Mathematica 11.3 Integration Test Results

Test results for the 227 problems in "5.1.2 (d x)^m (a+b arcsin(c x))^n.m"

Problem 121: Unable to integrate problem.

$$\int (b\,x)^m\,\text{ArcSin}[a\,x]^2\,dx$$

Optimal (type 5, 150 leaves, 2 steps):

$$\frac{(b\,x)^{1+m}\,\text{ArcSin}[a\,x]^2}{b\,(1+m)}-\frac{2\,a\,(b\,x)^{2+m}\,\text{ArcSin}[a\,x]\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},a^2\,x^2\right]}{b^2\,(1+m)\,(2+m)}+\left(2\,a^2\,(b\,x)^{3+m}\,\text{HypergeometricPFQ}\left[\left\{1,\frac{3}{2}+\frac{m}{2},\frac{3}{2}+\frac{m}{2}\right\},\left\{2+\frac{m}{2},\frac{5}{2}+\frac{m}{2}\right\},a^2\,x^2\right]\right)\Big/\left(b^3\,(1+m)\,(2+m)\,(3+m)\right)$$

Result (type 9, 143 leaves):

$$\frac{1}{(1+m)\,(2+m)}2^{-2-m}\,x\,(b\,x)^m\left(2^{2+m}\,\text{ArcSin}[a\,x]\left((2+m)\,\text{ArcSin}[a\,x]-2\,a\,x\,\sqrt{1-a^2\,x^2}\,\text{Hypergeometric2F1}\left[1,\frac{3+m}{2},\frac{4+m}{2},a^2\,x^2\right]\right)+a^2\,(2+m)\,\sqrt{\pi}\,x^2\,\text{Gamma}[2+m]\,\text{HypergeometricPFQRegularized}\left[\left\{1,\frac{3+m}{2},\frac{3+m}{2}\right\},\left\{\frac{4+m}{2},\frac{5+m}{2}\right\},a^2\,x^2\right]\right)$$

Problem 157: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+b\,\text{ArcSin}[c\,x])^3}{x^2}\,dx$$

Optimal (type 4, 137 leaves, 9 steps):

$$-\frac{(a+b\,\text{ArcSin}[c\,x])^3}{x}-6\,b\,c\,(a+b\,\text{ArcSin}[c\,x])^2\,\text{ArcTanh}\left[e^{i\,\text{ArcSin}[c\,x]}\right]+6\,i\,b^2\,c\,(a+b\,\text{ArcSin}[c\,x])\,\text{PolyLog}\left[2,-e^{i\,\text{ArcSin}[c\,x]}\right]-6\,i\,b^2\,c\,(a+b\,\text{ArcSin}[c\,x])\,\text{PolyLog}\left[2,e^{i\,\text{ArcSin}[c\,x]}\right]-6\,b^3\,c\,\text{PolyLog}\left[3,-e^{i\,\text{ArcSin}[c\,x]}\right]+6\,b^3\,c\,\text{PolyLog}\left[3,e^{i\,\text{ArcSin}[c\,x]}\right]$$

Result (type 4, 283 leaves):

$$-\frac{a^3}{x}-\frac{3 a^2 b \text{ArcSin}[c x]}{x}+3 a^2 b c \text{Log}[x]-3 a^2 b c \text{Log}\left[1+\sqrt{1-c^2 x^2}\right]+$$
$$3 a b^2 c \left(-\text{ArcSin}[c x] \left(\frac{\text{ArcSin}[c x]}{c x}-2 \text{Log}\left[1-e^{i \text{ArcSin}[c x]}\right]+2 \text{Log}\left[1+e^{i \text{ArcSin}[c x]}\right]\right)+\right.$$
$$\left.2 i \text{PolyLog}\left[2,-e^{i \text{ArcSin}[c x]}\right]-2 i \text{PolyLog}\left[2,e^{i \text{ArcSin}[c x]}\right]\right)+$$
$$b^3 c \left(-\frac{\text{ArcSin}[c x]^3}{c x}+3 \text{ArcSin}[c x]^2 \text{Log}\left[1-e^{i \text{ArcSin}[c x]}\right]-3 \text{ArcSin}[c x]^2 \text{Log}\left[1+e^{i \text{ArcSin}[c x]}\right]+\right.$$
$$6 i \text{ArcSin}[c x] \text{PolyLog}\left[2,-e^{i \text{ArcSin}[c x]}\right]-6 i \text{ArcSin}[c x] \text{PolyLog}\left[2,e^{i \text{ArcSin}[c x]}\right]-$$
$$\left.6 \text{PolyLog}\left[3,-e^{i \text{ArcSin}[c x]}\right]+6 \text{PolyLog}\left[3,e^{i \text{ArcSin}[c x]}\right]\right)$$

