

Mathematica 11.3 Integration Test Results

Test results for the 525 problems in "6.1.7 hyper^m (a+b sinh^n)^p.m"

Problem 6: Result more than twice size of optimal antiderivative.

$$\int \text{Csch}[c + d x] (a + b \text{Sinh}[c + d x]^2) dx$$

Optimal (type 3, 25 leaves, 2 steps):

$$-\frac{a \text{ArcTanh}[\text{Cosh}[c + d x]]}{d} + \frac{b \text{Cosh}[c + d x]}{d}$$

Result (type 3, 62 leaves):

$$\frac{b \text{Cosh}[c] \text{Cosh}[d x]}{d} - \frac{a \text{Log}[\text{Cosh}[\frac{c}{2} + \frac{d x}{2}]]}{d} + \frac{a \text{Log}[\text{Sinh}[\frac{c}{2} + \frac{d x}{2}]]}{d} + \frac{b \text{Sinh}[c] \text{Sinh}[d x]}{d}$$

Problem 8: Result more than twice size of optimal antiderivative.

$$\int \text{Csch}[c + d x]^3 (a + b \text{Sinh}[c + d x]^2) dx$$

Optimal (type 3, 40 leaves, 2 steps):

$$\frac{(a - 2 b) \text{ArcTanh}[\text{Cosh}[c + d x]]}{2 d} - \frac{a \text{Coth}[c + d x] \text{Csch}[c + d x]}{2 d}$$

Result (type 3, 118 leaves):

$$-\frac{a \text{Csch}[\frac{1}{2} (c + d x)]^2}{8 d} - \frac{b \text{Log}[\text{Cosh}[\frac{c}{2} + \frac{d x}{2}]]}{d} + \frac{a \text{Log}[\text{Cosh}[\frac{1}{2} (c + d x)]]}{2 d} + \frac{b \text{Log}[\text{Sinh}[\frac{c}{2} + \frac{d x}{2}]]}{d} - \frac{a \text{Log}[\text{Sinh}[\frac{1}{2} (c + d x)]]}{2 d} - \frac{a \text{Sech}[\frac{1}{2} (c + d x)]^2}{8 d}$$

Problem 17: Result more than twice size of optimal antiderivative.

$$\int \text{Csch}[c + d x]^3 (a + b \text{Sinh}[c + d x]^2)^2 dx$$

Optimal (type 3, 56 leaves, 5 steps):

$$\frac{a (a - 4 b) \text{ArcTanh}[\text{Cosh}[c + d x]]}{2 d} + \frac{b^2 \text{Cosh}[c + d x]}{d} - \frac{a^2 \text{Coth}[c + d x] \text{Csch}[c + d x]}{2 d}$$

Result (type 3, 155 leaves):

$$\frac{b^2 \operatorname{Cosh}[c] \operatorname{Cosh}[d x]}{d} - \frac{a^2 \operatorname{Csch}\left[\frac{1}{2}(c+d x)\right]^2}{8 d} - \frac{2 a b \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{c}{2}+\frac{d x}{2}\right]\right]}{d} + \frac{a^2 \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right]\right]}{2 d} + \frac{2 a b \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{c}{2}+\frac{d x}{2}\right]\right]}{d} - \frac{a^2 \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right]\right]}{2 d} - \frac{a^2 \operatorname{Sech}\left[\frac{1}{2}(c+d x)\right]^2}{8 d} + \frac{b^2 \operatorname{Sinh}[c] \operatorname{Sinh}[d x]}{d}$$

Problem 18: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Csch}[c+d x]^4 (a+b \operatorname{Sinh}[c+d x]^2)^2 d x$$

Optimal (type 3, 40 leaves, 4 steps):

$$b^2 x + \frac{a(a-2 b) \operatorname{Coth}[c+d x]}{d} - \frac{a^2 \operatorname{Coth}[c+d x]^3}{3 d}$$

Result (type 3, 85 leaves):

$$\left(4(b+a \operatorname{Csch}[c+d x]^2)^2(3 b^2(c+d x)-a \operatorname{Coth}[c+d x](-2 a+6 b+a \operatorname{Csch}[c+d x]^2)) \operatorname{Sinh}[c+d x]^4\right) / \left(3 d(2 a-b+b \operatorname{Cosh}[2(c+d x)])^2\right)$$

Problem 26: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Csch}[c+d x]^3 (a+b \operatorname{Sinh}[c+d x]^2)^3 d x$$

Optimal (type 3, 83 leaves, 5 steps):

$$\frac{a^2(a-6 b) \operatorname{ArcTanh}[\operatorname{Cosh}[c+d x]]}{2 d} + \frac{(3 a-b) b^2 \operatorname{Cosh}[c+d x]}{d} + \frac{b^3 \operatorname{Cosh}[c+d x]^3}{3 d} - \frac{a^3 \operatorname{Coth}[c+d x] \operatorname{Csch}[c+d x]}{2 d}$$

Result (type 3, 561 leaves):

$$\begin{aligned}
 & \left(6 (4a - b) b^2 \operatorname{Cosh}[c] \operatorname{Cosh}[dx] \operatorname{Sinh}[c + dx]^3 (a \operatorname{Csch}[c + dx] + b \operatorname{Sinh}[c + dx])^3 \right) / \\
 & \left(d (2a - b + b \operatorname{Cosh}[2c + 2dx])^3 \right) + \\
 & \left(2 b^3 \operatorname{Cosh}[3c] \operatorname{Cosh}[3dx] \operatorname{Sinh}[c + dx]^3 (a \operatorname{Csch}[c + dx] + b \operatorname{Sinh}[c + dx])^3 \right) / \\
 & \left(3 d (2a - b + b \operatorname{Cosh}[2c + 2dx])^3 \right) - \\
 & \frac{a^3 \operatorname{Csch}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Sinh}[c + dx]^3 (a \operatorname{Csch}[c + dx] + b \operatorname{Sinh}[c + dx])^3}{d (2a - b + b \operatorname{Cosh}[2c + 2dx])^3} + \\
 & \left(4 (a^3 - 6a^2b) \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sinh}[c + dx]^3 (a \operatorname{Csch}[c + dx] + b \operatorname{Sinh}[c + dx])^3 \right) / \\
 & \left(d (2a - b + b \operatorname{Cosh}[2c + 2dx])^3 \right) - \\
 & \left(4 (a^3 - 6a^2b) \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sinh}[c + dx]^3 (a \operatorname{Csch}[c + dx] + b \operatorname{Sinh}[c + dx])^3 \right) / \\
 & \left(d (2a - b + b \operatorname{Cosh}[2c + 2dx])^3 \right) - \\
 & \frac{a^3 \operatorname{Sech}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Sinh}[c + dx]^3 (a \operatorname{Csch}[c + dx] + b \operatorname{Sinh}[c + dx])^3}{d (2a - b + b \operatorname{Cosh}[2c + 2dx])^3} + \\
 & \left(6 (4a - b) b^2 \operatorname{Sinh}[c] \operatorname{Sinh}[dx] \operatorname{Sinh}[c + dx]^3 (a \operatorname{Csch}[c + dx] + b \operatorname{Sinh}[c + dx])^3 \right) / \\
 & \left(d (2a - b + b \operatorname{Cosh}[2c + 2dx])^3 \right) + \\
 & \left(2 b^3 \operatorname{Sinh}[3c] \operatorname{Sinh}[3dx] \operatorname{Sinh}[c + dx]^3 (a \operatorname{Csch}[c + dx] + b \operatorname{Sinh}[c + dx])^3 \right) / \\
 & \left(3 d (2a - b + b \operatorname{Cosh}[2c + 2dx])^3 \right)
 \end{aligned}$$

Problem 28: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Sinh}[c + dx]^7}{a + b \operatorname{Sinh}[c + dx]^2} dx$$

Optimal (type 3, 109 leaves, 4 steps):

$$-\frac{a^3 \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Cosh}[c + dx]}{\sqrt{a - b}}\right]}{\sqrt{a - b} b^{7/2} d} + \frac{(a^2 + a b + b^2) \operatorname{Cosh}[c + dx]}{b^3 d} - \frac{(a + 2b) \operatorname{Cosh}[c + dx]^3}{3 b^2 d} + \frac{\operatorname{Cosh}[c + dx]^5}{5 b d}$$

Result (type 3, 165 leaves):

$$\begin{aligned}
 & \frac{1}{240 b^{7/2} d} \\
 & \left(-\frac{1}{\sqrt{a - b}} 240 a^3 \left(\operatorname{ArcTan}\left[\frac{\sqrt{b} - i \sqrt{a} \operatorname{Tanh}\left[\frac{1}{2}(c + dx)\right]}{\sqrt{a - b}}\right] + \operatorname{ArcTan}\left[\frac{\sqrt{b} + i \sqrt{a} \operatorname{Tanh}\left[\frac{1}{2}(c + dx)\right]}{\sqrt{a - b}}\right] \right) + \right. \\
 & 30 \sqrt{b} (8a^2 + 6ab + 5b^2) \operatorname{Cosh}[c + dx] - \\
 & \left. 5 b^{3/2} (4a + 5b) \operatorname{Cosh}[3(c + dx)] + 3 b^{5/2} \operatorname{Cosh}[5(c + dx)] \right)
 \end{aligned}$$

Problem 30: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{Sinh}[c + d x]^5}{a + b \text{Sinh}[c + d x]^2} dx$$

Optimal (type 3, 79 leaves, 4 steps):

$$\frac{a^2 \text{ArcTan}\left[\frac{\sqrt{b} \text{Cosh}[c+dx]}{\sqrt{a-b}}\right]}{\sqrt{a-b} b^{5/2} d} - \frac{(a+b) \text{Cosh}[c+dx]}{b^2 d} + \frac{\text{Cosh}[c+dx]^3}{3 b d}$$

Result (type 3, 134 leaves):

$$\frac{1}{12 b^{5/2} d} \left(\frac{12 a^2 \left(\text{ArcTan}\left[\frac{\sqrt{b} - i \sqrt{a} \text{Tanh}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a-b}}\right] + \text{ArcTan}\left[\frac{\sqrt{b} + i \sqrt{a} \text{Tanh}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a-b}}\right] \right)}{\sqrt{a-b}} - \frac{3 \sqrt{b} (4 a + 3 b) \text{Cosh}[c + d x] + b^{3/2} \text{Cosh}[3 (c + d x)]}{\sqrt{a-b}} \right)$$

Problem 32: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{Sinh}[c + d x]^3}{a + b \text{Sinh}[c + d x]^2} dx$$

Optimal (type 3, 56 leaves, 3 steps):

$$-\frac{a \text{ArcTan}\left[\frac{\sqrt{b} \text{Cosh}[c+dx]}{\sqrt{a-b}}\right]}{\sqrt{a-b} b^{3/2} d} + \frac{\text{Cosh}[c+dx]}{b d}$$

Result (type 3, 107 leaves):

$$\frac{1}{b^{3/2} d} \left(-\frac{a \left(\text{ArcTan}\left[\frac{\sqrt{b} - i \sqrt{a} \text{Tanh}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a-b}}\right] + \text{ArcTan}\left[\frac{\sqrt{b} + i \sqrt{a} \text{Tanh}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a-b}}\right] \right)}{\sqrt{a-b}} + \sqrt{b} \text{Cosh}[c + d x] \right)$$

Problem 34: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Sinh}[c + d x]}{a + b \text{Sinh}[c + d x]^2} dx$$

Optimal (type 3, 40 leaves, 2 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{b} \text{Cosh}[c+dx]}{\sqrt{a-b}}\right]}{\sqrt{a-b} \sqrt{b} d}$$

Result (type 3, 91 leaves):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{b} - i \sqrt{a} \text{Tanh}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a-b}}\right] + \text{ArcTan}\left[\frac{\sqrt{b} + i \sqrt{a} \text{Tanh}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a-b}}\right]}{\sqrt{a-b} \sqrt{b} d}$$

Problem 36: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Csch}[c+dx]}{a+b \text{Sinh}[c+dx]^2} dx$$

Optimal (type 3, 60 leaves, 4 steps):

$$-\frac{\sqrt{b} \text{ArcTan}\left[\frac{\sqrt{b} \text{Cosh}[c+dx]}{\sqrt{a-b}}\right]}{a \sqrt{a-b} d} - \frac{\text{ArcTanh}[\text{Cosh}[c+dx]]}{a d}$$

Result (type 3, 135 leaves):

$$-\frac{1}{a d} \left(\frac{\sqrt{b} \text{ArcTan}\left[\frac{\sqrt{b} - i \sqrt{a} \text{Tanh}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a-b}}\right]}{\sqrt{a-b}} + \frac{\sqrt{b} \text{ArcTan}\left[\frac{\sqrt{b} + i \sqrt{a} \text{Tanh}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a-b}}\right]}{\sqrt{a-b}} + \text{Log}\left[\text{Cosh}\left[\frac{1}{2}(c+dx)\right]\right] - \text{Log}\left[\text{Sinh}\left[\frac{1}{2}(c+dx)\right]\right] \right)$$

Problem 38: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Csch}[c+dx]^3}{a+b \text{Sinh}[c+dx]^2} dx$$

Optimal (type 3, 88 leaves, 5 steps):

$$\frac{b^{3/2} \text{ArcTan}\left[\frac{\sqrt{b} \text{Cosh}[c+dx]}{\sqrt{a-b}}\right]}{a^2 \sqrt{a-b} d} + \frac{(a+2b) \text{ArcTanh}[\text{Cosh}[c+dx]]}{2 a^2 d} - \frac{\text{Coth}[c+dx] \text{Csch}[c+dx]}{2 a d}$$

Result (type 3, 220 leaves):

$$\left((2a - b + b \operatorname{Cosh}[2(c + dx)]) \operatorname{Csch}[c + dx]^2 \right. \\ \left. \frac{8b^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} - i\sqrt{a} \operatorname{Tanh}\left[\frac{1}{2}(c + dx)\right]}{\sqrt{a-b}}\right]}{\sqrt{a-b}} + \frac{8b^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} + i\sqrt{a} \operatorname{Tanh}\left[\frac{1}{2}(c + dx)\right]}{\sqrt{a-b}}\right]}{\sqrt{a-b}} - \right. \\ \left. a \operatorname{Csch}\left[\frac{1}{2}(c + dx)\right]^2 + 4(a + 2b) \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{1}{2}(c + dx)\right]\right] - \right. \\ \left. 4(a + 2b) \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{1}{2}(c + dx)\right]\right] - a \operatorname{Sech}\left[\frac{1}{2}(c + dx)\right]^2 \right) / (16a^2 d (b + a \operatorname{Csch}[c + dx]^2))$$

Problem 40: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Csch}[c + dx]^5}{a + b \operatorname{Sinh}[c + dx]^2} dx$$

Optimal (type 3, 130 leaves, 6 steps):

$$-\frac{b^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Cosh}[c + dx]}{\sqrt{a-b}}\right]}{a^3 \sqrt{a-b} d} - \frac{(3a^2 + 4ab + 8b^2) \operatorname{ArcTanh}[\operatorname{Cosh}[c + dx]]}{8a^3 d} + \\ \frac{(3a + 4b) \operatorname{Coth}[c + dx] \operatorname{Csch}[c + dx]}{8a^2 d} - \frac{\operatorname{Coth}[c + dx] \operatorname{Csch}[c + dx]^3}{4ad}$$

Result (type 3, 649 leaves):

$$\begin{aligned}
 & - \left(\left(b^{5/2} \operatorname{ArcTan} \left[\frac{1}{\sqrt{a-b}} \operatorname{Sech} \left[\frac{1}{2} (c+dx) \right] \left(\sqrt{b} \operatorname{Cosh} \left[\frac{1}{2} (c+dx) \right] - i \sqrt{a} \operatorname{Sinh} \left[\frac{1}{2} (c+dx) \right] \right) \right] \right. \right. \\
 & \quad \left. \left. (2a-b+b \operatorname{Cosh} [2(c+dx)]) \operatorname{Csch} [c+dx]^2 \right) / \left(2a^3 \sqrt{a-b} d (b+a \operatorname{Csch} [c+dx]^2) \right) \right) - \\
 & \left(b^{5/2} \operatorname{ArcTan} \left[\frac{1}{\sqrt{a-b}} \operatorname{Sech} \left[\frac{1}{2} (c+dx) \right] \left(\sqrt{b} \operatorname{Cosh} \left[\frac{1}{2} (c+dx) \right] + i \sqrt{a} \operatorname{Sinh} \left[\frac{1}{2} (c+dx) \right] \right) \right] \right. \\
 & \quad \left. (2a-b+b \operatorname{Cosh} [2(c+dx)]) \operatorname{Csch} [c+dx]^2 \right) / \left(2a^3 \sqrt{a-b} d (b+a \operatorname{Csch} [c+dx]^2) \right) + \\
 & \left((3a+4b) (2a-b+b \operatorname{Cosh} [2(c+dx)]) \operatorname{Csch} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Csch} [c+dx]^2 \right) / \\
 & \quad (64a^2 d (b+a \operatorname{Csch} [c+dx]^2)) - \\
 & \frac{(2a-b+b \operatorname{Cosh} [2(c+dx)]) \operatorname{Csch} \left[\frac{1}{2} (c+dx) \right]^4 \operatorname{Csch} [c+dx]^2}{128ad(b+a \operatorname{Csch} [c+dx]^2)} + \\
 & \left((-3a^2-4ab-8b^2) (2a-b+b \operatorname{Cosh} [2(c+dx)]) \operatorname{Csch} [c+dx]^2 \operatorname{Log} \left[\operatorname{Cosh} \left[\frac{1}{2} (c+dx) \right] \right] \right) / \\
 & \quad (16a^3 d (b+a \operatorname{Csch} [c+dx]^2)) + \\
 & \left((3a^2+4ab+8b^2) (2a-b+b \operatorname{Cosh} [2(c+dx)]) \operatorname{Csch} [c+dx]^2 \operatorname{Log} \left[\operatorname{Sinh} \left[\frac{1}{2} (c+dx) \right] \right] \right) / \\
 & \quad (16a^3 d (b+a \operatorname{Csch} [c+dx]^2)) + \\
 & \left((3a+4b) (2a-b+b \operatorname{Cosh} [2(c+dx)]) \operatorname{Csch} [c+dx]^2 \operatorname{Sech} \left[\frac{1}{2} (c+dx) \right]^2 \right) / \\
 & \quad (64a^2 d (b+a \operatorname{Csch} [c+dx]^2)) + \\
 & \frac{(2a-b+b \operatorname{Cosh} [2(c+dx)]) \operatorname{Csch} [c+dx]^2 \operatorname{Sech} \left[\frac{1}{2} (c+dx) \right]^4}{128ad(b+a \operatorname{Csch} [c+dx]^2)}
 \end{aligned}$$

Problem 43: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Sinh} [c+dx]^3}{(a+b \operatorname{Sinh} [c+dx]^2)^2} dx$$

Optimal (type 3, 90 leaves, 3 steps):

$$\frac{(a-2b) \operatorname{ArcTan} \left[\frac{\sqrt{b} \operatorname{Cosh} [c+dx]}{\sqrt{a-b}} \right]}{2(a-b)^{3/2} b^{3/2} d} - \frac{a \operatorname{Cosh} [c+dx]}{2(a-b) b d (a-b+b \operatorname{Cosh} [c+dx]^2)}$$

Result (type 3, 141 leaves):

$$\frac{1}{2 b^{3/2} d} \left(\frac{1}{(a-b)^{3/2}} \right. \\ \left. (a-2b) \left(\operatorname{ArcTan} \left[\frac{\sqrt{b}-i \sqrt{a} \operatorname{Tanh} \left[\frac{1}{2}(c+dx) \right]}{\sqrt{a-b}} \right] + \operatorname{ArcTan} \left[\frac{\sqrt{b}+i \sqrt{a} \operatorname{Tanh} \left[\frac{1}{2}(c+dx) \right]}{\sqrt{a-b}} \right] \right) - \right. \\ \left. \frac{2 a \sqrt{b} \operatorname{Cosh}[c+dx]}{(a-b) (2 a-b+b \operatorname{Cosh}[2(c+dx)])} \right)$$

Problem 45: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Sinh}[c+dx]}{(a+b \operatorname{Sinh}[c+dx]^2)^2} dx$$

Optimal (type 3, 81 leaves, 3 steps):

$$\frac{\operatorname{ArcTan} \left[\frac{\sqrt{b} \operatorname{Cosh}[c+dx]}{\sqrt{a-b}} \right]}{2 (a-b)^{3/2} \sqrt{b} d} + \frac{\operatorname{Cosh}[c+dx]}{2 (a-b) d (a-b+b \operatorname{Cosh}[c+dx]^2)}$$

Result (type 3, 130 leaves):

$$\frac{1}{2 d} \left(\frac{\operatorname{ArcTan} \left[\frac{\sqrt{b}-i \sqrt{a} \operatorname{Tanh} \left[\frac{1}{2}(c+dx) \right]}{\sqrt{a-b}} \right] + \operatorname{ArcTan} \left[\frac{\sqrt{b}+i \sqrt{a} \operatorname{Tanh} \left[\frac{1}{2}(c+dx) \right]}{\sqrt{a-b}} \right]}{(a-b)^{3/2} \sqrt{b}} + \right. \\ \left. \frac{2 \operatorname{Cosh}[c+dx]}{(a-b) (2 a-b+b \operatorname{Cosh}[2(c+dx)])} \right)$$

Problem 47: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Csch}[c+dx]}{(a+b \operatorname{Sinh}[c+dx]^2)^2} dx$$

Optimal (type 3, 110 leaves, 5 steps):

$$\frac{(3 a-2 b) \sqrt{b} \operatorname{ArcTan} \left[\frac{\sqrt{b} \operatorname{Cosh}[c+dx]}{\sqrt{a-b}} \right]}{2 a^2 (a-b)^{3/2} d} - \frac{\operatorname{ArcTanh}[\operatorname{Cosh}[c+dx]]}{a^2 d} - \frac{b \operatorname{Cosh}[c+dx]}{2 a (a-b) d (a-b+b \operatorname{Cosh}[c+dx]^2)}$$

Result (type 3, 189 leaves):

$$\frac{1}{2 a^2 d} \left(\frac{\sqrt{b} (-3 a + 2 b) \operatorname{ArcTan}\left[\frac{\sqrt{b} - i \sqrt{a} \operatorname{Tanh}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a-b}}\right]}{(a-b)^{3/2}} + \frac{\sqrt{b} (-3 a + 2 b) \operatorname{ArcTan}\left[\frac{\sqrt{b} + i \sqrt{a} \operatorname{Tanh}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a-b}}\right]}{(a-b)^{3/2}} - \frac{2 a b \operatorname{Cosh}[c+dx]}{(a-b)(2 a - b + b \operatorname{Cosh}[2(c+dx)])} - 2 \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right]\right] + 2 \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right]\right] \right)$$

Problem 49: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Csch}[c+dx]^3}{(a+b \operatorname{Sinh}[c+dx]^2)^2} dx$$

Optimal (type 3, 161 leaves, 6 steps):

$$\frac{(5 a - 4 b) b^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Cosh}[c+dx]}{\sqrt{a-b}}\right]}{2 a^3 (a-b)^{3/2} d} + \frac{(a+4 b) \operatorname{ArcTanh}[\operatorname{Cosh}[c+dx]]}{2 a^3 d} - \frac{(a-2 b) b \operatorname{Cosh}[c+dx]}{2 a^2 (a-b) d (a-b+b \operatorname{Cosh}[c+dx]^2)} - \frac{\operatorname{Coth}[c+dx] \operatorname{Csch}[c+dx]}{2 a d (a-b+b \operatorname{Cosh}[c+dx]^2)}$$

Result (type 3, 391 leaves):

$$\frac{1}{32 a^3 d (b+a \operatorname{Csch}[c+dx]^2)^2} \left((2 a - b + b \operatorname{Cosh}[2(c+dx)]) \operatorname{Csch}[c+dx]^3 \left(\frac{8 a b^2 \operatorname{Coth}[c+dx]}{a-b} + \frac{1}{(a-b)^{3/2}} 4 (5 a - 4 b) b^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} - i \sqrt{a} \operatorname{Tanh}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a-b}}\right] \right) (2 a - b + b \operatorname{Cosh}[2(c+dx)]) \operatorname{Csch}[c+dx] + \frac{1}{(a-b)^{3/2}} 4 (5 a - 4 b) b^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} + i \sqrt{a} \operatorname{Tanh}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a-b}}\right] (2 a - b + b \operatorname{Cosh}[2(c+dx)]) \right) \operatorname{Csch}[c+dx] - a (2 a - b + b \operatorname{Cosh}[2(c+dx)]) \operatorname{Csch}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Csch}[c+dx] + 4 (a+4 b) (2 a - b + b \operatorname{Cosh}[2(c+dx)]) \operatorname{Csch}[c+dx] \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right]\right] - 4 (a+4 b) (2 a - b + b \operatorname{Cosh}[2(c+dx)]) \operatorname{Csch}[c+dx] \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right]\right] - a (2 a - b + b \operatorname{Cosh}[2(c+dx)]) \operatorname{Csch}[c+dx] \operatorname{Sech}\left[\frac{1}{2}(c+dx)\right]^2 \right)$$

Problem 52: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{Sinh}[c + d x]^3}{(a + b \text{Sinh}[c + d x]^2)^3} dx$$

Optimal (type 3, 135 leaves, 4 steps):

$$\frac{(a - 4 b) \text{ArcTan}\left[\frac{\sqrt{b} \text{Cosh}[c + d x]}{\sqrt{a - b}}\right]}{8 (a - b)^{5/2} b^{3/2} d} - \frac{a \text{Cosh}[c + d x]}{4 (a - b) b d (a - b + b \text{Cosh}[c + d x]^2)^2} + \frac{(a - 4 b) \text{Cosh}[c + d x]}{8 (a - b)^2 b d (a - b + b \text{Cosh}[c + d x]^2)}$$

Result (type 3, 170 leaves):

$$\frac{1}{8 b^{3/2} d} \left(\frac{1}{(a - b)^{5/2}} (a - 4 b) \left(\text{ArcTan}\left[\frac{\sqrt{b} - i \sqrt{a} \text{Tanh}\left[\frac{1}{2} (c + d x)\right]}{\sqrt{a - b}}\right] + \text{ArcTan}\left[\frac{\sqrt{b} + i \sqrt{a} \text{Tanh}\left[\frac{1}{2} (c + d x)\right]}{\sqrt{a - b}}\right] \right) + \left(2 \sqrt{b} \text{Cosh}[c + d x] (-2 a^2 - 5 a b + 4 b^2 + (a - 4 b) b \text{Cosh}[2 (c + d x)]) \right) / \left((a - b)^2 (2 a - b + b \text{Cosh}[2 (c + d x)])^2 \right) \right)$$

Problem 54: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{Sinh}[c + d x]}{(a + b \text{Sinh}[c + d x]^2)^3} dx$$

Optimal (type 3, 118 leaves, 4 steps):

$$\frac{3 \text{ArcTan}\left[\frac{\sqrt{b} \text{Cosh}[c + d x]}{\sqrt{a - b}}\right]}{8 (a - b)^{5/2} \sqrt{b} d} + \frac{\text{Cosh}[c + d x]}{4 (a - b) d (a - b + b \text{Cosh}[c + d x]^2)^2} + \frac{3 \text{Cosh}[c + d x]}{8 (a - b)^2 d (a - b + b \text{Cosh}[c + d x]^2)}$$

Result (type 3, 149 leaves):

$$\frac{1}{8 d} \left(\frac{3 \left(\text{ArcTan}\left[\frac{\sqrt{b} - i \sqrt{a} \text{Tanh}\left[\frac{1}{2} (c + d x)\right]}{\sqrt{a - b}}\right] + \text{ArcTan}\left[\frac{\sqrt{b} + i \sqrt{a} \text{Tanh}\left[\frac{1}{2} (c + d x)\right]}{\sqrt{a - b}}\right] \right)}{(a - b)^{5/2} \sqrt{b}} + \frac{2 \text{Cosh}[c + d x] (10 a - 7 b + 3 b \text{Cosh}[2 (c + d x)])}{(a - b)^2 (2 a - b + b \text{Cosh}[2 (c + d x)])^2} \right)$$

Problem 56: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{Csch}[c + d x]}{(a + b \text{Sinh}[c + d x]^2)^3} dx$$

Optimal (type 3, 166 leaves, 6 steps):

$$\frac{\sqrt{b} (15 a^2 - 20 a b + 8 b^2) \text{ArcTan}\left[\frac{\sqrt{b} \text{Cosh}[c + d x]}{\sqrt{a - b}}\right] - \frac{\text{ArcTanh}[\text{Cosh}[c + d x]]}{a^3 d}}{8 a^3 (a - b)^{5/2} d} - \frac{b \text{Cosh}[c + d x]}{4 a (a - b) d (a - b + b \text{Cosh}[c + d x]^2)^2} - \frac{(7 a - 4 b) b \text{Cosh}[c + d x]}{8 a^2 (a - b)^2 d (a - b + b \text{Cosh}[c + d x]^2)^2}$$

Result (type 3, 329 leaves):

$$\begin{aligned} & - \frac{1}{8 a^3 (a - b)^{5/2} d} \sqrt{b} (15 a^2 - 20 a b + 8 b^2) \\ & \quad \text{ArcTan}\left[\frac{1}{\sqrt{a - b}} \text{Sech}\left[\frac{1}{2} (c + d x)\right]\right] \left(\sqrt{b} \text{Cosh}\left[\frac{1}{2} (c + d x)\right] - i \sqrt{a} \text{Sinh}\left[\frac{1}{2} (c + d x)\right]\right) - \\ & \frac{1}{8 a^3 (a - b)^{5/2} d} \sqrt{b} (15 a^2 - 20 a b + 8 b^2) \\ & \quad \text{ArcTan}\left[\frac{1}{\sqrt{a - b}} \text{Sech}\left[\frac{1}{2} (c + d x)\right]\right] \left(\sqrt{b} \text{Cosh}\left[\frac{1}{2} (c + d x)\right] + i \sqrt{a} \text{Sinh}\left[\frac{1}{2} (c + d x)\right]\right) - \\ & \frac{b \text{Cosh}[c + d x]}{a (a - b) d (2 a - b + b \text{Cosh}[2 (c + d x)])^2} + \frac{-7 a b \text{Cosh}[c + d x] + 4 b^2 \text{Cosh}[c + d x]}{4 a^2 (a - b)^2 d (2 a - b + b \text{Cosh}[2 (c + d x)])^2} - \\ & \frac{\text{Log}[\text{Cosh}[\frac{1}{2} (c + d x)]]}{a^3 d} + \frac{\text{Log}[\text{Sinh}[\frac{1}{2} (c + d x)]]}{a^3 d} \end{aligned}$$

Problem 58: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Csch}[c + d x]^3}{(a + b \text{Sinh}[c + d x]^2)^3} dx$$

Optimal (type 3, 224 leaves, 7 steps):

$$\frac{b^{3/2} (35 a^2 - 56 a b + 24 b^2) \text{ArcTan}\left[\frac{\sqrt{b} \text{Cosh}[c + d x]}{\sqrt{a - b}}\right] + \frac{(a + 6 b) \text{ArcTanh}[\text{Cosh}[c + d x]]}{2 a^4 d} - \frac{(2 a - 3 b) b \text{Cosh}[c + d x]}{4 a^2 (a - b) d (a - b + b \text{Cosh}[c + d x]^2)^2} - \frac{(a - 4 b) (4 a - 3 b) b \text{Cosh}[c + d x]}{8 a^3 (a - b)^2 d (a - b + b \text{Cosh}[c + d x]^2)^2} - \frac{\text{Coth}[c + d x] \text{Csch}[c + d x]}{2 a d (a - b + b \text{Cosh}[c + d x]^2)^2}}{8 a^4 (a - b)^{5/2} d}$$

Result (type 3, 462 leaves):

$$\begin{aligned}
& \frac{1}{64 a^4 d (b + a \operatorname{Csch}[c + d x]^2)^3} (2 a - b + b \operatorname{Cosh}[2 (c + d x)]) \operatorname{Csch}[c + d x]^5 \\
& \left(\frac{8 a^2 b^2 \operatorname{Coth}[c + d x]}{a - b} + \frac{2 a (11 a - 8 b) b^2 (2 a - b + b \operatorname{Cosh}[2 (c + d x)]) \operatorname{Coth}[c + d x]}{(a - b)^2} + \right. \\
& \frac{1}{(a - b)^{5/2}} b^{3/2} (35 a^2 - 56 a b + 24 b^2) \operatorname{ArcTan}\left[\frac{\sqrt{b} - i \sqrt{a} \operatorname{Tanh}\left[\frac{1}{2} (c + d x)\right]}{\sqrt{a - b}}\right] \\
& (2 a - b + b \operatorname{Cosh}[2 (c + d x)])^2 \operatorname{Csch}[c + d x] + \frac{1}{(a - b)^{5/2}} b^{3/2} (35 a^2 - 56 a b + 24 b^2) \\
& \operatorname{ArcTan}\left[\frac{\sqrt{b} + i \sqrt{a} \operatorname{Tanh}\left[\frac{1}{2} (c + d x)\right]}{\sqrt{a - b}}\right] (2 a - b + b \operatorname{Cosh}[2 (c + d x)])^2 \operatorname{Csch}[c + d x] - \\
& a (2 a - b + b \operatorname{Cosh}[2 (c + d x)])^2 \operatorname{Csch}\left[\frac{1}{2} (c + d x)\right]^2 \operatorname{Csch}[c + d x] + \\
& 4 (a + 6 b) (2 a - b + b \operatorname{Cosh}[2 (c + d x)])^2 \operatorname{Csch}[c + d x] \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{1}{2} (c + d x)\right]\right] - \\
& 4 (a + 6 b) (2 a - b + b \operatorname{Cosh}[2 (c + d x)])^2 \operatorname{Csch}[c + d x] \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{1}{2} (c + d x)\right]\right] - \\
& \left. a (2 a - b + b \operatorname{Cosh}[2 (c + d x)])^2 \operatorname{Csch}[c + d x] \operatorname{Sech}\left[\frac{1}{2} (c + d x)\right]^2 \right)
\end{aligned}$$

Problem 71: Result unnecessarily involves imaginary or complex numbers.

$$\int \operatorname{Sinh}[e + f x]^4 \sqrt{a + b \operatorname{Sinh}[e + f x]^2} dx$$

Optimal (type 4, 300 leaves, 7 steps):

$$\begin{aligned}
 & \frac{(a-4b) \operatorname{Cosh}[e+fx] \operatorname{Sinh}[e+fx] \sqrt{a+b \operatorname{Sinh}[e+fx]^2}}{15bf} + \\
 & \frac{\operatorname{Cosh}[e+fx] \operatorname{Sinh}[e+fx]^3 \sqrt{a+b \operatorname{Sinh}[e+fx]^2}}{5f} + \\
 & \left((2a^2+3ab-8b^2) \operatorname{EllipticE}[\operatorname{ArcTan}[\operatorname{Sinh}[e+fx]], 1-\frac{b}{a}] \operatorname{Sech}[e+fx] \right. \\
 & \left. \sqrt{a+b \operatorname{Sinh}[e+fx]^2} \right) / \left(15b^2f \sqrt{\frac{\operatorname{Sech}[e+fx]^2 (a+b \operatorname{Sinh}[e+fx]^2)}{a}} \right) - \\
 & \left((a-4b) \operatorname{EllipticF}[\operatorname{ArcTan}[\operatorname{Sinh}[e+fx]], 1-\frac{b}{a}] \operatorname{Sech}[e+fx] \sqrt{a+b \operatorname{Sinh}[e+fx]^2} \right) / \\
 & \left(15bf \sqrt{\frac{\operatorname{Sech}[e+fx]^2 (a+b \operatorname{Sinh}[e+fx]^2)}{a}} \right) - \\
 & \frac{(2a^2+3ab-8b^2) \sqrt{a+b \operatorname{Sinh}[e+fx]^2} \operatorname{Tanh}[e+fx]}{15b^2f}
 \end{aligned}$$

Result (type 4, 210 leaves):

$$\begin{aligned}
 & \left(16i a (2a^2+3ab-8b^2) \sqrt{\frac{2a-b+b \operatorname{Cosh}[2(e+fx)]}{a}} \operatorname{EllipticE}\left[i(e+fx), \frac{b}{a}\right] - \right. \\
 & 32i a (a^2+ab-2b^2) \sqrt{\frac{2a-b+b \operatorname{Cosh}[2(e+fx)]}{a}} \operatorname{EllipticF}\left[i(e+fx), \frac{b}{a}\right] + \\
 & \left. \sqrt{2} b (8a^2-48ab+25b^2+4(4a-7b)b \operatorname{Cosh}[2(e+fx)]+3b^2 \operatorname{Cosh}[4(e+fx)]) \right. \\
 & \left. \operatorname{Sinh}[2(e+fx)] \right) / \left(240b^2f \sqrt{2a-b+b \operatorname{Cosh}[2(e+fx)]} \right)
 \end{aligned}$$

Problem 74: Result unnecessarily involves imaginary or complex numbers.

$$\int \operatorname{Csch}[e+fx]^2 \sqrt{a+b \operatorname{Sinh}[e+fx]^2} dx$$

Optimal (type 4, 199 leaves, 7 steps):

$$\frac{\text{Coth}[e + f x] \sqrt{a + b \text{Sinh}[e + f x]^2}}{f} - \left(\text{EllipticE}\left[\text{ArcTan}[\text{Sinh}[e + f x]], 1 - \frac{b}{a}\right] \text{Sech}[e + f x] \sqrt{a + b \text{Sinh}[e + f x]^2} \right) / \left(f \sqrt{\frac{\text{Sech}[e + f x]^2 (a + b \text{Sinh}[e + f x]^2)}{a}} \right) + \left(b \text{EllipticF}\left[\text{ArcTan}[\text{Sinh}[e + f x]], 1 - \frac{b}{a}\right] \text{Sech}[e + f x] \sqrt{a + b \text{Sinh}[e + f x]^2} \right) / \left(a f \sqrt{\frac{\text{Sech}[e + f x]^2 (a + b \text{Sinh}[e + f x]^2)}{a}} \right) + \frac{\sqrt{a + b \text{Sinh}[e + f x]^2} \text{Tanh}[e + f x]}{f}$$

Result (type 4, 151 leaves):

$$\left(\sqrt{2} (-2a + b - b \text{Cosh}[2(e + f x)]) \text{Coth}[e + f x] - 2 i a \sqrt{\frac{2a - b + b \text{Cosh}[2(e + f x)]}{a}} \text{EllipticE}\left[i(e + f x), \frac{b}{a}\right] + 2 i (a - b) \sqrt{\frac{2a - b + b \text{Cosh}[2(e + f x)]}{a}} \text{EllipticF}\left[i(e + f x), \frac{b}{a}\right] \right) / \left(2 f \sqrt{2a - b + b \text{Cosh}[2(e + f x)]} \right)$$

Problem 75: Result unnecessarily involves imaginary or complex numbers.

$$\int \text{Csch}[e + f x]^4 \sqrt{a + b \text{Sinh}[e + f x]^2} dx$$

Optimal (type 4, 276 leaves, 7 steps):

$$\begin{aligned}
 & \frac{(2a-b) \operatorname{Coth}[e+fx] \sqrt{a+b \operatorname{Sinh}[e+fx]^2}}{3af} - \frac{\operatorname{Coth}[e+fx] \operatorname{Csch}[e+fx]^2 \sqrt{a+b \operatorname{Sinh}[e+fx]^2}}{3f} + \\
 & \left((2a-b) \operatorname{EllipticE}\left[\operatorname{ArcTan}[\operatorname{Sinh}[e+fx]], 1-\frac{b}{a}\right] \operatorname{Sech}[e+fx] \sqrt{a+b \operatorname{Sinh}[e+fx]^2} \right) / \\
 & \left(3af \sqrt{\frac{\operatorname{Sech}[e+fx]^2 (a+b \operatorname{Sinh}[e+fx]^2)}{a}} \right) - \\
 & \left(b \operatorname{EllipticF}\left[\operatorname{ArcTan}[\operatorname{Sinh}[e+fx]], 1-\frac{b}{a}\right] \operatorname{Sech}[e+fx] \sqrt{a+b \operatorname{Sinh}[e+fx]^2} \right) / \\
 & \left(3af \sqrt{\frac{\operatorname{Sech}[e+fx]^2 (a+b \operatorname{Sinh}[e+fx]^2)}{a}} \right) - \frac{(2a-b) \sqrt{a+b \operatorname{Sinh}[e+fx]^2} \operatorname{Tanh}[e+fx]}{3af}
 \end{aligned}$$

Result(type 4, 342 leaves):

$$\begin{aligned}
 & \frac{1}{f} \sqrt{2a-b+b \operatorname{Cosh}[2(e+fx)]} \\
 & \left(\frac{(2\sqrt{2}a \operatorname{Cosh}[e+fx] - \sqrt{2}b \operatorname{Cosh}[e+fx]) \operatorname{Csch}[e+fx]}{6a} - \frac{\operatorname{Coth}[e+fx] \operatorname{Csch}[e+fx]^2}{3\sqrt{2}} \right) + \\
 & \frac{1}{3af} b \left(\frac{i b \sqrt{\frac{2a-b+b \operatorname{Cosh}[2(e+fx)]}{a}} \operatorname{EllipticF}\left[i(e+fx), \frac{b}{a}\right]}{2\sqrt{2a-b+b \operatorname{Cosh}[2(e+fx)]}} - \frac{1}{2b} \right. \\
 & \left. i \left(-\sqrt{2}a + \frac{b}{\sqrt{2}} \right) \left(\frac{2\sqrt{2}a \sqrt{\frac{2a-b+b \operatorname{Cosh}[2(e+fx)]}{a}} \operatorname{EllipticE}\left[i(e+fx), \frac{b}{a}\right]}{\sqrt{2a-b+b \operatorname{Cosh}[2(e+fx)]}} - \right. \right. \\
 & \left. \left. \frac{\sqrt{2}(2a-b) \sqrt{\frac{2a-b+b \operatorname{Cosh}[2(e+fx)]}{a}} \operatorname{EllipticF}\left[i(e+fx), \frac{b}{a}\right]}{\sqrt{2a-b+b \operatorname{Cosh}[2(e+fx)]}} \right) \right)
 \end{aligned}$$

Problem 82: Result unnecessarily involves imaginary or complex numbers.

$$\int \operatorname{Sinh}[e+fx]^4 (a+b \operatorname{Sinh}[e+fx]^2)^{3/2} dx$$

Optimal (type 4, 367 leaves, 8 steps):

$$\begin{aligned}
 & \frac{(a^2 - 11 a b + 8 b^2) \operatorname{Cosh}[e + f x] \operatorname{Sinh}[e + f x] \sqrt{a + b \operatorname{Sinh}[e + f x]^2}}{35 b f} + \\
 & \frac{2 (4 a - 3 b) \operatorname{Cosh}[e + f x] \operatorname{Sinh}[e + f x]^3 \sqrt{a + b \operatorname{Sinh}[e + f x]^2}}{35 f} + \\
 & \frac{b \operatorname{Cosh}[e + f x] \operatorname{Sinh}[e + f x]^5 \sqrt{a + b \operatorname{Sinh}[e + f x]^2}}{7 f} + \\
 & \left(2 (a - 2 b) (a^2 + 4 a b - 4 b^2) \operatorname{EllipticE}[\operatorname{ArcTan}[\operatorname{Sinh}[e + f x]], 1 - \frac{b}{a}] \operatorname{Sech}[e + f x] \right. \\
 & \left. \sqrt{a + b \operatorname{Sinh}[e + f x]^2} \right) / \left(35 b^2 f \sqrt{\frac{\operatorname{Sech}[e + f x]^2 (a + b \operatorname{Sinh}[e + f x]^2)}{a}} \right) - \\
 & \left((a^2 - 11 a b + 8 b^2) \operatorname{EllipticF}[\operatorname{ArcTan}[\operatorname{Sinh}[e + f x]], 1 - \frac{b}{a}] \operatorname{Sech}[e + f x] \sqrt{a + b \operatorname{Sinh}[e + f x]^2} \right) / \\
 & \left(35 b f \sqrt{\frac{\operatorname{Sech}[e + f x]^2 (a + b \operatorname{Sinh}[e + f x]^2)}{a}} \right) - \\
 & \frac{2 (a - 2 b) (a^2 + 4 a b - 4 b^2) \sqrt{a + b \operatorname{Sinh}[e + f x]^2} \operatorname{Tanh}[e + f x]}{35 b^2 f}
 \end{aligned}$$

Result (type 4, 262 leaves):

$$\begin{aligned}
 & \frac{1}{2240 b^2 f \sqrt{2 a - b + b \operatorname{Cosh}[2 (e + f x)]}} \\
 & \left(128 i a (a^3 + 2 a^2 b - 12 a b^2 + 8 b^3) \sqrt{\frac{2 a - b + b \operatorname{Cosh}[2 (e + f x)]}{a}} \operatorname{EllipticE}\left[i (e + f x), \frac{b}{a}\right] - \right. \\
 & 64 i a (2 a^3 + 3 a^2 b - 13 a b^2 + 8 b^3) \sqrt{\frac{2 a - b + b \operatorname{Cosh}[2 (e + f x)]}{a}} \operatorname{EllipticF}\left[i (e + f x), \frac{b}{a}\right] + \\
 & \left. \sqrt{2} b (32 a^3 - 496 a^2 b + 684 a b^2 - 250 b^3 + b (144 a^2 - 480 a b + 299 b^2) \operatorname{Cosh}[2 (e + f x)] + \right. \\
 & \left. 2 (26 a - 27 b) b^2 \operatorname{Cosh}[4 (e + f x)] + 5 b^3 \operatorname{Cosh}[6 (e + f x)]) \operatorname{Sinh}[2 (e + f x)] \right)
 \end{aligned}$$

Problem 85: Result unnecessarily involves imaginary or complex numbers.

$$\int \operatorname{Csch}[e + f x]^2 (a + b \operatorname{Sinh}[e + f x]^2)^{3/2} dx$$

Optimal (type 4, 204 leaves, 6 steps):

$$\begin{aligned}
 & - \frac{a \operatorname{Coth}[e + f x] \sqrt{a + b \operatorname{Sinh}[e + f x]^2}}{f} - \\
 & \left((a + b) \operatorname{EllipticE}\left[\operatorname{ArcTan}[\operatorname{Sinh}[e + f x]], 1 - \frac{b}{a}\right] \operatorname{Sech}[e + f x] \sqrt{a + b \operatorname{Sinh}[e + f x]^2} \right) / \\
 & \left(f \sqrt{\frac{\operatorname{Sech}[e + f x]^2 (a + b \operatorname{Sinh}[e + f x]^2)}{a}} \right) + \\
 & \left(2 b \operatorname{EllipticF}\left[\operatorname{ArcTan}[\operatorname{Sinh}[e + f x]], 1 - \frac{b}{a}\right] \operatorname{Sech}[e + f x] \sqrt{a + b \operatorname{Sinh}[e + f x]^2} \right) / \\
 & \left(f \sqrt{\frac{\operatorname{Sech}[e + f x]^2 (a + b \operatorname{Sinh}[e + f x]^2)}{a}} \right) + \frac{(a + b) \sqrt{a + b \operatorname{Sinh}[e + f x]^2} \operatorname{Tanh}[e + f x]}{f}
 \end{aligned}$$

Result (type 4, 155 leaves):

$$\begin{aligned}
 & - \left(\left(a \sqrt{2} (2 a - b + b \operatorname{Cosh}[2 (e + f x)]) \operatorname{Coth}[e + f x] + \right. \right. \\
 & \quad 2 i (a + b) \sqrt{\frac{2 a - b + b \operatorname{Cosh}[2 (e + f x)]}{a}} \operatorname{EllipticE}\left[i (e + f x), \frac{b}{a}\right] - \\
 & \quad \left. \left. 2 i (a - b) \sqrt{\frac{2 a - b + b \operatorname{Cosh}[2 (e + f x)]}{a}} \operatorname{EllipticF}\left[i (e + f x), \frac{b}{a}\right] \right) \right) / \\
 & \left(2 f \sqrt{2 a - b + b \operatorname{Cosh}[2 (e + f x)]} \right)
 \end{aligned}$$

Problem 86: Result unnecessarily involves imaginary or complex numbers.

$$\int \operatorname{Csch}[e + f x]^4 (a + b \operatorname{Sinh}[e + f x]^2)^{3/2} dx$$

Optimal (type 4, 267 leaves, 7 steps):

$$\frac{2(a-2b) \operatorname{Coth}[e+fx] \sqrt{a+b \operatorname{Sinh}[e+fx]^2}}{3f} - \frac{a \operatorname{Coth}[e+fx] \operatorname{Csch}[e+fx]^2 \sqrt{a+b \operatorname{Sinh}[e+fx]^2}}{3f} + \left(2(a-2b) \operatorname{EllipticE}[\operatorname{ArcTan}[\operatorname{Sinh}[e+fx]], 1 - \frac{b}{a}] \operatorname{Sech}[e+fx] \sqrt{a+b \operatorname{Sinh}[e+fx]^2} \right) / \left(3f \sqrt{\frac{\operatorname{Sech}[e+fx]^2 (a+b \operatorname{Sinh}[e+fx]^2)}{a}} \right) - \left((a-3b) b \operatorname{EllipticF}[\operatorname{ArcTan}[\operatorname{Sinh}[e+fx]], 1 - \frac{b}{a}] \operatorname{Sech}[e+fx] \sqrt{a+b \operatorname{Sinh}[e+fx]^2} \right) / \left(3af \sqrt{\frac{\operatorname{Sech}[e+fx]^2 (a+b \operatorname{Sinh}[e+fx]^2)}{a}} \right) - \frac{2(a-2b) \sqrt{a+b \operatorname{Sinh}[e+fx]^2} \operatorname{Tanh}[e+fx]}{3f}$$

