

Mathematica 11.3 Integration Test Results

Test results for the 83 problems in "6.6.2 (e x)^m (a+b csch(c+d x^n))^p.m"

Problem 5: Result more than twice size of optimal antiderivative.

$$\int x (a + b \operatorname{Csch}[c + d x^2]) dx$$

Optimal (type 3, 26 leaves, 4 steps):

$$\frac{a x^2}{2} - \frac{b \operatorname{ArcTanh}[\operatorname{Cosh}[c + d x^2]]}{2 d}$$

Result (type 3, 57 leaves):

$$\frac{a x^2}{2} - \frac{b \operatorname{Log}[\operatorname{Cosh}[\frac{c}{2} + \frac{d x^2}{2}]]}{2 d} + \frac{b \operatorname{Log}[\operatorname{Sinh}[\frac{c}{2} + \frac{d x^2}{2}]]}{2 d}$$

Problem 10: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int x^3 (a + b \operatorname{Csch}[c + d x^2])^2 dx$$

Optimal (type 4, 108 leaves, 10 steps):

$$\frac{a^2 x^4}{4} - \frac{2 a b x^2 \operatorname{ArcTanh}[e^{c+d x^2}]}{d} - \frac{b^2 x^2 \operatorname{Coth}[c + d x^2]}{2 d} + \frac{b^2 \operatorname{Log}[\operatorname{Sinh}[c + d x^2]]}{2 d^2} - \frac{a b \operatorname{PolyLog}[2, -e^{c+d x^2}]}{d^2} + \frac{a b \operatorname{PolyLog}[2, e^{c+d x^2}]}{d^2}$$

Result (type 4, 598 leaves):

$$\begin{aligned}
 & \frac{b^2 x^2 \operatorname{Coth}[c] (a + b \operatorname{Csch}[c + d x^2])^2 \operatorname{Sinh}[c + d x^2]^2}{2 d (b + a \operatorname{Sinh}[c + d x^2])^2} + \\
 & \left(x^2 \operatorname{Csch}\left[\frac{c}{2}\right] (a + b \operatorname{Csch}[c + d x^2])^2 \operatorname{Sech}\left[\frac{c}{2}\right] (-2 b^2 \operatorname{Cosh}[c] + a^2 d x^2 \operatorname{Sinh}[c]) \operatorname{Sinh}[c + d x^2]^2 \right) / \\
 & \left(8 d (b + a \operatorname{Sinh}[c + d x^2])^2 - (b^2 \operatorname{Csch}[c] (a + b \operatorname{Csch}[c + d x^2])^2 \right. \\
 & \quad \left. (-d x^2 \operatorname{Cosh}[c] + \operatorname{Log}[\operatorname{Cosh}[d x^2] \operatorname{Sinh}[c] + \operatorname{Cosh}[c] \operatorname{Sinh}[d x^2]]) \operatorname{Sinh}[c]) \operatorname{Sinh}[c + d x^2]^2 \right) / \\
 & \left(2 d^2 (-\operatorname{Cosh}[c]^2 + \operatorname{Sinh}[c]^2) (b + a \operatorname{Sinh}[c + d x^2])^2 \right) + \\
 & \left(b^2 x^2 \operatorname{Csch}\left[\frac{c}{2}\right] \operatorname{Csch}\left[\frac{c}{2} + \frac{d x^2}{2}\right] (a + b \operatorname{Csch}[c + d x^2])^2 \operatorname{Sinh}\left[\frac{d x^2}{2}\right] \operatorname{Sinh}[c + d x^2]^2 \right) / \\
 & \left(4 d (b + a \operatorname{Sinh}[c + d x^2])^2 - \right. \\
 & \left. \left(b^2 x^2 (a + b \operatorname{Csch}[c + d x^2])^2 \operatorname{Sech}\left[\frac{c}{2}\right] \operatorname{Sech}\left[\frac{c}{2} + \frac{d x^2}{2}\right] \operatorname{Sinh}\left[\frac{d x^2}{2}\right] \operatorname{Sinh}[c + d x^2]^2 \right) / \right. \\
 & \left. \left(4 d (b + a \operatorname{Sinh}[c + d x^2])^2 \right) + \left(a b (a + b \operatorname{Csch}[c + d x^2])^2 \operatorname{Sinh}[c + d x^2]^2 \right. \right. \\
 & \left. \left. \left(-\frac{2 \operatorname{ArcTan}\left[\frac{\operatorname{Cosh}[c] + \operatorname{Sinh}[c] \operatorname{Tanh}\left[\frac{d x^2}{2}\right]}{\sqrt{-\operatorname{Cosh}[c]^2 + \operatorname{Sinh}[c]^2}}\right] \operatorname{ArcTanh}[\operatorname{Tanh}[c]]}{\sqrt{-\operatorname{Cosh}[c]^2 + \operatorname{Sinh}[c]^2}} - \left(\operatorname{I} \left(\operatorname{I} (d x^2 + \operatorname{ArcTanh}[\operatorname{Tanh}[c]]) \right) \right. \right. \right. \\
 & \quad \left. \left. \left(\operatorname{Log}\left[1 - e^{-d x^2 - \operatorname{ArcTanh}[\operatorname{Tanh}[c]]}\right] - \operatorname{Log}\left[1 + e^{-d x^2 - \operatorname{ArcTanh}[\operatorname{Tanh}[c]]}\right] \right) + \right. \right. \\
 & \quad \left. \left. \operatorname{I} \left(\operatorname{PolyLog}\left[2, -e^{-d x^2 - \operatorname{ArcTanh}[\operatorname{Tanh}[c]]}\right] - \operatorname{PolyLog}\left[2, e^{-d x^2 - \operatorname{ArcTanh}[\operatorname{Tanh}[c]]}\right] \right) \right) \operatorname{Sech}[c] \right) / \right. \\
 & \left. \left. \left(\sqrt{1 - \operatorname{Tanh}[c]^2} \right) \right) \right) / \left(d^2 (b + a \operatorname{Sinh}[c + d x^2])^2 \right)
 \end{aligned}$$

Problem 18: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{x^3}{a + b \operatorname{Csch}[c + d x^2]} dx$$

Optimal (type 4, 225 leaves, 11 steps):

$$\frac{x^4}{4 a} - \frac{b x^2 \operatorname{Log}\left[1 + \frac{a e^{c+d x^2}}{b - \sqrt{a^2+b^2}}\right]}{2 a \sqrt{a^2+b^2} d} + \frac{b x^2 \operatorname{Log}\left[1 + \frac{a e^{c+d x^2}}{b + \sqrt{a^2+b^2}}\right]}{2 a \sqrt{a^2+b^2} d} -$$

$$\frac{b \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x^2}}{b - \sqrt{a^2+b^2}}\right]}{2 a \sqrt{a^2+b^2} d^2} + \frac{b \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x^2}}{b + \sqrt{a^2+b^2}}\right]}{2 a \sqrt{a^2+b^2} d^2}$$

Result (type 4, 1321 leaves):

$$\frac{x^4 \operatorname{Csch}\left[c + d x^2\right] \left(b + a \operatorname{Sinh}\left[c + d x^2\right]\right)}{4 a \left(a + b \operatorname{Csch}\left[c + d x^2\right]\right)} +$$

$$\frac{1}{2 a d^2 \left(a + b \operatorname{Csch}\left[c + d x^2\right]\right)} b \operatorname{Csch}\left[c + d x^2\right] \left(\frac{i \pi \operatorname{ArcTanh}\left[\frac{-a + b \operatorname{Tanh}\left[\frac{1}{2} (c + d x^2)\right]}{\sqrt{a^2+b^2}}\right]}{\sqrt{a^2+b^2}} + \right.$$

$$\frac{1}{\sqrt{-a^2-b^2}} \left(2 \left(-i c + \frac{\pi}{2} - i d x^2\right) \operatorname{ArcTanh}\left[\frac{(-i a + b) \operatorname{Cot}\left[\frac{1}{2} \left(-i c + \frac{\pi}{2} - i d x^2\right)\right]}{\sqrt{-a^2-b^2}}\right] - \right.$$

$$2 \left(-i c + \operatorname{ArcCos}\left[-\frac{i b}{a}\right]\right) \operatorname{ArcTanh}\left[\frac{(-i a - b) \operatorname{Tan}\left[\frac{1}{2} \left(-i c + \frac{\pi}{2} - i d x^2\right)\right]}{\sqrt{-a^2-b^2}}\right] +$$

