

Mathematica 11.3 Integration Test Results

Test results for the 84 problems in "4.6.11 (e x)^m (a+b csc(c+d x^n))^p.m"

Problem 5: Result more than twice size of optimal antiderivative.

$$\int x (a + b \operatorname{Csc}[c + d x^2]) dx$$

Optimal (type 3, 26 leaves, 4 steps):

$$\frac{a x^2}{2} - \frac{b \operatorname{ArcTanh}[\operatorname{Cos}[c + d x^2]]}{2 d}$$

Result (type 3, 57 leaves):

$$\frac{a x^2}{2} - \frac{b \operatorname{Log}[\operatorname{Cos}[\frac{c}{2} + \frac{d x^2}{2}]]}{2 d} + \frac{b \operatorname{Log}[\operatorname{Sin}[\frac{c}{2} + \frac{d x^2}{2}]]}{2 d}$$

Problem 10: Result more than twice size of optimal antiderivative.

$$\int x^3 (a + b \operatorname{Csc}[c + d x^2])^2 dx$$

Optimal (type 4, 125 leaves, 10 steps):

$$\frac{a^2 x^4}{4} - \frac{2 a b x^2 \operatorname{ArcTanh}[e^{i(c+d x^2)}]}{d} - \frac{b^2 x^2 \operatorname{Cot}[c + d x^2]}{2 d} + \frac{b^2 \operatorname{Log}[\operatorname{Sin}[c + d x^2]]}{2 d^2} + \frac{i a b \operatorname{PolyLog}[2, -e^{i(c+d x^2)}]}{d^2} - \frac{i a b \operatorname{PolyLog}[2, e^{i(c+d x^2)}]}{d^2}$$

Result (type 4, 590 leaves):

Result (type 4, 1104 leaves):

$$\frac{x^4 \operatorname{Csc}[c+d x^2] (b+a \operatorname{Sin}[c+d x^2])}{4 a (a+b \operatorname{Csc}[c+d x^2])} -$$

$$\frac{1}{2 a d^2 (a+b \operatorname{Csc}[c+d x^2])} b \operatorname{Csc}[c+d x^2] \left(\frac{\pi \operatorname{ArcTan}\left[\frac{a+b \operatorname{Tan}\left[\frac{1}{2}(c+d x^2)\right]}{\sqrt{-a^2+b^2}}\right]}{\sqrt{-a^2+b^2}} + \right.$$

$$\frac{1}{\sqrt{a^2-b^2}} \left(2 \left(-c + \frac{\pi}{2} - d x^2\right) \operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - d x^2\right)\right]}{\sqrt{a^2-b^2}}\right] - \right.$$

$$2 \left(-c + \operatorname{ArcCos}\left[-\frac{b}{a}\right]\right) \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - d x^2\right)\right]}{\sqrt{a^2-b^2}}\right] +$$

$$\left(\operatorname{ArcCos}\left[-\frac{b}{a}\right] - 2 i \left(\operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - d x^2\right)\right]}{\sqrt{a^2-b^2}}\right] - \right.$$

$$\left. \left. \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - d x^2\right)\right]}{\sqrt{a^2-b^2}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{a^2-b^2} e^{-\frac{1}{2} i\left(-c + \frac{\pi}{2} - d x^2\right)}}{\sqrt{2} \sqrt{a} \sqrt{b+a \operatorname{Sin}[c+d x^2]}}\right] +$$

$$\left(\operatorname{ArcCos}\left[-\frac{b}{a}\right] + 2 i \left(\operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - d x^2\right)\right]}{\sqrt{a^2-b^2}}\right] - \right.$$

$$\left. \left. \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - d x^2\right)\right]}{\sqrt{a^2-b^2}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{a^2-b^2} e^{\frac{1}{2} i\left(-c + \frac{\pi}{2} - d x^2\right)}}{\sqrt{2} \sqrt{a} \sqrt{b+a \operatorname{Sin}[c+d x^2]}}\right] -$$

$$\left(\operatorname{ArcCos}\left[-\frac{b}{a}\right] + 2 i \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - d x^2\right)\right]}{\sqrt{a^2-b^2}}\right] \right)$$

$$\operatorname{Log}\left[1 - \left(\left(b - i \sqrt{a^2-b^2} \right) \left(a+b - \sqrt{a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - d x^2\right)\right] \right) \right) \right] /$$

$$\left(a \left(a+b + \sqrt{a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - d x^2\right)\right] \right) \right) +$$

$$\left(-\operatorname{ArcCos}\left[-\frac{b}{a}\right] + 2 i \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - d x^2\right)\right]}{\sqrt{a^2-b^2}}\right] \right)$$

$$\operatorname{Log}\left[1 - \left(\left(b + i \sqrt{a^2-b^2} \right) \left(a+b - \sqrt{a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - d x^2\right)\right] \right) \right) \right] /$$

$$\left(a \left(a+b + \sqrt{a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - d x^2\right)\right] \right) \right) +$$

$$i \left(\operatorname{PolyLog}\left[2, \left(\left(b - i \sqrt{a^2-b^2} \right) \left(a+b - \sqrt{a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - d x^2\right)\right] \right) \right) \right] \right) /$$

$$\left(a \left(a+b + \sqrt{a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - d x^2\right)\right] \right) \right) -$$

$$\text{PolyLog}\left[2, \left(\left(b + i \sqrt{a^2 - b^2} \right) \left(a + b - \sqrt{a^2 - b^2} \tan\left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x^2 \right)\right] \right) \right) / \left(a \left(a + b + \sqrt{a^2 - b^2} \tan\left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x^2 \right)\right] \right) \right) \right] \left(b + a \sin[c + d x^2] \right)$$

