# Mathematica 11.3 Integration Test Results

## Test results for the 63 problems in "4.3.10 (c+d x)^m (a+b tan)^n.m"

### Problem 3: Result more than twice size of optimal antiderivative.

$$\int x \tan(a+b x)\, dx$$

Optimal (type 4, 54 leaves, 4 steps):

$$\frac{i x^2}{2}-\frac{x \log\left[1+e^{2 i (a+b x)}\right]}{b}+\frac{i \text{PolyLog}\left[2,-e^{2 i (a+b x)}\right]}{2 b^2}$$

Result (type 4, 175 leaves):

$$-\left(\left(\text{Csc}[a]\left(b^2 e^{-i \text{ArcTan}[\text{Cot}[a]]} x^2-\frac{1}{\sqrt{1+\text{Cot}[a]^2}}\text{Cot}[a]\left(i b x\left(-\pi-2\text{ArcTan}[\text{Cot}[a]]\right)-\pi \text{Log}\left[1+e^{-2 i b x}\right]-2\left(b x-\text{ArcTan}[\text{Cot}[a]]\right)\text{Log}\left[1-e^{2 i (b x-\text{ArcTan}[\text{Cot}[a]])}\right]+\pi \text{Log}[\text{Cos}[b x]]-2\text{ArcTan}[\text{Cot}[a]]\text{Log}[\text{Sin}[b x-\text{ArcTan}[\text{Cot}[a]]]]+i \text{PolyLog}\left[2,e^{2 i (b x-\text{ArcTan}[\text{Cot}[a]])}\right]\right)\right)\text{Sec}[a]\right)\Big/\left(2 b^2\sqrt{\text{Csc}[a]^2\left(\text{Cos}[a]^2+\text{Sin}[a]^2\right)}\right)\right)+\frac{1}{2}x^2\text{Tan}[a]$$

### Problem 7: Result more than twice size of optimal antiderivative.

$$\int x^2 \tan(a+b x)^2\, dx$$

Optimal (type 4, 73 leaves, 6 steps):

$$-\frac{i x^2}{b}-\frac{x^3}{3}+\frac{2 x \log\left[1+e^{2 i (a+b x)}\right]}{b^2}-\frac{i \text{PolyLog}\left[2,-e^{2 i (a+b x)}\right]}{b^3}+\frac{x^2 \tan[a+b x]}{b}$$

Result (type 4, 189 leaves):

$$-\frac{x^3}{3}+\left(\text{Csc}[a]\left(b^2 e^{-i\,\text{ArcTan}[\text{Cot}[a]]}x^2-\frac{1}{\sqrt{1+\text{Cot}[a]^2}}\text{Cot}[a]\left(i\,b\,x\left(-\pi-2\,\text{ArcTan}[\text{Cot}[a]]\right)-\pi\,\text{Log}\left[1+e^{-2\,i\,b\,x}\right]-2\left(b\,x-\text{ArcTan}[\text{Cot}[a]]\right)\text{Log}\left[1-e^{2\,i\,(b\,x-\text{ArcTan}[\text{Cot}[a]])}\right]+\pi\,\text{Log}[\text{Cos}[b\,x]]-2\,\text{ArcTan}[\text{Cot}[a]]\,\text{Log}[\text{Sin}[b\,x-\text{ArcTan}[\text{Cot}[a]]]]+i\,\text{PolyLog}\left[2,e^{2\,i\,(b\,x-\text{ArcTan}[\text{Cot}[a]])}\right]\right)\right)\text{Sec}[a]\right)\Big/\left(b^3\sqrt{\text{Csc}[a]^2\left(\text{Cos}[a]^2+\text{Sin}[a]^2\right)}\right)+\frac{x^2\,\text{Sec}[a]\,\text{Sec}[a+b\,x]\,\text{Sin}[b\,x]}{b}$$

## Problem 13: Result more than twice size of optimal antiderivative.

$$\int x\,\text{Tan}[a+b\,x]^3\,dx$$

Optimal (type 4, 90 leaves, 7 steps):

$$\frac{x}{2\,b}-\frac{i\,x^2}{2}+\frac{x\,\text{Log}\left[1+e^{2\,i\,(a+b\,x)}\right]}{b}-\frac{i\,\text{PolyLog}\left[2,-e^{2\,i\,(a+b\,x)}\right]}{2\,b^2}-\frac{\text{Tan}[a+b\,x]}{2\,b^2}+\frac{x\,\text{Tan}[a+b\,x]^2}{2\,b}$$

Result (type 4, 210 leaves):

$$\frac{x\,\text{Sec}[a+b\,x]^2}{2\,b}+\left(\text{Csc}[a]\left(b^2 e^{-i\,\text{ArcTan}[\text{Cot}[a]]}x^2-\frac{1}{\sqrt{1+\text{Cot}[a]^2}}\text{Cot}[a]\left(i\,b\,x\left(-\pi-2\,\text{ArcTan}[\text{Cot}[a]]\right)-\pi\,\text{Log}\left[1+e^{-2\,i\,b\,x}\right]-2\left(b\,x-\text{ArcTan}[\text{Cot}[a]]\right)\text{Log}\left[1-e^{2\,i\,(b\,x-\text{ArcTan}[\text{Cot}[a]])}\right]+\pi\,\text{Log}[\text{Cos}[b\,x]]-2\,\text{ArcTan}[\text{Cot}[a]]\,\text{Log}[\text{Sin}[b\,x-\text{ArcTan}[\text{Cot}[a]]]]+i\,\text{PolyLog}\left[2,e^{2\,i\,(b\,x-\text{ArcTan}[\text{Cot}[a]])}\right]\right)\right)\text{Sec}[a]\right)\Big/\left(2\,b^2\sqrt{\text{Csc}[a]^2\left(\text{Cos}[a]^2+\text{Sin}[a]^2\right)}\right)-\frac{\text{Sec}[a]\,\text{Sec}[a+b\,x]\,\text{Sin}[b\,x]}{2\,b^2}-\frac{1}{2}x^2\,\text{Tan}[a]$$

## Problem 36: Result more than twice size of optimal antiderivative.

$$\int\frac{(c+d\,x)^m}{a+i\,a\,\text{Tan}[e+f\,x]}\,dx$$

Optimal (type 4, 98 leaves, 2 steps):

$$\frac{(c+d\,x)^{1+m}}{2\,a\,d\,(1+m)}+\frac{\mathrm{i}\,2^{-2-m}\,e^{-2\,\mathrm{i}\,\left(e-\frac{c\,f}{d}\right)}\,(c+d\,x)^m\,\left(\frac{\mathrm{i}\,f\,(c+d\,x)}{d}\right)^{-m}\,\mathrm{Gamma}\left[1+m,\,\frac{2\,\mathrm{i}\,f\,(c+d\,x)}{d}\right]}{a\,f}$$

Result (type 4, 205 leaves):

$$\left(2^{-2-m}\,(c+d\,x)^m\,\left(-\frac{\mathrm{i}\,f\,(c+d\,x)}{d}\right)^m\,\left(\frac{f^2\,(c+d\,x)^2}{d^2}\right)^{-m}\right.$$
$$\mathrm{Sec}[e+f\,x]\,\left(2^{1+m}\,f\,(c+d\,x)\,\left(\frac{\mathrm{i}\,f\,(c+d\,x)}{d}\right)^m\,\left(\mathrm{Cos}\left[e-\frac{c\,f}{d}\right]+\mathrm{i}\,\mathrm{Sin}\left[e-\frac{c\,f}{d}\right]\right)+\right.$$
$$\left.d\,(1+m)\,\mathrm{Gamma}\left[1+m,\,\frac{2\,\mathrm{i}\,f\,(c+d\,x)}{d}\right]\,\left(\mathrm{i}\,\mathrm{Cos}\left[e-\frac{c\,f}{d}\right]+\mathrm{Sin}\left[e-\frac{c\,f}{d}\right]\right)\right)$$
$$\left.\left(-\mathrm{i}\,\mathrm{Cos}\left[f\left(\frac{c}{d}+x\right)\right]+\mathrm{Sin}\left[f\left(\frac{c}{d}+x\right)\right]\right)\right)\Big/\left(a\,d\,f\,(1+m)\,(-\mathrm{i}+\mathrm{Tan}[e+f\,x])\right)$$

## Problem 37: Attempted integration timed out after 120 seconds.

$$\int\frac{(c+d\,x)^m}{(a+\mathrm{i}\,a\,\mathrm{Tan}[e+f\,x])^2}\,dx$$

Optimal (type 4, 171 leaves, 4 steps):

$$\frac{(c+d\,x)^{1+m}}{4\,a^2\,d\,(1+m)}+\frac{\mathrm{i}\,2^{-2-m}\,e^{-2\,\mathrm{i}\,\left(e-\frac{c\,f}{d}\right)}\,(c+d\,x)^m\,\left(\frac{\mathrm{i}\,f\,(c+d\,x)}{d}\right)^{-m}\,\mathrm{Gamma}\left[1+m,\,\frac{2\,\mathrm{i}\,f\,(c+d\,x)}{d}\right]}{a^2\,f}+$$
$$\frac{\mathrm{i}\,4^{-2-m}\,e^{-4\,\mathrm{i}\,\left(e-\frac{c\,f}{d}\right)}\,(c+d\,x)^m\,\left(\frac{\mathrm{i}\,f\,(c+d\,x)}{d}\right)^{-m}\,\mathrm{Gamma}\left[1+m,\,\frac{4\,\mathrm{i}\,f\,(c+d\,x)}{d}\right]}{a^2\,f}$$

Result (type 1, 1 leaves):

???

## Problem 39: Result more than twice size of optimal antiderivative.

$$\int(c+d\,x)^3\,(a+b\,\mathrm{Tan}[e+f\,x])\,dx$$

Optimal (type 4, 152 leaves, 8 steps):

$$\frac{a\,(c+d\,x)^4}{4\,d}+\frac{\mathrm{i}\,b\,(c+d\,x)^4}{4\,d}-\frac{b\,(c+d\,x)^3\,\mathrm{Log}\left[1+e^{2\,\mathrm{i}\,(e+f\,x)}\right]}{f}+$$
$$\frac{3\,\mathrm{i}\,b\,d\,(c+d\,x)^2\,\mathrm{PolyLog}\left[2,\,-e^{2\,\mathrm{i}\,(e+f\,x)}\right]}{2\,f^2}-$$
$$\frac{3\,b\,d^2\,(c+d\,x)\,\mathrm{PolyLog}\left[3,\,-e^{2\,\mathrm{i}\,(e+f\,x)}\right]}{2\,f^3}-\frac{3\,\mathrm{i}\,b\,d^3\,\mathrm{PolyLog}\left[4,\,-e^{2\,\mathrm{i}\,(e+f\,x)}\right]}{4\,f^4}$$

