

# Mathematica 11.3 Integration Test Results

Test results for the 51 problems in "4.3.9 trig^m (a+b tan^n+c tan^(2n))^p.m"

Problem 1: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \tan [d+e x]^5 \sqrt{a+b \tan [d+e x]+c \tan [d+e x]^2} d x$$

Optimal (type 3, 975 leaves, 21 steps):

$$\left( \sqrt{a^2+b^2+c\left(c+\sqrt{a^2+b^2-2ac+c^2}\right)}-a\left(2c+\sqrt{a^2+b^2-2ac+c^2}\right) \right. \\ \left. \operatorname{ArcTan}\left[\left(b^2+(a-c)\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)-b\sqrt{a^2+b^2-2ac+c^2} \tan [d+e x]\right) / \right. \right. \\ \left. \left. \left(\sqrt{2}\left(a^2+b^2-2ac+c^2\right)^{1 / 4}\right) \right. \right. \\ \left. \left. \sqrt{a^2+b^2+c\left(c+\sqrt{a^2+b^2-2ac+c^2}\right)}-a\left(2c+\sqrt{a^2+b^2-2ac+c^2}\right) \right. \right. \\ \left. \left. \sqrt{a+b \tan [d+e x]+c \tan [d+e x]^2}\right) \right] \Bigg) / \\ \left(\sqrt{2}\left(a^2+b^2-2ac+c^2\right)^{1 / 4} e\right)+\frac{b \operatorname{ArcTanh}\left[\frac{b+2 c \tan [d+e x]}{2 \sqrt{c} \sqrt{a+b \tan [d+e x]+c \tan [d+e x]^2}}\right]}{2 \sqrt{c} e}- \\ \frac{b\left(b^2-4 a c\right) \operatorname{ArcTanh}\left[\frac{b+2 c \tan [d+e x]}{2 \sqrt{c} \sqrt{a+b \tan [d+e x]+c \tan [d+e x]^2}}\right]}{16 c^{5 / 2} e}+ \\ \frac{b\left(7 b^2-12 a c\right)\left(b^2-4 a c\right) \operatorname{ArcTanh}\left[\frac{b+2 c \tan [d+e x]}{2 \sqrt{c} \sqrt{a+b \tan [d+e x]+c \tan [d+e x]^2}}\right]}{256 c^{9 / 2} e}- \\ \left(\sqrt{a^2+b^2+c\left(c-\sqrt{a^2+b^2-2ac+c^2}\right)}-a\left(2c-\sqrt{a^2+b^2-2ac+c^2}\right)\right)$$

$$\begin{aligned} & \text{ArcTanh} \left[ \left( b^2 + (a - c) \left( a - c + \sqrt{a^2 + b^2 - 2ac + c^2} \right) + b \sqrt{a^2 + b^2 - 2ac + c^2} \tan[d + ex] \right) \right. \\ & \left. \left( \sqrt{2} (a^2 + b^2 - 2ac + c^2)^{1/4} \right. \right. \\ & \left. \left. \sqrt{a^2 + b^2 + c \left( c - \sqrt{a^2 + b^2 - 2ac + c^2} \right) - a \left( 2c - \sqrt{a^2 + b^2 - 2ac + c^2} \right)} \right. \right. \\ & \left. \left. \sqrt{a + b \tan[d + ex] + c \tan[d + ex]^2} \right) \right] \Bigg/ \\ & \left( \sqrt{2} (a^2 + b^2 - 2ac + c^2)^{1/4} e \right) + \frac{\sqrt{a + b \tan[d + ex] + c \tan[d + ex]^2}}{e} + \\ & \frac{b (b + 2c \tan[d + ex]) \sqrt{a + b \tan[d + ex] + c \tan[d + ex]^2}}{8c^2 e} - \\ & \frac{b (7b^2 - 12ac) (b + 2c \tan[d + ex]) \sqrt{a + b \tan[d + ex] + c \tan[d + ex]^2}}{128c^4 e} - \\ & \frac{(a + b \tan[d + ex] + c \tan[d + ex]^2)^{3/2}}{3ce} + \\ & \frac{\tan[d + ex]^2 (a + b \tan[d + ex] + c \tan[d + ex]^2)^{3/2}}{5ce} + \\ & \frac{(35b^2 - 32ac - 42bc \tan[d + ex]) (a + b \tan[d + ex] + c \tan[d + ex]^2)^{3/2}}{240c^3 e} \end{aligned}$$

Result (type 3, 2599 leaves):

$$\begin{aligned} & \frac{1}{e} \sqrt{\frac{a + c + a \cos[2(d + ex)] - c \cos[2(d + ex)] + b \sin[2(d + ex)]}{1 + \cos[2(d + ex)]}} \\ & \left( \frac{-105b^4 + 460ab^2c - 256a^2c^2 + 296b^2c^2 - 768ac^3 + 2944c^4}{1920c^4} + \right. \\ & \left. \frac{(-7b^2 + 16ac - 176c^2) \sec[d + ex]^2}{240c^2} + \frac{1}{5} \sec[d + ex]^4 + \frac{1}{960c^3} \right. \\ & \left. \sec[d + ex] (35b^3 \sin[d + ex] - 116abc \sin[d + ex] - 104bc^2 \sin[d + ex]) + \right. \\ & \left. \frac{b \sec[d + ex]^2 \tan[d + ex]}{40c} \right) + \\ & \left( \left( -\frac{1}{2} \sqrt{a - ib - c} \log \left[ \left( 2a - 2ic \tan[d + ex] + b(-i + \tan[d + ex]) + 2\sqrt{a - ib - c} \right. \right. \right. \right. \\ & \left. \left. \left. \sqrt{a + \tan[d + ex] (b + c \tan[d + ex])} \right) \right] \right) \Bigg/ \left( 128(a - ib - c)^{3/2} c^4 (i + \tan[d + ex]) \right) \Bigg) - \\ & \frac{1}{2} \sqrt{a + ib - c} \log \left[ \left( 2a + 2ic \tan[d + ex] + b(i + \tan[d + ex]) + \right. \right. \end{aligned}$$

$$\begin{aligned}
 & 2 \sqrt{a + i b - c} \sqrt{a + \tan[d + e x] (b + c \tan[d + e x])} \Big/ \left( 128 (a + i b - c)^{3/2} c^4 \right. \\
 & \left. (-i + \tan[d + e x]) \right) + \frac{1}{256 c^{9/2}} b (7 b^4 - 8 b^2 c (5 a + 2 c) + 16 c^2 (3 a^2 + 4 a c + 8 c^2)) \\
 & \text{Log}[b + 2 c \tan[d + e x] + 2 \sqrt{c} \sqrt{a + \tan[d + e x] (b + c \tan[d + e x])}] \\
 & \left( - \left( \left( 7 b^5 \sqrt{\left( \frac{a}{1 + \cos[2 (d + e x)]} + \frac{c}{1 + \cos[2 (d + e x)]} + \frac{a \cos[2 (d + e x)]}{1 + \cos[2 (d + e x)]} - \right. \right. \right. \right. \\
 & \left. \left. \left. \frac{c \cos[2 (d + e x)]}{1 + \cos[2 (d + e x)]} + \frac{b \sin[2 (d + e x)]}{1 + \cos[2 (d + e x)]} \right) \right) \Big/ \right. \\
 & \left. \left. (128 c^4 (-a - c - a \cos[2 (d + e x)] + c \cos[2 (d + e x)] - b \sin[2 (d + e x)])) \right) \right) + \\
 & \left( 5 a b^3 \sqrt{\left( \frac{a}{1 + \cos[2 (d + e x)]} + \frac{c}{1 + \cos[2 (d + e x)]} + \frac{a \cos[2 (d + e x)]}{1 + \cos[2 (d + e x)]} - \right. \right. \\
 & \left. \left. \frac{c \cos[2 (d + e x)]}{1 + \cos[2 (d + e x)]} + \frac{b \sin[2 (d + e x)]}{1 + \cos[2 (d + e x)]} \right) \right) \Big/ \\
 & (16 c^3 (-a - c - a \cos[2 (d + e x)] + c \cos[2 (d + e x)] - b \sin[2 (d + e x)])) - \\
 & \left( 3 a^2 b \sqrt{\left( \frac{a}{1 + \cos[2 (d + e x)]} + \frac{c}{1 + \cos[2 (d + e x)]} + \frac{a \cos[2 (d + e x)]}{1 + \cos[2 (d + e x)]} - \right. \right. \\
 & \left. \left. \frac{c \cos[2 (d + e x)]}{1 + \cos[2 (d + e x)]} + \frac{b \sin[2 (d + e x)]}{1 + \cos[2 (d + e x)]} \right) \right) \Big/ \\
 & (8 c^2 (-a - c - a \cos[2 (d + e x)] + c \cos[2 (d + e x)] - b \sin[2 (d + e x)])) + \\
 & \left( b^3 \sqrt{\left( \frac{a}{1 + \cos[2 (d + e x)]} + \frac{c}{1 + \cos[2 (d + e x)]} + \frac{a \cos[2 (d + e x)]}{1 + \cos[2 (d + e x)]} - \right. \right. \\
 & \left. \left. \frac{c \cos[2 (d + e x)]}{1 + \cos[2 (d + e x)]} + \frac{b \sin[2 (d + e x)]}{1 + \cos[2 (d + e x)]} \right) \right) \Big/ \\
 & (8 c^2 (-a - c - a \cos[2 (d + e x)] + c \cos[2 (d + e x)] - b \sin[2 (d + e x)])) - \\
 & \left( a b \sqrt{\left( \frac{a}{1 + \cos[2 (d + e x)]} + \frac{c}{1 + \cos[2 (d + e x)]} + \frac{a \cos[2 (d + e x)]}{1 + \cos[2 (d + e x)]} - \right. \right. \\
 & \left. \left. \frac{c \cos[2 (d + e x)]}{1 + \cos[2 (d + e x)]} + \frac{b \sin[2 (d + e x)]}{1 + \cos[2 (d + e x)]} \right) \right) \Big/ \\
 & (2 c (-a - c - a \cos[2 (d + e x)] + c \cos[2 (d + e x)] - b \sin[2 (d + e x)])) + \\
 & \left( b \cos[2 (d + e x)] \sqrt{\left( \frac{a}{1 + \cos[2 (d + e x)]} + \frac{c}{1 + \cos[2 (d + e x)]} + \right. \right. \\
 & \left. \left. \frac{a \cos[2 (d + e x)]}{1 + \cos[2 (d + e x)]} - \frac{c \cos[2 (d + e x)]}{1 + \cos[2 (d + e x)]} + \frac{b \sin[2 (d + e x)]}{1 + \cos[2 (d + e x)]} \right) \right) \Big/ \\
 & (-a - c - a \cos[2 (d + e x)] + c \cos[2 (d + e x)] - b \sin[2 (d + e x)]) -
 \end{aligned}$$

$$\begin{aligned}
 & \left( a \sin [2 (d+e x)] \sqrt{\left( \frac{a}{1+\cos [2 (d+e x)]} + \frac{c}{1+\cos [2 (d+e x)]} + \right.} \right. \\
 & \quad \left. \left. \frac{a \cos [2 (d+e x)]}{1+\cos [2 (d+e x)]} - \frac{c \cos [2 (d+e x)]}{1+\cos [2 (d+e x)]} + \frac{b \sin [2 (d+e x)]}{1+\cos [2 (d+e x)]} \right)} \right) / \\
 & \quad (-a-c-a \cos [2 (d+e x)]+c \cos [2 (d+e x)]-b \sin [2 (d+e x)]) + \\
 & \left( c \sin [2 (d+e x)] \sqrt{\left( \frac{a}{1+\cos [2 (d+e x)]} + \frac{c}{1+\cos [2 (d+e x)]} + \right.} \right. \\
 & \quad \left. \left. \frac{a \cos [2 (d+e x)]}{1+\cos [2 (d+e x)]} - \frac{c \cos [2 (d+e x)]}{1+\cos [2 (d+e x)]} + \frac{b \sin [2 (d+e x)]}{1+\cos [2 (d+e x)]} \right)} \right) / \\
 & \quad (-a-c-a \cos [2 (d+e x)]+c \cos [2 (d+e x)]-b \sin [2 (d+e x)]) \Big) / \\
 & \left( e \left( \left( b \left( 7 b^4-8 b^2 c \left( 5 a+2 c \right)+16 c^2 \left( 3 a^2+4 a c+8 c^2 \right) \right) \left( 2 c \sec [d+e x]^2 + \right. \right. \right. \right. \\
 & \quad \left. \left. \left( \sqrt{c} \left( c \sec [d+e x]^2 \tan [d+e x]+\sec [d+e x]^2 \left( b+c \tan [d+e x] \right) \right) \right) \right) / \right. \right. \\
 & \quad \left. \left. \left( \sqrt{a+\tan [d+e x] \left( b+c \tan [d+e x] \right)} \right) \right) \right) / \right. \\
 & \quad \left. \left( 256 c^{9 / 2} \left( b+2 c \tan [d+e x]+2 \sqrt{c} \sqrt{a+\tan [d+e x] \left( b+c \tan [d+e x] \right)} \right) \right) - \right. \\
 & \quad \left. \left( 64 \left( a-i b-c \right)^2 c^4 \left( i+\tan [d+e x] \right) \left( \left( b \sec [d+e x]^2-2 i c \sec [d+e x]^2 + \right. \right. \right. \right. \\
 & \quad \left. \left. \left( \sqrt{a-i b-c} \left( c \sec [d+e x]^2 \tan [d+e x]+\sec [d+e x]^2 \left( b+c \tan [d+e x] \right) \right) \right) \right) / \right. \right. \\
 & \quad \left. \left. \left( \sqrt{a+\tan [d+e x] \left( b+c \tan [d+e x] \right)} \right) \right) \right) / \right. \\
 & \quad \left. \left( 128 \left( a-i b-c \right)^{3 / 2} c^4 \left( i+\tan [d+e x] \right) \right) - \left( \sec [d+e x]^2 \left( 2 a-2 i c \tan [d+e x] + \right. \right. \right. \\
 & \quad \left. \left. b \left( -i+\tan [d+e x] \right)+2 \sqrt{a-i b-c} \sqrt{a+\tan [d+e x] \left( b+c \tan [d+e x] \right)} \right) \right) \right) / \right. \\
 & \quad \left. \left( 128 \left( a-i b-c \right)^{3 / 2} c^4 \left( i+\tan [d+e x] \right)^2 \right) \right) / \left( 2 a-2 i c \tan [d+e x] + \right. \\
 & \quad \left. b \left( -i+\tan [d+e x] \right)+2 \sqrt{a-i b-c} \sqrt{a+\tan [d+e x] \left( b+c \tan [d+e x] \right)} \right) - \\
 & \quad \left( 64 \left( a+i b-c \right)^2 c^4 \left( -i+\tan [d+e x] \right) \left( \left( b \sec [d+e x]^2+2 i c \sec [d+e x]^2 + \right. \right. \right. \right. \\
 & \quad \left. \left. \left( \sqrt{a+i b-c} \left( c \sec [d+e x]^2 \tan [d+e x]+\sec [d+e x]^2 \left( b+c \tan [d+e x] \right) \right) \right) \right) / \right. \right. \\
 & \quad \left. \left. \left( \sqrt{a+\tan [d+e x] \left( b+c \tan [d+e x] \right)} \right) \right) \right) / \right. \\
 & \quad \left. \left( 128 \left( a+i b-c \right)^{3 / 2} c^4 \left( -i+\tan [d+e x] \right) \right) - \left( \sec [d+e x]^2 \left( 2 a+2 i c \tan [d+e x] + \right. \right. \right. \\
 & \quad \left. \left. b \left( i+\tan [d+e x] \right)+2 \sqrt{a+i b-c} \sqrt{a+\tan [d+e x] \left( b+c \tan [d+e x] \right)} \right) \right) \right) / \right. \\
 & \quad \left. \left( 128 \left( a+i b-c \right)^{3 / 2} c^4 \left( -i+\tan [d+e x] \right)^2 \right) \right) / \left( 2 a+2 i c \tan [d+e x] + \right. \\
 & \quad \left. b \left( i+\tan [d+e x] \right)+2 \sqrt{a+i b-c} \sqrt{a+\tan [d+e x] \left( b+c \tan [d+e x] \right)} \right) \Big) \Big)
 \end{aligned}$$

Problem 2: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \text{Tan}[d + e x]^4 \sqrt{a + b \text{Tan}[d + e x] + c \text{Tan}[d + e x]^2} dx$$

Optimal (type 3, 889 leaves, 19 steps):

$$\begin{aligned}
 & - \left( \left( \sqrt{a^2 + b^2 + c \left( c - \sqrt{a^2 + b^2 - 2ac + c^2} \right)} - a \left( 2c - \sqrt{a^2 + b^2 - 2ac + c^2} \right) \right) \right. \\
 & \quad \left. \text{ArcTan} \left[ \left( b \sqrt{a^2 + b^2 - 2ac + c^2} - \left( b^2 + (a - c) \left( a - c + \sqrt{a^2 + b^2 - 2ac + c^2} \right) \right) \right) \tan[d + ex] \right] \right) / \\
 & \quad \left( \sqrt{2} \left( a^2 + b^2 - 2ac + c^2 \right)^{1/4} \sqrt{a^2 + b^2 + c \left( c - \sqrt{a^2 + b^2 - 2ac + c^2} \right)} - \right. \\
 & \quad \left. a \left( 2c - \sqrt{a^2 + b^2 - 2ac + c^2} \right) \sqrt{a + b \tan[d + ex] + c \tan[d + ex]^2} \right) \Bigg) / \\
 & \quad \left( \sqrt{2} \left( a^2 + b^2 - 2ac + c^2 \right)^{1/4} e \right) + \frac{\sqrt{c} \text{ArcTanh} \left[ \frac{b + 2c \tan[d + ex]}{2\sqrt{c} \sqrt{a + b \tan[d + ex] + c \tan[d + ex]^2}} \right]}{e} + \\
 & \quad \frac{(b^2 - 4ac) \text{ArcTanh} \left[ \frac{b + 2c \tan[d + ex]}{2\sqrt{c} \sqrt{a + b \tan[d + ex] + c \tan[d + ex]^2}} \right]}{8c^{3/2} e} - \\
 & \quad \frac{(b^2 - 4ac) (5b^2 - 4ac) \text{ArcTanh} \left[ \frac{b + 2c \tan[d + ex]}{2\sqrt{c} \sqrt{a + b \tan[d + ex] + c \tan[d + ex]^2}} \right]}{128c^{7/2} e} - \\
 & \quad \left( \sqrt{a^2 + b^2 + c \left( c + \sqrt{a^2 + b^2 - 2ac + c^2} \right)} - a \left( 2c + \sqrt{a^2 + b^2 - 2ac + c^2} \right) \right) \\
 & \quad \text{ArcTanh} \left[ \left( b \sqrt{a^2 + b^2 - 2ac + c^2} + \left( b^2 + (a - c) \left( a - c - \sqrt{a^2 + b^2 - 2ac + c^2} \right) \right) \right) \tan[d + ex] \right] / \\
 & \quad \left( \sqrt{2} \left( a^2 + b^2 - 2ac + c^2 \right)^{1/4} \sqrt{a^2 + b^2 + c \left( c + \sqrt{a^2 + b^2 - 2ac + c^2} \right)} - \right. \\
 & \quad \left. a \left( 2c + \sqrt{a^2 + b^2 - 2ac + c^2} \right) \sqrt{a + b \tan[d + ex] + c \tan[d + ex]^2} \right) \Bigg) / \\
 & \quad \left( \sqrt{2} \left( a^2 + b^2 - 2ac + c^2 \right)^{1/4} e \right) - \frac{(b + 2c \tan[d + ex]) \sqrt{a + b \tan[d + ex] + c \tan[d + ex]^2}}{4ce} + \\
 & \quad \frac{1}{64c^3 e} \\
 & \quad \frac{(5b^2 - 4ac)}{(b + 2c \tan[d + ex])} \\
 & \quad \frac{\sqrt{a + b \tan[d + ex] + c \tan[d + ex]^2} - 5b (a + b \tan[d + ex] + c \tan[d + ex]^2)^{3/2}}{24c^2 e} + \\
 & \quad \frac{\tan[d + ex] (a + b \tan[d + ex] + c \tan[d + ex]^2)^{3/2}}{4ce}
 \end{aligned}$$

Result (type 3, 2537 leaves):

$$\begin{aligned}
 & \frac{1}{e} \sqrt{\frac{a + c + a \cos[2(d + ex)] - c \cos[2(d + ex)] + b \sin[2(d + ex)]}{1 + \cos[2(d + ex)]}} \\
 & \quad \left( \frac{b(15b^2 - 52ac - 56c^2)}{192c^3} + \frac{b \sec[d + ex]^2}{24c} + \frac{1}{96c^2} \sec[d + ex] \right)
 \end{aligned}$$

$$\begin{aligned}
 & \left( -5 b^2 \sin [d+e x]+12 a c \sin [d+e x]-72 c^2 \sin [d+e x] \right)+\frac{1}{4} \sec [d+e x]^2 \tan [d+e x] \Bigg) + \\
 & \left( \left( -64 i \sqrt{a-i b-c} \log \left[ -\left( \left( i \left( 2 a-2 i c \tan [d+e x]+b(-i+\tan [d+e x]) \right)+2 \right. \right. \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \left. \left. \sqrt{a-i b-c} \sqrt{a+\tan [d+e x] (b+c \tan [d+e x])} \right) \right) \right] \right) \right) +64 i \sqrt{a+i b-c} \\
 & \log \left[ \left( i \left( 2 a+2 i c \tan [d+e x]+b(i+\tan [d+e x]) \right)+2 \sqrt{a+i b-c} \right. \right. \\
 & \quad \left. \left. \sqrt{a+\tan [d+e x] (b+c \tan [d+e x])} \right) \right] \Bigg) / \left( 64 (a+i b-c)^{3 / 2} c^3 \right. \\
 & \quad \left. (-i+\tan [d+e x]) \right) \Bigg) +\frac{1}{c^{7 / 2}}\left(-5 b^4+8 b^2 c(3 a+2 c)-16 c^2\left(a^2+4 a c-8 c^2\right)\right) \\
 & \log \left[ b+2 c \tan [d+e x]+2 \sqrt{c} \sqrt{a+\tan [d+e x] (b+c \tan [d+e x])} \right] \Bigg) \\
 & \left( \left( 5 b^4 \sqrt{\left( \frac{a}{1+\cos [2(d+e x)]}+\frac{c}{1+\cos [2(d+e x)]}+\frac{a \cos [2(d+e x)]}{1+\cos [2(d+e x)]}-\right. \right. \right. \\
 & \quad \left. \left. \frac{c \cos [2(d+e x)]}{1+\cos [2(d+e x)]}+\frac{b \sin [2(d+e x)]}{1+\cos [2(d+e x)]} \right) \right) \Bigg) / \\
 & \quad \left( 64 c^3(-a-c-a \cos [2(d+e x)]+c \cos [2(d+e x)]-b \sin [2(d+e x)]) \right) - \\
 & \left( 3 a b^2 \sqrt{\left( \frac{a}{1+\cos [2(d+e x)]}+\frac{c}{1+\cos [2(d+e x)]}+\frac{a \cos [2(d+e x)]}{1+\cos [2(d+e x)]}-\right. \right. \\
 & \quad \left. \left. \frac{c \cos [2(d+e x)]}{1+\cos [2(d+e x)]}+\frac{b \sin [2(d+e x)]}{1+\cos [2(d+e x)]} \right) \right) \Bigg) / \\
 & \quad \left( 8 c^2(-a-c-a \cos [2(d+e x)]+c \cos [2(d+e x)]-b \sin [2(d+e x)]) \right) + \\
 & \left( a^2 \sqrt{\left( \frac{a}{1+\cos [2(d+e x)]}+\frac{c}{1+\cos [2(d+e x)]}+\frac{a \cos [2(d+e x)]}{1+\cos [2(d+e x)]}-\right. \right. \\
 & \quad \left. \left. \frac{c \cos [2(d+e x)]}{1+\cos [2(d+e x)]}+\frac{b \sin [2(d+e x)]}{1+\cos [2(d+e x)]} \right) \right) \Bigg) / \\
 & \quad \left( 4 c(-a-c-a \cos [2(d+e x)]+c \cos [2(d+e x)]-b \sin [2(d+e x)]) \right) - \\
 & \left( b^2 \sqrt{\left( \frac{a}{1+\cos [2(d+e x)]}+\frac{c}{1+\cos [2(d+e x)]}+\frac{a \cos [2(d+e x)]}{1+\cos [2(d+e x)]}-\right. \right. \\
 & \quad \left. \left. \frac{c \cos [2(d+e x)]}{1+\cos [2(d+e x)]}+\frac{b \sin [2(d+e x)]}{1+\cos [2(d+e x)]} \right) \right) \Bigg) / \\
 & \quad \left( 4 c(-a-c-a \cos [2(d+e x)]+c \cos [2(d+e x)]-b \sin [2(d+e x)]) \right) - \\
 & \left( c \sqrt{\left( \frac{a}{1+\cos [2(d+e x)]}+\frac{c}{1+\cos [2(d+e x)]}+\frac{a \cos [2(d+e x)]}{1+\cos [2(d+e x)]}-\right. \right. \\
 & \quad \left. \left. \frac{c \cos [2(d+e x)]}{1+\cos [2(d+e x)]}+\frac{b \sin [2(d+e x)]}{1+\cos [2(d+e x)]} \right) \right) \Bigg) / \\
 & \quad \left( -a-c-a \cos [2(d+e x)]+c \cos [2(d+e x)]-b \sin [2(d+e x)] \right) -
 \end{aligned}$$

