Mathematica 11.3 Integration Test Results

Test results for the 181 problems in "6.4.2 Hyperbolic cotangent functions.m"

Problem 70: Result unnecessarily involves imaginary or complex numbers.

$$\int (1+\text{Coth}[x])^{7/2}\, dx$$

Optimal (type 3, 57 leaves, 5 steps):

$$8\sqrt{2}\,\text{ArcTanh}\left[\frac{\sqrt{1+\text{Coth}[x]}}{\sqrt{2}}\right]-8\sqrt{1+\text{Coth}[x]}-\frac{4}{3}(1+\text{Coth}[x])^{3/2}-\frac{2}{5}(1+\text{Coth}[x])^{5/2}$$

Result (type 3, 101 leaves):

$$-\left(\left(2\,(1+\text{Coth}[x])^{7/2}\left(4\left((-15+15\,i)\,\text{ArcTan}\left[\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i\,(1+\text{Coth}[x])}\right]+19\sqrt{i\,(1+\text{Coth}[x])}\right)\text{Sinh}[x]^3+\sqrt{i\,(1+\text{Coth}[x])}\,\text{Sinh}[x]\,(3+8\,\text{Sinh}[2\,x])\right)\right)\Big/\left(15\sqrt{i\,(1+\text{Coth}[x])}\,(\text{Cosh}[x]+\text{Sinh}[x])^3\right)\right)$$

Problem 71: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int (1+\text{Coth}[x])^{5/2}\, dx$$

Optimal (type 3, 45 leaves, 4 steps):

$$4\sqrt{2}\,\text{ArcTanh}\left[\frac{\sqrt{1+\text{Coth}[x]}}{\sqrt{2}}\right]-4\sqrt{1+\text{Coth}[x]}-\frac{2}{3}(1+\text{Coth}[x])^{3/2}$$

Result (type 3, 92 leaves):

$$-\left(\left(2\,(1+\text{Coth}[x])^{5/2}\,\text{Sinh}[x]\left(\text{Cosh}[x]\sqrt{i\,(1+\text{Coth}[x])}+\left((-6+6\,i)\,\text{ArcTan}\left[\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i\,(1+\text{Coth}[x])}\right]+7\sqrt{i\,(1+\text{Coth}[x])}\right)\text{Sinh}[x]\right)\right)\Big/\left(3\sqrt{i\,(1+\text{Coth}[x])}\,(\text{Cosh}[x]+\text{Sinh}[x])^2\right)\right)$$

Problem 72: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int (1+\text{Coth}[x])^{3/2}\, dx$$

Optimal (type 3, 33 leaves, 3 steps):

$$2\sqrt{2}\ \text{ArcTanh}\left[\frac{\sqrt{1+\text{Coth}[x]}}{\sqrt{2}}\right]-2\sqrt{1+\text{Coth}[x]}$$

Result (type 3, 69 leaves):

$$-\left(\left(2\ (1+\text{Coth}[x])^{3/2}\left((-1+i)\ \text{ArcTan}\left[\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i\ (1+\text{Coth}[x])}\right]+\sqrt{i\ (1+\text{Coth}[x])}\right)\text{Sinh}[x]\right)\Big/\left(\sqrt{i\ (1+\text{Coth}[x])}\ (\text{Cosh}[x]+\text{Sinh}[x])\right)\right)$$

Problem 73: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \sqrt{1+\text{Coth}[x]}\, dx$$

Optimal (type 3, 21 leaves, 2 steps):

$$\sqrt{2}\ \text{ArcTanh}\left[\frac{\sqrt{1+\text{Coth}[x]}}{\sqrt{2}}\right]$$

Result (type 3, 45 leaves):

$$\frac{(1+i)\ \text{ArcTan}\left[\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i\ (1+\text{Coth}[x])}\right]\ (1+\text{Coth}[x])^{3/2}}{(i\ (1+\text{Coth}[x]))^{3/2}}$$

Problem 74: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{1+\text{Coth}[x]}}\, dx$$

Optimal (type 3, 32 leaves, 3 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{1+\text{Coth}[x]}}{\sqrt{2}}\right]}{\sqrt{2}}-\frac{1}{\sqrt{1+\text{Coth}[x]}}$$

Result (type 3, 51 leaves):

$$\frac{-2-(1+i)\ \text{ArcTan}\left[\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i\ (1+\text{Coth}[x])}\right]\sqrt{i\ (1+\text{Coth}[x])}}{2\sqrt{1+\text{Coth}[x]}}$$

Problem 75: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(1+\text{Coth}[x])^{3/2}}\, dx$$