Result (type 4, 546 leaves):

$$\frac{1}{4 f^3} b c d^2 e^{-i e} \left(2 i f^2 x^2 \left(2 e^{2 i e} f x + 3 i \left(1 + e^{2 i e}\right) \text{Log}\left[1 + e^{2 i (e+f x)}\right]\right) +\right.$$
$$\left. 6 i \left(1 + e^{2 i e}\right) f x \,\text{PolyLog}\left[2, -e^{2 i (e+f x)}\right] - 3 \left(1 + e^{2 i e}\right) \text{PolyLog}\left[3, -e^{2 i (e+f x)}\right]\right) \text{Sec}[e] -$$
$$\frac{1}{4} i b d^3 e^{i e} \left(-x^4 + \left(1 + e^{-2 i e}\right) x^4 - \frac{1}{2 f^4} e^{-2 i e} \left(1 + e^{2 i e}\right) \left(2 f^4 x^4 + 4 i f^3 x^3 \text{Log}\left[1 + e^{2 i (e+f x)}\right] + 6 f^2\right.\right.$$
$$\left.\left. x^2 \,\text{PolyLog}\left[2, -e^{2 i (e+f x)}\right] + 6 i f x \,\text{PolyLog}\left[3, -e^{2 i (e+f x)}\right] - 3 \,\text{PolyLog}\left[4, -e^{2 i (e+f x)}\right]\right)\right)$$
$$\text{Sec}[e] + \frac{1}{4} x \left(4 c^3 + 6 c^2 d x + 4 c d^2 x^2 + d^3 x^3\right) \text{Sec}[e] \left(a \,\text{Cos}[e] + b \,\text{Sin}[e]\right) -$$
$$\left(b c^3 \,\text{Sec}[e] \left(\text{Cos}[e] \,\text{Log}[\text{Cos}[e] \,\text{Cos}[f x] - \text{Sin}[e] \,\text{Sin}[f x]] + f x \,\text{Sin}[e]\right)\right) \Big/$$
$$\left(f \left(\text{Cos}[e]^2 + \text{Sin}[e]^2\right)\right) -$$
$$\left(3 b c^2 d \,\text{Csc}[e] \left(e^{-i \,\text{ArcTan}[\text{Cot}[e]]} f^2 x^2 - \frac{1}{\sqrt{1 + \text{Cot}[e]^2}} \text{Cot}[e] \left(i f x \left(-\pi - 2 \,\text{ArcTan}[\text{Cot}[e]]\right) -\right.\right.\right.$$
$$\pi \,\text{Log}\left[1 + e^{-2 i f x}\right] - 2 \left(f x - \text{ArcTan}[\text{Cot}[e]]\right) \text{Log}\left[1 - e^{2 i (f x - \text{ArcTan}[\text{Cot}[e]])}\right] +$$
$$\pi \,\text{Log}[\text{Cos}[f x]] - 2 \,\text{ArcTan}[\text{Cot}[e]] \,\text{Log}[\text{Sin}[f x - \text{ArcTan}[\text{Cot}[e]]]] +$$
$$\left.\left.\left. i \,\text{PolyLog}\left[2, e^{2 i (f x - \text{ArcTan}[\text{Cot}[e]])}\right]\right)\right) \text{Sec}[e]\right) \Big/ \left(2 f^2 \sqrt{\text{Csc}[e]^2 \left(\text{Cos}[e]^2 + \text{Sin}[e]^2\right)}\right)$$

## Problem 40: Result more than twice size of optimal antiderivative.

$$\int (c + d x)^2 (a + b \,\text{Tan}[e + f x]) \, dx$$

Optimal (type 4, 115 leaves, 7 steps):

$$\frac{a (c + d x)^3}{3 d} + \frac{i b (c + d x)^3}{3 d} - \frac{b (c + d x)^2 \,\text{Log}\left[1 + e^{2 i (e+f x)}\right]}{f} +$$
$$\frac{i b d (c + d x) \,\text{PolyLog}\left[2, -e^{2 i (e+f x)}\right]}{f^2} - \frac{b d^2 \,\text{PolyLog}\left[3, -e^{2 i (e+f x)}\right]}{2 f^3}$$

Result (type 4, 375 leaves):

$$\frac{1}{12 f^3} b d^2 e^{-i e} \left(2 i f^2 x^2 \left(2 e^{2 i e} f x + 3 i \left(1 + e^{2 i e}\right) \text{Log}\left[1 + e^{2 i (e+f x)}\right]\right) + 6 i \left(1 + e^{2 i e}\right) f x \,\text{PolyLog}\left[2, -e^{2 i (e+f x)}\right] - 3 \left(1 + e^{2 i e}\right) \text{PolyLog}\left[3, -e^{2 i (e+f x)}\right]\right) \text{Sec}[e] + \frac{1}{3} x \left(3 c^2 + 3 c d x + d^2 x^2\right) \text{Sec}[e] \left(a \,\text{Cos}[e] + b \,\text{Sin}[e]\right) - \left(b c^2 \,\text{Sec}[e] \left(\text{Cos}[e] \,\text{Log}[\text{Cos}[e] \,\text{Cos}[f x] - \text{Sin}[e] \,\text{Sin}[f x]] + f x \,\text{Sin}[e]\right)\right) \Big/ \left(f \left(\text{Cos}[e]^2 + \text{Sin}[e]^2\right)\right) - \left(b c d \,\text{Csc}[e] \left(e^{-i \text{ArcTan}[\text{Cot}[e]]} f^2 x^2 - \frac{1}{\sqrt{1 + \text{Cot}[e]^2}} \text{Cot}[e] \left(i f x \left(-\pi - 2 \,\text{ArcTan}[\text{Cot}[e]]\right) - \pi \,\text{Log}\left[1 + e^{-2 i f x}\right] - 2 \left(f x - \text{ArcTan}[\text{Cot}[e]]\right) \text{Log}\left[1 - e^{2 i (f x - \text{ArcTan}[\text{Cot}[e]])}\right] + \pi \,\text{Log}[\text{Cos}[f x]] - 2 \,\text{ArcTan}[\text{Cot}[e]] \,\text{Log}[\text{Sin}[f x - \text{ArcTan}[\text{Cot}[e]]]] + i \,\text{PolyLog}\left[2, e^{2 i (f x - \text{ArcTan}[\text{Cot}[e]])}\right]\right)\right) \text{Sec}[e]\right) \Big/ \left(f^2 \sqrt{\text{Csc}[e]^2 \left(\text{Cos}[e]^2 + \text{Sin}[e]^2\right)}\right)$$

## Problem 41: Result more than twice size of optimal antiderivative.

$$\int (c + d x) \left(a + b \,\text{Tan}[e + f x]\right) dx$$

Optimal (type 4, 84 leaves, 6 steps):

$$\frac{a (c + d x)^2}{2 d} + \frac{i b (c + d x)^2}{2 d} - \frac{b (c + d x) \,\text{Log}\left[1 + e^{2 i (e+f x)}\right]}{f} + \frac{i b d \,\text{PolyLog}\left[2, -e^{2 i (e+f x)}\right]}{2 f^2}$$

Result (type 4, 206 leaves):

$$a c x + \frac{1}{2} a d x^2 - \frac{b c \,\text{Log}[\text{Cos}[e + f x]]}{f} - \left(b d \,\text{Csc}[e] \left(e^{-i \text{ArcTan}[\text{Cot}[e]]} f^2 x^2 - \frac{1}{\sqrt{1 + \text{Cot}[e]^2}} \text{Cot}[e] \left(i f x \left(-\pi - 2 \,\text{ArcTan}[\text{Cot}[e]]\right) - \pi \,\text{Log}\left[1 + e^{-2 i f x}\right] - 2 \left(f x - \text{ArcTan}[\text{Cot}[e]]\right) \text{Log}\left[1 - e^{2 i (f x - \text{ArcTan}[\text{Cot}[e]])}\right] + \pi \,\text{Log}[\text{Cos}[f x]] - 2 \,\text{ArcTan}[\text{Cot}[e]] \,\text{Log}[\text{Sin}[f x - \text{ArcTan}[\text{Cot}[e]]]] + i \,\text{PolyLog}\left[2, e^{2 i (f x - \text{ArcTan}[\text{Cot}[e]])}\right]\right)\right) \text{Sec}[e]\right) \Big/ \left(2 f^2 \sqrt{\text{Csc}[e]^2 \left(\text{Cos}[e]^2 + \text{Sin}[e]^2\right)}\right) + \frac{1}{2} b d x^2 \,\text{Tan}[e]$$

## Problem 44: Result more than twice size of optimal antiderivative.

$$\int (c + d x)^3 \left(a + b \,\text{Tan}[e + f x]\right)^2 dx$$

Optimal (type 4, 300 leaves, 15 steps):

$$-\frac{i b^2 (c+d x)^3}{f}+\frac{a^2 (c+d x)^4}{4 d}+\frac{i a b (c+d x)^4}{2 d}-\frac{b^2 (c+d x)^4}{4 d}+$$
$$\frac{3 b^2 d (c+d x)^2 \text{Log}\left[1+e^{2 i (e+f x)}\right]}{f^2}-\frac{2 a b (c+d x)^3 \text{Log}\left[1+e^{2 i (e+f x)}\right]}{f}-$$
$$\frac{3 i b^2 d^2 (c+d x) \text{PolyLog}\left[2,-e^{2 i (e+f x)}\right]}{f^3}+\frac{3 i a b d (c+d x)^2 \text{PolyLog}\left[2,-e^{2 i (e+f x)}\right]}{f^2}+$$
$$\frac{3 b^2 d^3 \text{PolyLog}\left[3,-e^{2 i (e+f x)}\right]}{2 f^4}-\frac{3 a b d^2 (c+d x) \text{PolyLog}\left[3,-e^{2 i (e+f x)}\right]}{f^3}-$$
$$\frac{3 i a b d^3 \text{PolyLog}\left[4,-e^{2 i (e+f x)}\right]}{2 f^4}+\frac{b^2 (c+d x)^3 \text{Tan}[e+f x]}{f}$$

Result (type 4, 1347 leaves) :

$$-\frac{1}{4 f^4} b^2 d^3 e^{-i e} \left(2 i f^2 x^2 \left(2 e^{2 i e} f x+3 i \left(1+e^{2 i e}\right) \text{Log}\left[1+e^{2 i (e+f x)}\right]\right)+6 i \left(1+e^{2 i e}\right) f x \text{PolyLog}\left[2,-e^{2 i (e+f x)}\right]-3 \left(1+e^{2 i e}\right) \text{PolyLog}\left[3,-e^{2 i (e+f x)}\right]\right) \text{Sec}[e]+\frac{1}{2 f^3} a b c d^2 e^{-i e} \left(2 i f^2 x^2 \left(2 e^{2 i e} f x+3 i \left(1+e^{2 i e}\right) \text{Log}\left[1+e^{2 i (e+f x)}\right]\right)+6 i \left(1+e^{2 i e}\right) f x \text{PolyLog}\left[2,-e^{2 i (e+f x)}\right]-3 \left(1+e^{2 i e}\right) \text{PolyLog}\left[3,-e^{2 i (e+f x)}\right]\right) \text{Sec}[e]-\frac{1}{2} i a b d^3 e^{i e} \left(-x^4+\left(1+e^{-2 i e}\right) x^4-\frac{1}{2 f^4} e^{-2 i e} \left(1+e^{2 i e}\right) \left(2 f^4 x^4+4 i f^3 x^3 \text{Log}\left[1+e^{2 i (e+f x)}\right]+6 f^2 x^2 \text{PolyLog}\left[2,-e^{2 i (e+f x)}\right]+6 i f x \text{PolyLog}\left[3,-e^{2 i (e+f x)}\right]-3 \text{PolyLog}\left[4,-e^{2 i (e+f x)}\right]\right)\right) \text{Sec}[e]+\left(3 b^2 c^2 d \text{Sec}[e] \left(\text{Cos}[e] \text{Log}[\text{Cos}[e] \text{Cos}[f x]-\text{Sin}[e] \text{Sin}[f x]]+f x \text{Sin}[e]\right)\right)/\left(f^2 \left(\text{Cos}[e]^2+\text{Sin}[e]^2\right)\right)-\left(2 a b c^3 \text{Sec}[e] \left(\text{Cos}[e] \text{Log}[\text{Cos}[e] \text{Cos}[f x]-\text{Sin}[e] \text{Sin}[f x]]+f x \text{Sin}[e]\right)\right)/\left(f \left(\text{Cos}[e]^2+\text{Sin}[e]^2\right)\right)+$$

$$\left(3 b^2 c d^2 \text{Csc}[e] \left(e^{-i \text{ArcTan}[\text{Cot}[e]]} f^2 x^2-\frac{1}{\sqrt{1+\text{Cot}[e]^2}} \text{Cot}[e] \left(i f x \left(-\pi-2 \text{ArcTan}[\text{Cot}[e]]\right)-\pi \text{Log}\left[1+e^{-2 i f x}\right]-2 \left(f x-\text{ArcTan}[\text{Cot}[e]]\right) \text{Log}\left[1-e^{2 i (f x-\text{ArcTan}[\text{Cot}[e]])}\right]+\pi \text{Log}[\text{Cos}[f x]]-2 \text{ArcTan}[\text{Cot}[e]] \text{Log}[\text{Sin}[f x-\text{ArcTan}[\text{Cot}[e]]]]+i \text{PolyLog}\left[2,e^{2 i (f x-\text{ArcTan}[\text{Cot}[e]])}\right]\right)\right) \text{Sec}[e]\right)/\left(f^3 \sqrt{\text{Csc}[e]^2 \left(\text{Cos}[e]^2+\text{Sin}[e]^2\right)}\right)-$$

