

# Mathematica 11.3 Integration Test Results

Test results for the 183 problems in "6.2.1 (c+d x)^m (a+b cosh)^n.m"

Problem 28: Result more than twice size of optimal antiderivative.

$$\int (c + d x) \operatorname{Sech}[a + b x] dx$$

Optimal (type 4, 61 leaves, 5 steps):

$$\frac{2 (c + d x) \operatorname{ArcTan}\left[e^{a+bx}\right]}{b} - \frac{i d \operatorname{PolyLog}\left[2, -i e^{a+bx}\right]}{b^2} + \frac{i d \operatorname{PolyLog}\left[2, i e^{a+bx}\right]}{b^2}$$

Result (type 4, 132 leaves):

$$\begin{aligned} & \frac{1}{2 b^2} \left( 4 b c \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2}(a + b x)\right]\right] \right) - d (-2 i a + \pi - 2 i b x) \left( \operatorname{Log}\left[1 - i e^{a+bx}\right] - \operatorname{Log}\left[1 + i e^{a+bx}\right] \right) + \\ & d (-2 i a + \pi) \operatorname{Log}\left[\operatorname{Cot}\left[\frac{1}{4}(2 i a + \pi + 2 i b x)\right]\right] - \\ & 2 i d \left( \operatorname{PolyLog}\left[2, -i e^{a+bx}\right] - \operatorname{PolyLog}\left[2, i e^{a+bx}\right] \right) \end{aligned}$$

Problem 32: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int (c + d x)^2 \operatorname{Sech}[a + b x]^2 dx$$

Optimal (type 4, 73 leaves, 5 steps):

$$\frac{(c + d x)^2}{b} - \frac{2 d (c + d x) \operatorname{Log}\left[1 + e^{2(a+bx)}\right]}{b^2} - \frac{d^2 \operatorname{PolyLog}\left[2, -e^{2(a+bx)}\right]}{b^3} + \frac{(c + d x)^2 \operatorname{Tanh}[a + b x]}{b}$$

Result (type 4, 277 leaves):

$$\begin{aligned}
 & - \left( \left( 2 c d \operatorname{Sech}[a] \left( \operatorname{Cosh}[a] \operatorname{Log}[\operatorname{Cosh}[a] \operatorname{Cosh}[b x] + \operatorname{Sinh}[a] \operatorname{Sinh}[b x]] - b x \operatorname{Sinh}[a] \right) \right) / \right. \\
 & \quad \left. \left( b^2 \left( \operatorname{Cosh}[a]^2 - \operatorname{Sinh}[a]^2 \right) \right) \right) + \\
 & \left( d^2 \operatorname{Csch}[a] \left( -b^2 e^{-\operatorname{ArcTanh}[\operatorname{Coth}[a]]} x^2 + \left( i \operatorname{Coth}[a] \left( -b x \left( -\pi + 2 i \operatorname{ArcTanh}[\operatorname{Coth}[a]] \right) \right) - \right. \right. \right. \\
 & \quad \left. \left. \left. \pi \operatorname{Log}\left[1 + e^{2 b x}\right] - 2 \left( i b x + i \operatorname{ArcTanh}[\operatorname{Coth}[a]] \right) \operatorname{Log}\left[1 - e^{2 i \left( i b x + i \operatorname{ArcTanh}[\operatorname{Coth}[a]] \right)}\right] \right) \right) + \right. \\
 & \quad \left. \pi \operatorname{Log}[\operatorname{Cosh}[b x]] + 2 i \operatorname{ArcTanh}[\operatorname{Coth}[a]] \operatorname{Log}\left[ i \operatorname{Sinh}[b x + \operatorname{ArcTanh}[\operatorname{Coth}[a]]] \right] \right) + \\
 & \quad \left. i \operatorname{PolyLog}\left[2, e^{2 i \left( i b x + i \operatorname{ArcTanh}[\operatorname{Coth}[a]] \right)}\right] \right) / \left( \sqrt{1 - \operatorname{Coth}[a]^2} \right) \operatorname{Sech}[a] / \\
 & \left( b^3 \sqrt{\operatorname{Csch}[a]^2 \left( -\operatorname{Cosh}[a]^2 + \operatorname{Sinh}[a]^2 \right)} \right) + \frac{1}{b} \operatorname{Sech}[a] \operatorname{Sech}[ \\
 & \quad a + b x] \\
 & \left( c^2 \operatorname{Sinh}[b x] + 2 c d x \operatorname{Sinh}[b x] + d^2 x^2 \operatorname{Sinh}[b x] \right)
 \end{aligned}$$

**Problem 38: Result more than twice size of optimal antiderivative.**

$$\int (c + d x) \operatorname{Sech}[a + b x]^3 dx$$

Optimal (type 4, 102 leaves, 6 steps):

$$\begin{aligned}
 & \frac{(c + d x) \operatorname{ArcTan}\left[e^{a+b x}\right]}{b} - \frac{i d \operatorname{PolyLog}\left[2, -i e^{a+b x}\right]}{2 b^2} + \\
 & \frac{i d \operatorname{PolyLog}\left[2, i e^{a+b x}\right]}{2 b^2} + \frac{d \operatorname{Sech}[a + b x]}{2 b^2} + \frac{(c + d x) \operatorname{Sech}[a + b x] \operatorname{Tanh}[a + b x]}{2 b}
 \end{aligned}$$

Result (type 4, 263 leaves):

$$\begin{aligned}
 & \frac{c \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2}(a + b x)\right]\right]}{b} - \frac{1}{2 b^2} \\
 & d \left( \left( -i a + \frac{\pi}{2} - i b x \right) \left( \operatorname{Log}\left[1 - e^{i \left( -i a + \frac{\pi}{2} - i b x \right)}\right] - \operatorname{Log}\left[1 + e^{i \left( -i a + \frac{\pi}{2} - i b x \right)}\right] \right) - \right. \\
 & \quad \left( -i a + \frac{\pi}{2} \right) \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2}\left(-i a + \frac{\pi}{2} - i b x\right)\right]\right] + \\
 & \quad \left. i \left( \operatorname{PolyLog}\left[2, -e^{i \left( -i a + \frac{\pi}{2} - i b x \right)}\right] - \operatorname{PolyLog}\left[2, e^{i \left( -i a + \frac{\pi}{2} - i b x \right)}\right] \right) \right) + \\
 & \frac{d \operatorname{Sech}[a] \operatorname{Sech}[a + b x] \left( \operatorname{Cosh}[a] + b x \operatorname{Sinh}[a] \right)}{2 b^2} + \frac{d x \operatorname{Sech}[a] \operatorname{Sech}[a + b x]^2 \operatorname{Sinh}[b x]}{2 b} + \\
 & \frac{c \operatorname{Sech}[a + b x] \operatorname{Tanh}[a + b x]}{2 b}
 \end{aligned}$$

**Problem 39: Attempted integration timed out after 120 seconds.**

$$\int \frac{\operatorname{Sech}[a + b x]^3}{c + d x} dx$$

Optimal (type 8, 19 leaves, 0 steps):

$$\operatorname{Int}\left[\frac{\operatorname{Sech}[a + b x]^3}{c + d x}, x\right]$$

Result (type 1, 1 leaves):

???

**Problem 40: Attempted integration timed out after 120 seconds.**

$$\int \frac{\text{Sech}[a + b x]^3}{(c + d x)^2} dx$$

Optimal (type 8, 19 leaves, 0 steps):

$$\text{Int}\left[\frac{\text{Sech}[a + b x]^3}{(c + d x)^2}, x\right]$$

Result (type 1, 1 leaves):

???

**Problem 48: Result more than twice size of optimal antiderivative.**

$$\int (c + d x)^{5/2} \text{Cosh}[a + b x]^2 dx$$

Optimal (type 4, 239 leaves, 10 steps):

$$\begin{aligned} & \frac{5 d (c + d x)^{3/2}}{16 b^2} + \frac{(c + d x)^{7/2}}{7 d} - \frac{5 d (c + d x)^{3/2} \text{Cosh}[a + b x]^2}{8 b^2} + \\ & \frac{15 d^{5/2} e^{-2 a + \frac{2 b c}{d}} \sqrt{\frac{\pi}{2}} \text{Erf}\left[\frac{\sqrt{2} \sqrt{b} \sqrt{c + d x}}{\sqrt{d}}\right]}{256 b^{7/2}} - \frac{15 d^{5/2} e^{2 a - \frac{2 b c}{d}} \sqrt{\frac{\pi}{2}} \text{Erfi}\left[\frac{\sqrt{2} \sqrt{b} \sqrt{c + d x}}{\sqrt{d}}\right]}{256 b^{7/2}} + \\ & \frac{(c + d x)^{5/2} \text{Cosh}[a + b x] \text{Sinh}[a + b x]}{2 b} + \frac{15 d^2 \sqrt{c + d x} \text{Sinh}[2 a + 2 b x]}{64 b^3} \end{aligned}$$

Result (type 4, 3531 leaves):

$$\begin{aligned} & \frac{(c + d x)^{7/2}}{7 d} + \frac{1}{2} c^2 \text{Cosh}[2 a] \\ & \left( -\frac{1}{d} \left( \frac{d \sqrt{c + d x} \text{Cosh}\left[\frac{2 b (c + d x)}{d}\right]}{4 b} - \frac{d^{3/2} \sqrt{\pi} \left( \text{Erf}\left[\frac{\sqrt{2} \sqrt{b} \sqrt{c + d x}}{\sqrt{d}}\right] + \text{Erfi}\left[\frac{\sqrt{2} \sqrt{b} \sqrt{c + d x}}{\sqrt{d}}\right] \right)}{16 \sqrt{2} b^{3/2}} \right) \right. \\ & \quad \left. \text{Sinh}\left[\frac{2 b c}{d}\right] + \frac{1}{d} 2 \text{Cosh}\left[\frac{2 b c}{d}\right] \right. \\ & \quad \left. \left( -\frac{d^{3/2} \sqrt{\pi} \left( -\text{Erf}\left[\frac{\sqrt{2} \sqrt{b} \sqrt{c + d x}}{\sqrt{d}}\right] + \text{Erfi}\left[\frac{\sqrt{2} \sqrt{b} \sqrt{c + d x}}{\sqrt{d}}\right] \right)}{16 \sqrt{2} b^{3/2}} + \frac{d \sqrt{c + d x} \text{Sinh}\left[\frac{2 b (c + d x)}{d}\right]}{4 b} \right) \right) + \\ & c^2 \text{Cosh}[a] \text{Sinh}[a] \left( \frac{1}{d} 2 \text{Cosh}\left[\frac{2 b c}{d}\right] \left( \frac{d \sqrt{c + d x} \text{Cosh}\left[\frac{2 b (c + d x)}{d}\right]}{4 b} - \right. \right. \end{aligned}$$

