

Mathematica 11.3 Integration Test Results

Test results for the 299 problems in "4.5.0 (a sec)^m (b trg)^n.m"

Problem 1: Result more than twice size of optimal antiderivative.

$$\int \text{Sec}[a + b x] dx$$

Optimal (type 3, 11 leaves, 1 step):

$$\frac{\text{ArcTanh}[\text{Sin}[a + b x]]}{b}$$

Result (type 3, 68 leaves):

$$-\frac{\text{Log}\left[\text{Cos}\left[\frac{a}{2} + \frac{bx}{2}\right] - \text{Sin}\left[\frac{a}{2} + \frac{bx}{2}\right]\right]}{b} + \frac{\text{Log}\left[\text{Cos}\left[\frac{a}{2} + \frac{bx}{2}\right] + \text{Sin}\left[\frac{a}{2} + \frac{bx}{2}\right]\right]}{b}$$

Problem 3: Result more than twice size of optimal antiderivative.

$$\int \text{Sec}[a + b x]^3 dx$$

Optimal (type 3, 34 leaves, 2 steps):

$$\frac{\text{ArcTanh}[\text{Sin}[a + b x]]}{2b} + \frac{\text{Sec}[a + b x] \text{Tan}[a + b x]}{2b}$$

Result (type 3, 69 leaves):

$$\frac{1}{2b} \left(-\text{Log}\left[\text{Cos}\left[\frac{1}{2}(a + b x)\right] - \text{Sin}\left[\frac{1}{2}(a + b x)\right]\right] + \text{Log}\left[\text{Cos}\left[\frac{1}{2}(a + b x)\right] + \text{Sin}\left[\frac{1}{2}(a + b x)\right]\right] + \text{Sec}[a + b x] \text{Tan}[a + b x] \right)$$

Problem 41: Result more than twice size of optimal antiderivative.

$$\int (\text{Sec}[x]^2)^{3/2} dx$$

Optimal (type 3, 22 leaves, 3 steps):

$$\frac{1}{2} \text{ArcSinh}[\text{Tan}[x]] + \frac{1}{2} \sqrt{\text{Sec}[x]^2} \text{Tan}[x]$$

Result (type 3, 52 leaves):

$$\frac{1}{2} \cos [x] \sqrt{\sec [x]^2} \left(-\operatorname{Log}\left[\cos \left[\frac{x}{2}\right] - \sin \left[\frac{x}{2}\right]\right] + \operatorname{Log}\left[\cos \left[\frac{x}{2}\right] + \sin \left[\frac{x}{2}\right]\right] + \sec [x] \tan [x] \right)$$

Problem 42: Result more than twice size of optimal antiderivative.

$$\int \sqrt{\sec [x]^2} dx$$

Optimal (type 3, 3 leaves, 2 steps):

$$\operatorname{ArcSinh}[\tan [x]]$$

Result (type 3, 44 leaves):

$$\cos [x] \left(-\operatorname{Log}\left[\cos \left[\frac{x}{2}\right] - \sin \left[\frac{x}{2}\right]\right] + \operatorname{Log}\left[\cos \left[\frac{x}{2}\right] + \sin \left[\frac{x}{2}\right]\right] \right) \sqrt{\sec [x]^2}$$

Problem 136: Result more than twice size of optimal antiderivative.

$$\int \sqrt{\sec [c+d x]} \sqrt{b \sec [c+d x]} dx$$

Optimal (type 3, 33 leaves, 2 steps):

$$\frac{\operatorname{ArcTanh}[\sin [c+d x]] \sqrt{b \sec [c+d x]}}{d \sqrt{\sec [c+d x]}}$$

Result (type 3, 75 leaves):

$$\frac{1}{d \sqrt{\sec [c+d x]}} \left(-\operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right] - \sin \left[\frac{1}{2}(c+d x)\right]\right] + \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right] + \sin \left[\frac{1}{2}(c+d x)\right]\right] \right) \sqrt{b \sec [c+d x]}$$

Problem 146: Result more than twice size of optimal antiderivative.

$$\int \frac{(b \sec [c+d x])^{3/2}}{\sqrt{\sec [c+d x]}} dx$$

Optimal (type 3, 34 leaves, 2 steps):

$$\frac{b \operatorname{ArcTanh}[\sin [c+d x]] \sqrt{b \sec [c+d x]}}{d \sqrt{\sec [c+d x]}}$$

Result (type 3, 75 leaves):

$$\frac{1}{d \sec [c+d x]^{3/2}} \left(-\operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right] - \sin \left[\frac{1}{2}(c+d x)\right]\right] + \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right] + \sin \left[\frac{1}{2}(c+d x)\right]\right] \right) (b \sec [c+d x])^{3/2}$$

Problem 156: Result more than twice size of optimal antiderivative.

$$\int \frac{(b \operatorname{Sec}[c + d x])^{5/2}}{\operatorname{Sec}[c + d x]^{3/2}} dx$$

Optimal (type 3, 36 leaves, 2 steps):

$$\frac{b^2 \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]] \sqrt{b \operatorname{Sec}[c + d x]}}{d \sqrt{\operatorname{Sec}[c + d x]}}$$

Result (type 3, 75 leaves):

$$\frac{1}{d \operatorname{Sec}[c + d x]^{5/2}} \left(-\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \right) (b \operatorname{Sec}[c + d x])^{5/2}$$

Problem 163: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sec}[c + d x]^{3/2}}{\sqrt{b \operatorname{Sec}[c + d x]}} dx$$

Optimal (type 3, 33 leaves, 2 steps):

$$\frac{\operatorname{ArcTanh}[\operatorname{Sin}[c + d x]] \sqrt{\operatorname{Sec}[c + d x]}}{d \sqrt{b \operatorname{Sec}[c + d x]}}$$

Result (type 3, 75 leaves):

$$\left(\left(-\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \right) \sqrt{\operatorname{Sec}[c + d x]} \right) / (d \sqrt{b \operatorname{Sec}[c + d x]})$$

Problem 170: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sec}[c + d x]^{5/2}}{(b \operatorname{Sec}[c + d x])^{3/2}} dx$$

Optimal (type 3, 36 leaves, 2 steps):

$$\frac{\operatorname{ArcTanh}[\operatorname{Sin}[c + d x]] \sqrt{\operatorname{Sec}[c + d x]}}{b d \sqrt{b \operatorname{Sec}[c + d x]}}$$

Result (type 3, 75 leaves):

$$\left(\left(-\operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] + \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] \right) \operatorname{Sec}[c+dx]^{3/2} \right) / \left(d (b \operatorname{Sec}[c+dx])^{3/2} \right)$$

Problem 178: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sec}[c+dx]^{7/2}}{(b \operatorname{Sec}[c+dx])^{5/2}} dx$$

Optimal (type 3, 36 leaves, 2 steps):

$$\frac{\operatorname{ArcTanh}[\operatorname{Sin}[c+dx]] \sqrt{\operatorname{Sec}[c+dx]}}{b^2 d \sqrt{b \operatorname{Sec}[c+dx]}}$$

Result (type 3, 78 leaves):

$$\left(\left(-\operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] + \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] \right) \sqrt{\operatorname{Sec}[c+dx]} \right) / \left(b^2 d \sqrt{b \operatorname{Sec}[c+dx]} \right)$$

Problem 230: Result unnecessarily involves higher level functions.

$$\int (d \operatorname{Csc}[a+bx])^{9/2} \sqrt{c \operatorname{Sec}[a+bx]} dx$$

Optimal (type 4, 128 leaves, 5 steps):

$$-\frac{4 c d^3 (d \operatorname{Csc}[a+bx])^{3/2}}{7 b \sqrt{c \operatorname{Sec}[a+bx]}} - \frac{2 c d (d \operatorname{Csc}[a+bx])^{7/2}}{7 b \sqrt{c \operatorname{Sec}[a+bx]}} + \frac{1}{7 b} \\ 4 d^4 \sqrt{d \operatorname{Csc}[a+bx]} \operatorname{EllipticF}\left[a - \frac{\pi}{4} + bx, 2\right] \sqrt{c \operatorname{Sec}[a+bx]} \sqrt{\operatorname{Sin}[2a+2bx]}$$

Result (type 5, 122 leaves):

$$\left(2 d^4 \cos[2(a+bx)] \cot[a+bx] \sqrt{d \operatorname{Csc}[a+bx]} \sqrt{c \operatorname{Sec}[a+bx]} \left((-2 + \cos[2(a+bx)]) \operatorname{Csc}[a+bx]^4 - 2 (-\cot[a+bx]^2)^{3/4} \right. \right. \\ \left. \left. \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \operatorname{Csc}[a+bx]^2\right] \operatorname{Sec}[a+bx]^2 \right) \right) / \left(7 b (-2 + \operatorname{Csc}[a+bx]^2) \right)$$

Problem 232: Result unnecessarily involves higher level functions.

$$\int (d \operatorname{Csc}[a+bx])^{5/2} \sqrt{c \operatorname{Sec}[a+bx]} dx$$

Optimal (type 4, 93 leaves, 4 steps):

$$-\frac{2cd(d\operatorname{Csc}[a+bx])^{3/2}}{3b\sqrt{c\operatorname{Sec}[a+bx]}} + \frac{1}{3b}$$

$$2d^2\sqrt{d\operatorname{Csc}[a+bx]}\operatorname{EllipticF}\left[a-\frac{\pi}{4}+bx, 2\right]\sqrt{c\operatorname{Sec}[a+bx]}\sqrt{\operatorname{Sin}[2a+2bx]}$$

Result (type 5, 109 leaves):

$$-\left(\left(d(\operatorname{Cos}[a+bx]+\operatorname{Cos}[3(a+bx)])\right)(d\operatorname{Csc}[a+bx])^{3/2}\right.$$

$$\left.\left(\operatorname{Cot}[a+bx]^2+(-\operatorname{Cot}[a+bx]^2)^{3/4}\operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \operatorname{Csc}[a+bx]^2\right]\right)\right.$$

$$\left.\operatorname{Sec}[a+bx]^2\sqrt{c\operatorname{Sec}[a+bx]}\right)/\left(3b(-2+\operatorname{Csc}[a+bx]^2)\right)$$

Problem 234: Result unnecessarily involves higher level functions.

$$\int\sqrt{d\operatorname{Csc}[a+bx]}\sqrt{c\operatorname{Sec}[a+bx]}\,dx$$

Optimal (type 4, 53 leaves, 3 steps):

$$\frac{1}{b}\sqrt{d\operatorname{Csc}[a+bx]}\operatorname{EllipticF}\left[a-\frac{\pi}{4}+bx, 2\right]\sqrt{c\operatorname{Sec}[a+bx]}\sqrt{\operatorname{Sin}[2a+2bx]}$$

Result (type 5, 68 leaves):

$$\frac{1}{b}(-\operatorname{Cot}[a+bx]^2)^{7/4}\sqrt{d\operatorname{Csc}[a+bx]}$$

$$\operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \operatorname{Csc}[a+bx]^2\right]\sqrt{c\operatorname{Sec}[a+bx]}\operatorname{Tan}[a+bx]^3$$

Problem 235: Result unnecessarily involves higher level functions.

$$\int\frac{\sqrt{c\operatorname{Sec}[a+bx]}}{\sqrt{d\operatorname{Csc}[a+bx]}}\,dx$$

Optimal (type 3, 270 leaves, 12 steps):

$$-\frac{\operatorname{ArcTan}\left[1-\sqrt{2}\sqrt{\operatorname{Tan}[a+bx]}\right]\sqrt{c\operatorname{Sec}[a+bx]}}{\sqrt{2}b\sqrt{d\operatorname{Csc}[a+bx]}\sqrt{\operatorname{Tan}[a+bx]}} +$$

$$\frac{\operatorname{ArcTan}\left[1+\sqrt{2}\sqrt{\operatorname{Tan}[a+bx]}\right]\sqrt{c\operatorname{Sec}[a+bx]}}{\sqrt{2}b\sqrt{d\operatorname{Csc}[a+bx]}\sqrt{\operatorname{Tan}[a+bx]}} +$$

$$\frac{\operatorname{Log}\left[1-\sqrt{2}\sqrt{\operatorname{Tan}[a+bx]}+\operatorname{Tan}[a+bx]\right]\sqrt{c\operatorname{Sec}[a+bx]}}{2\sqrt{2}b\sqrt{d\operatorname{Csc}[a+bx]}\sqrt{\operatorname{Tan}[a+bx]}} -$$

$$\frac{\operatorname{Log}\left[1+\sqrt{2}\sqrt{\operatorname{Tan}[a+bx]}+\operatorname{Tan}[a+bx]\right]\sqrt{c\operatorname{Sec}[a+bx]}}{2\sqrt{2}b\sqrt{d\operatorname{Csc}[a+bx]}\sqrt{\operatorname{Tan}[a+bx]}}$$

Result (type 5, 69 leaves):

$$\left(\text{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \sin[a+bx]^2\right] \sqrt{c \sec[a+bx]} \sin[2(a+bx)] \right) / \left(3 b (\cos[a+bx]^2)^{1/4} \sqrt{d \csc[a+bx]} \right)$$

Problem 236: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{c \sec[a+bx]}}{(d \csc[a+bx])^{3/2}} dx$$

Optimal (type 4, 93 leaves, 4 steps):

$$-\frac{c}{b d \sqrt{d \csc[a+bx]} \sqrt{c \sec[a+bx]}} + \frac{1}{2 b d^2} \sqrt{d \csc[a+bx]} \text{EllipticF}\left[a - \frac{\pi}{4} + bx, 2\right] \sqrt{c \sec[a+bx]} \sqrt{\sin[2a+2bx]}$$

