Mathematica 11.3 Integration Test Results

Test results for the 61 problems in "6.4.1 (c+d x)^m (a+b coth)^n.m"

Problem 3: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int x \operatorname{Coth}[a+b x] \, dx$$

Optimal (type 4, 45 leaves, 4 steps):

$$-\frac{x^2}{2}+\frac{x \operatorname{Log}\left[1-e^{2(a+b x)}\right]}{b}+\frac{\operatorname{PolyLog}\left[2, e^{2(a+b x)}\right]}{2 b^2}$$

Result (type 4, 148 leaves):

$$\frac{1}{2 b^2}\Big(i b \pi x+b^2 x^2 \operatorname{Coth}[a]-i \pi \operatorname{Log}\left[1+e^{2 b x}\right]+ 2 b x \operatorname{Log}\left[1-e^{-2(b x+\operatorname{ArcTanh}[\operatorname{Tanh}[a]])}\right]+i \pi \operatorname{Log}[\operatorname{Cosh}[b x]]+2 \operatorname{ArcTanh}[\operatorname{Tanh}[a]] \left(b x+\operatorname{Log}\left[1-e^{-2(b x+\operatorname{ArcTanh}[\operatorname{Tanh}[a]])}\right]-\operatorname{Log}[i \operatorname{Sinh}[b x+\operatorname{ArcTanh}[\operatorname{Tanh}[a]]]]\right)- \operatorname{PolyLog}\left[2, e^{-2(b x+\operatorname{ArcTanh}[\operatorname{Tanh}[a]])}\right]-b^2 e^{-\operatorname{ArcTanh}[\operatorname{Tanh}[a]]} x^2 \operatorname{Coth}[a] \sqrt{\operatorname{Sech}[a]^2}\Big)$$

Problem 7: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int x^2 \operatorname{Coth}[a+b x]^2 \, dx$$

Optimal (type 4, 65 leaves, 6 steps):

$$-\frac{x^2}{b}+\frac{x^3}{3}-\frac{x^2 \operatorname{Coth}[a+b x]}{b}+\frac{2 x \operatorname{Log}\left[1-e^{2(a+b x)}\right]}{b^2}+\frac{\operatorname{PolyLog}\left[2, e^{2(a+b x)}\right]}{b^3}$$

Result (type 4, 211 leaves):

$$\frac{x^3}{3}+\frac{x^2\,\text{Csch}[a]\,\text{Csch}[a+b\,x]\,\text{Sinh}[b\,x]}{b}+\Big(\text{Csch}[a]\,\text{Sech}[a]\,\Big(-b^2\,e^{-\text{ArcTanh}[\text{Tanh}[a]]}\,x^2+\big(i\,\big(-b\,x\,(-\pi+2\,i\,\text{ArcTanh}[\text{Tanh}[a]])-\pi\,\text{Log}\big[1+e^{2\,b\,x}\big]-2\,(i\,b\,x+i\,\text{ArcTanh}[\text{Tanh}[a]])\,\text{Log}\big[1-e^{2\,i\,(i\,b\,x+i\,\text{ArcTanh}[\text{Tanh}[a]])}\big]+\pi\,\text{Log}[\text{Cosh}[b\,x]]+2\,i\,\text{ArcTanh}[\text{Tanh}[a]]\,\text{Log}[i\,\text{Sinh}[b\,x+\text{ArcTanh}[\text{Tanh}[a]]]]+i\,\text{PolyLog}\big[2,\,e^{2\,i\,(i\,b\,x+i\,\text{ArcTanh}[\text{Tanh}[a]])}\big]\big)\,\text{Tanh}[a]\big)\Big/\Big(\sqrt{1-\text{Tanh}[a]^2}\Big)\Big)\Big)\Big/\Big(b^3\,\sqrt{\text{Sech}[a]^2\,(\text{Cosh}[a]^2-\text{Sinh}[a]^2)}\Big)$$

Problem 13: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int x\,\text{Coth}[a+b\,x]^3\,dx$$

Optimal (type 4, 82 leaves, 7 steps):

$$\frac{x}{2\,b}-\frac{x^2}{2}-\frac{\text{Coth}[a+b\,x]}{2\,b^2}-\frac{x\,\text{Coth}[a+b\,x]^2}{2\,b}+\frac{x\,\text{Log}\big[1-e^{2\,(a+b\,x)}\big]}{b}+\frac{\text{PolyLog}\big[2,\,e^{2\,(a+b\,x)}\big]}{2\,b^2}$$

Result (type 4, 232 leaves):

$$\frac{1}{2}\,x^2\,\text{Coth}[a]-\frac{x\,\text{Csch}[a+b\,x]^2}{2\,b}+\frac{\text{Csch}[a]\,\text{Csch}[a+b\,x]\,\text{Sinh}[b\,x]}{2\,b^2}+\Big(\text{Csch}[a]\,\text{Sech}[a]\,\Big(-b^2\,e^{-\text{ArcTanh}[\text{Tanh}[a]]}\,x^2+\big(i\,\big(-b\,x\,(-\pi+2\,i\,\text{ArcTanh}[\text{Tanh}[a]])-\pi\,\text{Log}\big[1+e^{2\,b\,x}\big]-2\,(i\,b\,x+i\,\text{ArcTanh}[\text{Tanh}[a]])\,\text{Log}\big[1-e^{2\,i\,(i\,b\,x+i\,\text{ArcTanh}[\text{Tanh}[a]])}\big]+\pi\,\text{Log}[\text{Cosh}[b\,x]]+2\,i\,\text{ArcTanh}[\text{Tanh}[a]]\,\text{Log}[i\,\text{Sinh}[b\,x+\text{ArcTanh}[\text{Tanh}[a]]]]+i\,\text{PolyLog}\big[2,\,e^{2\,i\,(i\,b\,x+i\,\text{ArcTanh}[\text{Tanh}[a]])}\big]\big)\,\text{Tanh}[a]\big)\Big/\Big(\sqrt{1-\text{Tanh}[a]^2}\Big)\Big)\Big)\Big/\Big(2\,b^2\,\sqrt{\text{Sech}[a]^2\,(\text{Cosh}[a]^2-\text{Sinh}[a]^2)}\Big)$$

Problem 34: Result more than twice size of optimal antiderivative.

$$\int\frac{(c+d\,x)^m}{a+a\,\text{Coth}[e+f\,x]}\,dx$$

Optimal (type 4, 88 leaves, 2 steps):

$$\frac{(c+d\,x)^{1+m}}{2\,a\,d\,(1+m)}+\frac{2^{-2-m}\,e^{-2\,e+\frac{2\,c\,f}{d}}\,(c+d\,x)^m\,\left(\frac{f\,(c+d\,x)}{d}\right)^{-m}\,\text{Gamma}\big[1+m,\,\frac{2\,f\,(c+d\,x)}{d}\big]}{a\,f}$$

Result (type 4, 186 leaves):

$$\left(2^{-2-m} (c+d x)^m \left(-\frac{f (c+d x)}{d}\right)^m \left(-\frac{f^2 (c+d x)^2}{d^2}\right)^{-m} \text{Csch}[e+f x] \left(d (1+m) \text{Gamma}\left[1+m, \frac{2 f (c+d x)}{d}\right] \left(\text{Cosh}\left[e-\frac{c f}{d}\right]-\text{Sinh}\left[e-\frac{c f}{d}\right]\right)+2^{1+m} f \left(f \left(\frac{c}{d}+x\right)\right)^m (c+d x) \left(\text{Cosh}\left[e-\frac{c f}{d}\right]+\text{Sinh}\left[e-\frac{c f}{d}\right]\right)\right) \left(\text{Cosh}\left[f \left(\frac{c}{d}+x\right)\right]+\text{Sinh}\left[f \left(\frac{c}{d}+x\right)\right]\right)\right) \Big/ \left(a d f (1+m) (1+\text{Coth}[e+f x])\right)$$

Problem 35: Attempted integration timed out after 120 seconds.

$$\int \frac{(c+d x)^m}{(a+a \text{Coth}[e+f x])^2} \, dx$$

Optimal (type 4, 152 leaves, 4 steps):

$$\frac{(c+d x)^{1+m}}{4 a^2 d (1+m)}+\frac{2^{-2-m} e^{-2 e+\frac{2 c f}{d}} (c+d x)^m \left(\frac{f (c+d x)}{d}\right)^{-m} \text{Gamma}\left[1+m, \frac{2 f (c+d x)}{d}\right]}{a^2 f}-\frac{4^{-2-m} e^{-4 e+\frac{4 c f}{d}} (c+d x)^m \left(\frac{f (c+d x)}{d}\right)^{-m} \text{Gamma}\left[1+m, \frac{4 f (c+d x)}{d}\right]}{a^2 f}$$

Result (type 1, 1 leaves):

???