Optimal (type 3, 49 leaves, 4 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{1+\text{Coth}[x]}}{\sqrt{2}}\right]}{2\sqrt{2}}-\frac{1}{3\,(1+\text{Coth}[x])^{3/2}}-\frac{1}{2\sqrt{1+\text{Coth}[x]}}$$

Result (type 3, 86 leaves):

$$\left(\frac{1}{4}+\frac{i}{4}\right)\sqrt{1+\text{Coth}[x]}\left(-\frac{i\,\text{ArcTan}\left[\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i\,(1+\text{Coth}[x])}\right]}{\sqrt{i\,(1+\text{Coth}[x])}}+\left(\frac{1}{6}-\frac{i}{6}\right)(-4+5\,\text{Cosh}[2x]-\text{Cosh}[4x]-5\,\text{Sinh}[2x]+\text{Sinh}[4x])\right)$$

Problem 76: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(1+\text{Coth}[x])^{5/2}}\, dx$$

Optimal (type 3, 61 leaves, 5 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{1+\text{Coth}[x]}}{\sqrt{2}}\right]}{4\sqrt{2}}-\frac{1}{5\,(1+\text{Coth}[x])^{5/2}}-\frac{1}{6\,(1+\text{Coth}[x])^{3/2}}-\frac{1}{4\sqrt{1+\text{Coth}[x]}}$$

Result (type 3, 94 leaves):

$$\frac{\left(\frac{1}{8}+\frac{i}{8}\right)\text{ArcTan}\left[\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i\,(1+\text{Coth}[x])}\right](1+\text{Coth}[x])^{3/2}}{(i\,(1+\text{Coth}[x]))^{3/2}}-\frac{1}{60}\sqrt{1+\text{Coth}[x]}\,(\text{Cosh}[3x]-\text{Sinh}[3x])\,(-10\,\text{Cosh}[x]+10\,\text{Cosh}[3x]-24\,\text{Sinh}[x]+13\,\text{Sinh}[3x])$$

Problem 77: Result more than twice size of optimal antiderivative.

$$\int (a+b\,\text{Coth}[c+d\,x])^5\, dx$$

Optimal (type 3, 142 leaves, 5 steps):

$$a\,(a^4+10\,a^2\,b^2+5\,b^4)\,x-\frac{4\,a\,b^2\,(a^2+b^2)\,\text{Coth}[c+d\,x]}{d}-\frac{b\,(3\,a^2+b^2)\,(a+b\,\text{Coth}[c+d\,x])^2}{2\,d}-\frac{2\,a\,b\,(a+b\,\text{Coth}[c+d\,x])^3}{3\,d}-\frac{b\,(a+b\,\text{Coth}[c+d\,x])^4}{4\,d}+\frac{b\,(5\,a^4+10\,a^2\,b^2+b^4)\,\text{Log}[\text{Sinh}[c+d\,x]]}{d}$$

Result (type 3, 367 leaves):

$$-\frac{b^5 (a+b\,\text{Coth}[c+d\,x])^5\,\text{Sinh}[c+d\,x]}{4\,d\,(b\,\text{Cosh}[c+d\,x]+a\,\text{Sinh}[c+d\,x])^5}-\frac{5\,a\,b^4\,\text{Cosh}[c+d\,x]\,(a+b\,\text{Coth}[c+d\,x])^5\,\text{Sinh}[c+d\,x]^2}{3\,d\,(b\,\text{Cosh}[c+d\,x]+a\,\text{Sinh}[c+d\,x])^5}-\frac{b^3\,(5\,a^2+b^2)\,(a+b\,\text{Coth}[c+d\,x])^5\,\text{Sinh}[c+d\,x]^3}{d\,(b\,\text{Cosh}[c+d\,x]+a\,\text{Sinh}[c+d\,x])^5}-\left(10\,(3\,a^3\,b^2\,\text{Cosh}[c+d\,x]+2\,a\,b^4\,\text{Cosh}[c+d\,x])\,(a+b\,\text{Coth}[c+d\,x])^5\,\text{Sinh}[c+d\,x]^4\right)\Big/\left(3\,d\,(b\,\text{Cosh}[c+d\,x]+a\,\text{Sinh}[c+d\,x])^5\right)+\frac{a\,(a^4+10\,a^2\,b^2+5\,b^4)\,(c+d\,x)\,(a+b\,\text{Coth}[c+d\,x])^5\,\text{Sinh}[c+d\,x]^5}{d\,(b\,\text{Cosh}[c+d\,x]+a\,\text{Sinh}[c+d\,x])^5}+\left((5\,a^4\,b+10\,a^2\,b^3+b^5)\,(a+b\,\text{Coth}[c+d\,x])^5\,\text{Log}[\text{Sinh}[c+d\,x]]\,\text{Sinh}[c+d\,x]^5\right)\Big/\left(d\,(b\,\text{Cosh}[c+d\,x]+a\,\text{Sinh}[c+d\,x])^5\right)$$

