Mathematica 11.3 Integration Test Results

Test results for the 49 problems in "5.6.2 Inverse cosecant functions.m"

Problem 13: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcCsc}\left[\frac{a}{x}\right]}{x^2}\,dx$$

Optimal (type 3, 32 leaves, 5 steps):

$$-\frac{\text{ArcSin}\left[\frac{x}{a}\right]}{x}-\frac{\text{ArcTanh}\left[\sqrt{1-\frac{x^2}{a^2}}\right]}{a}$$

Result (type 3, 93 leaves):

$$-\frac{\text{ArcCsc}\left[\frac{a}{x}\right]}{x}-\frac{\sqrt{-1+\frac{a^2}{x^2}}\;x\left(-\text{Log}\left[1-\frac{a}{\sqrt{-1+\frac{a^2}{x^2}}\;x}\right]+\text{Log}\left[1+\frac{a}{\sqrt{-1+\frac{a^2}{x^2}}\;x}\right]\right)}{2\,a^2\sqrt{1-\frac{x^2}{a^2}}}$$

Problem 16: Result unnecessarily involves higher level functions.

$$\int \frac{\text{ArcCsc}\left[a\,x^n\right]}{x}\,dx$$

Optimal (type 4, 69 leaves, 7 steps):

$$\frac{i\,\text{ArcCsc}\left[a\,x^n\right]^2}{2\,n}-\frac{\text{ArcCsc}\left[a\,x^n\right]\,\text{Log}\left[1-e^{2\,i\,\text{ArcCsc}\left[a\,x^n\right]}\right]}{n}+\frac{i\,\text{PolyLog}\left[2,\,e^{2\,i\,\text{ArcCsc}\left[a\,x^n\right]}\right]}{2\,n}$$

Result (type 5, 63 leaves):

$$-\frac{x^{-n}\,\text{HypergeometricPFQ}\left[\left\{\frac{1}{2},\frac{1}{2},\frac{1}{2}\right\},\left\{\frac{3}{2},\frac{3}{2}\right\},\frac{x^{-2\,n}}{a^2}\right]}{a\,n}+\left(\text{ArcCsc}\left[a\,x^n\right]-\text{ArcSin}\left[\frac{x^{-n}}{a}\right]\right)\,\text{Log}[x]$$

Problem 21: Result more than twice size of optimal antiderivative.

$$\int \text{ArcCsc}[a+b\,x]\,dx$$

Optimal (type 3, 36 leaves, 5 steps):

$$\frac{(a+b\,x)\,\text{ArcCsc}[a+b\,x]}{b}+\frac{\text{ArcTanh}\left[\sqrt{1-\frac{1}{(a+b\,x)^2}}\right]}{b}$$

Result (type 3, 120 leaves):

$$x\,\text{ArcCsc}[a+b\,x]+\left((a+b\,x)\sqrt{\frac{-1+a^2+2\,a\,b\,x+b^2\,x^2}{(a+b\,x)^2}}\left(a\,\text{ArcTan}\left[\frac{1}{\sqrt{-1+a^2+2\,a\,b\,x+b^2\,x^2}}\right]+\text{Log}\left[a+b\,x+\sqrt{-1+a^2+2\,a\,b\,x+b^2\,x^2}\right]\right)\right)\Big/\left(b\sqrt{-1+a^2+2\,a\,b\,x+b^2\,x^2}\right)$$

Problem 23: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\text{ArcCsc}[a+b\,x]}{x^2}\,dx$$

Optimal (type 3, 69 leaves, 6 steps):

$$-\frac{b\,\text{ArcCsc}[a+b\,x]}{a}-\frac{\text{ArcCsc}[a+b\,x]}{x}-\frac{2\,b\,\text{ArcTan}\left[\frac{a-\text{Tan}\left[\frac{1}{2}\text{ArcCsc}[a+b\,x]\right]}{\sqrt{1-a^2}}\right]}{a\sqrt{1-a^2}}$$

Result (type 3, 115 leaves):

$$-\frac{\text{ArcCsc}[a+b\,x]}{x}+\frac{b\left(-\text{ArcSin}\left[\frac{1}{a+b\,x}\right]+\frac{i\,\text{Log}\left[\frac{2\left(-\frac{i\,a\,(-1+a^2+a\,b\,x)}{\sqrt{1-a^2}}-a\,(a+b\,x)\sqrt{\frac{-1+a^2+2\,a\,b\,x+b^2\,x^2}{(a+b\,x)^2}}\right)}{b\,x}\right]}{\sqrt{1-a^2}}\right)}{a}$$

