Mathematica 11.3 Integration Test Results

Test results for the 204 problems in "6.3.2 Hyperbolic tangent functions.m"

Problem 41: Result more than twice size of optimal antiderivative.

$$\int (a + a \operatorname{Tanh}[c + d x])^5 \, dx$$

Optimal (type 3, 100 leaves, 5 steps):

$$16 a^5 x + \frac{16 a^5 \operatorname{Log}[\operatorname{Cosh}[c + d x]]}{d} - \frac{8 a^5 \operatorname{Tanh}[c + d x]}{d} - \frac{2 a^2 (a + a \operatorname{Tanh}[c + d x])^3}{3 d} - \frac{a (a + a \operatorname{Tanh}[c + d x])^4}{4 d} - \frac{2 a (a^2 + a^2 \operatorname{Tanh}[c + d x])^2}{d}$$

Result (type 3, 202 leaves):

$$\frac{1}{12 d} a^5 \operatorname{Sech}[c] \operatorname{Sech}[c + d x]^4 \big(18 \operatorname{Cosh}[3 c + 2 d x] + 48 d x \operatorname{Cosh}[3 c + 2 d x] + 12 d x \operatorname{Cosh}[3 c + 4 d x] + 12 d x \operatorname{Cosh}[5 c + 4 d x] + 48 \operatorname{Cosh}[3 c + 2 d x] \operatorname{Log}[\operatorname{Cosh}[c + d x]] + 12 \operatorname{Cosh}[3 c + 4 d x] \operatorname{Log}[\operatorname{Cosh}[c + d x]] + 12 \operatorname{Cosh}[5 c + 4 d x] \operatorname{Log}[\operatorname{Cosh}[c + d x]] + 6 \operatorname{Cosh}[c + 2 d x] (3 + 8 d x + 8 \operatorname{Log}[\operatorname{Cosh}[c + d x]]) + \operatorname{Cosh}[c] (33 + 72 d x + 72 \operatorname{Log}[\operatorname{Cosh}[c + d x]]) + 75 \operatorname{Sinh}[c] - 70 \operatorname{Sinh}[c + 2 d x] + 30 \operatorname{Sinh}[3 c + 2 d x] - 25 \operatorname{Sinh}[3 c + 4 d x]\big)$$

Problem 42: Result more than twice size of optimal antiderivative.

$$\int (a + a \operatorname{Tanh}[c + d x])^4 \, dx$$

Optimal (type 3, 77 leaves, 4 steps):

$$8 a^4 x + \frac{8 a^4 \operatorname{Log}[\operatorname{Cosh}[c + d x]]}{d} - \frac{4 a^4 \operatorname{Tanh}[c + d x]}{d} - \frac{a (a + a \operatorname{Tanh}[c + d x])^3}{3 d} - \frac{(a^2 + a^2 \operatorname{Tanh}[c + d x])^2}{d}$$

Result (type 3, 178 leaves):

$$\frac{1}{6\,d\,\left(\text{Cosh}[d\,x]+\text{Sinh}[d\,x]\right)^4}\,a^4\,\text{Sech}[c]\,\text{Sech}[c+d\,x]^3\,\left(\text{Cosh}[4\,d\,x]+\text{Sinh}[4\,d\,x]\right)$$
$$\left(6\,d\,x\,\text{Cosh}[2\,c+3\,d\,x]+6\,d\,x\,\text{Cosh}[4\,c+3\,d\,x]+6\,\text{Cosh}[2\,c+3\,d\,x]\,\text{Log}[\text{Cosh}[c+d\,x]]+6\,\text{Cosh}[4\,c+3\,d\,x]\,\text{Log}[\text{Cosh}[c+d\,x]]+6\,\text{Cosh}[d\,x]\,\left(1+3\,d\,x+3\,\text{Log}[\text{Cosh}[c+d\,x]]\right)+6\,\text{Cosh}[2\,c+d\,x]\,\left(1+3\,d\,x+3\,\text{Log}[\text{Cosh}[c+d\,x]]\right)-21\,\text{Sinh}[d\,x]+12\,\text{Sinh}[2\,c+d\,x]-11\,\text{Sinh}[2\,c+3\,d\,x]\right)$$

Problem 57: Result more than twice size of optimal antiderivative.

$$\int (a+b\,\text{Tanh}[c+d\,x])^5\,dx$$

Optimal (type 3, 142 leaves, 5 steps):

$$a\,\left(a^4+10\,a^2\,b^2+5\,b^4\right)\,x+\frac{b\,\left(5\,a^4+10\,a^2\,b^2+b^4\right)\,\text{Log}[\text{Cosh}[c+d\,x]]}{d}-\frac{4\,a\,b^2\,\left(a^2+b^2\right)\,\text{Tanh}[c+d\,x]}{d}-\frac{b\,\left(3\,a^2+b^2\right)\,\left(a+b\,\text{Tanh}[c+d\,x]\right)^2}{2\,d}-\frac{2\,a\,b\,\left(a+b\,\text{Tanh}[c+d\,x]\right)^3}{3\,d}-\frac{b\,\left(a+b\,\text{Tanh}[c+d\,x]\right)^4}{4\,d}$$