Problem 61: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Csc}[\sqrt{x}]^3}{\sqrt{x}} dx$$

Optimal (type 3, 24 leaves, 3 steps):

$$-\text{ArcTanh}[\text{Cos}[\sqrt{x}]] - \text{Cot}[\sqrt{x}] \text{Csc}[\sqrt{x}]$$

Result (type 3, 57 leaves):

$$-\frac{1}{4} \text{Csc}\left[\frac{\sqrt{x}}{2}\right]^2 - \text{Log}\left[\text{Cos}\left[\frac{\sqrt{x}}{2}\right]\right] + \text{Log}\left[\text{Sin}\left[\frac{\sqrt{x}}{2}\right]\right] + \frac{1}{4} \text{Sec}\left[\frac{\sqrt{x}}{2}\right]^2$$

Problem 75: Unable to integrate problem.

$$\int (e x)^{-1+3 n} (a + b \text{Csc}[c + d x^n]) dx$$

Optimal (type 4, 221 leaves, 11 steps):

$$\frac{a (e x)^{3 n}}{3 e n} - \frac{2 b x^{-n} (e x)^{3 n} \text{ArcTanh}\left[e^{i(c+d x^n)}\right]}{d e n} + \frac{2 i b x^{-2 n} (e x)^{3 n} \text{PolyLog}\left[2, -e^{i(c+d x^n)}\right]}{d^2 e n} - \frac{2 i b x^{-2 n} (e x)^{3 n} \text{PolyLog}\left[2, e^{i(c+d x^n)}\right]}{d^2 e n} - \frac{2 b x^{-3 n} (e x)^{3 n} \text{PolyLog}\left[3, -e^{i(c+d x^n)}\right]}{d^3 e n} + \frac{2 b x^{-3 n} (e x)^{3 n} \text{PolyLog}\left[3, e^{i(c+d x^n)}\right]}{d^3 e n}$$

Result (type 8, 24 leaves):

$$\int (e x)^{-1+3 n} (a + b \text{Csc}[c + d x^n]) dx$$

Problem 77: Result more than twice size of optimal antiderivative.

$$\int (e x)^{-1+2 n} (a + b \text{Csc}[c + d x^n])^2 dx$$

Optimal (type 4, 214 leaves, 11 steps):

$$\frac{a^2 (e x)^{2n}}{2 e n} - \frac{4 a b x^{-n} (e x)^{2n} \operatorname{ArcTanh}\left[e^{i(c+d x^n)}\right]}{d e n} - \frac{b^2 x^{-n} (e x)^{2n} \operatorname{Cot}[c+d x^n]}{d e n} + \frac{b^2 x^{-2n} (e x)^{2n} \operatorname{Log}[\operatorname{Sin}[c+d x^n]]}{d^2 e n} + \frac{2 i a b x^{-2n} (e x)^{2n} \operatorname{PolyLog}\left[2, -e^{i(c+d x^n)}\right]}{d^2 e n} - \frac{2 i a b x^{-2n} (e x)^{2n} \operatorname{PolyLog}\left[2, e^{i(c+d x^n)}\right]}{d^2 e n}$$

Result (type 4, 687 leaves):