Result (type 5, 80 leaves):

$$-\left(\left((1 + \cos[2(a+bx)]) + (-\cot[a+bx]^2)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \csc[a+bx]^2\right] \right) (c \sec[a+bx])^{3/2} \right) / \left(2 b c d \sqrt{d \csc[a+bx]} \right)$$

Problem 237: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{c \sec[a+bx]}}{(d \csc[a+bx])^{5/2}} dx$$

Optimal (type 3, 322 leaves, 13 steps):

$$-\frac{c}{2 b d (d \csc[a+bx])^{3/2} \sqrt{c \sec[a+bx]}} - \frac{3 \text{ArcTan}\left[1 - \sqrt{2} \sqrt{\tan[a+bx]}\right] \sqrt{c \sec[a+bx]}}{4 \sqrt{2} b d^2 \sqrt{d \csc[a+bx]} \sqrt{\tan[a+bx]}} + \frac{3 \text{ArcTan}\left[1 + \sqrt{2} \sqrt{\tan[a+bx]}\right] \sqrt{c \sec[a+bx]}}{4 \sqrt{2} b d^2 \sqrt{d \csc[a+bx]} \sqrt{\tan[a+bx]}} + \frac{3 \text{Log}\left[1 - \sqrt{2} \sqrt{\tan[a+bx]} + \tan[a+bx]\right] \sqrt{c \sec[a+bx]}}{8 \sqrt{2} b d^2 \sqrt{d \csc[a+bx]} \sqrt{\tan[a+bx]}} - \frac{3 \text{Log}\left[1 + \sqrt{2} \sqrt{\tan[a+bx]} + \tan[a+bx]\right] \sqrt{c \sec[a+bx]}}{8 \sqrt{2} b d^2 \sqrt{d \csc[a+bx]} \sqrt{\tan[a+bx]}}$$

Result (type 5, 87 leaves):

$$-\left(\left((\cos[a+bx]^2)^{1/4} - \text{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \sin[a+bx]^2\right] \right) \sqrt{c \sec[a+bx]} \sin[2(a+bx)] \right) / \left(4 b d^2 (\cos[a+bx]^2)^{1/4} \sqrt{d \csc[a+bx]} \right)$$

Problem 239: Result unnecessarily involves higher level functions.

$$\int (d \operatorname{Csc}[a + b x])^{7/2} (c \operatorname{Sec}[a + b x])^{3/2} dx$$

Optimal (type 4, 166 leaves, 6 steps):

$$\frac{24 c d^5 \sqrt{c \operatorname{Sec}[a + b x]}}{5 b (d \operatorname{Csc}[a + b x])^{3/2}} - \frac{12 c d^3 \sqrt{d \operatorname{Csc}[a + b x]} \sqrt{c \operatorname{Sec}[a + b x]}}{5 b} - \frac{2 c d (d \operatorname{Csc}[a + b x])^{5/2} \sqrt{c \operatorname{Sec}[a + b x]}}{5 b} - \frac{24 c^2 d^4 \operatorname{EllipticE}\left[a - \frac{\pi}{4} + b x, 2\right]}{5 b \sqrt{d \operatorname{Csc}[a + b x]} \sqrt{c \operatorname{Sec}[a + b x]} \sqrt{\operatorname{Sin}[2 a + 2 b x]}}$$

Result (type 5, 87 leaves):

$$\frac{1}{5 b} 2 c d (d \operatorname{Csc}[a + b x])^{5/2} \sqrt{c \operatorname{Sec}[a + b x]} \left(2 - 3 \operatorname{Cos}[2(a + b x)]\right) - 6 (-\operatorname{Cot}[a + b x]^2)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, \operatorname{Csc}[a + b x]^2\right] \operatorname{Sin}[a + b x]^2$$

Problem 241: Result unnecessarily involves higher level functions.

$$\int (d \operatorname{Csc}[a + b x])^{3/2} (c \operatorname{Sec}[a + b x])^{3/2} dx$$

Optimal (type 4, 125 leaves, 5 steps):

$$\frac{4 c d^3 \sqrt{c \operatorname{Sec}[a + b x]}}{b (d \operatorname{Csc}[a + b x])^{3/2}} - \frac{2 c d \sqrt{d \operatorname{Csc}[a + b x]} \sqrt{c \operatorname{Sec}[a + b x]}}{b} - \frac{4 c^2 d^2 \operatorname{EllipticE}\left[a - \frac{\pi}{4} + b x, 2\right]}{b \sqrt{d \operatorname{Csc}[a + b x]} \sqrt{c \operatorname{Sec}[a + b x]} \sqrt{\operatorname{Sin}[2 a + 2 b x]}}$$

Result (type 5, 66 leaves):

$$-\frac{1}{b} 2 c d \sqrt{d \operatorname{Csc}[a + b x]} \left(-1 + (-\operatorname{Cot}[a + b x]^2)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, \operatorname{Csc}[a + b x]^2\right]\right) \sqrt{c \operatorname{Sec}[a + b x]}$$

Problem 243: Result unnecessarily involves higher level functions.

$$\int \frac{(c \operatorname{Sec}[a + b x])^{3/2}}{\sqrt{d \operatorname{Csc}[a + b x]}} dx$$

Optimal (type 4, 89 leaves, 4 steps):

$$\frac{2 c d \sqrt{c \operatorname{Sec}[a + b x]}}{b (d \operatorname{Csc}[a + b x])^{3/2}} - \frac{2 c^2 \operatorname{EllipticE}\left[a - \frac{\pi}{4} + b x, 2\right]}{b \sqrt{d \operatorname{Csc}[a + b x]} \sqrt{c \operatorname{Sec}[a + b x]} \sqrt{\operatorname{Sin}[2 a + 2 b x]}}$$

Result (type 5, 70 leaves):

$$-\left(\left(\cot [a + b x] \left(-2 + (-\cot [a + b x]^2)^{1/4} \operatorname{Hypergeometric2F1} \left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, \csc [a + b x]^2 \right] \right) \right. \right. \\ \left. \left. (c \operatorname{Sec} [a + b x])^{3/2} \right) / \left(b \sqrt{d \operatorname{Csc} [a + b x]} \right) \right)$$

Problem 244: Result unnecessarily involves higher level functions.

$$\int \frac{(c \operatorname{Sec} [a + b x])^{3/2}}{(d \operatorname{Csc} [a + b x])^{3/2}} dx$$

Optimal (type 3, 327 leaves, 13 steps):

$$\frac{2 c \sqrt{c \operatorname{Sec} [a + b x]}}{b d \sqrt{d \operatorname{Csc} [a + b x]}} + \frac{c^2 \operatorname{ArcTan} [1 - \sqrt{2} \sqrt{\operatorname{Tan} [a + b x]}] \sqrt{d \operatorname{Csc} [a + b x]} \sqrt{\operatorname{Tan} [a + b x]}}{\sqrt{2} b d^2 \sqrt{c \operatorname{Sec} [a + b x]}} - \\ \frac{c^2 \operatorname{ArcTan} [1 + \sqrt{2} \sqrt{\operatorname{Tan} [a + b x]}] \sqrt{d \operatorname{Csc} [a + b x]} \sqrt{\operatorname{Tan} [a + b x]}}{\sqrt{2} b d^2 \sqrt{c \operatorname{Sec} [a + b x]}} + \\ \left(c^2 \sqrt{d \operatorname{Csc} [a + b x]} \operatorname{Log} [1 - \sqrt{2} \sqrt{\operatorname{Tan} [a + b x]} + \operatorname{Tan} [a + b x]] \sqrt{\operatorname{Tan} [a + b x]} \right) / \\ \left(2 \sqrt{2} b d^2 \sqrt{c \operatorname{Sec} [a + b x]} \right) - \\ \left(c^2 \sqrt{d \operatorname{Csc} [a + b x]} \operatorname{Log} [1 + \sqrt{2} \sqrt{\operatorname{Tan} [a + b x]} + \operatorname{Tan} [a + b x]] \sqrt{\operatorname{Tan} [a + b x]} \right) / \\ \left(2 \sqrt{2} b d^2 \sqrt{c \operatorname{Sec} [a + b x]} \right)$$

Result (type 5, 86 leaves):

$$\left(2 c \left((\cos [a + b x]^2)^{3/4} - \cos [a + b x]^2 \operatorname{Hypergeometric2F1} \left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \sin [a + b x]^2 \right] \right) \right. \\ \left. \sqrt{c \operatorname{Sec} [a + b x]} \right) / \left(b d (\cos [a + b x]^2)^{3/4} \sqrt{d \operatorname{Csc} [a + b x]} \right)$$

Problem 245: Result unnecessarily involves higher level functions.

$$\int \frac{(c \operatorname{Sec} [a + b x])^{3/2}}{(d \operatorname{Csc} [a + b x])^{5/2}} dx$$

Optimal (type 4, 94 leaves, 4 steps):

$$\frac{2 c \sqrt{c \operatorname{Sec} [a + b x]}}{b d (d \operatorname{Csc} [a + b x])^{3/2}} - \frac{3 c^2 \operatorname{EllipticE} [a - \frac{\pi}{4} + b x, 2]}{b d^2 \sqrt{d \operatorname{Csc} [a + b x]} \sqrt{c \operatorname{Sec} [a + b x]} \sqrt{\sin [2 a + 2 b x]}}$$

Result (type 5, 79 leaves):

$$\frac{1}{2 b d^3} c \sqrt{d \operatorname{Csc} [a + b x]} \\ \left(5 + \cos [2 (a + b x)] - 3 (-\cot [a + b x]^2)^{1/4} \operatorname{Hypergeometric2F1} \left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, \csc [a + b x]^2 \right] \right) \\ \sqrt{c \operatorname{Sec} [a + b x]}$$

Problem 246: Result unnecessarily involves higher level functions.

$$\int (d \operatorname{Csc}[a + b x])^{9/2} (c \operatorname{Sec}[a + b x])^{5/2} dx$$

Optimal (type 4, 166 leaves, 6 steps):

$$\frac{40 c d^5 (c \operatorname{Sec}[a + b x])^{3/2}}{21 b \sqrt{d \operatorname{Csc}[a + b x]}} - \frac{20 c d^3 (d \operatorname{Csc}[a + b x])^{3/2} (c \operatorname{Sec}[a + b x])^{3/2}}{21 b} - \frac{2 c d (d \operatorname{Csc}[a + b x])^{7/2} (c \operatorname{Sec}[a + b x])^{3/2}}{7 b} + \frac{1}{21 b} \\ 40 c^2 d^4 \sqrt{d \operatorname{Csc}[a + b x]} \operatorname{EllipticF}\left[a - \frac{\pi}{4} + b x, 2\right] \sqrt{c \operatorname{Sec}[a + b x]} \sqrt{\operatorname{Sin}[2 a + 2 b x]}$$

Result (type 5, 92 leaves):

$$- \left(\left(2 c d^5 \left(-7 + \operatorname{Cot}[a + b x]^2 (13 + 3 \operatorname{Csc}[a + b x]^2) + 20 (-\operatorname{Cot}[a + b x]^2)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \operatorname{Csc}[a + b x]^2\right] \right) (c \operatorname{Sec}[a + b x])^{3/2} \right) / \left(21 b \sqrt{d \operatorname{Csc}[a + b x]} \right) \right)$$

Problem 248: Result unnecessarily involves higher level functions.

$$\int (d \operatorname{Csc}[a + b x])^{5/2} (c \operatorname{Sec}[a + b x])^{5/2} dx$$

Optimal (type 4, 131 leaves, 5 steps):

$$\frac{4 c d^3 (c \operatorname{Sec}[a + b x])^{3/2}}{3 b \sqrt{d \operatorname{Csc}[a + b x]}} - \frac{2 c d (d \operatorname{Csc}[a + b x])^{3/2} (c \operatorname{Sec}[a + b x])^{3/2}}{3 b} + \frac{1}{3 b} \\ 4 c^2 d^2 \sqrt{d \operatorname{Csc}[a + b x]} \operatorname{EllipticF}\left[a - \frac{\pi}{4} + b x, 2\right] \sqrt{c \operatorname{Sec}[a + b x]} \sqrt{\operatorname{Sin}[2 a + 2 b x]}$$

Result (type 5, 87 leaves):

$$- \left(\left(2 c^3 d (d \operatorname{Csc}[a + b x])^{3/2} \left(-1 + \operatorname{Cot}[a + b x]^2 + 2 (-\operatorname{Cot}[a + b x]^2)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \operatorname{Csc}[a + b x]^2\right] \right) \operatorname{Tan}[a + b x]^2 \right) / \left(3 b \sqrt{c \operatorname{Sec}[a + b x]} \right) \right)$$

Problem 250: Result unnecessarily involves higher level functions.

$$\int \sqrt{d \operatorname{Csc}[a + b x]} (c \operatorname{Sec}[a + b x])^{5/2} dx$$

Optimal (type 4, 93 leaves, 4 steps):

$$\frac{2 c d (c \operatorname{Sec}[a+b x])^{3/2}}{3 b \sqrt{d \operatorname{Csc}[a+b x]}} + \frac{1}{3 b}$$

$$2 c^2 \sqrt{d \operatorname{Csc}[a+b x]} \operatorname{EllipticF}\left[a - \frac{\pi}{4} + b x, 2\right] \sqrt{c \operatorname{Sec}[a+b x]} \sqrt{\operatorname{Sin}[2 a+2 b x]}$$

Result (type 5, 68 leaves):

$$-\left(\left(2 c d \left(-1 + (-\operatorname{Cot}[a+b x])^2\right)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \operatorname{Csc}[a+b x]^2\right]\right) (c \operatorname{Sec}[a+b x])^{3/2}\right) / \left(3 b \sqrt{d \operatorname{Csc}[a+b x]}\right)$$

Problem 252: Result unnecessarily involves higher level functions.