$$\begin{aligned}
 & \left. \frac{d^{3/2} \sqrt{\pi} \left( \operatorname{Erf} \left[ \frac{\sqrt{2} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right] + \operatorname{Erfi} \left[ \frac{\sqrt{2} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right] \right)}{16 \sqrt{2} b^{3/2}} - \frac{1}{d} 2 \operatorname{Sinh} \left[ \frac{2bc}{d} \right]}{2} \right. \\
 & \left. \left( - \frac{d^{3/2} \sqrt{\pi} \left( -\operatorname{Erf} \left[ \frac{\sqrt{2} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right] + \operatorname{Erfi} \left[ \frac{\sqrt{2} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right] \right)}{16 \sqrt{2} b^{3/2}} + \frac{d \sqrt{c+dx} \operatorname{Sinh} \left[ \frac{2b(c+dx)}{d} \right]}{4b} \right) \right) + \\
 & c d \operatorname{Cosh} [2a] \left( \frac{1}{d^2} 2c \left( \frac{d \sqrt{c+dx} \operatorname{Cosh} \left[ \frac{2b(c+dx)}{d} \right]}{4b} - \right. \right. \\
 & \left. \left. \frac{d^{3/2} \sqrt{\pi} \left( \operatorname{Erf} \left[ \frac{\sqrt{2} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right] + \operatorname{Erfi} \left[ \frac{\sqrt{2} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right] \right)}{16 \sqrt{2} b^{3/2}} \right) \operatorname{Sinh} \left[ \frac{2bc}{d} \right] - \right. \\
 & \left. \frac{1}{d^2} 2c \operatorname{Cosh} \left[ \frac{2bc}{d} \right] \left( - \frac{d^{3/2} \sqrt{\pi} \left( -\operatorname{Erf} \left[ \frac{\sqrt{2} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right] + \operatorname{Erfi} \left[ \frac{\sqrt{2} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right] \right)}{16 \sqrt{2} b^{3/2}} + \right. \right. \\
 & \left. \left. \frac{d \sqrt{c+dx} \operatorname{Sinh} \left[ \frac{2b(c+dx)}{d} \right]}{4b} \right) + \frac{1}{32 \sqrt{2} b^{5/2} d} \right. \\
 & \left. \operatorname{Sinh} \left[ \frac{2bc}{d} \right] \left( 3 d^{3/2} \sqrt{\pi} \operatorname{Erf} \left[ \frac{\sqrt{2} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right] - 3 d^{3/2} \sqrt{\pi} \operatorname{Erfi} \left[ \frac{\sqrt{2} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right] + 4 \sqrt{2} \right. \right. \\
 & \left. \left. \sqrt{b} \sqrt{c+dx} \left( -4b(c+dx) \operatorname{Cosh} \left[ \frac{2b(c+dx)}{d} \right] + 3d \operatorname{Sinh} \left[ \frac{2b(c+dx)}{d} \right] \right) \right) \right) + \frac{1}{32 \sqrt{2} b^{5/2} d} \\
 & \left. \operatorname{Cosh} \left[ \frac{2bc}{d} \right] \left( 3 d^{3/2} \sqrt{\pi} \operatorname{Erf} \left[ \frac{\sqrt{2} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right] + 3 d^{3/2} \sqrt{\pi} \operatorname{Erfi} \left[ \frac{\sqrt{2} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right] + \right. \right. \\
 & \left. \left. 4 \sqrt{2} \sqrt{b} \sqrt{c+dx} \left( -3d \operatorname{Cosh} \left[ \frac{2b(c+dx)}{d} \right] + 4b(c+dx) \operatorname{Sinh} \left[ \frac{2b(c+dx)}{d} \right] \right) \right) \right) \right) + \\
 & 2cd \operatorname{Cosh} [a] \operatorname{Sinh} [a] \left( - \frac{1}{d^2} 2c \operatorname{Cosh} \left[ \frac{2bc}{d} \right] \left( \frac{d \sqrt{c+dx} \operatorname{Cosh} \left[ \frac{2b(c+dx)}{d} \right]}{4b} - \right. \right. \\
 & \left. \left. \frac{d^{3/2} \sqrt{\pi} \left( \operatorname{Erf} \left[ \frac{\sqrt{2} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right] + \operatorname{Erfi} \left[ \frac{\sqrt{2} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right] \right)}{16 \sqrt{2} b^{3/2}} \right) + \frac{1}{d^2} \right. \\
 & \left. \left. 2c \operatorname{Sinh} \left[ \frac{2bc}{d} \right] \left( - \frac{d^{3/2} \sqrt{\pi} \left( -\operatorname{Erf} \left[ \frac{\sqrt{2} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right] + \operatorname{Erfi} \left[ \frac{\sqrt{2} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right] \right)}{16 \sqrt{2} b^{3/2}} + \right. \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & \left. \frac{d \sqrt{c+dx} \operatorname{Sinh}\left[\frac{2b(c+dx)}{d}\right]}{4b} \right) + \frac{1}{32\sqrt{2}b^{5/2}d} \\
 & \operatorname{Cosh}\left[\frac{2bc}{d}\right] \left( -3d^{3/2}\sqrt{\pi} \operatorname{Erf}\left[\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right] + 3d^{3/2}\sqrt{\pi} \operatorname{Erfi}\left[\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right] + 4\sqrt{2} \right. \\
 & \left. \sqrt{b}\sqrt{c+dx} \left( 4b(c+dx) \operatorname{Cosh}\left[\frac{2b(c+dx)}{d}\right] - 3d \operatorname{Sinh}\left[\frac{2b(c+dx)}{d}\right] \right) \right) - \frac{1}{32\sqrt{2}b^{5/2}d} \\
 & \operatorname{Sinh}\left[\frac{2bc}{d}\right] \left( 3d^{3/2}\sqrt{\pi} \operatorname{Erf}\left[\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right] + 3d^{3/2}\sqrt{\pi} \operatorname{Erfi}\left[\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right] + \right. \\
 & \left. 4\sqrt{2}\sqrt{b}\sqrt{c+dx} \left( -3d \operatorname{Cosh}\left[\frac{2b(c+dx)}{d}\right] + 4b(c+dx) \operatorname{Sinh}\left[\frac{2b(c+dx)}{d}\right] \right) \right) \left. \right) + \\
 & \frac{1}{2}d^2 \operatorname{Cosh}[2a] \left( -\frac{1}{d^3}2c^2 \left( \frac{d\sqrt{c+dx} \operatorname{Cosh}\left[\frac{2b(c+dx)}{d}\right]}{4b} - \right. \right. \\
 & \left. \left. \frac{d^{3/2}\sqrt{\pi} \left( \operatorname{Erf}\left[\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right] + \operatorname{Erfi}\left[\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right] \right)}{16\sqrt{2}b^{3/2}} \right) \operatorname{Sinh}\left[\frac{2bc}{d}\right] + \right. \\
 & \left. \frac{1}{d^3}2c^2 \operatorname{Cosh}\left[\frac{2bc}{d}\right] \left( -\frac{d^{3/2}\sqrt{\pi} \left( -\operatorname{Erf}\left[\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right] + \operatorname{Erfi}\left[\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right] \right)}{16\sqrt{2}b^{3/2}} + \right. \right. \\
 & \left. \left. \frac{d\sqrt{c+dx} \operatorname{Sinh}\left[\frac{2b(c+dx)}{d}\right]}{4b} \right) + \frac{1}{16\sqrt{2}b^{5/2}d^2} \right. \\
 & c \operatorname{Sinh}\left[\frac{2bc}{d}\right] \left( -3d^{3/2}\sqrt{\pi} \operatorname{Erf}\left[\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right] + 3d^{3/2}\sqrt{\pi} \operatorname{Erfi}\left[\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right] + 4\sqrt{2} \right. \\
 & \left. \sqrt{b}\sqrt{c+dx} \left( 4b(c+dx) \operatorname{Cosh}\left[\frac{2b(c+dx)}{d}\right] - 3d \operatorname{Sinh}\left[\frac{2b(c+dx)}{d}\right] \right) \right) - \frac{1}{16\sqrt{2}b^{5/2}d^2} \\
 & c \operatorname{Cosh}\left[\frac{2bc}{d}\right] \left( 3d^{3/2}\sqrt{\pi} \operatorname{Erf}\left[\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right] + 3d^{3/2}\sqrt{\pi} \operatorname{Erfi}\left[\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right] + \right. \\
 & \left. 4\sqrt{2}\sqrt{b}\sqrt{c+dx} \left( -3d \operatorname{Cosh}\left[\frac{2b(c+dx)}{d}\right] + 4b(c+dx) \operatorname{Sinh}\left[\frac{2b(c+dx)}{d}\right] \right) \right) - \\
 & \left( (c+dx)^{3/2} \operatorname{Sinh}\left[\frac{2bc}{d}\right] \left( -15d^2\sqrt{\pi} \operatorname{Erf}\left[\sqrt{2}\sqrt{\frac{b(c+dx)}{d}}\right] - \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & 15 d^2 \sqrt{\pi} \operatorname{Erfi} \left[ \sqrt{2} \sqrt{\frac{b(c+dx)}{d}} \right] + 4 \sqrt{2} \sqrt{\frac{b(c+dx)}{d}} \\
 & \left( \left( (15 d^2 + 16 b^2 (c+dx)^2) \operatorname{Cosh} \left[ \frac{2 b (c+dx)}{d} \right] - 20 b d (c+dx) \operatorname{Sinh} \left[ \frac{2 b (c+dx)}{d} \right] \right) \right) / \\
 & \left( 128 \sqrt{2} b^2 d^3 \left( \frac{b(c+dx)}{d} \right)^{3/2} \right) + \frac{1}{128 \sqrt{2} b^{7/2} d^2} \operatorname{Cosh} \left[ \frac{2 b c}{d} \right] \left( 15 d^{5/2} \sqrt{\pi} \right. \\
 & \operatorname{Erf} \left[ \frac{\sqrt{2} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right] - 15 d^{5/2} \sqrt{\pi} \operatorname{Erfi} \left[ \frac{\sqrt{2} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right] + 4 \sqrt{2} \sqrt{b} \sqrt{c+dx} \\
 & \left. \left( -20 b d (c+dx) \operatorname{Cosh} \left[ \frac{2 b (c+dx)}{d} \right] + (15 d^2 + 16 b^2 (c+dx)^2) \operatorname{Sinh} \left[ \frac{2 b (c+dx)}{d} \right] \right) \right) + \\
 & d^2 \operatorname{Cosh}[a] \operatorname{Sinh}[a] \left( \frac{1}{d^3} 2 c^2 \operatorname{Cosh} \left[ \frac{2 b c}{d} \right] \left( \frac{d \sqrt{c+dx} \operatorname{Cosh} \left[ \frac{2 b (c+dx)}{d} \right]}{4 b} - \right. \right. \\
 & \left. \left. \frac{d^{3/2} \sqrt{\pi} \left( \operatorname{Erf} \left[ \frac{\sqrt{2} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right] + \operatorname{Erfi} \left[ \frac{\sqrt{2} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right] \right)}{16 \sqrt{2} b^{3/2}} \right) - \frac{1}{d^3} \right. \\
 & \left. 2 c^2 \operatorname{Sinh} \left[ \frac{2 b c}{d} \right] \left( - \frac{d^{3/2} \sqrt{\pi} \left( - \operatorname{Erf} \left[ \frac{\sqrt{2} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right] + \operatorname{Erfi} \left[ \frac{\sqrt{2} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right] \right)}{16 \sqrt{2} b^{3/2}} + \right. \right. \\
 & \left. \left. \frac{d \sqrt{c+dx} \operatorname{Sinh} \left[ \frac{2 b (c+dx)}{d} \right]}{4 b} \right) + \frac{1}{16 \sqrt{2} b^{5/2} d^2} \right. \\
 & c \operatorname{Cosh} \left[ \frac{2 b c}{d} \right] \left( 3 d^{3/2} \sqrt{\pi} \operatorname{Erf} \left[ \frac{\sqrt{2} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right] - 3 d^{3/2} \sqrt{\pi} \operatorname{Erfi} \left[ \frac{\sqrt{2} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right] + 4 \sqrt{2} \right. \\
 & \left. \sqrt{b} \sqrt{c+dx} \left( -4 b (c+dx) \operatorname{Cosh} \left[ \frac{2 b (c+dx)}{d} \right] + 3 d \operatorname{Sinh} \left[ \frac{2 b (c+dx)}{d} \right] \right) \right) + \frac{1}{16 \sqrt{2} b^{5/2} d^2} \\
 & c \operatorname{Sinh} \left[ \frac{2 b c}{d} \right] \left( 3 d^{3/2} \sqrt{\pi} \operatorname{Erf} \left[ \frac{\sqrt{2} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right] + 3 d^{3/2} \sqrt{\pi} \operatorname{Erfi} \left[ \frac{\sqrt{2} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right] + \right. \\
 & \left. 4 \sqrt{2} \sqrt{b} \sqrt{c+dx} \left( -3 d \operatorname{Cosh} \left[ \frac{2 b (c+dx)}{d} \right] + 4 b (c+dx) \operatorname{Sinh} \left[ \frac{2 b (c+dx)}{d} \right] \right) \right) + \\
 & \frac{1}{128 \sqrt{2} b^{7/2} d^2} \operatorname{Cosh} \left[ \frac{2 b c}{d} \right] \left( -15 d^{5/2} \sqrt{\pi} \operatorname{Erf} \left[ \frac{\sqrt{2} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right] - \right. \\
 & \left. 15 d^{5/2} \sqrt{\pi} \operatorname{Erfi} \left[ \frac{\sqrt{2} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right] + 4 \sqrt{2} \sqrt{b} \sqrt{c+dx} \right)
 \end{aligned}$$

