# Mathematica 11.3 Integration Test Results

## Test results for the 49 problems in "7.3.3 (d+e x)^m (a+b arctanh(c x^n))^p.m"

### Problem 5: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{a + b\,\text{ArcTanh}[c\,x]}{d + e\,x}\,dx$$

Optimal (type 4, 114 leaves, 4 steps):

$$-\frac{(a + b\,\text{ArcTanh}[c\,x])\,\text{Log}\left[\frac{2}{1+c\,x}\right]}{e} + \frac{(a + b\,\text{ArcTanh}[c\,x])\,\text{Log}\left[\frac{2\,c\,(d+e\,x)}{(c\,d+e)\,(1+c\,x)}\right]}{e} + \frac{b\,\text{PolyLog}\left[2,\,1-\frac{2}{1+c\,x}\right]}{2\,e} - \frac{b\,\text{PolyLog}\left[2,\,1-\frac{2\,c\,(d+e\,x)}{(c\,d+e)\,(1+c\,x)}\right]}{2\,e}$$

Result (type 4, 257 leaves):

$$\frac{1}{e}\left(a\,\text{Log}[d+e\,x] + b\,\text{ArcTanh}[c\,x]\left(\frac{1}{2}\text{Log}\left[1-c^2\,x^2\right] + \text{Log}\left[i\,\text{Sinh}\left[\text{ArcTanh}\left[\frac{c\,d}{e}\right] + \text{ArcTanh}[c\,x]\right]\right]\right) - \frac{1}{2}\,i\,b\left(-\frac{1}{4}\,i\,(\pi - 2\,i\,\text{ArcTanh}[c\,x])^2 + i\left(\text{ArcTanh}\left[\frac{c\,d}{e}\right] + \text{ArcTanh}[c\,x]\right)^2 + (\pi - 2\,i\,\text{ArcTanh}[c\,x])\,\text{Log}\left[1 + e^{2\,\text{ArcTanh}[c\,x]}\right] + 2\,i\left(\text{ArcTanh}\left[\frac{c\,d}{e}\right] + \text{ArcTanh}[c\,x]\right)\text{Log}\left[1 - e^{-2\left(\text{ArcTanh}\left[\frac{c\,d}{e}\right]+\text{ArcTanh}[c\,x]\right)}\right] - (\pi - 2\,i\,\text{ArcTanh}[c\,x])\,\text{Log}\left[\frac{2}{\sqrt{1-c^2\,x^2}}\right] - 2\,i\left(\text{ArcTanh}\left[\frac{c\,d}{e}\right] + \text{ArcTanh}[c\,x]\right)\text{Log}\left[2\,i\,\text{Sinh}\left[\text{ArcTanh}\left[\frac{c\,d}{e}\right] + \text{ArcTanh}[c\,x]\right]\right] - i\,\text{PolyLog}\left[2,\,-e^{2\,\text{ArcTanh}[c\,x]}\right] - i\,\text{PolyLog}\left[2,\,e^{-2\left(\text{ArcTanh}\left[\frac{c\,d}{e}\right]+\text{ArcTanh}[c\,x]\right)}\right]\right)\right)$$

### Problem 12: Unable to integrate problem.

$$\int \frac{(a + b\,\text{ArcTanh}[c\,x])^2}{d + e\,x}\,dx$$

Optimal (type 4, 188 leaves, 1 step):

$$-\frac{(a+b\,\text{ArcTanh}[c\,x])^2\,\text{Log}\left[\frac{2}{1+c\,x}\right]}{e}+\frac{(a+b\,\text{ArcTanh}[c\,x])^2\,\text{Log}\left[\frac{2\,c\,(d+e\,x)}{(c\,d+e)\,(1+c\,x)}\right]}{e}+$$
$$\frac{b\,(a+b\,\text{ArcTanh}[c\,x])\,\text{PolyLog}\left[2,1-\frac{2}{1+c\,x}\right]}{e}-\frac{b\,(a+b\,\text{ArcTanh}[c\,x])\,\text{PolyLog}\left[2,1-\frac{2\,c\,(d+e\,x)}{(c\,d+e)\,(1+c\,x)}\right]}{e}+$$
$$\frac{b^2\,\text{PolyLog}\left[3,1-\frac{2}{1+c\,x}\right]}{2\,e}-\frac{b^2\,\text{PolyLog}\left[3,1-\frac{2\,c\,(d+e\,x)}{(c\,d+e)\,(1+c\,x)}\right]}{2\,e}$$

Result (type 8, 20 leaves) :

$$\int\frac{(a+b\,\text{ArcTanh}[c\,x])^2}{d+e\,x}\,dx$$

## Problem 13: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{(a+b\,\text{ArcTanh}[c\,x])^2}{(d+e\,x)^2}\,dx$$

Optimal (type 4, 321 leaves, 12 steps) :

$$-\frac{(a+b\,\text{ArcTanh}[c\,x])^2}{e\,(d+e\,x)}+\frac{b\,c\,(a+b\,\text{ArcTanh}[c\,x])\,\text{Log}\left[\frac{2}{1-c\,x}\right]}{e\,(c\,d+e)}-$$
$$\frac{b\,c\,(a+b\,\text{ArcTanh}[c\,x])\,\text{Log}\left[\frac{2}{1+c\,x}\right]}{(c\,d-e)\,e}+\frac{2\,b\,c\,(a+b\,\text{ArcTanh}[c\,x])\,\text{Log}\left[\frac{2}{1+c\,x}\right]}{c^2\,d^2-e^2}-$$
$$\frac{2\,b\,c\,(a+b\,\text{ArcTanh}[c\,x])\,\text{Log}\left[\frac{2\,c\,(d+e\,x)}{(c\,d+e)\,(1+c\,x)}\right]}{c^2\,d^2-e^2}+\frac{b^2\,c\,\text{PolyLog}\left[2,1-\frac{2}{1-c\,x}\right]}{2\,e\,(c\,d+e)}+$$
$$\frac{b^2\,c\,\text{PolyLog}\left[2,1-\frac{2}{1+c\,x}\right]}{2\,(c\,d-e)\,e}-\frac{b^2\,c\,\text{PolyLog}\left[2,1-\frac{2}{1+c\,x}\right]}{c^2\,d^2-e^2}+\frac{b^2\,c\,\text{PolyLog}\left[2,1-\frac{2\,c\,(d+e\,x)}{(c\,d+e)\,(1+c\,x)}\right]}{c^2\,d^2-e^2}$$

Result (type 4, 317 leaves) :

$$-\frac{a^2}{e\,(d+e\,x)}+\frac{a\,b\,c\left(-\frac{2\,\text{ArcTanh}[c\,x]}{c\,d+c\,e\,x}+\frac{(-c\,d+e)\,\text{Log}[1-c\,x]+(c\,d+e)\,\text{Log}[1+c\,x]-2\,e\,\text{Log}[c\,(d+e\,x)]}{(c\,d-e)\,(c\,d+e)}\right)}{e}+\frac{1}{d}$$

$$b^2\left(-\frac{e^{-\text{ArcTanh}\left[\frac{c\,d}{e}\right]}\,\text{ArcTanh}[c\,x]^2}{\sqrt{1-\frac{c^2\,d^2}{e^2}}\;e}+\frac{x\,\text{ArcTanh}[c\,x]^2}{d+e\,x}+\frac{1}{c^2\,d^2-e^2}c\,d\left(i\,\pi\,\text{Log}\left[1+e^{2\,\text{ArcTanh}[c\,x]}\right]-\right.\right.$$

$$2\,\text{ArcTanh}[c\,x]\,\text{Log}\left[1-e^{-2\left(\text{ArcTanh}\left[\frac{c\,d}{e}\right]+\text{ArcTanh}[c\,x]\right)}\right]-i\,\pi\left(\text{ArcTanh}[c\,x]-\frac{1}{2}\,\text{Log}\left[1-c^2\,x^2\right]\right)-$$

$$2\,\text{ArcTanh}\left[\frac{c\,d}{e}\right]\left(\text{ArcTanh}[c\,x]+\text{Log}\left[1-e^{-2\left(\text{ArcTanh}\left[\frac{c\,d}{e}\right]+\text{ArcTanh}[c\,x]\right)}\right]-\right.$$

$$\left.\left.\left.\text{Log}\left[i\,\text{Sinh}\left[\text{ArcTanh}\left[\frac{c\,d}{e}\right]+\text{ArcTanh}[c\,x]\right]\right]\right)+\text{PolyLog}\left[2,\,e^{-2\left(\text{ArcTanh}\left[\frac{c\,d}{e}\right]+\text{ArcTanh}[c\,x]\right)}\right]\right)\right)$$

