Mathematica 11.3 Integration Test Results

Test results for the 64 problems in "4.4.7 (d trig)^m (a+b (c cot)^n)^p.m"

Problem 8: Result more than twice size of optimal antiderivative.

$$\int \left(1+\text{Cot}[x]^2\right)^{3/2}\, dx$$

Optimal (type 3, 22 leaves, 4 steps):

$$-\frac{1}{2}\,\text{ArcSinh}[\text{Cot}[x]]-\frac{1}{2}\,\text{Cot}[x]\,\sqrt{\text{Csc}[x]^2}$$

Result (type 3, 51 leaves):

$$\frac{1}{8}\,\sqrt{\text{Csc}[x]^2}\,\left(-\text{Csc}\left[\frac{x}{2}\right]^2-4\,\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right]+4\,\text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right]+\text{Sec}\left[\frac{x}{2}\right]^2\right)\,\text{Sin}[x]$$

Problem 9: Result more than twice size of optimal antiderivative.

$$\int \sqrt{1+\text{Cot}[x]^2}\, dx$$

Optimal (type 3, 5 leaves, 3 steps):

$$-\text{ArcSinh}[\text{Cot}[x]]$$

Result (type 3, 28 leaves):

$$\sqrt{\text{Csc}[x]^2}\,\left(-\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right]+\text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right]\right)\,\text{Sin}[x]$$

Problem 12: Result more than twice size of optimal antiderivative.

$$\int \sqrt{-1-\text{Cot}[x]^2}\, dx$$

Optimal (type 3, 14 leaves, 4 steps):

$$\text{ArcTan}\left[\frac{\text{Cot}[x]}{\sqrt{-\text{Csc}[x]^2}}\right]$$

Result (type 3, 30 leaves):

$$\frac{\text{Csc}[x]\left(\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right]-\text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right]\right)}{\sqrt{-\text{Csc}[x]^2}}$$

Problem 19: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \text{Cot}[x]^3\sqrt{a+b\,\text{Cot}[x]^2}\,dx$$

Optimal (type 3, 66 leaves, 6 steps):

$$-\sqrt{a-b}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Cot}[x]^2}}{\sqrt{a-b}}\right]+\sqrt{a+b\,\text{Cot}[x]^2}-\frac{\left(a+b\,\text{Cot}[x]^2\right)^{3/2}}{3\,b}$$

Result (type 4, 505 leaves):

$$\sqrt{\frac{-a-b+a\,\text{Cos}[2x]-b\,\text{Cos}[2x]}{-1+\text{Cos}[2x]}}\left(\frac{-a+4b}{3b}-\frac{\text{Csc}[x]^2}{3}\right)+$$
$$\left(2\,i\,(a-b)\,(1+\text{Cos}[x])\sqrt{\frac{-1+\text{Cos}[2x]}{(1+\text{Cos}[x])^2}}\sqrt{\frac{-a-b+(a-b)\,\text{Cos}[2x]}{-1+\text{Cos}[2x]}}\right.$$
$$\left(\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{b}{2a+2\sqrt{a\,(a-b)}-b}}\,\text{Tan}\left[\frac{x}{2}\right]\right],\frac{-2a-2\sqrt{a\,(a-b)}+b}{-2a+2\sqrt{a\,(a-b)}+b}\right]-\right.$$
$$2\,\text{EllipticPi}\left[\frac{2a+2\sqrt{a\,(a-b)}-b}{b},\right.$$
$$\left.\left.i\,\text{ArcSinh}\left[\sqrt{\frac{b}{2a+2\sqrt{a\,(a-b)}-b}}\,\text{Tan}\left[\frac{x}{2}\right]\right],\frac{-2a-2\sqrt{a\,(a-b)}+b}{-2a+2\sqrt{a\,(a-b)}+b}\right]\right)$$
$$\left.\text{Tan}\left[\frac{x}{2}\right]\sqrt{1+\frac{b\,\text{Tan}\left[\frac{x}{2}\right]^2}{2a+2\sqrt{a\,(a-b)}-b}}\sqrt{1-\frac{b\,\text{Tan}\left[\frac{x}{2}\right]^2}{-2a+2\sqrt{a\,(a-b)}+b}}\right)\Bigg/$$
$$\left(\sqrt{\frac{b}{2a+2\sqrt{a\,(a-b)}-b}}\sqrt{-a-b+(a-b)\,\text{Cos}[2x]}\sqrt{-\text{Tan}\left[\frac{x}{2}\right]^2}\right.$$
$$\left.\left(1+\text{Tan}\left[\frac{x}{2}\right]^2\right)\sqrt{-\frac{4a\,\text{Tan}\left[\frac{x}{2}\right]^2+b\left(-1+\text{Tan}\left[\frac{x}{2}\right]^2\right)^2}{\left(1+\text{Tan}\left[\frac{x}{2}\right]^2\right)^2}}\right)$$

Problem 20: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \text{Cot}[x]\,\sqrt{a+b\,\text{Cot}[x]^2}\,dx$$

Optimal (type 3, 48 leaves, 5 steps):

$$\sqrt{a-b}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Cot}[x]^2}}{\sqrt{a-b}}\right]-\sqrt{a+b\,\text{Cot}[x]^2}$$

Result (type 4, 363 leaves):

$$\frac{1}{\sqrt{2}}\sqrt{-\left(-a-b+(a-b)\,\text{Cos}[2x]\right)\text{Csc}[x]^2}\left(-1+\left(8\,i\,(a-b)\,\text{Cos}\left[\frac{x}{2}\right]^3\left(\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{b}{2a+2\sqrt{a\,(a-b)}-b}}\,\text{Tan}\left[\frac{x}{2}\right]\right],\frac{-2a-2\sqrt{a\,(a-b)}+b}{-2a+2\sqrt{a\,(a-b)}+b}\right]-2\,\text{EllipticPi}\left[\frac{2a+2\sqrt{a\,(a-b)}-b}{b},\right.\right.\right.\right.$$

$$\left.\left.\left.i\,\text{ArcSinh}\left[\sqrt{\frac{b}{2a+2\sqrt{a\,(a-b)}-b}}\,\text{Tan}\left[\frac{x}{2}\right]\right],\frac{-2a-2\sqrt{a\,(a-b)}+b}{-2a+2\sqrt{a\,(a-b)}+b}\right]\right)$$

$$\left.\text{Sin}\left[\frac{x}{2}\right]\sqrt{1+\frac{b\,\text{Tan}\left[\frac{x}{2}\right]^2}{2a+2\sqrt{a\,(a-b)}-b}}\,\sqrt{1-\frac{b\,\text{Tan}\left[\frac{x}{2}\right]^2}{-2a+2\sqrt{a\,(a-b)}+b}}\right)\Bigg/$$

$$\left.\left(\sqrt{\frac{b}{2a+2\sqrt{a\,(a-b)}-b}}\,\left(a+b+(-a+b)\,\text{Cos}[2x]\right)\right)\right)$$

Problem 21: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \sqrt{a+b\,\text{Cot}[x]^2}\,\text{Tan}[x]\,dx$$

Optimal (type 3, 60 leaves, 7 steps):

$$\sqrt{a}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Cot}[x]^2}}{\sqrt{a}}\right]-\sqrt{a-b}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Cot}[x]^2}}{\sqrt{a-b}}\right]$$

Result (type 3, 197 leaves):

$$\left(\sqrt{a+b\,\mathrm{Cot}[x]^2}\left(2\sqrt{a}\sqrt{a-b}\,\mathrm{Log}\left[a\,\mathrm{Tan}[x]+\sqrt{a}\sqrt{b+a\,\mathrm{Tan}[x]^2}\right]+\right.\right.$$

$$(a-b)\left(\mathrm{Log}\left[\frac{4\left(b+i\,a\,\mathrm{Tan}[x]-i\sqrt{a-b}\sqrt{b+a\,\mathrm{Tan}[x]^2}\right)}{(a-b)^{3/2}\,(-i+\mathrm{Tan}[x])}\right]-\mathrm{Log}\left[\right.\right.$$

$$\left.\left.\left.\left.\frac{4\,i\left(i\,b+a\,\mathrm{Tan}[x]+\sqrt{a-b}\sqrt{b+a\,\mathrm{Tan}[x]^2}\right)}{(a-b)^{3/2}\,(i+\mathrm{Tan}[x])}\right]\right)\right)\mathrm{Tan}[x]\right)\Big/\left(2\sqrt{a-b}\sqrt{b+a\,\mathrm{Tan}[x]^2}\right)$$

Problem 22: Result more than twice size of optimal antiderivative.

$$\int \mathrm{Cot}[x]^2\sqrt{a+b\,\mathrm{Cot}[x]^2}\,dx$$

Optimal (type 3, 89 leaves, 7 steps):

$$\sqrt{a-b}\,\mathrm{ArcTan}\left[\frac{\sqrt{a-b}\,\mathrm{Cot}[x]}{\sqrt{a+b\,\mathrm{Cot}[x]^2}}\right]-\frac{(a-2b)\,\mathrm{ArcTanh}\left[\frac{\sqrt{b}\,\mathrm{Cot}[x]}{\sqrt{a+b\,\mathrm{Cot}[x]^2}}\right]}{2\sqrt{b}}-\frac{1}{2}\mathrm{Cot}[x]\sqrt{a+b\,\mathrm{Cot}[x]^2}$$

Result (type 3, 2937 leaves):

$$-\frac{1}{2}\sqrt{\frac{-a-b+a\,\mathrm{Cos}[2x]-b\,\mathrm{Cos}[2x]}{-1+\mathrm{Cos}[2x]}}\,\mathrm{Cot}[x]+$$

$$\left(\left(\frac{b\sqrt{-\frac{a}{-1+\mathrm{Cos}[2x]}-\frac{b}{-1+\mathrm{Cos}[2x]}+\frac{a\,\mathrm{Cos}[2x]}{-1+\mathrm{Cos}[2x]}-\frac{b\,\mathrm{Cos}[2x]}{-1+\mathrm{Cos}[2x]}}}{-a-b+a\,\mathrm{Cos}[2x]-b\,\mathrm{Cos}[2x]}-\right.\right.$$

$$\frac{a\,\mathrm{Cos}[2x]\sqrt{-\frac{a}{-1+\mathrm{Cos}[2x]}-\frac{b}{-1+\mathrm{Cos}[2x]}+\frac{a\,\mathrm{Cos}[2x]}{-1+\mathrm{Cos}[2x]}-\frac{b\,\mathrm{Cos}[2x]}{-1+\mathrm{Cos}[2x]}}}{-a-b+a\,\mathrm{Cos}[2x]-b\,\mathrm{Cos}[2x]}+$$

$$\left.\frac{b\,\mathrm{Cos}[2x]\sqrt{-\frac{a}{-1+\mathrm{Cos}[2x]}-\frac{b}{-1+\mathrm{Cos}[2x]}+\frac{a\,\mathrm{Cos}[2x]}{-1+\mathrm{Cos}[2x]}-\frac{b\,\mathrm{Cos}[2x]}{-1+\mathrm{Cos}[2x]}}}{-a-b+a\,\mathrm{Cos}[2x]-b\,\mathrm{Cos}[2x]}\right)$$

$$\sqrt{a+b\,\mathrm{Cot}[x]^2}\left(4\sqrt{b}\sqrt{-a+b}\,\mathrm{Log}\left[\mathrm{Sec}\left[\frac{x}{2}\right]^2\right]+(a-2b)\,\mathrm{Log}\left[\mathrm{Tan}\left[\frac{x}{2}\right]^2\right]-\right.$$

$$a\,\text{Log}\Big[b+(2a-b)\,\text{Tan}\Big[\frac{x}{2}\Big]^2+\sqrt{b}\,\sqrt{b\,\text{Cos}[x]^2\,\text{Sec}\Big[\frac{x}{2}\Big]^4+4a\,\text{Tan}\Big[\frac{x}{2}\Big]^2}\,\Big]+$$

$$2b\,\text{Log}\Big[b+(2a-b)\,\text{Tan}\Big[\frac{x}{2}\Big]^2+\sqrt{b}\,\sqrt{b\,\text{Cos}[x]^2\,\text{Sec}\Big[\frac{x}{2}\Big]^4+4a\,\text{Tan}\Big[\frac{x}{2}\Big]^2}\,\Big]+$$

$$a\,\text{Log}\Big[2a-b+b\,\text{Tan}\Big[\frac{x}{2}\Big]^2+\sqrt{b}\,\sqrt{b\,\text{Cos}[x]^2\,\text{Sec}\Big[\frac{x}{2}\Big]^4+4a\,\text{Tan}\Big[\frac{x}{2}\Big]^2}\,\Big]-$$

$$2b\,\text{Log}\Big[2a-b+b\,\text{Tan}\Big[\frac{x}{2}\Big]^2+\sqrt{b}\,\sqrt{b\,\text{Cos}[x]^2\,\text{Sec}\Big[\frac{x}{2}\Big]^4+4a\,\text{Tan}\Big[\frac{x}{2}\Big]^2}\,\Big]-$$

$$4\sqrt{b}\,\sqrt{-a+b}\,\text{Log}\Big[-a+b+(a-b)\,\text{Tan}\Big[\frac{x}{2}\Big]^2+\sqrt{-a+b}\,\sqrt{b\,\text{Cos}[x]^2\,\text{Sec}\Big[\frac{x}{2}\Big]^4+4a\,\text{Tan}\Big[\frac{x}{2}\Big]^2}\,\Big]\Bigg)$$

$$\text{Tan}\Big[\frac{x}{2}\Big]\Bigg)\Bigg/\left(\sqrt{2}\,\sqrt{b}\,\sqrt{\big(a+b+(-a+b)\,\text{Cos}[2x]\big)\,\text{Sec}\Big[\frac{x}{2}\Big]^4}\right.$$

$$\Bigg(\frac{1}{2\sqrt{2}\,\sqrt{b}\,\sqrt{\big(a+b+(-a+b)\,\text{Cos}[2x]\big)\,\text{Sec}\big[\frac{x}{2}\big]^4}}$$

$$\sqrt{a+b\,\text{Cot}[x]^2}\,\Bigg(4\sqrt{b}\,\sqrt{-a+b}\,\text{Log}\Big[\text{Sec}\Big[\frac{x}{2}\Big]^2\Big]+(a-2b)\,\text{Log}\Big[\text{Tan}\Big[\frac{x}{2}\Big]^2\Big]-$$

