Mathematica 11.3 Integration Test Results

Test results for the 59 problems in "4.6.1.2 (d csc)^n (a+b csc)^m.m"

Problem 1: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Csc}[x]^5}{a+a \operatorname{Csc}[x]} \, dx$$

Optimal (type 3, 55 leaves, 6 steps):

$$\frac{3 \operatorname{ArcTanh}[\operatorname{Cos}[x]]}{2 a}-\frac{4 \operatorname{Cot}[x]}{a}-\frac{4 \operatorname{Cot}[x]^3}{3 a}+\frac{3 \operatorname{Cot}[x] \operatorname{Csc}[x]}{2 a}+\frac{\operatorname{Cot}[x] \operatorname{Csc}[x]^3}{a+a \operatorname{Csc}[x]}$$

Result (type 3, 113 leaves):

$$\frac{1}{24 a}\left(-20 \operatorname{Cot}\left[\frac{x}{2}\right]+3 \operatorname{Csc}\left[\frac{x}{2}\right]^2+36 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right]\right]-36 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{x}{2}\right]\right]-\right.$$
$$\left.3 \operatorname{Sec}\left[\frac{x}{2}\right]^2+8 \operatorname{Csc}[x]^3 \operatorname{Sin}\left[\frac{x}{2}\right]^4+\frac{48 \operatorname{Sin}\left[\frac{x}{2}\right]}{\operatorname{Cos}\left[\frac{x}{2}\right]+\operatorname{Sin}\left[\frac{x}{2}\right]}-\frac{1}{2} \operatorname{Csc}\left[\frac{x}{2}\right]^4 \operatorname{Sin}[x]+20 \operatorname{Tan}\left[\frac{x}{2}\right]\right)$$

Problem 3: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Csc}[x]^3}{a+a \operatorname{Csc}[x]} \, dx$$

Optimal (type 3, 27 leaves, 4 steps):

$$\frac{\operatorname{ArcTanh}[\operatorname{Cos}[x]]}{a}-\frac{\operatorname{Cot}[x]}{a}-\frac{\operatorname{Cot}[x]}{a+a \operatorname{Csc}[x]}$$

Result (type 3, 63 leaves):

$$\frac{-\operatorname{Cot}\left[\frac{x}{2}\right]+2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right]\right]-2 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{x}{2}\right]\right]+\frac{4 \operatorname{Sin}\left[\frac{x}{2}\right]}{\operatorname{Cos}\left[\frac{x}{2}\right]+\operatorname{Sin}\left[\frac{x}{2}\right]}+\operatorname{Tan}\left[\frac{x}{2}\right]}{2 a}$$

Problem 4: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Csc}[x]^2}{a+a \operatorname{Csc}[x]} \, dx$$

Optimal (type 3, 20 leaves, 3 steps):

$$-\frac{\text{ArcTanh}[\text{Cos}[x]]}{a}+\frac{\text{Cot}[x]}{a+a\,\text{Csc}[x]}$$

Result (type 3, 44 leaves):

$$\frac{-\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right]+\text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right]-\frac{2\,\text{Sin}\left[\frac{x}{2}\right]}{\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]}}{a}$$

Problem 5: Result more than twice size of optimal antiderivative.

$$\int\frac{\text{Csc}[x]}{a+a\,\text{Csc}[x]}\,dx$$

Optimal (type 3, 12 leaves, 1 step):

$$-\frac{\text{Cot}[x]}{a+a\,\text{Csc}[x]}$$

Result (type 3, 26 leaves):

$$\frac{2\,\text{Sin}\left[\frac{x}{2}\right]}{a\left(\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]\right)}$$

Problem 17: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{(a+a\,\text{Csc}[x])^{3/2}}\,dx$$

Optimal (type 3, 81 leaves, 6 steps):

$$-\frac{2\,\text{ArcTan}\left[\frac{\sqrt{a}\,\text{Cot}[x]}{\sqrt{a+a\,\text{Csc}[x]}}\right]}{a^{3/2}}+\frac{5\,\text{ArcTan}\left[\frac{\sqrt{a}\,\text{Cot}[x]}{\sqrt{2}\,\sqrt{a+a\,\text{Csc}[x]}}\right]}{2\sqrt{2}\,a^{3/2}}+\frac{\text{Cot}[x]}{2\,(a+a\,\text{Csc}[x])^{3/2}}$$

Result (type 3, 165 leaves):

$$-\left(\left(\left(\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]\right)\left(2-2\,\text{Csc}[x]+4\,\text{ArcTan}\left[\frac{-2+\sqrt{1+\text{Csc}[x]}}{\sqrt{-1+\text{Csc}[x]}}\right]\sqrt{-1+\text{Csc}[x]}\,(1+\text{Csc}[x])-\right.\right.\right.$$
$$4\,\text{ArcTan}\left[\frac{2+\sqrt{1+\text{Csc}[x]}}{\sqrt{-1+\text{Csc}[x]}}\right]\sqrt{-1+\text{Csc}[x]}\,(1+\text{Csc}[x])+$$
$$\left.\left.5\sqrt{2}\,\text{ArcTan}\left[\frac{\sqrt{2}}{\sqrt{-1+\text{Csc}[x]}}\right]\sqrt{-1+\text{Csc}[x]}\,\text{Csc}[x]\left(\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]\right)^2\right)\right)\Big/$$
$$\left.\left(4\,(a\,(1+\text{Csc}[x]))^{3/2}\left(\text{Cos}\left[\frac{x}{2}\right]-\text{Sin}\left[\frac{x}{2}\right]\right)\right)\right)$$