Problem 203: Result unnecessarily involves imaginary or complex numbers.

$$\int (d x)^{5/2} (a+b \text{ArcSin}[c x]) \, dx$$

Optimal (type 4, 120 leaves, 5 steps):

$$\frac{20 b d^2 \sqrt{d x} \sqrt{1-c^2 x^2}}{147 c^3}+\frac{4 b (d x)^{5/2} \sqrt{1-c^2 x^2}}{49 c}+$$
$$\frac{2 (d x)^{7/2} (a+b \text{ArcSin}[c x])}{7 d}-\frac{20 b d^{5/2} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{d x}}{\sqrt{d}}\right],-1\right]}{147 c^{7/2}}$$

Result (type 4, 159 leaves):

$$\left(2 d^2 \sqrt{d x} \left(10 b-4 b c^2 x^2-6 b c^4 x^4+21 a c^3 x^3 \sqrt{1-c^2 x^2}+21 b c^3 x^3 \sqrt{1-c^2 x^2} \text{ArcSin}[c x]+\right.\right.$$
$$\left.\left.10 i b \sqrt{-\frac{1}{c}} c \sqrt{1-\frac{1}{c^2 x^2}} \sqrt{x} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\frac{1}{c}}}{\sqrt{x}}\right],-1\right]\right)\right) \Big/ \left(147 c^3 \sqrt{1-c^2 x^2}\right)$$

Problem 204: Result unnecessarily involves imaginary or complex numbers.

$$\int (d x)^{3/2} (a+b \text{ArcSin}[c x]) \, dx$$

Optimal (type 4, 124 leaves, 7 steps):

$$\frac{4 b (d x)^{3/2} \sqrt{1-c^2 x^2}}{25 c}+\frac{2 (d x)^{5/2} (a+b \text{ArcSin}[c x])}{5 d}-\frac{12 b d^{3/2} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{d x}}{\sqrt{d}}\right],-1\right]}{25 c^{5/2}}+\frac{12 b d^{3/2} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{d x}}{\sqrt{d}}\right],-1\right]}{25 c^{5/2}}$$

Result (type 4, 107 leaves):

$$\frac{1}{25 c^2 \sqrt{-c x}} 2 d \sqrt{d x} \left(c x \sqrt{-c x} \left(5 a c x+2 b \sqrt{1-c^2 x^2}+5 b c x \text{ArcSin}[c x]\right)+6 i b \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{-c x}\right],-1\right]-6 i b \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-c x}\right],-1\right]\right)$$

Problem 205: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{d x} (a+b \text{ArcSin}[c x]) \, dx$$

Optimal (type 4, 88 leaves, 4 steps):

$$\frac{4 b \sqrt{d x} \sqrt{1-c^2 x^2}}{9 c}+\frac{2 (d x)^{3/2} (a+b \text{ArcSin}[c x])}{3 d}-\frac{4 b \sqrt{d} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{d x}}{\sqrt{d}}\right],-1\right]}{9 c^{3/2}}$$

Result (type 4, 113 leaves):

$$\frac{2}{9} \sqrt{d x} \left(3 a x+\frac{2 b \sqrt{1-c^2 x^2}}{c}+3 b x \text{ArcSin}[c x]+\frac{2 i b \sqrt{-\frac{1}{c}} \sqrt{1-\frac{1}{c^2 x^2}} \sqrt{x} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\frac{1}{c}}}{\sqrt{x}}\right],-1\right]}{\sqrt{1-c^2 x^2}}\right)$$

Problem 206: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a+b \text{ArcSin}[c x]}{\sqrt{d x}} \, dx$$

