

# Mathematica 11.3 Integration Test Results

Test results for the 61 problems in "4.4.10 (c+d x)^m (a+b cot)^n.m"

Problem 3: Result more than twice size of optimal antiderivative.

$$\int x \cot [a + b x] dx$$

Optimal (type 4, 53 leaves, 4 steps):

$$-\frac{i x^2}{2} + \frac{x \operatorname{Log}\left[1 - e^{2 i (a+b x)}\right]}{b} - \frac{i \operatorname{PolyLog}\left[2, e^{2 i (a+b x)}\right]}{2 b^2}$$

Result (type 4, 166 leaves):

$$\frac{1}{2} x^2 \cot [a] - \left( \operatorname{Csc}[a] \operatorname{Sec}[a] \left( b^2 e^{i \operatorname{ArcTan}[\operatorname{Tan}[a]]} x^2 + \frac{1}{\sqrt{1 + \operatorname{Tan}[a]^2}} (i b x (-\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[a]]) - \pi \operatorname{Log}\left[1 + e^{-2 i b x}\right] - 2 (b x + \operatorname{ArcTan}[\operatorname{Tan}[a]]) \operatorname{Log}\left[1 - e^{2 i (b x + \operatorname{ArcTan}[\operatorname{Tan}[a])}\right]\right) + \pi \operatorname{Log}[\operatorname{Cos}[b x]] + 2 \operatorname{ArcTan}[\operatorname{Tan}[a]] \operatorname{Log}[\operatorname{Sin}[b x + \operatorname{ArcTan}[\operatorname{Tan}[a]]]] + i \operatorname{PolyLog}\left[2, e^{2 i (b x + \operatorname{ArcTan}[\operatorname{Tan}[a])}\right]) \operatorname{Tan}[a] \right) \Bigg/ \left( 2 b^2 \sqrt{\operatorname{Sec}[a]^2 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} \right)$$

Problem 7: Result more than twice size of optimal antiderivative.

$$\int x^2 \cot [a + b x]^2 dx$$

Optimal (type 4, 74 leaves, 6 steps):

$$-\frac{i x^2}{b} - \frac{x^3}{3} - \frac{x^2 \cot [a + b x]}{b} + \frac{2 x \operatorname{Log}\left[1 - e^{2 i (a+b x)}\right]}{b^2} - \frac{i \operatorname{PolyLog}\left[2, e^{2 i (a+b x)}\right]}{b^3}$$

Result (type 4, 181 leaves):

$$-\frac{x^3}{3} + \frac{x^2 \operatorname{Csc}[a] \operatorname{Csc}[a + b x] \operatorname{Sin}[b x]}{b} - \left( \operatorname{Csc}[a] \operatorname{Sec}[a] \left( b^2 e^{i \operatorname{ArcTan}[\operatorname{Tan}[a]]} x^2 + \frac{1}{\sqrt{1 + \operatorname{Tan}[a]^2}} (i b x (-\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[a]]) \right) - \pi \operatorname{Log}[1 + e^{-2 i b x}] - 2 (b x + \operatorname{ArcTan}[\operatorname{Tan}[a]]) \operatorname{Log}[1 - e^{2 i (b x + \operatorname{ArcTan}[\operatorname{Tan}[a])}] \right] + \pi \operatorname{Log}[\operatorname{Cos}[b x]] + 2 \operatorname{ArcTan}[\operatorname{Tan}[a]] \operatorname{Log}[\operatorname{Sin}[b x + \operatorname{ArcTan}[\operatorname{Tan}[a]]]] + i \operatorname{PolyLog}[2, e^{2 i (b x + \operatorname{ArcTan}[\operatorname{Tan}[a])}] \operatorname{Tan}[a] \right) \Bigg) / \left( b^3 \sqrt{\operatorname{Sec}[a]^2 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} \right)$$

**Problem 13: Result more than twice size of optimal antiderivative.**

$$\int x \operatorname{Cot}[a + b x]^3 dx$$

Optimal (type 4, 91 leaves, 7 steps):

$$-\frac{x}{2b} + \frac{i x^2}{2} - \frac{\operatorname{Cot}[a + b x]}{2b^2} - \frac{x \operatorname{Cot}[a + b x]^2}{2b} - \frac{x \operatorname{Log}[1 - e^{2 i (a + b x)}]}{b} + \frac{i \operatorname{PolyLog}[2, e^{2 i (a + b x)}]}{2b^2}$$

Result (type 4, 201 leaves):

$$-\frac{1}{2} x^2 \operatorname{Cot}[a] - \frac{x \operatorname{Csc}[a + b x]^2}{2b} + \frac{\operatorname{Csc}[a] \operatorname{Csc}[a + b x] \operatorname{Sin}[b x]}{2b^2} + \left( \operatorname{Csc}[a] \operatorname{Sec}[a] \left( b^2 e^{i \operatorname{ArcTan}[\operatorname{Tan}[a]]} x^2 + \frac{1}{\sqrt{1 + \operatorname{Tan}[a]^2}} (i b x (-\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[a]]) \right) - \pi \operatorname{Log}[1 + e^{-2 i b x}] - 2 (b x + \operatorname{ArcTan}[\operatorname{Tan}[a]]) \operatorname{Log}[1 - e^{2 i (b x + \operatorname{ArcTan}[\operatorname{Tan}[a])}] \right] + \pi \operatorname{Log}[\operatorname{Cos}[b x]] + 2 \operatorname{ArcTan}[\operatorname{Tan}[a]] \operatorname{Log}[\operatorname{Sin}[b x + \operatorname{ArcTan}[\operatorname{Tan}[a]]]] + i \operatorname{PolyLog}[2, e^{2 i (b x + \operatorname{ArcTan}[\operatorname{Tan}[a])}] \operatorname{Tan}[a] \right) \Bigg) / \left( 2b^2 \sqrt{\operatorname{Sec}[a]^2 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} \right)$$

**Problem 37: Result more than twice size of optimal antiderivative.**

$$\int (c + d x)^3 (a + b \operatorname{Cot}[e + f x]) dx$$

Optimal (type 4, 147 leaves, 8 steps):

$$\frac{a (c + d x)^4}{4d} - \frac{i b (c + d x)^4}{4d} + \frac{b (c + d x)^3 \operatorname{Log}[1 - e^{2 i (e + f x)}]}{f} - \frac{3 i b d (c + d x)^2 \operatorname{PolyLog}[2, e^{2 i (e + f x)}]}{2 f^2} + \frac{3 b d^2 (c + d x) \operatorname{PolyLog}[3, e^{2 i (e + f x)}]}{2 f^3} + \frac{3 i b d^3 \operatorname{PolyLog}[4, e^{2 i (e + f x)}]}{4 f^4}$$

Result (type 4, 524 leaves):

$$\begin{aligned}
 & -\frac{1}{4f^3} b c d^2 e^{-ie} \operatorname{Csc}[e] \left( 2f^2 x^2 \left( 2e^{2ie} f x + 3i(-1 + e^{2ie}) \operatorname{Log}[1 - e^{2i(e+fx)}] \right) + \right. \\
 & \quad \left. 6(-1 + e^{2ie}) f x \operatorname{PolyLog}[2, e^{2i(e+fx)}] + 3i(-1 + e^{2ie}) \operatorname{PolyLog}[3, e^{2i(e+fx)}] \right) - \\
 & \frac{1}{4} b d^3 e^{ie} \operatorname{Csc}[e] \left( x^4 + (-1 + e^{-2ie}) x^4 + \frac{1}{2f^4} e^{-2ie} (-1 + e^{2ie}) \left( 2f^4 x^4 + 4i f^3 x^3 \operatorname{Log}[1 - e^{2i(e+fx)}] + \right. \right. \\
 & \quad \left. \left. 6f^2 x^2 \operatorname{PolyLog}[2, e^{2i(e+fx)}] + 6i f x \operatorname{PolyLog}[3, e^{2i(e+fx)}] - 3 \operatorname{PolyLog}[4, e^{2i(e+fx)}] \right) \right) + \\
 & \frac{1}{4} x \left( 4c^3 + 6c^2 d x + 4c d^2 x^2 + d^3 x^3 \right) \operatorname{Csc}[e] \left( b \operatorname{Cos}[e] + a \operatorname{Sin}[e] \right) + \\
 & \left( b c^3 \operatorname{Csc}[e] \left( -f x \operatorname{Cos}[e] + \operatorname{Log}[\operatorname{Cos}[f x] \operatorname{Sin}[e] + \operatorname{Cos}[e] \operatorname{Sin}[f x]] \operatorname{Sin}[e] \right) \right) / \\
 & \left( f \left( \operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2 \right) \right) - \\
 & \left( 3 b c^2 d \operatorname{Csc}[e] \operatorname{Sec}[e] \left( e^{i \operatorname{ArcTan}[\operatorname{Tan}[e]]} f^2 x^2 + \frac{1}{\sqrt{1 + \operatorname{Tan}[e]^2}} \left( i f x \left( -\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[e]] \right) \right) - \right. \right. \\
 & \quad \left. \left. \pi \operatorname{Log}[1 + e^{-2ifx}] - 2 \left( f x + \operatorname{ArcTan}[\operatorname{Tan}[e]] \right) \operatorname{Log}[1 - e^{2i(fx + \operatorname{ArcTan}[\operatorname{Tan}[e]])}] \right) + \right. \\
 & \quad \left. \left. \pi \operatorname{Log}[\operatorname{Cos}[f x]] + 2 \operatorname{ArcTan}[\operatorname{Tan}[e]] \operatorname{Log}[\operatorname{Sin}[f x + \operatorname{ArcTan}[\operatorname{Tan}[e]]] \right] \right) + \right. \\
 & \quad \left. \left. i \operatorname{PolyLog}[2, e^{2i(fx + \operatorname{ArcTan}[\operatorname{Tan}[e]])}] \operatorname{Tan}[e] \right) \right) / \left( 2f^2 \sqrt{\operatorname{Sec}[e]^2 \left( \operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2 \right)} \right)
 \end{aligned}$$

**Problem 38: Result more than twice size of optimal antiderivative.**

$$\int (c + dx)^2 (a + b \operatorname{Cot}[e + fx]) dx$$

Optimal (type 4, 112 leaves, 7 steps):

$$\begin{aligned}
 & \frac{a(c+dx)^3}{3d} - \frac{ib(c+dx)^3}{3d} + \frac{b(c+dx)^2 \operatorname{Log}[1 - e^{2i(e+fx)}]}{f} - \\
 & \frac{id(c+dx) \operatorname{PolyLog}[2, e^{2i(e+fx)}]}{f^2} + \frac{bd^2 \operatorname{PolyLog}[3, e^{2i(e+fx)}]}{2f^3}
 \end{aligned}$$

