# Mathematica 11.3 Integration Test Results

## Test results for the 91 problems in "4.3.1.3 (d sin)^m (a+b tan)^n.m"

### Problem 7: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Csc}[x]^3}{i + \text{Tan}[x]} \, dx$$

Optimal (type 3, 24 leaves, 8 steps):

$$-\frac{1}{2} i \,\text{ArcTanh}[\text{Cos}[x]] - \text{Csc}[x] + \frac{1}{2} i \,\text{Cot}[x] \,\text{Csc}[x]$$

Result (type 3, 75 leaves):

$$-\frac{1}{2} \text{Cot}\left[\frac{x}{2}\right] + \frac{1}{8} i \,\text{Csc}\left[\frac{x}{2}\right]^2 - \frac{1}{2} i \,\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right] + \frac{1}{2} i \,\text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right] - \frac{1}{8} i \,\text{Sec}\left[\frac{x}{2}\right]^2 - \frac{1}{2} \text{Tan}\left[\frac{x}{2}\right]$$

### Problem 9: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Csc}[x]^5}{i + \text{Tan}[x]} \, dx$$

Optimal (type 3, 40 leaves, 9 steps):

$$-\frac{1}{8} i \,\text{ArcTanh}[\text{Cos}[x]] - \frac{1}{8} i \,\text{Cot}[x] \,\text{Csc}[x] - \frac{\text{Csc}[x]^3}{3} + \frac{1}{4} i \,\text{Cot}[x] \,\text{Csc}[x]^3$$

Result (type 3, 139 leaves):

$$-\frac{1}{12} \text{Cot}\left[\frac{x}{2}\right] - \frac{1}{32} i \,\text{Csc}\left[\frac{x}{2}\right]^2 - \frac{1}{24} \text{Cot}\left[\frac{x}{2}\right] \text{Csc}\left[\frac{x}{2}\right]^2 + \frac{1}{64} i \,\text{Csc}\left[\frac{x}{2}\right]^4 - \frac{1}{8} i \,\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right] + \frac{1}{8} i \,\text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right] + \frac{1}{32} i \,\text{Sec}\left[\frac{x}{2}\right]^2 - \frac{1}{64} i \,\text{Sec}\left[\frac{x}{2}\right]^4 - \frac{1}{12} \text{Tan}\left[\frac{x}{2}\right] - \frac{1}{24} \text{Sec}\left[\frac{x}{2}\right]^2 \text{Tan}\left[\frac{x}{2}\right]$$

### Problem 15: Result more than twice size of optimal antiderivative.

$$\int \text{Sin}[c + d x] \, (a + b \,\text{Tan}[c + d x]) \, dx$$

Optimal (type 3, 37 leaves, 6 steps):

$$\frac{b \,\text{ArcTanh}[\text{Sin}[c + d x]]}{d} - \frac{a \,\text{Cos}[c + d x]}{d} - \frac{b \,\text{Sin}[c + d x]}{d}$$

Result (type 3, 93 leaves):

$$-\frac{a\,\text{Cos}[c]\,\text{Cos}[d\,x]}{d}-\frac{b\,\text{Log}\left[\text{Cos}\left[\frac{1}{2}\,(c+d\,x)\right]-\text{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]\right]}{d}+\frac{b\,\text{Log}\left[\text{Cos}\left[\frac{1}{2}\,(c+d\,x)\right]+\text{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]\right]}{d}+\frac{a\,\text{Sin}[c]\,\text{Sin}[d\,x]}{d}-\frac{b\,\text{Sin}[c+d\,x]}{d}$$

## Problem 16: Result more than twice size of optimal antiderivative.

$$\int \text{Csc}[c+d\,x]\,(a+b\,\text{Tan}[c+d\,x])\,dx$$

Optimal (type 3, 26 leaves, 4 steps):

$$-\frac{a\,\text{ArcTanh}[\text{Cos}[c+d\,x]]}{d}+\frac{b\,\text{ArcTanh}[\text{Sin}[c+d\,x]]}{d}$$

Result (type 3, 109 leaves):

$$-\frac{a\,\text{Log}\left[\text{Cos}\left[\frac{c}{2}+\frac{d\,x}{2}\right]\right]}{d}-\frac{b\,\text{Log}\left[\text{Cos}\left[\frac{c}{2}+\frac{d\,x}{2}\right]-\text{Sin}\left[\frac{c}{2}+\frac{d\,x}{2}\right]\right]}{d}+\frac{a\,\text{Log}\left[\text{Sin}\left[\frac{c}{2}+\frac{d\,x}{2}\right]\right]}{d}+\frac{b\,\text{Log}\left[\text{Cos}\left[\frac{c}{2}+\frac{d\,x}{2}\right]+\text{Sin}\left[\frac{c}{2}+\frac{d\,x}{2}\right]\right]}{d}$$

## Problem 18: Result more than twice size of optimal antiderivative.

$$\int \text{Csc}[c+d\,x]^3\,(a+b\,\text{Tan}[c+d\,x])\,dx$$

Optimal (type 3, 60 leaves, 7 steps):

$$-\frac{a\,\text{ArcTanh}[\text{Cos}[c+d\,x]]}{2\,d}+\frac{b\,\text{ArcTanh}[\text{Sin}[c+d\,x]]}{d}-\frac{b\,\text{Csc}[c+d\,x]}{d}-\frac{a\,\text{Cot}[c+d\,x]\,\text{Csc}[c+d\,x]}{2\,d}$$

Result (type 3, 172 leaves):

$$-\frac{b\,\text{Cot}\left[\frac{1}{2}\,(c+d\,x)\right]}{2\,d}-\frac{a\,\text{Csc}\left[\frac{1}{2}\,(c+d\,x)\right]^2}{8\,d}-\frac{a\,\text{Log}\left[\text{Cos}\left[\frac{1}{2}\,(c+d\,x)\right]\right]}{2\,d}-\frac{b\,\text{Log}\left[\text{Cos}\left[\frac{1}{2}\,(c+d\,x)\right]-\text{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]\right]}{d}+\frac{a\,\text{Log}\left[\text{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]\right]}{2\,d}+\frac{b\,\text{Log}\left[\text{Cos}\left[\frac{1}{2}\,(c+d\,x)\right]+\text{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]\right]}{d}+\frac{a\,\text{Sec}\left[\frac{1}{2}\,(c+d\,x)\right]^2}{8\,d}-\frac{b\,\text{Tan}\left[\frac{1}{2}\,(c+d\,x)\right]}{2\,d}$$

## Problem 20: Result more than twice size of optimal antiderivative.

$$\int \text{Csc}[c+d\,x]^5\,(a+b\,\text{Tan}[c+d\,x])\,dx$$

Optimal (type 3, 98 leaves, 9 steps):

$$-\frac{3\,a\,\text{ArcTanh}[\text{Cos}[c+d\,x]]}{8\,d}+\frac{b\,\text{ArcTanh}[\text{Sin}[c+d\,x]]}{d}-\frac{b\,\text{Csc}[c+d\,x]}{d}-\frac{3\,a\,\text{Cot}[c+d\,x]\,\text{Csc}[c+d\,x]}{8\,d}-\frac{b\,\text{Csc}[c+d\,x]^3}{3\,d}-\frac{a\,\text{Cot}[c+d\,x]\,\text{Csc}[c+d\,x]^3}{4\,d}$$

Result (type 3, 272 leaves):

$$-\frac{7\,b\,\text{Cot}\left[\frac{1}{2}\,(c+d\,x)\right]}{12\,d}-\frac{3\,a\,\text{Csc}\left[\frac{1}{2}\,(c+d\,x)\right]^2}{32\,d}-\frac{b\,\text{Cot}\left[\frac{1}{2}\,(c+d\,x)\right]\,\text{Csc}\left[\frac{1}{2}\,(c+d\,x)\right]^2}{24\,d}-\frac{a\,\text{Csc}\left[\frac{1}{2}\,(c+d\,x)\right]^4}{64\,d}-\frac{3\,a\,\text{Log}\left[\text{Cos}\left[\frac{1}{2}\,(c+d\,x)\right]\right]}{8\,d}-\frac{b\,\text{Log}\left[\text{Cos}\left[\frac{1}{2}\,(c+d\,x)\right]-\text{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]\right]}{d}+\frac{3\,a\,\text{Log}\left[\text{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]\right]}{8\,d}+\frac{b\,\text{Log}\left[\text{Cos}\left[\frac{1}{2}\,(c+d\,x)\right]+\text{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]\right]}{d}+\frac{3\,a\,\text{Sec}\left[\frac{1}{2}\,(c+d\,x)\right]^2}{32\,d}+\frac{a\,\text{Sec}\left[\frac{1}{2}\,(c+d\,x)\right]^4}{64\,d}-\frac{7\,b\,\text{Tan}\left[\frac{1}{2}\,(c+d\,x)\right]}{12\,d}-\frac{b\,\text{Sec}\left[\frac{1}{2}\,(c+d\,x)\right]^2\,\text{Tan}\left[\frac{1}{2}\,(c+d\,x)\right]}{24\,d}$$

## Problem 26: Result more than twice size of optimal antiderivative.

$$\int \text{Csc}[c+d\,x]\,(a+b\,\text{Tan}[c+d\,x])^2\,dx$$

Optimal (type 3, 43 leaves, 6 steps):

$$-\frac{a^2\,\text{ArcTanh}[\text{Cos}[c+d\,x]]}{d}+\frac{2\,a\,b\,\text{ArcTanh}[\text{Sin}[c+d\,x]]}{d}+\frac{b^2\,\text{Sec}[c+d\,x]}{d}$$

Result (type 3, 97 leaves):

$$\frac{1}{d}\left(a\left(-a\,\text{Log}\left[\text{Cos}\left[\frac{1}{2}\,(c+d\,x)\right]\right]-2\,b\,\text{Log}\left[\text{Cos}\left[\frac{1}{2}\,(c+d\,x)\right]-\text{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]\right]+a\,\text{Log}\left[\text{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]\right]+2\,b\,\text{Log}\left[\text{Cos}\left[\frac{1}{2}\,(c+d\,x)\right]+\text{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]\right]\right)+b^2\,\text{Sec}[c+d\,x]\right)$$

## Problem 27: Result more than twice size of optimal antiderivative.

$$\int \text{Csc}[c+d\,x]^2\,(a+b\,\text{Tan}[c+d\,x])^2\,dx$$

Optimal (type 3, 42 leaves, 3 steps):

$$-\frac{a^2\,\text{Cot}[c+d\,x]}{d}+\frac{2\,a\,b\,\text{Log}[\text{Tan}[c+d\,x]]}{d}+\frac{b^2\,\text{Tan}[c+d\,x]}{d}$$

Result (type 3, 91 leaves):

$$-\Big(\Big(\text{Cos}[c+d\,x]\,\big(a\,\text{Cos}[c+d\,x]\,\big(a\,\text{Cot}[c+d\,x]+2\,b\,(\text{Log}[\text{Cos}[c+d\,x]]-\text{Log}[\text{Sin}[c+d\,x]])\big)-b^2\,\text{Sin}[c+d\,x]\big)\,(a+b\,\text{Tan}[c+d\,x])^2\Big)\Big/\Big(d\,(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x])^2\Big)\Big)$$

## Problem 28: Result more than twice size of optimal antiderivative.

$$\int \mathrm{Csc}[c+d\,x]^3\,(a+b\,\mathrm{Tan}[c+d\,x])^2\,dx$$

Optimal (type 3, 95 leaves, 10 steps):

$$-\frac{a^2\,\mathrm{ArcTanh}[\mathrm{Cos}[c+d\,x]]}{2\,d}-\frac{b^2\,\mathrm{ArcTanh}[\mathrm{Cos}[c+d\,x]]}{d}+\frac{2\,a\,b\,\mathrm{ArcTanh}[\mathrm{Sin}[c+d\,x]]}{d}-\frac{2\,a\,b\,\mathrm{Csc}[c+d\,x]}{d}-\frac{a^2\,\mathrm{Cot}[c+d\,x]\,\mathrm{Csc}[c+d\,x]}{2\,d}+\frac{b^2\,\mathrm{Sec}[c+d\,x]}{d}$$

Result (type 3, 250 leaves):

$$\frac{1}{8\,d}\left(8\,b^2-8\,a\,b\,\mathrm{Cot}\left[\frac{1}{2}\,(c+d\,x)\right]-a^2\,\mathrm{Csc}\left[\frac{1}{2}\,(c+d\,x)\right]^2-4\,(a^2+2\,b^2)\,\mathrm{Log}\left[\mathrm{Cos}\left[\frac{1}{2}\,(c+d\,x)\right]\right]-16\,a\,b\,\mathrm{Log}\left[\mathrm{Cos}\left[\frac{1}{2}\,(c+d\,x)\right]-\mathrm{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]\right]+4\,(a^2+2\,b^2)\,\mathrm{Log}\left[\mathrm{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]\right]+16\,a\,b\,\mathrm{Log}\left[\mathrm{Cos}\left[\frac{1}{2}\,(c+d\,x)\right]+\mathrm{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]\right]+a^2\,\mathrm{Sec}\left[\frac{1}{2}\,(c+d\,x)\right]^2+\frac{8\,b^2\,\mathrm{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]}{\mathrm{Cos}\left[\frac{1}{2}\,(c+d\,x)\right]-\mathrm{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]}-\frac{8\,b^2\,\mathrm{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]}{\mathrm{Cos}\left[\frac{1}{2}\,(c+d\,x)\right]+\mathrm{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]}-8\,a\,b\,\mathrm{Tan}\left[\frac{1}{2}\,(c+d\,x)\right]\right)$$

## Problem 30: Result more than twice size of optimal antiderivative.

$$\int \mathrm{Csc}[c+d\,x]^5\,(a+b\,\mathrm{Tan}[c+d\,x])^2\,dx$$

Optimal (type 3, 165 leaves, 13 steps):

$$-\frac{3\,a^2\,\mathrm{ArcTanh}[\mathrm{Cos}[c+d\,x]]}{8\,d}-\frac{3\,b^2\,\mathrm{ArcTanh}[\mathrm{Cos}[c+d\,x]]}{2\,d}+\frac{2\,a\,b\,\mathrm{ArcTanh}[\mathrm{Sin}[c+d\,x]]}{d}-\frac{2\,a\,b\,\mathrm{Csc}[c+d\,x]}{d}-\frac{3\,a^2\,\mathrm{Cot}[c+d\,x]\,\mathrm{Csc}[c+d\,x]}{8\,d}-\frac{2\,a\,b\,\mathrm{Csc}[c+d\,x]^3}{3\,d}-\frac{a^2\,\mathrm{Cot}[c+d\,x]\,\mathrm{Csc}[c+d\,x]^3}{4\,d}+\frac{3\,b^2\,\mathrm{Sec}[c+d\,x]}{2\,d}-\frac{b^2\,\mathrm{Csc}[c+d\,x]^2\,\mathrm{Sec}[c+d\,x]}{2\,d}$$

Result (type 3, 994 leaves):

$$\frac{b^2 \,\text{Cos}[c+d\,x]^2 \left(a+b\,\text{Tan}[c+d\,x]\right)^2}{d\left(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]\right)^2}-\frac{7\,a\,b\,\text{Cos}[c+d\,x]^2\,\text{Cot}\left[\frac{1}{2}(c+d\,x)\right]\left(a+b\,\text{Tan}[c+d\,x]\right)^2}{6\,d\left(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]\right)^2}+$$

$$\frac{\left(-3\,a^2-4\,b^2\right)\text{Cos}[c+d\,x]^2\,\text{Csc}\left[\frac{1}{2}(c+d\,x)\right]^2\left(a+b\,\text{Tan}[c+d\,x]\right)^2}{32\,d\left(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]\right)^2}-$$

$$\left(a\,b\,\text{Cos}[c+d\,x]^2\,\text{Cot}\left[\frac{1}{2}(c+d\,x)\right]\text{Csc}\left[\frac{1}{2}(c+d\,x)\right]^2\left(a+b\,\text{Tan}[c+d\,x]\right)^2\right)\Big/$$

$$\left(12\,d\left(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]\right)^2\right)-\frac{a^2\,\text{Cos}[c+d\,x]^2\,\text{Csc}\left[\frac{1}{2}(c+d\,x)\right]^4\left(a+b\,\text{Tan}[c+d\,x]\right)^2}{64\,d\left(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]\right)^2}-$$

$$\frac{3\left(a^2+4\,b^2\right)\text{Cos}[c+d\,x]^2\,\text{Log}\left[\text{Cos}\left[\frac{1}{2}(c+d\,x)\right]\right]\left(a+b\,\text{Tan}[c+d\,x]\right)^2}{8\,d\left(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]\right)^2}-$$

$$\left(2\,a\,b\,\text{Cos}[c+d\,x]^2\,\text{Log}\left[\text{Cos}\left[\frac{1}{2}(c+d\,x)\right]-\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right]\left(a+b\,\text{Tan}[c+d\,x]\right)^2\right)\Big/$$

$$\left(d\left(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]\right)^2\right)+$$

$$\frac{3\left(a^2+4\,b^2\right)\text{Cos}[c+d\,x]^2\,\text{Log}\left[\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right]\left(a+b\,\text{Tan}[c+d\,x]\right)^2}{8\,d\left(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]\right)^2}+$$

$$\left(2\,a\,b\,\text{Cos}[c+d\,x]^2\,\text{Log}\left[\text{Cos}\left[\frac{1}{2}(c+d\,x)\right]+\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right]\left(a+b\,\text{Tan}[c+d\,x]\right)^2\right)\Big/$$

$$\left(d\left(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]\right)^2\right)+$$

$$\frac{\left(3\,a^2+4\,b^2\right)\text{Cos}[c+d\,x]^2\,\text{Sec}\left[\frac{1}{2}(c+d\,x)\right]^2\left(a+b\,\text{Tan}[c+d\,x]\right)^2}{32\,d\left(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]\right)^2}+$$

$$\frac{a^2\,\text{Cos}[c+d\,x]^2\,\text{Sec}\left[\frac{1}{2}(c+d\,x)\right]^4\left(a+b\,\text{Tan}[c+d\,x]\right)^2}{64\,d\left(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]\right)^2}+$$

$$\left(b^2\,\text{Cos}[c+d\,x]^2\,\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\left(a+b\,\text{Tan}[c+d\,x]\right)^2\right)\Big/$$

$$\left(d\left(\text{Cos}\left[\frac{1}{2}(c+d\,x)\right]-\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right)\left(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]\right)^2\right)-$$

$$\left(b^2\,\text{Cos}[c+d\,x]^2\,\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\left(a+b\,\text{Tan}[c+d\,x]\right)^2\right)\Big/$$

$$\left(d\left(\text{Cos}\left[\frac{1}{2}(c+d\,x)\right]+\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right)\left(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]\right)^2\right)-$$

$$\frac{7\,a\,b\,\text{Cos}[c+d\,x]^2\,\text{Tan}\left[\frac{1}{2}(c+d\,x)\right]\left(a+b\,\text{Tan}[c+d\,x]\right)^2}{6\,d\left(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]\right)^2}-$$

$$\left(a\,b\,\text{Cos}[c+d\,x]^2\,\text{Sec}\left[\frac{1}{2}(c+d\,x)\right]^2\text{Tan}\left[\frac{1}{2}(c+d\,x)\right]\left(a+b\,\text{Tan}[c+d\,x]\right)^2\right)\Big/$$

$$\left(12\,d\left(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]\right)^2\right)$$

## Problem 32: Result more than twice size of optimal antiderivative.

$$\int \mathrm{Sin}[c+d\,x]^3\,(a+b\,\mathrm{Tan}[c+d\,x])^3\,dx$$

Optimal (type 3, 205 leaves, 16 steps):

$$\frac{3\,a^2\,b\,\mathrm{ArcTanh}[\mathrm{Sin}[c+d\,x]]}{d}-\frac{5\,b^3\,\mathrm{ArcTanh}[\mathrm{Sin}[c+d\,x]]}{2\,d}-\frac{a^3\,\mathrm{Cos}[c+d\,x]}{d}+\frac{6\,a\,b^2\,\mathrm{Cos}[c+d\,x]}{d}+\frac{a^3\,\mathrm{Cos}[c+d\,x]^3}{3\,d}-\frac{a\,b^2\,\mathrm{Cos}[c+d\,x]^3}{d}+\frac{3\,a\,b^2\,\mathrm{Sec}[c+d\,x]}{d}-\frac{3\,a^2\,b\,\mathrm{Sin}[c+d\,x]}{d}+\frac{5\,b^3\,\mathrm{Sin}[c+d\,x]}{2\,d}-\frac{a^2\,b\,\mathrm{Sin}[c+d\,x]^3}{d}+\frac{5\,b^3\,\mathrm{Sin}[c+d\,x]^3}{6\,d}+\frac{b^3\,\mathrm{Sin}[c+d\,x]^3\,\mathrm{Tan}[c+d\,x]^2}{2\,d}$$

Result (type 3, 771 leaves):

$$\frac{3\,a\,b^2\,\mathrm{Cos}[c+d\,x]^3\,(a+b\,\mathrm{Tan}[c+d\,x])^3}{d\,(a\,\mathrm{Cos}[c+d\,x]+b\,\mathrm{Sin}[c+d\,x])^3}-\frac{3\,a\,(a^2-7\,b^2)\,\mathrm{Cos}[c+d\,x]^4\,(a+b\,\mathrm{Tan}[c+d\,x])^3}{4\,d\,(a\,\mathrm{Cos}[c+d\,x]+b\,\mathrm{Sin}[c+d\,x])^3}+\frac{a\,(a^2-3\,b^2)\,\mathrm{Cos}[c+d\,x]^3\,\mathrm{Cos}[3\,(c+d\,x)]\,(a+b\,\mathrm{Tan}[c+d\,x])^3}{12\,d\,(a\,\mathrm{Cos}[c+d\,x]+b\,\mathrm{Sin}[c+d\,x])^3}+$$
$$\left((-6\,a^2\,b+5\,b^3)\,\mathrm{Cos}[c+d\,x]^3\,\mathrm{Log}\left[\mathrm{Cos}\left[\frac{1}{2}\,(c+d\,x)\right]-\mathrm{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]\right]\,(a+b\,\mathrm{Tan}[c+d\,x])^3\right)\Big/\left(2\,d\,(a\,\mathrm{Cos}[c+d\,x]+b\,\mathrm{Sin}[c+d\,x])^3\right)+$$
$$\left((6\,a^2\,b-5\,b^3)\,\mathrm{Cos}[c+d\,x]^3\,\mathrm{Log}\left[\mathrm{Cos}\left[\frac{1}{2}\,(c+d\,x)\right]+\mathrm{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]\right]\,(a+b\,\mathrm{Tan}[c+d\,x])^3\right)\Big/\left(2\,d\,(a\,\mathrm{Cos}[c+d\,x]+b\,\mathrm{Sin}[c+d\,x])^3\right)+\left(b^3\,\mathrm{Cos}[c+d\,x]^3\,(a+b\,\mathrm{Tan}[c+d\,x])^3\right)\Big/$$
$$\left(4\,d\,\left(\mathrm{Cos}\left[\frac{1}{2}\,(c+d\,x)\right]-\mathrm{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]\right)^2\,(a\,\mathrm{Cos}[c+d\,x]+b\,\mathrm{Sin}[c+d\,x])^3\right)+$$
$$\left(3\,a\,b^2\,\mathrm{Cos}[c+d\,x]^3\,\mathrm{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]\,(a+b\,\mathrm{Tan}[c+d\,x])^3\right)\Big/$$
$$\left(d\,\left(\mathrm{Cos}\left[\frac{1}{2}\,(c+d\,x)\right]-\mathrm{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]\right)\,(a\,\mathrm{Cos}[c+d\,x]+b\,\mathrm{Sin}[c+d\,x])^3\right)-$$
$$\left(b^3\,\mathrm{Cos}[c+d\,x]^3\,(a+b\,\mathrm{Tan}[c+d\,x])^3\right)\Big/$$
$$\left(4\,d\,\left(\mathrm{Cos}\left[\frac{1}{2}\,(c+d\,x)\right]+\mathrm{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]\right)^2\,(a\,\mathrm{Cos}[c+d\,x]+b\,\mathrm{Sin}[c+d\,x])^3\right)-$$
$$\left(3\,a\,b^2\,\mathrm{Cos}[c+d\,x]^3\,\mathrm{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]\,(a+b\,\mathrm{Tan}[c+d\,x])^3\right)\Big/$$
$$\left(d\,\left(\mathrm{Cos}\left[\frac{1}{2}\,(c+d\,x)\right]+\mathrm{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]\right)\,(a\,\mathrm{Cos}[c+d\,x]+b\,\mathrm{Sin}[c+d\,x])^3\right)-$$
$$\frac{3\,b\,(5\,a^2-3\,b^2)\,\mathrm{Cos}[c+d\,x]^3\,\mathrm{Sin}[c+d\,x]\,(a+b\,\mathrm{Tan}[c+d\,x])^3}{4\,d\,(a\,\mathrm{Cos}[c+d\,x]+b\,\mathrm{Sin}[c+d\,x])^3}+\frac{b\,(3\,a^2-b^2)\,\mathrm{Cos}[c+d\,x]^3\,\mathrm{Sin}[3\,(c+d\,x)]\,(a+b\,\mathrm{Tan}[c+d\,x])^3}{12\,d\,(a\,\mathrm{Cos}[c+d\,x]+b\,\mathrm{Sin}[c+d\,x])^3}$$

