Mathematica 11.3 Integration Test Results

Test results for the 70 problems in "4.5.4.1 (a+b sec)^m (A+B sec+C sec^2).m"

Problem 2: Result more than twice size of optimal antiderivative.

$$\int \text{Sec}[c+d x]^5 \left(A+C\,\text{Sec}[c+d x]^2\right) dx$$

Optimal (type 3, 98 leaves, 4 steps):

$$\frac{(6 A+5 C)\,\text{ArcTanh}[\text{Sin}[c+d x]]}{16 d}+\frac{(6 A+5 C)\,\text{Sec}[c+d x]\,\text{Tan}[c+d x]}{16 d}+\frac{(6 A+5 C)\,\text{Sec}[c+d x]^3\,\text{Tan}[c+d x]}{24 d}+\frac{C\,\text{Sec}[c+d x]^5\,\text{Tan}[c+d x]}{6 d}$$

Result (type 3, 445 leaves):

$$-\frac{3 A\,\text{Log}\left[\text{Cos}\left[\frac{1}{2}(c+d x)\right]-\text{Sin}\left[\frac{1}{2}(c+d x)\right]\right]}{8 d}-\frac{5 C\,\text{Log}\left[\text{Cos}\left[\frac{1}{2}(c+d x)\right]-\text{Sin}\left[\frac{1}{2}(c+d x)\right]\right]}{16 d}+\frac{3 A\,\text{Log}\left[\text{Cos}\left[\frac{1}{2}(c+d x)\right]+\text{Sin}\left[\frac{1}{2}(c+d x)\right]\right]}{8 d}+\frac{5 C\,\text{Log}\left[\text{Cos}\left[\frac{1}{2}(c+d x)\right]+\text{Sin}\left[\frac{1}{2}(c+d x)\right]\right]}{16 d}+\frac{C}{48 d\left(\text{Cos}\left[\frac{1}{2}(c+d x)\right]-\text{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^6}+\frac{A}{16 d\left(\text{Cos}\left[\frac{1}{2}(c+d x)\right]-\text{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^4}+\frac{C}{16 d\left(\text{Cos}\left[\frac{1}{2}(c+d x)\right]-\text{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^4}+\frac{3 A}{16 d\left(\text{Cos}\left[\frac{1}{2}(c+d x)\right]-\text{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^2}+\frac{5 C}{32 d\left(\text{Cos}\left[\frac{1}{2}(c+d x)\right]-\text{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^2}-\frac{C}{48 d\left(\text{Cos}\left[\frac{1}{2}(c+d x)\right]+\text{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^6}-\frac{A}{16 d\left(\text{Cos}\left[\frac{1}{2}(c+d x)\right]+\text{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^4}-\frac{C}{16 d\left(\text{Cos}\left[\frac{1}{2}(c+d x)\right]+\text{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^4}-\frac{3 A}{16 d\left(\text{Cos}\left[\frac{1}{2}(c+d x)\right]+\text{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^2}-\frac{5 C}{32 d\left(\text{Cos}\left[\frac{1}{2}(c+d x)\right]+\text{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^2}$$

Problem 8: Result more than twice size of optimal antiderivative.

$$\int \text{Cos}[c+d x] \left(A+C\,\text{Sec}[c+d x]^2\right) dx$$

Optimal (type 3, 24 leaves, 2 steps):

$$\frac{C\,\text{ArcTanh}[\text{Sin}[c+d\,x]]}{d}+\frac{A\,\text{Sin}[c+d\,x]}{d}$$

Result (type 3, 92 leaves):

$$-\frac{C\,\text{Log}\left[\text{Cos}\left[\frac{c}{2}+\frac{d\,x}{2}\right]-\text{Sin}\left[\frac{c}{2}+\frac{d\,x}{2}\right]\right]}{d}+\frac{C\,\text{Log}\left[\text{Cos}\left[\frac{c}{2}+\frac{d\,x}{2}\right]+\text{Sin}\left[\frac{c}{2}+\frac{d\,x}{2}\right]\right]}{d}+\frac{A\,\text{Cos}[d\,x]\,\text{Sin}[c]}{d}+\frac{A\,\text{Cos}[c]\,\text{Sin}[d\,x]}{d}$$

Problem 17: Result unnecessarily involves higher level functions.

$$\int (b\,\text{Sec}[c+d\,x])^{3/2}\,\left(A+C\,\text{Sec}[c+d\,x]^2\right)\,dx$$

Optimal (type 4, 110 leaves, 4 steps):

$$-\frac{2\,b^2\,(5\,A+3\,C)\,\text{EllipticE}\left[\frac{1}{2}\,(c+d\,x),\,2\right]}{5\,d\,\sqrt{\text{Cos}[c+d\,x]}\,\sqrt{b\,\text{Sec}[c+d\,x]}}+\frac{2\,b\,(5\,A+3\,C)\,\sqrt{b\,\text{Sec}[c+d\,x]}\,\text{Sin}[c+d\,x]}{5\,d}+\frac{2\,C\,(b\,\text{Sec}[c+d\,x])^{3/2}\,\text{Tan}[c+d\,x]}{5\,d}$$