Problem 24: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\text{ArcCsc}[a+b\,x]}{x^3}\,dx$$

Optimal (type 3, 123 leaves, 8 steps):

$$-\frac{b\,(a+b\,x)\sqrt{1-\frac{1}{(a+b\,x)^2}}}{2\,a\,(1-a^2)\,x}+\frac{b^2\,\text{ArcCsc}[a+b\,x]}{2\,a^2}-\frac{\text{ArcCsc}[a+b\,x]}{2\,x^2}+\frac{(1-2\,a^2)\,b^2\,\text{ArcTan}\left[\frac{a-\text{Tan}\left[\frac{1}{2}\text{ArcCsc}[a+b\,x]\right]}{\sqrt{1-a^2}}\right]}{a^2\,(1-a^2)^{3/2}}$$

Result (type 3, 199 leaves):

$$\frac{1}{2x^2}\left(\frac{b\,x\,(a+b\,x)\sqrt{\frac{-1+a^2+2\,a\,b\,x+b^2\,x^2}{(a+b\,x)^2}}}{a\,(-1+a^2)}-\text{ArcCsc}[a+b\,x]+\frac{b^2\,x^2\,\text{ArcSin}\left[\frac{1}{a+b\,x}\right]}{a^2}+\frac{1}{a^2\,(1-a^2)^{3/2}}\right.$$
$$\left.i\,(-1+2\,a^2)\,b^2\,x^2\,\text{Log}\left[\frac{4\,(-1+a)\,a^2\,(1+a)\left(\frac{i\,(-1+a^2+a\,b\,x)}{\sqrt{1-a^2}}+(a+b\,x)\sqrt{\frac{-1+a^2+2\,a\,b\,x+b^2\,x^2}{(a+b\,x)^2}}\right)}{(-1+2\,a^2)\,b^2\,x}\right]\right)$$

Problem 25: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\text{ArcCsc}[a+b\,x]}{x^4}\,dx$$

Optimal (type 3, 180 leaves, 9 steps):

$$-\frac{b\,(a+b\,x)\sqrt{1-\frac{1}{(a+b\,x)^2}}}{6\,a\,(1-a^2)\,x^2}+\frac{(2-5\,a^2)\,b^2\,(a+b\,x)\sqrt{1-\frac{1}{(a+b\,x)^2}}}{6\,a^2\,(1-a^2)^2\,x}-$$
$$\frac{b^3\,\text{ArcCsc}[a+b\,x]}{3\,a^3}-\frac{\text{ArcCsc}[a+b\,x]}{3\,x^3}-\frac{(2-5\,a^2+6\,a^4)\,b^3\,\text{ArcTan}\left[\frac{a-\text{Tan}\left[\frac{1}{2}\text{ArcCsc}[a+b\,x]\right]}{\sqrt{1-a^2}}\right]}{3\,a^3\,(1-a^2)^{5/2}}$$

Result (type 3, 241 leaves):

$$\frac{1}{6}\left(\frac{b\sqrt{\frac{-1+a^2+2\,a\,b\,x+b^2\,x^2}{(a+b\,x)^2}}\,\left(a^4+a\,b\,x-4\,a^3\,b\,x+2\,b^2\,x^2-a^2\,(1+5\,b^2\,x^2)\right)}{a^2\,(-1+a^2)^2\,x^2}-\right.$$
$$\frac{2\,\text{ArcCsc}[a+b\,x]}{x^3}-\frac{2\,b^3\,\text{ArcSin}\left[\frac{1}{a+b\,x}\right]}{a^3}+\frac{1}{a^3\,(1-a^2)^{5/2}}$$
$$\left.i\,(2-5\,a^2+6\,a^4)\,b^3\,\text{Log}\left[\frac{12\,a^3\,(-1+a^2)^2\left(-\frac{i\,(-1+a^2+a\,b\,x)}{\sqrt{1-a^2}}-(a+b\,x)\sqrt{\frac{-1+a^2+2\,a\,b\,x+b^2\,x^2}{(a+b\,x)^2}}\right)}{(2-5\,a^2+6\,a^4)\,b^3\,x}\right]\right)$$

Problem 26: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{ArcCsc}[a+b x]}{x^5} \, dx$$