## Problem 34: Result more than twice size of optimal antiderivative.

$$\int \text{Sin}[c+d x] \, (a+b \, \text{Tan}[c+d x])^3 \, dx$$

Optimal (type 3, 133 leaves, 13 steps):

$$\frac{3 a^2 b \, \text{ArcTanh}[\text{Sin}[c+d x]]}{d}-\frac{3 b^3 \, \text{ArcTanh}[\text{Sin}[c+d x]]}{2 d}-\frac{a^3 \, \text{Cos}[c+d x]}{d}+\frac{3 a b^2 \, \text{Cos}[c+d x]}{d}+\frac{3 a b^2 \, \text{Sec}[c+d x]}{d}-\frac{3 a^2 b \, \text{Sin}[c+d x]}{d}+\frac{3 b^3 \, \text{Sin}[c+d x]}{2 d}+\frac{b^3 \, \text{Sin}[c+d x] \, \text{Tan}[c+d x]^2}{2 d}$$

Result (type 3, 637 leaves):

$$\frac{3 a b^2 \, \text{Cos}[c+d x]^3 \, (a+b \, \text{Tan}[c+d x])^3}{d \, (a \, \text{Cos}[c+d x]+b \, \text{Sin}[c+d x])^3}-\frac{a \, (a^2-3 b^2) \, \text{Cos}[c+d x]^4 \, (a+b \, \text{Tan}[c+d x])^3}{d \, (a \, \text{Cos}[c+d x]+b \, \text{Sin}[c+d x])^3}-$$
$$\left(3 \, (2 a^2 b-b^3) \, \text{Cos}[c+d x]^3 \, \text{Log}\left[\text{Cos}\left[\frac{1}{2} (c+d x)\right]-\text{Sin}\left[\frac{1}{2} (c+d x)\right]\right] (a+b \, \text{Tan}[c+d x])^3\right) \Big/ \left(2 d \, (a \, \text{Cos}[c+d x]+b \, \text{Sin}[c+d x])^3\right)+$$
$$\left(3 \, (2 a^2 b-b^3) \, \text{Cos}[c+d x]^3 \, \text{Log}\left[\text{Cos}\left[\frac{1}{2} (c+d x)\right]+\text{Sin}\left[\frac{1}{2} (c+d x)\right]\right] (a+b \, \text{Tan}[c+d x])^3\right) \Big/ \left(2 d \, (a \, \text{Cos}[c+d x]+b \, \text{Sin}[c+d x])^3\right)+\left(b^3 \, \text{Cos}[c+d x]^3 \, (a+b \, \text{Tan}[c+d x])^3\right) \Big/$$
$$\left(4 d \left(\text{Cos}\left[\frac{1}{2} (c+d x)\right]-\text{Sin}\left[\frac{1}{2} (c+d x)\right]\right)^2 (a \, \text{Cos}[c+d x]+b \, \text{Sin}[c+d x])^3\right)+$$
$$\left(3 a b^2 \, \text{Cos}[c+d x]^3 \, \text{Sin}\left[\frac{1}{2} (c+d x)\right] (a+b \, \text{Tan}[c+d x])^3\right) \Big/ \left(d \left(\text{Cos}\left[\frac{1}{2} (c+d x)\right]-\text{Sin}\left[\frac{1}{2} (c+d x)\right]\right) (a \, \text{Cos}[c+d x]+b \, \text{Sin}[c+d x])^3\right)-$$
$$\left(b^3 \, \text{Cos}[c+d x]^3 \, (a+b \, \text{Tan}[c+d x])^3\right) \Big/ \left(4 d \left(\text{Cos}\left[\frac{1}{2} (c+d x)\right]+\text{Sin}\left[\frac{1}{2} (c+d x)\right]\right)^2 (a \, \text{Cos}[c+d x]+b \, \text{Sin}[c+d x])^3\right)-$$
$$\left(3 a b^2 \, \text{Cos}[c+d x]^3 \, \text{Sin}\left[\frac{1}{2} (c+d x)\right] (a+b \, \text{Tan}[c+d x])^3\right) \Big/ \left(d \left(\text{Cos}\left[\frac{1}{2} (c+d x)\right]+\text{Sin}\left[\frac{1}{2} (c+d x)\right]\right) (a \, \text{Cos}[c+d x]+b \, \text{Sin}[c+d x])^3\right)-$$
$$\frac{b \, (3 a^2-b^2) \, \text{Cos}[c+d x]^3 \, \text{Sin}[c+d x] \, (a+b \, \text{Tan}[c+d x])^3}{d \, (a \, \text{Cos}[c+d x]+b \, \text{Sin}[c+d x])^3}$$

## Problem 35: Result more than twice size of optimal antiderivative.

$$\int \text{Csc}[c+d x] \, (a+b \, \text{Tan}[c+d x])^3 \, dx$$

Optimal (type 3, 86 leaves, 8 steps):

$$-\frac{a^3\,\text{ArcTanh}[\text{Cos}[c+d\,x]]}{d}+\frac{3\,a^2\,b\,\text{ArcTanh}[\text{Sin}[c+d\,x]]}{d}-\frac{b^3\,\text{ArcTanh}[\text{Sin}[c+d\,x]]}{2\,d}+\frac{3\,a\,b^2\,\text{Sec}[c+d\,x]}{d}+\frac{b^3\,\text{Sec}[c+d\,x]\,\text{Tan}[c+d\,x]}{2\,d}$$

Result (type 3, 634 leaves):

$$\frac{3\,a\,b^2\,\text{Cos}[c+d\,x]^3\,(a+b\,\text{Tan}[c+d\,x])^3}{d\,(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x])^3}-\frac{a^3\,\text{Cos}[c+d\,x]^3\,\text{Log}\left[\text{Cos}\left[\frac{1}{2}(c+d\,x)\right]\right]\,(a+b\,\text{Tan}[c+d\,x])^3}{d\,(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x])^3}+$$
$$\left(\left(-6\,a^2\,b+b^3\right)\text{Cos}[c+d\,x]^3\,\text{Log}\left[\text{Cos}\left[\frac{1}{2}(c+d\,x)\right]-\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right](a+b\,\text{Tan}[c+d\,x])^3\right)\Big/$$
$$\left(2\,d\,(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x])^3\right)+$$
$$\frac{a^3\,\text{Cos}[c+d\,x]^3\,\text{Log}\left[\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right]\,(a+b\,\text{Tan}[c+d\,x])^3}{d\,(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x])^3}+$$
$$\left(\left(6\,a^2\,b-b^3\right)\text{Cos}[c+d\,x]^3\,\text{Log}\left[\text{Cos}\left[\frac{1}{2}(c+d\,x)\right]+\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right](a+b\,\text{Tan}[c+d\,x])^3\right)\Big/$$
$$\left(2\,d\,(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x])^3\right)+\left(b^3\,\text{Cos}[c+d\,x]^3\,(a+b\,\text{Tan}[c+d\,x])^3\right)\Big/$$
$$\left(4\,d\left(\text{Cos}\left[\frac{1}{2}(c+d\,x)\right]-\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right)^2(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x])^3\right)+$$
$$\left(3\,a\,b^2\,\text{Cos}[c+d\,x]^3\,\text{Sin}\left[\frac{1}{2}(c+d\,x)\right](a+b\,\text{Tan}[c+d\,x])^3\right)\Big/$$
$$\left(d\left(\text{Cos}\left[\frac{1}{2}(c+d\,x)\right]-\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right)(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x])^3\right)-$$
$$\left(b^3\,\text{Cos}[c+d\,x]^3\,(a+b\,\text{Tan}[c+d\,x])^3\right)\Big/$$
$$\left(4\,d\left(\text{Cos}\left[\frac{1}{2}(c+d\,x)\right]+\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right)^2(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x])^3\right)-$$
$$\left(3\,a\,b^2\,\text{Cos}[c+d\,x]^3\,\text{Sin}\left[\frac{1}{2}(c+d\,x)\right](a+b\,\text{Tan}[c+d\,x])^3\right)\Big/$$
$$\left(d\left(\text{Cos}\left[\frac{1}{2}(c+d\,x)\right]+\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right)(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x])^3\right)$$

## Problem 37: Result more than twice size of optimal antiderivative.

$$\int \text{Csc}[c+d\,x]^3\,(a+b\,\text{Tan}[c+d\,x])^3\,dx$$

Optimal (type 3, 141 leaves, 12 steps):

$$-\frac{a^3 \text{ArcTanh}[\text{Cos}[c+d x]]}{2 d}-\frac{3 a b^2 \text{ArcTanh}[\text{Cos}[c+d x]]}{d}+\frac{3 a^2 b \text{ArcTanh}[\text{Sin}[c+d x]]}{d}+\frac{b^3 \text{ArcTanh}[\text{Sin}[c+d x]]}{2 d}-\frac{3 a^2 b \text{Csc}[c+d x]}{d}-\frac{a^3 \text{Cot}[c+d x] \text{Csc}[c+d x]}{2 d}+\frac{3 a b^2 \text{Sec}[c+d x]}{d}+\frac{b^3 \text{Sec}[c+d x] \text{Tan}[c+d x]}{2 d}$$

Result (type 3, 897 leaves):

$$\frac{3 a b^2 \text{Cos}[c+d x]^3 (a+b \text{Tan}[c+d x])^3}{d (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^3}-\frac{3 a^2 b \text{Cos}[c+d x]^3 \text{Cot}\left[\frac{1}{2} (c+d x)\right] (a+b \text{Tan}[c+d x])^3}{2 d (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^3}-\frac{a^3 \text{Cos}[c+d x]^3 \text{Csc}\left[\frac{1}{2} (c+d x)\right]^2 (a+b \text{Tan}[c+d x])^3}{8 d (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^3}+$$

$$\left(\left(-a^3-6 a b^2\right) \text{Cos}[c+d x]^3 \text{Log}\left[\text{Cos}\left[\frac{1}{2} (c+d x)\right]\right] (a+b \text{Tan}[c+d x])^3\right) \Big/ \left(2 d (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^3\right)+$$

$$\left(\left(-6 a^2 b-b^3\right) \text{Cos}[c+d x]^3 \text{Log}\left[\text{Cos}\left[\frac{1}{2} (c+d x)\right]-\text{Sin}\left[\frac{1}{2} (c+d x)\right]\right] (a+b \text{Tan}[c+d x])^3\right) \Big/ \left(2 d (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^3\right)+$$

$$\frac{\left(a^3+6 a b^2\right) \text{Cos}[c+d x]^3 \text{Log}\left[\text{Sin}\left[\frac{1}{2} (c+d x)\right]\right] (a+b \text{Tan}[c+d x])^3}{2 d (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^3}+$$

$$\left(\left(6 a^2 b+b^3\right) \text{Cos}[c+d x]^3 \text{Log}\left[\text{Cos}\left[\frac{1}{2} (c+d x)\right]+\text{Sin}\left[\frac{1}{2} (c+d x)\right]\right] (a+b \text{Tan}[c+d x])^3\right) \Big/ \left(2 d (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^3\right)+$$

$$\frac{a^3 \text{Cos}[c+d x]^3 \text{Sec}\left[\frac{1}{2} (c+d x)\right]^2 (a+b \text{Tan}[c+d x])^3}{8 d (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^3}+\left(b^3 \text{Cos}[c+d x]^3 (a+b \text{Tan}[c+d x])^3\right) \Big/ \left(4 d \left(\text{Cos}\left[\frac{1}{2} (c+d x)\right]-\text{Sin}\left[\frac{1}{2} (c+d x)\right]\right)^2 (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^3\right)+$$

$$\left(3 a b^2 \text{Cos}[c+d x]^3 \text{Sin}\left[\frac{1}{2} (c+d x)\right] (a+b \text{Tan}[c+d x])^3\right) \Big/ \left(d \left(\text{Cos}\left[\frac{1}{2} (c+d x)\right]-\text{Sin}\left[\frac{1}{2} (c+d x)\right]\right) (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^3\right)-$$

$$\left(b^3 \text{Cos}[c+d x]^3 (a+b \text{Tan}[c+d x])^3\right) \Big/ \left(4 d \left(\text{Cos}\left[\frac{1}{2} (c+d x)\right]+\text{Sin}\left[\frac{1}{2} (c+d x)\right]\right)^2 (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^3\right)-$$

$$\left(3 a b^2 \text{Cos}[c+d x]^3 \text{Sin}\left[\frac{1}{2} (c+d x)\right] (a+b \text{Tan}[c+d x])^3\right) \Big/ \left(d \left(\text{Cos}\left[\frac{1}{2} (c+d x)\right]+\text{Sin}\left[\frac{1}{2} (c+d x)\right]\right) (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^3\right)-$$

$$\frac{3 a^2 b \text{Cos}[c+d x]^3 \text{Tan}\left[\frac{1}{2} (c+d x)\right] (a+b \text{Tan}[c+d x])^3}{2 d (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^3}$$

## Problem 39: Result more than twice size of optimal antiderivative.

$$\int \mathrm{Csc}[c+d\,x]^5\,(a+b\,\mathrm{Tan}[c+d\,x])^3\,dx$$

Optimal (type 3, 229 leaves, 17 steps):

$$-\frac{3\,a^3\,\mathrm{ArcTanh}[\mathrm{Cos}[c+d\,x]]}{8\,d}-\frac{9\,a\,b^2\,\mathrm{ArcTanh}[\mathrm{Cos}[c+d\,x]]}{2\,d}+\frac{3\,a^2\,b\,\mathrm{ArcTanh}[\mathrm{Sin}[c+d\,x]]}{d}+\frac{3\,b^3\,\mathrm{ArcTanh}[\mathrm{Sin}[c+d\,x]]}{2\,d}-\frac{3\,a^2\,b\,\mathrm{Csc}[c+d\,x]}{d}-\frac{3\,b^3\,\mathrm{Csc}[c+d\,x]}{2\,d}-\frac{3\,a^3\,\mathrm{Cot}[c+d\,x]\,\mathrm{Csc}[c+d\,x]}{8\,d}-\frac{a^2\,b\,\mathrm{Csc}[c+d\,x]^3}{d}-\frac{a^3\,\mathrm{Cot}[c+d\,x]\,\mathrm{Csc}[c+d\,x]^3}{4\,d}+\frac{9\,a\,b^2\,\mathrm{Sec}[c+d\,x]}{2\,d}-\frac{3\,a\,b^2\,\mathrm{Csc}[c+d\,x]^2\,\mathrm{Sec}[c+d\,x]}{2\,d}+\frac{b^3\,\mathrm{Csc}[c+d\,x]\,\mathrm{Sec}[c+d\,x]^2}{2\,d}$$

Result (type 3, 1229 leaves):

$$\frac{3\,a\,b^2\,\mathrm{Cos}[c+d\,x]^3\,(a+b\,\mathrm{Tan}[c+d\,x])^3}{d\,(a\,\mathrm{Cos}[c+d\,x]+b\,\mathrm{Sin}[c+d\,x])^3}+\left(\left(-7\,a^2\,b\,\mathrm{Cos}\left[\frac{1}{2}(c+d\,x)\right]-2\,b^3\,\mathrm{Cos}\left[\frac{1}{2}(c+d\,x)\right]\right)\mathrm{Cos}[c+d\,x]^3\,\mathrm{Csc}\left[\frac{1}{2}(c+d\,x)\right](a+b\,\mathrm{Tan}[c+d\,x])^3\right)\Big/\left(4\,d\,(a\,\mathrm{Cos}[c+d\,x]+b\,\mathrm{Sin}[c+d\,x])^3\right)-\frac{3\,(a^3+4\,a\,b^2)\,\mathrm{Cos}[c+d\,x]^3\,\mathrm{Csc}\left[\frac{1}{2}(c+d\,x)\right]^2(a+b\,\mathrm{Tan}[c+d\,x])^3}{32\,d\,(a\,\mathrm{Cos}[c+d\,x]+b\,\mathrm{Sin}[c+d\,x])^3}-\left(a^2\,b\,\mathrm{Cos}[c+d\,x]^3\,\mathrm{Cot}\left[\frac{1}{2}(c+d\,x)\right]\mathrm{Csc}\left[\frac{1}{2}(c+d\,x)\right]^2(a+b\,\mathrm{Tan}[c+d\,x])^3\right)\Big/\left(8\,d\,(a\,\mathrm{Cos}[c+d\,x]+b\,\mathrm{Sin}[c+d\,x])^3\right)-\frac{a^3\,\mathrm{Cos}[c+d\,x]^3\,\mathrm{Csc}\left[\frac{1}{2}(c+d\,x)\right]^4(a+b\,\mathrm{Tan}[c+d\,x])^3}{64\,d\,(a\,\mathrm{Cos}[c+d\,x]+b\,\mathrm{Sin}[c+d\,x])^3}-\left(3\,(a^3+12\,a\,b^2)\,\mathrm{Cos}[c+d\,x]^3\,\mathrm{Log}\left[\mathrm{Cos}\left[\frac{1}{2}(c+d\,x)\right]\right](a+b\,\mathrm{Tan}[c+d\,x])^3\right)\Big/\left(8\,d\,(a\,\mathrm{Cos}[c+d\,x]+b\,\mathrm{Sin}[c+d\,x])^3\right)-\left(3\,(2\,a^2\,b+b^3)\,\mathrm{Cos}[c+d\,x]^3\,\mathrm{Log}\left[\mathrm{Cos}\left[\frac{1}{2}(c+d\,x)\right]-\mathrm{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right](a+b\,\mathrm{Tan}[c+d\,x])^3\right)\Big/\left(2\,d\,(a\,\mathrm{Cos}[c+d\,x]+b\,\mathrm{Sin}[c+d\,x])^3\right)+\left(3\,(a^3+12\,a\,b^2)\,\mathrm{Cos}[c+d\,x]^3\,\mathrm{Log}\left[\mathrm{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right](a+b\,\mathrm{Tan}[c+d\,x])^3\right)\Big/\left(8\,d\,(a\,\mathrm{Cos}[c+d\,x]+b\,\mathrm{Sin}[c+d\,x])^3\right)+\left(3\,(2\,a^2\,b+b^3)\,\mathrm{Cos}[c+d\,x]^3\,\mathrm{Log}\left[\mathrm{Cos}\left[\frac{1}{2}(c+d\,x)\right]+\mathrm{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right](a+b\,\mathrm{Tan}[c+d\,x])^3\right)\Big/\left(2\,d\,(a\,\mathrm{Cos}[c+d\,x]+b\,\mathrm{Sin}[c+d\,x])^3\right)+\frac{3\,(a^3+4\,a\,b^2)\,\mathrm{Cos}[c+d\,x]^3\,\mathrm{Sec}\left[\frac{1}{2}(c+d\,x)\right]^2(a+b\,\mathrm{Tan}[c+d\,x])^3}{32\,d\,(a\,\mathrm{Cos}[c+d\,x]+b\,\mathrm{Sin}[c+d\,x])^3}+$$

$$\frac{a^3 \operatorname{Cos}[c+d x]^3 \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^4 (a+b \operatorname{Tan}[c+d x])^3}{64 d (a \operatorname{Cos}[c+d x]+b \operatorname{Sin}[c+d x])^3}+\left(b^3 \operatorname{Cos}[c+d x]^3 (a+b \operatorname{Tan}[c+d x])^3\right) \Big/ \left(4 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^2 (a \operatorname{Cos}[c+d x]+b \operatorname{Sin}[c+d x])^3\right)+$$
$$\left(3 a b^2 \operatorname{Cos}[c+d x]^3 \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right] (a+b \operatorname{Tan}[c+d x])^3\right) \Big/ \left(d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right) (a \operatorname{Cos}[c+d x]+b \operatorname{Sin}[c+d x])^3\right)-$$
$$\left(b^3 \operatorname{Cos}[c+d x]^3 (a+b \operatorname{Tan}[c+d x])^3\right) \Big/ \left(4 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^2 (a \operatorname{Cos}[c+d x]+b \operatorname{Sin}[c+d x])^3\right)-$$
$$\left(3 a b^2 \operatorname{Cos}[c+d x]^3 \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right] (a+b \operatorname{Tan}[c+d x])^3\right) \Big/ \left(d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right) (a \operatorname{Cos}[c+d x]+b \operatorname{Sin}[c+d x])^3\right)+$$
$$\left(\operatorname{Cos}[c+d x]^3 \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right] \left(-7 a^2 b \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]-2 b^3 \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right) (a+b \operatorname{Tan}[c+d x])^3\right) \Big/ \left(4 d (a \operatorname{Cos}[c+d x]+b \operatorname{Sin}[c+d x])^3\right)-$$
$$\left(a^2 b \operatorname{Cos}[c+d x]^3 \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] (a+b \operatorname{Tan}[c+d x])^3\right) \Big/ \left(8 d (a \operatorname{Cos}[c+d x]+b \operatorname{Sin}[c+d x])^3\right)$$

## Problem 40: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Csc}[c+d x]^6 (a+b \operatorname{Tan}[c+d x])^3 \, dx$$

Optimal (type 3, 167 leaves, 3 steps):

$$-\frac{a \left(a^2+6 b^2\right) \operatorname{Cot}[c+d x]}{d}-\frac{b \left(6 a^2+b^2\right) \operatorname{Cot}[c+d x]^2}{2 d}-\frac{a \left(2 a^2+3 b^2\right) \operatorname{Cot}[c+d x]^3}{3 d}-\frac{3 a^2 b \operatorname{Cot}[c+d x]^4}{4 d}-\frac{a^3 \operatorname{Cot}[c+d x]^5}{5 d}+\frac{b \left(3 a^2+2 b^2\right) \operatorname{Log}[\operatorname{Tan}[c+d x]]}{d}+\frac{3 a b^2 \operatorname{Tan}[c+d x]}{d}+\frac{b^3 \operatorname{Tan}[c+d x]^2}{2 d}$$

Result (type 3, 515 leaves):

$$-\frac{1}{960\,d}\text{Csc}[c+d\,x]^5\,\text{Sec}[c+d\,x]^2\,\Big(40\,a\,(5\,a^2+3\,b^2)\,\text{Cos}[c+d\,x]+8\,(a^3+15\,a\,b^2)\,\text{Cos}[3\,(c+d\,x)]-24\,a^3\,\text{Cos}[5\,(c+d\,x)]-360\,a\,b^2\,\text{Cos}[5\,(c+d\,x)]+8\,a^3\,\text{Cos}[7\,(c+d\,x)]+120\,a\,b^2\,\text{Cos}[7\,(c+d\,x)]+360\,a^2\,b\,\text{Sin}[c+d\,x]-240\,b^3\,\text{Sin}[c+d\,x]+225\,a^2\,b\,\text{Log}[\text{Cos}[c+d\,x]]\,\text{Sin}[c+d\,x]+150\,b^3\,\text{Log}[\text{Cos}[c+d\,x]]\,\text{Sin}[c+d\,x]-225\,a^2\,b\,\text{Log}[\text{Sin}[c+d\,x]]\,\text{Sin}[c+d\,x]-150\,b^3\,\text{Log}[\text{Sin}[c+d\,x]]\,\text{Sin}[c+d\,x]+270\,a^2\,b\,\text{Sin}[3\,(c+d\,x)]+180\,b^3\,\text{Sin}[3\,(c+d\,x)]+45\,a^2\,b\,\text{Log}[\text{Cos}[c+d\,x]]\,\text{Sin}[3\,(c+d\,x)]+30\,b^3\,\text{Log}[\text{Cos}[c+d\,x]]\,\text{Sin}[3\,(c+d\,x)]-45\,a^2\,b\,\text{Log}[\text{Sin}[c+d\,x]]\,\text{Sin}[3\,(c+d\,x)]-30\,b^3\,\text{Log}[\text{Sin}[c+d\,x]]\,\text{Sin}[3\,(c+d\,x)]-90\,a^2\,b\,\text{Sin}[5\,(c+d\,x)]-60\,b^3\,\text{Sin}[5\,(c+d\,x)]-135\,a^2\,b\,\text{Log}[\text{Cos}[c+d\,x]]\,\text{Sin}[5\,(c+d\,x)]-90\,b^3\,\text{Log}[\text{Cos}[c+d\,x]]\,\text{Sin}[5\,(c+d\,x)]+135\,a^2\,b\,\text{Log}[\text{Sin}[c+d\,x]]\,\text{Sin}[5\,(c+d\,x)]+90\,b^3\,\text{Log}[\text{Sin}[c+d\,x]]\,\text{Sin}[5\,(c+d\,x)]+45\,a^2\,b\,\text{Log}[\text{Cos}[c+d\,x]]\,\text{Sin}[7\,(c+d\,x)]+30\,b^3\,\text{Log}[\text{Cos}[c+d\,x]]\,\text{Sin}[7\,(c+d\,x)]-45\,a^2\,b\,\text{Log}[\text{Sin}[c+d\,x]]\,\text{Sin}[7\,(c+d\,x)]-30\,b^3\,\text{Log}[\text{Sin}[c+d\,x]]\,\text{Sin}[7\,(c+d\,x)]\Big)$$

