \left[\frac{x}{2}\right]]-\log [\sin \left[\frac{x}{2}\right]]\right)+2 (3 a^2+10 a b+7 b^2) \sec \left[\frac{x}{2}\right]^2+(a+b)^2 \sec \left[\frac{x}{2}\right]^4\right)\right)$$

Problem 32: Result more than twice size of optimal antiderivative.

$$\int \frac{\sec [x]}{a+b \cos [x]^2} dx$$

Optimal (type 3, 41 leaves, 4 steps) :

$$\frac{\operatorname{ArcTanh}[\sin [x]]}{a}-\frac{\sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sin [x]}{\sqrt{a+b}}\right]}{a \sqrt{a+b}}$$

Result (type 3, 93 leaves) :

$$\frac{1}{2 a} \left( -2 \operatorname{Log}[\cos(\frac{x}{2})] - \sin(\frac{x}{2}) + 2 \operatorname{Log}[\cos(\frac{x}{2})] + \sin(\frac{x}{2}) + \frac{\sqrt{b} (\operatorname{Log}[\sqrt{a+b} - \sqrt{b} \sin(x)] - \operatorname{Log}[\sqrt{a+b} + \sqrt{b} \sin(x)])}{\sqrt{a+b}} \right)$$

**Problem 33:** Result more than twice size of optimal antiderivative.

$$\int \frac{\sec^3 x}{a + b \cos^2 x} dx$$

Optimal (type 3, 59 leaves, 5 steps):

$$\frac{(a - 2 b) \operatorname{ArcTanh}[\sin(x)]}{2 a^2} + \frac{b^{3/2} \operatorname{ArcTanh}[\frac{\sqrt{b} \sin(x)}{\sqrt{a+b}}]}{a^2 \sqrt{a+b}} + \frac{\sec(x) \tan(x)}{2 a}$$

Result (type 3, 152 leaves):

$$\begin{aligned} & \frac{1}{4 a^2} \left( -2 (a - 2 b) \operatorname{Log}[\cos(\frac{x}{2})] - \sin(\frac{x}{2}) + \right. \\ & 2 (a - 2 b) \operatorname{Log}[\cos(\frac{x}{2}) + \sin(\frac{x}{2})] - \frac{2 b^{3/2} \operatorname{Log}[\sqrt{a+b} - \sqrt{b} \sin(x)]}{\sqrt{a+b}} + \\ & \left. \frac{2 b^{3/2} \operatorname{Log}[\sqrt{a+b} + \sqrt{b} \sin(x)]}{\sqrt{a+b}} + \frac{a}{(\cos(\frac{x}{2}) - \sin(\frac{x}{2}))^2} - \frac{a}{(\cos(\frac{x}{2}) + \sin(\frac{x}{2}))^2} \right) \end{aligned}$$

**Problem 34:** Result more than twice size of optimal antiderivative.

$$\int \frac{\sec^5 x}{a + b \cos^2 x} dx$$

Optimal (type 3, 90 leaves, 6 steps):

$$\begin{aligned} & \frac{(3 a^2 - 4 a b + 8 b^2) \operatorname{ArcTanh}[\sin(x)]}{8 a^3} - \\ & \frac{b^{5/2} \operatorname{ArcTanh}[\frac{\sqrt{b} \sin(x)}{\sqrt{a+b}}]}{a^3 \sqrt{a+b}} + \frac{(3 a - 4 b) \sec(x) \tan(x)}{8 a^2} + \frac{\sec(x)^3 \tan(x)}{4 a} \end{aligned}$$

Result (type 3, 215 leaves):

$$\frac{1}{16 a^3} \left( -2 (3 a^2 - 4 a b + 8 b^2) \operatorname{Log}[\cos[\frac{x}{2}] - \sin[\frac{x}{2}]] + 2 (3 a^2 - 4 a b + 8 b^2) \operatorname{Log}[\cos[\frac{x}{2}] + \sin[\frac{x}{2}]] + \frac{8 b^{5/2} \operatorname{Log}[\sqrt{a+b} - \sqrt{b} \sin[x]]}{\sqrt{a+b}} - \frac{8 b^{5/2} \operatorname{Log}[\sqrt{a+b} + \sqrt{b} \sin[x]]}{\sqrt{a+b}} + \frac{a^2}{(\cos[\frac{x}{2}] - \sin[\frac{x}{2}])^4} - \frac{a^2}{(\cos[\frac{x}{2}] + \sin[\frac{x}{2}])^4} + \frac{a (-3 a + 4 b)}{(\cos[\frac{x}{2}] + \sin[\frac{x}{2}])^2} + \frac{a (-3 a + 4 b)}{-1 + \sin[x]} \right)$$

**Problem 67:** Result more than twice size of optimal antiderivative.

$$\int \frac{\cos[x]}{\sqrt{1 + \cos[x]^2}} dx$$

Optimal (type 3, 9 leaves, 2 steps):

$$\text{ArcSin}\left[\frac{\sin[x]}{\sqrt{2}}\right]$$

Result (type 3, 19 leaves):

$$\text{ArcTan}\left[\frac{\sqrt{2} \sin[x]}{\sqrt{3 + \cos[2x]}}\right]$$

**Problem 68:** Result more than twice size of optimal antiderivative.

$$\int \frac{\cos[5+3x]}{\sqrt{3 + \cos[5+3x]^2}} dx$$

Optimal (type 3, 15 leaves, 2 steps):

$$\frac{1}{3} \text{ArcSin}\left[\frac{1}{2} \sin[5+3x]\right]$$

Result (type 3, 31 leaves):

$$\frac{1}{3} \text{ArcTan}\left[\frac{\sqrt{2} \sin[5+3x]}{\sqrt{7 + \cos[2(5+3x)]}}\right]$$