Problem 19: Result more than twice size of optimal antiderivative.

$$\int \sqrt{\text{Csc}[e+f x]}\ \sqrt{a+a\,\text{Csc}[e+f x]}\ dx$$

Optimal (type 3, 37 leaves, 2 steps):

$$-\frac{2\sqrt{a}\ \text{ArcSinh}\left[\frac{\sqrt{a}\ \text{Cot}[e+f x]}{\sqrt{a+a\,\text{Csc}[e+f x]}}\right]}{f}$$

Result (type 3, 108 leaves):

$$\left(2\,\text{Cot}[e+f x]\ \sqrt{a\left(1+\text{Csc}[e+f x]\right)}\ \left(\text{Log}[1+\text{Csc}[e+f x]]-\text{Log}\left[\sqrt{\text{Csc}[e+f x]}+\text{Csc}[e+f x]^{3/2}+\sqrt{\text{Cot}[e+f x]^2}\ \sqrt{1+\text{Csc}[e+f x]}\right]\right)\right)\Big/\left(f\,\sqrt{\text{Cot}[e+f x]^2}\ \sqrt{1+\text{Csc}[e+f x]}\right)$$

Problem 20: Result more than twice size of optimal antiderivative.

$$\int \sqrt{-\text{Csc}[e+f x]}\ \sqrt{a-a\,\text{Csc}[e+f x]}\ dx$$

Optimal (type 3, 38 leaves, 2 steps):

$$-\frac{2\sqrt{a}\ \text{ArcSinh}\left[\frac{\sqrt{a}\ \text{Cot}[e+f x]}{\sqrt{a-a\,\text{Csc}[e+f x]}}\right]}{f}$$

Result (type 3, 116 leaves):

$$\left(2\sqrt{-\text{Csc}[e+f x]}\ \sqrt{a-a\,\text{Csc}[e+f x]}\ \left(\text{ArcSinh}\left[\text{Tan}\left[\frac{1}{2}(e+f x)\right]\right]+\text{Log}\left[1+\sqrt{\text{Sec}\left[\frac{1}{2}(e+f x)\right]^2}\right]-\text{Log}\left[\text{Tan}\left[\frac{1}{2}(e+f x)\right]\right]\right)\ \text{Tan}\left[\frac{1}{2}(e+f x)\right]\right)\Big/\left(f\,\sqrt{\text{Sec}\left[\frac{1}{2}(e+f x)\right]^2}\ \left(-1+\text{Tan}\left[\frac{1}{2}(e+f x)\right]\right)\right)$$

Problem 21: Result unnecessarily involves higher level functions.

$$\int \text{Csc}[c+d x]^{4/3}\ \sqrt{a+a\,\text{Csc}[c+d x]}\ dx$$

Optimal (type 4, 254 leaves, 4 steps):

$$-\frac{6\,a\,\mathrm{Cos}[c+d\,x]\,\mathrm{Csc}[c+d\,x]^{4/3}}{5\,d\,\sqrt{a+a\,\mathrm{Csc}[c+d\,x]}}-\left(4\times 3^{3/4}\sqrt{2+\sqrt{3}}\;a^2\,\mathrm{Cot}[c+d\,x]\,\left(1-\mathrm{Csc}[c+d\,x]^{1/3}\right)\sqrt{\frac{1+\mathrm{Csc}[c+d\,x]^{1/3}+\mathrm{Csc}[c+d\,x]^{2/3}}{\left(1+\sqrt{3}-\mathrm{Csc}[c+d\,x]^{1/3}\right)^2}}\;\mathrm{EllipticF}\left[\mathrm{ArcSin}\left[\frac{1-\sqrt{3}-\mathrm{Csc}[c+d\,x]^{1/3}}{1+\sqrt{3}-\mathrm{Csc}[c+d\,x]^{1/3}}\right],-7-4\sqrt{3}\right]\right)\Bigg/\left(5\,d\,\sqrt{\frac{1-\mathrm{Csc}[c+d\,x]^{1/3}}{\left(1+\sqrt{3}-\mathrm{Csc}[c+d\,x]^{1/3}\right)^2}}\,\left(a-a\,\mathrm{Csc}[c+d\,x]\right)\sqrt{a+a\,\mathrm{Csc}[c+d\,x]}\right)$$

Result (type 5, 102 leaves):

$$-\left(\left(2\sqrt{a\,\left(1+\mathrm{Csc}[c+d\,x]\right)}\,\left(3\,\mathrm{Csc}[c+d\,x]^{1/3}+2\,\mathrm{Hypergeometric2F1}\left[\frac{1}{2},\frac{2}{3},\frac{3}{2},1-\mathrm{Csc}[c+d\,x]\right]\right)\left(\mathrm{Cos}\left[\frac{1}{2}\,(c+d\,x)\right]-\mathrm{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]\right)\right)\Bigg/\left(5\,d\,\left(\mathrm{Cos}\left[\frac{1}{2}\,(c+d\,x)\right]+\mathrm{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]\right)\right)\right)$$