## Problem 14: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{(a+b\,\text{ArcTanh}[c\,x])^2}{(d+e\,x)^3}\,dx$$

Optimal (type 4, 480 leaves, 18 steps):

$$\frac{b\,c\,(a+b\,\text{ArcTanh}[c\,x])}{(c^2\,d^2-e^2)\,(d+e\,x)}-\frac{(a+b\,\text{ArcTanh}[c\,x])^2}{2\,e\,(d+e\,x)^2}+\frac{b\,c^2\,(a+b\,\text{ArcTanh}[c\,x])\,\text{Log}\left[\frac{2}{1-c\,x}\right]}{2\,e\,(c\,d+e)^2}+$$

$$\frac{b^2\,c^2\,\text{Log}[1-c\,x]}{2\,(c\,d-e)\,(c\,d+e)^2}-\frac{b\,c^2\,(a+b\,\text{ArcTanh}[c\,x])\,\text{Log}\left[\frac{2}{1+c\,x}\right]}{2\,(c\,d-e)^2\,e}+$$

$$\frac{2\,b\,c^3\,d\,(a+b\,\text{ArcTanh}[c\,x])\,\text{Log}\left[\frac{2}{1+c\,x}\right]}{(c\,d-e)^2\,(c\,d+e)^2}-\frac{b^2\,c^2\,\text{Log}[1+c\,x]}{2\,(c\,d-e)^2\,(c\,d+e)}+\frac{b^2\,c^2\,e\,\text{Log}[d+e\,x]}{(c\,d-e)^2\,(c\,d+e)^2}-$$

$$\frac{2\,b\,c^3\,d\,(a+b\,\text{ArcTanh}[c\,x])\,\text{Log}\left[\frac{2\,c\,(d+e\,x)}{(c\,d+e)\,(1+c\,x)}\right]}{(c\,d-e)^2\,(c\,d+e)^2}+\frac{b^2\,c^2\,\text{PolyLog}\left[2,\,1-\frac{2}{1-c\,x}\right]}{4\,e\,(c\,d+e)^2}+$$

$$\frac{b^2\,c^2\,\text{PolyLog}\left[2,\,1-\frac{2}{1+c\,x}\right]}{4\,(c\,d-e)^2\,e}-\frac{b^2\,c^3\,d\,\text{PolyLog}\left[2,\,1-\frac{2}{1+c\,x}\right]}{(c\,d-e)^2\,(c\,d+e)^2}+\frac{b^2\,c^3\,d\,\text{PolyLog}\left[2,\,1-\frac{2\,c\,(d+e\,x)}{(c\,d+e)\,(1+c\,x)}\right]}{(c\,d-e)^2\,(c\,d+e)^2}$$

Result (type 4, 467 leaves):

$$-\frac{a^2}{2 e (d+e x)^2}-\frac{1}{2 e} a b c^2 \left(\frac{2 \operatorname{ArcTanh}[c x]}{(c d+c e x)^2}+\frac{\operatorname{Log}[1-c x]}{(c d+e)^2}+\frac{-\operatorname{Log}[1+c x]+\frac{2 e \left(-c^2 d^2+e^2+2 c^2 d (d+e x) \operatorname{Log}[c (d+e x)]\right)}{c (c d+e)^2 (d+e x)}}{(-c d+e)^2}\right)+$$

$$\frac{1}{2 (c d-e) (c d+e)} b^2 c^2 \left(\frac{e \left(1-c^2 x^2\right) \operatorname{ArcTanh}[c x]^2}{(c d+c e x)^2}+\right.$$

$$\frac{2 x \operatorname{ArcTanh}[c x] (-e+c d \operatorname{ArcTanh}[c x])}{c d (d+e x)}+\frac{2 e \left(-e \operatorname{ArcTanh}[c x]+c d \operatorname{Log}\left[\frac{c (d+e x)}{\sqrt{1-c^2 x^2}}\right]\right)}{c^3 d^3-c d e^2}+$$

$$\frac{1}{c^2 d^2-e^2} 2 \left(\sqrt{1-\frac{c^2 d^2}{e^2}}\, e\, e^{-\operatorname{ArcTanh}\left[\frac{c d}{e}\right]} \operatorname{ArcTanh}[c x]^2+\right.$$

$$i c d \left(-\left(\pi-2 i \operatorname{ArcTanh}\left[\frac{c d}{e}\right]\right) \operatorname{ArcTanh}[c x]+\pi \operatorname{Log}\left[1+e^{2 \operatorname{ArcTanh}[c x]}\right]+2 i\right.$$

$$\left(\operatorname{ArcTanh}\left[\frac{c d}{e}\right]+\operatorname{ArcTanh}[c x]\right) \operatorname{Log}\left[1-e^{-2 \left(\operatorname{ArcTanh}\left[\frac{c d}{e}\right]+\operatorname{ArcTanh}[c x]\right)}\right]+$$

$$\frac{1}{2} \pi \operatorname{Log}\left[1-c^2 x^2\right]-2 i \operatorname{ArcTanh}\left[\frac{c d}{e}\right] \operatorname{Log}\left[i \operatorname{Sinh}\left[\operatorname{ArcTanh}\left[\frac{c d}{e}\right]+\operatorname{ArcTanh}[c x]\right]\right]-$$

$$\left.\left.\left.i \operatorname{PolyLog}\left[2, e^{-2 \left(\operatorname{ArcTanh}\left[\frac{c d}{e}\right]+\operatorname{ArcTanh}[c x]\right)}\right]\right)\right)\right)$$

## Problem 18: Unable to integrate problem.

$$\int \frac{(a+b \operatorname{ArcTanh}[c x])^3}{d+e x} \, dx$$

Optimal (type 4, 272 leaves, 1 step):

$$-\frac{(a+b\,\text{ArcTanh}[c\,x])^3\,\text{Log}\left[\frac{2}{1+c\,x}\right]}{e}+\frac{(a+b\,\text{ArcTanh}[c\,x])^3\,\text{Log}\left[\frac{2\,c\,(d+e\,x)}{(c\,d+e)\,(1+c\,x)}\right]}{e}+\frac{3\,b\,(a+b\,\text{ArcTanh}[c\,x])^2\,\text{PolyLog}\left[2,1-\frac{2}{1+c\,x}\right]}{2\,e}-\frac{3\,b\,(a+b\,\text{ArcTanh}[c\,x])^2\,\text{PolyLog}\left[2,1-\frac{2\,c\,(d+e\,x)}{(c\,d+e)\,(1+c\,x)}\right]}{2\,e}+\frac{3\,b^2\,(a+b\,\text{ArcTanh}[c\,x])\,\text{PolyLog}\left[3,1-\frac{2}{1+c\,x}\right]}{2\,e}-\frac{3\,b^2\,(a+b\,\text{ArcTanh}[c\,x])\,\text{PolyLog}\left[3,1-\frac{2\,c\,(d+e\,x)}{(c\,d+e)\,(1+c\,x)}\right]}{2\,e}+\frac{3\,b^3\,\text{PolyLog}\left[4,1-\frac{2}{1+c\,x}\right]}{4\,e}-\frac{3\,b^3\,\text{PolyLog}\left[4,1-\frac{2\,c\,(d+e\,x)}{(c\,d+e)\,(1+c\,x)}\right]}{4\,e}$$

Result (type 8, 20 leaves):

$$\int\frac{(a+b\,\text{ArcTanh}[c\,x])^3}{d+e\,x}\,dx$$

## Problem 19: Unable to integrate problem.

$$\int\frac{(a+b\,\text{ArcTanh}[c\,x])^3}{(d+e\,x)^2}\,dx$$

Optimal (type 4, 517 leaves, 9 steps):

$$-\frac{(a+b\,\text{ArcTanh}[c\,x])^3}{e\,(d+e\,x)}+\frac{3\,b\,c\,(a+b\,\text{ArcTanh}[c\,x])^2\,\text{Log}\left[\frac{2}{1-c\,x}\right]}{2\,e\,(c\,d+e)}-\frac{3\,b\,c\,(a+b\,\text{ArcTanh}[c\,x])^2\,\text{Log}\left[\frac{2}{1+c\,x}\right]}{2\,(c\,d-e)\,e}+\frac{3\,b\,c\,(a+b\,\text{ArcTanh}[c\,x])^2\,\text{Log}\left[\frac{2}{1+c\,x}\right]}{c^2\,d^2-e^2}-\frac{3\,b\,c\,(a+b\,\text{ArcTanh}[c\,x])^2\,\text{Log}\left[\frac{2\,c\,(d+e\,x)}{(c\,d+e)\,(1+c\,x)}\right]}{c^2\,d^2-e^2}+\frac{3\,b^2\,c\,(a+b\,\text{ArcTanh}[c\,x])\,\text{PolyLog}\left[2,1-\frac{2}{1-c\,x}\right]}{2\,e\,(c\,d+e)}+\frac{3\,b^2\,c\,(a+b\,\text{ArcTanh}[c\,x])\,\text{PolyLog}\left[2,1-\frac{2}{1+c\,x}\right]}{2\,(c\,d-e)\,e}-\frac{3\,b^2\,c\,(a+b\,\text{ArcTanh}[c\,x])\,\text{PolyLog}\left[2,1-\frac{2}{1+c\,x}\right]}{c^2\,d^2-e^2}+\frac{3\,b^2\,c\,(a+b\,\text{ArcTanh}[c\,x])\,\text{PolyLog}\left[2,1-\frac{2\,c\,(d+e\,x)}{(c\,d+e)\,(1+c\,x)}\right]}{c^2\,d^2-e^2}-\frac{3\,b^3\,c\,\text{PolyLog}\left[3,1-\frac{2}{1-c\,x}\right]}{4\,e\,(c\,d+e)}+\frac{3\,b^3\,c\,\text{PolyLog}\left[3,1-\frac{2}{1+c\,x}\right]}{4\,(c\,d-e)\,e}-\frac{3\,b^3\,c\,\text{PolyLog}\left[3,1-\frac{2}{1+c\,x}\right]}{2\,(c^2\,d^2-e^2)}+\frac{3\,b^3\,c\,\text{PolyLog}\left[3,1-\frac{2\,c\,(d+e\,x)}{(c\,d+e)\,(1+c\,x)}\right]}{2\,(c^2\,d^2-e^2)}$$