## Problem 41: Result more than twice size of optimal antiderivative.

$$\int \text{Sin}[c+d\,x]^3\,(a+b\,\text{Tan}[c+d\,x])^4\,dx$$

Optimal (type 3, 275 leaves, 19 steps):

$$\frac{4\,a^3\,b\,\text{ArcTanh}[\text{Sin}[c+d\,x]]}{d}-\frac{10\,a\,b^3\,\text{ArcTanh}[\text{Sin}[c+d\,x]]}{d}-\frac{a^4\,\text{Cos}[c+d\,x]}{d}+\frac{12\,a^2\,b^2\,\text{Cos}[c+d\,x]}{d}-\frac{3\,b^4\,\text{Cos}[c+d\,x]}{d}+\frac{a^4\,\text{Cos}[c+d\,x]^3}{3\,d}-\frac{2\,a^2\,b^2\,\text{Cos}[c+d\,x]^3}{d}+\frac{b^4\,\text{Cos}[c+d\,x]^3}{3\,d}+\frac{6\,a^2\,b^2\,\text{Sec}[c+d\,x]}{d}-\frac{3\,b^4\,\text{Sec}[c+d\,x]}{d}+\frac{b^4\,\text{Sec}[c+d\,x]^3}{3\,d}-\frac{4\,a^3\,b\,\text{Sin}[c+d\,x]}{d}+\frac{10\,a\,b^3\,\text{Sin}[c+d\,x]}{d}-\frac{4\,a^3\,b\,\text{Sin}[c+d\,x]^3}{3\,d}+\frac{10\,a\,b^3\,\text{Sin}[c+d\,x]^3}{3\,d}+\frac{2\,a\,b^3\,\text{Sin}[c+d\,x]^3\,\text{Tan}[c+d\,x]^2}{d}$$

Result (type 3, 1017 leaves):

$$-\frac{b^2\left(-36a^2+17b^2\right)\text{Cos}[c+dx]^4\left(a+b\,\text{Tan}[c+dx]\right)^4}{6d\left(a\,\text{Cos}[c+dx]+b\,\text{Sin}[c+dx]\right)^4}-$$

$$\frac{\left(3a^4-42a^2b^2+11b^4\right)\text{Cos}[c+dx]^5\left(a+b\,\text{Tan}[c+dx]\right)^4}{4d\left(a\,\text{Cos}[c+dx]+b\,\text{Sin}[c+dx]\right)^4}+$$

$$\frac{\left(a^4-6a^2b^2+b^4\right)\text{Cos}[c+dx]^4\,\text{Cos}\left[3\left(c+dx\right)\right]\left(a+b\,\text{Tan}[c+dx]\right)^4}{12d\left(a\,\text{Cos}[c+dx]+b\,\text{Sin}[c+dx]\right)^4}-$$

$$\left(2\left(2a^3b-5ab^3\right)\text{Cos}[c+dx]^4\,\text{Log}\left[\text{Cos}\left[\frac{1}{2}\left(c+dx\right)\right]-\text{Sin}\left[\frac{1}{2}\left(c+dx\right)\right]\right]\left(a+b\,\text{Tan}[c+dx]\right)^4\right)\Big/$$
$$\left(d\left(a\,\text{Cos}[c+dx]+b\,\text{Sin}[c+dx]\right)^4\right)+$$

$$\left(2\left(2a^3b-5ab^3\right)\text{Cos}[c+dx]^4\,\text{Log}\left[\text{Cos}\left[\frac{1}{2}\left(c+dx\right)\right]+\text{Sin}\left[\frac{1}{2}\left(c+dx\right)\right]\right]\left(a+b\,\text{Tan}[c+dx]\right)^4\right)\Big/$$
$$\left(d\left(a\,\text{Cos}[c+dx]+b\,\text{Sin}[c+dx]\right)^4\right)+\left(\left(12ab^3+b^4\right)\text{Cos}[c+dx]^4\left(a+b\,\text{Tan}[c+dx]\right)^4\right)\Big/$$
$$\left(12d\left(\text{Cos}\left[\frac{1}{2}\left(c+dx\right)\right]-\text{Sin}\left[\frac{1}{2}\left(c+dx\right)\right]\right)^2\left(a\,\text{Cos}[c+dx]+b\,\text{Sin}[c+dx]\right)^4\right)+$$

$$\left(b^4\,\text{Cos}[c+dx]^4\,\text{Sin}\left[\frac{1}{2}\left(c+dx\right)\right]\left(a+b\,\text{Tan}[c+dx]\right)^4\right)\Big/$$
$$\left(6d\left(\text{Cos}\left[\frac{1}{2}\left(c+dx\right)\right]-\text{Sin}\left[\frac{1}{2}\left(c+dx\right)\right]\right)^3\left(a\,\text{Cos}[c+dx]+b\,\text{Sin}[c+dx]\right)^4\right)-$$

$$\left(b^4\,\text{Cos}[c+dx]^4\,\text{Sin}\left[\frac{1}{2}\left(c+dx\right)\right]\left(a+b\,\text{Tan}[c+dx]\right)^4\right)\Big/$$
$$\left(6d\left(\text{Cos}\left[\frac{1}{2}\left(c+dx\right)\right]+\text{Sin}\left[\frac{1}{2}\left(c+dx\right)\right]\right)^3\left(a\,\text{Cos}[c+dx]+b\,\text{Sin}[c+dx]\right)^4\right)+$$

$$\left(\left(-12ab^3+b^4\right)\text{Cos}[c+dx]^4\left(a+b\,\text{Tan}[c+dx]\right)^4\right)\Big/$$
$$\left(12d\left(\text{Cos}\left[\frac{1}{2}\left(c+dx\right)\right]+\text{Sin}\left[\frac{1}{2}\left(c+dx\right)\right]\right)^2\left(a\,\text{Cos}[c+dx]+b\,\text{Sin}[c+dx]\right)^4\right)+$$

$$\left(\text{Cos}[c+dx]^4\left(36a^2b^2\,\text{Sin}\left[\frac{1}{2}\left(c+dx\right)\right]-17b^4\,\text{Sin}\left[\frac{1}{2}\left(c+dx\right)\right]\right)\left(a+b\,\text{Tan}[c+dx]\right)^4\right)\Big/$$
$$\left(6d\left(\text{Cos}\left[\frac{1}{2}\left(c+dx\right)\right]-\text{Sin}\left[\frac{1}{2}\left(c+dx\right)\right]\right)\left(a\,\text{Cos}[c+dx]+b\,\text{Sin}[c+dx]\right)^4\right)+$$

$$\left(\text{Cos}[c+dx]^4\left(-36a^2b^2\,\text{Sin}\left[\frac{1}{2}\left(c+dx\right)\right]+17b^4\,\text{Sin}\left[\frac{1}{2}\left(c+dx\right)\right]\right)\left(a+b\,\text{Tan}[c+dx]\right)^4\right)\Big/$$
$$\left(6d\left(\text{Cos}\left[\frac{1}{2}\left(c+dx\right)\right]+\text{Sin}\left[\frac{1}{2}\left(c+dx\right)\right]\right)\left(a\,\text{Cos}[c+dx]+b\,\text{Sin}[c+dx]\right)^4\right)-$$

$$\frac{ab\left(5a^2-9b^2\right)\text{Cos}[c+dx]^4\,\text{Sin}[c+dx]\left(a+b\,\text{Tan}[c+dx]\right)^4}{d\left(a\,\text{Cos}[c+dx]+b\,\text{Sin}[c+dx]\right)^4}+$$

$$\frac{ab\left(a^2-b^2\right)\text{Cos}[c+dx]^4\,\text{Sin}\left[3\left(c+dx\right)\right]\left(a+b\,\text{Tan}[c+dx]\right)^4}{3d\left(a\,\text{Cos}[c+dx]+b\,\text{Sin}[c+dx]\right)^4}$$

## Problem 43: Result more than twice size of optimal antiderivative.

$$\int \text{Sin}[c+d\,x]\,\left(a+b\,\text{Tan}[c+d\,x]\right)^4\,dx$$

Optimal (type 3, 180 leaves, 16 steps):

$$\frac{4\,a^3\,b\,\text{ArcTanh}[\text{Sin}[c+d\,x]]}{d}-\frac{6\,a\,b^3\,\text{ArcTanh}[\text{Sin}[c+d\,x]]}{d}-\frac{a^4\,\text{Cos}[c+d\,x]}{d}+\frac{6\,a^2\,b^2\,\text{Cos}[c+d\,x]}{d}-\frac{b^4\,\text{Cos}[c+d\,x]}{d}+\frac{6\,a^2\,b^2\,\text{Sec}[c+d\,x]}{d}-\frac{2\,b^4\,\text{Sec}[c+d\,x]}{d}+\frac{b^4\,\text{Sec}[c+d\,x]^3}{3\,d}-\frac{4\,a^3\,b\,\text{Sin}[c+d\,x]}{d}+\frac{6\,a\,b^3\,\text{Sin}[c+d\,x]}{d}+\frac{2\,a\,b^3\,\text{Sin}[c+d\,x]\,\text{Tan}[c+d\,x]^2}{d}$$

Result (type 3, 875 leaves):

$$-\frac{b^2\left(-36 a^2+11 b^2\right) \text{Cos}[c+d x]^4 (a+b \text{Tan}[c+d x])^4}{6 d (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^4}-\frac{\left(a^4-6 a^2 b^2+b^4\right) \text{Cos}[c+d x]^5 (a+b \text{Tan}[c+d x])^4}{d (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^4}-$$
$$\left(2\left(2 a^3 b-3 a b^3\right) \text{Cos}[c+d x]^4 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c+d x)\right]-\text{Sin}\left[\frac{1}{2}(c+d x)\right]\right](a+b \text{Tan}[c+d x])^4\right)\Big/$$
$$\left(d (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^4\right)+$$
$$\left(2\left(2 a^3 b-3 a b^3\right) \text{Cos}[c+d x]^4 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c+d x)\right]+\text{Sin}\left[\frac{1}{2}(c+d x)\right]\right](a+b \text{Tan}[c+d x])^4\right)\Big/$$
$$\left(d (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^4\right)+\left(\left(12 a b^3+b^4\right) \text{Cos}[c+d x]^4 (a+b \text{Tan}[c+d x])^4\right)\Big/$$
$$\left(12 d\left(\text{Cos}\left[\frac{1}{2}(c+d x)\right]-\text{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^2 (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^4\right)+$$
$$\left(b^4 \text{Cos}[c+d x]^4 \text{Sin}\left[\frac{1}{2}(c+d x)\right](a+b \text{Tan}[c+d x])^4\right)\Big/$$
$$\left(6 d\left(\text{Cos}\left[\frac{1}{2}(c+d x)\right]-\text{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^3 (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^4\right)-$$
$$\left(b^4 \text{Cos}[c+d x]^4 \text{Sin}\left[\frac{1}{2}(c+d x)\right](a+b \text{Tan}[c+d x])^4\right)\Big/$$
$$\left(6 d\left(\text{Cos}\left[\frac{1}{2}(c+d x)\right]+\text{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^3 (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^4\right)+$$
$$\left(\left(-12 a b^3+b^4\right) \text{Cos}[c+d x]^4 (a+b \text{Tan}[c+d x])^4\right)\Big/$$
$$\left(12 d\left(\text{Cos}\left[\frac{1}{2}(c+d x)\right]+\text{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^2 (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^4\right)+$$
$$\left(\text{Cos}[c+d x]^4\left(36 a^2 b^2 \text{Sin}\left[\frac{1}{2}(c+d x)\right]-11 b^4 \text{Sin}\left[\frac{1}{2}(c+d x)\right]\right)(a+b \text{Tan}[c+d x])^4\right)\Big/$$
$$\left(6 d\left(\text{Cos}\left[\frac{1}{2}(c+d x)\right]-\text{Sin}\left[\frac{1}{2}(c+d x)\right]\right)(a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^4\right)+$$
$$\left(\text{Cos}[c+d x]^4\left(-36 a^2 b^2 \text{Sin}\left[\frac{1}{2}(c+d x)\right]+11 b^4 \text{Sin}\left[\frac{1}{2}(c+d x)\right]\right)(a+b \text{Tan}[c+d x])^4\right)\Big/$$
$$\left(6 d\left(\text{Cos}\left[\frac{1}{2}(c+d x)\right]+\text{Sin}\left[\frac{1}{2}(c+d x)\right]\right)(a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^4\right)-$$
$$\frac{4 a b\left(a^2-b^2\right) \text{Cos}[c+d x]^4 \text{Sin}[c+d x](a+b \text{Tan}[c+d x])^4}{d (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^4}$$

## Problem 44: Result more than twice size of optimal antiderivative.

$$\int \text{Csc}[c+d x] (a+b \text{Tan}[c+d x])^4 \, dx$$

Optimal (type 3, 118 leaves, 10 steps):

$$-\frac{a^4\,\text{ArcTanh}[\text{Cos}[c+d\,x]]}{d}+\frac{4\,a^3\,b\,\text{ArcTanh}[\text{Sin}[c+d\,x]]}{d}-\frac{2\,a\,b^3\,\text{ArcTanh}[\text{Sin}[c+d\,x]]}{d}+\frac{6\,a^2\,b^2\,\text{Sec}[c+d\,x]}{d}-\frac{b^4\,\text{Sec}[c+d\,x]}{d}+\frac{b^4\,\text{Sec}[c+d\,x]^3}{3\,d}+\frac{2\,a\,b^3\,\text{Sec}[c+d\,x]\,\text{Tan}[c+d\,x]}{d}$$

Result (type 3, 870 leaves):

$$\frac{b^2\left(36\,a^2-5\,b^2\right)\text{Cos}[c+d\,x]^4\,(a+b\,\text{Tan}[c+d\,x])^4}{6\,d\,(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x])^4}-\frac{a^4\,\text{Cos}[c+d\,x]^4\,\text{Log}\left[\text{Cos}\left[\frac{1}{2}(c+d\,x)\right]\right](a+b\,\text{Tan}[c+d\,x])^4}{d\,(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x])^4}-$$

$$\left(2\left(2\,a^3\,b-a\,b^3\right)\text{Cos}[c+d\,x]^4\,\text{Log}\left[\text{Cos}\left[\frac{1}{2}(c+d\,x)\right]-\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right](a+b\,\text{Tan}[c+d\,x])^4\right)\Big/\left(d\,(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x])^4\right)+$$

$$\frac{a^4\,\text{Cos}[c+d\,x]^4\,\text{Log}\left[\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right](a+b\,\text{Tan}[c+d\,x])^4}{d\,(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x])^4}+$$

$$\left(2\left(2\,a^3\,b-a\,b^3\right)\text{Cos}[c+d\,x]^4\,\text{Log}\left[\text{Cos}\left[\frac{1}{2}(c+d\,x)\right]+\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right](a+b\,\text{Tan}[c+d\,x])^4\right)\Big/\left(d\,(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x])^4\right)+\left(\left(12\,a\,b^3+b^4\right)\text{Cos}[c+d\,x]^4\,(a+b\,\text{Tan}[c+d\,x])^4\right)\Big/$$

$$\left(12\,d\left(\text{Cos}\left[\frac{1}{2}(c+d\,x)\right]-\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right)^2(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x])^4\right)+$$

$$\left(b^4\,\text{Cos}[c+d\,x]^4\,\text{Sin}\left[\frac{1}{2}(c+d\,x)\right](a+b\,\text{Tan}[c+d\,x])^4\right)\Big/$$

$$\left(6\,d\left(\text{Cos}\left[\frac{1}{2}(c+d\,x)\right]-\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right)^3(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x])^4\right)-$$

$$\left(b^4\,\text{Cos}[c+d\,x]^4\,\text{Sin}\left[\frac{1}{2}(c+d\,x)\right](a+b\,\text{Tan}[c+d\,x])^4\right)\Big/$$

$$\left(6\,d\left(\text{Cos}\left[\frac{1}{2}(c+d\,x)\right]+\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right)^3(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x])^4\right)+$$

$$\left(\left(-12\,a\,b^3+b^4\right)\text{Cos}[c+d\,x]^4\,(a+b\,\text{Tan}[c+d\,x])^4\right)\Big/$$

$$\left(12\,d\left(\text{Cos}\left[\frac{1}{2}(c+d\,x)\right]+\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right)^2(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x])^4\right)+$$

$$\left(\text{Cos}[c+d\,x]^4\left(36\,a^2\,b^2\,\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]-5\,b^4\,\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right)(a+b\,\text{Tan}[c+d\,x])^4\right)\Big/$$

$$\left(6\,d\left(\text{Cos}\left[\frac{1}{2}(c+d\,x)\right]-\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right)(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x])^4\right)+$$

$$\left(\text{Cos}[c+d\,x]^4\left(-36\,a^2\,b^2\,\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]+5\,b^4\,\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right)(a+b\,\text{Tan}[c+d\,x])^4\right)\Big/$$

$$\left(6\,d\left(\text{Cos}\left[\frac{1}{2}(c+d\,x)\right]+\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right)(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x])^4\right)$$

# Problem 46: Result more than twice size of optimal antiderivative.

$$\int \mathrm{Csc}[c+d x]^3 (a+b\,\mathrm{Tan}[c+d x])^4\, dx$$

Optimal (type 3, 161 leaves, 14 steps):

$$-\frac{a^4\,\mathrm{ArcTanh}[\mathrm{Cos}[c+d x]]}{2 d}-\frac{6 a^2 b^2\,\mathrm{ArcTanh}[\mathrm{Cos}[c+d x]]}{d}+\frac{4 a^3 b\,\mathrm{ArcTanh}[\mathrm{Sin}[c+d x]]}{d}+\frac{2 a b^3\,\mathrm{ArcTanh}[\mathrm{Sin}[c+d x]]}{d}-\frac{4 a^3 b\,\mathrm{Csc}[c+d x]}{d}-\frac{a^4\,\mathrm{Cot}[c+d x]\,\mathrm{Csc}[c+d x]}{2 d}+\frac{6 a^2 b^2\,\mathrm{Sec}[c+d x]}{d}+\frac{b^4\,\mathrm{Sec}[c+d x]^3}{3 d}+\frac{2 a b^3\,\mathrm{Sec}[c+d x]\,\mathrm{Tan}[c+d x]}{d}$$

Result (type 3, 1128 leaves):

$$\frac{b^2 (36 a^2+b^2)\,\mathrm{Cos}[c+d x]^4 (a+b\,\mathrm{Tan}[c+d x])^4}{6 d (a\,\mathrm{Cos}[c+d x]+b\,\mathrm{Sin}[c+d x])^4}-\frac{2 a^3 b\,\mathrm{Cos}[c+d x]^4\,\mathrm{Cot}\left[\frac{1}{2}(c+d x)\right] (a+b\,\mathrm{Tan}[c+d x])^4}{d (a\,\mathrm{Cos}[c+d x]+b\,\mathrm{Sin}[c+d x])^4}-\frac{a^4\,\mathrm{Cos}[c+d x]^4\,\mathrm{Csc}\left[\frac{1}{2}(c+d x)\right]^2 (a+b\,\mathrm{Tan}[c+d x])^4}{8 d (a\,\mathrm{Cos}[c+d x]+b\,\mathrm{Sin}[c+d x])^4}+$$

$$\left((-a^4-12 a^2 b^2)\,\mathrm{Cos}[c+d x]^4\,\mathrm{Log}\left[\mathrm{Cos}\left[\frac{1}{2}(c+d x)\right]\right] (a+b\,\mathrm{Tan}[c+d x])^4\right)\Big/\left(2 d (a\,\mathrm{Cos}[c+d x]+b\,\mathrm{Sin}[c+d x])^4\right)-$$

$$\left(2 (2 a^3 b+a b^3)\,\mathrm{Cos}[c+d x]^4\,\mathrm{Log}\left[\mathrm{Cos}\left[\frac{1}{2}(c+d x)\right]-\mathrm{Sin}\left[\frac{1}{2}(c+d x)\right]\right] (a+b\,\mathrm{Tan}[c+d x])^4\right)\Big/\left(d (a\,\mathrm{Cos}[c+d x]+b\,\mathrm{Sin}[c+d x])^4\right)+$$

$$\left((a^4+12 a^2 b^2)\,\mathrm{Cos}[c+d x]^4\,\mathrm{Log}\left[\mathrm{Sin}\left[\frac{1}{2}(c+d x)\right]\right] (a+b\,\mathrm{Tan}[c+d x])^4\right)\Big/\left(2 d (a\,\mathrm{Cos}[c+d x]+b\,\mathrm{Sin}[c+d x])^4\right)+$$

$$\left(2 (2 a^3 b+a b^3)\,\mathrm{Cos}[c+d x]^4\,\mathrm{Log}\left[\mathrm{Cos}\left[\frac{1}{2}(c+d x)\right]+\mathrm{Sin}\left[\frac{1}{2}(c+d x)\right]\right] (a+b\,\mathrm{Tan}[c+d x])^4\right)\Big/\left(d (a\,\mathrm{Cos}[c+d x]+b\,\mathrm{Sin}[c+d x])^4\right)+\frac{a^4\,\mathrm{Cos}[c+d x]^4\,\mathrm{Sec}\left[\frac{1}{2}(c+d x)\right]^2 (a+b\,\mathrm{Tan}[c+d x])^4}{8 d (a\,\mathrm{Cos}[c+d x]+b\,\mathrm{Sin}[c+d x])^4}+$$

$$\left((12 a b^3+b^4)\,\mathrm{Cos}[c+d x]^4 (a+b\,\mathrm{Tan}[c+d x])^4\right)\Big/\left(12 d \left(\mathrm{Cos}\left[\frac{1}{2}(c+d x)\right]-\mathrm{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^2 (a\,\mathrm{Cos}[c+d x]+b\,\mathrm{Sin}[c+d x])^4\right)+$$

$$\left(b^4\,\mathrm{Cos}[c+d x]^4\,\mathrm{Sin}\left[\frac{1}{2}(c+d x)\right] (a+b\,\mathrm{Tan}[c+d x])^4\right)\Big/\left(6 d \left(\mathrm{Cos}\left[\frac{1}{2}(c+d x)\right]-\mathrm{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^3 (a\,\mathrm{Cos}[c+d x]+b\,\mathrm{Sin}[c+d x])^4\right)-$$

$$\left(b^4 \operatorname{Cos}[c+d x]^4 \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right] (a+b \operatorname{Tan}[c+d x])^4\right) \Big/$$
$$\left(6 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^3 (a \operatorname{Cos}[c+d x]+b \operatorname{Sin}[c+d x])^4\right)+$$
$$\left(\left(-12 a b^3+b^4\right) \operatorname{Cos}[c+d x]^4 (a+b \operatorname{Tan}[c+d x])^4\right) \Big/$$
$$\left(12 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^2 (a \operatorname{Cos}[c+d x]+b \operatorname{Sin}[c+d x])^4\right)+$$
$$\left(\operatorname{Cos}[c+d x]^4 \left(-36 a^2 b^2 \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]-b^4 \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right) (a+b \operatorname{Tan}[c+d x])^4\right) \Big/$$
$$\left(6 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right) (a \operatorname{Cos}[c+d x]+b \operatorname{Sin}[c+d x])^4\right)+$$
$$\left(\operatorname{Cos}[c+d x]^4 \left(36 a^2 b^2 \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]+b^4 \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right) (a+b \operatorname{Tan}[c+d x])^4\right) \Big/$$
$$\left(6 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right) (a \operatorname{Cos}[c+d x]+b \operatorname{Sin}[c+d x])^4\right)-$$
$$\frac{2 a^3 b \operatorname{Cos}[c+d x]^4 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] (a+b \operatorname{Tan}[c+d x])^4}{d (a \operatorname{Cos}[c+d x]+b \operatorname{Sin}[c+d x])^4}$$