$$\left(\operatorname{ArcCos}\left[-\frac{i b}{a}\right] - 2 i \left(\operatorname{ArcTanh}\left[\frac{(-i a + b) \operatorname{Cot}\left[\frac{1}{2} \left(-i c + \frac{\pi}{2} - i d x^2\right)\right]}{\sqrt{-a^2-b^2}}\right] - \operatorname{ArcTanh}\left[\frac{(-i a - b) \operatorname{Tan}\left[\frac{1}{2} \left(-i c + \frac{\pi}{2} - i d x^2\right)\right]}{\sqrt{-a^2-b^2}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{-a^2-b^2} e^{-\frac{1}{2} i \left(-i c + \frac{\pi}{2} - i d x^2\right)}}{\sqrt{2} \sqrt{-i a} \sqrt{b + a \operatorname{Sinh}\left[c + d x^2\right]}}\right] +$$

$$\left(\operatorname{ArcCos}\left[-\frac{i b}{a}\right] + 2 i \left(\operatorname{ArcTanh}\left[\frac{(-i a + b) \operatorname{Cot}\left[\frac{1}{2} \left(-i c + \frac{\pi}{2} - i d x^2\right)\right]}{\sqrt{-a^2-b^2}}\right] - \operatorname{ArcTanh}\left[\frac{(-i a - b) \operatorname{Tan}\left[\frac{1}{2} \left(-i c + \frac{\pi}{2} - i d x^2\right)\right]}{\sqrt{-a^2-b^2}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{-a^2-b^2} e^{\frac{1}{2} i \left(-i c + \frac{\pi}{2} - i d x^2\right)}}{\sqrt{2} \sqrt{-i a} \sqrt{b + a \operatorname{Sinh}\left[c + d x^2\right]}}\right] -$$

$$\left(\operatorname{ArcCos}\left[-\frac{i b}{a}\right] + 2 i \operatorname{ArcTanh}\left[\frac{(-i a - b) \operatorname{Tan}\left[\frac{1}{2} \left(-i c + \frac{\pi}{2} - i d x^2\right)\right]}{\sqrt{-a^2-b^2}}\right] \right)$$

$$\operatorname{Log}\left[1 - \left(i \left(b - i \sqrt{-a^2-b^2}\right) \left(-i a + b - \sqrt{-a^2-b^2} \operatorname{Tan}\left[\frac{1}{2} \left(-i c + \frac{\pi}{2} - i d x^2\right)\right]\right)\right) \right] /$$

$$\left(a \left(-i a + b + \sqrt{-a^2-b^2} \operatorname{Tan}\left[\frac{1}{2} \left(-i c + \frac{\pi}{2} - i d x^2\right)\right]\right) \right) +$$

$$\left(-\operatorname{ArcCos}\left[-\frac{i b}{a}\right] + 2 i \operatorname{ArcTanh}\left[\frac{(-i a - b) \operatorname{Tan}\left[\frac{1}{2} \left(-i c + \frac{\pi}{2} - i d x^2\right)\right]}{\sqrt{-a^2-b^2}}\right] \right)$$

$$\begin{aligned} & \left(\frac{\text{Log}\left[1 - \left(i \left(b + i \sqrt{-a^2 - b^2}\right) \left(-i a + b - \sqrt{-a^2 - b^2} \text{Tan}\left[\frac{1}{2} \left(-i c + \frac{\pi}{2} - i d x^2\right)\right]\right)\right]}{\left(a \left(-i a + b + \sqrt{-a^2 - b^2} \text{Tan}\left[\frac{1}{2} \left(-i c + \frac{\pi}{2} - i d x^2\right)\right]\right)\right)} + \right. \\ & i \left(\frac{\text{PolyLog}\left[2, \left(i \left(b - i \sqrt{-a^2 - b^2}\right) \left(-i a + b - \sqrt{-a^2 - b^2} \text{Tan}\left[\frac{1}{2} \left(-i c + \frac{\pi}{2} - i d x^2\right)\right]\right)\right]}{\left(a \left(-i a + b + \sqrt{-a^2 - b^2} \text{Tan}\left[\frac{1}{2} \left(-i c + \frac{\pi}{2} - i d x^2\right)\right]\right)\right)} - \right. \\ & \left. \left. \frac{\text{PolyLog}\left[2, \left(i \left(b + i \sqrt{-a^2 - b^2}\right) \left(-i a + b - \sqrt{-a^2 - b^2} \text{Tan}\left[\frac{1}{2} \left(-i c + \frac{\pi}{2} - i d x^2\right)\right]\right)\right]}{\left(a \left(-i a + b + \sqrt{-a^2 - b^2} \text{Tan}\left[\frac{1}{2} \left(-i c + \frac{\pi}{2} - i d x^2\right)\right]\right)\right)} \right] \right) (b + a \text{Sinh}[c + d x^2]) \end{aligned}$$

Problem 24: Attempted integration timed out after 120 seconds.

$$\int \frac{x^4}{(a + b \text{Csch}[c + d x^2])^2} dx$$

Optimal (type 8, 21 leaves, 0 steps):

$$\text{Int}\left[\frac{x^4}{(a + b \text{Csch}[c + d x^2])^2}, x\right]$$

Result (type 1, 1 leaves):

???

Problem 26: Attempted integration timed out after 120 seconds.

$$\int \frac{x^2}{(a + b \text{Csch}[c + d x^2])^2} dx$$

Optimal (type 8, 21 leaves, 0 steps):

$$\text{Int}\left[\frac{x^2}{(a + b \text{Csch}[c + d x^2])^2}, x\right]$$

Result (type 1, 1 leaves):

???

Problem 28: Attempted integration timed out after 120 seconds.

$$\int \frac{1}{x (a + b \text{Csch}[c + d x^2])^2} dx$$

Optimal (type 8, 21 leaves, 0 steps):

$$\text{Int}\left[\frac{1}{x (a + b \operatorname{Csch}[c + d x^2])^2}, x\right]$$

Result (type 1, 1 leaves):

???

Problem 29: Attempted integration timed out after 120 seconds.

$$\int \frac{1}{x^2 (a + b \operatorname{Csch}[c + d x^2])^2} dx$$

Optimal (type 8, 21 leaves, 0 steps):

$$\text{Int}\left[\frac{1}{x^2 (a + b \operatorname{Csch}[c + d x^2])^2}, x\right]$$

Result (type 1, 1 leaves):

???

Problem 30: Attempted integration timed out after 120 seconds.

$$\int \frac{1}{x^3 (a + b \operatorname{Csch}[c + d x^2])^2} dx$$

Optimal (type 8, 21 leaves, 0 steps):

$$\text{Int}\left[\frac{1}{x^3 (a + b \operatorname{Csch}[c + d x^2])^2}, x\right]$$

Result (type 1, 1 leaves):

???

Problem 38: Result more than twice size of optimal antiderivative.

$$\int x (a + b \operatorname{Csch}[c + d \sqrt{x}])^2 dx$$

Optimal (type 4, 287 leaves, 18 steps):

$$\begin{aligned}
 & -\frac{2 b^2 x^{3/2}}{d} + \frac{a^2 x^2}{2} - \frac{8 a b x^{3/2} \operatorname{ArcTanh}\left[e^{c+d \sqrt{x}}\right]}{d} - \frac{2 b^2 x^{3/2} \operatorname{Coth}\left[c+d \sqrt{x}\right]}{d} + \\
 & \frac{6 b^2 x \operatorname{Log}\left[1-e^{2(c+d \sqrt{x})}\right]}{d^2} - \frac{12 a b x \operatorname{PolyLog}\left[2,-e^{c+d \sqrt{x}}\right]}{d^2} + \\
 & \frac{12 a b x \operatorname{PolyLog}\left[2,e^{c+d \sqrt{x}}\right]}{d^2} + \frac{6 b^2 \sqrt{x} \operatorname{PolyLog}\left[2,e^{2(c+d \sqrt{x})}\right]}{d^3} + \\
 & \frac{24 a b \sqrt{x} \operatorname{PolyLog}\left[3,-e^{c+d \sqrt{x}}\right]}{d^3} - \frac{24 a b \sqrt{x} \operatorname{PolyLog}\left[3,e^{c+d \sqrt{x}}\right]}{d^3} - \\
 & \frac{3 b^2 \operatorname{PolyLog}\left[3,e^{2(c+d \sqrt{x})}\right]}{d^4} - \frac{24 a b \operatorname{PolyLog}\left[4,-e^{c+d \sqrt{x}}\right]}{d^4} + \frac{24 a b \operatorname{PolyLog}\left[4,e^{c+d \sqrt{x}}\right]}{d^4}
 \end{aligned}$$

