

Mathematica 11.3 Integration Test Results

Test results for the 234 problems in "5.4.1 Inverse cotangent functions.m"

Problem 46: Result more than twice size of optimal antiderivative.

$$\int \frac{x^2 \operatorname{ArcCot}[c x]}{1+x^2} dx$$

Optimal (type 4, 206 leaves, 28 steps):

$$\begin{aligned} & x \operatorname{ArcCot}[c x] - \frac{1}{2} i \operatorname{ArcTan}[x] \operatorname{Log}\left[1 - \frac{i}{c x}\right] + \frac{1}{2} i \operatorname{ArcTan}[x] \operatorname{Log}\left[1 + \frac{i}{c x}\right] + \\ & \frac{1}{2} i \operatorname{ArcTan}[x] \operatorname{Log}\left[-\frac{2 i (i - c x)}{(1 - c) (1 - i x)}\right] - \frac{1}{2} i \operatorname{ArcTan}[x] \operatorname{Log}\left[-\frac{2 i (i + c x)}{(1 + c) (1 - i x)}\right] + \\ & \frac{\operatorname{Log}[1 + c^2 x^2]}{2 c} + \frac{1}{4} \operatorname{PolyLog}\left[2, 1 + \frac{2 i (i - c x)}{(1 - c) (1 - i x)}\right] - \frac{1}{4} \operatorname{PolyLog}\left[2, 1 + \frac{2 i (i + c x)}{(1 + c) (1 - i x)}\right] \end{aligned}$$

Result (type 4, 626 leaves):

$$\frac{1}{c} \left(c x \operatorname{ArcCot}[c x] - \operatorname{Log}\left[\frac{1}{c \sqrt{1 + \frac{1}{c^2 x^2}}}\right] + \right. \\
\left. \frac{1}{4} \sqrt{-c^2} \left(2 \operatorname{ArcCos}\left[\frac{1+c^2}{-1+c^2}\right] \operatorname{ArcTanh}\left[\frac{\sqrt{-c^2}}{c x}\right] - 4 \operatorname{ArcCot}[c x] \operatorname{ArcTanh}\left[\frac{c x}{\sqrt{-c^2}}\right] - \right. \right. \\
\left. \left(\operatorname{ArcCos}\left[\frac{1+c^2}{-1+c^2}\right] - 2 i \operatorname{ArcTanh}\left[\frac{\sqrt{-c^2}}{c x}\right] \right) \operatorname{Log}\left[-\frac{2(c^2 + i \sqrt{-c^2})(-i + c x)}{(-1+c^2)(\sqrt{-c^2} - c x)}\right] - \right. \\
\left. \left(\operatorname{ArcCos}\left[\frac{1+c^2}{-1+c^2}\right] + 2 i \operatorname{ArcTanh}\left[\frac{\sqrt{-c^2}}{c x}\right] \right) \operatorname{Log}\left[\frac{2 i(i c^2 + \sqrt{-c^2})(i + c x)}{(-1+c^2)(\sqrt{-c^2} - c x)}\right] + \right. \\
\left. \left(\operatorname{ArcCos}\left[\frac{1+c^2}{-1+c^2}\right] - 2 i \operatorname{ArcTanh}\left[\frac{\sqrt{-c^2}}{c x}\right] + 2 i \operatorname{ArcTanh}\left[\frac{c x}{\sqrt{-c^2}}\right] \right) \right. \\
\left. \operatorname{Log}\left[\frac{\sqrt{2} \sqrt{-c^2} e^{-i \operatorname{ArcCot}[c x]}}{\sqrt{-1+c^2} \sqrt{-1-c^2 + (-1+c^2) \operatorname{Cos}[2 \operatorname{ArcCot}[c x]]}}\right] + \right. \\
\left. \left(\operatorname{ArcCos}\left[\frac{1+c^2}{-1+c^2}\right] + 2 i \operatorname{ArcTanh}\left[\frac{\sqrt{-c^2}}{c x}\right] - 2 i \operatorname{ArcTanh}\left[\frac{c x}{\sqrt{-c^2}}\right] \right) \right. \\
\left. \operatorname{Log}\left[\frac{\sqrt{2} \sqrt{-c^2} e^{i \operatorname{ArcCot}[c x]}}{\sqrt{-1+c^2} \sqrt{-1-c^2 + (-1+c^2) \operatorname{Cos}[2 \operatorname{ArcCot}[c x]]}}\right] + \right. \\
\left. i \left(-\operatorname{PolyLog}\left[2, \frac{(1+c^2 - 2 i \sqrt{-c^2})(\sqrt{-c^2} + c x)}{(-1+c^2)(\sqrt{-c^2} - c x)}\right] + \right. \right. \\
\left. \left. \operatorname{PolyLog}\left[2, \frac{(1+c^2 + 2 i \sqrt{-c^2})(\sqrt{-c^2} + c x)}{(-1+c^2)(\sqrt{-c^2} - c x)}\right] \right) \right) \right)$$

Problem 48: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{ArcCot}[c x]}{1+x^2} dx$$

Optimal (type 4, 183 leaves, 25 steps):

$$\begin{aligned} & \frac{1}{2} i \operatorname{ArcTan}[x] \operatorname{Log}\left[1 - \frac{i}{c x}\right] - \frac{1}{2} i \operatorname{ArcTan}[x] \operatorname{Log}\left[1 + \frac{i}{c x}\right] - \\ & \frac{1}{2} i \operatorname{ArcTan}[x] \operatorname{Log}\left[-\frac{2 i (i - c x)}{(1 - c)(1 - i x)}\right] + \frac{1}{2} i \operatorname{ArcTan}[x] \operatorname{Log}\left[-\frac{2 i (i + c x)}{(1 + c)(1 - i x)}\right] - \\ & \frac{1}{4} \operatorname{PolyLog}\left[2, 1 + \frac{2 i (i - c x)}{(1 - c)(1 - i x)}\right] + \frac{1}{4} \operatorname{PolyLog}\left[2, 1 + \frac{2 i (i + c x)}{(1 + c)(1 - i x)}\right] \end{aligned}$$

Result (type 4, 592 leaves):

$$\begin{aligned} & \frac{1}{4 \sqrt{-c^2}} c \left(2 \operatorname{ArcCos}\left[\frac{1 + c^2}{-1 + c^2}\right] \operatorname{ArcTanh}\left[\frac{\sqrt{-c^2}}{c x}\right] - 4 \operatorname{ArcCot}[c x] \operatorname{ArcTanh}\left[\frac{c x}{\sqrt{-c^2}}\right] - \right. \\ & \left. \left(\operatorname{ArcCos}\left[\frac{1 + c^2}{-1 + c^2}\right] - 2 i \operatorname{ArcTanh}\left[\frac{\sqrt{-c^2}}{c x}\right] \right) \operatorname{Log}\left[-\frac{2 (c^2 + i \sqrt{-c^2}) (-i + c x)}{(-1 + c^2) (\sqrt{-c^2} - c x)}\right] - \right. \\ & \left. \left(\operatorname{ArcCos}\left[\frac{1 + c^2}{-1 + c^2}\right] + 2 i \operatorname{ArcTanh}\left[\frac{\sqrt{-c^2}}{c x}\right] \right) \operatorname{Log}\left[\frac{2 i (i c^2 + \sqrt{-c^2}) (i + c x)}{(-1 + c^2) (\sqrt{-c^2} - c x)}\right] + \right. \\ & \left. \left(\operatorname{ArcCos}\left[\frac{1 + c^2}{-1 + c^2}\right] - 2 i \operatorname{ArcTanh}\left[\frac{\sqrt{-c^2}}{c x}\right] + 2 i \operatorname{ArcTanh}\left[\frac{c x}{\sqrt{-c^2}}\right] \right) \right. \\ & \left. \operatorname{Log}\left[\frac{\sqrt{2} \sqrt{-c^2} e^{-i \operatorname{ArcCot}[c x]}}{\sqrt{-1 + c^2} \sqrt{-1 - c^2} + (-1 + c^2) \operatorname{Cos}[2 \operatorname{ArcCot}[c x]]}\right] + \right. \\ & \left. \left(\operatorname{ArcCos}\left[\frac{1 + c^2}{-1 + c^2}\right] + 2 i \operatorname{ArcTanh}\left[\frac{\sqrt{-c^2}}{c x}\right] - 2 i \operatorname{ArcTanh}\left[\frac{c x}{\sqrt{-c^2}}\right] \right) \right. \\ & \left. \operatorname{Log}\left[\frac{\sqrt{2} \sqrt{-c^2} e^{i \operatorname{ArcCot}[c x]}}{\sqrt{-1 + c^2} \sqrt{-1 - c^2} + (-1 + c^2) \operatorname{Cos}[2 \operatorname{ArcCot}[c x]]}\right] + \right. \\ & \left. i \left(-\operatorname{PolyLog}\left[2, \frac{(1 + c^2 - 2 i \sqrt{-c^2}) (\sqrt{-c^2} + c x)}{(-1 + c^2) (\sqrt{-c^2} - c x)}\right] + \right. \right. \\ & \left. \left. \operatorname{PolyLog}\left[2, \frac{(1 + c^2 + 2 i \sqrt{-c^2}) (\sqrt{-c^2} + c x)}{(-1 + c^2) (\sqrt{-c^2} - c x)}\right] \right) \right) \end{aligned}$$

Problem 50: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{ArcCot}[c x]}{x^2 (1 + x^2)} dx$$

Optimal (type 4, 212 leaves, 31 steps):

$$\begin{aligned}
 & -\frac{\text{ArcCot}[c x]}{x} - \frac{1}{2} i \text{ArcTan}[x] \text{Log}\left[1 - \frac{i}{c x}\right] + \frac{1}{2} i \text{ArcTan}[x] \text{Log}\left[1 + \frac{i}{c x}\right] - c \text{Log}[x] + \\
 & \frac{1}{2} i \text{ArcTan}[x] \text{Log}\left[-\frac{2 i (i - c x)}{(1 - c)(1 - i x)}\right] - \frac{1}{2} i \text{ArcTan}[x] \text{Log}\left[-\frac{2 i (i + c x)}{(1 + c)(1 - i x)}\right] + \\
 & \frac{1}{2} c \text{Log}[1 + c^2 x^2] + \frac{1}{4} \text{PolyLog}\left[2, 1 + \frac{2 i (i - c x)}{(1 - c)(1 - i x)}\right] - \frac{1}{4} \text{PolyLog}\left[2, 1 + \frac{2 i (i + c x)}{(1 + c)(1 - i x)}\right]
 \end{aligned}$$

Result (type 4, 619 leaves):

$$\begin{aligned}
 & -\frac{\text{ArcCot}[c x]}{x} - c \text{Log}\left[\frac{1}{\sqrt{1 + \frac{1}{c^2 x^2}}}\right] - \\
 & \frac{1}{4 \sqrt{-c^2}} c \left(2 \text{ArcCos}\left[\frac{1 + c^2}{-1 + c^2}\right] \text{ArcTanh}\left[\frac{\sqrt{-c^2}}{c x}\right] - 4 \text{ArcCot}[c x] \text{ArcTanh}\left[\frac{c x}{\sqrt{-c^2}}\right] - \right. \\
 & \left. \left(\text{ArcCos}\left[\frac{1 + c^2}{-1 + c^2}\right] - 2 i \text{ArcTanh}\left[\frac{\sqrt{-c^2}}{c x}\right] \right) \text{Log}\left[-\frac{2 (c^2 + i \sqrt{-c^2}) (-i + c x)}{(-1 + c^2) (\sqrt{-c^2} - c x)}\right] - \right. \\
 & \left. \left(\text{ArcCos}\left[\frac{1 + c^2}{-1 + c^2}\right] + 2 i \text{ArcTanh}\left[\frac{\sqrt{-c^2}}{c x}\right] \right) \text{Log}\left[\frac{2 i (i c^2 + \sqrt{-c^2}) (i + c x)}{(-1 + c^2) (\sqrt{-c^2} - c x)}\right] + \right. \\
 & \left. \left(\text{ArcCos}\left[\frac{1 + c^2}{-1 + c^2}\right] - 2 i \text{ArcTanh}\left[\frac{\sqrt{-c^2}}{c x}\right] + 2 i \text{ArcTanh}\left[\frac{c x}{\sqrt{-c^2}}\right] \right) \right. \\
 & \left. \text{Log}\left[\frac{\sqrt{2} \sqrt{-c^2} e^{-i \text{ArcCot}[c x]}}{\sqrt{-1 + c^2} \sqrt{-1 - c^2} + (-1 + c^2) \text{Cos}[2 \text{ArcCot}[c x]]}\right] + \right. \\
 & \left. \left(\text{ArcCos}\left[\frac{1 + c^2}{-1 + c^2}\right] + 2 i \text{ArcTanh}\left[\frac{\sqrt{-c^2}}{c x}\right] - 2 i \text{ArcTanh}\left[\frac{c x}{\sqrt{-c^2}}\right] \right) \right. \\
 & \left. \text{Log}\left[\frac{\sqrt{2} \sqrt{-c^2} e^{i \text{ArcCot}[c x]}}{\sqrt{-1 + c^2} \sqrt{-1 - c^2} + (-1 + c^2) \text{Cos}[2 \text{ArcCot}[c x]]}\right] + \right. \\
 & \left. i \left(-\text{PolyLog}\left[2, \frac{(1 + c^2 - 2 i \sqrt{-c^2}) (\sqrt{-c^2} + c x)}{(-1 + c^2) (\sqrt{-c^2} - c x)}\right] + \right. \right. \\
 & \left. \left. \text{PolyLog}\left[2, \frac{(1 + c^2 + 2 i \sqrt{-c^2}) (\sqrt{-c^2} + c x)}{(-1 + c^2) (\sqrt{-c^2} - c x)}\right] \right) \right)
 \end{aligned}$$

Problem 61: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcCot}[a x]}{(c + d x^2)^{3/2}} dx$$

Optimal (type 3, 66 leaves, 5 steps):

$$\frac{x \operatorname{ArcCot}[a x]}{c \sqrt{c+d x^2}} - \frac{\operatorname{ArcTanh}\left[\frac{a \sqrt{c+d x^2}}{\sqrt{a^2 c-d}}\right]}{c \sqrt{a^2 c-d}}$$

Result (type 3, 169 leaves):

$$\frac{1}{2 c} \left(\frac{2 x \operatorname{ArcCot}[a x]}{\sqrt{c+d x^2}} + \frac{-\operatorname{Log}\left[\frac{4 a c\left(a c-i d x+\sqrt{a^2 c-d} \sqrt{c+d x^2}\right)}{\sqrt{a^2 c-d}(i+a x)}\right] - \operatorname{Log}\left[\frac{4 a c\left(a c+i d x+\sqrt{a^2 c-d} \sqrt{c+d x^2}\right)}{\sqrt{a^2 c-d}(-i+a x)}\right]}{\sqrt{a^2 c-d}} \right)$$

Problem 62: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{ArcCot}[a x]}{(c+d x^2)^{5/2}} dx$$

Optimal (type 3, 134 leaves, 7 steps):

$$\frac{a}{3 c\left(a^2 c-d\right) \sqrt{c+d x^2}} + \frac{x \operatorname{ArcCot}[a x]}{3 c\left(c+d x^2\right)^{3/2}} + \frac{2 x \operatorname{ArcCot}[a x]}{3 c^2 \sqrt{c+d x^2}} - \frac{\left(3 a^2 c-2 d\right) \operatorname{ArcTanh}\left[\frac{a \sqrt{c+d x^2}}{\sqrt{a^2 c-d}}\right]}{3 c^2\left(a^2 c-d\right)^{3/2}}$$

Result (type 3, 262 leaves):

$$-\frac{1}{6 c^2} \left(-\frac{2 a c}{\left(a^2 c-d\right) \sqrt{c+d x^2}} - \frac{2 x\left(3 c+2 d x^2\right) \operatorname{ArcCot}[a x]}{\left(c+d x^2\right)^{3/2}} + \frac{\left(3 a^2 c-2 d\right) \operatorname{Log}\left[\frac{12 a c^2 \sqrt{a^2 c-d}\left(a c-i d x+\sqrt{a^2 c-d} \sqrt{c+d x^2}\right)}{\left(3 a^2 c-2 d\right)(i+a x)}\right]}{\left(a^2 c-d\right)^{3/2}} + \frac{\left(3 a^2 c-2 d\right) \operatorname{Log}\left[\frac{12 a c^2 \sqrt{a^2 c-d}\left(a c+i d x+\sqrt{a^2 c-d} \sqrt{c+d x^2}\right)}{\left(3 a^2 c-2 d\right)(-i+a x)}\right]}{\left(a^2 c-d\right)^{3/2}} \right)$$

Problem 63: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{ArcCot}[a x]}{(c+d x^2)^{7/2}} dx$$

Optimal (type 3, 208 leaves, 8 steps):

$$\frac{a}{15 c (a^2 c - d) (c + d x^2)^{3/2}} + \frac{a (7 a^2 c - 4 d)}{15 c^2 (a^2 c - d)^2 \sqrt{c + d x^2}} + \frac{x \operatorname{ArcCot}[a x]}{5 c (c + d x^2)^{5/2}} +$$

$$\frac{4 x \operatorname{ArcCot}[a x]}{15 c^2 (c + d x^2)^{3/2}} + \frac{8 x \operatorname{ArcCot}[a x]}{15 c^3 \sqrt{c + d x^2}} - \frac{(15 a^4 c^2 - 20 a^2 c d + 8 d^2) \operatorname{ArcTanh}\left[\frac{a \sqrt{c + d x^2}}{\sqrt{a^2 c - d}}\right]}{15 c^3 (a^2 c - d)^{5/2}}$$

Result (type 3, 345 leaves):

$$-\frac{1}{30 c^3} \left(-\frac{2 a c (-d (5 c + 4 d x^2) + a^2 c (8 c + 7 d x^2))}{(-a^2 c + d)^2 (c + d x^2)^{3/2}} - \right.$$

$$\frac{2 x (15 c^2 + 20 c d x^2 + 8 d^2 x^4) \operatorname{ArcCot}[a x]}{(c + d x^2)^{5/2}} + \frac{1}{(a^2 c - d)^{5/2}} (15 a^4 c^2 - 20 a^2 c d + 8 d^2)$$

$$\left. \operatorname{Log}\left[\frac{60 a c^3 (a^2 c - d)^{3/2} (a c - i d x + \sqrt{a^2 c - d} \sqrt{c + d x^2})}{(15 a^4 c^2 - 20 a^2 c d + 8 d^2) (i + a x)}\right] + \frac{1}{(a^2 c - d)^{5/2}} \right.$$

$$\left. (15 a^4 c^2 - 20 a^2 c d + 8 d^2) \operatorname{Log}\left[\frac{60 a c^3 (a^2 c - d)^{3/2} (a c + i d x + \sqrt{a^2 c - d} \sqrt{c + d x^2})}{(15 a^4 c^2 - 20 a^2 c d + 8 d^2) (-i + a x)}\right] \right)$$

Problem 64: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{ArcCot}[a x]}{(c + d x^2)^{9/2}} dx$$

Optimal (type 3, 293 leaves, 8 steps):

$$\frac{a}{35 c (a^2 c - d) (c + d x^2)^{5/2}} + \frac{a (11 a^2 c - 6 d)}{105 c^2 (a^2 c - d)^2 (c + d x^2)^{3/2}} +$$

$$\frac{a (19 a^4 c^2 - 22 a^2 c d + 8 d^2)}{35 c^3 (a^2 c - d)^3 \sqrt{c + d x^2}} + \frac{x \operatorname{ArcCot}[a x]}{7 c (c + d x^2)^{7/2}} + \frac{6 x \operatorname{ArcCot}[a x]}{35 c^2 (c + d x^2)^{5/2}} + \frac{8 x \operatorname{ArcCot}[a x]}{35 c^3 (c + d x^2)^{3/2}} +$$

$$\frac{16 x \operatorname{ArcCot}[a x]}{35 c^4 \sqrt{c + d x^2}} - \frac{(35 a^6 c^3 - 70 a^4 c^2 d + 56 a^2 c d^2 - 16 d^3) \operatorname{ArcTanh}\left[\frac{a \sqrt{c + d x^2}}{\sqrt{a^2 c - d}}\right]}{35 c^4 (a^2 c - d)^{7/2}}$$

