

Mathematica 11.3 Integration Test Results

Test results for the 175 problems in "6.6.3 Hyperbolic cosecant functions.m"

Problem 1: Result more than twice size of optimal antiderivative.

$$\int \text{Csch}[a + b x] dx$$

Optimal (type 3, 12 leaves, 1 step):

$$-\frac{\text{ArcTanh}[\text{Cosh}[a + b x]]}{b}$$

Result (type 3, 38 leaves):

$$-\frac{\text{Log}\left[\text{Cosh}\left[\frac{a}{2} + \frac{bx}{2}\right]\right]}{b} + \frac{\text{Log}\left[\text{Sinh}\left[\frac{a}{2} + \frac{bx}{2}\right]\right]}{b}$$

Problem 3: Result more than twice size of optimal antiderivative.

$$\int \text{Csch}[a + b x]^3 dx$$

Optimal (type 3, 34 leaves, 2 steps):

$$\frac{\text{ArcTanh}[\text{Cosh}[a + b x]]}{2 b} - \frac{\text{Coth}[a + b x] \text{Csch}[a + b x]}{2 b}$$

Result (type 3, 75 leaves):

$$-\frac{\text{Csch}\left[\frac{1}{2}(a + b x)\right]^2}{8 b} + \frac{\text{Log}\left[\text{Cosh}\left[\frac{1}{2}(a + b x)\right]\right]}{2 b} - \frac{\text{Log}\left[\text{Sinh}\left[\frac{1}{2}(a + b x)\right]\right]}{2 b} - \frac{\text{Sech}\left[\frac{1}{2}(a + b x)\right]^2}{8 b}$$

Problem 5: Result more than twice size of optimal antiderivative.

$$\int \text{Csch}[a + b x]^5 dx$$

Optimal (type 3, 55 leaves, 3 steps):

$$-\frac{3 \text{ArcTanh}[\text{Cosh}[a + b x]]}{8 b} + \frac{3 \text{Coth}[a + b x] \text{Csch}[a + b x]}{8 b} - \frac{\text{Coth}[a + b x] \text{Csch}[a + b x]^3}{4 b}$$

Result (type 3, 113 leaves):

$$\frac{3 \operatorname{Csch}\left[\frac{1}{2}(a+bx)\right]^2}{32b} - \frac{\operatorname{Csch}\left[\frac{1}{2}(a+bx)\right]^4}{64b} - \frac{3 \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{1}{2}(a+bx)\right]\right]}{8b} + \frac{3 \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{1}{2}(a+bx)\right]\right]}{8b} + \frac{3 \operatorname{Sech}\left[\frac{1}{2}(a+bx)\right]^2}{32b} + \frac{\operatorname{Sech}\left[\frac{1}{2}(a+bx)\right]^4}{64b}$$

Problem 23: Result more than twice size of optimal antiderivative.

$$\int (-\operatorname{Csch}[x]^2)^{3/2} dx$$

Optimal (type 3, 24 leaves, 3 steps):

$$\frac{1}{2} \operatorname{ArcSin}[\operatorname{Coth}[x]] + \frac{1}{2} \operatorname{Coth}[x] \sqrt{-\operatorname{Csch}[x]^2}$$

Result (type 3, 51 leaves):

$$\frac{1}{8} \sqrt{-\operatorname{Csch}[x]^2} \left(\operatorname{Csch}\left[\frac{x}{2}\right]^2 - 4 \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{x}{2}\right]\right] + 4 \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{x}{2}\right]\right] + \operatorname{Sech}\left[\frac{x}{2}\right]^2 \right) \operatorname{Sinh}[x]$$

Problem 24: Result more than twice size of optimal antiderivative.

$$\int \sqrt{-\operatorname{Csch}[x]^2} dx$$

Optimal (type 3, 3 leaves, 2 steps):

$$\operatorname{ArcSin}[\operatorname{Coth}[x]]$$

Result (type 3, 30 leaves):

$$\sqrt{-\operatorname{Csch}[x]^2} \left(-\operatorname{Log}\left[\operatorname{Cosh}\left[\frac{x}{2}\right]\right] + \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{x}{2}\right]\right] \right) \operatorname{Sinh}[x]$$

Problem 54: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{a+ia \operatorname{Csch}[c+dx]}} dx$$

Optimal (type 3, 91 leaves, 5 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Coth}[c+dx]}{\sqrt{a+ia \operatorname{Csch}[c+dx]}}\right]}{\sqrt{a} d} - \frac{\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Coth}[c+dx]}{\sqrt{2} \sqrt{a+ia \operatorname{Csch}[c+dx]}}\right]}{\sqrt{a} d}$$

Result (type 3, 254 leaves):

$$\begin{aligned}
 & \left(\sqrt{a} \operatorname{Coth}[c + dx] \left(\sqrt{2} \operatorname{ArcTan} \left[\frac{\sqrt{2} \sqrt{a}}{\sqrt{i a (i + \operatorname{Csch}[c + dx])}} \right] - \right. \right. \\
 & \quad \left. \left. i \left(\operatorname{Log} \left[- \left(\left(2 a \left(-2 i \sqrt{a} + \sqrt{i a (i + \operatorname{Csch}[c + dx])} \right) + i \sqrt{a + i a \operatorname{Csch}[c + dx]} \right) \right) \right] / \right. \right. \\
 & \quad \left. \left. \left(-\sqrt{a} + \sqrt{a + i a \operatorname{Csch}[c + dx]} \right) \right) \right] + \right. \\
 & \quad \left. \operatorname{Log} \left[\left(2 i a \left(2 \sqrt{a} + i \sqrt{i a (i + \operatorname{Csch}[c + dx])} + \sqrt{a + i a \operatorname{Csch}[c + dx]} \right) \right) \right] / \right. \\
 & \quad \left. \left. \left(\sqrt{a} + \sqrt{a + i a \operatorname{Csch}[c + dx]} \right) \right) \right] \right) \Bigg) / \\
 & \left(d \sqrt{i a (i + \operatorname{Csch}[c + dx])} \sqrt{a + i a \operatorname{Csch}[c + dx]} \right)
 \end{aligned}$$

Problem 55: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + i a \operatorname{Csch}[c + dx])^{3/2}} dx$$

Optimal (type 3, 123 leaves, 6 steps):

$$\frac{2 \operatorname{ArcTanh} \left[\frac{\sqrt{a} \operatorname{Coth}[c + dx]}{\sqrt{a + i a \operatorname{Csch}[c + dx]}} \right]}{a^{3/2} d} - \frac{5 \operatorname{ArcTanh} \left[\frac{\sqrt{a} \operatorname{Coth}[c + dx]}{\sqrt{2} \sqrt{a + i a \operatorname{Csch}[c + dx]}} \right]}{2 \sqrt{2} a^{3/2} d} - \frac{\operatorname{Coth}[c + dx]}{2 d (a + i a \operatorname{Csch}[c + dx])^{3/2}}$$