Result (type 4, 361 leaves):

$$\begin{aligned}
 & -\frac{1}{12 f^3} b d^2 e^{-i e} \operatorname{Csc}[e] \left( 2 f^2 x^2 \left( 2 e^{2 i e} f x + 3 i \left( -1 + e^{2 i e} \right) \operatorname{Log}\left[1 - e^{2 i (e+fx)}\right] \right) + \right. \\
 & \quad \left. 6 \left( -1 + e^{2 i e} \right) f x \operatorname{PolyLog}\left[2, e^{2 i (e+fx)}\right] + 3 i \left( -1 + e^{2 i e} \right) \operatorname{PolyLog}\left[3, e^{2 i (e+fx)}\right] \right) + \\
 & \frac{1}{3} x \left( 3 c^2 + 3 c d x + d^2 x^2 \right) \operatorname{Csc}[e] \left( b \operatorname{Cos}[e] + a \operatorname{Sin}[e] \right) + \\
 & \left( b c^2 \operatorname{Csc}[e] \left( -f x \operatorname{Cos}[e] + \operatorname{Log}\left[\operatorname{Cos}[f x] \operatorname{Sin}[e] + \operatorname{Cos}[e] \operatorname{Sin}[f x]\right] \operatorname{Sin}[e] \right) \right) / \\
 & \quad \left( f \left( \operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2 \right) \right) - \\
 & \left( b c d \operatorname{Csc}[e] \operatorname{Sec}[e] \left( e^{i \operatorname{ArcTan}[\operatorname{Tan}[e]]} f^2 x^2 + \frac{1}{\sqrt{1 + \operatorname{Tan}[e]^2}} \left( i f x \left( -\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[e]] \right) \right) - \right. \right. \\
 & \quad \left. \left. \pi \operatorname{Log}\left[1 + e^{-2 i f x}\right] - 2 \left( f x + \operatorname{ArcTan}[\operatorname{Tan}[e]] \right) \operatorname{Log}\left[1 - e^{2 i (f x + \operatorname{ArcTan}[\operatorname{Tan}[e]])}\right] \right) + \right. \\
 & \quad \left. \pi \operatorname{Log}\left[\operatorname{Cos}[f x]\right] + 2 \operatorname{ArcTan}[\operatorname{Tan}[e]] \operatorname{Log}\left[\operatorname{Sin}[f x + \operatorname{ArcTan}[\operatorname{Tan}[e]]]\right] \right) + \\
 & \quad \left. i \operatorname{PolyLog}\left[2, e^{2 i (f x + \operatorname{ArcTan}[\operatorname{Tan}[e]])}\right] \right) \operatorname{Tan}[e] \left. \right) / \left( f^2 \sqrt{\operatorname{Sec}[e]^2 \left( \operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2 \right)} \right)
 \end{aligned}$$

**Problem 39: Result more than twice size of optimal antiderivative.**

$$\int (c + d x) (a + b \operatorname{Cot}[e + f x]) dx$$

Optimal (type 4, 83 leaves, 6 steps):

$$\frac{a (c + d x)^2}{2 d} - \frac{i b (c + d x)^2}{2 d} + \frac{b (c + d x) \operatorname{Log}\left[1 - e^{2 i (e+fx)}\right]}{f} - \frac{i b d \operatorname{PolyLog}\left[2, e^{2 i (e+fx)}\right]}{2 f^2}$$

Result (type 4, 196 leaves):

$$\begin{aligned}
 & a c x + \frac{1}{2} a d x^2 + \frac{1}{2} b d x^2 \operatorname{Cot}[e] + \frac{b c \operatorname{Log}[\operatorname{Sin}[e + f x]]}{f} - \\
 & \left( b d \operatorname{Csc}[e] \operatorname{Sec}[e] \left( e^{i \operatorname{ArcTan}[\operatorname{Tan}[e]]} f^2 x^2 + \frac{1}{\sqrt{1 + \operatorname{Tan}[e]^2}} \left( i f x \left( -\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[e]] \right) \right) - \right. \right. \\
 & \quad \left. \left. \pi \operatorname{Log}\left[1 + e^{-2 i f x}\right] - 2 \left( f x + \operatorname{ArcTan}[\operatorname{Tan}[e]] \right) \operatorname{Log}\left[1 - e^{2 i (f x + \operatorname{ArcTan}[\operatorname{Tan}[e]])}\right] \right) + \right. \\
 & \quad \left. \pi \operatorname{Log}\left[\operatorname{Cos}[f x]\right] + 2 \operatorname{ArcTan}[\operatorname{Tan}[e]] \operatorname{Log}\left[\operatorname{Sin}[f x + \operatorname{ArcTan}[\operatorname{Tan}[e]]]\right] \right) + \\
 & \quad \left. i \operatorname{PolyLog}\left[2, e^{2 i (f x + \operatorname{ArcTan}[\operatorname{Tan}[e]])}\right] \right) \operatorname{Tan}[e] \left. \right) / \left( 2 f^2 \sqrt{\operatorname{Sec}[e]^2 \left( \operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2 \right)} \right)
 \end{aligned}$$

**Problem 42: Result more than twice size of optimal antiderivative.**

$$\int (c + d x)^3 (a + b \operatorname{Cot}[e + f x])^2 dx$$

Optimal (type 4, 295 leaves, 15 steps):

$$\begin{aligned}
 & -\frac{i b^2 (c+dx)^3}{f} + \frac{a^2 (c+dx)^4}{4d} - \frac{i a b (c+dx)^4}{2d} - \frac{b^2 (c+dx)^4}{4d} - \\
 & \frac{b^2 (c+dx)^3 \operatorname{Cot}[e+fx]}{f} + \frac{3 b^2 d (c+dx)^2 \operatorname{Log}[1 - e^{2i(e+fx)}]}{f^2} + \\
 & \frac{2 a b (c+dx)^3 \operatorname{Log}[1 - e^{2i(e+fx)}]}{f} - \frac{3 i b^2 d^2 (c+dx) \operatorname{PolyLog}[2, e^{2i(e+fx)}]}{f^3} - \\
 & \frac{3 i a b d (c+dx)^2 \operatorname{PolyLog}[2, e^{2i(e+fx)}]}{f^2} + \frac{3 b^2 d^3 \operatorname{PolyLog}[3, e^{2i(e+fx)}]}{2 f^4} + \\
 & \frac{3 a b d^2 (c+dx) \operatorname{PolyLog}[3, e^{2i(e+fx)}]}{f^3} + \frac{3 i a b d^3 \operatorname{PolyLog}[4, e^{2i(e+fx)}]}{2 f^4}
 \end{aligned}$$

Result (type 4, 1313 leaves):

$$\begin{aligned}
 & -\frac{1}{4 f^4} b^2 d^3 e^{-i e} \operatorname{Csc}[e] \left( 2 f^2 x^2 \left( 2 e^{2 i e} f x + 3 i \left( -1 + e^{2 i e} \right) \operatorname{Log}\left[1 - e^{2 i (e+f x)}\right] \right) + \right. \\
 & \quad \left. 6 \left( -1 + e^{2 i e} \right) f x \operatorname{PolyLog}\left[2, e^{2 i (e+f x)}\right] + 3 i \left( -1 + e^{2 i e} \right) \operatorname{PolyLog}\left[3, e^{2 i (e+f x)}\right] \right) - \\
 & \frac{1}{2 f^3} a b c d^2 e^{-i e} \operatorname{Csc}[e] \left( 2 f^2 x^2 \left( 2 e^{2 i e} f x + 3 i \left( -1 + e^{2 i e} \right) \operatorname{Log}\left[1 - e^{2 i (e+f x)}\right] \right) + \right. \\
 & \quad \left. 6 \left( -1 + e^{2 i e} \right) f x \operatorname{PolyLog}\left[2, e^{2 i (e+f x)}\right] + 3 i \left( -1 + e^{2 i e} \right) \operatorname{PolyLog}\left[3, e^{2 i (e+f x)}\right] \right) - \frac{1}{2} a b d^3 e^{i e} \\
 & \operatorname{Csc}[e] \left( x^4 + \left( -1 + e^{-2 i e} \right) x^4 + \frac{1}{2 f^4} e^{-2 i e} \left( -1 + e^{2 i e} \right) \left( 2 f^4 x^4 + 4 i f^3 x^3 \operatorname{Log}\left[1 - e^{2 i (e+f x)}\right] + \right. \right. \\
 & \quad \left. \left. 6 f^2 x^2 \operatorname{PolyLog}\left[2, e^{2 i (e+f x)}\right] + 6 i f x \operatorname{PolyLog}\left[3, e^{2 i (e+f x)}\right] - 3 \operatorname{PolyLog}\left[4, e^{2 i (e+f x)}\right] \right) \right) + \\
 & \left( 3 b^2 c^2 d \operatorname{Csc}[e] \left( -f x \operatorname{Cos}[e] + \operatorname{Log}\left[\operatorname{Cos}[f x] \operatorname{Sin}[e] + \operatorname{Cos}[e] \operatorname{Sin}[f x]\right] \operatorname{Sin}[e] \right) \right) / \\
 & \left( f^2 \left( \operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2 \right) \right) + \\
 & \left( 2 a b c^3 \operatorname{Csc}[e] \left( -f x \operatorname{Cos}[e] + \operatorname{Log}\left[\operatorname{Cos}[f x] \operatorname{Sin}[e] + \operatorname{Cos}[e] \operatorname{Sin}[f x]\right] \operatorname{Sin}[e] \right) \right) / \\
 & \left( f \left( \operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2 \right) \right) + \\
 & \frac{1}{8 f} \operatorname{Csc}[e] \operatorname{Csc}[e+f x] \left( 4 a^2 c^3 f x \operatorname{Cos}[f x] - 4 b^2 c^3 f x \operatorname{Cos}[f x] + 6 a^2 c^2 d f x^2 \operatorname{Cos}[f x] - \right. \\
 & \quad 6 b^2 c^2 d f x^2 \operatorname{Cos}[f x] + 4 a^2 c d^2 f x^3 \operatorname{Cos}[f x] - 4 b^2 c d^2 f x^3 \operatorname{Cos}[f x] + a^2 d^3 f x^4 \operatorname{Cos}[f x] - \\
 & \quad b^2 d^3 f x^4 \operatorname{Cos}[f x] - 4 a^2 c^3 f x \operatorname{Cos}[2 e+f x] + 4 b^2 c^3 f x \operatorname{Cos}[2 e+f x] - \\
 & \quad 6 a^2 c^2 d f x^2 \operatorname{Cos}[2 e+f x] + 6 b^2 c^2 d f x^2 \operatorname{Cos}[2 e+f x] - 4 a^2 c d^2 f x^3 \operatorname{Cos}[2 e+f x] + \\
 & \quad 4 b^2 c d^2 f x^3 \operatorname{Cos}[2 e+f x] - a^2 d^3 f x^4 \operatorname{Cos}[2 e+f x] + b^2 d^3 f x^4 \operatorname{Cos}[2 e+f x] + 8 b^2 c^3 \operatorname{Sin}[f x] + \\
 & \quad 24 b^2 c^2 d x \operatorname{Sin}[f x] + 8 a b c^3 f x \operatorname{Sin}[f x] + 24 b^2 c d^2 x^2 \operatorname{Sin}[f x] + 12 a b c^2 d f x^2 \operatorname{Sin}[f x] + \\
 & \quad 8 b^2 d^3 x^3 \operatorname{Sin}[f x] + 8 a b c d^2 f x^3 \operatorname{Sin}[f x] + 2 a b d^3 f x^4 \operatorname{Sin}[f x] + 8 a b c^3 f x \operatorname{Sin}[2 e+f x] + \\
 & \quad \left. 12 a b c^2 d f x^2 \operatorname{Sin}[2 e+f x] + 8 a b c d^2 f x^3 \operatorname{Sin}[2 e+f x] + 2 a b d^3 f x^4 \operatorname{Sin}[2 e+f x] \right) - \\
 & \left( 3 b^2 c d^2 \operatorname{Csc}[e] \operatorname{Sec}[e] \left( e^{i \operatorname{ArcTan}[\operatorname{Tan}[e]]} f^2 x^2 + \frac{1}{\sqrt{1 + \operatorname{Tan}[e]^2}} \right. \right. \\
 & \quad \left. \left. \left( i f x \left( -\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[e]] \right) \right) - \pi \operatorname{Log}\left[1 + e^{-2 i f x}\right] - 2 \left( f x + \operatorname{ArcTan}[\operatorname{Tan}[e]] \right) \right) \right. \\
 & \quad \left. \operatorname{Log}\left[1 - e^{2 i (f x + \operatorname{ArcTan}[\operatorname{Tan}[e]])}\right] + \pi \operatorname{Log}[\operatorname{Cos}[f x]] + 2 \operatorname{ArcTan}[\operatorname{Tan}[e]] \right) \\
 & \quad \left. \operatorname{Log}[\operatorname{Sin}[f x + \operatorname{ArcTan}[\operatorname{Tan}[e]]]] + i \operatorname{PolyLog}\left[2, e^{2 i (f x + \operatorname{ArcTan}[\operatorname{Tan}[e]])}\right] \right) \operatorname{Tan}[e] \right) \Bigg) / \\
 & \left( f^3 \sqrt{\operatorname{Sec}[e]^2 \left( \operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2 \right)} \right) - \left( 3 a b c^2 d \operatorname{Csc}[e] \operatorname{Sec}[e] \right. \\
 & \left. \left( e^{i \operatorname{ArcTan}[\operatorname{Tan}[e]]} f^2 x^2 + \frac{1}{\sqrt{1 + \operatorname{Tan}[e]^2}} \left( i f x \left( -\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[e]] \right) \right) - \pi \operatorname{Log}\left[1 + e^{-2 i f x}\right] - \right. \right. \\
 & \quad \left. \left. 2 \left( f x + \operatorname{ArcTan}[\operatorname{Tan}[e]] \right) \operatorname{Log}\left[1 - e^{2 i (f x + \operatorname{ArcTan}[\operatorname{Tan}[e]])}\right] + \pi \operatorname{Log}[\operatorname{Cos}[f x]] + \right. \right. \\
 & \quad \left. \left. 2 \operatorname{ArcTan}[\operatorname{Tan}[e]] \operatorname{Log}[\operatorname{Sin}[f x + \operatorname{ArcTan}[\operatorname{Tan}[e]]]] + i \operatorname{PolyLog}\left[2, e^{2 i (f x + \operatorname{ArcTan}[\operatorname{Tan}[e]])}\right] \right) \right) \right. \\
 & \quad \left. \operatorname{Tan}[e] \right) \Bigg) / \left( f^2 \sqrt{\operatorname{Sec}[e]^2 \left( \operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2 \right)} \right)
 \end{aligned}$$