$$\left(3 a b c^2 d \text{Csc}[e] \left(e^{-i \text{ArcTan}[\text{Cot}[e]]} f^2 x^2-\frac{1}{\sqrt{1+\text{Cot}[e]^2}} \text{Cot}[e] \left(i f x \left(-\pi-2 \text{ArcTan}[\text{Cot}[e]]\right)-\pi \text{Log}\left[1+e^{-2 i f x}\right]-2 \left(f x-\text{ArcTan}[\text{Cot}[e]]\right) \text{Log}\left[1-e^{2 i (f x-\text{ArcTan}[\text{Cot}[e]])}\right]+\pi \text{Log}[\text{Cos}[f x]]-2 \text{ArcTan}[\text{Cot}[e]] \text{Log}[\text{Sin}[f x-\text{ArcTan}[\text{Cot}[e]]]]+i \text{PolyLog}\left[2,e^{2 i (f x-\text{ArcTan}[\text{Cot}[e]])}\right]\right)\right) \text{Sec}[e]\right)/\left(f^2 \sqrt{\text{Csc}[e]^2 \left(\text{Cos}[e]^2+\text{Sin}[e]^2\right)}\right)+\frac{1}{8 f} \text{Sec}[e] \text{Sec}[e+f x]$$

$$\left(4 a^2 c^3 f x \text{Cos}[f x]-4 b^2 c^3 f x \text{Cos}[f x]+6 a^2 c^2 d f x^2 \text{Cos}[f x]-6 b^2 c^2 d f x^2 \text{Cos}[f x]+4 a^2 c d^2 f x^3 \text{Cos}[f x]-4 b^2 c d^2 f x^3 \text{Cos}[f x]+a^2 d^3 f x^4 \text{Cos}[f x]-b^2 d^3 f x^4 \text{Cos}[f x]+4 a^2 c^3 f x \text{Cos}[2 e+f x]-4 b^2 c^3 f x \text{Cos}[2 e+f x]+6 a^2 c^2 d f x^2 \text{Cos}[2 e+f x]-6 b^2 c^2 d f x^2 \text{Cos}[2 e+f x]+4 a^2 c d^2 f x^3 \text{Cos}[2 e+f x]-4 b^2 c d^2 f x^3 \text{Cos}[2 e+f x]+a^2 d^3 f x^4 \text{Cos}[2 e+f x]-b^2 d^3 f x^4 \text{Cos}[2 e+f x]+8 b^2 c^3 \text{Sin}[f x]+24 b^2 c^2 d x \text{Sin}[f x]-8 a b c^3 f x \text{Sin}[f x]+24 b^2 c d^2 x^2 \text{Sin}[f x]-12 a b c^2 d f x^2 \text{Sin}[f x]+8 b^2 d^3 x^3 \text{Sin}[f x]-8 a b c d^2 f x^3 \text{Sin}[f x]-2 a b d^3 f x^4 \text{Sin}[f x]+8 a b c^3 f x \text{Sin}[2 e+f x]+12 a b c^2 d f x^2 \text{Sin}[2 e+f x]+8 a b c d^2 f x^3 \text{Sin}[2 e+f x]+2 a b d^3 f x^4 \text{Sin}[2 e+f x]\right)$$

## Problem 45: Result more than twice size of optimal antiderivative.

$$\int (c+d x)^2 (a+b \text{Tan}[e+f x])^2 \, dx$$

Optimal (type 4, 229 leaves, 13 steps):

$$-\frac{i b^2 (c+d x)^2}{f}+\frac{a^2 (c+d x)^3}{3 d}+\frac{2 i a b (c+d x)^3}{3 d}-\frac{b^2 (c+d x)^3}{3 d}+\frac{2 b^2 d (c+d x) \text{Log}\left[1+e^{2 i (e+f x)}\right]}{f^2}-\frac{2 a b (c+d x)^2 \text{Log}\left[1+e^{2 i (e+f x)}\right]}{f}-\frac{i b^2 d^2 \text{PolyLog}\left[2,-e^{2 i (e+f x)}\right]}{f^3}+\frac{2 i a b d (c+d x) \text{PolyLog}\left[2,-e^{2 i (e+f x)}\right]}{f^2}-\frac{a b d^2 \text{PolyLog}\left[3,-e^{2 i (e+f x)}\right]}{f^3}+\frac{b^2 (c+d x)^2 \text{Tan}[e+f x]}{f}$$

Result (type 4, 656 leaves):

$$\frac{1}{6 f^3} a b d^2 e^{-i e} \left(2 i f^2 x^2 \left(2 e^{2 i e} f x+3 i \left(1+e^{2 i e}\right) \text{Log}\left[1+e^{2 i (e+f x)}\right]\right)+6 i \left(1+e^{2 i e}\right) f x \text{PolyLog}\left[2,-e^{2 i (e+f x)}\right]-3 \left(1+e^{2 i e}\right) \text{PolyLog}\left[3,-e^{2 i (e+f x)}\right]\right) \text{Sec}[e]+\frac{1}{3} x \left(3 c^2+3 c d x+d^2 x^2\right) \text{Sec}[e] \left(a^2 \text{Cos}[e]-b^2 \text{Cos}[e]+2 a b \text{Sin}[e]\right)+\left(2 b^2 c d \text{Sec}[e] (\text{Cos}[e] \text{Log}[\text{Cos}[e] \text{Cos}[f x]-\text{Sin}[e] \text{Sin}[f x]]+f x \text{Sin}[e])\right) \Big/ \left(f^2 \left(\text{Cos}[e]^2+\text{Sin}[e]^2\right)\right)-\left(2 a b c^2 \text{Sec}[e] (\text{Cos}[e] \text{Log}[\text{Cos}[e] \text{Cos}[f x]-\text{Sin}[e] \text{Sin}[f x]]+f x \text{Sin}[e])\right) \Big/ \left(f \left(\text{Cos}[e]^2+\text{Sin}[e]^2\right)\right)+\left(b^2 d^2 \text{Csc}[e] \left(e^{-i \text{ArcTan}[\text{Cot}[e]]} f^2 x^2-\frac{1}{\sqrt{1+\text{Cot}[e]^2}} \text{Cot}[e] \left(i f x (-\pi-2 \text{ArcTan}[\text{Cot}[e]])-\pi \text{Log}\left[1+e^{-2 i f x}\right]-2 (f x-\text{ArcTan}[\text{Cot}[e]]) \text{Log}\left[1-e^{2 i (f x-\text{ArcTan}[\text{Cot}[e]])}\right]+\pi \text{Log}[\text{Cos}[f x]]-2 \text{ArcTan}[\text{Cot}[e]] \text{Log}[\text{Sin}[f x-\text{ArcTan}[\text{Cot}[e]]]]+i \text{PolyLog}\left[2,e^{2 i (f x-\text{ArcTan}[\text{Cot}[e]])}\right]\right)\right) \text{Sec}[e]\right) \Big/ \left(f^3 \sqrt{\text{Csc}[e]^2 \left(\text{Cos}[e]^2+\text{Sin}[e]^2\right)}\right)-\left(2 a b c d \text{Csc}[e] \left(e^{-i \text{ArcTan}[\text{Cot}[e]]} f^2 x^2-\frac{1}{\sqrt{1+\text{Cot}[e]^2}} \text{Cot}[e] \left(i f x (-\pi-2 \text{ArcTan}[\text{Cot}[e]])-\pi \text{Log}\left[1+e^{-2 i f x}\right]-2 (f x-\text{ArcTan}[\text{Cot}[e]]) \text{Log}\left[1-e^{2 i (f x-\text{ArcTan}[\text{Cot}[e]])}\right]+\pi \text{Log}[\text{Cos}[f x]]-2 \text{ArcTan}[\text{Cot}[e]] \text{Log}[\text{Sin}[f x-\text{ArcTan}[\text{Cot}[e]]]]+i \text{PolyLog}\left[2,e^{2 i (f x-\text{ArcTan}[\text{Cot}[e]])}\right]\right)\right) \text{Sec}[e]\right) \Big/ \left(f^2 \sqrt{\text{Csc}[e]^2 \left(\text{Cos}[e]^2+\text{Sin}[e]^2\right)}\right)+\frac{1}{f} \text{Sec}[e] \text{Sec}[e+f x] \left(b^2 c^2 \text{Sin}[f x]+2 b^2 c d x \text{Sin}[f x]+b^2 d^2 x^2 \text{Sin}[f x]\right)$$

## Problem 49: Result more than twice size of optimal antiderivative.

$$\int (c+d x)^3 (a+b \text{Tan}[e+f x])^3 \, dx$$

Optimal (type 4, 612 leaves, 28 steps):

$$\frac{3\,i\,b^3\,d\,(c+d\,x)^2}{2\,f^2}-\frac{3\,i\,a\,b^2\,(c+d\,x)^3}{f}+\frac{b^3\,(c+d\,x)^3}{2\,f}+\frac{a^3\,(c+d\,x)^4}{4\,d}+$$
$$\frac{3\,i\,a^2\,b\,(c+d\,x)^4}{4\,d}-\frac{3\,a\,b^2\,(c+d\,x)^4}{4\,d}-\frac{i\,b^3\,(c+d\,x)^4}{4\,d}-\frac{3\,b^3\,d^2\,(c+d\,x)\,\text{Log}\left[1+e^{2\,i\,(e+f\,x)}\right]}{f^3}+$$
$$\frac{9\,a\,b^2\,d\,(c+d\,x)^2\,\text{Log}\left[1+e^{2\,i\,(e+f\,x)}\right]}{f^2}-\frac{3\,a^2\,b\,(c+d\,x)^3\,\text{Log}\left[1+e^{2\,i\,(e+f\,x)}\right]}{f}+$$
$$\frac{b^3\,(c+d\,x)^3\,\text{Log}\left[1+e^{2\,i\,(e+f\,x)}\right]}{f}+\frac{3\,i\,b^3\,d^3\,\text{PolyLog}\left[2,-e^{2\,i\,(e+f\,x)}\right]}{2\,f^4}-$$
$$\frac{9\,i\,a\,b^2\,d^2\,(c+d\,x)\,\text{PolyLog}\left[2,-e^{2\,i\,(e+f\,x)}\right]}{f^3}+\frac{9\,i\,a^2\,b\,d\,(c+d\,x)^2\,\text{PolyLog}\left[2,-e^{2\,i\,(e+f\,x)}\right]}{2\,f^2}-$$
$$\frac{3\,i\,b^3\,d\,(c+d\,x)^2\,\text{PolyLog}\left[2,-e^{2\,i\,(e+f\,x)}\right]}{2\,f^2}+\frac{9\,a\,b^2\,d^3\,\text{PolyLog}\left[3,-e^{2\,i\,(e+f\,x)}\right]}{2\,f^4}-$$
$$\frac{9\,a^2\,b\,d^2\,(c+d\,x)\,\text{PolyLog}\left[3,-e^{2\,i\,(e+f\,x)}\right]}{2\,f^3}+\frac{3\,b^3\,d^2\,(c+d\,x)\,\text{PolyLog}\left[3,-e^{2\,i\,(e+f\,x)}\right]}{2\,f^3}-$$
$$\frac{9\,i\,a^2\,b\,d^3\,\text{PolyLog}\left[4,-e^{2\,i\,(e+f\,x)}\right]}{4\,f^4}+\frac{3\,i\,b^3\,d^3\,\text{PolyLog}\left[4,-e^{2\,i\,(e+f\,x)}\right]}{4\,f^4}-$$
$$\frac{3\,b^3\,d\,(c+d\,x)^2\,\text{Tan}[e+f\,x]}{2\,f^2}+\frac{3\,a\,b^2\,(c+d\,x)^3\,\text{Tan}[e+f\,x]}{f}+\frac{b^3\,(c+d\,x)^3\,\text{Tan}[e+f\,x]^2}{2\,f}$$