Problem 22: Result unnecessarily involves higher level functions.

$$\int \mathrm{Csc}[c+d\,x]^{1/3}\sqrt{a+a\,\mathrm{Csc}[c+d\,x]}\,dx$$

Optimal (type 4, 213 leaves, 3 steps):

$$-\left(\left(2\times 3^{3/4}\sqrt{2+\sqrt{3}}\;a^2\,\mathrm{Cot}[c+d\,x]\,\left(1-\mathrm{Csc}[c+d\,x]^{1/3}\right)\sqrt{\frac{1+\mathrm{Csc}[c+d\,x]^{1/3}+\mathrm{Csc}[c+d\,x]^{2/3}}{\left(1+\sqrt{3}-\mathrm{Csc}[c+d\,x]^{1/3}\right)^2}}\;\mathrm{EllipticF}\left[\mathrm{ArcSin}\left[\frac{1-\sqrt{3}-\mathrm{Csc}[c+d\,x]^{1/3}}{1+\sqrt{3}-\mathrm{Csc}[c+d\,x]^{1/3}}\right],-7-4\sqrt{3}\right]\right)\Bigg/\left(d\,\sqrt{\frac{1-\mathrm{Csc}[c+d\,x]^{1/3}}{\left(1+\sqrt{3}-\mathrm{Csc}[c+d\,x]^{1/3}\right)^2}}\,\left(a-a\,\mathrm{Csc}[c+d\,x]\right)\sqrt{a+a\,\mathrm{Csc}[c+d\,x]}\right)\right)$$

Result (type 5, 46 leaves):

$$-\frac{2\,a\,\mathrm{Cot}[c+d\,x]\,\mathrm{Hypergeometric2F1}\left[\frac{1}{2},\frac{2}{3},\frac{3}{2},1-\mathrm{Csc}[c+d\,x]\right]}{d\,\sqrt{a\,\left(1+\mathrm{Csc}[c+d\,x]\right)}}$$

Problem 23: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{a+a\,\text{Csc}[c+d\,x]}}{\text{Csc}[c+d\,x]^{2/3}}\,dx$$

Optimal (type 4, 254 leaves, 4 steps):

$$-\frac{3\,a\,\text{Cos}[c+d\,x]\,\text{Csc}[c+d\,x]^{1/3}}{2\,d\,\sqrt{a+a\,\text{Csc}[c+d\,x]}}-\left(3^{3/4}\,\sqrt{2+\sqrt{3}}\;a^2\,\text{Cot}[c+d\,x]\,\left(1-\text{Csc}[c+d\,x]^{1/3}\right)\,\sqrt{\frac{1+\text{Csc}[c+d\,x]^{1/3}+\text{Csc}[c+d\,x]^{2/3}}{\left(1+\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}}{1+\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}}\right],\,-7-4\,\sqrt{3}\right]\right)\Bigg/\left(2\,d\,\sqrt{\frac{1-\text{Csc}[c+d\,x]^{1/3}}{\left(1+\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}\right)^2}}\;\left(a-a\,\text{Csc}[c+d\,x]\right)\,\sqrt{a+a\,\text{Csc}[c+d\,x]}\right)$$

Result (type 5, 110 leaves):

$$-\left(\left(\sqrt{a\,\left(1+\text{Csc}[c+d\,x]\right)}\;\left(3+\text{Csc}[c+d\,x]^{2/3}\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{2}{3},\frac{3}{2},1-\text{Csc}[c+d\,x]\right]\right)\left(\text{Cos}\left[\frac{1}{2}\,(c+d\,x)\right]-\text{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]\right)\right)\Bigg/\left(2\,d\,\text{Csc}[c+d\,x]^{2/3}\,\left(\text{Cos}\left[\frac{1}{2}\,(c+d\,x)\right]+\text{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]\right)\right)\right)$$

Problem 24: Result unnecessarily involves higher level functions.

$$\int \text{Csc}[c+d\,x]^{5/3}\,\sqrt{a+a\,\text{Csc}[c+d\,x]}\,dx$$

Optimal (type 4, 514 leaves, 6 steps):

$$\frac{24\,a\,\text{Cot}[c+d\,x]}{7\,d\left(1+\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}\right)\sqrt{a+a\,\text{Csc}[c+d\,x]}}-\frac{6\,a\,\text{Cos}[c+d\,x]\,\text{Csc}[c+d\,x]^{5/3}}{7\,d\,\sqrt{a+a\,\text{Csc}[c+d\,x]}}-$$

$$\left(12\times 3^{1/4}\sqrt{2-\sqrt{3}}\;a^2\,\text{Cot}[c+d\,x]\left(1-\text{Csc}[c+d\,x]^{1/3}\right)\sqrt{\frac{1+\text{Csc}[c+d\,x]^{1/3}+\text{Csc}[c+d\,x]^{2/3}}{\left(1+\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}}{1+\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}}\right],-7-4\sqrt{3}\right]\right)\bigg/$$