Result (type 5, 180 leaves):

$$-\left(\left(4\,i\,e^{-i\,(c+d\,x)}\,\text{Cos}[c+d\,x]^3\,\left(-5\,A\,\left(1+e^{2\,i\,(c+d\,x)}\right)^2-C\,\left(3+8\,e^{2\,i\,(c+d\,x)}+e^{4\,i\,(c+d\,x)}\right)+(5\,A+3\,C)\,\left(1+e^{2\,i\,(c+d\,x)}\right)^{5/2}\,\text{Hypergeometric2F1}\left[-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2\,i\,(c+d\,x)}\right]\right)\,(b\,\text{Sec}[c+d\,x])^{3/2}\,\left(A+C\,\text{Sec}[c+d\,x]^2\right)\right)\Big/\left(5\,d\,\left(1+e^{2\,i\,(c+d\,x)}\right)^2\,(A+2\,C+A\,\text{Cos}[2\,(c+d\,x)])\right)\right)$$

Problem 19: Result unnecessarily involves higher level functions.

$$\int \frac{A+C\,\text{Sec}[c+d\,x]^2}{\sqrt{b\,\text{Sec}[c+d\,x]}}\,dx$$

Optimal (type 4, 68 leaves, 3 steps):

$$\frac{2\,(A-C)\,\text{EllipticE}\left[\frac{1}{2}\,(c+d\,x),\,2\right]}{d\,\sqrt{\text{Cos}[c+d\,x]}\,\sqrt{b\,\text{Sec}[c+d\,x]}}+\frac{2\,C\,\text{Tan}[c+d\,x]}{d\,\sqrt{b\,\text{Sec}[c+d\,x]}}$$

Result (type 5, 99 leaves):

$$-\left(\left(2\,i\,\left(A-2\,C+A\,e^{2\,i\,(c+d\,x)}+2\,(-A+C)\,\sqrt{1+e^{2\,i\,(c+d\,x)}}\,\text{Hypergeometric2F1}\left[-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2\,i\,(c+d\,x)}\right]\right)\right)\Big/\left(d\,\left(1+e^{2\,i\,(c+d\,x)}\right)\,\sqrt{b\,\text{Sec}[c+d\,x]}\right)\right)$$

Problem 21: Result unnecessarily involves higher level functions.

$$\int \frac{A + C\,\text{Sec}[c + d\,x]^2}{(b\,\text{Sec}[c + d\,x])^{5/2}}\,dx$$

Optimal (type 4, 77 leaves, 3 steps):

$$\frac{2\,(3\,A + 5\,C)\,\text{EllipticE}\left[\frac{1}{2}\,(c + d\,x),\,2\right]}{5\,b^2\,d\,\sqrt{\text{Cos}[c + d\,x]}\,\sqrt{b\,\text{Sec}[c + d\,x]}} + \frac{2\,A\,\text{Tan}[c + d\,x]}{5\,d\,(b\,\text{Sec}[c + d\,x])^{5/2}}$$

Result (type 5, 135 leaves):

$$\left(e^{-i\,(2\,c+d\,x)}\,\text{Sec}[c + d\,x]^2\left(-4\,i\,(3\,A + 5\,C) + \frac{8\,i\,(3\,A + 5\,C)\,\text{Hypergeometric2F1}\left[-\frac{1}{4},\,\frac{1}{2},\,\frac{3}{4},\,-e^{2\,i\,(c+d\,x)}\right]}{\sqrt{1 + e^{2\,i\,(c+d\,x)}}} + 2\,A\,\text{Sin}[2\,(c + d\,x)]\right)(\text{Cos}[2\,c + d\,x] + i\,\text{Sin}[2\,c + d\,x])\right) \Big/ \left(10\,d\,(b\,\text{Sec}[c + d\,x])^{5/2}\right)$$

Problem 23: Result unnecessarily involves higher level functions.

$$\int \frac{A + C\,\text{Sec}[c + d\,x]^2}{(b\,\text{Sec}[c + d\,x])^{9/2}}\,dx$$

Optimal (type 4, 112 leaves, 4 steps):

$$\frac{2\,(7\,A + 9\,C)\,\text{EllipticE}\left[\frac{1}{2}\,(c + d\,x),\,2\right]}{15\,b^4\,d\,\sqrt{\text{Cos}[c + d\,x]}\,\sqrt{b\,\text{Sec}[c + d\,x]}} + \frac{2\,(7\,A + 9\,C)\,\text{Sin}[c + d\,x]}{45\,b^3\,d\,(b\,\text{Sec}[c + d\,x])^{3/2}} + \frac{2\,A\,\text{Tan}[c + d\,x]}{9\,d\,(b\,\text{Sec}[c + d\,x])^{9/2}}$$

Result (type 5, 145 leaves):

$$\left(e^{-i\,(2\,c+d\,x)}\left(-336\,i\,A - 432\,i\,C + \frac{96\,i\,(7\,A + 9\,C)\,\text{Hypergeometric2F1}\left[-\frac{1}{4},\,\frac{1}{2},\,\frac{3}{4},\,-e^{2\,i\,(c+d\,x)}\right]}{\sqrt{1 + e^{2\,i\,(c+d\,x)}}} + (76\,A + 72\,C)\,\text{Sin}[2\,(c + d\,x)] + 10\,A\,\text{Sin}[4\,(c + d\,x)]\right)(\text{Cos}[2\,c + d\,x] + i\,\text{Sin}[2\,c + d\,x])\right) \Big/ \left(360\,b^4\,d\,\sqrt{b\,\text{Sec}[c + d\,x]}\right)$$

Problem 32: Result more than twice size of optimal antiderivative.