$$a\,\text{Log}\Big[b+(2a-b)\,\text{Tan}\Big[\frac{x}{2}\Big]^2+\sqrt{b}\,\sqrt{b\,\text{Cos}[x]^2\,\text{Sec}\Big[\frac{x}{2}\Big]^4+4a\,\text{Tan}\Big[\frac{x}{2}\Big]^2}\,\Big]+$$

$$2b\,\text{Log}\Big[b+(2a-b)\,\text{Tan}\Big[\frac{x}{2}\Big]^2+\sqrt{b}\,\sqrt{b\,\text{Cos}[x]^2\,\text{Sec}\Big[\frac{x}{2}\Big]^4+4a\,\text{Tan}\Big[\frac{x}{2}\Big]^2}\,\Big]+$$

$$a\,\text{Log}\Big[2a-b+b\,\text{Tan}\Big[\frac{x}{2}\Big]^2+\sqrt{b}\,\sqrt{b\,\text{Cos}[x]^2\,\text{Sec}\Big[\frac{x}{2}\Big]^4+4a\,\text{Tan}\Big[\frac{x}{2}\Big]^2}\,\Big]-$$

$$2b\,\text{Log}\Big[2a-b+b\,\text{Tan}\Big[\frac{x}{2}\Big]^2+\sqrt{b}\,\sqrt{b\,\text{Cos}[x]^2\,\text{Sec}\Big[\frac{x}{2}\Big]^4+4a\,\text{Tan}\Big[\frac{x}{2}\Big]^2}\,\Big]-4\sqrt{b}\,\sqrt{-a+b}$$

$$\text{Log}\Big[-a+b+(a-b)\,\text{Tan}\Big[\frac{x}{2}\Big]^2+\sqrt{-a+b}\,\sqrt{b\,\text{Cos}[x]^2\,\text{Sec}\Big[\frac{x}{2}\Big]^4+4a\,\text{Tan}\Big[\frac{x}{2}\Big]^2}\,\Big]\Bigg)$$

$$\text{Sec}\Big[\frac{x}{2}\Big]^2-\frac{1}{\sqrt{2}\,\sqrt{a+b\,\text{Cot}[x]^2}\,\sqrt{\big(a+b+(-a+b)\,\text{Cos}[2x]\big)\,\text{Sec}\big[\frac{x}{2}\big]^4}}$$

$$\sqrt{b}\,\text{Cot}[x]\,\text{Csc}[x]^2\,\Bigg(4\sqrt{b}\,\sqrt{-a+b}\,\text{Log}\Big[\text{Sec}\Big[\frac{x}{2}\Big]^2\Big]+(a-2b)\,\text{Log}\Big[\text{Tan}\Big[\frac{x}{2}\Big]^2\Big]-$$

$$a\,\text{Log}\Big[b+(2a-b)\,\text{Tan}\Big[\frac{x}{2}\Big]^2+\sqrt{b}\,\sqrt{b\,\text{Cos}[x]^2\,\text{Sec}\Big[\frac{x}{2}\Big]^4+4a\,\text{Tan}\Big[\frac{x}{2}\Big]^2}\,\Big]+$$

$$2\,b\,\text{Log}\Big[b+(2\,a-b)\,\text{Tan}\big[\tfrac{x}{2}\big]^2+\sqrt{b}\,\sqrt{b\,\text{Cos}[x]^2\,\text{Sec}\big[\tfrac{x}{2}\big]^4+4\,a\,\text{Tan}\big[\tfrac{x}{2}\big]^2}\,\Big]+$$

$$a\,\text{Log}\Big[2\,a-b+b\,\text{Tan}\big[\tfrac{x}{2}\big]^2+\sqrt{b}\,\sqrt{b\,\text{Cos}[x]^2\,\text{Sec}\big[\tfrac{x}{2}\big]^4+4\,a\,\text{Tan}\big[\tfrac{x}{2}\big]^2}\,\Big]-$$

$$2\,b\,\text{Log}\Big[2\,a-b+b\,\text{Tan}\big[\tfrac{x}{2}\big]^2+\sqrt{b}\,\sqrt{b\,\text{Cos}[x]^2\,\text{Sec}\big[\tfrac{x}{2}\big]^4+4\,a\,\text{Tan}\big[\tfrac{x}{2}\big]^2}\,\Big]-4\,\sqrt{b}\,\sqrt{-a+b}$$

$$\text{Log}\Big[-a+b+(a-b)\,\text{Tan}\big[\tfrac{x}{2}\big]^2+\sqrt{-a+b}\,\sqrt{b\,\text{Cos}[x]^2\,\text{Sec}\big[\tfrac{x}{2}\big]^4+4\,a\,\text{Tan}\big[\tfrac{x}{2}\big]^2}\,\Big]\Bigg)\,\text{Tan}\big[\tfrac{x}{2}\big]-$$

$$\frac{1}{2\,\sqrt{2}\,\sqrt{b}\,\Big(\big(a+b+(-a+b)\,\text{Cos}[2\,x]\big)\,\text{Sec}\big[\tfrac{x}{2}\big]^4\Big)^{3/2}}\,\sqrt{a+b\,\text{Cot}[x]^2}$$

$$\Bigg(4\,\sqrt{b}\,\sqrt{-a+b}\,\text{Log}\Big[\text{Sec}\big[\tfrac{x}{2}\big]^2\Big]+(a-2\,b)\,\text{Log}\Big[\text{Tan}\big[\tfrac{x}{2}\big]^2\Big]-$$

$$a\,\text{Log}\Big[b+(2\,a-b)\,\text{Tan}\big[\tfrac{x}{2}\big]^2+\sqrt{b}\,\sqrt{b\,\text{Cos}[x]^2\,\text{Sec}\big[\tfrac{x}{2}\big]^4+4\,a\,\text{Tan}\big[\tfrac{x}{2}\big]^2}\,\Big]+$$

$$2\,b\,\text{Log}\Big[b+(2\,a-b)\,\text{Tan}\big[\tfrac{x}{2}\big]^2+\sqrt{b}\,\sqrt{b\,\text{Cos}[x]^2\,\text{Sec}\big[\tfrac{x}{2}\big]^4+4\,a\,\text{Tan}\big[\tfrac{x}{2}\big]^2}\,\Big]+$$

$$a\,\text{Log}\Big[2\,a-b+b\,\text{Tan}\big[\tfrac{x}{2}\big]^2+\sqrt{b}\,\sqrt{b\,\text{Cos}[x]^2\,\text{Sec}\big[\tfrac{x}{2}\big]^4+4\,a\,\text{Tan}\big[\tfrac{x}{2}\big]^2}\,\Big]-$$

$$2\,b\,\text{Log}\Big[2\,a-b+b\,\text{Tan}\big[\tfrac{x}{2}\big]^2+\sqrt{b}\,\sqrt{b\,\text{Cos}[x]^2\,\text{Sec}\big[\tfrac{x}{2}\big]^4+4\,a\,\text{Tan}\big[\tfrac{x}{2}\big]^2}\,\Big]-4\,\sqrt{b}\,\sqrt{-a+b}$$

$$\text{Log}\Big[-a+b+(a-b)\,\text{Tan}\big[\tfrac{x}{2}\big]^2+\sqrt{-a+b}\,\sqrt{b\,\text{Cos}[x]^2\,\text{Sec}\big[\tfrac{x}{2}\big]^4+4\,a\,\text{Tan}\big[\tfrac{x}{2}\big]^2}\,\Big]\Bigg)\,\text{Tan}\big[\tfrac{x}{2}\big]$$

$$\Big(-2\,(-a+b)\,\text{Sec}\big[\tfrac{x}{2}\big]^4\,\text{Sin}[2\,x]+2\,\big(a+b+(-a+b)\,\text{Cos}[2\,x]\big)\,\text{Sec}\big[\tfrac{x}{2}\big]^4\,\text{Tan}\big[\tfrac{x}{2}\big]\Big)+$$

$$\frac{1}{\sqrt{2}\,\sqrt{b}\,\sqrt{\big(a+b+(-a+b)\,\text{Cos}[2\,x]\big)\,\text{Sec}\big[\tfrac{x}{2}\big]^4}}\,\sqrt{a+b\,\text{Cot}[x]^2}\,\text{Tan}\big[\tfrac{x}{2}\big]$$

$$\Bigg((a-2\,b)\,\text{Csc}\big[\tfrac{x}{2}\big]\,\text{Sec}\big[\tfrac{x}{2}\big]+4\,\sqrt{b}\,\sqrt{-a+b}\,\text{Tan}\big[\tfrac{x}{2}\big]-\Bigg(a\,\Bigg((2\,a-b)\,\text{Sec}\big[\tfrac{x}{2}\big]^2\,\text{Tan}\big[\tfrac{x}{2}\big]+$$

$$\Big(\sqrt{b}\,\Big(-2\,b\,\text{Cos}[x]\,\text{Sec}\big[\tfrac{x}{2}\big]^4\,\text{Sin}[x]+4\,a\,\text{Sec}\big[\tfrac{x}{2}\big]^2\,\text{Tan}\big[\tfrac{x}{2}\big]+2\,b\,\text{Cos}[x]^2$$

$$\text{Sec}\big[\tfrac{x}{2}\big]^4\,\text{Tan}\big[\tfrac{x}{2}\big]\Big)\Big)\Big/\Big(2\,\sqrt{b\,\text{Cos}[x]^2\,\text{Sec}\big[\tfrac{x}{2}\big]^4+4\,a\,\text{Tan}\big[\tfrac{x}{2}\big]^2}\Big)\Bigg)\Bigg)\Big/$$

$$\Big(b+(2\,a-b)\,\text{Tan}\big[\tfrac{x}{2}\big]^2+\sqrt{b}\,\sqrt{b\,\text{Cos}[x]^2\,\text{Sec}\big[\tfrac{x}{2}\big]^4+4\,a\,\text{Tan}\big[\tfrac{x}{2}\big]^2}\Big)+\Bigg(2\,b\,\Bigg((2\,a-b)$$

$$\text{Sec}\big[\tfrac{x}{2}\big]^2\,\text{Tan}\big[\tfrac{x}{2}\big]+\Big(\sqrt{b}\,\Big(-2\,b\,\text{Cos}[x]\,\text{Sec}\big[\tfrac{x}{2}\big]^4\,\text{Sin}[x]+4\,a\,\text{Sec}\big[\tfrac{x}{2}\big]^2\,\text{Tan}\big[\tfrac{x}{2}\big]+2\,b$$

$$\text{Cos}[x]^2\,\text{Sec}\big[\tfrac{x}{2}\big]^4\,\text{Tan}\big[\tfrac{x}{2}\big]\Big)\Big)\Big/\Big(2\,\sqrt{b\,\text{Cos}[x]^2\,\text{Sec}\big[\tfrac{x}{2}\big]^4+4\,a\,\text{Tan}\big[\tfrac{x}{2}\big]^2}\Big)\Bigg)\Bigg)\Big/$$

$$\left(b+(2\,a-b)\,\text{Tan}\left[\frac{x}{2}\right]^2+\sqrt{b}\,\sqrt{b\,\text{Cos}[x]^2\,\text{Sec}\left[\frac{x}{2}\right]^4+4\,a\,\text{Tan}\left[\frac{x}{2}\right]^2}\right)+$$

$$\Bigg(a\,\Bigg(b\,\text{Sec}\left[\frac{x}{2}\right]^2\text{Tan}\left[\frac{x}{2}\right]+\Bigg(\sqrt{b}\,\Bigg(-2\,b\,\text{Cos}[x]\,\text{Sec}\left[\frac{x}{2}\right]^4\text{Sin}[x]+4\,a\,\text{Sec}\left[\frac{x}{2}\right]^2\text{Tan}\left[\frac{x}{2}\right]+2\,b$$

$$\text{Cos}[x]^2\,\text{Sec}\left[\frac{x}{2}\right]^4\text{Tan}\left[\frac{x}{2}\right]\Bigg)\Bigg)\Bigg/\left(2\,\sqrt{b\,\text{Cos}[x]^2\,\text{Sec}\left[\frac{x}{2}\right]^4+4\,a\,\text{Tan}\left[\frac{x}{2}\right]^2}\right)\Bigg)\Bigg)\Bigg/$$

$$\left(2\,a-b+b\,\text{Tan}\left[\frac{x}{2}\right]^2+\sqrt{b}\,\sqrt{b\,\text{Cos}[x]^2\,\text{Sec}\left[\frac{x}{2}\right]^4+4\,a\,\text{Tan}\left[\frac{x}{2}\right]^2}\right)-$$

$$\Bigg(2\,b\,\Bigg(b\,\text{Sec}\left[\frac{x}{2}\right]^2\text{Tan}\left[\frac{x}{2}\right]+\Bigg(\sqrt{b}\,\Bigg(-2\,b\,\text{Cos}[x]\,\text{Sec}\left[\frac{x}{2}\right]^4\text{Sin}[x]+4\,a\,\text{Sec}\left[\frac{x}{2}\right]^2\text{Tan}\left[\frac{x}{2}\right]+2\,b$$

$$\text{Cos}[x]^2\,\text{Sec}\left[\frac{x}{2}\right]^4\text{Tan}\left[\frac{x}{2}\right]\Bigg)\Bigg)\Bigg/\left(2\,\sqrt{b\,\text{Cos}[x]^2\,\text{Sec}\left[\frac{x}{2}\right]^4+4\,a\,\text{Tan}\left[\frac{x}{2}\right]^2}\right)\Bigg)\Bigg)\Bigg/$$

$$\left(2\,a-b+b\,\text{Tan}\left[\frac{x}{2}\right]^2+\sqrt{b}\,\sqrt{b\,\text{Cos}[x]^2\,\text{Sec}\left[\frac{x}{2}\right]^4+4\,a\,\text{Tan}\left[\frac{x}{2}\right]^2}\right)-$$

$$\Bigg(4\,\sqrt{b}\,\sqrt{-a+b}\,\Bigg((a-b)\,\text{Sec}\left[\frac{x}{2}\right]^2\text{Tan}\left[\frac{x}{2}\right]+$$

$$\Bigg(\sqrt{-a+b}\,\Bigg(-2\,b\,\text{Cos}[x]\,\text{Sec}\left[\frac{x}{2}\right]^4\text{Sin}[x]+4\,a\,\text{Sec}\left[\frac{x}{2}\right]^2\text{Tan}\left[\frac{x}{2}\right]+2\,b\,\text{Cos}[x]^2$$

$$\text{Sec}\left[\frac{x}{2}\right]^4\text{Tan}\left[\frac{x}{2}\right]\Bigg)\Bigg)\Bigg/\left(2\,\sqrt{b\,\text{Cos}[x]^2\,\text{Sec}\left[\frac{x}{2}\right]^4+4\,a\,\text{Tan}\left[\frac{x}{2}\right]^2}\right)\Bigg)\Bigg)\Bigg/$$

$$\left(-a+b+(a-b)\,\text{Tan}\left[\frac{x}{2}\right]^2+\sqrt{-a+b}\,\sqrt{b\,\text{Cos}[x]^2\,\text{Sec}\left[\frac{x}{2}\right]^4+4\,a\,\text{Tan}\left[\frac{x}{2}\right]^2}\right)\Bigg)\Bigg)$$