Problem 36: Attempted integration timed out after 120 seconds.

$$\int \frac{(c+d x)^m}{(a+a \text{Coth}[e+f x])^3} \, dx$$

Optimal (type 4, 223 leaves, 5 steps):

$$\frac{(c+d x)^{1+m}}{8 a^3 d (1+m)}+\frac{3\times 2^{-4-m} e^{-2 e+\frac{2 c f}{d}} (c+d x)^m \left(\frac{f (c+d x)}{d}\right)^{-m} \text{Gamma}\left[1+m, \frac{2 f (c+d x)}{d}\right]}{a^3 f}-\frac{3\times 2^{-5-2 m} e^{-4 e+\frac{4 c f}{d}} (c+d x)^m \left(\frac{f (c+d x)}{d}\right)^{-m} \text{Gamma}\left[1+m, \frac{4 f (c+d x)}{d}\right]}{a^3 f}+\frac{2^{-4-m}\times 3^{-1-m} e^{-6 e+\frac{6 c f}{d}} (c+d x)^m \left(\frac{f (c+d x)}{d}\right)^{-m} \text{Gamma}\left[1+m, \frac{6 f (c+d x)}{d}\right]}{a^3 f}$$

Result (type 1, 1 leaves):

???

Problem 39: Result unnecessarily involves complex numbers and more than

twice size of optimal antiderivative.

$$\int (c+d x) (a+b \operatorname{Coth}[e+f x]) \, dx$$

Optimal (type 4, 75 leaves, 6 steps):

$$\frac{a (c+d x)^2}{2 d}-\frac{b (c+d x)^2}{2 d}+\frac{b (c+d x) \operatorname{Log}\left[1-e^{2 (e+f x)}\right]}{f}+\frac{b d \operatorname{PolyLog}\left[2, e^{2 (e+f x)}\right]}{2 f^2}$$

Result (type 4, 227 leaves):

$$a c x+\frac{1}{2} a d x^2+\frac{1}{2} b d x^2 \operatorname{Coth}[e]+\frac{b c \operatorname{Log}[\operatorname{Sinh}[e+f x]]}{f}+$$
$$\left(b d \operatorname{Csch}[e] \operatorname{Sech}[e] \left(-e^{-\operatorname{ArcTanh}[\operatorname{Tanh}[e]]} f^2 x^2+\frac{1}{\sqrt{1-\operatorname{Tanh}[e]^2}}\right.\right.$$
$$i \left(-f x (-\pi+2 i \operatorname{ArcTanh}[\operatorname{Tanh}[e]])-\pi \operatorname{Log}\left[1+e^{2 f x}\right]-2 (i f x+i \operatorname{ArcTanh}[\operatorname{Tanh}[e]])\right.$$
$$\operatorname{Log}\left[1-e^{2 i (i f x+i \operatorname{ArcTanh}[\operatorname{Tanh}[e]])}\right]+\pi \operatorname{Log}[\operatorname{Cosh}[f x]]+2 i \operatorname{ArcTanh}[\operatorname{Tanh}[e]]$$
$$\left.\operatorname{Log}[i \operatorname{Sinh}[f x+\operatorname{ArcTanh}[\operatorname{Tanh}[e]]]]+i \operatorname{PolyLog}\left[2, e^{2 i (i f x+i \operatorname{ArcTanh}[\operatorname{Tanh}[e]])}\right]\right)$$
$$\left.\left.\operatorname{Tanh}[e]\right)\right) \Big/ \left(2 f^2 \sqrt{\operatorname{Sech}[e]^2 \left(\operatorname{Cosh}[e]^2-\operatorname{Sinh}[e]^2\right)}\right)$$

Problem 42: Result more than twice size of optimal antiderivative.

$$\int (c+d x)^3 (a+b \operatorname{Coth}[e+f x])^2 \, dx$$

Optimal (type 4, 271 leaves, 15 steps):

$$-\frac{b^2 (c+d x)^3}{f}+\frac{a^2 (c+d x)^4}{4 d}-\frac{a b (c+d x)^4}{2 d}+\frac{b^2 (c+d x)^4}{4 d}-\frac{b^2 (c+d x)^3 \operatorname{Coth}[e+f x]}{f}+$$
$$\frac{3 b^2 d (c+d x)^2 \operatorname{Log}\left[1-e^{2 (e+f x)}\right]}{f^2}+\frac{2 a b (c+d x)^3 \operatorname{Log}\left[1-e^{2 (e+f x)}\right]}{f}+$$
$$\frac{3 b^2 d^2 (c+d x) \operatorname{PolyLog}\left[2, e^{2 (e+f x)}\right]}{f^3}+\frac{3 a b d (c+d x)^2 \operatorname{PolyLog}\left[2, e^{2 (e+f x)}\right]}{f^2}-$$
$$\frac{3 b^2 d^3 \operatorname{PolyLog}\left[3, e^{2 (e+f x)}\right]}{2 f^4}-\frac{3 a b d^2 (c+d x) \operatorname{PolyLog}\left[3, e^{2 (e+f x)}\right]}{f^3}+\frac{3 a b d^3 \operatorname{PolyLog}\left[4, e^{2 (e+f x)}\right]}{2 f^4}$$

Result (type 4, 1084 leaves):