$$\int -\text{Cos}[e + f\,x]\,dx$$

Optimal (type 3, 11 leaves, 1 step):

$$-\frac{\text{Sin}[e+f x]}{f}$$

Result (type 3, 23 leaves):

$$-\frac{\text{Cos}[f x]\,\text{Sin}[e]}{f}-\frac{\text{Cos}[e]\,\text{Sin}[f x]}{f}$$

Problem 34: Result more than twice size of optimal antiderivative.

$$\int \text{Cos}[e+f x]^3\left(-3+2\,\text{Sec}[e+f x]^2\right)\,dx$$

Optimal (type 3, 19 leaves, 1 step):

$$-\frac{\text{Cos}[e+f x]^2\,\text{Sin}[e+f x]}{f}$$

Result (type 3, 51 leaves):

$$\frac{2\,\text{Cos}[f x]\,\text{Sin}[e]}{f}+\frac{2\,\text{Cos}[e]\,\text{Sin}[f x]}{f}-\frac{9\,\text{Sin}[e+f x]}{4 f}-\frac{\text{Sin}[3\,(e+f x)]}{4 f}$$

Problem 36: Result more than twice size of optimal antiderivative.

$$\int \text{Cos}[e+f x]^5\left(-5+4\,\text{Sec}[e+f x]^2\right)\,dx$$

Optimal (type 3, 19 leaves, 1 step):

$$-\frac{\text{Cos}[e+f x]^4\,\text{Sin}[e+f x]}{f}$$

Result (type 3, 44 leaves):

$$-\frac{\text{Sin}[e+f x]}{8 f}-\frac{3\,\text{Sin}[3\,(e+f x)]}{16 f}-\frac{\text{Sin}[5\,(e+f x)]}{16 f}$$

Problem 37: Result more than twice size of optimal antiderivative.

$$\int \text{Sec}[c+d x]^3\left(B\,\text{Sec}[c+d x]+C\,\text{Sec}[c+d x]^2\right)\,dx$$

Optimal (type 3, 85 leaves, 7 steps):

$$\frac{3\,C\,\text{ArcTanh}[\text{Sin}[c+d x]]}{8 d}+\frac{B\,\text{Tan}[c+d x]}{d}+\frac{3\,C\,\text{Sec}[c+d x]\,\text{Tan}[c+d x]}{8 d}+\frac{C\,\text{Sec}[c+d x]^3\,\text{Tan}[c+d x]}{4 d}+\frac{B\,\text{Tan}[c+d x]^3}{3 d}$$

Result (type 3, 227 leaves):

$$-\frac{3\,C\,\text{Log}\left[\text{Cos}\left[\frac{1}{2}\,(c+d\,x)\right]-\text{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]\right]}{8\,d}+\frac{3\,C\,\text{Log}\left[\text{Cos}\left[\frac{1}{2}\,(c+d\,x)\right]+\text{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]\right]}{8\,d}+\frac{C}{16\,d\,\left(\text{Cos}\left[\frac{1}{2}\,(c+d\,x)\right]-\text{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]\right)^4}+\frac{3\,C}{16\,d\,\left(\text{Cos}\left[\frac{1}{2}\,(c+d\,x)\right]-\text{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]\right)^2}-\frac{C}{16\,d\,\left(\text{Cos}\left[\frac{1}{2}\,(c+d\,x)\right]+\text{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]\right)^4}-\frac{3\,C}{16\,d\,\left(\text{Cos}\left[\frac{1}{2}\,(c+d\,x)\right]+\text{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]\right)^2}+\frac{2\,B\,\text{Tan}[c+d\,x]}{3\,d}+\frac{B\,\text{Sec}[c+d\,x]^2\,\text{Tan}[c+d\,x]}{3\,d}$$

Problem 40: Result more than twice size of optimal antiderivative.

$$\int \left(B\,\text{Sec}[c+d\,x]+C\,\text{Sec}[c+d\,x]^2\right)\,dx$$

Optimal (type 3, 24 leaves, 4 steps):

$$\frac{B\,\text{ArcTanh}[\text{Sin}[c+d\,x]]}{d}+\frac{C\,\text{Tan}[c+d\,x]}{d}$$

Result (type 3, 81 leaves):

$$-\frac{B\,\text{Log}\left[\text{Cos}\left[\frac{c}{2}+\frac{d\,x}{2}\right]-\text{Sin}\left[\frac{c}{2}+\frac{d\,x}{2}\right]\right]}{d}+\frac{B\,\text{Log}\left[\text{Cos}\left[\frac{c}{2}+\frac{d\,x}{2}\right]+\text{Sin}\left[\frac{c}{2}+\frac{d\,x}{2}\right]\right]}{d}+\frac{C\,\text{Tan}[c+d\,x]}{d}$$

Problem 41: Result more than twice size of optimal antiderivative.

$$\int \text{Cos}[c+d\,x]\,\left(B\,\text{Sec}[c+d\,x]+C\,\text{Sec}[c+d\,x]^2\right)\,dx$$

Optimal (type 3, 16 leaves, 4 steps):

$$B\,x+\frac{C\,\text{ArcTanh}[\text{Sin}[c+d\,x]]}{d}$$

Result (type 3, 73 leaves):

$$B\,x-\frac{C\,\text{Log}\left[\text{Cos}\left[\frac{c}{2}+\frac{d\,x}{2}\right]-\text{Sin}\left[\frac{c}{2}+\frac{d\,x}{2}\right]\right]}{d}+\frac{C\,\text{Log}\left[\text{Cos}\left[\frac{c}{2}+\frac{d\,x}{2}\right]+\text{Sin}\left[\frac{c}{2}+\frac{d\,x}{2}\right]\right]}{d}$$

