

# Mathematica 11.3 Integration Test Results

Test results for the 166 problems in "5.3.2 (d x)^m (a+b arctan(c x^n))^p.m"

Problem 81: Unable to integrate problem.

$$\int x^2 (a + b \operatorname{ArcTan}[c x^2])^2 dx$$

Optimal (type 4, 1393 leaves, 86 steps):

$$\begin{aligned} & -\frac{4 a b x}{3 c} + \frac{2}{9} i a b x^3 + \frac{4 (-1)^{3/4} b^2 \operatorname{ArcTan}\left[(-1)^{3/4} \sqrt{c} x\right]}{3 c^{3/2}} + \frac{(-1)^{1/4} b^2 \operatorname{ArcTan}\left[(-1)^{3/4} \sqrt{c} x\right]^2}{3 c^{3/2}} \\ & - \frac{2 (-1)^{1/4} a b \operatorname{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right]}{3 c^{3/2}} - \frac{4 (-1)^{3/4} b^2 \operatorname{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right]}{3 c^{3/2}} \\ & - \frac{(-1)^{3/4} b^2 \operatorname{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right]^2}{3 c^{3/2}} - \frac{2 (-1)^{3/4} b^2 \operatorname{ArcTan}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[\frac{2}{1-(-1)^{1/4} \sqrt{c} x}\right]}{3 c^{3/2}} + \\ & - \frac{2 (-1)^{3/4} b^2 \operatorname{ArcTan}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[\frac{2}{1+(-1)^{1/4} \sqrt{c} x}\right]}{3 c^{3/2}} \\ & - \frac{(-1)^{3/4} b^2 \operatorname{ArcTan}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[\frac{\sqrt{2}\left((-1)^{1/4}+\sqrt{c} x\right)}{1+(-1)^{1/4} \sqrt{c} x}\right]}{3 c^{3/2}} + \\ & - \frac{2 (-1)^{3/4} b^2 \operatorname{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[\frac{2}{1-(-1)^{3/4} \sqrt{c} x}\right]}{3 c^{3/2}} \\ & - \frac{2 (-1)^{3/4} b^2 \operatorname{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[\frac{2}{1+(-1)^{3/4} \sqrt{c} x}\right]}{3 c^{3/2}} + \\ & + \frac{(-1)^{3/4} b^2 \operatorname{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[-\frac{\sqrt{2}\left((-1)^{3/4}+\sqrt{c} x\right)}{1+(-1)^{3/4} \sqrt{c} x}\right]}{3 c^{3/2}} + \\ & - \frac{(-1)^{3/4} b^2 \operatorname{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[\frac{(1+i)\left(1+(-1)^{1/4} \sqrt{c} x\right)}{1+(-1)^{3/4} \sqrt{c} x}\right]}{3 c^{3/2}} \\ & - \frac{(-1)^{3/4} b^2 \operatorname{ArcTan}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[\frac{(1-i)\left(1+(-1)^{3/4} \sqrt{c} x\right)}{1+(-1)^{1/4} \sqrt{c} x}\right]}{3 c^{3/2}} - \frac{2 i b^2 x \operatorname{Log}\left[1-i c x^2\right]}{3 c} \\ & - \frac{1}{9} b^2 x^3 \operatorname{Log}\left[1-i c x^2\right] - \frac{(-1)^{3/4} b^2 \operatorname{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[1-i c x^2\right]}{3 c^{3/2}} \end{aligned}$$

$$\begin{aligned} & \frac{1}{9} i b x^3 (2 a + i b \operatorname{Log}[1 - i c x^2]) - \frac{(-1)^{1/4} b \operatorname{ArcTan}\left[(-1)^{3/4} \sqrt{c} x\right] (2 a + i b \operatorname{Log}[1 - i c x^2])}{3 c^{3/2}} + \\ & \frac{1}{12} x^3 (2 a + i b \operatorname{Log}[1 - i c x^2])^2 + \frac{2 i b^2 x \operatorname{Log}[1 + i c x^2]}{3 c} - \\ & \frac{1}{3} i a b x^3 \operatorname{Log}[1 + i c x^2] + \frac{(-1)^{3/4} b^2 \operatorname{ArcTan}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}[1 + i c x^2]}{3 c^{3/2}} + \\ & \frac{(-1)^{3/4} b^2 \operatorname{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}[1 + i c x^2]}{3 c^{3/2}} + \\ & \frac{1}{6} b^2 x^3 \operatorname{Log}[1 - i c x^2] \operatorname{Log}[1 + i c x^2] - \frac{1}{12} b^2 x^3 \operatorname{Log}[1 + i c x^2]^2 + \\ & \frac{(-1)^{1/4} b^2 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - (-1)^{1/4} \sqrt{c} x}\right]}{3 c^{3/2}} + \frac{(-1)^{1/4} b^2 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 + (-1)^{1/4} \sqrt{c} x}\right]}{3 c^{3/2}} - \\ & \frac{(-1)^{1/4} b^2 \operatorname{PolyLog}\left[2, 1 - \frac{\sqrt{2} \left((-1)^{1/4} + \sqrt{c} x\right)}{1 + (-1)^{1/4} \sqrt{c} x}\right]}{6 c^{3/2}} + \frac{(-1)^{3/4} b^2 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - (-1)^{3/4} \sqrt{c} x}\right]}{3 c^{3/2}} + \\ & \frac{(-1)^{3/4} b^2 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 + (-1)^{3/4} \sqrt{c} x}\right]}{3 c^{3/2}} - \frac{(-1)^{3/4} b^2 \operatorname{PolyLog}\left[2, 1 + \frac{\sqrt{2} \left((-1)^{3/4} + \sqrt{c} x\right)}{1 + (-1)^{3/4} \sqrt{c} x}\right]}{6 c^{3/2}} - \\ & \frac{(-1)^{3/4} b^2 \operatorname{PolyLog}\left[2, 1 - \frac{(1+i) \left(1 + (-1)^{1/4} \sqrt{c} x\right)}{1 + (-1)^{3/4} \sqrt{c} x}\right]}{6 c^{3/2}} - \frac{(-1)^{1/4} b^2 \operatorname{PolyLog}\left[2, 1 - \frac{(1-i) \left(1 + (-1)^{3/4} \sqrt{c} x\right)}{1 + (-1)^{1/4} \sqrt{c} x}\right]}{6 c^{3/2}} \end{aligned}$$

Result (type 8, 18 leaves):

$$\int x^2 (a + b \operatorname{ArcTan}[c x^2])^2 dx$$

**Problem 82: Result more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{ArcTan}[c x^2])^2 dx$$

Optimal (type 4, 1191 leaves, 69 steps):

$$\begin{aligned} & a^2 x - \frac{2 (-1)^{3/4} a b \operatorname{ArcTan}\left[(-1)^{3/4} \sqrt{c} x\right]}{\sqrt{c}} + \frac{(-1)^{3/4} b^2 \operatorname{ArcTan}\left[(-1)^{3/4} \sqrt{c} x\right]^2}{\sqrt{c}} + \\ & \frac{2 (-1)^{3/4} a b \operatorname{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right]}{\sqrt{c}} - \frac{(-1)^{1/4} b^2 \operatorname{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right]^2}{\sqrt{c}} + \\ & \frac{2 (-1)^{1/4} b^2 \operatorname{ArcTan}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[\frac{2}{1 - (-1)^{1/4} \sqrt{c} x}\right]}{\sqrt{c}} - \\ & \frac{2 (-1)^{1/4} b^2 \operatorname{ArcTan}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[\frac{2}{1 + (-1)^{1/4} \sqrt{c} x}\right]}{\sqrt{c}} + \end{aligned}$$