$$-\frac{1}{2\left(-1+e^{2e}\right)f}b\,e^{2e}\left(12 b c^2 d x+8 a c^3 f x+12 b c d^2 x^2+12 a c^2 d f x^2+4 b d^3 x^3+8 a c d^2 f x^3+2 a d^3 f x^4-\right.$$
$$4 a c^3 \text{Log}\left[1-e^{2(e+f x)}\right]+4 a c^3 e^{-2e} \text{Log}\left[1-e^{2(e+f x)}\right]-\frac{6 b c^2 d \text{Log}\left[1-e^{2(e+f x)}\right]}{f}+$$
$$\frac{6 b c^2 d e^{-2e} \text{Log}\left[1-e^{2(e+f x)}\right]}{f}-12 a c^2 d x \text{Log}\left[1-e^{2(e+f x)}\right]+12 a c^2 d e^{-2e} x$$
$$\text{Log}\left[1-e^{2(e+f x)}\right]-\frac{12 b c d^2 x \text{Log}\left[1-e^{2(e+f x)}\right]}{f}+\frac{12 b c d^2 e^{-2e} x \text{Log}\left[1-e^{2(e+f x)}\right]}{f}-$$
$$12 a c d^2 x^2 \text{Log}\left[1-e^{2(e+f x)}\right]+12 a c d^2 e^{-2e} x^2 \text{Log}\left[1-e^{2(e+f x)}\right]-\frac{6 b d^3 x^2 \text{Log}\left[1-e^{2(e+f x)}\right]}{f}+$$
$$\frac{6 b d^3 e^{-2e} x^2 \text{Log}\left[1-e^{2(e+f x)}\right]}{f}-4 a d^3 x^3 \text{Log}\left[1-e^{2(e+f x)}\right]+4 a d^3 e^{-2e} x^3 \text{Log}\left[1-e^{2(e+f x)}\right]-$$
$$\frac{1}{f^2}6 d e^{-2e}\left(-1+e^{2e}\right)(c+d x)\left(b d+a f(c+d x)\right)\text{PolyLog}\left[2,e^{2(e+f x)}\right]+$$
$$\frac{3 d^2 e^{-2e}\left(-1+e^{2e}\right)\left(b d+2 a f(c+d x)\right)\text{PolyLog}\left[3,e^{2(e+f x)}\right]}{f^3}-$$
$$\left.\frac{3 a d^3 \text{PolyLog}\left[4,e^{2(e+f x)}\right]}{f^3}+\frac{3 a d^3 e^{-2e}\text{PolyLog}\left[4,e^{2(e+f x)}\right]}{f^3}\right)+$$
$$\frac{1}{8f}\text{Csch}[e]\,\text{Csch}[e+f x]\left(-4 a^2 c^3 f x \text{Cosh}[f x]-4 b^2 c^3 f x \text{Cosh}[f x]-6 a^2 c^2 d f x^2 \text{Cosh}[f x]-\right.$$
$$6 b^2 c^2 d f x^2 \text{Cosh}[f x]-4 a^2 c d^2 f x^3 \text{Cosh}[f x]-4 b^2 c d^2 f x^3 \text{Cosh}[f x]-$$
$$a^2 d^3 f x^4 \text{Cosh}[f x]-b^2 d^3 f x^4 \text{Cosh}[f x]+4 a^2 c^3 f x \text{Cosh}[2e+f x]+$$
$$4 b^2 c^3 f x \text{Cosh}[2e+f x]+6 a^2 c^2 d f x^2 \text{Cosh}[2e+f x]+6 b^2 c^2 d f x^2 \text{Cosh}[2e+f x]+$$
$$4 a^2 c d^2 f x^3 \text{Cosh}[2e+f x]+4 b^2 c d^2 f x^3 \text{Cosh}[2e+f x]+a^2 d^3 f x^4 \text{Cosh}[2e+f x]+$$
$$b^2 d^3 f x^4 \text{Cosh}[2e+f x]+8 b^2 c^3 \text{Sinh}[f x]+24 b^2 c^2 d x \text{Sinh}[f x]+8 a b c^3 f x \text{Sinh}[f x]+$$
$$24 b^2 c d^2 x^2 \text{Sinh}[f x]+12 a b c^2 d f x^2 \text{Sinh}[f x]+8 b^2 d^3 x^3 \text{Sinh}[f x]+$$
$$8 a b c d^2 f x^3 \text{Sinh}[f x]+2 a b d^3 f x^4 \text{Sinh}[f x]+8 a b c^3 f x \text{Sinh}[2e+f x]+$$
$$\left.12 a b c^2 d f x^2 \text{Sinh}[2e+f x]+8 a b c d^2 f x^3 \text{Sinh}[2e+f x]+2 a b d^3 f x^4 \text{Sinh}[2e+f x]\right)$$

Problem 47: Result more than twice size of optimal antiderivative.

$$\int (c+d x)^3 \left(a+b \text{Coth}[e+f x]\right)^3 dx$$

Optimal (type 4, 556 leaves, 28 steps):

$$-\frac{3 b^3 d (c+d x)^2}{2 f^2}-\frac{3 a b^2 (c+d x)^3}{f}+\frac{b^3 (c+d x)^3}{2 f}+\frac{a^3 (c+d x)^4}{4 d}-$$
$$\frac{3 a^2 b (c+d x)^4}{4 d}+\frac{3 a b^2 (c+d x)^4}{4 d}-\frac{b^3 (c+d x)^4}{4 d}-\frac{3 b^3 d (c+d x)^2 \text{Coth}[e+f x]}{2 f^2}-$$
$$\frac{3 a b^2 (c+d x)^3 \text{Coth}[e+f x]}{f}-\frac{b^3 (c+d x)^3 \text{Coth}[e+f x]^2}{2 f}+\frac{3 b^3 d^2 (c+d x) \text{Log}\left[1-e^{2 (e+f x)}\right]}{f^3}+$$
$$\frac{9 a b^2 d (c+d x)^2 \text{Log}\left[1-e^{2 (e+f x)}\right]}{f^2}+\frac{3 a^2 b (c+d x)^3 \text{Log}\left[1-e^{2 (e+f x)}\right]}{f}+$$
$$\frac{b^3 (c+d x)^3 \text{Log}\left[1-e^{2 (e+f x)}\right]}{f}+\frac{3 b^3 d^3 \text{PolyLog}\left[2, e^{2 (e+f x)}\right]}{2 f^4}+$$
$$\frac{9 a b^2 d^2 (c+d x) \text{PolyLog}\left[2, e^{2 (e+f x)}\right]}{f^3}+\frac{9 a^2 b d (c+d x)^2 \text{PolyLog}\left[2, e^{2 (e+f x)}\right]}{2 f^2}+$$
$$\frac{3 b^3 d (c+d x)^2 \text{PolyLog}\left[2, e^{2 (e+f x)}\right]}{2 f^2}-\frac{9 a b^2 d^3 \text{PolyLog}\left[3, e^{2 (e+f x)}\right]}{2 f^4}-$$
$$\frac{9 a^2 b d^2 (c+d x) \text{PolyLog}\left[3, e^{2 (e+f x)}\right]}{2 f^3}-\frac{3 b^3 d^2 (c+d x) \text{PolyLog}\left[3, e^{2 (e+f x)}\right]}{2 f^3}+$$
$$\frac{9 a^2 b d^3 \text{PolyLog}\left[4, e^{2 (e+f x)}\right]}{4 f^4}+\frac{3 b^3 d^3 \text{PolyLog}\left[4, e^{2 (e+f x)}\right]}{4 f^4}$$

Result (type 4, 2043 leaves):

$$\frac{\left(-b^3 c^3-3 b^3 c^2 d x-3 b^3 c d^2 x^2-b^3 d^3 x^3\right) \text{Csch}[e+f x]^2}{2 f}-$$
$$\frac{1}{4 \left(-1+e^{2 e}\right) f^2} b\, e^{2 e} \left(24 b^2 c d^2 x+72 a b c^2 d f x+24 a^2 c^3 f^2 x+8 b^2 c^3 f^2 x+12 b^2 d^3 x^2+\right.$$
$$72 a b c d^2 f x^2+36 a^2 c^2 d f^2 x^2+12 b^2 c^2 d f^2 x^2+24 a b d^3 f x^3+24 a^2 c d^2 f^2 x^3+8 b^2 c d^2 f^2 x^3+$$
$$6 a^2 d^3 f^2 x^4+2 b^2 d^3 f^2 x^4-36 a b c^2 d \text{Log}\left[1-e^{2 (e+f x)}\right]+36 a b c^2 d e^{-2 e} \text{Log}\left[1-e^{2 (e+f x)}\right]-$$
$$\frac{12 b^2 c d^2 \text{Log}\left[1-e^{2 (e+f x)}\right]}{f}+\frac{12 b^2 c d^2 e^{-2 e} \text{Log}\left[1-e^{2 (e+f x)}\right]}{f}-12 a^2 c^3 f \text{Log}\left[1-e^{2 (e+f x)}\right]-$$
$$4 b^2 c^3 f \text{Log}\left[1-e^{2 (e+f x)}\right]+12 a^2 c^3 e^{-2 e} f \text{Log}\left[1-e^{2 (e+f x)}\right]+4 b^2 c^3 e^{-2 e} f \text{Log}\left[1-e^{2 (e+f x)}\right]-$$
$$72 a b c d^2 x \text{Log}\left[1-e^{2 (e+f x)}\right]+72 a b c d^2 e^{-2 e} x \text{Log}\left[1-e^{2 (e+f x)}\right]-$$
$$\frac{12 b^2 d^3 x \text{Log}\left[1-e^{2 (e+f x)}\right]}{f}+\frac{12 b^2 d^3 e^{-2 e} x \text{Log}\left[1-e^{2 (e+f x)}\right]}{f}-$$
$$36 a^2 c^2 d f x \text{Log}\left[1-e^{2 (e+f x)}\right]-12 b^2 c^2 d f x \text{Log}\left[1-e^{2 (e+f x)}\right]+$$
$$36 a^2 c^2 d e^{-2 e} f x \text{Log}\left[1-e^{2 (e+f x)}\right]+12 b^2 c^2 d e^{-2 e} f x \text{Log}\left[1-e^{2 (e+f x)}\right]-$$
$$36 a b d^3 x^2 \text{Log}\left[1-e^{2 (e+f x)}\right]+36 a b d^3 e^{-2 e} x^2 \text{Log}\left[1-e^{2 (e+f x)}\right]-$$
$$36 a^2 c d^2 f x^2 \text{Log}\left[1-e^{2 (e+f x)}\right]-12 b^2 c d^2 f x^2 \text{Log}\left[1-e^{2 (e+f x)}\right]+$$
$$36 a^2 c d^2 e^{-2 e} f x^2 \text{Log}\left[1-e^{2 (e+f x)}\right]+12 b^2 c d^2 e^{-2 e} f x^2 \text{Log}\left[1-e^{2 (e+f x)}\right]-$$
$$12 a^2 d^3 f x^3 \text{Log}\left[1-e^{2 (e+f x)}\right]-4 b^2 d^3 f x^3 \text{Log}\left[1-e^{2 (e+f x)}\right]+$$
$$12 a^2 d^3 e^{-2 e} f x^3 \text{Log}\left[1-e^{2 (e+f x)}\right]+4 b^2 d^3 e^{-2 e} f x^3 \text{Log}\left[1-e^{2 (e+f x)}\right]-\frac{1}{f^2}$$
$$6 d e^{-2 e} \left(-1+e^{2 e}\right) \left(6 a b d f (c+d x)+3 a^2 f^2 (c+d x)^2+b^2 \left(d^2+c^2 f^2+2 c d f^2 x+d^2 f^2 x^2\right)\right)$$
$$\text{PolyLog}\left[2, e^{2 (e+f x)}\right]+\frac{1}{f^2} 6 d^2 e^{-2 e} \left(-1+e^{2 e}\right) \left(3 a b d+3 a^2 f (c+d x)+b^2 f (c+d x)\right)$$