Problem 54: Result more than twice size of optimal antiderivative.

$$\int \text{Sec}[c+d\,x]^4\,\left(A+B\,\text{Sec}[c+d\,x]+C\,\text{Sec}[c+d\,x]^2\right)\,dx$$

Optimal (type 3, 122 leaves, 7 steps):

$$\frac{3\,B\,\text{ArcTanh}[\text{Sin}[c+d\,x]]}{8\,d}+\frac{(5\,A+4\,C)\,\text{Tan}[c+d\,x]}{5\,d}+\frac{3\,B\,\text{Sec}[c+d\,x]\,\text{Tan}[c+d\,x]}{8\,d}+\frac{B\,\text{Sec}[c+d\,x]^3\,\text{Tan}[c+d\,x]}{4\,d}+\frac{C\,\text{Sec}[c+d\,x]^4\,\text{Tan}[c+d\,x]}{5\,d}+\frac{(5\,A+4\,C)\,\text{Tan}[c+d\,x]^3}{15\,d}$$

Result (type 3, 285 leaves):

$$-\frac{3\,B\,\text{Log}\left[\text{Cos}\left[\frac{1}{2}(c+d\,x)\right]-\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right]}{8\,d}+\frac{3\,B\,\text{Log}\left[\text{Cos}\left[\frac{1}{2}(c+d\,x)\right]+\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right]}{8\,d}+\frac{B}{16\,d\left(\text{Cos}\left[\frac{1}{2}(c+d\,x)\right]-\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right)^4}+\frac{3\,B}{16\,d\left(\text{Cos}\left[\frac{1}{2}(c+d\,x)\right]-\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right)^2}-\frac{B}{16\,d\left(\text{Cos}\left[\frac{1}{2}(c+d\,x)\right]+\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right)^4}-\frac{3\,B}{16\,d\left(\text{Cos}\left[\frac{1}{2}(c+d\,x)\right]+\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right)^2}+\frac{2\,A\,\text{Tan}[c+d\,x]}{3\,d}+\frac{8\,C\,\text{Tan}[c+d\,x]}{15\,d}+\frac{A\,\text{Sec}[c+d\,x]^2\,\text{Tan}[c+d\,x]}{3\,d}+\frac{4\,C\,\text{Sec}[c+d\,x]^2\,\text{Tan}[c+d\,x]}{15\,d}+\frac{C\,\text{Sec}[c+d\,x]^4\,\text{Tan}[c+d\,x]}{5\,d}$$

Problem 55: Result more than twice size of optimal antiderivative.

$$\int \text{Sec}[c+d\,x]^3\left(A+B\,\text{Sec}[c+d\,x]+C\,\text{Sec}[c+d\,x]^2\right)\,dx$$

Optimal (type 3, 97 leaves, 6 steps):

$$\frac{(4\,A+3\,C)\,\text{ArcTanh}[\text{Sin}[c+d\,x]]}{8\,d}+\frac{B\,\text{Tan}[c+d\,x]}{d}+\frac{(4\,A+3\,C)\,\text{Sec}[c+d\,x]\,\text{Tan}[c+d\,x]}{8\,d}+\frac{C\,\text{Sec}[c+d\,x]^3\,\text{Tan}[c+d\,x]}{4\,d}+\frac{B\,\text{Tan}[c+d\,x]^3}{3\,d}$$

Result (type 3, 353 leaves):

$$-\frac{A\,\mathrm{Log}\left[\mathrm{Cos}\left[\frac{1}{2}(c+d\,x)\right]-\mathrm{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right]}{2\,d}-\frac{3\,C\,\mathrm{Log}\left[\mathrm{Cos}\left[\frac{1}{2}(c+d\,x)\right]-\mathrm{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right]}{8\,d}+\frac{A\,\mathrm{Log}\left[\mathrm{Cos}\left[\frac{1}{2}(c+d\,x)\right]+\mathrm{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right]}{2\,d}+\frac{3\,C\,\mathrm{Log}\left[\mathrm{Cos}\left[\frac{1}{2}(c+d\,x)\right]+\mathrm{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right]}{8\,d}+\frac{C}{16\,d\left(\mathrm{Cos}\left[\frac{1}{2}(c+d\,x)\right]-\mathrm{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right)^4}+\frac{A}{4\,d\left(\mathrm{Cos}\left[\frac{1}{2}(c+d\,x)\right]-\mathrm{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right)^2}+\frac{3\,C}{16\,d\left(\mathrm{Cos}\left[\frac{1}{2}(c+d\,x)\right]-\mathrm{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right)^2}-\frac{C}{16\,d\left(\mathrm{Cos}\left[\frac{1}{2}(c+d\,x)\right]+\mathrm{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right)^4}-\frac{A}{4\,d\left(\mathrm{Cos}\left[\frac{1}{2}(c+d\,x)\right]+\mathrm{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right)^2}-\frac{3\,C}{16\,d\left(\mathrm{Cos}\left[\frac{1}{2}(c+d\,x)\right]+\mathrm{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right)^2}+\frac{2\,B\,\mathrm{Tan}[c+d\,x]}{3\,d}+\frac{B\,\mathrm{Sec}[c+d\,x]^2\,\mathrm{Tan}[c+d\,x]}{3\,d}$$

Problem 57: Result more than twice size of optimal antiderivative.