$$\frac{b^2 x^{1-n} (e x)^{-1+2n} \operatorname{Cot}[c] (a+b \operatorname{Csc}[c+d x^n])^2 \operatorname{Sin}[c+d x^n]^2}{d n (b+a \operatorname{Sin}[c+d x^n])^2} + \left(x^{1-n} (e x)^{-1+2n} \operatorname{Csc}\left[\frac{c}{2}\right] (a+b \operatorname{Csc}[c+d x^n])^2 \operatorname{Sec}\left[\frac{c}{2}\right] (-2 b^2 \operatorname{Cos}[c] + a^2 d x^n \operatorname{Sin}[c]) \operatorname{Sin}[c+d x^n]^2 \right) / \left(4 d n (b+a \operatorname{Sin}[c+d x^n])^2 + (b^2 x^{1-2n} (e x)^{-1+2n} \operatorname{Csc}[c] (a+b \operatorname{Csc}[c+d x^n])^2 (-d x^n \operatorname{Cos}[c] + \operatorname{Log}[\operatorname{Cos}[d x^n] \operatorname{Sin}[c] + \operatorname{Cos}[c] \operatorname{Sin}[d x^n]] \operatorname{Sin}[c]) \operatorname{Sin}[c+d x^n]^2) / (d^2 n (\operatorname{Cos}[c]^2 + \operatorname{Sin}[c]^2) (b+a \operatorname{Sin}[c+d x^n])^2) + (b^2 x^{1-n} (e x)^{-1+2n} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Csc}\left[\frac{c}{2} + \frac{d x^n}{2}\right] (a+b \operatorname{Csc}[c+d x^n])^2 \operatorname{Sin}\left[\frac{d x^n}{2}\right] \operatorname{Sin}[c+d x^n]^2) / (2 d n (b+a \operatorname{Sin}[c+d x^n])^2) + (b^2 x^{1-n} (e x)^{-1+2n} (a+b \operatorname{Csc}[c+d x^n])^2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x^n}{2}\right] \operatorname{Sin}\left[\frac{d x^n}{2}\right] \operatorname{Sin}[c+d x^n]^2) / (2 d n (b+a \operatorname{Sin}[c+d x^n])^2) + \left(2 a b x^{1-2n} (e x)^{-1+2n} (a+b \operatorname{Csc}[c+d x^n])^2 \operatorname{Sin}[c+d x^n]^2 \right) \left(\frac{2 \operatorname{ArcTan}[\operatorname{Tan}[c]] \operatorname{ArcTanh}\left[\frac{-\operatorname{Cos}[c] + \operatorname{Sin}[c] \operatorname{Tan}\left[\frac{d x^n}{2}\right]}{\sqrt{\operatorname{Cos}[c]^2 + \operatorname{Sin}[c]^2}}\right]}{\sqrt{\operatorname{Cos}[c]^2 + \operatorname{Sin}[c]^2}} + \frac{1}{\sqrt{1 + \operatorname{Tan}[c]^2}} \right) + \left((d x^n + \operatorname{ArcTan}[\operatorname{Tan}[c]]) (\operatorname{Log}[1 - e^{i(d x^n + \operatorname{ArcTan}[\operatorname{Tan}[c])}] - \operatorname{Log}[1 + e^{i(d x^n + \operatorname{ArcTan}[\operatorname{Tan}[c])}]] + i (\operatorname{PolyLog}[2, -e^{i(d x^n + \operatorname{ArcTan}[\operatorname{Tan}[c])}] - \operatorname{PolyLog}[2, e^{i(d x^n + \operatorname{ArcTan}[\operatorname{Tan}[c])}]])) \operatorname{Sec}[c] \right) / (d^2 n (b+a \operatorname{Sin}[c+d x^n])^2$$

Problem 78: Unable to integrate problem.

$$\int (e x)^{-1+3n} (a+b \operatorname{Csc}[c+d x^n])^2 dx$$

Optimal (type 4, 377 leaves, 16 steps):

$$\frac{a^2 (e x)^{3 n}}{3 e n} - \frac{i b^2 x^{-n} (e x)^{3 n}}{d e n} - \frac{4 a b x^{-n} (e x)^{3 n} \operatorname{ArcTanh}\left[e^{i(c+d x^n)}\right]}{d e n} - \frac{b^2 x^{-n} (e x)^{3 n} \operatorname{Cot}\left[c+d x^n\right]}{d e n} +$$

$$\frac{2 b^2 x^{-2 n} (e x)^{3 n} \operatorname{Log}\left[1-e^{2 i(c+d x^n)}\right]}{d^2 e n} + \frac{4 i a b x^{-2 n} (e x)^{3 n} \operatorname{PolyLog}\left[2,-e^{i(c+d x^n)}\right]}{d^2 e n} -$$

$$\frac{4 i a b x^{-2 n} (e x)^{3 n} \operatorname{PolyLog}\left[2,e^{i(c+d x^n)}\right]}{d^2 e n} - \frac{i b^2 x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[2,e^{2 i(c+d x^n)}\right]}{d^3 e n} -$$

$$\frac{4 a b x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[3,-e^{i(c+d x^n)}\right]}{d^3 e n} + \frac{4 a b x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[3,e^{i(c+d x^n)}\right]}{d^3 e n}$$

Result (type 8, 26 leaves):

$$\int (e x)^{-1+3 n} (a+b \operatorname{Csc}\left[c+d x^n\right])^2 d x$$

Problem 80: Result more than twice size of optimal antiderivative.

$$\int \frac{(e x)^{-1+2 n}}{a+b \operatorname{Csc}\left[c+d x^n\right]} d x$$

Optimal (type 4, 338 leaves, 12 steps):

$$\frac{(e x)^{2 n}}{2 a e n} + \frac{i b x^{-n} (e x)^{2 n} \operatorname{Log}\left[1-\frac{i a e^{i(c+d x^n)}}{b-\sqrt{-a^2+b^2}}\right]}{a \sqrt{-a^2+b^2} d e n} - \frac{i b x^{-n} (e x)^{2 n} \operatorname{Log}\left[1-\frac{i a e^{i(c+d x^n)}}{b+\sqrt{-a^2+b^2}}\right]}{a \sqrt{-a^2+b^2} d e n} +$$

$$\frac{b x^{-2 n} (e x)^{2 n} \operatorname{PolyLog}\left[2,\frac{i a e^{i(c+d x^n)}}{b-\sqrt{-a^2+b^2}}\right]}{a \sqrt{-a^2+b^2} d^2 e n} - \frac{b x^{-2 n} (e x)^{2 n} \operatorname{PolyLog}\left[2,\frac{i a e^{i(c+d x^n)}}{b+\sqrt{-a^2+b^2}}\right]}{a \sqrt{-a^2+b^2} d^2 e n}$$