Result (type 3, 380 leaves):

$$\begin{aligned}
 & \left(i \left(\left(a^{3/2} \operatorname{Coth}[c + dx] \left(-4 i \sqrt{2} \operatorname{ArcTan} \left[\frac{\sqrt{2} \sqrt{a}}{\sqrt{i a (i + \operatorname{Csch}[c + dx])}} \right] + \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \sqrt{2} \operatorname{Log} \left[- \frac{2 \left(-i \sqrt{2} \sqrt{a} + \sqrt{i a (i + \operatorname{Csch}[c + dx])} \right)}{\sqrt{a + i a \operatorname{Csch}[c + dx]}} \right] - \right. \right. \right. \\
 & \quad \left. \left. \left. 4 \left(\operatorname{Log} \left[- \left(\left(2 a \left(-2 i \sqrt{a} + \sqrt{i a (i + \operatorname{Csch}[c + dx])} \right) + i \sqrt{a + i a \operatorname{Csch}[c + dx]} \right) \right) \right] / \right. \right. \right. \\
 & \quad \left. \left. \left. \left(-\sqrt{a} + \sqrt{a + i a \operatorname{Csch}[c + dx]} \right) \right) \right] + \right. \right. \\
 & \quad \left. \left. \left. \operatorname{Log} \left[\left(2 i a \left(2 \sqrt{a} + i \sqrt{i a (i + \operatorname{Csch}[c + dx])} + \sqrt{a + i a \operatorname{Csch}[c + dx]} \right) \right) \right] / \right. \right. \right. \\
 & \quad \left. \left. \left. \left(\sqrt{a} + \sqrt{a + i a \operatorname{Csch}[c + dx]} \right) \right) \right] \right) \Bigg) \Bigg) / \\
 & \left(\sqrt{i a (i + \operatorname{Csch}[c + dx])} + \frac{2 a \left(\operatorname{Cosh} \left[\frac{1}{2} (c + dx) \right] + i \operatorname{Sinh} \left[\frac{1}{2} (c + dx) \right] \right)}{\operatorname{Cosh} \left[\frac{1}{2} (c + dx) \right] - i \operatorname{Sinh} \left[\frac{1}{2} (c + dx) \right]} \right) \Bigg) / \\
 & \left(4 a^2 d \sqrt{a + i a \operatorname{Csch}[c + dx]} \right)
 \end{aligned}$$

Problem 57: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{a - i a \operatorname{Csch}[c + d x]}} dx$$

Optimal (type 3, 91 leaves, 5 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Coth}[c + d x]}{\sqrt{a - i a \operatorname{Csch}[c + d x]}}\right]}{\sqrt{a} d} - \frac{\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Coth}[c + d x]}{\sqrt{2} \sqrt{a - i a \operatorname{Csch}[c + d x]}}\right]}{\sqrt{a} d}$$

Result (type 3, 253 leaves):

$$\left(\sqrt{a} \operatorname{Coth}[c + d x] \left(\sqrt{2} \operatorname{ArcTan}\left[\frac{\sqrt{2} \sqrt{a}}{\sqrt{-i a (-i + \operatorname{Csch}[c + d x])}} \right] - \right. \right. \\ \left. \left. i \left(\operatorname{Log}\left[- \left(\left(2 a \left(-2 i \sqrt{a} + \sqrt{-i a (-i + \operatorname{Csch}[c + d x])} \right) + i \sqrt{a - i a \operatorname{Csch}[c + d x]} \right) \right] \right) / \right. \right. \\ \left. \left. \left(-\sqrt{a} + \sqrt{a - i a \operatorname{Csch}[c + d x]} \right) \right] \right) + \\ \left. \operatorname{Log}\left[\left(2 i a \left(2 \sqrt{a} + i \sqrt{-i a (-i + \operatorname{Csch}[c + d x])} \right) + \sqrt{a - i a \operatorname{Csch}[c + d x]} \right) \right] \right) / \\ \left. \left(\sqrt{a} + \sqrt{a - i a \operatorname{Csch}[c + d x]} \right) \right] \right) / \\ \left(d \sqrt{a (-1 - i \operatorname{Csch}[c + d x])} \sqrt{a - i a \operatorname{Csch}[c + d x]} \right)$$

Problem 60: Result more than twice size of optimal antiderivative.

$$\int \sqrt{-3 + 3 i \operatorname{Csch}[x]} dx$$

Optimal (type 3, 23 leaves, 2 steps):

$$-2 \sqrt{3} \operatorname{ArcTan}\left[\frac{\operatorname{Coth}[x]}{\sqrt{-1 + i \operatorname{Csch}[x]}}\right]$$

Result (type 3, 67 leaves):

$$\frac{\sqrt{3} \operatorname{Coth}[x] \left(\operatorname{Log}\left[1 - \sqrt{1 + i \operatorname{Csch}[x]} \right] - \operatorname{Log}\left[1 + \sqrt{1 + i \operatorname{Csch}[x]} \right] \right)}{\sqrt{-1 + i \operatorname{Csch}[x]} \sqrt{1 + i \operatorname{Csch}[x]}}$$

Problem 61: Result more than twice size of optimal antiderivative.

$$\int \sqrt{-3 - 3 i \operatorname{Csch}[x]} dx$$

Optimal (type 3, 23 leaves, 2 steps):

$$-2\sqrt{3} \operatorname{ArcTan}\left[\frac{\operatorname{Coth}[x]}{\sqrt{-1-i\operatorname{Csch}[x]}}\right]$$

Result (type 3, 67 leaves):

$$\frac{\sqrt{3} \operatorname{Coth}[x] \left(\operatorname{Log}\left[1 - \sqrt{1 - i\operatorname{Csch}[x]}\right] - \operatorname{Log}\left[1 + \sqrt{1 - i\operatorname{Csch}[x]}\right] \right)}{\sqrt{-1 - i\operatorname{Csch}[x]} \sqrt{1 - i\operatorname{Csch}[x]}}$$

Problem 67: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Csch}[x]^2}{i + \operatorname{Csch}[x]} dx$$

Optimal (type 3, 17 leaves, 3 steps):

$$-\operatorname{ArcTanh}[\operatorname{Cosh}[x]] + \frac{\operatorname{Coth}[x]}{i + \operatorname{Csch}[x]}$$

Result (type 3, 46 leaves):

$$-\operatorname{Log}\left[\operatorname{Cosh}\left[\frac{x}{2}\right]\right] + \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{x}{2}\right]\right] - \frac{2i\operatorname{Sinh}\left[\frac{x}{2}\right]}{\operatorname{Cosh}\left[\frac{x}{2}\right] + i\operatorname{Sinh}\left[\frac{x}{2}\right]}$$

Problem 68: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Csch}[x]^3}{i + \operatorname{Csch}[x]} dx$$

Optimal (type 3, 26 leaves, 4 steps):

$$i\operatorname{ArcTanh}[\operatorname{Cosh}[x]] - \operatorname{Coth}[x] - \frac{i\operatorname{Coth}[x]}{i + \operatorname{Csch}[x]}$$