Problem 84: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{(a+b\,\text{Coth}[c+d\,x])^4}\,dx$$

Optimal (type 3, 169 leaves, 5 steps):

$$\frac{(a^4+6\,a^2\,b^2+b^4)\,x}{(a^2-b^2)^4}+\frac{b}{3\,(a^2-b^2)\,d\,(a+b\,\text{Coth}[c+d\,x])^3}+\frac{a\,b}{(a^2-b^2)^2\,d\,(a+b\,\text{Coth}[c+d\,x])^2}+\frac{b\,(3\,a^2+b^2)}{(a^2-b^2)^3\,d\,(a+b\,\text{Coth}[c+d\,x])}-\frac{4\,a\,b\,(a^2+b^2)\,\text{Log}[b\,\text{Cosh}[c+d\,x]+a\,\text{Sinh}[c+d\,x]]}{(a^2-b^2)^4\,d}$$

Result (type 3, 440 leaves):

$$\frac{1}{3\,(a-b)^4\,(a+b)^4\,d\,(a+b\,\text{Coth}[c+d\,x])^3}\Big(b^3\,(6\,a^4-7\,a^2\,b^2+b^4)\,\text{Csch}[c+d\,x]^2+3\,b^3\,\text{Coth}[c+d\,x]^3\,\big((a^4+6\,a^2\,b^2+b^4)\,(c+d\,x)-4\,a\,b\,(a^2+b^2)\,\text{Log}[b\,\text{Cosh}[c+d\,x]+a\,\text{Sinh}[c+d\,x]]\big)+b^2\,\text{Coth}[c+d\,x]^2\,\big(18\,a^4\,b-14\,a^2\,b^3-4\,b^5+9\,a^5\,(c+d\,x)+54\,a^3\,b^2\,(c+d\,x)+9\,a\,b^4\,(c+d\,x)-36\,a^2\,b\,(a^2+b^2)\,\text{Log}[b\,\text{Cosh}[c+d\,x]+a\,\text{Sinh}[c+d\,x]]\big)+a\,b\,\text{Coth}[c+d\,x]\,\big(36\,a^4\,b-28\,a^2\,b^3-8\,b^5+9\,a^5\,c+54\,a^3\,b^2\,c+9\,a\,b^4\,c+9\,a^5\,d\,x+54\,a^3\,b^2\,d\,x+9\,a\,b^4\,d\,x+5\,b^3\,(a^2-b^2)\,\text{Csch}[c+d\,x]^2-36\,a^2\,b\,(a^2+b^2)\,\text{Log}[b\,\text{Cosh}[c+d\,x]+a\,\text{Sinh}[c+d\,x]]\big)+a^2\,\big(18\,a^4\,b-14\,a^2\,b^3-4\,b^5+3\,a^5\,(c+d\,x)+18\,a^3\,b^2\,(c+d\,x)+3\,a\,b^4\,(c+d\,x)-12\,a^2\,b\,(a^2+b^2)\,\text{Log}[b\,\text{Cosh}[c+d\,x]+a\,\text{Sinh}[c+d\,x]]\big)\Big)$$

Problem 87: Result unnecessarily involves imaginary or complex numbers.

$$\int\sqrt{a+b\,\text{Coth}[c+d\,x]}\,dx$$

Optimal (type 3, 74 leaves, 5 steps):

$$-\frac{\sqrt{a-b}\ \text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Coth}[c+d\,x]}}{\sqrt{a-b}}\right]}{d}+\frac{\sqrt{a+b}\ \text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Coth}[c+d\,x]}}{\sqrt{a+b}}\right]}{d}$$

Result (type 3, 128 leaves):

$$\left(\left(-\sqrt{i\,(a-b)}\ \text{ArcTanh}\left[\frac{\sqrt{i\,(a+b\,\text{Coth}[c+d\,x])}}{\sqrt{i\,(a-b)}}\right]+\sqrt{i\,(a+b)}\ \text{ArcTanh}\left[\frac{\sqrt{i\,(a+b\,\text{Coth}[c+d\,x])}}{\sqrt{i\,(a+b)}}\right]\right)\sqrt{a+b\,\text{Coth}[c+d\,x]}\right)\Big/\left(d\,\sqrt{i\,(a+b\,\text{Coth}[c+d\,x])}\right)$$