Result (type 3, 450 leaves):

$$\frac{1}{210 c^4} \left(\left(2 a c \left(3 c^2 \left(-a^2 c + d \right)^2 + c \left(11 a^2 c - 6 d \right) \left(a^2 c - d \right) \left(c + d x^2 \right) + \right. \right. \right. \\ \left. \left. \left. 3 \left(19 a^4 c^2 - 22 a^2 c d + 8 d^2 \right) \left(c + d x^2 \right)^2 \right) \right) / \left(\left(a^2 c - d \right)^3 \left(c + d x^2 \right)^{5/2} \right) + \right. \\ \left. \frac{6 x \left(35 c^3 + 70 c^2 d x^2 + 56 c d^2 x^4 + 16 d^3 x^6 \right) \text{ArcCot}[a x]}{\left(c + d x^2 \right)^{7/2}} - \frac{1}{\left(a^2 c - d \right)^{7/2}} \right. \\ \left. \frac{3 \left(35 a^6 c^3 - 70 a^4 c^2 d + 56 a^2 c d^2 - 16 d^3 \right)}{\left(a^2 c - d \right)^{7/2}} \right. \\ \left. \text{Log} \left[\frac{140 a c^4 \left(a^2 c - d \right)^{5/2} \left(a c - i d x + \sqrt{a^2 c - d} \sqrt{c + d x^2} \right)}{\left(35 a^6 c^3 - 70 a^4 c^2 d + 56 a^2 c d^2 - 16 d^3 \right) \left(i + a x \right)} \right] - \right. \\ \left. \frac{1}{\left(a^2 c - d \right)^{7/2}} \frac{3 \left(35 a^6 c^3 - 70 a^4 c^2 d + 56 a^2 c d^2 - 16 d^3 \right)}{\left(a^2 c - d \right)^{7/2}} \right. \\ \left. \text{Log} \left[\frac{140 a c^4 \left(a^2 c - d \right)^{5/2} \left(a c + i d x + \sqrt{a^2 c - d} \sqrt{c + d x^2} \right)}{\left(35 a^6 c^3 - 70 a^4 c^2 d + 56 a^2 c d^2 - 16 d^3 \right) \left(-i + a x \right)} \right] \right)$$

Problem 97: Result unnecessarily involves higher level functions.

$$\int \frac{\text{ArcCot}[a x^n]}{x} dx$$

Optimal (type 4, 47 leaves, 4 steps):

$$-\frac{i \text{PolyLog}\left[2, -\frac{i x^n}{a}\right]}{2 n} + \frac{i \text{PolyLog}\left[2, \frac{i x^n}{a}\right]}{2 n}$$

Result (type 5, 52 leaves):

$$-\frac{a x^n \text{HypergeometricPFQ}\left[\left\{\frac{1}{2}, \frac{1}{2}, 1\right\}, \left\{\frac{3}{2}, \frac{3}{2}\right\}, -a^2 x^{2 n}\right]}{n} + \left(\text{ArcCot}[a x^n] + \text{ArcTan}[a x^n]\right) \text{Log}[x]$$

Problem 103: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcCot}[a + b x]}{x} dx$$

Optimal (type 4, 120 leaves, 5 steps):

$$-\text{ArcCot}[a + b x] \text{Log}\left[\frac{2}{1 - i(a + b x)}\right] + \text{ArcCot}[a + b x] \text{Log}\left[\frac{2 b x}{(i - a)(1 - i(a + b x))}\right] - \\ \frac{1}{2} i \text{PolyLog}\left[2, 1 - \frac{2}{1 - i(a + b x)}\right] + \frac{1}{2} i \text{PolyLog}\left[2, 1 - \frac{2 b x}{(i - a)(1 - i(a + b x))}\right]$$

Result (type 4, 256 leaves):

$$\begin{aligned}
 & (\text{ArcCot}[a + b x] + \text{ArcTan}[a + b x]) \text{Log}[x] + \\
 & \text{ArcTan}[a + b x] \left(\text{Log}\left[\frac{1}{\sqrt{1 + (a + b x)^2}}\right] - \text{Log}[-\text{Sin}[\text{ArcTan}[a] - \text{ArcTan}[a + b x]]] \right) + \\
 & \frac{1}{2} \left(\frac{1}{4} i (\pi - 2 \text{ArcTan}[a + b x])^2 + i (\text{ArcTan}[a] - \text{ArcTan}[a + b x])^2 - \right. \\
 & (\pi - 2 \text{ArcTan}[a + b x]) \text{Log}[1 + e^{-2 i \text{ArcTan}[a + b x]}] + 2 (\text{ArcTan}[a] - \text{ArcTan}[a + b x]) \\
 & \left. \text{Log}[1 - e^{2 i (-\text{ArcTan}[a] + \text{ArcTan}[a + b x])}] + (\pi - 2 \text{ArcTan}[a + b x]) \text{Log}\left[\frac{2}{\sqrt{1 + (a + b x)^2}}\right] + \right. \\
 & 2 (-\text{ArcTan}[a] + \text{ArcTan}[a + b x]) \text{Log}[-2 \text{Sin}[\text{ArcTan}[a] - \text{ArcTan}[a + b x]]] + \\
 & \left. i \text{PolyLog}[2, -e^{-2 i \text{ArcTan}[a + b x]}] + i \text{PolyLog}[2, e^{2 i (-\text{ArcTan}[a] + \text{ArcTan}[a + b x])}] \right)
 \end{aligned}$$

Problem 107: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcCot}[a + b x]}{c + d x^2} dx$$

Optimal (type 4, 642 leaves, 15 steps):

$$\begin{aligned}
 & - \frac{\text{Log}\left[\frac{i+a+bx}{a+bx}\right] \text{Log}\left[-\frac{b(i\sqrt{c}-\sqrt{d}x)}{(b\sqrt{c}+(1-i)a\sqrt{d})(a+bx)}\right]}{4\sqrt{c}\sqrt{d}} + \frac{\text{Log}\left[-\frac{i-a-bx}{a+bx}\right] \text{Log}\left[\frac{ib(\sqrt{c}+i\sqrt{d}x)}{(b\sqrt{c}-(1+i)a\sqrt{d})(a+bx)}\right]}{4\sqrt{c}\sqrt{d}} - \\
 & \frac{\text{Log}\left[-\frac{i-a-bx}{a+bx}\right] \text{Log}\left[\frac{b(i\sqrt{c}+\sqrt{d}x)}{(b\sqrt{c}+(1+i)a\sqrt{d})(a+bx)}\right]}{4\sqrt{c}\sqrt{d}} + \frac{\text{Log}\left[\frac{i+a+bx}{a+bx}\right] \text{Log}\left[-\frac{b(i\sqrt{c}+\sqrt{d}x)}{(b\sqrt{c}+i(i+a)\sqrt{d})(a+bx)}\right]}{4\sqrt{c}\sqrt{d}} + \\
 & \frac{\text{PolyLog}\left[2, -\frac{(b\sqrt{c}-ia\sqrt{d})(i-a-bx)}{(b\sqrt{c}-(1+i)a\sqrt{d})(a+bx)}\right]}{4\sqrt{c}\sqrt{d}} - \frac{\text{PolyLog}\left[2, -\frac{(b\sqrt{c}+ia\sqrt{d})(i-a-bx)}{(b\sqrt{c}+(1+i)a\sqrt{d})(a+bx)}\right]}{4\sqrt{c}\sqrt{d}} - \\
 & \frac{\text{PolyLog}\left[2, \frac{(b\sqrt{c}-ia\sqrt{d})(i+a+bx)}{(b\sqrt{c}+(1-i)a\sqrt{d})(a+bx)}\right]}{4\sqrt{c}\sqrt{d}} + \frac{\text{PolyLog}\left[2, \frac{(b\sqrt{c}+ia\sqrt{d})(i+a+bx)}{(b\sqrt{c}+i(i+a)\sqrt{d})(a+bx)}\right]}{4\sqrt{c}\sqrt{d}}
 \end{aligned}$$

Result (type 4, 1530 leaves):

$$\begin{aligned}
 & \frac{1}{4(1+a^2)\sqrt{c}d(a+bx)^2\left(1+\frac{1}{(a+bx)^2}\right)} \\
 & \left(1+(a+bx)^2\right) \left(4(1+a^2)\sqrt{d}\text{ArcCot}[a+bx]\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right] + \right.
 \end{aligned}$$

$$\begin{aligned}
 & 2\sqrt{d} \operatorname{ArcTan}\left[\frac{(-i+a)\sqrt{d}}{b\sqrt{c}}\right] \operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right] + 2a^2\sqrt{d} \operatorname{ArcTan}\left[\frac{(-i+a)\sqrt{d}}{b\sqrt{c}}\right] \operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right] - \\
 & 2\sqrt{d} \operatorname{ArcTan}\left[\frac{(i+a)\sqrt{d}}{b\sqrt{c}}\right] \operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right] - 2a^2\sqrt{d} \operatorname{ArcTan}\left[\frac{(i+a)\sqrt{d}}{b\sqrt{c}}\right] \operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right] + \\
 & 2b\sqrt{c} \operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right]^2 - b\sqrt{c} \sqrt{\frac{b^2c + (-i+a)^2d}{b^2c}} e^{-i \operatorname{ArcTan}\left[\frac{(-i+a)\sqrt{d}}{b\sqrt{c}}\right]} \operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right]^2 + \\
 & iab\sqrt{c} \sqrt{\frac{b^2c + (-i+a)^2d}{b^2c}} e^{-i \operatorname{ArcTan}\left[\frac{(-i+a)\sqrt{d}}{b\sqrt{c}}\right]} \operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right]^2 - \\
 & b\sqrt{c} \sqrt{\frac{b^2c + (i+a)^2d}{b^2c}} e^{-i \operatorname{ArcTan}\left[\frac{(i+a)\sqrt{d}}{b\sqrt{c}}\right]} \operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right]^2 - \\
 & iab\sqrt{c} \sqrt{\frac{b^2c + (i+a)^2d}{b^2c}} e^{-i \operatorname{ArcTan}\left[\frac{(i+a)\sqrt{d}}{b\sqrt{c}}\right]} \operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right]^2 - \\
 & 2i\sqrt{d} \operatorname{ArcTan}\left[\frac{(-i+a)\sqrt{d}}{b\sqrt{c}}\right] \operatorname{Log}\left[1 - e^{-2i\left(\operatorname{ArcTan}\left[\frac{(-i+a)\sqrt{d}}{b\sqrt{c}}\right] + \operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right]\right)}\right] - \\
 & 2ia^2\sqrt{d} \operatorname{ArcTan}\left[\frac{(-i+a)\sqrt{d}}{b\sqrt{c}}\right] \operatorname{Log}\left[1 - e^{-2i\left(\operatorname{ArcTan}\left[\frac{(-i+a)\sqrt{d}}{b\sqrt{c}}\right] + \operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right]\right)}\right] - \\
 & 2i\sqrt{d} \operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right] \operatorname{Log}\left[1 - e^{-2i\left(\operatorname{ArcTan}\left[\frac{(-i+a)\sqrt{d}}{b\sqrt{c}}\right] + \operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right]\right)}\right] - \\
 & 2ia^2\sqrt{d} \operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right] \operatorname{Log}\left[1 - e^{-2i\left(\operatorname{ArcTan}\left[\frac{(-i+a)\sqrt{d}}{b\sqrt{c}}\right] + \operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right]\right)}\right] + \\
 & 2i\sqrt{d} \operatorname{ArcTan}\left[\frac{(i+a)\sqrt{d}}{b\sqrt{c}}\right] \operatorname{Log}\left[1 - e^{-2i\left(\operatorname{ArcTan}\left[\frac{(i+a)\sqrt{d}}{b\sqrt{c}}\right] + \operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right]\right)}\right] + \\
 & 2ia^2\sqrt{d} \operatorname{ArcTan}\left[\frac{(i+a)\sqrt{d}}{b\sqrt{c}}\right] \operatorname{Log}\left[1 - e^{-2i\left(\operatorname{ArcTan}\left[\frac{(i+a)\sqrt{d}}{b\sqrt{c}}\right] + \operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right]\right)}\right] + \\
 & 2i\sqrt{d} \operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right] \operatorname{Log}\left[1 - e^{-2i\left(\operatorname{ArcTan}\left[\frac{(i+a)\sqrt{d}}{b\sqrt{c}}\right] + \operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right]\right)}\right] + \\
 & 2ia^2\sqrt{d} \operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right] \operatorname{Log}\left[1 - e^{-2i\left(\operatorname{ArcTan}\left[\frac{(i+a)\sqrt{d}}{b\sqrt{c}}\right] + \operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right]\right)}\right] + \\
 & 2i\sqrt{d} \operatorname{ArcTan}\left[\frac{(-i+a)\sqrt{d}}{b\sqrt{c}}\right] \operatorname{Log}\left[-\operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{(-i+a)\sqrt{d}}{b\sqrt{c}}\right] + \operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right]\right]\right] + \\
 & 2ia^2\sqrt{d} \operatorname{ArcTan}\left[\frac{(-i+a)\sqrt{d}}{b\sqrt{c}}\right] \operatorname{Log}\left[-\operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{(-i+a)\sqrt{d}}{b\sqrt{c}}\right] + \operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right]\right]\right] -
 \end{aligned}$$

$$\begin{aligned}
 & 2 i \sqrt{d} \operatorname{ArcTan}\left[\frac{(i+a) \sqrt{d}}{b \sqrt{c}}\right] \operatorname{Log}\left[-\operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{(i+a) \sqrt{d}}{b \sqrt{c}}\right]+\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]\right]\right]- \\
 & 2 i a^2 \sqrt{d} \operatorname{ArcTan}\left[\frac{(i+a) \sqrt{d}}{b \sqrt{c}}\right] \operatorname{Log}\left[-\operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{(i+a) \sqrt{d}}{b \sqrt{c}}\right]+\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]\right]\right]+ \\
 & (1+a^2) \sqrt{d} \operatorname{PolyLog}\left[2, e^{-2 i\left(\operatorname{ArcTan}\left[\frac{(i+a) \sqrt{d}}{b \sqrt{c}}\right]+\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]\right)}\right]- \\
 & (1+a^2) \sqrt{d} \operatorname{PolyLog}\left[2, e^{-2 i\left(\operatorname{ArcTan}\left[\frac{(i+a) \sqrt{d}}{b \sqrt{c}}\right]+\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]\right)}\right]
 \end{aligned}$$

Problem 108: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{ArcCot}[a+b x]}{c+d x} d x$$

Optimal (type 4, 152 leaves, 5 steps):

$$\begin{aligned}
 & -\frac{\operatorname{ArcCot}[a+b x] \operatorname{Log}\left[\frac{2}{1-i(a+b x)}\right]}{d} + \frac{\operatorname{ArcCot}[a+b x] \operatorname{Log}\left[\frac{2 b(c+d x)}{(b c+i d-a d)(1-i(a+b x))}\right]}{d} - \\
 & \frac{i \operatorname{PolyLog}\left[2, 1-\frac{2}{1-i(a+b x)}\right]}{2 d} + \frac{i \operatorname{PolyLog}\left[2, 1-\frac{2 b(c+d x)}{(b c+i d-a d)(1-i(a+b x))}\right]}{2 d}
 \end{aligned}$$

Result (type 4, 325 leaves):

$$\begin{aligned}
 & \frac{1}{d} \left((\operatorname{ArcCot}[a+b x] + \operatorname{ArcTan}[a+b x]) \operatorname{Log}[c+d x] + \right. \\
 & \left. \operatorname{ArcTan}[a+b x] \left(\operatorname{Log}\left[\frac{1}{\sqrt{1+(a+b x)^2}}\right] - \operatorname{Log}\left[\operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{b c-a d}{d}\right] + \operatorname{ArcTan}[a+b x]\right]\right] \right) \right) + \\
 & \frac{1}{2} \left(\frac{1}{4} i (\pi - 2 \operatorname{ArcTan}[a+b x])^2 + i \left(\operatorname{ArcTan}\left[\frac{b c-a d}{d}\right] + \operatorname{ArcTan}[a+b x] \right)^2 - \right. \\
 & \left. (\pi - 2 \operatorname{ArcTan}[a+b x]) \operatorname{Log}\left[1 + e^{-2 i \operatorname{ArcTan}[a+b x]}\right] - 2 \left(\operatorname{ArcTan}\left[\frac{b c-a d}{d}\right] + \operatorname{ArcTan}[a+b x] \right) \right. \\
 & \left. \operatorname{Log}\left[1 - e^{2 i \left(\operatorname{ArcTan}\left[\frac{b c-a d}{d}\right] + \operatorname{ArcTan}[a+b x]\right)}\right] + (\pi - 2 \operatorname{ArcTan}[a+b x]) \operatorname{Log}\left[\frac{2}{\sqrt{1+(a+b x)^2}}\right] + \right. \\
 & \left. 2 \left(\operatorname{ArcTan}\left[\frac{b c-a d}{d}\right] + \operatorname{ArcTan}[a+b x] \right) \operatorname{Log}\left[2 \operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{b c-a d}{d}\right] + \operatorname{ArcTan}[a+b x]\right]\right] + \right. \\
 & \left. i \operatorname{PolyLog}\left[2, -e^{-2 i \operatorname{ArcTan}[a+b x]}\right] + i \operatorname{PolyLog}\left[2, e^{2 i \left(\operatorname{ArcTan}\left[\frac{b c-a d}{d}\right] + \operatorname{ArcTan}[a+b x]\right)}\right] \right)
 \end{aligned}$$

Problem 110: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcCot}[a + b x]}{c + \frac{d}{x^2}} dx$$

Optimal (type 4, 735 leaves, 57 steps):

$$\begin{aligned} & \frac{\text{Log}[i - a - b x]}{2 b c} + \frac{i (a + b x) \text{Log}\left[-\frac{i - a - b x}{a + b x}\right]}{2 b c} - \frac{i \sqrt{d} \text{ArcTan}\left[\frac{\sqrt{c} x}{\sqrt{d}}\right] \text{Log}\left[-\frac{i - a - b x}{a + b x}\right]}{2 c^{3/2}} + \\ & \frac{\text{Log}[i + a + b x]}{2 b c} - \frac{i (a + b x) \text{Log}\left[\frac{i + a + b x}{a + b x}\right]}{2 b c} + \frac{i \sqrt{d} \text{ArcTan}\left[\frac{\sqrt{c} x}{\sqrt{d}}\right] \text{Log}\left[\frac{i + a + b x}{a + b x}\right]}{2 c^{3/2}} - \\ & \frac{\sqrt{d} \text{Log}\left[\frac{-\sqrt{c} (i - a - b x)}{(i - a) \sqrt{c} + i b \sqrt{d}}\right] \text{Log}\left[1 - \frac{i \sqrt{c} x}{\sqrt{d}}\right]}{4 c^{3/2}} + \frac{\sqrt{d} \text{Log}\left[\frac{-\sqrt{c} (i + a + b x)}{(i + a) \sqrt{c} - i b \sqrt{d}}\right] \text{Log}\left[1 - \frac{i \sqrt{c} x}{\sqrt{d}}\right]}{4 c^{3/2}} + \\ & \frac{\sqrt{d} \text{Log}\left[\frac{-\sqrt{c} (i - a - b x)}{(i - a) \sqrt{c} - i b \sqrt{d}}\right] \text{Log}\left[1 + \frac{i \sqrt{c} x}{\sqrt{d}}\right]}{4 c^{3/2}} - \frac{\sqrt{d} \text{Log}\left[\frac{-\sqrt{c} (i + a + b x)}{(i + a) \sqrt{c} + i b \sqrt{d}}\right] \text{Log}\left[1 + \frac{i \sqrt{c} x}{\sqrt{d}}\right]}{4 c^{3/2}} - \\ & \frac{\sqrt{d} \text{PolyLog}\left[2, \frac{b (\sqrt{d} - i \sqrt{c} x)}{(1 + i a) \sqrt{c} + b \sqrt{d}}\right]}{4 c^{3/2}} + \frac{\sqrt{d} \text{PolyLog}\left[2, \frac{b (\sqrt{d} - i \sqrt{c} x)}{i (i + a) \sqrt{c} + b \sqrt{d}}\right]}{4 c^{3/2}} + \\ & \frac{\sqrt{d} \text{PolyLog}\left[2, -\frac{b (\sqrt{d} + i \sqrt{c} x)}{(1 + i a) \sqrt{c} - b \sqrt{d}}\right]}{4 c^{3/2}} - \frac{\sqrt{d} \text{PolyLog}\left[2, \frac{b (\sqrt{d} + i \sqrt{c} x)}{(1 - i a) \sqrt{c} + b \sqrt{d}}\right]}{4 c^{3/2}} \end{aligned}$$