$$\left( \left( 15 d^2 + 16 b^2 (c + d x)^2 \right) \operatorname{Cosh} \left[ \frac{2 b (c + d x)}{d} \right] - 20 b d (c + d x) \operatorname{Sinh} \left[ \frac{2 b (c + d x)}{d} \right] \right) - \frac{1}{128 \sqrt{2} b^{7/2} d^2} \operatorname{Sinh} \left[ \frac{2 b c}{d} \right] \left( 15 d^{5/2} \sqrt{\pi} \operatorname{Erf} \left[ \frac{\sqrt{2} \sqrt{b} \sqrt{c + d x}}{\sqrt{d}} \right] - 15 d^{5/2} \sqrt{\pi} \operatorname{Erfi} \left[ \frac{\sqrt{2} \sqrt{b} \sqrt{c + d x}}{\sqrt{d}} \right] + 4 \sqrt{2} \sqrt{b} \sqrt{c + d x} \right) \left( -20 b d (c + d x) \operatorname{Cosh} \left[ \frac{2 b (c + d x)}{d} \right] + (15 d^2 + 16 b^2 (c + d x)^2) \operatorname{Sinh} \left[ \frac{2 b (c + d x)}{d} \right] \right) \right)$$

**Problem 61: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cosh}[a + b x]^3}{(c + d x)^{5/2}} dx$$

Optimal (type 4, 277 leaves, 18 steps):

$$\begin{aligned} & -\frac{2 \operatorname{Cosh}[a + b x]^3}{3 d (c + d x)^{3/2}} + \frac{b^{3/2} e^{-a + \frac{bc}{d}} \sqrt{\pi} \operatorname{Erf} \left[ \frac{\sqrt{b} \sqrt{c + d x}}{\sqrt{d}} \right]}{2 d^{5/2}} + \\ & \frac{b^{3/2} e^{-3a + \frac{3bc}{d}} \sqrt{3\pi} \operatorname{Erf} \left[ \frac{\sqrt{3} \sqrt{b} \sqrt{c + d x}}{\sqrt{d}} \right]}{2 d^{5/2}} + \frac{b^{3/2} e^{a - \frac{bc}{d}} \sqrt{\pi} \operatorname{Erfi} \left[ \frac{\sqrt{b} \sqrt{c + d x}}{\sqrt{d}} \right]}{2 d^{5/2}} + \\ & \frac{b^{3/2} e^{3a - \frac{3bc}{d}} \sqrt{3\pi} \operatorname{Erfi} \left[ \frac{\sqrt{3} \sqrt{b} \sqrt{c + d x}}{\sqrt{d}} \right]}{2 d^{5/2}} - \frac{4 b \operatorname{Cosh}[a + b x]^2 \operatorname{Sinh}[a + b x]}{d^2 \sqrt{c + d x}} \end{aligned}$$

Result (type 4, 716 leaves):

$$\frac{1}{6 d^{5/2} (c+dx)^{3/2}} \left( -3 d^{3/2} \operatorname{Cosh}[a+bx] - \right.$$

$$d^{3/2} \operatorname{Cosh}\left[3(a+bx)\right] + 3 b^{3/2} c \sqrt{\pi} \sqrt{c+dx} \operatorname{Cosh}\left[a-\frac{bc}{d}\right] \operatorname{Erfi}\left[\frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right] +$$

$$3 b^{3/2} d \sqrt{\pi} x \sqrt{c+dx} \operatorname{Cosh}\left[a-\frac{bc}{d}\right] \operatorname{Erfi}\left[\frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right] +$$

$$3 b^{3/2} c \sqrt{3 \pi} \sqrt{c+dx} \operatorname{Cosh}\left[3a-\frac{3bc}{d}\right] \operatorname{Erfi}\left[\frac{\sqrt{3} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right] +$$

$$3 b^{3/2} d \sqrt{3 \pi} x \sqrt{c+dx} \operatorname{Cosh}\left[3a-\frac{3bc}{d}\right] \operatorname{Erfi}\left[\frac{\sqrt{3} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right] +$$

$$3 b^{3/2} \sqrt{3 \pi} (c+dx)^{3/2} \operatorname{Erf}\left[\frac{\sqrt{3} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right] \left( \operatorname{Cosh}\left[3a-\frac{3bc}{d}\right] - \operatorname{Sinh}\left[3a-\frac{3bc}{d}\right] \right) +$$

$$3 b^{3/2} c \sqrt{3 \pi} \sqrt{c+dx} \operatorname{Erfi}\left[\frac{\sqrt{3} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right] \operatorname{Sinh}\left[3a-\frac{3bc}{d}\right] +$$

$$3 b^{3/2} d \sqrt{3 \pi} x \sqrt{c+dx} \operatorname{Erfi}\left[\frac{\sqrt{3} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right] \operatorname{Sinh}\left[3a-\frac{3bc}{d}\right] +$$

$$3 b^{3/2} \sqrt{\pi} (c+dx)^{3/2} \operatorname{Erf}\left[\frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right] \left( \operatorname{Cosh}\left[a-\frac{bc}{d}\right] - \operatorname{Sinh}\left[a-\frac{bc}{d}\right] \right) +$$

$$3 b^{3/2} c \sqrt{\pi} \sqrt{c+dx} \operatorname{Erfi}\left[\frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right] \operatorname{Sinh}\left[a-\frac{bc}{d}\right] +$$

$$3 b^{3/2} d \sqrt{\pi} x \sqrt{c+dx} \operatorname{Erfi}\left[\frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right] \operatorname{Sinh}\left[a-\frac{bc}{d}\right] - 6 bc \sqrt{d} \operatorname{Sinh}[a+bx] -$$

$$6 b d^{3/2} x \operatorname{Sinh}[a+bx] - 6 bc \sqrt{d} \operatorname{Sinh}[3(a+bx)] - 6 b d^{3/2} x \operatorname{Sinh}[3(a+bx)] \left. \right)$$

**Problem 62: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cosh}[a+bx]^3}{(c+dx)^{7/2}} dx$$

Optimal (type 4, 331 leaves, 19 steps):



$$\begin{aligned}
 & \frac{16 b^2 \operatorname{Cosh}[a+b x]}{5 d^3 \sqrt{c+d x}} - \frac{2 \operatorname{Cosh}[a+b x]^3}{5 d (c+d x)^{5/2}} - \frac{24 b^2 \operatorname{Cosh}[a+b x]^3}{5 d^3 \sqrt{c+d x}} - \frac{b^{5/2} e^{-a+\frac{bc}{d}} \sqrt{\pi} \operatorname{Erf}\left[\frac{\sqrt{b} \sqrt{c+d x}}{\sqrt{d}}\right]}{5 d^{7/2}} - \\
 & \frac{3 b^{5/2} e^{-3 a+\frac{3bc}{d}} \sqrt{3 \pi} \operatorname{Erf}\left[\frac{\sqrt{3} \sqrt{b} \sqrt{c+d x}}{\sqrt{d}}\right]}{5 d^{7/2}} + \frac{b^{5/2} e^{a-\frac{bc}{d}} \sqrt{\pi} \operatorname{Erfi}\left[\frac{\sqrt{b} \sqrt{c+d x}}{\sqrt{d}}\right]}{5 d^{7/2}} + \\
 & \frac{3 b^{5/2} e^{3 a-\frac{3bc}{d}} \sqrt{3 \pi} \operatorname{Erfi}\left[\frac{\sqrt{3} \sqrt{b} \sqrt{c+d x}}{\sqrt{d}}\right]}{5 d^{7/2}} - \frac{4 b \operatorname{Cosh}[a+b x]^2 \operatorname{Sinh}[a+b x]}{5 d^2 (c+d x)^{3/2}}
 \end{aligned}$$