Result (type 3, 70 leaves):

$$-\frac{1}{2}\operatorname{Coth}\left[\frac{x}{2}\right] + i\operatorname{Log}\left[\operatorname{Cosh}\left[\frac{x}{2}\right]\right] - i\operatorname{Log}\left[\operatorname{Sinh}\left[\frac{x}{2}\right]\right] - \frac{2\operatorname{Sinh}\left[\frac{x}{2}\right]}{\operatorname{Cosh}\left[\frac{x}{2}\right] + i\operatorname{Sinh}\left[\frac{x}{2}\right]} - \frac{1}{2}\operatorname{Tanh}\left[\frac{x}{2}\right]$$

Problem 69: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Csch}[x]^4}{i + \operatorname{Csch}[x]} dx$$

Optimal (type 3, 37 leaves, 6 steps):

$$\frac{3}{2}\operatorname{ArcTanh}[\operatorname{Cosh}[x]] + 2i\operatorname{Coth}[x] - \frac{3}{2}\operatorname{Coth}[x]\operatorname{Csch}[x] + \frac{\operatorname{Coth}[x]\operatorname{Csch}[x]^2}{i + \operatorname{Csch}[x]}$$

Result (type 3, 90 leaves):

$$\frac{1}{8} \left(4 i \operatorname{Coth}\left[\frac{x}{2}\right] - \operatorname{Csch}\left[\frac{x}{2}\right]^2 + 12 \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{x}{2}\right]\right] - \right. \\ \left. 12 \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{x}{2}\right]\right] - \operatorname{Sech}\left[\frac{x}{2}\right]^2 + \frac{16 \operatorname{Sinh}\left[\frac{x}{2}\right]}{-i \operatorname{Cosh}\left[\frac{x}{2}\right] + \operatorname{Sinh}\left[\frac{x}{2}\right]} + 4 i \operatorname{Tanh}\left[\frac{x}{2}\right] \right)$$

Problem 70: Result more than twice size of optimal antiderivative.

$$\int (a + b \operatorname{Csch}[c + d x])^4 dx$$

Optimal (type 3, 109 leaves, 6 steps):

$$a^4 x - \frac{2 a b (2 a^2 - b^2) \operatorname{ArcTanh}[\operatorname{Cosh}[c + d x]]}{d} - \frac{b^2 (17 a^2 - 2 b^2) \operatorname{Coth}[c + d x]}{3 d} - \\ \frac{4 a b^3 \operatorname{Coth}[c + d x] \operatorname{Csch}[c + d x]}{3 d} - \frac{b^2 \operatorname{Coth}[c + d x] (a + b \operatorname{Csch}[c + d x])^2}{3 d}$$

Result (type 3, 567 leaves):

$$\begin{aligned}
 & \frac{a^4 (c + d x) (a + b \operatorname{Csch}[c + d x])^4 \operatorname{Sinh}[c + d x]^4}{d (b + a \operatorname{Sinh}[c + d x])^4} + \\
 & \left(\left(-9 a^2 b^2 \operatorname{Cosh}\left[\frac{1}{2}(c + d x)\right] + b^4 \operatorname{Cosh}\left[\frac{1}{2}(c + d x)\right] \right) \operatorname{Csch}\left[\frac{1}{2}(c + d x)\right] \right. \\
 & \quad \left. (a + b \operatorname{Csch}[c + d x])^4 \operatorname{Sinh}[c + d x]^4 \right) / (3 d (b + a \operatorname{Sinh}[c + d x])^4) - \\
 & \frac{a b^3 \operatorname{Csch}\left[\frac{1}{2}(c + d x)\right]^2 (a + b \operatorname{Csch}[c + d x])^4 \operatorname{Sinh}[c + d x]^4}{2 d (b + a \operatorname{Sinh}[c + d x])^4} - \\
 & \left(b^4 \operatorname{Coth}\left[\frac{1}{2}(c + d x)\right] \operatorname{Csch}\left[\frac{1}{2}(c + d x)\right]^2 (a + b \operatorname{Csch}[c + d x])^4 \operatorname{Sinh}[c + d x]^4 \right) / \\
 & \quad (24 d (b + a \operatorname{Sinh}[c + d x])^4) + \\
 & \left(2 (-2 a^3 b + a b^3) (a + b \operatorname{Csch}[c + d x])^4 \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{1}{2}(c + d x)\right]\right] \operatorname{Sinh}[c + d x]^4 \right) / \\
 & \quad (d (b + a \operatorname{Sinh}[c + d x])^4) - \\
 & \left(2 (-2 a^3 b + a b^3) (a + b \operatorname{Csch}[c + d x])^4 \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{1}{2}(c + d x)\right]\right] \operatorname{Sinh}[c + d x]^4 \right) / \\
 & \quad (d (b + a \operatorname{Sinh}[c + d x])^4) - \frac{a b^3 (a + b \operatorname{Csch}[c + d x])^4 \operatorname{Sech}\left[\frac{1}{2}(c + d x)\right]^2 \operatorname{Sinh}[c + d x]^4}{2 d (b + a \operatorname{Sinh}[c + d x])^4} + \\
 & \left((a + b \operatorname{Csch}[c + d x])^4 \operatorname{Sech}\left[\frac{1}{2}(c + d x)\right] \left(-9 a^2 b^2 \operatorname{Sinh}\left[\frac{1}{2}(c + d x)\right] + b^4 \operatorname{Sinh}\left[\frac{1}{2}(c + d x)\right] \right) \right. \\
 & \quad \left. \operatorname{Sinh}[c + d x]^4 \right) / (3 d (b + a \operatorname{Sinh}[c + d x])^4) + \\
 & \left(b^4 (a + b \operatorname{Csch}[c + d x])^4 \operatorname{Sech}\left[\frac{1}{2}(c + d x)\right]^2 \operatorname{Sinh}[c + d x]^4 \operatorname{Tanh}\left[\frac{1}{2}(c + d x)\right] \right) / \\
 & \quad (24 d (b + a \operatorname{Sinh}[c + d x])^4)
 \end{aligned}$$

Problem 71: Result more than twice size of optimal antiderivative.

$$\int (a + b \operatorname{Csch}[c + d x])^3 dx$$

Optimal (type 3, 75 leaves, 5 steps):

$$\begin{aligned}
 & a^3 x - \frac{b (6 a^2 - b^2) \operatorname{ArcTanh}[\operatorname{Cosh}[c + d x]]}{2 d} - \\
 & \frac{5 a b^2 \operatorname{Coth}[c + d x]}{2 d} - \frac{b^2 \operatorname{Coth}[c + d x] (a + b \operatorname{Csch}[c + d x])}{2 d}
 \end{aligned}$$

Result (type 3, 151 leaves):

$$-\frac{1}{8d} \left(-8a^3c - 8a^3dx + 12ab^2 \operatorname{Coth} \left[\frac{1}{2}(c+dx) \right] + b^3 \operatorname{Csch} \left[\frac{1}{2}(c+dx) \right]^2 + \right. \\ \left. 24a^2b \operatorname{Log} \left[\operatorname{Cosh} \left[\frac{1}{2}(c+dx) \right] \right] - 4b^3 \operatorname{Log} \left[\operatorname{Cosh} \left[\frac{1}{2}(c+dx) \right] \right] - 24a^2b \operatorname{Log} \left[\operatorname{Sinh} \left[\frac{1}{2}(c+dx) \right] \right] + \right. \\ \left. 4b^3 \operatorname{Log} \left[\operatorname{Sinh} \left[\frac{1}{2}(c+dx) \right] \right] + b^3 \operatorname{Sech} \left[\frac{1}{2}(c+dx) \right]^2 + 12ab^2 \operatorname{Tanh} \left[\frac{1}{2}(c+dx) \right] \right)$$