**Problem 69:** Result more than twice size of optimal antiderivative.

$$\int \frac{\cos[x]}{\sqrt{4 - \cos[x]^2}} dx$$

Optimal (type 3, 9 leaves, 2 steps):

$$\text{ArcSinh}\left[\frac{\text{Sin}[x]}{\sqrt{3}}\right]$$

Result (type 3, 21 leaves):

$$\text{ArcTanh}\left[\frac{\sqrt{2} \text{Sin}[x]}{\sqrt{7-\text{Cos}[2 x]}}\right]$$

Problem 70: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{a+b \cos[x]^4} dx$$

Optimal (type 3, 487 leaves, 10 steps):

$$\begin{aligned} & \frac{\left(\sqrt{a} + \sqrt{a+b}\right) \text{ArcTan}\left[\frac{a^{1/4} \sqrt{a+b-\sqrt{a} \sqrt{a+b}} - \sqrt{2} (a+b)^{3/4} \text{Cot}[x]}{a^{1/4} \sqrt{a+b+\sqrt{a} \sqrt{a+b}}}\right]}{2 \sqrt{2} a^{3/4} (a+b)^{1/4} \sqrt{a+b+\sqrt{a} \sqrt{a+b}}} - \\ & \frac{\left(\sqrt{a} + \sqrt{a+b}\right) \text{ArcTan}\left[\frac{a^{1/4} \sqrt{a+b-\sqrt{a} \sqrt{a+b}} + \sqrt{2} (a+b)^{3/4} \text{Cot}[x]}{a^{1/4} \sqrt{a+b+\sqrt{a} \sqrt{a+b}}}\right]}{2 \sqrt{2} a^{3/4} (a+b)^{1/4} \sqrt{a+b+\sqrt{a} \sqrt{a+b}}} - \\ & \left(\left(\sqrt{a} - \sqrt{a+b}\right) \text{Log}\left[\sqrt{a} (a+b)^{1/4} - \sqrt{2} a^{1/4} \sqrt{a+b-\sqrt{a} \sqrt{a+b}} \text{Cot}[x] + (a+b)^{3/4} \text{Cot}[x]^2\right]\right) / \\ & \left(4 \sqrt{2} a^{3/4} (a+b)^{1/4} \sqrt{a+b-\sqrt{a} \sqrt{a+b}}\right) + \\ & \left(\left(\sqrt{a} - \sqrt{a+b}\right) \text{Log}\left[\sqrt{a} (a+b)^{1/4} + \sqrt{2} a^{1/4} \sqrt{a+b-\sqrt{a} \sqrt{a+b}} \text{Cot}[x] + (a+b)^{3/4} \text{Cot}[x]^2\right]\right) / \\ & \left(4 \sqrt{2} a^{3/4} (a+b)^{1/4} \sqrt{a+b-\sqrt{a} \sqrt{a+b}}\right) \end{aligned}$$

Result (type 3, 121 leaves):

$$\begin{aligned} & \frac{\text{ArcTan}\left[\frac{\sqrt{a} \tan[x]}{\sqrt{a+i} \sqrt{a} \sqrt{b}}\right]}{2 \sqrt{a} \sqrt{a+\frac{i}{2}} \sqrt{a} \sqrt{b}} - \frac{\text{ArcTanh}\left[\frac{\sqrt{a} \tan[x]}{\sqrt{-a+i} \sqrt{a} \sqrt{b}}\right]}{2 \sqrt{a} \sqrt{-a+\frac{i}{2}} \sqrt{a} \sqrt{b}} \end{aligned}$$

Problem 72: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{1+\cos[x]^4} dx$$

Optimal (type 3, 292 leaves, 10 steps):

$$\begin{aligned}
& \frac{x}{2\sqrt{-1+\sqrt{2}}} + \frac{\operatorname{ArcTan}\left[\frac{(-2+\sqrt{2}) \cos[x] \sin[x]+\sqrt{-1+\sqrt{2}} (1-2 \sin[x]^2)}{2+\sqrt{1+\sqrt{2}}+2 \sqrt{-1+\sqrt{2}} \cos[x] \sin[x]+(-2+\sqrt{2}) \sin[x]^2}\right]}{4\sqrt{-1+\sqrt{2}}} + \\
& \frac{\operatorname{ArcTan}\left[\frac{(-2+\sqrt{2}) \cos[x] \sin[x]+\sqrt{-1+\sqrt{2}} (-1+2 \sin[x]^2)}{2+\sqrt{1+\sqrt{2}}-2 \sqrt{-1+\sqrt{2}} \cos[x] \sin[x]+(-2+\sqrt{2}) \sin[x]^2}\right]}{4\sqrt{-1+\sqrt{2}}} + \\
& \frac{\frac{1}{8} \sqrt{-1+\sqrt{2}} \log \left[\sqrt{2}-2 \sqrt{-1+\sqrt{2}} \cot[x]+2 \cot[x]^2\right]-}{\frac{1}{8} \sqrt{-1+\sqrt{2}} \log \left[1+\sqrt{2 \left(-1+\sqrt{2}\right)} \cot[x]+\sqrt{2} \cot[x]^2\right]}
\end{aligned}$$

Result (type 3, 45 leaves):

$$\frac{\operatorname{ArcTan}\left[\frac{\tan[x]}{\sqrt{1-i}}\right]}{2\sqrt{1-i}} + \frac{\operatorname{ArcTan}\left[\frac{\tan[x]}{\sqrt{1+i}}\right]}{2\sqrt{1+i}}$$