Optimal (type 3, 239 leaves, 10 steps):

$$-\frac{b (a+b x) \sqrt{1-\frac{1}{(a+b x)^2}}}{12 a (1-a^2) x^3}+\frac{(3-8 a^2) b^2 (a+b x) \sqrt{1-\frac{1}{(a+b x)^2}}}{24 a^2 (1-a^2)^2 x^2}-\frac{(6-17 a^2+26 a^4) b^3 (a+b x) \sqrt{1-\frac{1}{(a+b x)^2}}}{24 a^3 (1-a^2)^3 x}+\frac{b^4 \text{ArcCsc}[a+b x]}{4 a^4}-\frac{\text{ArcCsc}[a+b x]}{4 x^4}+\frac{(2-7 a^2+8 a^4-8 a^6) b^4 \text{ArcTan}\left[\frac{a-\text{Tan}\left[\frac{1}{2} \text{ArcCsc}[a+b x]\right]}{\sqrt{1-a^2}}\right]}{4 a^4 (1-a^2)^{7/2}}$$

Result (type 3, 307 leaves):

$$\frac{1}{8} \left(\left(b \sqrt{\frac{-1+a^2+2 a b x+b^2 x^2}{(a+b x)^2}} \left(2 a^7-6 a^6 b x+3 a b^2 x^2+6 b^3 x^3+a^3 (2-6 b^2 x^2)+2 a^5 (-2+9 b^2 x^2)+a^4 b x (7+26 b^2 x^2)-a^2 (b x+17 b^3 x^3)\right)\right) \Big/ \left(3 a^3 (-1+a^2)^3 x^3\right)-\frac{2 \text{ArcCsc}[a+b x]}{x^4}+\frac{2 b^4 \text{ArcSin}\left[\frac{1}{a+b x}\right]}{a^4}+\frac{1}{a^4 (1-a^2)^{7/2}} i (-2+7 a^2-8 a^4+8 a^6) b^4 \text{Log}\left[\frac{16 a^4 (-1+a^2)^3 \left(\frac{i (-1+a^2+a b x)}{\sqrt{1-a^2}}+(a+b x) \sqrt{\frac{-1+a^2+2 a b x+b^2 x^2}{(a+b x)^2}}\right)}{(-2+7 a^2-8 a^4+8 a^6) b^4 x}\right]\right)$$

Problem 31: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcCsc}[a+b x]^2}{x} \, dx$$

Optimal (type 4, 324 leaves, 17 steps):

$$\text{ArcCsc}[a+b x]^2 \text{Log}\left[1+\frac{i a e^{i \text{ArcCsc}[a+b x]}}{1-\sqrt{1-a^2}}\right]+\text{ArcCsc}[a+b x]^2 \text{Log}\left[1+\frac{i a e^{i \text{ArcCsc}[a+b x]}}{1+\sqrt{1-a^2}}\right]-$$
$$\text{ArcCsc}[a+b x]^2 \text{Log}\left[1-e^{2 i \text{ArcCsc}[a+b x]}\right]-2 i \text{ArcCsc}[a+b x] \text{PolyLog}\left[2,-\frac{i a e^{i \text{ArcCsc}[a+b x]}}{1-\sqrt{1-a^2}}\right]-$$
$$2 i \text{ArcCsc}[a+b x] \text{PolyLog}\left[2,-\frac{i a e^{i \text{ArcCsc}[a+b x]}}{1+\sqrt{1-a^2}}\right]+i \text{ArcCsc}[a+b x] \text{PolyLog}\left[2, e^{2 i \text{ArcCsc}[a+b x]}\right]+$$
$$2 \text{PolyLog}\left[3,-\frac{i a e^{i \text{ArcCsc}[a+b x]}}{1-\sqrt{1-a^2}}\right]+2 \text{PolyLog}\left[3,-\frac{i a e^{i \text{ArcCsc}[a+b x]}}{1+\sqrt{1-a^2}}\right]-\frac{1}{2} \text{PolyLog}\left[3, e^{2 i \text{ArcCsc}[a+b x]}\right]$$