### Problem 43: Result more than twice size of optimal antiderivative.

$$\int (c+dx)^2 (a+b \cot [e+fx])^2 dx$$

Optimal (type 4, 227 leaves, 13 steps):

$$\begin{aligned} & -\frac{i b^2 (c+dx)^2}{f} + \frac{a^2 (c+dx)^3}{3d} - \frac{2 i a b (c+dx)^3}{3d} - \frac{b^2 (c+dx)^3}{3d} - \\ & \frac{b^2 (c+dx)^2 \cot [e+fx]}{f} + \frac{2 b^2 d (c+dx) \operatorname{Log}[1 - e^{2i(e+fx)}]}{f^2} + \\ & \frac{2 a b (c+dx)^2 \operatorname{Log}[1 - e^{2i(e+fx)}]}{f} - \frac{i b^2 d^2 \operatorname{PolyLog}[2, e^{2i(e+fx)}]}{f^3} - \\ & \frac{2 i a b d (c+dx) \operatorname{PolyLog}[2, e^{2i(e+fx)}]}{f^2} + \frac{a b d^2 \operatorname{PolyLog}[3, e^{2i(e+fx)}]}{f^3} \end{aligned}$$

Result (type 4, 635 leaves):

$$\begin{aligned}
 & -\frac{1}{6 f^3} a b d^2 e^{-i e} \operatorname{Csc}[e] \left( 2 f^2 x^2 \left( 2 e^{2 i e} f x + 3 i \left( -1 + e^{2 i e} \right) \operatorname{Log}\left[ 1 - e^{2 i (e+f x)} \right] \right) + \right. \\
 & \quad \left. 6 \left( -1 + e^{2 i e} \right) f x \operatorname{PolyLog}\left[ 2, e^{2 i (e+f x)} \right] + 3 i \left( -1 + e^{2 i e} \right) \operatorname{PolyLog}\left[ 3, e^{2 i (e+f x)} \right] \right) + \\
 & \frac{1}{3} x \left( 3 c^2 + 3 c d x + d^2 x^2 \right) \operatorname{Csc}[e] \left( 2 a b \operatorname{Cos}[e] + a^2 \operatorname{Sin}[e] - b^2 \operatorname{Sin}[e] \right) + \\
 & \left( 2 b^2 c d \operatorname{Csc}[e] \left( -f x \operatorname{Cos}[e] + \operatorname{Log}\left[ \operatorname{Cos}[f x] \operatorname{Sin}[e] + \operatorname{Cos}[e] \operatorname{Sin}[f x] \right] \operatorname{Sin}[e] \right) \right) / \\
 & \left( f^2 \left( \operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2 \right) \right) + \\
 & \left( 2 a b c^2 \operatorname{Csc}[e] \left( -f x \operatorname{Cos}[e] + \operatorname{Log}\left[ \operatorname{Cos}[f x] \operatorname{Sin}[e] + \operatorname{Cos}[e] \operatorname{Sin}[f x] \right] \operatorname{Sin}[e] \right) \right) / \\
 & \left( f \left( \operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2 \right) \right) + \frac{1}{f} \\
 & \operatorname{Csc}[e] \operatorname{Csc}[e+f x] \left( b^2 c^2 \operatorname{Sin}[f x] + 2 b^2 c d x \operatorname{Sin}[f x] + b^2 d^2 x^2 \operatorname{Sin}[f x] \right) - \\
 & \left( b^2 d^2 \operatorname{Csc}[e] \operatorname{Sec}[e] \left( e^{i \operatorname{ArcTan}[\operatorname{Tan}[e]]} f^2 x^2 + \frac{1}{\sqrt{1 + \operatorname{Tan}[e]^2}} \right. \right. \\
 & \quad \left. \left( i f x \left( -\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[e]] \right) - \pi \operatorname{Log}\left[ 1 + e^{-2 i f x} \right] - 2 \left( f x + \operatorname{ArcTan}[\operatorname{Tan}[e]] \right) \right) \right. \\
 & \quad \left. \left. \operatorname{Log}\left[ 1 - e^{2 i (f x + \operatorname{ArcTan}[\operatorname{Tan}[e]])} \right] + \pi \operatorname{Log}\left[ \operatorname{Cos}[f x] \right] + 2 \operatorname{ArcTan}[\operatorname{Tan}[e]] \right) \right. \\
 & \quad \left. \left. \operatorname{Log}\left[ \operatorname{Sin}[f x + \operatorname{ArcTan}[\operatorname{Tan}[e]]] \right] + i \operatorname{PolyLog}\left[ 2, e^{2 i (f x + \operatorname{ArcTan}[\operatorname{Tan}[e]])} \right] \right) \operatorname{Tan}[e] \right) \right) / \\
 & \left( f^3 \sqrt{\operatorname{Sec}[e]^2 \left( \operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2 \right)} \right) - \left( 2 a b c d \operatorname{Csc}[e] \operatorname{Sec}[e] \right. \\
 & \left. \left( e^{i \operatorname{ArcTan}[\operatorname{Tan}[e]]} f^2 x^2 + \frac{1}{\sqrt{1 + \operatorname{Tan}[e]^2}} \left( i f x \left( -\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[e]] \right) - \pi \operatorname{Log}\left[ 1 + e^{-2 i f x} \right] - \right. \right. \right. \\
 & \quad \left. \left. 2 \left( f x + \operatorname{ArcTan}[\operatorname{Tan}[e]] \right) \operatorname{Log}\left[ 1 - e^{2 i (f x + \operatorname{ArcTan}[\operatorname{Tan}[e]])} \right] + \pi \operatorname{Log}\left[ \operatorname{Cos}[f x] \right] + \right. \right. \\
 & \quad \left. \left. 2 \operatorname{ArcTan}[\operatorname{Tan}[e]] \operatorname{Log}\left[ \operatorname{Sin}[f x + \operatorname{ArcTan}[\operatorname{Tan}[e]]] \right] + i \operatorname{PolyLog}\left[ 2, e^{2 i (f x + \operatorname{ArcTan}[\operatorname{Tan}[e]])} \right] \right) \right. \\
 & \quad \left. \left. \operatorname{Tan}[e] \right) \right) / \left( f^2 \sqrt{\operatorname{Sec}[e]^2 \left( \operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2 \right)} \right)
 \end{aligned}$$

**Problem 47: Result more than twice size of optimal antiderivative.**

$$\int (c+d x)^3 (a+b \operatorname{Cot}[e+f x])^3 dx$$

Optimal (type 4, 603 leaves, 28 steps):



$$\begin{aligned}
 & -\frac{3 i b^3 d (c+d x)^2}{2 f^2} - \frac{3 i a b^2 (c+d x)^3}{f} - \frac{b^3 (c+d x)^3}{2 f} + \frac{a^3 (c+d x)^4}{4 d} - \\
 & \frac{3 i a^2 b (c+d x)^4}{4 d} - \frac{3 a b^2 (c+d x)^4}{4 d} + \frac{i b^3 (c+d x)^4}{4 d} - \frac{3 b^3 d (c+d x)^2 \operatorname{Cot}[e+f x]}{2 f^2} - \\
 & \frac{3 a b^2 (c+d x)^3 \operatorname{Cot}[e+f x]}{f} - \frac{b^3 (c+d x)^3 \operatorname{Cot}[e+f x]^2}{2 f} + \frac{3 b^3 d^2 (c+d x) \operatorname{Log}\left[1-e^{2 i (e+f x)}\right]}{f^3} + \\
 & \frac{9 a b^2 d (c+d x)^2 \operatorname{Log}\left[1-e^{2 i (e+f x)}\right]}{f^2} + \frac{3 a^2 b (c+d x)^3 \operatorname{Log}\left[1-e^{2 i (e+f x)}\right]}{f} - \\
 & \frac{b^3 (c+d x)^3 \operatorname{Log}\left[1-e^{2 i (e+f x)}\right]}{f} - \frac{3 i b^3 d^3 \operatorname{PolyLog}\left[2, e^{2 i (e+f x)}\right]}{2 f^4} - \\
 & \frac{9 i a b^2 d^2 (c+d x) \operatorname{PolyLog}\left[2, e^{2 i (e+f x)}\right]}{f^3} - \frac{9 i a^2 b d (c+d x)^2 \operatorname{PolyLog}\left[2, e^{2 i (e+f x)}\right]}{2 f^2} + \\
 & \frac{3 i b^3 d (c+d x)^2 \operatorname{PolyLog}\left[2, e^{2 i (e+f x)}\right]}{2 f^2} + \frac{9 a b^2 d^3 \operatorname{PolyLog}\left[3, e^{2 i (e+f x)}\right]}{2 f^4} + \\
 & \frac{9 a^2 b d^2 (c+d x) \operatorname{PolyLog}\left[3, e^{2 i (e+f x)}\right]}{2 f^3} - \frac{3 b^3 d^2 (c+d x) \operatorname{PolyLog}\left[3, e^{2 i (e+f x)}\right]}{2 f^3} + \\
 & \frac{9 i a^2 b d^3 \operatorname{PolyLog}\left[4, e^{2 i (e+f x)}\right]}{4 f^4} - \frac{3 i b^3 d^3 \operatorname{PolyLog}\left[4, e^{2 i (e+f x)}\right]}{4 f^4}
 \end{aligned}$$