$$\int \frac{(c \operatorname{Sec}[a+b x])^{5/2}}{(d \operatorname{Csc}[a+b x])^{3/2}} dx$$

Optimal (type 4, 98 leaves, 4 steps):

$$\frac{2 c (c \operatorname{Sec}[a+b x])^{3/2}}{3 b d \sqrt{d \operatorname{Csc}[a+b x]}} - \frac{1}{3 b d^2}$$

$$c^2 \sqrt{d \operatorname{Csc}[a+b x]} \operatorname{EllipticF}\left[a - \frac{\pi}{4} + b x, 2\right] \sqrt{c \operatorname{Sec}[a+b x]} \sqrt{\operatorname{Sin}[2 a+2 b x]}$$

Result (type 5, 70 leaves):

$$\left(c \left(2 + (-\operatorname{Cot}[a+b x])^2\right)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \operatorname{Csc}[a+b x]^2\right]\right) (c \operatorname{Sec}[a+b x])^{3/2} / \left(3 b d \sqrt{d \operatorname{Csc}[a+b x]}\right)$$

Problem 253: Result unnecessarily involves higher level functions.

$$\int \frac{(c \operatorname{Sec}[a+b x])^{5/2}}{(d \operatorname{Csc}[a+b x])^{5/2}} dx$$

Optimal (type 3, 329 leaves, 13 steps):

$$\frac{2 c (c \operatorname{Sec}[a+b x])^{3/2}}{3 b d (d \operatorname{Csc}[a+b x])^{3/2}} + \frac{c^2 \operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{\operatorname{Tan}[a+b x]}\right] \sqrt{c \operatorname{Sec}[a+b x]}}{\sqrt{2} b d^2 \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{\operatorname{Tan}[a+b x]}} -$$

$$\frac{c^2 \operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{\operatorname{Tan}[a+b x]}\right] \sqrt{c \operatorname{Sec}[a+b x]}}{\sqrt{2} b d^2 \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{\operatorname{Tan}[a+b x]}} -$$

$$\frac{c^2 \operatorname{Log}\left[1 - \sqrt{2} \sqrt{\operatorname{Tan}[a+b x]} + \operatorname{Tan}[a+b x]\right] \sqrt{c \operatorname{Sec}[a+b x]}}{2 \sqrt{2} b d^2 \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{\operatorname{Tan}[a+b x]}} +$$

$$\frac{c^2 \operatorname{Log}\left[1 + \sqrt{2} \sqrt{\operatorname{Tan}[a+b x]} + \operatorname{Tan}[a+b x]\right] \sqrt{c \operatorname{Sec}[a+b x]}}{2 \sqrt{2} b d^2 \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{\operatorname{Tan}[a+b x]}}$$

Result (type 5, 88 leaves):

$$\left(2c \left((\cos[a+bx]^2)^{1/4} - \cos[a+bx]^2 \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \sin[a+bx]^2\right] \right) \right. \\ \left. (c \operatorname{Sec}[a+bx])^{3/2} \right) / \left(3bd (\cos[a+bx]^2)^{1/4} (d \operatorname{Csc}[a+bx])^{3/2} \right)$$

Problem 255: Result unnecessarily involves higher level functions.

$$\int \frac{(d \operatorname{Csc}[a+bx])^{7/2}}{\sqrt{c \operatorname{Sec}[a+bx]}} dx$$

Optimal (type 4, 128 leaves, 5 steps):

$$\frac{4cd^3 \sqrt{d \operatorname{Csc}[a+bx]}}{5b(c \operatorname{Sec}[a+bx])^{3/2}} - \frac{2cd(d \operatorname{Csc}[a+bx])^{5/2}}{5b(c \operatorname{Sec}[a+bx])^{3/2}} - \\ \frac{4d^4 \operatorname{EllipticE}\left[a - \frac{\pi}{4} + bx, 2\right]}{5b \sqrt{d \operatorname{Csc}[a+bx]} \sqrt{c \operatorname{Sec}[a+bx]} \sqrt{\sin[2a+2bx]}}$$

Result (type 5, 79 leaves):

$$-\frac{1}{5bc} 2d^3 \sqrt{d \operatorname{Csc}[a+bx]} \\ \left(\cot[a+bx]^2 + (-\cot[a+bx]^2)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, \operatorname{Csc}[a+bx]^2\right] \right) \sqrt{c \operatorname{Sec}[a+bx]}$$

Problem 257: Result unnecessarily involves higher level functions.

$$\int \frac{(d \operatorname{Csc}[a+bx])^{3/2}}{\sqrt{c \operatorname{Sec}[a+bx]}} dx$$

Optimal (type 4, 89 leaves, 4 steps):

$$\frac{2cd \sqrt{d \operatorname{Csc}[a+bx]}}{b(c \operatorname{Sec}[a+bx])^{3/2}} - \frac{2d^2 \operatorname{EllipticE}\left[a - \frac{\pi}{4} + bx, 2\right]}{b \sqrt{d \operatorname{Csc}[a+bx]} \sqrt{c \operatorname{Sec}[a+bx]} \sqrt{\sin[2a+2bx]}}$$

Result (type 5, 65 leaves):

$$-\frac{1}{bc} \\ d (-\cot[a+bx]^2)^{1/4} \sqrt{d \operatorname{Csc}[a+bx]} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, \operatorname{Csc}[a+bx]^2\right] \sqrt{c \operatorname{Sec}[a+bx]}$$

Problem 258: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{d \operatorname{Csc}[a+bx]}}{\sqrt{c \operatorname{Sec}[a+bx]}} dx$$

Optimal (type 3, 270 leaves, 12 steps):

$$\frac{\text{ArcTan}\left[1 - \sqrt{2} \sqrt{\tan[a + bx]}\right] \sqrt{d \csc[a + bx]} \sqrt{\tan[a + bx]}}{\sqrt{2} b \sqrt{c \sec[a + bx]}} + \frac{\text{ArcTan}\left[1 + \sqrt{2} \sqrt{\tan[a + bx]}\right] \sqrt{d \csc[a + bx]} \sqrt{\tan[a + bx]}}{\sqrt{2} b \sqrt{c \sec[a + bx]}} - \left(\frac{\sqrt{d \csc[a + bx]} \log\left[1 - \sqrt{2} \sqrt{\tan[a + bx]} + \tan[a + bx]\right] \sqrt{\tan[a + bx]}}{2 \sqrt{2} b \sqrt{c \sec[a + bx]}}\right) + \left(\frac{\sqrt{d \csc[a + bx]} \log\left[1 + \sqrt{2} \sqrt{\tan[a + bx]} + \tan[a + bx]\right] \sqrt{\tan[a + bx]}}{2 \sqrt{2} b \sqrt{c \sec[a + bx]}}\right)$$

Result (type 5, 66 leaves):

$$\left(\frac{\sqrt{d \csc[a + bx]} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \sin[a + bx]^2\right] \sin[2(a + bx)]}{b (\cos[a + bx]^2)^{3/4} \sqrt{c \sec[a + bx]}}\right) /$$

Problem 259: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\sqrt{d \csc[a + bx]} \sqrt{c \sec[a + bx]}} dx$$

Optimal (type 4, 53 leaves, 3 steps):

$$\frac{\text{EllipticE}\left[a - \frac{\pi}{4} + bx, 2\right]}{b \sqrt{d \csc[a + bx]} \sqrt{c \sec[a + bx]} \sqrt{\sin[2a + 2bx]}}$$

Result (type 5, 81 leaves):

$$-\frac{1}{2 b c d} \sqrt{d \csc[a + bx]} \left(1 + \cos[2(a + bx)] - (-\cot[a + bx]^2)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, \csc[a + bx]^2\right]\right) \sqrt{c \sec[a + bx]}$$

Problem 260: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(d \csc[a + bx])^{3/2} \sqrt{c \sec[a + bx]}} dx$$

Optimal (type 3, 322 leaves, 13 steps):

$$\begin{aligned}
 & - \frac{c}{2 b d \sqrt{d \operatorname{Csc}[a+b x]} (c \operatorname{Sec}[a+b x])^{3/2}} - \\
 & \frac{\operatorname{ArcTan}\left[1-\sqrt{2} \sqrt{\operatorname{Tan}[a+b x]}\right] \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{\operatorname{Tan}[a+b x]}}{4 \sqrt{2} b d^2 \sqrt{c \operatorname{Sec}[a+b x]}} + \\
 & \frac{\operatorname{ArcTan}\left[1+\sqrt{2} \sqrt{\operatorname{Tan}[a+b x]}\right] \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{\operatorname{Tan}[a+b x]}}{4 \sqrt{2} b d^2 \sqrt{c \operatorname{Sec}[a+b x]}} - \\
 & \left(\sqrt{d \operatorname{Csc}[a+b x]} \operatorname{Log}\left[1-\sqrt{2} \sqrt{\operatorname{Tan}[a+b x]}+\operatorname{Tan}[a+b x]\right] \sqrt{\operatorname{Tan}[a+b x]}\right) / \\
 & \left(8 \sqrt{2} b d^2 \sqrt{c \operatorname{Sec}[a+b x]}\right) + \\
 & \left(\sqrt{d \operatorname{Csc}[a+b x]} \operatorname{Log}\left[1+\sqrt{2} \sqrt{\operatorname{Tan}[a+b x]}+\operatorname{Tan}[a+b x]\right] \sqrt{\operatorname{Tan}[a+b x]}\right) / \\
 & \left(8 \sqrt{2} b d^2 \sqrt{c \operatorname{Sec}[a+b x]}\right)
 \end{aligned}$$

Result (type 5, 82 leaves):

$$\begin{aligned}
 & - \left(\left(\operatorname{Cot}[a+b x] \left((\operatorname{Cos}[a+b x])^{3/4} - \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \operatorname{Sin}[a+b x]^2\right] \right) \right) \right) / \\
 & \left(2 b (\operatorname{Cos}[a+b x])^{3/4} (d \operatorname{Csc}[a+b x])^{3/2} \sqrt{c \operatorname{Sec}[a+b x]} \right)
 \end{aligned}$$

Problem 261: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(d \operatorname{Csc}[a+b x])^{5/2} \sqrt{c \operatorname{Sec}[a+b x]}} dx$$

Optimal (type 4, 95 leaves, 4 steps):

$$\begin{aligned}
 & - \frac{c}{3 b d (d \operatorname{Csc}[a+b x])^{3/2} (c \operatorname{Sec}[a+b x])^{3/2}} + \\
 & \frac{\operatorname{EllipticE}\left[a-\frac{\pi}{4}+b x, 2\right]}{2 b d^2 \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{c \operatorname{Sec}[a+b x]} \sqrt{\operatorname{Sin}[2 a+2 b x]}}
 \end{aligned}$$

Result (type 5, 99 leaves):

$$\begin{aligned}
 & \left(2 (-4 + \operatorname{Cos}[2(a+b x)]) \operatorname{Cot}[a+b x] + \right. \\
 & \left. 3 (-\operatorname{Cot}[a+b x])^{1/4} \operatorname{Csc}[a+b x] \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, \operatorname{Csc}[a+b x]^2\right] \operatorname{Sec}[a+b x] \right) / \\
 & \left(12 b d^2 \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{c \operatorname{Sec}[a+b x]} \right)
 \end{aligned}$$

Problem 263: Result unnecessarily involves higher level functions.

$$\int \frac{(d \operatorname{Csc}[a+b x])^{9/2}}{(c \operatorname{Sec}[a+b x])^{3/2}} dx$$

Optimal (type 4, 135 leaves, 5 steps):

$$\frac{2 d^3 (d \operatorname{Csc}[a + b x])^{3/2}}{21 b c \sqrt{c \operatorname{Sec}[a + b x]}} - \frac{2 d (d \operatorname{Csc}[a + b x])^{7/2}}{7 b c \sqrt{c \operatorname{Sec}[a + b x]}} - \frac{1}{21 b c^2} \\ 2 d^4 \sqrt{d \operatorname{Csc}[a + b x]} \operatorname{EllipticF}\left[a - \frac{\pi}{4} + b x, 2\right] \sqrt{c \operatorname{Sec}[a + b x]} \sqrt{\operatorname{Sin}[2 a + 2 b x]}$$

Result (type 5, 119 leaves):

$$- \left(\left(d^3 \operatorname{Cos}[2 (a + b x)] (d \operatorname{Csc}[a + b x])^{3/2} \left((5 + \operatorname{Cos}[2 (a + b x)]) \operatorname{Csc}[a + b x]^4 - \right. \right. \right. \\ \left. \left. \left. 2 (-\operatorname{Cot}[a + b x]^2)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \operatorname{Csc}[a + b x]^2\right] \operatorname{Sec}[a + b x]^2 \right) \right) \right) / \\ \left(21 b c (-2 + \operatorname{Csc}[a + b x]^2) \sqrt{c \operatorname{Sec}[a + b x]} \right)$$

Problem 265: Result unnecessarily involves higher level functions.

$$\int \frac{(d \operatorname{Csc}[a + b x])^{5/2}}{(c \operatorname{Sec}[a + b x])^{3/2}} dx$$

Optimal (type 4, 98 leaves, 4 steps):

$$- \frac{2 d (d \operatorname{Csc}[a + b x])^{3/2}}{3 b c \sqrt{c \operatorname{Sec}[a + b x]}} - \frac{1}{3 b c^2} \\ d^2 \sqrt{d \operatorname{Csc}[a + b x]} \operatorname{EllipticF}\left[a - \frac{\pi}{4} + b x, 2\right] \sqrt{c \operatorname{Sec}[a + b x]} \sqrt{\operatorname{Sin}[2 a + 2 b x]}$$