$$\begin{aligned}
 & \frac{(-1)^{1/4} b^2 \operatorname{ArcTan}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[\frac{\sqrt{2}\left((-1)^{1/4}+\sqrt{c} x\right)}{1+(-1)^{1/4} \sqrt{c} x}\right]}{\sqrt{c}} + \\
 & \frac{2(-1)^{1/4} b^2 \operatorname{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[\frac{2}{1-(-1)^{3/4} \sqrt{c} x}\right]}{\sqrt{c}} - \\
 & \frac{2(-1)^{1/4} b^2 \operatorname{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[\frac{2}{1+(-1)^{3/4} \sqrt{c} x}\right]}{\sqrt{c}} + \\
 & \frac{(-1)^{1/4} b^2 \operatorname{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[-\frac{\sqrt{2}\left((-1)^{3/4}+\sqrt{c} x\right)}{1+(-1)^{3/4} \sqrt{c} x}\right]}{\sqrt{c}} + \\
 & \frac{(-1)^{1/4} b^2 \operatorname{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[\frac{(1+i)\left(1+(-1)^{1/4} \sqrt{c} x\right)}{1+(-1)^{3/4} \sqrt{c} x}\right]}{\sqrt{c}} + \\
 & \frac{(-1)^{1/4} b^2 \operatorname{ArcTan}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[\frac{(1-i)\left(1+(-1)^{3/4} \sqrt{c} x\right)}{1+(-1)^{1/4} \sqrt{c} x}\right]}{\sqrt{c}} + \\
 & i a b x \operatorname{Log}\left[1-i c x^2\right] + \frac{(-1)^{1/4} b^2 \operatorname{ArcTan}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[1-i c x^2\right]}{\sqrt{c}} - \\
 & \frac{(-1)^{1/4} b^2 \operatorname{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[1-i c x^2\right]}{\sqrt{c}} - \frac{1}{4} b^2 x \operatorname{Log}\left[1-i c x^2\right]^2 - \\
 & i a b x \operatorname{Log}\left[1+i c x^2\right] - \frac{(-1)^{1/4} b^2 \operatorname{ArcTan}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[1+i c x^2\right]}{\sqrt{c}} + \\
 & \frac{(-1)^{1/4} b^2 \operatorname{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[1+i c x^2\right]}{\sqrt{c}} + \\
 & \frac{1}{2} b^2 x \operatorname{Log}\left[1-i c x^2\right] \operatorname{Log}\left[1+i c x^2\right] - \frac{1}{4} b^2 x \operatorname{Log}\left[1+i c x^2\right]^2 + \\
 & \frac{(-1)^{3/4} b^2 \operatorname{PolyLog}\left[2, 1-\frac{2}{1-(-1)^{1/4} \sqrt{c} x}\right]}{\sqrt{c}} + \frac{(-1)^{3/4} b^2 \operatorname{PolyLog}\left[2, 1-\frac{2}{1+(-1)^{1/4} \sqrt{c} x}\right]}{\sqrt{c}} - \\
 & \frac{(-1)^{3/4} b^2 \operatorname{PolyLog}\left[2, 1-\frac{\sqrt{2}\left((-1)^{1/4}+\sqrt{c} x\right)}{1+(-1)^{1/4} \sqrt{c} x}\right]}{2 \sqrt{c}} + \frac{(-1)^{1/4} b^2 \operatorname{PolyLog}\left[2, 1-\frac{2}{1-(-1)^{3/4} \sqrt{c} x}\right]}{\sqrt{c}} + \\
 & \frac{(-1)^{1/4} b^2 \operatorname{PolyLog}\left[2, 1-\frac{2}{1+(-1)^{3/4} \sqrt{c} x}\right]}{\sqrt{c}} - \frac{(-1)^{1/4} b^2 \operatorname{PolyLog}\left[2, 1+\frac{\sqrt{2}\left((-1)^{3/4}+\sqrt{c} x\right)}{1+(-1)^{3/4} \sqrt{c} x}\right]}{2 \sqrt{c}} - \\
 & \frac{(-1)^{1/4} b^2 \operatorname{PolyLog}\left[2, 1-\frac{(1+i)\left(1+(-1)^{1/4} \sqrt{c} x\right)}{1+(-1)^{3/4} \sqrt{c} x}\right]}{2 \sqrt{c}} - \frac{(-1)^{3/4} b^2 \operatorname{PolyLog}\left[2, 1-\frac{(1-i)\left(1+(-1)^{3/4} \sqrt{c} x\right)}{1+(-1)^{1/4} \sqrt{c} x}\right]}{2 \sqrt{c}}
 \end{aligned}$$

Result (type 4, 5620 leaves):

$$\begin{aligned}
 & a^2 x + \frac{1}{c x} \\
 & a b \sqrt{c x^2} \left( 2 \sqrt{c x^2} \operatorname{ArcTan}[c x^2] - \frac{1}{\sqrt{2}} \left( -2 \operatorname{ArcTan}[1 - \sqrt{2} \sqrt{c x^2}] + 2 \operatorname{ArcTan}[1 + \sqrt{2} \sqrt{c x^2}] + \right. \right. \\
 & \quad \left. \left. \operatorname{Log}[1 + c x^2 - \sqrt{2} \sqrt{c x^2}] - \operatorname{Log}[1 + c x^2 + \sqrt{2} \sqrt{c x^2}] \right) \right) + \\
 & \frac{1}{2 c x} b^2 \sqrt{c x^2} \left( 2 \sqrt{c x^2} \operatorname{ArcTan}[c x^2]^2 - 4 \left( \frac{1}{2 \sqrt{2}} \operatorname{ArcTan}[c x^2] \left( -2 \operatorname{ArcTan}[1 - \sqrt{2} \sqrt{c x^2}] + \right. \right. \right. \\
 & \quad \left. \left. 2 \operatorname{ArcTan}[1 + \sqrt{2} \sqrt{c x^2}] + \operatorname{Log}[1 + c x^2 - \sqrt{2} \sqrt{c x^2}] - \operatorname{Log}[1 + c x^2 + \sqrt{2} \sqrt{c x^2}] \right) - \right. \\
 & \quad \left. \frac{1}{2 \sqrt{2}} \left( - \left( \operatorname{ArcTan}[1 - \sqrt{2} \sqrt{c x^2}] + \operatorname{ArcTan}[1 + \sqrt{2} \sqrt{c x^2}] \right) \operatorname{Log}[1 + c x^2 - \sqrt{2} \sqrt{c x^2}] + \right. \right. \\
 & \quad \left. \left( \operatorname{ArcTan}[1 - \sqrt{2} \sqrt{c x^2}] + \operatorname{ArcTan}[1 + \sqrt{2} \sqrt{c x^2}] \right) \operatorname{Log}[1 + c x^2 + \sqrt{2} \sqrt{c x^2}] - \right. \\
 & \quad \left. \left( \sqrt{c x^2} \left( 1 + \left( 1 - \sqrt{2} \sqrt{c x^2} \right)^2 \right)^{3/2} \right. \right. \\
 & \quad \left. \left( 2 \left( -5 \operatorname{ArcTan}[2 + i] \operatorname{ArcTan}[1 - \sqrt{2} \sqrt{c x^2}] + 4 \operatorname{ArcTan}[1 - \sqrt{2} \sqrt{c x^2}]^2 + \right. \right. \\
 & \quad \left. \left( 1 + 2 i \right) \sqrt{1 + i} e^{-i \operatorname{ArcTan}[2 + i]} \operatorname{ArcTan}[1 - \sqrt{2} \sqrt{c x^2}]^2 + \left( 1 - 2 i \right) \sqrt{1 - i} \right. \\
 & \quad \left. e^{-\operatorname{ArcTan}[1 + 2 i]} \operatorname{ArcTan}[1 - \sqrt{2} \sqrt{c x^2}]^2 - 5 i \operatorname{ArcTan}[1 - \sqrt{2} \sqrt{c x^2}] \right. \\
 & \quad \left. \operatorname{ArcTanh}[1 + 2 i] + 5 i \left( -\operatorname{ArcTan}[2 + i] + \operatorname{ArcTan}[1 - \sqrt{2} \sqrt{c x^2}] \right) \right. \\
 & \quad \left. \operatorname{Log}\left[1 - e^{2 i \left( -\operatorname{ArcTan}[2 + i] + \operatorname{ArcTan}[1 - \sqrt{2} \sqrt{c x^2}] \right)} \right] + 5 \left( -i \operatorname{ArcTan}[1 - \sqrt{2} \sqrt{c x^2}] + \right. \right. \\
 & \quad \left. \left. \operatorname{ArcTanh}[1 + 2 i] \right) \operatorname{Log}\left[1 - e^{2 i \operatorname{ArcTan}[1 - \sqrt{2} \sqrt{c x^2}] - 2 \operatorname{ArcTanh}[1 + 2 i]} \right] + \right. \\
 & \quad \left. 5 i \operatorname{ArcTan}[2 + i] \operatorname{Log}\left[-\operatorname{Sin}\left[\operatorname{ArcTan}[2 + i] - \operatorname{ArcTan}[1 - \sqrt{2} \sqrt{c x^2}]\right]\right] - \right. \\
 & \quad \left. 5 \operatorname{ArcTanh}[1 + 2 i] \operatorname{Log}\left[\operatorname{Sin}\left[\operatorname{ArcTan}[1 - \sqrt{2} \sqrt{c x^2}] + i \operatorname{ArcTanh}[1 + 2 i]\right]\right] \right) \right) + \\
 & 5 \operatorname{PolyLog}\left[2, e^{2 i \left( -\operatorname{ArcTan}[2 + i] + \operatorname{ArcTan}[1 - \sqrt{2} \sqrt{c x^2}] \right)} \right] - 5 \operatorname{PolyLog}\left[2, \right. \\
 & \quad \left. e^{2 i \operatorname{ArcTan}[1 - \sqrt{2} \sqrt{c x^2}] - 2 \operatorname{ArcTanh}[1 + 2 i]} \right] \left( 3 + 2 \operatorname{Cos}\left[2 \operatorname{ArcTan}[1 - \sqrt{2} \sqrt{c x^2}]\right] \right) -
 \end{aligned}$$