Result (type 4, 680 leaves):

$$\begin{aligned}
 & -\frac{1}{10 d^{7/2} (c+d x)^{5/2}} \left( 4 b^2 c^2 \sqrt{d} \operatorname{Cosh}[a+b x] + \right. \\
 & 3 d^{5/2} \operatorname{Cosh}[a+b x] + 8 b^2 c d^{3/2} x \operatorname{Cosh}[a+b x] + 4 b^2 d^{5/2} x^2 \operatorname{Cosh}[a+b x] + \\
 & 12 b^2 c^2 \sqrt{d} \operatorname{Cosh}[3(a+b x)] + d^{5/2} \operatorname{Cosh}[3(a+b x)] + 24 b^2 c d^{3/2} x \operatorname{Cosh}[3(a+b x)] + \\
 & 12 b^2 d^{5/2} x^2 \operatorname{Cosh}[3(a+b x)] + 2 b^{5/2} \sqrt{\pi} (c+d x)^{5/2} \operatorname{Cosh}\left[a-\frac{bc}{d}\right] \operatorname{Erf}\left[\frac{\sqrt{b} \sqrt{c+d x}}{\sqrt{d}}\right] + \\
 & 6 b^{5/2} \sqrt{3 \pi} (c+d x)^{5/2} \operatorname{Cosh}\left[3 a-\frac{3bc}{d}\right] \operatorname{Erf}\left[\frac{\sqrt{3} \sqrt{b} \sqrt{c+d x}}{\sqrt{d}}\right] - \\
 & 2 b^{5/2} \sqrt{\pi} (c+d x)^{5/2} \operatorname{Cosh}\left[a-\frac{bc}{d}\right] \operatorname{Erfi}\left[\frac{\sqrt{b} \sqrt{c+d x}}{\sqrt{d}}\right] - \\
 & 6 b^{5/2} \sqrt{3 \pi} (c+d x)^{5/2} \operatorname{Cosh}\left[3 a-\frac{3bc}{d}\right] \operatorname{Erfi}\left[\frac{\sqrt{3} \sqrt{b} \sqrt{c+d x}}{\sqrt{d}}\right] - \\
 & 6 b^{5/2} \sqrt{3 \pi} (c+d x)^{5/2} \operatorname{Erf}\left[\frac{\sqrt{3} \sqrt{b} \sqrt{c+d x}}{\sqrt{d}}\right] \operatorname{Sinh}\left[3 a-\frac{3bc}{d}\right] - \\
 & 6 b^{5/2} \sqrt{3 \pi} (c+d x)^{5/2} \operatorname{Erfi}\left[\frac{\sqrt{3} \sqrt{b} \sqrt{c+d x}}{\sqrt{d}}\right] \operatorname{Sinh}\left[3 a-\frac{3bc}{d}\right] - \\
 & 2 b^{5/2} \sqrt{\pi} (c+d x)^{5/2} \operatorname{Erf}\left[\frac{\sqrt{b} \sqrt{c+d x}}{\sqrt{d}}\right] \operatorname{Sinh}\left[a-\frac{bc}{d}\right] - \\
 & 2 b^{5/2} \sqrt{\pi} (c+d x)^{5/2} \operatorname{Erfi}\left[\frac{\sqrt{b} \sqrt{c+d x}}{\sqrt{d}}\right] \operatorname{Sinh}\left[a-\frac{bc}{d}\right] + 2 b c d^{3/2} \operatorname{Sinh}[a+b x] + \\
 & \left. 2 b d^{5/2} x \operatorname{Sinh}[a+b x] + 2 b c d^{3/2} \operatorname{Sinh}[3(a+b x)] + 2 b d^{5/2} x \operatorname{Sinh}[3(a+b x)] \right)
 \end{aligned}$$

**Problem 71: Result more than twice size of optimal antiderivative.**

$$\int \left( \frac{x}{\operatorname{Cosh}[x]^{3/2}} + x \sqrt{\operatorname{Cosh}[x]} \right) dx$$

Optimal (type 3, 20 leaves, 2 steps):

$$-4 \sqrt{\text{Cosh}[x]} + \frac{2 x \text{Sinh}[x]}{\sqrt{\text{Cosh}[x]}}$$

Result (type 3, 46 leaves):

$$\frac{2 \text{Sinh}[x] \left( x - \frac{2 \text{Cosh}[x] \text{Sinh}[x] \sqrt{\text{Tanh}\left[\frac{x}{2}\right]^2}}{(-1+\text{Cosh}[x])^{3/2} \sqrt{1+\text{Cosh}[x]}} \right)}{\sqrt{\text{Cosh}[x]}}$$

**Problem 74: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \left( \frac{x^2}{\text{Cosh}[x]^{3/2}} + x^2 \sqrt{\text{Cosh}[x]} \right) dx$$

Optimal (type 4, 36 leaves, 3 steps):

$$-8 x \sqrt{\text{Cosh}[x]} - 16 \text{EllipticE}\left[\frac{i x}{2}, 2\right] + \frac{2 x^2 \text{Sinh}[x]}{\sqrt{\text{Cosh}[x]}}$$

Result (type 5, 76 leaves):

$$\frac{1}{1 + e^{2x}} 4 \sqrt{\text{Cosh}[x]} (\text{Cosh}[x] + \text{Sinh}[x]) \left( -4 (-2 + x) \text{Cosh}[x] + x^2 \text{Sinh}[x] + 8 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2x}\right] (-\text{Cosh}[x] + \text{Sinh}[x]) \sqrt{1 + \text{Cosh}[2x] + \text{Sinh}[2x]} \right)$$

**Problem 76: Attempted integration timed out after 120 seconds.**

$$\int (c + d x)^m \text{Cosh}[a + b x]^3 dx$$

Optimal (type 4, 237 leaves, 8 steps):

$$\frac{3^{-1-m} e^{3a - \frac{3bc}{d}} (c + d x)^m \left(-\frac{b(c+dx)}{d}\right)^{-m} \text{Gamma}\left[1 + m, -\frac{3b(c+dx)}{d}\right]}{8 b} + \frac{3 e^{a - \frac{bc}{d}} (c + d x)^m \left(-\frac{b(c+dx)}{d}\right)^{-m} \text{Gamma}\left[1 + m, -\frac{b(c+dx)}{d}\right]}{8 b} - \frac{3 e^{-a + \frac{bc}{d}} (c + d x)^m \left(\frac{b(c+dx)}{d}\right)^{-m} \text{Gamma}\left[1 + m, \frac{b(c+dx)}{d}\right]}{8 b} - \frac{3^{-1-m} e^{-3a + \frac{3bc}{d}} (c + d x)^m \left(\frac{b(c+dx)}{d}\right)^{-m} \text{Gamma}\left[1 + m, \frac{3b(c+dx)}{d}\right]}{8 b}$$

Result (type 1, 1 leaves):

???

**Problem 112: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(c+dx)^2}{a+a \cosh[ex+fx]} dx$$

Optimal (type 4, 88 leaves, 6 steps):

$$\frac{(c+dx)^2}{af} - \frac{4d(c+dx) \operatorname{Log}[1+e^{e+fx}]}{af^2} - \frac{4d^2 \operatorname{PolyLog}[2, -e^{e+fx}]}{af^3} + \frac{(c+dx)^2 \operatorname{Tanh}\left[\frac{e}{2} + \frac{fx}{2}\right]}{af}$$

Result (type 4, 472 leaves):

$$\begin{aligned} & - \left( \left( 8cd \operatorname{Cosh}\left[\frac{e}{2} + \frac{fx}{2}\right]^2 \operatorname{Sech}\left[\frac{e}{2}\right] \right. \right. \\ & \quad \left. \left( \operatorname{Cosh}\left[\frac{e}{2}\right] \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{e}{2}\right] \operatorname{Cosh}\left[\frac{fx}{2}\right] + \operatorname{Sinh}\left[\frac{e}{2}\right] \operatorname{Sinh}\left[\frac{fx}{2}\right]\right] - \frac{1}{2}fx \operatorname{Sinh}\left[\frac{e}{2}\right] \right) \right) / \\ & \left( f^2 (a+a \cosh[ex+fx]) \left( \operatorname{Cosh}\left[\frac{e}{2}\right]^2 - \operatorname{Sinh}\left[\frac{e}{2}\right]^2 \right) \right) + \left( 8d^2 \operatorname{Cosh}\left[\frac{e}{2} + \frac{fx}{2}\right]^2 \operatorname{Csch}\left[\frac{e}{2}\right] \right. \\ & \left. \left( -\frac{1}{4}e^{-\operatorname{ArcTanh}\left[\operatorname{Coth}\left[\frac{e}{2}\right]\right]} f^2 x^2 + \left( i \operatorname{Coth}\left[\frac{e}{2}\right] \left( -\frac{1}{2}fx \left( -\pi + 2i \operatorname{ArcTanh}\left[\operatorname{Coth}\left[\frac{e}{2}\right]\right] \right) - \right. \right. \right. \\ & \quad \left. \left. \left. \pi \operatorname{Log}\left[1+e^{fx}\right] - 2 \left( \frac{ifx}{2} + i \operatorname{ArcTanh}\left[\operatorname{Coth}\left[\frac{e}{2}\right]\right] \right) \operatorname{Log}\left[1 - e^{2i\left(\frac{ifx}{2} + i \operatorname{ArcTanh}\left[\operatorname{Coth}\left[\frac{e}{2}\right]\right)}\right]} \right) \right) + \right. \right. \\ & \quad \left. \left. \pi \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{fx}{2}\right]\right] + 2i \operatorname{ArcTanh}\left[\operatorname{Coth}\left[\frac{e}{2}\right]\right] \operatorname{Log}\left[i \operatorname{Sinh}\left[\frac{fx}{2}\right] + \operatorname{ArcTanh}\left[\operatorname{Coth}\left[\frac{e}{2}\right]\right] \right] \right) \right) + \\ & \quad \left. i \operatorname{PolyLog}\left[2, e^{2i\left(\frac{ifx}{2} + i \operatorname{ArcTanh}\left[\operatorname{Coth}\left[\frac{e}{2}\right]\right)}\right]} \right) \right) / \left( \sqrt{1 - \operatorname{Coth}\left[\frac{e}{2}\right]^2} \right) \operatorname{Sech}\left[\frac{e}{2}\right] \Big/ \\ & \left( f^3 (a+a \cosh[ex+fx]) \sqrt{\operatorname{Csch}\left[\frac{e}{2}\right]^2 \left( -\operatorname{Cosh}\left[\frac{e}{2}\right]^2 + \operatorname{Sinh}\left[\frac{e}{2}\right]^2 \right)} \right) + \\ & \left( 2 \operatorname{Cosh}\left[\frac{e}{2} + \frac{fx}{2}\right] \operatorname{Sech}\left[\frac{e}{2}\right] \right. \\ & \quad \left. \left( c^2 \operatorname{Sinh}\left[\frac{fx}{2}\right] + 2cdx \operatorname{Sinh}\left[\frac{fx}{2}\right] + d^2 x^2 \operatorname{Sinh}\left[\frac{fx}{2}\right] \right) \right) / (f \\ & \quad (a+a \cosh[ex+fx])) \end{aligned}$$

**Problem 117: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(c+dx)^2}{(a+a \cosh[ex+fx])^2} dx$$

Optimal (type 4, 200 leaves, 9 steps):

$$\frac{(c + d x)^2}{3 a^2 f} - \frac{4 d (c + d x) \operatorname{Log}[1 + e^{e+f x}]}{3 a^2 f^2} - \frac{4 d^2 \operatorname{PolyLog}[2, -e^{e+f x}]}{3 a^2 f^3} + \frac{d (c + d x) \operatorname{Sech}\left[\frac{e}{2} + \frac{f x}{2}\right]^2}{3 a^2 f^2} - \frac{2 d^2 \operatorname{Tanh}\left[\frac{e}{2} + \frac{f x}{2}\right]}{3 a^2 f^3} + \frac{(c + d x)^2 \operatorname{Tanh}\left[\frac{e}{2} + \frac{f x}{2}\right]}{3 a^2 f} + \frac{(c + d x)^2 \operatorname{Sech}\left[\frac{e}{2} + \frac{f x}{2}\right]^2 \operatorname{Tanh}\left[\frac{e}{2} + \frac{f x}{2}\right]}{6 a^2 f}$$