Result (type 5, 105 leaves):

$$- \left(\left(d \operatorname{Cos}[2 (a + b x)] (d \operatorname{Csc}[a + b x])^{3/2} \right. \right. \\ \left. \left. \left(2 \operatorname{Cot}[a + b x]^2 - (-\operatorname{Cot}[a + b x]^2)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \operatorname{Csc}[a + b x]^2\right] \right) \right. \right. \\ \left. \left. \operatorname{Sec}[a + b x]^3 \right) \right) / \left(3 b (-2 + \operatorname{Csc}[a + b x]^2) (c \operatorname{Sec}[a + b x])^{3/2} \right)$$

Problem 266: Result unnecessarily involves higher level functions.

$$\int \frac{(d \operatorname{Csc}[a + b x])^{3/2}}{(c \operatorname{Sec}[a + b x])^{3/2}} dx$$

Optimal (type 3, 327 leaves, 13 steps):

$$\begin{aligned}
 & - \frac{2 d \sqrt{d \operatorname{Csc}[a+b x]}}{b c \sqrt{c \operatorname{Sec}[a+b x]}} + \frac{d^2 \operatorname{ArcTan}\left[1-\sqrt{2} \sqrt{\operatorname{Tan}[a+b x]}\right] \sqrt{c \operatorname{Sec}[a+b x]}}{\sqrt{2} b c^2 \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{\operatorname{Tan}[a+b x]}} \\
 & - \frac{d^2 \operatorname{ArcTan}\left[1+\sqrt{2} \sqrt{\operatorname{Tan}[a+b x]}\right] \sqrt{c \operatorname{Sec}[a+b x]}}{\sqrt{2} b c^2 \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{\operatorname{Tan}[a+b x]}} \\
 & + \frac{d^2 \operatorname{Log}\left[1-\sqrt{2} \sqrt{\operatorname{Tan}[a+b x]}+\operatorname{Tan}[a+b x]\right] \sqrt{c \operatorname{Sec}[a+b x]}}{2 \sqrt{2} b c^2 \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{\operatorname{Tan}[a+b x]}} \\
 & + \frac{d^2 \operatorname{Log}\left[1+\sqrt{2} \sqrt{\operatorname{Tan}[a+b x]}+\operatorname{Tan}[a+b x]\right] \sqrt{c \operatorname{Sec}[a+b x]}}{2 \sqrt{2} b c^2 \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{\operatorname{Tan}[a+b x]}}
 \end{aligned}$$

Result (type 5, 95 leaves):

$$\begin{aligned}
 & - \left(\left(2 (d \operatorname{Csc}[a+b x])^{3/2} \right. \right. \\
 & \quad \left. \left. \left(3 (\operatorname{Cos}[a+b x])^{1/4} \operatorname{Csc}[a+b x]^2 + \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \operatorname{Sin}[a+b x]^2\right] \right) \right. \right. \\
 & \quad \left. \left. \operatorname{Sin}[a+b x]^3 \right) / \left(3 b c (\operatorname{Cos}[a+b x])^{1/4} \sqrt{c \operatorname{Sec}[a+b x]} \right) \right)
 \end{aligned}$$

Problem 267: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{d \operatorname{Csc}[a+b x]}}{(c \operatorname{Sec}[a+b x])^{3/2}} dx$$

Optimal (type 4, 92 leaves, 4 steps):

$$\begin{aligned}
 & \frac{d}{b c \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{c \operatorname{Sec}[a+b x]}} + \frac{1}{2 b c^2} \\
 & \sqrt{d \operatorname{Csc}[a+b x]} \operatorname{EllipticF}\left[a-\frac{\pi}{4}+b x, 2\right] \sqrt{c \operatorname{Sec}[a+b x]} \sqrt{\operatorname{Sin}[2 a+2 b x]}
 \end{aligned}$$

Result (type 5, 84 leaves):

$$\begin{aligned}
 & \left(d \left(1 + \operatorname{Cos}[2(a+b x)] \right) - (-\operatorname{Cot}[a+b x]^2)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \operatorname{Csc}[a+b x]^2\right] \right) \\
 & \operatorname{Sec}[a+b x]^3 \Big/ \left(2 b \sqrt{d \operatorname{Csc}[a+b x]} (c \operatorname{Sec}[a+b x])^{3/2} \right)
 \end{aligned}$$

Problem 268: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\sqrt{d \operatorname{Csc}[a+b x]} (c \operatorname{Sec}[a+b x])^{3/2}} dx$$

Optimal (type 3, 322 leaves, 13 steps):

$$\frac{d}{2 b c (d \operatorname{Csc}[a+b x])^{3/2} \sqrt{c \operatorname{Sec}[a+b x]}} - \frac{\operatorname{ArcTan}\left[1-\sqrt{2} \sqrt{\operatorname{Tan}[a+b x]}\right] \sqrt{c \operatorname{Sec}[a+b x]}}{4 \sqrt{2} b c^2 \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{\operatorname{Tan}[a+b x]}} +$$

$$\frac{\operatorname{ArcTan}\left[1+\sqrt{2} \sqrt{\operatorname{Tan}[a+b x]}\right] \sqrt{c \operatorname{Sec}[a+b x]}}{4 \sqrt{2} b c^2 \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{\operatorname{Tan}[a+b x]}} +$$

$$\frac{\operatorname{Log}\left[1-\sqrt{2} \sqrt{\operatorname{Tan}[a+b x]}+\operatorname{Tan}[a+b x]\right] \sqrt{c \operatorname{Sec}[a+b x]}}{8 \sqrt{2} b c^2 \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{\operatorname{Tan}[a+b x]}} -$$

$$\frac{\operatorname{Log}\left[1+\sqrt{2} \sqrt{\operatorname{Tan}[a+b x]}+\operatorname{Tan}[a+b x]\right] \sqrt{c \operatorname{Sec}[a+b x]}}{8 \sqrt{2} b c^2 \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{\operatorname{Tan}[a+b x]}}$$

Result (type 5, 80 leaves):

$$\frac{d \left(3 (\operatorname{Cos}[a+b x]^2)^{1/4} + \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \operatorname{Sin}[a+b x]^2\right]\right)}{6 b c (\operatorname{Cos}[a+b x]^2)^{1/4} (d \operatorname{Csc}[a+b x])^{3/2} \sqrt{c \operatorname{Sec}[a+b x]}}$$

Problem 269: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(d \operatorname{Csc}[a+b x])^{3/2} (c \operatorname{Sec}[a+b x])^{3/2}} dx$$

Optimal (type 4, 135 leaves, 5 steps):

$$-\frac{c}{3 b d \sqrt{d \operatorname{Csc}[a+b x]} (c \operatorname{Sec}[a+b x])^{5/2}} + \frac{1}{6 b c d \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{c \operatorname{Sec}[a+b x]}} +$$

$$\frac{1}{12 b c^2 d^2} \sqrt{d \operatorname{Csc}[a+b x]} \operatorname{EllipticF}\left[a-\frac{\pi}{4}+b x, 2\right] \sqrt{c \operatorname{Sec}[a+b x]} \sqrt{\operatorname{Sin}[2 a+2 b x]}$$

Result (type 5, 89 leaves):

$$\frac{-2 \operatorname{Cos}[2(a+b x)] + \frac{\operatorname{Csc}[a+b x]^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \operatorname{Csc}[a+b x]^2\right]}{(-\operatorname{Cot}[a+b x]^2)^{1/4}}}{12 b c d \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{c \operatorname{Sec}[a+b x]}}$$

Problem 270: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(d \operatorname{Csc}[a+b x])^{5/2} (c \operatorname{Sec}[a+b x])^{3/2}} dx$$

Optimal (type 3, 371 leaves, 14 steps):

$$\begin{aligned}
 & - \frac{c}{4 b d (d \operatorname{Csc}[a+b x])^{3/2} (c \operatorname{Sec}[a+b x])^{5/2}} + \\
 & \frac{3}{16 b c d (d \operatorname{Csc}[a+b x])^{3/2} \sqrt{c \operatorname{Sec}[a+b x]}} - \frac{3 \operatorname{ArcTan}\left[1-\sqrt{2} \sqrt{\operatorname{Tan}[a+b x]}\right] \sqrt{c \operatorname{Sec}[a+b x]}}{32 \sqrt{2} b c^2 d^2 \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{\operatorname{Tan}[a+b x]}} + \\
 & \frac{3 \operatorname{ArcTan}\left[1+\sqrt{2} \sqrt{\operatorname{Tan}[a+b x]}\right] \sqrt{c \operatorname{Sec}[a+b x]}}{32 \sqrt{2} b c^2 d^2 \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{\operatorname{Tan}[a+b x]}} + \\
 & \frac{3 \operatorname{Log}\left[1-\sqrt{2} \sqrt{\operatorname{Tan}[a+b x]}+\operatorname{Tan}[a+b x]\right] \sqrt{c \operatorname{Sec}[a+b x]}}{64 \sqrt{2} b c^2 d^2 \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{\operatorname{Tan}[a+b x]}} - \\
 & \frac{3 \operatorname{Log}\left[1+\sqrt{2} \sqrt{\operatorname{Tan}[a+b x]}+\operatorname{Tan}[a+b x]\right] \sqrt{c \operatorname{Sec}[a+b x]}}{64 \sqrt{2} b c^2 d^2 \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{\operatorname{Tan}[a+b x]}}
 \end{aligned}$$

Result (type 5, 93 leaves):

$$\begin{aligned}
 & \left((\operatorname{Cos}[a+b x]^2)^{1/4} (1-2 \operatorname{Cos}[2(a+b x)]) + \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \operatorname{Sin}[a+b x]^2\right] \right) / \\
 & \left(16 b c d (\operatorname{Cos}[a+b x]^2)^{1/4} (d \operatorname{Csc}[a+b x])^{3/2} \sqrt{c \operatorname{Sec}[a+b x]} \right)
 \end{aligned}$$

Problem 272: Result unnecessarily involves higher level functions.

$$\int \frac{(d \operatorname{Csc}[a+b x])^{7/2}}{(c \operatorname{Sec}[a+b x])^{5/2}} dx$$

Optimal (type 4, 135 leaves, 5 steps):

$$\begin{aligned}
 & \frac{6 d^3 \sqrt{d \operatorname{Csc}[a+b x]}}{5 b c (c \operatorname{Sec}[a+b x])^{3/2}} - \frac{2 d (d \operatorname{Csc}[a+b x])^{5/2}}{5 b c (c \operatorname{Sec}[a+b x])^{3/2}} + \\
 & \frac{6 d^4 \operatorname{EllipticE}\left[a-\frac{\pi}{4}+b x, 2\right]}{5 b c^2 \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{c \operatorname{Sec}[a+b x]} \sqrt{\operatorname{Sin}[2 a+2 b x]}}
 \end{aligned}$$

Result (type 5, 82 leaves):

$$\begin{aligned}
 & - \frac{1}{5 b c^3} d^3 \sqrt{d \operatorname{Csc}[a+b x]} \\
 & \left(2 \operatorname{Cot}[a+b x]^2 - 3 (-\operatorname{Cot}[a+b x]^2)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{3}{2}, \operatorname{Csc}[a+b x]^2\right] \right) \\
 & \sqrt{c \operatorname{Sec}[a+b x]}
 \end{aligned}$$

Problem 273: Result unnecessarily involves higher level functions.

$$\int \frac{(d \operatorname{Csc}[a+b x])^{5/2}}{(c \operatorname{Sec}[a+b x])^{5/2}} dx$$

Optimal (type 3, 329 leaves, 13 steps):

$$\begin{aligned}
 & - \frac{2 d (d \operatorname{Csc}[a+b x])^{3/2}}{3 b c (c \operatorname{Sec}[a+b x])^{3/2}} + \frac{d^2 \operatorname{ArcTan}\left[1-\sqrt{2} \sqrt{\operatorname{Tan}[a+b x]}\right] \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{\operatorname{Tan}[a+b x]}}{\sqrt{2} b c^2 \sqrt{c \operatorname{Sec}[a+b x]}} \\
 & + \frac{d^2 \operatorname{ArcTan}\left[1+\sqrt{2} \sqrt{\operatorname{Tan}[a+b x]}\right] \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{\operatorname{Tan}[a+b x]}}{\sqrt{2} b c^2 \sqrt{c \operatorname{Sec}[a+b x]}} + \\
 & \left(d^2 \sqrt{d \operatorname{Csc}[a+b x]} \operatorname{Log}\left[1-\sqrt{2} \sqrt{\operatorname{Tan}[a+b x]}+\operatorname{Tan}[a+b x]\right] \sqrt{\operatorname{Tan}[a+b x]}\right) / \\
 & \left(2 \sqrt{2} b c^2 \sqrt{c \operatorname{Sec}[a+b x]}\right) - \\
 & \left(d^2 \sqrt{d \operatorname{Csc}[a+b x]} \operatorname{Log}\left[1+\sqrt{2} \sqrt{\operatorname{Tan}[a+b x]}+\operatorname{Tan}[a+b x]\right] \sqrt{\operatorname{Tan}[a+b x]}\right) / \\
 & \left(2 \sqrt{2} b c^2 \sqrt{c \operatorname{Sec}[a+b x]}\right)
 \end{aligned}$$

Result (type 5, 88 leaves):

$$\begin{aligned}
 & - \left(\left(2 d (d \operatorname{Csc}[a+b x])^{3/2} \right. \right. \\
 & \left. \left. \left(\left(\operatorname{Cos}[a+b x]^2 \right)^{3/4} + 3 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \operatorname{Sin}[a+b x]^2\right] \operatorname{Sin}[a+b x]^2 \right) \right) \right) / \\
 & \left(3 b c (\operatorname{Cos}[a+b x]^2)^{3/4} (c \operatorname{Sec}[a+b x])^{3/2} \right)
 \end{aligned}$$