Optimal (type 4, 89 leaves, 6 steps):

$$\frac{2 \sqrt{d x} (a+b \text{ArcSin}[c x])}{d}-\frac{4 b \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{d x}}{\sqrt{d}}\right],-1\right]}{\sqrt{c} \sqrt{d}}+\frac{4 b \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{d x}}{\sqrt{d}}\right],-1\right]}{\sqrt{c} \sqrt{d}}$$

Result (type 4, 76 leaves):

$$\frac{1}{\sqrt{-c\,x}\;\sqrt{d\,x}}2\,x\left(\sqrt{-c\,x}\;\left(a+b\,\text{ArcSin}[c\,x]\right)+\right.$$
$$\left.2\,i\,b\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-c\,x}\right],-1\right]-2\,i\,b\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-c\,x}\right],-1\right]\right)$$

Problem 207: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{a+b\,\text{ArcSin}[c\,x]}{(d\,x)^{3/2}}\,dx$$

Optimal (type 4, 55 leaves, 3 steps):

$$-\frac{2\left(a+b\,\text{ArcSin}[c\,x]\right)}{d\,\sqrt{d\,x}}+\frac{4\,b\,\sqrt{c}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c}\;\sqrt{d\,x}}{\sqrt{d}}\right],-1\right]}{d^{3/2}}$$

Result (type 4, 91 leaves):

$$\frac{2\,x\left(-a-b\,\text{ArcSin}[c\,x]+\frac{2\,i\,b\,c\,\sqrt{1-\frac{1}{c^2x^2}}\;x^{3/2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\frac{1}{c}}}{\sqrt{x}}\right],-1\right]}{\sqrt{-\frac{1}{c}}\;\sqrt{1-c^2x^2}}\right)}{(d\,x)^{3/2}}$$

Problem 208: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{a+b\,\text{ArcSin}[c\,x]}{(d\,x)^{5/2}}\,dx$$

Optimal (type 4, 125 leaves, 7 steps):

$$-\frac{4\,b\,c\,\sqrt{1-c^2x^2}}{3\,d^2\,\sqrt{d\,x}}-\frac{2\left(a+b\,\text{ArcSin}[c\,x]\right)}{3\,d\,(d\,x)^{3/2}}-$$
$$\frac{4\,b\,c^{3/2}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c}\;\sqrt{d\,x}}{\sqrt{d}}\right],-1\right]}{3\,d^{5/2}}+\frac{4\,b\,c^{3/2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c}\;\sqrt{d\,x}}{\sqrt{d}}\right],-1\right]}{3\,d^{5/2}}$$

Result (type 4, 110 leaves):

$$\frac{1}{3\,\sqrt{-c\,x}\;(d\,x)^{5/2}}x\left(-2\,\sqrt{-c\,x}\;\left(a+2\,b\,c\,x\,\sqrt{1-c^2x^2}\;+b\,\text{ArcSin}[c\,x]\right)+\right.$$
$$\left.4\,i\,b\,c^2\,x^2\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-c\,x}\right],-1\right]-4\,i\,b\,c^2\,x^2\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-c\,x}\right],-1\right]\right)$$

Problem 209: Result more than twice size of optimal antiderivative.

$$\int(d\,x)^{5/2}\left(a+b\,\text{ArcSin}[c\,x]\right)^2\,dx$$

Optimal (type 5, 109 leaves, 2 steps):

$$\frac{2\,(d\,x)^{7/2}\,(a+b\,\text{ArcSin}[c\,x])^2}{7\,d}-\frac{1}{63\,d^2}8\,b\,c\,(d\,x)^{9/2}\,(a+b\,\text{ArcSin}[c\,x])\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{9}{4},\frac{13}{4},c^2\,x^2\right]+\frac{16\,b^2\,c^2\,(d\,x)^{11/2}\,\text{HypergeometricPFQ}\left[\left\{1,\frac{11}{4},\frac{11}{4}\right\},\left\{\frac{13}{4},\frac{15}{4}\right\},c^2\,x^2\right]}{693\,d^3}$$