$$\begin{aligned}
 & \left( a \cos [2 (d+e x)] \sqrt{\left( \frac{a}{1+\cos [2 (d+e x)]} + \frac{c}{1+\cos [2 (d+e x)]} + \right.} \right. \\
 & \quad \left. \frac{a \cos [2 (d+e x)]}{1+\cos [2 (d+e x)]} - \frac{c \cos [2 (d+e x)]}{1+\cos [2 (d+e x)]} + \frac{b \sin [2 (d+e x)]}{1+\cos [2 (d+e x)]} \right) / \\
 & \quad (-a-c-a \cos [2 (d+e x)]+c \cos [2 (d+e x)]-b \sin [2 (d+e x)]) + \\
 & \left( c \cos [2 (d+e x)] \sqrt{\left( \frac{a}{1+\cos [2 (d+e x)]} + \frac{c}{1+\cos [2 (d+e x)]} + \right.} \right. \\
 & \quad \left. \frac{a \cos [2 (d+e x)]}{1+\cos [2 (d+e x)]} - \frac{c \cos [2 (d+e x)]}{1+\cos [2 (d+e x)]} + \frac{b \sin [2 (d+e x)]}{1+\cos [2 (d+e x)]} \right) / \\
 & \quad (-a-c-a \cos [2 (d+e x)]+c \cos [2 (d+e x)]-b \sin [2 (d+e x)]) - \\
 & \left( b \sin [2 (d+e x)] \sqrt{\left( \frac{a}{1+\cos [2 (d+e x)]} + \frac{c}{1+\cos [2 (d+e x)]} + \right.} \right. \\
 & \quad \left. \frac{a \cos [2 (d+e x)]}{1+\cos [2 (d+e x)]} - \frac{c \cos [2 (d+e x)]}{1+\cos [2 (d+e x)]} + \frac{b \sin [2 (d+e x)]}{1+\cos [2 (d+e x)]} \right) / \\
 & \quad (-a-c-a \cos [2 (d+e x)]+c \cos [2 (d+e x)]-b \sin [2 (d+e x)]) \Big) / \\
 & \left( e \left( \left( (-5 b^4+8 b^2 c (3 a+2 c)-16 c^2 (a^2+4 a c-8 c^2)) \left( 2 c \sec [d+e x]^2 + \right. \right. \right. \right. \\
 & \quad \left. \left. \left( \sqrt{c} (c \sec [d+e x]^2 \tan [d+e x]+ \sec [d+e x]^2 (b+c \tan [d+e x])) \right) \right) \right) / \\
 & \quad \left( \sqrt{a+\tan [d+e x]} (b+c \tan [d+e x]) \right) \Big) / \\
 & \left( c^{7/2} (b+2 c \tan [d+e x]+2 \sqrt{c} \sqrt{a+\tan [d+e x]} (b+c \tan [d+e x])) \right) + \\
 & \left( 4096 (a-i b-c)^2 c^3 (i+\tan [d+e x]) \left( - \left( \left( i (b \sec [d+e x]^2-2 i c \sec [d+e x]^2 + \right. \right. \right. \right. \right. \\
 & \quad \left. \left. \left( \sqrt{a-i b-c} (c \sec [d+e x]^2 \tan [d+e x]+ \sec [d+e x]^2 (b+ \right. \right. \right. \right. \\
 & \quad \left. \left. \left. c \tan [d+e x]) \right) \right) \right) / \left( \sqrt{a+\tan [d+e x]} (b+c \tan [d+e x]) \right) \right) \Big) / (64 \\
 & \quad (a-i b-c)^{3/2} c^3 (i+\tan [d+e x])) + \left( i \sec [d+e x]^2 (2 a-2 i c \tan [d+e x]+ \right. \\
 & \quad \left. b (-i+\tan [d+e x])+2 \sqrt{a-i b-c} \sqrt{a+\tan [d+e x]} (b+c \tan [d+e x]) \right) \Big) / \\
 & \left( 64 (a-i b-c)^{3/2} c^3 (i+\tan [d+e x])^2 \right) \Big) / \left( 2 a-2 i c \tan [d+e x]+ \right. \\
 & \quad \left. b (-i+\tan [d+e x])+2 \sqrt{a-i b-c} \sqrt{a+\tan [d+e x]} (b+c \tan [d+e x]) \right) + \\
 & \left( 4096 (a+i b-c)^2 c^3 (-i+\tan [d+e x]) \left( \left( i (b \sec [d+e x]^2+2 i c \sec [d+e x]^2 + \right. \right. \right. \right. \right. \\
 & \quad \left. \left. \left( \sqrt{a+i b-c} (c \sec [d+e x]^2 \tan [d+e x]+ \sec [d+e x]^2 (b+c \tan [d+e x])) \right) \right) \right) / \\
 & \quad \left( \sqrt{a+\tan [d+e x]} (b+c \tan [d+e x]) \right) \Big) \Big) / \\
 & \left( 64 (a+i b-c)^{3/2} c^3 (-i+\tan [d+e x]) \right) - \left( i \sec [d+e x]^2 (2 a+2 i c \tan [d+e x]+ \right.
 \end{aligned}$$



$$\frac{b \left( i + \tan [d + e x] \right) + 2 \sqrt{a + i b - c} \sqrt{a + \tan [d + e x] \left( b + c \tan [d + e x] \right)}}{\left( 64 \left( a + i b - c \right)^{3/2} c^3 \left( -i + \tan [d + e x] \right)^2 \right)} \left( 2 a + 2 i c \tan [d + e x] + \right. \\ \left. b \left( i + \tan [d + e x] \right) + 2 \sqrt{a + i b - c} \sqrt{a + \tan [d + e x] \left( b + c \tan [d + e x] \right)} \right)$$

**Problem 3: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \tan [d + e x]^3 \sqrt{a + b \tan [d + e x] + c \tan [d + e x]^2} dx$$

Optimal (type 3, 748 leaves, 16 steps):

$$\frac{- \left( \left( \sqrt{a^2 + b^2 + c} \left( c + \sqrt{a^2 + b^2 - 2 a c + c^2} \right) - a \left( 2 c + \sqrt{a^2 + b^2 - 2 a c + c^2} \right) \right) \right. \\ \left. \text{ArcTan} \left[ \left( b^2 + (a - c) \left( a - c - \sqrt{a^2 + b^2 - 2 a c + c^2} \right) - b \sqrt{a^2 + b^2 - 2 a c + c^2} \tan [d + e x] \right) \right] \right. \\ \left. \left( \sqrt{2} \left( a^2 + b^2 - 2 a c + c^2 \right)^{1/4} \sqrt{a^2 + b^2 + c} \left( c + \sqrt{a^2 + b^2 - 2 a c + c^2} \right) - \right. \right. \\ \left. \left. a \left( 2 c + \sqrt{a^2 + b^2 - 2 a c + c^2} \right) \sqrt{a + b \tan [d + e x] + c \tan [d + e x]^2} \right) \right] \right) / \\ \left( \sqrt{2} \left( a^2 + b^2 - 2 a c + c^2 \right)^{1/4} e \right) - \frac{b \text{ArcTanh} \left[ \frac{b + 2 c \tan [d + e x]}{2 \sqrt{c} \sqrt{a + b \tan [d + e x] + c \tan [d + e x]^2}} \right]}{2 \sqrt{c} e} + \\ \frac{b \left( b^2 - 4 a c \right) \text{ArcTanh} \left[ \frac{b + 2 c \tan [d + e x]}{2 \sqrt{c} \sqrt{a + b \tan [d + e x] + c \tan [d + e x]^2}} \right]}{16 c^{5/2} e} + \\ \left( \left( \sqrt{a^2 + b^2 + c} \left( c - \sqrt{a^2 + b^2 - 2 a c + c^2} \right) - a \left( 2 c - \sqrt{a^2 + b^2 - 2 a c + c^2} \right) \right) \right. \\ \left. \text{ArcTanh} \left[ \left( b^2 + (a - c) \left( a - c + \sqrt{a^2 + b^2 - 2 a c + c^2} \right) + b \sqrt{a^2 + b^2 - 2 a c + c^2} \tan [d + e x] \right) \right] \right. \\ \left. \left( \sqrt{2} \left( a^2 + b^2 - 2 a c + c^2 \right)^{1/4} \sqrt{a^2 + b^2 + c} \left( c - \sqrt{a^2 + b^2 - 2 a c + c^2} \right) - \right. \right. \\ \left. \left. a \left( 2 c - \sqrt{a^2 + b^2 - 2 a c + c^2} \right) \sqrt{a + b \tan [d + e x] + c \tan [d + e x]^2} \right) \right] \right) / \\ \left( \sqrt{2} \left( a^2 + b^2 - 2 a c + c^2 \right)^{1/4} e \right) - \frac{\sqrt{a + b \tan [d + e x] + c \tan [d + e x]^2}}{e} - \\ \frac{b \left( b + 2 c \tan [d + e x] \right) \sqrt{a + b \tan [d + e x] + c \tan [d + e x]^2}}{8 c^2 e} + \\ \frac{\left( a + b \tan [d + e x] + c \tan [d + e x]^2 \right)^{3/2}}{3 c e}$$

Result (type 3, 1960 leaves):

$$\begin{aligned}
 & \frac{1}{e} \sqrt{\frac{a+c+a \cos [2(d+e x)]-c \cos [2(d+e x)]+b \sin [2(d+e x)]}{1+\cos [2(d+e x)]}} \\
 & \left( -\frac{3 b^2-8 a c+32 c^2}{24 c^2} + \frac{1}{3} \sec [d+e x]^2 + \frac{b \tan [d+e x]}{12 c} \right) + \\
 & \left( \left( 8 \sqrt{a+i b-c} \operatorname{Log}\left[(-2 a-i b-(b+2 i c) \tan [d+e x]-2 \sqrt{a+i b-c} \right. \right. \right. \\
 & \quad \left. \left. \left. \sqrt{a+b \tan [d+e x]+c \tan [d+e x]^2}\right) / \left( 8(a+i b-c)^{3 / 2} c^2(-i+\tan [d+e x])\right) \right] \right) + \\
 & \quad 8 \sqrt{a-i b-c} \operatorname{Log}\left[(-2 a+i b-b \tan [d+e x]+2 i c \tan [d+e x]- \right. \\
 & \quad \left. 2 \sqrt{a-i b-c} \sqrt{a+\tan [d+e x](b+c \tan [d+e x])}\right) / \\
 & \quad \left( 8(a-i b-c)^{3 / 2} c^2(i+\tan [d+e x])\right) \left. \right] + \frac{1}{c^{5 / 2}} b\left(b^2-4 c(a+2 c)\right) \\
 & \quad \left. \operatorname{Log}\left[b+2 c \tan [d+e x]+2 \sqrt{c} \sqrt{a+\tan [d+e x](b+c \tan [d+e x])}\right] \right) \\
 & \left( -\left( \left( b^3 \sqrt{\left( \frac{a}{1+\cos [2(d+e x)]} + \frac{c}{1+\cos [2(d+e x)]} + \frac{a \cos [2(d+e x)]}{1+\cos [2(d+e x)]} - \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \frac{c \cos [2(d+e x)]}{1+\cos [2(d+e x)]} + \frac{b \sin [2(d+e x)]}{1+\cos [2(d+e x)]} \right) \right) / \right. \\
 & \quad \left. \left. \left. \left( 8 c^2(-a-c-a \cos [2(d+e x)]+c \cos [2(d+e x)]-b \sin [2(d+e x)]) \right) \right) \right) + \\
 & \quad \left( a b \sqrt{\left( \frac{a}{1+\cos [2(d+e x)]} + \frac{c}{1+\cos [2(d+e x)]} + \frac{a \cos [2(d+e x)]}{1+\cos [2(d+e x)]} - \right. \right. \\
 & \quad \left. \left. \frac{c \cos [2(d+e x)]}{1+\cos [2(d+e x)]} + \frac{b \sin [2(d+e x)]}{1+\cos [2(d+e x)]} \right) \right) / \\
 & \quad \left. \left. \left. \left( 2 c(-a-c-a \cos [2(d+e x)]+c \cos [2(d+e x)]-b \sin [2(d+e x)]) \right) \right) - \right. \\
 & \quad \left( b \cos [2(d+e x)] \sqrt{\left( \frac{a}{1+\cos [2(d+e x)]} + \frac{c}{1+\cos [2(d+e x)]} + \right. \right. \\
 & \quad \left. \left. \frac{a \cos [2(d+e x)]}{1+\cos [2(d+e x)]} - \frac{c \cos [2(d+e x)]}{1+\cos [2(d+e x)]} + \frac{b \sin [2(d+e x)]}{1+\cos [2(d+e x)]} \right) \right) / \\
 & \quad \left. \left. \left. \left( -a-c-a \cos [2(d+e x)]+c \cos [2(d+e x)]-b \sin [2(d+e x)] \right) \right) + \right. \\
 & \quad \left( a \sin [2(d+e x)] \sqrt{\left( \frac{a}{1+\cos [2(d+e x)]} + \frac{c}{1+\cos [2(d+e x)]} + \right. \right. \\
 & \quad \left. \left. \frac{a \cos [2(d+e x)]}{1+\cos [2(d+e x)]} - \frac{c \cos [2(d+e x)]}{1+\cos [2(d+e x)]} + \frac{b \sin [2(d+e x)]}{1+\cos [2(d+e x)]} \right) \right) / \\
 & \quad \left. \left. \left. \left( -a-c-a \cos [2(d+e x)]+c \cos [2(d+e x)]-b \sin [2(d+e x)] \right) \right) - \right. \\
 & \quad \left( c \sin [2(d+e x)] \sqrt{\left( \frac{a}{1+\cos [2(d+e x)]} + \frac{c}{1+\cos [2(d+e x)]} + \right. \right. \\
 & \quad \left. \left. \frac{a \cos [2(d+e x)]}{1+\cos [2(d+e x)]} - \frac{c \cos [2(d+e x)]}{1+\cos [2(d+e x)]} + \frac{b \sin [2(d+e x)]}{1+\cos [2(d+e x)]} \right) \right) / \\
 & \quad \left. \left. \left. \left( -a-c-a \cos [2(d+e x)]+c \cos [2(d+e x)]-b \sin [2(d+e x)] \right) \right) \right)
 \end{aligned}$$

$$\begin{aligned}
 & \frac{a \cos[2(d+ex)]}{1 + \cos[2(d+ex)]} - \frac{c \cos[2(d+ex)]}{1 + \cos[2(d+ex)]} + \frac{b \sin[2(d+ex)]}{1 + \cos[2(d+ex)]} \bigg) \bigg/ \\
 & (-a - c - a \cos[2(d+ex)] + c \cos[2(d+ex)] - b \sin[2(d+ex)]) \bigg) \bigg/ \\
 & \left( e \left( \left( b(b^2 - 4c(a+2c)) \left( 2c \sec[d+ex]^2 + \left( \sqrt{c} (c \sec[d+ex]^2 \tan[d+ex] + \sec[d+ex]^2 \right. \right. \right. \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \left. (b + c \tan[d+ex]) \right) \right) \right) \right) \bigg/ \left( \sqrt{a + \tan[d+ex] (b + c \tan[d+ex])} \right) \right) \bigg) \bigg/ \\
 & \left( c^{5/2} \left( b + 2c \tan[d+ex] + 2\sqrt{c} \sqrt{a + \tan[d+ex] (b + c \tan[d+ex])} \right) \right) + \\
 & \left( 64(a + ib - c)^2 c^2 (-i + \tan[d+ex]) \right. \\
 & \quad \left( \left( -(b + 2ic) \sec[d+ex]^2 - \left( \sqrt{a + ib - c} (b \sec[d+ex]^2 + \right. \right. \right. \\
 & \quad \left. \left. \left. 2c \sec[d+ex]^2 \tan[d+ex]) \right) \right) \bigg/ \left( \sqrt{a + b \tan[d+ex] + c \tan[d+ex]^2} \right) \right) \bigg) \bigg/ \\
 & \left( 8(a + ib - c)^{3/2} c^2 (-i + \tan[d+ex]) \right) - \left( \sec[d+ex]^2 \left( -2a - ib - \right. \right. \\
 & \quad \left. \left. (b + 2ic) \tan[d+ex] - 2\sqrt{a + ib - c} \sqrt{a + b \tan[d+ex] + c \tan[d+ex]^2} \right) \right) \bigg) \bigg/ \\
 & \left( 8(a + ib - c)^{3/2} c^2 (-i + \tan[d+ex])^2 \right) \bigg) \bigg/ \\
 & \left( -2a - ib - (b + 2ic) \tan[d+ex] - 2\sqrt{a + ib - c} \sqrt{a + b \tan[d+ex] + c \tan[d+ex]^2} \right) + \\
 & \left( 64(a - ib - c)^2 c^2 (i + \tan[d+ex]) \left( \left( -b \sec[d+ex]^2 + 2ic \sec[d+ex]^2 - \right. \right. \right. \\
 & \quad \left. \left. \left. \left( \sqrt{a - ib - c} (c \sec[d+ex]^2 \tan[d+ex] + \sec[d+ex]^2 (b + c \tan[d+ex])) \right) \right) \right) \bigg) \bigg/ \\
 & \left( \sqrt{a + \tan[d+ex] (b + c \tan[d+ex])} \right) \bigg) \bigg/ \\
 & \left( 8(a - ib - c)^{3/2} c^2 (i + \tan[d+ex]) \right) - \left( \sec[d+ex]^2 \left( -2a + ib - b \tan[d+ex] + \right. \right. \\
 & \quad \left. \left. 2ic \tan[d+ex] - 2\sqrt{a - ib - c} \sqrt{a + \tan[d+ex] (b + c \tan[d+ex])} \right) \right) \bigg) \bigg/ \\
 & \left( 8(a - ib - c)^{3/2} c^2 (i + \tan[d+ex])^2 \right) \bigg) \bigg/ \left( -2a + ib - b \tan[d+ex] + \right. \\
 & \quad \left. 2ic \tan[d+ex] - 2\sqrt{a - ib - c} \sqrt{a + \tan[d+ex] (b + c \tan[d+ex])} \right) \bigg) \bigg)
 \end{aligned}$$

**Problem 4: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \tan[d+ex]^2 \sqrt{a + b \tan[d+ex] + c \tan[d+ex]^2} dx$$

Optimal (type 3, 676 leaves, 10 steps):

$$\begin{aligned}
 & \left( \sqrt{\left( a^2 + b^2 + c \left( c - \sqrt{a^2 + b^2 - 2 a c + c^2} \right) - a \left( 2 c - \sqrt{a^2 + b^2 - 2 a c + c^2} \right) \right)} \right. \\
 & \quad \text{ArcTan} \left[ \left( b \sqrt{a^2 + b^2 - 2 a c + c^2} - \right. \right. \\
 & \quad \quad \left. \left. \left( a^2 + b^2 + c \left( c - \sqrt{a^2 + b^2 - 2 a c + c^2} \right) - a \left( 2 c - \sqrt{a^2 + b^2 - 2 a c + c^2} \right) \right) \text{Tan} [d + e x] \right) \right] / \\
 & \quad \left( \sqrt{2} \left( a^2 + b^2 - 2 a c + c^2 \right)^{1/4} \sqrt{\left( a^2 + b^2 + c \left( c - \sqrt{a^2 + b^2 - 2 a c + c^2} \right) - \right. \right. \\
 & \quad \quad \left. \left. a \left( 2 c - \sqrt{a^2 + b^2 - 2 a c + c^2} \right) \right) \sqrt{a + b \text{Tan} [d + e x] + c \text{Tan} [d + e x]^2} \right) \right] / \\
 & \quad \left( \sqrt{2} \left( a^2 + b^2 - 2 a c + c^2 \right)^{1/4} e \right) - \frac{\left( b^2 - 4 (a - 2 c) c \right) \text{ArcTanh} \left[ \frac{b + 2 c \text{Tan} [d + e x]}{2 \sqrt{c} \sqrt{a + b \text{Tan} [d + e x] + c \text{Tan} [d + e x]^2}} \right]}{8 c^{3/2} e} + \\
 & \left( \sqrt{\left( a^2 + b^2 + c \left( c + \sqrt{a^2 + b^2 - 2 a c + c^2} \right) - a \left( 2 c + \sqrt{a^2 + b^2 - 2 a c + c^2} \right) \right)} \right) \\
 & \quad \text{ArcTanh} \left[ \left( b \sqrt{a^2 + b^2 - 2 a c + c^2} + \right. \right. \\
 & \quad \quad \left. \left. \left( a^2 + b^2 + c \left( c + \sqrt{a^2 + b^2 - 2 a c + c^2} \right) - a \left( 2 c + \sqrt{a^2 + b^2 - 2 a c + c^2} \right) \right) \text{Tan} [d + e x] \right) \right] / \\
 & \quad \left( \sqrt{2} \left( a^2 + b^2 - 2 a c + c^2 \right)^{1/4} \sqrt{\left( a^2 + b^2 + c \left( c + \sqrt{a^2 + b^2 - 2 a c + c^2} \right) - \right. \right. \\
 & \quad \quad \left. \left. a \left( 2 c + \sqrt{a^2 + b^2 - 2 a c + c^2} \right) \right) \sqrt{a + b \text{Tan} [d + e x] + c \text{Tan} [d + e x]^2} \right) \right] / \\
 & \quad \left( \sqrt{2} \left( a^2 + b^2 - 2 a c + c^2 \right)^{1/4} e \right) + \frac{\left( b + 2 c \text{Tan} [d + e x] \right) \sqrt{a + b \text{Tan} [d + e x] + c \text{Tan} [d + e x]^2}}{4 c e}
 \end{aligned}$$

Result (type 3, 1958 leaves):

$$\begin{aligned}
 & \frac{\sqrt{\frac{a + c + a \text{Cos} [2 (d + e x)] - c \text{Cos} [2 (d + e x)] + b \text{Sin} [2 (d + e x)]}{1 + \text{Cos} [2 (d + e x)]}} \left( \frac{b}{4 c} + \frac{1}{2} \text{Tan} [d + e x] \right)}{e} + \\
 & \left( \left( 4 \text{i} \sqrt{a - \text{i} b - c} \text{Log} \left[ \left( 2 \text{i} a + b + \text{i} b \text{Tan} [d + e x] + 2 c \text{Tan} [d + e x] + 2 \text{i} \sqrt{a - \text{i} b - c} \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \sqrt{a + \text{Tan} [d + e x] (b + c \text{Tan} [d + e x])} \right) \right] / \left( 4 (a - \text{i} b - c)^{3/2} c (\text{i} + \text{Tan} [d + e x]) \right) \right) - \\
 & \quad 4 \text{i} \sqrt{a + \text{i} b - c} \text{Log} \left[ \left( -2 \text{i} a + b - \text{i} b \text{Tan} [d + e x] + 2 c \text{Tan} [d + e x] - 2 \text{i} \sqrt{a + \text{i} b - c} \right. \right. \\
 & \quad \left. \left. \sqrt{a + \text{Tan} [d + e x] (b + c \text{Tan} [d + e x])} \right) \right] / \left( 4 (a + \text{i} b - c)^{3/2} c (-\text{i} + \text{Tan} [d + e x]) \right) \right) - \\
 & \quad \frac{1}{c^{3/2}} (b^2 - 4 a c + 8 c^2) \text{Log} [b + 2 c \text{Tan} [d + e x] + 2 \sqrt{c} \sqrt{a + \text{Tan} [d + e x] (b + c \text{Tan} [d + e x])}] \right) \\
 & \left( \left( b^2 \sqrt{\left( \frac{a}{1 + \text{Cos} [2 (d + e x)]} + \frac{c}{1 + \text{Cos} [2 (d + e x)]} + \frac{a \text{Cos} [2 (d + e x)]}{1 + \text{Cos} [2 (d + e x)]} \right)} - \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & \left. \left. \left. \frac{c \cos[2(d+ex)]}{1+\cos[2(d+ex)]} + \frac{b \sin[2(d+ex)]}{1+\cos[2(d+ex)]} \right) \right) / \\
 & (4c(-a-c-a\cos[2(d+ex)]+c\cos[2(d+ex)]-b\sin[2(d+ex)])) + \\
 & \left( c \sqrt{\left( \frac{a}{1+\cos[2(d+ex)]} + \frac{c}{1+\cos[2(d+ex)]} + \frac{a\cos[2(d+ex)]}{1+\cos[2(d+ex)]} - \right. \right. \\
 & \left. \left. \frac{c \cos[2(d+ex)]}{1+\cos[2(d+ex)]} + \frac{b \sin[2(d+ex)]}{1+\cos[2(d+ex)]} \right) \right) / \\
 & (-a-c-a\cos[2(d+ex)]+c\cos[2(d+ex)]-b\sin[2(d+ex)]) + \\
 & \left( a \cos[2(d+ex)] \sqrt{\left( \frac{a}{1+\cos[2(d+ex)]} + \frac{c}{1+\cos[2(d+ex)]} + \right. \right. \\
 & \left. \left. \frac{a\cos[2(d+ex)]}{1+\cos[2(d+ex)]} - \frac{c\cos[2(d+ex)]}{1+\cos[2(d+ex)]} + \frac{b\sin[2(d+ex)]}{1+\cos[2(d+ex)]} \right) \right) / \\
 & (-a-c-a\cos[2(d+ex)]+c\cos[2(d+ex)]-b\sin[2(d+ex)]) - \\
 & \left( c \cos[2(d+ex)] \sqrt{\left( \frac{a}{1+\cos[2(d+ex)]} + \frac{c}{1+\cos[2(d+ex)]} + \right. \right. \\
 & \left. \left. \frac{a\cos[2(d+ex)]}{1+\cos[2(d+ex)]} - \frac{c\cos[2(d+ex)]}{1+\cos[2(d+ex)]} + \frac{b\sin[2(d+ex)]}{1+\cos[2(d+ex)]} \right) \right) / \\
 & (-a-c-a\cos[2(d+ex)]+c\cos[2(d+ex)]-b\sin[2(d+ex)]) + \\
 & \left( b \sin[2(d+ex)] \sqrt{\left( \frac{a}{1+\cos[2(d+ex)]} + \frac{c}{1+\cos[2(d+ex)]} + \right. \right. \\
 & \left. \left. \frac{a\cos[2(d+ex)]}{1+\cos[2(d+ex)]} - \frac{c\cos[2(d+ex)]}{1+\cos[2(d+ex)]} + \frac{b\sin[2(d+ex)]}{1+\cos[2(d+ex)]} \right) \right) / \\
 & (-a-c-a\cos[2(d+ex)]+c\cos[2(d+ex)]-b\sin[2(d+ex)]) \Big) / \\
 & \left( e \left( - \left( \left( (b^2 - 4ac + 8c^2) \left( 2c \sec[d+ex]^2 + (\sqrt{c} (c \sec[d+ex]^2 \tan[d+ex] + \sec[d+ex])^2 \right. \right. \right. \right. \right. \right. \\
 & \left. \left. \left. \left. \left. (b+c \tan[d+ex]) \right) \right) \right) / \left( \sqrt{a + \tan[d+ex]} (b+c \tan[d+ex]) \right) \right) \right) \right) / \\
 & \left( c^{3/2} \left( b + 2c \tan[d+ex] + 2\sqrt{c} \sqrt{a + \tan[d+ex]} (b+c \tan[d+ex]) \right) \right) + \\
 & \left( 16i(a-ib-c)^2 c (i + \tan[d+ex]) \left( (ib \sec[d+ex]^2 + 2c \sec[d+ex]^2 + \right. \right. \\
 & \left. \left. (i \sqrt{a-ib-c} (c \sec[d+ex]^2 \tan[d+ex] + \sec[d+ex]^2 (b+c \tan[d+ex]))) \right) \right) / \\
 & \left( \sqrt{a + \tan[d+ex]} (b+c \tan[d+ex]) \right) \Big) / \\
 & \left( 4(a-ib-c)^{3/2} c (i + \tan[d+ex]) \right) - \left( \sec[d+ex]^2 \left( 2ia + b + ib \tan[d+ex] + \right. \right. \\
 & \left. \left. 2c \tan[d+ex] + 2i \sqrt{a-ib-c} \sqrt{a + \tan[d+ex]} (b+c \tan[d+ex]) \right) \right) / \\
 & \left( 4(a-ib-c)^{3/2} c (i + \tan[d+ex])^2 \right) \Big) / \left( 2ia + b + ib \tan[d+ex] + \right.
 \end{aligned}$$

$$\begin{aligned}
 & 2 c \operatorname{Tan}[d+e x]+2 i \sqrt{a-i b-c} \sqrt{a+\operatorname{Tan}[d+e x](b+c \operatorname{Tan}[d+e x])}- \\
 & \left(16 i(a+i b-c)^2 c(-i+\operatorname{Tan}[d+e x])\left(\left(-i b \operatorname{Sec}[d+e x]^2+2 c \operatorname{Sec}[d+e x]^2-\right.\right.\right. \\
 & \left.\left.\left(i \sqrt{a+i b-c}\left(c \operatorname{Sec}[d+e x]^2 \operatorname{Tan}[d+e x]+\operatorname{Sec}[d+e x]^2(b+c \operatorname{Tan}[d+e x])\right)\right)\right)\right) / \\
 & \left(\sqrt{a+\operatorname{Tan}[d+e x](b+c \operatorname{Tan}[d+e x])}\right) / \\
 & \left(4(a+i b-c)^{3 / 2} c(-i+\operatorname{Tan}[d+e x])\right)-\left(\operatorname{Sec}[d+e x]^2\left(-2 i a+b-i b \operatorname{Tan}[d+e x]+\right.\right. \\
 & \left.\left.2 c \operatorname{Tan}[d+e x]-2 i \sqrt{a+i b-c} \sqrt{a+\operatorname{Tan}[d+e x](b+c \operatorname{Tan}[d+e x])}\right)\right) / \\
 & \left(4(a+i b-c)^{3 / 2} c(-i+\operatorname{Tan}[d+e x])^2\right) / \left(-2 i a+b-i b \operatorname{Tan}[d+e x]+\right. \\
 & \left.2 c \operatorname{Tan}[d+e x]-2 i \sqrt{a+i b-c} \sqrt{a+\operatorname{Tan}[d+e x](b+c \operatorname{Tan}[d+e x])}\right)
 \end{aligned}$$