Problem 72: Result more than twice size of optimal antiderivative.

$$\int (a + b \operatorname{Csch}[c + dx])^2 dx$$

Optimal (type 3, 34 leaves, 4 steps):

$$a^2 x - \frac{2ab \operatorname{ArcTanh}[\operatorname{Cosh}[c + dx]]}{d} - \frac{b^2 \operatorname{Coth}[c + dx]}{d}$$

Result (type 3, 75 leaves):

$$-\frac{1}{2d} \left(b^2 \operatorname{Coth} \left[\frac{1}{2}(c+dx) \right] - \right. \\ \left. 2a \left(ac + adx - 2b \operatorname{Log} \left[\operatorname{Cosh} \left[\frac{1}{2}(c+dx) \right] \right] + 2b \operatorname{Log} \left[\operatorname{Sinh} \left[\frac{1}{2}(c+dx) \right] \right] \right) + b^2 \operatorname{Tanh} \left[\frac{1}{2}(c+dx) \right] \right)$$

Problem 73: Result more than twice size of optimal antiderivative.

$$\int (a + b \operatorname{Csch}[c + dx]) dx$$

Optimal (type 3, 17 leaves, 2 steps):

$$ax - \frac{b \operatorname{ArcTanh}[\operatorname{Cosh}[c + dx]]}{d}$$

Result (type 3, 43 leaves):

$$ax - \frac{b \operatorname{Log} \left[\operatorname{Cosh} \left[\frac{c}{2} + \frac{dx}{2} \right] \right]}{d} + \frac{b \operatorname{Log} \left[\operatorname{Sinh} \left[\frac{c}{2} + \frac{dx}{2} \right] \right]}{d}$$

Problem 89: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sech}[x]^2}{i + \operatorname{Csch}[x]} dx$$

Optimal (type 3, 19 leaves, 6 steps):

$$-\frac{1}{3} \operatorname{Sech}[x]^3 - \frac{1}{3} i \operatorname{Tanh}[x]^3$$

Result (type 3, 64 leaves):

$$\frac{-3 + \operatorname{Cosh}[x] + \operatorname{Cosh}[2x] - 2i \operatorname{Sinh}[x] + i \operatorname{Cosh}[x] \operatorname{Sinh}[x]}{6 \left(\operatorname{Cosh} \left[\frac{x}{2} \right] - i \operatorname{Sinh} \left[\frac{x}{2} \right] \right) \left(\operatorname{Cosh} \left[\frac{x}{2} \right] + i \operatorname{Sinh} \left[\frac{x}{2} \right] \right)^3}$$

Problem 91: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sech}[x]^4}{i + \operatorname{Csch}[x]} dx$$

Optimal (type 3, 29 leaves, 7 steps):

$$-\frac{1}{5} \operatorname{Sech}[x]^5 - \frac{1}{3} i \operatorname{Tanh}[x]^3 + \frac{1}{5} i \operatorname{Tanh}[x]^5$$

Result (type 3, 96 leaves):

$$\begin{aligned} & (-240 + 54 \operatorname{Cosh}[x] + 32 \operatorname{Cosh}[2x] + 18 \operatorname{Cosh}[3x] + \\ & 16 \operatorname{Cosh}[4x] - 96 i \operatorname{Sinh}[x] + 18 i \operatorname{Sinh}[2x] - 32 i \operatorname{Sinh}[3x] + 9 i \operatorname{Sinh}[4x]) / \\ & \left(960 \left(\operatorname{Cosh}\left[\frac{x}{2}\right] - i \operatorname{Sinh}\left[\frac{x}{2}\right] \right)^3 \left(\operatorname{Cosh}\left[\frac{x}{2}\right] + i \operatorname{Sinh}\left[\frac{x}{2}\right] \right)^5 \right) \end{aligned}$$

Problem 103: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Tanh}[x]^4}{i + \operatorname{Csch}[x]} dx$$

Optimal (type 3, 52 leaves, 5 steps):

$$-i x + \frac{1}{15} (15 i - 8 \operatorname{Csch}[x]) \operatorname{Tanh}[x] + \frac{1}{15} (5 i - 4 \operatorname{Csch}[x]) \operatorname{Tanh}[x]^3 + \frac{1}{5} (i - \operatorname{Csch}[x]) \operatorname{Tanh}[x]^5$$

Result (type 3, 126 leaves):

$$\begin{aligned} & (-200 + 6 (89 - 120 i x) \operatorname{Cosh}[x] - 128 \operatorname{Cosh}[2x] + 178 \operatorname{Cosh}[3x] - \\ & 240 i x \operatorname{Cosh}[3x] - 184 \operatorname{Cosh}[4x] + 64 i \operatorname{Sinh}[x] + 178 i \operatorname{Sinh}[2x] + \\ & 240 x \operatorname{Sinh}[2x] + 128 i \operatorname{Sinh}[3x] + 89 i \operatorname{Sinh}[4x] + 120 x \operatorname{Sinh}[4x]) / \\ & \left(960 \left(\operatorname{Cosh}\left[\frac{x}{2}\right] - i \operatorname{Sinh}\left[\frac{x}{2}\right] \right)^3 \left(\operatorname{Cosh}\left[\frac{x}{2}\right] + i \operatorname{Sinh}\left[\frac{x}{2}\right] \right)^5 \right) \end{aligned}$$

Problem 109: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Coth}[x]^3}{i + \operatorname{Csch}[x]} dx$$

Optimal (type 3, 12 leaves, 3 steps):

$$-\operatorname{Csch}[x] - i \operatorname{Log}[\operatorname{Sinh}[x]]$$

Result (type 3, 28 leaves):

$$-\frac{1}{2} \operatorname{Coth}\left[\frac{x}{2}\right] - i \operatorname{Log}[\operatorname{Sinh}[x]] + \frac{1}{2} \operatorname{Tanh}\left[\frac{x}{2}\right]$$

Problem 110: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Coth}[x]^4}{i + \operatorname{Csch}[x]} dx$$

Optimal (type 3, 27 leaves, 4 steps):

$$-i x - \frac{1}{2} \operatorname{ArcTanh}[\operatorname{Cosh}[x]] + \frac{1}{2} \operatorname{Coth}[x] (2i - \operatorname{Csch}[x])$$

Result (type 3, 76 leaves):

$$-i x + \frac{1}{2} i \operatorname{Coth}\left[\frac{x}{2}\right] - \frac{1}{8} \operatorname{Csch}\left[\frac{x}{2}\right]^2 - \frac{1}{2} \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{x}{2}\right]\right] + \frac{1}{2} \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{x}{2}\right]\right] - \frac{1}{8} \operatorname{Sech}\left[\frac{x}{2}\right]^2 + \frac{1}{2} i \operatorname{Tanh}\left[\frac{x}{2}\right]$$