Result (type 5, 269 leaves):

$$\frac{1}{6174}(d\,x)^{5/2}\Bigg(1764\,a^2\,x+3528\,a\,b\,x\,\text{ArcSin}[c\,x]-\left(336\,a\,b\,x\left(\sqrt{c\,x}\,\left(-5+2\,c^2\,x^2+3\,c^4\,x^4\right)-5\,c\,\sqrt{1-\frac{1}{c^2\,x^2}}\;x\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1}{\sqrt{c\,x}}\right],-1\right]\right)\right)\Big/\left((c\,x)^{7/2}\sqrt{1-c^2\,x^2}\right)+\left(b^2\left(210\sqrt{2}\,c\,\pi\,x\,\text{HypergeometricPFQ}\left[\left\{\frac{3}{4},\frac{3}{4},1\right\},\left\{\frac{5}{4},\frac{7}{4}\right\},c^2\,x^2\right]+4\,\text{Gamma}\left[\frac{5}{4}\right]\text{Gamma}\left[\frac{7}{4}\right]\left(-334\,c\,x+441\,c^3\,x^3\,\text{ArcSin}[c\,x]^2+21\,\text{ArcSin}[c\,x]\left(23\sqrt{1-c^2\,x^2}-3\,\text{Cos}[3\,\text{ArcSin}[c\,x]]\right)-420\sqrt{1-c^2\,x^2}\,\text{ArcSin}[c\,x]\,\text{Hypergeometric2F1}\left[\frac{3}{4},1,\frac{5}{4},c^2\,x^2\right]+18\,\text{Sin}[3\,\text{ArcSin}[c\,x]]\right)\right)\right)\Big/\left(c^3\,x^2\,\text{Gamma}\left[\frac{5}{4}\right]\text{Gamma}\left[\frac{7}{4}\right]\right)\Bigg)$$

Problem 211: Result more than twice size of optimal antiderivative.

$$\int\sqrt{d\,x}\,(a+b\,\text{ArcSin}[c\,x])^2\,dx$$

Optimal (type 5, 109 leaves, 2 steps):

$$\frac{2\,(d\,x)^{3/2}\,(a+b\,\text{ArcSin}[c\,x])^2}{3\,d}-\frac{8\,b\,c\,(d\,x)^{5/2}\,(a+b\,\text{ArcSin}[c\,x])\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{5}{4},\frac{9}{4},c^2\,x^2\right]}{15\,d^2}+\frac{16\,b^2\,c^2\,(d\,x)^{7/2}\,\text{HypergeometricPFQ}\left[\left\{1,\frac{7}{4},\frac{7}{4}\right\},\left\{\frac{9}{4},\frac{11}{4}\right\},c^2\,x^2\right]}{105\,d^3}$$

Result (type 5, 228 leaves):

$$\frac{1}{27}\sqrt{d x}\left(18 a^2 x+36 a b x \operatorname{ArcSin}[c x]+\frac{24 b^2\sqrt{1-c^2 x^2}\operatorname{ArcSin}[c x]}{c}+2 b^2 x\left(-8+9 \operatorname{ArcSin}[c x]^2\right)-\right.$$
$$\left(24 a b x\left(-\sqrt{c x}+(c x)^{5/2}-c\sqrt{1-\frac{1}{c^2 x^2}}\, x \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1}{\sqrt{c x}}\right],-1\right]\right)\right)\Big/$$
$$\left((c x)^{3/2}\sqrt{1-c^2 x^2}\right)-\frac{24 b^2\sqrt{1-c^2 x^2}\operatorname{ArcSin}[c x]\operatorname{Hypergeometric2F1}\left[\frac{3}{4},1,\frac{5}{4},c^2 x^2\right]}{c}+$$
$$\left.\frac{3\sqrt{2}\, b^2 \pi x \operatorname{HypergeometricPFQ}\left[\left\{\frac{3}{4},\frac{3}{4},1\right\},\left\{\frac{5}{4},\frac{7}{4}\right\},c^2 x^2\right]}{\operatorname{Gamma}\left[\frac{5}{4}\right]\operatorname{Gamma}\left[\frac{7}{4}\right]}\right)$$

Problem 214: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+b \operatorname{ArcSin}[c x])^2}{(d x)^{5/2}}\, dx$$