Result (type 3, 366 leaves):

$$-\frac{b^5\,\text{Cosh}[c+d\,x]\,\left(a+b\,\text{Tanh}[c+d\,x]\right)^5}{4\,d\,\left(a\,\text{Cosh}[c+d\,x]+b\,\text{Sinh}[c+d\,x]\right)^5}+\frac{b^3\,\left(5\,a^2+b^2\right)\,\text{Cosh}[c+d\,x]^3\,\left(a+b\,\text{Tanh}[c+d\,x]\right)^5}{d\,\left(a\,\text{Cosh}[c+d\,x]+b\,\text{Sinh}[c+d\,x]\right)^5}+$$
$$\left(a\,\left(a^4+10\,a^2\,b^2+5\,b^4\right)\,(c+d\,x)\,\text{Cosh}[c+d\,x]^5\,\left(a+b\,\text{Tanh}[c+d\,x]\right)^5\right)\Big/\left(d\,\left(a\,\text{Cosh}[c+d\,x]+b\,\text{Sinh}[c+d\,x]\right)^5\right)+$$
$$\left(\left(5\,a^4\,b+10\,a^2\,b^3+b^5\right)\,\text{Cosh}[c+d\,x]^5\,\text{Log}[\text{Cosh}[c+d\,x]]\,\left(a+b\,\text{Tanh}[c+d\,x]\right)^5\right)\Big/\left(d\,\left(a\,\text{Cosh}[c+d\,x]+b\,\text{Sinh}[c+d\,x]\right)^5\right)+\frac{5\,a\,b^4\,\text{Cosh}[c+d\,x]^2\,\text{Sinh}[c+d\,x]\,\left(a+b\,\text{Tanh}[c+d\,x]\right)^5}{3\,d\,\left(a\,\text{Cosh}[c+d\,x]+b\,\text{Sinh}[c+d\,x]\right)^5}-$$
$$\left(10\,\text{Cosh}[c+d\,x]^4\,\left(3\,a^3\,b^2\,\text{Sinh}[c+d\,x]+2\,a\,b^4\,\text{Sinh}[c+d\,x]\right)\,\left(a+b\,\text{Tanh}[c+d\,x]\right)^5\right)\Big/\left(3\,d\,\left(a\,\text{Cosh}[c+d\,x]+b\,\text{Sinh}[c+d\,x]\right)^5\right)$$

Problem 73: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Csch}[x]}{1+\text{Tanh}[x]}\,dx$$

Optimal (type 3, 12 leaves, 8 steps):

$$-\text{ArcTanh}[\text{Cosh}[x]]+\text{Cosh}[x]-\text{Sinh}[x]$$

Result (type 3, 49 leaves):

$$\frac{1}{1+\text{Tanh}[x]}\left(\text{Cosh}[x]-\text{Log}\left[\text{Cosh}\left[\frac{x}{2}\right]\right]+\text{Log}\left[\text{Sinh}\left[\frac{x}{2}\right]\right]-\left(\text{Log}\left[\text{Cosh}\left[\frac{x}{2}\right]\right]-\text{Log}\left[\text{Sinh}\left[\frac{x}{2}\right]\right]+\text{Sinh}[x]\right)\,\text{Tanh}[x]\right)$$

Problem 75: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Csch}[x]^3}{1+\text{Tanh}[x]}\,dx$$

Optimal (type 3, 18 leaves, 8 steps):

$$-\frac{1}{2}\,\text{ArcTanh}[\text{Cosh}[x]]+\text{Csch}[x]-\frac{1}{2}\,\text{Coth}[x]\,\text{Csch}[x]$$

Result (type 3, 59 leaves):

$$\frac{1}{8}\left(4\,\text{Coth}\left[\frac{x}{2}\right]-\text{Csch}\left[\frac{x}{2}\right]^2-4\,\text{Log}\left[\text{Cosh}\left[\frac{x}{2}\right]\right]+4\,\text{Log}\left[\text{Sinh}\left[\frac{x}{2}\right]\right]-\text{Sech}\left[\frac{x}{2}\right]^2-4\,\text{Tanh}\left[\frac{x}{2}\right]\right)$$

Problem 77: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Csch}[x]^5}{1+\text{Tanh}[x]}\,dx$$

Optimal (type 3, 34 leaves, 9 steps):

$$\frac{1}{8}\,\text{ArcTanh}[\text{Cosh}[x]]-\frac{1}{8}\,\text{Coth}[x]\,\text{Csch}[x]+\frac{\text{Csch}[x]^3}{3}-\frac{1}{4}\,\text{Coth}[x]\,\text{Csch}[x]^3$$

Result (type 3, 69 leaves):

$$\frac{1}{192}\,\text{Csch}[x]^4\left(-42\,\text{Cosh}[x]-6\,\text{Cosh}[3\,x]+2\,\text{Sinh}[x]\left(32-9\left(\text{Log}\left[\text{Cosh}\left[\frac{x}{2}\right]\right]-\text{Log}\left[\text{Sinh}\left[\frac{x}{2}\right]\right]\right)\text{Sinh}[x]+3\left(\text{Log}\left[\text{Cosh}\left[\frac{x}{2}\right]\right]-\text{Log}\left[\text{Sinh}\left[\frac{x}{2}\right]\right]\right)\text{Sinh}[3\,x]\right)\right)$$

Problem 79: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Csch}[x]^7}{1+\text{Tanh}[x]}\,dx$$

Optimal (type 3, 44 leaves, 10 steps):

$$-\frac{1}{16}\,\text{ArcTanh}[\text{Cosh}[x]]+\frac{1}{16}\,\text{Coth}[x]\,\text{Csch}[x]-\frac{1}{24}\,\text{Coth}[x]\,\text{Csch}[x]^3+\frac{\text{Csch}[x]^5}{5}-\frac{1}{6}\,\text{Coth}[x]\,\text{Csch}[x]^5$$