Result (type 4, 1217 leaves):

$$\frac{i \pi^3}{6}-\frac{1}{3} i \text{ArcCsc}[a+b x]^3+$$
$$8 i \text{ArcCsc}[a+b x] \text{ArcSin}\left[\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right] \text{ArcTan}\left[\frac{(1+a) \text{Cot}\left[\frac{1}{4}(\pi+2 \text{ArcCsc}[a+b x])\right]}{\sqrt{1-a^2}}\right]-$$
$$8 i \text{ArcCsc}[a+b x] \text{ArcSin}\left[\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right]$$
$$\text{ArcTan}\left[\frac{(1+a)\left(\text{Cos}\left[\frac{1}{2} \text{ArcCsc}[a+b x]\right]-\text{Sin}\left[\frac{1}{2} \text{ArcCsc}[a+b x]\right]\right)}{\sqrt{1-a^2}\left(\text{Cos}\left[\frac{1}{2} \text{ArcCsc}[a+b x]\right]+\text{Sin}\left[\frac{1}{2} \text{ArcCsc}[a+b x]\right]\right)}\right]-$$
$$\text{ArcCsc}[a+b x]^2 \text{Log}\left[1-e^{-i \text{ArcCsc}[a+b x]}\right]-$$
$$\pi \text{ArcCsc}[a+b x] \text{Log}\left[1+\frac{i\left(-1+\sqrt{1-a^2}\right) e^{-i \text{ArcCsc}[a+b x]}}{a}\right]+$$
$$\text{ArcCsc}[a+b x]^2 \text{Log}\left[1+\frac{i\left(-1+\sqrt{1-a^2}\right) e^{-i \text{ArcCsc}[a+b x]}}{a}\right]+$$
$$4 \text{ArcCsc}[a+b x] \text{ArcSin}\left[\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right] \text{Log}\left[1+\frac{i\left(-1+\sqrt{1-a^2}\right) e^{-i \text{ArcCsc}[a+b x]}}{a}\right]-$$
$$\pi \text{ArcCsc}[a+b x] \text{Log}\left[1-\frac{i\left(1+\sqrt{1-a^2}\right) e^{-i \text{ArcCsc}[a+b x]}}{a}\right]+$$
$$\text{ArcCsc}[a+b x]^2 \text{Log}\left[1-\frac{i\left(1+\sqrt{1-a^2}\right) e^{-i \text{ArcCsc}[a+b x]}}{a}\right]-$$
$$4 \text{ArcCsc}[a+b x] \text{ArcSin}\left[\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right] \text{Log}\left[1-\frac{i\left(1+\sqrt{1-a^2}\right) e^{-i \text{ArcCsc}[a+b x]}}{a}\right]-$$
$$\text{ArcCsc}[a+b x]^2 \text{Log}\left[1+e^{i \text{ArcCsc}[a+b x]}\right]+\text{ArcCsc}[a+b x]^2 \text{Log}\left[1+\frac{a e^{i \text{ArcCsc}[a+b x]}}{-i+\sqrt{-1+a^2}}\right]+$$

$$\text{ArcCsc}[a+b\,x]^2\,\text{Log}\left[1-\frac{a\,e^{i\,\text{ArcCsc}[a+b\,x]}}{i+\sqrt{-1+a^2}}\right]+$$

$$\pi\,\text{ArcCsc}[a+b\,x]\,\text{Log}\left[1+\frac{\left(-1+\sqrt{1-a^2}\right)\left(\frac{1}{a+b\,x}+i\,\sqrt{1-\frac{1}{(a+b\,x)^2}}\right)}{a}\right]-$$

$$\text{ArcCsc}[a+b\,x]^2\,\text{Log}\left[1+\frac{\left(-1+\sqrt{1-a^2}\right)\left(\frac{1}{a+b\,x}+i\,\sqrt{1-\frac{1}{(a+b\,x)^2}}\right)}{a}\right]-$$

$$4\,\text{ArcCsc}[a+b\,x]\,\text{ArcSin}\left[\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{\left(-1+\sqrt{1-a^2}\right)\left(\frac{1}{a+b\,x}+i\,\sqrt{1-\frac{1}{(a+b\,x)^2}}\right)}{a}\right]+$$

$$\pi\,\text{ArcCsc}[a+b\,x]\,\text{Log}\left[1-\frac{\left(1+\sqrt{1-a^2}\right)\left(\frac{1}{a+b\,x}+i\,\sqrt{1-\frac{1}{(a+b\,x)^2}}\right)}{a}\right]-$$

$$\text{ArcCsc}[a+b\,x]^2\,\text{Log}\left[1-\frac{\left(1+\sqrt{1-a^2}\right)\left(\frac{1}{a+b\,x}+i\,\sqrt{1-\frac{1}{(a+b\,x)^2}}\right)}{a}\right]+$$

$$4\,\text{ArcCsc}[a+b\,x]\,\text{ArcSin}\left[\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{\left(1+\sqrt{1-a^2}\right)\left(\frac{1}{a+b\,x}+i\,\sqrt{1-\frac{1}{(a+b\,x)^2}}\right)}{a}\right]-$$