Problem 274: Result unnecessarily involves higher level functions.

$$\int \frac{(d \operatorname{Csc}[a+b x])^{3/2}}{(c \operatorname{Sec}[a+b x])^{5/2}} dx$$

Optimal (type 4, 94 leaves, 4 steps):

$$\frac{2 d \sqrt{d \operatorname{Csc}[a+b x]}}{b c (c \operatorname{Sec}[a+b x])^{3/2}} - \frac{3 d^2 \operatorname{EllipticE}\left[a-\frac{\pi}{4}+b x, 2\right]}{b c^2 \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{c \operatorname{Sec}[a+b x]} \sqrt{\operatorname{Sin}[2 a+2 b x]}}$$

Result (type 5, 79 leaves):

$$\begin{aligned}
 & \frac{1}{2 b c^3} d \sqrt{d \operatorname{Csc}[a+b x]} \\
 & \left(1 + \operatorname{Cos}[2(a+b x)] - 3 (-\operatorname{Cot}[a+b x]^2)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, \operatorname{Csc}[a+b x]^2\right] \right) \\
 & \sqrt{c \operatorname{Sec}[a+b x]}
 \end{aligned}$$

Problem 275: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{d \operatorname{Csc}[a+b x]}}{(c \operatorname{Sec}[a+b x])^{5/2}} dx$$

Optimal (type 3, 322 leaves, 13 steps):

$$\frac{d}{2 b c \sqrt{d \operatorname{Csc}[a+b x]} (c \operatorname{Sec}[a+b x])^{3/2}} -$$

$$\frac{3 \operatorname{ArcTan}\left[1-\sqrt{2} \sqrt{\operatorname{Tan}[a+b x]}\right] \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{\operatorname{Tan}[a+b x]}}{4 \sqrt{2} b c^2 \sqrt{c \operatorname{Sec}[a+b x]}} +$$

$$\frac{3 \operatorname{ArcTan}\left[1+\sqrt{2} \sqrt{\operatorname{Tan}[a+b x]}\right] \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{\operatorname{Tan}[a+b x]}}{4 \sqrt{2} b c^2 \sqrt{c \operatorname{Sec}[a+b x]}} -$$

$$\left(3 \sqrt{d \operatorname{Csc}[a+b x]} \operatorname{Log}\left[1-\sqrt{2} \sqrt{\operatorname{Tan}[a+b x]}+\operatorname{Tan}[a+b x]\right] \sqrt{\operatorname{Tan}[a+b x]}\right) /$$

$$\left(8 \sqrt{2} b c^2 \sqrt{c \operatorname{Sec}[a+b x]}\right) +$$

$$\left(3 \sqrt{d \operatorname{Csc}[a+b x]} \operatorname{Log}\left[1+\sqrt{2} \sqrt{\operatorname{Tan}[a+b x]}+\operatorname{Tan}[a+b x]\right] \sqrt{\operatorname{Tan}[a+b x]}\right) /$$

$$\left(8 \sqrt{2} b c^2 \sqrt{c \operatorname{Sec}[a+b x]}\right)$$

Result (type 5, 87 leaves):

$$\left(\sqrt{d \operatorname{Csc}[a+b x]} \left((\operatorname{Cos}[a+b x]^2)^{3/4}+3 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \operatorname{Sin}[a+b x]^2\right]\right)\right.$$

$$\left.\operatorname{Sin}[2(a+b x)]\right) / \left(4 b c^2 (\operatorname{Cos}[a+b x]^2)^{3/4} \sqrt{c \operatorname{Sec}[a+b x]}\right)$$

Problem 276: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\sqrt{d \operatorname{Csc}[a+b x]} (c \operatorname{Sec}[a+b x])^{5/2}} dx$$

Optimal (type 4, 95 leaves, 4 steps):

$$\frac{d}{3 b c (d \operatorname{Csc}[a+b x])^{3/2} (c \operatorname{Sec}[a+b x])^{3/2}} +$$

$$\frac{\operatorname{EllipticE}\left[a-\frac{\pi}{4}+b x, 2\right]}{2 b c^2 \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{c \operatorname{Sec}[a+b x]} \sqrt{\operatorname{Sin}[2 a+2 b x]}}$$

Result (type 5, 91 leaves):

$$-\frac{1}{24 b c^3 d} \sqrt{d \operatorname{Csc}[a+b x]} \left(5+6 \operatorname{Cos}[2(a+b x)]+\operatorname{Cos}[4(a+b x)]-\right.$$

$$\left.6(-\operatorname{Cot}[a+b x]^2)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, \operatorname{Csc}[a+b x]^2\right]\right) \sqrt{c \operatorname{Sec}[a+b x]}$$

Problem 277: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(d \operatorname{Csc}[a+b x])^{3/2} (c \operatorname{Sec}[a+b x])^{5/2}} dx$$

Optimal (type 3, 371 leaves, 14 steps):

$$\begin{aligned}
 & - \frac{c}{4 b d \sqrt{d \operatorname{Csc}[a+b x]} (c \operatorname{Sec}[a+b x])^{7/2}} + \frac{1}{16 b c d \sqrt{d \operatorname{Csc}[a+b x]} (c \operatorname{Sec}[a+b x])^{3/2}} - \\
 & \frac{3 \operatorname{ArcTan}\left[1-\sqrt{2} \sqrt{\operatorname{Tan}[a+b x]}\right] \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{\operatorname{Tan}[a+b x]}}{32 \sqrt{2} b c^2 d^2 \sqrt{c \operatorname{Sec}[a+b x]}} + \\
 & \frac{3 \operatorname{ArcTan}\left[1+\sqrt{2} \sqrt{\operatorname{Tan}[a+b x]}\right] \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{\operatorname{Tan}[a+b x]}}{32 \sqrt{2} b c^2 d^2 \sqrt{c \operatorname{Sec}[a+b x]}} - \\
 & \left(3 \sqrt{d \operatorname{Csc}[a+b x]} \operatorname{Log}\left[1-\sqrt{2} \sqrt{\operatorname{Tan}[a+b x]}+\operatorname{Tan}[a+b x]\right] \sqrt{\operatorname{Tan}[a+b x]}\right) / \\
 & \left(64 \sqrt{2} b c^2 d^2 \sqrt{c \operatorname{Sec}[a+b x]}\right) + \\
 & \left(3 \sqrt{d \operatorname{Csc}[a+b x]} \operatorname{Log}\left[1+\sqrt{2} \sqrt{\operatorname{Tan}[a+b x]}+\operatorname{Tan}[a+b x]\right] \sqrt{\operatorname{Tan}[a+b x]}\right) / \\
 & \left(64 \sqrt{2} b c^2 d^2 \sqrt{c \operatorname{Sec}[a+b x]}\right)
 \end{aligned}$$

Result (type 5, 98 leaves):

$$\begin{aligned}
 & - \left(\left(\operatorname{Cot}[a+b x] \left((\operatorname{Cos}[a+b x])^{3/4} (1+2 \operatorname{Cos}[2(a+b x)]) \right) - \right. \right. \\
 & \quad \left. \left. 3 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \operatorname{Sin}[a+b x]^2\right] \right) \right) / \\
 & \left(16 b c^2 (\operatorname{Cos}[a+b x]^2)^{3/4} (d \operatorname{Csc}[a+b x])^{3/2} \sqrt{c \operatorname{Sec}[a+b x]} \right)
 \end{aligned}$$

Problem 278: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(d \operatorname{Csc}[a+b x])^{5/2} (c \operatorname{Sec}[a+b x])^{5/2}} dx$$

Optimal (type 4, 135 leaves, 5 steps):

$$\begin{aligned}
 & - \frac{c}{5 b d (d \operatorname{Csc}[a+b x])^{3/2} (c \operatorname{Sec}[a+b x])^{7/2}} + \frac{1}{10 b c d (d \operatorname{Csc}[a+b x])^{3/2} (c \operatorname{Sec}[a+b x])^{3/2}} + \\
 & \frac{3 \operatorname{EllipticE}\left[a-\frac{\pi}{4}+b x, 2\right]}{20 b c^2 d^2 \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{c \operatorname{Sec}[a+b x]} \sqrt{\operatorname{Sin}[2 a+2 b x]}}
 \end{aligned}$$

Result (type 5, 91 leaves):

$$\begin{aligned}
 & \frac{1}{160 b c^3 d^3} \sqrt{d \operatorname{Csc}[a+b x]} \left(-12 - 13 \operatorname{Cos}[2(a+b x)] + \operatorname{Cos}[6(a+b x)] + \right. \\
 & \quad \left. 12 (-\operatorname{Cot}[a+b x]^2)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, \operatorname{Csc}[a+b x]^2\right] \right) \sqrt{c \operatorname{Sec}[a+b x]}
 \end{aligned}$$

Problem 279: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(d \operatorname{Csc}[a+b x])^{7/2} (c \operatorname{Sec}[a+b x])^{5/2}} dx$$

Optimal (type 3, 406 leaves, 15 steps):

$$\begin{aligned}
 & - \frac{c}{6 b d (d \operatorname{Csc}[a+b x])^{5/2} (c \operatorname{Sec}[a+b x])^{7/2}} - \\
 & \frac{5 c}{48 b d^3 \sqrt{d \operatorname{Csc}[a+b x]} (c \operatorname{Sec}[a+b x])^{7/2}} + \frac{5}{192 b c d^3 \sqrt{d \operatorname{Csc}[a+b x]} (c \operatorname{Sec}[a+b x])^{3/2}} - \\
 & \frac{5 \operatorname{ArcTan}\left[1-\sqrt{2} \sqrt{\operatorname{Tan}[a+b x]}\right] \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{\operatorname{Tan}[a+b x]}}{128 \sqrt{2} b c^2 d^4 \sqrt{c \operatorname{Sec}[a+b x]}} + \\
 & \frac{5 \operatorname{ArcTan}\left[1+\sqrt{2} \sqrt{\operatorname{Tan}[a+b x]}\right] \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{\operatorname{Tan}[a+b x]}}{128 \sqrt{2} b c^2 d^4 \sqrt{c \operatorname{Sec}[a+b x]}} - \\
 & \left(5 \sqrt{d \operatorname{Csc}[a+b x]} \operatorname{Log}\left[1-\sqrt{2} \sqrt{\operatorname{Tan}[a+b x]}+\operatorname{Tan}[a+b x]\right] \sqrt{\operatorname{Tan}[a+b x]}\right) / \\
 & \left(256 \sqrt{2} b c^2 d^4 \sqrt{c \operatorname{Sec}[a+b x]}\right) + \\
 & \left(5 \sqrt{d \operatorname{Csc}[a+b x]} \operatorname{Log}\left[1+\sqrt{2} \sqrt{\operatorname{Tan}[a+b x]}+\operatorname{Tan}[a+b x]\right] \sqrt{\operatorname{Tan}[a+b x]}\right) / \\
 & \left(256 \sqrt{2} b c^2 d^4 \sqrt{c \operatorname{Sec}[a+b x]}\right)
 \end{aligned}$$

Result (type 5, 106 leaves):

$$\begin{aligned}
 & \left(-2 (\operatorname{Cos}[a+b x]^2)^{3/4} (9+10 \operatorname{Cos}[2(a+b x)]-4 \operatorname{Cos}[4(a+b x)]) + \right. \\
 & \quad \left. 30 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \operatorname{Sin}[a+b x]^2\right]\right) / \\
 & \left(384 b c d^3 (\operatorname{Cos}[a+b x]^2)^{3/4} \sqrt{d \operatorname{Csc}[a+b x]} (c \operatorname{Sec}[a+b x])^{3/2}\right)
 \end{aligned}$$

Problem 280: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \operatorname{Csc}[e+f x]^n \operatorname{Sec}[e+f x]^m dx$$

Optimal (type 5, 81 leaves, 2 steps):

$$\begin{aligned}
 & \frac{1}{f(1-n)} (\operatorname{Cos}[e+f x]^2)^{\frac{1+m}{2}} \operatorname{Csc}[e+f x]^{-1+n} \\
 & \operatorname{Hypergeometric2F1}\left[\frac{1+m}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \operatorname{Sin}[e+f x]^2\right] \operatorname{Sec}[e+f x]^{1+m}
 \end{aligned}$$