$$\text{PolyLog}\left[3, e^{2(e+fx)}\right] - \frac{9 a^2 d^3 \text{PolyLog}\left[4, e^{2(e+fx)}\right]}{f^2} - \frac{3 b^2 d^3 \text{PolyLog}\left[4, e^{2(e+fx)}\right]}{f^2} + \frac{9 a^2 d^3 e^{-2e} \text{PolyLog}\left[4, e^{2(e+fx)}\right]}{f^2} + \frac{3 b^2 d^3 e^{-2e} \text{PolyLog}\left[4, e^{2(e+fx)}\right]}{f^2}\Bigg) +$$

$$\Big(3 x^2 \big(-a^3 c^2 d + 3 a^2 b c^2 d - 3 a b^2 c^2 d + b^3 c^2 d + a^3 c^2 d \text{Cosh}[2e] + 3 a^2 b c^2 d \text{Cosh}[2e] + 3 a b^2 c^2 d \text{Cosh}[2e] + b^3 c^2 d \text{Cosh}[2e] + a^3 c^2 d \text{Sinh}[2e] + 3 a^2 b c^2 d \text{Sinh}[2e] + 3 a b^2 c^2 d \text{Sinh}[2e] + b^3 c^2 d \text{Sinh}[2e]\big)\Big) \Big/ \big(2 (-1 + \text{Cosh}[2e] + \text{Sinh}[2e])\big) +$$

$$\Big(x^3 \big(-a^3 c d^2 + 3 a^2 b c d^2 - 3 a b^2 c d^2 + b^3 c d^2 + a^3 c d^2 \text{Cosh}[2e] + 3 a^2 b c d^2 \text{Cosh}[2e] + 3 a b^2 c d^2 \text{Cosh}[2e] + b^3 c d^2 \text{Cosh}[2e] + a^3 c d^2 \text{Sinh}[2e] + 3 a^2 b c d^2 \text{Sinh}[2e] + 3 a b^2 c d^2 \text{Sinh}[2e] + b^3 c d^2 \text{Sinh}[2e]\big)\Big) \Big/ \big(-1 + \text{Cosh}[2e] + \text{Sinh}[2e]\big) +$$

$$\Big(x^4 \big(-a^3 d^3 + 3 a^2 b d^3 - 3 a b^2 d^3 + b^3 d^3 + a^3 d^3 \text{Cosh}[2e] + 3 a^2 b d^3 \text{Cosh}[2e] + 3 a b^2 d^3 \text{Cosh}[2e] + b^3 d^3 \text{Cosh}[2e] + a^3 d^3 \text{Sinh}[2e] + 3 a^2 b d^3 \text{Sinh}[2e] + 3 a b^2 d^3 \text{Sinh}[2e] + b^3 d^3 \text{Sinh}[2e]\big)\Big) \Big/$$

$$\big(4 (-1 + \text{Cosh}[2e] + \text{Sinh}[2e])\big) + x \Bigg(a^3 c^3 + 3 a b^2 c^3 + \frac{3 a^2 b c^3}{-1 + \text{Cosh}[2e] + \text{Sinh}[2e]} + \frac{3 a^2 b c^3 \text{Cosh}[2e] + 3 a^2 b c^3 \text{Sinh}[2e]}{-1 + \text{Cosh}[2e] + \text{Sinh}[2e]} + \big(2 b^3 c^3 \text{Cosh}[2e] + 2 b^3 c^3 \text{Sinh}[2e]\big) \Big/$$

$$\Big(\big(-1 + \text{Cosh}[2e] + \text{Sinh}[2e]\big) \big(1 + \text{Cosh}[2e] + \text{Cosh}[4e] + \text{Sinh}[2e] + \text{Sinh}[4e]\big)\Big) + \big(2 b^3 c^3 \text{Cosh}[4e] + 2 b^3 c^3 \text{Sinh}[4e]\big) \Big/$$

$$\Big(\big(-1 + \text{Cosh}[2e] + \text{Sinh}[2e]\big) \big(1 + \text{Cosh}[2e] + \text{Cosh}[4e] + \text{Sinh}[2e] + \text{Sinh}[4e]\big)\Big) + \frac{b^3 c^3}{-1 + \text{Cosh}[6e] + \text{Sinh}[6e]} + \frac{b^3 c^3 \text{Cosh}[6e] + b^3 c^3 \text{Sinh}[6e]}{-1 + \text{Cosh}[6e] + \text{Sinh}[6e]}\Bigg) + \frac{1}{2 f^2}$$

$$3 \text{Csch}[e] \text{Csch}[e + f x] \big(b^3 c^2 d \text{Sinh}[f x] + 2 a b^2 c^3 f \text{Sinh}[f x] + 2 b^3 c d^2 x \text{Sinh}[f x] + 6 a b^2 c^2 d f x \text{Sinh}[f x] + b^3 d^3 x^2 \text{Sinh}[f x] + 6 a b^2 c d^2 f x^2 \text{Sinh}[f x] + 2 a b^2 d^3 f x^3 \text{Sinh}[f x]\big)$$

Problem 48: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int (c + d x)^2 (a + b \text{Coth}[e + f x])^3 \, dx$$

Optimal (type 4, 401 leaves, 22 steps):

$$\frac{b^3 c d x}{f} + \frac{b^3 d^2 x^2}{2 f} - \frac{3 a b^2 (c + d x)^2}{f} + \frac{a^3 (c + d x)^3}{3 d} - \frac{a^2 b (c + d x)^3}{d} + \frac{a b^2 (c + d x)^3}{d} - \frac{b^3 (c + d x)^3}{3 d} - \frac{b^3 d (c + d x) \text{Coth}[e + f x]}{f^2} - \frac{3 a b^2 (c + d x)^2 \text{Coth}[e + f x]}{f} - \frac{b^3 (c + d x)^2 \text{Coth}[e + f x]^2}{2 f} + \frac{6 a b^2 d (c + d x) \text{Log}\left[1 - e^{2(e+fx)}\right]}{f^2} + \frac{3 a^2 b (c + d x)^2 \text{Log}\left[1 - e^{2(e+fx)}\right]}{f} + \frac{b^3 (c + d x)^2 \text{Log}\left[1 - e^{2(e+fx)}\right]}{f} + \frac{b^3 d^2 \text{Log}[\text{Sinh}[e + f x]]}{f^3} + \frac{3 a b^2 d^2 \text{PolyLog}\left[2, e^{2(e+fx)}\right]}{f^3} + \frac{3 a^2 b d (c + d x) \text{PolyLog}\left[2, e^{2(e+fx)}\right]}{f^2} + \frac{b^3 d (c + d x) \text{PolyLog}\left[2, e^{2(e+fx)}\right]}{f^2} - \frac{3 a^2 b d^2 \text{PolyLog}\left[3, e^{2(e+fx)}\right]}{2 f^3} - \frac{b^3 d^2 \text{PolyLog}\left[3, e^{2(e+fx)}\right]}{2 f^3}$$