Result(type 4, 335 leaves):

$$\frac{1}{f} \sqrt{2a-b+b \operatorname{Cosh}[2(e+fx)]} \left(\frac{1}{3} (\sqrt{2} a \operatorname{Cosh}[e+fx] - 2\sqrt{2} b \operatorname{Cosh}[e+fx]) \operatorname{Csch}[e+fx] - \frac{a \operatorname{Coth}[e+fx] \operatorname{Csch}[e+fx]^2}{3\sqrt{2}} \right) + \frac{1}{3f} \sqrt{2} b \left(- \frac{i b \sqrt{\frac{2a-b+b \operatorname{Cosh}[2(e+fx)]}{a}} \operatorname{EllipticF}[i(e+fx), \frac{b}{a}]}{\sqrt{2} \sqrt{2a-b+b \operatorname{Cosh}[2(e+fx)]}} - \frac{1}{2b} \right) + i(-a+2b) \left(\frac{2\sqrt{2} a \sqrt{\frac{2a-b+b \operatorname{Cosh}[2(e+fx)]}{a}} \operatorname{EllipticE}[i(e+fx), \frac{b}{a}]}{\sqrt{2a-b+b \operatorname{Cosh}[2(e+fx)]}} - \frac{\sqrt{2} (2a-b) \sqrt{\frac{2a-b+b \operatorname{Cosh}[2(e+fx)]}{a}} \operatorname{EllipticF}[i(e+fx), \frac{b}{a}]}{\sqrt{2a-b+b \operatorname{Cosh}[2(e+fx)]}} \right)$$

Problem 102: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Sinh}[e+fx]^4}{\sqrt{a+b \operatorname{Sinh}[e+fx]^2}} dx$$

Optimal (type 4, 229 leaves, 6 steps):

$$\frac{\cosh[e + f x] \sinh[e + f x] \sqrt{a + b \sinh[e + f x]^2}}{3 b f} + \left(2 (a + b) \operatorname{EllipticE}\left[\operatorname{ArcTan}[\sinh[e + f x]], 1 - \frac{b}{a}\right] \operatorname{sech}[e + f x] \sqrt{a + b \sinh[e + f x]^2} \right) / \left(3 b^2 f \sqrt{\frac{\operatorname{sech}[e + f x]^2 (a + b \sinh[e + f x]^2)}{a}} \right) - \left(\operatorname{EllipticF}\left[\operatorname{ArcTan}[\sinh[e + f x]], 1 - \frac{b}{a}\right] \operatorname{sech}[e + f x] \sqrt{a + b \sinh[e + f x]^2} \right) / \left(3 b f \sqrt{\frac{\operatorname{sech}[e + f x]^2 (a + b \sinh[e + f x]^2)}{a}} \right) - \frac{2 (a + b) \sqrt{a + b \sinh[e + f x]^2} \tanh[e + f x]}{3 b^2 f}$$

Result (type 4, 168 leaves):

$$\left(4 i \sqrt{2} a (a + b) \sqrt{\frac{2 a - b + b \cosh[2 (e + f x)]}{a}} \operatorname{EllipticE}\left[i (e + f x), \frac{b}{a}\right] - 2 i \sqrt{2} a (2 a + b) \sqrt{\frac{2 a - b + b \cosh[2 (e + f x)]}{a}} \operatorname{EllipticF}\left[i (e + f x), \frac{b}{a}\right] + b (2 a - b + b \cosh[2 (e + f x)]) \sinh[2 (e + f x)] \right) / \left(6 b^2 f \sqrt{4 a - 2 b + 2 b \cosh[2 (e + f x)]} \right)$$

Problem 105: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{csch}[e + f x]^2}{\sqrt{a + b \sinh[e + f x]^2}} dx$$

Optimal (type 4, 134 leaves, 5 steps):

$$-\frac{\operatorname{coth}[e + f x] \sqrt{a + b \sinh[e + f x]^2}}{a f} - \left(\operatorname{EllipticE}\left[\operatorname{ArcTan}[\sinh[e + f x]], 1 - \frac{b}{a}\right] \operatorname{sech}[e + f x] \sqrt{a + b \sinh[e + f x]^2} \right) / \left(a f \sqrt{\frac{\operatorname{sech}[e + f x]^2 (a + b \sinh[e + f x]^2)}{a}} \right) + \frac{\sqrt{a + b \sinh[e + f x]^2} \tanh[e + f x]}{a f}$$

Result (type 4, 150 leaves):

$$\left(\sqrt{2} (-2a + b - b \operatorname{Cosh}[2(e + fx)]) \operatorname{Coth}[e + fx] - \right. \\ \left. 2i a \sqrt{\frac{2a - b + b \operatorname{Cosh}[2(e + fx)]}{a}} \operatorname{EllipticE}\left[i(e + fx), \frac{b}{a}\right] + \right. \\ \left. 2i a \sqrt{\frac{2a - b + b \operatorname{Cosh}[2(e + fx)]}{a}} \operatorname{EllipticF}\left[i(e + fx), \frac{b}{a}\right] \right) / \\ (2af \sqrt{2a - b + b \operatorname{Cosh}[2(e + fx)]})$$

Problem 106: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Csch}[e + fx]^4}{\sqrt{a + b \operatorname{Sinh}[e + fx]^2}} dx$$

Optimal (type 4, 267 leaves, 7 steps):

$$\frac{2(a+b) \operatorname{Coth}[e + fx] \sqrt{a + b \operatorname{Sinh}[e + fx]^2}}{3a^2 f} - \frac{\operatorname{Coth}[e + fx] \operatorname{Csch}[e + fx]^2 \sqrt{a + b \operatorname{Sinh}[e + fx]^2}}{3af} + \\ \left(2(a+b) \operatorname{EllipticE}\left[\operatorname{ArcTan}[\operatorname{Sinh}[e + fx]], 1 - \frac{b}{a}\right] \operatorname{Sech}[e + fx] \sqrt{a + b \operatorname{Sinh}[e + fx]^2} \right) / \\ \left(3a^2 f \sqrt{\frac{\operatorname{Sech}[e + fx]^2 (a + b \operatorname{Sinh}[e + fx]^2)}{a}} \right) - \\ \left(b \operatorname{EllipticF}\left[\operatorname{ArcTan}[\operatorname{Sinh}[e + fx]], 1 - \frac{b}{a}\right] \operatorname{Sech}[e + fx] \sqrt{a + b \operatorname{Sinh}[e + fx]^2} \right) / \\ \left(3a^2 f \sqrt{\frac{\operatorname{Sech}[e + fx]^2 (a + b \operatorname{Sinh}[e + fx]^2)}{a}} \right) - \frac{2(a+b) \sqrt{a + b \operatorname{Sinh}[e + fx]^2} \operatorname{Tanh}[e + fx]}{3a^2 f}$$

Result (type 4, 338 leaves):

$$\begin{aligned}
 & \frac{1}{f} \sqrt{2a - b + b \operatorname{Cosh}[2(e + fx)]} \\
 & \left(\frac{(\sqrt{2} a \operatorname{Cosh}[e + fx] + \sqrt{2} b \operatorname{Cosh}[e + fx]) \operatorname{Csch}[e + fx]}{3a^2} - \frac{\operatorname{Coth}[e + fx] \operatorname{Csch}[e + fx]^2}{3\sqrt{2} a} \right) - \\
 & \frac{1}{3a^2 f} \sqrt{2} b \left(\frac{i b \sqrt{\frac{2a - b + b \operatorname{Cosh}[2(e + fx)]}{a}} \operatorname{EllipticF}\left[i(e + fx), \frac{b}{a}\right]}{\sqrt{2} \sqrt{2a - b + b \operatorname{Cosh}[2(e + fx)]}} - \frac{1}{2b} \right. \\
 & \left. i(a + b) \left(\frac{2\sqrt{2} a \sqrt{\frac{2a - b + b \operatorname{Cosh}[2(e + fx)]}{a}} \operatorname{EllipticE}\left[i(e + fx), \frac{b}{a}\right]}{\sqrt{2a - b + b \operatorname{Cosh}[2(e + fx)]}} - \right. \right. \\
 & \left. \left. \frac{\sqrt{2}(2a - b) \sqrt{\frac{2a - b + b \operatorname{Cosh}[2(e + fx)]}{a}} \operatorname{EllipticF}\left[i(e + fx), \frac{b}{a}\right]}{\sqrt{2a - b + b \operatorname{Cosh}[2(e + fx)]}} \right) \right)
 \end{aligned}$$

Problem 111: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Sinh}[e + fx]^6}{(a + b \operatorname{Sinh}[e + fx]^2)^{3/2}} dx$$

Optimal (type 4, 341 leaves, 7 steps):

$$\begin{aligned}
 & - \frac{a \operatorname{Cosh}[e + fx] \operatorname{Sinh}[e + fx]^3}{(a - b) b f \sqrt{a + b \operatorname{Sinh}[e + fx]^2}} + \frac{(4a - b) \operatorname{Cosh}[e + fx] \operatorname{Sinh}[e + fx] \sqrt{a + b \operatorname{Sinh}[e + fx]^2}}{3(a - b) b^2 f} + \\
 & \left((8a^2 - 3ab - 2b^2) \operatorname{EllipticE}\left[\operatorname{ArcTan}[\operatorname{Sinh}[e + fx]], 1 - \frac{b}{a}\right] \operatorname{Sech}[e + fx] \right. \\
 & \left. \sqrt{a + b \operatorname{Sinh}[e + fx]^2} \right) / \left(3(a - b) b^3 f \sqrt{\frac{\operatorname{Sech}[e + fx]^2 (a + b \operatorname{Sinh}[e + fx]^2)}{a}} \right) - \\
 & \left((4a - b) \operatorname{EllipticF}\left[\operatorname{ArcTan}[\operatorname{Sinh}[e + fx]], 1 - \frac{b}{a}\right] \operatorname{Sech}[e + fx] \sqrt{a + b \operatorname{Sinh}[e + fx]^2} \right) / \\
 & \left(3(a - b) b^2 f \sqrt{\frac{\operatorname{Sech}[e + fx]^2 (a + b \operatorname{Sinh}[e + fx]^2)}{a}} \right) - \\
 & \frac{(8a^2 - 3ab - 2b^2) \sqrt{a + b \operatorname{Sinh}[e + fx]^2} \operatorname{Tanh}[e + fx]}{3(a - b) b^3 f}
 \end{aligned}$$

Result (type 4, 211 leaves):

$$\left(2 i \sqrt{2} a (8 a^2 - 3 a b - 2 b^2) \sqrt{\frac{2 a - b + b \operatorname{Cosh}[2 (e + f x)]}{a}} \operatorname{EllipticE}\left[i (e + f x), \frac{b}{a}\right] - \right. \\ \left. 2 i \sqrt{2} a (8 a^2 - 7 a b - b^2) \sqrt{\frac{2 a - b + b \operatorname{Cosh}[2 (e + f x)]}{a}} \operatorname{EllipticF}\left[i (e + f x), \frac{b}{a}\right] - \right. \\ \left. b (-8 a^2 + 3 a b - b^2 + b (-a + b) \operatorname{Cosh}[2 (e + f x)]) \operatorname{Sinh}[2 (e + f x)] \right) / \\ \left(6 (a - b) b^3 f \sqrt{4 a - 2 b + 2 b \operatorname{Cosh}[2 (e + f x)]} \right)$$

Problem 112: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Sinh}[e + f x]^4}{(a + b \operatorname{Sinh}[e + f x]^2)^{3/2}} dx$$

Optimal (type 4, 256 leaves, 6 steps):

$$- \frac{a \operatorname{Cosh}[e + f x] \operatorname{Sinh}[e + f x]}{(a - b) b f \sqrt{a + b \operatorname{Sinh}[e + f x]^2}} - \\ \left((2 a - b) \operatorname{EllipticE}\left[\operatorname{ArcTan}[\operatorname{Sinh}[e + f x]], 1 - \frac{b}{a}\right] \operatorname{Sech}[e + f x] \sqrt{a + b \operatorname{Sinh}[e + f x]^2} \right) / \\ \left((a - b) b^2 f \sqrt{\frac{\operatorname{Sech}[e + f x]^2 (a + b \operatorname{Sinh}[e + f x]^2)}{a}} \right) + \\ \left(\operatorname{EllipticF}\left[\operatorname{ArcTan}[\operatorname{Sinh}[e + f x]], 1 - \frac{b}{a}\right] \operatorname{Sech}[e + f x] \sqrt{a + b \operatorname{Sinh}[e + f x]^2} \right) / \\ \left((a - b) b f \sqrt{\frac{\operatorname{Sech}[e + f x]^2 (a + b \operatorname{Sinh}[e + f x]^2)}{a}} \right) + \\ \frac{(2 a - b) \sqrt{a + b \operatorname{Sinh}[e + f x]^2} \operatorname{Tanh}[e + f x]}{(a - b) b^2 f}$$

Result (type 4, 156 leaves):

$$\left(a \left(-2 i (2 a - b) \sqrt{\frac{2 a - b + b \operatorname{Cosh}[2 (e + f x)]}{a}} \operatorname{EllipticE}\left[i (e + f x), \frac{b}{a}\right] + 4 i (a - b) \sqrt{\frac{2 a - b + b \operatorname{Cosh}[2 (e + f x)]}{a}} \operatorname{EllipticF}\left[i (e + f x), \frac{b}{a}\right] - \sqrt{2} b \operatorname{Sinh}[2 (e + f x)] \right) \right) / \left(2 (a - b) b^2 f \sqrt{2 a - b + b \operatorname{Cosh}[2 (e + f x)]} \right)$$

Problem 115: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Csch}[e + f x]^2}{(a + b \operatorname{Sinh}[e + f x]^2)^{3/2}} dx$$

Optimal (type 4, 290 leaves, 7 steps):

$$\begin{aligned} & - \frac{b \operatorname{Coth}[e + f x]}{a (a - b) f \sqrt{a + b \operatorname{Sinh}[e + f x]^2}} - \frac{(a - 2 b) \operatorname{Coth}[e + f x] \sqrt{a + b \operatorname{Sinh}[e + f x]^2}}{a^2 (a - b) f} \\ & \left((a - 2 b) \operatorname{EllipticE}\left[\operatorname{ArcTan}[\operatorname{Sinh}[e + f x]], 1 - \frac{b}{a}\right] \operatorname{Sech}[e + f x] \sqrt{a + b \operatorname{Sinh}[e + f x]^2} \right) / \\ & \left(a^2 (a - b) f \sqrt{\frac{\operatorname{Sech}[e + f x]^2 (a + b \operatorname{Sinh}[e + f x]^2)}{a}} \right) - \\ & \left(b \operatorname{EllipticF}\left[\operatorname{ArcTan}[\operatorname{Sinh}[e + f x]], 1 - \frac{b}{a}\right] \operatorname{Sech}[e + f x] \sqrt{a + b \operatorname{Sinh}[e + f x]^2} \right) / \\ & \left(a^2 (a - b) f \sqrt{\frac{\operatorname{Sech}[e + f x]^2 (a + b \operatorname{Sinh}[e + f x]^2)}{a}} \right) + \\ & \frac{(a - 2 b) \sqrt{a + b \operatorname{Sinh}[e + f x]^2} \operatorname{Tanh}[e + f x]}{a^2 (a - b) f} \end{aligned}$$

Result (type 4, 185 leaves):

$$\left(- (2 a^2 - 3 a b + 2 b^2 + (a - 2 b) b \operatorname{Cosh}[2 (e + f x)]) \operatorname{Coth}[e + f x] - \right. \\ \left. i \sqrt{2} a (a - 2 b) \sqrt{\frac{2 a - b + b \operatorname{Cosh}[2 (e + f x)]}{a}} \operatorname{EllipticE}\left[i (e + f x), \frac{b}{a}\right] + \right. \\ \left. i \sqrt{2} a (a - b) \sqrt{\frac{2 a - b + b \operatorname{Cosh}[2 (e + f x)]}{a}} \operatorname{EllipticF}\left[i (e + f x), \frac{b}{a}\right] \right) / \\ \left(a^2 (a - b) f \sqrt{4 a - 2 b + 2 b \operatorname{Cosh}[2 (e + f x)]} \right)$$

Problem 120: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Sinh}[e + f x]^6}{(a + b \operatorname{Sinh}[e + f x]^2)^{5/2}} dx$$

Optimal (type 4, 344 leaves, 7 steps):

$$\frac{a \operatorname{Cosh}[e + f x] \operatorname{Sinh}[e + f x]^3}{3 (a - b) b f (a + b \operatorname{Sinh}[e + f x]^2)^{3/2}} - \frac{2 a (2 a - 3 b) \operatorname{Cosh}[e + f x] \operatorname{Sinh}[e + f x]}{3 (a - b)^2 b^2 f \sqrt{a + b \operatorname{Sinh}[e + f x]^2}} - \\ \left((8 a^2 - 13 a b + 3 b^2) \operatorname{EllipticE}\left[\operatorname{ArcTan}[\operatorname{Sinh}[e + f x]], 1 - \frac{b}{a}\right] \operatorname{Sech}[e + f x] \right. \\ \left. \sqrt{a + b \operatorname{Sinh}[e + f x]^2} \right) / \left(3 (a - b)^2 b^3 f \sqrt{\frac{\operatorname{Sech}[e + f x]^2 (a + b \operatorname{Sinh}[e + f x]^2)}{a}} \right) + \\ \left(2 (2 a - 3 b) \operatorname{EllipticF}\left[\operatorname{ArcTan}[\operatorname{Sinh}[e + f x]], 1 - \frac{b}{a}\right] \operatorname{Sech}[e + f x] \sqrt{a + b \operatorname{Sinh}[e + f x]^2} \right) / \\ \left(3 (a - b)^2 b^2 f \sqrt{\frac{\operatorname{Sech}[e + f x]^2 (a + b \operatorname{Sinh}[e + f x]^2)}{a}} \right) + \\ \frac{(8 a^2 - 13 a b + 3 b^2) \sqrt{a + b \operatorname{Sinh}[e + f x]^2} \operatorname{Tanh}[e + f x]}{3 (a - b)^2 b^3 f}$$

Result (type 4, 207 leaves):

$$\left(a \left(-2 i a (8 a^2 - 13 a b + 3 b^2) \left(\frac{2 a - b + b \operatorname{Cosh}[2 (e + f x)]}{a} \right)^{3/2} \operatorname{EllipticE}\left[i (e + f x), \frac{b}{a} \right] + \right. \right. \\ \left. \left. 2 i a (8 a^2 - 17 a b + 9 b^2) \left(\frac{2 a - b + b \operatorname{Cosh}[2 (e + f x)]}{a} \right)^{3/2} \operatorname{EllipticF}\left[i (e + f x), \frac{b}{a} \right] + \right. \right. \\ \left. \left. \sqrt{2} b (-8 a^2 + 17 a b - 7 b^2 + b (-5 a + 7 b) \operatorname{Cosh}[2 (e + f x)]) \operatorname{Sinh}[2 (e + f x)] \right) \right) / \\ \left(6 (a - b)^2 b^3 f (2 a - b + b \operatorname{Cosh}[2 (e + f x)]) \right)^{3/2}$$

Problem 121: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Sinh}[e + f x]^4}{(a + b \operatorname{Sinh}[e + f x]^2)^{5/2}} dx$$

Optimal (type 4, 244 leaves, 5 steps):

$$- \frac{a \operatorname{Cosh}[e + f x] \operatorname{Sinh}[e + f x]}{3 (a - b) b f (a + b \operatorname{Sinh}[e + f x]^2)^{3/2}} + \\ \frac{2 \sqrt{a} (a - 2 b) \operatorname{Cosh}[e + f x] \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Sinh}[e + f x]}{\sqrt{a}} \right], 1 - \frac{a}{b} \right]}{3 (a - b)^2 b^{3/2} f \sqrt{\frac{a \operatorname{Cosh}[e + f x]^2}{a + b \operatorname{Sinh}[e + f x]^2}} \sqrt{a + b \operatorname{Sinh}[e + f x]^2}} - \\ \left((a - 3 b) \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\operatorname{Sinh}[e + f x] \right], 1 - \frac{b}{a} \right] \operatorname{Sech}[e + f x] \sqrt{a + b \operatorname{Sinh}[e + f x]^2} \right) / \\ \left(3 a (a - b)^2 b f \sqrt{\frac{\operatorname{Sech}[e + f x]^2 (a + b \operatorname{Sinh}[e + f x]^2)}{a}} \right)$$

Result (type 4, 198 leaves):

$$\left(2 i a^2 (a - 2 b) \left(\frac{2 a - b + b \operatorname{Cosh}[2 (e + f x)]}{a} \right)^{3/2} \operatorname{EllipticE}\left[i (e + f x), \frac{b}{a} \right] - \right. \\ \left. i a (2 a^2 - 5 a b + 3 b^2) \left(\frac{2 a - b + b \operatorname{Cosh}[2 (e + f x)]}{a} \right)^{3/2} \operatorname{EllipticF}\left[i (e + f x), \frac{b}{a} \right] - \right. \\ \left. \sqrt{2} b (-a^2 + 4 a b - 2 b^2 - (a - 2 b) b \operatorname{Cosh}[2 (e + f x)]) \operatorname{Sinh}[2 (e + f x)] \right) / \\ \left(3 (a - b)^2 b^2 f (2 a - b + b \operatorname{Cosh}[2 (e + f x)]) \right)^{3/2}$$

Problem 124: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Csch}[e + f x]^2}{(a + b \operatorname{Sinh}[e + f x]^2)^{5/2}} dx$$

Optimal (type 4, 385 leaves, 8 steps):

$$\begin{aligned} & -\frac{b \operatorname{Coth}[e+f x]}{3 a(a-b) f(a+b \operatorname{Sinh}[e+f x]^2)^{3/2}} - \frac{2(3 a-2 b) b \operatorname{Coth}[e+f x]}{3 a^2(a-b)^2 f \sqrt{a+b \operatorname{Sinh}[e+f x]^2}} - \\ & \frac{(3 a^2-13 a b+8 b^2) \operatorname{Coth}[e+f x] \sqrt{a+b \operatorname{Sinh}[e+f x]^2}}{3 a^3(a-b)^2 f} - \\ & \left((3 a^2-13 a b+8 b^2) \operatorname{EllipticE}\left[\operatorname{ArcTan}[\operatorname{Sinh}[e+f x]], 1-\frac{b}{a}\right] \operatorname{Sech}[e+f x] \right. \\ & \left. \sqrt{a+b \operatorname{Sinh}[e+f x]^2} \right) / \left(3 a^3(a-b)^2 f \sqrt{\frac{\operatorname{Sech}[e+f x]^2(a+b \operatorname{Sinh}[e+f x]^2)}{a}} \right) - \\ & \left(2(3 a-2 b) b \operatorname{EllipticF}\left[\operatorname{ArcTan}[\operatorname{Sinh}[e+f x]], 1-\frac{b}{a}\right] \operatorname{Sech}[e+f x] \sqrt{a+b \operatorname{Sinh}[e+f x]^2} \right) / \\ & \left(3 a^3(a-b)^2 f \sqrt{\frac{\operatorname{Sech}[e+f x]^2(a+b \operatorname{Sinh}[e+f x]^2)}{a}} \right) + \\ & \frac{(3 a^2-13 a b+8 b^2) \sqrt{a+b \operatorname{Sinh}[e+f x]^2} \operatorname{Tanh}[e+f x]}{3 a^3(a-b)^2 f} \end{aligned}$$

Result (type 4, 234 leaves):

$$\begin{aligned} & \frac{1}{12 a^3(a-b)^2 f(2 a-b+b \operatorname{Cosh}[2(e+f x)])^{3/2}} \\ & i \left(4 a^2 \left(\frac{2 a-b+b \operatorname{Cosh}[2(e+f x)]}{a} \right)^{3/2} \left((-3 a^2+13 a b-8 b^2) \operatorname{EllipticE}\left[i(e+f x), \frac{b}{a}\right] + \right. \right. \\ & \left. \left. (3 a^2-7 a b+4 b^2) \operatorname{EllipticF}\left[i(e+f x), \frac{b}{a}\right] \right) + \right. \\ & \left. 2 i \sqrt{2} \left(3(a-b)^2(2 a-b+b \operatorname{Cosh}[2(e+f x)])^2 \operatorname{Coth}[e+f x] - 2 a(a-b) b^2 \operatorname{Sinh}[2(e+f x)] - \right. \right. \\ & \left. \left. (7 a-5 b) b^2(2 a-b+b \operatorname{Cosh}[2(e+f x)]) \operatorname{Sinh}[2(e+f x)] \right) \right) \end{aligned}$$

Problem 130: Unable to integrate problem.

$$\int (d \operatorname{Sinh}[e+f x])^m (a+b \operatorname{Sinh}[e+f x]^2)^p dx$$

Optimal (type 6, 128 leaves, 3 steps):

$$\begin{aligned} & \frac{1}{f} d \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1-m}{2}, -p, \frac{3}{2}, \operatorname{Cosh}[e+f x]^2, -\frac{b \operatorname{Cosh}[e+f x]^2}{a-b}\right] \operatorname{Cosh}[e+f x] \\ & (a-b+b \operatorname{Cosh}[e+f x]^2)^p \left(1 + \frac{b \operatorname{Cosh}[e+f x]^2}{a-b} \right)^{-p} (d \operatorname{Sinh}[e+f x])^{-1+m} (-\operatorname{Sinh}[e+f x]^2)^{\frac{1-m}{2}} \end{aligned}$$

Result (type 8, 27 leaves):

$$\int (d \operatorname{Sinh}[e + f x])^m (a + b \operatorname{Sinh}[e + f x]^2)^p dx$$

Problem 131: Unable to integrate problem.

$$\int \operatorname{Sinh}[e + f x]^5 (a + b \operatorname{Sinh}[e + f x]^2)^p dx$$

Optimal (type 5, 226 leaves, 5 steps):

$$\begin{aligned} & - \frac{(3 a + 2 b (2 + p)) \operatorname{Cosh}[e + f x] (a - b + b \operatorname{Cosh}[e + f x]^2)^{1+p}}{b^2 f (3 + 2 p) (5 + 2 p)} + \\ & \left((3 a^2 + 4 a b (1 + p) + 4 b^2 (2 + 3 p + p^2)) \operatorname{Cosh}[e + f x] (a - b + b \operatorname{Cosh}[e + f x]^2)^p \right. \\ & \quad \left. \left(1 + \frac{b \operatorname{Cosh}[e + f x]^2}{a - b} \right)^{-p} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -p, \frac{3}{2}, -\frac{b \operatorname{Cosh}[e + f x]^2}{a - b}\right] \right) / \\ & (b^2 f (3 + 2 p) (5 + 2 p)) + \frac{\operatorname{Cosh}[e + f x] (a - b + b \operatorname{Cosh}[e + f x]^2)^{1+p} \operatorname{Sinh}[e + f x]^2}{b f (5 + 2 p)} \end{aligned}$$

Result (type 8, 25 leaves):

$$\int \operatorname{Sinh}[e + f x]^5 (a + b \operatorname{Sinh}[e + f x]^2)^p dx$$

Problem 132: Unable to integrate problem.

$$\int \operatorname{Sinh}[e + f x]^3 (a + b \operatorname{Sinh}[e + f x]^2)^p dx$$

Optimal (type 5, 137 leaves, 4 steps):

$$\begin{aligned} & \frac{\operatorname{Cosh}[e + f x] (a - b + b \operatorname{Cosh}[e + f x]^2)^{1+p}}{b f (3 + 2 p)} - \frac{1}{b f (3 + 2 p)} \\ & (a + 2 b (1 + p)) \operatorname{Cosh}[e + f x] (a - b + b \operatorname{Cosh}[e + f x]^2)^p \\ & \left(1 + \frac{b \operatorname{Cosh}[e + f x]^2}{a - b} \right)^{-p} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -p, \frac{3}{2}, -\frac{b \operatorname{Cosh}[e + f x]^2}{a - b}\right] \end{aligned}$$

Result (type 8, 25 leaves):

$$\int \operatorname{Sinh}[e + f x]^3 (a + b \operatorname{Sinh}[e + f x]^2)^p dx$$

Problem 134: Unable to integrate problem.

$$\int \operatorname{Csch}[e + f x] (a + b \operatorname{Sinh}[e + f x]^2)^p dx$$

Optimal (type 6, 88 leaves, 3 steps):

$$-\frac{1}{f} \text{AppellF1}\left[\frac{1}{2}, 1, -p, \frac{3}{2}, \text{Cosh}[e + f x]^2, -\frac{b \text{Cosh}[e + f x]^2}{a - b}\right] \\ \text{Cosh}[e + f x] (a - b + b \text{Cosh}[e + f x]^2)^p \left(1 + \frac{b \text{Cosh}[e + f x]^2}{a - b}\right)^{-p}$$

Result (type 8, 23 leaves):

$$\int \text{Csch}[e + f x] (a + b \text{Sinh}[e + f x]^2)^p dx$$

Problem 135: Unable to integrate problem.

$$\int \text{Csch}[e + f x]^3 (a + b \text{Sinh}[e + f x]^2)^p dx$$

Optimal (type 6, 87 leaves, 3 steps):

$$\frac{1}{f} \text{AppellF1}\left[\frac{1}{2}, 2, -p, \frac{3}{2}, \text{Cosh}[e + f x]^2, -\frac{b \text{Cosh}[e + f x]^2}{a - b}\right] \\ \text{Cosh}[e + f x] (a - b + b \text{Cosh}[e + f x]^2)^p \left(1 + \frac{b \text{Cosh}[e + f x]^2}{a - b}\right)^{-p}$$

Result (type 8, 25 leaves):

$$\int \text{Csch}[e + f x]^3 (a + b \text{Sinh}[e + f x]^2)^p dx$$

Problem 136: Unable to integrate problem.

$$\int \text{Csch}[e + f x]^5 (a + b \text{Sinh}[e + f x]^2)^p dx$$

Optimal (type 6, 88 leaves, 3 steps):

$$-\frac{1}{f} \text{AppellF1}\left[\frac{1}{2}, 3, -p, \frac{3}{2}, \text{Cosh}[e + f x]^2, -\frac{b \text{Cosh}[e + f x]^2}{a - b}\right] \\ \text{Cosh}[e + f x] (a - b + b \text{Cosh}[e + f x]^2)^p \left(1 + \frac{b \text{Cosh}[e + f x]^2}{a - b}\right)^{-p}$$

Result (type 8, 25 leaves):

$$\int \text{Csch}[e + f x]^5 (a + b \text{Sinh}[e + f x]^2)^p dx$$

Problem 137: Unable to integrate problem.

$$\int \text{Sinh}[e + f x]^4 (a + b \text{Sinh}[e + f x]^2)^p dx$$

Optimal (type 6, 103 leaves, 3 steps):

$$\frac{1}{5f} \text{AppellF1}\left[\frac{5}{2}, \frac{1}{2}, -p, \frac{7}{2}, -\text{Sinh}[e+fx]^2, -\frac{b \text{Sinh}[e+fx]^2}{a}\right] \sqrt{\text{Cosh}[e+fx]^2} \\ \text{Sinh}[e+fx]^4 (a+b \text{Sinh}[e+fx]^2)^p \left(1 + \frac{b \text{Sinh}[e+fx]^2}{a}\right)^{-p} \text{Tanh}[e+fx]$$

Result (type 8, 25 leaves):

$$\int \text{Sinh}[e+fx]^4 (a+b \text{Sinh}[e+fx]^2)^p dx$$

Problem 138: Result more than twice size of optimal antiderivative.

$$\int \text{Sinh}[e+fx]^2 (a+b \text{Sinh}[e+fx]^2)^p dx$$

Optimal (type 6, 101 leaves, 3 steps):

$$\frac{1}{3f} \text{AppellF1}\left[\frac{3}{2}, 2+p, -p, \frac{5}{2}, \text{Tanh}[e+fx]^2, \frac{(a-b) \text{Tanh}[e+fx]^2}{a}\right] \\ (\text{Sech}[e+fx]^2)^p (a+b \text{Sinh}[e+fx]^2)^p \text{Tanh}[e+fx]^3 \left(1 - \frac{(a-b) \text{Tanh}[e+fx]^2}{a}\right)^{-p}$$

Result (type 6, 250 leaves):

$$\left(2^{-2-p} \sqrt{\frac{b \text{Cosh}[e+fx]^2}{-a+b}} (2a-b+b \text{Cosh}[2(e+fx)])^{1+p} \right. \\ \left. \left(-2a(2+p) \text{AppellF1}\left[1+p, \frac{1}{2}, \frac{1}{2}, 2+p, \frac{2a-b+b \text{Cosh}[2(e+fx)]}{2a}\right], \right. \right. \\ \left. \frac{2a-b+b \text{Cosh}[2(e+fx)]}{2(a-b)}\right) + (1+p) \text{AppellF1}\left[2+p, \frac{1}{2}, \frac{1}{2}, 3+p, \right. \\ \left. \frac{2a-b+b \text{Cosh}[2(e+fx)]}{2a}, \frac{2a-b+b \text{Cosh}[2(e+fx)]}{2(a-b)}\right] (2a-b+b \text{Cosh}[2(e+fx)]) \left. \right) \\ \text{Csch}[2(e+fx)] \sqrt{-\frac{b \text{Sinh}[e+fx]^2}{a}} \Big/ (b^2 f (1+p) (2+p))$$

Problem 139: Unable to integrate problem.

$$\int \text{Csch}[e+fx]^2 (a+b \text{Sinh}[e+fx]^2)^p dx$$

Optimal (type 6, 99 leaves, 3 steps):

$$-\frac{1}{f} \text{AppellF1}\left[-\frac{1}{2}, \frac{1}{2}, -p, \frac{1}{2}, -\text{Sinh}[e+fx]^2, -\frac{b \text{Sinh}[e+fx]^2}{a}\right] \sqrt{\text{Cosh}[e+fx]^2} \\ \text{Csch}[e+fx] \text{Sech}[e+fx] (a+b \text{Sinh}[e+fx]^2)^p \left(1 + \frac{b \text{Sinh}[e+fx]^2}{a}\right)^{-p}$$

Result (type 8, 25 leaves):

$$\int \text{Csch}[e + f x]^2 (a + b \text{Sinh}[e + f x]^2)^p dx$$

Problem 140: Unable to integrate problem.

$$\int \text{Csch}[e + f x]^4 (a + b \text{Sinh}[e + f x]^2)^p dx$$

Optimal (type 6, 103 leaves, 3 steps):

$$-\frac{1}{3f} \text{AppellF1}\left[-\frac{3}{2}, \frac{1}{2}, -p, -\frac{1}{2}, -\text{Sinh}[e + f x]^2, -\frac{b \text{Sinh}[e + f x]^2}{a}\right] \sqrt{\text{Cosh}[e + f x]^2} \text{Csch}[e + f x]^3 \text{Sech}[e + f x] (a + b \text{Sinh}[e + f x]^2)^p \left(1 + \frac{b \text{Sinh}[e + f x]^2}{a}\right)^{-p}$$

Result (type 8, 25 leaves):

$$\int \text{Csch}[e + f x]^4 (a + b \text{Sinh}[e + f x]^2)^p dx$$

Problem 148: Result more than twice size of optimal antiderivative.

$$\int \text{Csch}[c + d x]^3 (a + b \text{Sinh}[c + d x]^3) dx$$

Optimal (type 3, 39 leaves, 4 steps):

$$b x + \frac{a \text{ArcTanh}[\text{Cosh}[c + d x]]}{2 d} - \frac{a \text{Coth}[c + d x] \text{Csch}[c + d x]}{2 d}$$

Result (type 3, 82 leaves):

$$b x - \frac{a \text{Csch}\left[\frac{1}{2}(c + d x)\right]^2}{8 d} + \frac{a \text{Log}[\text{Cosh}\left[\frac{1}{2}(c + d x)\right]]}{2 d} - \frac{a \text{Log}[\text{Sinh}\left[\frac{1}{2}(c + d x)\right]]}{2 d} - \frac{a \text{Sech}\left[\frac{1}{2}(c + d x)\right]^2}{8 d}$$

Problem 159: Result more than twice size of optimal antiderivative.

$$\int \text{Csch}[c + d x]^6 (a + b \text{Sinh}[c + d x]^3)^2 dx$$

Optimal (type 3, 88 leaves, 6 steps):

$$b^2 x + \frac{a b \text{ArcTanh}[\text{Cosh}[c + d x]]}{d} - \frac{a^2 \text{Coth}[c + d x]}{d} + \frac{2 a^2 \text{Coth}[c + d x]^3}{3 d} - \frac{a^2 \text{Coth}[c + d x]^5}{5 d} - \frac{a b \text{Coth}[c + d x] \text{Csch}[c + d x]}{d}$$

Result (type 3, 216 leaves):

$$\begin{aligned} & \frac{1}{480 d} \left(-128 a^2 \operatorname{Coth} \left[\frac{1}{2} (c + d x) \right] - 120 a b \operatorname{Csch} \left[\frac{1}{2} (c + d x) \right]^2 + \right. \\ & \quad \frac{19}{2} a^2 \operatorname{Csch} \left[\frac{1}{2} (c + d x) \right]^4 \operatorname{Sinh} [c + d x] - \frac{3}{2} a^2 \operatorname{Csch} \left[\frac{1}{2} (c + d x) \right]^6 \operatorname{Sinh} [c + d x] + \\ & \quad 8 \left(60 b^2 c + 60 b^2 d x + 60 a b \operatorname{Log} \left[\operatorname{Cosh} \left[\frac{1}{2} (c + d x) \right] \right] - 60 a b \operatorname{Log} \left[\operatorname{Sinh} \left[\frac{1}{2} (c + d x) \right] \right] \right) - \\ & \quad 15 a b \operatorname{Sech} \left[\frac{1}{2} (c + d x) \right]^2 - 19 a^2 \operatorname{Csch} [c + d x]^3 \operatorname{Sinh} \left[\frac{1}{2} (c + d x) \right]^4 - \\ & \quad \left. 12 a^2 \operatorname{Csch} [c + d x]^5 \operatorname{Sinh} \left[\frac{1}{2} (c + d x) \right]^6 - 16 a^2 \operatorname{Tanh} \left[\frac{1}{2} (c + d x) \right] \right) \end{aligned}$$

Problem 171: Result is not expressed in closed-form.

$$\int \frac{\operatorname{Sinh} [c + d x]^6}{a + b \operatorname{Sinh} [c + d x]^3} dx$$

Optimal (type 3, 328 leaves, 15 steps):

$$\begin{aligned} & -\frac{a x}{b^2} - \frac{2 (-1)^{2/3} a^{4/3} \operatorname{ArcTan} \left[\frac{(-1)^{1/6} \left((-1)^{1/6} b^{1/3} + i a^{1/3} \operatorname{Tanh} \left[\frac{1}{2} (c + d x) \right] \right)}{\sqrt{(-1)^{1/3} a^{2/3} - (-1)^{2/3} b^{2/3}}} \right]}{3 \sqrt{(-1)^{1/3} a^{2/3} - (-1)^{2/3} b^{2/3}} b^2 d} - \\ & \frac{2 (-1)^{2/3} a^{4/3} \operatorname{ArcTan} \left[\frac{(-1)^{1/6} \left((-1)^{5/6} b^{1/3} + i a^{1/3} \operatorname{Tanh} \left[\frac{1}{2} (c + d x) \right] \right)}{\sqrt{(-1)^{1/3} a^{2/3} - b^{2/3}}} \right]}{3 \sqrt{(-1)^{1/3} a^{2/3} - b^{2/3}} b^2 d} - \\ & \frac{2 a^{4/3} \operatorname{ArcTanh} \left[\frac{b^{1/3} - a^{1/3} \operatorname{Tanh} \left[\frac{1}{2} (c + d x) \right]}{\sqrt{a^{2/3} + b^{2/3}}} \right]}{3 \sqrt{a^{2/3} + b^{2/3}} b^2 d} - \frac{\operatorname{Cosh} [c + d x]}{b d} + \frac{\operatorname{Cosh} [c + d x]^3}{3 b d} \end{aligned}$$

Result (type 7, 168 leaves):

$$\begin{aligned} & \frac{1}{12 b^2 d} \left(-12 a c - 12 a d x - 9 b \operatorname{Cosh} [c + d x] + \right. \\ & \quad b \operatorname{Cosh} [3 (c + d x)] + 8 a^2 \operatorname{RootSum} \left[-b + 3 b \#1^2 + 8 a \#1^3 - 3 b \#1^4 + b \#1^6 \&, \right. \\ & \quad \left. \left(c \#1 + d x \#1 + 2 \operatorname{Log} \left[-\operatorname{Cosh} \left[\frac{1}{2} (c + d x) \right] - \operatorname{Sinh} \left[\frac{1}{2} (c + d x) \right] + \operatorname{Cosh} \left[\frac{1}{2} (c + d x) \right] \#1 - \right. \right. \\ & \quad \left. \left. \operatorname{Sinh} \left[\frac{1}{2} (c + d x) \right] \#1 \right] \right) / (b + 4 a \#1 - 2 b \#1^2 + b \#1^4) \& \right) \end{aligned}$$

Problem 172: Result is not expressed in closed-form.

$$\int \frac{\operatorname{Sinh} [c + d x]^5}{a + b \operatorname{Sinh} [c + d x]^3} dx$$

Optimal (type 3, 295 leaves, 15 steps):

$$\begin{aligned}
 & -\frac{x}{2b} + \frac{2a \operatorname{ArcTan}\left[\frac{(-1)^{5/6}\left((-1)^{1/6}b^{1/3}+i a^{1/3}\operatorname{Tanh}\left[\frac{1}{2}(c+dx)\right]\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right]}{3\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}b^{5/3}d} + \\
 & \frac{2a \operatorname{ArcTan}\left[\frac{(-1)^{1/6}\left((-1)^{5/6}b^{1/3}+i a^{1/3}\operatorname{Tanh}\left[\frac{1}{2}(c+dx)\right]\right)}{\sqrt{(-1)^{1/3}a^{2/3}-b^{2/3}}}\right]}{3\sqrt{(-1)^{1/3}a^{2/3}-b^{2/3}}b^{5/3}d} + \\
 & \frac{2a \operatorname{ArcTanh}\left[\frac{b^{1/3}-a^{1/3}\operatorname{Tanh}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3}+b^{2/3}}}\right]}{3\sqrt{a^{2/3}+b^{2/3}}b^{5/3}d} + \frac{\operatorname{Cosh}[c+dx]\operatorname{Sinh}[c+dx]}{2bd}
 \end{aligned}$$

Result (type 7, 299 leaves):

$$\begin{aligned}
 & \frac{1}{12bd} \\
 & \left(-6(c+dx) - 2a \operatorname{RootSum}\left[-b + 3b\#1^2 + 8a\#1^3 - 3b\#1^4 + b\#1^6 \&, \frac{1}{b\#1 + 4a\#1^2 - 2b\#1^3 + b\#1^5}\right.\right. \\
 & \left.\left.(c+dx + 2\operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right]\#1 - \right.\right.\right. \\
 & \left.\left.\left.\operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right]\#1 - 2c\#1^2 - 2dx\#1^2 - 4\operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right] - \right.\right.\right.\right. \\
 & \left.\left.\left.\operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right]\#1 - \operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right]\#1\right]\#1^2 + c\#1^4 + \right.\right. \\
 & \left.\left.\left.dx\#1^4 + 2\operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right]\#1 - \right.\right.\right.\right. \\
 & \left.\left.\left.\operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right]\#1\right]\#1^4 \& \right) + 3\operatorname{Sinh}\left[2(c+dx)\right]\right)
 \end{aligned}$$

Problem 173: Result is not expressed in closed-form.

$$\int \frac{\operatorname{Sinh}[c+dx]^4}{a+b\operatorname{Sinh}[c+dx]^3} dx$$

Optimal (type 3, 303 leaves, 14 steps):

$$\begin{aligned}
 & -\frac{2a^{2/3}\operatorname{ArcTan}\left[\frac{(-1)^{1/6}\left((-1)^{1/6}b^{1/3}+i a^{1/3}\operatorname{Tanh}\left[\frac{1}{2}(c+dx)\right]\right)}{\sqrt{(-1)^{1/3}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right]}{3\sqrt{(-1)^{1/3}a^{2/3}-(-1)^{2/3}b^{2/3}}b^{4/3}d} + \\
 & \frac{2(-1)^{1/3}a^{2/3}\operatorname{ArcTan}\left[\frac{(-1)^{1/6}\left((-1)^{5/6}b^{1/3}+i a^{1/3}\operatorname{Tanh}\left[\frac{1}{2}(c+dx)\right]\right)}{\sqrt{(-1)^{1/3}a^{2/3}-b^{2/3}}}\right]}{3\sqrt{(-1)^{1/3}a^{2/3}-b^{2/3}}b^{4/3}d} - \\
 & \frac{2a^{2/3}\operatorname{ArcTanh}\left[\frac{b^{1/3}-a^{1/3}\operatorname{Tanh}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3}+b^{2/3}}}\right]}{3\sqrt{a^{2/3}+b^{2/3}}b^{4/3}d} + \frac{\operatorname{Cosh}[c+dx]}{bd}
 \end{aligned}$$

Result (type 7, 214 leaves):

$$\frac{1}{3 b d} \left(3 \operatorname{Cosh}[c + d x] - a \operatorname{RootSum}\left[-b + 3 b \#1^2 + 8 a \#1^3 - 3 b \#1^4 + b \#1^6 \&, \left(-c - d x - 2 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sinh}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Cosh}\left[\frac{1}{2}(c + d x)\right] \#1 - \operatorname{Sinh}\left[\frac{1}{2}(c + d x)\right] \#1\right] + c \#1^2 + d x \#1^2 + 2 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sinh}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Cosh}\left[\frac{1}{2}(c + d x)\right] \#1 - \operatorname{Sinh}\left[\frac{1}{2}(c + d x)\right] \#1\right] \#1\right] \#1^2\right) / (b + 4 a \#1 - 2 b \#1^2 + b \#1^4) \& \right)$$

Problem 174: Result is not expressed in closed-form.

$$\int \frac{\operatorname{Sinh}[c + d x]^3}{a + b \operatorname{Sinh}[c + d x]^3} dx$$

Optimal (type 3, 294 leaves, 13 steps):

$$\frac{x}{b} + \frac{2 (-1)^{2/3} a^{1/3} \operatorname{ArcTan}\left[\frac{(-1)^{1/6} \left((-1)^{1/6} b^{1/3} + i a^{1/3} \operatorname{Tanh}\left[\frac{1}{2}(c + d x)\right]\right)}{\sqrt{(-1)^{1/3} a^{2/3} - (-1)^{2/3} b^{2/3}}}\right]}{3 \sqrt{(-1)^{1/3} a^{2/3} - (-1)^{2/3} b^{2/3}} b d} + \frac{2 (-1)^{2/3} a^{1/3} \operatorname{ArcTan}\left[\frac{(-1)^{1/6} \left((-1)^{5/6} b^{1/3} + i a^{1/3} \operatorname{Tanh}\left[\frac{1}{2}(c + d x)\right]\right)}{\sqrt{(-1)^{1/3} a^{2/3} - b^{2/3}}}\right]}{3 \sqrt{(-1)^{1/3} a^{2/3} - b^{2/3}} b d} + \frac{2 a^{1/3} \operatorname{ArcTanh}\left[\frac{b^{1/3} - a^{1/3} \operatorname{Tanh}\left[\frac{1}{2}(c + d x)\right]}{\sqrt{a^{2/3} + b^{2/3}}}\right]}{3 \sqrt{a^{2/3} + b^{2/3}} b d}$$

Result (type 7, 145 leaves):

$$\frac{1}{3 b d} \left(3 c + 3 d x - 2 a \operatorname{RootSum}\left[-b + 3 b \#1^2 + 8 a \#1^3 - 3 b \#1^4 + b \#1^6 \&, \left(c \#1 + d x \#1 + 2 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sinh}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Cosh}\left[\frac{1}{2}(c + d x)\right] \#1 - \operatorname{Sinh}\left[\frac{1}{2}(c + d x)\right] \#1\right] \#1\right] \#1\right) / (b + 4 a \#1 - 2 b \#1^2 + b \#1^4) \& \right)$$

Problem 175: Result is not expressed in closed-form.

$$\int \frac{\operatorname{Sinh}[c + d x]^2}{a + b \operatorname{Sinh}[c + d x]^3} dx$$