$$\int \mathrm{Sec}[c+d\,x]\left(A+B\,\mathrm{Sec}[c+d\,x]+C\,\mathrm{Sec}[c+d\,x]^2\right)dx$$

Optimal (type 3, 51 leaves, 5 steps):

$$\frac{(2\,A+C)\,\mathrm{ArcTanh}[\mathrm{Sin}[c+d\,x]]}{2\,d}+\frac{B\,\mathrm{Tan}[c+d\,x]}{d}+\frac{C\,\mathrm{Sec}[c+d\,x]\,\mathrm{Tan}[c+d\,x]}{2\,d}$$

Result (type 3, 151 leaves):

$$\frac{1}{4\,d}\left(-2\,(2\,A+C)\,\mathrm{Log}\left[\mathrm{Cos}\left[\frac{1}{2}(c+d\,x)\right]-\mathrm{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right]+4\,A\,\mathrm{Log}\left[\mathrm{Cos}\left[\frac{1}{2}(c+d\,x)\right]+\mathrm{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right]+2\,C\,\mathrm{Log}\left[\mathrm{Cos}\left[\frac{1}{2}(c+d\,x)\right]+\mathrm{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right]+\frac{C}{\left(\mathrm{Cos}\left[\frac{1}{2}(c+d\,x)\right]-\mathrm{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right)^2}-\frac{C}{\left(\mathrm{Cos}\left[\frac{1}{2}(c+d\,x)\right]+\mathrm{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right)^2}+4\,B\,\mathrm{Tan}[c+d\,x]\right)$$

Problem 58: Result more than twice size of optimal antiderivative.

$$\int \left(A+B\,\mathrm{Sec}[c+d\,x]+C\,\mathrm{Sec}[c+d\,x]^2\right)dx$$

Optimal (type 3, 27 leaves, 4 steps):

$$A\,x+\frac{B\,\mathrm{ArcTanh}[\mathrm{Sin}[c+d\,x]]}{d}+\frac{C\,\mathrm{Tan}[c+d\,x]}{d}$$

Result (type 3, 84 leaves):

$$A x - \frac{B \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2}+\frac{d x}{2}\right]-\operatorname{Sin}\left[\frac{c}{2}+\frac{d x}{2}\right]\right]}{d} + \frac{B \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2}+\frac{d x}{2}\right]+\operatorname{Sin}\left[\frac{c}{2}+\frac{d x}{2}\right]\right]}{d} + \frac{C \operatorname{Tan}[c+d x]}{d}$$

Problem 59: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Cos}[c+d x] \left(A + B \operatorname{Sec}[c+d x] + C \operatorname{Sec}[c+d x]^2\right) dx$$

Optimal (type 3, 27 leaves, 4 steps):

$$B x + \frac{C \operatorname{ArcTanh}[\operatorname{Sin}[c+d x]]}{d} + \frac{A \operatorname{Sin}[c+d x]}{d}$$

Result (type 3, 95 leaves):

$$B x - \frac{C \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2}+\frac{d x}{2}\right]-\operatorname{Sin}\left[\frac{c}{2}+\frac{d x}{2}\right]\right]}{d} + \frac{C \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2}+\frac{d x}{2}\right]+\operatorname{Sin}\left[\frac{c}{2}+\frac{d x}{2}\right]\right]}{d} + \frac{A \operatorname{Cos}[d x] \operatorname{Sin}[c]}{d} + \frac{A \operatorname{Cos}[c] \operatorname{Sin}[d x]}{d}$$

Problem 65: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (b \operatorname{Sec}[c+d x])^{3/2} \left(A + B \operatorname{Sec}[c+d x] + C \operatorname{Sec}[c+d x]^2\right) dx$$

Optimal (type 4, 178 leaves, 8 steps):

$$-\frac{2 b^2 (5 A + 3 C) \operatorname{EllipticE}\left[\frac{1}{2} (c+d x), 2\right]}{5 d \sqrt{\operatorname{Cos}[c+d x]} \sqrt{b \operatorname{Sec}[c+d x]}} + \frac{2 b B \sqrt{\operatorname{Cos}[c+d x]} \operatorname{EllipticF}\left[\frac{1}{2} (c+d x), 2\right] \sqrt{b \operatorname{Sec}[c+d x]}}{3 d} + \frac{2 b (5 A + 3 C) \sqrt{b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{5 d} + \frac{2 B (b \operatorname{Sec}[c+d x])^{3/2} \operatorname{Sin}[c+d x]}{3 d} + \frac{2 C (b \operatorname{Sec}[c+d x])^{3/2} \operatorname{Tan}[c+d x]}{5 d}$$

Result (type 5, 618 leaves):

$$\left(4\,B\,\mathrm{Cos}[c+d\,x]^{7/2}\,\mathrm{EllipticF}\left[\frac{1}{2}\,(c+d\,x),\,2\right]\,(b\,\mathrm{Sec}[c+d\,x])^{3/2}\,\left(A+B\,\mathrm{Sec}[c+d\,x]+C\,\mathrm{Sec}[c+d\,x]^2\right)\right)\Big/$$

$$\left(3\,d\,(A+2\,C+2\,B\,\mathrm{Cos}[c+d\,x]+A\,\mathrm{Cos}[2\,c+2\,d\,x])\right)-\left(2\,\sqrt{2}\,A\,e^{-i\,(2\,c+d\,x)}\,\sqrt{\frac{e^{i\,(c+d\,x)}}{1+e^{2\,i\,(c+d\,x)}}}\,\mathrm{Csc}[c]\right.$$