Problem 111: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Coth}[x]^5}{i + \operatorname{Csch}[x]} dx$$

Optimal (type 3, 30 leaves, 3 steps):

$$-\operatorname{Csch}[x] + \frac{1}{2} i \operatorname{Csch}[x]^2 - \frac{\operatorname{Csch}[x]^3}{3} - i \operatorname{Log}[\operatorname{Sinh}[x]]$$

Result (type 3, 92 leaves):

$$-\frac{5}{12} \operatorname{Coth}\left[\frac{x}{2}\right] + \frac{1}{8} i \operatorname{Csch}\left[\frac{x}{2}\right]^2 - \frac{1}{24} \operatorname{Coth}\left[\frac{x}{2}\right] \operatorname{Csch}\left[\frac{x}{2}\right]^2 - i \operatorname{Log}[\operatorname{Sinh}[x]] - \frac{1}{8} i \operatorname{Sech}\left[\frac{x}{2}\right]^2 + \frac{5}{12} \operatorname{Tanh}\left[\frac{x}{2}\right] - \frac{1}{24} \operatorname{Sech}\left[\frac{x}{2}\right]^2 \operatorname{Tanh}\left[\frac{x}{2}\right]$$

Problem 112: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Coth}[x]^6}{i + \operatorname{Csch}[x]} dx$$

Optimal (type 3, 43 leaves, 5 steps):

$$-i x - \frac{3}{8} \operatorname{ArcTanh}[\operatorname{Cosh}[x]] + \frac{1}{12} \operatorname{Coth}[x]^3 (4i - 3 \operatorname{Csch}[x]) + \frac{1}{8} \operatorname{Coth}[x] (8i - 3 \operatorname{Csch}[x])$$

Result (type 3, 140 leaves):

$$-i x + \frac{2}{3} i \operatorname{Coth}\left[\frac{x}{2}\right] - \frac{5}{32} \operatorname{Csch}\left[\frac{x}{2}\right]^2 + \frac{1}{24} i \operatorname{Coth}\left[\frac{x}{2}\right] \operatorname{Csch}\left[\frac{x}{2}\right]^2 - \frac{1}{64} \operatorname{Csch}\left[\frac{x}{2}\right]^4 - \frac{3}{8} \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{x}{2}\right]\right] + \frac{3}{8} \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{x}{2}\right]\right] - \frac{5}{32} \operatorname{Sech}\left[\frac{x}{2}\right]^2 + \frac{1}{64} \operatorname{Sech}\left[\frac{x}{2}\right]^4 + \frac{2}{3} i \operatorname{Tanh}\left[\frac{x}{2}\right] - \frac{1}{24} i \operatorname{Sech}\left[\frac{x}{2}\right]^2 \operatorname{Tanh}\left[\frac{x}{2}\right]$$

Problem 122: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Coth}[x]^5}{a + b \operatorname{Csch}[x]} dx$$

Optimal (type 3, 70 leaves, 3 steps):

$$-\frac{(a^2 + 2b^2) \operatorname{Csch}[x]}{b^3} + \frac{a \operatorname{Csch}[x]^2}{2b^2} - \frac{\operatorname{Csch}[x]^3}{3b} + \frac{(a^2 + b^2)^2 \operatorname{Log}[a + b \operatorname{Csch}[x]]}{ab^4} + \frac{\operatorname{Log}[\operatorname{Sinh}[x]]}{a}$$

Result (type 3, 180 leaves):

$$\begin{aligned} & \frac{1}{48ab^4} \left(-4ab(6a^2 + 11b^2) \operatorname{Coth}\left[\frac{x}{2}\right] + 6a^2b^2 \operatorname{Csch}\left[\frac{x}{2}\right]^2 - \right. \\ & 48a^4 \operatorname{Log}[\operatorname{Sinh}[x]] - 96a^2b^2 \operatorname{Log}[\operatorname{Sinh}[x]] + 48a^4 \operatorname{Log}[b + a \operatorname{Sinh}[x]] + \\ & 96a^2b^2 \operatorname{Log}[b + a \operatorname{Sinh}[x]] + 48b^4 \operatorname{Log}[b + a \operatorname{Sinh}[x]] - 6a^2b^2 \operatorname{Sech}\left[\frac{x}{2}\right]^2 - \\ & \left. 16a^3 \operatorname{Csch}[x]^3 \operatorname{Sinh}\left[\frac{x}{2}\right]^4 - a^3 \operatorname{Csch}\left[\frac{x}{2}\right]^4 \operatorname{Sinh}[x] + 24a^3b \operatorname{Tanh}\left[\frac{x}{2}\right] + 44a^3b \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \end{aligned}$$

Problem 124: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Coth}[x]^7}{a + b \operatorname{Csch}[x]} dx$$

Optimal (type 3, 119 leaves, 3 steps):

$$-\frac{(a^4 + 3a^2b^2 + 3b^4) \operatorname{Csch}[x]}{b^5} + \frac{a(a^2 + 3b^2) \operatorname{Csch}[x]^2}{2b^4} - \frac{(a^2 + 3b^2) \operatorname{Csch}[x]^3}{3b^3} + \frac{a \operatorname{Csch}[x]^4}{4b^2} - \frac{\operatorname{Csch}[x]^5}{5b} + \frac{(a^2 + b^2)^3 \operatorname{Log}[a + b \operatorname{Csch}[x]]}{ab^6} + \frac{\operatorname{Log}[\operatorname{Sinh}[x]]}{a}$$

Result (type 3, 344 leaves):

$$\begin{aligned} & \frac{1}{960ab^6} \left(-4ab(120a^4 + 340a^2b^2 + 309b^4) \operatorname{Coth}\left[\frac{x}{2}\right] + 30a^2b^2(4a^2 + 11b^2) \operatorname{Csch}\left[\frac{x}{2}\right]^2 - \right. \\ & 960a^6 \operatorname{Log}[\operatorname{Sinh}[x]] - 2880a^4b^2 \operatorname{Log}[\operatorname{Sinh}[x]] - 2880a^2b^4 \operatorname{Log}[\operatorname{Sinh}[x]] + \\ & 960a^6 \operatorname{Log}[b + a \operatorname{Sinh}[x]] + 2880a^4b^2 \operatorname{Log}[b + a \operatorname{Sinh}[x]] + 2880a^2b^4 \operatorname{Log}[b + a \operatorname{Sinh}[x]] + \\ & 960b^6 \operatorname{Log}[b + a \operatorname{Sinh}[x]] - 120a^4b^2 \operatorname{Sech}\left[\frac{x}{2}\right]^2 - 330a^2b^4 \operatorname{Sech}\left[\frac{x}{2}\right]^2 + \\ & 15a^2b^4 \operatorname{Sech}\left[\frac{x}{2}\right]^4 - 320a^3b^3 \operatorname{Csch}[x]^3 \operatorname{Sinh}\left[\frac{x}{2}\right]^4 - 816ab^5 \operatorname{Csch}[x]^3 \operatorname{Sinh}\left[\frac{x}{2}\right]^4 - \\ & 3ab^5 \operatorname{Csch}\left[\frac{x}{2}\right]^6 \operatorname{Sinh}[x] - a^3 \operatorname{Csch}\left[\frac{x}{2}\right]^4 (-15ab + 20a^2 \operatorname{Sinh}[x] + 51b^2 \operatorname{Sinh}[x]) + \\ & \left. 480a^5b \operatorname{Tanh}\left[\frac{x}{2}\right] + 1360a^3b^3 \operatorname{Tanh}\left[\frac{x}{2}\right] + 1236ab^5 \operatorname{Tanh}\left[\frac{x}{2}\right] + 6ab^5 \operatorname{Sech}\left[\frac{x}{2}\right]^4 \operatorname{Tanh}\left[\frac{x}{2}\right] \right) \end{aligned}$$