Optimal (type 3, 262 leaves, 11 steps):

$$\frac{2 \operatorname{ArcTan}\left[\frac{(-1)^{5/6}\left((-1)^{1/6} b^{1/3} + i a^{1/3} \operatorname{Tanh}\left[\frac{1}{2}(c+dx)\right]\right)}{\sqrt{-(-1)^{2/3} a^{2/3} - b^{2/3}}}\right]}{3 \sqrt{-(-1)^{2/3} a^{2/3} - b^{2/3}} b^{2/3} d} - \frac{2 \operatorname{ArcTan}\left[\frac{(-1)^{1/6}\left((-1)^{5/6} b^{1/3} + i a^{1/3} \operatorname{Tanh}\left[\frac{1}{2}(c+dx)\right]\right)}{\sqrt{(-1)^{1/3} a^{2/3} - b^{2/3}}}\right]}{3 \sqrt{(-1)^{1/3} a^{2/3} - b^{2/3}} b^{2/3} d} - \frac{2 \operatorname{ArcTanh}\left[\frac{b^{1/3} - a^{1/3} \operatorname{Tanh}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} + b^{2/3}}}\right]}{3 \sqrt{a^{2/3} + b^{2/3}} b^{2/3} d}$$

Result (type 7, 275 leaves):

$$\frac{1}{6d} \operatorname{RootSum}\left[-b + 3 b \#1^2 + 8 a \#1^3 - 3 b \#1^4 + b \#1^6 \&, \left(c + dx + 2 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right]\right] + \operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right] \#1 - \operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right] \#1 - 2 c \#1^2 - 2 dx \#1^2 - 4 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right] \#1 - \operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right] \#1\right] \#1^2 + c \#1^4 + dx \#1^4 + 2 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right]\right] + \operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right] \#1 - \operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right] \#1\right] \#1^4\right) / (b \#1 + 4 a \#1^2 - 2 b \#1^3 + b \#1^5) \&]$$

Problem 176: Result is not expressed in closed-form.

$$\int \frac{\operatorname{Sinh}[c+dx]}{a + b \operatorname{Sinh}[c+dx]^3} dx$$

Optimal (type 3, 290 leaves, 11 steps):

$$\frac{2 \operatorname{ArcTan}\left[\frac{(-1)^{1/6}\left((-1)^{1/6} b^{1/3} + i a^{1/3} \operatorname{Tanh}\left[\frac{1}{2}(c+dx)\right]\right)}{\sqrt{(-1)^{1/3} a^{2/3} - (-1)^{2/3} b^{2/3}}}\right]}{3 a^{1/3} \sqrt{(-1)^{1/3} a^{2/3} - (-1)^{2/3} b^{2/3}} b^{1/3} d} - \frac{2 (-1)^{1/3} \operatorname{ArcTan}\left[\frac{(-1)^{1/6}\left((-1)^{5/6} b^{1/3} + i a^{1/3} \operatorname{Tanh}\left[\frac{1}{2}(c+dx)\right]\right)}{\sqrt{(-1)^{1/3} a^{2/3} - b^{2/3}}}\right]}{3 a^{1/3} \sqrt{(-1)^{1/3} a^{2/3} - b^{2/3}} b^{1/3} d} + \frac{2 \operatorname{ArcTanh}\left[\frac{b^{1/3} - a^{1/3} \operatorname{Tanh}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} + b^{2/3}}}\right]}{3 a^{1/3} \sqrt{a^{2/3} + b^{2/3}} b^{1/3} d}$$

Result (type 7, 199 leaves):

$$\frac{1}{3d} \text{RootSum}[-b + 3 b \#1^2 + 8 a \#1^3 - 3 b \#1^4 + b \#1^6 \&, \\ \left(-c - d x - 2 \text{Log} \left[-\text{Cosh} \left[\frac{1}{2} (c + d x) \right] - \text{Sinh} \left[\frac{1}{2} (c + d x) \right] \right] + \right. \\ \left. \text{Cosh} \left[\frac{1}{2} (c + d x) \right] \#1 - \text{Sinh} \left[\frac{1}{2} (c + d x) \right] \#1 \right] + c \#1^2 + d x \#1^2 + \\ 2 \text{Log} \left[-\text{Cosh} \left[\frac{1}{2} (c + d x) \right] - \text{Sinh} \left[\frac{1}{2} (c + d x) \right] \right] + \text{Cosh} \left[\frac{1}{2} (c + d x) \right] \#1 - \text{Sinh} \left[\frac{1}{2} (c + d x) \right] \#1 \right] \\ \#1^2) / (b + 4 a \#1 - 2 b \#1^2 + b \#1^4) \&$$

Problem 177: Result is not expressed in closed-form.

$$\int \frac{1}{a + b \text{Sinh}[c + d x]^3} dx$$

Optimal (type 3, 280 leaves, 11 steps):

$$\frac{2 (-1)^{2/3} \text{ArcTan} \left[\frac{(-1)^{1/6} \left((-1)^{1/6} b^{1/3} + i a^{1/3} \text{Tanh} \left[\frac{1}{2} (c + d x) \right] \right)}{\sqrt{(-1)^{1/3} a^{2/3} - (-1)^{2/3} b^{2/3}}} \right]}{3 a^{2/3} \sqrt{(-1)^{1/3} a^{2/3} - (-1)^{2/3} b^{2/3}} d} - \frac{2 (-1)^{2/3} \text{ArcTan} \left[\frac{(-1)^{1/6} \left((-1)^{5/6} b^{1/3} + i a^{1/3} \text{Tanh} \left[\frac{1}{2} (c + d x) \right] \right)}{\sqrt{(-1)^{1/3} a^{2/3} - b^{2/3}}} \right]}{3 a^{2/3} \sqrt{(-1)^{1/3} a^{2/3} - b^{2/3}} d} - \frac{2 \text{ArcTanh} \left[\frac{b^{1/3} - a^{1/3} \text{Tanh} \left[\frac{1}{2} (c + d x) \right]}{\sqrt{a^{2/3} + b^{2/3}}} \right]}{3 a^{2/3} \sqrt{a^{2/3} + b^{2/3}} d}$$

Result (type 7, 131 leaves):

$$\frac{1}{3d} 2 \text{RootSum}[-b + 3 b \#1^2 + 8 a \#1^3 - 3 b \#1^4 + b \#1^6 \&, \\ \left(c \#1 + d x \#1 + 2 \text{Log} \left[-\text{Cosh} \left[\frac{1}{2} (c + d x) \right] - \text{Sinh} \left[\frac{1}{2} (c + d x) \right] \right] + \right. \\ \left. \text{Cosh} \left[\frac{1}{2} (c + d x) \right] \#1 - \text{Sinh} \left[\frac{1}{2} (c + d x) \right] \#1 \right] \#1) / (b + 4 a \#1 - 2 b \#1^2 + b \#1^4) \&$$

Problem 178: Result is not expressed in closed-form.

$$\int \frac{\text{Csch}[c + d x]}{a + b \text{Sinh}[c + d x]^3} dx$$

Optimal (type 3, 286 leaves, 14 steps):

$$\frac{2 b^{1/3} \operatorname{ArcTan}\left[\frac{(-1)^{5/6}\left((-1)^{1/6} b^{1/3}+i a^{1/3} \operatorname{Tanh}\left[\frac{1}{2}(c+d x)\right]\right)}{\sqrt{-(-1)^{2/3} a^{2/3}-b^{2/3}}}\right]}{3 a \sqrt{-(-1)^{2/3} a^{2/3}-b^{2/3}} d} +$$

$$\frac{2 b^{1/3} \operatorname{ArcTan}\left[\frac{(-1)^{1/6}\left((-1)^{5/6} b^{1/3}+i a^{1/3} \operatorname{Tanh}\left[\frac{1}{2}(c+d x)\right]\right)}{\sqrt{(-1)^{1/3} a^{2/3}-b^{2/3}}}\right]}{3 a \sqrt{(-1)^{1/3} a^{2/3}-b^{2/3}} d} -$$

$$\frac{\operatorname{ArcTanh}[\operatorname{Cosh}[c+d x]]}{a d} + \frac{2 b^{1/3} \operatorname{ArcTanh}\left[\frac{b^{1/3}-a^{1/3} \operatorname{Tanh}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^{2/3}+b^{2/3}}}\right]}{3 a \sqrt{a^{2/3}+b^{2/3}} d}$$

Result (type 7, 307 leaves):

$$-\frac{1}{6 a d}\left(6 \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right]\right]\right) -$$

$$6 \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right]\right]+b \operatorname{RootSum}\left[-b+3 b \#1^2+8 a \#1^3-3 b \#1^4+b \#1^6 \&, \right.$$

$$\frac{1}{b \#1+4 a \#1^2-2 b \#1^3+b \#1^5}\left(c+d x+2 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right]\right]+ \right.$$

$$\left.\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \#1-\operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1\right]-2 c \#1^2-2 d x \#1^2 -$$

$$4 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \#1 -$$

$$\operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1\right] \#1^2+c \#1^4+d x \#1^4+2 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right]-\right.$$

$$\left.\operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \#1-\operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1\right] \#1^4 \& \left.)\right]$$

Problem 179: Result is not expressed in closed-form.

$$\int \frac{\operatorname{Csch}[c+d x]^2}{a+b \operatorname{Sinh}[c+d x]^3} d x$$

Optimal (type 3, 304 leaves, 15 steps):

$$-\frac{2 b^{2/3} \operatorname{ArcTan}\left[\frac{(-1)^{1/6}\left((-1)^{1/6} b^{1/3}+i a^{1/3} \operatorname{Tanh}\left[\frac{1}{2}(c+d x)\right]\right)}{\sqrt{(-1)^{1/3} a^{2/3}-(-1)^{2/3} b^{2/3}}}\right]}{3 a^{4/3} \sqrt{(-1)^{1/3} a^{2/3}-(-1)^{2/3} b^{2/3}} d} +$$

$$\frac{2(-1)^{1/3} b^{2/3} \operatorname{ArcTan}\left[\frac{(-1)^{1/6}\left((-1)^{5/6} b^{1/3}+i a^{1/3} \operatorname{Tanh}\left[\frac{1}{2}(c+d x)\right]\right)}{\sqrt{(-1)^{1/3} a^{2/3}-b^{2/3}}}\right]}{3 a^{4/3} \sqrt{(-1)^{1/3} a^{2/3}-b^{2/3}} d} -$$

$$\frac{2 b^{2/3} \operatorname{ArcTanh}\left[\frac{b^{1/3}-a^{1/3} \operatorname{Tanh}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^{2/3}+b^{2/3}}}\right]}{3 a^{4/3} \sqrt{a^{2/3}+b^{2/3}} d} - \frac{\operatorname{Coth}[c+d x]}{a d}$$

Result (type 7, 230 leaves):

$$\begin{aligned}
 & -\frac{1}{6ad} \left(3 \operatorname{Coth} \left[\frac{1}{2} (c+dx) \right] + \right. \\
 & \quad 2b \operatorname{RootSum} \left[-b + 3b\#1^2 + 8a\#1^3 - 3b\#1^4 + b\#1^6 \&, \left(-c - dx - 2 \operatorname{Log} \left[-\operatorname{Cosh} \left[\frac{1}{2} (c+dx) \right] \right] - \right. \right. \\
 & \quad \quad \left. \left. \operatorname{Sinh} \left[\frac{1}{2} (c+dx) \right] + \operatorname{Cosh} \left[\frac{1}{2} (c+dx) \right] \#1 - \operatorname{Sinh} \left[\frac{1}{2} (c+dx) \right] \#1 + c\#1^2 + \right. \right. \\
 & \quad \quad \left. \left. dx\#1^2 + 2 \operatorname{Log} \left[-\operatorname{Cosh} \left[\frac{1}{2} (c+dx) \right] \right] - \operatorname{Sinh} \left[\frac{1}{2} (c+dx) \right] + \operatorname{Cosh} \left[\frac{1}{2} (c+dx) \right] \#1 - \right. \right. \\
 & \quad \quad \left. \left. \operatorname{Sinh} \left[\frac{1}{2} (c+dx) \right] \#1 \right) \right] \Big/ (b + 4a\#1 - 2b\#1^2 + b\#1^4) \& + 3 \operatorname{Tanh} \left[\frac{1}{2} (c+dx) \right] \Big)
 \end{aligned}$$

Problem 180: Result is not expressed in closed-form.

$$\int \frac{\operatorname{Csch}[c+dx]^3}{a+b \operatorname{Sinh}[c+dx]^3} dx$$

Optimal (type 3, 322 leaves, 15 steps):

$$\begin{aligned}
 & \frac{2(-1)^{2/3} b \operatorname{ArcTan} \left[\frac{(-1)^{1/6} \left((-1)^{1/6} b^{1/3} + i a^{1/3} \operatorname{Tanh} \left[\frac{1}{2} (c+dx) \right] \right)}{\sqrt{(-1)^{1/3} a^{2/3} - (-1)^{2/3} b^{2/3}}} \right]}{3 a^{5/3} \sqrt{(-1)^{1/3} a^{2/3} - (-1)^{2/3} b^{2/3}} d} + \\
 & \frac{2(-1)^{2/3} b \operatorname{ArcTan} \left[\frac{(-1)^{1/6} \left((-1)^{5/6} b^{1/3} + i a^{1/3} \operatorname{Tanh} \left[\frac{1}{2} (c+dx) \right] \right)}{\sqrt{(-1)^{1/3} a^{2/3} - b^{2/3}}} \right]}{3 a^{5/3} \sqrt{(-1)^{1/3} a^{2/3} - b^{2/3}} d} + \frac{\operatorname{ArcTanh}[\operatorname{Cosh}[c+dx]]}{2ad} + \\
 & \frac{2b \operatorname{ArcTanh} \left[\frac{b^{1/3} - a^{1/3} \operatorname{Tanh} \left[\frac{1}{2} (c+dx) \right]}{\sqrt{a^{2/3} + b^{2/3}}} \right]}{3 a^{5/3} \sqrt{a^{2/3} + b^{2/3}} d} - \frac{\operatorname{Coth}[c+dx] \operatorname{Csch}[c+dx]}{2ad}
 \end{aligned}$$

Result (type 7, 191 leaves):

$$\begin{aligned}
 & -\frac{1}{24ad} \left(16b \operatorname{RootSum} \left[-b + 3b\#1^2 + 8a\#1^3 - 3b\#1^4 + b\#1^6 \&, \right. \right. \\
 & \quad \left(c\#1 + dx\#1 + 2 \operatorname{Log} \left[-\operatorname{Cosh} \left[\frac{1}{2} (c+dx) \right] \right] - \operatorname{Sinh} \left[\frac{1}{2} (c+dx) \right] + \operatorname{Cosh} \left[\frac{1}{2} (c+dx) \right] \#1 - \right. \\
 & \quad \quad \left. \left. \operatorname{Sinh} \left[\frac{1}{2} (c+dx) \right] \#1 \right) \right] \Big/ (b + 4a\#1 - 2b\#1^2 + b\#1^4) \& + \\
 & \quad 3 \left(\operatorname{Csch} \left[\frac{1}{2} (c+dx) \right] \right)^2 - 4 \operatorname{Log} \left[\operatorname{Cosh} \left[\frac{1}{2} (c+dx) \right] \right] + 4 \operatorname{Log} \left[\operatorname{Sinh} \left[\frac{1}{2} (c+dx) \right] \right] \Big) \\
 & \quad \operatorname{Sech} \left[\frac{1}{2} (c+dx) \right]^2 \Big)
 \end{aligned}$$

Problem 181: Result is not expressed in closed-form.

$$\int \frac{\text{Csch}[c + d x]^4}{a + b \text{Sinh}[c + d x]^3} dx$$

Optimal (type 3, 317 leaves, 16 steps):

$$\frac{2 b^{4/3} \text{ArcTan}\left[\frac{(-1)^{5/6} \left((-1)^{1/6} b^{1/3} + i a^{1/3} \text{Tanh}\left[\frac{1}{2}(c + d x)\right]\right)}{\sqrt{-(-1)^{2/3} a^{2/3} - b^{2/3}}}\right]}{3 a^2 \sqrt{-(-1)^{2/3} a^{2/3} - b^{2/3}} d} - \frac{2 b^{4/3} \text{ArcTan}\left[\frac{(-1)^{1/6} \left((-1)^{5/6} b^{1/3} + i a^{1/3} \text{Tanh}\left[\frac{1}{2}(c + d x)\right]\right)}{\sqrt{(-1)^{1/3} a^{2/3} - b^{2/3}}}\right]}{3 a^2 \sqrt{(-1)^{1/3} a^{2/3} - b^{2/3}} d} + \frac{b \text{ArcTanh}[\text{Cosh}[c + d x]]}{a^2 d} - \frac{2 b^{4/3} \text{ArcTanh}\left[\frac{b^{1/3} - a^{1/3} \text{Tanh}\left[\frac{1}{2}(c + d x)\right]}{\sqrt{a^{2/3} + b^{2/3}}}\right]}{3 a^2 \sqrt{a^{2/3} + b^{2/3}} d} + \frac{\text{Coth}[c + d x]}{a d} - \frac{\text{Coth}[c + d x]^3}{3 a d}$$

Result (type 7, 450 leaves):

$$\frac{\text{Coth}\left[\frac{1}{2}(c + d x)\right]}{3 a d} - \frac{\text{Coth}\left[\frac{1}{2}(c + d x)\right] \text{Csch}\left[\frac{1}{2}(c + d x)\right]^2}{24 a d} + \frac{b \text{Log}[\text{Cosh}\left[\frac{1}{2}(c + d x)\right]]}{a^2 d} - \frac{b \text{Log}[\text{Sinh}\left[\frac{1}{2}(c + d x)\right]]}{a^2 d} + \frac{1}{6 a^2 d} + \frac{\text{RootSum}\left[-b + 3 b \#1^2 + 8 a \#1^3 - 3 b \#1^4 + b \#1^6 \&, \left(b^2 c + b^2 d x + 2 b^2 \text{Log}\left[-\text{Cosh}\left[\frac{1}{2}(c + d x)\right] - \text{Sinh}\left[\frac{1}{2}(c + d x)\right] + \text{Cosh}\left[\frac{1}{2}(c + d x)\right] \#1 - \text{Sinh}\left[\frac{1}{2}(c + d x)\right] \#1\right] - 2 b^2 c \#1^2 - 2 b^2 d x \#1^2 - 4 b^2 \text{Log}\left[-\text{Cosh}\left[\frac{1}{2}(c + d x)\right] - \text{Sinh}\left[\frac{1}{2}(c + d x)\right] + \text{Cosh}\left[\frac{1}{2}(c + d x)\right] \#1 - \text{Sinh}\left[\frac{1}{2}(c + d x)\right] \#1\right] \#1^2 + b^2 c \#1^4 + b^2 d x \#1^4 + 2 b^2 \text{Log}\left[-\text{Cosh}\left[\frac{1}{2}(c + d x)\right] - \text{Sinh}\left[\frac{1}{2}(c + d x)\right] + \text{Cosh}\left[\frac{1}{2}(c + d x)\right] \#1 - \text{Sinh}\left[\frac{1}{2}(c + d x)\right] \#1\right] \#1^4\right)}{\left(b \#1 + 4 a \#1^2 - 2 b \#1^3 + b \#1^5\right) \&} + \frac{\text{Tanh}\left[\frac{1}{2}(c + d x)\right]}{3 a d} + \frac{\text{Sech}\left[\frac{1}{2}(c + d x)\right]^2 \text{Tanh}\left[\frac{1}{2}(c + d x)\right]}{24 a d}$$

Problem 191: Result more than twice size of optimal antiderivative.

$$\int \text{Csch}[c + d x]^3 (a + b \text{Sinh}[c + d x]^4) dx$$

Optimal (type 3, 47 leaves, 4 steps):

$$\frac{a \operatorname{ArcTanh}[\operatorname{Cosh}[c + d x]]}{2 d} + \frac{b \operatorname{Cosh}[c + d x]}{d} - \frac{a \operatorname{Coth}[c + d x] \operatorname{Csch}[c + d x]}{2 d}$$

Result (type 3, 101 leaves):

$$\frac{b \operatorname{Cosh}[c] \operatorname{Cosh}[d x]}{d} - \frac{a \operatorname{Csch}\left[\frac{1}{2}(c + d x)\right]^2}{8 d} + \frac{a \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{1}{2}(c + d x)\right]\right]}{2 d} - \frac{a \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{1}{2}(c + d x)\right]\right]}{2 d} - \frac{a \operatorname{Sech}\left[\frac{1}{2}(c + d x)\right]^2}{8 d} + \frac{b \operatorname{Sinh}[c] \operatorname{Sinh}[d x]}{d}$$

Problem 193: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Csch}[c + d x]^5 (a + b \operatorname{Sinh}[c + d x]^4) dx$$

Optimal (type 3, 64 leaves, 4 steps):

$$-\frac{(3 a + 8 b) \operatorname{ArcTanh}[\operatorname{Cosh}[c + d x]]}{8 d} + \frac{3 a \operatorname{Coth}[c + d x] \operatorname{Csch}[c + d x]}{8 d} - \frac{a \operatorname{Coth}[c + d x] \operatorname{Csch}[c + d x]^3}{4 d}$$

Result (type 3, 158 leaves):

$$\frac{3 a \operatorname{Csch}\left[\frac{1}{2}(c + d x)\right]^2}{32 d} - \frac{a \operatorname{Csch}\left[\frac{1}{2}(c + d x)\right]^4}{64 d} - \frac{b \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{c}{2} + \frac{d x}{2}\right]\right]}{d} - \frac{3 a \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{1}{2}(c + d x)\right]\right]}{8 d} + \frac{b \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{c}{2} + \frac{d x}{2}\right]\right]}{d} + \frac{3 a \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{1}{2}(c + d x)\right]\right]}{8 d} + \frac{3 a \operatorname{Sech}\left[\frac{1}{2}(c + d x)\right]^2}{32 d} + \frac{a \operatorname{Sech}\left[\frac{1}{2}(c + d x)\right]^4}{64 d}$$

Problem 195: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Csch}[c + d x]^7 (a + b \operatorname{Sinh}[c + d x]^4) dx$$

Optimal (type 3, 92 leaves, 5 steps):

$$\frac{(5 a + 8 b) \operatorname{ArcTanh}[\operatorname{Cosh}[c + d x]]}{16 d} - \frac{(5 a + 8 b) \operatorname{Coth}[c + d x] \operatorname{Csch}[c + d x]}{16 d} + \frac{5 a \operatorname{Coth}[c + d x] \operatorname{Csch}[c + d x]^3}{24 d} - \frac{a \operatorname{Coth}[c + d x] \operatorname{Csch}[c + d x]^5}{6 d}$$

Result (type 3, 237 leaves):

$$\begin{aligned}
 & - \frac{5 a \operatorname{Csch}\left[\frac{1}{2}(c+d x)\right]^2}{64 d} - \frac{b \operatorname{Csch}\left[\frac{1}{2}(c+d x)\right]^2}{8 d} + \frac{a \operatorname{Csch}\left[\frac{1}{2}(c+d x)\right]^4}{64 d} - \\
 & \frac{a \operatorname{Csch}\left[\frac{1}{2}(c+d x)\right]^6}{384 d} + \frac{5 a \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right]\right]}{16 d} + \frac{b \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right]\right]}{2 d} - \\
 & \frac{5 a \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right]\right]}{16 d} - \frac{b \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right]\right]}{2 d} - \frac{5 a \operatorname{Sech}\left[\frac{1}{2}(c+d x)\right]^2}{64 d} - \\
 & \frac{b \operatorname{Sech}\left[\frac{1}{2}(c+d x)\right]^2}{8 d} - \frac{a \operatorname{Sech}\left[\frac{1}{2}(c+d x)\right]^4}{64 d} - \frac{a \operatorname{Sech}\left[\frac{1}{2}(c+d x)\right]^6}{384 d}
 \end{aligned}$$

Problem 204: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Csch}[c+d x]^5 (a+b \operatorname{Sinh}[c+d x]^4)^2 dx$$

Optimal (type 3, 101 leaves, 6 steps):

$$\begin{aligned}
 & - \frac{a(3 a+16 b) \operatorname{ArcTanh}\left[\operatorname{Cosh}[c+d x]\right]}{8 d} - \frac{b^2 \operatorname{Cosh}[c+d x]}{d} + \\
 & \frac{b^2 \operatorname{Cosh}[c+d x]^3}{3 d} + \frac{3 a^2 \operatorname{Coth}[c+d x] \operatorname{Csch}[c+d x]}{8 d} - \frac{a^2 \operatorname{Coth}[c+d x] \operatorname{Csch}[c+d x]^3}{4 d}
 \end{aligned}$$

Result (type 3, 207 leaves):

$$\begin{aligned}
 & - \frac{3 b^2 \operatorname{Cosh}[c+d x]}{4 d} + \frac{b^2 \operatorname{Cosh}\left[3(c+d x)\right]}{12 d} + \frac{3 a^2 \operatorname{Csch}\left[\frac{1}{2}(c+d x)\right]^2}{32 d} - \frac{a^2 \operatorname{Csch}\left[\frac{1}{2}(c+d x)\right]^4}{64 d} - \\
 & \frac{2 a b \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{c}{2}+\frac{d x}{2}\right]\right]}{d} - \frac{3 a^2 \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right]\right]}{8 d} + \frac{2 a b \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{c}{2}+\frac{d x}{2}\right]\right]}{d} + \\
 & \frac{3 a^2 \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right]\right]}{8 d} + \frac{3 a^2 \operatorname{Sech}\left[\frac{1}{2}(c+d x)\right]^2}{32 d} + \frac{a^2 \operatorname{Sech}\left[\frac{1}{2}(c+d x)\right]^4}{64 d}
 \end{aligned}$$

Problem 206: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Csch}[c+d x]^7 (a+b \operatorname{Sinh}[c+d x]^4)^2 dx$$

Optimal (type 3, 111 leaves, 6 steps):

$$\begin{aligned}
 & \frac{a(5 a+16 b) \operatorname{ArcTanh}\left[\operatorname{Cosh}[c+d x]\right]}{16 d} + \frac{b^2 \operatorname{Cosh}[c+d x]}{d} - \frac{a(5 a+16 b) \operatorname{Coth}[c+d x] \operatorname{Csch}[c+d x]}{16 d} + \\
 & \frac{5 a^2 \operatorname{Coth}[c+d x] \operatorname{Csch}[c+d x]^3}{24 d} - \frac{a^2 \operatorname{Coth}[c+d x] \operatorname{Csch}[c+d x]^5}{6 d}
 \end{aligned}$$

Result (type 3, 278 leaves):

$$\begin{aligned}
 & \frac{b^2 \operatorname{Cosh}[c] \operatorname{Cosh}[d x]}{d} - \frac{5 a^2 \operatorname{Csch}\left[\frac{1}{2}(c+d x)\right]^2}{64 d} - \frac{a b \operatorname{Csch}\left[\frac{1}{2}(c+d x)\right]^2}{4 d} + \frac{a^2 \operatorname{Csch}\left[\frac{1}{2}(c+d x)\right]^4}{64 d} - \\
 & \frac{a^2 \operatorname{Csch}\left[\frac{1}{2}(c+d x)\right]^6}{384 d} + \frac{5 a^2 \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right]\right]}{16 d} + \frac{a b \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right]\right]}{d} - \\
 & \frac{5 a^2 \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right]\right]}{16 d} - \frac{a b \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right]\right]}{d} - \frac{5 a^2 \operatorname{Sech}\left[\frac{1}{2}(c+d x)\right]^2}{64 d} - \\
 & \frac{a b \operatorname{Sech}\left[\frac{1}{2}(c+d x)\right]^2}{4 d} - \frac{a^2 \operatorname{Sech}\left[\frac{1}{2}(c+d x)\right]^4}{64 d} - \frac{a^2 \operatorname{Sech}\left[\frac{1}{2}(c+d x)\right]^6}{384 d} + \frac{b^2 \operatorname{Sinh}[c] \operatorname{Sinh}[d x]}{d}
 \end{aligned}$$

Problem 225: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Csch}[c+d x]^{14} (a+b \operatorname{Sinh}[c+d x]^4)^3 dx$$

Optimal (type 3, 144 leaves, 3 steps):

$$\begin{aligned}
 & -\frac{(a+b)^3 \operatorname{Coth}[c+d x]}{d} + \frac{2 a (a+b)^2 \operatorname{Coth}[c+d x]^3}{d} - \frac{3 a (a+b) (5 a+b) \operatorname{Coth}[c+d x]^5}{5 d} + \\
 & \frac{4 a^2 (5 a+3 b) \operatorname{Coth}[c+d x]^7}{7 d} - \frac{a^2 (5 a+b) \operatorname{Coth}[c+d x]^9}{3 d} + \frac{6 a^3 \operatorname{Coth}[c+d x]^{11}}{11 d} - \frac{a^3 \operatorname{Coth}[c+d x]^{13}}{13 d}
 \end{aligned}$$

Result (type 3, 386 leaves):

$$\begin{aligned}
 & \frac{1}{61501440 d} \left(-8785920 a^3 \operatorname{Cosh}[c+d x] - 9884160 a^2 b \operatorname{Cosh}[c+d x] - 7207200 a b^2 \operatorname{Cosh}[c+d x] - \right. \\
 & 1981980 b^3 \operatorname{Cosh}[c+d x] + 6589440 a^3 \operatorname{Cosh}[3(c+d x)] + 18944640 a^2 b \operatorname{Cosh}[3(c+d x)] + \\
 & 15495480 a b^2 \operatorname{Cosh}[3(c+d x)] + 4459455 b^3 \operatorname{Cosh}[3(c+d x)] - 3660800 a^3 \operatorname{Cosh}[5(c+d x)] - \\
 & 13087360 a^2 b \operatorname{Cosh}[5(c+d x)] - 13093080 a b^2 \operatorname{Cosh}[5(c+d x)] - \\
 & 4129125 b^3 \operatorname{Cosh}[5(c+d x)] + 1464320 a^3 \operatorname{Cosh}[7(c+d x)] + 5234944 a^2 b \operatorname{Cosh}[7(c+d x)] + \\
 & 6390384 a b^2 \operatorname{Cosh}[7(c+d x)] + 2312310 b^3 \operatorname{Cosh}[7(c+d x)] - 399360 a^3 \operatorname{Cosh}[9(c+d x)] - \\
 & 1427712 a^2 b \operatorname{Cosh}[9(c+d x)] - 1873872 a b^2 \operatorname{Cosh}[9(c+d x)] - 810810 b^3 \operatorname{Cosh}[9(c+d x)] + \\
 & 66560 a^3 \operatorname{Cosh}[11(c+d x)] + 237952 a^2 b \operatorname{Cosh}[11(c+d x)] + 312312 a b^2 \operatorname{Cosh}[11(c+d x)] + \\
 & 165165 b^3 \operatorname{Cosh}[11(c+d x)] - 5120 a^3 \operatorname{Cosh}[13(c+d x)] - 18304 a^2 b \operatorname{Cosh}[13(c+d x)] - \\
 & 24024 a b^2 \operatorname{Cosh}[13(c+d x)] - 15015 b^3 \operatorname{Cosh}[13(c+d x)] \left. \right) \operatorname{Csch}[c+d x]^{13}
 \end{aligned}$$

Problem 226: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Csch}[c+d x]^{16} (a+b \operatorname{Sinh}[c+d x]^4)^3 dx$$

Optimal (type 3, 182 leaves, 3 steps):

$$\frac{(a+b)^3 \operatorname{Coth}[c+dx]}{d} - \frac{(a+b)^2 (7a+b) \operatorname{Coth}[c+dx]^3}{3d} + \frac{3a(a+b)(7a+3b) \operatorname{Coth}[c+dx]^5}{5d} - \frac{a(35a^2+30ab+3b^2) \operatorname{Coth}[c+dx]^7}{7d} + \frac{5a^2(7a+3b) \operatorname{Coth}[c+dx]^9}{9d} - \frac{3a^2(7a+b) \operatorname{Coth}[c+dx]^{11}}{11d} + \frac{7a^3 \operatorname{Coth}[c+dx]^{13}}{13d} - \frac{a^3 \operatorname{Coth}[c+dx]^{15}}{15d}$$

Result (type 3, 440 leaves):

$$\frac{1}{369008640d} (-46126080a^3 \operatorname{Cosh}[c+dx] - 51891840a^2b \operatorname{Cosh}[c+dx] - 37837800a^2b^2 \operatorname{Cosh}[c+dx] - 10405395b^3 \operatorname{Cosh}[c+dx] + 35875840a^3 \operatorname{Cosh}[3(c+dx)] + 101861760a^2b \operatorname{Cosh}[3(c+dx)] + 83243160a^2b^2 \operatorname{Cosh}[3(c+dx)] + 23948925b^3 \operatorname{Cosh}[3(c+dx)] - 21525504a^3 \operatorname{Cosh}[5(c+dx)] - 74954880a^2b \operatorname{Cosh}[5(c+dx)] - 74162088a^2b^2 \operatorname{Cosh}[5(c+dx)] - 23288265b^3 \operatorname{Cosh}[5(c+dx)] + 9784320a^3 \operatorname{Cosh}[7(c+dx)] + 34070400a^2b \operatorname{Cosh}[7(c+dx)] + 39999960a^2b^2 \operatorname{Cosh}[7(c+dx)] + 14189175b^3 \operatorname{Cosh}[7(c+dx)] - 3261440a^3 \operatorname{Cosh}[9(c+dx)] - 11356800a^2b \operatorname{Cosh}[9(c+dx)] - 14054040a^2b^2 \operatorname{Cosh}[9(c+dx)] - 5720715b^3 \operatorname{Cosh}[9(c+dx)] + 752640a^3 \operatorname{Cosh}[11(c+dx)] + 2620800a^2b \operatorname{Cosh}[11(c+dx)] + 3243240a^2b^2 \operatorname{Cosh}[11(c+dx)] + 1486485b^3 \operatorname{Cosh}[11(c+dx)] - 107520a^3 \operatorname{Cosh}[13(c+dx)] - 374400a^2b \operatorname{Cosh}[13(c+dx)] - 463320a^2b^2 \operatorname{Cosh}[13(c+dx)] - 225225b^3 \operatorname{Cosh}[13(c+dx)] + 7168a^3 \operatorname{Cosh}[15(c+dx)] + 24960a^2b \operatorname{Cosh}[15(c+dx)] + 30888a^2b^2 \operatorname{Cosh}[15(c+dx)] + 15015b^3 \operatorname{Cosh}[15(c+dx)]) \operatorname{Csch}[c+dx]^{15}$$

Problem 227: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Csch}[c+dx]^{18} (a+b \operatorname{Sinh}[c+dx]^4)^3 dx$$

Optimal (type 3, 221 leaves, 3 steps):

$$\frac{(a+b)^3 \operatorname{Coth}[c+dx]}{d} + \frac{2(a+b)^2(4a+b) \operatorname{Coth}[c+dx]^3}{3d} - \frac{(a+b)(28a^2+17ab+b^2) \operatorname{Coth}[c+dx]^5}{5d} + \frac{4a(14a^2+15ab+3b^2) \operatorname{Coth}[c+dx]^7}{7d} - \frac{a(70a^2+45ab+3b^2) \operatorname{Coth}[c+dx]^9}{9d} + \frac{2a^2(28a+9b) \operatorname{Coth}[c+dx]^{11}}{11d} - \frac{a^2(28a+3b) \operatorname{Coth}[c+dx]^{13}}{13d} + \frac{8a^3 \operatorname{Coth}[c+dx]^{15}}{15d} - \frac{a^3 \operatorname{Coth}[c+dx]^{17}}{17d}$$

Result (type 3, 494 leaves):

1

6 273 146 880 d

$$\begin{aligned}
 & (-697\,016\,320\,a^3 \operatorname{Cosh}[c+dx] - 784\,143\,360\,a^2\,b \operatorname{Cosh}[c+dx] - 571\,771\,200\,a\,b^2 \operatorname{Cosh}[c+dx] - \\
 & 157\,237\,080\,b^3 \operatorname{Cosh}[c+dx] + 557\,613\,056\,a^3 \operatorname{Cosh}[3(c+dx)] + \\
 & 1\,568\,286\,720\,a^2\,b \operatorname{Cosh}[3(c+dx)] + 1\,280\,767\,488\,a\,b^2 \operatorname{Cosh}[3(c+dx)] + \\
 & 368\,384\,016\,b^3 \operatorname{Cosh}[3(c+dx)] - 354\,844\,672\,a^3 \operatorname{Cosh}[5(c+dx)] - \\
 & 1\,211\,857\,920\,a^2\,b \operatorname{Cosh}[5(c+dx)] - 1\,189\,284\,096\,a\,b^2 \operatorname{Cosh}[5(c+dx)] - \\
 & 372\,263\,892\,b^3 \operatorname{Cosh}[5(c+dx)] + 177\,422\,336\,a^3 \operatorname{Cosh}[7(c+dx)] + \\
 & 605\,928\,960\,a^2\,b \operatorname{Cosh}[7(c+dx)] + 692\,659\,968\,a\,b^2 \operatorname{Cosh}[7(c+dx)] + \\
 & 242\,288\,046\,b^3 \operatorname{Cosh}[7(c+dx)] - 68\,239\,360\,a^3 \operatorname{Cosh}[9(c+dx)] - \\
 & 233\,049\,600\,a^2\,b \operatorname{Cosh}[9(c+dx)] - 277\,717\,440\,a\,b^2 \operatorname{Cosh}[9(c+dx)] - \\
 & 108\,738\,630\,b^3 \operatorname{Cosh}[9(c+dx)] + 19\,496\,960\,a^3 \operatorname{Cosh}[11(c+dx)] + \\
 & 66\,585\,600\,a^2\,b \operatorname{Cosh}[11(c+dx)] + 79\,347\,840\,a\,b^2 \operatorname{Cosh}[11(c+dx)] + \\
 & 33\,693\,660\,b^3 \operatorname{Cosh}[11(c+dx)] - 3\,899\,392\,a^3 \operatorname{Cosh}[13(c+dx)] - \\
 & 13\,317\,120\,a^2\,b \operatorname{Cosh}[13(c+dx)] - 15\,869\,568\,a\,b^2 \operatorname{Cosh}[13(c+dx)] - \\
 & 6\,942\,936\,b^3 \operatorname{Cosh}[13(c+dx)] + 487\,424\,a^3 \operatorname{Cosh}[15(c+dx)] + \\
 & 1\,664\,640\,a^2\,b \operatorname{Cosh}[15(c+dx)] + 1\,983\,696\,a\,b^2 \operatorname{Cosh}[15(c+dx)] + \\
 & 867\,867\,b^3 \operatorname{Cosh}[15(c+dx)] - 28\,672\,a^3 \operatorname{Cosh}[17(c+dx)] - 97\,920\,a^2\,b \operatorname{Cosh}[17(c+dx)] - \\
 & 116\,688\,a\,b^2 \operatorname{Cosh}[17(c+dx)] - 51\,051\,b^3 \operatorname{Cosh}[17(c+dx)]) \operatorname{Csch}[c+dx]^{17}
 \end{aligned}$$

Problem 228: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Csch}[c+dx]^{20} (a+b \operatorname{Sinh}[c+dx]^4)^3 dx$$

Optimal (type 3, 248 leaves, 3 steps):

$$\begin{aligned}
 & \frac{(a+b)^3 \operatorname{Coth}[c+dx]}{d} - \frac{(a+b)^2 (3a+b) \operatorname{Coth}[c+dx]^3}{d} + \frac{3(a+b)(12a^2+9ab+b^2) \operatorname{Coth}[c+dx]^5}{5d} - \\
 & \frac{(84a^3+105a^2b+30ab^2+b^3) \operatorname{Coth}[c+dx]^7}{7d} + \frac{a(42a^2+35ab+5b^2) \operatorname{Coth}[c+dx]^9}{3d} - \\
 & \frac{3a(42a^2+21ab+b^2) \operatorname{Coth}[c+dx]^{11}}{11d} + \frac{21a^2(4a+b) \operatorname{Coth}[c+dx]^{13}}{13d} - \\
 & \frac{a^2(12a+b) \operatorname{Coth}[c+dx]^{15}}{5d} + \frac{9a^3 \operatorname{Coth}[c+dx]^{17}}{17d} - \frac{a^3 \operatorname{Coth}[c+dx]^{19}}{19d}
 \end{aligned}$$

Result (type 3, 548 leaves):

$$\begin{aligned}
 & \frac{1}{79459860480d} \\
 & (-7945986048a^3 \operatorname{Cosh}[c+dx] - 8939234304a^2b \operatorname{Cosh}[c+dx] - 6518191680ab^2 \operatorname{Cosh}[c+dx] - \\
 & 1792502712b^3 \operatorname{Cosh}[c+dx] + 6501261312a^3 \operatorname{Cosh}[3(c+dx)] + \\
 & 18149354496a^2b \operatorname{Cosh}[3(c+dx)] + 14814072000ab^2 \operatorname{Cosh}[3(c+dx)] + \\
 & 4260103848b^3 \operatorname{Cosh}[3(c+dx)] - 4334174208a^3 \operatorname{Cosh}[5(c+dx)] - \\
 & 14582690304a^2b \operatorname{Cosh}[5(c+dx)] - 14221509120ab^2 \operatorname{Cosh}[5(c+dx)] - \\
 & 4440518082b^3 \operatorname{Cosh}[5(c+dx)] + 2333786112a^3 \operatorname{Cosh}[7(c+dx)] + \\
 & 7852217856a^2b \operatorname{Cosh}[7(c+dx)] + 8803791360ab^2 \operatorname{Cosh}[7(c+dx)] + \\
 & 3047642598b^3 \operatorname{Cosh}[7(c+dx)] - 1000194048a^3 \operatorname{Cosh}[9(c+dx)] - \\
 & 3365236224a^2b \operatorname{Cosh}[9(c+dx)] - 3906077760ab^2 \operatorname{Cosh}[9(c+dx)] - \\
 & 1489040982b^3 \operatorname{Cosh}[9(c+dx)] + 333398016a^3 \operatorname{Cosh}[11(c+dx)] + \\
 & 1121745408a^2b \operatorname{Cosh}[11(c+dx)] + 1302025920ab^2 \operatorname{Cosh}[11(c+dx)] + \\
 & 527386002b^3 \operatorname{Cosh}[11(c+dx)] - 83349504a^3 \operatorname{Cosh}[13(c+dx)] - \\
 & 280436352a^2b \operatorname{Cosh}[13(c+dx)] - 325506480ab^2 \operatorname{Cosh}[13(c+dx)] - \\
 & 134271423b^3 \operatorname{Cosh}[13(c+dx)] + 14708736a^3 \operatorname{Cosh}[15(c+dx)] + \\
 & 49488768a^2b \operatorname{Cosh}[15(c+dx)] + 57442320ab^2 \operatorname{Cosh}[15(c+dx)] + \\
 & 23694957b^3 \operatorname{Cosh}[15(c+dx)] - 1634304a^3 \operatorname{Cosh}[17(c+dx)] - \\
 & 5498752a^2b \operatorname{Cosh}[17(c+dx)] - 6382480ab^2 \operatorname{Cosh}[17(c+dx)] - \\
 & 2632773b^3 \operatorname{Cosh}[17(c+dx)] + 86016a^3 \operatorname{Cosh}[19(c+dx)] + 289408a^2b \operatorname{Cosh}[19(c+dx)] + \\
 & 335920ab^2 \operatorname{Cosh}[19(c+dx)] + 138567b^3 \operatorname{Cosh}[19(c+dx)] \operatorname{Csch}[c+dx]^{19}
 \end{aligned}$$

Problem 229: Result is not expressed in closed-form.

$$\int \frac{\operatorname{Sinh}[c+dx]^7}{a-b \operatorname{Sinh}[c+dx]^4} dx$$

Optimal (type 3, 148 leaves, 6 steps):

$$-\frac{a \operatorname{ArcTan}\left[\frac{b^{1/4} \operatorname{Cosh}[c+dx]}{\sqrt{\sqrt{a}-\sqrt{b}}}\right]}{2\sqrt{\sqrt{a}-\sqrt{b}}b^{7/4}d} + \frac{a \operatorname{ArcTanh}\left[\frac{b^{1/4} \operatorname{Cosh}[c+dx]}{\sqrt{\sqrt{a}+\sqrt{b}}}\right]}{2\sqrt{\sqrt{a}+\sqrt{b}}b^{7/4}d} + \frac{\operatorname{Cosh}[c+dx]}{bd} - \frac{\operatorname{Cosh}[c+dx]^3}{3bd}$$

Result (type 7, 390 leaves):

$$\frac{1}{24 b d} \left(18 \operatorname{Cosh}[c+d x] - 2 \operatorname{Cosh}\left[3(c+d x)\right] - 3 a \operatorname{RootSum}\left[b - 4 b \#1^2 - 16 a \#1^4 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \right. \right. \\ \left. \left. \frac{1}{-b \#1 - 8 a \#1^3 + 3 b \#1^3 - 3 b \#1^5 + b \#1^7} \left(-c - d x - 2 \operatorname{Log}\left[\right. \right. \right. \\ \left. \left. \left. -\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \#1 - \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1\right] + \right. \right. \\ \left. \left. 3 c \#1^2 + 3 d x \#1^2 + 6 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right]\right] + \right. \right. \\ \left. \left. \operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \#1 - \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1\right] \#1^2 - 3 c \#1^4 - 3 d x \#1^4 - \right. \\ \left. \left. 6 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \#1 - \right. \right. \\ \left. \left. \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1\right] \#1^4 + c \#1^6 + d x \#1^6 + 2 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] - \right. \right. \\ \left. \left. \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \#1 - \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1\right] \#1^6 \right) \& \left. \right)$$

Problem 230: Result is not expressed in closed-form.

$$\int \frac{\operatorname{Sinh}[c+d x]^5}{a-b \operatorname{Sinh}[c+d x]^4} dx$$

Optimal (type 3, 139 leaves, 6 steps):

$$\frac{\sqrt{a} \operatorname{ArcTan}\left[\frac{b^{1/4} \operatorname{Cosh}[c+d x]}{\sqrt{\sqrt{a}-\sqrt{b}}}\right]}{2 \sqrt{\sqrt{a}-\sqrt{b}} b^{5/4} d} + \frac{\sqrt{a} \operatorname{ArcTanh}\left[\frac{b^{1/4} \operatorname{Cosh}[c+d x]}{\sqrt{\sqrt{a}+\sqrt{b}}}\right]}{2 \sqrt{\sqrt{a}+\sqrt{b}} b^{5/4} d} - \frac{\operatorname{Cosh}[c+d x]}{b d}$$

Result (type 7, 235 leaves):

$$-\frac{1}{2 b d} \left(2 \operatorname{Cosh}[c+d x] + a \operatorname{RootSum}\left[b - 4 b \#1^2 - 16 a \#1^4 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \left(-c \#1 - d x \#1 - 2 \operatorname{Log}\left[\right. \right. \right. \\ \left. \left. \left. -\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \#1 - \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1\right] \right. \right. \\ \left. \left. \#1 + c \#1^3 + d x \#1^3 + 2 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \right. \right. \\ \left. \left. \#1 - \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1\right] \#1^3 \right) / \left(-b - 8 a \#1^2 + 3 b \#1^2 - 3 b \#1^4 + b \#1^6 \right) \& \right)$$

Problem 231: Result is not expressed in closed-form.

$$\int \frac{\operatorname{Sinh}[c+d x]^3}{a-b \operatorname{Sinh}[c+d x]^4} dx$$

Optimal (type 3, 115 leaves, 4 steps):

$$-\frac{\text{ArcTan}\left[\frac{b^{1/4} \text{Cosh}[c+dx]}{\sqrt{\sqrt{a}-\sqrt{b}}}\right]}{2\sqrt{\sqrt{a}-\sqrt{b}} b^{3/4} d} + \frac{\text{ArcTanh}\left[\frac{b^{1/4} \text{Cosh}[c+dx]}{\sqrt{\sqrt{a}+\sqrt{b}}}\right]}{2\sqrt{\sqrt{a}+\sqrt{b}} b^{3/4} d}$$

Result (type 7, 365 leaves):

$$-\frac{1}{8d} \text{RootSum}\left[b - 4 b \#1^2 - 16 a \#1^4 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \frac{1}{-b \#1 - 8 a \#1^3 + 3 b \#1^3 - 3 b \#1^5 + b \#1^7} \left(-c - dx - 2 \text{Log}\left[-\text{Cosh}\left[\frac{1}{2}(c+dx)\right] - \text{Sinh}\left[\frac{1}{2}(c+dx)\right] + \text{Cosh}\left[\frac{1}{2}(c+dx)\right] \#1 - \text{Sinh}\left[\frac{1}{2}(c+dx)\right] \#1\right] + 3 c \#1^2 + 3 dx \#1^2 + 6 \text{Log}\left[-\text{Cosh}\left[\frac{1}{2}(c+dx)\right] - \text{Sinh}\left[\frac{1}{2}(c+dx)\right]\right] + \text{Cosh}\left[\frac{1}{2}(c+dx)\right] \#1 - \text{Sinh}\left[\frac{1}{2}(c+dx)\right] \#1\right] \#1^2 - 3 c \#1^4 - 3 dx \#1^4 - 6 \text{Log}\left[-\text{Cosh}\left[\frac{1}{2}(c+dx)\right] - \text{Sinh}\left[\frac{1}{2}(c+dx)\right] + \text{Cosh}\left[\frac{1}{2}(c+dx)\right] \#1 - \text{Sinh}\left[\frac{1}{2}(c+dx)\right] \#1\right] \#1^4 + c \#1^6 + dx \#1^6 + 2 \text{Log}\left[-\text{Cosh}\left[\frac{1}{2}(c+dx)\right] - \text{Sinh}\left[\frac{1}{2}(c+dx)\right]\right] + \text{Cosh}\left[\frac{1}{2}(c+dx)\right] \#1 - \text{Sinh}\left[\frac{1}{2}(c+dx)\right] \#1\right] \#1^6 \right) \&]$$