Result (type 4, 1131 leaves):

$$\frac{x (e x)^{-1+2 n} \operatorname{Csc}\left[c+d x^n\right] (b+a \operatorname{Sin}\left[c+d x^n\right])}{2 a n (a+b \operatorname{Csc}\left[c+d x^n\right])} -$$

$$\frac{1}{a d^2 n (a+b \operatorname{Csc}\left[c+d x^n\right])} b x^{1-2 n} (e x)^{-1+2 n} \operatorname{Csc}\left[c+d x^n\right] \left(\frac{\pi \operatorname{ArcTan}\left[\frac{a+b \operatorname{Tan}\left[\frac{1}{2}(c+d x^n)\right]}{\sqrt{-a^2+b^2}}\right]}{\sqrt{-a^2+b^2}} + \right.$$

$$\left. \frac{1}{\sqrt{a^2-b^2}} \left(2 \left(-c + \frac{\pi}{2} - d x^n \right) \operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - d x^n\right)\right]}{\sqrt{a^2-b^2}}\right] - \right.$$

$$\left. 2 \left(-c + \operatorname{ArcCos}\left[-\frac{b}{a}\right] \right) \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - d x^n\right)\right]}{\sqrt{a^2-b^2}}\right] + \right.$$

$$\left. \left(\operatorname{ArcCos}\left[-\frac{b}{a}\right] - 2 i \left(\operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}\left(-c + \frac{\pi}{2} - d x^n\right)\right]}{\sqrt{a^2-b^2}}\right] - \right.$$

$$\frac{(e x)^{3 n}}{3 a e n} + \frac{i b x^{-n} (e x)^{3 n} \operatorname{Log}\left[1 - \frac{i a e^{i(c+d x^n)}}{b-\sqrt{-a^2+b^2}}\right]}{a \sqrt{-a^2+b^2} d e n} - \frac{i b x^{-n} (e x)^{3 n} \operatorname{Log}\left[1 - \frac{i a e^{i(c+d x^n)}}{b+\sqrt{-a^2+b^2}}\right]}{a \sqrt{-a^2+b^2} d e n} +$$

$$\frac{2 b x^{-2 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, \frac{i a e^{i(c+d x^n)}}{b-\sqrt{-a^2+b^2}}\right]}{a \sqrt{-a^2+b^2} d^2 e n} - \frac{2 b x^{-2 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, \frac{i a e^{i(c+d x^n)}}{b+\sqrt{-a^2+b^2}}\right]}{a \sqrt{-a^2+b^2} d^2 e n} +$$

$$\frac{2 i b x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[3, \frac{i a e^{i(c+d x^n)}}{b-\sqrt{-a^2+b^2}}\right]}{a \sqrt{-a^2+b^2} d^3 e n} - \frac{2 i b x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[3, \frac{i a e^{i(c+d x^n)}}{b+\sqrt{-a^2+b^2}}\right]}{a \sqrt{-a^2+b^2} d^3 e n}$$

Result (type 8, 26 leaves):

$$\int \frac{(e x)^{-1+3 n}}{a+b \operatorname{Csc}[c+d x^n]} dx$$

Problem 83: Result more than twice size of optimal antiderivative.

$$\int \frac{(e x)^{-1+2 n}}{(a+b \operatorname{Csc}[c+d x^n])^2} dx$$

Optimal (type 4, 778 leaves, 23 steps):

$$\frac{(e x)^{2 n}}{2 a^2 e n} - \frac{i b^3 x^{-n} (e x)^{2 n} \operatorname{Log}\left[1 - \frac{i a e^{i(c+d x^n)}}{b-\sqrt{-a^2+b^2}}\right]}{a^2 (-a^2+b^2)^{3/2} d e n} + \frac{2 i b x^{-n} (e x)^{2 n} \operatorname{Log}\left[1 - \frac{i a e^{i(c+d x^n)}}{b-\sqrt{-a^2+b^2}}\right]}{a^2 \sqrt{-a^2+b^2} d e n} +$$

$$\frac{i b^3 x^{-n} (e x)^{2 n} \operatorname{Log}\left[1 - \frac{i a e^{i(c+d x^n)}}{b+\sqrt{-a^2+b^2}}\right]}{a^2 (-a^2+b^2)^{3/2} d e n} - \frac{2 i b x^{-n} (e x)^{2 n} \operatorname{Log}\left[1 - \frac{i a e^{i(c+d x^n)}}{b+\sqrt{-a^2+b^2}}\right]}{a^2 \sqrt{-a^2+b^2} d e n} +$$

$$\frac{b^2 x^{-2 n} (e x)^{2 n} \operatorname{Log}[b+a \operatorname{Sin}[c+d x^n]]}{a^2 (a^2-b^2) d^2 e n} - \frac{b^3 x^{-2 n} (e x)^{2 n} \operatorname{PolyLog}\left[2, \frac{i a e^{i(c+d x^n)}}{b-\sqrt{-a^2+b^2}}\right]}{a^2 (-a^2+b^2)^{3/2} d^2 e n} +$$