Result (type 3, 124 leaves):

$$\frac{1}{1920}\left(72\,\text{Coth}\left[\frac{x}{2}\right]+30\,\text{Csch}\left[\frac{x}{2}\right]^2-120\,\text{Log}\left[\text{Cosh}\left[\frac{x}{2}\right]\right]+120\,\text{Log}\left[\text{Sinh}\left[\frac{x}{2}\right]\right]+30\,\text{Sech}\left[\frac{x}{2}\right]^2-5\,\text{Sech}\left[\frac{x}{2}\right]^6-288\,\text{Csch}[x]^3\,\text{Sinh}\left[\frac{x}{2}\right]^4-384\,\text{Csch}[x]^5\,\text{Sinh}\left[\frac{x}{2}\right]^6-18\,\text{Csch}\left[\frac{x}{2}\right]^4\,\text{Sinh}[x]+\text{Csch}\left[\frac{x}{2}\right]^6\left(-5+6\,\text{Sinh}[x]\right)-72\,\text{Tanh}\left[\frac{x}{2}\right]\right)$$

Problem 144: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{x\,\text{Sech}[c+d\,x]^2}{a+b\,\text{Tanh}[c+d\,x]^2}\,dx$$

Optimal (type 4, 231 leaves, 9 steps):

$$\frac{x\,\text{Log}\left[1+\frac{(a+b)\,e^{2c+2dx}}{a-2\sqrt{-a}\,\sqrt{b}-b}\right]}{2\sqrt{-a}\,\sqrt{b}\,d}-\frac{x\,\text{Log}\left[1+\frac{(a+b)\,e^{2c+2dx}}{a+2\sqrt{-a}\,\sqrt{b}-b}\right]}{2\sqrt{-a}\,\sqrt{b}\,d}+\frac{\text{PolyLog}\left[2,-\frac{(a+b)\,e^{2c+2dx}}{a-2\sqrt{-a}\,\sqrt{b}-b}\right]}{4\sqrt{-a}\,\sqrt{b}\,d^2}-\frac{\text{PolyLog}\left[2,-\frac{(a+b)\,e^{2c+2dx}}{a+2\sqrt{-a}\,\sqrt{b}-b}\right]}{4\sqrt{-a}\,\sqrt{b}\,d^2}$$

Result (type 4, 690 leaves):

$$\left(2\left(2\,i\,c\,\text{ArcTan}\left[\frac{1}{\sqrt{a}\,\sqrt{b}}\left(\text{Cosh}[c+d\,x]+\text{Sinh}[c+d\,x]\right)\left(a\,\text{Cosh}[c+d\,x]+b\,\text{Sinh}[c+d\,x]\right)\right]-\right.\right.$$

$$(c+d\,x)\,\text{Log}\left[1-\frac{\text{Cosh}[c+d\,x]+\text{Sinh}[c+d\,x]}{\sqrt{-\frac{\sqrt{a}-i\sqrt{b}}{\sqrt{a}+i\sqrt{b}}}}\right]-$$

$$(c+d\,x)\,\text{Log}\left[1+\frac{\text{Cosh}[c+d\,x]+\text{Sinh}[c+d\,x]}{\sqrt{-\frac{\sqrt{a}-i\sqrt{b}}{\sqrt{a}+i\sqrt{b}}}}\right]+(c+d\,x)$$

$$\text{Log}\left[1-\frac{\text{Cosh}[c+d\,x]+\text{Sinh}[c+d\,x]}{\sqrt{-\frac{\sqrt{a}+i\sqrt{b}}{\sqrt{a}-i\sqrt{b}}}}\right]+(c+d\,x)\,\text{Log}\left[1+\frac{\text{Cosh}[c+d\,x]+\text{Sinh}[c+d\,x]}{\sqrt{-\frac{\sqrt{a}+i\sqrt{b}}{\sqrt{a}-i\sqrt{b}}}}\right]-$$

$$\text{PolyLog}\left[2,-\frac{\text{Cosh}[c+d\,x]+\text{Sinh}[c+d\,x]}{\sqrt{-\frac{\sqrt{a}-i\sqrt{b}}{\sqrt{a}+i\sqrt{b}}}}\right]-\text{PolyLog}\left[2,\frac{\text{Cosh}[c+d\,x]+\text{Sinh}[c+d\,x]}{\sqrt{-\frac{\sqrt{a}-i\sqrt{b}}{\sqrt{a}+i\sqrt{b}}}}\right]+$$

$$\left.\left.\text{PolyLog}\left[2,-\frac{\text{Cosh}[c+d\,x]+\text{Sinh}[c+d\,x]}{\sqrt{-\frac{\sqrt{a}+i\sqrt{b}}{\sqrt{a}-i\sqrt{b}}}}\right]+\text{PolyLog}\left[2,\frac{\text{Cosh}[c+d\,x]+\text{Sinh}[c+d\,x]}{\sqrt{-\frac{\sqrt{a}+i\sqrt{b}}{\sqrt{a}-i\sqrt{b}}}}\right]\right)\right)\Bigg/$$

$$\left(\left(-\sqrt{\frac{-\sqrt{a}+i\sqrt{b}}{\sqrt{a}+i\sqrt{b}}}+\sqrt{-\frac{\sqrt{a}+i\sqrt{b}}{\sqrt{a}-i\sqrt{b}}}\right)\left(\sqrt{\frac{-\sqrt{a}+i\sqrt{b}}{\sqrt{a}+i\sqrt{b}}}+\sqrt{-\frac{\sqrt{a}+i\sqrt{b}}{\sqrt{a}-i\sqrt{b}}}\right)(a+b)\,d^2\right)$$