**Problem 74:** Result is not expressed in closed-form.

$$\int \frac{1}{a+b \cos[x]^5} dx$$

Optimal (type 3, 494 leaves, 12 steps):

$$\begin{aligned}
& \frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{a^{1/5}-b^{1/5}} \tan \left[\frac{x}{2}\right]}{\sqrt{a^{1/5}+b^{1/5}}}\right]}{5 a^{4/5} \sqrt{a^{1/5}-b^{1/5}} \sqrt{a^{1/5}+b^{1/5}}} + \frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{a^{1/5}+(-1)^{1/5} b^{1/5}} \tan \left[\frac{x}{2}\right]}{\sqrt{a^{1/5}-(-1)^{1/5} b^{1/5}}}\right]}{5 a^{4/5} \sqrt{a^{1/5}-(-1)^{1/5} b^{1/5}} \sqrt{a^{1/5}+(-1)^{1/5} b^{1/5}}} + \\
& \frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{a^{1/5}-(-1)^{2/5} b^{1/5}} \tan \left[\frac{x}{2}\right]}{\sqrt{a^{1/5}+(-1)^{2/5} b^{1/5}}}\right]}{5 a^{4/5} \sqrt{a^{1/5}-(-1)^{2/5} b^{1/5}} \sqrt{a^{1/5}+(-1)^{2/5} b^{1/5}}} + \\
& \frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{a^{1/5}+(-1)^{3/5} b^{1/5}} \tan \left[\frac{x}{2}\right]}{\sqrt{a^{1/5}-(-1)^{3/5} b^{1/5}}}\right]}{5 a^{4/5} \sqrt{a^{1/5}-(-1)^{3/5} b^{1/5}} \sqrt{a^{1/5}+(-1)^{3/5} b^{1/5}}} + \frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{a^{1/5}-(-1)^{4/5} b^{1/5}} \tan \left[\frac{x}{2}\right]}{\sqrt{a^{1/5}+(-1)^{4/5} b^{1/5}}}\right]}{5 a^{4/5} \sqrt{a^{1/5}-(-1)^{4/5} b^{1/5}} \sqrt{a^{1/5}+(-1)^{4/5} b^{1/5}}}
\end{aligned}$$

Result (type 7, 130 leaves):

$$\begin{aligned}
& \frac{8}{5} \operatorname{RootSum}\left[b+5 b \#1^2+10 b \#1^4+32 a \#1^5+10 b \#1^6+5 b \#1^8+b \#1^{10}, \&\right. \\
& \left. \frac{2 \operatorname{ArcTan}\left[\frac{\sin[x]}{\cos[x]-\#1}\right] \#1^3-\#1 \log \left[1-2 \cos[x] \#1+\#1^2\right] \#1^3}{b+4 b \#1^2+16 a \#1^3+6 b \#1^4+4 b \#1^6+b \#1^8} \&\right]
\end{aligned}$$

### Problem 75: Result is not expressed in closed-form.

$$\int \frac{1}{a + b \cos[x]^6} dx$$

Optimal (type 3, 171 leaves, 7 steps) :

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{a^{1/3}+b^{1/3}} \cot[x]}{a^{1/6}}\right]}{3 a^{5/6} \sqrt{a^{1/3}+b^{1/3}}} - \frac{\text{ArcTan}\left[\frac{\sqrt{a^{1/3}-(-1)^{1/3} b^{1/3}} \cot[x]}{a^{1/6}}\right]}{3 a^{5/6} \sqrt{a^{1/3}-(-1)^{1/3} b^{1/3}}} - \frac{\text{ArcTan}\left[\frac{\sqrt{a^{1/3}+(-1)^{2/3} b^{1/3}} \cot[x]}{a^{1/6}}\right]}{3 a^{5/6} \sqrt{a^{1/3}+(-1)^{2/3} b^{1/3}}}$$

Result (type 7, 146 leaves) :

$$\begin{aligned} & \frac{8}{3} \text{RootSum}\left[b+6 b \#1+15 b \#1^2+64 a \#1^3+20 b \#1^3+15 b \#1^4+6 b \#1^5+b \#1^6 \&, \right. \\ & \left. \frac{2 \text{ArcTan}\left[\frac{\sin[2x]}{\cos[2x]-\#1}\right] \#1^2-\text{i} \log\left[1-2 \cos[2x] \#1+\#1^2\right] \#1^2}{b+5 b \#1+32 a \#1^2+10 b \#1^2+10 b \#1^3+5 b \#1^4+b \#1^5} \& \right] \end{aligned}$$

### Problem 76: Result is not expressed in closed-form.

$$\int \frac{1}{a + b \cos[x]^8} dx$$

Optimal (type 3, 245 leaves, 9 steps) :

$$\begin{aligned} & \frac{\text{ArcTan}\left[\frac{\sqrt{(-a)^{1/4}-b^{1/4}} \cot[x]}{(-a)^{1/8}}\right]}{4 (-a)^{7/8} \sqrt{(-a)^{1/4}-b^{1/4}}} + \frac{\text{ArcTan}\left[\frac{\sqrt{(-a)^{1/4}-\frac{1}{2} b^{1/4}} \cot[x]}{(-a)^{1/8}}\right]}{4 (-a)^{7/8} \sqrt{(-a)^{1/4}-\frac{1}{2} b^{1/4}}} + \\ & \frac{\text{ArcTan}\left[\frac{\sqrt{(-a)^{1/4}+\frac{1}{2} b^{1/4}} \cot[x]}{(-a)^{1/8}}\right]}{4 (-a)^{7/8} \sqrt{(-a)^{1/4}+\frac{1}{2} b^{1/4}}} + \frac{\text{ArcTan}\left[\frac{\sqrt{(-a)^{1/4}+b^{1/4}} \cot[x]}{(-a)^{1/8}}\right]}{4 (-a)^{7/8} \sqrt{(-a)^{1/4}+b^{1/4}}} \end{aligned}$$