Result (type 4, 2539 leaves):

$$\begin{aligned}
 & \left(-b^3 c^3 - 3 b^3 c^2 d x - 3 b^3 c d^2 x^2 - b^3 d^3 x^3\right) \operatorname{Csc}[e+f x]^2 - \frac{1}{4 f^4} \\
 & \frac{3 a b^2 d^3 e^{-i e} \operatorname{Csc}[e] \left(2 f^2 x^2 \left(2 e^{2 i e} f x + 3 i \left(-1 + e^{2 i e}\right) \operatorname{Log}\left[1 - e^{2 i (e+f x)}\right]\right) + \right. \\
 & \left. 6 \left(-1 + e^{2 i e}\right) f x \operatorname{PolyLog}\left[2, e^{2 i (e+f x)}\right] + 3 i \left(-1 + e^{2 i e}\right) \operatorname{PolyLog}\left[3, e^{2 i (e+f x)}\right]\right) - \\
 & \frac{1}{4 f^3} 3 a^2 b c d^2 e^{-i e} \operatorname{Csc}[e] \left(2 f^2 x^2 \left(2 e^{2 i e} f x + 3 i \left(-1 + e^{2 i e}\right) \operatorname{Log}\left[1 - e^{2 i (e+f x)}\right]\right) + \right. \\
 & \left. 6 \left(-1 + e^{2 i e}\right) f x \operatorname{PolyLog}\left[2, e^{2 i (e+f x)}\right] + 3 i \left(-1 + e^{2 i e}\right) \operatorname{PolyLog}\left[3, e^{2 i (e+f x)}\right]\right) + \\
 & \frac{1}{4 f^3} b^3 c d^2 e^{-i e} \operatorname{Csc}[e] \left(2 f^2 x^2 \left(2 e^{2 i e} f x + 3 i \left(-1 + e^{2 i e}\right) \operatorname{Log}\left[1 - e^{2 i (e+f x)}\right]\right) + \right. \\
 & \left. 6 \left(-1 + e^{2 i e}\right) f x \operatorname{PolyLog}\left[2, e^{2 i (e+f x)}\right] + 3 i \left(-1 + e^{2 i e}\right) \operatorname{PolyLog}\left[3, e^{2 i (e+f x)}\right]\right) - \frac{3}{4} a^2 b d^3 e^{i e} \\
 & \operatorname{Csc}[e] \left(x^4 + \left(-1 + e^{-2 i e}\right) x^4 + \frac{1}{2 f^4} e^{-2 i e} \left(-1 + e^{2 i e}\right) \left(2 f^4 x^4 + 4 i f^3 x^3 \operatorname{Log}\left[1 - e^{2 i (e+f x)}\right] + 6 f^2 x^2 \right. \right. \\
 & \left. \left. \operatorname{PolyLog}\left[2, e^{2 i (e+f x)}\right] + 6 i f x \operatorname{PolyLog}\left[3, e^{2 i (e+f x)}\right] - 3 \operatorname{PolyLog}\left[4, e^{2 i (e+f x)}\right]\right)\right) + \frac{1}{4} b^3 d^3 \\
 & e^{i e} \operatorname{Csc}[e] \left(x^4 + \left(-1 + e^{-2 i e}\right) x^4 + \frac{1}{2 f^4} e^{-2 i e} \left(-1 + e^{2 i e}\right) \left(2 f^4 x^4 + 4 i f^3 x^3 \operatorname{Log}\left[1 - e^{2 i (e+f x)}\right] + \right. \right. \\
 & \left. \left. 6 f^2 x^2 \operatorname{PolyLog}\left[2, e^{2 i (e+f x)}\right] + 6 i f x \operatorname{PolyLog}\left[3, e^{2 i (e+f x)}\right] - 3 \operatorname{PolyLog}\left[4, e^{2 i (e+f x)}\right]\right)\right) + \\
 & \left(3 b^3 c d^2 \operatorname{Csc}[e] \left(-f x \operatorname{Cos}[e] + \operatorname{Log}\left[\operatorname{Cos}[f x] \operatorname{Sin}[e] + \operatorname{Cos}[e] \operatorname{Sin}[f x]\right] \operatorname{Sin}[e]\right)\right) / \\
 & \left(f^3 \left(\operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2\right)\right) + \\
 & \left(9 a b^2 c^2 d \operatorname{Csc}[e] \left(-f x \operatorname{Cos}[e] + \operatorname{Log}\left[\operatorname{Cos}[f x] \operatorname{Sin}[e] + \operatorname{Cos}[e] \operatorname{Sin}[f x]\right] \operatorname{Sin}[e]\right)\right) / \\
 & \left(f^2 \left(\operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2\right)\right) + \\
 & \left(3 a^2 b c^3 \operatorname{Csc}[e] \left(-f x \operatorname{Cos}[e] + \operatorname{Log}\left[\operatorname{Cos}[f x] \operatorname{Sin}[e] + \operatorname{Cos}[e] \operatorname{Sin}[f x]\right] \operatorname{Sin}[e]\right)\right) /
 \end{aligned}$$

$$\begin{aligned}
 & \left( f \left( \cos[e]^2 + \sin[e]^2 \right) \right) - \\
 & \left( b^3 c^3 \operatorname{Csc}[e] \left( -f x \cos[e] + \operatorname{Log}[\cos[f x] \sin[e] + \cos[e] \sin[f x]] \sin[e] \right) \right) / \\
 & \left( f \left( \cos[e]^2 + \sin[e]^2 \right) \right) + \\
 & \left( 3 x^2 \left( -a^3 c^2 d + 3 i a^2 b c^2 d + 3 a b^2 c^2 d - i b^3 c^2 d + a^3 c^2 d \cos[2 e] + 3 i a^2 b c^2 d \cos[2 e] - \right. \right. \\
 & \quad \left. \left. 3 a b^2 c^2 d \cos[2 e] - i b^3 c^2 d \cos[2 e] + i a^3 c^2 d \sin[2 e] - 3 a^2 b c^2 d \sin[2 e] - \right. \right. \\
 & \quad \left. \left. 3 i a b^2 c^2 d \sin[2 e] + b^3 c^2 d \sin[2 e] \right) \right) / \left( 2 \left( -1 + \cos[2 e] + i \sin[2 e] \right) \right) + \\
 & \left( x^3 \left( -a^3 c d^2 + 3 i a^2 b c d^2 + 3 a b^2 c d^2 - i b^3 c d^2 + a^3 c d^2 \cos[2 e] + 3 i a^2 b c d^2 \cos[2 e] - \right. \right. \\
 & \quad \left. \left. 3 a b^2 c d^2 \cos[2 e] - i b^3 c d^2 \cos[2 e] + i a^3 c d^2 \sin[2 e] - 3 a^2 b c d^2 \sin[2 e] - \right. \right. \\
 & \quad \left. \left. 3 i a b^2 c d^2 \sin[2 e] + b^3 c d^2 \sin[2 e] \right) \right) / \left( -1 + \cos[2 e] + i \sin[2 e] \right) + \\
 & \left( x^4 \left( -a^3 d^3 + 3 i a^2 b d^3 + 3 a b^2 d^3 - i b^3 d^3 + a^3 d^3 \cos[2 e] + 3 i a^2 b d^3 \cos[2 e] - 3 a b^2 d^3 \cos[2 e] - \right. \right. \\
 & \quad \left. \left. i b^3 d^3 \cos[2 e] + i a^3 d^3 \sin[2 e] - 3 a^2 b d^3 \sin[2 e] - 3 i a b^2 d^3 \sin[2 e] + b^3 d^3 \sin[2 e] \right) \right) / \\
 & \left( 4 \left( -1 + \cos[2 e] + i \sin[2 e] \right) \right) + x \left( a^3 c^3 - 3 a b^2 c^3 + \frac{3 i a^2 b c^3}{-1 + \cos[2 e] + i \sin[2 e]} + \right. \\
 & \quad \left. \frac{3 i a^2 b c^3 \cos[2 e] - 3 a^2 b c^3 \sin[2 e]}{-1 + \cos[2 e] + i \sin[2 e]} + \left( -2 i b^3 c^3 \cos[2 e] + 2 b^3 c^3 \sin[2 e] \right) / \right. \\
 & \quad \left( \left( -1 + \cos[2 e] + i \sin[2 e] \right) \left( 1 + \cos[2 e] + \cos[4 e] + i \sin[2 e] + i \sin[4 e] \right) \right) + \\
 & \quad \left( -2 i b^3 c^3 \cos[4 e] + 2 b^3 c^3 \sin[4 e] \right) / \\
 & \quad \left( \left( -1 + \cos[2 e] + i \sin[2 e] \right) \left( 1 + \cos[2 e] + \cos[4 e] + i \sin[2 e] + i \sin[4 e] \right) \right) - \\
 & \quad \frac{i b^3 c^3}{-1 + \cos[6 e] + i \sin[6 e]} + \frac{-i b^3 c^3 \cos[6 e] + b^3 c^3 \sin[6 e]}{-1 + \cos[6 e] + i \sin[6 e]} \Bigg) + \frac{1}{2 f^2} \\
 & 3 \operatorname{Csc}[e] \operatorname{Csc}[e + f x] \left( b^3 c^2 d \sin[f x] + 2 a b^2 c^3 f \sin[f x] + 2 b^3 c d^2 x \sin[f x] + \right. \\
 & \quad \left. 6 a b^2 c^2 d f x \sin[f x] + b^3 d^3 x^2 \sin[f x] + 6 a b^2 c d^2 f x^2 \sin[f x] + 2 a b^2 d^3 f x^3 \sin[f x] \right) - \\
 & \left( 3 b^3 d^3 \operatorname{Csc}[e] \operatorname{Sec}[e] \left( e^{i \operatorname{ArcTan}[\tan[e]]} f^2 x^2 + \frac{1}{\sqrt{1 + \tan[e]^2}} \right. \right. \\
 & \quad \left. \left( i f x \left( -\pi + 2 \operatorname{ArcTan}[\tan[e]] \right) - \pi \operatorname{Log}\left[1 + e^{-2 i f x}\right] - 2 \left( f x + \operatorname{ArcTan}[\tan[e]] \right) \right) \right. \\
 & \quad \left. \operatorname{Log}\left[1 - e^{2 i \left( f x + \operatorname{ArcTan}[\tan[e]] \right)}\right] + \pi \operatorname{Log}[\cos[f x]] + 2 \operatorname{ArcTan}[\tan[e]] \right) \right. \\
 & \quad \left. \left. \operatorname{Log}[\sin[f x + \operatorname{ArcTan}[\tan[e]]] \right] + i \operatorname{PolyLog}\left[2, e^{2 i \left( f x + \operatorname{ArcTan}[\tan[e]] \right)}\right] \right) \tan[e] \Bigg) / \\
 & \left( 2 f^4 \sqrt{\sec[e]^2 \left( \cos[e]^2 + \sin[e]^2 \right)} \right) - \left( 9 a b^2 c d^2 \operatorname{Csc}[e] \operatorname{Sec}[e] \left( e^{i \operatorname{ArcTan}[\tan[e]]} f^2 x^2 + \right. \right. \\
 & \quad \frac{1}{\sqrt{1 + \tan[e]^2}} \left( i f x \left( -\pi + 2 \operatorname{ArcTan}[\tan[e]] \right) - \pi \operatorname{Log}\left[1 + e^{-2 i f x}\right] - 2 \left( f x + \operatorname{ArcTan}[\tan[e]] \right) \right) \right. \\
 & \quad \left. \operatorname{Log}\left[1 - e^{2 i \left( f x + \operatorname{ArcTan}[\tan[e]] \right)}\right] + \pi \operatorname{Log}[\cos[f x]] + 2 \operatorname{ArcTan}[\tan[e]] \right) \right. \\
 & \quad \left. \left. \operatorname{Log}[\sin[f x + \operatorname{ArcTan}[\tan[e]]] \right] + i \operatorname{PolyLog}\left[2, e^{2 i \left( f x + \operatorname{ArcTan}[\tan[e]] \right)}\right] \right) \tan[e] \Bigg) / \\
 & \left( f^3 \sqrt{\sec[e]^2 \left( \cos[e]^2 + \sin[e]^2 \right)} \right) - \left( 9 a^2 b c^2 d \operatorname{Csc}[e] \operatorname{Sec}[e] \left( e^{i \operatorname{ArcTan}[\tan[e]]} f^2 x^2 + \right. \right. \\
 & \quad \frac{1}{\sqrt{1 + \tan[e]^2}} \left( i f x \left( -\pi + 2 \operatorname{ArcTan}[\tan[e]] \right) - \pi \operatorname{Log}\left[1 + e^{-2 i f x}\right] - 2 \left( f x + \operatorname{ArcTan}[\tan[e]] \right) \right) \right. \\
 & \quad \left. \operatorname{Log}\left[1 - e^{2 i \left( f x + \operatorname{ArcTan}[\tan[e]] \right)}\right] + \pi \operatorname{Log}[\cos[f x]] + 2 \operatorname{ArcTan}[\tan[e]] \right) \right. \\
 & \quad \left. \left. \operatorname{Log}[\sin[f x + \operatorname{ArcTan}[\tan[e]]] \right] + i \operatorname{PolyLog}\left[2, e^{2 i \left( f x + \operatorname{ArcTan}[\tan[e]] \right)}\right] \right) \tan[e] \Bigg) /
 \end{aligned}$$