Result (type 4, 2607 leaves):

$$-\frac{1}{4\,f^4}3\,a\,b^2\,d^3\,e^{-i\,e}\,\Big(2\,i\,f^2\,x^2\,\big(2\,e^{2\,i\,e}\,f\,x+3\,i\,(1+e^{2\,i\,e})\,\text{Log}\left[1+e^{2\,i\,(e+f\,x)}\right]\big)+$$
$$6\,i\,(1+e^{2\,i\,e})\,f\,x\,\text{PolyLog}\left[2,-e^{2\,i\,(e+f\,x)}\right]-3\,(1+e^{2\,i\,e})\,\text{PolyLog}\left[3,-e^{2\,i\,(e+f\,x)}\right]\Big)\,\text{Sec}[e]+$$
$$\frac{1}{4\,f^3}3\,a^2\,b\,c\,d^2\,e^{-i\,e}\,\Big(2\,i\,f^2\,x^2\,\big(2\,e^{2\,i\,e}\,f\,x+3\,i\,(1+e^{2\,i\,e})\,\text{Log}\left[1+e^{2\,i\,(e+f\,x)}\right]\big)+$$
$$6\,i\,(1+e^{2\,i\,e})\,f\,x\,\text{PolyLog}\left[2,-e^{2\,i\,(e+f\,x)}\right]-3\,(1+e^{2\,i\,e})\,\text{PolyLog}\left[3,-e^{2\,i\,(e+f\,x)}\right]\Big)\,\text{Sec}[e]-$$
$$\frac{1}{4\,f^3}b^3\,c\,d^2\,e^{-i\,e}\,\Big(2\,i\,f^2\,x^2\,\big(2\,e^{2\,i\,e}\,f\,x+3\,i\,(1+e^{2\,i\,e})\,\text{Log}\left[1+e^{2\,i\,(e+f\,x)}\right]\big)+$$
$$6\,i\,(1+e^{2\,i\,e})\,f\,x\,\text{PolyLog}\left[2,-e^{2\,i\,(e+f\,x)}\right]-3\,(1+e^{2\,i\,e})\,\text{PolyLog}\left[3,-e^{2\,i\,(e+f\,x)}\right]\Big)$$
$$\text{Sec}[e]-\frac{3}{4}\,i\,a^2\,b\,d^3\,e^{i\,e}\,\Bigg(-x^4+(1+e^{-2\,i\,e})\,x^4-\frac{1}{2\,f^4}$$
$$e^{-2\,i\,e}\,(1+e^{2\,i\,e})\,\Big(2\,f^4\,x^4+4\,i\,f^3\,x^3\,\text{Log}\left[1+e^{2\,i\,(e+f\,x)}\right]+6\,f^2\,x^2\,\text{PolyLog}\left[2,-e^{2\,i\,(e+f\,x)}\right]+$$
$$6\,i\,f\,x\,\text{PolyLog}\left[3,-e^{2\,i\,(e+f\,x)}\right]-3\,\text{PolyLog}\left[4,-e^{2\,i\,(e+f\,x)}\right]\Big)\Bigg)\,\text{Sec}[e]+$$
$$\frac{1}{4}\,i\,b^3\,d^3\,e^{i\,e}\,\Bigg(-x^4+(1+e^{-2\,i\,e})\,x^4-\frac{1}{2\,f^4}e^{-2\,i\,e}\,(1+e^{2\,i\,e})\,\Big(2\,f^4\,x^4+4\,i\,f^3\,x^3\,\text{Log}\left[1+e^{2\,i\,(e+f\,x)}\right]+6\,f^2$$
$$x^2\,\text{PolyLog}\left[2,-e^{2\,i\,(e+f\,x)}\right]+6\,i\,f\,x\,\text{PolyLog}\left[3,-e^{2\,i\,(e+f\,x)}\right]-3\,\text{PolyLog}\left[4,-e^{2\,i\,(e+f\,x)}\right]\Big)\Bigg)$$
$$\text{Sec}[e]+\frac{(b^3\,c^3+3\,b^3\,c^2\,d\,x+3\,b^3\,c\,d^2\,x^2+b^3\,d^3\,x^3)\,\text{Sec}[e+f\,x]^2}{2\,f}-$$
$$\big(3\,b^3\,c\,d^2\,\text{Sec}[e]\,(\text{Cos}[e]\,\text{Log}[\text{Cos}[e]\,\text{Cos}[f\,x]-\text{Sin}[e]\,\text{Sin}[f\,x]]+f\,x\,\text{Sin}[e])\big)\,/$$
$$\big(f^3\,(\text{Cos}[e]^2+\text{Sin}[e]^2)\big)+$$
$$\big(9\,a\,b^2\,c^2\,d\,\text{Sec}[e]\,(\text{Cos}[e]\,\text{Log}[\text{Cos}[e]\,\text{Cos}[f\,x]-\text{Sin}[e]\,\text{Sin}[f\,x]]+f\,x\,\text{Sin}[e])\big)\,/$$
$$\big(f^2\,(\text{Cos}[e]^2+\text{Sin}[e]^2)\big)-$$

$$\left(3 a^2 b c^3 \operatorname{Sec}[e] \left(\operatorname{Cos}[e] \operatorname{Log}[\operatorname{Cos}[e] \operatorname{Cos}[f x] - \operatorname{Sin}[e] \operatorname{Sin}[f x]] + f x \operatorname{Sin}[e]\right)\right) \Big/ \left(f \left(\operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2\right)\right) +$$

$$\left(b^3 c^3 \operatorname{Sec}[e] \left(\operatorname{Cos}[e] \operatorname{Log}[\operatorname{Cos}[e] \operatorname{Cos}[f x] - \operatorname{Sin}[e] \operatorname{Sin}[f x]] + f x \operatorname{Sin}[e]\right)\right) \Big/ \left(f \left(\operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2\right)\right) -$$

$$\left(3 b^3 d^3 \operatorname{Csc}[e] \left(e^{-i \operatorname{ArcTan}[\operatorname{Cot}[e]]} f^2 x^2 - \frac{1}{\sqrt{1 + \operatorname{Cot}[e]^2}} \operatorname{Cot}[e] \left(i f x \left(-\pi - 2 \operatorname{ArcTan}[\operatorname{Cot}[e]]\right) - \pi \operatorname{Log}\left[1 + e^{-2 i f x}\right] - 2 \left(f x - \operatorname{ArcTan}[\operatorname{Cot}[e]]\right) \operatorname{Log}\left[1 - e^{2 i (f x - \operatorname{ArcTan}[\operatorname{Cot}[e]])}\right] + \pi \operatorname{Log}[\operatorname{Cos}[f x]] - 2 \operatorname{ArcTan}[\operatorname{Cot}[e]] \operatorname{Log}[\operatorname{Sin}[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]]]] + i \operatorname{PolyLog}\left[2, e^{2 i (f x - \operatorname{ArcTan}[\operatorname{Cot}[e]])}\right]\right)\right) \operatorname{Sec}[e]\right) \Big/ \left(2 f^4 \sqrt{\operatorname{Csc}[e]^2 \left(\operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2\right)}\right) +$$

$$\left(9 a b^2 c d^2 \operatorname{Csc}[e] \left(e^{-i \operatorname{ArcTan}[\operatorname{Cot}[e]]} f^2 x^2 - \frac{1}{\sqrt{1 + \operatorname{Cot}[e]^2}} \operatorname{Cot}[e] \left(i f x \left(-\pi - 2 \operatorname{ArcTan}[\operatorname{Cot}[e]]\right) - \pi \operatorname{Log}\left[1 + e^{-2 i f x}\right] - 2 \left(f x - \operatorname{ArcTan}[\operatorname{Cot}[e]]\right) \operatorname{Log}\left[1 - e^{2 i (f x - \operatorname{ArcTan}[\operatorname{Cot}[e]])}\right] + \pi \operatorname{Log}[\operatorname{Cos}[f x]] - 2 \operatorname{ArcTan}[\operatorname{Cot}[e]] \operatorname{Log}[\operatorname{Sin}[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]]]] + i \operatorname{PolyLog}\left[2, e^{2 i (f x - \operatorname{ArcTan}[\operatorname{Cot}[e]])}\right]\right)\right) \operatorname{Sec}[e]\right) \Big/ \left(f^3 \sqrt{\operatorname{Csc}[e]^2 \left(\operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2\right)}\right) -$$

$$\left(9 a^2 b c^2 d \operatorname{Csc}[e] \left(e^{-i \operatorname{ArcTan}[\operatorname{Cot}[e]]} f^2 x^2 - \frac{1}{\sqrt{1 + \operatorname{Cot}[e]^2}} \operatorname{Cot}[e] \left(i f x \left(-\pi - 2 \operatorname{ArcTan}[\operatorname{Cot}[e]]\right) - \pi \operatorname{Log}\left[1 + e^{-2 i f x}\right] - 2 \left(f x - \operatorname{ArcTan}[\operatorname{Cot}[e]]\right) \operatorname{Log}\left[1 - e^{2 i (f x - \operatorname{ArcTan}[\operatorname{Cot}[e]])}\right] + \pi \operatorname{Log}[\operatorname{Cos}[f x]] - 2 \operatorname{ArcTan}[\operatorname{Cot}[e]] \operatorname{Log}[\operatorname{Sin}[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]]]] + i \operatorname{PolyLog}\left[2, e^{2 i (f x - \operatorname{ArcTan}[\operatorname{Cot}[e]])}\right]\right)\right) \operatorname{Sec}[e]\right) \Big/ \left(2 f^2 \sqrt{\operatorname{Csc}[e]^2 \left(\operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2\right)}\right) +$$

$$\left(3 b^3 c^2 d \operatorname{Csc}[e] \left(e^{-i \operatorname{ArcTan}[\operatorname{Cot}[e]]} f^2 x^2 - \frac{1}{\sqrt{1 + \operatorname{Cot}[e]^2}} \operatorname{Cot}[e] \left(i f x \left(-\pi - 2 \operatorname{ArcTan}[\operatorname{Cot}[e]]\right) - \pi \operatorname{Log}\left[1 + e^{-2 i f x}\right] - 2 \left(f x - \operatorname{ArcTan}[\operatorname{Cot}[e]]\right) \operatorname{Log}\left[1 - e^{2 i (f x - \operatorname{ArcTan}[\operatorname{Cot}[e]])}\right] + \pi \operatorname{Log}[\operatorname{Cos}[f x]] - 2 \operatorname{ArcTan}[\operatorname{Cot}[e]] \operatorname{Log}[\operatorname{Sin}[f x - \operatorname{ArcTan}[\operatorname{Cot}[e]]]] + i \operatorname{PolyLog}\left[2, e^{2 i (f x - \operatorname{ArcTan}[\operatorname{Cot}[e]])}\right]\right)\right) \operatorname{Sec}[e]\right) \Big/ \left(2 f^2 \sqrt{\operatorname{Csc}[e]^2 \left(\operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2\right)}\right) +$$

$$\left(3 x^2 \left(a^3 c^2 d + 3 i a^2 b c^2 d - 3 a b^2 c^2 d - i b^3 c^2 d + a^3 c^2 d \operatorname{Cos}[2 e] - 3 i a^2 b c^2 d \operatorname{Cos}[2 e] - 3 a b^2 c^2 d \operatorname{Cos}[2 e] + i b^3 c^2 d \operatorname{Cos}[2 e] + i a^3 c^2 d \operatorname{Sin}[2 e] + 3 a^2 b c^2 d \operatorname{Sin}[2 e] - 3 i a b^2 c^2 d \operatorname{Sin}[2 e] - b^3 c^2 d \operatorname{Sin}[2 e]\right)\right) \Big/ \left(2 \left(1 + \operatorname{Cos}[2 e] + i \operatorname{Sin}[2 e]\right)\right) +$$

$$\left(x^3 \left(a^3 c d^2 + 3 i a^2 b c d^2 - 3 a b^2 c d^2 - i b^3 c d^2 + a^3 c d^2 \operatorname{Cos}[2 e] - 3 i a^2 b c d^2 \operatorname{Cos}[2 e] - 3 a b^2 c d^2 \operatorname{Cos}[2 e] + i b^3 c d^2 \operatorname{Cos}[2 e] + i a^3 c d^2 \operatorname{Sin}[2 e] + 3 a^2 b c d^2 \operatorname{Sin}[2 e] - 3 i a b^2 c d^2 \operatorname{Sin}[2 e] - b^3 c d^2 \operatorname{Sin}[2 e]\right)\right) \Big/ \left(1 + \operatorname{Cos}[2 e] + i \operatorname{Sin}[2 e]\right) +$$

