# Mathematica 11.3 Integration Test Results

## Test results for the 66 problems in "4.3.11 (e x)^m (a+b tan(c+d x^n))^p.m"

### Problem 1: Result more than twice size of optimal antiderivative.

$$\int x^3 \left(a+b \operatorname{Tan}\left[c+d x^2\right]\right) dx$$

Optimal (type 4, 73 leaves, 7 steps):

$$\frac{a x^4}{4}+\frac{1}{4} i b x^4-\frac{b x^2 \operatorname{Log}\left[1+e^{2 i \left(c+d x^2\right)}\right]}{2 d}+\frac{i b \operatorname{PolyLog}\left[2,-e^{2 i \left(c+d x^2\right)}\right]}{4 d^2}$$

Result (type 4, 199 leaves):

$$\frac{a x^4}{4}-\left(b \operatorname{Csc}[c]\left(d^2 e^{-i \operatorname{ArcTan}[\operatorname{Cot}[c]]} x^4-\frac{1}{\sqrt{1+\operatorname{Cot}[c]^2}}\operatorname{Cot}[c]\left(i d x^2\left(-\pi-2 \operatorname{ArcTan}[\operatorname{Cot}[c]]\right)-\pi \operatorname{Log}\left[1+e^{-2 i d x^2}\right]-2\left(d x^2-\operatorname{ArcTan}[\operatorname{Cot}[c]]\right) \operatorname{Log}\left[1-e^{2 i \left(d x^2-\operatorname{ArcTan}[\operatorname{Cot}[c]]\right)}\right]+\pi \operatorname{Log}\left[\operatorname{Cos}\left[d x^2\right]\right]-2 \operatorname{ArcTan}[\operatorname{Cot}[c]] \operatorname{Log}\left[\operatorname{Sin}\left[d x^2-\operatorname{ArcTan}[\operatorname{Cot}[c]]\right]\right]+i \operatorname{PolyLog}\left[2, e^{2 i \left(d x^2-\operatorname{ArcTan}[\operatorname{Cot}[c]]\right)}\right]\right)\right) \operatorname{Sec}[c]\right) \Big/ \left(4 d^2 \sqrt{\operatorname{Csc}[c]^2\left(\operatorname{Cos}[c]^2+\operatorname{Sin}[c]^2\right)}\right)+\frac{1}{4} b x^4 \operatorname{Tan}[c]$$

### Problem 7: Result more than twice size of optimal antiderivative.

$$\int x^3 \left(a+b \operatorname{Tan}\left[c+d x^2\right]\right)^2 dx$$

Optimal (type 4, 126 leaves, 10 steps):

$$\frac{a^2 x^4}{4}+\frac{1}{2} i a b x^4-\frac{b^2 x^4}{4}-\frac{a b x^2 \operatorname{Log}\left[1+e^{2 i \left(c+d x^2\right)}\right]}{d}+\frac{b^2 \operatorname{Log}\left[\operatorname{Cos}\left[c+d x^2\right]\right]}{2 d^2}+\frac{i a b \operatorname{PolyLog}\left[2,-e^{2 i \left(c+d x^2\right)}\right]}{2 d^2}+\frac{b^2 x^2 \operatorname{Tan}\left[c+d x^2\right]}{2 d}$$

Result (type 4, 295 leaves):