Problem 88: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{a+b\,\text{Coth}[c+d\,x]}}\,dx$$

Optimal (type 3, 74 leaves, 5 steps):

$$-\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Coth}[c+d\,x]}}{\sqrt{a-b}}\right]}{\sqrt{a-b}\ d}+\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Coth}[c+d\,x]}}{\sqrt{a+b}}\right]}{\sqrt{a+b}\ d}$$

Result (type 3, 129 leaves):

$$-\left(\left(\left(\frac{\text{ArcTanh}\left[\frac{\sqrt{i\,(a+b\,\text{Coth}[c+d\,x])}}{\sqrt{i\,(a-b)}}\right]}{\sqrt{i\,(a-b)}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{i\,(a+b\,\text{Coth}[c+d\,x])}}{\sqrt{i\,(a+b)}}\right]}{\sqrt{i\,(a+b)}}\right)\sqrt{i\,(a+b\,\text{Coth}[c+d\,x])}\right)\Big/\left(d\,\sqrt{a+b\,\text{Coth}[c+d\,x]}\right)\right)$$

Problem 95: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Csch}[x]^3}{1+\text{Coth}[x]}\,dx$$

Optimal (type 3, 8 leaves, 2 steps):

$$\text{ArcTanh}[\text{Cosh}[x]]-\text{Csch}[x]$$

Result (type 3, 21 leaves):

$$-\text{Csch}[x]+\text{Log}\left[\text{Cosh}\left[\frac{x}{2}\right]\right]-\text{Log}\left[\text{Sinh}\left[\frac{x}{2}\right]\right]$$

Problem 113: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\sqrt{1+\text{Coth}[x]}\,\text{Sech}[x]^2\,dx$$

Optimal (type 3, 21 leaves, 4 steps):

$$\text{ArcTanh}\left[\sqrt{1+\text{Coth}[x]}\right]+\sqrt{1+\text{Coth}[x]}\,\text{Tanh}[x]$$

Result (type 3, 675 leaves):

$$\frac{1}{2}\sqrt{1+\text{Coth}[x]}$$

$$\Bigg(\frac{(1-i)\,\text{ArcTan}\left[\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i\,(1+\text{Coth}[x])}\right]}{\sqrt{i\,(1+\text{Coth}[x])}}+\frac{1}{2\sqrt{\text{Tanh}\left[\frac{x}{2}\right]}}\Bigg((2+2\,i)\,(-1)^{1/4}\,\text{ArcTan}\Big[$$

$$\Big((2+i)+2\sqrt{-1-i}\left(1+(-1)^{1/4}\sqrt{\text{Tanh}\left[\frac{x}{2}\right]}\right)\sqrt{-1+\text{Tanh}\left[\frac{x}{2}\right]}+2\,(-1)^{1/4}\sqrt{\text{Tanh}\left[\frac{x}{2}\right]}-$$

$$\text{Tanh}\left[\frac{x}{2}\right]\Big)\Big/\left((-2-i)-2\sqrt{-1-i}\sqrt{-1+\text{Tanh}\left[\frac{x}{2}\right]}+(1+2\,i)\,\text{Tanh}\left[\frac{x}{2}\right]\right)\Big]+$$

$$(2+2\,i)\,(-1)^{1/4}\,\text{ArcTan}\Big[\Big((2+i)+(-1-i)^{3/2}\left((1-i)+\sqrt{2}\sqrt{\text{Tanh}\left[\frac{x}{2}\right]}\right)$$

$$\sqrt{-1+\text{Tanh}\left[\frac{x}{2}\right]}+2\,(-1)^{1/4}\sqrt{\text{Tanh}\left[\frac{x}{2}\right]}-\text{Tanh}\left[\frac{x}{2}\right]\Big)\Big/$$

$$\left((-2-i)+2\sqrt{-1-i}\sqrt{-1+\text{Tanh}\left[\frac{x}{2}\right]}+(1+2\,i)\,\text{Tanh}\left[\frac{x}{2}\right]\right)\Big]+$$

$$\sqrt{2}\,\text{Log}\Big[(1-i)\left(1+\sqrt{2}\sqrt{\text{Tanh}\left[\frac{x}{2}\right]}\right)-\frac{2\left((1+i)+i\sqrt{2}\sqrt{\text{Tanh}\left[\frac{x}{2}\right]}\right)\sqrt{-1+\text{Tanh}\left[\frac{x}{2}\right]}}{\sqrt{-1+i}}+$$