Result (type 6, 2840 leaves):

$$\begin{aligned}
 & - \left(\left((-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, m, 1-m-n, \frac{3}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] \right. \right. \\
 & \quad \left. \left. \operatorname{Csc}[e+f x]^{-1+2n} \left(\operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2\right)^m \operatorname{Sec}[e+f x]^m \right. \right. \\
 & \quad \left. \left. \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]^2 \operatorname{Sec}[e+f x]\right)^m \right) / \left(f(-1+n) \right. \right. \\
 & \quad \left. \left. \left((-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, m, 1-m-n, \frac{3}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] - \right. \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & 2 \left((-1+m+n) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, m, 2-m-n, \frac{5}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \\
 & \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + m \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1+m, 1-m-n, \frac{5}{2}-\frac{n}{2}, \right. \right. \\
 & \quad \left. \left. \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right]\right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \\
 & \left(\left((-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, m, 1-m-n, \frac{3}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \right. \\
 & \quad \left. \operatorname{Cos}[e+fx] \operatorname{Csc}[e+fx]^n \left(\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right)^m \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx] \right)^m \right) / \\
 & \left((-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, m, 1-m-n, \frac{3}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) - \\
 & 2 \left((-1+m+n) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, m, 2-m-n, \frac{5}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \\
 & \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + m \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1+m, 1-m-n, \frac{5}{2}-\right. \right. \\
 & \quad \left. \left. \frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right]\right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 - \\
 & \left(m (-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, m, 1-m-n, \frac{3}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \\
 & \quad \operatorname{Csc}[e+fx]^{-1+n} \left(\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right)^m \\
 & \quad \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx] \right)^m \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) / \\
 & \left((-1+n) \left((-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, m, 1-m-n, \frac{3}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
 & \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] - 2 \left((-1+m+n) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, m, 2-m-n, \frac{5}{2}-\frac{n}{2}, \right. \right. \right. \\
 & \quad \left. \left. \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + m \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1+m, 1-m-n, \right. \right. \\
 & \quad \left. \left. \frac{5}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right]\right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) - \\
 & \left((-3+n) \operatorname{Csc}[e+fx]^{-1+n} \left(\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right)^m \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx] \right)^m \right) \\
 & \left(-\frac{1}{\frac{3}{2}-\frac{n}{2}} (1-m-n) \left(\frac{1}{2}-\frac{n}{2} \right) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, m, 2-m-n, \frac{5}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \\
 & \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \frac{1}{\frac{3}{2}-\frac{n}{2}} \right. \\
 & \quad \left. m \left(\frac{1}{2}-\frac{n}{2} \right) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1+m, 1-m-n, \frac{5}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \\
 & \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \right) /
 \end{aligned}$$

$$\begin{aligned}
 & \left((-1+n) \left((-3+n) \operatorname{AppellF1} \left[\frac{1}{2} - \frac{n}{2}, m, 1-m-n, \frac{3}{2} - \frac{n}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, \right. \right. \right. \\
 & \quad \left. \left. - \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] - 2 \left((-1+m+n) \operatorname{AppellF1} \left[\frac{3}{2} - \frac{n}{2}, m, 2-m-n, \frac{5}{2} - \frac{n}{2}, \right. \right. \right. \\
 & \quad \left. \left. \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] + m \operatorname{AppellF1} \left[\frac{3}{2} - \frac{n}{2}, 1+m, 1-m-n, \right. \right. \\
 & \quad \left. \left. \frac{5}{2} - \frac{n}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right) + \\
 & \left((-3+n) \operatorname{AppellF1} \left[\frac{1}{2} - \frac{n}{2}, m, 1-m-n, \frac{3}{2} - \frac{n}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \\
 & \operatorname{Csc} [e+fx]^{-1+n} \left(\operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \right)^m \left(\operatorname{Cos} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Sec} [e+fx] \right)^m \\
 & \left(-2 \left((-1+m+n) \operatorname{AppellF1} \left[\frac{3}{2} - \frac{n}{2}, m, 2-m-n, \frac{5}{2} - \frac{n}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, \right. \right. \right. \\
 & \quad \left. \left. -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] + m \operatorname{AppellF1} \left[\frac{3}{2} - \frac{n}{2}, 1+m, 1-m-n, \frac{5}{2} - \frac{n}{2}, \right. \right. \\
 & \quad \left. \left. \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] + \\
 & (-3+n) \left(-\frac{1}{\frac{3}{2} - \frac{n}{2}} (1-m-n) \left(\frac{1}{2} - \frac{n}{2} \right) \operatorname{AppellF1} \left[\frac{3}{2} - \frac{n}{2}, m, 2-m-n, \frac{5}{2} - \frac{n}{2}, \right. \right. \\
 & \quad \left. \left. \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] + \right. \\
 & \quad \left. \frac{1}{\frac{3}{2} - \frac{n}{2}} m \left(\frac{1}{2} - \frac{n}{2} \right) \operatorname{AppellF1} \left[\frac{3}{2} - \frac{n}{2}, 1+m, 1-m-n, \frac{5}{2} - \frac{n}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, \right. \right. \\
 & \quad \left. \left. -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] \right) - 2 \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \\
 & \left((-1+m+n) \left(-\frac{1}{\frac{5}{2} - \frac{n}{2}} (2-m-n) \left(\frac{3}{2} - \frac{n}{2} \right) \operatorname{AppellF1} \left[\frac{5}{2} - \frac{n}{2}, m, 3-m-n, \frac{7}{2} - \frac{n}{2}, \operatorname{Tan} \left[\right. \right. \right. \right. \\
 & \quad \left. \left. \frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] + \right. \\
 & \quad \left. \frac{1}{\frac{5}{2} - \frac{n}{2}} m \left(\frac{3}{2} - \frac{n}{2} \right) \operatorname{AppellF1} \left[\frac{5}{2} - \frac{n}{2}, 1+m, 2-m-n, \frac{7}{2} - \frac{n}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, \right. \right. \\
 & \quad \left. \left. -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] \right) + m \left(-\frac{1}{\frac{5}{2} - \frac{n}{2}} (1-m- \right. \\
 & \quad \left. n) \left(\frac{3}{2} - \frac{n}{2} \right) \operatorname{AppellF1} \left[\frac{5}{2} - \frac{n}{2}, 1+m, 2-m-n, \frac{7}{2} - \frac{n}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, \right. \right. \\
 & \quad \left. \left. -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] + \frac{1}{\frac{5}{2} - \frac{n}{2}} \right)
 \end{aligned}$$

$$\begin{aligned}
 & \left((1+m) \left(\frac{3}{2} - \frac{n}{2} \right) \text{AppellF1} \left[\frac{5}{2} - \frac{n}{2}, 2+m, 1-m-n, \frac{7}{2} - \frac{n}{2}, \text{Tan} \left[\frac{1}{2} (e+fx) \right]^2, \right. \right. \\
 & \quad \left. \left. - \text{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \text{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \text{Tan} \left[\frac{1}{2} (e+fx) \right] \right) \Bigg) \Bigg) \Bigg) \Bigg) / \\
 & \left((-1+n) \left((-3+n) \text{AppellF1} \left[\frac{1}{2} - \frac{n}{2}, m, 1-m-n, \frac{3}{2} - \frac{n}{2}, \text{Tan} \left[\frac{1}{2} (e+fx) \right]^2, \right. \right. \right. \right. \\
 & \quad \left. \left. - \text{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] - 2 \left((-1+m+n) \text{AppellF1} \left[\frac{3}{2} - \frac{n}{2}, m, 2-m-n, \frac{5}{2} - \frac{n}{2}, \right. \right. \right. \right. \\
 & \quad \left. \left. \text{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\text{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] + m \text{AppellF1} \left[\frac{3}{2} - \frac{n}{2}, 1+m, 1-m-n, \right. \right. \right. \\
 & \quad \left. \left. \frac{5}{2} - \frac{n}{2}, \text{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\text{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \text{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right) \Bigg) - \\
 & \left(m (-3+n) \text{AppellF1} \left[\frac{1}{2} - \frac{n}{2}, m, 1-m-n, \frac{3}{2} - \frac{n}{2}, \text{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\text{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \\
 & \quad \text{Csc} [e+fx]^{-1+n} \left(\text{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \right)^m \left(\text{Cos} \left[\frac{1}{2} (e+fx) \right]^2 \text{Sec} [e+fx] \right)^{-1+m} \\
 & \quad \left(-\text{Cos} \left[\frac{1}{2} (e+fx) \right] \text{Sec} [e+fx] \text{Sin} \left[\frac{1}{2} (e+fx) \right] + \right. \\
 & \quad \left. \text{Cos} \left[\frac{1}{2} (e+fx) \right]^2 \text{Sec} [e+fx] \text{Tan} [e+fx] \right) \Bigg) \Bigg) / \\
 & \left((-1+n) \left((-3+n) \text{AppellF1} \left[\frac{1}{2} - \frac{n}{2}, m, 1-m-n, \frac{3}{2} - \frac{n}{2}, \text{Tan} \left[\frac{1}{2} (e+fx) \right]^2, \right. \right. \right. \right. \\
 & \quad \left. \left. - \text{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] - 2 \left((-1+m+n) \text{AppellF1} \left[\frac{3}{2} - \frac{n}{2}, m, 2-m-n, \frac{5}{2} - \frac{n}{2}, \right. \right. \right. \right. \\
 & \quad \left. \left. \text{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\text{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] + m \text{AppellF1} \left[\frac{3}{2} - \frac{n}{2}, 1+m, 1-m-n, \right. \right. \right. \\
 & \quad \left. \left. \frac{5}{2} - \frac{n}{2}, \text{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\text{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \text{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right) \Bigg) \Bigg) \Bigg) \Bigg)
 \end{aligned}$$

Problem 281: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \text{Csc} [e+fx]^n (a \text{Sec} [e+fx])^m dx$$

Optimal (type 5, 86 leaves, 2 steps):

$$\frac{1}{af(1-n)} (\text{Cos} [e+fx]^2)^{\frac{1+m}{2}} \text{Csc} [e+fx]^{-1+n}$$

$$\text{Hypergeometric2F1} \left[\frac{1+m}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \text{Sin} [e+fx]^2 \right] (a \text{Sec} [e+fx])^{1+m}$$

Result (type 6, 2842 leaves):

$$- \left(\left((-3+n) \text{AppellF1} \left[\frac{1}{2} - \frac{n}{2}, m, 1-m-n, \frac{3}{2} - \frac{n}{2}, \text{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\text{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \right)$$

$$\begin{aligned}
 & \text{Csc}[e + f x]^{-1+2n} \left(\text{Sec}\left[\frac{1}{2}(e + f x)\right]^2 \right)^m (a \text{Sec}[e + f x])^m \\
 & \left(\text{Cos}\left[\frac{1}{2}(e + f x)\right]^2 \text{Sec}[e + f x] \right)^m / \left(f(-1+n) \right. \\
 & \left. \left((-3+n) \text{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, m, 1-m-n, \frac{3}{2}-\frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] - \right. \right. \\
 & \quad 2 \left((-1+m+n) \text{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, m, 2-m-n, \frac{5}{2}-\frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, \right. \right. \\
 & \quad \quad \left. \left. -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] + m \text{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1+m, 1-m-n, \frac{5}{2}-\frac{n}{2}, \right. \right. \\
 & \quad \quad \left. \left. \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] \right) \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right) \\
 & \left(\left((-3+n) \text{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, m, 1-m-n, \frac{3}{2}-\frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] \right) \right. \\
 & \quad \left. \text{Cos}[e + f x] \text{Csc}[e + f x]^n \left(\text{Sec}\left[\frac{1}{2}(e + f x)\right]^2 \right)^m \left(\text{Cos}\left[\frac{1}{2}(e + f x)\right]^2 \text{Sec}[e + f x] \right)^m \right) / \\
 & \left((-3+n) \text{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, m, 1-m-n, \frac{3}{2}-\frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] - \right. \\
 & \quad 2 \left((-1+m+n) \text{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, m, 2-m-n, \frac{5}{2}-\frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, \right. \right. \\
 & \quad \quad \left. \left. -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] + m \text{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1+m, 1-m-n, \frac{5}{2}-\frac{n}{2}, \right. \right. \\
 & \quad \quad \left. \left. \frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] \right) \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right) - \\
 & \left(m(-3+n) \text{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, m, 1-m-n, \frac{3}{2}-\frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] \right) \\
 & \quad \text{Csc}[e + f x]^{-1+n} \left(\text{Sec}\left[\frac{1}{2}(e + f x)\right]^2 \right)^m \\
 & \quad \left(\text{Cos}\left[\frac{1}{2}(e + f x)\right]^2 \text{Sec}[e + f x] \right)^m \text{Tan}\left[\frac{1}{2}(e + f x)\right] / \\
 & \left((-1+n) \left((-3+n) \text{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, m, 1-m-n, \frac{3}{2}-\frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, \right. \right. \right. \\
 & \quad \left. \left. -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] - 2 \left((-1+m+n) \text{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, m, 2-m-n, \frac{5}{2}-\frac{n}{2}, \right. \right. \right. \\
 & \quad \quad \left. \left. \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] + m \text{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1+m, 1-m-n, \right. \right. \\
 & \quad \quad \left. \left. \frac{5}{2}-\frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] \right) \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right) \right) - \\
 & \left((-3+n) \text{Csc}[e + f x]^{-1+n} \left(\text{Sec}\left[\frac{1}{2}(e + f x)\right]^2 \right)^m \left(\text{Cos}\left[\frac{1}{2}(e + f x)\right]^2 \text{Sec}[e + f x] \right)^m \right. \\
 & \quad \left. \left(-\frac{1}{\frac{3}{2}-\frac{n}{2}} (1-m-n) \left(\frac{1}{2}-\frac{n}{2} \right) \text{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, m, 2-m-n, \frac{5}{2}-\frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, \right. \right. \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \frac{1}{\frac{3}{2}-\frac{n}{2}} \\
 & m\left(\frac{1}{2}-\frac{n}{2}\right) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1+m, 1-m-n, \frac{5}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \\
 & \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \Big) / \\
 & \left((-1+n) \left((-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, m, 1-m-n, \frac{3}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
 & \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] - 2 \left((-1+m+n) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, m, 2-m-n, \frac{5}{2}-\frac{n}{2}, \right. \right. \right. \\
 & \left. \left. \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + m \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1+m, 1-m-n, \right. \right. \\
 & \left. \left. \frac{5}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \Big) + \\
 & \left((-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, m, 1-m-n, \frac{3}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \\
 & \operatorname{Csc}[e+fx]^{-1+n} \left(\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right)^m \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx] \right)^m \\
 & \left(-2 \left((-1+m+n) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, m, 2-m-n, \frac{5}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
 & \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + m \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1+m, 1-m-n, \frac{5}{2}-\frac{n}{2}, \right. \right. \\
 & \left. \left. \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \\
 & (-3+n) \left(-\frac{1}{\frac{3}{2}-\frac{n}{2}} (1-m-n) \left(\frac{1}{2}-\frac{n}{2}\right) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, m, 2-m-n, \frac{5}{2}-\frac{n}{2}, \right. \right. \\
 & \left. \left. \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \right. \\
 & \left. \frac{1}{\frac{3}{2}-\frac{n}{2}} m \left(\frac{1}{2}-\frac{n}{2}\right) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1+m, 1-m-n, \frac{5}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \\
 & \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) - 2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \\
 & \left((-1+m+n) \left(-\frac{1}{\frac{5}{2}-\frac{n}{2}} (2-m-n) \left(\frac{3}{2}-\frac{n}{2}\right) \operatorname{AppellF1}\left[\frac{5}{2}-\frac{n}{2}, m, 3-m-n, \frac{7}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
 & \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \right. \\
 & \left. \frac{1}{\frac{5}{2}-\frac{n}{2}} m \left(\frac{3}{2}-\frac{n}{2}\right) \operatorname{AppellF1}\left[\frac{5}{2}-\frac{n}{2}, 1+m, 2-m-n, \frac{7}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \\
 & \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right)
 \end{aligned}$$