$$\begin{aligned}
 & \left. 2 \operatorname{Sin}\left[2 \operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{cx^2}\right]\right]\right) \Bigg/ \left(20 \sqrt{2} \left(-1 - cx^2 + \sqrt{2} \sqrt{cx^2}\right)\right) \\
 & \left(1 + cx^2 + \sqrt{2} \sqrt{cx^2}\right) \left(\frac{1}{\sqrt{1 + \left(1 - \sqrt{2} \sqrt{cx^2}\right)^2}} - \frac{1 - \sqrt{2} \sqrt{cx^2}}{\sqrt{1 + \left(1 - \sqrt{2} \sqrt{cx^2}\right)^2}}\right) + \\
 & \frac{1}{1 + cx^2 + \sqrt{2} \sqrt{cx^2}} \left(\frac{1}{20} + \frac{i}{20}\right) e^{-i \operatorname{ArcTan}[2+i] - \operatorname{ArcTanh}[1+2i]} \left(-1 - cx^2 + \sqrt{2} \sqrt{cx^2}\right) \\
 & \left( (5 + 5i) e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2i]} \pi \operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{cx^2}\right] + 10i \right. \\
 & e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2i]} \operatorname{ArcTan}[2+i] \operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{cx^2}\right] + (2 - 4i) \sqrt{1-i} \\
 & e^{i \operatorname{ArcTan}[2+i]} \operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{cx^2}\right]^2 + (4 - 2i) \sqrt{1+i} e^{\operatorname{ArcTanh}[1+2i]} \operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{cx^2}\right]^2 - \\
 & (8 - 8i) e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2i]} \operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{cx^2}\right]^2 - \\
 & 10i e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2i]} \operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{cx^2}\right] \operatorname{ArcTanh}[1+2i] + \\
 & (5 - 5i) e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2i]} \pi \operatorname{Log}\left[1 + e^{-2i \operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{cx^2}\right]}\right] - 10 \\
 & e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2i]} \operatorname{ArcTan}[2+i] \operatorname{Log}\left[1 - e^{2i \left(-\operatorname{ArcTan}[2+i] + \operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{cx^2}\right]\right)}\right] + \\
 & 10 e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2i]} \operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{cx^2}\right] \\
 & \operatorname{Log}\left[1 - e^{2i \left(-\operatorname{ArcTan}[2+i] + \operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{cx^2}\right]\right)}\right] - 10i e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2i]} \\
 & \operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{cx^2}\right] \operatorname{Log}\left[1 - e^{2i \operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{cx^2}\right] - 2 \operatorname{ArcTanh}[1+2i]}\right] + 10 \\
 & e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2i]} \operatorname{ArcTanh}[1+2i] \\
 & \operatorname{Log}\left[1 - e^{2i \operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{cx^2}\right] - 2 \operatorname{ArcTanh}[1+2i]}\right] - (5 - 5i) e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2i]} \\
 & \pi \operatorname{Log}\left[\frac{1}{\sqrt{1 + \left(1 - \sqrt{2} \sqrt{cx^2}\right)^2}}\right] + 10 e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2i]} \operatorname{ArcTan}[2+i] \\
 & \operatorname{Log}\left[-\operatorname{Sin}\left[\operatorname{ArcTan}[2+i] - \operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{cx^2}\right]\right]\right] - 10 e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2i]} \\
 & \operatorname{ArcTanh}[1+2i] \operatorname{Log}\left[\operatorname{Sin}\left[\operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{cx^2}\right] + i \operatorname{ArcTanh}[1+2i]\right]\right] - 5 \\
 & i e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2i]} \operatorname{PolyLog}\left[2, e^{2i \left(-\operatorname{ArcTan}[2+i] + \operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{cx^2}\right]\right)}\right] - 5
 \end{aligned}$$

$$\left. e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2i]} \operatorname{PolyLog}\left[2, e^{2i \operatorname{ArcTan}\left[1-\sqrt{2}\sqrt{cx^2}\right] - 2\operatorname{ArcTanh}[1+2i]}\right]\right)$$

$$\left(3 + 2 \operatorname{Cos}\left[2 \operatorname{ArcTan}\left[1-\sqrt{2}\sqrt{cx^2}\right]\right] - 2 \operatorname{Sin}\left[2 \operatorname{ArcTan}\left[1-\sqrt{2}\sqrt{cx^2}\right]\right]\right) +$$

$$\left(\frac{1}{40} + \frac{i}{40}\right) c e^{-i \operatorname{ArcTan}[2+i] - \operatorname{ArcTanh}[1+2i]} x^2 \left(1 + \left(1 - \sqrt{2}\sqrt{cx^2}\right)^2\right)$$

$$\left(5 + 5i\right) e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2i]} \pi \operatorname{ArcTan}\left[1 - \sqrt{2}\sqrt{cx^2}\right] +$$

$$10 e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2i]} \operatorname{ArcTan}[2+i] \operatorname{ArcTan}\left[1 - \sqrt{2}\sqrt{cx^2}\right] +$$

$$(4 + 2i) \sqrt{1-i} e^{i \operatorname{ArcTan}[2+i]} \operatorname{ArcTan}\left[1 - \sqrt{2}\sqrt{cx^2}\right]^2 -$$