Result (type 4, 637 leaves):

$$\begin{aligned} & - \left( \left( 16 c d \operatorname{Cosh}\left[\frac{e}{2} + \frac{f x}{2}\right]^4 \operatorname{Sech}\left[\frac{e}{2}\right] \right. \right. \\ & \quad \left. \left( \operatorname{Cosh}\left[\frac{e}{2}\right] \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{e}{2}\right] \operatorname{Cosh}\left[\frac{f x}{2}\right] + \operatorname{Sinh}\left[\frac{e}{2}\right] \operatorname{Sinh}\left[\frac{f x}{2}\right]\right] - \frac{1}{2} f x \operatorname{Sinh}\left[\frac{e}{2}\right] \right) \right) / \\ & \left( 3 f^2 (a + a \operatorname{Cosh}[e + f x])^2 \left( \operatorname{Cosh}\left[\frac{e}{2}\right]^2 - \operatorname{Sinh}\left[\frac{e}{2}\right]^2 \right) \right) + \left( 16 d^2 \operatorname{Cosh}\left[\frac{e}{2} + \frac{f x}{2}\right]^4 \right. \\ & \operatorname{Csch}\left[\frac{e}{2}\right] \left( -\frac{1}{4} e^{-\operatorname{ArcTanh}\left[\operatorname{Coth}\left[\frac{e}{2}\right]\right]} f^2 x^2 + \left( i \operatorname{Coth}\left[\frac{e}{2}\right] \left( -\frac{1}{2} f x \left( -\pi + 2 i \operatorname{ArcTanh}\left[\operatorname{Coth}\left[\frac{e}{2}\right]\right] \right) - \right. \right. \right. \\ & \quad \left. \left. \left. \pi \operatorname{Log}\left[1 + e^{f x}\right] - 2 \left( \frac{i f x}{2} + i \operatorname{ArcTanh}\left[\operatorname{Coth}\left[\frac{e}{2}\right]\right] \right) \operatorname{Log}\left[1 - e^{2 i \left( \frac{i f x}{2} + i \operatorname{ArcTanh}\left[\operatorname{Coth}\left[\frac{e}{2}\right]\right] \right)} \right] \right) + \right. \right. \\ & \quad \left. \left. \pi \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{f x}{2}\right]\right] + 2 i \operatorname{ArcTanh}\left[\operatorname{Coth}\left[\frac{e}{2}\right]\right] \operatorname{Log}\left[i \operatorname{Sinh}\left[\frac{f x}{2}\right] + \operatorname{ArcTanh}\left[\operatorname{Coth}\left[\frac{e}{2}\right]\right] \right] \right) + \right. \\ & \quad \left. i \operatorname{PolyLog}\left[2, e^{2 i \left( \frac{i f x}{2} + i \operatorname{ArcTanh}\left[\operatorname{Coth}\left[\frac{e}{2}\right]\right] \right)} \right] \right) / \left( \sqrt{1 - \operatorname{Coth}\left[\frac{e}{2}\right]^2} \right) \operatorname{Sech}\left[\frac{e}{2}\right] \right) / \\ & \left( 3 f^3 (a + a \operatorname{Cosh}[e + f x])^2 \sqrt{\operatorname{Csch}\left[\frac{e}{2}\right]^2 \left( -\operatorname{Cosh}\left[\frac{e}{2}\right]^2 + \operatorname{Sinh}\left[\frac{e}{2}\right]^2 \right)} \right) + \\ & \frac{1}{3 f^3 (a + a \operatorname{Cosh}[e + f x])^2} \\ & \operatorname{Cosh}\left[\frac{e}{2} + \frac{f x}{2}\right] \\ & \operatorname{Sech}\left[\frac{e}{2}\right] \\ & \left( 2 c d f \operatorname{Cosh}\left[\frac{f x}{2}\right] + 2 d^2 f x \operatorname{Cosh}\left[\frac{f x}{2}\right] + 2 c d f \operatorname{Cosh}\left[e + \frac{f x}{2}\right] + \right. \\ & \quad 2 d^2 f x \operatorname{Cosh}\left[e + \frac{f x}{2}\right] - 4 d^2 \operatorname{Sinh}\left[\frac{f x}{2}\right] + 3 c^2 f^2 \operatorname{Sinh}\left[\frac{f x}{2}\right] + 6 c d f^2 x \operatorname{Sinh}\left[\frac{f x}{2}\right] + \\ & \quad 3 d^2 f^2 x^2 \operatorname{Sinh}\left[\frac{f x}{2}\right] + 2 d^2 \operatorname{Sinh}\left[e + \frac{f x}{2}\right] - 2 d^2 \operatorname{Sinh}\left[e + \frac{3 f x}{2}\right] + \\ & \quad \left. c^2 f^2 \operatorname{Sinh}\left[e + \frac{3 f x}{2}\right] + 2 c d f^2 x \operatorname{Sinh}\left[e + \frac{3 f x}{2}\right] + d^2 f^2 x^2 \operatorname{Sinh}\left[e + \frac{3 f x}{2}\right] \right) \end{aligned}$$

**Problem 168: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c + d x)^3}{a + b \operatorname{Cosh}[e + f x]} dx$$

Optimal (type 4, 436 leaves, 12 steps):

$$\begin{aligned}
 & \frac{(c+dx)^3 \operatorname{Log}\left[1 + \frac{b e^{e+fx}}{a - \sqrt{a^2 - b^2}}\right]}{\sqrt{a^2 - b^2} f} - \frac{(c+dx)^3 \operatorname{Log}\left[1 + \frac{b e^{e+fx}}{a + \sqrt{a^2 - b^2}}\right]}{\sqrt{a^2 - b^2} f} + \frac{3d (c+dx)^2 \operatorname{PolyLog}\left[2, -\frac{b e^{e+fx}}{a - \sqrt{a^2 - b^2}}\right]}{\sqrt{a^2 - b^2} f^2} - \\
 & \frac{3d (c+dx)^2 \operatorname{PolyLog}\left[2, -\frac{b e^{e+fx}}{a + \sqrt{a^2 - b^2}}\right]}{\sqrt{a^2 - b^2} f^2} - \frac{6d^2 (c+dx) \operatorname{PolyLog}\left[3, -\frac{b e^{e+fx}}{a - \sqrt{a^2 - b^2}}\right]}{\sqrt{a^2 - b^2} f^3} + \\
 & \frac{6d^2 (c+dx) \operatorname{PolyLog}\left[3, -\frac{b e^{e+fx}}{a + \sqrt{a^2 - b^2}}\right]}{\sqrt{a^2 - b^2} f^3} + \frac{6d^3 \operatorname{PolyLog}\left[4, -\frac{b e^{e+fx}}{a - \sqrt{a^2 - b^2}}\right]}{\sqrt{a^2 - b^2} f^4} - \frac{6d^3 \operatorname{PolyLog}\left[4, -\frac{b e^{e+fx}}{a + \sqrt{a^2 - b^2}}\right]}{\sqrt{a^2 - b^2} f^4}
 \end{aligned}$$

Result (type 4, 1031 leaves):

$$\frac{1}{\sqrt{-a^2+b^2} \sqrt{(a^2-b^2) e^{2e}} f^4} \left( 2 c^3 \sqrt{(a^2-b^2) e^{2e}} f^3 \operatorname{ArcTan}\left[\frac{a+b e^{e+fx}}{\sqrt{-a^2+b^2}}\right] + \right.$$

$$3 \sqrt{-a^2+b^2} c^2 d e^e f^3 x \operatorname{Log}\left[1 + \frac{b e^{2e+fx}}{a e^e - \sqrt{(a^2-b^2) e^{2e}}}\right] + 3 \sqrt{-a^2+b^2} c d^2 e^e f^3 x^2$$

$$\operatorname{Log}\left[1 + \frac{b e^{2e+fx}}{a e^e - \sqrt{(a^2-b^2) e^{2e}}}\right] + \sqrt{-a^2+b^2} d^3 e^e f^3 x^3 \operatorname{Log}\left[1 + \frac{b e^{2e+fx}}{a e^e - \sqrt{(a^2-b^2) e^{2e}}}\right] -$$

$$3 \sqrt{-a^2+b^2} c^2 d e^e f^3 x \operatorname{Log}\left[1 + \frac{b e^{2e+fx}}{a e^e + \sqrt{(a^2-b^2) e^{2e}}}\right] - 3 \sqrt{-a^2+b^2} c d^2 e^e f^3 x^2$$

$$\operatorname{Log}\left[1 + \frac{b e^{2e+fx}}{a e^e + \sqrt{(a^2-b^2) e^{2e}}}\right] - \sqrt{-a^2+b^2} d^3 e^e f^3 x^3 \operatorname{Log}\left[1 + \frac{b e^{2e+fx}}{a e^e + \sqrt{(a^2-b^2) e^{2e}}}\right] +$$

$$3 \sqrt{-a^2+b^2} d e^e f^2 (c+dx)^2 \operatorname{PolyLog}\left[2, -\frac{b e^{2e+fx}}{a e^e - \sqrt{(a^2-b^2) e^{2e}}}\right] -$$

$$3 \sqrt{-a^2+b^2} d e^e f^2 (c+dx)^2 \operatorname{PolyLog}\left[2, -\frac{b e^{2e+fx}}{a e^e + \sqrt{(a^2-b^2) e^{2e}}}\right] -$$

$$6 \sqrt{-a^2+b^2} c d^2 e^e f \operatorname{PolyLog}\left[3, -\frac{b e^{2e+fx}}{a e^e - \sqrt{(a^2-b^2) e^{2e}}}\right] -$$

$$6 \sqrt{-a^2+b^2} d^3 e^e f x \operatorname{PolyLog}\left[3, -\frac{b e^{2e+fx}}{a e^e - \sqrt{(a^2-b^2) e^{2e}}}\right] +$$

$$6 \sqrt{-a^2+b^2} c d^2 e^e f \operatorname{PolyLog}\left[3, -\frac{b e^{2e+fx}}{a e^e + \sqrt{(a^2-b^2) e^{2e}}}\right] +$$

$$6 \sqrt{-a^2+b^2} d^3 e^e f x \operatorname{PolyLog}\left[3, -\frac{b e^{2e+fx}}{a e^e + \sqrt{(a^2-b^2) e^{2e}}}\right] +$$

$$6 \sqrt{-a^2+b^2} d^3 e^e \operatorname{PolyLog}\left[4, -\frac{b e^{2e+fx}}{a e^e - \sqrt{(a^2-b^2) e^{2e}}}\right] -$$

$$6 \sqrt{-a^2+b^2} d^3 e^e \operatorname{PolyLog}\left[4, -\frac{b e^{2e+fx}}{a e^e + \sqrt{(a^2-b^2) e^{2e}}}\right] \Bigg)$$

Problem 173: Attempted integration timed out after 120 seconds.

$$\int \frac{(c+dx)^3}{(a+b \operatorname{Cosh}[e+fx])^2} dx$$

Optimal (type 4, 823 leaves, 22 steps):

$$\begin{aligned}
 & - \frac{(c+dx)^3}{(a^2-b^2)f} + \frac{3d(c+dx)^2 \operatorname{Log}\left[1 + \frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right]}{(a^2-b^2)f^2} + \frac{a(c+dx)^3 \operatorname{Log}\left[1 + \frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{3/2}f} + \\
 & \frac{3d(c+dx)^2 \operatorname{Log}\left[1 + \frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right]}{(a^2-b^2)f^2} - \frac{a(c+dx)^3 \operatorname{Log}\left[1 + \frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{3/2}f} + \\
 & \frac{6d^2(c+dx) \operatorname{PolyLog}\left[2, -\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right]}{(a^2-b^2)f^3} + \frac{3ad(c+dx)^2 \operatorname{PolyLog}\left[2, -\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{3/2}f^2} + \\
 & \frac{6d^2(c+dx) \operatorname{PolyLog}\left[2, -\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right]}{(a^2-b^2)f^3} - \frac{3ad(c+dx)^2 \operatorname{PolyLog}\left[2, -\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{3/2}f^2} - \\
 & \frac{6d^3 \operatorname{PolyLog}\left[3, -\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right]}{(a^2-b^2)f^4} - \frac{6ad^2(c+dx) \operatorname{PolyLog}\left[3, -\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{3/2}f^3} - \\
 & \frac{6d^3 \operatorname{PolyLog}\left[3, -\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right]}{(a^2-b^2)f^4} + \frac{6ad^2(c+dx) \operatorname{PolyLog}\left[3, -\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{3/2}f^3} + \\
 & \frac{6ad^3 \operatorname{PolyLog}\left[4, -\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{3/2}f^4} - \frac{6ad^3 \operatorname{PolyLog}\left[4, -\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{3/2}f^4} - \frac{b(c+dx)^3 \operatorname{Sinh}[e+fx]}{(a^2-b^2)f(a+b \operatorname{Cosh}[e+fx])}
 \end{aligned}$$

Result (type 1, 1 leaves):

???