Problem 232: Result is not expressed in closed-form.

$$\int \frac{\text{Sinh}[c+dx]}{a - b \text{Sinh}[c+dx]^4} dx$$

Optimal (type 3, 125 leaves, 4 steps):

$$\frac{\text{ArcTan}\left[\frac{b^{1/4} \text{Cosh}[c+dx]}{\sqrt{\sqrt{a}-\sqrt{b}}}\right]}{2\sqrt{a} \sqrt{\sqrt{a}-\sqrt{b}} b^{1/4} d} + \frac{\text{ArcTanh}\left[\frac{b^{1/4} \text{Cosh}[c+dx]}{\sqrt{\sqrt{a}+\sqrt{b}}}\right]}{2\sqrt{a} \sqrt{\sqrt{a}+\sqrt{b}} b^{1/4} d}$$

Result (type 7, 221 leaves):

$$-\frac{1}{2d} \text{RootSum}\left[b - 4 b \#1^2 - 16 a \#1^4 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \left(-c \#1 - dx \#1 - 2 \text{Log}\left[-\text{Cosh}\left[\frac{1}{2}(c+dx)\right] - \text{Sinh}\left[\frac{1}{2}(c+dx)\right]\right] + \text{Cosh}\left[\frac{1}{2}(c+dx)\right] \#1 - \text{Sinh}\left[\frac{1}{2}(c+dx)\right] \#1\right] \#1 + c \#1^3 + dx \#1^3 + 2 \text{Log}\left[-\text{Cosh}\left[\frac{1}{2}(c+dx)\right] - \text{Sinh}\left[\frac{1}{2}(c+dx)\right] + \text{Cosh}\left[\frac{1}{2}(c+dx)\right] \#1 - \text{Sinh}\left[\frac{1}{2}(c+dx)\right] \#1\right] \#1^3 \right) / \left(-b - 8 a \#1^2 + 3 b \#1^2 - 3 b \#1^4 + b \#1^6 \right) \&]$$

Problem 233: Result is not expressed in closed-form.

$$\int \frac{\text{Csch}[c + d x]}{a - b \text{Sinh}[c + d x]^4} dx$$

Optimal (type 3, 136 leaves, 7 steps):

$$-\frac{b^{1/4} \text{ArcTan}\left[\frac{b^{1/4} \text{Cosh}[c+dx]}{\sqrt{\sqrt{a}-\sqrt{b}}}\right]}{2 a \sqrt{\sqrt{a}-\sqrt{b}} d} - \frac{\text{ArcTanh}[\text{Cosh}[c + d x]]}{a d} + \frac{b^{1/4} \text{ArcTanh}\left[\frac{b^{1/4} \text{Cosh}[c+dx]}{\sqrt{\sqrt{a}+\sqrt{b}}}\right]}{2 a \sqrt{\sqrt{a}+\sqrt{b}} d}$$

Result (type 7, 397 leaves):

$$-\frac{1}{8 a d} \left(8 \text{Log}[\text{Cosh}\left[\frac{1}{2}(c + d x)\right]] - 8 \text{Log}[\text{Sinh}\left[\frac{1}{2}(c + d x)\right]] + \right. \\ \left. b \text{RootSum}\left[b - 4 b \#1^2 - 16 a \#1^4 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \right. \right. \\ \left. \frac{1}{-b \#1 - 8 a \#1^3 + 3 b \#1^3 - 3 b \#1^5 + b \#1^7} \left(-c - d x - 2 \text{Log}\left[-\text{Cosh}\left[\frac{1}{2}(c + d x)\right] - \right. \right. \right. \\ \left. \left. \left. \text{Sinh}\left[\frac{1}{2}(c + d x)\right] + \text{Cosh}\left[\frac{1}{2}(c + d x)\right] \#1 - \text{Sinh}\left[\frac{1}{2}(c + d x)\right] \#1\right] + \right. \right. \\ \left. \left. 3 c \#1^2 + 3 d x \#1^2 + 6 \text{Log}\left[-\text{Cosh}\left[\frac{1}{2}(c + d x)\right] - \text{Sinh}\left[\frac{1}{2}(c + d x)\right] + \right. \right. \right. \\ \left. \left. \left. \text{Cosh}\left[\frac{1}{2}(c + d x)\right] \#1 - \text{Sinh}\left[\frac{1}{2}(c + d x)\right] \#1\right] \#1^2 - 3 c \#1^4 - 3 d x \#1^4 - \right. \right. \\ \left. \left. 6 \text{Log}\left[-\text{Cosh}\left[\frac{1}{2}(c + d x)\right] - \text{Sinh}\left[\frac{1}{2}(c + d x)\right] + \text{Cosh}\left[\frac{1}{2}(c + d x)\right] \#1 - \right. \right. \right. \\ \left. \left. \left. \text{Sinh}\left[\frac{1}{2}(c + d x)\right] \#1\right] \#1^4 + c \#1^6 + d x \#1^6 + 2 \text{Log}\left[-\text{Cosh}\left[\frac{1}{2}(c + d x)\right] - \right. \right. \right. \\ \left. \left. \left. \text{Sinh}\left[\frac{1}{2}(c + d x)\right] + \text{Cosh}\left[\frac{1}{2}(c + d x)\right] \#1 - \text{Sinh}\left[\frac{1}{2}(c + d x)\right] \#1\right] \#1^6\right] \& \right) \left. \right)$$

Problem 234: Result is not expressed in closed-form.

$$\int \frac{\text{Csch}[c + d x]^3}{a - b \text{Sinh}[c + d x]^4} dx$$

Optimal (type 3, 184 leaves, 7 steps):

$$\frac{b^{3/4} \text{ArcTan}\left[\frac{b^{1/4} \text{Cosh}[c+dx]}{\sqrt{\sqrt{a}-\sqrt{b}}}\right]}{2 a^{3/2} \sqrt{\sqrt{a}-\sqrt{b}} d} + \frac{\text{ArcTanh}[\text{Cosh}[c + d x]]}{2 a d} + \\ \frac{b^{3/4} \text{ArcTanh}\left[\frac{b^{1/4} \text{Cosh}[c+dx]}{\sqrt{\sqrt{a}+\sqrt{b}}}\right]}{2 a^{3/2} \sqrt{\sqrt{a}+\sqrt{b}} d} + \frac{1}{4 a d (1 - \text{Cosh}[c + d x])} - \frac{1}{4 a d (1 + \text{Cosh}[c + d x])}$$

Result (type 7, 278 leaves):

$$\begin{aligned}
 & -\frac{1}{8 a d} \left(\operatorname{Csch}\left[\frac{1}{2}(c+d x)\right]^2 - 4 \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right]\right] \right) + \\
 & 4 \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right]\right] + 4 b \operatorname{RootSum}\left[b - 4 b \#1^2 - 16 a \#1^4 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \right. \\
 & \left. \left(-c \#1 - d x \#1 - 2 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right]\right] - \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \#1 - \right. \right. \\
 & \left. \left. \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1\right] \#1 + c \#1^3 + d x \#1^3 + 2 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right]\right] - \right. \\
 & \left. \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \#1 - \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1\right] \#1^3 \Big/ \\
 & \left. \left(-b - 8 a \#1^2 + 3 b \#1^2 - 3 b \#1^4 + b \#1^6\right) \& \right] + \operatorname{Sech}\left[\frac{1}{2}(c+d x)\right]^2 \Big)
 \end{aligned}$$

Problem 241: Result is not expressed in closed-form.

$$\int \frac{\operatorname{Sinh}[c+d x]^9}{(a-b \operatorname{Sinh}[c+d x]^4)^2} dx$$

Optimal (type 3, 235 leaves, 7 steps):

$$\begin{aligned}
 & \frac{\sqrt{a} \left(5 \sqrt{a} - 6 \sqrt{b}\right) \operatorname{ArcTan}\left[\frac{b^{1/4} \operatorname{Cosh}[c+d x]}{\sqrt{\sqrt{a}-\sqrt{b}}}\right]}{8 \left(\sqrt{a}-\sqrt{b}\right)^{3/2} b^{9/4} d} - \frac{\sqrt{a} \left(5 \sqrt{a} + 6 \sqrt{b}\right) \operatorname{ArcTanh}\left[\frac{b^{1/4} \operatorname{Cosh}[c+d x]}{\sqrt{\sqrt{a}+\sqrt{b}}}\right]}{8 \left(\sqrt{a}+\sqrt{b}\right)^{3/2} b^{9/4} d} + \\
 & \frac{\operatorname{Cosh}[c+d x]}{b^2 d} + \frac{a \operatorname{Cosh}[c+d x] \left(a+b-b \operatorname{Cosh}[c+d x]^2\right)}{4(a-b) b^2 d \left(a-b+2 b \operatorname{Cosh}[c+d x]^2-b \operatorname{Cosh}[c+d x]^4\right)}
 \end{aligned}$$

Result (type 7, 615 leaves):

$$\frac{1}{32 b^2 d} \left(32 \operatorname{Cosh}[c + d x] + \frac{32 a \operatorname{Cosh}[c + d x] (2 a + b - b \operatorname{Cosh}[2 (c + d x)])}{(a - b) (8 a - 3 b + 4 b \operatorname{Cosh}[2 (c + d x)] - b \operatorname{Cosh}[4 (c + d x)])} + \frac{1}{a - b} a \operatorname{RootSum} \left[\right. \right.$$

$$b - 4 b \#1^2 - 16 a \#1^4 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \frac{1}{-b \#1 - 8 a \#1^3 + 3 b \#1^3 - 3 b \#1^5 + b \#1^7}$$

$$\left. \left. \begin{aligned} & \left(-b c - b d x - 2 b \operatorname{Log} \left[-\operatorname{Cosh} \left[\frac{1}{2} (c + d x) \right] - \operatorname{Sinh} \left[\frac{1}{2} (c + d x) \right] + \operatorname{Cosh} \left[\frac{1}{2} (c + d x) \right] \#1 - \right. \right. \\ & \quad \operatorname{Sinh} \left[\frac{1}{2} (c + d x) \right] \#1 \right] - 20 a c \#1^2 + 27 b c \#1^2 - 20 a d x \#1^2 + 27 b d x \#1^2 - 40 a \operatorname{Log} \left[\right. \\ & \quad \left. -\operatorname{Cosh} \left[\frac{1}{2} (c + d x) \right] - \operatorname{Sinh} \left[\frac{1}{2} (c + d x) \right] + \operatorname{Cosh} \left[\frac{1}{2} (c + d x) \right] \#1 - \operatorname{Sinh} \left[\frac{1}{2} (c + d x) \right] \#1 \right] \\ & \quad \#1^2 + 54 b \operatorname{Log} \left[-\operatorname{Cosh} \left[\frac{1}{2} (c + d x) \right] - \operatorname{Sinh} \left[\frac{1}{2} (c + d x) \right] + \operatorname{Cosh} \left[\frac{1}{2} (c + d x) \right] \#1 - \right. \\ & \quad \left. \operatorname{Sinh} \left[\frac{1}{2} (c + d x) \right] \#1 \right] \#1^2 + 20 a c \#1^4 - 27 b c \#1^4 + 20 a d x \#1^4 - 27 b d x \#1^4 + 40 a \operatorname{Log} \left[\right. \\ & \quad \left. -\operatorname{Cosh} \left[\frac{1}{2} (c + d x) \right] - \operatorname{Sinh} \left[\frac{1}{2} (c + d x) \right] + \operatorname{Cosh} \left[\frac{1}{2} (c + d x) \right] \#1 - \operatorname{Sinh} \left[\frac{1}{2} (c + d x) \right] \#1 \right] \\ & \quad \#1^4 - 54 b \operatorname{Log} \left[-\operatorname{Cosh} \left[\frac{1}{2} (c + d x) \right] - \operatorname{Sinh} \left[\frac{1}{2} (c + d x) \right] + \operatorname{Cosh} \left[\frac{1}{2} (c + d x) \right] \#1 - \right. \\ & \quad \left. \operatorname{Sinh} \left[\frac{1}{2} (c + d x) \right] \#1 \right] \#1^4 + b c \#1^6 + b d x \#1^6 + 2 b \operatorname{Log} \left[-\operatorname{Cosh} \left[\frac{1}{2} (c + d x) \right] - \right. \\ & \quad \left. \operatorname{Sinh} \left[\frac{1}{2} (c + d x) \right] + \operatorname{Cosh} \left[\frac{1}{2} (c + d x) \right] \#1 - \operatorname{Sinh} \left[\frac{1}{2} (c + d x) \right] \#1 \right] \#1^6 \left. \right) \& \left. \right) \end{aligned} \right)$$

Problem 242: Result is not expressed in closed-form.

$$\int \frac{\operatorname{Sinh}[c + d x]^7}{(a - b \operatorname{Sinh}[c + d x]^4)^2} dx$$

Optimal (type 3, 210 leaves, 5 steps):

$$\frac{(3 \sqrt{a} - 4 \sqrt{b}) \operatorname{ArcTan} \left[\frac{b^{1/4} \operatorname{Cosh}[c + d x]}{\sqrt{\sqrt{a} - \sqrt{b}}} \right]}{8 (\sqrt{a} - \sqrt{b})^{3/2} b^{7/4} d} - \frac{(3 \sqrt{a} + 4 \sqrt{b}) \operatorname{ArcTanh} \left[\frac{b^{1/4} \operatorname{Cosh}[c + d x]}{\sqrt{\sqrt{a} + \sqrt{b}}} \right]}{8 (\sqrt{a} + \sqrt{b})^{3/2} b^{7/4} d} - \frac{a \operatorname{Cosh}[c + d x] (2 - \operatorname{Cosh}[c + d x]^2)}{4 (a - b) b d (a - b + 2 b \operatorname{Cosh}[c + d x]^2 - b \operatorname{Cosh}[c + d x]^4)}$$

Result (type 7, 737 leaves):

$$\begin{aligned}
 & - \frac{1}{32 (a-b) b d} \left(- \frac{16 a (-5 \operatorname{Cosh}[c+d x] + \operatorname{Cosh}[3(c+d x)])}{8 a - 3 b + 4 b \operatorname{Cosh}[2(c+d x)] - b \operatorname{Cosh}[4(c+d x)]} + \right. \\
 & \quad \operatorname{RootSum}\left[b - 4 b \#1^2 - 16 a \#1^4 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \right. \\
 & \quad \left. \frac{1}{-b \#1 - 8 a \#1^3 + 3 b \#1^3 - 3 b \#1^5 + b \#1^7} \left(3 a c - 4 b c + 3 a d x - 4 b d x + 6 a \operatorname{Log}\left[\right. \right. \right. \\
 & \quad \left. \left. \left. - \operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \#1 - \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1\right] - \right. \right. \\
 & \quad \left. \left. 8 b \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \#1 - \right. \right. \right. \\
 & \quad \left. \left. \left. \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1\right] - 5 a c \#1^2 + 12 b c \#1^2 - 5 a d x \#1^2 + 12 b d x \#1^2 - \right. \right. \\
 & \quad \left. \left. 10 a \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \#1 - \right. \right. \right. \\
 & \quad \left. \left. \left. \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1\right] \#1^2 + 24 b \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] + \right. \right. \right. \\
 & \quad \left. \left. \left. \operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \#1 - \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1\right] \#1^2 + 5 a c \#1^4 - 12 b c \#1^4 + \right. \right. \\
 & \quad \left. \left. 5 a d x \#1^4 - 12 b d x \#1^4 + 10 a \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] + \right. \right. \right. \\
 & \quad \left. \left. \left. \operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \#1 - \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1\right] \#1^4 - 24 b \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] - \right. \right. \right. \\
 & \quad \left. \left. \left. \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \#1 - \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1\right] \#1^4 - 3 a c \#1^6 + \right. \right. \\
 & \quad \left. \left. 4 b c \#1^6 - 3 a d x \#1^6 + 4 b d x \#1^6 - 6 a \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] + \right. \right. \right. \\
 & \quad \left. \left. \left. \operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \#1 - \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1\right] \#1^6 + 8 b \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] - \right. \right. \right. \\
 & \quad \left. \left. \left. \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \#1 - \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1\right] \#1^6 \right) \& \right) \left. \right)
 \end{aligned}$$

Problem 243: Result is not expressed in closed-form.

$$\int \frac{\operatorname{Sinh}[c+d x]^5}{(a-b \operatorname{Sinh}[c+d x]^4)^2} dx$$

Optimal (type 3, 217 leaves, 5 steps):

$$\begin{aligned}
 & \frac{(\sqrt{a}-2 \sqrt{b}) \operatorname{ArcTan}\left[\frac{b^{1/4} \operatorname{Cosh}[c+d x]}{\sqrt{\sqrt{a}-\sqrt{b}}}\right] - (\sqrt{a}+2 \sqrt{b}) \operatorname{ArcTanh}\left[\frac{b^{1/4} \operatorname{Cosh}[c+d x]}{\sqrt{\sqrt{a}+\sqrt{b}}}\right]}{8 \sqrt{a}(\sqrt{a}-\sqrt{b})^{3/2} b^{5/4} d} - \frac{8 \sqrt{a}(\sqrt{a}+\sqrt{b})^{3/2} b^{5/4} d}{8 \sqrt{a}(\sqrt{a}-\sqrt{b})^{3/2} b^{5/4} d} + \\
 & \frac{\operatorname{Cosh}[c+d x](a+b-b \operatorname{Cosh}[c+d x]^2)}{4(a-b) b d(a-b+2 b \operatorname{Cosh}[c+d x]^2-b \operatorname{Cosh}[c+d x]^4)}
 \end{aligned}$$

Result (type 7, 597 leaves):

$$\begin{aligned}
 & \frac{1}{32 (a-b) b d} \left(\frac{32 \operatorname{Cosh}[c+d x] (2 a+b-b \operatorname{Cosh}[2(c+d x)])}{8 a-3 b+4 b \operatorname{Cosh}[2(c+d x)]-b \operatorname{Cosh}[4(c+d x)]} + \right. \\
 & \operatorname{RootSum}\left[b-4 b \#1^2-16 a \#1^4+6 b \#1^4-4 b \#1^6+b \#1^8 \&, \right. \\
 & \left. \frac{1}{-b \#1-8 a \#1^3+3 b \#1^3-3 b \#1^5+b \#1^7} \left(-b c-b d x-2 b \operatorname{Log}\left[\right. \right. \right. \\
 & \left. \left. \left. -\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \#1-\operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1\right] - \right. \right. \\
 & \left. \left. 4 a c \#1^2+11 b c \#1^2-4 a d x \#1^2+11 b d x \#1^2-8 a \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] - \right. \right. \right. \\
 & \left. \left. \left. \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \#1-\operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1\right] \#1^2+22 b \operatorname{Log}\left[\right. \right. \right. \\
 & \left. \left. \left. -\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \#1-\operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1\right] \right. \right. \\
 & \left. \left. \#1^2+4 a c \#1^4-11 b c \#1^4+4 a d x \#1^4-11 b d x \#1^4+8 a \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] - \right. \right. \right. \\
 & \left. \left. \left. \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \#1-\operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1\right] \#1^4 - \right. \right. \\
 & \left. \left. 22 b \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \#1 - \right. \right. \right. \\
 & \left. \left. \left. \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1\right] \#1^4+b c \#1^6+b d x \#1^6+2 b \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] - \right. \right. \right. \\
 & \left. \left. \left. \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \#1-\operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1\right] \#1^6\right] \& \right) \left. \right)
 \end{aligned}$$

Problem 244: Result is not expressed in closed-form.

$$\int \frac{\operatorname{Sinh}[c+d x]^3}{(a-b \operatorname{Sinh}[c+d x]^4)^2} dx$$

Optimal (type 3, 186 leaves, 5 steps):

$$\begin{aligned}
 & -\frac{\operatorname{ArcTan}\left[\frac{b^{1/4} \operatorname{Cosh}[c+d x]}{\sqrt{\sqrt{a}-\sqrt{b}}}\right]}{8 \sqrt{a} (\sqrt{a}-\sqrt{b})^{3/2} b^{3/4} d} + \frac{\operatorname{ArcTanh}\left[\frac{b^{1/4} \operatorname{Cosh}[c+d x]}{\sqrt{\sqrt{a}+\sqrt{b}}}\right]}{8 \sqrt{a} (\sqrt{a}+\sqrt{b})^{3/2} b^{3/4} d} - \\
 & \frac{\operatorname{Cosh}[c+d x] (2-\operatorname{Cosh}[c+d x]^2)}{4 (a-b) d (a-b+2 b \operatorname{Cosh}[c+d x]^2-b \operatorname{Cosh}[c+d x]^4)}
 \end{aligned}$$

Result (type 7, 422 leaves):

$$\begin{aligned}
 & - \frac{1}{32 (a - b) d} \left(\frac{16 (-5 \operatorname{Cosh}[c + d x] + \operatorname{Cosh}[3 (c + d x)])}{-8 a + 3 b - 4 b \operatorname{Cosh}[2 (c + d x)] + b \operatorname{Cosh}[4 (c + d x)]} + \right. \\
 & \quad \operatorname{RootSum}\left[b - 4 b \#1^2 - 16 a \#1^4 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \right. \\
 & \quad \left. \frac{1}{-b \#1 - 8 a \#1^3 + 3 b \#1^3 - 3 b \#1^5 + b \#1^7} \left(-c - d x - 2 \operatorname{Log}\left[\right. \right. \\
 & \quad \left. \left. -\operatorname{Cosh}\left[\frac{1}{2} (c + d x)\right] - \operatorname{Sinh}\left[\frac{1}{2} (c + d x)\right] + \operatorname{Cosh}\left[\frac{1}{2} (c + d x)\right] \#1 - \operatorname{Sinh}\left[\frac{1}{2} (c + d x)\right] \#1\right] + \right. \\
 & \quad \left. 7 c \#1^2 + 7 d x \#1^2 + 14 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2} (c + d x)\right] - \operatorname{Sinh}\left[\frac{1}{2} (c + d x)\right] + \right. \right. \\
 & \quad \left. \left. \operatorname{Cosh}\left[\frac{1}{2} (c + d x)\right] \#1 - \operatorname{Sinh}\left[\frac{1}{2} (c + d x)\right] \#1\right] \#1^2 - 7 c \#1^4 - 7 d x \#1^4 - \right. \\
 & \quad \left. 14 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2} (c + d x)\right] - \operatorname{Sinh}\left[\frac{1}{2} (c + d x)\right] + \operatorname{Cosh}\left[\frac{1}{2} (c + d x)\right] \#1 - \right. \right. \\
 & \quad \left. \left. \operatorname{Sinh}\left[\frac{1}{2} (c + d x)\right] \#1\right] \#1^4 + c \#1^6 + d x \#1^6 + 2 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2} (c + d x)\right] - \right. \right. \\
 & \quad \left. \left. \operatorname{Sinh}\left[\frac{1}{2} (c + d x)\right] + \operatorname{Cosh}\left[\frac{1}{2} (c + d x)\right] \#1 - \operatorname{Sinh}\left[\frac{1}{2} (c + d x)\right] \#1\right] \#1^6 \right) \& \left. \right)
 \end{aligned}$$

Problem 245: Result is not expressed in closed-form.

$$\int \frac{\operatorname{Sinh}[c + d x]}{(a - b \operatorname{Sinh}[c + d x]^4)^2} dx$$

Optimal (type 3, 221 leaves, 5 steps):

$$\begin{aligned}
 & \frac{\left(3 \sqrt{a} - 2 \sqrt{b}\right) \operatorname{ArcTan}\left[\frac{b^{1/4} \operatorname{Cosh}[c+d x]}{\sqrt{\sqrt{a}-\sqrt{b}}}\right]}{8 a^{3/2} \left(\sqrt{a}-\sqrt{b}\right)^{3/2} b^{1/4} d} + \frac{\left(3 \sqrt{a} + 2 \sqrt{b}\right) \operatorname{ArcTanh}\left[\frac{b^{1/4} \operatorname{Cosh}[c+d x]}{\sqrt{\sqrt{a}+\sqrt{b}}}\right]}{8 a^{3/2} \left(\sqrt{a}+\sqrt{b}\right)^{3/2} b^{1/4} d} + \\
 & \frac{\operatorname{Cosh}[c+d x] (a+b-b \operatorname{Cosh}[c+d x]^2)}{4 a (a-b) d (a-b+2 b \operatorname{Cosh}[c+d x]^2-b \operatorname{Cosh}[c+d x]^4)}
 \end{aligned}$$

Result (type 7, 597 leaves):

$$\begin{aligned}
 & \frac{1}{32 a (a-b) d} \left(\frac{32 \operatorname{Cosh}[c+d x] (2 a+b-b \operatorname{Cosh}[2(c+d x)])}{8 a-3 b+4 b \operatorname{Cosh}[2(c+d x)]-b \operatorname{Cosh}[4(c+d x)]} + \right. \\
 & \operatorname{RootSum}\left[b-4 b \#1^2-16 a \#1^4+6 b \#1^4-4 b \#1^6+b \#1^8 \&, \right. \\
 & \left. \frac{1}{-b \#1-8 a \#1^3+3 b \#1^3-3 b \#1^5+b \#1^7} \left(-b c-b d x-2 b \operatorname{Log}\left[\right. \right. \right. \\
 & \left. \left. \left. -\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \#1-\operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1\right] + \right. \right. \\
 & 12 a c \#1^2-5 b c \#1^2+12 a d x \#1^2-5 b d x \#1^2+24 a \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right]-\right. \\
 & \left. \left. \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \#1-\operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1\right] \#1^2-10 b \operatorname{Log}\left[\right. \right. \\
 & \left. \left. \left. -\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \#1-\operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1\right] \right. \right. \\
 & \left. \#1^2-12 a c \#1^4+5 b c \#1^4-12 a d x \#1^4+5 b d x \#1^4-24 a \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right]-\right. \\
 & \left. \left. \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \#1-\operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1\right] \#1^4 + \right. \\
 & \left. 10 b \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \#1-\right. \right. \\
 & \left. \left. \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1\right] \#1^4+b c \#1^6+b d x \#1^6+2 b \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right]-\right. \right. \\
 & \left. \left. \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \#1-\operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1\right] \#1^6 \right) \& \left. \right)
 \end{aligned}$$

Problem 246: Result is not expressed in closed-form.

$$\int \frac{\operatorname{Csch}[c+d x]}{(a-b \operatorname{Sinh}[c+d x]^4)^2} dx$$

Optimal (type 3, 325 leaves, 11 steps):

$$\begin{aligned}
 & -\frac{b^{1/4} \operatorname{ArcTan}\left[\frac{b^{1/4} \operatorname{Cosh}[c+d x]}{\sqrt{\sqrt{a}-\sqrt{b}}}\right]}{8 a^{3/2} (\sqrt{a}-\sqrt{b})^{3/2} d} - \frac{b^{1/4} \operatorname{ArcTan}\left[\frac{b^{1/4} \operatorname{Cosh}[c+d x]}{\sqrt{\sqrt{a}-\sqrt{b}}}\right]}{2 a^2 \sqrt{\sqrt{a}-\sqrt{b}} d} - \\
 & \frac{\operatorname{ArcTanh}[\operatorname{Cosh}[c+d x]]}{a^2 d} + \frac{b^{1/4} \operatorname{ArcTanh}\left[\frac{b^{1/4} \operatorname{Cosh}[c+d x]}{\sqrt{\sqrt{a}+\sqrt{b}}}\right]}{8 a^{3/2} (\sqrt{a}+\sqrt{b})^{3/2} d} + \frac{b^{1/4} \operatorname{ArcTanh}\left[\frac{b^{1/4} \operatorname{Cosh}[c+d x]}{\sqrt{\sqrt{a}+\sqrt{b}}}\right]}{2 a^2 \sqrt{\sqrt{a}+\sqrt{b}} d} - \\
 & \frac{b \operatorname{Cosh}[c+d x] (2-\operatorname{Cosh}[c+d x]^2)}{4 a (a-b) d (a-b+2 b \operatorname{Cosh}[c+d x]^2-b \operatorname{Cosh}[c+d x]^4)}
 \end{aligned}$$

Result (type 7, 774 leaves):

$$\frac{1}{32 a^2 d} \left(\frac{16 a b (-5 \operatorname{Cosh}[c+d x] + \operatorname{Cosh}[3(c+d x)])}{(a-b)(8 a-3 b+4 b \operatorname{Cosh}[2(c+d x)]-b \operatorname{Cosh}[4(c+d x)])} - \right. \\
 32 \operatorname{Log}[\operatorname{Cosh}[\frac{1}{2}(c+d x)]] + 32 \operatorname{Log}[\operatorname{Sinh}[\frac{1}{2}(c+d x)]] - \frac{1}{a-b} b \operatorname{RootSum}[\\
 b-4 b \#1^2-16 a \#1^4+6 b \#1^4-4 b \#1^6+b \#1^8 \&, \frac{1}{-b \#1-8 a \#1^3+3 b \#1^3-3 b \#1^5+b \#1^7} \\
 (-5 a c+4 b c-5 a d x+4 b d x-10 a \operatorname{Log}[-\operatorname{Cosh}[\frac{1}{2}(c+d x)]-\operatorname{Sinh}[\frac{1}{2}(c+d x)]] + \\
 \operatorname{Cosh}[\frac{1}{2}(c+d x)] \#1-\operatorname{Sinh}[\frac{1}{2}(c+d x)] \#1]+8 b \operatorname{Log}[-\operatorname{Cosh}[\frac{1}{2}(c+d x)]- \\
 \operatorname{Sinh}[\frac{1}{2}(c+d x)]+\operatorname{Cosh}[\frac{1}{2}(c+d x)] \#1-\operatorname{Sinh}[\frac{1}{2}(c+d x)] \#1]+ \\
 19 a c \#1^2-12 b c \#1^2+19 a d x \#1^2-12 b d x \#1^2+38 a \operatorname{Log}[-\operatorname{Cosh}[\frac{1}{2}(c+d x)]- \\
 \operatorname{Sinh}[\frac{1}{2}(c+d x)]+\operatorname{Cosh}[\frac{1}{2}(c+d x)] \#1-\operatorname{Sinh}[\frac{1}{2}(c+d x)] \#1] \#1^2- \\
 24 b \operatorname{Log}[-\operatorname{Cosh}[\frac{1}{2}(c+d x)]-\operatorname{Sinh}[\frac{1}{2}(c+d x)]+\operatorname{Cosh}[\frac{1}{2}(c+d x)] \#1- \\
 \operatorname{Sinh}[\frac{1}{2}(c+d x)] \#1] \#1^2-19 a c \#1^4+12 b c \#1^4-19 a d x \#1^4+12 b d x \#1^4- \\
 38 a \operatorname{Log}[-\operatorname{Cosh}[\frac{1}{2}(c+d x)]-\operatorname{Sinh}[\frac{1}{2}(c+d x)]+\operatorname{Cosh}[\frac{1}{2}(c+d x)] \#1- \\
 \operatorname{Sinh}[\frac{1}{2}(c+d x)] \#1] \#1^4+24 b \operatorname{Log}[-\operatorname{Cosh}[\frac{1}{2}(c+d x)]-\operatorname{Sinh}[\frac{1}{2}(c+d x)]+ \\
 \operatorname{Cosh}[\frac{1}{2}(c+d x)] \#1-\operatorname{Sinh}[\frac{1}{2}(c+d x)] \#1] \#1^4+5 a c \#1^6-4 b c \#1^6+ \\
 5 a d x \#1^6-4 b d x \#1^6+10 a \operatorname{Log}[-\operatorname{Cosh}[\frac{1}{2}(c+d x)]-\operatorname{Sinh}[\frac{1}{2}(c+d x)]+ \\
 \operatorname{Cosh}[\frac{1}{2}(c+d x)] \#1-\operatorname{Sinh}[\frac{1}{2}(c+d x)] \#1] \#1^6-8 b \operatorname{Log}[-\operatorname{Cosh}[\frac{1}{2}(c+d x)]- \\
 \operatorname{Sinh}[\frac{1}{2}(c+d x)]+\operatorname{Cosh}[\frac{1}{2}(c+d x)] \#1-\operatorname{Sinh}[\frac{1}{2}(c+d x)] \#1] \#1^6) \& \left. \right)$$

Problem 253: Result is not expressed in closed-form.

$$\int \frac{\operatorname{Sinh}[c+d x]^9}{(a-b \operatorname{Sinh}[c+d x]^4)^3} dx$$

Optimal (type 3, 315 leaves, 6 steps):

$$\frac{(5a - 14\sqrt{a}\sqrt{b} + 12b) \operatorname{ArcTan}\left[\frac{b^{1/4} \operatorname{Cosh}[c+dx]}{\sqrt{\sqrt{a}-\sqrt{b}}}\right]}{64\sqrt{a}(\sqrt{a}-\sqrt{b})^{5/2}b^{9/4}d} + \frac{(5a + 14\sqrt{a}\sqrt{b} + 12b) \operatorname{ArcTanh}\left[\frac{b^{1/4} \operatorname{Cosh}[c+dx]}{\sqrt{\sqrt{a}+\sqrt{b}}}\right]}{64\sqrt{a}(\sqrt{a}+\sqrt{b})^{5/2}b^{9/4}d} +$$

$$\frac{a \operatorname{Cosh}[c+dx](a+b-b \operatorname{Cosh}[c+dx]^2)}{8(a-b)b^2d(a-b+2b \operatorname{Cosh}[c+dx]^2-b \operatorname{Cosh}[c+dx]^4)^2} -$$

$$\frac{\operatorname{Cosh}[c+dx](9a^2-11ab-10b^2-2(2a-5b)b \operatorname{Cosh}[c+dx]^2)}{32(a-b)^2b^2d(a-b+2b \operatorname{Cosh}[c+dx]^2-b \operatorname{Cosh}[c+dx]^4)}$$

Result (type 7, 1021 leaves):

$$\begin{aligned}
& \frac{1}{128 (a-b)^2 b^2 d} \left((32 \operatorname{Cosh}[c+dx] (-9a^2 + 13ab + 5b^2 + (2a-5b)b \operatorname{Cosh}[2(c+dx)])) / \right. \\
& \quad (8a-3b+4b \operatorname{Cosh}[2(c+dx)] - b \operatorname{Cosh}[4(c+dx)]) + \\
& \quad \left. \frac{512a(a-b) \operatorname{Cosh}[c+dx] (2a+b-b \operatorname{Cosh}[2(c+dx)])}{(-8a+3b-4b \operatorname{Cosh}[2(c+dx)] + b \operatorname{Cosh}[4(c+dx)])^2} \right. \\
& \quad \left. \operatorname{RootSum}[b-4b \#1^2 - 16a \#1^4 + 6b \#1^4 - 4b \#1^6 + b \#1^8 \&, \right. \\
& \quad \left. \frac{1}{-b \#1 - 8a \#1^3 + 3b \#1^3 - 3b \#1^5 + b \#1^7} \left(-2abc + 5b^2c - 2abd x + 5b^2dx - 4ab \operatorname{Log} \left[\right. \right. \right. \\
& \quad \left. \left. \left. -\operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right] \#1 - \operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right] \#1 \right] + \right. \right. \\
& \quad \left. \left. 10b^2 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right] \#1 - \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right] \#1\right] - 10a^2c \#1^2 + 28abc \#1^2 - 39b^2c \#1^2 - 10a^2dx \#1^2 + \right. \right. \\
& \quad \left. \left. 28abd x \#1^2 - 39b^2dx \#1^2 - 20a^2 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right] + \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right] \#1 - \operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right] \#1\right] \#1^2 + 56ab \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right] - \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right] \#1 - \operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right] \#1\right] \#1^2 - \right. \right. \\
& \quad \left. \left. 78b^2 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right] \#1 - \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right] \#1\right] \#1^2 + 10a^2c \#1^4 - 28abc \#1^4 + 39b^2c \#1^4 + \right. \right. \\
& \quad \left. \left. 10a^2dx \#1^4 - 28abd x \#1^4 + 39b^2dx \#1^4 + 20a^2 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right] - \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right] \#1 - \operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right] \#1\right] \#1^4 - \right. \right. \\
& \quad \left. \left. 56ab \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right] \#1 - \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right] \#1\right] \#1^4 + 78b^2 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right] + \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right] \#1 - \operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right] \#1\right] \#1^4 + 2abc \#1^6 - 5b^2c \#1^6 + \right. \right. \\
& \quad \left. \left. 2abd x \#1^6 - 5b^2dx \#1^6 + 4ab \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right] + \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right] \#1 - \operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right] \#1\right] \#1^6 - 10b^2 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right] - \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right] \#1 - \operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right] \#1\right] \#1^6 \right) \& \left. \right) \left. \right)
\end{aligned}$$

Problem 254: Result is not expressed in closed-form.

$$\int \frac{\text{Sinh}[c + d x]^7}{(a - b \text{Sinh}[c + d x]^4)^3} dx$$

Optimal (type 3, 290 leaves, 6 steps):

$$\frac{3 \left(\sqrt{a} - 2 \sqrt{b} \right) \text{ArcTan} \left[\frac{b^{1/4} \text{Cosh}[c + d x]}{\sqrt{\sqrt{a} - \sqrt{b}}} \right]}{64 \sqrt{a} \left(\sqrt{a} - \sqrt{b} \right)^{5/2} b^{7/4} d} - \frac{3 \left(\sqrt{a} + 2 \sqrt{b} \right) \text{ArcTanh} \left[\frac{b^{1/4} \text{Cosh}[c + d x]}{\sqrt{\sqrt{a} + \sqrt{b}}} \right]}{64 \sqrt{a} \left(\sqrt{a} + \sqrt{b} \right)^{5/2} b^{7/4} d} +$$

$$\frac{a \text{Cosh}[c + d x] \left(2 - \text{Cosh}[c + d x]^2 \right)}{8 (a - b) b d \left(a - b + 2 b \text{Cosh}[c + d x]^2 - b \text{Cosh}[c + d x]^4 \right)^2} +$$

$$\frac{\text{Cosh}[c + d x] \left(5 a - 17 b - 3 (a - 3 b) \text{Cosh}[c + d x]^2 \right)}{32 (a - b)^2 b d \left(a - b + 2 b \text{Cosh}[c + d x]^2 - b \text{Cosh}[c + d x]^4 \right)}$$

Result (type 7, 802 leaves):

$$\frac{1}{256 (a - b)^2 b d} \left(-\frac{32 \operatorname{Cosh}[c + d x] (-7 a + 25 b + 3 (a - 3 b) \operatorname{Cosh}[2 (c + d x)])}{8 a - 3 b + 4 b \operatorname{Cosh}[2 (c + d x)] - b \operatorname{Cosh}[4 (c + d x)]} + \frac{512 a (a - b) (-5 \operatorname{Cosh}[c + d x] + \operatorname{Cosh}[3 (c + d x)])}{(-8 a + 3 b - 4 b \operatorname{Cosh}[2 (c + d x)] + b \operatorname{Cosh}[4 (c + d x)])^2} - \right. \\ \left. 3 \operatorname{RootSum}\left[b - 4 b \#1^2 - 16 a \#1^4 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \frac{1}{-b \#1 - 8 a \#1^3 + 3 b \#1^3 - 3 b \#1^5 + b \#1^7} \left(a c - 3 b c + a d x - 3 b d x + 2 a \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2} (c + d x)\right] - \operatorname{Sinh}\left[\frac{1}{2} (c + d x)\right] + \operatorname{Cosh}\left[\frac{1}{2} (c + d x)\right] \#1 - \operatorname{Sinh}\left[\frac{1}{2} (c + d x)\right] \#1\right] - \right. \right. \right. \\ \left. \left. 6 b \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2} (c + d x)\right] - \operatorname{Sinh}\left[\frac{1}{2} (c + d x)\right] + \operatorname{Cosh}\left[\frac{1}{2} (c + d x)\right] \#1 - \operatorname{Sinh}\left[\frac{1}{2} (c + d x)\right] \#1\right] - 3 a c \#1^2 + 17 b c \#1^2 - 3 a d x \#1^2 + 17 b d x \#1^2 - \right. \right. \\ \left. \left. 6 a \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2} (c + d x)\right] - \operatorname{Sinh}\left[\frac{1}{2} (c + d x)\right] + \operatorname{Cosh}\left[\frac{1}{2} (c + d x)\right] \#1 - \operatorname{Sinh}\left[\frac{1}{2} (c + d x)\right] \#1\right] \#1^2 + 34 b \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2} (c + d x)\right] - \operatorname{Sinh}\left[\frac{1}{2} (c + d x)\right] + \right. \right. \\ \left. \left. \operatorname{Cosh}\left[\frac{1}{2} (c + d x)\right] \#1 - \operatorname{Sinh}\left[\frac{1}{2} (c + d x)\right] \#1\right] \#1^2 + 3 a c \#1^4 - 17 b c \#1^4 + 3 a d x \#1^4 - 17 b d x \#1^4 + 6 a \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2} (c + d x)\right] - \operatorname{Sinh}\left[\frac{1}{2} (c + d x)\right] + \right. \right. \\ \left. \left. \operatorname{Cosh}\left[\frac{1}{2} (c + d x)\right] \#1 - \operatorname{Sinh}\left[\frac{1}{2} (c + d x)\right] \#1\right] \#1^4 - 34 b \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2} (c + d x)\right] - \operatorname{Sinh}\left[\frac{1}{2} (c + d x)\right] + \operatorname{Cosh}\left[\frac{1}{2} (c + d x)\right] \#1 - \operatorname{Sinh}\left[\frac{1}{2} (c + d x)\right] \#1\right] \#1^4 - a c \#1^6 + \right. \\ \left. \left. 3 b c \#1^6 - a d x \#1^6 + 3 b d x \#1^6 - 2 a \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2} (c + d x)\right] - \operatorname{Sinh}\left[\frac{1}{2} (c + d x)\right] + \operatorname{Cosh}\left[\frac{1}{2} (c + d x)\right] \#1 - \operatorname{Sinh}\left[\frac{1}{2} (c + d x)\right] \#1\right] \#1^6 + 6 b \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2} (c + d x)\right] - \right. \right. \\ \left. \left. \operatorname{Sinh}\left[\frac{1}{2} (c + d x)\right] + \operatorname{Cosh}\left[\frac{1}{2} (c + d x)\right] \#1 - \operatorname{Sinh}\left[\frac{1}{2} (c + d x)\right] \#1\right] \#1^6 \right) \& \left. \right)$$

Problem 255: Result is not expressed in closed-form.

$$\int \frac{\operatorname{Sinh}[c + d x]^5}{(a - b \operatorname{Sinh}[c + d x]^4)^3} dx$$

Optimal (type 3, 313 leaves, 6 steps):

$$\begin{aligned}
 & - \frac{(3a - 10\sqrt{a}\sqrt{b} + 4b) \operatorname{ArcTan}\left[\frac{b^{1/4} \operatorname{Cosh}[c+dx]}{\sqrt{\sqrt{a}-\sqrt{b}}}\right]}{64 a^{3/2} (\sqrt{a} - \sqrt{b})^{5/2} b^{5/4} d} - \frac{(3a + 10\sqrt{a}\sqrt{b} + 4b) \operatorname{ArcTanh}\left[\frac{b^{1/4} \operatorname{Cosh}[c+dx]}{\sqrt{\sqrt{a}+\sqrt{b}}}\right]}{64 a^{3/2} (\sqrt{a} + \sqrt{b})^{5/2} b^{5/4} d} + \\
 & \frac{\operatorname{Cosh}[c+dx] (a+b-b \operatorname{Cosh}[c+dx]^2)}{8(a-b)bd(a-b+2b \operatorname{Cosh}[c+dx]^2-b \operatorname{Cosh}[c+dx]^4)^2} - \\
 & \frac{\operatorname{Cosh}[c+dx] (a^2-11ab-2b^2+2b(2a+b) \operatorname{Cosh}[c+dx]^2)}{32a(a-b)^2bd(a-b+2b \operatorname{Cosh}[c+dx]^2-b \operatorname{Cosh}[c+dx]^4)}
 \end{aligned}$$

Result (type 7, 1019 leaves):

$$\begin{aligned}
& - \frac{1}{128 (a-b)^2 b d} \left(\frac{32 \operatorname{Cosh}[c+d x] (a^2 - 9 a b - b^2 + b (2 a + b) \operatorname{Cosh}[2 (c+d x)])}{a (8 a - 3 b + 4 b \operatorname{Cosh}[2 (c+d x)] - b \operatorname{Cosh}[4 (c+d x)])} - \right. \\
& \quad \frac{512 (a-b) \operatorname{Cosh}[c+d x] (2 a + b - b \operatorname{Cosh}[2 (c+d x)])}{(-8 a + 3 b - 4 b \operatorname{Cosh}[2 (c+d x)] + b \operatorname{Cosh}[4 (c+d x)])^2} + \\
& \quad \frac{1}{a} \operatorname{RootSum}[b - 4 b \#1^2 - 16 a \#1^4 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \\
& \quad \frac{1}{-b \#1 - 8 a \#1^3 + 3 b \#1^5 - 3 b \#1^5 + b \#1^7} \left(2 a b c + b^2 c + 2 a b d x + b^2 d x + \right. \\
& \quad 4 a b \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \#1 - \right. \\
& \quad \quad \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1 + 2 b^2 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] + \right. \\
& \quad \quad \operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \#1 - \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1 + 6 a^2 c \#1^2 - 32 a b c \#1^2 + 5 b^2 c \#1^2 + \\
& \quad 6 a^2 d x \#1^2 - 32 a b d x \#1^2 + 5 b^2 d x \#1^2 + 12 a^2 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] - \right. \\
& \quad \quad \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \#1 - \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1 \#1^2 - \\
& \quad 64 a b \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \#1 - \right. \\
& \quad \quad \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1 \#1^2 + 10 b^2 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] + \right. \\
& \quad \quad \operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \#1 - \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1 \#1^2 - 6 a^2 c \#1^4 + 32 a b c \#1^4 - \\
& \quad 5 b^2 c \#1^4 - 6 a^2 d x \#1^4 + 32 a b d x \#1^4 - 5 b^2 d x \#1^4 - 12 a^2 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] - \right. \\
& \quad \quad \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \#1 - \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1 \#1^4 + \\
& \quad 64 a b \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \#1 - \right. \\
& \quad \quad \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1 \#1^4 - 10 b^2 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] + \right. \\
& \quad \quad \operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \#1 - \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1 \#1^4 - 2 a b c \#1^6 - b^2 c \#1^6 - \\
& \quad 2 a b d x \#1^6 - b^2 d x \#1^6 - 4 a b \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] + \right. \\
& \quad \quad \operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \#1 - \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1 \#1^6 - 2 b^2 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] - \right. \\
& \quad \quad \left. \left. \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \#1 - \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1 \#1^6\right] \& \right) \left. \right)
\end{aligned}$$

Problem 256: Result is not expressed in closed-form.

$$\int \frac{\text{Sinh}[c + d x]^3}{(a - b \text{Sinh}[c + d x]^4)^3} dx$$

Optimal (type 3, 288 leaves, 6 steps):

$$\begin{aligned} & - \frac{(5\sqrt{a} - 2\sqrt{b}) \text{ArcTan}\left[\frac{b^{1/4} \text{Cosh}[c+dx]}{\sqrt{\sqrt{a}-\sqrt{b}}}\right]}{64 a^{3/2} (\sqrt{a} - \sqrt{b})^{5/2} b^{3/4} d} + \frac{(5\sqrt{a} + 2\sqrt{b}) \text{ArcTanh}\left[\frac{b^{1/4} \text{Cosh}[c+dx]}{\sqrt{\sqrt{a}+\sqrt{b}}}\right]}{64 a^{3/2} (\sqrt{a} + \sqrt{b})^{5/2} b^{3/4} d} \\ & - \frac{\text{Cosh}[c + d x] (2 - \text{Cosh}[c + d x]^2)}{8 (a - b) d (a - b + 2 b \text{Cosh}[c + d x]^2 - b \text{Cosh}[c + d x]^4)^2} \\ & - \frac{\text{Cosh}[c + d x] (11 a + b - (5 a + b) \text{Cosh}[c + d x]^2)}{32 a (a - b)^2 d (a - b + 2 b \text{Cosh}[c + d x]^2 - b \text{Cosh}[c + d x]^4)} \end{aligned}$$