$$\left(x^4 \left(a^3 d^3 + 3 i a^2 b d^3 - 3 a b^2 d^3 - i b^3 d^3 + a^3 d^3 \operatorname{Cos}[2 e] - 3 i a^2 b d^3 \operatorname{Cos}[2 e] - 3 a b^2 d^3 \operatorname{Cos}[2 e] + i b^3 d^3 \operatorname{Cos}[2 e] + i a^3 d^3 \operatorname{Sin}[2 e] + 3 a^2 b d^3 \operatorname{Sin}[2 e] - 3 i a b^2 d^3 \operatorname{Sin}[2 e] - b^3 d^3 \operatorname{Sin}[2 e]\right)\right) \Big/$$

$$\left(4\left(1+\text{Cos}[2e]+i\,\text{Sin}[2e]\right)\right)+x\left(a^3c^3-3ab^2c^3+\frac{3\,i\,a^2bc^3}{1+\text{Cos}[2e]+i\,\text{Sin}[2e]}+\frac{-3\,i\,a^2bc^3\,\text{Cos}[2e]+3a^2bc^3\,\text{Sin}[2e]}{1+\text{Cos}[2e]+i\,\text{Sin}[2e]}+\left(2\,i\,b^3c^3\,\text{Cos}[2e]-2b^3c^3\,\text{Sin}[2e]\right)\Big/\left(\left(1+\text{Cos}[2e]+i\,\text{Sin}[2e]\right)\left(1-\text{Cos}[2e]+\text{Cos}[4e]-i\,\text{Sin}[2e]+i\,\text{Sin}[4e]\right)\right)+\left(-2\,i\,b^3c^3\,\text{Cos}[4e]+2b^3c^3\,\text{Sin}[4e]\right)\Big/\left(\left(1+\text{Cos}[2e]+i\,\text{Sin}[2e]\right)\left(1-\text{Cos}[2e]+\text{Cos}[4e]-i\,\text{Sin}[2e]+i\,\text{Sin}[4e]\right)\right)-\frac{i\,b^3c^3}{1+\text{Cos}[6e]+i\,\text{Sin}[6e]}+\frac{i\,b^3c^3\,\text{Cos}[6e]-b^3c^3\,\text{Sin}[6e]}{1+\text{Cos}[6e]+i\,\text{Sin}[6e]}\right)+\frac{1}{2f^2}$$
$$3\,\text{Sec}[e]\,\text{Sec}[e+fx]\left(-b^3c^2d\,\text{Sin}[fx]+2ab^2c^3f\,\text{Sin}[fx]-2b^3cd^2x\,\text{Sin}[fx]+6ab^2c^2dfx\,\text{Sin}[fx]-b^3d^3x^2\,\text{Sin}[fx]+6ab^2cd^2fx^2\,\text{Sin}[fx]+2ab^2d^3fx^3\,\text{Sin}[fx]\right)$$

## Problem 50: Result more than twice size of optimal antiderivative.

$$\int (c+dx)^2\,(a+b\,\text{Tan}[e+fx])^3\,dx$$

Optimal (type 4, 436 leaves, 22 steps):

$$\frac{b^3cdx}{f}+\frac{b^3d^2x^2}{2f}-\frac{3\,i\,ab^2(c+dx)^2}{f}+\frac{a^3(c+dx)^3}{3d}+\frac{i\,a^2b(c+dx)^3}{d}-\frac{ab^2(c+dx)^3}{d}-\frac{i\,b^3(c+dx)^3}{3d}+\frac{6ab^2d(c+dx)\,\text{Log}\left[1+e^{2i(e+fx)}\right]}{f^2}-\frac{3a^2b(c+dx)^2\,\text{Log}\left[1+e^{2i(e+fx)}\right]}{f}+\frac{b^3(c+dx)^2\,\text{Log}\left[1+e^{2i(e+fx)}\right]}{f}-\frac{b^3d^2\,\text{Log}[\text{Cos}[e+fx]]}{f^3}-\frac{3\,i\,ab^2d^2\,\text{PolyLog}\left[2,-e^{2i(e+fx)}\right]}{f^3}+\frac{3\,i\,a^2bd(c+dx)\,\text{PolyLog}\left[2,-e^{2i(e+fx)}\right]}{f^2}-\frac{i\,b^3d(c+dx)\,\text{PolyLog}\left[2,-e^{2i(e+fx)}\right]}{f^2}-\frac{3a^2bd^2\,\text{PolyLog}\left[3,-e^{2i(e+fx)}\right]}{2f^3}+\frac{b^3d^2\,\text{PolyLog}\left[3,-e^{2i(e+fx)}\right]}{2f^3}-\frac{b^3d(c+dx)\,\text{Tan}[e+fx]}{f^2}+\frac{3ab^2(c+dx)^2\,\text{Tan}[e+fx]}{f}+\frac{b^3(c+dx)^2\,\text{Tan}[e+fx]^2}{2f}$$

Result (type 4, 1860 leaves):

$$\frac{1}{4f^3}a^2bd^2e^{-ie}\left(2\,i\,f^2x^2\left(2e^{2ie}fx+3\,i\left(1+e^{2ie}\right)\text{Log}\left[1+e^{2i(e+fx)}\right]\right)+6\,i\left(1+e^{2ie}\right)fx\,\text{PolyLog}\left[2,-e^{2i(e+fx)}\right]-3\left(1+e^{2ie}\right)\text{PolyLog}\left[3,-e^{2i(e+fx)}\right]\right)\text{Sec}[e]-$$
$$\frac{1}{12f^3}b^3d^2e^{-ie}\left(2\,i\,f^2x^2\left(2e^{2ie}fx+3\,i\left(1+e^{2ie}\right)\text{Log}\left[1+e^{2i(e+fx)}\right]\right)+6\,i\left(1+e^{2ie}\right)fx\,\text{PolyLog}\left[2,-e^{2i(e+fx)}\right]-3\left(1+e^{2ie}\right)\text{PolyLog}\left[3,-e^{2i(e+fx)}\right]\right)\text{Sec}[e]-$$
$$\left(b^3d^2\,\text{Sec}[e]\left(\text{Cos}[e]\,\text{Log}[\text{Cos}[e]\,\text{Cos}[fx]-\text{Sin}[e]\,\text{Sin}[fx]]+fx\,\text{Sin}[e]\right)\right)\Big/\left(f^3\left(\text{Cos}[e]^2+\text{Sin}[e]^2\right)\right)+$$
$$\left(6ab^2cd\,\text{Sec}[e]\left(\text{Cos}[e]\,\text{Log}[\text{Cos}[e]\,\text{Cos}[fx]-\text{Sin}[e]\,\text{Sin}[fx]]+fx\,\text{Sin}[e]\right)\right)\Big/\left(f^2\left(\text{Cos}[e]^2+\text{Sin}[e]^2\right)\right)-$$
$$\left(3a^2bc^2\,\text{Sec}[e]\left(\text{Cos}[e]\,\text{Log}[\text{Cos}[e]\,\text{Cos}[fx]-\text{Sin}[e]\,\text{Sin}[fx]]+fx\,\text{Sin}[e]\right)\right)\Big/\left(f\left(\text{Cos}[e]^2+\text{Sin}[e]^2\right)\right)+$$
$$\left(b^3c^2\,\text{Sec}[e]\left(\text{Cos}[e]\,\text{Log}[\text{Cos}[e]\,\text{Cos}[fx]-\text{Sin}[e]\,\text{Sin}[fx]]+fx\,\text{Sin}[e]\right)\right)\Big/$$

$$\left(f\left(\text{Cos}[e]^2+\text{Sin}[e]^2\right)\right)+$$

$$\left(3\,a\,b^2\,d^2\,\text{Csc}[e]\left(e^{-i\,\text{ArcTan}[\text{Cot}[e]]}\,f^2\,x^2-\frac{1}{\sqrt{1+\text{Cot}[e]^2}}\text{Cot}[e]\left(i\,f\,x\left(-\pi-2\,\text{ArcTan}[\text{Cot}[e]]\right)-\pi\,\text{Log}\left[1+e^{-2\,i\,f\,x}\right]-2\left(f\,x-\text{ArcTan}[\text{Cot}[e]]\right)\text{Log}\left[1-e^{2\,i\,(f\,x-\text{ArcTan}[\text{Cot}[e]])}\right]+\pi\,\text{Log}[\text{Cos}[f\,x]]-2\,\text{ArcTan}[\text{Cot}[e]]\,\text{Log}[\text{Sin}[f\,x-\text{ArcTan}[\text{Cot}[e]]]]+i\,\text{PolyLog}\left[2,\,e^{2\,i\,(f\,x-\text{ArcTan}[\text{Cot}[e]])}\right]\right)\right)\text{Sec}[e]\right)\Big/\left(f^3\sqrt{\text{Csc}[e]^2\left(\text{Cos}[e]^2+\text{Sin}[e]^2\right)}\right)-$$

$$\left(3\,a^2\,b\,c\,d\,\text{Csc}[e]\left(e^{-i\,\text{ArcTan}[\text{Cot}[e]]}\,f^2\,x^2-\frac{1}{\sqrt{1+\text{Cot}[e]^2}}\text{Cot}[e]\left(i\,f\,x\left(-\pi-2\,\text{ArcTan}[\text{Cot}[e]]\right)-\pi\,\text{Log}\left[1+e^{-2\,i\,f\,x}\right]-2\left(f\,x-\text{ArcTan}[\text{Cot}[e]]\right)\text{Log}\left[1-e^{2\,i\,(f\,x-\text{ArcTan}[\text{Cot}[e]])}\right]+\pi\,\text{Log}[\text{Cos}[f\,x]]-2\,\text{ArcTan}[\text{Cot}[e]]\,\text{Log}[\text{Sin}[f\,x-\text{ArcTan}[\text{Cot}[e]]]]+i\,\text{PolyLog}\left[2,\,e^{2\,i\,(f\,x-\text{ArcTan}[\text{Cot}[e]])}\right]\right)\right)\text{Sec}[e]\right)\Big/\left(f^2\sqrt{\text{Csc}[e]^2\left(\text{Cos}[e]^2+\text{Sin}[e]^2\right)}\right)+$$

$$\left(b^3\,c\,d\,\text{Csc}[e]\left(e^{-i\,\text{ArcTan}[\text{Cot}[e]]}\,f^2\,x^2-\frac{1}{\sqrt{1+\text{Cot}[e]^2}}\text{Cot}[e]\left(i\,f\,x\left(-\pi-2\,\text{ArcTan}[\text{Cot}[e]]\right)-\pi\,\text{Log}\left[1+e^{-2\,i\,f\,x}\right]-2\left(f\,x-\text{ArcTan}[\text{Cot}[e]]\right)\text{Log}\left[1-e^{2\,i\,(f\,x-\text{ArcTan}[\text{Cot}[e]])}\right]+\pi\,\text{Log}[\text{Cos}[f\,x]]-2\,\text{ArcTan}[\text{Cot}[e]]\,\text{Log}[\text{Sin}[f\,x-\text{ArcTan}[\text{Cot}[e]]]]+i\,\text{PolyLog}\left[2,\,e^{2\,i\,(f\,x-\text{ArcTan}[\text{Cot}[e]])}\right]\right)\right)\text{Sec}[e]\right)\Big/\left(f^2\sqrt{\text{Csc}[e]^2\left(\text{Cos}[e]^2+\text{Sin}[e]^2\right)}\right)+$$