$$\begin{aligned}
 & -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \Bigg) + m \left(-\frac{1}{\frac{5}{2}-\frac{n}{2}} (1-m-n) \right. \\
 & \quad n) \left(\frac{3}{2}-\frac{n}{2} \right) \operatorname{AppellF1}\left[\frac{5}{2}-\frac{n}{2}, 1+m, 2-m-n, \frac{7}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \\
 & \quad \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \frac{1}{\frac{5}{2}-\frac{n}{2}} \\
 & (1+m) \left(\frac{3}{2}-\frac{n}{2} \right) \operatorname{AppellF1}\left[\frac{5}{2}-\frac{n}{2}, 2+m, 1-m-n, \frac{7}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \\
 & \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \Bigg) \Bigg) \Bigg) \Bigg) / \\
 & \left((-1+n) \left((-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, m, 1-m-n, \frac{3}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \right. \\
 & \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] - 2 \left((-1+m+n) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, m, 2-m-n, \frac{5}{2}-\frac{n}{2}, \right. \right. \right. \right. \\
 & \quad \left. \left. \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + m \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1+m, 1-m-n, \right. \right. \\
 & \quad \left. \left. \frac{5}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right)^2 \Bigg) - \\
 & \left(m (-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, m, 1-m-n, \frac{3}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right. \\
 & \quad \left. \operatorname{Csc}[e+fx]^{-1+n} \left(\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right)^m \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx] \right)^{-1+m} \right. \\
 & \quad \left. \left(-\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right] + \right. \right. \\
 & \quad \left. \left. \operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx] \operatorname{Tan}[e+fx] \right) \right) \Bigg) \Bigg) / \\
 & \left((-1+n) \left((-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, m, 1-m-n, \frac{3}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \right. \\
 & \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] - 2 \left((-1+m+n) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, m, 2-m-n, \frac{5}{2}-\frac{n}{2}, \right. \right. \right. \right. \\
 & \quad \left. \left. \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + m \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1+m, 1-m-n, \right. \right. \\
 & \quad \left. \left. \frac{5}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \Bigg) \Bigg) \Bigg) \Bigg)
 \end{aligned}$$

Problem 282: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (b \operatorname{Csc}[e+fx])^n \operatorname{Sec}[e+fx]^m dx$$

Optimal (type 5, 84 leaves, 2 steps):

$$\frac{1}{f(1-n)} b (\cos [e+fx]^2)^{\frac{1+m}{2}} (b \operatorname{Csc}[e+fx])^{-1+n}$$

$$\operatorname{Hypergeometric2F1}\left[\frac{1+m}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \sin [e+fx]^2\right] \operatorname{Sec}[e+fx]^{1+m}$$

Result (type 6, 2848 leaves):

$$-\left(\left((-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, m, 1-m-n, \frac{3}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right]\right.\right.$$

$$\operatorname{Csc}[e+fx]^{-1+n} (b \operatorname{Csc}[e+fx])^n \left(\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2\right)^m$$

$$\operatorname{Sec}[e+fx]^m \left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx]\right)^m\bigg) / \left(f(-1+n)\right.$$

$$\left.\left((-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, m, 1-m-n, \frac{3}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] -\right.\right.$$

$$2\left(\left(-1+m+n\right) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, m, 2-m-n, \frac{5}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2,\right.\right.$$

$$\left.-\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + m \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1+m, 1-m-n, \frac{5}{2}-\frac{n}{2},\right.$$

$$\left.\left.\tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right]\right) \tan\left[\frac{1}{2}(e+fx)\right]^2\right)$$

$$\left(\left((-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, m, 1-m-n, \frac{3}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right]\right.\right.$$

$$\cos [e+fx] \operatorname{Csc}[e+fx]^n \left(\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2\right)^m \left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx]\right)^m\bigg) /$$

$$\left(\left((-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, m, 1-m-n, \frac{3}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] -\right.\right.$$

$$2\left(\left(-1+m+n\right) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, m, 2-m-n, \frac{5}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2,\right.\right.$$

$$\left.-\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + m \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1+m, 1-m-n, \frac{5}{2}-\right.$$

$$\left.\left.\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right]\right) \tan\left[\frac{1}{2}(e+fx)\right]^2\right) -$$

$$\left(m(-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, m, 1-m-n, \frac{3}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right]\right.$$

$$\operatorname{Csc}[e+fx]^{-1+n} \left(\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2\right)^m$$

$$\left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx]\right)^m \tan\left[\frac{1}{2}(e+fx)\right]\bigg) /$$

$$\left((-1+n)\left(\left((-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, m, 1-m-n, \frac{3}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2,\right.\right.\right.\right.$$

$$\left.-\tan\left[\frac{1}{2}(e+fx)\right]^2\right] - 2\left(\left(-1+m+n\right) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, m, 2-m-n, \frac{5}{2}-\frac{n}{2},\right.\right.$$

$$\left.\left.\tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + m \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1+m, 1-m-n,\right.\right.$$

$$\begin{aligned}
 & \left. \left(\frac{5}{2} - \frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) - \\
 & \left((-3+n) \operatorname{Csc}[e+fx]^{-1+n} \left(\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right)^m \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx] \right)^m \right. \\
 & \left(-\frac{1}{\frac{3}{2}-\frac{n}{2}} (1-m-n) \left(\frac{1}{2} - \frac{n}{2} \right) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, m, 2-m-n, \frac{5}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \\
 & \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \frac{1}{\frac{3}{2}-\frac{n}{2}} \right. \right. \\
 & \quad \left. \left. m \left(\frac{1}{2} - \frac{n}{2} \right) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1+m, 1-m-n, \frac{5}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
 & \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \right) / \\
 & \left((-1+n) \left((-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, m, 1-m-n, \frac{3}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \right. \\
 & \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right] - 2 \left((-1+m+n) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, m, 2-m-n, \frac{5}{2}-\frac{n}{2}, \right. \right. \right. \right. \\
 & \quad \left. \left. \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right] + m \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1+m, 1-m-n, \right. \right. \right. \\
 & \quad \left. \left. \frac{5}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) + \\
 & \left((-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, m, 1-m-n, \frac{3}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right] \right. \\
 & \quad \left. \operatorname{Csc}[e+fx]^{-1+n} \left(\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right)^m \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx] \right)^m \right. \\
 & \left(-2 \left((-1+m+n) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, m, 2-m-n, \frac{5}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \right. \\
 & \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right] + m \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1+m, 1-m-n, \frac{5}{2}-\frac{n}{2}, \right. \right. \right. \\
 & \quad \left. \left. \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right] \right) \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \\
 & \quad (-3+n) \left(-\frac{1}{\frac{3}{2}-\frac{n}{2}} (1-m-n) \left(\frac{1}{2} - \frac{n}{2} \right) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, m, 2-m-n, \frac{5}{2}-\frac{n}{2}, \right. \right. \right. \\
 & \quad \left. \left. \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \right. \\
 & \quad \left. \frac{1}{\frac{3}{2}-\frac{n}{2}} m \left(\frac{1}{2} - \frac{n}{2} \right) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1+m, 1-m-n, \frac{5}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
 & \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) - 2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2
 \end{aligned}$$

$$\begin{aligned}
 & \left((-1+m+n) \left(-\frac{1}{\frac{5}{2}-\frac{n}{2}} (2-m-n) \left(\frac{3}{2}-\frac{n}{2} \right) \text{AppellF1} \left[\frac{5}{2}-\frac{n}{2}, m, 3-m-n, \frac{7}{2}-\frac{n}{2}, \text{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\text{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \text{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \text{Tan} \left[\frac{1}{2} (e+fx) \right] + \right. \right. \\
 & \quad \left. \frac{1}{\frac{5}{2}-\frac{n}{2}} m \left(\frac{3}{2}-\frac{n}{2} \right) \text{AppellF1} \left[\frac{5}{2}-\frac{n}{2}, 1+m, 2-m-n, \frac{7}{2}-\frac{n}{2}, \text{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\text{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \text{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \text{Tan} \left[\frac{1}{2} (e+fx) \right] + m \left(-\frac{1}{\frac{5}{2}-\frac{n}{2}} (1-m-n) \right) \left(\frac{3}{2}-\frac{n}{2} \right) \text{AppellF1} \left[\frac{5}{2}-\frac{n}{2}, 1+m, 2-m-n, \frac{7}{2}-\frac{n}{2}, \text{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\text{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \text{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \text{Tan} \left[\frac{1}{2} (e+fx) \right] + \frac{1}{\frac{5}{2}-\frac{n}{2}} (1+m) \left(\frac{3}{2}-\frac{n}{2} \right) \text{AppellF1} \left[\frac{5}{2}-\frac{n}{2}, 2+m, 1-m-n, \frac{7}{2}-\frac{n}{2}, \text{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\text{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \text{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \text{Tan} \left[\frac{1}{2} (e+fx) \right] \right) \right) / \\
 & \left((-1+n) \left((-3+n) \text{AppellF1} \left[\frac{1}{2}-\frac{n}{2}, m, 1-m-n, \frac{3}{2}-\frac{n}{2}, \text{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\text{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] - 2 \left((-1+m+n) \text{AppellF1} \left[\frac{3}{2}-\frac{n}{2}, m, 2-m-n, \frac{5}{2}-\frac{n}{2}, \text{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\text{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] + m \text{AppellF1} \left[\frac{3}{2}-\frac{n}{2}, 1+m, 1-m-n, \frac{5}{2}-\frac{n}{2}, \text{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\text{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \text{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right) - \right. \\
 & \left(m (-3+n) \text{AppellF1} \left[\frac{1}{2}-\frac{n}{2}, m, 1-m-n, \frac{3}{2}-\frac{n}{2}, \text{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\text{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \text{Csc} [e+fx]^{-1+n} \left(\text{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \right)^m \left(\text{Cos} \left[\frac{1}{2} (e+fx) \right]^2 \text{Sec} [e+fx] \right)^{-1+m} \right. \\
 & \quad \left. \left(-\text{Cos} \left[\frac{1}{2} (e+fx) \right] \text{Sec} [e+fx] \text{Sin} \left[\frac{1}{2} (e+fx) \right] + \text{Cos} \left[\frac{1}{2} (e+fx) \right]^2 \text{Sec} [e+fx] \text{Tan} [e+fx] \right) \right) / \\
 & \left((-1+n) \left((-3+n) \text{AppellF1} \left[\frac{1}{2}-\frac{n}{2}, m, 1-m-n, \frac{3}{2}-\frac{n}{2}, \text{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\text{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] - 2 \left((-1+m+n) \text{AppellF1} \left[\frac{3}{2}-\frac{n}{2}, m, 2-m-n, \frac{5}{2}-\frac{n}{2}, \text{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\text{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] + m \text{AppellF1} \left[\frac{3}{2}-\frac{n}{2}, 1+m, 1-m-n, \frac{5}{2}-\frac{n}{2}, \text{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\text{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \text{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right) \right) \right) /
 \end{aligned}$$

Problem 283: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (b \operatorname{Csc}[e + f x])^n (a \operatorname{Sec}[e + f x])^m dx$$

Optimal (type 5, 89 leaves, 2 steps):

$$\frac{1}{a f (1-n)} b (\operatorname{Cos}[e + f x]^2)^{\frac{1+m}{2}} (b \operatorname{Csc}[e + f x])^{-1+n}$$

$$\operatorname{Hypergeometric2F1}\left[\frac{1+m}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \operatorname{Sin}[e + f x]^2\right] (a \operatorname{Sec}[e + f x])^{1+m}$$