$$(2 + 4i) \sqrt{1+i} e^{\operatorname{ArcTanh}[1+2i]} \operatorname{ArcTan}\left[1 - \sqrt{2}\sqrt{cx^2}\right]^2 + (4 - 4i)$$

$$e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2i]} \operatorname{ArcTan}\left[1 - \sqrt{2}\sqrt{cx^2}\right]^2 + 10 e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2i]}$$

$$\operatorname{ArcTan}\left[1 - \sqrt{2}\sqrt{cx^2}\right] \operatorname{ArcTanh}[1+2i] + (5 - 5i) e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2i]}$$

$$\pi \operatorname{Log}\left[1 + e^{-2i \operatorname{ArcTan}\left[1-\sqrt{2}\sqrt{cx^2}\right]}\right] + 10i e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2i]} \operatorname{ArcTan}[2+i]$$

$$\operatorname{Log}\left[1 - e^{2i\left(-\operatorname{ArcTan}[2+i] + \operatorname{ArcTan}\left[1-\sqrt{2}\sqrt{cx^2}\right]\right)}\right] - 10i e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2i]}$$

$$\operatorname{ArcTan}\left[1 - \sqrt{2}\sqrt{cx^2}\right] \operatorname{Log}\left[1 - e^{2i\left(-\operatorname{ArcTan}[2+i] + \operatorname{ArcTan}\left[1-\sqrt{2}\sqrt{cx^2}\right]\right)}\right] +$$

$$10 e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2i]} \operatorname{ArcTan}\left[1 - \sqrt{2}\sqrt{cx^2}\right]$$

$$\operatorname{Log}\left[1 - e^{2i \operatorname{ArcTan}\left[1-\sqrt{2}\sqrt{cx^2}\right] - 2\operatorname{ArcTanh}[1+2i]}\right] + 10i e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2i]}$$

$$\operatorname{ArcTanh}[1+2i] \operatorname{Log}\left[1 - e^{2i \operatorname{ArcTan}\left[1-\sqrt{2}\sqrt{cx^2}\right] - 2\operatorname{ArcTanh}[1+2i]}\right] -$$

$$(5 - 5i) e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2i]} \pi \operatorname{Log}\left[\frac{1}{\sqrt{1 + \left(1 - \sqrt{2}\sqrt{cx^2}\right)^2}}\right] -$$

$$10i e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2i]} \operatorname{ArcTan}[2+i] \operatorname{Log}\left[$$

$$-\operatorname{Sin}\left[\operatorname{ArcTan}[2+i] - \operatorname{ArcTan}\left[1 - \sqrt{2}\sqrt{cx^2}\right]\right]\right] - 10i e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2i]}$$

$$\operatorname{ArcTanh}[1+2i] \operatorname{Log}\left[\operatorname{Sin}\left[\operatorname{ArcTan}\left[1 - \sqrt{2}\sqrt{cx^2}\right] + i \operatorname{ArcTanh}[1+2i]\right]\right] -$$

$$5 e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2i]} \operatorname{PolyLog}\left[2, e^{2i\left(-\operatorname{ArcTan}[2+i] + \operatorname{ArcTan}\left[1-\sqrt{2}\sqrt{cx^2}\right]\right)}\right] -$$

$$\left. \begin{aligned}
 & 5i e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2i]} \operatorname{PolyLog}\left[2, e^{2i \operatorname{ArcTan}\left[1-\sqrt{2}\sqrt{cx^2}\right] - 2\operatorname{ArcTanh}[1+2i]}\right] \\
 & \left(3 + 2 \operatorname{Cos}\left[2 \operatorname{ArcTan}\left[1-\sqrt{2}\sqrt{cx^2}\right]\right] - 2 \operatorname{Sin}\left[2 \operatorname{ArcTan}\left[1-\sqrt{2}\sqrt{cx^2}\right]\right]\right) / \\
 & \left( \left(-1 - cx^2 + \sqrt{2}\sqrt{cx^2}\right) \left(1 + cx^2 + \sqrt{2}\sqrt{cx^2}\right) \left( \frac{1}{\sqrt{1 + \left(1 - \sqrt{2}\sqrt{cx^2}\right)^2}} - \right. \right. \\
 & \left. \left. \frac{1 - \sqrt{2}\sqrt{cx^2}}{\sqrt{1 + \left(1 - \sqrt{2}\sqrt{cx^2}\right)^2}} \right)^2 \right) - \left(\sqrt{cx^2} \left(1 + \left(1 + \sqrt{2}\sqrt{cx^2}\right)^2\right)^{3/2}\right) \\
 & \left(2 \left(-5 \operatorname{ArcTan}[2+i] \operatorname{ArcTan}\left[1 + \sqrt{2}\sqrt{cx^2}\right] + 4 \operatorname{ArcTan}\left[1 + \sqrt{2}\sqrt{cx^2}\right]^2 + \right. \right. \\
 & \left. \left(1 + 2i\right) \sqrt{1+i} e^{-i \operatorname{ArcTan}[2+i]} \operatorname{ArcTan}\left[1 + \sqrt{2}\sqrt{cx^2}\right]^2 + \left(1 - 2i\right) \sqrt{1-i} e^{-\operatorname{ArcTanh}[1+2i]} \operatorname{ArcTan}\left[1 + \sqrt{2}\sqrt{cx^2}\right]^2 - 5i \operatorname{ArcTan}\left[1 + \sqrt{2}\sqrt{cx^2}\right] \right. \\
 & \left. \operatorname{ArcTanh}[1+2i] + 5i \left(-\operatorname{ArcTan}[2+i] + \operatorname{ArcTan}\left[1 + \sqrt{2}\sqrt{cx^2}\right]\right) \right. \\
 & \left. \operatorname{Log}\left[1 - e^{2i \left(-\operatorname{ArcTan}[2+i] + \operatorname{ArcTan}\left[1 + \sqrt{2}\sqrt{cx^2}\right]\right)}\right] + 5 \left(-i \operatorname{ArcTan}\left[1 + \sqrt{2}\sqrt{cx^2}\right] + \right. \right. \\
 & \left. \left. \operatorname{ArcTanh}[1+2i]\right) \operatorname{Log}\left[1 - e^{2i \operatorname{ArcTan}\left[1 + \sqrt{2}\sqrt{cx^2}\right] - 2\operatorname{ArcTanh}[1+2i]}\right] + \right. \\
 & \left. 5i \operatorname{ArcTan}[2+i] \operatorname{Log}\left[-\operatorname{Sin}\left[\operatorname{ArcTan}[2+i] - \operatorname{ArcTan}\left[1 + \sqrt{2}\sqrt{cx^2}\right]\right]\right] - \right. \\
 & \left. 5 \operatorname{ArcTanh}[1+2i] \operatorname{Log}\left[\operatorname{Sin}\left[\operatorname{ArcTan}\left[1 + \sqrt{2}\sqrt{cx^2}\right] + i \operatorname{ArcTanh}[1+2i]\right]\right] \right) + \\
 & 5 \operatorname{PolyLog}\left[2, e^{2i \left(-\operatorname{ArcTan}[2+i] + \operatorname{ArcTan}\left[1 + \sqrt{2}\sqrt{cx^2}\right]\right)}\right] - \\
 & 5 \operatorname{PolyLog}\left[2, e^{2i \operatorname{ArcTan}\left[1 + \sqrt{2}\sqrt{cx^2}\right] - 2\operatorname{ArcTanh}[1+2i]}\right] \\
 & \left(3 + 2 \operatorname{Cos}\left[2 \operatorname{ArcTan}\left[1 + \sqrt{2}\sqrt{cx^2}\right]\right] - 2 \operatorname{Sin}\left[2 \operatorname{ArcTan}\left[1 + \sqrt{2}\sqrt{cx^2}\right]\right]\right) /
 \end{aligned} \right)$$