Result (type 8, 20 leaves):

$$\int \frac{(a+b\,\text{ArcTanh}[c\,x])^3}{(d+e\,x)^2}\,dx$$

## Problem 20: Attempted integration timed out after 120 seconds.

$$\int \frac{(a+b\,\text{ArcTanh}[c\,x])^3}{(d+e\,x)^3}\,dx$$

Optimal (type 4, 953 leaves, 21 steps):

$$\frac{3\,b\,c\,(a+b\,\text{ArcTanh}[c\,x])^2}{2\,(c^2 d^2-e^2)\,(d+e\,x)}-\frac{(a+b\,\text{ArcTanh}[c\,x])^3}{2\,e\,(d+e\,x)^2}-\frac{3\,b^2 c^2\,(a+b\,\text{ArcTanh}[c\,x])\,\text{Log}\left[\frac{2}{1-c\,x}\right]}{2\,(c\,d-e)\,(c\,d+e)^2}+$$
$$\frac{3\,b\,c^2\,(a+b\,\text{ArcTanh}[c\,x])^2\,\text{Log}\left[\frac{2}{1-c\,x}\right]}{4\,e\,(c\,d+e)^2}-\frac{3\,b^2 c^2\,e\,(a+b\,\text{ArcTanh}[c\,x])\,\text{Log}\left[\frac{2}{1+c\,x}\right]}{(c\,d-e)^2\,(c\,d+e)^2}+$$
$$\frac{3\,b^2 c^2\,(a+b\,\text{ArcTanh}[c\,x])\,\text{Log}\left[\frac{2}{1+c\,x}\right]}{2\,(c\,d-e)^2\,(c\,d+e)}-\frac{3\,b\,c^2\,(a+b\,\text{ArcTanh}[c\,x])^2\,\text{Log}\left[\frac{2}{1+c\,x}\right]}{4\,(c\,d-e)^2\,e}+$$
$$\frac{3\,b\,c^3\,d\,(a+b\,\text{ArcTanh}[c\,x])^2\,\text{Log}\left[\frac{2}{1+c\,x}\right]}{(c\,d-e)^2\,(c\,d+e)^2}+\frac{3\,b^2 c^2\,e\,(a+b\,\text{ArcTanh}[c\,x])\,\text{Log}\left[\frac{2\,c\,(d+e\,x)}{(c\,d+e)\,(1+c\,x)}\right]}{(c\,d-e)^2\,(c\,d+e)^2}-$$
$$\frac{3\,b\,c^3\,d\,(a+b\,\text{ArcTanh}[c\,x])^2\,\text{Log}\left[\frac{2\,c\,(d+e\,x)}{(c\,d+e)\,(1+c\,x)}\right]}{(c\,d-e)^2\,(c\,d+e)^2}-\frac{3\,b^3 c^2\,\text{PolyLog}\left[2,1-\frac{2}{1-c\,x}\right]}{4\,(c\,d-e)\,(c\,d+e)^2}+$$
$$\frac{3\,b^2 c^2\,(a+b\,\text{ArcTanh}[c\,x])\,\text{PolyLog}\left[2,1-\frac{2}{1-c\,x}\right]}{4\,e\,(c\,d+e)^2}+\frac{3\,b^3 c^2\,e\,\text{PolyLog}\left[2,1-\frac{2}{1+c\,x}\right]}{2\,(c\,d-e)^2\,(c\,d+e)^2}-$$
$$\frac{3\,b^3 c^2\,\text{PolyLog}\left[2,1-\frac{2}{1+c\,x}\right]}{4\,(c\,d-e)^2\,(c\,d+e)}+\frac{3\,b^2 c^2\,(a+b\,\text{ArcTanh}[c\,x])\,\text{PolyLog}\left[2,1-\frac{2}{1+c\,x}\right]}{4\,(c\,d-e)^2\,e}-$$
$$\frac{3\,b^2 c^3\,d\,(a+b\,\text{ArcTanh}[c\,x])\,\text{PolyLog}\left[2,1-\frac{2}{1+c\,x}\right]}{(c\,d-e)^2\,(c\,d+e)^2}-\frac{3\,b^3 c^2\,e\,\text{PolyLog}\left[2,1-\frac{2\,c\,(d+e\,x)}{(c\,d+e)\,(1+c\,x)}\right]}{2\,(c\,d-e)^2\,(c\,d+e)^2}+$$
$$\frac{3\,b^2 c^3\,d\,(a+b\,\text{ArcTanh}[c\,x])\,\text{PolyLog}\left[2,1-\frac{2\,c\,(d+e\,x)}{(c\,d+e)\,(1+c\,x)}\right]}{(c\,d-e)^2\,(c\,d+e)^2}-\frac{3\,b^3 c^2\,\text{PolyLog}\left[3,1-\frac{2}{1-c\,x}\right]}{8\,e\,(c\,d+e)^2}+$$
$$\frac{3\,b^3 c^2\,\text{PolyLog}\left[3,1-\frac{2}{1+c\,x}\right]}{8\,(c\,d-e)^2\,e}-\frac{3\,b^3 c^3\,d\,\text{PolyLog}\left[3,1-\frac{2}{1+c\,x}\right]}{2\,(c\,d-e)^2\,(c\,d+e)^2}+\frac{3\,b^3 c^3\,d\,\text{PolyLog}\left[3,1-\frac{2\,c\,(d+e\,x)}{(c\,d+e)\,(1+c\,x)}\right]}{2\,(c\,d-e)^2\,(c\,d+e)^2}$$

Result (type 1, 1 leaves):

???

## Problem 21: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{a + b\,\text{ArcTanh}[c\,x]}{1 + 2\,c\,x}\,dx$$

Optimal (type 4, 67 leaves, 4 steps):

$$\frac{\left(a - b\,\text{ArcTanh}\left[\frac{1}{2}\right]\right)\,\text{Log}\left[-\frac{1+2\,c\,x}{2\,d}\right]}{2\,c} - \frac{b\,\text{PolyLog}[2,\,-1-2\,c\,x]}{4\,c} + \frac{b\,\text{PolyLog}\left[2,\,\frac{1}{3}\,(1+2\,c\,x)\right]}{4\,c}$$

Result (type 4, 240 leaves):

$$\frac{1}{2\,c}\left(a\,\text{Log}[1+2\,c\,x] + b\,\text{ArcTanh}[c\,x]\,\left(\frac{1}{2}\,\text{Log}\left[1-c^2\,x^2\right] + \text{Log}\left[i\,\text{Sinh}\left[\text{ArcTanh}\left[\frac{1}{2}\right] + \text{ArcTanh}[c\,x]\right]\right]\right) - \frac{1}{2}\,i\,b\,\left(-\frac{1}{4}\,i\,(\pi - 2\,i\,\text{ArcTanh}[c\,x])^2 + i\,\left(\text{ArcTanh}\left[\frac{1}{2}\right] + \text{ArcTanh}[c\,x]\right)^2 + (\pi - 2\,i\,\text{ArcTanh}[c\,x])\,\text{Log}\left[1+e^{2\,\text{ArcTanh}[c\,x]}\right] + 2\,i\,\left(\text{ArcTanh}\left[\frac{1}{2}\right] + \text{ArcTanh}[c\,x]\right)\,\text{Log}\left[1-e^{-2\left(\text{ArcTanh}\left[\frac{1}{2}\right]+\text{ArcTanh}[c\,x]\right)}\right] - (\pi - 2\,i\,\text{ArcTanh}[c\,x])\,\text{Log}\left[\frac{2}{\sqrt{1-c^2\,x^2}}\right] - 2\,i\,\left(\text{ArcTanh}\left[\frac{1}{2}\right] + \text{ArcTanh}[c\,x]\right)\,\text{Log}\left[2\,i\,\text{Sinh}\left[\text{ArcTanh}\left[\frac{1}{2}\right] + \text{ArcTanh}[c\,x]\right]\right] - i\,\text{PolyLog}\left[2,\,-e^{2\,\text{ArcTanh}[c\,x]}\right] - i\,\text{PolyLog}\left[2,\,e^{-2\left(\text{ArcTanh}\left[\frac{1}{2}\right]+\text{ArcTanh}[c\,x]\right)}\right]\right)\right)$$