$$\frac{1}{4} x^4 \text{Sec}[c] \left(a^2 \text{Cos}[c] - b^2 \text{Cos}[c] + 2 a b \text{Sin}[c]\right) + \frac{b^2 \text{Sec}[c] \left(\text{Cos}[c] \text{Log}\left[\text{Cos}[c] \text{Cos}\left[d x^2\right] - \text{Sin}[c] \text{Sin}\left[d x^2\right]\right] + d x^2 \text{Sin}[c]\right)}{2 d^2 \left(\text{Cos}[c]^2 + \text{Sin}[c]^2\right)} - \left(a b \text{Csc}[c] \left(d^2 e^{-i \text{ArcTan}[\text{Cot}[c]]} x^4 - \frac{1}{\sqrt{1 + \text{Cot}[c]^2}} \text{Cot}[c] \left(i d x^2 \left(-\pi - 2 \text{ArcTan}[\text{Cot}[c]]\right) - \pi \text{Log}\left[1 + e^{-2 i d x^2}\right] - 2 \left(d x^2 - \text{ArcTan}[\text{Cot}[c]]\right) \text{Log}\left[1 - e^{2 i \left(d x^2 - \text{ArcTan}[\text{Cot}[c]]\right)}\right] + \pi \text{Log}\left[\text{Cos}\left[d x^2\right]\right] - 2 \text{ArcTan}[\text{Cot}[c]] \text{Log}\left[\text{Sin}\left[d x^2 - \text{ArcTan}[\text{Cot}[c]]\right]\right] + i \text{PolyLog}\left[2, e^{2 i \left(d x^2 - \text{ArcTan}[\text{Cot}[c]]\right)}\right]\right)\right) \text{Sec}[c]\right) \Big/ \left(2 d^2 \sqrt{\text{Csc}[c]^2 \left(\text{Cos}[c]^2 + \text{Sin}[c]^2\right)}\right) + \frac{b^2 x^2 \text{Sec}[c] \text{Sec}\left[c + d x^2\right] \text{Sin}\left[d x^2\right]}{2 d}$$

## Problem 19: Result more than twice size of optimal antiderivative.

$$\int \frac{x^3}{\left(a + b \text{Tan}\left[c + d x^2\right]\right)^2} \, dx$$

Optimal (type 4, 202 leaves, 6 steps):

$$-\frac{x^4}{4 \left(a^2 + b^2\right)} + \frac{\left(b + 2 a d x^2\right)^2}{8 a (a + i b) \left(a^2 + b^2\right) d^2} + \frac{b \left(b + 2 a d x^2\right) \text{Log}\left[1 + \frac{\left(a^2+b^2\right) e^{2 i \left(c+d x^2\right)}}{(a+i b)^2}\right]}{2 \left(a^2 + b^2\right)^2 d^2} - \frac{i a b \text{PolyLog}\left[2, -\frac{\left(a^2+b^2\right) e^{2 i \left(c+d x^2\right)}}{(a+i b)^2}\right]}{2 \left(a^2 + b^2\right)^2 d^2} - \frac{b x^2}{2 \left(a^2 + b^2\right) d \left(a + b \text{Tan}\left[c + d x^2\right]\right)}$$

Result (type 4, 703 leaves):

$$\frac{\left(-c+d x^2\right)\left(c+d x^2\right)\operatorname{Sec}\left[c+d x^2\right]^2\left(a\operatorname{Cos}\left[c+d x^2\right]+b\operatorname{Sin}\left[c+d x^2\right]\right)^2}{4\,(a-i b)\,(a+i b)\,d^2\left(a+b\operatorname{Tan}\left[c+d x^2\right]\right)^2}+$$

$$\left(b^2\left(-b\left(c+d x^2\right)+a\operatorname{Log}\left[a\operatorname{Cos}\left[c+d x^2\right]+b\operatorname{Sin}\left[c+d x^2\right]\right]\right)\operatorname{Sec}\left[c+d x^2\right]^2\right.$$
$$\left.\left(a\operatorname{Cos}\left[c+d x^2\right]+b\operatorname{Sin}\left[c+d x^2\right]\right)^2\right)\Big/\left(2 a\,(a-i b)\,(a+i b)\left(a^2+b^2\right)d^2\left(a+b\operatorname{Tan}\left[c+d x^2\right]\right)^2\right)-$$

$$\left(b c\left(-b\left(c+d x^2\right)+a\operatorname{Log}\left[a\operatorname{Cos}\left[c+d x^2\right]+b\operatorname{Sin}\left[c+d x^2\right]\right]\right)\operatorname{Sec}\left[c+d x^2\right]^2\right.$$
$$\left.\left(a\operatorname{Cos}\left[c+d x^2\right]+b\operatorname{Sin}\left[c+d x^2\right]\right)^2\right)\Big/\left((a-i b)\,(a+i b)\left(a^2+b^2\right)d^2\left(a+b\operatorname{Tan}\left[c+d x^2\right]\right)^2\right)-$$