## Problem 47: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Csc}[c+d x]^4 (a+b \operatorname{Tan}[c+d x])^4 \, dx$$

Optimal (type 3, 137 leaves, 3 steps):

$$-\frac{a^2 \left(a^2+6 b^2\right) \operatorname{Cot}[c+d x]}{d}-\frac{2 a^3 b \operatorname{Cot}[c+d x]^2}{d}-\frac{a^4 \operatorname{Cot}[c+d x]^3}{3 d}+$$
$$\frac{4 a b \left(a^2+b^2\right) \operatorname{Log}[\operatorname{Tan}[c+d x]]}{d}+\frac{b^2 \left(6 a^2+b^2\right) \operatorname{Tan}[c+d x]}{d}+\frac{2 a b^3 \operatorname{Tan}[c+d x]^2}{d}+\frac{b^4 \operatorname{Tan}[c+d x]^3}{3 d}$$

Result (type 3, 487 leaves):

$$\frac{2\,a\,b^3\,\text{Cos}[c+d\,x]^2\,\left(a+b\,\text{Tan}[c+d\,x]\right)^4}{d\,\left(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]\right)^4}-\left(2\,\text{Cos}[c+d\,x]^3\,\left(a^4\,\text{Cos}[c+d\,x]+9\,a^2\,b^2\,\text{Cos}[c+d\,x]\right)\,\text{Cot}[c+d\,x]\,\left(a+b\,\text{Tan}[c+d\,x]\right)^4\right)\Big/\left(3\,d\,\left(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]\right)^4\right)-\frac{2\,a^3\,b\,\text{Cos}[c+d\,x]^2\,\text{Cot}[c+d\,x]^2\,\left(a+b\,\text{Tan}[c+d\,x]\right)^4}{d\,\left(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]\right)^4}-\frac{a^4\,\text{Cos}[c+d\,x]^2\,\text{Cot}[c+d\,x]^3\,\left(a+b\,\text{Tan}[c+d\,x]\right)^4}{3\,d\,\left(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]\right)^4}-\frac{4\,\left(a^3\,b+a\,b^3\right)\,\text{Cos}[c+d\,x]^4\,\text{Log}[\text{Cos}[c+d\,x]]\,\left(a+b\,\text{Tan}[c+d\,x]\right)^4}{d\,\left(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]\right)^4}+\frac{4\,\left(a^3\,b+a\,b^3\right)\,\text{Cos}[c+d\,x]^4\,\text{Log}[\text{Sin}[c+d\,x]]\,\left(a+b\,\text{Tan}[c+d\,x]\right)^4}{d\,\left(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]\right)^4}+\frac{b^4\,\text{Cos}[c+d\,x]\,\text{Sin}[c+d\,x]\,\left(a+b\,\text{Tan}[c+d\,x]\right)^4}{3\,d\,\left(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]\right)^4}+\left(2\,\text{Cos}[c+d\,x]^3\,\left(9\,a^2\,b^2\,\text{Sin}[c+d\,x]+b^4\,\text{Sin}[c+d\,x]\right)\,\left(a+b\,\text{Tan}[c+d\,x]\right)^4\right)\Big/\left(3\,d\,\left(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]\right)^4\right)$$

## Problem 48: Result more than twice size of optimal antiderivative.

$$\int \text{Csc}[c+d\,x]^5\,\left(a+b\,\text{Tan}[c+d\,x]\right)^4\,dx$$

Optimal (type 3, 274 leaves, 21 steps):

$$-\frac{3\,a^4\,\text{ArcTanh}[\text{Cos}[c+d\,x]]}{8\,d}-\frac{9\,a^2\,b^2\,\text{ArcTanh}[\text{Cos}[c+d\,x]]}{d}-\frac{b^4\,\text{ArcTanh}[\text{Cos}[c+d\,x]]}{d}+\frac{4\,a^3\,b\,\text{ArcTanh}[\text{Sin}[c+d\,x]]}{d}+\frac{6\,a\,b^3\,\text{ArcTanh}[\text{Sin}[c+d\,x]]}{d}-\frac{4\,a^3\,b\,\text{Csc}[c+d\,x]}{d}-\frac{6\,a\,b^3\,\text{Csc}[c+d\,x]}{d}-\frac{3\,a^4\,\text{Cot}[c+d\,x]\,\text{Csc}[c+d\,x]}{8\,d}-\frac{4\,a^3\,b\,\text{Csc}[c+d\,x]^3}{3\,d}-\frac{a^4\,\text{Cot}[c+d\,x]\,\text{Csc}[c+d\,x]^3}{4\,d}+\frac{9\,a^2\,b^2\,\text{Sec}[c+d\,x]}{d}+\frac{b^4\,\text{Sec}[c+d\,x]}{d}-\frac{3\,a^2\,b^2\,\text{Csc}[c+d\,x]^2\,\text{Sec}[c+d\,x]}{d}+\frac{2\,a\,b^3\,\text{Csc}[c+d\,x]\,\text{Sec}[c+d\,x]^2}{d}+\frac{b^4\,\text{Sec}[c+d\,x]^3}{3\,d}$$

Result (type 3, 1491 leaves):

$$\frac{b^2\,\left(36\,a^2+7\,b^2\right)\,\text{Cos}[c+d\,x]^4\,\left(a+b\,\text{Tan}[c+d\,x]\right)^4}{6\,d\,\left(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]\right)^4}+\left(\left(-7\,a^3\,b\,\text{Cos}\left[\frac{1}{2}\,(c+d\,x)\right]-6\,a\,b^3\,\text{Cos}\left[\frac{1}{2}\,(c+d\,x)\right]\right)\,\text{Cos}[c+d\,x]^4\,\text{Csc}\left[\frac{1}{2}\,(c+d\,x)\right]\,\left(a+b\,\text{Tan}[c+d\,x]\right)^4\right)\Big/\left(3\,d\,\left(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]\right)^4\right)-$$

$$\frac{3\left(a^4+8 a^2 b^2\right) \operatorname{Cos}[c+d x]^4 \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2(a+b \operatorname{Tan}[c+d x])^4}{32 d(a \operatorname{Cos}[c+d x]+b \operatorname{Sin}[c+d x])^4}-$$

$$\left(a^3 b \operatorname{Cos}[c+d x]^4 \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2(a+b \operatorname{Tan}[c+d x])^4\right) \Big/$$

$$\left(6 d(a \operatorname{Cos}[c+d x]+b \operatorname{Sin}[c+d x])^4\right)-\frac{a^4 \operatorname{Cos}[c+d x]^4 \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^4(a+b \operatorname{Tan}[c+d x])^4}{64 d(a \operatorname{Cos}[c+d x]+b \operatorname{Sin}[c+d x])^4}+$$

$$\left(\left(-3 a^4-72 a^2 b^2-8 b^4\right) \operatorname{Cos}[c+d x]^4 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]\right](a+b \operatorname{Tan}[c+d x])^4\right) \Big/$$

$$\left(8 d(a \operatorname{Cos}[c+d x]+b \operatorname{Sin}[c+d x])^4\right)-$$

$$\left(2\left(2 a^3 b+3 a b^3\right) \operatorname{Cos}[c+d x]^4 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right](a+b \operatorname{Tan}[c+d x])^4\right) \Big/$$

$$\left(d(a \operatorname{Cos}[c+d x]+b \operatorname{Sin}[c+d x])^4\right)+$$

$$\left(\left(3 a^4+72 a^2 b^2+8 b^4\right) \operatorname{Cos}[c+d x]^4 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right](a+b \operatorname{Tan}[c+d x])^4\right) \Big/$$

$$\left(8 d(a \operatorname{Cos}[c+d x]+b \operatorname{Sin}[c+d x])^4\right)+$$

$$\left(2\left(2 a^3 b+3 a b^3\right) \operatorname{Cos}[c+d x]^4 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right](a+b \operatorname{Tan}[c+d x])^4\right) \Big/$$

$$\left(d(a \operatorname{Cos}[c+d x]+b \operatorname{Sin}[c+d x])^4\right)+$$

$$\frac{3\left(a^4+8 a^2 b^2\right) \operatorname{Cos}[c+d x]^4 \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2(a+b \operatorname{Tan}[c+d x])^4}{32 d(a \operatorname{Cos}[c+d x]+b \operatorname{Sin}[c+d x])^4}+$$

$$\frac{a^4 \operatorname{Cos}[c+d x]^4 \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^4(a+b \operatorname{Tan}[c+d x])^4}{64 d(a \operatorname{Cos}[c+d x]+b \operatorname{Sin}[c+d x])^4}+$$

$$\left(\left(12 a b^3+b^4\right) \operatorname{Cos}[c+d x]^4(a+b \operatorname{Tan}[c+d x])^4\right) \Big/$$

$$\left(12 d\left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^2(a \operatorname{Cos}[c+d x]+b \operatorname{Sin}[c+d x])^4\right)+$$

$$\left(b^4 \operatorname{Cos}[c+d x]^4 \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right](a+b \operatorname{Tan}[c+d x])^4\right) \Big/$$

$$\left(6 d\left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^3(a \operatorname{Cos}[c+d x]+b \operatorname{Sin}[c+d x])^4\right)-$$

$$\left(b^4 \operatorname{Cos}[c+d x]^4 \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right](a+b \operatorname{Tan}[c+d x])^4\right) \Big/$$

$$\left(6 d\left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^3(a \operatorname{Cos}[c+d x]+b \operatorname{Sin}[c+d x])^4\right)+$$

$$\left(\left(-12 a b^3+b^4\right) \operatorname{Cos}[c+d x]^4(a+b \operatorname{Tan}[c+d x])^4\right) \Big/$$

$$\left(12 d\left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^2(a \operatorname{Cos}[c+d x]+b \operatorname{Sin}[c+d x])^4\right)+$$

$$\left(\operatorname{Cos}[c+d x]^4 \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]\left(-7 a^3 b \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]-6 a b^3 \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)\right.$$

$$\left.(a+b \operatorname{Tan}[c+d x])^4\right) \Big/\left(3 d(a \operatorname{Cos}[c+d x]+b \operatorname{Sin}[c+d x])^4\right)+$$

$$\left(\text{Cos}[c+d x]^4 \left(-36 a^2 b^2 \text{Sin}\left[\frac{1}{2} (c+d x)\right]-7 b^4 \text{Sin}\left[\frac{1}{2} (c+d x)\right]\right) (a+b \text{Tan}[c+d x])^4\right) \Big/ \left(6 d \left(\text{Cos}\left[\frac{1}{2} (c+d x)\right]+\text{Sin}\left[\frac{1}{2} (c+d x)\right]\right) (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^4\right)+ \left(\text{Cos}[c+d x]^4 \left(36 a^2 b^2 \text{Sin}\left[\frac{1}{2} (c+d x)\right]+7 b^4 \text{Sin}\left[\frac{1}{2} (c+d x)\right]\right) (a+b \text{Tan}[c+d x])^4\right) \Big/ \left(6 d \left(\text{Cos}\left[\frac{1}{2} (c+d x)\right]-\text{Sin}\left[\frac{1}{2} (c+d x)\right]\right) (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^4\right)- \left(a^3 b \text{Cos}[c+d x]^4 \text{Sec}\left[\frac{1}{2} (c+d x)\right]^2 \text{Tan}\left[\frac{1}{2} (c+d x)\right] (a+b \text{Tan}[c+d x])^4\right) \Big/ \left(6 d (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^4\right)$$

## Problem 49: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \text{Csc}[c+d x]^6 (a+b \text{Tan}[c+d x])^4 \, dx$$

Optimal (type 3, 194 leaves, 3 steps):

$$-\frac{\left(a^4+12 a^2 b^2+b^4\right) \text{Cot}[c+d x]}{d}-\frac{2 a b \left(2 a^2+b^2\right) \text{Cot}[c+d x]^2}{d}-\frac{2 a^2 \left(a^2+3 b^2\right) \text{Cot}[c+d x]^3}{3 d}-\frac{a^3 b \text{Cot}[c+d x]^4}{d}-\frac{a^4 \text{Cot}[c+d x]^5}{5 d}+\frac{4 a b \left(a^2+2 b^2\right) \text{Log}[\text{Tan}[c+d x]]}{d}+\frac{2 b^2 \left(3 a^2+b^2\right) \text{Tan}[c+d x]}{d}+\frac{2 a b^3 \text{Tan}[c+d x]^2}{d}+\frac{b^4 \text{Tan}[c+d x]^3}{3 d}$$

Result (type 3, 632 leaves):

$$\frac{2\,a\,b^3\,\text{Cos}[c+d\,x]^2\,\left(a+b\,\text{Tan}[c+d\,x]\right)^4}{d\,\left(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]\right)^4}+\left(\text{Cos}[c+d\,x]^3\,\left(-8\,a^4\,\text{Cos}[c+d\,x]-150\,a^2\,b^2\,\text{Cos}[c+d\,x]-15\,b^4\,\text{Cos}[c+d\,x]\right)\,\text{Cot}[c+d\,x]\,\left(a+b\,\text{Tan}[c+d\,x]\right)^4\right)\Big/\left(15\,d\,\left(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]\right)^4\right)-\left(2\,a\,\left(a-\text{i}\,b\right)\,\left(a+\text{i}\,b\right)\,b\,\text{Cos}[c+d\,x]^2\,\text{Cot}[c+d\,x]^2\,\left(a+b\,\text{Tan}[c+d\,x]\right)^4\right)\Big/\left(d\,\left(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]\right)^4\right)-\left(2\,\text{Cos}[c+d\,x]\,\left(2\,a^4\,\text{Cos}[c+d\,x]+15\,a^2\,b^2\,\text{Cos}[c+d\,x]\right)\,\text{Cot}[c+d\,x]^3\,\left(a+b\,\text{Tan}[c+d\,x]\right)^4\right)\Big/\left(15\,d\,\left(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]\right)^4\right)-\frac{a^3\,b\,\text{Cot}[c+d\,x]^4\,\left(a+b\,\text{Tan}[c+d\,x]\right)^4}{d\,\left(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]\right)^4}-\frac{a^4\,\text{Cot}[c+d\,x]^5\,\left(a+b\,\text{Tan}[c+d\,x]\right)^4}{5\,d\,\left(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]\right)^4}-\frac{4\,\left(a^3\,b+2\,a\,b^3\right)\,\text{Cos}[c+d\,x]^4\,\text{Log}[\text{Cos}[c+d\,x]]\,\left(a+b\,\text{Tan}[c+d\,x]\right)^4}{d\,\left(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]\right)^4}+\frac{4\,\left(a^3\,b+2\,a\,b^3\right)\,\text{Cos}[c+d\,x]^4\,\text{Log}[\text{Sin}[c+d\,x]]\,\left(a+b\,\text{Tan}[c+d\,x]\right)^4}{d\,\left(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]\right)^4}+\frac{b^4\,\text{Cos}[c+d\,x]\,\text{Sin}[c+d\,x]\,\left(a+b\,\text{Tan}[c+d\,x]\right)^4}{3\,d\,\left(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]\right)^4}+\left(\text{Cos}[c+d\,x]^3\,\left(18\,a^2\,b^2\,\text{Sin}[c+d\,x]+5\,b^4\,\text{Sin}[c+d\,x]\right)\,\left(a+b\,\text{Tan}[c+d\,x]\right)^4\right)\Big/\left(3\,d\,\left(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]\right)^4\right)$$

## Problem 52: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Sin}[c+d\,x]^4}{a+b\,\text{Tan}[c+d\,x]}\,dx$$

Optimal (type 3, 158 leaves, 8 steps):

$$\frac{a\,\left(3\,a^4-6\,a^2\,b^2-b^4\right)\,x}{8\,\left(a^2+b^2\right)^3}+\frac{a^4\,b\,\text{Log}[a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]]}{\left(a^2+b^2\right)^3\,d}+\frac{\text{Cos}[c+d\,x]^4\,\left(b+a\,\text{Tan}[c+d\,x]\right)}{4\,\left(a^2+b^2\right)\,d}-\frac{\text{Cos}[c+d\,x]^2\,\left(4\,b\,\left(2\,a^2+b^2\right)+a\,\left(5\,a^2+b^2\right)\,\text{Tan}[c+d\,x]\right)}{8\,\left(a^2+b^2\right)^2\,d}$$

Result (type 3, 443 leaves):

$$\frac{1}{32\left(a^2+b^2\right)^3 d}\left(12 a^5 c+28 i a^4 b c-24 a^3 b^2 c-8 i a^2 b^3 c-4 a b^4 c-4 i b^5 c+12 a^5 d x+28 i a^4 b d x-24 a^3 b^2 d x-8 i a^2 b^3 d x-4 a b^4 d x-4 i b^5 d x+4 i b\left(-7 a^4+2 a^2 b^2+b^4\right) \text{ArcTan}[\text{Tan}[c+d x]]-4\left(3 a^4 b+4 a^2 b^3+b^5\right) \text{Cos}[2(c+d x)]+a^4 b \text{Cos}[4(c+d x)]+2 a^2 b^3 \text{Cos}[4(c+d x)]+b^5 \text{Cos}[4(c+d x)]+4 a^4 b \text{Log}[a \text{Cos}[c+d x]+b \text{Sin}[c+d x]]+8 a^2 b^3 \text{Log}[a \text{Cos}[c+d x]+b \text{Sin}[c+d x]]+4 b^5 \text{Log}[a \text{Cos}[c+d x]+b \text{Sin}[c+d x]]+14 a^4 b \text{Log}\left[(a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^2\right]-4 a^2 b^3 \text{Log}\left[(a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^2\right]-2 b^5 \text{Log}\left[(a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^2\right]-8 a^5 \text{Sin}[2(c+d x)]-8 a^3 b^2 \text{Sin}[2(c+d x)]+a^5 \text{Sin}[4(c+d x)]+2 a^3 b^2 \text{Sin}[4(c+d x)]+a b^4 \text{Sin}[4(c+d x)]\right)$$

## Problem 54: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Sin}[c+d x]^2}{a+b \text{Tan}[c+d x]} \, dx$$

Optimal (type 3, 94 leaves, 7 steps):

$$\frac{a\left(a^2-b^2\right) x}{2\left(a^2+b^2\right)^2}+\frac{a^2 b \text{Log}[a \text{Cos}[c+d x]+b \text{Sin}[c+d x]]}{\left(a^2+b^2\right)^2 d}-\frac{\text{Cos}[c+d x]^2(b+a \text{Tan}[c+d x])}{2\left(a^2+b^2\right) d}$$

Result (type 3, 245 leaves):

$$\frac{1}{8\left(a^2+b^2\right)^2 d}\left(4 a^3 c+6 i a^2 b c-4 a b^2 c-2 i b^3 c+4 a^3 d x+6 i a^2 b d x-4 a b^2 d x-2 i b^3 d x+2 i b\left(-3 a^2+b^2\right) \text{ArcTan}[\text{Tan}[c+d x]]-2 b\left(a^2+b^2\right) \text{Cos}[2(c+d x)]+2 a^2 b \text{Log}[a \text{Cos}[c+d x]+b \text{Sin}[c+d x]]+2 b^3 \text{Log}[a \text{Cos}[c+d x]+b \text{Sin}[c+d x]]+3 a^2 b \text{Log}\left[(a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^2\right]-b^3 \text{Log}\left[(a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^2\right]-2 a^3 \text{Sin}[2(c+d x)]-2 a b^2 \text{Sin}[2(c+d x)]\right)$$

## Problem 61: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Sin}[c+d x]^6}{(a+b \text{Tan}[c+d x])^2} \, dx$$

Optimal (type 3, 297 leaves, 9 steps):

$$\frac{\left(5 a^8-80 a^6 b^2+50 a^4 b^4+8 a^2 b^6+b^8\right) x}{16 \left(a^2+b^2\right)^5}+\frac{2 a^5 b \left(a^2-3 b^2\right) \text{Log}[a \text{Cos}[c+d x]+b \text{Sin}[c+d x]]}{\left(a^2+b^2\right)^5 d}-\frac{a^6 b}{\left(a^2+b^2\right)^4 d (a+b \text{Tan}[c+d x])}-\frac{\text{Cos}[c+d x]^6 \left(2 a b+\left(a^2-b^2\right) \text{Tan}[c+d x]\right)}{6 \left(a^2+b^2\right)^2 d}+\frac{1}{24 \left(a^2+b^2\right)^3 d}\text{Cos}[c+d x]^4 \left(12 a b \left(3 a^2+b^2\right)+\left(13 a^4-18 a^2 b^2-7 b^4\right) \text{Tan}[c+d x]\right)-\frac{\text{Cos}[c+d x]^2 \left(48 a^5 b+\left(11 a^6-43 a^4 b^2-7 a^2 b^4-b^6\right) \text{Tan}[c+d x]\right)}{16 \left(a^2+b^2\right)^4 d}$$

Result (type 3, 1916 leaves):

$$-\frac{1}{32 a \left(a^2+b^2\right)^2 d (a+b \text{Tan}[c+d x])^2}\text{Sec}[c+d x]^2 (a \text{Cos}[c+d x]+b \text{Sin}[c+d x]) \Big(2 a^2 \text{Cos}[c+d x] \left((a+i b)^2 (c+d x)+a b \text{Log}\left[(a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^2\right]\right)+\Big(-a^4+b^4+2 a^3 b (c+d x)+4 i a^2 b^2 (c+d x)-2 a b^3 (c+d x)+2 a^2 b^2 \text{Log}\left[(a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^2\right]\Big) \text{Sin}[c+d x]-4 i a^2 b \text{ArcTan}[\text{Tan}[c+d x]] (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])\Big)-$$

$$\left(\text{Sec}[c+d x]^2 (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^2 \left(-4 \left(a^4-6 a^2 b^2+b^4\right) (c+d x)+4 a b \left(a^2+b^2\right) \text{Cos}[2 (c+d x)]-16 a b \left(a^2-b^2\right) \text{Log}[a \text{Cos}[c+d x]+b \text{Sin}[c+d x]]+\frac{\left(a^2+b^2\right) \left(a^4-6 a^2 b^2+b^4\right) \text{Sin}[c+d x]}{a (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])}+2 \left(a^4-b^4\right) \text{Sin}[2 (c+d x)]\right)\right)\Big/\left(32 \left(a^2+b^2\right)^3 d (a+b \text{Tan}[c+d x])^2\right)+\frac{1}{32 \left(a^2+b^2\right)^4 d (a+b \text{Tan}[c+d x])^2}$$

$$\text{Sec}[c+d x]^2 (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^2 \Big(6 (a-b) (a+b) \left(a^2-4 a b+b^2\right) \left(a^2+4 a b+b^2\right) (c+d x)+12 i \left(3 a^5 b-10 a^3 b^3+3 a b^5\right) (c+d x)-12 i a b \left(3 a^4-10 a^2 b^2+3 b^4\right) \text{ArcTan}[\text{Tan}[c+d x]]+16 a b \left(-a^4+b^4\right) \text{Cos}[2 (c+d x)]+2 a b \left(a^2+b^2\right)^2 \text{Cos}[4 (c+d x)]+6 a b \left(3 a^4-10 a^2 b^2+3 b^4\right) \text{Log}\left[(a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^2\right]+\frac{\left(-a^8+14 a^6 b^2-14 a^2 b^6+b^8\right) \text{Sin}[c+d x]}{a (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])}-4 \left(a^2+b^2\right) \left(a^4-6 a^2 b^2+b^4\right) \text{Sin}[2 (c+d x)]+\left(a^2-b^2\right) \left(a^2+b^2\right)^2 \text{Sin}[4 (c+d x)]\Big)-$$

$$\frac{1}{384 a \left(a^2+b^2\right)^5 d (a+b \text{Tan}[c+d x])^2}\text{Sec}[c+d x]^2 (a \text{Cos}[c+d x]+b \text{Sin}[c+d x]) \Big(30 a^9 b \text{Cos}[3 (c+d x)]-84 a^5 b^5 \text{Cos}[3 (c+d x)]-48 a^3 b^7 \text{Cos}[3 (c+d x)]+6 a b^9 \text{Cos}[3 (c+d x)]-6 a^9 b \text{Cos}[5 (c+d x)]-16 a^7 b^3 \text{Cos}[5 (c+d x)]-12 a^5 b^5 \text{Cos}[5 (c+d x)]+2 a b^9 \text{Cos}[5 (c+d x)]+a^9 b \text{Cos}[7 (c+d x)]+4 a^7 b^3 \text{Cos}[7 (c+d x)]+6 a^5 b^5 \text{Cos}[7 (c+d x)]+4 a^3 b^7 \text{Cos}[7 (c+d x)]+a b^9 \text{Cos}[7 (c+d x)]-3 a \text{Cos}[c+d x] \Big(3 b^9+8 a^9 (c+d x)-224 a^7 b^2 (c+d x)+560 a^5 b^4 (c+d x)-224 a^3 b^6 (c+d x)+8 a b^8 (c+d x)+4 a^2 b^7 (-15-16 i (c+d x))+$$