## Problem 22: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcTanh}[x]}{1 - \sqrt{2}\,x}\,dx$$

Optimal (type 4, 88 leaves, 4 steps):

$$-\frac{\text{ArcTanh}\left[\frac{1}{\sqrt{2}}\right]\,\text{Log}\left[1-\sqrt{2}\,x\right]}{\sqrt{2}} - \frac{\text{PolyLog}\left[2,\,-\frac{\sqrt{2}-2\,x}{2-\sqrt{2}}\right]}{2\,\sqrt{2}} + \frac{\text{PolyLog}\left[2,\,\frac{\sqrt{2}-2\,x}{2+\sqrt{2}}\right]}{2\,\sqrt{2}}$$

Result (type 4, 272 leaves):

$$\frac{1}{8\sqrt{2}}\left(\pi^2 - 4\,\text{ArcTanh}\left[\frac{1}{\sqrt{2}}\right]^2 - 4\,i\,\pi\,\text{ArcTanh}[x] + 8\,\text{ArcTanh}\left[\frac{1}{\sqrt{2}}\right]\text{ArcTanh}[x] - 8\,\text{ArcTanh}[x]^2 + 8\,\text{ArcTanh}\left[\frac{1}{\sqrt{2}}\right]\text{Log}\left[1 - e^{2\,\text{ArcTanh}\left[\frac{1}{\sqrt{2}}\right] - 2\,\text{ArcTanh}[x]}\right] - 8\,\text{ArcTanh}[x]\,\text{Log}\left[1 - e^{2\,\text{ArcTanh}\left[\frac{1}{\sqrt{2}}\right] - 2\,\text{ArcTanh}[x]}\right] + 4\,i\,\pi\,\text{Log}\left[1 + e^{2\,\text{ArcTanh}[x]}\right] + 8\,\text{ArcTanh}[x]\,\text{Log}\left[1 + e^{2\,\text{ArcTanh}[x]}\right] - 4\,i\,\pi\,\text{Log}\left[\frac{2}{\sqrt{1-x^2}}\right] - 8\,\text{ArcTanh}[x]\,\text{Log}\left[\frac{2}{\sqrt{1-x^2}}\right] - 4\,\text{ArcTanh}[x]\,\text{Log}\left[1-x^2\right] - 8\,\text{ArcTanh}[x]\,\text{Log}\left[-i\,\text{Sinh}\left[\text{ArcTanh}\left[\frac{1}{\sqrt{2}}\right] - \text{ArcTanh}[x]\right]\right] - 8\,\text{ArcTanh}\left[\frac{1}{\sqrt{2}}\right]\text{Log}\left[-2\,i\,\text{Sinh}\left[\text{ArcTanh}\left[\frac{1}{\sqrt{2}}\right] - \text{ArcTanh}[x]\right]\right] + 8\,\text{ArcTanh}[x]\,\text{Log}\left[-2\,i\,\text{Sinh}\left[\text{ArcTanh}\left[\frac{1}{\sqrt{2}}\right] - \text{ArcTanh}[x]\right]\right] + 4\,\text{PolyLog}\left[2, e^{2\,\text{ArcTanh}\left[\frac{1}{\sqrt{2}}\right] - 2\,\text{ArcTanh}[x]}\right] + 4\,\text{PolyLog}\left[2, -e^{2\,\text{ArcTanh}[x]}\right]\right)$$

## Problem 26: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a + b\,\text{ArcTanh}\left[c\,x^2\right]}{d + e\,x}\,dx$$

Optimal (type 4, 325 leaves, 19 steps):

$$\frac{\left(a + b\,\text{ArcTanh}\left[c\,x^2\right]\right)\text{Log}[d+e\,x]}{e} - \frac{b\,\text{Log}\left[\frac{e\left(1-\sqrt{-c}\,x\right)}{\sqrt{-c}\,d+e}\right]\text{Log}[d+e\,x]}{2\,e} - \frac{b\,\text{Log}\left[-\frac{e\left(1+\sqrt{-c}\,x\right)}{\sqrt{-c}\,d-e}\right]\text{Log}[d+e\,x]}{2\,e} + \frac{b\,\text{Log}\left[\frac{e\left(1-\sqrt{c}\,x\right)}{\sqrt{c}\,d+e}\right]\text{Log}[d+e\,x]}{2\,e} + \frac{b\,\text{Log}\left[-\frac{e\left(1+\sqrt{c}\,x\right)}{\sqrt{c}\,d-e}\right]\text{Log}[d+e\,x]}{2\,e} - \frac{b\,\text{PolyLog}\left[2, \frac{\sqrt{-c}\,(d+e\,x)}{\sqrt{-c}\,d-e}\right]}{2\,e} + \frac{b\,\text{PolyLog}\left[2, \frac{\sqrt{c}\,(d+e\,x)}{\sqrt{c}\,d-e}\right]}{2\,e} - \frac{b\,\text{PolyLog}\left[2, \frac{\sqrt{-c}\,(d+e\,x)}{\sqrt{-c}\,d+e}\right]}{2\,e} + \frac{b\,\text{PolyLog}\left[2, \frac{\sqrt{c}\,(d+e\,x)}{\sqrt{c}\,d+e}\right]}{2\,e}$$

Result (type 4, 285 leaves):

$$\frac{a\,\text{Log}[d+e\,x]}{e}+\frac{1}{2\,e}b\left(2\,\text{ArcTanh}\left[c\,x^2\right]\,\text{Log}[d+e\,x]-\text{Log}\left[\frac{e\left(i-\sqrt{c}\,x\right)}{\sqrt{c}\,d+i\,e}\right]\,\text{Log}[d+e\,x]-\text{Log}\left[-\frac{e\left(i+\sqrt{c}\,x\right)}{\sqrt{c}\,d-i\,e}\right]\,\text{Log}[d+e\,x]+\text{Log}\left[-\frac{e\left(1+\sqrt{c}\,x\right)}{\sqrt{c}\,d-e}\right]\,\text{Log}[d+e\,x]+\text{Log}[d+e\,x]\,\text{Log}\left[\frac{e-\sqrt{c}\,e\,x}{\sqrt{c}\,d+e}\right]+\text{PolyLog}\left[2,\frac{\sqrt{c}\,(d+e\,x)}{\sqrt{c}\,d-e}\right]-\text{PolyLog}\left[2,\frac{\sqrt{c}\,(d+e\,x)}{\sqrt{c}\,d-i\,e}\right]-\text{PolyLog}\left[2,\frac{\sqrt{c}\,(d+e\,x)}{\sqrt{c}\,d+i\,e}\right]+\text{PolyLog}\left[2,\frac{\sqrt{c}\,(d+e\,x)}{\sqrt{c}\,d+e}\right]\right)$$

## Problem 30: Attempted integration timed out after 120 seconds.

$$\int\frac{\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)^2}{d+e\,x}\,dx$$

Optimal (type 8, 23 leaves, 0 steps):

$$\text{Int}\left[\frac{\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)^2}{d+e\,x},x\right]$$

Result (type 1, 1 leaves):

???

## Problem 34: Attempted integration timed out after 120 seconds.

$$\int\frac{a+b\,\text{ArcTanh}\left[c\,x^3\right]}{d+e\,x}\,dx$$

Optimal (type 4, 523 leaves, 25 steps):

$$\frac{\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)\,\text{Log}[d+e\,x]}{e}+\frac{b\,\text{Log}\left[\frac{e\left(1-c^{1/3}x\right)}{c^{1/3}d+e}\right]\,\text{Log}[d+e\,x]}{2\,e}-\frac{b\,\text{Log}\left[-\frac{e\left(1+c^{1/3}x\right)}{c^{1/3}d-e}\right]\,\text{Log}[d+e\,x]}{2\,e}+\frac{b\,\text{Log}\left[-\frac{e\left((-1)^{1/3}+c^{1/3}x\right)}{c^{1/3}d-(-1)^{1/3}e}\right]\,\text{Log}[d+e\,x]}{2\,e}-\frac{b\,\text{Log}\left[-\frac{e\left((-1)^{2/3}+c^{1/3}x\right)}{c^{1/3}d-(-1)^{2/3}e}\right]\,\text{Log}[d+e\,x]}{2\,e}+\frac{b\,\text{Log}\left[\frac{(-1)^{2/3}e\left(1+(-1)^{1/3}c^{1/3}x\right)}{c^{1/3}d+(-1)^{2/3}e}\right]\,\text{Log}[d+e\,x]}{2\,e}-\frac{b\,\text{Log}\left[\frac{(-1)^{1/3}e\left(1+(-1)^{2/3}c^{1/3}x\right)}{c^{1/3}d+(-1)^{1/3}e}\right]\,\text{Log}[d+e\,x]}{2\,e}-\frac{b\,\text{PolyLog}\left[2,\frac{c^{1/3}(d+e\,x)}{c^{1/3}d-e}\right]}{2\,e}+\frac{b\,\text{PolyLog}\left[2,\frac{c^{1/3}(d+e\,x)}{c^{1/3}d+e}\right]}{2\,e}+\frac{b\,\text{PolyLog}\left[2,\frac{c^{1/3}(d+e\,x)}{c^{1/3}d-(-1)^{1/3}e}\right]}{2\,e}-\frac{b\,\text{PolyLog}\left[2,\frac{c^{1/3}(d+e\,x)}{c^{1/3}d+(-1)^{1/3}e}\right]}{2\,e}-\frac{b\,\text{PolyLog}\left[2,\frac{c^{1/3}(d+e\,x)}{c^{1/3}d-(-1)^{2/3}e}\right]}{2\,e}+\frac{b\,\text{PolyLog}\left[2,\frac{c^{1/3}(d+e\,x)}{c^{1/3}d+(-1)^{2/3}e}\right]}{2\,e}$$

Result (type 1, 1 leaves):

???