**Problem 5: Result unnecessarily involves imaginary or complex numbers.**

$$\int \operatorname{Tan}[d+e x] \sqrt{a+b \operatorname{Tan}[d+e x]+c \operatorname{Tan}[d+e x]^2} d x$$

Optimal (type 3, 601 leaves, 10 steps):

$$\begin{aligned}
 & \left(\sqrt{\left(a^2+b^2+c\left(c+\sqrt{a^2+b^2-2 a c+c^2}\right)-a\left(2 c+\sqrt{a^2+b^2-2 a c+c^2}\right)\right)}\right. \\
 & \left.\operatorname{ArcTan}\left[\left(b^2+(a-c)\left(a-c-\sqrt{a^2+b^2-2 a c+c^2}\right)-b \sqrt{a^2+b^2-2 a c+c^2} \operatorname{Tan}[d+e x]\right) / \right.\right. \\
 & \left.\left(\sqrt{2}\left(a^2+b^2-2 a c+c^2\right)^{1 / 4} \sqrt{\left(a^2+b^2+c\left(c+\sqrt{a^2+b^2-2 a c+c^2}\right)-\right.\right.\right. \\
 & \left.\left.\left.a\left(2 c+\sqrt{a^2+b^2-2 a c+c^2}\right)\right)\right) \sqrt{a+b \operatorname{Tan}[d+e x]+c \operatorname{Tan}[d+e x]^2}\right)\right] / \\
 & \left(\sqrt{2}\left(a^2+b^2-2 a c+c^2\right)^{1 / 4} e\right)+\frac{b \operatorname{ArcTanh}\left[\frac{b+2 c \operatorname{Tan}[d+e x]}{2 \sqrt{c} \sqrt{a+b \operatorname{Tan}[d+e x]+c \operatorname{Tan}[d+e x]^2}}\right]}{2 \sqrt{c} e}- \\
 & \left(\sqrt{\left(a^2+b^2+c\left(c-\sqrt{a^2+b^2-2 a c+c^2}\right)-a\left(2 c-\sqrt{a^2+b^2-2 a c+c^2}\right)\right)}\right. \\
 & \left.\operatorname{ArcTanh}\left[\left(b^2+(a-c)\left(a-c+\sqrt{a^2+b^2-2 a c+c^2}\right)+b \sqrt{a^2+b^2-2 a c+c^2} \operatorname{Tan}[d+e x]\right) / \right.\right. \\
 & \left.\left(\sqrt{2}\left(a^2+b^2-2 a c+c^2\right)^{1 / 4} \sqrt{\left(a^2+b^2+c\left(c-\sqrt{a^2+b^2-2 a c+c^2}\right)-\right.\right.\right. \\
 & \left.\left.\left.a\left(2 c-\sqrt{a^2+b^2-2 a c+c^2}\right)\right)\right) \sqrt{a+b \operatorname{Tan}[d+e x]+c \operatorname{Tan}[d+e x]^2}\right)\right] / \\
 & \left(\sqrt{2}\left(a^2+b^2-2 a c+c^2\right)^{1 / 4} e\right)+\frac{\sqrt{a+b \operatorname{Tan}[d+e x]+c \operatorname{Tan}[d+e x]^2}}{e}
 \end{aligned}$$

Result (type 3, 333 leaves):

$$\frac{1}{2e} \left( -\sqrt{a-ib-c} \operatorname{Log} \left[ \left( 2a-2ic \operatorname{Tan}[d+ex] + b(-i+\operatorname{Tan}[d+ex]) + 2\sqrt{a-ib-c} \right. \right. \right. \\ \left. \left. \left. \sqrt{a+\operatorname{Tan}[d+ex](b+c \operatorname{Tan}[d+ex])} \right) \right] / \left( (a-ib-c)^{3/2} (i+\operatorname{Tan}[d+ex]) \right) \right) - \\ \sqrt{a+ib-c} \operatorname{Log} \left[ \left( 2a+2ic \operatorname{Tan}[d+ex] + b(i+\operatorname{Tan}[d+ex]) + 2\sqrt{a+ib-c} \right. \right. \\ \left. \left. \left. \sqrt{a+\operatorname{Tan}[d+ex](b+c \operatorname{Tan}[d+ex])} \right) \right] / \left( (a+ib-c)^{3/2} (-i+\operatorname{Tan}[d+ex]) \right) \right] + \\ \frac{1}{\sqrt{c}} b \operatorname{Log} \left[ b+2c \operatorname{Tan}[d+ex] + 2\sqrt{c} \sqrt{a+\operatorname{Tan}[d+ex](b+c \operatorname{Tan}[d+ex])} \right] + \\ \frac{\sqrt{\frac{a+c+a \operatorname{Cos}[2(d+ex)]-c \operatorname{Cos}[2(d+ex)]+b \operatorname{Sin}[2(d+ex)]}{1+\operatorname{Cos}[2(d+ex)]}}}{e}$$

**Problem 6: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{a+b \operatorname{Tan}[d+ex]+c \operatorname{Tan}[d+ex]^2} dx$$

Optimal (type 3, 574 leaves, 9 steps):

$$- \left( \left( \sqrt{a^2+b^2+c(c-\sqrt{a^2+b^2-2ac+c^2})} - a(2c-\sqrt{a^2+b^2-2ac+c^2}) \right) \right. \\ \left. \operatorname{ArcTan} \left[ \left( b\sqrt{a^2+b^2-2ac+c^2} - (b^2+(a-c)(a-c+\sqrt{a^2+b^2-2ac+c^2})) \operatorname{Tan}[d+ex] \right) \right] / \right. \\ \left. \left( \sqrt{2(a^2+b^2-2ac+c^2)}^{1/4} \sqrt{a^2+b^2+c(c-\sqrt{a^2+b^2-2ac+c^2})} - \right. \right. \\ \left. \left. a(2c-\sqrt{a^2+b^2-2ac+c^2}) \sqrt{a+b \operatorname{Tan}[d+ex]+c \operatorname{Tan}[d+ex]^2} \right) \right] / \\ \left( \sqrt{2(a^2+b^2-2ac+c^2)}^{1/4} e \right) + \frac{\sqrt{c} \operatorname{ArcTanh} \left[ \frac{b+2c \operatorname{Tan}[d+ex]}{2\sqrt{c} \sqrt{a+b \operatorname{Tan}[d+ex]+c \operatorname{Tan}[d+ex]^2}} \right]}{e} - \\ \left( \sqrt{a^2+b^2+c(c+\sqrt{a^2+b^2-2ac+c^2})} - a(2c+\sqrt{a^2+b^2-2ac+c^2}) \right) \\ \operatorname{ArcTanh} \left[ \left( b\sqrt{a^2+b^2-2ac+c^2} + (b^2+(a-c)(a-c-\sqrt{a^2+b^2-2ac+c^2})) \operatorname{Tan}[d+ex] \right) \right] / \\ \left( \sqrt{2(a^2+b^2-2ac+c^2)}^{1/4} \right. \\ \left. \sqrt{a^2+b^2+c(c+\sqrt{a^2+b^2-2ac+c^2})} - a(2c+\sqrt{a^2+b^2-2ac+c^2}) \right) \\ \left. \sqrt{a+b \operatorname{Tan}[d+ex]+c \operatorname{Tan}[d+ex]^2} \right] / \left( \sqrt{2(a^2+b^2-2ac+c^2)}^{1/4} e \right)$$

Result (type 3, 282 leaves):

$$\frac{1}{2e} \left( -i \sqrt{a-ib-c} \operatorname{Log} \left[ - \left( 2i \left( 2a-ib + (b-2ic) \operatorname{Tan}[d+ex] + 2\sqrt{a-ib-c} \sqrt{a+b \operatorname{Tan}[d+ex] + c \operatorname{Tan}[d+ex]^2} \right) \right) / \left( (a-ib-c)^{3/2} (i + \operatorname{Tan}[d+ex]) \right) \right] + i \sqrt{a+ib-c} \operatorname{Log} \left[ 2i \left( 2a+ib + (b+2ic) \operatorname{Tan}[d+ex] + 2\sqrt{a+ib-c} \sqrt{a+b \operatorname{Tan}[d+ex] + c \operatorname{Tan}[d+ex]^2} \right) \right) / \left( (a+ib-c)^{3/2} (-i + \operatorname{Tan}[d+ex]) \right) \right] + 2\sqrt{c} \operatorname{Log} [b+2c \operatorname{Tan}[d+ex] + 2\sqrt{c} \sqrt{a+b \operatorname{Tan}[d+ex] + c \operatorname{Tan}[d+ex]^2}] \right)$$

**Problem 7: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Cot}[d+ex] \sqrt{a+b \operatorname{Tan}[d+ex] + c \operatorname{Tan}[d+ex]^2} dx$$

Optimal (type 3, 571 leaves, 18 steps):

$$\begin{aligned} & - \left( \left( \sqrt{a^2+b^2+c} \left( c + \sqrt{a^2+b^2-2ac+c^2} \right) - a \left( 2c + \sqrt{a^2+b^2-2ac+c^2} \right) \right) \right. \\ & \quad \left. \operatorname{ArcTan} \left[ \left( b^2 + (a-c) \left( a-c - \sqrt{a^2+b^2-2ac+c^2} \right) - b \sqrt{a^2+b^2-2ac+c^2} \operatorname{Tan}[d+ex] \right) / \right. \right. \\ & \quad \left. \left( \sqrt{2} (a^2+b^2-2ac+c^2)^{1/4} \sqrt{a^2+b^2+c} \left( c + \sqrt{a^2+b^2-2ac+c^2} \right) - a \left( 2c + \sqrt{a^2+b^2-2ac+c^2} \right) \sqrt{a+b \operatorname{Tan}[d+ex] + c \operatorname{Tan}[d+ex]^2} \right) \right] \right) / \\ & \quad \left( \sqrt{2} (a^2+b^2-2ac+c^2)^{1/4} e \right) - \frac{\sqrt{a} \operatorname{ArcTanh} \left[ \frac{2a+b \operatorname{Tan}[d+ex]}{2\sqrt{a} \sqrt{a+b \operatorname{Tan}[d+ex] + c \operatorname{Tan}[d+ex]^2}} \right]}{e} + \\ & \quad \left( \sqrt{a^2+b^2+c} \left( c - \sqrt{a^2+b^2-2ac+c^2} \right) - a \left( 2c - \sqrt{a^2+b^2-2ac+c^2} \right) \right) \\ & \quad \operatorname{ArcTanh} \left[ \left( b^2 + (a-c) \left( a-c + \sqrt{a^2+b^2-2ac+c^2} \right) + b \sqrt{a^2+b^2-2ac+c^2} \operatorname{Tan}[d+ex] \right) / \right. \\ & \quad \left. \left( \sqrt{2} (a^2+b^2-2ac+c^2)^{1/4} \sqrt{a^2+b^2+c} \left( c - \sqrt{a^2+b^2-2ac+c^2} \right) - a \left( 2c - \sqrt{a^2+b^2-2ac+c^2} \right) \sqrt{a+b \operatorname{Tan}[d+ex] + c \operatorname{Tan}[d+ex]^2} \right) \right] \right) / \left( \sqrt{2} (a^2+b^2-2ac+c^2)^{1/4} e \right) \end{aligned}$$

Result (type 3, 1193 leaves):

$$\left( \operatorname{Cot}[d+ex] \left( 2\sqrt{a} \operatorname{Log}[\operatorname{Tan}[d+ex]] - \right. \right.$$



$$\begin{aligned}
& 2\sqrt{a} \operatorname{Log} [2 a + b \operatorname{Tan}[d+e x] + 2\sqrt{a} \sqrt{a + \operatorname{Tan}[d+e x] (b + c \operatorname{Tan}[d+e x])}] + \\
& \sqrt{a-i b-c} \operatorname{Log} \left[ \left( -4 a + 2 i b - 2 b \operatorname{Tan}[d+e x] + 4 i c \operatorname{Tan}[d+e x] - 4 \sqrt{a-i b-c} \right. \right. \\
& \quad \left. \left. \sqrt{a + \operatorname{Tan}[d+e x] (b + c \operatorname{Tan}[d+e x])} \right) / \left( (a-i b-c)^{3/2} (i + \operatorname{Tan}[d+e x]) \right) \right] + \\
& \sqrt{a+i b-c} \operatorname{Log} \left[ - \left( \left( 2 \left( 2 a + 2 i c \operatorname{Tan}[d+e x] + b (i + \operatorname{Tan}[d+e x]) \right. \right. \right. \right. \\
& \quad \left. \left. \left. 2 \sqrt{a+i b-c} \sqrt{a + \operatorname{Tan}[d+e x] (b + c \operatorname{Tan}[d+e x])} \right) \right) / \right. \\
& \quad \left. \left( (a+i b-c)^{3/2} (-i + \operatorname{Tan}[d+e x]) \right) \right] \left. \right) \\
& \sqrt{\left( \frac{a}{1 + \operatorname{Cos}[2(d+e x)]} + \frac{c}{1 + \operatorname{Cos}[2(d+e x)]} + \frac{a \operatorname{Cos}[2(d+e x)]}{1 + \operatorname{Cos}[2(d+e x)]} - \right. \\
& \quad \left. \frac{c \operatorname{Cos}[2(d+e x)]}{1 + \operatorname{Cos}[2(d+e x)]} + \frac{b \operatorname{Sin}[2(d+e x)]}{1 + \operatorname{Cos}[2(d+e x)]} \right) / \\
& \left( e \left( 2\sqrt{a} \operatorname{Csc}[d+e x] \operatorname{Sec}[d+e x] - \left( 2\sqrt{a} (b \operatorname{Sec}[d+e x]^2 + (\sqrt{a} (c \operatorname{Sec}[d+e x]^2 \operatorname{Tan}[d+e x] + \right. \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{Sec}[d+e x]^2 (b + c \operatorname{Tan}[d+e x])) \right) / \left( \sqrt{a + \operatorname{Tan}[d+e x] (b + c \operatorname{Tan}[d+e x])} \right) \right) \right) / \\
& \quad \left( 2 a + b \operatorname{Tan}[d+e x] + 2\sqrt{a} \sqrt{a + \operatorname{Tan}[d+e x] (b + c \operatorname{Tan}[d+e x])} \right) + \\
& \quad \left( (a-i b-c)^2 (i + \operatorname{Tan}[d+e x]) \left( \left( -2 b \operatorname{Sec}[d+e x]^2 + 4 i c \operatorname{Sec}[d+e x]^2 - \right. \right. \right. \\
& \quad \left. \left. \left( 2\sqrt{a-i b-c} (c \operatorname{Sec}[d+e x]^2 \operatorname{Tan}[d+e x] + \operatorname{Sec}[d+e x]^2 (b + c \operatorname{Tan}[d+e x])) \right) \right) / \right. \\
& \quad \left. \left( \sqrt{a + \operatorname{Tan}[d+e x] (b + c \operatorname{Tan}[d+e x])} \right) \right) / \left( (a-i b-c)^{3/2} (i + \operatorname{Tan}[d+e x]) \right) - \\
& \quad \left( \operatorname{Sec}[d+e x]^2 \left( -4 a + 2 i b - 2 b \operatorname{Tan}[d+e x] + 4 i c \operatorname{Tan}[d+e x] - 4 \right. \right. \\
& \quad \left. \left. \sqrt{a-i b-c} \sqrt{a + \operatorname{Tan}[d+e x] (b + c \operatorname{Tan}[d+e x])} \right) \right) / \\
& \quad \left( (a-i b-c)^{3/2} (i + \operatorname{Tan}[d+e x])^2 \right) \left. \right) / \left( -4 a + 2 i b - 2 b \operatorname{Tan}[d+e x] + \right. \\
& \quad \left. 4 i c \operatorname{Tan}[d+e x] - 4 \sqrt{a-i b-c} \sqrt{a + \operatorname{Tan}[d+e x] (b + c \operatorname{Tan}[d+e x])} \right) - \\
& \quad \left( (a+i b-c)^2 (-i + \operatorname{Tan}[d+e x]) \left( - \left( \left( 2 \left( b \operatorname{Sec}[d+e x]^2 + 2 i c \operatorname{Sec}[d+e x]^2 + \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left( \sqrt{a+i b-c} (c \operatorname{Sec}[d+e x]^2 \operatorname{Tan}[d+e x] + \operatorname{Sec}[d+e x]^2 (b + c \operatorname{Tan}[d+e x]) \right) \right) \right) / \right. \right. \\
& \quad \left. \left. \left( \sqrt{a + \operatorname{Tan}[d+e x] (b + c \operatorname{Tan}[d+e x])} \right) \right) \right) \left. \right) / \\
& \quad \left( (a+i b-c)^{3/2} (-i + \operatorname{Tan}[d+e x]) \right) + \left( 2 \operatorname{Sec}[d+e x]^2 \left( 2 a + 2 i c \operatorname{Tan}[d+e x] + \right. \right. \\
& \quad \left. \left. b (i + \operatorname{Tan}[d+e x]) + 2 \sqrt{a+i b-c} \sqrt{a + \operatorname{Tan}[d+e x] (b + c \operatorname{Tan}[d+e x])} \right) \right) / \\
& \quad \left( (a+i b-c)^{3/2} (-i + \operatorname{Tan}[d+e x])^2 \right) \left. \right) / \left( 2 \left( 2 a + 2 i c \operatorname{Tan}[d+e x] + \right. \right. \\
& \quad \left. \left. b (i + \operatorname{Tan}[d+e x]) + 2 \sqrt{a+i b-c} \sqrt{a + \operatorname{Tan}[d+e x] (b + c \operatorname{Tan}[d+e x])} \right) \right) \left. \right) \left. \right)
\end{aligned}$$

**Problem 8: Humongous result has more than 200000 leaves.**

$$\int \text{Cot}[d + e x]^2 \sqrt{a + b \text{Tan}[d + e x] + c \text{Tan}[d + e x]^2} dx$$

Optimal (type 3, 612 leaves, 17 steps):

$$\left( \sqrt{\left( a^2 + b^2 + c \left( c - \sqrt{a^2 + b^2 - 2 a c + c^2} \right) - a \left( 2 c - \sqrt{a^2 + b^2 - 2 a c + c^2} \right) \right)} \right. \\ \left. \text{ArcTan} \left[ \left( b \sqrt{a^2 + b^2 - 2 a c + c^2} - \left( b^2 + (a - c) \left( a - c + \sqrt{a^2 + b^2 - 2 a c + c^2} \right) \right) \text{Tan}[d + e x] \right) / \right. \right. \\ \left. \left. \left( \sqrt{2} \left( a^2 + b^2 - 2 a c + c^2 \right)^{1/4} \sqrt{\left( a^2 + b^2 + c \left( c - \sqrt{a^2 + b^2 - 2 a c + c^2} \right) - \right. \right. \right. \right. \right. \\ \left. \left. \left. a \left( 2 c - \sqrt{a^2 + b^2 - 2 a c + c^2} \right) \right) \sqrt{a + b \text{Tan}[d + e x] + c \text{Tan}[d + e x]^2} \right) \right] \right) / \\ \left( \sqrt{2} \left( a^2 + b^2 - 2 a c + c^2 \right)^{1/4} e \right) - \frac{b \text{ArcTanh} \left[ \frac{2 a + b \text{Tan}[d + e x]}{2 \sqrt{a} \sqrt{a + b \text{Tan}[d + e x] + c \text{Tan}[d + e x]^2}} \right]}{2 \sqrt{a} e} + \\ \left( \sqrt{\left( a^2 + b^2 + c \left( c + \sqrt{a^2 + b^2 - 2 a c + c^2} \right) - a \left( 2 c + \sqrt{a^2 + b^2 - 2 a c + c^2} \right) \right)} \right. \\ \left. \text{ArcTanh} \left[ \left( b \sqrt{a^2 + b^2 - 2 a c + c^2} + \left( b^2 + (a - c) \left( a - c - \sqrt{a^2 + b^2 - 2 a c + c^2} \right) \right) \text{Tan}[d + e x] \right) / \right. \right. \\ \left. \left. \left( \sqrt{2} \left( a^2 + b^2 - 2 a c + c^2 \right)^{1/4} \sqrt{\left( a^2 + b^2 + c \left( c + \sqrt{a^2 + b^2 - 2 a c + c^2} \right) - \right. \right. \right. \right. \right. \\ \left. \left. \left. a \left( 2 c + \sqrt{a^2 + b^2 - 2 a c + c^2} \right) \right) \sqrt{a + b \text{Tan}[d + e x] + c \text{Tan}[d + e x]^2} \right) \right] \right) / \\ \left( \sqrt{2} \left( a^2 + b^2 - 2 a c + c^2 \right)^{1/4} e \right) - \frac{\text{Cot}[d + e x] \sqrt{a + b \text{Tan}[d + e x] + c \text{Tan}[d + e x]^2}}{e}$$

Result (type ?, 325 908 leaves): Display of huge result suppressed!

**Problem 9: Humongous result has more than 200000 leaves.**

$$\int \text{Cot}[d + e x]^3 \sqrt{a + b \text{Tan}[d + e x] + c \text{Tan}[d + e x]^2} dx$$

Optimal (type 3, 690 leaves, 21 steps):

$$\begin{aligned}
 & \left( \sqrt{\left( a^2 + b^2 + c \left( c + \sqrt{a^2 + b^2 - 2ac + c^2} \right) - a \left( 2c + \sqrt{a^2 + b^2 - 2ac + c^2} \right) \right)} \right. \\
 & \quad \left. \text{ArcTan} \left[ \left( b^2 + (a - c) \left( a - c - \sqrt{a^2 + b^2 - 2ac + c^2} \right) - b \sqrt{a^2 + b^2 - 2ac + c^2} \tan[d + ex] \right) / \right. \right. \\
 & \quad \left. \left. \left( \sqrt{2} \left( a^2 + b^2 - 2ac + c^2 \right)^{1/4} \sqrt{\left( a^2 + b^2 + c \left( c + \sqrt{a^2 + b^2 - 2ac + c^2} \right) - a \left( 2c + \sqrt{a^2 + b^2 - 2ac + c^2} \right) \right)} \sqrt{a + b \tan[d + ex] + c \tan[d + ex]^2} \right) \right] \right) / \\
 & \quad \left( \sqrt{2} \left( a^2 + b^2 - 2ac + c^2 \right)^{1/4} e \right) + \frac{\sqrt{a} \text{ArcTanh} \left[ \frac{2a + b \tan[d + ex]}{2\sqrt{a} \sqrt{a + b \tan[d + ex] + c \tan[d + ex]^2}} \right]}{e} + \\
 & \quad \frac{(b^2 - 4ac) \text{ArcTanh} \left[ \frac{2a + b \tan[d + ex]}{2\sqrt{a} \sqrt{a + b \tan[d + ex] + c \tan[d + ex]^2}} \right]}{8a^{3/2}e} - \\
 & \left( \sqrt{\left( a^2 + b^2 + c \left( c - \sqrt{a^2 + b^2 - 2ac + c^2} \right) - a \left( 2c - \sqrt{a^2 + b^2 - 2ac + c^2} \right) \right)} \right. \\
 & \quad \left. \text{ArcTanh} \left[ \left( b^2 + (a - c) \left( a - c + \sqrt{a^2 + b^2 - 2ac + c^2} \right) + b \sqrt{a^2 + b^2 - 2ac + c^2} \tan[d + ex] \right) / \right. \right. \\
 & \quad \left. \left. \left( \sqrt{2} \left( a^2 + b^2 - 2ac + c^2 \right)^{1/4} \sqrt{\left( a^2 + b^2 + c \left( c - \sqrt{a^2 + b^2 - 2ac + c^2} \right) - a \left( 2c - \sqrt{a^2 + b^2 - 2ac + c^2} \right) \right)} \sqrt{a + b \tan[d + ex] + c \tan[d + ex]^2} \right) \right] \right) / \\
 & \quad \left( \sqrt{2} \left( a^2 + b^2 - 2ac + c^2 \right)^{1/4} e \right) - \frac{1}{4ae} \text{Cot}[d + ex]^2 (2a + b \tan[d + ex]) \\
 & \quad \sqrt{a + b \tan[d + ex] + c \tan[d + ex]^2}
 \end{aligned}$$

Result (type ?, 439306 leaves): Display of huge result suppressed!