$$2\,i\,\text{ArcCsc}[a+b\,x]\,\text{PolyLog}\left[2,\,e^{-i\,\text{ArcCsc}[a+b\,x]}\right]+2\,i\,\text{ArcCsc}[a+b\,x]\,\text{PolyLog}\left[2,\,-e^{i\,\text{ArcCsc}[a+b\,x]}\right]-$$

$$2\,i\,\text{ArcCsc}[a+b\,x]\,\text{PolyLog}\left[2,\,-\frac{a\,e^{i\,\text{ArcCsc}[a+b\,x]}}{-i+\sqrt{-1+a^2}}\right]-2\,i\,\text{ArcCsc}[a+b\,x]\,\text{PolyLog}\left[2,\,\frac{a\,e^{i\,\text{ArcCsc}[a+b\,x]}}{i+\sqrt{-1+a^2}}\right]-$$

$$2\,\text{PolyLog}\left[3,\,e^{-i\,\text{ArcCsc}[a+b\,x]}\right]-2\,\text{PolyLog}\left[3,\,-e^{i\,\text{ArcCsc}[a+b\,x]}\right]+$$

$$2\,\text{PolyLog}\left[3,\,-\frac{a\,e^{i\,\text{ArcCsc}[a+b\,x]}}{-i+\sqrt{-1+a^2}}\right]+2\,\text{PolyLog}\left[3,\,\frac{a\,e^{i\,\text{ArcCsc}[a+b\,x]}}{i+\sqrt{-1+a^2}}\right]$$

Problem 32: Result more than twice size of optimal antiderivative.

$$\int\frac{\text{ArcCsc}[a+b\,x]^2}{x^2}\,dx$$

Optimal (type 4, 254 leaves, 12 steps) :

$$-\frac{b\,\text{ArcCsc}[a+b\,x]^2}{a}-\frac{\text{ArcCsc}[a+b\,x]^2}{x}-\frac{2\,i\,b\,\text{ArcCsc}[a+b\,x]\,\text{Log}\left[1+\frac{i\,a\,e^{i\,\text{ArcCsc}[a+b\,x]}}{1-\sqrt{1-a^2}}\right]}{a\,\sqrt{1-a^2}}+\frac{2\,i\,b\,\text{ArcCsc}[a+b\,x]\,\text{Log}\left[1+\frac{i\,a\,e^{i\,\text{ArcCsc}[a+b\,x]}}{1+\sqrt{1-a^2}}\right]}{a\,\sqrt{1-a^2}}-$$

$$\frac{2\,b\,\text{PolyLog}\left[2,-\frac{i\,a\,e^{i\,\text{ArcCsc}[a+b\,x]}}{1-\sqrt{1-a^2}}\right]}{a\,\sqrt{1-a^2}}+\frac{2\,b\,\text{PolyLog}\left[2,-\frac{i\,a\,e^{i\,\text{ArcCsc}[a+b\,x]}}{1+\sqrt{1-a^2}}\right]}{a\,\sqrt{1-a^2}}$$

Result (type 4, 804 leaves):

$$-\frac{1}{a}\,b\left(\frac{(a+b\,x)\,\text{ArcCsc}[a+b\,x]^2}{b\,x}+\frac{2\,\pi\,\text{ArcTan}\left[\frac{a-\text{Tan}\left[\frac{1}{2}\text{ArcCsc}[a+b\,x]\right]}{\sqrt{1-a^2}}\right]}{\sqrt{1-a^2}}+\right.$$

$$\frac{1}{\sqrt{-1+a^2}}\,2\left(-2\,\text{ArcCos}\left[\frac{1}{a}\right]\text{ArcTanh}\left[\frac{(1+a)\,\text{Cot}\left[\frac{1}{4}\left(\pi+2\,\text{ArcCsc}[a+b\,x]\right)\right]}{\sqrt{-1+a^2}}\right]+\right.$$

$$\left(\pi-2\,\text{ArcCsc}[a+b\,x]\right)\text{ArcTanh}\left[\frac{(-1+a)\,\text{Tan}\left[\frac{1}{4}\left(\pi+2\,\text{ArcCsc}[a+b\,x]\right)\right]}{\sqrt{-1+a^2}}\right]+$$