$28 a^6 b^3 (3 - 16 i (c + d x)) + 14 a^4 b^5 (3 + 32 i (c + d x)) + a^8 b (-21 + 64 i (c + d x)) +$
$32 a^2 b (a^6 - 7 a^4 b^2 + 7 a^2 b^4 - b^6) \text{Log}[(a \text{Cos}[c + d x] + b \text{Sin}[c + d x])^2]) +$
$12 a^{10} \text{Sin}[c + d x] - 261 a^8 b^2 \text{Sin}[c + d x] + 252 a^6 b^4 \text{Sin}[c + d x] + 378 a^4 b^6 \text{Sin}[c + d x] -$
$144 a^2 b^8 \text{Sin}[c + d x] + 3 b^{10} \text{Sin}[c + d x] - 24 a^9 b (c + d x) \text{Sin}[c + d x] -$
$192 i a^8 b^2 (c + d x) \text{Sin}[c + d x] + 672 a^7 b^3 (c + d x) \text{Sin}[c + d x] +$
$1344 i a^6 b^4 (c + d x) \text{Sin}[c + d x] - 1680 a^5 b^5 (c + d x) \text{Sin}[c + d x] -$
$1344 i a^4 b^6 (c + d x) \text{Sin}[c + d x] + 672 a^3 b^7 (c + d x) \text{Sin}[c + d x] +$
$192 i a^2 b^8 (c + d x) \text{Sin}[c + d x] - 24 a b^9 (c + d x) \text{Sin}[c + d x] -$
$96 a^8 b^2 \text{Log}[(a \text{Cos}[c + d x] + b \text{Sin}[c + d x])^2] \text{Sin}[c + d x] +$
$672 a^6 b^4 \text{Log}[(a \text{Cos}[c + d x] + b \text{Sin}[c + d x])^2] \text{Sin}[c + d x] -$
$672 a^4 b^6 \text{Log}[(a \text{Cos}[c + d x] + b \text{Sin}[c + d x])^2] \text{Sin}[c + d x] +$
$96 a^2 b^8 \text{Log}[(a \text{Cos}[c + d x] + b \text{Sin}[c + d x])^2] \text{Sin}[c + d x] +$
$192 i a^2 b (a^6 - 7 a^4 b^2 + 7 a^2 b^4 - b^6) \text{ArcTan}[\text{Tan}[c + d x]] (a \text{Cos}[c + d x] + b \text{Sin}[c + d x]) +$
$6 a^{10} \text{Sin}[3 (c + d x)] - 48 a^8 b^2 \text{Sin}[3 (c + d x)] - 84 a^6 b^4 \text{Sin}[3 (c + d x)] +$
$30 a^2 b^8 \text{Sin}[3 (c + d x)] - 2 a^{10} \text{Sin}[5 (c + d x)] + 12 a^6 b^4 \text{Sin}[5 (c + d x)] + 16 a^4 b^6$
$\text{Sin}[5 (c + d x)] + 6 a^2 b^8 \text{Sin}[5 (c + d x)] + a^{10} \text{Sin}[7 (c + d x)] + 4 a^8 b^2 \text{Sin}[7 (c + d x)] +$
$6 a^6 b^4 \text{Sin}[7 (c + d x)] + 4 a^4 b^6 \text{Sin}[7 (c + d x)] + a^2 b^8 \text{Sin}[7 (c + d x)]) +$

$$\frac{5 \text{Sec}[c + d x] (a \text{Cos}[c + d x] + b \text{Sin}[c + d x]) \text{Tan}[c + d x]}{128 a d (a + b \text{Tan}[c + d x])^2}$$

## Problem 62: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Sin}[c + d x]^4}{(a + b \text{Tan}[c + d x])^2} \, dx$$

Optimal (type 3, 217 leaves, 8 steps):

$$\frac{(3 a^6 - 33 a^4 b^2 + 13 a^2 b^4 + b^6) x}{8 (a^2 + b^2)^4} + \frac{2 a^3 b (a^2 - 2 b^2) \text{Log}[a \text{Cos}[c + d x] + b \text{Sin}[c + d x]]}{(a^2 + b^2)^4 d} -$$
$$\frac{a^4 b}{(a^2 + b^2)^3 d (a + b \text{Tan}[c + d x])} + \frac{\text{Cos}[c + d x]^4 (2 a b + (a^2 - b^2) \text{Tan}[c + d x])}{4 (a^2 + b^2)^2 d} -$$
$$\frac{\text{Cos}[c + d x]^2 (16 a^3 b + (5 a^4 - 12 a^2 b^2 - b^4) \text{Tan}[c + d x])}{8 (a^2 + b^2)^3 d}$$

Result (type 3, 823 leaves):

$$-\frac{1}{32\,a\,(a^2+b^2)^2\,d\,(a+b\,\text{Tan}[c+d\,x])^2}\,\text{Sec}[c+d\,x]^2\,(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x])$$
$$\Big(2\,a^2\,\text{Cos}[c+d\,x]\,\big((a+i\,b)^2\,(c+d\,x)+a\,b\,\text{Log}\big[(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x])^2\big]\big)+$$
$$\big(-a^4+b^4+2\,a^3\,b\,(c+d\,x)+4\,i\,a^2\,b^2\,(c+d\,x)-2\,a\,b^3\,(c+d\,x)+$$
$$2\,a^2\,b^2\,\text{Log}\big[(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x])^2\big]\big)\,\text{Sin}[c+d\,x]-$$
$$4\,i\,a^2\,b\,\text{ArcTan}[\text{Tan}[c+d\,x]]\,(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x])\Big)-$$
$$\Big(\text{Sec}[c+d\,x]^2\,(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x])^2\,\Big(-4\,(a^4-6\,a^2\,b^2+b^4)\,(c+d\,x)+$$
$$4\,a\,b\,(a^2+b^2)\,\text{Cos}[2\,(c+d\,x)]-16\,a\,b\,(a^2-b^2)\,\text{Log}[a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]]+$$
$$\frac{(a^2+b^2)\,(a^4-6\,a^2\,b^2+b^4)\,\text{Sin}[c+d\,x]}{a\,(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x])}+2\,(a^4-b^4)\,\text{Sin}[2\,(c+d\,x)]\Big)\Big)\Big/$$
$$\big(16\,(a^2+b^2)^3\,d\,(a+b\,\text{Tan}[c+d\,x])^2\big)+\frac{1}{32\,(a^2+b^2)^4\,d\,(a+b\,\text{Tan}[c+d\,x])^2}$$
$$\text{Sec}[c+d\,x]^2\,(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x])^2$$
$$\Big(6\,(a-b)\,(a+b)\,(a^2-4\,a\,b+b^2)\,(a^2+4\,a\,b+b^2)\,(c+d\,x)+$$
$$12\,i\,(3\,a^5\,b-10\,a^3\,b^3+3\,a\,b^5)\,(c+d\,x)-12\,i\,a\,b\,(3\,a^4-10\,a^2\,b^2+3\,b^4)\,\text{ArcTan}[\text{Tan}[c+d\,x]]+$$
$$16\,a\,b\,(-a^4+b^4)\,\text{Cos}[2\,(c+d\,x)]+2\,a\,b\,(a^2+b^2)^2\,\text{Cos}[4\,(c+d\,x)]+6\,a\,b\,(3\,a^4-10\,a^2\,b^2+3\,b^4)$$
$$\text{Log}\big[(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x])^2\big]+\frac{(-a^8+14\,a^6\,b^2-14\,a^2\,b^6+b^8)\,\text{Sin}[c+d\,x]}{a\,(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x])}-$$
$$4\,(a^2+b^2)\,(a^4-6\,a^2\,b^2+b^4)\,\text{Sin}[2\,(c+d\,x)]+(a^2-b^2)\,(a^2+b^2)^2\,\text{Sin}[4\,(c+d\,x)]\Big)+$$
$$\frac{\text{Sec}[c+d\,x]\,(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x])\,\text{Tan}[c+d\,x]}{16\,a\,d\,(a+b\,\text{Tan}[c+d\,x])^2}$$

## Problem 66: Result more than twice size of optimal antiderivative.

$$\int\frac{\text{Csc}[c+d\,x]^6}{(a+b\,\text{Tan}[c+d\,x])^2}\,dx$$

Optimal (type 3, 219 leaves, 3 steps):

$$-\frac{(a^2+b^2)\,(a^2+5\,b^2)\,\text{Cot}[c+d\,x]}{a^6\,d}+\frac{2\,b\,(a^2+b^2)\,\text{Cot}[c+d\,x]^2}{a^5\,d}-\frac{(2\,a^2+3\,b^2)\,\text{Cot}[c+d\,x]^3}{3\,a^4\,d}+$$
$$\frac{b\,\text{Cot}[c+d\,x]^4}{2\,a^3\,d}-\frac{\text{Cot}[c+d\,x]^5}{5\,a^2\,d}-\frac{2\,b\,(a^2+b^2)\,(a^2+3\,b^2)\,\text{Log}[\text{Tan}[c+d\,x]]}{a^7\,d}+$$
$$\frac{2\,b\,(a^2+b^2)\,(a^2+3\,b^2)\,\text{Log}[a+b\,\text{Tan}[c+d\,x]]}{a^7\,d}-\frac{b\,(a^2+b^2)^2}{a^6\,d\,(a+b\,\text{Tan}[c+d\,x])}$$

Result (type 3, 589 leaves):

$$-\frac{\text{Csc}[c+d x]^5 \,\text{Sec}[c+d x] \,(a \,\text{Cos}[c+d x]+b \,\text{Sin}[c+d x])^2}{5 a^2 d \,(a+b \,\text{Tan}[c+d x])^2}+$$
$$\left(\left(-8 a^4 \text{Cos}[c+d x]-75 a^2 b^2 \text{Cos}[c+d x]-75 b^4 \text{Cos}[c+d x]\right) \text{Csc}[c+d x]\right.$$
$$\left.\text{Sec}[c+d x]^2 \left(a \,\text{Cos}[c+d x]+b \,\text{Sin}[c+d x]\right)^2\right) \Big/ \left(15 a^6 d \left(a+b \,\text{Tan}[c+d x]\right)^2\right)+$$
$$\left(b \left(a^2+2 b^2\right) \text{Csc}[c+d x]^2 \,\text{Sec}[c+d x]^2 \left(a \,\text{Cos}[c+d x]+b \,\text{Sin}[c+d x]\right)^2\right) \Big/$$
$$\left(a^5 d \left(a+b \,\text{Tan}[c+d x]\right)^2\right)+\left(\left(-4 a^2 \text{Cos}[c+d x]-15 b^2 \text{Cos}[c+d x]\right) \text{Csc}[c+d x]^3\right.$$
$$\left.\text{Sec}[c+d x]^2 \left(a \,\text{Cos}[c+d x]+b \,\text{Sin}[c+d x]\right)^2\right) \Big/ \left(15 a^4 d \left(a+b \,\text{Tan}[c+d x]\right)^2\right)+$$
$$\frac{b \,\text{Csc}[c+d x]^4 \,\text{Sec}[c+d x]^2 \left(a \,\text{Cos}[c+d x]+b \,\text{Sin}[c+d x]\right)^2}{2 a^3 d \left(a+b \,\text{Tan}[c+d x]\right)^2}-$$
$$\left(2 \left(a^4 b+4 a^2 b^3+3 b^5\right) \text{Log}[\text{Sin}[c+d x]] \,\text{Sec}[c+d x]^2 \left(a \,\text{Cos}[c+d x]+b \,\text{Sin}[c+d x]\right)^2\right) \Big/$$
$$\left(a^7 d \left(a+b \,\text{Tan}[c+d x]\right)^2\right)+\left(2 \left(a^4 b+4 a^2 b^3+3 b^5\right)\right.$$
$$\left.\text{Log}[a \,\text{Cos}[c+d x]+b \,\text{Sin}[c+d x]] \,\text{Sec}[c+d x]^2 \left(a \,\text{Cos}[c+d x]+b \,\text{Sin}[c+d x]\right)^2\right) \Big/$$
$$\left(a^7 d \left(a+b \,\text{Tan}[c+d x]\right)^2\right)+\left(\text{Sec}[c+d x]^2 \left(a \,\text{Cos}[c+d x]+b \,\text{Sin}[c+d x]\right)\right.$$
$$\left.\left(a^4 b^2 \text{Sin}[c+d x]+2 a^2 b^4 \text{Sin}[c+d x]+b^6 \text{Sin}[c+d x]\right)\right) \Big/ \left(a^7 d \left(a+b \,\text{Tan}[c+d x]\right)^2\right)$$

## Problem 67: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Sin}[c+d x]^6}{(a+b \,\text{Tan}[c+d x])^3} \, dx$$

Optimal (type 3, 382 leaves, 9 steps):

$$\frac{a \left(5 a^8-180 a^6 b^2+390 a^4 b^4-68 a^2 b^6-3 b^8\right) x}{16 \left(a^2+b^2\right)^6}+$$
$$\frac{a^4 b \left(3 a^4-22 a^2 b^2+15 b^4\right) \text{Log}[a \,\text{Cos}[c+d x]+b \,\text{Sin}[c+d x]]}{\left(a^2+b^2\right)^6 d}-$$
$$\frac{a^6 b}{2 \left(a^2+b^2\right)^4 d \left(a+b \,\text{Tan}[c+d x]\right)^2}-\frac{2 a^5 b \left(a^2-3 b^2\right)}{\left(a^2+b^2\right)^5 d \left(a+b \,\text{Tan}[c+d x]\right)}-$$
$$\frac{\text{Cos}[c+d x]^6 \left(b \left(3 a^2-b^2\right)+a \left(a^2-3 b^2\right) \text{Tan}[c+d x]\right)}{6 \left(a^2+b^2\right)^3 d}+\frac{1}{24 \left(a^2+b^2\right)^4 d}$$
$$\text{Cos}[c+d x]^4 \left(6 b \left(9 a^4-4 a^2 b^2-b^4\right)+a \left(13 a^4-62 a^2 b^2-3 b^4\right) \text{Tan}[c+d x]\right)-\frac{1}{16 \left(a^2+b^2\right)^5 d}$$
$$a \,\text{Cos}[c+d x]^2 \left(24 a^3 b \left(3 a^2-5 b^2\right)+\left(11 a^6-119 a^4 b^2+65 a^2 b^4+3 b^6\right) \text{Tan}[c+d x]\right)$$

Result (type 3, 3335 leaves):

$$\left(\text{Sec}[c+d x]^3 \left(a \,\text{Cos}[c+d x]+b \,\text{Sin}[c+d x]\right)^3\right.$$

$$\left(-\frac{4 a \left(a^2-3 b^2\right) (c+d x)}{\left(a^2+b^2\right)^3}+\frac{4 b \left(-3 a^2+b^2\right) \text{Log}[a \text{Cos}[c+d x]+b \text{Sin}[c+d x]]}{\left(a^2+b^2\right)^3}-\right.$$
$$\frac{b \left(3 a^2-b^2\right)}{2 (a-\text{i} b)^2 (a+\text{i} b)^2 (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^2}+$$
$$\left.\left.\frac{3 \left(a^2-3 b^2\right) \text{Sin}[c+d x]}{\left(a^2+b^2\right)^2 (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])}\right)\right)\Big/\left(32 d (a+b \text{Tan}[c+d x])^3\right)+$$
$$\left(3 \text{Sec}[c+d x]^3 (a \text{Cos}[c+d x]+b \text{Sin}[c+d x]) (-b \text{Cos}[2 (c+d x)]+a \text{Sin}[2 (c+d x)])\right)\Big/$$
$$\left(256 \left(a^2+b^2\right) d (a+b \text{Tan}[c+d x])^3\right)+$$
$$\frac{1}{512 \left(a^2+b^2\right)^5 d (a+b \text{Tan}[c+d x])^3} 3 \text{Sec}[c+d x]^3 (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])$$
$$\left(-42 a^8 b+280 a^6 b^3-28 a^4 b^5-296 a^2 b^7+54 b^9+24 a^9 (c+d x)+168 \text{i} a^8 b (c+d x)-\right.$$
$$480 a^7 b^2 (c+d x)-672 \text{i} a^6 b^3 (c+d x)+336 a^5 b^4 (c+d x)-336 \text{i} a^4 b^5 (c+d x)+$$
$$672 a^3 b^6 (c+d x)+480 \text{i} a^2 b^7 (c+d x)-168 a b^8 (c+d x)-24 \text{i} b^9 (c+d x)-$$
$$12 a^8 b \text{Cos}[4 (c+d x)]-32 a^6 b^3 \text{Cos}[4 (c+d x)]-24 a^4 b^5 \text{Cos}[4 (c+d x)]+$$
$$4 b^9 \text{Cos}[4 (c+d x)]+a^8 b \text{Cos}[6 (c+d x)]+4 a^6 b^3 \text{Cos}[6 (c+d x)]+6 a^4 b^5 \text{Cos}[6 (c+d x)]+$$
$$4 a^2 b^7 \text{Cos}[6 (c+d x)]+b^9 \text{Cos}[6 (c+d x)]+84 a^8 b \text{Log}\left[(a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^2\right]-$$
$$336 a^6 b^3 \text{Log}\left[(a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^2\right]-$$
$$168 a^4 b^5 \text{Log}\left[(a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^2\right]+$$
$$240 a^2 b^7 \text{Log}\left[(a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^2\right]-12 b^9 \text{Log}\left[(a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^2\right]+$$
$$4 \text{Cos}[2 (c+d x)] \left(6 a^9 (c+d x)-132 a^7 b^2 (c+d x)+336 a^5 b^4 (c+d x)-252 a^3 b^6 (c+d x)+\right.$$
$$42 a b^8 (c+d x)+3 b^9 (-5+2 \text{i} (c+d x))+21 a^4 b^5 (3+16 \text{i} (c+d x))+$$
$$7 a^6 b^3 (-5-36 \text{i} (c+d x))+a^2 b^7 (71-132 \text{i} (c+d x))+6 \text{i} a^8 b (2 \text{i}+7 (c+d x))+$$
$$\left.3 b \left(7 a^8-42 a^6 b^2+56 a^4 b^4-22 a^2 b^6+b^8\right) \text{Log}\left[(a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^2\right]\right)+$$
$$48 \text{i} b \left(-7 a^6+35 a^4 b^2-21 a^2 b^4+b^6\right) \text{ArcTan}[\text{Tan}[c+d x]] (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^2-$$
$$18 a^9 \text{Sin}[2 (c+d x)]+228 a^7 b^2 \text{Sin}[2 (c+d x)]+56 a^5 b^4 \text{Sin}[2 (c+d x)]-$$
$$196 a^3 b^6 \text{Sin}[2 (c+d x)]-6 a b^8 \text{Sin}[2 (c+d x)]+$$
$$48 a^8 b (c+d x) \text{Sin}[2 (c+d x)]+336 \text{i} a^7 b^2 (c+d x) \text{Sin}[2 (c+d x)]-$$
$$1008 a^6 b^3 (c+d x) \text{Sin}[2 (c+d x)]-1680 \text{i} a^5 b^4 (c+d x) \text{Sin}[2 (c+d x)]+$$
$$1680 a^4 b^5 (c+d x) \text{Sin}[2 (c+d x)]+1008 \text{i} a^3 b^6 (c+d x) \text{Sin}[2 (c+d x)]-$$
$$336 a^2 b^7 (c+d x) \text{Sin}[2 (c+d x)]-48 \text{i} a b^8 (c+d x) \text{Sin}[2 (c+d x)]+$$
$$168 a^7 b^2 \text{Log}\left[(a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^2\right] \text{Sin}[2 (c+d x)]-$$
$$840 a^5 b^4 \text{Log}\left[(a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^2\right] \text{Sin}[2 (c+d x)]+$$
$$504 a^3 b^6 \text{Log}\left[(a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^2\right] \text{Sin}[2 (c+d x)]-$$
$$24 a b^8 \text{Log}\left[(a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^2\right] \text{Sin}[2 (c+d x)]-$$
$$4 a^9 \text{Sin}[4 (c+d x)]+24 a^5 b^4 \text{Sin}[4 (c+d x)]+32 a^3 b^6 \text{Sin}[4 (c+d x)]+$$
$$12 a b^8 \text{Sin}[4 (c+d x)]+a^9 \text{Sin}[6 (c+d x)]+4 a^7 b^2 \text{Sin}[6 (c+d x)]+$$
$$\left.6 a^5 b^4 \text{Sin}[6 (c+d x)]+4 a^3 b^6 \text{Sin}[6 (c+d x)]+a b^8 \text{Sin}[6 (c+d x)]\right)-$$
$$\frac{1}{1536 \left(a^2+b^2\right)^6 d (a+b \text{Tan}[c+d x])^3} \text{Sec}[c+d x]^3 (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])$$
$$\left(324 a^{10} b-3420 a^8 b^3+3816 a^6 b^5+4104 a^4 b^7-3180 a^2 b^9+276 b^{11}-\right.$$
$$120 a^{11} (c+d x)-1080 \text{i} a^{10} b (c+d x)+4200 a^9 b^2 (c+d x)+9000 \text{i} a^8 b^3 (c+d x)-$$
$$10800 a^7 b^4 (c+d x)-5040 \text{i} a^6 b^5 (c+d x)-5040 a^5 b^6 (c+d x)-$$