Result (type 7, 172 leaves) :

$$\begin{aligned} & 8 \text{RootSum}\left[b+8 b \#1+28 b \#1^2+56 b \#1^3+256 a \#1^4+70 b \#1^4+56 b \#1^5+28 b \#1^6+8 b \#1^7+b \#1^8 \&, \right. \\ & \left. \left(2 \text{ArcTan}\left[\frac{\sin[2x]}{\cos[2x]-\#1}\right] \#1^3-\text{i} \log\left[1-2 \cos[2x] \#1+\#1^2\right] \#1^3\right) \middle/ \right. \\ & \left. \left(b+7 b \#1+21 b \#1^2+128 a \#1^3+35 b \#1^3+35 b \#1^4+21 b \#1^5+7 b \#1^6+b \#1^7\right) \& \right] \end{aligned}$$

### Problem 77: Result is not expressed in closed-form.

$$\int \frac{1}{a - b \cos[x]^5} dx$$

Optimal (type 3, 494 leaves, 12 steps) :

$$\begin{aligned}
& \frac{2 \operatorname{ArcTan} \left[ \frac{\sqrt{a^{1/5} + b^{1/5}} \tan \left[ \frac{x}{2} \right]}{\sqrt{a^{1/5} - b^{1/5}}} \right]}{5 a^{4/5} \sqrt{a^{1/5} - b^{1/5}} \sqrt{a^{1/5} + b^{1/5}}} + \frac{2 \operatorname{ArcTan} \left[ \frac{\sqrt{a^{1/5} - (-1)^{1/5} b^{1/5}} \tan \left[ \frac{x}{2} \right]}{\sqrt{a^{1/5} + (-1)^{1/5} b^{1/5}}} \right]}{5 a^{4/5} \sqrt{a^{1/5} - (-1)^{1/5} b^{1/5}} \sqrt{a^{1/5} + (-1)^{1/5} b^{1/5}}} + \\
& \frac{2 \operatorname{ArcTan} \left[ \frac{\sqrt{a^{1/5} + (-1)^{2/5} b^{1/5}} \tan \left[ \frac{x}{2} \right]}{\sqrt{a^{1/5} - (-1)^{2/5} b^{1/5}}} \right]}{5 a^{4/5} \sqrt{a^{1/5} - (-1)^{2/5} b^{1/5}} \sqrt{a^{1/5} + (-1)^{2/5} b^{1/5}}} + \\
& \frac{2 \operatorname{ArcTan} \left[ \frac{\sqrt{a^{1/5} - (-1)^{3/5} b^{1/5}} \tan \left[ \frac{x}{2} \right]}{\sqrt{a^{1/5} + (-1)^{3/5} b^{1/5}}} \right]}{5 a^{4/5} \sqrt{a^{1/5} - (-1)^{3/5} b^{1/5}} \sqrt{a^{1/5} + (-1)^{3/5} b^{1/5}}} + \frac{2 \operatorname{ArcTan} \left[ \frac{\sqrt{a^{1/5} + (-1)^{4/5} b^{1/5}} \tan \left[ \frac{x}{2} \right]}{\sqrt{a^{1/5} - (-1)^{4/5} b^{1/5}}} \right]}{5 a^{4/5} \sqrt{a^{1/5} - (-1)^{4/5} b^{1/5}} \sqrt{a^{1/5} + (-1)^{4/5} b^{1/5}}}
\end{aligned}$$

Result (type 7, 130 leaves) :

$$\begin{aligned}
& -\frac{8}{5} \operatorname{RootSum} [b + 5 b \#1^2 + 10 b \#1^4 - 32 a \#1^5 + 10 b \#1^6 + 5 b \#1^8 + b \#1^{10} \&, \\
& 2 \operatorname{ArcTan} \left[ \frac{\sin[x]}{\cos[x] - \#1} \right] \#1^3 - \operatorname{Log} [1 - 2 \cos[x] \#1 + \#1^2] \#1^3 \\
& \quad \#1^3] \\
& b + 4 b \#1^2 - 16 a \#1^3 + 6 b \#1^4 + 4 b \#1^6 + b \#1^8
\end{aligned}$$

Problem 78: Result is not expressed in closed-form.

$$\int \frac{1}{a - b \cos[x]^6} dx$$

Optimal (type 3, 175 leaves, 7 steps) :

$$\begin{aligned}
& -\frac{\operatorname{ArcTan} \left[ \frac{\sqrt{a^{1/3} - b^{1/3}} \cot[x]}{a^{1/6}} \right]}{3 a^{5/6} \sqrt{a^{1/3} - b^{1/3}}} - \frac{\operatorname{ArcTan} \left[ \frac{\sqrt{a^{1/3} + (-1)^{1/3} b^{1/3}} \cot[x]}{a^{1/6}} \right]}{3 a^{5/6} \sqrt{a^{1/3} + (-1)^{1/3} b^{1/3}}} - \frac{\operatorname{ArcTan} \left[ \frac{\sqrt{a^{1/3} - (-1)^{2/3} b^{1/3}} \cot[x]}{a^{1/6}} \right]}{3 a^{5/6} \sqrt{a^{1/3} - (-1)^{2/3} b^{1/3}}}
\end{aligned}$$

Result (type 7, 146 leaves) :

$$\begin{aligned}
& -\frac{8}{3} \operatorname{RootSum} [b + 6 b \#1 + 15 b \#1^2 - 64 a \#1^3 + 20 b \#1^3 + 15 b \#1^4 + 6 b \#1^5 + b \#1^6 \&, \\
& 2 \operatorname{ArcTan} \left[ \frac{\sin[2x]}{\cos[2x] - \#1} \right] \#1^2 - \operatorname{Log} [1 - 2 \cos[2x] \#1 + \#1^2] \#1^2 \\
& \quad \#1^2] \\
& b + 5 b \#1^2 - 32 a \#1^2 + 10 b \#1^2 + 10 b \#1^3 + 5 b \#1^4 + b \#1^5
\end{aligned}$$