Problem 23: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\sqrt{a+b\,\text{Cot}[x]^2}\,dx$$

Optimal (type 3, 65 leaves, 6 steps):

$$-\sqrt{a-b}\,\text{ArcTan}\left[\frac{\sqrt{a-b}\,\text{Cot}[x]}{\sqrt{a+b\,\text{Cot}[x]^2}}\right]-\sqrt{b}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,\text{Cot}[x]}{\sqrt{a+b\,\text{Cot}[x]^2}}\right]$$

Result (type 3, 167 leaves):

$$\frac{1}{2} i \left(\sqrt{a-b}\ \text{Log}\left[-\frac{4 i \left(a - i b \text{Cot}[x] + \sqrt{a-b} \sqrt{a + b \text{Cot}[x]^2}\right)}{(a-b)^{3/2} (i + \text{Cot}[x])}\right] - \sqrt{a-b}\ \text{Log}\left[\frac{4 i \left(a + i b \text{Cot}[x] + \sqrt{a-b} \sqrt{a + b \text{Cot}[x]^2}\right)}{(a-b)^{3/2} (-i + \text{Cot}[x])}\right] + 2 i \sqrt{b}\ \text{Log}\left[b \text{Cot}[x] + \sqrt{b} \sqrt{a + b \text{Cot}[x]^2}\right]\right)$$

Problem 24: Result more than twice size of optimal antiderivative.

$$\int \sqrt{a + b \text{Cot}[x]^2}\ \text{Tan}[x]^2\, dx$$

Optimal (type 3, 51 leaves, 5 steps):

$$\sqrt{a-b}\ \text{ArcTan}\left[\frac{\sqrt{a-b}\ \text{Cot}[x]}{\sqrt{a + b \text{Cot}[x]^2}}\right] + \sqrt{a + b \text{Cot}[x]^2}\ \text{Tan}[x]$$

Result (type 3, 129 leaves):

$$\left(\sqrt{-(-a - b + (a-b) \text{Cos}[2 x]) \text{Csc}[x]^2} \left(-2 \sqrt{a-b}\ \text{ArcTanh}\left[\frac{\sqrt{2} \sqrt{a-b}\ \text{Cos}[x]}{\sqrt{-a - b + (a-b) \text{Cos}[2 x]}}\right] + \sqrt{-2 (a+b) + 2 (a-b) \text{Cos}[2 x]}\ \text{Sec}[x]\right) \text{Sin}[x]\right) \Big/ \left(2 \sqrt{-a - b + (a-b) \text{Cos}[2 x]}\right)$$

Problem 26: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \text{Cot}[x]^3 \left(a + b \text{Cot}[x]^2\right)^{3/2} dx$$

Optimal (type 3, 88 leaves, 7 steps):

$$-(a-b)^{3/2} \text{ArcTanh}\left[\frac{\sqrt{a + b \text{Cot}[x]^2}}{\sqrt{a-b}}\right] + (a-b) \sqrt{a + b \text{Cot}[x]^2} + \frac{1}{3} \left(a + b \text{Cot}[x]^2\right)^{3/2} - \frac{\left(a + b \text{Cot}[x]^2\right)^{5/2}}{5 b}$$

Result (type 4, 531 leaves):

$$\sqrt{\frac{-a-b+a\,\text{Cos}[2x]-b\,\text{Cos}[2x]}{-1+\text{Cos}[2x]}}\left(-\frac{3a^2-26ab+23b^2}{15b}+\frac{1}{15}(-6a+11b)\,\text{Csc}[x]^2-\frac{1}{5}b\,\text{Csc}[x]^4\right)+$$

$$\left(2\,\text{i}\,(a-b)^2\,(1+\text{Cos}[x])\,\sqrt{\frac{-1+\text{Cos}[2x]}{(1+\text{Cos}[x])^2}}\,\sqrt{\frac{-a-b+(a-b)\,\text{Cos}[2x]}{-1+\text{Cos}[2x]}}\right.$$

$$\left(\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[\sqrt{\frac{b}{2a+2\sqrt{a(a-b)}-b}}\,\text{Tan}\left[\frac{x}{2}\right]\right],\frac{-2a-2\sqrt{a(a-b)}+b}{-2a+2\sqrt{a(a-b)}+b}\right]-\right.$$

$$2\,\text{EllipticPi}\left[\frac{2a+2\sqrt{a(a-b)}-b}{b},\right.$$

$$\left.\left.\text{i}\,\text{ArcSinh}\left[\sqrt{\frac{b}{2a+2\sqrt{a(a-b)}-b}}\,\text{Tan}\left[\frac{x}{2}\right]\right],\frac{-2a-2\sqrt{a(a-b)}+b}{-2a+2\sqrt{a(a-b)}+b}\right]\right)$$

$$\left.\text{Tan}\left[\frac{x}{2}\right]\sqrt{1+\frac{b\,\text{Tan}\left[\frac{x}{2}\right]^2}{2a+2\sqrt{a(a-b)}-b}}\,\sqrt{1-\frac{b\,\text{Tan}\left[\frac{x}{2}\right]^2}{-2a+2\sqrt{a(a-b)}+b}}\right)\Bigg/$$

$$\left(\sqrt{\frac{b}{2a+2\sqrt{a(a-b)}-b}}\,\sqrt{-a-b+(a-b)\,\text{Cos}[2x]}\,\sqrt{-\text{Tan}\left[\frac{x}{2}\right]^2}\right.$$

$$\left.\left(1+\text{Tan}\left[\frac{x}{2}\right]^2\right)\sqrt{-\frac{4a\,\text{Tan}\left[\frac{x}{2}\right]^2+b\left(-1+\text{Tan}\left[\frac{x}{2}\right]^2\right)^2}{\left(1+\text{Tan}\left[\frac{x}{2}\right]^2\right)^2}}\right)$$

Problem 28: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \text{Cot}[x]\,\left(a+b\,\text{Cot}[x]^2\right)^{3/2}\,dx$$

Optimal (type 3, 69 leaves, 6 steps):

$$(a-b)^{3/2}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Cot}[x]^2}}{\sqrt{a-b}}\right]-(a-b)\,\sqrt{a+b\,\text{Cot}[x]^2}-\frac{1}{3}\left(a+b\,\text{Cot}[x]^2\right)^{3/2}$$

Result (type 4, 503 leaves):

$$\sqrt{\frac{-a-b+a\,\text{Cos}[2\,x]-b\,\text{Cos}[2\,x]}{-1+\text{Cos}[2\,x]}}\left(-\frac{4}{3}(a-b)-\frac{1}{3}b\,\text{Csc}[x]^2\right)-$$

$$\left(2\,i\,(a-b)^2\,(1+\text{Cos}[x])\sqrt{\frac{-1+\text{Cos}[2\,x]}{(1+\text{Cos}[x])^2}}\sqrt{\frac{-a-b+(a-b)\,\text{Cos}[2\,x]}{-1+\text{Cos}[2\,x]}}\right.$$

$$\left(\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{b}{2\,a+2\sqrt{a\,(a-b)}-b}}\,\text{Tan}\left[\frac{x}{2}\right]\right],\frac{-2\,a-2\sqrt{a\,(a-b)}+b}{-2\,a+2\sqrt{a\,(a-b)}+b}\right]-\right.$$

$$2\,\text{EllipticPi}\left[\frac{2\,a+2\sqrt{a\,(a-b)}-b}{b},\right.$$

$$\left.\left.i\,\text{ArcSinh}\left[\sqrt{\frac{b}{2\,a+2\sqrt{a\,(a-b)}-b}}\,\text{Tan}\left[\frac{x}{2}\right]\right],\frac{-2\,a-2\sqrt{a\,(a-b)}+b}{-2\,a+2\sqrt{a\,(a-b)}+b}\right]\right)$$

$$\left.\text{Tan}\left[\frac{x}{2}\right]\sqrt{1+\frac{b\,\text{Tan}\left[\frac{x}{2}\right]^2}{2\,a+2\sqrt{a\,(a-b)}-b}}\sqrt{1-\frac{b\,\text{Tan}\left[\frac{x}{2}\right]^2}{-2\,a+2\sqrt{a\,(a-b)}+b}}\right)\Big/$$

$$\left(\sqrt{\frac{b}{2\,a+2\sqrt{a\,(a-b)}-b}}\sqrt{-a-b+(a-b)\,\text{Cos}[2\,x]}\sqrt{-\text{Tan}\left[\frac{x}{2}\right]^2}\right.$$

$$\left.\left(1+\text{Tan}\left[\frac{x}{2}\right]^2\right)\sqrt{-\frac{4\,a\,\text{Tan}\left[\frac{x}{2}\right]^2+b\left(-1+\text{Tan}\left[\frac{x}{2}\right]^2\right)^2}{\left(1+\text{Tan}\left[\frac{x}{2}\right]^2\right)^2}}\right)$$

Problem 29: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \left(a+b\,\text{Cot}[x]^2\right)^{3/2}\,\text{Tan}[x]\,dx$$

Optimal (type 3, 75 leaves, 8 steps):

$$a^{3/2}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Cot}[x]^2}}{\sqrt{a}}\right]-(a-b)^{3/2}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Cot}[x]^2}}{\sqrt{a-b}}\right]-b\sqrt{a+b\,\text{Cot}[x]^2}$$

Result (type 3, 230 leaves):

$$-\frac{b\sqrt{\left(a+b+(-a+b)\,\text{Cos}[2\,x]\right)\text{Csc}[x]^2}}{\sqrt{2}}+$$

$$\left(\sqrt{a+b\,\text{Cot}[x]^2}\left(2\,a^{3/2}\sqrt{a-b}\,\text{Log}\left[a\,\text{Tan}[x]+\sqrt{a}\,\sqrt{b+a\,\text{Tan}[x]^2}\right]+\right.\right.$$

$$(a-b)^2\left(\text{Log}\left[\frac{4\left(b+\mathrm{i}\,a\,\text{Tan}[x]-\mathrm{i}\sqrt{a-b}\,\sqrt{b+a\,\text{Tan}[x]^2}\right)}{(a-b)^{5/2}\,(-\mathrm{i}+\text{Tan}[x])}\right]-\text{Log}\left[\right.\right.$$

$$\left.\left.\left.\left.\frac{4\,\mathrm{i}\left(\mathrm{i}\,b+a\,\text{Tan}[x]+\sqrt{a-b}\,\sqrt{b+a\,\text{Tan}[x]^2}\right)}{(a-b)^{5/2}\,(\mathrm{i}+\text{Tan}[x])}\right]\right)\right)\text{Tan}[x]\right)\Big/\left(2\sqrt{a-b}\,\sqrt{b+a\,\text{Tan}[x]^2}\right)$$

Problem 30: Result more than twice size of optimal antiderivative.

$$\int\left(a+b\,\text{Cot}[x]^2\right)^{3/2}\text{Tan}[x]^2\,dx$$

Optimal (type 3, 80 leaves, 7 steps):

$$(a-b)^{3/2}\,\text{ArcTan}\left[\frac{\sqrt{a-b}\,\text{Cot}[x]}{\sqrt{a+b\,\text{Cot}[x]^2}}\right]-b^{3/2}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,\text{Cot}[x]}{\sqrt{a+b\,\text{Cot}[x]^2}}\right]+a\sqrt{a+b\,\text{Cot}[x]^2}\,\text{Tan}[x]$$

Result (type 3, 222 leaves):

$$\left(\sqrt{-\left(-a-b+(a-b)\,\text{Cos}[2\,x]\right)\text{Csc}[x]^2}\right.$$

$$\left(-\sqrt{2}\,(a-b)^2\sqrt{-b}\,\text{ArcTanh}\left[\frac{\sqrt{2}\,\sqrt{a-b}\,\text{Cos}[x]}{\sqrt{-a-b+(a-b)\,\text{Cos}[2\,x]}}\right]+\sqrt{a-b}\right.$$

$$\left.\left(\sqrt{2}\,b^2\,\text{ArcTanh}\left[\frac{\sqrt{2}\,\sqrt{-b}\,\text{Cos}[x]}{\sqrt{-a-b+(a-b)\,\text{Cos}[2\,x]}}\right]+a\sqrt{-b}\,\sqrt{-a-b+(a-b)\,\text{Cos}[2\,x]}\,\text{Sec}[x]\right)\right)$$

$$\left.\text{Sin}[x]\right)\Big/\left(\sqrt{2}\,\sqrt{a-b}\,\sqrt{-b}\,\sqrt{-a-b+(a-b)\,\text{Cos}[2\,x]}\right)$$

Problem 31: Result unnecessarily involves imaginary or complex numbers.