$$\left( 20 \sqrt{2} \left( -1 - c x^2 + \sqrt{2} \sqrt{c x^2} \right) \left( 1 + c x^2 + \sqrt{2} \sqrt{c x^2} \right) \right.$$

$$\left. \left( \frac{1}{\sqrt{1 + \left( 1 + \sqrt{2} \sqrt{c x^2} \right)^2}} - \frac{1 + \sqrt{2} \sqrt{c x^2}}{\sqrt{1 + \left( 1 + \sqrt{2} \sqrt{c x^2} \right)^2}} \right) \right) -$$

$$\frac{1}{-1 - c x^2 + \sqrt{2} \sqrt{c x^2}} \left( \frac{1}{20} + \frac{i}{20} \right) e^{-i \operatorname{ArcTan}[2+i] - \operatorname{ArcTanh}[1+2 i]} \left( 1 + c x^2 + \sqrt{2} \sqrt{c x^2} \right)$$

$$\left( (5 + 5 i) e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2 i]} \pi \operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{c x^2}\right] + 10 i \right.$$

$$e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2 i]} \operatorname{ArcTan}[2 + i] \operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{c x^2}\right] + (2 - 4 i)$$

$$\sqrt{1 - i} e^{i \operatorname{ArcTan}[2+i]} \operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{c x^2}\right]^2 + (4 - 2 i) \sqrt{1 + i}$$

$$e^{\operatorname{ArcTanh}[1+2 i]} \operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{c x^2}\right]^2 - (8 - 8 i) e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2 i]}$$

$$\operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{c x^2}\right]^2 - 10 i e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2 i]}$$

$$\operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{c x^2}\right] \operatorname{ArcTanh}[1 + 2 i] + (5 - 5 i) e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2 i]}$$

$$\pi \operatorname{Log}\left[1 + e^{-2 i \operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{c x^2}\right]}\right] - 10 e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2 i]} \operatorname{ArcTan}[2 + i]$$

$$\operatorname{Log}\left[1 - e^{2 i \left(-\operatorname{ArcTan}[2+i] + \operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{c x^2}\right]\right)}\right] + 10 e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2 i]}$$

$$\operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{c x^2}\right] \operatorname{Log}\left[1 - e^{2 i \left(-\operatorname{ArcTan}[2+i] + \operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{c x^2}\right]\right)}\right] - 10$$

$$i e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2 i]} \operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{c x^2}\right]$$

$$\operatorname{Log}\left[1 - e^{2 i \operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{c x^2}\right] - 2 \operatorname{ArcTanh}[1+2 i]}\right] + 10 e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2 i]}$$

$$\operatorname{ArcTanh}[1 + 2 i] \operatorname{Log}\left[1 - e^{2 i \operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{c x^2}\right] - 2 \operatorname{ArcTanh}[1+2 i]}\right] - (5 - 5 i)$$

$$e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2 i]} \pi \operatorname{Log}\left[\frac{1}{\sqrt{1 + \left( 1 + \sqrt{2} \sqrt{c x^2} \right)^2}}\right] + 10$$

$$e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2 i]} \operatorname{ArcTan}[2 + i]$$

$$\operatorname{Log}\left[-\operatorname{Sin}\left[\operatorname{ArcTan}[2 + i] - \operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{c x^2}\right]\right]\right] - 10 e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2 i]}$$

$$\operatorname{ArcTanh}[1 + 2 i] \operatorname{Log}\left[\operatorname{Sin}\left[\operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{c x^2}\right] + i \operatorname{ArcTanh}[1 + 2 i]\right]\right] - 5$$

$$i e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2 i]} \operatorname{PolyLog}\left[2, e^{2 i \left(-\operatorname{ArcTan}[2+i] + \operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{c x^2}\right]\right)}\right] - 5$$



$$\left. e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2i]} \operatorname{PolyLog}\left[2, e^{2i \operatorname{ArcTan}\left[1+\sqrt{2}\sqrt{cx^2}\right] - 2\operatorname{ArcTanh}[1+2i]}\right]\right)$$

$$\left(3 + 2 \operatorname{Cos}\left[2 \operatorname{ArcTan}\left[1 + \sqrt{2}\sqrt{cx^2}\right]\right] - 2 \operatorname{Sin}\left[2 \operatorname{ArcTan}\left[1 + \sqrt{2}\sqrt{cx^2}\right]\right]\right) -$$

$$\left(\frac{1}{40} + \frac{i}{40}\right) c e^{-i \operatorname{ArcTan}[2+i] - \operatorname{ArcTanh}[1+2i]} x^2 \left(1 + \left(1 + \sqrt{2}\sqrt{cx^2}\right)^2\right)$$

$$\left( (5 + 5i) e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2i]} \pi \operatorname{ArcTan}\left[1 + \sqrt{2}\sqrt{cx^2}\right] + \right.$$

$$10 e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2i]} \operatorname{ArcTan}[2+i] \operatorname{ArcTan}\left[1 + \sqrt{2}\sqrt{cx^2}\right] +$$

$$(4 + 2i) \sqrt{1-i} e^{i \operatorname{ArcTan}[2+i]} \operatorname{ArcTan}\left[1 + \sqrt{2}\sqrt{cx^2}\right]^2 -$$

$$(2 + 4i) \sqrt{1+i} e^{\operatorname{ArcTanh}[1+2i]} \operatorname{ArcTan}\left[1 + \sqrt{2}\sqrt{cx^2}\right]^2 +$$

$$(4 - 4i) e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2i]} \operatorname{ArcTan}\left[1 + \sqrt{2}\sqrt{cx^2}\right]^2 +$$

$$10 e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2i]} \operatorname{ArcTan}\left[1 + \sqrt{2}\sqrt{cx^2}\right] \operatorname{ArcTanh}[1+2i] +$$

$$(5 - 5i) e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2i]} \pi \operatorname{Log}\left[1 + e^{-2i \operatorname{ArcTan}\left[1 + \sqrt{2}\sqrt{cx^2}\right]}\right] +$$

$$10i e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2i]} \operatorname{ArcTan}[2+i]$$

$$\operatorname{Log}\left[1 - e^{2i \left(-\operatorname{ArcTan}[2+i] + \operatorname{ArcTan}\left[1 + \sqrt{2}\sqrt{cx^2}\right]\right)}\right] - 10i e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2i]}$$

$$\operatorname{ArcTan}\left[1 + \sqrt{2}\sqrt{cx^2}\right] \operatorname{Log}\left[1 - e^{2i \left(-\operatorname{ArcTan}[2+i] + \operatorname{ArcTan}\left[1 + \sqrt{2}\sqrt{cx^2}\right]\right)}\right] +$$

$$10 e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2i]} \operatorname{ArcTan}\left[1 + \sqrt{2}\sqrt{cx^2}\right]$$

$$\operatorname{Log}\left[1 - e^{2i \operatorname{ArcTan}\left[1 + \sqrt{2}\sqrt{cx^2}\right] - 2\operatorname{ArcTanh}[1+2i]}\right] + 10i e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2i]}$$

$$\operatorname{ArcTanh}[1+2i] \operatorname{Log}\left[1 - e^{2i \operatorname{ArcTan}\left[1 + \sqrt{2}\sqrt{cx^2}\right] - 2\operatorname{ArcTanh}[1+2i]}\right] -$$

$$(5 - 5i) e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2i]} \pi \operatorname{Log}\left[\frac{1}{\sqrt{1 + \left(1 + \sqrt{2}\sqrt{cx^2}\right)^2}}\right] -$$

$$10i e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2i]} \operatorname{ArcTan}[2+i] \operatorname{Log}\left[$$

$$-\operatorname{Sin}\left[\operatorname{ArcTan}[2+i] - \operatorname{ArcTan}\left[1 + \sqrt{2}\sqrt{cx^2}\right]\right]\right] - 10i e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2i]}$$