$$\frac{1}{12\,f^2}\text{Sec}[e]\,\text{Sec}[e+f\,x]^2\left(6\,b^3\,c^2\,f\,\text{Cos}[e]+12\,b^3\,c\,d\,f\,x\,\text{Cos}[e]+6\,a^3\,c^2\,f^2\,x\,\text{Cos}[e]-18\,a\,b^2\,c^2\,f^2\,x\,\text{Cos}[e]+6\,b^3\,d^2\,f\,x^2\,\text{Cos}[e]+6\,a^3\,c\,d\,f^2\,x^2\,\text{Cos}[e]-18\,a\,b^2\,c\,d\,f^2\,x^2\,\text{Cos}[e]+2\,a^3\,d^2\,f^2\,x^3\,\text{Cos}[e]-6\,a\,b^2\,d^2\,f^2\,x^3\,\text{Cos}[e]+3\,a^3\,c^2\,f^2\,x\,\text{Cos}[e+2\,f\,x]-9\,a\,b^2\,c^2\,f^2\,x\,\text{Cos}[e+2\,f\,x]+3\,a^3\,c\,d\,f^2\,x^2\,\text{Cos}[e+2\,f\,x]-9\,a\,b^2\,c\,d\,f^2\,x^2\,\text{Cos}[e+2\,f\,x]+a^3\,d^2\,f^2\,x^3\,\text{Cos}[e+2\,f\,x]-3\,a\,b^2\,d^2\,f^2\,x^3\,\text{Cos}[e+2\,f\,x]+3\,a^3\,c^2\,f^2\,x\,\text{Cos}[3\,e+2\,f\,x]-9\,a\,b^2\,c^2\,f^2\,x\,\text{Cos}[3\,e+2\,f\,x]+3\,a^3\,c\,d\,f^2\,x^2\,\text{Cos}[3\,e+2\,f\,x]-9\,a\,b^2\,c\,d\,f^2\,x^2\,\text{Cos}[3\,e+2\,f\,x]+a^3\,d^2\,f^2\,x^3\,\text{Cos}[3\,e+2\,f\,x]-3\,a\,b^2\,d^2\,f^2\,x^3\,\text{Cos}[3\,e+2\,f\,x]+6\,b^3\,c\,d\,\text{Sin}[e]-18\,a\,b^2\,c^2\,f\,\text{Sin}[e]+6\,b^3\,d^2\,x\,\text{Sin}[e]-36\,a\,b^2\,c\,d\,f\,x\,\text{Sin}[e]+18\,a^2\,b\,c^2\,f^2\,x\,\text{Sin}[e]-6\,b^3\,c^2\,f^2\,x\,\text{Sin}[e]-18\,a\,b^2\,d^2\,f\,x^2\,\text{Sin}[e]+18\,a^2\,b\,c\,d\,f^2\,x^2\,\text{Sin}[e]-6\,b^3\,c\,d\,f^2\,x^2\,\text{Sin}[e]+6\,a^2\,b\,d^2\,f^2\,x^3\,\text{Sin}[e]-2\,b^3\,d^2\,f^2\,x^3\,\text{Sin}[e]-6\,b^3\,c\,d\,\text{Sin}[e+2\,f\,x]+18\,a\,b^2\,c^2\,f\,\text{Sin}[e+2\,f\,x]-6\,b^3\,d^2\,x\,\text{Sin}[e+2\,f\,x]+36\,a\,b^2\,c\,d\,f\,x\,\text{Sin}[e+2\,f\,x]-9\,a^2\,b\,c^2\,f^2\,x\,\text{Sin}[e+2\,f\,x]+3\,b^3\,c^2\,f^2\,x\,\text{Sin}[e+2\,f\,x]+18\,a\,b^2\,d^2\,f\,x^2\,\text{Sin}[e+2\,f\,x]-9\,a^2\,b\,c\,d\,f^2\,x^2\,\text{Sin}[e+2\,f\,x]+3\,b^3\,c\,d\,f^2\,x^2\,\text{Sin}[e+2\,f\,x]-3\,a^2\,b\,d^2\,f^2\,x^3\,\text{Sin}[e+2\,f\,x]+b^3\,d^2\,f^2\,x^3\,\text{Sin}[e+2\,f\,x]+9\,a^2\,b\,c^2\,f^2\,x\,\text{Sin}[3\,e+2\,f\,x]-3\,b^3\,c^2\,f^2\,x\,\text{Sin}[3\,e+2\,f\,x]+9\,a^2\,b\,c\,d\,f^2\,x^2\,\text{Sin}[3\,e+2\,f\,x]-3\,b^3\,c\,d\,f^2\,x^2\,\text{Sin}[3\,e+2\,f\,x]+3\,a^2\,b\,d^2\,f^2\,x^3\,\text{Sin}[3\,e+2\,f\,x]-b^3\,d^2\,f^2\,x^3\,\text{Sin}[3\,e+2\,f\,x]\right)$$

## Problem 59: Result more than twice size of optimal antiderivative.

$$\int\frac{(c+d\,x)^3}{(a+b\,\text{Tan}[e+f\,x])^2}\,dx$$

Optimal (type 4, 848 leaves, 21 steps):

$$-\frac{2\,i\,b^2\,(c+d\,x)^3}{(a^2+b^2)^2\,f}+\frac{2\,b^2\,(c+d\,x)^3}{(a+i\,b)\,(i\,a+b)^2\,\left(i\,a-b+(i\,a+b)\,e^{2\,i\,e+2\,i\,f\,x}\right)\,f}+\frac{(c+d\,x)^4}{4\,(a-i\,b)^2\,d}+$$
$$\frac{b\,(c+d\,x)^4}{(i\,a-b)\,(a-i\,b)^2\,d}-\frac{b^2\,(c+d\,x)^4}{(a^2+b^2)^2\,d}+\frac{3\,b^2\,d\,(c+d\,x)^2\,\text{Log}\left[1+\frac{(a-i\,b)\,e^{2\,i\,e+2\,i\,f\,x}}{a+i\,b}\right]}{(a^2+b^2)^2\,f^2}+$$
$$\frac{2\,b\,(c+d\,x)^3\,\text{Log}\left[1+\frac{(a-i\,b)\,e^{2\,i\,e+2\,i\,f\,x}}{a+i\,b}\right]}{(a-i\,b)^2\,(a+i\,b)\,f}-\frac{2\,i\,b^2\,(c+d\,x)^3\,\text{Log}\left[1+\frac{(a-i\,b)\,e^{2\,i\,e+2\,i\,f\,x}}{a+i\,b}\right]}{(a^2+b^2)^2\,f}-$$
$$\frac{3\,i\,b^2\,d^2\,(c+d\,x)\,\text{PolyLog}\left[2,-\frac{(a-i\,b)\,e^{2\,i\,e+2\,i\,f\,x}}{a+i\,b}\right]}{(a^2+b^2)^2\,f^3}+\frac{3\,b\,d\,(c+d\,x)^2\,\text{PolyLog}\left[2,-\frac{(a-i\,b)\,e^{2\,i\,e+2\,i\,f\,x}}{a+i\,b}\right]}{(i\,a-b)\,(a-i\,b)^2\,f^2}-$$
$$\frac{3\,b^2\,d\,(c+d\,x)^2\,\text{PolyLog}\left[2,-\frac{(a-i\,b)\,e^{2\,i\,e+2\,i\,f\,x}}{a+i\,b}\right]}{(a^2+b^2)^2\,f^2}+\frac{3\,b^2\,d^3\,\text{PolyLog}\left[3,-\frac{(a-i\,b)\,e^{2\,i\,e+2\,i\,f\,x}}{a+i\,b}\right]}{2\,(a^2+b^2)^2\,f^4}+$$
$$\frac{3\,b\,d^2\,(c+d\,x)\,\text{PolyLog}\left[3,-\frac{(a-i\,b)\,e^{2\,i\,e+2\,i\,f\,x}}{a+i\,b}\right]}{(a-i\,b)^2\,(a+i\,b)\,f^3}-\frac{3\,i\,b^2\,d^2\,(c+d\,x)\,\text{PolyLog}\left[3,-\frac{(a-i\,b)\,e^{2\,i\,e+2\,i\,f\,x}}{a+i\,b}\right]}{(a^2+b^2)^2\,f^3}-$$
$$\frac{3\,b\,d^3\,\text{PolyLog}\left[4,-\frac{(a-i\,b)\,e^{2\,i\,e+2\,i\,f\,x}}{a+i\,b}\right]}{2\,(i\,a-b)\,(a-i\,b)^2\,f^4}+\frac{3\,b^2\,d^3\,\text{PolyLog}\left[4,-\frac{(a-i\,b)\,e^{2\,i\,e+2\,i\,f\,x}}{a+i\,b}\right]}{2\,(a^2+b^2)^2\,f^4}$$

Result (type 4, 2713 leaves):

$$\frac{1}{2\,(a-i\,b)^2\,(a+i\,b)^3\,\left(-i\,b\,\left(-1+e^{2\,i\,e}\right)+a\,\left(1+e^{2\,i\,e}\right)\right)\,f^4}$$
$$b\,e^{2\,i\,e}\left(4\,(a-i\,b)\,(-i\,a+b)\,c^2\,f^3\,(3\,b\,d+2\,a\,c\,f)\,x+\right.$$
$$4\,(a+i\,b)\,c^2\,e^{-2\,i\,e}\left(b\,\left(-1+e^{2\,i\,e}\right)+i\,a\,\left(1+e^{2\,i\,e}\right)\right)\,f^3\,(3\,b\,d+2\,a\,c\,f)\,x+$$
$$12\,i\,a\,(a+i\,b)\,b\,c\,d^2\,f^3\,x^2+12\,(a+i\,b)\,b^2\,c\,d^2\,f^3\,x^2+12\,i\,a\,(a+i\,b)\,b\,c\,d^2\,e^{-2\,i\,e}\,f^3\,x^2-$$
$$12\,i\,b^2\,(-i\,a+b)\,c\,d^2\,e^{-2\,i\,e}\,f^3\,x^2+12\,i\,a^2\,(a+i\,b)\,c^2\,d\,f^4\,x^2+12\,a\,(a+i\,b)\,b\,c^2\,d\,f^4\,x^2+$$
$$12\,i\,a^2\,(a+i\,b)\,c^2\,d\,e^{-2\,i\,e}\,f^4\,x^2-12\,a\,(a+i\,b)\,b\,c^2\,d\,e^{-2\,i\,e}\,f^4\,x^2+$$
$$12\,(a-i\,b)\,(-i\,a+b)\,c\,d\,f^3\,(b\,d+a\,c\,f)\,x^2+4\,i\,a\,(a+i\,b)\,b\,d^3\,f^3\,x^3+4\,(a+i\,b)\,b^2\,d^3\,f^3\,x^3+$$
$$4\,i\,a\,(a+i\,b)\,b\,d^3\,e^{-2\,i\,e}\,f^3\,x^3-4\,i\,b^2\,(-i\,a+b)\,d^3\,e^{-2\,i\,e}\,f^3\,x^3+8\,i\,a^2\,(a+i\,b)\,c\,d^2\,f^4\,x^3+$$
$$8\,a\,(a+i\,b)\,b\,c\,d^2\,f^4\,x^3+8\,i\,a^2\,(a+i\,b)\,c\,d^2\,e^{-2\,i\,e}\,f^4\,x^3-8\,a\,(a+i\,b)\,b\,c\,d^2\,e^{-2\,i\,e}\,f^4\,x^3+$$
$$4\,(a-i\,b)\,(-i\,a+b)\,d^2\,f^3\,(b\,d+2\,a\,c\,f)\,x^3+2\,i\,a^2\,(a+i\,b)\,d^3\,f^4\,x^4+$$
$$2\,a\,(a+i\,b)\,b\,d^3\,f^4\,x^4+2\,a\,(a-i\,b)\,(-i\,a+b)\,d^3\,f^4\,x^4+2\,i\,a^2\,(a+i\,b)\,d^3\,e^{-2\,i\,e}\,f^4\,x^4-$$
$$2\,a\,(a+i\,b)\,b\,d^3\,e^{-2\,i\,e}\,f^4\,x^4+3\,b\,(-i\,a+b)\,c^2\,d\,e^{-2\,i\,e}\left(b\,\left(-1+e^{2\,i\,e}\right)+i\,a\,\left(1+e^{2\,i\,e}\right)\right)$$
$$f^2\left(-4\,i\,f\,x-2\,i\,\text{ArcTan}\left[\frac{2\,a\,b\,e^{2\,i\,(e+f\,x)}}{-b^2\,\left(-1+e^{2\,i\,(e+f\,x)}\right)+a^2\,\left(1+e^{2\,i\,(e+f\,x)}\right)}\right]+\right.$$
$$\left.\text{Log}\left[b^2\,\left(-1+e^{2\,i\,(e+f\,x)}\right)^2+a^2\,\left(1+e^{2\,i\,(e+f\,x)}\right)^2\right]\right)+$$
$$2\,a\,(a+i\,b)\,c^3\,e^{-2\,i\,e}\left(-i\,b\,\left(-1+e^{2\,i\,e}\right)+a\,\left(1+e^{2\,i\,e}\right)\right)\,f^3$$
$$\left(-4\,i\,f\,x-2\,i\,\text{ArcTan}\left[\frac{2\,a\,b\,e^{2\,i\,(e+f\,x)}}{-b^2\,\left(-1+e^{2\,i\,(e+f\,x)}\right)+a^2\,\left(1+e^{2\,i\,(e+f\,x)}\right)}\right]+\right.$$