$$\left(\left(e^{i\operatorname{ArcTan}\left[\frac{a}{b}\right]}\left(c+d x^2\right)^2+\frac{1}{\sqrt{1+\frac{a^2}{b^2}}\;b}a\left(i\left(c+d x^2\right)\left(-\pi+2\operatorname{ArcTan}\left[\frac{a}{b}\right]\right)-\pi\operatorname{Log}\left[1+e^{-2 i\left(c+d x^2\right)}\right]-\right.\right.\right.$$
$$2\left(c+d x^2+\operatorname{ArcTan}\left[\frac{a}{b}\right]\right)\operatorname{Log}\left[1-e^{2 i\left(c+d x^2+\operatorname{ArcTan}\left[\frac{a}{b}\right]\right)}\right]+\pi\operatorname{Log}\left[\operatorname{Cos}\left[c+d x^2\right]\right]+$$
$$\left.\left.2\operatorname{ArcTan}\left[\frac{a}{b}\right]\operatorname{Log}\left[\operatorname{Sin}\left[c+d x^2+\operatorname{ArcTan}\left[\frac{a}{b}\right]\right]\right]+i\operatorname{PolyLog}\left[2,e^{2 i\left(c+d x^2+\operatorname{ArcTan}\left[\frac{a}{b}\right]\right)}\right]\right)\right)$$
$$\left.\operatorname{Sec}\left[c+d x^2\right]^2\left(a\operatorname{Cos}\left[c+d x^2\right]+b\operatorname{Sin}\left[c+d x^2\right]\right)^2\right)\Big/$$
$$\left(2\,(a-i b)\,(a+i b)\sqrt{\frac{a^2+b^2}{b^2}}\;d^2\left(a+b\operatorname{Tan}\left[c+d x^2\right]\right)^2\right)+$$

$$\left(\operatorname{Sec}\left[c+d x^2\right]^2\left(a\operatorname{Cos}\left[c+d x^2\right]+b\operatorname{Sin}\left[c+d x^2\right]\right)\left(-b^2 c\operatorname{Sin}\left[c+d x^2\right]+b^2\left(c+d x^2\right)\operatorname{Sin}\left[c+d x^2\right]\right)\right)\Big/$$
$$\left(2 a\,(a-i b)\,(a+i b)\,d^2\left(a+b\operatorname{Tan}\left[c+d x^2\right]\right)^2\right)$$

## Problem 21: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{x}{\left(a+b\operatorname{Tan}\left[c+d x^2\right]\right)^2}\,dx$$

Optimal (type 3, 94 leaves, 4 steps):

$$\frac{\left(a^2-b^2\right)x^2}{2\left(a^2+b^2\right)^2}+\frac{a b\operatorname{Log}\left[a\operatorname{Cos}\left[c+d x^2\right]+b\operatorname{Sin}\left[c+d x^2\right]\right]}{\left(a^2+b^2\right)^2 d}-\frac{b}{2\left(a^2+b^2\right)d\left(a+b\operatorname{Tan}\left[c+d x^2\right]\right)}$$

Result (type 3, 197 leaves):

$$\left(a^2\left((a+i b)^2\left(c+d x^2\right)+a b \operatorname{Log}\left[\left(a \operatorname{Cos}\left[c+d x^2\right]+b \operatorname{Sin}\left[c+d x^2\right]\right)^2\right]\right)+b\left((a+i b)\left(-i b^2+a b\left(1+i c+i d x^2\right)+a^2\left(c+d x^2\right)\right)+a^2 b \operatorname{Log}\left[\left(a \operatorname{Cos}\left[c+d x^2\right]+b \operatorname{Sin}\left[c+d x^2\right]\right)^2\right]\right) \operatorname{Tan}\left[c+d x^2\right]-2 i a^2 b \operatorname{ArcTan}\left[\operatorname{Tan}\left[c+d x^2\right]\right]\left(a+b \operatorname{Tan}\left[c+d x^2\right]\right)\right) \Big/ \left(2 a\left(a^2+b^2\right)^2 d\left(a+b \operatorname{Tan}\left[c+d x^2\right]\right)\right)$$