$$\left(1+e^{2\,i\,(c+d\,x)}+\left(-1+e^{2\,i\,c}\right)\sqrt{1+e^{2\,i\,(c+d\,x)}}\,\mathrm{Hypergeometric2F1}\left[-\frac{1}{4},\,\frac{1}{2},\,\frac{3}{4},\,-e^{2\,i\,(c+d\,x)}\right]\right)$$

$$\left.(b\,\mathrm{Sec}[c+d\,x])^{3/2}\,\left(A+B\,\mathrm{Sec}[c+d\,x]+C\,\mathrm{Sec}[c+d\,x]^2\right)\right)\Big/$$

$$\left(d\,(A+2\,C+2\,B\,\mathrm{Cos}[c+d\,x]+A\,\mathrm{Cos}[2\,c+2\,d\,x])\,\mathrm{Sec}[c+d\,x]^{7/2}\right)-$$

$$\left(6\,\sqrt{2}\,C\,e^{-i\,(2\,c+d\,x)}\,\sqrt{\frac{e^{i\,(c+d\,x)}}{1+e^{2\,i\,(c+d\,x)}}}\,\mathrm{Csc}[c]\right.$$

$$\left(1+e^{2\,i\,(c+d\,x)}+\left(-1+e^{2\,i\,c}\right)\sqrt{1+e^{2\,i\,(c+d\,x)}}\,\mathrm{Hypergeometric2F1}\left[-\frac{1}{4},\,\frac{1}{2},\,\frac{3}{4},\,-e^{2\,i\,(c+d\,x)}\right]\right)$$

$$\left.(b\,\mathrm{Sec}[c+d\,x])^{3/2}\,\left(A+B\,\mathrm{Sec}[c+d\,x]+C\,\mathrm{Sec}[c+d\,x]^2\right)\right)\Big/$$

$$\left(5\,d\,(A+2\,C+2\,B\,\mathrm{Cos}[c+d\,x]+A\,\mathrm{Cos}[2\,c+2\,d\,x])\,\mathrm{Sec}[c+d\,x]^{7/2}\right)+$$

$$\left(\mathrm{Cos}[c+d\,x]^3\,(b\,\mathrm{Sec}[c+d\,x])^{3/2}\,\left(A+B\,\mathrm{Sec}[c+d\,x]+C\,\mathrm{Sec}[c+d\,x]^2\right)\right.$$

$$\left(\frac{4\,(5\,A+3\,C)\,\mathrm{Cos}[d\,x]\,\mathrm{Csc}[c]}{5\,d}+\frac{4\,C\,\mathrm{Sec}[c]\,\mathrm{Sec}[c+d\,x]^2\,\mathrm{Sin}[d\,x]}{5\,d}+\right.$$

$$\left.\left.\frac{4\,\mathrm{Sec}[c]\,\mathrm{Sec}[c+d\,x]\,(3\,C\,\mathrm{Sin}[c]+5\,B\,\mathrm{Sin}[d\,x])}{15\,d}+\frac{4\,B\,\mathrm{Tan}[c]}{3\,d}\right)\right)\Big/$$

$$(A+2\,C+2\,B\,\mathrm{Cos}[c+d\,x]+A\,\mathrm{Cos}[2\,c+2\,d\,x])$$

Problem 66: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\sqrt{b\,\mathrm{Sec}[c+d\,x]}\,\left(A+B\,\mathrm{Sec}[c+d\,x]+C\,\mathrm{Sec}[c+d\,x]^2\right)\,dx$$

Optimal (type 4, 136 leaves, 7 steps):

$$-\frac{2\,b\,B\,\text{EllipticE}\left[\frac{1}{2}\,(c+d\,x),\,2\right]}{d\,\sqrt{\text{Cos}[c+d\,x]}\,\sqrt{b\,\text{Sec}[c+d\,x]}}+\frac{2\,(3\,A+C)\,\sqrt{\text{Cos}[c+d\,x]}\,\text{EllipticF}\left[\frac{1}{2}\,(c+d\,x),\,2\right]\,\sqrt{b\,\text{Sec}[c+d\,x]}}{3\,d}+\frac{2\,B\,\sqrt{b\,\text{Sec}[c+d\,x]}\,\text{Sin}[c+d\,x]}{d}+\frac{2\,C\,\sqrt{b\,\text{Sec}[c+d\,x]}\,\text{Tan}[c+d\,x]}{3\,d}$$

Result (type 5, 302 leaves):

$$\left(4\,\sqrt{b\,\text{Sec}[c+d\,x]}\,\left(A+B\,\text{Sec}[c+d\,x]+C\,\text{Sec}[c+d\,x]^2\right)\left(-\frac{1}{-1+e^{2\,i\,c}}\,i\,\sqrt{2}\,e^{-i\,(c+d\,x)}\,\sqrt{\frac{e^{i\,(c+d\,x)}}{1+e^{2\,i\,(c+d\,x)}}}\,\left(3\,B\,\left(1+e^{2\,i\,(c+d\,x)}\right)+3\,B\,\left(-1+e^{2\,i\,c}\right)\sqrt{1+e^{2\,i\,(c+d\,x)}}\,\text{Hypergeometric2F1}\left[-\frac{1}{4},\,\frac{1}{2},\,\frac{3}{4},\,-e^{2\,i\,(c+d\,x)}\right]+(3\,A+C)\,e^{i\,(c+d\,x)}\,\left(-1+e^{2\,i\,c}\right)\sqrt{1+e^{2\,i\,(c+d\,x)}}\,\text{Hypergeometric2F1}\left[\frac{1}{4},\,\frac{1}{2},\,\frac{5}{4},\,-e^{2\,i\,(c+d\,x)}\right]\right)+\sqrt{\text{Sec}[c+d\,x]}\,\left(3\,B\,\text{Cos}[d\,x]\,\text{Csc}[c]+C\,\text{Tan}[c+d\,x]\right)\right)\right)\Big/\left(3\,d\,\left(A+2\,C+2\,B\,\text{Cos}[c+d\,x]+A\,\text{Cos}\left[2\,(c+d\,x)\right]\right)\,\text{Sec}[c+d\,x]^{5/2}\right)$$