Result (type 7, 802 leaves):

$$\frac{1}{256 (a-b)^2 d} \left(\frac{32 \operatorname{Cosh}[c+dx] (-17a-b+(5a+b) \operatorname{Cosh}[2(c+dx)])}{a(8a-3b+4b \operatorname{Cosh}[2(c+dx)]-b \operatorname{Cosh}[4(c+dx)])} + \frac{512(a-b)(-5 \operatorname{Cosh}[c+dx]+\operatorname{Cosh}[3(c+dx)])}{(-8a+3b-4b \operatorname{Cosh}[2(c+dx)]+b \operatorname{Cosh}[4(c+dx)])^2} + \frac{1}{a} \operatorname{RootSum}\left[b-4b \#1^2-16a \#1^4+6b \#1^4-4b \#1^6+b \#1^8 \&, \frac{1}{-b \#1-8a \#1^3+3b \#1^3-3b \#1^5+b \#1^7} \left(5ac+bc+5adx+bdx+10a \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right]-\operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right]+\operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right] \#1-\operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right] \#1\right] + 2b \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right]-\operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right]+\operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right] \#1-\operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right] \#1\right]-47ac \#1^2+5bc \#1^2-47adx \#1^2+5bdx \#1^2-94a \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right]-\operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right]+\operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right] \#1-\operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right] \#1\right] \#1^2+10b \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right]-\operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right]+\operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right] \#1-\operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right] \#1\right] \#1^2+47ac \#1^4-5bc \#1^4+47adx \#1^4-5bdx \#1^4+94a \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right]-\operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right]+\operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right] \#1-\operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right] \#1\right] \#1^4-10b \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right]-\operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right]+\operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right] \#1-\operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right] \#1\right] \#1^4-5ac \#1^6-bc \#1^6-5adx \#1^6-bdx \#1^6-10a \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right]-\operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right]+\operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right] \#1-\operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right] \#1\right] \#1^6-2b \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right]-\operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right]+\operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right] \#1-\operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right] \#1\right] \#1^6 \& \right) \right)$$

Problem 257: Result is not expressed in closed-form.

$$\int \frac{\operatorname{Sinh}[c+dx]}{(a-b \operatorname{Sinh}[c+dx]^4)^3} dx$$

Optimal (type 3, 313 leaves, 6 steps):

$$\frac{3 \left(7 a - 10 \sqrt{a} \sqrt{b} + 4 b \right) \text{ArcTan} \left[\frac{b^{1/4} \text{Cosh}[c+dx]}{\sqrt{\sqrt{a}-\sqrt{b}}} \right]}{64 a^{5/2} \left(\sqrt{a} - \sqrt{b} \right)^{5/2} b^{1/4} d} + \frac{3 \left(7 a + 10 \sqrt{a} \sqrt{b} + 4 b \right) \text{ArcTanh} \left[\frac{b^{1/4} \text{Cosh}[c+dx]}{\sqrt{\sqrt{a}+\sqrt{b}}} \right]}{64 a^{5/2} \left(\sqrt{a} + \sqrt{b} \right)^{5/2} b^{1/4} d} +$$

$$\frac{\text{Cosh}[c+dx] \left(a + b - b \text{Cosh}[c+dx]^2 \right)}{8 a (a-b) d \left(a - b + 2 b \text{Cosh}[c+dx]^2 - b \text{Cosh}[c+dx]^4 \right)^2} +$$

$$\frac{\text{Cosh}[c+dx] \left((7 a - 3 b) (a + 2 b) - 6 (2 a - b) b \text{Cosh}[c+dx]^2 \right)}{32 a^2 (a-b)^2 d \left(a - b + 2 b \text{Cosh}[c+dx]^2 - b \text{Cosh}[c+dx]^4 \right)}$$

Result (type 7, 1018 leaves):

$$\frac{1}{128 a^2 (a-b)^2 d} \left(\frac{32 \operatorname{Cosh}[c+d x] (7 a^2+5 a b-3 b^2+3 b(-2 a+b) \operatorname{Cosh}[2(c+d x)])}{8 a-3 b+4 b \operatorname{Cosh}[2(c+d x)]-b \operatorname{Cosh}[4(c+d x)]} + \right. \\ \frac{512 a(a-b) \operatorname{Cosh}[c+d x] (2 a+b-b \operatorname{Cosh}[2(c+d x)])}{(-8 a+3 b-4 b \operatorname{Cosh}[2(c+d x)]+b \operatorname{Cosh}[4(c+d x)])^2} + \\ 3 \operatorname{RootSum}\left[b-4 b \#1^2-16 a \#1^4+6 b \#1^4-4 b \#1^6+b \#1^8 \&, \right. \\ \frac{1}{-b \#1-8 a \#1^3+3 b \#1^3-3 b \#1^5+b \#1^7} \left(-2 a b c+b^2 c-2 a b d x+b^2 d x-4 a b \operatorname{Log}\left[\right. \right. \\ \left. \left. -\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \#1-\operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1\right] + \right. \\ \left. 2 b^2 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \#1-\right. \right. \\ \left. \left. \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1\right]+14 a^2 c \#1^2-12 a b c \#1^2+5 b^2 c \#1^2+14 a^2 d x \#1^2- \right. \\ \left. 12 a b d x \#1^2+5 b^2 d x \#1^2+28 a^2 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right]\right]+ \right. \\ \left. \operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \#1-\operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1\right] \#1^2-24 a b \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right]- \right. \\ \left. \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \#1-\operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1\right] \#1^2+ \\ \left. 10 b^2 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \#1- \right. \right. \\ \left. \left. \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1\right] \#1^2-14 a^2 c \#1^4+12 a b c \#1^4-5 b^2 c \#1^4- \right. \\ \left. 14 a^2 d x \#1^4+12 a b d x \#1^4-5 b^2 d x \#1^4-28 a^2 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right]- \right. \right. \\ \left. \left. \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \#1-\operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1\right] \#1^4+ \right. \\ \left. 24 a b \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \#1- \right. \right. \\ \left. \left. \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1\right] \#1^4-10 b^2 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right]- \right. \right. \\ \left. \left. \operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \#1-\operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1\right] \#1^4+2 a b c \#1^6-b^2 c \#1^6+ \right. \\ \left. 2 a b d x \#1^6-b^2 d x \#1^6+4 a b \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right]+ \right. \right. \\ \left. \left. \operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \#1-\operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1\right] \#1^6-2 b^2 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right]- \right. \right. \\ \left. \left. \operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right] \#1-\operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right] \#1\right] \#1^6 \right. \left. \right) \left. \right) \right]$$

Problem 258: Result is not expressed in closed-form.

$$\int \frac{\operatorname{Csch}[c+d x]}{(a-b \operatorname{Sinh}[c+d x]^4)^3} dx$$

Optimal (type 3, 617 leaves, 16 steps):

$$\begin{aligned}
 & - \frac{(5\sqrt{a} - 2\sqrt{b}) b^{1/4} \operatorname{ArcTan}\left[\frac{b^{1/4} \operatorname{Cosh}[c+dx]}{\sqrt{\sqrt{a}-\sqrt{b}}}\right]}{64 a^{5/2} (\sqrt{a} - \sqrt{b})^{5/2} d} - \frac{b^{1/4} \operatorname{ArcTan}\left[\frac{b^{1/4} \operatorname{Cosh}[c+dx]}{\sqrt{\sqrt{a}-\sqrt{b}}}\right]}{8 a^{5/2} (\sqrt{a} - \sqrt{b})^{3/2} d} \\
 & - \frac{b^{1/4} \operatorname{ArcTan}\left[\frac{b^{1/4} \operatorname{Cosh}[c+dx]}{\sqrt{\sqrt{a}-\sqrt{b}}}\right]}{2 a^3 \sqrt{\sqrt{a} - \sqrt{b}} d} - \frac{\operatorname{ArcTanh}[\operatorname{Cosh}[c+dx]]}{a^3 d} + \frac{b^{1/4} \operatorname{ArcTanh}\left[\frac{b^{1/4} \operatorname{Cosh}[c+dx]}{\sqrt{\sqrt{a}+\sqrt{b}}}\right]}{8 a^{5/2} (\sqrt{a} + \sqrt{b})^{3/2} d} + \\
 & \frac{b^{1/4} \operatorname{ArcTanh}\left[\frac{b^{1/4} \operatorname{Cosh}[c+dx]}{\sqrt{\sqrt{a}+\sqrt{b}}}\right]}{2 a^3 \sqrt{\sqrt{a} + \sqrt{b}} d} + \frac{(5\sqrt{a} + 2\sqrt{b}) b^{1/4} \operatorname{ArcTanh}\left[\frac{b^{1/4} \operatorname{Cosh}[c+dx]}{\sqrt{\sqrt{a}+\sqrt{b}}}\right]}{64 a^{5/2} (\sqrt{a} + \sqrt{b})^{5/2} d} - \\
 & \frac{b \operatorname{Cosh}[c+dx] (2 - \operatorname{Cosh}[c+dx]^2)}{8 a (a-b) d (a-b+2b \operatorname{Cosh}[c+dx]^2 - b \operatorname{Cosh}[c+dx]^4)^2} - \\
 & \frac{b \operatorname{Cosh}[c+dx] (2 - \operatorname{Cosh}[c+dx]^2)}{4 a^2 (a-b) d (a-b+2b \operatorname{Cosh}[c+dx]^2 - b \operatorname{Cosh}[c+dx]^4)} - \\
 & \frac{b \operatorname{Cosh}[c+dx] (11a+b - (5a+b) \operatorname{Cosh}[c+dx]^2)}{32 a^2 (a-b)^2 d (a-b+2b \operatorname{Cosh}[c+dx]^2 - b \operatorname{Cosh}[c+dx]^4)}
 \end{aligned}$$

Result (type 7, 1274 leaves):

$$\begin{aligned}
 & \frac{2 \left(-5 b \operatorname{Cosh}[c + d x] + b \operatorname{Cosh}[3 (c + d x)] \right)}{a (a - b) d \left(-8 a + 3 b - 4 b \operatorname{Cosh}[2 (c + d x)] + b \operatorname{Cosh}[4 (c + d x)] \right)^2} + \\
 & \frac{\left(69 a b \operatorname{Cosh}[c + d x] - 39 b^2 \operatorname{Cosh}[c + d x] - 13 a b \operatorname{Cosh}[3 (c + d x)] + 7 b^2 \operatorname{Cosh}[3 (c + d x)] \right) /}{\left(16 a^2 (a - b)^2 d \left(-8 a + 3 b - 4 b \operatorname{Cosh}[2 (c + d x)] + b \operatorname{Cosh}[4 (c + d x)] \right) \right) -} \\
 & \frac{\operatorname{Log}\left[\operatorname{Cosh}\left[\frac{1}{2} (c + d x)\right]\right]}{a^3 d} + \frac{\operatorname{Log}\left[\operatorname{Sinh}\left[\frac{1}{2} (c + d x)\right]\right]}{a^3 d} + \\
 & \frac{1}{256 a^3 (a - b)^2 d} \operatorname{RootSum}\left[b - 4 b \#1^2 - 16 a \#1^4 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \right. \\
 & \frac{1}{-b \#1 - 8 a \#1^3 + 3 b \#1^3 - 3 b \#1^5 + b \#1^7} \left(45 a^2 b c - 71 a b^2 c + 32 b^3 c + 45 a^2 b d x - 71 a b^2 d x + \right. \\
 & 32 b^3 d x + 90 a^2 b \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2} (c + d x)\right] - \operatorname{Sinh}\left[\frac{1}{2} (c + d x)\right] + \operatorname{Cosh}\left[\frac{1}{2} (c + d x)\right] \#1 - \right. \\
 & \operatorname{Sinh}\left[\frac{1}{2} (c + d x)\right] \#1 - 142 a b^2 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2} (c + d x)\right] - \operatorname{Sinh}\left[\frac{1}{2} (c + d x)\right] + \right. \\
 & \operatorname{Cosh}\left[\frac{1}{2} (c + d x)\right] \#1 - \operatorname{Sinh}\left[\frac{1}{2} (c + d x)\right] \#1 + 64 b^3 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2} (c + d x)\right] - \right. \\
 & \operatorname{Sinh}\left[\frac{1}{2} (c + d x)\right] + \operatorname{Cosh}\left[\frac{1}{2} (c + d x)\right] \#1 - \operatorname{Sinh}\left[\frac{1}{2} (c + d x)\right] \#1 - 199 a^2 b c \#1^2 + \\
 & 253 a b^2 c \#1^2 - 96 b^3 c \#1^2 - 199 a^2 b d x \#1^2 + 253 a b^2 d x \#1^2 - 96 b^3 d x \#1^2 - 398 a^2 b \\
 & \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2} (c + d x)\right] - \operatorname{Sinh}\left[\frac{1}{2} (c + d x)\right] + \operatorname{Cosh}\left[\frac{1}{2} (c + d x)\right] \#1 - \operatorname{Sinh}\left[\frac{1}{2} (c + d x)\right] \#1 \\
 & \#1^2 + 506 a b^2 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2} (c + d x)\right] - \operatorname{Sinh}\left[\frac{1}{2} (c + d x)\right] + \operatorname{Cosh}\left[\frac{1}{2} (c + d x)\right] \#1 - \right. \\
 & \operatorname{Sinh}\left[\frac{1}{2} (c + d x)\right] \#1 \#1^2 - 192 b^3 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2} (c + d x)\right] - \operatorname{Sinh}\left[\frac{1}{2} (c + d x)\right] + \right. \\
 & \operatorname{Cosh}\left[\frac{1}{2} (c + d x)\right] \#1 - \operatorname{Sinh}\left[\frac{1}{2} (c + d x)\right] \#1 \#1^2 + 199 a^2 b c \#1^4 - \\
 & 253 a b^2 c \#1^4 + 96 b^3 c \#1^4 + 199 a^2 b d x \#1^4 - 253 a b^2 d x \#1^4 + 96 b^3 d x \#1^4 + 398 a^2 b \\
 & \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2} (c + d x)\right] - \operatorname{Sinh}\left[\frac{1}{2} (c + d x)\right] + \operatorname{Cosh}\left[\frac{1}{2} (c + d x)\right] \#1 - \operatorname{Sinh}\left[\frac{1}{2} (c + d x)\right] \#1 \\
 & \#1^4 - 506 a b^2 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2} (c + d x)\right] - \operatorname{Sinh}\left[\frac{1}{2} (c + d x)\right] + \operatorname{Cosh}\left[\frac{1}{2} (c + d x)\right] \#1 - \right. \\
 & \operatorname{Sinh}\left[\frac{1}{2} (c + d x)\right] \#1 \#1^4 + 192 b^3 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2} (c + d x)\right] - \operatorname{Sinh}\left[\frac{1}{2} (c + d x)\right] + \right. \\
 & \operatorname{Cosh}\left[\frac{1}{2} (c + d x)\right] \#1 - \operatorname{Sinh}\left[\frac{1}{2} (c + d x)\right] \#1 \#1^4 - 45 a^2 b c \#1^6 + \\
 & 71 a b^2 c \#1^6 - 32 b^3 c \#1^6 - 45 a^2 b d x \#1^6 + 71 a b^2 d x \#1^6 - 32 b^3 d x \#1^6 - 90 a^2 b \\
 & \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2} (c + d x)\right] - \operatorname{Sinh}\left[\frac{1}{2} (c + d x)\right] + \operatorname{Cosh}\left[\frac{1}{2} (c + d x)\right] \#1 - \operatorname{Sinh}\left[\frac{1}{2} (c + d x)\right] \#1 \\
 & \#1^6 + 142 a b^2 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2} (c + d x)\right] - \operatorname{Sinh}\left[\frac{1}{2} (c + d x)\right] + \operatorname{Cosh}\left[\frac{1}{2} (c + d x)\right] \#1 - \right. \\
 & \operatorname{Sinh}\left[\frac{1}{2} (c + d x)\right] \#1 \#1^6 - 64 b^3 \operatorname{Log}\left[-\operatorname{Cosh}\left[\frac{1}{2} (c + d x)\right] - \right. \\
 & \left. \left. \operatorname{Sinh}\left[\frac{1}{2} (c + d x)\right] + \operatorname{Cosh}\left[\frac{1}{2} (c + d x)\right] \#1 - \operatorname{Sinh}\left[\frac{1}{2} (c + d x)\right] \#1 \#1^6 \right) \& \right]
 \end{aligned}$$

Problem 266: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{1 + \text{Sinh}[x]^4} dx$$

Optimal (type 3, 176 leaves, 10 steps):

$$\begin{aligned} & -\frac{\text{ArcTan}\left[\frac{\sqrt{1+\sqrt{2}}-2\text{Tanh}[x]}{\sqrt{-1+\sqrt{2}}}\right]}{4\sqrt{1+\sqrt{2}}} + \frac{\text{ArcTan}\left[\frac{\sqrt{1+\sqrt{2}}+2\text{Tanh}[x]}{\sqrt{-1+\sqrt{2}}}\right]}{4\sqrt{1+\sqrt{2}}} - \\ & \frac{1}{8}\sqrt{1+\sqrt{2}}\text{Log}\left[\sqrt{2}-2\sqrt{1+\sqrt{2}}\text{Tanh}[x]+2\text{Tanh}[x]^2\right] + \\ & \frac{1}{8}\sqrt{1+\sqrt{2}}\text{Log}\left[1+\sqrt{2}\left(1+\sqrt{2}\right)\text{Tanh}[x]+\sqrt{2}\text{Tanh}[x]^2\right] \end{aligned}$$

Result (type 3, 45 leaves):

$$\frac{\text{ArcTanh}\left[\sqrt{1-i}\text{Tanh}[x]\right]}{2\sqrt{1-i}} + \frac{\text{ArcTanh}\left[\sqrt{1+i}\text{Tanh}[x]\right]}{2\sqrt{1+i}}$$

Problem 267: Result is not expressed in closed-form.

$$\int \frac{1}{a + b \text{Sinh}[x]^5} dx$$

Optimal (type 3, 435 leaves, 17 steps):

$$\begin{aligned} & -\frac{2\text{ArcTanh}\left[\frac{b^{1/5}-a^{1/5}\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{a^{2/5}+b^{2/5}}}\right]}{5a^{4/5}\sqrt{a^{2/5}+b^{2/5}}} + \frac{2(-1)^{9/10}\text{ArcTanh}\left[\frac{(-1)^{9/10}\left((-1)^{1/5}b^{1/5}+a^{1/5}\text{Tanh}\left[\frac{x}{2}\right]\right)}{\sqrt{-(-1)^{4/5}a^{2/5}+(-1)^{1/5}b^{2/5}}}\right]}{5a^{4/5}\sqrt{-(-1)^{4/5}a^{2/5}+(-1)^{1/5}b^{2/5}}} + \\ & \frac{2(-1)^{1/5}\text{ArcTanh}\left[\frac{b^{1/5}+(-1)^{1/5}a^{1/5}\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{(-1)^{2/5}a^{2/5}+b^{2/5}}}\right]}{5a^{4/5}\sqrt{(-1)^{2/5}a^{2/5}+b^{2/5}}} + \frac{2(-1)^{9/10}\text{ArcTanh}\left[\frac{(-1)^{3/10}\left(b^{1/5}+(-1)^{3/5}a^{1/5}\text{Tanh}\left[\frac{x}{2}\right]\right)}{\sqrt{-(-1)^{4/5}a^{2/5}+(-1)^{3/5}b^{2/5}}}\right]}{5a^{4/5}\sqrt{-(-1)^{4/5}a^{2/5}+(-1)^{3/5}b^{2/5}}} - \\ & \frac{2(-1)^{9/10}\text{ArcTanh}\left[\frac{i b^{1/5}-(-1)^{9/10}a^{1/5}\text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{-(-1)^{4/5}a^{2/5}-b^{2/5}}}\right]}{5a^{4/5}\sqrt{-(-1)^{4/5}a^{2/5}-b^{2/5}}} \end{aligned}$$

Result (type 7, 141 leaves):

$$\begin{aligned} & \frac{8}{5}\text{RootSum}\left[-b+5b\#1^2-10b\#1^4+32a\#1^5+10b\#1^6-5b\#1^8+b\#1^{10}\&, \right. \\ & \left. \frac{x\#1^3+2\text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right]-\text{Sinh}\left[\frac{x}{2}\right]+\text{Cosh}\left[\frac{x}{2}\right]\#1-\text{Sinh}\left[\frac{x}{2}\right]\#1\right]\#1^3}{b-4b\#1^2+16a\#1^3+6b\#1^4-4b\#1^6+b\#1^8}\&] \end{aligned}$$

Problem 268: Result is not expressed in closed-form.

$$\int \frac{1}{a + b \operatorname{Sinh}[x]^6} dx$$

Optimal (type 3, 175 leaves, 7 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a^{1/3}-b^{1/3}} \operatorname{Tanh}[x]}{a^{1/6}}\right]}{3 a^{5/6} \sqrt{a^{1/3}-b^{1/3}}} + \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a^{1/3}+(-1)^{1/3} b^{1/3}} \operatorname{Tanh}[x]}{a^{1/6}}\right]}{3 a^{5/6} \sqrt{a^{1/3}+(-1)^{1/3} b^{1/3}}} + \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a^{1/3}-(-1)^{2/3} b^{1/3}} \operatorname{Tanh}[x]}{a^{1/6}}\right]}{3 a^{5/6} \sqrt{a^{1/3}-(-1)^{2/3} b^{1/3}}}$$

Result (type 7, 134 leaves):

$$\frac{16}{3} \operatorname{RootSum}\left[b - 6 b \#1 + 15 b \#1^2 + 64 a \#1^3 - 20 b \#1^3 + 15 b \#1^4 - 6 b \#1^5 + b \#1^6 \&, \right. \\ \left. \frac{x \#1^2 + \operatorname{Log}\left[-\operatorname{Cosh}[x] - \operatorname{Sinh}[x] + \operatorname{Cosh}[x] \#1 - \operatorname{Sinh}[x] \#1\right] \#1^2}{-b + 5 b \#1 + 32 a \#1^2 - 10 b \#1^2 + 10 b \#1^3 - 5 b \#1^4 + b \#1^5} \&\right]$$

Problem 269: Result is not expressed in closed-form.

$$\int \frac{1}{a + b \operatorname{Sinh}[x]^8} dx$$

Optimal (type 3, 245 leaves, 9 steps):

$$-\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{(-a)^{1/4}-b^{1/4}} \operatorname{Tanh}[x]}{(-a)^{1/8}}\right]}{4 (-a)^{7/8} \sqrt{(-a)^{1/4}-b^{1/4}}} - \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{(-a)^{1/4}-i b^{1/4}} \operatorname{Tanh}[x]}{(-a)^{1/8}}\right]}{4 (-a)^{7/8} \sqrt{(-a)^{1/4}-i b^{1/4}}} - \\ \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{(-a)^{1/4}+i b^{1/4}} \operatorname{Tanh}[x]}{(-a)^{1/8}}\right]}{4 (-a)^{7/8} \sqrt{(-a)^{1/4}+i b^{1/4}}} - \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{(-a)^{1/4}+b^{1/4}} \operatorname{Tanh}[x]}{(-a)^{1/8}}\right]}{4 (-a)^{7/8} \sqrt{(-a)^{1/4}+b^{1/4}}}$$

Result (type 7, 160 leaves):

$$16 \operatorname{RootSum}\left[b - 8 b \#1 + 28 b \#1^2 - 56 b \#1^3 + 256 a \#1^4 + 70 b \#1^4 - 56 b \#1^5 + 28 b \#1^6 - 8 b \#1^7 + b \#1^8 \&, \right. \\ \left. \left(x \#1^3 + \operatorname{Log}\left[-\operatorname{Cosh}[x] - \operatorname{Sinh}[x] + \operatorname{Cosh}[x] \#1 - \operatorname{Sinh}[x] \#1\right] \#1^3\right) / \right. \\ \left. \left(-b + 7 b \#1 - 21 b \#1^2 + 128 a \#1^3 + 35 b \#1^3 - 35 b \#1^4 + 21 b \#1^5 - 7 b \#1^6 + b \#1^7\right) \&\right]$$

Problem 270: Result is not expressed in closed-form.

$$\int \frac{1}{1 + \operatorname{Sinh}[x]^5} dx$$

Optimal (type 3, 242 leaves, 17 steps):

$$\begin{aligned}
 & - \frac{2 (-1)^{3/5} \operatorname{ArcTan} \left[\frac{1 + (-1)^{3/5} \operatorname{Tanh} \left[\frac{x}{2} \right]}{\sqrt{-1 + (-1)^{1/5}}} \right]}{5 \sqrt{-1 + (-1)^{1/5}}} + \\
 & \frac{2 (-1)^{9/10} \operatorname{ArcTan} \left[\frac{i - (-1)^{9/10} \operatorname{Tanh} \left[\frac{x}{2} \right]}{\sqrt{1 + (-1)^{4/5}}} \right]}{5 \sqrt{1 + (-1)^{4/5}}} - \frac{1}{5} \sqrt{2} \operatorname{ArcTanh} \left[\frac{1 - \operatorname{Tanh} \left[\frac{x}{2} \right]}{\sqrt{2}} \right] + \\
 & \frac{2 (-1)^{9/10} \operatorname{ArcTanh} \left[\frac{(-1)^{7/10} (1 + (-1)^{1/5} \operatorname{Tanh} \left[\frac{x}{2} \right])}{\sqrt{-(-1)^{2/5} (1 + (-1)^{2/5})}} \right]}{5 \sqrt{-(-1)^{2/5} (1 + (-1)^{2/5})}} - \frac{2 (-1)^{4/5} \operatorname{ArcTanh} \left[\frac{1 - (-1)^{4/5} \operatorname{Tanh} \left[\frac{x}{2} \right]}{\sqrt{1 - (-1)^{3/5}}} \right]}{5 \sqrt{1 - (-1)^{3/5}}}
 \end{aligned}$$

Result (type 7, 439 leaves):

$$\begin{aligned}
 & \frac{1}{10} \left(2 \sqrt{2} \operatorname{ArcTanh} \left[\frac{-1 + \operatorname{Tanh} \left[\frac{x}{2} \right]}{\sqrt{2}} \right] - \right. \\
 & \operatorname{RootSum} \left[1 + 2 \#1 + 2 \#1^3 + 14 \#1^4 - 2 \#1^5 - 2 \#1^7 + \#1^8 \&, \frac{1}{1 + 3 \#1^2 + 28 \#1^3 - 5 \#1^4 - 7 \#1^6 + 4 \#1^7} \right. \\
 & \left. \left(-x - 2 \operatorname{Log} \left[-\operatorname{Cosh} \left[\frac{x}{2} \right] - \operatorname{Sinh} \left[\frac{x}{2} \right] + \operatorname{Cosh} \left[\frac{x}{2} \right] \#1 - \operatorname{Sinh} \left[\frac{x}{2} \right] \#1 \right] - 4 x \#1 - \right. \\
 & 8 \operatorname{Log} \left[-\operatorname{Cosh} \left[\frac{x}{2} \right] - \operatorname{Sinh} \left[\frac{x}{2} \right] + \operatorname{Cosh} \left[\frac{x}{2} \right] \#1 - \operatorname{Sinh} \left[\frac{x}{2} \right] \#1 \right] \#1 - 9 x \#1^2 - \\
 & 18 \operatorname{Log} \left[-\operatorname{Cosh} \left[\frac{x}{2} \right] - \operatorname{Sinh} \left[\frac{x}{2} \right] + \operatorname{Cosh} \left[\frac{x}{2} \right] \#1 - \operatorname{Sinh} \left[\frac{x}{2} \right] \#1 \right] \#1^2 - 24 x \#1^3 - \\
 & 48 \operatorname{Log} \left[-\operatorname{Cosh} \left[\frac{x}{2} \right] - \operatorname{Sinh} \left[\frac{x}{2} \right] + \operatorname{Cosh} \left[\frac{x}{2} \right] \#1 - \operatorname{Sinh} \left[\frac{x}{2} \right] \#1 \right] \#1^3 + 9 x \#1^4 + \\
 & 18 \operatorname{Log} \left[-\operatorname{Cosh} \left[\frac{x}{2} \right] - \operatorname{Sinh} \left[\frac{x}{2} \right] + \operatorname{Cosh} \left[\frac{x}{2} \right] \#1 - \operatorname{Sinh} \left[\frac{x}{2} \right] \#1 \right] \#1^4 - 4 x \#1^5 - \\
 & 8 \operatorname{Log} \left[-\operatorname{Cosh} \left[\frac{x}{2} \right] - \operatorname{Sinh} \left[\frac{x}{2} \right] + \operatorname{Cosh} \left[\frac{x}{2} \right] \#1 - \operatorname{Sinh} \left[\frac{x}{2} \right] \#1 \right] \#1^5 + x \#1^6 + \\
 & \left. \left. 2 \operatorname{Log} \left[-\operatorname{Cosh} \left[\frac{x}{2} \right] - \operatorname{Sinh} \left[\frac{x}{2} \right] + \operatorname{Cosh} \left[\frac{x}{2} \right] \#1 - \operatorname{Sinh} \left[\frac{x}{2} \right] \#1 \right] \#1^6 \right) \& \right)
 \end{aligned}$$

Problem 272: Result is not expressed in closed-form.

$$\int \frac{1}{1 + \operatorname{Sinh}[x]^8} dx$$

Optimal (type 3, 129 leaves, 9 steps):

$$\frac{\text{ArcTanh}\left[\sqrt{1 - (-1)^{1/4}} \text{Tanh}[x]\right]}{4\sqrt{1 - (-1)^{1/4}}} + \frac{\text{ArcTanh}\left[\sqrt{1 + (-1)^{1/4}} \text{Tanh}[x]\right]}{4\sqrt{1 + (-1)^{1/4}}} +$$

$$\frac{\text{ArcTanh}\left[\sqrt{1 - (-1)^{3/4}} \text{Tanh}[x]\right]}{4\sqrt{1 - (-1)^{3/4}}} + \frac{\text{ArcTanh}\left[\sqrt{1 + (-1)^{3/4}} \text{Tanh}[x]\right]}{4\sqrt{1 + (-1)^{3/4}}}$$

Result (type 7, 127 leaves):

$$16 \text{RootSum}\left[1 - 8 \#1 + 28 \#1^2 - 56 \#1^3 + 326 \#1^4 - 56 \#1^5 + 28 \#1^6 - 8 \#1^7 + \#1^8 \&, \right.$$

$$\left. \frac{x \#1^3 + \text{Log}\left[-\text{Cosh}[x] - \text{Sinh}[x] + \text{Cosh}[x] \#1 - \text{Sinh}[x] \#1\right] \#1^3}{-1 + 7 \#1 - 21 \#1^2 + 163 \#1^3 - 35 \#1^4 + 21 \#1^5 - 7 \#1^6 + \#1^7} \&\right]$$

Problem 273: Result is not expressed in closed-form.

$$\int \frac{1}{1 - \text{Sinh}[x]^5} dx$$

Optimal (type 3, 228 leaves, 17 steps):

$$-\frac{2(-1)^{1/10} \text{ArcTan}\left[\frac{i+(-1)^{1/10} \text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{1-(-1)^{1/5}}}\right]}{5\sqrt{1-(-1)^{1/5}}} - \frac{2 \text{ArcTanh}\left[\frac{(-1)^{3/5} - \text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{1-(-1)^{1/5}}}\right]}{5\sqrt{1-(-1)^{1/5}}} + \frac{1}{5} \sqrt{2} \text{ArcTanh}\left[\frac{1 + \text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{2}}\right] +$$

$$\frac{2 \text{ArcTanh}\left[\frac{(-1)^{4/5} + \text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{1-(-1)^{3/5}}}\right]}{5\sqrt{1-(-1)^{3/5}}} - \frac{2(-1)^{1/10} \text{ArcTanh}\left[\frac{(-1)^{3/10} \left(1 + (-1)^{4/5} \text{Tanh}\left[\frac{x}{2}\right]\right)}{\sqrt{(-1)^{1/5} + (-1)^{3/5}}}\right]}{5\sqrt{(-1)^{1/5} + (-1)^{3/5}}}$$

Result (type 7, 437 leaves):

$$\frac{1}{10} \left(2 \sqrt{2} \text{ArcTanh}\left[\frac{1 + \text{Tanh}\left[\frac{x}{2}\right]}{\sqrt{2}}\right] + \right.$$

$$\left. \text{RootSum}\left[1 - 2 \#1 - 2 \#1^3 + 14 \#1^4 + 2 \#1^5 + 2 \#1^7 + \#1^8 \&, \frac{1}{-1 - 3 \#1^2 + 28 \#1^3 + 5 \#1^4 + 7 \#1^6 + 4 \#1^7} \right.\right.$$

$$\left. \left. \left(-x - 2 \text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right] - \text{Sinh}\left[\frac{x}{2}\right] + \text{Cosh}\left[\frac{x}{2}\right] \#1 - \text{Sinh}\left[\frac{x}{2}\right] \#1\right] + 4 x \#1 + \right.\right.$$

$$8 \text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right] - \text{Sinh}\left[\frac{x}{2}\right] + \text{Cosh}\left[\frac{x}{2}\right] \#1 - \text{Sinh}\left[\frac{x}{2}\right] \#1\right] \#1 - 9 x \#1^2 -$$

$$18 \text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right] - \text{Sinh}\left[\frac{x}{2}\right] + \text{Cosh}\left[\frac{x}{2}\right] \#1 - \text{Sinh}\left[\frac{x}{2}\right] \#1\right] \#1^2 + 24 x \#1^3 +$$

$$48 \text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right] - \text{Sinh}\left[\frac{x}{2}\right] + \text{Cosh}\left[\frac{x}{2}\right] \#1 - \text{Sinh}\left[\frac{x}{2}\right] \#1\right] \#1^3 + 9 x \#1^4 +$$

$$18 \text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right] - \text{Sinh}\left[\frac{x}{2}\right] + \text{Cosh}\left[\frac{x}{2}\right] \#1 - \text{Sinh}\left[\frac{x}{2}\right] \#1\right] \#1^4 + 4 x \#1^5 +$$

$$8 \text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right] - \text{Sinh}\left[\frac{x}{2}\right] + \text{Cosh}\left[\frac{x}{2}\right] \#1 - \text{Sinh}\left[\frac{x}{2}\right] \#1\right] \#1^5 + x \#1^6 +$$

$$\left. \left. 2 \text{Log}\left[-\text{Cosh}\left[\frac{x}{2}\right] - \text{Sinh}\left[\frac{x}{2}\right] + \text{Cosh}\left[\frac{x}{2}\right] \#1 - \text{Sinh}\left[\frac{x}{2}\right] \#1\right] \#1^6 \right) \&\right)$$

Problem 292: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Sech}[c+dx]^6 (a+b \operatorname{Sinh}[c+dx]^2) dx$$

Optimal (type 3, 54 leaves, 3 steps):

$$\frac{a \operatorname{Tanh}[c+dx]}{d} - \frac{(2a-b) \operatorname{Tanh}[c+dx]^3}{3d} + \frac{(a-b) \operatorname{Tanh}[c+dx]^5}{5d}$$

Result (type 3, 117 leaves):

$$\frac{8a \operatorname{Tanh}[c+dx]}{15d} + \frac{2b \operatorname{Tanh}[c+dx]}{15d} + \frac{4a \operatorname{Sech}[c+dx]^2 \operatorname{Tanh}[c+dx]}{15d} + \frac{b \operatorname{Sech}[c+dx]^2 \operatorname{Tanh}[c+dx]}{15d} + \frac{a \operatorname{Sech}[c+dx]^4 \operatorname{Tanh}[c+dx]}{5d} - \frac{b \operatorname{Sech}[c+dx]^4 \operatorname{Tanh}[c+dx]}{5d}$$

Problem 315: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Sech}[c+dx]^8 (a+b \operatorname{Sinh}[c+dx]^2)^3 dx$$

Optimal (type 3, 80 leaves, 3 steps):

$$\frac{a^3 \operatorname{Tanh}[c+dx]}{d} - \frac{a^2 (a-b) \operatorname{Tanh}[c+dx]^3}{d} + \frac{3a(a-b)^2 \operatorname{Tanh}[c+dx]^5}{5d} - \frac{(a-b)^3 \operatorname{Tanh}[c+dx]^7}{7d}$$

Result (type 3, 163 leaves):

$$\frac{1}{1120d} (512a^3 - 304a^2b + 192ab^2 - 50b^3 + (464a^3 + 232a^2b - 246ab^2 + 75b^3) \operatorname{Cosh}[2(c+dx)] + 2(64a^3 + 32a^2b + 24ab^2 - 15b^3) \operatorname{Cosh}[4(c+dx)] + 16a^3 \operatorname{Cosh}[6(c+dx)] + 8a^2b \operatorname{Cosh}[6(c+dx)] + 6ab^2 \operatorname{Cosh}[6(c+dx)] + 5b^3 \operatorname{Cosh}[6(c+dx)]) \operatorname{Sech}[c+dx]^6 \operatorname{Tanh}[c+dx]$$

Problem 345: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sech}[c+dx]}{(a+b \operatorname{Sinh}[c+dx]^2)^3} dx$$

Optimal (type 3, 159 leaves, 6 steps):

$$\frac{\operatorname{ArcTan}[\operatorname{Sinh}[c+dx]]}{(a-b)^3 d} - \frac{\sqrt{b} (15a^2 - 10ab + 3b^2) \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Sinh}[c+dx]}{\sqrt{a}}\right]}{8a^{5/2} (a-b)^3 d} - \frac{b \operatorname{Sinh}[c+dx]}{4a(a-b)d(a+b \operatorname{Sinh}[c+dx]^2)^2} - \frac{(7a-3b)b \operatorname{Sinh}[c+dx]}{8a^2(a-b)^2 d(a+b \operatorname{Sinh}[c+dx]^2)}$$

Result (type 3, 321 leaves):

$$\frac{1}{8 a^{5/2} (a-b)^3 d (2 a-b+b \operatorname{Cosh}[2(c+d x)])^2} \left((-2 a+b)^2 \right. \\ \left. \left(\sqrt{b} (15 a^2-10 a b+3 b^2) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Csch}[c+d x]}{\sqrt{b}}\right] + 16 a^{5/2} \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2}(c+d x)\right]\right] \right) + \right. \\ \left. \left(b^{5/2} (15 a^2-10 a b+3 b^2) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Csch}[c+d x]}{\sqrt{b}}\right] + 16 a^{5/2} b^2 \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2}(c+d x)\right]\right] \right) \right. \\ \left. \operatorname{Cosh}[2(c+d x)]^2 - 2 \sqrt{a} b (18 a^3-35 a^2 b+20 a b^2-3 b^3) \operatorname{Sinh}[c+d x] - \right. \\ \left. 2 b \operatorname{Cosh}[2(c+d x)] \left(- (2 a-b) \left(\sqrt{b} (15 a^2-10 a b+3 b^2) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Csch}[c+d x]}{\sqrt{b}}\right] + \right. \right. \right. \\ \left. \left. \left. 16 a^{5/2} \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2}(c+d x)\right]\right] \right) + \sqrt{a} b (7 a^2-10 a b+3 b^2) \operatorname{Sinh}[c+d x] \right) \right) \right)$$

Problem 350: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Cosh}[x]^3}{1-\operatorname{Sinh}[x]^2} dx$$

Optimal (type 3, 10 leaves, 3 steps):

$$2 \operatorname{ArcTanh}[\operatorname{Sinh}[x]] - \operatorname{Sinh}[x]$$

Result (type 3, 29 leaves):

$$-2 \left(\frac{1}{2} \operatorname{Log}[1-\operatorname{Sinh}[x]] - \frac{1}{2} \operatorname{Log}[1+\operatorname{Sinh}[x]] + \frac{\operatorname{Sinh}[x]}{2} \right)$$

Problem 357: Result unnecessarily involves imaginary or complex numbers.

$$\int \operatorname{Cosh}[e+f x]^4 \sqrt{a+b \operatorname{Sinh}[e+f x]^2} dx$$

Optimal (type 4, 301 leaves, 7 steps):

$$\begin{aligned}
 & - \frac{2(a-3b) \operatorname{Cosh}[e+fx] \operatorname{Sinh}[e+fx] \sqrt{a+b \operatorname{Sinh}[e+fx]^2}}{15bf} + \\
 & \frac{\operatorname{Cosh}[e+fx] \operatorname{Sinh}[e+fx] (a+b \operatorname{Sinh}[e+fx]^2)^{3/2}}{5bf} + \\
 & \left((2a^2 - 7ab - 3b^2) \operatorname{EllipticE}[\operatorname{ArcTan}[\operatorname{Sinh}[e+fx]], 1 - \frac{b}{a}] \operatorname{Sech}[e+fx] \right. \\
 & \left. \sqrt{a+b \operatorname{Sinh}[e+fx]^2} \right) / \left(15b^2f \sqrt{\frac{\operatorname{Sech}[e+fx]^2 (a+b \operatorname{Sinh}[e+fx]^2)}{a}} \right) - \\
 & \left((a-9b) \operatorname{EllipticF}[\operatorname{ArcTan}[\operatorname{Sinh}[e+fx]], 1 - \frac{b}{a}] \operatorname{Sech}[e+fx] \sqrt{a+b \operatorname{Sinh}[e+fx]^2} \right) / \\
 & \left(15bf \sqrt{\frac{\operatorname{Sech}[e+fx]^2 (a+b \operatorname{Sinh}[e+fx]^2)}{a}} \right) - \\
 & \frac{(2a^2 - 7ab - 3b^2) \sqrt{a+b \operatorname{Sinh}[e+fx]^2} \operatorname{Tanh}[e+fx]}{15b^2f}
 \end{aligned}$$

Result (type 4, 211 leaves):

$$\begin{aligned}
 & \left(16ia(2a^2 - 7ab - 3b^2) \sqrt{\frac{2a-b+b \operatorname{Cosh}[2(e+fx)]}{a}} \operatorname{EllipticE}\left[i(e+fx), \frac{b}{a}\right] - \right. \\
 & 32ia(a^2 - 4ab + 3b^2) \sqrt{\frac{2a-b+b \operatorname{Cosh}[2(e+fx)]}{a}} \operatorname{EllipticF}\left[i(e+fx), \frac{b}{a}\right] + \\
 & \left. \sqrt{2}b(8a^2 + 32ab - 15b^2 + 4b(4a+3b) \operatorname{Cosh}[2(e+fx)] + 3b^2 \operatorname{Cosh}[4(e+fx)]) \right. \\
 & \left. \operatorname{Sinh}[2(e+fx)] \right) / \left(240b^2f \sqrt{2a-b+b \operatorname{Cosh}[2(e+fx)]} \right)
 \end{aligned}$$

Problem 358: Result unnecessarily involves imaginary or complex numbers.

$$\int \operatorname{Cosh}[e+fx]^2 \sqrt{a+b \operatorname{Sinh}[e+fx]^2} dx$$

Optimal (type 4, 223 leaves, 6 steps):

$$\frac{\text{Cosh}[e + f x] \text{Sinh}[e + f x] \sqrt{a + b \text{Sinh}[e + f x]^2}}{3 f} - \left((a + b) \text{EllipticE}\left[\text{ArcTan}[\text{Sinh}[e + f x]], 1 - \frac{b}{a}\right] \text{Sech}[e + f x] \sqrt{a + b \text{Sinh}[e + f x]^2} \right) / \left(3 b f \sqrt{\frac{\text{Sech}[e + f x]^2 (a + b \text{Sinh}[e + f x]^2)}{a}} \right) + \left(2 \text{EllipticF}\left[\text{ArcTan}[\text{Sinh}[e + f x]], 1 - \frac{b}{a}\right] \text{Sech}[e + f x] \sqrt{a + b \text{Sinh}[e + f x]^2} \right) / \left(3 f \sqrt{\frac{\text{Sech}[e + f x]^2 (a + b \text{Sinh}[e + f x]^2)}{a}} \right) + \frac{(a + b) \sqrt{a + b \text{Sinh}[e + f x]^2} \text{Tanh}[e + f x]}{3 b f}$$

Result(type 4, 168 leaves):

$$\left(-2 i \sqrt{2} a (a + b) \sqrt{\frac{2 a - b + b \text{Cosh}[2 (e + f x)]}{a}} \text{EllipticE}\left[i (e + f x), \frac{b}{a}\right] + 2 i \sqrt{2} a (a - b) \sqrt{\frac{2 a - b + b \text{Cosh}[2 (e + f x)]}{a}} \text{EllipticF}\left[i (e + f x), \frac{b}{a}\right] + b (2 a - b + b \text{Cosh}[2 (e + f x)]) \text{Sinh}[2 (e + f x)] \right) / \left(6 b f \sqrt{4 a - 2 b + 2 b \text{Cosh}[2 (e + f x)]} \right)$$

Problem 360: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \text{Sech}[e + f x]^2 \sqrt{a + b \text{Sinh}[e + f x]^2} dx$$

Optimal (type 4, 70 leaves, 2 steps):

$$\left(\text{EllipticE}\left[\text{ArcTan}[\text{Sinh}[e + f x]], 1 - \frac{b}{a}\right] \text{Sech}[e + f x] \sqrt{a + b \text{Sinh}[e + f x]^2} \right) / \left(f \sqrt{\frac{\text{Sech}[e + f x]^2 (a + b \text{Sinh}[e + f x]^2)}{a}} \right)$$

Result(type 4, 148 leaves):

$$\left(2 i a \sqrt{\frac{2 a - b + b \operatorname{Cosh}[2 (e + f x)]}{a}} \operatorname{EllipticE}\left[i (e + f x), \frac{b}{a}\right] - \right. \\ \left. 2 i a \sqrt{\frac{2 a - b + b \operatorname{Cosh}[2 (e + f x)]}{a}} \operatorname{EllipticF}\left[i (e + f x), \frac{b}{a}\right] + \right. \\ \left. \sqrt{2} (2 a - b + b \operatorname{Cosh}[2 (e + f x)]) \operatorname{Tanh}[e + f x] \right) / \left(2 f \sqrt{2 a - b + b \operatorname{Cosh}[2 (e + f x)]} \right)$$

Problem 361: Result unnecessarily involves imaginary or complex numbers.

$$\int \operatorname{Sech}[e + f x]^4 \sqrt{a + b \operatorname{Sinh}[e + f x]^2} dx$$

Optimal (type 4, 206 leaves, 5 steps):

$$\left((2 a - b) \operatorname{EllipticE}\left[\operatorname{ArcTan}[\operatorname{Sinh}[e + f x]], 1 - \frac{b}{a}\right] \operatorname{Sech}[e + f x] \sqrt{a + b \operatorname{Sinh}[e + f x]^2} \right) / \\ \left(3 (a - b) f \sqrt{\frac{\operatorname{Sech}[e + f x]^2 (a + b \operatorname{Sinh}[e + f x]^2)}{a}} \right) - \\ \left(b \operatorname{EllipticF}\left[\operatorname{ArcTan}[\operatorname{Sinh}[e + f x]], 1 - \frac{b}{a}\right] \operatorname{Sech}[e + f x] \sqrt{a + b \operatorname{Sinh}[e + f x]^2} \right) / \\ \left(3 (a - b) f \sqrt{\frac{\operatorname{Sech}[e + f x]^2 (a + b \operatorname{Sinh}[e + f x]^2)}{a}} \right) + \\ \frac{\operatorname{Sech}[e + f x]^2 \sqrt{a + b \operatorname{Sinh}[e + f x]^2} \operatorname{Tanh}[e + f x]}{3 f}$$

Result (type 4, 204 leaves):

$$\left(8 i a (2 a - b) \sqrt{\frac{2 a - b + b \operatorname{Cosh}[2 (e + f x)]}{a}} \operatorname{EllipticE}\left[i (e + f x), \frac{b}{a}\right] - \right. \\ \left. 16 i a (a - b) \sqrt{\frac{2 a - b + b \operatorname{Cosh}[2 (e + f x)]}{a}} \operatorname{EllipticF}\left[i (e + f x), \frac{b}{a}\right] + \right. \\ \left. \sqrt{2} \left((8 a^2 - 4 b^2) \operatorname{Cosh}[2 (e + f x)] + (2 a - b) (8 a - 5 b + b \operatorname{Cosh}[4 (e + f x)]) \right) \right) \\ \operatorname{Sech}[e + f x]^2 \operatorname{Tanh}[e + f x] \left. \right) / \left(24 (a - b) f \sqrt{2 a - b + b \operatorname{Cosh}[2 (e + f x)]} \right)$$

Problem 368: Result unnecessarily involves imaginary or complex numbers.

$$\int \text{Cosh}[e + f x]^4 (a + b \text{Sinh}[e + f x]^2)^{3/2} dx$$

Optimal (type 4, 357 leaves, 8 steps):

$$\begin{aligned} & \frac{(a^2 + 9 a b - 2 b^2) \text{Cosh}[e + f x] \text{Sinh}[e + f x] \sqrt{a + b \text{Sinh}[e + f x]^2}}{35 b f} + \\ & \frac{2 (4 a - b) \text{Cosh}[e + f x]^3 \text{Sinh}[e + f x] \sqrt{a + b \text{Sinh}[e + f x]^2}}{35 f} + \\ & \frac{b \text{Cosh}[e + f x]^5 \text{Sinh}[e + f x] \sqrt{a + b \text{Sinh}[e + f x]^2}}{7 f} + \\ & \left(2 (a + b) (a^2 - 6 a b + b^2) \text{EllipticE}\left[\text{ArcTan}[\text{Sinh}[e + f x]], 1 - \frac{b}{a}\right] \text{Sech}[e + f x] \right. \\ & \quad \left. \sqrt{a + b \text{Sinh}[e + f x]^2} \right) / \left(35 b^2 f \sqrt{\frac{\text{Sech}[e + f x]^2 (a + b \text{Sinh}[e + f x]^2)}{a}} \right) - \\ & \left((a^2 - 18 a b + b^2) \text{EllipticF}\left[\text{ArcTan}[\text{Sinh}[e + f x]], 1 - \frac{b}{a}\right] \text{Sech}[e + f x] \sqrt{a + b \text{Sinh}[e + f x]^2} \right) / \\ & \left(35 b f \sqrt{\frac{\text{Sech}[e + f x]^2 (a + b \text{Sinh}[e + f x]^2)}{a}} \right) - \\ & \frac{2 (a + b) (a^2 - 6 a b + b^2) \sqrt{a + b \text{Sinh}[e + f x]^2} \text{Tanh}[e + f x]}{35 b^2 f} \end{aligned}$$