$$(2+i)\left(-1+\text{Tanh}\left[\frac{x}{2}\right]\right)\Big]+\sqrt{2}\,\text{Log}\Big[(1-i)\left(1+\sqrt{2}\sqrt{\text{Tanh}\left[\frac{x}{2}\right]}\right)-$$

$$(-1+i)^{3/2}\left((1-i)+\sqrt{2}\sqrt{\text{Tanh}\left[\frac{x}{2}\right]}\right)\sqrt{-1+\text{Tanh}\left[\frac{x}{2}\right]}+(2+i)\left(-1+\text{Tanh}\left[\frac{x}{2}\right]\right)\Big]-$$

$$8\,\text{Log}\left[1+\sqrt{\text{Tanh}\left[\frac{x}{2}\right]}\right]+\sqrt{2}\,\text{Log}\left[(2+i)+2\sqrt{-1-i}\sqrt{-1+\text{Tanh}\left[\frac{x}{2}\right]}-\text{Tanh}\left[\frac{x}{2}\right]\right]+$$

$$4\,\text{Log}\left[-1+\text{Tanh}\left[\frac{x}{2}\right]\right]-\sqrt{2}\,\text{Log}\left[(-2-\text{i})-2\sqrt{-1-\text{i}}\,\sqrt{-1+\text{Tanh}\left[\frac{x}{2}\right]}+\text{Tanh}\left[\frac{x}{2}\right]\right]-$$

$$\sqrt{2}\,\text{Log}\left[(-2+\text{i})-2\sqrt{-1+\text{i}}\,\sqrt{-1+\text{Tanh}\left[\frac{x}{2}\right]}+\text{Tanh}\left[\frac{x}{2}\right]\right]-$$

$$\left.\sqrt{2}\,\text{Log}\left[(-2+\text{i})+2\sqrt{-1+\text{i}}\,\sqrt{-1+\text{Tanh}\left[\frac{x}{2}\right]}+\text{Tanh}\left[\frac{x}{2}\right]\right]\right)$$

$$\left.\left(\text{Cosh}\left[\frac{x}{2}\right]-\text{Sinh}\left[\frac{x}{2}\right]\right)\text{Sinh}\left[\frac{x}{2}\right]+2\,\text{Tanh}[x]\right)$$

Problem 132: Result unnecessarily involves imaginary or complex numbers.

$$\int \text{Coth}[x]\,(1+\text{Coth}[x])^{3/2}\,dx$$

Optimal (type 3, 45 leaves, 4 steps):

$$2\sqrt{2}\,\text{ArcTanh}\left[\frac{\sqrt{1+\text{Coth}[x]}}{\sqrt{2}}\right]-2\sqrt{1+\text{Coth}[x]}-\frac{2}{3}\,(1+\text{Coth}[x])^{3/2}$$

Result (type 3, 90 leaves):

$$-\left(\left(2\,(1+\text{Coth}[x])^{3/2}\left(\text{Cosh}[x]\sqrt{\text{i}\,(1+\text{Coth}[x])}-(3-3\,\text{i})\,\text{ArcTanh}\left[\left(\frac{1}{2}+\frac{\text{i}}{2}\right)\sqrt{\text{i}\,(1+\text{Coth}[x])}\right]\text{Sinh}[x]+4\sqrt{\text{i}\,(1+\text{Coth}[x])}\,\text{Sinh}[x]\right)\right)\Big/\left(3\sqrt{\text{i}\,(1+\text{Coth}[x])}\,(\text{Cosh}[x]+\text{Sinh}[x])\right)\right)$$

Problem 133: Result unnecessarily involves imaginary or complex numbers.

$$\int \text{Coth}[x]\,\sqrt{1+\text{Coth}[x]}\,dx$$

Optimal (type 3, 32 leaves, 3 steps):

$$\sqrt{2}\,\text{ArcTanh}\left[\frac{\sqrt{1+\text{Coth}[x]}}{\sqrt{2}}\right]-2\sqrt{1+\text{Coth}[x]}$$

Result (type 3, 53 leaves):

$$(1+\text{i})\sqrt{1+\text{Coth}[x]}\left((-1+\text{i})-\frac{\text{i}\,\text{ArcTan}\left[\left(\frac{1}{2}+\frac{\text{i}}{2}\right)\sqrt{\text{i}\,(1+\text{Coth}[x])}\right]}{\sqrt{\text{i}\,(1+\text{Coth}[x])}}\right)$$