Problem 145: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^2\,\text{Sech}[c+d\,x]^2}{a+b\,\text{Tanh}[c+d\,x]^2}\,dx$$

Optimal (type 4, 351 leaves, 11 steps):

$$\frac{x^2\,\text{Log}\left[1+\frac{(a+b)\,e^{2c+2dx}}{a-2\sqrt{-a}\sqrt{b}-b}\right]}{2\sqrt{-a}\sqrt{b}\,d}-\frac{x^2\,\text{Log}\left[1+\frac{(a+b)\,e^{2c+2dx}}{a+2\sqrt{-a}\sqrt{b}-b}\right]}{2\sqrt{-a}\sqrt{b}\,d}+\frac{x\,\text{PolyLog}\left[2,-\frac{(a+b)\,e^{2c+2dx}}{a-2\sqrt{-a}\sqrt{b}-b}\right]}{2\sqrt{-a}\sqrt{b}\,d^2}-$$

$$\frac{x\,\text{PolyLog}\left[2,-\frac{(a+b)\,e^{2c+2dx}}{a+2\sqrt{-a}\sqrt{b}-b}\right]}{2\sqrt{-a}\sqrt{b}\,d^2}-\frac{\text{PolyLog}\left[3,-\frac{(a+b)\,e^{2c+2dx}}{a-2\sqrt{-a}\sqrt{b}-b}\right]}{4\sqrt{-a}\sqrt{b}\,d^3}+\frac{\text{PolyLog}\left[3,-\frac{(a+b)\,e^{2c+2dx}}{a+2\sqrt{-a}\sqrt{b}-b}\right]}{4\sqrt{-a}\sqrt{b}\,d^3}$$

Result (type 4, 316 leaves):

$$\frac{1}{4\sqrt{a}\,\sqrt{b}\,d^3}\,\mathbb{i}\left(2\,d^2\,x^2\,\text{Log}\left[1+\frac{\left(\sqrt{a}-\mathbb{i}\,\sqrt{b}\right)\,e^{2\,(c+d\,x)}}{\sqrt{a}+\mathbb{i}\,\sqrt{b}}\right]-2\,d^2\,x^2\,\text{Log}\left[1+\frac{\left(\sqrt{a}+\mathbb{i}\,\sqrt{b}\right)\,e^{2\,(c+d\,x)}}{\sqrt{a}-\mathbb{i}\,\sqrt{b}}\right]+\right.$$
$$2\,d\,x\,\text{PolyLog}\left[2,-\frac{\left(\sqrt{a}-\mathbb{i}\,\sqrt{b}\right)\,e^{2\,(c+d\,x)}}{\sqrt{a}+\mathbb{i}\,\sqrt{b}}\right]-2\,d\,x\,\text{PolyLog}\left[2,-\frac{\left(\sqrt{a}+\mathbb{i}\,\sqrt{b}\right)\,e^{2\,(c+d\,x)}}{\sqrt{a}-\mathbb{i}\,\sqrt{b}}\right]-$$
$$\left.\text{PolyLog}\left[3,-\frac{\left(\sqrt{a}-\mathbb{i}\,\sqrt{b}\right)\,e^{2\,(c+d\,x)}}{\sqrt{a}+\mathbb{i}\,\sqrt{b}}\right]+\text{PolyLog}\left[3,-\frac{\left(\sqrt{a}+\mathbb{i}\,\sqrt{b}\right)\,e^{2\,(c+d\,x)}}{\sqrt{a}-\mathbb{i}\,\sqrt{b}}\right]\right)$$

Problem 157: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Tanh}[x]^5}{\sqrt{a+b\,\text{Tanh}[x]^2+c\,\text{Tanh}[x]^4}}\,dx$$

Optimal (type 3, 135 leaves, 8 steps):

$$\frac{(b-2\,c)\,\text{ArcTanh}\left[\frac{b+2\,c\,\text{Tanh}[x]^2}{2\,\sqrt{c}\,\sqrt{a+b\,\text{Tanh}[x]^2+c\,\text{Tanh}[x]^4}}\right]}{4\,c^{3/2}}+\frac{\text{ArcTanh}\left[\frac{2\,a+b+(b+2\,c)\,\text{Tanh}[x]^2}{2\,\sqrt{a+b+c}\,\sqrt{a+b\,\text{Tanh}[x]^2+c\,\text{Tanh}[x]^4}}\right]}{2\,\sqrt{a+b+c}}-\frac{\sqrt{a+b\,\text{Tanh}[x]^2+c\,\text{Tanh}[x]^4}}{2\,c}$$

Result (type 3, 42 734 leaves): Display of huge result suppressed!

Problem 158: Attempted integration timed out after 120 seconds.

$$\int \frac{\text{Tanh}[x]^3}{\sqrt{a+b\,\text{Tanh}[x]^2+c\,\text{Tanh}[x]^4}}\,dx$$

Optimal (type 3, 105 leaves, 7 steps):

$$-\frac{\text{ArcTanh}\left[\frac{b+2\,c\,\text{Tanh}[x]^2}{2\,\sqrt{c}\,\sqrt{a+b\,\text{Tanh}[x]^2+c\,\text{Tanh}[x]^4}}\right]}{2\,\sqrt{c}}+\frac{\text{ArcTanh}\left[\frac{2\,a+b+(b+2\,c)\,\text{Tanh}[x]^2}{2\,\sqrt{a+b+c}\,\sqrt{a+b\,\text{Tanh}[x]^2+c\,\text{Tanh}[x]^4}}\right]}{2\,\sqrt{a+b+c}}$$

Result (type 1, 1 leaves):

???