$$\int\left(a+b\,\text{Cot}[c+d\,x]^2\right)^{5/2}dx$$

Optimal (type 3, 171 leaves, 8 steps):

$$-\frac{(a-b)^{5/2}\,\text{ArcTan}\left[\frac{\sqrt{a-b}\,\text{Cot}[c+d\,x]}{\sqrt{a+b\,\text{Cot}[c+d\,x]^2}}\right]}{d}-\frac{\sqrt{b}\,\left(15\,a^2-20\,a\,b+8\,b^2\right)\,\text{ArcTanh}\left[\frac{\sqrt{b}\,\text{Cot}[c+d\,x]}{\sqrt{a+b\,\text{Cot}[c+d\,x]^2}}\right]}{8\,d}-$$

$$\frac{(7\,a-4\,b)\,b\,\text{Cot}[c+d\,x]\,\sqrt{a+b\,\text{Cot}[c+d\,x]^2}}{8\,d}-\frac{b\,\text{Cot}[c+d\,x]\,\left(a+b\,\text{Cot}[c+d\,x]^2\right)^{3/2}}{4\,d}$$

Result (type 3, 259 leaves) :

$$-\frac{1}{8\,d}\left(b\,\text{Cot}[c+d\,x]\,\sqrt{a+b\,\text{Cot}[c+d\,x]^2}\,\left(9\,a-4\,b+2\,b\,\text{Cot}[c+d\,x]^2\right)-\right.$$

$$4\,\mathbb{i}\,(a-b)^{5/2}\,\text{Log}\left[-\frac{4\,\mathbb{i}\,\left(a-\mathbb{i}\,b\,\text{Cot}[c+d\,x]+\sqrt{a-b}\,\sqrt{a+b\,\text{Cot}[c+d\,x]^2}\right)}{(a-b)^{7/2}\,(\mathbb{i}+\text{Cot}[c+d\,x])}\right]+$$

$$4\,\mathbb{i}\,(a-b)^{5/2}\,\text{Log}\left[\frac{4\,\mathbb{i}\,\left(a+\mathbb{i}\,b\,\text{Cot}[c+d\,x]+\sqrt{a-b}\,\sqrt{a+b\,\text{Cot}[c+d\,x]^2}\right)}{(a-b)^{7/2}\,(-\mathbb{i}+\text{Cot}[c+d\,x])}\right]+$$

$$\left.\sqrt{b}\,\left(15\,a^2-20\,a\,b+8\,b^2\right)\,\text{Log}\left[b\,\text{Cot}[c+d\,x]+\sqrt{b}\,\sqrt{a+b\,\text{Cot}[c+d\,x]^2}\right]\right)$$

Problem 32: Result unnecessarily involves imaginary or complex numbers.

$$\int\left(a+b\,\text{Cot}[c+d\,x]^2\right)^{3/2}\,dx$$

Optimal (type 3, 126 leaves, 7 steps) :

$$-\frac{(a-b)^{3/2}\,\text{ArcTan}\left[\frac{\sqrt{a-b}\,\text{Cot}[c+d\,x]}{\sqrt{a+b\,\text{Cot}[c+d\,x]^2}}\right]}{d}-$$

$$\frac{(3\,a-2\,b)\,\sqrt{b}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,\text{Cot}[c+d\,x]}{\sqrt{a+b\,\text{Cot}[c+d\,x]^2}}\right]}{2\,d}-\frac{b\,\text{Cot}[c+d\,x]\,\sqrt{a+b\,\text{Cot}[c+d\,x]^2}}{2\,d}$$

Result (type 3, 234 leaves) :

$$\frac{1}{2 d}\left(-b \operatorname{Cot}[c+d x] \sqrt{a+b \operatorname{Cot}[c+d x]^2}+\right.$$
$$i (a-b)^{3/2} \operatorname{Log}\left[-\frac{4 i \left(a-i b \operatorname{Cot}[c+d x]+\sqrt{a-b} \sqrt{a+b \operatorname{Cot}[c+d x]^2}\right)}{(a-b)^{5/2} (i+\operatorname{Cot}[c+d x])}\right]-$$
$$i (a-b)^{3/2} \operatorname{Log}\left[\frac{4 i \left(a+i b \operatorname{Cot}[c+d x]+\sqrt{a-b} \sqrt{a+b \operatorname{Cot}[c+d x]^2}\right)}{(a-b)^{5/2} (-i+\operatorname{Cot}[c+d x])}\right]+$$
$$\left.\sqrt{b} (-3 a+2 b) \operatorname{Log}\left[b \operatorname{Cot}[c+d x]+\sqrt{b} \sqrt{a+b \operatorname{Cot}[c+d x]^2}\right]\right)$$

Problem 33: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \sqrt{a+b \operatorname{Cot}[c+d x]^2}\, dx$$

Optimal (type 3, 87 leaves, 6 steps):

$$-\frac{\sqrt{a-b} \operatorname{ArcTan}\left[\frac{\sqrt{a-b} \operatorname{Cot}[c+d x]}{\sqrt{a+b \operatorname{Cot}[c+d x]^2}}\right]}{d}-\frac{\sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Cot}[c+d x]}{\sqrt{a+b \operatorname{Cot}[c+d x]^2}}\right]}{d}$$

Result (type 3, 202 leaves):

$$\frac{1}{2 d} i \left(\sqrt{a-b} \operatorname{Log}\left[-\frac{4 i \left(a-i b \operatorname{Cot}[c+d x]+\sqrt{a-b} \sqrt{a+b \operatorname{Cot}[c+d x]^2}\right)}{(a-b)^{3/2} (i+\operatorname{Cot}[c+d x])}\right]-\right.$$
$$\sqrt{a-b} \operatorname{Log}\left[\frac{4 i \left(a+i b \operatorname{Cot}[c+d x]+\sqrt{a-b} \sqrt{a+b \operatorname{Cot}[c+d x]^2}\right)}{(a-b)^{3/2} (-i+\operatorname{Cot}[c+d x])}\right]+$$
$$\left.2 i \sqrt{b} \operatorname{Log}\left[b \operatorname{Cot}[c+d x]+\sqrt{b} \sqrt{a+b \operatorname{Cot}[c+d x]^2}\right]\right)$$

Problem 34: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{a + b \,\mathrm{Cot}[c + d\,x]^2}}\,dx$$

Optimal (type 3, 47 leaves, 3 steps):

$$-\frac{\mathrm{ArcTan}\left[\frac{\sqrt{a-b}\,\mathrm{Cot}[c+d\,x]}{\sqrt{a+b\,\mathrm{Cot}[c+d\,x]^2}}\right]}{\sqrt{a-b}\,d}$$

Result (type 3, 151 leaves):

$$\frac{1}{2\sqrt{a-b}\,d}\,i\left(\mathrm{Log}\left[-\frac{4\,i\left(a - i\,b\,\mathrm{Cot}[c+d\,x] + \sqrt{a-b}\,\sqrt{a+b\,\mathrm{Cot}[c+d\,x]^2}\right)}{\sqrt{a-b}\,\left(i + \mathrm{Cot}[c+d\,x]\right)}\right] - \mathrm{Log}\left[\frac{4\,i\left(a + i\,b\,\mathrm{Cot}[c+d\,x] + \sqrt{a-b}\,\sqrt{a+b\,\mathrm{Cot}[c+d\,x]^2}\right)}{\sqrt{a-b}\,\left(-i + \mathrm{Cot}[c+d\,x]\right)}\right]\right)$$

Problem 35: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(a + b\,\mathrm{Cot}[c + d\,x]^2\right)^{3/2}}\,dx$$

Optimal (type 3, 85 leaves, 4 steps):

$$-\frac{\mathrm{ArcTan}\left[\frac{\sqrt{a-b}\,\mathrm{Cot}[c+d\,x]}{\sqrt{a+b\,\mathrm{Cot}[c+d\,x]^2}}\right]}{(a-b)^{3/2}\,d} + \frac{b\,\mathrm{Cot}[c+d\,x]}{a\,(a-b)\,d\,\sqrt{a+b\,\mathrm{Cot}[c+d\,x]^2}}$$

Result (type 3, 189 leaves):

$$\frac{1}{2d}\left(\frac{2b\,\text{Cot}[c+dx]}{a(a-b)\sqrt{a+b\,\text{Cot}[c+dx]^2}}+\frac{1}{(a-b)^{3/2}}\right.$$
$$i\left(\text{Log}\left[-\frac{4 i\sqrt{a-b}\left(a-i b\,\text{Cot}[c+dx]+\sqrt{a-b}\sqrt{a+b\,\text{Cot}[c+dx]^2}\right)}{i+\text{Cot}[c+dx]}\right]-\right.$$
$$\left.\left.\text{Log}\left[\frac{4 i\sqrt{a-b}\left(a+i b\,\text{Cot}[c+dx]+\sqrt{a-b}\sqrt{a+b\,\text{Cot}[c+dx]^2}\right)}{-i+\text{Cot}[c+dx]}\right]\right)\right)$$

Problem 36: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(a+b\,\text{Cot}[c+dx]^2\right)^{5/2}}\,dx$$

Optimal (type 3, 135 leaves, 6 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{a-b}\ \text{Cot}[c+dx]}{\sqrt{a+b\,\text{Cot}[c+dx]^2}}\right]}{(a-b)^{5/2}d}+\frac{b\,\text{Cot}[c+dx]}{3a(a-b)d\left(a+b\,\text{Cot}[c+dx]^2\right)^{3/2}}+\frac{(5a-2b)\,b\,\text{Cot}[c+dx]}{3a^2(a-b)^2d\sqrt{a+b\,\text{Cot}[c+dx]^2}}$$

Result (type 3, 229 leaves):

$$\frac{1}{2d}\left(\frac{2b\,\text{Cot}[c+dx]\left(3a(2a-b)+(5a-2b)\,b\,\text{Cot}[c+dx]^2\right)}{3a^2(a-b)^2\left(a+b\,\text{Cot}[c+dx]^2\right)^{3/2}}+\right.$$
$$\frac{i\,\text{Log}\left[-\frac{4 i (a-b)^{3/2}\left(a-i b\,\text{Cot}[c+dx]+\sqrt{a-b}\sqrt{a+b\,\text{Cot}[c+dx]^2}\right)}{i+\text{Cot}[c+dx]}\right]}{(a-b)^{5/2}}-$$
$$\left.\frac{i\,\text{Log}\left[\frac{4 i (a-b)^{3/2}\left(a+i b\,\text{Cot}[c+dx]+\sqrt{a-b}\sqrt{a+b\,\text{Cot}[c+dx]^2}\right)}{-i+\text{Cot}[c+dx]}\right]}{(a-b)^{5/2}}\right)$$

Problem 37: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{1}{\left(a+b\,\text{Cot}[c+dx]^2\right)^{7/2}}\,dx$$

Optimal (type 3, 190 leaves, 7 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{a-b}\,\text{Cot}[c+d\,x]}{\sqrt{a+b\,\text{Cot}[c+d\,x]^2}}\right]}{(a-b)^{7/2}\,d}+\frac{b\,\text{Cot}[c+d\,x]}{5\,a\,(a-b)\,d\,\left(a+b\,\text{Cot}[c+d\,x]^2\right)^{5/2}}+\frac{(9\,a-4\,b)\,b\,\text{Cot}[c+d\,x]}{15\,a^2\,(a-b)^2\,d\,\left(a+b\,\text{Cot}[c+d\,x]^2\right)^{3/2}}+\frac{b\,\left(33\,a^2-26\,a\,b+8\,b^2\right)\,\text{Cot}[c+d\,x]}{15\,a^3\,(a-b)^3\,d\,\sqrt{a+b\,\text{Cot}[c+d\,x]^2}}$$

Result (type 3, 478 leaves):

$$-\frac{1}{d}\sqrt{a+b\,\text{Cot}[c+d\,x]^2}\,\left(-\frac{b\,\text{Cot}[c+d\,x]}{5\,a\,(a-b)\,\left(a+b\,\text{Cot}[c+d\,x]^2\right)^3}-\frac{(9\,a-4\,b)\,b\,\text{Cot}[c+d\,x]}{15\,a^2\,(a-b)^2\,\left(a+b\,\text{Cot}[c+d\,x]^2\right)^2}-\frac{b\,\left(33\,a^2-26\,a\,b+8\,b^2\right)\,\text{Cot}[c+d\,x]}{15\,a^3\,(a-b)^3\,\left(a+b\,\text{Cot}[c+d\,x]^2\right)}\right)-$$
$$\frac{1}{2\,(a-b)^{7/2}\,d}\,i\,\text{Log}\Big[\left(4\,\left(i\,a^4-3\,i\,a^3\,b+3\,i\,a^2\,b^2-i\,a\,b^3-a^3\,b\,\text{Cot}[c+d\,x]+3\,a^2\,b^2\,\text{Cot}[c+d\,x]-3\,a\,b^3\,\text{Cot}[c+d\,x]+b^4\,\text{Cot}[c+d\,x]\right)\right)\Big/\left(\sqrt{a-b}\,\left(-i+\text{Cot}[c+d\,x]\right)\right)+\frac{4\,i\,(a-b)^3\,\sqrt{a+b\,\text{Cot}[c+d\,x]^2}}{-i+\text{Cot}[c+d\,x]}\Big]+$$
$$\frac{1}{2\,(a-b)^{7/2}\,d}\,i\,\text{Log}\Big[\left(4\,\left(-i\,a^4+3\,i\,a^3\,b-3\,i\,a^2\,b^2+i\,a\,b^3-a^3\,b\,\text{Cot}[c+d\,x]+3\,a^2\,b^2\,\text{Cot}[c+d\,x]-3\,a\,b^3\,\text{Cot}[c+d\,x]+b^4\,\text{Cot}[c+d\,x]\right)\right)\Big/\left(\sqrt{a-b}\,\left(i+\text{Cot}[c+d\,x]\right)\right)-\frac{4\,i\,(a-b)^3\,\sqrt{a+b\,\text{Cot}[c+d\,x]^2}}{i+\text{Cot}[c+d\,x]}\Big]$$

Problem 38: Result more than twice size of optimal antiderivative.

$$\int\left(1-\text{Cot}[x]^2\right)^{3/2}\,dx$$

Optimal (type 3, 54 leaves, 6 steps):

$$\frac{5}{2}\,\text{ArcSin}[\text{Cot}[x]]-2\,\sqrt{2}\,\text{ArcTan}\left[\frac{\sqrt{2}\,\text{Cot}[x]}{\sqrt{1-\text{Cot}[x]^2}}\right]+\frac{1}{2}\,\text{Cot}[x]\,\sqrt{1-\text{Cot}[x]^2}$$