## Problem 44: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2 \left(a + b\, \text{ArcTanh}\left[c \sqrt{x}\right]\right)}{d + e x} \, dx$$

Optimal (type 4, 460 leaves, 20 steps):

$$-\frac{b d \sqrt{x}}{c e^2} + \frac{b \sqrt{x}}{2 c^3 e} + \frac{b x^{3/2}}{6 c e} + \frac{b d\, \text{ArcTanh}\left[c \sqrt{x}\right]}{c^2 e^2} - \frac{b\, \text{ArcTanh}\left[c \sqrt{x}\right]}{2 c^4 e} - \frac{d x \left(a + b\, \text{ArcTanh}\left[c \sqrt{x}\right]\right)}{e^2} + \frac{x^2 \left(a + b\, \text{ArcTanh}\left[c \sqrt{x}\right]\right)}{2 e} - \frac{2 d^2 \left(a + b\, \text{ArcTanh}\left[c \sqrt{x}\right]\right) \text{Log}\left[\frac{2}{1+c \sqrt{x}}\right]}{e^3} + \frac{d^2 \left(a + b\, \text{ArcTanh}\left[c \sqrt{x}\right]\right) \text{Log}\left[\frac{2 c \left(\sqrt{-d} - \sqrt{e} \sqrt{x}\right)}{\left(c \sqrt{-d} - \sqrt{e}\right) \left(1+c \sqrt{x}\right)}\right]}{e^3} + \frac{d^2 \left(a + b\, \text{ArcTanh}\left[c \sqrt{x}\right]\right) \text{Log}\left[\frac{2 c \left(\sqrt{-d} + \sqrt{e} \sqrt{x}\right)}{\left(c \sqrt{-d} + \sqrt{e}\right) \left(1+c \sqrt{x}\right)}\right]}{e^3} + \frac{b d^2\, \text{PolyLog}\left[2, 1 - \frac{2}{1+c \sqrt{x}}\right]}{e^3} - \frac{b d^2\, \text{PolyLog}\left[2, 1 - \frac{2 c \left(\sqrt{-d} - \sqrt{e} \sqrt{x}\right)}{\left(c \sqrt{-d} - \sqrt{e}\right) \left(1+c \sqrt{x}\right)}\right]}{2 e^3} - \frac{b d^2\, \text{PolyLog}\left[2, 1 - \frac{2 c \left(\sqrt{-d} + \sqrt{e} \sqrt{x}\right)}{\left(c \sqrt{-d} + \sqrt{e}\right) \left(1+c \sqrt{x}\right)}\right]}{2 e^3}$$

Result (type 4, 558 leaves):

$$\frac{1}{6 e^3}\left(-6 a d e x+3 a e^2 x^2+6 a d^2 \text{Log}[d+e x]+\frac{1}{c^4} b\left(2 c e\left(-3 c^2 d+2 e\right)\sqrt{x}+c e^2\sqrt{x}\left(-1+c^2 x\right)-6\left(c^2 d-e\right) e\left(-1+c^2 x\right)\text{ArcTanh}\left[c\sqrt{x}\right]+3 e^2\left(-1+c^2 x\right)^2 \text{ArcTanh}\left[c\sqrt{x}\right]-6 c^4 d^2\left(\text{ArcTanh}\left[c\sqrt{x}\right]\left(\text{ArcTanh}\left[c\sqrt{x}\right]+2\,\text{Log}\left[1+e^{-2\,\text{ArcTanh}\left[c\sqrt{x}\right]}\right]\right)-\text{PolyLog}\left[2,-e^{-2\,\text{ArcTanh}\left[c\sqrt{x}\right]}\right]\right)+3 c^4 d^2\left(2\,\text{ArcTanh}\left[c\sqrt{x}\right]^2-4\,i\,\text{ArcSin}\left[\sqrt{\frac{c^2 d}{c^2 d+e}}\right]\text{ArcTanh}\left[\frac{c e\sqrt{x}}{\sqrt{-c^2 d e}}\right]+2\left(-i\,\text{ArcSin}\left[\sqrt{\frac{c^2 d}{c^2 d+e}}\right]+\text{ArcTanh}\left[c\sqrt{x}\right]\right)\text{Log}\left[\frac{1}{c^2 d+e}e^{-2\,\text{ArcTanh}\left[c\sqrt{x}\right]}\left(-2\sqrt{-c^2 d e}+e\left(-1+e^{2\,\text{ArcTanh}\left[c\sqrt{x}\right]}\right)+c^2 d\left(1+e^{2\,\text{ArcTanh}\left[c\sqrt{x}\right]}\right)\right)\right]+2\left(i\,\text{ArcSin}\left[\sqrt{\frac{c^2 d}{c^2 d+e}}\right]+\text{ArcTanh}\left[c\sqrt{x}\right]\right)\text{Log}\left[\frac{1}{c^2 d+e}e^{-2\,\text{ArcTanh}\left[c\sqrt{x}\right]}\left(2\sqrt{-c^2 d e}+e\left(-1+e^{2\,\text{ArcTanh}\left[c\sqrt{x}\right]}\right)+c^2 d\left(1+e^{2\,\text{ArcTanh}\left[c\sqrt{x}\right]}\right)\right)\right]-\text{PolyLog}\left[2,\frac{\left(-c^2 d+e-2\sqrt{-c^2 d e}\right)e^{-2\,\text{ArcTanh}\left[c\sqrt{x}\right]}}{c^2 d+e}\right]-\text{PolyLog}\left[2,\frac{\left(-c^2 d+e+2\sqrt{-c^2 d e}\right)e^{-2\,\text{ArcTanh}\left[c\sqrt{x}\right]}}{c^2 d+e}\right]\right)\right)\right)$$

## Problem 45: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x\left(a+b\,\text{ArcTanh}\left[c\sqrt{x}\right]\right)}{d+e x}\,dx$$

Optimal (type 4, 374 leaves, 15 steps):

$$\frac{b\sqrt{x}}{c\,e}-\frac{b\,\text{ArcTanh}\left[c\sqrt{x}\right]}{c^2\,e}+\frac{x\left(a+b\,\text{ArcTanh}\left[c\sqrt{x}\right]\right)}{e}+\frac{2\,d\left(a+b\,\text{ArcTanh}\left[c\sqrt{x}\right]\right)\text{Log}\left[\frac{2}{1+c\sqrt{x}}\right]}{e^2}-\frac{d\left(a+b\,\text{ArcTanh}\left[c\sqrt{x}\right]\right)\text{Log}\left[\frac{2\,c\left(\sqrt{-d}-\sqrt{e}\,\sqrt{x}\right)}{\left(c\sqrt{-d}-\sqrt{e}\right)\left(1+c\sqrt{x}\right)}\right]}{e^2}-\frac{d\left(a+b\,\text{ArcTanh}\left[c\sqrt{x}\right]\right)\text{Log}\left[\frac{2\,c\left(\sqrt{-d}+\sqrt{e}\,\sqrt{x}\right)}{\left(c\sqrt{-d}+\sqrt{e}\right)\left(1+c\sqrt{x}\right)}\right]}{e^2}-\frac{b\,d\,\text{PolyLog}\left[2,1-\frac{2}{1+c\sqrt{x}}\right]}{e^2}+\frac{b\,d\,\text{PolyLog}\left[2,1-\frac{2\,c\left(\sqrt{-d}-\sqrt{e}\,\sqrt{x}\right)}{\left(c\sqrt{-d}-\sqrt{e}\right)\left(1+c\sqrt{x}\right)}\right]}{2\,e^2}+\frac{b\,d\,\text{PolyLog}\left[2,1-\frac{2\,c\left(\sqrt{-d}+\sqrt{e}\,\sqrt{x}\right)}{\left(c\sqrt{-d}+\sqrt{e}\right)\left(1+c\sqrt{x}\right)}\right]}{2\,e^2}$$