Problem 67: Result unnecessarily involves higher level functions.

$$\int\frac{A+B\,\text{Sec}[c+d\,x]+C\,\text{Sec}[c+d\,x]^2}{\sqrt{b\,\text{Sec}[c+d\,x]}}\,dx$$

Optimal (type 4, 110 leaves, 6 steps):

$$\frac{2\,(A-C)\,\text{EllipticE}\left[\frac{1}{2}\,(c+d\,x),\,2\right]}{d\,\sqrt{\text{Cos}[c+d\,x]}\,\sqrt{b\,\text{Sec}[c+d\,x]}}+\frac{2\,B\,\sqrt{\text{Cos}[c+d\,x]}\,\text{EllipticF}\left[\frac{1}{2}\,(c+d\,x),\,2\right]\,\sqrt{b\,\text{Sec}[c+d\,x]}}{b\,d}+\frac{2\,C\,\text{Tan}[c+d\,x]}{d\,\sqrt{b\,\text{Sec}[c+d\,x]}}$$

Result (type 5, 135 leaves):

$$\frac{1}{b\,d}e^{-i\,(c+d\,x)}\left(2\,B\,e^{i\,(c+d\,x)}\,\sqrt{\text{Cos}[c+d\,x]}\,\text{EllipticF}\left[\frac{1}{2}\,(c+d\,x),\,2\right]-i\,\left(A-2\,C+A\,e^{2\,i\,(c+d\,x)}+2\,(-A+C)\,\sqrt{1+e^{2\,i\,(c+d\,x)}}\,\text{Hypergeometric2F1}\left[-\frac{1}{4},\,\frac{1}{2},\,\frac{3}{4},\,-e^{2\,i\,(c+d\,x)}\right]\right)\right)\sqrt{b\,\text{Sec}[c+d\,x]}$$

Problem 68: Result unnecessarily involves higher level functions.

$$\int \frac{A+B\,\mathrm{Sec}[c+d\,x]+C\,\mathrm{Sec}[c+d\,x]^2}{(b\,\mathrm{Sec}[c+d\,x])^{3/2}}\,dx$$

Optimal (type 4, 117 leaves, 6 steps):

$$\frac{2\,B\,\mathrm{EllipticE}\left[\frac{1}{2}\,(c+d\,x),\,2\right]}{b\,d\,\sqrt{\mathrm{Cos}[c+d\,x]}\,\sqrt{b\,\mathrm{Sec}[c+d\,x]}}+\frac{2\,(A+3\,C)\,\sqrt{\mathrm{Cos}[c+d\,x]}\,\mathrm{EllipticF}\left[\frac{1}{2}\,(c+d\,x),\,2\right]\,\sqrt{b\,\mathrm{Sec}[c+d\,x]}}{3\,b^2\,d}+\frac{2\,A\,\mathrm{Tan}[c+d\,x]}{3\,d\,(b\,\mathrm{Sec}[c+d\,x])^{3/2}}$$

Result (type 5, 143 leaves):

$$\left(2\left(6\,i\,B\,\mathrm{Hypergeometric2F1}\left[-\frac{1}{4},\,\frac{1}{2},\,\frac{3}{4},\,-e^{2\,i\,(c+d\,x)}\right]-2\,i\,(A+3\,C)\,e^{i\,(c+d\,x)}\,\mathrm{Hypergeometric2F1}\left[\frac{1}{4},\,\frac{1}{2},\,\frac{5}{4},\,-e^{2\,i\,(c+d\,x)}\right]+\sqrt{1+e^{2\,i\,(c+d\,x)}}\,(-3\,i\,B+A\,\mathrm{Sin}[c+d\,x])\right)\right)\Big/\left(3\,b\,d\,\sqrt{1+e^{2\,i\,(c+d\,x)}}\,\sqrt{b\,\mathrm{Sec}[c+d\,x]}\right)$$

Problem 69: Result unnecessarily involves higher level functions.

$$\int \frac{A+B\,\mathrm{Sec}[c+d\,x]+C\,\mathrm{Sec}[c+d\,x]^2}{(b\,\mathrm{Sec}[c+d\,x])^{5/2}}\,dx$$

Optimal (type 4, 150 leaves, 7 steps):

$$\frac{2\,(3\,A+5\,C)\,\mathrm{EllipticE}\left[\frac{1}{2}\,(c+d\,x),\,2\right]}{5\,b^2\,d\,\sqrt{\mathrm{Cos}[c+d\,x]}\,\sqrt{b\,\mathrm{Sec}[c+d\,x]}}+\frac{2\,B\,\sqrt{\mathrm{Cos}[c+d\,x]}\,\mathrm{EllipticF}\left[\frac{1}{2}\,(c+d\,x),\,2\right]\,\sqrt{b\,\mathrm{Sec}[c+d\,x]}}{3\,b^3\,d}+\frac{2\,B\,\mathrm{Sin}[c+d\,x]}{3\,b^2\,d\,\sqrt{b\,\mathrm{Sec}[c+d\,x]}}+\frac{2\,A\,\mathrm{Tan}[c+d\,x]}{5\,d\,(b\,\mathrm{Sec}[c+d\,x])^{5/2}}$$