Problem 159: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Tanh}[x]}{\sqrt{a+b\,\text{Tanh}[x]^2+c\,\text{Tanh}[x]^4}}\,dx$$

Optimal (type 3, 58 leaves, 4 steps):

$$\frac{\text{ArcTanh}\left[\frac{2 a+b+(b+2 c) \text{Tanh}[x]^2}{2 \sqrt{a+b+c} \sqrt{a+b \text{Tanh}[x]^2+c \text{Tanh}[x]^4}}\right]}{2 \sqrt{a+b+c}}$$

Result (type 3, 59 564 leaves): Display of huge result suppressed!

Problem 160: Unable to integrate problem.

$$\int \frac{\text{Coth}[x]}{\sqrt{a+b \text{Tanh}[x]^2+c \text{Tanh}[x]^4}} \, dx$$

Optimal (type 3, 106 leaves, 8 steps):

$$-\frac{\text{ArcTanh}\left[\frac{2 a+b \text{Tanh}[x]^2}{2 \sqrt{a} \sqrt{a+b \text{Tanh}[x]^2+c \text{Tanh}[x]^4}}\right]}{2 \sqrt{a}}+\frac{\text{ArcTanh}\left[\frac{2 a+b+(b+2 c) \text{Tanh}[x]^2}{2 \sqrt{a+b+c} \sqrt{a+b \text{Tanh}[x]^2+c \text{Tanh}[x]^4}}\right]}{2 \sqrt{a+b+c}}$$

Result (type 8, 23 leaves):

$$\int \frac{\text{Coth}[x]}{\sqrt{a+b \text{Tanh}[x]^2+c \text{Tanh}[x]^4}} \, dx$$

Problem 161: Unable to integrate problem.

$$\int \frac{\text{Coth}[x]^3}{\sqrt{a+b \text{Tanh}[x]^2+c \text{Tanh}[x]^4}} \, dx$$

Optimal (type 3, 183 leaves, 11 steps):

$$-\frac{\text{ArcTanh}\left[\frac{2 a+b \text{Tanh}[x]^2}{2 \sqrt{a} \sqrt{a+b \text{Tanh}[x]^2+c \text{Tanh}[x]^4}}\right]}{2 \sqrt{a}}+\frac{b \,\text{ArcTanh}\left[\frac{2 a+b \text{Tanh}[x]^2}{2 \sqrt{a} \sqrt{a+b \text{Tanh}[x]^2+c \text{Tanh}[x]^4}}\right]}{4 a^{3/2}}+$$

$$\frac{\text{ArcTanh}\left[\frac{2 a+b+(b+2 c) \text{Tanh}[x]^2}{2 \sqrt{a+b+c} \sqrt{a+b \text{Tanh}[x]^2+c \text{Tanh}[x]^4}}\right]}{2 \sqrt{a+b+c}}-\frac{\text{Coth}[x]^2 \sqrt{a+b \text{Tanh}[x]^2+c \text{Tanh}[x]^4}}{2 a}$$

Result (type 8, 25 leaves):

$$\int \frac{\text{Coth}[x]^3}{\sqrt{a+b \text{Tanh}[x]^2+c \text{Tanh}[x]^4}} \, dx$$

Problem 162: Result more than twice size of optimal antiderivative.

$$\int \text{Tanh}[x] \sqrt{a+b \text{Tanh}[x]^2+c \text{Tanh}[x]^4} \, dx$$

Optimal (type 3, 132 leaves, 8 steps):

$$-\frac{\left(b+2c\right)\text{ArcTanh}\left[\frac{b+2c\,\text{Tanh}[x]^2}{2\sqrt{c}\,\sqrt{a+b\,\text{Tanh}[x]^2+c\,\text{Tanh}[x]^4}}\right]}{4\sqrt{c}}+$$
$$\frac{1}{2}\sqrt{a+b+c}\,\text{ArcTanh}\left[\frac{2a+b+\left(b+2c\right)\text{Tanh}[x]^2}{2\sqrt{a+b+c}\,\sqrt{a+b\,\text{Tanh}[x]^2+c\,\text{Tanh}[x]^4}}\right]-\frac{1}{2}\sqrt{a+b\,\text{Tanh}[x]^2+c\,\text{Tanh}[x]^4}$$

Result (type 3, 178 715 leaves) : Display of huge result suppressed!

Problem 171: Result is not expressed in closed-form.

$$\int e^x\,\text{Tanh}[2x]^2\,dx$$

Optimal (type 3, 113 leaves, 13 steps) :

$$e^x+\frac{e^x}{1+e^{4x}}+\frac{\text{ArcTan}\left[1-\sqrt{2}\,e^x\right]}{2\sqrt{2}}-$$
$$\frac{\text{ArcTan}\left[1+\sqrt{2}\,e^x\right]}{2\sqrt{2}}+\frac{\text{Log}\left[1-\sqrt{2}\,e^x+e^{2x}\right]}{4\sqrt{2}}-\frac{\text{Log}\left[1+\sqrt{2}\,e^x+e^{2x}\right]}{4\sqrt{2}}$$

Result (type 7, 48 leaves) :

$$e^x+\frac{e^x}{1+e^{4x}}+\frac{1}{4}\text{RootSum}\left[1+\#1^4\,\&,\,\frac{x-\text{Log}\left[e^x-\#1\right]}{\#1^3}\,\&\right]$$