Result (type 3, 123 leaves):

$$\frac{1}{2}\left(1-\text{Cot}[x]^2\right)^{3/2}\,\text{Sec}[2\,x]^2\,\left(\text{ArcTan}\left[\frac{\text{Cos}[x]}{\sqrt{-\text{Cos}[2\,x]}}\right]\sqrt{-\text{Cos}[2\,x]}\,\text{Sin}[x]^3+4\,\text{ArcTanh}\left[\frac{\text{Cos}[x]}{\sqrt{\text{Cos}[2\,x]}}\right]\sqrt{\text{Cos}[2\,x]}\,\text{Sin}[x]^3-4\,\sqrt{2}\,\sqrt{\text{Cos}[2\,x]}\,\text{Log}\left[\sqrt{2}\,\text{Cos}[x]+\sqrt{\text{Cos}[2\,x]}\right]\text{Sin}[x]^3-\frac{1}{4}\,\text{Sin}[4\,x]\right)$$

Problem 44: Result unnecessarily involves higher level functions and more than

twice size of optimal antiderivative.

$$\int \frac{\text{Cot}[x]^3}{\sqrt{a+b\,\text{Cot}[x]^2}}\,dx$$

Optimal (type 3, 52 leaves, 5 steps):

$$-\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Cot}[x]^2}}{\sqrt{a-b}}\right]}{\sqrt{a-b}}-\frac{\sqrt{a+b\,\text{Cot}[x]^2}}{b}$$

Result (type 4, 481 leaves):

$$-\frac{\sqrt{\frac{-a-b+a\,\text{Cos}[2x]-b\,\text{Cos}[2x]}{-1+\text{Cos}[2x]}}}{b}+\left(2\,i\,(1+\text{Cos}[x])\sqrt{\frac{-1+\text{Cos}[2x]}{(1+\text{Cos}[x])^2}}\sqrt{\frac{-a-b+(a-b)\,\text{Cos}[2x]}{-1+\text{Cos}[2x]}}\right.$$
$$\left(\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{b}{2a+2\sqrt{a(a-b)}-b}}\,\text{Tan}\left[\frac{x}{2}\right]\right],\frac{-2a-2\sqrt{a(a-b)}+b}{-2a+2\sqrt{a(a-b)}+b}\right]-\right.$$
$$2\,\text{EllipticPi}\left[\frac{2a+2\sqrt{a(a-b)}-b}{b},\right.$$
$$\left.\left.i\,\text{ArcSinh}\left[\sqrt{\frac{b}{2a+2\sqrt{a(a-b)}-b}}\,\text{Tan}\left[\frac{x}{2}\right]\right],\frac{-2a-2\sqrt{a(a-b)}+b}{-2a+2\sqrt{a(a-b)}+b}\right]\right)$$
$$\left.\text{Tan}\left[\frac{x}{2}\right]\sqrt{1+\frac{b\,\text{Tan}\left[\frac{x}{2}\right]^2}{2a+2\sqrt{a(a-b)}-b}}\sqrt{1-\frac{b\,\text{Tan}\left[\frac{x}{2}\right]^2}{-2a+2\sqrt{a(a-b)}+b}}\right)\Bigg/$$
$$\left(\sqrt{\frac{b}{2a+2\sqrt{a(a-b)}-b}}\sqrt{-a-b+(a-b)\,\text{Cos}[2x]}\sqrt{-\text{Tan}\left[\frac{x}{2}\right]^2}\right.$$
$$\left.\left(1+\text{Tan}\left[\frac{x}{2}\right]^2\right)\sqrt{-\frac{4a\,\text{Tan}\left[\frac{x}{2}\right]^2+b\left(-1+\text{Tan}\left[\frac{x}{2}\right]^2\right)^2}{\left(1+\text{Tan}\left[\frac{x}{2}\right]^2\right)^2}}\right)$$

Problem 45: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Cot}[x]^2}{\sqrt{a+b\,\text{Cot}[x]^2}}\,dx$$

Optimal (type 3, 64 leaves, 6 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{a-b}\ \text{Cot}[x]}{\sqrt{a+b\,\text{Cot}[x]^2}}\right]}{\sqrt{a-b}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{b}\ \text{Cot}[x]}{\sqrt{a+b\,\text{Cot}[x]^2}}\right]}{\sqrt{b}}$$

Result (type 3, 158 leaves):

$$\left(\left(-\sqrt{-b}\ \text{ArcTanh}\left[\frac{\sqrt{2}\ \sqrt{a-b}\ \text{Cos}[x]}{\sqrt{-a-b+(a-b)\ \text{Cos}[2x]}}\right]+\sqrt{a-b}\ \text{ArcTanh}\left[\frac{\sqrt{2}\ \sqrt{-b}\ \text{Cos}[x]}{\sqrt{-a-b+(a-b)\ \text{Cos}[2x]}}\right]\right)\right.$$
$$\left.\sqrt{(a+b+(-a+b)\ \text{Cos}[2x])\ \text{Csc}[x]^2}\ \text{Sin}[x]\right)\Big/\left(\sqrt{a-b}\ \sqrt{-b}\ \sqrt{-a-b+(a-b)\ \text{Cos}[2x]}\right)$$

Problem 46: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{\text{Cot}[x]}{\sqrt{a+b\,\text{Cot}[x]^2}}\,dx$$

Optimal (type 3, 33 leaves, 4 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Cot}[x]^2}}{\sqrt{a-b}}\right]}{\sqrt{a-b}}$$

Result (type 4, 352 leaves):

$$\Bigg(2\,i\,\mathrm{Cos}\left[\frac{x}{2}\right]\,(1+\mathrm{Cos}[x])\,\sqrt{-(-a-b+(a-b)\,\mathrm{Cos}[2x])\,\mathrm{Csc}[x]^2}$$

$$\Bigg(\mathrm{EllipticF}\left[i\,\mathrm{ArcSinh}\left[\sqrt{\frac{b}{2a+2\sqrt{a\,(a-b)}-b}}\,\mathrm{Tan}\left[\frac{x}{2}\right]\right],\frac{-2a-2\sqrt{a\,(a-b)}+b}{-2a+2\sqrt{a\,(a-b)}+b}\right]-$$

$$2\,\mathrm{EllipticPi}\left[\frac{2a+2\sqrt{a\,(a-b)}-b}{b},\right.$$

$$\left.i\,\mathrm{ArcSinh}\left[\sqrt{\frac{b}{2a+2\sqrt{a\,(a-b)}-b}}\,\mathrm{Tan}\left[\frac{x}{2}\right]\right],\frac{-2a-2\sqrt{a\,(a-b)}+b}{-2a+2\sqrt{a\,(a-b)}+b}\right]\Bigg)$$

$$\mathrm{Sin}\left[\frac{x}{2}\right]\sqrt{1+\frac{b\,\mathrm{Tan}\left[\frac{x}{2}\right]^2}{2a+2\sqrt{a\,(a-b)}-b}}\,\sqrt{1-\frac{b\,\mathrm{Tan}\left[\frac{x}{2}\right]^2}{-2a+2\sqrt{a\,(a-b)}+b}}\Bigg)\Bigg/$$

$$\left(\sqrt{\frac{b}{4a+4\sqrt{a\,(a-b)}-2b}}\,(a+b+(-a+b)\,\mathrm{Cos}[2x])\right)$$

Problem 47: Result more than twice size of optimal antiderivative.

$$\int\frac{\mathrm{Tan}[x]}{\sqrt{a+b\,\mathrm{Cot}[x]^2}}\,dx$$

Optimal (type 3, 60 leaves, 7 steps):

$$\frac{\mathrm{ArcTanh}\left[\frac{\sqrt{a+b\,\mathrm{Cot}[x]^2}}{\sqrt{a}}\right]}{\sqrt{a}}-\frac{\mathrm{ArcTanh}\left[\frac{\sqrt{a+b\,\mathrm{Cot}[x]^2}}{\sqrt{a-b}}\right]}{\sqrt{a-b}}$$

Result (type 3, 204 leaves):

$$\Bigg(2\sqrt{\mathrm{Cos}[x]^2}\,\sqrt{-(-a-b+(a-b)\,\mathrm{Cos}[2x])\,\mathrm{Csc}[x]^2}\,\Bigg(\sqrt{a-b}\,\mathrm{ArcTanh}\left[\frac{\sqrt{a}\,\sqrt{-\mathrm{Sin}[x]^2}}{\sqrt{-b\,\mathrm{Cos}[x]^2-a\,\mathrm{Sin}[x]^2}}\right]-$$

$$\sqrt{a}\,\mathrm{Log}\left[a\sqrt{-1+\mathrm{Cos}[2x]}-b\sqrt{-1+\mathrm{Cos}[2x]}+\sqrt{a-b}\,\sqrt{-a-b+(a-b)\,\mathrm{Cos}[2x]}\right]\Bigg)$$

$$\sqrt{-\mathrm{Sin}[x]^4}\Bigg)\Bigg/\left(\sqrt{a}\,\sqrt{a-b}\,\sqrt{-a-b+(a-b)\,\mathrm{Cos}[2x]}\,\sqrt{\mathrm{Sin}[2x]^2}\right)$$

Problem 48: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Tan}[x]^2}{\sqrt{a+b\,\text{Cot}[x]^2}}\,dx$$

Optimal (type 3, 54 leaves, 5 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{a-b}\,\text{Cot}[x]}{\sqrt{a+b\,\text{Cot}[x]^2}}\right]}{\sqrt{a-b}}+\frac{\sqrt{a+b\,\text{Cot}[x]^2}\,\text{Tan}[x]}{a}$$

Result (type 3, 149 leaves):

$$\left(\sqrt{-\left(-a-b+(a-b)\,\text{Cos}[2x]\right)\text{Csc}[x]^2}\left(-\sqrt{2}\,a\,\text{ArcTanh}\left[\frac{\sqrt{2}\,\sqrt{a-b}\,\text{Cos}[x]}{\sqrt{-a-b+(a-b)\,\text{Cos}[2x]}}\right]\text{Sin}[x]+\sqrt{a-b}\,\sqrt{-a-b+(a-b)\,\text{Cos}[2x]}\,\text{Tan}[x]\right)\right)\Big/\left(\sqrt{2}\,a\,\sqrt{a-b}\,\sqrt{-a-b+(a-b)\,\text{Cos}[2x]}\right)$$

Problem 49: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\text{Cot}[x]^3}{\left(a+b\,\text{Cot}[x]^2\right)^{3/2}}\,dx$$

Optimal (type 3, 59 leaves, 5 steps):

$$-\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Cot}[x]^2}}{\sqrt{a-b}}\right]}{(a-b)^{3/2}}+\frac{a}{(a-b)\,b\,\sqrt{a+b\,\text{Cot}[x]^2}}$$

Result (type 4, 489 leaves):

$$-\frac{1}{(a-b)\,b\,\sqrt{\frac{b}{4a+4\sqrt{a\,(a-b)}-2b}}\,\left(a+b+(-a+b)\,\mathrm{Cos}[2x]\right)}4\,\mathrm{i}\,\mathrm{Cos}\left[\frac{x}{2}\right]^2$$

$$\sqrt{-\left(-a-b+(a-b)\,\mathrm{Cos}[2x]\right)\,\mathrm{Csc}[x]^2}\;\mathrm{Sin}\left[\frac{x}{2}\right]\left(\mathrm{i}\,a\,\sqrt{\frac{b}{2a+2\sqrt{a\,(a-b)}-b}}\;\mathrm{Sin}\left[\frac{x}{2}\right]+\right.$$

$$b\,\mathrm{Cos}\left[\frac{x}{2}\right]\,\mathrm{EllipticF}\left[\mathrm{i}\,\mathrm{ArcSinh}\left[\sqrt{\frac{b}{2a+2\sqrt{a\,(a-b)}-b}}\;\mathrm{Tan}\left[\frac{x}{2}\right]\right],\frac{-2a-2\sqrt{a\,(a-b)}+b}{-2a+2\sqrt{a\,(a-b)}+b}\right]$$

$$\sqrt{1+\frac{b\,\mathrm{Tan}\left[\frac{x}{2}\right]^2}{2a+2\sqrt{a\,(a-b)}-b}}\;\sqrt{1-\frac{b\,\mathrm{Tan}\left[\frac{x}{2}\right]^2}{-2a+2\sqrt{a\,(a-b)}+b}}\;-$$

$$2\,b\,\mathrm{Cos}\left[\frac{x}{2}\right]\,\mathrm{EllipticPi}\left[\frac{2a+2\sqrt{a\,(a-b)}-b}{b},\,\mathrm{i}\,\mathrm{ArcSinh}\left[\sqrt{\frac{b}{2a+2\sqrt{a\,(a-b)}-b}}\;\mathrm{Tan}\left[\frac{x}{2}\right]\right],\right.$$

$$\left.\left.\frac{-2a-2\sqrt{a\,(a-b)}+b}{-2a+2\sqrt{a\,(a-b)}+b}\right]\sqrt{1+\frac{b\,\mathrm{Tan}\left[\frac{x}{2}\right]^2}{2a+2\sqrt{a\,(a-b)}-b}}\;\sqrt{1-\frac{b\,\mathrm{Tan}\left[\frac{x}{2}\right]^2}{-2a+2\sqrt{a\,(a-b)}+b}}\right)$$

Problem 50: Result more than twice size of optimal antiderivative.

$$\int\frac{\mathrm{Cot}[x]^2}{\left(a+b\,\mathrm{Cot}[x]^2\right)^{3/2}}\,dx$$

Optimal (type 3, 59 leaves, 4 steps):

$$\frac{\mathrm{ArcTan}\left[\frac{\sqrt{a-b}\;\mathrm{Cot}[x]}{\sqrt{a+b\,\mathrm{Cot}[x]^2}}\right]}{(a-b)^{3/2}}-\frac{\mathrm{Cot}[x]}{(a-b)\,\sqrt{a+b\,\mathrm{Cot}[x]^2}}$$