Problem 79: Result is not expressed in closed-form.

$$\int \frac{1}{a - b \cos[x]^8} dx$$

Optimal (type 3, 213 leaves, 9 steps) :

$$\begin{aligned}
 & -\frac{\operatorname{ArcTan}\left[\frac{\sqrt{a^{1/4}-b^{1/4}} \cot [x]}{a^{1/8}}\right]}{4 a^{7/8} \sqrt{a^{1/4}-b^{1/4}}} - \frac{\operatorname{ArcTan}\left[\frac{\sqrt{a^{1/4}-i b^{1/4}} \cot [x]}{a^{1/8}}\right]}{4 a^{7/8} \sqrt{a^{1/4}-i b^{1/4}}} - \\
 & \frac{\operatorname{ArcTan}\left[\frac{\sqrt{a^{1/4}+i b^{1/4}} \cot [x]}{a^{1/8}}\right]}{4 a^{7/8} \sqrt{a^{1/4}+i b^{1/4}}} - \frac{\operatorname{ArcTan}\left[\frac{\sqrt{a^{1/4}+b^{1/4}} \cot [x]}{a^{1/8}}\right]}{4 a^{7/8} \sqrt{a^{1/4}+b^{1/4}}}
 \end{aligned}$$

Result (type 7, 172 leaves):

$$\begin{aligned}
 & -8 \operatorname{RootSum}\left[b+8 b \# 1+28 b \# 1^2+56 b \# 1^3-256 a \# 1^4+70 b \# 1^4+56 b \# 1^5+28 b \# 1^6+8 b \# 1^7+b \# 1^8 \&, \right. \\
 & \left.\left(2 \operatorname{ArcTan}\left[\frac{\sin [2 x]}{\cos [2 x]-\# 1}\right] \# 1^3-i \log \left[1-2 \cos [2 x] \# 1+\# 1^2\right] \# 1^3\right) / \right. \\
 & \left.\left(b+7 b \# 1+21 b \# 1^2-128 a \# 1^3+35 b \# 1^3+35 b \# 1^4+21 b \# 1^5+7 b \# 1^6+b \# 1^7\right)\right] \&
 \end{aligned}$$

Problem 80: Result is not expressed in closed-form.

$$\int \frac{1}{1+\cos [x]^5} dx$$

Optimal (type 3, 223 leaves, 11 steps):

$$\begin{aligned}
 & \frac{2 \operatorname{ArcTan}\left[\sqrt{\frac{1-(-1)^{2/5}}{1+(-1)^{2/5}}} \tan \left[\frac{x}{2}\right]\right]}{5 \sqrt{1-(-1)^{4/5}}}+\frac{2 \operatorname{ArcTan}\left[\sqrt{\frac{1-(-1)^{4/5}}{1+(-1)^{4/5}}} \tan \left[\frac{x}{2}\right]\right]}{5 \sqrt{1+(-1)^{3/5}}}- \\
 & \frac{2 \operatorname{ArcTanh}\left[\frac{\tan \left[\frac{x}{2}\right]}{\sqrt{\frac{1-(-1)^{1/5}}{1+(-1)^{1/5}}}}\right]}{5 \sqrt{-1+(-1)^{2/5}}}-\frac{2 \sqrt{\frac{1+(-1)^{3/5}}{1-(-1)^{3/5}}} \operatorname{ArcTanh}\left[\sqrt{-\frac{1+(-1)^{3/5}}{1-(-1)^{3/5}}} \tan \left[\frac{x}{2}\right]\right]}{5 \left(1+(-1)^{3/5}\right)}+\frac{\sin [x]}{5 \left(1+\cos [x]\right)}
 \end{aligned}$$

Result (type 7, 378 leaves):

$$\begin{aligned}
& -\frac{1}{10} \operatorname{RootSum}\left[1-2 \#1+8 \#1^2-14 \#1^3+30 \#1^4-14 \#1^5+8 \#1^6-2 \#1^7+\#1^8 \&, \right. \\
& \quad \frac{1}{-1+8 \#1-21 \#1^2+60 \#1^3-35 \#1^4+24 \#1^5-7 \#1^6+4 \#1^7} \\
& \quad \left(2 \operatorname{ArcTan}\left[\frac{\sin[x]}{\cos[x]-\#1}\right]-i \log \left[1-2 \cos[x] \#1+\#1^2\right]-8 \operatorname{ArcTan}\left[\frac{\sin[x]}{\cos[x]-\#1}\right] \#1+ \right. \\
& \quad 4 i \log \left[1-2 \cos[x] \#1+\#1^2\right] \#1+30 \operatorname{ArcTan}\left[\frac{\sin[x]}{\cos[x]-\#1}\right] \#1^2- \\
& \quad 15 i \log \left[1-2 \cos[x] \#1+\#1^2\right] \#1^2-80 \operatorname{ArcTan}\left[\frac{\sin[x]}{\cos[x]-\#1}\right] \#1^3+ \\
& \quad 40 i \log \left[1-2 \cos[x] \#1+\#1^2\right] \#1^3+30 \operatorname{ArcTan}\left[\frac{\sin[x]}{\cos[x]-\#1}\right] \#1^4- \\
& \quad 15 i \log \left[1-2 \cos[x] \#1+\#1^2\right] \#1^4-8 \operatorname{ArcTan}\left[\frac{\sin[x]}{\cos[x]-\#1}\right] \#1^5+ \\
& \quad 4 i \log \left[1-2 \cos[x] \#1+\#1^2\right] \#1^5+2 \operatorname{ArcTan}\left[\frac{\sin[x]}{\cos[x]-\#1}\right] \#1^6- \\
& \quad \left.i \log \left[1-2 \cos[x] \#1+\#1^2\right] \#1^6\right)\&]+\frac{1}{5} \tan \left[\frac{x}{2}\right]
\end{aligned}$$