$$\left(7\,d\sqrt{\frac{1-\text{Csc}[c+d\,x]^{1/3}}{\left(1+\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}\right)^2}}\left(a-a\,\text{Csc}[c+d\,x]\right)\sqrt{a+a\,\text{Csc}[c+d\,x]}\right)+$$

$$\left(8\sqrt{2}\;3^{3/4}\,a^2\,\text{Cot}[c+d\,x]\left(1-\text{Csc}[c+d\,x]^{1/3}\right)\sqrt{\frac{1+\text{Csc}[c+d\,x]^{1/3}+\text{Csc}[c+d\,x]^{2/3}}{\left(1+\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}}{1+\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}}\right],-7-4\sqrt{3}\right]\right)\bigg/$$

$$\left(7\,d\sqrt{\frac{1-\text{Csc}[c+d\,x]^{1/3}}{\left(1+\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}\right)^2}}\left(a-a\,\text{Csc}[c+d\,x]\right)\sqrt{a+a\,\text{Csc}[c+d\,x]}\right)$$

Result (type 5, 102 leaves):

$$-\left(\left(2\sqrt{a\left(1+\text{Csc}[c+d\,x]\right)}\left(3\,\text{Csc}[c+d\,x]^{2/3}+4\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{2},\frac{3}{2},1-\text{Csc}[c+d\,x]\right]\right)\right.\right.$$

$$\left.\left.\left(\text{Cos}\left[\frac{1}{2}(c+d\,x)\right]-\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right)\right)\bigg/\left(7\,d\left(\text{Cos}\left[\frac{1}{2}(c+d\,x)\right]+\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right)\right)\right)$$

Problem 25: Result unnecessarily involves higher level functions.

$$\int \text{Csc}[c+d\,x]^{2/3}\sqrt{a+a\,\text{Csc}[c+d\,x]}\;dx$$

Optimal (type 4, 470 leaves, 5 steps):

$$\frac{6\,a\,\text{Cot}[c+d\,x]}{d\left(1+\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}\right)\sqrt{a+a\,\text{Csc}[c+d\,x]}}-$$

$$\left(3\times 3^{1/4}\sqrt{2-\sqrt{3}}\;a^2\,\text{Cot}[c+d\,x]\left(1-\text{Csc}[c+d\,x]^{1/3}\right)\sqrt{\frac{1+\text{Csc}[c+d\,x]^{1/3}+\text{Csc}[c+d\,x]^{2/3}}{\left(1+\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}}{1+\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(d\sqrt{\frac{1-\text{Csc}[c+d\,x]^{1/3}}{\left(1+\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}\right)^2}}\left(a-a\,\text{Csc}[c+d\,x]\right)\sqrt{a+a\,\text{Csc}[c+d\,x]}\right)+$$

$$\left(2\sqrt{2}\;3^{3/4}\,a^2\,\text{Cot}[c+d\,x]\left(1-\text{Csc}[c+d\,x]^{1/3}\right)\sqrt{\frac{1+\text{Csc}[c+d\,x]^{1/3}+\text{Csc}[c+d\,x]^{2/3}}{\left(1+\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}}{1+\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(d\sqrt{\frac{1-\text{Csc}[c+d\,x]^{1/3}}{\left(1+\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}\right)^2}}\left(a-a\,\text{Csc}[c+d\,x]\right)\sqrt{a+a\,\text{Csc}[c+d\,x]}\right)$$

Result (type 5, 85 leaves):

$$-\left(\left(2\sqrt{a\left(1+\text{Csc}[c+d\,x]\right)}\;\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{2},\frac{3}{2},1-\text{Csc}[c+d\,x]\right]\right.\right.$$

$$\left.\left.\left(\text{Cos}\left[\frac{1}{2}(c+d\,x)\right]-\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right)\right)\Big/\left(d\left(\text{Cos}\left[\frac{1}{2}(c+d\,x)\right]+\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right)\right)\right)$$

Problem 26: Result unnecessarily involves higher level functions.

$$\int\frac{\sqrt{a+a\,\text{Csc}[c+d\,x]}}{\text{Csc}[c+d\,x]^{1/3}}\,dx$$

Optimal (type 4, 508 leaves, 6 steps):

$$-\frac{3\,a\,\mathrm{Cot}[c+d\,x]}{d\left(1+\sqrt{3}-\mathrm{Csc}[c+d\,x]^{1/3}\right)\sqrt{a+a\,\mathrm{Csc}[c+d\,x]}}-\frac{3\,a\,\mathrm{Cos}[c+d\,x]\,\mathrm{Csc}[c+d\,x]^{2/3}}{d\sqrt{a+a\,\mathrm{Csc}[c+d\,x]}}+$$

$$\left(3\times 3^{1/4}\sqrt{2-\sqrt{3}}\,a^2\,\mathrm{Cot}[c+d\,x]\left(1-\mathrm{Csc}[c+d\,x]^{1/3}\right)\sqrt{\frac{1+\mathrm{Csc}[c+d\,x]^{1/3}+\mathrm{Csc}[c+d\,x]^{2/3}}{\left(1+\sqrt{3}-\mathrm{Csc}[c+d\,x]^{1/3}\right)^2}}\right.$$