Problem 134: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Coth}[x]}{\sqrt{1+\text{Coth}[x]}}\,dx$$

Optimal (type 3, 30 leaves, 3 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{1+\text{Coth}[x]}}{\sqrt{2}}\right]}{\sqrt{2}}+\frac{1}{\sqrt{1+\text{Coth}[x]}}$$

Result (type 3, 97 leaves):

$$\left(\left(\frac{1}{2}-\frac{i}{2}\right)\text{ArcTan}\left[\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i+i\,\text{Coth}[x]}\right]\text{Csch}[x]\left(\text{Cosh}[x]+\text{Sinh}[x]\right)\right)\Big/\left(\sqrt{i+i\,\text{Coth}[x]}\,\sqrt{1+\text{Coth}[x]}\right)+\frac{\text{Csch}[x]\left(\text{Cosh}[x]+\text{Sinh}[x]\right)\left(\frac{1}{2}-\frac{1}{2}\text{Cosh}[2x]+\frac{1}{2}\text{Sinh}[2x]\right)}{\sqrt{1+\text{Coth}[x]}}$$

Problem 135: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{Coth}[x]}{\left(1+\text{Coth}[x]\right)^{3/2}}\,dx$$

Optimal (type 3, 49 leaves, 4 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{1+\text{Coth}[x]}}{\sqrt{2}}\right]}{2\sqrt{2}}+\frac{1}{3\left(1+\text{Coth}[x]\right)^{3/2}}-\frac{1}{2\sqrt{1+\text{Coth}[x]}}$$

Result (type 3, 84 leaves):

$$\left(\frac{1}{4}+\frac{i}{4}\right)\sqrt{1+\text{Coth}[x]}\left(-\frac{i\,\text{ArcTan}\left[\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i\left(1+\text{Coth}[x]\right)}\right]}{\sqrt{i\left(1+\text{Coth}[x]\right)}}+\left(\frac{1}{6}-\frac{i}{6}\right)\left(-2+\text{Cosh}[2x]+\text{Cosh}[4x]-\text{Sinh}[2x]-\text{Sinh}[4x]\right)\right)$$

Problem 136: Result unnecessarily involves imaginary or complex numbers.

$$\int \text{Coth}[x]^2\left(1+\text{Coth}[x]\right)^{3/2}\,dx$$

Optimal (type 3, 45 leaves, 4 steps):

$$2\sqrt{2}\,\text{ArcTanh}\left[\frac{\sqrt{1+\text{Coth}[x]}}{\sqrt{2}}\right]-2\sqrt{1+\text{Coth}[x]}-\frac{2}{5}\left(1+\text{Coth}[x]\right)^{5/2}$$

Result (type 3, 70 leaves):

$$-\frac{1}{5\sqrt{1+\text{Coth}[x]}}2\left(7+2\,\text{Coth}[x]^2+(5+5\,i)\,\text{ArcTan}\left[\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i\,(1+\text{Coth}[x])}\right]\sqrt{i\,(1+\text{Coth}[x])}+\text{Csch}[x]^2+\text{Coth}[x]\,\left(9+\text{Csch}[x]^2\right)\right)$$

Problem 137: Result unnecessarily involves imaginary or complex numbers.

$$\int \text{Coth}[x]^2\sqrt{1+\text{Coth}[x]}\,dx$$

Optimal (type 3, 34 leaves, 3 steps):

$$\sqrt{2}\,\text{ArcTanh}\left[\frac{\sqrt{1+\text{Coth}[x]}}{\sqrt{2}}\right]-\frac{2}{3}\,(1+\text{Coth}[x])^{3/2}$$

Result (type 3, 61 leaves):

$$\frac{1}{3\sqrt{1+\text{Coth}[x]}}\left(-2-4\,\text{Coth}[x]-2\,\text{Coth}[x]^2-(3+3\,i)\,\text{ArcTan}\left[\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i\,(1+\text{Coth}[x])}\right]\sqrt{i\,(1+\text{Coth}[x])}\right)$$

Problem 138: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{Coth}[x]^2}{\sqrt{1+\text{Coth}[x]}}\,dx$$

Optimal (type 3, 42 leaves, 4 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{1+\text{Coth}[x]}}{\sqrt{2}}\right]}{\sqrt{2}}-\frac{1}{\sqrt{1+\text{Coth}[x]}}-2\sqrt{1+\text{Coth}[x]}$$