$$\left(\text{ArcCos}\left[\frac{1}{a}\right]+2\,i\left(-\text{ArcTanh}\left[\frac{(1+a)\,\text{Cot}\left[\frac{1}{4}\left(\pi+2\,\text{ArcCsc}[a+b\,x]\right)\right]}{\sqrt{-1+a^2}}\right]+\text{ArcTanh}\left[\right.\right.\right.$$

$$\left.\left.\left.\frac{(-1+a)\,\text{Tan}\left[\frac{1}{4}\left(\pi+2\,\text{ArcCsc}[a+b\,x]\right)\right]}{\sqrt{-1+a^2}}\right]\right)\right)\text{Log}\left[\frac{\sqrt{-1+a^2}\;e^{\frac{1}{4}i\left(\pi-2\,\text{ArcCsc}[a+b\,x]\right)}}{\sqrt{2}\,\sqrt{a}\,\sqrt{-\frac{b\,x}{a+b\,x}}}\right]+$$

$$\left(\text{ArcCos}\left[\frac{1}{a}\right]+2\,i\,\text{ArcTanh}\left[\frac{(1+a)\,\text{Cot}\left[\frac{1}{4}\left(\pi+2\,\text{ArcCsc}[a+b\,x]\right)\right]}{\sqrt{-1+a^2}}\right]-2\,i\,\text{ArcTanh}\left[\right.\right.$$

$$\left.\left.\frac{(-1+a)\,\text{Tan}\left[\frac{1}{4}\left(\pi+2\,\text{ArcCsc}[a+b\,x]\right)\right]}{\sqrt{-1+a^2}}\right]\right)\text{Log}\left[\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{-1+a^2}\;e^{\frac{1}{2}i\,\text{ArcCsc}[a+b\,x]}}{\sqrt{a}\,\sqrt{-\frac{b\,x}{a+b\,x}}}\right]-$$

$$\left(\text{ArcCos}\left[\frac{1}{a}\right]-2\,i\,\text{ArcTanh}\left[\frac{(1+a)\,\text{Cot}\left[\frac{1}{4}\left(\pi+2\,\text{ArcCsc}[a+b\,x]\right)\right]}{\sqrt{-1+a^2}}\right]\right)$$

$$\text{Log}\left[\left((-1+a)\left(i+i\,a+\sqrt{-1+a^2}\right)\left(-i+\text{Cot}\left[\frac{1}{4}\left(\pi+2\,\text{ArcCsc}[a+b\,x]\right)\right]\right)\right)\Big/\right.$$

$$\left.\left(a\left(-1+a+\sqrt{-1+a^2}\;\text{Cot}\left[\frac{1}{4}\left(\pi+2\,\text{ArcCsc}[a+b\,x]\right)\right]\right)\right)\right]-$$

$$\left(\text{ArcCos}\left[\frac{1}{a}\right]+2\,i\,\text{ArcTanh}\left[\frac{(1+a)\,\text{Cot}\left[\frac{1}{4}\,(\pi+2\,\text{ArcCsc}[a+b\,x])\right]}{\sqrt{-1+a^2}}\right]\right)$$

$$\text{Log}\left[\left((-1+a)\,\left(-i-i\,a+\sqrt{-1+a^2}\right)\,\left(i+\text{Cot}\left[\frac{1}{4}\,(\pi+2\,\text{ArcCsc}[a+b\,x])\right]\right)\right)\Big/\left(a\,\left(-1+a+\sqrt{-1+a^2}\,\text{Cot}\left[\frac{1}{4}\,(\pi+2\,\text{ArcCsc}[a+b\,x])\right]\right)\right)\right]+$$

$$i\,\left(-\text{PolyLog}\left[2,\,\left(\left(1-i\,\sqrt{-1+a^2}\right)\,\left(1-a+\sqrt{-1+a^2}\,\text{Cot}\left[\frac{1}{4}\,(\pi+2\,\text{ArcCsc}[a+b\,x])\right]\right)\right)\Big/\left(a\,\left(-1+a+\sqrt{-1+a^2}\,\text{Cot}\left[\frac{1}{4}\,(\pi+2\,\text{ArcCsc}[a+b\,x])\right]\right)\right)\right]+\text{PolyLog}\left[2,\,\left(\left(1+i\,\sqrt{-1+a^2}\right)\,\left(1-a+\sqrt{-1+a^2}\,\text{Cot}\left[\frac{1}{4}\,(\pi+2\,\text{ArcCsc}[a+b\,x])\right]\right)\right)\Big/\left(a\,\left(-1+a+\sqrt{-1+a^2}\,\text{Cot}\left[\frac{1}{4}\,(\pi+2\,\text{ArcCsc}[a+b\,x])\right]\right)\right)\right]\right)\Bigg)\Bigg)$$