$$10800\,i\,a^4 b^7 (c+d x)+9000 a^3 b^8 (c+d x)+4200\,i\,a^2 b^9 (c+d x)-1080 a b^{10} (c+d x)-$$
$$120\,i\,b^{11} (c+d x)+100 a^{10} b \,\text{Cos}[4 (c+d x)]+100 a^8 b^3 \,\text{Cos}[4 (c+d x)]-$$
$$280 a^6 b^5 \,\text{Cos}[4 (c+d x)]-440 a^4 b^7 \,\text{Cos}[4 (c+d x)]-140 a^2 b^9 \,\text{Cos}[4 (c+d x)]+$$
$$20 b^{11} \,\text{Cos}[4 (c+d x)]-15 a^{10} b \,\text{Cos}[6 (c+d x)]-55 a^8 b^3 \,\text{Cos}[6 (c+d x)]-$$
$$70 a^6 b^5 \,\text{Cos}[6 (c+d x)]-30 a^4 b^7 \,\text{Cos}[6 (c+d x)]+5 a^2 b^9 \,\text{Cos}[6 (c+d x)]+$$
$$5 b^{11} \,\text{Cos}[6 (c+d x)]+2 a^{10} b \,\text{Cos}[8 (c+d x)]+10 a^8 b^3 \,\text{Cos}[8 (c+d x)]+$$
$$20 a^6 b^5 \,\text{Cos}[8 (c+d x)]+20 a^4 b^7 \,\text{Cos}[8 (c+d x)]+10 a^2 b^9 \,\text{Cos}[8 (c+d x)]+$$
$$2 b^{11} \,\text{Cos}[8 (c+d x)]-540 a^{10} b \,\text{Log}\left[(a \,\text{Cos}[c+d x]+b \,\text{Sin}[c+d x])^2\right]+$$
$$4500 a^8 b^3 \,\text{Log}\left[(a \,\text{Cos}[c+d x]+b \,\text{Sin}[c+d x])^2\right]-$$
$$2520 a^6 b^5 \,\text{Log}\left[(a \,\text{Cos}[c+d x]+b \,\text{Sin}[c+d x])^2\right]-5400 a^4 b^7$$
$$\text{Log}\left[(a \,\text{Cos}[c+d x]+b \,\text{Sin}[c+d x])^2\right]+2100 a^2 b^9 \,\text{Log}\left[(a \,\text{Cos}[c+d x]+b \,\text{Sin}[c+d x])^2\right]-$$
$$60 b^{11} \,\text{Log}\left[(a \,\text{Cos}[c+d x]+b \,\text{Sin}[c+d x])^2\right]-6 \,\text{Cos}[2 (c+d x)]$$
$$\Big(20 a^{11} (c+d x)-740 a^9 b^2 (c+d x)+3240 a^7 b^4 (c+d x)-4200 a^5 b^6 (c+d x)+1860 a^3 b^8$$
$$(c+d x)-180 a b^{10} (c+d x)+b^{11} (51-20\,i\,(c+d x))+3 a^{10} b (-23+60\,i\,(c+d x))+$$
$$18 a^4 b^7 (7-180\,i\,(c+d x))+3 a^8 b^3 (21-620\,i\,(c+d x))+6 a^6 b^5 (141+700\,i\,(c+d x))+$$
$$a^2 b^9 (-537+740\,i\,(c+d x))-10 b \left(-9 a^{10}+93 a^8 b^2-210 a^6 b^4+162 a^4 b^6-37 a^2 b^8+b^{10}\right)$$
$$\text{Log}\left[(a \,\text{Cos}[c+d x]+b \,\text{Sin}[c+d x])^2\right]\Big)+240\,i\,b \left(9 a^8-84 a^6 b^2+126 a^4 b^4-36 a^2 b^6+b^8\right)$$
$$\text{ArcTan}[\text{Tan}[c+d x]] (a \,\text{Cos}[c+d x]+b \,\text{Sin}[c+d x])^2+90 a^{11} \,\text{Sin}[2 (c+d x)]-$$
$$2142 a^9 b^2 \,\text{Sin}[2 (c+d x)]+2052 a^7 b^4 \,\text{Sin}[2 (c+d x)]+3780 a^5 b^6 \,\text{Sin}[2 (c+d x)]-$$
$$702 a^3 b^8 \,\text{Sin}[2 (c+d x)]-198 a b^{10} \,\text{Sin}[2 (c+d x)]-$$
$$240 a^{10} b (c+d x) \,\text{Sin}[2 (c+d x)]-2160\,i\,a^9 b^2 (c+d x) \,\text{Sin}[2 (c+d x)]+$$
$$8640 a^8 b^3 (c+d x) \,\text{Sin}[2 (c+d x)]+20160\,i\,a^7 b^4 (c+d x) \,\text{Sin}[2 (c+d x)]-$$
$$30240 a^6 b^5 (c+d x) \,\text{Sin}[2 (c+d x)]-30240\,i\,a^5 b^6 (c+d x) \,\text{Sin}[2 (c+d x)]+$$
$$20160 a^4 b^7 (c+d x) \,\text{Sin}[2 (c+d x)]+8640\,i\,a^3 b^8 (c+d x) \,\text{Sin}[2 (c+d x)]-$$
$$2160 a^2 b^9 (c+d x) \,\text{Sin}[2 (c+d x)]-240\,i\,a b^{10} (c+d x) \,\text{Sin}[2 (c+d x)]-$$
$$1080 a^9 b^2 \,\text{Log}\left[(a \,\text{Cos}[c+d x]+b \,\text{Sin}[c+d x])^2\right] \text{Sin}[2 (c+d x)]+$$
$$10080 a^7 b^4 \,\text{Log}\left[(a \,\text{Cos}[c+d x]+b \,\text{Sin}[c+d x])^2\right] \text{Sin}[2 (c+d x)]-$$
$$15120 a^5 b^6 \,\text{Log}\left[(a \,\text{Cos}[c+d x]+b \,\text{Sin}[c+d x])^2\right] \text{Sin}[2 (c+d x)]+$$
$$4320 a^3 b^8 \,\text{Log}\left[(a \,\text{Cos}[c+d x]+b \,\text{Sin}[c+d x])^2\right] \text{Sin}[2 (c+d x)]-$$
$$120 a b^{10} \,\text{Log}\left[(a \,\text{Cos}[c+d x]+b \,\text{Sin}[c+d x])^2\right] \text{Sin}[2 (c+d x)]+$$
$$20 a^{11} \,\text{Sin}[4 (c+d x)]-140 a^9 b^2 \,\text{Sin}[4 (c+d x)]-440 a^7 b^4 \,\text{Sin}[4 (c+d x)]-$$
$$280 a^5 b^6 \,\text{Sin}[4 (c+d x)]+100 a^3 b^8 \,\text{Sin}[4 (c+d x)]+100 a b^{10} \,\text{Sin}[4 (c+d x)]-$$
$$5 a^{11} \,\text{Sin}[6 (c+d x)]-5 a^9 b^2 \,\text{Sin}[6 (c+d x)]+30 a^7 b^4 \,\text{Sin}[6 (c+d x)]+$$
$$70 a^5 b^6 \,\text{Sin}[6 (c+d x)]+55 a^3 b^8 \,\text{Sin}[6 (c+d x)]+15 a b^{10} \,\text{Sin}[6 (c+d x)]+$$
$$2 a^{11} \,\text{Sin}[8 (c+d x)]+10 a^9 b^2 \,\text{Sin}[8 (c+d x)]+20 a^7 b^4 \,\text{Sin}[8 (c+d x)]+$$
$$20 a^5 b^6 \,\text{Sin}[8 (c+d x)]+10 a^3 b^8 \,\text{Sin}[8 (c+d x)]+2 a b^{10} \,\text{Sin}[8 (c+d x)]\Big)\Big)$$

## Problem 68: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Sin}[c+d x]^4}{(a+b \,\text{Tan}[c+d x])^3}\, dx$$

Optimal (type 3, 285 leaves, 8 steps):

$$\frac{3 a \left(a^6-25 a^4 b^2+35 a^2 b^4-3 b^6\right) x}{8 \left(a^2+b^2\right)^5}+\frac{3 a^2 b \left(a^4-5 a^2 b^2+2 b^4\right) \text{Log}[a \text{Cos}[c+d x]+b \text{Sin}[c+d x]]}{\left(a^2+b^2\right)^5 d}-\frac{a^4 b}{2 \left(a^2+b^2\right)^3 d (a+b \text{Tan}[c+d x])^2}-\frac{2 a^3 b \left(a^2-2 b^2\right)}{\left(a^2+b^2\right)^4 d (a+b \text{Tan}[c+d x])}+\frac{\text{Cos}[c+d x]^4 \left(b \left(3 a^2-b^2\right)+a \left(a^2-3 b^2\right) \text{Tan}[c+d x]\right)}{4 \left(a^2+b^2\right)^3 d}-\frac{a \text{Cos}[c+d x]^2 \left(24 a b \left(a^2-b^2\right)+\left(5 a^4-34 a^2 b^2+9 b^4\right) \text{Tan}[c+d x]\right)}{8 \left(a^2+b^2\right)^4 d}$$

Result (type 3, 1894 leaves):

$$-\left(\left(3 \text{Sec}[c+d x]^3 (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^3 \left(\frac{4 a \left(a^2-3 b^2\right) (c+d x)}{\left(a^2+b^2\right)^3}-\frac{4 b \left(-3 a^2+b^2\right) \text{Log}[a \text{Cos}[c+d x]+b \text{Sin}[c+d x]]}{\left(a^2+b^2\right)^3}+\frac{b \left(3 a^2-b^2\right)}{2 (a-\mathrm{i} b)^2 (a+\mathrm{i} b)^2 (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^2}-\frac{3 \left(a^2-3 b^2\right) \text{Sin}[c+d x]}{\left(a^2+b^2\right)^2 (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])}\right)\right)\Big/\left(64 d (a+b \text{Tan}[c+d x])^3\right)\right)+$$

$$\left(3 \text{Sec}[c+d x]^3 (a \text{Cos}[c+d x]+b \text{Sin}[c+d x]) (-b \text{Cos}[2 (c+d x)]+a \text{Sin}[2 (c+d x)])\right)\Big/\left(128 \left(a^2+b^2\right) d (a+b \text{Tan}[c+d x])^3\right)+$$

$$\frac{1}{128 \left(a^2+b^2\right)^4 d (a+b \text{Tan}[c+d x])^3} \text{Sec}[c+d x]^3 (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^3 \Big(24 a \left(a^4-10 a^2 b^2+5 b^4\right) (c+d x)+24 \mathrm{i} \left(5 a^4 b-10 a^2 b^3+b^5\right) (c+d x)-24 \mathrm{i} b \left(5 a^4-10 a^2 b^2+b^4\right) \text{ArcTan}[\text{Tan}[c+d x]]+8 b \left(-3 a^2+b^2\right) \left(a^2+b^2\right) \text{Cos}[2 (c+d x)]+12 b \left(5 a^4-10 a^2 b^2+b^4\right) \text{Log}\left[(a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^2\right]+\frac{b \left(a^2+b^2\right) \left(5 a^4-10 a^2 b^2+b^4\right)}{(a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^2}-\frac{10 \left(a^2+b^2\right) \left(a^4-10 a^2 b^2+5 b^4\right) \text{Sin}[c+d x]}{a \text{Cos}[c+d x]+b \text{Sin}[c+d x]}-8 a \left(a^2-3 b^2\right) \left(a^2+b^2\right) \text{Sin}[2 (c+d x)]\Big)+$$

$$\frac{1}{128 \left(a^2+b^2\right)^5 d (a+b \text{Tan}[c+d x])^3} \text{Sec}[c+d x]^3 (a \text{Cos}[c+d x]+b \text{Sin}[c+d x]) \Big(-42 a^8 b+280 a^6 b^3-28 a^4 b^5-296 a^2 b^7+54 b^9+24 a^9 (c+d x)+168 \mathrm{i} a^8 b (c+d x)-480 a^7 b^2 (c+d x)-672 \mathrm{i} a^6 b^3 (c+d x)+336 a^5 b^4 (c+d x)-336 \mathrm{i} a^4 b^5 (c+d x)+672 a^3 b^6 (c+d x)+480 \mathrm{i} a^2 b^7 (c+d x)-168 a b^8 (c+d x)-24 \mathrm{i} b^9 (c+d x)-12 a^8 b \text{Cos}[4 (c+d x)]-32 a^6 b^3 \text{Cos}[4 (c+d x)]-24 a^4 b^5 \text{Cos}[4 (c+d x)]+4 b^9 \text{Cos}[4 (c+d x)]+a^8 b \text{Cos}[6 (c+d x)]+4 a^6 b^3 \text{Cos}[6 (c+d x)]+6 a^4 b^5 \text{Cos}[6 (c+d x)]+4 a^2 b^7 \text{Cos}[6 (c+d x)]+b^9 \text{Cos}[6 (c+d x)]+84 a^8 b \text{Log}\left[(a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^2\right]-$$

$$336\,a^6 b^3 \,\text{Log}\left[\left(a \,\text{Cos}[c+d x]+b \,\text{Sin}[c+d x]\right)^2\right]-$$
$$168\,a^4 b^5 \,\text{Log}\left[\left(a \,\text{Cos}[c+d x]+b \,\text{Sin}[c+d x]\right)^2\right]+$$
$$240\,a^2 b^7 \,\text{Log}\left[\left(a \,\text{Cos}[c+d x]+b \,\text{Sin}[c+d x]\right)^2\right]-12\,b^9 \,\text{Log}\left[\left(a \,\text{Cos}[c+d x]+b \,\text{Sin}[c+d x]\right)^2\right]+$$
$$4 \,\text{Cos}[2\,(c+d x)]\,\Big(6\,a^9\,(c+d x)-132\,a^7 b^2\,(c+d x)+336\,a^5 b^4\,(c+d x)-252\,a^3 b^6\,(c+d x)+$$
$$42\,a\,b^8\,(c+d x)+3\,b^9\,(-5+2\,i\,(c+d x))+21\,a^4 b^5\,(3+16\,i\,(c+d x))+$$
$$7\,a^6 b^3\,(-5-36\,i\,(c+d x))+a^2 b^7\,(71-132\,i\,(c+d x))+6\,i\,a^8 b\,(2\,i+7\,(c+d x))+$$
$$3\,b\,\left(7\,a^8-42\,a^6 b^2+56\,a^4 b^4-22\,a^2 b^6+b^8\right)\,\text{Log}\left[\left(a \,\text{Cos}[c+d x]+b \,\text{Sin}[c+d x]\right)^2\right]\Big)+$$
$$48\,i\,b\,\left(-7\,a^6+35\,a^4 b^2-21\,a^2 b^4+b^6\right)\,\text{ArcTan}[\text{Tan}[c+d x]]\,\left(a \,\text{Cos}[c+d x]+b \,\text{Sin}[c+d x]\right)^2-$$
$$18\,a^9 \,\text{Sin}[2\,(c+d x)]+228\,a^7 b^2 \,\text{Sin}[2\,(c+d x)]+56\,a^5 b^4 \,\text{Sin}[2\,(c+d x)]-$$
$$196\,a^3 b^6 \,\text{Sin}[2\,(c+d x)]-6\,a\,b^8 \,\text{Sin}[2\,(c+d x)]+$$
$$48\,a^8 b\,(c+d x)\,\text{Sin}[2\,(c+d x)]+336\,i\,a^7 b^2\,(c+d x)\,\text{Sin}[2\,(c+d x)]-$$
$$1008\,a^6 b^3\,(c+d x)\,\text{Sin}[2\,(c+d x)]-1680\,i\,a^5 b^4\,(c+d x)\,\text{Sin}[2\,(c+d x)]+$$
$$1680\,a^4 b^5\,(c+d x)\,\text{Sin}[2\,(c+d x)]+1008\,i\,a^3 b^6\,(c+d x)\,\text{Sin}[2\,(c+d x)]-$$
$$336\,a^2 b^7\,(c+d x)\,\text{Sin}[2\,(c+d x)]-48\,i\,a\,b^8\,(c+d x)\,\text{Sin}[2\,(c+d x)]+$$
$$168\,a^7 b^2 \,\text{Log}\left[\left(a \,\text{Cos}[c+d x]+b \,\text{Sin}[c+d x]\right)^2\right]\,\text{Sin}[2\,(c+d x)]-$$
$$840\,a^5 b^4 \,\text{Log}\left[\left(a \,\text{Cos}[c+d x]+b \,\text{Sin}[c+d x]\right)^2\right]\,\text{Sin}[2\,(c+d x)]+$$
$$504\,a^3 b^6 \,\text{Log}\left[\left(a \,\text{Cos}[c+d x]+b \,\text{Sin}[c+d x]\right)^2\right]\,\text{Sin}[2\,(c+d x)]-$$
$$24\,a\,b^8 \,\text{Log}\left[\left(a \,\text{Cos}[c+d x]+b \,\text{Sin}[c+d x]\right)^2\right]\,\text{Sin}[2\,(c+d x)]-$$
$$4\,a^9 \,\text{Sin}[4\,(c+d x)]+24\,a^5 b^4 \,\text{Sin}[4\,(c+d x)]+32\,a^3 b^6 \,\text{Sin}[4\,(c+d x)]+$$
$$12\,a\,b^8 \,\text{Sin}[4\,(c+d x)]+a^9 \,\text{Sin}[6\,(c+d x)]+4\,a^7 b^2 \,\text{Sin}[6\,(c+d x)]+$$
$$6\,a^5 b^4 \,\text{Sin}[6\,(c+d x)]+4\,a^3 b^6 \,\text{Sin}[6\,(c+d x)]+a\,b^8 \,\text{Sin}[6\,(c+d x)]\Big]\Big)$$

## Problem 69: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Sin}[c+d x]^2}{(a+b \,\text{Tan}[c+d x])^3}\,dx$$

Optimal (type 3, 206 leaves, 7 steps):

$$\frac{a\left(a^4-14\,a^2 b^2+9\,b^4\right) x}{2\left(a^2+b^2\right)^4}+\frac{b\left(3\,a^4-8\,a^2 b^2+b^4\right)\text{Log}[a \,\text{Cos}[c+d x]+b \,\text{Sin}[c+d x]]}{\left(a^2+b^2\right)^4 d}-$$
$$\frac{a^2 b}{2\left(a^2+b^2\right)^2 d\,(a+b \,\text{Tan}[c+d x])^2}-\frac{2\,a\,b\left(a^2-b^2\right)}{\left(a^2+b^2\right)^3 d\,(a+b \,\text{Tan}[c+d x])}-$$
$$\frac{\text{Cos}[c+d x]^2\left(b\left(3\,a^2-b^2\right)+a\left(a^2-3\,b^2\right)\text{Tan}[c+d x]\right)}{2\left(a^2+b^2\right)^3 d}$$

Result (type 3, 613 leaves):

$$\left(\text{Sec}[c+d x]^3 (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^3 \left(-\frac{4 a (a^2-3 b^2) (c+d x)}{(a^2+b^2)^3}+\frac{4 b (-3 a^2+b^2) \text{Log}[a \text{Cos}[c+d x]+b \text{Sin}[c+d x]]}{(a^2+b^2)^3}-\frac{b (3 a^2-b^2)}{2 (a-\mathrm{i} b)^2 (a+\mathrm{i} b)^2 (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^2}+\frac{3 (a^2-3 b^2) \text{Sin}[c+d x]}{(a^2+b^2)^2 (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])}\right)\right) \Big/ \left(16 d (a+b \text{Tan}[c+d x])^3\right)+$$

$$\left(\text{Sec}[c+d x]^3 (a \text{Cos}[c+d x]+b \text{Sin}[c+d x]) (-b \text{Cos}[2 (c+d x)]+a \text{Sin}[2 (c+d x)])\right) \Big/ \left(16 (a^2+b^2) d (a+b \text{Tan}[c+d x])^3\right)+$$

$$\frac{1}{32 (a^2+b^2)^4 d (a+b \text{Tan}[c+d x])^3} \text{Sec}[c+d x]^3 (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^3 \Big(24 a (a^4-10 a^2 b^2+5 b^4) (c+d x)+24 \mathrm{i} (5 a^4 b-10 a^2 b^3+b^5) (c+d x)-$$
$$24 \mathrm{i} b (5 a^4-10 a^2 b^2+b^4) \text{ArcTan}[\text{Tan}[c+d x]]+8 b (-3 a^2+b^2) (a^2+b^2) \text{Cos}[2 (c+d x)]+$$
$$12 b (5 a^4-10 a^2 b^2+b^4) \text{Log}\left[(a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^2\right]+$$
$$\frac{b (a^2+b^2) (5 a^4-10 a^2 b^2+b^4)}{(a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^2}-\frac{10 (a^2+b^2) (a^4-10 a^2 b^2+5 b^4) \text{Sin}[c+d x]}{a \text{Cos}[c+d x]+b \text{Sin}[c+d x]}-$$
$$8 a (a^2-3 b^2) (a^2+b^2) \text{Sin}[2 (c+d x)]\Big)$$

## Problem 70: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Csc}[c+d x]^2}{(a+b \text{Tan}[c+d x])^3} \, dx$$

Optimal (type 3, 95 leaves, 3 steps):

$$-\frac{\text{Cot}[c+d x]}{a^3 d}-\frac{3 b \text{Log}[\text{Tan}[c+d x]]}{a^4 d}+\frac{3 b \text{Log}[a+b \text{Tan}[c+d x]]}{a^4 d}-\frac{b}{2 a^2 d (a+b \text{Tan}[c+d x])^2}-\frac{2 b}{a^3 d (a+b \text{Tan}[c+d x])}$$

Result (type 3, 241 leaves):

$$\frac{1}{2 a^4 (a^2+b^2) d (a+b \text{Tan}[c+d x])^2} \Big(-2 a^3 (a^2+b^2) \text{Cot}[c+d x]+$$
$$b \big(-2 a^2 (a^2+b^2) (2+3 \text{Log}[\text{Sin}[c+d x]]-3 \text{Log}[a \text{Cos}[c+d x]+b \text{Sin}[c+d x]])-$$
$$a^2 b^2 \text{Sec}[c+d x]^2+$$
$$2 a b \left(2 a^2+b^2-6 (a^2+b^2) \text{Log}[\text{Sin}[c+d x]]+6 (a^2+b^2) \text{Log}[a \text{Cos}[c+d x]+b \text{Sin}[c+d x]]\right)$$
$$\text{Tan}[c+d x]-2 b^2 \left(-3 a^2-2 b^2+3 (a^2+b^2) \text{Log}[\text{Sin}[c+d x]]-\right.$$
$$\left.3 (a^2+b^2) \text{Log}[a \text{Cos}[c+d x]+b \text{Sin}[c+d x]]\right) \text{Tan}[c+d x]^2\big)\Big)$$

## Problem 71: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Csc}[c+d x]^4}{(a+b \,\text{Tan}[c+d x])^3}\, dx$$

Optimal (type 3, 178 leaves, 3 steps):

$$-\frac{(a^2+6 b^2)\,\text{Cot}[c+d x]}{a^5 d}+\frac{3 b\,\text{Cot}[c+d x]^2}{2 a^4 d}-\frac{\text{Cot}[c+d x]^3}{3 a^3 d}-\frac{b\,(3 a^2+10 b^2)\,\text{Log}[\text{Tan}[c+d x]]}{a^6 d}+\frac{b\,(3 a^2+10 b^2)\,\text{Log}[a+b\,\text{Tan}[c+d x]]}{a^6 d}-\frac{b\,(a^2+b^2)}{2 a^4 d\,(a+b\,\text{Tan}[c+d x])^2}-\frac{2 b\,(a^2+2 b^2)}{a^5 d\,(a+b\,\text{Tan}[c+d x])}$$

Result (type 3, 456 leaves):

$$-\frac{b^3\,\text{Sec}[c+d x]^3\,(a\,\text{Cos}[c+d x]+b\,\text{Sin}[c+d x])}{2 a^4 d\,(a+b\,\text{Tan}[c+d x])^3}-\frac{\text{Csc}[c+d x]^3\,\text{Sec}[c+d x]^2\,(a\,\text{Cos}[c+d x]+b\,\text{Sin}[c+d x])^3}{3 a^3 d\,(a+b\,\text{Tan}[c+d x])^3}-\left(2\,(a^2\,\text{Cos}[c+d x]+9 b^2\,\text{Cos}[c+d x])\,\text{Csc}[c+d x]\,\text{Sec}[c+d x]^3\,(a\,\text{Cos}[c+d x]+b\,\text{Sin}[c+d x])^3\right)\Big/\left(3 a^5 d\,(a+b\,\text{Tan}[c+d x])^3\right)+\frac{3 b\,\text{Csc}[c+d x]^2\,\text{Sec}[c+d x]^3\,(a\,\text{Cos}[c+d x]+b\,\text{Sin}[c+d x])^3}{2 a^4 d\,(a+b\,\text{Tan}[c+d x])^3}+\left((-3 a^2 b-10 b^3)\,\text{Log}[\text{Sin}[c+d x]]\,\text{Sec}[c+d x]^3\,(a\,\text{Cos}[c+d x]+b\,\text{Sin}[c+d x])^3\right)\Big/\left(a^6 d\,(a+b\,\text{Tan}[c+d x])^3\right)+\left((3 a^2 b+10 b^3)\,\text{Log}[a\,\text{Cos}[c+d x]+b\,\text{Sin}[c+d x]]\,\text{Sec}[c+d x]^3\,(a\,\text{Cos}[c+d x]+b\,\text{Sin}[c+d x])^3\right)\Big/\left(a^6 d\,(a+b\,\text{Tan}[c+d x])^3\right)+\left(\text{Sec}[c+d x]^3\,(a\,\text{Cos}[c+d x]+b\,\text{Sin}[c+d x])^2\,(3 a^2 b^2\,\text{Sin}[c+d x]+4 b^4\,\text{Sin}[c+d x])\right)\Big/\left(a^6 d\,(a+b\,\text{Tan}[c+d x])^3\right)$$

## Problem 72: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Csc}[c+d x]^6}{(a+b\,\text{Tan}[c+d x])^3}\, dx$$

Optimal (type 3, 265 leaves, 3 steps):

$$-\frac{(a^4+12 a^2 b^2+15 b^4)\,\text{Cot}[c+d x]}{a^7 d}+\frac{b\,(3 a^2+5 b^2)\,\text{Cot}[c+d x]^2}{a^6 d}-\frac{2\,(a^2+3 b^2)\,\text{Cot}[c+d x]^3}{3 a^5 d}+\frac{3 b\,\text{Cot}[c+d x]^4}{4 a^4 d}-\frac{\text{Cot}[c+d x]^5}{5 a^3 d}-\frac{b\,(3 a^4+20 a^2 b^2+21 b^4)\,\text{Log}[\text{Tan}[c+d x]]}{a^8 d}+\frac{b\,(3 a^4+20 a^2 b^2+21 b^4)\,\text{Log}[a+b\,\text{Tan}[c+d x]]}{a^8 d}-\frac{b\,(a^2+b^2)^2}{2 a^6 d\,(a+b\,\text{Tan}[c+d x])^2}-\frac{2 b\,(a^2+b^2)\,(a^2+3 b^2)}{a^7 d\,(a+b\,\text{Tan}[c+d x])}$$