Result (type 4, 256 leaves):

$$\begin{aligned} & \frac{1}{2240 b^2 f \sqrt{2 a - b + b \text{Cosh}[2 (e + f x)]}} \\ & \left(128 i a (a^3 - 5 a^2 b - 5 a b^2 + b^3) \sqrt{\frac{2 a - b + b \text{Cosh}[2 (e + f x)]}{a}} \text{EllipticE}\left[i (e + f x), \frac{b}{a}\right] - \right. \\ & \quad 64 i a (2 a^3 - 11 a^2 b + 8 a b^2 + b^3) \sqrt{\frac{2 a - b + b \text{Cosh}[2 (e + f x)]}{a}} \text{EllipticF}\left[i (e + f x), \frac{b}{a}\right] + \\ & \quad \sqrt{2} b (32 a^3 + 400 a^2 b - 212 a b^2 + 30 b^3 + b (144 a^2 + 192 a b - 37 b^2) \text{Cosh}[2 (e + f x)] + \\ & \quad \left. 2 b^2 (26 a + b) \text{Cosh}[4 (e + f x)] + 5 b^3 \text{Cosh}[6 (e + f x)]\right) \text{Sinh}[2 (e + f x)] \left. \right) \end{aligned}$$

Problem 369: Result unnecessarily involves imaginary or complex numbers.

$$\int \text{Cosh}[e + f x]^2 (a + b \text{Sinh}[e + f x]^2)^{3/2} dx$$

Optimal (type 4, 299 leaves, 7 steps):

$$\begin{aligned} & \frac{2 (3 a - b) \text{Cosh}[e + f x] \text{Sinh}[e + f x] \sqrt{a + b \text{Sinh}[e + f x]^2}}{15 f} + \\ & \frac{b \text{Cosh}[e + f x]^3 \text{Sinh}[e + f x] \sqrt{a + b \text{Sinh}[e + f x]^2}}{5 f} - \\ & \left((3 a^2 + 7 a b - 2 b^2) \text{EllipticE}\left[\text{ArcTan}[\text{Sinh}[e + f x]], 1 - \frac{b}{a}\right] \text{Sech}[e + f x] \right. \\ & \quad \left. \sqrt{a + b \text{Sinh}[e + f x]^2} \right) / \left(15 b f \sqrt{\frac{\text{Sech}[e + f x]^2 (a + b \text{Sinh}[e + f x]^2)}{a}} \right) + \\ & \left((9 a - b) \text{EllipticF}\left[\text{ArcTan}[\text{Sinh}[e + f x]], 1 - \frac{b}{a}\right] \text{Sech}[e + f x] \sqrt{a + b \text{Sinh}[e + f x]^2} \right) / \\ & \left(15 f \sqrt{\frac{\text{Sech}[e + f x]^2 (a + b \text{Sinh}[e + f x]^2)}{a}} \right) + \\ & \frac{(3 a^2 + 7 a b - 2 b^2) \sqrt{a + b \text{Sinh}[e + f x]^2} \text{Tanh}[e + f x]}{15 b f} \end{aligned}$$

Result (type 4, 213 leaves):

$$\begin{aligned} & \left(-16 i a (3 a^2 + 7 a b - 2 b^2) \sqrt{\frac{2 a - b + b \text{Cosh}[2 (e + f x)]}{a}} \text{EllipticE}\left[i (e + f x), \frac{b}{a}\right] + \right. \\ & 16 i a (3 a^2 - 2 a b - b^2) \sqrt{\frac{2 a - b + b \text{Cosh}[2 (e + f x)]}{a}} \text{EllipticF}\left[i (e + f x), \frac{b}{a}\right] + \\ & \sqrt{2} b (48 a^2 - 28 a b + 5 b^2 + 4 (9 a - 2 b) b \text{Cosh}[2 (e + f x)] + 3 b^2 \text{Cosh}[4 (e + f x)]) \\ & \left. \text{Sinh}[2 (e + f x)] \right) / \left(240 b f \sqrt{2 a - b + b \text{Cosh}[2 (e + f x)]} \right) \end{aligned}$$

Problem 371: Result unnecessarily involves imaginary or complex numbers.

$$\int \text{Sech}[e + f x]^2 (a + b \text{Sinh}[e + f x]^2)^{3/2} dx$$

Optimal (type 4, 210 leaves, 6 steps):

$$\left((a - 2b) \operatorname{EllipticE}\left[\operatorname{ArcTan}[\operatorname{Sinh}[e + fx]], 1 - \frac{b}{a}\right] \operatorname{Sech}[e + fx] \sqrt{a + b \operatorname{Sinh}[e + fx]^2} \right) /$$

$$\left(f \sqrt{\frac{\operatorname{Sech}[e + fx]^2 (a + b \operatorname{Sinh}[e + fx]^2)}{a}} \right) +$$

$$\left(b \operatorname{EllipticF}\left[\operatorname{ArcTan}[\operatorname{Sinh}[e + fx]], 1 - \frac{b}{a}\right] \operatorname{Sech}[e + fx] \sqrt{a + b \operatorname{Sinh}[e + fx]^2} \right) /$$

$$\left(f \sqrt{\frac{\operatorname{Sech}[e + fx]^2 (a + b \operatorname{Sinh}[e + fx]^2)}{a}} \right) -$$

$$\frac{(a - 2b) \sqrt{a + b \operatorname{Sinh}[e + fx]^2} \operatorname{Tanh}[e + fx]}{f} + \frac{(a - b) \sqrt{a + b \operatorname{Sinh}[e + fx]^2} \operatorname{Tanh}[e + fx]}{f}$$

Result (type 4, 160 leaves):

$$\left(2 i a (a - 2b) \sqrt{\frac{2a - b + b \operatorname{Cosh}[2(e + fx)]}{a}} \operatorname{EllipticE}\left[i(e + fx), \frac{b}{a}\right] + \right.$$

$$(a - b) \left(-2 i a \sqrt{\frac{2a - b + b \operatorname{Cosh}[2(e + fx)]}{a}} \operatorname{EllipticF}\left[i(e + fx), \frac{b}{a}\right] + \right.$$

$$\left. \left. \sqrt{2} (2a - b + b \operatorname{Cosh}[2(e + fx)]) \operatorname{Tanh}[e + fx] \right) \right) / \left(2 f \sqrt{2a - b + b \operatorname{Cosh}[2(e + fx)]} \right)$$

Problem 372: Result unnecessarily involves imaginary or complex numbers.

$$\int \operatorname{Sech}[e + fx]^4 (a + b \operatorname{Sinh}[e + fx]^2)^{3/2} dx$$

Optimal (type 4, 193 leaves, 5 steps):

$$\begin{aligned}
 & \left(2 (a+b) \operatorname{EllipticE} \left[\operatorname{ArcTan} [\operatorname{Sinh} [e+f x]], 1 - \frac{b}{a} \right] \operatorname{Sech} [e+f x] \sqrt{a+b \operatorname{Sinh} [e+f x]^2} \right) / \\
 & \left(3 f \sqrt{\frac{\operatorname{Sech} [e+f x]^2 (a+b \operatorname{Sinh} [e+f x]^2)}{a}} \right) - \\
 & \left(b \operatorname{EllipticF} \left[\operatorname{ArcTan} [\operatorname{Sinh} [e+f x]], 1 - \frac{b}{a} \right] \operatorname{Sech} [e+f x] \sqrt{a+b \operatorname{Sinh} [e+f x]^2} \right) / \\
 & \left(3 f \sqrt{\frac{\operatorname{Sech} [e+f x]^2 (a+b \operatorname{Sinh} [e+f x]^2)}{a}} \right) + \\
 & \frac{(a-b) \operatorname{Sech} [e+f x]^2 \sqrt{a+b \operatorname{Sinh} [e+f x]^2} \operatorname{Tanh} [e+f x]}{3 f}
 \end{aligned}$$

Result (type 4, 197 leaves):

$$\begin{aligned}
 & \left(4 i a (a+b) \sqrt{\frac{2 a-b+b \operatorname{Cosh} [2 (e+f x)]}{a}} \operatorname{EllipticE} \left[i (e+f x), \frac{b}{a} \right] - \right. \\
 & 2 i a (2 a+b) \sqrt{\frac{2 a-b+b \operatorname{Cosh} [2 (e+f x)]}{a}} \operatorname{EllipticF} \left[i (e+f x), \frac{b}{a} \right] + \frac{1}{\sqrt{2}} \\
 & \left. (8 a^2 - 3 a b + b^2 + (4 a^2 + 6 a b - 2 b^2) \operatorname{Cosh} [2 (e+f x)] + b (a+b) \operatorname{Cosh} [4 (e+f x)]) \right) \\
 & \left. \operatorname{Sech} [e+f x]^2 \operatorname{Tanh} [e+f x] \right) / \left(6 f \sqrt{2 a-b+b \operatorname{Cosh} [2 (e+f x)]} \right)
 \end{aligned}$$

Problem 377: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Cosh} [e+f x]^4}{\sqrt{a+b \operatorname{Sinh} [e+f x]^2}} dx$$

Optimal (type 4, 241 leaves, 6 steps):

$$\frac{\text{Cosh}[e + f x] \text{Sinh}[e + f x] \sqrt{a + b \text{Sinh}[e + f x]^2}}{3 b f} +$$

$$\left(2 (a - 2 b) \text{EllipticE}[\text{ArcTan}[\text{Sinh}[e + f x]], 1 - \frac{b}{a}] \text{Sech}[e + f x] \sqrt{a + b \text{Sinh}[e + f x]^2} \right) /$$

$$\left(3 b^2 f \sqrt{\frac{\text{Sech}[e + f x]^2 (a + b \text{Sinh}[e + f x]^2)}{a}} \right) -$$

$$\left((a - 3 b) \text{EllipticF}[\text{ArcTan}[\text{Sinh}[e + f x]], 1 - \frac{b}{a}] \text{Sech}[e + f x] \sqrt{a + b \text{Sinh}[e + f x]^2} \right) /$$

$$\left(3 a b f \sqrt{\frac{\text{Sech}[e + f x]^2 (a + b \text{Sinh}[e + f x]^2)}{a}} \right) - \frac{2 (a - 2 b) \sqrt{a + b \text{Sinh}[e + f x]^2} \text{Tanh}[e + f x]}{3 b^2 f}$$

Result (type 4, 179 leaves):

$$\left(4 i \sqrt{2} a (a - 2 b) \sqrt{\frac{2 a - b + b \text{Cosh}[2 (e + f x)]}{a}} \text{EllipticE}\left[i (e + f x), \frac{b}{a}\right] - \right.$$

$$2 i \sqrt{2} (2 a^2 - 5 a b + 3 b^2) \sqrt{\frac{2 a - b + b \text{Cosh}[2 (e + f x)]}{a}} \text{EllipticF}\left[i (e + f x), \frac{b}{a}\right] +$$

$$\left. b (2 a - b + b \text{Cosh}[2 (e + f x)]) \text{Sinh}[2 (e + f x)] \right) / \left(6 b^2 f \sqrt{4 a - 2 b + 2 b \text{Cosh}[2 (e + f x)]} \right)$$

Problem 378: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{Cosh}[e + f x]^2}{\sqrt{a + b \text{Sinh}[e + f x]^2}} dx$$

Optimal (type 4, 177 leaves, 5 steps):

$$- \left(\text{EllipticE}[\text{ArcTan}[\text{Sinh}[e + f x]], 1 - \frac{b}{a}] \text{Sech}[e + f x] \sqrt{a + b \text{Sinh}[e + f x]^2} \right) /$$

$$\left(b f \sqrt{\frac{\text{Sech}[e + f x]^2 (a + b \text{Sinh}[e + f x]^2)}{a}} \right) +$$

$$\left(\text{EllipticF}[\text{ArcTan}[\text{Sinh}[e + f x]], 1 - \frac{b}{a}] \text{Sech}[e + f x] \sqrt{a + b \text{Sinh}[e + f x]^2} \right) /$$

$$\left(a f \sqrt{\frac{\text{Sech}[e + f x]^2 (a + b \text{Sinh}[e + f x]^2)}{a}} \right) + \frac{\sqrt{a + b \text{Sinh}[e + f x]^2} \text{Tanh}[e + f x]}{b f}$$

Result (type 4, 95 leaves):

$$\begin{aligned}
 & - \left(\left(i \sqrt{\frac{2a - b + b \operatorname{Cosh}[2(e + fx)]}{a}} \right. \right. \\
 & \quad \left. \left(a \operatorname{EllipticE}\left[i(e + fx), \frac{b}{a}\right] + (-a + b) \operatorname{EllipticF}\left[i(e + fx), \frac{b}{a}\right] \right) \right) / \\
 & \quad \left. \left(bf \sqrt{2a - b + b \operatorname{Cosh}[2(e + fx)]} \right) \right)
 \end{aligned}$$

Problem 380: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Sech}[e + fx]^2}{\sqrt{a + b \operatorname{Sinh}[e + fx]^2}} dx$$

Optimal (type 4, 160 leaves, 7 steps):

$$\begin{aligned}
 & \left(\operatorname{EllipticE}\left[\operatorname{ArcTan}[\operatorname{Sinh}[e + fx]], 1 - \frac{b}{a}\right] \operatorname{Sech}[e + fx] \sqrt{a + b \operatorname{Sinh}[e + fx]^2} \right) / \\
 & \quad \left((a - b) f \sqrt{\frac{\operatorname{Sech}[e + fx]^2 (a + b \operatorname{Sinh}[e + fx]^2)}{a}} \right) - \\
 & \quad \left(b \operatorname{EllipticF}\left[\operatorname{ArcTan}[\operatorname{Sinh}[e + fx]], 1 - \frac{b}{a}\right] \operatorname{Sech}[e + fx] \sqrt{a + b \operatorname{Sinh}[e + fx]^2} \right) / \\
 & \quad \left(a (a - b) f \sqrt{\frac{\operatorname{Sech}[e + fx]^2 (a + b \operatorname{Sinh}[e + fx]^2)}{a}} \right)
 \end{aligned}$$

Result (type 4, 159 leaves):

$$\begin{aligned}
 & \left(2i a \sqrt{\frac{2a - b + b \operatorname{Cosh}[2(e + fx)]}{a}} \operatorname{EllipticE}\left[i(e + fx), \frac{b}{a}\right] - \right. \\
 & \quad 2i (a - b) \sqrt{\frac{2a - b + b \operatorname{Cosh}[2(e + fx)]}{a}} \operatorname{EllipticF}\left[i(e + fx), \frac{b}{a}\right] + \\
 & \quad \left. \sqrt{2} (2a - b + b \operatorname{Cosh}[2(e + fx)]) \operatorname{Tanh}[e + fx] \right) / \left(2(a - b) f \sqrt{2a - b + b \operatorname{Cosh}[2(e + fx)]} \right)
 \end{aligned}$$

Problem 381: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Sech}[e + f x]^4}{\sqrt{a + b \operatorname{Sinh}[e + f x]^2}} dx$$

Optimal (type 4, 219 leaves, 5 steps):

$$\left(2 (a - 2 b) \operatorname{EllipticE}\left[\operatorname{ArcTan}[\operatorname{Sinh}[e + f x]], 1 - \frac{b}{a}\right] \operatorname{Sech}[e + f x] \sqrt{a + b \operatorname{Sinh}[e + f x]^2} \right) /$$

$$\left(3 (a - b)^2 f \sqrt{\frac{\operatorname{Sech}[e + f x]^2 (a + b \operatorname{Sinh}[e + f x]^2)}{a}} \right) -$$

$$\left((a - 3 b) b \operatorname{EllipticF}\left[\operatorname{ArcTan}[\operatorname{Sinh}[e + f x]], 1 - \frac{b}{a}\right] \operatorname{Sech}[e + f x] \sqrt{a + b \operatorname{Sinh}[e + f x]^2} \right) /$$

$$\left(3 a (a - b)^2 f \sqrt{\frac{\operatorname{Sech}[e + f x]^2 (a + b \operatorname{Sinh}[e + f x]^2)}{a}} \right) +$$

$$\frac{\operatorname{Sech}[e + f x]^2 \sqrt{a + b \operatorname{Sinh}[e + f x]^2} \operatorname{Tanh}[e + f x]}{3 (a - b) f}$$

Result (type 4, 219 leaves):

$$\left(4 i a (a - 2 b) \sqrt{\frac{2 a - b + b \operatorname{Cosh}[2 (e + f x)]}{a}} \operatorname{EllipticE}\left[i (e + f x), \frac{b}{a}\right] - \right.$$

$$2 i (2 a^2 - 5 a b + 3 b^2) \sqrt{\frac{2 a - b + b \operatorname{Cosh}[2 (e + f x)]}{a}} \operatorname{EllipticF}\left[i (e + f x), \frac{b}{a}\right] + \frac{1}{\sqrt{2}}$$

$$(8 a^2 - 15 a b + 4 b^2 + (4 a^2 - 6 a b - 2 b^2) \operatorname{Cosh}[2 (e + f x)] + (a - 2 b) b \operatorname{Cosh}[4 (e + f x)])$$

$$\left. \operatorname{Sech}[e + f x]^2 \operatorname{Tanh}[e + f x] \right) / \left(6 (a - b)^2 f \sqrt{2 a - b + b \operatorname{Cosh}[2 (e + f x)]} \right)$$

Problem 386: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Cosh}[e + f x]^6}{(a + b \operatorname{Sinh}[e + f x]^2)^{3/2}} dx$$

Optimal (type 4, 325 leaves, 7 steps):

$$\begin{aligned}
 & - \frac{(a-b) \operatorname{Cosh}[e+fx]^3 \operatorname{Sinh}[e+fx]}{abf \sqrt{a+b \operatorname{Sinh}[e+fx]^2}} + \frac{(4a-3b) \operatorname{Cosh}[e+fx] \operatorname{Sinh}[e+fx] \sqrt{a+b \operatorname{Sinh}[e+fx]^2}}{3ab^2f} + \\
 & \left((8a^2 - 13ab + 3b^2) \operatorname{EllipticE}[\operatorname{ArcTan}[\operatorname{Sinh}[e+fx]], 1 - \frac{b}{a}] \operatorname{Sech}[e+fx] \sqrt{a+b \operatorname{Sinh}[e+fx]^2} \right) / \\
 & \left(3ab^3f \sqrt{\frac{\operatorname{Sech}[e+fx]^2 (a+b \operatorname{Sinh}[e+fx]^2)}{a}} \right) - \\
 & \left(2(2a-3b) \operatorname{EllipticF}[\operatorname{ArcTan}[\operatorname{Sinh}[e+fx]], 1 - \frac{b}{a}] \operatorname{Sech}[e+fx] \sqrt{a+b \operatorname{Sinh}[e+fx]^2} \right) / \\
 & \left(3ab^2f \sqrt{\frac{\operatorname{Sech}[e+fx]^2 (a+b \operatorname{Sinh}[e+fx]^2)}{a}} \right) - \\
 & \frac{(8a^2 - 13ab + 3b^2) \sqrt{a+b \operatorname{Sinh}[e+fx]^2} \operatorname{Tanh}[e+fx]}{3ab^3f}
 \end{aligned}$$

Result (type 4, 196 leaves):

$$\begin{aligned}
 & \left(4i a (8a^2 - 13ab + 3b^2) \sqrt{\frac{2a-b+b \operatorname{Cosh}[2(e+fx)]}{a}} \operatorname{EllipticE}[i(e+fx), \frac{b}{a}] - \right. \\
 & 4i a (8a^2 - 17ab + 9b^2) \sqrt{\frac{2a-b+b \operatorname{Cosh}[2(e+fx)]}{a}} \operatorname{EllipticF}[i(e+fx), \frac{b}{a}] + \\
 & \left. \sqrt{2} b (8a^2 - 13ab + 6b^2 + ab \operatorname{Cosh}[2(e+fx)]) \operatorname{Sinh}[2(e+fx)] \right) / \\
 & (12ab^3f \sqrt{2a-b+b \operatorname{Cosh}[2(e+fx)]})
 \end{aligned}$$

Problem 387: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Cosh}[e+fx]^4}{(a+b \operatorname{Sinh}[e+fx]^2)^{3/2}} dx$$

Optimal (type 4, 244 leaves, 6 steps):

$$\begin{aligned}
 & - \frac{(a-b) \operatorname{Cosh}[e+fx] \operatorname{Sinh}[e+fx]}{a b f \sqrt{a+b \operatorname{Sinh}[e+fx]^2}} - \\
 & \left((2a-b) \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\operatorname{Sinh}[e+fx]\right], 1-\frac{b}{a}\right] \operatorname{Sech}[e+fx] \sqrt{a+b \operatorname{Sinh}[e+fx]^2} \right) / \\
 & \left(a b^2 f \sqrt{\frac{\operatorname{Sech}[e+fx]^2 (a+b \operatorname{Sinh}[e+fx]^2)}{a}} \right) + \\
 & \left(\operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\operatorname{Sinh}[e+fx]\right], 1-\frac{b}{a}\right] \operatorname{Sech}[e+fx] \sqrt{a+b \operatorname{Sinh}[e+fx]^2} \right) / \\
 & \left(a b f \sqrt{\frac{\operatorname{Sech}[e+fx]^2 (a+b \operatorname{Sinh}[e+fx]^2)}{a}} \right) + \frac{(2a-b) \sqrt{a+b \operatorname{Sinh}[e+fx]^2} \operatorname{Tanh}[e+fx]}{a b^2 f}
 \end{aligned}$$

Result (type 4, 155 leaves):

$$\begin{aligned}
 & \left(-2 i a (2a-b) \sqrt{\frac{2a-b+b \operatorname{Cosh}[2(e+fx)]}{a}} \operatorname{EllipticE}\left[i(e+fx), \frac{b}{a}\right] + (a-b) \right. \\
 & \left. \left(4 i a \sqrt{\frac{2a-b+b \operatorname{Cosh}[2(e+fx)]}{a}} \operatorname{EllipticF}\left[i(e+fx), \frac{b}{a}\right] - \sqrt{2} b \operatorname{Sinh}[2(e+fx)] \right) \right) / \\
 & (2 a b^2 f \sqrt{2 a-b+b \operatorname{Cosh}[2(e+fx)]})
 \end{aligned}$$

Problem 388: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Cosh}[e+fx]^2}{(a+b \operatorname{Sinh}[e+fx]^2)^{3/2}} dx$$

Optimal (type 4, 91 leaves, 2 steps):

$$\frac{\operatorname{Cosh}[e+fx] \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Sinh}[e+fx]}{\sqrt{a}}\right], 1-\frac{a}{b}\right]}{\sqrt{a} \sqrt{b} f \sqrt{\frac{a \operatorname{Cosh}[e+fx]^2}{a+b \operatorname{Sinh}[e+fx]^2}} \sqrt{a+b \operatorname{Sinh}[e+fx]^2}}$$

Result (type 4, 143 leaves):

$$\left(i \sqrt{2} a \sqrt{\frac{2a - b + b \operatorname{Cosh}[2(e + fx)]}{a}} \operatorname{EllipticE}\left[i(e + fx), \frac{b}{a}\right] - \right. \\ \left. i \sqrt{2} a \sqrt{\frac{2a - b + b \operatorname{Cosh}[2(e + fx)]}{a}} \operatorname{EllipticF}\left[i(e + fx), \frac{b}{a}\right] + b \operatorname{Sinh}[2(e + fx)] \right) / \\ \left(a b f \sqrt{4a - 2b + 2b \operatorname{Cosh}[2(e + fx)]} \right)$$

Problem 390: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Sech}[e + fx]^2}{(a + b \operatorname{Sinh}[e + fx]^2)^{3/2}} dx$$

Optimal (type 4, 217 leaves, 5 steps):

$$\frac{\sqrt{b} (a + b) \operatorname{Cosh}[e + fx] \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Sinh}[e + fx]}{\sqrt{a}}\right], 1 - \frac{a}{b}\right]}{\sqrt{a} (a - b)^2 f \sqrt{\frac{a \operatorname{Cosh}[e + fx]^2}{a + b \operatorname{Sinh}[e + fx]^2}} \sqrt{a + b \operatorname{Sinh}[e + fx]^2}} \\ \left(2 b \operatorname{EllipticF}\left[\operatorname{ArcTan}[\operatorname{Sinh}[e + fx]], 1 - \frac{b}{a}\right] \operatorname{Sech}[e + fx] \sqrt{a + b \operatorname{Sinh}[e + fx]^2} \right) / \\ \left(a (a - b)^2 f \sqrt{\frac{\operatorname{Sech}[e + fx]^2 (a + b \operatorname{Sinh}[e + fx]^2)}{a}} \right) + \frac{\operatorname{Tanh}[e + fx]}{(a - b) f \sqrt{a + b \operatorname{Sinh}[e + fx]^2}}$$

Result (type 4, 178 leaves):

$$\left(i \sqrt{2} a (a + b) \sqrt{\frac{2a - b + b \operatorname{Cosh}[2(e + fx)]}{a}} \operatorname{EllipticE}\left[i(e + fx), \frac{b}{a}\right] - \right. \\ \left. i \sqrt{2} a (a - b) \sqrt{\frac{2a - b + b \operatorname{Cosh}[2(e + fx)]}{a}} \operatorname{EllipticF}\left[i(e + fx), \frac{b}{a}\right] + \right. \\ \left. (2 a^2 - a b + b^2 + b (a + b) \operatorname{Cosh}[2(e + fx)]) \operatorname{Tanh}[e + fx] \right) / \\ \left(a (a - b)^2 f \sqrt{4a - 2b + 2b \operatorname{Cosh}[2(e + fx)]} \right)$$

Problem 395: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Cosh}[e + fx]^6}{(a + b \operatorname{Sinh}[e + fx]^2)^{5/2}} dx$$

Optimal (type 4, 330 leaves, 7 steps):

$$\begin{aligned} & - \frac{(a-b) \operatorname{Cosh}[e+fx]^3 \operatorname{Sinh}[e+fx]}{3abf(a+b\operatorname{Sinh}[e+fx]^2)^{3/2}} - \frac{2(a-b)(2a+b) \operatorname{Cosh}[e+fx] \operatorname{Sinh}[e+fx]}{3a^2b^2f\sqrt{a+b\operatorname{Sinh}[e+fx]^2}} - \\ & \left((8a^2-3ab-2b^2) \operatorname{EllipticE}\left[\operatorname{ArcTan}[\operatorname{Sinh}[e+fx]], 1-\frac{b}{a}\right] \operatorname{Sech}[e+fx] \right. \\ & \left. \sqrt{a+b\operatorname{Sinh}[e+fx]^2} \right) / \left(3a^2b^3f \sqrt{\frac{\operatorname{Sech}[e+fx]^2(a+b\operatorname{Sinh}[e+fx]^2)}{a}} \right) + \\ & \left((4a-b) \operatorname{EllipticF}\left[\operatorname{ArcTan}[\operatorname{Sinh}[e+fx]], 1-\frac{b}{a}\right] \operatorname{Sech}[e+fx] \sqrt{a+b\operatorname{Sinh}[e+fx]^2} \right) / \\ & \left(3a^2b^2f \sqrt{\frac{\operatorname{Sech}[e+fx]^2(a+b\operatorname{Sinh}[e+fx]^2)}{a}} \right) + \\ & \frac{(8a^2-3ab-2b^2) \sqrt{a+b\operatorname{Sinh}[e+fx]^2} \operatorname{Tanh}[e+fx]}{3a^2b^3f} \end{aligned}$$

Result (type 4, 206 leaves):

$$\begin{aligned} & \left(-2ia^2(8a^2-3ab-2b^2) \left(\frac{2a-b+b\operatorname{Cosh}[2(e+fx)]}{a} \right)^{3/2} \operatorname{EllipticE}\left[i(e+fx), \frac{b}{a}\right] + \right. \\ & \left. \frac{1}{2}(a-b) \left(4ia^2(8a+b) \left(\frac{2a-b+b\operatorname{Cosh}[2(e+fx)]}{a} \right)^{3/2} \operatorname{EllipticF}\left[i(e+fx), \frac{b}{a}\right] - \right. \right. \\ & \left. \left. 2\sqrt{2}b(8a^2+ab-2b^2+b(5a+2b)\operatorname{Cosh}[2(e+fx)]) \operatorname{Sinh}[2(e+fx)] \right) \right) / \\ & \left(6a^2b^3f(2a-b+b\operatorname{Cosh}[2(e+fx)])^{3/2} \right) \end{aligned}$$

Problem 396: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Cosh}[e+fx]^4}{(a+b\operatorname{Sinh}[e+fx]^2)^{5/2}} dx$$

Optimal (type 4, 223 leaves, 5 steps):

$$\begin{aligned}
 & - \frac{(a-b) \operatorname{Cosh}[e+fx] \operatorname{Sinh}[e+fx]}{3abf(a+b\operatorname{Sinh}[e+fx]^2)^{3/2}} + \\
 & \frac{2(a+b) \operatorname{Cosh}[e+fx] \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{b}\operatorname{Sinh}[e+fx]}{\sqrt{a}}\right], 1-\frac{a}{b}\right]}{3a^{3/2}b^{3/2}f\sqrt{\frac{a\operatorname{Cosh}[e+fx]^2}{a+b\operatorname{Sinh}[e+fx]^2}}\sqrt{a+b\operatorname{Sinh}[e+fx]^2}} \\
 & \left(\operatorname{EllipticF}\left[\operatorname{ArcTan}[\operatorname{Sinh}[e+fx]], 1-\frac{b}{a}\right] \operatorname{Sech}[e+fx] \sqrt{a+b\operatorname{Sinh}[e+fx]^2}\right) / \\
 & \left(3a^2bf\sqrt{\frac{\operatorname{Sech}[e+fx]^2(a+b\operatorname{Sinh}[e+fx]^2)}{a}}\right)
 \end{aligned}$$

Result (type 4, 178 leaves):

$$\begin{aligned}
 & \left(2ia^2(a+b)\left(\frac{2a-b+b\operatorname{Cosh}[2(e+fx)]}{a}\right)^{3/2}\operatorname{EllipticE}\left[i(e+fx), \frac{b}{a}\right] - \right. \\
 & \left. ia^2(2a+b)\left(\frac{2a-b+b\operatorname{Cosh}[2(e+fx)]}{a}\right)^{3/2}\operatorname{EllipticF}\left[i(e+fx), \frac{b}{a}\right] + \right. \\
 & \left. \sqrt{2}b(a^2+2ab-b^2+b(a+b)\operatorname{Cosh}[2(e+fx)])\operatorname{Sinh}[2(e+fx)]\right) / \\
 & (3a^2b^2f(2a-b+b\operatorname{Cosh}[2(e+fx)])^{3/2})
 \end{aligned}$$

Problem 397: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Cosh}[e+fx]^2}{(a+b\operatorname{Sinh}[e+fx]^2)^{5/2}} dx$$

Optimal (type 4, 228 leaves, 5 steps):

$$\begin{aligned}
 & \frac{\operatorname{Cosh}[e+fx] \operatorname{Sinh}[e+fx]}{3af(a+b\operatorname{Sinh}[e+fx]^2)^{3/2}} + \frac{(a-2b) \operatorname{Cosh}[e+fx] \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{b}\operatorname{Sinh}[e+fx]}{\sqrt{a}}\right], 1-\frac{a}{b}\right]}{3a^{3/2}(a-b)\sqrt{b}f\sqrt{\frac{a\operatorname{Cosh}[e+fx]^2}{a+b\operatorname{Sinh}[e+fx]^2}}\sqrt{a+b\operatorname{Sinh}[e+fx]^2}} + \\
 & \left(\operatorname{EllipticF}\left[\operatorname{ArcTan}[\operatorname{Sinh}[e+fx]], 1-\frac{b}{a}\right] \operatorname{Sech}[e+fx] \sqrt{a+b\operatorname{Sinh}[e+fx]^2}\right) / \\
 & \left(3a^2(a-b)f\sqrt{\frac{\operatorname{Sech}[e+fx]^2(a+b\operatorname{Sinh}[e+fx]^2)}{a}}\right)
 \end{aligned}$$

Result (type 4, 193 leaves):

$$\left(2 i a^2 (a - 2 b) \left(\frac{2 a - b + b \operatorname{Cosh}[2 (e + f x)]}{a} \right)^{3/2} \operatorname{EllipticE}\left[i (e + f x), \frac{b}{a}\right] - \right. \\ \left. 2 i a^2 (a - b) \left(\frac{2 a - b + b \operatorname{Cosh}[2 (e + f x)]}{a} \right)^{3/2} \operatorname{EllipticF}\left[i (e + f x), \frac{b}{a}\right] - \right. \\ \left. \sqrt{2} b (-4 a^2 + 7 a b - 2 b^2 - (a - 2 b) b \operatorname{Cosh}[2 (e + f x)]) \operatorname{Sinh}[2 (e + f x)] \right) / \\ (6 a^2 (a - b) b f (2 a - b + b \operatorname{Cosh}[2 (e + f x)])^{3/2})$$

Problem 399: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Sech}[e + f x]^2}{(a + b \operatorname{Sinh}[e + f x]^2)^{5/2}} dx$$

Optimal (type 4, 292 leaves, 6 steps):

$$\frac{b (3 a + b) \operatorname{Cosh}[e + f x] \operatorname{Sinh}[e + f x]}{3 a (a - b)^2 f (a + b \operatorname{Sinh}[e + f x]^2)^{3/2}} + \\ \left(\sqrt{b} (3 a^2 + 7 a b - 2 b^2) \operatorname{Cosh}[e + f x] \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Sinh}[e + f x]}{\sqrt{a}}\right], 1 - \frac{a}{b}\right] \right) / \\ \left(3 a^{3/2} (a - b)^3 f \sqrt{\frac{a \operatorname{Cosh}[e + f x]^2}{a + b \operatorname{Sinh}[e + f x]^2}} \sqrt{a + b \operatorname{Sinh}[e + f x]^2} \right) - \\ \left((9 a - b) b \operatorname{EllipticF}\left[\operatorname{ArcTan}[\operatorname{Sinh}[e + f x]], 1 - \frac{b}{a}\right] \operatorname{Sech}[e + f x] \sqrt{a + b \operatorname{Sinh}[e + f x]^2} \right) / \\ \left(3 a^2 (a - b)^3 f \sqrt{\frac{\operatorname{Sech}[e + f x]^2 (a + b \operatorname{Sinh}[e + f x]^2)}{a}} \right) + \frac{\operatorname{Tanh}[e + f x]}{(a - b) f (a + b \operatorname{Sinh}[e + f x]^2)^{3/2}}$$

Result (type 4, 468 leaves):

$$\begin{aligned}
 & -\frac{1}{3 a^2 (a-b)^3 f} \\
 & b \left(-\left(\left(\left(\frac{15 a^2}{\sqrt{2}} - \frac{9 a b}{\sqrt{2}} + \sqrt{2} b^2 \right) \sqrt{\frac{2 a - b + b \operatorname{Cosh}[2 (e + f x)]}{a}} \operatorname{EllipticF}\left[\frac{i}{2} (e + f x), \frac{b}{a}\right] \right) \right. \right. \\
 & \quad \left. \left. \left(\sqrt{2} \sqrt{2 a - b + b \operatorname{Cosh}[2 (e + f x)]} \right) \right) - \frac{1}{2 b} \right. \\
 & \quad \left. \frac{i \left(\frac{3 a^2}{\sqrt{2}} + \frac{7 a b}{\sqrt{2}} - \sqrt{2} b^2 \right) \left(\frac{2 \sqrt{2} a \sqrt{\frac{2 a - b + b \operatorname{Cosh}[2 (e + f x)]}{a}} \operatorname{EllipticE}\left[\frac{i}{2} (e + f x), \frac{b}{a}\right]}{\sqrt{2 a - b + b \operatorname{Cosh}[2 (e + f x)]}} \right. \right. \\
 & \quad \left. \left. \frac{\sqrt{2} (2 a - b) \sqrt{\frac{2 a - b + b \operatorname{Cosh}[2 (e + f x)]}{a}} \operatorname{EllipticF}\left[\frac{i}{2} (e + f x), \frac{b}{a}\right]}{\sqrt{2 a - b + b \operatorname{Cosh}[2 (e + f x)]}} \right) \right) + \\
 & \quad \frac{1}{f} \sqrt{2 a - b + b \operatorname{Cosh}[2 (e + f x)]} \left(\frac{\sqrt{2} b^2 \operatorname{Sinh}[2 (e + f x)]}{3 a (a - b)^2 (2 a - b + b \operatorname{Cosh}[2 (e + f x)])^2} + \right. \\
 & \quad \left. \frac{7 \sqrt{2} a b^2 \operatorname{Sinh}[2 (e + f x)] - 2 \sqrt{2} b^3 \operatorname{Sinh}[2 (e + f x)]}{6 a^2 (a - b)^3 (2 a - b + b \operatorname{Cosh}[2 (e + f x)])} + \frac{\operatorname{Tanh}[e + f x]}{\sqrt{2} (a - b)^3} \right)
 \end{aligned}$$

Problem 400: Unable to integrate problem.

$$\int (d \operatorname{Cosh}[e + f x])^m (a + b \operatorname{Sinh}[e + f x]^2)^p dx$$

Optimal (type 6, 117 leaves, 3 steps):

$$\begin{aligned}
 & \frac{1}{f} d \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1-m}{2}, -p, \frac{3}{2}, -\operatorname{Sinh}[e + f x]^2, -\frac{b \operatorname{Sinh}[e + f x]^2}{a}\right] (d \operatorname{Cosh}[e + f x])^{-1+m} \\
 & \left(\operatorname{Cosh}[e + f x]^2 \right)^{\frac{1-m}{2}} \operatorname{Sinh}[e + f x] (a + b \operatorname{Sinh}[e + f x]^2)^p \left(1 + \frac{b \operatorname{Sinh}[e + f x]^2}{a} \right)^{-p}
 \end{aligned}$$

Result (type 8, 27 leaves):

$$\int (d \operatorname{Cosh}[e + f x])^m (a + b \operatorname{Sinh}[e + f x]^2)^p dx$$

Problem 401: Unable to integrate problem.

$$\int \text{Cosh}[e + f x]^5 (a + b \text{Sinh}[e + f x]^2)^p dx$$

Optimal (type 5, 214 leaves, 5 steps):

$$\begin{aligned} & - \frac{(3a - b(7 + 2p)) \text{Sinh}[e + f x] (a + b \text{Sinh}[e + f x]^2)^{1+p}}{b^2 f (3 + 2p) (5 + 2p)} + \\ & \frac{\text{Cosh}[e + f x]^2 \text{Sinh}[e + f x] (a + b \text{Sinh}[e + f x]^2)^{1+p}}{b f (5 + 2p)} + \\ & \left((3a^2 - 2ab(5 + 2p) + b^2(15 + 16p + 4p^2)) \text{Hypergeometric2F1}\left[\frac{1}{2}, -p, \frac{3}{2}, -\frac{b \text{Sinh}[e + f x]^2}{a}\right] \right. \\ & \left. \text{Sinh}[e + f x] (a + b \text{Sinh}[e + f x]^2)^p \left(1 + \frac{b \text{Sinh}[e + f x]^2}{a}\right)^{-p} \right) / (b^2 f (3 + 2p) (5 + 2p)) \end{aligned}$$

Result (type 8, 25 leaves):

$$\int \text{Cosh}[e + f x]^5 (a + b \text{Sinh}[e + f x]^2)^p dx$$

Problem 402: Unable to integrate problem.

$$\int \text{Cosh}[e + f x]^3 (a + b \text{Sinh}[e + f x]^2)^p dx$$

Optimal (type 5, 125 leaves, 4 steps):

$$\begin{aligned} & \frac{\text{Sinh}[e + f x] (a + b \text{Sinh}[e + f x]^2)^{1+p}}{b f (3 + 2p)} - \frac{1}{b f (3 + 2p)} \\ & (a - b(3 + 2p)) \text{Hypergeometric2F1}\left[\frac{1}{2}, -p, \frac{3}{2}, -\frac{b \text{Sinh}[e + f x]^2}{a}\right] \\ & \text{Sinh}[e + f x] (a + b \text{Sinh}[e + f x]^2)^p \left(1 + \frac{b \text{Sinh}[e + f x]^2}{a}\right)^{-p} \end{aligned}$$

Result (type 8, 25 leaves):

$$\int \text{Cosh}[e + f x]^3 (a + b \text{Sinh}[e + f x]^2)^p dx$$

Problem 404: Unable to integrate problem.

$$\int \text{Sech}[e + f x] (a + b \text{Sinh}[e + f x]^2)^p dx$$

Optimal (type 6, 78 leaves, 3 steps):

$$\frac{1}{f} \text{AppellF1}\left[\frac{1}{2}, 1, -p, \frac{3}{2}, -\text{Sinh}[e + f x]^2, -\frac{b \text{Sinh}[e + f x]^2}{a}\right]$$

$$\text{Sinh}[e + f x] (a + b \text{Sinh}[e + f x]^2)^p \left(1 + \frac{b \text{Sinh}[e + f x]^2}{a}\right)^{-p}$$

Result (type 8, 23 leaves):

$$\int \text{Sech}[e + f x] (a + b \text{Sinh}[e + f x]^2)^p dx$$

Problem 405: Unable to integrate problem.

$$\int \text{Sech}[e + f x]^3 (a + b \text{Sinh}[e + f x]^2)^p dx$$

Optimal (type 6, 78 leaves, 3 steps):

$$\frac{1}{f} \text{AppellF1}\left[\frac{1}{2}, 2, -p, \frac{3}{2}, -\text{Sinh}[e + f x]^2, -\frac{b \text{Sinh}[e + f x]^2}{a}\right]$$

$$\text{Sinh}[e + f x] (a + b \text{Sinh}[e + f x]^2)^p \left(1 + \frac{b \text{Sinh}[e + f x]^2}{a}\right)^{-p}$$

Result (type 8, 25 leaves):

$$\int \text{Sech}[e + f x]^3 (a + b \text{Sinh}[e + f x]^2)^p dx$$

Problem 406: Unable to integrate problem.

$$\int \text{Cosh}[e + f x]^4 (a + b \text{Sinh}[e + f x]^2)^p dx$$

Optimal (type 6, 92 leaves, 3 steps):

$$\frac{1}{f} \text{AppellF1}\left[\frac{1}{2}, -\frac{3}{2}, -p, \frac{3}{2}, -\text{Sinh}[e + f x]^2, -\frac{b \text{Sinh}[e + f x]^2}{a}\right]$$

$$\sqrt{\text{Cosh}[e + f x]^2} (a + b \text{Sinh}[e + f x]^2)^p \left(1 + \frac{b \text{Sinh}[e + f x]^2}{a}\right)^{-p} \text{Tanh}[e + f x]$$

Result (type 8, 25 leaves):

$$\int \text{Cosh}[e + f x]^4 (a + b \text{Sinh}[e + f x]^2)^p dx$$

Problem 407: Unable to integrate problem.

$$\int \text{Cosh}[e + f x]^2 (a + b \text{Sinh}[e + f x]^2)^p dx$$

Optimal (type 6, 92 leaves, 3 steps):

$$\frac{1}{f} \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, -p, \frac{3}{2}, -\text{Sinh}[e + f x]^2, -\frac{b \text{Sinh}[e + f x]^2}{a}\right] \sqrt{\text{Cosh}[e + f x]^2} (a + b \text{Sinh}[e + f x]^2)^p \left(1 + \frac{b \text{Sinh}[e + f x]^2}{a}\right)^{-p} \text{Tanh}[e + f x]$$

Result (type 8, 25 leaves):

$$\int \text{Cosh}[e + f x]^2 (a + b \text{Sinh}[e + f x]^2)^p dx$$

Problem 408: Unable to integrate problem.

$$\int (a + b \text{Sinh}[e + f x]^2)^p dx$$

Optimal (type 6, 92 leaves, 3 steps):

$$\frac{1}{f} \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -p, \frac{3}{2}, -\text{Sinh}[e + f x]^2, -\frac{b \text{Sinh}[e + f x]^2}{a}\right] \sqrt{\text{Cosh}[e + f x]^2} (a + b \text{Sinh}[e + f x]^2)^p \left(1 + \frac{b \text{Sinh}[e + f x]^2}{a}\right)^{-p} \text{Tanh}[e + f x]$$

Result (type 8, 16 leaves):

$$\int (a + b \text{Sinh}[e + f x]^2)^p dx$$

Problem 409: Unable to integrate problem.

$$\int \text{Sech}[e + f x]^2 (a + b \text{Sinh}[e + f x]^2)^p dx$$

Optimal (type 6, 92 leaves, 3 steps):

$$\frac{1}{f} \text{AppellF1}\left[\frac{1}{2}, \frac{3}{2}, -p, \frac{3}{2}, -\text{Sinh}[e + f x]^2, -\frac{b \text{Sinh}[e + f x]^2}{a}\right] \sqrt{\text{Cosh}[e + f x]^2} (a + b \text{Sinh}[e + f x]^2)^p \left(1 + \frac{b \text{Sinh}[e + f x]^2}{a}\right)^{-p} \text{Tanh}[e + f x]$$

Result (type 8, 25 leaves):

$$\int \text{Sech}[e + f x]^2 (a + b \text{Sinh}[e + f x]^2)^p dx$$

Problem 410: Unable to integrate problem.

$$\int \text{Sech}[e + f x]^4 (a + b \text{Sinh}[e + f x]^2)^p dx$$

Optimal (type 6, 92 leaves, 3 steps):

$$\frac{1}{f} \text{AppellF1}\left[\frac{1}{2}, \frac{5}{2}, -p, \frac{3}{2}, -\text{Sinh}[e+fx]^2, -\frac{b \text{Sinh}[e+fx]^2}{a}\right] \sqrt{\text{Cosh}[e+fx]^2} (a+b \text{Sinh}[e+fx]^2)^p \left(1 + \frac{b \text{Sinh}[e+fx]^2}{a}\right)^{-p} \text{Tanh}[e+fx]$$

Result (type 8, 25 leaves):

$$\int \text{Sech}[e+fx]^4 (a+b \text{Sinh}[e+fx]^2)^p dx$$

Problem 412: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Cosh}[c+dx]^3}{a+b \sqrt{\text{Sinh}[c+dx]}} dx$$

Optimal (type 3, 136 leaves, 4 steps):

$$\frac{2a(a^4+b^4) \text{Log}[a+b \sqrt{\text{Sinh}[c+dx]}]}{b^6 d} + \frac{2(a^4+b^4) \sqrt{\text{Sinh}[c+dx]}}{b^5 d} - \frac{a^3 \text{Sinh}[c+dx]}{b^4 d} + \frac{2a^2 \text{Sinh}[c+dx]^{3/2}}{3b^3 d} - \frac{a \text{Sinh}[c+dx]^2}{2b^2 d} + \frac{2 \text{Sinh}[c+dx]^{5/2}}{5b d}$$

Result (type 3, 311 leaves):

$$\frac{a \text{Cosh}[2(c+dx)]}{4b^2 d} + \frac{(-a^5 - a b^4) \text{Log}[a^2 - b^2 \text{Sinh}[c+dx]]}{b^6 d} - \frac{a^3 \text{Sinh}[c+dx]}{b^4 d} + \frac{\sqrt{\text{Sinh}[c+dx]} \left(\frac{\text{Cosh}[2(c+dx)]}{5b} + \frac{2a^2 \text{Sinh}[c+dx]}{3b^3} \right)}{d} - \frac{1}{20b^3 d} \left(\left(\left(4ab \text{ArcTanh}\left[\frac{b \sqrt{\text{Sinh}[c+dx]}}{a}\right] \text{Cosh}[c+dx]^2 (-a^2 + b^2 \text{Sinh}[c+dx]) \right) \right) / \left((a^2 - b^2 \text{Sinh}[c+dx]) (1 + \text{Sinh}[c+dx]^2) \right) \right) - \left(2(10a^4 + 9b^4) \text{Coth}[c+dx] \left(\frac{a \text{ArcTanh}\left[\frac{b \sqrt{\text{Sinh}[c+dx]}}{a}\right]}{b^3} - \frac{\sqrt{\text{Sinh}[c+dx]}}{b^2} \right) (-a^2 + b^2 \text{Sinh}[c+dx]) \text{Sinh}[2(c+dx)] \right) / \left((a^2 - b^2 \text{Sinh}[c+dx]) (1 + \text{Sinh}[c+dx]^2) \right) \right)$$

Problem 418: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{Sech}[c+dx]}{(a+b \sqrt{\text{Sinh}[c+dx]})^2} dx$$