**Problem 10: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\tan[d + ex]^5}{\sqrt{a + b \tan[d + ex] + c \tan[d + ex]^2}} dx$$

Optimal (type 3, 548 leaves, 15 steps):

$$\begin{aligned}
 & \left( \sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}} \operatorname{ArcTanh}\left[\left(a-c-\sqrt{a^2+b^2-2ac+c^2}+b \tan [d+e x]\right)\right] / \right. \\
 & \quad \left. \left(\sqrt{2} \sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}} \sqrt{a+b \tan [d+e x]+c \tan [d+e x]^2}\right)\right) / \\
 & \left(\sqrt{2} \sqrt{a^2+b^2-2ac+c^2} e\right)-\left(\sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}} \right. \\
 & \quad \left. \operatorname{ArcTanh}\left[\left(a-c+\sqrt{a^2+b^2-2ac+c^2}+b \tan [d+e x]\right)\right] / \right. \\
 & \quad \left. \left(\sqrt{2} \sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}} \sqrt{a+b \tan [d+e x]+c \tan [d+e x]^2}\right)\right) / \\
 & \left(\sqrt{2} \sqrt{a^2+b^2-2ac+c^2} e\right)+\frac{b \operatorname{ArcTanh}\left[\frac{b+2 c \tan [d+e x]}{2 \sqrt{c} \sqrt{a+b \tan [d+e x]+c \tan [d+e x]^2}}\right]}{2 c^{3 / 2} e}- \\
 & \frac{b\left(5 b^2-12 a c\right) \operatorname{ArcTanh}\left[\frac{b+2 c \tan [d+e x]}{2 \sqrt{c} \sqrt{a+b \tan [d+e x]+c \tan [d+e x]^2}}\right]}{16 c^{7 / 2} e}- \\
 & \frac{\sqrt{a+b \tan [d+e x]+c \tan [d+e x]^2}}{c e}+ \\
 & \frac{\tan [d+e x]^2 \sqrt{a+b \tan [d+e x]+c \tan [d+e x]^2}}{3 c e}+ \\
 & \frac{1}{24 c^3 e} \\
 & \left(15 b^2-16 a c-10 b c \tan [d+e x]\right) \sqrt{a+b \tan [d+e x]+c \tan [d+e x]^2}
 \end{aligned}$$

Result(type 3, 389 leaves):

$$\begin{aligned}
 & \frac{1}{16 e} \left( - \frac{8 \operatorname{Log} \left[ \frac{2 a-i b+(b-2 i c) \operatorname{Tan}[d+e x]+2 \sqrt{a-i b-c} \sqrt{a+b \operatorname{Tan}[d+e x]+c \operatorname{Tan}[d+e x]^2}}{8 \sqrt{a-i b-c} c^3 (i+\operatorname{Tan}[d+e x])} \right]}{\sqrt{a-i b-c}} - \right. \\
 & \left. \frac{8 \operatorname{Log} \left[ \frac{2 a+i b+(b+2 i c) \operatorname{Tan}[d+e x]+2 \sqrt{a+i b-c} \sqrt{a+b \operatorname{Tan}[d+e x]+c \operatorname{Tan}[d+e x]^2}}{8 \sqrt{a+i b-c} c^3 (-i+\operatorname{Tan}[d+e x])} \right]}{\sqrt{a+i b-c}} + \frac{1}{c^{7/2}} \right. \\
 & \left. b \left( -5 b^2+4 c(3 a+2 c) \right) \operatorname{Log} \left[ b+2 c \operatorname{Tan}[d+e x]+2 \sqrt{c} \sqrt{a+\operatorname{Tan}[d+e x] (b+c \operatorname{Tan}[d+e x])} \right] + \right. \\
 & \left. \frac{1}{3 c^3} \sqrt{2} \sqrt{\left( \operatorname{Sec}[d+e x]^2 (a+c+(a-c) \operatorname{Cos}[2(d+e x)]+b \operatorname{Sin}[2(d+e x)]) \right)} \right. \\
 & \left. \left. \left( 15 b^2-16 a c-32 c^2+8 c^2 \operatorname{Sec}[d+e x]^2-10 b c \operatorname{Tan}[d+e x] \right) \right) \right]
 \end{aligned}$$

**Problem 11: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Tan}[d+e x]^4}{\sqrt{a+b \operatorname{Tan}[d+e x]+c \operatorname{Tan}[d+e x]^2}} dx$$

Optimal (type 3, 495 leaves, 14 steps):

$$\left( \sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}} \text{ArcTan}\left[\left(b-\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\text{Tan}[d+ex]\right)\right] \right) /$$

$$\left( \sqrt{2} \sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}} \sqrt{a+b \text{Tan}[d+ex]+c \text{Tan}[d+ex]^2} \right) /$$

$$\left( \sqrt{2} \sqrt{a^2+b^2-2ac+c^2} e \right) - \left( \sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}} \right.$$

$$\left. \text{ArcTan}\left[\left(b-\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)\text{Tan}[d+ex]\right)\right] \right) /$$

$$\left( \sqrt{2} \sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}} \sqrt{a+b \text{Tan}[d+ex]+c \text{Tan}[d+ex]^2} \right) /$$

$$\left( \sqrt{2} \sqrt{a^2+b^2-2ac+c^2} e \right) - \frac{\text{ArcTanh}\left[\frac{b+2c \text{Tan}[d+ex]}{2\sqrt{c} \sqrt{a+b \text{Tan}[d+ex]+c \text{Tan}[d+ex]^2}}\right]}{\sqrt{c} e} +$$

$$\frac{(3b^2-4ac) \text{ArcTanh}\left[\frac{b+2c \text{Tan}[d+ex]}{2\sqrt{c} \sqrt{a+b \text{Tan}[d+ex]+c \text{Tan}[d+ex]^2}}\right]}{8c^{5/2} e} -$$

$$\frac{3b \sqrt{a+b \text{Tan}[d+ex]+c \text{Tan}[d+ex]^2}}{4c^2 e} +$$

$$\frac{\text{Tan}[d+ex] \sqrt{a+b \text{Tan}[d+ex]+c \text{Tan}[d+ex]^2}}{2ce}$$

Result (type 3, 388 leaves):

$$\frac{1}{8e} \left( -\frac{1}{\sqrt{a-ib-c}} 4i \text{Log}\left[-\left(i\left(2a-2ic \text{Tan}[d+ex]+b(-i+\text{Tan}[d+ex])\right)+2\sqrt{a-ib-c}\right.\right.\right.$$

$$\left.\left.\left.\sqrt{a+\text{Tan}[d+ex](b+c \text{Tan}[d+ex])}\right)\right)\right] / \left(4\sqrt{a-ib-c} c^2 (i+\text{Tan}[d+ex])\right) \right) +$$

$$\frac{1}{\sqrt{a+ib-c}} 4i \text{Log}\left[\left(i\left(2a+2ic \text{Tan}[d+ex]+b(i+\text{Tan}[d+ex])\right)+2\sqrt{a+ib-c}\right.\right.$$

$$\left.\left.\left.\sqrt{a+\text{Tan}[d+ex](b+c \text{Tan}[d+ex])}\right)\right)\right] / \left(4\sqrt{a+ib-c} c^2 (-i+\text{Tan}[d+ex])\right) \right) +$$

$$\frac{1}{c^{5/2}} (3b^2-4c(a+2c)) \text{Log}\left[b+2c \text{Tan}[d+ex]+2\sqrt{c} \sqrt{a+\text{Tan}[d+ex](b+c \text{Tan}[d+ex])}\right] \right) +$$

$$\frac{\sqrt{\frac{a+c+a \text{Cos}[2(d+ex)]-c \text{Cos}[2(d+ex)]+b \text{Sin}[2(d+ex)]}{1+\text{Cos}[2(d+ex)]}} \left(-\frac{3b}{4c^2} + \frac{\text{Tan}[d+ex]}{2c}\right)}{e}$$

Problem 12: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\tan [d+e x]^3}{\sqrt{a+b \tan [d+e x]+c \tan [d+e x]^2}} dx$$

Optimal (type 3, 383 leaves, 11 steps):

$$\begin{aligned} & - \left( \left( \sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}} \operatorname{ArcTanh} \left[ \left( a-c-\sqrt{a^2+b^2-2ac+c^2}+b \tan [d+e x] \right) \right] \right) \right. \\ & \quad \left. \left( \sqrt{2} \sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}} \sqrt{a+b \tan [d+e x]+c \tan [d+e x]^2} \right) \right) / \\ & \quad \left( \sqrt{2} \sqrt{a^2+b^2-2ac+c^2} e \right) + \left( \sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}} \right. \\ & \quad \left. \operatorname{ArcTanh} \left[ \left( a-c+\sqrt{a^2+b^2-2ac+c^2}+b \tan [d+e x] \right) \right] \right) / \\ & \quad \left( \sqrt{2} \sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}} \sqrt{a+b \tan [d+e x]+c \tan [d+e x]^2} \right) / \\ & \quad \left( \sqrt{2} \sqrt{a^2+b^2-2ac+c^2} e \right) - \frac{b \operatorname{ArcTanh} \left[ \frac{b+2c \tan [d+e x]}{2\sqrt{c} \sqrt{a+b \tan [d+e x]+c \tan [d+e x]^2}} \right]}{2c^{3/2}e} + \\ & \quad \frac{\sqrt{a+b \tan [d+e x]+c \tan [d+e x]^2}}{ce} \end{aligned}$$

Result (type 3, 325 leaves):

$$\frac{1}{2e} \left( \frac{\text{Log} \left[ \frac{2a-i b+(b-2i c) \text{Tan}[d+e x]+2 \sqrt{a+\text{Tan}[d+e x] (b+c \text{Tan}[d+e x])}}{\sqrt{a-i b-c} c (i+\text{Tan}[d+e x])} \right]}{\sqrt{a-i b-c}} + \right.$$

$$\left. \frac{\text{Log} \left[ \frac{2 a+2 i c \text{Tan}[d+e x]+b (i+\text{Tan}[d+e x])+2 \sqrt{a+i b-c} \sqrt{a+\text{Tan}[d+e x] (b+c \text{Tan}[d+e x])}}{\sqrt{a+i b-c} c (-i+\text{Tan}[d+e x])} \right]}{\sqrt{a+i b-c}} - \frac{1}{c^{3/2}} \right.$$

$$\left. b \text{Log} \left[ b+2 c \text{Tan}[d+e x]+2 \sqrt{c} \sqrt{a+\text{Tan}[d+e x] (b+c \text{Tan}[d+e x])} \right] + \right.$$

$$\left. \frac{\sqrt{\frac{a+c+a \text{Cos}[2 (d+e x)]-c \text{Cos}[2 (d+e x)]+b \text{Sin}[2 (d+e x)]}{1+\text{Cos}[2 (d+e x)]}}}{c e} \right)$$

Problem 13: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{Tan}[d+e x]^2}{\sqrt{a+b \text{Tan}[d+e x]+c \text{Tan}[d+e x]^2}} dx$$

Optimal (type 3, 352 leaves, 9 steps):

$$- \left( \left( \sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}} \text{ArcTan} \left[ \left( b - \left( a-c-\sqrt{a^2+b^2-2ac+c^2} \right) \text{Tan}[d+e x] \right) \right] / \right. \right.$$

$$\left. \left( \sqrt{2} \sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}} \sqrt{a+b \text{Tan}[d+e x]+c \text{Tan}[d+e x]^2} \right) \right) /$$

$$\left( \sqrt{2} \sqrt{a^2+b^2-2ac+c^2} e \right) + \left( \sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}} \right.$$

$$\text{ArcTan} \left[ \left( b - \left( a-c+\sqrt{a^2+b^2-2ac+c^2} \right) \text{Tan}[d+e x] \right) \right] /$$

$$\left( \sqrt{2} \sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}} \sqrt{a+b \text{Tan}[d+e x]+c \text{Tan}[d+e x]^2} \right) \right) /$$

$$\left( \sqrt{2} \sqrt{a^2+b^2-2ac+c^2} e \right) + \frac{\text{ArcTanh} \left[ \frac{b+2c \text{Tan}[d+e x]}{2 \sqrt{c} \sqrt{a+b \text{Tan}[d+e x]+c \text{Tan}[d+e x]^2}} \right]}{\sqrt{c} e}$$

Result (type 3, 255 leaves):



$$\frac{1}{2e} \left( \frac{i \operatorname{Log} \left[ \frac{2 \left( \frac{-2ia-b+(-ib+2c)\operatorname{Tan}[d+ex]}{\sqrt{a-ib-c}} - 2i\sqrt{a+\operatorname{Tan}[d+ex]}(b+c\operatorname{Tan}[d+ex]) \right)}{-i+\operatorname{Tan}[d+ex]} \right]}{\sqrt{a+ib-c}} + \frac{i \operatorname{Log} \left[ \frac{2 \left( \frac{2ia+b+(ib+2c)\operatorname{Tan}[d+ex]}{\sqrt{a-ib-c}} + 2i\sqrt{a+\operatorname{Tan}[d+ex]}(b+c\operatorname{Tan}[d+ex]) \right)}{i+\operatorname{Tan}[d+ex]} \right]}{\sqrt{a-ib-c}} + \frac{1}{\sqrt{c}} + 2 \operatorname{Log} [b + 2c \operatorname{Tan}[d + ex] + 2\sqrt{c} \sqrt{a + \operatorname{Tan}[d + ex]}(b + c \operatorname{Tan}[d + ex])] \right)$$

**Problem 14: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Tan}[d + ex]}{\sqrt{a + b \operatorname{Tan}[d + ex] + c \operatorname{Tan}[d + ex]^2}} dx$$

Optimal (type 3, 294 leaves, 6 steps):

$$\left( \sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}} \operatorname{ArcTanh} \left[ \left( a-c-\sqrt{a^2+b^2-2ac+c^2} + b \operatorname{Tan}[d+ex] \right) / \left( \sqrt{2} \sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}} \sqrt{a+b \operatorname{Tan}[d+ex] + c \operatorname{Tan}[d+ex]^2} \right) \right] \right) / \left( \sqrt{2} \sqrt{a^2+b^2-2ac+c^2} e \right) - \left( \sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}} \operatorname{ArcTanh} \left[ \left( a-c+\sqrt{a^2+b^2-2ac+c^2} + b \operatorname{Tan}[d+ex] \right) / \left( \sqrt{2} \sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}} \sqrt{a+b \operatorname{Tan}[d+ex] + c \operatorname{Tan}[d+ex]^2} \right) \right] \right) / \left( \sqrt{2} \sqrt{a^2+b^2-2ac+c^2} e \right)$$

Result (type 3, 196 leaves):

$$-\frac{1}{2e} \left( \frac{\text{Log} \left[ \frac{2 \left( \frac{2a-i b+(b-2i c) \text{Tan}[d+e x]}{\sqrt{a-i b-c}}+2 \sqrt{a+\text{Tan}[d+e x] (b+c \text{Tan}[d+e x])} \right)}{i+\text{Tan}[d+e x]} \right]}{\sqrt{a-i b-c}} + \frac{\text{Log} \left[ \frac{2 \left( \frac{2a+i b+(b+2i c) \text{Tan}[d+e x]}{\sqrt{a+i b-c}}+2 \sqrt{a+\text{Tan}[d+e x] (b+c \text{Tan}[d+e x])} \right)}{-i+\text{Tan}[d+e x]} \right]}{\sqrt{a+i b-c}} \right)$$

**Problem 15: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\sqrt{a+b \text{Tan}[d+e x]+c \text{Tan}[d+e x]^2}} dx$$

Optimal (type 3, 298 leaves, 6 steps):

$$\left( \sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}} \text{ArcTan} \left[ \left( b-\left( a-c-\sqrt{a^2+b^2-2ac+c^2} \right) \text{Tan}[d+e x] \right) / \left( \sqrt{2} \sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}} \sqrt{a+b \text{Tan}[d+e x]+c \text{Tan}[d+e x]^2} \right) \right] \right) / \left( \sqrt{2} \sqrt{a^2+b^2-2ac+c^2} e \right) - \left( \sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}} \text{ArcTan} \left[ \left( b-\left( a-c+\sqrt{a^2+b^2-2ac+c^2} \right) \text{Tan}[d+e x] \right) / \left( \sqrt{2} \sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}} \sqrt{a+b \text{Tan}[d+e x]+c \text{Tan}[d+e x]^2} \right) \right] \right) / \left( \sqrt{2} \sqrt{a^2+b^2-2ac+c^2} e \right)$$

Result (type 3, 229 leaves):

$$\frac{1}{2e} i \left( \frac{\text{Log} \left[ -\frac{2i \left( 2a-i b+(b-2i c) \text{Tan}[d+e x]+2 \sqrt{a-i b-c} \sqrt{a+b \text{Tan}[d+e x]+c \text{Tan}[d+e x]^2} \right)}{\sqrt{a-i b-c} (i+\text{Tan}[d+e x])} \right]}{\sqrt{a-i b-c}} + \frac{\text{Log} \left[ \frac{2i \left( 2a+i b+(b+2i c) \text{Tan}[d+e x]+2 \sqrt{a+i b-c} \sqrt{a+b \text{Tan}[d+e x]+c \text{Tan}[d+e x]^2} \right)}{\sqrt{a+i b-c} (-i+\text{Tan}[d+e x])} \right]}{\sqrt{a+i b-c}} \right)$$

Problem 16: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\text{Cot}[d + e x]}{\sqrt{a + b \text{Tan}[d + e x] + c \text{Tan}[d + e x]^2}} dx$$

Optimal (type 3, 350 leaves, 10 steps):

$$\frac{\text{ArcTanh}\left[\frac{2 a + b \text{Tan}[d + e x]}{2 \sqrt{a} \sqrt{a + b \text{Tan}[d + e x] + c \text{Tan}[d + e x]^2}}\right]}{\sqrt{a} e} - \left( \sqrt{a - c - \sqrt{a^2 + b^2 - 2 a c + c^2}} \text{ArcTanh}\left[\left(a - c - \sqrt{a^2 + b^2 - 2 a c + c^2} + b \text{Tan}[d + e x]\right) / \left(\sqrt{2} \sqrt{a - c - \sqrt{a^2 + b^2 - 2 a c + c^2}} \sqrt{a + b \text{Tan}[d + e x] + c \text{Tan}[d + e x]^2}\right)\right] \right) / \left( \sqrt{2} \sqrt{a^2 + b^2 - 2 a c + c^2} e \right) + \left( \sqrt{a - c + \sqrt{a^2 + b^2 - 2 a c + c^2}} \text{ArcTanh}\left[\left(a - c + \sqrt{a^2 + b^2 - 2 a c + c^2} + b \text{Tan}[d + e x]\right) / \left(\sqrt{2} \sqrt{a - c + \sqrt{a^2 + b^2 - 2 a c + c^2}} \sqrt{a + b \text{Tan}[d + e x] + c \text{Tan}[d + e x]^2}\right)\right] \right) / \left( \sqrt{2} \sqrt{a^2 + b^2 - 2 a c + c^2} e \right)$$

Result (type 4, 154 575 leaves): Display of huge result suppressed!

Problem 17: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\text{Cot}[d + e x]^2}{\sqrt{a + b \text{Tan}[d + e x] + c \text{Tan}[d + e x]^2}} dx$$

Optimal (type 3, 395 leaves, 11 steps):

$$\begin{aligned}
 & - \left( \left( \sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}} \operatorname{ArcTan} \left[ \left( b - \left( a-c-\sqrt{a^2+b^2-2ac+c^2} \right) \tan [d+ex] \right) \right] \right) / \right. \\
 & \quad \left. \left( \sqrt{2} \sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}} \sqrt{a+b \tan [d+ex]+c \tan [d+ex]^2} \right) \right) / \\
 & \quad \left( \sqrt{2} \sqrt{a^2+b^2-2ac+c^2} e \right) + \left( \sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}} \right. \\
 & \quad \left. \operatorname{ArcTan} \left[ \left( b - \left( a-c+\sqrt{a^2+b^2-2ac+c^2} \right) \tan [d+ex] \right) \right] \right) / \\
 & \quad \left( \sqrt{2} \sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}} \sqrt{a+b \tan [d+ex]+c \tan [d+ex]^2} \right) \right) / \\
 & \quad \left( \sqrt{2} \sqrt{a^2+b^2-2ac+c^2} e \right) + \frac{b \operatorname{ArcTanh} \left[ \frac{2a+b \tan [d+ex]}{2\sqrt{a} \sqrt{a+b \tan [d+ex]+c \tan [d+ex]^2}} \right]}{2a^{3/2} e} - \\
 & \quad \frac{\operatorname{Cot} [d+ex] \sqrt{a+b \tan [d+ex]+c \tan [d+ex]^2}}{ae}
 \end{aligned}$$

Result (type 4, 167 080 leaves): Display of huge result suppressed!

**Problem 18: Humongous result has more than 200000 leaves.**

$$\int \frac{\operatorname{Cot} [d+ex]^3}{\sqrt{a+b \tan [d+ex]+c \tan [d+ex]^2}} dx$$

Optimal (type 3, 500 leaves, 14 steps):

$$\begin{aligned}
 & \frac{\text{ArcTanh}\left[\frac{2a+b \tan[d+ex]}{2\sqrt{a}\sqrt{a+b \tan[d+ex]+c \tan[d+ex]^2}}\right]}{\sqrt{a} e} - \frac{(3b^2-4ac) \text{ArcTanh}\left[\frac{2a+b \tan[d+ex]}{2\sqrt{a}\sqrt{a+b \tan[d+ex]+c \tan[d+ex]^2}}\right]}{8a^{5/2} e} + \\
 & \left( \sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}} \text{ArcTanh}\left[\left(a-c-\sqrt{a^2+b^2-2ac+c^2}+b \tan[d+ex]\right)\right] / \right. \\
 & \left. \left( \sqrt{2}\sqrt{a-c-\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a+b \tan[d+ex]+c \tan[d+ex]^2} \right) \right) / \\
 & \left( \sqrt{2}\sqrt{a^2+b^2-2ac+c^2} e \right) - \left( \sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}} \right. \\
 & \left. \text{ArcTanh}\left[\left(a-c+\sqrt{a^2+b^2-2ac+c^2}+b \tan[d+ex]\right)\right] / \right. \\
 & \left. \left( \sqrt{2}\sqrt{a-c+\sqrt{a^2+b^2-2ac+c^2}}\sqrt{a+b \tan[d+ex]+c \tan[d+ex]^2} \right) \right) / \\
 & \left( \sqrt{2}\sqrt{a^2+b^2-2ac+c^2} e \right) + \frac{3b \cot[d+ex] \sqrt{a+b \tan[d+ex]+c \tan[d+ex]^2}}{4a^2 e} - \\
 & \frac{\cot[d+ex]^2 \sqrt{a+b \tan[d+ex]+c \tan[d+ex]^2}}{2a e}
 \end{aligned}$$

Result (type ?, 281691 leaves): Display of huge result suppressed!

**Problem 19: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\tan[d+ex]^7}{(a+b \tan[d+ex]+c \tan[d+ex]^2)^{3/2}} dx$$

Optimal (type 3, 1190 leaves, 20 steps):

$$\begin{aligned}
 & \frac{3b \text{ArcTanh}\left[\frac{b+2c \tan[d+ex]}{2\sqrt{c}\sqrt{a+b \tan[d+ex]+c \tan[d+ex]^2}}\right]}{2c^{5/2} e} - \\
 & \frac{5b(7b^2-12ac) \text{ArcTanh}\left[\frac{b+2c \tan[d+ex]}{2\sqrt{c}\sqrt{a+b \tan[d+ex]+c \tan[d+ex]^2}}\right]}{16c^{9/2} e} - \\
 & \left( \sqrt{2a-2c-\sqrt{a^2+b^2-2ac+c^2}} \sqrt{a^2-b^2-2ac+c^2} + (a-c) \sqrt{a^2+b^2-2ac+c^2} \text{ArcTanh}\left[ \right. \right. \\
 & \left. \left. \left( b^2 - (a-c) \left( a-c+\sqrt{a^2+b^2-2ac+c^2} \right) - b \left( 2a-2c-\sqrt{a^2+b^2-2ac+c^2} \right) \tan[d+ex] \right) \right] / \right. \\
 & \left. \left( \sqrt{2}\sqrt{2a-2c-\sqrt{a^2+b^2-2ac+c^2}} \sqrt{a^2-b^2-2ac+c^2} + (a-c) \sqrt{a^2+b^2-2ac+c^2} \right) \right) /
 \end{aligned}$$

$$\begin{aligned}
 & \left. \left. \left. \sqrt{a + b \operatorname{Tan}[d + e x] + c \operatorname{Tan}[d + e x]^2} \right) \right) \Big/ \left( \sqrt{2} (a^2 + b^2 - 2 a c + c^2)^{3/2} e \right) + \\
 & \left( \sqrt{2 a - 2 c + \sqrt{a^2 + b^2 - 2 a c + c^2}} \sqrt{a^2 - b^2 - 2 a c + c^2} - (a - c) \sqrt{a^2 + b^2 - 2 a c + c^2} \operatorname{ArcTanh} \left[ \right. \right. \\
 & \left. \left. \left( b^2 - (a - c) \left( a - c - \sqrt{a^2 + b^2 - 2 a c + c^2} \right) - b \left( 2 a - 2 c + \sqrt{a^2 + b^2 - 2 a c + c^2} \right) \operatorname{Tan}[d + e x] \right) \right] \right) \Big/ \\
 & \left( \sqrt{2} \sqrt{2 a - 2 c + \sqrt{a^2 + b^2 - 2 a c + c^2}} \sqrt{a^2 - b^2 - 2 a c + c^2} - (a - c) \sqrt{a^2 + b^2 - 2 a c + c^2} \right. \\
 & \left. \left. \sqrt{a + b \operatorname{Tan}[d + e x] + c \operatorname{Tan}[d + e x]^2} \right) \right) \Big/ \\
 & \left( \sqrt{2} (a^2 + b^2 - 2 a c + c^2)^{3/2} e \right) + \frac{2 (2 a + b \operatorname{Tan}[d + e x])}{(b^2 - 4 a c) e \sqrt{a + b \operatorname{Tan}[d + e x] + c \operatorname{Tan}[d + e x]^2}} - \\
 & \frac{2 \operatorname{Tan}[d + e x]^2 (2 a + b \operatorname{Tan}[d + e x])}{(b^2 - 4 a c) e \sqrt{a + b \operatorname{Tan}[d + e x] + c \operatorname{Tan}[d + e x]^2}} + \\
 & \frac{2 \operatorname{Tan}[d + e x]^4 (2 a + b \operatorname{Tan}[d + e x])}{(b^2 - 4 a c) e \sqrt{a + b \operatorname{Tan}[d + e x] + c \operatorname{Tan}[d + e x]^2}} - \\
 & \frac{2 (a (b^2 - 2 (a - c) c) + b c (a + c) \operatorname{Tan}[d + e x])}{(b^2 + (a - c)^2) (b^2 - 4 a c) e \sqrt{a + b \operatorname{Tan}[d + e x] + c \operatorname{Tan}[d + e x]^2}} + \\
 & \frac{(7 b^2 - 16 a c) \operatorname{Tan}[d + e x]^2 \sqrt{a + b \operatorname{Tan}[d + e x] + c \operatorname{Tan}[d + e x]^2}}{3 c^2 (b^2 - 4 a c) e} - \\
 & \frac{2 b \operatorname{Tan}[d + e x]^3 \sqrt{a + b \operatorname{Tan}[d + e x] + c \operatorname{Tan}[d + e x]^2}}{c (b^2 - 4 a c) e} - \\
 & \frac{(3 b^2 - 8 a c - 2 b c \operatorname{Tan}[d + e x]) \sqrt{a + b \operatorname{Tan}[d + e x] + c \operatorname{Tan}[d + e x]^2}}{c^2 (b^2 - 4 a c) e} + \\
 & \left( (105 b^4 - 460 a b^2 c + 256 a^2 c^2 - 2 b c (35 b^2 - 116 a c) \operatorname{Tan}[d + e x]) \right. \\
 & \left. \sqrt{a + b \operatorname{Tan}[d + e x] + c \operatorname{Tan}[d + e x]^2} \right) \Big/ (24 c^4 (b^2 - 4 a c) e)
 \end{aligned}$$