Result (type 4, 568 leaves):

$$\frac{1}{2\,e^2}\left(2\,a\,e\,x-2\,a\,d\,\text{Log}[d+e\,x]+\frac{1}{c^2}2\,b\left(c\,e\sqrt{x}+c^2\,d\,\text{ArcTanh}\left[c\sqrt{x}\right]^2+\text{ArcTanh}\left[c\sqrt{x}\right]\left(-e+c^2\,e\,x+2\,c^2\,d\,\text{Log}\left[1+e^{-2\,\text{ArcTanh}\left[c\sqrt{x}\right]}\right]\right)-c^2\,d\,\text{PolyLog}\left[2,-e^{-2\,\text{ArcTanh}\left[c\sqrt{x}\right]}\right]\right)+b\,d\left(-2\left(\text{ArcTanh}\left[c\sqrt{x}\right]^2-i\,\text{ArcSin}\left[\sqrt{\frac{c^2\,d}{c^2\,d+e}}\right]\left(2\,\text{ArcTanh}\left[\frac{c\,e\sqrt{x}}{\sqrt{-c^2\,d\,e}}\right]+\text{Log}\left[\frac{1}{c^2\,d+e}e^{-2\,\text{ArcTanh}\left[c\sqrt{x}\right]}\left(-2\sqrt{-c^2\,d\,e}+e\left(-1+e^{2\,\text{ArcTanh}\left[c\sqrt{x}\right]}\right)+c^2\,d\left(1+e^{2\,\text{ArcTanh}\left[c\sqrt{x}\right]}\right)\right)\right]-\text{Log}\left[\frac{1}{c^2\,d+e}e^{-2\,\text{ArcTanh}\left[c\sqrt{x}\right]}\left(2\sqrt{-c^2\,d\,e}+e\left(-1+e^{2\,\text{ArcTanh}\left[c\sqrt{x}\right]}\right)+c^2\,d\left(1+e^{2\,\text{ArcTanh}\left[c\sqrt{x}\right]}\right)\right)\right]\right)+\text{ArcTanh}\left[c\sqrt{x}\right]\left(\text{Log}\left[\frac{1}{c^2\,d+e}e^{-2\,\text{ArcTanh}\left[c\sqrt{x}\right]}\left(-2\sqrt{-c^2\,d\,e}+e\left(-1+e^{2\,\text{ArcTanh}\left[c\sqrt{x}\right]}\right)+c^2\,d\left(1+e^{2\,\text{ArcTanh}\left[c\sqrt{x}\right]}\right)\right)\right]+\text{Log}\left[\frac{1}{c^2\,d+e}e^{-2\,\text{ArcTanh}\left[c\sqrt{x}\right]}\left(2\sqrt{-c^2\,d\,e}+e\left(-1+e^{2\,\text{ArcTanh}\left[c\sqrt{x}\right]}\right)+c^2\,d\left(1+e^{2\,\text{ArcTanh}\left[c\sqrt{x}\right]}\right)\right)\right]\right)\right)+\text{PolyLog}\left[2,\frac{\left(-c^2\,d+e-2\sqrt{-c^2\,d\,e}\right)e^{-2\,\text{ArcTanh}\left[c\sqrt{x}\right]}}{c^2\,d+e}\right]+\text{PolyLog}\left[2,\frac{\left(-c^2\,d+e+2\sqrt{-c^2\,d\,e}\right)e^{-2\,\text{ArcTanh}\left[c\sqrt{x}\right]}}{c^2\,d+e}\right]\right)\right)$$

## Problem 46: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a+b\,\text{ArcTanh}\left[c\sqrt{x}\right]}{d+e\,x}\,dx$$

Optimal (type 4, 318 leaves, 11 steps):

$$-\frac{2\left(a+b\,\text{ArcTanh}\left[c\sqrt{x}\right]\right)\text{Log}\left[\frac{2}{1+c\sqrt{x}}\right]}{e}+\frac{\left(a+b\,\text{ArcTanh}\left[c\sqrt{x}\right]\right)\text{Log}\left[\frac{2c\left(\sqrt{-d}-\sqrt{e}\sqrt{x}\right)}{\left(c\sqrt{-d}-\sqrt{e}\right)\left(1+c\sqrt{x}\right)}\right]}{e}+\frac{\left(a+b\,\text{ArcTanh}\left[c\sqrt{x}\right]\right)\text{Log}\left[\frac{2c\left(\sqrt{-d}+\sqrt{e}\sqrt{x}\right)}{\left(c\sqrt{-d}+\sqrt{e}\right)\left(1+c\sqrt{x}\right)}\right]}{e}+\frac{b\,\text{PolyLog}\left[2,1-\frac{2}{1+c\sqrt{x}}\right]}{e}-\frac{b\,\text{PolyLog}\left[2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}\sqrt{x}\right)}{\left(c\sqrt{-d}-\sqrt{e}\right)\left(1+c\sqrt{x}\right)}\right]}{2e}-\frac{b\,\text{PolyLog}\left[2,1-\frac{2c\left(\sqrt{-d}+\sqrt{e}\sqrt{x}\right)}{\left(c\sqrt{-d}+\sqrt{e}\right)\left(1+c\sqrt{x}\right)}\right]}{2e}$$

Result (type 4, 551 leaves):

$$\frac{a\,\text{Log}[d+e\,x]}{e}-\frac{1}{2e}b\left(4\,i\,\text{ArcSin}\left[\sqrt{\frac{c^2 d}{c^2 d+e}}\right]\text{ArcTanh}\left[\frac{c\,e\sqrt{x}}{\sqrt{-c^2 d\,e}}\right]+4\,\text{ArcTanh}\left[c\sqrt{x}\right]\text{Log}\left[1+e^{-2\,\text{ArcTanh}\left[c\sqrt{x}\right]}\right]+\right.$$
$$2\,i\,\text{ArcSin}\left[\sqrt{\frac{c^2 d}{c^2 d+e}}\right]\text{Log}\left[\frac{1}{c^2 d+e}e^{-2\,\text{ArcTanh}\left[c\sqrt{x}\right]}\right.$$
$$\left.\left(-2\sqrt{-c^2 d\,e}+e\left(-1+e^{2\,\text{ArcTanh}\left[c\sqrt{x}\right]}\right)+c^2 d\left(1+e^{2\,\text{ArcTanh}\left[c\sqrt{x}\right]}\right)\right)\right]-2\,\text{ArcTanh}\left[c\sqrt{x}\right]\text{Log}\left[\right.$$
$$\left.\frac{1}{c^2 d+e}e^{-2\,\text{ArcTanh}\left[c\sqrt{x}\right]}\left(-2\sqrt{-c^2 d\,e}+e\left(-1+e^{2\,\text{ArcTanh}\left[c\sqrt{x}\right]}\right)+c^2 d\left(1+e^{2\,\text{ArcTanh}\left[c\sqrt{x}\right]}\right)\right)\right]-$$
$$2\,i\,\text{ArcSin}\left[\sqrt{\frac{c^2 d}{c^2 d+e}}\right]\text{Log}\left[\frac{1}{c^2 d+e}e^{-2\,\text{ArcTanh}\left[c\sqrt{x}\right]}\right.$$
$$\left.\left(2\sqrt{-c^2 d\,e}+e\left(-1+e^{2\,\text{ArcTanh}\left[c\sqrt{x}\right]}\right)+c^2 d\left(1+e^{2\,\text{ArcTanh}\left[c\sqrt{x}\right]}\right)\right)\right]-2\,\text{ArcTanh}\left[c\sqrt{x}\right]\text{Log}\left[\right.$$
$$\left.\frac{1}{c^2 d+e}e^{-2\,\text{ArcTanh}\left[c\sqrt{x}\right]}\left(2\sqrt{-c^2 d\,e}+e\left(-1+e^{2\,\text{ArcTanh}\left[c\sqrt{x}\right]}\right)+c^2 d\left(1+e^{2\,\text{ArcTanh}\left[c\sqrt{x}\right]}\right)\right)\right]-$$
$$2\,\text{PolyLog}\left[2,-e^{-2\,\text{ArcTanh}\left[c\sqrt{x}\right]}\right]+\text{PolyLog}\left[2,\frac{\left(-c^2 d+e-2\sqrt{-c^2 d\,e}\right)e^{-2\,\text{ArcTanh}\left[c\sqrt{x}\right]}}{c^2 d+e}\right]+$$
$$\left.\text{PolyLog}\left[2,\frac{\left(-c^2 d+e+2\sqrt{-c^2 d\,e}\right)e^{-2\,\text{ArcTanh}\left[c\sqrt{x}\right]}}{c^2 d+e}\right]\right)$$