Result (type 4, 591 leaves):

$$\begin{aligned}
 & \frac{a^2 x^2 \left(a+b \operatorname{Csch}\left[c+d \sqrt{x}\right]\right)^2 \operatorname{Sinh}\left[c+d \sqrt{x}\right]^2}{2\left(b+a \operatorname{Sinh}\left[c+d \sqrt{x}\right]\right)^2} + \\
 & \frac{1}{d^4\left(b+a \operatorname{Sinh}\left[c+d \sqrt{x}\right]\right)^2} b\left(a+b \operatorname{Csch}\left[c+d \sqrt{x}\right]\right)^2 \\
 & \left(-\frac{4 b d^3 e^{2 c} x^{3/2}}{-1+e^{2 c}}+12 b d^2 x \operatorname{Log}\left[1-e^{c+d \sqrt{x}}\right]+4 a d^3 x^{3/2} \operatorname{Log}\left[1-e^{c+d \sqrt{x}}\right]+ \right. \\
 & 12 b d^2 x \operatorname{Log}\left[1+e^{c+d \sqrt{x}}\right]-4 a d^3 x^{3/2} \operatorname{Log}\left[1+e^{c+d \sqrt{x}}\right]-6 b d^2 x \operatorname{Log}\left[-1+e^{2(c+d \sqrt{x})}\right]- \\
 & 12\left(-b d \sqrt{x}+a d^2 x\right) \operatorname{PolyLog}\left[2,-e^{c+d \sqrt{x}}\right]+12\left(b d \sqrt{x}+a d^2 x\right) \operatorname{PolyLog}\left[2,e^{c+d \sqrt{x}}\right]+ \\
 & 24 a d \sqrt{x} \operatorname{PolyLog}\left[3,-e^{c+d \sqrt{x}}\right]-24 a d \sqrt{x} \operatorname{PolyLog}\left[3,e^{c+d \sqrt{x}}\right]-3 b \operatorname{PolyLog}\left[3,e^{2(c+d \sqrt{x})}\right]- \\
 & \left. 24 a \operatorname{PolyLog}\left[4,-e^{c+d \sqrt{x}}\right]+24 a \operatorname{PolyLog}\left[4,e^{c+d \sqrt{x}}\right]\right) \operatorname{Sinh}\left[c+d \sqrt{x}\right]^2 + \\
 & \left(b^2 x^{3/2} \operatorname{Csch}\left[\frac{c}{2}\right] \operatorname{Csch}\left[\frac{c}{2}+\frac{d \sqrt{x}}{2}\right]\left(a+b \operatorname{Csch}\left[c+d \sqrt{x}\right]\right)^2 \operatorname{Sinh}\left[c+d \sqrt{x}\right]^2 \operatorname{Sinh}\left[\frac{d \sqrt{x}}{2}\right]\right) / \\
 & \left(d\left(b+a \operatorname{Sinh}\left[c+d \sqrt{x}\right]\right)^2\right)- \\
 & \left(b^2 x^{3/2}\left(a+b \operatorname{Csch}\left[c+d \sqrt{x}\right]\right)^2 \operatorname{Sech}\left[\frac{c}{2}\right] \operatorname{Sech}\left[\frac{c}{2}+\frac{d \sqrt{x}}{2}\right] \operatorname{Sinh}\left[c+d \sqrt{x}\right]^2 \operatorname{Sinh}\left[\frac{d \sqrt{x}}{2}\right]\right) / \\
 & \left(d\left(b+a \operatorname{Sinh}\left[c+d \sqrt{x}\right]\right)^2\right)
 \end{aligned}$$

Problem 39: Attempted integration timed out after 120 seconds.

$$\int \frac{\left(a+b \operatorname{Csch}\left[c+d \sqrt{x}\right]\right)^2}{x} d x$$

Optimal (type 8, 23 leaves, 0 steps):

$$\operatorname{Int}\left[\frac{(a+b \operatorname{Csch}[c+d \sqrt{x}])^2}{x}, x\right]$$

Result (type 1, 1 leaves):

???

Problem 49: Attempted integration timed out after 120 seconds.

$$\int \frac{1}{x (a+b \operatorname{Csch}[c+d \sqrt{x}])^2} dx$$

Optimal (type 8, 23 leaves, 0 steps):

$$\operatorname{Int}\left[\frac{1}{x (a+b \operatorname{Csch}[c+d \sqrt{x}])^2}, x\right]$$

Result (type 1, 1 leaves):

???

Problem 50: Attempted integration timed out after 120 seconds.

$$\int \frac{1}{x^2 (a+b \operatorname{Csch}[c+d \sqrt{x}])^2} dx$$

Optimal (type 8, 23 leaves, 0 steps):

$$\operatorname{Int}\left[\frac{1}{x^2 (a+b \operatorname{Csch}[c+d \sqrt{x}])^2}, x\right]$$

Result (type 1, 1 leaves):

???

Problem 57: Result more than twice size of optimal antiderivative.

$$\int \sqrt{x} (a+b \operatorname{Csch}[c+d \sqrt{x}])^2 dx$$

Optimal (type 4, 209 leaves, 15 steps):

$$\begin{aligned} & -\frac{2 b^2 x}{d} + \frac{2}{3} a^2 x^{3/2} - \frac{8 a b x \operatorname{ArcTanh}\left[e^{c+d \sqrt{x}}\right]}{d} - \frac{2 b^2 x \operatorname{Coth}\left[c+d \sqrt{x}\right]}{d} + \\ & \frac{4 b^2 \sqrt{x} \operatorname{Log}\left[1-e^{2(c+d \sqrt{x})}\right]}{d^2} - \frac{8 a b \sqrt{x} \operatorname{PolyLog}\left[2,-e^{c+d \sqrt{x}}\right]}{d^2} + \frac{8 a b \sqrt{x} \operatorname{PolyLog}\left[2,e^{c+d \sqrt{x}}\right]}{d^2} + \\ & \frac{2 b^2 \operatorname{PolyLog}\left[2,e^{2(c+d \sqrt{x})}\right]}{d^3} + \frac{8 a b \operatorname{PolyLog}\left[3,-e^{c+d \sqrt{x}}\right]}{d^3} - \frac{8 a b \operatorname{PolyLog}\left[3,e^{c+d \sqrt{x}}\right]}{d^3} \end{aligned}$$

Result (type 4, 470 leaves):

$$\frac{2 a^2 x^{3/2} \left(a + b \operatorname{Csch} [c + d \sqrt{x}] \right)^2 \operatorname{Sinh} [c + d \sqrt{x}]^2}{3 \left(b + a \operatorname{Sinh} [c + d \sqrt{x}] \right)^2} +$$

$$\frac{1}{d^3 \left(b + a \operatorname{Sinh} [c + d \sqrt{x}] \right)^2} 2 b \left(a + b \operatorname{Csch} [c + d \sqrt{x}] \right)^2$$

$$\left(-\frac{2 b d^2 e^{2 c} x}{-1 + e^{2 c}} + 2 a d^2 x \operatorname{Log} [1 - e^{c+d \sqrt{x}}] - 2 a d^2 x \operatorname{Log} [1 + e^{c+d \sqrt{x}}] + 2 b d \sqrt{x} \operatorname{Log} [1 - e^{2 (c+d \sqrt{x})}] - \right.$$

$$4 a d \sqrt{x} \operatorname{PolyLog} [2, -e^{c+d \sqrt{x}}] + 4 a d \sqrt{x} \operatorname{PolyLog} [2, e^{c+d \sqrt{x}}] + b \operatorname{PolyLog} [2, e^{2 (c+d \sqrt{x})}] +$$

$$\left. 4 a \operatorname{PolyLog} [3, -e^{c+d \sqrt{x}}] - 4 a \operatorname{PolyLog} [3, e^{c+d \sqrt{x}}] \right) \operatorname{Sinh} [c + d \sqrt{x}]^2 +$$

$$\left(b^2 x \operatorname{Csch} \left[\frac{c}{2} \right] \operatorname{Csch} \left[\frac{c}{2} + \frac{d \sqrt{x}}{2} \right] \left(a + b \operatorname{Csch} [c + d \sqrt{x}] \right)^2 \operatorname{Sinh} [c + d \sqrt{x}]^2 \operatorname{Sinh} \left[\frac{d \sqrt{x}}{2} \right] \right) /$$

$$\left(d \left(b + a \operatorname{Sinh} [c + d \sqrt{x}] \right)^2 \right) -$$

$$\left(b^2 x \left(a + b \operatorname{Csch} [c + d \sqrt{x}] \right)^2 \operatorname{Sech} \left[\frac{c}{2} \right] \operatorname{Sech} \left[\frac{c}{2} + \frac{d \sqrt{x}}{2} \right] \operatorname{Sinh} [c + d \sqrt{x}]^2 \operatorname{Sinh} \left[\frac{d \sqrt{x}}{2} \right] \right) /$$

$$\left(d \left(b + a \operatorname{Sinh} [c + d \sqrt{x}] \right)^2 \right)$$

Problem 69: Attempted integration timed out after 120 seconds.