Problem 132: Result unnecessarily involves higher level functions.

$$\int \frac{x^5}{\sqrt{\operatorname{Csch}[2 \operatorname{Log}[cx]]}} dx$$

Optimal (type 4, 81 leaves, 6 steps):

$$-\frac{2x^2}{21c^4\sqrt{\text{Csch}[2\text{Log}[cx]]}} + \frac{x^6}{7\sqrt{\text{Csch}[2\text{Log}[cx]]}} + \frac{2\text{EllipticF}[\text{ArcCsc}[cx], -1]}{21c^7\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\text{Csch}[2\text{Log}[cx]]}}$$

Result (type 5, 81 leaves):

$$\frac{1}{21c^6}\sqrt{\frac{c^2x^2}{-2+2c^4x^4}}\left(2-5c^4x^4+3c^8x^8-2\sqrt{1-c^4x^4}\text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, c^4x^4\right]\right)$$

Problem 134: Result unnecessarily involves higher level functions.

$$\int \frac{x^3}{\sqrt{\text{Csch}[2\text{Log}[cx]]}} dx$$

Optimal (type 4, 119 leaves, 9 steps):

$$-\frac{2}{5c^4\sqrt{\text{Csch}[2\text{Log}[cx]]}} + \frac{x^4}{5\sqrt{\text{Csch}[2\text{Log}[cx]]}} - \frac{2\text{EllipticE}[\text{ArcCsc}[cx], -1]}{5c^5\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\text{Csch}[2\text{Log}[cx]]}} + \frac{2\text{EllipticF}[\text{ArcCsc}[cx], -1]}{5c^5\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\text{Csch}[2\text{Log}[cx]]}}$$

Result (type 5, 76 leaves):

$$\frac{1}{15c^2}x^2\sqrt{\frac{c^2x^2}{-2+2c^4x^4}}\left(-3+3c^4x^4-2\sqrt{1-c^4x^4}\text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, c^4x^4\right]\right)$$

Problem 136: Result unnecessarily involves higher level functions.

$$\int \frac{x}{\sqrt{\text{Csch}[2\text{Log}[cx]]}} dx$$

Optimal (type 4, 60 leaves, 5 steps):

$$\frac{x^2}{3\sqrt{\text{Csch}[2\text{Log}[cx]]}} + \frac{2\text{EllipticF}[\text{ArcCsc}[cx], -1]}{3c^3\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\text{Csch}[2\text{Log}[cx]]}}$$

Result (type 5, 72 leaves):

$$\frac{1}{3c^2}\sqrt{\frac{c^2x^2}{-2+2c^4x^4}}\left(-1+c^4x^4-2\sqrt{1-c^4x^4}\text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, c^4x^4\right]\right)$$

Problem 140: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{\text{Csch}[2 \text{Log}[c x]]}}{x^3} dx$$

Optimal (type 4, 74 leaves, 7 steps):

$$-c^3 \sqrt{1 - \frac{1}{c^4 x^4}} x \sqrt{\text{Csch}[2 \text{Log}[c x]]} \text{EllipticE}[\text{ArcCsc}[c x], -1] +$$

$$c^3 \sqrt{1 - \frac{1}{c^4 x^4}} x \sqrt{\text{Csch}[2 \text{Log}[c x]]} \text{EllipticF}[\text{ArcCsc}[c x], -1]$$

Result (type 4, 56 leaves):

$$c^2 \sqrt{\text{Csch}[2 \text{Log}[c x]]} \left(-\text{EllipticE}\left[\frac{\pi}{4} - i \text{Log}[c x], 2\right] \sqrt{i \text{Sinh}[2 \text{Log}[c x]]} + \text{Sinh}[2 \text{Log}[c x]] \right)$$

Problem 142: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{\text{Csch}[2 \text{Log}[c x]]}}{x^5} dx$$

Optimal (type 4, 64 leaves, 5 steps):

$$\frac{1}{3} \left(c^4 - \frac{1}{x^4} \right) \sqrt{\text{Csch}[2 \text{Log}[c x]]} -$$

$$\frac{1}{3} c^5 \sqrt{1 - \frac{1}{c^4 x^4}} x \sqrt{\text{Csch}[2 \text{Log}[c x]]} \text{EllipticF}[\text{ArcCsc}[c x], -1]$$

Result (type 5, 81 leaves):

$$\frac{1}{3 x^4} \sqrt{2} \sqrt{\frac{c^2 x^2}{-1 + c^4 x^4}} \left(-1 + c^4 x^4 + c^4 x^4 \sqrt{1 - c^4 x^4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, c^4 x^4\right] \right)$$

Problem 144: Result unnecessarily involves higher level functions.

$$\int \frac{x^7}{\text{Csch}[2 \text{Log}[c x]]^{3/2}} dx$$

Optimal (type 4, 118 leaves, 7 steps):

$$\frac{4}{77 c^4 \left(c^4 - \frac{1}{x^4} \right) \text{Csch}[2 \text{Log}[c x]]^{3/2}} - \frac{6 x^4}{77 \left(c^4 - \frac{1}{x^4} \right) \text{Csch}[2 \text{Log}[c x]]^{3/2}} +$$

$$\frac{x^8}{11 \text{Csch}[2 \text{Log}[c x]]^{3/2}} - \frac{4 \text{EllipticF}[\text{ArcCsc}[c x], -1]}{77 c^{11} \left(1 - \frac{1}{c^4 x^4} \right)^{3/2} x^3 \text{Csch}[2 \text{Log}[c x]]^{3/2}}$$

Result (type 5, 89 leaves):

$$\frac{1}{154 c^8} \sqrt{\frac{c^2 x^2}{-2 + 2 c^4 x^4}} \left(-4 + 17 c^4 x^4 - 20 c^8 x^8 + 7 c^{12} x^{12} + 4 \sqrt{1 - c^4 x^4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, c^4 x^4\right] \right)$$

Problem 146: Result unnecessarily involves higher level functions.