Result (type 3, 884 leaves):

$\frac{1}{16e}$

$$\left( \frac{8 \operatorname{Log} \left[ \frac{-2a - ib - (b+2ic) \operatorname{Tan}[d+ex] - 2\sqrt{a+ib-c} \sqrt{a+b \operatorname{Tan}[d+ex] + c \operatorname{Tan}[d+ex]^2}}{8(a-ib-c)\sqrt{a+ib-c}c^4(-i+\operatorname{Tan}[d+ex])} \right]}{(a+ib-c)^{3/2}} + \frac{1}{(a-ib-c)^{3/2}} 8 \operatorname{Log} \left[ \left( -2a + ib - \right. \right. \right. \\ \left. \left. \left. b \operatorname{Tan}[d+ex] + 2ic \operatorname{Tan}[d+ex] - 2\sqrt{a-ib-c} \sqrt{a+\operatorname{Tan}[d+ex] (b+c \operatorname{Tan}[d+ex])} \right) \right] \right) / \\ \left( 8\sqrt{a-ib-c} (a+ib-c) c^4 (i+\operatorname{Tan}[d+ex]) \right) + \frac{1}{c^{9/2}} b (-35b^2 + 60ac + 24c^2) \\ \left. \operatorname{Log} \left[ b + 2c \operatorname{Tan}[d+ex] + 2\sqrt{c} \sqrt{a+\operatorname{Tan}[d+ex] (b+c \operatorname{Tan}[d+ex])} \right] \right) + \\ \frac{1}{e} \sqrt{\frac{a+c+a \operatorname{Cos}[2(d+ex)] - c \operatorname{Cos}[2(d+ex)] + b \operatorname{Sin}[2(d+ex)]}{1+\operatorname{Cos}[2(d+ex)]}} \\ \left( - \left( (105a^3b^4 + 105ab^6 - 460a^4b^2c - 727a^2b^4c - 57b^6c + 256a^5c^2 + 1364a^3b^2c^2 + 407ab^4c^2 - \right. \right. \\ \left. \left. 448a^4c^3 - 740a^2b^2c^3 - 25b^4c^3 + 96a^3c^4 + 44ab^2c^4 + 224a^2c^5 + 32b^2c^5 - 128ac^6) / \right. \right. \\ \left. \left. (24(a-c)(a-ib-c)(a+ib-c)c^4(-b^2+4ac)) \right) + \frac{\operatorname{Sec}[d+ex]^2}{3c^2} + \right. \\ \left. (2(2a^3b^4 + 2ab^6 - 8a^4b^2c - 12a^2b^4c + 4a^5c^2 + 18a^3b^2c^2 - 4a^4c^3 + \right. \\ \left. a^4b^3 \operatorname{Sin}[2(d+ex)] + 2a^2b^5 \operatorname{Sin}[2(d+ex)] + b^7 \operatorname{Sin}[2(d+ex)] - \right. \\ \left. 3a^5bc \operatorname{Sin}[2(d+ex)] - 10a^3b^3c \operatorname{Sin}[2(d+ex)] - 7ab^5c \operatorname{Sin}[2(d+ex)] + \right. \\ \left. 10a^4bc^2 \operatorname{Sin}[2(d+ex)] + 14a^2b^3c^2 \operatorname{Sin}[2(d+ex)] - 7a^3bc^3 \operatorname{Sin}[2(d+ex)] \right) / \\ \left. ((a-c)(a-ib-c)(a+ib-c)c^3(-b^2+4ac)(a+c+a \operatorname{Cos}[2(d+ex)] - \right. \\ \left. c \operatorname{Cos}[2(d+ex)] + b \operatorname{Sin}[2(d+ex)])) - \frac{11b \operatorname{Tan}[d+ex]}{12c^3} \right)$$

**Problem 20: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Tan}[d+ex]^5}{(a+b \operatorname{Tan}[d+ex] + c \operatorname{Tan}[d+ex]^2)^{3/2}} dx$$

Optimal (type 3, 864 leaves, 14 steps):

$$\begin{aligned}
 & - \frac{3 b \operatorname{ArcTanh}\left[\frac{b+2 c \operatorname{Tan}[d+e x]}{2 \sqrt{c} \sqrt{a+b \operatorname{Tan}[d+e x]+c \operatorname{Tan}[d+e x]^2}}\right]}{2 c^{5/2} e} + \\
 & \left( \sqrt{2 a-2 c-\sqrt{a^2+b^2-2 a c+c^2}} \sqrt{a^2-b^2-2 a c+c^2+(a-c) \sqrt{a^2+b^2-2 a c+c^2}} \operatorname{ArcTanh}\left[ \right. \right. \\
 & \quad \left. \left. \left( b^2-(a-c)\left(a-c+\sqrt{a^2+b^2-2 a c+c^2}\right)-b\left(2 a-2 c-\sqrt{a^2+b^2-2 a c+c^2}\right) \operatorname{Tan}[d+e x]\right) / \right. \right. \\
 & \quad \left. \left. \left( \sqrt{2} \sqrt{2 a-2 c-\sqrt{a^2+b^2-2 a c+c^2}} \sqrt{a^2-b^2-2 a c+c^2+(a-c) \sqrt{a^2+b^2-2 a c+c^2}} \right. \right. \right. \\
 & \quad \left. \left. \left. \sqrt{a+b \operatorname{Tan}[d+e x]+c \operatorname{Tan}[d+e x]^2} \right) \right] \right) / \left( \sqrt{2}\left(a^2+b^2-2 a c+c^2\right)^{3/2} e \right) - \\
 & \left( \sqrt{2 a-2 c+\sqrt{a^2+b^2-2 a c+c^2}} \sqrt{a^2-b^2-2 a c+c^2-(a-c) \sqrt{a^2+b^2-2 a c+c^2}} \operatorname{ArcTanh}\left[ \right. \right. \\
 & \quad \left. \left. \left( b^2-(a-c)\left(a-c-\sqrt{a^2+b^2-2 a c+c^2}\right)-b\left(2 a-2 c+\sqrt{a^2+b^2-2 a c+c^2}\right) \operatorname{Tan}[d+e x]\right) / \right. \right. \\
 & \quad \left. \left. \left( \sqrt{2} \sqrt{2 a-2 c+\sqrt{a^2+b^2-2 a c+c^2}} \sqrt{a^2-b^2-2 a c+c^2-(a-c) \sqrt{a^2+b^2-2 a c+c^2}} \right. \right. \right. \\
 & \quad \left. \left. \left. \sqrt{a+b \operatorname{Tan}[d+e x]+c \operatorname{Tan}[d+e x]^2} \right) \right] \right) / \\
 & \left( \sqrt{2}\left(a^2+b^2-2 a c+c^2\right)^{3/2} e \right) - \frac{2\left(2 a+b \operatorname{Tan}[d+e x]\right)}{\left(b^2-4 a c\right) e \sqrt{a+b \operatorname{Tan}[d+e x]+c \operatorname{Tan}[d+e x]^2}} + \\
 & \frac{2 \operatorname{Tan}[d+e x]^2\left(2 a+b \operatorname{Tan}[d+e x]\right)}{\left(b^2-4 a c\right) e \sqrt{a+b \operatorname{Tan}[d+e x]+c \operatorname{Tan}[d+e x]^2}} + \\
 & \frac{2\left(a\left(b^2-2(a-c) c\right)+b c(a+c) \operatorname{Tan}[d+e x]\right)}{\left(b^2+(a-c)^2\right)\left(b^2-4 a c\right) e \sqrt{a+b \operatorname{Tan}[d+e x]+c \operatorname{Tan}[d+e x]^2}} + \\
 & \frac{\left(3 b^2-8 a c-2 b c \operatorname{Tan}[d+e x]\right) \sqrt{a+b \operatorname{Tan}[d+e x]+c \operatorname{Tan}[d+e x]^2}}{c^2\left(b^2-4 a c\right) e}
 \end{aligned}$$

Result (type 3, 697 leaves):



$$\begin{aligned}
 & \frac{1}{2e} \left( -\frac{1}{(a-ib-c)^{3/2}} \operatorname{Log} \left[ \left( 2a - 2ib \operatorname{Tan}[d+ex] + \right. \right. \right. \\
 & \quad \left. \left. \left. b(-ib + \operatorname{Tan}[d+ex]) + 2\sqrt{a-ib-c} \sqrt{a + \operatorname{Tan}[d+ex] (b+c \operatorname{Tan}[d+ex])} \right) \right] / \right. \\
 & \quad \left. \left( \sqrt{a-ib-c} (a+ib-c) c^2 (ib + \operatorname{Tan}[d+ex]) \right) \right] - \\
 & \quad \frac{\operatorname{Log} \left[ \frac{2a+2ib \operatorname{Tan}[d+ex]+b(ib+\operatorname{Tan}[d+ex])+2\sqrt{a+ib-c} \sqrt{a+\operatorname{Tan}[d+ex](b+c \operatorname{Tan}[d+ex])}}{(a-ib-c) \sqrt{a+ib-c} c^2 (-ib+\operatorname{Tan}[d+ex])} \right]}{(a+ib-c)^{3/2}} - \frac{1}{c^{5/2}} \\
 & \quad \left. \left. \left. 3b \operatorname{Log} \left[ b + 2c \operatorname{Tan}[d+ex] + 2\sqrt{c} \sqrt{a + \operatorname{Tan}[d+ex] (b+c \operatorname{Tan}[d+ex])} \right] \right] \right) + \\
 & \frac{1}{e} \sqrt{\frac{a+c+a \operatorname{Cos}[2(d+ex)] - c \operatorname{Cos}[2(d+ex)] + b \operatorname{Sin}[2(d+ex)]}{1+\operatorname{Cos}[2(d+ex)]}} \\
 & \left( (-3a^3b^2 - 3ab^4 + 8a^4c + 15a^2b^2c + b^4c - 16a^3c^2 - 7ab^2c^2 + 12a^2c^3 + b^2c^3 - 4ac^4) / \right. \\
 & \quad \left( (a-c)(a-ib-c)(a+ib-c)c^2(-b^2+4ac) \right) - \\
 & \quad \left( 2(-2a^3b^2 - 2ab^4 + 4a^4c + 8a^2b^2c - 4a^3c^2 - a^4b \operatorname{Sin}[2(d+ex)] - 2a^2b^3 \operatorname{Sin}[2(d+ex)] - \right. \\
 & \quad \left. b^5 \operatorname{Sin}[2(d+ex)] + 6a^3bc \operatorname{Sin}[2(d+ex)] + 5ab^3c \operatorname{Sin}[2(d+ex)] - \right. \\
 & \quad \left. 5a^2b^2c^2 \operatorname{Sin}[2(d+ex)]) \right) / \left( (a-c)(a-ib-c)(a+ib-c)c \right. \\
 & \quad \left. (-b^2+4ac)(a+c+a \operatorname{Cos}[2(d+ex)] - c \operatorname{Cos}[2(d+ex)] + b \operatorname{Sin}[2(d+ex)]) \right) \left. \right)
 \end{aligned}$$

**Problem 21: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Tan}[d+ex]^3}{(a+b \operatorname{Tan}[d+ex] + c \operatorname{Tan}[d+ex]^2)^{3/2}} dx$$

Optimal (type 3, 686 leaves, 10 steps):

$$\begin{aligned}
 & - \left( \left( \sqrt{2a-2c-\sqrt{a^2+b^2-2ac+c^2}} \sqrt{a^2-b^2-2ac+c^2+(a-c)\sqrt{a^2+b^2-2ac+c^2}} \operatorname{ArcTanh} \left[ \right. \right. \right. \\
 & \quad \left. \left. \left( b^2-(a-c) \left( a-c+\sqrt{a^2+b^2-2ac+c^2} \right) - b \left( 2a-2c-\sqrt{a^2+b^2-2ac+c^2} \right) \operatorname{Tan}[d+ex] \right) \right] \right) / \\
 & \quad \left( \sqrt{2} \sqrt{2a-2c-\sqrt{a^2+b^2-2ac+c^2}} \sqrt{a^2-b^2-2ac+c^2+(a-c)\sqrt{a^2+b^2-2ac+c^2}} \right. \\
 & \quad \left. \sqrt{a+b \operatorname{Tan}[d+ex]+c \operatorname{Tan}[d+ex]^2} \right) \left. \right) / \left( \sqrt{2} (a^2+b^2-2ac+c^2)^{3/2} e \right) + \\
 & \left( \sqrt{2a-2c+\sqrt{a^2+b^2-2ac+c^2}} \sqrt{a^2-b^2-2ac+c^2-(a-c)\sqrt{a^2+b^2-2ac+c^2}} \operatorname{ArcTanh} \left[ \right. \right. \\
 & \quad \left. \left. \left( b^2-(a-c) \left( a-c-\sqrt{a^2+b^2-2ac+c^2} \right) - b \left( 2a-2c+\sqrt{a^2+b^2-2ac+c^2} \right) \operatorname{Tan}[d+ex] \right) \right] \right) / \\
 & \quad \left( \sqrt{2} \sqrt{2a-2c+\sqrt{a^2+b^2-2ac+c^2}} \sqrt{a^2-b^2-2ac+c^2-(a-c)\sqrt{a^2+b^2-2ac+c^2}} \right. \\
 & \quad \left. \sqrt{a+b \operatorname{Tan}[d+ex]+c \operatorname{Tan}[d+ex]^2} \right) \left. \right) / \\
 & \left( \sqrt{2} (a^2+b^2-2ac+c^2)^{3/2} e \right) + \frac{2(2a+b \operatorname{Tan}[d+ex])}{(b^2-4ac) e \sqrt{a+b \operatorname{Tan}[d+ex]+c \operatorname{Tan}[d+ex]^2}} - \\
 & \frac{2(a(b^2-2(a-c)c)+bc(a+c) \operatorname{Tan}[d+ex])}{(b^2+(a-c)^2)(b^2-4ac) e \sqrt{a+b \operatorname{Tan}[d+ex]+c \operatorname{Tan}[d+ex]^2}}
 \end{aligned}$$

Result (type 3, 738 leaves):

$$\begin{aligned}
 & \frac{1}{2e} \left( \frac{\text{Log} \left[ \frac{4a-4ic \tan[d+ex]+2b(-i+\tan[d+ex])+4\sqrt{a-ib-c} \sqrt{a+\tan[d+ex]}(b+c \tan[d+ex])}{\sqrt{a-ib-c}(a+ib-c)(i+\tan[d+ex])} \right]}{(a-ib-c)^{3/2}} \right) + \\
 & \frac{\text{Log} \left[ \frac{4a+4ic \tan[d+ex]+2b(i+\tan[d+ex])+4\sqrt{a+ib-c} \sqrt{a+\tan[d+ex]}(b+c \tan[d+ex])}{(a-ib-c)\sqrt{a+ib-c}(-i+\tan[d+ex])} \right]}{(a+ib-c)^{3/2}} \right) + \\
 & \frac{1}{e} \sqrt{\frac{a+c+a \cos[2(d+ex)]-c \cos[2(d+ex)]+b \sin[2(d+ex)]}{1+\cos[2(d+ex)]}} \\
 & \left( - \left( \left( 2a(2a^2+b^2-2ac) \right) / \left( (a-c)(a-ib-c)(-ab^2-ib^3+4a^2c+4iab+c+b^2c-4a^2c^2) \right) \right) + \right. \\
 & \left( \left( \cos[2(d+ex)]-i \sin[2(d+ex)] \right) \left( ia^3b+2ia^2bc+ib^3c-3iab+c^2+ \right. \right. \\
 & \quad 8a^3c \cos[2(d+ex)]+4ab^2c \cos[2(d+ex)]-8a^2c^2 \cos[2(d+ex)]- \\
 & \quad ia^3b \cos[4(d+ex)]-2ia^2bc \cos[4(d+ex)]-ib^3c \cos[4(d+ex)]+ \\
 & \quad 3iab+c^2 \cos[4(d+ex)]+8ia^3c \sin[2(d+ex)]+4iab^2c \sin[2(d+ex)]- \\
 & \quad 8ia^2c^2 \sin[2(d+ex)]+a^3b \sin[4(d+ex)]+2a^2bc \sin[4(d+ex)]+ \\
 & \quad \left. \left. b^3c \sin[4(d+ex)]-3abc^2 \sin[4(d+ex)] \right) \right) / \left( (a-c)(a-ib-c)(a+ib-c) \right) \\
 & \left. \left( -b^2+4ac \right) \left( a+c+a \cos[2(d+ex)]-c \cos[2(d+ex)]+b \sin[2(d+ex)] \right) \right) \right)
 \end{aligned}$$

**Problem 22: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\tan[d+ex]^2}{(a+b \tan[d+ex]+c \tan[d+ex]^2)^{3/2}} dx$$

Optimal (type 3, 638 leaves, 7 steps):

$$\begin{aligned}
 & - \left( \left( \sqrt{2 a - 2 c + \sqrt{a^2 + b^2 - 2 a c + c^2}} \right. \right. \\
 & \quad \left. \sqrt{a^2 - b^2 - 2 a c + c^2} - (a - c) \sqrt{a^2 + b^2 - 2 a c + c^2} \text{ArcTan} \left[ \left( b \left( 2 a - 2 c + \sqrt{a^2 + b^2 - 2 a c + c^2} \right) + \right. \right. \right. \\
 & \quad \left. \left. \left( b^2 - (a - c) \left( a - c - \sqrt{a^2 + b^2 - 2 a c + c^2} \right) \right) \text{Tan}[d + e x] \right] \right) / \\
 & \quad \left( \sqrt{2} \sqrt{2 a - 2 c + \sqrt{a^2 + b^2 - 2 a c + c^2}} \sqrt{a^2 - b^2 - 2 a c + c^2} - (a - c) \sqrt{a^2 + b^2 - 2 a c + c^2} \right. \\
 & \quad \left. \sqrt{a + b \text{Tan}[d + e x] + c \text{Tan}[d + e x]^2} \right) \Bigg) / \left( \sqrt{2} (a^2 + b^2 - 2 a c + c^2)^{3/2} e \right) + \\
 & \left( \sqrt{2 a - 2 c - \sqrt{a^2 + b^2 - 2 a c + c^2}} \sqrt{a^2 - b^2 - 2 a c + c^2} + (a - c) \sqrt{a^2 + b^2 - 2 a c + c^2} \right. \\
 & \quad \text{ArcTan} \left[ \left( b \left( 2 a - 2 c - \sqrt{a^2 + b^2 - 2 a c + c^2} \right) + \left( b^2 - (a - c) \left( a - c + \sqrt{a^2 + b^2 - 2 a c + c^2} \right) \right) \text{Tan}[d + e x] \right) / \right. \\
 & \quad \left. \left( \sqrt{2} \sqrt{2 a - 2 c - \sqrt{a^2 + b^2 - 2 a c + c^2}} \sqrt{a^2 - b^2 - 2 a c + c^2} + (a - c) \sqrt{a^2 + b^2 - 2 a c + c^2} \right. \right. \\
 & \quad \left. \left. \sqrt{a + b \text{Tan}[d + e x] + c \text{Tan}[d + e x]^2} \right) \right] \Bigg) / \left( \sqrt{2} (a^2 + b^2 - 2 a c + c^2)^{3/2} e \right) - \\
 & \frac{2 (a b (a + c) + c (2 a^2 + b^2 - 2 a c) \text{Tan}[d + e x])}{(b^2 + (a - c)^2) (b^2 - 4 a c) e \sqrt{a + b \text{Tan}[d + e x] + c \text{Tan}[d + e x]^2}}
 \end{aligned}$$

Result(type 3, 538 leaves):

$$\frac{1}{2e} i \left( \frac{1}{(a - i b - c)^{3/2}} \text{Log} \left[ \left( 2 \left( 2 i a + b + i b \text{Tan}[d + e x] + \right. \right. \right. \right. \\ \left. \left. \left. 2 c \text{Tan}[d + e x] + 2 i \sqrt{a - i b - c} \sqrt{a + \text{Tan}[d + e x] (b + c \text{Tan}[d + e x])} \right) \right) \right] / \\ \left( \sqrt{a - i b - c} (a + i b - c) (i + \text{Tan}[d + e x]) \right) \right] - \frac{1}{(a + i b - c)^{3/2}} \\ \text{Log} \left[ \left( 2 \left( -2 i a + b - i b \text{Tan}[d + e x] + 2 c \text{Tan}[d + e x] - \right. \right. \right. \\ \left. \left. \left. 2 i \sqrt{a + i b - c} \sqrt{a + \text{Tan}[d + e x] (b + c \text{Tan}[d + e x])} \right) \right) \right] / \\ \left( (a - i b - c) \sqrt{a + i b - c} (-i + \text{Tan}[d + e x]) \right) \right] + \frac{1}{e} \\ \sqrt{\frac{a + c + a \text{Cos}[2(d + e x)] - c \text{Cos}[2(d + e x)] + b \text{Sin}[2(d + e x)]}{1 + \text{Cos}[2(d + e x)]}} \\ \left( \frac{2 a b (a + c)}{(a - c) (a - i b - c) (a + i b - c) (-b^2 + 4 a c)} + \right. \\ \left. \frac{(2 (-2 a^2 b c - 2 a b c^2 - a^2 b^2 \text{Sin}[2(d + e x)] + 2 a^3 c \text{Sin}[2(d + e x)] - 4 a^2 c^2 \text{Sin}[2(d + e x)] - \right. \\ \left. b^2 c^2 \text{Sin}[2(d + e x)] + 2 a c^3 \text{Sin}[2(d + e x)]))}{(a - c) (a - i b - c) (a + i b - c)} \right) / \\ \left. (-b^2 + 4 a c) (a + c + a \text{Cos}[2(d + e x)] - c \text{Cos}[2(d + e x)] + b \text{Sin}[2(d + e x)]) \right)$$

**Problem 23: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\text{Tan}[d + e x]}{(a + b \text{Tan}[d + e x] + c \text{Tan}[d + e x]^2)^{3/2}} dx$$

Optimal (type 3, 635 leaves, 7 steps):

$$\left( \sqrt{2 a-2 c-\sqrt{a^2+b^2-2 a c+c^2}} \sqrt{a^2-b^2-2 a c+c^2+(a-c) \sqrt{a^2+b^2-2 a c+c^2}} \operatorname{ArcTanh}\left[\frac{\left(b^2-(a-c)\left(a-c+\sqrt{a^2+b^2-2 a c+c^2}\right)-b\left(2 a-2 c-\sqrt{a^2+b^2-2 a c+c^2}\right) \operatorname{Tan}[d+e x]\right)}{\left(\sqrt{2} \sqrt{2 a-2 c-\sqrt{a^2+b^2-2 a c+c^2}} \sqrt{a^2-b^2-2 a c+c^2+(a-c) \sqrt{a^2+b^2-2 a c+c^2}}\right)}\right]\right) / \left(\sqrt{2} \sqrt{a^2+b^2-2 a c+c^2}\right)^{3 / 2} e -$$

$$\left( \sqrt{2 a-2 c+\sqrt{a^2+b^2-2 a c+c^2}} \sqrt{a^2-b^2-2 a c+c^2-(a-c) \sqrt{a^2+b^2-2 a c+c^2}} \operatorname{ArcTanh}\left[\frac{\left(b^2-(a-c)\left(a-c-\sqrt{a^2+b^2-2 a c+c^2}\right)-b\left(2 a-2 c+\sqrt{a^2+b^2-2 a c+c^2}\right) \operatorname{Tan}[d+e x]\right)}{\left(\sqrt{2} \sqrt{2 a-2 c+\sqrt{a^2+b^2-2 a c+c^2}} \sqrt{a^2-b^2-2 a c+c^2-(a-c) \sqrt{a^2+b^2-2 a c+c^2}}\right)}\right]\right) / \left(\sqrt{2} \sqrt{a^2+b^2-2 a c+c^2}\right)^{3 / 2} e +$$

$$\frac{2(a(b^2-2(a-c)c)+bc(a+c)\operatorname{Tan}[d+ex])}{(b^2+(a-c)^2)(b^2-4ac)e\sqrt{a+b\operatorname{Tan}[d+ex]+c\operatorname{Tan}[d+ex]^2}}$$

Result (type 3, 535 leaves):

$$\frac{\operatorname{Log}\left[\frac{4 a-4 i c \operatorname{Tan}[d+e x]+2 b(-i+\operatorname{Tan}[d+e x])+4 \sqrt{a-i b-c} \sqrt{a+\operatorname{Tan}[d+e x](b+c \operatorname{Tan}[d+e x])}}{\sqrt{a-i b-c}(a+i b-c)(i+\operatorname{Tan}[d+e x])}\right]}{2(a-i b-c)^{3 / 2} e} -$$

$$\frac{\operatorname{Log}\left[\frac{4 a+4 i c \operatorname{Tan}[d+e x]+2 b(i+\operatorname{Tan}[d+e x])+4 \sqrt{a+i b-c} \sqrt{a+\operatorname{Tan}[d+e x](b+c \operatorname{Tan}[d+e x])}}{(a-i b-c) \sqrt{a+i b-c}(-i+\operatorname{Tan}[d+e x])}\right]}{2(a+i b-c)^{3 / 2} e} +$$

$$\frac{1}{e} \sqrt{\frac{a+c+a \operatorname{Cos}[2(d+e x)]-c \operatorname{Cos}[2(d+e x)]+b \operatorname{Sin}[2(d+e x)]}{1+\operatorname{Cos}[2(d+e x)]}}$$

$$\left(\frac{2 a\left(-b^2+2 a c-2 c^2\right)}{(a-c)(a-i b-c)(a+i b-c)\left(-b^2+4 a c\right)} - \frac{\left(2\left(-2 a b^2 c+4 a^2 c^2-4 a c^3-a b^3 \operatorname{Sin}[2(d+e x)]+3 a^2 b c \operatorname{Sin}[2(d+e x)]-2 a b c^2 \operatorname{Sin}[2(d+e x)]-b c^3 \operatorname{Sin}[2(d+e x)]\right)\right)}{(a-c)(a-i b-c)(a+i b-c)\left(-b^2+4 a c\right)\left(a+c+a \operatorname{Cos}[2(d+e x)]-c \operatorname{Cos}[2(d+e x)]+b \operatorname{Sin}[2(d+e x)]\right)}\right)$$

**Problem 24: Humongous result has more than 200000 leaves.**

$$\int \frac{\text{Cot}[d + e x]}{(a + b \text{Tan}[d + e x] + c \text{Tan}[d + e x]^2)^{3/2}} dx$$

Optimal (type 3, 750 leaves, 13 steps):

$$\frac{\text{ArcTanh}\left[\frac{2 a+b \text{Tan}[d+e x]}{2 \sqrt{a} \sqrt{a+b \text{Tan}[d+e x]+c \text{Tan}[d+e x]^2}}\right]}{a^{3/2} e} - \left( \sqrt{2 a-2 c-\sqrt{a^2+b^2-2 a c+c^2}} \sqrt{a^2-b^2-2 a c+c^2+(a-c) \sqrt{a^2+b^2-2 a c+c^2}} \text{ArcTanh}\left[\frac{\left(b^2-(a-c)\left(a-c+\sqrt{a^2+b^2-2 a c+c^2}\right)-b\left(2 a-2 c-\sqrt{a^2+b^2-2 a c+c^2}\right) \text{Tan}[d+e x]\right)}{\left(\sqrt{2} \sqrt{2 a-2 c-\sqrt{a^2+b^2-2 a c+c^2}} \sqrt{a^2-b^2-2 a c+c^2+(a-c) \sqrt{a^2+b^2-2 a c+c^2}} \sqrt{a+b \text{Tan}[d+e x]+c \text{Tan}[d+e x]^2}\right)}\right] \right) / \left(\sqrt{2}\left(a^2+b^2-2 a c+c^2\right)^{3/2} e\right) + \left( \sqrt{2 a-2 c+\sqrt{a^2+b^2-2 a c+c^2}} \sqrt{a^2-b^2-2 a c+c^2-(a-c) \sqrt{a^2+b^2-2 a c+c^2}} \text{ArcTanh}\left[\frac{\left(b^2-(a-c)\left(a-c-\sqrt{a^2+b^2-2 a c+c^2}\right)-b\left(2 a-2 c+\sqrt{a^2+b^2-2 a c+c^2}\right) \text{Tan}[d+e x]\right)}{\left(\sqrt{2} \sqrt{2 a-2 c+\sqrt{a^2+b^2-2 a c+c^2}} \sqrt{a^2-b^2-2 a c+c^2-(a-c) \sqrt{a^2+b^2-2 a c+c^2}} \sqrt{a+b \text{Tan}[d+e x]+c \text{Tan}[d+e x]^2}\right)}\right] \right) / \left(\sqrt{2}\left(a^2+b^2-2 a c+c^2\right)^{3/2} e\right) + \frac{2\left(b^2-2 a c+b c \text{Tan}[d+e x]\right)}{a\left(b^2-4 a c\right) e \sqrt{a+b \text{Tan}[d+e x]+c \text{Tan}[d+e x]^2}} - \frac{2\left(a\left(b^2-2(a-c) c\right)+b c(a+c) \text{Tan}[d+e x]\right)}{\left(b^2+(a-c)^2\right)\left(b^2-4 a c\right) e \sqrt{a+b \text{Tan}[d+e x]+c \text{Tan}[d+e x]^2}}$$

Result (type ?, 512551 leaves): Display of huge result suppressed!