Result (type 4, 16412 leaves):

$$\begin{aligned} & \frac{1}{(a + b x)^2 \left(1 + \frac{1}{(a + b x)^2}\right)} \left(1 + (a + b x)^2\right) \\ & \left(\frac{(a + b x) \text{ArcCot}[a + b x] - \text{Log}\left[\frac{1}{(a + b x) \sqrt{1 + \frac{1}{(a + b x)^2}}}\right]}{b c} - \frac{1}{c} 2 b d \left(-\frac{\text{ArcCot}[a + b x] \text{ArcTan}\left[\frac{-a c + \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]}{2 b \sqrt{c} \sqrt{d}} + \right. \right. \\ & \left. \left. \frac{1}{2 (a^2 c + b^2 d) \left(1 + \frac{1}{(a + b x)^2}\right)} \left(1 + \frac{c \left(a \sqrt{c} - b \sqrt{d} \left(\frac{a \sqrt{c}}{b \sqrt{d}} - \frac{a^2 c + b^2 d}{b \sqrt{c} \sqrt{d} (a + b x)}\right)\right)^2}{(a^2 c + b^2 d)^2} \right) \right) \end{aligned}$$

$$\begin{aligned}
 & \frac{1}{4(-i a c + a^2 c + b^2 d) \sqrt{1 - \frac{(-i a c + a^2 c + b^2 d)^2}{b^2 c d}}} \left(3 a^4 c \operatorname{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] \right. \\
 & \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]^2 - \frac{1}{b \sqrt{c} \sqrt{d} \sqrt{1 - \frac{(-i a c + a^2 c + b^2 d)^2}{b^2 c d}}} \\
 & \left. (-i a c + a^2 c + b^2 d) \left(\pi \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] - i \pi \operatorname{Log}\left[1 + e^{-2 i \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]}\right] - \right. \right. \\
 & \left. \left. 2 i \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] \operatorname{Log}\left[1 - e^{2 \left(i \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] + \operatorname{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]}\right]\right] + \right. \\
 & \left. i \pi \operatorname{Log}\left[\frac{1}{\sqrt{\frac{(a^2 c + b^2 d) \left(c + \frac{a^2 c + b^2 d}{(a + b x)^2} - \frac{2 a c}{a + b x}\right)}{b^2 c d}}}\right] + 2 \operatorname{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] \right. \\
 & \left. \left(i \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] - \operatorname{Log}\left[1 - e^{2 \left(i \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] + \operatorname{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]}\right]\right) + \right. \\
 & \left. \operatorname{Log}\left[\operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] - i \operatorname{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]\right]\right] \right) - \\
 & \left. \operatorname{PolyLog}\left[2, e^{2 \left(i \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] + \operatorname{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]}\right)}\right] \right) \right) -
 \end{aligned}$$

$$\begin{aligned}
 & \frac{1}{4 b^2 d (-i a c + a^2 c + b^2 d) \sqrt{1 - \frac{(-i a c + a^2 c + b^2 d)^2}{b^2 c d}}} a^4 c^2 \left(e^{\operatorname{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]} \right. \\
 & \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]^2 - \frac{1}{b \sqrt{c} \sqrt{d} \sqrt{1 - \frac{(-i a c + a^2 c + b^2 d)^2}{b^2 c d}}} \\
 & \left. (-i a c + a^2 c + b^2 d) \left(\pi \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] - i \pi \operatorname{Log}\left[1 + e^{-2 i \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]}\right] - \right. \right. \\
 & \left. \left. 2 i \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] \operatorname{Log}\left[1 - e^{2 \left(i \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] + \operatorname{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]}\right)}\right] + \right. \\
 & \left. i \pi \operatorname{Log}\left[\frac{1}{\sqrt{\frac{(a^2 c + b^2 d) \left(c + \frac{a^2 c + b^2 d}{(a - b x)^2} - \frac{2 a c}{a - b x}\right)}{b^2 c d}}}\right] + 2 \operatorname{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] \right. \\
 & \left. \left(i \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] - \operatorname{Log}\left[1 - e^{2 \left(i \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] + \operatorname{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]}\right)}\right] \right) + \\
 & \left. \operatorname{Log}\left[\operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] - i \operatorname{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]\right]\right] \right) - \\
 & \left. \operatorname{PolyLog}\left[2, e^{2 \left(i \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] + \operatorname{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]}\right)}\right] \right) \right) -
 \end{aligned}$$

$$\begin{aligned}
 & \frac{1}{2 b^2 d (-i a c + a^2 c + b^2 d) \sqrt{1 - \frac{(-i a c + a^2 c + b^2 d)^2}{b^2 c d}}} i a^5 c^2 \left(e^{\operatorname{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]} \right. \\
 & \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]^2 - \frac{1}{b \sqrt{c} \sqrt{d} \sqrt{1 - \frac{(-i a c + a^2 c + b^2 d)^2}{b^2 c d}}} \\
 & \left. (-i a c + a^2 c + b^2 d) \left(\pi \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] - i \pi \operatorname{Log}\left[1 + e^{-2 i \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]}\right] - \right. \right. \\
 & \left. \left. 2 i \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] \operatorname{Log}\left[1 - e^{2 \left(i \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] + \operatorname{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]\right)}\right] + \right. \\
 & \left. i \pi \operatorname{Log}\left[\frac{1}{\sqrt{\frac{(a^2 c + b^2 d) \left(c + \frac{a^2 c + b^2 d}{(a - b x)^2} - \frac{2 a c}{a - b x}\right)}{b^2 c d}}}\right] + 2 \operatorname{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] \right. \\
 & \left. \left(i \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] - \operatorname{Log}\left[1 - e^{2 \left(i \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] + \operatorname{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]\right)}\right] \right) + \\
 & \left. \operatorname{Log}\left[\operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] - i \operatorname{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]\right]\right] \right) - \\
 & \left. \operatorname{PolyLog}\left[2, e^{2 \left(i \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] + \operatorname{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]\right)}\right] \right) + \left. \right)
 \end{aligned}$$

$$\begin{aligned}
 & \frac{1}{4 b^2 d (-i a c + a^2 c + b^2 d) \sqrt{1 - \frac{(-i a c + a^2 c + b^2 d)^2}{b^2 c d}}} a^6 c^2 \left(e^{\operatorname{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]} \right. \\
 & \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]^2 - \frac{1}{b \sqrt{c} \sqrt{d} \sqrt{1 - \frac{(-i a c + a^2 c + b^2 d)^2}{b^2 c d}}} \\
 & \left. (-i a c + a^2 c + b^2 d) \left(\pi \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] - i \pi \operatorname{Log}\left[1 + e^{-2 i \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]}\right] - \right. \right. \\
 & \left. \left. 2 i \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] \operatorname{Log}\left[1 - e^{2 \left(i \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] + \operatorname{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]}\right)}\right] + \right. \\
 & \left. i \pi \operatorname{Log}\left[\frac{1}{\sqrt{\frac{(a^2 c + b^2 d) \left(c + \frac{a^2 c + b^2 d}{(a - b x)^2} - \frac{2 a c}{a - b x}\right)}{b^2 c d}}}\right] + 2 \operatorname{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] \right. \\
 & \left. \left(i \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] - \operatorname{Log}\left[1 - e^{2 \left(i \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] + \operatorname{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]}\right)}\right] + \right. \\
 & \left. \left. \operatorname{Log}\left[\operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] - i \operatorname{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]\right]\right] \right) - \right. \\
 & \left. \left. \operatorname{PolyLog}\left[2, e^{2 \left(i \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] + \operatorname{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]}\right)}\right] \right) \right) -
 \end{aligned}$$

$$\begin{aligned}
 & \frac{1}{4(-i a c + a^2 c + b^2 d) \sqrt{1 - \frac{(-i a c + a^2 c + b^2 d)^2}{b^2 c d}}} b^2 d \left(e^{\text{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]} \right. \\
 & \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]^2 - \frac{1}{b \sqrt{c} \sqrt{d} \sqrt{1 - \frac{(-i a c + a^2 c + b^2 d)^2}{b^2 c d}}} \\
 & \left. (-i a c + a^2 c + b^2 d) \left(\pi \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] - i \pi \text{Log}\left[1 + e^{-2 i \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]}\right] - \right. \right. \\
 & \left. \left. 2 i \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] \text{Log}\left[1 - e^{2 \left(i \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] + \text{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]\right)}\right] + \right. \\
 & \left. i \pi \text{Log}\left[\frac{1}{\sqrt{\frac{(a^2 c + b^2 d) \left(c + \frac{a^2 c + b^2 d}{(a - b x)^2} - \frac{2 a c}{a - b x}\right)}{b^2 c d}}}\right] + 2 \text{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] \right. \\
 & \left. \left(i \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] - \text{Log}\left[1 - e^{2 \left(i \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] + \text{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]\right)}\right] \right) + \\
 & \left. \text{Log}\left[\text{Sin}\left[\text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] - i \text{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]\right]\right] \right) - \\
 & \left. \left. \text{PolyLog}\left[2, e^{2 \left(i \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] + \text{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]\right)}\right] \right) \right) -
 \end{aligned}$$

$$\begin{aligned}
 & \frac{1}{2(-i a c + a^2 c + b^2 d) \sqrt{1 - \frac{(-i a c + a^2 c + b^2 d)^2}{b^2 c d}}} i a b^2 d \left(e^{\text{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]} \right. \\
 & \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]^2 - \frac{1}{b \sqrt{c} \sqrt{d} \sqrt{1 - \frac{(-i a c + a^2 c + b^2 d)^2}{b^2 c d}}} \\
 & \left. (-i a c + a^2 c + b^2 d) \left(\pi \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] - i \pi \text{Log}\left[1 + e^{-2 i \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]}\right] - \right. \right. \\
 & \left. \left. 2 i \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] \text{Log}\left[1 - e^{2 \left(i \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] + \text{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]}\right)}\right] + \right. \\
 & \left. i \pi \text{Log}\left[\frac{1}{\sqrt{\frac{(a^2 c + b^2 d) \left(c + \frac{a^2 c + b^2 d}{(a - b x)^2} - \frac{2 a c}{a - b x}\right)}{b^2 c d}}}\right] + 2 \text{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] \right. \\
 & \left. \left(i \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] - \text{Log}\left[1 - e^{2 \left(i \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] + \text{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]}\right)}\right] + \right. \\
 & \left. \left. \text{Log}\left[\text{Sin}\left[\text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] - i \text{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]\right]\right] \right) - \right. \\
 & \left. \left. \text{PolyLog}\left[2, e^{2 \left(i \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] + \text{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]}\right)}\right] \right) \right) +
 \end{aligned}$$

$$\begin{aligned}
 & \frac{1}{4(-i a c + a^2 c + b^2 d) \sqrt{1 - \frac{(-i a c + a^2 c + b^2 d)^2}{b^2 c d}}} - 3 a^2 b^2 d \left(e^{\operatorname{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]} \right. \\
 & \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]^2 - \frac{1}{b \sqrt{c} \sqrt{d} \sqrt{1 - \frac{(-i a c + a^2 c + b^2 d)^2}{b^2 c d}}} \\
 & \left. (-i a c + a^2 c + b^2 d) \left(\pi \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] - i \pi \operatorname{Log}\left[1 + e^{-2 i \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]}\right] - \right. \right. \\
 & \left. \left. 2 i \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] \operatorname{Log}\left[1 - e^{2 \left(i \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] + \operatorname{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]\right)}\right] + \right. \\
 & \left. i \pi \operatorname{Log}\left[\frac{1}{\sqrt{\frac{(a^2 c + b^2 d) \left(c + \frac{a^2 c + b^2 d}{(a + b x)^2} - \frac{2 a c}{a + b x}\right)}}{b^2 c d}}\right] + 2 \operatorname{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] \right. \\
 & \left. \left(i \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] - \operatorname{Log}\left[1 - e^{2 \left(i \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] + \operatorname{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]\right)}\right] \right) + \\
 & \left. \operatorname{Log}\left[\operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] - i \operatorname{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]\right]\right] \right) - \\
 & \left. \operatorname{PolyLog}\left[2, e^{2 \left(i \operatorname{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] + \operatorname{ArcTanh}\left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]\right)}\right] \right) + \left. \right)
 \end{aligned}$$

$$\begin{aligned}
 & \frac{1}{4c(-ia + a^2c + b^2d)\sqrt{1 - \frac{(-ia + a^2c + b^2d)^2}{b^2cd}}} b^4 d^2 \left(e^{\text{ArcTanh}\left[\frac{-ia + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]} \right. \\
 & \text{ArcTan}\left[\frac{ac - \frac{a^2c + b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right]^2 - \frac{1}{b\sqrt{c}\sqrt{d}\sqrt{1 - \frac{(-ia + a^2c + b^2d)^2}{b^2cd}}} \\
 & \left. (-ia + a^2c + b^2d) \left(\pi \text{ArcTan}\left[\frac{ac - \frac{a^2c + b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] - i\pi \text{Log}\left[1 + e^{-2i \text{ArcTan}\left[\frac{ac - \frac{a^2c + b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right]}\right] - \right. \right. \\
 & \left. \left. 2i \text{ArcTan}\left[\frac{ac - \frac{a^2c + b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] \text{Log}\left[1 - e^{2\left(i \text{ArcTan}\left[\frac{ac - \frac{a^2c + b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] + \text{ArcTanh}\left[\frac{-ia + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]}\right]\right] + \right. \\
 & \left. i\pi \text{Log}\left[\frac{1}{\sqrt{\frac{(a^2c + b^2d)\left(c + \frac{a^2c + b^2d}{(a-bx)^2} - \frac{2ac}{a-bx}\right)}}{b^2cd}}\right] + 2 \text{ArcTanh}\left[\frac{-ia + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] \right. \\
 & \left. \left(i \text{ArcTan}\left[\frac{ac - \frac{a^2c + b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] - \text{Log}\left[1 - e^{2\left(i \text{ArcTan}\left[\frac{ac - \frac{a^2c + b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] + \text{ArcTanh}\left[\frac{-ia + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]}\right]\right) \right) + \\
 & \left. \text{Log}\left[\text{Sin}\left[\text{ArcTan}\left[\frac{ac - \frac{a^2c + b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] - i \text{ArcTanh}\left[\frac{-ia + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]\right]\right] \right) - \\
 & \left. \left. \left. \text{PolyLog}\left[2, e^{2\left(i \text{ArcTan}\left[\frac{ac - \frac{a^2c + b^2d}{a+bx}}{b\sqrt{c}\sqrt{d}}\right] + \text{ArcTanh}\left[\frac{-ia + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]}\right)}\right] \right) \right) \right) +
 \end{aligned}$$

$$\begin{aligned}
 & \frac{1}{2 b \sqrt{d} \left(1 - \frac{(-i a c + a^2 c + b^2 d)^2}{b^2 c d}\right)} a^2 \sqrt{c} \left(\pi \operatorname{ArcTan} \left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] - \right. \\
 & i \pi \operatorname{Log} \left[1 + e^{-2 i \operatorname{ArcTan} \left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right]} \right] - 2 i \operatorname{ArcTan} \left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] \\
 & \operatorname{Log} \left[1 - e^{2 \left(i \operatorname{ArcTan} \left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] + \operatorname{ArcTanh} \left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \right)} \right] + \\
 & i \pi \operatorname{Log} \left[\frac{1}{\sqrt{\frac{(a^2 c + b^2 d) \left(c + \frac{a^2 c + b^2 d}{(a + b x)^2} - \frac{2 a c}{a + b x} \right)}{b^2 c d}}} \right] + 2 \operatorname{ArcTanh} \left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \\
 & \left(i \operatorname{ArcTan} \left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] - \operatorname{Log} \left[1 - e^{2 \left(i \operatorname{ArcTan} \left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] + \operatorname{ArcTanh} \left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \right)} \right] \right) + \\
 & \operatorname{Log} \left[\operatorname{Sin} \left[\operatorname{ArcTan} \left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] - i \operatorname{ArcTanh} \left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \right] \right] \left. \right) - \\
 & \operatorname{PolyLog} \left[2, e^{2 \left(i \operatorname{ArcTan} \left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] + \operatorname{ArcTanh} \left[\frac{-i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \right)} \right] - \\
 & \frac{1}{2 (i a c + a^2 c + b^2 d) \sqrt{-\frac{-b^2 c d + (i a c + a^2 c + b^2 d)^2}{b^2 c d}}} a^2 c \left(e^{-\operatorname{ArcTanh} \left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right]} \right. \\
 & \left. \operatorname{ArcTan} \left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right]^2 + \frac{1}{b \sqrt{c} \sqrt{d} \sqrt{1 - \frac{(i a c + a^2 c + b^2 d)^2}{b^2 c d}}} i (i a c + a^2 c + b^2 d) \right)
 \end{aligned}$$