$$\int \frac{1}{x^{3/2} \left(a + b \operatorname{Csch} [c + d \sqrt{x}] \right)^2} dx$$

Optimal (type 8, 25 leaves, 0 steps):

$$\operatorname{Int} \left[\frac{1}{x^{3/2} \left(a + b \operatorname{Csch} [c + d \sqrt{x}] \right)^2}, x \right]$$

Result (type 1, 1 leaves):

???

Problem 70: Attempted integration timed out after 120 seconds.

$$\int \frac{1}{x^{5/2} \left(a + b \operatorname{Csch} [c + d \sqrt{x}] \right)^2} dx$$

Optimal (type 8, 25 leaves, 0 steps):

$$\operatorname{Int} \left[\frac{1}{x^{5/2} \left(a + b \operatorname{Csch} [c + d \sqrt{x}] \right)^2}, x \right]$$

Result (type 1, 1 leaves):

???

Problem 74: Unable to integrate problem.

$$\int (e x)^{-1+3 n} (a+b \operatorname{Csch}[c+d x^n]) dx$$

Optimal (type 4, 197 leaves, 11 steps):

$$\frac{a (e x)^{3 n}}{3 e n} - \frac{2 b x^{-n} (e x)^{3 n} \operatorname{ArcTanh}\left[e^{c+d x^n}\right]}{d e n} - \frac{2 b x^{-2 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, -e^{c+d x^n}\right]}{d^2 e n} + \frac{2 b x^{-2 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, e^{c+d x^n}\right]}{d^2 e n} + \frac{2 b x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[3, -e^{c+d x^n}\right]}{d^3 e n} - \frac{2 b x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[3, e^{c+d x^n}\right]}{d^3 e n}$$

Result (type 8, 24 leaves):

$$\int (e x)^{-1+3 n} (a+b \operatorname{Csch}[c+d x^n]) dx$$

Problem 76: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int (e x)^{-1+2 n} (a+b \operatorname{Csch}[c+d x^n])^2 dx$$

Optimal (type 4, 198 leaves, 11 steps):

$$\frac{a^2 (e x)^{2 n}}{2 e n} - \frac{4 a b x^{-n} (e x)^{2 n} \operatorname{ArcTanh}\left[e^{c+d x^n}\right]}{d e n} - \frac{b^2 x^{-n} (e x)^{2 n} \operatorname{Coth}[c+d x^n]}{d e n} + \frac{b^2 x^{-2 n} (e x)^{2 n} \operatorname{Log}[\operatorname{Sinh}[c+d x^n]]}{d^2 e n} - \frac{2 a b x^{-2 n} (e x)^{2 n} \operatorname{PolyLog}\left[2, -e^{c+d x^n}\right]}{d^2 e n} + \frac{2 a b x^{-2 n} (e x)^{2 n} \operatorname{PolyLog}\left[2, e^{c+d x^n}\right]}{d^2 e n}$$

Result (type 4, 696 leaves):

$$\begin{aligned}
 & \frac{b^2 x^{1-n} (e x)^{-1+2n} \operatorname{Coth}[c] (a + b \operatorname{Csch}[c + d x^n])^2 \operatorname{Sinh}[c + d x^n]^2}{d n (b + a \operatorname{Sinh}[c + d x^n])^2} + \\
 & \left(x^{1-n} (e x)^{-1+2n} \operatorname{Csch}\left[\frac{c}{2}\right] (a + b \operatorname{Csch}[c + d x^n])^2 \right. \\
 & \quad \left. \operatorname{Sech}\left[\frac{c}{2}\right] (-2 b^2 \operatorname{Cosh}[c] + a^2 d x^n \operatorname{Sinh}[c]) \operatorname{Sinh}[c + d x^n]^2 \right) / \\
 & \left(4 d n (b + a \operatorname{Sinh}[c + d x^n])^2 \right) - \left(b^2 x^{1-2n} (e x)^{-1+2n} \operatorname{Csch}[c] (a + b \operatorname{Csch}[c + d x^n])^2 \right. \\
 & \quad \left. (-d x^n \operatorname{Cosh}[c] + \operatorname{Log}[\operatorname{Cosh}[d x^n] \operatorname{Sinh}[c] + \operatorname{Cosh}[c] \operatorname{Sinh}[d x^n]] \operatorname{Sinh}[c]) \operatorname{Sinh}[c + d x^n]^2 \right) / \\
 & \left(d^2 n (-\operatorname{Cosh}[c]^2 + \operatorname{Sinh}[c]^2) (b + a \operatorname{Sinh}[c + d x^n])^2 \right) + \\
 & \left(b^2 x^{1-n} (e x)^{-1+2n} \operatorname{Csch}\left[\frac{c}{2}\right] \operatorname{Csch}\left[\frac{c}{2} + \frac{d x^n}{2}\right] (a + b \operatorname{Csch}[c + d x^n])^2 \operatorname{Sinh}\left[\frac{d x^n}{2}\right] \operatorname{Sinh}[c + d x^n]^2 \right) / \\
 & \left(2 d n (b + a \operatorname{Sinh}[c + d x^n])^2 \right) - \\
 & \left(b^2 x^{1-n} (e x)^{-1+2n} (a + b \operatorname{Csch}[c + d x^n])^2 \operatorname{Sech}\left[\frac{c}{2}\right] \operatorname{Sech}\left[\frac{c}{2} + \frac{d x^n}{2}\right] \operatorname{Sinh}\left[\frac{d x^n}{2}\right] \operatorname{Sinh}[c + d x^n]^2 \right) / \\
 & \left(2 d n (b + a \operatorname{Sinh}[c + d x^n])^2 \right) + \left(2 a b x^{1-2n} (e x)^{-1+2n} (a + b \operatorname{Csch}[c + d x^n])^2 \right. \\
 & \quad \left. \operatorname{Sinh}[c + d x^n]^2 \left(-\frac{2 \operatorname{ArcTan}\left[\frac{\operatorname{Cosh}[c] + \operatorname{Sinh}[c] \operatorname{Tanh}\left[\frac{d x^n}{2}\right]}{\sqrt{-\operatorname{Cosh}[c]^2 + \operatorname{Sinh}[c]^2}}\right]}{\sqrt{-\operatorname{Cosh}[c]^2 + \operatorname{Sinh}[c]^2}} \operatorname{ArcTanh}[\operatorname{Tanh}[c]] \right. \right. \\
 & \quad \left. \left. - \frac{1}{\sqrt{1 - \operatorname{Tanh}[c]^2}} \right) \right. \\
 & \quad \left. \left(\operatorname{I}\left(\operatorname{I}\left(d x^n + \operatorname{ArcTanh}[\operatorname{Tanh}[c]]\right) \left(\operatorname{Log}\left[1 - e^{-d x^n - \operatorname{ArcTanh}[\operatorname{Tanh}[c]]}\right] - \operatorname{Log}\left[1 + e^{-d x^n - \operatorname{ArcTanh}[\operatorname{Tanh}[c]]}\right]\right) \right) \right. \right. \\
 & \quad \left. \left. \operatorname{I}\left(\operatorname{PolyLog}\left[2, -e^{-d x^n - \operatorname{ArcTanh}[\operatorname{Tanh}[c]}\right] - \operatorname{PolyLog}\left[2, e^{-d x^n - \operatorname{ArcTanh}[\operatorname{Tanh}[c]}\right]\right] \right) \right) \right) \\
 & \left. \left. \operatorname{Sech}[c] \right) \right) / \left(d^2 n (b + a \operatorname{Sinh}[c + d x^n])^2 \right)
 \end{aligned}$$