$$\frac{2 b x^{-2 n} (e x)^{2 n} \operatorname{PolyLog}\left[2, \frac{i a e^{i(c+d x^n)}}{b-\sqrt{-a^2+b^2}}\right]}{a^2 \sqrt{-a^2+b^2} d^2 e n} + \frac{b^3 x^{-2 n} (e x)^{2 n} \operatorname{PolyLog}\left[2, \frac{i a e^{i(c+d x^n)}}{b+\sqrt{-a^2+b^2}}\right]}{a^2 (-a^2+b^2)^{3/2} d^2 e n} -$$

$$\frac{2 b x^{-2 n} (e x)^{2 n} \operatorname{PolyLog}\left[2, \frac{i a e^{i(c+d x^n)}}{b+\sqrt{-a^2+b^2}}\right]}{a^2 \sqrt{-a^2+b^2} d^2 e n} - \frac{b^2 x^{-n} (e x)^{2 n} \operatorname{Cos}[c+d x^n]}{a (a^2-b^2) d e n (b+a \operatorname{Sin}[c+d x^n])}$$

Result (type 4, 2850 leaves):

$$-\left(\left(b^2 x^{1-n} (e x)^{-1+2 n} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Csc}[c+d x^n]^2 \operatorname{Sec}\left[\frac{c}{2}\right] (b \operatorname{Cos}[c]+a \operatorname{Sin}[d x^n]) (b+a \operatorname{Sin}[c+d x^n])\right)\right) /$$

$$\left(2 a^2 (-a+b) (a+b) d n (a+b \operatorname{Csc}[c+d x^n])^2\right) -$$

$$\begin{aligned}
 & \frac{b^2 x^{1-n} (e x)^{-1+2n} \cot [c] \csc [c+d x^n]^2 (b+a \sin [c+d x^n])^2}{a^2 (-a^2+b^2) d n (a+b \csc [c+d x^n])^2} - \\
 & \left(2 b^3 x^{1-2n} (e x)^{-1+2n} \operatorname{ArcTan} \left[\frac{a \cos [c+d x^n] + i (b+a \sin [c+d x^n])}{\sqrt{-a^2+b^2}} \right] \cot [c] \csc [c+d x^n]^2 \right. \\
 & \quad \left. (b+a \sin [c+d x^n])^2 \right) / \left(a^2 (a^2-b^2) \sqrt{-a^2+b^2} d^2 n (a+b \csc [c+d x^n])^2 \right) - \\
 & \frac{1}{(a^2-b^2) d^2 n (a+b \csc [c+d x^n])^2} 2 b x^{1-2n} (e x)^{-1+2n} \csc [c+d x^n]^2 \left(\frac{\pi \operatorname{ArcTan} \left[\frac{a+b \tan \left[\frac{1}{2} (c+d x^n) \right]}{\sqrt{-a^2+b^2}} \right]}{\sqrt{-a^2+b^2}} + \right. \\
 & \quad \frac{1}{\sqrt{a^2-b^2}} \left(2 \left(-c + \frac{\pi}{2} - d x^n \right) \operatorname{ArcTanh} \left[\frac{(a+b) \cot \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x^n \right) \right]}{\sqrt{a^2-b^2}} \right] - \right. \\
 & \quad \left. 2 \left(-c + \operatorname{ArcCos} \left[-\frac{b}{a} \right] \right) \operatorname{ArcTanh} \left[\frac{(a-b) \tan \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x^n \right) \right]}{\sqrt{a^2-b^2}} \right] + \right. \\
 & \quad \left. \left(\operatorname{ArcCos} \left[-\frac{b}{a} \right] - 2 i \left(\operatorname{ArcTanh} \left[\frac{(a+b) \cot \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x^n \right) \right]}{\sqrt{a^2-b^2}} \right] - \right. \right. \right. \\
 & \quad \left. \left. \left. \operatorname{ArcTanh} \left[\frac{(a-b) \tan \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x^n \right) \right]}{\sqrt{a^2-b^2}} \right] \right) \right) \right) \operatorname{Log} \left[\frac{\sqrt{a^2-b^2} e^{-\frac{1}{2} i \left(-c + \frac{\pi}{2} - d x^n \right)}}{\sqrt{2} \sqrt{a} \sqrt{b+a \sin [c+d x^n]}} \right] + \\
 & \quad \left(\operatorname{ArcCos} \left[-\frac{b}{a} \right] + 2 i \left(\operatorname{ArcTanh} \left[\frac{(a+b) \cot \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x^n \right) \right]}{\sqrt{a^2-b^2}} \right] - \right. \right. \\
 & \quad \left. \left. \operatorname{ArcTanh} \left[\frac{(a-b) \tan \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x^n \right) \right]}{\sqrt{a^2-b^2}} \right] \right) \right) \right) \operatorname{Log} \left[\frac{\sqrt{a^2-b^2} e^{\frac{1}{2} i \left(-c + \frac{\pi}{2} - d x^n \right)}}{\sqrt{2} \sqrt{a} \sqrt{b+a \sin [c+d x^n]}} \right] - \\
 & \quad \left(\operatorname{ArcCos} \left[-\frac{b}{a} \right] + 2 i \operatorname{ArcTanh} \left[\frac{(a-b) \tan \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x^n \right) \right]}{\sqrt{a^2-b^2}} \right] \right) \\
 & \quad \operatorname{Log} \left[1 - \left(\left(b - i \sqrt{a^2-b^2} \right) \left(a+b - \sqrt{a^2-b^2} \tan \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x^n \right) \right] \right) \right) \right) / \\
 & \quad \left(a \left(a+b + \sqrt{a^2-b^2} \tan \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x^n \right) \right] \right) \right) \right) + \\
 & \quad \left(-\operatorname{ArcCos} \left[-\frac{b}{a} \right] + 2 i \operatorname{ArcTanh} \left[\frac{(a-b) \tan \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x^n \right) \right]}{\sqrt{a^2-b^2}} \right] \right) \\
 & \quad \operatorname{Log} \left[1 - \left(\left(b + i \sqrt{a^2-b^2} \right) \left(a+b - \sqrt{a^2-b^2} \tan \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x^n \right) \right] \right) \right) \right) / \\
 & \quad \left(a \left(a+b + \sqrt{a^2-b^2} \tan \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x^n \right) \right] \right) \right) \right) \right) +
 \end{aligned}$$