## Problem 47: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]}{x\,(d+e\,x)}\,dx$$

Optimal (type 4, 358 leaves, 15 steps):

$$\frac{2\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)\text{Log}\left[\frac{2}{1+c\,\sqrt{x}}\right]}{d}-\frac{\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)\text{Log}\left[\frac{2\,c\,\left(\sqrt{-d}-\sqrt{e}\,\sqrt{x}\right)}{\left(c\,\sqrt{-d}-\sqrt{e}\right)\left(1+c\,\sqrt{x}\right)}\right]}{d}-$$
$$\frac{\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)\text{Log}\left[\frac{2\,c\,\left(\sqrt{-d}+\sqrt{e}\,\sqrt{x}\right)}{\left(c\,\sqrt{-d}+\sqrt{e}\right)\left(1+c\,\sqrt{x}\right)}\right]}{d}+\frac{a\,\text{Log}[x]}{d}-$$
$$\frac{b\,\text{PolyLog}\left[2,\,1-\frac{2}{1+c\,\sqrt{x}}\right]}{d}+\frac{b\,\text{PolyLog}\left[2,\,1-\frac{2\,c\,\left(\sqrt{-d}-\sqrt{e}\,\sqrt{x}\right)}{\left(c\,\sqrt{-d}-\sqrt{e}\right)\left(1+c\,\sqrt{x}\right)}\right]}{2\,d}+$$
$$\frac{b\,\text{PolyLog}\left[2,\,1-\frac{2\,c\,\left(\sqrt{-d}+\sqrt{e}\,\sqrt{x}\right)}{\left(c\,\sqrt{-d}+\sqrt{e}\right)\left(1+c\,\sqrt{x}\right)}\right]}{2\,d}-\frac{b\,\text{PolyLog}\left[2,\,-c\,\sqrt{x}\,\right]}{d}+\frac{b\,\text{PolyLog}\left[2,\,c\,\sqrt{x}\,\right]}{d}$$

Result (type 4, 563 leaves):

$$\frac{1}{2d}\left(4\,i\,b\,\text{ArcSin}\left[\sqrt{\frac{c^2 d}{c^2 d+e}}\right]\text{ArcTanh}\left[\frac{c\,e\,\sqrt{x}}{\sqrt{-c^2 d\,e}}\right]+\right.$$

$$4\,b\,\text{ArcTanh}\left[c\,\sqrt{x}\right]\text{Log}\left[1-e^{-2\,\text{ArcTanh}\left[c\,\sqrt{x}\right]}\right]+2\,i\,b\,\text{ArcSin}\left[\sqrt{\frac{c^2 d}{c^2 d+e}}\right]$$

$$\text{Log}\left[\frac{1}{c^2 d+e}e^{-2\,\text{ArcTanh}\left[c\,\sqrt{x}\right]}\left(-2\,\sqrt{-c^2 d\,e}+e\left(-1+e^{2\,\text{ArcTanh}\left[c\,\sqrt{x}\right]}\right)+c^2 d\left(1+e^{2\,\text{ArcTanh}\left[c\,\sqrt{x}\right]}\right)\right)\right]-$$

$$2\,b\,\text{ArcTanh}\left[c\,\sqrt{x}\right]\text{Log}\left[\frac{1}{c^2 d+e}\right.$$

$$\left.e^{-2\,\text{ArcTanh}\left[c\,\sqrt{x}\right]}\left(-2\,\sqrt{-c^2 d\,e}+e\left(-1+e^{2\,\text{ArcTanh}\left[c\,\sqrt{x}\right]}\right)+c^2 d\left(1+e^{2\,\text{ArcTanh}\left[c\,\sqrt{x}\right]}\right)\right)\right]-$$

$$2\,i\,b\,\text{ArcSin}\left[\sqrt{\frac{c^2 d}{c^2 d+e}}\right]\text{Log}\left[\frac{1}{c^2 d+e}e^{-2\,\text{ArcTanh}\left[c\,\sqrt{x}\right]}\right.$$

$$\left.\left(2\,\sqrt{-c^2 d\,e}+e\left(-1+e^{2\,\text{ArcTanh}\left[c\,\sqrt{x}\right]}\right)+c^2 d\left(1+e^{2\,\text{ArcTanh}\left[c\,\sqrt{x}\right]}\right)\right)\right]-2\,b\,\text{ArcTanh}\left[c\,\sqrt{x}\right]$$

$$\text{Log}\left[\frac{1}{c^2 d+e}e^{-2\,\text{ArcTanh}\left[c\,\sqrt{x}\right]}\left(2\,\sqrt{-c^2 d\,e}+e\left(-1+e^{2\,\text{ArcTanh}\left[c\,\sqrt{x}\right]}\right)+c^2 d\left(1+e^{2\,\text{ArcTanh}\left[c\,\sqrt{x}\right]}\right)\right)\right]+$$

$$2\,a\,\text{Log}[x]-2\,a\,\text{Log}[d+e\,x]-2\,b\,\text{PolyLog}\left[2,\,e^{-2\,\text{ArcTanh}\left[c\,\sqrt{x}\right]}\right]+$$

$$b\,\text{PolyLog}\left[2,\,\frac{\left(-c^2 d+e-2\,\sqrt{-c^2 d\,e}\right)e^{-2\,\text{ArcTanh}\left[c\,\sqrt{x}\right]}}{c^2 d+e}\right]+$$

$$\left.b\,\text{PolyLog}\left[2,\,\frac{\left(-c^2 d+e+2\,\sqrt{-c^2 d\,e}\right)e^{-2\,\text{ArcTanh}\left[c\,\sqrt{x}\right]}}{c^2 d+e}\right]\right)$$

## Problem 48: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\right]}{x^2\,(d+e\,x)}\,dx$$

Optimal (type 4, 413 leaves, 19 steps) :

$$-\frac{b\,c}{d\,\sqrt{x}}+\frac{b\,c^2\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]}{d}-\frac{a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]}{d\,x}-$$

$$\frac{2\,e\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)\,\text{Log}\left[\frac{2}{1+c\,\sqrt{x}}\right]}{d^2}+\frac{e\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)\,\text{Log}\left[\frac{2\,c\,\left(\sqrt{-d}-\sqrt{e}\,\sqrt{x}\right)}{\left(c\,\sqrt{-d}-\sqrt{e}\right)\,\left(1+c\,\sqrt{x}\right)}\right]}{d^2}+$$

$$\frac{e\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)\,\text{Log}\left[\frac{2\,c\,\left(\sqrt{-d}+\sqrt{e}\,\sqrt{x}\right)}{\left(c\,\sqrt{-d}+\sqrt{e}\right)\,\left(1+c\,\sqrt{x}\right)}\right]}{d^2}-\frac{a\,e\,\text{Log}[x]}{d^2}+$$

$$\frac{b\,e\,\text{PolyLog}\left[2,\,1-\frac{2}{1+c\,\sqrt{x}}\right]}{d^2}-\frac{b\,e\,\text{PolyLog}\left[2,\,1-\frac{2\,c\,\left(\sqrt{-d}-\sqrt{e}\,\sqrt{x}\right)}{\left(c\,\sqrt{-d}-\sqrt{e}\right)\,\left(1+c\,\sqrt{x}\right)}\right]}{2\,d^2}-$$

$$\frac{b\,e\,\text{PolyLog}\left[2,\,1-\frac{2\,c\,\left(\sqrt{-d}+\sqrt{e}\,\sqrt{x}\right)}{\left(c\,\sqrt{-d}+\sqrt{e}\right)\,\left(1+c\,\sqrt{x}\right)}\right]}{2\,d^2}+\frac{b\,e\,\text{PolyLog}\left[2,\,-c\,\sqrt{x}\,\right]}{d^2}-\frac{b\,e\,\text{PolyLog}\left[2,\,c\,\sqrt{x}\,\right]}{d^2}$$

Result (type 4, 567 leaves) :

$$-\frac{1}{2 d^2 x}\left(2 a d+2 a e x \operatorname{Log}[x]-2 a e x \operatorname{Log}[d+e x]+\right.$$
$$2 b\left(c d \sqrt{x}+\operatorname{ArcTanh}\left[c \sqrt{x}\right]\left(d-c^2 d x+e x \operatorname{ArcTanh}\left[c \sqrt{x}\right]+2 e x \operatorname{Log}\left[1-e^{-2 \operatorname{ArcTanh}\left[c \sqrt{x}\right]}\right]\right)-\right.$$
$$\left.e x \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcTanh}\left[c \sqrt{x}\right]}\right]\right)+$$
$$b e x\left(-2\left(\operatorname{ArcTanh}\left[c \sqrt{x}\right]^2-i \operatorname{ArcSin}\left[\sqrt{\frac{c^2 d}{c^2 d+e}}\right]\left(2 \operatorname{ArcTanh}\left[\frac{c e \sqrt{x}}{\sqrt{-c^2 d e}}\right]+\right.\right.\right.$$
$$\operatorname{Log}\left[\frac{1}{c^2 d+e} e^{-2 \operatorname{ArcTanh}\left[c \sqrt{x}\right]}\left(-2 \sqrt{-c^2 d e}+e\left(-1+e^{2 \operatorname{ArcTanh}\left[c \sqrt{x}\right]}\right)+\right.\right.$$
$$\left.\left.c^2 d\left(1+e^{2 \operatorname{ArcTanh}\left[c \sqrt{x}\right]}\right)\right)\right]-\operatorname{Log}\left[\frac{1}{c^2 d+e} e^{-2 \operatorname{ArcTanh}\left[c \sqrt{x}\right]}\right.$$
$$\left.\left.\left(2 \sqrt{-c^2 d e}+e\left(-1+e^{2 \operatorname{ArcTanh}\left[c \sqrt{x}\right]}\right)+c^2 d\left(1+e^{2 \operatorname{ArcTanh}\left[c \sqrt{x}\right]}\right)\right)\right]\right)+$$
$$\operatorname{ArcTanh}\left[c \sqrt{x}\right]\left(\operatorname{Log}\left[\frac{1}{c^2 d+e} e^{-2 \operatorname{ArcTanh}\left[c \sqrt{x}\right]}\left(-2 \sqrt{-c^2 d e}+e\left(-1+e^{2 \operatorname{ArcTanh}\left[c \sqrt{x}\right]}\right)+\right.\right.\right.$$
$$\left.\left.c^2 d\left(1+e^{2 \operatorname{ArcTanh}\left[c \sqrt{x}\right]}\right)\right)\right]+\operatorname{Log}\left[\frac{1}{c^2 d+e} e^{-2 \operatorname{ArcTanh}\left[c \sqrt{x}\right]}\right.$$
$$\left.\left.\left.\left(2 \sqrt{-c^2 d e}+e\left(-1+e^{2 \operatorname{ArcTanh}\left[c \sqrt{x}\right]}\right)+c^2 d\left(1+e^{2 \operatorname{ArcTanh}\left[c \sqrt{x}\right]}\right)\right)\right]\right)\right)+$$
$$\operatorname{PolyLog}\left[2, \frac{\left(-c^2 d+e-2 \sqrt{-c^2 d e}\right) e^{-2 \operatorname{ArcTanh}\left[c \sqrt{x}\right]}}{c^2 d+e}\right]+\operatorname{PolyLog}\left[2,\right.$$
$$\left.\left.\left.\frac{\left(-c^2 d+e+2 \sqrt{-c^2 d e}\right) e^{-2 \operatorname{ArcTanh}\left[c \sqrt{x}\right]}}{c^2 d+e}\right]\right)\right)$$