Problem 77: Attempted integration timed out after 120 seconds.

$$\int (e x)^{-1+3n} (a + b \operatorname{Csch}[c + d x^n])^2 dx$$

Optimal (type 4, 344 leaves, 16 steps):

$$\frac{a^2 (e x)^{3 n}}{3 e n} - \frac{b^2 x^{-n} (e x)^{3 n}}{d e n} - \frac{4 a b x^{-n} (e x)^{3 n} \operatorname{ArcTanh}\left[e^{c+d x^n}\right]}{d e n} - \frac{b^2 x^{-n} (e x)^{3 n} \operatorname{Coth}\left[c+d x^n\right]}{d e n} +$$

$$\frac{2 b^2 x^{-2 n} (e x)^{3 n} \operatorname{Log}\left[1-e^{2(c+d x^n)}\right]}{d^2 e n} - \frac{4 a b x^{-2 n} (e x)^{3 n} \operatorname{PolyLog}\left[2,-e^{c+d x^n}\right]}{d^2 e n} +$$

$$\frac{4 a b x^{-2 n} (e x)^{3 n} \operatorname{PolyLog}\left[2,e^{c+d x^n}\right]}{d^2 e n} + \frac{b^2 x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[2,e^{2(c+d x^n)}\right]}{d^3 e n} +$$

$$\frac{4 a b x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[3,-e^{c+d x^n}\right]}{d^3 e n} - \frac{4 a b x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[3,e^{c+d x^n}\right]}{d^3 e n}$$

Result(type 1, 1 leaves):

???

Problem 79: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(e x)^{-1+2 n}}{a+b \operatorname{Csch}\left[c+d x^n\right]} d x$$

Optimal (type 4, 291 leaves, 12 steps):

$$\frac{(e x)^{2 n}}{2 a e n} - \frac{b x^{-n} (e x)^{2 n} \operatorname{Log}\left[1+\frac{a e^{c+d x^n}}{b-\sqrt{a^2+b^2}}\right]}{a \sqrt{a^2+b^2} d e n} + \frac{b x^{-n} (e x)^{2 n} \operatorname{Log}\left[1+\frac{a e^{c+d x^n}}{b+\sqrt{a^2+b^2}}\right]}{a \sqrt{a^2+b^2} d e n} -$$

$$\frac{b x^{-2 n} (e x)^{2 n} \operatorname{PolyLog}\left[2,-\frac{a e^{c+d x^n}}{b-\sqrt{a^2+b^2}}\right]}{a \sqrt{a^2+b^2} d^2 e n} + \frac{b x^{-2 n} (e x)^{2 n} \operatorname{PolyLog}\left[2,-\frac{a e^{c+d x^n}}{b+\sqrt{a^2+b^2}}\right]}{a \sqrt{a^2+b^2} d^2 e n}$$

Result (type 4, 1347 leaves):

$$\frac{x (e x)^{-1+2 n} \operatorname{Csch}\left[c+d x^n\right] \left(b+a \operatorname{Sinh}\left[c+d x^n\right]\right)}{2 a n \left(a+b \operatorname{Csch}\left[c+d x^n\right]\right)} +$$

$$\frac{1}{a d^2 n \left(a+b \operatorname{Csch}\left[c+d x^n\right]\right)} b x^{1-2 n} (e x)^{-1+2 n} \operatorname{Csch}\left[c+d x^n\right] \left(\frac{i \pi \operatorname{ArcTanh}\left[\frac{-a+b \operatorname{Tanh}\left[\frac{1}{2}(c+d x^n)\right]}{\sqrt{a^2+b^2}}\right]}{\sqrt{a^2+b^2}} +\right.$$

$$\left.\frac{1}{\sqrt{-a^2-b^2}} \left(2\left(-i c+\frac{\pi}{2}-i d x^n\right) \operatorname{ArcTanh}\left[\frac{(-i a+b) \operatorname{Cot}\left[\frac{1}{2}\left(-i c+\frac{\pi}{2}-i d x^n\right)\right]}{\sqrt{-a^2-b^2}}\right]\right) -\right.$$

$$\left.2\left(-i c+\operatorname{ArcCos}\left[-\frac{i b}{a}\right]\right) \operatorname{ArcTanh}\left[\frac{(-i a-b) \operatorname{Tan}\left[\frac{1}{2}\left(-i c+\frac{\pi}{2}-i d x^n\right)\right]}{\sqrt{-a^2-b^2}}\right]\right) +$$

$$\left(\operatorname{ArcCos}\left[-\frac{i b}{a}\right]-2 i\left(\operatorname{ArcTanh}\left[\frac{(-i a+b) \operatorname{Cot}\left[\frac{1}{2}\left(-i c+\frac{\pi}{2}-i d x^n\right)\right]}{\sqrt{-a^2-b^2}}\right]\right)-\operatorname{ArcTanh}\left[\right.$$

$$\begin{aligned}
 & \left. \left(\frac{(-i a - b) \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right]}{\sqrt{-a^2 - b^2}} \right) \right) \operatorname{Log}\left[\frac{\sqrt{-a^2 - b^2} e^{-\frac{1}{2} i\left(-i c + \frac{\pi}{2} - i d x^n\right)}}{\sqrt{2} \sqrt{-i a} \sqrt{b + a \operatorname{Sinh}[c + d x^n]}}\right] + \\
 & \left(\operatorname{ArcCos}\left[-\frac{i b}{a}\right] + 2 i \left(\operatorname{ArcTanh}\left[\frac{(-i a + b) \operatorname{Cot}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right]}{\sqrt{-a^2 - b^2}}\right] - \operatorname{ArcTanh}\left[\frac{(-i a - b) \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right]}{\sqrt{-a^2 - b^2}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{-a^2 - b^2} e^{\frac{1}{2} i\left(-i c + \frac{\pi}{2} - i d x^n\right)}}{\sqrt{2} \sqrt{-i a} \sqrt{b + a \operatorname{Sinh}[c + d x^n]}}\right] - \\
 & \left(\operatorname{ArcCos}\left[-\frac{i b}{a}\right] + 2 i \operatorname{ArcTanh}\left[\frac{(-i a - b) \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right]}{\sqrt{-a^2 - b^2}}\right] \right) \\
 & \operatorname{Log}\left[1 - \left(i\left(b - i \sqrt{-a^2 - b^2}\right)\left(-i a + b - \sqrt{-a^2 - b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right]\right)\right) / \\
 & \left(a\left(-i a + b + \sqrt{-a^2 - b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right]\right)\right) + \\
 & \left(-\operatorname{ArcCos}\left[-\frac{i b}{a}\right] + 2 i \operatorname{ArcTanh}\left[\frac{(-i a - b) \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right]}{\sqrt{-a^2 - b^2}}\right] \right) \\
 & \operatorname{Log}\left[1 - \left(i\left(b + i \sqrt{-a^2 - b^2}\right)\left(-i a + b - \sqrt{-a^2 - b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right]\right)\right) / \\
 & \left(a\left(-i a + b + \sqrt{-a^2 - b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right]\right)\right) + \\
 & i \left(\operatorname{PolyLog}\left[2, \left(i\left(b - i \sqrt{-a^2 - b^2}\right)\left(-i a + b - \sqrt{-a^2 - b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right]\right)\right] \right) / \\
 & \left(a\left(-i a + b + \sqrt{-a^2 - b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right]\right)\right) - \\
 & \operatorname{PolyLog}\left[2, \left(i\left(b + i \sqrt{-a^2 - b^2}\right)\left(-i a + b - \sqrt{-a^2 - b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right]\right)\right) \right) / \\
 & \left(a\left(-i a + b + \sqrt{-a^2 - b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right]\right)\right) \right] \left(b + a \operatorname{Sinh}[c + d x^n] \right)
 \end{aligned}$$

Problem 80: Unable to integrate problem.