$$\left.\mathrm{EllipticE}\left[\mathrm{ArcSin}\left[\frac{1-\sqrt{3}-\mathrm{Csc}[c+d\,x]^{1/3}}{1+\sqrt{3}-\mathrm{Csc}[c+d\,x]^{1/3}}\right],-7-4\sqrt{3}\right]\right)\Bigg/$$

$$\left(2\,d\sqrt{\frac{1-\mathrm{Csc}[c+d\,x]^{1/3}}{\left(1+\sqrt{3}-\mathrm{Csc}[c+d\,x]^{1/3}\right)^2}}\left(a-a\,\mathrm{Csc}[c+d\,x]\right)\sqrt{a+a\,\mathrm{Csc}[c+d\,x]}\right)-$$

$$\left(\sqrt{2}\,3^{3/4}\,a^2\,\mathrm{Cot}[c+d\,x]\left(1-\mathrm{Csc}[c+d\,x]^{1/3}\right)\sqrt{\frac{1+\mathrm{Csc}[c+d\,x]^{1/3}+\mathrm{Csc}[c+d\,x]^{2/3}}{\left(1+\sqrt{3}-\mathrm{Csc}[c+d\,x]^{1/3}\right)^2}}\right.$$

$$\left.\mathrm{EllipticF}\left[\mathrm{ArcSin}\left[\frac{1-\sqrt{3}-\mathrm{Csc}[c+d\,x]^{1/3}}{1+\sqrt{3}-\mathrm{Csc}[c+d\,x]^{1/3}}\right],-7-4\sqrt{3}\right]\right)\Bigg/$$

$$\left(d\sqrt{\frac{1-\mathrm{Csc}[c+d\,x]^{1/3}}{\left(1+\sqrt{3}-\mathrm{Csc}[c+d\,x]^{1/3}\right)^2}}\left(a-a\,\mathrm{Csc}[c+d\,x]\right)\sqrt{a+a\,\mathrm{Csc}[c+d\,x]}\right)$$

Result (type 5, 66 leaves):

$$\left(-3\,a\,\mathrm{Cos}[c+d\,x]\,\mathrm{Csc}[c+d\,x]^{2/3}+a\,\mathrm{Cot}[c+d\,x]\,\mathrm{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{2},\frac{3}{2},1-\mathrm{Csc}[c+d\,x]\right]\right)\Bigg/$$

$$\left(d\sqrt{a\left(1+\mathrm{Csc}[c+d\,x]\right)}\right)$$

Problem 27: Result unnecessarily involves higher level functions.

$$\int\frac{\sqrt{a+a\,\mathrm{Csc}[c+d\,x]}}{\mathrm{Csc}[c+d\,x]^{4/3}}\,dx$$

Optimal (type 4, 552 leaves, 7 steps):

$$-\frac{15\,a\,\text{Cot}[c+d\,x]}{8\,d\left(1+\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}\right)\sqrt{a+a\,\text{Csc}[c+d\,x]}}-\frac{3\,a\,\text{Cos}[c+d\,x]}{4\,d\,\text{Csc}[c+d\,x]^{1/3}\sqrt{a+a\,\text{Csc}[c+d\,x]}}-\frac{15\,a\,\text{Cos}[c+d\,x]\,\text{Csc}[c+d\,x]^{2/3}}{8\,d\sqrt{a+a\,\text{Csc}[c+d\,x]}}+$$

$$\left(15\times 3^{1/4}\sqrt{2-\sqrt{3}}\;a^2\,\text{Cot}[c+d\,x]\left(1-\text{Csc}[c+d\,x]^{1/3}\right)\sqrt{\frac{1+\text{Csc}[c+d\,x]^{1/3}+\text{Csc}[c+d\,x]^{2/3}}{\left(1+\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}}{1+\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(16\,d\sqrt{\frac{1-\text{Csc}[c+d\,x]^{1/3}}{\left(1+\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}\right)^2}}\left(a-a\,\text{Csc}[c+d\,x]\right)\sqrt{a+a\,\text{Csc}[c+d\,x]}\right)-$$

$$\left(5\times 3^{3/4}\,a^2\,\text{Cot}[c+d\,x]\left(1-\text{Csc}[c+d\,x]^{1/3}\right)\sqrt{\frac{1+\text{Csc}[c+d\,x]^{1/3}+\text{Csc}[c+d\,x]^{2/3}}{\left(1+\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}}{1+\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(4\sqrt{2}\,d\sqrt{\frac{1-\text{Csc}[c+d\,x]^{1/3}}{\left(1+\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}\right)^2}}\left(a-a\,\text{Csc}[c+d\,x]\right)\sqrt{a+a\,\text{Csc}[c+d\,x]}\right)$$