**Problem 174: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c+dx)^2}{(a+b \operatorname{Cosh}[e+fx])^2} dx$$

Optimal (type 4, 593 leaves, 18 steps):

$$\begin{aligned}
 & - \frac{(c+d x)^2}{(a^2-b^2) f} + \frac{2 d (c+d x) \operatorname{Log}\left[1+\frac{b e^{e+f x}}{a-\sqrt{a^2-b^2}}\right]}{(a^2-b^2) f^2} + \frac{a (c+d x)^2 \operatorname{Log}\left[1+\frac{b e^{e+f x}}{a-\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{3/2} f} + \\
 & \frac{2 d (c+d x) \operatorname{Log}\left[1+\frac{b e^{e+f x}}{a+\sqrt{a^2-b^2}}\right]}{(a^2-b^2) f^2} - \frac{a (c+d x)^2 \operatorname{Log}\left[1+\frac{b e^{e+f x}}{a+\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{3/2} f} + \frac{2 d^2 \operatorname{PolyLog}\left[2,-\frac{b e^{e+f x}}{a-\sqrt{a^2-b^2}}\right]}{(a^2-b^2) f^3} + \\
 & \frac{2 a d (c+d x) \operatorname{PolyLog}\left[2,-\frac{b e^{e+f x}}{a-\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{3/2} f^2} + \frac{2 d^2 \operatorname{PolyLog}\left[2,-\frac{b e^{e+f x}}{a+\sqrt{a^2-b^2}}\right]}{(a^2-b^2) f^3} - \\
 & \frac{2 a d (c+d x) \operatorname{PolyLog}\left[2,-\frac{b e^{e+f x}}{a+\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{3/2} f^2} - \frac{2 a d^2 \operatorname{PolyLog}\left[3,-\frac{b e^{e+f x}}{a-\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{3/2} f^3} + \\
 & \frac{2 a d^2 \operatorname{PolyLog}\left[3,-\frac{b e^{e+f x}}{a+\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{3/2} f^3} - \frac{b (c+d x)^2 \operatorname{Sinh}[e+f x]}{(a^2-b^2) f (a+b \operatorname{Cosh}[e+f x])}
 \end{aligned}$$

Result(type 4, 6016 leaves):

$$\begin{aligned}
 & \frac{1}{(a^2-b^2) (1+e^{2e}) f} \\
 & 2 e^e \left( -2 c d e^e x + 2 c d e^{-e} (1+e^{2e}) x - d^2 e^e x^2 + d^2 e^{-e} (1+e^{2e}) x^2 + \frac{a c^2 e^{-e} \operatorname{ArcTan}\left[\frac{a+b e^{e+f x}}{\sqrt{-a^2+b^2}}\right]}{\sqrt{-a^2+b^2}} \right) + \\
 & \frac{a c^2 e^e \operatorname{ArcTan}\left[\frac{a+b e^{e+f x}}{\sqrt{-a^2+b^2}}\right]}{\sqrt{-a^2+b^2}} - \frac{2 a c d e^{-e} \operatorname{ArcTan}\left[\frac{a+b e^{e+f x}}{\sqrt{-a^2+b^2}}\right]}{\sqrt{-a^2+b^2} f} - \frac{2 a c d e^e \operatorname{ArcTan}\left[\frac{a+b e^{e+f x}}{\sqrt{-a^2+b^2}}\right]}{\sqrt{-a^2+b^2} f} + \\
 & c d e^{-e} \left( -2 x + \frac{2 a \operatorname{ArcTan}\left[\frac{a+b e^{e+f x}}{\sqrt{-a^2+b^2}}\right]}{\sqrt{-a^2+b^2} f} + \frac{\operatorname{Log}\left[b+2 a e^{e+f x}+b e^{2(e+f x)}\right]}{f} \right) + \\
 & c d e^e \left( -2 x + \frac{2 a \operatorname{ArcTan}\left[\frac{a+b e^{e+f x}}{\sqrt{-a^2+b^2}}\right]}{\sqrt{-a^2+b^2} f} + \frac{\operatorname{Log}\left[b+2 a e^{e+f x}+b e^{2(e+f x)}\right]}{f} \right) - \\
 & 2 b d^2 e^{-e} \left( \left( \frac{x^2}{2 \left(a e^e - \sqrt{-(-a^2+b^2) e^{2e}}\right)} - \frac{x \operatorname{Log}\left[1+\frac{b e^{2e+f x}}{a e^e - \sqrt{-(-a^2+b^2) e^{2e}}}\right]}{\left(a e^e - \sqrt{-(-a^2+b^2) e^{2e}}\right) f} \right. \right. \\
 & \left. \left. \frac{\operatorname{PolyLog}\left[2,-\frac{b e^{2e+f x}}{a e^e - \sqrt{-(-a^2+b^2) e^{2e}}}\right]}{\left(a e^e - \sqrt{-(-a^2+b^2) e^{2e}}\right) f^2} \right) \right) / \left( \frac{-a e^{-e} - e^{-2e} \sqrt{a^2 e^{2e} - b^2 e^{2e}}}{b} - \right.
 \end{aligned}$$



$$\begin{aligned}
 & \left. \left. \frac{-a e^{-e} + e^{-2e} \sqrt{a^2 e^{2e} - b^2 e^{2e}}}{b} \right) \right) + \left( \frac{x^2}{2 \left( a e^e + \sqrt{-(-a^2 + b^2) e^{2e}} \right)} - \right. \\
 & \left. \frac{x \operatorname{Log} \left[ 1 + \frac{b e^{2e+fx}}{a e^e + \sqrt{-(-a^2 + b^2) e^{2e}}} \right]}{\left( a e^e + \sqrt{-(-a^2 + b^2) e^{2e}} \right) f} - \frac{\operatorname{PolyLog} \left[ 2, -\frac{b e^{2e+fx}}{a e^e + \sqrt{-(-a^2 + b^2) e^{2e}}} \right]}{\left( a e^e + \sqrt{-(-a^2 + b^2) e^{2e}} \right) f^2} \right) / \\
 & \left( \frac{-a e^{-e} - e^{-2e} \sqrt{a^2 e^{2e} - b^2 e^{2e}}}{b} - \frac{-a e^{-e} + e^{-2e} \sqrt{a^2 e^{2e} - b^2 e^{2e}}}{b} \right) \Bigg) - \\
 2 b d^2 e^e & \left( - \left( \left( \frac{x^2}{2 \left( a e^e - \sqrt{-(-a^2 + b^2) e^{2e}} \right)} - \frac{x \operatorname{Log} \left[ 1 + \frac{b e^{2e+fx}}{a e^e - \sqrt{-(-a^2 + b^2) e^{2e}}} \right]}{\left( a e^e - \sqrt{-(-a^2 + b^2) e^{2e}} \right) f} - \right. \right. \right. \\
 & \left. \left. \frac{\operatorname{PolyLog} \left[ 2, -\frac{b e^{2e+fx}}{a e^e - \sqrt{-(-a^2 + b^2) e^{2e}}} \right]}{\left( a e^e - \sqrt{-(-a^2 + b^2) e^{2e}} \right) f^2} \right) \right) / \left( \frac{-a e^{-e} - e^{-2e} \sqrt{a^2 e^{2e} - b^2 e^{2e}}}{b} - \right. \\
 & \left. \frac{-a e^{-e} + e^{-2e} \sqrt{a^2 e^{2e} - b^2 e^{2e}}}{b} \right) \right) + \left( \frac{x^2}{2 \left( a e^e + \sqrt{-(-a^2 + b^2) e^{2e}} \right)} - \right. \\
 & \left. \frac{x \operatorname{Log} \left[ 1 + \frac{b e^{2e+fx}}{a e^e + \sqrt{-(-a^2 + b^2) e^{2e}}} \right]}{\left( a e^e + \sqrt{-(-a^2 + b^2) e^{2e}} \right) f} - \frac{\operatorname{PolyLog} \left[ 2, -\frac{b e^{2e+fx}}{a e^e + \sqrt{-(-a^2 + b^2) e^{2e}}} \right]}{\left( a e^e + \sqrt{-(-a^2 + b^2) e^{2e}} \right) f^2} \right) / \\
 & \left( \frac{-a e^{-e} - e^{-2e} \sqrt{a^2 e^{2e} - b^2 e^{2e}}}{b} - \frac{-a e^{-e} + e^{-2e} \sqrt{a^2 e^{2e} - b^2 e^{2e}}}{b} \right) \Bigg) - \\
 2 a d^2 & \left( - \left( \left( \left( -a e^{-e} + e^{-2e} \sqrt{a^2 e^{2e} - b^2 e^{2e}} \right) \left( \frac{x^2}{2 \left( a e^e - \sqrt{-(-a^2 + b^2) e^{2e}} \right)} - \right. \right. \right. \right. \\
 & \left. \left. \frac{x \operatorname{Log} \left[ 1 + \frac{b e^{2e+fx}}{a e^e - \sqrt{-(-a^2 + b^2) e^{2e}}} \right]}{\left( a e^e - \sqrt{-(-a^2 + b^2) e^{2e}} \right) f} - \frac{\operatorname{PolyLog} \left[ 2, -\frac{b e^{2e+fx}}{a e^e - \sqrt{-(-a^2 + b^2) e^{2e}}} \right]}{\left( a e^e - \sqrt{-(-a^2 + b^2) e^{2e}} \right) f^2} \right) \right) \right) / \\
 & \left( b \left( \frac{-a e^{-e} - e^{-2e} \sqrt{a^2 e^{2e} - b^2 e^{2e}}}{b} - \frac{-a e^{-e} + e^{-2e} \sqrt{a^2 e^{2e} - b^2 e^{2e}}}{b} \right) \right) \Bigg) +
 \end{aligned}$$