$$\mathrm{Log}\left[b^2 \left(-1+e^{2 i (e+f x)}\right)^2+a^2 \left(1+e^{2 i (e+f x)}\right)^2\right]\Big) -$$

$$6 i b (-i a+b) c d^2 e^{-2 i e} \left(b \left(-1+e^{2 i e}\right)+i a \left(1+e^{2 i e}\right)\right) f$$

$$\left(2 f x \left(f x+i\, \mathrm{Log}\left[1+\frac{(a-i b) e^{2 i (e+f x)}}{a+i b}\right]\right)+\mathrm{PolyLog}\left[2,-\frac{(a-i b) e^{2 i (e+f x)}}{a+i b}\right]\right) -$$

$$6 i a (a+i b) c^2 d e^{-2 i e} \left(-i b \left(-1+e^{2 i e}\right)+a \left(1+e^{2 i e}\right)\right) f^2$$

$$\left(2 f x \left(f x+i\, \mathrm{Log}\left[1+\frac{(a-i b) e^{2 i (e+f x)}}{a+i b}\right]\right)+\mathrm{PolyLog}\left[2,-\frac{(a-i b) e^{2 i (e+f x)}}{a+i b}\right]\right) +$$

$$b (-i a+b) d^3 e^{-2 i e} \left(b \left(-1+e^{2 i e}\right)+i a \left(1+e^{2 i e}\right)\right)$$

$$\Big(2 f^2 x^2 \left(-2 i f x+3\, \mathrm{Log}\left[1+\frac{(a-i b) e^{2 i (e+f x)}}{a+i b}\right]\right) -$$

$$6 i f x\, \mathrm{PolyLog}\left[2,-\frac{(a-i b) e^{2 i (e+f x)}}{a+i b}\right]+3\, \mathrm{PolyLog}\left[3,-\frac{(a-i b) e^{2 i (e+f x)}}{a+i b}\right]\Big) +$$

$$2 a (a+i b) c d^2 e^{-2 i e} \left(-i b \left(-1+e^{2 i e}\right)+a \left(1+e^{2 i e}\right)\right) f$$

$$\Big(2 f^2 x^2 \left(-2 i f x+3\, \mathrm{Log}\left[1+\frac{(a-i b) e^{2 i (e+f x)}}{a+i b}\right]\right)-6 i f x\, \mathrm{PolyLog}\left[2,-\frac{(a-i b) e^{2 i (e+f x)}}{a+i b}\right] +$$

$$3\, \mathrm{PolyLog}\left[3,-\frac{(a-i b) e^{2 i (e+f x)}}{a+i b}\right]\Big)+a (a+i b) d^3 e^{-2 i e} \left(-i b \left(-1+e^{2 i e}\right)+a \left(1+e^{2 i e}\right)\right)$$

$$\Big(-2 i f^4 x^4+4 f^3 x^3\, \mathrm{Log}\left[1+\frac{(a-i b) e^{2 i (e+f x)}}{a+i b}\right]-6 i f^2 x^2\, \mathrm{PolyLog}\left[2,-\frac{(a-i b) e^{2 i (e+f x)}}{a+i b}\right] +$$

$$6 f x\, \mathrm{PolyLog}\left[3,-\frac{(a-i b) e^{2 i (e+f x)}}{a+i b}\right]+3 i\, \mathrm{PolyLog}\left[4,-\frac{(a-i b) e^{2 i (e+f x)}}{a+i b}\right]\Big)\Big) +$$

$$\left(3 x^2 \left(a c^2 d-i b c^2 d+a c^2 d \,\mathrm{Cos}[2 e]+i b c^2 d \,\mathrm{Cos}[2 e]+i a c^2 d \,\mathrm{Sin}[2 e]-b c^2 d \,\mathrm{Sin}[2 e]\right)\right) /$$

$$\Big(2 (a-i b) (a+i b)$$

$$\left(a+i b+a \,\mathrm{Cos}[2 e]-i b \,\mathrm{Cos}[2 e]+i a \,\mathrm{Sin}[2 e]+b \,\mathrm{Sin}[2 e]\right)\Big) +$$

$$\left(x^3 \left(a c d^2-i b c d^2+a c d^2 \,\mathrm{Cos}[2 e]+i b c d^2 \,\mathrm{Cos}[2 e]+i a c d^2 \,\mathrm{Sin}[2 e]-b c d^2 \,\mathrm{Sin}[2 e]\right)\right) /$$

$$\left((a-i b) (a+i b) \left(a+i b+a \,\mathrm{Cos}[2 e]-i b \,\mathrm{Cos}[2 e]+i a \,\mathrm{Sin}[2 e]+b \,\mathrm{Sin}[2 e]\right)\right) +$$

$$\left(x^4 \left(a d^3-i b d^3+a d^3 \,\mathrm{Cos}[2 e]+i b d^3 \,\mathrm{Cos}[2 e]+i a d^3 \,\mathrm{Sin}[2 e]-b d^3 \,\mathrm{Sin}[2 e]\right)\right) /$$

$$\left(4 (a-i b) (a+i b) \left(a+i b+a \,\mathrm{Cos}[2 e]-i b \,\mathrm{Cos}[2 e]+i a \,\mathrm{Sin}[2 e]+b \,\mathrm{Sin}[2 e]\right)\right) +$$

$$x \Big(c^3 \Big/ \Big(a^2+2 i a b-b^2+a^2 \,\mathrm{Cos}[4 e]-2 i a b \,\mathrm{Cos}[4 e]-b^2 \,\mathrm{Cos}[4 e]+i a^2 \,\mathrm{Sin}[4 e]+2 a b \,\mathrm{Sin}[4 e] -$$

$$i b^2 \,\mathrm{Sin}[4 e]\Big)+\Big(\left(-a-i b+a \,\mathrm{Cos}[2 e]-i b \,\mathrm{Cos}[2 e]+i a \,\mathrm{Sin}[2 e]+b \,\mathrm{Sin}[2 e]\right)$$

$$\left(-4 i a b c^3 \,\mathrm{Cos}[2 e]+4 a b c^3 \,\mathrm{Sin}[2 e]\right)\Big) \Big/ \Big((a-i b) (a+i b)$$

$$\left(a+i b+a \,\mathrm{Cos}[2 e]-i b \,\mathrm{Cos}[2 e]+i a \,\mathrm{Sin}[2 e]+b \,\mathrm{Sin}[2 e]\right) \Big(a^2+2 i a b-b^2+a^2 \,\mathrm{Cos}[4 e] -$$

$$2 i a b \,\mathrm{Cos}[4 e]-b^2 \,\mathrm{Cos}[4 e]+i a^2 \,\mathrm{Sin}[4 e]+2 a b \,\mathrm{Sin}[4 e]-i b^2 \,\mathrm{Sin}[4 e]\Big)\Big) +$$

$$\left(c^3 \,\mathrm{Cos}[4 e]+i c^3 \,\mathrm{Sin}[4 e]\right) \Big/ \Big(a^2+2 i a b-b^2+a^2 \,\mathrm{Cos}[4 e]-2 i a b \,\mathrm{Cos}[4 e] -$$

$$b^2 \,\mathrm{Cos}[4 e]+i a^2 \,\mathrm{Sin}[4 e]+2 a b \,\mathrm{Sin}[4 e]-i b^2 \,\mathrm{Sin}[4 e]\Big)\Big) +$$

$$\left(b^2 c^3 \,\mathrm{Sin}[f x]+3 b^2 c^2 d x \,\mathrm{Sin}[f x]+3 b^2 c d^2 x^2 \,\mathrm{Sin}[f x]+b^2 d^3 x^3 \,\mathrm{Sin}[f x]\right) /$$

$$\left((a-i b) (a+i b) f \left(a \,\mathrm{Cos}[e]+b \,\mathrm{Sin}[e]\right) \left(a \,\mathrm{Cos}[e+f x]+b \,\mathrm{Sin}[e+f x]\right)\right)$$

## Problem 60: Result more than twice size of optimal antiderivative.

$$\int \frac{(c+d\,x)^2}{(a+b\,\mathrm{Tan}[e+f\,x])^2}\,dx$$

Optimal (type 4, 654 leaves, 18 steps):

$$-\frac{2\,i\,b^2\,(c+d\,x)^2}{(a^2+b^2)^2\,f}+\frac{2\,b^2\,(c+d\,x)^2}{(a+i\,b)\,(i\,a+b)^2\,\left(i\,a-b+(i\,a+b)\,e^{2\,i\,e+2\,i\,f\,x}\right)\,f}+$$
$$\frac{(c+d\,x)^3}{3\,(a-i\,b)^2\,d}+\frac{4\,b\,(c+d\,x)^3}{3\,(i\,a-b)\,(a-i\,b)^2\,d}-\frac{4\,b^2\,(c+d\,x)^3}{3\,(a^2+b^2)^2\,d}+$$
$$\frac{2\,b^2\,d\,(c+d\,x)\,\mathrm{Log}\left[1+\frac{(a-i\,b)\,e^{2\,i\,e+2\,i\,f\,x}}{a+i\,b}\right]}{(a^2+b^2)^2\,f^2}+\frac{2\,b\,(c+d\,x)^2\,\mathrm{Log}\left[1+\frac{(a-i\,b)\,e^{2\,i\,e+2\,i\,f\,x}}{a+i\,b}\right]}{(a-i\,b)^2\,(a+i\,b)\,f}-$$
$$\frac{2\,i\,b^2\,(c+d\,x)^2\,\mathrm{Log}\left[1+\frac{(a-i\,b)\,e^{2\,i\,e+2\,i\,f\,x}}{a+i\,b}\right]}{(a^2+b^2)^2\,f}-\frac{i\,b^2\,d^2\,\mathrm{PolyLog}\left[2,-\frac{(a-i\,b)\,e^{2\,i\,e+2\,i\,f\,x}}{a+i\,b}\right]}{(a^2+b^2)^2\,f^3}+$$
$$\frac{2\,b\,d\,(c+d\,x)\,\mathrm{PolyLog}\left[2,-\frac{(a-i\,b)\,e^{2\,i\,e+2\,i\,f\,x}}{a+i\,b}\right]}{(i\,a-b)\,(a-i\,b)^2\,f^2}-\frac{2\,b^2\,d\,(c+d\,x)\,\mathrm{PolyLog}\left[2,-\frac{(a-i\,b)\,e^{2\,i\,e+2\,i\,f\,x}}{a+i\,b}\right]}{(a^2+b^2)^2\,f^2}+$$
$$\frac{b\,d^2\,\mathrm{PolyLog}\left[3,-\frac{(a-i\,b)\,e^{2\,i\,e+2\,i\,f\,x}}{a+i\,b}\right]}{(a-i\,b)^2\,(a+i\,b)\,f^3}-\frac{i\,b^2\,d^2\,\mathrm{PolyLog}\left[3,-\frac{(a-i\,b)\,e^{2\,i\,e+2\,i\,f\,x}}{a+i\,b}\right]}{(a^2+b^2)^2\,f^3}$$