$$\int \frac{(e x)^{-1+3 n}}{a + b \operatorname{Csch}[c + d x^n]} dx$$

Optimal (type 4, 428 leaves, 14 steps):

$$\frac{(e x)^{3 n}}{3 a e n} - \frac{b x^{-n} (e x)^{3 n} \operatorname{Log}\left[1 + \frac{a e^{c+d x^n}}{b-\sqrt{a^2+b^2}}\right]}{a \sqrt{a^2+b^2} d e n} + \frac{b x^{-n} (e x)^{3 n} \operatorname{Log}\left[1 + \frac{a e^{c+d x^n}}{b+\sqrt{a^2+b^2}}\right]}{a \sqrt{a^2+b^2} d e n} -$$

$$\frac{2 b x^{-2 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x^n}}{b-\sqrt{a^2+b^2}}\right]}{a \sqrt{a^2+b^2} d^2 e n} + \frac{2 b x^{-2 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x^n}}{b+\sqrt{a^2+b^2}}\right]}{a \sqrt{a^2+b^2} d^2 e n} +$$

$$\frac{2 b x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[3, -\frac{a e^{c+d x^n}}{b-\sqrt{a^2+b^2}}\right]}{a \sqrt{a^2+b^2} d^3 e n} - \frac{2 b x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[3, -\frac{a e^{c+d x^n}}{b+\sqrt{a^2+b^2}}\right]}{a \sqrt{a^2+b^2} d^3 e n}$$

Result (type 8, 26 leaves):

$$\int \frac{(e x)^{-1+3 n}}{a+b \operatorname{Csch}[c+d x^n]} dx$$

Problem 82: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(e x)^{-1+2 n}}{(a+b \operatorname{Csch}[c+d x^n])^2} dx$$

Optimal (type 4, 681 leaves, 23 steps):

$$\frac{(e x)^{2 n}}{2 a^2 e n} + \frac{b^3 x^{-n} (e x)^{2 n} \operatorname{Log}\left[1 + \frac{a e^{c+d x^n}}{b-\sqrt{a^2+b^2}}\right]}{a^2 (a^2+b^2)^{3/2} d e n} - \frac{2 b x^{-n} (e x)^{2 n} \operatorname{Log}\left[1 + \frac{a e^{c+d x^n}}{b-\sqrt{a^2+b^2}}\right]}{a^2 \sqrt{a^2+b^2} d e n} -$$

$$\frac{b^3 x^{-n} (e x)^{2 n} \operatorname{Log}\left[1 + \frac{a e^{c+d x^n}}{b+\sqrt{a^2+b^2}}\right]}{a^2 (a^2+b^2)^{3/2} d e n} + \frac{2 b x^{-n} (e x)^{2 n} \operatorname{Log}\left[1 + \frac{a e^{c+d x^n}}{b+\sqrt{a^2+b^2}}\right]}{a^2 \sqrt{a^2+b^2} d e n} +$$

$$\frac{b^2 x^{-2 n} (e x)^{2 n} \operatorname{Log}[b+a \operatorname{Sinh}[c+d x^n]]}{a^2 (a^2+b^2) d^2 e n} + \frac{b^3 x^{-2 n} (e x)^{2 n} \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x^n}}{b-\sqrt{a^2+b^2}}\right]}{a^2 (a^2+b^2)^{3/2} d^2 e n} -$$

$$\frac{2 b x^{-2 n} (e x)^{2 n} \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x^n}}{b-\sqrt{a^2+b^2}}\right]}{a^2 \sqrt{a^2+b^2} d^2 e n} - \frac{b^3 x^{-2 n} (e x)^{2 n} \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x^n}}{b+\sqrt{a^2+b^2}}\right]}{a^2 (a^2+b^2)^{3/2} d^2 e n} +$$

$$\frac{2 b x^{-2 n} (e x)^{2 n} \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x^n}}{b+\sqrt{a^2+b^2}}\right]}{a^2 \sqrt{a^2+b^2} d^2 e n} - \frac{b^2 x^{-n} (e x)^{2 n} \operatorname{Cosh}[c+d x^n]}{a (a^2+b^2) d e n (b+a \operatorname{Sinh}[c+d x^n])}$$

Result (type 4, 3256 leaves):

$$\left(b^2 x^{1-n} (e x)^{-1+2 n} \operatorname{Csch}\left[\frac{c}{2}\right] \operatorname{Csch}[c+d x^n]^2 \operatorname{Sech}\left[\frac{c}{2}\right] (b \operatorname{Cosh}[c] + a \operatorname{Sinh}[d x^n]) \right. \\ \left. (b+a \operatorname{Sinh}[c+d x^n]) \right) / \left(2 a^2 (a^2+b^2) d n (a+b \operatorname{Csch}[c+d x^n])^2 \right) +$$

$$\begin{aligned}
 & \frac{b^2 x^{1-n} (e x)^{-1+2n} \operatorname{Coth}[c] \operatorname{Csch}[c+d x^n]^2 (b+a \operatorname{Sinh}[c+d x^n])^2}{a^2 (a^2+b^2) d n (a+b \operatorname{Csch}[c+d x^n])^2} - \\
 & \left(2 b^3 x^{1-2n} (e x)^{-1+2n} \operatorname{ArcTan}\left[\frac{a-b \operatorname{Tanh}\left[\frac{1}{2}(c+d x^n)\right]}{\sqrt{-a^2-b^2}}\right] \right. \\
 & \quad \left. \operatorname{Coth}[c] \operatorname{Csch}[c+d x^n]^2 (b+a \operatorname{Sinh}[c+d x^n])^2 \right) / \\
 & \left(a^2 \sqrt{-a^2-b^2} (a^2+b^2) d^2 n (a+b \operatorname{Csch}[c+d x^n])^2 \right) + \frac{1}{(a^2+b^2) d^2 n (a+b \operatorname{Csch}[c+d x^n])^2} \\
 & 2 b x^{1-2n} (e x)^{-1+2n} \operatorname{Csch}[c+d x^n]^2 \left(\frac{i \pi \operatorname{ArcTanh}\left[\frac{-a+b \operatorname{Tanh}\left[\frac{1}{2}(c+d x^n)\right]}{\sqrt{a^2+b^2}}\right]}{\sqrt{a^2+b^2}} + \right. \\
 & \quad \left. \frac{1}{\sqrt{-a^2-b^2}} \left(2 \left(-i c + \frac{\pi}{2} - i d x^n \right) \operatorname{ArcTanh}\left[\frac{(-i a+b) \operatorname{Cot}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right]}{\sqrt{-a^2-b^2}}\right] - \right. \right. \\
 & \quad \left. \left. 2 \left(-i c + \operatorname{ArcCos}\left[-\frac{i b}{a}\right]\right) \operatorname{ArcTanh}\left[\frac{(-i a-b) \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right]}{\sqrt{-a^2-b^2}}\right] + \right. \right. \\
 & \quad \left. \left. \left(\operatorname{ArcCos}\left[-\frac{i b}{a}\right] - 2 i \left(\operatorname{ArcTanh}\left[\frac{(-i a+b) \operatorname{Cot}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right]}{\sqrt{-a^2-b^2}}\right]} - \operatorname{ArcTanh}\left[\frac{(-i a-b) \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right]}{\sqrt{-a^2-b^2}}\right]} \right) \right) \right) \operatorname{Log}\left[\frac{\sqrt{-a^2-b^2} e^{-\frac{1}{2} i \left(-i c + \frac{\pi}{2} - i d x^n\right)}}{\sqrt{2} \sqrt{-i a} \sqrt{b+a \operatorname{Sinh}[c+d x^n]}}\right] + \right. \\
 & \quad \left. \left(\operatorname{ArcCos}\left[-\frac{i b}{a}\right] + 2 i \left(\operatorname{ArcTanh}\left[\frac{(-i a+b) \operatorname{Cot}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right]}{\sqrt{-a^2-b^2}}\right]} - \operatorname{ArcTanh}\left[\frac{(-i a-b) \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right]}{\sqrt{-a^2-b^2}}\right]} \right) \right) \right) \operatorname{Log}\left[\frac{\sqrt{-a^2-b^2} e^{\frac{1}{2} i \left(-i c + \frac{\pi}{2} - i d x^n\right)}}{\sqrt{2} \sqrt{-i a} \sqrt{b+a \operatorname{Sinh}[c+d x^n]}}\right] - \\
 & \quad \left(\operatorname{ArcCos}\left[-\frac{i b}{a}\right] + 2 i \operatorname{ArcTanh}\left[\frac{(-i a-b) \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right]}{\sqrt{-a^2-b^2}}\right] \right) \\
 & \quad \operatorname{Log}\left[1 - \left(i \left(b - i \sqrt{-a^2-b^2} \right) \left(-i a + b - \sqrt{-a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right] \right) \right) \right] / \\
 & \quad \left(a \left(-i a + b + \sqrt{-a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right] \right) \right) + \\
 & \quad \left(-\operatorname{ArcCos}\left[-\frac{i b}{a}\right] + 2 i \operatorname{ArcTanh}\left[\frac{(-i a-b) \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right]}{\sqrt{-a^2-b^2}}\right] \right) \\
 & \quad \operatorname{Log}\left[1 - \left(i \left(b + i \sqrt{-a^2-b^2} \right) \left(-i a + b - \sqrt{-a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-i c + \frac{\pi}{2} - i d x^n\right)\right] \right) \right) \right] /
 \end{aligned}$$