Result (type 3, 670 leaves):

$$\left(\left(-3 a^4 b-20 a^2 b^3-21 b^5\right) \text{Log}[\text{Sin}[c+d x]] \text{Sec}[c+d x]^3 (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^3\right) \Big/ \left(a^8 d (a+b \text{Tan}[c+d x])^3\right)+\left(\left(3 a^4 b+20 a^2 b^3+21 b^5\right) \text{Log}[a \text{Cos}[c+d x]+b \text{Sin}[c+d x]] \text{Sec}[c+d x]^3 (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^3\right) \Big/ \left(a^8 d (a+b \text{Tan}[c+d x])^3\right)+\frac{1}{960 a^8 d (a+b \text{Tan}[c+d x])^3} \text{Csc}[c+d x]^5 \text{Sec}[c+d x]^3 (a \text{Cos}[c+d x]+b \text{Sin}[c+d x]) \left(-200 a^7 \text{Cos}[c+d x]+135 a^5 b^2 \text{Cos}[c+d x]+210 a^3 b^4 \text{Cos}[c+d x]-675 a b^6 \text{Cos}[c+d x]-8 a^7 \text{Cos}[3 (c+d x)]-567 a^5 b^2 \text{Cos}[3 (c+d x)]-630 a^3 b^4 \text{Cos}[3 (c+d x)]+1215 a b^6 \text{Cos}[3 (c+d x)]+24 a^7 \text{Cos}[5 (c+d x)]+619 a^5 b^2 \text{Cos}[5 (c+d x)]+630 a^3 b^4 \text{Cos}[5 (c+d x)]-675 a b^6 \text{Cos}[5 (c+d x)]-8 a^7 \text{Cos}[7 (c+d x)]-187 a^5 b^2 \text{Cos}[7 (c+d x)]-210 a^3 b^4 \text{Cos}[7 (c+d x)]+135 a b^6 \text{Cos}[7 (c+d x)]+120 a^6 b \text{Sin}[c+d x]+1335 a^4 b^3 \text{Sin}[c+d x]+5175 a^2 b^5 \text{Sin}[c+d x]+3150 b^7 \text{Sin}[c+d x]+126 a^6 b \text{Sin}[3 (c+d x)]-1665 a^4 b^3 \text{Sin}[3 (c+d x)]-4635 a^2 b^5 \text{Sin}[3 (c+d x)]-1890 b^7 \text{Sin}[3 (c+d x)]-10 a^6 b \text{Sin}[5 (c+d x)]+1215 a^4 b^3 \text{Sin}[5 (c+d x)]+2565 a^2 b^5 \text{Sin}[5 (c+d x)]+630 b^7 \text{Sin}[5 (c+d x)]-16 a^6 b \text{Sin}[7 (c+d x)]-345 a^4 b^3 \text{Sin}[7 (c+d x)]-585 a^2 b^5 \text{Sin}[7 (c+d x)]-90 b^7 \text{Sin}[7 (c+d x)]\right)$$

## Problem 73: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Sin}[c+d x]^4}{(a+b \text{Tan}[c+d x])^4} \, dx$$

Optimal (type 3, 366 leaves, 8 steps):

$$\frac{\left(3 a^8-132 a^6 b^2+370 a^4 b^4-132 a^2 b^6+3 b^8\right) x}{8 \left(a^2+b^2\right)^6}+\frac{4 a b \left(a^2-b^2\right) \left(a^4-8 a^2 b^2+b^4\right) \text{Log}[a \text{Cos}[c+d x]+b \text{Sin}[c+d x]]}{\left(a^2+b^2\right)^6 d}-\frac{a^4 b}{3 \left(a^2+b^2\right)^3 d (a+b \text{Tan}[c+d x])^3}-\frac{a^3 b \left(a^2-2 b^2\right)}{\left(a^2+b^2\right)^4 d (a+b \text{Tan}[c+d x])^2}-\frac{3 a^2 b \left(a^4-5 a^2 b^2+2 b^4\right)}{\left(a^2+b^2\right)^5 d (a+b \text{Tan}[c+d x])}+\frac{\text{Cos}[c+d x]^4 \left(4 a b \left(a^2-b^2\right)+\left(a^4-6 a^2 b^2+b^4\right) \text{Tan}[c+d x]\right)}{4 \left(a^2+b^2\right)^4 d}-\frac{1}{8 \left(a^2+b^2\right)^5 d}\text{Cos}[c+d x]^2 \left(16 a b \left(2 a^4-5 a^2 b^2+b^4\right)+\left(5 a^6-65 a^4 b^2+55 a^2 b^4-3 b^6\right) \text{Tan}[c+d x]\right)$$

Result (type 3, 2613 leaves):

$$\frac{1}{768 a \left(a^2+b^2\right)^6 d (a+b \text{Tan}[c+d x])^4} \text{Sec}[c+d x]^4 (a \text{Cos}[c+d x]+b \text{Sin}[c+d x]) \left(-221 a^{11} b \text{Cos}[3 (c+d x)]-2853 a^9 b^3 \text{Cos}[3 (c+d x)]+4830 a^7 b^5 \text{Cos}[3 (c+d x)]+\right.$$

$$5334\,a^5\,b^7\,\text{Cos}\left[3\,\left(c+d\,x\right)\right]-2097\,a^3\,b^9\,\text{Cos}\left[3\,\left(c+d\,x\right)\right]+31\,a\,b^{11}\,\text{Cos}\left[3\,\left(c+d\,x\right)\right]+$$
$$120\,a^{12}\,\left(c+d\,x\right)\,\text{Cos}\left[3\,\left(c+d\,x\right)\right]+960\,\text{i}\,a^{11}\,b\,\left(c+d\,x\right)\,\text{Cos}\left[3\,\left(c+d\,x\right)\right]-$$
$$3720\,a^{10}\,b^2\,\left(c+d\,x\right)\,\text{Cos}\left[3\,\left(c+d\,x\right)\right]-9600\,\text{i}\,a^9\,b^3\,\left(c+d\,x\right)\,\text{Cos}\left[3\,\left(c+d\,x\right)\right]+$$
$$18480\,a^8\,b^4\,\left(c+d\,x\right)\,\text{Cos}\left[3\,\left(c+d\,x\right)\right]+26880\,\text{i}\,a^7\,b^5\,\left(c+d\,x\right)\,\text{Cos}\left[3\,\left(c+d\,x\right)\right]-$$
$$28560\,a^6\,b^6\,\left(c+d\,x\right)\,\text{Cos}\left[3\,\left(c+d\,x\right)\right]-21120\,\text{i}\,a^5\,b^7\,\left(c+d\,x\right)\,\text{Cos}\left[3\,\left(c+d\,x\right)\right]+$$
$$10200\,a^4\,b^8\,\left(c+d\,x\right)\,\text{Cos}\left[3\,\left(c+d\,x\right)\right]+2880\,\text{i}\,a^3\,b^9\,\left(c+d\,x\right)\,\text{Cos}\left[3\,\left(c+d\,x\right)\right]-$$
$$360\,a^2\,b^{10}\,\left(c+d\,x\right)\,\text{Cos}\left[3\,\left(c+d\,x\right)\right]-45\,a^{11}\,b\,\text{Cos}\left[5\,\left(c+d\,x\right)\right]-165\,a^9\,b^3\,\text{Cos}\left[5\,\left(c+d\,x\right)\right]-$$
$$210\,a^7\,b^5\,\text{Cos}\left[5\,\left(c+d\,x\right)\right]-90\,a^5\,b^7\,\text{Cos}\left[5\,\left(c+d\,x\right)\right]+15\,a^3\,b^9\,\text{Cos}\left[5\,\left(c+d\,x\right)\right]+$$
$$15\,a\,b^{11}\,\text{Cos}\left[5\,\left(c+d\,x\right)\right]+3\,a^{11}\,b\,\text{Cos}\left[7\,\left(c+d\,x\right)\right]+15\,a^9\,b^3\,\text{Cos}\left[7\,\left(c+d\,x\right)\right]+$$
$$30\,a^7\,b^5\,\text{Cos}\left[7\,\left(c+d\,x\right)\right]+30\,a^5\,b^7\,\text{Cos}\left[7\,\left(c+d\,x\right)\right]+15\,a^3\,b^9\,\text{Cos}\left[7\,\left(c+d\,x\right)\right]+$$
$$3\,a\,b^{11}\,\text{Cos}\left[7\,\left(c+d\,x\right)\right]+480\,a^{11}\,b\,\text{Cos}\left[3\,\left(c+d\,x\right)\right]\,\text{Log}\left[\left(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]\right)^2\right]-$$
$$4800\,a^9\,b^3\,\text{Cos}\left[3\,\left(c+d\,x\right)\right]\,\text{Log}\left[\left(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]\right)^2\right]+$$
$$13440\,a^7\,b^5\,\text{Cos}\left[3\,\left(c+d\,x\right)\right]\,\text{Log}\left[\left(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]\right)^2\right]-$$
$$10560\,a^5\,b^7\,\text{Cos}\left[3\,\left(c+d\,x\right)\right]\,\text{Log}\left[\left(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]\right)^2\right]+$$
$$1440\,a^3\,b^9\,\text{Cos}\left[3\,\left(c+d\,x\right)\right]\,\text{Log}\left[\left(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]\right)^2\right]+$$
$$3\,a\,\left(a^2+b^2\right)\,\text{Cos}[c+d\,x]\,\left(-17\,b^9+120\,a^9\,\left(c+d\,x\right)-3360\,a^7\,b^2\,\left(c+d\,x\right)+8400\,a^5\,b^4\,\left(c+d\,x\right)-\right.$$
$$3360\,a^3\,b^6\,\left(c+d\,x\right)+120\,a\,b^8\,\left(c+d\,x\right)+60\,a^2\,b^7\,\left(11-16\,\text{i}\,\left(c+d\,x\right)\right)+$$
$$84\,a^6\,b^3\,\left(21-80\,\text{i}\,\left(c+d\,x\right)\right)+3\,a^8\,b\,\left(-67+320\,\text{i}\,\left(c+d\,x\right)\right)+42\,\text{i}\,a^4\,b^5\,\left(59\,\text{i}+160\,\left(c+d\,x\right)\right)+$$
$$\left.480\,a^2\,b\,\left(a^6-7\,a^4\,b^2+7\,a^2\,b^4-b^6\right)\,\text{Log}\left[\left(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]\right)^2\right]\right)-$$
$$90\,a^{12}\,\text{Sin}[c+d\,x]+1737\,a^{10}\,b^2\,\text{Sin}[c+d\,x]+3339\,a^8\,b^4\,\text{Sin}[c+d\,x]-$$
$$9702\,a^6\,b^6\,\text{Sin}[c+d\,x]-6804\,a^4\,b^8\,\text{Sin}[c+d\,x]+4269\,a^2\,b^{10}\,\text{Sin}[c+d\,x]-$$
$$141\,b^{12}\,\text{Sin}[c+d\,x]+360\,a^{11}\,b\,\left(c+d\,x\right)\,\text{Sin}[c+d\,x]+2880\,\text{i}\,a^{10}\,b^2\,\left(c+d\,x\right)\,\text{Sin}[c+d\,x]-$$
$$9720\,a^9\,b^3\,\left(c+d\,x\right)\,\text{Sin}[c+d\,x]-17280\,\text{i}\,a^8\,b^4\,\left(c+d\,x\right)\,\text{Sin}[c+d\,x]+$$
$$15120\,a^7\,b^5\,\left(c+d\,x\right)\,\text{Sin}[c+d\,x]+15120\,a^5\,b^7\,\left(c+d\,x\right)\,\text{Sin}[c+d\,x]+$$
$$17280\,\text{i}\,a^4\,b^8\,\left(c+d\,x\right)\,\text{Sin}[c+d\,x]-9720\,a^3\,b^9\,\left(c+d\,x\right)\,\text{Sin}[c+d\,x]-$$
$$2880\,\text{i}\,a^2\,b^{10}\,\left(c+d\,x\right)\,\text{Sin}[c+d\,x]+360\,a\,b^{11}\,\left(c+d\,x\right)\,\text{Sin}[c+d\,x]+$$
$$1440\,a^{10}\,b^2\,\text{Log}\left[\left(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]\right)^2\right]\,\text{Sin}[c+d\,x]-$$
$$8640\,a^8\,b^4\,\text{Log}\left[\left(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]\right)^2\right]\,\text{Sin}[c+d\,x]+$$
$$8640\,a^4\,b^8\,\text{Log}\left[\left(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]\right)^2\right]\,\text{Sin}[c+d\,x]-$$
$$1440\,a^2\,b^{10}\,\text{Log}\left[\left(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]\right)^2\right]\,\text{Sin}[c+d\,x]-$$
$$3840\,\text{i}\,a^2\,b\,\left(a^6-7\,a^4\,b^2+7\,a^2\,b^4-b^6\right)\,\text{ArcTan}[\text{Tan}[c+d\,x]]\,\left(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]\right)^3+$$
$$6\,\left(a^2+b^2\right)^5\,\left(-a\,b\,\text{Cos}\left[3\,\left(c+d\,x\right)\right]+\left(2\,a^2+b^2+\left(a^2-b^2\right)\,\text{Cos}\left[2\,\left(c+d\,x\right)\right]\right)\,\text{Sin}[c+d\,x]\right)-$$
$$8\,\left(a^2+b^2\right)^2\,\left(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]\right)^3$$
$$\left(96\,\text{i}\,a^2\,b\,\left(a^2-b^2\right)\,\left(c+d\,x\right)-96\,\text{i}\,a^2\,b\,\left(a^2-b^2\right)\,\text{ArcTan}[\text{Tan}[c+d\,x]]+\right.$$
$$48\,a^2\,b\,\left(a^2-b^2\right)\,\text{Log}\left[\left(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]\right)^2\right]+\frac{1}{\left(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]\right)^3}$$
$$\left(6\,a\,\left(a^2+b^2\right)\,\left(2\,a^4\,b+8\,a^2\,b^3-2\,b^5+3\,a^5\,\left(c+d\,x\right)-18\,a^3\,b^2\,\left(c+d\,x\right)+3\,a\,b^4\,\left(c+d\,x\right)\right)\right.$$
$$\text{Cos}[c+d\,x]+a\,\left(a^4-6\,a^2\,b^2+b^4\right)\,\left(11\,a^2\,b+11\,b^3+6\,a^3\,\left(c+d\,x\right)-18\,a\,b^2\,\left(c+d\,x\right)\right)$$
$$\text{Cos}\left[3\,\left(c+d\,x\right)\right]-\left(10\,a^8-63\,a^6\,b^2-105\,a^4\,b^4-21\,a^2\,b^6+11\,b^8-36\,a^7\,b\,\left(c+d\,x\right)+\right.$$
$$204\,a^5\,b^3\,\left(c+d\,x\right)+36\,a^3\,b^5\,\left(c+d\,x\right)-12\,a\,b^7\,\left(c+d\,x\right)+\left(a^4-6\,a^2\,b^2+b^4\right)$$
$$\left.\left.\left.\left(11\,a^4-11\,b^4-36\,a^3\,b\,\left(c+d\,x\right)+12\,a\,b^3\,\left(c+d\,x\right)\right)\,\text{Cos}\left[2\,\left(c+d\,x\right)\right]\right)\,\text{Sin}[c+d\,x]\right)\right)-$$
$$110\,a^{12}\,\text{Sin}\left[3\,\left(c+d\,x\right)\right]+1757\,a^{10}\,b^2\,\text{Sin}\left[3\,\left(c+d\,x\right)\right]-1857\,a^8\,b^4\,\text{Sin}\left[3\,\left(c+d\,x\right)\right]+$$

$$882 a^6 b^6 \text{Sin}[3 (c+d x)] + 2928 a^4 b^8 \text{Sin}[3 (c+d x)] - 1631 a^2 b^{10} \text{Sin}[3 (c+d x)] + 47 b^{12} \text{Sin}[3 (c+d x)] + 360 a^{11} b (c+d x) \text{Sin}[3 (c+d x)] + 2880 i a^{10} b^2 (c+d x) \text{Sin}[3 (c+d x)] - 10200 a^9 b^3 (c+d x) \text{Sin}[3 (c+d x)] - 21120 i a^8 b^4 (c+d x) \text{Sin}[3 (c+d x)] + 28560 a^7 b^5 (c+d x) \text{Sin}[3 (c+d x)] + 26880 i a^6 b^6 (c+d x) \text{Sin}[3 (c+d x)] - 18480 a^5 b^7 (c+d x) \text{Sin}[3 (c+d x)] - 9600 i a^4 b^8 (c+d x) \text{Sin}[3 (c+d x)] + 3720 a^3 b^9 (c+d x) \text{Sin}[3 (c+d x)] + 960 i a^2 b^{10} (c+d x) \text{Sin}[3 (c+d x)] - 120 a b^{11} (c+d x) \text{Sin}[3 (c+d x)] + 1440 a^{10} b^2 \text{Log}[(a \text{Cos}[c+d x] + b \text{Sin}[c+d x])^2] \text{Sin}[3 (c+d x)] - 10560 a^8 b^4 \text{Log}[(a \text{Cos}[c+d x] + b \text{Sin}[c+d x])^2] \text{Sin}[3 (c+d x)] + 13440 a^6 b^6 \text{Log}[(a \text{Cos}[c+d x] + b \text{Sin}[c+d x])^2] \text{Sin}[3 (c+d x)] - 4800 a^4 b^8 \text{Log}[(a \text{Cos}[c+d x] + b \text{Sin}[c+d x])^2] \text{Sin}[3 (c+d x)] + 480 a^2 b^{10} \text{Log}[(a \text{Cos}[c+d x] + b \text{Sin}[c+d x])^2] \text{Sin}[3 (c+d x)] - 15 a^{12} \text{Sin}[5 (c+d x)] - 15 a^{10} b^2 \text{Sin}[5 (c+d x)] + 90 a^8 b^4 \text{Sin}[5 (c+d x)] + 210 a^6 b^6 \text{Sin}[5 (c+d x)] + 165 a^4 b^8 \text{Sin}[5 (c+d x)] + 45 a^2 b^{10} \text{Sin}[5 (c+d x)] + 3 a^{12} \text{Sin}[7 (c+d x)] + 15 a^{10} b^2 \text{Sin}[7 (c+d x)] + 30 a^8 b^4 \text{Sin}[7 (c+d x)] + 30 a^6 b^6 \text{Sin}[7 (c+d x)] + 15 a^4 b^8 \text{Sin}[7 (c+d x)] + 3 a^2 b^{10} \text{Sin}[7 (c+d x)] \Big)$$

## Problem 74: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Sin}[c+d x]^2}{(a+b \text{Tan}[c+d x])^4} \, dx$$

Optimal (type 3, 264 leaves, 7 steps):

$$\frac{(a^6 - 25 a^4 b^2 + 35 a^2 b^4 - 3 b^6) x}{2 (a^2+b^2)^5} + \frac{4 a b (a^4 - 5 a^2 b^2 + 2 b^4) \text{Log}[a \text{Cos}[c+d x] + b \text{Sin}[c+d x]]}{(a^2+b^2)^5 d} - \frac{a^2 b}{3 (a^2+b^2)^2 d (a+b \text{Tan}[c+d x])^3} - \frac{a b (a^2-b^2)}{(a^2+b^2)^3 d (a+b \text{Tan}[c+d x])^2} - \frac{b (3 a^4 - 8 a^2 b^2 + b^4)}{(a^2+b^2)^4 d (a+b \text{Tan}[c+d x])} - \frac{\text{Cos}[c+d x]^2 (4 a b (a^2-b^2) + (a^4 - 6 a^2 b^2 + b^4) \text{Tan}[c+d x])}{2 (a^2+b^2)^4 d}$$

Result (type 3, 2340 leaves):

$$\Big(\text{Sec}[c+d x]^4 (a \text{Cos}[c+d x] + b \text{Sin}[c+d x]) \left(-a b \text{Cos}[3 (c+d x)] + (2 a^2 + b^2 + (a^2-b^2) \text{Cos}[2 (c+d x)]) \text{Sin}[c+d x]\right)\Big) \Big/ \left(48 a (a^2+b^2) d (a+b \text{Tan}[c+d x])^4\right) - \frac{1}{48 a (a^2+b^2)^4 d (a+b \text{Tan}[c+d x])^4}$$

$$\text{Sec}[c+d x]^4 (a \text{Cos}[c+d x] + b \text{Sin}[c+d x])^4 \Big(96 i a^2 b (a^2-b^2) (c+d x) - 96 i a^2 b (a^2-b^2) \text{ArcTan}[\text{Tan}[c+d x]] + 48 a^2 b (a^2-b^2) \text{Log}[(a \text{Cos}[c+d x] + b \text{Sin}[c+d x])^2] + \frac{1}{(a \text{Cos}[c+d x] + b \text{Sin}[c+d x])^3}$$

$$\left(6 a \left(a^2+b^2\right) \left(2 a^4 b+8 a^2 b^3-2 b^5+3 a^5 (c+d x)-18 a^3 b^2 (c+d x)+3 a b^4 (c+d x)\right) \text{Cos}[c+d x]+a \left(a^4-6 a^2 b^2+b^4\right) \left(11 a^2 b+11 b^3+6 a^3 (c+d x)-18 a b^2 (c+d x)\right) \text{Cos}[3 (c+d x)]-\left(10 a^8-63 a^6 b^2-105 a^4 b^4-21 a^2 b^6+11 b^8-36 a^7 b (c+d x)+204 a^5 b^3 (c+d x)+36 a^3 b^5 (c+d x)-12 a b^7 (c+d x)+\left(a^4-6 a^2 b^2+b^4\right) \left(11 a^4-11 b^4-36 a^3 b (c+d x)+12 a b^3 (c+d x)\right) \text{Cos}[2 (c+d x)]\right) \text{Sin}[c+d x]\right)+$$

$$\left(\text{Sec}[c+d x]^4 (a \text{Cos}[c+d x]+b \text{Sin}[c+d x]) \left(-3 a b \left(a^2+b^2\right) \text{Cos}[c+d x]-2 a b \left(a^2-b^2\right) \text{Cos}[3 (c+d x)]-3 a^2 b^2 \text{Sin}[c+d x]-3 b^4 \text{Sin}[c+d x]+a^4 \text{Sin}[3 (c+d x)]-2 a^2 b^2 \text{Sin}[3 (c+d x)]+b^4 \text{Sin}[3 (c+d x)]\right)\right) \Big/ \left(96 a \left(a^2+b^2\right)^2 d (a+b \text{Tan}[c+d x])^4\right)-\frac{1}{96 a \left(a^2+b^2\right)^5 d (a+b \text{Tan}[c+d x])^4}$$