Result (type 3, 157 leaves):

$$\left(-2\sqrt{a-b}\,\sqrt{-a-b+(a-b)\,\mathrm{Cos}[2x]}\;\mathrm{Cot}[x]+\right.$$

$$\left.\sqrt{2}\;\mathrm{ArcTanh}\left[\frac{\sqrt{2}\,\sqrt{a-b}\;\mathrm{Cos}[x]}{\sqrt{-a-b+(a-b)\,\mathrm{Cos}[2x]}}\right]\left(-a-b+(a-b)\,\mathrm{Cos}[2x]\right)\mathrm{Csc}[x]\right)\Big/$$

$$\left((a-b)^{3/2}\,\sqrt{-2\,(a+b)+2\,(a-b)\,\mathrm{Cos}[2x]}\;\sqrt{-\left(-a-b+(a-b)\,\mathrm{Cos}[2x]\right)\,\mathrm{Csc}[x]^2}\right)$$

Problem 51: Result unnecessarily involves higher level functions and more than

twice size of optimal antiderivative.

$$\int \frac{\text{Cot}[x]}{\left(a+b\,\text{Cot}[x]^2\right)^{3/2}}\,dx$$

Optimal (type 3, 55 leaves, 5 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Cot}[x]^2}}{\sqrt{a-b}}\right]}{(a-b)^{3/2}}-\frac{1}{(a-b)\sqrt{a+b\,\text{Cot}[x]^2}}$$

Result (type 4, 483 leaves):

$$-\frac{1}{(a-b)\sqrt{\frac{b}{4a+4\sqrt{a(a-b)}-2b}}\,\left(a+b+(-a+b)\,\text{Cos}[2x]\right)}$$

$$4\,\text{Cos}\left[\frac{x}{2}\right]^2\sqrt{-\left(-a-b+(a-b)\,\text{Cos}[2x]\right)\text{Csc}[x]^2}\;\text{Sin}\left[\frac{x}{2}\right]\left(\sqrt{\frac{b}{2a+2\sqrt{a(a-b)}-b}}\;\text{Sin}\left[\frac{x}{2}\right]-\right.$$

$$i\,\text{Cos}\left[\frac{x}{2}\right]\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{b}{2a+2\sqrt{a(a-b)}-b}}\;\text{Tan}\left[\frac{x}{2}\right]\right],\frac{-2a-2\sqrt{a(a-b)}+b}{-2a+2\sqrt{a(a-b)}+b}\right]$$

$$\sqrt{1+\frac{b\,\text{Tan}\left[\frac{x}{2}\right]^2}{2a+2\sqrt{a(a-b)}-b}}\;\sqrt{1-\frac{b\,\text{Tan}\left[\frac{x}{2}\right]^2}{-2a+2\sqrt{a(a-b)}+b}}+$$

$$2\,i\,\text{Cos}\left[\frac{x}{2}\right]\text{EllipticPi}\left[\frac{2a+2\sqrt{a(a-b)}-b}{b},\;i\,\text{ArcSinh}\left[\sqrt{\frac{b}{2a+2\sqrt{a(a-b)}-b}}\;\text{Tan}\left[\frac{x}{2}\right]\right],\right.$$

$$\left.\left.\frac{-2a-2\sqrt{a(a-b)}+b}{-2a+2\sqrt{a(a-b)}+b}\right]\sqrt{1+\frac{b\,\text{Tan}\left[\frac{x}{2}\right]^2}{2a+2\sqrt{a(a-b)}-b}}\;\sqrt{1-\frac{b\,\text{Tan}\left[\frac{x}{2}\right]^2}{-2a+2\sqrt{a(a-b)}+b}}\right)$$

Problem 52: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Tan}[x]}{\left(a+b\,\text{Cot}[x]^2\right)^{3/2}}\,dx$$

Optimal (type 3, 84 leaves, 8 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Cot}[x]^2}}{\sqrt{a}}\right]}{a^{3/2}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Cot}[x]^2}}{\sqrt{a-b}}\right]}{(a-b)^{3/2}}+\frac{b}{a\,(a-b)\sqrt{a+b\,\text{Cot}[x]^2}}$$

Result (type 3, 243 leaves):

$$\frac{\sqrt{2}\, b}{a (a-b) \sqrt{(a+b+(-a+b) \text{Cos}[2 x]) \text{Csc}[x]^2}}+\left(\text{Cot}[x] \left(2 (a-b)^{3/2} \text{Log}\left[a \text{Tan}[x]+\sqrt{a} \sqrt{b+a \text{Tan}[x]^2}\right]+a^{3/2} \left(\text{Log}\left[\frac{4 i \left(i b-a \text{Tan}[x]+\sqrt{a-b} \sqrt{b+a \text{Tan}[x]^2}\right)}{a \sqrt{a-b} (-i+\text{Tan}[x])}\right]-\text{Log}\left[\frac{4 \left(b-i \left(a \text{Tan}[x]+\sqrt{a-b} \sqrt{b+a \text{Tan}[x]^2}\right)\right)}{a \sqrt{a-b} (i+\text{Tan}[x])}\right]\right)\right) \sqrt{b+a \text{Tan}[x]^2}\right) \Big/ \left(2 a^{3/2} (a-b)^{3/2} \sqrt{a+b \text{Cot}[x]^2}\right)$$

Problem 54: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\text{Cot}[x]^3}{\left(a+b \text{Cot}[x]^2\right)^{5/2}} \, dx$$

Optimal (type 3, 82 leaves, 6 steps):

$$-\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b \text{Cot}[x]^2}}{\sqrt{a-b}}\right]}{(a-b)^{5/2}}+\frac{a}{3 (a-b) b \left(a+b \text{Cot}[x]^2\right)^{3/2}}+\frac{1}{(a-b)^2 \sqrt{a+b \text{Cot}[x]^2}}$$

Result (type 4, 579 leaves):

$$\sqrt{\frac{-a-b+a\,\mathrm{Cos}[2\,x]-b\,\mathrm{Cos}[2\,x]}{-1+\mathrm{Cos}[2\,x]}}\left(\frac{a+3\,b}{3\,(a-b)^3\,b}+\right.$$

$$\left.\frac{4\,a\,b}{3\,(a-b)^3\,(-a-b+a\,\mathrm{Cos}[2\,x]-b\,\mathrm{Cos}[2\,x])^2}+\frac{2\,(2\,a+3\,b)}{3\,(a-b)^3\,(-a-b+a\,\mathrm{Cos}[2\,x]-b\,\mathrm{Cos}[2\,x])}\right)+$$

$$\left(2\,\mathrm{i}\,(1+\mathrm{Cos}[x])\,\sqrt{\frac{-1+\mathrm{Cos}[2\,x]}{(1+\mathrm{Cos}[x])^2}}\,\sqrt{\frac{-a-b+(a-b)\,\mathrm{Cos}[2\,x]}{-1+\mathrm{Cos}[2\,x]}}\right.$$

$$\left(\mathrm{EllipticF}\left[\mathrm{i}\,\mathrm{ArcSinh}\left[\sqrt{\frac{b}{2\,a+2\,\sqrt{a\,(a-b)}-b}}\,\mathrm{Tan}\left[\frac{x}{2}\right]\right],\frac{-2\,a-2\,\sqrt{a\,(a-b)}+b}{-2\,a+2\,\sqrt{a\,(a-b)}+b}\right]-\right.$$

$$2\,\mathrm{EllipticPi}\left[\frac{2\,a+2\,\sqrt{a\,(a-b)}-b}{b},\right.$$

$$\left.\left.\mathrm{i}\,\mathrm{ArcSinh}\left[\sqrt{\frac{b}{2\,a+2\,\sqrt{a\,(a-b)}-b}}\,\mathrm{Tan}\left[\frac{x}{2}\right]\right],\frac{-2\,a-2\,\sqrt{a\,(a-b)}+b}{-2\,a+2\,\sqrt{a\,(a-b)}+b}\right]\right)$$

$$\left.\mathrm{Tan}\left[\frac{x}{2}\right]\sqrt{1+\frac{b\,\mathrm{Tan}\left[\frac{x}{2}\right]^2}{2\,a+2\,\sqrt{a\,(a-b)}-b}}\,\sqrt{1-\frac{b\,\mathrm{Tan}\left[\frac{x}{2}\right]^2}{-2\,a+2\,\sqrt{a\,(a-b)}+b}}\right)\Big/$$

$$\left((a-b)^2\sqrt{\frac{b}{2\,a+2\,\sqrt{a\,(a-b)}-b}}\,\sqrt{-a-b+(a-b)\,\mathrm{Cos}[2\,x]}\,\sqrt{-\mathrm{Tan}\left[\frac{x}{2}\right]^2}\right.$$

$$\left.\left(1+\mathrm{Tan}\left[\frac{x}{2}\right]^2\right)\sqrt{-\frac{4\,a\,\mathrm{Tan}\left[\frac{x}{2}\right]^2+b\left(-1+\mathrm{Tan}\left[\frac{x}{2}\right]^2\right)^2}{\left(1+\mathrm{Tan}\left[\frac{x}{2}\right]^2\right)^2}}\right)$$

Problem 55: Result more than twice size of optimal antiderivative.

$$\int\frac{\mathrm{Cot}[x]^2}{\left(a+b\,\mathrm{Cot}[x]^2\right)^{5/2}}\,dx$$

Optimal (type 3, 94 leaves, 6 steps):

$$\frac{\mathrm{ArcTan}\left[\frac{\sqrt{a-b}\,\mathrm{Cot}[x]}{\sqrt{a+b\,\mathrm{Cot}[x]^2}}\right]}{(a-b)^{5/2}}-\frac{\mathrm{Cot}[x]}{3\,(a-b)\,\left(a+b\,\mathrm{Cot}[x]^2\right)^{3/2}}-\frac{(2\,a+b)\,\mathrm{Cot}[x]}{3\,a\,(a-b)^2\,\sqrt{a+b\,\mathrm{Cot}[x]^2}}$$

Result (type 3, 194 leaves):

$$-\left(\left(\left(6\sqrt{2}\ a\,\text{ArcTanh}\left[\frac{\sqrt{2}\ \sqrt{a-b}\ \text{Cos}[x]}{\sqrt{-a-b+(a-b)\ \text{Cos}[2\,x]}}\right]\ (a+b+(-a+b)\ \text{Cos}[2\,x])^2+2\sqrt{a-b}\ \sqrt{-a-b+(a-b)\ \text{Cos}[2\,x]}\ \left(3\ (a+b)^2\ \text{Cos}[x]+\left(-3\,a^2+2\,a\,b+b^2\right)\ \text{Cos}[3\,x]\right)\right)\sqrt{-(-a-b+(a-b)\ \text{Cos}[2\,x])\ \text{Csc}[x]^2}\ \text{Sin}[x]\right)\Big/\left(6\sqrt{2}\ a\ (a-b)^{5/2}\ (-a-b+(a-b)\ \text{Cos}[2\,x])^{5/2}\right)\right)$$

Problem 56: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\text{Cot}[x]}{\left(a+b\,\text{Cot}[x]^2\right)^{5/2}}\,dx$$

Optimal (type 3, 78 leaves, 6 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Cot}[x]^2}}{\sqrt{a-b}}\right]}{(a-b)^{5/2}}-\frac{1}{3\ (a-b)\ \left(a+b\,\text{Cot}[x]^2\right)^{3/2}}-\frac{1}{(a-b)^2\sqrt{a+b\,\text{Cot}[x]^2}}$$

Result (type 4, 566 leaves):

$$\sqrt{\frac{-a-b+a\,\text{Cos}[2\,x]-b\,\text{Cos}[2\,x]}{-1+\text{Cos}[2\,x]}}\left(-\frac{4}{3\,(a-b)^3}-\frac{4\,b^2}{3\,(a-b)^3\,(-a-b+a\,\text{Cos}[2\,x]-b\,\text{Cos}[2\,x])^2}-\frac{10\,b}{3\,(a-b)^3\,(-a-b+a\,\text{Cos}[2\,x]-b\,\text{Cos}[2\,x])}\right)-$$

$$\left(2\,\mathrm{i}\,(1+\text{Cos}[x])\sqrt{\frac{-1+\text{Cos}[2\,x]}{(1+\text{Cos}[x])^2}}\sqrt{\frac{-a-b+(a-b)\,\text{Cos}[2\,x]}{-1+\text{Cos}[2\,x]}}\right.$$

$$\left(\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{b}{2\,a+2\,\sqrt{a\,(a-b)}-b}}\,\text{Tan}\left[\frac{x}{2}\right]\right],\frac{-2\,a-2\,\sqrt{a\,(a-b)}+b}{-2\,a+2\,\sqrt{a\,(a-b)}+b}\right]-\right.$$

$$2\,\text{EllipticPi}\left[\frac{2\,a+2\,\sqrt{a\,(a-b)}-b}{b},\right.$$

$$\left.\left.\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{b}{2\,a+2\,\sqrt{a\,(a-b)}-b}}\,\text{Tan}\left[\frac{x}{2}\right]\right],\frac{-2\,a-2\,\sqrt{a\,(a-b)}+b}{-2\,a+2\,\sqrt{a\,(a-b)}+b}\right]\right)$$

$$\left.\text{Tan}\left[\frac{x}{2}\right]\sqrt{1+\frac{b\,\text{Tan}\left[\frac{x}{2}\right]^2}{2\,a+2\,\sqrt{a\,(a-b)}-b}}\sqrt{1-\frac{b\,\text{Tan}\left[\frac{x}{2}\right]^2}{-2\,a+2\,\sqrt{a\,(a-b)}+b}}\right)\Bigg/$$

$$\left((a-b)^2\sqrt{\frac{b}{2\,a+2\,\sqrt{a\,(a-b)}-b}}\sqrt{-a-b+(a-b)\,\text{Cos}[2\,x]}\sqrt{-\text{Tan}\left[\frac{x}{2}\right]^2}\right.$$

$$\left.\left(1+\text{Tan}\left[\frac{x}{2}\right]^2\right)\sqrt{-\frac{4\,a\,\text{Tan}\left[\frac{x}{2}\right]^2+b\,\left(-1+\text{Tan}\left[\frac{x}{2}\right]^2\right)^2}{\left(1+\text{Tan}\left[\frac{x}{2}\right]^2\right)^2}}\right)$$

Problem 57: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{\text{Tan}[x]}{\left(a+b\,\text{Cot}[x]^2\right)^{5/2}}\,dx$$