$$\operatorname{ArcTanh}[1+2i] \operatorname{Log}\left[\operatorname{Sin}\left[\operatorname{ArcTan}\left[1 + \sqrt{2}\sqrt{cx^2}\right] + i \operatorname{ArcTanh}[1+2i]\right]\right] -$$

$$5 e^{i \operatorname{ArcTan}[2+i] + \operatorname{ArcTanh}[1+2i]} \operatorname{PolyLog}\left[2, e^{2i \left(-\operatorname{ArcTan}[2+i] + \operatorname{ArcTan}\left[1 + \sqrt{2}\sqrt{cx^2}\right]\right)}\right] -$$

$$\left. \begin{aligned}
 & 5 i e^{i \operatorname{ArcTan}[2+i]+\operatorname{ArcTanh}[1+2 i]} \operatorname{PolyLog}\left[2, e^{2 i \operatorname{ArcTan}\left[1+\sqrt{2} \sqrt{c x^2}\right]-2 \operatorname{ArcTanh}[1+2 i]}\right] \\
 & \left(3+2 \operatorname{Cos}\left[2 \operatorname{ArcTan}\left[1+\sqrt{2} \sqrt{c x^2}\right]\right]-2 \operatorname{Sin}\left[2 \operatorname{ArcTan}\left[1+\sqrt{2} \sqrt{c x^2}\right]\right]\right) / \\
 & \left(\left(-1-c x^2+\sqrt{2} \sqrt{c x^2}\right)\left(1+c x^2+\sqrt{2} \sqrt{c x^2}\right)\right. \\
 & \left.\left(\frac{1}{\sqrt{1+\left(1+\sqrt{2} \sqrt{c x^2}\right)^2}}-\frac{1+\sqrt{2} \sqrt{c x^2}}{\sqrt{1+\left(1+\sqrt{2} \sqrt{c x^2}\right)^2}}\right)\right) \right)
 \end{aligned}
 \right)$$

**Problem 83: Attempted integration timed out after 120 seconds.**

$$\int \frac{(a+b \operatorname{ArcTan}[c x^2])^2}{x^2} dx$$

Optimal (type 4, 1164 leaves, 47 steps):

$$\begin{aligned}
 & (-1)^{1/4} b^2 \sqrt{c} \operatorname{ArcTan}\left[(-1)^{3/4} \sqrt{c} x\right]^2 - \\
 & 2(-1)^{1/4} a b \sqrt{c} \operatorname{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right] - (-1)^{3/4} b^2 \sqrt{c} \operatorname{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right]^2 - \\
 & 2(-1)^{3/4} b^2 \sqrt{c} \operatorname{ArcTan}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[\frac{2}{1-(-1)^{1/4} \sqrt{c} x}\right] + \\
 & 2(-1)^{3/4} b^2 \sqrt{c} \operatorname{ArcTan}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[\frac{2}{1+(-1)^{1/4} \sqrt{c} x}\right] - \\
 & (-1)^{3/4} b^2 \sqrt{c} \operatorname{ArcTan}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[\frac{\sqrt{2}\left((-1)^{1/4}+\sqrt{c} x\right)}{1+(-1)^{1/4} \sqrt{c} x}\right] + \\
 & 2(-1)^{3/4} b^2 \sqrt{c} \operatorname{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[\frac{2}{1-(-1)^{3/4} \sqrt{c} x}\right] - \\
 & 2(-1)^{3/4} b^2 \sqrt{c} \operatorname{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[\frac{2}{1+(-1)^{3/4} \sqrt{c} x}\right] + \\
 & (-1)^{3/4} b^2 \sqrt{c} \operatorname{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[-\frac{\sqrt{2}\left((-1)^{3/4}+\sqrt{c} x\right)}{1+(-1)^{3/4} \sqrt{c} x}\right] +
 \end{aligned}$$

$$\begin{aligned}
 & (-1)^{3/4} b^2 \sqrt{c} \operatorname{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[\frac{(1+i)\left(1+(-1)^{1/4} \sqrt{c} x\right)}{1+(-1)^{3/4} \sqrt{c} x}\right] - \\
 & (-1)^{3/4} b^2 \sqrt{c} \operatorname{ArcTan}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[\frac{(1-i)\left(1+(-1)^{3/4} \sqrt{c} x\right)}{1+(-1)^{1/4} \sqrt{c} x}\right] - \\
 & (-1)^{3/4} b^2 \sqrt{c} \operatorname{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[1-icx^2\right] - \\
 & (-1)^{1/4} b \sqrt{c} \operatorname{ArcTan}\left[(-1)^{3/4} \sqrt{c} x\right] \left(2a+ib \operatorname{Log}\left[1-icx^2\right]\right) - \frac{(2a+ib \operatorname{Log}\left[1-icx^2\right])^2}{4x} + \\
 & \frac{iab \operatorname{Log}\left[1+icx^2\right]}{x} + (-1)^{3/4} b^2 \sqrt{c} \operatorname{ArcTan}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[1+icx^2\right] + \\
 & (-1)^{3/4} b^2 \sqrt{c} \operatorname{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[1+icx^2\right] - \frac{b^2 \operatorname{Log}\left[1-icx^2\right] \operatorname{Log}\left[1+icx^2\right]}{2x} + \\
 & \frac{b^2 \operatorname{Log}\left[1+icx^2\right]^2}{4x} + (-1)^{1/4} b^2 \sqrt{c} \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+(-1)^{1/4} \sqrt{c} x}\right] + \\
 & (-1)^{1/4} b^2 \sqrt{c} \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+(-1)^{1/4} \sqrt{c} x}\right] - \\
 & \frac{1}{2} (-1)^{1/4} b^2 \sqrt{c} \operatorname{PolyLog}\left[2, 1 - \frac{\sqrt{2}\left((-1)^{1/4} + \sqrt{c} x\right)}{1+(-1)^{1/4} \sqrt{c} x}\right] + \\
 & (-1)^{3/4} b^2 \sqrt{c} \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+(-1)^{3/4} \sqrt{c} x}\right] + \\
 & (-1)^{3/4} b^2 \sqrt{c} \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+(-1)^{3/4} \sqrt{c} x}\right] - \\
 & \frac{1}{2} (-1)^{3/4} b^2 \sqrt{c} \operatorname{PolyLog}\left[2, 1 + \frac{\sqrt{2}\left((-1)^{3/4} + \sqrt{c} x\right)}{1+(-1)^{3/4} \sqrt{c} x}\right] - \\
 & \frac{1}{2} (-1)^{3/4} b^2 \sqrt{c} \operatorname{PolyLog}\left[2, 1 - \frac{(1+i)\left(1+(-1)^{1/4} \sqrt{c} x\right)}{1+(-1)^{3/4} \sqrt{c} x}\right] - \\
 & \frac{1}{2} (-1)^{1/4} b^2 \sqrt{c} \operatorname{PolyLog}\left[2, 1 - \frac{(1-i)\left(1+(-1)^{3/4} \sqrt{c} x\right)}{1+(-1)^{1/4} \sqrt{c} x}\right]
 \end{aligned}$$

Result(type 1, 1 leaves):

???