## Problem 49: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a+b \operatorname{ArcTanh}\left[c \sqrt{x}\right]}{x^3 (d+e x)} \, dx$$

Optimal (type 4, 506 leaves, 24 steps):

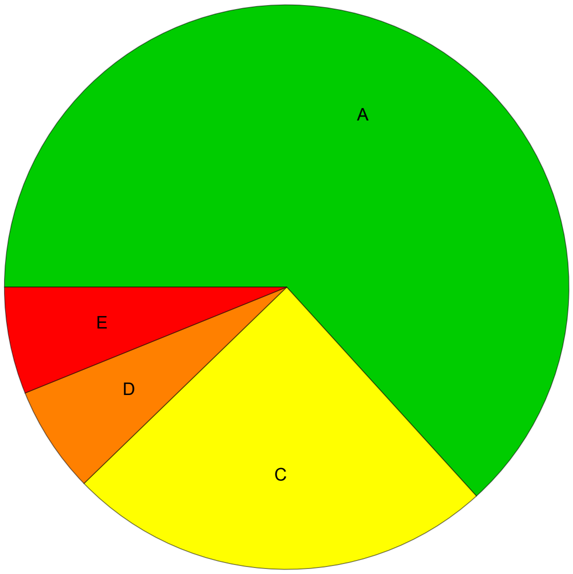
$$-\frac{b c}{6 d x^{3/2}}-\frac{b c^3}{2 d \sqrt{x}}+\frac{b c e}{d^2 \sqrt{x}}+\frac{b c^4 \text{ArcTanh}\left[c \sqrt{x}\right]}{2 d}-\frac{b c^2 e \text{ArcTanh}\left[c \sqrt{x}\right]}{d^2}-\frac{a+b \text{ArcTanh}\left[c \sqrt{x}\right]}{2 d x^2}+\frac{e \left(a+b \text{ArcTanh}\left[c \sqrt{x}\right]\right)}{d^2 x}+\frac{2 e^2 \left(a+b \text{ArcTanh}\left[c \sqrt{x}\right]\right) \text{Log}\left[\frac{2}{1+c \sqrt{x}}\right]}{d^3}-\frac{e^2 \left(a+b \text{ArcTanh}\left[c \sqrt{x}\right]\right) \text{Log}\left[\frac{2 c \left(\sqrt{-d}-\sqrt{e} \sqrt{x}\right)}{\left(c \sqrt{-d}-\sqrt{e}\right) \left(1+c \sqrt{x}\right)}\right]}{d^3}-\frac{e^2 \left(a+b \text{ArcTanh}\left[c \sqrt{x}\right]\right) \text{Log}\left[\frac{2 c \left(\sqrt{-d}+\sqrt{e} \sqrt{x}\right)}{\left(c \sqrt{-d}+\sqrt{e}\right) \left(1+c \sqrt{x}\right)}\right]}{d^3}+\frac{a e^2 \text{Log}[x]}{d^3}-\frac{b e^2 \text{PolyLog}\left[2,1-\frac{2}{1+c \sqrt{x}}\right]}{d^3}+\frac{b e^2 \text{PolyLog}\left[2,1-\frac{2 c \left(\sqrt{-d}-\sqrt{e} \sqrt{x}\right)}{\left(c \sqrt{-d}-\sqrt{e}\right) \left(1+c \sqrt{x}\right)}\right]}{2 d^3}+\frac{b e^2 \text{PolyLog}\left[2,1-\frac{2 c \left(\sqrt{-d}+\sqrt{e} \sqrt{x}\right)}{\left(c \sqrt{-d}+\sqrt{e}\right) \left(1+c \sqrt{x}\right)}\right]}{2 d^3}-\frac{b e^2 \text{PolyLog}\left[2,-c \sqrt{x}\right]}{d^3}+\frac{b e^2 \text{PolyLog}\left[2,c \sqrt{x}\right]}{d^3}$$

Result (type 4, 626 leaves):

$$-\frac{1}{6 d^3 x^2}\left(3 a d^2 - 6 a d e x - 6 a e^2 x^2 \text{Log}[x] + 6 a e^2 x^2 \text{Log}[d + e x] + b\left(c d \sqrt{x}\,(d + 3 c^2 d x - 6 e x) - 3 \text{ArcTanh}[c\sqrt{x}]\left(d\,(-1 + c^2 x)\,(d + c^2 d x - 2 e x) + 2 e^2 x^2 \text{ArcTanh}[c\sqrt{x}] + 4 e^2 x^2 \text{Log}\left[1 - e^{-2 \text{ArcTanh}[c\sqrt{x}]}\right]\right) + 6 e^2 x^2 \text{PolyLog}\left[2, e^{-2 \text{ArcTanh}[c\sqrt{x}]}\right] - 3 e^2 x^2\left(-2\left(\text{ArcTanh}[c\sqrt{x}]^2 - i \text{ArcSin}\left[\sqrt{\frac{c^2 d}{c^2 d + e}}\right]\left(2 \text{ArcTanh}\left[\frac{c e \sqrt{x}}{\sqrt{-c^2 d e}}\right] + \text{Log}\left[\frac{1}{c^2 d + e} e^{-2 \text{ArcTanh}[c\sqrt{x}]}\left(-2\sqrt{-c^2 d e} + e\left(-1 + e^{2 \text{ArcTanh}[c\sqrt{x}]}\right) + c^2 d\left(1 + e^{2 \text{ArcTanh}[c\sqrt{x}]}\right)\right)\right] - \text{Log}\left[\frac{1}{c^2 d + e} e^{-2 \text{ArcTanh}[c\sqrt{x}]}\left(2\sqrt{-c^2 d e} + e\left(-1 + e^{2 \text{ArcTanh}[c\sqrt{x}]}\right) + c^2 d\left(1 + e^{2 \text{ArcTanh}[c\sqrt{x}]}\right)\right)\right]\right) + \text{ArcTanh}[c\sqrt{x}]\left(\text{Log}\left[\frac{1}{c^2 d + e} e^{-2 \text{ArcTanh}[c\sqrt{x}]}\left(-2\sqrt{-c^2 d e} + e\left(-1 + e^{2 \text{ArcTanh}[c\sqrt{x}]}\right) + c^2 d\left(1 + e^{2 \text{ArcTanh}[c\sqrt{x}]}\right)\right)\right] + \text{Log}\left[\frac{1}{c^2 d + e} e^{-2 \text{ArcTanh}[c\sqrt{x}]}\left(2\sqrt{-c^2 d e} + e\left(-1 + e^{2 \text{ArcTanh}[c\sqrt{x}]}\right) + c^2 d\left(1 + e^{2 \text{ArcTanh}[c\sqrt{x}]}\right)\right)\right]\right)\right) + \text{PolyLog}\left[2, \frac{\left(-c^2 d + e - 2\sqrt{-c^2 d e}\right) e^{-2 \text{ArcTanh}[c\sqrt{x}]}}{c^2 d + e}\right] + \text{PolyLog}\left[2, \frac{\left(-c^2 d + e + 2\sqrt{-c^2 d e}\right) e^{-2 \text{ArcTanh}[c\sqrt{x}]}}{c^2 d + e}\right]\right)\right)\right)$$

# Summary of Integration Test Results

49 integration problems




A - 31 optimal antiderivatives

B - 0 more than twice size of optimal antiderivatives

C - 12 unnecessarily complex antiderivatives

D - 3 unable to integrate problems

E - 3 integration timeouts