**Problem 82:** Result is not expressed in closed-form.

$$\int \frac{1}{1 + \cos[x]^8} dx$$

Optimal (type 3, 129 leaves, 9 steps) :

$$\begin{aligned}
& -\frac{\operatorname{ArcTan}\left[\sqrt{1-\left(-1\right)^{1/4}} \cot [x]\right]}{4 \sqrt{1-\left(-1\right)^{1/4}}}-\frac{\operatorname{ArcTan}\left[\sqrt{1+\left(-1\right)^{1/4}} \cot [x]\right]}{4 \sqrt{1+\left(-1\right)^{1/4}}}- \\
& \frac{\operatorname{ArcTan}\left[\sqrt{1-\left(-1\right)^{3/4}} \cot [x]\right]}{4 \sqrt{1-\left(-1\right)^{3/4}}}-\frac{\operatorname{ArcTan}\left[\sqrt{1+\left(-1\right)^{3/4}} \cot [x]\right]}{4 \sqrt{1+\left(-1\right)^{3/4}}}
\end{aligned}$$

Result (type 7, 141 leaves) :

$$\begin{aligned}
& 8 \operatorname{RootSum}\left[1+8 \#1+28 \#1^2+56 \#1^3+326 \#1^4+56 \#1^5+28 \#1^6+8 \#1^7+\#1^8 \&, \right. \\
& \quad \left.\frac{2 \operatorname{ArcTan}\left[\frac{\sin [2 x]}{\cos [2 x]-\#1}\right] \#1^3-i \log \left[1-2 \cos [2 x] \#1+\#1^2\right] \#1^3}{1+7 \#1+21 \#1^2+163 \#1^3+35 \#1^4+21 \#1^5+7 \#1^6+\#1^7}\right)\&
\end{aligned}$$

**Problem 83:** Result is not expressed in closed-form.

$$\int \frac{1}{1 - \cos[x]^5} dx$$

Optimal (type 3, 205 leaves, 11 steps) :

$$\frac{2 \operatorname{ArcTan}\left[\sqrt{\frac{1-(-1)^{1/5}}{1+(-1)^{1/5}}} \tan\left[\frac{x}{2}\right]\right]}{5 \sqrt{1-\left(-1\right)^{2/5}}}+\frac{2 \operatorname{ArcTan}\left[\sqrt{\frac{1-(-1)^{3/5}}{1+(-1)^{3/5}}} \tan\left[\frac{x}{2}\right]\right]}{5 \sqrt{1+\left(-1\right)^{1/5}}}-$$

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\tan\left[\frac{x}{2}\right]}{\sqrt{\frac{1-(-1)^{2/5}}{1+(-1)^{2/5}}}}\right]}{5 \sqrt{-1+\left(-1\right)^{4/5}}}+\frac{2 \operatorname{ArcTanh}\left[\sqrt{\frac{1+(-1)^{4/5}}{1-(-1)^{4/5}}} \tan\left[\frac{x}{2}\right]\right]}{5 \sqrt{-1-\left(-1\right)^{3/5}}}-\frac{\sin [x]}{5 (1-\cos [x])}$$

Result (type 7, 378 leaves):

$$-\frac{1}{5} \cot\left[\frac{x}{2}\right]+\frac{1}{10} \operatorname{RootSum}\left[1+2 \# 1+8 \# 1^2+14 \# 1^3+30 \# 1^4+14 \# 1^5+8 \# 1^6+2 \# 1^7+\# 1^8 \&,\right.$$

$$\frac{1}{1+8 \# 1+21 \# 1^2+60 \# 1^3+35 \# 1^4+24 \# 1^5+7 \# 1^6+4 \# 1^7}\left(2 \operatorname{ArcTan}\left[\frac{\sin [x]}{\cos [x]-\# 1}\right]-\right.$$

$$\left.\# 1 \log \left[1-2 \cos [x] \# 1+\# 1^2\right]+8 \operatorname{ArcTan}\left[\frac{\sin [x]}{\cos [x]-\# 1}\right] \# 1-4 \# 1 \log \left[1-2 \cos [x] \# 1+\# 1^2\right] \# 1+\right.$$

$$30 \operatorname{ArcTan}\left[\frac{\sin [x]}{\cos [x]-\# 1}\right] \# 1^2-15 \# 1 \log \left[1-2 \cos [x] \# 1+\# 1^2\right] \# 1^2+$$

$$80 \operatorname{ArcTan}\left[\frac{\sin [x]}{\cos [x]-\# 1}\right] \# 1^3-40 \# 1 \log \left[1-2 \cos [x] \# 1+\# 1^2\right] \# 1^3+$$

$$30 \operatorname{ArcTan}\left[\frac{\sin [x]}{\cos [x]-\# 1}\right] \# 1^4-15 \# 1 \log \left[1-2 \cos [x] \# 1+\# 1^2\right] \# 1^4+$$

$$8 \operatorname{ArcTan}\left[\frac{\sin [x]}{\cos [x]-\# 1}\right] \# 1^5-4 \# 1 \log \left[1-2 \cos [x] \# 1+\# 1^2\right] \# 1^5+$$

$$\left.2 \operatorname{ArcTan}\left[\frac{\sin [x]}{\cos [x]-\# 1}\right] \# 1^6-\# 1 \log \left[1-2 \cos [x] \# 1+\# 1^2\right] \# 1^6\right)\&]$$

**Problem 88:** Result more than twice size of optimal antiderivative.

$$\int \sqrt{1-\cos [x]^2} \tan [x] dx$$

Optimal (type 3, 20 leaves, 5 steps):

$$\operatorname{ArcTanh}\left[\sqrt{\sin [x]^2}\right]-\sqrt{\sin [x]^2}$$

Result (type 3, 47 leaves):

$$-\csc [x] \sqrt{\sin [x]^2}\left(\log \left[\cos \left[\frac{x}{2}\right]-\sin \left[\frac{x}{2}\right]\right]-\log \left[\cos \left[\frac{x}{2}\right]+\sin \left[\frac{x}{2}\right]\right]+\sin [x]\right)$$