Result (type 6, 2850 leaves):

$$\begin{aligned} & - \left(\left((-3+n) \operatorname{AppellF1}\left[\frac{1}{2} - \frac{n}{2}, m, 1-m-n, \frac{3}{2} - \frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right. \right. \\ & \quad \left. \left. \operatorname{Csc}[e+fx]^{-1+n} (b \operatorname{Csc}[e+fx])^n \left(\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2\right)^m \right. \right. \\ & \quad \left. \left. (a \operatorname{Sec}[e+fx])^m \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx]\right)^m \right) \right) / \left(f(-1+n) \right. \\ & \quad \left((-3+n) \operatorname{AppellF1}\left[\frac{1}{2} - \frac{n}{2}, m, 1-m-n, \frac{3}{2} - \frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] - \right. \\ & \quad \left. 2 \left((-1+m+n) \operatorname{AppellF1}\left[\frac{3}{2} - \frac{n}{2}, m, 2-m-n, \frac{5}{2} - \frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\ & \quad \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + m \operatorname{AppellF1}\left[\frac{3}{2} - \frac{n}{2}, 1+m, 1-m-n, \frac{5}{2} - \frac{n}{2}, \right. \right. \\ & \quad \quad \left. \left. \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \\ & \quad \left(\left((-3+n) \operatorname{AppellF1}\left[\frac{1}{2} - \frac{n}{2}, m, 1-m-n, \frac{3}{2} - \frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \right. \\ & \quad \left. \operatorname{Cos}[e+fx] \operatorname{Csc}[e+fx]^n \left(\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2\right)^m \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx]\right)^m \right) / \right. \\ & \quad \left((-3+n) \operatorname{AppellF1}\left[\frac{1}{2} - \frac{n}{2}, m, 1-m-n, \frac{3}{2} - \frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] - \right. \\ & \quad \left. 2 \left((-1+m+n) \operatorname{AppellF1}\left[\frac{3}{2} - \frac{n}{2}, m, 2-m-n, \frac{5}{2} - \frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\ & \quad \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + m \operatorname{AppellF1}\left[\frac{3}{2} - \frac{n}{2}, 1+m, 1-m-n, \frac{5}{2} - \frac{n}{2}, \right. \right. \\ & \quad \quad \left. \left. \frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) - \\ & \quad \left(m(-3+n) \operatorname{AppellF1}\left[\frac{1}{2} - \frac{n}{2}, m, 1-m-n, \frac{3}{2} - \frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \\ & \quad \left. \operatorname{Csc}[e+fx]^{-1+n} \left(\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2\right)^m \right) \end{aligned}$$

$$\begin{aligned}
& \left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx] \right)^m \tan\left[\frac{1}{2}(e+fx)\right] \Big/ \\
& \left((-1+n) \left((-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, m, 1-m-n, \frac{3}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] - 2 \left((-1+m+n) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, m, 2-m-n, \frac{5}{2}-\frac{n}{2}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + m \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1+m, 1-m-n, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \frac{5}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) - \\
& \left((-3+n) \operatorname{Csc}[e+fx]^{-1+n} \left(\sec\left[\frac{1}{2}(e+fx)\right]^2 \right)^m \left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx] \right)^m \right. \\
& \quad \left(-\frac{1}{\frac{3}{2}-\frac{n}{2}} (1-m-n) \left(\frac{1}{2}-\frac{n}{2}\right) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, m, 2-m-n, \frac{5}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \\
& \quad \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \frac{1}{\frac{3}{2}-\frac{n}{2}} \right. \right. \\
& \quad \left. \left. m \left(\frac{1}{2}-\frac{n}{2}\right) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1+m, 1-m-n, \frac{5}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) \Big/ \\
& \left((-1+n) \left((-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, m, 1-m-n, \frac{3}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] - 2 \left((-1+m+n) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, m, 2-m-n, \frac{5}{2}-\frac{n}{2}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + m \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1+m, 1-m-n, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \frac{5}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) + \\
& \left((-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, m, 1-m-n, \frac{3}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \\
& \operatorname{Csc}[e+fx]^{-1+n} \left(\sec\left[\frac{1}{2}(e+fx)\right]^2 \right)^m \left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx] \right)^m \\
& \quad \left(-2 \left((-1+m+n) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, m, 2-m-n, \frac{5}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + m \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1+m, 1-m-n, \frac{5}{2}-\frac{n}{2}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \right. \\
& \quad \left. (-3+n) \left(-\frac{1}{\frac{3}{2}-\frac{n}{2}} (1-m-n) \left(\frac{1}{2}-\frac{n}{2}\right) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, m, 2-m-n, \frac{5}{2}-\frac{n}{2}, \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \begin{aligned}
& \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \\
& \frac{1}{\frac{3}{2}-\frac{n}{2}}m\left(\frac{1}{2}-\frac{n}{2}\right)\text{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1+m, 1-m-n, \frac{5}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \\
& \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2 \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right]\right] - 2\tan\left[\frac{1}{2}(e+fx)\right]^2 \\
& \left((-1+m+n) \left(-\frac{1}{\frac{5}{2}-\frac{n}{2}}(2-m-n) \left(\frac{3}{2}-\frac{n}{2} \right) \text{AppellF1}\left[\frac{5}{2}-\frac{n}{2}, m, 3-m-n, \frac{7}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \left. \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2 \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right]\right] + \right. \right. \\
& \left. \frac{1}{\frac{5}{2}-\frac{n}{2}}m\left(\frac{3}{2}-\frac{n}{2}\right)\text{AppellF1}\left[\frac{5}{2}-\frac{n}{2}, 1+m, 2-m-n, \frac{7}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \\
& \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2 \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right]\right] + m \left(-\frac{1}{\frac{5}{2}-\frac{n}{2}}(1-m- \right. \right. \\
& \left. \left. n) \left(\frac{3}{2}-\frac{n}{2} \right) \text{AppellF1}\left[\frac{5}{2}-\frac{n}{2}, 1+m, 2-m-n, \frac{7}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2 \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \frac{1}{\frac{5}{2}-\frac{n}{2}} \right. \right. \\
& \left. \left. (1+m) \left(\frac{3}{2}-\frac{n}{2} \right) \text{AppellF1}\left[\frac{5}{2}-\frac{n}{2}, 2+m, 1-m-n, \frac{7}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \left. \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2 \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right]\right] \right) \right) \right) \right) \right) \Big/ \\
& \left((-1+n) \left((-3+n) \text{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, m, 1-m-n, \frac{3}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \right. \\
& \left. \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] - 2 \left((-1+m+n) \text{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, m, 2-m-n, \frac{5}{2}-\frac{n}{2}, \right. \right. \right. \right. \\
& \left. \left. \left. \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] + m \text{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1+m, 1-m-n, \right. \right. \right. \\
& \left. \left. \left. \frac{5}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) - \\
& \left(m (-3+n) \text{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, m, 1-m-n, \frac{3}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2 \right] \right) \\
& \text{Csc}[e+fx]^{-1+n} \left(\text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right)^m \left(\text{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \text{Sec}[e+fx] \right)^{-1+m} \\
& \left(-\text{Cos}\left[\frac{1}{2}(e+fx)\right] \text{Sec}[e+fx] \text{Sin}\left[\frac{1}{2}(e+fx)\right] + \right. \\
& \left. \text{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \text{Sec}[e+fx] \tan[e+fx] \right) \Big/ \\
& \left((-1+n) \left((-3+n) \text{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, m, 1-m-n, \frac{3}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
 & -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 - 2\left(\left(-1+m+n\right)\operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, m, 2-m-n, \frac{5}{2}-\frac{n}{2},\right.\right. \\
 & \quad \left.\left.\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right]+m\operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1+m, 1-m-n,\right.\right. \\
 & \quad \left.\left.\frac{5}{2}-\frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right]\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right)\right)
 \end{aligned}$$

Problem 284: Result more than twice size of optimal antiderivative.

$$\int (b \operatorname{Csc}[e+fx])^n \operatorname{Sec}[e+fx]^5 dx$$

Optimal (type 5, 48 leaves, 2 steps):

$$\frac{(b \operatorname{Csc}[e+fx])^{5+n} \operatorname{Hypergeometric2F1}\left[3, \frac{5+n}{2}, \frac{7+n}{2}, \operatorname{Csc}[e+fx]^2\right]}{b^5 f (5+n)}$$

Result (type 5, 139 leaves):

$$\begin{aligned}
 & -\left(\left(b (b \operatorname{Csc}[e+fx])^{-1+n} (\operatorname{Sec}[e+fx]^2)^{\frac{1-n}{2}}\right.\right. \\
 & \quad \left.\left(\left(-3+n\right)\operatorname{Hypergeometric2F1}\left[\frac{1}{2}(-1-n), \frac{1-n}{2}, \frac{3-n}{2}, -\operatorname{Tan}[e+fx]^2\right]+(-1+n)\right.\right. \\
 & \quad \left.\left.\operatorname{Hypergeometric2F1}\left[\frac{1}{2}(-1-n), \frac{3-n}{2}, \frac{5-n}{2}, -\operatorname{Tan}[e+fx]^2\right]\operatorname{Tan}[e+fx]^2\right)\right) / (f(-3+n)(-1+n))
 \end{aligned}$$

Problem 290: Result more than twice size of optimal antiderivative.

$$\int (b \operatorname{Csc}[e+fx])^n \operatorname{Sec}[e+fx]^6 dx$$

Optimal (type 5, 72 leaves, 2 steps):

$$\frac{1}{f(1-n)} b \sqrt{\operatorname{Cos}[e+fx]^2} (b \operatorname{Csc}[e+fx])^{-1+n} \operatorname{Hypergeometric2F1}\left[\frac{7}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \operatorname{Sin}[e+fx]^2\right] \operatorname{Sec}[e+fx]$$

Result (type 5, 192 leaves):

$$\begin{aligned}
 & -\frac{1}{f(-5+n)(-3+n)(-1+n)} (b \operatorname{Csc}[e+fx])^n (\operatorname{Sec}[e+fx]^2)^{-n/2} \operatorname{Tan}[e+fx] \\
 & \quad \left(\left((15-8n+n^2)\operatorname{Hypergeometric2F1}\left[\frac{1}{2}-\frac{n}{2}, -\frac{n}{2}, \frac{3}{2}-\frac{n}{2}, -\operatorname{Tan}[e+fx]^2\right]+(-1+n)\right.\right. \\
 & \quad \left.\left.\operatorname{Tan}[e+fx]^2\left(2(-5+n)\operatorname{Hypergeometric2F1}\left[\frac{3}{2}-\frac{n}{2}, -\frac{n}{2}, \frac{5}{2}-\frac{n}{2}, -\operatorname{Tan}[e+fx]^2\right]+(-3+n)\right.\right.\right. \\
 & \quad \left.\left.\left.\operatorname{Hypergeometric2F1}\left[\frac{5}{2}-\frac{n}{2}, -\frac{n}{2}, \frac{7}{2}-\frac{n}{2}, -\operatorname{Tan}[e+fx]^2\right]\operatorname{Tan}[e+fx]^2\right)\right)\right)
 \end{aligned}$$

Problem 294: Result more than twice size of optimal antiderivative.

$$\int \cos[e + f x]^2 (b \operatorname{Csc}[e + f x])^n dx$$

Optimal (type 5, 72 leaves, 2 steps):

$$\left(b \cos[e + f x] (b \operatorname{Csc}[e + f x])^{-1+n} \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \sin[e + f x]^2\right] \right) / \left(f(1-n) \sqrt{\cos[e + f x]^2} \right)$$

Result (type 5, 165 leaves):

$$-\frac{1}{f(-1+n)} 2 (b \operatorname{Csc}[e + f x])^n \left(\operatorname{Hypergeometric2F1}\left[1-n, \frac{1}{2} - \frac{n}{2}, \frac{3}{2} - \frac{n}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] - 4 \operatorname{Hypergeometric2F1}\left[2-n, \frac{1}{2} - \frac{n}{2}, \frac{3}{2} - \frac{n}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] + 4 \operatorname{Hypergeometric2F1}\left[3-n, \frac{1}{2} - \frac{n}{2}, \frac{3}{2} - \frac{n}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] \right) \left(\operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^2 \right)^{-n} \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]$$

Problem 295: Result more than twice size of optimal antiderivative.

$$\int \cos[e + f x]^4 (b \operatorname{Csc}[e + f x])^n dx$$

Optimal (type 5, 72 leaves, 2 steps):

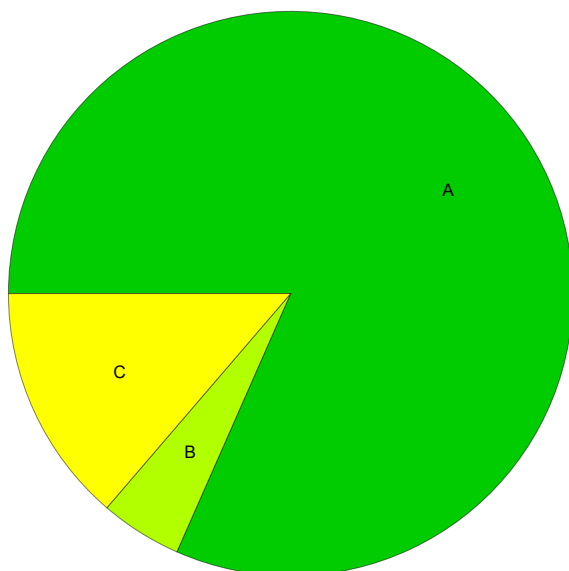
$$\left(b \cos[e + f x] (b \operatorname{Csc}[e + f x])^{-1+n} \operatorname{Hypergeometric2F1}\left[-\frac{3}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \sin[e + f x]^2\right] \right) / \left(f(1-n) \sqrt{\cos[e + f x]^2} \right)$$

Result (type 5, 246 leaves):

$$-\frac{1}{f(-1+n)} 2 (b \operatorname{Csc}[e + f x])^n \left(\operatorname{Hypergeometric2F1}\left[1-n, \frac{1}{2} - \frac{n}{2}, \frac{3}{2} - \frac{n}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] - 8 \left(\operatorname{Hypergeometric2F1}\left[2-n, \frac{1}{2} - \frac{n}{2}, \frac{3}{2} - \frac{n}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] - 3 \operatorname{Hypergeometric2F1}\left[3-n, \frac{1}{2} - \frac{n}{2}, \frac{3}{2} - \frac{n}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] + 4 \operatorname{Hypergeometric2F1}\left[4-n, \frac{1}{2} - \frac{n}{2}, \frac{3}{2} - \frac{n}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] - 2 \operatorname{Hypergeometric2F1}\left[5-n, \frac{1}{2} - \frac{n}{2}, \frac{3}{2} - \frac{n}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] \right) \right) \left(\operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^2 \right)^{-n} \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]$$

Summary of Integration Test Results

299 integration problems



A - 244 optimal antiderivatives

B - 14 more than twice size of optimal antiderivatives

C - 41 unnecessarily complex antiderivatives

D - 0 unable to integrate problems

E - 0 integration timeouts