**Problem 25: Humongous result has more than 200000 leaves.**

$$\int \frac{\text{Cot}[d + e x]^2}{(a + b \text{Tan}[d + e x] + c \text{Tan}[d + e x]^2)^{3/2}} dx$$

Optimal (type 3, 829 leaves, 13 steps):

$$\begin{aligned}
 & - \left( \left( \sqrt{2 a - 2 c + \sqrt{a^2 + b^2 - 2 a c + c^2}} \right. \right. \\
 & \quad \left. \left. \sqrt{a^2 - b^2 - 2 a c + c^2} - (a - c) \sqrt{a^2 + b^2 - 2 a c + c^2} \operatorname{ArcTan} \left[ \left( b \left( 2 a - 2 c + \sqrt{a^2 + b^2 - 2 a c + c^2} \right) + \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \left( b^2 - (a - c) \left( a - c - \sqrt{a^2 + b^2 - 2 a c + c^2} \right) \right) \operatorname{Tan}[d + e x] \right) \right] \right) / \right. \\
 & \quad \left. \left( \sqrt{2} \sqrt{2 a - 2 c + \sqrt{a^2 + b^2 - 2 a c + c^2}} \sqrt{a^2 - b^2 - 2 a c + c^2} - (a - c) \sqrt{a^2 + b^2 - 2 a c + c^2} \right. \right. \\
 & \quad \left. \left. \sqrt{a + b \operatorname{Tan}[d + e x] + c \operatorname{Tan}[d + e x]^2} \right) \right) / \left( \sqrt{2} \left( a^2 + b^2 - 2 a c + c^2 \right)^{3/2} e \right) + \\
 & \left( \sqrt{2 a - 2 c - \sqrt{a^2 + b^2 - 2 a c + c^2}} \sqrt{a^2 - b^2 - 2 a c + c^2} + (a - c) \sqrt{a^2 + b^2 - 2 a c + c^2} \right. \\
 & \quad \left. \operatorname{ArcTan} \left[ \left( b \left( 2 a - 2 c - \sqrt{a^2 + b^2 - 2 a c + c^2} \right) + \left( b^2 - (a - c) \left( a - c + \sqrt{a^2 + b^2 - 2 a c + c^2} \right) \right) \operatorname{Tan}[d + e x] \right) \right] / \right. \\
 & \quad \left. \left( \sqrt{2} \sqrt{2 a - 2 c - \sqrt{a^2 + b^2 - 2 a c + c^2}} \sqrt{a^2 - b^2 - 2 a c + c^2} + (a - c) \sqrt{a^2 + b^2 - 2 a c + c^2} \right. \right. \\
 & \quad \left. \left. \sqrt{a + b \operatorname{Tan}[d + e x] + c \operatorname{Tan}[d + e x]^2} \right) \right) / \right. \\
 & \quad \left. \left( \sqrt{2} \left( a^2 + b^2 - 2 a c + c^2 \right)^{3/2} e \right) + \frac{3 b \operatorname{ArcTanh} \left[ \frac{2 a + b \operatorname{Tan}[d + e x]}{2 \sqrt{a} \sqrt{a + b \operatorname{Tan}[d + e x] + c \operatorname{Tan}[d + e x]^2}} \right]}{2 a^{5/2} e} + \right. \\
 & \quad \frac{2 \operatorname{Cot}[d + e x] \left( b^2 - 2 a c + b c \operatorname{Tan}[d + e x] \right)}{a \left( b^2 - 4 a c \right) e \sqrt{a + b \operatorname{Tan}[d + e x] + c \operatorname{Tan}[d + e x]^2}} + \\
 & \quad \frac{2 \left( b \left( b^2 - \left( 3 a - c \right) c \right) + c \left( b^2 - 2 \left( a - c \right) c \right) \operatorname{Tan}[d + e x] \right)}{\left( b^2 + \left( a - c \right)^2 \right) \left( b^2 - 4 a c \right) e \sqrt{a + b \operatorname{Tan}[d + e x] + c \operatorname{Tan}[d + e x]^2}} - \\
 & \quad \frac{\left( 3 b^2 - 8 a c \right) \operatorname{Cot}[d + e x] \sqrt{a + b \operatorname{Tan}[d + e x] + c \operatorname{Tan}[d + e x]^2}}{a^2 \left( b^2 - 4 a c \right) e}
 \end{aligned}$$

Result (type ?, 536928 leaves): Display of huge result suppressed!



Problem 26: Humongous result has more than 200000 leaves.

$$\int \frac{\text{Cot}[d + e x]^3}{(a + b \text{Tan}[d + e x] + c \text{Tan}[d + e x]^2)^{3/2}} dx$$

Optimal (type 3, 1007 leaves, 18 steps):

$$\begin{aligned}
 & \frac{\text{ArcTanh}\left[\frac{2a+b \tan[d+ex]}{2\sqrt{a}\sqrt{a+b \tan[d+ex]+c \tan[d+ex]^2}}\right]}{a^{3/2}e} - \frac{3(5b^2-4ac)\text{ArcTanh}\left[\frac{2a+b \tan[d+ex]}{2\sqrt{a}\sqrt{a+b \tan[d+ex]+c \tan[d+ex]^2}}\right]}{8a^{7/2}e} + \\
 & \left( \sqrt{2a-2c-\sqrt{a^2+b^2-2ac+c^2}} \sqrt{a^2-b^2-2ac+c^2+(a-c)\sqrt{a^2+b^2-2ac+c^2}} \text{ArcTanh}\left[ \right. \right. \\
 & \quad \left. \left. \left( b^2-(a-c)\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)-b\left(2a-2c-\sqrt{a^2+b^2-2ac+c^2}\right)\tan[d+ex] \right) \right] \right) / \\
 & \left( \sqrt{2}\sqrt{2a-2c-\sqrt{a^2+b^2-2ac+c^2}} \sqrt{a^2-b^2-2ac+c^2+(a-c)\sqrt{a^2+b^2-2ac+c^2}} \right. \\
 & \quad \left. \sqrt{a+b \tan[d+ex]+c \tan[d+ex]^2} \right) \Bigg) / \left( \sqrt{2}\left(a^2+b^2-2ac+c^2\right)^{3/2}e \right) - \\
 & \left( \sqrt{2a-2c+\sqrt{a^2+b^2-2ac+c^2}} \sqrt{a^2-b^2-2ac+c^2-(a-c)\sqrt{a^2+b^2-2ac+c^2}} \text{ArcTanh}\left[ \right. \right. \\
 & \quad \left. \left. \left( b^2-(a-c)\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)-b\left(2a-2c+\sqrt{a^2+b^2-2ac+c^2}\right)\tan[d+ex] \right) \right] \right) / \\
 & \left( \sqrt{2}\sqrt{2a-2c+\sqrt{a^2+b^2-2ac+c^2}} \sqrt{a^2-b^2-2ac+c^2-(a-c)\sqrt{a^2+b^2-2ac+c^2}} \right. \\
 & \quad \left. \sqrt{a+b \tan[d+ex]+c \tan[d+ex]^2} \right) \Bigg) / \\
 & \left( \sqrt{2}\left(a^2+b^2-2ac+c^2\right)^{3/2}e \right) - \frac{2\left(b^2-2ac+bc \tan[d+ex]\right)}{a\left(b^2-4ac\right)e\sqrt{a+b \tan[d+ex]+c \tan[d+ex]^2}} + \\
 & \frac{2\text{Cot}[d+ex]^2\left(b^2-2ac+bc \tan[d+ex]\right)}{a\left(b^2-4ac\right)e\sqrt{a+b \tan[d+ex]+c \tan[d+ex]^2}} + \\
 & \frac{2\left(a\left(b^2-2(a-c)c\right)+bc(a+c)\tan[d+ex]\right)}{\left(b^2+(a-c)^2\right)\left(b^2-4ac\right)e\sqrt{a+b \tan[d+ex]+c \tan[d+ex]^2}} + \\
 & \frac{b\left(15b^2-52ac\right)\text{Cot}[d+ex]\sqrt{a+b \tan[d+ex]+c \tan[d+ex]^2}}{4a^3\left(b^2-4ac\right)e} - \\
 & \frac{\left(5b^2-12ac\right)\text{Cot}[d+ex]^2\sqrt{a+b \tan[d+ex]+c \tan[d+ex]^2}}{2a^2\left(b^2-4ac\right)e}
 \end{aligned}$$

Result(type ?, 788811 leaves): Display of huge result suppressed!

### Problem 27: Humongous result has more than 200000 leaves.

$$\int \tan [d+e x]^5 \sqrt{a+b \tan [d+e x]^2+c \tan [d+e x]^4} d x$$

Optimal (type 3, 270 leaves, 9 steps):

$$\frac{\sqrt{a-b+c} \operatorname{ArcTanh}\left[\frac{2 a-b+(b-2 c) \tan [d+e x]^2}{2 \sqrt{a-b+c} \sqrt{a+b \tan [d+e x]^2+c \tan [d+e x]^4}}\right]}{2 e} + \frac{1}{32 c^{5/2} e}$$

$$\frac{(b^3+2 b^2 c-4 b(a-2 c) c-8 c^2(a+2 c)) \operatorname{ArcTanh}\left[\frac{b+2 c \tan [d+e x]^2}{2 \sqrt{c} \sqrt{a+b \tan [d+e x]^2+c \tan [d+e x]^4}}\right]}{2 \sqrt{c} \sqrt{a+b \tan [d+e x]^2+c \tan [d+e x]^4}} -$$

$$\frac{1}{16 c^2 e} \left( (b-2 c)(b+4 c)+2 c(b+2 c) \tan [d+e x]^2 \right) \sqrt{a+b \tan [d+e x]^2+c \tan [d+e x]^4} +$$

$$\frac{(a+b \tan [d+e x]^2+c \tan [d+e x]^4)^{3/2}}{6 c e}$$

Result (type ?, 421511 leaves): Display of huge result suppressed!

### Problem 28: Humongous result has more than 200000 leaves.

$$\int \tan [d+e x]^3 \sqrt{a+b \tan [d+e x]^2+c \tan [d+e x]^4} d x$$

Optimal (type 3, 209 leaves, 8 steps):

$$\frac{\sqrt{a-b+c} \operatorname{ArcTanh}\left[\frac{2 a-b+(b-2 c) \tan [d+e x]^2}{2 \sqrt{a-b+c} \sqrt{a+b \tan [d+e x]^2+c \tan [d+e x]^4}}\right]}{2 e} -$$

$$\frac{(b^2+4 b c-4 c(a+2 c)) \operatorname{ArcTanh}\left[\frac{b+2 c \tan [d+e x]^2}{2 \sqrt{c} \sqrt{a+b \tan [d+e x]^2+c \tan [d+e x]^4}}\right]}{16 c^{3/2} e} +$$

$$\frac{(b-4 c+2 c \tan [d+e x]^2) \sqrt{a+b \tan [d+e x]^2+c \tan [d+e x]^4}}{8 c e}$$

Result (type ?, 307606 leaves): Display of huge result suppressed!

### Problem 29: Humongous result has more than 200000 leaves.

$$\int \tan [d+e x] \sqrt{a+b \tan [d+e x]^2+c \tan [d+e x]^4} d x$$

Optimal (type 3, 179 leaves, 8 steps):

$$\begin{aligned}
 & - \frac{\sqrt{a-b+c} \operatorname{ArcTanh}\left[\frac{2a-b+(b-2c)\operatorname{Tan}[d+ex]^2}{2\sqrt{a-b+c}\sqrt{a+b\operatorname{Tan}[d+ex]^2+c\operatorname{Tan}[d+ex]^4}}\right]}{2e} + \\
 & \frac{(b-2c)\operatorname{ArcTanh}\left[\frac{b+2c\operatorname{Tan}[d+ex]^2}{2\sqrt{c}\sqrt{a+b\operatorname{Tan}[d+ex]^2+c\operatorname{Tan}[d+ex]^4}}\right]}{4\sqrt{c}e} + \frac{\sqrt{a+b\operatorname{Tan}[d+ex]^2+c\operatorname{Tan}[d+ex]^4}}{2e}
 \end{aligned}$$

Result (type ?, 216968 leaves): Display of huge result suppressed!

**Problem 32: Result unnecessarily involves imaginary or complex numbers.**

$$\int \operatorname{Tan}[d+ex]^2 \sqrt{a+b\operatorname{Tan}[d+ex]^2+c\operatorname{Tan}[d+ex]^4} dx$$

Optimal (type 4, 1254 leaves, 14 steps):

$$\begin{aligned}
 & - \frac{\sqrt{a-b+c} \operatorname{ArcTan}\left[\frac{\sqrt{a-b+c} \operatorname{Tan}[d+ex]}{\sqrt{a+b\operatorname{Tan}[d+ex]^2+c\operatorname{Tan}[d+ex]^4}}\right]}{2e} + \frac{\operatorname{Tan}[d+ex] \sqrt{a+b\operatorname{Tan}[d+ex]^2+c\operatorname{Tan}[d+ex]^4}}{3e} + \\
 & \frac{b \operatorname{Tan}[d+ex] \sqrt{a+b\operatorname{Tan}[d+ex]^2+c\operatorname{Tan}[d+ex]^4}}{3\sqrt{c}e(\sqrt{a}+\sqrt{c}\operatorname{Tan}[d+ex]^2)} - \\
 & \frac{\sqrt{c} \operatorname{Tan}[d+ex] \sqrt{a+b\operatorname{Tan}[d+ex]^2+c\operatorname{Tan}[d+ex]^4}}{e(\sqrt{a}+\sqrt{c}\operatorname{Tan}[d+ex]^2)} - \\
 & \left( a^{1/4} b \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} \operatorname{Tan}[d+ex]}{a^{1/4}}\right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a}\sqrt{c}}\right)\right] (\sqrt{a}+\sqrt{c}\operatorname{Tan}[d+ex]^2) \right. \\
 & \left. \sqrt{\frac{a+b\operatorname{Tan}[d+ex]^2+c\operatorname{Tan}[d+ex]^4}{(\sqrt{a}+\sqrt{c}\operatorname{Tan}[d+ex]^2)^2}} \right) / \left( 3c^{3/4}e\sqrt{a+b\operatorname{Tan}[d+ex]^2+c\operatorname{Tan}[d+ex]^4} \right) + \\
 & \left( a^{1/4} c^{1/4} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} \operatorname{Tan}[d+ex]}{a^{1/4}}\right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a}\sqrt{c}}\right)\right] (\sqrt{a}+\sqrt{c}\operatorname{Tan}[d+ex]^2) \right. \\
 & \left. \sqrt{\frac{a+b\operatorname{Tan}[d+ex]^2+c\operatorname{Tan}[d+ex]^4}{(\sqrt{a}+\sqrt{c}\operatorname{Tan}[d+ex]^2)^2}} \right) / \left( e\sqrt{a+b\operatorname{Tan}[d+ex]^2+c\operatorname{Tan}[d+ex]^4} \right) + \\
 & \left( a^{1/4} (b+2\sqrt{a}\sqrt{c}) \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} \operatorname{Tan}[d+ex]}{a^{1/4}}\right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a}\sqrt{c}}\right)\right] \right. \\
 & \left. (\sqrt{a}+\sqrt{c}\operatorname{Tan}[d+ex]^2) \sqrt{\frac{a+b\operatorname{Tan}[d+ex]^2+c\operatorname{Tan}[d+ex]^4}{(\sqrt{a}+\sqrt{c}\operatorname{Tan}[d+ex]^2)^2}} \right) /
 \end{aligned}$$

$$\begin{aligned}
 & \left( 6 c^{3/4} e^{\sqrt{a+b \tan [d+e x]^2+c \tan [d+e x]^4}} \right) - \\
 & \left( (b+\sqrt{a} \sqrt{c}-c) \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \tan [d+e x]}{a^{1/4}}\right], \frac{1}{4}\left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right] \right. \\
 & \quad \left. (\sqrt{a}+\sqrt{c} \tan [d+e x]^2) \sqrt{\frac{a+b \tan [d+e x]^2+c \tan [d+e x]^4}{(\sqrt{a}+\sqrt{c} \tan [d+e x]^2)^2}} \right) / \\
 & \left( 2 a^{1/4} c^{1/4} e^{\sqrt{a+b \tan [d+e x]^2+c \tan [d+e x]^4}} \right) + \\
 & \left( c^{1/4} (a-b+c) \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \tan [d+e x]}{a^{1/4}}\right], \frac{1}{4}\left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right] \right. \\
 & \quad \left. (\sqrt{a}+\sqrt{c} \tan [d+e x]^2) \sqrt{\frac{a+b \tan [d+e x]^2+c \tan [d+e x]^4}{(\sqrt{a}+\sqrt{c} \tan [d+e x]^2)^2}} \right) / \\
 & \left( 2 a^{1/4} (\sqrt{a}-\sqrt{c}) e^{\sqrt{a+b \tan [d+e x]^2+c \tan [d+e x]^4}} \right) - \\
 & \left( (\sqrt{a}+\sqrt{c})(a-b+c) \text{EllipticPi}\left[-\frac{(\sqrt{a}-\sqrt{c})^2}{4 \sqrt{a} \sqrt{c}}, 2 \text{ArcTan}\left[\frac{c^{1/4} \tan [d+e x]}{a^{1/4}}\right], \right. \right. \\
 & \quad \left. \left. \frac{1}{4}\left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right] (\sqrt{a}+\sqrt{c} \tan [d+e x]^2) \sqrt{\frac{a+b \tan [d+e x]^2+c \tan [d+e x]^4}{(\sqrt{a}+\sqrt{c} \tan [d+e x]^2)^2}} \right) / \\
 & \left( 4 a^{1/4} (\sqrt{a}-\sqrt{c}) c^{1/4} e^{\sqrt{a+b \tan [d+e x]^2+c \tan [d+e x]^4}} \right)
 \end{aligned}$$

Result(type 4, 639 leaves):

$$\frac{1}{e} \sqrt{\left( (3a + b + 3c + 4a \cos[2(d+ex)] - 4c \cos[2(d+ex)] + a \cos[4(d+ex)] - b \cos[4(d+ex)] + c \cos[4(d+ex)]) / (3 + 4 \cos[2(d+ex)] + \cos[4(d+ex)]) \right) \left( \frac{(b-3c) \sin[2(d+ex)]}{6c} + \frac{1}{3} \tan[d+ex] \right) + \frac{1}{12ce \sqrt{a+b \tan[d+ex]^2+c \tan[d+ex]^4}} \left( \frac{1}{\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}} \right. \\ \left. i \sqrt{2} \left( (b-3c) \left( -b+\sqrt{b^2-4ac} \right) \text{EllipticE} \left[ i \text{ArcSinh} \left[ \sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4ac}}} \tan[d+ex] \right], \frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}} \right] + \left( b^2-b \left( -3c+\sqrt{b^2-4ac} \right) + c \left( -4a-6c+3\sqrt{b^2-4ac} \right) \right) \right. \right. \\ \left. \left. \text{EllipticF} \left[ i \text{ArcSinh} \left[ \sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4ac}}} \tan[d+ex] \right], \frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}} \right] + 6c(a-b+c) \text{EllipticPi} \left[ \frac{b+\sqrt{b^2-4ac}}{2c}, i \text{ArcSinh} \left[ \sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4ac}}} \tan[d+ex] \right], \frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}} \right] \right) \sqrt{\frac{b+\sqrt{b^2-4ac}+2c \tan[d+ex]^2}{b+\sqrt{b^2-4ac}}} \sqrt{1+\frac{2c \tan[d+ex]^2}{b-\sqrt{b^2-4ac}}} - \frac{4(b-3c) \tan[d+ex] (a+b \tan[d+ex]^2+c \tan[d+ex]^4)}{1+\tan[d+ex]^2} \right)$$

**Problem 33: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{a+b \tan [d+e x]^2+c \tan [d+e x]^4} d x$$

Optimal (type 4, 829 leaves, 8 steps):

$$\begin{aligned}
 & \frac{\sqrt{a-b+c} \operatorname{ArcTan}\left[\frac{\sqrt{a-b+c} \operatorname{Tan}[d+ex]}{\sqrt{a+b \operatorname{Tan}[d+ex]^2+c \operatorname{Tan}[d+ex]^4}}\right]}{2e} + \\
 & \frac{\sqrt{c} \operatorname{Tan}[d+ex] \sqrt{a+b \operatorname{Tan}[d+ex]^2+c \operatorname{Tan}[d+ex]^4}}{e(\sqrt{a}+\sqrt{c} \operatorname{Tan}[d+ex]^2)} - \\
 & \left( a^{1/4} c^{1/4} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} \operatorname{Tan}[d+ex]}{a^{1/4}}\right], \frac{1}{4}\left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right] (\sqrt{a}+\sqrt{c} \operatorname{Tan}[d+ex]^2) \right. \\
 & \left. \sqrt{\frac{a+b \operatorname{Tan}[d+ex]^2+c \operatorname{Tan}[d+ex]^4}{(\sqrt{a}+\sqrt{c} \operatorname{Tan}[d+ex]^2)^2}} \right) / \left( e \sqrt{a+b \operatorname{Tan}[d+ex]^2+c \operatorname{Tan}[d+ex]^4} \right) + \\
 & \left( (b+\sqrt{a} \sqrt{c}-c) \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} \operatorname{Tan}[d+ex]}{a^{1/4}}\right], \frac{1}{4}\left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right] \right. \\
 & \left. (\sqrt{a}+\sqrt{c} \operatorname{Tan}[d+ex]^2) \sqrt{\frac{a+b \operatorname{Tan}[d+ex]^2+c \operatorname{Tan}[d+ex]^4}{(\sqrt{a}+\sqrt{c} \operatorname{Tan}[d+ex]^2)^2}} \right) / \\
 & \left( 2 a^{1/4} c^{1/4} e \sqrt{a+b \operatorname{Tan}[d+ex]^2+c \operatorname{Tan}[d+ex]^4} \right) - \\
 & \left( c^{1/4} (a-b+c) \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} \operatorname{Tan}[d+ex]}{a^{1/4}}\right], \frac{1}{4}\left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right] \right. \\
 & \left. (\sqrt{a}+\sqrt{c} \operatorname{Tan}[d+ex]^2) \sqrt{\frac{a+b \operatorname{Tan}[d+ex]^2+c \operatorname{Tan}[d+ex]^4}{(\sqrt{a}+\sqrt{c} \operatorname{Tan}[d+ex]^2)^2}} \right) / \\
 & \left( 2 a^{1/4} (\sqrt{a}-\sqrt{c}) e \sqrt{a+b \operatorname{Tan}[d+ex]^2+c \operatorname{Tan}[d+ex]^4} \right) + \\
 & \left( (\sqrt{a}+\sqrt{c})(a-b+c) \operatorname{EllipticPi}\left[-\frac{(\sqrt{a}-\sqrt{c})^2}{4 \sqrt{a} \sqrt{c}}, 2 \operatorname{ArcTan}\left[\frac{c^{1/4} \operatorname{Tan}[d+ex]}{a^{1/4}}\right], \right. \right. \\
 & \left. \left. \frac{1}{4}\left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right] (\sqrt{a}+\sqrt{c} \operatorname{Tan}[d+ex]^2) \sqrt{\frac{a+b \operatorname{Tan}[d+ex]^2+c \operatorname{Tan}[d+ex]^4}{(\sqrt{a}+\sqrt{c} \operatorname{Tan}[d+ex]^2)^2}} \right) / \\
 & \left( 4 a^{1/4} (\sqrt{a}-\sqrt{c}) c^{1/4} e \sqrt{a+b \operatorname{Tan}[d+ex]^2+c \operatorname{Tan}[d+ex]^4} \right)
 \end{aligned}$$

Result(type 4, 428 leaves):

$$\frac{1}{2 \sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4ac}}} e^{\sqrt{a+b \tan [d+e x]^2+c \tan [d+e x]^4}}$$

$$i \left( \left( -b+\sqrt{b^2-4ac} \right) \text{EllipticE} \left[ i \text{ArcSinh} \left[ \sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4ac}}} \tan [d+e x] \right], \frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}} \right] - \right.$$

$$\left. \left( b-2c+\sqrt{b^2-4ac} \right) \text{EllipticF} \left[ i \text{ArcSinh} \left[ \sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4ac}}} \tan [d+e x] \right], \right.$$

$$\left. \frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}} \right] - 2(a-b+c) \text{EllipticPi} \left[ \frac{b+\sqrt{b^2-4ac}}{2c}, \right.$$

$$\left. i \text{ArcSinh} \left[ \sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4ac}}} \tan [d+e x] \right], \frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}} \right]$$

$$\sqrt{\frac{b+\sqrt{b^2-4ac}+2c \tan [d+e x]^2}{b+\sqrt{b^2-4ac}}} \sqrt{1-\frac{2c \tan [d+e x]^2}{-b+\sqrt{b^2-4ac}}}$$