Problem 172: Result is not expressed in closed-form.

$$\int e^x\,\text{Tanh}[2x]\,dx$$

Optimal (type 3, 95 leaves, 11 steps) :

$$e^x+\frac{\text{ArcTan}\left[1-\sqrt{2}\,e^x\right]}{\sqrt{2}}-\frac{\text{ArcTan}\left[1+\sqrt{2}\,e^x\right]}{\sqrt{2}}+\frac{\text{Log}\left[1-\sqrt{2}\,e^x+e^{2x}\right]}{2\sqrt{2}}-\frac{\text{Log}\left[1+\sqrt{2}\,e^x+e^{2x}\right]}{2\sqrt{2}}$$

Result (type 7, 35 leaves) :

$$e^x+\frac{1}{2}\text{RootSum}\left[1+\#1^4\,\&,\,\frac{x-\text{Log}\left[e^x-\#1\right]}{\#1^3}\,\&\right]$$

Problem 175: Result is not expressed in closed-form.

$$\int e^x\,\text{Tanh}[3x]^2\,dx$$

Optimal (type 3, 113 leaves, 14 steps) :

$$e^x + \frac{2 e^x}{3 \left(1 + e^{6 x}\right)} - \frac{2 \text{ArcTan}[e^x]}{9} + \frac{1}{9} \text{ArcTan}\left[\sqrt{3} - 2 e^x\right] - \frac{1}{9} \text{ArcTan}\left[\sqrt{3} + 2 e^x\right] + \frac{\text{Log}\left[1 - \sqrt{3}\, e^x + e^{2 x}\right]}{6 \sqrt{3}} - \frac{\text{Log}\left[1 + \sqrt{3}\, e^x + e^{2 x}\right]}{6 \sqrt{3}}$$

Result (type 7, 97 leaves):

$$e^x + \frac{2 e^x}{3 \left(1 + e^{6 x}\right)} - \frac{2 \text{ArcTan}[e^x]}{9} - \frac{1}{9} \text{RootSum}\left[1 - \#1^2 + \#1^4 \&, \frac{-2 x + 2 \text{Log}[e^x - \#1] + x \#1^2 - \text{Log}[e^x - \#1] \#1^2}{-\#1 + 2 \#1^3} \&\right]$$

Problem 176: Result is not expressed in closed-form.

$$\int e^x \text{Tanh}[3 x] \, dx$$

Optimal (type 3, 97 leaves, 12 steps):

$$e^x - \frac{2 \text{ArcTan}[e^x]}{3} + \frac{1}{3} \text{ArcTan}\left[\sqrt{3} - 2 e^x\right] - \frac{1}{3} \text{ArcTan}\left[\sqrt{3} + 2 e^x\right] + \frac{\text{Log}\left[1 - \sqrt{3}\, e^x + e^{2 x}\right]}{2 \sqrt{3}} - \frac{\text{Log}\left[1 + \sqrt{3}\, e^x + e^{2 x}\right]}{2 \sqrt{3}}$$

Result (type 7, 81 leaves):

$$e^x - \frac{2 \text{ArcTan}[e^x]}{3} - \frac{1}{3} \text{RootSum}\left[1 - \#1^2 + \#1^4 \&, \frac{-2 x + 2 \text{Log}[e^x - \#1] + x \#1^2 - \text{Log}[e^x - \#1] \#1^2}{-\#1 + 2 \#1^3} \&\right]$$

Problem 179: Result is not expressed in closed-form.

$$\int e^x \text{Tanh}[4 x]^2 \, dx$$

Optimal (type 3, 382 leaves, 23 steps):

$$e^x+\frac{e^x}{2\left(1+e^{8x}\right)}+\frac{\text{ArcTan}\left[\frac{\sqrt{2-\sqrt{2}}-2e^x}{\sqrt{2+\sqrt{2}}}\right]}{8\sqrt{2\left(2-\sqrt{2}\right)}}+$$

$$\frac{\text{ArcTan}\left[\frac{\sqrt{2+\sqrt{2}}-2e^x}{\sqrt{2-\sqrt{2}}}\right]}{8\sqrt{2\left(2+\sqrt{2}\right)}}-\frac{\text{ArcTan}\left[\frac{\sqrt{2-\sqrt{2}}+2e^x}{\sqrt{2+\sqrt{2}}}\right]}{8\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\text{ArcTan}\left[\frac{\sqrt{2+\sqrt{2}}+2e^x}{\sqrt{2-\sqrt{2}}}\right]}{8\sqrt{2\left(2+\sqrt{2}\right)}}+$$

$$\frac{1}{32}\sqrt{2-\sqrt{2}}\,\text{Log}\left[1-\sqrt{2-\sqrt{2}}\,e^x+e^{2x}\right]-\frac{1}{32}\sqrt{2-\sqrt{2}}\,\text{Log}\left[1+\sqrt{2-\sqrt{2}}\,e^x+e^{2x}\right]+$$

$$\frac{1}{32}\sqrt{2+\sqrt{2}}\,\text{Log}\left[1-\sqrt{2+\sqrt{2}}\,e^x+e^{2x}\right]-\frac{1}{32}\sqrt{2+\sqrt{2}}\,\text{Log}\left[1+\sqrt{2+\sqrt{2}}\,e^x+e^{2x}\right]$$