Result (type 5, 118 leaves):

$$\left(a\,\text{Csc}[c+d\,x]^{2/3}\left(\text{Cos}\left[\frac{1}{2}(c+d\,x)\right]-\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right)\left(\text{Cos}\left[\frac{1}{2}(c+d\,x)\right]+\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right)\right.$$

$$\left.\left(-15+5\,\text{Csc}[c+d\,x]^{1/3}\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{2},\frac{3}{2},1-\text{Csc}[c+d\,x]\right]-6\,\text{Sin}[c+d\,x]\right)\right)\Big/$$

$$\left(8\,d\sqrt{a\left(1+\text{Csc}[c+d\,x]\right)}\right)$$

Problem 33: Unable to integrate problem.

$$\int\left(a+a\,\text{Csc}[e+f\,x]\right)^m\,dx$$

Optimal (type 6, 84 leaves, 3 steps):

$$-\left(\left(\sqrt{2}\;\text{AppellF1}\left[\frac{1}{2}+m,\frac{1}{2},1,\frac{3}{2}+m,\frac{1}{2}\left(1+\text{Csc}[e+f\,x]\right),1+\text{Csc}[e+f\,x]\right]\right.\right.$$

$$\left.\left.\text{Cot}[e+f\,x]\left(a+a\,\text{Csc}[e+f\,x]\right)^m\right)\Big/\left(f\,(1+2\,m)\sqrt{1-\text{Csc}[e+f\,x]}\right)\right)$$

Result (type 8, 14 leaves):

$$\int (a + a\,\mathrm{Csc}[e + f x])^m \, dx$$

Problem 34: Unable to integrate problem.

$$\int (a + a\,\mathrm{Csc}[e + f x])^m \,\mathrm{Sin}[e + f x] \, dx$$

Optimal (type 6, 83 leaves, 3 steps):

$$\left(\sqrt{2}\ \mathrm{AppellF1}\left[\frac{1}{2} + m, \frac{1}{2}, 2, \frac{3}{2} + m, \frac{1}{2}\,(1 + \mathrm{Csc}[e + f x]), 1 + \mathrm{Csc}[e + f x]\right] \mathrm{Cot}[e + f x]\,(a + a\,\mathrm{Csc}[e + f x])^m\right) \Big/ \left(f\,(1 + 2 m)\,\sqrt{1 - \mathrm{Csc}[e + f x]}\right)$$

Result (type 8, 21 leaves):

$$\int (a + a\,\mathrm{Csc}[e + f x])^m \,\mathrm{Sin}[e + f x] \, dx$$

Problem 35: Unable to integrate problem.

$$\int (a + a\,\mathrm{Csc}[e + f x])^m \,\mathrm{Sin}[e + f x]^2 \, dx$$

Optimal (type 6, 84 leaves, 3 steps):

$$-\left(\left(\sqrt{2}\ \mathrm{AppellF1}\left[\frac{1}{2} + m, \frac{1}{2}, 3, \frac{3}{2} + m, \frac{1}{2}\,(1 + \mathrm{Csc}[e + f x]), 1 + \mathrm{Csc}[e + f x]\right] \mathrm{Cot}[e + f x]\,(a + a\,\mathrm{Csc}[e + f x])^m\right) \Big/ \left(f\,(1 + 2 m)\,\sqrt{1 - \mathrm{Csc}[e + f x]}\right)\right)$$

Result (type 8, 23 leaves):

$$\int (a + a\,\mathrm{Csc}[e + f x])^m \,\mathrm{Sin}[e + f x]^2 \, dx$$

Problem 36: Result more than twice size of optimal antiderivative.

$$\int (a + b\,\mathrm{Csc}[c + d x])^4 \, dx$$

Optimal (type 3, 107 leaves, 6 steps):

$$a^4 x - \frac{2 a b\,(2 a^2 + b^2)\,\mathrm{ArcTanh}[\mathrm{Cos}[c + d x]]}{d} - \frac{b^2\,(17 a^2 + 2 b^2)\,\mathrm{Cot}[c + d x]}{3 d} - \frac{4 a b^3\,\mathrm{Cot}[c + d x]\,\mathrm{Csc}[c + d x]}{3 d} - \frac{b^2\,\mathrm{Cot}[c + d x]\,(a + b\,\mathrm{Csc}[c + d x])^2}{3 d}$$

Result (type 3, 568 leaves):