Optimal (type 3, 118 leaves, 9 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Cot}[x]^2}}{\sqrt{a}}\right]}{a^{5/2}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Cot}[x]^2}}{\sqrt{a-b}}\right]}{(a-b)^{5/2}}+\frac{b}{3\,a\,(a-b)\,\left(a+b\,\text{Cot}[x]^2\right)^{3/2}}+\frac{(2\,a-b)\,b}{a^2\,(a-b)^2\,\sqrt{a+b\,\text{Cot}[x]^2}}$$

Result (type 3, 982 leaves) :

$$\sqrt{\frac{-a-b+a\,\text{Cos}[2\,x]-b\,\text{Cos}[2\,x]}{-1+\text{Cos}[2\,x]}}\left(\frac{(7\,a-3\,b)\,b}{3\,a^2\,(a-b)^3}+\frac{4\,b^3}{3\,a\,(a-b)^3\,(-a-b+a\,\text{Cos}[2\,x]-b\,\text{Cos}[2\,x])^2}+\frac{2\,(8\,a-3\,b)\,b^2}{3\,a^2\,(a-b)^3\,(-a-b+a\,\text{Cos}[2\,x]-b\,\text{Cos}[2\,x])}\right)+$$

$$\Bigg(\sqrt{\frac{-a-b+a\,\text{Cos}[2\,x]-b\,\text{Cos}[2\,x]}{-1+\text{Cos}[2\,x]}}\,(-i+\text{Cot}[x])\,(i+\text{Cot}[x])\,\left(a+b\,\text{Cot}[x]^2\right)$$

$$\Bigg(2\,(a-b)^{5/2}\,\text{Log}\left[a\,\text{Tan}[x]+\sqrt{a}\,\sqrt{b+a\,\text{Tan}[x]^2}\right]+$$

$$a^{5/2}\left(\text{Log}\left[\frac{4\left(b+i\,a\,\text{Tan}[x]-i\,\sqrt{a-b}\,\sqrt{b+a\,\text{Tan}[x]^2}\right)}{a^2\,\sqrt{a-b}\,(-i+\text{Tan}[x])}\right]-\right.$$

$$\left.\left.\text{Log}\left[\frac{4\,i\left(i\,b+a\,\text{Tan}[x]+\sqrt{a-b}\,\sqrt{b+a\,\text{Tan}[x]^2}\right)}{a^2\,\sqrt{a-b}\,(i+\text{Tan}[x])}\right]\right)\right)$$

$$\left(-3\,a^2+8\,a\,b-4\,b^2+a^2\,\text{Csc}[x]\,\text{Sin}[3\,x]\right)\,\text{Tan}[x]\,\Bigg(-a+i\,b\,\text{Cot}[x]+$$

$$\left.\sqrt{a-b}\,\text{Cot}[x]\,\sqrt{b+a\,\text{Tan}[x]^2}\right)\left(a+i\,b\,\text{Cot}[x]+\sqrt{a-b}\,\text{Cot}[x]\,\sqrt{b+a\,\text{Tan}[x]^2}\right)\Bigg)\Bigg/$$

$$\Big(4\,a^{5/2}\,(a-b)^2\,(-a-b+a\,\text{Cos}[2\,x]-b\,\text{Cos}[2\,x])\,\Big(2\,i\,a^4\,b\,\text{Csc}[x]^2-6\,i\,a^3\,b^2\,\text{Csc}[x]^2+$$
$$6\,i\,a^2\,b^3\,\text{Csc}[x]^2-2\,i\,a\,b^4\,\text{Csc}[x]^2-2\,i\,a^3\,b^2\,\text{Cot}[x]^2\,\text{Csc}[x]^2+$$
$$4\,i\,a\,b^4\,\text{Cot}[x]^2\,\text{Csc}[x]^2-2\,i\,b^5\,\text{Cot}[x]^2\,\text{Csc}[x]^2-4\,i\,a^2\,b^3\,\text{Cot}[x]^4\,\text{Csc}[x]^2+$$
$$6\,i\,a\,b^4\,\text{Cot}[x]^4\,\text{Csc}[x]^2-2\,i\,b^5\,\text{Cot}[x]^4\,\text{Csc}[x]^2-a^3\,\sqrt{a-b}\,b\,\text{Csc}[x]^2\,\sqrt{b+a\,\text{Tan}[x]^2}+$$
$$2\,a^2\,\sqrt{a-b}\,b^2\,\text{Csc}[x]^2\,\sqrt{b+a\,\text{Tan}[x]^2}-a\,\sqrt{a-b}\,b^3\,\text{Csc}[x]^2\,\sqrt{b+a\,\text{Tan}[x]^2}+a^3\,\sqrt{a-b}$$
$$b\,\text{Cot}[x]^2\,\text{Csc}[x]^2\,\sqrt{b+a\,\text{Tan}[x]^2}-2\,a^2\,\sqrt{a-b}\,b^2\,\text{Cot}[x]^2\,\text{Csc}[x]^2\,\sqrt{b+a\,\text{Tan}[x]^2}+$$
$$4\,a\,\sqrt{a-b}\,b^3\,\text{Cot}[x]^2\,\text{Csc}[x]^2\,\sqrt{b+a\,\text{Tan}[x]^2}-2\,\sqrt{a-b}\,b^4\,\text{Cot}[x]^2\,\text{Csc}[x]^2\,\sqrt{b+a\,\text{Tan}[x]^2}-$$
$$2\,a^2\,\sqrt{a-b}\,b^2\,\text{Cot}[x]^4\,\text{Csc}[x]^2\,\sqrt{b+a\,\text{Tan}[x]^2}+$$
$$5\,a\,\sqrt{a-b}\,b^3\,\text{Cot}[x]^4\,\text{Csc}[x]^2\,\sqrt{b+a\,\text{Tan}[x]^2}-2\,\sqrt{a-b}\,b^4\,\text{Cot}[x]^4\,\text{Csc}[x]^2\,\sqrt{b+a\,\text{Tan}[x]^2}\Big)\Big)$$

Problem 60: Result more than twice size of optimal antiderivative.

$$\int \text{Cot}[x]\,\sqrt{a+b\,\text{Cot}[x]^4}\,dx$$

Optimal (type 3, 90 leaves, 8 steps):

$$\frac{1}{2}\sqrt{b}\ \text{ArcTanh}\left[\frac{\sqrt{b}\ \text{Cot}[x]^2}{\sqrt{a+b\,\text{Cot}[x]^4}}\right]+\frac{1}{2}\sqrt{a+b}\ \text{ArcTanh}\left[\frac{a-b\,\text{Cot}[x]^2}{\sqrt{a+b}\ \sqrt{a+b\,\text{Cot}[x]^4}}\right]-\frac{1}{2}\sqrt{a+b\,\text{Cot}[x]^4}$$

Result (type 3, 1081 leaves):

$$-\frac{1}{2}\sqrt{\frac{3a+3b-4a\,\text{Cos}[2x]+4b\,\text{Cos}[2x]+a\,\text{Cos}[4x]+b\,\text{Cos}[4x]}{3-4\,\text{Cos}[2x]+\text{Cos}[4x]}}+$$
$$\left(\sqrt{\frac{-3a-3b+4a\,\text{Cos}[2x]-4b\,\text{Cos}[2x]-a\,\text{Cos}[4x]-b\,\text{Cos}[4x]}{-3+4\,\text{Cos}[2x]-\text{Cos}[4x]}}\right.$$
$$\text{Cot}[x]^3\left(a+b\,\text{Cot}[x]^4\right)\left(-\sqrt{a+b}\ \text{Log}\left[\text{Sec}[x]^2\right]+\sqrt{b}\ \text{Log}\left[\text{Tan}[x]^2\right]-\right.$$
$$\left.\sqrt{b}\ \text{Log}\left[b+\sqrt{b}\ \sqrt{b+a\,\text{Tan}[x]^4}\right]+\sqrt{a+b}\ \text{Log}\left[b-a\,\text{Tan}[x]^2+\sqrt{a+b}\ \sqrt{b+a\,\text{Tan}[x]^4}\right]\right)$$
$$\left(2a\,\text{Sin}[2x]-2b\,\text{Sin}[2x]-a\,\text{Sin}[4x]-b\,\text{Sin}[4x]\right)\left(\sqrt{b}+\sqrt{b+a\,\text{Tan}[x]^4}\right)$$
$$\left.\left(a-b\,\text{Cot}[x]^2-\sqrt{a+b}\ \text{Cot}[x]^2\sqrt{b+a\,\text{Tan}[x]^4}\right)\right)\Big/$$
$$\left(2\left(-3a-3b+4a\,\text{Cos}[2x]-4b\,\text{Cos}[2x]-a\,\text{Cos}[4x]-b\,\text{Cos}[4x]\right)\right.$$
$$\left(-a^3-a^2b+a^2\sqrt{b}\ \sqrt{a+b}\ \text{Cot}[x]^2-2a^2b\,\text{Cot}[x]^4-2ab^2\,\text{Cot}[x]^4-ab^{3/2}\sqrt{a+b}\ \text{Cot}[x]^4+\right.$$
$$ab^{3/2}\sqrt{a+b}\ \text{Cot}[x]^6-ab^2\,\text{Cot}[x]^8-b^3\,\text{Cot}[x]^8-b^{5/2}\sqrt{a+b}\ \text{Cot}[x]^8+a^3\,\text{Csc}[x]^2+$$
$$a^2b\,\text{Csc}[x]^2-a^2b\,\text{Cot}[x]^2\,\text{Csc}[x]^2-a^2\sqrt{b}\ \sqrt{a+b}\ \text{Cot}[x]^2\,\text{Csc}[x]^2+a^2b\,\text{Cot}[x]^4\,\text{Csc}[x]^2+$$
$$2ab^2\,\text{Cot}[x]^4\,\text{Csc}[x]^2+ab^{3/2}\sqrt{a+b}\ \text{Cot}[x]^4\,\text{Csc}[x]^2-ab^2\,\text{Cot}[x]^6\,\text{Csc}[x]^2-$$
$$ab^{3/2}\sqrt{a+b}\ \text{Cot}[x]^6\,\text{Csc}[x]^2+b^3\,\text{Cot}[x]^8\,\text{Csc}[x]^2+b^{5/2}\sqrt{a+b}\ \text{Cot}[x]^8\,\text{Csc}[x]^2+$$
$$a^2\sqrt{a+b}\ \text{Cot}[x]^2\sqrt{b+a\,\text{Tan}[x]^4}-a^2\sqrt{b}\ \text{Cot}[x]^4\sqrt{b+a\,\text{Tan}[x]^4}-$$
$$ab^{3/2}\,\text{Cot}[x]^4\sqrt{b+a\,\text{Tan}[x]^4}-ab\sqrt{a+b}\ \text{Cot}[x]^4\sqrt{b+a\,\text{Tan}[x]^4}+$$
$$ab\sqrt{a+b}\ \text{Cot}[x]^6\sqrt{b+a\,\text{Tan}[x]^4}-ab^{3/2}\,\text{Cot}[x]^8\sqrt{b+a\,\text{Tan}[x]^4}-$$
$$b^{5/2}\,\text{Cot}[x]^8\sqrt{b+a\,\text{Tan}[x]^4}-b^2\sqrt{a+b}\ \text{Cot}[x]^8\sqrt{b+a\,\text{Tan}[x]^4}-$$
$$a^2\sqrt{a+b}\ \text{Cot}[x]^2\,\text{Csc}[x]^2\sqrt{b+a\,\text{Tan}[x]^4}+a^2\sqrt{b}\ \text{Cot}[x]^4\,\text{Csc}[x]^2\sqrt{b+a\,\text{Tan}[x]^4}+$$
$$ab^{3/2}\,\text{Cot}[x]^4\,\text{Csc}[x]^2\sqrt{b+a\,\text{Tan}[x]^4}+ab\sqrt{a+b}\ \text{Cot}[x]^4\,\text{Csc}[x]^2\sqrt{b+a\,\text{Tan}[x]^4}-$$
$$ab^{3/2}\,\text{Cot}[x]^6\,\text{Csc}[x]^2\sqrt{b+a\,\text{Tan}[x]^4}-ab\sqrt{a+b}\ \text{Cot}[x]^6\,\text{Csc}[x]^2\sqrt{b+a\,\text{Tan}[x]^4}+$$
$$\left.\left.b^{5/2}\,\text{Cot}[x]^8\,\text{Csc}[x]^2\sqrt{b+a\,\text{Tan}[x]^4}+b^2\sqrt{a+b}\ \text{Cot}[x]^8\,\text{Csc}[x]^2\sqrt{b+a\,\text{Tan}[x]^4}\right)\right)$$

Problem 61: Result more than twice size of optimal antiderivative.

$$\int \text{Cot}[x]\left(a+b\,\text{Cot}[x]^4\right)^{3/2}dx$$

Optimal (type 3, 126 leaves, 9 steps):

$$\frac{1}{4}\sqrt{b}\,(3a+2b)\,\text{ArcTanh}\left[\frac{\sqrt{b}\,\text{Cot}[x]^2}{\sqrt{a+b\,\text{Cot}[x]^4}}\right]+\frac{1}{2}(a+b)^{3/2}\,\text{ArcTanh}\left[\frac{a-b\,\text{Cot}[x]^2}{\sqrt{a+b}\,\sqrt{a+b\,\text{Cot}[x]^4}}\right]-\frac{1}{4}\left(2(a+b)-b\,\text{Cot}[x]^2\right)\sqrt{a+b\,\text{Cot}[x]^4}-\frac{1}{6}\left(a+b\,\text{Cot}[x]^4\right)^{3/2}$$