Result (type 7, 51 leaves):

$$e^x+\frac{e^x}{2\left(1+e^{8x}\right)}+\frac{1}{16}\text{RootSum}\left[1+\#1^8\,\&,\frac{x-\text{Log}\left[e^x-\#1\right]}{\#1^7}\,\&\right]$$

Problem 180: Result is not expressed in closed-form.

$$\int e^x\,\text{Tanh}[4x]\,dx$$

Optimal (type 3, 366 leaves, 21 steps):

$$e^x+\frac{\text{ArcTan}\left[\frac{\sqrt{2-\sqrt{2}}-2e^x}{\sqrt{2+\sqrt{2}}}\right]}{2\sqrt{2\left(2-\sqrt{2}\right)}}+\frac{\text{ArcTan}\left[\frac{\sqrt{2+\sqrt{2}}-2e^x}{\sqrt{2-\sqrt{2}}}\right]}{2\sqrt{2\left(2+\sqrt{2}\right)}}-\frac{\text{ArcTan}\left[\frac{\sqrt{2-\sqrt{2}}+2e^x}{\sqrt{2+\sqrt{2}}}\right]}{2\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\text{ArcTan}\left[\frac{\sqrt{2+\sqrt{2}}+2e^x}{\sqrt{2-\sqrt{2}}}\right]}{2\sqrt{2\left(2+\sqrt{2}\right)}}+$$

$$\frac{1}{8}\sqrt{2-\sqrt{2}}\,\text{Log}\left[1-\sqrt{2-\sqrt{2}}\,e^x+e^{2x}\right]-\frac{1}{8}\sqrt{2-\sqrt{2}}\,\text{Log}\left[1+\sqrt{2-\sqrt{2}}\,e^x+e^{2x}\right]+$$

$$\frac{1}{8}\sqrt{2+\sqrt{2}}\,\text{Log}\left[1-\sqrt{2+\sqrt{2}}\,e^x+e^{2x}\right]-\frac{1}{8}\sqrt{2+\sqrt{2}}\,\text{Log}\left[1+\sqrt{2+\sqrt{2}}\,e^x+e^{2x}\right]$$

Result (type 7, 35 leaves):

$$e^x+\frac{1}{4}\text{RootSum}\left[1+\#1^8\,\&,\frac{x-\text{Log}\left[e^x-\#1\right]}{\#1^7}\,\&\right]$$

Problem 181: Result is not expressed in closed-form.

$$\int e^x\,\text{Coth}[4x]\,dx$$

Optimal (type 3, 116 leaves, 15 steps):

$$e^x-\frac{\text{ArcTan}[e^x]}{2}+\frac{\text{ArcTan}\left[1-\sqrt{2}\,e^x\right]}{2\sqrt{2}}-\frac{\text{ArcTan}\left[1+\sqrt{2}\,e^x\right]}{2\sqrt{2}}-\frac{\text{ArcTanh}[e^x]}{2}+\frac{\text{Log}\left[1-\sqrt{2}\,e^x+e^{2x}\right]}{4\sqrt{2}}-\frac{\text{Log}\left[1+\sqrt{2}\,e^x+e^{2x}\right]}{4\sqrt{2}}$$

Result (type 7, 59 leaves):

$$\frac{1}{4}\left(4\,e^x-2\,\text{ArcTan}\left[e^x\right]+\text{Log}\left[1-e^x\right]-\text{Log}\left[1+e^x\right]+\text{RootSum}\left[1+\#1^4\,\&,\,\frac{x-\text{Log}\left[e^x-\#1\right]}{\#1^3}\,\&\right]\right)$$

Problem 182: Result is not expressed in closed-form.

$$\int e^x\,\text{Coth}[4\,x]^2\,dx$$

Optimal (type 3, 134 leaves, 17 steps):

$$e^x+\frac{e^x}{2\left(1-e^{8x}\right)}-\frac{\text{ArcTan}[e^x]}{8}+\frac{\text{ArcTan}\left[1-\sqrt{2}\,e^x\right]}{8\sqrt{2}}-\frac{\text{ArcTan}\left[1+\sqrt{2}\,e^x\right]}{8\sqrt{2}}-\frac{\text{ArcTanh}[e^x]}{8}+\frac{\text{Log}\left[1-\sqrt{2}\,e^x+e^{2x}\right]}{16\sqrt{2}}-\frac{\text{Log}\left[1+\sqrt{2}\,e^x+e^{2x}\right]}{16\sqrt{2}}$$

Result (type 7, 73 leaves):

$$\frac{1}{16}\left(16\,e^x-\frac{8\,e^x}{-1+e^{8x}}-2\,\text{ArcTan}\left[e^x\right]+\text{Log}\left[1-e^x\right]-\text{Log}\left[1+e^x\right]+\text{RootSum}\left[1+\#1^4\,\&,\,\frac{x-\text{Log}\left[e^x-\#1\right]}{\#1^3}\,\&\right]\right)$$

Problem 183: Result is not expressed in closed-form.

$$\int\frac{e^x}{a-\text{Tanh}[2\,x]}\,dx$$

Optimal (type 3, 107 leaves, 5 steps):

$$-\frac{e^x}{1-a}+\frac{\text{ArcTan}\left[\frac{(1-a)^{1/4}\,e^x}{(1+a)^{1/4}}\right]}{(1-a)\,\sqrt{1+a}\,\left(1-a^2\right)^{1/4}}+\frac{\text{ArcTanh}\left[\frac{(1-a)^{1/4}\,e^x}{(1+a)^{1/4}}\right]}{(1-a)\,\sqrt{1+a}\,\left(1-a^2\right)^{1/4}}$$