Problem 36: Unable to integrate problem.

$$\int\frac{\text{ArcCsc}[a+b\,x]^3}{x}\,dx$$

Optimal (type 4, 448 leaves, 20 steps):

$$\text{ArcCsc}[a+b\,x]^3\,\text{Log}\left[1+\frac{i\,a\,e^{i\,\text{ArcCsc}[a+b\,x]}}{1-\sqrt{1-a^2}}\right]+\text{ArcCsc}[a+b\,x]^3\,\text{Log}\left[1+\frac{i\,a\,e^{i\,\text{ArcCsc}[a+b\,x]}}{1+\sqrt{1-a^2}}\right]-\text{ArcCsc}[a+b\,x]^3\,\text{Log}\left[1-e^{2\,i\,\text{ArcCsc}[a+b\,x]}\right]-3\,i\,\text{ArcCsc}[a+b\,x]^2\,\text{PolyLog}\left[2,\,-\frac{i\,a\,e^{i\,\text{ArcCsc}[a+b\,x]}}{1-\sqrt{1-a^2}}\right]-3\,i\,\text{ArcCsc}[a+b\,x]^2\,\text{PolyLog}\left[2,\,-\frac{i\,a\,e^{i\,\text{ArcCsc}[a+b\,x]}}{1+\sqrt{1-a^2}}\right]+\frac{3}{2}\,i\,\text{ArcCsc}[a+b\,x]^2\,\text{PolyLog}\left[2,\,e^{2\,i\,\text{ArcCsc}[a+b\,x]}\right]+6\,\text{ArcCsc}[a+b\,x]\,\text{PolyLog}\left[3,\,-\frac{i\,a\,e^{i\,\text{ArcCsc}[a+b\,x]}}{1-\sqrt{1-a^2}}\right]+6\,\text{ArcCsc}[a+b\,x]\,\text{PolyLog}\left[3,\,-\frac{i\,a\,e^{i\,\text{ArcCsc}[a+b\,x]}}{1+\sqrt{1-a^2}}\right]-\frac{3}{2}\,\text{ArcCsc}[a+b\,x]\,\text{PolyLog}\left[3,\,e^{2\,i\,\text{ArcCsc}[a+b\,x]}\right]+6\,i\,\text{PolyLog}\left[4,\,-\frac{i\,a\,e^{i\,\text{ArcCsc}[a+b\,x]}}{1-\sqrt{1-a^2}}\right]+6\,i\,\text{PolyLog}\left[4,\,-\frac{i\,a\,e^{i\,\text{ArcCsc}[a+b\,x]}}{1+\sqrt{1-a^2}}\right]-\frac{3}{4}\,i\,\text{PolyLog}\left[4,\,e^{2\,i\,\text{ArcCsc}[a+b\,x]}\right]$$

Result (type 8, 14 leaves):

$$\int \frac{\text{ArcCsc}[a+b x]^3}{x} \, dx$$

Problem 37: Unable to integrate problem.

$$\int \frac{\text{ArcCsc}[a+b x]^3}{x^2} \, dx$$

Optimal (type 4, 378 leaves, 14 steps):

$$-\frac{b\,\text{ArcCsc}[a+b x]^3}{a}-\frac{\text{ArcCsc}[a+b x]^3}{x}-\frac{3\,i\,b\,\text{ArcCsc}[a+b x]^2\,\text{Log}\left[1+\frac{i\,a\,e^{i\,\text{ArcCsc}[a+b x]}}{1-\sqrt{1-a^2}}\right]}{a\sqrt{1-a^2}}+\frac{3\,i\,b\,\text{ArcCsc}[a+b x]^2\,\text{Log}\left[1+\frac{i\,a\,e^{i\,\text{ArcCsc}[a+b x]}}{1+\sqrt{1-a^2}}\right]}{a\sqrt{1-a^2}}-\frac{6\,b\,\text{ArcCsc}[a+b x]\,\text{PolyLog}\left[2,-\frac{i\,a\,e^{i\,\text{ArcCsc}[a+b x]}}{1-\sqrt{1-a^2}}\right]}{a\sqrt{1-a^2}}+\frac{6\,b\,\text{ArcCsc}[a+b x]\,\text{PolyLog}\left[2,-\frac{i\,a\,e^{i\,\text{ArcCsc}[a+b x]}}{1+\sqrt{1-a^2}}\right]}{a\sqrt{1-a^2}}-\frac{6\,i\,b\,\text{PolyLog}\left[3,-\frac{i\,a\,e^{i\,\text{ArcCsc}[a+b x]}}{1-\sqrt{1-a^2}}\right]}{a\sqrt{1-a^2}}+\frac{6\,i\,b\,\text{PolyLog}\left[3,-\frac{i\,a\,e^{i\,\text{ArcCsc}[a+b x]}}{1+\sqrt{1-a^2}}\right]}{a\sqrt{1-a^2}}$$