Result (type 4, 1887 leaves) :

$$-\frac{1}{4 f^3}a^2 b d^2 e^{-e} \text{Csch}[e] \left(2 f^2 x^2 \left(2 e^{2 e} f x-3 \left(-1+e^{2 e}\right) \text{Log}\left[1-e^{2 (e+f x)}\right]\right)-\right.$$
$$\left.6 \left(-1+e^{2 e}\right) f x \text{PolyLog}\left[2, e^{2 (e+f x)}\right]+3 \left(-1+e^{2 e}\right) \text{PolyLog}\left[3, e^{2 (e+f x)}\right]\right)-$$
$$\frac{1}{12 f^3}b^3 d^2 e^{-e} \text{Csch}[e] \left(2 f^2 x^2 \left(2 e^{2 e} f x-3 \left(-1+e^{2 e}\right) \text{Log}\left[1-e^{2 (e+f x)}\right]\right)-\right.$$
$$\left.6 \left(-1+e^{2 e}\right) f x \text{PolyLog}\left[2, e^{2 (e+f x)}\right]+3 \left(-1+e^{2 e}\right) \text{PolyLog}\left[3, e^{2 (e+f x)}\right]\right)-$$
$$\left(b^3 d^2 \text{Csch}[e] \left(-f x \text{Cosh}[e]+\text{Log}[\text{Cosh}[f x] \text{Sinh}[e]+\text{Cosh}[e] \text{Sinh}[f x]] \text{Sinh}[e]\right)\right) /$$
$$\left(f^3 \left(-\text{Cosh}[e]^2+\text{Sinh}[e]^2\right)\right)-$$
$$\left(6 a b^2 c d \text{Csch}[e] \left(-f x \text{Cosh}[e]+\text{Log}[\text{Cosh}[f x] \text{Sinh}[e]+\text{Cosh}[e] \text{Sinh}[f x]] \text{Sinh}[e]\right)\right) /$$
$$\left(f^2 \left(-\text{Cosh}[e]^2+\text{Sinh}[e]^2\right)\right)-$$
$$\left(3 a^2 b c^2 \text{Csch}[e] \left(-f x \text{Cosh}[e]+\text{Log}[\text{Cosh}[f x] \text{Sinh}[e]+\text{Cosh}[e] \text{Sinh}[f x]] \text{Sinh}[e]\right)\right) /$$
$$\left(f \left(-\text{Cosh}[e]^2+\text{Sinh}[e]^2\right)\right)-$$
$$\left(b^3 c^2 \text{Csch}[e] \left(-f x \text{Cosh}[e]+\text{Log}[\text{Cosh}[f x] \text{Sinh}[e]+\text{Cosh}[e] \text{Sinh}[f x]] \text{Sinh}[e]\right)\right) /$$
$$\left(f \left(-\text{Cosh}[e]^2+\text{Sinh}[e]^2\right)\right)+$$
$$\frac{1}{12 f^2}\text{Csch}[e] \text{Csch}[e+f x]^2 \left(-6 b^3 c d \text{Cosh}[e]-18 a b^2 c^2 f \text{Cosh}[e]-6 b^3 d^2 x \text{Cosh}[e]-\right.$$
$$36 a b^2 c d f x \text{Cosh}[e]-18 a^2 b c^2 f^2 x \text{Cosh}[e]-6 b^3 c^2 f^2 x \text{Cosh}[e]-18 a b^2 d^2 f x^2 \text{Cosh}[e]-$$
$$18 a^2 b c d f^2 x^2 \text{Cosh}[e]-6 b^3 c d f^2 x^2 \text{Cosh}[e]-6 a^2 b d^2 f^2 x^3 \text{Cosh}[e]-2 b^3 d^2 f^2 x^3 \text{Cosh}[e]+$$
$$6 b^3 c d \text{Cosh}[e+2 f x]+18 a b^2 c^2 f \text{Cosh}[e+2 f x]+6 b^3 d^2 x \text{Cosh}[e+2 f x]+$$
$$36 a b^2 c d f x \text{Cosh}[e+2 f x]+9 a^2 b c^2 f^2 x \text{Cosh}[e+2 f x]+3 b^3 c^2 f^2 x \text{Cosh}[e+2 f x]+$$
$$18 a b^2 d^2 f x^2 \text{Cosh}[e+2 f x]+9 a^2 b c d f^2 x^2 \text{Cosh}[e+2 f x]+3 b^3 c d f^2 x^2 \text{Cosh}[e+2 f x]+$$
$$3 a^2 b d^2 f^2 x^3 \text{Cosh}[e+2 f x]+b^3 d^2 f^2 x^3 \text{Cosh}[e+2 f x]+9 a^2 b c^2 f^2 x \text{Cosh}[3 e+2 f x]+$$
$$3 b^3 c^2 f^2 x \text{Cosh}[3 e+2 f x]+9 a^2 b c d f^2 x^2 \text{Cosh}[3 e+2 f x]+3 b^3 c d f^2 x^2 \text{Cosh}[3 e+2 f x]+$$
$$3 a^2 b d^2 f^2 x^3 \text{Cosh}[3 e+2 f x]+b^3 d^2 f^2 x^3 \text{Cosh}[3 e+2 f x]-6 b^3 c^2 f \text{Sinh}[e]-$$
$$12 b^3 c d f x \text{Sinh}[e]-6 a^3 c^2 f^2 x \text{Sinh}[e]-18 a b^2 c^2 f^2 x \text{Sinh}[e]-6 b^3 d^2 f x^2 \text{Sinh}[e]-$$
$$6 a^3 c d f^2 x^2 \text{Sinh}[e]-18 a b^2 c d f^2 x^2 \text{Sinh}[e]-2 a^3 d^2 f^2 x^3 \text{Sinh}[e]-6 a b^2 d^2 f^2 x^3 \text{Sinh}[e]-$$
$$3 a^3 c^2 f^2 x \text{Sinh}[e+2 f x]-9 a b^2 c^2 f^2 x \text{Sinh}[e+2 f x]-3 a^3 c d f^2 x^2 \text{Sinh}[e+2 f x]-$$
$$9 a b^2 c d f^2 x^2 \text{Sinh}[e+2 f x]-a^3 d^2 f^2 x^3 \text{Sinh}[e+2 f x]-3 a b^2 d^2 f^2 x^3 \text{Sinh}[e+2 f x]+$$
$$3 a^3 c^2 f^2 x \text{Sinh}[3 e+2 f x]+9 a b^2 c^2 f^2 x \text{Sinh}[3 e+2 f x]+3 a^3 c d f^2 x^2 \text{Sinh}[3 e+2 f x]+$$
$$\left.9 a b^2 c d f^2 x^2 \text{Sinh}[3 e+2 f x]+a^3 d^2 f^2 x^3 \text{Sinh}[3 e+2 f x]+3 a b^2 d^2 f^2 x^3 \text{Sinh}[3 e+2 f x]\right)+$$
$$\left(3 a b^2 d^2 \text{Csch}[e] \text{Sech}[e] \left(-e^{-\text{ArcTanh}[\text{Tanh}[e]]} f^2 x^2+\frac{1}{\sqrt{1-\text{Tanh}[e]^2}}\right.\right.$$
$$i \left(-f x \left(-\pi+2 i \text{ArcTanh}[\text{Tanh}[e]]\right)-\pi \text{Log}\left[1+e^{2 f x}\right]-2 \left(i f x+i \text{ArcTanh}[\text{Tanh}[e]]\right)\right.$$
$$\text{Log}\left[1-e^{2 i (i f x+i \text{ArcTanh}[\text{Tanh}[e]])}\right]+\pi \text{Log}[\text{Cosh}[f x]]+2 i \text{ArcTanh}[\text{Tanh}[e]] \text{Log}[i \text{Sinh}[$$
$$\left.\left.\left.f x+\text{ArcTanh}[\text{Tanh}[e]]]]+i \text{PolyLog}\left[2, e^{2 i (i f x+i \text{ArcTanh}[\text{Tanh}[e]])}\right]\right) \text{Tanh}[e]\right)\right) /$$
$$\left(f^3 \sqrt{\text{Sech}[e]^2 \left(\text{Cosh}[e]^2-\text{Sinh}[e]^2\right)}\right)+\left(3 a^2 b c d \text{Csch}[e] \text{Sech}[e]\right.$$
$$\left(-e^{-\text{ArcTanh}[\text{Tanh}[e]]} f^2 x^2+\frac{1}{\sqrt{1-\text{Tanh}[e]^2}} i \left(-f x \left(-\pi+2 i \text{ArcTanh}[\text{Tanh}[e]]\right)-\right.\right.$$
$$\pi \text{Log}\left[1+e^{2 f x}\right]-2 \left(i f x+i \text{ArcTanh}[\text{Tanh}[e]]\right) \text{Log}\left[1-e^{2 i (i f x+i \text{ArcTanh}[\text{Tanh}[e]])}\right]+$$
$$\pi \text{Log}[\text{Cosh}[f x]]+2 i \text{ArcTanh}[\text{Tanh}[e]] \text{Log}[i \text{Sinh}[f x+\text{ArcTanh}[\text{Tanh}[e]]]]+$$