## Problem 28: Result more than twice size of optimal antiderivative.

$$\int\left(a+b \operatorname{Tan}\left[c+d \sqrt{x}\right]\right) dx$$

Optimal (type 4, 66 leaves, 6 steps):

$$a x+i b x-\frac{2 b \sqrt{x} \operatorname{Log}\left[1+e^{2 i\left(c+d \sqrt{x}\right)}\right]}{d}+\frac{i b \operatorname{PolyLog}\left[2,-e^{2 i\left(c+d \sqrt{x}\right)}\right]}{d^2}$$

Result (type 4, 199 leaves):

$$a x-\left(b \operatorname{Csc}[c]\left(d^2 e^{-i \operatorname{ArcTan}[\operatorname{Cot}[c]]} x-\frac{1}{\sqrt{1+\operatorname{Cot}[c]^2}} \operatorname{Cot}[c]\left(i d \sqrt{x}\left(-\pi-2 \operatorname{ArcTan}[\operatorname{Cot}[c]]\right)-\pi \operatorname{Log}\left[1+e^{-2 i d \sqrt{x}}\right]-2\left(d \sqrt{x}-\operatorname{ArcTan}[\operatorname{Cot}[c]]\right) \operatorname{Log}\left[1-e^{2 i\left(d \sqrt{x}-\operatorname{ArcTan}[\operatorname{Cot}[c]]\right)}\right]+\pi \operatorname{Log}\left[\operatorname{Cos}\left[d \sqrt{x}\right]\right]-2 \operatorname{ArcTan}[\operatorname{Cot}[c]] \operatorname{Log}\left[\operatorname{Sin}\left[d \sqrt{x}-\operatorname{ArcTan}[\operatorname{Cot}[c]]\right]\right]+i \operatorname{PolyLog}\left[2, e^{2 i\left(d \sqrt{x}-\operatorname{ArcTan}[\operatorname{Cot}[c]]\right)}\right]\right)\right) \operatorname{Sec}[c]\right) \Big/ \left(d^2 \sqrt{\operatorname{Csc}[c]^2\left(\operatorname{Cos}[c]^2+\operatorname{Sin}[c]^2\right)}\right)+b x \operatorname{Tan}[c]$$

## Problem 33: Result more than twice size of optimal antiderivative.

$$\int\left(a+b \operatorname{Tan}\left[c+d \sqrt{x}\right]\right)^2 dx$$

Optimal (type 4, 119 leaves, 10 steps):

$$a^2 x+2 i a b x-b^2 x-\frac{4 a b \sqrt{x} \operatorname{Log}\left[1+e^{2 i\left(c+d \sqrt{x}\right)}\right]}{d}+\frac{2 b^2 \operatorname{Log}\left[\operatorname{Cos}\left[c+d \sqrt{x}\right]\right]}{d^2}+\frac{2 i a b \operatorname{PolyLog}\left[2,-e^{2 i\left(c+d \sqrt{x}\right)}\right]}{d^2}+\frac{2 b^2 \sqrt{x} \operatorname{Tan}\left[c+d \sqrt{x}\right]}{d}$$

Result (type 4, 308 leaves):

$$x\,\text{Sec}[c]\left(a^2\,\text{Cos}[c]-b^2\,\text{Cos}[c]+2\,a\,b\,\text{Sin}[c]\right)+\left(2\,b^2\,\text{Sec}[c]\left(\text{Cos}[c]\,\text{Log}\left[\text{Cos}[c]\,\text{Cos}\left[d\,\sqrt{x}\right]-\text{Sin}[c]\,\text{Sin}\left[d\,\sqrt{x}\right]\right]+d\,\sqrt{x}\,\text{Sin}[c]\right)\right)\Big/\left(d^2\left(\text{Cos}[c]^2+\text{Sin}[c]^2\right)\right)-\left(2\,a\,b\,\text{Csc}[c]\left(d^2\,e^{-i\,\text{ArcTan}[\text{Cot}[c]]}\,x-\frac{1}{\sqrt{1+\text{Cot}[c]^2}}\text{Cot}[c]\left(i\,d\,\sqrt{x}\left(-\pi-2\,\text{ArcTan}[\text{Cot}[c]]\right)-\pi\,\text{Log}\left[1+e^{-2\,i\,d\,\sqrt{x}}\right]-2\left(d\,\sqrt{x}-\text{ArcTan}[\text{Cot}[c]]\right)\text{Log}\left[1-e^{2\,i\left(d\,\sqrt{x}-\text{ArcTan}[\text{Cot}[c]]\right)}\right]+\pi\,\text{Log}\left[\text{Cos}\left[d\,\sqrt{x}\right]\right]-2\,\text{ArcTan}[\text{Cot}[c]]\,\text{Log}\left[\text{Sin}\left[d\,\sqrt{x}-\text{ArcTan}[\text{Cot}[c]]\right]\right]+i\,\text{PolyLog}\left[2,\,e^{2\,i\left(d\,\sqrt{x}-\text{ArcTan}[\text{Cot}[c]]\right)}\right]\right)\right)\text{Sec}[c]\right)\Big/\left(d^2\,\sqrt{\text{Csc}[c]^2\left(\text{Cos}[c]^2+\text{Sin}[c]^2\right)}\right)+\frac{2\,b^2\,\sqrt{x}\,\text{Sec}[c]\,\text{Sec}\left[c+d\,\sqrt{x}\right]\,\text{Sin}\left[d\,\sqrt{x}\right]}{d}$$