Result (type 3, 81 leaves):

$$\frac{1}{\sqrt{1+\text{Coth}[x]}}\left(\frac{1}{2}+\frac{i}{2}\right)\text{Csch}[x]\,(\text{Cosh}[x]+\text{Sinh}[x])\left(-\frac{i\,\text{ArcTan}\left[\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i\,(1+\text{Coth}[x])}\right]}{\sqrt{i\,(1+\text{Coth}[x])}}+\left(\frac{1}{2}-\frac{i}{2}\right)(-5+\text{Cosh}[2\,x]-\text{Sinh}[2\,x])\right)$$

Problem 139: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{Coth}[x]^2}{(1+\text{Coth}[x])^{3/2}}\,dx$$

Optimal (type 3, 49 leaves, 4 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{1+\text{Coth}[x]}}{\sqrt{2}}\right]}{2\sqrt{2}}-\frac{1}{3\,(1+\text{Coth}[x])^{3/2}}+\frac{3}{2\sqrt{1+\text{Coth}[x]}}$$

Result (type 3, 86 leaves):

$$\left(\frac{1}{4}+\frac{i}{4}\right)\sqrt{1+\text{Coth}[x]}\left(-\frac{i\,\text{ArcTan}\left[\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i\,(1+\text{Coth}[x])}\right]}{\sqrt{i\,(1+\text{Coth}[x])}}-\left(\frac{1}{6}-\frac{i}{6}\right)(-8+7\,\text{Cosh}[2\,x]+\text{Cosh}[4\,x]-7\,\text{Sinh}[2\,x]-\text{Sinh}[4\,x])\right)$$

Problem 162: Result more than twice size of optimal antiderivative.

$$\int\frac{\text{Coth}[x]^5}{\sqrt{a+b\,\text{Coth}[x]^2+c\,\text{Coth}[x]^4}}\,dx$$

Optimal (type 3, 135 leaves, 8 steps):

$$\frac{(b-2\,c)\,\text{ArcTanh}\left[\frac{b+2\,c\,\text{Coth}[x]^2}{2\sqrt{c}\,\sqrt{a+b\,\text{Coth}[x]^2+c\,\text{Coth}[x]^4}}\right]}{4\,c^{3/2}}+\frac{\text{ArcTanh}\left[\frac{2\,a+b+(b+2\,c)\,\text{Coth}[x]^2}{2\sqrt{a+b+c}\,\sqrt{a+b\,\text{Coth}[x]^2+c\,\text{Coth}[x]^4}}\right]}{2\sqrt{a+b+c}}-\frac{\sqrt{a+b\,\text{Coth}[x]^2+c\,\text{Coth}[x]^4}}{2\,c}$$

Result (type 3, 42 946 leaves): Display of huge result suppressed!

Problem 163: Result more than twice size of optimal antiderivative.

$$\int\frac{\text{Coth}[x]^3}{\sqrt{a+b\,\text{Coth}[x]^2+c\,\text{Coth}[x]^4}}\,dx$$

Optimal (type 3, 105 leaves, 7 steps):

$$-\frac{\text{ArcTanh}\left[\frac{b+2\,c\,\text{Coth}[x]^2}{2\sqrt{c}\,\sqrt{a+b\,\text{Coth}[x]^2+c\,\text{Coth}[x]^4}}\right]}{2\sqrt{c}}+\frac{\text{ArcTanh}\left[\frac{2\,a+b+(b+2\,c)\,\text{Coth}[x]^2}{2\sqrt{a+b+c}\,\sqrt{a+b\,\text{Coth}[x]^2+c\,\text{Coth}[x]^4}}\right]}{2\sqrt{a+b+c}}$$

Result (type 3, 27 092 leaves): Display of huge result suppressed!

Problem 164: Result more than twice size of optimal antiderivative.

$$\int\frac{\text{Coth}[x]}{\sqrt{a+b\,\text{Coth}[x]^2+c\,\text{Coth}[x]^4}}\,dx$$

Optimal (type 3, 58 leaves, 4 steps):

$$\frac{\text{ArcTanh}\left[\frac{2\,a+b+(b+2\,c)\,\text{Coth}[x]^2}{2\,\sqrt{a+b+c}\,\sqrt{a+b\,\text{Coth}[x]^2+c\,\text{Coth}[x]^4}}\right]}{2\,\sqrt{a+b+c}}$$

Result (type 3, 27 092 leaves): Display of huge result suppressed!