$$\begin{aligned}
 & \left. \begin{aligned}
 & \text{Log}\left[1 - e^{2i(fx + \text{ArcTan}[\text{Tan}[e]])}\right] + \pi \text{Log}[\text{Cos}[fx]] + 2 \text{ArcTan}[\text{Tan}[e]] \\
 & \text{Log}[\text{Sin}[fx + \text{ArcTan}[\text{Tan}[e]]] + i \text{PolyLog}\left[2, e^{2i(fx + \text{ArcTan}[\text{Tan}[e]])}\right] \text{Tan}[e] \right) \Bigg/ \\
 & \left(2f^2 \sqrt{\text{Sec}[e]^2 (\text{Cos}[e]^2 + \text{Sin}[e]^2)}\right) + \left(3b^3 c^2 d \text{Csc}[e] \text{Sec}[e] \right. \\
 & \left. \left( e^{i \text{ArcTan}[\text{Tan}[e]} f^2 x^2 + \frac{1}{\sqrt{1 + \text{Tan}[e]^2}} (i f x (-\pi + 2 \text{ArcTan}[\text{Tan}[e]]) - \pi \text{Log}[1 + e^{-2i f x}] - \right. \right. \\
 & \left. \left. 2(fx + \text{ArcTan}[\text{Tan}[e]]) \text{Log}[1 - e^{2i(fx + \text{ArcTan}[\text{Tan}[e]])}] + \pi \text{Log}[\text{Cos}[fx]] + \right. \right. \\
 & \left. \left. 2 \text{ArcTan}[\text{Tan}[e]] \text{Log}[\text{Sin}[fx + \text{ArcTan}[\text{Tan}[e]]] + i \text{PolyLog}\left[2, e^{2i(fx + \text{ArcTan}[\text{Tan}[e]])}\right] \right) \right) \\
 & \left. \text{Tan}[e] \right) \Bigg/ \left(2f^2 \sqrt{\text{Sec}[e]^2 (\text{Cos}[e]^2 + \text{Sin}[e]^2)}\right)
 \end{aligned}
 \end{aligned}$$

### Problem 48: Result more than twice size of optimal antiderivative.

$$\int (c+dx)^2 (a+b \cot[e+fx])^3 dx$$

Optimal (type 4, 433 leaves, 22 steps):

$$\begin{aligned}
 & -\frac{b^3 c d x}{f} - \frac{b^3 d^2 x^2}{2f} - \frac{3 i a b^2 (c+dx)^2}{f} + \frac{a^3 (c+dx)^3}{3d} - \frac{i a^2 b (c+dx)^3}{d} - \\
 & \frac{a b^2 (c+dx)^3}{d} + \frac{i b^3 (c+dx)^3}{3d} - \frac{b^3 d (c+dx) \text{Cot}[e+fx]}{f^2} - \frac{3 a b^2 (c+dx)^2 \text{Cot}[e+fx]}{f} - \\
 & \frac{b^3 (c+dx)^2 \text{Cot}[e+fx]^2}{2f} + \frac{6 a b^2 d (c+dx) \text{Log}[1 - e^{2i(e+fx)}]}{f^2} + \\
 & \frac{3 a^2 b (c+dx)^2 \text{Log}[1 - e^{2i(e+fx)}]}{f} - \frac{b^3 (c+dx)^2 \text{Log}[1 - e^{2i(e+fx)}]}{f} + \\
 & \frac{b^3 d^2 \text{Log}[\text{Sin}[e+fx]]}{f^3} - \frac{3 i a b^2 d^2 \text{PolyLog}[2, e^{2i(e+fx)}]}{f^3} - \\
 & \frac{3 i a^2 b d (c+dx) \text{PolyLog}[2, e^{2i(e+fx)}]}{f^2} + \frac{i b^3 d (c+dx) \text{PolyLog}[2, e^{2i(e+fx)}]}{f^2} + \\
 & \frac{3 a^2 b d^2 \text{PolyLog}[3, e^{2i(e+fx)}]}{2 f^3} - \frac{b^3 d^2 \text{PolyLog}[3, e^{2i(e+fx)}]}{2 f^3}
 \end{aligned}$$

Result (type 4, 1825 leaves):

$$\begin{aligned}
 & -\frac{1}{4 f^3} a^2 b d^2 e^{-ie} \text{Csc}[e] \left(2 f^2 x^2 \left(2 e^{2ie} f x + 3 i (-1 + e^{2ie}) \text{Log}[1 - e^{2i(e+fx)}]\right) + \right. \\
 & \left. 6 (-1 + e^{2ie}) f x \text{PolyLog}[2, e^{2i(e+fx)}] + 3 i (-1 + e^{2ie}) \text{PolyLog}[3, e^{2i(e+fx)}]\right) + \\
 & \frac{1}{12 f^3} b^3 d^2 e^{-ie} \text{Csc}[e] \left(2 f^2 x^2 \left(2 e^{2ie} f x + 3 i (-1 + e^{2ie}) \text{Log}[1 - e^{2i(e+fx)}]\right) + \right. \\
 & \left. 6 (-1 + e^{2ie}) f x \text{PolyLog}[2, e^{2i(e+fx)}] + 3 i (-1 + e^{2ie}) \text{PolyLog}[3, e^{2i(e+fx)}]\right) +
 \end{aligned}$$