$$\begin{aligned}
 & \left(i \operatorname{ArcTan} \left[\frac{a c - a^2 c + b^2 d}{a + b x} \right] \left(-\pi + 2 i \operatorname{ArcTanh} \left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \right) - \right. \\
 & \pi \operatorname{Log} \left[1 + e^{-2 i \operatorname{ArcTan} \left[\frac{a c - a^2 c + b^2 d}{a + b x} \right]} \right] - 2 \left(\operatorname{ArcTan} \left[\frac{a c - a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] + i \operatorname{ArcTanh} \left[\right. \right. \\
 & \left. \left. \frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \right) \operatorname{Log} \left[1 - e^{2 i \left(\operatorname{ArcTan} \left[\frac{a c - a^2 c + b^2 d}{a + b x} \right] + i \operatorname{ArcTanh} \left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \right)} \right] + \\
 & \pi \operatorname{Log} \left[\frac{1}{\sqrt{\frac{(a^2 c + b^2 d) \left(c + \frac{a^2 c + b^2 d}{(a + b x)^2} \frac{2 a c}{a + b x} \right)}{b^2 c d}}} \right] + 2 i \operatorname{ArcTanh} \left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \\
 & \operatorname{Log} \left[\operatorname{Sin} \left[\operatorname{ArcTan} \left[\frac{a c - a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] + i \operatorname{ArcTanh} \left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \right] \right] + \\
 & \left. i \operatorname{PolyLog} \left[2, e^{2 i \left(\operatorname{ArcTan} \left[\frac{a c - a^2 c + b^2 d}{a + b x} \right] + i \operatorname{ArcTanh} \left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \right)} \right] \right) + \\
 & \frac{1}{(i a c + a^2 c + b^2 d) \sqrt{-\frac{b^2 c d + (i a c + a^2 c + b^2 d)^2}{b^2 c d}}} i a^3 c \left(e^{-\operatorname{ArcTanh} \left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right]} \right. \\
 & \operatorname{ArcTan} \left[\frac{a c - a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right]^2 + \frac{1}{b \sqrt{c} \sqrt{d} \sqrt{1 - \frac{(i a c + a^2 c + b^2 d)^2}{b^2 c d}}} i (i a c + a^2 c + b^2 d) \\
 & \left. \left(i \operatorname{ArcTan} \left[\frac{a c - a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \left(-\pi + 2 i \operatorname{ArcTanh} \left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \right) - \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & \pi \operatorname{Log}\left[1 + e^{-2i \operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]}\right] - 2 \left(\operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] + i \operatorname{ArcTanh}\left[\frac{iac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] \right) \\
 & \left. \operatorname{Log}\left[1 - e^{2i \left(\operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] + i \operatorname{ArcTanh}\left[\frac{iac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] \right)}\right] \right) + \\
 & \pi \operatorname{Log}\left[\frac{1}{\sqrt{\frac{(a^2c + b^2d)\left(c + \frac{a^2c + b^2d}{(a+bx)^2} - \frac{2ac}{a+bx}\right)}{b^2cd}}}\right] + 2i \operatorname{ArcTanh}\left[\frac{iac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] \\
 & \operatorname{Log}\left[\operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] + i \operatorname{ArcTanh}\left[\frac{iac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]\right]\right] + \\
 & \left. i \operatorname{PolyLog}\left[2, e^{2i \left(\operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] + i \operatorname{ArcTanh}\left[\frac{iac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] \right)}\right] \right) \right) + \\
 & \frac{1}{4(iac + a^2c + b^2d)\sqrt{-\frac{-b^2cd + (iac + a^2c + b^2d)^2}{b^2cd}}} 3a^4c \left(e^{-\operatorname{ArcTanh}\left[\frac{iac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]} \right. \\
 & \left. \operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]^2 + \frac{1}{b\sqrt{c}\sqrt{d}\sqrt{1 - \frac{(iac + a^2c + b^2d)^2}{b^2cd}}} i(iac + a^2c + b^2d) \right. \\
 & \left. \left(i \operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] \left(-\pi + 2i \operatorname{ArcTanh}\left[\frac{iac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] \right) \right) - \right. \\
 & \left. \pi \operatorname{Log}\left[1 + e^{-2i \operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]}\right] - 2 \left(\operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] + i \operatorname{ArcTanh}\left[\frac{iac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] \right) \right. \\
 & \left. \operatorname{Log}\left[1 - e^{2i \left(\operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] + i \operatorname{ArcTanh}\left[\frac{iac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] \right)}\right] \right) +
 \end{aligned}$$

$$\begin{aligned}
 & \pi \operatorname{Log} \left[\frac{1}{\sqrt{\frac{(a^2 c + b^2 d) \left(c + \frac{a^2 c + b^2 d}{(a + b x)^2} - \frac{2 a c}{a + b x} \right)}{b^2 c d}}} \right] + 2 i \operatorname{ArcTanh} \left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \\
 & \operatorname{Log} \left[\operatorname{Sin} \left[\operatorname{ArcTan} \left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] + i \operatorname{ArcTanh} \left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \right] \right] + \\
 & i \operatorname{PolyLog} \left[2, e^{2 i \left(\operatorname{ArcTan} \left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] + i \operatorname{ArcTanh} \left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \right)} \right] \Bigg) - \\
 & \frac{1}{4 b^2 d (i a c + a^2 c + b^2 d) \sqrt{-\frac{-b^2 c d + (i a c + a^2 c + b^2 d)^2}{b^2 c d}}} a^4 c^2 \left(e^{-\operatorname{ArcTanh} \left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right]} \right. \\
 & \operatorname{ArcTan} \left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right]^2 + \frac{1}{b \sqrt{c} \sqrt{d} \sqrt{1 - \frac{(i a c + a^2 c + b^2 d)^2}{b^2 c d}}} i (i a c + a^2 c + b^2 d) \\
 & \left. \left(i \operatorname{ArcTan} \left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] \left(-\pi + 2 i \operatorname{ArcTanh} \left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \right) - \right. \right. \\
 & \left. \left. \pi \operatorname{Log} \left[1 + e^{-2 i \operatorname{ArcTan} \left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right]} \right] - 2 \left(\operatorname{ArcTan} \left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] + i \operatorname{ArcTanh} \left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \right) \right. \right. \\
 & \left. \left. \frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right) \operatorname{Log} \left[1 - e^{2 i \left(\operatorname{ArcTan} \left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] + i \operatorname{ArcTanh} \left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \right)} \right] \right] + \\
 & \pi \operatorname{Log} \left[\frac{1}{\sqrt{\frac{(a^2 c + b^2 d) \left(c + \frac{a^2 c + b^2 d}{(a + b x)^2} - \frac{2 a c}{a + b x} \right)}{b^2 c d}}} \right] + 2 i \operatorname{ArcTanh} \left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right]
 \end{aligned}$$

$$\begin{aligned}
 & \frac{1}{4 (i a c + a^2 c + b^2 d) \sqrt{-\frac{-b^2 c d + (i a c + a^2 c + b^2 d)^2}{b^2 c d}}} b^2 d \left(e^{-\text{ArcTanh}\left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]} \right. \\
 & \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]^2 + \frac{1}{b \sqrt{c} \sqrt{d} \sqrt{1 - \frac{(i a c + a^2 c + b^2 d)^2}{b^2 c d}}} i (i a c + a^2 c + b^2 d) \\
 & \left(i \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] \left(-\pi + 2 i \text{ArcTanh}\left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] \right) - \right. \\
 & \pi \text{Log}\left[1 + e^{-2 i \text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right]}\right] - 2 \left(\text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] + i \text{ArcTanh}\left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] \right) \\
 & \left. \left. \text{Log}\left[1 - e^{2 i \left(\text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] + i \text{ArcTanh}\left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] \right)}\right] \right) + \right. \\
 & \pi \text{Log}\left[\frac{1}{\sqrt{\frac{(a^2 c + b^2 d) \left(c + \frac{a^2 c + b^2 d}{(a + b x)^2} - \frac{2 a c}{a + b x} \right)}{b^2 c d}}}\right] + 2 i \text{ArcTanh}\left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] \\
 & \text{Log}\left[\text{Sin}\left[\text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] + i \text{ArcTanh}\left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]\right]\right] + \\
 & i \text{PolyLog}\left[2, e^{2 i \left(\text{ArcTan}\left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}}\right] + i \text{ArcTanh}\left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] \right)}\right] \left. \right) + \\
 & \frac{1}{2 (i a c + a^2 c + b^2 d) \sqrt{-\frac{-b^2 c d + (i a c + a^2 c + b^2 d)^2}{b^2 c d}}} i a b^2 d \left(e^{-\text{ArcTanh}\left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]} \right.
 \end{aligned}$$

$$\begin{aligned}
 & \text{ArcTan}\left[\frac{a c - a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]^2 + \frac{1}{b \sqrt{c} \sqrt{d} \sqrt{1 - \frac{(i a c + a^2 c + b^2 d)^2}{b^2 c d}}} i (i a c + a^2 c + b^2 d) \\
 & \left(i \text{ArcTan}\left[\frac{a c - a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] \left(-\pi + 2 i \text{ArcTanh}\left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] \right) - \right. \\
 & \pi \text{Log}\left[1 + e^{-2 i \text{ArcTan}\left[\frac{a c - a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]}\right] - 2 \left(\text{ArcTan}\left[\frac{a c - a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] + i \text{ArcTanh}\left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] \right) \\
 & \left. \text{Log}\left[1 - e^{2 i \left(\text{ArcTan}\left[\frac{a c - a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] + i \text{ArcTanh}\left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] \right)}\right] \right) + \\
 & \pi \text{Log}\left[\frac{1}{\sqrt{\frac{(a^2 c + b^2 d) \left(c + \frac{a^2 c + b^2 d}{(a + b x)^2} - \frac{2 a c}{a + b x} \right)}{b^2 c d}}}\right] + 2 i \text{ArcTanh}\left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] \\
 & \text{Log}\left[\text{Sin}\left[\text{ArcTan}\left[\frac{a c - a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] + i \text{ArcTanh}\left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]\right]\right] + \\
 & i \text{PolyLog}\left[2, e^{2 i \left(\text{ArcTan}\left[\frac{a c - a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] + i \text{ArcTanh}\left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right] \right)}\right] \left. \right) + \\
 & \frac{1}{4 (i a c + a^2 c + b^2 d) \sqrt{-\frac{-b^2 c d + (i a c + a^2 c + b^2 d)^2}{b^2 c d}}} 3 a^2 b^2 d \left(e^{-\text{ArcTanh}\left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]} \right. \\
 & \left. \text{ArcTan}\left[\frac{a c - a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}}\right]^2 + \frac{1}{b \sqrt{c} \sqrt{d} \sqrt{1 - \frac{(i a c + a^2 c + b^2 d)^2}{b^2 c d}}} i (i a c + a^2 c + b^2 d) \right)
 \end{aligned}$$

$$\left(\begin{aligned}
 & i \operatorname{ArcTan} \left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] \left(-\pi + 2 i \operatorname{ArcTanh} \left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \right) - \\
 & \pi \operatorname{Log} \left[1 + e^{-2 i \operatorname{ArcTan} \left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right]} \right] - 2 \left(\operatorname{ArcTan} \left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] + i \operatorname{ArcTanh} \left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \right) \\
 & \left. \operatorname{Log} \left[1 - e^{2 i \left(\operatorname{ArcTan} \left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] + i \operatorname{ArcTanh} \left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \right)} \right] \right) + \\
 & \pi \operatorname{Log} \left[\frac{1}{\sqrt{\frac{(a^2 c + b^2 d) \left(c + \frac{a^2 c + b^2 d}{(a + b x)^2} - \frac{2 a c}{a + b x} \right)}{b^2 c d}}} \right] + 2 i \operatorname{ArcTanh} \left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \\
 & \operatorname{Log} \left[\operatorname{Sin} \left[\operatorname{ArcTan} \left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] + i \operatorname{ArcTanh} \left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \right] \right] + \\
 & \left. i \operatorname{PolyLog} \left[2, e^{2 i \left(\operatorname{ArcTan} \left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] + i \operatorname{ArcTanh} \left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \right)} \right] \right) \right) + \\
 & \frac{1}{4 c (i a c + a^2 c + b^2 d) \sqrt{-\frac{b^2 c d + (i a c + a^2 c + b^2 d)^2}{b^2 c d}}} b^4 d^2 \left(e^{-\operatorname{ArcTanh} \left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right]} \right. \\
 & \left. \operatorname{ArcTan} \left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right]^2 + \frac{1}{b \sqrt{c} \sqrt{d} \sqrt{1 - \frac{(i a c + a^2 c + b^2 d)^2}{b^2 c d}}} i (i a c + a^2 c + b^2 d) \right) \\
 & \left(\begin{aligned}
 & i \operatorname{ArcTan} \left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] \left(-\pi + 2 i \operatorname{ArcTanh} \left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \right) - \\
 & \pi \operatorname{Log} \left[1 + e^{-2 i \operatorname{ArcTan} \left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right]} \right] - 2 \left(\operatorname{ArcTan} \left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] + i \operatorname{ArcTanh} \left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \right) \\
 & \left. \operatorname{Log} \left[1 - e^{2 i \left(\operatorname{ArcTan} \left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] + i \operatorname{ArcTanh} \left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \right)} \right] \right) + \\
 & \pi \operatorname{Log} \left[\frac{1}{\sqrt{\frac{(a^2 c + b^2 d) \left(c + \frac{a^2 c + b^2 d}{(a + b x)^2} - \frac{2 a c}{a + b x} \right)}{b^2 c d}}} \right] + 2 i \operatorname{ArcTanh} \left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \\
 & \operatorname{Log} \left[\operatorname{Sin} \left[\operatorname{ArcTan} \left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] + i \operatorname{ArcTanh} \left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \right] \right] + \\
 & \left. i \operatorname{PolyLog} \left[2, e^{2 i \left(\operatorname{ArcTan} \left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right] + i \operatorname{ArcTanh} \left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right] \right)} \right] \right) \right) + \\
 & \frac{1}{4 c (i a c + a^2 c + b^2 d) \sqrt{-\frac{b^2 c d + (i a c + a^2 c + b^2 d)^2}{b^2 c d}}} b^4 d^2 \left(e^{-\operatorname{ArcTanh} \left[\frac{i a c + a^2 c + b^2 d}{b \sqrt{c} \sqrt{d}} \right]} \right. \\
 & \left. \operatorname{ArcTan} \left[\frac{a c - \frac{a^2 c + b^2 d}{a + b x}}{b \sqrt{c} \sqrt{d}} \right]^2 + \frac{1}{b \sqrt{c} \sqrt{d} \sqrt{1 - \frac{(i a c + a^2 c + b^2 d)^2}{b^2 c d}}} i (i a c + a^2 c + b^2 d) \right)
 \end{aligned} \right)$$

$$\begin{aligned} & \pi \operatorname{Log}\left[1 + e^{-2i \operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]}\right] - 2 \left(\operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] + i \operatorname{ArcTanh}\left[\frac{i ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] \right) \operatorname{Log}\left[1 - e^{2i \left(\operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] + i \operatorname{ArcTanh}\left[\frac{i ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] \right)}\right] + \\ & \pi \operatorname{Log}\left[\frac{1}{\sqrt{\frac{(a^2c + b^2d)\left(c + \frac{a^2c + b^2d}{(a+bx)^2} - \frac{2ac}{a+bx}\right)}{b^2cd}}}\right] + 2i \operatorname{ArcTanh}\left[\frac{i ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] \\ & \operatorname{Log}\left[\operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] + i \operatorname{ArcTanh}\left[\frac{i ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right]\right]\right] + \\ & i \operatorname{PolyLog}\left[2, e^{2i \left(\operatorname{ArcTan}\left[\frac{ac - a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] + i \operatorname{ArcTanh}\left[\frac{i ac + a^2c + b^2d}{b\sqrt{c}\sqrt{d}}\right] \right)}\right] \end{aligned}$$

Problem 111: Result is not expressed in closed-form.

$$\int \frac{\operatorname{ArcCot}[a + bx]}{c + d\sqrt{x}} dx$$

Optimal (type 4, 693 leaves, 55 steps):

$$\begin{aligned} & -\frac{2i\sqrt{i+a}\operatorname{ArcTan}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right]}{\sqrt{b}d} + \frac{2i\sqrt{i-a}\operatorname{ArcTanh}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right]}{\sqrt{b}d} - \\ & \frac{i c \operatorname{Log}\left[\frac{d(\sqrt{-i-a}-\sqrt{b}\sqrt{x})}{\sqrt{b}c+\sqrt{-i-a}d}\right] \operatorname{Log}[c+d\sqrt{x}]}{d^2} + \frac{i c \operatorname{Log}\left[\frac{d(\sqrt{i-a}-\sqrt{b}\sqrt{x})}{\sqrt{b}c+\sqrt{i-a}d}\right] \operatorname{Log}[c+d\sqrt{x}]}{d^2} - \\ & \frac{i c \operatorname{Log}\left[-\frac{d(\sqrt{-i-a}+\sqrt{b}\sqrt{x})}{\sqrt{b}c-\sqrt{-i-a}d}\right] \operatorname{Log}[c+d\sqrt{x}]}{d^2} + \frac{i c \operatorname{Log}\left[-\frac{d(\sqrt{i-a}+\sqrt{b}\sqrt{x})}{\sqrt{b}c-\sqrt{i-a}d}\right] \operatorname{Log}[c+d\sqrt{x}]}{d^2} + \\ & \frac{i\sqrt{x}\operatorname{Log}\left[-\frac{i-a-bx}{a+bx}\right]}{d} - \frac{i c \operatorname{Log}[c+d\sqrt{x}]\operatorname{Log}\left[-\frac{i-a-bx}{a+bx}\right]}{d^2} - \frac{i\sqrt{x}\operatorname{Log}\left[\frac{i+a+bx}{a+bx}\right]}{d} + \\ & \frac{i c \operatorname{Log}[c+d\sqrt{x}]\operatorname{Log}\left[\frac{i+a+bx}{a+bx}\right]}{d^2} - \frac{i c \operatorname{PolyLog}\left[2, \frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c-\sqrt{-i-a}d}\right]}{d^2} - \\ & \frac{i c \operatorname{PolyLog}\left[2, \frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c+\sqrt{-i-a}d}\right]}{d^2} + \frac{i c \operatorname{PolyLog}\left[2, \frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c-\sqrt{i-a}d}\right]}{d^2} + \frac{i c \operatorname{PolyLog}\left[2, \frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c+\sqrt{i-a}d}\right]}{d^2} \end{aligned}$$

Result (type 7, 313 leaves):

$$\frac{1}{2 d^2} \left(4 \operatorname{ArcCot}[a + b x] \left(d \sqrt{x} - c \operatorname{Log}[c + d \sqrt{x}] \right) + \right. \\ \left. \frac{1}{\sqrt{b}} d \left(\frac{4 (1 + i a) \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{x}}{\sqrt{-i+a}}\right]}{\sqrt{-i+a}} + \frac{4 (1 - i a) \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{x}}{\sqrt{i+a}}\right]}{\sqrt{i+a}} - \right. \right. \\ \left. \left. \sqrt{b} c d \operatorname{RootSum}\left[b^2 c^4 + 2 a b c^2 d^2 + d^4 + a^2 d^4 - 4 b^2 c^3 \#1 - 4 a b c d^2 \#1 + 6 b^2 c^2 \#1^2 + 2 a b d^2 \#1^2 - \right. \right. \\ \left. \left. 4 b^2 c \#1^3 + b^2 \#1^4 \&, \left(-\operatorname{Log}[c + d \sqrt{x}]^2 + 2 \operatorname{Log}[c + d \sqrt{x}] \operatorname{Log}\left[1 - \frac{c + d \sqrt{x}}{\#1}\right] + \right. \right. \\ \left. \left. 2 \operatorname{PolyLog}\left[2, \frac{c + d \sqrt{x}}{\#1}\right]\right) / \left(b c^2 + a d^2 - 2 b c \#1 + b \#1^2 \right) \& \right) \right)$$

Problem 112: Unable to integrate problem.