## Problem 44: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(a+b\,\text{Tan}\left[c+d\,\sqrt{x}\right]\right)^2}\,dx$$

Optimal (type 4, 204 leaves, 6 steps):

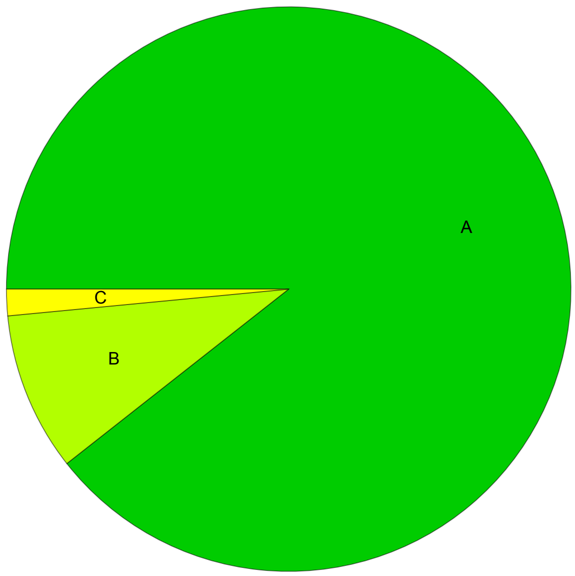
$$\frac{\left(b+2\,a\,d\,\sqrt{x}\right)^2}{2\,a\,(a+i\,b)\left(a^2+b^2\right)d^2}-\frac{x}{a^2+b^2}+\frac{2\,b\left(b+2\,a\,d\,\sqrt{x}\right)\text{Log}\left[1+\frac{\left(a^2+b^2\right)e^{2\,i\left(c+d\,\sqrt{x}\right)}}{(a+i\,b)^2}\right]}{\left(a^2+b^2\right)^2 d^2}-\frac{2\,i\,a\,b\,\text{PolyLog}\left[2,\,-\frac{\left(a^2+b^2\right)e^{2\,i\left(c+d\,\sqrt{x}\right)}}{(a+i\,b)^2}\right]}{\left(a^2+b^2\right)^2 d^2}-\frac{2\,b\,\sqrt{x}}{\left(a^2+b^2\right)d\left(a+b\,\text{Tan}\left[c+d\,\sqrt{x}\right]\right)}$$

Result (type 4, 772 leaves):