$$\text{Sec}[c+d x]^4 (a \text{Cos}[c+d x]+b \text{Sin}[c+d x]) \Big(a^9 b \text{Cos}[3 (c+d x)]+436 a^7 b^3 \text{Cos}[3 (c+d x)]+$$
$$54 a^5 b^5 \text{Cos}[3 (c+d x)]-364 a^3 b^7 \text{Cos}[3 (c+d x)]+17 a b^9 \text{Cos}[3 (c+d x)]-$$
$$24 a^{10} (c+d x) \text{Cos}[3 (c+d x)]-144 i a^9 b (c+d x) \text{Cos}[3 (c+d x)]+$$
$$432 a^8 b^2 (c+d x) \text{Cos}[3 (c+d x)]+912 i a^7 b^3 (c+d x) \text{Cos}[3 (c+d x)]-$$
$$1440 a^6 b^4 (c+d x) \text{Cos}[3 (c+d x)]-1584 i a^5 b^5 (c+d x) \text{Cos}[3 (c+d x)]+$$
$$1104 a^4 b^6 (c+d x) \text{Cos}[3 (c+d x)]+432 i a^3 b^7 (c+d x) \text{Cos}[3 (c+d x)]-$$
$$72 a^2 b^8 (c+d x) \text{Cos}[3 (c+d x)]+3 a^9 b \text{Cos}[5 (c+d x)]+12 a^7 b^3 \text{Cos}[5 (c+d x)]+$$
$$18 a^5 b^5 \text{Cos}[5 (c+d x)]+12 a^3 b^7 \text{Cos}[5 (c+d x)]+3 a b^9 \text{Cos}[5 (c+d x)]-$$
$$72 a^9 b \text{Cos}[3 (c+d x)] \text{Log}\left[(a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^2\right]+$$
$$456 a^7 b^3 \text{Cos}[3 (c+d x)] \text{Log}\left[(a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^2\right]-$$
$$792 a^5 b^5 \text{Cos}[3 (c+d x)] \text{Log}\left[(a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^2\right]+$$
$$216 a^3 b^7 \text{Cos}[3 (c+d x)] \text{Log}\left[(a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^2\right]-3 a \left(a^2+b^2\right) \text{Cos}[c+d x]$$
$$\Big(7 b^7+24 a^7 (c+d x)-360 a^5 b^2 (c+d x)+360 a^3 b^4 (c+d x)-24 a b^6 (c+d x)+$$
$$5 a^4 b^3 (25-96 i (c+d x))+a^6 b (-13+144 i (c+d x))+3 i a^2 b^5 (37 i+48 (c+d x))+$$
$$24 a^2 b \left(3 a^4-10 a^2 b^2+3 b^4\right) \text{Log}\left[(a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^2\right]\Big)+$$
$$18 a^{10} \text{Sin}[c+d x]-195 a^8 b^2 \text{Sin}[c+d x]-588 a^6 b^4 \text{Sin}[c+d x]+210 a^4 b^6 \text{Sin}[c+d x]+$$
$$546 a^2 b^8 \text{Sin}[c+d x]-39 b^{10} \text{Sin}[c+d x]-72 a^9 b (c+d x) \text{Sin}[c+d x]-$$
$$432 i a^8 b^2 (c+d x) \text{Sin}[c+d x]+1008 a^7 b^3 (c+d x) \text{Sin}[c+d x]+$$
$$1008 i a^6 b^4 (c+d x) \text{Sin}[c+d x]+1008 i a^4 b^6 (c+d x) \text{Sin}[c+d x]-$$
$$1008 a^3 b^7 (c+d x) \text{Sin}[c+d x]-432 i a^2 b^8 (c+d x) \text{Sin}[c+d x]+$$
$$72 a b^9 (c+d x) \text{Sin}[c+d x]-216 a^8 b^2 \text{Log}\left[(a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^2\right] \text{Sin}[c+d x]+$$
$$504 a^6 b^4 \text{Log}\left[(a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^2\right] \text{Sin}[c+d x]+$$
$$504 a^4 b^6 \text{Log}\left[(a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^2\right] \text{Sin}[c+d x]-$$
$$216 a^2 b^8 \text{Log}\left[(a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^2\right] \text{Sin}[c+d x]+$$
$$192 i a^2 b \left(3 a^4-10 a^2 b^2+3 b^4\right) \text{ArcTan}[\text{Tan}[c+d x]] (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^3+$$
$$22 a^{10} \text{Sin}[3 (c+d x)]-195 a^8 b^2 \text{Sin}[3 (c+d x)]+128 a^6 b^4 \text{Sin}[3 (c+d x)]+$$
$$110 a^4 b^6 \text{Sin}[3 (c+d x)]-222 a^2 b^8 \text{Sin}[3 (c+d x)]+$$
$$13 b^{10} \text{Sin}[3 (c+d x)]-72 a^9 b (c+d x) \text{Sin}[3 (c+d x)]-$$
$$432 i a^8 b^2 (c+d x) \text{Sin}[3 (c+d x)]+1104 a^7 b^3 (c+d x) \text{Sin}[3 (c+d x)]+$$
$$1584 i a^6 b^4 (c+d x) \text{Sin}[3 (c+d x)]-1440 a^5 b^5 (c+d x) \text{Sin}[3 (c+d x)]-$$
$$912 i a^4 b^6 (c+d x) \text{Sin}[3 (c+d x)]+432 a^3 b^7 (c+d x) \text{Sin}[3 (c+d x)]+$$

$$144\,i\,a^2 b^8 (c+d x) \text{Sin}[3 (c+d x)] - 24 a b^9 (c+d x) \text{Sin}[3 (c+d x)] - 216 a^8 b^2 \text{Log}\left[(a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^2\right] \text{Sin}[3 (c+d x)] + 792 a^6 b^4 \text{Log}\left[(a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^2\right] \text{Sin}[3 (c+d x)] - 456 a^4 b^6 \text{Log}\left[(a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^2\right] \text{Sin}[3 (c+d x)] + 72 a^2 b^8 \text{Log}\left[(a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^2\right] \text{Sin}[3 (c+d x)] + 3 a^{10} \text{Sin}[5 (c+d x)] + 12 a^8 b^2 \text{Sin}[5 (c+d x)] + 18 a^6 b^4 \text{Sin}[5 (c+d x)] + 12 a^4 b^6 \text{Sin}[5 (c+d x)] + 3 a^2 b^8 \text{Sin}[5 (c+d x)]\Big)$$

## Problem 75: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Csc}[c+d x]^2}{(a+b \text{Tan}[c+d x])^4} \, dx$$

Optimal (type 3, 116 leaves, 3 steps):

$$-\frac{\text{Cot}[c+d x]}{a^4 d}-\frac{4 b \text{Log}[\text{Tan}[c+d x]]}{a^5 d}+\frac{4 b \text{Log}[a+b \text{Tan}[c+d x]]}{a^5 d}-\frac{b}{3 a^2 d (a+b \text{Tan}[c+d x])^3}-\frac{b}{a^3 d (a+b \text{Tan}[c+d x])^2}-\frac{3 b}{a^4 d (a+b \text{Tan}[c+d x])}$$

Result (type 3, 259 leaves):

$$\frac{1}{3 a^5 d (a+b \text{Tan}[c+d x])^4} \text{Sec}[c+d x]^3 (a \text{Cos}[c+d x]+b \text{Sin}[c+d x]) \left(-3 a (b+a \text{Cot}[c+d x])^3 \text{Sin}[c+d x]^2+\frac{a^2 b^4 \text{Tan}[c+d x]}{a^2+b^2}+\frac{1}{(a^2+b^2)^2} b^2 (18 a^4+23 a^2 b^2+9 b^4) (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^2 \text{Tan}[c+d x]-\frac{2 a^2 b^3 (3 a^2+2 b^2) (a+b \text{Tan}[c+d x])}{(a^2+b^2)^2}-12 b \text{Cos}[c+d x]^2 \text{Log}[\text{Sin}[c+d x]] (a+b \text{Tan}[c+d x])^3+12 b \text{Cos}[c+d x]^2 \text{Log}[a \text{Cos}[c+d x]+b \text{Sin}[c+d x]] (a+b \text{Tan}[c+d x])^3\right)$$

## Problem 76: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Csc}[c+d x]^4}{(a+b \text{Tan}[c+d x])^4} \, dx$$

Optimal (type 3, 205 leaves, 3 steps):

$$-\frac{(a^2+10 b^2) \text{Cot}[c+d x]}{a^6 d}+\frac{2 b \text{Cot}[c+d x]^2}{a^5 d}-\frac{\text{Cot}[c+d x]^3}{3 a^4 d}-\frac{4 b (a^2+5 b^2) \text{Log}[\text{Tan}[c+d x]]}{a^7 d}+\frac{4 b (a^2+5 b^2) \text{Log}[a+b \text{Tan}[c+d x]]}{a^7 d}-\frac{b (a^2+b^2)}{3 a^4 d (a+b \text{Tan}[c+d x])^3}-\frac{b (a^2+2 b^2)}{a^5 d (a+b \text{Tan}[c+d x])^2}-\frac{b (3 a^2+10 b^2)}{a^6 d (a+b \text{Tan}[c+d x])}$$

Result (type 3, 528 leaves) :

$$\frac{1}{48 a^7 d (a+b \text{Tan}[c+d x])^4} \text{Sec}[c+d x]^4 (a \text{Cos}[c+d x]+b \text{Sin}[c+d x]) \Big(-192 b (a^2+5 b^2) \text{Log}[\text{Sin}[c+d x]] (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^3+ 192 b (a^2+5 b^2) \text{Log}[a \text{Cos}[c+d x]+b \text{Sin}[c+d x]] (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^3- \frac{1}{a^2+b^2} \text{Csc}[c+d x]^3 (8 a^8-4 a^6 b^2-50 a^4 b^4-190 a^2 b^6-150 b^8+ 3 (3 a^8+10 a^6 b^2+45 a^4 b^4+115 a^2 b^6+75 b^8) \text{Cos}[2 (c+d x)]+ 6 (2 a^6 b^2-17 a^4 b^4-35 a^2 b^6-15 b^8) \text{Cos}[4 (c+d x)]-a^8 \text{Cos}[6 (c+d x)]- 22 a^6 b^2 \text{Cos}[6 (c+d x)]+17 a^4 b^4 \text{Cos}[6 (c+d x)]+55 a^2 b^6 \text{Cos}[6 (c+d x)]+ 15 b^8 \text{Cos}[6 (c+d x)]-3 a^7 b \text{Sin}[2 (c+d x)]+3 a^5 b^3 \text{Sin}[2 (c+d x)]-75 a^3 b^5 \text{Sin}[2 (c+d x)]-75 a b^7 \text{Sin}[2 (c+d x)]-6 a^7 b \text{Sin}[4 (c+d x)]+84 a^5 b^3 \text{Sin}[4 (c+d x)]+ 156 a^3 b^5 \text{Sin}[4 (c+d x)]+60 a b^7 \text{Sin}[4 (c+d x)]-3 a^7 b \text{Sin}[6 (c+d x)]- 65 a^5 b^3 \text{Sin}[6 (c+d x)]-79 a^3 b^5 \text{Sin}[6 (c+d x)]-15 a b^7 \text{Sin}[6 (c+d x)])\Big)$$

## Problem 77: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Csc}[c+d x]^6}{(a+b \text{Tan}[c+d x])^4} \, dx$$

Optimal (type 3, 300 leaves, 3 steps) :

$$-\frac{(a^4+20 a^2 b^2+35 b^4) \text{Cot}[c+d x]}{a^8 d}+\frac{2 b (2 a^2+5 b^2) \text{Cot}[c+d x]^2}{a^7 d}- \frac{2 (a^2+5 b^2) \text{Cot}[c+d x]^3}{3 a^6 d}+\frac{b \text{Cot}[c+d x]^4}{a^5 d}-\frac{\text{Cot}[c+d x]^5}{5 a^4 d}- \frac{4 b (a^4+10 a^2 b^2+14 b^4) \text{Log}[\text{Tan}[c+d x]]}{a^9 d}+\frac{4 b (a^4+10 a^2 b^2+14 b^4) \text{Log}[a+b \text{Tan}[c+d x]]}{a^9 d}- \frac{b (a^2+b^2)^2}{3 a^6 d (a+b \text{Tan}[c+d x])^3}-\frac{b (a^2+b^2) (a^2+3 b^2)}{a^7 d (a+b \text{Tan}[c+d x])^2}-\frac{b (3 a^4+20 a^2 b^2+21 b^4)}{a^8 d (a+b \text{Tan}[c+d x])}$$

Result (type 3, 673 leaves) :

$$\frac{1}{1920\, a^9\, d\, (a+b\,\text{Tan}[c+d\,x])^4}\,\text{Sec}[c+d\,x]^4\,(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x])$$
$$\Big(-7680\, b\, (a^4+10\, a^2\, b^2+14\, b^4)\,\text{Log}[\text{Sin}[c+d\,x]]\,(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x])^3+7680\, b$$
$$(a^4+10\, a^2\, b^2+14\, b^4)\,\text{Log}[a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x]]\,(a\,\text{Cos}[c+d\,x]+b\,\text{Sin}[c+d\,x])^3+$$
$$\text{Csc}[c+d\,x]^5\,\big(-200\, a^8+380\, a^6\, b^2+3070\, a^4\, b^4+11375\, a^2\, b^6+11025\, b^8-$$
$$4\,(52\, a^8+194\, a^6\, b^2+1510\, a^4\, b^4+5705\, a^2\, b^6+4410\, b^8)\,\text{Cos}[2\,(c+d\,x)]+$$
$$4\,(4\, a^8-16\, a^6\, b^2+1010\, a^4\, b^4+4585\, a^2\, b^6+2205\, b^8)\,\text{Cos}[4\,(c+d\,x)]+$$
$$16\, a^8\,\text{Cos}[6\,(c+d\,x)]+776\, a^6\, b^2\,\text{Cos}[6\,(c+d\,x)]-1000\, a^4\, b^4\,\text{Cos}[6\,(c+d\,x)]-$$
$$8540\, a^2\, b^6\,\text{Cos}[6\,(c+d\,x)]-2520\, b^8\,\text{Cos}[6\,(c+d\,x)]-8\, a^8\,\text{Cos}[8\,(c+d\,x)]-$$
$$316\, a^6\, b^2\,\text{Cos}[8\,(c+d\,x)]-70\, a^4\, b^4\,\text{Cos}[8\,(c+d\,x)]+1645\, a^2\, b^6\,\text{Cos}[8\,(c+d\,x)]+$$
$$315\, b^8\,\text{Cos}[8\,(c+d\,x)]+264\, a^7\, b\,\text{Sin}[2\,(c+d\,x)]+372\, a^5\, b^3\,\text{Sin}[2\,(c+d\,x)]+$$
$$4830\, a^3\, b^5\,\text{Sin}[2\,(c+d\,x)]+1470\, a\, b^7\,\text{Sin}[2\,(c+d\,x)]+144\, a^7\, b\,\text{Sin}[4\,(c+d\,x)]-$$
$$2476\, a^5\, b^3\,\text{Sin}[4\,(c+d\,x)]-9730\, a^3\, b^5\,\text{Sin}[4\,(c+d\,x)]-$$
$$1470\, a\, b^7\,\text{Sin}[4\,(c+d\,x)]-24\, a^7\, b\,\text{Sin}[6\,(c+d\,x)]+2756\, a^5\, b^3\,\text{Sin}[6\,(c+d\,x)]+$$
$$7670\, a^3\, b^5\,\text{Sin}[6\,(c+d\,x)]+630\, a\, b^7\,\text{Sin}[6\,(c+d\,x)]-24\, a^7\, b\,\text{Sin}[8\,(c+d\,x)]-$$
$$922\, a^5\, b^3\,\text{Sin}[8\,(c+d\,x)]-2095\, a^3\, b^5\,\text{Sin}[8\,(c+d\,x)]-105\, a\, b^7\,\text{Sin}[8\,(c+d\,x)]\big)\Big)$$

## Problem 78: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{Csc}[x]}{1+\text{Tan}[x]}\,dx$$

Optimal (type 3, 26 leaves, 6 steps):

$$-\text{ArcTanh}[\text{Cos}[x]]+\frac{\text{ArcTanh}\left[\frac{\text{Cos}[x]-\text{Sin}[x]}{\sqrt{2}}\right]}{\sqrt{2}}$$

Result (type 3, 41 leaves):

$$(1+i)\,(-1)^{3/4}\,\text{ArcTanh}\left[\frac{-1+\text{Tan}\left[\frac{x}{2}\right]}{\sqrt{2}}\right]-\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right]+\text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right]$$

## Problem 82: Unable to integrate problem.

$$\int \frac{\text{Sin}[c+d\,x]^m}{a+b\,\text{Tan}[c+d\,x]}\,dx$$

Optimal (type 6, 765 leaves, 14 steps):

$$\frac{1}{a\,d\,(1+m)}2^{1+m}\,\text{Hypergeometric2F1}\left[\frac{1+m}{2},1+m,\frac{3+m}{2},-\text{Tan}\left[\frac{1}{2}(c+d\,x)\right]^2\right]$$
$$\text{Tan}\left[\frac{1}{2}(c+d\,x)\right]\left(\frac{\text{Tan}\left[\frac{1}{2}(c+d\,x)\right]}{1+\text{Tan}\left[\frac{1}{2}(c+d\,x)\right]^2}\right)^m\left(1+\text{Tan}\left[\frac{1}{2}(c+d\,x)\right]^2\right)^m+$$
$$\left(2^{1+m}\,b\,\text{AppellF1}\left[\frac{2+m}{2},1+m,1,\frac{4+m}{2},-\text{Tan}\left[\frac{1}{2}(c+d\,x)\right]^2,\frac{a^2\,\text{Tan}\left[\frac{1}{2}(c+d\,x)\right]^2}{\left(b-\sqrt{a^2+b^2}\right)^2}\right]\right.$$
$$\left.\text{Tan}\left[\frac{1}{2}(c+d\,x)\right]^2\left(\frac{\text{Tan}\left[\frac{1}{2}(c+d\,x)\right]}{1+\text{Tan}\left[\frac{1}{2}(c+d\,x)\right]^2}\right)^m\left(1+\text{Tan}\left[\frac{1}{2}(c+d\,x)\right]^2\right)^m\right)\Big/$$
$$\left(\sqrt{a^2+b^2}\left(b-\sqrt{a^2+b^2}\right)d\,(2+m)\right)-$$
$$\left(2^{1+m}\,b\,\text{AppellF1}\left[\frac{2+m}{2},1+m,1,\frac{4+m}{2},-\text{Tan}\left[\frac{1}{2}(c+d\,x)\right]^2,\frac{a^2\,\text{Tan}\left[\frac{1}{2}(c+d\,x)\right]^2}{\left(b+\sqrt{a^2+b^2}\right)^2}\right]\right.$$
$$\left.\text{Tan}\left[\frac{1}{2}(c+d\,x)\right]^2\left(\frac{\text{Tan}\left[\frac{1}{2}(c+d\,x)\right]}{1+\text{Tan}\left[\frac{1}{2}(c+d\,x)\right]^2}\right)^m\left(1+\text{Tan}\left[\frac{1}{2}(c+d\,x)\right]^2\right)^m\right)\Big/$$
$$\left(\sqrt{a^2+b^2}\left(b+\sqrt{a^2+b^2}\right)d\,(2+m)\right)+$$
$$\left(2^{1+m}\,a\,b\,\text{AppellF1}\left[\frac{3+m}{2},1+m,1,\frac{5+m}{2},-\text{Tan}\left[\frac{1}{2}(c+d\,x)\right]^2,\frac{a^2\,\text{Tan}\left[\frac{1}{2}(c+d\,x)\right]^2}{\left(b-\sqrt{a^2+b^2}\right)^2}\right]\right.$$
$$\left.\text{Tan}\left[\frac{1}{2}(c+d\,x)\right]^3\left(\frac{\text{Tan}\left[\frac{1}{2}(c+d\,x)\right]}{1+\text{Tan}\left[\frac{1}{2}(c+d\,x)\right]^2}\right)^m\left(1+\text{Tan}\left[\frac{1}{2}(c+d\,x)\right]^2\right)^m\right)\Big/$$
$$\left(\sqrt{a^2+b^2}\left(b-\sqrt{a^2+b^2}\right)^2 d\,(3+m)\right)-$$
$$\left(2^{1+m}\,a\,b\,\text{AppellF1}\left[\frac{3+m}{2},1+m,1,\frac{5+m}{2},-\text{Tan}\left[\frac{1}{2}(c+d\,x)\right]^2,\frac{a^2\,\text{Tan}\left[\frac{1}{2}(c+d\,x)\right]^2}{\left(b+\sqrt{a^2+b^2}\right)^2}\right]\right.$$
$$\left.\text{Tan}\left[\frac{1}{2}(c+d\,x)\right]^3\left(\frac{\text{Tan}\left[\frac{1}{2}(c+d\,x)\right]}{1+\text{Tan}\left[\frac{1}{2}(c+d\,x)\right]^2}\right)^m\left(1+\text{Tan}\left[\frac{1}{2}(c+d\,x)\right]^2\right)^m\right)\Big/$$
$$\left(\sqrt{a^2+b^2}\left(b+\sqrt{a^2+b^2}\right)^2 d\,(3+m)\right)$$

Result (type 8, 23 leaves):

$$\int\frac{\text{Sin}[c+d\,x]^m}{a+b\,\text{Tan}[c+d\,x]}\,dx$$

## Problem 84: Unable to integrate problem.

$$\int \mathrm{Sin}[c+d\,x]^4\,(a+b\,\mathrm{Tan}[c+d\,x])^n\,dx$$

Optimal (type 5, 435 leaves, 7 steps):

$$-\Big(\Big(\Big(a\,b^2\,n\,(5\,a^2+b^2\,(3+2\,n))+\sqrt{-b^2}\,(3\,a^4+a^2\,b^2\,(6+6\,n-n^2)+b^4\,(3+4\,n+n^2))\Big)\,\mathrm{Hypergeometric2F1}\Big[1,1+n,2+n,\frac{a+b\,\mathrm{Tan}[c+d\,x]}{a-\sqrt{-b^2}}\Big]\,(a+b\,\mathrm{Tan}[c+d\,x])^{1+n}\Big)\Big/\Big(16\,b\,(a^2+b^2)^2\,\Big(a-\sqrt{-b^2}\Big)\,d\,(1+n)\Big)\Big)-\Big(\Big(a\,b^2\,n\,(5\,a^2+b^2\,(3+2\,n))-\sqrt{-b^2}\,(3\,a^4+a^2\,b^2\,(6+6\,n-n^2)+b^4\,(3+4\,n+n^2))\Big)\,\mathrm{Hypergeometric2F1}\Big[1,1+n,2+n,\frac{a+b\,\mathrm{Tan}[c+d\,x]}{a+\sqrt{-b^2}}\Big]\,(a+b\,\mathrm{Tan}[c+d\,x])^{1+n}\Big)\Big/\Big(16\,b\,(a^2+b^2)^2\,\Big(a+\sqrt{-b^2}\Big)\,d\,(1+n)\Big)+\frac{\mathrm{Cos}[c+d\,x]^4\,(b+a\,\mathrm{Tan}[c+d\,x])\,(a+b\,\mathrm{Tan}[c+d\,x])^{1+n}}{4\,(a^2+b^2)\,d}-\frac{1}{8\,(a^2+b^2)^2\,d}\mathrm{Cos}[c+d\,x]^2\,(a+b\,\mathrm{Tan}[c+d\,x])^{1+n}\,\Big(b\,\big(a^2\,(7-n)+b^2\,(5+n)\big)+a\,\big(5\,a^2+b^2\,(3+2\,n)\big)\,\mathrm{Tan}[c+d\,x]\Big)$$

Result (type 8, 23 leaves):

$$\int \mathrm{Sin}[c+d\,x]^4\,(a+b\,\mathrm{Tan}[c+d\,x])^n\,dx$$

## Problem 85: Unable to integrate problem.

$$\int \mathrm{Sin}[c+d\,x]^2\,(a+b\,\mathrm{Tan}[c+d\,x])^n\,dx$$

Optimal (type 5, 276 leaves, 6 steps):

$$-\left(\left(\left(\left(a\,b^2\,n+\sqrt{-b^2}\,\left(a^2+b^2\,(1+n)\right)\right)\text{Hypergeometric2F1}\left[1,\,1+n,\,2+n,\,\frac{a+b\,\text{Tan}[c+d\,x]}{a-\sqrt{-b^2}}\right]\right.\right.\right.$$

$$\left.\left.\left.(a+b\,\text{Tan}[c+d\,x])^{1+n}\right)\Big/\left(4\,b\,\left(a^2+b^2\right)\left(a-\sqrt{-b^2}\right)d\,(1+n)\right)\right)-$$

$$\left(\left(a\,b^2\,n-\sqrt{-b^2}\,\left(a^2+b^2\,(1+n)\right)\right)\text{Hypergeometric2F1}\left[1,\,1+n,\,2+n,\,\frac{a+b\,\text{Tan}[c+d\,x]}{a+\sqrt{-b^2}}\right]\right.$$

$$\left.(a+b\,\text{Tan}[c+d\,x])^{1+n}\right)\Big/\left(4\,b\,\left(a^2+b^2\right)\left(a+\sqrt{-b^2}\right)d\,(1+n)\right)-$$

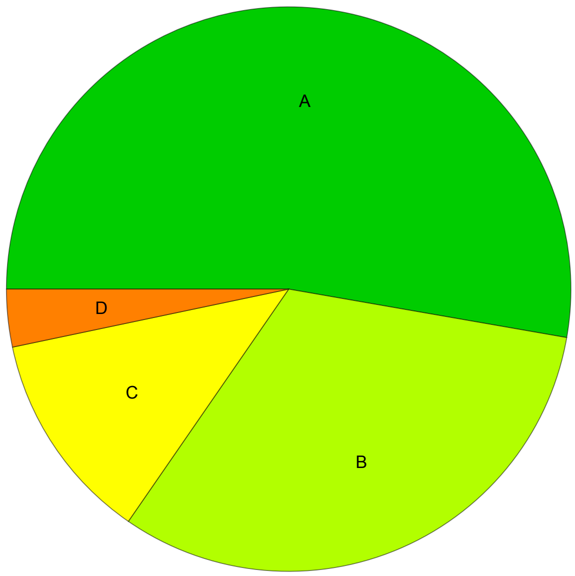
$$\frac{\text{Cos}[c+d\,x]^2\,(b+a\,\text{Tan}[c+d\,x])\,(a+b\,\text{Tan}[c+d\,x])^{1+n}}{2\,\left(a^2+b^2\right)d}$$

Result (type 8, 23 leaves) :

$$\int \text{Sin}[c+d\,x]^2\,(a+b\,\text{Tan}[c+d\,x])^n\,dx$$

# Summary of Integration Test Results

91 integration problems




A - 48 optimal antiderivatives

B - 29 more than twice size of optimal antiderivatives

C - 11 unnecessarily complex antiderivatives

D - 3 unable to integrate problems

E - 0 integration timeouts