$$\int \frac{x^5}{\operatorname{Csch}[2 \operatorname{Log}[c x]]^{3/2}} dx$$

Optimal (type 4, 162 leaves, 10 steps):

$$\frac{4}{15 c^4 \left(c^4 - \frac{1}{x^4} \right) x^2 \operatorname{Csch}[2 \operatorname{Log}[c x]]^{3/2}} - \frac{2 x^2}{15 \left(c^4 - \frac{1}{x^4} \right) \operatorname{Csch}[2 \operatorname{Log}[c x]]^{3/2}} + \frac{x^6}{9 \operatorname{Csch}[2 \operatorname{Log}[c x]]^{3/2}} + \frac{4 \operatorname{EllipticE}[\operatorname{ArcCsc}[c x], -1]}{15 c^9 \left(1 - \frac{1}{c^4 x^4} \right)^{3/2} x^3 \operatorname{Csch}[2 \operatorname{Log}[c x]]^{3/2}} - \frac{4 \operatorname{EllipticF}[\operatorname{ArcCsc}[c x], -1]}{15 c^9 \left(1 - \frac{1}{c^4 x^4} \right)^{3/2} x^3 \operatorname{Csch}[2 \operatorname{Log}[c x]]^{3/2}}$$

Result (type 5, 84 leaves):

$$\frac{1}{90 c^4} x^2 \sqrt{\frac{c^2 x^2}{-2 + 2 c^4 x^4}} \left(11 - 16 c^4 x^4 + 5 c^8 x^8 + 4 \sqrt{1 - c^4 x^4} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, c^4 x^4\right] \right)$$

Problem 148: Result unnecessarily involves higher level functions.

$$\int \frac{x^3}{\operatorname{Csch}[2 \operatorname{Log}[c x]]^{3/2}} dx$$

Optimal (type 4, 86 leaves, 6 steps):

$$-\frac{2}{7 \left(c^4 - \frac{1}{x^4} \right) \operatorname{Csch}[2 \operatorname{Log}[c x]]^{3/2}} + \frac{x^4}{7 \operatorname{Csch}[2 \operatorname{Log}[c x]]^{3/2}} - \frac{4 \operatorname{EllipticF}[\operatorname{ArcCsc}[c x], -1]}{7 c^7 \left(1 - \frac{1}{c^4 x^4} \right)^{3/2} x^3 \operatorname{Csch}[2 \operatorname{Log}[c x]]^{3/2}}$$

Result (type 5, 80 leaves):

$$\frac{1}{14 c^4} \sqrt{\frac{c^2 x^2}{-2 + 2 c^4 x^4}} \left(3 - 4 c^4 x^4 + c^8 x^8 + 4 \sqrt{1 - c^4 x^4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, c^4 x^4\right] \right)$$

Problem 150: Result unnecessarily involves higher level functions.

$$\int \frac{x}{\operatorname{Csch}[2 \operatorname{Log}[c x]]^{3/2}} dx$$

Optimal (type 4, 130 leaves, 9 steps):

$$\begin{aligned}
 & -\frac{6}{5\left(c^4 - \frac{1}{x^4}\right)x^2 \operatorname{Csch}[2 \operatorname{Log}[c x]]^{3/2}} + \frac{x^2}{5 \operatorname{Csch}[2 \operatorname{Log}[c x]]^{3/2}} - \\
 & \frac{12 \operatorname{EllipticE}[\operatorname{ArcCsc}[c x], -1]}{5 c^5 \left(1 - \frac{1}{c^4 x^4}\right)^{3/2} x^3 \operatorname{Csch}[2 \operatorname{Log}[c x]]^{3/2}} + \frac{12 \operatorname{EllipticF}[\operatorname{ArcCsc}[c x], -1]}{5 c^5 \left(1 - \frac{1}{c^4 x^4}\right)^{3/2} x^3 \operatorname{Csch}[2 \operatorname{Log}[c x]]^{3/2}}
 \end{aligned}$$

Result (type 5, 83 leaves):

$$\frac{1}{10 c^4 x^2} \sqrt{\frac{c^2 x^2}{-2 + 2 c^4 x^4}} \left(7 - 8 c^4 x^4 + c^8 x^8 - 12 \sqrt{1 - c^4 x^4} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, c^4 x^4\right] \right)$$

Problem 154: Result unnecessarily involves higher level functions.

$$\int \frac{\operatorname{Csch}[2 \operatorname{Log}[c x]]^{3/2}}{x^3} dx$$

Optimal (type 4, 69 leaves, 5 steps):

$$\begin{aligned}
 & -\frac{1}{2} \left(c^4 - \frac{1}{x^4}\right) x^2 \operatorname{Csch}[2 \operatorname{Log}[c x]]^{3/2} + \\
 & \frac{1}{2} c^5 \left(1 - \frac{1}{c^4 x^4}\right)^{3/2} x^3 \operatorname{Csch}[2 \operatorname{Log}[c x]]^{3/2} \operatorname{EllipticF}[\operatorname{ArcCsc}[c x], -1]
 \end{aligned}$$

Result (type 5, 66 leaves):

$$-\sqrt{2} c^2 \sqrt{\frac{c^2 x^2}{-1 + c^4 x^4}} \left(1 + \sqrt{1 - c^4 x^4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, c^4 x^4\right] \right)$$

Problem 159: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Csch}[a + b \operatorname{Log}[c x^n]]^4 dx$$

Optimal (type 5, 68 leaves, 4 steps):

$$\frac{1}{1 + 4 b n} 16 e^{4a} x (c x^n)^{4b} \operatorname{Hypergeometric2F1}\left[4, \frac{1}{2} \left(4 + \frac{1}{b n}\right), \frac{1}{2} \left(6 + \frac{1}{b n}\right), e^{2a} (c x^n)^{2b}\right]$$

Result (type 5, 488 leaves):