Optimal (type 3, 384 leaves, 19 steps):

$$\frac{\sqrt{2} a b (a^4 - 2 a^2 b^2 - b^4) \operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{\operatorname{Sinh}[c + d x]}\right]}{(a^4 + b^4)^2 d} -$$

$$\frac{\sqrt{2} a b (a^4 - 2 a^2 b^2 - b^4) \operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{\operatorname{Sinh}[c + d x]}\right]}{(a^4 + b^4)^2 d} + \frac{a^2 (a^4 - 3 b^4) \operatorname{ArcTan}[\operatorname{Sinh}[c + d x]]}{(a^4 + b^4)^2 d} +$$

$$\frac{b^2 (3 a^4 - b^4) \operatorname{Log}[\operatorname{Cosh}[c + d x]]}{(a^4 + b^4)^2 d} - \frac{2 b^2 (3 a^4 - b^4) \operatorname{Log}\left[a + b \sqrt{\operatorname{Sinh}[c + d x]}\right]}{(a^4 + b^4)^2 d} -$$

$$\frac{a b (a^4 + 2 a^2 b^2 - b^4) \operatorname{Log}\left[1 - \sqrt{2} \sqrt{\operatorname{Sinh}[c + d x]} + \operatorname{Sinh}[c + d x]\right]}{\sqrt{2} (a^4 + b^4)^2 d} +$$

$$\frac{a b (a^4 + 2 a^2 b^2 - b^4) \operatorname{Log}\left[1 + \sqrt{2} \sqrt{\operatorname{Sinh}[c + d x]} + \operatorname{Sinh}[c + d x]\right]}{\sqrt{2} (a^4 + b^4)^2 d} +$$

$$\frac{2 a b^2}{(a^4 + b^4) d \left(a + b \sqrt{\operatorname{Sinh}[c + d x]}\right)}$$

Result (type 3, 708 leaves):

$$\begin{aligned}
 & \frac{1}{2d} \left(\frac{2\sqrt{2} a^3 b (a^2 - b^2) \operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{\operatorname{Sinh}[c + dx]}\right]}{(a^4 + b^4)^2} - \right. \\
 & \frac{2\sqrt{2} a b^3 (a^2 + b^2) \operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{\operatorname{Sinh}[c + dx]}\right]}{(a^4 + b^4)^2} - \\
 & \frac{2\sqrt{2} a^3 b (a^2 - b^2) \operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{\operatorname{Sinh}[c + dx]}\right]}{(a^4 + b^4)^2} + \\
 & \frac{2\sqrt{2} a b^3 (a^2 + b^2) \operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{\operatorname{Sinh}[c + dx]}\right]}{(a^4 + b^4)^2} + \frac{2(a^2 - ib^2) \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2}(c + dx)\right]\right]}{(a^2 + ib^2)^2} + \\
 & \frac{2(a^2 + ib^2) \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2}(c + dx)\right]\right]}{(a^2 - ib^2)^2} - \frac{10a^4 b^2 \operatorname{ArcTanh}\left[\frac{b\sqrt{\operatorname{Sinh}[c+dx]}}{a}\right]}{(a^4 + b^4)^2} + \\
 & \frac{6b^6 \operatorname{ArcTanh}\left[\frac{b\sqrt{\operatorname{Sinh}[c+dx]}}{a}\right]}{(a^4 + b^4)^2} - \frac{2b^2 \operatorname{ArcTanh}\left[\frac{b\sqrt{\operatorname{Sinh}[c+dx]}}{a}\right]}{a^4 + b^4} + \frac{(-ia^2 + b^2) \operatorname{Log}[\operatorname{Cosh}[c + dx]]}{(a^2 - ib^2)^2} + \\
 & \frac{(ia^2 + b^2) \operatorname{Log}[\operatorname{Cosh}[c + dx]]}{(a^2 + ib^2)^2} - \frac{1}{(a^4 + b^4)^2} \sqrt{2} a b^3 (a^2 - b^2) \\
 & \left(\operatorname{Log}\left[1 - \sqrt{2} \sqrt{\operatorname{Sinh}[c + dx]} + \operatorname{Sinh}[c + dx]\right] - \operatorname{Log}\left[1 + \sqrt{2} \sqrt{\operatorname{Sinh}[c + dx]} + \operatorname{Sinh}[c + dx]\right] \right) - \\
 & \frac{1}{(a^4 + b^4)^2} \sqrt{2} a^3 b (a^2 + b^2) \left(\operatorname{Log}\left[1 - \sqrt{2} \sqrt{\operatorname{Sinh}[c + dx]} + \operatorname{Sinh}[c + dx]\right] - \right. \\
 & \left. \operatorname{Log}\left[1 + \sqrt{2} \sqrt{\operatorname{Sinh}[c + dx]} + \operatorname{Sinh}[c + dx]\right] \right) + \frac{2(-3a^4 b^2 + b^6) \operatorname{Log}\left[a^2 - b^2 \operatorname{Sinh}[c + dx]\right]}{(a^4 + b^4)^2} + \\
 & \left. \frac{4a^2 b^2}{(a^4 + b^4)(a^2 - b^2 \operatorname{Sinh}[c + dx])} - \frac{4ab^3 \sqrt{\operatorname{Sinh}[c + dx]}}{(a^4 + b^4)(a^2 - b^2 \operatorname{Sinh}[c + dx])} \right)
 \end{aligned}$$

Problem 419: Unable to integrate problem.

$$\int \frac{\operatorname{Cosh}[c + dx]^5}{a + b \operatorname{Sinh}[c + dx]^n} dx$$

Optimal (type 5, 130 leaves, 6 steps):

$$\begin{aligned}
 & \frac{\operatorname{Hypergeometric2F1}\left[1, \frac{1}{n}, 1 + \frac{1}{n}, -\frac{b \operatorname{Sinh}[c+dx]^n}{a}\right] \operatorname{Sinh}[c + dx]}{ad} + \\
 & \frac{2 \operatorname{Hypergeometric2F1}\left[1, \frac{3}{n}, \frac{3+n}{n}, -\frac{b \operatorname{Sinh}[c+dx]^n}{a}\right] \operatorname{Sinh}[c + dx]^3}{3ad} + \\
 & \frac{\operatorname{Hypergeometric2F1}\left[1, \frac{5}{n}, \frac{5+n}{n}, -\frac{b \operatorname{Sinh}[c+dx]^n}{a}\right] \operatorname{Sinh}[c + dx]^5}{5ad}
 \end{aligned}$$

Result (type 8, 25 leaves):

$$\int \frac{\text{Cosh}[c + d x]^5}{a + b \text{Sinh}[c + d x]^n} dx$$

Problem 420: Unable to integrate problem.

$$\int \frac{\text{Cosh}[c + d x]^3}{a + b \text{Sinh}[c + d x]^n} dx$$

Optimal (type 5, 84 leaves, 5 steps):

$$\frac{\text{Hypergeometric2F1}\left[1, \frac{1}{n}, 1 + \frac{1}{n}, -\frac{b \text{Sinh}[c+dx]^n}{a}\right] \text{Sinh}[c + d x]}{a d} + \frac{\text{Hypergeometric2F1}\left[1, \frac{3}{n}, \frac{3+n}{n}, -\frac{b \text{Sinh}[c+dx]^n}{a}\right] \text{Sinh}[c + d x]^3}{3 a d}$$

Result (type 8, 25 leaves):

$$\int \frac{\text{Cosh}[c + d x]^3}{a + b \text{Sinh}[c + d x]^n} dx$$

Problem 422: Attempted integration timed out after 120 seconds.

$$\int \frac{\text{Cosh}[c + d x]^5}{(a + b \text{Sinh}[c + d x]^n)^2} dx$$

Optimal (type 5, 130 leaves, 6 steps):

$$\frac{\text{Hypergeometric2F1}\left[2, \frac{1}{n}, 1 + \frac{1}{n}, -\frac{b \text{Sinh}[c+dx]^n}{a}\right] \text{Sinh}[c + d x]}{a^2 d} + \frac{2 \text{Hypergeometric2F1}\left[2, \frac{3}{n}, \frac{3+n}{n}, -\frac{b \text{Sinh}[c+dx]^n}{a}\right] \text{Sinh}[c + d x]^3}{3 a^2 d} + \frac{\text{Hypergeometric2F1}\left[2, \frac{5}{n}, \frac{5+n}{n}, -\frac{b \text{Sinh}[c+dx]^n}{a}\right] \text{Sinh}[c + d x]^5}{5 a^2 d}$$

Result (type 1, 1 leaves):

???

Problem 423: Attempted integration timed out after 120 seconds.

$$\int \frac{\text{Cosh}[c + d x]^3}{(a + b \text{Sinh}[c + d x]^n)^2} dx$$

Optimal (type 5, 84 leaves, 5 steps):

$$\frac{\text{Hypergeometric2F1}\left[2, \frac{1}{n}, 1 + \frac{1}{n}, -\frac{b \text{Sinh}[c+dx]^n}{a}\right] \text{Sinh}[c+dx]}{a^2 d} + \frac{\text{Hypergeometric2F1}\left[2, \frac{3}{n}, \frac{3+n}{n}, -\frac{b \text{Sinh}[c+dx]^n}{a}\right] \text{Sinh}[c+dx]^3}{3 a^2 d}$$

Result (type 1, 1 leaves):

???

Problem 457: Result more than twice size of optimal antiderivative.

$$\int \sqrt{a + b \text{Sinh}[e + f x]^2} \text{Tanh}[e + f x]^5 dx$$

Optimal (type 3, 187 leaves, 6 steps):

$$-\frac{(8 a^2 - 24 a b + 15 b^2) \text{ArcTanh}\left[\frac{\sqrt{a+b \text{Sinh}[e+f x]^2}}{\sqrt{a-b}}\right]}{8 (a-b)^{3/2} f} + \frac{(8 a^2 - 24 a b + 15 b^2) \sqrt{a + b \text{Sinh}[e + f x]^2}}{8 (a-b)^2 f} + \frac{(8 a - 7 b) \text{Sech}[e + f x]^2 (a + b \text{Sinh}[e + f x]^2)^{3/2}}{8 (a-b)^2 f} - \frac{\text{Sech}[e + f x]^4 (a + b \text{Sinh}[e + f x]^2)^{3/2}}{4 (a-b) f}$$

Result (type 3, 631 leaves):

$$\frac{\sqrt{2 a-b+b \operatorname{Cosh}[2(e+f x)]}}{f} \left(\frac{(8 a-9 b) \operatorname{Sech}[e+f x]^2}{8 \sqrt{2}(a-b)} - \frac{\operatorname{Sech}[e+f x]^4}{4 \sqrt{2}} \right) + \frac{1}{4(a-b) f}$$

$$\left(-\frac{1}{\sqrt{2 a-2 b}} \left(4 \sqrt{2} a^2 - \frac{a b}{\sqrt{2}} - 11 \sqrt{2} a b + 7 \sqrt{2} b^2 \right) \operatorname{ArcTanh}\left[\frac{\sqrt{2 a-b+b \operatorname{Cosh}[2(e+f x)]}}{\sqrt{2 a-2 b}} \right] + \right.$$

$$\left. \left(4 \sqrt{2} \left(\frac{3 a b}{\sqrt{2}} - \frac{3 b^2}{\sqrt{2}} \right) (1+\operatorname{Cosh}[e+f x]) \sqrt{\frac{2 a-b+b \operatorname{Cosh}[2(e+f x)]}{(1+\operatorname{Cosh}[e+f x])^2}} \right. \right.$$

$$\left. \left. \sqrt{\left(a-2 a \operatorname{Tanh}\left[\frac{1}{2}(e+f x) \right]^2 + 4 b \operatorname{Tanh}\left[\frac{1}{2}(e+f x) \right]^2 + a \operatorname{Tanh}\left[\frac{1}{2}(e+f x) \right]^4 \right)} \right) / \right.$$

$$\left. \left(\sqrt{2 a-b+b \operatorname{Cosh}[2(e+f x)]} \left(4 b-4 b \operatorname{Tanh}\left[\frac{1}{2}(e+f x) \right]^2 \right) \right) + \right.$$

$$\left. \left(1 / \left(\sqrt{2} \sqrt{a-b} b \sqrt{2 a-b+b \operatorname{Cosh}[2(e+f x)]} \left(-1+\operatorname{Tanh}\left[\frac{1}{2}(e+f x) \right]^2 \right) \right) \right) \left(\frac{a b}{\sqrt{2}} - \frac{b^2}{\sqrt{2}} \right) \right.$$

$$\left. (1+\operatorname{Cosh}[e+f x]) \sqrt{\frac{2 a-b+b \operatorname{Cosh}[2(e+f x)]}{(1+\operatorname{Cosh}[e+f x])^2}} \left(b \operatorname{Log}\left[a-b-a \operatorname{Tanh}\left[\frac{1}{2}(e+f x) \right]^2 + \right. \right. \right.$$

$$\left. \left. b \operatorname{Tanh}\left[\frac{1}{2}(e+f x) \right]^2 + \sqrt{a-b} \sqrt{4 b \operatorname{Tanh}\left[\frac{1}{2}(e+f x) \right]^2 + a \left(-1+\operatorname{Tanh}\left[\frac{1}{2}(e+f x) \right]^2 \right)^2} \right)^2 \right.$$

$$\left. \left(-1+\operatorname{Tanh}\left[\frac{1}{2}(e+f x) \right]^2 \right) + \operatorname{Log}\left[1+\operatorname{Tanh}\left[\frac{1}{2}(e+f x) \right]^2 \right] \left(b-b \operatorname{Tanh}\left[\frac{1}{2}(e+f x) \right]^2 \right) - \right.$$

$$\left. \left. 2 \sqrt{a-b} \sqrt{4 b \operatorname{Tanh}\left[\frac{1}{2}(e+f x) \right]^2 + a \left(-1+\operatorname{Tanh}\left[\frac{1}{2}(e+f x) \right]^2 \right)^2} \right) \right)$$

Problem 458: Result more than twice size of optimal antiderivative.

$$\int \sqrt{a+b \operatorname{Sinh}[e+f x]^2} \operatorname{Tanh}[e+f x]^3 dx$$

Optimal (type 3, 126 leaves, 5 steps):

$$-\frac{(2 a-3 b) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sinh}[e+f x]^2}}{\sqrt{a-b}} \right]}{2 \sqrt{a-b} f} +$$

$$\frac{(2 a-3 b) \sqrt{a+b \operatorname{Sinh}[e+f x]^2}}{2(a-b) f} + \frac{\operatorname{Sech}[e+f x]^2 (a+b \operatorname{Sinh}[e+f x]^2)^{3/2}}{2(a-b) f}$$

Result (type 3, 523 leaves):

$$\begin{aligned}
 & \frac{\sqrt{2a-b+b \operatorname{Cosh}[2(e+fx)]} \operatorname{Sech}[e+fx]^2}{2\sqrt{2}f} + \\
 & \frac{1}{2f} \left(-\frac{\left(2\sqrt{2}a - \frac{11b}{2\sqrt{2}}\right) \operatorname{ArcTanh}\left[\frac{\sqrt{2a-b+b \operatorname{Cosh}[2(e+fx)]}}{\sqrt{2a-2b}}\right]}{\sqrt{2a-2b}} + \right. \\
 & \left. \left(6b(1+\operatorname{Cosh}[e+fx]) \sqrt{\frac{2a-b+b \operatorname{Cosh}[2(e+fx)]}{(1+\operatorname{Cosh}[e+fx])^2}} \right. \right. \\
 & \left. \left. \sqrt{\left(a-2a \operatorname{Tanh}\left[\frac{1}{2}(e+fx)\right]^2 + 4b \operatorname{Tanh}\left[\frac{1}{2}(e+fx)\right]^2 + a \operatorname{Tanh}\left[\frac{1}{2}(e+fx)\right]^4\right)} \right) \right) / \\
 & \left(\sqrt{2a-b+b \operatorname{Cosh}[2(e+fx)]} \left(4b-4b \operatorname{Tanh}\left[\frac{1}{2}(e+fx)\right]^2\right) \right) + \\
 & \frac{1}{4\sqrt{a-b} \sqrt{2a-b+b \operatorname{Cosh}[2(e+fx)]} \left(-1+\operatorname{Tanh}\left[\frac{1}{2}(e+fx)\right]^2\right)} \\
 & \left((1+\operatorname{Cosh}[e+fx]) \sqrt{\frac{2a-b+b \operatorname{Cosh}[2(e+fx)]}{(1+\operatorname{Cosh}[e+fx])^2}} \left(b \operatorname{Log}\left[a-b-a \operatorname{Tanh}\left[\frac{1}{2}(e+fx)\right]^2 + \right. \right. \right. \\
 & \left. \left. \left. b \operatorname{Tanh}\left[\frac{1}{2}(e+fx)\right]^2 + \sqrt{a-b} \sqrt{4b \operatorname{Tanh}\left[\frac{1}{2}(e+fx)\right]^2 + a \left(-1+\operatorname{Tanh}\left[\frac{1}{2}(e+fx)\right]^2\right)^2} \right] \right) \right. \\
 & \left. \left(-1+\operatorname{Tanh}\left[\frac{1}{2}(e+fx)\right]^2\right) + \operatorname{Log}\left[1+\operatorname{Tanh}\left[\frac{1}{2}(e+fx)\right]^2\right] \left(b-b \operatorname{Tanh}\left[\frac{1}{2}(e+fx)\right]^2\right) - \right. \\
 & \left. \left. 2\sqrt{a-b} \sqrt{4b \operatorname{Tanh}\left[\frac{1}{2}(e+fx)\right]^2 + a \left(-1+\operatorname{Tanh}\left[\frac{1}{2}(e+fx)\right]^2\right)^2} \right) \right) \right)
 \end{aligned}$$

Problem 463: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{a+b \operatorname{Sinh}[e+fx]^2} \operatorname{Tanh}[e+fx]^4 dx$$

Optimal (type 4, 292 leaves, 7 steps):

$$\begin{aligned}
 & - \left(\left((7a - 8b) \operatorname{EllipticE}[\operatorname{ArcTan}[\operatorname{Sinh}[e + fx]], 1 - \frac{b}{a}] \operatorname{Sech}[e + fx] \sqrt{a + b \operatorname{Sinh}[e + fx]^2} \right) / \right. \\
 & \quad \left. \left(3(a - b) f \sqrt{\frac{\operatorname{Sech}[e + fx]^2 (a + b \operatorname{Sinh}[e + fx]^2)}{a}} \right) \right) + \\
 & \left((3a - 4b) \operatorname{EllipticF}[\operatorname{ArcTan}[\operatorname{Sinh}[e + fx]], 1 - \frac{b}{a}] \operatorname{Sech}[e + fx] \sqrt{a + b \operatorname{Sinh}[e + fx]^2} \right) / \\
 & \quad \left(3(a - b) f \sqrt{\frac{\operatorname{Sech}[e + fx]^2 (a + b \operatorname{Sinh}[e + fx]^2)}{a}} \right) + \\
 & \frac{(7a - 8b) \sqrt{a + b \operatorname{Sinh}[e + fx]^2} \operatorname{Tanh}[e + fx]}{3(a - b) f} - \frac{(3a - 4b) \sqrt{a + b \operatorname{Sinh}[e + fx]^2} \operatorname{Tanh}[e + fx]}{3(a - b) f} - \\
 & \frac{\sqrt{a + b \operatorname{Sinh}[e + fx]^2} \operatorname{Tanh}[e + fx]^3}{3f}
 \end{aligned}$$

Result (type 4, 214 leaves):

$$\begin{aligned}
 & \left(-2ia(7a - 8b) \sqrt{\frac{2a - b + b \operatorname{Cosh}[2(e + fx)]}{a}} \operatorname{EllipticE}\left[i(e + fx), \frac{b}{a}\right] + \right. \\
 & \quad 8ia(a - b) \sqrt{\frac{2a - b + b \operatorname{Cosh}[2(e + fx)]}{a}} \operatorname{EllipticF}\left[i(e + fx), \frac{b}{a}\right] - \frac{1}{2\sqrt{2}} \\
 & \quad \left. (8a^2 - 12ab + b^2 + 4(4a^2 - 6ab + b^2) \operatorname{Cosh}[2(e + fx)] + (4a - 5b)b \operatorname{Cosh}[4(e + fx)]) \right) \\
 & \quad \left. \operatorname{Sech}[e + fx]^2 \operatorname{Tanh}[e + fx] \right) / \left(6(a - b) f \sqrt{2a - b + b \operatorname{Cosh}[2(e + fx)]} \right)
 \end{aligned}$$

Problem 464: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{a + b \operatorname{Sinh}[e + fx]^2} \operatorname{Tanh}[e + fx]^2 dx$$

Optimal (type 4, 168 leaves, 6 steps):

$$\begin{aligned}
 & - \left(2 \operatorname{EllipticE} \left[\operatorname{ArcTan}[\operatorname{Sinh}[e + f x]], 1 - \frac{b}{a} \right] \operatorname{Sech}[e + f x] \sqrt{a + b \operatorname{Sinh}[e + f x]^2} \right) / \\
 & \left(f \sqrt{\frac{\operatorname{Sech}[e + f x]^2 (a + b \operatorname{Sinh}[e + f x]^2)}{a}} \right) + \\
 & \left(\operatorname{EllipticF} \left[\operatorname{ArcTan}[\operatorname{Sinh}[e + f x]], 1 - \frac{b}{a} \right] \operatorname{Sech}[e + f x] \sqrt{a + b \operatorname{Sinh}[e + f x]^2} \right) / \\
 & \left(f \sqrt{\frac{\operatorname{Sech}[e + f x]^2 (a + b \operatorname{Sinh}[e + f x]^2)}{a}} \right) + \frac{\sqrt{a + b \operatorname{Sinh}[e + f x]^2} \operatorname{Tanh}[e + f x]}{f}
 \end{aligned}$$

Result (type 4, 150 leaves):

$$\begin{aligned}
 & \left(-2 i \sqrt{2} a \sqrt{\frac{2 a - b + b \operatorname{Cosh}[2 (e + f x)]}{a}} \operatorname{EllipticE} \left[i (e + f x), \frac{b}{a} \right] + \right. \\
 & \left. i \sqrt{2} a \sqrt{\frac{2 a - b + b \operatorname{Cosh}[2 (e + f x)]}{a}} \operatorname{EllipticF} \left[i (e + f x), \frac{b}{a} \right] + \right. \\
 & \left. (-2 a + b - b \operatorname{Cosh}[2 (e + f x)]) \operatorname{Tanh}[e + f x] \right) / \left(f \sqrt{4 a - 2 b + 2 b \operatorname{Cosh}[2 (e + f x)]} \right)
 \end{aligned}$$

Problem 466: Result unnecessarily involves imaginary or complex numbers.

$$\int \operatorname{Coth}[e + f x]^2 \sqrt{a + b \operatorname{Sinh}[e + f x]^2} dx$$

Optimal (type 4, 202 leaves, 6 steps):

$$\begin{aligned}
 & \frac{\operatorname{Coth}[e + f x] \sqrt{a + b \operatorname{Sinh}[e + f x]^2}}{f} - \\
 & \left(2 \operatorname{EllipticE} \left[\operatorname{ArcTan}[\operatorname{Sinh}[e + f x]], 1 - \frac{b}{a} \right] \operatorname{Sech}[e + f x] \sqrt{a + b \operatorname{Sinh}[e + f x]^2} \right) / \\
 & \left(f \sqrt{\frac{\operatorname{Sech}[e + f x]^2 (a + b \operatorname{Sinh}[e + f x]^2)}{a}} \right) + \\
 & \left((a + b) \operatorname{EllipticF} \left[\operatorname{ArcTan}[\operatorname{Sinh}[e + f x]], 1 - \frac{b}{a} \right] \operatorname{Sech}[e + f x] \sqrt{a + b \operatorname{Sinh}[e + f x]^2} \right) / \\
 & \left(a f \sqrt{\frac{\operatorname{Sech}[e + f x]^2 (a + b \operatorname{Sinh}[e + f x]^2)}{a}} \right) + \frac{2 \sqrt{a + b \operatorname{Sinh}[e + f x]^2} \operatorname{Tanh}[e + f x]}{f}
 \end{aligned}$$

Result (type 4, 154 leaves):

$$\left((-2a + b - b \operatorname{Cosh}[2(e + fx)]) \operatorname{Coth}[e + fx] - \right. \\ \left. 2i\sqrt{2} a \sqrt{\frac{2a - b + b \operatorname{Cosh}[2(e + fx)]}{a}} \operatorname{EllipticE}\left[i(e + fx), \frac{b}{a}\right] + \right. \\ \left. i\sqrt{2} (a - b) \sqrt{\frac{2a - b + b \operatorname{Cosh}[2(e + fx)]}{a}} \operatorname{EllipticF}\left[i(e + fx), \frac{b}{a}\right] \right) / \\ \left(f \sqrt{4a - 2b + 2b \operatorname{Cosh}[2(e + fx)]} \right)$$

Problem 467: Result unnecessarily involves imaginary or complex numbers.

$$\int \operatorname{Coth}[e + fx]^4 \sqrt{a + b \operatorname{Sinh}[e + fx]^2} dx$$

Optimal (type 4, 270 leaves, 7 steps):

$$\frac{(3a + b) \operatorname{Coth}[e + fx] \sqrt{a + b \operatorname{Sinh}[e + fx]^2}}{3af} - \frac{\operatorname{Coth}[e + fx]^3 \sqrt{a + b \operatorname{Sinh}[e + fx]^2}}{3f} - \\ \left((7a + b) \operatorname{EllipticE}\left[\operatorname{ArcTan}[\operatorname{Sinh}[e + fx]], 1 - \frac{b}{a}\right] \operatorname{Sech}[e + fx] \sqrt{a + b \operatorname{Sinh}[e + fx]^2} \right) / \\ \left(3af \sqrt{\frac{\operatorname{Sech}[e + fx]^2 (a + b \operatorname{Sinh}[e + fx]^2)}{a}} \right) + \\ \left((3a + 5b) \operatorname{EllipticF}\left[\operatorname{ArcTan}[\operatorname{Sinh}[e + fx]], 1 - \frac{b}{a}\right] \operatorname{Sech}[e + fx] \sqrt{a + b \operatorname{Sinh}[e + fx]^2} \right) / \\ \left(3af \sqrt{\frac{\operatorname{Sech}[e + fx]^2 (a + b \operatorname{Sinh}[e + fx]^2)}{a}} \right) + \frac{(7a + b) \sqrt{a + b \operatorname{Sinh}[e + fx]^2} \operatorname{Tanh}[e + fx]}{3af}$$

Result (type 4, 376 leaves):

$$\begin{aligned}
 & \frac{1}{f} \sqrt{2a - b + b \operatorname{Cosh}[2(e + fx)]} \\
 & \left(\frac{(-4\sqrt{2} a \operatorname{Cosh}[e + fx] - \sqrt{2} b \operatorname{Cosh}[e + fx]) \operatorname{Csch}[e + fx]}{6a} - \frac{\operatorname{Coth}[e + fx] \operatorname{Csch}[e + fx]^2}{3\sqrt{2}} \right) + \\
 & \frac{1}{3af} \left(\left(\left(i \left(3\sqrt{2} a^2 + \frac{3ab}{\sqrt{2}} - \frac{b^2}{\sqrt{2}} \right) \sqrt{\frac{2a - b + b \operatorname{Cosh}[2(e + fx)]}{a}} \operatorname{EllipticF}\left[i(e + fx), \frac{b}{a}\right] \right) / \right. \right. \\
 & \left. \left. \left(\sqrt{2} \sqrt{2a - b + b \operatorname{Cosh}[2(e + fx)]} \right) \right) - \frac{1}{2b} \right. \\
 & \left. i \left(\frac{7ab}{\sqrt{2}} + \frac{b^2}{\sqrt{2}} \right) \left(\frac{2\sqrt{2} a \sqrt{\frac{2a - b + b \operatorname{Cosh}[2(e + fx)]}{a}} \operatorname{EllipticE}\left[i(e + fx), \frac{b}{a}\right]}{\sqrt{2a - b + b \operatorname{Cosh}[2(e + fx)]}} - \right. \right. \\
 & \left. \left. \frac{\sqrt{2}(2a - b) \sqrt{\frac{2a - b + b \operatorname{Cosh}[2(e + fx)]}{a}} \operatorname{EllipticF}\left[i(e + fx), \frac{b}{a}\right]}{\sqrt{2a - b + b \operatorname{Cosh}[2(e + fx)]}} \right) \right)
 \end{aligned}$$

Problem 468: Result more than twice size of optimal antiderivative.

$$\int (a + b \operatorname{Sinh}[e + fx]^2)^{3/2} \operatorname{Tanh}[e + fx]^5 dx$$

Optimal (type 3, 232 leaves, 7 steps):

$$\begin{aligned}
 & - \frac{(8a^2 - 40ab + 35b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{a + b \operatorname{Sinh}[e + fx]^2}}{\sqrt{a - b}}\right]}{8\sqrt{a - b} f} + \\
 & \frac{(8a^2 - 40ab + 35b^2) \sqrt{a + b \operatorname{Sinh}[e + fx]^2}}{8(a - b) f} + \frac{(8a^2 - 40ab + 35b^2) (a + b \operatorname{Sinh}[e + fx]^2)^{3/2}}{24(a - b)^2 f} + \\
 & \frac{(8a - 9b) \operatorname{Sech}[e + fx]^2 (a + b \operatorname{Sinh}[e + fx]^2)^{5/2}}{8(a - b)^2 f} - \frac{\operatorname{Sech}[e + fx]^4 (a + b \operatorname{Sinh}[e + fx]^2)^{5/2}}{4(a - b) f}
 \end{aligned}$$

Result (type 3, 648 leaves):

$$\frac{1}{f} \sqrt{2a - b + b \operatorname{Cosh}[2(e + fx)]}$$

$$\left(\frac{b \operatorname{Cosh}[2(e + fx)]}{6\sqrt{2}} + \frac{(8a - 13b) \operatorname{Sech}[e + fx]^2}{8\sqrt{2}} - \frac{(a - b) \operatorname{Sech}[e + fx]^4}{4\sqrt{2}} \right) + \frac{1}{12f} \left(-\frac{1}{\sqrt{2a - 2b}} \right.$$

$$\left. \left(12\sqrt{2} a^2 - 58\sqrt{2} ab + \frac{19b^2}{2\sqrt{2}} + 43\sqrt{2} b^2 \right) \operatorname{ArcTanh} \left[\frac{\sqrt{2a - b + b \operatorname{Cosh}[2(e + fx)]}}{\sqrt{2a - 2b}} \right] + \right.$$

$$\left. \left(4\sqrt{2} \left(6\sqrt{2} ab - \frac{57b^2}{2\sqrt{2}} \right) (1 + \operatorname{Cosh}[e + fx]) \sqrt{\frac{2a - b + b \operatorname{Cosh}[2(e + fx)]}{(1 + \operatorname{Cosh}[e + fx])^2}} \right. \right.$$

$$\left. \left. \sqrt{\left(a - 2a \operatorname{Tanh} \left[\frac{1}{2}(e + fx) \right]^2 + 4b \operatorname{Tanh} \left[\frac{1}{2}(e + fx) \right]^2 + a \operatorname{Tanh} \left[\frac{1}{2}(e + fx) \right]^4 \right)} \right) / \right.$$

$$\left. \left(\sqrt{2a - b + b \operatorname{Cosh}[2(e + fx)]} \left(4b - 4b \operatorname{Tanh} \left[\frac{1}{2}(e + fx) \right]^2 \right) \right) + \right.$$

$$\left. \left(1 / \left(\sqrt{2} \sqrt{a - b} b \sqrt{2a - b + b \operatorname{Cosh}[2(e + fx)]} \left(-1 + \operatorname{Tanh} \left[\frac{1}{2}(e + fx) \right]^2 \right) \right) \right) \right)$$

$$\left(2\sqrt{2} ab - \frac{19b^2}{2\sqrt{2}} \right) (1 + \operatorname{Cosh}[e + fx]) \sqrt{\frac{2a - b + b \operatorname{Cosh}[2(e + fx)]}{(1 + \operatorname{Cosh}[e + fx])^2}}$$

$$\left(b \operatorname{Log} \left[a - b - a \operatorname{Tanh} \left[\frac{1}{2}(e + fx) \right]^2 + b \operatorname{Tanh} \left[\frac{1}{2}(e + fx) \right]^2 + \right. \right.$$

$$\left. \sqrt{a - b} \sqrt{4b \operatorname{Tanh} \left[\frac{1}{2}(e + fx) \right]^2 + a \left(-1 + \operatorname{Tanh} \left[\frac{1}{2}(e + fx) \right]^2 \right)^2} \right]$$

$$\left(-1 + \operatorname{Tanh} \left[\frac{1}{2}(e + fx) \right]^2 \right) + \operatorname{Log} \left[1 + \operatorname{Tanh} \left[\frac{1}{2}(e + fx) \right]^2 \right] \left(b - b \operatorname{Tanh} \left[\frac{1}{2}(e + fx) \right]^2 \right) - \right.$$

$$\left. \left. 2\sqrt{a - b} \sqrt{4b \operatorname{Tanh} \left[\frac{1}{2}(e + fx) \right]^2 + a \left(-1 + \operatorname{Tanh} \left[\frac{1}{2}(e + fx) \right]^2 \right)^2} \right) \right)$$

Problem 469: Result more than twice size of optimal antiderivative.

$$\int (a + b \operatorname{Sinh}[e + fx]^2)^{3/2} \operatorname{Tanh}[e + fx]^3 dx$$

Optimal (type 3, 156 leaves, 6 steps):

$$\begin{aligned}
 & - \frac{(2a-5b)\sqrt{a-b} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sinh}[e+fx]^2}}{\sqrt{a-b}}\right]}{2f} + \frac{(2a-5b)\sqrt{a+b \operatorname{Sinh}[e+fx]^2}}{2f} + \\
 & \frac{(2a-5b)(a+b \operatorname{Sinh}[e+fx]^2)^{3/2}}{6(a-b)f} + \frac{\operatorname{Sech}[e+fx]^2(a+b \operatorname{Sinh}[e+fx]^2)^{5/2}}{2(a-b)f}
 \end{aligned}$$

Result (type 3, 614 leaves):

$$\begin{aligned}
 & \frac{\sqrt{2a-b+b \operatorname{Cosh}[2(e+fx)]}}{f} \left(\frac{b \operatorname{Cosh}[2(e+fx)]}{6\sqrt{2}} + \frac{(a-b) \operatorname{Sech}[e+fx]^2}{2\sqrt{2}} \right) + \\
 & \frac{1}{12f} \left(- \frac{\left(12\sqrt{2}a^2 - 40\sqrt{2}ab + \frac{107b^2}{2\sqrt{2}}\right) \operatorname{ArcTanh}\left[\frac{\sqrt{2a-b+b \operatorname{Cosh}[2(e+fx)]}}{\sqrt{2a-2b}}\right]}{\sqrt{2a-2b}} + \right. \\
 & \left. \left(4\sqrt{2}\left(6\sqrt{2}ab - \frac{39b^2}{2\sqrt{2}}\right)(1+\operatorname{Cosh}[e+fx])\sqrt{\frac{2a-b+b \operatorname{Cosh}[2(e+fx)]}{(1+\operatorname{Cosh}[e+fx])^2}} \right. \right. \\
 & \left. \left. \sqrt{\left(a-2a \operatorname{Tanh}\left[\frac{1}{2}(e+fx)\right]^2 + 4b \operatorname{Tanh}\left[\frac{1}{2}(e+fx)\right]^2 + a \operatorname{Tanh}\left[\frac{1}{2}(e+fx)\right]^4\right)} \right) / \right. \\
 & \left. \left(\sqrt{2a-b+b \operatorname{Cosh}[2(e+fx)]}\left(4b-4b \operatorname{Tanh}\left[\frac{1}{2}(e+fx)\right]^2\right)\right) + \right. \\
 & \left. \left(1/\left(\sqrt{2}\sqrt{a-b}b\sqrt{2a-b+b \operatorname{Cosh}[2(e+fx)]}\left(-1+\operatorname{Tanh}\left[\frac{1}{2}(e+fx)\right]^2\right)\right)\right) \right) \\
 & \left(2\sqrt{2}ab - \frac{13b^2}{2\sqrt{2}}\right)(1+\operatorname{Cosh}[e+fx])\sqrt{\frac{2a-b+b \operatorname{Cosh}[2(e+fx)]}{(1+\operatorname{Cosh}[e+fx])^2}} \\
 & \left(b \operatorname{Log}\left[a-b-a \operatorname{Tanh}\left[\frac{1}{2}(e+fx)\right]^2 + b \operatorname{Tanh}\left[\frac{1}{2}(e+fx)\right]^2 + \right. \right. \\
 & \left. \left. \sqrt{a-b}\sqrt{4b \operatorname{Tanh}\left[\frac{1}{2}(e+fx)\right]^2 + a\left(-1+\operatorname{Tanh}\left[\frac{1}{2}(e+fx)\right]^2\right)^2}\right] \right. \\
 & \left. \left(-1+\operatorname{Tanh}\left[\frac{1}{2}(e+fx)\right]^2\right) + \operatorname{Log}\left[1+\operatorname{Tanh}\left[\frac{1}{2}(e+fx)\right]^2\right]\left(b-b \operatorname{Tanh}\left[\frac{1}{2}(e+fx)\right]^2\right) - \right. \\
 & \left. \left. 2\sqrt{a-b}\sqrt{4b \operatorname{Tanh}\left[\frac{1}{2}(e+fx)\right]^2 + a\left(-1+\operatorname{Tanh}\left[\frac{1}{2}(e+fx)\right]^2\right)^2}\right) \right)
 \end{aligned}$$

Problem 470: Result more than twice size of optimal antiderivative.

$$\int (a + b \operatorname{Sinh}[e + f x]^2)^{3/2} \operatorname{Tanh}[e + f x] \, dx$$

Optimal (type 3, 90 leaves, 5 steps):

$$-\frac{(a-b)^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sinh}[e+fx]^2}}{\sqrt{a-b}}\right]}{f} + \frac{(a-b) \sqrt{a+b \operatorname{Sinh}[e+fx]^2}}{f} + \frac{(a+b \operatorname{Sinh}[e+fx]^2)^{3/2}}{3f}$$

Result (type 3, 590 leaves):

$$\frac{b \operatorname{Cosh}[2(e+fx)] \sqrt{2a-b+b \operatorname{Cosh}[2(e+fx)]}}{6\sqrt{2}f} + \frac{1}{12f} \left(-\frac{\left(12\sqrt{2}a^2 - 22\sqrt{2}ab + \frac{41b^2}{2\sqrt{2}}\right) \operatorname{ArcTanh}\left[\frac{\sqrt{2a-b+b \operatorname{Cosh}[2(e+fx)]}}{\sqrt{2a-2b}}\right]}{\sqrt{2a-2b}} + \left(4\sqrt{2}\left(6\sqrt{2}ab - \frac{21b^2}{2\sqrt{2}}\right)(1+\operatorname{Cosh}[e+fx]) \sqrt{\frac{2a-b+b \operatorname{Cosh}[2(e+fx)]}{(1+\operatorname{Cosh}[e+fx])^2}} \right. \right. \\ \left. \left. \sqrt{\left(a-2a \operatorname{Tanh}\left[\frac{1}{2}(e+fx)\right]^2 + 4b \operatorname{Tanh}\left[\frac{1}{2}(e+fx)\right]^2 + a \operatorname{Tanh}\left[\frac{1}{2}(e+fx)\right]^4\right)} \right) / \left(\sqrt{2a-b+b \operatorname{Cosh}[2(e+fx)]} \left(4b-4b \operatorname{Tanh}\left[\frac{1}{2}(e+fx)\right]^2\right) \right) + \left(1 / \left(\sqrt{2}\sqrt{a-b}b \sqrt{2a-b+b \operatorname{Cosh}[2(e+fx)]} \left(-1+\operatorname{Tanh}\left[\frac{1}{2}(e+fx)\right]^2\right)\right) \right) \right. \\ \left. \left(2\sqrt{2}ab - \frac{7b^2}{2\sqrt{2}}\right)(1+\operatorname{Cosh}[e+fx]) \sqrt{\frac{2a-b+b \operatorname{Cosh}[2(e+fx)]}{(1+\operatorname{Cosh}[e+fx])^2}} \right. \right. \\ \left. \left. \left(b \operatorname{Log}\left[a-b-a \operatorname{Tanh}\left[\frac{1}{2}(e+fx)\right]^2 + b \operatorname{Tanh}\left[\frac{1}{2}(e+fx)\right]^2 + \sqrt{a-b} \sqrt{4b \operatorname{Tanh}\left[\frac{1}{2}(e+fx)\right]^2 + a \left(-1+\operatorname{Tanh}\left[\frac{1}{2}(e+fx)\right]^2\right)^2} \right] \right. \right. \right. \\ \left. \left. \left(-1+\operatorname{Tanh}\left[\frac{1}{2}(e+fx)\right]^2\right) + \operatorname{Log}\left[1+\operatorname{Tanh}\left[\frac{1}{2}(e+fx)\right]^2\right] \left(b-b \operatorname{Tanh}\left[\frac{1}{2}(e+fx)\right]^2\right) - 2\sqrt{a-b} \sqrt{4b \operatorname{Tanh}\left[\frac{1}{2}(e+fx)\right]^2 + a \left(-1+\operatorname{Tanh}\left[\frac{1}{2}(e+fx)\right]^2\right)^2} \right) \right) \right)$$

Problem 474: Result unnecessarily involves imaginary or complex numbers.

$$\int (a + b \operatorname{Sinh}[e + f x]^2)^{3/2} \operatorname{Tanh}[e + f x]^4 dx$$

Optimal (type 4, 305 leaves, 8 steps):

$$\begin{aligned} & - \frac{(3a - 8b) \operatorname{Cosh}[e + f x] \operatorname{Sinh}[e + f x] \sqrt{a + b \operatorname{Sinh}[e + f x]^2}}{3f} - \\ & \left(8(a - 2b) \operatorname{EllipticE}\left[\operatorname{ArcTan}[\operatorname{Sinh}[e + f x]], 1 - \frac{b}{a}\right] \operatorname{Sech}[e + f x] \sqrt{a + b \operatorname{Sinh}[e + f x]^2} \right) / \\ & \left(3f \sqrt{\frac{\operatorname{Sech}[e + f x]^2 (a + b \operatorname{Sinh}[e + f x]^2)}{a}} \right) + \\ & \left((3a - 8b) \operatorname{EllipticF}\left[\operatorname{ArcTan}[\operatorname{Sinh}[e + f x]], 1 - \frac{b}{a}\right] \operatorname{Sech}[e + f x] \sqrt{a + b \operatorname{Sinh}[e + f x]^2} \right) / \\ & \left(3f \sqrt{\frac{\operatorname{Sech}[e + f x]^2 (a + b \operatorname{Sinh}[e + f x]^2)}{a}} \right) + \frac{8(a - 2b) \sqrt{a + b \operatorname{Sinh}[e + f x]^2} \operatorname{Tanh}[e + f x]}{3f} + \\ & \frac{(a - 2b) \operatorname{Sinh}[e + f x]^2 \sqrt{a + b \operatorname{Sinh}[e + f x]^2} \operatorname{Tanh}[e + f x]}{f} - \frac{(a + b \operatorname{Sinh}[e + f x]^2)^{3/2} \operatorname{Tanh}[e + f x]^3}{3f} \end{aligned}$$

Result (type 4, 224 leaves):

$$\begin{aligned} & \left(-32i a (a - 2b) \sqrt{\frac{2a - b + b \operatorname{Cosh}[2(e + f x)]}{a}} \operatorname{EllipticE}\left[i(e + f x), \frac{b}{a}\right] + \right. \\ & 4i a (5a - 8b) \sqrt{\frac{2a - b + b \operatorname{Cosh}[2(e + f x)]}{a}} \operatorname{EllipticF}\left[i(e + f x), \frac{b}{a}\right] - \\ & \frac{1}{4\sqrt{2}} (32a^2 - 108ab + 18b^2 + (64a^2 - 160ab + 17b^2) \operatorname{Cosh}[2(e + f x)] + \\ & \left. 2(6a - 17b) b \operatorname{Cosh}[4(e + f x)] - b^2 \operatorname{Cosh}[6(e + f x)]) \right) \\ & \left. \operatorname{Sech}[e + f x]^2 \operatorname{Tanh}[e + f x] \right) / \left(12f \sqrt{2a - b + b \operatorname{Cosh}[2(e + f x)]} \right) \end{aligned}$$

Problem 475: Result unnecessarily involves imaginary or complex numbers.

$$\int (a + b \operatorname{Sinh}[e + f x]^2)^{3/2} \operatorname{Tanh}[e + f x]^2 dx$$

Optimal (type 4, 260 leaves, 7 steps):

$$\frac{4 b \operatorname{Cosh}[e+f x] \operatorname{Sinh}[e+f x] \sqrt{a+b \operatorname{Sinh}[e+f x]^2}}{3 f} - \left((7 a-8 b) \operatorname{EllipticE}\left[\operatorname{ArcTan}[\operatorname{Sinh}[e+f x]], 1-\frac{b}{a}\right] \operatorname{Sech}[e+f x] \sqrt{a+b \operatorname{Sinh}[e+f x]^2} \right) / \left(3 f \sqrt{\frac{\operatorname{Sech}[e+f x]^2 (a+b \operatorname{Sinh}[e+f x]^2)}{a}} \right) + \left((3 a-4 b) \operatorname{EllipticF}\left[\operatorname{ArcTan}[\operatorname{Sinh}[e+f x]], 1-\frac{b}{a}\right] \operatorname{Sech}[e+f x] \sqrt{a+b \operatorname{Sinh}[e+f x]^2} \right) / \left(3 f \sqrt{\frac{\operatorname{Sech}[e+f x]^2 (a+b \operatorname{Sinh}[e+f x]^2)}{a}} \right) + \frac{(7 a-8 b) \sqrt{a+b \operatorname{Sinh}[e+f x]^2} \operatorname{Tanh}[e+f x]}{3 f} - \frac{(a+b \operatorname{Sinh}[e+f x]^2)^{3/2} \operatorname{Tanh}[e+f x]}{f}$$

Result (type 4, 188 leaves):

$$\left(-8 i a (7 a-8 b) \sqrt{\frac{2 a-b+b \operatorname{Cosh}[2(e+f x)]}{a}} \operatorname{EllipticE}\left[i(e+f x), \frac{b}{a}\right] + 32 i a (a-b) \sqrt{\frac{2 a-b+b \operatorname{Cosh}[2(e+f x)]}{a}} \operatorname{EllipticF}\left[i(e+f x), \frac{b}{a}\right] + \sqrt{2} (-24 a^2+40 a b-13 b^2-4(2 a-3 b) b \operatorname{Cosh}[2(e+f x)]+b^2 \operatorname{Cosh}[4(e+f x)]) \operatorname{Tanh}[e+f x] \right) / \left(24 f \sqrt{2 a-b+b \operatorname{Cosh}[2(e+f x)]} \right)$$

Problem 477: Result unnecessarily involves imaginary or complex numbers.

$$\int \operatorname{Coth}[e+f x]^2 (a+b \operatorname{Sinh}[e+f x]^2)^{3/2} dx$$

Optimal (type 4, 256 leaves, 7 steps):

$$\frac{4 b \operatorname{Cosh}[e+f x] \operatorname{Sinh}[e+f x] \sqrt{a+b \operatorname{Sinh}[e+f x]^2}}{3 f} - \frac{\operatorname{Coth}[e+f x] (a+b \operatorname{Sinh}[e+f x]^2)^{3/2}}{f} -$$

$$\left((7 a+b) \operatorname{EllipticE}\left[\operatorname{ArcTan}[\operatorname{Sinh}[e+f x]], 1-\frac{b}{a}\right] \operatorname{Sech}[e+f x] \sqrt{a+b \operatorname{Sinh}[e+f x]^2} \right) /$$

$$\left(3 f \sqrt{\frac{\operatorname{Sech}[e+f x]^2 (a+b \operatorname{Sinh}[e+f x]^2)}{a}} \right) +$$

$$\left((3 a+5 b) \operatorname{EllipticF}\left[\operatorname{ArcTan}[\operatorname{Sinh}[e+f x]], 1-\frac{b}{a}\right] \operatorname{Sech}[e+f x] \sqrt{a+b \operatorname{Sinh}[e+f x]^2} \right) /$$

$$\left(3 f \sqrt{\frac{\operatorname{Sech}[e+f x]^2 (a+b \operatorname{Sinh}[e+f x]^2)}{a}} \right) + \frac{(7 a+b) \sqrt{a+b \operatorname{Sinh}[e+f x]^2} \operatorname{Tanh}[e+f x]}{3 f}$$