$$\left(\left(-c+d\sqrt{x}\right)\left(c+d\sqrt{x}\right)\operatorname{Sec}\left[c+d\sqrt{x}\right]^2\left(a\operatorname{Cos}\left[c+d\sqrt{x}\right]+b\operatorname{Sin}\left[c+d\sqrt{x}\right]\right)^2\right)\Big/$$
$$\left(\left(a-i\,b\right)\left(a+i\,b\right)d^2\left(a+b\operatorname{Tan}\left[c+d\sqrt{x}\right]\right)^2\right)+$$
$$\left(2\,b^2\left(-b\left(c+d\sqrt{x}\right)+a\operatorname{Log}\left[a\operatorname{Cos}\left[c+d\sqrt{x}\right]+b\operatorname{Sin}\left[c+d\sqrt{x}\right]\right]\right)\right.$$
$$\left.\operatorname{Sec}\left[c+d\sqrt{x}\right]^2\left(a\operatorname{Cos}\left[c+d\sqrt{x}\right]+b\operatorname{Sin}\left[c+d\sqrt{x}\right]\right)^2\right)\Big/$$
$$\left(a\left(a-i\,b\right)\left(a+i\,b\right)\left(a^2+b^2\right)d^2\left(a+b\operatorname{Tan}\left[c+d\sqrt{x}\right]\right)^2\right)-$$
$$\left(4\,b\,c\left(-b\left(c+d\sqrt{x}\right)+a\operatorname{Log}\left[a\operatorname{Cos}\left[c+d\sqrt{x}\right]+b\operatorname{Sin}\left[c+d\sqrt{x}\right]\right]\right)\right.$$
$$\left.\operatorname{Sec}\left[c+d\sqrt{x}\right]^2\left(a\operatorname{Cos}\left[c+d\sqrt{x}\right]+b\operatorname{Sin}\left[c+d\sqrt{x}\right]\right)^2\right)\Big/$$
$$\left(\left(a-i\,b\right)\left(a+i\,b\right)\left(a^2+b^2\right)d^2\left(a+b\operatorname{Tan}\left[c+d\sqrt{x}\right]\right)^2\right)-\left(2\left(e^{i\operatorname{ArcTan}\left[\frac{a}{b}\right]}\left(c+d\sqrt{x}\right)^2+\right.\right.$$
$$\frac{1}{\sqrt{1+\frac{a^2}{b^2}}\,b}a\left(i\left(c+d\sqrt{x}\right)\left(-\pi+2\operatorname{ArcTan}\left[\frac{a}{b}\right]\right)-\pi\operatorname{Log}\left[1+e^{-2\,i\left(c+d\sqrt{x}\right)}\right]-\right.$$
$$2\left(c+d\sqrt{x}+\operatorname{ArcTan}\left[\frac{a}{b}\right]\right)\operatorname{Log}\left[1-e^{2\,i\left(c+d\sqrt{x}+\operatorname{ArcTan}\left[\frac{a}{b}\right]\right)}\right]+\pi\operatorname{Log}\left[\operatorname{Cos}\left[c+d\sqrt{x}\right]\right]+$$
$$\left.\left.2\operatorname{ArcTan}\left[\frac{a}{b}\right]\operatorname{Log}\left[\operatorname{Sin}\left[c+d\sqrt{x}+\operatorname{ArcTan}\left[\frac{a}{b}\right]\right]\right]+i\operatorname{PolyLog}\left[2,e^{2\,i\left(c+d\sqrt{x}+\operatorname{ArcTan}\left[\frac{a}{b}\right]\right)}\right]\right)\right)$$
$$\left.\operatorname{Sec}\left[c+d\sqrt{x}\right]^2\left(a\operatorname{Cos}\left[c+d\sqrt{x}\right]+b\operatorname{Sin}\left[c+d\sqrt{x}\right]\right)^2\right)\Big/$$
$$\left(\left(a-i\,b\right)\left(a+i\,b\right)\sqrt{\frac{a^2+b^2}{b^2}}\,d^2\left(a+b\operatorname{Tan}\left[c+d\sqrt{x}\right]\right)^2\right)+$$
$$\left(2\operatorname{Sec}\left[c+d\sqrt{x}\right]^2\left(a\operatorname{Cos}\left[c+d\sqrt{x}\right]+b\operatorname{Sin}\left[c+d\sqrt{x}\right]\right)\right.$$
$$\left.\left(-b^2\,c\operatorname{Sin}\left[c+d\sqrt{x}\right]+b^2\left(c+d\sqrt{x}\right)\operatorname{Sin}\left[c+d\sqrt{x}\right]\right)\right)\Big/$$
$$\left(a\left(a-i\,b\right)\left(a+i\,b\right)d^2\left(a+b\operatorname{Tan}\left[c+d\sqrt{x}\right]\right)^2\right)$$

# Summary of Integration Test Results

66 integration problems




A - 59 optimal antiderivatives

B - 6 more than twice size of optimal antiderivatives

C - 1 unnecessarily complex antiderivatives

D - 0 unable to integrate problems

E - 0 integration timeouts