$$\int \frac{\operatorname{ArcCot}[a + b x]}{c + \frac{d}{\sqrt{x}}} dx$$

Optimal (type 4, 830 leaves, 65 steps):

$$\frac{2 i \sqrt{i+a} d \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{x}}{\sqrt{i+a}}\right]}{\sqrt{b} c^2} - \frac{2 i \sqrt{i-a} d \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{x}}{\sqrt{i-a}}\right]}{\sqrt{b} c^2} + \\ \frac{i d^2 \operatorname{Log}\left[\frac{c(\sqrt{-i-a}-\sqrt{b} \sqrt{x})}{\sqrt{-i-a} c+\sqrt{b} d}\right] \operatorname{Log}[d+c \sqrt{x}]}{c^3} - \frac{i d^2 \operatorname{Log}\left[\frac{c(\sqrt{i-a}-\sqrt{b} \sqrt{x})}{\sqrt{i-a} c+\sqrt{b} d}\right] \operatorname{Log}[d+c \sqrt{x}]}{c^3} + \\ \frac{i d^2 \operatorname{Log}\left[\frac{c(\sqrt{-i-a}+\sqrt{b} \sqrt{x})}{\sqrt{-i-a} c-\sqrt{b} d}\right] \operatorname{Log}[d+c \sqrt{x}]}{c^3} - \frac{i d^2 \operatorname{Log}\left[\frac{c(\sqrt{i-a}+\sqrt{b} \sqrt{x})}{\sqrt{i-a} c-\sqrt{b} d}\right] \operatorname{Log}[d+c \sqrt{x}]}{c^3} + \\ \frac{(1+i a) \operatorname{Log}[i-a-b x]}{2 b c} - \frac{i d \sqrt{x} \operatorname{Log}\left[-\frac{i-a-b x}{a+b x}\right]}{c^2} + \frac{i x \operatorname{Log}\left[-\frac{i-a-b x}{a+b x}\right]}{2 c} + \\ \frac{i d^2 \operatorname{Log}[d+c \sqrt{x}] \operatorname{Log}\left[-\frac{i-a-b x}{a+b x}\right]}{c^3} + \frac{(1-i a) \operatorname{Log}[i+a+b x]}{2 b c} + \frac{i d \sqrt{x} \operatorname{Log}\left[\frac{i+a+b x}{a+b x}\right]}{c^2} - \\ \frac{i x \operatorname{Log}\left[\frac{i+a+b x}{a+b x}\right]}{2 c} - \frac{i d^2 \operatorname{Log}[d+c \sqrt{x}] \operatorname{Log}\left[\frac{i+a+b x}{a+b x}\right]}{c^3} + \frac{i d^2 \operatorname{PolyLog}\left[2, -\frac{\sqrt{b}(d+c \sqrt{x})}{\sqrt{-i-a} c-\sqrt{b} d}\right]}{c^3} - \\ \frac{i d^2 \operatorname{PolyLog}\left[2, -\frac{\sqrt{b}(d+c \sqrt{x})}{\sqrt{i-a} c+\sqrt{b} d}\right]}{c^3} + \frac{i d^2 \operatorname{PolyLog}\left[2, \frac{\sqrt{b}(d+c \sqrt{x})}{\sqrt{-i-a} c+\sqrt{b} d}\right]}{c^3} - \frac{i d^2 \operatorname{PolyLog}\left[2, \frac{\sqrt{b}(d+c \sqrt{x})}{\sqrt{i-a} c-\sqrt{b} d}\right]}{c^3}$$

Result (type 8, 20 leaves):

$$\int \frac{\text{ArcCot}[a + b x]}{c + \frac{d}{\sqrt{x}}} dx$$

Problem 113: Attempted integration timed out after 120 seconds.

$$\int \frac{\text{ArcCot}[d + e x]}{a + b x + c x^2} dx$$

Optimal (type 4, 367 leaves, 12 steps):

$$\frac{\text{ArcCot}[d + e x] \text{Log}\left[\frac{2 e \left(b - \sqrt{b^2 - 4 a c} + 2 c x\right)}{\left(2 c (i - d) + \left(b - \sqrt{b^2 - 4 a c}\right) e\right) (1 - i (d + e x))}\right]}{\sqrt{b^2 - 4 a c}} -$$

$$\frac{\text{ArcCot}[d + e x] \text{Log}\left[\frac{2 e \left(b + \sqrt{b^2 - 4 a c} + 2 c x\right)}{\left(2 c (i - d) + \left(b + \sqrt{b^2 - 4 a c}\right) e\right) (1 - i (d + e x))}\right]}{\sqrt{b^2 - 4 a c}} +$$

$$\frac{i \text{PolyLog}\left[2, 1 + \frac{2 \left(2 c d - \left(b - \sqrt{b^2 - 4 a c}\right) e - 2 c (d + e x)\right)}{\left(2 i c - 2 c d + b e - \sqrt{b^2 - 4 a c} e\right) (1 - i (d + e x))}\right]}{2 \sqrt{b^2 - 4 a c}} -$$

$$\frac{i \text{PolyLog}\left[2, 1 + \frac{2 \left(2 c d - \left(b + \sqrt{b^2 - 4 a c}\right) e - 2 c (d + e x)\right)}{\left(2 c (i - d) + \left(b + \sqrt{b^2 - 4 a c}\right) e\right) (1 - i (d + e x))}\right]}{2 \sqrt{b^2 - 4 a c}}$$

Result (type 1, 1 leaves):

???

Problem 126: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcCot}[1 + x]}{2 + 2 x} dx$$

Optimal (type 4, 35 leaves, 5 steps):

$$-\frac{1}{4} i \text{PolyLog}\left[2, -\frac{i}{1 + x}\right] + \frac{1}{4} i \text{PolyLog}\left[2, \frac{i}{1 + x}\right]$$

Result (type 4, 157 leaves):

$$\frac{1}{16} \left(i \pi^2 - 4 i \pi \text{ArcTan}[1 + x] + 8 i \text{ArcTan}[1 + x]^2 + \pi \text{Log}[16] - 4 \pi \text{Log}\left[1 + e^{-2 i \text{ArcTan}[1 + x]}\right] + \right.$$

$$8 \text{ArcTan}[1 + x] \text{Log}\left[1 + e^{-2 i \text{ArcTan}[1 + x]}\right] - 8 \text{ArcTan}[1 + x] \text{Log}\left[1 - e^{2 i \text{ArcTan}[1 + x]}\right] +$$

$$8 \text{ArcCot}[1 + x] \text{Log}[1 + x] + 8 \text{ArcTan}[1 + x] \text{Log}[1 + x] - 2 \pi \text{Log}\left[2 + 2 x + x^2\right] +$$

$$\left. 4 i \text{PolyLog}\left[2, -e^{-2 i \text{ArcTan}[1 + x]}\right] + 4 i \text{PolyLog}\left[2, e^{2 i \text{ArcTan}[1 + x]}\right] \right)$$

Problem 127: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcCot}[a + b x]}{\frac{a d}{b} + d x} dx$$

Optimal (type 4, 45 leaves, 5 steps):

$$-\frac{i \text{PolyLog}\left[2, -\frac{i}{a+bx}\right]}{2 d} + \frac{i \text{PolyLog}\left[2, \frac{i}{a+bx}\right]}{2 d}$$

Result (type 4, 195 leaves):

$$\begin{aligned} & \frac{1}{8 d} \left(i \pi^2 - 4 i \pi \text{ArcTan}[a + b x] + 8 i \text{ArcTan}[a + b x]^2 + \pi \text{Log}[16] - 4 \pi \text{Log}\left[1 + e^{-2 i \text{ArcTan}[a+bx]}\right] + \right. \\ & 8 \text{ArcTan}[a + b x] \text{Log}\left[1 + e^{-2 i \text{ArcTan}[a+bx]}\right] - 8 \text{ArcTan}[a + b x] \text{Log}\left[1 - e^{2 i \text{ArcTan}[a+bx]}\right] + \\ & 8 \text{ArcCot}[a + b x] \text{Log}[a + b x] + 8 \text{ArcTan}[a + b x] \text{Log}[a + b x] - 2 \pi \text{Log}\left[1 + a^2 + 2 a b x + b^2 x^2\right] + \\ & \left. 4 i \text{PolyLog}\left[2, -e^{-2 i \text{ArcTan}[a+bx]}\right] + 4 i \text{PolyLog}\left[2, e^{2 i \text{ArcTan}[a+bx]}\right] \right) \end{aligned}$$

Problem 133: Result more than twice size of optimal antiderivative.

$$\int \frac{a + b \text{ArcCot}[c + d x]}{e + f x} dx$$

Optimal (type 4, 162 leaves, 5 steps):

$$\begin{aligned} & -\frac{(a + b \text{ArcCot}[c + d x]) \text{Log}\left[\frac{2}{1-i(c+dx)}\right]}{f} + \frac{(a + b \text{ArcCot}[c + d x]) \text{Log}\left[\frac{2 d (e+fx)}{(de+if-cf)(1-i(c+dx))}\right]}{f} \\ & -\frac{i b \text{PolyLog}\left[2, 1 - \frac{2}{1-i(c+dx)}\right]}{2 f} + \frac{i b \text{PolyLog}\left[2, 1 - \frac{2 d (e+fx)}{(de+if-cf)(1-i(c+dx))}\right]}{2 f} \end{aligned}$$

Result (type 4, 336 leaves):

$$\frac{1}{f} \left(a \operatorname{Log}[e + f x] + b \left(\operatorname{ArcCot}[c + d x] + \operatorname{ArcTan}[c + d x] \right) \operatorname{Log}[e + f x] + \operatorname{ArcTan}[c + d x] \left(\operatorname{Log}\left[\frac{1}{\sqrt{1 + (c + d x)^2}} \right] - \operatorname{Log}\left[\operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{d e - c f}{f} \right] + \operatorname{ArcTan}[c + d x] \right] \right] \right) \right) + \frac{1}{2} \left(\frac{1}{4} i (\pi - 2 \operatorname{ArcTan}[c + d x])^2 + i \left(\operatorname{ArcTan}\left[\frac{d e - c f}{f} \right] + \operatorname{ArcTan}[c + d x] \right)^2 - (\pi - 2 \operatorname{ArcTan}[c + d x]) \operatorname{Log}\left[1 + e^{-2 i \operatorname{ArcTan}[c + d x]} \right] - 2 \left(\operatorname{ArcTan}\left[\frac{d e - c f}{f} \right] + \operatorname{ArcTan}[c + d x] \right) \operatorname{Log}\left[1 - e^{2 i \left(\operatorname{ArcTan}\left[\frac{d e - c f}{f} \right] + \operatorname{ArcTan}[c + d x] \right)} \right] + (\pi - 2 \operatorname{ArcTan}[c + d x]) \operatorname{Log}\left[\frac{2}{\sqrt{1 + (c + d x)^2}} \right] + 2 \left(\operatorname{ArcTan}\left[\frac{d e - c f}{f} \right] + \operatorname{ArcTan}[c + d x] \right) \operatorname{Log}\left[2 \operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{d e - c f}{f} \right] + \operatorname{ArcTan}[c + d x] \right] \right] + i \operatorname{PolyLog}\left[2, -e^{-2 i \operatorname{ArcTan}[c + d x]} \right] + i \operatorname{PolyLog}\left[2, e^{2 i \left(\operatorname{ArcTan}\left[\frac{d e - c f}{f} \right] + \operatorname{ArcTan}[c + d x] \right)} \right] \right) \right)$$

Problem 139: Attempted integration timed out after 120 seconds.

$$\int \frac{(a + b \operatorname{ArcCot}[c + d x])^2}{e + f x} dx$$

Optimal (type 4, 261 leaves, 2 steps):

$$\begin{aligned} & - \frac{(a + b \operatorname{ArcCot}[c + d x])^2 \operatorname{Log}\left[\frac{2}{1 - i(c + d x)} \right]}{f} + \frac{(a + b \operatorname{ArcCot}[c + d x])^2 \operatorname{Log}\left[\frac{2 d (e + f x)}{(d e + i f - c f)(1 - i(c + d x))} \right]}{f} \\ & + \frac{i b (a + b \operatorname{ArcCot}[c + d x]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - i(c + d x)} \right]}{f} \\ & - \frac{i b (a + b \operatorname{ArcCot}[c + d x]) \operatorname{PolyLog}\left[2, 1 - \frac{2 d (e + f x)}{(d e + i f - c f)(1 - i(c + d x))} \right]}{f} \\ & + \frac{b^2 \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 - i(c + d x)} \right]}{2 f} + \frac{b^2 \operatorname{PolyLog}\left[3, 1 - \frac{2 d (e + f x)}{(d e + i f - c f)(1 - i(c + d x))} \right]}{2 f} \end{aligned}$$

Result (type 1, 1 leaves):

???

Problem 140: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b \operatorname{ArcCot}[c + d x])^2}{(e + f x)^2} dx$$

Optimal (type 4, 567 leaves, 25 steps):

$$\begin{aligned} & \frac{i b^2 d \operatorname{ArcCot}[c+d x]^2}{d^2 e^2-2 c d e f+(1+c^2) f^2}+\frac{b^2 d(d e-c f) \operatorname{ArcCot}[c+d x]^2}{f\left(d^2 e^2-2 c d e f+(1+c^2) f^2\right)}- \\ & \frac{(a+b \operatorname{ArcCot}[c+d x])^2}{f(e+f x)}-\frac{2 a b d(d e-c f) \operatorname{ArcTan}[c+d x]}{f\left(f^2+(d e-c f)^2\right)}-\frac{2 a b d \operatorname{Log}[e+f x]}{f^2+(d e-c f)^2}+ \\ & \frac{2 b^2 d \operatorname{ArcCot}[c+d x] \operatorname{Log}\left[\frac{2}{1-i(c+d x)}\right]}{d^2 e^2-2 c d e f+(1+c^2) f^2}-\frac{2 b^2 d \operatorname{ArcCot}[c+d x] \operatorname{Log}\left[\frac{2 d(e+f x)}{(d e+i f-c f)(1-i(c+d x))}\right]}{d^2 e^2-2 c d e f+(1+c^2) f^2}- \\ & \frac{2 b^2 d \operatorname{ArcCot}[c+d x] \operatorname{Log}\left[\frac{2}{1+i(c+d x)}\right]}{d^2 e^2-2 c d e f+(1+c^2) f^2}+\frac{a b d \operatorname{Log}\left[1+(c+d x)^2\right]}{f^2+(d e-c f)^2}+\frac{i b^2 d \operatorname{PolyLog}\left[2,1-\frac{2}{1-i(c+d x)}\right]}{d^2 e^2-2 c d e f+(1+c^2) f^2}- \\ & \frac{i b^2 d \operatorname{PolyLog}\left[2,1-\frac{2 d(e+f x)}{(d e+i f-c f)(1-i(c+d x))}\right]}{d^2 e^2-2 c d e f+(1+c^2) f^2}+\frac{i b^2 d \operatorname{PolyLog}\left[2,1-\frac{2}{1+i(c+d x)}\right]}{d^2 e^2-2 c d e f+(1+c^2) f^2} \end{aligned}$$

Result (type 4, 1188 leaves):

$$\begin{aligned} & -\frac{a^2}{f(e+f x)}-\frac{1}{d f(e+f x)^2} 2 a b\left(1+(c+d x)^2\right) \\ & \left(\frac{f}{\sqrt{1+\frac{1}{(c+d x)^2}}+\frac{d e-c f}{(c+d x) \sqrt{1+\frac{1}{(c+d x)^2}}}\right)^2\left(\frac{\operatorname{ArcCot}[c+d x]}{(c+d x) \sqrt{1+\frac{1}{(c+d x)^2}}\left(\frac{f}{\sqrt{1+\frac{1}{(c+d x)^2}}+\frac{d e-c f}{(c+d x) \sqrt{1+\frac{1}{(c+d x)^2}}}\right)}+\right. \\ & \left.\left(-d e \operatorname{ArcCot}[c+d x]+c f \operatorname{ArcCot}[c+d x]+f \operatorname{Log}\left[-\frac{f}{\sqrt{1+\frac{1}{(c+d x)^2}}}\right]-\right. \right. \\ & \left. \left.\frac{d e}{(c+d x) \sqrt{1+\frac{1}{(c+d x)^2}}+\frac{c f}{(c+d x) \sqrt{1+\frac{1}{(c+d x)^2}}}\right) / \left(d^2 e^2-2 c d e f+(1+c^2) f^2\right)-\right. \\ & \left.\frac{1}{d(e+f x)^2} b^2\left(1+(c+d x)^2\right)\left(\frac{f}{\sqrt{1+\frac{1}{(c+d x)^2}}+\frac{d e-c f}{(c+d x) \sqrt{1+\frac{1}{(c+d x)^2}}}\right)^2\right) \end{aligned}$$

$$\begin{aligned}
 & \left(- \left(\text{ArcCot}[c + d x]^2 / \left(f (c + d x) \sqrt{1 + \frac{1}{(c + d x)^2}} \right. \right. \right. \\
 & \left. \left. \left. \left(- \frac{f}{\sqrt{1 + \frac{1}{(c + d x)^2}}} - \frac{d e}{(c + d x) \sqrt{1 + \frac{1}{(c + d x)^2}}} + \frac{c f}{(c + d x) \sqrt{1 + \frac{1}{(c + d x)^2}} \right) \right) \right) + \right. \\
 & \frac{1}{f} 2 \left(\frac{d e \text{ArcCot}[c + d x]^2}{2 (d^2 e^2 - 2 c d e f + f^2 + c^2 f^2)} - \frac{i f \text{ArcCot}[c + d x]^2}{2 (d^2 e^2 - 2 c d e f + f^2 + c^2 f^2)} - \right. \\
 & \left. \frac{c f \text{ArcCot}[c + d x]^2}{2 (d^2 e^2 - 2 c d e f + f^2 + c^2 f^2)} - \right. \\
 & \left. \left(\text{ArcCot}[c + d x] \left(2 (d e - i f - c f) \text{ArcCot}[c + d x] + 2 i f \text{ArcTan}\left[\frac{1}{c + d x}\right] - \right. \right. \right. \\
 & \left. \left. \left. f \text{Log}\left[\frac{f}{\sqrt{1 + \frac{1}{(c + d x)^2}}} + \frac{d e}{(c + d x) \sqrt{1 + \frac{1}{(c + d x)^2}}} - \frac{c f}{(c + d x) \sqrt{1 + \frac{1}{(c + d x)^2}} \right] \right) \right) \right) / \\
 & \left(2 (d^2 e^2 - 2 c d e f + (1 + c^2) f^2) \right) - \frac{1}{2 (d^2 e^2 - 2 c d e f + (1 + c^2) f^2)} \\
 & \left(- i \pi \text{ArcCot}[c + d x] + c \text{ArcCot}[c + d x]^2 - \frac{d e \text{ArcCot}[c + d x]^2}{f} - \right. \\
 & \left. c e^{i \text{ArcTan}\left[\frac{f}{d e - c f}\right]} \sqrt{\frac{d^2 e^2 - 2 c d e f + (1 + c^2) f^2}{(d e - c f)^2}} \text{ArcCot}[c + d x]^2 + \frac{1}{f} d e e^{i \text{ArcTan}\left[\frac{f}{d e - c f}\right]} \right. \\
 & \left. \sqrt{\frac{d^2 e^2 - 2 c d e f + (1 + c^2) f^2}{(d e - c f)^2}} \text{ArcCot}[c + d x]^2 - i \text{ArcTan}\left[\frac{1}{c + d x}\right]^2 - \right. \\
 & \left. \pi \text{Log}\left[1 + e^{-2 i \text{ArcCot}[c + d x]}\right] - 2 \text{ArcCot}[c + d x] \text{Log}\left[1 - e^{2 i (\text{ArcCot}[c + d x] + \text{ArcTan}\left[\frac{f}{d e - c f}\right])}\right] \right) +
 \end{aligned}$$

$$\begin{aligned}
 & 2 \operatorname{ArcTan}\left[\frac{f}{-d e + c f}\right] \operatorname{Log}\left[1 - e^{2 i \left(\operatorname{ArcCot}[c+d x] + \operatorname{ArcTan}\left[\frac{f}{d e - c f}\right]\right)}\right] + \pi \operatorname{Log}\left[\frac{1}{\sqrt{1 + \frac{1}{(c+d x)^2}}}\right] + \\
 & \operatorname{ArcCot}[c+d x] \operatorname{Log}\left[\left(\frac{f}{\sqrt{1 + \frac{1}{(c+d x)^2}} + \frac{d e - c f}{(c+d x) \sqrt{1 + \frac{1}{(c+d x)^2}}}\right)^2\right] + 2 \operatorname{ArcTan}\left[\frac{f}{d e - c f}\right] \\
 & \left(i \operatorname{ArcCot}[c+d x] + \operatorname{Log}\left[\operatorname{Sin}\left[\operatorname{ArcCot}[c+d x] + \operatorname{ArcTan}\left[\frac{f}{d e - c f}\right]\right]\right]\right) + \\
 & \left(i \operatorname{PolyLog}\left[2, e^{2 i \left(\operatorname{ArcCot}[c+d x] + \operatorname{ArcTan}\left[\frac{f}{d e - c f}\right]\right)}\right]\right)
 \end{aligned}$$