$$\begin{aligned}
 & \left(a \left(-i a + b + \sqrt{-a^2 - b^2} \operatorname{Tan} \left[\frac{1}{2} \left(-i c + \frac{\pi}{2} - i d x^n \right) \right] \right) \right) + \\
 & i \left(\operatorname{PolyLog} \left[2, \left(i \left(b - i \sqrt{-a^2 - b^2} \right) \left(-i a + b - \sqrt{-a^2 - b^2} \operatorname{Tan} \left[\frac{1}{2} \left(-i c + \frac{\pi}{2} - i d x^n \right) \right] \right) \right] \right) / \\
 & \left(a \left(-i a + b + \sqrt{-a^2 - b^2} \operatorname{Tan} \left[\frac{1}{2} \left(-i c + \frac{\pi}{2} - i d x^n \right) \right] \right) \right) - \\
 & \operatorname{PolyLog} \left[2, \left(i \left(b + i \sqrt{-a^2 - b^2} \right) \left(-i a + b - \sqrt{-a^2 - b^2} \operatorname{Tan} \left[\frac{1}{2} \left(-i c + \frac{\pi}{2} - i d x^n \right) \right] \right) \right) \right] / \\
 & \left(a \left(-i a + b + \sqrt{-a^2 - b^2} \operatorname{Tan} \left[\frac{1}{2} \left(-i c + \frac{\pi}{2} - i d x^n \right) \right] \right) \right) \right) \\
 & (b + a \operatorname{Sinh}[c + d x^n])^2 + \frac{1}{a^2 (a^2 + b^2) d^2 n (a + b \operatorname{Csch}[c + d x^n])^2} \\
 & b^3 \\
 & x^{1-2n} \\
 & (e x)^{-1+2n} \\
 & \operatorname{Csch}[c + d x^n]^2 \\
 & \left(\frac{i \pi \operatorname{ArcTanh} \left[\frac{-a+b \operatorname{Tanh} \left[\frac{1}{2} (c+d x^n) \right]}{\sqrt{a^2+b^2}} \right]}{\sqrt{a^2+b^2}} + \right. \\
 & \frac{1}{\sqrt{-a^2-b^2}} \left(2 \left(-i c + \frac{\pi}{2} - i d x^n \right) \operatorname{ArcTanh} \left[\frac{(-i a + b) \operatorname{Cot} \left[\frac{1}{2} \left(-i c + \frac{\pi}{2} - i d x^n \right) \right]}{\sqrt{-a^2-b^2}} \right] - \right. \\
 & 2 \left(-i c + \operatorname{ArcCos} \left[-\frac{i b}{a} \right] \right) \operatorname{ArcTanh} \left[\frac{(-i a - b) \operatorname{Tan} \left[\frac{1}{2} \left(-i c + \frac{\pi}{2} - i d x^n \right) \right]}{\sqrt{-a^2-b^2}} \right] + \\
 & \left. \left(\operatorname{ArcCos} \left[-\frac{i b}{a} \right] - 2 i \left(\operatorname{ArcTanh} \left[\frac{(-i a + b) \operatorname{Cot} \left[\frac{1}{2} \left(-i c + \frac{\pi}{2} - i d x^n \right) \right]}{\sqrt{-a^2-b^2}} \right] - \operatorname{ArcTanh} \left[\right. \right. \right. \\
 & \left. \left. \left. \frac{(-i a - b) \operatorname{Tan} \left[\frac{1}{2} \left(-i c + \frac{\pi}{2} - i d x^n \right) \right]}{\sqrt{-a^2-b^2}} \right] \right) \right) \operatorname{Log} \left[\frac{\sqrt{-a^2-b^2} e^{-\frac{1}{2} i \left(-i c + \frac{\pi}{2} - i d x^n \right)}}{\sqrt{2} \sqrt{-i a} \sqrt{b + a \operatorname{Sinh}[c + d x^n]}} \right] + \\
 & \left(\operatorname{ArcCos} \left[-\frac{i b}{a} \right] + 2 i \left(\operatorname{ArcTanh} \left[\frac{(-i a + b) \operatorname{Cot} \left[\frac{1}{2} \left(-i c + \frac{\pi}{2} - i d x^n \right) \right]}{\sqrt{-a^2-b^2}} \right] - \operatorname{ArcTanh} \left[\right. \right. \right. \\
 & \left. \left. \left. \frac{(-i a - b) \operatorname{Tan} \left[\frac{1}{2} \left(-i c + \frac{\pi}{2} - i d x^n \right) \right]}{\sqrt{-a^2-b^2}} \right] \right) \right) \operatorname{Log} \left[\frac{\sqrt{-a^2-b^2} e^{\frac{1}{2} i \left(-i c + \frac{\pi}{2} - i d x^n \right)}}{\sqrt{2} \sqrt{-i a} \sqrt{b + a \operatorname{Sinh}[c + d x^n]}} \right] - \\
 & \left. \left(\operatorname{ArcCos} \left[-\frac{i b}{a} \right] + 2 i \operatorname{ArcTanh} \left[\frac{(-i a - b) \operatorname{Tan} \left[\frac{1}{2} \left(-i c + \frac{\pi}{2} - i d x^n \right) \right]}{\sqrt{-a^2-b^2}} \right] \right) \right)
 \end{aligned}$$

$$\begin{aligned} & \left(\frac{\text{Log}\left[1 - \left(i \left(b - i \sqrt{-a^2 - b^2}\right) \left(-i a + b - \sqrt{-a^2 - b^2} \text{Tan}\left[\frac{1}{2} \left(-i c + \frac{\pi}{2} - i d x^n\right)\right]\right)\right]}{\left(a \left(-i a + b + \sqrt{-a^2 - b^2} \text{Tan}\left[\frac{1}{2} \left(-i c + \frac{\pi}{2} - i d x^n\right)\right]\right)\right)} + \right. \\ & \left. \left(-\text{ArcCos}\left[-\frac{i b}{a}\right] + 2 i \text{ArcTanh}\left[\frac{\left(-i a - b\right) \text{Tan}\left[\frac{1}{2} \left(-i c + \frac{\pi}{2} - i d x^n\right)\right]}{\sqrt{-a^2 - b^2}}\right]\right) \right) \\ & \left(\frac{\text{Log}\left[1 - \left(i \left(b + i \sqrt{-a^2 - b^2}\right) \left(-i a + b - \sqrt{-a^2 - b^2} \text{Tan}\left[\frac{1}{2} \left(-i c + \frac{\pi}{2} - i d x^n\right)\right]\right)\right]}{\left(a \left(-i a + b + \sqrt{-a^2 - b^2} \text{Tan}\left[\frac{1}{2} \left(-i c + \frac{\pi}{2} - i d x^n\right)\right]\right)\right)} + \right. \\ & \left. i \left(\text{PolyLog}\left[2, \left(i \left(b - i \sqrt{-a^2 - b^2}\right) \left(-i a + b - \sqrt{-a^2 - b^2} \text{Tan}\left[\frac{1}{2} \left(-i c + \frac{\pi}{2} - i d x^n\right)\right]\right)\right]\right) \right. \\ & \left. \left(a \left(-i a + b + \sqrt{-a^2 - b^2} \text{Tan}\left[\frac{1}{2} \left(-i c + \frac{\pi}{2} - i d x^n\right)\right]\right)\right) - \right. \\ & \left. \text{PolyLog}\left[2, \left(i \left(b + i \sqrt{-a^2 - b^2}\right) \left(-i a + b - \sqrt{-a^2 - b^2} \text{Tan}\left[\frac{1}{2} \left(-i c + \frac{\pi}{2} - i d x^n\right)\right]\right)\right]\right] \right) \right) \\ & \left. \left(a \left(-i a + b + \sqrt{-a^2 - b^2} \text{Tan}\left[\frac{1}{2} \left(-i c + \frac{\pi}{2} - i d x^n\right)\right]\right)\right) \right) \right) \end{aligned}$$