Result (type 3, 1837 leaves):

$$\sqrt{\frac{3a+3b-4a\,\text{Cos}[2x]+4b\,\text{Cos}[2x]+a\,\text{Cos}[4x]+b\,\text{Cos}[4x]}{3-4\,\text{Cos}[2x]+\text{Cos}[4x]}}\left(\frac{1}{12}(-8a-11b)+\frac{7}{12}b\,\text{Csc}[x]^2-\frac{1}{6}b\,\text{Csc}[x]^4\right)+$$

$$\left(\sqrt{a+b\,\text{Cot}[x]^4}\left(2(a+b)^{3/2}\,\text{Log}\left[\text{Sec}[x]^2\right]-\sqrt{b}\,(3a+2b)\,\text{Log}\left[\text{Tan}[x]^2\right]+\sqrt{b}\,(3a+2b)\right.\right.$$

$$\left.\text{Log}\left[b+\sqrt{b}\,\sqrt{b+a\,\text{Tan}[x]^4}\,\right]-2(a+b)^{3/2}\,\text{Log}\left[b-a\,\text{Tan}[x]^2+\sqrt{a+b}\,\sqrt{b+a\,\text{Tan}[x]^4}\,\right]\right)\Bigg/$$

$$\left(\left(2a^2\sqrt{\left(\frac{3a}{3-4\,\text{Cos}[2x]+\text{Cos}[4x]}+\frac{3b}{3-4\,\text{Cos}[2x]+\text{Cos}[4x]}-\frac{4a\,\text{Cos}[2x]}{3-4\,\text{Cos}[2x]+\text{Cos}[4x]}+\frac{4b\,\text{Cos}[2x]}{3-4\,\text{Cos}[2x]+\text{Cos}[4x]}+\frac{a\,\text{Cos}[4x]}{3-4\,\text{Cos}[2x]+\text{Cos}[4x]}+\frac{b\,\text{Cos}[4x]}{3-4\,\text{Cos}[2x]+\text{Cos}[4x]}\right)}\,\text{Sin}[2x]\right)\Big/\left(3a+3b-4a\,\text{Cos}[2x]+4b\,\text{Cos}[2x]+a\,\text{Cos}[4x]+b\,\text{Cos}[4x]\right)-\right.$$

$$\left(2ab\sqrt{\left(\frac{3a}{3-4\,\text{Cos}[2x]+\text{Cos}[4x]}+\frac{3b}{3-4\,\text{Cos}[2x]+\text{Cos}[4x]}-\frac{4a\,\text{Cos}[2x]}{3-4\,\text{Cos}[2x]+\text{Cos}[4x]}+\frac{4b\,\text{Cos}[2x]}{3-4\,\text{Cos}[2x]+\text{Cos}[4x]}+\frac{a\,\text{Cos}[4x]}{3-4\,\text{Cos}[2x]+\text{Cos}[4x]}+\frac{b\,\text{Cos}[4x]}{3-4\,\text{Cos}[2x]+\text{Cos}[4x]}\right)}\,\text{Sin}[2x]\right)\Big/\left(3a+3b-4a\,\text{Cos}[2x]+4b\,\text{Cos}[2x]+a\,\text{Cos}[4x]+b\,\text{Cos}[4x]\right)-$$

$$\left(2b^2\sqrt{\left(\frac{3a}{3-4\,\text{Cos}[2x]+\text{Cos}[4x]}+\frac{3b}{3-4\,\text{Cos}[2x]+\text{Cos}[4x]}-\frac{4a\,\text{Cos}[2x]}{3-4\,\text{Cos}[2x]+\text{Cos}[4x]}+\frac{4b\,\text{Cos}[2x]}{3-4\,\text{Cos}[2x]+\text{Cos}[4x]}+\frac{a\,\text{Cos}[4x]}{3-4\,\text{Cos}[2x]+\text{Cos}[4x]}+\frac{b\,\text{Cos}[4x]}{3-4\,\text{Cos}[2x]+\text{Cos}[4x]}\right)}\,\text{Sin}[2x]\right)\Big/\left(3a+3b-4a\,\text{Cos}[2x]+4b\,\text{Cos}[2x]+a\,\text{Cos}[4x]+b\,\text{Cos}[4x]\right)-$$

$$\left(a^2\sqrt{\left(\frac{3a}{3-4\,\text{Cos}[2x]+\text{Cos}[4x]}+\frac{3b}{3-4\,\text{Cos}[2x]+\text{Cos}[4x]}-\frac{4a\,\text{Cos}[2x]}{3-4\,\text{Cos}[2x]+\text{Cos}[4x]}+\frac{4b\,\text{Cos}[2x]}{3-4\,\text{Cos}[2x]+\text{Cos}[4x]}+\frac{a\,\text{Cos}[4x]}{3-4\,\text{Cos}[2x]+\text{Cos}[4x]}+\frac{b\,\text{Cos}[4x]}{3-4\,\text{Cos}[2x]+\text{Cos}[4x]}\right)}\,\text{Sin}[4x]\right)\Big/\left(3a+3b-4a\,\text{Cos}[2x]+4b\,\text{Cos}[2x]+a\,\text{Cos}[4x]+b\,\text{Cos}[4x]\right)-$$

$$\left(2ab\sqrt{\left(\frac{3a}{3-4\,\text{Cos}[2x]+\text{Cos}[4x]}+\frac{3b}{3-4\,\text{Cos}[2x]+\text{Cos}[4x]}-\frac{4a\,\text{Cos}[2x]}{3-4\,\text{Cos}[2x]+\text{Cos}[4x]}+\right.\right.$$

$$\frac{4\,b\,\text{Cos}[2\,x]}{3-4\,\text{Cos}[2\,x]+\text{Cos}[4\,x]}+\frac{a\,\text{Cos}[4\,x]}{3-4\,\text{Cos}[2\,x]+\text{Cos}[4\,x]}+\frac{b\,\text{Cos}[4\,x]}{3-4\,\text{Cos}[2\,x]+\text{Cos}[4\,x]}\Bigg)$$

$$\text{Sin}[4\,x]\Bigg)\Big/\left(3\,a+3\,b-4\,a\,\text{Cos}[2\,x]+4\,b\,\text{Cos}[2\,x]+a\,\text{Cos}[4\,x]+b\,\text{Cos}[4\,x]\right)-$$

$$\Bigg(b^2\Bigg/\Bigg(\frac{3\,a}{3-4\,\text{Cos}[2\,x]+\text{Cos}[4\,x]}+\frac{3\,b}{3-4\,\text{Cos}[2\,x]+\text{Cos}[4\,x]}-\frac{4\,a\,\text{Cos}[2\,x]}{3-4\,\text{Cos}[2\,x]+\text{Cos}[4\,x]}+$$

$$\frac{4\,b\,\text{Cos}[2\,x]}{3-4\,\text{Cos}[2\,x]+\text{Cos}[4\,x]}+\frac{a\,\text{Cos}[4\,x]}{3-4\,\text{Cos}[2\,x]+\text{Cos}[4\,x]}+\frac{b\,\text{Cos}[4\,x]}{3-4\,\text{Cos}[2\,x]+\text{Cos}[4\,x]}\Bigg)$$

$$\text{Sin}[4\,x]\Bigg)\Big/\left(3\,a+3\,b-4\,a\,\text{Cos}[2\,x]+4\,b\,\text{Cos}[2\,x]+a\,\text{Cos}[4\,x]+b\,\text{Cos}[4\,x]\right)\Bigg)\,\text{Tan}[x]^2\Bigg)\Big/$$

$$\Bigg(4\,\sqrt{b+a\,\text{Tan}[x]^4}\,\Bigg(-\Bigg(\Bigg(a\,\sqrt{a+b\,\text{Cot}[x]^4}\,\Big(2\,(a+b)^{3/2}\,\text{Log}\big[\text{Sec}[x]^2\big]-$$

$$\sqrt{b}\,(3\,a+2\,b)\,\text{Log}\big[\text{Tan}[x]^2\big]+\sqrt{b}\,(3\,a+2\,b)\,\text{Log}\Big[b+\sqrt{b}\,\sqrt{b+a\,\text{Tan}[x]^4}\Big]-$$

$$2\,(a+b)^{3/2}\,\text{Log}\Big[b-a\,\text{Tan}[x]^2+\sqrt{a+b}\,\sqrt{b+a\,\text{Tan}[x]^4}\Big]\Big)\,\text{Sec}[x]^2\,\text{Tan}[x]^5\Bigg)\Big/$$

$$\left(2\,\left(b+a\,\text{Tan}[x]^4\right)^{3/2}\right)\Bigg)-\Bigg(b\,\text{Cot}[x]\,\text{Csc}[x]^2\,\Big(2\,(a+b)^{3/2}\,\text{Log}\big[\text{Sec}[x]^2\big]-$$

$$\sqrt{b}\,(3\,a+2\,b)\,\text{Log}\big[\text{Tan}[x]^2\big]+\sqrt{b}\,(3\,a+2\,b)\,\text{Log}\Big[b+\sqrt{b}\,\sqrt{b+a\,\text{Tan}[x]^4}\Big]-$$

$$2\,(a+b)^{3/2}\,\text{Log}\Big[b-a\,\text{Tan}[x]^2+\sqrt{a+b}\,\sqrt{b+a\,\text{Tan}[x]^4}\Big]\Big)\Bigg)\Big/$$

$$\left(2\,\sqrt{a+b\,\text{Cot}[x]^4}\,\sqrt{b+a\,\text{Tan}[x]^4}\right)+\Bigg(\sqrt{a+b\,\text{Cot}[x]^4}\,\Big(2\,(a+b)^{3/2}\,\text{Log}\big[\text{Sec}[x]^2\big]-$$

$$\sqrt{b}\,(3\,a+2\,b)\,\text{Log}\big[\text{Tan}[x]^2\big]+\sqrt{b}\,(3\,a+2\,b)\,\text{Log}\Big[b+\sqrt{b}\,\sqrt{b+a\,\text{Tan}[x]^4}\Big]-$$

$$2\,(a+b)^{3/2}\,\text{Log}\Big[b-a\,\text{Tan}[x]^2+\sqrt{a+b}\,\sqrt{b+a\,\text{Tan}[x]^4}\Big]\Big)\,\text{Sec}[x]^2\,\text{Tan}[x]\Bigg)\Big/$$

$$\left(2\,\sqrt{b+a\,\text{Tan}[x]^4}\right)+\Bigg(\sqrt{a+b\,\text{Cot}[x]^4}\,\text{Tan}[x]^2\,\Bigg(-2\,\sqrt{b}\,(3\,a+2\,b)\,\text{Csc}[x]\,\text{Sec}[x]+$$

$$4\,(a+b)^{3/2}\,\text{Tan}[x]+\frac{2\,a\,b\,(3\,a+2\,b)\,\text{Sec}[x]^2\,\text{Tan}[x]^3}{\sqrt{b+a\,\text{Tan}[x]^4}\,\left(b+\sqrt{b}\,\sqrt{b+a\,\text{Tan}[x]^4}\right)}-$$

$$\frac{2\,(a+b)^{3/2}\,\left(-2\,a\,\text{Sec}[x]^2\,\text{Tan}[x]+\frac{2\,a\,\sqrt{a+b}\,\text{Sec}[x]^2\,\text{Tan}[x]^3}{\sqrt{b+a\,\text{Tan}[x]^4}}\right)}{b-a\,\text{Tan}[x]^2+\sqrt{a+b}\,\sqrt{b+a\,\text{Tan}[x]^4}}\Bigg)\Bigg)\Big/\left(4\,\sqrt{b+a\,\text{Tan}[x]^4}\right)\Bigg)\Bigg)$$

Problem 62: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\text{Cot}[x]}{\sqrt{a + b\,\text{Cot}[x]^4}}\,dx$$

Optimal (type 3, 41 leaves, 4 steps):

$$\frac{\text{ArcTanh}\left[\frac{a-b\,\text{Cot}[x]^2}{\sqrt{a+b}\,\sqrt{a+b\,\text{Cot}[x]^4}}\right]}{2\sqrt{a+b}}$$

Result (type 4, 72 807 leaves): Display of huge result suppressed!

Problem 63: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\text{Cot}[x]}{\left(a + b\,\text{Cot}[x]^4\right)^{3/2}}\,dx$$

Optimal (type 3, 74 leaves, 6 steps):

$$\frac{\text{ArcTanh}\left[\frac{a-b\,\text{Cot}[x]^2}{\sqrt{a+b}\,\sqrt{a+b\,\text{Cot}[x]^4}}\right]}{2\,(a+b)^{3/2}} - \frac{a + b\,\text{Cot}[x]^2}{2\,a\,(a+b)\,\sqrt{a+b\,\text{Cot}[x]^4}}$$

Result (type 4, 61 450 leaves): Display of huge result suppressed!

Problem 64: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\text{Cot}[x]}{\left(a + b\,\text{Cot}[x]^4\right)^{5/2}}\,dx$$

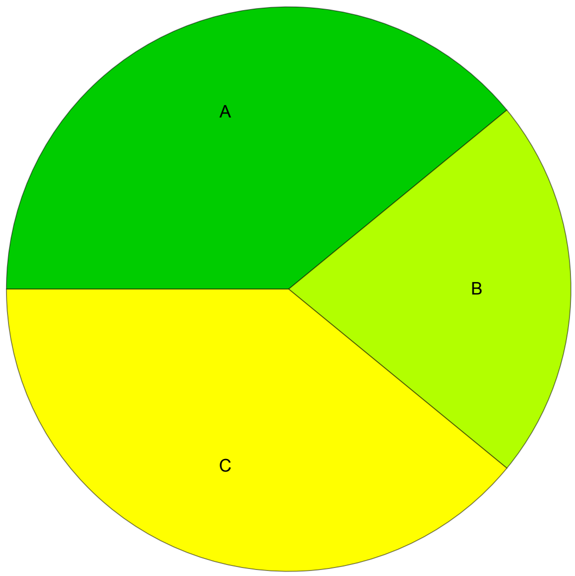
Optimal (type 3, 117 leaves, 7 steps):

$$\frac{\text{ArcTanh}\left[\frac{a-b\,\text{Cot}[x]^2}{\sqrt{a+b}\,\sqrt{a+b\,\text{Cot}[x]^4}}\right]}{2\,(a+b)^{5/2}} - \frac{a + b\,\text{Cot}[x]^2}{6\,a\,(a+b)\,\left(a+b\,\text{Cot}[x]^4\right)^{3/2}} - \frac{3\,a^2 + b\,(5\,a + 2\,b)\,\text{Cot}[x]^2}{6\,a^2\,(a+b)^2\,\sqrt{a+b\,\text{Cot}[x]^4}}$$

Result (type 4, 73 108 leaves): Display of huge result suppressed!

Summary of Integration Test Results

64 integration problems



A - 25 optimal antiderivatives

B - 14 more than twice size of optimal antiderivatives

C - 25 unnecessarily complex antiderivatives

D - 0 unable to integrate problems

E - 0 integration timeouts