Problem 141: Result more than twice size of optimal antiderivative.

$$\int (e + f x)^2 (a + b \operatorname{ArcCot}[c + d x])^3 dx$$

Optimal (type 4, 565 leaves, 21 steps):

$$\begin{aligned}
 & \frac{a b^2 f^2 x}{d^2} + \frac{b^3 f^2 (c + d x) \operatorname{ArcCot}[c + d x]}{d^3} + \\
 & \frac{b f^2 (a + b \operatorname{ArcCot}[c + d x])^2}{2 d^3} + \frac{3 i b f (d e - c f) (a + b \operatorname{ArcCot}[c + d x])^2}{d^3} + \\
 & \frac{3 b f (d e - c f) (c + d x) (a + b \operatorname{ArcCot}[c + d x])^2}{d^3} + \frac{b f^2 (c + d x)^2 (a + b \operatorname{ArcCot}[c + d x])^2}{2 d^3} + \\
 & \frac{i (3 d^2 e^2 - 6 c d e f - (1 - 3 c^2) f^2) (a + b \operatorname{ArcCot}[c + d x])^3}{3 d^3} - \\
 & \frac{(d e - c f) (d^2 e^2 - 2 c d e f - (3 - c^2) f^2) (a + b \operatorname{ArcCot}[c + d x])^3}{3 d^3 f} + \\
 & \frac{(e + f x)^3 (a + b \operatorname{ArcCot}[c + d x])^3}{3 f} - \frac{6 b^2 f (d e - c f) (a + b \operatorname{ArcCot}[c + d x]) \operatorname{Log}\left[\frac{2}{1+i(c+dx)}\right]}{d^3} - \\
 & \frac{1}{d^3} b (3 d^2 e^2 - 6 c d e f - (1 - 3 c^2) f^2) (a + b \operatorname{ArcCot}[c + d x])^2 \operatorname{Log}\left[\frac{2}{1+i(c+dx)}\right] + \\
 & \frac{b^3 f^2 \operatorname{Log}\left[1 + (c + d x)^2\right]}{2 d^3} + \frac{3 i b^3 f (d e - c f) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+i(c+dx)}\right]}{d^3} + \frac{1}{d^3} \\
 & i b^2 (3 d^2 e^2 - 6 c d e f - (1 - 3 c^2) f^2) (a + b \operatorname{ArcCot}[c + d x]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+i(c+dx)}\right] - \\
 & \frac{b^3 (3 d^2 e^2 - 6 c d e f - (1 - 3 c^2) f^2) \operatorname{PolyLog}\left[3, 1 - \frac{2}{1+i(c+dx)}\right]}{2 d^3}
 \end{aligned}$$

Result (type 4, 2336 leaves):

$$\begin{aligned}
 & \frac{a^2 (a d^2 e^2 + 3 b d e f - 2 b c f^2) x}{d^2} + \frac{a^2 f (2 a d e + b f) x^2}{2 d} + \\
 & \frac{1}{3} a^3 f^2 x^3 + a^2 b x (3 e^2 + 3 e f x + f^2 x^2) \operatorname{ArcCot}[c + d x] + \frac{1}{d^3} \\
 & (-3 a^2 b c d^2 e^2 - 3 a^2 b d e f + 3 a^2 b c^2 d e f + 3 a^2 b c f^2 - a^2 b c^3 f^2) \operatorname{ArcTan}[c + d x] + \\
 & \frac{1}{2 d^3} (3 a^2 b d^2 e^2 - 6 a^2 b c d e f - a^2 b f^2 + 3 a^2 b c^2 f^2) \operatorname{Log}\left[1 + c^2 + 2 c d x + d^2 x^2\right] + \\
 & \frac{1}{4 d (c + d x)^2 \left(1 + \frac{1}{(c + d x)^2}\right) \left(\frac{1}{\sqrt{1 + \frac{1}{(c + d x)^2}}} - \frac{c}{(c + d x) \sqrt{1 + \frac{1}{(c + d x)^2}}}\right)^2} a b^2 f^2 x^2 (1 + (c + d x)^2) \\
 & \left((c + d x) (1 - 6 c \operatorname{ArcCot}[c + d x] + 3 \operatorname{ArcCot}[c + d x]^2 + 3 c^2 \operatorname{ArcCot}[c + d x]^2) - \right. \\
 & \left. (c + d x) \sqrt{1 + \frac{1}{(c + d x)^2}} (1 - 6 c \operatorname{ArcCot}[c + d x] - \operatorname{ArcCot}[c + d x]^2 + 3 c^2 \operatorname{ArcCot}[c + d x]^2) \right)
 \end{aligned}$$

$$\begin{aligned}
 & \cos [3 \operatorname{ArcCot}[c+d x]] - 2 \left(-2 \operatorname{ArcCot}[c+d x] + i \operatorname{ArcCot}[c+d x]^2 + 6 c \operatorname{ArcCot}[c+d x]^2 - \right. \\
 & 3 i c^2 \operatorname{ArcCot}[c+d x]^2 + 2(-1+3 c^2) \operatorname{ArcCot}[c+d x] \operatorname{Log}\left[1 - e^{2 i \operatorname{ArcCot}[c+d x]}\right] - \\
 & 6 c \operatorname{Log}\left[\frac{1}{(c+d x) \sqrt{1 + \frac{1}{(c+d x)^2}}}\right] + \cos [2 \operatorname{ArcCot}[c+d x]] \left(i(-1+3 c^2) \operatorname{ArcCot}[c+d x]^2 + \right. \\
 & \left. \left. (2-6 c^2) \operatorname{ArcCot}[c+d x] \operatorname{Log}\left[1 - e^{2 i \operatorname{ArcCot}[c+d x]}\right] + 6 c \operatorname{Log}\left[\frac{1}{(c+d x) \sqrt{1 + \frac{1}{(c+d x)^2}}}\right]\right) \right) + \\
 & \left. \frac{4 i(-1+3 c^2) \operatorname{PolyLog}\left[2, e^{2 i \operatorname{ArcCot}[c+d x]}\right]}{(c+d x)^2 \left(1 + \frac{1}{(c+d x)^2}\right)} \right) - \left(3 a b^2 e^2 \left(1 + (c+d x)^2\right) \right. \\
 & \left. (- (c+d x) \operatorname{ArcCot}[c+d x]^2 + 2 \operatorname{ArcCot}[c+d x] \operatorname{Log}\left[1 - e^{2 i \operatorname{ArcCot}[c+d x]}\right] - \right. \\
 & \left. i(\operatorname{ArcCot}[c+d x]^2 + \operatorname{PolyLog}\left[2, e^{2 i \operatorname{ArcCot}[c+d x]}\right])\right) \right) / \left(d (c+d x)^2 \left(1 + \frac{1}{(c+d x)^2}\right) \right) + \\
 & \left(6 a b^2 e f \left(1 + (c+d x)^2\right) \left(\frac{(c+d x) \operatorname{ArcCot}[c+d x]}{d^2} - \frac{c (c+d x) \operatorname{ArcCot}[c+d x]^2}{d^2} + \right. \right. \\
 & \left. \frac{(c+d x)^2 \left(1 + \frac{1}{(c+d x)^2}\right) \operatorname{ArcCot}[c+d x]^2}{2 d^2} - \frac{\operatorname{Log}\left[\frac{1}{(c+d x) \sqrt{1 + \frac{1}{(c+d x)^2}}}\right]}{d^2} + \frac{1}{d^2} 2 c \left(\operatorname{ArcCot}[c+d x] \right. \right. \\
 & \left. \left. \operatorname{Log}\left[1 - e^{2 i \operatorname{ArcCot}[c+d x]}\right] - \frac{1}{2} i(\operatorname{ArcCot}[c+d x]^2 + \operatorname{PolyLog}\left[2, e^{2 i \operatorname{ArcCot}[c+d x]}\right])\right) \right) \right) / \\
 & \left((c+d x)^2 \left(1 + \frac{1}{(c+d x)^2}\right) \right) - \left(b^3 e^2 \left(1 + (c+d x)^2\right) \left(-\frac{i \pi^3}{8} + i \operatorname{ArcCot}[c+d x]^3 - \right. \right. \\
 & \left. (c+d x) \operatorname{ArcCot}[c+d x]^3 + 3 \operatorname{ArcCot}[c+d x]^2 \operatorname{Log}\left[1 - e^{-2 i \operatorname{ArcCot}[c+d x]}\right] + \right. \\
 & \left. \left. 3 i \operatorname{ArcCot}[c+d x] \operatorname{PolyLog}\left[2, e^{-2 i \operatorname{ArcCot}[c+d x]}\right] + \frac{3}{2} \operatorname{PolyLog}\left[3, e^{-2 i \operatorname{ArcCot}[c+d x]}\right] \right) \right) /
 \end{aligned}$$

$$\begin{aligned}
& \left(d (c + d x)^2 \left(1 + \frac{1}{(c + d x)^2} \right) \right) + \frac{1}{4 d^2 (c + d x)^2 \left(1 + \frac{1}{(c + d x)^2} \right)} \\
& b^3 e f \left(1 + (c + d x)^2 \right) \\
& \left(-i c \pi^3 + 12 i \operatorname{ArcCot}[c + d x]^2 + 12 (c + d x) \operatorname{ArcCot}[c + d x]^2 + 8 i c \operatorname{ArcCot}[c + d x]^3 - \right. \\
& 8 c (c + d x) \operatorname{ArcCot}[c + d x]^3 + 4 (c + d x)^2 \left(1 + \frac{1}{(c + d x)^2} \right) \operatorname{ArcCot}[c + d x]^3 + \\
& 24 c \operatorname{ArcCot}[c + d x]^2 \operatorname{Log}\left[1 - e^{-2 i \operatorname{ArcCot}[c + d x]}\right] - 24 \operatorname{ArcCot}[c + d x] \operatorname{Log}\left[1 - e^{2 i \operatorname{ArcCot}[c + d x]}\right] + \\
& 24 i c \operatorname{ArcCot}[c + d x] \operatorname{PolyLog}\left[2, e^{-2 i \operatorname{ArcCot}[c + d x]}\right] + 12 i \operatorname{PolyLog}\left[2, e^{2 i \operatorname{ArcCot}[c + d x]}\right] + \\
& \left. 12 c \operatorname{PolyLog}\left[3, e^{-2 i \operatorname{ArcCot}[c + d x]}\right] \right) - \frac{1}{d^3 (c + d x)^2 \left(1 + \frac{1}{(c + d x)^2} \right)} \\
& b^3 f^2 \left(1 + (c + d x)^2 \right) \left(i (-1 + 3 c^2) \operatorname{ArcCot}[c + d x] \operatorname{PolyLog}\left[2, e^{-2 i \operatorname{ArcCot}[c + d x]}\right] + \right. \\
& \left. \frac{1}{96} (c + d x)^3 \left(1 + \frac{1}{(c + d x)^2} \right)^{3/2} \left(\frac{3 i \pi^3}{(c + d x) \sqrt{1 + \frac{1}{(c + d x)^2}}} - \frac{9 i c^2 \pi^3}{(c + d x) \sqrt{1 + \frac{1}{(c + d x)^2}}} - \right. \right. \\
& \frac{24 \operatorname{ArcCot}[c + d x]}{\sqrt{1 + \frac{1}{(c + d x)^2}}} + \frac{72 c \operatorname{ArcCot}[c + d x]^2}{\sqrt{1 + \frac{1}{(c + d x)^2}}} - \frac{48 \operatorname{ArcCot}[c + d x]^2}{(c + d x) \sqrt{1 + \frac{1}{(c + d x)^2}}} + \\
& \frac{216 i c \operatorname{ArcCot}[c + d x]^2}{(c + d x) \sqrt{1 + \frac{1}{(c + d x)^2}}} - \frac{24 \operatorname{ArcCot}[c + d x]^3}{\sqrt{1 + \frac{1}{(c + d x)^2}}} - \frac{24 c^2 \operatorname{ArcCot}[c + d x]^3}{\sqrt{1 + \frac{1}{(c + d x)^2}}} - \\
& \frac{24 i \operatorname{ArcCot}[c + d x]^3}{(c + d x) \sqrt{1 + \frac{1}{(c + d x)^2}}} + \frac{96 c \operatorname{ArcCot}[c + d x]^3}{(c + d x) \sqrt{1 + \frac{1}{(c + d x)^2}}} + \frac{72 i c^2 \operatorname{ArcCot}[c + d x]^3}{(c + d x) \sqrt{1 + \frac{1}{(c + d x)^2}}} + \\
& \frac{24 \operatorname{ArcCot}[c + d x] \operatorname{Cos}\left[3 \operatorname{ArcCot}[c + d x]\right] - 72 c \operatorname{ArcCot}[c + d x]^2 \operatorname{Cos}\left[3 \operatorname{ArcCot}[c + d x]\right] - 8 \operatorname{ArcCot}[c + d x]^3 \operatorname{Cos}\left[3 \operatorname{ArcCot}[c + d x]\right] + 24 c^2 \operatorname{ArcCot}[c + d x]^3 \operatorname{Cos}\left[3 \operatorname{ArcCot}[c + d x]\right] - 72 \operatorname{ArcCot}[c + d x]^2 \operatorname{Log}\left[1 - e^{-2 i \operatorname{ArcCot}[c + d x]}\right]}{(c + d x) \sqrt{1 + \frac{1}{(c + d x)^2}}} + \\
& \left. \frac{216 c^2 \operatorname{ArcCot}[c + d x]^2 \operatorname{Log}\left[1 - e^{-2 i \operatorname{ArcCot}[c + d x]}\right]}{(c + d x) \sqrt{1 + \frac{1}{(c + d x)^2}}} \right) -
\end{aligned}$$

$$\begin{aligned}
 & \frac{432 c \operatorname{ArcCot}[c+d x] \operatorname{Log}\left[1-e^{2 i \operatorname{ArcCot}[c+d x]}\right]}{(c+d x) \sqrt{1+\frac{1}{(c+d x)^2}}} + \frac{72 \operatorname{Log}\left[\frac{1}{(c+d x) \sqrt{1+\frac{1}{(c+d x)^2}}}\right]}{(c+d x) \sqrt{1+\frac{1}{(c+d x)^2}}} + \\
 & \frac{288 i c \operatorname{PolyLog}\left[2, e^{2 i \operatorname{ArcCot}[c+d x]}\right]}{(c+d x)^3 \left(1+\frac{1}{(c+d x)^2}\right)^{3/2}} + \frac{48(-1+3 c^2) \operatorname{PolyLog}\left[3, e^{-2 i \operatorname{ArcCot}[c+d x]}\right]}{(c+d x)^3 \left(1+\frac{1}{(c+d x)^2}\right)^{3/2}} - \\
 & i \pi^3 \operatorname{Sin}\left[3 \operatorname{ArcCot}[c+d x]\right] + 3 i c^2 \pi^3 \operatorname{Sin}\left[3 \operatorname{ArcCot}[c+d x]\right] - \\
 & 72 i c \operatorname{ArcCot}[c+d x]^2 \operatorname{Sin}\left[3 \operatorname{ArcCot}[c+d x]\right] + 8 i \operatorname{ArcCot}[c+d x]^3 \\
 & \operatorname{Sin}\left[3 \operatorname{ArcCot}[c+d x]\right] - 24 i c^2 \operatorname{ArcCot}[c+d x]^3 \operatorname{Sin}\left[3 \operatorname{ArcCot}[c+d x]\right] + \\
 & 24 \operatorname{ArcCot}[c+d x]^2 \operatorname{Log}\left[1-e^{-2 i \operatorname{ArcCot}[c+d x]}\right] \operatorname{Sin}\left[3 \operatorname{ArcCot}[c+d x]\right] - \\
 & 72 c^2 \operatorname{ArcCot}[c+d x]^2 \operatorname{Log}\left[1-e^{-2 i \operatorname{ArcCot}[c+d x]}\right] \operatorname{Sin}\left[3 \operatorname{ArcCot}[c+d x]\right] + \\
 & 144 c \operatorname{ArcCot}[c+d x] \operatorname{Log}\left[1-e^{2 i \operatorname{ArcCot}[c+d x]}\right] \operatorname{Sin}\left[3 \operatorname{ArcCot}[c+d x]\right] - \\
 & \left. 24 \operatorname{Log}\left[\frac{1}{(c+d x) \sqrt{1+\frac{1}{(c+d x)^2}}}\right] \operatorname{Sin}\left[3 \operatorname{ArcCot}[c+d x]\right]\right)
 \end{aligned}$$

Problem 144: Attempted integration timed out after 120 seconds.

$$\int \frac{(a+b \operatorname{ArcCot}[c+d x])^3}{e+f x} dx$$

Optimal (type 4, 372 leaves, 2 steps):

$$\begin{aligned}
 & -\frac{(a+b \operatorname{ArcCot}[c+d x])^3 \operatorname{Log}\left[\frac{2}{1-i(c+d x)}\right]}{f} + \frac{(a+b \operatorname{ArcCot}[c+d x])^3 \operatorname{Log}\left[\frac{2 d(e+f x)}{(d e+i f-c f)(1-i(c+d x))}\right]}{f} - \\
 & \frac{3 i b(a+b \operatorname{ArcCot}[c+d x])^2 \operatorname{PolyLog}\left[2, 1-\frac{2}{1-i(c+d x)}\right]}{2 f} + \\
 & \frac{3 i b(a+b \operatorname{ArcCot}[c+d x])^2 \operatorname{PolyLog}\left[2, 1-\frac{2 d(e+f x)}{(d e+i f-c f)(1-i(c+d x))}\right]}{2 f} - \\
 & \frac{3 b^2(a+b \operatorname{ArcCot}[c+d x]) \operatorname{PolyLog}\left[3, 1-\frac{2}{1-i(c+d x)}\right]}{2 f} + \\
 & \frac{3 b^2(a+b \operatorname{ArcCot}[c+d x]) \operatorname{PolyLog}\left[3, 1-\frac{2 d(e+f x)}{(d e+i f-c f)(1-i(c+d x))}\right]}{2 f} + \\
 & \frac{3 i b^3 \operatorname{PolyLog}\left[4, 1-\frac{2}{1-i(c+d x)}\right]}{4 f} - \frac{3 i b^3 \operatorname{PolyLog}\left[4, 1-\frac{2 d(e+f x)}{(d e+i f-c f)(1-i(c+d x))}\right]}{4 f}
 \end{aligned}$$

Result (type 1, 1 leaves):

???