**Problem 34: Result unnecessarily involves imaginary or complex numbers.**

$$\int \cot [d+e x]^2 \sqrt{a+b \tan [d+e x]^2+c \tan [d+e x]^4} dx$$

Optimal (type 4, 861 leaves, 9 steps):



$$\begin{aligned}
 & - \frac{\sqrt{a-b+c} \operatorname{ArcTan}\left[\frac{\sqrt{a-b+c} \operatorname{Tan}[d+e x]}{\sqrt{a+b \operatorname{Tan}[d+e x]^2+c \operatorname{Tan}[d+e x]^4}}\right]}{2 e} - \frac{\operatorname{Cot}[d+e x] \sqrt{a+b \operatorname{Tan}[d+e x]^2+c \operatorname{Tan}[d+e x]^4}}{e} + \\
 & \frac{\sqrt{c} \operatorname{Tan}[d+e x] \sqrt{a+b \operatorname{Tan}[d+e x]^2+c \operatorname{Tan}[d+e x]^4}}{e\left(\sqrt{a}+\sqrt{c} \operatorname{Tan}[d+e x]^2\right)} - \\
 & \left(a^{1/4} c^{1/4} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} \operatorname{Tan}[d+e x]}{a^{1/4}}\right], \frac{1}{4}\left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right]\left(\sqrt{a}+\sqrt{c} \operatorname{Tan}[d+e x]^2\right)\right. \\
 & \left.\sqrt{\frac{a+b \operatorname{Tan}[d+e x]^2+c \operatorname{Tan}[d+e x]^4}{\left(\sqrt{a}+\sqrt{c} \operatorname{Tan}[d+e x]^2\right)^2}}\right) / \left(e \sqrt{a+b \operatorname{Tan}[d+e x]^2+c \operatorname{Tan}[d+e x]^4}\right) + \\
 & \left(\left(\sqrt{a}+\sqrt{c}\right) c^{1/4} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} \operatorname{Tan}[d+e x]}{a^{1/4}}\right], \frac{1}{4}\left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right]\right. \\
 & \left.\left(\sqrt{a}+\sqrt{c} \operatorname{Tan}[d+e x]^2\right) \sqrt{\frac{a+b \operatorname{Tan}[d+e x]^2+c \operatorname{Tan}[d+e x]^4}{\left(\sqrt{a}+\sqrt{c} \operatorname{Tan}[d+e x]^2\right)^2}}\right) / \\
 & \left(2 a^{1/4} e \sqrt{a+b \operatorname{Tan}[d+e x]^2+c \operatorname{Tan}[d+e x]^4}\right) + \\
 & \left(c^{1/4}(a-b+c) \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} \operatorname{Tan}[d+e x]}{a^{1/4}}\right], \frac{1}{4}\left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right]\right. \\
 & \left.\left(\sqrt{a}+\sqrt{c} \operatorname{Tan}[d+e x]^2\right) \sqrt{\frac{a+b \operatorname{Tan}[d+e x]^2+c \operatorname{Tan}[d+e x]^4}{\left(\sqrt{a}+\sqrt{c} \operatorname{Tan}[d+e x]^2\right)^2}}\right) / \\
 & \left(2 a^{1/4}\left(\sqrt{a}-\sqrt{c}\right) e \sqrt{a+b \operatorname{Tan}[d+e x]^2+c \operatorname{Tan}[d+e x]^4}\right) - \\
 & \left(\left(\sqrt{a}+\sqrt{c}\right)(a-b+c) \operatorname{EllipticPi}\left[-\frac{\left(\sqrt{a}-\sqrt{c}\right)^2}{4 \sqrt{a} \sqrt{c}}, 2 \operatorname{ArcTan}\left[\frac{c^{1/4} \operatorname{Tan}[d+e x]}{a^{1/4}}\right],\right.\right. \\
 & \left.\left.\frac{1}{4}\left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right]\left(\sqrt{a}+\sqrt{c} \operatorname{Tan}[d+e x]^2\right) \sqrt{\frac{a+b \operatorname{Tan}[d+e x]^2+c \operatorname{Tan}[d+e x]^4}{\left(\sqrt{a}+\sqrt{c} \operatorname{Tan}[d+e x]^2\right)^2}}\right) / \\
 & \left(4 a^{1/4}\left(\sqrt{a}-\sqrt{c}\right) c^{1/4} e \sqrt{a+b \operatorname{Tan}[d+e x]^2+c \operatorname{Tan}[d+e x]^4}\right)
 \end{aligned}$$

Result (type 4, 1258 leaves):

$$\frac{1}{e} \sqrt{\left(\left(3 a+b+3 c+4 a \operatorname{Cos}\left[2(d+e x)\right]-4 c \operatorname{Cos}\left[2(d+e x)\right]+a \operatorname{Cos}\left[4(d+e x)\right]-\right.\right.}$$

$$\left.\left. b \operatorname{Cos}\left[4(d+e x)\right]+c \operatorname{Cos}\left[4(d+e x)\right]\right) / \left(3+4 \operatorname{Cos}\left[2(d+e x)\right]+\operatorname{Cos}\left[4(d+e x)\right]\right)}$$

$$\begin{aligned}
 & \left( -\text{Cot}[d+ex] + \frac{1}{2} \text{Sin}[2(d+ex)] \right) + \left( i \sqrt{2} \left( -b + \sqrt{b^2 - 4ac} \right) \right. \\
 & \left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4ac}}} \text{Tan}[d+ex] \right], \frac{b + \sqrt{b^2 - 4ac}}{b - \sqrt{b^2 - 4ac}} \right] - \right. \\
 & \left. \text{EllipticF} \left[ i \text{ArcSinh} \left[ \sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4ac}}} \text{Tan}[d+ex] \right], \frac{b + \sqrt{b^2 - 4ac}}{b - \sqrt{b^2 - 4ac}} \right] \right) \\
 & (1 + \text{Tan}[d+ex]^2) \sqrt{\frac{b + \sqrt{b^2 - 4ac} + 2c \text{Tan}[d+ex]^2}{b + \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2c \text{Tan}[d+ex]^2}{b - \sqrt{b^2 - 4ac}}} - \\
 & 2i \sqrt{2} c \text{EllipticF} \left[ i \text{ArcSinh} \left[ \sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4ac}}} \text{Tan}[d+ex] \right], \frac{b + \sqrt{b^2 - 4ac}}{b - \sqrt{b^2 - 4ac}} \right] \\
 & (1 + \text{Tan}[d+ex]^2) \sqrt{\frac{b + \sqrt{b^2 - 4ac} + 2c \text{Tan}[d+ex]^2}{b + \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2c \text{Tan}[d+ex]^2}{b - \sqrt{b^2 - 4ac}}} + 2i \sqrt{2} a \\
 & \text{EllipticPi} \left[ \frac{b + \sqrt{b^2 - 4ac}}{2c}, i \text{ArcSinh} \left[ \sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4ac}}} \text{Tan}[d+ex] \right], \frac{b + \sqrt{b^2 - 4ac}}{b - \sqrt{b^2 - 4ac}} \right] \\
 & (1 + \text{Tan}[d+ex]^2) \sqrt{\frac{b + \sqrt{b^2 - 4ac} + 2c \text{Tan}[d+ex]^2}{b + \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2c \text{Tan}[d+ex]^2}{b - \sqrt{b^2 - 4ac}}} - 2i \sqrt{2} b \\
 & \text{EllipticPi} \left[ \frac{b + \sqrt{b^2 - 4ac}}{2c}, i \text{ArcSinh} \left[ \sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4ac}}} \text{Tan}[d+ex] \right], \frac{b + \sqrt{b^2 - 4ac}}{b - \sqrt{b^2 - 4ac}} \right] \\
 & (1 + \text{Tan}[d+ex]^2) \sqrt{\frac{b + \sqrt{b^2 - 4ac} + 2c \text{Tan}[d+ex]^2}{b + \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2c \text{Tan}[d+ex]^2}{b - \sqrt{b^2 - 4ac}}} + 2i \sqrt{2} c \\
 & \text{EllipticPi} \left[ \frac{b + \sqrt{b^2 - 4ac}}{2c}, i \text{ArcSinh} \left[ \sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4ac}}} \text{Tan}[d+ex] \right], \frac{b + \sqrt{b^2 - 4ac}}{b - \sqrt{b^2 - 4ac}} \right] \\
 & (1 + \text{Tan}[d+ex]^2) \sqrt{\frac{b + \sqrt{b^2 - 4ac} + 2c \text{Tan}[d+ex]^2}{b + \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2c \text{Tan}[d+ex]^2}{b - \sqrt{b^2 - 4ac}}} - \\
 & 4 \sqrt{\frac{c}{b + \sqrt{b^2 - 4ac}}} \text{Tan}[d+ex] (a + b \text{Tan}[d+ex]^2 + c \text{Tan}[d+ex]^4) \Big/ \\
 & \left( 4 \sqrt{\frac{c}{b + \sqrt{b^2 - 4ac}}} e (1 + \text{Tan}[d+ex]^2) \sqrt{a + b \text{Tan}[d+ex]^2 + c \text{Tan}[d+ex]^4} \right)
 \end{aligned}$$

**Problem 35: Result unnecessarily involves imaginary or complex numbers.**

$$\int \text{Cot}[d+ex]^4 \sqrt{a+b \text{Tan}[d+ex]^2+c \text{Tan}[d+ex]^4} dx$$

Optimal (type 4, 943 leaves, 10 steps):

$$\frac{\sqrt{a-b+c} \text{ArcTan}\left[\frac{\sqrt{a-b+c} \text{Tan}[d+ex]}{\sqrt{a+b \text{Tan}[d+ex]^2+c \text{Tan}[d+ex]^4}}\right]}{2e} +$$

$$\frac{(3a-b) \text{Cot}[d+ex] \sqrt{a+b \text{Tan}[d+ex]^2+c \text{Tan}[d+ex]^4}}{3ae} -$$

$$\frac{\text{Cot}[d+ex]^3 \sqrt{a+b \text{Tan}[d+ex]^2+c \text{Tan}[d+ex]^4}}{3e} -$$

$$\frac{(3a-b) \sqrt{c} \text{Tan}[d+ex] \sqrt{a+b \text{Tan}[d+ex]^2+c \text{Tan}[d+ex]^4}}{3ae (\sqrt{a} + \sqrt{c} \text{Tan}[d+ex]^2)} +$$

$$\left( (3a-b) c^{1/4} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \text{Tan}[d+ex]}{a^{1/4}}\right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a} \sqrt{c}}\right)\right] (\sqrt{a} + \sqrt{c} \text{Tan}[d+ex]^2) \right.$$

$$\left. \sqrt{\frac{a+b \text{Tan}[d+ex]^2+c \text{Tan}[d+ex]^4}{(\sqrt{a} + \sqrt{c} \text{Tan}[d+ex]^2)^2}} \right) / \left( 3a^{3/4} e \sqrt{a+b \text{Tan}[d+ex]^2+c \text{Tan}[d+ex]^4} \right) -$$

$$\left( (3a-b+\sqrt{a} \sqrt{c}) c^{1/4} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \text{Tan}[d+ex]}{a^{1/4}}\right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a} \sqrt{c}}\right)\right] \right.$$

$$\left. (\sqrt{a} + \sqrt{c} \text{Tan}[d+ex]^2) \sqrt{\frac{a+b \text{Tan}[d+ex]^2+c \text{Tan}[d+ex]^4}{(\sqrt{a} + \sqrt{c} \text{Tan}[d+ex]^2)^2}} \right) /$$

$$\left( 6a^{3/4} e \sqrt{a+b \text{Tan}[d+ex]^2+c \text{Tan}[d+ex]^4} \right) -$$

$$\left( c^{1/4} (a-b+c) \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \text{Tan}[d+ex]}{a^{1/4}}\right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a} \sqrt{c}}\right)\right] \right.$$

$$\left. (\sqrt{a} + \sqrt{c} \text{Tan}[d+ex]^2) \sqrt{\frac{a+b \text{Tan}[d+ex]^2+c \text{Tan}[d+ex]^4}{(\sqrt{a} + \sqrt{c} \text{Tan}[d+ex]^2)^2}} \right) /$$

$$\left( 2a^{1/4} (\sqrt{a} - \sqrt{c}) e \sqrt{a+b \text{Tan}[d+ex]^2+c \text{Tan}[d+ex]^4} \right) +$$

$$\left( (\sqrt{a} + \sqrt{c}) (a - b + c) \text{EllipticPi}\left[-\frac{(\sqrt{a} - \sqrt{c})^2}{4\sqrt{a}\sqrt{c}}, 2 \text{ArcTan}\left[\frac{c^{1/4} \text{Tan}[d + e x]}{a^{1/4}}\right], \right. \right. \\ \left. \left. \frac{1}{4} \left(2 - \frac{b}{\sqrt{a}\sqrt{c}}\right) \right] (\sqrt{a} + \sqrt{c} \text{Tan}[d + e x]^2) \sqrt{\frac{a + b \text{Tan}[d + e x]^2 + c \text{Tan}[d + e x]^4}{(\sqrt{a} + \sqrt{c} \text{Tan}[d + e x]^2)^2}} \right) / \\ \left( 4 a^{1/4} (\sqrt{a} - \sqrt{c}) c^{1/4} e \sqrt{a + b \text{Tan}[d + e x]^2 + c \text{Tan}[d + e x]^4} \right)$$

Result (type 4, 1590 leaves):

$$\frac{1}{e} \sqrt{\left( (3 a + b + 3 c + 4 a \text{Cos}[2 (d + e x)] - 4 c \text{Cos}[2 (d + e x)] + a \text{Cos}[4 (d + e x)] - \right. \\ \left. b \text{Cos}[4 (d + e x)] + c \text{Cos}[4 (d + e x)]) / (3 + 4 \text{Cos}[2 (d + e x)] + \text{Cos}[4 (d + e x)]) \right) \\ \left( \frac{(4 a \text{Cos}[d + e x] - b \text{Cos}[d + e x]) \text{Csc}[d + e x]}{3 a} - \frac{1}{3} \text{Cot}[d + e x] \text{Csc}[d + e x]^2 - \right. \\ \left. \frac{(3 a - b) \text{Sin}[2 (d + e x)]}{6 a} \right) + \left( 3 i \sqrt{2} a (b - \sqrt{b^2 - 4 a c}) \right. \\ \left( \text{EllipticE}\left[ i \text{ArcSinh}\left[ \sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} \text{Tan}[d + e x] \right], \frac{b + \sqrt{b^2 - 4 a c}}{b - \sqrt{b^2 - 4 a c}} \right] - \text{EllipticF}\left[ \right. \right. \\ \left. \left. i \text{ArcSinh}\left[ \sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} \text{Tan}[d + e x] \right], \frac{b + \sqrt{b^2 - 4 a c}}{b - \sqrt{b^2 - 4 a c}} \right] \right) (1 + \text{Tan}[d + e x]^2) \\ \sqrt{\frac{b + \sqrt{b^2 - 4 a c} + 2 c \text{Tan}[d + e x]^2}{b + \sqrt{b^2 - 4 a c}}} \sqrt{1 + \frac{2 c \text{Tan}[d + e x]^2}{b - \sqrt{b^2 - 4 a c}}} + i \sqrt{2} b (-b + \sqrt{b^2 - 4 a c}) \\ \left( \text{EllipticE}\left[ i \text{ArcSinh}\left[ \sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} \text{Tan}[d + e x] \right], \frac{b + \sqrt{b^2 - 4 a c}}{b - \sqrt{b^2 - 4 a c}} \right] - \right. \\ \left. \text{EllipticF}\left[ i \text{ArcSinh}\left[ \sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} \text{Tan}[d + e x] \right], \frac{b + \sqrt{b^2 - 4 a c}}{b - \sqrt{b^2 - 4 a c}} \right] \right) \\ (1 + \text{Tan}[d + e x]^2) \sqrt{\frac{b + \sqrt{b^2 - 4 a c} + 2 c \text{Tan}[d + e x]^2}{b + \sqrt{b^2 - 4 a c}}} \sqrt{1 + \frac{2 c \text{Tan}[d + e x]^2}{b - \sqrt{b^2 - 4 a c}}} + \\ 2 i \sqrt{2} a c \text{EllipticF}\left[ i \text{ArcSinh}\left[ \sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} \text{Tan}[d + e x] \right], \frac{b + \sqrt{b^2 - 4 a c}}{b - \sqrt{b^2 - 4 a c}} \right] \\ (1 + \text{Tan}[d + e x]^2) \sqrt{\frac{b + \sqrt{b^2 - 4 a c} + 2 c \text{Tan}[d + e x]^2}{b + \sqrt{b^2 - 4 a c}}} \sqrt{1 + \frac{2 c \text{Tan}[d + e x]^2}{b - \sqrt{b^2 - 4 a c}}} - 6 i \sqrt{2} a^2$$

$$\begin{aligned}
 & \text{EllipticPi}\left[\frac{b + \sqrt{b^2 - 4ac}}{2c}, i \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4ac}}} \tan[d + ex]\right], \frac{b + \sqrt{b^2 - 4ac}}{b - \sqrt{b^2 - 4ac}}\right] \\
 & (1 + \tan[d + ex]^2) \sqrt{\frac{b + \sqrt{b^2 - 4ac} + 2c \tan[d + ex]^2}{b + \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2c \tan[d + ex]^2}{b - \sqrt{b^2 - 4ac}}} + 6i\sqrt{2}ab \\
 & \text{EllipticPi}\left[\frac{b + \sqrt{b^2 - 4ac}}{2c}, i \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4ac}}} \tan[d + ex]\right], \frac{b + \sqrt{b^2 - 4ac}}{b - \sqrt{b^2 - 4ac}}\right] \\
 & (1 + \tan[d + ex]^2) \sqrt{\frac{b + \sqrt{b^2 - 4ac} + 2c \tan[d + ex]^2}{b + \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2c \tan[d + ex]^2}{b - \sqrt{b^2 - 4ac}}} - 6i\sqrt{2}ac \\
 & \text{EllipticPi}\left[\frac{b + \sqrt{b^2 - 4ac}}{2c}, i \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4ac}}} \tan[d + ex]\right], \frac{b + \sqrt{b^2 - 4ac}}{b - \sqrt{b^2 - 4ac}}\right] \\
 & (1 + \tan[d + ex]^2) \sqrt{\frac{b + \sqrt{b^2 - 4ac} + 2c \tan[d + ex]^2}{b + \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2c \tan[d + ex]^2}{b - \sqrt{b^2 - 4ac}}} - \\
 & 4(-3a + b) \sqrt{\frac{c}{b + \sqrt{b^2 - 4ac}}} \tan[d + ex] (a + b \tan[d + ex]^2 + c \tan[d + ex]^4) \Big/ \\
 & \left(12a \sqrt{\frac{c}{b + \sqrt{b^2 - 4ac}}} e (1 + \tan[d + ex]^2) \sqrt{a + b \tan[d + ex]^2 + c \tan[d + ex]^4}\right)
 \end{aligned}$$

Problem 36: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\tan[d + ex]^5}{\sqrt{a + b \tan[d + ex]^2 + c \tan[d + ex]^4}} dx$$

Optimal (type 3, 182 leaves, 8 steps):

$$\begin{aligned}
 & \frac{\text{ArcTanh}\left[\frac{2a - b + (b - 2c) \tan[d + ex]^2}{2\sqrt{a - b + c} \sqrt{a + b \tan[d + ex]^2 + c \tan[d + ex]^4}}\right]}{2\sqrt{a - b + c} e} \\
 & \frac{(b + 2c) \text{ArcTanh}\left[\frac{b + 2c \tan[d + ex]^2}{2\sqrt{c} \sqrt{a + b \tan[d + ex]^2 + c \tan[d + ex]^4}}\right]}{4c^{3/2} e} + \frac{\sqrt{a + b \tan[d + ex]^2 + c \tan[d + ex]^4}}{2ce}
 \end{aligned}$$

Result (type 4, 125619 leaves): Display of huge result suppressed!

Problem 37: Result unnecessarily involves higher level functions and more than

twice size of optimal antiderivative.

$$\int \frac{\text{Tan}[d + e x]^3}{\sqrt{a + b \text{Tan}[d + e x]^2 + c \text{Tan}[d + e x]^4}} dx$$

Optimal (type 3, 141 leaves, 7 steps):

$$\frac{\text{ArcTanh}\left[\frac{2a - b + (b - 2c) \text{Tan}[d + e x]^2}{2\sqrt{a - b + c} \sqrt{a + b \text{Tan}[d + e x]^2 + c \text{Tan}[d + e x]^4}}\right]}{2\sqrt{a - b + c} e} + \frac{\text{ArcTanh}\left[\frac{b + 2c \text{Tan}[d + e x]^2}{2\sqrt{c} \sqrt{a + b \text{Tan}[d + e x]^2 + c \text{Tan}[d + e x]^4}}\right]}{2\sqrt{c} e}$$

Result (type 4, 80416 leaves): Display of huge result suppressed!

Problem 38: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\text{Tan}[d + e x]}{\sqrt{a + b \text{Tan}[d + e x]^2 + c \text{Tan}[d + e x]^4}} dx$$

Optimal (type 3, 79 leaves, 4 steps):

$$\frac{\text{ArcTanh}\left[\frac{2a - b + (b - 2c) \text{Tan}[d + e x]^2}{2\sqrt{a - b + c} \sqrt{a + b \text{Tan}[d + e x]^2 + c \text{Tan}[d + e x]^4}}\right]}{2\sqrt{a - b + c} e}$$

Result (type 4, 57267 leaves): Display of huge result suppressed!

Problem 41: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{Tan}[d + e x]^4}{\sqrt{a + b \text{Tan}[d + e x]^2 + c \text{Tan}[d + e x]^4}} dx$$

Optimal (type 4, 662 leaves, 5 steps):

$$\begin{aligned}
 & \frac{\text{ArcTan}\left[\frac{\sqrt{a-b+c}\tan[d+ex]}{\sqrt{a+b\tan[d+ex]^2+c\tan[d+ex]^4}}\right]}{2\sqrt{a-b+c}e} + \frac{\tan[d+ex]\sqrt{a+b\tan[d+ex]^2+c\tan[d+ex]^4}}{\sqrt{c}e\left(\sqrt{a}+\sqrt{c}\tan[d+ex]^2\right)} - \\
 & \left(a^{1/4}\text{EllipticE}\left[2\text{ArcTan}\left[\frac{c^{1/4}\tan[d+ex]}{a^{1/4}}\right], \frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]\left(\sqrt{a}+\sqrt{c}\tan[d+ex]^2\right)\right. \\
 & \left.\sqrt{\frac{a+b\tan[d+ex]^2+c\tan[d+ex]^4}{\left(\sqrt{a}+\sqrt{c}\tan[d+ex]^2\right)^2}}\right) / \left(c^{3/4}e\sqrt{a+b\tan[d+ex]^2+c\tan[d+ex]^4}\right) + \\
 & \left(a^{1/4}\left(\sqrt{a}-2\sqrt{c}\right)\text{EllipticF}\left[2\text{ArcTan}\left[\frac{c^{1/4}\tan[d+ex]}{a^{1/4}}\right], \frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]\right. \\
 & \left.\left(\sqrt{a}+\sqrt{c}\tan[d+ex]^2\right)\sqrt{\frac{a+b\tan[d+ex]^2+c\tan[d+ex]^4}{\left(\sqrt{a}+\sqrt{c}\tan[d+ex]^2\right)^2}}\right) / \\
 & \left(2\left(\sqrt{a}-\sqrt{c}\right)c^{3/4}e\sqrt{a+b\tan[d+ex]^2+c\tan[d+ex]^4}\right) + \\
 & \left(\left(\sqrt{a}+\sqrt{c}\right)\text{EllipticPi}\left[-\frac{\left(\sqrt{a}-\sqrt{c}\right)^2}{4\sqrt{a}\sqrt{c}}, 2\text{ArcTan}\left[\frac{c^{1/4}\tan[d+ex]}{a^{1/4}}\right], \frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]\right. \\
 & \left.\left(\sqrt{a}+\sqrt{c}\tan[d+ex]^2\right)\sqrt{\frac{a+b\tan[d+ex]^2+c\tan[d+ex]^4}{\left(\sqrt{a}+\sqrt{c}\tan[d+ex]^2\right)^2}}\right) / \\
 & \left(4a^{1/4}\left(\sqrt{a}-\sqrt{c}\right)c^{1/4}e\sqrt{a+b\tan[d+ex]^2+c\tan[d+ex]^4}\right)
 \end{aligned}$$

Result (type 4, 579 leaves):

$$\frac{1}{2 c e} \sqrt{\left( (3 a + b + 3 c + 4 a \cos [2 (d + e x)] - 4 c \cos [2 (d + e x)] + a \cos [4 (d + e x)] - b \cos [4 (d + e x)] + c \cos [4 (d + e x)]) / (3 + 4 \cos [2 (d + e x)] + \cos [4 (d + e x)]) \right)}$$

$$\sin [2 (d + e x)] + \left( \frac{1}{\sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}}} i \sqrt{2} \left( (-b + \sqrt{b^2 - 4 a c}) \text{EllipticE} \left[ i \text{ArcSinh} \left[ \sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} \tan [d + e x] \right], \frac{b + \sqrt{b^2 - 4 a c}}{b - \sqrt{b^2 - 4 a c}} \right] + (b + 2 c - \sqrt{b^2 - 4 a c}) \text{EllipticF} \left[ i \text{ArcSinh} \left[ \sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} \tan [d + e x] \right], \frac{b + \sqrt{b^2 - 4 a c}}{b - \sqrt{b^2 - 4 a c}} \right] - 2 c \text{EllipticPi} \left[ \frac{b + \sqrt{b^2 - 4 a c}}{2 c}, i \text{ArcSinh} \left[ \sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} \tan [d + e x] \right], \frac{b + \sqrt{b^2 - 4 a c}}{b - \sqrt{b^2 - 4 a c}} \right] \sqrt{\frac{b + \sqrt{b^2 - 4 a c} + 2 c \tan [d + e x]^2}{b + \sqrt{b^2 - 4 a c}}} \right) \sqrt{1 + \frac{2 c \tan [d + e x]^2}{b - \sqrt{b^2 - 4 a c}}} - \frac{4 \tan [d + e x] (a + b \tan [d + e x]^2 + c \tan [d + e x]^4)}{1 + \tan [d + e x]^2} \right) / (4 c e \sqrt{a + b \tan [d + e x]^2 + c \tan [d + e x]^4})$$

**Problem 42: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\tan [d + e x]^2}{\sqrt{a + b \tan [d + e x]^2 + c \tan [d + e x]^4}} dx$$