**Problem 91:** Result more than twice size of optimal antiderivative.

$$\int \frac{\tan [x]}{\sqrt{1-\cos [x]^2}} dx$$

Optimal (type 3, 9 leaves, 4 steps):

$$\text{ArcTanh}[\sqrt{\sin[x]^2}]$$

Result (type 3, 44 leaves):

$$\frac{\left(-\text{Log}[\cos[\frac{x}{2}]-\sin[\frac{x}{2}]]+\text{Log}[\cos[\frac{x}{2}]+\sin[\frac{x}{2}]]\right) \sin[x]}{\sqrt{\sin[x]^2}}$$

**Problem 92:** Result is not expressed in closed-form.

$$\int \frac{\tan[x]^3}{a+b \cos[x]^3} dx$$

Optimal (type 3, 153 leaves, 11 steps):

$$\begin{aligned} & -\frac{b^{2/3} \text{ArcTan}\left[\frac{a^{1/3}-2 b^{1/3} \cos[x]}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} a^{5/3}}+\frac{\text{Log}[\cos[x]]}{a}+\frac{b^{2/3} \log \left[a^{1/3}+b^{1/3} \cos[x]\right]}{3 a^{5/3}}- \\ & \frac{b^{2/3} \log \left[a^{2/3}-a^{1/3} b^{1/3} \cos[x]+b^{2/3} \cos[x]^2\right]}{6 a^{5/3}}-\frac{\text{Log}[a+b \cos[x]^3]}{3 a}+\frac{\sec [x]^2}{2 a} \end{aligned}$$

Result (type 7, 217 leaves):

$$\begin{aligned} & \frac{1}{6 a} \\ & \left(6 \left(\text{Log}[\cos[x]]+\text{Log}\left[\sec \left[\frac{x}{2}\right]^2\right]\right)-2 \text{RootSum}\left[a+b+3 a \# 1-3 b \# 1+3 a \# 1^2+3 b \# 1^2+a \# 1^3-b \# 1^3 \&, \right.\right. \\ & \left(a \text{Log}\left[-\# 1+\tan \left[\frac{x}{2}\right]^2\right]+b \text{Log}\left[-\# 1+\tan \left[\frac{x}{2}\right]^2\right]+2 a \text{Log}\left[-\# 1+\tan \left[\frac{x}{2}\right]^2\right] \# 1+\right. \\ & \left.4 b \text{Log}\left[-\# 1+\tan \left[\frac{x}{2}\right]^2\right] \# 1+a \text{Log}\left[-\# 1+\tan \left[\frac{x}{2}\right]^2\right] \# 1^2-b \text{Log}\left[-\# 1+\tan \left[\frac{x}{2}\right]^2\right] \# 1^2\right)\Big/ \\ & \left.\left.(a-b+2 a \# 1+2 b \# 1+a \# 1^2-b \# 1^2) \&+3 \sec [x]^2\right)\right) \end{aligned}$$

**Problem 93:** Result more than twice size of optimal antiderivative.

$$\int \sqrt{a+b \cos[x]^3} \tan[x] dx$$

Optimal (type 3, 45 leaves, 5 steps):

$$\frac{2}{3} \sqrt{a} \text{ArcTanh}\left[\frac{\sqrt{a+b \cos[x]^3}}{\sqrt{a}}\right]-\frac{2}{3} \sqrt{a+b \cos[x]^3}$$

Result (type 3, 668 leaves):

$$\begin{aligned}
& - \left( \sqrt{4 a + 3 b \cos[x] + b \cos[3x]} \right. \\
& \left. \left( b + a (\sec[x]^2)^{3/2} - \sqrt{a} \sqrt{b} \operatorname{ArcSinh}\left[\frac{\sqrt{a} (\sec[x]^2)^{3/4}}{\sqrt{b}}\right] (\sec[x]^2)^{3/4} \sqrt{1 + \frac{a (\sec[x]^2)^{3/2}}{b}} \right) \right. \\
& \left. \left. \tan[x] \sqrt{\cos[x]^4 \left(a + b \sqrt{\sec[x]^2} + 2 a \tan[x]^2 + a \tan[x]^4\right)} \right) / \right. \\
& \left. \left( 3 \left(b + a (\sec[x]^2)^{3/2}\right) \left( 2 a (\sec[x]^2)^{3/2} \left( b + a (\sec[x]^2)^{3/2} - \right. \right. \right. \right. \\
& \left. \left. \left. \left. \sqrt{a} \sqrt{b} \operatorname{ArcSinh}\left[\frac{\sqrt{a} (\sec[x]^2)^{3/4}}{\sqrt{b}}\right] (\sec[x]^2)^{3/4} \sqrt{1 + \frac{a (\sec[x]^2)^{3/2}}{b}} \right) \tan[x] \right. \right. \\
& \left. \left. \left. \left. \sqrt{\cos[x]^4 \left(a + b \sqrt{\sec[x]^2} + 2 a \tan[x]^2 + a \tan[x]^4\right)} \right) / \left( b + a (\sec[x]^2)^{3/2} \right)^2 - \right. \right. \\
& \left. \left. \left. \left. 2 \left( \frac{3}{2} a (\sec[x]^2)^{3/2} \tan[x] - \frac{3 a^{3/2} \operatorname{ArcSinh}\left[\frac{\sqrt{a} (\sec[x]^2)^{3/4}}{\sqrt{b}}\right] (\sec[x]^2)^{9/4} \tan[x]}{2 \sqrt{b} \sqrt{1 + \frac{a (\sec[x]^2)^{3/2}}{b}}} \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \frac{3}{2} \sqrt{a} \sqrt{b} \operatorname{ArcSinh}\left[\frac{\sqrt{a} (\sec[x]^2)^{3/4}}{\sqrt{b}}\right] (\sec[x]^2)^{3/4} \sqrt{1 + \frac{a (\sec[x]^2)^{3/2}}{b}} \tan[x] \right) \right) \right. \right. \\
& \left. \left. \left. \left. \left. \sqrt{\cos[x]^4 \left(a + b \sqrt{\sec[x]^2} + 2 a \tan[x]^2 + a \tan[x]^4\right)} \right) / \left( 3 \left(b + a (\sec[x]^2)^{3/2}\right) \right) - \right. \right. \\
& \left. \left. \left. \left. \left. \left( b + a (\sec[x]^2)^{3/2} - \sqrt{a} \sqrt{b} \operatorname{ArcSinh}\left[\frac{\sqrt{a} (\sec[x]^2)^{3/4}}{\sqrt{b}}\right] \right) \right. \right. \right. \right. \\
\end{aligned}$$