Problem 165: Attempted integration timed out after 120 seconds.

$$\int \frac{\text{Tanh}[x]}{\sqrt{a+b\,\text{Coth}[x]^2+c\,\text{Coth}[x]^4}}\,dx$$

Optimal (type 3, 106 leaves, 8 steps):

$$-\frac{\text{ArcTanh}\left[\frac{2\,a+b\,\text{Coth}[x]^2}{2\,\sqrt{a}\,\sqrt{a+b\,\text{Coth}[x]^2+c\,\text{Coth}[x]^4}}\right]}{2\,\sqrt{a}}+\frac{\text{ArcTanh}\left[\frac{2\,a+b+(b+2\,c)\,\text{Coth}[x]^2}{2\,\sqrt{a+b+c}\,\sqrt{a+b\,\text{Coth}[x]^2+c\,\text{Coth}[x]^4}}\right]}{2\,\sqrt{a+b+c}}$$

Result (type 1, 1 leaves):

???

Problem 166: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Tanh}[x]^3}{\sqrt{a+b\,\text{Coth}[x]^2+c\,\text{Coth}[x]^4}}\,dx$$

Optimal (type 3, 183 leaves, 11 steps):

$$-\frac{\text{ArcTanh}\left[\frac{2\,a+b\,\text{Coth}[x]^2}{2\,\sqrt{a}\,\sqrt{a+b\,\text{Coth}[x]^2+c\,\text{Coth}[x]^4}}\right]}{2\,\sqrt{a}}+\frac{b\,\text{ArcTanh}\left[\frac{2\,a+b\,\text{Coth}[x]^2}{2\,\sqrt{a}\,\sqrt{a+b\,\text{Coth}[x]^2+c\,\text{Coth}[x]^4}}\right]}{4\,a^{3/2}}+$$
$$\frac{\text{ArcTanh}\left[\frac{2\,a+b+(b+2\,c)\,\text{Coth}[x]^2}{2\,\sqrt{a+b+c}\,\sqrt{a+b\,\text{Coth}[x]^2+c\,\text{Coth}[x]^4}}\right]}{2\,\sqrt{a+b+c}}-\frac{\sqrt{a+b\,\text{Coth}[x]^2+c\,\text{Coth}[x]^4}\;\text{Tanh}[x]^2}{2\,a}$$

Result (type 3, 42 369 leaves): Display of huge result suppressed!

Problem 167: Result more than twice size of optimal antiderivative.

$$\int \text{Coth}[x]\,\sqrt{a+b\,\text{Coth}[x]^2+c\,\text{Coth}[x]^4}\,dx$$

Optimal (type 3, 132 leaves, 8 steps):

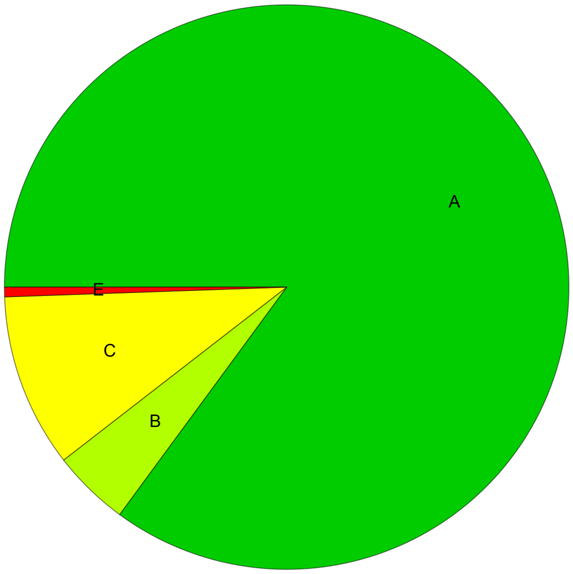
$$-\frac{(b+2c)\,\text{ArcTanh}\left[\frac{b+2c\,\text{Coth}[x]^2}{2\sqrt{c}\,\sqrt{a+b\,\text{Coth}[x]^2+c\,\text{Coth}[x]^4}}\right]}{4\sqrt{c}}+$$

$$\frac{1}{2}\sqrt{a+b+c}\,\text{ArcTanh}\left[\frac{2a+b+(b+2c)\,\text{Coth}[x]^2}{2\sqrt{a+b+c}\,\sqrt{a+b\,\text{Coth}[x]^2+c\,\text{Coth}[x]^4}}\right]-\frac{1}{2}\sqrt{a+b\,\text{Coth}[x]^2+c\,\text{Coth}[x]^4}$$

Result (type 3, 81 208 leaves) : Display of huge result suppressed!

Summary of Integration Test Results

181 integration problems



A - 154 optimal antiderivatives

B - 8 more than twice size of optimal antiderivatives

C - 18 unnecessarily complex antiderivatives

D - 0 unable to integrate problems

E - 1 integration timeouts