$$\begin{aligned}
 & i \left(\text{PolyLog}\left[2, \left(\left(b - i \sqrt{a^2 - b^2} \right) \left(a + b - \sqrt{a^2 - b^2} \tan\left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x^n \right) \right] \right) \right) \right] / \right. \\
 & \quad \left. \left(a \left(a + b + \sqrt{a^2 - b^2} \tan\left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x^n \right) \right] \right) \right) \right] - \right. \\
 & \quad \left. \text{PolyLog}\left[2, \left(\left(b + i \sqrt{a^2 - b^2} \right) \left(a + b - \sqrt{a^2 - b^2} \tan\left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x^n \right) \right] \right) \right) \right] / \right. \\
 & \quad \left. \left(a \left(a + b + \sqrt{a^2 - b^2} \tan\left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x^n \right) \right] \right) \right) \right] \right) \\
 & (b + a \sin[c + d x^n])^2 + \frac{1}{a^2 (a^2 - b^2) d^2 n (a + b \csc[c + d x^n])^2} \\
 & b^3 \\
 & x^{1-2n} \\
 & (e x)^{-1+2n} \\
 & \csc[c + d x^n]^2 \\
 & \left(\frac{\pi \operatorname{ArcTan}\left[\frac{a+b \tan\left[\frac{1}{2}(c+d x^n)\right]}{\sqrt{-a^2+b^2}}\right]}{\sqrt{-a^2+b^2}} + \right. \\
 & \frac{1}{\sqrt{a^2-b^2}} \left(2 \left(-c + \frac{\pi}{2} - d x^n \right) \operatorname{ArcTanh}\left[\frac{(a+b) \cot\left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x^n \right) \right]}{\sqrt{a^2-b^2}}\right] - \right. \\
 & \quad \left. 2 \left(-c + \operatorname{ArcCos}\left[-\frac{b}{a}\right] \right) \operatorname{ArcTanh}\left[\frac{(a-b) \tan\left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x^n \right) \right]}{\sqrt{a^2-b^2}}\right] + \right. \\
 & \quad \left. \left(\operatorname{ArcCos}\left[-\frac{b}{a}\right] - 2 i \left(\operatorname{ArcTanh}\left[\frac{(a+b) \cot\left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x^n \right) \right]}{\sqrt{a^2-b^2}}\right] - \right. \right. \right. \\
 & \quad \left. \left. \left. \operatorname{ArcTanh}\left[\frac{(a-b) \tan\left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x^n \right) \right]}{\sqrt{a^2-b^2}}\right] \right) \right) \right) \operatorname{Log}\left[\frac{\sqrt{a^2-b^2} e^{-\frac{1}{2} i \left(-c + \frac{\pi}{2} - d x^n \right)}}{\sqrt{2} \sqrt{a} \sqrt{b+a \sin[c+d x^n]}}\right] + \right. \\
 & \quad \left. \left(\operatorname{ArcCos}\left[-\frac{b}{a}\right] + 2 i \left(\operatorname{ArcTanh}\left[\frac{(a+b) \cot\left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x^n \right) \right]}{\sqrt{a^2-b^2}}\right] - \right. \right. \right. \\
 & \quad \left. \left. \left. \operatorname{ArcTanh}\left[\frac{(a-b) \tan\left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x^n \right) \right]}{\sqrt{a^2-b^2}}\right] \right) \right) \right) \operatorname{Log}\left[\frac{\sqrt{a^2-b^2} e^{\frac{1}{2} i \left(-c + \frac{\pi}{2} - d x^n \right)}}{\sqrt{2} \sqrt{a} \sqrt{b+a \sin[c+d x^n]}}\right] - \right. \\
 & \quad \left. \left(\operatorname{ArcCos}\left[-\frac{b}{a}\right] + 2 i \operatorname{ArcTanh}\left[\frac{(a-b) \tan\left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x^n \right) \right]}{\sqrt{a^2-b^2}}\right] \right) \right) \\
 & \left. \operatorname{Log}\left[1 - \left(\left(b - i \sqrt{a^2 - b^2} \right) \left(a + b - \sqrt{a^2 - b^2} \tan\left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x^n \right) \right] \right) \right) \right] \right) /
 \end{aligned}$$