Problem 145: Attempted integration timed out after 120 seconds.

$$\int \frac{(a + b \operatorname{ArcCot}[c + d x])^3}{(e + f x)^2} dx$$

Optimal (type 4, 1233 leaves, 35 steps):

$$\begin{aligned} & \frac{3 i a b^2 d \operatorname{ArcCot}[c + d x]^2}{d^2 e^2 - 2 c d e f + (1 + c^2) f^2} + \frac{3 a b^2 d (d e - c f) \operatorname{ArcCot}[c + d x]^2}{f (d^2 e^2 - 2 c d e f + (1 + c^2) f^2)} + \\ & \frac{i b^3 d \operatorname{ArcCot}[c + d x]^3}{d^2 e^2 - 2 c d e f + (1 + c^2) f^2} + \frac{b^3 d (d e - c f) \operatorname{ArcCot}[c + d x]^3}{f (d^2 e^2 - 2 c d e f + (1 + c^2) f^2)} - \frac{(a + b \operatorname{ArcCot}[c + d x])^3}{f (e + f x)} - \\ & \frac{3 a^2 b d (d e - c f) \operatorname{ArcTan}[c + d x]}{f (f^2 + (d e - c f)^2)} - \frac{3 a^2 b d \operatorname{Log}[e + f x]}{f^2 + (d e - c f)^2} + \frac{6 a b^2 d \operatorname{ArcCot}[c + d x] \operatorname{Log}\left[\frac{2}{1 - i (c + d x)}\right]}{d^2 e^2 - 2 c d e f + (1 + c^2) f^2} + \\ & \frac{3 b^3 d \operatorname{ArcCot}[c + d x]^2 \operatorname{Log}\left[\frac{2}{1 - i (c + d x)}\right]}{d^2 e^2 - 2 c d e f + (1 + c^2) f^2} - \frac{6 a b^2 d \operatorname{ArcCot}[c + d x] \operatorname{Log}\left[\frac{2 d (e + f x)}{(d e + i f - c f) (1 - i (c + d x))}\right]}{d^2 e^2 - 2 c d e f + (1 + c^2) f^2} - \\ & \frac{3 b^3 d \operatorname{ArcCot}[c + d x]^2 \operatorname{Log}\left[\frac{2 d (e + f x)}{(d e + i f - c f) (1 - i (c + d x))}\right]}{d^2 e^2 - 2 c d e f + (1 + c^2) f^2} - \frac{6 a b^2 d \operatorname{ArcCot}[c + d x] \operatorname{Log}\left[\frac{2}{1 + i (c + d x)}\right]}{d^2 e^2 - 2 c d e f + (1 + c^2) f^2} - \\ & \frac{3 b^3 d \operatorname{ArcCot}[c + d x]^2 \operatorname{Log}\left[\frac{2}{1 + i (c + d x)}\right]}{d^2 e^2 - 2 c d e f + (1 + c^2) f^2} + \frac{3 a^2 b d \operatorname{Log}[1 + (c + d x)^2]}{2 (f^2 + (d e - c f)^2)} + \\ & \frac{3 i a b^2 d \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - i (c + d x)}\right]}{d^2 e^2 - 2 c d e f + (1 + c^2) f^2} + \frac{3 i b^3 d \operatorname{ArcCot}[c + d x] \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - i (c + d x)}\right]}{d^2 e^2 - 2 c d e f + (1 + c^2) f^2} - \\ & \frac{3 i a b^2 d \operatorname{PolyLog}\left[2, 1 - \frac{2 d (e + f x)}{(d e + i f - c f) (1 - i (c + d x))}\right]}{d^2 e^2 - 2 c d e f + (1 + c^2) f^2} - \\ & \frac{3 i b^3 d \operatorname{ArcCot}[c + d x] \operatorname{PolyLog}\left[2, 1 - \frac{2 d (e + f x)}{(d e + i f - c f) (1 - i (c + d x))}\right]}{d^2 e^2 - 2 c d e f + (1 + c^2) f^2} + \frac{3 i a b^2 d \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 + i (c + d x)}\right]}{d^2 e^2 - 2 c d e f + (1 + c^2) f^2} + \\ & \frac{3 i b^3 d \operatorname{ArcCot}[c + d x] \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 + i (c + d x)}\right]}{d^2 e^2 - 2 c d e f + (1 + c^2) f^2} + \frac{3 b^3 d \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 - i (c + d x)}\right]}{2 (d^2 e^2 - 2 c d e f + (1 + c^2) f^2)} - \\ & \frac{3 b^3 d \operatorname{PolyLog}\left[3, 1 - \frac{2 d (e + f x)}{(d e + i f - c f) (1 - i (c + d x))}\right]}{2 (d^2 e^2 - 2 c d e f + (1 + c^2) f^2)} - \frac{3 b^3 d \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 + i (c + d x)}\right]}{2 (d^2 e^2 - 2 c d e f + (1 + c^2) f^2)} - \\ & \frac{3 b^3 d \operatorname{PolyLog}\left[3, 1 - \frac{2 d (e + f x)}{(d e + i f - c f) (1 - i (c + d x))}\right]}{2 (d^2 e^2 - 2 c d e f + (1 + c^2) f^2)} - \frac{3 b^3 d \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 + i (c + d x)}\right]}{2 (d^2 e^2 - 2 c d e f + (1 + c^2) f^2)} \end{aligned}$$

Result (type 1, 1 leaves):

???

Problem 146: Unable to integrate problem.

$$\int (e + f x)^m (a + b \operatorname{ArcCot}[c + d x]) dx$$

Optimal (type 5, 177 leaves, 6 steps):

$$\frac{(e + f x)^{1+m} (a + b \operatorname{ArcCot}[c + d x])}{f (1+m)} + \frac{i b d (e + f x)^{2+m} \operatorname{Hypergeometric2F1}\left[1, 2+m, 3+m, \frac{d(e+fx)}{de+if-cf}\right]}{2 f (d e + (i - c) f) (1+m) (2+m)}$$

$$\frac{i b d (e + f x)^{2+m} \operatorname{Hypergeometric2F1}\left[1, 2+m, 3+m, \frac{d(e+fx)}{de-(i+c)f}\right]}{2 f (d e - (i + c) f) (1+m) (2+m)}$$

Result (type 8, 20 leaves):

$$\int (e + f x)^m (a + b \operatorname{ArcCot}[c + d x]) dx$$

Problem 152: Unable to integrate problem.

$$\int \frac{\left(a + b \operatorname{ArcCot}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^3}{1 - c^2 x^2} dx$$

Optimal (type 4, 488 leaves, 9 steps):

$$-\frac{2 \left(a + b \operatorname{ArcCot}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^3 \operatorname{ArcCoth}\left[1 - \frac{2}{1 + \frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right]}{c} +$$

$$\frac{3 i b \left(a + b \operatorname{ArcCot}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^2 \operatorname{PolyLog}\left[2, 1 - \frac{2 i}{i + \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right]}{2 c} -$$

$$\frac{3 i b \left(a + b \operatorname{ArcCot}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^2 \operatorname{PolyLog}\left[2, 1 - \frac{2\sqrt{1-cx}}{\sqrt{1+cx} \left(i + \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right]}{2 c} +$$

$$\frac{3 b^2 \left(a + b \operatorname{ArcCot}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right) \operatorname{PolyLog}\left[3, 1 - \frac{2 i}{i + \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right]}{2 c} -$$

$$\frac{3 b^2 \left(a + b \operatorname{ArcCot}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right) \operatorname{PolyLog}\left[3, 1 - \frac{2\sqrt{1-cx}}{\sqrt{1+cx} \left(i + \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right]}{2 c} -$$

$$\frac{3 i b^3 \operatorname{PolyLog}\left[4, 1 - \frac{2 i}{i + \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right]}{4 c} + \frac{3 i b^3 \operatorname{PolyLog}\left[4, 1 - \frac{2\sqrt{1-cx}}{\sqrt{1+cx} \left(i + \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right]}{4 c}$$

Result (type 8, 42 leaves):

$$\int \frac{\left(a + b \operatorname{ArcCot}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^3}{1 - c^2 x^2} dx$$

Problem 153: Unable to integrate problem.

$$\int \frac{\left(a + b \operatorname{ArcCot}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^2}{1 - c^2 x^2} dx$$

Optimal (type 4, 321 leaves, 7 steps):

$$\begin{aligned} & \frac{2 \left(a + b \operatorname{ArcCot}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^2 \operatorname{ArcCoth}\left[1 - \frac{2}{1 + \frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right]}{c} + \\ & \frac{i b \left(a + b \operatorname{ArcCot}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right) \operatorname{PolyLog}\left[2, 1 - \frac{2i}{i + \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right]}{c} - \\ & \frac{i b \left(a + b \operatorname{ArcCot}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right) \operatorname{PolyLog}\left[2, 1 - \frac{2\sqrt{1-cx}}{\sqrt{1+cx} \left(i + \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right]}{c} + \\ & \frac{b^2 \operatorname{PolyLog}\left[3, 1 - \frac{2i}{i + \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right]}{2c} - \frac{b^2 \operatorname{PolyLog}\left[3, 1 - \frac{2\sqrt{1-cx}}{\sqrt{1+cx} \left(i + \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right]}{2c} \end{aligned}$$

Result (type 8, 42 leaves):

$$\int \frac{\left(a + b \operatorname{ArcCot}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^2}{1 - c^2 x^2} dx$$

Problem 160: Result more than twice size of optimal antiderivative.

$$\int \operatorname{ArcCot}[c + d \operatorname{Tan}[a + b x]] dx$$

Optimal (type 4, 198 leaves, 7 steps):

$$\begin{aligned} & x \operatorname{ArcCot}[c + d \operatorname{Tan}[a + b x]] - \\ & \frac{1}{2} i x \operatorname{Log}\left[1 + \frac{(1 + i c + d) e^{2ia + 2ibx}}{1 + i c - d}\right] + \frac{1}{2} i x \operatorname{Log}\left[1 + \frac{(c + i(1 - d)) e^{2ia + 2ibx}}{c + i(1 + d)}\right] - \\ & \frac{\operatorname{PolyLog}\left[2, -\frac{(1 + i c + d) e^{2ia + 2ibx}}{1 + i c - d}\right]}{4b} + \frac{\operatorname{PolyLog}\left[2, -\frac{(c + i(1 - d)) e^{2ia + 2ibx}}{c + i(1 + d)}\right]}{4b} \end{aligned}$$

Result (type 4, 418 leaves):

$$\begin{aligned}
 & x \operatorname{ArcCot}[c + d \operatorname{Tan}[a + b x]] - \\
 & \frac{1}{4b} \left(2a \operatorname{ArcTan}\left[\frac{c(1 + e^{2i(a+bx)})}{1 + d + e^{2i(a+bx)} - d e^{2i(a+bx)}}\right] + 2a \operatorname{ArcTan}\left[\frac{c(1 + e^{2i(a+bx)})}{1 + e^{2i(a+bx)} + d(-1 + e^{2i(a+bx)})}\right] \right) + \\
 & 2i(a+bx) \operatorname{Log}\left[1 + \frac{(c - i(1+d))e^{2i(a+bx)}}{c + i(-1+d)}\right] - 2i(a+bx) \operatorname{Log}\left[1 + \frac{(i + c - id)e^{2i(a+bx)}}{c + i(1+d)}\right] + \\
 & ia \operatorname{Log}\left[e^{-4i(a+bx)} \left(c^2 (1 + e^{2i(a+bx)})^2 + (1 + d + e^{2i(a+bx)} - d e^{2i(a+bx)})^2 \right) \right] - \\
 & ia \operatorname{Log}\left[e^{-4i(a+bx)} \left(c^2 (1 + e^{2i(a+bx)})^2 + (1 + e^{2i(a+bx)} + d(-1 + e^{2i(a+bx)}))^2 \right) \right] + \\
 & \operatorname{PolyLog}\left[2, -\frac{(c - i(1+d))e^{2i(a+bx)}}{c + i(-1+d)}\right] - \operatorname{PolyLog}\left[2, -\frac{(i + c - id)e^{2i(a+bx)}}{c + i(1+d)}\right] \Big)
 \end{aligned}$$

Problem 173: Result more than twice size of optimal antiderivative.

$$\int \operatorname{ArcCot}[c + d \operatorname{Cot}[a + b x]] dx$$

Optimal (type 4, 198 leaves, 7 steps):

$$\begin{aligned}
 & x \operatorname{ArcCot}[c + d \operatorname{Cot}[a + b x]] - \\
 & \frac{1}{2} i x \operatorname{Log}\left[1 - \frac{(1 + ic - d)e^{2ia+2ibx}}{1 + ic + d}\right] + \frac{1}{2} i x \operatorname{Log}\left[1 - \frac{(c + i(1+d))e^{2ia+2ibx}}{c + i(1-d)}\right] - \\
 & \frac{\operatorname{PolyLog}\left[2, \frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right]}{4b} + \frac{\operatorname{PolyLog}\left[2, \frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right]}{4b}
 \end{aligned}$$

Result (type 4, 416 leaves):

$$\begin{aligned}
 & x \operatorname{ArcCot}[c + d \operatorname{Cot}[a + b x]] - \frac{1}{4b} \\
 & \left(2a \operatorname{ArcTan}\left[\frac{c(-1 + e^{-2i(a+bx)})}{-1 + d + e^{-2i(a+bx)} + d e^{-2i(a+bx)}}\right] + 2a \operatorname{ArcTan}\left[\frac{c(-1 + e^{2i(a+bx)})}{-1 + d + e^{2i(a+bx)} + d e^{2i(a+bx)}}\right] + 2i \right. \\
 & (a+bx) \operatorname{Log}\left[1 - \frac{(c + i(-1+d))e^{2i(a+bx)}}{c - i(1+d)}\right] - 2i(a+bx) \operatorname{Log}\left[1 - \frac{(c + i(1+d))e^{2i(a+bx)}}{i + c - id}\right] - \\
 & ia \operatorname{Log}\left[e^{-4i(a+bx)} \left(c^2 (-1 + e^{2i(a+bx)})^2 + (1 + d - e^{2i(a+bx)} + d e^{2i(a+bx)})^2 \right) \right] + \\
 & ia \operatorname{Log}\left[e^{-4i(a+bx)} \left(c^2 (-1 + e^{2i(a+bx)})^2 + (-1 + d + e^{2i(a+bx)} + d e^{2i(a+bx)})^2 \right) \right] + \\
 & \left. \operatorname{PolyLog}\left[2, \frac{(c + i(-1+d))e^{2i(a+bx)}}{c - i(1+d)}\right] - \operatorname{PolyLog}\left[2, \frac{(c + i(1+d))e^{2i(a+bx)}}{i + c - id}\right] \right)
 \end{aligned}$$

Problem 183: Result more than twice size of optimal antiderivative.

$$\int (e + f x)^3 \operatorname{ArcCot}[\operatorname{Tanh}[a + b x]] dx$$

Optimal (type 4, 299 leaves, 12 steps):

$$\begin{aligned} & \frac{(e+fx)^4 \text{ArcCot}[\text{Tanh}[a+bx]]}{4f} + \frac{(e+fx)^4 \text{ArcTan}[e^{2a+2bx}]}{4f} - \\ & \frac{i(e+fx)^3 \text{PolyLog}[2, -ie^{2a+2bx}]}{4b} + \frac{i(e+fx)^3 \text{PolyLog}[2, ie^{2a+2bx}]}{4b} + \\ & \frac{3if(e+fx)^2 \text{PolyLog}[3, -ie^{2a+2bx}]}{8b^2} - \frac{3if(e+fx)^2 \text{PolyLog}[3, ie^{2a+2bx}]}{8b^2} - \\ & \frac{3if^2(e+fx) \text{PolyLog}[4, -ie^{2a+2bx}]}{8b^3} + \frac{3if^2(e+fx) \text{PolyLog}[4, ie^{2a+2bx}]}{8b^3} + \\ & \frac{3if^3 \text{PolyLog}[5, -ie^{2a+2bx}]}{16b^4} - \frac{3if^3 \text{PolyLog}[5, ie^{2a+2bx}]}{16b^4} \end{aligned}$$

Result (type 4, 600 leaves):

$$\begin{aligned} & \frac{1}{4} x (4e^3 + 6e^2fx + 4ef^2x^2 + f^3x^3) \text{ArcCot}[\text{Tanh}[a+bx]] + \\ & \frac{1}{16b^4} i \left(8b^4 e^3 x \text{Log}[1 - ie^{2(a+bx)}] + 12b^4 e^2 f x^2 \text{Log}[1 - ie^{2(a+bx)}] + \right. \\ & \quad 8b^4 e f^2 x^3 \text{Log}[1 - ie^{2(a+bx)}] + 2b^4 f^3 x^4 \text{Log}[1 - ie^{2(a+bx)}] - 8b^4 e^3 x \text{Log}[1 + ie^{2(a+bx)}] - \\ & \quad 12b^4 e^2 f x^2 \text{Log}[1 + ie^{2(a+bx)}] - 8b^4 e f^2 x^3 \text{Log}[1 + ie^{2(a+bx)}] - \\ & \quad 2b^4 f^3 x^4 \text{Log}[1 + ie^{2(a+bx)}] - 4b^3 (e+fx)^3 \text{PolyLog}[2, -ie^{2(a+bx)}] + \\ & \quad 4b^3 (e+fx)^3 \text{PolyLog}[2, ie^{2(a+bx)}] + 6b^2 e^2 f \text{PolyLog}[3, -ie^{2(a+bx)}] + \\ & \quad 12b^2 e f^2 x \text{PolyLog}[3, -ie^{2(a+bx)}] + 6b^2 f^3 x^2 \text{PolyLog}[3, -ie^{2(a+bx)}] - \\ & \quad 6b^2 e^2 f \text{PolyLog}[3, ie^{2(a+bx)}] - 12b^2 e f^2 x \text{PolyLog}[3, ie^{2(a+bx)}] - \\ & \quad 6b^2 f^3 x^2 \text{PolyLog}[3, ie^{2(a+bx)}] - 6b e f^2 \text{PolyLog}[4, -ie^{2(a+bx)}] - \\ & \quad 6b f^3 x \text{PolyLog}[4, -ie^{2(a+bx)}] + 6b e f^2 \text{PolyLog}[4, ie^{2(a+bx)}] + \\ & \quad \left. 6b f^3 x \text{PolyLog}[4, ie^{2(a+bx)}] + 3f^3 \text{PolyLog}[5, -ie^{2(a+bx)}] - 3f^3 \text{PolyLog}[5, ie^{2(a+bx)}] \right) \end{aligned}$$

Problem 190: Result more than twice size of optimal antiderivative.

$$\int \text{ArcCot}[c+d \text{Tanh}[a+bx]] dx$$

Optimal (type 4, 174 leaves, 7 steps):

$$\begin{aligned} & x \text{ArcCot}[c+d \text{Tanh}[a+bx]] - \frac{1}{2} i x \text{Log}\left[1 + \frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right] + \\ & \frac{1}{2} i x \text{Log}\left[1 + \frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right] - \frac{i \text{PolyLog}\left[2, -\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right]}{4b} + \frac{i \text{PolyLog}\left[2, -\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right]}{4b} \end{aligned}$$

Result (type 4, 365 leaves):

$$\begin{aligned}
 & x \operatorname{ArcCot}[c + d \operatorname{Tanh}[a + b x]] - \frac{1}{2b} \\
 & i \left(2 i a \operatorname{ArcTan}\left[\frac{1 + e^{2(a+bx)}}{c - d + c e^{2(a+bx)} + d e^{2(a+bx)}}\right] + (a + b x) \operatorname{Log}\left[1 - \frac{\sqrt{-i + c + d} e^{a+bx}}{\sqrt{i - c + d}}\right] + \right. \\
 & (a + b x) \operatorname{Log}\left[1 + \frac{\sqrt{-i + c + d} e^{a+bx}}{\sqrt{i - c + d}}\right] - (a + b x) \operatorname{Log}\left[1 - \frac{\sqrt{i + c + d} e^{a+bx}}{\sqrt{-i - c + d}}\right] - \\
 & (a + b x) \operatorname{Log}\left[1 + \frac{\sqrt{i + c + d} e^{a+bx}}{\sqrt{-i - c + d}}\right] + \operatorname{PolyLog}\left[2, -\frac{\sqrt{-i + c + d} e^{a+bx}}{\sqrt{i - c + d}}\right] + \operatorname{PolyLog}\left[2, \right. \\
 & \left. \frac{\sqrt{-i + c + d} e^{a+bx}}{\sqrt{i - c + d}}\right] - \operatorname{PolyLog}\left[2, -\frac{\sqrt{i + c + d} e^{a+bx}}{\sqrt{-i - c + d}}\right] - \operatorname{PolyLog}\left[2, \frac{\sqrt{i + c + d} e^{a+bx}}{\sqrt{-i - c + d}}\right] \left. \right)
 \end{aligned}$$