**Problem 84: Unable to integrate problem.**

$$\int \frac{(a+b \operatorname{ArcTan}\left[cx^2\right])^2}{x^4} dx$$

Optimal (type 4, 1360 leaves, 64 steps):

$$\begin{aligned}
& -\frac{2abc}{3x} - \frac{4}{3} (-1)^{1/4} b^2 c^{3/2} \text{ArcTan}\left[(-1)^{3/4} \sqrt{c} x\right] + \\
& \frac{1}{3} (-1)^{3/4} b^2 c^{3/2} \text{ArcTan}\left[(-1)^{3/4} \sqrt{c} x\right]^2 + \frac{2}{3} (-1)^{3/4} abc^{3/2} \text{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right] - \\
& \frac{4}{3} (-1)^{1/4} b^2 c^{3/2} \text{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right] - \frac{1}{3} (-1)^{1/4} b^2 c^{3/2} \text{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right]^2 + \\
& \frac{2}{3} (-1)^{1/4} b^2 c^{3/2} \text{ArcTan}\left[(-1)^{3/4} \sqrt{c} x\right] \text{Log}\left[\frac{2}{1 - (-1)^{1/4} \sqrt{c} x}\right] - \\
& \frac{2}{3} (-1)^{1/4} b^2 c^{3/2} \text{ArcTan}\left[(-1)^{3/4} \sqrt{c} x\right] \text{Log}\left[\frac{2}{1 + (-1)^{1/4} \sqrt{c} x}\right] + \\
& \frac{1}{3} (-1)^{1/4} b^2 c^{3/2} \text{ArcTan}\left[(-1)^{3/4} \sqrt{c} x\right] \text{Log}\left[\frac{\sqrt{2} \left((-1)^{1/4} + \sqrt{c} x\right)}{1 + (-1)^{1/4} \sqrt{c} x}\right] + \\
& \frac{2}{3} (-1)^{1/4} b^2 c^{3/2} \text{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right] \text{Log}\left[\frac{2}{1 - (-1)^{3/4} \sqrt{c} x}\right] - \\
& \frac{2}{3} (-1)^{1/4} b^2 c^{3/2} \text{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right] \text{Log}\left[\frac{2}{1 + (-1)^{3/4} \sqrt{c} x}\right] + \\
& \frac{1}{3} (-1)^{1/4} b^2 c^{3/2} \text{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right] \text{Log}\left[-\frac{\sqrt{2} \left((-1)^{3/4} + \sqrt{c} x\right)}{1 + (-1)^{3/4} \sqrt{c} x}\right] + \\
& \frac{1}{3} (-1)^{1/4} b^2 c^{3/2} \text{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right] \text{Log}\left[\frac{(1+i) \left(1 + (-1)^{1/4} \sqrt{c} x\right)}{1 + (-1)^{3/4} \sqrt{c} x}\right] + \\
& \frac{1}{3} (-1)^{1/4} b^2 c^{3/2} \text{ArcTan}\left[(-1)^{3/4} \sqrt{c} x\right] \text{Log}\left[\frac{(1-i) \left(1 + (-1)^{3/4} \sqrt{c} x\right)}{1 + (-1)^{1/4} \sqrt{c} x}\right] - \\
& \frac{i b^2 c \text{Log}\left[1 - i c x^2\right]}{3x} - \frac{1}{3} (-1)^{1/4} b^2 c^{3/2} \text{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right] \text{Log}\left[1 - i c x^2\right] - \\
& \frac{bc \left(2a + i b \text{Log}\left[1 - i c x^2\right]\right)}{3x} - \frac{1}{3} (-1)^{3/4} b c^{3/2} \text{ArcTan}\left[(-1)^{3/4} \sqrt{c} x\right] \left(2a + i b \text{Log}\left[1 - i c x^2\right]\right) - \\
& \frac{\left(2a + i b \text{Log}\left[1 - i c x^2\right]\right)^2}{12x^3} + \frac{i a b \text{Log}\left[1 + i c x^2\right]}{3x^3} + \frac{2 i b^2 c \text{Log}\left[1 + i c x^2\right]}{3x} - \\
& \frac{1}{3} (-1)^{1/4} b^2 c^{3/2} \text{ArcTan}\left[(-1)^{3/4} \sqrt{c} x\right] \text{Log}\left[1 + i c x^2\right] + \\
& \frac{1}{3} (-1)^{1/4} b^2 c^{3/2} \text{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right] \text{Log}\left[1 + i c x^2\right] - \frac{b^2 \text{Log}\left[1 - i c x^2\right] \text{Log}\left[1 + i c x^2\right]}{6x^3} + \\
& \frac{b^2 \text{Log}\left[1 + i c x^2\right]^2}{12x^3} + \frac{1}{3} (-1)^{3/4} b^2 c^{3/2} \text{PolyLog}\left[2, 1 - \frac{2}{1 - (-1)^{1/4} \sqrt{c} x}\right] + \\
& \frac{1}{3} (-1)^{3/4} b^2 c^{3/2} \text{PolyLog}\left[2, 1 - \frac{2}{1 + (-1)^{1/4} \sqrt{c} x}\right] -
\end{aligned}$$

$$\begin{aligned} & \frac{1}{6} (-1)^{3/4} b^2 c^{3/2} \text{PolyLog}\left[2, 1 - \frac{\sqrt{2} \left( (-1)^{1/4} + \sqrt{c} x \right)}{1 + (-1)^{1/4} \sqrt{c} x}\right] + \\ & \frac{1}{3} (-1)^{1/4} b^2 c^{3/2} \text{PolyLog}\left[2, 1 - \frac{2}{1 - (-1)^{3/4} \sqrt{c} x}\right] + \\ & \frac{1}{3} (-1)^{1/4} b^2 c^{3/2} \text{PolyLog}\left[2, 1 - \frac{2}{1 + (-1)^{3/4} \sqrt{c} x}\right] - \\ & \frac{1}{6} (-1)^{1/4} b^2 c^{3/2} \text{PolyLog}\left[2, 1 + \frac{\sqrt{2} \left( (-1)^{3/4} + \sqrt{c} x \right)}{1 + (-1)^{3/4} \sqrt{c} x}\right] - \\ & \frac{1}{6} (-1)^{1/4} b^2 c^{3/2} \text{PolyLog}\left[2, 1 - \frac{(1+i) \left( 1 + (-1)^{1/4} \sqrt{c} x \right)}{1 + (-1)^{3/4} \sqrt{c} x}\right] - \\ & \frac{1}{6} (-1)^{3/4} b^2 c^{3/2} \text{PolyLog}\left[2, 1 - \frac{(1-i) \left( 1 + (-1)^{3/4} \sqrt{c} x \right)}{1 + (-1)^{1/4} \sqrt{c} x}\right] \end{aligned}$$

Result (type 8, 18 leaves):

$$\int \frac{(a + b \text{ArcTan}[c x^2])^2}{x^4} dx$$

**Problem 85: Unable to integrate problem.**

$$\int \frac{(a + b \text{ArcTan}[c x^2])^2}{x^6} dx$$

Optimal (type 4, 1444 leaves, 77 steps):

$$\begin{aligned} & -\frac{2 a b c}{15 x^3} + \frac{2 i a b c^2}{5 x} - \frac{8 b^2 c^2}{15 x} - \frac{4}{15} (-1)^{3/4} b^2 c^{5/2} \text{ArcTan}\left[(-1)^{3/4} \sqrt{c} x\right] - \\ & \frac{1}{5} (-1)^{1/4} b^2 c^{5/2} \text{ArcTan}\left[(-1)^{3/4} \sqrt{c} x\right]^2 + \frac{2}{5} (-1)^{1/4} a b c^{5/2} \text{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right] + \\ & \frac{4}{15} (-1)^{3/4} b^2 c^{5/2} \text{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right] + \frac{1}{5} (-1)^{3/4} b^2 c^{5/2} \text{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right]^2 + \\ & \frac{2}{5} (-1)^{3/4} b^2 c^{5/2} \text{ArcTan}\left[(-1)^{3/4} \sqrt{c} x\right] \text{Log}\left[\frac{2}{1 - (-1)^{1/4} \sqrt{c} x}\right] - \\ & \frac{2}{5} (-1)^{3/4} b^2 c^{5/2} \text{ArcTan}\left[(-1)^{3/4} \sqrt{c} x\right] \text{Log}\left[\frac{2}{1 + (-1)^{1/4} \sqrt{c} x}\right] + \\ & \frac{1}{5} (-1)^{3/4} b^2 c^{5/2} \text{ArcTan}\left[(-1)^{3/4} \sqrt{c} x\right] \text{Log}\left[\frac{\sqrt{2} \left( (-1)^{1/4} + \sqrt{c} x \right)}{1 + (-1)^{1/4} \sqrt{c} x}\right] - \\ & \frac{2}{5} (-1)^{3/4} b^2 c^{5/2} \text{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right] \text{Log}\left[\frac{2}{1 - (-1)^{3/4} \sqrt{c} x}\right] + \end{aligned}$$