$$\begin{aligned}
 & \left(a \left(a + b + \sqrt{a^2 - b^2} \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x^n \right) \right] \right) \right) + \\
 & \left(-\operatorname{ArcCos} \left[-\frac{b}{a} \right] + 2 i \operatorname{ArcTanh} \left[\frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x^n \right) \right]}{\sqrt{a^2 - b^2}} \right] \right) \\
 & \operatorname{Log} \left[1 - \left(\left(b + i \sqrt{a^2 - b^2} \right) \left(a + b - \sqrt{a^2 - b^2} \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x^n \right) \right] \right) \right) \right] / \\
 & \left(a \left(a + b + \sqrt{a^2 - b^2} \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x^n \right) \right] \right) \right) + \\
 & i \left(\operatorname{PolyLog} \left[2, \left(\left(b - i \sqrt{a^2 - b^2} \right) \left(a + b - \sqrt{a^2 - b^2} \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x^n \right) \right] \right) \right) \right] \right) / \\
 & \left(a \left(a + b + \sqrt{a^2 - b^2} \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x^n \right) \right] \right) \right) - \\
 & \operatorname{PolyLog} \left[2, \left(\left(b + i \sqrt{a^2 - b^2} \right) \left(a + b - \sqrt{a^2 - b^2} \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x^n \right) \right] \right) \right) \right] / \\
 & \left(a \left(a + b + \sqrt{a^2 - b^2} \operatorname{Tan} \left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x^n \right) \right] \right) \right) \right) \\
 & (b + a \operatorname{Sin}[c + d x^n])^2 + \left(x^{1-n} (e x)^{-1+2n} \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{Csc}[c + d x^n]^2 \right. \\
 & \operatorname{Sec} \left[\frac{c}{2} \right] \\
 & (-2 b^2 \operatorname{Cos}[c] + a^2 d x^n \operatorname{Sin}[c] - b^2 d x^n \operatorname{Sin}[c]) \\
 & \left. (b + a \operatorname{Sin}[c + d x^n])^2 \right) / (4 \\
 & a^2 \\
 & (a - b) \\
 & (a + b) \\
 & d \\
 & n \\
 & (a + b \operatorname{Csc}[c + d x^n])^2 + \left(b^2 \right. \\
 & x^{1-2n} \\
 & (e x)^{-1+2n} \\
 & \operatorname{Csc}[c] \\
 & \operatorname{Csc}[c + d x^n]^2 \\
 & \left. -a d x^n \operatorname{Cos}[c] + a \operatorname{Log}[b + a \operatorname{Cos}[d x^n] \operatorname{Sin}[c] + a \operatorname{Cos}[c] \operatorname{Sin}[d x^n]] \operatorname{Sin}[c] + \right.
 \end{aligned}$$

$$\left. \frac{2 \operatorname{Im} a b \operatorname{ArcTan} \left[\frac{i a \operatorname{Cos}[c] - i (-b+a \operatorname{Sin}[c]) \operatorname{Tan} \left[\frac{d x^n}{2} \right]}{\sqrt{-b^2+a^2 \operatorname{Cos}[c]^2+a^2 \operatorname{Sin}[c]^2}} \right] \operatorname{Cos}[c]}{\sqrt{-b^2+a^2 \operatorname{Cos}[c]^2+a^2 \operatorname{Sin}[c]^2}} \right)$$

$$\left. \right) \left/ (a (a^2 - b^2)) \right.$$

$$\frac{d^2}{n} \left((a + b \operatorname{Csc}[c + d x^n])^2 (a^2 \operatorname{Cos}[c]^2 + a^2 \operatorname{Sin}[c]^2) \right)$$

Problem 84: Unable to integrate problem.

$$\int \frac{(e x)^{-1+3 n}}{(a + b \operatorname{Csc}[c + d x^n])^2} dx$$

Optimal (type 4, 1417 leaves, 32 steps):