$$\left.\left.\left.i\,\text{PolyLog}\left[2,\, e^{2 i (i f x+i \text{ArcTanh}[\text{Tanh}[e]])}\right]\right)\text{Tanh}[e]\right)\right)\Big/$$

$$\left(f^2 \sqrt{\text{Sech}[e]^2 \left(\text{Cosh}[e]^2-\text{Sinh}[e]^2\right)}\right)+\left(b^3\, c\, d\, \text{Csch}[e]\, \text{Sech}[e]\right.$$

$$\left(-e^{-\text{ArcTanh}[\text{Tanh}[e]]} f^2 x^2+\frac{1}{\sqrt{1-\text{Tanh}[e]^2}}\right.$$

$$i\left(-f x\left(-\pi+2 i\,\text{ArcTanh}[\text{Tanh}[e]]\right)-\pi\,\text{Log}\left[1+e^{2 f x}\right]-2\left(i f x+i\,\text{ArcTanh}[\text{Tanh}[e]]\right)\right.$$
$$\text{Log}\left[1-e^{2 i (i f x+i \text{ArcTanh}[\text{Tanh}[e]])}\right]+\pi\,\text{Log}[\text{Cosh}[f x]]+2 i\,\text{ArcTanh}[\text{Tanh}[e]]$$
$$\left.\text{Log}[i\,\text{Sinh}[f x+\text{ArcTanh}[\text{Tanh}[e]]]]+i\,\text{PolyLog}\left[2,\, e^{2 i (i f x+i \text{ArcTanh}[\text{Tanh}[e]])}\right]\right)$$

$$\left.\left.\text{Tanh}[e]\right)\right)\Big/\left(f^2 \sqrt{\text{Sech}[e]^2 \left(\text{Cosh}[e]^2-\text{Sinh}[e]^2\right)}\right)$$

Problem 57: Result more than twice size of optimal antiderivative.

$$\int \frac{(c+d x)^3}{(a+b\,\text{Coth}[e+f x])^2}\, dx$$

Optimal (type 4, 638 leaves, 28 steps):

$$-\frac{2 b^2 (c+d x)^3}{\left(a^2-b^2\right)^2 f}+\frac{2 b^2 (c+d x)^3}{(a-b)(a+b)^2\left(a-b-(a+b)\, e^{2 e+2 f x}\right) f}+$$
$$\frac{(c+d x)^4}{4 (a-b)^2 d}+\frac{3 b^2 d (c+d x)^2 \text{Log}\left[1-\frac{(a+b)\, e^{2 e+2 f x}}{a-b}\right]}{\left(a^2-b^2\right)^2 f^2}-$$
$$\frac{2 b (c+d x)^3 \text{Log}\left[1-\frac{(a+b)\, e^{2 e+2 f x}}{a-b}\right]}{(a-b)^2 (a+b) f}+\frac{2 b^2 (c+d x)^3 \text{Log}\left[1-\frac{(a+b)\, e^{2 e+2 f x}}{a-b}\right]}{\left(a^2-b^2\right)^2 f}+$$
$$\frac{3 b^2 d^2 (c+d x)\,\text{PolyLog}\left[2,\frac{(a+b)\, e^{2 e+2 f x}}{a-b}\right]}{\left(a^2-b^2\right)^2 f^3}-\frac{3 b d (c+d x)^2\,\text{PolyLog}\left[2,\frac{(a+b)\, e^{2 e+2 f x}}{a-b}\right]}{(a-b)^2 (a+b) f^2}+$$
$$\frac{3 b^2 d (c+d x)^2\,\text{PolyLog}\left[2,\frac{(a+b)\, e^{2 e+2 f x}}{a-b}\right]}{\left(a^2-b^2\right)^2 f^2}-\frac{3 b^2 d^3\,\text{PolyLog}\left[3,\frac{(a+b)\, e^{2 e+2 f x}}{a-b}\right]}{2\left(a^2-b^2\right)^2 f^4}+$$
$$\frac{3 b d^2 (c+d x)\,\text{PolyLog}\left[3,\frac{(a+b)\, e^{2 e+2 f x}}{a-b}\right]}{(a-b)^2 (a+b) f^3}-\frac{3 b^2 d^2 (c+d x)\,\text{PolyLog}\left[3,\frac{(a+b)\, e^{2 e+2 f x}}{a-b}\right]}{\left(a^2-b^2\right)^2 f^3}-$$
$$\frac{3 b d^3\,\text{PolyLog}\left[4,\frac{(a+b)\, e^{2 e+2 f x}}{a-b}\right]}{2 (a-b)^2 (a+b) f^4}+\frac{3 b^2 d^3\,\text{PolyLog}\left[4,\frac{(a+b)\, e^{2 e+2 f x}}{a-b}\right]}{2\left(a^2-b^2\right)^2 f^4}$$

Result (type 4, 2115 leaves):

$$-\frac{1}{2\,(a-b)^2\,(a+b)^2\,\left(a\,\left(-1+e^{2\,e}\right)+b\,\left(1+e^{2\,e}\right)\right)\,f^4}$$

$$b\,\Big(12\,a\,b\,c^2\,d\,e^{2\,e}\,f^3\,x+12\,b^2\,c^2\,d\,e^{2\,e}\,f^3\,x-8\,a^2\,c^3\,e^{2\,e}\,f^4\,x-8\,a\,b\,c^3\,e^{2\,e}\,f^4\,x+12\,a\,b\,c\,d^2\,e^{2\,e}\,f^3\,x^2+$$

$$12\,b^2\,c\,d^2\,e^{2\,e}\,f^3\,x^2-12\,a^2\,c^2\,d\,e^{2\,e}\,f^4\,x^2-12\,a\,b\,c^2\,d\,e^{2\,e}\,f^4\,x^2+4\,a\,b\,d^3\,e^{2\,e}\,f^3\,x^3+$$

$$4\,b^2\,d^3\,e^{2\,e}\,f^3\,x^3-8\,a^2\,c\,d^2\,e^{2\,e}\,f^4\,x^3-8\,a\,b\,c\,d^2\,e^{2\,e}\,f^4\,x^3-2\,a^2\,d^3\,e^{2\,e}\,f^4\,x^4-2\,a\,b\,d^3\,e^{2\,e}\,f^4\,x^4+$$

$$12\,a\,b\,c\,d^2\,f^2\,x\,\text{Log}\Big[1+\frac{(a+b)\,e^{2\,(e+f\,x)}}{-a+b}\Big]-12\,b^2\,c\,d^2\,f^2\,x\,\text{Log}\Big[1+\frac{(a+b)\,e^{2\,(e+f\,x)}}{-a+b}\Big]-$$