$$\frac{\left(\sec^2 x\right)^{3/4} \sqrt{1 + \frac{a (\sec^2 x)^{3/2}}{b}}}{\left(\cos^4 x \left(4 a \sec^2 x \tan x + b \sqrt{\sec^2 x} \tan x + 4 a \sec^2 x \tan^3 x\right) - 4 \cos^3 x \sin x \left(a + b \sqrt{\sec^2 x} + 2 a \tan^2 x + a \tan^4 x\right)\right) / \left(3 \left(b + a (\sec^2 x)^{3/2}\right) \sqrt{\cos^4 x \left(a + b \sqrt{\sec^2 x} + 2 a \tan^2 x + a \tan^4 x\right)}\right)}$$

**Problem 94:** Result more than twice size of optimal antiderivative.

$$\int \frac{\tan x}{\sqrt{a + b \cos^3 x}} dx$$

Optimal (type 3, 28 leaves, 4 steps):

$$\frac{2 \operatorname{ArcTanh} \left[ \frac{\sqrt{a+b \cos [x]}^3}{\sqrt{a}} \right]}{3 \sqrt{a}}$$

### Result (type 3, 207 leaves):

$$\left( 128 \sqrt{b} \operatorname{ArcSinh} \left[ \frac{\sqrt{a} (\operatorname{Sec}[x]^2)^{3/4}}{\sqrt{b}} \right] \operatorname{Cos}[x]^4 (\operatorname{Sec}[x]^2)^{3/4} \left( b \operatorname{Cos}[x]^2 + a \sqrt{\operatorname{Sec}[x]^2} \right) \right.$$

$$\left. \sqrt{1 + \frac{a (\operatorname{Sec}[x]^2)^{3/2}}{b}} \left( a + 2 a \operatorname{Cot}[x]^2 + \operatorname{Cot}[x]^4 \left( a + b \sqrt{\operatorname{Sec}[x]^2} \right) \right) \operatorname{Sin}[x]^4 \right) /$$

$$\left( 3 \sqrt{a} \sqrt{4 a + 3 b \operatorname{Cos}[x] + b \operatorname{Cos}[3 x]} \left( 32 a^2 + 10 b^2 + b^2 \operatorname{Cos}[6 x] + 24 a b \sqrt{\operatorname{Sec}[x]^2} + 2 b \operatorname{Cos}[4 x] \left( 3 b + 4 a \sqrt{\operatorname{Sec}[x]^2} \right) + b \operatorname{Cos}[2 x] \left( 15 b + 32 a \sqrt{\operatorname{Sec}[x]^2} \right) \right) \right)$$

**Problem 95:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \sqrt{a + b \cos[x]^4} \tan[x] dx$$

Optimal (type 3, 45 leaves, 5 steps):

$$\frac{1}{2} \sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \cos [x]^4}}{\sqrt{a}}\right]-\frac{1}{2} \sqrt{a+b \cos [x]^4}$$

Result (type 4, 47 997 leaves) : Display of huge result suppressed!

**Problem 96: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\tan [x]}{\sqrt{a+b \cos [x]^4}} dx$$

Optimal (type 3, 28 leaves, 4 steps) :

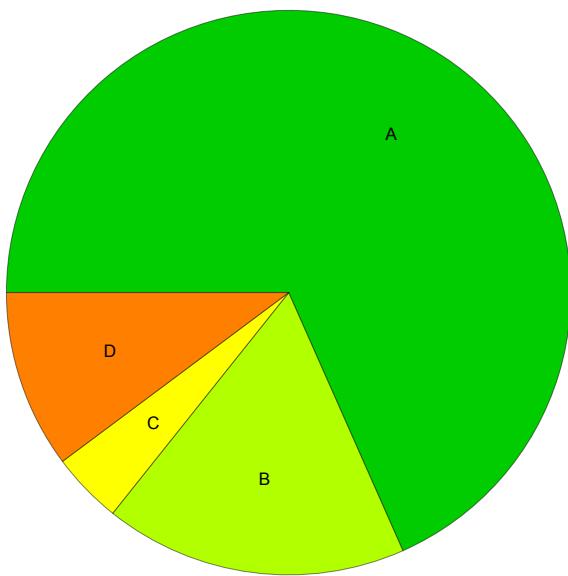
$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \cos [x]^4}}{\sqrt{a}}\right]}{2 \sqrt{a}}$$

Result (type 4, 48 584 leaves) : Display of huge result suppressed!

---

## Summary of Integration Test Results

98 integration problems



A - 67 optimal antiderivatives

B - 17 more than twice size of optimal antiderivatives

C - 4 unnecessarily complex antiderivatives

D - 10 unable to integrate problems

E - 0 integration timeouts