Result (type 7, 54 leaves):

$$\frac{2\,(-1+a)\,e^x+\text{RootSum}\left[1+a-\#1^4+a\,\#1^4\,\&,\,\frac{x-\text{Log}\left[e^x-\#1\right]}{\#1^3}\,\&\right]}{2\,(-1+a)^2}$$

Problem 184: Result is not expressed in closed-form.

$$\int\frac{e^x}{(a-\text{Tanh}[2\,x])^2}\,dx$$

Optimal (type 3, 152 leaves, 7 steps):

$$\frac{e^x}{(1-a)^2}+\frac{e^x}{(1-a)^2 (1+a) \left(1+a+(-1+a) e^{4 x}\right)}-\frac{(1+4 a) \operatorname{ArcTan}\left[\frac{(1-a)^{1/4} e^x}{(1+a)^{1/4}}\right]}{2 (1-a)^2 (1+a)^{3/2} \left(1-a^2\right)^{1/4}}-\frac{(1+4 a) \operatorname{ArcTanh}\left[\frac{(1-a)^{1/4} e^x}{(1+a)^{1/4}}\right]}{2 (1-a)^2 (1+a)^{3/2} \left(1-a^2\right)^{1/4}}$$

Result (type 7, 107 leaves):

$$\left(\frac{4 (-1+a) e^x \left(2+2 a-e^{4 x}+a^2 \left(1+e^{4 x}\right)\right)}{1+a-e^{4 x}+a e^{4 x}}+(1+4 a) \operatorname{RootSum}\left[1+a-\#1^4+a \#1^4 \&, \frac{x-\operatorname{Log}\left[e^x-\#1\right]}{\#1^3} \&\right]\right) \Big/ \left(4 (-1+a)^3 (1+a)\right)$$

Problem 187: Result more than twice size of optimal antiderivative.

$$\int e^{c (a+b x)} \operatorname{Tanh}[d+e x] \, dx$$

Optimal (type 5, 67 leaves, 4 steps):

$$\frac{e^{c (a+b x)}}{b c}-\frac{2 e^{c (a+b x)} \operatorname{Hypergeometric2F1}\left[1, \frac{b c}{2 e}, 1+\frac{b c}{2 e}, -e^{2 (d+e x)}\right]}{b c}$$

Result (type 5, 141 leaves):

$$\left(e^{c (a+b x)} \left(2 b c e^{2 (d+e x)} \operatorname{Hypergeometric2F1}\left[1, 1+\frac{b c}{2 e}, 2+\frac{b c}{2 e}, -e^{2 (d+e x)}\right]-(b c+2 e) \left(1-e^{2 d}+2 e^{2 d} \operatorname{Hypergeometric2F1}\left[1, \frac{b c}{2 e}, 1+\frac{b c}{2 e}, -e^{2 (d+e x)}\right]\right)\right)\right) \Big/ \left(b c (b c+2 e) \left(1+e^{2 d}\right)\right)$$

Problem 188: Result more than twice size of optimal antiderivative.

$$\int e^{c (a+b x)} \operatorname{Coth}[d+e x] \, dx$$

Optimal (type 5, 65 leaves, 4 steps):

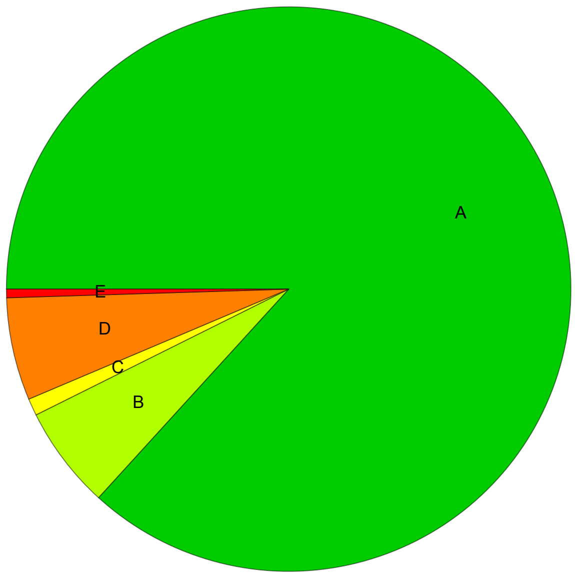
$$\frac{e^{c (a+b x)}}{b c}-\frac{2 e^{c (a+b x)} \operatorname{Hypergeometric2F1}\left[1, \frac{b c}{2 e}, 1+\frac{b c}{2 e}, e^{2 (d+e x)}\right]}{b c}$$

Result (type 5, 134 leaves):

$$\left(e^{c (a+b x)} \left(2 b c e^{2 (d+e x)} \operatorname{Hypergeometric2F1}\left[1, 1+\frac{b c}{2 e}, 2+\frac{b c}{2 e}, e^{2 (d+e x)}\right]+(b c+2 e) \left(1+e^{2 d}-2 e^{2 d} \operatorname{Hypergeometric2F1}\left[1, \frac{b c}{2 e}, 1+\frac{b c}{2 e}, e^{2 (d+e x)}\right]\right)\right)\right) \Big/ \left(b c (b c+2 e) \left(-1+e^{2 d}\right)\right)$$

Summary of Integration Test Results

204 integration problems



A - 177 optimal antiderivatives

B - 12 more than twice size of optimal antiderivatives

C - 2 unnecessarily complex antiderivatives

D - 12 unable to integrate problems

E - 1 integration timeouts