$$12\,a\,b\,c\,d^2\,e^{2\,e}\,f^2\,x\,\text{Log}\Big[1+\frac{(a+b)\,e^{2\,(e+f\,x)}}{-a+b}\Big]-12\,b^2\,c\,d^2\,e^{2\,e}\,f^2\,x\,\text{Log}\Big[1+\frac{(a+b)\,e^{2\,(e+f\,x)}}{-a+b}\Big]-$$

$$12\,a^2\,c^2\,d\,f^3\,x\,\text{Log}\Big[1+\frac{(a+b)\,e^{2\,(e+f\,x)}}{-a+b}\Big]+12\,a\,b\,c^2\,d\,f^3\,x\,\text{Log}\Big[1+\frac{(a+b)\,e^{2\,(e+f\,x)}}{-a+b}\Big]+$$

$$12\,a^2\,c^2\,d\,e^{2\,e}\,f^3\,x\,\text{Log}\Big[1+\frac{(a+b)\,e^{2\,(e+f\,x)}}{-a+b}\Big]+12\,a\,b\,c^2\,d\,e^{2\,e}\,f^3\,x\,\text{Log}\Big[1+\frac{(a+b)\,e^{2\,(e+f\,x)}}{-a+b}\Big]+$$

$$6\,a\,b\,d^3\,f^2\,x^2\,\text{Log}\Big[1+\frac{(a+b)\,e^{2\,(e+f\,x)}}{-a+b}\Big]-6\,b^2\,d^3\,f^2\,x^2\,\text{Log}\Big[1+\frac{(a+b)\,e^{2\,(e+f\,x)}}{-a+b}\Big]-$$

$$6\,a\,b\,d^3\,e^{2\,e}\,f^2\,x^2\,\text{Log}\Big[1+\frac{(a+b)\,e^{2\,(e+f\,x)}}{-a+b}\Big]-6\,b^2\,d^3\,e^{2\,e}\,f^2\,x^2\,\text{Log}\Big[1+\frac{(a+b)\,e^{2\,(e+f\,x)}}{-a+b}\Big]-$$

$$12\,a^2\,c\,d^2\,f^3\,x^2\,\text{Log}\Big[1+\frac{(a+b)\,e^{2\,(e+f\,x)}}{-a+b}\Big]+12\,a\,b\,c\,d^2\,f^3\,x^2\,\text{Log}\Big[1+\frac{(a+b)\,e^{2\,(e+f\,x)}}{-a+b}\Big]+$$

$$12\,a^2\,c\,d^2\,e^{2\,e}\,f^3\,x^2\,\text{Log}\Big[1+\frac{(a+b)\,e^{2\,(e+f\,x)}}{-a+b}\Big]+12\,a\,b\,c\,d^2\,e^{2\,e}\,f^3\,x^2\,\text{Log}\Big[1+\frac{(a+b)\,e^{2\,(e+f\,x)}}{-a+b}\Big]-$$

$$4\,a^2\,d^3\,f^3\,x^3\,\text{Log}\Big[1+\frac{(a+b)\,e^{2\,(e+f\,x)}}{-a+b}\Big]+4\,a\,b\,d^3\,f^3\,x^3\,\text{Log}\Big[1+\frac{(a+b)\,e^{2\,(e+f\,x)}}{-a+b}\Big]+$$

$$4\,a^2\,d^3\,e^{2\,e}\,f^3\,x^3\,\text{Log}\Big[1+\frac{(a+b)\,e^{2\,(e+f\,x)}}{-a+b}\Big]+4\,a\,b\,d^3\,e^{2\,e}\,f^3\,x^3\,\text{Log}\Big[1+\frac{(a+b)\,e^{2\,(e+f\,x)}}{-a+b}\Big]+$$

$$6\,a\,b\,c^2\,d\,f^2\,\text{Log}\Big[a\,\left(-1+e^{2\,(e+f\,x)}\right)+b\,\left(1+e^{2\,(e+f\,x)}\right)\Big]-$$

$$6\,b^2\,c^2\,d\,f^2\,\text{Log}\Big[a\,\left(-1+e^{2\,(e+f\,x)}\right)+b\,\left(1+e^{2\,(e+f\,x)}\right)\Big]-$$

$$6\,a\,b\,c^2\,d\,e^{2\,e}\,f^2\,\text{Log}\Big[a\,\left(-1+e^{2\,(e+f\,x)}\right)+b\,\left(1+e^{2\,(e+f\,x)}\right)\Big]-6\,b^2\,c^2\,d\,e^{2\,e}\,f^2$$

$$\text{Log}\Big[a\,\left(-1+e^{2\,(e+f\,x)}\right)+b\,\left(1+e^{2\,(e+f\,x)}\right)\Big]-4\,a^2\,c^3\,f^3\,\text{Log}\Big[a\,\left(-1+e^{2\,(e+f\,x)}\right)+b\,\left(1+e^{2\,(e+f\,x)}\right)\Big]+$$

$$4\,a\,b\,c^3\,f^3\,\text{Log}\Big[a\,\left(-1+e^{2\,(e+f\,x)}\right)+b\,\left(1+e^{2\,(e+f\,x)}\right)\Big]+$$

$$4\,a^2\,c^3\,e^{2\,e}\,f^3\,\text{Log}\Big[a\,\left(-1+e^{2\,(e+f\,x)}\right)+b\,\left(1+e^{2\,(e+f\,x)}\right)\Big]+$$

$$4\,a\,b\,c^3\,e^{2\,e}\,f^3\,\text{Log}\Big[a\,\left(-1+e^{2\,(e+f\,x)}\right)+b\,\left(1+e^{2\,(e+f\,x)}\right)\Big]+$$

$$6\,d\,\left(a\,\left(-1+e^{2\,e}\right)+b\,\left(1+e^{2\,e}\right)\right)\,f\,(c+d\,x)\,\left(-b\,d+a\,f\,(c+d\,x)\right)\,\text{PolyLog}\Big[2,\,\frac{(a+b)\,e^{2\,(e+f\,x)}}{a-b}\Big]-$$

$$3\,d^2\,\left(a\,\left(-1+e^{2\,e}\right)+b\,\left(1+e^{2\,e}\right)\right)\,\left(-b\,d+2\,a\,f\,(c+d\,x)\right)\,\text{PolyLog}\Big[3,\,\frac{(a+b)\,e^{2\,(e+f\,x)}}{a-b}\Big]-$$

$$3\,a^2\,d^3\,\text{PolyLog}\Big[4,\,\frac{(a+b)\,e^{2\,(e+f\,x)}}{a-b}\Big]+3\,a\,b\,d^3\,\text{PolyLog}\Big[4,\,\frac{(a+b)\,e^{2\,(e+f\,x)}}{a-b}\Big]+$$

$$3\,a^2\,d^3\,e^{2\,e}\,\text{PolyLog}\Big[4,\,\frac{(a+b)\,e^{2\,(e+f\,x)}}{a-b}\Big]+3\,a\,b\,d^3\,e^{2\,e}\,\text{PolyLog}\Big[4,\,\frac{(a+b)\,e^{2\,(e+f\,x)}}{a-b}\Big]\Big)+$$

$$\Big(-4\,a^2\,c^3\,f\,x\,\text{Cosh}[f\,x]-4\,b^2\,c^3\,f\,x\,\text{Cosh}[f\,x]-6\,a^2\,c^2\,d\,f\,x^2\,\text{Cosh}[f\,x]-$$

$$6\,b^2\,c^2\,d\,f\,x^2\,\text{Cosh}[f\,x]-4\,a^2\,c\,d^2\,f\,x^3\,\text{Cosh}[f\,x]-$$

$$4\,b^2\,c\,d^2\,f\,x^3\,\text{Cosh}[f\,x]-a^2\,d^3\,f\,x^4\,\text{Cosh}[f\,x]-b^2\,d^3\,f\,x^4\,\text{Cosh}[f\,x]+$$