$$\frac{a^4 (c+d x) (a+b \operatorname{Csc}[c+d x])^4 \operatorname{Sin}[c+d x]^4}{d (b+a \operatorname{Sin}[c+d x])^4}+$$
$$\left(\left(-9 a^2 b^2 \operatorname{Cos}\left[\frac{1}{2} (c+d x)\right]-b^4 \operatorname{Cos}\left[\frac{1}{2} (c+d x)\right]\right) \operatorname{Csc}\left[\frac{1}{2} (c+d x)\right] (a+b \operatorname{Csc}[c+d x])^4 \operatorname{Sin}[c+d x]^4\right) \Big/ \left(3 d (b+a \operatorname{Sin}[c+d x])^4\right)-$$
$$\frac{a b^3 \operatorname{Csc}\left[\frac{1}{2} (c+d x)\right]^2 (a+b \operatorname{Csc}[c+d x])^4 \operatorname{Sin}[c+d x]^4}{2 d (b+a \operatorname{Sin}[c+d x])^4}-$$
$$\left(b^4 \operatorname{Cot}\left[\frac{1}{2} (c+d x)\right] \operatorname{Csc}\left[\frac{1}{2} (c+d x)\right]^2 (a+b \operatorname{Csc}[c+d x])^4 \operatorname{Sin}[c+d x]^4\right) \Big/ \left(24 d (b+a \operatorname{Sin}[c+d x])^4\right)-$$
$$\left(2 \left(2 a^3 b+a b^3\right) (a+b \operatorname{Csc}[c+d x])^4 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} (c+d x)\right]\right] \operatorname{Sin}[c+d x]^4\right) \Big/ \left(d (b+a \operatorname{Sin}[c+d x])^4\right)+$$
$$\left(2 \left(2 a^3 b+a b^3\right) (a+b \operatorname{Csc}[c+d x])^4 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2} (c+d x)\right]\right] \operatorname{Sin}[c+d x]^4\right) \Big/ \left(d (b+a \operatorname{Sin}[c+d x])^4\right)+\frac{a b^3 (a+b \operatorname{Csc}[c+d x])^4 \operatorname{Sec}\left[\frac{1}{2} (c+d x)\right]^2 \operatorname{Sin}[c+d x]^4}{2 d (b+a \operatorname{Sin}[c+d x])^4}+$$
$$\left((a+b \operatorname{Csc}[c+d x])^4 \operatorname{Sec}\left[\frac{1}{2} (c+d x)\right] \left(9 a^2 b^2 \operatorname{Sin}\left[\frac{1}{2} (c+d x)\right]+b^4 \operatorname{Sin}\left[\frac{1}{2} (c+d x)\right]\right) \operatorname{Sin}[c+d x]^4\right) \Big/ \left(3 d (b+a \operatorname{Sin}[c+d x])^4\right)+$$
$$\left(b^4 (a+b \operatorname{Csc}[c+d x])^4 \operatorname{Sec}\left[\frac{1}{2} (c+d x)\right]^2 \operatorname{Sin}[c+d x]^4 \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]\right) \Big/ \left(24 d (b+a \operatorname{Sin}[c+d x])^4\right)$$

Problem 37: Result more than twice size of optimal antiderivative.

$$\int (a+b \operatorname{Csc}[c+d x])^3 \, dx$$

Optimal (type 3, 73 leaves, 5 steps):

$$a^3 x-\frac{b \left(6 a^2+b^2\right) \operatorname{ArcTanh}[\operatorname{Cos}[c+d x]]}{2 d}-\frac{5 a b^2 \operatorname{Cot}[c+d x]}{2 d}-\frac{b^2 \operatorname{Cot}[c+d x] (a+b \operatorname{Csc}[c+d x])}{2 d}$$

Result (type 3, 152 leaves):

$$\frac{1}{8 d}\left(8 a^3 c+8 a^3 d x-12 a b^2 \operatorname{Cot}\left[\frac{1}{2} (c+d x)\right]-b^3 \operatorname{Csc}\left[\frac{1}{2} (c+d x)\right]^2-24 a^2 b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} (c+d x)\right]\right]-4 b^3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} (c+d x)\right]\right]+24 a^2 b \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2} (c+d x)\right]\right]+4 b^3 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2} (c+d x)\right]\right]+b^3 \operatorname{Sec}\left[\frac{1}{2} (c+d x)\right]^2+12 a b^2 \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]\right)$$

Problem 38: Result more than twice size of optimal antiderivative.

$$\int (a + b \operatorname{Csc}[c + d x])^2 \, dx$$

Optimal (type 3, 34 leaves, 4 steps):

$$a^2 x - \frac{2 a b \operatorname{ArcTanh}[\operatorname{Cos}[c + d x]]}{d} - \frac{b^2 \operatorname{Cot}[c + d x]}{d}$$

Result (type 3, 76 leaves):

$$\frac{1}{2 d} \left(-b^2 \operatorname{Cot}\left[\frac{1}{2} (c + d x)\right] + 2 a \left(a c + a d x - 2 b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} (c + d x)\right]\right] + 2 b \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]\right] \right) + b^2 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] \right)$$

Problem 52: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{3 + 5 \operatorname{Csc}[c + d x]} \, dx$$

Optimal (type 3, 31 leaves, 2 steps):

$$-\frac{x}{12} - \frac{5 \operatorname{ArcTan}\left[\frac{\operatorname{Cos}[c+d x]}{3+\operatorname{Sin}[c+d x]}\right]}{6 d}$$

Result (type 3, 66 leaves):

$$\frac{2 (c + d x) - 5 \operatorname{ArcTan}\left[\frac{2 \left(\operatorname{Cos}\left[\frac{1}{2} (c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2} (c+d x)\right]\right)}{\operatorname{Cos}\left[\frac{1}{2} (c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2} (c+d x)\right]}\right]}{6 d}$$

Problem 54: Unable to integrate problem.

$$\int \operatorname{Csc}[e + f x]^3 (a + b \operatorname{Csc}[e + f x])^m \, dx$$