$$\begin{aligned}
 & -\frac{1}{6 b^3 n^3} (-1 + 4 b^2 n^2) x \operatorname{Csch}\left[a + b \left(-n \operatorname{Log}[x] + \operatorname{Log}\left[c x^n\right]\right)\right] \\
 & \operatorname{Csch}\left[a + b n \operatorname{Log}[x] + b \left(-n \operatorname{Log}[x] + \operatorname{Log}\left[c x^n\right]\right)\right] \operatorname{Sinh}\left[b n \operatorname{Log}[x]\right] + \frac{1}{3 b n} \\
 & x \operatorname{Csch}\left[a + b \left(-n \operatorname{Log}[x] + \operatorname{Log}\left[c x^n\right]\right)\right] \operatorname{Csch}\left[a + b n \operatorname{Log}[x] + b \left(-n \operatorname{Log}[x] + \operatorname{Log}\left[c x^n\right]\right)\right]^3 \\
 & \operatorname{Sinh}\left[b n \operatorname{Log}[x]\right] - \frac{1}{6 b^2 n^2} \\
 & x \operatorname{Csch}\left[a + b \left(-n \operatorname{Log}[x] + \operatorname{Log}\left[c x^n\right]\right)\right] \operatorname{Csch}\left[a + b n \operatorname{Log}[x] + b \left(-n \operatorname{Log}[x] + \operatorname{Log}\left[c x^n\right]\right)\right]^2 \\
 & \left(2 b n \operatorname{Cosh}\left[a + b \left(-n \operatorname{Log}[x] + \operatorname{Log}\left[c x^n\right]\right)\right] + \operatorname{Sinh}\left[a + b \left(-n \operatorname{Log}[x] + \operatorname{Log}\left[c x^n\right]\right)\right]\right) + \\
 & \frac{1}{6 b^3 n^3 (1 + 2 b n)} e^{-\frac{a+b \left(-n \operatorname{Log}[x] + \operatorname{Log}\left[c x^n\right]\right)}{b n}} (-1 + 4 b^2 n^2) \operatorname{Csch}\left[a + b \left(-n \operatorname{Log}[x] + \operatorname{Log}\left[c x^n\right]\right)\right] \\
 & \left(e^{\left(2 + \frac{1}{b n}\right) (a+b \operatorname{Log}\left[c x^n\right])} \operatorname{Hypergeometric2F1}\left[1, 1 + \frac{1}{2 b n}, 2 + \frac{1}{2 b n}, e^{2 (a+b \operatorname{Log}\left[c x^n\right])}\right] \right. \\
 & \left. \operatorname{Sinh}\left[a + b \left(-n \operatorname{Log}[x] + \operatorname{Log}\left[c x^n\right]\right)\right] + \right. \\
 & \left. e^{\frac{a}{b n} + \frac{-n \operatorname{Log}[x] + \operatorname{Log}\left[c x^n\right]}{n}} (1 + 2 b n) x \left(\operatorname{Cosh}\left[a + b \left(-n \operatorname{Log}[x] + \operatorname{Log}\left[c x^n\right]\right)\right] + \operatorname{Hypergeometric2F1}\left[1, \right. \right. \right. \\
 & \left. \left. \left. \frac{1}{2 b n}, 1 + \frac{1}{2 b n}, e^{2 (a+b n \operatorname{Log}[x] + b \left(-n \operatorname{Log}[x] + \operatorname{Log}\left[c x^n\right])\right)}\right] \operatorname{Sinh}\left[a + b \left(-n \operatorname{Log}[x] + \operatorname{Log}\left[c x^n\right]\right)\right]\right)\right)
 \end{aligned}$$

Problem 161: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Csch}\left[a + 2 \operatorname{Log}\left[c \sqrt{x}\right]\right]^3 dx$$

Optimal (type 1, 26 leaves, 3 steps):

$$-\frac{2 c^6 e^{-a}}{\left(c^4 - \frac{e^{-2 a}}{x^2}\right)^2}$$

Result (type 1, 62 leaves):

$$\frac{\left(2 \left(\operatorname{Cosh}[a] - \operatorname{Sinh}[a]\right) \left(-2 c^4 x^2 + \operatorname{Cosh}[a]^2 - 2 \operatorname{Cosh}[a] \operatorname{Sinh}[a] + \operatorname{Sinh}[a]^2\right)\right) / \left(c^2 \left((-1 + c^4 x^2) \operatorname{Cosh}[a] + (1 + c^4 x^2) \operatorname{Sinh}[a]\right)^2\right)}{1}$$

Problem 162: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Csch}\left[a + 2 \operatorname{Log}\left[\frac{c}{\sqrt{x}}\right]\right]^3 dx$$

Optimal (type 1, 26 leaves, 4 steps):

$$\frac{2 c^2 e^{-3 a}}{\left(e^{-2 a} - \frac{c^4}{x^2}\right)^2}$$

Result (type 1, 65 leaves):

$$-\left(\left(2 c^6 \left((c^4 - 2 x^2) \operatorname{Cosh}[a] + (c^4 + 2 x^2) \operatorname{Sinh}[a]\right) \left(\operatorname{Cosh}[2 a] + \operatorname{Sinh}[2 a]\right)\right) / \left((-c^4 + x^2) \operatorname{Cosh}[a] - (c^4 + x^2) \operatorname{Sinh}[a]\right)^2\right)$$

Problem 164: Result more than twice size of optimal antiderivative.

$$\int \text{Csch} \left[a - \frac{\text{Log}[c x^n]}{n(-2+p)} \right]^p dx$$

Optimal (type 3, 66 leaves, 3 steps):

$$\frac{(2-p) x \left(1 - e^{-2a} (c x^n)^{-\frac{2}{n(2-p)}} \right) \text{Csch} \left[a + \frac{\text{Log}[c x^n]}{n(2-p)} \right]^p}{2(1-p)}$$

Result (type 3, 140 leaves):

$$\frac{1}{-1+p} 2^{-1+p} e^{-\frac{2ap}{-2+p}} (-2+p) x \left(e^{\frac{2ap}{-2+p}} - e^{\frac{4a}{-2+p}} (c x^n)^{\frac{2}{n(-2+p)}} \right) \left(-\frac{e^{\frac{a(2+p)}{-2+p}} (c x^n)^{\frac{1}{n(-2+p)}}}{-e^{-\frac{2ap}{-2+p}} + e^{\frac{4a}{-2+p}} (c x^n)^{\frac{2}{n(-2+p)}}} \right)^p$$

Problem 165: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Csch}[a + b \text{Log}[c x^n]]}{x} dx$$

Optimal (type 3, 20 leaves, 2 steps):

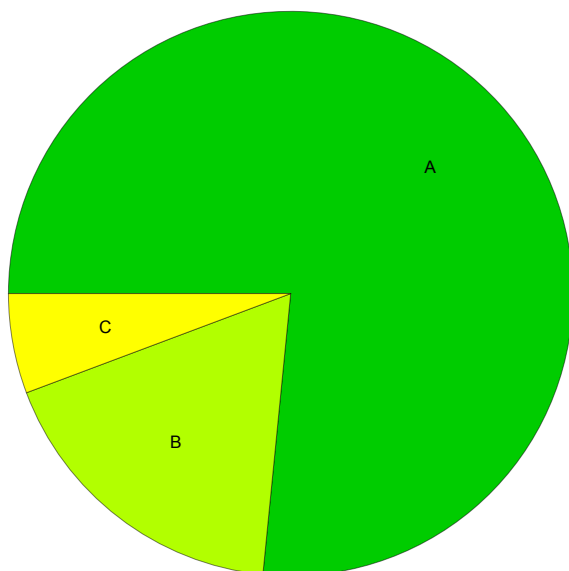
$$\frac{\text{ArcTanh}[\text{Cosh}[a + b \text{Log}[c x^n]]]}{bn}$$

Result (type 3, 54 leaves):

$$-\frac{\text{Log}\left[\text{Cosh}\left[\frac{a}{2} + \frac{1}{2} b \text{Log}[c x^n]\right]\right]}{bn} + \frac{\text{Log}\left[\text{Sinh}\left[\frac{a}{2} + \frac{1}{2} b \text{Log}[c x^n]\right]\right]}{bn}$$

Summary of Integration Test Results

175 integration problems



- A - 134 optimal antiderivatives
- B - 31 more than twice size of optimal antiderivatives
- C - 10 unnecessarily complex antiderivatives
- D - 0 unable to integrate problems
- E - 0 integration timeouts