$$\begin{aligned}
 & \left( \left( -a e^{-e} - e^{-2e} \sqrt{a^2 e^{2e} - b^2 e^{2e}} \right) \left( \frac{x^2}{2 \left( a e^e + \sqrt{-(-a^2 + b^2) e^{2e}} \right)} - \right. \right. \\
 & \quad \left. \left. \frac{x \operatorname{Log} \left[ 1 + \frac{b e^{2e+fx}}{a e^e + \sqrt{-(-a^2 + b^2) e^{2e}}} \right]}{\left( a e^e + \sqrt{-(-a^2 + b^2) e^{2e}} \right) f} - \frac{\operatorname{PolyLog} \left[ 2, -\frac{b e^{2e+fx}}{a e^e + \sqrt{-(-a^2 + b^2) e^{2e}}} \right]}{\left( a e^e + \sqrt{-(-a^2 + b^2) e^{2e}} \right) f^2} \right] \right) / \\
 & \left( b \left( \frac{-a e^{-e} - e^{-2e} \sqrt{a^2 e^{2e} - b^2 e^{2e}}}{b} - \frac{-a e^{-e} + e^{-2e} \sqrt{a^2 e^{2e} - b^2 e^{2e}}}{b} \right) \right) + \\
 2 a c d f & \left( - \left( \left( -a e^{-e} + e^{-2e} \sqrt{a^2 e^{2e} - b^2 e^{2e}} \right) \left( \frac{x^2}{2 \left( a e^e - \sqrt{-(-a^2 + b^2) e^{2e}} \right)} - \right. \right. \right. \\
 & \quad \left. \left. \frac{x \operatorname{Log} \left[ 1 + \frac{b e^{2e+fx}}{a e^e - \sqrt{-(-a^2 + b^2) e^{2e}}} \right]}{\left( a e^e - \sqrt{-(-a^2 + b^2) e^{2e}} \right) f} - \frac{\operatorname{PolyLog} \left[ 2, -\frac{b e^{2e+fx}}{a e^e - \sqrt{-(-a^2 + b^2) e^{2e}}} \right]}{\left( a e^e - \sqrt{-(-a^2 + b^2) e^{2e}} \right) f^2} \right] \right) / \\
 & \left( b \left( \frac{-a e^{-e} - e^{-2e} \sqrt{a^2 e^{2e} - b^2 e^{2e}}}{b} - \frac{-a e^{-e} + e^{-2e} \sqrt{a^2 e^{2e} - b^2 e^{2e}}}{b} \right) \right) + \\
 & \left( -a e^{-e} - e^{-2e} \sqrt{a^2 e^{2e} - b^2 e^{2e}} \right) \left( \frac{x^2}{2 \left( a e^e + \sqrt{-(-a^2 + b^2) e^{2e}} \right)} - \right. \\
 & \quad \left. \frac{x \operatorname{Log} \left[ 1 + \frac{b e^{2e+fx}}{a e^e + \sqrt{-(-a^2 + b^2) e^{2e}}} \right]}{\left( a e^e + \sqrt{-(-a^2 + b^2) e^{2e}} \right) f} - \frac{\operatorname{PolyLog} \left[ 2, -\frac{b e^{2e+fx}}{a e^e + \sqrt{-(-a^2 + b^2) e^{2e}}} \right]}{\left( a e^e + \sqrt{-(-a^2 + b^2) e^{2e}} \right) f^2} \right) / \\
 & \left( b \left( \frac{-a e^{-e} - e^{-2e} \sqrt{a^2 e^{2e} - b^2 e^{2e}}}{b} - \frac{-a e^{-e} + e^{-2e} \sqrt{a^2 e^{2e} - b^2 e^{2e}}}{b} \right) \right) - \\
 2 a d^2 & \left( - \left( \left( e^{2e} \left( -a e^{-e} + e^{-2e} \sqrt{a^2 e^{2e} - b^2 e^{2e}} \right) \left( \frac{x^2}{2 \left( a e^e - \sqrt{-(-a^2 + b^2) e^{2e}} \right)} - \right. \right. \right. \right. \\
 & \quad \left. \left. \frac{x \operatorname{Log} \left[ 1 + \frac{b e^{2e+fx}}{a e^e - \sqrt{-(-a^2 + b^2) e^{2e}}} \right]}{\left( a e^e - \sqrt{-(-a^2 + b^2) e^{2e}} \right) f} - \frac{\operatorname{PolyLog} \left[ 2, -\frac{b e^{2e+fx}}{a e^e - \sqrt{-(-a^2 + b^2) e^{2e}}} \right]}{\left( a e^e - \sqrt{-(-a^2 + b^2) e^{2e}} \right) f^2} \right] \right) / \\
 & \left( b \left( \frac{-a e^{-e} - e^{-2e} \sqrt{a^2 e^{2e} - b^2 e^{2e}}}{b} - \frac{-a e^{-e} + e^{-2e} \sqrt{a^2 e^{2e} - b^2 e^{2e}}}{b} \right) \right) -
 \end{aligned}$$

$$\begin{aligned}
 & \left( b \left( \frac{-a e^{-e} - e^{-2e} \sqrt{a^2 e^{2e} - b^2 e^{2e}}}{b} - \frac{-a e^{-e} + e^{-2e} \sqrt{a^2 e^{2e} - b^2 e^{2e}}}{b} \right) \right) + \\
 & \left( e^{2e} \left( -a e^{-e} - e^{-2e} \sqrt{a^2 e^{2e} - b^2 e^{2e}} \right) \left( \frac{x^2}{2 \left( a e^e + \sqrt{-(-a^2 + b^2) e^{2e}} \right)} - \right. \right. \\
 & \left. \left. \frac{x \operatorname{Log} \left[ 1 + \frac{b e^{2e+fx}}{a e^e + \sqrt{-(-a^2 + b^2) e^{2e}}} \right]}{\left( a e^e + \sqrt{-(-a^2 + b^2) e^{2e}} \right) f} - \frac{\operatorname{PolyLog} \left[ 2, -\frac{b e^{2e+fx}}{a e^e + \sqrt{-(-a^2 + b^2) e^{2e}}} \right]}{\left( a e^e + \sqrt{-(-a^2 + b^2) e^{2e}} \right) f^2} \right] \right) / \\
 & \left( b \left( \frac{-a e^{-e} - e^{-2e} \sqrt{a^2 e^{2e} - b^2 e^{2e}}}{b} - \frac{-a e^{-e} + e^{-2e} \sqrt{a^2 e^{2e} - b^2 e^{2e}}}{b} \right) \right) + \\
 2 a c d f & \left( - \left( \left( e^{2e} \left( -a e^{-e} + e^{-2e} \sqrt{a^2 e^{2e} - b^2 e^{2e}} \right) \left( \frac{x^2}{2 \left( a e^e - \sqrt{-(-a^2 + b^2) e^{2e}} \right)} - \right. \right. \right. \right. \\
 & \left. \left. \frac{x \operatorname{Log} \left[ 1 + \frac{b e^{2e+fx}}{a e^e - \sqrt{-(-a^2 + b^2) e^{2e}}} \right]}{\left( a e^e - \sqrt{-(-a^2 + b^2) e^{2e}} \right) f} - \frac{\operatorname{PolyLog} \left[ 2, -\frac{b e^{2e+fx}}{a e^e - \sqrt{-(-a^2 + b^2) e^{2e}}} \right]}{\left( a e^e - \sqrt{-(-a^2 + b^2) e^{2e}} \right) f^2} \right] \right) / \\
 & \left( b \left( \frac{-a e^{-e} - e^{-2e} \sqrt{a^2 e^{2e} - b^2 e^{2e}}}{b} - \frac{-a e^{-e} + e^{-2e} \sqrt{a^2 e^{2e} - b^2 e^{2e}}}{b} \right) \right) + \\
 & \left( e^{2e} \left( -a e^{-e} - e^{-2e} \sqrt{a^2 e^{2e} - b^2 e^{2e}} \right) \left( \frac{x^2}{2 \left( a e^e + \sqrt{-(-a^2 + b^2) e^{2e}} \right)} - \right. \right. \\
 & \left. \left. \frac{x \operatorname{Log} \left[ 1 + \frac{b e^{2e+fx}}{a e^e + \sqrt{-(-a^2 + b^2) e^{2e}}} \right]}{\left( a e^e + \sqrt{-(-a^2 + b^2) e^{2e}} \right) f} - \frac{\operatorname{PolyLog} \left[ 2, -\frac{b e^{2e+fx}}{a e^e + \sqrt{-(-a^2 + b^2) e^{2e}}} \right]}{\left( a e^e + \sqrt{-(-a^2 + b^2) e^{2e}} \right) f^2} \right] \right) / \\
 & \left( b \left( \frac{-a e^{-e} - e^{-2e} \sqrt{a^2 e^{2e} - b^2 e^{2e}}}{b} - \frac{-a e^{-e} + e^{-2e} \sqrt{a^2 e^{2e} - b^2 e^{2e}}}{b} \right) \right) + \\
 a d^2 f & \left( - \left( \left( \left( -a e^{-e} + e^{-2e} \sqrt{a^2 e^{2e} - b^2 e^{2e}} \right) \left( \frac{x^3}{3 \left( a e^e - \sqrt{-(-a^2 + b^2) e^{2e}} \right)} - \right. \right. \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & \left. \left( \frac{x^2 \operatorname{Log}\left[1 + \frac{b e^{2e+fx}}{a e^e - \sqrt{-(-a^2+b^2)} e^{2e}}\right]}{\left(a e^e - \sqrt{-(-a^2+b^2)} e^{2e}\right) f} - \frac{2 x \operatorname{PolyLog}\left[2, -\frac{b e^{2e+fx}}{a e^e - \sqrt{-(-a^2+b^2)} e^{2e}}\right]}{\left(a e^e - \sqrt{-(-a^2+b^2)} e^{2e}\right) f^2} + \right. \right. \\
 & \left. \left. \frac{2 \operatorname{PolyLog}\left[3, -\frac{b e^{2e+fx}}{a e^e - \sqrt{-(-a^2+b^2)} e^{2e}}\right]}{\left(a e^e - \sqrt{-(-a^2+b^2)} e^{2e}\right) f^3} \right) \right) / \\
 & \left( b \left( \frac{-a e^{-e} - e^{-2e} \sqrt{a^2 e^{2e} - b^2 e^{2e}}}{b} - \frac{-a e^{-e} + e^{-2e} \sqrt{a^2 e^{2e} - b^2 e^{2e}}}{b} \right) \right) + \\
 & \left( -a e^{-e} - e^{-2e} \sqrt{a^2 e^{2e} - b^2 e^{2e}} \right) \left( \frac{x^3}{3 \left(a e^e + \sqrt{-(-a^2+b^2)} e^{2e}\right)} - \right. \\
 & \left. \frac{x^2 \operatorname{Log}\left[1 + \frac{b e^{2e+fx}}{a e^e + \sqrt{-(-a^2+b^2)} e^{2e}}\right]}{\left(a e^e + \sqrt{-(-a^2+b^2)} e^{2e}\right) f} - \frac{2 x \operatorname{PolyLog}\left[2, -\frac{b e^{2e+fx}}{a e^e + \sqrt{-(-a^2+b^2)} e^{2e}}\right]}{\left(a e^e + \sqrt{-(-a^2+b^2)} e^{2e}\right) f^2} + \right. \\
 & \left. \left. \frac{2 \operatorname{PolyLog}\left[3, -\frac{b e^{2e+fx}}{a e^e + \sqrt{-(-a^2+b^2)} e^{2e}}\right]}{\left(a e^e + \sqrt{-(-a^2+b^2)} e^{2e}\right) f^3} \right) \right) / \\
 & \left( b \left( \frac{-a e^{-e} - e^{-2e} \sqrt{a^2 e^{2e} - b^2 e^{2e}}}{b} - \frac{-a e^{-e} + e^{-2e} \sqrt{a^2 e^{2e} - b^2 e^{2e}}}{b} \right) \right) + \\
 & a d^2 f \left( - \left( \left( e^{2e} \left( -a e^{-e} + e^{-2e} \sqrt{a^2 e^{2e} - b^2 e^{2e}} \right) \left( \frac{x^3}{3 \left(a e^e - \sqrt{-(-a^2+b^2)} e^{2e}\right)} - \right. \right. \right. \right. \\
 & \left. \left. \frac{x^2 \operatorname{Log}\left[1 + \frac{b e^{2e+fx}}{a e^e - \sqrt{-(-a^2+b^2)} e^{2e}}\right]}{\left(a e^e - \sqrt{-(-a^2+b^2)} e^{2e}\right) f} - \frac{2 x \operatorname{PolyLog}\left[2, -\frac{b e^{2e+fx}}{a e^e - \sqrt{-(-a^2+b^2)} e^{2e}}\right]}{\left(a e^e - \sqrt{-(-a^2+b^2)} e^{2e}\right) f^2} + \right. \right. \\
 & \left. \left. \frac{2 \operatorname{PolyLog}\left[3, -\frac{b e^{2e+fx}}{a e^e - \sqrt{-(-a^2+b^2)} e^{2e}}\right]}{\left(a e^e - \sqrt{-(-a^2+b^2)} e^{2e}\right) f^3} \right) \right) \right) / \\
 & \left( b \left( \frac{-a e^{-e} - e^{-2e} \sqrt{a^2 e^{2e} - b^2 e^{2e}}}{b} - \frac{-a e^{-e} + e^{-2e} \sqrt{a^2 e^{2e} - b^2 e^{2e}}}{b} \right) \right) +
 \end{aligned}$$