Optimal (type 6, 274 leaves, 8 steps):

$$-\frac{\text{Cot}[e+f x]\,(a+b\,\text{Csc}[e+f x])^{1+m}}{b\,f\,(2+m)}+\left(\sqrt{2}\,a\,(a+b)\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{2},-1-m,\frac{3}{2},\frac{1}{2}\,(1-\text{Csc}[e+f x]),\frac{b\,(1-\text{Csc}[e+f x])}{a+b}\right]\text{Cot}[e+f x]\,(a+b\,\text{Csc}[e+f x])^m\left(\frac{a+b\,\text{Csc}[e+f x]}{a+b}\right)^{-m}\right)\Big/\left(b^2\,f\,(2+m)\,\sqrt{1+\text{Csc}[e+f x]}\right)-\left(\sqrt{2}\,\left(a^2+b^2\,(1+m)\right)\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}\,(1-\text{Csc}[e+f x]),\frac{b\,(1-\text{Csc}[e+f x])}{a+b}\right]\text{Cot}[e+f x]\,(a+b\,\text{Csc}[e+f x])^m\left(\frac{a+b\,\text{Csc}[e+f x]}{a+b}\right)^{-m}\right)\Big/\left(b^2\,f\,(2+m)\,\sqrt{1+\text{Csc}[e+f x]}\right)$$

Result (type 8, 23 leaves):

$$\int \text{Csc}[e+f x]^3\,(a+b\,\text{Csc}[e+f x])^m\,dx$$

Problem 55: Unable to integrate problem.

$$\int \text{Csc}[e+f x]^2\,(a+b\,\text{Csc}[e+f x])^m\,dx$$

Optimal (type 6, 220 leaves, 7 steps):

$$-\left(\left(\sqrt{2}\,(a+b)\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{2},-1-m,\frac{3}{2},\frac{1}{2}\,(1-\text{Csc}[e+f x]),\frac{b\,(1-\text{Csc}[e+f x])}{a+b}\right]\text{Cot}[e+f x]\,(a+b\,\text{Csc}[e+f x])^m\left(\frac{a+b\,\text{Csc}[e+f x]}{a+b}\right)^{-m}\right)\Big/\left(b\,f\,\sqrt{1+\text{Csc}[e+f x]}\right)\right)+\left(\sqrt{2}\,a\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}\,(1-\text{Csc}[e+f x]),\frac{b\,(1-\text{Csc}[e+f x])}{a+b}\right]\text{Cot}[e+f x]\,(a+b\,\text{Csc}[e+f x])^m\left(\frac{a+b\,\text{Csc}[e+f x]}{a+b}\right)^{-m}\right)\Big/\left(b\,f\,\sqrt{1+\text{Csc}[e+f x]}\right)$$

Result (type 8, 23 leaves):

$$\int \text{Csc}[e+f x]^2\,(a+b\,\text{Csc}[e+f x])^m\,dx$$

Problem 56: Unable to integrate problem.

$$\int \text{Csc}[e+f x]\,(a+b\,\text{Csc}[e+f x])^m\,dx$$

Optimal (type 6, 104 leaves, 3 steps):

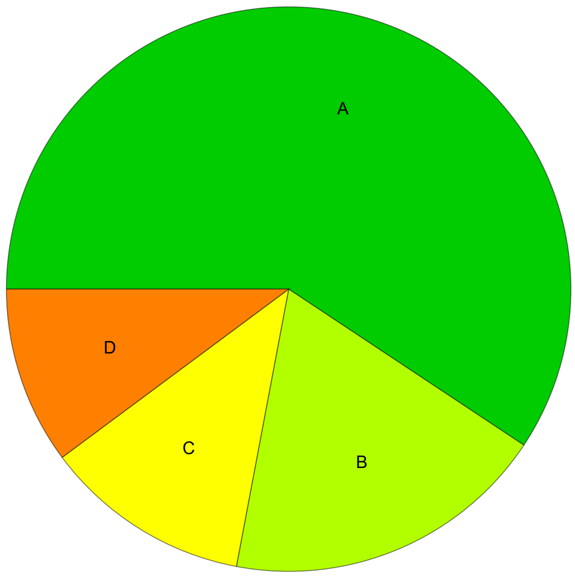
$$-\left(\left(\sqrt{2}\,\text{AppellF1}\left[\frac{1}{2},\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}\,(1-\text{Csc}[e+f x]),\frac{b\,(1-\text{Csc}[e+f x])}{a+b}\right]\text{Cot}[e+f x]\,(a+b\,\text{Csc}[e+f x])^m\left(\frac{a+b\,\text{Csc}[e+f x]}{a+b}\right)^{-m}\right)\Big/\left(f\,\sqrt{1+\text{Csc}[e+f x]}\right)\right)$$

Result (type 8, 21 leaves):

$$\int \text{Csc}[e+f x]\,(a+b\,\text{Csc}[e+f x])^m\,dx$$

Summary of Integration Test Results

59 integration problems



A - 35 optimal antiderivatives

B - 11 more than twice size of optimal antiderivatives

C - 7 unnecessarily complex antiderivatives

D - 6 unable to integrate problems

E - 0 integration timeouts