$$(b + a \text{Sinh}[c + d x^n])^2 + (x^{1-n} (e x)^{-1+2n} \text{Csch}\left[\frac{c}{2}\right] \text{Csch}[c + d x^n])^2$$

$$\text{Sech}\left[\frac{c}{2}\right]$$

$$(-2 b^2 \text{Cosh}[c] + a^2 d x^n \text{Sinh}[c] + b^2 d x^n \text{Sinh}[c])$$

$$(b + a \text{Sinh}[c + d x^n])^2 \Big/ (4$$

$$\frac{a^2}{(a^2 + b^2)}$$

$$d$$

$$n$$

$$(a + b \text{Csch}[c + d x^n])^2 - b^2$$

$$x^{1-2n}$$

$$(e x)^{-1+2n}$$

$$\text{Csch}[c]$$

$$\text{Csch}[c + d x^n]^2$$

$$\left(-a d x^n \text{Cosh}[c] + a \text{Log}\left[b + a \text{Cosh}[d x^n] \text{Sinh}[c] + a \text{Cosh}[c] \text{Sinh}[d x^n]\right] \text{Sinh}[c] + \right.$$

$$\left. \frac{2 a b \operatorname{ArcTan}\left[\frac{a \operatorname{Cosh}[c]+(-b+a \operatorname{Sinh}[c]) \operatorname{Tanh}\left[\frac{d x^n}{2}\right]}{\sqrt{-b^2-a^2 \operatorname{Cosh}[c]^2+a^2 \operatorname{Sinh}[c]^2}}\right] \operatorname{Cosh}[c]}{\sqrt{-b^2-a^2 \operatorname{Cosh}[c]^2+a^2 \operatorname{Sinh}[c]^2}}\right)$$

$$\left. \left. (b+a \operatorname{Sinh}[c+d x^n])^2 \right) / \left(a (a^2+b^2) \right.$$

$$\left. \left. \begin{array}{l} d^2 \\ n \\ (a+b \operatorname{Csch}[c+d x^n])^2 \\ (-a^2 \operatorname{Cosh}[c]^2+a^2 \operatorname{Sinh}[c]^2) \end{array} \right)$$

Problem 83: Attempted integration timed out after 120 seconds.

$$\int \frac{(e x)^{-1+3 n}}{(a+b \operatorname{Csch}[c+d x^n])^2} dx$$

Optimal (type 4, 1218 leaves, 32 steps):

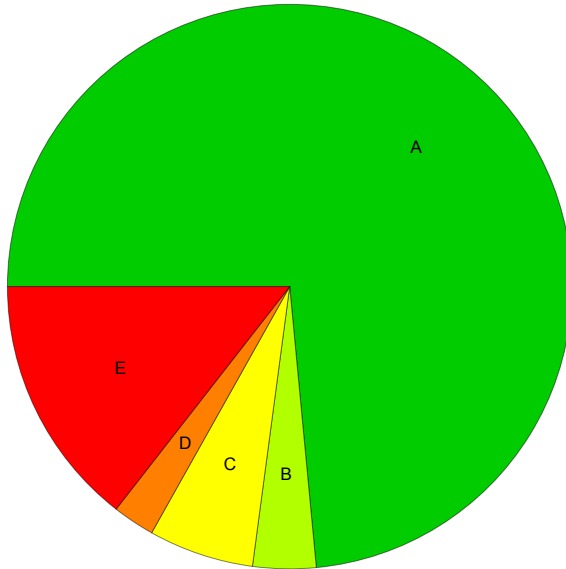
$$\begin{aligned}
 & \frac{(e x)^{3 n}}{3 a^2 e n} - \frac{b^2 x^{-n} (e x)^{3 n}}{a^2 (a^2 + b^2) d e n} + \frac{2 b^2 x^{-2 n} (e x)^{3 n} \operatorname{Log}\left[1 + \frac{a e^{c+d x^n}}{b-\sqrt{a^2+b^2}}\right]}{a^2 (a^2 + b^2) d^2 e n} + \\
 & \frac{b^3 x^{-n} (e x)^{3 n} \operatorname{Log}\left[1 + \frac{a e^{c+d x^n}}{b-\sqrt{a^2+b^2}}\right]}{a^2 (a^2 + b^2)^{3/2} d e n} - \frac{2 b x^{-n} (e x)^{3 n} \operatorname{Log}\left[1 + \frac{a e^{c+d x^n}}{b-\sqrt{a^2+b^2}}\right]}{a^2 \sqrt{a^2 + b^2} d e n} + \\
 & \frac{2 b^2 x^{-2 n} (e x)^{3 n} \operatorname{Log}\left[1 + \frac{a e^{c+d x^n}}{b+\sqrt{a^2+b^2}}\right]}{a^2 (a^2 + b^2) d^2 e n} - \frac{b^3 x^{-n} (e x)^{3 n} \operatorname{Log}\left[1 + \frac{a e^{c+d x^n}}{b+\sqrt{a^2+b^2}}\right]}{a^2 (a^2 + b^2)^{3/2} d e n} + \\
 & \frac{2 b x^{-n} (e x)^{3 n} \operatorname{Log}\left[1 + \frac{a e^{c+d x^n}}{b+\sqrt{a^2+b^2}}\right]}{a^2 \sqrt{a^2 + b^2} d e n} + \frac{2 b^2 x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x^n}}{b-\sqrt{a^2+b^2}}\right]}{a^2 (a^2 + b^2) d^3 e n} + \\
 & \frac{2 b^3 x^{-2 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x^n}}{b-\sqrt{a^2+b^2}}\right]}{a^2 (a^2 + b^2)^{3/2} d^2 e n} - \frac{4 b x^{-2 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x^n}}{b-\sqrt{a^2+b^2}}\right]}{a^2 \sqrt{a^2 + b^2} d^2 e n} + \\
 & \frac{2 b^2 x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x^n}}{b+\sqrt{a^2+b^2}}\right]}{a^2 (a^2 + b^2) d^3 e n} - \frac{2 b^3 x^{-2 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x^n}}{b+\sqrt{a^2+b^2}}\right]}{a^2 (a^2 + b^2)^{3/2} d^2 e n} + \\
 & \frac{4 b x^{-2 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, -\frac{a e^{c+d x^n}}{b+\sqrt{a^2+b^2}}\right]}{a^2 \sqrt{a^2 + b^2} d^2 e n} - \frac{2 b^3 x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[3, -\frac{a e^{c+d x^n}}{b-\sqrt{a^2+b^2}}\right]}{a^2 (a^2 + b^2)^{3/2} d^3 e n} + \\
 & \frac{4 b x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[3, -\frac{a e^{c+d x^n}}{b-\sqrt{a^2+b^2}}\right]}{a^2 \sqrt{a^2 + b^2} d^3 e n} + \frac{2 b^3 x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[3, -\frac{a e^{c+d x^n}}{b+\sqrt{a^2+b^2}}\right]}{a^2 (a^2 + b^2)^{3/2} d^3 e n} - \\
 & \frac{4 b x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[3, -\frac{a e^{c+d x^n}}{b+\sqrt{a^2+b^2}}\right]}{a^2 \sqrt{a^2 + b^2} d^3 e n} - \frac{b^2 x^{-n} (e x)^{3 n} \operatorname{Cosh}[c + d x^n]}{a (a^2 + b^2) d e n (b + a \operatorname{Sinh}[c + d x^n])}
 \end{aligned}$$

Result(type 1, 1 leaves):

???

Summary of Integration Test Results

83 integration problems



- A - 61 optimal antiderivatives
- B - 3 more than twice size of optimal antiderivatives
- C - 5 unnecessarily complex antiderivatives
- D - 2 unable to integrate problems
- E - 12 integration timeouts