$$\left( e^{2e} \left( -a e^{-e} - e^{-2e} \sqrt{a^2 e^{2e} - b^2 e^{2e}} \right) \left( \frac{x^3}{3 \left( a e^e + \sqrt{-(-a^2 + b^2) e^{2e}} \right)} - \frac{x^2 \operatorname{Log} \left[ 1 + \frac{b e^{2e+fx}}{a e^e + \sqrt{-(-a^2 + b^2) e^{2e}}} \right]}{\left( a e^e + \sqrt{-(-a^2 + b^2) e^{2e}} \right) f} - \frac{2 \operatorname{PolyLog} \left[ 2, -\frac{b e^{2e+fx}}{a e^e + \sqrt{-(-a^2 + b^2) e^{2e}}} \right]}{\left( a e^e + \sqrt{-(-a^2 + b^2) e^{2e}} \right) f^2} + \frac{2 \operatorname{PolyLog} \left[ 3, -\frac{b e^{2e+fx}}{a e^e + \sqrt{-(-a^2 + b^2) e^{2e}}} \right]}{\left( a e^e + \sqrt{-(-a^2 + b^2) e^{2e}} \right) f^3} \right) \right) / \left( b \left( \frac{-a e^{-e} - e^{-2e} \sqrt{a^2 e^{2e} - b^2 e^{2e}}}{b} - \frac{-a e^{-e} + e^{-2e} \sqrt{a^2 e^{2e} - b^2 e^{2e}}}{b} \right) \right) + \left( \operatorname{Sech}[e] \left( a c^2 \operatorname{Sinh}[e] + 2 a c d x \operatorname{Sinh}[e] + a d^2 x^2 \operatorname{Sinh}[e] - b c^2 \operatorname{Sinh}[f x] - 2 b c d x \operatorname{Sinh}[f x] - b d^2 x^2 \operatorname{Sinh}[f x] \right) \right) / \left( (a-b)(a+b) f (a+b \operatorname{Cosh}[e+fx]) \right)$$

### Problem 180: Result more than twice size of optimal antiderivative.

$$\int (c+dx)^m (a+b \operatorname{Cosh}[e+fx])^2 dx$$

Optimal (type 4, 282 leaves, 10 steps):

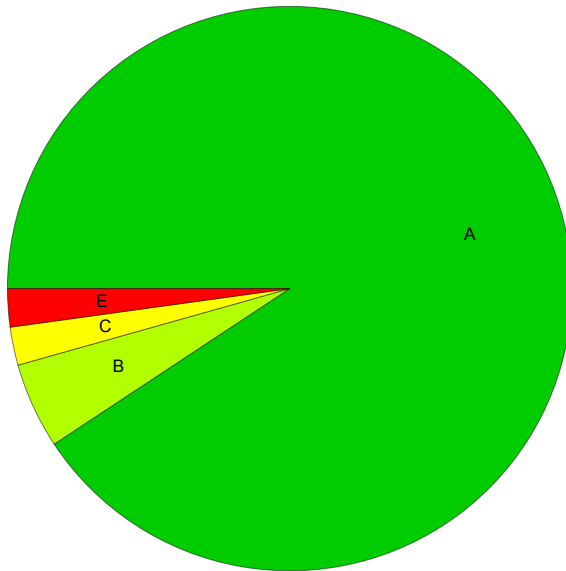
$$\frac{a^2 (c+dx)^{1+m}}{d(1+m)} + \frac{b^2 (c+dx)^{1+m}}{2d(1+m)} + \frac{2^{-3-m} b^2 e^{2e-\frac{2cf}{d}} (c+dx)^m \left( -\frac{f(c+dx)}{d} \right)^{-m} \operatorname{Gamma} \left[ 1+m, -\frac{2f(c+dx)}{d} \right]}{f} + \frac{a b e^{e-\frac{cf}{d}} (c+dx)^m \left( -\frac{f(c+dx)}{d} \right)^{-m} \operatorname{Gamma} \left[ 1+m, -\frac{f(c+dx)}{d} \right]}{f} - \frac{a b e^{-e+\frac{cf}{d}} (c+dx)^m \left( \frac{f(c+dx)}{d} \right)^{-m} \operatorname{Gamma} \left[ 1+m, \frac{f(c+dx)}{d} \right]}{f} - \frac{2^{-3-m} b^2 e^{-2e+\frac{2cf}{d}} (c+dx)^m \left( \frac{f(c+dx)}{d} \right)^{-m} \operatorname{Gamma} \left[ 1+m, \frac{2f(c+dx)}{d} \right]}{f}$$

Result (type 4, 650 leaves):

$$\begin{aligned}
 & \frac{1}{d f (1+m)} 2^{-3-m} (c+dx)^m \left( -\frac{f^2 (c+dx)^2}{d^2} \right)^{-m} \\
 & \left( 2^{3+m} a^2 c f \left( -\frac{f^2 (c+dx)^2}{d^2} \right)^m + 2^{2+m} b^2 c f \left( -\frac{f^2 (c+dx)^2}{d^2} \right)^m + 2^{3+m} a^2 d f x \left( -\frac{f^2 (c+dx)^2}{d^2} \right)^m + \right. \\
 & \left. 2^{2+m} b^2 d f x \left( -\frac{f^2 (c+dx)^2}{d^2} \right)^m - 2^{3+m} a b d \left( -\frac{f (c+dx)}{d} \right)^m \operatorname{Cosh} \left[ e - \frac{c f}{d} \right] \right. \\
 & \left. \operatorname{Gamma} \left[ 1+m, \frac{f (c+dx)}{d} \right] - 2^{3+m} a b d m \left( -\frac{f (c+dx)}{d} \right)^m \operatorname{Cosh} \left[ e - \frac{c f}{d} \right] \operatorname{Gamma} \left[ 1+m, \frac{f (c+dx)}{d} \right] - \right. \\
 & \left. b^2 d \left( -\frac{f (c+dx)}{d} \right)^m \operatorname{Cosh} \left[ 2 e - \frac{2 c f}{d} \right] \operatorname{Gamma} \left[ 1+m, \frac{2 f (c+dx)}{d} \right] - \right. \\
 & \left. b^2 d m \left( -\frac{f (c+dx)}{d} \right)^m \operatorname{Cosh} \left[ 2 e - \frac{2 c f}{d} \right] \operatorname{Gamma} \left[ 1+m, \frac{2 f (c+dx)}{d} \right] + \right. \\
 & \left. b^2 d \left( -\frac{f (c+dx)}{d} \right)^m \operatorname{Gamma} \left[ 1+m, \frac{2 f (c+dx)}{d} \right] \operatorname{Sinh} \left[ 2 e - \frac{2 c f}{d} \right] + \right. \\
 & \left. b^2 d m \left( -\frac{f (c+dx)}{d} \right)^m \operatorname{Gamma} \left[ 1+m, \frac{2 f (c+dx)}{d} \right] \operatorname{Sinh} \left[ 2 e - \frac{2 c f}{d} \right] + \right. \\
 & \left. b^2 d (1+m) \left( f \left( \frac{c}{d} + x \right) \right)^m \operatorname{Gamma} \left[ 1+m, -\frac{2 f (c+dx)}{d} \right] \left( \operatorname{Cosh} \left[ 2 e - \frac{2 c f}{d} \right] + \operatorname{Sinh} \left[ 2 e - \frac{2 c f}{d} \right] \right) + \right. \\
 & \left. 2^{3+m} a b d \left( -\frac{f (c+dx)}{d} \right)^m \operatorname{Gamma} \left[ 1+m, \frac{f (c+dx)}{d} \right] \operatorname{Sinh} \left[ e - \frac{c f}{d} \right] + \right. \\
 & \left. 2^{3+m} a b d m \left( -\frac{f (c+dx)}{d} \right)^m \operatorname{Gamma} \left[ 1+m, \frac{f (c+dx)}{d} \right] \operatorname{Sinh} \left[ e - \frac{c f}{d} \right] + \right. \\
 & \left. 2^{3+m} a b d (1+m) \left( f \left( \frac{c}{d} + x \right) \right)^m \operatorname{Gamma} \left[ 1+m, -\frac{f (c+dx)}{d} \right] \left( \operatorname{Cosh} \left[ e - \frac{c f}{d} \right] + \operatorname{Sinh} \left[ e - \frac{c f}{d} \right] \right) \right)
 \end{aligned}$$

## Summary of Integration Test Results

183 integration problems



- A - 166 optimal antiderivatives
- B - 9 more than twice size of optimal antiderivatives
- C - 4 unnecessarily complex antiderivatives
- D - 0 unable to integrate problems
- E - 4 integration timeouts