$4 a^2 c^3 f x \text{Cosh}[2 e + f x] - 4 b^2 c^3 f x \text{Cosh}[2 e + f x] +$
$6 a^2 c^2 d f x^2 \text{Cosh}[2 e + f x] - 6 b^2 c^2 d f x^2 \text{Cosh}[2 e + f x] +$
$4 a^2 c d^2 f x^3 \text{Cosh}[2 e + f x] - 4 b^2 c d^2 f x^3 \text{Cosh}[2 e + f x] +$
$a^2 d^3 f x^4 \text{Cosh}[2 e + f x] - b^2 d^3 f x^4 \text{Cosh}[2 e + f x] +$
$8 b^2 c^3 \text{Sinh}[f x] + 24 b^2 c^2 d x \text{Sinh}[f x] - 8 a b c^3 f x \text{Sinh}[f x] +$
$24 b^2 c d^2 x^2 \text{Sinh}[f x] - 12 a b c^2 d f x^2 \text{Sinh}[f x] +$
$8 b^2 d^3 x^3 \text{Sinh}[f x] - 8 a b c d^2 f x^3 \text{Sinh}[f x] - 2 a b d^3 f x^4 \text{Sinh}[f x]\Big) \Big/$
$\Big(8 (a - b) (a + b) f (b \text{Cosh}[e] + a \text{Sinh}[e]) (b \text{Cosh}[e + f x] + a \text{Sinh}[e + f x])\Big)$

Problem 59: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{c + d x}{(a + b \text{Coth}[e + f x])^2} \, dx$$

Optimal (type 4, 196 leaves, 5 steps):

$$-\frac{(c + d x)^2}{2 (a^2 - b^2) d} + \frac{(b d - 2 a c f - 2 a d f x)^2}{4 a (a - b) (a + b)^2 d f^2} + \frac{b (c + d x)}{(a^2 - b^2) f (a + b \text{Coth}[e + f x])} +$$
$$\frac{b (b d - 2 a c f - 2 a d f x) \text{Log}\left[1 - \frac{(a-b) e^{-2 (e+f x)}}{a+b}\right]}{(a^2 - b^2)^2 f^2} + \frac{a b d \text{PolyLog}\left[2, \frac{(a-b) e^{-2 (e+f x)}}{a+b}\right]}{(a^2 - b^2)^2 f^2}$$

Result (type 4, 737 leaves):

$$\left((e+f x)\left(-2 d e+2 c f+d (e+f x)\right) \text{Csch}[e+f x]^2 \left(b \text{Cosh}[e+f x]+a \text{Sinh}[e+f x]\right)^2\right) \Big/ \left(2 (-a+b) (a+b) f^2 \left(a+b \text{Coth}[e+f x]\right)^2\right)+$$
$$\left(b d \text{Csch}[e+f x]^2 \left(-a (e+f x)+b \text{Log}[b \text{Cosh}[e+f x]+a \text{Sinh}[e+f x]]\right) \left(b \text{Cosh}[e+f x]+a \text{Sinh}[e+f x]\right)^2\right) \Big/ \left((-a+b)(a+b)(-a^2+b^2) f^2 \left(a+b \text{Coth}[e+f x]\right)^2\right)+$$
$$\left(2 a d e \text{Csch}[e+f x]^2 \left(-a (e+f x)+b \text{Log}[b \text{Cosh}[e+f x]+a \text{Sinh}[e+f x]]\right) \left(b \text{Cosh}[e+f x]+a \text{Sinh}[e+f x]\right)^2\right) \Big/ \left((-a+b)(a+b)(-a^2+b^2) f^2 \left(a+b \text{Coth}[e+f x]\right)^2\right)-$$
$$\left(2 a c \text{Csch}[e+f x]^2 \left(-a (e+f x)+b \text{Log}[b \text{Cosh}[e+f x]+a \text{Sinh}[e+f x]]\right) \left(b \text{Cosh}[e+f x]+a \text{Sinh}[e+f x]\right)^2\right) \Big/ \left((-a+b)(a+b)(-a^2+b^2) f \left(a+b \text{Coth}[e+f x]\right)^2\right)+$$
$$\left(d \text{Csch}[e+f x]^2 \left(-e^{-\text{ArcTanh}\left[\frac{b}{a}\right]} (e+f x)^2+\frac{1}{a \sqrt{1-\frac{b^2}{a^2}}} i b \left(-(e+f x)\left(-\pi+2 i \text{ArcTanh}\left[\frac{b}{a}\right]\right)-\pi \text{Log}\left[1+e^{2 (e+f x)}\right]-2 \left(i (e+f x)+i \text{ArcTanh}\left[\frac{b}{a}\right]\right) \text{Log}\left[1-e^{2 i \left(i (e+f x)+i \text{ArcTanh}\left[\frac{b}{a}\right]\right)}\right]+\pi \text{Log}[\text{Cosh}[e+f x]]+2 i \text{ArcTanh}\left[\frac{b}{a}\right] \text{Log}\left[i \text{Sinh}\left[e+f x+\text{ArcTanh}\left[\frac{b}{a}\right]\right]\right]+i \text{PolyLog}\left[2, e^{2 i \left(i (e+f x)+i \text{ArcTanh}\left[\frac{b}{a}\right]\right)}\right]\right)\right) \left(b \text{Cosh}[e+f x]+a \text{Sinh}[e+f x]\right)^2\right) \Big/ \left((-a+b)(a+b)\sqrt{\frac{a^2-b^2}{a^2}} f^2 \left(a+b \text{Coth}[e+f x]\right)^2\right)+$$
$$\left(\text{Csch}[e+f x]^2 \left(b \text{Cosh}[e+f x]+a \text{Sinh}[e+f x]\right) \left(b d e \text{Sinh}[e+f x]-b c f \text{Sinh}[e+f x]-b d (e+f x) \text{Sinh}[e+f x]\right)\right) \Big/ \left((-a+b)(a+b) f^2 \left(a+b \text{Coth}[e+f x]\right)^2\right)$$

Problem 60: Attempted integration timed out after 120 seconds.

$$\int \frac{1}{(c+d x)\left(a+b \text{Coth}[e+f x]\right)^2} \, dx$$

Optimal (type 8, 23 leaves, 0 steps):

$$\text{Int}\left[\frac{1}{(c+d x)\left(a+b \text{Coth}[e+f x]\right)^2}, x\right]$$

Result (type 1, 1 leaves):

???

Problem 61: Attempted integration timed out after 120 seconds.

$$\int \frac{1}{(c+d\,x)^2\,(a+b\,\mathrm{Coth}[e+f\,x])^2}\,dx$$

Optimal (type 8, 23 leaves, 0 steps):

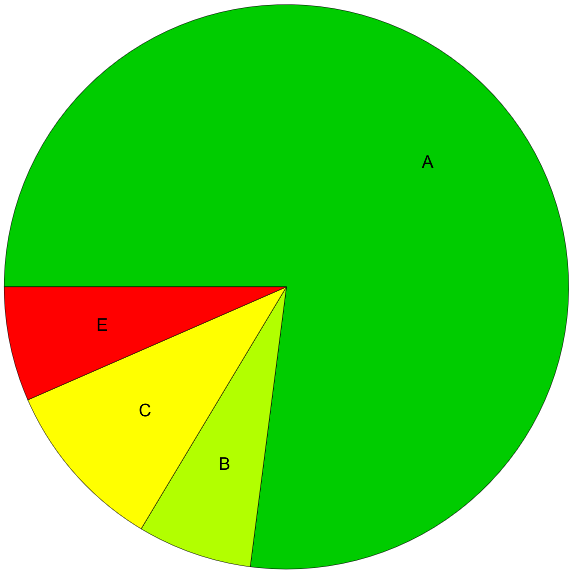
$$\mathrm{Int}\left[\frac{1}{(c+d\,x)^2\,(a+b\,\mathrm{Coth}[e+f\,x])^2},\,x\right]$$

Result (type 1, 1 leaves):

???

Summary of Integration Test Results

61 integration problems



A - 47 optimal antiderivatives

B - 4 more than twice size of optimal antiderivatives

C - 6 unnecessarily complex antiderivatives

D - 0 unable to integrate problems

E - 4 integration timeouts