Result (type 4, 1320 leaves):

$$\frac{1}{3\,(a-i\,b)\,(a+i\,b)\,\left(a^2+b^2\right)\,\left(b-b\,e^{2\,i\,e}-i\,a\,\left(1+e^{2\,i\,e}\right)\right)\,f^3}$$

$$b\,\Bigg(-f\,\Bigg(12\,a\,b\,c\,d\,e^{2\,i\,e}\,f\,x-12\,i\,b^2\,c\,d\,e^{2\,i\,e}\,f\,x+12\,a^2\,c^2\,e^{2\,i\,e}\,f^2\,x-12\,i\,a\,b\,c^2\,e^{2\,i\,e}\,f^2\,x+$$

$$6\,a\,b\,d^2\,e^{2\,i\,e}\,f\,x^2-6\,i\,b^2\,d^2\,e^{2\,i\,e}\,f\,x^2+12\,a^2\,c\,d\,e^{2\,i\,e}\,f^2\,x^2-12\,i\,a\,b\,c\,d\,e^{2\,i\,e}\,f^2\,x^2+$$

$$4\,a^2\,d^2\,e^{2\,i\,e}\,f^2\,x^3-4\,i\,a\,b\,d^2\,e^{2\,i\,e}\,f^2\,x^3+6\,c\,\left(-i\,b\,\left(-1+e^{2\,i\,e}\right)+a\,\left(1+e^{2\,i\,e}\right)\right)$$

$$(b\,d+a\,c\,f)\,\mathrm{ArcTan}\left[\frac{2\,a\,b\,e^{2\,i\,(e+f\,x)}}{-b^2\,\left(-1+e^{2\,i\,(e+f\,x)}\right)+a^2\,\left(1+e^{2\,i\,(e+f\,x)}\right)}\right]+$$

$$6\,d\,\left(b\,\left(-1+e^{2\,i\,e}\right)+i\,a\,\left(1+e^{2\,i\,e}\right)\right)\,x\,(b\,d+a\,f\,(2\,c+d\,x))\,\mathrm{Log}\left[1+\frac{(a-i\,b)\,e^{2\,i\,(e+f\,x)}}{a+i\,b}\right]+$$

$$3\,i\,a\,b\,c\,d\,\mathrm{Log}\left[b^2\,\left(-1+e^{2\,i\,(e+f\,x)}\right)^2+a^2\,\left(1+e^{2\,i\,(e+f\,x)}\right)^2\right]-$$

$$3\,b^2\,c\,d\,\mathrm{Log}\left[b^2\,\left(-1+e^{2\,i\,(e+f\,x)}\right)^2+a^2\,\left(1+e^{2\,i\,(e+f\,x)}\right)^2\right]+$$

$$3\,i\,a\,b\,c\,d\,e^{2\,i\,e}\,\mathrm{Log}\left[b^2\,\left(-1+e^{2\,i\,(e+f\,x)}\right)^2+a^2\,\left(1+e^{2\,i\,(e+f\,x)}\right)^2\right]+$$

$$3\,b^2\,c\,d\,e^{2\,i\,e}\,\mathrm{Log}\left[b^2\,\left(-1+e^{2\,i\,(e+f\,x)}\right)^2+a^2\,\left(1+e^{2\,i\,(e+f\,x)}\right)^2\right]+$$

$$3\,i\,a^2\,c^2\,f\,\mathrm{Log}\left[b^2\,\left(-1+e^{2\,i\,(e+f\,x)}\right)^2+a^2\,\left(1+e^{2\,i\,(e+f\,x)}\right)^2\right]-$$

$$3\,a\,b\,c^2\,f\,\mathrm{Log}\left[b^2\,\left(-1+e^{2\,i\,(e+f\,x)}\right)^2+a^2\,\left(1+e^{2\,i\,(e+f\,x)}\right)^2\right]+$$

$$3\,i\,a^2\,c^2\,e^{2\,i\,e}\,f\,\mathrm{Log}\left[b^2\,\left(-1+e^{2\,i\,(e+f\,x)}\right)^2+a^2\,\left(1+e^{2\,i\,(e+f\,x)}\right)^2\right]+$$

$$3\,a\,b\,c^2\,e^{2\,i\,e}\,f\,\mathrm{Log}\left[b^2\,\left(-1+e^{2\,i\,(e+f\,x)}\right)^2+a^2\,\left(1+e^{2\,i\,(e+f\,x)}\right)^2\right]\Bigg)-$$

$$3\,d\,\left(-i\,b\,\left(-1+e^{2\,i\,e}\right)+a\,\left(1+e^{2\,i\,e}\right)\right)\,(b\,d+2\,a\,f\,(c+d\,x))\,\mathrm{PolyLog}\left[2,-\frac{(a-i\,b)\,e^{2\,i\,(e+f\,x)}}{a+i\,b}\right]+$$

$$3\,a\,d^2\,\left(b-b\,e^{2\,i\,e}-i\,a\,\left(1+e^{2\,i\,e}\right)\right)\,\mathrm{PolyLog}\left[3,-\frac{(a-i\,b)\,e^{2\,i\,(e+f\,x)}}{a+i\,b}\right]\Bigg)+$$

$$\Big(3\,a^2\,c^2\,f\,x\,\mathrm{Cos}[f\,x]-3\,b^2\,c^2\,f\,x\,\mathrm{Cos}[f\,x]+3\,a^2\,c\,d\,f\,x^2\,\mathrm{Cos}[f\,x]-$$
$$3\,b^2\,c\,d\,f\,x^2\,\mathrm{Cos}[f\,x]+$$
$$a^2\,d^2\,f\,x^3\,\mathrm{Cos}[f\,x]-$$
$$b^2\,d^2\,f\,x^3\,\mathrm{Cos}[f\,x]+$$
$$3\,a^2\,c^2\,f\,x\,\mathrm{Cos}[2\,e+f\,x]+$$
$$3\,b^2\,c^2\,f\,x\,\mathrm{Cos}[2\,e+f\,x]+$$
$$3\,a^2\,c\,d\,f\,x^2\,\mathrm{Cos}[2\,e+f\,x]+$$
$$3\,b^2\,c\,d\,f\,x^2\,\mathrm{Cos}[2\,e+f\,x]+$$
$$a^2\,d^2\,f\,x^3\,\mathrm{Cos}[2\,e+f\,x]+b^2\,d^2\,f\,x^3\,\mathrm{Cos}[2\,e+f\,x]+$$
$$6\,b^2\,c^2\,\mathrm{Sin}[f\,x]+12\,b^2\,c\,d\,x\,\mathrm{Sin}[f\,x]+$$
$$6\,a\,b\,c^2\,f\,x\,\mathrm{Sin}[f\,x]+6\,b^2\,d^2\,x^2\,\mathrm{Sin}[f\,x]+$$
$$6\,a\,b\,c\,d\,f\,x^2\,\mathrm{Sin}[f\,x]+2\,a\,b\,d^2\,f\,x^3\,\mathrm{Sin}[f\,x]\Big)\Big/$$
$$\big(6\,(a-i\,b)\,(a+i\,b)\,f\,(a\,\mathrm{Cos}[e]+b\,\mathrm{Sin}[e])\,(a\,\mathrm{Cos}[e+f\,x]+b\,\mathrm{Sin}[e+f\,x])\big)$$

## Problem 61: Result more than twice size of optimal antiderivative.

$$\int\frac{c+d\,x}{(a+b\,\mathrm{Tan}[e+f\,x])^2}\,dx$$

Optimal (type 4, 214 leaves, 5 steps):

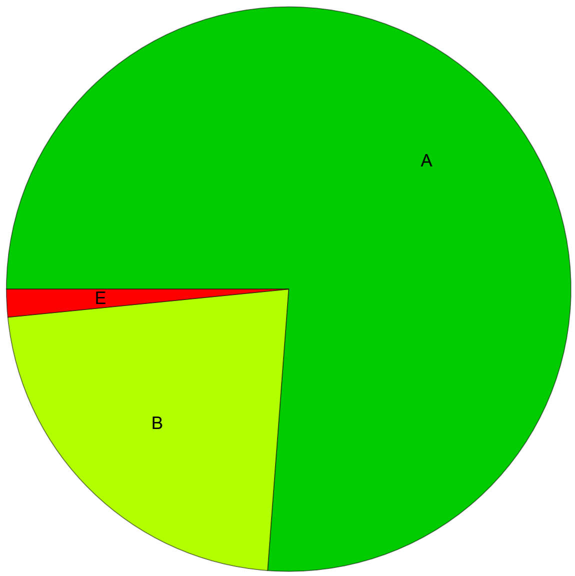
$$-\frac{(c+d x)^2}{2 \left(a^2+b^2\right) d}+\frac{(b d+2 a c f+2 a d f x)^2}{4 a (a+i b) \left(a^2+b^2\right) d f^2}+\frac{b (b d+2 a c f+2 a d f x) \text{Log}\left[1+\frac{\left(a^2+b^2\right) e^{2 i (e+f x)}}{(a+i b)^2}\right]}{\left(a^2+b^2\right)^2 f^2}-\frac{i a b d \text{PolyLog}\left[2,-\frac{\left(a^2+b^2\right) e^{2 i (e+f x)}}{(a+i b)^2}\right]}{\left(a^2+b^2\right)^2 f^2}-\frac{b (c+d x)}{\left(a^2+b^2\right) f (a+b \text{Tan}[e+f x])}$$

Result (type 4, 745 leaves) :

$$\left((e+f x) (-2 d e+2 c f+d (e+f x)) \text{Sec}[e+f x]^2 (a \text{Cos}[e+f x]+b \text{Sin}[e+f x])^2\right) \Big/ \left(2 (a-i b) (a+i b) f^2 (a+b \text{Tan}[e+f x])^2\right)+$$
$$\left(b^2 d (-b (e+f x)+a \text{Log}[a \text{Cos}[e+f x]+b \text{Sin}[e+f x]]) \text{Sec}[e+f x]^2 (a \text{Cos}[e+f x]+b \text{Sin}[e+f x])^2\right) \Big/ \left(a (a-i b) (a+i b) \left(a^2+b^2\right) f^2 (a+b \text{Tan}[e+f x])^2\right)-$$
$$\left(2 b d e (-b (e+f x)+a \text{Log}[a \text{Cos}[e+f x]+b \text{Sin}[e+f x]]) \text{Sec}[e+f x]^2 (a \text{Cos}[e+f x]+b \text{Sin}[e+f x])^2\right) \Big/ \left((a-i b) (a+i b) \left(a^2+b^2\right) f^2 (a+b \text{Tan}[e+f x])^2\right)+$$
$$\left(2 b c (-b (e+f x)+a \text{Log}[a \text{Cos}[e+f x]+b \text{Sin}[e+f x]]) \text{Sec}[e+f x]^2 (a \text{Cos}[e+f x]+b \text{Sin}[e+f x])^2\right) \Big/ \left((a-i b) (a+i b) \left(a^2+b^2\right) f (a+b \text{Tan}[e+f x])^2\right)-$$
$$\left(d \left(e^{i \text{ArcTan}\left[\frac{a}{b}\right]} (e+f x)^2+\frac{1}{\sqrt{1+\frac{a^2}{b^2}}\, b} a \left(i (e+f x) \left(-\pi+2 \text{ArcTan}\left[\frac{a}{b}\right]\right)-\pi \text{Log}\left[1+e^{-2 i (e+f x)}\right]-\right.\right.\right.$$
$$2 \left(e+f x+\text{ArcTan}\left[\frac{a}{b}\right]\right) \text{Log}\left[1-e^{2 i \left(e+f x+\text{ArcTan}\left[\frac{a}{b}\right]\right)}\right]+\pi \text{Log}[\text{Cos}[e+f x]]+$$
$$\left.\left.\left.2 \text{ArcTan}\left[\frac{a}{b}\right] \text{Log}\left[\text{Sin}\left[e+f x+\text{ArcTan}\left[\frac{a}{b}\right]\right]\right]+i \text{PolyLog}\left[2, e^{2 i \left(e+f x+\text{ArcTan}\left[\frac{a}{b}\right]\right)}\right]\right)\right)$$
$$\left.\text{Sec}[e+f x]^2 (a \text{Cos}[e+f x]+b \text{Sin}[e+f x])^2\right) \Big/ \left((a-i b) (a+i b) \sqrt{\frac{a^2+b^2}{b^2}}\right.$$
$$\left.f^2 (a+b \text{Tan}[e+f x])^2\right)+\left(\text{Sec}[e+f x]^2 (a \text{Cos}[e+f x]+b \text{Sin}[e+f x])\right.$$
$$\left.\left(-b^2 d e \text{Sin}[e+f x]+b^2 c f \text{Sin}[e+f x]+b^2 d (e+f x) \text{Sin}[e+f x]\right)\right) \Big/$$
$$\left(a (a-i b) (a+i b) f^2 (a+b \text{Tan}[e+f x])^2\right)$$

# Summary of Integration Test Results

63 integration problems




A - 48 optimal antiderivatives

B - 14 more than twice size of optimal antiderivatives

C - 0 unnecessarily complex antiderivatives

D - 0 unable to integrate problems

E - 1 integration timeouts