Problem 200: Result more than twice size of optimal antiderivative.

$$\int (e + f x)^3 \operatorname{ArcCot}[\operatorname{Coth}[a + b x]] dx$$

Optimal (type 4, 299 leaves, 12 steps):

$$\begin{aligned}
 & \frac{(e + f x)^4 \operatorname{ArcCot}[\operatorname{Coth}[a + b x]]}{4f} - \frac{(e + f x)^4 \operatorname{ArcTan}[e^{2a+2bx}]}{4f} + \\
 & \frac{i(e + f x)^3 \operatorname{PolyLog}[2, -i e^{2a+2bx}]}{4b} - \frac{i(e + f x)^3 \operatorname{PolyLog}[2, i e^{2a+2bx}]}{4b} - \\
 & \frac{3 i f (e + f x)^2 \operatorname{PolyLog}[3, -i e^{2a+2bx}]}{8 b^2} + \frac{3 i f (e + f x)^2 \operatorname{PolyLog}[3, i e^{2a+2bx}]}{8 b^2} + \\
 & \frac{3 i f^2 (e + f x) \operatorname{PolyLog}[4, -i e^{2a+2bx}]}{8 b^3} - \frac{3 i f^2 (e + f x) \operatorname{PolyLog}[4, i e^{2a+2bx}]}{8 b^3} - \\
 & \frac{3 i f^3 \operatorname{PolyLog}[5, -i e^{2a+2bx}]}{16 b^4} + \frac{3 i f^3 \operatorname{PolyLog}[5, i e^{2a+2bx}]}{16 b^4}
 \end{aligned}$$

Result (type 4, 600 leaves):

$$\begin{aligned}
 & \frac{1}{4} x (4 e^3 + 6 e^2 f x + 4 e f^2 x^2 + f^3 x^3) \operatorname{ArcCot}[\operatorname{Coth}[a + b x]] - \\
 & \frac{1}{16 b^4} i \left(8 b^4 e^3 x \operatorname{Log}[1 - i e^{2(a+bx)}] + 12 b^4 e^2 f x^2 \operatorname{Log}[1 - i e^{2(a+bx)}] + \right. \\
 & 8 b^4 e f^2 x^3 \operatorname{Log}[1 - i e^{2(a+bx)}] + 2 b^4 f^3 x^4 \operatorname{Log}[1 - i e^{2(a+bx)}] - 8 b^4 e^3 x \operatorname{Log}[1 + i e^{2(a+bx)}] - \\
 & 12 b^4 e^2 f x^2 \operatorname{Log}[1 + i e^{2(a+bx)}] - 8 b^4 e f^2 x^3 \operatorname{Log}[1 + i e^{2(a+bx)}] - \\
 & 2 b^4 f^3 x^4 \operatorname{Log}[1 + i e^{2(a+bx)}] - 4 b^3 (e + f x)^3 \operatorname{PolyLog}[2, -i e^{2(a+bx)}] + \\
 & 4 b^3 (e + f x)^3 \operatorname{PolyLog}[2, i e^{2(a+bx)}] + 6 b^2 e^2 f \operatorname{PolyLog}[3, -i e^{2(a+bx)}] + \\
 & 12 b^2 e f^2 x \operatorname{PolyLog}[3, -i e^{2(a+bx)}] + 6 b^2 f^3 x^2 \operatorname{PolyLog}[3, -i e^{2(a+bx)}] - \\
 & 6 b^2 e^2 f \operatorname{PolyLog}[3, i e^{2(a+bx)}] - 12 b^2 e f^2 x \operatorname{PolyLog}[3, i e^{2(a+bx)}] - \\
 & 6 b^2 f^3 x^2 \operatorname{PolyLog}[3, i e^{2(a+bx)}] - 6 b e f^2 \operatorname{PolyLog}[4, -i e^{2(a+bx)}] - \\
 & 6 b f^3 x \operatorname{PolyLog}[4, -i e^{2(a+bx)}] + 6 b e f^2 \operatorname{PolyLog}[4, i e^{2(a+bx)}] + \\
 & 6 b f^3 x \operatorname{PolyLog}[4, i e^{2(a+bx)}] + 3 f^3 \operatorname{PolyLog}[5, -i e^{2(a+bx)}] - 3 f^3 \operatorname{PolyLog}[5, i e^{2(a+bx)}] \left. \right)
 \end{aligned}$$

Problem 207: Result more than twice size of optimal antiderivative.

$$\int \text{ArcCot}[c + d \text{Coth}[a + b x]] dx$$

Optimal (type 4, 174 leaves, 7 steps):

$$x \text{ArcCot}[c + d \text{Coth}[a + b x]] - \frac{1}{2} i x \text{Log}\left[1 - \frac{(i - c - d) e^{2a+2bx}}{i - c + d}\right] + \frac{1}{2} i x \text{Log}\left[1 - \frac{(i + c + d) e^{2a+2bx}}{i + c - d}\right] - \frac{i \text{PolyLog}\left[2, \frac{(i - c - d) e^{2a+2bx}}{i - c + d}\right]}{4b} + \frac{i \text{PolyLog}\left[2, \frac{(i + c + d) e^{2a+2bx}}{i + c - d}\right]}{4b}$$

Result (type 4, 365 leaves):

$$x \text{ArcCot}[c + d \text{Coth}[a + b x]] - \frac{1}{2b} i \left(2 i a \text{ArcTan}\left[\frac{-1 + e^{2(a+bx)}}{-c + d + c e^{2(a+bx)} + d e^{2(a+bx)}}\right] + (a + b x) \text{Log}\left[1 - \frac{\sqrt{-i + c + d} e^{a+bx}}{\sqrt{-i + c - d}}\right] + (a + b x) \text{Log}\left[1 + \frac{\sqrt{-i + c + d} e^{a+bx}}{\sqrt{-i + c - d}}\right] - (a + b x) \text{Log}\left[1 - \frac{\sqrt{i + c + d} e^{a+bx}}{\sqrt{i + c - d}}\right] - (a + b x) \text{Log}\left[1 + \frac{\sqrt{i + c + d} e^{a+bx}}{\sqrt{i + c - d}}\right] + \text{PolyLog}\left[2, -\frac{\sqrt{-i + c + d} e^{a+bx}}{\sqrt{-i + c - d}}\right] + \text{PolyLog}\left[2, \frac{\sqrt{-i + c + d} e^{a+bx}}{\sqrt{-i + c - d}}\right] - \text{PolyLog}\left[2, -\frac{\sqrt{i + c + d} e^{a+bx}}{\sqrt{i + c - d}}\right] - \text{PolyLog}\left[2, \frac{\sqrt{i + c + d} e^{a+bx}}{\sqrt{i + c - d}}\right] \right)$$

Problem 217: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b \text{ArcCot}[c x^n]) (d + e \text{Log}[f x^m])}{x} dx$$

Optimal (type 4, 187 leaves, 13 steps):

$$a d \text{Log}[x] + \frac{a e \text{Log}[f x^m]^2}{2m} - \frac{i b d \text{PolyLog}\left[2, -\frac{i x^{-n}}{c}\right]}{2n} - \frac{i b e \text{Log}[f x^m] \text{PolyLog}\left[2, -\frac{i x^{-n}}{c}\right]}{2n} + \frac{i b d \text{PolyLog}\left[2, \frac{i x^{-n}}{c}\right]}{2n} + \frac{i b e \text{Log}[f x^m] \text{PolyLog}\left[2, \frac{i x^{-n}}{c}\right]}{2n} - \frac{i b e m \text{PolyLog}\left[3, -\frac{i x^{-n}}{c}\right]}{2n^2} + \frac{i b e m \text{PolyLog}\left[3, \frac{i x^{-n}}{c}\right]}{2n^2}$$

Result (type 5, 132 leaves):

$$\frac{b c e m x^n \text{HypergeometricPFQ}\left[\left\{\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 1\right\}, \left\{\frac{3}{2}, \frac{3}{2}, \frac{3}{2}\right\}, -c^2 x^{2n}\right] - \frac{1}{n^2}}{b c x^n \text{HypergeometricPFQ}\left[\left\{\frac{1}{2}, \frac{1}{2}, 1\right\}, \left\{\frac{3}{2}, \frac{3}{2}\right\}, -c^2 x^{2n}\right] (d + e \text{Log}[f x^m])} - \frac{1}{n}$$

$$\frac{1}{2} (a + b \text{ArcCot}[c x^n] + b \text{ArcTan}[c x^n]) \text{Log}[x] (e m \text{Log}[x] - 2 (d + e \text{Log}[f x^m]))$$

Problem 224: Attempted integration timed out after 120 seconds.

$$\int \text{ArcCot}[a + b f^{c+dx}] dx$$

Optimal (type 4, 196 leaves, 6 steps):

$$-\frac{\text{ArcCot}[a + b f^{c+dx}] \text{Log}\left[\frac{2}{1-i(a+b f^{c+dx})}\right]}{d \text{Log}[f]} + \frac{\text{ArcCot}[a + b f^{c+dx}] \text{Log}\left[\frac{2 b f^{c+dx}}{(i-a)(1-i(a+b f^{c+dx}))}\right]}{d \text{Log}[f]}$$

$$-\frac{i \text{PolyLog}\left[2, 1 - \frac{2}{1-i(a+b f^{c+dx})}\right]}{2 d \text{Log}[f]} + \frac{i \text{PolyLog}\left[2, 1 - \frac{2 b f^{c+dx}}{(i-a)(1-i(a+b f^{c+dx}))}\right]}{2 d \text{Log}[f]}$$

Result (type 1, 1 leaves):

???

Problem 225: Unable to integrate problem.

$$\int x \text{ArcCot}[a + b f^{c+dx}] dx$$

Optimal (type 4, 250 leaves, 25 steps):

$$-\frac{1}{4} i x^2 \text{Log}\left[1 - \frac{b f^{c+dx}}{i-a}\right] + \frac{1}{4} i x^2 \text{Log}\left[1 + \frac{b f^{c+dx}}{i+a}\right] +$$

$$\frac{1}{4} i x^2 \text{Log}\left[1 - \frac{i}{a + b f^{c+dx}}\right] - \frac{1}{4} i x^2 \text{Log}\left[1 + \frac{i}{a + b f^{c+dx}}\right] - \frac{i x \text{PolyLog}\left[2, \frac{b f^{c+dx}}{i-a}\right]}{2 d \text{Log}[f]} +$$

$$\frac{i x \text{PolyLog}\left[2, -\frac{b f^{c+dx}}{i+a}\right]}{2 d \text{Log}[f]} + \frac{i \text{PolyLog}\left[3, \frac{b f^{c+dx}}{i-a}\right]}{2 d^2 \text{Log}[f]^2} - \frac{i \text{PolyLog}\left[3, -\frac{b f^{c+dx}}{i+a}\right]}{2 d^2 \text{Log}[f]^2}$$

Result (type 8, 16 leaves):

$$\int x \text{ArcCot}[a + b f^{c+dx}] dx$$

Problem 226: Unable to integrate problem.

$$\int x^2 \text{ArcCot}[a + b f^{c+dx}] dx$$

Optimal (type 4, 313 leaves, 29 steps):

$$\begin{aligned}
& -\frac{1}{6} i x^3 \operatorname{Log}\left[1 - \frac{b f^{c+d x}}{i-a}\right] + \frac{1}{6} i x^3 \operatorname{Log}\left[1 + \frac{b f^{c+d x}}{i+a}\right] + \frac{1}{6} i x^3 \operatorname{Log}\left[1 - \frac{i}{a+b f^{c+d x}}\right] - \\
& \frac{1}{6} i x^3 \operatorname{Log}\left[1 + \frac{i}{a+b f^{c+d x}}\right] - \frac{i x^2 \operatorname{PolyLog}\left[2, \frac{b f^{c+d x}}{i-a}\right]}{2 d \operatorname{Log}[f]} + \frac{i x^2 \operatorname{PolyLog}\left[2, -\frac{b f^{c+d x}}{i+a}\right]}{2 d \operatorname{Log}[f]} + \\
& \frac{i x \operatorname{PolyLog}\left[3, \frac{b f^{c+d x}}{i-a}\right]}{d^2 \operatorname{Log}[f]^2} - \frac{i x \operatorname{PolyLog}\left[3, -\frac{b f^{c+d x}}{i+a}\right]}{d^2 \operatorname{Log}[f]^2} - \frac{i \operatorname{PolyLog}\left[4, \frac{b f^{c+d x}}{i-a}\right]}{d^3 \operatorname{Log}[f]^3} + \frac{i \operatorname{PolyLog}\left[4, -\frac{b f^{c+d x}}{i+a}\right]}{d^3 \operatorname{Log}[f]^3}
\end{aligned}$$

Result (type 8, 18 leaves):

$$\int x^2 \operatorname{ArcCot}\left[a+b f^{c+d x}\right] dx$$

Problem 230: Result is not expressed in closed-form.

$$\int e^{c(a+b x)} \operatorname{ArcCot}\left[\operatorname{Cosh}\left[ac+bcx\right]\right] dx$$

Optimal (type 3, 103 leaves, 8 steps):

$$\begin{aligned}
& \frac{e^{ac+bcx} \operatorname{ArcCot}\left[\operatorname{Cosh}\left[ac+bcx\right]\right]}{bc} + \\
& \frac{\left(1-\sqrt{2}\right) \operatorname{Log}\left[3-2\sqrt{2}+e^{2c(a+bx)}\right]}{2bc} + \frac{\left(1+\sqrt{2}\right) \operatorname{Log}\left[3+2\sqrt{2}+e^{2c(a+bx)}\right]}{2bc}
\end{aligned}$$

Result (type 7, 146 leaves):

$$\begin{aligned}
& \frac{1}{2bc} \left(4c(a+bx) + 2e^{c(a+bx)} \operatorname{ArcCot}\left[\frac{1}{2}e^{-c(a+bx)}\left(1+e^{2c(a+bx)}\right)\right] + \operatorname{RootSum}\left[1+6\#1^2+\#1^4 \&, \right. \right. \\
& \left. \left. \frac{1}{1+3\#1^2}\left(-ac-bcx + \operatorname{Log}\left[e^{c(a+bx)}-\#1\right] - 7ac\#1^2 - 7bcx\#1^2 + 7\operatorname{Log}\left[e^{c(a+bx)}-\#1\right]\#1^2\right) \&\right]
\end{aligned}$$

Problem 231: Result is not expressed in closed-form.

$$\int e^{c(a+b x)} \operatorname{ArcCot}\left[\operatorname{Tanh}\left[ac+bcx\right]\right] dx$$

Optimal (type 3, 180 leaves, 13 steps):

$$\begin{aligned}
& \frac{e^{ac+bcx} \operatorname{ArcCot}\left[\operatorname{Tanh}\left[ac+bcx\right]\right]}{bc} - \frac{\operatorname{ArcTan}\left[1-\sqrt{2}e^{ac+bcx}\right]}{\sqrt{2}bc} + \frac{\operatorname{ArcTan}\left[1+\sqrt{2}e^{ac+bcx}\right]}{\sqrt{2}bc} + \\
& \frac{\operatorname{Log}\left[1+e^{2c(a+bx)}-\sqrt{2}e^{ac+bcx}\right]}{2\sqrt{2}bc} - \frac{\operatorname{Log}\left[1+e^{2c(a+bx)}+\sqrt{2}e^{ac+bcx}\right]}{2\sqrt{2}bc}
\end{aligned}$$

Result (type 7, 89 leaves):

$$\frac{1}{2bc} \left(2e^{c(a+bx)} \operatorname{ArcCot}\left[\frac{-1+e^{2c(a+bx)}}{1+e^{2c(a+bx)}}\right] + \operatorname{RootSum}\left[1+\#1^4 \&, \frac{-ac-bcx + \operatorname{Log}\left[e^{c(a+bx)}-\#1\right]}{\#1} \&\right]
\right)$$

Problem 232: Result is not expressed in closed-form.

$$\int e^{c(a+bx)} \operatorname{ArcCot}[\operatorname{Coth}[ac + bcx]] dx$$

Optimal (type 3, 180 leaves, 13 steps):

$$\frac{e^{ac+bcx} \operatorname{ArcCot}[\operatorname{Coth}[c(a+bx)]]}{bc} + \frac{\operatorname{ArcTan}[1 - \sqrt{2} e^{ac+bcx}]}{\sqrt{2} bc} - \frac{\operatorname{ArcTan}[1 + \sqrt{2} e^{ac+bcx}]}{\sqrt{2} bc} - \frac{\operatorname{Log}[1 + e^{2c(a+bx)} - \sqrt{2} e^{ac+bcx}]}{2\sqrt{2} bc} + \frac{\operatorname{Log}[1 + e^{2c(a+bx)} + \sqrt{2} e^{ac+bcx}]}{2\sqrt{2} bc}$$

Result (type 7, 89 leaves):

$$\frac{1}{2bc} \left(2 e^{c(a+bx)} \operatorname{ArcCot}\left[\frac{1 + e^{2c(a+bx)}}{-1 + e^{2c(a+bx)}}\right] + \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{ac + bcx - \operatorname{Log}[e^{c(a+bx)} - \#1]}{\#1} \&\right] \right)$$

Problem 233: Result is not expressed in closed-form.

$$\int e^{c(a+bx)} \operatorname{ArcCot}[\operatorname{Sech}[ac + bcx]] dx$$

Optimal (type 3, 103 leaves, 8 steps):

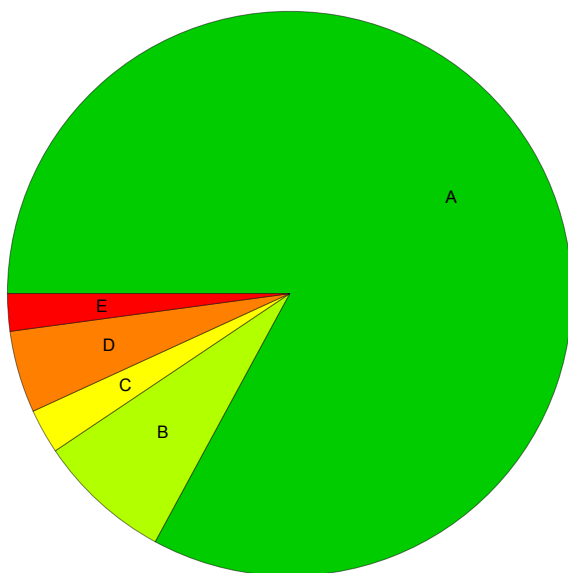
$$\frac{e^{ac+bcx} \operatorname{ArcCot}[\operatorname{Sech}[c(a+bx)]]}{bc} - \frac{(1 - \sqrt{2}) \operatorname{Log}[3 - 2\sqrt{2} + e^{2c(a+bx)}]}{2bc} - \frac{(1 + \sqrt{2}) \operatorname{Log}[3 + 2\sqrt{2} + e^{2c(a+bx)}]}{2bc}$$

Result (type 7, 145 leaves):

$$\frac{1}{2bc} \left(-4c(a+bx) + 2 e^{c(a+bx)} \operatorname{ArcCot}\left[\frac{2 e^{c(a+bx)}}{1 + e^{2c(a+bx)}}\right] + \operatorname{RootSum}\left[1 + 6\#1^2 + \#1^4 \&, \frac{1}{1 + 3\#1^2} (ac + bcx - \operatorname{Log}[e^{c(a+bx)} - \#1] + 7ac\#1^2 + 7bcx\#1^2 - 7\operatorname{Log}[e^{c(a+bx)} - \#1]\#1^2) \&\right] \right)$$

Summary of Integration Test Results

234 integration problems



A - 194 optimal antiderivatives

B - 18 more than twice size of optimal antiderivatives

C - 6 unnecessarily complex antiderivatives

D - 11 unable to integrate problems

E - 5 integration timeouts