Result (type 8, 14 leaves):

$$\int \frac{\text{ArcCsc}[a+b x]^3}{x^2} \, dx$$

Problem 38: Result more than twice size of optimal antiderivative.

$$\int x^3\,\text{ArcCsc}\left[a+b x^4\right] \, dx$$

Optimal (type 3, 48 leaves, 6 steps):

$$\frac{\left(a+b x^4\right)\text{ArcCsc}\left[a+b x^4\right]}{4 b}+\frac{\text{ArcTanh}\left[\sqrt{1-\frac{1}{\left(a+b x^4\right)^2}}\right]}{4 b}$$

Result (type 3, 127 leaves):

$$\frac{\left(a+b x^4\right)\text{ArcCsc}\left[a+b x^4\right]}{4 b}+\left(\sqrt{-1+\left(a+b x^4\right)^2}\left(-\text{Log}\left[1-\frac{a+b x^4}{\sqrt{-1+\left(a+b x^4\right)^2}}\right]+\text{Log}\left[1+\frac{a+b x^4}{\sqrt{-1+\left(a+b x^4\right)^2}}\right]\right)\right)\Big/\left(8 b\left(a+b x^4\right)\sqrt{1-\frac{1}{\left(a+b x^4\right)^2}}\right)$$

Problem 39: Result more than twice size of optimal antiderivative.

$$\int x^{-1+n} \text{ArcCsc}\left[a+b x^n\right] dx$$

Optimal (type 3, 48 leaves, 6 steps):

$$\frac{\left(a+b x^n\right) \text{ArcCsc}\left[a+b x^n\right]}{b n}+\frac{\text{ArcTanh}\left[\sqrt{1-\frac{1}{\left(a+b x^n\right)^2}}\right]}{b n}$$

Result (type 3, 130 leaves):

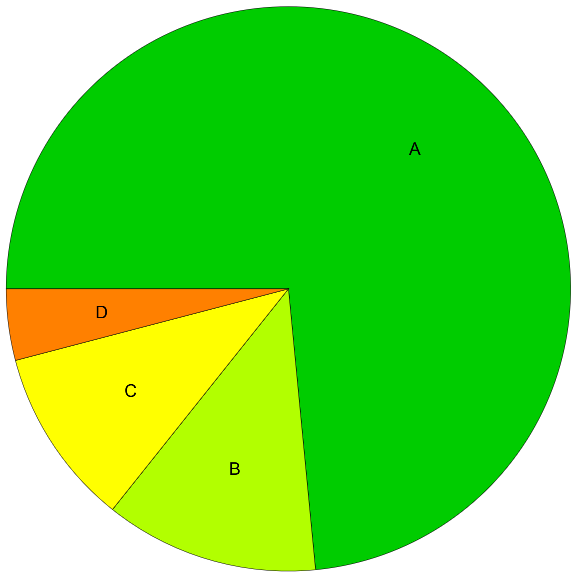
$$\frac{\left(a+b x^n\right) \text{ArcCsc}\left[a+b x^n\right]}{b n}+$$

$$\left(\sqrt{-1+\left(a+b x^n\right)^2} \left(-\text{Log}\left[1-\frac{a+b x^n}{\sqrt{-1+\left(a+b x^n\right)^2}}\right]+\text{Log}\left[1+\frac{a+b x^n}{\sqrt{-1+\left(a+b x^n\right)^2}}\right]\right)\right) \Bigg/$$

$$\left(2 b n \left(a+b x^n\right) \sqrt{1-\frac{1}{\left(a+b x^n\right)^2}}\right)$$

Summary of Integration Test Results

49 integration problems



A - 36 optimal antiderivatives

B - 6 more than twice size of optimal antiderivatives

C - 5 unnecessarily complex antiderivatives

D - 2 unable to integrate problems

E - 0 integration timeouts