$$\begin{aligned}
 & \left( b^3 d^2 \operatorname{Csc}[e] \left( -f x \operatorname{Cos}[e] + \operatorname{Log}[\operatorname{Cos}[f x] \operatorname{Sin}[e] + \operatorname{Cos}[e] \operatorname{Sin}[f x]] \operatorname{Sin}[e] \right) \right) / \\
 & \left( f^3 \left( \operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2 \right) \right) + \\
 & \left( 6 a b^2 c d \operatorname{Csc}[e] \left( -f x \operatorname{Cos}[e] + \operatorname{Log}[\operatorname{Cos}[f x] \operatorname{Sin}[e] + \operatorname{Cos}[e] \operatorname{Sin}[f x]] \operatorname{Sin}[e] \right) \right) / \\
 & \left( f^2 \left( \operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2 \right) \right) + \\
 & \left( 3 a^2 b c^2 \operatorname{Csc}[e] \left( -f x \operatorname{Cos}[e] + \operatorname{Log}[\operatorname{Cos}[f x] \operatorname{Sin}[e] + \operatorname{Cos}[e] \operatorname{Sin}[f x]] \operatorname{Sin}[e] \right) \right) / \\
 & \left( f \left( \operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2 \right) \right) - \\
 & \left( b^3 c^2 \operatorname{Csc}[e] \left( -f x \operatorname{Cos}[e] + \operatorname{Log}[\operatorname{Cos}[f x] \operatorname{Sin}[e] + \operatorname{Cos}[e] \operatorname{Sin}[f x]] \operatorname{Sin}[e] \right) \right) / \\
 & \left( f \left( \operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2 \right) \right) + \\
 & \frac{1}{12 f^2} \operatorname{Csc}[e] \operatorname{Csc}[e + f x]^2 \left( 6 b^3 c d \operatorname{Cos}[e] + 18 a b^2 c^2 f \operatorname{Cos}[e] + 6 b^3 d^2 x \operatorname{Cos}[e] + \right. \\
 & 36 a b^2 c d f x \operatorname{Cos}[e] + 18 a^2 b c^2 f^2 x \operatorname{Cos}[e] - 6 b^3 c^2 f^2 x \operatorname{Cos}[e] + 18 a b^2 d^2 f x^2 \operatorname{Cos}[e] + \\
 & 18 a^2 b c d f^2 x^2 \operatorname{Cos}[e] - 6 b^3 c d f^2 x^2 \operatorname{Cos}[e] + 6 a^2 b d^2 f^2 x^3 \operatorname{Cos}[e] - 2 b^3 d^2 f^2 x^3 \operatorname{Cos}[e] - \\
 & 6 b^3 c d \operatorname{Cos}[e + 2 f x] - 18 a b^2 c^2 f \operatorname{Cos}[e + 2 f x] - 6 b^3 d^2 x \operatorname{Cos}[e + 2 f x] - \\
 & 36 a b^2 c d f x \operatorname{Cos}[e + 2 f x] - 9 a^2 b c^2 f^2 x \operatorname{Cos}[e + 2 f x] + 3 b^3 c^2 f^2 x \operatorname{Cos}[e + 2 f x] - \\
 & 18 a b^2 d^2 f x^2 \operatorname{Cos}[e + 2 f x] - 9 a^2 b c d f^2 x^2 \operatorname{Cos}[e + 2 f x] + 3 b^3 c d f^2 x^2 \operatorname{Cos}[e + 2 f x] - \\
 & 3 a^2 b d^2 f^2 x^3 \operatorname{Cos}[e + 2 f x] + b^3 d^2 f^2 x^3 \operatorname{Cos}[e + 2 f x] - 9 a^2 b c^2 f^2 x \operatorname{Cos}[3 e + 2 f x] + \\
 & 3 b^3 c^2 f^2 x \operatorname{Cos}[3 e + 2 f x] - 9 a^2 b c d f^2 x^2 \operatorname{Cos}[3 e + 2 f x] + 3 b^3 c d f^2 x^2 \operatorname{Cos}[3 e + 2 f x] - \\
 & 3 a^2 b d^2 f^2 x^3 \operatorname{Cos}[3 e + 2 f x] + b^3 d^2 f^2 x^3 \operatorname{Cos}[3 e + 2 f x] - 6 b^3 c^2 f \operatorname{Sin}[e] - \\
 & 12 b^3 c d f x \operatorname{Sin}[e] + 6 a^3 c^2 f^2 x \operatorname{Sin}[e] - 18 a b^2 c^2 f^2 x \operatorname{Sin}[e] - 6 b^3 d^2 f x^2 \operatorname{Sin}[e] + \\
 & 6 a^3 c d f^2 x^2 \operatorname{Sin}[e] - 18 a b^2 c d f^2 x^2 \operatorname{Sin}[e] + 2 a^3 d^2 f^2 x^3 \operatorname{Sin}[e] - 6 a b^2 d^2 f^2 x^3 \operatorname{Sin}[e] + \\
 & 3 a^3 c^2 f^2 x \operatorname{Sin}[e + 2 f x] - 9 a b^2 c^2 f^2 x \operatorname{Sin}[e + 2 f x] + 3 a^3 c d f^2 x^2 \operatorname{Sin}[e + 2 f x] - \\
 & 9 a b^2 c d f^2 x^2 \operatorname{Sin}[e + 2 f x] + a^3 d^2 f^2 x^3 \operatorname{Sin}[e + 2 f x] - 3 a b^2 d^2 f^2 x^3 \operatorname{Sin}[e + 2 f x] - \\
 & 3 a^3 c^2 f^2 x \operatorname{Sin}[3 e + 2 f x] + 9 a b^2 c^2 f^2 x \operatorname{Sin}[3 e + 2 f x] - 3 a^3 c d f^2 x^2 \operatorname{Sin}[3 e + 2 f x] + \\
 & 9 a b^2 c d f^2 x^2 \operatorname{Sin}[3 e + 2 f x] - a^3 d^2 f^2 x^3 \operatorname{Sin}[3 e + 2 f x] + 3 a b^2 d^2 f^2 x^3 \operatorname{Sin}[3 e + 2 f x] \left. \right) - \\
 & \left( 3 a b^2 d^2 \operatorname{Csc}[e] \operatorname{Sec}[e] \left( e^{i \operatorname{ArcTan}[\operatorname{Tan}[e]]} f^2 x^2 + \frac{1}{\sqrt{1 + \operatorname{Tan}[e]^2}} \right. \right. \\
 & \left. \left. \left( i f x \left( -\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[e]] \right) - \pi \operatorname{Log}\left[1 + e^{-2 i f x}\right] - 2 \left( f x + \operatorname{ArcTan}[\operatorname{Tan}[e]] \right) \right) \right. \right. \\
 & \left. \left. \operatorname{Log}\left[1 - e^{2 i \left( f x + \operatorname{ArcTan}[\operatorname{Tan}[e]] \right)}\right] + \pi \operatorname{Log}[\operatorname{Cos}[f x]] + 2 \operatorname{ArcTan}[\operatorname{Tan}[e]] \right) \right) \right) \left. \right) / \\
 & \left. \left. \operatorname{Log}[\operatorname{Sin}[f x + \operatorname{ArcTan}[\operatorname{Tan}[e]]]] + i \operatorname{PolyLog}\left[2, e^{2 i \left( f x + \operatorname{ArcTan}[\operatorname{Tan}[e]] \right)}\right] \right) \operatorname{Tan}[e] \right) \right) / \\
 & \left( f^3 \sqrt{\operatorname{Sec}[e]^2 \left( \operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2 \right)} \right) - \left( 3 a^2 b c d \operatorname{Csc}[e] \operatorname{Sec}[e] \right. \\
 & \left. \left( e^{i \operatorname{ArcTan}[\operatorname{Tan}[e]]} f^2 x^2 + \frac{1}{\sqrt{1 + \operatorname{Tan}[e]^2}} \left( i f x \left( -\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[e]] \right) - \pi \operatorname{Log}\left[1 + e^{-2 i f x}\right] - \right. \right. \right. \\
 & \left. \left. 2 \left( f x + \operatorname{ArcTan}[\operatorname{Tan}[e]] \right) \operatorname{Log}\left[1 - e^{2 i \left( f x + \operatorname{ArcTan}[\operatorname{Tan}[e]] \right)}\right] + \pi \operatorname{Log}[\operatorname{Cos}[f x]] + \right. \right. \\
 & \left. \left. 2 \operatorname{ArcTan}[\operatorname{Tan}[e]] \operatorname{Log}[\operatorname{Sin}[f x + \operatorname{ArcTan}[\operatorname{Tan}[e]]]] + i \operatorname{PolyLog}\left[2, e^{2 i \left( f x + \operatorname{ArcTan}[\operatorname{Tan}[e]] \right)}\right] \right) \right) \right) \left. \right) / \\
 & \left. \left. \left( f^2 \sqrt{\operatorname{Sec}[e]^2 \left( \operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2 \right)} \right) + \right. \right. \\
 & \left. \left. \left( b^3 c d \operatorname{Csc}[e] \operatorname{Sec}[e] \left( e^{i \operatorname{ArcTan}[\operatorname{Tan}[e]]} f^2 x^2 + \frac{1}{\sqrt{1 + \operatorname{Tan}[e]^2}} \left( i f x \left( -\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[e]] \right) - \right. \right. \right. \right. \right. \right.
 \end{aligned}$$

$$\begin{aligned} & \pi \operatorname{Log}\left[1 + e^{-2i f x}\right] - 2\left(f x + \operatorname{ArcTan}\left[\operatorname{Tan}[e]\right]\right) \operatorname{Log}\left[1 - e^{2i\left(f x + \operatorname{ArcTan}\left[\operatorname{Tan}[e]\right]\right)}\right] + \\ & \pi \operatorname{Log}\left[\operatorname{Cos}[f x]\right] + 2 \operatorname{ArcTan}\left[\operatorname{Tan}[e]\right] \operatorname{Log}\left[\operatorname{Sin}\left[f x + \operatorname{ArcTan}\left[\operatorname{Tan}[e]\right]\right]\right] + \\ & i \operatorname{PolyLog}\left[2, e^{2i\left(f x + \operatorname{ArcTan}\left[\operatorname{Tan}[e]\right]\right)}\right] \operatorname{Tan}[e] \Bigg) \Bigg/ \left(f^2 \sqrt{\operatorname{Sec}[e]^2\left(\operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2\right)}\right) \end{aligned}$$

**Problem 57: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c+dx)^3}{(a+b \cot [e+fx])^2} dx$$

Optimal (type 4, 839 leaves, 21 steps):

$$\begin{aligned} & -\frac{2 i b^2 (c+d x)^3}{\left(a^2+b^2\right)^2 f}-\frac{2 b^2 (c+d x)^3}{(a-i b)(a+i b)^2(i a+b-(i a-b) e^{2 i e+2 i f x}) f}+\frac{(c+d x)^4}{4(a+i b)^2 d} \\ & -\frac{b(c+d x)^4}{(a+i b)^2(i a+b) d}-\frac{b^2(c+d x)^4}{\left(a^2+b^2\right)^2 d}+\frac{3 b^2 d(c+d x)^2 \operatorname{Log}\left[1-\frac{(a+i b) e^{2 i e+2 i f x}}{a-i b}\right]}{\left(a^2+b^2\right)^2 f^2}- \\ & \frac{2 b(c+d x)^3 \operatorname{Log}\left[1-\frac{(a+i b) e^{2 i e+2 i f x}}{a-i b}\right]}{(a-i b)(a+i b)^2 f}-\frac{2 i b^2(c+d x)^3 \operatorname{Log}\left[1-\frac{(a+i b) e^{2 i e+2 i f x}}{a-i b}\right]}{\left(a^2+b^2\right)^2 f}- \\ & \frac{3 i b^2 d^2(c+d x) \operatorname{PolyLog}\left[2, \frac{(a+i b) e^{2 i e+2 i f x}}{a-i b}\right]}{\left(a^2+b^2\right)^2 f^3}-\frac{3 b d(c+d x)^2 \operatorname{PolyLog}\left[2, \frac{(a+i b) e^{2 i e+2 i f x}}{a-i b}\right]}{(a+i b)^2(i a+b) f^2}- \\ & \frac{3 b^2 d(c+d x)^2 \operatorname{PolyLog}\left[2, \frac{(a+i b) e^{2 i e+2 i f x}}{a-i b}\right]}{\left(a^2+b^2\right)^2 f^2}+\frac{3 b^2 d^3 \operatorname{PolyLog}\left[3, \frac{(a+i b) e^{2 i e+2 i f x}}{a-i b}\right]}{2\left(a^2+b^2\right)^2 f^4}- \\ & \frac{3 b d^2(c+d x) \operatorname{PolyLog}\left[3, \frac{(a+i b) e^{2 i e+2 i f x}}{a-i b}\right]}{(a-i b)(a+i b)^2 f^3}-\frac{3 i b^2 d^2(c+d x) \operatorname{PolyLog}\left[3, \frac{(a+i b) e^{2 i e+2 i f x}}{a-i b}\right]}{\left(a^2+b^2\right)^2 f^3}+ \\ & \frac{3 b d^3 \operatorname{PolyLog}\left[4, \frac{(a+i b) e^{2 i e+2 i f x}}{a-i b}\right]}{2(a+i b)^2(i a+b) f^4}+\frac{3 b^2 d^3 \operatorname{PolyLog}\left[4, \frac{(a+i b) e^{2 i e+2 i f x}}{a-i b}\right]}{2\left(a^2+b^2\right)^2 f^4} \end{aligned}$$

Result (type 4, 2706 leaves):

$$\begin{aligned} & \frac{1}{2(a-i b)^3(a+i b)^2(-i a(-1+e^{2 i e})+b(1+e^{2 i e})) f^4} \\ & b e^{2 i e}\left(4(a-i b)(a+i b) c^2 f^3(-3 b d+2 a c f) x- \right. \\ & 4(a-i b) c^2 e^{-2 i e}(a(-1+e^{2 i e})+i b(1+e^{2 i e})) f^3(-3 b d+2 a c f) x+ \\ & 12 a(a-i b) b c d^2 f^3 x^2+12 b^2(i a+b) c d^2 f^3 x^2-12 a(a-i b) b c d^2 e^{-2 i e} f^3 x^2+ \\ & 12 b^2(i a+b) c d^2 e^{-2 i e} f^3 x^2-12 a^2(a-i b) c^2 d f^4 x^2-12 i a(a-i b) b c^2 d f^4 x^2+ \\ & 12 a^2(a-i b) c^2 d e^{-2 i e} f^4 x^2-12 i a(a-i b) b c^2 d e^{-2 i e} f^4 x^2+ \\ & 12(a-i b)(a+i b) c d f^3(-b d+a c f) x^2+4 a(a-i b) b d^3 f^3 x^3+ \\ & 4 b^2(i a+b) d^3 f^3 x^3-4 a(a-i b) b d^3 e^{-2 i e} f^3 x^3+4 b^2(i a+b) d^3 e^{-2 i e} f^3 x^3- \\ & 8 a^2(a-i b) c d^2 f^4 x^3-8 i a(a-i b) b c d^2 f^4 x^3+8 a^2(a-i b) c d^2 e^{-2 i e} f^4 x^3- \end{aligned}$$