Result (type 5, 183 leaves):

$$\frac{1}{30\,b^3\,d}e^{-i\,(2\,c+d\,x)}\,\sqrt{b\,\mathrm{Sec}[c+d\,x]}\,\left(20\,B\,\sqrt{\mathrm{Cos}[c+d\,x]}\,\mathrm{EllipticF}\left[\frac{1}{2}\,(c+d\,x),\,2\right]+12\,i\,(3\,A+5\,C)\,e^{-i\,(c+d\,x)}\,\sqrt{1+e^{2\,i\,(c+d\,x)}}\,\mathrm{Hypergeometric2F1}\left[-\frac{1}{4},\,\frac{1}{2},\,\frac{3}{4},\,-e^{2\,i\,(c+d\,x)}\right]+2\,\mathrm{Cos}[c+d\,x]\,(-6\,i\,(3\,A+5\,C)+10\,B\,\mathrm{Sin}[c+d\,x]+3\,A\,\mathrm{Sin}[2\,(c+d\,x)])\right)(\mathrm{Cos}[2\,c+d\,x]+i\,\mathrm{Sin}[2\,c+d\,x])$$

Problem 70: Result unnecessarily involves higher level functions.

$$\int \frac{A + B\,\text{Sec}[c+d\,x] + C\,\text{Sec}[c+d\,x]^2}{\left(b\,\text{Sec}[c+d\,x]\right)^{7/2}}\,dx$$

Optimal (type 4, 185 leaves, 8 steps):

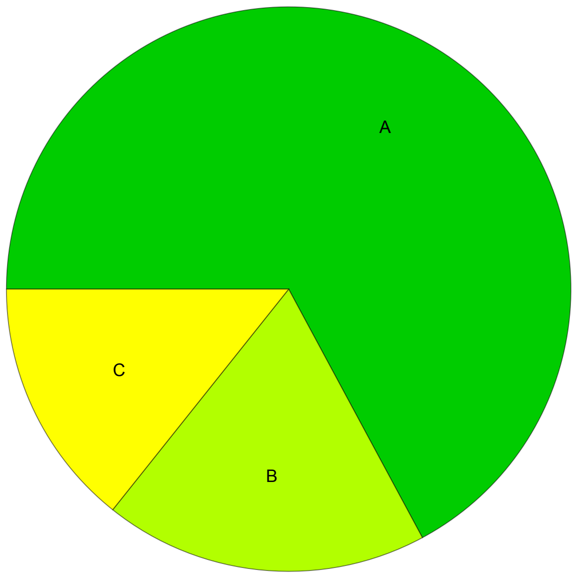
$$\frac{6\,B\,\text{EllipticE}\left[\frac{1}{2}\,(c+d\,x),\,2\right]}{5\,b^3\,d\,\sqrt{\text{Cos}[c+d\,x]}\,\sqrt{b\,\text{Sec}[c+d\,x]}} + \frac{2\,(5\,A+7\,C)\,\sqrt{\text{Cos}[c+d\,x]}\,\text{EllipticF}\left[\frac{1}{2}\,(c+d\,x),\,2\right]\,\sqrt{b\,\text{Sec}[c+d\,x]}}{21\,b^4\,d} + \frac{2\,B\,\text{Sin}[c+d\,x]}{5\,b^2\,d\,\left(b\,\text{Sec}[c+d\,x]\right)^{3/2}} + \frac{2\,(5\,A+7\,C)\,\text{Sin}[c+d\,x]}{21\,b^3\,d\,\sqrt{b\,\text{Sec}[c+d\,x]}} + \frac{2\,A\,\text{Tan}[c+d\,x]}{7\,d\,\left(b\,\text{Sec}[c+d\,x]\right)^{7/2}}$$

Result (type 5, 177 leaves):

$$\left(504\,i\,B\,\text{Hypergeometric2F1}\left[-\frac{1}{4},\,\frac{1}{2},\,\frac{3}{4},\,-e^{2\,i\,(c+d\,x)}\right] - 40\,i\,(5\,A+7\,C)\,e^{i\,(c+d\,x)}\,\text{Hypergeometric2F1}\left[\frac{1}{4},\,\frac{1}{2},\,\frac{5}{4},\,-e^{2\,i\,(c+d\,x)}\right] + \sqrt{1+e^{2\,i\,(c+d\,x)}}\,\left(5\,(23\,A+28\,C)\,\text{Sin}[c+d\,x] + 3\,\left(-84\,i\,B + 14\,B\,\text{Sin}[2\,(c+d\,x)] + 5\,A\,\text{Sin}[3\,(c+d\,x)]\right)\right)\right) \Big/ \left(210\,b^3\,d\,\sqrt{1+e^{2\,i\,(c+d\,x)}}\,\sqrt{b\,\text{Sec}[c+d\,x]}\right)$$

Summary of Integration Test Results

70 integration problems



A - 47 optimal antiderivatives

B - 13 more than twice size of optimal antiderivatives

C - 10 unnecessarily complex antiderivatives

D - 0 unable to integrate problems

E - 0 integration timeouts