Optimal (type 5, 109 leaves, 2 steps):

$$-\frac{2(a+b \operatorname{ArcSin}[c x])^2}{3 d (d x)^{3/2}}-\frac{8 b c (a+b \operatorname{ArcSin}[c x])\operatorname{Hypergeometric2F1}\left[-\frac{1}{4},\frac{1}{2},\frac{3}{4},c^2 x^2\right]}{3 d^2\sqrt{d x}}+$$
$$\frac{16 b^2 c^2\sqrt{d x}\operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4},\frac{1}{4},1\right\},\left\{\frac{3}{4},\frac{5}{4}\right\},c^2 x^2\right]}{3 d^3}$$

Result (type 5, 242 leaves):

$$\frac{1}{36 (d x)^{5/2}\operatorname{Gamma}\left[\frac{7}{4}\right]\operatorname{Gamma}\left[\frac{9}{4}\right]}$$
$$x\left(-8 \operatorname{Gamma}\left[\frac{7}{4}\right]\operatorname{Gamma}\left[\frac{9}{4}\right]\left(3 a^2-24 b^2 c^2 x^2+12 a b c x\sqrt{1-c^2 x^2}+6 a b \operatorname{ArcSin}[c x]+\right.\right.$$
$$12 b^2 c x\sqrt{1-c^2 x^2}\operatorname{ArcSin}[c x]+3 b^2\operatorname{ArcSin}[c x]^2+12 a b (c x)^{3/2}$$
$$\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{c x}\right],-1\right]-12 a b (c x)^{3/2}\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{c x}\right],-1\right]+$$
$$\left.4 b^2 c^3 x^3\sqrt{1-c^2 x^2}\operatorname{ArcSin}[c x]\operatorname{Hypergeometric2F1}\left[1,\frac{5}{4},\frac{7}{4},c^2 x^2\right]\right)+$$
$$\left.3\sqrt{2}\, b^2 c^4 \pi x^4\operatorname{HypergeometricPFQ}\left[\left\{1,\frac{5}{4},\frac{5}{4}\right\},\left\{\frac{7}{4},\frac{9}{4}\right\},c^2 x^2\right]\right)$$

Problem 216: Attempted integration timed out after 120 seconds.

$$\int \sqrt{d x}\,(a+b \operatorname{ArcSin}[c x])^3\, dx$$

Optimal (type 8, 67 leaves, 1 step):

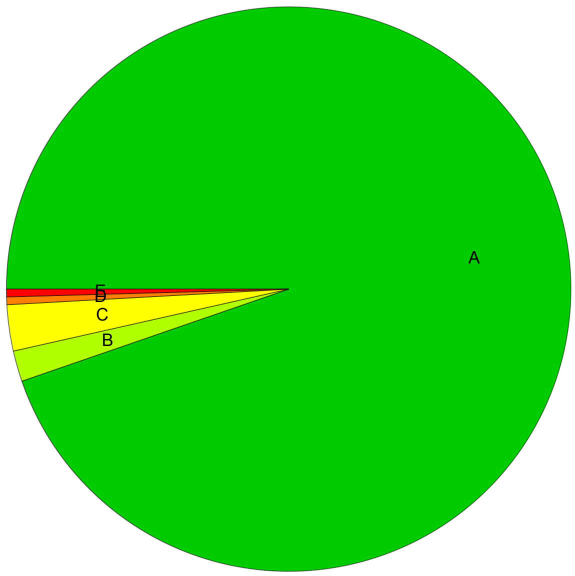
$$\frac{2\,(d\,x)^{3/2}\,(a+b\,\text{ArcSin}[c\,x])^3}{3\,d}-\frac{2\,b\,c\,\text{Int}\left[\frac{(d\,x)^{3/2}\,(a+b\,\text{ArcSin}[c\,x])^2}{\sqrt{1-c^2\,x^2}},\,x\right]}{d}$$

Result (type 1, 1 leaves):

???

Summary of Integration Test Results

227 integration problems



A - 215 optimal antiderivatives

B - 4 more than twice size of optimal antiderivatives

C - 6 unnecessarily complex antiderivatives

D - 1 unable to integrate problems

E - 1 integration timeouts