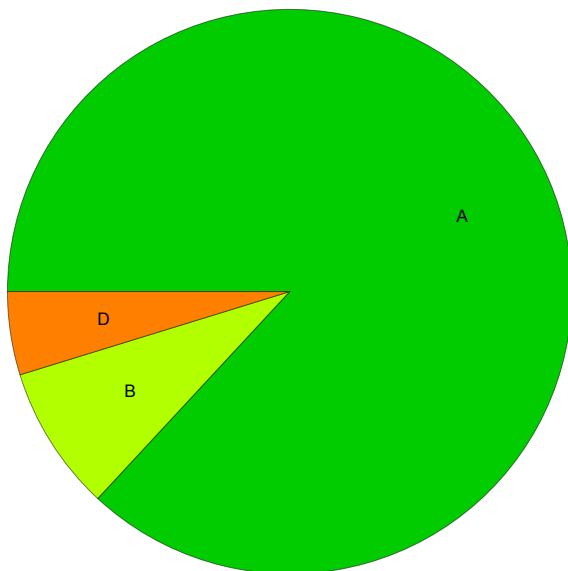
$$\begin{aligned}
 & \frac{(e x)^{3 n}}{3 a^2 e n} - \frac{i b^2 x^{-n} (e x)^{3 n}}{a^2 (a^2 - b^2) d e n} + \frac{2 b^2 x^{-2 n} (e x)^{3 n} \operatorname{Log}\left[1 + \frac{a e^{i(c+d x^n)}}{i b - \sqrt{a^2 - b^2}}\right]}{a^2 (a^2 - b^2) d^2 e n} + \\
 & \frac{2 b^2 x^{-2 n} (e x)^{3 n} \operatorname{Log}\left[1 + \frac{a e^{i(c+d x^n)}}{i b + \sqrt{a^2 - b^2}}\right]}{a^2 (a^2 - b^2) d^2 e n} - \frac{i b^3 x^{-n} (e x)^{3 n} \operatorname{Log}\left[1 - \frac{i a e^{i(c+d x^n)}}{b - \sqrt{-a^2 + b^2}}\right]}{a^2 (-a^2 + b^2)^{3/2} d e n} + \\
 & \frac{2 i b x^{-n} (e x)^{3 n} \operatorname{Log}\left[1 - \frac{i a e^{i(c+d x^n)}}{b - \sqrt{-a^2 + b^2}}\right]}{a^2 \sqrt{-a^2 + b^2} d e n} + \frac{i b^3 x^{-n} (e x)^{3 n} \operatorname{Log}\left[1 - \frac{i a e^{i(c+d x^n)}}{b + \sqrt{-a^2 + b^2}}\right]}{a^2 (-a^2 + b^2)^{3/2} d e n} - \\
 & \frac{2 i b x^{-n} (e x)^{3 n} \operatorname{Log}\left[1 - \frac{i a e^{i(c+d x^n)}}{b + \sqrt{-a^2 + b^2}}\right]}{a^2 \sqrt{-a^2 + b^2} d e n} - \frac{2 i b^2 x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, -\frac{a e^{i(c+d x^n)}}{i b - \sqrt{a^2 - b^2}}\right]}{a^2 (a^2 - b^2) d^3 e n} - \\
 & \frac{2 i b^2 x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, -\frac{a e^{i(c+d x^n)}}{i b + \sqrt{a^2 - b^2}}\right]}{a^2 (a^2 - b^2) d^3 e n} - \frac{2 b^3 x^{-2 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, \frac{i a e^{i(c+d x^n)}}{b - \sqrt{-a^2 + b^2}}\right]}{a^2 (-a^2 + b^2)^{3/2} d^2 e n} + \\
 & \frac{4 b x^{-2 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, \frac{i a e^{i(c+d x^n)}}{b - \sqrt{-a^2 + b^2}}\right]}{a^2 \sqrt{-a^2 + b^2} d^2 e n} + \frac{2 b^3 x^{-2 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, \frac{i a e^{i(c+d x^n)}}{b + \sqrt{-a^2 + b^2}}\right]}{a^2 (-a^2 + b^2)^{3/2} d^2 e n} - \\
 & \frac{4 b x^{-2 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, \frac{i a e^{i(c+d x^n)}}{b + \sqrt{-a^2 + b^2}}\right]}{a^2 \sqrt{-a^2 + b^2} d^2 e n} - \frac{2 i b^3 x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[3, \frac{i a e^{i(c+d x^n)}}{b - \sqrt{-a^2 + b^2}}\right]}{a^2 (-a^2 + b^2)^{3/2} d^3 e n} + \\
 & \frac{4 i b x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[3, \frac{i a e^{i(c+d x^n)}}{b - \sqrt{-a^2 + b^2}}\right]}{a^2 \sqrt{-a^2 + b^2} d^3 e n} + \frac{2 i b^3 x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[3, \frac{i a e^{i(c+d x^n)}}{b + \sqrt{-a^2 + b^2}}\right]}{a^2 (-a^2 + b^2)^{3/2} d^3 e n} - \\
 & \frac{4 i b x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[3, \frac{i a e^{i(c+d x^n)}}{b + \sqrt{-a^2 + b^2}}\right]}{a^2 \sqrt{-a^2 + b^2} d^3 e n} - \frac{b^2 x^{-n} (e x)^{3 n} \operatorname{Cos}[c + d x^n]}{a (a^2 - b^2) d e n (b + a \operatorname{Sin}[c + d x^n])}
 \end{aligned}$$

Result (type 8, 26 leaves):

$$\int \frac{(e x)^{-1+3 n}}{(a + b \operatorname{Csc}[c + d x^n])^2} dx$$

Summary of Integration Test Results

84 integration problems



A - 73 optimal antiderivatives

B - 7 more than twice size of optimal antiderivatives

C - 0 unnecessarily complex antiderivatives

D - 4 unable to integrate problems

E - 0 integration timeouts