$$\begin{aligned}
& \frac{2}{5} (-1)^{3/4} b^2 c^{5/2} \operatorname{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[\frac{2}{1+(-1)^{3/4} \sqrt{c} x}\right] - \\
& \frac{1}{5} (-1)^{3/4} b^2 c^{5/2} \operatorname{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[-\frac{\sqrt{2}\left((-1)^{3/4} + \sqrt{c} x\right)}{1+(-1)^{3/4} \sqrt{c} x}\right] - \\
& \frac{1}{5} (-1)^{3/4} b^2 c^{5/2} \operatorname{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[\frac{(1+i)\left(1+(-1)^{1/4} \sqrt{c} x\right)}{1+(-1)^{3/4} \sqrt{c} x}\right] + \\
& \frac{1}{5} (-1)^{3/4} b^2 c^{5/2} \operatorname{ArcTan}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[\frac{(1-i)\left(1+(-1)^{3/4} \sqrt{c} x\right)}{1+(-1)^{1/4} \sqrt{c} x}\right] - \frac{i b^2 c \operatorname{Log}\left[1-i c x^2\right]}{15 x^3} - \\
& \frac{b^2 c^2 \operatorname{Log}\left[1-i c x^2\right]}{5 x} + \frac{1}{5} (-1)^{3/4} b^2 c^{5/2} \operatorname{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[1-i c x^2\right] - \\
& \frac{b c\left(2 a+i b \operatorname{Log}\left[1-i c x^2\right]\right)}{15 x^3} - \frac{i b c^2\left(2 a+i b \operatorname{Log}\left[1-i c x^2\right]\right)}{5 x} + \\
& \frac{1}{5} (-1)^{1/4} b c^{5/2} \operatorname{ArcTan}\left[(-1)^{3/4} \sqrt{c} x\right] \left(2 a+i b \operatorname{Log}\left[1-i c x^2\right]\right) - \\
& \frac{\left(2 a+i b \operatorname{Log}\left[1-i c x^2\right]\right)^2}{20 x^5} + \frac{i a b \operatorname{Log}\left[1+i c x^2\right]}{5 x^5} + \frac{2 i b^2 c \operatorname{Log}\left[1+i c x^2\right]}{15 x^3} - \\
& \frac{1}{5} (-1)^{3/4} b^2 c^{5/2} \operatorname{ArcTan}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[1+i c x^2\right] - \\
& \frac{1}{5} (-1)^{3/4} b^2 c^{5/2} \operatorname{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[1+i c x^2\right] - \frac{b^2 \operatorname{Log}\left[1-i c x^2\right] \operatorname{Log}\left[1+i c x^2\right]}{10 x^5} + \\
& \frac{b^2 \operatorname{Log}\left[1+i c x^2\right]^2}{20 x^5} - \frac{1}{5} (-1)^{1/4} b^2 c^{5/2} \operatorname{PolyLog}\left[2, 1-\frac{2}{1-(-1)^{1/4} \sqrt{c} x}\right] - \\
& \frac{1}{5} (-1)^{1/4} b^2 c^{5/2} \operatorname{PolyLog}\left[2, 1-\frac{2}{1+(-1)^{1/4} \sqrt{c} x}\right] + \\
& \frac{1}{10} (-1)^{1/4} b^2 c^{5/2} \operatorname{PolyLog}\left[2, 1-\frac{\sqrt{2}\left((-1)^{1/4} + \sqrt{c} x\right)}{1+(-1)^{1/4} \sqrt{c} x}\right] - \\
& \frac{1}{5} (-1)^{3/4} b^2 c^{5/2} \operatorname{PolyLog}\left[2, 1-\frac{2}{1-(-1)^{3/4} \sqrt{c} x}\right] - \\
& \frac{1}{5} (-1)^{3/4} b^2 c^{5/2} \operatorname{PolyLog}\left[2, 1-\frac{2}{1+(-1)^{3/4} \sqrt{c} x}\right] + \\
& \frac{1}{10} (-1)^{3/4} b^2 c^{5/2} \operatorname{PolyLog}\left[2, 1+\frac{\sqrt{2}\left((-1)^{3/4} + \sqrt{c} x\right)}{1+(-1)^{3/4} \sqrt{c} x}\right] + \\
& \frac{1}{10} (-1)^{3/4} b^2 c^{5/2} \operatorname{PolyLog}\left[2, 1-\frac{(1+i)\left(1+(-1)^{1/4} \sqrt{c} x\right)}{1+(-1)^{3/4} \sqrt{c} x}\right] +
\end{aligned}$$

$$\frac{1}{10} (-1)^{1/4} b^2 c^{5/2} \text{PolyLog}\left[2, 1 - \frac{(1-i) \left(1 + (-1)^{3/4} \sqrt{c} x\right)}{1 + (-1)^{1/4} \sqrt{c} x}\right]$$

Result (type 8, 18 leaves):

$$\int \frac{(a + b \text{ArcTan}[c x^2])^2}{x^6} dx$$

**Problem 166: Result unnecessarily involves higher level functions.**

$$\int \frac{\text{ArcTan}[a x^n]}{x} dx$$

Optimal (type 4, 39 leaves, 4 steps):

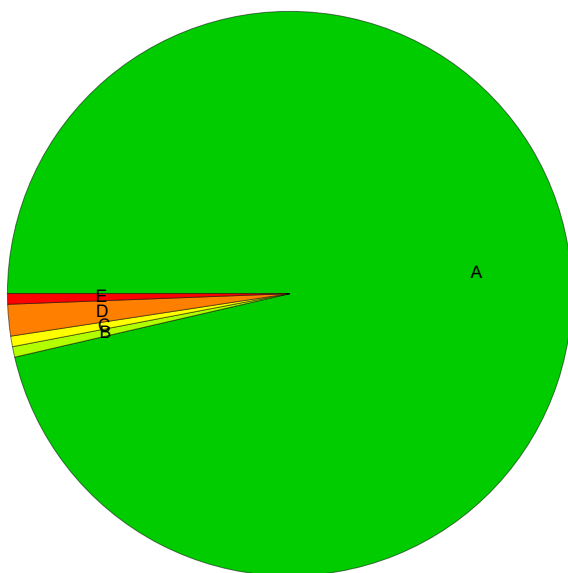
$$\frac{i \text{PolyLog}[2, -i a x^n]}{2 n} - \frac{i \text{PolyLog}[2, i a x^n]}{2 n}$$

Result (type 5, 34 leaves):

$$\frac{a x^n \text{HypergeometricPFQ}\left[\left\{\frac{1}{2}, \frac{1}{2}, 1\right\}, \left\{\frac{3}{2}, \frac{3}{2}\right\}, -a^2 x^{2 n}\right]}{n}$$

## Summary of Integration Test Results

166 integration problems



- A - 160 optimal antiderivatives
- B - 1 more than twice size of optimal antiderivatives
- C - 1 unnecessarily complex antiderivatives
- D - 3 unable to integrate problems
- E - 1 integration timeouts