Result (type 4, 184 leaves):

$$\left(\sqrt{2} (-24 a^2 + 8 a b + 3 b^2 - 4 b (2 a + b) \operatorname{Cosh}[2 (e+f x)] + b^2 \operatorname{Cosh}[4 (e+f x)]) \operatorname{Coth}[e+f x] - \right.$$

$$8 i a (7 a+b) \sqrt{\frac{2 a-b+b \operatorname{Cosh}[2 (e+f x)]}{a}} \operatorname{EllipticE}\left[i (e+f x), \frac{b}{a}\right] +$$

$$\left. 32 i a (a-b) \sqrt{\frac{2 a-b+b \operatorname{Cosh}[2 (e+f x)]}{a}} \operatorname{EllipticF}\left[i (e+f x), \frac{b}{a}\right] \right) /$$

$$\left(24 f \sqrt{2 a-b+b \operatorname{Cosh}[2 (e+f x)]} \right)$$

Problem 478: Result unnecessarily involves imaginary or complex numbers.

$$\int \operatorname{Coth}[e+f x]^4 (a+b \operatorname{Sinh}[e+f x]^2)^{3/2} dx$$

Optimal (type 4, 306 leaves, 8 steps):

$$\begin{aligned}
 & - \frac{(a+b) \operatorname{Cosh}[e+fx]^2 \operatorname{Coth}[e+fx] \sqrt{a+b \operatorname{Sinh}[e+fx]^2}}{f} + \\
 & \frac{(3a+5b) \operatorname{Cosh}[e+fx] \operatorname{Sinh}[e+fx] \sqrt{a+b \operatorname{Sinh}[e+fx]^2}}{3f} - \\
 & \frac{\operatorname{Coth}[e+fx]^3 (a+b \operatorname{Sinh}[e+fx]^2)^{3/2}}{3f} - \\
 & \left(8(a+b) \operatorname{EllipticE}[\operatorname{ArcTan}[\operatorname{Sinh}[e+fx]], 1 - \frac{b}{a}] \operatorname{Sech}[e+fx] \sqrt{a+b \operatorname{Sinh}[e+fx]^2} \right) / \\
 & \left(3f \sqrt{\frac{\operatorname{Sech}[e+fx]^2 (a+b \operatorname{Sinh}[e+fx]^2)}{a}} \right) + \\
 & \left((3a+b)(a+3b) \operatorname{EllipticF}[\operatorname{ArcTan}[\operatorname{Sinh}[e+fx]], 1 - \frac{b}{a}] \operatorname{Sech}[e+fx] \sqrt{a+b \operatorname{Sinh}[e+fx]^2} \right) / \\
 & \left(3af \sqrt{\frac{\operatorname{Sech}[e+fx]^2 (a+b \operatorname{Sinh}[e+fx]^2)}{a}} \right) + \frac{8(a+b) \sqrt{a+b \operatorname{Sinh}[e+fx]^2} \operatorname{Tanh}[e+fx]}{3f}
 \end{aligned}$$

Result (type 4, 368 leaves):

$$\begin{aligned}
 & \frac{1}{3f} \sqrt{2} \left(- \left(\left(i (3a^2 + 6ab - b^2) \sqrt{\frac{2a-b+b \operatorname{Cosh}[2(e+fx)]}{a}} \operatorname{EllipticF}\left[i(e+fx), \frac{b}{a}\right] \right) / \right. \right. \\
 & \left. \left. \left(\sqrt{2} \sqrt{2a-b+b \operatorname{Cosh}[2(e+fx)]} \right) \right) - \frac{1}{2b} \right. \\
 & \left. i(4ab+4b^2) \left(\frac{2\sqrt{2}a \sqrt{\frac{2a-b+b \operatorname{Cosh}[2(e+fx)]}{a}} \operatorname{EllipticE}\left[i(e+fx), \frac{b}{a}\right]}{\sqrt{2a-b+b \operatorname{Cosh}[2(e+fx)]}} - \right. \right. \\
 & \left. \left. \frac{\sqrt{2}(2a-b) \sqrt{\frac{2a-b+b \operatorname{Cosh}[2(e+fx)]}{a}} \operatorname{EllipticF}\left[i(e+fx), \frac{b}{a}\right]}{\sqrt{2a-b+b \operatorname{Cosh}[2(e+fx)]}} \right) \right) + \frac{1}{f} \\
 & \sqrt{2a-b+b \operatorname{Cosh}[2(e+fx)]} \left(-\frac{2}{3} \left(\sqrt{2}a \operatorname{Cosh}[e+fx] + \sqrt{2}b \operatorname{Cosh}[e+fx] \right) \operatorname{Csch}[e+fx] - \right. \\
 & \left. \frac{a \operatorname{Coth}[e+fx] \operatorname{Csch}[e+fx]^2}{3\sqrt{2}} + \frac{b \operatorname{Sinh}[2(e+fx)]}{6\sqrt{2}} \right)
 \end{aligned}$$

Problem 485: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{Tanh}[e + f x]^4}{\sqrt{a + b \text{Sinh}[e + f x]^2}} dx$$

Optimal (type 4, 219 leaves, 5 steps):

$$\begin{aligned} & - \left(\left(2 (2 a - b) \text{EllipticE}[\text{ArcTan}[\text{Sinh}[e + f x]], 1 - \frac{b}{a}] \text{Sech}[e + f x] \sqrt{a + b \text{Sinh}[e + f x]^2} \right) / \right. \\ & \quad \left. \left(3 (a - b)^2 f \sqrt{\frac{\text{Sech}[e + f x]^2 (a + b \text{Sinh}[e + f x]^2)}{a}} \right) \right) + \\ & \left((3 a - b) \text{EllipticF}[\text{ArcTan}[\text{Sinh}[e + f x]], 1 - \frac{b}{a}] \text{Sech}[e + f x] \sqrt{a + b \text{Sinh}[e + f x]^2} \right) / \\ & \left(3 (a - b)^2 f \sqrt{\frac{\text{Sech}[e + f x]^2 (a + b \text{Sinh}[e + f x]^2)}{a}} \right) + \\ & \frac{\text{Sech}[e + f x]^2 \sqrt{a + b \text{Sinh}[e + f x]^2} \text{Tanh}[e + f x]}{3 (a - b) f} \end{aligned}$$

Result (type 4, 206 leaves):

$$\begin{aligned} & \left(-4 i a (2 a - b) \sqrt{\frac{2 a - b + b \text{Cosh}[2 (e + f x)]}{a}} \text{EllipticE}[i (e + f x), \frac{b}{a}] + \right. \\ & \quad 2 i a (a - b) \sqrt{\frac{2 a - b + b \text{Cosh}[2 (e + f x)]}{a}} \text{EllipticF}[i (e + f x), \frac{b}{a}] - \frac{1}{\sqrt{2}} \\ & \quad \left. (2 (4 a^2 - 3 a b + b^2) \text{Cosh}[2 (e + f x)] + (2 a - b) (2 a + b + b \text{Cosh}[4 (e + f x)])) \right) \\ & \quad \left. \text{Sech}[e + f x]^2 \text{Tanh}[e + f x] \right) / \left(6 (a - b)^2 f \sqrt{2 a - b + b \text{Cosh}[2 (e + f x)]} \right) \end{aligned}$$

Problem 486: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{Tanh}[e + f x]^2}{\sqrt{a + b \text{Sinh}[e + f x]^2}} dx$$

Optimal (type 4, 156 leaves, 6 steps):

$$\begin{aligned}
 & - \left(\text{EllipticE} \left[\text{ArcTan}[\text{Sinh}[e + f x]], 1 - \frac{b}{a} \right] \text{Sech}[e + f x] \sqrt{a + b \text{Sinh}[e + f x]^2} \right) / \\
 & \left((a - b) f \sqrt{\frac{\text{Sech}[e + f x]^2 (a + b \text{Sinh}[e + f x]^2)}{a}} \right) + \\
 & \left(\text{EllipticF} \left[\text{ArcTan}[\text{Sinh}[e + f x]], 1 - \frac{b}{a} \right] \text{Sech}[e + f x] \sqrt{a + b \text{Sinh}[e + f x]^2} \right) / \\
 & \left((a - b) f \sqrt{\frac{\text{Sech}[e + f x]^2 (a + b \text{Sinh}[e + f x]^2)}{a}} \right)
 \end{aligned}$$

Result (type 4, 109 leaves):

$$\begin{aligned}
 & \left(-2 i a \sqrt{\frac{2 a - b + b \text{Cosh}[2 (e + f x)]}{a}} \text{EllipticE} \left[i (e + f x), \frac{b}{a} \right] + \right. \\
 & \left. \sqrt{2} (-2 a + b - b \text{Cosh}[2 (e + f x)]) \text{Tanh}[e + f x] \right) / \left(2 (a - b) f \sqrt{2 a - b + b \text{Cosh}[2 (e + f x)]} \right)
 \end{aligned}$$

Problem 488: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{Coth}[e + f x]^2}{\sqrt{a + b \text{Sinh}[e + f x]^2}} dx$$

Optimal (type 4, 207 leaves, 6 steps):

$$\begin{aligned}
 & \frac{\text{Coth}[e + f x] \sqrt{a + b \text{Sinh}[e + f x]^2}}{a f} - \\
 & \left(\text{EllipticE} \left[\text{ArcTan}[\text{Sinh}[e + f x]], 1 - \frac{b}{a} \right] \text{Sech}[e + f x] \sqrt{a + b \text{Sinh}[e + f x]^2} \right) / \\
 & \left(a f \sqrt{\frac{\text{Sech}[e + f x]^2 (a + b \text{Sinh}[e + f x]^2)}{a}} \right) + \\
 & \left(\text{EllipticF} \left[\text{ArcTan}[\text{Sinh}[e + f x]], 1 - \frac{b}{a} \right] \text{Sech}[e + f x] \sqrt{a + b \text{Sinh}[e + f x]^2} \right) / \\
 & \left(a f \sqrt{\frac{\text{Sech}[e + f x]^2 (a + b \text{Sinh}[e + f x]^2)}{a}} \right) + \frac{\sqrt{a + b \text{Sinh}[e + f x]^2} \text{Tanh}[e + f x]}{a f}
 \end{aligned}$$

Result (type 4, 105 leaves):

$$\left(\sqrt{2} (-2a + b - b \operatorname{Cosh}[2(e + fx)]) \operatorname{Coth}[e + fx] - \right. \\ \left. 2ia \sqrt{\frac{2a - b + b \operatorname{Cosh}[2(e + fx)]}{a}} \operatorname{EllipticE}\left[i(e + fx), \frac{b}{a}\right] \right) / \\ (2af \sqrt{2a - b + b \operatorname{Cosh}[2(e + fx)]})$$

Problem 489: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Coth}[e + fx]^4}{\sqrt{a + b \operatorname{Sinh}[e + fx]^2}} dx$$

Optimal (type 4, 285 leaves, 7 steps):

$$\frac{2(2a - b) \operatorname{Coth}[e + fx] \sqrt{a + b \operatorname{Sinh}[e + fx]^2}}{3a^2 f} - \\ \frac{\operatorname{Coth}[e + fx] \operatorname{Csch}[e + fx]^2 \sqrt{a + b \operatorname{Sinh}[e + fx]^2}}{3af} - \\ \left(2(2a - b) \operatorname{EllipticE}\left[\operatorname{ArcTan}[\operatorname{Sinh}[e + fx]], 1 - \frac{b}{a}\right] \operatorname{Sech}[e + fx] \sqrt{a + b \operatorname{Sinh}[e + fx]^2} \right) / \\ \left(3a^2 f \sqrt{\frac{\operatorname{Sech}[e + fx]^2 (a + b \operatorname{Sinh}[e + fx]^2)}{a}} \right) + \\ \left((3a - b) \operatorname{EllipticF}\left[\operatorname{ArcTan}[\operatorname{Sinh}[e + fx]], 1 - \frac{b}{a}\right] \operatorname{Sech}[e + fx] \sqrt{a + b \operatorname{Sinh}[e + fx]^2} \right) / \\ \left(3a^2 f \sqrt{\frac{\operatorname{Sech}[e + fx]^2 (a + b \operatorname{Sinh}[e + fx]^2)}{a}} \right) + \frac{2(2a - b) \sqrt{a + b \operatorname{Sinh}[e + fx]^2} \operatorname{Tanh}[e + fx]}{3a^2 f}$$

Result (type 4, 357 leaves):

$$\frac{1}{f} \sqrt{2a - b + b \operatorname{Cosh}[2(e + fx)]}$$

$$\left(\frac{(-2\sqrt{2} a \operatorname{Cosh}[e + fx] + \sqrt{2} b \operatorname{Cosh}[e + fx]) \operatorname{Csch}[e + fx]}{3a^2} - \frac{\operatorname{Coth}[e + fx] \operatorname{Csch}[e + fx]^2}{3\sqrt{2} a} \right) +$$

$$\frac{1}{3a^2 f} \sqrt{2} \left(\left(\left(i (3a^2 - 3ab + b^2) \sqrt{\frac{2a - b + b \operatorname{Cosh}[2(e + fx)]}{a}} \operatorname{EllipticF}\left[i(e + fx), \frac{b}{a}\right] \right) / \right. \right.$$

$$\left. \left. \left(\sqrt{2} \sqrt{2a - b + b \operatorname{Cosh}[2(e + fx)]} \right) \right) - \frac{1}{2b}$$

$$i(2ab - b^2) \left(\frac{2\sqrt{2} a \sqrt{\frac{2a - b + b \operatorname{Cosh}[2(e + fx)]}{a}} \operatorname{EllipticE}\left[i(e + fx), \frac{b}{a}\right]}{\sqrt{2a - b + b \operatorname{Cosh}[2(e + fx)]}} - \right.$$

$$\left. \left. \frac{\sqrt{2}(2a - b) \sqrt{\frac{2a - b + b \operatorname{Cosh}[2(e + fx)]}{a}} \operatorname{EllipticF}\left[i(e + fx), \frac{b}{a}\right]}{\sqrt{2a - b + b \operatorname{Cosh}[2(e + fx)]}} \right) \right)$$

Problem 496: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Tanh}[e + fx]^4}{(a + b \operatorname{Sinh}[e + fx]^2)^{3/2}} dx$$

Optimal (type 4, 275 leaves, 6 steps):

$$\begin{aligned}
 & - \left(\left(\sqrt{a} \sqrt{b} (7a+b) \operatorname{Cosh}[e+fx] \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Sinh}[e+fx]}{\sqrt{a}}\right], 1 - \frac{a}{b}\right] \right) / \right. \\
 & \quad \left. \left(3(a-b)^3 f \sqrt{\frac{a \operatorname{Cosh}[e+fx]^2}{a+b \operatorname{Sinh}[e+fx]^2}} \sqrt{a+b \operatorname{Sinh}[e+fx]^2} \right) \right) + \\
 & \quad \left((3a+5b) \operatorname{EllipticF}\left[\operatorname{ArcTan}[\operatorname{Sinh}[e+fx]], 1 - \frac{b}{a}\right] \operatorname{Sech}[e+fx] \sqrt{a+b \operatorname{Sinh}[e+fx]^2} \right) / \\
 & \quad \left(3(a-b)^3 f \sqrt{\frac{\operatorname{Sech}[e+fx]^2 (a+b \operatorname{Sinh}[e+fx]^2)}{a}} \right) - \\
 & \quad \frac{4a \operatorname{Tanh}[e+fx]}{3(a-b)^2 f \sqrt{a+b \operatorname{Sinh}[e+fx]^2}} + \frac{\operatorname{Sech}[e+fx]^2 \operatorname{Tanh}[e+fx]}{3(a-b) f \sqrt{a+b \operatorname{Sinh}[e+fx]^2}}
 \end{aligned}$$

Result (type 4, 212 leaves):

$$\begin{aligned}
 & \left(-2i a (7a+b) \sqrt{\frac{2a-b+b \operatorname{Cosh}[2(e+fx)]}{a}} \operatorname{EllipticE}\left[i(e+fx), \frac{b}{a}\right] + \right. \\
 & \quad 8i a (a-b) \sqrt{\frac{2a-b+b \operatorname{Cosh}[2(e+fx)]}{a}} \operatorname{EllipticF}\left[i(e+fx), \frac{b}{a}\right] - \frac{1}{2\sqrt{2}} \\
 & \quad \left. (8a^2 + 21ab - 5b^2 + 4(4a^2 + 3ab + b^2) \operatorname{Cosh}[2(e+fx)] + b(7a+b) \operatorname{Cosh}[4(e+fx)]) \right) \\
 & \quad \left. \operatorname{Sech}[e+fx]^2 \operatorname{Tanh}[e+fx] \right) / \left(6(a-b)^3 f \sqrt{2a-b+b \operatorname{Cosh}[2(e+fx)]} \right)
 \end{aligned}$$

Problem 497: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Tanh}[e+fx]^2}{(a+b \operatorname{Sinh}[e+fx]^2)^{3/2}} dx$$

Optimal (type 4, 217 leaves, 5 steps):

$$\begin{aligned}
 & - \frac{2\sqrt{a} \sqrt{b} \operatorname{Cosh}[e+fx] \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Sinh}[e+fx]}{\sqrt{a}}\right], 1 - \frac{a}{b}\right]}{(a-b)^2 f \sqrt{\frac{a \operatorname{Cosh}[e+fx]^2}{a+b \operatorname{Sinh}[e+fx]^2}} \sqrt{a+b \operatorname{Sinh}[e+fx]^2}} + \\
 & \quad \left((a+b) \operatorname{EllipticF}\left[\operatorname{ArcTan}[\operatorname{Sinh}[e+fx]], 1 - \frac{b}{a}\right] \operatorname{Sech}[e+fx] \sqrt{a+b \operatorname{Sinh}[e+fx]^2} \right) / \\
 & \quad \left(a(a-b)^2 f \sqrt{\frac{\operatorname{Sech}[e+fx]^2 (a+b \operatorname{Sinh}[e+fx]^2)}{a}} \right) - \frac{\operatorname{Tanh}[e+fx]}{(a-b) f \sqrt{a+b \operatorname{Sinh}[e+fx]^2}}
 \end{aligned}$$

Result (type 4, 158 leaves):

$$\left(-2 i \sqrt{2} a \sqrt{\frac{2 a - b + b \operatorname{Cosh}[2 (e + f x)]}{a}} \operatorname{EllipticE}\left[i (e + f x), \frac{b}{a}\right] + \right. \\ \left. i \sqrt{2} (a - b) \sqrt{\frac{2 a - b + b \operatorname{Cosh}[2 (e + f x)]}{a}} \operatorname{EllipticF}\left[i (e + f x), \frac{b}{a}\right] - \right. \\ \left. 2 (a + b \operatorname{Cosh}[2 (e + f x)]) \operatorname{Tanh}[e + f x] \right) / \left((a - b)^2 f \sqrt{4 a - 2 b + 2 b \operatorname{Cosh}[2 (e + f x)]} \right)$$

Problem 499: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Coth}[e + f x]^2}{(a + b \operatorname{Sinh}[e + f x]^2)^{3/2}} dx$$

Optimal (type 4, 237 leaves, 7 steps):

$$\frac{\operatorname{Coth}[e + f x]}{a f \sqrt{a + b \operatorname{Sinh}[e + f x]^2}} - \frac{2 \operatorname{Coth}[e + f x] \sqrt{a + b \operatorname{Sinh}[e + f x]^2}}{a^2 f} - \\ \left(2 \operatorname{EllipticE}\left[\operatorname{ArcTan}[\operatorname{Sinh}[e + f x]], 1 - \frac{b}{a}\right] \operatorname{Sech}[e + f x] \sqrt{a + b \operatorname{Sinh}[e + f x]^2} \right) / \\ \left(a^2 f \sqrt{\frac{\operatorname{Sech}[e + f x]^2 (a + b \operatorname{Sinh}[e + f x]^2)}{a}} \right) + \\ \left(\operatorname{EllipticF}\left[\operatorname{ArcTan}[\operatorname{Sinh}[e + f x]], 1 - \frac{b}{a}\right] \operatorname{Sech}[e + f x] \sqrt{a + b \operatorname{Sinh}[e + f x]^2} \right) / \\ \left(a^2 f \sqrt{\frac{\operatorname{Sech}[e + f x]^2 (a + b \operatorname{Sinh}[e + f x]^2)}{a}} \right) + \frac{2 \sqrt{a + b \operatorname{Sinh}[e + f x]^2} \operatorname{Tanh}[e + f x]}{a^2 f}$$

Result (type 4, 153 leaves):

$$\left(-2 (a - b + b \operatorname{Cosh}[2 (e + f x)]) \operatorname{Coth}[e + f x] - \right. \\ \left. 2 i \sqrt{2} a \sqrt{\frac{2 a - b + b \operatorname{Cosh}[2 (e + f x)]}{a}} \operatorname{EllipticE}\left[i (e + f x), \frac{b}{a}\right] + \right. \\ \left. i \sqrt{2} a \sqrt{\frac{2 a - b + b \operatorname{Cosh}[2 (e + f x)]}{a}} \operatorname{EllipticF}\left[i (e + f x), \frac{b}{a}\right] \right) / \\ \left(a^2 f \sqrt{4 a - 2 b + 2 b \operatorname{Cosh}[2 (e + f x)]} \right)$$

Problem 500: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Coth}[e + f x]^4}{(a + b \operatorname{Sinh}[e + f x]^2)^{3/2}} dx$$

Optimal (type 4, 341 leaves, 8 steps):

$$\begin{aligned} & - \frac{(a - b) \operatorname{Coth}[e + f x] \operatorname{Csch}[e + f x]^2}{a b f \sqrt{a + b \operatorname{Sinh}[e + f x]^2}} - \frac{(7 a - 8 b) \operatorname{Coth}[e + f x] \sqrt{a + b \operatorname{Sinh}[e + f x]^2}}{3 a^3 f} + \\ & \frac{(3 a - 4 b) \operatorname{Coth}[e + f x] \operatorname{Csch}[e + f x]^2 \sqrt{a + b \operatorname{Sinh}[e + f x]^2}}{3 a^2 b f} - \\ & \left((7 a - 8 b) \operatorname{EllipticE}\left[\operatorname{ArcTan}[\operatorname{Sinh}[e + f x]], 1 - \frac{b}{a}\right] \operatorname{Sech}[e + f x] \sqrt{a + b \operatorname{Sinh}[e + f x]^2} \right) / \\ & \left(3 a^3 f \sqrt{\frac{\operatorname{Sech}[e + f x]^2 (a + b \operatorname{Sinh}[e + f x]^2)}{a}} \right) + \\ & \left((3 a - 4 b) \operatorname{EllipticF}\left[\operatorname{ArcTan}[\operatorname{Sinh}[e + f x]], 1 - \frac{b}{a}\right] \operatorname{Sech}[e + f x] \sqrt{a + b \operatorname{Sinh}[e + f x]^2} \right) / \\ & \left(3 a^3 f \sqrt{\frac{\operatorname{Sech}[e + f x]^2 (a + b \operatorname{Sinh}[e + f x]^2)}{a}} \right) + \frac{(7 a - 8 b) \sqrt{a + b \operatorname{Sinh}[e + f x]^2} \operatorname{Tanh}[e + f x]}{3 a^3 f} \end{aligned}$$

Result (type 4, 441 leaves):

$$\frac{1}{3 a^3 f} \left(\left(\left(i \left(3 \sqrt{2} a^2 - \frac{15 a b}{\sqrt{2}} + 4 \sqrt{2} b^2 \right) \sqrt{\frac{2 a - b + b \operatorname{Cosh}[2 (e + f x)]}{a}} \operatorname{EllipticF}\left[i (e + f x), \frac{b}{a}\right] \right) / \left(\sqrt{2} \sqrt{2 a - b + b \operatorname{Cosh}[2 (e + f x)]} \right) \right) - \frac{1}{2 b} \right. \\ \left. i \left(\frac{7 a b}{\sqrt{2}} - 4 \sqrt{2} b^2 \right) \left(\frac{2 \sqrt{2} a \sqrt{\frac{2 a - b + b \operatorname{Cosh}[2 (e + f x)]}{a}} \operatorname{EllipticE}\left[i (e + f x), \frac{b}{a}\right]}{\sqrt{2 a - b + b \operatorname{Cosh}[2 (e + f x)]}} - \frac{\sqrt{2} (2 a - b) \sqrt{\frac{2 a - b + b \operatorname{Cosh}[2 (e + f x)]}{a}} \operatorname{EllipticF}\left[i (e + f x), \frac{b}{a}\right]}{\sqrt{2 a - b + b \operatorname{Cosh}[2 (e + f x)]}} \right) \right) + \frac{1}{f} \right. \\ \left. \sqrt{2 a - b + b \operatorname{Cosh}[2 (e + f x)]} \left(\frac{(-4 \sqrt{2} a \operatorname{Cosh}[e + f x] + 5 \sqrt{2} b \operatorname{Cosh}[e + f x]) \operatorname{Csch}[e + f x]}{6 a^3} - \frac{\operatorname{Coth}[e + f x] \operatorname{Csch}[e + f x]^2}{3 \sqrt{2} a^2} + \frac{-\sqrt{2} a b \operatorname{Sinh}[2 (e + f x)] + \sqrt{2} b^2 \operatorname{Sinh}[2 (e + f x)]}{2 a^3 (2 a - b + b \operatorname{Cosh}[2 (e + f x)])} \right) \right)$$

Problem 507: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Tanh}[e + f x]^4}{(a + b \operatorname{Sinh}[e + f x]^2)^{5/2}} dx$$

Optimal (type 4, 333 leaves, 7 steps):

$$\begin{aligned}
 & - \frac{b (5 a + 3 b) \operatorname{Cosh}[e + f x] \operatorname{Sinh}[e + f x]}{3 (a - b)^3 f (a + b \operatorname{Sinh}[e + f x]^2)^{3/2}} - \\
 & \left(8 \sqrt{a} \sqrt{b} (a + b) \operatorname{Cosh}[e + f x] \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Sinh}[e + f x]}{\sqrt{a}}\right], 1 - \frac{a}{b}\right] \right) / \\
 & \left(3 (a - b)^4 f \sqrt{\frac{a \operatorname{Cosh}[e + f x]^2}{a + b \operatorname{Sinh}[e + f x]^2} \sqrt{a + b \operatorname{Sinh}[e + f x]^2}} \right) + \\
 & \left((3 a + b) (a + 3 b) \operatorname{EllipticF}\left[\operatorname{ArcTan}[\operatorname{Sinh}[e + f x]], 1 - \frac{b}{a}\right] \operatorname{Sech}[e + f x] \sqrt{a + b \operatorname{Sinh}[e + f x]^2} \right) / \\
 & \left(3 a (a - b)^4 f \sqrt{\frac{\operatorname{Sech}[e + f x]^2 (a + b \operatorname{Sinh}[e + f x]^2)}{a}} \right) - \\
 & \frac{2 (2 a + b) \operatorname{Tanh}[e + f x]}{3 (a - b)^2 f (a + b \operatorname{Sinh}[e + f x]^2)^{3/2}} + \frac{\operatorname{Sech}[e + f x]^2 \operatorname{Tanh}[e + f x]}{3 (a - b) f (a + b \operatorname{Sinh}[e + f x]^2)^{3/2}}
 \end{aligned}$$

Result(type 4, 479 leaves):

$$\frac{1}{3(a-b)^4 f} \sqrt{2} \left(- \left(\left(i (3a^2 + 6ab - b^2) \sqrt{\frac{2a-b+b \operatorname{Cosh}[2(e+fx)]}{a}} \operatorname{EllipticF}\left[i(e+fx), \frac{b}{a}\right] \right) / \right. \right. \\ \left. \left. \left(\sqrt{2} \sqrt{2a-b+b \operatorname{Cosh}[2(e+fx)]} \right) \right) - \frac{1}{2b} \right. \\ \left. i(4ab + 4b^2) \left(\frac{2\sqrt{2} a \sqrt{\frac{2a-b+b \operatorname{Cosh}[2(e+fx)]}{a}} \operatorname{EllipticE}\left[i(e+fx), \frac{b}{a}\right]}{\sqrt{2a-b+b \operatorname{Cosh}[2(e+fx)]}} - \right. \right. \\ \left. \left. \frac{\sqrt{2}(2a-b) \sqrt{\frac{2a-b+b \operatorname{Cosh}[2(e+fx)]}{a}} \operatorname{EllipticF}\left[i(e+fx), \frac{b}{a}\right]}{\sqrt{2a-b+b \operatorname{Cosh}[2(e+fx)]}} \right) \right) + \frac{1}{f} \\ \sqrt{2a-b+b \operatorname{Cosh}[2(e+fx)]} \left(- \frac{2 \operatorname{Sech}[e+fx] (\sqrt{2} a \operatorname{Sinh}[e+fx] + \sqrt{2} b \operatorname{Sinh}[e+fx])}{3(a-b)^4} - \right. \\ \left. \frac{\sqrt{2} a b \operatorname{Sinh}[2(e+fx)]}{3(a-b)^3 (2a-b+b \operatorname{Cosh}[2(e+fx)])^2} - \right. \\ \left. \frac{2(\sqrt{2} a b \operatorname{Sinh}[2(e+fx)] + \sqrt{2} b^2 \operatorname{Sinh}[2(e+fx)])}{3(a-b)^4 (2a-b+b \operatorname{Cosh}[2(e+fx)])} + \frac{\operatorname{Sech}[e+fx]^2 \operatorname{Tanh}[e+fx]}{3\sqrt{2}(a-b)^3} \right)$$

Problem 508: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Tanh}[e+fx]^2}{(a+b \operatorname{Sinh}[e+fx]^2)^{5/2}} dx$$

Optimal (type 4, 274 leaves, 6 steps):

$$\begin{aligned}
 & - \frac{4 b \operatorname{Cosh}[e+f x] \operatorname{Sinh}[e+f x]}{3 (a-b)^2 f (a+b \operatorname{Sinh}[e+f x]^2)^{3/2}} - \\
 & \frac{\sqrt{b} (7 a+b) \operatorname{Cosh}[e+f x] \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Sinh}[e+f x]}{\sqrt{a}}\right], 1-\frac{a}{b}\right]}{3 \sqrt{a} (a-b)^3 f \sqrt{\frac{a \operatorname{Cosh}[e+f x]^2}{a+b \operatorname{Sinh}[e+f x]^2}} \sqrt{a+b \operatorname{Sinh}[e+f x]^2}} + \\
 & \left((3 a+5 b) \operatorname{EllipticF}\left[\operatorname{ArcTan}[\operatorname{Sinh}[e+f x]], 1-\frac{b}{a}\right] \operatorname{Sech}[e+f x] \sqrt{a+b \operatorname{Sinh}[e+f x]^2} \right) / \\
 & \left(3 a (a-b)^3 f \sqrt{\frac{\operatorname{Sech}[e+f x]^2 (a+b \operatorname{Sinh}[e+f x]^2)}{a}} \right) - \frac{\operatorname{Tanh}[e+f x]}{(a-b) f (a+b \operatorname{Sinh}[e+f x]^2)^{3/2}}
 \end{aligned}$$

Result (type 4, 215 leaves):

$$\begin{aligned}
 & \left(-2 i a^2 (7 a+b) \left(\frac{2 a-b+b \operatorname{Cosh}[2(e+f x)]}{a} \right)^{3/2} \operatorname{EllipticE}\left[i(e+f x), \frac{b}{a}\right] + \right. \\
 & \left. 8 i a^2 (a-b) \left(\frac{2 a-b+b \operatorname{Cosh}[2(e+f x)]}{a} \right)^{3/2} \operatorname{EllipticF}\left[i(e+f x), \frac{b}{a}\right] - \frac{1}{\sqrt{2}} \right. \\
 & \left. (24 a^3 - 4 a^2 b + 5 a b^2 - b^3 + 4 a (11 a - 3 b) b \operatorname{Cosh}[2(e+f x)] + b^2 (7 a+b) \operatorname{Cosh}[4(e+f x)]) \right. \\
 & \left. \operatorname{Tanh}[e+f x] \right) / \left(6 a (a-b)^3 f (2 a-b+b \operatorname{Cosh}[2(e+f x)])^{3/2} \right)
 \end{aligned}$$

Problem 510: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Coth}[e+f x]^2}{(a+b \operatorname{Sinh}[e+f x]^2)^{5/2}} dx$$

Optimal (type 4, 351 leaves, 8 steps):

$$\frac{\operatorname{Coth}[e + f x]}{3 a f (a + b \operatorname{Sinh}[e + f x]^2)^{3/2}} + \frac{(3 a - 4 b) \operatorname{Coth}[e + f x]}{3 a^2 (a - b) f \sqrt{a + b \operatorname{Sinh}[e + f x]^2}} -$$

$$\frac{(7 a - 8 b) \operatorname{Coth}[e + f x] \sqrt{a + b \operatorname{Sinh}[e + f x]^2}}{3 a^3 (a - b) f} -$$

$$\left((7 a - 8 b) \operatorname{EllipticE}\left[\operatorname{ArcTan}[\operatorname{Sinh}[e + f x]], 1 - \frac{b}{a}\right] \operatorname{Sech}[e + f x] \sqrt{a + b \operatorname{Sinh}[e + f x]^2} \right) /$$

$$\left(3 a^3 (a - b) f \sqrt{\frac{\operatorname{Sech}[e + f x]^2 (a + b \operatorname{Sinh}[e + f x]^2)}{a}} \right) +$$

$$\left((3 a - 4 b) \operatorname{EllipticF}\left[\operatorname{ArcTan}[\operatorname{Sinh}[e + f x]], 1 - \frac{b}{a}\right] \operatorname{Sech}[e + f x] \sqrt{a + b \operatorname{Sinh}[e + f x]^2} \right) /$$

$$\left(3 a^3 (a - b) f \sqrt{\frac{\operatorname{Sech}[e + f x]^2 (a + b \operatorname{Sinh}[e + f x]^2)}{a}} \right) +$$

$$\frac{(7 a - 8 b) \sqrt{a + b \operatorname{Sinh}[e + f x]^2} \operatorname{Tanh}[e + f x]}{3 a^3 (a - b) f}$$

Result (type 4, 226 leaves):

$$\left(-\frac{1}{\sqrt{2}} (24 a^3 - 68 a^2 b + 69 a b^2 - 24 b^3 +$$

$$4 b (11 a^2 - 19 a b + 8 b^2) \operatorname{Cosh}[2 (e + f x)] + (7 a - 8 b) b^2 \operatorname{Cosh}[4 (e + f x)] \right) \operatorname{Coth}[e + f x] -$$

$$2 i a^2 (7 a - 8 b) \left(\frac{2 a - b + b \operatorname{Cosh}[2 (e + f x)]}{a} \right)^{3/2} \operatorname{EllipticE}\left[i (e + f x), \frac{b}{a}\right] +$$

$$8 i a^2 (a - b) \left(\frac{2 a - b + b \operatorname{Cosh}[2 (e + f x)]}{a} \right)^{3/2} \operatorname{EllipticF}\left[i (e + f x), \frac{b}{a}\right] \right) /$$

$$(6 a^3 (a - b) f (2 a - b + b \operatorname{Cosh}[2 (e + f x)])^{3/2})$$

Problem 511: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Coth}[e + f x]^4}{(a + b \operatorname{Sinh}[e + f x]^2)^{5/2}} dx$$

Optimal (type 4, 385 leaves, 9 steps):

$$\begin{aligned}
 & - \frac{(a-b) \operatorname{Coth}[e+fx] \operatorname{Csch}[e+fx]^2}{3abf(a+b\operatorname{Sinh}[e+fx]^2)^{3/2}} - \\
 & \frac{2(a-3b) \operatorname{Coth}[e+fx] \operatorname{Csch}[e+fx]^2}{3a^2bf\sqrt{a+b\operatorname{Sinh}[e+fx]^2}} - \frac{8(a-2b) \operatorname{Coth}[e+fx] \sqrt{a+b\operatorname{Sinh}[e+fx]^2}}{3a^4f} + \\
 & \frac{(3a-8b) \operatorname{Coth}[e+fx] \operatorname{Csch}[e+fx]^2 \sqrt{a+b\operatorname{Sinh}[e+fx]^2}}{3a^3bf} - \\
 & \left(\frac{8(a-2b) \operatorname{EllipticE}[\operatorname{ArcTan}[\operatorname{Sinh}[e+fx]], 1-\frac{b}{a}] \operatorname{Sech}[e+fx] \sqrt{a+b\operatorname{Sinh}[e+fx]^2}}{3a^4f} \right) / \\
 & \left(\frac{3a^4f \sqrt{\frac{\operatorname{Sech}[e+fx]^2(a+b\operatorname{Sinh}[e+fx]^2)}{a}}}{3a^4f} \right) + \\
 & \left(\frac{(3a-8b) \operatorname{EllipticF}[\operatorname{ArcTan}[\operatorname{Sinh}[e+fx]], 1-\frac{b}{a}] \operatorname{Sech}[e+fx] \sqrt{a+b\operatorname{Sinh}[e+fx]^2}}{3a^4f} \right) / \\
 & \left(\frac{3a^4f \sqrt{\frac{\operatorname{Sech}[e+fx]^2(a+b\operatorname{Sinh}[e+fx]^2)}{a}}}{3a^4f} \right) + \frac{8(a-2b) \sqrt{a+b\operatorname{Sinh}[e+fx]^2} \operatorname{Tanh}[e+fx]}{3a^4f}
 \end{aligned}$$

Result (type 4, 247 leaves):

$$\begin{aligned}
 & - \left(\left(\frac{1}{\sqrt{2}} i b (8a^3 - 63a^2b + 92ab^2 - 40b^3 - 2(8a^3 - 38a^2b + 63ab^2 - 30b^3) \operatorname{Cosh}[2(e+fx)] - \right. \right. \\
 & \quad b(13a^2 - 36ab + 24b^2) \operatorname{Cosh}[4(e+fx)] - 2ab^2 \operatorname{Cosh}[6(e+fx)] + 4b^3 \\
 & \quad \left. \left. \operatorname{Cosh}[6(e+fx)]) \operatorname{Coth}[e+fx] \operatorname{Csch}[e+fx]^2 + 2a^2b \left(\frac{2a-b+b\operatorname{Cosh}[2(e+fx)]}{a} \right)^{3/2} \right. \right. \\
 & \quad \left. \left. \left(8(a-2b) \operatorname{EllipticE}[i(e+fx), \frac{b}{a}] + (-5a+8b) \operatorname{EllipticF}[i(e+fx), \frac{b}{a}] \right) \right) \right) / \\
 & \left(6a^4bf(2a-b+b\operatorname{Cosh}[2(e+fx)])^{3/2} \right)
 \end{aligned}$$

Problem 512: Unable to integrate problem.

$$\int (a+b\operatorname{Sinh}[e+fx]^2)^p (d\operatorname{Tanh}[e+fx])^m dx$$

Optimal (type 6, 122 leaves, 3 steps):

$$\begin{aligned}
 & \frac{1}{df(1+m)} \operatorname{AppellF1}\left[\frac{1+m}{2}, \frac{1+m}{2}, -p, \frac{3+m}{2}, -\operatorname{Sinh}[e+fx]^2, -\frac{b\operatorname{Sinh}[e+fx]^2}{a}\right] \\
 & (\operatorname{Cosh}[e+fx]^2)^{\frac{1+m}{2}} (a+b\operatorname{Sinh}[e+fx]^2)^p \left(1 + \frac{b\operatorname{Sinh}[e+fx]^2}{a}\right)^{-p} (d\operatorname{Tanh}[e+fx])^{1+m}
 \end{aligned}$$

Result (type 8, 27 leaves):

$$\int (a + b \operatorname{Sinh}[e + f x]^2)^p (d \operatorname{Tanh}[e + f x])^m dx$$

Problem 513: Unable to integrate problem.

$$\int (a + b \operatorname{Sinh}[c + d x]^2)^p \operatorname{Tanh}[c + d x]^3 dx$$

Optimal (type 5, 110 leaves, 3 steps):

$$-\left((a - b(1 + p)) \operatorname{Hypergeometric2F1}\left[1, 1 + p, 2 + p, \frac{a + b \operatorname{Sinh}[c + d x]^2}{a - b}\right] (a + b \operatorname{Sinh}[c + d x]^2)^{1+p} \right) / \left(2(a - b)^2 d(1 + p) \right) + \frac{\operatorname{Sech}[c + d x]^2 (a + b \operatorname{Sinh}[c + d x]^2)^{1+p}}{2(a - b)d}$$

Result (type 8, 25 leaves):

$$\int (a + b \operatorname{Sinh}[c + d x]^2)^p \operatorname{Tanh}[c + d x]^3 dx$$

Problem 514: Unable to integrate problem.

$$\int (a + b \operatorname{Sinh}[c + d x]^2)^p \operatorname{Tanh}[c + d x] dx$$

Optimal (type 5, 63 leaves, 2 steps):

$$-\left(\left(\operatorname{Hypergeometric2F1}\left[1, 1 + p, 2 + p, \frac{a + b \operatorname{Sinh}[c + d x]^2}{a - b}\right] (a + b \operatorname{Sinh}[c + d x]^2)^{1+p} \right) / \left(2(a - b)d(1 + p) \right) \right)$$

Result (type 8, 23 leaves):

$$\int (a + b \operatorname{Sinh}[c + d x]^2)^p \operatorname{Tanh}[c + d x] dx$$

Problem 516: Unable to integrate problem.

$$\int \operatorname{Coth}[c + d x]^3 (a + b \operatorname{Sinh}[c + d x]^2)^p dx$$

Optimal (type 5, 94 leaves, 3 steps):

$$-\frac{\operatorname{Csch}[c + d x]^2 (a + b \operatorname{Sinh}[c + d x]^2)^{1+p}}{2ad} - \frac{1}{2a^2 d(1 + p)} (a + b p) \operatorname{Hypergeometric2F1}\left[1, 1 + p, 2 + p, 1 + \frac{b \operatorname{Sinh}[c + d x]^2}{a}\right] (a + b \operatorname{Sinh}[c + d x]^2)^{1+p}$$

Result (type 8, 25 leaves):

$$\int \operatorname{Coth}[c + d x]^3 (a + b \operatorname{Sinh}[c + d x]^2)^p dx$$

Problem 517: Unable to integrate problem.

$$\int (a + b \operatorname{Sinh}[c + d x]^2)^p \operatorname{Tanh}[c + d x]^4 dx$$

Optimal (type 6, 103 leaves, 3 steps):

$$\frac{1}{5d} \operatorname{AppellF1}\left[\frac{5}{2}, \frac{5}{2}, -p, \frac{7}{2}, -\operatorname{Sinh}[c + d x]^2, -\frac{b \operatorname{Sinh}[c + d x]^2}{a}\right] \sqrt{\operatorname{Cosh}[c + d x]^2} \operatorname{Sinh}[c + d x]^4 (a + b \operatorname{Sinh}[c + d x]^2)^p \left(1 + \frac{b \operatorname{Sinh}[c + d x]^2}{a}\right)^{-p} \operatorname{Tanh}[c + d x]$$

Result (type 8, 25 leaves):

$$\int (a + b \operatorname{Sinh}[c + d x]^2)^p \operatorname{Tanh}[c + d x]^4 dx$$

Problem 518: Unable to integrate problem.

$$\int (a + b \operatorname{Sinh}[c + d x]^2)^p \operatorname{Tanh}[c + d x]^2 dx$$

Optimal (type 6, 103 leaves, 3 steps):

$$\frac{1}{3d} \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2}, -p, \frac{5}{2}, -\operatorname{Sinh}[c + d x]^2, -\frac{b \operatorname{Sinh}[c + d x]^2}{a}\right] \sqrt{\operatorname{Cosh}[c + d x]^2} \operatorname{Sinh}[c + d x]^2 (a + b \operatorname{Sinh}[c + d x]^2)^p \left(1 + \frac{b \operatorname{Sinh}[c + d x]^2}{a}\right)^{-p} \operatorname{Tanh}[c + d x]$$

Result (type 8, 25 leaves):

$$\int (a + b \operatorname{Sinh}[c + d x]^2)^p \operatorname{Tanh}[c + d x]^2 dx$$

Problem 519: Unable to integrate problem.

$$\int \operatorname{Coth}[c + d x]^2 (a + b \operatorname{Sinh}[c + d x]^2)^p dx$$

Optimal (type 6, 99 leaves, 3 steps):

$$-\frac{1}{d} \operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -p, \frac{1}{2}, -\operatorname{Sinh}[c + d x]^2, -\frac{b \operatorname{Sinh}[c + d x]^2}{a}\right] \sqrt{\operatorname{Cosh}[c + d x]^2} \operatorname{Csch}[c + d x] \operatorname{Sech}[c + d x] (a + b \operatorname{Sinh}[c + d x]^2)^p \left(1 + \frac{b \operatorname{Sinh}[c + d x]^2}{a}\right)^{-p}$$

Result (type 8, 25 leaves):

$$\int \operatorname{Coth}[c + d x]^2 (a + b \operatorname{Sinh}[c + d x]^2)^p dx$$

Problem 520: Unable to integrate problem.

$$\int \text{Coth}[c + d x]^4 (a + b \text{Sinh}[c + d x]^2)^p dx$$

Optimal (type 6, 103 leaves, 3 steps):

$$-\frac{1}{3d} \text{AppellF1}\left[-\frac{3}{2}, -\frac{3}{2}, -p, -\frac{1}{2}, -\text{Sinh}[c + d x]^2, -\frac{b \text{Sinh}[c + d x]^2}{a}\right] \sqrt{\text{Cosh}[c + d x]^2} \\ \text{Csch}[c + d x]^3 \text{Sech}[c + d x] (a + b \text{Sinh}[c + d x]^2)^p \left(1 + \frac{b \text{Sinh}[c + d x]^2}{a}\right)^{-p}$$

Result (type 8, 25 leaves):

$$\int \text{Coth}[c + d x]^4 (a + b \text{Sinh}[c + d x]^2)^p dx$$

Problem 521: Result is not expressed in closed-form.

$$\int \frac{\text{Coth}[x]^3}{a + b \text{Sinh}[x]^3} dx$$

Optimal (type 3, 152 leaves, 12 steps):

$$\frac{b^{2/3} \text{ArcTan}\left[\frac{a^{1/3} - 2b^{1/3} \text{Sinh}[x]}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} a^{5/3}} - \frac{\text{Csch}[x]^2}{2a} + \frac{\text{Log}[\text{Sinh}[x]]}{a} - \frac{b^{2/3} \text{Log}[a^{1/3} + b^{1/3} \text{Sinh}[x]]}{3 a^{5/3}} + \\ \frac{b^{2/3} \text{Log}[a^{2/3} - a^{1/3} b^{1/3} \text{Sinh}[x] + b^{2/3} \text{Sinh}[x]^2]}{6 a^{5/3}} - \frac{\text{Log}[a + b \text{Sinh}[x]^3]}{3a}$$

Result (type 7, 162 leaves):

$$-\frac{1}{24a} \left(8 \text{RootSum}[-b + 3b \#1^2 + 8a \#1^3 - 3b \#1^4 + b \#1^6 \&, \right. \\ \left. (-bx + b \text{Log}[e^x - \#1] + 4ax \#1^3 - 4a \text{Log}[e^x - \#1] \#1^3 - 3bx \#1^4 + 3b \text{Log}[e^x - \#1] \#1^4) / \right. \\ \left. (b - 2b \#1^2 - 4a \#1^3 + b \#1^4) \& \right] + 3 \left(8x + \text{Csch}\left[\frac{x}{2}\right]^2 - 8 \text{Log}[\text{Sinh}[x]] - \text{Sech}\left[\frac{x}{2}\right]^2 \right) \right)$$

Problem 522: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Coth}[x]}{\sqrt{a + b \text{Sinh}[x]^3}} dx$$

Optimal (type 3, 28 leaves, 4 steps):

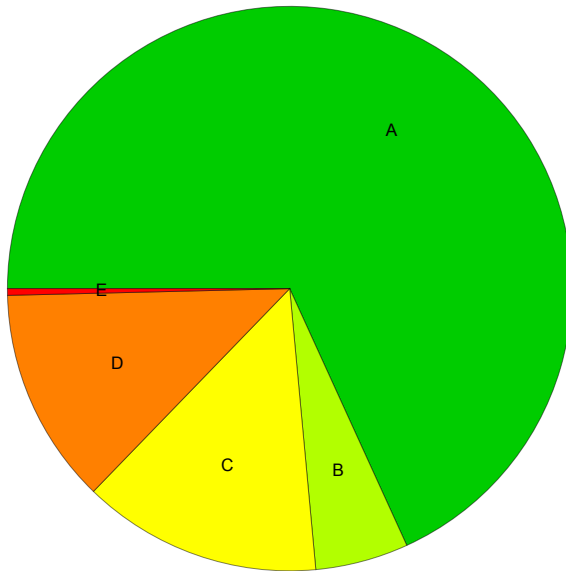
$$-\frac{2 \text{ArcTanh}\left[\frac{\sqrt{a + b \text{Sinh}[x]^3}}{\sqrt{a}}\right]}{3 \sqrt{a}}$$

Result (type 3, 66 leaves):

$$\frac{2 \sqrt{b} \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Csch}[x]^{3/2}}{\sqrt{b}}\right] \sqrt{\frac{b+a \operatorname{Csch}[x]^3}{b}}}{3 \sqrt{a} \operatorname{Csch}[x]^{3/2} \sqrt{a+b \operatorname{Sinh}[x]^3}}$$

Summary of Integration Test Results

525 integration problems



A - 358 optimal antiderivatives

B - 28 more than twice size of optimal antiderivatives

C - 72 unnecessarily complex antiderivatives

D - 65 unable to integrate problems

E - 2 integration timeouts