$$\begin{aligned}
 & 8 i a (a - i b) b c d^2 e^{-2 i e} f^4 x^3 + 4 (a - i b) (a + i b) d^2 f^3 (-b d + 2 a c f) x^3 - \\
 & 2 a^2 (a - i b) d^3 f^4 x^4 + 2 a (a - i b) (a + i b) d^3 f^4 x^4 - 2 i a (a - i b) b d^3 f^4 x^4 + \\
 & 2 a^2 (a - i b) d^3 e^{-2 i e} f^4 x^4 - 2 i a (a - i b) b d^3 e^{-2 i e} f^4 x^4 - 3 (a - i b) b c^2 d e^{-2 i e} \\
 & \left( (a (-1 + e^{2 i e}) + i b (1 + e^{2 i e})) f^2 \left( 4 f x - 2 \operatorname{ArcTan} \left[ \frac{2 a b e^{2 i (e+f x)}}{a^2 (-1 + e^{2 i (e+f x)}) - b^2 (1 + e^{2 i (e+f x)})} \right] \right) + \right. \\
 & \left. i \operatorname{Log} \left[ a^2 (-1 + e^{2 i (e+f x)})^2 + b^2 (1 + e^{2 i (e+f x)})^2 \right] \right) + 2 a (a - i b) c^3 e^{-2 i e} \\
 & \left( (a (-1 + e^{2 i e}) + i b (1 + e^{2 i e})) f^3 \left( 4 f x - 2 \operatorname{ArcTan} \left[ \frac{2 a b e^{2 i (e+f x)}}{a^2 (-1 + e^{2 i (e+f x)}) - b^2 (1 + e^{2 i (e+f x)})} \right] \right) + \right. \\
 & \left. i \operatorname{Log} \left[ a^2 (-1 + e^{2 i (e+f x)})^2 + b^2 (1 + e^{2 i (e+f x)})^2 \right] \right) - \\
 & 6 b (i a + b) c d^2 e^{-2 i e} (-i a (-1 + e^{2 i e}) + b (1 + e^{2 i e})) f \\
 & \left( 2 f x \left( f x + i \operatorname{Log} \left[ 1 - \frac{(a + i b) e^{2 i (e+f x)}}{a - i b} \right] \right) + \operatorname{PolyLog} \left[ 2, \frac{(a + i b) e^{2 i (e+f x)}}{a - i b} \right] \right) + \\
 & 6 a (a - i b) c^2 d e^{-2 i e} (a (-1 + e^{2 i e}) + i b (1 + e^{2 i e})) f^2 \\
 & \left( 2 f x \left( f x + i \operatorname{Log} \left[ 1 - \frac{(a + i b) e^{2 i (e+f x)}}{a - i b} \right] \right) + \operatorname{PolyLog} \left[ 2, \frac{(a + i b) e^{2 i (e+f x)}}{a - i b} \right] \right) - b (i a + b) \\
 & d^3 e^{-2 i e} (-i a (-1 + e^{2 i e}) + b (1 + e^{2 i e})) \left( 2 f^2 x^2 \left( 2 f x + 3 i \operatorname{Log} \left[ 1 - \frac{(a + i b) e^{2 i (e+f x)}}{a - i b} \right] \right) + \right. \\
 & \left. 6 f x \operatorname{PolyLog} \left[ 2, \frac{(a + i b) e^{2 i (e+f x)}}{a - i b} \right] + 3 i \operatorname{PolyLog} \left[ 3, \frac{(a + i b) e^{2 i (e+f x)}}{a - i b} \right] \right) + \\
 & 2 a (a - i b) c d^2 e^{-2 i e} (a (-1 + e^{2 i e}) + i b (1 + e^{2 i e})) f \\
 & \left( 2 f^2 x^2 \left( 2 f x + 3 i \operatorname{Log} \left[ 1 - \frac{(a + i b) e^{2 i (e+f x)}}{a - i b} \right] \right) + 6 f x \operatorname{PolyLog} \left[ 2, \frac{(a + i b) e^{2 i (e+f x)}}{a - i b} \right] + \right. \\
 & \left. 3 i \operatorname{PolyLog} \left[ 3, \frac{(a + i b) e^{2 i (e+f x)}}{a - i b} \right] \right) + a (a - i b) d^3 e^{-2 i e} (a (-1 + e^{2 i e}) + i b (1 + e^{2 i e})) \\
 & \left( 2 f^4 x^4 + 4 i f^3 x^3 \operatorname{Log} \left[ 1 - \frac{(a + i b) e^{2 i (e+f x)}}{a - i b} \right] + 6 f^2 x^2 \operatorname{PolyLog} \left[ 2, \frac{(a + i b) e^{2 i (e+f x)}}{a - i b} \right] + \right. \\
 & \left. 6 i f x \operatorname{PolyLog} \left[ 3, \frac{(a + i b) e^{2 i (e+f x)}}{a - i b} \right] - 3 \operatorname{PolyLog} \left[ 4, \frac{(a + i b) e^{2 i (e+f x)}}{a - i b} \right] \right) + \\
 & \left( 3 x^2 (-a c^2 d - i b c^2 d + a c^2 d \operatorname{Cos}[2 e] - i b c^2 d \operatorname{Cos}[2 e] + i a c^2 d \operatorname{Sin}[2 e] + b c^2 d \operatorname{Sin}[2 e]) \right) / \\
 & \left( 2 (a - i b) (a + i b) \right. \\
 & \left. (-a + i b + a \operatorname{Cos}[2 e] + i b \operatorname{Cos}[2 e] + i a \operatorname{Sin}[2 e] - b \operatorname{Sin}[2 e]) \right) + \\
 & \left( x^3 (-a c d^2 - i b c d^2 + a c d^2 \operatorname{Cos}[2 e] - i b c d^2 \operatorname{Cos}[2 e] + i a c d^2 \operatorname{Sin}[2 e] + b c d^2 \operatorname{Sin}[2 e]) \right) / \\
 & \left( (a - i b) (a + i b) (-a + i b + a \operatorname{Cos}[2 e] + i b \operatorname{Cos}[2 e] + i a \operatorname{Sin}[2 e] - b \operatorname{Sin}[2 e]) \right) + \\
 & \left( x^4 (-a d^3 - i b d^3 + a d^3 \operatorname{Cos}[2 e] - i b d^3 \operatorname{Cos}[2 e] + i a d^3 \operatorname{Sin}[2 e] + b d^3 \operatorname{Sin}[2 e]) \right) / \\
 & \left( 4 (a - i b) (a + i b) (-a + i b + a \operatorname{Cos}[2 e] + i b \operatorname{Cos}[2 e] + i a \operatorname{Sin}[2 e] - b \operatorname{Sin}[2 e]) \right) + \\
 & x \left( c^3 / (a^2 - 2 i a b - b^2 + a^2 \operatorname{Cos}[4 e] + 2 i a b \operatorname{Cos}[4 e] - b^2 \operatorname{Cos}[4 e] + i a^2 \operatorname{Sin}[4 e] - 2 a b \operatorname{Sin}[4 e] - \right. \\
 & \left. i b^2 \operatorname{Sin}[4 e]) + ((-i a - b - i a \operatorname{Cos}[2 e] + b \operatorname{Cos}[2 e] + a \operatorname{Sin}[2 e] + i b \operatorname{Sin}[2 e]) \right. \\
 & \left. (4 a b c^3 \operatorname{Cos}[2 e] + 4 i a b c^3 \operatorname{Sin}[2 e])) / ((a - i b) (a + i b)) \right. \\
 & \left. (-a + i b + a \operatorname{Cos}[2 e] + i b \operatorname{Cos}[2 e] + i a \operatorname{Sin}[2 e] - b \operatorname{Sin}[2 e]) (a^2 - 2 i a b - b^2 + a^2 \right. \\
 & \left. \operatorname{Cos}[4 e] + 2 i a b \operatorname{Cos}[4 e] - b^2 \operatorname{Cos}[4 e] + i a^2 \operatorname{Sin}[4 e] - 2 a b \operatorname{Sin}[4 e] - i b^2 \operatorname{Sin}[4 e]) \right) +
 \end{aligned}$$

$$\frac{(c^3 \cos[4e] + i c^3 \sin[4e]) / (a^2 - 2 i a b - b^2 + a^2 \cos[4e] + 2 i a b \cos[4e] - b^2 \cos[4e] + i a^2 \sin[4e] - 2 a b \sin[4e] - i b^2 \sin[4e]) + (b^2 c^3 \sin[fx] + 3 b^2 c^2 d x \sin[fx] + 3 b^2 c d^2 x^2 \sin[fx] + b^2 d^3 x^3 \sin[fx]) / ((a - i b) (a + i b) f (b \cos[e] + a \sin[e]) (b \cos[e + fx] + a \sin[e + fx]))}{}$$

**Problem 58: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c + dx)^2}{(a + b \cot[e + fx])^2} dx$$

Optimal (type 4, 650 leaves, 18 steps):

$$\begin{aligned} & - \frac{2 i b^2 (c + dx)^2}{(a^2 + b^2)^2 f} - \frac{2 b^2 (c + dx)^2}{(a - i b) (a + i b)^2 (i a + b - (i a - b) e^{2 i e + 2 i f x}) f} + \\ & \frac{(c + dx)^3}{3 (a + i b)^2 d} - \frac{4 b (c + dx)^3}{3 (a + i b)^2 (i a + b) d} - \frac{4 b^2 (c + dx)^3}{3 (a^2 + b^2)^2 d} + \\ & \frac{2 b^2 d (c + dx) \operatorname{Log}\left[1 - \frac{(a + i b) e^{2 i e + 2 i f x}}{a - i b}\right]}{(a^2 + b^2)^2 f^2} - \frac{2 b (c + dx)^2 \operatorname{Log}\left[1 - \frac{(a + i b) e^{2 i e + 2 i f x}}{a - i b}\right]}{(a - i b) (a + i b)^2 f} - \\ & \frac{2 i b^2 (c + dx)^2 \operatorname{Log}\left[1 - \frac{(a + i b) e^{2 i e + 2 i f x}}{a - i b}\right]}{(a^2 + b^2)^2 f} - \frac{i b^2 d^2 \operatorname{PolyLog}\left[2, \frac{(a + i b) e^{2 i e + 2 i f x}}{a - i b}\right]}{(a^2 + b^2)^2 f^3} - \\ & \frac{2 b d (c + dx) \operatorname{PolyLog}\left[2, \frac{(a + i b) e^{2 i e + 2 i f x}}{a - i b}\right]}{(a + i b)^2 (i a + b) f^2} - \frac{2 b^2 d (c + dx) \operatorname{PolyLog}\left[2, \frac{(a + i b) e^{2 i e + 2 i f x}}{a - i b}\right]}{(a^2 + b^2)^2 f^2} - \\ & \frac{b d^2 \operatorname{PolyLog}\left[3, \frac{(a + i b) e^{2 i e + 2 i f x}}{a - i b}\right]}{(a - i b) (a + i b)^2 f^3} - \frac{i b^2 d^2 \operatorname{PolyLog}\left[3, \frac{(a + i b) e^{2 i e + 2 i f x}}{a - i b}\right]}{(a^2 + b^2)^2 f^3} \end{aligned}$$