Optimal (type 4, 436 leaves, 4 steps):



$$\begin{aligned}
 & - \frac{\text{ArcTan}\left[\frac{\sqrt{a-b+c} \text{Tan}[d+e x]}{\sqrt{a+b \text{Tan}[d+e x]^2+c \text{Tan}[d+e x]^4}}\right]}{2 \sqrt{a-b+c} e} + \\
 & \left( a^{1/4} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \text{Tan}[d+e x]}{a^{1/4}}\right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a} \sqrt{c}}\right)\right] \right. \\
 & \quad \left. \left(\sqrt{a} + \sqrt{c} \text{Tan}[d+e x]^2\right) \sqrt{\frac{a+b \text{Tan}[d+e x]^2+c \text{Tan}[d+e x]^4}{\left(\sqrt{a} + \sqrt{c} \text{Tan}[d+e x]^2\right)^2}}\right) / \\
 & \quad \left(2 \left(\sqrt{a} - \sqrt{c}\right) c^{1/4} e \sqrt{a+b \text{Tan}[d+e x]^2+c \text{Tan}[d+e x]^4}\right) - \\
 & \quad \left(\left(\sqrt{a} + \sqrt{c}\right) \text{EllipticPi}\left[-\frac{\left(\sqrt{a} - \sqrt{c}\right)^2}{4 \sqrt{a} \sqrt{c}}, 2 \text{ArcTan}\left[\frac{c^{1/4} \text{Tan}[d+e x]}{a^{1/4}}\right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a} \sqrt{c}}\right)\right] \right. \\
 & \quad \left. \left(\sqrt{a} + \sqrt{c} \text{Tan}[d+e x]^2\right) \sqrt{\frac{a+b \text{Tan}[d+e x]^2+c \text{Tan}[d+e x]^4}{\left(\sqrt{a} + \sqrt{c} \text{Tan}[d+e x]^2\right)^2}}\right) / \\
 & \quad \left(4 a^{1/4} \left(\sqrt{a} - \sqrt{c}\right) c^{1/4} e \sqrt{a+b \text{Tan}[d+e x]^2+c \text{Tan}[d+e x]^4}\right)
 \end{aligned}$$

Result (type 4, 311 leaves):

$$\begin{aligned}
 & - \left( \left( \text{EllipticF}\left[\text{i ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4ac}}} \text{Tan}[d+e x]\right], \frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right] - \text{EllipticPi}\left[\right. \right. \\
 & \quad \left. \left. \frac{b+\sqrt{b^2-4ac}}{2c}, \text{i ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4ac}}} \text{Tan}[d+e x]\right], \frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right] \right) \\
 & \quad \left. \sqrt{\frac{b+\sqrt{b^2-4ac}+2c \text{Tan}[d+e x]^2}{b+\sqrt{b^2-4ac}}} \sqrt{1+\frac{2c \text{Tan}[d+e x]^2}{b-\sqrt{b^2-4ac}}}\right) / \\
 & \quad \left( \sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4ac}}} e \sqrt{a+b \text{Tan}[d+e x]^2+c \text{Tan}[d+e x]^4} \right)
 \end{aligned}$$

**Problem 43: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\sqrt{a+b \text{Tan}[d+e x]^2+c \text{Tan}[d+e x]^4}} dx$$

Optimal (type 4, 436 leaves, 4 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{a-b+c}\tan[d+ex]}{\sqrt{a+b\tan[d+ex]^2+c\tan[d+ex]^4}}\right]}{2\sqrt{a-b+c}e} - \left( c^{1/4} \text{EllipticF}\left[2\text{ArcTan}\left[\frac{c^{1/4}\tan[d+ex]}{a^{1/4}}\right], \frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right] (\sqrt{a}+\sqrt{c}\tan[d+ex]^2) \sqrt{\frac{a+b\tan[d+ex]^2+c\tan[d+ex]^4}{(\sqrt{a}+\sqrt{c}\tan[d+ex]^2)^2}} \right) / \left( 2a^{1/4}(\sqrt{a}-\sqrt{c})e\sqrt{a+b\tan[d+ex]^2+c\tan[d+ex]^4} + (\sqrt{a}+\sqrt{c})\text{EllipticPi}\left[-\frac{(\sqrt{a}-\sqrt{c})^2}{4\sqrt{a}\sqrt{c}}, 2\text{ArcTan}\left[\frac{c^{1/4}\tan[d+ex]}{a^{1/4}}\right], \frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right] (\sqrt{a}+\sqrt{c}\tan[d+ex]^2) \sqrt{\frac{a+b\tan[d+ex]^2+c\tan[d+ex]^4}{(\sqrt{a}+\sqrt{c}\tan[d+ex]^2)^2}} \right) / \left( 4a^{1/4}(\sqrt{a}-\sqrt{c})c^{1/4}e\sqrt{a+b\tan[d+ex]^2+c\tan[d+ex]^4} \right)$$

Result (type 4, 235 leaves):

$$- \left( \left( \text{EllipticPi}\left[\frac{b+\sqrt{b^2-4ac}}{2c}, \text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\tan[d+ex]\right], \frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}} \right] \sqrt{\frac{b+\sqrt{b^2-4ac}+2c\tan[d+ex]^2}{b+\sqrt{b^2-4ac}}} \sqrt{1-\frac{2c\tan[d+ex]^2}{-b+\sqrt{b^2-4ac}}} \right) / \left( \sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}e\sqrt{a+b\tan[d+ex]^2+c\tan[d+ex]^4} \right) \right)$$

**Problem 44: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\text{Cot}[d+ex]^2}{\sqrt{a+b\tan[d+ex]^2+c\tan[d+ex]^4}} dx$$

Optimal (type 4, 707 leaves, 7 steps):

$$\begin{aligned}
 & - \frac{\text{ArcTan}\left[\frac{\sqrt{a-b+c} \tan[d+ex]}{\sqrt{a+b \tan[d+ex]^2+c \tan[d+ex]^4}}\right]}{2 \sqrt{a-b+c} e} - \frac{\text{Cot}[d+ex] \sqrt{a+b \tan[d+ex]^2+c \tan[d+ex]^4}}{a e} + \\
 & \frac{\sqrt{c} \tan[d+ex] \sqrt{a+b \tan[d+ex]^2+c \tan[d+ex]^4}}{a e (\sqrt{a} + \sqrt{c} \tan[d+ex]^2)} - \\
 & \left( c^{1/4} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \tan[d+ex]}{a^{1/4}}\right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a} \sqrt{c}}\right)\right] (\sqrt{a} + \sqrt{c} \tan[d+ex]^2) \right. \\
 & \left. \sqrt{\frac{a+b \tan[d+ex]^2+c \tan[d+ex]^4}{(\sqrt{a} + \sqrt{c} \tan[d+ex]^2)^2}} \right) / \left( a^{3/4} e \sqrt{a+b \tan[d+ex]^2+c \tan[d+ex]^4} \right) + \\
 & \left( (2 \sqrt{a} - \sqrt{c}) c^{1/4} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \tan[d+ex]}{a^{1/4}}\right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a} \sqrt{c}}\right)\right] \right. \\
 & \left. (\sqrt{a} + \sqrt{c} \tan[d+ex]^2) \sqrt{\frac{a+b \tan[d+ex]^2+c \tan[d+ex]^4}{(\sqrt{a} + \sqrt{c} \tan[d+ex]^2)^2}} \right) / \\
 & \left( 2 a^{3/4} (\sqrt{a} - \sqrt{c}) e \sqrt{a+b \tan[d+ex]^2+c \tan[d+ex]^4} \right) - \\
 & \left( (\sqrt{a} + \sqrt{c}) \text{EllipticPi}\left[-\frac{(\sqrt{a} - \sqrt{c})^2}{4 \sqrt{a} \sqrt{c}}, 2 \text{ArcTan}\left[\frac{c^{1/4} \tan[d+ex]}{a^{1/4}}\right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a} \sqrt{c}}\right)\right] \right. \\
 & \left. (\sqrt{a} + \sqrt{c} \tan[d+ex]^2) \sqrt{\frac{a+b \tan[d+ex]^2+c \tan[d+ex]^4}{(\sqrt{a} + \sqrt{c} \tan[d+ex]^2)^2}} \right) / \\
 & \left( 4 a^{1/4} (\sqrt{a} - \sqrt{c}) c^{1/4} e \sqrt{a+b \tan[d+ex]^2+c \tan[d+ex]^4} \right)
 \end{aligned}$$

Result(type 4, 735 leaves):

$$\frac{1}{e} \sqrt{\left( (3a + b + 3c + 4a \cos[2(d+ex)] - 4c \cos[2(d+ex)] + a \cos[4(d+ex)] - b \cos[4(d+ex)] + c \cos[4(d+ex)]) / (3 + 4 \cos[2(d+ex)] + \cos[4(d+ex)]) \right) \left( -\frac{\cot[d+ex]}{a} + \frac{\sin[2(d+ex)]}{2a} \right) + \frac{1}{ae} \left( \left( i \left( -b + \sqrt{b^2 - 4ac} \right) \left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \sqrt{2} \sqrt{-\frac{c}{-b - \sqrt{b^2 - 4ac}}} \tan[d+ex]} \right], \frac{-b - \sqrt{b^2 - 4ac}}{-b + \sqrt{b^2 - 4ac}} \right] - \text{EllipticF} \left[ i \text{ArcSinh} \left[ \sqrt{2} \sqrt{-\frac{c}{-b - \sqrt{b^2 - 4ac}}} \tan[d+ex]} \right], \frac{-b - \sqrt{b^2 - 4ac}}{-b + \sqrt{b^2 - 4ac}} \right] \right) \sqrt{1 - \frac{2c \tan[d+ex]^2}{-b - \sqrt{b^2 - 4ac}}} \sqrt{1 - \frac{2c \tan[d+ex]^2}{-b + \sqrt{b^2 - 4ac}}} \right) / \left( 2 \sqrt{2} \sqrt{-\frac{c}{-b - \sqrt{b^2 - 4ac}}} \sqrt{a + b \tan[d+ex]^2 + c \tan[d+ex]^4} \right) + \left( i a \text{EllipticPi} \left[ -\frac{-b - \sqrt{b^2 - 4ac}}{2c}, i \text{ArcSinh} \left[ \sqrt{2} \sqrt{-\frac{c}{-b - \sqrt{b^2 - 4ac}}} \tan[d+ex]} \right], \frac{-b - \sqrt{b^2 - 4ac}}{-b + \sqrt{b^2 - 4ac}} \right] \sqrt{1 - \frac{2c \tan[d+ex]^2}{-b - \sqrt{b^2 - 4ac}}} \sqrt{1 - \frac{2c \tan[d+ex]^2}{-b + \sqrt{b^2 - 4ac}}} \right) / \left( \sqrt{2} \sqrt{-\frac{c}{-b - \sqrt{b^2 - 4ac}}} \sqrt{a + b \tan[d+ex]^2 + c \tan[d+ex]^4} \right) - \frac{\tan[d+ex] \sqrt{a + b \tan[d+ex]^2 + c \tan[d+ex]^4}}{1 + \tan[d+ex]^2} \right)$$

Problem 45: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\tan[d+ex]^7}{(a + b \tan[d+ex]^2 + c \tan[d+ex]^4)^{3/2}} dx$$

Optimal (type 3, 235 leaves, 8 steps):

$$\frac{\text{ArcTanh} \left[ \frac{2a - b + (b - 2c) \tan[d+ex]^2}{2\sqrt{a-b+c} \sqrt{a+b \tan[d+ex]^2 + c \tan[d+ex]^4}} \right]}{2(a-b+c)^{3/2} e} + \frac{\text{ArcTanh} \left[ \frac{b + 2c \tan[d+ex]^2}{2\sqrt{c} \sqrt{a+b \tan[d+ex]^2 + c \tan[d+ex]^4}} \right]}{2c^{3/2} e} + \frac{a(b^2 - a(b+2c)) + (b^3 + 2a^2c - ab(b+3c)) \tan[d+ex]^2}{c(a-b+c)(b^2 - 4ac) e \sqrt{a+b \tan[d+ex]^2 + c \tan[d+ex]^4}}$$

Result (type 4, 182 725 leaves): Display of huge result suppressed!

**Problem 46: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Tan}[d + e x]^5}{(a + b \text{Tan}[d + e x]^2 + c \text{Tan}[d + e x]^4)^{3/2}} dx$$

Optimal (type 3, 159 leaves, 6 steps):

$$\frac{\text{ArcTanh}\left[\frac{2 a-b+(b-2 c) \text{Tan}[d+e x]^2}{2 \sqrt{a-b+c} \sqrt{a+b \text{Tan}[d+e x]^2+c \text{Tan}[d+e x]^4}}\right]}{2(a-b+c)^{3/2} e} + \frac{a(2 a-b)+((a-b) b+2 a c) \text{Tan}[d+e x]^2}{(a-b+c)\left(b^2-4 a c\right) e \sqrt{a+b \text{Tan}[d+e x]^2+c \text{Tan}[d+e x]^4}}$$

Result (type 4, 57 597 leaves): Display of huge result suppressed!

**Problem 47: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Tan}[d + e x]^3}{(a + b \text{Tan}[d + e x]^2 + c \text{Tan}[d + e x]^4)^{3/2}} dx$$

Optimal (type 3, 154 leaves, 6 steps):

$$\frac{\text{ArcTanh}\left[\frac{2 a-b+(b-2 c) \text{Tan}[d+e x]^2}{2 \sqrt{a-b+c} \sqrt{a+b \text{Tan}[d+e x]^2+c \text{Tan}[d+e x]^4}}\right]}{2(a-b+c)^{3/2} e} - \frac{a(b-2 c)+(2 a-b) c \text{Tan}[d+e x]^2}{(a-b+c)\left(b^2-4 a c\right) e \sqrt{a+b \text{Tan}[d+e x]^2+c \text{Tan}[d+e x]^4}}$$

Result (type 4, 57 592 leaves): Display of huge result suppressed!

**Problem 48: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Tan}[d + e x]}{(a + b \text{Tan}[d + e x]^2 + c \text{Tan}[d + e x]^4)^{3/2}} dx$$

Optimal (type 3, 155 leaves, 6 steps):

$$\begin{aligned}
 & - \frac{\text{ArcTanh}\left[\frac{2a-b+(b-2c)\text{Tan}[d+ex]^2}{2\sqrt{a-b+c}\sqrt{a+b\text{Tan}[d+ex]^2+c\text{Tan}[d+ex]^4}}\right]}{2(a-b+c)^{3/2}e} + \\
 & \frac{b^2-2ac-bc+(b-2c)c\text{Tan}[d+ex]^2}{(a-b+c)(b^2-4ac)e\sqrt{a+b\text{Tan}[d+ex]^2+c\text{Tan}[d+ex]^4}}
 \end{aligned}$$

Result (type 4, 57615 leaves): Display of huge result suppressed!

**Problem 49: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Cot}[d+ex]}{(a+b\text{Tan}[d+ex]^2+c\text{Tan}[d+ex]^4)^{3/2}} dx$$

Optimal (type 3, 280 leaves, 12 steps):

$$\begin{aligned}
 & - \frac{\text{ArcTanh}\left[\frac{2a+b\text{Tan}[d+ex]^2}{2\sqrt{a}\sqrt{a+b\text{Tan}[d+ex]^2+c\text{Tan}[d+ex]^4}}\right]}{2a^{3/2}e} + \frac{\text{ArcTanh}\left[\frac{2a-b+(b-2c)\text{Tan}[d+ex]^2}{2\sqrt{a-b+c}\sqrt{a+b\text{Tan}[d+ex]^2+c\text{Tan}[d+ex]^4}}\right]}{2(a-b+c)^{3/2}e} + \\
 & \frac{b^2-2ac+bc\text{Tan}[d+ex]^2}{a(b^2-4ac)e\sqrt{a+b\text{Tan}[d+ex]^2+c\text{Tan}[d+ex]^4}} - \\
 & \frac{b^2-2ac-bc+(b-2c)c\text{Tan}[d+ex]^2}{(a-b+c)(b^2-4ac)e\sqrt{a+b\text{Tan}[d+ex]^2+c\text{Tan}[d+ex]^4}}
 \end{aligned}$$

Result (type 3, 694 leaves):

$$\begin{aligned}
 & \frac{1}{e} \sqrt{\left( (3a + b + 3c + 4a \cos[2(dx)]) - \right. \\
 & \quad \left. 4c \cos[2(dx)] + a \cos[4(dx)] - b \cos[4(dx)] + c \cos[4(dx)] \right) /} \\
 & \quad \left( (3 + 4 \cos[2(dx)] + \cos[4(dx)]) \right) \left( - \frac{-b^3 + 3abc + 2b^2c - 4ac^2 - bc^2}{a(a-b+c)^2(-b^2+4ac)} - \right. \\
 & \quad \left. (4(b^4 - 4ab^2c - b^3c + 2a^2c^2 + 3abc^2 - b^2c^2 + 2ac^3 + bc^3 + b^4 \cos[2(dx)] - \right. \\
 & \quad \quad \left. 4ab^2c \cos[2(dx)] - 3b^3c \cos[2(dx)] + 2a^2c^2 \cos[2(dx)] + 9abc^2 \cos[ \right. \\
 & \quad \quad \left. 2(dx)] + 3b^2c^2 \cos[2(dx)] - 6ac^3 \cos[2(dx)] - bc^3 \cos[2(dx)]) \right) /} \\
 & \quad \left( a(a-b+c)^2(-b^2+4ac) (3a + b + 3c + 4a \cos[2(dx)] - 4c \cos[2(dx)] + \right. \\
 & \quad \quad \left. a \cos[4(dx)] - b \cos[4(dx)] + c \cos[4(dx)]) \right) \Bigg) + \\
 & \frac{1}{2a^{3/2}(a-b+c)e} \left( - \frac{a^{3/2} \operatorname{Log}[\operatorname{Sec}[dx]^2]}{\sqrt{a-b+c}} + (a-b+c) \operatorname{Log}[\operatorname{Tan}[dx]^2] - \right. \\
 & \quad a \operatorname{Log}\left[2a + b \operatorname{Tan}[dx]^2 + 2\sqrt{a} \sqrt{a + b \operatorname{Tan}[dx]^2 + c \operatorname{Tan}[dx]^4}\right] + \\
 & \quad b \operatorname{Log}\left[2a + b \operatorname{Tan}[dx]^2 + 2\sqrt{a} \sqrt{a + b \operatorname{Tan}[dx]^2 + c \operatorname{Tan}[dx]^4}\right] - \\
 & \quad c \operatorname{Log}\left[2a + b \operatorname{Tan}[dx]^2 + 2\sqrt{a} \sqrt{a + b \operatorname{Tan}[dx]^2 + c \operatorname{Tan}[dx]^4}\right] + \frac{1}{\sqrt{a-b+c}} \\
 & \quad \left. a^{3/2} \operatorname{Log}\left[2a - b + (b - 2c) \operatorname{Tan}[dx]^2 + 2\sqrt{a-b+c} \sqrt{a + b \operatorname{Tan}[dx]^2 + c \operatorname{Tan}[dx]^4}\right] \right)
 \end{aligned}$$

**Problem 51: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Tan}[dx]^2}{(a + b \operatorname{Tan}[dx]^2 + c \operatorname{Tan}[dx]^4)^{3/2}} dx$$

Optimal (type 4, 981 leaves, 9 steps):

$$\begin{aligned}
 & - \frac{\text{ArcTan}\left[\frac{\sqrt{a-b+c} \text{Tan}[d+ex]}{\sqrt{a+b \text{Tan}[d+ex]^2+c \text{Tan}[d+ex]^4}}\right]}{2(a-b+c)^{3/2} e} + \frac{\text{Tan}[d+ex] (b^2-2ac-bc+(b-2c)c \text{Tan}[d+ex]^2)}{(a-b+c)(b^2-4ac) e \sqrt{a+b \text{Tan}[d+ex]^2+c \text{Tan}[d+ex]^4}} - \\
 & \frac{(b-2c)\sqrt{c} \text{Tan}[d+ex] \sqrt{a+b \text{Tan}[d+ex]^2+c \text{Tan}[d+ex]^4}}{(a-b+c)(b^2-4ac) e (\sqrt{a}+\sqrt{c} \text{Tan}[d+ex]^2)} + \\
 & \left[ a^{1/4} (b-2c) c^{1/4} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \text{Tan}[d+ex]}{a^{1/4}}\right], \frac{1}{4} \left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right] \right. \\
 & \left. (\sqrt{a}+\sqrt{c} \text{Tan}[d+ex]^2) \sqrt{\frac{a+b \text{Tan}[d+ex]^2+c \text{Tan}[d+ex]^4}{(\sqrt{a}+\sqrt{c} \text{Tan}[d+ex]^2)^2}} \right] / \\
 & \left( (a-b+c)(b^2-4ac) e \sqrt{a+b \text{Tan}[d+ex]^2+c \text{Tan}[d+ex]^4} \right) + \\
 & \left[ c^{1/4} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \text{Tan}[d+ex]}{a^{1/4}}\right], \frac{1}{4} \left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right] \right. \\
 & \left. (\sqrt{a}+\sqrt{c} \text{Tan}[d+ex]^2) \sqrt{\frac{a+b \text{Tan}[d+ex]^2+c \text{Tan}[d+ex]^4}{(\sqrt{a}+\sqrt{c} \text{Tan}[d+ex]^2)^2}} \right] / \\
 & \left( 2 a^{1/4} (\sqrt{a}-\sqrt{c})(a-b+c) e \sqrt{a+b \text{Tan}[d+ex]^2+c \text{Tan}[d+ex]^4} \right) - \\
 & \left[ (\sqrt{a}-\sqrt{c}) c^{1/4} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \text{Tan}[d+ex]}{a^{1/4}}\right], \frac{1}{4} \left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right] \right. \\
 & \left. (\sqrt{a}+\sqrt{c} \text{Tan}[d+ex]^2) \sqrt{\frac{a+b \text{Tan}[d+ex]^2+c \text{Tan}[d+ex]^4}{(\sqrt{a}+\sqrt{c} \text{Tan}[d+ex]^2)^2}} \right] / \\
 & \left( 2 a^{1/4} (b-2\sqrt{a}\sqrt{c})(a-b+c) e \sqrt{a+b \text{Tan}[d+ex]^2+c \text{Tan}[d+ex]^4} \right) - \\
 & \left[ (\sqrt{a}+\sqrt{c}) \text{EllipticPi}\left[-\frac{(\sqrt{a}-\sqrt{c})^2}{4\sqrt{a}\sqrt{c}}, 2 \text{ArcTan}\left[\frac{c^{1/4} \text{Tan}[d+ex]}{a^{1/4}}\right], \frac{1}{4} \left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right] \right. \\
 & \left. (\sqrt{a}+\sqrt{c} \text{Tan}[d+ex]^2) \sqrt{\frac{a+b \text{Tan}[d+ex]^2+c \text{Tan}[d+ex]^4}{(\sqrt{a}+\sqrt{c} \text{Tan}[d+ex]^2)^2}} \right] / \\
 & \left( 4 a^{1/4} (\sqrt{a}-\sqrt{c}) c^{1/4} (a-b+c) e \sqrt{a+b \text{Tan}[d+ex]^2+c \text{Tan}[d+ex]^4} \right)
 \end{aligned}$$

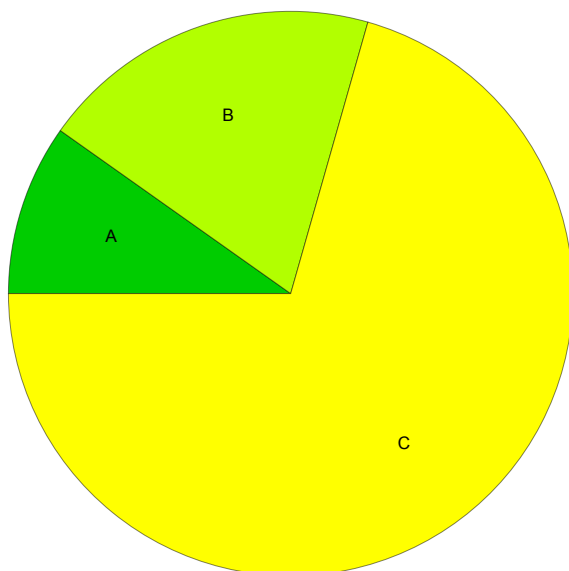
Result(type 4, 831 leaves):



$$\begin{aligned}
 & \frac{1}{e} \sqrt{\left( (3a + b + 3c + 4a \cos[2(d+ex)] - 4c \cos[2(d+ex)] + a \cos[4(d+ex)] - \right. \\
 & \quad \left. b \cos[4(d+ex)] + c \cos[4(d+ex)]) / (3 + 4 \cos[2(d+ex)] + \cos[4(d+ex)]) \right)} \\
 & \left( \frac{(b-2c) \sin[2(d+ex)]}{2(-a+b-c)(b^2-4ac)} + (2b^2 \sin[2(d+ex)] - 4ac \sin[2(d+ex)] - 4c^2 \sin[2(d+ex)] + \right. \\
 & \quad \left. b^2 \sin[4(d+ex)] - 2ac \sin[4(d+ex)] - 2bc \sin[4(d+ex)] + 2c^2 \sin[4(d+ex)]) / \right. \\
 & \quad \left. ((a-b+c)(-b^2+4ac)(-3a-b-3c-4a \cos[2(d+ex)] + 4c \cos[2(d+ex)] - \right. \\
 & \quad \left. a \cos[4(d+ex)] + b \cos[4(d+ex)] - c \cos[4(d+ex)])) \right) + \\
 & \frac{1}{4(a-b+c)(-b^2+4ac)e \sqrt{a+b \tan[d+ex]^2+c \tan[d+ex]^4}} \\
 & \left( \frac{1}{\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}} \right. \\
 & \quad \left. i \sqrt{2} \left( (b-2c) \left( -b + \sqrt{b^2-4ac} \right) \text{EllipticE} \left[ i \text{ArcSinh} \left[ \sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4ac}}} \tan[d+ex] \right], \right. \right. \right. \\
 & \quad \left. \left. \frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}} \right] + \left( b^2 - b \sqrt{b^2-4ac} + 2c \left( -2a + \sqrt{b^2-4ac} \right) \right) \right. \\
 & \quad \left. \text{EllipticF} \left[ i \text{ArcSinh} \left[ \sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4ac}}} \tan[d+ex] \right], \frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}} \right] - \right. \\
 & \quad \left. 2(b^2-4ac) \text{EllipticPi} \left[ \frac{b+\sqrt{b^2-4ac}}{2c}, i \text{ArcSinh} \left[ \sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4ac}}} \tan[d+ex] \right], \right. \right. \\
 & \quad \left. \left. \frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}} \right] \right) \sqrt{\frac{b+\sqrt{b^2-4ac}+2c \tan[d+ex]^2}{b+\sqrt{b^2-4ac}}} \sqrt{1 + \frac{2c \tan[d+ex]^2}{b-\sqrt{b^2-4ac}}} - \\
 & \left. \frac{4(b-2c) \tan[d+ex] (a+b \tan[d+ex]^2+c \tan[d+ex]^4)}{1+\tan[d+ex]^2} \right)
 \end{aligned}$$

## Summary of Integration Test Results

51 integration problems



A - 5 optimal antiderivatives

B - 10 more than twice size of optimal antiderivatives

C - 36 unnecessarily complex antiderivatives

D - 0 unable to integrate problems

E - 0 integration timeouts