Result (type 4, 1309 leaves):

$$\begin{aligned}
 & \frac{1}{3 (a^2 + b^2)^2 (-i a (-1 + e^{2ie}) + b (1 + e^{2ie})) f^3} \\
 & b \left( f \left( -12 a b c d e^{2ie} f x - 12 i b^2 c d e^{2ie} f x + 12 a^2 c^2 e^{2ie} f^2 x + 12 i a b c^2 e^{2ie} f^2 x - \right. \right. \\
 & \quad 6 a b d^2 e^{2ie} f x^2 - 6 i b^2 d^2 e^{2ie} f x^2 + 12 a^2 c d e^{2ie} f^2 x^2 + 12 i a b c d e^{2ie} f^2 x^2 + \\
 & \quad \left. \left. 4 a^2 d^2 e^{2ie} f^2 x^3 + 4 i a b d^2 e^{2ie} f^2 x^3 - 6 c (a (-1 + e^{2ie}) + i b (1 + e^{2ie})) \right) \right. \\
 & \quad \left. (-b d + a c f) \operatorname{ArcTan} \left[ \frac{2 a b e^{2i (e+fx)}}{a^2 (-1 + e^{2i (e+fx)}) - b^2 (1 + e^{2i (e+fx)})} \right] + \right. \\
 & \quad \left. 6 i d (a (-1 + e^{2ie}) + i b (1 + e^{2ie})) x (-b d + a f (2 c + d x)) \operatorname{Log} \left[ 1 - \frac{(a + i b) e^{2i (e+fx)}}{a - i b} \right] + \right. \\
 & \quad \left. 3 i a b c d \operatorname{Log} \left[ a^2 (-1 + e^{2i (e+fx)})^2 + b^2 (1 + e^{2i (e+fx)})^2 \right] + \right. \\
 & \quad \left. 3 b^2 c d \operatorname{Log} \left[ a^2 (-1 + e^{2i (e+fx)})^2 + b^2 (1 + e^{2i (e+fx)})^2 \right] - \right. \\
 & \quad \left. 3 i a b c d e^{2ie} \operatorname{Log} \left[ a^2 (-1 + e^{2i (e+fx)})^2 + b^2 (1 + e^{2i (e+fx)})^2 \right] + \right. \\
 & \quad \left. 3 b^2 c d e^{2ie} \operatorname{Log} \left[ a^2 (-1 + e^{2i (e+fx)})^2 + b^2 (1 + e^{2i (e+fx)})^2 \right] - \right. \\
 & \quad \left. 3 i a^2 c^2 f \operatorname{Log} \left[ a^2 (-1 + e^{2i (e+fx)})^2 + b^2 (1 + e^{2i (e+fx)})^2 \right] - \right. \\
 & \quad \left. 3 a b c^2 f \operatorname{Log} \left[ a^2 (-1 + e^{2i (e+fx)})^2 + b^2 (1 + e^{2i (e+fx)})^2 \right] + \right. \\
 & \quad \left. 3 i a^2 c^2 e^{2ie} f \operatorname{Log} \left[ a^2 (-1 + e^{2i (e+fx)})^2 + b^2 (1 + e^{2i (e+fx)})^2 \right] - \right. \\
 & \quad \left. 3 a b c^2 e^{2ie} f \operatorname{Log} \left[ a^2 (-1 + e^{2i (e+fx)})^2 + b^2 (1 + e^{2i (e+fx)})^2 \right] \right) + \\
 & \quad 3 d (a (-1 + e^{2ie}) + i b (1 + e^{2ie})) (-b d + 2 a f (c + d x)) \operatorname{PolyLog} \left[ 2, \frac{(a + i b) e^{2i (e+fx)}}{a - i b} \right] + \\
 & \quad 3 i a d^2 (a (-1 + e^{2ie}) + i b (1 + e^{2ie})) \operatorname{PolyLog} \left[ 3, \frac{(a + i b) e^{2i (e+fx)}}{a - i b} \right] \Big) + \\
 & (3 a^2 c^2 f x \operatorname{Cos}[f x] - 3 b^2 c^2 f x \operatorname{Cos}[f x] + 3 a^2 c d f x^2 \operatorname{Cos}[f x] - \\
 & \quad 3 b^2 c d f x^2 \operatorname{Cos}[f x] + \\
 & \quad a^2 d^2 f x^3 \operatorname{Cos}[f x] - \\
 & \quad b^2 d^2 f x^3 \operatorname{Cos}[f x] - \\
 & \quad 3 a^2 c^2 f x \operatorname{Cos}[2 e + f x] - \\
 & \quad 3 b^2 c^2 f x \operatorname{Cos}[2 e + f x] - \\
 & \quad 3 a^2 c d f x^2 \operatorname{Cos}[2 e + f x] - \\
 & \quad 3 b^2 c d f x^2 \operatorname{Cos}[2 e + f x] - \\
 & \quad a^2 d^2 f x^3 \operatorname{Cos}[2 e + f x] - b^2 d^2 f x^3 \operatorname{Cos}[2 e + f x] + \\
 & \quad 6 b^2 c^2 \operatorname{Sin}[f x] + 12 b^2 c d x \operatorname{Sin}[f x] - \\
 & \quad 6 a b c^2 f x \operatorname{Sin}[f x] + 6 b^2 d^2 x^2 \operatorname{Sin}[f x] - \\
 & \quad 6 a b c d f x^2 \operatorname{Sin}[f x] - 2 a b d^2 f x^3 \operatorname{Sin}[f x]) / \\
 & (6 (a - i b) (a + i b) f (b \operatorname{Cos}[e] + a \operatorname{Sin}[e]) (b \operatorname{Cos}[e + f x] + a \operatorname{Sin}[e + f x]))
 \end{aligned}$$

**Problem 59: Result more than twice size of optimal antiderivative.**

$$\int \frac{c + d x}{(a + b \operatorname{Cot}[e + f x])^2} dx$$

Optimal (type 4, 213 leaves, 5 steps):



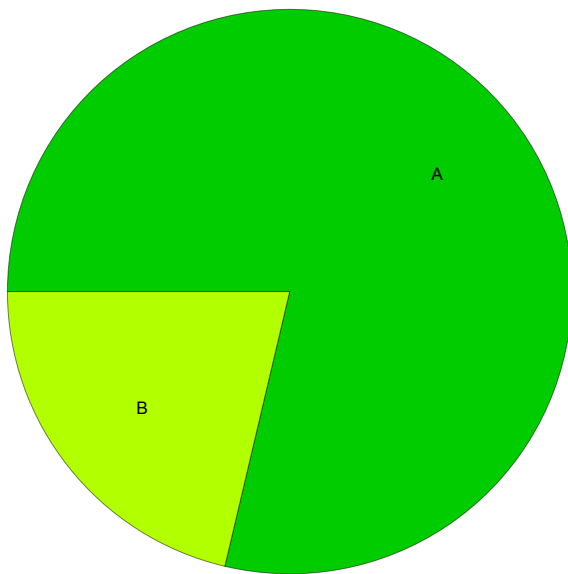
$$\begin{aligned}
 & -\frac{(c+dx)^2}{2(a^2+b^2)d} + \frac{(bd-2acf-2adf x)^2}{4a(a-ib)^2(a+ib)df^2} + \frac{b(c+dx)}{(a^2+b^2)f(a+b \cot[e+fx])} + \\
 & \frac{b(bd-2acf-2adf x) \operatorname{Log}\left[1 - \frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right]}{(a^2+b^2)^2 f^2} + \frac{ibad \operatorname{PolyLog}\left[2, \frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right]}{(a^2+b^2)^2 f^2}
 \end{aligned}$$

Result (type 4, 730 leaves):

$$\begin{aligned}
 & -\left( \left( (e+fx)(-2de+2cf+d(e+fx)) \operatorname{Csc}[e+fx]^2 (b \operatorname{Cos}[e+fx] + a \operatorname{Sin}[e+fx])^2 \right) / \right. \\
 & \quad \left. \left( 2(-ia+b)(ia+b)f^2(a+b \cot[e+fx])^2 \right) \right) + \\
 & \left( bd \operatorname{Csc}[e+fx]^2 (-a(e+fx) + b \operatorname{Log}[b \operatorname{Cos}[e+fx] + a \operatorname{Sin}[e+fx]]) \right. \\
 & \quad \left. (b \operatorname{Cos}[e+fx] + a \operatorname{Sin}[e+fx])^2 \right) / \left( (-ia+b)(ia+b)(a^2+b^2)f^2(a+b \cot[e+fx])^2 \right) + \\
 & \left( 2ade \operatorname{Csc}[e+fx]^2 (-a(e+fx) + b \operatorname{Log}[b \operatorname{Cos}[e+fx] + a \operatorname{Sin}[e+fx]]) \right. \\
 & \quad \left. (b \operatorname{Cos}[e+fx] + a \operatorname{Sin}[e+fx])^2 \right) / \left( (-ia+b)(ia+b)(a^2+b^2)f^2(a+b \cot[e+fx])^2 \right) - \\
 & \left( 2ac \operatorname{Csc}[e+fx]^2 (-a(e+fx) + b \operatorname{Log}[b \operatorname{Cos}[e+fx] + a \operatorname{Sin}[e+fx]]) \right. \\
 & \quad \left. (b \operatorname{Cos}[e+fx] + a \operatorname{Sin}[e+fx])^2 \right) / \\
 & \left( (-ia+b)(ia+b)(a^2+b^2)f(a+b \cot[e+fx])^2 \right) + \left( d \operatorname{Csc}[e+fx]^2 \right. \\
 & \left. \left( \operatorname{E}^{i \operatorname{ArcTan}\left[\frac{b}{a}\right]} (e+fx)^2 + \frac{1}{a \sqrt{1+\frac{b^2}{a^2}}} b \left( i(e+fx) \left( -\pi + 2 \operatorname{ArcTan}\left[\frac{b}{a}\right] \right) - \pi \operatorname{Log}\left[1 + e^{-2i(e+fx)}\right] - \right. \right. \right. \\
 & \quad \left. \left. \left. 2 \left( e+fx + \operatorname{ArcTan}\left[\frac{b}{a}\right] \right) \operatorname{Log}\left[1 - e^{2i(e+fx+\operatorname{ArcTan}\left[\frac{b}{a}\right])}\right] + \pi \operatorname{Log}\left[\operatorname{Cos}[e+fx]\right] + \right. \right. \right. \\
 & \quad \left. \left. \left. 2 \operatorname{ArcTan}\left[\frac{b}{a}\right] \operatorname{Log}\left[\operatorname{Sin}\left[e+fx + \operatorname{ArcTan}\left[\frac{b}{a}\right]\right]\right] + i \operatorname{PolyLog}\left[2, e^{2i(e+fx+\operatorname{ArcTan}\left[\frac{b}{a}\right])}\right] \right) \right) \right) \\
 & \left. (b \operatorname{Cos}[e+fx] + a \operatorname{Sin}[e+fx])^2 \right) / \left( (-ia+b)(ia+b) \sqrt{\frac{a^2+b^2}{a^2}} f^2 (a+b \cot[e+fx])^2 \right) + \\
 & \left( \operatorname{Csc}[e+fx]^2 (b \operatorname{Cos}[e+fx] + a \operatorname{Sin}[e+fx]) \right. \\
 & \quad \left. (-bde \operatorname{Sin}[e+fx] + bcf \operatorname{Sin}[e+fx] + bd(e+fx) \operatorname{Sin}[e+fx]) \right) / \\
 & \left( (-ia+b)(ia+b)f^2(a+b \cot[e+fx])^2 \right)
 \end{aligned}$$

## Summary of Integration Test Results

61 integration problems



A - 48 optimal antiderivatives

B - 13 more than twice size of optimal antiderivatives

C - 0 unnecessarily complex antiderivatives

D - 0 unable to integrate problems

E - 0 integration timeouts